FORECASTING THE MONEY STOCK

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ABSTRACT

FORCASTING THE MONEY STOCK

by

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A number of economic models exist which have the money stock as the endogenous variable. However, these models have not been systematically exposed to what many maintain is the ultimate test of an economic model: predictive performance. The subject of this dissertation is the examination of the predictive performance of a cross-section of money stock models.

The models examined ranged from mechanistic models to two equation models estimated with TSLS estimation techniques. Some models examined had explanatory variables which primarily reflected the behavior of the banking system, e.g. reserves and the discount rate, while other models had explanatory variables which primarily reflected the private non-bank sector, e.g. interest rates and income. The mechanistic models were either autoregressive models of the money stock or money multiplier models which assumed that the multiplier either did not change or changed at a constant rate over the forecast period.

Predictive performance of the models for the 1961-1970 period was examined. Where parameter estimation was necessary, 1947-1960 data was used. In order to reduce the role of judgment in using the models, all models forecast ex post, i.e. the actual values of the exogenous variables were used. The root mean square error (RMSE) statistic was used as the measure of predictive performance. Other prediction evaluation statistics were used, but their ranking of predicitve performance differed little from the RMSE ranking.

The autoregressive seasonally adjusted money stock and the no-change money multiplier model were the two models which forecast with the lowest RMSE. The strong time trend of the money stock data was one explanation of the good predictive performance of the autoregressive model while the relative stability over time of the multiplier contributed to the low RMSE of the multiplier model.

The best performing economic models, i.e. models with expanatory variables in addition to the lagged values of the dependent variable, were a number of single equation models. One had a short-term interest rate, income (or permanent income), and the lagged dependent variable as explanatory variables with all quantity variables expressed in nominal or real terms. Another had total reserves plus reserves released through changes in the reserve requirement, the short-term interest rate, and the discount rate as explanatory variables. No two-equation model performed better than these single equation models just mentioned.

Predictive performance was improved in this dissertation a number of ways. Including the lagged dependent variable in a forecasting equation almost without exception improved prediction performance. Linear combinations of the residuals of prior period(s) when added to the constant term of the equation also lowered the RMSE. Correction for first-order autocorrelation also improved predictive performance.

Where possible, the models were tested for the existence of structural shift. In this study it was observed that structural stability was neither a necessary or a sufficient condition for a low RMSE forecast, i.e. in some cases where the hypothesis that structural shift had occurred could not be rejected, the model forecast with a low RMSE; and in other cases where the hypothesis was rejected the model forecast with a relatively high RMSE.

The impressive performance of the mechanistic models vis-a-vis economic models in forecasting the money stock should serve as a challenge to the econometric model builder. It also indicates that such mechanistic models provide a tough standard of comparison for conventional economic models.

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CHAPTER I

INTRODUCTION

Many economists view the predictive performance of an economic model as its ultimate test. In light of this, in recent years increased attention has been paid to the predictive performance of econometric models. Most of this interest, however, has been focused on forecasts of GNP, or its principle components, while little attention has been paid to models that can be used to forecast the money stock.

This dissertation is an attempt to begin filling in this lucuna. This project will inevitably go beyond the question of which money stock model forecasts best. For example, it will have implications for the question of the importance of the role of money in the economy. The dissertation will also briefly discuss a variety of prediction evaluation statistics, and examine how well they perform in actual practice. But the basic question will remain: how satisfied can the econometric model builder be with the current models of the money stock.

The dissertation will be organized in the following fashion. Econometric forecasting and its evaluation will be surveyed in Chapter II. An introduction to the various techniques of evaluating the performance of predictions will also be part of this chapter. Various money stock models which can be used for forecasting are then described in Chapter III. The next three chapters discuss the application of these models to forecasts of the money stock for the 1961-1970 period. Data from the 1947-1960 period are used to estimate the parameters of the models. Chapter IV will deal with mechanistic money stock models, Chapter V with single-equation models and Chapter VI with simple simultaneous equation models. Chapter VII will tie together the results derived from these chapters.

CHAPTER II

FORECASTING WITH AN ECONOMETRIC MODEL

Until recently economic forecasting has been an "artistic, subjective, and personal" endeavor [Klein, 1968, p 9], but with the development of theory--both economic and econometric--and the availability of reasonably reliable and extensive data and machines capable of manipulating this data, economic forecasting has become more objective. Most forecasting models used by economists still require the use of the forecaster's judgment in order to make forecasts with small errors. Nevertheless the results of most forecasting models can be replicated and the forecast analyzed. It may, in fact, be possible to ascertain, at least partially, the source of a forecast error. The development of a number of different models each explaining the same economic variable (e.g. GNP) has allowed us to compare the forecasting ability of particular models (as opposed to comparing economic forecasters who would usually have difficulty consistently replicating their forecasts).

In this chapter econometric forecasting and its evaluation will be discussed. In Chapter III econometric forecasting of the money stock will be discussed.

A. FORECASTING MODELS

1. Non-econometric forecasting models

The business cycle attracted the early economic forecasters. For example, Wesley Mitchell and the National Bureau of Economic Research (NBER) attempted to forecast the behavior of economic aggregates such as GNP and the national income components by tracking leading indicators: economic events which usually preceeded turning points in the business cycle.[1,2] Since World War II, forecasts based on data from surveys of the buying intentions of consumers have been made. These forecasts using anticipations data have been used to predict either consumption (usually of durables) or investment. While on the whole such forecasts are unreliable [Evans, 1969, p 494], they are frequently more reliable than mechanistic forecasts,[3] and so this forecasting technique continues to be used.[4]

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1. Use of the leading indicator methodology was not limited to the NBER. For an example a bit removed from the NBER mainstream, see Roos, 1955, pp 369-379.

2. For a review of early attempts at economic forecasting, see Zarnowitz, 1968, 1968; Roos, 1955. For a recent discussion of leading indicators and NBER methodology, see Evans, 1969, pp 445-460.

3. Mechanistic forecasts depend solely on the statistical characteristics of the data series, e.g. an autoregressive model. A good example of such a mechanistic model is Juster's, an autoregressive model with up to 8 lagged values of the dependent variable. Juster, 1969, pp 226-229.

4. This grossly oversimplifies the case for forecasting with anticipations data. It is, however, an example of a widely used forecasting technique which forecasts

Economists (usually along with businessmen and government officials) have been frequently surveyed to determine what they feel certain economic aggregates will be in the future. Since the forecast aggregate depends upon the judgment of the participants in the survey, this forecast is frequently called a judgmental forecast. Zarnowitz [1967] examined the prediction performance of these forecasts and found that judgmental forecasts performed better than did no-change and same-change extrapolations of the series being forecast. Other studies have confirmed that judgmental forecasts predict better than mechanistic forecasts.[5] Of course the success of recent judgmental forecasts must be viewed with caution since the judgments of many individual economists (and others) may have been influenced by forecasts made with econometric models.

2. Econometric forecasting models

The econometric model has greatly increased the objectivity of the econometric forecast.[6] While there were

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variables of interest to the economist, but which makes little use of econometric techniques or the more conventional economic data series.

5. For example, see Stekler, 1970, pp 74-91. Smyth, 1966 summarizes the performance of Australian judgmental forecasts and compares them with judgmental forecasts for other countries.

6. Forecasting it seems will never be completely objective. Much judgment is required to successfully specify, estimate, and use even (or especially?) the largest econometric forecasting models.

earlier macro-econometric models, the Klein-Goldburger model is the water-shed in the use of macro models by the economist.[7] Christ's review article of the Klein-Goldburger model [Christ, 1956] included one of the first evaluations of the predictive performance of an econometric model. The shift in attitude towards econometric models which occurred around this time can be seen in two articles which discussed econometric methods of forecasting. Bassie, for example, saw too many inflexibilities introduced by the econometric approach to forecasting, and maintained that such an approach would remain impractical. [Bassie, 1955, p 33]. Roos [1955], on the other hand, gave an optimistic prediction of the future role of econometric models in forecasting.[8]

Econometric models have come a long way since the Klein-Goldburger model. Many are quite small, consisting of one or two equations; while others are quite large.[9] Some have been designed explicitly for forecasting,[10] while others have been designed to examine the behavior of a certain

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7. The review article by Fox, 1956 brings this out most strongly.

8. Roos stated that "the modern econometric forecaster, viewing the economy as an elastic membrane in disequilibrium, is forever on the altert to identify new impressed forces that might reinforece or negate his forecasts." Roos, p 395.

9. For example, The Brookings model. Duesenberry, 1965.

10. For example, the Wharton-EFU model. Evans, 1968.

sector of the economy.[11] There are, in fact, a number of models which can be used to forecast the money stock some of which will be discussed in the next chapter.

3. Judgment and forecasting with an econometric model

While the objective nature of forecasts generated by an econometric model will be emphasized in this study, judgment is necessary for successful econometric forecasting. For example, in the actual forecasting situation the values of the exogenous variables must be chosen. In many cases the constant terms of the equations are adjusted to reflect the forecaster's estimation of the size and impact of structural shifts in the economy, or the behavior of the residual term of the pervious periods. Further adjustments may be made if, after making a preliminary forecast, the resultant prediction of the endogenous variables simply do not look correct to the forecaster.[12]

Since we want to compare the predictive ability of various econometric models, and not the various forecasters, the role of judgment in using a forecasting model must be minimized. Thus in evaluating forecasts actual values of exogenous variables for the periods forecast (henceforth, ex post values) will be used rather than the values of the

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11. The FRB-MIT model, for example, emphasized the role of the financial sector of the economy.

12. For a description of this process, see Evans, 1972, pp 1126-1128, and Klein, 1968, pp 50-51.

exogenous variables which are themselves predicted when making forecasts (henceforth, ex ante values), and while constant adjustment terms may be used to improve the performance of the models, the adjustments will be mechanical. This approach rules out evaluation of forecasts using ex ante data and equation adjustments based on the forecaster's feel of the economy.[13] Forecasts using ex ante data frequently predict better than forecasts using ex post data even though both forecasts make use of constant adjustments.[14]

B. FORECASTING ERRORS IN ECONOMETRIC MODELS

1. Sources of forecasting error

The world is stocastic, not deterministic, and therefore we will always be confronted with forecasting error. The relationship between the dependent variable (Y_t) and the independent variable (X_t) can be expressed in the form (assuming that the relationship between the two is linear):

$$(2.1) \qquad Y_t = B_0 + B_1 X_t + e_t$$

where e_t is a random disturbance term. Standard

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13. Klein, 1968, p 42 maintains that such judgmental adjustments are an important element in any realistic forecasting situation. So a loss of realism is the price of using the ex post approach.

14. See Haitovsky, 1970 and Su, 1971. Possible explanation of why the use of ex ante data leads to better forecasts than the use of ex post data is discussed in these articles.

econometric theory is that the total variance of the forecasting error of the prediction[15]

$$\hat{Y}_t = \hat{B}_0 + \hat{B}_1 X_0 \qquad (2.2)$$

is given by the expression[16]

$$s_F^2 = s^2 \left[1 + \frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum_{t=1}^{n} (X_t - \bar{X})^2} \right] \qquad (2.3)$$

This suggests that there is a lower limit to forecasting accuracy.[17] Estimating the forecasting error for more complex situations is difficult. Klein [1968, pp 27-28] has developed a numerical technique for determining the standard error of forecast for equations with lagged endogenous variables, while Feldstein [1971] has constructed an estimator for the interval of the forecast error when the exogenous variables themselves are stochastic.[18] But except for the most elementary models,

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15. $\hat{Y}_t$ is the predicted value of Y_t given X_0. $\hat{B}_0$ and $\hat{B}_1$ are estimated values of B_0 and B_1. $(Y_t - \hat{Y}_t)$ is the forecast error, and s_F is an estimate of that error.

16. Kmenta, 1971, pp 240-241. s^2 is an estimate of the variance of $(Y_t - (B_0 + B_1 X))$. For the multiple regression case, see Kmenta, 1971, p 375.

17. This expression tells us explicitly that the further away the independent variable is from its mean, the larger the standard error of forecast.

18. But three rather restrictive assumptions underlie Feldstein's estimation of the interval: the stochastic disturbance is not autocorrelated, lagged endogenous variables are not used, and only linear models are considered.

estimation of the forecast interval is difficult. Even when the forecasting error can be estimated, however, the error is usually greater than the accuracy requirement of the forecaster [Klein, 1968, p 40], and therefore the forecaster is forced to resort to an activity called "fine tuning." [Evans, 1972, p 952]. This, of course, implies the application of judgment to forecasting with econometric models.

The forecaster frequently must use preliminary data which is to be subsequently revised. There is a trade-off between data accuracy and reporting speed [Stekler, 1970, pp 102-121], although in recent years the quality of preliminary data has improved. [Zellner, 1958; Cole, 1969]. While this is a problem for the forecaster,[19] the evaluator of forecasting performance usually has the revised data available.[20]

An econometric model assumes that during the period of the fit, the relationship between the variables in the model remain fixed (or at least changes are allowed for by dummy variables). If the model is used for forecasting, the relationship is assumed to remain fixed. But the world

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19. Klein, 1968, pp 68, 42. Klein sees it as an important research task.

20. Poor data may influence the construction and use of a econometric model. Do adjustments to the constant term compensate for poor data, or for structural shifts? This question is not dealt with here.

is not that neat. Changes do occur.[21] Such change may be compensated for by adjustment of the constant term although frequently change is either not noticed, or is difficult to quantify. Cooper maintains that statistically significant shifts do occur even within a short time horizon.[22] Usually the constant term is adjusted in the short run and the model is periodically reestimated. Klein [1968, p 51], however, has pointed out that a freshly estimated system needs adjustment as much after a year or two as it does after four or five years. This adjustments, unfortunately, are usually based on the forecaster's feel for the economy and for how expected changes will appear in the model. Since we are interested in examining the behavior of the forecasting model and not the forecaster, as mentioned above, adjustments of this type will not be considered.

A forecasting model may be misspecified, and if the misspecification biases the disturbance variance term, forecasting error is unnecessarily high. The three misspecifications which do increase forecasting error are omitted variables, incorrect functional forms, and

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21. Another possibility is that the model was misspecified. In this case, while the world may have remained fixed, the hypothesis that this was the case could be rejected by a test for structural stability.

22. Cooper, 1972, pp 900-915. Cooper fitted his models over the period 1949-1 to 1960-4, and found that there was a significant shift of many of the equations in the models during the 1961-1 to 1965-4 period.

heteroskedasticity of the disturbance terms. Careful examination for misspecification is usually not part of the model estimation procedure.

Another source of forecast error is incorrect estimation of the values of the exogenous variables. Evans [1972, pp 954-956] maintains that the only correct way of evaluating the forecasting ability of a model is by examining its ex ante forecasting ability. But as Cooper [1972, p 816] has pointed out, only by comparing the accuracy of ex post forecasts is the judgmental element held constant.[23] In this study, therefore, the ex post values of the exogenous variables will be used.

2. Reducing forecasting error

When a forecasting model has autocorrelated residuals, adjustment of the constant term is explicitly specified by the model. The adjustment for first order autocorrelation for a T period forecast is

$$A_{t+T} = \rho^{T} e_{t} \tag{2.4}$$

where A is the constant term correction factor, ρ is the autocorrelation coefficient, t is the period when the forecast was made, T the length of the forecast, and e is the observed residual for that period.[24] A refinement

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23. Stekler [Evans, 1972, p 1141] in his discussion of the Evans approach makes the same observation.

24. Goldberger, 1962. See also Klein, 1968, pp 68, 51-55.

of this adjustment is[25]

$$(2.5) \qquad A'_{t+T} = \rho^T \frac{e_t + \rho e_{t-1}}{2}$$

The over-adjustment that results from exceptionally large residuals in the latest observed quarter is thereby reduced [Evans, 1972, p 966], and, where this adjustment has been used, the forecasting performance of the model has been improved. Predictive performance of models not corrected for autocorrelation is frequently improved by introducing adjustments based on the residuals of prior periods. One such correction factor was used by Haitovsky [1970, pp 504-505]:[26]

$$(2.6) \qquad A''_t = \frac{e_{t-1} + e_{t-2}}{2} .$$

The desired properties of estimated models, e.g. minimum variance of the forecast endogenous variable, depend upon error specification of the estimated model meeting the assumptions of the classical linear regression model. Otherwise the model is said to be misspecified. The restrictive nature of these assumptions is usually acknowledged in the course of estimating the model, but whether the residuals actually meet these assumptions is rarely

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25. This adjustment is frequently called the Goldberger-Green adjustment. Evans, 1972, p 966; Haitovsky, 1972B, pp 319-320.

26. Such a correction term, of course, implies that the model is autocorrelated, and assumes that the best correction technique is a second-order correction scheme with $\rho_1 = .5$ and $\rho_2 = .5$.

asked. Four tests for specification error have been developed by Ramsey [1971] and used by Gilbert [1969] to determine the proper specification for the demand for money function.

C. EVALUATION OF THE PERFORMANCE OF A FORECASTING MODEL

When a number of econometric models have the same dependent variable, comparison of the different models is inevitable. There are a number of approaches to making such comparisons.[27] For example, a model can be evaluated on its use of a priori knowledge, or its use of appropriate estimation techniques,[28] but the ultimate performance criterion is frequently seen to be the model's forecasting ability.[29] It is this aspect of the performance of econometric models of the money stock which is the focal point of this study.

Goodness-of-fit statistics have sometimes served as a proxy for direct examination of forecasting performance. The forecaster usually wants maximum mileage from his data, and so is reluctant to reduce his sample size. This is

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27. See Naylor, 1966, pp 311-315; and 1971, pp 154, 158 for a discussion of the four methodological positions regarding this question. See also Zecher, 1971.

28. McCarthy in Cooper, 1972, p 934.

29. Naylor, 1966, p 318 maintains that prediction is the ultimate test of a model.

particularly true with large models.[30] But since a model may perform quite differently outside its estimation period, goodness-of-fit statistics do not necessarily indicate good forecasting ability. Nelson [1972], for example, has shown that within the estimation period the consumption-investment block of the Penn-FRB-MIT model explains a high proportion of the variation of the block's endogenous variables, while outside the estimation period a significantly higher proportion of that variation is explained by a mechanistic model. Such results are not uncommon. The goodness-of-fit criteria may also promote "data mining," i.e. estimating a wide range of possible models and using the model with the best fit statistics. In such a case goodness-of-fit criteria provides little indication of the validity of the model.[31]

It is generally agreed, therefore, that the more rigorous test of a model is to forecast with the model, and evaluate its predictive performance. [Jorgenson, 1970, p 214]. Using either ex ante [Evans, 1972] or ex post [Cooper, 1972] values for the exogenous variables, the endogenous variables are predicted and then compared to actual values. Occasionally prediction evaluation stops

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30. Of course once a model has been estimated and used for forecasting, after a while it is possible to study its predictive performance. This has been done in Evans [1972] for the Wharton-EFU model and the OBE model.

31. Jorgenson, 1970, p 214 elaborates on this problem.

there[32] but usually summary statistics are calculated to aid in the evaluation of the prediction performance of a particular model. These statistics are the subject of this section.

1. Conventional test statistics

The two principal prediction evaluation statistics are the root mean square error (RMSE) statistic

$$(2.7) \qquad RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\hat{Y}_t - Y_t)^2}$$

and the mean absolute error (MAE) statistic

$$(2.8) \qquad MAE = \frac{1}{n}\sum_{t=1}^{n}\left|\hat{Y}_t - Y_t\right|$$

The MAE statistic is preferred by Klein because of its simplicity and ease of understanding [Klein, 1968, p 40], while others prefer the RMSE because it has a quadratic loss function.[33] Two variations of these two statistics are the mean squared error statistic (which, however, does not have the same dimension as the error term), and the mean percent absolute error statistic. Occasionally the

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32. For example, Christ, 1956, where the predictive performance of the Klein-Goldburger model is tabulated.

33. The loss function of the MAE statistic in linear. Proponents of the RMSE statistic assume, at least implicitly, that they become more concerned about an error as it increases in magnitude. For more on this, see Fromm, 1964.

rnage of the prediction error is used [Fromm, 1964, p14]. Naylor [1971, pp 159-160] suggests the possible use of spectral analysis and various non-parametric tests.

2. Other test statistics

Another approach, frequently proposed[34] but rarely used, is to regress the forecast values on the actual values of the prediction model, and to examine the test statistics associated with the regression. In other words, the R^2 of the regression

$$(2.9) \qquad \hat{Y}_t = \hat{B}_0 + \hat{B}_1 Y_t + e_t$$

would be examined along with the standard error of the regression and the closeness of B_0 to zero, B_1 to one. This approach will be used in Chapter IV.

The model can also be examined to determine whether structural shift has occurred between the estimation period and the forecast period.[35] This is the most powerful of the univariant tests with the same level of Type I error (the incorrect rejection of the null hypothesis).[36] The

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34. See Klein, 1968, p 39; Mincer, 1969, pp 9-10; and Naylor, 1971, pp 159-160.

35. An implicit assumption of any test for structural stability is that the model is correctly specified. Rejection of the hypothesis that no structural shift has occured may indicate that the model is in fact misspecified. Ramsey, 1969, has developed tests of misspecification.

36. Jorgenson, 1970, p 218. The test is from Chow, 1960.

structural shift statistic as described by Chow[37] has an F distribution. Cooper described a less powerful alternative structural shift statistic, i.e., that of the adjusted ratio of the sum of squared residuals over the forecast period to the reduced form mean squared error over the fitted period, which has a Chi-square distribution.

While stability may be desirable in a forecasting model, a clear-cut relationship does not necessarily exist between tests for stability and the forecasting ability of a model. [Cooper, 1972, p 919]. The techniques described ignore the interdependence in simultaneous equation models. Also coefficients may change over time in such a way that forecasting performance of the model improves. And, of course, the test for structural change may fail (i.e. a Type I error is made).[38]

Another approach to the evaluation of predictive performance is to ask whether the model forecasts better than a naive or mechanistic model. The simplest naive models are the no-change model

$$(2.10) \quad \hat{Y}_t = Y_{t-1}$$

and the same change model

$$(2.11) \quad \hat{Y}_t = 2\, Y_{t-1} - Y_{t-2}.$$

37. This test is described by Kmenta, 1971, p 370.

38. See p 26 where we evaluate the actual performance of the test.

There are, of course, other forms for naive models.

In recent years a growing use has been made of the autoregressive model[39]

(2.12) $\hat{Y}_t = \sum_{i=1}^{p} B_i Y_{t-i}$

as a standard of comparison for econometric models.[40] A more sophisticated mechanistic model is the Box-Jenkins model. If the process being specified is stationary, i.e. the process repeatedly returns to the neighborhood of the mean, then the process can be discribed by a combination of an autoregressive equation (such as just described) and a moving average equation:[41]

(2.13) $\hat{Y}_t = A_0 + \sum_{i=1}^{p} B_i Y_{t-i} + u_t - \sum_{j=1}^{q} A_j u_{t-j}.$

The criteria for proper identification (i.e. what are the optimum values of p and q) are minimization of the number of parameters and minimization of the mean squared residuals. When the process is not stationary (e.g. GNP, prices), successive differences are taken until the model exhibits stationarity. First differences are usually sufficient to bring about a stationary situation with economic data [Nelson, 1973, chapter IV, section 1]. A model whose

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39. For a description of such models, see Klein, 1968, p 43; or Cooper, 1972, pp 830-831.

40. p is usually chosen so that standard error of the model is minimized.

41. $u_{t-j} = Y_{t-j} - \hat{Y}_{t-j}$. A_0 is the mean of Y.

values of p and q are both 1, and where stationarity results from taking the first difference would be

$$(2.14) \qquad Y_t = B_1 \Delta Y_{t-1} + A_0 + u_t - A_1 u_{t-1}.$$

Identification and estimation of the model requires iterative techniques for which computer programs exist. [Nelson, 1973].[42]

Another statistic used to evaluate the predictive performance of a model is the Theil inequality coefficient [Theil, 1966, pp 27-28]--the ratio of the MSE statistic of the model to the MSE of the no-change naive model[43]

$$(2.15) \qquad U^2 = \frac{\sum_{t=1}^{n} (\hat{Y}_t - Y_t)^2}{\sum_{t=1}^{n} Y_t^2} .$$

The value of U^2 will be a positive number. The model which predicts perfectly has $U^2 = 0$; if the forecast performed only as well as the no-change model, then $U^2 = 1$.

A variation of this statistic is the Janus coefficient[44]

$$(2.16) \qquad J^2 = \frac{(1/m) \sum_{t=n+1}^{n+m} (\hat{Y}_t - Y_t)^2}{(1/n) \sum_{t=1}^{n} (\hat{Y}_t - Y_t)^2}$$

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42. Box, 1970 presents this model in all its rigor. Nelson, 1972A, pp 11-14 has a brief summary of the above while Nelson, 1973 gives a rather complete explanation of the technique.

43. In some write-ups of the coefficient, $\hat{Y}_t$ and Y_t are first differences. See Theil, 1961 and Theil, 1966.

44. Gadd, 1964.

This coefficient resembles the F ratio, but since time series data usually exhibit autocorrelation and the forecast values in the numerator are obtained by extrapolation from those in the denominator, it does not have a F distribution.

The numerator of the Theil and the Janus coefficients are sometimes decomposed and these decompositions are said to measure the degree of bias, unequal variation, and incomplete variation in the forecasting model. [Theil, 1966]. But Jorgenson [1970] has questioned the usefulness of the decomposition. This issue will be discussed in the Appendix of this chapter.

An evaluation technique for single period ex ante forecasts made with simultaneous equation models has been outlined by Haitovsky [1970]. Haitovskey examined the error of the individual equations when ex ante and ex post data are used, as well as the effect of three forms of constant adjustment.[45] The effects that different data sets and different constant adjustments have on the entire system is also examined. This technique revealed that contant adjustments did improve the 1966-3 Wharton-EFU forecast [Haitovsky, 1970], and the 1968-3 OBE forecast performed better with ex ante values of the exogenous than with ex post values. [Su, 1971].

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45. The three forms of constant adjustment were no adjustment, average residual adjustment ($A = [u_{t-1} + u_{t-2}] / 2$), and the adjustment actually used by the forecaster.

D. STUDIES OF FORECASTING PERFORMANCE

Studies of the forecasting ability of money stock models, unfortunately, are rare. With the exception of work done by the Research Department of the Federal Reserve Bank of St. Louis,[46] the predictive performance of monetary models is at best a side issue. Tobin and Swan [Tobin, 1969], for example, maintained that the forecasting superiority of a naive model and a trend model over two examples of a Friedman-Swartz money model lessened the significance of the relationship between the money stock and Friedman's permanent income. Christ's [1971] comparison of models of the financial sector described the predictive performance of the models, but concentrated on the model's structure and theoretical foundation rather than its actual performance.

This section describes the measurement of forecasting performance of econometric models other than money stock models as a means of illustrating forecasting evaluation techniques.

1. Comparison with mechanistic models

Frequently the prediction performance of an econometric model is compared to that of a naive model.[47] Christ [1956],

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46. This research will be discussed in the following chapters.

47. There are test statistics which make implicit use of such criteria. The R^2 statistic, for example, compares the explanatory power of the regression against the average value of the dependent variable.

for example, compared a no-change naive model with the Klein-Goldburger model and found that the naive model frequently forecast certain components of GNP better than the econometric model. Evans [1972, p 967] in his large-scale study used both a no-change and a same-change model as standards of comparison.[48] A variety of naive models has been used. Smyth [1966] compared three specifications of the same-change naive model to forecast the components of Australia's GNP[49] along with an average (for the entire period) change model. In this case, only the average change model[50] occasionally outperformed the econometric model. Some studies of judgmental forecasts have also used naive models as a standard of comparison. [Zarnowitz, 1967, pp 83-88].

The autoregressive model has come into favor as a standard of comparison for forecasting models.[51] For example, Green [1972, p 51] compared autoregressive models with 2 and 4 lagged dependent variables to various equations of

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48. His same-change model for forecasts up to 6 months was

$$\hat{Y}_{t+T} = Y_t + T\,(Y_t - Y_{t-1})$$

where T is the length of the forecast.

49. These were

$$\%\Delta Y_t = \%\Delta Y_{t-1};$$

$$\%\Delta Y_t = (1/3)\textstyle\sum_{i=1}^{3} \%\Delta Y_{t-i}; \text{ and}$$

$$\%\Delta Y_t = (1/5)\textstyle\sum_{i=1}^{5} \%\Delta Y_{t-i}.$$

50. This of course is an ex post concept.

51. Equation 2.12.

the OBE model in predicting components of GNP. The OBE model proved to be superior to the autoregressive models. Jorgenson [1970, pp 203, 208] found that a fourth order autoregressive model for different investment components predicted better than the poorer performing econometric models in his study. But this was an arbitrary way to choose the number of lagged terms for the autoregressive model of each investment component. Cooper [1972, pp 830-831] chose the number of variables which would minimize the RMSE of the equations and found that the autoregressive model frequently outperformed (i.e. had smaller RMSE) the equations of some large econometric models of the U. S. economy.[52]

The Box-Jenkins generalization of the mechanistic model[53] was used in Nelson's [1972A] study of the term structure of interesst rates, and again [Nelson, 1972B] in his study of the Penn-FRB-MIT model.[54] In the latter case, within the

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52. Cooper, 1972, p 917 summarized his results by choosing the best performing model for 33 endogenous variables. No single model was superior. Of the 33 variables, the econometric model performed best for both the period of fit and of forecast 6 times, the autoregressive model performed best 13 times, and the autoregressive model performed best for the period of fit 8 times, and for the forecast period 6 times.

53. Equations 2.12 and 2.13.

54. Two examples from the Penn-FRB-MIT model study (Nelson, 1972B, p 915) are the equations for gross national product

$$GNP_t = GNP_{t-1} + .615(GNP_{t-1} - GNP_{t-2}) + 2.76 + u_t$$

and non-farm inventory investment

estimation period the econometric model itself contributed much towards minimizing forecasting error; but outside the estimation period the Box-Jenkins model tended to perform better.[55] It seems probable that this model will supercede the other mechanistic models in those cases where time or expense is not a constraint.

2. Other prediction evaluation statistics

The Theil inequality statistic has been widely used for prediction evaluation. For an example, see much of the work of Stekler [1970]. Use of the decomposition of the Theil inequality statistic is also widespread. Theil has used the decomposition on a range of projects [Theil, 1961, 1966] as had Kuh [1963] in his study of capital investment, Smyth [1966] in his examination of Australian judgmental forecasts, and Evans [1972] in his analysis of large scale models of the U.S. economy. But nowhere in these investigations was an attempt made to discuss the Jorgenson critique of the approach.

The test for structural shift has been used, but the

$$I_t = .581\, I_{t-1} + 1.69 + u_t + .0013\, u_{t-1} + .742\, u_{t-2}.$$

The first equation illustrates the difference and autoregressive forms of the model, the second the autoregressive and moving average forms of the model.

55. Nelson also used a composit prediction using both models. The composit model was better than the Penn-FRB-MIT model for 12 out of 14 variables while it was better than the Box-Jenkins model for only 7 out of 14 cases. Nelson, 1972B, p 915.

test results at times conflict with other prediction evaluation statistics. For example, the MSE for real total consumer expenditures over the forecast period in Cooper [1972, p 875] study is lowest for the OBE model yet this model exhibited the greatest degree of structural shift [p 902]. This pattern, however is not consistent. Current plant and equipment expenditure, for example, in the Fromm model has the lowest MSE [p 877] and least structural shift [p 903]. Such inconsistancy is also seen in Stekler's [1970, pp 65, 69] comparison of inventory forecasts. Ranking forecasts by the Theil inequality statistic differs from ranking by the structural shift test statistic. For example, See Table 2.1 where ten models are ranked by relative lack of structural shift[56] and the inequality statistic for two different time periods. While structural stability is a desirable characteristic for a forecasting model, it is obviously not necessary for good forecasting performance.

TABLE 2.1[57]

RANKING OF PREDICTION TEST STATISTICS

Lack of Shift	1	2	3	4	5	6	7	8	9	10
U1	1	9	6	3	5	2	4	7	8	10
U2	1	10	2	4	9	3	5	9	8	6

56. Since the degrees of freedom associated with each of the F ratios are different, it is the significance level which is ranked.

57. From Stekler, 1970, p 65, Table 3-1, and p 69, Table 3-4. The first row is the ranking of each model based on the Chow test for structural shift. The other two rows give the rank of the model cited in the first row based on the Theil

E. SUMMARY

This chapter has presented a range of statistics which can be used to measure the predictive performance of econometric models. These statistics will be used to evaluate the forecasts made by the money stock models which are discussed in the next chapter.

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inequality statistic. U1 is for the period 1948-3 to 1964-4; U2 is for the 1953-1 to 1964-4 period.

APPENDIX TO CHAPTER II

THE THEIL INEQUALITY COEFFICIENT

Theil attempted to evaluate the predictive performance of an econometric model by means of what he called an inequality coefficient: the scaled[1] mean squared error[2]

$$(2A.1) \qquad (1/n) \sum_{i=1}^{n} (P_i - A_i)^2$$

The most useful characteristic of the coefficient, Theil maintained, was the decomposed form of the coefficient which measured three attributes of the forecasting model.

The usual form of the decomposition was

$$(2A.2) \qquad (1/n)\sum_{i=1}^{n}(P_i - A_i)^2 = (\bar{P} - \bar{A})^2 + (s_P - s_A)^2 + (1 - r^2)\, s_P\, s_A$$

where

$$s_P^2 = (1/n) \sum_{i=1}^{n} (P_i - \bar{P})^2,$$

$$s_A^2 = (1/n) \sum_{i=1}^{n} (A_i - \bar{A})^2, \text{ and}$$

---

1. Theil used at least two different scaling factors:

$\left(\sqrt{(1/n)\Sigma P_i^2} + \sqrt{(1/n)\Sigma A_i^2} \right)^2$ (Theil, 1961, p 32), and

$(1/n)\Sigma A_i^2$ (Theil, 1966, p 28).

2. P_i is the predicted value and A_i the actual value of the predicted variable.

$$r = \frac{(1/n)\ \Sigma\ (P_i - \bar{P})\ (A_i - \bar{A})}{s_P\ s_A} .$$

The first term of the decomposition according to Theil measured bias, the second measured unequal variance, and the third measured unequal covariance.[3] Theil maintained that if the values of the first two decomposed terms approached zero, this spoke highly of the predictive performance of the model.[4] Theil's only defense of this argument was graphical.[5] In spite of this rather casual approach towards validating this method of prediction evaluation, the inequality coefficient and its decomposition have been widely used.[6]

Jorgenson, however, in a recent article questioned the meaningfulness of the decomposed statistics by examining the expected values of the first two decomposed terms.[7] The expected values of the first decomposed term was found to be[8]

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3. Theil, 1961, p 34-35; Theil, 1966, pp 30-32. An alternative decomposition is given in Theil, 1966, p 33; but this does not change the argument which follows.

4. This, of course, implied that the third term would in such a case approach the value of the scaling factor.

5. Theil, 1961, p 36; Theil, 1966, p 31.

6. For example, see Stekler, 1970; and Cooper, 1972.

7. Jorgenson, 1970.

8. $\Sigma x_i'^2 = \Sigma (X_i - \bar{X})^2$ where X_i is the value of the explanatory variable at the time of prediction and $\bar{X}$ is the

$$\text{(2A.3)} \qquad E(\bar{P} - \bar{A})^2 = \sigma^2 \left(\frac{1}{n} + \frac{(X_i - \bar{X})^2}{\Sigma x_i'^2} \right)$$

This term clearly does not approach zero, but rather σ^2/n from above, and it increases as the difference between the actual value of the explanatory variable in the forecasting equation and the average value of the explanatory variable during the estimation period increases.

The expected value of the second decomposed term can be written:

$$\text{(2A.4)} \qquad E(s_P - s_A)^2 = E(s_P^2 + s_A^2 - 2\, s_P\, s_A)$$

Then term by term:

$$\text{(2A.5)} \qquad E(s_P^2) = E[\hat{P}_i - E(P_i)]^2$$

$$= E[(\hat{\alpha} + \hat{\beta}X_i) - (\alpha - \beta X_i)]^2$$

$$= E(\hat{\alpha} - \alpha)^2 + E(\hat{\beta} - \beta)^2 X_i^2 + 2E(\hat{\alpha} - \alpha)(\hat{\beta} - \beta)X_i$$

$$= \text{Var}(\hat{\alpha}) + X_i^2\text{Var}(\hat{\beta}) + 2X_i\text{Cov}(\hat{\alpha},\hat{\beta})$$

$$= \sigma^2 \; [(1/n) + (\bar{X}^2/\Sigma x_i'^2)] + X_i[\sigma^2/\Sigma x_i'^2] - 2X_i\bar{X}\; [\sigma^2/\Sigma x_i'\;]$$

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average value of the explanatory variable during the estimation period. See Kmenta, 1971, p 228. We are using Theil's notation.

$$= \sigma^2 \; [(1/n) + (X_i - \overline{X})^2/\Sigma x_i'^2]$$

(2A.5a) $E(s_A^2) = E[A_i - E(A_i)]^2 = \sigma^2$

(2A.5b) $E(s_P s_A) = E[\hat{P}_i - E(P_i)][\hat{A}_i - E(A_i)]$

$$= E(\hat{\alpha} + \hat{\beta}X_i - \alpha - \beta X_i)(\varepsilon_o).$$

but ε_o is independent of sample disturbances of the observations used to estimate α and β, therefore

(2A.5b') $E(s_P s_A) = 0.$

Therefore

(2A.6) $E(s_P - s_A)^2 = \sigma^2[1 + (1/n) + (X_i - \overline{X})^2/\Sigma x_i'^2]$

which is the total variance of the forecast error.[9]

Therefore the predictionevaluation approach proposed by Theil can be misleading. The expected value of the squared mean error of prediction is not zero, nor is the variance of the predictor and the variance of the observations to be predicted the same. Other methods of prediction evaluation are called for.

9. Kmenta, 1971, pp 228, 240.

CHAPTER III

ECONOMETRIC MODELS OF THE MONEY STOCK

Models which attempt to specify the behavior of the money stock range from naive models to the large econometric models constructed by committee. Since naive models lack economic content, they will be ignored in this chapter. A cross-section of other money stock models which have the money stock as the endogenous variable and, therefore, can be used to forecast the money stock will be presented in this chapter. Money in this dissertation is define to be M_1, i.e. currency and demand deposits held by the non-bank public.

A. MONEY MULTIPLIER MODELS

The relationship between the money stock and the money base can be specified by the money multiplier:

$$(3.1) \qquad M = mB$$

where m is the money multiplier and B the monetary base. There are a number of ways of defining the monetary base. Brunner and Meltzer have defined three different base concepts. The economists of the Federal Reserve Bank of St.

Louis refer to one concept as the source base,[1] and a second as the monetary base.[2] In this study the net source base, the sum of unborrowed reserves and currency held by the public or not included in reserves, will be used as the monetary base.[3] It is felt that this aggregate is most subject to control by the Federal Reserve, and for this reason has been the more widely used base concept.[4] The use of the net source base will facilitate comparison with other multiplier models.

The money multiplier can be decomposed into a combination of ratios which reflect the behavior of the public, the banking system, and the monetary authorities. The multiplier decomposition for the net source base can be written:

$$(3.2) \qquad m = (1 + k) / [(r - b)(1 + t + d) + k].$$

k is the ratio of currency held by the public to demand deposits held by the public, r the ratio of total reserves to total deposits, b the ratio of borrowed reserves to total deposits, t the ratio of time deposits to demand deposits held by the public, d the ratio of Treasury

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1. The sum of member bank reserves, currency held by member banks (not already included in reserves), and currency held by the public.

2. The monetary base is the source base plus a reserve adjustment which compensates for changes in the reserve requirement. See Andersen, 1968.

3. This also is the terminology of the economists of the Federal Reserve Bank of St. Louis.

4. For example, see Burger, 1972; and Pierce, 1973.

deposits to demand deposits held by the public. Brunner and Meltzer constructed their non-linear money supply hypothesis with this equation. The ratio reflecting the behavior of the U. S. Treasury (d) they assumed to be exogenous while the currency ratio (k), they maintained, influences, but is not influenced by, the current monetary process. [Brunner, 1966, p 155]. The remaining ratios can then be specified by the following relationships:[5]

(3.3) r = r(RRD, RRT, RMTB, RMFR)

(3.4) b = b(RMTB, RMCP, RMFR), and

(3.5) t = t(RMTD, RMTB, RMCP, YP).

Brunner and Meltzer never published estimated coefficents for these three equations. In their published work on the non-linear money supply model, they used as their explanatory variable the linear combination of the logarithms of the base and the ratios each weighted by its appropriate elasticity. [Brunner, 1964, pp 274-282]. While this was a clever way of avoiding the practical difficulties associated with estimating the ratios, it avoided empirically testing the validity of Equations 3.3 to 3.5, and the linear combination variable would seem to be imappropriate within a forecasting situation.

A model where the ratios were considered to be endogenous

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5. Notation is listed in the Notation Appendix.

was estimated by Hosek [1970]. His multiplier decomposition

$$(3.6) \qquad m = (1 + k) / [(r + e)(1 + t) + k]$$

where r is the ratio of required reserves to total deposits and e is the ratio of excess reserves to total deposits. (The balance of the notation is the same as above). Since the r ratio is determined by the Federal Reserve,[6] only three equations need to be estimated:

$$(3.7) \qquad k = \beta_{10} + \beta_{11}[RMTB/(RMTB+RMDD)] + \beta_{12}YP + u_1$$

$$(3.8) \qquad t = \beta_{20} + \beta_{21}[(RMTB-RMTD)/(RMTB+RMDD)] + \beta_{22}YP + u_2$$

$$(3.9) \qquad e = \beta_{30} + \beta_{31}[RMTB-RMFR] + \beta_{32}t + \sum_{i=5}^{5} \beta_{3i}S_i + u_3$$

Hosek, however, made no attempt to determine how well his model would forecast the multiplier, let alone the money stock.[7] Kalish [1970] ruled out using the multiplier decomposition to forecast the money stock because he maintained its use would require forecasting expectations and the interest rate.[8] Kalish, instead, used various verisons

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6. Hosek showed that r = [(RRD + RRT*t) / (+ t)]. The statistical properties of the model were never discussed.

7. Hosek's published test statistics looked respectable, but the unpublished results show that the three equations had low Durbin-Watson statistics.

8. An early version of this paper (St. Louis Federal Reserve Working Paper No. 11) omitted this explanation for abandoning the decomposition approach.

of a naive model to forecast the multiplier, but with poor results. Burger, a colleague of Kalish's at the Federal Reserve Bank of St. Louis, asserted that the multiplier could be determined by a moving average of previous multipliers, the percent change of the Treasury bill rate, and seasonal factors.[9] The model was run twice using two different money base concepts: the net source base and reserves held against privately held deposits (RPD's)--the present target variable for the Open Market Committee of the Federal Reserve System. Burger observed that the RPD multiplier model displayed larger standard errors then did the net source model.[10]

B. MONEY SUPPLY MODELS

To talk about single equation money supply or money demand models is not really proper. Such models at best will be reduced form equations of the money stock. Nevertheless those models using exogenous variables which appear to reflect the behavior of the banking system will be called money suuply models,[11] and those models whose exogenous

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9. The equation was also corrected for first order autocorrelation.

10. Burger's results will be discussed in more detail in Chapter IV.

11. A free reserves equation is another approach to specifying the money supply process. For example, see the money supply equation of Teigen's model described below. But within a single-equation money stock forecasting model context, such an appreach is inappropriate.

variables reflect primarily the behavior of the public will be called money demand models. Since the Federal Reserve and the banking system see themselves merely responding to the needs of the economy, the money supply models can be viewed as the reaction function of the banking system. The money demand model resembles the conventional demand model in that it has both price and income as independent variables.

The money stock in most models which include a financial sector is assumed to be exogenous and when at equilibrium is equal to the demand for money. Little attention has been given to the money supply function. This is a neglect of long standing,[12] and is a condition that seems to persist.[13]

Brunner and Meltzer have paid the most attention to the money supply function. They have developed a linear and a non-linear money supply hypothesis. In the linear hypothesis the money supply is explained by the public's relative demand for currency and time deposits, and a monetary base. The three monetary base concepts they use are total reserves plus currency (the St. Louis source base), total reserves plus currency plus an adjustment for reserves

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12. Brunner, 1964, pp 242-243 summarized what little work was done up to 1964 in an annotated bibliography in a relatively brief footnote.

13. For example, based on the title of a article by Starleaf, 1970 ("The specification of money demand-supply models..."), one would expect an explicitly stated money supply function, but instead M_s was exogenous and equal to M_d.

liberated through changes in the required reserve ratios (the St. Louis monetary base), and unborrowed reserves plus currency plus the liberated reserves adjustment (the net source base plus liberated reserves). Equations for estimating the money stock using the different bases are:[14]

$$(3.10a)\quad M = \beta_{10} + \beta_{11}(CURN + RST) - \beta_{12}k - \beta_{13}t - \beta_{14}r + e_1$$

$$(3.10b)\quad M = \beta_{20} + \beta_{21}(CURN + RST + RSL) - \beta_{22}k - \beta_{23}t + e_2$$

$$(3.10c)\quad M = \beta_{30} + \beta_{31}(CURN + RSU + RSL) - \beta_{32}k - \beta_{33}t + e_3$$

This model will be examined more fully in Chapter V. The other Brunner-Meltzer money supply hypothesis, their non-linear model, with the decomposed multiplier as its focal point was discussed above.

Other models which have tended to view the money stock from the suppliers side are few. But examples do exist. Teigen [1964] and Gibson [1972] in models which will be discussed below[15] estimated money supply equations where the money stock was the function of an interest rate and for Gibson a reserve variable. Hendershott [1970] in a

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14. The ratios r, k, and t are as defined above. The balance of the notation can be found in the Notation Appendix.

15. See pp 43-46.

study of the relationship between free reserves and interest rates developed an equation which had demand deposits being a function of the discount rate, the Treasury bill rate, lagged free reserves, changes in unborrowed reserves, changes in commercial loans, and borrowed reserves with all the explanatory variables divided by the ratio of required reserves to total commercial bank deposits. [Hendershott, 1970, p 606]. This is essentially a demand deposits supply function. Weintraub [1970], in a very simple model investigating the "better" definition of money, M_1 or M_2, maintained that money was a linear function of the money base (defined as currency held by the public plus reserves deflated by the CPI) and the reciprocal of the interest rate. [Weintraub, 1970, p 115]. Even Weintraub admitted that such a model was a bit austere.

C. MONEY DEMAND MODELS

The literature on the demand for money is voluminous and the range of models considered is wide. Some of the models are simply regressions of various specifications of income (or wealth) and interest rates on money [Heller, 1965] while others have an elaborate structure of expectations and adjustments. [Feige, 1967]. Some have an estimated polynominal lag structure for each of the explanatory variables [Dickson, 1972], others attempt to reveal structural stability of the functions by being estimated over different time periods. [Laidler, 1966]. Even the conventional linear equation form was by-passed to investigate the proper structural form of the money demand function. [Zarambka, 1968].

However within a forecasting context much of this is not relevant. The theoretical justification of the models is not an important consideration although the models used should make economic sense. Also the variables used in the models should be available to the forecaster[16] and not be excessively difficult to obtain.[17] Preferably, the functional form of the equations should not be complex.

Within this context, it seems appropriate to view money as a function of an economic activity variable and of some interest rate. The question of proper functional form for the equation must also be answered. The Zarembka study mentioned above provides an answer to the functional form question. He estimated the model[18]

$$(3.11) \qquad M_t^{\lambda} = \beta_0 + \beta_1 Y_t^{\lambda} + \beta_2 RMCP_t^{\lambda} + \beta_3 M_{t-1}^{\lambda} + e_t$$

and found that the standard error of estimate was minimum when λ was close to zero. [Zarembka, 1968, p 509].

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16. Certain data such as GNP and the other national income series are available only on a quarterly basis, and therefore can not be used in a monthly forecasting model. This for the most part restricts the study to quarterly models.

17. In a forecasting model, ideally the exogenous variables should be either controlable or easier to forecast than the endogenous variable.

18. Y in this case is Kuznet's estimate of net national product. Annual data was used in estimating the equation.

This suggests that the proper functional form for a money demand model is log-log. The specification error tests used by Gilbert, moreover, revealed that the log-log form of the demand function was less likely to display specification error than the linear form of the function. [Gilbert, 1969, pp 80-81].

Two economic activity variables, widely used in money demand equations, will be used here: GNP and permanent income: the latter is derivable as an exponentially weighted sum of past GNP and is usually viewed as a wealth variable. Two interest rates will also be used --a short term rate (the 90-day commercial paper rate) and a long-term rate (Moody's Aaa 20-year corporate bond index). A lagged dependent variable will also be included in the model. The theoretical justification for this would be that the money stock does not adjust within the quarter to changes in income or the interest rate. Moreover, it has been observed that the inclusion of the lagged variable in a forecasting model usually makes a substantial improvement in forecasting performance. Gilbert in his study also observed that money demand equations with the lagged dependent variable were less subject to specification error. [Gilbert, 1969, p 73].

The quantity variables in money demand functions are usually expressed in aggregate nominal terms. The Gilbert analysis suggests that scaling the quantity variables by either a price index or population will improve the specification of the model.[19] [Gilbert, 1969, p 80]. In fact, most of the models which were not rejected by the Ramsey specification error tests were expressed in real, per capita terms.[20] In Chapter V money demand functions expressed in real or in per capita terms will be presented.

D. SIMULTANEOUS EQUATION MODELS

1. Money supply and demand models

An econometric money stock forecasting model, if limited to two or three equations, should have at least a money supply equation and a money demand equation. This, however, is either rarely done, or, as is usually the case, the money supply is assumed to be exogenous and to equal the money demand as a condition of equilibrium.

One small rudimentary money stock model with both a money supply and demand equation is that of Weintraub. [Weintraub, 1970]. Briefly the model is[21]

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19. Scaling by population can be justified if the per capita income elasticity is other than one.

20. All but one of Gilbert's accepted functions were expressed in real terms, all but two in per capita terms.

21. Permanent income in this case is defined as

$$Y_p = (4Y_t + 2Y_{t-1} + Y_{t-2}) / 7.$$

$$(3.12)\quad M_d = \beta_{10} + \beta_{11}(1 / RMCP) + \beta_{12}YP + e_1$$

$$(3.13)\quad M_s = \beta_{20} + \beta_{21}B + \beta_{22}(1 / RMCP) + e_2$$

where B is the real money base: currency held by the public and reserves deflated by the CPI. This model while never intended to be used as a forecasting model of the money stock does illustrate rather starkly the supply and demand aspects of a money stock model.

Brunner and Meltzer specified both money supply and money demand equations in their empirical work. Their money supply functions are discribed above.[22] Money in their typical money demand function depends on the bond yield, real wealth,[23] the national income price deflator, and in some cases a measure of transitory income--the ratio of current net income to Friedman's permanent income.[24] The complexity of the Brunner-Meltzer approach and its use of annual data rule it out for forecasting purposes.

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22. See p 39.

23. They used an adjusted Goldsmith measure of the public's tangible and non-human wealth deflated according to an appropriate price index supplied by Goldsmith. Brunner, 1964, p 266. This series is an annual series.

24. Brunner, 1964, pp 266-267, 276-277. The transitory measure was never used with the non-linear money supply equation. Brunner estimated this model by means of TSLS.

2. The Teigen model

Teigen [1964] constructed a simultaneous equation model where money stock, short-term interest rate and income were the endogenous variables. The three structural equations were a demand for money function, a free reserves function, and an income equation. The dependent variable in the free reserves function was a bit unorthodox: the ratio of the money stock to that segment of the money stock that was based on supplied (exogenous) reserves, or using Teigen's notation, M/M*.[25] The model was

$$(3.14)\qquad M = \beta_{10} + \beta_{11}RMCP * Y + \beta_{12}Y + \beta_{13}M_{t-1} + \textstyle\sum_{i=4}^{6}\beta_{1i}S_i + e_1$$

$$(3.15)\qquad M/M^* = \beta_{20} + \beta_{21}(RMCP - RMFR) + \beta_{22}S_4 + \beta_{23}S_5 + e_2$$

$$(3.16)\qquad Y = \beta_{30} + \beta_{31}E + \beta_{32}NW + \beta_{33}Y + \textstyle\sum_{i=4}^{6}\beta_{3i}S_i + e_3$$

where Y is GNP, E exogenous expenditure and NW net worth all three variables measured at current prices. The model was estimated using 1946-4 to 1959-4 data with the

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25. $M^* = [(k)/(1 - c - h)](R^S)$ where k is the ratio of private demand deposits of member banks to reserves required for those demand deposits, c the ratio of currency held by the public to the money stock, h the ratio of private demand deposits held by non-member banks to the money stock, and R^S is unborrowed reserves. Teigen, 1964, p 479.

stock variables, measured at each quarterly call date. [Teigen, 1964, p 488, f 18]. While some of the variables used in this model are not conventional, they are relatively stable and are readily available; therefore they are useful for forecasting purposes.

Gibson [1972], in a critique of the Teigen study, criticized that Teigen's money supply equation was in fact a free reserves equation[26] and that his demand equation incorporated the product of GNP and the interest rate. Gibson maintained that a more meaningful and useful way of expressing the first two equations of the model was:[27]

$$(3.17) \quad \ln M_d = \beta_{10} + \beta_{11} \ln RMCP + \beta_{12} \ln Y + \beta_{13} \ln M_{t-1} + \sum_{i=4}^{6} \beta_{1i\ i} + e_1$$

$$(3.18) \quad \ln M_s = \beta_{20} + \beta_{21}(RST + RSL) + \beta_{22} RMCP + \beta_{23} RMFR + \beta_{24} D_1 + e_2$$

In estimating these regressions Gibson used average data for the quarter rather than the call report data used by Teigen. Gibson reestimated the Teigen model with the average data, and obtained estimated coefficients with respectable goodness-of-fit statistics which were quite

26. Teigen titled the section where his free reserves equation was developed "The Supply-of-Money Function." He was well in the mainstream doing this.

27. Gibson left Equation 3.16 the same.

different than Teigen's coefficients.[28] In Chapter V and Chapter VI the money supply functions will be reestimated.[29]

3. Large econometric models

In recent years econometric model builders have begun to intensively study the financial sector. Early simultaneous econometric models dealt with the real sector and ignored the financial sector since, it was maintained, money and interest rates contributed little to the models.[30] But in recent years models have been developed explicitly to deal with the financial sector.[31] The structure or theoretical foundations of such models will not be discussed here,[32] nor can the money stock forecasting record

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28. This was likely the most significant of Gibson's observations. Teigen used the constraint on interest rates seen in Equation 3.15 because the signs of both coefficients when the interest rates were not so constrained were opposite to what economic theory would expect, and the coefficient of RMCP was insignificant. Gibson had similar results with both coefficients being insignificant.

29. Equations 3.15 and 3.18 will be estimated by OLS in Chapter V and Equation 3.18 by TSLS in Chapter VI.

30. Klein, a leading figure in the development of large econometric macro-models, once stated that "I have tried hard over the years, in several models, to give the benefit of every doubt to money and interest rates when making statistical estimates. My empirical verdict, thus far, is that little evidence can be found for the actual influence of money or interest on real activity." Klein, 1964, p 56.

31. The standard example of this is the FRB-MIT model.

32. See Christ, 1971. He takes a close look at nine financial sector models: four versions of the FRB-MIT

of any of these models be discussed, since such a record does not exist.

E. SUMMARY

The cross-section of money stock models presented in this chapter will be used as forecasting models in subsequent chapters. They are the best known of the small money stock models that can be used for forecasting. The multiplier models as well as the naive models discussed in Chapter II is the subject of Chapter IV, the one-equation models the subject of Chapter V, and the two-equation models the subject of Chapter VI.

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model, the Wharton-EFU model, the OBE model, the Suits (quarterly) Michigan model, the deLeeuw model in the condensed Brookings model, and the original Brookings model. This study of Christ's was based on the published results of the various models and their use.

CHAPTER IV

PREDICTIVE PERFORMANCE OF NAIVE MONEY STOCK MODELS

A. THE MODELS

Mechanistic models which use only the money stock time series are the most elementary money stock models. While these models lack economic content,[1] they are capable of making reasonably accurate forecasts, and for this reason they are frequently used as a yardstick to determine how much has been gained by using more sophisticated models.

Various mechanistic models capable of forecasting the money stock will be described in this chapter. After outlining the prediction evaluation statistics, the predictive performance of the various models described will be examined. In subsequent chapters the mechanistic model with the best prediction record will be compared to the other money stock forecasting models considered in this study.

1. Mechanistic models of the money stock

The various mechanistic models have been described in Chapter II. Since the autoregressive model is a

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1. Mechanistic models have no explanatory variables other than lagged values of the dependent variable. Such models, therefore, are of little interest to the economist except as a forecasting tool.

generalization of the no-change and the same-change naive models, the autoregressive model will be used as the mechanistic forecasting model of the money stock.[2]

2. The money multiplier models

The frequently postulated tight relationship between the money stock and the monetary base is shown by the money multiplier which, while not constant, is rather stable. A mechanistic money multiplier model is, therefore, an alternative method of forecasting the money stock. Following the spirit of Burger's study,[3] the multiplier model using two monetary base concepts will be examined.

The first money multiplier model will use the net source base,[4] while the second model will use the new reserves target of the Federal Reserve System's Open Market Operations Committee: RPD's. Since the base is considered to be an exogenous variable and assumed to be known when forecasting, problems of data errors, data revisions, and the incorrect estimation of the exogenous variables are avoided. No-change, same-percent-change, and autoregressive models will be used to forecast the multiplier, and then by multiplying that forecast by the base, to forecast the money stock.

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2. The only exception to this is the decomposed money multiplier model which will use a same-change naive model. This will be discussed below.

3. See p 36.

4. This base is defined on p 33.

3. The decomposed money multiplier models

The money multiplier can also be expressed in terms of various ratios which are seen as reflecting the behavior of the various economic actors who jointly determine the size of the money stock. Following an approach used by Brunner and Meltzer[5] the expressions for the monetary base multiplier (m_1) and the RPD base multiplier (m_2) can be written as follows:

$$(4.1) \qquad m_1 \equiv \frac{1 + k}{r(a + t) + (1 + c)k}$$

$$(4.2 \qquad m_2 \equiv \frac{1 + k}{r'(a' + a''t)}$$

where m is the multiplier, k the ratio of currency to deposits, r the ratio of reserves to deposits, r' the ratio of RPD's to deposits, and t the ratio of time deposits to demand deposits. The a variables and c are blowup factors which scale the deposits and currency series to conform to the multiplier identity.[6] Since the no-change, decomposed multiplier model will make forecasts identical to those made by the no-change multiplier model, only the same-change decomposed multiplier will be examined for its forecasting ability.[7]

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5. For an example of this approach, see Brunner, 1966.

6. The Appendix to Chapter IV has the derivation of Equation 4.1 and 4.2, and a more precise definition of the variables.

7. The components of the decomposed models will probabily each change at a different rate, and, therefore, the

4. Summary

From this spectrum of mechanistic money stock models, the forecasting characteristics of six different models will be examined. Two will be autoregressive money stock models, one using seasonally adjusted money stock data and the other using not-seasonally adjusted money stock data with seasonal dummy variables. The other four models will be money multiplier models: a no-change naive model of the multiplier, a constant-percent-change naive model of the multiplier, a constant-percent-change naive model of the decomposed multiplier, and an autoregressive money multiplier model. Three different base concepts will be used with the first two multiplier models: the net source base, RPD's seasonally adjusted, and RPD's not seasonally adjusted.

Since in later chapters the seasonally adjusted money stock will be forecast, it is necessary to forecast the seasonally adjusted money stock in this chapter. Since some data series such as reserves do not exist in seasonally adjusted form,[8] a multiplicative seasonaly adjustment factor, the ratio of the seasonally adjusted money stock

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forecast of the same-change decomposed multiplier model will be different than the not-decomposed same-change model. This, of course, is not the case with the no-change decomposed model.

8. An exception is the RPD data series which is both seasonally adjusted and not seasonally adjusted. These series, however, only go back to 1960.

to the not-seasonally adjusted money stock, will be used to adjust the not-seasonally adjusted money stock which will be forecast by the multiplier model. The RPD multiplier models will provide an opportunity to compare forecasts seasonally adjusted as just described to forecasts using the seasonally adjusted base to directly forecast the seasonally adjusted money stock.

In the description of the models that follows, the subscript j represents a forecast of j months or quarters beyond time t . Values of M_{t+k} where k is 1 or greater are determined by the model, i.e. future money stock quantities are predicted values, not actual values. But the values of the monetary base, whether past or future, are actual values. The seasonal adjustment technique is included in the description. The autoregressive parameters are estimated using data outside the forecasting period.

1. Autoregressive SA money stock model.[9]

$$(4.3) \qquad M^{SA}_{t+j} = \sum_{i=1}^{p} \hat{\beta}_i M^{SA}_{t-i+j} + e_{t+j}$$

2. Autoregressive NSA money stock with fixed seasonal adjustments.

$$(4.4) \qquad M_{t+j} = \sum_{i=1}^{p} \hat{\beta}_i M^{NSA}_{t-i+j} + \sum_{k=1}^{A} \hat{\gamma}_k D_k + e_{t+j}$$

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9. The notation and data series to be used with the following models are given in the Notation Appendix. The superscript SA stands for seasonally adjusted, NSA for not-seasonally adjusted.

where $D_k = 1$ for month (or quarter) k, otherwise $= 0$; $A = 12$ for the monthly model, $A = 4$ for the quarterly model.[10]

3. Money multiplier no-change model.[11]

$$(4.5) \qquad M^{SA}_{t+j} = m_t (B^{NSA}_{t+j}) (M^{SA}_{t+j} \; / \; M^{NSA}_{t+j})$$

where

$$m_t = M^{NSA}_t \; / \; B^{NSA}_t .$$

4. Money multiplier same percent change model.[12]

$$(4.6) \qquad M^{SA}_{t+j} = (1 + \alpha_t)^j \; m_t \; (B^{NSA}_{t+j}) (M^{SA}_{t+j} \; / \; M^{NSA}_{t+j})$$

where

$$\alpha_t = (m_t - m_{t-1}) \; / \; m_{t-1} .$$

This model can also be written

$$(4.6a) \qquad M \quad = (m_t / m_{t-1})^j \; m_t \; (B^{NSA}_{t+j}) (M^{SA}_{t+j} \; / \; M^{NSA}_{t+j})$$

5. Ratio version of the money multiplier model, same-percent-change model (i.e. the decomposed multiplier model).

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10. The autoregressive equation does not have a constant term, and so there may be 4 seasonal dummies for the quarterly model, 12 for the monthly model.

11. Three different base concepts will be used with this model: the net source base and reserves against private deposits--both seasonally adjusted and not-seasonally adjusted.

12. The three base concepts will be used with this model.

$$(4.7) \qquad M_{t+j} = \big(\{1 + k_t(k_t/k_{t-1})^j\} \,/\, \{r_t(r_t/r_{t-1})^j [a_t(a_t/a_{t-1})^j + t_t(t_t/t_{t-1})^j] + [1 + c_t(c_t/c_{t-1})^j]\ k_t(k_t/k_{t-1})\}\big)\ B^{NSA}_{t+k}\ (M^{SA}_{t+k} \,/\, M^{NSA}_{t+k}).$$

6. The autoregressive money multiplier model.[13]

$$(4.8) \qquad m_{t+j} = \textstyle\sum_{i=1}^{p} \hat{\beta}_i m_{t-i+j} + e_{t+j}$$

B. EVALUATING PREDICTION PERFORMANCE

Ideally there should be a common basis for the comparisom of forecasting models. In this study, models with estimated parameters will, to the extent feasible, be estimated with 1947-1960 data, and forecasts will be for the 1961-1970 period. Since the RPD series only goes back to 1960, an RPD base autoregressive multiplier model is ruled out, but all other data series extend back to 1947. Therefore, autoregressive versions of the other models are possible.

There are a plethora of prediction evaluation statistics. A set of commonly used prediction evaluation statistics will be used to evaluate prediction performance over a one year

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13. This model will be estimated only for the net source base multiplier model.

time horizon. 1, 2, 3, 6, and 12 month forecasts and quarterly forecasts up to four quarters will be evaluated.

The primary prediction evaluation statistic used in this project will be the root mean squared error (RMSE) statistic.[14] The RMSE will be computed for the entire 1960 to 1970 period and for each year of the forecast period. The variability and trends exhibited by the annual RMSE statistic will serve to show the degree of stability of the tested models.

Another approach to prediction evaluation is to regress the predicted values on the actual values.[15] Since this approach is infrequently used, whether the implications of these statistics differ from the more conventional prediction evaluation statistics will be of interest.

There are also a number of mean error prediction statistics. The mean error itself is not computed since offsetting errors can make predictions look better than they actually are. This problem is avoided with the mean absolute error (MAE) statistic.[16]

C. RESULTS

In order to facilitate discussion about and comparison of the different models, the following notation will be

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14. See Equation 2.7, p 16.

15. See Equation 2.9.

16. See Equation 2.8.

used throughout this chapter. Each model will be identified by the letter M to indicate that the model is monthly or by the letter Q to indicate that it is a quarterly model. There will then be a number corresponding to the listing given below; and in the case of the money multiplier models (Models 3 and 4), the letters A, B, or C indicate the monetary base used. The numbers refer to the following models:

1. the autoregressive SA money stock model;
2. the autoregressive NSA money stock model with seasonal dummies;
3. multiplier model, the no-change version;
4. the multiplier model, the constant-percent-change version;
5. the decomposed multiplier model with constant-percent-change of ratios; and
6. the autoregressive multiplier model.

The letters, in turn, refer to the following base concepts:

A. the net source base;
B. RPD's seasonally adjusted; and
C. NSA RPD's with multiplicative seasonal adjustment factors.

1. Autoregressive models

Six autoregressive models will be estimated, three with monthly data, the remainder with quarterly data. The seasonally adjusted and the not-seasonally adjusted money stock and the net source base money multiplier autoregressive models will be estimated.[17] While an autoregressive version of the decomposed money multiplier model could have been estimated, since it would have required five qutoregressive equations, it was not estimated. Instead the same-percent-change decomposed multiplier model was used to give a feel for such a model. The disappointing results of the decomposed model[18] would seem to indicate that the decision not to estimate a decomposed autoregressive money multiplier model was correct.

The parameters of the autoregressive models were estimated for the 1947-1960 period.[19] The t-ratios for the coefficients are given in parenthesis below the estimated coefficients.

M1. Monthly autoregressive model of the SA money stock.

$$(4.9) \quad M_t = \underset{(15.1634)}{1.1905}\, M_{t-1} + \underset{(0.8903)}{0.1101}\, M_{t-2} - \underset{(-0.5222)}{0.0642}\, M_{t-3} - \underset{(-3.0063)}{0.2360}\, M_{t-4}$$

$$SE = 0.2913; \; \bar{R}^2 = .9993.$$

17. The model numbers for these three models are 1, 2, and 6.

18. These results are discussed on pp 62-67.

19. Because lagged values of the money stock were used in the equations, the actual period of estimation begins in 1948 for the monthly models, 1948-3 for the quarterly models.

M2. Monthly autoregressive model of the NSA money stock.[20]

$$(4.10)\quad M_t = \underset{(11.4338)}{0.9554}\, M_{t-1} + \underset{(0.7267)}{0.0837}\, M_{t-2} + \underset{(2.1796)}{0.2479}\, M_{t-3} - \underset{(-1.6164)}{0.1852}\, M_{t-4} - \underset{(-1.2651)}{0.1051}\, M_{t-5} + \sum_{i=1}^{12} b_i S_i$$

$$SE = 0.5415;\ \bar{R}^2 = .9976.$$

M6. Monthly autoregressive model of the monetary base money multiplier.

$$(4.11)\quad m_t = \underset{(1353.6030)}{1.0010}\, m_{t-1}$$

$$SE = 0.02473;\ \bar{R}^2 = .9721.$$

Q1. Quarterly autoregressive model of the SA money stock.

$$(4.12)\quad M_t = \underset{(11.1088)}{1.7268}\, M_{t-1} - \underset{(-2.7758)}{0.8715}\, M_{t-2} + \underset{(1.2030)}{0.4084}\, M_{t-3} - \underset{(-1.8607)}{0.5801}\, M_{t-4} + \underset{(2.0570)}{0.3189}\, M_{t-5}$$

$$SE = 0.5629,\ \bar{R}^2 = .9971.$$

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20. $S_i = 1$ when i equals the numerical value of the month, otherwise zero. b_i is the seasonal dummy. See Table 4.1 for estimated values of b_i.

21. The estimated coefficient was 1.00096.

Q2. Quarterly autoregressive model of the NSA money stock.

$$(4.13)\quad M_t = \underset{(10.2906)}{1.4382}\, M_{t-1} - \underset{(-3.1511)}{0.4449}\, M_{t-2} - \underset{(-2.7445)}{4.2023}\, S_1 + \underset{(1.7853)}{2.8856}\, S_2 + \underset{(1.5181)}{2.2175}\, S_3 + \underset{(2.7812)}{4.0465}\, S_4$$

$SE = 0.7955;\ \bar{R}^2 = .9944.$

Q6. Quarterly autoregressive model of the money multiplier.

$$(4.14)\quad m_t = \underset{(10.3362)}{1.4323}\, m_{t-1} - \underset{(-2.9037)}{0.6816}\, m_{t-2} + \underset{(0.1122)}{0.0277}\, m_{t-3} + \underset{(2.0914)}{0.5160}\, m_{t-4} - \underset{(-3.0447)}{0.7290}\, m_{t-5} + \underset{(3.0658)}{0.4387}\, m_{t-6}$$

$SE = 0.0337;\ \bar{R}^2 = .9461.$

These regressions were chosen by first estimating a series of autoregressive equations with as many as 8 lagged variables.[22] The regressions with the lowest standard error are given above. The difference in the standard error of the regressions as the number of lagged terms change is small as can be seen in Table 4.2. In order to acquire a feeling for the validity of the minimum standard error criterion, the predictive performance was evaluated of an alternative version of Model M6:[23]

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22. This was the case with the monthly models. The quarterly models were estimated with up to 5 or 6 lagged variables.

23. The standard error of estimate for this model was 0.024755; $\bar{R}^2$ was .9720.

TABLE 4.1

ESTIMATED COEFFICIENTS OF SEASONAL DUMMIES FOR EQUATION 4.10

Month	Estimated Coefficient	t-ratio
1	0.1846	0.3198
2	-2.5468	-4.4256
3	-1.3465	-2.2025
4	-0.1876	-0.3095
5	0.8444	1.4725
6	1.8605	3.2265
7	0.6412	1.1656
8	0.7266	1.3481
9	1.2039	2.2417
10	1.2411	2.2874
11	1.4374	2.6145
12	2.4210	4.3458

TABLE 4.2

STANDARD ERROR'S OF AUTOREGRESSIVE MODELS AS THE NUMBER OF LAGGED TERMS VARY

Lagged Terms	Model M1	Model M2	Model M6	Model Q1	Model Q2	Model Q6
1	0.3527	0.5623	0.02473	0.7490	0.8779	0.0388
2	0.3195	0.5643	0.02478	0.5731	0.7955	0.0373
3	0.2988	0.5655	0.02475	0.5775	0.8034	0.0366
4	0.2913	0.5427	0.02477	0.5841	0.8136	0.0367
5	0.2918	0.5415	0.02484	0.5629	0.8123	0.0367
6	0.2922	0.5418	0.02492			0.0337
7	0.2925	0.5432				
8	0.2932	0.5446				

$$(4.15) \quad m_t = \underset{(12.5782)}{1.0448}\, m_{t-1} + \underset{(0.4302)}{0.0515}\, m_{t-2} - \underset{(-1.1525)}{0.0954}\, m_{t-3}$$

The difference in the standard error in this series of regressions was so small that it was not observed until the 4th digit past the decimal point. In all cases, the RMSE of the Equation 4.11 version of Model M6 was slightly lower than the Equation 4.15 version. This implies that for money stock forecasting models, autoregressive models with minimum standard error will forecast with minimum RMSE.

2. Evaluating predictive performance

One criterion of a good forecasting model which has been proposed is structural stability. One test for this, proposed by Cooper, uses as its test statistic the sum of the squared residuals overthe forecast period to the mean squared error over the fitted period. [Cooper, 1972, pp 827-828]. This statistic has a χ^2 distribution; and if large values of the statistic are obtained, this necessitates rejection of the hypothesis that no structural change has occurred. This test will examine whether the trend relationship of the money stock remains the same for the 1947-1960 and the 1961-1970 periods.[24]

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24. In the case of the decomposed money multiplier, this test would show whether the trend relationship held for the various ratios included in the model remained the same.

The Cooper test statistics, displayed in Table 4.3 and 4.4, show that it is impossible to reject the hypothesis that no structural change has occurred.[25] The only exception to this statement is for 6 to 12 month forecasts made by the no-change multiplier model.[26] If the test for structural stability is indeed a good predictor of forecasting performance, it would be expected that Model M3A would outperform the other money multiplier models which were subjected to this test.[27]

If the RMSE performance of the various models is examined, it is found that the autoregressive SA money stock models (M1 and Q1) had the lowest RMSE for forecasts up to six months, and that some of the no-change multiplier models (M3A, M3C, and Q3) had the lowest RMSE in the longer forecasts. The RMSE statistics for all the models are given in Table 4.5. The difference between forecasts made by the monthly and the corresponding quarterly models was slight. The superior performance of the money multiplier model for longer period forecasting is not surprising since the multiplier is quite stable, and the monetary base is assumed to

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25. With monthly models the degrees of freedom for the test statistic is 108, for for a critical region of 5% (using a one-tail test), $\chi^2 = 133$.

26. This implies that the only trend relationship which held between the 1947-1960 and the 1960-1970 periods was that the value of the money multiplier remained the same for both periods.

27. Since RPD data prior to 1960 does not exist, we can not apply the Cooper test to the RPD multiplier models.

TABLE 4.3

COOPER TEST STATISTICS
FOR AUTOREGRESSIVE MONEY STOCK MODELS

Forecast Length	Model M1	Model M2
1	603	1073
2	302	988
3	269	787
6	318	214
12	362	139

TABLE 4.4

COOPER TEST STATISTICS
FOR MONEY MULTIPLIER MODELS

Forecast Length	Model M3A	Model M4A	Model M5
1	278	404	444
2	171	357	545
3	123	276	1105
6	104	298	685
12	39	265	363

TABLE 4.5

PREDICTIVE PERFORMANCE OF MECHANISTIC MONEY STOCK MODELS

(RMSE of 1 to 12 month forecasts)

Model	1 month	2 month	3 month	6 month	9 month	12 month
M1	0.4615	0.7545	1.0102	2.1032		4.3810
M3B	0.9984	1.5304	1.8778	2.5127		4.4373
M4B	1.2988	2.4845	3.6888	6.8665		14.6002
M2	1.4720	2.5144	3.2493	5.2443		7.4997
M3A	1.8864	2.1763	2.3456	3.2926		2.5513
M6	1.9025	2.2268	2.4568	3.6226		3.9897
M3C	2.3673	2.7052	2.9331	3.4814		4.2155
M4A	3.0892	4.9853	6.1716	10.6607		23.1651
M4C	4.0750	6.6023	9.2806	17.1590		35.8358
M5	3.2188	6.0821	12.4562	18.6912		25.6654
Q1			1.0863	2.1982	3.4247	4.5915
Q3			2.2898	3.2188	2.7896	2.4133
Q2			3.4263	5.0004	6.3291	6.4198
Q4			3.1217	6.8295	9.0707	10.0093
Q5			5.3535	10.0490	14.5390	18.5063

be known over the forecasting period. The fact that the autoregressive model performed as well as it did over the full twelve months shows the forecasting power of such a model. Since seasonal adjustment of the money stock series is done after-the-fact, even the SA autoregressive money stock model is partially ex post. The only model using no information from the forecast period itself is the autoregressive NSA money stock model, and even this model performed well. Its RMSE for the one month forecast was three times that of the M1 model, but less than twice the M1 model for the 12 month forecast.

As the estimated models predict further away from the 1947-1960 estimation period, it would be expected that prediction performance would decline. The annual RMSE statistics of 12 month forecasts, displayed in Table 4.6, show that such is the case. The autoregressive multiplier model, however, for forecasts of 3 or more months had a uniform RMSE over the entire 1961 to 1969 period. The performance of the money multiplier models whose forecasts depended only upon immediately prior data[28] deteriorated in the late 1960's. While this was not unexpected in light of the results of the Cooper test, it also suggested that the money crunches and the impact of Regulation Q increased the variance of the time trend of the money stock in the late 1960's.

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28. Use of the no-change and the same-change multiplier models does not require the estimation of coefficients which are based on data from prior periods.

TABLE 4.6

ANNUAL PREDICTIVE PERFORMANCE OF MECHANISTIC MONEY STOCK MODELS

(RMSE of 12 month forecasts)

Year	M1	M2	M3A	M3B	M3C	M4A	M4B	M4C	M5	M6
1961	0.5323	3.0406	1.6072	0.4437	1.8654	4.5090	1.3884	7.3548	3.1179	1.6536
1962	0.7701	2.8835	3.2923	2.9180	3.0781	4.2515	5.7514	8.1479	4.1382	3.3104
1963	0.5631	2.8259	2.5982	0.5394	2.0612	6.3953	1.4265	7.6218	14.1363	2.7522
1964	0.7849	2.9199	2.2302	0.5631	2.2017	5.0793	1.5689	7.9110	5.0405	2.3393
1965	0.8291	2.7903	2.4695	0.5391	2.2339	6.3099	0.9623	8.7292	6.2993	2.5595
1966	1.1885	3.6315	2.7116	2.7291	4.2186	7.3376	3.6219	10.7270	7.2789	3.0424
1967	1.5986	3.9689	1.6659	2.5689	3.9017	5.1033	5.1481	10.0520	5.1021	1.6689
1968	1.4771	3.7373	2.3041	1.5655	2.2197	7.1673	3.4615	9.1455	7.1423	2.2477
1969	0.7339	3.2026	1.6831	2.3724	3.5367	8.1620	5.5166	12.5722	7.8010	1.9842
1961-1969	1.0102	3.2493	2.3456	1.8778	2.9331	6.1716	3.6888	9.2806	10.4562	2.4568

This could account for the poor prediction performance of most of our models during this period.

The ranking of the RMSE, the MAE, and the MPAE of the different models are the same for one month forecasts and change little for 12 month forecasts. (The rankings of the monthly models are shown in Table 4.7). The rankings remained the same for quarterly models over the entire forecast period. The differences were due to the fact that the mean error measures have a linear loss function while the RMSE statistic has a quadratic loss function and so is more strongly influenced by outliers.

The regression of predicted values on actual values gave little statistically meaningful information. The intercept term of the regression did at times differ from zero, but the difference was rarely statistically significant. The slope in most cases remained close to one, and R^2 close to .99. Only the standard error of estimate seemed to indicate anything, and that was not much different than what the more conventional measures suggested. Therefore, this prediction evaluation method will not be used in subsequent chapters. The ranking of the standard error of these regressions for the various models is shown in Table 4.7.

3. The Burger multiplier model

In recent years economists at the Federal Reserve Bank of St. Louis have studied the possible use of multiplier

TABLE 4.7

RANKING OF THE PREDICTIVE PERFORMANCE OF MECHANISTIC MODELS

(Rankings based on 12 month forecasts)

Rank	RMSE	Value	MPAE	Value	SE of $\hat{Y}$ on Y	Value
1	M3A	2.5513	M3A	1.1131	M3A	2.2335
2	M6	3.9897	M3B	1.6453	M6	2.2594
3	M3C	4.2155	M1	1.9361	M2	2.8916
4	M1	4.3810	M6	1.9412	M1	2.9884
5	M3B	4.4373	M3C	2.06.8	M3C	4.2013
6	M2	7.4997	M2	3.5001	M3B	4.4440
7	M4B	14.6002	M4B	4.8557	M4B	14.7280
8	M4A	23.1651	M5	10.2336	M4A	23.2814
9	M5	25.6654	M4A	10.4939	M5	25.9279
10	M4C	35.8358	M4C	15.2384	M4C	36.0477

TABLE 4.8

ANNUAL PREDICTIVE PERFORMANCE OF SELECTED MODELS

(RMSE of 1 month forecasts)

Year	1964	1965	1966	1967	1968	1969
Burger	0.6890	0.6602	1.4993	1.6307	0.9333	0.0456
M1	0.2842	0.3263	0.5798	0.8581	0.5324	0.3517
M2	1.3784	1.2369	1.5459	1.8937	1.5273	1.4578
M3A	1.5968	2.0097	2.2468	1.5754	2.1761	2.5995
M6	1.6116	2.0221	2.0221	2.2851	1.5784	2.6253

models for forecasting.[29] The most successful multiplier forecasting model was constructed by Burger [1972] who used the following monthly model:[30]

$$(4.16) \quad m = \beta_0 + \beta_1 \left[(1/3) \textstyle\sum_{i=1}^{3} m_{t-i}\right] + \beta_2 \, TB + \textstyle\sum_{i=1}^{11} \gamma_i D_i + \rho u_{t-1}$$

The coefficients of the regression used to forecast each month's multiplier were estimated by OLS using the previous 36 months' observations. Each forecast, therefore, depended only on the data of the preceeding 3 year period.[31] The multiplier was then used to forecast the SA money stock exactly as shown in Equation 4.8.

The use of the Treasury bill rate removes this model from the mechanistic model catagory. However, of the various models considered, it would seem most appropriate to compare the Burger model to the multiplier models. Table 4.8 lists the annual RMSE statistics for this model along with the RMSE statistics for the no-change multiplier model and the three monthly autoregressive models.

In comparing the models we find that the RMSE of the autoregressive SA money stock model is consistently quite

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29. A review of this work can be found in the Burger, 1972 article.

30. TB is the lagged percentage change in the Treasury bill rate, the D_i's are seasonal dummies.

31. In this study, the forecasting equation was estimated once. Burger's approach would have required the estimation of 108 forecasting equations if forecasts were made for the 1961-1970 period.

smaller than the RMSE of the Burger model. The Burger model, however, except for 1967, outperformed the other models considered in this chapter. Considering the complexity of the Burger model (especially when compared to the autoregressive money stock model), and its inability to outperform a mechanistic model, it is difficult to become particularly enthusiastic about its forecasting abilities.[32]

D. SUMMARY

The results of this chapter can be summarized briefly. (1). The autoregressive SA money stock model forecast best for time periods up to 6 months. For 12 month forecasts the no-change multiplier model forecast with smaller error. For all models, the RMSE usually increased as the period of the forecast lengthened. (2). The performance of the autoregressive NSA money stock model, the only model which did not to some degree use ex post information, forecast poorer than the two models mentioned above, but its relative forecasting ability did improve as the length of the forecast increased. (3). The minimum standard error of estimate is an appropriate statistic for selecting the proper number of lagged terms for the autoregressive forecasting model.

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32. In his article, Burger showed that the use of the RPD base gives inferior predictions in comparison to the use of the monetary base. Our data above (compare Model M3B and M3C to M3A) confirms Burger's conclusion.

(4). The difference which exists between the various prediction evaluation statistics (RMSE, MAE, and MPAE) can be accounted for by the loss function implicit in each statistic. The test statistics of the regression of predicted on actual values tended to statistically insignificant.

APPENDIX TO CHAPTER IV

THE RATIO MULTIPLIER MODELS

A. THE RATIOS[33]

$a = (DTM - TDM) / DDP$

$a' = DDO / DDP$

$a'' = TD / TDM$

$c = (CURN - CURR - CURB) / CURR$

$k = CURR / DDP$

$r = RSU / (DTM - TDM + TD)$

$r' = RSRPD / (DDO + TD)$

$t = TDM / DDP$

B. DEFINITIONS

$M = CURR + DDP$

$B_1 = RSU + (CURN - CURR - CURB) + CURR$

$\quad = r\,(a*DDP + t*DDP) + (1 + c)\,k*DDP$

$B_2 = RSRPD = r'\,(a'*DDP + a''*t*DDP)$

C. THE DECOMPOSED MULTIPLIER

$m_1 = M/B_1 = (1 + k) / [r(a + t) + (1 + c)k]$

$m_2 = M/B_2 = (1 + k) / [r'(a' + a''t)]$

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33. Notation is listed in the Notation Appendix.

CHAPTER V

FORECASTING WITH SINGLE EQUATION MODELS OF THE MONEY STOCK

In this chapter a number of single equation models that can be used to forecast the money stock will be evaluated. In the first section the different models will be described and the coefficients, estimated over the period 1947-3 to 1960-4, will be presented. The various prediction evaluation statistics and the use of the constant adjustment term will be discussed in the second section. The forecasting performance of the various models over the 1961-1970 decade will be the subject of the third section.

A. THE MODELS

The coefficients of a number of version of six different money stock models will be estimated. Three of these six models can be described as money supply models; the other three will be called money demand models.[1] The money demand function will be first specified in conventional form: the nominal money stock regressed on nominal values of income or wealth and either a short or a long term interest rate.

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1. See pp 36-37 where the meaning of money demand and money supply models within the forecasting context is discussed.

The other two money demand functions will have the quantity variables scaled by either population or the GNP price deflator. These models are in log-log form. The money supply models will include three versions of the Brunner linear money supply model, the Teigen free reserves equation, and the equation Gibson proposed as an alternative to the Teigen equation.

The notation used in this chapter is summarized in Table 5.1. Each of the six models will be identified by a letter. The variables included in each model will be identified by the letters which follow the model identifier.

TABLE 5.1

SUMMARY OF NOTATION FOR CHAPTER V

A	Log-log nominal money demand model.
B	Log-log per capita money demand model.
C	Log-log real money demand model.
D	The Teigen money supply model.
E	The Brunner linear money supply models.
F	The Gibson model.
S	Yield on commercial paper.
L	Yield on long-term Aaa bonds.
F	The Federal Reserve discount rate.
X	The difference between S and F.
Y	Gross National Product.
W	Permanent income (defined in footnote 2 below).
RRSL	Total reserves plus reserves released through changes in the reserve requirements.
BA, BB, BC	The three versions of the Brunner model.
D	Dummy variables in the Teigen model.
S	Seasonal dummy variables.
-L	Model includes a lagged dependent variable.
-A	Model corrected for first order autocorrelation.

The letters which follow a hyphen will indicate whether the model had either a lagged dependent variable and/or has been corrected for first order autocorrelation.

1. The conventional money demand function

The conventional single equation money demand function is

$$(5.1) \qquad \ln M = \beta_0 + \beta_1 \ln R + \beta_2 \ln Y + e$$

where R is either the commercial paper rate or the long-term bond rate, and Y is either GNP or permanent income.[2] The estimated coefficients for different versions of this model are given in Table 5.2.[3] (The t-ratios are given underneath the coefficients).

The signs of the estimated coefficients conformed to conventional economic theory.[4] The value of R^2 was high with the lowest value being .96. But at the same time the values of the Durbin-Watson statistics were low, e.g. for regressions not corrected for autocorrelation or with a lagged dependent variable, they were between 0.17 and 0.21.

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2. The following expression was used to compute the permanent income:

$$YP_{t+1} = 0.114 \sum_{i=0}^{19} (0.9)^i \, Y_{t-i}$$

This expression was developed by deLeeuw, 1969.

3. Notation heading the columns are given in the Notation Appendix.

4. The only exceptions were models ALW-A and Blw-A, where the estimated coefficient for the interest rate variables were positive. However these coefficients were not significantly different than zero.

TABLE 5.2

ESTIMATED COEFFICIENTS OF THE MONEY DEMAND MODEL

Model	RMCP	RMLB	Y	YP	Lag	Const	Rho	R^2/SE	D.W.
ASY	-0.0345		.4271			2.3616		.9700	.2140
	-3.4041		23.9443			24.0774		.0153	
ASW	-0.0049			.3591		2.7760		.9667	.1654
	-0.5083			22.6158		32.4503		.0161	
ALY		-0.0344	.3982			2.5442		.9643	.1907
		-1.2302	18.6082			25.8986		.0167	
ALW		-0.0293		.3710		2.7374		.9673	.1815
		-1.1105		19.5880		32.7377		.0160	
ASY-L	-0.0180		.0833		.8462	.2733		.9967	.6531
	-5.2234		4.6825		20.4712	2.5536		.0050	
ASW-L	-0.0108			.0281	.9553	.0673		.9955	.5378
	-3.0381			1.4644	18.0959	.4400		.0059	
ALY-L		-0.0417	.0624		.9045	.1482		.9965	.7555
		-4.7865	3.7263		21.8499	1.3016		.0052	
ALW-L		-0.0367		.0390	.9488	.0697		.9959	.6584
		-3.9434		2.0815	18.9722	.4854		.0056	
ASY-A	-0.0153		.2490				1.00		.7701
	-2.4751		6.2567					.0059	
ASW-A	-0.0095			.1742			1.00		.5139
	-1.3166			3.7022				.0069	
ALY-A		-0.0219	.2249				1.00		.7750
		-0.8305	5.0995					.0062	
ALW-A		0.0235		.1356			1.00		.5481
		0.8759		2.9865				.0070	
ASY-LA	-0.0189		.1013		.8039	.3719	0.65	.9982	1.7774
	-4.8237		3.7748		12.2279	2.1111		.0037	
ASW-LA	-0.0179			.0663	.8812	.2045	0.75	.9980	1.7468
	-4.2946			2.5712	13.3468	1.0566		.0039	
ALY-LA		-0.0557	.0999		.8394	.2581	0.65	.9980	1.8220
		-4.0003	3.4974		12.1638	1.3514		.0039	
ALW-LA		-0.0459		.0523	.9368	.0573	0.65	.9978	1.6683
		-3.1931		2.0956	14.2495	.2929		.0042	

When these regressions were corrected for first order autocorrelation, it was found that a first difference equation minimized the standard error for those regressions without a lagged dependent variable.[5] Some individual coefficients of the various models were not statistically significant, but there is little pattern to this situation. Only those money demand models which had GNP and the short-term interest rates as explanatory variables had t-ratios of above 2.0 for all the estimated coefficients. This was the case whether or not the model had a lagged dependent variable, and whether or not it was corrected for autocorrelation.

2. The per capita money demand function

The quantity variables (as opposed to the interest rate variables) of the money demand function can be expressed in per capita form. This transformation in some cases caused a considerable difference in the values of the estimated coefficients. The estimated parameters for the per capita regressions (and t-ratios for the coefficients) are given in Table 5.3.

The values of R^2 for the models without a lagged variable and not corrected for autocorrelation ranged from 0.51 to

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5. A scanning technique was used to estimate the value of ρ, i.e. various values of ρ were used to estimate the parameters, and that value of ρ which gave the lowest standard error was choosen as the estimate for ρ. See Hildreth, 1960.

TABLE 5.3

ESTIMATED COEFFICIENTS OF THE PER CAPITA MONEY DEMAND MODEL

Model	RMCP	RMLB	Y	YP	Lag	Const	Rho	R^2/SE	D.W.
BSY	-0.0276		.1790			5.3098		.5496	.1256
	-2.2311		5.8328			23.2958		.0182	
BSW	-0.0225			.1581		5.4716		.5053	.1328
	-1.7407			5.1392		24.0815		.0190	
BLY		-0.0882	.2024			5.2102		.5892	.1486
		-3.2198	6.8483			25.9745		.0174	
BLW		-0.0763		.1804		5.3711		.5388	.1707
		-2.6370		6.0177		26.6514		.0184	
BSY-L	-0.0172		.0445		.9270	.1574		.9573	.7638
	-4.4814		3.9600		22.1001	.6466		.0056	
BSW-L	-0.0143			.0321	.9523	.0832		.9519	.6829
	-3.5378			2.8676	21.7761	.3231		.0059	
BLY-L		-0.0443	.0521		.9022	.3027		.9605	.8671
		-5.0769	4.5511		21.9321	1.3035		.0054	
BLW-L		-0.0366		.0378	.9328	.2004		.9540	.7668
		-3.9314		3.2712	21.4870	.8053		.0058	
BSY-A	-0.0121		.1341				1.00		.7901
	-1.8759		2.8067					.0061	
BSW-A	-0.0071			.0723		6.1061	0.95	.9426	.6178
	-1.0188			1.2283		13.2266		.0065	
BLY-A		-0.0194	.1122				1.00		.8304
		-0.7232	2.1872					.0063	
BLW-A		0.0074		.0437		6.3132	0.95	.9415	.7010
		0.2967		.7757		14.4380		.0065	
BSY-LA	-0.0183		.0586		.8247	.7310	0.70	.9741	2.0811
	-3.8337		3.1042		10.2235	1.5259		.0044	
BSW-LA	-0.0180			.0529	.8380	.6876	0.75	.9738	1.9778
	-3.7200			2.8275	10.2018	1.3480		.0044	
BLY-LA		-0.0636	.0856		.8062	.7063	0.65	.9754	2.0359
		-4.3107	4.0893		11.0888	1.6640		.0042	
BLW-LA		-0.0527		.0623	.8594	.5200	0.65	.9733	1.8090
		-3.6763		3.3968	12.0509	1.1956		.0044	

0.55. These values, relatively low for time series data, are partly due to the value of the per capita money stock essentially remaining unchanged over the estimation period. The natural logarithm of the per capita money stock was 6.66, at both the beginning and the end of the estimation period, and ranged between 6.61 and 6.70.

The estimated coefficients of three of the four regressions with neither a lagged variable nor corrected for autocorrelation had t-ratios above 2.0.[6] If the estimated constand term was ignored, all the regressions with a lagged dependent variable whether or not corrected for autocorrelation had coefficients with t-ratios above 2.0. The Durbin-Watson statistics behaved in the per capita money demand function as they did for the nominal money demand functions, i.e. the statistic was close to 2.0 only when the regression was corrected for autocorrelation and included a lagged dependent variable.[7]

3. The real money demand function

A money demand function with the quantity variables divided by the implicit GNP deflator was also estimated. The estimated coefficients and their t-ratios are shown in Table 5.4. The R^2 values of the regressions ranged from 0.15 to 0.32 for regressions without a lagged dependent

6. The exception was model BSW.

7. The inclusion of the lagged dependent variable, of course biases the Durbin-Watson statistic towards 2.0.

TABLE 5.4

ESTIMATED COEFFICIENTS OF THE REAL MONEY DEMAND MODEL

Model	RMCP	RMLB	Y	YP	Lag	Const	Rho	R^2/SE	D.W.
CSY	-0.0487		0.0900			4.4592		.1769	.2194
	-3.3595		2.1226			18.2401		.0222	
CSW	-0.0433			0.0683		4.5870		.1505	.2085
	-2.9488			1.6676		19.5556		.0226	
CLY		-0.1549	0.1353			4.3290		.3590	.2740
		-5.3836	3.6969			22.5537		.0196	
CLW		-0.1436		0.1128		4.4541		.3185	.2770
		-4.8387		3.1344		23.8945		.0202	
CSY-L	-0.0216		0.0539		0.8495	0.4393		.8828	.8960
	-3.8014		3.3383		17.5519	1.7790		.0084	
CSW-L	-0.0235			0.0582	0.8633	0.3472		.8910	.9223
	-4.3652			3.9667	18.6359	1.4316		.0081	
CLY-L		-0.0588	0.0613		0.7979	0.7013		.8913	.9966
		-4.4175	3.8869		15.8350	2.8938		.0081	
CLW-L		-0.0613		0.0633	0.8127	0.6212		.8988	1.0446
		-4.9437		4.4675	17.1339	2.6441		.0078	
CSY-A	-0.0229		0.2290			3.5387	0.95	.8710	.8429
	-2.4371		2.5418			6.4242		.0088	
CSW-A	-0.0173			0.0937		4.3915	0.90	.8619	.7331
	-1.8009			1.3262		10.3446		.0091	
CLY-A		-0.0898	0.2129			3.7605	0.90	.8715	.9263
		-2.3998	2.5698			7.8712		.0088	
CLW-A		-0.0605		0.0754		4.5682	0.85	.8588	.8106
		-1.8149		1.2601		13.3659		.0092	
CSY-LA	-0.0217		0.0553		0.7428	0.9592	0.60	.9190	1.7687
	-2.9993		2.2135		9.4154	2.2988		.0070	
CSW-LA	-0.0234			0.0615	0.7691	0.7942	0.60	.9222	1.8191
	-3.3452			2.6663	9.8693	1.8700		.0068	
CLY-LA		-0.0748	0.0824		0.6835	1.1604	0.55	.9232	1.7842
		-3.6082	3.0400		8.8146	3.0694		.0068	
CLW-LA		-0.0703		0.0751	0.7353	0.9440	0.55	.9251	1.7877
		-3.9486		3.4304	10.4954	2.6514		.0067	

variable, but when the lagged dependent variable was included in the regression, the value of R^2 approached .90. The value of the dependent variable declined over the estimation period and the range of the natural logarithm of the real money stock was small, from 4.91 to 5.03. The latter fact especially may partially explain the low value of the R^2 term.

The coefficients of most models had t-ratios of above 2.0.[8] The autocorrelation coefficient was less than 1.0 in all cases, and the pattern of the Durbin-Watson statistic was similar to that of our previous two cases.

4. Evaluation of money demand models

The estimated interest rate and economic activity coefficients of the money demand models discussed above are low. The highest interest rate coefficient is 0.15; the highest income coefficient is 0.43, and the highest permanent income coefficient is 0.36. Table 5.5 lists the results of some other studies of the money demand function. The estimated coefficients of other studies listed in this table are higher than those obtained in this study with similar variables. These studies, however, were all estimated over time periods which were different than that used in this study, and none of the other studies use the same

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8. The exceptions were Model CSY-L, Model CLW-A, and all models which have both the short-term interest rate and permanent income as explanatory variables.

TABLE 5.5 (cont'd)

Heller: Quarterly data. See text for comment on money stock series Heller used. [Heller, 1965].

Chow: Annual data. Data from *Historical Statistics of the United States*. Money stock is of June 30th, or nearest available date. The bond yield is the Durand Series, and income is net national product. [Chow, 1966, p 119].

Laidler: Annual data. Friedman's per capita expected real income series is used for YP. [Laidler, 1966, p 552].

Gilbert: Annual data. All quantities in the first regression are real and per capita; real in the second. [Gilbert, 1969, p 73].

TABLE 5.5

SELECTED ESTIMATED MONEY DEMAND MODELS

Model	Time Period	lnRMCP	lnRMLB	lnRMLG	lnY	lnYP	Lag	Const	R^2/SE
Heller	1947-58	-0.104			1.076			2.123	.956
		-3.4667			19.2143				--
				0.023	0.900			2.526	.972
				0.2190	12.000				--
Chow	1897-1958		-0.5408		0.9891			-0.2381	.9929
			-8.3328		75.3684				.008178
			-0.3109		0.3630		0.6096	0.3158	.9988
			-9.6854		8.1573		14.0138		.001367
Laidler	1946-1960	-0.176				0.044		6.030	.844
		-5.867				0.132			.039
			-0.516			0.523		3.145	.787
			-4.649			1.081			.045
Gilbert	1915-1958		-0.2902		0.1551		0.7469	-0.4540	.989
			-5.30		2.49		14.73	-4.40	--
			-0.2624		0.1714		0.7745	-0.1162	.994
			-4.95		2.84		15.01	0.595	--

series as was used here. Heller's regressions were closest to the regressions whose estimates appear in Table 5.2, but he apparently used an unusual money stock series to obtain his estimates.[9] Even so, his interest rate coefficients were close to those in Table 5.2.

All the models other than Heller's model listed in Table 5.5 were estimated with annual data. In order to determine what impact this would have on the values of the coefficients in Tables 5.2, 5.3, and 5.4, the money demand equations were reestimated with annual data. Table 5.6 gives the coefficients and the t-ratios for the annual money demand functions without a lagged dependent variable.[10] There is little difference between the annual and quarterly models. The coefficients for both the interest rate variable and the economic activity variable continue to be low. The largest interest rate coefficient is -.15[11] while the highest economic activity coefficient was .43.[12]

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9. It is impossible to determine from Heller's description of his data what money stock series he used. He cited as the source of his data The International Monetary Fund, *International Financial Statistics*. It would appear that the data series he used from there was derived from Flow-of-funds data. He never explained why he used this source for his data. Interestingly, he did use the Federal Reserve *Bulletin*--the best source for money stock data--for his interest rate data.

10. These regressions were estimated with average data for the year.

11. This was for Model CLY. Model CLW had an interest rate coefficient of -.14. The next highest coefficient was -.08.

12. This was for Model ASY. The two models cited in footnote 11 had estimated coefficients of .097 and .086 for the

TABLE 5.6

ESTIMATED COEFFICIENTS OF ANNUAL MONEY DEMAND MODELS WITHOUT LAGGED VARIABLES

MODEL	RMCP	RMLB	Y	YP	Const	R^2/SE	D.W.
ASY	-0.0452		.4345		2.3282	.9672	1.2577
	-1.6360		9.3172		9.1637	.0168	
ASW	-0.0406			.4151	2.4500	.9634	1.5552
	-1.4070			8.7557	9.5531	.0177	
ALY		-0.0152	.3745		2.6627	.9594	1.0668
		-.02325	7.9122		12.5292	.0187	
ALW		-0.0168		.3647	2.7317	.9570	1.3391
		-0.2490		7.6523	12.9572	.0192	
BSY	-0.0367		.1879		5.2502	.4846	1.1017
	-1.1320		2.4364		9.2127	.0191	
BSW	-0.0325			.1706	5.3843	.4623	1.1863
	-0.9958			2.2880	9.8207	.0195	
BLY		-0.0780	.1777		5.3901	.4976	.9885
		-1.2647	2.8243		12.7647	.0189	
BLW		-0.0773		.1696	5.4563	.4840	1.1397
		-1.2235		2.7348	13.2465	.0191	
CSY	-0.0684		.1195		4.3005	.1865	1.5809
	-1.8225		1.1265		7.0689	.0236	
CSW	-0.0622			.0959	4.4390	.1599	1.5663
	-1.6434			.9387	7.6210	.0240	
CLY		-0.1458	.0968		4.5514	.2420	1.3719
		-2.0908	1.1435		10.3668	.0228	
CLW		-0.1416		.0864	4.6111	.2277	1.4344
		-1.9758		1.0392	10.8368	.0230	

The top values are the estimated coefficients, the bottom values are the t-ratios.

Log-log models with a lagged dependent variable yield not only an impact elasticity but also the steady state elasticity. Table 5.7 gives these elasticities for the interest and activity variables for the quarterly money demand functions. Only one model has a steady state elasticity greater than 1.0; the nominal money demand regression with the long-term rate and permanent income. But it takes 25 quarters before the elasticity reaches one-half its steady state value.[13] The adjustment time is also long for the rest of the nominal money demand models and the per capita money demand models. The real money demand models, however, in all cases reach at least one-half of their steady state elasticities by the fifth quarter. Correction for autocorrelation of all three sets of models lowers the value of the steady state elasticity and shortens the half-life of the adjustment process.

The estimated interest rate and economic activity coefficients of the annula money demand models with lagged coefficients were approximately the same size as the quarterly models. The coefficients for the annual money stock models with a lagged dependent variable are given in Table 5.8. As would be expected, the coefficients of the lagged

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activity coefficient. The latter was the lowest of the economic activity coefficients.

13. The first and second quarter elasticities are also given in Table 5.7 as well as the half-life of the adjustment, i.e. the length of time necessary to reach one-half of the steady state elasticity.

TABLE 5.7

INTEREST AND INCOME ELASTICITY
OF QUARTERLY MONEY DEMAND MODELS

Model	Elasticity Impact	Quarter 1	Quarter 2	Steady State	Half Life
ASY-L	-.0180	-.0332	-.0461	-.1170	5
	.0833	.1538	.2134	.5416	
ASW-L	-.0145	-.0279	-.0404	-.1989	10
	.0452	.0873	.1262	.6214	
ALY-L	-.0417	-.0794	-.1135	-.4366	7
	.0624	.1188	.1699	.6534	
ALW-L	-.0369	-.0728	-.1076	-1.3132	25
	.0319	.0629	.0930	1.1352	
BSY-L	-.0172	-.0331	-.0479	-.2356	10
	.0445	.0858	.1240	.6096	
BSW-L	-.0143	-.0279	-.0409	-.2998	15
	.0321	.0627	.0918	.6730	
BLY-L	-.0443	-.0843	-.1203	-.4530	7
	.0521	.0991	.1415	.5327	
BLW-L	-.0366	-.0707	-.1026	-.5446	10
	.0378	.0731	.1060	.5625	
CSY-L	-.0216	-.0399	-.0555	-.1435	5
	.0539	.0997	.1386	.3581	
CSW-L	-.0235	-.0438	-.0613	-.1719	5
	.0582	.1084	.1518	.4257	
CLY-L	-.0588	-.1057	-.1432	-.2909	4
	.0613	.1102	.1492	.3033	
CLW-L	-.0613	-.1111	-.1516	-.3273	4
	.0633	.1147	.1566	.3380	

The top quantity is the interest rate elasticity, and the bottom line the economic activity variable elasticity. Half-life is the time (in quarters) it takes for the elasticity to reach at least one-half its steady-state value.

TABLE 5.8

ANNUAL MONEY DEMAND MODELS WITH LAGGED VARIABLES

Model	RMCP	RMLB	Y	YP	Lag	Const	R^2/SE	D.W.
ASY-L	-0.0306		0.3277		0.2760	1.6097	.9808	1.5537
	-1.4733		4.1586		1.6652	3.7471	.0123	
ASW-L	-0.0271			0.3436	0.1949	1.9151	.9729	2.0671
	-1.0790			3.0967	0.8032	3.1548	.0146	
ALY-L		-0.1024	0.3078		0.3983	1.2307	.9861	2.1536
		-2.5324	5.2593		2.8129	3.0004	.0105	
ALW-L		-0.0944		0.3219	0.3214	1.5176	.9778	2.6712
		-1.8527		3.7385	1.5192	2.5634	.0132	
BSY-L	-0.0282		0.1451		0.4011	2.8968	.6353	1.2224
	-0.9891		1.8544		1.5570	1.9220	.0167	
BSW-L	-0.0204			0.1221	0.3835	3.1885	.5891	1.3007
	-0.6714			1.4279	1.2919	1.9115	.0177	
BLY-L		-0.1346	0.2161		0.4326	2.2736	.8096	2.0529
		-3.1794	4.0589		2.3335	2.0572	.0120	
BLW-L		-0.1273		0.2075	0.3583	2.8320	.7510	2.3594
		-2.5683		3.2377	1.5736	2.1677	.0138	
CSY-L	-0.0586		0.1471		0.3293	2.4881	.1347	1.6851
	-1.7455		1.5532		1.2932	1.7552	.0209	
CSW-L	-0.0544			0.1288	0.3398	2.5460	.0985	1.7075
	-1.6014			1.3976	1.3027	1.7503	.0213	
CLY-L		-0.1904	0.2161		0.1966	2.9073	.5937	2.3538
		-4.0815	3.5414		1.1051	3.1374	.0143	
CLW-L		-0.1918		0.2084	0.2154	2.8673	.5750	2.8383
		-3.9408		3.4044	1.1854	3.0102	.0146	

The top values are the estimated coefficients, the bottom values are the t-ratios.

dependent variables of the annual models are lower than in the corresponding quarterly model. However when the steady state elasticities for the annual models (given in Table 5.9) are examined, the elasticities are found to always be at least one-half the steady-state value within one year, and the steady state elasticities to be lower than the quarterly steady-state elasticity. The use of annual data merely exacerbates the problem of low coefficients.[14]

Thus the money demand models seem to imply a world where a change in income or a weighted average of past income had little initial effect on the money stock, and even in those cases where the effect is sizable, the impact on the money stock occurs slowly. The interest rate elasticity also specifies a world where a change in the interest rate does not have a sizable impact on the money stock.

It should be noted that the period over which the money demand equation was estimated was a time of slow money stock growth. The natural logarithm of the SA money stock in 1947 was 4.72; in 1960, 4.95 while by 1970 it was 5.43. This situation can be observed in comparing the regressions for the nominal money stock for the 1947-1960 period, the 1947-1970 period, and the 1961-1970 period. Table 5.10 shows low coefficients (and therefore low elasticities)

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14. The annual money stock models were estimated with both the average of daily data for the entire year and the average of daily data for the two months around the first quarter. There was little difference between the two sets of estimates.

TABLE 5.9

INTEREST AND INCOME ELASTICITY
OF ANNUAL MONEY DEMAND MODELS

Model	Elasticity Impact	Elasticity Quarter 1	Elasticity Quarter 2	Elasticity Steady State	Half Life
ASY-L	-.0306	-.0390	-.0414	-.0423	1
	.3277	.4181	.4431	.4526	
ASW-L	-.0271	-.0324	-.0334	-.0336	1
	.3436	.4105	.4235	.4266	
ALY-L	-.1024	-.1432	-.1594	-.1702	1
	.3078	.4304	.4792	.5116	
ALW-L	-.0944	-.1247	-.1345	-.1391	1
	.3219	.4254	.4586	.4744	
BSY-L	-.0282	-.0395	-.0440	-.0471	1
	.1451	.2033	.2266	.2423	
BSW-L	-.0204	-.0282	-.0312	-.0331	1
	.1221	.1689	.1869	.1981	
BLY-L	-.1346	-.1928	-.2180	-.2372	1
	.2161	.3096	.3500	.3809	
BLW-L	-.1273	-.1729	-.1893	-.1984	1
	.2075	.2818	.3085	.3234	
CSY-L	-.0584	-.0779	-.0843	-.0874	1
	.1471	.1955	.2115	.2193	
CSW-L	-.0544	-.0729	-.0792	-.0824	1
	.1288	.1726	.1874	.1951	
CLY-L	-.1904	-.2278	-.2352	-.2370	1
	.2161	.2586	.2669	.2690	
CLW-L	-.1918	-.2331	-.2420	-.2445	1
	.2084	.2533	.2630	.2656	

The top quantity is the interest rate elasticity, and the bottom line the economic activity variable elasticity. Half-life is the time (in quarters) it takes for the elasticity to reach at least one-half its steady-state value.

TABLE 5.10

QUARTERLY NOMINAL MONEY DEMAND MODEL
FITTED OVER THREE TIME PERIODS

Estimation Period	Model	RMCP	RMLB	Y	YP	Const	R^2/SE	D.W.
1947- 1960	ASY	-0.0345 -3.4041		0.4271 23.9443		2.3616 24.0774	.9700 .0153	.2140
	ASW	-0.0049 -0.5083			0.3591 22.6158	2.7760 32.4503	.9667 .0161	.1654
	ALY		-0.0344 -1.2302	0.3982 18.6084		2.5442 24.8986	.9643 .0167	.1907
	ALW		-0.0293 -1.1105		0.3710 19.5880	2.7374 32.7377	.9673 .0160	.1815
1961- 1970	ASY	-0.0654 -3.8911		0.7151 30.7192		0.5441 4.1381	.9866 .0171	.2088
	ASW	-0.0692 -4.3214			0.7237 32.4332	0.5158 4.1126	.9879 .0162	.3226
	ALY		0.1437 4.4681	0.4987 15.3069		1.6291 9.8670	.9877 .0164	.2604
	ALW		0.1232 3.4884		0.5205 14.4815	1.5362 8.4926	.9865 .0172	.2869
1947- 1970	ASY	-0.0537 -2.9739		0.5151 22.9416		1.8543 15.3878	.9678 .0356	.0502
	ASW	-0.0554 -2.9012			0.5163 21.7542	1.8636 14.7142	.9648 .0372	.0659
	ALY		0.1873 4.8724	0.3147 10.6747		2.7750 21.0393	.9718 .0333	.0974
	ALW		0.1998 5.0017		0.3046 9.9442	2.8288 20.7770	.9696 .0345	.0980

The top values are the estimated coefficients, the bottom values are the t-ratios.

for the early period, and considerably higher coefficients for the latter period.[15] This situation should go a long way towards explaining the difference between the money demand regressions and those of other studies.

5. Money supply: the Brunner model

The estimated regressions of the Brunner linear money supply models in all cases had high R^2 values, and in two of the three cases (EBA and EBB) Durbin-Watson statistics which rejected the hypothesis of statistically significant autocorrelation. The results of these regressions are shown in Table 5.11. The coefficients in some cases were not significant. The best performing base concept (Model EBA) was currency in circulation plus member bank's total reserves, but even this model had an estimated coefficient which was not significantly different than zero. Compensating for reserves liberated through changes in the required reserve ratio (Model EBB) and subtracting out excess reserves (Model EBC) did not improve the goodness-of-fit statistics for the 1947-1960 estimation period.

These results are quite close to the results Brunner obtained when he estimated his model. Brunner's results

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15. This data will be used below in the Chow test for structural stability. The interest rate coefficients for the 1947-1970 and the 1961-1970 regressions were all significant (while only one of the four interest rate coefficients for the 1947-1960 regressions was significant). However the sign of the long run interest rate coefficients was positive in all cases.

TABLE 5.11

ESTIMATED COEFFICIENTS OF THE BRUNNER MODEL

Model	Base	k	t	r	Lag	Const	Rho	R^2/SE	D.W.
EBA	2.2716	-152.5442	4.9391	-266.3015		95.4967		.9828	1.9259
	14.0831	-5.5610	0.6280	-7.2756		8.7582		1.4577	
EBB	2.2403	-148.1212	13.8706			54.8503		.9767	1.7383
	11.5512	-4.7848	2.4158			3.5397		1.6992	
EBC	1.4200	-202.4563	5.3492			113.7570		.9477	.8312
	5.6390	-4.0675	0.4638			5.4666		2.5445	
EBA-L	0.1490	-65.6796	1.2498	-12.3102	0.8573	30.5305		.9962	1.1653
	0.8367	-4.5341	0.3370	-0.4764	13.1762	4.2921		.6855	
EBB-L	0.1247	-61.7415	0.5338		0.8696	27.7223		.9963	1.1630
	0.8217	-4.5790	0.2188		16.2242	4.3165		.6800	
EBC-L	0.0168	-61.2058	0.2662		0.9031	28.5883		.9962	1.0862
	0.1916	-4.2207	0.0856		25.3409	4.3790		.6844	
EBA-LA	0.3930	-38.9235	26.1953	-41.1362	0.6559	28.5464	0.95	.9977	1.5365
	2.4667	-1.6808	3.3312	-1.5700	7.2508	2.1813		.5376	
EBB-LA	0.2303	-37.7841	22.8320		0.7025	26.4842	0.93	.9976	1.6160
	1.7851	-1.7019	3.3806		8.4565	2.1085		.5463	
EBC-LA	0.0609	-42.8964	18.2831		0.7661	30.2481	0.90	.9975	1.7348
	0.6964	-1.8271	2.9728		10.3489	2.6413		.5591	

TABLE 5.12

BRUNNER'S REGRESSIONS

Model	Base	k	t	r	Const
EBA	2.28	-146.70	-31.39	-332.39	119.55
	57.00	-30.88	-5.27	-55.77	
EBB	2.62	-105.04	-5.94		26.42
	52.40	-18.36	-1.03		
EBC	2.57	-102.22	-8.94		31.32
	51.40	-17.50	-1.51		

Brunner, 1961, p 109. Brunner did not cite the goodness-of-fit statistics for these regressions. The bottom figures are t-ratios.

are given in Table 5.12. Brunner estimated his model using TSLS estimation methods and quarterly data from 1943-2 to 1957-4.[16] Brunner found all coefficients significant in his first equation, but he had some borderline cases in his other two models.

6. Money supply: the Teigen and Gibson models

In his study of money demand and supply, Teigen used the ratio of the money stock to the potential money stock as the dependent variable in the money supply equation. For explanatory variables, he used the difference between the

16. The coefficients in Table 5.12 were estimated by TSLS by Brunner over the 1947-3 to 1960-4 period. There are TSLS estimates of this model in Chapter VI, but the other equation in the system was not the Brunner money demand equation. Brunner, 1961, p 109.

commercial paper rate and the discount rate, along with two dummy variables; one of these delineated the end of the Korean War, the other delineated the beginning of the balance of payments problem for the United States. The constrained interest rate variable was used because when the restriction was removed, the coefficients for both interest variables were opposite to what economic theory indicates, and the coefficient for the commercial paper rate was not significantly different from zero. This model was re-estimated over the 1947-1960 period with both the constrained and unconstrained interest variables. Various combinations of Teigen's dummies were also used in obtaining the estimates.[17] The results of the regressions are shown in Table 5.13. Teigen found that the coefficients of the seasonal dummies were not significant. Similar results are shown in Table 5.13. A lagged interest difference variable was also included in some of the equations, but in all cases where it was used, the value of the adjusted R^2 declines. The lagged dependent variable, however, did improve the goodness-of-fit statistics for the model.

On the whole, the results of the Teigen model were disappointing. The R^2 for the regression without lagged dependent variables was between .23 and .46. The Durbin-Watson statistics were higher than was the case with the money

17. Since NSA money stock was the money variable, seasonal dummies were included in the model. They were not significant in the money supply equation.

TABLE5.13

ESTIMATED COEFFICIENTS OF THE TEIGEN MODEL

›del	RMDF	RMCP	RMFR	D1	D2	S1	S2	S3	Lag	Const	R^2/SE	D.W.
DXD	0.0433			0.0195	0.0307					1.0734	.3031	.9081
	3.1769			2.1555	2.4783					164.9898	.0294	
DXS	0.0584					0.0175	0.0159	0.0202		1.0682	.2339	.7947
	4.2464					1.4761	1.3319	1.7109		111.9110	.0308	
DXDS	0.0463			0.0193	0.0300	0.0178	0.0157	0.0185		1.0598	.3124	.8113
	3.3595			2.1372	2.4379	1.5809	1.3890	1.6499		110.1836	.0292	
DXDS-L	-0.0036			0.0057	0.0034	0.0246	0.0090	0.0143	0.7938	0.2133	.6431	1.9127
	-0.2896			0.8430	0.3513	3.0085	1.0976	1.7702	6.6752	1.6798	.0210	
DSFD		0.0137	0.0217	0.0007	-0.0301					1.0226	.4443	.8958
		0.9380	1.0143	0.0761	-1.5201					68.5905	.0263	
DSFS		0.0268	-0.0035			0.0187	0.0161	0.0163		1.0278	.4366	.7497
		1.9281	-0.1988			1.8305	1.5758	1.6050		82.7289	.0264	
DSFDS		0.0151	0.0207	0.0006	-0.0314	0.0197	0.0169	0.0150		1.0086	.4638	.7611
		1.0271	0.9629	0.0623	-1.6053	1.9825	1.6946	1.5077		62.9907	.0258	
DSFDS-L		-0.0089	0.0223	0.0005	-0.0162	0.0244	0.0103	0.0136	0.6908	0.3038	.6533	1.8130
		-0.6958	1.2895	0.0622	-1.0131	3.0330	1.2705	1.6951	5.1129	2.1949	.0207	

demand equations, but statistically significant autocorrelation still existed. The interest rate coefficient was statistically significant only when the difference of the interest rates was used as an explanatory variable. The dummy variables in the equation where the interest rate coefficient is not constrained were also not significant.

The results obtained in all cases fell between Teigen's estimates and Gibson's reestimation of the Teigen model. This comparison, shown in Table 5.14, includes a reestimate of the Teigen model for the 1947-1 to 1958-4 period--the period over which Gibson estimated the Teigen regression. Gibson maintained that the difference between the Teigen coefficients and his reestimated coefficients were due to the difference in the data series used. Teigen used Call Report data which is for one day each quarter while Gibson used quarterly averages of daily figures.[18] Data for the reestimated regressions was the average of monthly averages of daily figures of the two months that straddle the quarters. This concept is quite close to Gibson's, and the similarity of the reestimated coefficients to Gibson's coefficients would tend to confirm the hypothesis that the difference between the various estimates of this model is due to differences in the data series.

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18. Since banks almost always know when they must submit the Call Report data, window dressing of the data can occur. Nor is the Call Report data for the same day each year. Moreover neither December 31 or June 30, two common Call Report dates, are typical days for the financial sector.

TABLE 5.14

COMPARISON OF THE TEIGEN MODEL

Model	Estimation Method/ Period	RMDF	D1	D2	Const	R^2/SE	D.W.
Teigen	TSLS	0.0751	0.0328	0.1254	0.8522	.726	1.536
	46-4 - 59-4	4.723	3.4526	7.1657	123.3235	--	
Gibson	TSLS	0.0395	0.0112		0.7065	.2895	--
	47-1 - 58-4	4.0448	2.2358		190.5854	.0003	
DXD	OLS	0.0446	0.0182		1.0714	.2025	1.4754
	47-1 - 58-4	2.6207	1.6760		142.4473	.0354	
DXD	OLS	0.0433	0.0195	0.0307	1.0734	.3031	.9081
	47-3 - 60-4	3.1769	2.1555	2.4783	164.9898	.0294	

As Gibson observed, the difference in the regressions can be quite striking. Teigen obtained $R^2 = .73$ for his money supply regression, Gibson on the other hand obtained $R^2 = .29$. The reestimated regressions were similar to Gibson's. The t-ratios for the dummy variables in the reestimated regressions were quite a bit lower. They were never above 2.50 and one was 1.68. The poor performance of the Teigen model suggest that it would not be useful as a forecasting model.

The alternative money supply model used by Gibson in his study of the Teigen model was also estimated. These regressions are given in Table 5.15. Gibson added another variable to the Teigen model: total reserves plus reserves released through changes in the reserve requirement. He then estimated the regression with the constrained and unconstrained Teigen interest variable.

The estimated coefficients of the interest rate variables did not have signs economic theory would suggest, nor were they significantly different from zero. The lagged dependent variable had an estimated coefficient of .99 for one model (FRSF-L), .93 for the other two models. The constant term in the three cases where there was a lagged dependent variable was statistically indistinguishable from zero. The t-ratios for the coefficients were similar whether the variables were expressed in terms of levels or logarithms. The value of R^2 in all cases was high, but the Durbin-Watson statistic was low (between .64 and .89).

TABLE 5.15

ESTIMATED COEFFICIENTS OF THE GIBSON MODEL

Model	RSL	RMCP	RMFR	RMDF	Lag	Const	Rho	R^2/SE	D.W.
FRSF	4.5664	0.9501	0.9685			41.3822		.9418	.7966
	16.7832	0.6840	0.5527			10.5188		2.6830	
FRSF-L	0.2953	-0.3198	-0.4807		0.9911	-1.8243		.9962	.6977
	1.7004	-0.8947	-1.0677		26.8254	-0.9614		.6843	
FRX	5.0444			1.9998		36.5859		.9353	.8905
	23.8475			1.4263		9.9818		2.8304	
FRX-L	0.4233			-0.5753	0.9255	2.5822		.9952	.6436
	2.2140			-1.4607	25.3430	1.5478		.7682	
F'RSF	0.6584	0.0145	0.0175			2.9253		.9537	.8942
	17.9999	0.6707	0.6721			29.5929		.0190	
F'RSF-L	0.0742	-0.0018	-010094		0.9344	0.1162		.9959	.7394
	2.6748	-0.2801	-1.2090		22.8587	0.9196		.0056	
FRX-A	0.4571			-0.0545		129.0837	0.95	.9931	.5089
	1.8521			-0.1554		21.8204		.9228	
FRX-LA	0.0488			-0.8875	1.0393	-4.1054	0.70	.9980	1.8854
	0.3393			-4.3146	26.6734	-1.2473		.5033	

As can be seen from Table 5.16, the reestimation of the Gibson model was quite close to Gibson's results. The coefficients which were insignificant in the reestimated regressions also tended to be insignificant in Gibson's regressions. The standard error of his regressions were larger and his R^2 smaller, but Gibson used NSA money stock while the reestimated regressions used the SA money stock. The incorporation of the reserves variable into the money supply equation according to the t-ratios for this variable was a significant addition, but this was not the case where the interest difference term was replaced by the individual interest rates. The Gibson model with the Teigen difference of interest rates variable appears to be the most promising forecasting model examined in this section.

B. EVALUATION OF PREDICTIVE PERFORMANCE

While there is a wide range of possible prediction evaluation statistics, actually there is little difference between most of them. In this section will be described those statistics which will be used to evaluate the predictive performance of single equation models used to predict the money stock.

1. General evaluation

The primary prediction evaluation statistic will continue to be the root mean squared error for the entire forecast period and annually. The period of prediction will be up

TABLE 5.16

COMPARISON OF THE GIBSON MODEL

Model	Estimation Method/ Period	RSL	RMDF	RMCP	RMFR	D1	Const	R^2/SE
Gibson	TSLS	6.030		1.559	0.6338	1.848	23.04	.8956
	47-1 - 58-4	5.7593		0.3385	0.1009	0.7008	1.4023	11.60
Gibson	TSLS	5.488	6.246			3.787	33.69	.8935
	47-1 - 58-4	4.9982	1.9320			1.6191	2.0861	13.64
FRD	OLS	4.6062	1.7630			2.9854	43.0578	.9756
	47-1 - 58-4	16.5992	1.9392			3.2312	9.8581	1.6192
FRSFD	OLS	4.5128		1.1487	-0.2617	2.7514	43.2819	.9761
	47-1 - 58-4	15.9690		1.1470	-0.1866	2.9605	10.0087	1.6020

to six quarters. The Chow test for structural stability will be used to determine whether there has been a significant shift in the estimated values of the coefficients of the regressions between 1947-1960 and 1960-1970.[19] Except for the lagged terms and the constant adjustment terms, ex post values of the exogenous variables will be used in evaluating predictive performance.

When there is no lagged dependent variable in the model and ex post values of the exogenous variables are used, the prediction error does not deteriorate as the prediction period is lengthened; for example, the error of the second quarter forecast at time T is exactly the same as the error for the first quarter forcast at time $T + 1$. In other words, the one period forecast at time T is

$$(5.1) \qquad \hat{Y}_{T+1} = \hat{\beta}_0 + \hat{\beta}_1 X_{T+1}$$

while a two period forecast at time T is

$$(5.2) \qquad \hat{Y}_{T+2} = \hat{\beta}_0 + \hat{\beta}_1 X_{T+2}$$

But the second expression is also the one period forecast at time $T + 1$.

In the actual forecasting situation, predictive ability of a forecasting model deteriorates as the forecasting period is extended since the values of the exogenous variables must also be forecast; presumably the forecasts of the

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19. This is discussed in section 3 below.

exogenous variables would deteriorate as the forecast period was extended. This error buildup is observed in this study, however, only in those models which include a lagged dependent variable since the forecasting is ex post. Therefore prediction evaluation statistics for forecasts beyond the first quarter are informative only for those models which include a lagged variable.

2. Constant adjustments

A mechanical constant adjustment term will be incorporated into the model to determine whether this improves the predictive performance of the model. The error of the first quarter prediction made at time T - 1 will be used in forecasts made in period T. In other words, $\hat{e}_{T-1}$ is first determined,

$$(5.3) \qquad \hat{e}_{T-1} = Y_{T-1} - \hat{Y}_{T-1}$$

and then four constant adjustment terms will be computed:

$$(5.4) \qquad A_1 = 0.0$$

$$(5.5) \qquad A_2 = \hat{e}_{T-1}$$

$$(5.6) \qquad A_3 = 1/2\ [\hat{e}_{T-1} + \hat{e}_{T-2}]$$

$$(5.7) \qquad A_4 = 1/4 \sum_{i=1}^{4} (\hat{e}_{T-i})$$

The error term for forecasts made in period T is then computed from

$$\hat{e}'_{T+i} = Y_{T+i} - A_j - \hat{Y}_{T+i}; \quad i = 0, 1 \ldots 5; \quad j = 1, 2, 3, 4. \tag{5.8}$$

This error term will be used for computing the RMSE for models with a constant adjustment term.

3. The Chow test

The Chow test for structural stability will determine whether the estimated coefficients for the 1961 to 1970 period are significantly different from the estimated coefficients for the 1947 to 1960 period.[20] To compute the F-ratio for this test, each regression was estimated for the 1947-1960 period, the 1961-1970 period, and the 1947-1970 period. Then the sum of squared error terms for the three equations was computed. If SSE_A is that sum for the 1947-1960 period, SSE_B for the 1961-1970 period, and SSE_C for the 1947-1970 period, then the F test statistic is

$$\frac{(SSE_C - SSE_B - SSE_A) / k}{(SSE_A + SSE_B) / (n + m - 2k)}$$

where k is the number of estimated coefficients, and $(n + m)$ is the total number of observations. Table 5.17 gives the F ratio for the different values of k. If the test statistic is below the value indicated in the table, the hypothesis can not be rejected that no structural

20. Chow, 1960.

TABLE 5.17

SELECTED F-RATIOS FOR (n+m) = 100

Level of Significance	k=2	k=3	k=4	k=5	k=6
.95	3.1	2.7	2.5	2.3	2.2
.99	4.9	4.1	3.6	3.3	3.1

shift has occurred.

The occurrence of structural shift does not necessarily rule out the use of that model for forecasting. If the regression had a small standard error when estimated, and also had a small but different standard error in the forecasting period, the Chow test could indicate that structural shift has occurred. But this model could still have a smaller RMSE when forecasting than a model which because of a larger standard error during both the fit and forecast period has a low Chow statistic and thereby giving the appearance of a more stable model.[21] The Chow test, therefore, serves as a supplement to the conventional prediction evaluation statistics. Actual use of the test will reveal just how useful it can be.[22]

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21. The statistician faces a similar problem when he must choose between a biased estimator with a small variance and an unbiased estimator with a large variance.

22. At the end of Chapter II, the occurrence of both structural shift and good forecasting performance in the

C. PREDICTIVE PERFORMANCE

1. Structural stability of single equation models

The Chow test was used to test whether structural shift occurred between the 1947-1960 and the 1961-1970 periods. With the exception of the first of the Brunner models (EBA) and the Teigen models, the hypothesis that no structural shift has occurred in models without a lagged variable can be rejected. See Table 5.18 for a tabulation of these results. The lagged dependent variable in all cases improved the stability of the models.

The first two of the lagged Brunner models (EBA-L and EBB-L) and two of the lagged Gibson models (FRX-L and F'RSF-L) possessed test statistics low enough that it was not possible to reject the hypothesis that no structural shift had occurred.[23] The same can be said about the two real money demand equations with the short-term interest rate (CSY-L and CSW-L). Of the lagged models, the nominal money demand equations possessed the largest Chow test statistics, but even these statistics were reasonably close to the F-statistic which marked the beginning of the rejection region.[24]

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23. The prime indicates the log-log version of the model.

24. The largest test statistic for the lagged nominal money demand equations was 5.32. The rejection region at a 99% level of significance begins at 3.6 where there are four explanatory variables.

TABLE 5.18

CHOW TEST STATISTICS

Money Demand Models:

Model	A	B	C	k
SY	35.93	174	78	3
SW	35.83	152	75	3
LY	29.49	194	113	3
LW	27.23	175	100	3
SY-L	5.32	2.76	0.71	4
SW-L	3.74	3.88	0.01	4
LY-L	4.53	3.37	3.76	4
LW-L	4.01	2.77	3.84	4

Money Supply Models:[25]

Model	Ratio	k	Model	Ratio	k
DXD	3.05	4	FRSF	75.99	4
DXD-L	0.94	5	FRX	79.77	3
EBA	1.90	5	FRSF-L	3.78	5
EBB	5.68	4	FRX-L	1.67	4
EBC	22.18	4			
			F'RSF	74.84	4
EBA-L	0.88	6	F'RSF-L	2.52	5
EBB-L	1.71	5			
EBC-L	3.89	5			

This evidence strongly suggests that the forecasting record of the non-lagged models (except for model EBA) should be inferior to that of the lagged models. The models which appear particularly promising are the first lagged Brunner model, the Gibson model with the constrained

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25. The prime indicates the log-log version of the model.

interest rate coefficients (Model FRX-L), and the two real money demand models with the short-term interest rates. The Teigen model also had low Chow statistics.

2. The conventional prediction evaluation statistics of the money demand models

The RMSE statistics for the 1961-1970 forecasting period are displayed in Table 5.19 for the various money demand models. One quarter forecasts by money demand models with the short term interest rate had smaller RMSE statistics than did models with the long term rate. Inclusion of the lagged dependent variable and the correcting of the regressions for first order autocorrelation considerably reduced the size of the forecast error.[26] Except for those models which used the short term interest rate and income as the independent variables, correcting the non-lagged regressions for autocorrelation reduced the forecast error more than including a lagged variable in the regression would have.

Because of the linear relationship between the error terms,[27] only models with a lagged dependent variable or an autocorrelation correction term will be examined beyond the first quarter's forecast. As shown in Table 5.19 the

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26. This was not an unexpected result since the significance of the lagged variable in the model is due to the autocorrelated nature of the data.

27. See p 103 above.

TABLE 5.19

PREDICTIVE PERFORMANCE OF MONEY DEMAND MODELS

(RMSE of 1 and 6 quarter forecasts)

	1 Quarter Forecasts			6 Quarter Forecasts		
Models	A	B	C	A	B	C
SY	11.8575	16.6762	12.3338			
SW	12.5212	17.6531	13.1509			
LY	13.6014	18.8902	16.5875			
LW	13.0967	19.6003	17.1548			
SY-L	2.6654	2.0679	1.5729	11.9895	10.4605	6.0848
SW-L	1.8443	2.0789	1.5172	9.8346	11.1161	5.9358
LY-L	3.4943	3.8551	4.2630	18.1732	19.9882	17.7959
LW-L	2.8592	3.6676	4.1256	16.2371	20.1775	17.8275
SY-A	1.5914	1.5684	1.0852	9.1924	8.9438	4.1939
SW-A	1.8513	2.7925	1.7049	10.6847	15.9900	8.0297
LY-A	1.7320	1.6817	1.5510	9.9497	9.5648	6.9519
LW-A	1.9485	2.8950	2.7332	11.0892	16.5062	12.4722
SY-LA	1.4312	1.5143	1.3750	11.0444	13.0821	8.3737
SW-LA	1.1693	1.4079	1.2390	9.7408	12.7634	7.2712
LY-LA	1.8050	2.2877	2.7785	15.5693	19.4119	16.9412
LW-LA	1.5235	2.1474	2.8111	14.3395	19.6369	17.5128

expected error buildup occurs. However the relative performance of the various models tend to remain the same, i.e. the model with the lowest RMSE in the first quarter also tends to have the lowest RMSE in the sixth quarter forecast. The error buildup occurs over the entire forecasting period, not just the first couple of quarters or the last couple of quarters.

Table 5.20 presents the year-by-year RMSE statistics for the money demand models. All specifications of money demand displayed low RMSE's in the early 1960's while the RMSE's deteriorated markedly in the late 1960's. The worse forecasting year for most of the specifications of money demand occurred in 1970. For most of the specifications, the year with the lowest RMSE was 1963, with 1962 being a runner-up. This pattern remained constant whether there was a lagged dependent variable in the forecasting regression, or the regression was corrected for first order autocorrelation. In light of the results of the Chow test, these results are not surprising.

3. The Brunner model

The predictive performance of the three Brunner money supply models can be ranked in the order in which they were described with the predictive performance of the first two models being quite close. Introduction of a lagged dependent variable in the first two models had little effect on the size of the RMSE, but in the case of the third model,

TABLE 5.20

ANNUAL PREDICTIVE PERFORMANCE OF QUARTERLY MONEY DEMAND MODELS

Year	ASY	ASW	ALY	ALW	ASY-L	ASW-L	ALY-L	ALW-L
1961	3.5405	4.7022	1.8002	4.3779	0.6908	0.3104	0.6281	0.4522
1962	4.8282	6.1786	3.3294	6.1571	1.3164	0.9240	0.6304	0.6981
1963	2.6885	3.9096	1.5065	4.0503	0.2657	0.1891	0.3774	0.2531
1964	0.8660	1.3604	1.1304	1.5267	0.5111	0.7498	0.9053	0.7804
1965	2.1352	2.6460	3.2190	2.4497	1.2786	1.4430	1.4980	1.4025
1966	2.4097	2.0000	3.4451	1.9324	1.0144	1.0755	1.1609	1.0598
1967	7.4411	8.1432	9.9893	8.6182	2.3188	2.0796	3.5215	3.0331
1968	15.4564	16.7115	18.0051	17.3176	4.1969	3.2217	5.1819	4.4447
1969	20.6019	20.3136	22.6290	21.2954	4.0312	2.1155	4.9406	3.7615
1970	25.1617	26.8544	29.5607	28.3202	5.1573	3.1699	7.2864	5.8471

Year	BSY	BSW	BLY	BLW	BSY-L	BSW-L	BLY-L	BLW-L
1961	5.1264	4.4644	3.1547	2.7392	0.4929	0.2240	0.4318	0.7298
1962	5.9072	5.3342	4.7705	4.3496	1.0778	0.8507	0.6914	0.6024
1963	3.6036	2.9549	3.0274	2.4309	0.1209	0.3105	0.2625	0.6297
1964	1.0376	1.0873	0.8289	1.1731	0.6472	0.9076	0.8471	1.2145
1965	3.9287	5.2602	4.0718	5.4653	1.3314	1.6690	1.5523	2.0129
1966	5.6094	6.7554	6.2501	7.3783	1.0884	1.1546	1.2827	1.4719
1967	12.0381	13.4528	14.5222	15.6187	2.1445	2.3718	3.8672	3.9599
1968	22.3179	24.0778	25.0320	26.4797	3.4984	3.5977	5.6993	5.5925
1969	28.6901	30.0917	32.0637	33.0671	2.8649	2.6286	5.5987	5.0530
1970	34.5354	36.3108	40.0574	41.0506	3.5946	3.4538	8.0206	7.2828

TABLE 5.20 (cont'd)

Year	CSY	CSW	CLY	CLW	CSY-L	CSW-L	CLY-L	CLW-L
1961	5.0767	4.5409	1.6552	1.4402	0.7843	0.5867	0.6453	0.7316
1962	4.3131	3.7294	2.3470	1.9431	1.2230	1.1148	0.4582	0.4537
1963	1.3144	1.0890	0.6991	1.0707	0.1799	0.0957	0.5021	0.4309
1964	3.1802	3.9481	3.6135	4.3821	0.5252	0.5827	1.2228	1.1242
1965	7.7395	8.8201	7.8773	9.0727	1.3997	1.5170	2.2841	2.2470
1966	8.5708	9.5432	9.6890	10.6614	1.0263	0.9657	1.5756	1.4082
1967	12.0423	13.2297	16.5111	17.3889	1.6223	1.5250	4.5274	4.3388
1968	19.3372	20.6686	24.4857	25.5473	2.9635	2.9246	6.6543	6.4961
1969	20.9956	22.0067	27.6977	28.3247	1.6603	1.5390	6.0053	5.7322
1970	19.2845	20.5619	30.5680	31.0383	2.2667	2.1452	8.4154	8.1916

TABLE 5.21

PREDICTIVE PERFORMANCE OF THE BRUNNER MODEL

Model	RMSE(1)	MAE(1)	RMSE(6)	MAE(6)
EBA	4.1393	2.1277		
EBB	5.8542	5.2229		
EBC	14.6770	10.7060		
EBA-L	5.3046	4.4634	24.9462	21.7454
EBB-L	5.7147	4.7397	28.0052	24.0918
EBC-L	6.4384	5.4012	34.0910	29.5378

the lagged dependent variable reduced the RMSE for the forecasting decade to bring it closer to that of the other Brunner models.

Examination of the annual RMSE statistics (shown in Table 5.22) reveals that the RMSE statistics for the first two models remained quite stable over the entire forecasting period. Brunner's third model displays the same deterioration of the RMSE statistic in the late 1960's as displayed by the non-Brunner models discussed above.

4. The Teigen and Gibson models

While the Teigen model RMSE statistics for the entire forecasting period (shown in Table 5.23) compares favorably with those for the other models, annual RMSE statistics (shown in Table 5.22) reveal that the Teigen model tends to be quite unstable. In some years the RMSE statistic was close to $1 billion; but in 1970 it was as high as $58

TABLE 5.22

ANNUAL PREDICTIVE PERFORMANCE OF QUARTERLY MONEY SUPPLY MODELS

Year	DXD	DXS	DXDS	DSFS	EBA	EBB	EBC	EBA-L
1961	2.8456	1.4708	2.7841	3.6170	1.5225	3.2452	3.8166	0.7977
1962	2.1694	1.3775	2.1204	1.4738	3.0131	5.4924	4.0760	1.0913
1963	2.6608	3.1359	2.3326	1.9220	3.5758	6.2067	2.2820	2.1236
1964	0.9152	2.9112	1.0395	1.8454	4.0194	6.5112	1.3799	3.1258
1965	2.8628	4.8425	3.0293	3.9274	4.5143	7.0208	4.2811	4.2997
1966	7.3200	6.8124	7.4549	4.3133	6.4652	8.3459	7.0978	3.8732
1967	5.8202	4.9780	5.9068	5.5843	5.8600	8.0230	10.9491	6.2084
1968	0.8885	3.0776	1.9668	5.9796	4.0412	4.9777	18.6456	7.7051
1969	9.8455	12.1922	10.7247	3.3792	2.5048	1.7861	26.1805	7.0061
1970	57.4170	58.5544	57.3794	54.2983	3.4496	3.1337	29.9028	9.3161

Year	EBB-L	EBC-L	FRSF	FRX	F'RSF	FRSF-L	FRX-L	F'RSFL
1961	0.6177	0.7357	10.3993	11.2423	10.1822	0.5229	0.8992	0.8708
1962	0.9639	1.1909	11.6768	13.1646	11.1838	1.1436	1.3271	1.4876
1963	2.0627	2.4375	10.7031	11.7097	9.8073	0.1784	0.5299	0.6094
1964	3.1653	3.6594	9.2419	10.1169	7.8438	0.4862	0.5598	0.4597
1965	4.4114	5.0489	7.7979	8.2537	5.7757	1.1433	0.7599	0.7670
1966	4.1118	4.8507	7.2820	8.6934	4.3454	1.0629	1.1281	1.3106
1967	6.5043	7.4119	4.1289	5.8719	1.3536	1.4798	1.6761	1.2006
1968	8.1803	9.1460	2.5695	2.5465	6.7569	2.6430	2.5850	2.3182
1969	7.8044	8.7172	7.7798	6.8987	14.0721	1.9103	2.5778	1.5685
1970	10.2222	11.3861	12.5083	11.0211	18.7489	2.3276	3.4228	2.5742

TABLE 5.23

PREDICTIVE PERFORMANCE OF THE TEIGEN MODEL

(1 quarter forecasts)

Statistic	DXD	DXS	DXDS	DSFS
RMSE	18.7372	19.2451	18.7875	17.5586
MAE	6.6685	7.1941	6.8172	6.2040
SE of MAE	17.7334	18.0773	17.7300	16.6353

billion, and in 1971 approximately $4 billion. No other model displayed such erratic behavior. The standard deviation of the MAE statistic confirms this observation --it is about three times the value of the MAE.[28]

The Gibson alternative to the Teigen model outperformed the Teigen model in the area of prediction. (In the summary of the predictive performance of this model--Table

TABLE 5.24

PREDICTIVE PERFORMANCE OF THE GIBSON MODEL

Model	RMSE(1)	MAE(1)	RMSE(6)	MAE(6)
FRSF	8.9383	8.1890		
FRSF-L	1.5010	1.1715	7.5353	6.1326
FRX	9.4493	8.6965		
FRX-L	1.8129	1.4588	8.9923	7.0113
F'RSF	10.1771	8.8360		
F'RSF-L	1.4752	1.2118		

28. Except in this case, the standard error of the MAE is less than the actual value of the MAE itself.

5.24--the prime indicates a log-log version of the Gibson model, D the Teigen structural shift dummy variable).[29] In all cases the decade RMSE statistics and the MAE statistics are quite low. Within the decade (Table 5.22), no strong trend stood out. Predictive performance declined in 1970 as it did for most models, but not as radically as it did in the Teigen model.

5. Adjustments of the constant term

In order to examine the effect on a forecast that a constant adjustment term based on the residuals generated by the regression has, three different constant adjustment terms were studied. Table 5.25 tabulates the RMSE for the first quarter forecasts of the 1962-1970 period when no constant term was used, and when one of the three constant adjustment terms described above was used.[30] In all cases the constant adjustment term reduced the RMSE over the entire period. The larger the error without constant adjustment, the greater the proportionate reduction of the RMSE when the constant adjustment term is used. The constant adjustment term also reduced the RMSE of models which included

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29. Gibson included the Teigen dummy which marked the end of the Korean War, but dropped the dummy indicating the beginning of the U.S. balance of payments problems. Gibson did this because his estimation period was shorter.

30. See p 104-105. The 1962-1970 period was used so that RMSE statistics could also be computed for the triannual periods 1962-1964, 1965-1967, and 1968-1970. These results will be discussed in the next paragraph.

TABLE 5.25

PREDICTIVE PERFORMANCE OF SELECTED MODELS WITH CONSTANT ADJUSTMENT TERMS

(RMSE of 1 quarter forecasts)

Model	No Correction	Adjustment A	Adjustment B	Adjustment C
ASY	9.2670	1.2569	1.7192	2.5782
ASW	9.6991	1.3688	1.9096	2.8481
ALY	10.4144	1.3647	1.8571	2.7130
ALW	10.0731	1.4192	1.9780	2.9523
ASY-L	2.2223	.8874	.9536	1.0123
ASW-L	1.6318	.9342	1.0069	1.0418
ALY-L	2.7690	.9664	1.0576	1.1367
ALW-L	2.3403	.9646	1.0517	1.2070
CSY-L	1.4758	.9155	1.0087	1.0332
CSW-L	1.4305	.9030	.9946	1.0287
CLY-L	3.5105	.9835	1.1097	1.2126
CLW-L	3.3847	.9769	1.1066	1.2260
EBA	4.2089	2.3467	2.2196	1.7566
EBB	6.0818	2.2883	2.1137	1.9122
EBC	11.8320	2.3325	2.5647	3.0148
EBA-L	4.6764	.9871	1.1036	1.1364
EBB-L	4.9675	.9887	1.1076	1.1541
EBC-L	5.6262	.0023	1.1206	1.2026
FRX	9.2581	2.6416	2.6044	2.7177
FRSF	8.4490	2.4738	2.3790	2.5218
F'RSF	8.7192	2.3507	2.3954	2.8458
FRX-L	1.5330	.9122	.9676	.9600
FRSF-L	1.3788	.9370	.9970	.9890
F'RSF-L	1.2968	.9535	1.0318	1.0298

a lagged dependent variable. The constant adjustment term which worked best for all the money demand models was the error of the previous period's prediction. The average of the previous period, however, reduced the forecasting error of two (EBA and EBB) of the three Brunner non-lagged models better than did the previous period's residual. The latter adjustment worked best in the remaining of the Brunner models.

The triannual RMSE statistics showed that constant adjustment did not always improve the prediction performance of the model. In many money demand models, especially those where the lagged dependent variable was included in the model, the RMSE for the 1965-1967 period was lower when no adjustment term was used. This was not the case in the Brunner models where in all cases constant adjustment reduced the RMSE. The triannual data also showed that the error terms for the 1968-1970 triannual period was the highest of the three periods.

Longer forecasts increased the RMSE and lessened the effect of the constant adjustment. However the relative relationships described above held constant, i.e. if a particular constant adjustment term improved the predictive performance of the model in a one quarter forecast; then the predictive performance of the model in a six quarter forecast would also be improved, but to a lesser degree. This relationship held whether the constant term improved or worsened the performance of the model. The superiority of the single period adjustment term was not reported in other studies where this approach was tried, but there is no instrinsic reason why one particular constant adjustment term should usually work better than another.

D. SUMMARY

In this chapter a set of money demand and money supply models were estimated over the 1947-1960 period and then used to forecast the 1961-1970 money stock. The estimated coefficients of the various models were compared to estimates made by others of similar models, and where the results were different an attempt was made to explain the differences.

The principle observations of this chapter were as follows. (1) The Gibson models with the lagged dependent variable predicted with lowest RMSE. The real money demand function with the short-term interest rate and the lagged dependent variable was a close runner-up. (2) The hypothesis that structural shift occurred could be rejected by the Chow test for the models mentioned above. But this hypothesis was also rejected by the Chow test for a number of Brunner money supply models which had inferior RMSE statistics. (3) Adjustment of the constant term improved the predictive performance of the models. The previous period's error usually worked best. (4) The impact and steady state elasticities of the money demand model were low relative to other estimates of the money demand model.

CHAPTER VI

TWO-EQUATION MODELS

A. INTRODUCTION

In Chapter IV the predictive performance of naive and mechanistic money stock models were examined while in Chapter V the predictive performance of a number of single equation money stock forecasting models were examined. In this chapter the forecasting performance of some two-equation money stock forecasting models will be studie . The selection of models to be examined is based to a great extent on the results of the previous two chapters.

Using the convention of the previous chapter, each model will consist of a money supply equation and a money demand equation.[1] It is unlikely, of course, that two equations adequately specify the complexities of the money stock determination process. The primary concern, however, is the predictive ability of various money stock models, and not this more basic question. The criteria of correct specification and good forecasting ability are not in conflict, and ideally a model should have both characteristics.

If the equations estimated in Chapter V are incorrectly

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1. See pp 36-37.

specified reduced-form equations, and more than one endogenous variable of the equation system is included in an equation, unless the system is recursive, the equations estimated in Chapter V are biased and inconsistent. Therefore, to the extent that the two-equation models used in this chapter reflect the actual structure of the money stock determination process, the used of system estimation techniques such as two-stage least squares (TSLS) will give estimated equation parameters which are at least consistent.

The coefficients of the simultaneous equation models will be estimated using the appropriate estimation techniques for the 1947-2 to 1960-4 period.[2] While statistics such as the canonical correlation coefficient measure the goodness-of-fit of the simultaneous system; if the structure of the system changes, such statistics are of little value. Instead, consistent with the approach used in the earlier chapters, the estimated models will be used to generate six quarter simulations for the 1961-1 to 1972-2 period, and the forecasting ability of the models will be evaluated based on these simulations.

Two equation models imply that there are two endogenous variables in the system. Obviously one of these variables is the money stock. In this chapter the commercial paper rate shall also be treated as the other endogenous

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2. Only quarterly forecasts will be generated since the data for many of the exogenous variables are available only in quarterly form, e.g. GNP.

variable.[3] While the primary focus of this study remains the forecasting of the money stock, in the simultaneous equation context, the forecasts of the other endogenous variables can not be ignored. This situation will be discussed in more detail in Section D of this chapter.

B. THE MODELS

Three different classes of money stock models will be examined in this chapter. Each has a money supply equation and, where warrented, three different money demand equations. That is, there may be as many as three versions of a single model. The better performing versions of the various types of models which were examined in Chapter IV and Chapter V will be considered as components for the models to be examined in this chapter.

1. The money multiplier model

The supply side of the multiplier model is specified by the money multiplier identity discussed and evaluated in Chapter IV. There it was observed that the multiplier identity which forecast with the lowest RMSE was the

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3. The conventional second endogenous variable is the interest rate, but prior to the Treasury-Federal Reserve Accord, the Federal Reserve attempted to fix the rate at a low level. This convention, then, results in specification error. But this difficulty would exist for any variable the Federal Reserve viewed as a target variable. Therefore, if the interest rate is used as an endogenous variable in the 1947-1951 period, the question is not whether specification error exists, but how serious the problem really is.

no-change naive version of the multiplier.[4] This version of the multiplier, i.e. the value of the multiplier is assumed to be fixed at its current value over the period of the forecast, will be used in the multiplier model.

The demand side of this model will be specified by the nominal money demand equation with income, the commercial paper rate, and the lagged dependent variable as explanatory variables. Since this equation is the only equation which contains both endogenous variables--the money stock and the commercial paper rate--the system is recursive, and, therefore, ordinary least square estimates of the coefficients of the money demand equation will be consistent. Since the money supply equation is an identity, the OLS coefficients of the money demand equation will also be asymptotically efficient.

2. The Brunner-Meltzer model

The money supply equation in the second simultaneous equation model will be the first of the Brunner-Meltzer linear money supply equations.[5] Since the predictive performance of most models is improved by including a lagged dependent variable in the model, the equation will also include the lagged variable. However, as observed in Chapter IV, the predictive performance of this particular

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4. Equation 4.8, p 54.

5. Equation 3.10a, p 38.

equation did not improve when the lagged variable was used.[6] This situation suggests that the Brunner-Meltzer money supply equation without the lagged variable could also be considered as an alternative formulation of the model. However as we shall bring out in Section D, since this equation does not contain the endogenous interest rate variable, and is, therefore, part of a recursive equation system, both versions of this model can be evaluated with data from only one version of the simultaneous system.

As with the multiplier model, the nominal money demand model with the short-term interest rate, income, and the lagged dependent variable will be used as the money demand equation. From the recursive nature of the system, the OLS estimates of the coefficients of the two equations will be consistent, but asymptotically efficient only if the variance-covariance matrix is diagonal.[7]

3. The Teigen-Gibson models

Two versions of the Teigen-Gibson model will be used as money supply equations. In both cases reserves adjusted for reserve liberations due to changes in the required reserves ratios and the difference between the commercial paper rate

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6. See Table 5.21, p 114. The deterioration in predictive performance was small. The RMSE for a one period forecast was 4.1393 for the equation without the lagged variable, 5.3046 when the variable was included.

7. Kmenta, 1971, p 586. The correlation coefficient of the estimated residuals of the two equations is 0.65.

and the Federal Reserve discount rate will be used as explanatory variables. In one version of the model the lagged dependent variable will be excluded, in the other it will be included.

Three versions of the money demand equation will also be used in the model. The first will be the demand equation used in the above models. The second demand equation will use permanent income rather than GNP as an explanatory variable while the third equation will express all quantity variables in real terms. The economic activity variable in this last equation is real income. This set of demand equations will be expanded to six by having three with the lagged dependent variable, three without the lagged variable.

Since both equations of this model contain both endogenous variables, to obtain consistent estimates of the coefficients, a simultaneous equation estimation method must be used. Therefore, the coefficients will be estimated by means of TSLS and limited information single-equation (LISE) estimation techniques. Full information estimation techniques are ruled out because of the probability of equation misspecification and the complexity of such estimation techniques for non-linear equation systems.

4. The estimated models

In order to facilitate the discussion of these model, the following discriptors for the various models shall be used.

The multiplier model will be referred to as Model A, and the Brunner-Meltzer model as Model B. The Teigen-Gibson models will have a two letter identifier. The first letter C will identify the model with the lagged dependent variable in the demand equation, and without the lagged varialbe in the supply equation. The letter D identifies those models where both equations have the lagged variable, and the letter E models with the lagged variable in the supply equation but not in the demand equation. The second letter specifies the money demand equation used in the model. A represents the nominal money demand equation with income as an explanatory variable, B represents the equation with permanent income, and C represents the real money demand equation.

The OLS regressions used for Models A and B have been estimated in Chapter V. The estimated coefficients for the money demand equation for both Model A and Model B can be found in Table 6.1 as the OLS estimates for the demand equation of Model CA[8] while the Brunner-Meltzer money supply coefficients are given in Table 5.11.[9] The OLS, TSLS, and LISE coefficients for the three C models are given in Table 6.1. The demand equation coefficients for the D models are the same as the C model coefficients since the exogenous variables are the same for the two models. The

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8. These coefficients are also given in Table 5.2, Model ASY-L.

9. Equation EAA-L.

TABLE 6.1

ESTIMATED COEFFICIENTS OF THE C-SERIES MODELS

	Dependent Variable	Estimation Method	RMCP	Y	Lag	Const	R^2/SE	D.W.
Model CA	Demand ln M	OLS	-0.0180 -5.2234	0.0833 4.6825	0.8462 20.4712	0.2733 2.5536	.9967 .0050	.6531
		TSLS	-0.0234 -2.9987	0.0876 3.4396	0.8526 18.3208	0.2215 2.0092	.9964 .0053	.5953
		LISE	-0.0276 -3.3804	0.0985 3.6965	0.8407 17.2731	0.2188 1.8981	.9961 .0056	.5724
			RMCP	YP	Lag	Const	R^2/SE	D.W.
Model CB	Demand ln M	OLS	-0.0180 -3.0381	0.0281 1.4644	0.9553 18.0959	0.0673 0.4400	.9955 .0059	.5378
		TSLS	-0.0275 -2.9593	0.0699 2.5049	0.9093 17.4756	0.0566 0.4333	.9949 .0063	.4632
		LISE	-0.0317 -3.2025	0.0797 2.6846	0.8993 16.2335	0.0511 0.3672	.9943 .0068	.4449

TABLE 6.1 (cont'd)

	Dependent Variable	Estimation Method	RMDF	RSL	Const	R^2/SE	D.W.
Model CB	Supply	OLS	1.9998	5.0444	36.5859	.9353	.8905
			1.4263	23.8475	9.9818	2.8304	
	M	TSLS	9.6247	4.4668	44.8213	.9003	.9546
			2.9123	13.6765	8.5274	3.5459	
		LISE	33.3249	2.7931	68.0771	0.3257	.8822
			3.8773	3.2883	4.9801	9.2219	
			RMDF	RSL	Const	R^2/SE	D.W.
Model CC	Supply	OLS	1.9998	5.0444	36.5859	.9353	.8905
	M		1.4263	23.8475	9.9818	2.8304	
		TSLS	17.7720	3.8914	52.8158	.7816	.9266
			2.6251	6.6450	5.8460	5.2490	
		LISE	36.6959	2.5550	71.3848	.1873	.8765
			2.8102	2.2619	4.0964	10.1245	

TABLE 6.1 (cont'd)

	Dependent Variable	Estimation Method	RMCP	Y/P	Lag	Const	R^2/SE	D.W.
Model CC	Demand	OLS	-0.0216	0.0539	0.8495	0.4393	.8828	.8960
			-3.8014	3.3383	17.5519	1.7790	.0084	
	ln M/P							
		TSLS	-0.0065	0.0152	0.9000	0.0179	.8783	.8757
			-0.4362	0.4110	15.1185	0.5122	.0090	
		LISE	-0.0033	0.0075	0.9075	0.0234	.8714	.8568
			-0.2153	0.1970	14.8311	0.6510	.0093	
			RMDF	RSL		Const	R^2/SE	D.W.
Model CA	Supply	OLS	1.9998	5.0444		36.5859	.9353	.8905
			1.4263	23.8475		9.9818	2.8304	
	M							
		TSLS	13.3684	4.2024		48.4948	.8560	.9447
			2.9388	9.9439		7.2582	4.2619	
		LISE	32.4743	2.8532		67.2424	.3584	.8838
			3.3833	3.1986		4.7681	8.9955	

OLS, TSLS, and the LISE estimates for the money supply equations, however, are given in Table 6.2. Likewise the estimates for the coefficients of the money demand equation without the lagged variable (Model E) are given in Table 6.3. The dependent variable of the money demand equation of the C, D, and E models is the logarithm of the money stock.[10] The dependent variable of the money supply equation, the Teigen-Gibson money supply function, is the nominal money stock.

The coefficients of the different estimates of the money demand equations in models CA, CB, DA, and DB are quite close whether estimated by OLS, TSLS, or LISE; and the increase in the standard error for these regressions when the coefficients are estimated by TSLS or LISE is small. The difference between the OLS estimates and the TSLS and LISE estimates of demand equations for Model CC and DC is striking. But the coefficients which change the most have t-ratios of above 2.0 when estimated by OLS, and less than 0.6 when estimated by TSLS or LISE. The decrease in the standard error is slight when the demand equation is not estimated by OLS.

The coefficients for the demand equations without the lagged variable, however are affected by the estimation method used. The coefficients for the two variables are

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10. For Models CA, CB, DA, DB, EA, and EB it is the logarithm of the nominal money stock, for Model CC, DC, and EC it is the logarithm of the real money stock.

TABLE 6.2

ESTIMATED COEFFICIENTS OF THE MONEY SUPPLY EQUATIONS
OF THE D-SERIES MODELS

Model	Estimation Method	RMDF	RSL	Lag	Const	R^2/SE	D.W.
DA	OLS	-0.5753	0.4233	0.9255	2.5852	.9952	.6436
		-1.4607	2.2140	25.3430	1.5478	.7682	
	TSLS	2.3205	0.5408	0.8604	7.9564	.9902	.8469
		1.6349	1.9202	14.1255	2.3708	1.1098	
	LISE	2.7395	0.5623	0.8505	8.7094	.9887	.8606
		1.7928	1.8549	12.9706	2.4107	1.1947	
DB	TSLS	0.8634	0.4658	0.8948	5.3373	.9940	.7607
		0.9344	2.1343	19.7067	2.2406	.8691	
	LISE	1.8209	0.5151	0.8722	7.0582	.9918	.8248
		1.6855	2.0186	16.4288	2.5423	1.0161	
DC	TSLS	1.0976	0.4778	0.8893	5.7583	.9936	.7791
		0.7053	2.0358	16.0911	1.7399	.9003	
	LISE	3.8711	0.6205	0.8237	10.7434	.9835	.8825
		1.5502	1.6476	9.2886	2.0230	1.4446	

The money demand equation coefficients are the same as found in Table 6.1.

TABLE 6.3

ESTIMATED COEFFICIENTS OF THE MONEY DEMAND EQUATIONS
OF THE E-SERIES MODELS

Model	Estimation Method	RMCP	Y (or Y/P)	YP	Const	R^2/SE	D.W.
EA	OLS	-0.0345	0.4271		2.3616	.9700	.2140
		-3.4041	23.9443		24.0774	.0153	
	TSLS	-0.0920	0.5090		1.9265	.9506	.3532
		-3.5935	12.4844		8.7459	.0198	
	LISE	-0.2979	0.8242		0.2374	.5904	.4486
		-4.0411	7.0169		0.3741	.0571	
EB	OLS	-0.0049		0.3591	2.7760	.9667	.1654
		-0.5083		22.6158	32.4503	.0161	
	TSLS	-0.0973		0.5014	1.9894	.0356	.3705
		-3.2489		10.8360	8.0225	.0226	
	LISE	-0.3417		0.8646	0.0578	.4288	.4509
		-3.8319		6.2763	0.0783	.0674	
EC	OLS	-0.0487	0.0900		4.4592	.1769	.2194
		-3.3595	2.1226		18.2401	.0222	
	TSLS	-0.1369	0.3016		0.0458	.0893	.3958
		-3.5023	2.9282		0.4035	.0293	
	LISE	-0.4212	1.0223		-0.7291	.0188	.4440
		-3.8461	3.5432		-2.2924	.0822	

smallest in magnitude when estimated OLS, largest when estimated LISE. The size of the standard error is increased when the simultaneous estimation techniques are used --especially LISE.

The estimation of the money supply equations of the C and D models are also sensitive to the method of estimation. The standard error of estimate for the TSLS regression is in some cases almost double that of the OLS estimates, and the LISE standard error double that of the TSLS regression. The t-ratio for the interest rate variable for the C models is below 1.5 when estimated OLS, but is around 3.0 when estimated RSLS or LISE. But when the lagged variable is introduced into the money supply equation, the interest rate t-ratio drops to below 2.0, and the lagged variable t-ratio becomes quite high--at least four times the value of any other-t-ratio in the equation. The Durbin-Watson statistics are low whether or not the lagged variable is included in the model. This again affirms the strong time-trend of the money stock series.

C. EVALUATION OF PREDICTION PERFORMANCE

The use of a simultaneous forecasting model implies that the endogenous variables will be jointly forecast. One way of forecasting in such a situation is to determine the reduced form of the estimated regressions, and, as has been done in the previous chapters, place the exogenous variables in the reduced form equations and compute forecasts of the money stock and the interest rate.[11]

But there is a way to avoid computing the reduced form version of the models.[12] A simulation program can be used to jointly determine the value of the two endogenous variables. In this chapter a simulation routine will be used to make six quarter forecasts using actual exogenous variables and the forecast endogenous variables. In other words, at time T the money stock and the interest rate will be forecast for time T + i where i = 1, 2, ...6. Actual values of the exogenous variables will be used at time T + i, but the endogenous variables on the right hand side of the equation will use the forecast for the T + i period. The process will be repeated until forecasts for the entire 1961-1970 period are obtained.

The simulation program used required that each endogenous variable appeared once by itself on the left hand side of the equation. The regressions, however, were estimated such that only the money variable appeared on the left hand side of the equation. In the case of the C and D models, the money supply equation was left unchanged, but the money demand regression

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11. Since the two-equation models are non-linear, the reduced form equations could not be derived directly. The equations would first have to be linearized by means of a Taylor series expansion.

12. Actually this is not a problem for Model A or Model B.

$$(6.1) \qquad \ln \hat{M} = \hat{\beta}_0 + \hat{\beta}_1 \ln RMCP + \hat{\beta}_2 \ln Y + \hat{\beta}_3 \ln M_{t-1}$$

was rewritten

$$(6.1a) \qquad \ln RMCP = - \frac{\hat{\beta}_0}{\hat{\beta}_1} - \frac{\hat{\beta}_2}{\hat{\beta}_1} \ln Y - \frac{\hat{\beta}_3}{\hat{\beta}_1} \ln M_{t-1} + \frac{1}{\hat{\beta}_1} \ln M_t .$$

The other two versions of the money demand equations for MOdel C were rewritten with the appropriate changes. The transformation shown in Equation 6.1a was also used for the simulations of Model A and Model B.

Since we have both TSLS and LISE estimates of the model, the question arises as to which should be used for forecasting. In all cases, the standard error of estimate was lower for the TSLS regressions. This suggests the use of the TSLS coefficients. Moreover Theil maintains that when a difference in the values of the estimated coefficients is observed, that the TSLS estimates should be used in preference to the LISE. [Theil, 1971, p 532]. The TSLS coefficients, therefore, will be used in the forecasting equations.

D. THE RESULTS

1. Forecasting the money stock with recursive models

The results of the multiplier model (Model A) and the Brunner-Meltzer model (Model B) are identical to that of the single equation models.[13] Since there is no lagged term in the multiplier model, predictive ability does not decline as the forecasting period is lengthened.[14] The lagged term, however, is included in the Brunner-Meltzer equation so that forecasts longer than one quarter can be generated.

The overall RMSE statistics for one through six quarter forecasts for these two models are given in Table 6.4 while the mean percent absolute error data is given in Table 6.6. The error associated with forecasting the money stock in recursive models is the same as for the single equation models with the same endogenous variables. Therefore the ability to forecast the money stock by recursive models can be determined by examining the predictive performance of the appropriate single equation models.[15]

2. Forecasting the money stock with non-recusrive models

Unlike Models A and B, the Teigen-Gibson money demand models are not recursive, and so the money stock and the

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13. For Model B, see Table 5.21. The RMSE statistics for the multiplier model are a bit different since they were computed over the slightly different time period. Comparison of the annual RMSE data, however, shows the identity of the results.

14. When a lagged term is not included in an ex post forecasting model, all forecasts are essentially one period forecasts. See p 103.

15. For example, the predictive performance of the not-lagged first Brunner model can be found in Table 5.11, Model EBA.

TABLE 6.4

PREDICTIVE PERFORMANCE OF TWO-EQUATION MODELS OF THE MONEY STOCK

Model	Quarter 1	Quarter 2	Quarter 3	Quarter 4	Quarter 5	Quarter 6
A	2.2380	2.2245	2.2294	2.2270	2.2489	2.2292
B	5.3047	10.1477	14.3957	18.1238	21.6497	24.9463
CA	2.6365	4.9729	6.8511	8.3591	9.7970	11.1019
CB	3.1680	6.0131	8.3720	10.3227	12.1529	13.8141
CC	1.9134	3.6563	5.1022	6.3747	7.6729	9.0093
DA	2.4253	4.6765	6.5591	8.1445	9.7159	11.2000
DB	2.0239	3.9690	5.6142	7.0363	8.5071	9.9419
DC	1.9200	3.4675	4.7988	5.8844	6.9840	8.1252
EA	5.3809	8.6880	10.6554	11.8599	12.8755	13.7668
EB	3.6542	6.7601	9.0871	11.0575	12.7402	14.2245
EC	3.2573	5.8414	7.7314	9.0897	10.2491	11.2196

TABLE 6.5

PREDICTIVE PERFORMANCE OF TWO-EQUATION MODELS OF THE INTEREST RATE

Model	Quarter 1	Quarter 2	Quarter 3	Quarter 4	Quarter 5	Quarter 6
A	4.4144	6.6897	6.6651	6.6306	6.6508	6.6743
B	9.5640	10.0182	10.3759	10.5826	10.9316	11.3324
CA	0.5427	0.5730	0.5856	0.6088	0.6701	0.6990
CB	0.7537	0.8306	0.9070	0.9856	1.0742	1.1226
CC	0.5174	0.5664	0.5854	0.6256	0.6971	0.7410
DA	0.6010	0.6156	0.6145	0.6235	0.6836	0.7137
DB	0.5629	0.5970	0.6182	0.6541	0.7502	0.8043
DC	2.1404	1.9838	1.9504	1.9972	2.0446	2.0589
EA	1.7657	1.1202	0.8126	0.7036	0.7318	0.7642
EB	2.2249	1.8731	1.5012	1.1862	0.9570	0.8187
EC	1.5824	1.2159	0.9741	0.8668	0.9290	1.0093

TABLE 6.6

COMPARISON OF THE PREDICTION OF THE MONEY STOCK
AND THE INTEREST RATE

(Mean Percent Absolute Error of 1 to 6 quarter forecasts)

Model	Var-ible	Quarter 1	Quarter 2	Quarter 3	Quarter 4	Quarter 5	Quarter 6
A	M	1.1064	1.0855	1.0937	1.0805	1.0850	0.0545
	RMCP	341.0984	486.3145	482.4153	472.3153	477.1770	483.5724
B	M	2.4016	4.5519	6.4459	8.1175	9.6476	11.0543
	RMCP	846.2194	887.4291	922.6329	949.8376	983.9110	1018.660
CA	M	1.0942	1.9787	2.6774	3.2316	3.7246	4.1565
	RMCP	40.4304	41.9595	43.5137	45.7172	49.5123	51.7881
CB	M	1.2435	2.3235	3.2548	4.0475	4.7703	5.4011
	RMCP	62.6529	74.8008	84.1521	89.2568	95.4925	98.2608
CC	M	0.8220	1.5581	2.1977	2.7478	3.3071	3.8625
	RMCP	36.1223	39.7880	42.7585	47.4642	51.8774	56.9353
DA	M	0.9594	1.7912	2.4899	3.0973	3.6489	4.1768
	RMCP	41.3806	42.6238	42.5931	43.7603	47.6267	50.2393
DB	M	0.8170	1.5529	2.1917	2.7547	3.2930	3.8307
	RMCP	34.7272	37.0629	39.3253	42.8063	48.6164	52.8889
DC	M	0.8100	1.4396	2.0023	2.4070	2.8334	4.4933
	RMCP	101.0370	92.5165	90.5429	94.9068	100.1531	101.8735
EA	M	2.0959	3.2785	3.9573	4.3761	4.7017	4.9634
	RMCP	129.5495	80.2648	58.8731	50.7998	52.4478	54.6762
EB	M	1.4267	2.5643	3.4482	4.1110	4.6970	5.1871
	RMCP	160.6466	132.6093	103.5653	79.1343	62.6211	54.4618
EC	M	1.3672	2.4038	3.1593	3.6926	4.1328	4.4737
	RMCP	121.0287	92.2134	71.3931	62.1599	62.2277	64.4639

interest rate are jointly determined. The forecast quantities, therefore, will be differerent than the single equation forecasts. Ignoring the inclusion or exclusion of the lagged variable for a moment, the best performing models in general had the real money demand equation.[16] In two cases the models with the money demand equation with permanent income as an explanatory variable were ranked second.[17] The model with the lowest RMSE was Model DC where both equations had lagged variables and the demand equation was in real terms.[18]

The incorporation of the lagged dependent variable in a single equation model usually reduced the forecasting error for short forecasts. This gain in forecasting performance with the lagged variable, however, deteriorated as the forecast was lengthened since the model now included a forecast independent variable: the lagged variable. This caused error buildup to occur as the period of the forecast increased. Both of the equations in the D series models have lagged variables while the C series models only the money demand equations have a lagged variable. In the E series it is the money supply equations models which have the lagged variable. Single equation models without the

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16. Models CC, DC, and EC.

17. Models DB and EB.

18. In the first quarter Model CC had a lower RMSE, but this was by a very small margin.

lagged variable tend to have a relatively high, but constant error term over the forecast period while single equation models with the lagged variable tend to have an initially low error which builds as the forecast is extended. To what extent does this pattern carry over to the simultaneous equation models?

The data presented in Tables 6.4 show the D series models which have lagged dependent variables in both equations forecasting with a lower RMSE than models where at least one equation lacks the lagged variable.[19] Moreover error buildup occurs in all the forecasts, and with a pattern and magnitude similar to that of the single equation models.[20] The series with the highest overall RMSE statistics of the non-recursive models was the E series models which had the demand equation without the lagged variable.

The annual RMSE statistics for the C, D, and E series models are given in Table 6.5. These statistics reveal that part of the lower overall RMSE associated with the D series models is due to the fact that the annual RMSE statistics for these models deteriorate less in the late 1960's than

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19. The exceptions to this statement are the one-quarter forecasts of Model CC and the six quarter forecasts of Model CA. The difference in RMSE in each of these cases was small.

20. See Table 5.19 for the single equation money demand RMSE statistics. The RMSE statistics for the Teigen-Gibson model with the lagged variable over the six quarters are 1.8129, 3.5459, 5.0301, 6.3308, 7.6775, and 8.9922.

do those of other models. In the early 1960's other models frequently outperformed the D series models, but around 1967 when the quality of the 6 quarter forecasts declined, except for Model CA, the decline was less for the D series models.

Some of the RMSE statistics of the single equation model are given in Table 6.5 in order to compare the relative forecasting power of one and two equation models. Examination of the overall RMSE statistics show that except for Model DC, the RMSE for the six-quarter forecast of the Teigen-Gibson model with lagged variables was lower than any of the other two-equation models. The real money demand equation also had a lower RMSE than any of the two equation models. When two superior forecasting models are combined to make a two-equation forecasting model, that model is characterized by low RMSE statistics although in these cases the predictive performance of the single equation model is superior.[21] The RMSE statistics for the two equation model tends to be the average of the two individual equation models.

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21. For example, the RMSE statistics for Models FRD-L and ASW-L are close--8.9922 and 9.8346; but when combined to become Model DB, the RMSE is slightly higher--9.9414. This, of course, would be expected unless there were a strong negative covariance between the error terms of the two equations.

3. Forecasting the interest rate

While forecasting the interest rate is not a matter of concern in this study, nevertheless it may be worthwhile to examine these predictions. Table 6.7 gives the RMSE statistics for six quarter forecasts of the interest rate. The RMSE statistics for Models A and B are high. This situation follows from the recursive nature of the system: the money stock is first forecast, and only then is the interest rate forecast.

Since the average value of the interest rate is different from the average value of the money stock, a more realistic statistic to examine is the mean percent absolute error (MPAE) for both the money stock and the interest rate forecast. These statistics are given in Table 6.6. They bring out dramatically the inferior quality of the recursive model forecasts of the interest rate. This table, however, also shows that the interest rate forecasts of the non-recursive simultaneous equations models are inferior to that of the forecasts of the money stock. The forecasting performance over the six quarters remained constant for the D series models, deteriorated over the forecast period for the C series models, and actually improved over the forecast period for the E series models. It appears, therefore, that the money demand equation contributed more to the determination of the interest rate than the money supply

TABLE 6.7

ANNUAL PREDICTIVE PERFORMANCE OF SELECTED MODELS

(RMSE of 6 quarter forecasts)

Year	CA	CB	CC	DA	DB	DC	EA	EB	EC
1961	5.7126	3.0652	0.6661	3.8919	3.6908	0.8997	5.9860	5.0101	5.9980
1962	3.4401	1.0451	2.3528	1.7803	1.7096	2.0502	3.6460	2.8559	3.2970
1963	0.7915	2.6767	4.6703	0.9676	1.1578	4.3096	1.2220	0.3543	0.4369
1964	2,0812	6.1738	7.3238	3.4884	3.7421	6.8873	1.5756	3.4154	3.5787
1965	0.9868	4.9718	4.5602	1.8785	2.1380	4.3518	1.2464	2.2584	2.6890
1966	6.0297	9.8062	9.0068	7.2087	7.1598	8.9697	6.8002	8.1566	7.7727
1967	13.1051	17.7391	13.4648	14.0483	13.4689	13.5093	14.5598	16.0745	14.7374
1968	13.8236	18.6517	8.4712	13.7253	12.0992	5.6978	17.6337	18.0811	15.2669
1969	18.5918	22.4626	10.6037	18.3410	15.7224	6.0092	23.3508	23.8124	17.9028
1970	20.9111	23.7592	16.2120	21.0788	18.0444	8.1252	26.9474	27.4650	11.2196
61-70	11.1019	13.8141	9.0093	11.2000	9.9414	8.1252	13.7668	14.2245	11.2196

equation.[22]

The larger forecasting error for the interest rate can be explained by the fact that the interest rate series itself is a more volatile series than the money stock series. Moreover these equations were not conceived as having the interest rate as the endogenous variable.

D. SUMMARY

The results of this chapter can be summarized very briefly. (1). The money stock forecasting error in a two-equation recursive system will be the same as for the single equation model without the system's other endogenous variable. (2) The forecasting error of a model where the two endogenous variables are jointly determined, the money stock error statistic will tend to be the average of the results of the two single equation models.[23] (3). The models which were used in this study were poor forecasters of the interest rate.

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22. In order for error buildup to occur in a model over the forecast period, the lagged version of the model must be used. Error build-up occurred when lagged versions of the money demand equations were used, and the money supply equations had no lagged term (the C series models), and the opposite phenomenon occurred when the not-lagged version of the money demand equation and the lagged version of the money supply equations were used.

23. See footnote 18 for a caveat on this observation.

CHAPTER VII

CONCLUSION

This chapter is an outline of the principle results of this study. Because of the nature of the study, emphasis will be on the preceeding three chapters. Of immediate interest, of course, is the answer to the question: Which of the money stock models considered in this study forecast best? In addition, the various means of improving predictive performance and the evaluation of predictive performance will also be discussed in this chapter.

A. FORECASTING PERFORMANCE

The models which forecast best in this study were two mechanistic models: the autoregressive seasonally adjusted money stock model and the no-change money multiplier model.[1] The autoregressive model had the lowest RMSE statistic for short forecasts while the multiplier model had the lowest RMSE statistic for longer forecasts. The predictive performance of the autoregressive multiplier

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1. Since the monetary base must be forecast when using the multiplier models, they are not true mechanistic models. It may, in fact, be almost as difficult to forecast the base as it is to forecast the money stock. The value of the multiplier itself, however, is forecast by means of a mechanistic model.

model was also quite respectable, but examination of the estimated coefficients for this model show that it is essentially a no-change model.[2]

Three models are next in terms of forecasting performance. Two are single-equation real money demand models with the lagged dependent variable, the short-term interest rate, and either income or permanent income.[3] The other model is the single-equation Gibson model with the lagged dependent variable.[4] The RMSE statistics for these models were quite close for the short forecasts, but the real money demand models had better prediction statistics than the Gibson models in the longer forecasts. The next level of predictive performance would include many of the other money demand equations with the lagged variable. Also included in such a list would be the better performing of the two-equation forecasting models.

B. IMPROVEMENT OF FORECASTING PERFORMANCE

Since time series data were used in this study, the improvement of forecasting performance which occurred when a lagged variable was included in the model was not

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2. See Equation 4.11 and Equation 4.14.

3. The real money demand model with permanent income as the explanatory variable had a slightly lower RMSE over the forecast interval.

4. The Gibson model with the interest coefficient not constrained to be equal in magnitude had somewhat lower RMSE statistics than did the model with the constrained interest rates.

unexpected. Only once did forecasting performance of single-equation models deteriorate when such a variable was added.[5] The Gibson model was one example where the improvement in predictive ability of a model was quite marked.

One way of correcting for the temporal interrelatedness of the variables is to include the lagged variable in the model; another way is correcting for first-order autocorrelation. When the money demand equations without lagged variables were corrected for first-order autocorrelation, the RMSE of a forecast was reduced even more than when the lagged variable was included in the equations. When the money demand equations with the lagged terms were corrected for autocorrelation, the RMSE was also usually reduced but by a smaller proportion.

While no attempt was made in this study to use or evaluate judgmental corrections of the constant term, various mechanical constant adjustment techniques were used, all of which used the residual from the forecast of the previous period. The RMSE in most cases was reduced when these adjustments were made. The RMSE of models without the lagged term, when adjusted, was lower than that of the lagged model not adjusted. Adjustment of the lagged model, however, also improved forecasting performance.

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5. This case was the first Brunner-Meltzer model, Equation 3.10a.

The best performing constant adjustment term in this study was the previous period's residual.

C. TWO-EQUATION MODELS

Two types of the two-equation forecasting models, recursive and non-recursive, were examined. When the system was recursive, the money stock RMSE statistics were the same as the statistics for the one equation model which contained both endogenous variables. The other equation would forecast the other endogenous variable which in this case was the interest rate. The RMSE statistics for the interest variable were quite high.

The non-recursive two-equation models, where both endogenous variables were in both equations, consisted of the Gibson equation and three forms of the money demand equation. The models with the real money demand equation with the lagged dependent variable and the Gibson model with or without the lagged variable had the lowest RMSE.[6] These models, however, had higher RMSE's than did the better single equation models. Forecasts of the interest rate by the non-recursive models, while better than those by the recursive models, still had high RMSE statistics.

Comparison of the coefficients estimated by means of OLS and TSLS reveals that the two nominal money demand equations with a lagged variable were insenstive to

6. The Gibson equation with the lagged variable had a slightly lower RMSE over most of the forecast period.

estimation model. But the estimated coefficients of the Gibson model and the real money demand model whether or not the models include a lagged variable were quite sensitive to estimation method. This was also the situation for the nominal demand models without the lagged variable. In practicially all cases for these models, the TSLS estimates differed from the OLS estimates, and the LISE estimates differed even more so. The sensitivity of the lagged Gibson equation to the method of estimation was particularly interesting in light of the forecasting performance of the single-equation Gibson models.

D. EVALUATION OF PREDICTIONS

The RMSE statistic was the main prediction evaluation statistic used in this study. When it was necessary to compare the forecasts of different variables, the mean percent absolute error statistic was used. Little if anything seemed to be gained by using other prediction evaluation statistics. Regressions of the predicted quantity on the actual quantity showed themselves incapable of being a measure of forecasting performance. Only the standard error of estimate appeared to be of interest, but it revealed nothing that was not already revealed in a more meaningful fashion by the RMSE statistic.

Both the mechanistic and the one-equation models were tested to determine whether a significant structural shift occurred between 1947-1960 and 1961-1970. The hypothesis that no structural shift occurred was rejected for four of the five mechanistic models examined. The exception was the no-change multiplier model which for forecasts of over six months was the best performing mechanistic model. Among the single equation models, the hypothesis that structural shift had not occurred could be rejected for only two models without a lagged dependent variable: the first Brunner-Meltzer model and the Teigen model. Inclusion of the lagged term in the equations improved the structural stability of the models, and increased the number of models for which the hypothesis could be rejected. Included in a listing of these models would be the money demand models and the lagged Gibson model. It should also be noted that the Teigen model, with and without the lagged dependent variable, did not exhibit significant structural shift, but also had the worst prediction record of the single equation models examined. Structural stability, therefore, is not a sufficient condition nor, based on the evidence from this study, a necessary condition for forecasting with low RMSE.

E. INCOME AND INTEREST ELASTICITIES

The coefficients of the money demand equations indicated low impact and steady state income and interest elasticities.[7] The elasticities derived from TSLS estimates of

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7. In only one case were the elasticities above 1.0, and then it took 25 quarters for the elasticities to reach

the money demand equations with the lagged dependent variable were close to those derived from OLS estimates, but TSLS estimates of the demand equations without the lagged variables indicated elasticities of somewhat greater magnitude. Compared to other studies, however, the values of both the interest and income elasticities remained low.

F. SUMMARY

The results of this study can be seen as presenting a challenge to the econometric model builder. Two mechanistic models forecast with lower forecasting error than any of the one- or two-equation economic models. It should be noted, however, that the best performing economic models forecast as well as the autoregressive money stock model in the longer forecasts while the monetary base, the exogenous variable in the no-change multiplier model, may itself be quite difficult to forecast. Moreover, the strong time trend of the money stock data series may help explain the high level of performance of the mechanistic models, and indicate why such models provide such a tough performance standard for the economic models.

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one-half its steady-state value.

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NOTATION APPENDIX

a — Ratio of demand deposits subject to reserve requirement to demand deposits held by the non-bank public.

a' — Ratio of demand deposits held by the non-bank public and subject to reserve requirements to demand deposits held by the non-bank public.

a" — Ratio of time deposits held by the non-bank public to time deposits of member banks.

B — Monetary base. Usually the base is the net source base: the sum of unborrowed reserves plus currency held by the non-bank public or not included in reserves.

b — Ratio of borrowed reserves to total deposits subject to the reserve requirement.

CURB — Currency and coin held by member banks and included in reserves.

CURN — Currency in circulation.

CURR — Currency held by the public, i.e. currency component of the money stock series.

c — 1. The multiplier model. Ratio of currency held by banks but not part of their reserves to currency held by the non-bank public.

2. The Teigen model. Ratio of currency held by the non-bank public to the money stock.

DDO — Non-Federal government and non-bank demand deposits held at member banks subject to the reserve requirement.

DDP — Demand deposits held by the non-bank public, i.e. the demand deposits component of the money stock series.

DDR — Adjusted net demand deposits at all member banks.

DTM Total member bank deposits subject to the reserve requirement.

D Structural shift dummies for the Teigen model.

d The ratio of Treasury deposits to demand deposits held by the public.

E Exogeneous expenditure (as used in the Teigen model).

h Ratio of private demand deposits held by non-member banks to the money stock.

k 1. Multiplier models and the Brunner-Meltzer models. Ratio of currency held by the public to demand deposits held by the non-bank public.

2. The Teigen model. Ratio of private demand deposits of member banks to reserves required for those demand deposits.

M Money stock held by the public, i.e. currency and demand deposits held by the non-bank public.

M* Money stock component that is based on supplied (exogeneous) reserves (as specified in the Teigen model).

m Money multiplier.

N Population of the United States including the armed foreces overseas.

NW Net worth (as used in the Teigen model).

P Implicit GNP price deflator.

RMCP Commercial paper rate on 4-6 month prime paper.

RMDD Implicit yield on demand deposits.

RMDF Difference between the commercial paper rate and the Federal Reserve discount rate.

RMFR Federal Reserve discount rate.

RMLB Yield on domestic corporate bonds, Moody's Aaa rated.

RMLG Yield on long-term government bonds.

RMTB Yield on 90-day Treasury bills.

RMTD	Effective yield on pass-book savings deposits at commercial banks.
RRD	Implicit reserve requirement against demand deposits subject to the reserve requirement.
RRT	Implicit reserve requirement against time deposits of member banks.
RSF	Free reserves.
RSL	Total reserves plus reserves released through changes in the reserve requirement.
RSR	Required member bank reserves.
RSREL	Reserves released through changes in the reserve requirement.
RSRPD	Reserves available to support private non-bank deposits (RPD's).
RST	Total member bank reserves.
RSU	Unborrowed reserves.
r	Ratio of reserves to total deposits.
S_i	Seasonal dummies.
TD	Time deposits held by the public.
TDM	Time deposits of member banks subject to the reserve requirement.
t	Ratio of time deposits held by the non-bank public to demand deposits held by the public.
Y	Gross National Product.
YP	Permanent income, an exponentially weighted average of GNP. See footnote 2, p 75.

DATA APPENDIX

The sources of data used in this study were quite standard and are listed below. Where possible seasonally adjusted data was used.

Reserves and member bank deposits data was taken from Board of Governors, Federal Reserve System, revision of "Aggregate Reserves and Member Bank Deposits," Statistical Release H.3, April 1972. Some early data (prior to 1958) was obtained from the Federal Reserve *Bulletin*.

Reserves released by changes in the reserve requirement data series was from the Economic Research Department, Federal Reserve Bank of St. Louis.

The money stock series (i.e. demand deposits and currency held by the non-bank public) was from the Federal Reserve *Bulletin*, December 1970, pp 895-898.

Other data such as income and the interest rate series were taken from *Business Statistics*, 1971. In some cases, earlier volumes of this series were consulted.