ABSTRACT

EFFECT OF WALLS ON
STRUCTURAL RESPONSE TO EARTHQUAKES

By

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The development of an optimal numerical model for filler walls in building frames is described. A method of elasto-plastic dynamic analysis of plane building frames with filler walls is presented. A computer program written in FORTRAN IV for use on the Michigan State University CDC 6500 computer system was prepared to accomplish the dynamic solution of such structures subjected to earthquake ground motion.

The filler walls are treated as finite elements in plane stress, interacting with the moment-resisting frame such that the translational displacements of the joints are compatible. Both triangular and rectangular elements are considered. The effect of mesh size and modelling of the cracking phenomenon are studied. Computed results of load-displacement relations, crack propagation pattern and ultimate load capacity have been compared with known experimental results.

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In the dynamic analysis the mass of the system is handled by a lumping procedure, which accounts for rotary as well as translational inertia. Mass-proportional viscous damping has been taken into account. The equations of motion have been formulated in terms of joint displacements relative to supports. Two methods of analysis have been derived. In the first, all the three degrees of freedom of a joint are considered. In the second (modified) method the degrees of freedom associated with the axial deformation of frame members and rotation of joints are eliminated. The modified method makes possible the use of a much larger time step for the numerical integration procedure, resulting in a considerable saving of computation time.

Numerical results of the study of a three story steel frame with concrete filler walls subjected to selected portions of the El Centro Earthquake of 1940, are presented. The results of the study highlight the importance of the effect of walls on the lateral stiffness and dynamic response of infilled frames. It is also shown that there is no significant loss of accuracy in using the modified method.

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To my parents

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# CHAPTER I

# INTRODUCTION

## 1.1 General

The effect of frame-wall interaction in a structure subjected to lateral loads, such as those resulting from earthquake shocks, has received much attention in recent years. In order to design structures that are safe and economical, the importance of more accurate analysis has long been recognized. Although the loading under consideration is dynamic in nature, earlier investigations were based on statics only and often in conjunction with gross simplifying assumptions leading to a crude modelling of the structure. Nevertheless, those investigations were significant contributions considering the importance of the problem and the limited computational facilities available then. With the availability of modern high speed digital computers a more exact analysis of the problem seems warranted. The work reported in this thesis represents an effort in this direction.

It has often been claimed that the aseismic design of framed structures without considering the effect of walls would produce a conservative design. If this is true, considering the effect of walls would lead to a more economical design. However such an observation is made purely from a statics point of view. It is well known that the fundamental frequency of a structure is a major factor in its response to earthquakes. Corresponding to an increase in the fundamental frequency of a structure, the Code (16)* calls for an increase in the percentage of its gravity load to be applied as seismic lateral load. The fundamental frequency is substantially increased by the presence of walls even if cracked. This according to the Code would increase the lateral loads on the structure. Thus, a design neglecting the effect of walls might not be conservative after all. It must, however, be recognized that the inclusion of the effect of walls adds substantially to the lateral stiffness of the structure.

## 1.2 Scope and Outline of the Investigation

The purpose of this investigation is to study, using a more accurate formulation of the stiffness characteristics of structures, the effect of walls on the dynamic response of infilled frames subjected to earthquakes. The analysis extends beyond the elastic range. The nonlinearity of the structural response, due to the formation of plastic hinges in the frame members and crack propagation in the wall elements, is taken into account.

Only plane frames are considered. The bending moment-curvature relationship is assumed to be elastic-perfectly plastic. The filler walls, treated as finite elements

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* Numbers refer to items listed in the List of References.

in plane stress, interact with the frame such that the translational displacements of the joints are compatible. It is also assumed that the wall elements can not transmit moments at the joints. A wall element is assumed to have cracked when the principal tensile stress exceeds a prescribed "cracking stress". The overall stiffness of a cracked wall element is approximated by that of the original uncracked element except that the elastic modulus is reduced.

In the dynamic analysis the structure is treated as a discrete system of masses lumped at the joints. The equations of motion are formulated using the displacements of the joints relative to the supports as variables. The transient response is obtained by a numerical integration of the equations of motion. A modified procedure in which the axial and the rotational modes are eliminated, is also derived. This makes possible the use of a much larger time step for the numerical integration, saving considerable computation time without significant loss of accuracy.

## 1.3 Literature Review

Smith (13) presented a procedure for predicting the approximate lateral stiffness of infilled frames, by assuming an equivalent diagonal strut to replace the infill. The effective width of the strut was derived theoretically and checked by model experiments. Frischman, Prabhu and Toppler (5) used the influence coefficient method and the equivalent column method for the analysis of multistory frames with interconnected shear walls. The frame was replaced by a column whose stiffness equals the sum of all column stiffnesses and restraints were applied at each floor level equivalent to the beam stiffnesses. Beck (1) presented an approximate method of analysis, replacing the discontinuous frame system by a continuous system. He took into account shear wall deformations due to normal forces.

Cardan (2) proposed that the problem of flexural deformations of the wall, could be reduced to a second degree differential equation, with a few simplifying assumptions. Roseman (11) presented an approximate method of analysis of walls of multistory buildings with openings. The basic idea of his approach consisted in the replacement of the connecting beams with a continuous connection. To obtain the distribution of lateral forces on the elements of a frame-wall system, Khan and Sbarounis (8) considered the structure as separate parts with certain conditions for compatibility.

Gould (7) studied shear wall-frame interaction by reducing the problem, with some simplifying assumptions, to that of a cantilever beam supported by concentrated elastic reactions. He represented each story of the frame as an elastic spring connected to the shear wall at each floor by a rigid bar and a rotational spring, and connected to each of the adjacent floors through a rigid joint.

Kokinopoulos (9) investigated the seismic response of a multistory system, treating it as a cantilever with masses concentrated at the floor levels. Coull and Choudhury (3) analyzed coupled shear walls by replacing the discrete system of connecting beams by an equivalent continuous medium. Fedorkiw and Sozen (4) proposed a lumped parameter model to simulate the response of reinforced concrete frames with concrete filler walls. Their study included load-displacement characteristics of infilled frames and crack propagation in the filler walls.

Experimental work on the ultimate lateral load capacity of concrete frames with masonry filler walls has been reported by Sachanski (12). Yorulmaz and Sozen (15) made experimental studies on the lateral load capacity and load-displacement characteristics of reinforced concrete frames with concrete filler walls.

The preceding works were related to the static analysis of frame-wall systems. Goldberg and Herness (6) studied the vibration of multistory buildings. The floor and wall deformations were studied by use of a generalized slope-deflection equation. Saghera (17) investigated the effect of shear walls on the frequencies of vibration of a structure. His study included experimental verification of computed results. The results of his study indicate that the filler walls, as expected, increase the stiffness of a frame, resulting in considerably higher frequencies.

## 1.4 Organization of this Report

The formulation of the joint stiffness matrix of the structure is presented in Chapter II. A comparative study is also made, treating the walls by the finite element method. Computed load-displacement characteristics are compared with known experimental results. The effect of wall openings on the lateral stiffness of infilled frames is also discussed. Chapter III deals with the numerical solution of the governing differential equations of motion. Two methods of analysis are derived. In the first, all the three degrees of freedom of a joint are considered. In the second (modified) method, the degrees of freedom associated with the axial deformation of frame members and rotation of joints are eliminated. In this chapter a section on the computer program has also been included. In Chapter IV comparative results of dynamic analysis of a three story steel frame with concrete filler walls, with and without the effect of walls are presented. Comparative responses of the structure and the computation time involved, with and without the axial and rotational modes are discussed. A summary and conclusions are presented in Chapter V.

## 1.5 General Definitions

The joints of the structure consist of supports and free joints. The free joints are classified as frame joints and interior wall joints (see Figure 1). At a frame joint two or more frame members are incident. At an interior wall joint only wall elements are incident. At a frame joint three components of forces or displacements can be specified and at an interior wall joint only the two translational displacements or the corresponding forces can be specified since it is presumed that the wall elements can not transmit moments at the joints. The joints of the structure are numbered consecutively starting with 1. The ordering is arbitrary, but once assigned it remains fixed during the analysis.

The incidence of a frame or wall element is defined by the Joint Numbers of its ends or corners. A frame member whose incidence is IJ has its initial or positive end at I and its final or negative end at J. The incidence of a triangular wall element is given by IJK where I, J and K are the Joint Numbers of its corners. The choice of the initial end I is arbitrary but IJK is ordered counter-clockwise. The incidence of a rectangular wall element with its corners at Joints I, J, K and L is given by IJKL. I is the left hand bottom corner of the rectangle and IJKL is ordered clockwise.

Three coordinate systems are used in the analysis. (See Figures 1,2 and 3):

1) Structure Global Coordinate System:
   This system consists of a single set of cartesian axes with origin at any chosen point. Since the analysis is confined to plane frames, the set of cartesian axes implies a two dimensional system

with the X axis horizontal and Y axis vertical. Rotation is defined to be positive in the counter-clockwise sense.

2) Joint Coordinate System:

This system consists of one set of cartesian axes for each joint with its axes parallel to the structure global axes. This system is used to describe the displacements of the joint and forces acting on it. At a frame joint the displacements are represented by a vector with three elements, the first two elements being the translational displacements in the X and Y directions and the third element being the rotation in the XY plane. The force vector is made up of forces corresponding to these displacements. In the case of an interior wall joint, this vector is made up of two elements, the rotation or the moment component being absent.

3) Member Coordinate System:

This consists of a set of cartesian axes for each end of a frame member and for each corner of a wall member. The origins are located at the ends or corners as the case may be, and the axes are parallel to the global axes. These axes are used to describe the end or corner displacements of the elements, and also the forces acting at the ends or corners of the elements. For the frame members there will be three components of displacements or forces at each end and for the wall elements there will be two components of displacements or forces at each corner.

## 1.6 Notation

The notation shown below has been used in this report:

a, b = the dimensions of the rectangular wall element in the x and y directions;

A = area of the triangular wall element;

$B_r$ = the matrix relating strain and corner displacements of a rectangular wall element;

$B_t$ = the matrix relating strain and corner displacements of a triangular wall element;

C = damping matrix;

c = damping constant;

D = the matrix relating stress and strain in a plane stress formulation;

dt = time interval for the numerical integration procedure;

e = the strain vector at any point in the wall element;

E = Young's Modulus;

f = vector of forces acting at the ends of a frame element or the corners of a wall element;

$f_{ij}$ = element of the joint flexibility matrix F;

$F$ = joint flexibility matrix of the unreduced system;

$F^*$ = joint flexibility matrix of the reduced system;

$J_{ij}$ = a submatrix of K;

$K$ = joint stiffness matrix of the unreduced system;

$K^*$ = joint stiffness matrix of the reduced system;

$K_f$ = stiffness matrix of the frame element;

$K_{wr}$ = stiffness matrix of the rectangular wall element;

$K_{wt}$ = stiffness matrix of the triangular wall element;

$M$ = mass matrix of the unreduced system;

$M^*$ = mass matrix of the reduced system;

$n$ = Poisson's ratio of the wall material;

$N$ = number of free joints;

$P$ = vector of joint loads of the unreduced system;

$r$ = the ratio of the height to breadth of a rectangular wall element;

$R$ = internal joint resistance vector of the unreduced system;

$R^*$ = internal joint resistance vector of the reduced system;

$u$ = vector of end or corner displacements of a frame or wall element;

$U$ = joint displacement vector of the unreduced system;

$\dot{U}$ = joint velocity vector of the unreduced system;

$\ddot{U}$ = joint acceleration vector of the unreduced system;

$U^*$ = joint displacement vector of the reduced system;

$\dot{U}^*$ = joint velocity vector of the reduced system;

$\ddot{U}^*$ = joint acceleration vector of the reduced system;

$x_i$, $y_i$ = the coordinates of Joint i;

$\ddot{U}_g$ = a vector formed from prescribed ground accelerations;

$\alpha$, $\beta$ = dimensionless member coordinates of the rectangular wall element; and

$\sigma$ = stress vector at any point in the wall element.

# CHAPTER II

# STATIC LOAD-DISPLACEMENT RELATIONS

## 2.1 General

The main objective of this first phase of the investigation is to determine an "optimal" numerical model to represent the stiffness characteristics of infilled frames. The following approach is used.

1. Only plane frames are considered. Failure of frame elements is assumed to occur due to flexure only. The bending moment-curvature relationship is assumed to be elastic-perfectly plastic. It is assumed that when the yield moment is reached an abrupt transition takes place from a purely elastic state to a state where all the fibres are stressed to the yield limit and unrestricted plastic deformation can occur. This is tantamount to assuming a shape factor equal to unity.
2. The wall elements are treated as finite elements in plane stress, interacting with the frame such that the translational displacements of the joints are compatible. Both triangular and rectangular elements are considered.
3. Cracking is assumed to occur in the wall material when the principal tensile stress exceeds a prescribed "cracking stress". The stiffness characteristics of a wall element after cracking are computed as usual except

that the value of the elastic modulus for that element is reduced.

## 2.2 Member Stiffness Matrix

The member stiffness matrix of a frame element is a 6 x 6 symmetric matrix which relates the end displacements to the forces acting at the ends of the member. The member stiffness matrix of a wall element is also a symmetric matrix which relates the corner displacements to the forces acting at the corners of the wall element. Since it is presumed that the wall elements can not transmit moments at the joints, only the two translational components of the displacements or the corresponding forces can be specified at the corners. Thus the member stiffness matrix has a dimension of 6 x 6 for a triangular element and 8 x 8 for a rectangular element.

The member stiffness matrices are generated with their axes of reference parallel to the global axes. This has a distinct advantage in that the joint stiffness matrix of the structure can be assembled in a very efficient and quick manner by the direct summation of the appropriate stiffness coefficients of the member stiffness matrices.

2.2.1 Frame Elements.--The relation between the end displacements and the forces acting at the ends of a frame member is given by

$$K_f \cdot \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{Bmatrix} \qquad \text{....(1a)}$$

or in matrix notation

$$K_f \, u = f \qquad \text{....(1b)}$$

where $K_f$ is the member stiffness matrix of the frame element, f the vector of end forces and u the vector of end displacements. For the analysis a consistent procedure has been adopted to specify the incidence of the frame elements. The lower end of the vertical members and the left end of horizontal members are taken as positive ends. The member stiffness matrices of frame elements for different cases of moment releases are given in Appendix I.

2.2.2 Triangular Wall Elements.--The corner displacements u and forces f acting at the corners of a triangular wall element are related by

$$K_{wt} \, u = f \qquad \text{....(2)}$$

where $K_{wt}$ is the stiffness matrix of the triangular wall element, u the vector of corner displacements and f the vector of forces acting at the corners of the element. The vector u contains only the two translational components

of the displacements at each corner and f the corresponding force components. The matrix $K_{wt}$ is computed by a method suggested by Zienkiewicz and Cheung (15).

Consider a typical triangular wall element shown in Figure 4. A displacement function is chosen such that the displacement components are compatible at the interfaces of the various finite elements. This condition is satisfied by assuming linearly varying boundary displacements of the form

$$u_x = k_1 + k_2 x + k_3 y \qquad \text{....(3a)}$$

$$u_y = k_4 + k_5 x + k_6 y \qquad \text{....(3b)}$$

where $u_x$ and $u_y$ are the displacements in the x and y directions at any point (x,y) and $k_1$, $k_2$ etc. are constants which can be determined from the known values of the corner displacements. Solving, the following relations are obtained.

$$u_x = \frac{1}{2A} \sum_{i=1}^{3} (a_i + b_i x + c_i y) u_{2i-1} \qquad \text{....(4a)}$$

$$u_y = \frac{1}{2A} \sum_{i=1}^{3} (a_i + b_i x + c_i y) u_{2i} \qquad \text{....(4b)}$$

where A = the area of the triangular element,

$$a_1 = x_2 y_3 - x_3 y_2 \qquad \text{....(5a)}$$

$$b_1 = y_2 - y_3 \qquad \text{....(5b)}$$

$$c_1 = x_3 - x_2 \qquad \text{....(5c)}$$

$x_1, y_1$, $x_2, y_2$ and $x_3, y_3$ being the coordinates of the corners. Other constants $a_2$, $b_2$, $c_2$ etc. can be obtained by cyclic permutations of the relations given above.

The relation between strains and displacements is given by

$$e = \begin{Bmatrix} e_{xx} \\ e_{yy} \\ e_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_x}{\partial x} \\ \dfrac{\partial u_y}{\partial y} \\ \dfrac{\partial u_x}{\partial y} + \dfrac{\partial u_y}{\partial x} \end{Bmatrix} \qquad \ldots.(6)$$

where e is the strain vector, $e_{xx}$, $e_{yy}$, and $e_{xy}$ are the components of direct strain in the x direction, direct strain in the y direction and the shear strain in the xy plane respectively. Using Eq.(4) for $u_x$ and $u_y$ the following relation is obtained.

$$e = B_t \, u \qquad \ldots.(7)$$

where

$$B_t = \begin{bmatrix} b_1 & 0 & b_2 & 0 & b_3 & 0 \\ 0 & c_1 & 0 & c_2 & 0 & c_3 \\ c_1 & b_1 & c_2 & b_2 & c_3 & b_3 \end{bmatrix} \qquad \ldots.(8)$$

....

It is obvious that the strain field is constant within a given triangular element. Since a state of plane stress has been assumed, the stress components can now be computed from the following relationship.

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{1-n^2} \begin{bmatrix} 1 & n & 0 \\ n & 1 & 0 \\ 0 & 0 & \frac{1-n}{2} \end{bmatrix} \begin{Bmatrix} e_{xx} \\ e_{yy} \\ e_{xy} \end{Bmatrix} \qquad \text{....(9)}$$

where $\sigma_{xx}$ and $\sigma_{yy}$ are the normal stress components in the x and y directions and $\sigma_{xy}$ the shear stress component in the xy plane. E is the Young's modulus and n is the Poisson's ratio of the wall material. In matrix notation we have

$$\sigma = De \qquad \text{....(10)}$$

where

$$D = \begin{bmatrix} 1 & n & 0 \\ n & 1 & 0 \\ 0 & 0 & \frac{1-n}{2} \end{bmatrix} \frac{E}{1-n^2} \qquad \text{....(11)}$$

and $\sigma$ is the stress vector.

By the principle of virtual work, equating the work done by the external forces to the work accomplished by the internal stresses the following relationship is obtained.

$$u^T f = e^T \sigma A t \qquad \text{....(12)}$$

where t is the thickness of the wall element and the superscript T denotes the transpose of the matrix.

Substituting for e and $\sigma$ and grouping terms to one side of the equation, we obtain,

$$u^T(f - B_t^T D B_t Atu) = 0. \quad ....(13)$$

Since the corner displacements u are arbitrary,

$$f = B_t^T D B_t Atu = K_{wt} u \quad ....(14)$$

where

$$K_{wt} = B_t^T D B_t At. \quad ....(15)$$

2.2.3 Rectangular Wall Elements.--Consider a typical rectangular element shown in Figure 4. The origin of the local coordinate system is taken at the lower left corner of the rectangle. The following dimensionless coordinates are introduced.

$$\alpha = \frac{x}{a} \quad \text{and} \quad \beta = \frac{y}{b} \quad ....(16)$$

where a and b are the dimensions of the element and, $\alpha$ and $\beta$ are the dimensionless coordinates corresponding to the x and y directions. Let u be the vector of corner displacements again; it contains eight components, i.e. two for each of the four corners. Simple displacement functions, assuming linearly varying boundary displacements are given by the relations,

$$u_x = c_1\alpha + c_2\alpha\beta + c_3\beta + c_4 \quad ....(17a)$$

$$u_y = c_5\alpha + c_6\alpha\beta + c_7\beta + c_8 \quad ....(17b)$$

where the constants $c_1$, $c_2$ etc. can be determined from the known values of displacements at the four corners. Solving, the displacements at any point (x,y) are given by

$$u_x = (1-\alpha)(1-\beta)u_1 + (1-\alpha)\beta u_3 + \alpha\beta u_5 + \alpha(1-\beta)u_7 \quad \text{....(18a)}$$

$$u_y = (1-\alpha)(1-\beta)u_2 + (1-\alpha)\beta u_4 + \alpha\beta u_6 + \alpha(1-\beta)u_8 \quad \text{....(18b)}$$

From the known relations between the strain and displacement components, we obtain

$$e = B_r\, u \qquad \text{....(19)}$$

where

$$B_r = \begin{bmatrix} -\frac{(1-\beta)}{a} & 0 & -\frac{\beta}{a} & 0 & -\frac{\beta}{a} & 0 & \frac{1-\beta}{a} & 0 \\ 0 & -\frac{(1-\alpha)}{b} & 0 & \frac{1-\alpha}{b} & 0 & \frac{\alpha}{b} & 0 & -\frac{\alpha}{b} \\ -\frac{(1-\alpha)}{b} & -\frac{(1-\beta)}{a} & \frac{(1-\alpha)}{b} & -\frac{\beta}{a} & \frac{\alpha}{b} & \frac{\beta}{a} & -\frac{\alpha}{b} & \frac{1-\beta}{a} \end{bmatrix} \quad \text{..(20)}$$

The corner displacements and forces acting at the corners of the wall element are given by the relationship

$$K_{wr}\, u = f \qquad \text{....(21)}$$

where $K_{wr}$ is the member stiffness matrix of the rectangular wall element. To obtain $K_{wr}$ we proceed in a very similar manner and equate the work done by the external forces to the work accomplished by the internal stresses. It should be noted that the matrix $B_r$, unlike $B_t$ is a function of the position variables, and as such, the work accomplished by the internal stresses should be obtained by integration. The internal stress components are determined by using Eq.(9). The member stiffness matrix $K_{wr}$ is given in Appendix I. The method described above for computing the stiffness matrix of a rectangular element is according to Przemieniecki (10).

The assumption of linear edge displacements ensures compatibility of displacements at the interfaces of the finite elements. It can be observed that the displacement components represented by Eqs.(17) are second degree functions similar to a hyperbolic paraboloid and, when compared with the previously treated case of a triangular element, are suggestive of a better representation of the state of stress. This, in fact, is verified during the course of this investigation.

## 2.3 Joint Stiffness Matrix

The equilibrium equation for a statically loaded framed structure can be expressed in matrix form as

$$KU = P \qquad \ldots.(22)$$

where K is defined as the joint stiffness matrix of the structure, U the vector of joint displacements and P the joint load vector.

In partitioned form

$$K = \begin{bmatrix} J_{1,1} & & J_{1,N} \\ & J_{i,j} & \\ J_{N,1} & & J_{N,N} \end{bmatrix} \qquad \ldots.(23)$$

where the J's are the submatrices corresponding to the various free joints and N the number of free joints. Since the member stiffness matrices of the frame and wall elements have been generated with reference to global axes, the submatrices $J_{i,j}$ are easily computed by the direct summation of the contributions by the frame and/or wall members incident at Joint i and Joint j. Since wall elements can not transmit moments at the joints, displacement or force components corresponding to joint rotations can not be specified at the corners of a wall element. As such, the number of rows in a typical submatrix $J_{i,j}$ will be 3 or 2 according as i is a frame joint or an interior wall joint and the number of columns in $J_{i,j}$ will be 3 or 2 according as j is a frame joint or an interior wall joint. Thus $J_{i,j}$ will be a rectangular matrix when i and j are different type of joints and the extra row or column will be made up of zero entries, to make the matrix multiplication defined in Eq. (22) possible. It may be noted that $J_{i,j} = J_{j,i}^T$.

Referring to Figure 1, the submatrix $J_{1,9}$ would have three rows and two columns, since Joint 1 is a frame joint and Joint 9 is an interior wall joint. By definition, $J_{1,9}$ multiplied by the column vector of displacements at Joint 9, gives the force components at Joint 1 contributed by members incident at both Joints 1 and 9. Since wall elements can not transmit moments at the joints, irrespective of the magnitude of the translational displacements of Joint 9, the contribution from $J_{1,9}$ to the moment at Joint 1 is zero. Therefore the third row of $J_{1,9}$ consists of zero elements. Also, any rotation of Joint 1 can not affect the force components at Joint 9. Therefore the submatrix $J_{9,1}$ which has two rows and three columns, contains all zero elements in its third column.

## 2.4 Static Analysis

For a given set of static joint loads, the joint displacements are given by

$$U = K^{-1}P. \qquad ....(24)$$

A computer program was written to obtain the load-displacement relation and crack propagation pattern in the filler wall for steadily and linearly increasing joint loads. Initially, loads are specified at one or more joints. The magnitude of the loads are such that no cracks develop in the wall elements and no plastic hinges form in the frame members. The maximum tensile stress and the wall element in which this is developed are determined. The loads are increased proportionately such that this wall element just cracks. The joint stiffness matrix of the structure is now modified as explained in section 2.1, and the procedure is continued until the entire wall panel is cracked. The sequence in which the wall elements crack gives the crack propagation. At every step it is also checked whether plastic hinges form in any frame member. If any plastic hinge forms, the joint stiffness matrix is modified accordingly.

2.4.1 Effect of Reduction Factor.--The model structure considered for this part of the study is a single story reinforced concrete frame with concrete filler walls, experimental work on which has been reported by Yorulmaz and Sozen (14). Steadily increasing horizontal loads were applied at the quarter points of the beam. The load-displacement relation and crack propagation pattern in the wall were obtained for various values of a "reduction factor," which is defined to be the ratio of the elastic moduli of the wall material after and before cracking. Figure 5 shows the computed and experimental load-displacement plots. In this analysis the wall panel was divided into an 8 x 8 mesh with 64 triangular elements as shown in Figure 6. Results are given for two extreme values of the reduction factor assumed for the analysis. The crack propagation pattern is also shown in Figure 6. The location of the initial crack and the general pattern of crack propagation agreed well with the results reported by Fedorkiw and Sozen (4).

2.4.2 <u>Ultimate Load Capacity</u>.--A second type of model structure considered is a concrete frame with masonry filler walls, experimental work on which has been reported by Sachanski (12). The wall panel was divided into 8 triangular elements and a steadily increasing load was applied at the story level. The load capacity is given by the magnitude of the applied load which causes all the wall elements to crack. It may be mentioned here that this load will be far greater than the load capacity of the same frame without filler walls, the ultimate load in this case corresponding to the collapse mechanism. Table 1 shows the experimental values of load capacity and computed values for a reduction factor of 0.01. It is seen that the computed values agree well with the experimental ones. Results of Figure 5 and Table 1 would indicate that a reduction factor of 0.01 is a reasonable value for concrete and masonry filler walls.

2.4.3 <u>Effect of mesh size</u>.--The horizontal displacements of the single story structure referred to in Figure 5, under horizontally applied loads of 10 kips each at the ends of the beam, for several mesh sizes are shown in Tables 2 and 3, respectively, for triangular and rectangular elements. It is observed that there is convergence of results with increasing mesh size, which is an essential requirement of any finite difference or finite element formulation. It is also observed that the size of the mesh does not affect the displacement a great deal. Comparing the results corresponding to the finest and coarsest mesh in Table 3, the variation in the value of displacements is about 15%. On the other hand the computation time required is 45 seconds for the finest mesh and 0.16 second for the coarsest mesh. It may also be noted that for the same amount of computation effort, which depends on the size of the joint stiffness matrix, rectangular elements are superior to triangular elements.

## 2.5 An Optimal Numerical Model

It is obvious that there are bound to be some uncertain elements due to the complex nature of frame-wall interaction. Based on the data presented here and on the assumptions made in the course of the analytical formulation, it is assumed that an optimal numerical model to represent the lateral stiffness characteristics of a frame wall system could be obtained by treating the wall panel as a single rectangular element. This minimizes computation effort and seems to yield reasonably accurate results.

## 2.6 Effect of openings in the wall panel

The presence of openings in the wall panel reduces the lateral stiffness of infilled frames. Figure 7 shows the effect of openings on the lateral stiffness of a single story frame. A constant horizontal load is applied at story level. For various dimensions of the openings, the horizontal displacement at the story level has been plotted. It may be noted that the lateral stiffness is not appreciably affected for normal sized openings, i.e., for values of h/H between 0.45 to 0.55 and b/B between 0.2 and 0.3. (See Figure 7 for definition of symbols used here.) In the dynamic analysis the effect of openings has not been considered. If desired this can be easily included by a suitable modification of the computer program.

# CHAPTER III

# METHOD OF DYNAMIC ANALYSIS

## 3.1 General

The structure considered here is a multistoried plane frame with filler walls in plane stress. The dynamic loading consists of the horizontal and vertical components of ground motion in the plane of the frame. In the modified method of dynamic analysis described later in this chapter, dynamic loading consists of horizontal components of ground motion only.

## 3.2 Lumping of Masses

In the formulation of the equations of motion the structure is treated as a discrete system of masses lumped at the joints. Dead and live loads from each floor are assumed to be uniformly distributed over the beams. The mass of a beam or column is lumped equally at the ends of the member. The mass of a wall panel is lumped equally at the corners. The rotary inertia of a joint is taken to be equal to the sum of the moments of inertia about the joint as contributed by the columns and beams incident at that joint. The contribution of each member is taken to be that of a rigid bar with half of the mass of the member distributed uniformly over a quarter of its length. Walls are ignored in the calculation of the rotary moment of inertia since they do not add to the rotary stiffness of the joints. In the modified procedure, the mass elements are first computed as indicated above and then all those mass elements which correspond to the joints of the same floor are summed up and lumped at one point. Rotary moment of inertia is ignored in the modified procedure.

## 3.3 Numerical Integration Procedure

The Analysis is formulated in terms of the joint displacements relative to the supports. The transient response of the structure is obtained by a numerical integration of the equations of motion of the joint masses. Assuming a velocity damping, proportional to mass inertia, the equations of motion are given in matrix form by

$$M\ddot{U} + C\dot{U} + KU = P \qquad \text{....(25a)}$$

In modified form this can be written as

$$\ddot{U} + c\dot{U} + M^{-1}KU = -\ddot{U}_g \qquad \text{....(25b)}$$

where M is the diagonal mass matrix, $\ddot{U}$, $\dot{U}$ and U are the vectors of joint accelerations, velocities and displacements respectively, K is the joint stiffness matrix, c is a damping constant and $\ddot{U}_g$ is a vector formed from the prescribed ground accelerations. In Eq. (25a) C is the damping matrix. Elements of $\ddot{U}_g$ corresponding to the horizontal motion of joints will be equal to the horizontal component of ground acceleration, those corresponding to the vertical motion of joints will be equal to the vertical component of ground acceleration and those corresponding to rotation of joints will be equal to zero.

The initial values of the displacements and velocities of the joints are taken as zero. The initial values of the forces in the various members are those corresponding to the static loads before the commencement of dynamic loading. The initial accelerations of the joints are readily computed from Eq. (25b). The displacements, velocities and accelerations of the joints at any time after the ground motion commences, are computed by a step by step numerical integration procedure, using the following relations.

$$U_1 = U_0 + dt\dot{U}_0 + \frac{(dt)^2}{2}\ddot{U}_0 \qquad \text{....(26a)}$$

$$\ddot{U}_1 = -M^{-1}KU_1 - c\dot{U}_0 - \ddot{U}_{g_1} \qquad \text{....(26b)}$$

$$\dot{U}_1 = \dot{U}_0 + 0.5dt(\ddot{U}_0 + \ddot{U}_1) \qquad \text{....(26c)}$$

where dt is the time interval used, the subscript 0 denotes known conditions at the beginning of any given time step and subscript 1 denotes quantities to be computed at the end of the time step.

To minimize computation time, the largest possible time interval should be used. To obtain a stable solution this is limited to a certain fraction of the smallest period of the system. The theoretical value of this fraction is of the order $1/\pi$, and for the present investigation a value of 1/6 is used. The smallest period is easily estimated from the largest eigenvalue of the matrix $M^{-1}K$. Due to the nonlinear nature of the response, K may vary with time and so would the optimum time interval. Whenever a crack forms in a wall element there is a substantial increase in the value of the optimum time interval. The computer program automatically calculates the best time interval whenever there is a change in K due to formation of cracks in wall elements and uses it for further analysis.

For any step in the numerical integration procedure, the changes in the joint displacements cause changes in the stresses in the frame and wall elements. For given values of joint displacements these are easily computed from known stiffness properties of the elements. If the stress in any member at the end of a given time step exceeds the elastic limit values, causing a wall element to crack or a plastic hinge to form in a frame member, a smaller time interval estimated by interpolation, is used for the present step so that the crack in the wall element or the plastic hinge in the frame member forms just at the end of the time step. The joint stiffness matrix K is now modified accordingly and the procedure continued. At the end of each time interval, the energy absorption due to the incremental rotation of each plastic hinge is also checked. If it is negative, the elastic state is reinstated for the concerned hinge.

## 3.4 Modified Procedure With Reduced Degrees of Freedom

In the procedure discussed before there are three degrees of freedom for each of the N free joints and the structure has 3N degrees of freedom. In this section a modified procedure is presented. In this procedure the axial and rotational modes are eliminated and consequently the smallest period of the modified system would be much larger than that of the previous system with 3N degrees of freedom. This would make possible the use of a much larger time interval for the numerical integration and a considerable saving of computation time. For the sake of simplicity, the previous system with 3N degrees of freedom and the modified system will henceforth be called "the unreduced system" and "the reduced system", respectively.

The number of degrees of freedom of the reduced system is equal to the number of floors. The equations of motion are given by

$$\ddot{U}^* + c\dot{U}^* + M^{*-1}K^*U^* = -\ddot{U}_g^* \qquad \ldots.(27)$$

where the superscript * refers to the reduced system. The generalized coordinates of this system correspond to the horizontal displacements of the joints along a line of exterior columns. Consequently, there is one coordinate for each floor. The elements of $M^*$ are the sum of the elements of M corresponding to all joints at the same floor level. The joint stiffness matrix of the reduced system, $K^*$, can be computed from K as follows.

Let F and $F^*$ be the joint flexibility matrix of the unreduced and reduced systems respectively; i.e., $F = K^{-1}$ and $F^* = K^{*-1}$. From physical considerations it is apparent that $F^*$ is a submatrix of F. Thus to obtain $K^*$, F is first computed by inverting K. The appropriate elements of F are picked to form $F^*$, the inversion of which yields $K^*$. To illustrate this, the structure shown in Figure 8 is now considered. The unreduced system corresponding to this structure has eighteen degrees of freedom and the joint flexibility matrix is given by

$$F = \begin{bmatrix} f_{1,1} & \cdot & \cdot & \cdot & \cdot & \cdot & f_{1,18} \\ \cdot & & & & & & \cdot \\ \cdot & & f_{i,j} & & & & \cdot \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ \cdot & & & \cdot & & & \cdot \\ f_{18,1} & \cdot & \cdot & \cdot & \cdot & \cdot & f_{18,18} \end{bmatrix} \quad \ldots.(28)$$

and the joint flexibility matrix of the reduced system is given by

$$F^* = \begin{bmatrix} f^*_{1,1} & f^*_{1,2} & f^*_{1,3} \\ f^*_{2,1} & f^*_{2,2} & f^*_{2,3} \\ f^*_{3,1} & f^*_{3,2} & f^*_{3,3} \end{bmatrix} = \begin{bmatrix} f_{1,1} & f_{1,4} & f_{1,7} \\ f_{4,1} & f_{4,4} & f_{4,7} \\ f_{7,1} & f_{7,4} & f_{7,7} \end{bmatrix} \quad \ldots.(29)$$

The damping constant c is assumed to be the same in both systems. The displacements, velocities and accelerations of the joints of the reduced system are computed by a step by step numerical integration procedure using the following relations which are similar to Eqs. (26).

$$U_1^* = U_0^* + dt\dot{U}_0^* + 0.5(dt)^2\ddot{U}_0^* \qquad \text{....(30a)}$$

$$\ddot{U}_1^* = -M^{*-1}K^*U_1^* - c\dot{U}_0^* - \ddot{U}_{g_1}^* \qquad \text{....(30b)}$$

$$\dot{U}_1^* = \dot{U}_0^* + 0.5dt(\ddot{U}_0^* + \ddot{U}_1^*). \qquad \text{....(30c)}$$

Because of the nonlinear nature of the response, it is necessary to compute the internal forces in each element at every step of the numerical integration procedure. This requires the determination of the vector U, the displacements of the unreduced system. These displacements are computed as follows. The vector $R^*$, defined as the internal resistance at the joints due to displacements $U^*$, is given by the relation

$$R^* = K^*U^*. \qquad \text{....(31)}$$

Let R be the unreduced resistance vector corresponding to U. The vector R may be constructed from $R^*$ by setting each element equal to zero except those corresponding to the horizontal displacements of the joints of the unreduced system; and those corresponding to the horizontal displacements will be taken to be the same as those of $R^*$. This is tantamount to assuming that the inertia and damping forces in the axial and the rotational coordinates are negligible. Subsequent numerical results show that this is a reasonable assumptions. The vector U can now be computed from

$$U = K^{-1}R \qquad \text{....(32)}$$

The rest of the procedure is the same as before. Whenever there is a change in the stiffness properties of an element, due to cracking of a wall element, or the formation or disappearance of a plastic hinge in a frame member, $K^*$ must be recomputed.

It should be noted that the computation of $K^*$ involves the inversion of K, and as such the procedure will not operate once the joint stiffness matrix K becomes singular. However this does not appear to be a serious disadvantage, since in most cases, if a structure becomes statically unstable, collapse would often follow shortly, even in dynamic response. Besides, if necessary, one can always proceed with the analysis by reverting to the original procedure when K becomes singular.

### 3.5 Computer Program

An outline of the program developed for the study is presented in this section, the program itself is given in Appendix II. The important steps in the program are described in the order in which they are executed.

1) Input information includes location of joints, the incidence, geometrical and physical properties of all structural elements, number of floors, number of degrees of freedom, time limit up to which analysis is to be continued, etc. Both methods of analysis discussed in the preceding sections are incorporated in the

program. For a given structure, the value of the number of degrees of freedom specified directs the program to select the appropriate method of solution.

2) From the information input in 1, the stiffness matrices of the frame and wall elements are computed. The joint stiffness matrix K is next assembled. If necessary, $K^*$ is computed. The elements of the member stiffness matrices and the joint stiffness matrix K are stored as one dimensional arrays. Only the upper or lower triangular elements are stored. This makes possible a considerable saving of computer memory.

3) The mass matrix is next assembled using the procedure outlined in section 3.2.

4) The optimum value of the time interval namely 1/6th of the smallest period of the system is computed from the largest eigenvalue of the matrix $M^{-1}K$ obtained by an iteration procedure.

5) Information regarding earthquake loading is input. At any desired time the ground acceleration components are computed by a straight line interpolation between the appropriate discrete points. The input motion of supports is adopted from the two components (North-South and Up-Down) of ground acceleration records of the El Centro

Earthquake of May 18, 1940. This is accomplished by a subroutine which can also be easily modified to include dynamic loading on the joints.

6) The dynamic analysis is accomplished using the procedure outlined in sections 3.3 and 3.4. Information is output at designated intervals to permit a running record of joint displacements, forces in the frame members, maximum tensile stress in the wall elements and location of plastic hinges. In addition, the above information is given as output, whenever there is a change in the structural properties of an element, i.e., whenever there is a crack formation in a wall element or the formation or disappearance of a plastic hinge in a frame member.

7) The program checks joint displacements at every time increment against prescribed maximum values and will exit once the prescribed maximum values are exceeded. At the end, the final status of joint displacements, velocities and accelerations, forces in the frame members, maximum tensile stress in the wall elements, location of plastic hinges and the energy absorbed due to the rotation of plastic hinges are furnished as output. This data can be used to continue the analysis. The displacement of the top story and the corresponding time are furnished as punched

output. This information is used to plot the horizontal displacement of the top story versus time in the computer plotter CALCOMP 643. This procedure was necessary because the program has been written for the CDC 6500 computer system and the plotter forms part of the CDC 3600 system.

3.5.1 Limitations of the Program.--The program has been dimensioned to analyse structures with a maximum of 40 joints, 40 frame members and 40 wall elements. The frame should also be rectangular and there is no restriction to the number of bays or number of stories. Dynamic loading consists of components of ground accelerations due to earthquake motion. However, the program can be modified easily to handle different types of plane frames, more number of joints, frame members and wall elements and also other types of dynamic loading.

# CHAPTER IV

# NUMERICAL RESULTS

## 4.1 The Structure

The structure considered for the present study is a three story steel frame with concrete filler walls, six inches thick. The dimensions and sectional properties are shown in Figure 8. Values of other parameters are: elastic modulus of steel $30 \times 10^6$ psi, elastic modulus of wall material $2.5 \times 10^6$ psi before cracking and $2.5 \times 10^4$ psi after cracking, Poisson's ratio of concrete zero, yield stress for steel 33000 psi and cracking stress for concrete 150 psi. A 10% critical damping (based on the fundamental mode) is assumed in obtaining the following results.

### 4.1.1 Loading.

--For initial static loading, a uniformly distributed load of 2400 lb./ft. is assumed on each beam. Dynamic loading consists of selected portions of the El Centro Earthquake of May, 1940. Failure of the structure is assumed to occur if the horizontal displacement exceeds 2" per story height or the rotation of any joint exceeds 0.2 radians.

### 4.1.2 Time Increment.

--As mentioned previously, for the numerical integration procedure, the choice of time increment depends on the smallest period of the system. For the unreduced system the time interval used is 0.002 second.

For the reduced system the time step used is 0.004 second before the first floor wall cracks and 0.01 second afterwards. These values were estimated from the largest eigenvalue of the matrix $M^{-1}K$. In the final version of the computer program given in the Appendix, the optimum time interval to be used for the analysis is computed by the program itself and modified whenever a wall element develops cracks.

## 4.2 Numerical Examples

The results presented in this chapter may be divided into two parts. In the first part, four examples of earthquake loading are considered for the study of the effect of walls on the dynamic response of the structure. The responses with and without wall-stiffness are presented in the form of graphs, with the horizontal displacement of the top story plotted against time. In the second part, the dynamic response is studied by the modified procedure in which the axial and rotational modes are eliminated. The results are compared with those obtained by the previous procedure in which all the modes are retained. Items compared include the horizontal displacement of the top story and the computation time.

## 4.3 Effect of Walls

The linear responses of the structure with and without the effect of wall-stiffness are shown in Figure 9. The structure is subjected to the ground motion of El Centro

Earthquake of May, 1940, from 1.5 to 5.0 seconds with a scaling factor of 0.005 to ensure that the moments in the frame members and the maximum tensile stress in the wall elements do not exceed the elastic limit and that the response would be entirely in the linear range. It is seen that, if the wall-stiffness is taken into account, the structure vibrates with a high frequency and low amplitude. Without wall-stiffness the response has a large amplitude and low frequency.

Figure 10 represents the first example of nonlinear responses with and without wall-stiffness. The loading is the ground motion of the same El Centro Earthquake from 1.5 to 4.0 seconds, with a scaling factor of 0.5. If the wall-stiffness is considered, the first floor wall cracks soon after loading commences. The response has a relatively low amplitude and high frequency. The vibration is damped out after the ground motion ceases and the structure remains stable. When wall stiffness is neglected the response has a large amplitude and low frequency, and the structure fails due to excessive rotation of Joint 5.

In the next example shown in Figure 11 the nonlinear response to a more severe earthquake is considered. The ground motion of the same El Centro Earthquake from 1.5 to 4.0 seconds with a scaling factor of 1.0 is used. With wall-stiffness taken into account, soon after loading commences, the walls crack one by one starting from the ground floor. The vibrations are damped out after the ground motion ceases and the structure remains stable. The amplitude is substantially larger and the frequency lower when compared with the previous example in which two of the three walls did not develop cracks. When the wall-stiffness is ignored the structure fails within 1.5 seconds of loading, due to excessive rotation of Joint 3.

Figure 12 shows the last example of nonlinear response, the loading being the ground motion of the same earthquake from 24.0 to 30.0 seconds with a scaling factor of 1.0. With wall-stiffness, the walls crack one by one starting from the ground floor soon after loading commences. The response is quite similar to the previous example, except that the ground motion is present during the entire period for which the response is plotted. Without wall-stiffness the structure vibrates with a relatively large amplitude and low frequency. For a given interval of time the energy absorption due to the rotation of plastic hinges is also greater with wall-stiffness ignored than that with wall-stiffness considered.

The above results indicate that even when all the wall elements are cracked, the contribution of walls to the lateral stiffness of the structure and its dynamic response is too considerable to be neglected. The computed fundamental periods of vibration of the structure with and without wall-stiffness are shown in Table 4. The fundamental periods, in seconds, of the structure with the effect of walls ignored, with all walls cracked and with all walls intact are 2.170, 0.855 and 0.200, respectively. Expressed as a percentage of the fundamental period with walls ignored, the fundamental period with all walls cracked and with all walls intact are, 40% and 9%, respectively.

## 4.4 Effect of Axial and Rotational Modes

Figure 13 shows the dynamic responses of the structure without considering wall-stiffness by the modified procedure with three degrees of freedom and by the previous procedure with eighteen degrees of freedom. It may be seen that there is hardly any difference in the response. The loading conditions are the same as for the previous example, i.e., Figure 12. The computation time required by the modified method is only about 25% of that required by the previous procedure.

The graphs shown in Figure 14 represent the responses of the structure with wall-stiffness by the two methods, the conditions of loading being the same as in the previous example. This is representative of cases in which somewhat larger discrepencies are found between the responses computed by the two methods. In the unreduced system cracks develop in all the walls, whereas in the reduced system cracks develop in the first and second floor walls only. (Actually the maximum tensile stress developed in the third floor wall falls just short of the cracking stress so that it remains intact.) Thus the modified procedure would appear to be slightly less conservative.

It would seem reasonable to expect this considering the fact that the axial and rotational modes have been eliminated and also the structure is not subjected to the vertical components of ground motion. There is no appreciable difference in the amplitudes and the general trends of the response. With one wall still uncracked the response by the modified procedure has a slightly higher frequency. The computation time required by the modified procedure is again only about 25% of that required by the other procedure with eighteen degrees of freedom. The periods of the responses in the various graphs presented above agree well with the computed values given in Table 4.

## 4.5 Iteration Procedure for Acceleration of Joints

From the relation specified in Eq.(25b) the acceleration vector at a given time, say, $t_1$ should be

$$\ddot{U}_1 = -M^{-1}KU_1 - \ddot{U}_{g_1} - c\dot{U}_1. \qquad \text{....(33)}$$

In the numerical integration procedure, Eq.(26b) was actually used, the difference being, $U_0$ was used in place of $U_1$ in order to avoid iterations. This procedure amounted to assuming that the damping force is proportional to the velocity at the previous time step in stead of the current velocity. It was anticipated that this would be a justifiable assumption because the time interval was usually so small that it would hardly make any difference in the solution. Two numerical problems were actually solved with and without iteration. The time-displacements were almost identical, thus confirming the validity of the use of Eq.(26b) in stead of Eq.(33).

# CHAPTER V

## SUMMARY AND CONCLUSIONS

The development of an "optimal" numerical model and its application to the elasto-plastic dynamic analysis of building frames with filler walls have been presented. The nonlinearity of the structural response, due to the formation of plastic hinges in the frame members and/or crack propagation in the wall elements has been taken into account. The dynamic analysis has been embodied in a computer program written in FORTRAN IV. This has been used to study the dynamic response of a three story steel frame with concrete filler walls subjected to an earthquake loading. The study also includes the effect of the axial and rotational modes on the dynamic response.

For the study of the modelling of the stiffness characteristics of the structure, the filler walls are treated as finite elements in plane stress, interacting with the frame such that the translational displacements of the joints are compatible. After formation of crack in a wall element, the stiffness properties of the structure are computed by assuming a reduced elastic modulus for that wall element. Both triangular and rectangular elements are considered. The effect of mesh size and the cracking phenomenon in the wall panel are studied.

Computed results of load-displacement relations, crack propagation pattern and ultimate load capacity have been compared with known experimental results.

In the dynamic analysis the mass of the system is handled by a lumping procedure. Two methods of analysis have been derived. In the first, all the three degrees of freedom of a joint are considered. In the second (modified) method, the degrees of freedom associated with the axial deformation of frame members and the rotation of joints are eliminated. Mass-proportional viscous damping has been included in the analysis which is formulated using the joint displacements relative to supports as variables. The transient response of the structure is obtained by a numerical integration of the equations of motion of the joint masses. The time interval used for the numerical integration procedure is based on the smallest period of the system at any instant. For the dynamic analysis, the initial values of the forces in the various members are those corresponding to the static loads before the commencement of dynamic loading.

Based on the assumptions made during the course of analysis and on the limited numerical results presented in this report, the following observations may be made.

1. The modelling of the wall panel of a floor as a single rectangular finite element interacting with the moment-resisting frame could reasonably satisfactorily account for the contribution of walls to the overall

lateral stiffness of the structure.

2. The contribution of walls to the lateral stiffness and consequently to the fundamental frequency and the dynamic response of a structure appears to be too considerable to be ignored, even when the walls are cracked. In general it would be appear that the inclusion of the effect of walls would lead to a substantially conservative design.
3. Elimination of the axial and rotational modes makes possible the use of a much larger time interval for the numerical integration procedure, and thus resulting in a considerable saving of computation time without any appreciable loss accuracy.

Future extensions of this investigation could include: (1) experimental study of the dynamic response of frame-wall systems in the laboratory to form a basis of comparison: (2) contribution of floors to the rotational stiffness of joints and the effect of floors on the dynamic response of the structure.

Table 1. Ultimate load capacity of model infilled frame.

| Frame | Length of beam in centimeters | Height of columns in centimeters | Cross section of frame in centimeters | Thickness of wall in centimeters | Collapse load in tons | |
|---|---|---|---|---|---|---|
| | | | | | Sachanski's tests | Computed values |
| 1 | 350 | 280 | 15 x 60 | 30 | 23.2 | 22.70 |
| 2 | 350 | 280 | 15 x 60 | 15 | 8.6 | 8.12 |

Table 2. Effect of mesh size on load-displacement relation using triangular elements

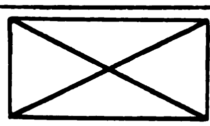
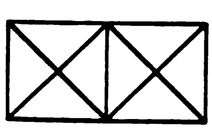
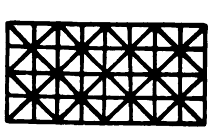
| Pattern | Dimension of joint stiffness matrix | Number of frame members | Number of wall members | Horizontal displacement of deck in inches x $10^2$ |
|---|---|---|---|---|
|  | 8 | 3 | 4 | 0.11965 |
|  | 13 | 4 | 8 | 0.13025 |
|  | 87 | 16 | 64 | 0.13884 |

Table 3. Effect of mesh size on load-displacement relation using rectangular elements

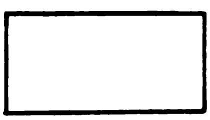
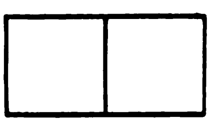
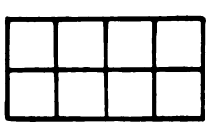
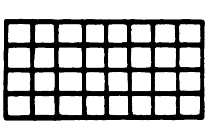
| Pattern | Dimension of joint stiffness matrix | Number of frame members | Number of wall members | Horizontal displacement of deck in inches x $10^2$ |
|---|---|---|---|---|
|  | 6 | 3 | 1 | 0.12074 |
|  | 9 | 4 | 2 | 0.13091 |
|  | 27 | 8 | 8 | 0.13960 |
|  | 87 | 16 | 32 | 0.14017 |

Table 4. Computed values of fundamental periods.

| Condition of Walls | Fundamental Period |
|---|---|
| 1. All walls intact | 0.200 sec. |
| 2. First floor wall cracked | 0.636 sec. |
| 3. First and second floor walls cracked | 0.806 sec. |
| 4. All walls cracked | 0.855 sec. |
| 5. Effect of walls ignored | 2.170 sec. |

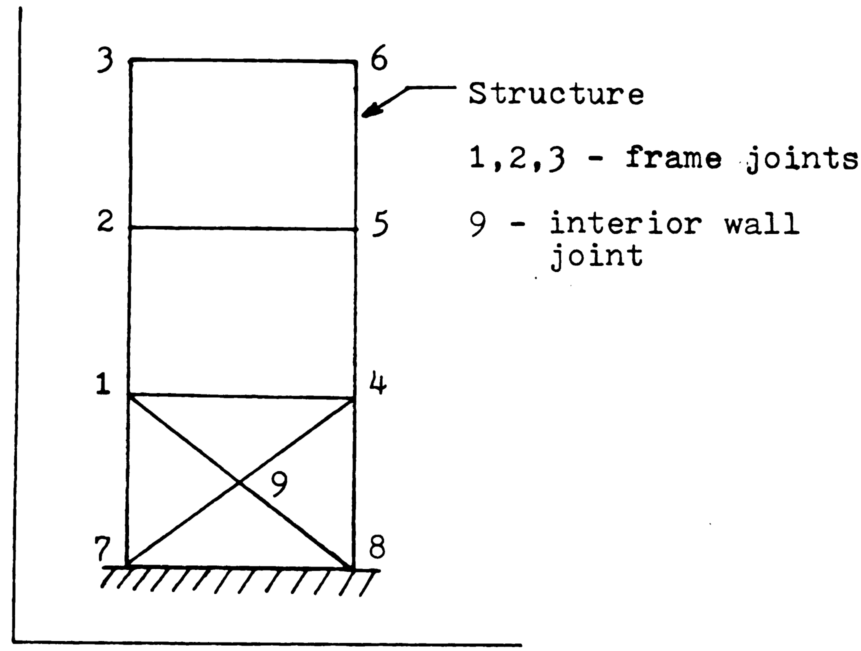



**FIG.1 STRUCTURE GLOBAL COORDINATES**

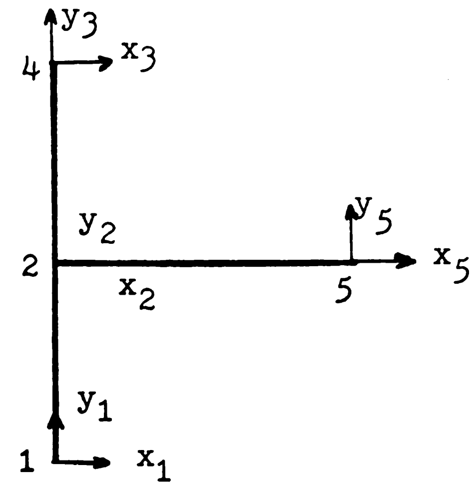
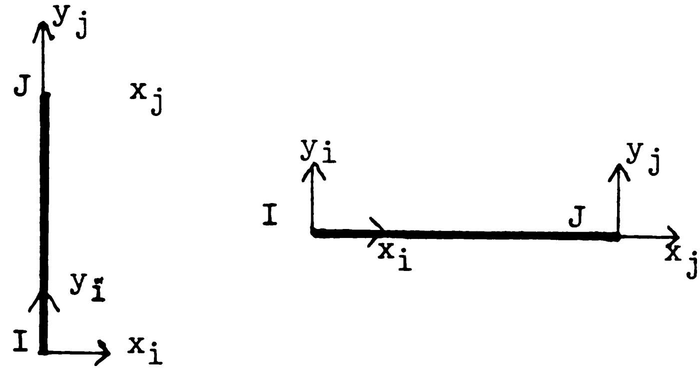
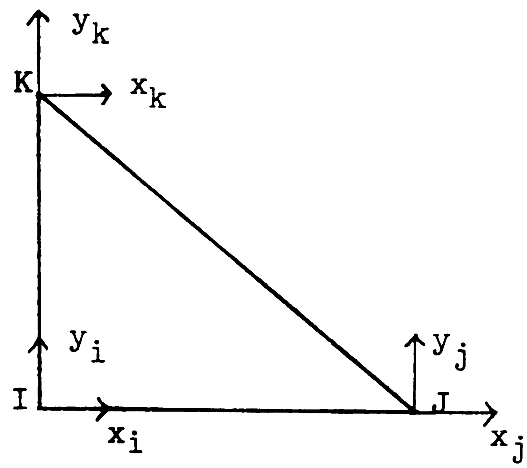


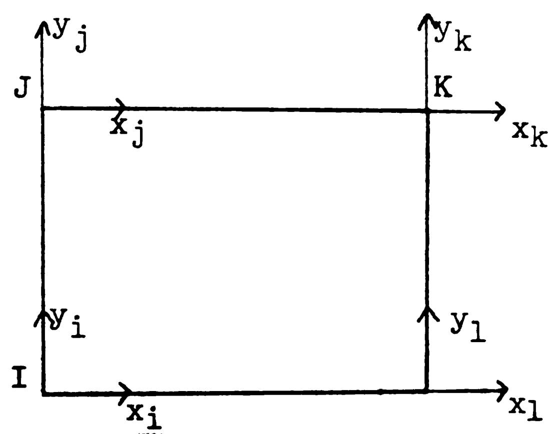

**FIG.2 JOINT COORDINATES**




Frame Elements: (Incidence IJ)

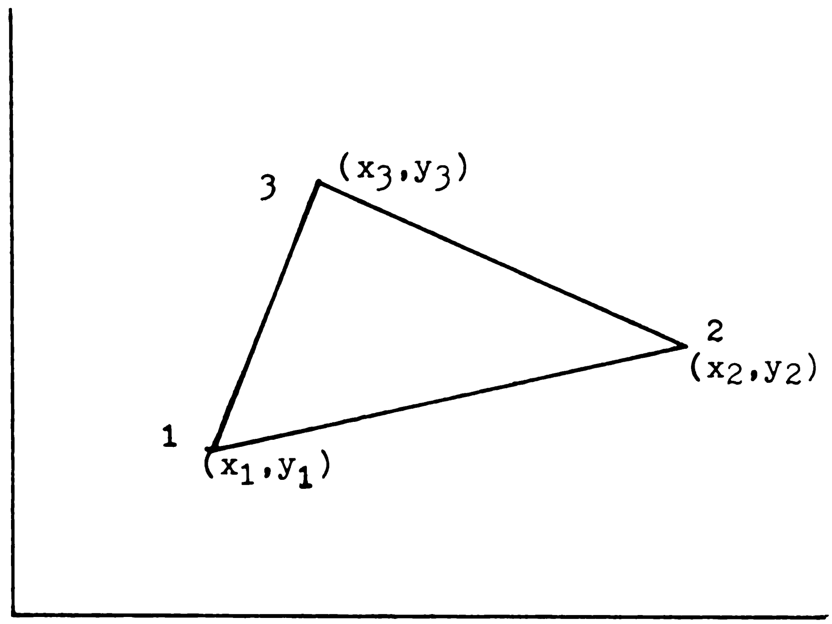



Triangular Wall Elements: (Incidence IJK)

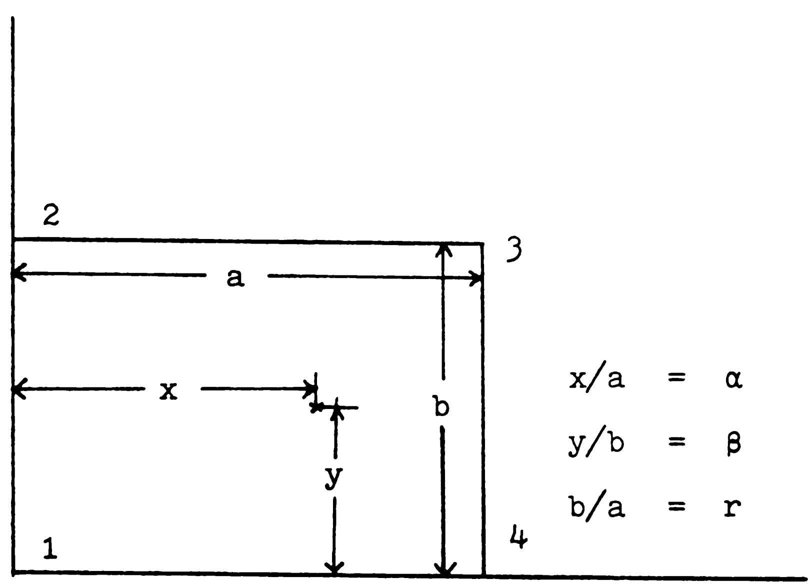



Rectangular Wall Elements: (Incidence IJKL)

**FIG.3 MEMBER COORDINATES**




Triangular Finite Element: (Incidence 123)




Rectangular Finite Element: (Incidence 1234)

**FIG. 4 WALL ELEMENTS**

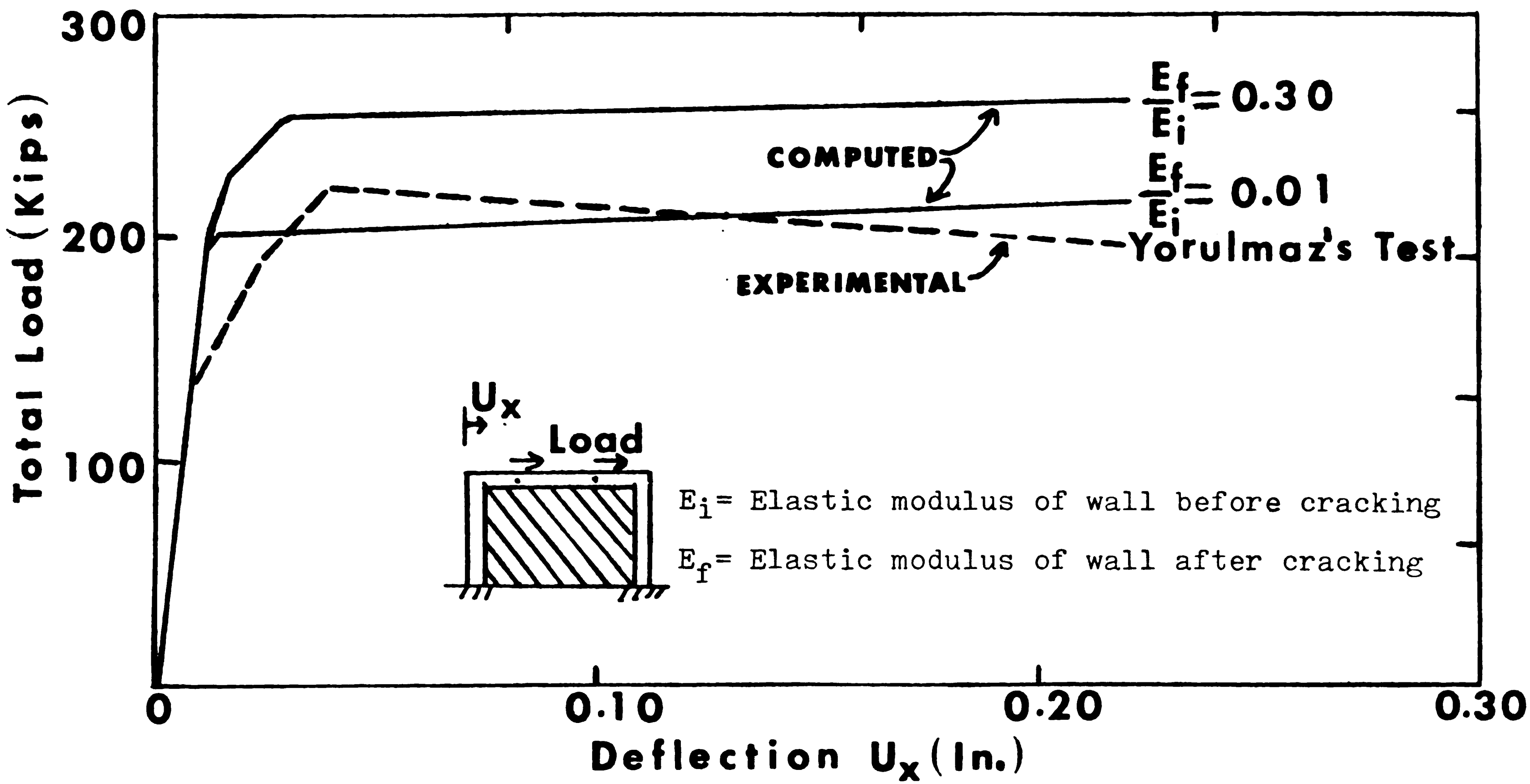
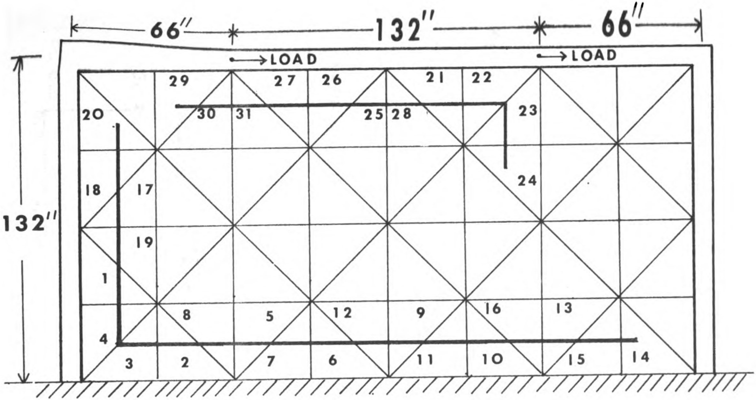



FIG.5 STATIC LOAD–DISPLACEMENT RELATIONS




Numbers indicate the order in which the wall elements crack

**FIG.6 CRACK PROPAGATION IN WALL**

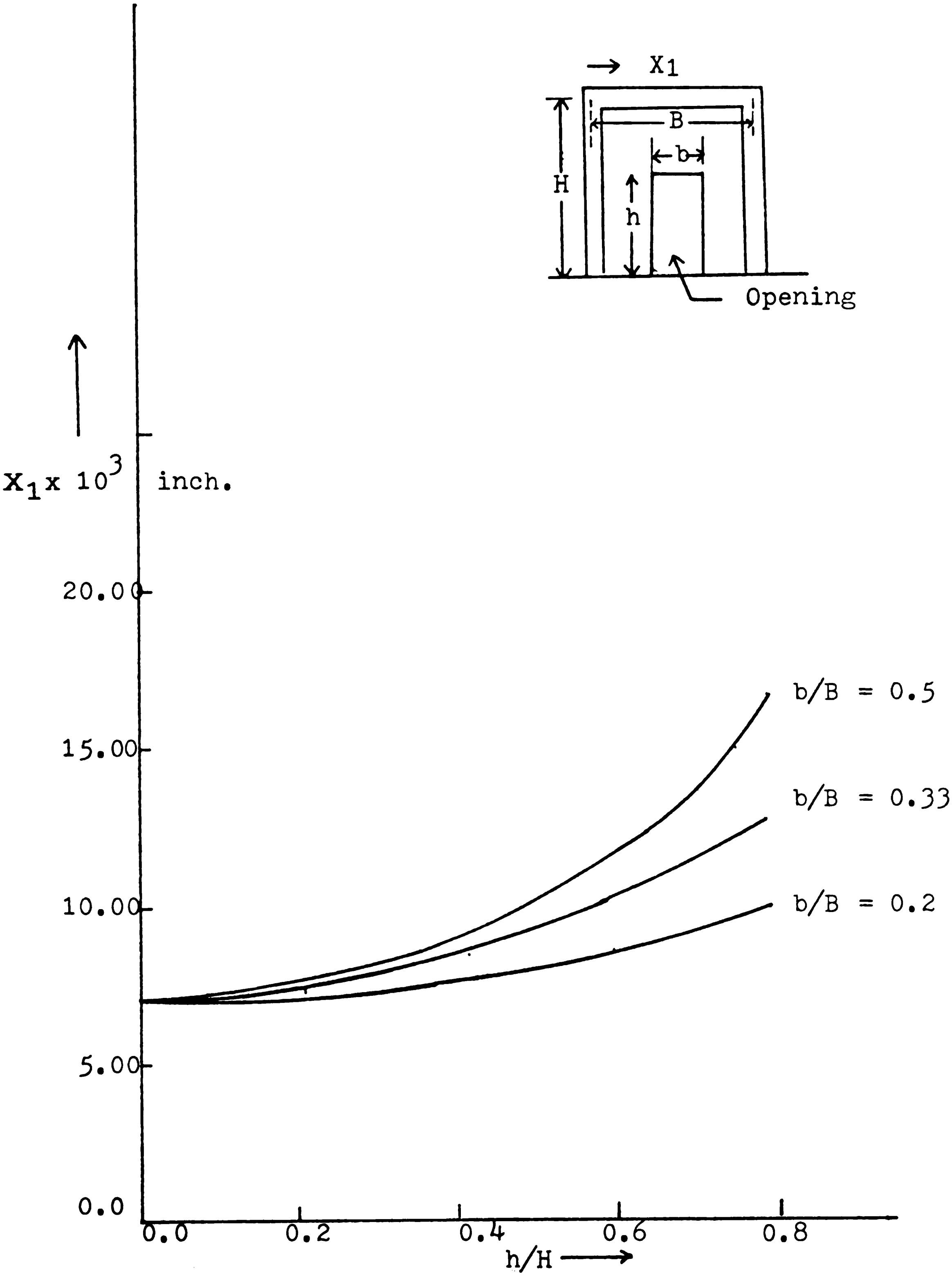



**FIG.7 EFFECT OF WALL OPENINGS**

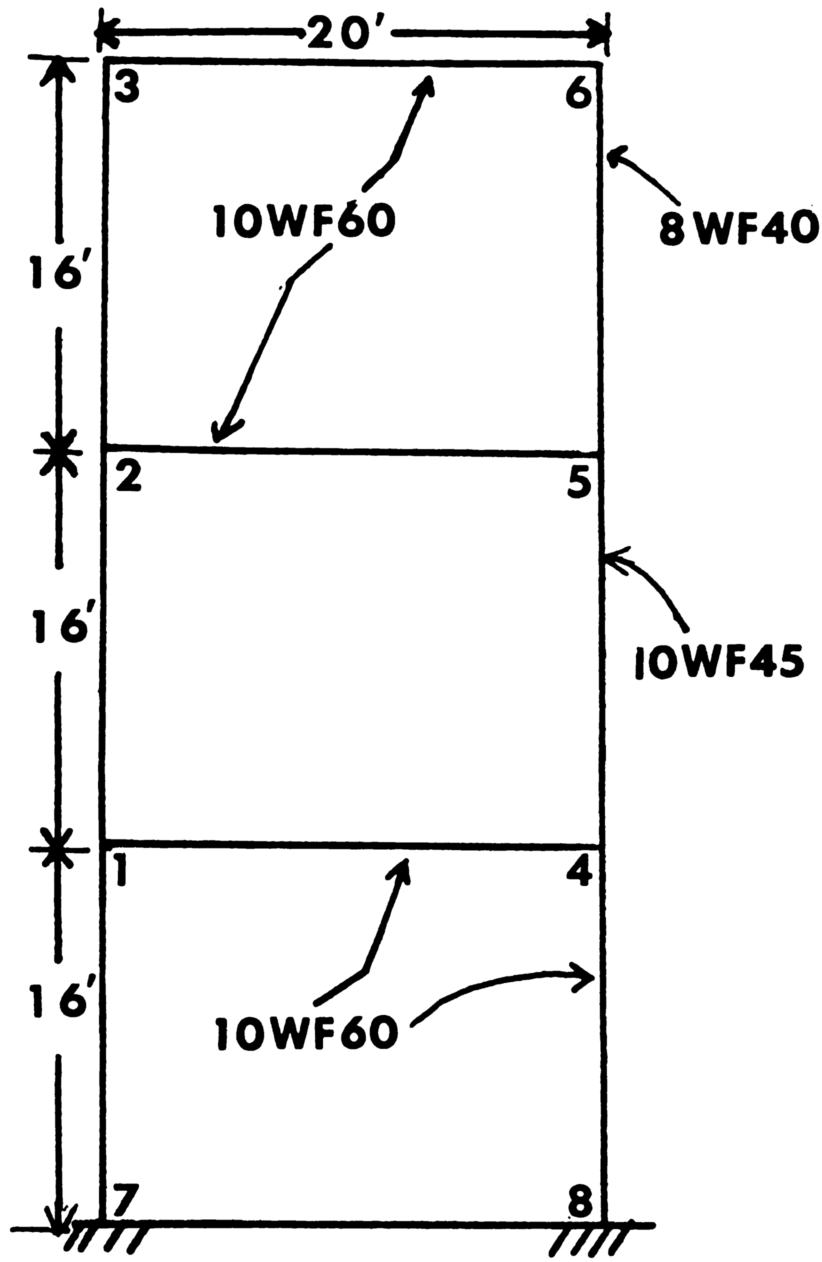



FIG.8 MODEL THREE STORY STRUCTURE

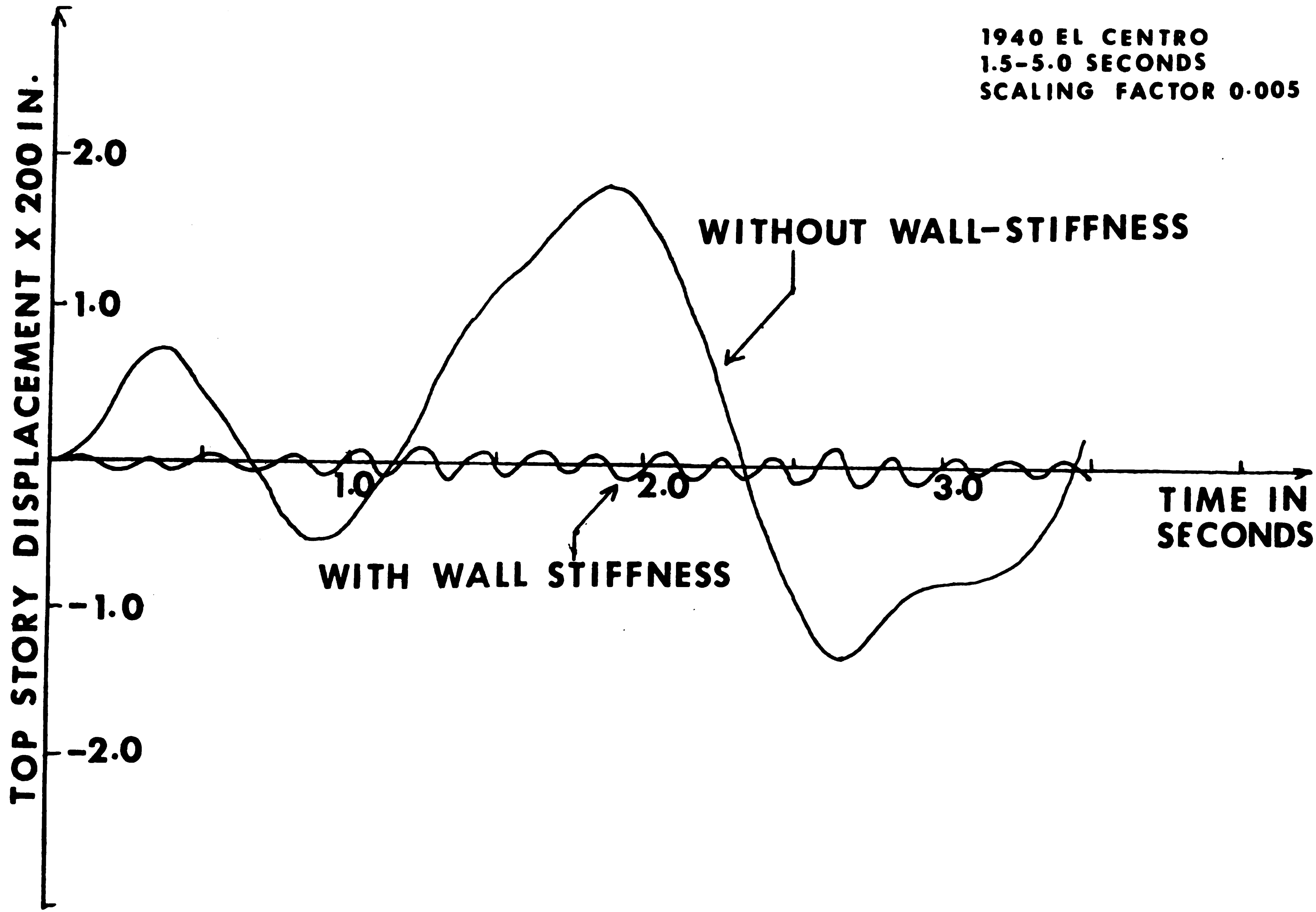



FIG.9 LINEAR RESPONSE

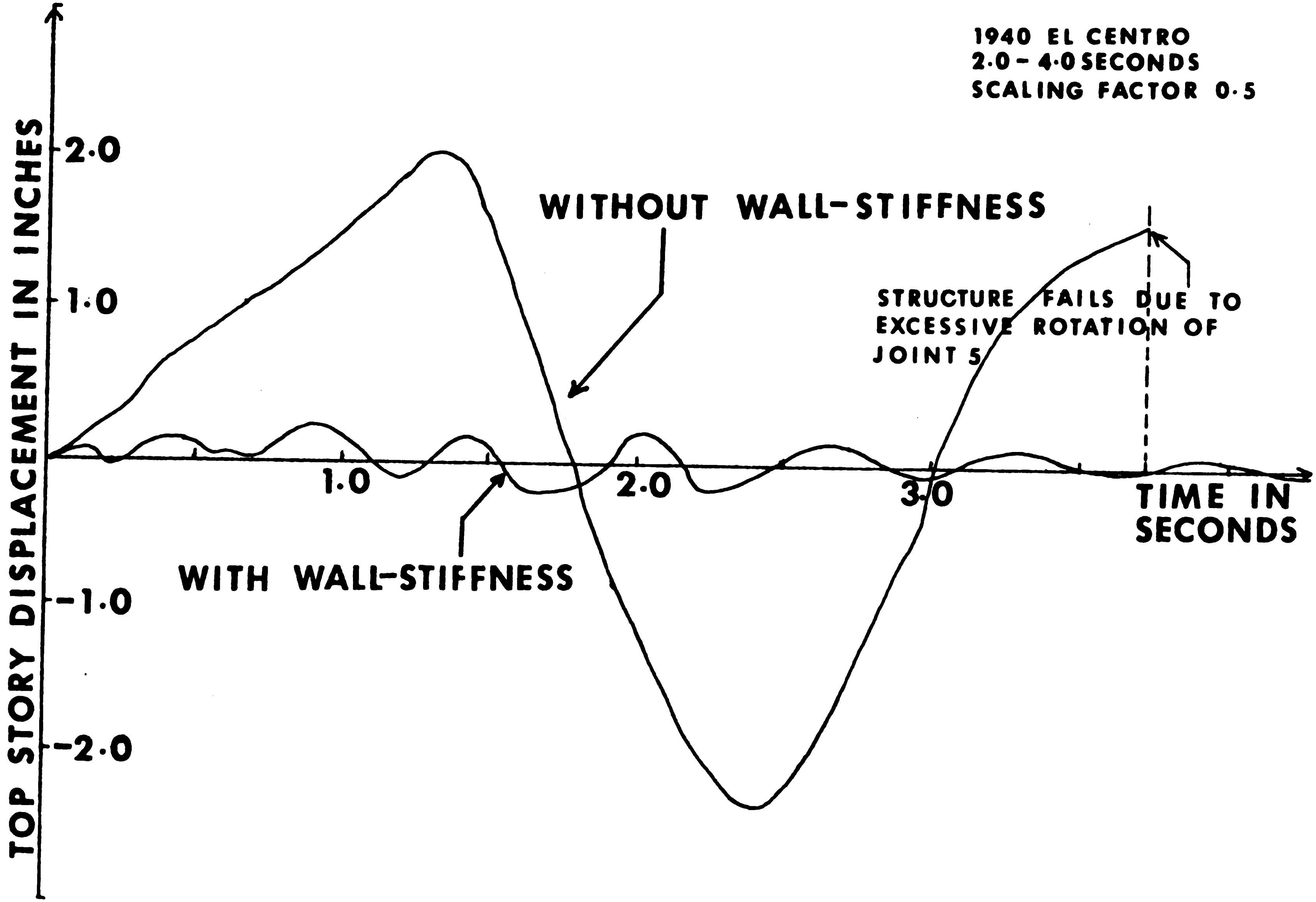



FIG. 10 NONLINEAR RESPONSE– GROUND MOTION 1

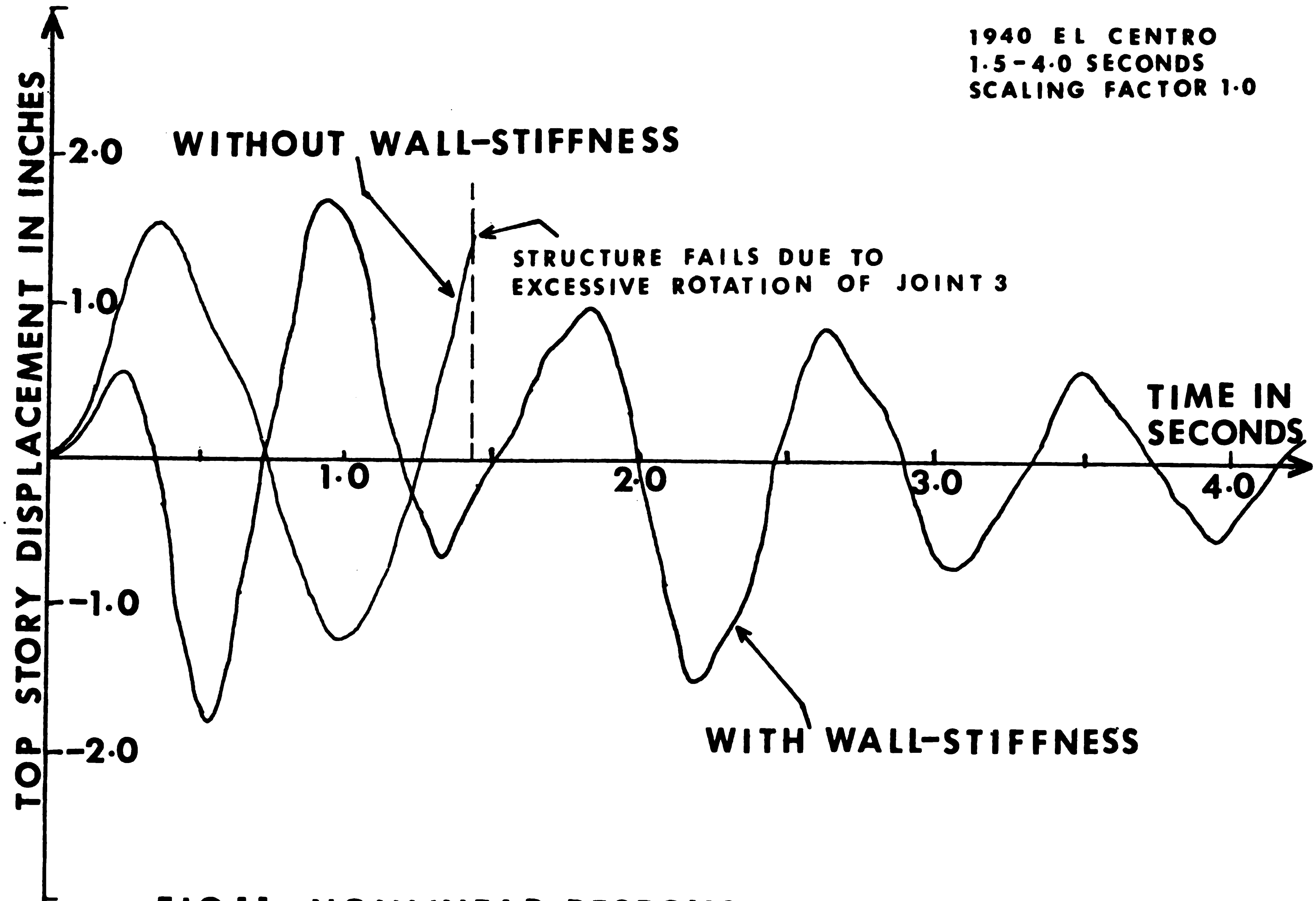



FIG.11 NONLINEAR RESPONSE—GROUND MOTION 2

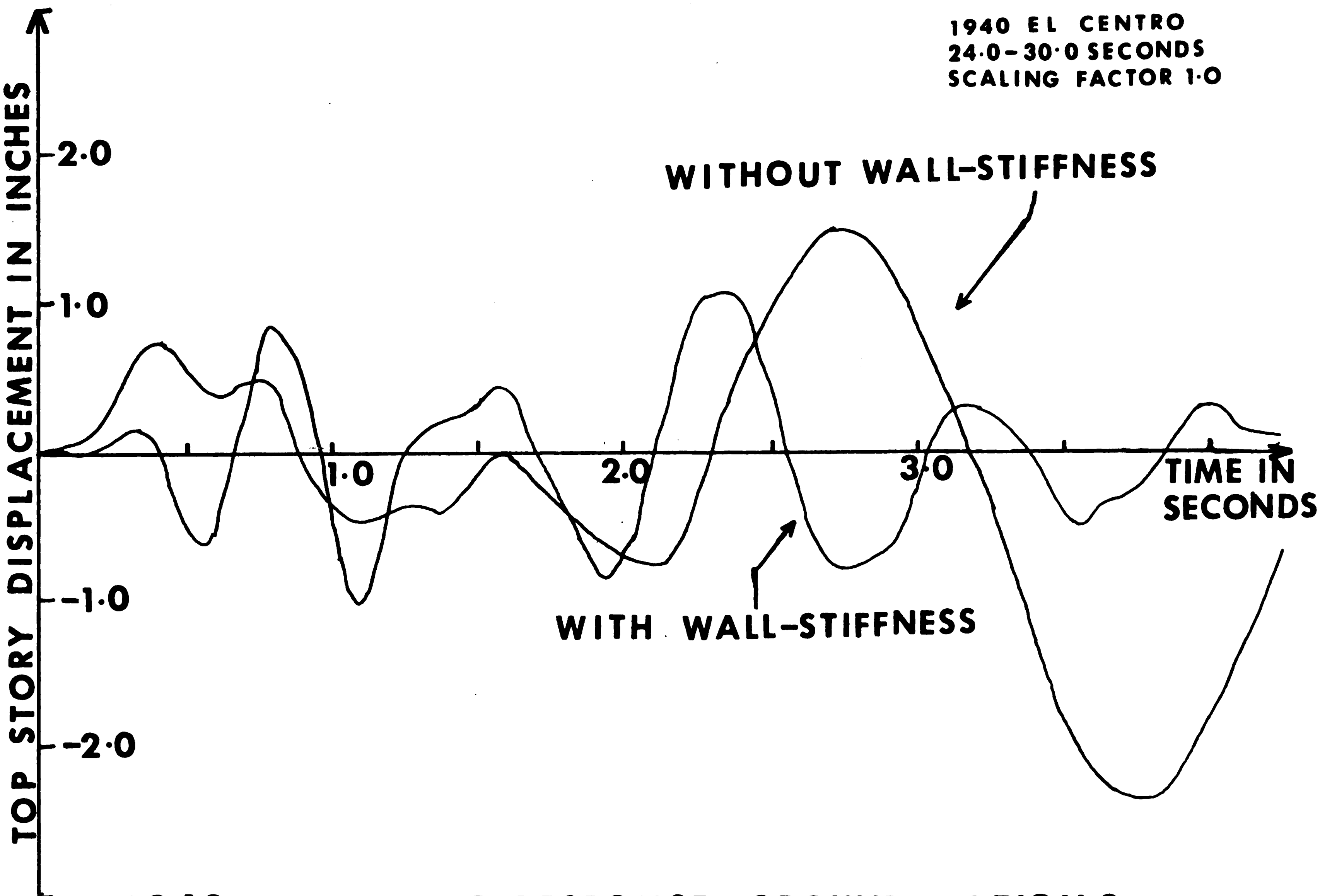



FIG.12 NONLINEAR RESPONSE – GROUND MOTION 3

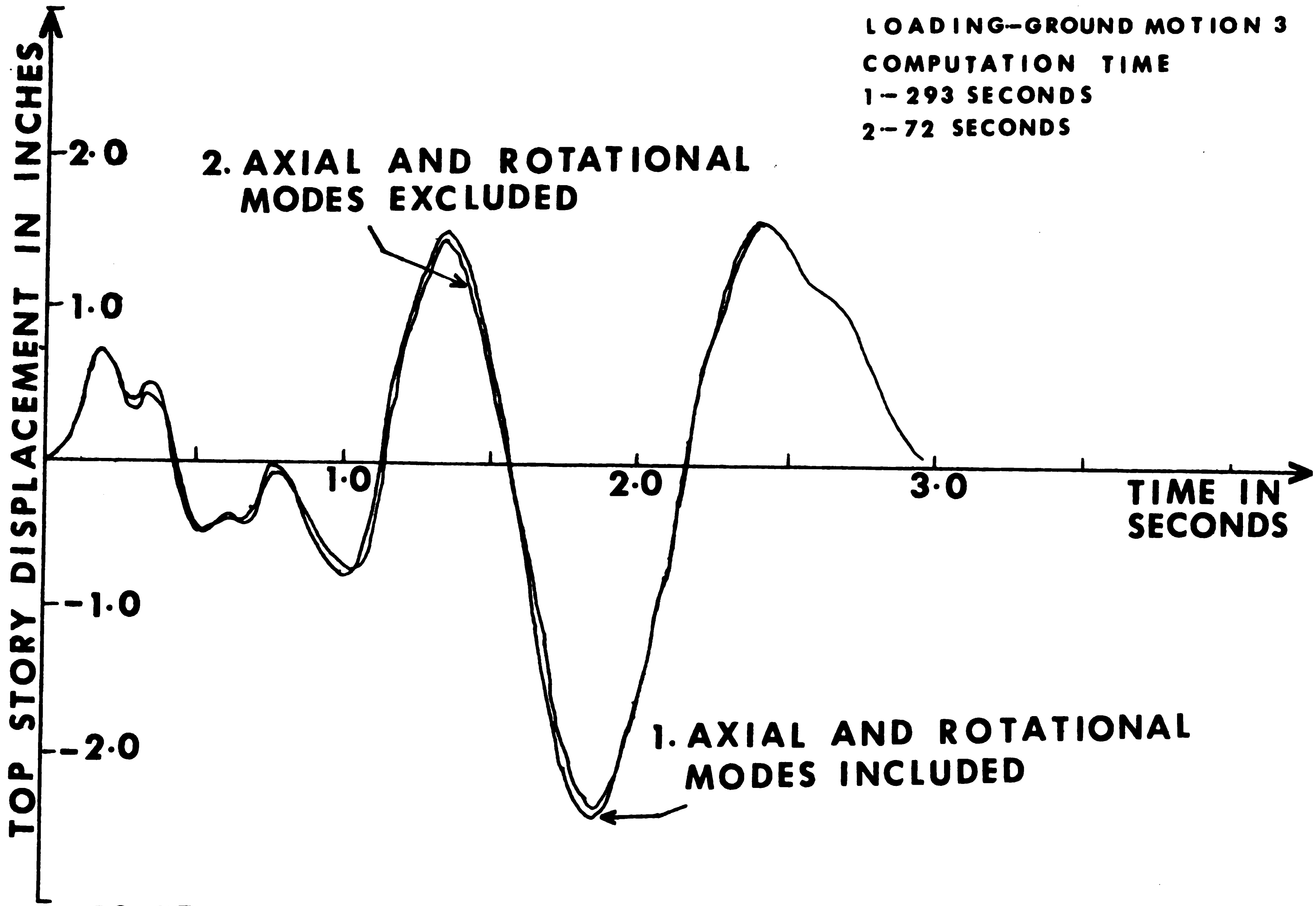



FIG.13 EFFECT OF AXIAL AND ROTATIONAL MODES — WITHOUT WALL-STIFFNESS

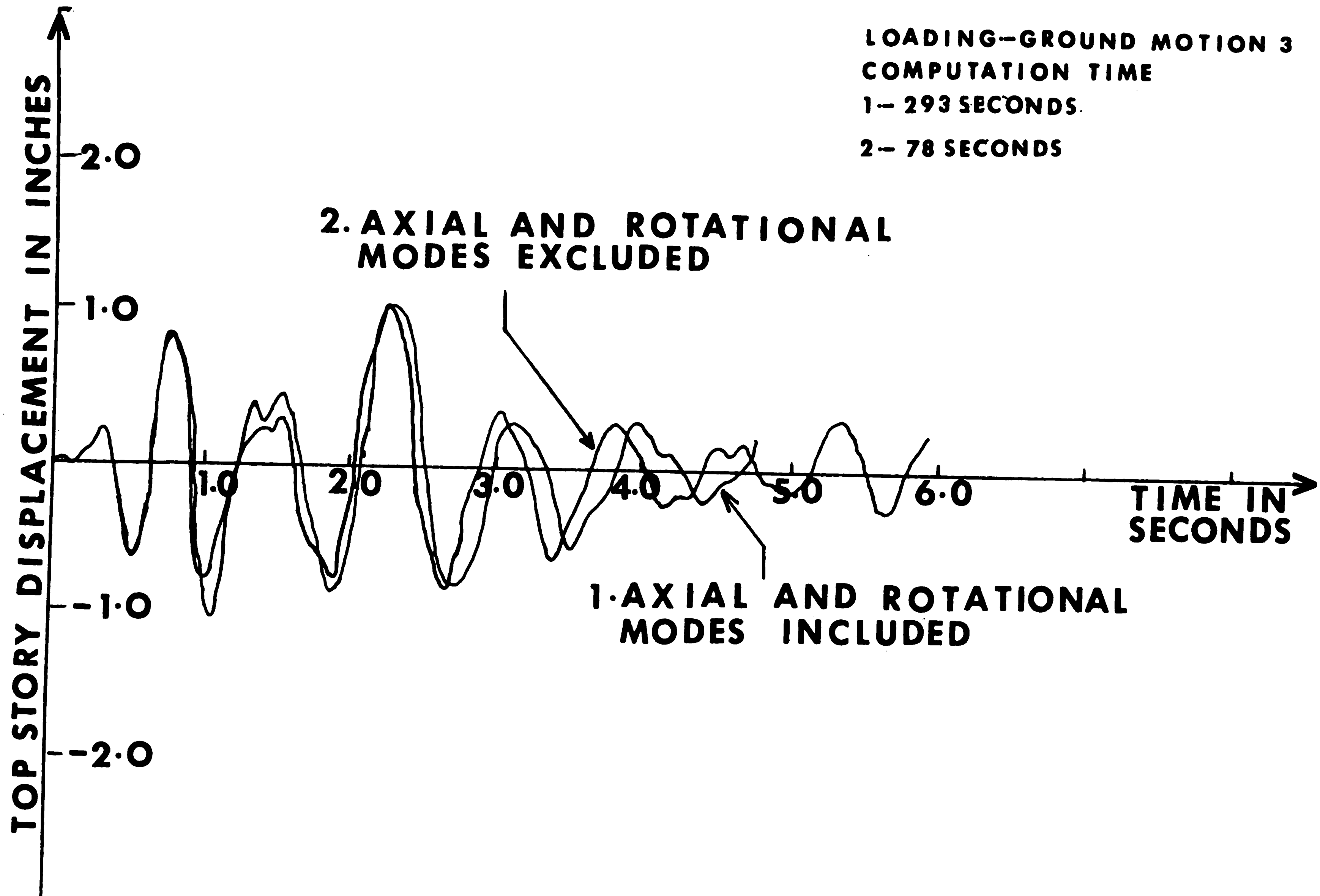



FIG. 14 EFFECT OF AXIAL AND ROTATIONAL MODES—WITH WALL-STIFFNESS

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APPENDIX I

MEMBER STIFFNESS MATRICES

## A1.1 Stiffness Matrix of a Frame Element

A = Area of cross section

L = Length of member

E = Young's Modulus

I = Moment of Inertia


$$K_f = \begin{bmatrix} \frac{AE}{L} & & & & & \\ 0 & \frac{12EI}{L^3} & & & \text{Symmetric} & \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & & & \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & & \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

Case I: No moment release at either end.

$$K_f = \begin{bmatrix} \frac{AE}{L} & & & & & \\ 0 & \frac{3EI}{L^3} & & & \text{Symmetric} & \\ 0 & 0 & 0 & & & \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & & \\ 0 & -\frac{3EI}{L^3} & 0 & 0 & \frac{3EI}{L^3} & \\ 0 & \frac{3EI}{L^2} & 0 & 0 & -\frac{3EI}{L^2} & \frac{3EI}{L} \end{bmatrix}$$

Case II: Moment release at positive end i.

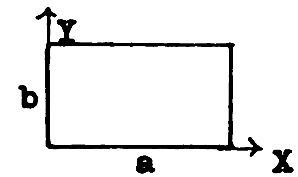
$$K_f = \begin{bmatrix} \frac{AE}{L} & & & & & \\ 0 & \frac{3EI}{L^3} & & & \text{Symmetric} & \\ 0 & \frac{3EI}{L^2} & \frac{3EI}{L} & & & \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & & \\ 0 & -\frac{3EI}{L^3} & -\frac{3EI}{L^2} & 0 & \frac{3EI}{L^3} & \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Case III: Moment release at negative end j.

$$K_f = \begin{bmatrix} \frac{AE}{L} & & & & & \\ 0 & 0 & & & & \\ 0 & 0 & 0 & & & \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Case IV: Moment releases at both ends.

A1.2 **Member Stiffness Matrix of a Rectangular Wall Element:** $K_{wr}$




$r = b/a$

$n$ = Poisson's ratio

$t$ = thickness

$E$ = Young's Modulus

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $4r+2\frac{(1-n)}{r}$ | | | | | | | |
| $3\frac{(1+n)}{2}$ | $\frac{4}{r}+2(1-n)r$ | | | | Symmetric | | |
| $2r-2\frac{(1-n)}{r}$ | $-3\frac{(1-3n)}{2}$ | $4r+2\frac{(1-n)}{r}$ | | | | | |
| $3\frac{(1-3n)}{2}$ | $+\frac{4}{r}+(1-n)r$ | $-3\frac{(1+n)}{2}$ | $\frac{4}{r}+2(1-n)r$ | | | | |
| $-2r-\frac{(1-n)}{r}$ | $-3\frac{(1+n)}{2}$ | $-4r+\frac{(1-n)}{r}$ | $-3\frac{(1-3n)}{2}$ | $4r+2\frac{(1-n)}{r}$ | | | |
| $-3\frac{(1+n)}{2}$ | $-\frac{2}{r}-(1-n)r$ | $3\frac{(1-3n)}{2}$ | $\frac{2}{r}-2(1-n)r$ | $3\frac{(1+n)}{2}$ | $\frac{4}{r}+2(1-n)r$ | | |
| $-4r+\frac{(1-n)}{r}$ | $3\frac{(1-3n)}{2}$ | $-2r-\frac{(1-n)}{r}$ | $3\frac{(1+n)}{2}$ | $2r-2\frac{(1-n)}{r}$ | $-3\frac{(1-3n)}{2}$ | $4r+2\frac{(1-n)}{r}$ | |
| $-3\frac{(1-3n)}{2}$ | $\frac{2}{r}-2(1-n)r$ | $3\frac{(1+n)}{2}$ | $-\frac{2}{r}-(1-n)r$ | $3\frac{(1-3n)}{2}$ | $-\frac{4}{r}+(1-n)r$ | $-3\frac{(1+n)}{2}$ | $\frac{4}{r}+2(1-n)r$ |

The elements shown above should be multiplied by a factor $= \frac{Et}{12(1-n^2)}$

APPENDIX II

COMPUTER PROGRAM

## A2.1 General

The analysis is accomplished by a main program called FINELEM, twelve subroutines and two function subprograms. All the basic information concerning the structure is read by the main program. The subroutine FRSTIF generates the stiffness matrices of the frame members and the subroutine WSTIFF generates the stiffness matrices of the wall elements. The subroutine JSTIFF assembles the joint stiffness matrix of the structure. As already explained in Chapter III the choice of the method of analysis is made by the number of degrees of freedom declared in the input. In case the modified method of analysis is chosen, the joint stiffness matrix assembled by JSTIFF is inverted by the subroutine INVERSE to yield the joint flexibility matrix of the unreduced system. The subroutine FLEX then forms the joint flexibility matrix of the reduced system and the subroutine MATIN computes the joint stiffness matrix of the reduced system by inversion.

The subroutine MASS reads data regarding dead and live loads on the structure and generates the mass matrix of the unreduced or reduced system as the case may be. Data pertaining to ground motion is read by the subroutine

GROUND which generates the vector $U_g$ or $U_g^*$ as the case may be, at any desired time, by linear interpolation between discrete points. The subroutine EIGEN and function TSTEP determine the best time interval to be used for the numerical integration procedure. All matrix multiplications are accomplished by the subroutine MATMULT. Function IPOS maps elements of a symmetric matrix to an one dimensional array and vice versa. The joint stiffness matrix of the unreduced system, and the member stiffness matrices of the frame and wall elements are stored as one dimensional arrays to save computer memory.

The subroutine FRAMST computes the incremental forces in the frame members due to incremental displacements during any time interval. The maximum tensile stress in the wall elements at any time is computed by the subroutine WALST.

The dynamic solution is accomplished by a step by step numerical integration procedure as already described in the section on computer program in Chapter III.

A2.2 Variables used in the Computer Program

The variable names used in the program are listed below in the alphabetical order:

Program FINELEM

AFR(J) = Area of section of frame member J;

COEF = ratio of elastic modulus of wall after and before cracking;

CRSTW = cracking stress for wall;

CTIME = computer time elapsed in seconds;

DAMP = damping constant;

DPN = incremental rotation of the J end of a frame member;

DPS = incremental rotation of the I end of a frame member;

DWL = computed value of maximum tensile stress in a wall element during any time step;

EF = elastic modulus of frame material;

EW = initial value of elastic modulus of wall material;

FACT = scaling factor for earthquake loading;

FACTOR = interpolating factor used when crack develops in a wall element;

FFR(I,J) = Jth element of internal force vector of frame member I;

FLONG = length of a frame member considered;

FRAMEK(I,J) = Jth element of the member stiffness matrix of frame member I stored as an one dimensional array;

FRMI(J) = moment of inertia of frame member J;

FWL(I) = maximum tensile stress in wall element I;

ICOUNT = counter to keep track of endless looping;

IDEXN = variable identifying transition of plastic to elastic state of J end of a frame member;

IDEXP = variable identifying transition from plastic to elastic state of I end of frame member;

IDFN(I) = variable identifying transition from plastic to elastic state of J end of frame member I;

IDFP(I) = variable identifying transition from plastic to elastic state of I end of frame member I;

IFLAG = variable identifying wall element when crack develops;

IFR(J) = joint number of I end of frame member J;

INDEX = variable controlling printing of results;

INQ = a common parameter for the main program and subroutine ground which causes earthquake data to be read on the first call only;

INTX = variable identifying a frame or wall element when structral properties change during any time interval;

IWALL(J) = joint number of I end of wall element J;

JFLAGN = variable identifying frame element when plastic hinge develops at the J end;

JFLAGP = variable identifying frame element when plastic hinge develops at the I end;

JFR(J) = joint number of J end of frame member J;

JWALL(J) = joint number of J end of wall element J;

KWALL(J) = joint number of K end of wall element J;

LWALL(J) = joint number of L end of wall element J;

MODEF(J) = variable identifying moment releases in frame member J; MODEF = 1 for no releases, 2 for release at I end, 3 for release at J end and 4 for releases at both ends;

NDEG = number of degrees of freedom;

NFLOOR = number of floors;

NFRAME = number of frame members;

NI = number of time steps after which results are printed;

NJF = number of frame joints;

NJFREE = number of free joints;

NJOINT = number of joints including supports;

NJW = number of interior wall joints;

NN = number of degrees of freedom of the unreduced system;

NSUP = number of supports;

NWALL = number of wall elements;

PLINCN(I) = incremental energy absorbed due to rotation of plastic hinge at the J end of frame member I;

PLINCP(I) = incremental energy absorbed due to rotation of plastic hinge at the I end of frame member I;

PLMOM(J) = yield moment in frame member J;

POISS = Poisson's ratio of wall material;

PROOF(J) = bending energy stored in frame member J at yield;

PWORKN(I) = energy absorbed due to rotation of plastic hinge at J end of frame member I;

PWORKP(I) = energy absorbed due to rotation of plastic hinge at the I end of frame member I;

RU = incremental joint displacement vector of the reduced system;

RUDF = joint displacement vector of the reduced system at the end of the time step;

RUDI = joint displacement vector of the reduced system at the beginning of the time step;

RUVF = joint velocity vector of the reduced system at the end of the time step;

RUVI = joint velocity vector of the reduced system at the beginning of the time step;

RUXF = joint acceleration vector of the reduced system at the end of the time step;

RUXI = joint acceleration vector of the reduced system at the beginning of the time step;

S = mass matrix;

SMJ = stiffness matrix of the unreduced system in one dimensional array; (in case the modified method is chosen, this is inverted and stored as joint flexibility matrix of the unreduced system)

STARK = stiffness matrix of the reduced system;

TINT = time interval used for the current step of numerical integration;

TIME = time at any stage of the dynamic analysis;

TLIMIT = time upto which analysis is to be carried out;

TMH = time interval for the numerical intergration;

TSTART = starting time of earthquake loading;

U = incremental joint displacement vector of the unreduced system;

UDF = joint displacement vector of the unreduced system at the end of the time interval;

UDI = joint displacement vector of the unreduced system at the beginning of the time step;

UVF = joint velocity vector of unreduced system at the end of the time interval;

UVI = joint velocity vector of the unreduced system at the beginning of the time step;

UXF = joint acceleration vector of the unreduced system at the end of the time interval;

UXI = joint acceleration vector of the unreduced system at the beginning of the time step;

UXLIM = maximum permissible horizontal displacement of joints;

UXLG = vector of accelerations $U_g$;

UZLIM = maximum permissible rotation of joints;

WALLK(I,J) = Jth element of the member stiffness matrix of wall element I, stored as

WT = thickness of wall in inches;

XJ(J) = X coordinate of joint J;

YJ(J) = Y coordinate of joint J;

ZW(J) = ratio of current value of elastic modulus to the initial value of elastic modulus of wall element J; (for wall openings this value is declared as zero).

Subroutine IPOS

IPOS(I,J) = Integer value defining the position of the I,J element of a symmetric matrix mapped on to an one dimensional array.

Subroutine FRSTIF

A = Matrix used for temporary storage of stiffness matrix during computation;

I = variable identifying the frame member whose stiffness matrix is being currently computed;

MODE = variable identifying moment releases;

R = rotation matrix;

RT = transpose of the rotation matrix.

Subroutine WSTIFF

BATA = Ratio of the height to the breadth of a rectangular wall element;

I = variable identifying the wall element whose stiffness matrix is being currently computed;

W(I,J) = Jth element of the member stiffness matrix of the wall element I, stored as one dimensional array.

Subroutine MASS

ALPHA = Fractional length of frame member over which one half of the mass of the frame member is lumped;

DENSE = density of frame material;

DNS = density of wall material;

NBAY = number of bays in the frame;

WLOAD = total dead and live load on beams in lb./in.

Subroutine EIGEN

B = Derived normalized eigen vector;

C = derived eigen vector;

EPSI = tolerence specified for the iteration; procedure;

INDEX = counter for the number of iterations;

TMH = time interval computed from the largest eigen value.

Subroutine GROUND

AH = Interpolated value of horizontal component of ground acceleration;

AV = interpolated value of vertical component of ground acceleration;

AXLH = horizontal component of ground acceleration;

AXLV = vertical component of ground acceleration;

INQ = a parameter which causes the subroutine to read data cards on first call only;

NOH = number of data cards for horizontal ground acceleration;

NOV = number of data cards for vertical ground acceleration;

QTH = time corresponding to any given value of AXLH;

QTV = time corresponding to any given value of AXLV;

TIME = time at which $U_g$ or $U_g^*$ is to be generated by linear interpolation between discrete points;

UXLG = vector $U_g$ or $U_g^*$ .

Subroutine FRAMST

X = Incremental end displacement vector.

Subroutine WALST

ALFA = Dimensional x coordinate of a point under consideration in the wall element;

BETA = dimensional y coordinate of a point under consideration in the wall element;

PMAXT = variable used for temporary storage of maximum stensile stress;

UU = vector of corner displacements in a wall element.

Subroutine MATIN

N = Dimension of S or STARF;

S = stiffness matrix of the reduced system;

STARF = flexibility matrix of the reduced system.

Subroutine INVERSE

A = Joint stiffness matrix of the unreduced system stored as one dimensional array;

N = dimension of the matrix to be inverted.

## A2.2 Computer Program

```
      PROGRAM FINELEM (INPUT,OUTPUT,PUNCH)
      COMMON IFR(40),JFR(40),IWALL(40),JWALL(40),KWALL(40),
     1FRAMEK(40,21),WALLK(40,36),NWALL,NFRAME,NJF,NJW,NSUP,
     2LWALL(40),ZW(40)
      COMMON/1/SMJ(7320),F(150),U(150),X(6),Y(6),
     1GG(3,8),HH(3,8)
      COMMON/2/STARF(20,20),STARK(20,20),S(120),NDEG
      COMMON/3/ XJ(50),YJ(50),AFR(40),FRMI(40),R(6,6),RT(6,6),
     1B(6,6),BB(3,6),BT(6,3),BC(6,3),DD(3,3),EF,EW,WT,POISS,
     2A(6,6),NMAX,PMAXT
      COMMON/4/DFR(40,4),DWL(40),FFR(40,4),FWL(40)
      COMMON/GRND/UXLG(120),AH,AV,INQ,FACT
      DIMENSION UDI(120),UVI(120),UXI(120),UDF(120),UVF(120),
     1UXF(120),PLMOM(40),MODEF(40),PWORKP(40),PWORKN(40),
     2PLINCP(40),PLINCN(40),IDFP(40),IDFN(40),PROOF(40)
     3,RU(20), RUDI(20),RUVI(20),RUXI(20),RUDF(20),RUVF(20),
     4RUXF(20)
C
C     READ NO. OF FRAME AND WALL JOINTS AND SUPPORTS
   90 FORMAT(16F5.0)
C
  100 READ 110,NJF,NJW,NSUP,NDEG,NI,NFLOOR
  110 FORMAT(16I5)
      PRINT 111,NJF,NJW,NSUP
  111 FORMAT(1H1,* NO. OF FRAME JOINTS =*,I3,*  NO. OF INTER*
     1*IOR WALL JOINTS =*,I3,*   NO. OF SUPPORTS =*,I3/)
      PRINT 112,NDEG,NFLOOR
  112 FORMAT(*  NO. OF DEGREES OF FREEDOM =*,I3,*   NO. OF*
     1* FLOORS =*I3/)
      NPUNCH = NFLOOR * 3 - 2
      NJOINT=NJF+NJW+NSUP
      NJFREE = NJF + NJW
      NN = 3*NJF + 2*NJW
C
C     READ JOINT COORDINATES
C
      DO 120 I=1,NJOINT
  120 READ 130,J,XJ(J),YJ(J)
  130 FORMAT(I5,2F10.0)
C
C     PRINT JOINT NO. AND COORDINATES
C
      PRINT 140
  140 FORMAT( // *   JOINT NO.         X COORD.    Y COORD. *
     1//)
      DO 150 I=1,NJOINT
  150 PRINT 160,I,XJ(I),YJ(I)
```

```
  160 FORMAT(7X,I3,7X,2F12.2/)
C
C     READ NO. OF FRAME AND WALL ELEMENTS
C
      READ 110,NFRAME,NWALL
C
C     READ FRAME ELEMENT NO.,INCIDENCE, ETC.
C
      DO 165 I=1,NFRAME
  165 READ 170,J,IFR(J),JFR(J),MODEF(J),AFR(J),FRMI(J),PLMOM(J)
  170 FORMAT(4I5,3F10.0)
C
C     READ WALL ELEMENT NO., AND INCIDENCE
C
      DO 180 I=1,NWALL
  180 READ 110,J,IWALL(J),JWALL(J),KWALL(J),LWALL(J)
      READ 90, (ZW(I),I=1,NWALL)
C
C     READ E OF FRAME AND WALL, THICKNESS OF WALL ETC.
C
      READ 190,EF,EW,WT,POISS,CRSTW,TLIMIT,TMH,COEF
      READ 190,FACT,DAMP,TSTART
      READ 190,UXLIM,UZLIM
  190 FORMAT(8F10.0)
C
C     PRINT FRAME AND WALL MEMBER INCIDENCE
C
      PRINT 191
  191 FORMAT(/*  FRAME ELEMENT          I NODE         J NODE    *
     1* AREA              M.I       MAX.MOM.   */)
      DO 192 I=1,NFRAME
  192 PRINT 193,I,IFR(I),JFR(I),AFR(I),FRMI(I),PLMOM(I)
  193 FORMAT(3(10X,I3),3F12.2/)
      PRINT 194
  194 FORMAT(/*  WALL ELEMENT        I NODE        J NODE        *
     1*K NODE          L NODE*//)
      DO 195 I=1,NWALL
  195 PRINT 196,I,IWALL(I),JWALL(I),KWALL(I),LWALL(I)
  196 FORMAT(5(9X,I3)/)
      PRINT 197
  197 FORMAT(//*  E OF FRAME     E OF WALL     WALL THICKNESS *)
      PRINT 198,EF,EW,WT
  198 FORMAT(/3(E12.5,2X))
      PRINT 199,TMH,TLIMIT,POISS
  199 FORMAT(/* TIME INTERVAL =*,F8.5,*   TIME LIMIT =*,F8.5,
     1*    POISSONS RATIO =*,F8.5/)
      PRINT 200,FACT,DAMP
  200 FORMAT(/* AMPLIFICATION FACTOR =*,F8.5,*   DAMPING *
     1*COEFFICIENT =*,F8.5/)
      PRINT 201
```

```
  201 FORMAT(/* LIMITING VALUES OF JOINT DISPLACEMENTS FOR *
     1*COLLAPSE*/)
      PRINT 202,UXLIM,UZLIM
  202 FORMAT(* MAX.HORL. JOINT DISPLACEMENT IN INCHES =*,F8.4,
     1 *  MAX. JOINT ROTATION IN RADIANS =*,F8.4/)
      PRINT 206,CRSTW
  206 FORMAT(/* CRACKING STRESS FOR WALL IN PSI*,F8.2/)
C     GENERATE MASS MATRIX
      CALL MASS(0.25)
C
C     GENERATE STIFFNESS MATRIX OF FRAME AND WALL ELEMENTS
C
      DO 240 I=1,NFRAME
      CALL FRSTIF(I,MODEF(I))
      PROOF(I) = PLMOM(I)**2/FRAMEK(I,21) / 2.0
  240 CONTINUE
      DO 250 I=1,NWALL
  250 CALL WSTIFF(I)
      DO 300 I=1,3
      DO 300 J=1,3
  300 DD(I,J) = 0.0
      DD(1,1) = 1.0
      DD(2,2) = 1.0
      DD(1,2) = POISS
      DD(2,1) = POISS
      DD(3,3) = 0.5*(1.0 - POISS)
      DO 320 I=1,3
      DO 320 J=1,3
  320 DD(I,J) = DD(I,J)* EW/(1.0 - POISS)
C
C     GENERATE JOINT STIFFNESS MATRIX
C
      CALL JSTIFF
C
C     COMPUTE BEST TIME INTERVAL TO BE USED
C
      CALL EIGEN(NN,TMH)
C     INTIALIZE VARIABLES
      ICOUNT = 0
      IDEXP = 0 $  IDEXN = 0
      TIME = TSTART
      INQ=0
      INDEX=1
      DO 500 I=1,120
      F(I) = 0.0
      U(I)=0.0
      UVI(I)=0.0
  500 UDI(I)=0.0
      DO 501 I=1,20
      RU(I) = 0.  $  RUDI(I) = 0.  $ RUVI(I) = 0.
```

```
      RUXI(I) = 0.0
      RUDF(I)=0.0 $ RUVF(I)=0.0 $ RUXF(I)=0.0
  501 CONTINUE
      DO 510 I=1,40
      IDFP(I) = 0 $ IDFN(I) = 0
      FWL(I)=0.0
      PWORKP(I)=0.0 $ PWORKN(I)=0.0
      DO 510 J=1,4
  510 FFR(I,J)=0.0
      DO 511 I=1,NFRAME
  511 READ 190,(FFR(I,J),J=1,4)
C
C     SET INTIAL ACCELERATION
C
      CALL GROUND(TIME)
      IF(NN.GT.NDEG) GO TO 521
      DO 520 I=1,NN
  520 UXI(I)=-UXLG(I)
      GO TO 523
  521 DO 522 I=1,NFLOOR
  522 RUXI(I) = -UXLG(I)
  523 CONTINUE
  550 TINT = TMH
  560 TIME=TIME+TINT
  570 CONTINUE
      FACTOR = 1.0
      IFLAG=0
      JFLAGP=0
      JFLAGN=0
  575 CONTINUE
C
C     COMPUTE INCREMENTAL DISPLACEMENTS
C
      IF(NN.GT.NDEG) GO TO 581
      DO 580 I = 1,NN
      U(I)=TINT*UVI(I)+0.5*TINT*TINT*UXI(I)
  580 F(I) = UDI(I) + U(I)
      GO TO 590
  581 DO 582 I=1,NFLOOR
  582 RU(I) = TINT*RUVI(I) + 0.5*TINT*TINT*RUXI(I)
      DO 585 I=1,NFLOOR
      SUM = 0.
      DO 584 J=1,NFLOOR
  584 SUM = SUM + STARK(I,J) * RU(J)
      K = 3*I - 2
  585 UVI(K) = SUM
      DO 587 I=1,NN
      SUM = 0.
      DO 586 J=1,NN
  586 SUM = SUM + SMJ(IPOS(I,J)) * UVI(J)
```

```
      U(I) = SUM
  587 F(I) = UDI(I) + U(I)
  590 CONTINUE
C
C     COMPUTE FORCES IN FRAME AND WALL ELEMENTS
C     CHECK FOR CRACKING OF WALL ELEMENTS, FORMATION OF
C     PLASTIC HINGES IN FRAME ELEMENTS
C     ADJUST TIME INTERVAL IF NECESSARY
C
      CALL WALST
      CALL FRAMST
      SUM=0.0
      DO 600 I=1,NWALL
      IF(DWL(I).LE.0.0) GO TO 600
      QNT = DWL(I)
      IF(SUM.GT.QNT) GO TO 600
      SUM=QNT
      INTX=I
  600 CONTINUE
      IF(SUM.LT.CRSTW) GO TO 650
      IFLAG=INTX
      INDEX = NI
      TIME=TIME-TINT
      FACTOR = CRSTW/SUM
      TINT = TINT*(F(1)*FACTOR - UDI(1))/U(1)
      IF(TINT.GT.TMH) GO TO 610
      IF(TINT.LT.0.0) GO TO 620
      GO TO 630
  610 TINT=TMH
      GO TO 630
  620 TINT=0.0
      FACTOR=1.0
  630 TIME=TIME+TINT
  650 CONTINUE
      TTIN = TMH
      DO 700 I=1,NFRAME
      IF(MODEF(I).EQ.2.OR.MODEF(I).EQ.4) GO TO 700
      QNT=ABS(FFR(I,3)+DFR(I,3))
      IF(QNT.LE.PLMOM(I)) GO TO 700
      STIME=TMH*(PLMOM(I) - ABS(FFR(I,3)))/(QNT-ABS(FFR(I,3)))
      IF(STIME.GT.TTIN) GO TO 700
      TTIN = STIME
      INTX=I
  700 CONTINUE
      IF(TTIN.GE.TINT) GO TO 750
      IFLAG=0
      JFLAGP=INTX
      INDEX = NI
      TIME=TIME-TINT+TTIN
      TINT=TTIN
```

```
  750 CONTINUE
      TTIN=TMH
      DO 800 I=1,NFRAME
      IF(MODEF(I).EQ.3.OR.MODEF(I).EQ.4) GO TO 800
      QNT=ABS(FFR(I,4)+DFR(I,4))
      IF(QNT.LE.PLMOM(I)) GO TO 800
      STIME=TMH*(PLMOM(I) - ABS(FFR(I,4)))/(QNT-ABS(FFR(I,4)))
      IF(STIME.GT.TTIN) GO TO 800
      TTIN=STIME
      INTX=I
  800 CONTINUE
      IF(TTIN.GE.TINT) GO TO 850
      IFLAG=0
      JFLAGP=0
      JFLAGN=INTX
      INDEX = NI
      TIME=TIME-TINT+TTIN
      TINT=TTIN
  850 CONTINUE
      IF(IDEXP.NE.0.AND.IDEXP.EQ.JFLAGP) GO TO 860
      IF(IDEXN.NE.0.AND.IDEXN.EQ.JFLAGN) GO TO 870
      GO TO 890
  860 PRINT 1035,IDEXP
      IDFP(IDEXP) = 1
      IF(MODEF(IDEXP).EQ.1) MODEF(IDEXP) = 2
      IF(MODEF(IDEXP).EQ.3) MODEF(IDEXP) = 4
      CALL FRSTIF(IDEXP,MODEF(IDEXP))
      GO TO 880
  870 PRINT 1065,IDEXN
      IDFN(IDEXN) = 1
      IF(MODEF(IDEXN).EQ.1) MODEF(IDEXN) = 3
      IF(MODEF(IDEXN).EQ.2) MODEF(IDEXN) = 4
      CALL FRSTIF(IDEXN,MODEF(IDEXN))
  880 CALL JSTIFF
      TINT = TMH
      IDEXP = 0 $ IDEXN = 0 $ JFLAGP = 0 $ JFLAGN = 0
      TIME = TIME + TINT
      PRINT 885
  885 FORMAT(/* ASSUME PLASTIC HINGE TO AVOID ENDLESS LOOPING*/)
      GO TO 575
  890 CONTINUE
C
C     CHECK WHETHER ENERGY ABSORBED DUE TO ROTATION OF PLASTIC
C     HINGE IS NEGATIVE. RESTORE ELASTIC STATE IF WARRENTED.
C
      DO 1000 I=1,NFRAME
      PLINCP(I)=0.0 $ PLINCN(I)=0.0
      IF(IDFP(I).NE.0.OR.IDFN(I).NE.0) GO TO 1000
      IF(MODEF(I).EQ.1) GO TO 1000
      IP = IFP(I)
```

```
      IN = JFR(I)
      FLONG = SQRT((XJ(IN)-XJ(IP))**2+(YJ(IN)-YJ(IP))**2)
      CP=XJ(IFR(I))-XJ(JFR(I))
      ICP=3*IFR(I)-2
      ICN=3*JFR(I)-2
      IF(CP.EQ.0.0) GO TO 900
      DPS=(U(ICP+1) - U(ICN+1))/FLONG + U(ICP+2)
      DPN=(U(ICP+1) - U(ICN+1))/FLONG + U(ICN+2)
      GO TO 910
  900 DPS=-(U(ICP)-U(ICN))/FLONG + U(ICP+2)
      DPN=-(U(ICP)-U(ICN))/FLONG + U(ICN+2)
  910 IF(MODEF(I).NE.2) GO TO 930
      QN=DPS*FFR(I,3)
      IF(QN.GE.0.0) PLINCP(I)=QN
      IF(QN.GE.-0.10) GO TO 1000
      IDEXP = I
      PRINT 915,I
  915 FORMAT(/* PLASTIC HINGE DISAPPEARS AT + END OF FRAME *
     1*MEMBER *,I4)
      INDEX = NI
  920 MODEF(I)=1
  925 CALL FRSTIF(I,MODEF(I))
      TIME=TIME-TINT
      CALL JSTIFF
      GO TO 550
  930 IF(MODEF(I).NE.3) GO TO 950
      QN=DPN*FFR(I,4)
      IF(QN.GE.0.0) PLINCN(I)=QN
      IF(QN.GE.-0.10) GO TO 1000
      IDEXN = I
      PRINT 935,I
  935 FORMAT(/* PLASTIC HINGE DISAPPEARS AT - END OF FRAME *
     1*MEMBER *,I4)
      INDEX = NI
      GO TO 920
  950 QN=DPS*FFR(I,3)
      IF(QN.GE.0.0) PLINCP(I)=QN
      IF(QN.GE.-0.10) GO TO 1000
      IDEXP = I
      PRINT 915,I
      INDEX = NI
      MODEF(I)=3
      GO TO 925
  960 QN=DPN*FFR(I,4)
      IF(QN.GE.0.0) PLINCN(I)=QN
      IF(QN.GE.-0.10) GO TO 1000
      IDEXN = I
      PRINT 935,I
      INDEX = NI
      MODEF(I)=2
```

```
      GO TO 925
 1000 CONTINUE
 1010 IF(IFLAG.EQ.0) GO TO 1030
      PRINT 1015,IFLAG
 1015 FORMAT(/*  CRACK FORMS IN WALL ELEMENT NO. *,I5/)
      ZW(IFLAG) = COEF*ZW(IFLAG)
      DWL(IFLAG) = COEF * DWL(IFLAG)
      DO 1020 I=1,36
 1020 WALLK(IFLAG,I) = COFF * WALLK(IFLAG,I)
 1030 IF(JFLAGP.EQ.0) GO TO 1060
      PRINT 1035,JFLAGP
 1035 FORMAT(/* PLASTIC HINGE FORMS AT + END OF FRAME MEMBER*
     1I4)
      IF(MODEF(JFLAGP).NE.1) GO TO 1040
      MODEF(JFLAGP)=2
      GO TO 1050
 1040 MODEF(JFLAGP)=4
 1050 CALL FRSTIF(JFLAGP,MODEF(JFLAGP))
      GO TO 1100
 1060 IF(JFLAGN.EQ.0) GO TO 1100
      PRINT 1065,JFLAGN
 1065 FORMAT(/* PLASTIC HINGE FORMS AT - END OF FRAME MEMBER*
     1I4)
      IF(MODEF(JFLAGN).NE.1) GO TO 1080
      MODEF(JFLAGN) = 3
      GO TO 1090
 1080 MODEF(JFLAGN)=4
 1090 CALL FRSTIF(JFLAGN,MODEF(JFLAGN))
 1100 CONTINUE
      IDEXP = 0 $ IDEXN = 0
      DO 1115 I=1,NFRAME
      IDFP(I) = 0
 1115 IDFN(I) = 0
      QNTY=(F(1)-U(1)+TINT/TMH*U(1))/F(1)
      IF(IFLAG.EQ.0) FACTOR=QNTY
      DO 1150 I=1,NWALL
      DWL(I) = DWL(I)*FACTOR
 1150 FWL(I) = DWL(I)
      DO 1160 I=1,NFRAME
      DO 1160 J=1,4
      DFR(I,J) = DFR(I,J) * TINT/TMH
 1160 FFR(I,J)=FFR(I,J)+DFR(I,J)
C
C     BEGIN NUMERICAL INTEGRATION
C
      DO 1170 I=1,NN
 1170 UDF(I)=UDI(I)+U(I)* TINT/TMH
      IF(NN.EQ.NDEG) GO TO 1173
 1171 DO 1172 I=1,NFLOOR
 1172 RUDF(I) = RUDI(I) + RU(I)*TINT/TMH
```

```
 1173 CONTINUE
      CALL GROUND(TIME)
      IF(NN.GT.NDEG) GO TO 1181
      DO 1190 I=1,NN
      SUM=0.0
      DO 1180 J=1,NN
 1180 SUM=SUM+SMJ(IPOS(I,J))*UDF(J)
      UXF(I)= -SUM/S(I) - UXLG(I) - UVI(I)*DAMP
 1190 UVF(I)=UVI(I)+0.5*TINT*(UXI(I)+UXF(I))
      GO TO 1195
 1181 DO 1191 I=1,NFLOOR
      SUM = 0.
      DO 1182 J=1,NFLOOR
 1182 SUM = SUM + STARK(I,J) * RUDF(J)
      RUXF(I) = -SUM/S(I) - UXLG(I) -RUVI(I)*DAMP
 1191 RUVF(I) = RUVI(I) + 0.5*TINT*(RUXI(I)+RUXF(I))
 1195 CONTINUE
C
C     NUMERICAL INTEGRATION COMPLETED FOR ONE STEP
C
 1300 FORMAT(1H1,*  TIME = *,F10.5,*  COMPUTER TIME ELAPSED =*
     1F10.5/)
      DO 1400 I=1,NN
      UDI(I)=UDF(I)
      IF(NN.GT.NDEG) GO TO 1400
      UVI(I)=UVF(I)
      UXI(I)=UXF(I)
 1400 CONTINUE
      IF(NN.EQ.NDEG) GO TO 1405
      DO 1401 I=1,NFLOOR
      RUDI(I) = RUDF(I)
      RUVI(I) = RUVF(I)
 1401 RUXI(I) = RUXF(I)
 1405 CONTINUE
      DO 1410 I=1,NFRAME
      PWORKP(I) = PWORKP(I) + PLINCP(I) * TINT/TMH
 1410 PWORKN(I) = PWORKN(I) + PLINCN(I) * TINT/TMH
      IF(INDEX.NE.NI) GO TO 2100
      CTIME = SECOND(0)
      INDEX = 0
      PUNCH 1449,TIME,UDF(NPUNCH)
 1449 FORMAT(2F15.5)
 1450 PRINT 1300,TIME,CTIME
C     LOOK FOR ENDLESS LOOPING AND EXIT  IF NECESSARY
      IF(TINT.GT.0.0002) ICOUNT = 0
      IF(TINT.LE.0.00001) ICOUNT = ICOUNT  + 1
      IF(ICOUNT.EQ.5) GO TO 2200
      PRINT 1950
 1950 FORMAT(/// * JOINT NO.   DISPL. X         DISPL. Y      *
     1*ROTATION Z */)
```

```
      DO 1960 I=1,NJF
      K1 = 3*I - 2
      K2 = K1 + 1
      K3 = K1 + 2
 1960 PRINT 1970,I,UDF(K1),UDF(K2),UDF(K3)
 1970 FORMAT(/2X,I3,2X,3F15.5)
      PRINT 1975
 1975 FORMAT(/* WALL ELEM.  MAX.TENSILE STRESS      *
     1*COEFFICIENT */)
      DO 1990 I=1,NWALL
      PRINT 1985,I,FWL(I),ZW(I)
 1985 FORMAT(4X,I3,6X,2F15.5/)
 1990 CONTINUE
      PRINT 1995
 1995 FORMAT(//* END FORCES IN FRAME ELEMENTS */)
      PRINT 2000
 2000 FORMAT(*   NO.   AXIAL FORCE       SHEAR          MOM. AT *
     1*+END   MOM. AT -END   MODE */)
      DO 2010 I=1,NFRAME
      FFR1 = -FFR(I,1) $ FFR2 = -FFR(I,2)
      CP = XJ(IFR(I)) - XJ(JFR(I))
      IF(CP.EQ.0.0) GO TO 2004
      PRINT 2005,I,FFR1,FFR(I,2),FFR(I,3),FFR(I,4),MODEF(I)
      GO TO 2010
 2004 PRINT 2005,I,FFR2,FFR1,FFR(I,3),FFR(I,4),MODEF(I)
 2005 FORMAT(2X,I3,4E15.5,I6/)
 2010 CONTINUE
      DO 2011 I=1,NJF
      J = 3*I - 2
      K = 3 * I
      IF(ABS(UDF(J)).GT.UXLIM) GO TO 2013
      IF(ABS(UDF(K)).GT.UZLIM) GO TO 2015
 2011 CONTINUE
      GO TO 2018
 2013 PRINT 2014,I
 2014 FORMAT(/* COLLAPSE DUE TO EXCESSIVE HORL. DISPLACEMENT OF
     1 JOINT NO. *,I4/)
      GO TO 2200
 2015 PRINT 2016,I
 2016 FORMAT(/* COLLAPSE DUE TO EXCESSIVE ROTATION OF JOINT
     1NO.*,I4/)
      GO TO 2200
 2018 CONTINUE
      IF(CTIME.GT.290.0) GO TO 2200
      IF(TIME.GT.TLIMIT) GO TO 2200
 2100 CONTINUE
      CTIME = SECOND(O)
      IF(CTIME.GT.290.0) GO TO 1450
      IF(TIME.GT.TLIMIT) GO TO 1450
      IF(IFLAG.NE.0) GO TO 2110
```

```
      IF(JFLAGP.NE.0) GO TO 2110
      IF(JFLAGN.NE.0) GO TO 2110
      GO TO 2120
 2110 CALL JSTIFF
      IF(NJF.EQ.0) GO TO 2200
      IF(IFLAG.NE.0) GO TO 2112
      GO TO 2120
C
C     RECOMPUTE TIME INTERVAL IF A WALL ELEMENT CRACKS
C
 2112 CALL EIGEN(NN,TMH)
 2120 CONTINUE
      INDEX=INDEX+1
      GO TO 550
 2200 CONTINUE
      IF(NN.GT.NDEG) GO TO 2236
      PRINT 2210
 2210 FORMAT(1H1,*  FINAL STATUS OF JOINT VELOCITY ,ACCELERA*
     1*TION ETC. */)
      PRINT 2220
 2220 FORMAT(/// * JOINT NO.    VEL. X             VEL. Y      *
     1*ANG. VEL. Z*/)
      DO 2225 I=1,NJF
      K1 = 3*I - 2
      K2 = K1 + 1
      K3 = K1 + 2
 2225 PRINT 1970,I,UVF(K1),UVF(K2),UVF(K3)
      PRINT 2230
 2230 FORMAT(/// * JOINT NO.    ACLN. X            ACLN. Y     *
     1*ACLN. Z*/)
      DO 2235 I=1,NJF
      K1 = 3*I - 2
      K2 = K1 + 1
      K3 = K1 + 2
 2235 PRINT 1970,I,UXF(K1),UXF(K2),UXF(K3)
 2236 CONTINUE
      PRINT 2240
 2240 FORMAT(/* ENERGY ABSORBTION DUE TO ROTATION OF PLASTIC*
     1* HINGES IN INCH KIPS* )
      PRINT 2241
 2241 FORMAT(/* FRAME MEMBER  POSITIVE END  PERCENTAGE OF  *
     1* NEGATIVE END  PERCENTAGE OF*)
      PRINT 2242
 2242 FORMAT(29X,*PROOF RESILIENCE*14X,*PROOF RESILIENCE*)
      DO 2245 I=1,NFRAME
      PRCP =    PWORKP(I) /PROOF(I) * 100.
      PRCN =    PWORKN(I) /PROOF(I) * 100.
      PWORKP(I) = PWORKP(I) / 1000.
      PWORKN(I) = PWORKN(I) / 1000.
 2245 PRINT 2250,I,PWORKP(I),PRCP,PWORKN(I),PRCN
```

```
 2250 FORMAT(/4X,I3,3X,F10.3,6X,F10.3,5X,F10.3,5X,F10.3)
      IF(NN.EQ.NDEG) GO TO 2500
      PRINT 2320
 2320 FORMAT(// * JOINT NO.    VEL. X */)
      DO 2325 I=1,NFLOOR
 2325 PRINT 1970,I,RUVF(I)
      PRINT 2330
 2330 FORMAT(/// * JOINT NO.    ACLN. X */)
      DO 2335 I=1,NFLOOR
 2335 PRINT 1970,I,RUXF(I)
 2500 CONTINUE
      END
```

```
      FUNCTION IPOS(J,K)
C     THIS MAPS ELEMENTS OF A SYMMETRIC MATRIX ON TO AN ONE
C     DIMENSIONAL ARRAY
      IF(J-K) 10,10,11
   10 IPOS=(K*(K-1))/2 +J
      RETURN
   11 IPOS = (J*(J-1))/2+K
      RETURN
      END
```

```
      FUNCTION TSTEP(X)
C     CALCULATE TIME INTERVAL FROM EIGENVALUES
      X= 1.0/SQRT(X) * 6.28318/6.0
C     ROUND OFF TO TWO SIGNIFICANT DIGITS
      Z=1000.0
      DO 5 I=1,20 $ Y=10.0/Z $ IF(X.GE.Z) GO TO 10 $ Z=Z*0.10
    5 CONTINUE
   10 X=Y*X $ I=X $ X=I $ X=X/Y $ TSTEP=X
      RETURN
      END
```

```
      SUBROUTINE FRSTIF(I,MODE)
C     THIS COMPUTES THE STIFFNESS MATRIX OF THE FRAME ELEMENTS
      COMMON IFR(40),JFR(40),IWALL(40),JWALL(40),KWALL(40),
     1FRAMEK(40,21),WALLK(40,36),NWALL,NFRAME,NJF,NJW,NSUP,
     2LWALL(40),ZW(40)
      COMMON/3/ XJ(50),YJ(50),AFR(40),FRMI(40),R(6,6),RT(6,6),
     1B(6,6),BB(3,6),BT(6,3),BC(6,3),DD(3,3),EF,EW,WT,POISS,
     2A(6,6),NMAX
C
      DO 200 II=1,6
```

```
      DO 200 JJ=1,6
      R(II,JJ)=0.
      A(II,JJ)=0. $ B(II,JJ)=0. $ FRAMFK(I,IPOS(II,JJ))=0.
  200 RT(II,JJ)=0.
      IP = IFR(I)
      IN = JFR(I)
      FLONG = SQRT((XJ(IN)-XJ(IP))**2+(YJ(IN)-YJ(IP))**2)
      R(1,1) = (XJ(IN) - XJ(IP))/ FLONG
      R(2,2)=R(1,1)
      R(4,4)=R(1,1)
      R(5,5)=R(1,1)
      R(1,2)=(YJ(IN)-YJ(IP))/ FLONG
      R(2,1)= -R(1,2)
      R(4,5)=R(1,2)
      R(5,4)= -R(4,5)
      R(3,3)=1.0
      R(6,6)=1.0
      DO 210 II=1,6
      DO 210 JJ=1,6
  210 RT(JJ,II)=R(II,JJ)
      GO TO (211,212,213,214) MODE
  211 A(1,1)=AFR(I)*EF/FLONG
      A(1,4)= -A(1,1)
      A(4,4)=A(1,1)
      A(2,2)=(12.0*EF*FRMI(I))/(FLONG**3)
      A(5,5)=A(2,2)
      A(2,5)= -A(2,2)
      A(2,3)=(6.0*EF*FRMI(I))/(FLONG**2)
      A(2,6) = A(2,3)
      A(3,5) = -A(2,3)
      A(5,6)= -A(2,3)
      A(3,3)=(4.0*EF*FRMI(I))/FLONG
      A(6,6)=A(3,3)
      A(3,6)=A(3,3)*0.5
      GO TO 215
  212 A(1,1) = AFR(I)*EF/FLONG
      A(1,4) = -A(1,1)
      A(4,4) = A(1,1)
      A(2,2) = 3.0*EF*FRMI(I)/(FLONG**3)
      A(2,5) = -A(2,2)
      A(5,5) = A(2,2)
      A(2,6) = 3.0*EF*FRMI(I)/(FLONG**2)
      A(5,6) = -A(2,6)
      A(6,6) = 3.0*EF*FRMI(I)/FLONG
      GO TO 215
  213 A(1,1) = AFR(I)*EF/FLONG
      A(1,4) = -A(1,1)
      A(4,4) = A(1,1)
      A(2,2) = 3.0*EF*FRMI(I)/(FLONG**3)
      A(2,5) = -A(2,2)
```

```
      A(5,5) = A(2,2)
      A(2,3) = 3.0*EF*FRMI(I)/(FLONG**2)
      A(3,5) = -A(2,3)
      A(3,3) = 3.0*EF*FRMI(I)/FLONG
      GO TO 215
  214 A(1,1) = AFR(I)*EF/FLONG
      A(1,4) = -A(1,1)
      A(4,4) = A(1,1)
  215 DO 220 II=1,6
      DO 220 JJ=1,6
  220 A(JJ,II)=A(II,JJ)
      CALL MATMULT(RT,A,B,6,6,6)
      CALL MATMULT(B,R,A,6,6,6)
      DO 230 II=1,6
      DO 230 JJ=1,II
  230 FRAMEK(I,IPOS(II,JJ))= A(II,JJ)
      RETURN
      END



      SUBROUTINE WSTIFF(I)
C     THIS COMPUTES THE STIFFNESS MATRIX OF RECTANGULAR
C     WALL ELEMENTS.
      COMMON IFR(40),JFR(40),IWALL(40),JWALL(40),KWALL(40),
     1FRAMEK(40,21),W     (40,36),NWALL,NFRAME,NJF,NJW,NSUP,
     2LWALL(40),ZW(40)
      COMMON/3/ XJ(50),YJ(50),AFR(40),FRMI(40),R(6,6),RT(6,6),
     1B(6,6),BB(3,6),BT(6,3),BC(6,3),DD(3,3),EF,EW,WT,POISS,
     2A(6,6),NMAX
      BETA=(YJ(JWALL(I))-YJ(IWALL(I)))/(XJ(LWALL(I))-
     1XJ(IWALL(I)))
      BETIN = 1.0/BETA
      GAMA = (1.0 - POISS) * BETA
      GAMIN = (1.0 - POISS) * BETIN
   50 W(I,1) = 4.0*BETA + 2.0*GAMIN
      W(I,2) = 1.5 * (1.0+POISS)
      W(I,3) = 4.0*BETIN + 2.0*GAMA
      W(I,4) = 2.0*BETA- 2.0*GAMIN
      W(I,5) = -1.5*(1.0-3.0*POISS)
      W(I,6) = W(I,1)
      W(I,7) = -W(I,5)
      W(I,8) = -4.0*BETIN + GAMA
      W(I,9) =-W(I,2)
      W(I,10) = W(I,3)
      W(I,11) = -2.0*BETA - GAMIN
      W(I,12) =-W(I,2)
      W(I,13) = -4.0*BETA + GAMIN
      W(I,14) = W(I,5)
      W(I,15) = W(I,1)
```

```
      W(I,16) = -W(I,2)
      W(I,17) = -2.0*BETIN - GAMA
      W(I,18) = -W(I,5)
      W(I,19) =  2.0*BETIN - 2.0*GAMA
      W(I,20) = W(I,2)
      W(I,21) = W(I,3)
      W(I,22) = -4.0*BETA + GAMIN
      W(I,23) = -W(I,5)
      W(I,24) = -2.0*BETA - GAMIN
      W(I,25) = W(I,2)
      W(I,26) = 2.0*BETA - 2.0*GAMIN
      W(I,27) = W(I,5)
      W(I,28) = W(I,1)
      W(I,29) = W(I,5)
      W(I,30) = 2.0*BETIN - 2.0*GAMA
      W(I,31) = W(I,2)
      W(I,32)=-2.0*BETIN - GAMA
      W(I,33) = -W(I,5)
      W(I,34) = -4.0*BETIN + GAMA
      W(I,35) = -W(I,2)
      W(I,36) = W(I,3)
      DO 60 J=1,36
   60 W(I,J) = W(I,J) * EW*WT/(12.0*(1.0-POISS**2)) * ZW(I)
      RETURN
      END


      SUBROUTINE JSTIFF
C     THIS COMPUTES THE JOINT STIFFNESS MATRIX
      COMMON IFR(40),JFR(40),IWALL(40),JWALL(40),KWALL(40),
     1FRAMEK(40,21),WALLK(40,36),NWALL,NFRAME,NJF,NJW,NSUP,
     2LWALL(40),ZW(40)
      COMMON/1/SMJ(7320),F(150),U(150),X(6),Y(6)
      COMMON/2/STARE(20,20),STARK(20,20),S(120),NDEG
      COMMON/3/ XJ(50),YJ(50),AFR(40),FRMI(40),R(6,6),RT(6,6),
     1B(6,6),BB(3,6),BT(6,3),BC(6,3),DD(3,3),EF,EW,WT,POISS,
     2A(6,6),NMAX
      NN=3*NJF + 2*NJW
      NJFREE=NJF+NJW
      DO 350 I=1,NN
      DO 350 J=1,I
  350 SMJ(IPOS(I,J))=0.0
      DO 810 I=1,NJF
      DO 810 J=1,NJF
      I1 = 3*I - 2
      I3 = I1 + 2
      J1 = 3*J - 2
      J3 = J1 + 2
      IF(I.GT.J) GO TO 810
```

```
660 DO 730 II=1,NFRAME
    KK = 0
    DO 730 K=I1,I3
    KK = KK+1
    LL=0
    DO 730 L=J1,J3
    LL = LL+1
    IF(L.LT.K) GO TO 730
    IF(I.EQ.IFR(II)) GO TO 670
    IF(I.NE.JFR(II)) GO TO 730
    KK1 = KK + 3
    GO TO 680
670 KK1 = KK
680 IF(J.EQ.IFR(II)) GO TO 690
    IF(J.NE.JFR(II)) GO TO 730
    LL1 = LL + 3
    GO TO 700
690 LL1 = LL
700 SMJ(IPOS(K,L)) = SMJ(IPOS(K,L)) + FRAMEK(II,IPOS(KK1,
   1LL1))
730 CONTINUE
810 CONTINUE
     DO 1100 I=1,NJFREE
     DO 1100 J=1,NJFREE
    IF(I.GT.J) GO TO 1100
    IF(I.GT.NJF) GO TO 950
    I1 = 3*I - 2
    I2 = I1 + 1
    GO TO 960
950 I1 = NJF + 2*I - 1
    I2 = I1 + 1
960 IF(J.GT.NJF) GO TO 970
    J1 = 3*J - 2
    J2 = J1 + 1
    GO TO 980
970 J1 = NJF + 2*J - 1
    J2 = J1 + 1
980 DO 1090 II=1,NWALL
    KK = 0
    DO 1090 K=I1,I2
    KK = KK + 1
    LL = 0
    DO 1090 L=J1,J2
    LL = LL + 1
    IF(L.LT.K) GO TO 1090
    IF(I.EQ.IWALL(II)) GO TO 1000
    IF(I.EQ.JWALL(II)) GO TO 1010
    IF(I.EQ.KWALL(II)) GO TO 1020
    IF(I.NE.LWALL(II)) GO TO 1090
    KK1 = KK+6  $ GO TO 1030
```

```
1000 KK1 = KK     $ GO TO 1030
1010 KK1 = KK+2   $ GO TO 1030
1020 KK1 = KK+4
1030 IF(J.EQ.IWALL(II)) GO TO 1040
     IF(J.EQ.JWALL(II)) GO TO 1050
     IF(J.EQ.KWALL(II)) GO TO 1060
     IF(J.NE.LWALL(II)) GO TO 1090
     LL1 = LL+6   $ GO TO 1070
1040 LL1 = LL     $ GO TO 1070
1050 LL1 = LL+2   $ GO TO 1070
1060 LL1 = LL+4
1070 SMJ(IPOS(K,L)) = SMJ(IPOS(K,L)) + WALLK(II,IPOS(KK1,
    1LL1))
1090 CONTINUE
1100 CONTINUE
     IF(NN.GT.NDEG) GO TO 1200
     RETURN
1200 CALL INVERSE(NN)
     IF(NN.EQ.0) NJF=0
     IF(NN.EQ.0) RETURN
     CALL FLEX
     CALL MATIN(NDEG)
     RETURN
     END


      SUBROUTINE MASS(ALPHA)
C
C     THIS COMPUTES  THE MASS MATRIX
C
      COMMON IFR(40),JFR(40),IWALL(40),JWALL(40),KWALL(40),
     1FRAMEK(40,21),WALLK(40,36),NWALL,NFRAME,NJF,NJW,NSUP,
     2LWALL(40),ZW(40)
      COMMON/2/STARF(20,20),STARK(20,20),S(120),NDEG
      COMMON/3/ XJ(50),YJ(50),AFR(40),FRMI(40),R(6,6),RT(6,6),
     1B(6,6),BB(3,6),BT(6,3),BC(6,3),DD(3,3),EF,EW,WT,POISS,
     2A(6,6),NMAX
      NN=3*NJF+2*NJW
   10 READ 20,WLOAD,DENSE,DNS,NBAY
   20 FORMAT(3F10.0,I5)
      DO 30 I=1,120
   30 S(I)=0.0
      DO 80 I=1,NJF
      J=3*I-2
      DO 70 II=1,NFRAME
      IF(IFR(II).NE.I.AND.JFR(II).NE.I) GO TO 70
      FLON=SQRT((XJ(IFR(II))-XJ(JFR(II)))**2+(YJ(IFR(II))-
     1YJ(JFR(II)))**2)
      IW=IFR(II)
```

```
      JW=JFR(II)
      IF(XJ(IW)-XJ(JW)) 40,50,40
   40 DLOAD=0.5*FLON*(AFR(II)*DENSE + WLOAD)/386.4
      GO TO 60
   50 DLOAD=0.5*FLON*AFR(II)*DENSE/386.4
   60 S(J)=DLOAD+S(J)
      S(J+1)=DLOAD+S(J+1)
      S(J+2)=(DLOAD/3.0)*(FLON**2)*(ALPHA**2) + S(J+2)
   70 CONTINUE
   80 CONTINUE
      DO 85 I=1,NWALL
      II=3*IWALL(I)-2
      JJ=3*JWALL(I)-2
      KK=3*KWALL(I)-2
      LL=3*LWALL(I)-2
      WLD=(YJ(JWALL(I))-YJ(IWALL(I)))*(XJ(KWALL(I))-XJ(IWALL
     1(I)))*WT*DNS*0.25/386.4
      IF(II.GT.NN) GO TO 81
      S(II)=S(II)+WLD
      S(II+1)=S(II+1)+WLD
   81 IF(LL.GT.NN) GO TO 82
      S(LL)=S(LL)+WLD
      S(LL+1)=S(LL+1)+WLD
   82 S(JJ)=S(JJ)+WLD
      S(JJ+1)=S(JJ+1)+WLD
      S(KK)=S(KK)+WLD
      S(KK+1)=S(KK+1)+WLD
   85 CONTINUE
      IF(NN.GT.NDEG) GO TO 90
      GO TO 120
   90 N = NDEG*(NBAY+1)
      DO 100 I=1,N
      J = 3*I-2
  100 S(I)=S(J)
      DO 110 I=1,NBAY
      DO 110 J=1,NDEG
      K=J+I*NDEG
  110 S(J)=S(J)+S(K)
  120 PRINT 121
  121 FORMAT(/* MASS MATRIX*)
      PRINT 122,(S(I),I=1,NDEG)
  122 FORMAT(/10E12.5)
      RETURN
      END


      SUBROUTINE EIGEN(NN,TMH)
C     THIS COMPUTES THE BEST TIME INTERVAL TO BE USED
      COMMON/1/SMJ(7320)
```

```
      COMMON/2/STARE(20,20),STARK(20,20),S(120),NDEG
      DIMENSION B(120),C(120),X(120)
    1 N=NDEG
      INDEX =0
C     THE ITERATION PROCEDURE FOR EIGENVALUES
   11 CONTINUE
      DO 20 I=1,N
   20 X(I)=1.0
C     CALCULATE COMPONENTS OF EIGENVECTORS
   21 DO 22 I=1,N
      C(I) = 0.0
      DO 22 J=1,N
      IF(NN.NE.NDEG) GO TO 201
      C(I) = C(I) + SMJ(IPOS(I,J)) * X(J) / S(I)
      GO TO 22
  201 C(I) = C(I) + STARK(I,J) * X(J) / S(I)
   22 CONTINUE
      DO 23 I=1,N
   23 B(I) = C(I) / C(1)
C     CHECK FOR ACCURACY
      EPSI = 0.000001
      DO 24 I=1,N
      DIFF=X(I) - B(I)
      IF(ABS(DIFF)-EPSI) 24,25,25
   24 CONTINUE
      GO TO 27
   25 DO 26 I=1,N
   26 X(I) = B(I)
      INDEX = INDEX + 1
      IF(INDEX.GT.50) GO TO 27
      GO TO 21
   27 TMH=TSTEP(C(1))
   28 PRINT 29,TMH
   29 FORMAT(//*  TIME INTERVAL = *F8.5/)
      RETURN
      END


      SUBROUTINE GROUND(TIME)
C     THIS COMPUTES THE ACCELERATION DUE TO GROUND MOTION AT(TIME)
      COMMON IFR(40),JFR(40),IWALL(40),JWALL(40),KWALL(40),
     1FRAMEK(40,21),WALLK(40,36),NWALL,NFRAME,NJF,NJW,NSUP,
     2LWALL(40),ZW(40)
      COMMON/2/STARE(20,20),STARK(20,20),S(120),NDEG
      COMMON/GRND/UXLG(120),AH,AV,INQ,FACT
      DIMENSION AXLH(400),AXLV(400),QTH(400),QTV(400)
      NN=3*NJF + 2*NJW
      IF(INQ.EQ.1) GO TO 140
      READ 10,NOH,NOV
```

```
   10 FORMAT(2I5)
      DO 80 I=1,NOH
      J1=4*I-3 $ J2=J1+1 $ J3=J1+2 $ J4=J1+3
   80 READ 100,QTH(J1),AXLH(J1),QTH(J2),AXLH(J2),QTH(J3),
     1AXLH(J3),QTH(J4),AXLH(J4)
      DO 90 I=1,NOV
      J1=4*I-3 $ J2=J1+1 $ J3=J1+2 $ J4=J1+3
   90 READ 100,QTV(J1),AXLV(J1),QTV(J2),AXLV(J2),QTV(J3),
     1AXLV(J3),QTV(J4),AXLV(J4)
  100 FORMAT(3X,4(F8.0,F9.0))
  140 DO 200 I=1,400
      IF(QTH(I).GT.TIME) GO TO 210
  200 CONTINUE
  210 TG=QTH(I)
      AG=AXLH(I)
      TL=QTH(I-1)
      AL=AXLH(I-1)
      CF=(TIME-TL)/(TG-TL)
      AH=AL+CF*(AG-AL)
      DO 300 I=1,400
      IF(QTV(I).GT.TIME) GO TO 310
  300 CONTINUE
  310 TG=QTV(I)
      AG=AXLV(I)
      TL=QTV(I-1)
      AL=AXLV(I-1)
      CF=(TIME-TL)/(TG-TL)
      AV=AL+CF*(AG-AL)
      IF(NDEG.LT.NN) GO TO 500
      DO 400 I=1,NJF
      J=3*I-2
      UXLG(J)=AH*FACT * 386.4
      UXLG(J+1)=AV*FACT * 386.4
  400 UXLG(J+2)=0.0
      GO TO 600
  500 DO 550 I=1,NDEG
  550 UXLG(I) = AH*FACT*386.4
  600 CONTINUE
      INQ=1
      RETURN
      END


      SUBROUTINE FRAMST
C     THIS COMPUTES THE INCREMENTAL FORCES IN FRAME ELEMENTS
      COMMON IFR(40),JFR(40),IWALL(40),JWALL(40),KWALL(40),
     1FRAMEK(40,21),WALLK(40,36),NWALL,NFRAME,NJF,NJW,NSUP,
     2LWALL(40),ZW(40)
      COMMON/1/SMJ(7320),F(150),U(150),X(6),Y(6)
```

```
      COMMON/3/ XJ(50),YJ(50),AFR(40),FRMI(40),R(6,6),RT(6,6),
     1B(6,6),BB(3,6),BT(6,3),BC(6,3),DD(3,3),EF,EW,WT,POISS,
     2A(6,6),NMAX
      COMMON/4/DFR(40,4),DWL(40),FFR(40,4),FWL(40)
      DO 300 I=1,NFRAME
      K1=3*IFR(I) - 2  $  K2=K1+1 $  K3 = K1 + 2
      K4 = 3*JFR(I) - 2  $  K5 = K4 + 1  $  K6 = K4 + 2
      X(1)=U(K1) $ X(2)=U(K2) $ X(3)=U(K3) $ X(4)=U(K4)
      X(5) = U(K5)  $ X(6) = U(K6)
      DO 200 II=1,6
      DO 200 JJ=1,6
  200 A(II,JJ)=FRAMEK(I,IPOS(II,JJ))
      CALL MATMULT (A,X,Y,6,6,1)
      DFR(I,1)=Y(1)
      DFR(I,2)=Y(2)
      DFR(I,3)=Y(3)
      DFR(I,4)=Y(6)
  300 CONTINUE
      RETURN
      END



      SUBROUTINE WALST
C     THIS COMPUTES THE MAX TENSILE STRESS IN THE WALL ELEMENTS
      DIMENSION UU(8),FF(3),K(8)
      COMMON IFR(40),JFR(40),IWALL(40),JWALL(40),KWALL(40),
     1FRAMEK(40,21),WALLK(40,36),NWALL,NFRAME,NJF,NJW,NSUP,
     2LWALL(40),ZW(40)
      COMMON/1/SMJ(7320),U(120),F(120),X(6),Y(6),
     1GG(3,8),HH(3,8)
      COMMON/3/ XJ(50),YJ(50),AFR(40),FRMI(40),R(6,6),RT(6,6),
     1B(6,6),BB(3,6),BT(6,3),BC(6,3),DD(3,3),EF,EW,WT,POISS,
     2A(6,6),NMAX,PMAXT
      COMMON/4/DFR(40,4),DWL(40),FFR(40,4),FWL(40)
      DO 600 N=1,NWALL
      PMAXT = 0.0
      ALFA = XJ(LWALL(N)) - XJ(IWALL(N))
      BETA = YJ(JWALL(N)) - YJ(IWALL(N))
      IF (IWALL(N).GT.NJF) GO TO 100
      K(1) = 3*IWALL(N) - 2
      GO TO 110
  100 K(1) = NJF + 2*IWALL(N) - 1
  110 K(2) = K(1) + 1
      IF (JWALL(N).GT.NJF) GO TO 120
      K(3) = 3*JWALL(N) - 2
      GO TO 130
  120 K(3) = NJF + 2*JWALL(N) - 1
  130 K(4) = K(3) + 1
      IF (KWALL(N) . GT.NJF) GO TO 140
```

```
      K(5) = 3*KWALL(N) - 2
      GO TO 150
  140 K(5) = NJF + 2*KWALL(N) - 1
  150 K(6) = K(5) + 1
      IF (LWALL(N).GT.NJF) GO TO 160
      K(7) = 3*LWALL(N) - 2
      GO TO 170
  160 K(7) = NJF + 2*LWALL(N) - 1
  170 K(8) = K(7) + 1
      DO 200 I=1,8
  200 UU(I) = U(K(I))
      DO 500 II=1,2
      DO 500 JJ=1,2
      XEE = (II-1)
      ETA = (JJ-1)
      DO 330 I=1,3
      DO 330 J=1,8
  330 GG(I,J) = 0.0
      GG(1,1) = -(1.0 - ETA)/ALFA
      GG(1,3) = -ETA/ALFA
      GG(1,5) = -GG(1,3)
      GG(1,7) = -GG(1,1)
      GG(2,2) = -(1.0 - XEE)/BETA
      GG(2,4) = -GG(2,2)
      GG(2,6) = XEE/BETA
      GG(2,8) = -GG(2,6)
      GG(3,1) = GG(2,2)
      GG(3,2) = GG(1,1)
      GG(3,3) =-GG(2,2)
      GG(3,4) = GG(1,3)
      GG(3,5) = GG(2,6)
      GG(3,6) = - GG(1,3)
      GG(3,7) = -GG(2,6)
      GG(3,8) = -GG(1,1)
      CALL MATMULT(DD,GG,HH,3,3,8)
  350 CALL MATMULT(HH,UU,FF,3,8,1)
      S1 = 0.5*(FF(1) + FF(2)+ SQRT((FF(1)-FF(2))**2 + 4.0*
     1FF(3)**2))
      IF(S1.LT.PMAXT) GO TO 400
      PMAXT = S1
  400 CONTINUE
  500 CONTINUE
      DWL(N) = PMAXT * ZW(N)
  600 CONTINUE
      RETURN
      END


      SUBROUTINE MATMULT (A,B,C,L,M,N)
```

```
C     THIS MULTIPLIES MATRICES A AND B AND STORES IN C
      DIMENSION A(L,M),B(M,N),C(L,N)
      DO 10 I=1,L $ DO 10 K=1,N $ C(I,K)=0. $ DO 10 J=1,M
   10 C(I,K)=C(I,K)+A(I,J)*B(J,K)
      RETURN
      END


      SUBROUTINE FLEX
C     THIS PICKS OUT THE PERTINENT ELEMENTS OF THE JOINT
C     FLEXIBILITY MATRIX OF THE UNREDUCED SYSTEM AND FORMS
C     THE JOINT FLEXIBILITY MATRIX OF THE REDUCED SYSTEM
      COMMON/1/SMJ(7320),F(150),U(150),X(6),Y(6)
      COMMON/2/A(20,20),STARK(20,20),S(120),NDEG
   20 DO 30 I=1,NDEG
      DO 30 J=1,NDEG
      NI = 3*I - 2
      NJ = 3*J - 2
   30 A(I,J)=SMJ(IPOS(NI,NJ))
      RETURN
      END


      SUBROUTINE MATIN(N)
C     THIS COMPUTES THE STIFFNESS MATRIX OF THE REDUCED SYSTEM
      COMMON/2/STARF(20,20),S(20,20)
      DO 300 I=1,N
      DO 300 J=1,N
  300 S(I,J)=STARF(I,J)
  400 DO 401 I=1,N $XX=S(I,I) $ S(I,I)=1.0 $ DO 402 J=1,N
  402 S(I,J)=S(I,J)/XX$ DO 401 K=1,N $ IF (K-I) 403,401,403
  403 XX=S(K,I)$ S(K,I)=0.0 $ DO 404 J=1,N
  404 S(K,J)=S(K,J) -XX*S(I,J)
  401 CONTINUE
      RETURN
      END


      SUBROUTINE INVERSE(N)
C     THIS INVERTS THE JOINT STIFFNESS MATRIX OF THE
C     UNREDUCED SYSTEM
      DIMENSION P(120),Q(120),IR(120)
      COMMON/1/A(7320)
      IF (N-1) 14,15,16
   15 A(1)=1./A(1)
      RETURN
   16 N4=(N*(N+1))/2
```

```
      DO 10 I=1,N
   10 IR(I)=0
C     GRAND LOOP STARTS
      DO 100 I=1,N
      BIGAJ =0.
      DO 20 J=1,N
      IF (IR(J)) 20,201,20
  201 M=(J*(J+1))/2
      Z=ABS (A(M))
      IF (Z-BIGAJ) 20,20,202
  202 BIGAJ=Z
      K=J
   20 CONTINUE
      IF (BIGAJ)  21,211,21
  211 PRINT 6
    6 FORMAT(///* STRUCTURE COLLAPSES*//)
      N = 0
      RETURN
C     PREPARATION OF ELIMINATION STEP 1
   21 IR(K)=1
      M=(K*(K+1))/2
      Q(K)=1./A(M)
      P(K)=1.
      A(M)=0.
      L=K-1
      IF (L) 35,35,351
  351 M=(K*(K-1))/2
      DO 30 J=1,L
      M=M+1
      P(J)=A(M)
      Q(J)=A(M)*Q(K)
      IF (IR(J)) 30,301,30
  301 Q(J)=-Q(J)
   30 A(M)=0.
      IF (K+1-N)  35,35,50
   35 L=K+1
      DO 45 J=L,N
      M=(J*(J-1))/2+K
      P(J)=A(M)
      IF (IR(J)) 471,47,471
  471 P(J)=-P(J)
   47 Q(J)=-A(M)*Q(K)
   45 A(M)=0.
   50 DO 100 J=1,N
      DO 100 K=J,N
      M=(K*(K-1))/2+J
  100 A(M)=A(M)+P(J)*Q(K)
C     END OF GRAND LOOP
   14 RETURN
      END
```