ABSTRACT

MECHANICAL CHECKING OF NAVY BEANS

by Chris V. Narayan

A stability analysis was developed to compute the stability modulus EI of navy beans loaded quasi-statically on end. Critical loads for bean cotyledons were obtained on an Instron Testing Machine at the point of instability as signified by the onset of checking or cracking of the seed coat.

Values of EI and elastic modulus, E, were computed for various moisture contents in the range of 11.5 to 28.2 percent w.b.

Dynamic studies in the form of low velocity impact of beans by a falling weight and high velocity impact by a rotating arm were also conducted. Impact forces to cause checking were measured, and the corresponding impact energies computed. Comparisons of the energy obtained by the two types of dynamic tests were made.

The results of the dynamic tests were extrapolated to field conditions and compared with previously made field observations on bean harvesting and handling.

The optimum moisture content range for resisting checking, or cracking of the seed coat, was found to be

13.4 to 15.6 percent w.b. For beans in this moisture content range, an impact velocity of 55 fps was required to cause checking, when the beans were struck with a rotating arm. Beans at 11.5 percent moisture were found to be the most susceptible to checking, under both static and dynamic loading conditions.

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Major Professor

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MECHANICAL CHECKING OF NAVY BEANS

By

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I. INTRODUCTION

Navy beans, like all agricultural products, suffer some mechanical damage during harvesting and handling. For navy beans this damage is manifested in two ways, internal damage to the cotyledons with no visible external damage, and visible damage.

Visible damage to the beans varies in severity from breaking and splitting of the cotyledons to checking, or cracking of the seed coat. The latter case has become increasingly important in recent years because of the large percentage of the bean crop which is pre-cooked and canned.

1.1 Objectives

This study was undertaken to determine the mechanical causes of the cracking of the seed coat with a view to reducing the frequency of its occurrence.

1.2 The Thesis Problem

The work reported in this thesis may be divided into three parts:

1. The measurement of the quasi-static forces required to cause cracking, in accordance with

a stability analysis developed for bean cotyledons.

2. The measurement of impact forces and energy required to cause cracking of the seed coat.
3. Extrapolation of the test results to field conditions.

II. BACKGROUND NOTES

Beans are classified in North America under the genus Phaseolus. Most of the common, dry edible beans, such as the navy bean (also called white or pea bean), the lima bean, the great northern, and kidney beans are contained in the varietal classification Phaseolus vulgaris.

The bean is a dicotyledonous seed, varying by variety from ellipsoidal to kidney shaped. The two cotyledons and the embryonic axis, which upon germination produce the seedling, are encased in a relatively impervious seed coat. The seed while still in the pod, is nourished through the hylum, which is the only discontinuity in the seed coat. The seed coat itself is bicellular in thickness except in the hylar area where, for strength purposes, an extra layer of cells exists.

The hylum does not cease to function after the seed leaves the pod. Hyde (1954) undertook a detailed investigation into the function of the hylum in Leguminosae (a British classification rather broader than Phaseolus) and found that the hylum acted like an hygroscopically activated valve, discouraging the entry of water, but allowing the outward flow of moisture and gases from the seed,* thus allowing the seed to harden rapidly (a phenomenon which Maddex (1953), upon encountering it while artifically drying navy beans, described as "case hardening"). This selectivity on the part of the hylum is probably the cause of the relative difficulty encountered by researchers, Bakker-Arkema, _et al_. (1966), in rewetting navy beans.

The cellular texture, impermeability and pigmentation of the seed coat, the closeness of the cotyledons to each other as well as to the coat, and the germination ability of beans vary between and within varieties. Some of these characteristics could, as will be discussed later, play an important role in the susceptibility of the seed to physical damage.

2.1 _Nutrition_

Edible beans, and peas (the two commonly referred to jointly as pulses) have long been an important food supply. Beans contain not only high percentages of energy compounds such as fats and starches but also large amounts of protein, although no amino acids. In addition beans contain the important vitamin thiamin. Thus the ratio of food value to bulk is very high for beans. For this reason, as Bracken and Rasmussen (1944) pointed out the United States Government called for sharp increases in

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*When the hylum was blocked Hyde found a reduced rate of drying.

bean production during both world wars, for supply to the troops.

The high protein content of pulses is an important consideration in the food supply of countries where there might be a shortage of animal proteins. Finally, beans are versatile as a food, having many modes of consumption. Beans are presently canned as well as milled into flour. Recently Bakker-Arkema, et al. (1969) has reported some success in processing a pre-cooked bean puree.

2.2 Historical Notes

Beans, as Beagle (1949) pointed out, have had a long and eventful history as a human food. He goes so far as to speculate that because of the relatively large seed and brief period of germination, the bean may well have been one of man's earliest cultivated plants. Thompson (1950) pointed to evidence of the existence of beans and lentils in the Nile Valley ca. 2000 B.C., while Hutchins (1947) has written of beans being grown in Switzerland and Northern Italy as early as the Bronze Age.

Beans have long been cultivated in Latin America, as well as Asia, and indeed Brazil and Mexico are respectively the world's first and second largest producers of beans [Anon (1966)].

In the United States the particular type of bean grown varies with location. Among the bean raising states, Michigan leads in the production of navy beans.

Beagle (1950) speculates that the term "navy" may have arisen because of the supply of beans to Commodore Perry on Lake Erie in 1812. Regardless of the accuracy of that speculation the military may well have played a major role in the development of bean production. The bean acreage in the United States did increase sharply during the two world wars. During the second world war, however, agricultural extension specialists such as Mercer (1945) were advising farmers to increase not only acreage but mechanization as well. Thus, after the second world war and on into the fifties not only was mechanization of bean production well entrenched, but as Andersen (1960) points out genetic breeding of disease resistant high yield lines were well underway. In addition, as Thompson (1950) reported to the producers, there was an expanding European market for dry edible beans; a market which has persisted to the present.

The increased production in the United States, accompanied by increased mechanization and bulk handling led to the problem of mechanical injury to the bean seed; injury which not only affected market value of the product, but also impaired germination. Solorio (1959) states that the three causes of impaired germination are mechanical injury, bacteria and insects. Thus there began in the late forties and through the fifties an increased interest in mechanical injury to seeds, especially by harvesting and handling equipment. Beagle (1949) described the type of equipment available at that time for pulling and threshing beans. He discussed many of the current problems but did not dwell for any length on injuries. McDow (1949) described the problem of "splits" occurring with mechanized handling of pea beans. The term "splits" indicates seeds with the cotyledons split apart or with at least one-quarter of a cotyledon completely broken off. He attributed the onset of splits to poor machine adjustments or low moisture content and developed the following regression equation for percent splits (Y) as a function of percent moisture content (x),

$$Y = 35.5 - 4.15x + 0.1256x^2$$

with the optimum handling moisture content being 16.5 percent w.b.

Harter (1930) and Borthwick (1931) had previously noted the effect of moisture content on thresher injury to snap beans and lima beans respectively. They were mainly interested in germination, and both noted the production of retarded seedlings ("baldheads") after mechanical threshing of very dry beans. Borthwick noted that a common injury was the detachment of the cotyledons. Toole, et al. (1951) working with navy beans found breaking damage in threshing tests as high as 20 percent for low moisture contents. Germination tests proved, through the occurrence of baldheads, that further damage had been done internally. McCollum (1953) investigated cotyledon injury with snap-beans and found marked varietal differences which obscured side conclusions.

The conclusions about moisture content as being an important factor in seed damage, as well as the increased production, led to investigations of damage in storage and during drying. The latter was fairly important since the harvesting tests results indicated an optimum harvest moisture of about 18 percent. Maddex (1953) in drying tests with heated air found a high onset of cracking near the bottom of the test bin when the moisture content of the beans dropped below 16 percent. He was able to reduce this by decreasing the air temperature (below 130° C) and adjusting the RH (to 15 percent), in effect by slowing the drying rate. Wang (1958) used intermittent application of dry air at 100° F in a test to dry navy beans. He was interested in seed coat cracking (as opposed to most of the previous investigators) as well as splitting. Wang tried to determine the stresses involved in the cracking of the seed coat but his assumption of sphericity of the pea bean probably accounted for his limited success, since the stress distribution over an ellipsoidal shaped body and a uniform sphere are very different. In addition, he did not take the significance of the discontinuity at the hylum into account.

Brown (1955) was apparently the first to work with individual navy beans. He reported forces of twelve to forty-two pounds required to break beans (6.9 percent to 9.2 percent moisture) on the "flat" with various pressure heads and two to forty-two pounds to break similar beans positioned on edge. His loading rate was in the quasi-static range. Alkin (1958) working on the susceptibility of snap beans to mechanical injury of the seed noted that the coats of resistant varieties adhered more tightly to the cotyledons than did others. He concluded that the tight seed coat and closely fitting cotyledons must inhibit movement and therefore protect the embryo better. Solorio (1959) struck individual beans by dropping them into the path of a rotating (777 rpm) paddle wheel, then examined them for cracks in seed coat (checks) and "splits." At 15.5 percent moisture he found 7.2 percent visible damage of which 6 percent was "checks" and 1.2 percent "splits." At 9.7 percent moisture he found 70.3 percent visible damage of which there was 27.3 percent "checks" and 43.0 percent "splits." Germination tests proved that very few of the "splits" produced healthy seedings. He concluded that when a cotyledon has a transverse crack across it, although the outer end is still attached, the food supply in the loosely attached part is not used. He also found a higher than normal incidence of "baldheads" in germination tests with the "checks" which he attributed to minute cracks. Faust (1955) pointed out the possibility of damage (splits) when pea beans are dropped into deep silos. He attributed 30 to 40 percent of total splits to this.

Needless to say this type of work was done on other grains as well. Øygard (1962), for example, evaluated visible kernel damage to wheat by flails and recommended a peripheral speed of 90 fps as an upper limit. Also threshing operations are being continually evaluated and will be as long as it is possible to improve existing machinery, existing genetic lines and existing practices. Green (1966), for example, recommended 13 percent moisture and 900 rpm cylinder speeds as respective minimum and maximum values in combining soybeans.

The characteristic features of the above types of testing are (a) the lack of measurements of force and energy (except where quoted) which cause the mechanical damage, and (b) the greater interest in splits. Indeed, with respect to the latter, Perry (1959) quotes the U. S. D. A. specifications for Grade No. 1 beans as 2 percent or less "splits, damaged beans, contrasting classes and foreign material." Presumably checking, unless so severe as to account for damaged beans, was not a specific problem.

As far as splitting and breakage is concerned, therefore, the above tests can be regarded as being relatively successful in reducing this type of major damage and loss, although the adherence to their recommendations has led to such practices as rewetting of the beans prior to major handling and transportation operations [Thompson (1962), Bakker-Arkema, et al. (1966)].

With the increased amount of canning, the checking of seed coats has, however, become a serious problem. Thompson (1950) reported that pea beans were principally used for canning since they have the tendency to hold their shape when baked. Perry (1959) pointed out that the canning industry which uses 90 to 95 percent of the pea bean crop annually dislikes checked beans because during the processing the checked beans may split, and also because checked seed coats may separate from the cotyledons and float at the top of the cans. French (1962) stated that cracked seed coats admit oxygen and moisture to the cotyledon, which is bad for storage, and permit the entry of bacteria and fungi which causes quality deterioration. He was very successful in developing a technique for determining minute cracks in seed coats rapidly, using Indol Acetate. This chemical enters the crack in the coat and stains the undersides of the coat a deep blue. It does not affect the exterior of the coat. For beans with relatively transparent coats, such as the navy bean, the results of dumping a handful into a solution of Indol Acetate is claimed to be remarkably effective in identifying checked beans. Kannenberg and Allard (1964) studied lignin formation in bean seed coats. They found small amounts of lignin in nonpigmented (white) seed coats. Noting that the function of lignin is primarily one of strength and protection, they concluded that low lignin content of white seed coats could account in part for their susceptibility to injury.

Adding to the above the implication of Solorio's (1959) findings, that the process which causes the checking could also cause internal damage to the cotyledons, the importance of checking cannot be discounted.

III. REVIEW OF LITERATURE

In the field of Physical Properties of Agricultural Products, the investigator is basically attempting to use the entire field of Theoretical and Applied Mechanics merely as a tool kit. Hopefully, he simply selects the tool most applicable to his problem and uses it. This requires a sound knowledge of mechanics. In addition, he must be aware of the biological factors which influence his work. Genetics for example as implied by the findings of Liu (1949) working on genetic inheritance of damage resistance to soybeans, could produce premature obsolescence of a mechanical study. Again, the chemical makeup of the product studied could have important effects on the physical properties of that produced, as evidenced by Kannenberg and Allard (1964) and Dorrell (1968). Although the Agricultural Engineer in the physical properties field could not possibly keep up to date in the biological fields, he should know the tools of his own field very well.

3.1 Mechanics

Lazan (1962) listed three approaches for studying rheological properties: (1) micromechanistic--solid state physics, (2) macroanalytical--applied science or engineering, and (3) *ad hoc* or simulated testing--the "practical" approach. Of the three approaches, the last two are most commonly used by engineers. Lazan admits the need for the practical approach in which one has to work with the actual object and loading conditions rather than specimens and models. However he does caution:

> . . . *ad hoc* property data are generally not extendable, in the absence of a more basic understanding, to future problems which lie in different regimes of stress and environment.

He notes however that:

> Although the *ad hoc* approach adds relatively little per unit effort to the store of basic information, it can often provide engineering answers in a relatively short time and is sometimes "the only way out."

The macroanalytical approach involves the determination of the properties of materials by idealization, simplified conditions and test specimen geometries. By idealizing is meant formulating ideal constitutive equations for the material. The theories of elasticity, plasticity and viscoelasticity are examples of this approach. The macroanalytical approach assumes continuity, isotropy and homegeneity. As Malvern (1965) states these quantities are difficult to establish with some materials. However, since by this approach one merely attempts to model the material behavior as it is observed for a given range of stress (or strain) then, provided that the model can predict the behavior of the material for a different stress regime in this range, the attainment of such a model can be considered the solution to the problem. The actual explanation of the behavior becomes immaterial.

The macroanalytical approach works best when the material stress-strain behavior is linear, when the load conditions are quasi-static and when the material is indeed homogeneous. When these conditions change however, for non-linear responses for example, or dynamic loads, or composite materials, much more work is required. When one is faced with all three of these complications, the ad hoc approach is in most cases "the only way out." Kerwin (1965) found that he could drastically change (dampen) the vibration of steel structural members by using viscoelastic material bonded between them. For his tests on beams, arches and base members, he had to use actual members and scale models.

In the study of the physical properties of agricultural products, investigators are often faced with all three of these complications.

3.2 Physical Properties of Agricultural Products

The application of the theory of mechanics to the testing of agricultural products on a large scale is a relatively recent development. It is largely based on the assumption that most agricultural products are viscoelastic in their load-deformation behavior. Zoerb (1958) applied this technique in the study of the physical properties of selected grains. He used both the ad hoc and the analytical approaches. Using core samples of pea beans at 36.4 percent moisture d.b., he plotted stress-strain curves, and calculated maximum strengths, for relatively low rates of loading (0.267 ipm). He evaluated shear strengths of bean "slabs" at four moisture contents. For his ad hoc tests, he ran quasi-static tests on beans on the flat as well as on edge and found linear relationships between load and deformation at the low moisture content value of 10.6 percent. To compare with his static shear tests, he ran impact shear tests using a pendulum impact tester. The energy required by the static test for shear failure was higher than that for the dynamic tests up to about 20 percent moisture. The situation reversed itself at higher moisture contents. Zoerb also investigated the rheological properties of pea beans using relaxation tests, in which an instantaneously applied deformation is held constant and the load relaxation is measured with time. He proposed a model of two parallel Maxwell units to represent navy bean tissue.

Perry (1959) extended Zoerb's work on navy beans. His approach, however, was from the dynamic point of view and was therefore of the practical or ad hoc type. His first tests consisted of dropping beans at various moisture contents and temperatures through three heights of drop, 11.25 ft, 22.5 ft and 45 ft. Maximum visible damage was found to occur at the 45 ft drop height and 12 percent moisture, w.b. Minimum visible damage occurred at 18 percent moisture w.b., for all drop heights. Beans were dropped individually, in small samples and poured slowly. Beans were visually examined for damage and germination tests were also used. Checking and splitting increased with increasing drop height and decreasing moisture content. Some temperature interaction was evident in the germination tests. In a second set of tests Perry used a wooden faced bar to strike beans which were partially confined by a small movable wooden block. The velocity of the bar was calculated to simulate bean velocity in dropping during commercial handling. A high speed movie camera was used to measure velocities before and after impact. Energy balance equations yielded impact energy. Newton's Law for the bean-block system was used to calculate maximum force of impact. This impact force was compared with Zoerb's data on whole beans. Coefficients of restitution were calculated for the four moisture contents. The kinetic energy range was 0.006 ft-lbs to 0.017 ft-lbs. Perry noted in examining the checked beans that the cracks "seemed to radiate from the hylum," creating a common check pattern.

Perry and Hall (1966) extended the above work to include auger transport of pea beans in a 21-foot screw auger at three slopes, horizontal (0 percent damage), 15 degrees (1.1 percent damage), and 28 degrees (1.5 percent damage). They also investigated the deterioration of bean quality in storage.

After Zoerb's application of mechanics principles to agricultural product testing, several other researchers applied viscoelastic principles to quasi-static testing of various products. Finney (1963) applied linear viscoelastic theory to the potato, to find material constants. Mohsenin (1963) developed a tester to perform quasi-static tests. Mohsenin, *et al*. (1963), (1965) extended elastic and viscoelastic theory to several fruits and vegetables, as did Timbers (1965) and Arnold (1966).

These tests, however, did not produce data that could be immediately applied. They are mentioned here only for the reason that they led to dynamic testing by the same authors, and by others.

3.3 Dynamic Tests

Most of the dynamic tests in this field are *ad hoc* or practical tests. Except for large products, they were performed on the whole product. The most important factor from the point of view of a literature survey is the apparatus and the parameters measured.

In discussing dynamic testing, Alfrey (1948) stresses the importance of energy to cause failure rather than maximum stress, when investigating impact strength. For linear elastic and viscoelastic materials, this energy is proportional to the square of the impact strength [Kolsky (1952)]. The development of shock waves, variations in texture and geometry, complicate the relationship.

For these reasons impact testing has been confined to measuring maximum impact energies. Bilanski (1966) subjected various types of seeds individually to impact testing at two loading rates. The slower testing was done with a pendulum. An energy balance equation was used to calculate impact energy absorbed by the test seed. The higher speed testing was done by dropping the seed in the path of a rotating paddle. Orientation of the seed was impossible in this case.

Fridley, et al. (1964) impacted peaches with a flat-plate plunger with a pendulum-type arrangement and computed bruising energy. Mohsenin and Hammerle (1965) devised a drop tester to measure bruising energy and impact forces on stationary supported apples struck by a falling weight. Photocells were used to measure impact velocity of the weight, and a quartz accelerometer to measure its acceleration. They plotted volume of bruise versus impact energy. Wright and Splinter (1968) used a drop tester similar to the one devised by Mohsenin and Hammerle to measure impact rupture energy of sweet potato samples. They compared this to static rupture energy. For the varieties of sweet potatoes used the rupture energies (about 2 in.-lbs) were between 4.5 and 3.5 times smaller than the static. Wright's analysis of the drop test was used in this study to design a drop tester and a series of low velocity impact tests for determining the rupture energy of navy beans under dynamic conditions. The analysis is discussed in section 5.2.

Mitchell and Rounthwaite (1964) impacted individual grains of wheat by moving them slowly into the path of a rotating hammer. Hammer speeds up to 120 fps were used for three moisture contents. A regression equation, not considering moisture contents was derived between undamaged grain and hammer speeds. Clark, et al. (1967) used a rotating arm to strike individual cotton seeds. The seeds were free to move after impact, and were tracked with a stroboscope and exposed photographic film. Energy balance equations were used again to compute the energy absorbed by the impacted seeds.

The last two studies discussed have the obvious advantage of approximating, in the laboratory, the forces on the seeds in a combine. Lamp (1959) ran threshing tests on wheat using a modified combine cylinder. He used the restitution equations to calculate energy imparted to the grain by the combine cylinder bars, and attempted to compute threshing forces. Peripheral speeds of up to 25.0 mph (36.7 fps) were found to be sufficient for complete threshing. Grain damage was recorded but not analyzed. Tabiszewski (1968) ran threshing tests on pea beans using a modified peanut combine. Peripheral speed of the cylinder was varied between 17 and 37 fps. The amount of visibly damaged grains was found to increase with peripheral speed and with decreasing moisture content. In the range of 10.6 to 12.3 percent moisture w.b., the maximum amount of visible damage was 31 percent.

Other dynamic tests have been devised to measure different parameters. Diener (1968) used steady state dynamic tests to measure the complex modulus of cherry bark. The principles of dynamic viscoelasticity [Ferry (1961)] were used. The direct application of these results is, however, not yet possible. Finney and Norris (1967) have used resonance characteristics of selected fruits as an indicator of fruit quality with some success.

3.4 Elastic Stability

The possibility of structural failure well within the elastic regime of stress exists for structures of given geometry. The classical problems of the slender column and the flag pole [Timoshenko and Gere (1961)] are well known. This stability problem is, in general, governed by the general fourth order differential equation of equilibrium

$$\frac{d^2}{dx^2}(EI\frac{d^2y}{dx^2}) + \frac{d}{dx}(p\frac{dy}{dx}) = q$$

where p is the axial compressive load,

EI is the flexural stiffness of the section

and q is the transverse load.

When q is absent, the equation becomes homogenous and the solution is simpler. For a varying cross-section the problem becomes more complicated although symmetrical variation lessens the work required.

Moustafa, et al. (1966) used the methods of stability to model the wheat plant under axial and lateral loads. Viscoelastic stability principles and large deformation theory were used.

For short columns with varying cross-section, the equilibrium equation is best solved by approximate methods. The finite difference method is the easiest to apply. This method is simply one of approximating derivatives with algebraic quantities. For example, the slope of a curved line between two points y_1 and y_2 at a distance 2λ apart is approximated by

$$\frac{dy}{dx} = \frac{y_1 - y_2}{2\lambda}$$

This method leads to an Eigenvalue problem with the Eigenvalues being the parameter of $\frac{p}{EI}$ required to cause buckling into the fundamental buckling modes. With p known, the EI values can be computed.

IV. QUASI-STATIC THEORY

Preliminary tests involving the loading of individual beans with a Valve-Air Loading Unit indicated that cracking of the seed coat occurred at lowest loads, when the bean was loaded on end (Figures 1 and 13).

Examination of the above phenomenon in greater detail indicated that under the end loading conditions the two cotyledons tended to separate, subjecting the coat to a tensile stress. Ultimately, this led to failure of the coat in the form of cracking. Because of the greater strength of the coat in the hylar region of the bean, the cracks tended to turn away from this region, producing the various check patterns often observed on damaged beans.

It was apparent that under these conditions of end loading, the failure could be characterized as buckling on the part of the cotyledons.

In the following sections the principles of stability and buckling are discussed, leading to an analysis of the bean failure under end loading.

4.1 Elementary Buckling Theories

4.1a The Equilibrium Formulation--Bifurcation Theory

Consider the system shown in Figure 2a. It consists of a rigid bar of length L, pinned at the lower end and free at the upper end. It is loaded at the free end by an axial force P, and constrained in the vertical position by a coil spring of strength k at the pinned end. The vertical is the equilibrium position.

The critical load P_{cr} can be defined as that load which is just sufficient to keep the bar in some deformed position characterized by the angle θ (Figure 2b). Under P_{cr} the bar neither collapses nor returns to the vertical position.

Considering the equilibrium conditions for the bar loaded by P_{cr}, spring moment $M = k\theta$

$\therefore$ for equilibrium,

$$P_{cr} L \sin\theta = k\theta$$

whence,

$$P_{cr} = \frac{k\theta}{L\sin\theta} \qquad (1)$$

Equation (1) can be linearized by assuming θ is small, in which case $\sin\theta = \theta$.

Thus,
$$P_{cr} = \frac{k}{L} \qquad (2)$$

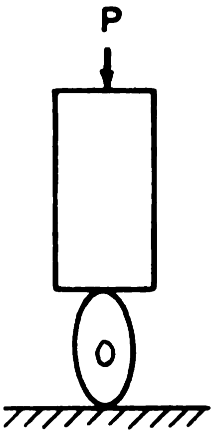


Figure 1.--Bean loaded on end.

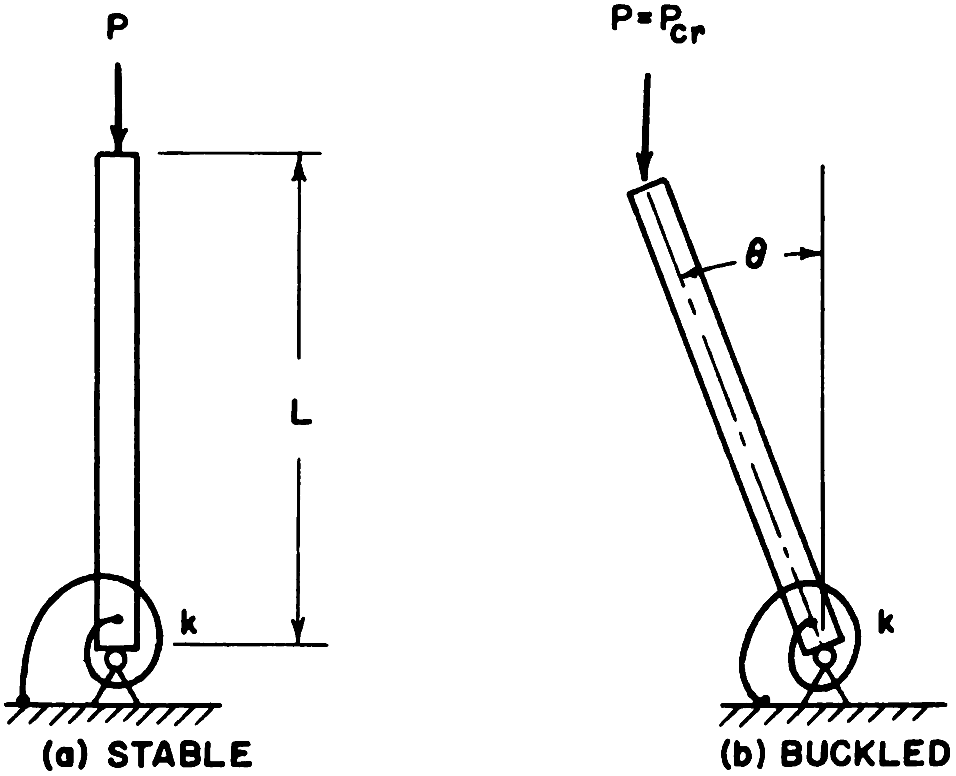


Figure 2.--Simple elastic system buckling under a critical load.

Under this formulation the bar supposedly remains vertical under load until this load P reaches the value of P_{cr}, at which point the bar bifurcates. In practice this need not be so. Other theories have been developed to illustrate equation (2) more realistically.

4.1b Equilibrium Formulation--Imperfection Theory

There are three types of imperfections:

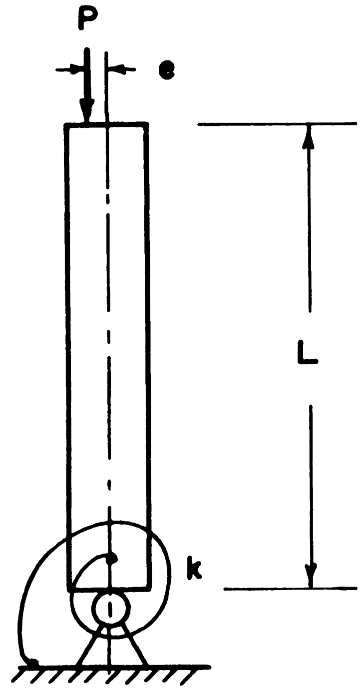
i. Initial deformation.
ii. Load not perfectly axial--eccentric loading.
iii. The presence of a small lateral load on the bar.

Of these, the eccentric load imperfection will be discussed, since, in considering cotyledon buckling, an end load axially applied to a bean will be slightly offset with respect to each individual cotyledon.

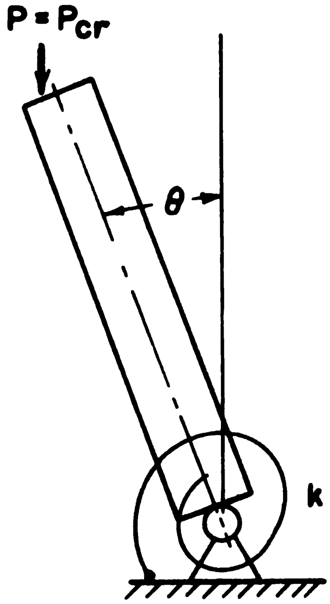
Consider now the bar-spring system with the load P applied a distance e from the geometric axis of the bar (Figure 3). The buckling criterion now applied is that when $P = P_{cr}$ the deformation θ will become unbounded.

Now, for equilibrium at any angle θ, for some loading P, the sum of the moments about the pinned end is zero. Thus,

$$P\,(L \sin \theta + e \cos \theta) = k\theta \qquad (3)$$



(a) STABLE



(b) BUCKLED

Figure 3.--Instability due to offset loading.

For small values of θ, equation (3) becomes,

$$P\,(L\theta + e) = k\theta$$

whence

$$\theta = \frac{Pe}{k - PL} = \frac{e}{k/P - L} \qquad (4)$$

Applying the buckling criterion to equation (4), it can be seen that θ will become unbounded, regardless of the magnitude of e, when

$$P = P_{cr} = \frac{k}{L}\ ,$$

the same result as given in equation (2).

4.1c Energy Formulation-- The Principle of Stationary Potential Energy

For any force-deformation system, the above principle states: "Of all possible configurations consistent with the constraints, (or boundary conditions), that one which satisfies the equilibrium conditions is the one for which the total potential energy of the system has a stationary value."

Thus the energy formulation is also based on equilibrium.

Consider Figure 2 again. The total potential energy, π, of the system is that of the load plus that of the spring, and is a function of θ. The potential energy of the external force P is given by

$$V = -PL\ (L - \cos\theta)$$

The potential energy of the spring, (strain energy), is given by

$$U = \frac{1}{2} k\theta^2$$

The total potential energy of the system is

$$\pi = V + U$$

Thus,

$$\pi = -PL\ (1 - \cos\theta) + \frac{1}{2} k\theta^2 \qquad (5)$$

Now, for the configuration for which π is constant, $(d\pi/d\theta) = 0$.

$$\therefore \frac{d\pi}{d\theta} = 0 = -PL \sin\theta + k\theta \qquad (6)$$

whence,

$$P = \frac{k\theta}{L \sin\theta},$$

which is again the equilibrium equation. Linearizing as before, by letting θ be small,

$$P = P_{cr} = \frac{k}{L},$$

which again is the same results as given in equation (2).

4.2 Beam-Column Theory

Consider a simply supported beam as shown in Figure 4a. It is subjected to an axial force P and a distributed lateral load q(x). Under these loads, the beam will bend as shown by the dotted line representing the deformed center line. Taking the x and y directions as shown in Figure 4a as positive, the radius of curvature, R, can be expressed, assuming small deflection, as:

$$\frac{1}{R} = -\frac{M}{EI} = \frac{d^2y}{dx^2} \qquad (7)$$

Now consider the free body element shown in Figure 4b. For equilibrium, $\Sigma M_0 = 0$

$$\therefore \qquad -M + (M + dM) - Vdx + qdx\,\frac{dx}{2} - Pdy = 0$$

Dropping powers of differentials higher than one, the above equation becomes

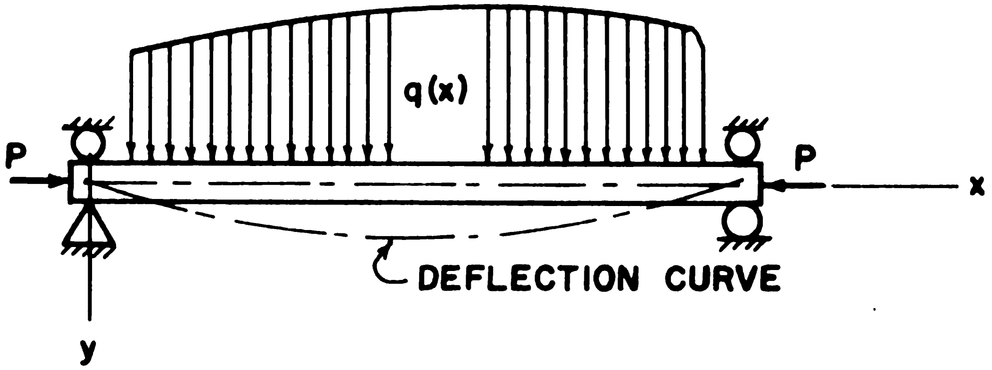
$$-M + M + dM - Vdx - Pdy = 0$$

$$\therefore \qquad dM - Vdx - Pdy = 0$$

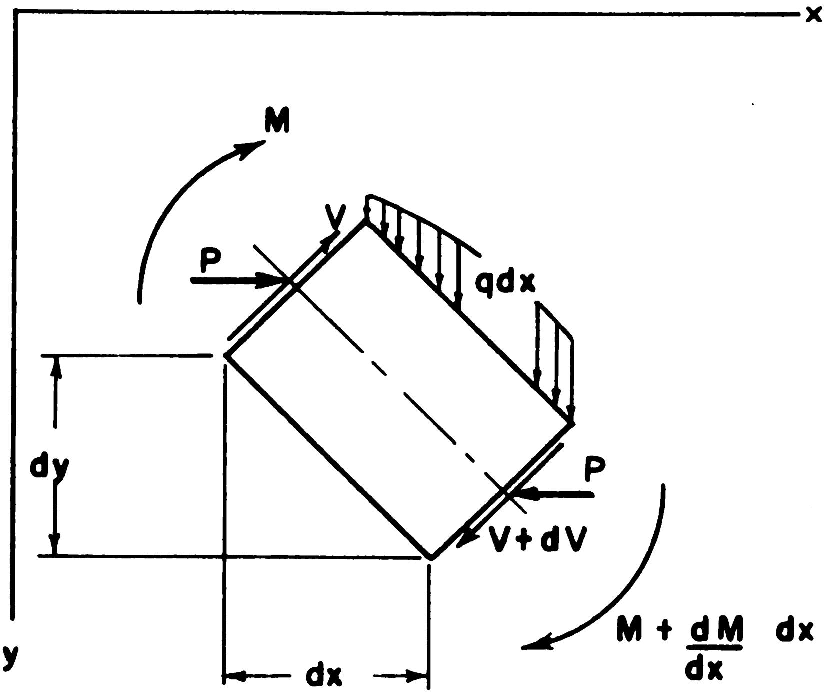
$$\therefore \qquad V = \frac{dM}{dx} - \frac{Pdy}{dx} \qquad (8)$$

Substituting for M in (8) using (7),

$$\frac{d}{dx}\left(-EI\,\frac{d^2y}{dx^2}\right) - \frac{Pdy}{dx} = V \qquad (8a)$$



(a) BEAM COLUMN UNDER LOAD



(b) FREE BODY ELEMENT OF THE DEFLECTED BEAM-COLUMN

Figure 4.--Simply supported beam-column under axial and lateral loads.

Referring again to the free body element, $\Sigma F_Y = 0$

$$\therefore \qquad -V + (V + dV) + q\,dx = 0$$

$$\therefore \qquad \frac{dV}{dx} = -q$$

Differentiating equation (8a), and substituting for dV/dx,

$$\frac{d^2}{dx^2}\left(EI\,\frac{d^2y}{dx^2}\right) + \frac{d}{dx}\left(P\,\frac{dy}{dx}\right) = q \qquad (9)$$

For constant E and I, and no lateral load, equation (9) takes on its well known form

$$EI\,\frac{d^4y}{dx^4} + P\,\frac{d^2y}{dx^2} = 0 \qquad (9a)$$

The solution of equation (9a) with the appropriate boundary conditions on y and its derivatives yields the critical load for the beam-column in question.

4.3 Complicating Factors

4.3a Short Columns

The Bernoulli-Euler theory, as the Beam-Column theory developed in the previous section is called, has certain limitations. Its accuracy in predicting critical loads depends on how well the actual boundary conditions can be made to suit the theoretical or mathematical boundary conditions. In addition, experience has shown that good agreement between actual critical loads and theoretical or Euler loads depends on the slenderness ratio of the column under test. The slenderness ratio is defined mathematically as L/r, where L is the column length and r is the minimum radius of gyration of its cross-section. For steels, depending on how well the boundary conditions can be satisfied, Timoshenko and Gere report good agreement with the Euler theory for slenderness ratios greater than 70.

For slenderness ratios lower than 70, empirical correction factors are applied to the Euler theory to match the actual results. These factors are often used for empirical design formulae for columns.

4.3b Columns with Varying Cross-sections

Variation in the cross-section of the column means that the moment of inertia, I, of the column will vary, i.e., $I = I(x)$.

Now, if the variation of I is a simple one, e.g. linear, then $I(x)$ can be based on some reference value, I_0, of the moment of inertia measured at a particular section $x = x_0$. In such a case the moment of inertia can be written as

$$I(x) = \Phi(x)\, I_0$$

The above expression can then be used with equation (9)

$$\frac{d^2}{dx^2}\left(EI(x)\frac{d^2y}{dx^2}\right) + P\frac{d^2y}{dx^2} = q ,$$

and P can be obtained, for the given boundary conditions, by solving the differential equation.

4.3c Approximate Solutions

Obviously, when $\Phi(x)$ is not a simple function, the resulting differential equation becomes very difficult to solve. In such a case, an approximate solution of the differential equation, by a numerical procedure, is often the best approach. One such numerical procedure is the finite difference method. By this method, the differential equation is approximated by an algebraic equation at each of a finite number of chosen points along the column. The resulting system of equations, with the appropriate boundary conditions, is then solved to yield the critical load.

The procedure is outlined in the following section.

4.4 Finite Differences

4.4a Approximation of Derivatives

Consider the curve $y = f(x)$ given in Figure 5. Suppose the derivatives of y are required for x equal to a value corresponding to the point 0. Two points are chosen on the x-axis, on either side of the point 0 and labelled as shown. The "E" and "W" signs have no mathematical significance; they serve only to indicate which side of point 0 the points they label lie. Any two successive points are separated by a finite distance λ on the x-axis.

Let q be defined as shown. The first derivative of y(x) at point 0 can be approximated by the following expression:

$$(y^{I})_0 = \left(\frac{dy}{dx}\right) = \frac{q}{2\lambda} = \frac{y_E - y_W}{2\lambda} \qquad (10a)$$

The second derivative:

$$(y^{II})_0 = \frac{y_E - 2y_0 + y_W}{\lambda^2} \qquad (10b)$$

The third and fourth derivatives at point 0 can be shown to be:

$$(y^{III})_0 = \frac{y_{EE} - 2y_E + 2y_W - y_{WW}}{2\lambda^3} \qquad (10c)$$

$$(y^{IV})_0 = \frac{y_{EE} - 4y_E + 6y_0 - 4y_W + y_{WW}}{\lambda^4} \qquad (10d)$$

Thus, a differential equation can be approximated by a system of algebraic equations set up at the points chosen (node points), and the system can be solved for the approximate shape of the curve y(x).

4.4b Error Involved

Consider Figure 5 again. A Taylor series expansion about point 0 yields

$$y(0 + \lambda) = y_E = (y)_0 + \lambda (y^I)_0 + \frac{\lambda^2}{2} (y^{II})_0 + \frac{\lambda^3}{3!} (y^{III})_0 + \ldots$$

$$y(0-\lambda) = y_W = (y)_0 - \lambda (y^I)_0 + \frac{\lambda^2}{2} (y^{II})_0 - \frac{\lambda^3}{3!} (y^{III})_0 + \ldots$$

Subtracting the above yields

$$(y_E - y_W) = 2\lambda (y^I)_0 + \frac{2\lambda^3}{3!} (y^{III})_0 + \ldots$$

$$\therefore \quad (y^I)_0 = \frac{y_E - y_W}{2\lambda} - \frac{\lambda^2}{3!} (y^{III})_0 \qquad (11a)$$

Comparing equations (10a) and (11a), it can be seen that the error involved is of the order of λ^2.

Similarly, by adding the two equations, it can be shown that the error involved in the finite difference approximation of the second derivative is also of the order of λ^2.

By expanding $y(x + 2\lambda)$ and $y(x - 2\lambda)$, the same order of magnitude of error is obtained for the third and fourth derivatives.

Thus the accuracy of the approximate solution can be increased by decreasing λ, i.e., by using a large number of node points. However at each node point, one algebraic equation is obtained. The greater the number of node points therefore, the more time consuming the operation becomes.

An easier method of increasing the accuracy of the procedure is by Richardson's extrapolation.

4.4c Richardson's Extrapolation

Recall that the problem at hand is to solve a given differential equation to find a function y(x), such as the one shown in Figure 5.

Now, if y_c is a finite difference approximation of y(x) obtained by using a coarse grid spacing λ_c, and y_f an approximation using a finer grid λ_f, then by the argument of the previous section it is known that:

$$y(x) = y_c + k\lambda^2_c$$

and also,

$$y(x) = y_f + k\lambda^2_f$$

Since k is unknown, it can be eliminated in the following manner

$$k = \frac{1}{\lambda^2_c} (y(x) - y_c)$$

$$k = \frac{1}{\lambda^2_f} (y(x) - y_f)$$

$$\therefore \qquad y(x)\left(\frac{1}{\lambda^2_c} - \frac{1}{\lambda^2_f}\right) = \frac{y_c}{\lambda^2_c} - \frac{y_f}{\lambda^2_f}$$

$$\therefore \qquad y(x) = \frac{\lambda^2_f}{\lambda^2_f - \lambda^2_c}\, y_c - \frac{\lambda^2_c}{\lambda^2_f - \lambda^2_c}\, y_f$$

Now let $n_c = \frac{L}{\lambda_c}$ and $n_f = \frac{L}{\lambda_f}$

where L is the total length over which the function y(x) is defined. The function can now be written

$$y(x) = \left(\frac{n^2_f}{n^2_f - n^2_c}\right) y_f - \left(\frac{n^2_c}{n^2_f - n^2_c}\right) y_c \qquad (12)$$

Note that a very good approximation of y(x) would be obtained even when two relatively coarse grids are used since the major portion of the error has been eliminated.

4.4d The Eigenvalue Problem

Consider the simple pin ended beam-column of Figure 6, with constant E and I. The second order differential equation of equilibrium is

$$EI\,\frac{d^2y}{dx^2} + Py = 0 \qquad (9)$$

Apply a very coarse grid, $\lambda = \frac{L}{4}$ as shown. The node points beyond the span, a and b are included so that the finite difference approximation of equation (9) can be made at points 0 and 3. Values of y are assigned to these

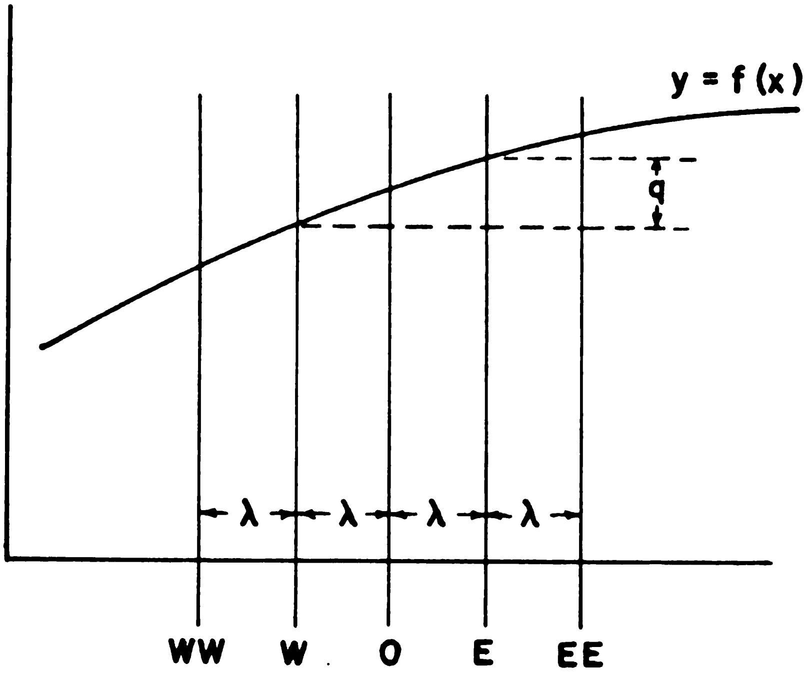


Figure 5.--Grid for approximating the derivative of a function y = f(x) at a given point 0.

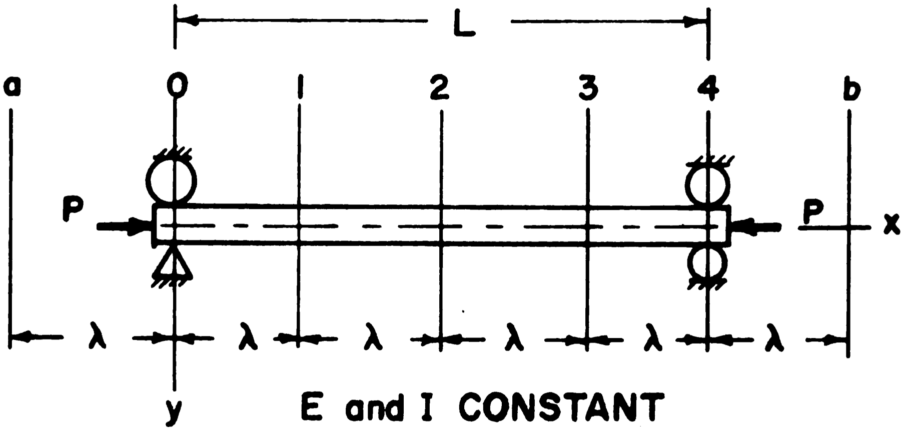


Figure 6.--Pin-ended column with a grid of λ = L/4.

imaginary node points a and b by applying the boundary conditions.

Now for node point 0 the boundary condition is

$$y_0 = 0$$

similarly, $y_4 = 0$

Also, since the column is pin ended, the moments at the ends are each zero;

$$\therefore \quad \left(\frac{d^2y}{dx^2}\right)_0 = \left(\frac{d^2y}{dx^2}\right)_4 = 0$$

$$\therefore \quad \frac{y_a - 2y_0 + y_1}{\lambda^2} = 0 = \frac{y_3 - 2y_4 + y_b}{\lambda^2}$$

$$\therefore \quad y_a = - y_1$$

and $y_b = - y_3$

Finally owing to symmetry $y_1 = y_3$. Now, applying the finite difference procedure to point 1 yields, (since $y_0 = 0$),

$$\frac{y_2 - 2y_1}{\lambda^2} + \frac{P_{cr}}{EI} y_2 = 0 \tag{13a}$$

and to point 2:

$$\frac{y_3 - 2y_2 + y_1}{\lambda^2} + \frac{P_{cr}}{EI} y_2 = 0 \tag{13b}$$

writing equations (13a) and (13b) in matrix form, and letting $\frac{P_{cr}\lambda^2}{EI} = k$,

$$\begin{bmatrix} -2 + k & 1 \\ 2 & 2 + k \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = 0$$

But this system is set up for a solution for y, while the critical load P_{cr} is really what is required. P_{cr} can be obtained because the above is an Eigenvalue problem. The system of equations will have non trivial solutions for existing values of y if and only if the determinant of the coefficient matrix is zero. Setting this determinant equal to zero fixes k and thus fixes P_{cr}. Thus y is never solved. Solving for k:

$$\begin{vmatrix} -2 + k & 1 \\ 2 & -2 + k \end{vmatrix} = 0$$

$$\therefore \quad 4 - 4k + k^2 - 2 = 0$$

$$\therefore \quad k = 0.586,\ 3.141.$$

Using only the lower value, to get the lowest value of P, i.e. P_{cr},

$$P_{cr} = \frac{kEI}{\lambda^2} = \frac{kEI4^2}{L^2} = \frac{9.37EI}{L^2}$$

The exact solution to this problem is

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{9.84EI}{L^2}$$

$\therefore$ The result from the coarse grid contains an error of only 4 percent. If a grid of $\lambda = \frac{L}{2}$ is used, the result is

$$P_{cr} = \frac{8EI}{L^2}$$

Using Richardson's extrapolation for the results with the two grids yields

$$P_{cr} = \frac{9.6EI}{L^2}$$

which is an even more accurate approximation.

4.4e Finite Difference Approximation of the Fourth Order Differential Equation with Variable I

Consider the pinned column of Figure 7. The conditions are similar to those of Figure 6 except that the cross section, and hence I, is not constant. In this case the differential equation given in equation (9a) is the one that is approximated.

$$\frac{d^2}{dx^2}\left(EI\,\frac{d^2y}{dx^2}\right) + P\,\frac{d^2y}{dx^2} = 0 \qquad (9)$$

Referring to Figure 7, the approximation to this equation at point 1 is

$$\frac{1}{\lambda^2}\left\{(EI)_0 \frac{y_a - 2y_0 + y_1}{\lambda^2} - 2(EI) \frac{y_0 - 2y_1 + y_2}{\lambda^2}\right.$$

$$\left. + (EI)_2 \frac{y_1 - 2y_2 + y_b}{\lambda^2}\right\} + P \frac{y_0 - 2y_1 + y_2}{\lambda^2} = 0 \qquad (9b)$$

Note that in this approximation EI is assumed constant over each node point ± $\lambda/2$.

4.5 Finite Difference Solution for a Bean Cotyledon

Consider the single cotyledon schematically represented in Figure 8. It is considered to be simply supported and end loaded with force P. Let the smaller principal moment of inertia of the cross-section be I_M.

For a finite difference analysis, a grid of $\lambda = L/6$ has been applied to Figure 8.

Assume symmetry about point 3. Assume also that there is a simply supported boundary condition at each end.

The finite difference procedure is as follows:

Boundary conditions:

$y_0 = y_6 = 0$

$y_a = -y_1$

$y_b = -y_5$

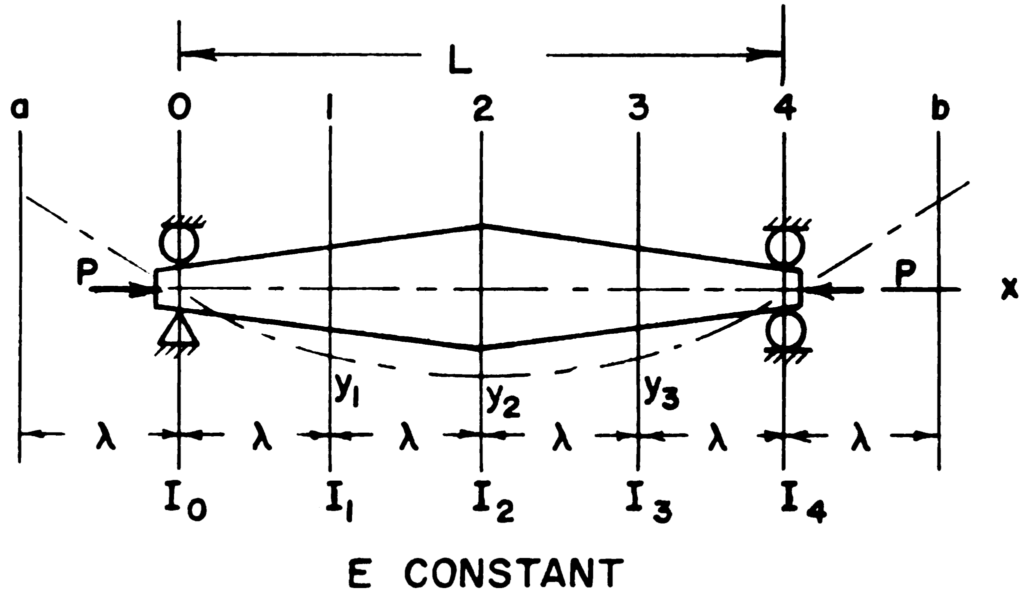


Figure 7.--Pin-ended column with variable moment of inertia.

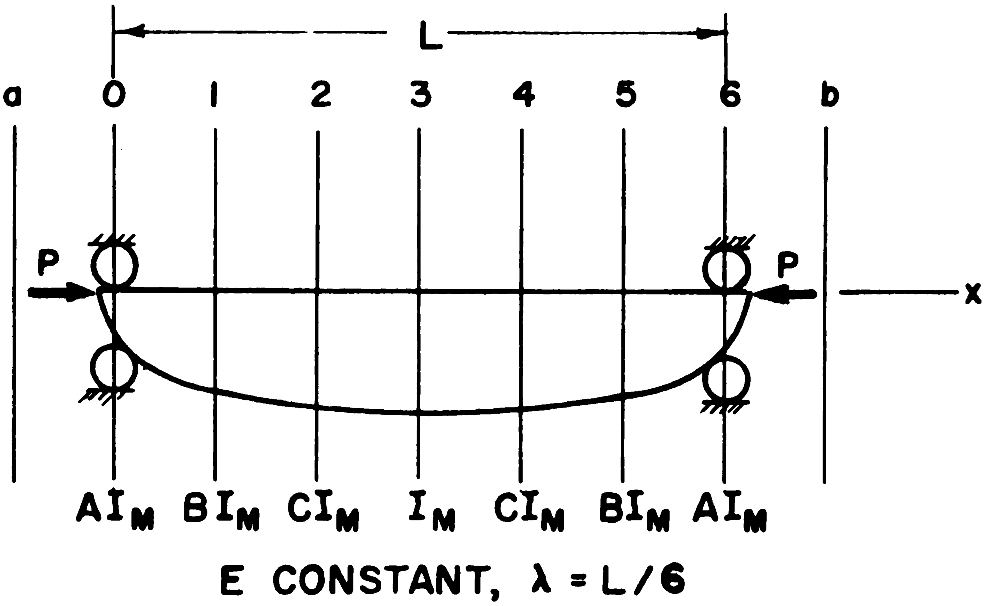


Figure 8.--Bean cotyledon considered as a pin-ended column.

Symmetry Conditions:

$y_1 = y_5$

$y_2 = y_4$

Moment of Inertia:

Let $I_3 = I_M$

$I_0 = I_6 = AI_M$

$I_1 = I_5 = BI_M$

$I_2 = I_4 = CI_M$

Substituting these values into equation (9b) and letting $(P\lambda^2)/(EI_M) = k$, the following equations are obtained.

Node Point 1.

$$y_1\,(2B + C - 2k) - y_2\,(2B + 2C + k) + Cy_3 = 0$$

Node Point 2.

$$y_1\,(-2B - 2C + k) + y_2\,(B + 4C + 2 - 2k)$$

$$+ y_3\,(-2C - 2 + k) = 0$$

Node Point 3.

$$y_1\,(2C) + y_2\,(-2C - 4 - 2C + 2k)$$

$$+ y_3\,(2C + 4 - 2k) = 0$$

In matrix form these equations can be expressed as a single system as,

$$\begin{bmatrix} (2B + C - 2k) & (-2B - 2C - k) & + C \\ (-2B - 2C + k) & (B + 4C + 2 - 2k) & (-2C - 2 + k) \\ 2C & 2(-2C - 2 + k) & 2(C + 2 - k) \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = 0$$

At this point the determinant of the coefficient matrix is set equal to zero to obtain the values of k. However B and C will be determined for the beans at the three stages of maturity used, then substituted in directly to save much of the bookkeeping required. With these values, the coefficient matrix will be simplified and its determinant easier to compute.

For extrapolation purposes a second, coarser grid, in which $\lambda = \frac{L}{4}$, will be used.

For this grid, the boundary conditions and symmetry conditions are the same as for the finer grid. The inertias will be,

$$I_1 = DI_M = I_3$$

$$I_2 = I_M$$

Again using equation (9b) and letting $(p\lambda^2)/EI_M$ be k, the finite difference equations become:

Node Point 1.

$$y_1 (4D + 2 - 2k) + y_2 (-2D - 2 + k) = 0$$

Node Point 2.

$$y_1 \ (-4D - 4 + 2k) + y_2 \ (2D + 4 - 2k) = 0$$

$$\therefore \begin{bmatrix} (2D + 1 - k) & (-2D - 2 + k) \\ (-2D - 2 + k) & 2(2D + 2 - k) \end{bmatrix} \begin{bmatrix} 2y_1 \\ y_2 \end{bmatrix} = 0$$

Again, values of D will be determined and substituted before the coefficient matrix will be set equal to zero.

From both grids, the values of P_{cr}, the lowest value in each case will be computed. Extrapolation will give the final value of P_{cr}

$$\text{But} \quad k = \frac{P_{cr}\lambda^2}{EI_M}$$

$\therefore$ P_{cr} will be computed as

$$P_{cr} = K \frac{EI_M}{L^2}$$

where K will be known from the calculations.

Now the actual value of P_{cr} will be measured on the Instron Testing Machine. Thus the value of EI_M can be computed.

V. DYNAMIC THEORY

As with most agricultural materials, beans are more often damaged by dynamic rather than quasi-static loads. Thus, while the preceding analysis serves to determine some of the mechanical properties of the bean, it could not answer many of the questions on damage to beans under actual harvesting and handling conditions.

The following analyses consider two types of impact conditions, a low velocity impact on a supported bean, by a falling weight, and a high velocity impact of a bean which is free to move after impact. In both cases, the loading is on end as in the static analysis (Figures 9 and 12).

5.1 Impact by a Falling Weight

Suppose a weight W, falling under gravity with a constant acceleration g were to strike a stationary bean (Figure 9) with an impact velocity V_i, and were to compress the bean from its original length L to a length y at which it ruptures.

The energy, at impact, of the system is:

$$E_i = \frac{1}{2}\frac{W}{g}V_i^2 + WL$$

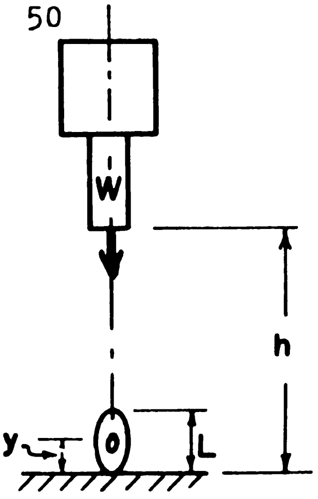


Figure 9.--Impact of a bean by a falling weight.

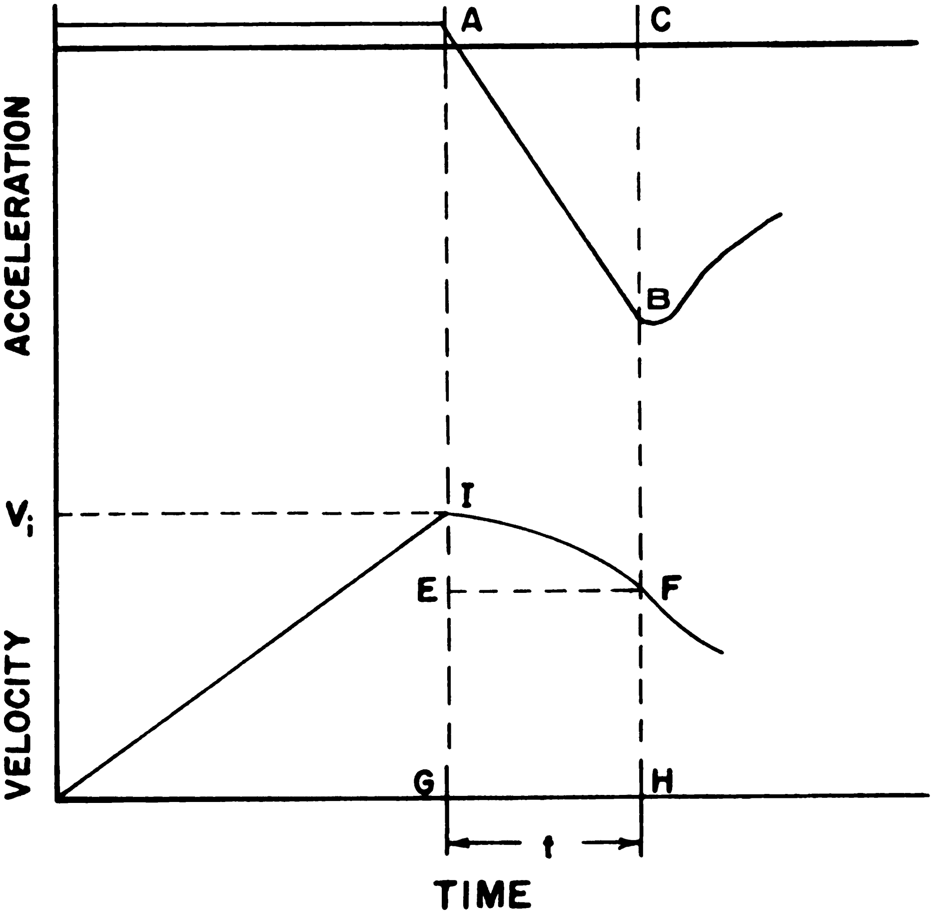


Figure 10.--Hypothetical acceleration-time and velocity-time curves [After Wright (1968)].

The energy at the rupture point, y, is

$$E_f = \frac{1}{2}\frac{W}{g} V_f^2 + Wy$$

Where V_f is the velocity of W at y. Then the energy dissipated during impact is

$$E_R = \frac{1}{2}\frac{W}{g} (V_i^2 - V_f^2) + mg\ (L-y)$$

Now suppose, as Wright (1968) suggested, the deceleration of the weight W is linear during impact up to the point of rupture. Then the decrease in velocity is parabolic.

Using an analysis similar to that of Wright, hypothetical impact curves can be drawn for acceleration and velocity (Figure 10). At time t = 0, the weight is released from a height h. At point A impact occurs; the bean is of length L. The deceleration of the weight is linear up to point B, at which point the bean has been reduced to length y.

The displacement of the weight during the impact from A to B is (L-y) and can be computed from the area under the velocity-time curve between A and B.

The change in velocity of the weight as it traverses the impact length (L-y) can be computed from the constant deceleration of the weight and the time of impact, i.e. the time between points A and C.

These calculations are as follows:

From Figure 10, the required area under the velocity curve is made up of a rectangle EFGH and half of a parabola EIF.

$$\text{Area EFGH} = V_f t = (V_i - \Delta V)t$$

$$\text{Area EIF} = \left(\frac{2}{3}\right) \quad (V_i - V_f)t = \frac{2}{3}\,(\Delta V)t$$

Adding these two equations yields the displacement of W during impact.

$$(L - y) = t(V_i - \frac{1}{3}\,\Delta V)$$

Thus the energy dissipated during impact is

$$E_R = \frac{W}{2g}\,(V_i^{\,2} - (V_i - \Delta V)^2) + Wt(V_i - \frac{1}{3}\,\Delta V)$$

$$= \frac{W}{2g}\,(2V_i\,\Delta V - \Delta V^2) + Wt(V_i - \frac{1}{3}\,\Delta V)$$

$$\therefore \quad E_R = \frac{W}{2g}\quad \Delta V(2V_i - \Delta V) + Wt(V_i - \frac{1}{3}\,\Delta V)$$

Where t is the duration of impact up to rupture and ΔV is the change in velocity of the falling weight during t.

Now let a_i be the constant rate of deceleration of the weight during impact up to the point of rupture. Beyond this rupture point the deceleration will no longer be linear, but this is of no consequence here. Letting t again be the duration of impact, then it can be measured as the horizontal distance from the initiation of impact to the point of rupture on the deceleration curve. The change in velocity ΔV is the area under the deceleration curve up to the point of rupture.

Thus the rupture energy E_R of the bean can be computed with suitable instrumentation to measure V_i, a_i and t.

The maximum force transmitted to the bean by the weight to cause rupture can either be computed or, with suitable instrumentation, be measured directly.

5.2 High Velocity Impact

When two deformable bodies in motion collide along a common line of action there is an exchange of energy consistent with the law of conservation of energy. During the first period of the impact the bodies come closer into contact with one another through a compression or deformation experienced by each, resulting in a fitting together of the two surfaces over a finite area. Because of the elastic properties of the bodies, a mutual force is called into action between them and tends to separate them. Lord Kelvin (1912), citing Newton's work in this area, states that provided the impact is not so violent as to destroy either body, the relative velocity of separation after the impact bears a proportion to their previous relative velocity of approach, which is constant for the same two bodies. This proportion, always less than unity, approaches unity the harder and more elastic the bodies are. The proportion, denoted by e is called the coefficient of restitution. If two bodies (Figure 11) moving with velocities V_i and V_i^1 respectively, collide and separate at velocities V_f and V_f^1 respectively, then by Newton's definition.

Coefficient of restitution $e = \frac{V_f^1 - V_f}{V_i - V_i^1}$

Now consider the case shown in Figure 12 where a mass on a rotating arm is about to strike a stationary bean. The following assumptions and definitions will be made.

V_{ai} = initial velocity (peripheral) of the mass on the arm.

V_{af} = final velocity after impact of the same mass.

V_{bi} and V_{bf} = initial and final velocities of the bean.

m = mass of the bean.

e = coefficient of restitution between the bean and the steel mass.

Assuming that the impact does not slow down the rotating arm, the arguments used by Lamp (1959) are valid here.

$$\therefore \quad V_{ai} = V_{af} = V_a$$

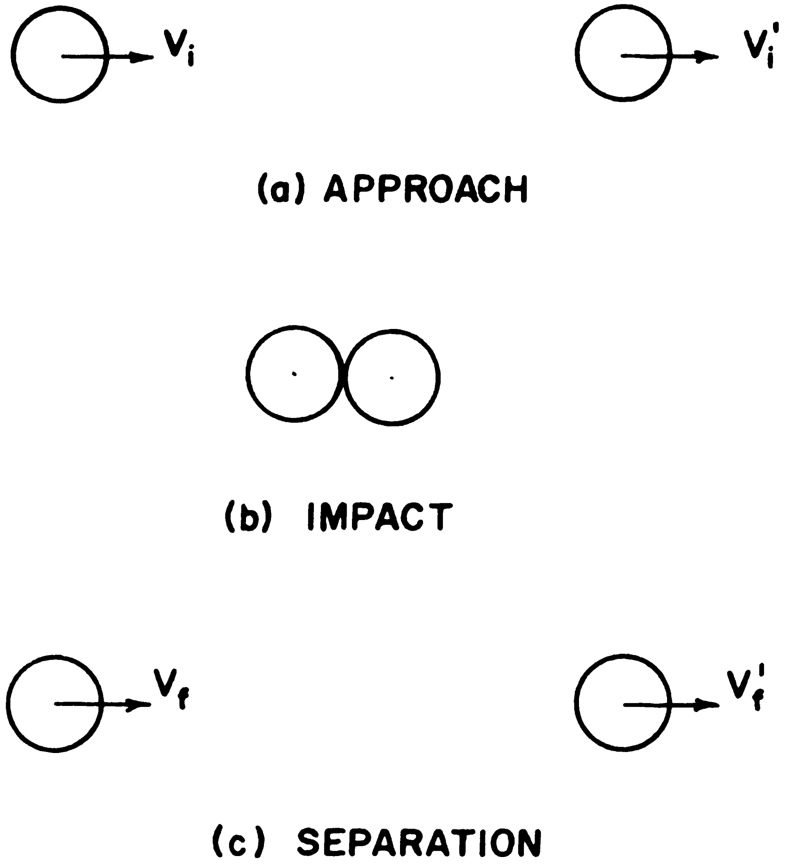


Figure 11.--Impact of two moving bodies.

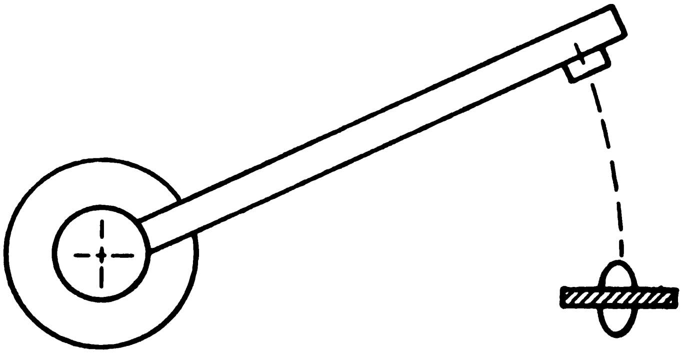


Figure 12.--Impact of a lightly held bean by a heavy rotating arm.

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Assume that the bean is originally at rest.

$$\therefore \quad V_{bi} = 0$$

Let $V_{bf} = V_b$

By the restitution equation

$$V_{af} - V_{bf} = -e(V_{ai} - V_{bi})$$

Applying the above assumptions the restitution equation becomes

$$V_b = V_a\ (1 + e)$$

If the bean were hard and perfectly elastic it would return all its internally absorbed energy; its coefficient of restitution would be unity. In this case the final velocity of the bean would be

$$V_b{}^1 = 2\ V_a$$

$$\therefore \quad V_b{}^1 - V_b = V_a(1 - e)$$

and $$V_b{}^1 + V_b = V_a(3 + e)$$

$$\therefore \quad (V_b{}^1)^2 - (V_b)^2 = V_a{}^2(1-e)\ (3 + e)$$

and the energy absorbed by the bean and not returned is thus

$$E = \frac{1}{2} m\{(V_b^1)^2 - (V_b)^2\}$$

$$\therefore \quad E = \frac{1}{2} m V_a^2(1 - e)(3 + e)$$

In his research on beans, Perry (1959) determined e for bean-steel impact. He gives a value of 0.57 which he states is quite independent of the moisture content of the bean within the range of 11 percent to 18 percent moisture, w.b. Using Perry's value for e then, the energy absorbed by the bean is

$$E = \frac{1}{2} m(0.43)\ (3.57)\ V_a^2$$

If V_a is just sufficient to cause cracking of the seed coat of the bean then E can be considered to be the rupture energy E_R

$$\therefore \quad E_R = \frac{1}{2} m(1.535)\ V_a^2$$

$$\therefore \quad E_R = 0.767 m V_a^2$$

where V_a is the peripheral velocity of the rotating arm of Figure 12, and m is the mass of the impacted bean.

Thus rupture energy can be quite easily obtained for the high velocity tests. As in the low velocity tests, impact force can be measured directly with suitable instrumentation.

VI. SUMMARY OF EXPERIMENTAL REQUIREMENTS

Because of the differences in the quasi-static and dynamic formulations, as well as those in the experimental instrumentation required, three sets of tests are required.

6.1 Quasi-Static Tests

6.1a Type of Analysis

Stability of end-loaded beans under the influence of an applied load sufficient to cause buckling.

6.1b Working Equations

1. $$\begin{vmatrix} (2B + C - 2k) & (-2B - 2C - k) & C \\ (-2b - 2C + k) & (B + 4C - 2k + 2) & (-2C - 2 + k) \\ 2C & 2(-2C + k - 2) & 2(C + 2 - k) \end{vmatrix} = 0$$

where B and C are the fractions relating the moments of inertia of the cotyledon at 1/6 L, and 1/3 L along the cotyledon to the moment of inertia, I_{M_1} of the center, and

$$k = \frac{P_{cr}\lambda^2}{EI_M}$$

where P_{cr} is the critical applied load

λ is the grid, $\lambda = L/6$

L is the length of the cotyledon and E is the modulus of elasticity of the bean.

$$2. \quad \begin{vmatrix} (2D + 1 - k) & (-2D - 2 + k) \\ (-2d - 2 + k) & 2(2D - 2 - 2k) \end{vmatrix} = 0$$

where D is the fraction relating the moments of inertia at $\frac{1}{4}$ L and $\frac{1}{2}$ L along the cotyledon and k is defined as before except that $\lambda = L/4$.

Finally the extrapolation equation will be used.

$$k = \frac{k_f \, n_f^2}{n_f^2 - n_c^2} - \frac{k_c \, n_c^2}{n_f^2 - n_c^2}$$

where the f and c refer to fine and coarse grids.

6.1c Quantities To Be Measured

These are: B, C, D, I_M, L and P_{cr}.

6.1d Objectives

Computation of EI_M and E. Compilation of the critical loads for beans of various moisture contents.

6.2 Low Velocity Impact Tests

6.2a Type of Analysis

Energy balance analysis of a weight falling under gravity and impinging upon a stationary, supported bean.

6.2b Working Equations

$$E_R = \frac{W}{2g} \Delta V(2V_i - \Delta V) + Wt(V_i - \frac{1}{3} \Delta V)$$

where W is the falling weight,

g is the acceleration due to gravity,

ΔV is the change in velocity of the weight during impact,

E_R is the rupture energy of the bean.

V_i is the impact velocity

t is the duration of impact up to rupture.

6.2c Quantities To Be Measured

These are: V_i, ΔV, t, assuming that W and g are known.

6.2d Objective

Computation of rupture energy, E_R, for beans of various moisture contents, and compilation of the maximum impact force F_R, which can be measured directly.

6.3 High Velocity Impact Tests

6.3a Type of Analysis

Energy balance analysis using the known restitution properties of beans.

6.3b Working Equations

$$E_R = 0.77\ m\ V_a^2$$

where E_R is the rupture energy,

m is the mass of the impacted bean,

V_a is the peripheral velocity of the impacting arm, and,

0.77 is a constant arrived at taking the coefficient of restitution of beans into account.

6.3c Quantities To Be Measured

V_a, m.

6.3d Objective

Computation of E_R for various moisture contents, and compilation of the maximum force F_R which can be measured directly.

VII. APPARATUS

For preliminary qualitative testing a Valve-Air Unit of the type developed by Mohsenin (1963) was used. The applied quasi-static loads were measured by a Sanborn 50-pound strain gage load cell. The unit is shown in Figure 13 (left).

7.1 Quasi-Static Tests

As the theory of Section 5.2 was being developed, the experimental procedure for determining the required variables was designed around an Instron Table Model Testing Machine. Figure 13 shows the general layout of the apparatus. The cross-head of the Instron was slightly modified so that the test beans could be loaded with a 3/8 inch diameter cylindrical probe. In this way the test bean could be positioned in the upright position with a small disc as shown in Figure 14. The cross-head would then be lowered manually just enough to hold the bean upright for the start of the test. Test loads and deformations were recorded on the Instron chart in the normal way.

During each test, the bean was kept under constant observation with the help of a Bausch & Lomb stereo

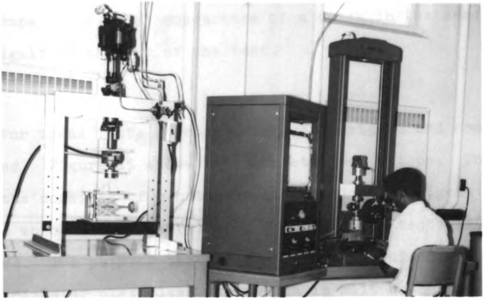

Figure 13.--General layout of the testing area with the Instron machine (right) and the Valve-Air unit (left).

Figure 14.--Location of the test bean.

microscope. The first appearance of a crack in the seed coat signified the end of the test.

7.2 Drop Tests

For these tests a Drop Tester was designed and constructed. Figure 15 shows the complete test set up. The apparatus consists of a hollow cylindrical drop weight (2)* to which the probe (3) used in the static tests is attached. The path of the falling weight is guided by a 2-inch diameter plexiglass tube which is slit down one side to allow the passage of a cable connected to an accelerometer in the drop cylinder. An electromagnet (1), powered by a d-c supply (6), was used to raise the weight to the required drop height. A Sigma 8P3 Photorelay, (5), energized by a Sigma 8L3 Light Source, (4), was used to trigger the Oscilloscope as well as to indicate the time taken for the cylinder to go by. The bean to be impacted was positioned on a load sensing device which consists of a quartz load cell sandwiched between two thick mild steel discs (Figure 16).

The impact deceleration of the falling weight was measured by a Piezotron Model 818 quartz accelerometer, which requires no charge amplifier. The impact force was measured by a Kistler Model 912 quartz load cell. The signal from the load cell was fed into a Kistler Model

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*The numbers in parentheses refer to those in Figure 15.

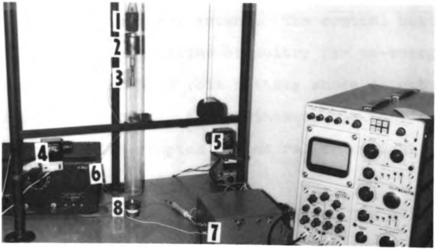

Figure 15.--The drop test apparatus.

1. Electromagnet
2. Drop weight
3. Probe
4. Light source for photorelay
5. Photorelay
6. D-C power supply
7. Trigger control box
8. Load cell assembly

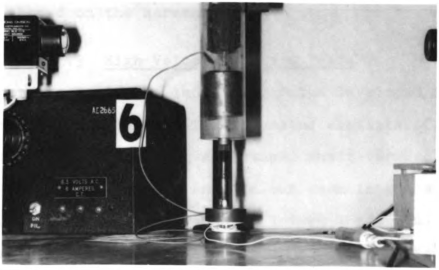

Figure 16.--Location of the test bean.

503M15 charge amplifier (not shown). The control box (7) shown in Figure 15 contains circuitry for de-energizing the electromagnet, and a 15 volt battery whose output passed through the normally open points of the photorelay, when the light was interrupted by the falling weight, into channel No. 1 of the oscilloscope.

The oscilloscope used was a Tektronix Type 549 storage model, set to trigger on the battery input into channel No. 1. The accelerometer and amplified load cell outputs were fed into channels No. 3 and No. 2 respectively. The length of the battery trace on channel No. 1 indicated the time taken by the 2-inch long cylinder to pass the photorelay, thus providing a measure of the impact velocity. A Hewlett Packard Oscilloscope camera was used, with a special adapter, to photograph the stored traces for some tests. Because of the storage capability however, the measurements were mostly taken directly from the patterns stored on the screen.

7.3 High Velocity Impact Tests

For these tests the impact apparatus developed by Burkhardt (1969) was used. The apparatus consists of a 5 hp electric motor powering a flywheel shaft through a variable speed belt drive. An aluminum beam impact arm, mounted on a shaft in line with the driven shaft could be engaged with the latter by means of an electric clutch.

The system was designed such that the impact arm attains the speed of the driven shaft in about 270 degrees of motion.

At the extremity of the impact arm, which is 18 inches long, a Kistler Model 901A load washer mounted between two steel discs (Figures 17, 18) is used to measure the impact force. The test bean was held upright between two thin strips of tape as shown. After impact, the bean was caught in a well padded catch box.

Impact takes place after.about 280 degrees of arm rotation. About 10 degrees after impact, a cam operated switch disengages the electric clutch and energizes a brake which stops the arm. The total arm rotation in a test is a little more than two revolutions. Thus the signal cable is run through the arm and out through a hole near the shaft and thence to a Kistler Model 503 M15 charge amplifier. No slip rings are used.

The amplified signal is fed into the Tektronix Storage Oscilloscope. The latter is triggered by the output of a Piezotron accelerometer mounted on the impact arm, near the load washer.

7.4 Auxiliary Equipment

A Gillings-Hamco thin-sectioning machine was used to section beans at the required positions for computing values of the inertia factors B, C and D.

Moisture contents were determined by oven drying at 104° C for 48 hours. For this, a Freas Model 625 forced

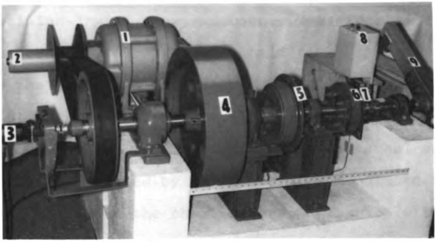

Figure 17.--High velocity impact apparatus.

1. 5 hp electric switch
2. Variable speed pulley
3. Tachometer
4. Flywheel
5. Electric clutch
6. Electric brake
7. Cam actuated limit switch
8. Brake release switch
9. Impact arm

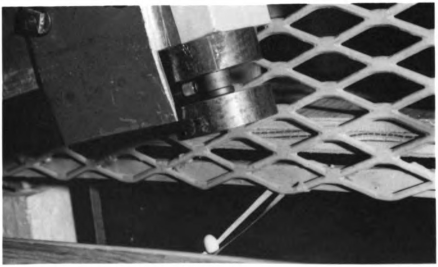

Figure 18.--Location of the test bean.

draft precision drying oven was used. Weighings were made on a Seederer-Kohlbusch precision balance.

For the high velocity impact tests, some rewetting of the test beans was necessary. For this, a small conditioning chamber was constructed, and its internal environment was controlled by an Aminco-Aire conditioner. The conditions within the chamber were monitoried with a Hygrodynamics Model 51-3001 hygrometer indicator.

7.5 Calibration of Equipment

The Instron load cell as well as the dynamic load cells were calibrated in place before and after each test run using the Instron calibration weights.

The accelerometer calibration was checked by bolting it to a Wilcoxin Model SR-44 quartz accelerometer and vibrating the two in the range of 1000 to 2000 hz. on a small shaker. The Wilcoxin accelerometer had previously (5/1/68) been sent back to the manufacturer for recalibration.

VIII. EXPERIMENTAL TECHNIQUE

In September, 1968, a series of quasi-static tests were conducted on two lines of navy beans which were developed by the Michigan State University Crop Science Department. These lines were designated No. 70 and No. 74. Dynamic tests were conducted on a commercial variety, Seafarer navy beans, during September and October of 1968.

8.1 Quasi-Static Tests

8.1a Harvest

Four harvests, two for each line were made on four separate dates. The period between the first harvest of line 74 and that of line 70 was one day. The same applied to the second harvests. The period between the first and second harvest for each line was two weeks.

Because of staggered planting dates the first set of harvests yielded beans with green, yellow and white (ripe) pods, making a wide variety of moisture contents available. Between the time of actual harvest and the start of a test the beans were kept in the pods in a refrigerator. The second set of harvests yielded only yellow pods and ripe beans.

8.1b Test Procedure (Figure 19)

For each line and each pod color, nine beans were selected and tested as soon as possible after harvesting and shelling. The nine beans were considered as three 3-bean samples. Each 3-bean sample was tested at one of three Instron cross-head speeds. These speeds were 0.05, 0.1, and 0.2 ipm. The use of three cross-head speeds was necessitated by the possibility that time dependent properties of the beans could affect the critical loads in the quasi-static range.

The rest of the shelled beans, in their labelled containers, were left out to dry in the laboratory. After 72 hours, another set of samples was chosen and tested. Another test was conducted after 144 hours.

Each 3-bean sample, after being tested at its designated cross-head speed was sealed immediately in a small plastic bag and was used at the end of the entire test for that particular line, color and time, for moisture content determination.

For each bean, a test consisted of loading the bean on end at its designated loading rate until a crack appeared in the seed coat. At this point the loading was stopped. The load thus obtained was considered the critical load for the bean, and therefore, twice the critical load, P_{cr}, for each cotyledon. The strength of the coat was considered as part of the cotyledon stiffness.

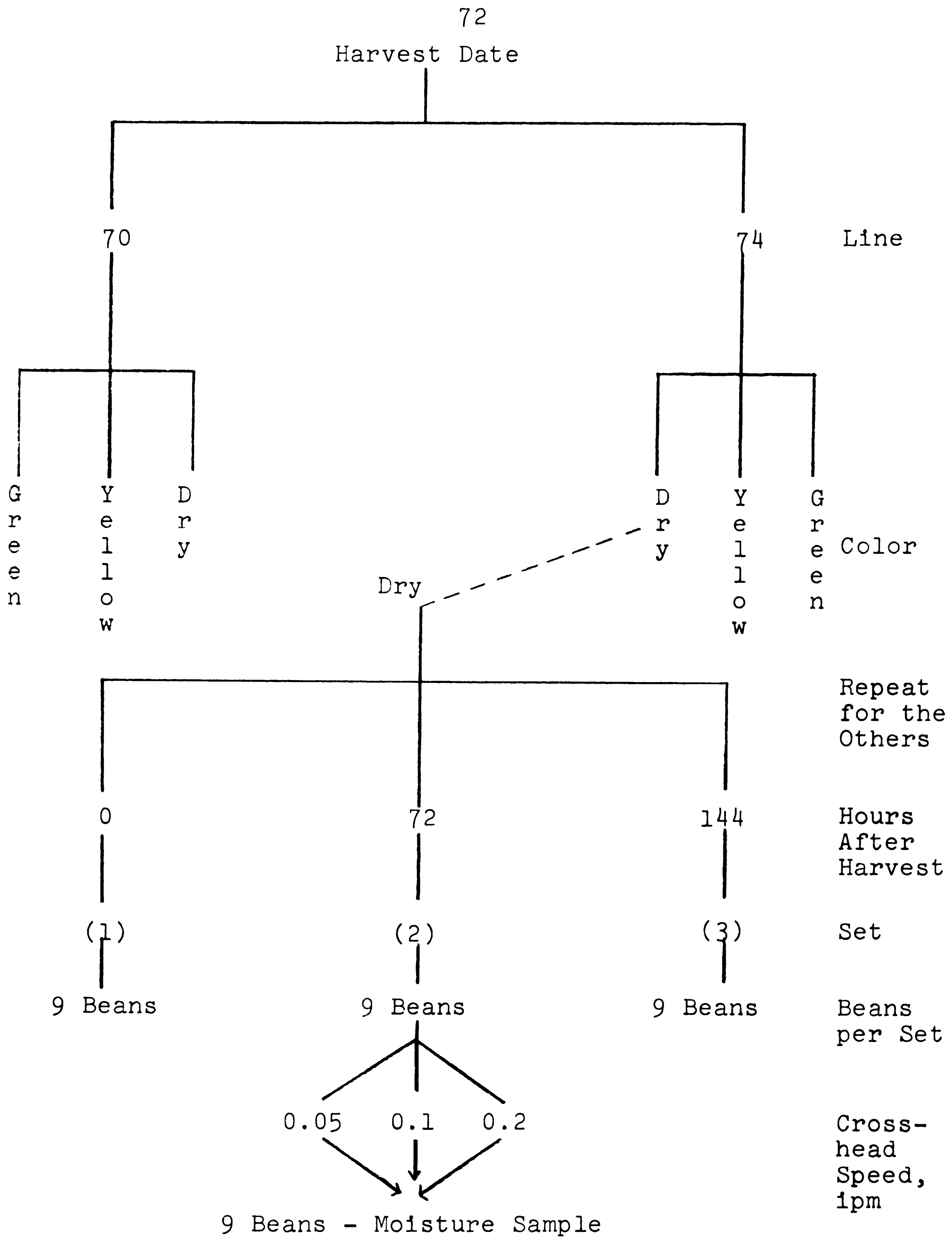

Figure 19.--Schematic of the experimental design.

8.1c Moment of Inertia Factors

For each line of beans, considering the mature beans only, three samples of five beans each were sectioned at distances of $\frac{L}{6}$, $\frac{L}{4}$, $\frac{L}{3}$ and $\frac{L}{2}$ from one end; L being the length of each bean. The major and minor diameters of each section was measured. The values were averaged over the 15 measurements for each position of sectioning. The average moment of inertia was computed for each position of sectioning of each line. The ratios B, C and D were then computed for the cotyledons. Thus for the beans used for the Instron tests only the lengths and the diameters of the center were measured.

8.1d Follow-up Tests

Impact tests were conducted during September and October of 1968 on a commercial line of Seafarer beans. For comparison purposes, Instron tests, using only one loading rate, 0.2 ipm, and two moisture contents were made on beans of this line also. For each moisture content a sample size of ten beans was used.

8.2 Low Velocity Impact Tests

Beans of the Seafarer variety were used for these tests. Because of weather problems during harvest, only one harvest was made. By allowing the beans to dry in the pods in the laboratory, however, four moisture contents were obtained for testing. In addition, beans that had been allowed to dry down to 11 percent moisture were re-conditioned to 17 percent to see whether this reconditioning had any effect on the impact resistance of the beans. This was done primarily because much of the high velocity impact testing was done on rewetted beans.

For each test, at the given moisture content, a drop height was found, by trial and error, at which cracking of the coat took place. Ten beans were then selected and tested. Acceleration, impact force and impact velocity were measured for each bean and the results averaged. Impact energy to cause rupture was computed.

At the higher moisture contents, because the checking was not severe, the tested beans were then used for moisture content determination. At the very dry condition (11 percent) however there was complete splitting of some of the test beans. For this case ten other beans, kept under identical conditions but not tested, had to be used to determine the moisture content.

8.3 High Velocity Impact Tests

Because of the difficulty in relating the two previously described tests, high velocity tests were performed. By the time the tests were designed only very dry beans were available for testing. Accordingly, in addition to the tests on the dry beans, rewetting was carried out using the Aminco-Aire Unit in order to attain two other moisture content conditions.

The tests were carried out in a manner to those for low velocity impact. For each moisture content, the velocity of the impact arm required to initiate seed coat cracking was found by trial and error. Then a ten-bean sample was tested and the impact force measured. Checks were made to ensure that the tape supporting the test bean was not affecting the test results by impacting the tape only. In all cases, the tape broke free from its attachments without eliciting a response from the load cell.

For the very dry beans, even the lowest velocities at which the apparatus could operate were sufficient to cause severe checking and splitting of the beans. Thus tests were carried out at three moisture levels only. As was the case with the dry beans of the low velocity impact tests, dry beans, because of the severe checking were not used to determine moisture content. Instead, ten beans which had been stored under the same conditions but which were not themselves impacted, were used to determine the moisture content.

IX. RESULTS AND DISCUSSION

9.1 Moment of Inertia Factors

The dimensions measured for the three 5-bean samples were averaged and the moment of inertia was then computed for each position or station along the bean. This value was then divided by two, assuming equal sized cotyledons, and the parallel Axis theorem was then used to compute the moment of inertia of each cotyledon.

The value of the factors B, C and D were then computed. These are presented in Table 1. The beans used for this determination were at 13.5 percent w.b. moisture level which was found to be the best moisture content for sectioning the beans without tearing or shattering. The assumption made here is that the bean will maintain its shape, even though it changes slightly in size, as the moisture content changes.

TABLE 1.--Average values of inertia factors B, C and D for bean cotyledons.

Station along bean	Moment of inertia of cotyledon in.^{4} x 10^5	Ratio I/I_M
L/6	1.17	B = 0.37
L/3	2.67	C = 0.84
L/4	1.53	D = 0.48
Center	3.18	1.00

Thus these values of B, C, and D were used for all the computations involved in the quasi-static data reduction.

9.2 Critical Loads and Stability and Elastic Moduli for Beans Loaded Quasi-Statically

For each moisture content, no effect of rate of loading was evident. This is in agreement with Zoerb's (1958) conclusion that there is very little viscoelastic effect within the range of loading rates used in this experiment.

Thus the results for each line, maturity (pod color) and moisture content, i.e., each 9-bean sample shown in Figure 19, were averaged. Using the measured value of critical load P_{cr} (half the load which caused the seed coat to crack), the major dimensions of the beans, and the values given in Table 1, the maximum stability modulus EI_M and the values of the apparent elastic modulus E for the cotyledon was then computed for each moisture content.

These values are presented in Table 2 and illustrated in Figures 20 through 27.

9.2a Critical Loads P_{cr}

Figures 20 and 21 show the effect of moisture content and maturity on the critical load P_{cr}. For line 74 the load increases rapidly with decreasing moisture content, peaks in the range of 12.5 - 13.5 percent moisture range but as can be seen in Figure 21, these differences

TABLE 2.--Critical loads, elastic and stability moduli for two lines of navy beans.

Harvest No.	Hours of drying	Maturity*	Line	Moisture percent	P_{cr} lbs	EI_M lb-in.^{2}	E psi x 10^{-4}
1	~6	G	74	20.1	7.7	0.221	0.98
		Y	74	18.2	11.5	0.342	1.54
		R	74	16.1	16.8	0.531	1.90
	72	G	74	13.5	18.1	0.474	2.84
		Y	74	13.2	20.0	0.544	2.64
		R	74	13.5	18.8	0.621	2.68
	144	G	74	12.0	16.8	0.414	2.26
		Y	74	11.8	21.9	0.547	2.16
		R	74	12.0	19.3	0.606	2.13
4	~2	Y	74	16.2	17.3	0.487	1.66
		R	74	13.4	19.3	0.636	1.67
	48	Y	74	12.8	19.2	0.552	1.86
		R	74	12.2	19.8	0.650	1.76
	144	Y	74	10.8	16.9	0.464	1.48
		R	74	10.8	16.7	0.508	1.35
2	~0	G	70	28.2	2.8	0.115	0.23
		Y	70	26.3	3.6	0.167	0.26
		R	70	26.0	3.8	0.165	0.30
	72	G	70	14.0	18.0	0.409	1.59
		Y	70	13.5	18.5	0.449	1.42
		R	70	13.5	18.2	0.454	1.51
	144	G	70	11.8	18.8	0.413	1.42
		Y	70	11.7	18.4	0.465	1.45
		R	70	11.7	18.8	0.466	1.48
3	~0	Y	70	16.4	15.4	0.346	1.50
		R	70	13.2	19.8	0.502	1.49
	48	Y	70	13.5	17.0	0.379	1.43
		R	70	12.5	19.0	0.482	1.43
	144	Y	70	10.5	17.0	0.329	1.40
		R	70	10.5	17.5	0.406	1.31

*The letters G, Y and R designate beans from green, yellow and ripe pods, respectively.

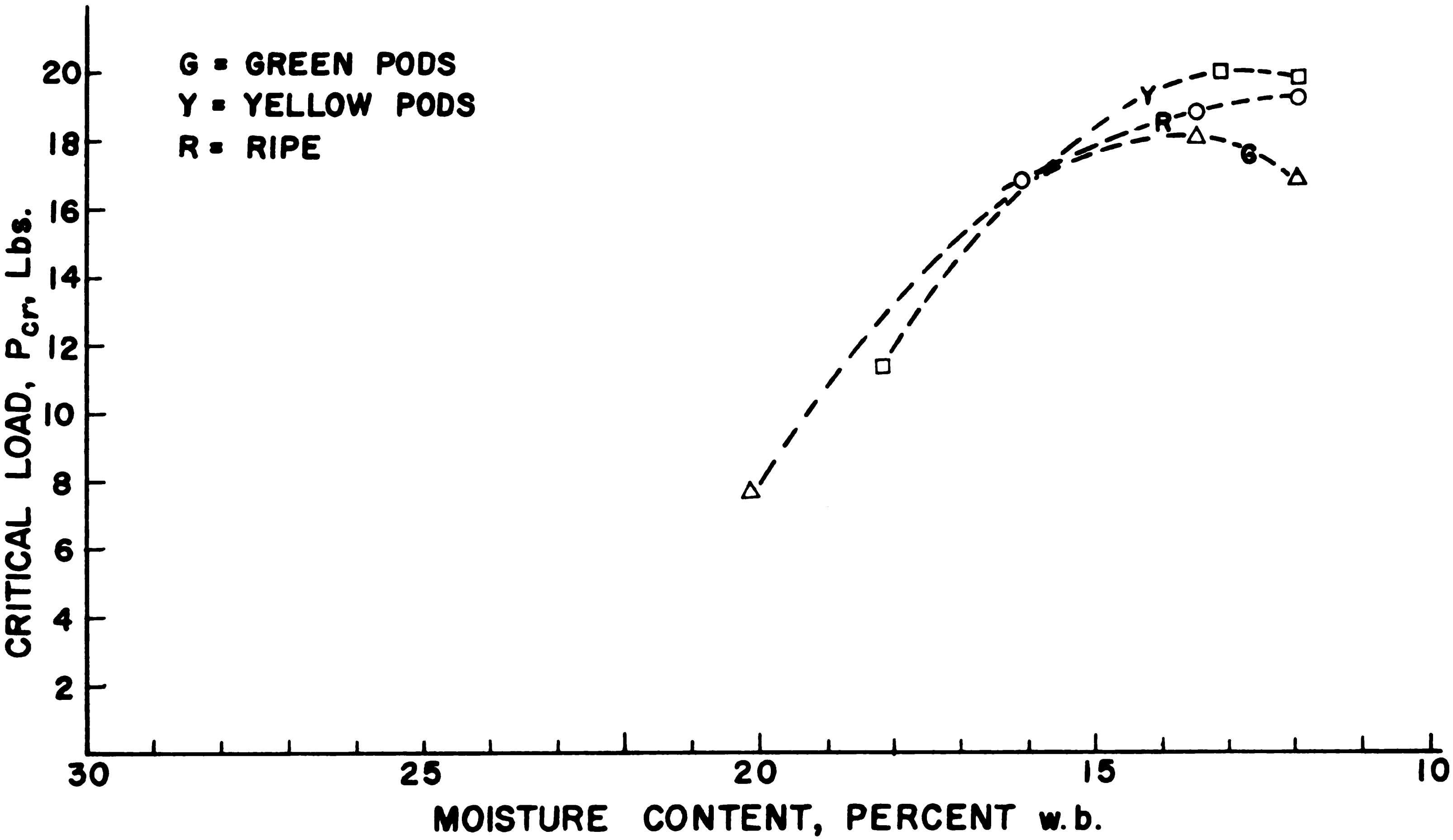

Figure 20.--Variation of P_{cr} with maturity and moisture content for navy beans. First harvest, quasi-static loading, line 74.

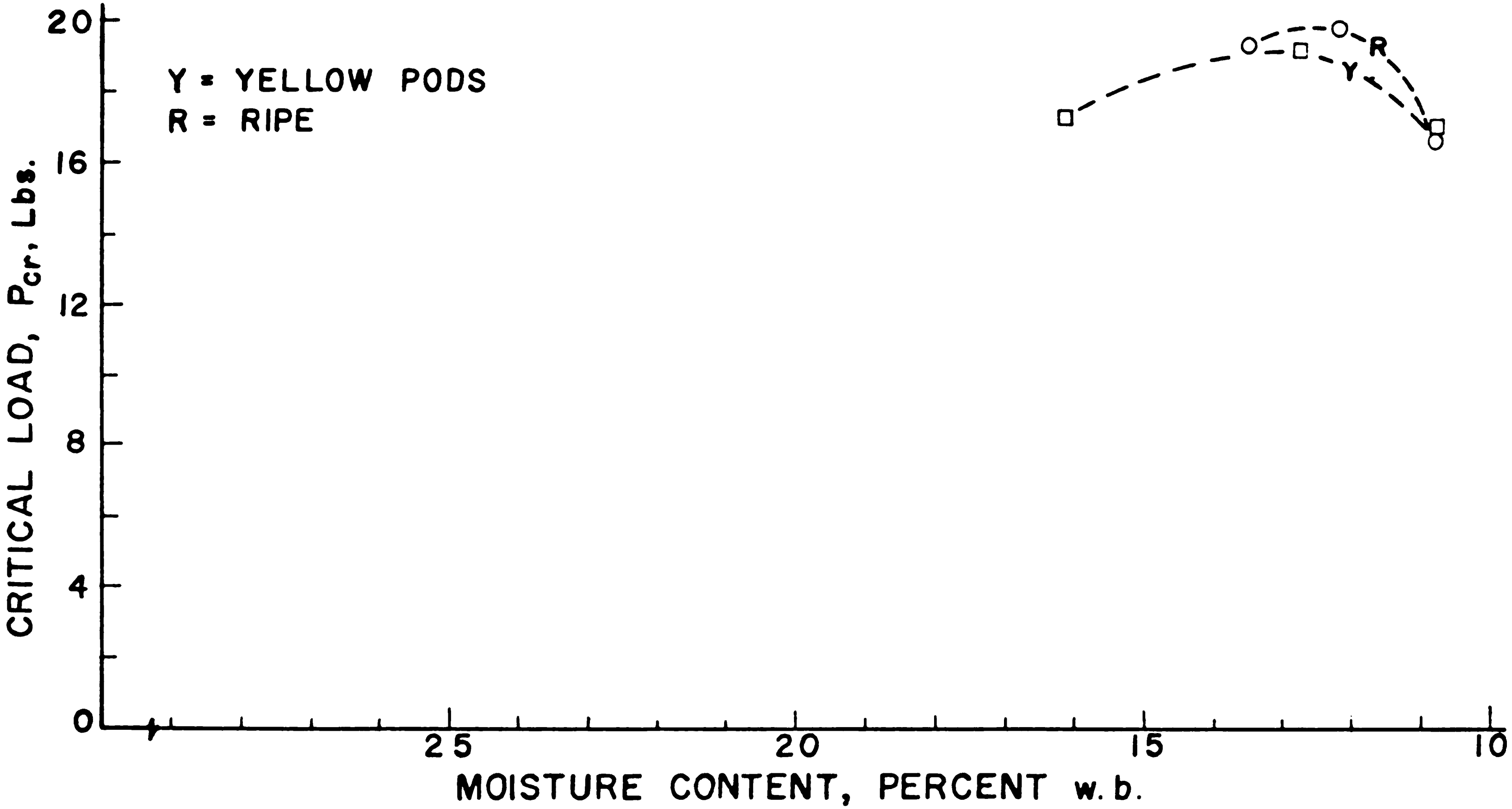

Figure 21.--Variation of P_{cr} with maturity and moisture content for navy beans. Second harvest, quasi-static loading, line 74.

tend to disappear as the beans dry out further, for the yellow and ripe beans. The beans from the green pods become much weaker at the low moisture contents. The similarity of results shown in Figures 20 and 21 for the low moisture range indicates that the extra two weeks of maturing on the plant had little effect on the critical load.

The beans of line 70 were much wetter than those of line 74 at the first harvest, and at the high moisture contents, yielded a different result. Figures 22 and 23 show the variation of P_{cr} for the two harvests. The critical loads start from a much lower point but eventually attain approximately the same maximum value, 19 - 20 lbs, in the 12.5 - 13.5 percent moisture range. Thus lower rates of increase of P_{cr} were found.

It is possible, however, that had the harvest been made one week later that the curves may have been more similar to those for line 74, i.e., an extra week of maturing on the plant may be critical at that stage. The second harvest yielded beans of which those in yellow pods were found to be weaker than on the first harvest, at about 13 percent moisture. This was the case also, but to a lesser extent with the beans of line 74.

For both lines of beans the optimum moisture content for withstanding axially compressive loads was 13 percent w.b. At this moisture level the critical load

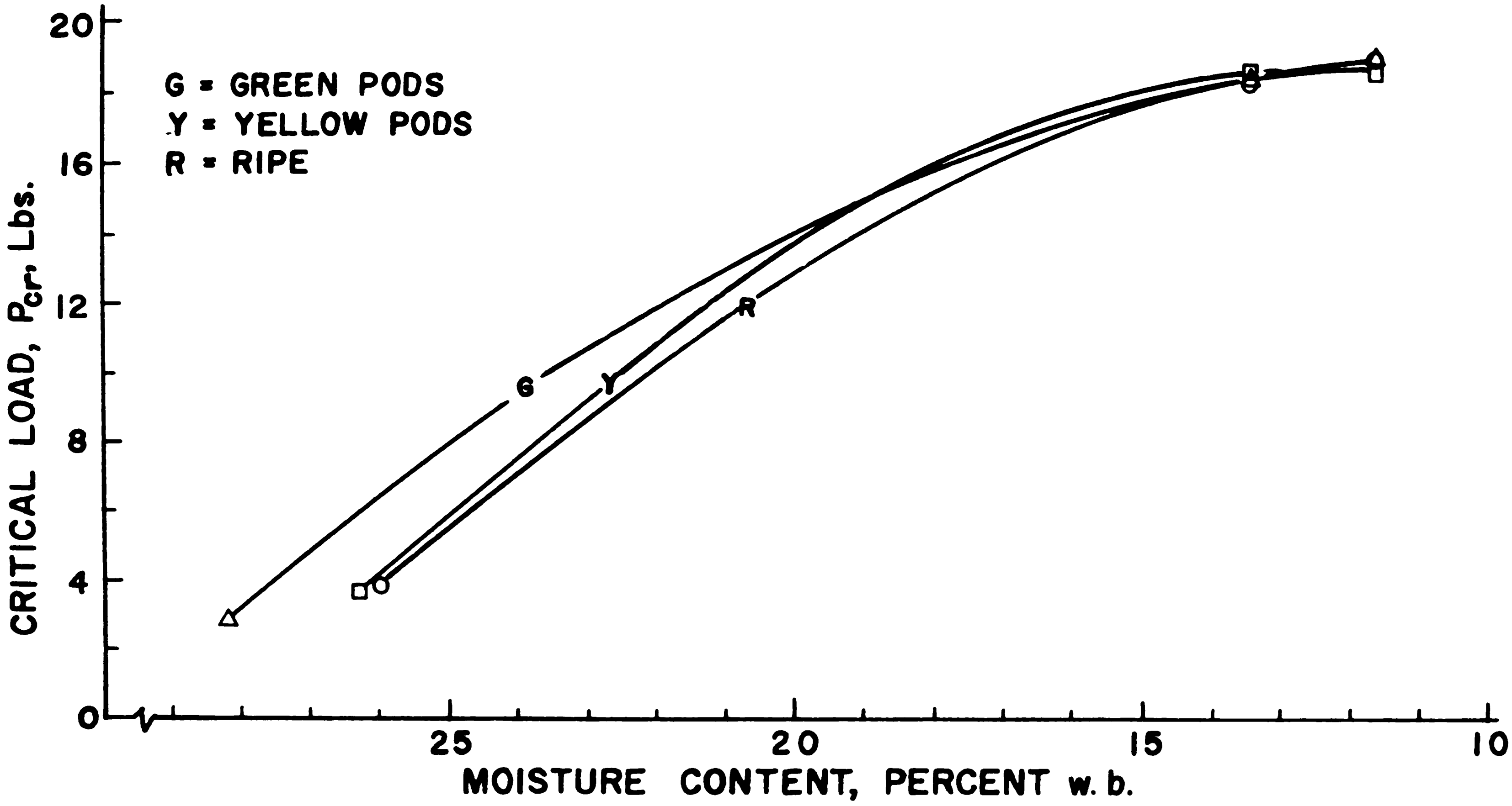

Figure 22.--Variation of P_{cr} with maturity and moisture content for navy beans. First harvest, quasi-static loading, line 70.

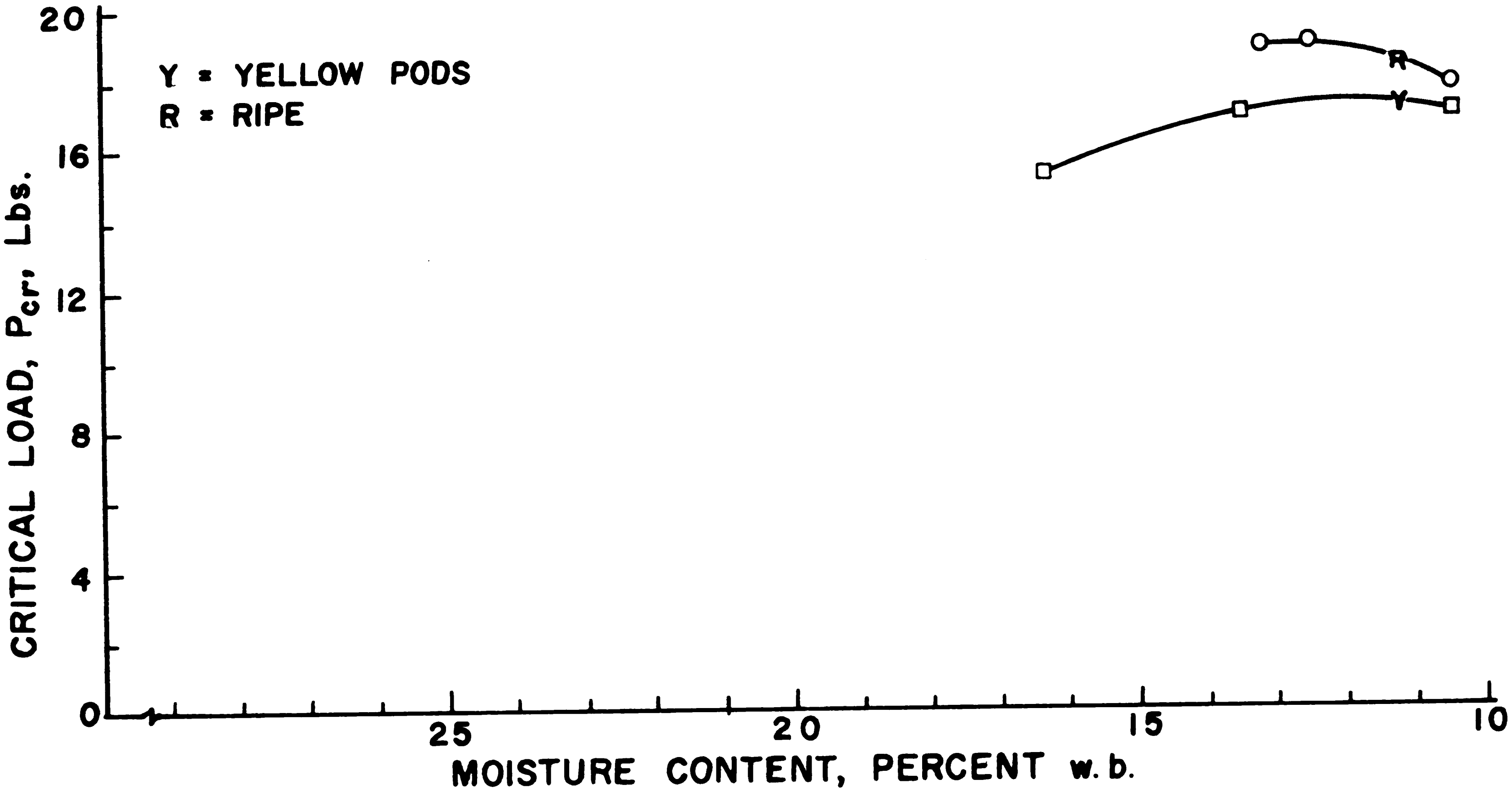

Figure 23.--Variation of P_{cr} with maturity and moisture content for navy beans. Second harvest, quasi-static loading, line 70.

to produce cracking of the seed coat was between 18 - 20 pounds for a single cotyledon, i.e., 36 - 40 pounds for a bean.

In most cases the cracks on the seed coat started at one extremity of the bean and progressed around the hylum. This is because the coat is thickest near the hylum and less susceptible to cracking there. For the very wet beans however (26 percent w.b.), the tissue in the center of the hylum appeared to be very soft and the cracks started there.

For each moisture level some beans were loaded on the side as a check. At the lowest moisture level, 11 percent, the side loaded beans were found to break at loads very little lower than those required to crack the coats of end loaded beans. Thus, when very dry, beans become susceptible to mechanical damage from both side and end loads.

9.2b Stability and Elastic Moduli

The variation in EI_M values with moisture content, line and maturity are shown in Figures 24 and 25. This variation is very similar to that for critical loads, except that at the lower moisture contents there is a definite ranking according to maturity, with the ripe beans exhibiting the highest values and those from green pods, the lowest values. Also, line 74 beans were found

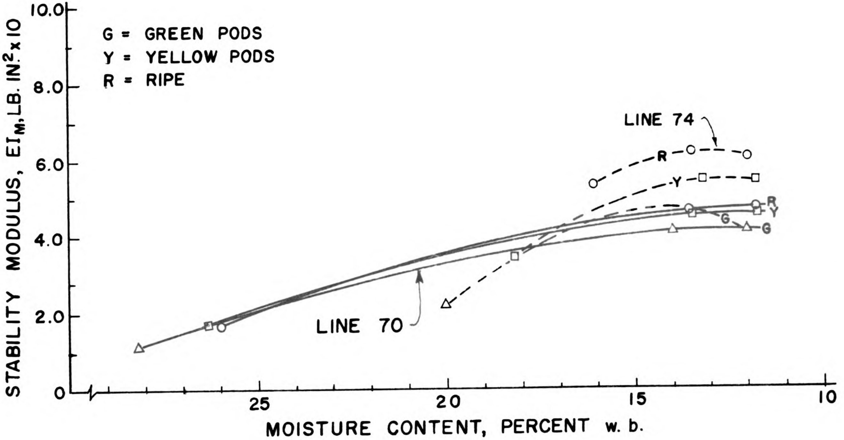

Figure 24.--Variation of EI_M with maturity and moisture content for navy beans. First harvest.

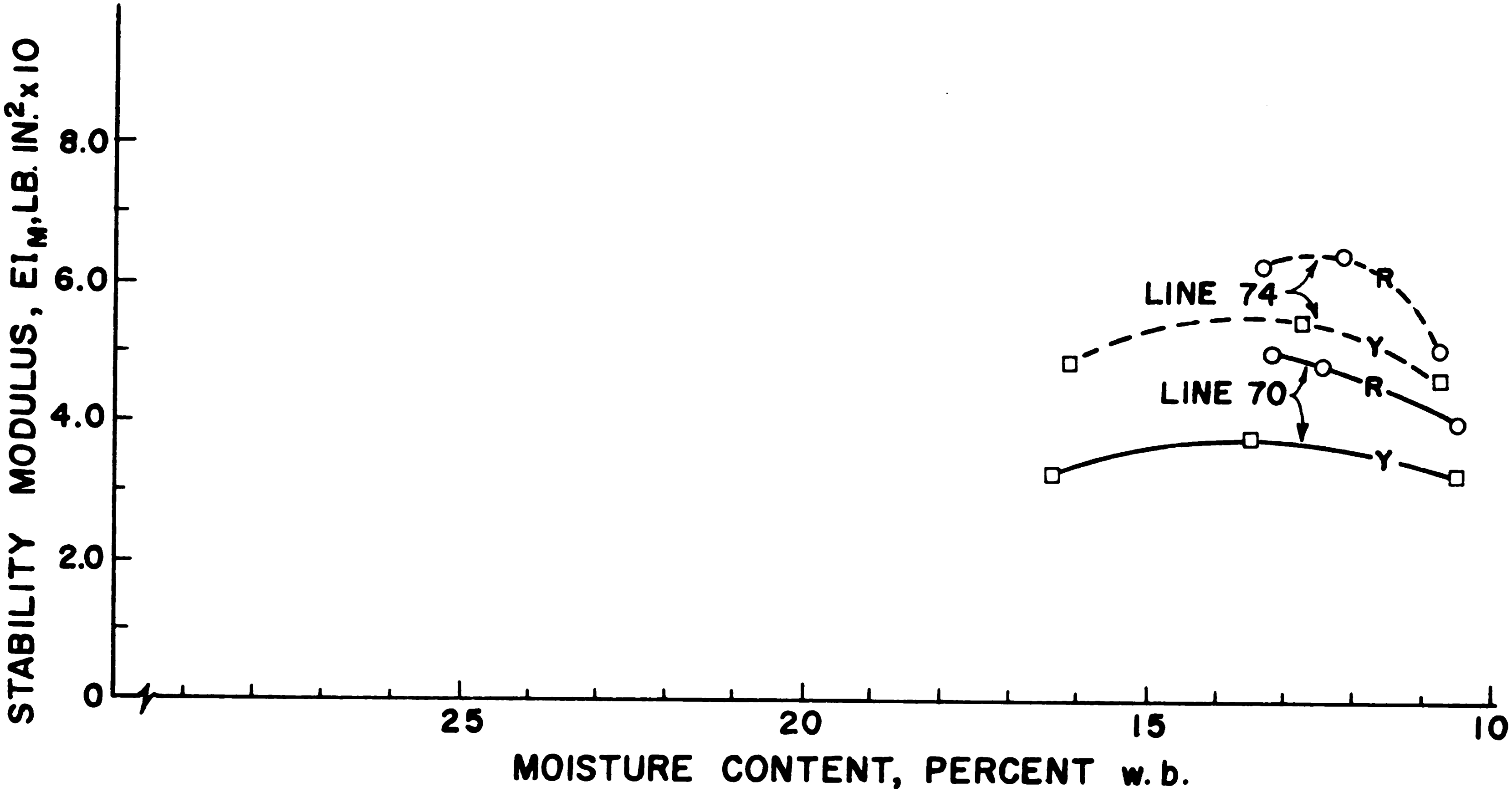

Figure 25.--Variation of EI_M with maturity and moisture content for navy beans. Second harvest.

to have higher EI_M values in the optimum moisture content range of 12.5 - 13.5 percent w.b.

This difference between lines becomes very apparent upon examination of the curves for elastic modulus E, shown in Figures 26 and 27.

The optimum moisture content in this case was found to be about 14 percent w.b., at which point the average E value for line 74 beans is 27,500 psi., almost double the 15,000 psi. value for line 70.

This difference in E values arises out of the differences in moment of inertia caused by small differences in major dimensions of the two lines.

The relative shapes of the two curves are similar to those of EI_M and P_{cr} except that the ranking according to maturity is no longer definite. The decrease at low moisture content is preserved.

Of the two quantities EI_M and E, the former is the more significant measure of the beans ability to withstand a buckling type of failure while the latter is a material constant which would be more useful for computing deformations of flat-loaded beans.

Because of the close mathematical relationship between P_{cr} and EI_M, the similarity of these two sets of curves was expected. Any differences in shape would be due to small differences in the lengths of the test beans.

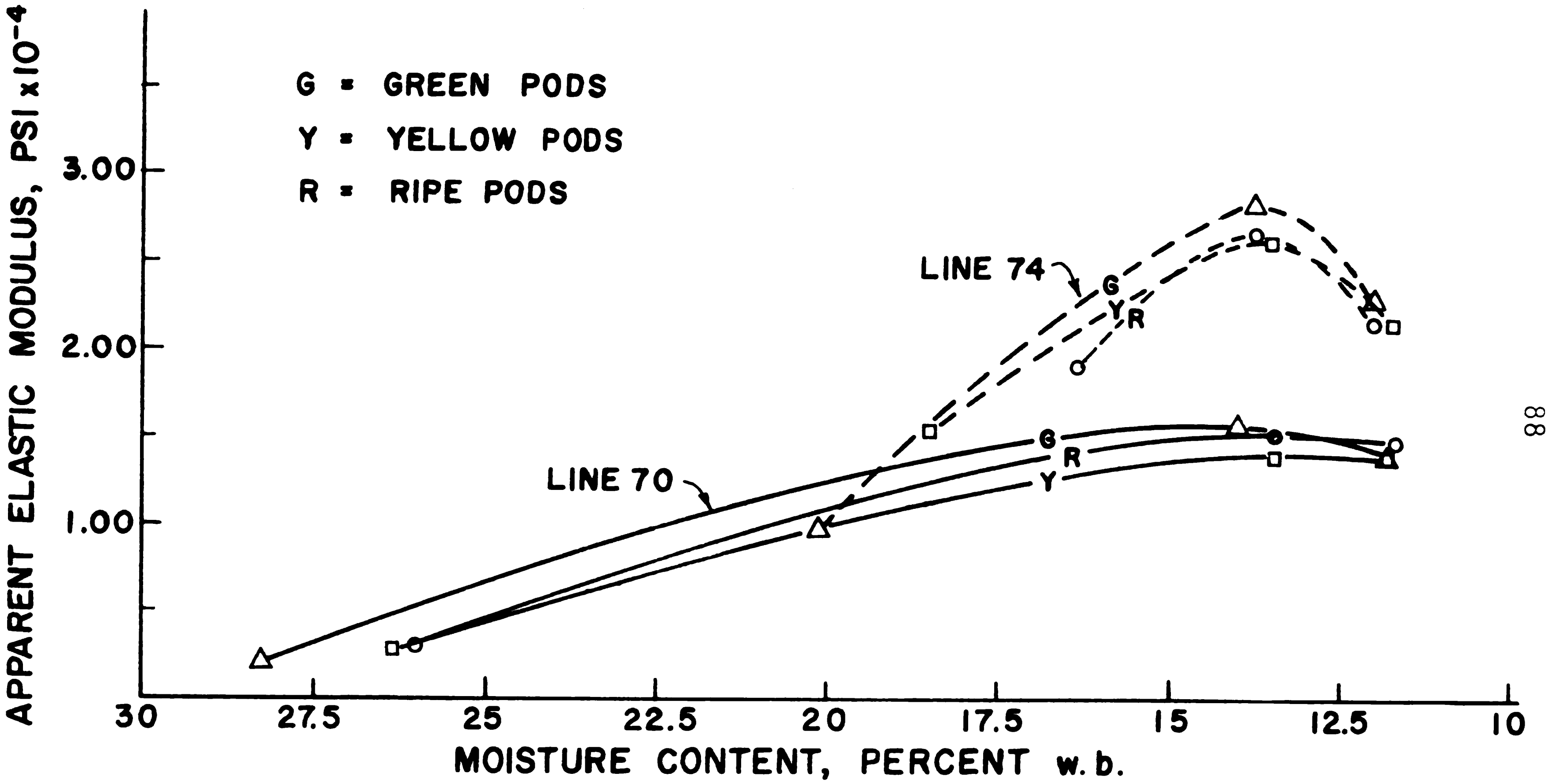

Figure 26.--Variation of E with maturity and moisture content for navy beans. First harvest.

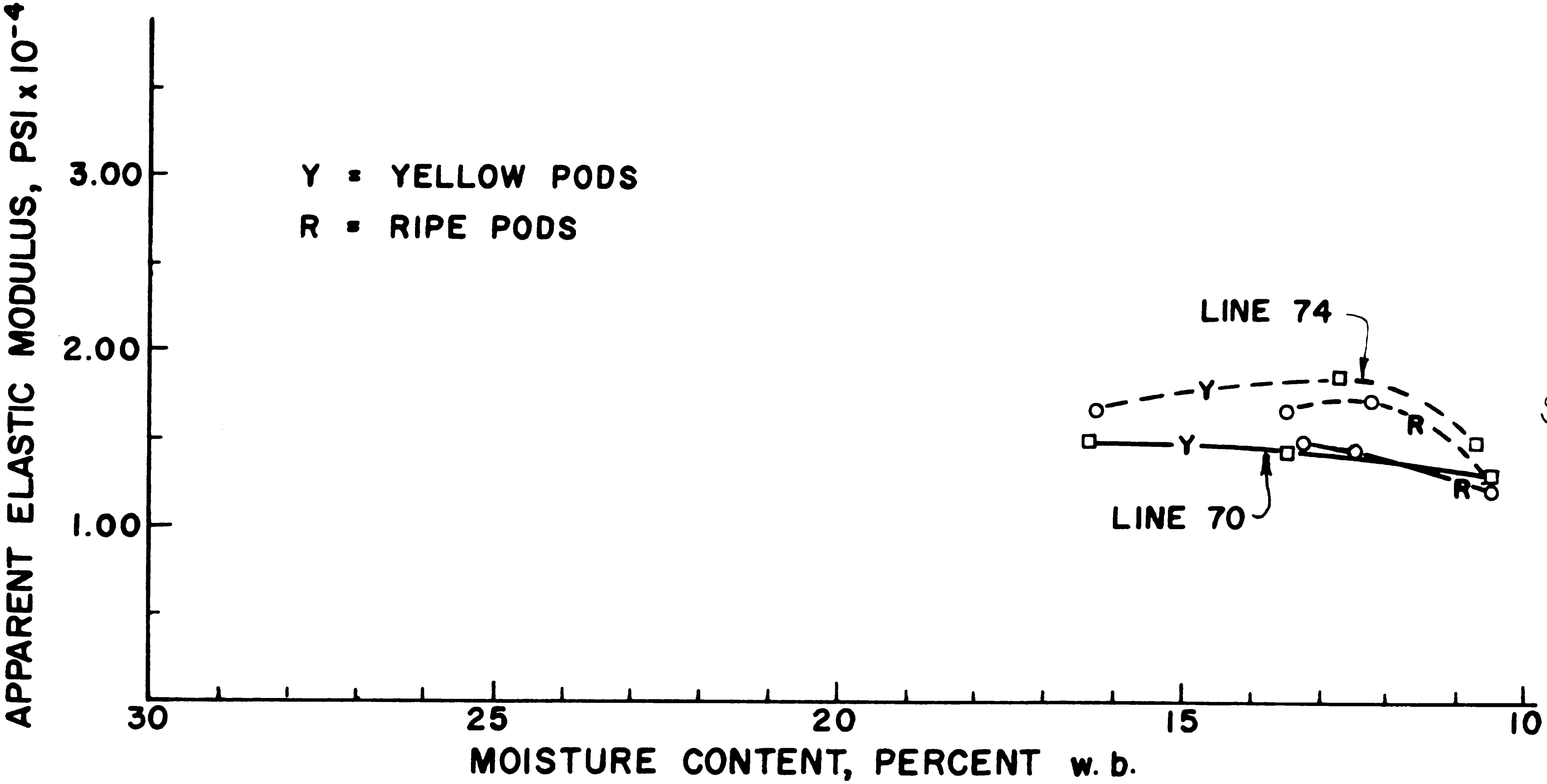

Figure 27.--Variation of E with maturity and moisture content for navy beans. Second harvest.

9.2 c Follow up Tests

Two tests were run each on 10-beam samples of Seafarer variety navy beans. Two moisture contents, 15.0 and 13.3 percent w.b. were used. Only critical loads P_{cr} were measured and these averaged out to be 17.1 lbs. at 15.0 percent w.b., and 18.5 lbs. at 13.5 percent w.b. These results are in good agreement with those for the beans tested the previous year (September, 1967).

9.3 Dynamic Forces and Energy

The dynamic rupture force F_R and energy E_R are presented in Tables 3 and 4 for the drop tests and high velocity tests, respectively. They are the values required to cause cracking for the whole bean.

As expected, the dynamic forces to cause cracking were lower than the quasi-static forces at corresponding moisture contents. Because of the fact that for the drop tests the beans were restrained at their bases, the drop test rupture forces were very much lower than those for the high velocity impact (Figure 28).

As can be seen in Figure 28, the variation of dynamic rupture force with moisture content was somewhat similar to that for critical quasi-static loads, with lower forces required to cause skin rupture at both the high and low extremities of the moisture content range. The optimum moisture content, deduced from Figure 28, was found to be about 14.5 percent w.b., for both dynamic

TABLE 3.--Impact velocities, rupture forces and energy for navy beans subjected to drop tests.

Moisture percent w.b.	Drop height in.	Velocity fps	Energy E_R ft-lbs	Force F_R lbs
16.9	8.5	6.6	0.050	9.0
15.2	7.5	5.5	0.055	10.1
13.5	6.0	3.7	0.044	10.0
11.5	3.0	2.4	0.015	7.2
15.5 (rewetted)	7.5	5.4	0.053	10.0

TABLE 4.--Impact velocities, rupture forces and energy for navy beans subjected to free, high velocity impact.

Moisture percent w.b.	Av. bean weight lbs x 10^{-}	Velocity fps	Energy E_R ft-lbs	Force F_R lbs
17.4	0.595	47.1	0.032	21.4
15.6	0.577	55.0	0.042	23.6
13.4	0.560	55.0	0.041	23.2

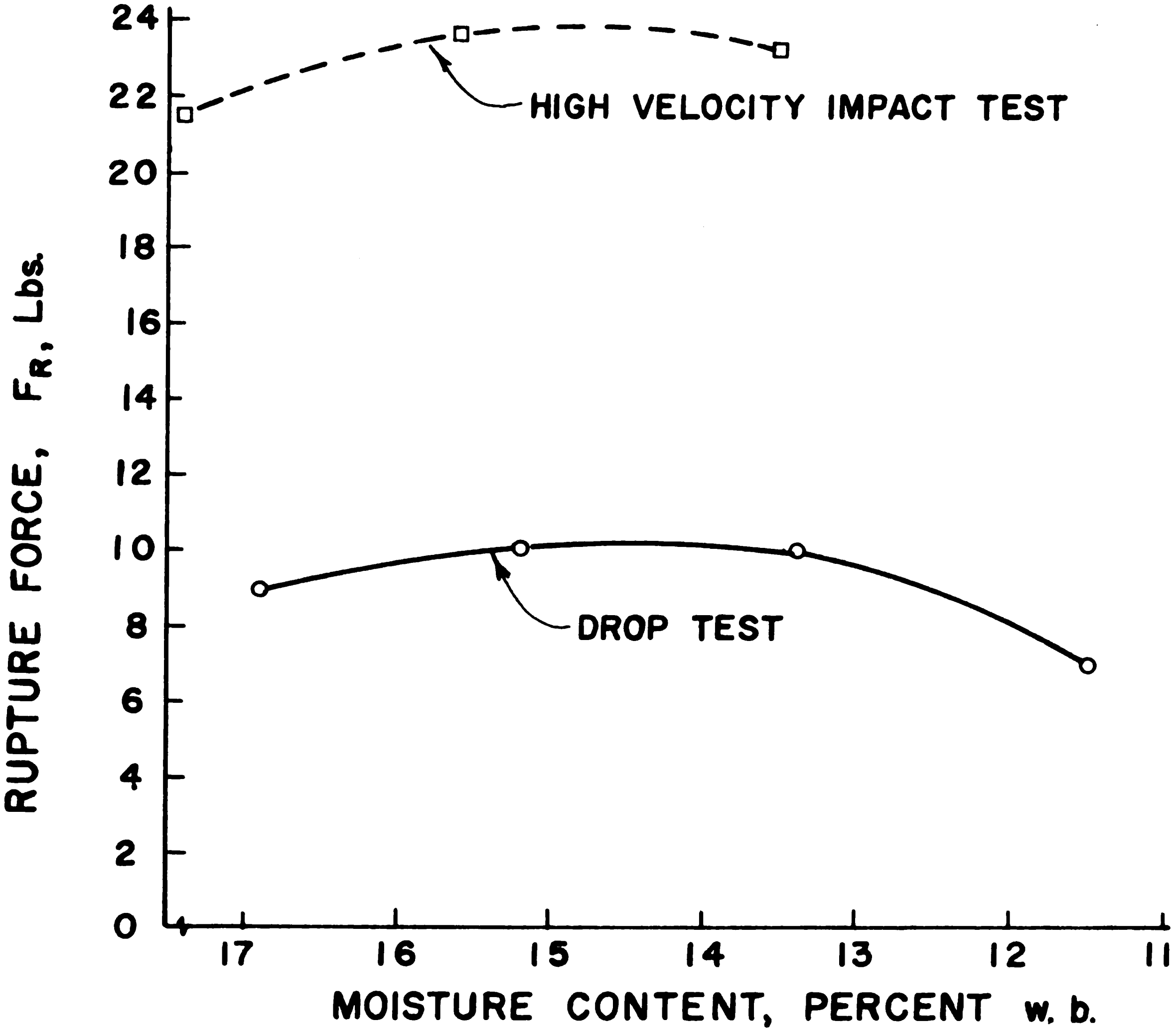

Figure 28.--Comparison of dynamic rupture force F_R for navy beans, determined from low and high velocity impact tests.

tests. This value is somewhat lower than the 17.5 percent w.b. predicted by Solorio (1959).

The maximum rupture forces at this optimum moisture content would be 10.4 lbs and 23.8 lbs for the drop test and the high velocity impact test respectively. The maximum impact velocities were 6.60 and 54.97 fps. The minimum impact velocities required to cause rupture were 2.44 fps with the drop tester for beans at 11.5 percent moisture and 47.12 fps for the rotating arm for beans at 13.4 percent moisture.

The values of forces and velocities given above are those sufficient to cause cracking of the seed coats of more than 50 percent of the impacted beans.

Figure 29 gives the variation of rupture energy E_R with moisture content for the two dynamic tests. Since the strain energy required to cause rupture under dynamic conditions should be invariable, there should be close agreement between the rupture energies measured by the two tests. It does appear, from Figure 29, that there is reasonable agreement at the lower moisture contents (12.5 percent difference at 13.6 percent moisture). There are, however, large differences in the two results at higher moisture contents (25 percent difference at 16 percent moisture). The two most likely causes of these differences are (a) strain rate or viscoelastic effect

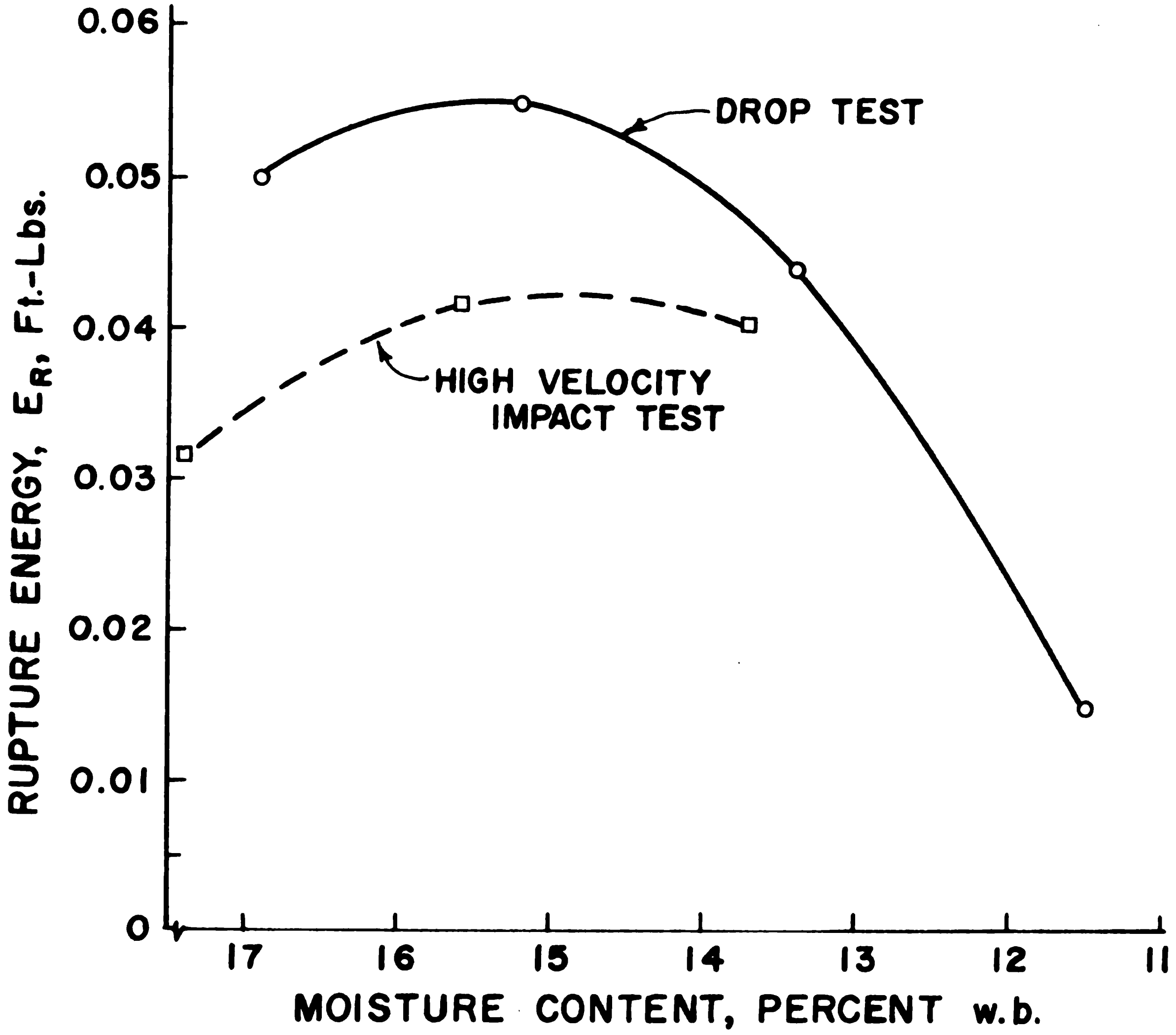

Figure 29.--Comparison of rupture energy E_R for navy beans, determined from low and high velocity impact tests.

exhibited by the beans at high moisture contents, and (b) inaccuracies in the drop test apparatus.

Viscoelastic behavior of the beans under load, although negligible under quasi-static conditions, could occur under dynamic conditions. Such behavior would be manifest by a greater resistance to deformation under high loading rates, and would result in lower strain energy in the high velocity test.

The second reason for the discrepancy shown in Figure 29 lies in the nature of the drop test apparatus. In a drop test, unlike the quasi-static or high velocity test, the load is still being applied after cracking has occurred. When the beans are dry and thus have a high EI_M value, this is no problem since the bean withstands this load with no further damage. When the beans are wet however, (16 percent moisture and above), the checked beans continue to deform under the continued load. In this case there is some difficulty in determining from the deceleration curve the exact point at which cracking of the coat occurred. Hence an important source of possible error.

A smaller diameter drop tube, with a shorter and lighter drop weight would probably have yielded results in closer agreement with those of the rotating arm tester.

As was the case with impact rupture forces, the rupture energy decreased with decreasing moisture content, in the range of 11.5 to 14.5 percent moisture. This extends Tabiszewski's (1968) findings in bean threshing tests in which visible damage increased, at a given combine cylinder speed, with decreasing moisture content in the range of 10.6 to 12.3 percent moisture.

9.4 Extrapolation of the Results to Field Conditions

The results of the quasi-static tests cannot be immediately applied to actual conditions since the distribution of forces between individual beans stored in deep silos or in sacks is not yet known. The quasi-static tests have, however, demonstrated that beans above 11 percent moisture content w.b. are most susceptible to checking, or cracking of the seed coat, when the load is applied on end. This fact was used in the dynamic tests to determine the minimum impact forces and energy required to cause checking. On the basis of the results obtained from the dynamic tests, deductions concerning bean damage can be made for two field conditions.

9.4a Threshing

From the results of the high velocity tests, as shown in Table 4, the impact velocity between individual beans and a rigid structure such as a cylinder bar must be below 55 fps when the moisture content of the beans is between 13.4 and 15.6 percent w.b., and below 47 fps when the beans are at 17.4 percent w.b. For beans whose moisture content is of the order of 11.5 percent w.b., the limiting velocity of such impacts can be computed using the rupture energy obtained at this moisture content by the drop test (0.015 ft-lbs), and the restitution equation developed in section 5.2. This procedure is valid here since there is reasonable agreement between the two dynamic tests at the lower moisture levels. The computation yields 30 fps as the limiting impact velocity between a cylinder bar and beans at 11.5 percent moisture.

Lamp (1959), in a threshing study, concluded that a combine cylinder speed of 37 fps was sufficient for complete threshing of grains. While this is a safe speed for beans above 13.4 percent moisture, harvesting of beans at 11.5 percent moisture would require a compromise between checking of the beans on the one hand, caused by speeds above 30 fps, and threshing losses on the other, caused by speeds below 37 fps.

9.4b Handling

The limiting velocities discussed above also apply to the case where moving beans impact rigid, stationary surfaces. Such conditions arise when beans are poured into deep silos. Perry (1959) found that beans, dropped from rest, attained velocities of about 25 fps after a free fall of 11 ft, and about 46 fps after a free fall of 45 ft.

Combining Perry's results with those of the two dynamic tests conducted in this study, it may be concluded that beans may be safely dropped into silos 45 ft deep, when the moisture content is between 13.4 and 15.6 percent w.b. When the moisture content of the beans is higher (17.4 percent w.b.), or lower (11.5 percent), there will be cracking caused by the impact after the 45 ft drop.

X. SUMMARY AND CONCLUSIONS

10.1 Summary

Three types of tests were conducted on navy beans in order to determine the loads and energies required to produce checking or cracking of the seed coat.

A series of quasi-static tests were run on two varietal lines of beans at three stages of maturity. The beans were loaded individually on end and the critical loads required to cause cotyledon buckling were measured on an Instron Testing Machine. With the critical loads a finite difference stability analysis was used to compute elastic and stability moduli for the test beans.

In order to increase the range of test moisture contents the beans were harvested twice at high moisture contents and allowed to dry in the laboratory for periods of 0, 48, 72 and 144 hours before testing.

An optimum moisture content range of 12.5 to 13.5 percent moisture w.b. was found for which the critical loads to cause cotyledon buckling was between 18 and 20 lbs.

At very low moisture contents, 10.5 and 11 percent w.b., not only did the critical loads decrease but the beans became susceptible to damage from side loads as well.

Two series of dynamic tests, a drop test and a high velocity impact test were conducted on individual beans which were restrained for the former test and free to move in the latter.

Forces, F_R, required to cause checking were measured with dynamic load cells, and the accompanying energy E_R absorption by the impacted beans was computed.

An optimum moisture content of 14.5 percent was found at which the dynamic loads required to cause checking were at their maximum values for both sets of tests.

10.2 Conclusions

1. In the range of 11.5 to 28 percent moisture content w.b., no discernible differences arise in the critical loads required to cause cotyledon buckling of end loaded beans when the rate of loading is varied from 0.05 to 0.2 ipm.

2. At moisture contents above 11.5 percent w.b., checking of bean seed coats is more likely to be caused by end loads causing outward buckling of the cotyledons and consequent tensile rupture of the seed coat.

3. The optimum moisture content at which beans can resist buckling under end loads is 13 percent w.b.

4. Beans, when very dry (below 11.5 percent w.b.) become very susceptible to mechanical damage to both seed coat and cotyledons caused by static end and side loads and by dynamic loads.

5. For dynamic load conditions similar to those existing in a combine cylinder, the optimum moisture content for lowest incidence of checking is 14.5 percent w.b. At this moisture content the velocity of the rigid body impacting the bean should be kept below 50 fps, and the strain energy imparted to the bean should be less than 0.04 ft-lbs.

6. For beans below 11.5 percent moisture content impact velocities as low as 30 fps will cause checking and splitting.

7. In theory a simple drop test with an instrumented falling weight impacting a stationary bean should be able to give the energy required to rupture the seed coat. This energy could then be used with the simple restitution equations to obtain limiting velocities for high velocity, free impacts. Complications arise however when the drop weight is heavy enough to cause continuing deformation of the bean cotyledons after checking has occurred. This places a serious limitation on a drop test apparatus especially when the beans are at high moisture levels.

8. Beans may be safely poured into deep silos (45 ft), or threshed at the cylinder periperal speed found to be the best for seed separation, when their moisture content is in the range of 13 - 15 percent w.b. At a

moisture content of 11.5 percent or lower however checking will occur in both cases, and some compromise will have to be made between combine efficiency and checking.

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