SIMULATION, DECOMPOSITION AND CONTROL OF A MULTI-FREQUENCY DYNAMIC SYSTEM: THE UNITED STATES HOG PRODUCTION CYCLE

Dissertation for the Degree of Ph. D.

MICHIGAN STATE UNIVERSITY

HOVAV TALPAZ

1973

This is to certify that the

thesis entitled

SIMULATION, DECOMPOSITION AND
CONTROL OF A MULTI-FREQUENCY
DYNAMIC SYSTEM:
THE UNITED STATES HOG
PRODUCTION CYCLE

presented by

HOVAV TALPAZ

has been accepted towards fulfillment
of the requirements for

Ph.D. degree in Agricultural Economics

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Major professor

Date 10/22/73

O-7639

ABSTRACT

SIMULATION, DECOMPOSITION AND CONTROL OF A MULTI-FREQUENCY DYNAMIC SYSTEM: THE UNITED STATES HOG PRODUCTION CYCLE

By

Hovav Talpaz

The hog production industry experiences large fluctuations in its production activities, inventories, sales, purchases, capacities and earnings. It is a dynamic industry characterized by cyclic, seasonal, and regular as well as irregular patterns of behavior. This phenomenon has a harmful impact on consumers and marginal producers, and, in the long run, it may decrease the competitive nature of the industry.

The problem can be divided into two areas. The first relates to improving the capability of the industry to supply its end products in an acceptable manner from the standpoint of price and stability, while retaining characteristics which encourage efficient performance, and guard against emerging market power which would be likely to introduce rigidities, unduly high prices, and other undesirable performance patterns. Damping the hog cycle or some components of it should provide an indirect, partial solution. The second area relates to improving the theoretical framework used in examining hog production and making it more useful in analyzing such a

dynamic industry. These improvements, dealing with manageable and controllable variables, must be incorporated into a quantitative framework. A better public policy can emerge if both problem areas are successfully attacked.

The main objectives of this study are:

1. To develop a U.S. hog supply response model subject to biological and economic constraints and, in the process, to identify the causal relationships, the feedback structure and the connections of the production subsystem to the marketing subsystem. The model would be capable of explaining and rationalizing the hog cycle phenomenon.
2. To construct a computer simulation model which reflects the above mentioned objectives.
3. To evaluate the production response to an alternative set of policy input control measures designed to eliminate or to restrict certain aspects of the hog cycle.

This thesis is a contribution to a larger, coordinated study entitled,"Systems Analysis of the U.S. Hog-Pork Subsector". It studies the production of hogs from breeding to slaughtering with the economic behavior of the industry as a whole subject to biological constraints. The demand function for hogs and the distribution and marketing subsystem are beyond the scope of the study, although they remain compatible with the study and allow for future integrated subsector research.

In analyzing the forces responsible for the creation of the hog cycle, a model was developed in which the hog-corn price ratio and the volume of sows farrowing are represented by a trigonometric time function, which is an application of the Fourier Series. Using hypothetical demand and supply functions for hogs, under the assumptions of the familiar Cobweb Model a single frequency cycle should have emerged. Extension of the basic Cobweb Model, which includes short, intermediate, and long run demand and supply curves operating simultaneously led to the development of the Multi-Frequency Cobweb Model. The model was then econometrically estimated, empirically tested and accepted. The results suggest that there are six hog cycles, which differ by their amplitude and frequencies. The existence of production delays and significant feedback influences were also confirmed.

Based on these findings and assumptions about production technology, a production-supply response model was hypothesized, estimated, tested and accepted. This model includes economic and technological causal relationships. A two-stage computer system was constructed. Stage I uses a variable parameter selector, designed to identify and estimate elements of the system's state and output vectors, using stepwise recursive econometric procedures. In Stage II, using a time-variant mixed difference equations system, the model includes a state hog population vector with differential age groups, a transfer matrix, composed of death loss, and farm slaughter rates, biological growth rates, a lagged price vector representing market forces in relation to breeding, and sales behavior. A unique slaughter allocation model to yield age and weight distributions of market hogs was developed based on a normal distribution approximation adjusted for seasonal skewness.

Simulation of the hog production industry, using this model, shows a very close relationship with the reported, real world data during the sample time period as well as beyond it.

Next, an open-loop policy control scheme, which removes all cycles with time periods longer than one year from the hog-corn cycle, was applied. It demonstrated how the hog cycle could be damped, yielding potential benefits for producers and consumers.

Specific conclusions based on the study include the following: 1) the hog cycle is composed of six cycles with time durations of four, two and one years, sixteen, six and four months; 2) this phenomenon is explained by a Multi-Frequency Cobweb Model, which assumes long and short run demand and supply functions operating simultaneously; 3) the sows farrowing variable represents the major state variable set, where the hog-corn price ratio represents the major input signal for the production system; 4) the output of the production system (sales) is more sensitive to the price policy application than the state variable--sows farrowing; 5) the model can assist in evaluating policy control alternatives, and complete evaluation is possible, using the entire hog-pork subsector model.

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A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Agricultural Economics

1973

Dedicated to Nechama, Israel, Dorit and Yaron

ACKNOWLEDGMENTS

The guidance, intellectual stimulation, encouragement and advice which Dr. Warren H. Vincent gave me throughout my stay at Michigan State University, as my major professor and adviser, is gratefully acknowledged. His suggestions in preparing and conducting my entire study program, and the freedom he allowed me in the preparation of this thesis made it a challenging and vastly rewarding experience. Due to his constructive comments, many obstacles have been overcome, and the road toward the eventual solution was paved. His and Mrs. Vincent's friendship and understanding greatly assisted my family and me in adjusting to and living in the Michigan State University environment.

The contributions to my educational program and professional development, made both jointly and separately by Dr. Lester Manderscheid, Dr. Larry Connor, Dr. Robert Gustafson, Dr. Robert Barr and Dr. Gerald Park, as members of my guidance and thesis committees, are fully recognized with sincere appreciation.

The author is grateful to the U.S. Department of Agriculture for its financial support and to Dr. Richard Crom who served as the project leader, for his encouragement and contributions to the study.

I would like to thank Dr. Glynn McBride and Dr. George Kessler for their great contributions in editing the final draft of this thesis and for their assistance and support in many aspects of my program.

The author is grateful to the Agricultural Economics personnel located at the Computer Center for their crucial contribution in carrying out this study.

Thanks are extended to Mrs. Denise Wootton who typed the early drafts of the manuscript in an excellent manner and to Mrs. LaNita Campbell who capably typed the final draft and followed up with the technical details.

Finally, but not the least, my deep gratitude to my wife, Nechama, for her personal contribution, sacrifice and encouragement and to my children Israel, Dorit and Yaron, who waited patiently for their student father. It is to them that this thesis is dedicated.

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Laws are like cobwebs, which may catch small flies, but let wasps and hornets break through.

Jonathan Swift
1667-1745

CHAPTER I

THE PROBLEM, OBJECTIVES AND SCOPE

The Problem Setting

The hog production industry experiences large fluctuations in its production activities, inventories, sales, purchases, capacities and earnings. It is a dynamic industry characterized by cyclical, seasonal, and regular, as well as irregular patterns of behavior. Basically, five groups should be concerned with these fluctuations: hog producers, input suppliers, processors, wholesalers and retailers, and consumers. These groups do not share equally the rewards and penalties during the changing phases of the cycle. To the contrary, other things being equal, high commodity prices penalize the consumer while benefiting the producer, and vice versa. The cycle may not even affect all producers in the same way during any given phase of the cycle.

As an industry working under fairly competitive conditions and a wide range of production efficiency, severe price fluctuations are likely to go below the average variable cost of many farmers, who may then be forced to terminate production temporarily or leave the industry entirely. The economic forces also create an opposite movement between the quantity supplied and market prices which affects everyone along the commodity "assembly line" including the consumer. The oscillatory path of pork supply and prices (shown in Figure 1.1) severely increases the problem of resource allocation, resulting in over- or under-utilized capacity.

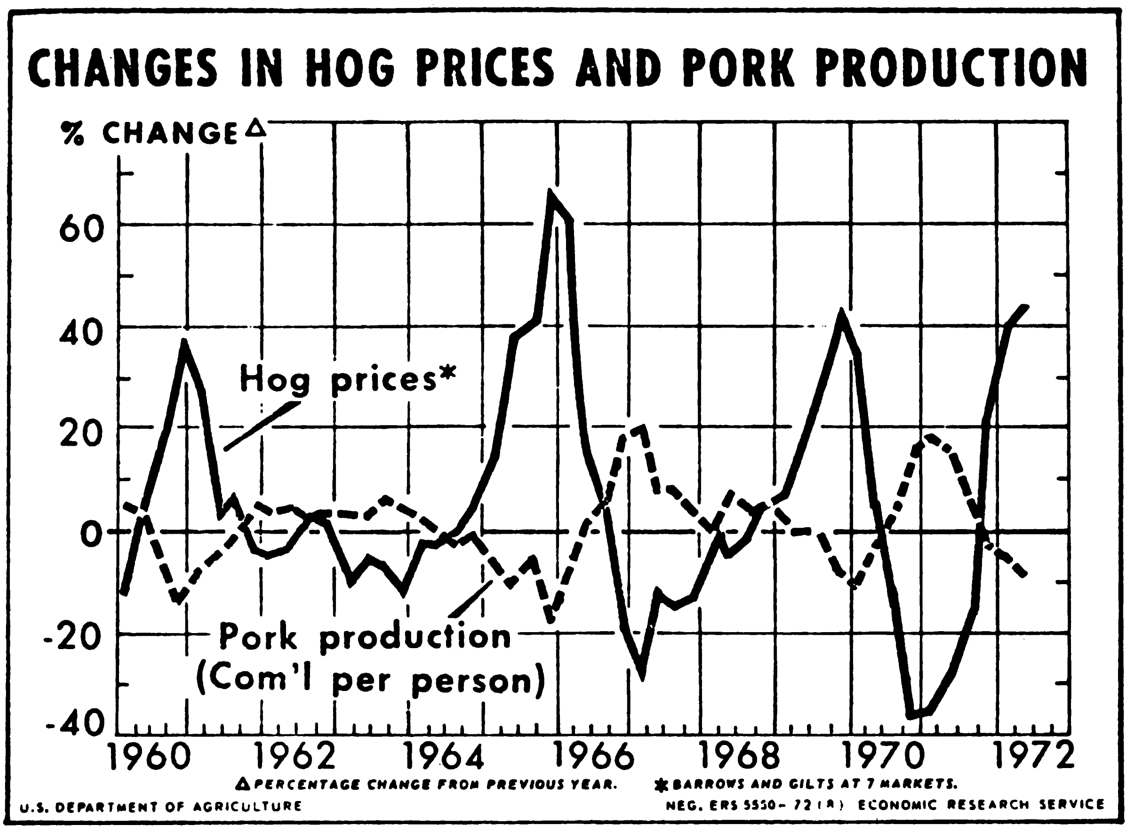

Figure 1.1.--Changes in hog prices and pork production.

Conditions of risk and uncertainty play a substantial role in the farmer's decision making process. Those producers that are forced out of operation during low price periods may leave behind a less flexible and less competitive industry than before. The way may be left open for a growing market power leading to higher consumer prices by restricting output, politically imposing import quotas, or a combination of these and other means. On the other hand, the remaining hog producers may become more efficient with their larger scale of production, provided economies of scale really exist. One also needs to consider the adverse effects of external circumstances such as pollution caused by large concentrated production units. The final outcome is unclear.

These oscillations are a concern to the consumer because they affect consumption patterns. In periods of high pork prices, consumers look for substitutes for pork. Having become accustomed to the substitutes a return to pork consumption is not assured when prices fall. Furthermore, where the housewife is looking for a readily available commodity with a stable price, she is likely to abandon pork under these conditions, especially if it is of lower quality or lower status than the commodities in the substitution group. To overcome this problem, meat packers and wholesalers tend to build expensive inventories of frozen pork beyond the capacity required, just to overcome short run seasonal variation in production.

What can be done? In the current debate over the likelihood of bold changes in farm programs, some economists suggest transferring some support programs from crop industries to livestock industries.[1] What would be the impact of such

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[1]Leo Meyer, of the Council of Economic Advisors, in a recent seminar held in the Department of Agricultural Economics at Michigan State University.

a change, and how would the industry respond to it? Given alternative policy input controls, what are the prospects of moderating or nearly eliminating the cyclical behavior of the industry (or some particular aspects of it)? These are some of the problems being addressed by this research. To answer these questions one immediately confronts a different set of problems and issues. These problems are associated with the knowledge of economic theory about dynamics of a competitive industry in general and the hog industry in particular.

As it will be shown in Chapters IV and V, the current theories seem unable to explain satisfactorily the hog industry dynamics, nor can they serve as a basis for predicting, projecting or evaluating alternative policy measures applied by partial controls. These theories fail to fully recognize certain features of dynamic systems such as feedback signals, adjustment processes on a continuous rather than discrete basis, long run versus short run moving equilibrium positions, and variable time delays function on different stages and activities of the production process.

In summary, the problem rests in two areas. The first is to improve the capability of the industry to supply its end products in a stable low price fashion while retaining competitive characteristics to guarantee efficient performance and a safeguard against emerging market power, which would be likely to introduce rigidities, higher prices and other undesirable performance patterns. The second, is to improve our theories governing a dynamic industry. The improvement of these theories must be converted into a quantitative framework dealing with manageable and controllable variables. This will be necessary before any achievement can be made toward a better public policy aimed at improving the industry's performance while keeping hog producers and other participants satisfied.

This research is an attempt to contribute some new ideas and knowledge toward solving these problems.

Objectives

The objectives of the study are:

1. To develop a U.S. hog supply response model subject to biological and economic constraints. In doing this, the following specific procedures will be emphasized:
    a. Past studies will be reviewed and constructively criticized.
    b. The dominant feedback structures which determine the pork supply behavior will be identified.
    c. The production subsystem will be coupled to the marketing subsystem.
    d. The working lag structure will be identified and converted into an equivalent delay structure.
2. Convert the hog supply response model into a computer simulation system, contributing to a better understanding of the interrelationships among the major variables and the industry's objectives.

3. Test and validate the simulation model.
4. Design the computer simulation of the hog supply response subsystem so that it is compatible with an overall hog-pork subsector simulation system being developed under contract with the U.S. Department of Agriculture (USDA).
5. Develop a model of the hog cycle as a multi-frequency cycles model with short, intermediate and long run dynamic disequilibrium positions. This model should give us some insight into the components and the pattern of the hog cycle.
6. Evaluate the production response to an alternative set of policy input control measures designed to eliminate or to restrict certain aspects of the hog cycle.

Scope

This study is a contribution to the larger coordinated study entitled "Systems Analysis of the U.S. Hog-Pork Subsector", sponsored by a USDA grant in cooperation with Michigan State University. It studies the production of hogs from farrowing to slaughtering, along with the economic behavior of the industry as a whole, subject to biological constraints.

Single producer issues are not investigated here but are implicitly recognized; that is, considerations of single farms are kept compatible with the logic of the microeconomic analysis known to date. The U.S. hog production industry is used as a case study, with linkages made to both sides of the commodity "assembly line", and with the input supply industry and the marketing industry both considered as given industries. A descriptive-predictive quantitatively analyzed type of research is done for modeling the structure of the industry and its behavior. Previous theories of the hog cycle are investigated, partially invalidated, and an improved theoretical-empirical model is offered. The impact of possible public policies which could serve as behavioral stimuli to the industry are explored.

The study is done for the U.S. hog production industry on a national basis, using monthly national statistics during the period 1964-1971. Some naive and controlled projections for policy evaluation are made with the models developed. The primary focus of the study is an examination of the behavior of the system under imposed policy control.

Study Design and Format

Given the scope and objectives of the study, the availability of data, and the current state of the art, the U.S. hog production industry was selected as the subject of the study. Computer simulation techniques were used for extending the economic and theoretical bases and then used for the simulation and evaluation of the hog supply response.

The first portion of the study (Chapter II) consists of a brief review of the relevant economic theories and models of the hog-pork industry, including those which explain the hog-cycle phenomena such as the distributed lag, cobweb and the harmonic motion models. This is an attempt to lay the foundation for this study by critically examining some of the shortcomings of the theories in relationship to the current state of the arts.

Chapter III consists of a discussion of selected methodological issues faced in describing the dynamics of the hog industry. The strategy and the framework of conducting the research is developed there; but the question of model validation and verification is delayed until Chapter VII, for reasons which will become apparent later.

Chapter IV contributes to the extension of the Cobweb Theorem by considering a continuous-time model with simultaneously interacting cycles. This helps to build the simulation system for the hog production industry and the supply response, which is discussed in Chapter V.

Chapter VI evaluates the consequences of imposing an implicit policy control for improving the industry's performance using the theory developed in Chapter IV, and the production-supply model constructed in Chapter V.

Chapter VII completes the discussion on the methodology, and sets the rationale for the selection of model verification in conjunction with the particular situations and the techniques employed. A summary and the conclusions from the study are also included in this chapter.

CHAPTER II

LITERATURE REVIEW

The hog-pork economy has been investigated considerably in the last three decades. The oscillatory behavior of the prices and quantities have attracted many researchers attempting to establish a workable economic theory explaining this phenomenon. In order to put this study into perspective with previous work, several selected, recent studies related to this work are briefly reviewed in this chapter.

Models of the Hog-Pork Industry: Selected Studies

A comprehensive study of the hog-pork subsector was completed in 1962 by Harlow.[2] Applying the Cobweb Theorem (which will be discussed later) as a theoretical basis for analyzing the hog production process, he built a recursive estimating model to fit six behavioral relationships capable of explaining the subsector behavior. The six explained variables were number of sows farrowing, number of hogs slaughtered, quantity of pork produced, quantity of pork in cold storage, retail price of pork and farm price of hogs. He used quarterly data for 1949 through 1959.

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[2] Harlow, A. A., Factors Affecting the Price and Supply of Hogs, ERS, U.S.D.A., Technical Bulletin 1274, Washington, December, 1962.

Implicitly, Harlow relies on the idea of production delays by stating that the number of pigs farrowed are fixed by the number of sows bred approximately four months previously; and that the numbers of hogs slaughtered are determined by the farrowing approximately six months previously. This process would lead to a recursive model eligible for a single-equation ordinary least-square (OLS) as an estimating method. Other variables such as the storage holding of pork are drafted in similar fashion. Harlow's model fits the quarterly hog-pork economy relatively well with over 90 percent of the variation in the dependent variables being explained by the set of explanatory variables. However, it lacks the ability to incorporate nonrecursive short run factors influencing the decision of when and how heavy to market hogs, given that they have already been produced. In other words, the recursive type model implies that, once the sow is bred, no other major decisions regarding the quantity of hogs slaughtered can be made. Of course, the recursive influence is very substantial, but further consideration should be given to important short run market influences affecting slaughter decisions.

An indirect contribution to this study through the overall hog-pork subsector study can be considered in Maki's work involving the quarterly interrelationships between market levels for pork and beef.[3] Maki showed the effect of

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[3]Maki, W. R. <u>Forecasting Beef Cattle and Hog Prices by Quarter-Years</u>, Iowa Agricultural Experiment Station Research Bulletin 473, December 1959.

beef price changes on the hog price and quantity changes, as well as the determination of the hog price at the farm level, once the retail, then wholesale price level have been determined. Maki's results indicate that quarterly pork and beef margins cannot be considered fixed and that the effect of volume and price changes on margins should be fully explored in a structural model involving either the beef or pork economy.

In his Ph.D. dissertation, Sappington presents a similar quarterly production model with some modifications of the explanatory variable set.[4] Perhaps unintentionally, the feed-back characteristics of the farrowing process is treated there, but analysis and short period feedback is impossible.

A remarkable step forward was taken by Myers, Havelick and Henderson.[5] Using a model of eight behavioral equations and two identity relationships, they analyzed the monthly structure of the hog-pork subsector. The dependent variables included monthly supplies of live hogs and cattle for slaughter, farm-retail margins for beef and pork and consumption demands for pork, beef and broilers. Applying two-stage least squares, all variables were estimated simultaneously. The explanatory

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[4]Sappington, C. B.,"The Dynamics of Supply Response for Hog Producers," unpublished Ph.D. thesis, University of Illinois, Urbana, Illinois, 1967.

[5]Myers, L. H., Joseph Havelick, Jr., and P. L. Henderson, Short-term Price Structure of the Hog-Pork Sector of the U.S., Purdue University Agricultural Experiment Station Research Bulletin 855, February, 1970. The bulletin was based on a previous L. H. Myers Ph. D. thesis at Purdue.

variables included live hog prices, inventory of live hogs on farms, interest rates, corn prices, a measure of cyclical behavior of the hog production, and a linear trend variable plus eleven monthly dummy variables. The interest rate and the measure of cyclical production variables had been hypothesized, but statistically rejected. The authors seem to have a reasonable predictive model, although the cyclical measure was not investigated as to its properties, including frequencies and amplitude, and its rejection may cause misinterpretation. Finally, the authors seem to go to other extremes, by applying two-stage least squares, they assume simultaneous determination of the endogenous variables, where the production delay structure being studies suggests strong recursive relationships with, of course, short run adjustments.[6] For example the authors estimate simultaneously the inventory of live hogs on farm and the supply of hogs for slaughter, where it would seem more logical to estimate the sows farrowing and the supply of hogs, and then let the live inventory be the residual elements. This approach proves to be more related to the decision-making process and much more straightforward with regard to data availability.

Several other studies investigate the price-demand framework of the livestock industries mainly from the demand point of view, looking at the production sector as a black box.

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[6]Nevertheless, the application of the two-stage-least-square could not cause any harm in this case.

Among them are G. H. Hoffman,[7] Prato,[8] Bullock,[9] Leuthold,[10] and Hacklander.[11] An approach to explaining hog production behavior via its cost analysis was presented briefly by Blosser.[12] The author computed the break-even price of hogs for the average west-dentral Ohio hog producers. Holding other production costs constant at their 1962 levels, different hypothetical prices for corn were used. From this break-even price of hogs and from the various postulated prices of corn, the break-even hog-corn price ratio was computed for each price of corn. Blosser commented:

> "The corn-hog ratio is a poorer indicator of hog profits today than it was 40 years ago because corn has become less important from the standpoint of the total cost of producing hogs."[13]

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[7]Hoffman, G. H., "A Short Run Price Forecasting Model for Beef," unpublished M.S. thesis, Colorado State University, 1968.

[8]Prato, A. A., "An Econometric Analysis of the Monthly Farm Level Demand for Beef Cattle," unpublished M.S. thesis, Purdue University, 1966.

[9]Bullock, J. B., "Cattle Feedlot Marketing Decisions Under Uncertainty," unpublished Ph.D. thesis, University of California, Berkeley, 1968.

[10]Leuthold, R. M., Economic Analysis and Predictions of Short-Run Hog Pirce and Quality Fluctuations. University of Illinois Agricultural Experiment Station, AERR 104, June 1970.

[11]Hacklander, D., "Price Relationships Among Selected Wholesale Beef and Pork Cuts," unpublished Ph.D. thesis, Michigan State University, 1971.

[12]Blosser, R. H., "Corn-Hog Ratio is Poor Indicator of Hog Profit," Journal of Farm Economics, May 1965, pp. 467-468.

[13]Ibid., pp. 468.

While this may be true, even Blosser does not suggest excluding either hog or corn prices as explanatory variables since both are very significant in explaining farrowing decisions. Furthermore, a thorough econometric interpretation leaves at least the decision on the form of using these two variables as inconclusive and probably equally justified.

Theoretical Economic Models of the Hog Cycle

Three basic economic models have been used to explain the hog cycle: The Cobweb Theorem, Distributed Lag Models and the Harmonic Motion Model. These models and some modifications are reviewed here through a sample of studies.

The Cobweb Model

Since the formal presentation in 1938 by M. Ezekiel, the Cobweb Model has been used in most studies concerning theoretical economic models for the hog industry.[14] A general formulation of the Cobweb hypotheses includes three assumptions about production and consumption within a commodity system:

1. Production is an increasing function of the expected price envisioned by producers.
2. The lag between initiation of production and availability of the resulting commodity is one period of time.
3. Consumption is a decreasing function of the price recognized by consumers.

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[14] Ezekiel, Mordecai, "The Cobweb Theorem," Quarterly Journal of Economics, Volume 53, February 1938.

These assumptions were suggested by Ezekiel and some followers.[15] Meadows offers further assumptions which are added to the above, in order to make the Cobweb Model more tractable analytically,[16]

4. Producers act as if their decisions will not influence future prices.

5. Producers will always expect the existing market price to continue indefinitely into the future.

6. Production, consumption and inventory decisions can be summarized by supply and demand curves, which are both functions only of price.

7. The continuous evolution of a system may usefully be divided into segments, each equal in length to the lag between initiation of production and ultimate availability of the commodity.

8. The price adjusts in each period so that supply and demand are equated for the period.

9. One irrevocable production decision is made in each period on the basis of current expected price.

The geometric model exposed by Ezekiel[17] and the mathematical analysis rederived by Nerlove[18] conclude that there are three

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[15] Ibid.

[16] Meadows, D. L., Dynamics of Commodity Production Cycles. Wright-Allen Press, Inc., Massachusetts, 1970.

[17] Op. cit.

[18] Nerlove, M., "Adaptive Expectations and Cobweb Phenomena," Quarterly Journal of Economics, Volume 75, May 1958, pp. 227-240.

possible behavior modes for price and production: divergent, sustained or convergent oscillators, with the two parameters 180° out of phase. The period of commodity oscillation should be twice the length of the production delay. The commodity systems will be convergent, sustained or divergent, since the elasticities of supply is less than, equal to or greater than the elasticities of demand, respectively.

In a derivation adapted from Nerlove,[19] it is assumed that the supply and demand functions may be represented in the effective vicinity of their intersection by the linear functions:

Demand: $Q_t^D = a + bP_t$

Supply: $Q_t^S = c + dP_{t-1}$

In equilibrium (assumption 8 above) $Q_t^D = Q_t^S$ then

(i) $$P_t = \frac{c - a}{b} + \frac{d}{b} P_{t-1}$$

Consequently:

(ii) $$Q_t^S = c + d \left[\frac{c - a}{b} + \frac{d}{b} P_{t-1}\right]$$

In steady-state (equilibrium)

$$P_t = P_{t-1} \equiv P_e = \frac{c - a}{b - d}$$

$$Q_t^S = Q_t^D \equiv Q_e = c + d \left[\frac{c - a}{b - d}\right]$$

Assuming that the initial price P_o is something other than P_e equation (i) gives the sequence of prices P_t.

$$P_t = P_e \left(\frac{d}{b}\right)^t [P_o - P_e]$$

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[19] Ibid.

and the production sequence is thus:

$$Q_t^S = c + dP_e + d(\frac{d}{b})^{t-1}[P_o - P]$$

where by assumption: $d > o$, $b < o$. Accordingly the oscilation will be

Convergent	where	$o > d/b > -1$
Sustained	where	$d/b = -1$
Divergent	where	$d/b < -1$

Arrow and Nerlove proposed an important revision by introducing the "adaptive expectation" model for the price adjustment between two successive time periods.[20]

$$\text{(iii)} \qquad P_t^* = P_{t-1}^* + B(P_{t-1} - P_{t-1}^*), \; o < B \leq 1$$

where:

P_t^* = price expected to obtain in period t.

P_t = market price at period t.

B = coefficient of expectations.

Nerlove has incorporated this assumption in an otherwise static cobweb model to determine its implications for the relationship of relative supply and demand elasticity to stability of the commodity system.

Although this model proved satisfactory as far as producer's forecast is concerned, it may be improved by removing the constraint on B to remain a constant. Since it is a

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[20] Arrow, K. J., and M. Nerlove, "A Note on Expectations and Stability," Econometrica, Volume 26, April, 1958, pp. 296-305.

difficult task to express B as a function of time or anything at all, this model was not used in this study; but the objective was achieved (see Chapter IV) by converting the model from discrete to a continuous time basis and dealing directly with the actual production, making the price expectation somewhat redundant.

The original cobweb model assumes the so-called naive expectation model, under which the producers expect the price at the next period to be the same as the current prices, and to adjust their production volume. Ackerman[21] suggested considering a short run supply curve against Ezekiel's longer run. By allowing the supply curve to shift, an important flexibility is implemented. Henderson and Quant[22] pushed Acherman's suggestion somewhat further. They presented corn production, using Ezekiel's formulation with a converging cycle and a one year lag in the production process. As the corn price changes from one period to the next period, the short run (one year) hog supply curve also shifts. This shift of the hog supply may violate the clockwise motion of the cobweb model, but it was an important contribution in explaining a departure from the original model.

In relaxing further the basic assumptions of the original model, Ferris used the expectation model of Nerlove

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[21] Ackerman, Gustav, "The Cobweb Theorem: A Reconsideration," Quarterly Journal of Economics, February 1957, pp. 151-160.

[22] Henderson, J. M. and R. E. Quant, Microeconomic Theory. McGraw-Hill Book Co., New York, 1958, p. 122.

rather than the naive model.[23] Dean and Heady relaxed the vertical market supply curve and reached a basically four year cycle.[24] In two different studies, Harlow[25] modified the Cobweb Model to reconcile the emerging four year cycle with the theoretical two year cycle. He explained that by including the increase of pigs raised per sow, additional flexibility was achieved in introducing short run demand shifts, so as to reflect changes in the prices of competing products and income. A significant conceptual contribution and some extension of the cobweb model was offered in 1964 by F. V. Waugh when he introduced some important generalizations.[26] A cobweb model operating under price support programs was considered, which led to some distortions of the traditional cobweb path. Then, a multi-commodity influence on the individual commodity supply response was used. Waugh also modified the basic concepts of disequilibrium between the demand and the supply curves by redefining them to represent the "price" curve, and showed how current production affected current price. This was used to replace the Cournot-Marshall

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[23]Ferris, J. N.,"Dynamics of the Hog Market with Emphasis on Distributed Lags in Supply Response,"unpublished Ph.D. thesis, Michigan State University, E. Lansing, 1960.

[24]Dean, Gerald W., and Earl O. Heady, "Changes in Supply Response and Elasticity for Hogs," Journal of Farm Economics, November 1958, pp. 345-860.

[25]Harlow, Arthur A., "The Hog Cycle and Cobweb Theorem," Journal of Farm Economics, November 1960, pp. 842-853. Also, Factors Affecting the Price and Supply of Hogs, op. cit.

[26]Waugh, Fredrick V., "Cobweb Models," Journal of Farm Economics. November 1964, pp. 732-750.

demand curve, and the "lagged output" curve, representing the current supply as a response of past prices, was used to replace the traditional supply curve.

Despite all of these modifications and improvements, the cobweb model was an incomplete tool in explaining the hog cycle for two main reasons: first, it is still a discrete model where the hog industry and other livestock industries are better represented by continuous models, or at least with very short time increments. Second, the assumption of a single time lag on the prices, in creating the lagged supply response, is too strong where other price lags will be shown to be too crucial to be ignored.

Distributed Lag Models

The Cobweb models assume a single time lag. This is fairly adequate in the case of some annual crops, but it also implies that the hog producer is influenced by only the last period's price when he adjusts this year's production.

But the work of Fisher,[27] Koyck,[28] and Nerlove[29] indicate that the output of most commodities is likely to be affected by prices over several periods in the past.

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[27]Fisher, Irving, "Our Unstable Dollar and the So-called Business Cycle," Journal of American Statistical Association, Volume 20, 1925.

[28]Koyck, L. M., Distributed Lags and Investment Analysis. Amsterdam: North-Holland, 1954.

[29]Nerlove, Marc, The Dynamics of Supply: Estimation of Farmers Response to Price. Johns Hopkins Press, Baltimore 1958.

Briefly reviewing Kmenta's [30] work, the general formulation which would allow for the current as well as the past values of X to affect Y, would be written as:

$$(2.1) \qquad Y_t = \alpha + \beta_o X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \cdot \cdot \cdot + \beta_m X_{t-m} + C_t$$

This equation is called the Distributed Lag Model because the influence of the explanatory variable on the expected value of $Y_t(E(Y_t))$ is distributed in undefined lagged values of X. This number, m, may be finite or infinite. C_t is attributed to the effect of the disturbances.

Although equation (2.1) becomes the model used for some theoretical aspects of this study, this form of the distributed lag model is very rarely used because m may be greater than the number of observations. However, even if enough observations are available, it is likely that a high degree of multicollinearity will be encountered which tends to be with large standard errors of the estimated coefficients (a way to overcome the multicollinearity will be discussed in conjunction with the development of the Multi-Frequency Cobweb Model, in Chapter VI). Most frequently, some _a priori_ restrictions are placed on the regression coefficients β_o, β_1,. . . β_m, so that the number of the regression parameters becomes substantially reduced.

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[30] Kmenta, Jan, _Elements of Econometrics_, MacMillan, Co., New York, 1971, pp. 473-495.

A popular variant of a distributed lag is that of a <u>Geometric Lag Distribution</u> characterized as:

$$(2.2) \qquad Y_t = \alpha + \beta_o (X_t + \lambda X_{t-1} + \lambda^2 X_{t-2} + \ldots) + e_t$$

where:

$o \leq \lambda < 1$

Here the effect of X on $E(Y_t)$ extends indefinitely into the past (i.e., $m \to \infty$), but the coefficients decline in a geometric progression so that the effect of the distant value of X eventually becomes negligible. The parameter reduction is carried out by appropriate transformations and substitutions according to the specific economic model.

With the <u>Pascal Lag</u> models, also called Inverted-V model the assumption is made that the weights attached to the lagged variables would first rise, and then decline, instead of declining all the way. The regression equation for this model becomes:

$$(2.3) \qquad Y_t = \infty + \beta(1-2)^r [X_t + r\lambda X_{t-1} + \frac{r(r+1)}{2!} \lambda^2 X_{t-2} + \ldots] + e_t$$

Again, by way of substitution followed by transformation, the number of parameters or variables is reduced. A more flexible weight structure can be achieved by the <u>Polynominal Lag</u> model in a situation in which it is assumed that the weigts w_i in:

$$(2.4) \qquad Y_t = \alpha + \beta(W_o X_t + W_1 X_{t-1} + \ldots + W_m X_{t-m}) + e_t$$

follow a polynomial of a given degree. The additional flexibility in this formulation permits presupposing practically any desired weight structure with relatively inexpensive ways of increasing the degree of the polynomial. If K is chosen to be the polynomial degree then:

(2.5) $W_i = \eta_o + \eta_1 i^2 + \ldots + \eta_k i^k$ $(i = -1, 0, 0, \ldots, m+1)$

Where the η's are the polynomial parameters.

A more detailed discussion is beyond the scope of this study. Many applications of these models or some of their modifications have been made in economic systems[31] with relatively high levels of success from the predictability point of view. However, it seems that a common weakness always occurs with these applications because of the necessity of the *a priori* weight function which is a subject of the researcher's choice. Regardless of the form this function may take, an objective search for the "best" weight function is usually overlooked.

An attempt in this direction, rather simple and straightforward, will be carried out in Chapter IV.

The Harmonic Motion Model

In an exceptional article, A. B. Larson[32] presented a theory of the hog cycle as true harmonic motion, arising

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[31] Griliches, Zvi, "Distributed Lags: A Survey," *Econometrica*, Volume 35, January 1967, pp. 16-49.

[32] Larson, Arnold B., "The Hog Cycle as Harmonic Motion," *Journal of Farm Economics*, Volume 53, February 1938.

from feedback, the mechanism of which produces sinusoidal fluctuations in servomechanism control systems. In reviewing briefly his article, the hog industry can be formulated as a set of three equations. Let the demand curve be linear

$$(2.6)\quad P_t = A - bQ_t$$

with price dependent on current output only. Next, a perfectly rigid production process is postulated with a 12-month lag between planned and realized production

$$(2.7)\quad Q_{t+12} = cB_t$$

where B is the number of sows bred. It is assumed that the rate of change of breeding is proportional to the deviation of price from equilibrium

$$(2.8)\quad \frac{d}{dt} B_t = e(P_t - \bar{P})$$

where e is a parameter, and $\bar{P}$ is the price trend. This price is viewed as the signal effecting a response in terms of change in rate of planned production via equation (2.8). The production response alters the price signal via equation (2.6), when it is fed back after the fixed delay embodied in equation (2.7). The set of equations can be solved simultaneously by first combining (2.7) and (2.8) into a single supply response by setting $K = c \cdot e$, obtaining

$$(2.9)\quad \frac{d}{dt} Q_{t+12} = K(P_t - \bar{P})$$

Then (2.6) and (2.9) can be combined to yield autoregressive differential equations in either price or quantity;

$$(2.10)\quad \frac{d}{dt} P_{t+12} = -bKP_t$$

or

$$(2.11) \quad \frac{d}{dt} q_{t+12} = -bKq_t$$

where the lower case p and q denote deviations from trends. To simplify the exposition it is assumed that the two parameters b and k satisfy b·k = 1, then the solutions for (2.10) and (2.11) are

$$(2.12) \quad P_t - \text{COS}\ (\frac{2\pi t}{48} + e_p)$$

$$(2.13) \quad q_t = \text{COS}\ (\frac{e\pi t}{48} + e_q)$$

where e_p and e_q are phase angles depending on initial conditions, and the period 48 months is four times the production lag, and is determined from equation (2.10) by its correspondence to

$$(2.14) \quad \frac{d}{dt} \text{COS}(t + \frac{tc}{2}) = -\text{SIN}(t + \frac{tc}{2}) = -\text{COS}\ (t)$$

so that the production lag of 12 months equals $\frac{tc}{2}$ radians or one-fourth of the period of the cycle. The idea that the period of the cycle is four times the production lag is rather central here. The above formulation should bring a motion following a circle in a price-quantity plane.

Despite Larson's[33] apparent objection, it seems that the harmonic motion may become another crucial extension-modification of the cobweb model. (This can be achieved by redefinition of the variables of the hog model.) The main contribution

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[33] Ibid.

can be viewed as the set up of the disequilibrium situation under a continuous time basis against a discrete time basis, traditionally formulated under the cobweb model.

The Harmonic Motion model may be found lacking on two points: first, the erroneous assumption that a single price lag is the only signal which sets in motion the adjustment process. Secondly, it does not show how to account for a cycle which is significantly different from a circle, an ellipse for example.

In summary, this chapter reviews the current state-of-the-art, which shows the need for a better theoretical explanation of the hog cycle, as well as a working model for evaluattion of the production subsystem, and possible public policies to steer it.

CHAPTER III

METHODOLOGY

Introduction

In the analysis of any dynamic system, it becomes necessary at an early stage to identify its basic characteristics. Answers to the following questions are sought: Is it a closed or open loop system; what state variables, input-output relationships, feedback loops will serve to define the system's components.[34]

Lacking a universally agreed upon procedure, the task of mathematical modeling is challenging.[35] A theoretical model describing the forces which interact with the decision rules and pilot the system's direction can benefit the modeling task. Based on the theories just reviewed, the modeling process begins. The methodology for this work included a study of complex systems in general with the intent of using the hog production features of the total hog-pork subsector as the empirical basis for such study.

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[34]In this section, words in the technical language of systems science are underlined and defined in the glossary, Appendix A.

[35]See Asimov, M., Introduction to Design. Prentice Hall, 1962.

The Hog-Pork Subsector as a Dynamic Servomechanic System

Consider the dynamics of the subsector in terms of the following continuous differential equations:

$$(3.1)\quad \frac{d\vec{X}(t)}{dt} = \underline{A}\vec{X}(t) + \underline{B}\vec{U}(t)$$

$$(3.2)\quad \frac{d\vec{y}(t)}{dt} = \underline{C}\vec{X}(t) + \underline{D}\vec{U}(t)$$

where:

$\vec{X}(t)$ = the state vector

$\vec{U}(t)$ = the input vector

$\vec{y}(t)$ = the output vector

$\underline{A}$, $\underline{B}$, $\underline{C}$, and $\underline{D}$ are the system's parameter matrices (for simplicity, these parameters are given here as constants. In the case of time-variant systems these matrices are functions of time, i.e., $\underline{A}(t)$, $\underline{B}(t)$, $\underline{C}(t)$, $\underline{D}(t)$). Specific identification will be given later. The (3.1) and (3.2) equations represent a situation where the change in the state vector is a function of the state of the system and the inputs. The change in outputs is a function of the state and input vectors. In other words, a portion of the output is independent of the state of the system. Due to the absence of national data instantaneously collected, an equivalent difference equation matrix is required. Modifying (3.1) and (3.2) accordingly the discrete time representation is obtained

$$(3.3)\quad \vec{X}(k+1) = \underline{A}\vec{X}(k) + \underline{B}\vec{U}(k)$$

$$\text{(3.4)} \quad \vec{Y}(k+1) = C\vec{X}(k) + D\vec{U}(k)$$

where the interpretation of $\vec{X}$, $\vec{Y}$ and $\vec{U}$ vectors is the same as in (3.1) and (3.2), but the parameter's matrices $\underline{A}$, $\underline{B}$, $\underline{C}$, $\underline{D}$ are adjusted to fit the discrete time domain. It is assumed here that the system behaves linearly.

To facilitate discussion, consider the block-diagram depicted in Figure 3.1. It is convenient to begin with the demand side, although any other point can serve as the starting point. The demand vector (demand for different cuts of pork) is determined by both exogenous and endogenous stimuli. Among the major exogenous determinants are income per capita, price of substitutes and other demand shifters such as change in tastes due to weather behavior, holidays and long run preferential shifts. At the same time a decisive impact is caused by the expected and realized pork prices, which are determined endogenously by the system.

Supply determination is much more complex. It involves the production process, with a distributed delay function, the production input parameters which are subject to uncertainties and significant disturbances caused by weather conditions, international relations and the like. Being a storable commodity, the total volume is distributed by the industry between immediate consumption and change of stored inventory. With the determination of the quantity of exports minus imports, the total supply versus demand has been determined. Under equilibrium conditions, the <u>ex post</u> identity demand $\equiv$ supply is achieved by the uncontrolled market price adjustment.

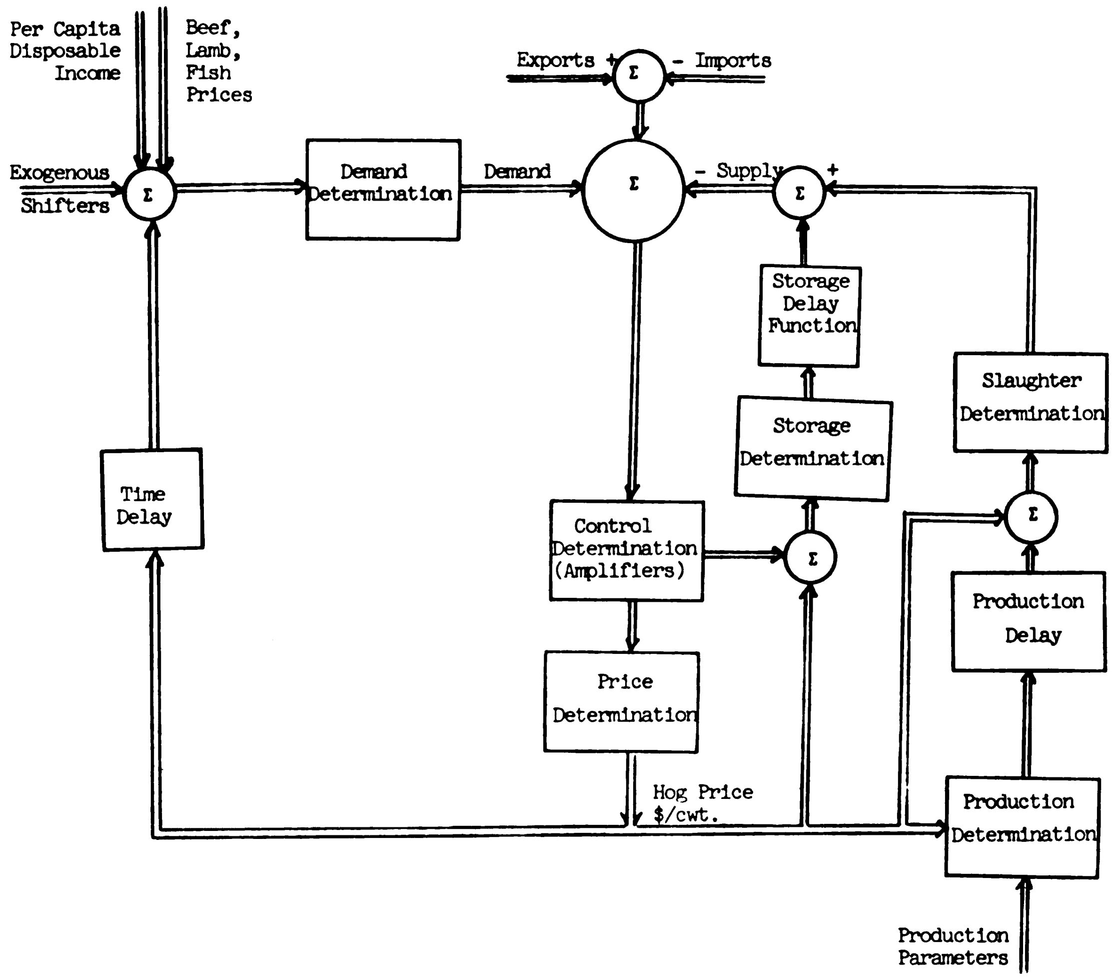

Figure 3.1.--A dynamic model of the hog-pork subsector

In the controlled case, the above identity can be achieved by other means such as further inventory adjustments and price policies. It sufficies at this stage to note that the control mechanism may be internal, exogenous, or a combination of the two. The system is servomechanic because the uncontrolled market price becomes the signal to the control subsystem which then amplifies it to activate the appropriate policy measure. When the control is exogenous, it may include the production costs and the international trade component within the policy choice set. In this case, the block diagram in Figure 3.1 should be adjusted as well as the modification of (3.3) and (3.4). (This adjustment is logically straightforward, but would make the picture too complex. It was omitted for simplicity of exposition.)

The above block diagram represents the framework for the subsector's economic behavior. The solution (the behavior of the system over time) for this dynamic system is not unique. On the contrary, the time path behavior may take many appropriate forms. One of the objectives of this study is to find the particular solution and then to develop an economic model capable of explaining this time path. This economic model is the subject of the next section. It serves, in turn the purpose of developing the model of the hog supply response.

The System Research Framework

The overall System Research Framework can be shown in the following diagram.

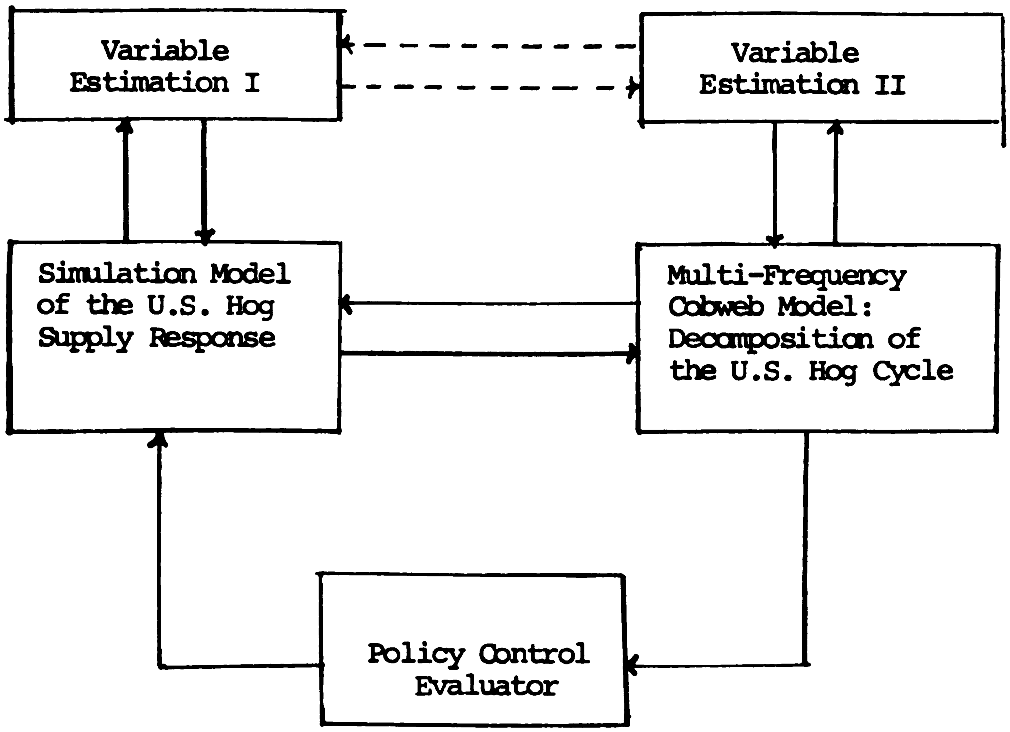

Figure 3.3.--The system research framework.

The Variable Estimation I package was used to identify the structure and estimate the variables and parameters of the U.S. hog supply response model. Basically, this is a set of computer programs designed to read in the input-output data, and to transform and manipulate them as needed for the estimation procedures. The Variable Estimation II package is another set, related to the Multi-Frequency Cobwebb Model, for the same purpose.

In developing the supply response model and multi-frequency model, information collected in either of them has been used for the benefit of developing the other (both models are described separately in the enclosed two papers).

The Policy Control Evaluator was built as a conclusion drawn from the Multi-Frequency Model. As it will become clear, this conclusion suggests that the hog cycle is a total summation of five or six decomposible cycles. Analytically, we can easily control any one, or even a combination of these particular cycles; but it is beyond the scope of this study to establish the policy measure to cause such a control. It is simply assumed that these are administratively feasible.

Hence, the Policy Control Evaluator is a computer Subsystem designed to control the input signals exogenously to meet predefined characteristics. Imposing these controls on the inputs to the production system, the supply response was projected and simulated for our evaluation.

Although this research framework was built and generated sequentially, it was improved simultaneously while in the stage of evaluating alternative policies. Additional specific methodological issues will be discussed in conjunction with each model.

CHAPTER IV

MULTI-FREQUENCY-COBWEB MODEL

Introduction

The primary objective of this chapter is to develop and apply a theory of the hog cycle as a Multi-Frequency-Cobweb Model arising from feedback signals; short, intermediate and long run disequilibrium positions. As will be shown, none of the models discussed in Chapter II (briefly mentioned here) will be dismissed, since all of them are important in building the model, and/or explaining its rationale.

Since the formulation of the Cobweb Theorem (for a detailed review of economic theories about the hog cycle, see Chapter II) by Ezekiel[36] in 1938, attempts have been made to explain the hog cycle in the Cobweb framework.[37] Harlow[38] modified the Cobweb Model to reconcile the emerging four-year cycle with the theoretical two-year cycle. An important

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[36] Ezekiel, Mordecai. op. cit.

[37] See works by (1) Dean, Gerald W., and Earl O. Heady, "Changes in Supply Response and Elasticity for Hogs," Journal of Farm Economics, November 1958, pp. 845-860. (2) Harlow, A. A., "The Hog Cycle and the Cobweb Theorem," Journal of Farm Economics, November 1960, pp. 842-853. and (3) Shepherd, G.S., and G. A. Futrell, Marketing Farm Products, Fifth Edition, Iowa State Press, 1969.

[38] Harlow, A.A. Ibid.

approach was later suggested by Larson[39] who considered the hog cycle as true harmonic motion arising from feedback, and closely related to the theory of inventory cycles. Nerlove's distributed lag approach was another significant contribution for the understanding of the dynamic behavior of the supply response.[40] Each of the above three models, standing alone, fails to satisfy the qualifications of a realistic, flexible explanatory, descriptive and accurate model.

The Model

The basic structure of the Multi-Frequency-Cobweb Model is shown in Figure 4.1. On a Price-Quantity plane, linear long run demand-supply curves are drawn (D_1 and S_1). Following F. V. Waugh[41] the following interpretation is offered: the D_i curve shows how current prices are related to current market output. The S_i curve ("lagged output") shows how current output is related to past prices.[42] This convention can be distinguished from the demand-supply relationships of the static theory formulated by Cournot and Marshall.[43] For

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[39]Larson, Arnold B. "The Hog Cycle as Harmonic Motion," Journal of Farm Economics, May 1964, pp. 375-386.

[40]Nerlove, M. "Adaptive Expectations and Cobweb Phenomena," Quarterly Journal of Economics, Vol. 75, May 1958, pp. 227-240.

[41]Waugh, F. V., "Cobweb Models," Journal of Farm Economics, November 1964, pp. 732-750.

[42]Ibid., pp. 733.

[43]However this distinction is not critical for the present argument.

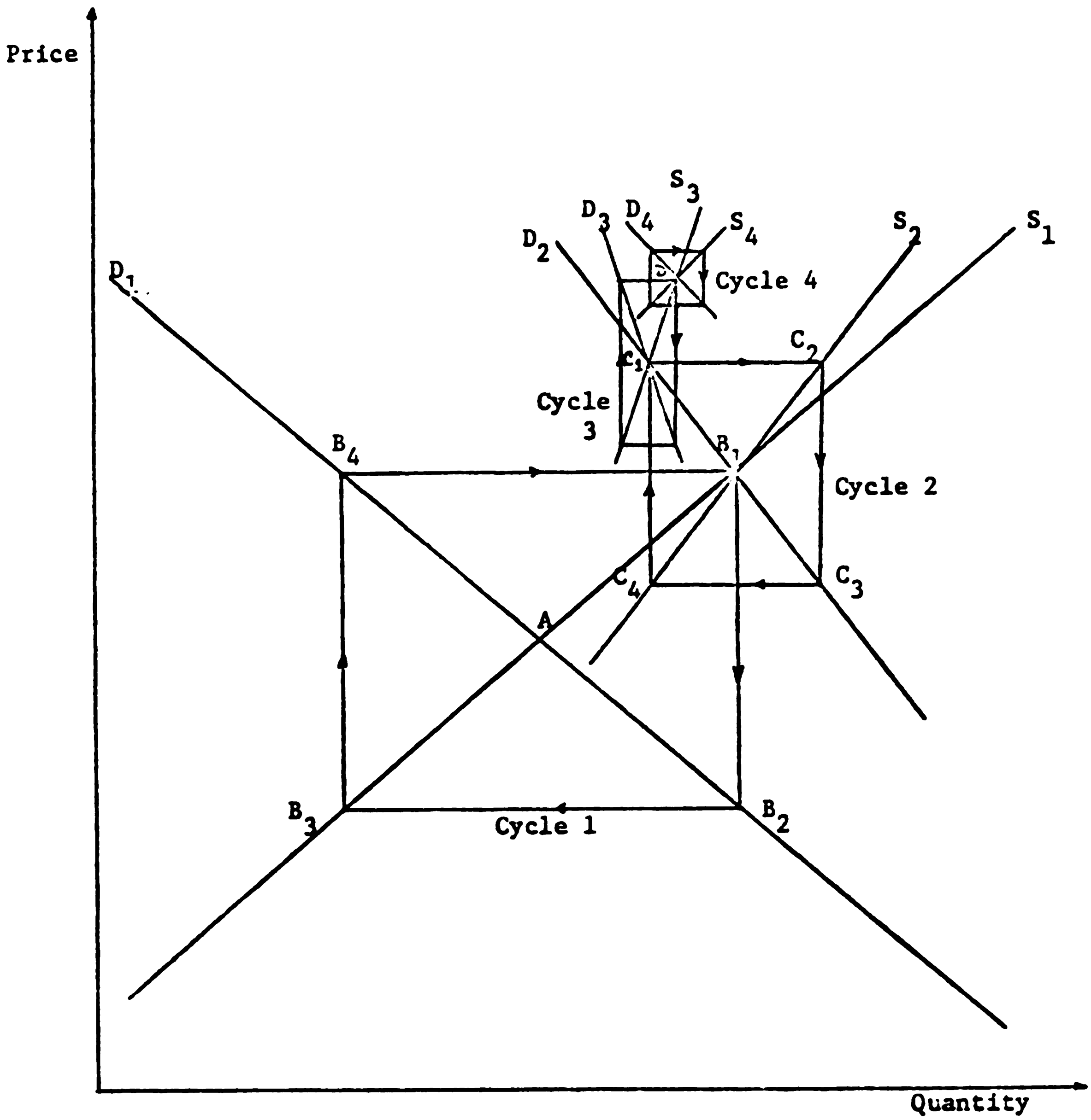

Figure 4.1.--The multi-frequency-cobweb model with four frequencies or cycles.

present purposes, it becomes necessary to extend the Waugh definitions. Anticipating cyclical behavior, as suggested by the Cobweb Theorem, "current" refers to one-half the time span required to complete one cycle. For example, D refers to a time span of two years (one-half of a four-year cycle). S, represents the lagged supply response for the corresponding two-year period.

Point A is the long run equilibrium position. Points B_1, B_2, B_3 and B_4 are the traditional Cobweb price-quantity coordinates as developed by Ezekiel[44] and his followers. The rectangle B_1, B_2, B_3, B_4 represents the assumption of a continuous Cobweb locus (neither converging nor explosive) which simplifies the exposition. The corners of the rectangle can be interpreted as the "intended" price-quantity coordinates or the long run disequilibrium points, if no other forces are imposed, and Ezekiel's three conditions hold.[45] One of these conditions implies a price lagged at a single and fixed time interval. If this time interval is exactly two years, then cycle 1 should be completed in exactly four years.

Let us now introduce a second lagged price as another stimulus, and let these two signals operate separately with appropriate weights corresponding to their influence on the producers' decision process. It should be clear that D_2 and S_2 are the intermediate demand-supply curves intersecting

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[44] *Op. cit.*

[45] *Ibid.*

each other at the long run disequilibrium point B_1. The arbitrary rectangle C_1, C_2, C_3, C_4 forms a two-year Cobweb cycle (cycle 2).[46]

Stated alternatively, if the two year price lag stimulus did not exist, then the traditional Cobweb Model is left with a locus about the equilibrium point B_1. Postponing the discussion of the simultaneous time lags influence, let it suffice to say at the moment that, in the simultaneous case, neither cycles 1 nor 2 will remain the same as it is presented in Figure 4.1.

Proceeding in the same fashion, additional price stimuli, lagged six and three months, will generate cycle 3 and cycle 4 about points C_1 and D, respectively. Notice that the time period for completion of a cycle is exactly one-half the time of the preceding cycle. Hence, by the time cycle 1 is completed, two are completed for cycle 2, four for cycle 3 and eight for cycle 4.

The number of cycles, and their duration for a particular system, is determined by the behavior of that system. The geometric properties of each rectangle are fully determined by the system behavior. The size and shape of the rectangle have a crucial influence on the final overall locus of the price-lagged quantity intersection points. There is a direct relationship between the rectangle's size and its contribution to the final locus. As in the Cobweb Theorem case, the direction

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[46] The corner B was selected for convenience. Any other point on the rectangle could serve as well.

of the motion is clockwise simply because the output is lagged after the price, and not vice-versa.

The Cobweb Theorem is somewhat inflexible when it suggests that the time path between any two corners (B_1 and B_2 for example) should be a straight line. While this assumption might be true in situations involving some annual cash crops, it is completely inadequate in livestock production where the adjustment process is a continuous one, generating a very interesting time path of too great an importance to be ignored. In fact, under a continuous adjustment process, the traditional Cobweb with its four rectangular corners (B_1, B_2, B_3, B_4) are replaced by a circle or ellipse. As proposed by Larson[47], the adjustment may be described as a process of harmonic motion.[48]

To see this point, let us formally consider the Cobweb formulation. In the simplified Cobweb case there are two basic functions for each cycle, considered separately and independently.

(4.1a) Price: $P_{it} = a_i - b_i Q_{it}$

(4.1b) Lagged output: $Q_{i(t+k_i)} = C_i + d_i P_{it}$

$i = 1, 2, \ldots, n$ cycles

b_i and d_i are the slopes of the demand and supply curves, respectively, for the cycle i where $d_i = 1/b_i$ for continuously oscillating model, since for the linear case a necessary

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[47]Larson, A. B., "The Quidity of the Cobweb Theorem," Food Research Institute Studies, Number 2, 1967.

[48]Not necessarily true harmonic motion. (See Bullock, op. cit., pp. 379-381).

condition is given by $(b_i d_i)^2 = 1$. k_i is the time period required to complete one-half of cycle i. It is clear from (4.1a) and (4.1b) that, if there is interest in what is happening on time intervals between 5 and $t + k_i$, the Cobweb formulation is incapable of providing this information.

Larson[49] (see Chapter II) suggests using the trigonometric function cosine to express the price and quantity:

$$(4.2a) \quad P_t = \cos \frac{2\pi t}{48} + e_p$$

$$(4.2b) \quad Q_t = \cos \frac{2\pi t}{48} + e_q$$

where the 48 stands for a four-year cycle, with months as units and e_p and e_q are phase angles depending on initial conditions. This set of equations gives a fixed amplitude four-year cycle. The contributions of this model are its power to explain intuitively, and the fact that the rate of change with respect to time or the first derivative is another similar trigonometric function, which is actually the same function phased in time and different in amplitude--a very important property possessed by systems with feedback. However, for practical purposes, Larson's model falls short as a sufficient and workable model.[50] There are two basic shortcomings: first, he implied that the frequency and amplitude of price and quantity are fixed and equal to each

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[49] *Ibid.*, pp. 378.

[50] It is quite possible that Larson did not intend to go beyond the basic theoretical features of his model.

each other. Second, his model dismisses the existency of other shorter cycles (higher frequency cycles). These two points can be seen to be wrong, even by a quick look at Figures 4.3 and 4.5 (see pages 48 and 49). There is not a pure single cosine curve in either the hog-corn price ratio or the number of sows farrowed variables.

The Econometric-Mathematical Model Representation

For representation and approximation of the price and quantity over time, the Fourier Series was chose because of its nice, convenient properties, some of which will be discussed here. The Fourier Series[51] may take several different forms, from which the following is selected for the stochastic case:[52]

$$(4.3) \quad \phi_m(t) = \sum_{n=0}^{m} (a_n \operatorname{COS}(nw_o t) + b_n \operatorname{SIN}(nw_o t)) + e$$

where: $\Phi_m(t)$ is the time variable to be approximated

m = integer, the maximum number of terms in the series.

$w_o = 2\pi/T$ is the fundamental radian frequency related to the base period T.

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[51]Thomas, George B. Jr., Calculus and Analytic Geometry. 4th Edition, Eddison-Wesley Publishing Co., Reading, Mass., 1968.

[52]Ibid. and Gilbert, E. G., "Controllability and Observability in Multi-Variable Control Systems," Journal SIAM Control Series A, Volume 2, No. 1, and Manetsch, T. J., and Park, G. L., Systems Analysis and Simulation with Application to Economic and Social Systems, Preliminary Edition, Michigan State University, September 1972 will be followed for the mathematical properties of the Fourier Series.

T = the time period needed to complete the major cycle; in our case T = 48 months, also called the base period.

t = time count in month units.

e = the error term.

Using least square method to estimate $\phi_m(t)$ we have

$$(4.4) \quad a_n = \frac{2}{T}\int_{t_o}^{t_o + T} (\phi\ (t)\ \mathrm{COS}(nw_o t))dt$$

$$b_n = \frac{2}{T}\int_{t_o}^{t_o + T} (\phi\ (t)\ \mathrm{SIN}(nw_o t))dt$$

This property also guarantees orthogonality, which provides the following convenience: if the function $\phi_m(t)$ is approximated by the trigonometric polynomial $\phi_n(t)$ substituting (4.4) into (4.3) and then another approximation $\phi_k(t)$ using more terms (k > n) is taken, more terms may be added to (4.4) without changing any of the coefficients $a_o, \ldots, b_n$ used in the first approximation. In the stochastic case, the following conditions must hold:

$$E(a_j a_k) = E(a_j b_k) = E(b_j b_k) = 0 \text{ for } j \neq k \text{ and } E(a_j^2)) = E(b_j^2)$$

The orthogonal property is of extreme importance because where n = o, . . . ,m we get what Fisher and Ando[53] called

[53]Fisher, F. M., and A. Ando, "Two Theorems on Ceteris Paribus in the Analysis of Dynamic Systems," American Political Science Review, Volume 56, pp. 108-113, 1962.

different "completely decomposable" sets of variables or absolutely <u>ceteris paribus</u> conditions. This is the property which makes it possible to decompose the hog cycle, to be elaborated later. In conclusion, something is gained by taking the Fourier Series model versus Larson's Harmonic Motion model <u>without losing</u> any of its representative and exploratory attributes. Making use of this characteristic, the value of n is capable of representing cycles with frequency equal to T/n. The motion of each cycle is totally independent of the motion in the other cycles. The coordinates of the price-quantity is given by solving (4.4) for both price and quantity, and substituting into (4.3). This implies that the ultimate location is determined by a summation over all the cycles and their corresponding time coordinates.

Methodology, Procedures and Data

To apply the model to the hog-pork subsector, variables were selected to represent the "price" and "quantity" discussed above.

For "quantity" a policy variable was sought with an impact which is crucially decisive in determining the quantity supplied to the market, but yet reflecting short, intermediate and long run considerations and minimally affected by past decisions. A direct approach might lead to the selection of total quantity marketed (head or pounds), but this variable is too much a result of past breeding rate decisions. The rate of sows bred out of the herd could meet the qualification satisfactorily. It is this breeding rate that is the major operative (contrary to the strategic) decision made by the producers concerned with market price conditions. Unfortunately, neither breeding rate nor accurate breeding herd population data are available on a national basis. However, the number of sows farrowing (SF)[54] is available and may serve as a good proxy for the breeding rate decision made almost 4 months earlier.

For the "price" the Hog-Corn Price Ratio (HCPR) was selected to reflect both the product and input prices. Hog Price (HP) is the barrows and gilts average price/cwt. received by farmers in the seven main markets. Corn Price (CP) is Corn No. 2 Price received by farmers at Omaha[55] (monthly time series). Figures 4.3 and 4.5 (see pages 48 and 49) show the nationally reported HCPR and SF, respectively. By Ordinary Least Square (OLS), the coefficients a_n and b_n were estimated for both variables, and using a Step-wise Delete Route[56] applied on the statistical-equivalent model

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[54] Source: USDA Hog and Pigs 1968 to 1970, Pig Crop Report 1964 to 1967. Monthly farrowing for 1968-1970 computed using 1955-1967 average percent of quarterly total on reported quarterly data (see Harlow, A.A. "The Hog Cycle and the Cobweb Theorem," *op. cit.* and Henderson, J.M. and Quant, R.E., *op. cit.*) reported in units of 1,000 head.

[55] Hogs and Pigs, U.S.D.A., SRS, Crop Reporting Board, Washington, D.C., 1968-1970 and Pig Crop Report, U.S.D.A., SRS, Crop Reporting Board, Washington, D.C., 1964-1967.

[56] Ruble, W. L.,"Improving the Computation of Simultaneous Stochastic Linear Equations Estimates," Agricutlural Economics Report No. 116 and Econometrics Special Report No. 1, Department of Agricultural Economics, Michigan State University, East Lansing, October, 1968. In Least Square Step-wise Deletion, an initial least square equation is obtained using all of the independent variables. One variable is then

to equation (4.3), all cosine or sine terms were deleted which did not satisfy a 5 percent significance level. The results are given in Table 4.1, for the period 1964-71 on a monthly basis.

Equations (4.5) and (4.6) below are the prediction equations for HCPR and SF as functions of time only. A special purpose computer simulation program was designed to map the price-quantity or, more particularly, HCPR-SF coordinates on the HCPR-SF plane. As mentioned above, equations (4.5) and (4.6) represent a linear summation of "completely decomposable" independent variables sets or cycles. Using this attribute, a "filter" was imposed on (4.5) to yield (4.6) capable of filtering through each individual cycle, and by the same simulation program tracing out the time path of each cycle combination of two or more cycles working simultaneously, and finally, all of them together. Methodology and procedure will be discussed in the next section when the empirical results are interpreted.

Empirical Results and Interpretations

Estimated-Predictable Equations

Applying equation (4.3) as a set up system for the Step-wise Delete Route[57], we let n take the values n = 0.5, 1,

deleted from the equation and a new least-squares equation is estimated. A second variable is deleted and the least-square equation recalculated. The selection of a variable to be deleted is based on the F-test where the least significant variable is to be deleted. The procedure continues until a variable selected as a candidate for deletion meets the stopping criteria, chosen here to be in terms of minimum significant value to be specified at each case (see above).

[57] Ibid.

TABLE 4.1. Regression Coefficients for the Sows Farrowing (SF) Estimation in Units of 1,000 Head

Independent Variable	Cycle Period (Months)	Frequency In Four Years	Regression Coefficient	Standard Error of Coefficient	T-Value	Significance Level
Constant	--	--	1029.88	10.31	99.86	<0.0005
$COS(4w_o t)$	12	4	-207.91	14.36	-14.48	<0.0005
$COS(8w_o t)$	6	8	-372.03	14.33	-25.96	<0.0005
$COS(12w_o t)$	4	12	48.70	14.33	3.39	0.001
$COS(16w_o t)$	3	16	99.09	14.33	6.91	<0.0005
$SIN(1w_o t)$	48	1	-46.44	14.51	-3.20	0.002
$SIN(2w_o t)$	24	2	55.15	14.62	3.77	<0.0005
$SIN(4w_o t)$	6	4	130.71	14.34	9.11	<0.0005
$SIN(12w_o t)$	4	12	57.72	14.33	4.03	<0.0005
$SIN(16w_o t)$	3	16	54.24	14.33	3.79	<0.0005

$R^2 = 0.9363$; $\bar{R}^2 = 0.9286$; F-value = 120.9; Significance Level <0.0005; Standard Error of Estimate = 92.85. D.W. = 1.30*

*D.W. is the Durbin-Watson statistic, See Durbin, J. and Watson, G. S. "Testing for Serial Correlation in Least-Squares Regression," Biometrika, Volume 38, 1951, pp. 159-177.

2,. . .,18, and the threshold significance level for including variables was 2 percent.[58] Table 4.1 shows the estimated coefficients for the SF, the rest of the coefficients are not significantly different from zero.

Table 4.1 and the other statistical measurements suggest that the SF variable is highly predictable and explainable by the independent variables set. The amplitude of each cycle is given by the absolute value of its regression coefficient, if only one trigonometric function is involved at this frequency. For example, the amplitude of the four-year cycle is 46.44. If both sine and cosine variables are involved at a particular frequency, the amplitude is equal to a phase combination of the two coefficients.

Figure 4.2 shows the predicted SF using Table 4.1 coefficients. The success of the prediction is evident when it is compared with the reported SF (Figure 4.3).

Table 4.2 shows the estimated regression coefficients for the HCPR, including only those significantly different from zero.

Figure 4.4 shows the predicted HCPR, using Table 4.2 coefficients. Five cycles were discovered for HCPR, namely: 48, 24, 16, 12, and 6 months cycles. Six cycles were ascertained for SF, namely: 48, 24, 12, 6, 4 and 3 month cycles.

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[58] All a_n, b_n coefficients will be set to zero unless the t-test with 1 percent critical area on each side of zero indicates rejection of the null hypothesis that a_n or $b_n = 0$.

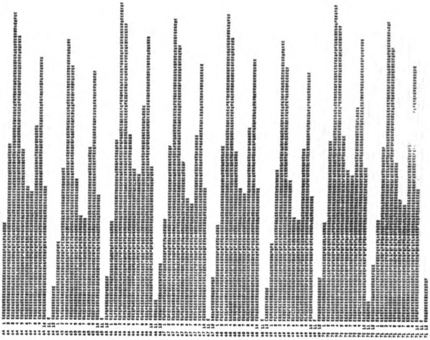

Figure 4.2.--Estimated U.S. farrowing by months, 1964-1971.

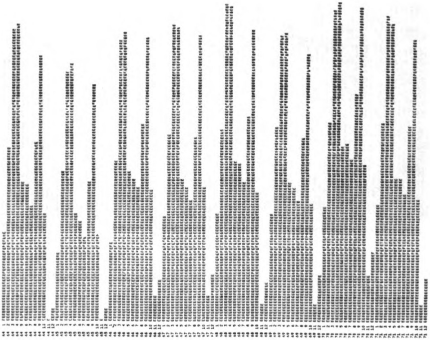

Figure 4.3.--Reported U.S. sows farrowing by months, 1964-1971.

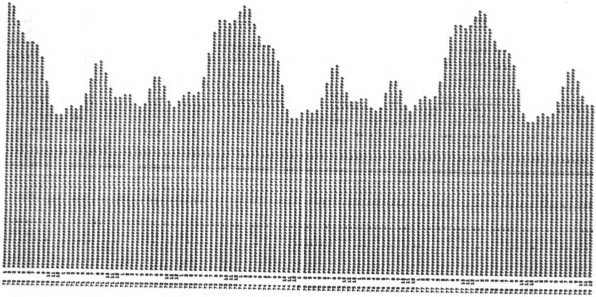

Figure 4.4.--Estimated hog-corn price ratio, 1964-1971.

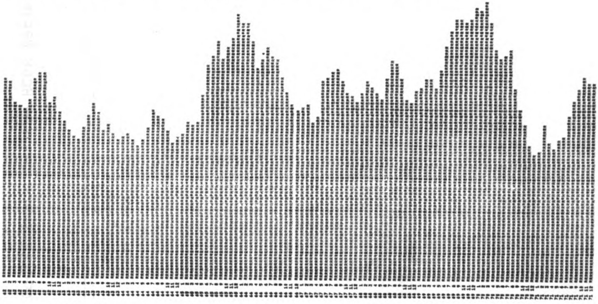

Figure 4.5.--Reported hog-corn price ratio, 1964-1971.

TABLE 4.2. Regression Coefficients for the HCPR Estimation (Unit: Bushels of Corn/cwt. Hogs)

Independent Variable	Cycle Period (Months)	Frequency in Four Years	Regression Coefficient	Standard Error of Coefficient	T-Value	Significance Level
Constant	--	--	16.63	0.18	94.95	<0.0005
$COS(1w_ot)$	48	1	-2.80	0.25	-11.13	<0.0005
$COS(2w_ot)$	24	2	2.27	0.25	0.01	<0.0005
$COS(3w_ot)$	16	3	-0.74	0.25	-2.93	<0.0005
$COS(4w_ot)$	12	4	-0.63	0.25	-2.57	0.012
$SIN(3w_ot)$	16	3	-0.60	0.25	-2.41	0.019
$SIN(4w_ot)$	12	4	-0.67	0.25	-2.66	0.009
$SIN(8w_ot)$	6	8	0.82	0.25	3.36	0.001

$R^2 = 0.7831$; $\bar{R}^2 = 0.7631$; F-value = 39.20; Significance Level <0.0005; Standard Error of Estimation = 1.58; D. W. = 0.222.

Graphical Decomposition of the Estimated Hog Cycle

In a matrix formulation we can rewrite (4.3) as follows:

$$(4.5) \quad \phi_m(t) = [a_o, \ldots, a_m, b_1, \ldots, b_m] \begin{bmatrix} 1 \\ \cos(1w_o t) \\ \cdot \\ \cdot \\ \cdot \\ \cos(mw_o t) \\ \sin(1w_o t) \\ \cdot \\ \cdot \\ \cdot \\ \sin(mw_o t) \end{bmatrix}$$

The coefficients row vector is a 1 x (2m + 1) matrix and the trigonometric column vector is a (2m + 1) x 1 matrix. If we insert an identity matrix dimensioned (2m + 1) between the two vectors the value of $\phi_m(t)$ will not be altered.[59]

$$(4.6) \quad \phi_m(t) = [a_o, \ldots, a_m, b_1, \ldots, b_m]$$

$$\underset{(2m+1) \times (2m+1)}{\begin{bmatrix} 1 & & & & \\ & \cdot & & 0 & \\ & & \cdot & & \\ & & & \cdot & \\ & 0 & & & \cdot \\ & & & & 1 \end{bmatrix}} \begin{bmatrix} 1 \\ \cos(1w_o t) \\ \cdot \\ \cdot \\ \cdot \\ \cos(mw_o t) \\ \sin(1w_o t) \\ \cdot \\ \cdot \\ \cdot \\ \sin(mw_o t) \end{bmatrix}$$

The identity matrix is easily converted into a "filter matrix by (4.7):

---

[59]Obviously, the last two matrices are first multiplied and only then the product vector by the first one.

(4.7) The "filter" matrix $\equiv$
$$\begin{bmatrix} 1 & & & & & & & & & 0 \\ & \alpha_1 & & & & & & & & \\ & & \alpha_2 & & & & & & & \\ & & & \ddots & & & & & & \\ & & & & \alpha_m & & & & & \\ & & & & & \beta_1 & & & & \\ & & & & & & \beta_2 & & & \\ 0 & & & & & & & \ddots & & \\ & & & & & & & & & \beta_m \end{bmatrix}$$

Where $\alpha_1, \ldots, \alpha_m, \beta_1, \ldots, \beta_m$ are binary zero-one variables. By using values of either 1 or 0, we can "filter out" any particular cycle or combinations of them. For example, if the four-year cycle is desired, we would set to 0 all α's and β's except $\alpha_1 = \beta_1 = 1$ and so on. This idea was translated into a computer simulation program with the additional capability of locating the coordinates on the price-quantity plane. The results are shown in Figures 4.6 through 4.18. In each figure the starting point is indicated by a small arrow called t_0. The plane coordinates are mapped in alphabetic order by the computer; the connecting lines are manually drawn. The time path of the first year in most cases is marked by a continuous straight line.[60] For the second year broken lines were used, and back to continuous lines for the third and so on. Where the cycle

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[60] It is assumed here that a straight line is as good as any other. Shorter intervals are of no interest here.

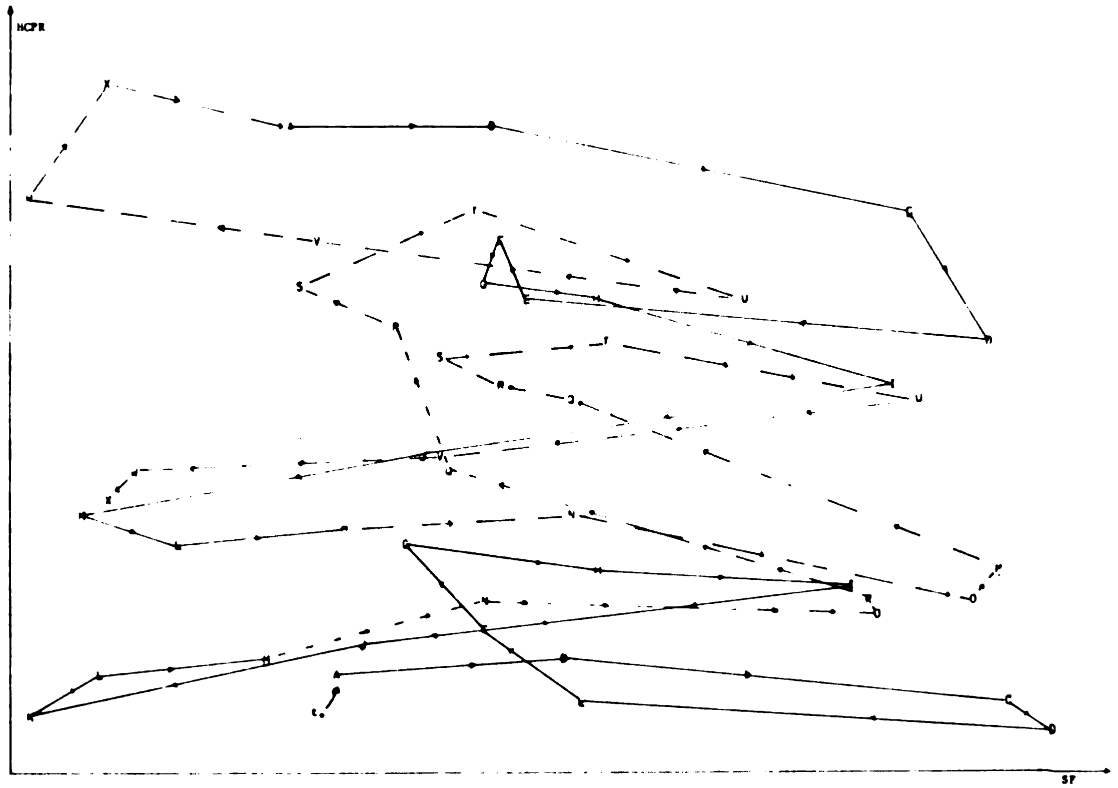

Figure 4.6.--HCPR versus SF, January 1964-December 1967, reported data. Source: Hogs and Pigs, USDA SRS, Crop Reporting Board, Washington, D.C. 1968-1970; Pig Crop Report, USDA, SRS, Crop Reporting Board, Washington, D.C., 1964-1967; Livestock and Meat Statistics, USDA, ERS, SRS, Washington, D.C., Supplement for 1970, Statistical Bulletin 333, and Grain Market News, USDA, Consumer and Marketing Service, Independence Missouri, 1964-1970.

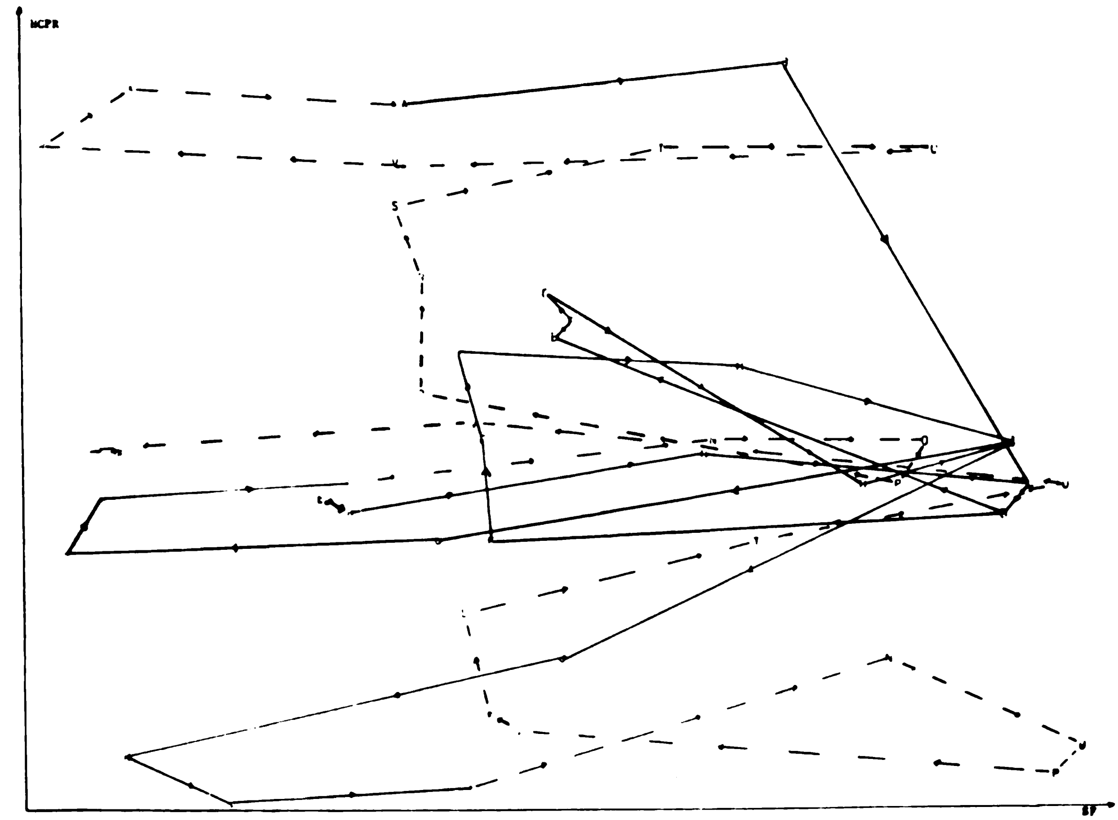

Figure 4.7.--HCPR versus SF, January 1968-December 1971, reported data. Source: Hogs and Pigs, USDA SRS, Crop Reporting Board, Washington, D.C., 1968-1970; Pig Crop Report, USDA, SRS, Crop Reporting Board, Washington, D.C., 1964-1967; Livestock and Meat Statistics, USDA, ERS, SRS, Washington, D.C., Supplement for 1970, Statistical Bulletin 333, and Grain Market News, USDA, Consumer and Marketing Service, Independence Missouri, 1964-1970.

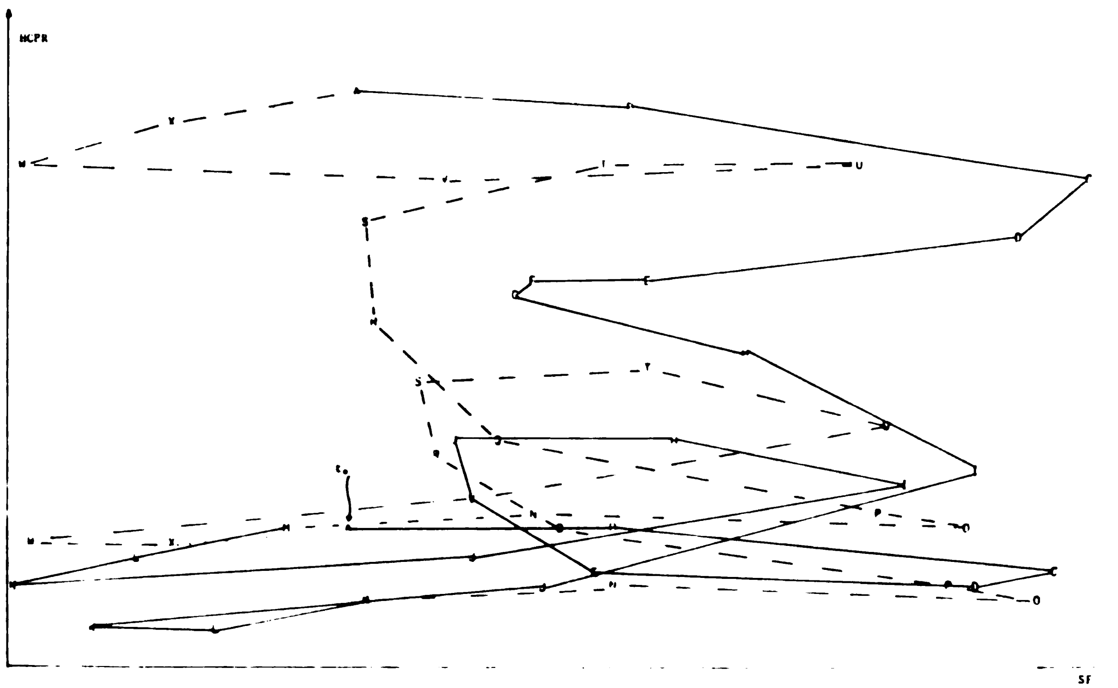

Figure 4.8.--Four-Years Estimated HCPR versus SF.

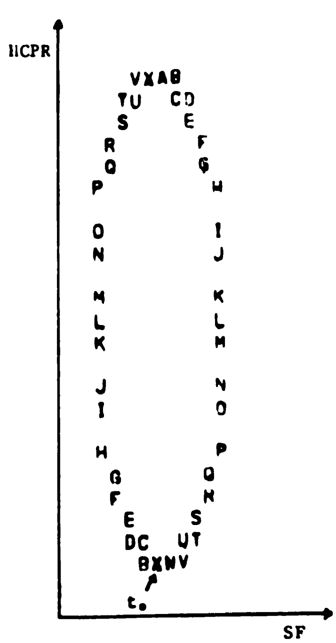

Figure 4.9.--Four-year cycle.

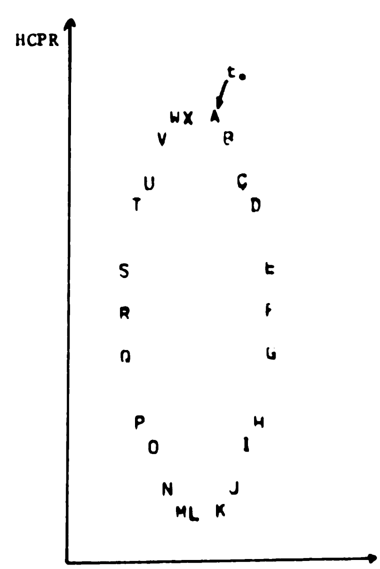

Figure 4.10.-- Two-year cycle.

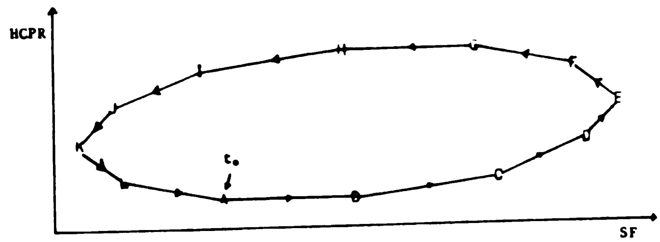

Figure 4.11.--One-year cycle.

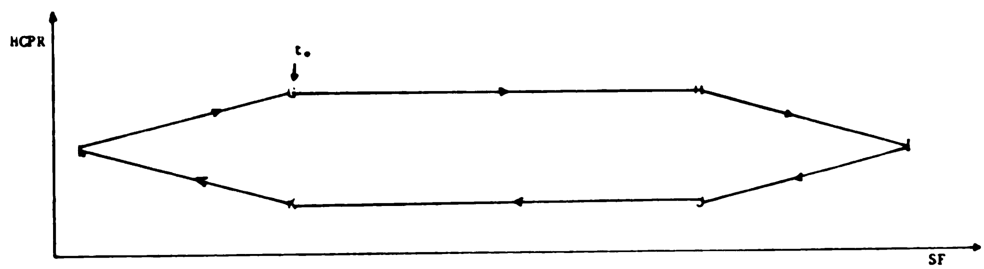

Figure 4.12.--Six-months cycle.

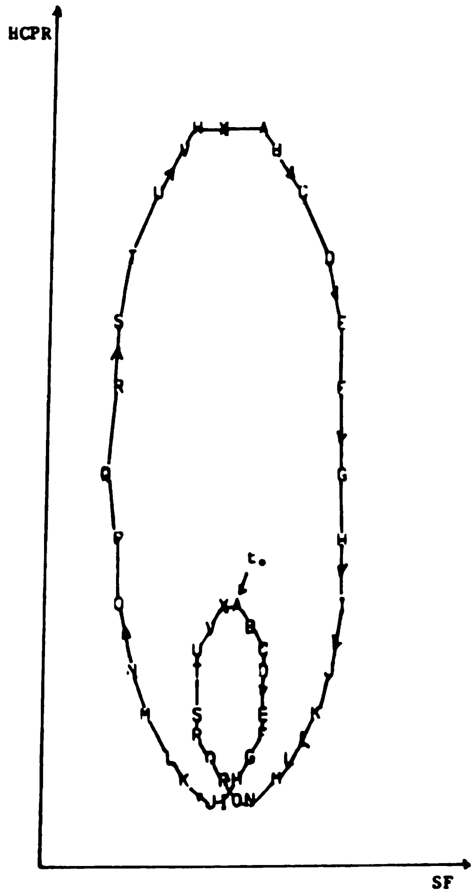

Figure 4.13.--Four and two years cycles.

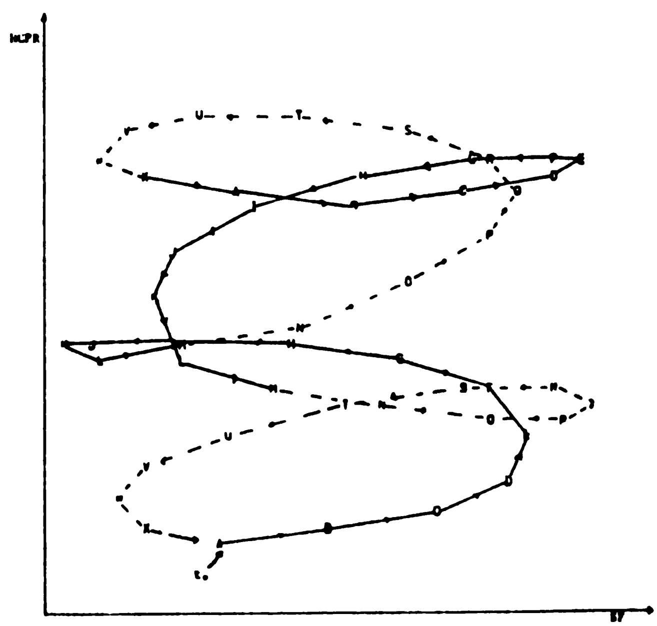

Figure 4.14.--Four and one-year cycles.

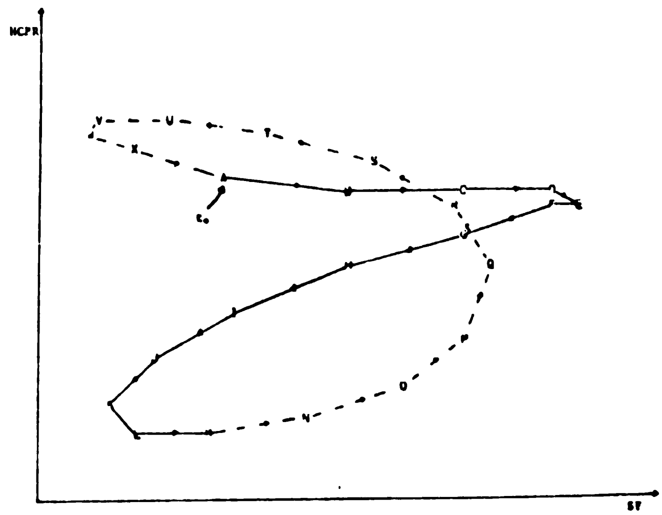

Figure 4.15.--Two and one-year cycles.

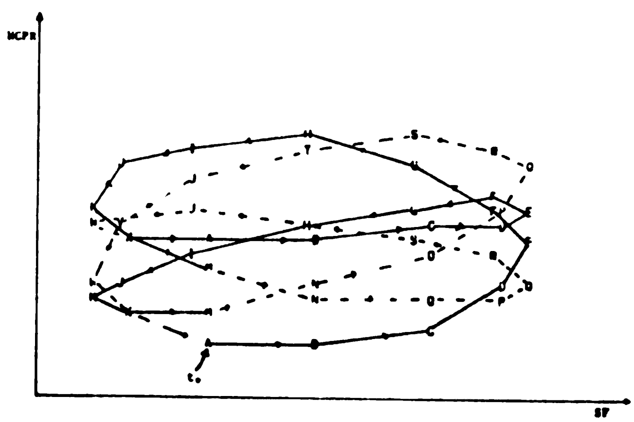

Figure 4.16.--One-year and 16-months cycles.

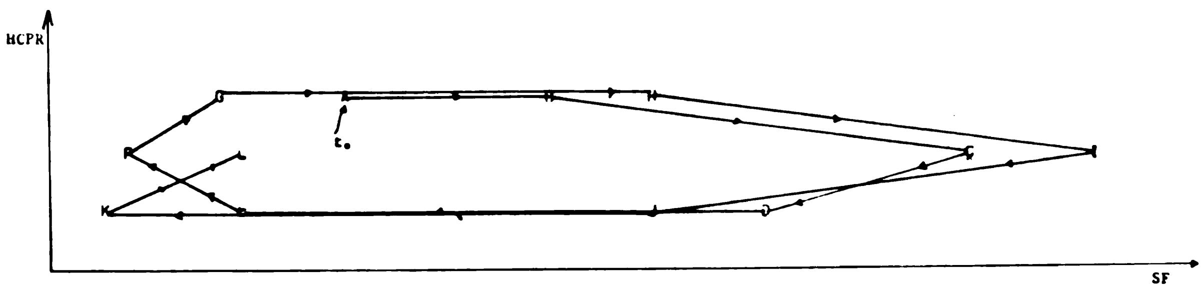

Figure 4.17.--Six, four and three months cycles.

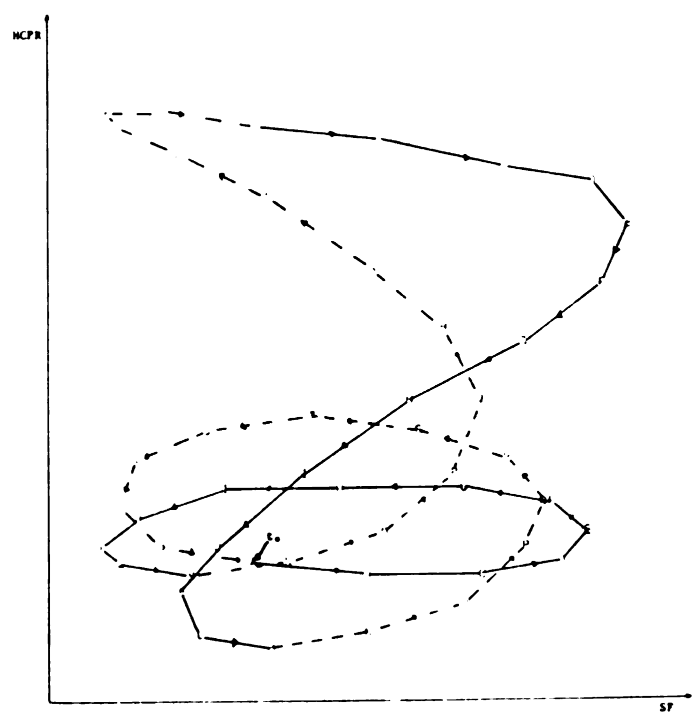

Figure 4.18.--Four, two and one-years and 16-months cycles.

is shorter than one year, or the sequential points are close to each other, only a continuous line is used and the time borders can be inferred by the alphabetic order.

Figures 4.6 and 4.7 show the reported time paths of two consecutive four-years HCPR-SF intersections. It can be seen that the year 1964 is similar to 1968, and that 1965 is similar to 1969. Resemblance is seen in the comparison of 1966 with 1970 and 1967 with 1971, even though they differ in their detailed time path. This observation by itself should suggest that some particular multi-frequency cycles are present.

By using the coefficients of the regressions equations summarized in Tables 4.1 and 4.2, a time path of the relationship between estimated SF and estimated HCPR was derived and diagrammed in Figure 4.8. By comparing this diagram with those derived from reported data, it can be seen that the predictive equations yield similar turning points on the cycle, as well as similar slopes on the corresponding connecting lines in the diagrams. As would be expected with different patterns for the reported four-year periods, the estimating equations trace a time path somewhere between the two. It is appropriate now to use this estimating equation, with the filter process described, to observe the time paths traced by systematically decomposing the cycle.

Figure 4.9 depicts the <u>four-year cycle</u> as an ellipse with a <u>clockwise</u> motion. This time path clearly reminds us of a cobweb continuous motion or the Larson's harmonic motion.

Referring to the two-year cycle shown in Figure 4.10, we see that it looks very much like the four-year cycle with twice the velocity. Note that the starting points are not in the same position, simply indicating that at t_0 the two cycles are not in the same cycle phase. Again, we have a clockwise motion. Some important points should be observed about the one-year cycle depicted in Figure 4.11. First, it appears that the ellipse's major axis has been rotated approximately 90 degrees. This fact leads to the following proposition: Where the four and two-year cycles are oscillating more widely about the price with a minor variation on the quantity, the shorter period cycles fluctuate more widely about the quantity than the price. This proposition indicates the impact of the national business cycle and other outside forces operating in the longer run versus the internal industry supply-demand interaction operating in the shorter run. Secondly, the motion direction is counterclockwise, perhaps violating, in this particular case, both theories--Cobweb and Harmonic Motion--advocating an opposite direction. It is beyond the scope of this study to establish a theory explaining this phenomenon. Nevertheless, a few alternative or complementary explanations may be provocative enough to encourage more investigation of this phenomenon: a) pork is a storable item for periods of less than a year and subject to consumer tastes and the costs of inventory maintenance. Assuming a clockwise motion on the inventory control process with relation to prices, then a counterclockwise one-year cycle will adequately refill the storage; b) Production cost differentials during the same period, may be unrelated to each other in causing this motion; c) After viewing the shorter cycles below, which again possess a clockwise motion, one may wonder if the one-year cycle is a kind of "overtone" caused by the industry, and crucial to keep balance among the other cycles; d) With regard to consumer preference for pork, such phenomena as holidays, religious custom, and weather conditions (with annual periodicity) may contradict the long run elasticity expectations. Figure 4.11 shows an ellipse which is very clearly the one-year cycle, but with a clockwise motion. The sharp variation on the quantity axis represent the two-peak, spring and fall farrowing, with a relatively low farrowing in the summer and winter.

Figures 4.13 through 4.18 present the combined motions of two or more cycles. In the discussion to follow, perhaps it will help to keep in mind that as the total cycle is decomposed, interest lies in analyzing the relative frequencies of its components. Figure 4.13 is a combination of the four and the two-year cycles. It shows two years of rapidly changing prices and two years of slowly changing prices and quantities which correspond with them respectively. With reference to the Cobweb and Harmonic Motion Theories which propose either a two-year or a four-year cycle, respectively, this figure demonstrates that both frequencies are required for an adequate explanation of the true cyclical behavior. Figure 4.14 is another excellent example of the combination of high and low frequency cycles. As the four-year cycle moves slowly to complete its time path in 48 months, shown as a smooth ellipse in Figure 4.9, it is seriously disturbed by the aggressive one-year cycle, which also determines the motions' direction. Figure 4.15 shows the interaction of the one and the two-year cycles, which represents roughly one-half the behavior shown in Figure 4.14. An interesting reaction between the one-year and 16-months cycles is revealed in Figure 4.16. The impact of the 16-months cycle can be interpreted as a vertical rotation of the axis of the one-year ellipse's within the four-year period. Essentially it allows for the different slopes and modifications of each of the one-year cycles.

The high speed short-run cycles presented at Figure 4.17 emphasize the relatively oscillatory quantity behavior versus the moderate variation on prices in a one-year period. All of the long and intermediate run cycles are being depicted in Figure 4.18. Here, the same pattern as shown in Figure 4.18 is generally discovered, which expresses the estimated time path in the hog production industry, where it is found that two out of four years with low hog prices and one year for each (upturn and downturn) trace two different paths. Particularly noteworthy is the observation that the downturn path is to the right of the supply curve in the case of expansion-contraction, when substantial variable and fixed costs are involved in the decision making process.[61]

Distributed Lag Estimation

Having statistically tested and accepted the existence of the combined series of cycles operating simultaneously, it is appropriate to show a linkage with the Cobweb model. According to the Cobweb model, the completion of the cycle by a price lag equal to one-half the cycle period is to be expected. Furthermore, these lagged price ratios are expected to be statistically significant in explaining the Sows Farrowing variable. To test this hypothesis, a modified special case of Koyck distributed lag model[62] was chosen. Let the structural equation express the SF as a linear function[63] of lagged price ratio variables as follows:

$$(4.8) \quad SF_t = \beta_0 + \beta_1 \, HCPR_{t-1} + \ldots + \beta_i HCPR_{t-1} + e_{SF}$$
$$(i = 1, 2, \ldots, 50)$$

where: $\beta_0, \beta_1, \ldots, \beta_i$ are constants to be estimated by Least Square procedure.[64] i goes from 1 to 50 to include 48

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[61]For an excellent treatment of the supply response under the condition of resource fixity see G. L. Johnson and C. L. Quance, Overproduction Trap in U.S. Agriculture, Michigan State University.

[62]Koyck, L. M., Distributed Lags and Investment Analysis. Amsterdam: North-Holland, 1954.

[63]Even though other mathematical transformations may be more suitable, linear transformation was used here for simplicity.

[64]Applying ordinary-least-square on (4.8) results in violations of the assumptions underlying multiple regressions using least-square techniques. But here we are interested in the relative size, and the sign. Therefore, these violations

months delay corresponding to an eight-year cycle, which has been statistically rejected by the frequency analysis, but is standing again for a test. e_{SF} is the disturbance term.

Applying a least-square-stepwise-add routine[65] the results summarized in Table 4.3 are reached. The variable selection procedure was to add a variable, if it was the best candidate and was significantly different from zero at the five percent level, and to reject any variable previously included, if it was no longer significant at the ten percent level.[66] The time lag in the price ratio was set at 4 time units (months) earlier than the sows farrowing time lag, to correspond to the approximate time of breeding. Since the time of breeding may be distributed throughout the month, it is safe to conclude that the actual farrowing could occur in a ± one month deviation from the time hypothesized. With this in mind, Table 4.3 gives a great deal of support to the Cobweb Theory behind each individual cycle. The first lagged

especially heteroscedosticity, if they occur, should not rule out least-square procedure. At this stage, multicollinearity is quite high, but after the stepwise selection is completed, the correlation coefficients do not exceed the value of .40 among the variables of the final equation.

[65]Ruble, W. L., Improving the Computation of Simultaneous Stochastic Linear Equations Estimates, Agricultural Economics Report No. 116 and Econometrics. Special Report No. 1, Department of Agricultural Economics, Michigan State University, East Lansing, October 1968.

[66]Such a situation may occur when a single independent variable, previously selected, becomes insignificant where a combination of later variables better "explain" the dependent variable and are linearly correlated with the single independent variable.

TABLE 4.3. Sows Farrowing Estimation Using Lagged Hog-Corn Price Ratio, by LS, Step-wise Add Method

Independent	Time Lag About Breeding = i	Regression Coefficient	Standard Error of Coefficient	T-Value	Signif. Level	Cycle Period	
						Selected Lag * 2	Hypothesized 2 * i
Constant	--	101.2	219.1	4.6	<0.0005		
HCPR(t-5)	1	-92.3	13.3	-6.9	<0.0005	2	3
HCPR(t-7)	3	135.1	15.3	8.8	<0.0005	6	6
HCPR(t-10)	6	-19.8	11.6	-1.7	0.091	12	12
HCPR(t-17)	13	-82.2	11.9	-6.9	<0.0005	26	24
HCPR(t-20)	16	93.2	12.0	7.8	<0.0005	32	--
HCPR(t-29)	25	-32.1	8.3	3.8	<0.0005	50	48

price should be considered in agreement with the hypothesized lags simply because there are no fractions of lag units, so the 1.5 lag is impossible. The next two time lags are exactly in agreement and the fourth and sixth are within the range allowed for the time range in actual farrowing. The 32-months cycle "estimated" here is exactly twice the 16-month cycle in the frequency analysis above. It may be simply because it is not compatible with a 48-month cycle. With regard to the 8-month cycle, although initially selected at an earlier step, it lost its level of significance in the stepwise-add routine by the dominance of other cycles.

Theoretical Implications

A theory of the hog cycle as a Multi-Frequency Cobweb Model, or as a linear combination of decomposable hog cycles, has been presented. This model reflects an integrated multi-frequency decision process resulting from the feedback of the production response to the price ratio signal through fixed multiple production lags. Long, intermediate and short run decisions are continuously made, and their impacts are projected to future decision and production process. The equilibrium and disequilibrium positions are under continuous attempts to adjust because of the existence of many simultaneous decision-response relationships. During periods of expansion, the hog producer builds or remodels facilities, and invests in a larger breeding herd. These investments have different time spans in their consequences associated with the long term investment made during expansion. This may explain the four-year cycle. A traditional Cobweb adjustment process may involve the two-year cycle. The 16-month cycle may be the producers' evaluation of the profit prospects, based on the relation between the corn supply of the current year and that of the previous year. The short run cycles of 6, 4, and 3 months may be explained by seasonal, weather and market signals coupled with capacity utilization of building and equipment subject to biological and technical constraints. Interactions with inventory control management are affecting the short cycles much more than they do to the long ones.[67]

Theoretically the model incorporates three basic models: The Cobweb Theorem, the Harmonic Motion and the Distributed Lags, Model, each of which was unable to explain satisfactorily the supply response in the hog industry when used individually. The Cobweb and Harmonic Motion models are special cases of the present model.

Econometrically, the Fourier Series appears to be an excellent mathematical representation of a dynamic disequilibrium phenomenon. Inertia of adjustments is preserved, as in cases where overshooting a target is gradually corrected. The characteristics of correction coming from previous motions preserves properties of macro-production systems with distributed delay behavior. The orthogonality property of the Fourier

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[67]Larson, Arnold B., "The Hog Cycle as Harmonic Motion," Journal of Farm Economics, May 1964, pp. 375-386.

Series improves the estimation equality, and allows for frequency analysis and decomposition of the cycle.

Finally, but by no means least important, the Multi-Frequency Cobweb Model in the above formulation provides support to the hypothesis that feedback signals exist which have a decisive effect on the overall industry time-path behavior. If this model is accepted as validated, then to show the existance of feedback, let us consider the discrete case. Substituting (4.1a) into (4.1b) to get

$$(4.1c) \quad Q_{i(t + K_i)} = C_i + d_i[a_i - b_iQ_{it}] = C_i + d_iA_i - d_ib_iQ_{it}$$

by redefinition

$$(4.2d) \quad Q_{i(t + K_i)} = C_i^* + \alpha_iQ_{it}(i\text{-}1,\ 2,\ldots,\ n \text{ cycles})$$

where:

$$C_i^* = C_i + d_iA_i$$

$$\alpha_i = d_ib_i$$

When α_i is unitary for all the i's (4.2d) represents a special case of a closed-loop feedback system independent of any input influence. It is obvious that this case is unrealistic, because inputs have their impacts on the hog industry; nevertheless, the feedback loops are playing both a theoretical and empirical role. This conclusion was implied in formulating the hog supply response model, theoretically, and via the simulation format. Further conclusions are postponed at this stage and will be discussed in Chapter VII.

CHAPTER V

THE HOG SUPPLY RESPONSE:
THE PRODUCTION SIMULATION MODEL

Introduction

The objectives of this chapter are: to model; simulate; and validate the hog production industry's behavior as a subsystem of the overall hog-pork subsector as shown in Figure 3.1. Based on the Multi-Frequency Cobweb Model developed in Chapter IV we have established some theoretical basis about the relationships between the price input signals, and the output response with the information on the feedback formulation assisting in building the lag structure and state variable of the system.

Later, before proceeding on the main track, a review of a past attempt to simulate the hog industry is presented. This study represents a breakthrough in simulation of an agricultural commodity industry for the purpose of research, where its apparent shortcomings inspired and directed the building of our model.

Basically, it has been inferred from the Multi-Frequency model that the following conditions are necessary for the study of the supply response:

1. The hog-corn price ratio is a crucial stimulus for the production decision making process.

2. The sows farrowing as a state variable reflects the producers decisions to steer their volume of production.
3. Decisions concerning the volume of the sows farrowing are mainly influenced by two factors: the hog-corn price ratio in the past, expressed in some form of lag structure, and feedback signals in terms of the number of sows farrowing at some point in the past.

These characteristics are being incorporated into the model of the hog-supply response.

Past Attempts: The Dyanmic Commodity Commodity Cycle Model

Studies have been made of the hog industry using some sort of simulation basically in an implicit way. A brief review of some of these studies was offered in Chapter II. An explicit use of simulation as an educational vehicle to investigate the hog-pork subsector, has been done by Meadows.[68] Capitalizing on J. W. Forrester's previous work in the area of Industrial Dynamics (the M.I.T. Press, 1961), he uses a simulation approach to build the dynamic hog model. The essentials of Meadow's[69] model are two coupled negative-feedback loops, consumption and production, each acting to adjust

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[68]Meadows, D. L., Dyanmics of Commodity Production Cycles. Wright-Allen Press, Inc., Massachusets, 1970.

[69]*Ibid*., pp. 19-60.

inventory coverage to the desired level. He suggests that the objective of the commodity system is to maintain the inventory at a particular level. It would appear that the goal of the system's participants is to minimize the product price. This objective is desirable from the consumers' point of view and it is a necessity for the producers to improve their competitive efforts. Nineteen different assumptions expressed in functional form are explicitly imposed. These include: desired and relative inventory coverage, price expectation and delayed price adjustments, desired production capacity, capacity transfer function, production capacity, production rate, and consumption rate. These assumptions are translated into a set of mathematical relationships (nonlinear functions, integration and distributed delay functions) and are simulated over time. In testing his model, Meadows points out three major differences between the actual and the simulated time series. First, fluctuations exhibited by the model are damped; the damping factor appears to be about 0.6, while the fluctuations in the real world appear to be sustained. Second, the real-world parameters are more erratic than those in the model. Simulated price changes quite smoothly over time. Third, the period of the model fluctuations is only about 60 percent as large as in the actual system; i.e., 27 months versus 48 months. Later attempts improve these discrepancies somewhat, but do not simulate the real world, and his time path behavior for all the variables exhibits sinosuidal curves which exclude what he called erratic behavior and what we know by now to be the existence of additional intermediate and short run cycles in the industry behavior. The source of the unrealistic system behavior, in the writer's judgment, lies in the fact that the production response is based on a single distributed delay function, which can be solved using a sine or cosine trigonometric function with or without a damping factor.[70]

Despite the above mentioned shortcoming, Meadow's work is a pioneering one in the sense of applying the simulation methodology as an approach to study the hog industry's dynamics, and to incorporate many important decision-making features into a dynamic system.

The simulation model developed in this study is narrower in scope than the Meadows work. It simulates the farrowing-to-slaughtering production process only. But an attempt has been made to capture as many details as possible leaving only a small proportion of the system's behavior to be characterized as erratic.

The Hogs Supply Response

Model: Background

In the static microeconomic theory, the supply curve of an industry under competition is associated with the

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[70]Distefano, J. J., Stubberud, A. R., and Williams, I. J., Theory and Problems of Feedback and Control Systems. Schaum's Outline Series, McGraw-Hill Book Co., New York, 1967.

industry's marginal cost (MC) curve, which is under <u>ceteris paribus</u> conditions equivalent to the summation of the firm's MC curve. That is, as the commodity price changes, the industry produces the quantity corresponding to the intersection of the price with the aggregated MC curve of individual firms. In the dynamic situation, it is difficult to define the MC curve, because it decisively depends on the time dimension. Delays between planning and executions, physical and biological delays make the supply as a response to past decision, conditions and commitments.

If a liquidation of the breeding stock to meet a sudden hog price increase in the market (although this response does take place on a very limited basis) is ruled out because this course of action might be very costly in the long run, then the duration of the production delay ranges from 10-12 months, corresponding to the <u>Gestation-Maturation Delay</u>. The delays involved in obtaining mature stock for slaughter are illustrated in Figure 5.1.

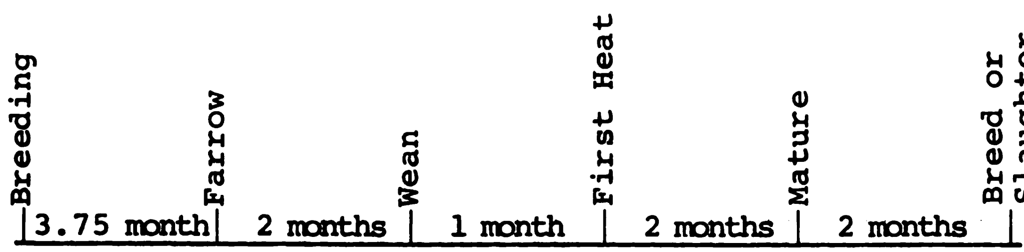

Figure 5.1.—Gestation-maturation delay.

Ninety percent of all farrowings take place between 111-119 days after breeding.[71] Pigs are weaned at about two months, and gilts come into heat for the first time about one month later. The best practice, however, is to wait until the gilt has reached around 250 pounds before first breeding it. This takes about five months from weaning. Hogs are commercially slaughtered within a liveweight range between 180-300 pounds corresponding to a range of 4.5 to 9.0 monhts old.

The implications are that the producers are responding to an expected price rather than the actual price. Price expectations, no matter how they are being determined, introduce errors due to uncertaintites. Attempts to correct these errors are logically only partially successful and are subject to the introduction of additional errors, and so on. The existence of several dalys, which has been demonstrated in Chapter IV, brings some flexibilities into the operation on one hand, and some complexities on the other hand. Decisions to produce at capacity or less are overlapped with decisions to alter the capacities themselves, where all of these occur under conditions of competition (horizontal as well as vertical) and partial knowledge. To cope with this complicated situation the following model was developed.

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[71] Carmichael, C. H., and Rice, J. R., "Variations in Farrow: With Special Reference to the Birth Weight of Pigs," University of Illinois Agricutlural Experiment Station Bulletin, No. 226, May 1920.

The Model

Being a dynamic production system, the process can be described starting at any desired point on the production cycle. It is convenient to begin with the breeding-farrowing subsystem.

Sows Farrowing

The breeding decision is the basis for the determination of the pig crop volume. The number of females bred is heavily influenced by the expected market conditions in the relevant future. As in Chapter IV, the breeding volume is approximated by the number of sows farrowing (SF in 1,000's head), and the market conditions by the hog-corn price ratio (HCPR--the number of corn bushels equal in value to 100 pounds of live hogs). Therefore, the HCPR is introduced with some lag structure under implicit (at the moment) function. The hog producers are facing another problem derived from the existence of variable cost as well as fixed cost curves on the farm. For some inputs, acquisition prices are substantially greater than their salvage values. Hence, the supply curve is more elastic where the commodity price is going up than where the supply curve corresponds to a decreasing commodity price. The reason for this phenomena is as follows: during the upswing of the price, the producers will respond with increasing production to full capacity, with additional investments in buildings, equipment and other assets to extend this capacity. This investment becomes fixed cost, and the down turn of the prices will not lead to an output contraction as long as the variable costs only can be covered, resulting in an inelastic supply at the later case. Another cause in the determination of the SF value is a feedback signal. As a conclusion of the existence of the multi-frequency-cobweb cycles, the significance of these feedback signals has been established. However, another consideration with reference to the expected hog-corn price ratio might be stressed. No part of the original cobweb model has received more criticism than the assumption that producers will continue to formulate their price forecasts on the basis of recent prices, even after several complete cycles in hog prices. As early as 1939 Buchanan warned: ". . .the inviolable assumption that people never learn from experience, no matter how protected, is at least debatable."[72] More recently Mills addressed this specific assumption.

> ". . .it is probably not reasonable to suppose that a decision-maker ignores information which is easy to obtain, particularly information which he can obtain by observing his own past behavior. If by observing his own past expectational errors, he can perceive a simple mechanical pattern in these errors, the economist should probably not assume that the decision-maker ignores this information."[73]

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[72]Buchanan, N. S., "A Reconsideration of the Cobweb Theorem," Journal of Political Economics, Volume 47, February 1939, p. 81.

[73]Mills, E. S., "The Use of Adaptive Expectations in Stability Analysis: Comment," Quarterly Journal of Economics, Volume 75, May 1961, p. 331.

The assumption is that the feedback information or, in particular, the sows farrowing at some past period of time, is an important determinant in the decision making process making up the breeding rate, which in turn gives the current SF level.

Finally, the manner in which weather conditions causes seasonal variations in costs of production must also be counted. These effects are operating more strongly on those producers who raise their hogs in less than the environment controlled confinement systems; but even the latter producers must purchase more energy during the extreme weather conditions. These and other seasonal causes make no one immune from cost differentials imposed on them.

Time, as a variable, enters the equation for additional reasons. Some of the cycle causes are not internal to the production subsystem, and even not to the more extended system of the hog-pork subsector. Effects of the business cycle, exogenous demand shifters, etc. all are exogenous to the production process. Myers, Havelock and Henderson[74] introduced a somewhat cumbersome measure of cycle phase to express this effect. Having included the price and the feedback signal in some lag structure, certain aspects of the cycle are already getting their voice. So, what is

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[74]Myers, L. H., Havelick, Joseph Jr., and Henderson, P. L., Short-Term Price Structure of the Hog-Pork Sector of the U.S., Purdue University, Agricultural Experiment Station Research Bulletin Number 855, February 1970.

needed here, is to include only parts of the entire cycle[75] which are unexpressed through the other factors mentioned above.

To summarize the determination of the sows farrowing in a given month 5, the implicit function can be written

$$(5.1) \quad SF_t = f_1(HCPR_{t-1}, \ldots, HCPR_{t-m}, SF_{t-1}, \ldots, SF_{t-k}, T_{st}, T_c)$$

where m and k are the maximum lags determined by assumption

T_{st} = seasonal factors depend on t

T_c = exogenous cyclical factors

f_1 = the implicit function which later becomes explicit under the mathematical modeling employed below.

e_{it} = the disturbance term normally distributed, $N(0,\alpha^2)$ and follows the regular independence conditions.

Pig Crop

Equation (5.1) forecasts SF_t, the Seasonal-Pigs-Per-Litter function (SPPL) provides the number of two day old pigs saved. SPPL--with the precise definition given below--is a time variant function including seasonal variation. A time trend to allow for technological improvements is included. The following identity computes the pig crop or weaning pigs (WPA).

$$(5.2) \quad WPA_t \equiv SF_t * f(t, SPPL)$$

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[75]In terms of the Multi-Frequency Cobweb Model of Chapter IV we want those cycles with the particular frequency which are unaccounted for by the other independent variables.

Age Transfer Assumptions

Pigs raised on the farm are subject to the following "Selections" made by nature and human conduct:

1. Diseases and other natural fatal causes amount to mortality rate expressed as a variable percent of each age group.
2. Farm slaughter, as a variable percent, depends on age and season of the year.
3. Market deliveries of hogs, will be discussed below.

Combining (a) and (b), the age-transfer matrix gives the following matrix equation

(5.3) $\vec{X}(t+1) = \underline{A}(t) * \vec{X}(t)$

where:

$\vec{X}(t)$ = the state vector composed by live inventory of groups of hogs in terms of their age, sex and function.

A(t) = the coefficients matrix combining the mortality rates with the farm slaughter rates, the matrix coefficients are time dependent.

Old Sows and Matured Gilts Sales (SGS in 1000's head)

Female breeding stock may consist of sows and/or mature gilts. Females are normally bred until their productivity begins to decrease. Any fixed assumption about the female disposal rate appears unrealistic. Meadows assumed an average productive life of sows to be three years.[76] This

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[76]<u>Loc</u>. <u>cit</u>., p. 53.

assumption appears inadequate, because it does not allow for different management procedures during expansion or contraction, short time market price effects, or seasonal variations.

Alternatively, it is hypothesized here that SGS is a function of similar characteristics in its implicit form

$$(5.4)\quad SGS_t = f_2(HCPR_{t-1}, \ldots, HCPR_{t-m}, SF_{t-1}, \ldots, SF_{t-k}, T_{st}, T_c)$$

where:

f_2 = the implicit function to become explicit under the mathematical modeling employed below.

The remaining variables should be interpreted the same as in equation (5.1). Notice that the HCPR variable must be lagged by at least one month to allow for the delay between planning and execution.

Barrows and Gilts Sold (BGS in 1000's head)

The number of barrows and gilts sold depends upon past farrowing, market prices of output-inputs, seasonal variation and the cycle's phase, subject to availability of pigs by age-weight distribution. Feed expenses constitute about three-fourths of total costs in swine production, and efficiency of feeding goes down with weight. Since heavier hogs also sell at a discount, producers cannot economically delay slaughter by more than two months.[77] Thus, once again the BGS's implicit function has the same general form

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[77] Loc. cit., p. 55.

$$(5.5) \quad BGS_t = f_3(HCPR_{t-1}, \ldots, HCPR_{t-m}, SF_{t-1}, \ldots, SF_{t-k^*}, T_{st}, T_{ct}) + e_{3t}$$

with the same variable interpretation as in (5.3).

Age-Weight Distribution of the BGS

Equation (5.5) provides the total number of barrows and gilts without specifying the age or weight distribution for each month. This information is needed to complete the live inventory adjustments on the farm for each age group. Another objective of obtaining this distribution is the extended hog-pork subsector model where the hog price is an endogenous variable, being a function of the pigs marketed quantities under every commercial liveweight group.

It is sufficient to state at this stage, that the age-weight distribution is related to the average liveweight for the total BGS. Hence, the average liveweight (AVLW measured in cwt./head) for marketed hogs is again an implicit function similar to (5.3).

$$(5.6) \quad AVLW_t = f_4(HCPR_{t-1}, \ldots, HCPR_{t-m}, SF_{t-1}, \ldots, SF_{t-k}, T_{st}, T_{ct}) + e_{4t}$$

with identical interpretations are the variables.

Equations (5.3),. . .,(5.6) represent the economic framework of the hog production industry and form the implicit set of hypothesis. Next, it is necessary to convert the implicit functions into the explicit forecast set of equations. For that purpose, let us now turn to the model estimation procedure.

Estimation Procedure and Data

All the independent and dependent variables appearing in equations (5.3),. . .,(5.6) are observable and being reported monthly by the USDA publications.[78] Data from January 1963 to December 1970 were used for the estimation procedure where 1971-1972 became the time period for extrapolating forecasts, with the possibility of validating the model performance versus the real world.

The Variable Estimation, previously mentioned in Chapter III is a computer program designed to prepare the independent variable set with the necessary lag structure and then to estimate each equation using ordinary-least-square stepwise delete routine.[79]

A priori, there are reasons to believe that the independent variable set is composed of only perdeterminant variables. This requirement is easy to establish. The independent variable set belongs either to an exogenous subset or to a lag endogenous subset. Therefore, the disturbance vector is

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[78] *Hogs and Pigs*, U.S.D.A., SRS, Crop Reporting Board, Washington, D.C., 1968-1970; *Pig Crop Report*, U.S.D.A., SRS, Crop Reporting Board, Washington, D.C., 1964-1967; *Livestock and Meat Statistics*, U.S.D.A., ERS, SRS, Washington, D.C., *Supplement for 1970*, Statistical Bulletin #333; *Grain Market News*, U.S.D.A., Consumer and Marketing Service, Independence Missouri, 1964-1970.

[79] Ruble, W. L., *Improving the Computation of Simultaneous Stochastic Linear Equations Estimates*, Agricultural Economics Report No. 116 and Econometrics Special Report No. 1, Department of Agricultural Economics, Michigan State University, East Lansing, October 1968.

uncorrelated with any individual independent variable.[80] Homoscedasticity can be assumed, since no cross-section data is involved, and there is no reason to believe otherwise. Hence, the estimation procedure using OLS method is appropriate to provide consistent, unbiased estimated coefficients.[81]

Computer memory size constraints limited the lag-subset for the HCPR and SF variables. The SF variable was lagged 1, 2, . . .,13 months, HCPR variable was lagged 1, 2,. . .,26 months. This procedure allowed for the SF feedback signals to arrive from as far as 13 months in the past, with the price ratio up to 26 months in the past; and so, covering the effective lag structure has been established in Chapter IV by the Multi-Frequency Cobweb Model. The seasonal variations represented by a set of relevant (twelve months minus one) binary dummy variables as the intercept shifter related to the particular month in a year. The residual cyclical aspects of the hog cycle, unexplained by the independent variables mentioned above, are expressed by a finite Fourier Series of the type defined by Equation (4.3) with ten terms of the series.

Equations (5.3), . . .,(5.6) represent the primary model hypotheses, in its implicit form. Acceptance of these hypotheses is now subject to statistical tests which

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[80]As a result the system is recursive.

[81]Kmenta, Jan. Elements of Econometrics, MacMillan Co., New York, 1971, pp. 197-408.

simultaneously convert them into a set of explicit hypotheses.

Applying the statistical estimation procedure described above, the following four equations have been obtained. They will be given in table form in Tables 5.2 through 5.5.

Estimated Equations and Their Properties

Sows Farrowing

Table 5.1 presents the estimated equation and its estimated coefficients of sows farrowing (SF). The independent variables selected include: SF_{t-6} which represents the feedback signal of the primary state variable. The six months lag was expected, since it reflects the relations between the spring-fall-spring. . .farrowing which corresponds to the same time lag. $HCPR_{t-5}$ and $HCPR_{t-21}$[82] reflect the short and long run breeding decisions as a response to the price conditions. The five months lag, with respect to farrowing, is equivalent to only one month lag with respect to breeding. If it is assumed that it takes some time before realized hog and corn prices become available to the producers, it can safely be said that this is a last minute change in planned production as a result of market conditions assessment based on the current situation. The twenty-one months lag which is equivalent to a seventeen months lag with respect to breeding

[82]In terms of lag in months, this corresponds to a 1:3 and 18:21 months lag which is very similar to what we have here. It is conceivable to assume a direct relationship between prices and sales as well as to consider breeding as an investment. For an excellent discussion see M. K. Evans.

TABLE 5.1. Estimation of Sows Farrowing (SF) ('000 heads)

Independent Variable	Regression Coefficient	Standard Error of Coefficient	T-Value	Significance Level
Constant	-231.32	58.86	- 3.93	<0.0005
SF(t-6	0.397	0.045	8.87	<0.0005
HCPR(t-5)	12.98	1.91	6.79	<0.0005
HCPR(t-21)	11.73	2.28	5.14	<0.0005
Sine($2w_o t$)	39.75	8.66	4.59	<0.0005
Sine($4w_o t$)	1145.61	33.95	33.75	<0.0005
Dummy, Jan.*	-312.98	24.19	-12.93	<0.0005
" Feb.	-506.85	33.66	-15.06	<0.0005
" Mar.	-380.99	45.10	- 8.45	<0.0005
" June	529.45	30.83	17.17	<0.0005
" July	948.29	39.96	23.73	<0.0005
" Aug.	1515.78	48.72	31.11	<0.0005
" Sept.	1855.26	56.91	32.60	<0.0005
" Oct.	1096.06	53.64	20.43	<0.0005
" Nov.	478.74	37.62	12.73	<0.0005
" Price Change*	34.52	12.88	2.68	0.009

R^2 = .9827; $\bar{R}^2$ = .9788; F = 256.93; Significance Level = 0.0005;

SF mean = 1030.93; Standard Error of Estimation = 50.55; DW = 1.176

*In the entire study the month of December served as the *a priori* excluded month. This footnote, thus, refers to all of the Tables in this chapter.

decisions, reflect the longer run investment-disinvestment decisions. These decisions, influenced by the output-input price ratio, may be realized into a complete investment project only after a considerable time delay. New building or equipment present additional potential capacity of production, which under a fixed cost situation induces the producer to change the volume of breeding. It is interesting to compare this double-peak price lag with Evans' business investment as a function of sales. He concluded that with time lags of one-quarter of a year and with seven quarters results in sales which effect the business investment function at the macro level.[83]

With respect to seasonal variations data indicate that during eight months in a year farrowings are significantly different from the "basic" month, December. The interpretation of the regression coefficients shows, for example, that farrowings in January are 324,400 litters less than the December figure, or 1,534,020 more than in August.

Cycle Effects

The significance of the sine($2w_o t$) and sine($4w_o t$) variable suggests that the 2-year cycle and 1-year cycle effects are not carried by the independent variables described so far. As already seen in Chapter IV, the effect of the 1-year cycle is much stronger than the effect of the 2-year cycle.

From the econometric point of view, the coefficients are efficiently estimated. All of the variables are very significant, with high "goodness of fit", and small standard

[83]Evans, M. K., Macroeconomic Activity: Theory, Forecasting, and Control. Harper and Row, Publishers, New York, 1969, pp. 95-106.

error of estimation. Nevertheless, one comment seems to be in order.[84]

The Durbin-Watson test for serial correlation[85] suggests that some relatively weak serial correlation exists. Is this a source of concern? Kmenta[86] discusses extensively this problem and concludes:

> ". . .if the disturbances are autoregressive and we persist in using a conventional least squares formula, [it is a fairly common situation in the economic time series data, in which the autoregression factor $\zeta > 0$] the calculated acceptance regions or confidence intervals will be often <u>narrower</u> than they should be for the specified level of significance or confidence."[87]

Essentially, the existence of serial correlation still allows for consistent, unbiased estimations with underestimated standard errors of the estimated coefficients, either caused by $\zeta > 0$, or by the fact that we actually have less independent observations or, in turn, less number of degrees of freedom. This implies that some of the coefficients are less significant than what is reported; but due to the high level of t-values (estimated) and only weak serial correlation, the estimation results can be safely accepted.

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[84]This comment is common to all four equations in Tables 5.2,...,5.5. The reader may skip this comment without any loss of continuity.

[85]Durbin, J., and Watson, G.S., "Testing for Serial Correlation in Least-Squares Regression," <u>Biometrika</u>, Vol. 38, 1951, pp. 159-177.

[86]<u>Loc</u>. <u>cit</u>., pp. 269-297.

[87]<u>Loc</u>. <u>cit</u>., p. 282.

Barrows and Gilts Sold (BGS)

Table 5.2 presents the estimated equation and its parameters for the Barrows and Gilts Sold (BGS). The independent variables in the equation are shown and were obtained by the same estimation procedure as described above. Note that the BGS, exceeding 90 percent of the total output, is the major output of the production system.[88] SF_{t-6}, SF_{t-8}, SF_{t-10} are the independent variables representing the impact of the state variables on the system's output. This shows the relative size of age-group-pigs six to ten months old.

$HCPR_{t-12}$ expresses the effect of the intermediate run market conditions, reflecting also the two-year cycle as explained by the Multi-Frequency Cobweb Model.

Cycle Effects

Only the one-year cycle effect is included in the BGS equation. All other cycles are explained by the included expanatory variables.

Seasonal Variations

Significant seasonal variations are expressed in terms of the binary-dummy monthly variables. Notice that the interpretation of these last coefficients must be done very carefully because of the substantial effect of the one-year cycle with a relatively large amplitude. For example, the

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[88] Livestock and Meat Statistics, USDA, ERS, SRS, Washington, D.C., Supplement for 1970, Statistical Bulletin #333.

TABLE 5.2. Estimation of Barrows and Gilts Sold (BGS) ('000 heads)

Independent Variable	Regression Coefficient	Standard Error of Coefficient	T-Value	Significance Level
Constant	-2507.03	614.94	- 4.07	<0.0005
SF(t-6)	1.41	0.22	6.39	<0.0005
SF(t-8)	3.25	0.31	10.44	<0.0005
SF(t-10)	2.02	0.22	9.24	<0.0005
HCPR(t-12)	43.73	10.60	4.12	<0.0005
Sine($4w_o t$)	1896.32	244.45	7.76	<0.0005
Dummy Feb.	-1551.41	154.25	-10.06	<0.0005
" May	-437.08	186.57	-2.34	0.022
" June	1203.43	248.39	4.84	<0.0005
" July	2242.10	429.66	5.22	<0.0005
" Aug.	3915.05	557.85	7.02	<0.0005
" Sept.	4388.93	602.09	7.28	<0.0005
" Oct.	3630.47	490.45	7.40	<0.0005
" Nov.	1460.19	253.36	5.76	<0.0005

R^2 = .9105; $\bar{R}^2$ = .8939; F = 54.77; Significance Level <0.0005;

BGS mean = 6237.17; Standard Error of Estimation - 269.37 DW = 1.69

September dummy coefficient amounts to 4,388,930 above December, but this is misleading somewhat because the sine($4w_ot$) reaches its lowest value approximately at the same period, namely close to -1.0. Multiplying it by its coefficient of 1,899,320, and subtracting it from the September intercept shifter, the remainder is only 2,489,610. In essence, this tells us that there is some interaction between the one-year cycle and the monthly shifter, or to put it differently, the interaction between different cycle frequencies as was seen in Chapter IV.

Econometrically, the Table 5.2 coefficients are satisfactorily estimated with high R^2, high level of significance, as a whole and for each parameter, low standard error of estimation, and low serial correlation.

Sows Sold (SGS in '000's Heads)

Table 5.3 presents the estimated equation and its parameters for the sows and gilts sold (SGS). It is obvious that this equation expresses the short run mature females disposal function. The SF_{t-3} simply means that, among the SGS, we can find a great many sows who have just finished weaning their newborn pigs. Short run price conditions represented by the $HCPR_{t-2}$ is another determinant operating on this function. The sign on the $HCPR_{t-2}$ coefficient is negative, and can be explained by the following consideration: When the market price conditions are favorable, farmers will try to expand their breeding herd; hence, will keep more females for

TABLE 5.3. Estimation of Sows Sold (SGS) ('000's heads)

Independent Variable	Regression Coefficient	Standard Error of Coefficient	T-Value	Significance Level
Constant	620.22	51.67	12.00	<0.0005
SF(t-3)	0.06	0.03	1.97	0.052
HCPR(t-2)	-10.37	1.80	-5.76	<0.0005
Dummy Jan.	-75.82	23.25	-3.26	0.002
" Feb.	-145.78	29.93	-4.87	<0.0005
" Mar.	-127.86	28.51	-4.48	<0.0005
" Apr.	-77.21	24.08	-3.21	0.002
" July	50.33	23.40	2.54	0.013
" Aug.	92.32	21.75	4.25	<0.0005

R^2 = .7689; $\bar{R}^2$ = .7442; F = 31.19; Significance Level <0.0005

SGS mean = 483.08; Standard error of estimation = 51.45;

DW = 1.30

the next breeding rather than to sell them for slaughter. The negative sign appropriately reflects this relationship. The seasonal variations are present but the cyclical effects are excluded. The goodness-of-fit is lower than in previous equations, but the F-value is still high enough to give a high level of significance for the entire equation, where in only one case (SF_{t-3}) the level of significance is just above 5 percent. However, with some weak serial correlation and a probable overestimation of the standard error of the coefficients, the problem is to be ignored.

Average Liveweight (AVLW) (Cwt per BGS Head)

Table 5.4 presents the estimated equation and its parameters for the average liveweight of barrows and gilts sold (AVLW). As expected, the lagged SF's contribute substantially to the determination of AVLW, simply because it reflects the corresponding volume farrowed with its age-weight distribution. The HP (hog price $/cwt.) lagged variables represent the sales response to the price changes (the hog-corn price ratio gave nonsignificant parameters, thus it was replaced by the hog-price variables). Different frequency cycles affect the AVLW, which along with the seasonal variations, explicitly introduce the effects of the time variables.

The results are a high R^2, high significant levels for the explanatory variables, and only weak serial correlation.

With the above four highly satisfactory basic estimated equations, the framework of the simulation model is ready to be constructed in detail.

The Structure of the Simulation Model

The objective of the simulation model is to test and validate the primary hypothesis of the production-supply response, and study the interrelationships among the objects of the hog production cycle.

The simulation system begins with a forecast of the SF using an algorithm based on Table 5.1 coefficients, and shown in Figure 5.2. To forecast dynamically the SF_t via simulation,

TABLE 5.4. Estimation of Average Liveweight (AVLW) (lbs. per head)

Independent Variable	Regression Coefficient	Standard Error of Coefficient	T-Value	Significance Level
Constant	217.36	2.73	79.49	<0.0005
SF(t-7)	0.006	0.001	5.52	<0.0005
SF(t-9)	0.01	0.0009	10.99	<0.0005
SF(t-10)	0.005	0.001	4.76	<0.0005
HP(t-2)	0.80	0.09	9.09	<0.0005
HP(t-8)	-1.17	0.14	-8.36	<0.0005
HP(t-25)	0.34	0.05	6.77	<0.0005
Cosine($2w_o t$)	-0.95	0.33	-2.84	0.006
Cosine($4w_o t$)	-4.81	0.44	-10.94	<0.0005
Cosine($5w_o t$)	0.59	0.26	2.76	0.007
Sine($w_o t$)	-5.78	0.56	-10.29	<0.0005
Sine($2w_o t$)	2.17	0.24	9.05	<0.0005
Dummy June	-5.65	0.88	-6.45	<0.0005
" July	-9.96	1.05	-9.52	<0.0005
" Aug.	-8.55	0.95	-9.05	<0.0005
" Sept.	-4.62	0.81	-5.67	<0.0005

R^2 = .9399; $\bar{R}^2$ = .9256; F - 65.49; Significance Level <0.0005

AVLW mean = 234.83; Standard Error of Estimation = 1.33 DW = 1.69

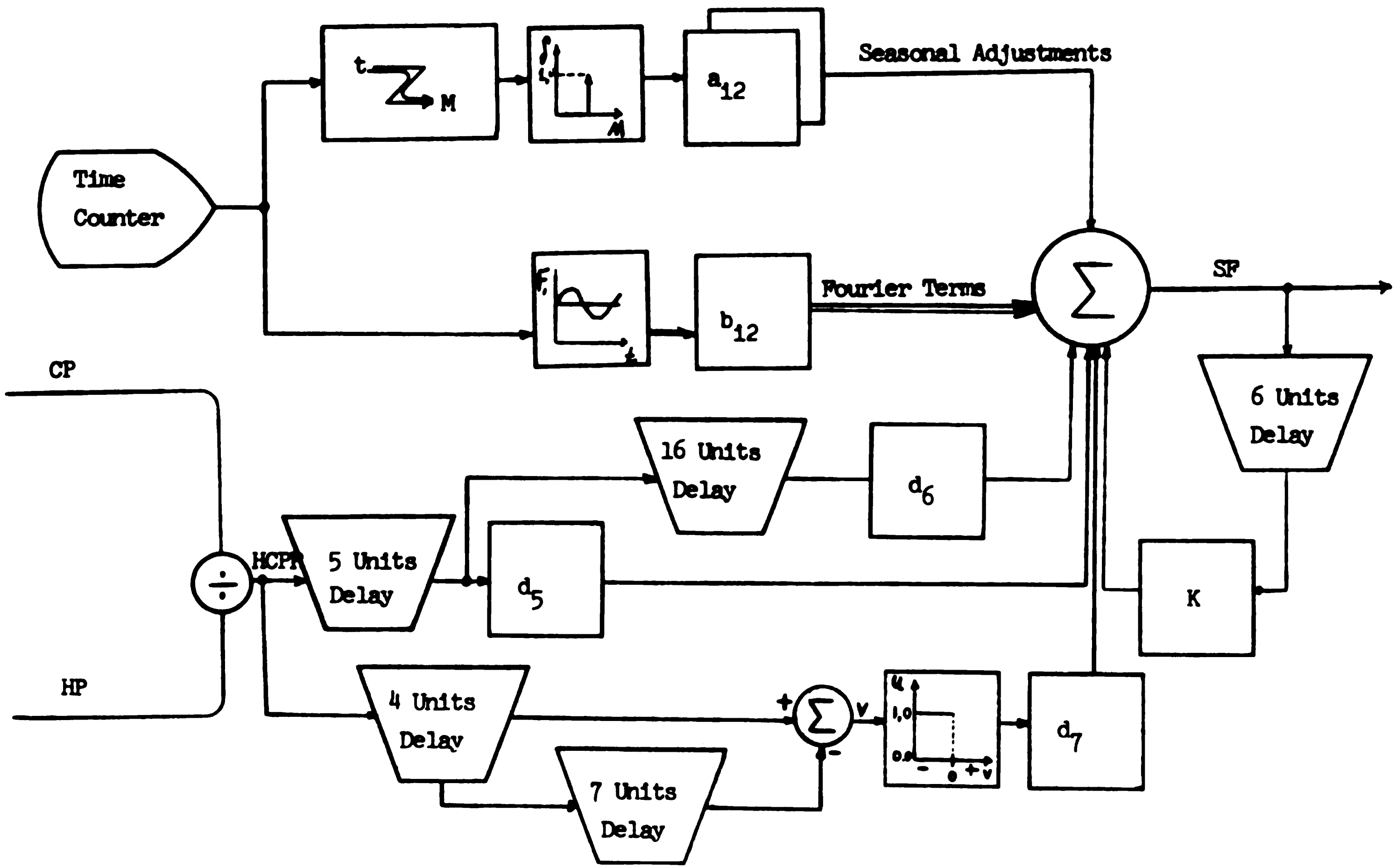

Figure 5.2.--Sows farrowing determination block diagram.

the $HCPR_t$ was computed from the two exogenous variables CP and HP. Then, the required lag structure was converted into the corresponding impulse delay function and multipled by the appropriate coefficients.[89] The bi-shaped supply response is due to asset fixity situations expressed in terms of an impulse function. The feedback signals expressed by SF_{t-6} is again an impulse delay function of the SF_t variable. Seasonal shifters are accomplished by another set of impulse functions and the cyclical effects become a series of trigonometric functions (sine and cosine) with their corresponding time variant arguments. In a similar way, Figure 5.3 shows the simulation algorithm for the forecasting of the BGS_t values. The $AVLW_t$ and BGS_t are forecast in a similar way. Figure 5.4 shows the aggregate structural block diagram of the entire dynamic simulation model. Appendix 2 includes the simulation model in its computer program (FORTRAN IV) form. Figure 5.7 shows the same thing in terms of a program flow chart, on its 34 program blocks.[90] These three forms should

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[89] Cooper, G. R., and McGillem, C.D., Methods of Signal and System Analysis. Holt, Rinehart and Winston, Inc., New York, 1967, pp. 45-69.

[90] Reference to the simulation forecast of the four basic variables is as follows:

SF_t -	Block	Number	6	in	Fig.	5.6;	Lines	147-150	in Appendix	2.
BGS_t -	"	"	15	"	"	"	"	228-231	" "	2.
SGS_t -	"	"	10	"	"	"	"	196-197	" "	2.
$AVLW_t$ -	"	"	16	"	"	"	"	243-247	" "	2.

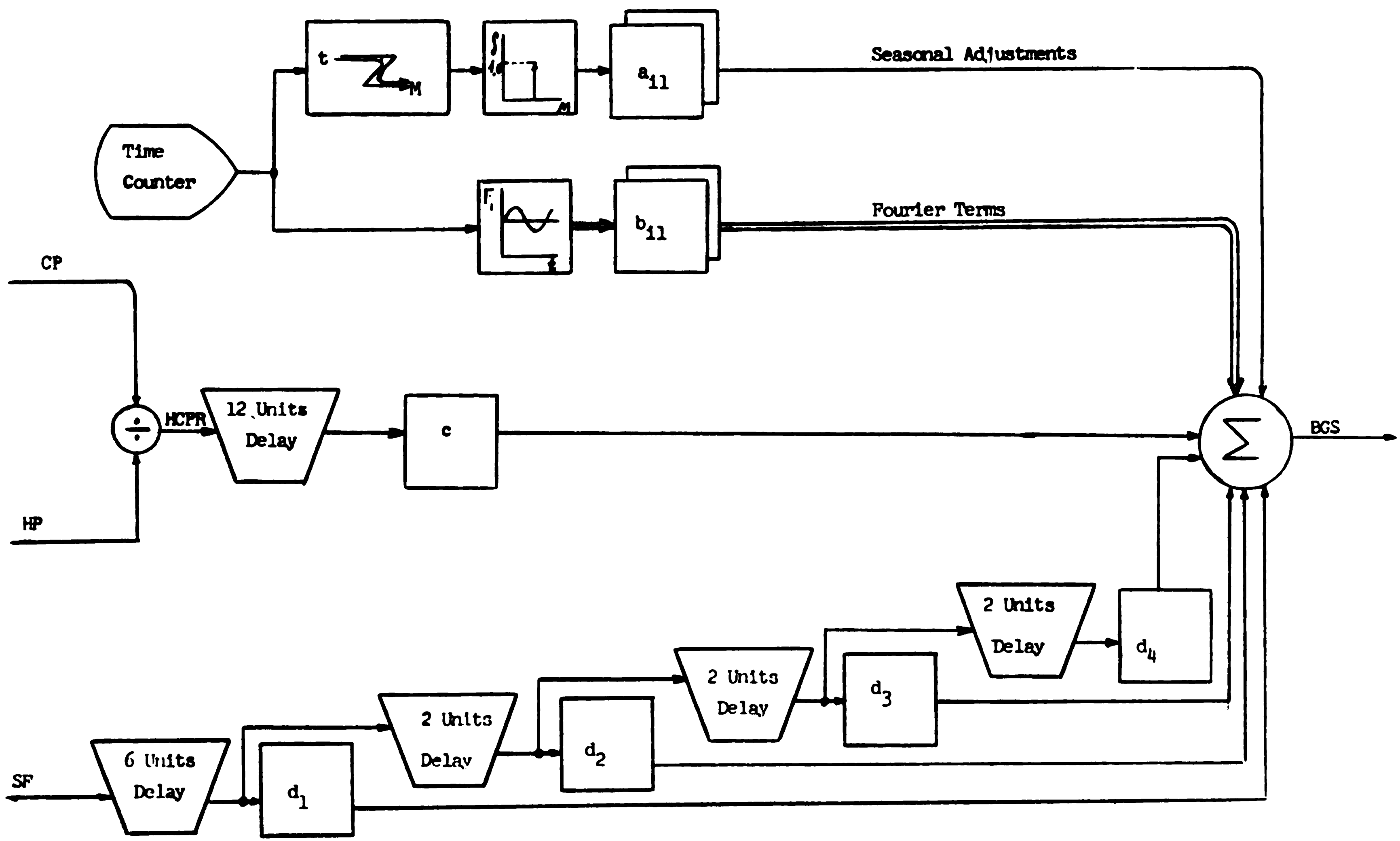

Figure 5.3.--Barrows and gilts slaughtered determination block diagram.

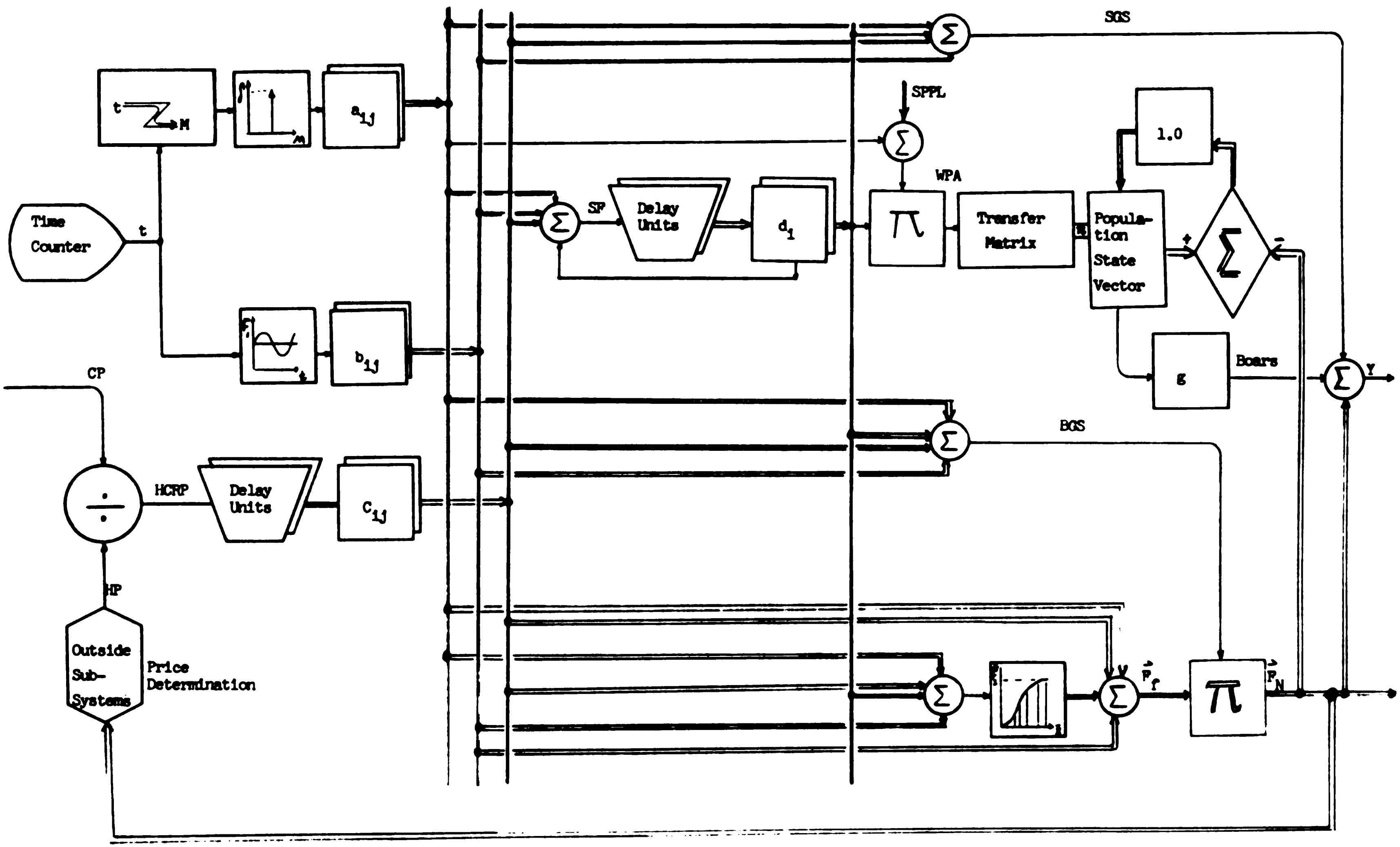

Figure 5.4.--Aggregate structural block diagram in the simulation of the hog production system.

help in following the description and discussion of the model's structure. Portions of the structure are self-explanatory. Elaboration of only the important procedures is needed.

Having forecast the four basic equations, what is left is the difficult task of filling up the gaps of the "black box" of the production process by constructing a reasonable technological-economic structure; namely, production stations which, while putting them in the right order, will transfer the inputs into the outputs. The first step has been achieved--the forecasting of two basic systems' status (SF, AVLW), plus the forecasting of the output BGS and SGS. Let us now turn to the next step--to build the rest of the simulation model around this basic framework.

WPA[91] Pig Crop

The pig-crop is computed by the following identity (see also Figure 5.4):

(5.7) WPA = SF * AK

where AK is the pigs per litter rate, computed:

(5.8) AK = 7.38 + 0.01 NYR + SPPL

NYR = number of years after 1963

SPPL = seasonal adjustment on the pigs per litter rate.

---

[91]WPA stands for weaning pigs in their first month. (The variables names were selected such that full computability is restored among the text and the appendices.

The annual linear improvement factor is based on the conclusion that technology will improve with time. The limited available information about it doesn't allow reaching a better estimate.

The Transfer Matrix

In an early stage of the development of this system, the transfer matrix was a diagonal matrix with zero values off the diagonal, with death and farm slaughter rates for each age group pigs on the diagonal. Later, it was found necessary to deal with pigs born on a weekly basis so the matrix was enlarged correspondingly. The sum of the death and farm slaughter assumed is as follows: 2.5 percent in the first month; 1.5 percent in the second; 1.3 percent in the third; 1.0 percent fourth and fifth; 0.7 percent sixth; and 0.5 percent in both the seventh and eighth month of age (see statements 158-169 in Appendix 2). The transfer matrix shifts the population state vector after the marketing allocation took place last month into the current month state vector but before the marketing procedure is to be worked out.

At eight months of age, the surviving pigs join the breeding herd in the ratio of 92 percent females to 8 percent males. There are data available once a year about the Total Female Held (TFH), which means the total female eligible for breeding. The reliability of these data based on the published national statistics is questionable. Nevertheless, an accounting of TFH made in the simulation reached the relationship of 92 percent female to 8 percent male, by trial and error. This relationship is compatible with the values of TFH_t and the marketed hogs.

The Market Allocation Scheme

Total Hogs Marketed (Y) can be computed by the identity:

(5.8) $Y \equiv BHS = SGS + \text{Boars Sold}$

So far, BGS and SGS have been predicted. Boars sold is computed as the disposal of the slack between the required 8.3 percent of the breeding herd and the actual number on hand (Statements 211-215, Appendix 2).[92] Next, the feasibility of Y predicted should be checked. In other words, is the inventory on hand capable of delivering these predicted hogs? To answer this critical question, only a check in connection with SGS and BGS values is needed. As far as SGS is concerned, the problem is relatively simple. It is necessary to make sure that the Total Female Held (TFH) is large enough to insure as many as Sows Farrowing at four months later. Taking all this under consideration, the minimum number of TFH is given by:

(5.9) $TFH_{min} - SF_{t+4} * 1.2987$[93]

Notice that SF_{t+4} can be computed because its predetermined

---

[92]Currently the hog producers keep one boar per twelve matured females.

[93]The assumption here is that percent of "open sows" has always exceeded 29.8 percent. See USDA, SRS, Livestock and Meat Statistics.

variables are lagged at least five months. (See statements 175-206, Appendix 2.) In fact, this check was negative throughout, apparently because the hog producers tend to keep quite a large slack--up to 50 percent "open sows"--in the breeding herd.

When answering the feasibility issue for the BGS, the problem becomes very complex. For one, not only BGS as a total of all borrows and gilts in different ages should be satisfied, but the live inventory balance of each age group for itself cannot be violated. Furthermore, the tolerances are much tighter than that. If too many pigs are drawn from one age group, it may "empty" the future heavy pigs later on. If too few are drawn, a huge number of boars and gilts will enter the breeding herd followed by very small number to make it unduly oscillatory. To summarize, it is necessary to estimate and predict the age-distribution under the BGS. There is very limtied available data to support a direct approach, thus the researcher is left with several alternatives to cope with the problem. Since this assignment is both crucial and complex, some elaboration is needed.

Among the several approaches available, three were seriously considered:

1. <u>Retention function.</u> At any point in time, the total number of hogs equals the number to be delivered to the market plus the number to be retained at the farm. So, if the retention function for each

age can be generated, the problem can be solved. In the absence of relevant information and a sound theory, a limited hypothetical model was built. This model generated results which could not be reconciled with the available market statistics, without seriously negating the underlying assumptions. Thus, this approach was dropped.

2. Loop Search Allocation Scheme. Realizing the need to reconcile the number of marketed pigs over time with inventory on hand and the total pigs marketed, a way was sought to directly determine the total of pigs delivered subject to this constraint and to the inventory of each age-group by small iterative steps. On every pass of a loop, the barrows and gilts were drawn from the heaviest four age groups under the growing ratio of the inventory on hand. Doing it in this way, a negative inventory can never be reached, and it smoothly keeps the different age-groups in relatively close ratios. In every pass, a check was conducted to find if the total pigs drawn so far has reached the quota, i.e., the BGS "target" figure predicted for the current month. An inventory adjustment was then made. In an attempt to hit the target, the process continued until the target was exceeded. At this point a movement backward was made, thus adjusting

to the exact figure. A safety feature against an infinite loop was introduced since the target BGS and the SF feeding the system with pigs are independently predicted figures with the possibility of having lagged SF underestimated for several consecutive months and BGS overestimated later on. The safety device was a counter with a prespecified tolerance which would stop the process automatically even if the quota had not yet been reached. The print out of this counter was a very important design tool in building the system.

The advantage of this approach was to permit going ahead with the construction of the program with a limited but satisfactory allocation scheme, in terms of inventory control logic and filling out the overall market "quotas".[94] When these are the objectives, this "black box" algorithm could fulfill all requirements. But when the overall Hog-Pork subsector simulation operates, hog prices become endogenous and functionally related to the age-weight distribution of the barrows and gilts sold. At the same time, the solution satisfying the limited objectives is not unique. Therefore it became necessary to pursue a solution which satisfies the broader set of objectives. Let us now discuss and describe this scheme.

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[94] A system operating with this feature is available, but is not included here.

3. The Age-Weight Distribution Allocation Scheme. It is necessary to know the relation between the age of pigs and the attained weight of the pigs. Feeding systems must have additional causal effects, but without a decision-behavioral feeding model available, an implicit feeding scheme unchanged with time and market condition was assumed.

Figure 5.5 shows the growth function used here. It gives the expected as well as the minimum-maximum days required to qualify a pig into one of the five commercial weight groups.

TABLE 5.5. Days Required for Hogs to Reach Selected Weights

Weight	Expected	Minimum	Maximum
180	143	133	153
200	155	144	166
220	170	158	182
240	184	172	196
270	208	195	221
300	233	219	247

Source: Estimated from graphic data, presented in Figure 5.5, reported by Dr. Elwyn Miller, Animal Science Department, Michigan State University, 1972. Assumes one standard deviation from the mean number of days. Prepared by Dr. Warren H. Vincent, Agricultural Economics Department, Michigan State University.

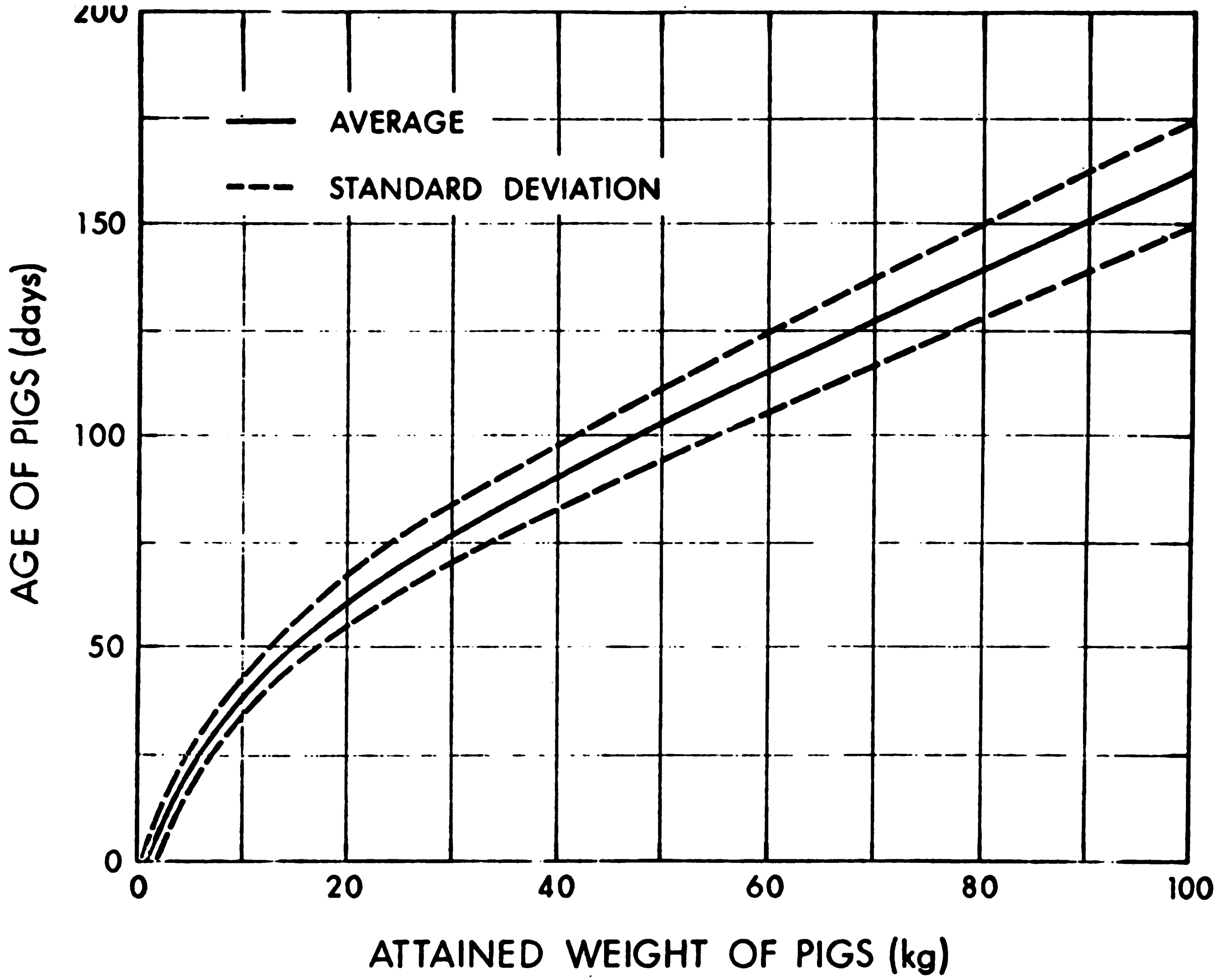


Figure 5.5.--Attained weight of pigs (kg head).

Source: Unpublished report by Dr. Elwyn Miller, Animal Science Department, Michigan State University, 1972.

The hypothesis is that the weight distribution is some nonsymetric distribution function which depends on the average liveweight.

Based on an unpublished study made in the major midwest hog markets, which sampled the weight of hogs, a picture of the "real" distribution over time was approximated. (Appendix 3)

With the $AVLW_t$ predicted, a first approximation was made by computing the area under the normal distribution curve corresponding to each weight group. The following diagram may explain the procedure. Assume that the standard deviation about $AVLW_t$ is 19.5 pounds. A specific algorithm, NDTR was developed by IBM[95] to compute the area under the normal distribution, assuming that the curve remains normal and symetric about $\bar{X}$, ($\bar{X} \equiv AVLW_t$) As shown in Figure 5.6, the following commercial weight groups would be estimated: 180-200, 201-220, 221-240, 241-270, and 271-300 lbs/head. The procedure in estimating these commercial groups is as follows: Step one: estimate $\bar{X} \equiv AVLW_t$. Compute the above five commercial weight groups as the area under the normal distribution. Step two: adjust the proportional group's size by the seasonal

[95]IBM, System/360 Scientific Subroutine Package, White Plains, N.Y., 1968, p. 78.

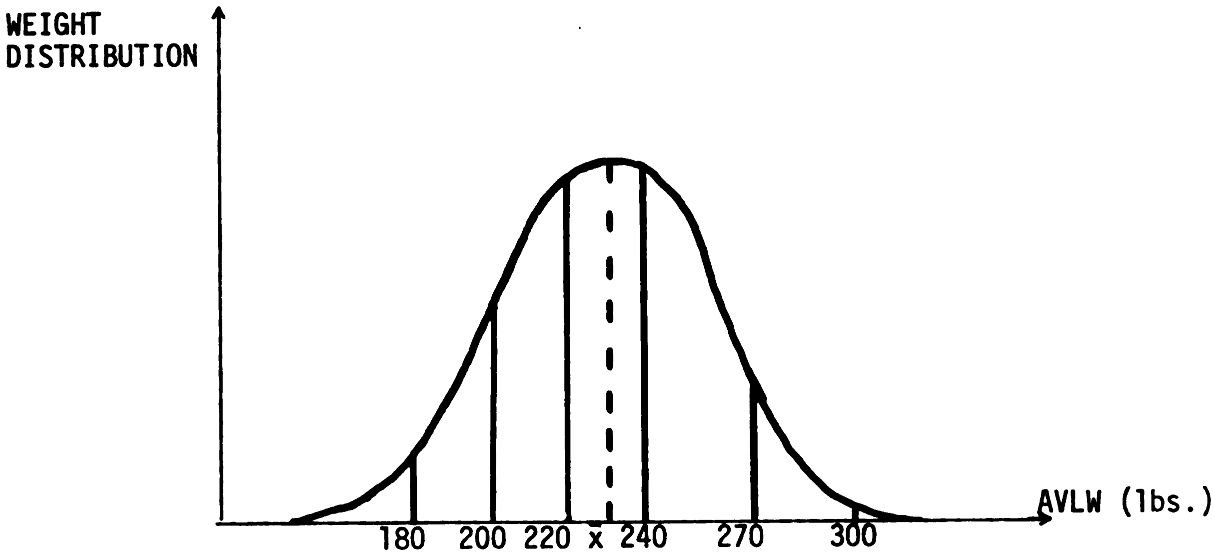


Figure 5.6.--An approximated normal weight distribution of barrows and gilts.

adjustment according to the particular month. These adjustments are based on the unpublished survey carried out jointly by the USDA and Purdue University during 1970-72 and summarized in Appendix 3. The completed estimated equations are given in Table 5.6 and obtained by applying ordinary-least squares procedure to the normal distribution approximation and the survey values of Appendix 3.

With an estimation of the number of BGS distributed among the five commercial weight groups, next it was necessary to make the age-inventory groups eligible for "sale" by the respective weight groups. To achieve this, it was assumed that the sows farrow throughout the month with a

TABLE 5.6. Estimation Equations for the Weight Groups as Fractions of Total Barrows and Gilts Sold.

Independent Variable	Dependent Variables F_1*		F_2		F_4		F_5	
	Reg. Coeff.	Sig. Level	Reg. Coeff.	Sig. Level	Reg. Coeff.	Sig. Level	Reg. Coeff.	Sig. Level
Constant	0.036	<0.0005	0.153	0.0005	0.103	0.025	0.023	0.001
Normal equivalent**	0.572	<0.0005	0.569	0.0005	0.387	0.002	0.874	0.005
Dummy Jan.***	−0.014	0.016						
" Feb.								
" March	−0.018	0.002					−0.014	0.101
" April	−0.025	<0.0005	−0.033	0.02	0.054	0.006	0.019	0.036
" May	−0.014	0.018	−0.036	0.02	0.024	0.226		
" June							0.016	0.072
" July			0.055	<0.0005	−0.053	0.007	−0.021	0.019
" Aug.			0.045	0.006	−0.041	0.070		
" Sept.	−0.015	0.015			0.040	0.065		
" Oct.	−0.03	<0.0005			0.029	0.098	−0.012	0.154
" Nov.	−0.019	0.002	−0.027	0.045	0.020	0.233		
R^2	0.916		0.9288		0.9069		0.9014	
$\bar{R}^2$	0.871		0.9037		0.8572		0.8666	
F value	20.471		36.972		18.254		25.907	
Sig. Level	0.0005		0.0005		0.0005		0.0005	
DW Stat.	1.58		1.95		1.74		1.47	

$F_3 = 1 - (F_1 + F_2 + F_4 + F_5)$

* F_i $(i = 1,\ldots,5)$ are the estimated fractions of total BGS. $i = 1$ means the 180-200 lbs./cwt./commercial group, $i = 2$ means the 201-220 lbs./head and so on.

** The equivalent fraction computed as the area under the normal distribution curve.

***The month of December was *a priori* deleted. Zero coefficients have been deleted.

uniform distribution. Beginning at age 4 months, each month is divided into four equal periods, and the pig inventory is carried accordingly. The eligibility of the expanded number of groups, 16, is determined with the help of Table 5.5, where one-third of each weight group is drawn only from the "expected" age groups, and the rest from the "expected" plus one standard deviation on both sides. The exact number of pigs to be drawn out of each weekly born pigs group is directly related to their group's size relative to that of the eligible groups. This scheme insures inventory control constraints and seems to resemble the real world pattern. The iterative process is retained as a stand-by option to be activated at a user's wish for further investigative purposes. The summation-decision box in Figure 5.4, page 98, represents this option.

The Computer Program

The above system has been converted into a FORTRAN IV computer program. Figure 5.7 reveals the program flow chart, and Appendix 2 the program code-edit. Both are self-explanatory. The program generates much more information than needed for only the simulation goal itself. To permit an evaluation of the system performance and to validate its results, several reports were constructed as well.

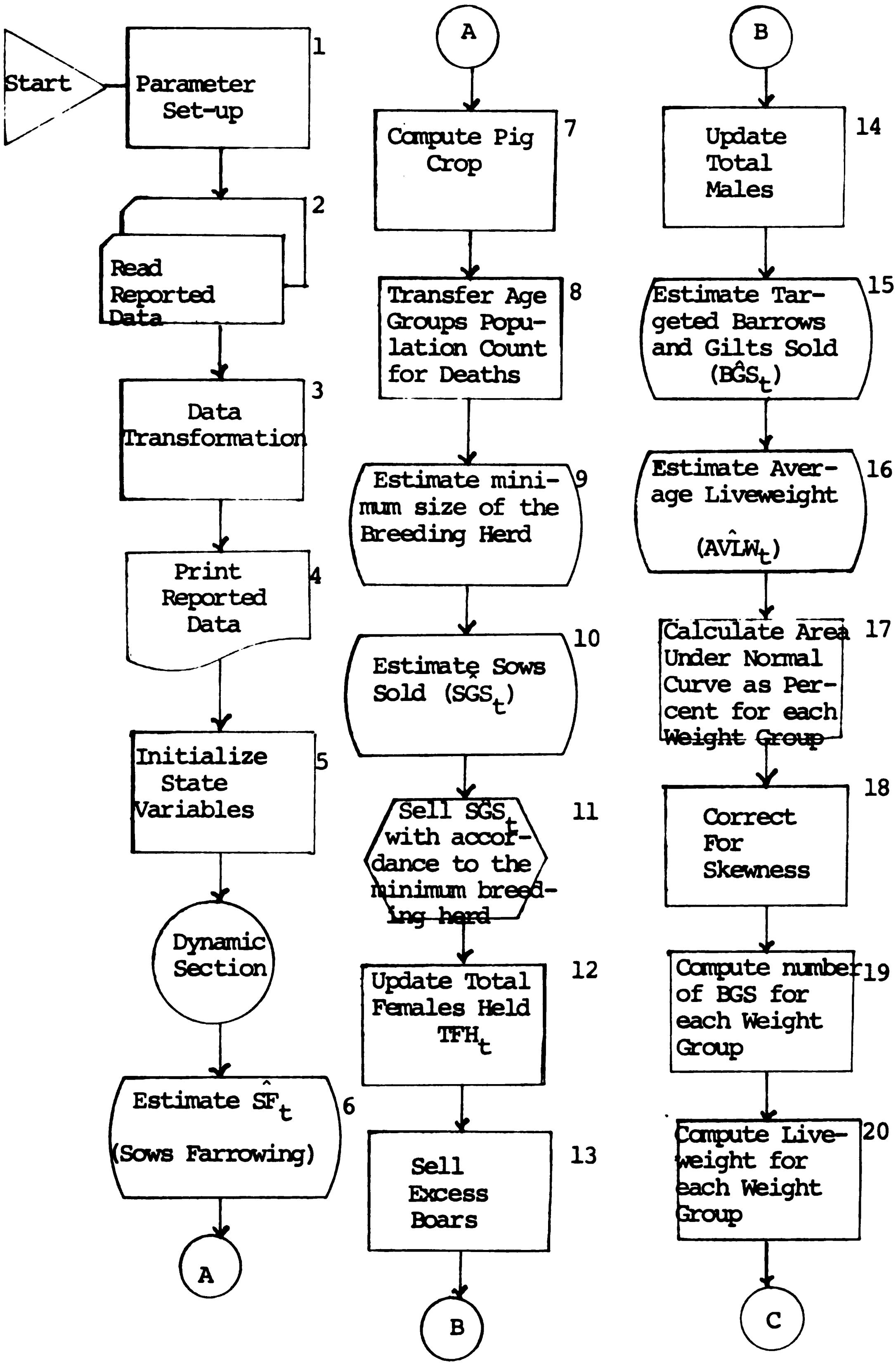
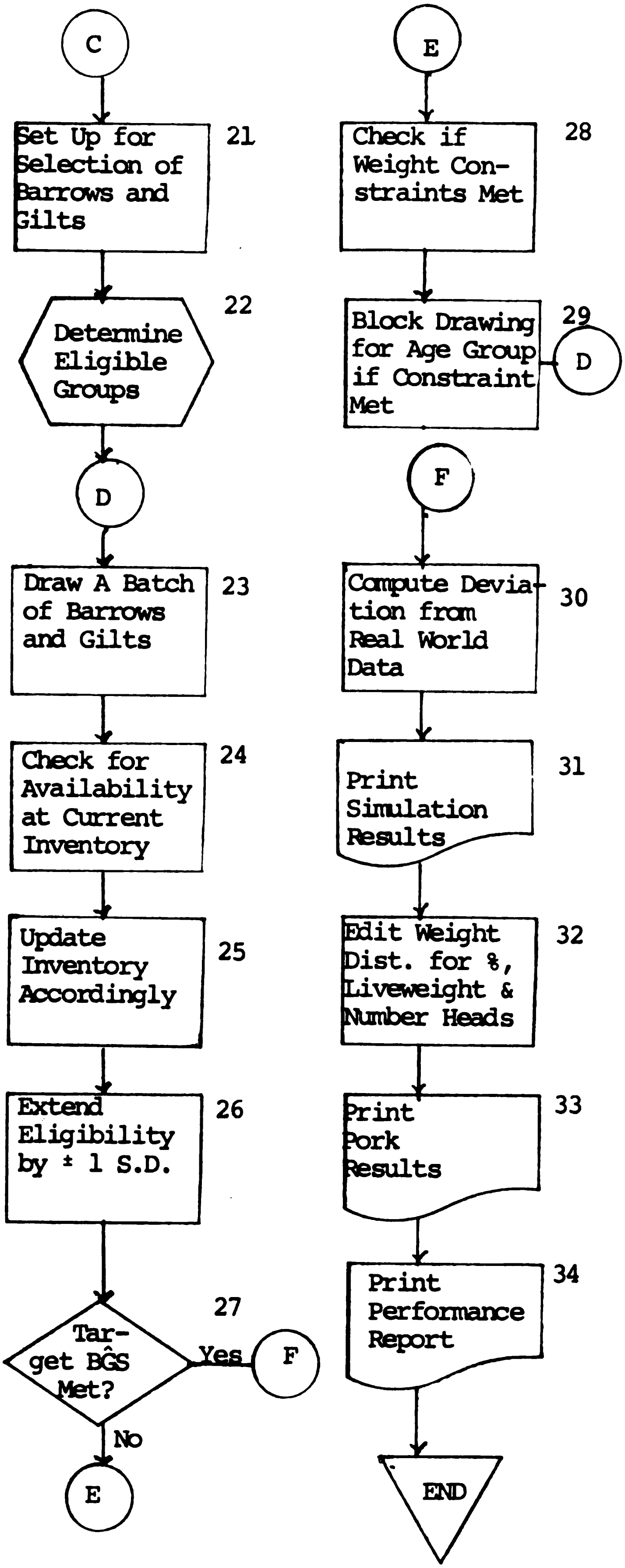


Figure 5.7.—The simulation program flowchart.


C
Set Up for Selection of Barrows and Gilts
21
Determine Eligible Groups
22
D
Draw A Batch of Barrows and Gilts
23
Check for Availability at Current Inventory
24
Update Inventory Accordingly
25
Extend Eligibility by ± 1 S.D.
26
Target BĜS Met?
27
Yes
F
No
E
E
Check if Weight Constraints Met
28
Block Drawing for Age Group if Constraint Met
29
D
F
Compute Deviation from Real World Data
30
Print Simulation Results
31
Edit Weight Dist. for %, Liveweight & Number Heads
32
Print Pork Results
33
Print Performance Report
34
END

Tests and Model Validation

The model derived and constructed in this chapter, together with the specifications of the exogenous price inputs, has been simulated on a monthly basis by the computer.[96] The time period of January, 1964 to December 1973 served as the test period. The 1963 reported data were used to initiate the state variables, and supplied the lagged variables needed for the forecasting algorithms. The test was conducted in such a way that the system remained nonrecursive throughout 1964 and 1965 to avoid any discrepancies caused by the initialization. From January 1966 to December 1970, the test was conducted on the same data base used for estimating the basic equations. The January 1971-December 1971 period represents a test where the model is extrapolated into a period beyond the original data base. The same computer program (Appendix 2) is designed to conduct the test by comparing the simulated data to the reported data, and calculating the error in absolute terms, percentagewise and by plotting it out. Figures 5.8, 5.9 and 5.10 reveal the results with relation to the SF, BGS and AVLW variables, respectively. Notice the marked similarities between the simulated and reported data. This means that not only the estimation's quality is fully satisfactory, but also under the model sturcture and sequence, the forecasted variables do not add

[96] CDC-6500 computer at the Michigan State University Computer Center.

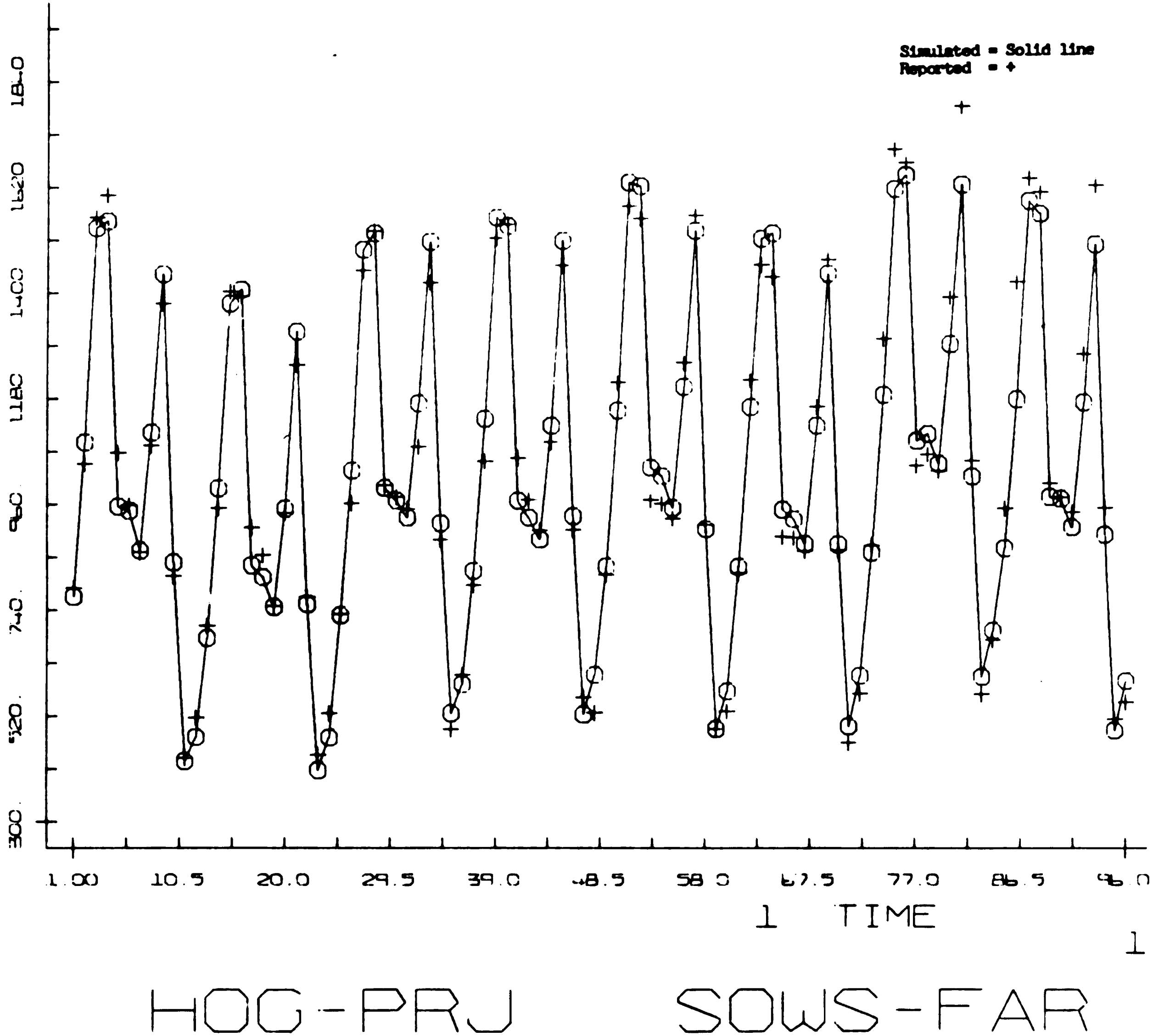


Figure 5.8.--Sows farrowed simulated versus reported 1964-1971.

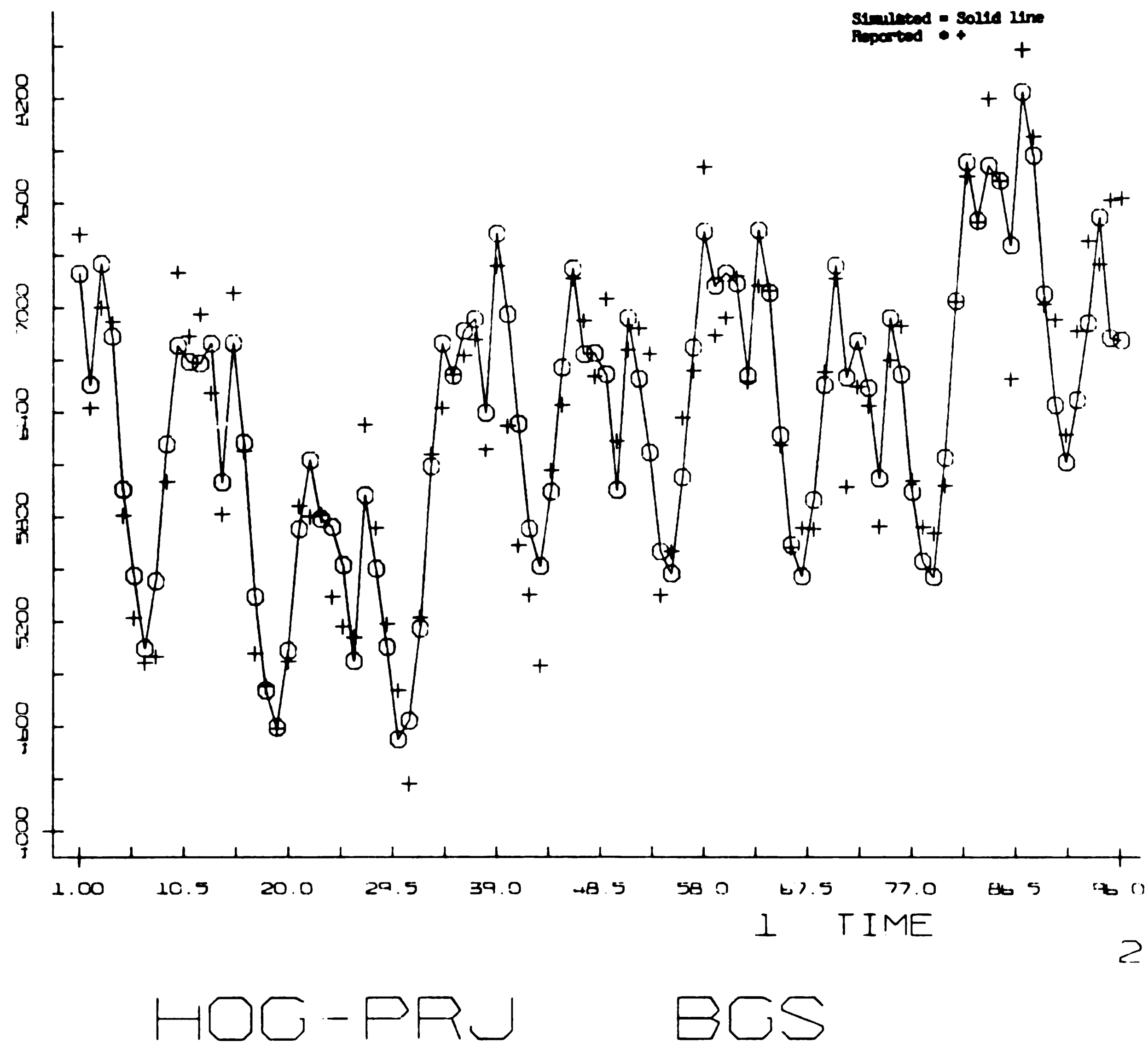


Figure 5.9.--Barrows and gilts sold simulated versus reported.

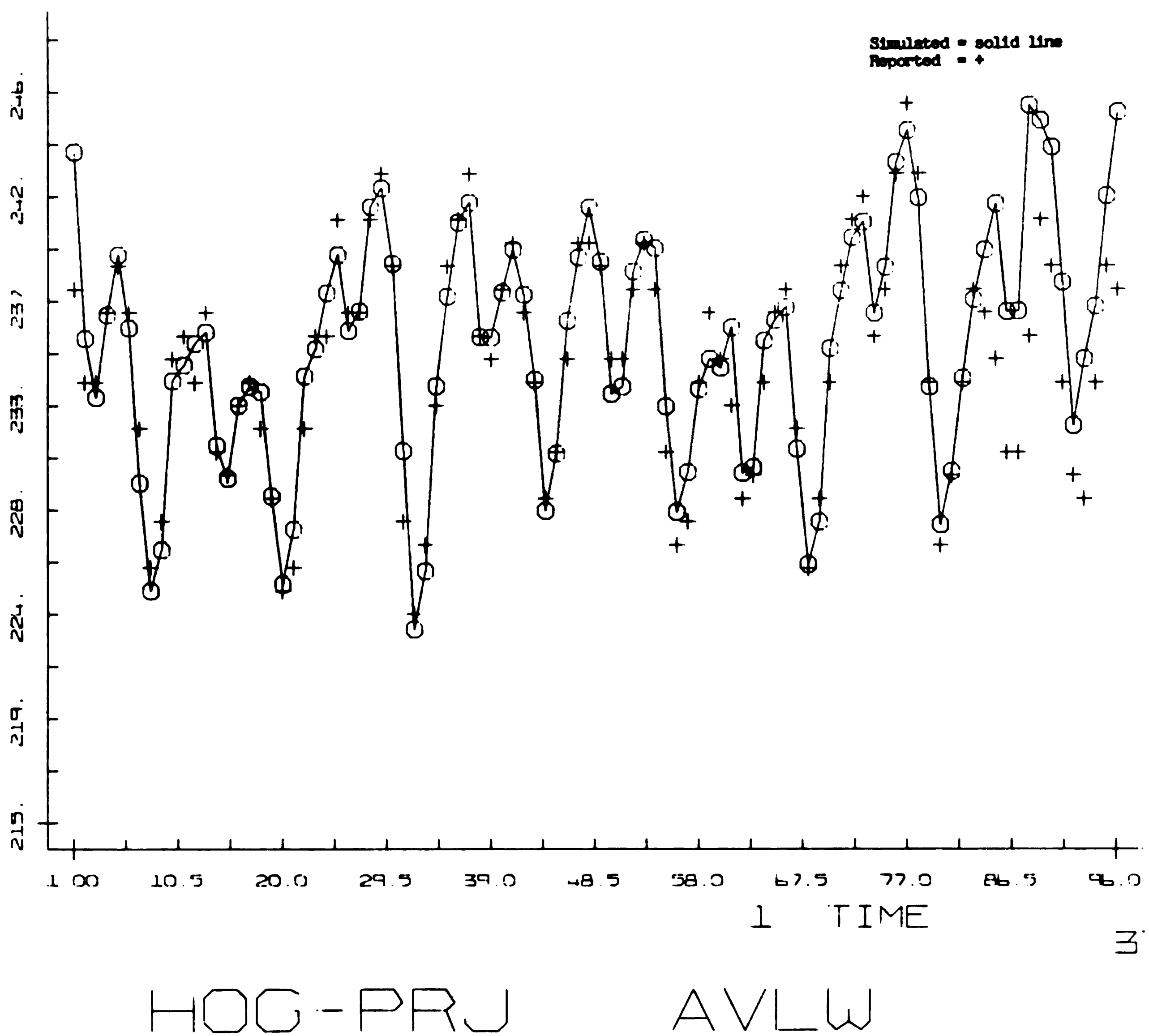


Figure 5.10.--Average liveweight simulated versus reported.

to the error term. This indicates that all the "targets" are actually being achieved. Appendix 7 demonstrates this point.

An inspection of Table 5.7, Figures 5.8, 5.9 and 5.10 or a detailed investigation of Appendix 5 clearly suggest that the model is satisfactorily verified and validated. Despite the existence of wide fluctuations, the model's natural response does follow it accurately with all turning points correctly predicted. The multiple frequency hog cyclical behavior has been modeled, including sufficient relationships to explain the production response in a dynamic setup. Furthermore, it appears that the model is inherently stable and convergent--important properties in simulation of stochastic systems. These properties guarantee that the error term, or the disturbance, closely resembles a random noise, therefore avoiding undershooting followed by overshooting, and so on, thus avoiding instability and explosion.

The interested reader may elect to examine closely the production process. This examination becomes possible with the assistance of the computer output available in the following order:[97]

Appendix 4: Reported Data: 1964-1971 presents the real-world data for the period simulated. These data are used only for purposes of comparison. Only the hog and corn

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[97]The titles of the Appendices and the abbreviations are compatible with the computer outputs which is required to be concise.

TABLE 5.7. Monthly Average Absolute Percentage Deviations Between Simulated Results and Reported Data

Year	Sows Farrowed	Total Hogs Marketed	Barrows and Gilts Marketed
1965	4.2	4.2	3.8
1966	3.5	4.7	5.2
1967	5.2	5.7	5.8
1968	3.8	4.7	4.8
1969	4.2	3.1	3.2
1970	4.1	3.5	4.6
1971	6.1	5.0	5.3

prices are used as exogenous variables.

<u>Appendix 5: Simulation Results</u>: shows the SF, state population vector, sales vector and the percent devision from reported data of sow farrowing (DS), total hogs sold (DY), and barrows and gilts sold (DB).

<u>Appendix 6: Pork Results Include</u>: Average Liveweight, fraction of each weight group, ratio between number of pigs delivered versus pigs predicted for each group, and the actual weight group distribution in absolute numbers.

<u>Appendix 7: Age-Weight Distribution</u>: The weekly age distribution of pigs delivered out of the total barrows and gilts.

<u>Appendix 8: B & G Performance</u>: A comparison among the total BGS reported--estimated-simulated.

Implications

The conclusions will be discussed in Chapter VII in detail, but some implications seem to be in order now.

In Chapter IV, the hog cycle was decomposed into six cycles as functions of the time only. In this chapter, the production supply response model, along with its economic and technological reasoning, replaced the time variable with economic ones to explain a great portion of the total cycle. Some aspects of the cycle still remain unaccounted for on the production supply side. Presumably, their resolution should be found in the demand-consumption side and/or by the interaction of the two sides. This aspect is an area for further research.

The number of sows bred and/or farrowed <u>is the most crucial state variable</u> to watch. Breeding or farrowing decisions have great implications later on concerning marketing rates and other farrowings. The feedback characteristics are very crucial throughout the producers decision making process. These relationships are quantitatively measured.

The market allocation scheme is an integral subsystem, closely interacting with the production subsystem and mutually affecting each other. Hence, the two must be analyzed together.

Having successfully identified, modeled, and simulated the production subsystem, we turn to the policy implications of the work. The issues under examination are those related to the extent to which selected controls might _steer_ hog production response. These issues are the subject of the next chapter.

CHAPTER VI

PRODUCTION RESPONSE TO POLICY CONTROL

Basic Principles of Price Policy

Since the appearance of D. G. Johnson's classic book Forward Prices for Agriculture,[98] agricultural price policy has been influenced substantially by his guidelines. Therefore, before setting the explicit goals of price policy applied in this study, it is worthwhile to revisit Johnson's thoughts.

The basic aspects of a forward-price system are few and simple:

1. The prices should be announced sufficiently far in advance to enable farmers to adjust their programs to the prices.
2. The price should cover a sufficient period of time to permit farmers to complete their production plans with considerable certainty.
3. The price announcements should be sufficiently clear and precise so that farmers can readily interpret their implications for him.

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[98] Johnson, D. Gale. Forward Prices for Agriculture. University of Chicago Press, Chicago, 1949.

4. The prices adopted should be those that achieve the desired output.[99]

Some immediate conclusions evolve: "Prices should not be used as goals to be achieved."[100] Although this assertion is a debatable issue and may be violated in practice, it merely suggests that prices should be viewed as the means to achieve the ultimate policy goals in terms of income and output.

The following principles of price policy are also very crucial:

> "Prices should be used as directives in the economy. . .price policy, as an adjunct of general monetary-fiscal policy, should be utilized to reduce fluctuations in farm prices and incomes. . .price policy should be utilized to attain a considerable degree of stability in the output of individual crops and livestock products. . .price policy should be utilized to reduce price uncertainty confronting farmers by consolidating and transferring uncertainty to the economy as a whole. . ."[101]

These guidelines determined the price policy employed here. Now, it seems appropriate to clarify a few basic characteristics of policy control from the system science point of view.

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[99] *Ibid.*, p. 11.

[100] *Ibid.*, p. 31.

[101] *Ibid.*, p. 31-37. Also pp. 37-120 for an elaborated reasoning.

Open-Loop Versus Closed-Loop Policy Control

By definition,[102] control systems are classified into two general categories: open-loop and closed-loop systems. The distinction is determined by the control action, which is that signal responsible for activating the system to produce the output.

An Open-Loop control system is one in which the control action is independent of the output.

A Closed-Loop control system is one in which the control action is somehow dependent on the output.

Generally, the Closed-Loop control system has a major advantage in steering the system's output under uncertain conditions. For if a significant disturbance affects the system, an appropriate adjustment of the control should be undertaken. In a perfect knowledge situation (unrealistic, of course), the two systems should yield the same results.

The Open-Loop control system, on the other hand, has some advantages in certain situations. It is relatively simple in nature, and predictable in the sense that every participant would know in advance the control time path, making the inputs' time path more certain. There is no need to observe the feedback and adjust the control application

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[102] Distefano, J. J., Stubberud, A. R., and Williams, I. J., Theory and Problems of Feedback and Control Systems. Schaum's Outline Series, McGraw-Hill Book Company, New York, 1967.

accordingly. Therefore, the cost of operating the Open-Loop control is much less than for a Closed-Loop. These considerations must be faced when a decision on open versus closed loop control is needed. Nevertheless, no such decision was needed here, because the hog production industry, as a subsystem of the hog-pork subsector, cannot "observe" the consequences of its output. The output effect on the forthcoming hog prices can be determined only via the demand-consumption subsystems within the overall subsector study. Therefore, it was necessarily postponed until the entire subsector simulation model could be completed.

Alternatively, the Open-Loop control policy might be feasible to apply. Are there necessary conditions to be fulfilled? Following Manetsch and Park[103] three basic conditions must be met:

a) The inputs to the system can be practically steered.

b) The system is "controllable".

c) The system is "observable".

The first condition is being met by assumption with regard to feasibility. It is beyond the scope of this study to demonstrate conclusively that price policy measures can be applied effectively. It is simply assumed here that there is enough evidence and experience in using price support,

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[103]Manetsch, T. J., and Park, G. L. System Analysis and Simulation with Application to Economic and Social Systems. Preliminary edition, Michigan State University, September 1972.

subsidy programs and the like, since the mid-1930's, to make price policy feasible.

The second condition requires that a system is completely state controllable if <u>all</u> components of the state vector are affected by or connected directly or indirectly to the input vector.[104]

The third condition requires that a system be observable if all components of the state vector are connected to the output vector.[105]

To establish the proper necessary conditions for the open-loop price policy, the next logical step is to question the feasibility of enlarging the policy control interval beyond the price policies. Can producers individually or collectively control their production directly, in the long run, under a carefully and coordinated plan? True, some

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[104]To test for controllability, let a linear model be:

(i) $\dot{\vec{X}} = \underline{A}\vec{X} + B\vec{u}$	$\underline{A}$ is n by n, $\underline{B}$ is n by m matrices
(ii) $\vec{Y} = \underline{H}\vec{X}$	$\vec{X}$ is n by 1 state vector and $\vec{u}$ is n by 1 input vector.
	$\underline{H}$ is q by n matrix and $\bar{Y}$ is the output vector.

The input to output transfer matrix is

(iii) $\underline{H}(sI-\underline{A})^{-1}\,\underline{B}$

The system (i), (ii), (iii) is completely state controllable if the n by m matrix $P = [\underline{B}\ \underline{AB}\ \underline{A}^2\underline{B}\ .\ .\ .\underline{A}^{n-1}\underline{B}]$ has rank n. For detailed discussion, see E. G. Gilbert, "Controllability and Observability in Multi-Variable Control System," <u>Journal SIAM Control</u>, Series A, Volume 2, No. 1.

[105]Similar test is available in <u>Journal SIAM Control</u> as noted above.

control on the quantities produced and marketed is being exercised on a short-run basis, where changes in delivery timing of pigs to the market, or effective use of the storage facilities, might be pursued by the hog producers and wholesalers, respectively. So, the question becomes a bit narrower, namely, can the system's participants move into the long run control area? There is no final answer available, but there is enough evidence to suggest that any quantity manipulation at the marketing level, on a long run basis, is not in sight. The storage capacity would have to be increased enormously to enable a quantity control on a four-year span. This would mean a substantial and perhaps prohibitive increase in the storage cost. What is left is an attempt to control production at the basic production level, namely, at the farrowing stage. Under perfect competition production controls may have harmful aspects, but with the growing degree of vertical and horizontal integration and coordination, it becomes possible to plan, contract, and execute long run production control on a limited scale at the present time, and on a more substantial scale in the foreseeable future.

The Policy Control Set-Up

The models developed in Chapter IV and V enable us to exercise different sets of policy controls, and to evaluate the consequences. To demonstrate this important potential, let us apply a particular policy scheme and trace the results.[106]

[106]It is beyond our scope here to seek any specific policy scheme, but to show that this can be potentially done. The policy control applied here is by no means claimed to represent an optimal one.

The objective is to remove the long run cycles from the hog production industry in terms of physical units produced and marketed. Long run cycles, defined as any cycle with a duration longer than one year, have to be damped completely within a period of very few years,[107] while the short run oscillations expressed in cycles with duration of one year or less, or by seasonal monthly shifters, remain unchanged or reduced in amplitude.

Technically, let us assume that there is a price policy applied appropriately on both the hog price and corn price such that all the long run cycles of the hog-corn price ratio will be completely eliminated (the four and two years and sixteen month cycles). In addition, a relatively minor coordinated policy control on the two year sows farrowing cycle is exercised successfully.

Explicitly, this open-loop control takes the following form in terms of the predictive equations established previously in Chapters IV and V.

For the hog-corn price ratio, consider the variable $HCPR_t$ according to the general equation (4.6) subject to the coefficients of Table 4.2. Let us now set the "filter" matrix in equation (4.7) equal to the identity matrix, and simulate $HCPR_t$ for $t = 1, . . . , 96$, generating the predicted

[107]The effect of the control policy will only gradually take place due to the production delay structure.

hog-corn price ratio for two complete cycles, representing the noncontrolled input to the hog production industry. Then, extract from the production system the main elements of the state vector, described in Chapter V, namely, the sows farrowing (SF_t), as given by Equation 5.1 subject to Table 5.2's coefficients. Similarly, the main elements of the output vector would be represented by the barrows and gilts sold (BGS_t), which account for about 90 percent of total output. The BGS_t variable will be generated by Equation 5.5 subject to the coefficients of Table 5.3. A sensitivity analysis done for the entire hog production system has shown that even extreme values of the input time path, beyond the past interval, still leave the model within the logical tolerances. In other words, the internal relationships like live inventory adjustments, and marketing of pigs in certain ages are satisfied. This fact allows us to simplify the simulation's operation and save computer time, rather than dealing with the entire detailed production model.

So far, we have set up the noncontrolled simulation run for comparative purposes. For the controlled run, the open-loop controlled input would be achieved by "removing" the four and two year cycles along with the 16 month cycles. (Technically, this is done by setting the appropriate elements of the diagonal "filter" matrix of Equation 4.7 to be zeroes.) In a similar fashion, a direct but weak control is imposed on the SF_t by taking the sine($2w_ot$) term out of the Equation 5.1 to eliminate the residual two year cycle from the SF_t time path.

Results of the Open-Loop Control Imposition

To facilitate the evaluation, attention is directed to Figures 6.1, 6.2, and 6.3. Figure 6.1 reveals the controlled versus noncontrolled $HCPR_t$. (Appendix 9 shows the numerical results.) The fluctuations have been reduced substantially and systematically by the removal of the low-frequency cycles. Explicitly, the controlled $HCPR_t$ ($HCPR_t^c$) has reduced the sampled coefficient of variation[108] $\hat{V}$ from 0.1678 to (for the noncontrolled $HCPR_t^n$) to 0.0525.

The impact of these two separate input streams can be shown by the effect on the state variable SF_t in Figure 6.2, and on the output variable BGS_t in Figure 6.3. The complete impact of this controlled $HCPR_t$ cannot be sensed during the first 24 months because the system becomes recursive only after that period.[109] The change of SF_t behavior is small, as can be seen from Figure 6.2. Nevertheless, the controlled SF_t is no longer subject to low frequency cyclical patterns. This is true especially at the turning points where the controlled SF_t always moderates the more extreme peaks and troughs of the low frequency cycles (four and two years).

The coefficient of variation has been increased from 0.3197 (noncontrolled SF_t) to 0.3271 (controlled SF_t) for

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[108]The coefficient of variation in the sample is defined by $\hat{V} = \hat{\sigma} / \hat{\mu}$ where $\hat{\sigma}$ is the sample standard error and $\hat{\mu}$ is the sample mean.

[109]Due to the lag structure the reported data is used rather than the simulated values during the first two years of simulation.

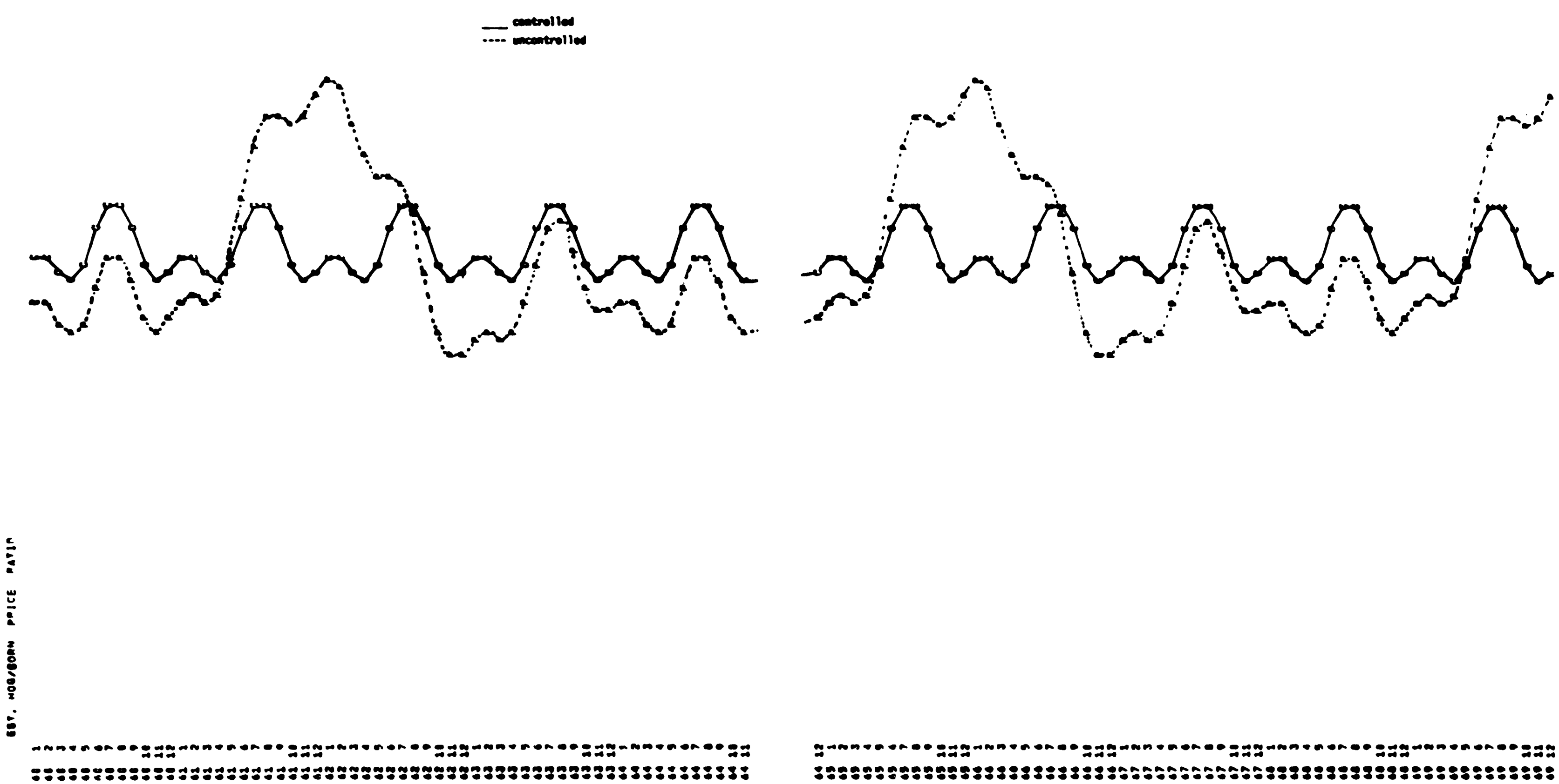

Figure 6.1.--The controlled and noncontrolled hog-corn price ratio.

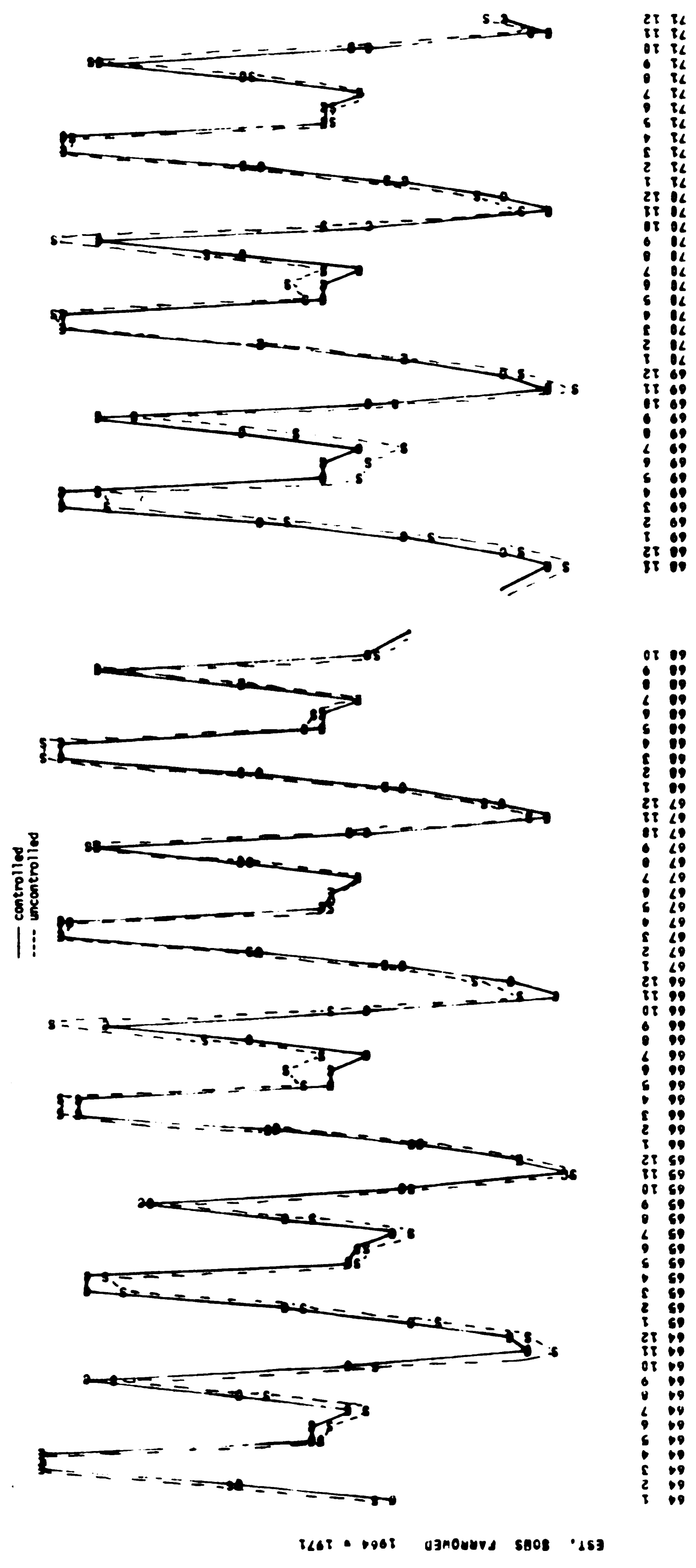

Figure 6.2.--The controlled and uncontrolled sows farrowing (SF_t)

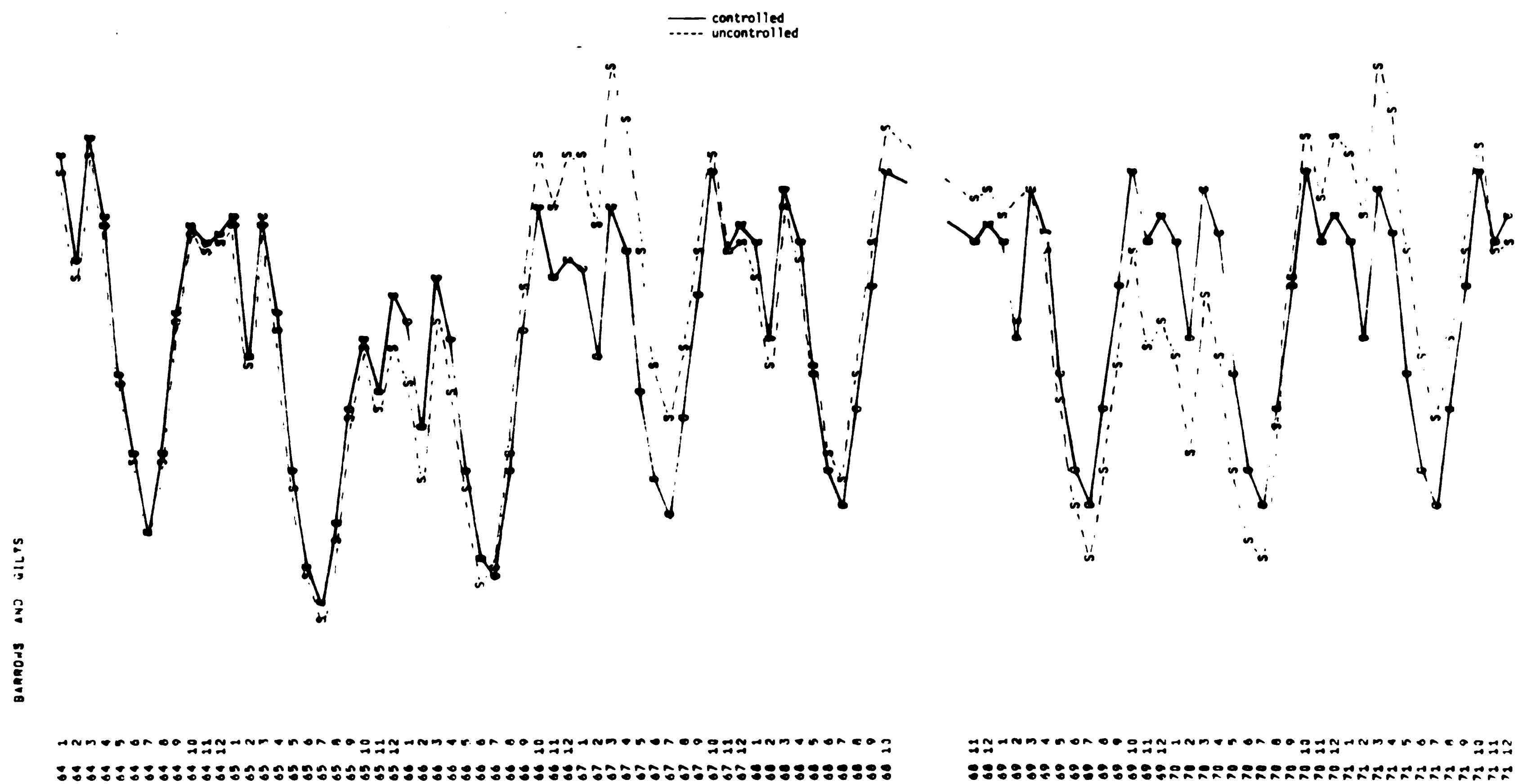

Figure 6.3.--The controlled and uncontrolled barrows and gilts sold (BGS_t).

t = 25,. . .,96. This increase is probably due to some decrease in $\hat{\mu}$ for the controlled SF_t with little decrease in the variance.

Yet, the impact on the output-BGS_t is rather substantial, simply because BGS_t is very sensitive to the level of past SF_t's. As soon as the system becomes recursive (in January 1966), the controlled BGS_t's time path looses the strong four and two year cyclical behavior, and the reduction in fluctuations is substantial too, as indicated by the decrease in the coefficient of variation from 0.1150 to 0.0992 for t = 25, . . .,96. The ultimate contribution from the policy control is achieved towards the end of the simulation run, as the extremely high output peak has been cut very significantly.

In conclusion, it seems that the open-loop control policy provides the necessary input signals to the production system to damp completely the short period cycles--the objective set forth has been met! It is the high sensitivity of the BGS_t to the past SF_t which finally makes the pursued response, where even a small change of the SF_t in the right direction causes the magnified change of the BGS_t. Nevertheless, these results are subject to some strong assumptions--namely, no effective disturbances should occur--shifting substantially the output out-of-phase, for if this happens, the control being "open" can not be "corrected" by the feedback, and may cause even higher fluctuations than the noncontrollable system which has the natural production feedback.

CHAPTER VII

SUMMARY AND CONCLUSIONS

This chapter begins with some of the methodology issues revisited, of particular interest are the problems of validity and inference in the simulation of computer models. The major conclusions of this study are discussed next, followed by a brief summary of this research effort and some implications, and finally, some recommendations for further study.

Model Verification

Decomposition--Reassembly Heuristics

The rationale for discussing the verification issue at this late stage arises from the existence of multiple scientific philosophies which are self-contained, but usually in some conflict with each other.[110] It seemed most appropriate to discuss and evaluate, in retrospect, the verification methodology described above while presenting the alternative methodologies.

M. Pfaff,[111] in presenting the need for modeling and

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[110] Naylor, T. H., Balintfy, J.L., Burdick, D.S., Chu, Kong. Computer Simulation Techniques. John Wiley and Sons, Inc., New York, 1966.

[111] Pfaff, Martin. "Complex Organizational Processes," in The Design of Computer Simulation Experiment, edited by T. H. Naylor, Duke University Press, Durham, 1969, pp 391-410.

simulating real-world systems for research and training purposes, observes that this move, although enabling inference and study of important relationships, introduces a necessary step before accepting the model, namely, model verification.

> ". . .the trade-off between 'validity and inference'--or, in other terminology, that between realism and formalism--implies the need for a strategic decision on the part of the model builder in the design phase of a simulation experiment."[112]

To assist inference and learning capacity in studying complex real systems, like ours, a set of research heuristics was formulated. In their most simple description, they may be termed "decomposition and reassembly heutristics".[113]

In Chapter IV the highly complex hog cycle phenomena was decomposed into sub-cycles, and then reassembled. During this process, by expanding our inference capacity, newly revised theory of the Cobweb Model was formulated. A more detailed, piece-by-piece reassembly has been performed in building the computer simulation of the hog supply response in Chapter V. The process of decomposition assisted in eliminating the real cause and effect relationships among the inputs, systems states and outputs, such that a meaningful causal interrelationship can be applied through the reassembly process. This is in contradiction to the pure positivistic

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[112] Ibid., p. 391.

[113] Ibid., p. 400.

"black box" approach. The behavior of the system is inferred from the behavior of its components. The cost of this Decomposition-Reassembly procedure is very much like the process used in automobile assembly, where there is a need to test and verify each component as well as the whole car system. Let us briefly review the verification stage.

Methodology Positions on Verification

Naylor and associates[114] name four basic methodological positions on verification:

1. Synthetic *a priorism*. Any theory is merely a system of logical deductions from a series of synthetic premises of unquestionable truth, not themselves open to empirical verification or general appeal to objective experience.

Since pure logic and mathematics represent the only eligible fields in which synthetic *a priorism* holds, the use of this approach is limited, although it contributes substantially to our way of reasoning and thinking.

2. Ultraempiricism. stands at the complete opposite position. This position regards that sense observation is the primary source and the ultimate judge of knowledge. It refuses to admit any postulates or assumptions that can not independently be verified by sense observation.

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[114] Naylor, T. H., Blaintfy, J.L., Burdick, D.S., Chu, Kong. *Computer Simulation Techniques*. John Wiley and Sons, Inc., New York, 1966.

3. Positive Economics. M. Friedman who is a leader of this camp claims that the validity of an economic model depends not on the validity of the assumptions on which the model rests, but rather on the ability of the model to predict the behavior of the endogenous variables that are treated by the model.[115]

4. Multi-stage Verification. says that each of the aforementioned methodological positions is a necessary procedure for validating simulation experiments, but none of them is a sufficient procedure for solving the problem of verification.

The first stage calls for the formulation of a set of consistent postulates or hypotheses describing the system behavior. The consistency requirements calls for logical verification.

At the second stage, the proper criterion[116] for falsity is applied. That is, a postulate or a model is scientifically meaningful if, and only if, it is possible to refute the postulate by empirical observation.

The third stage of this verification consists of testing the model's ability to predict the behavior of the system in the future.

The methodology employed in this study resembles the

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[115] Friedman, Milton. Essays in Positive Economics. University of Chicago Press, Chicago, 1953.

[116] *Loc. cit.*, p. 314.

multi-stage verification procedure, with one important modification regarding the third stage. The mandatory requirement to predict the future is too vaguely stated in terms of how long should the test go into the future and the indiscriminatory invalidation, in the case of failure to predict. For, if a distant future has been determined then logically we cannot accept the model until that point in time has been reached. But, from a policy point of view, to wait may be unwise. On the second point, we always allow for some error or unexplained disturbance to "explain" the discrepancies between the simulated results and real-world data. Since this disturbance is a random variable, it is possible that, at some point in time, the disturbance assumes an extreme value (wars, national disaster, etc.) with corresponding extreme consequences. In such an event, the options open to the systems analyst include rejecting the model outright, analyzing the situation carefully to decide whether to remodel the system or to leave the system as it is.

Alternatively, on this issue, the author's preference is for stability as the acceptance criteria. In particular the model is acceptable if, under a shock, which is an element of the disturbance vector (by definition), the system behavior is convergent to the pre-shock path.

Under this methodological rule, it can be safely concluded that the entire hog production and supply response model, as well as its sub-components and the economic reasoning of the hog cycle, has been verified, as shown in Chapter IV's and V's statistical measures and graphical representations.

Implications

It appears, based on the methodology employed above, that the thesis objectives have been successfully met. Verifying the Multi-Frequency model along with the production simulation model enables us to draw conclusions and implications in two basic areas: Theoretic economical behavior of the industry under dynamic situations, and in conducting an investigation of the impact of some policy alternatives on the industry. More specifically, the following are the set of the important conclusions and implications:

1. The so-called hog cycle is a combination of six cycles differing from each other by their amplitude and frequency. On the possible range of the cycle's time duration between eight years through two months, the following cycles have been identified: four, two and one year, sixteen, six and four month cycles.
2. An extended cobweb model has been developed, showing how moving long and short run equilibria can create the above cycles. As they move together, generate the total hog cycle.

3. The Multi-Frequency Cobweb Model requires for each subcycle a price input signal with a lag equal to one-half the period needed for completion of the particular subcycle. This requirement was fulfilled empirically by utilizing a special format of distributed hog-corn-price ratio lag model. These different time lags, apparently, represent decisions on investments with different lifetimes under effective degrees of asset fixity.
4. The Fourier Series proved to be an excellent mathematical representation of a dynamic disequilibrium phenomenon, retaining the inertia of adjustment effects.
5. A production simulation model has been constructed and has successfully estimated the industry behavior during 1964-1970 period, and has been tested during 1971. The sows farrowing variable represents the major state variables set. The hog-corn price ratio with some lag structure is the major input signal for the production system. Decisions made about the volume of actual breeding are fed back to influence further production decisions.
6. For simulating the market allocation scheme, it is necessary to couple it with the production system, and to keep adjusting the inventory of live hogs. Some unknown production parameters have been

detected as the necessary logical rates to fill the "gaps" between sets of the observed or predictable variables. A striking example of this kind is the finding about progressive male pig selection for slaughter as the pig's age increases.

7. The open-loop control policy imposed on the production model to eliminate the low frequency cycle components from the hog-corn price ratio time path, damps the low frequency four and two year cycle components from the barrows and gilts sold for slaughter cycle. The fluctuation of the output can be significantly moderated.

8. The breeding volume shows a small response to the changes in the price ratio, as reflected in the behavior of sows farrowing. But the output is more sensitive to these changes primarily due to the consequent change in the sow's farrowing behavior, which is amplified in determining the output. In other words, what appears to be insignificant response at the state variable level, becomes highly responsive impact at the output level.

Need for Further Research

Further research is needed in basically three areas:

1. Multi-Frequency Cobweb Model. Further investigation in the possibility for the existence of very low frequency cycles with twelve or sixteen year durations.

The specific method in applying the distributed lag model to support this extension of the Cobweb model should be revisited by applying series of different hog-corn price ratio polynomial lags[117] models. A search procedure with high degree polynomials may better verify the proper price ratio lag structure required to confirm the model.

2. The practical policy problem is the construction of the closed-loop policy control and its impact on the industry. This subject can be pursued only after the overall hog-pork subsector study is completed with the output feedbacks and determines the future prices.
3. The policy controls required either for the open-loop or the closed-loop control system should be converted into realistic policy measures feasible to administer and legislate. A benefit-cost analysis is required to accept the best policy under the real world constraints. The simulation models developed here may assist the researcher to pursue these important issues.

[117] Kmenta, Jan. Elements of Econometrics. MacMillan Company, New York, 1971, p. 492-495.

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APPENDICES

APPENDIX 1

GLOSSARY

The following glossary defines some terms usually used in system science, see Distefano, J. J., et. al., Theory and Problems of Feedback and Control Systems.

SYSTEM, an arrangement, set, or collection of things connected or related in such a manner as to form an entirety or whole.

CONTROL SYSTEM, an arrangement of physical components connected or related in such a manner as to command, direct, or regulate itself or another system.

OPEN-LOOP CONTROL SYSTEM, a system in which the control action is independent of the output.

CLOSED-LOOP CONTROL SYSTEM, a system in which the control action is somehow dependent on the output.

FEEDBACK, is the property of a closed-loop system which permits the output (or some other controlled variable of the system) to be compared with the input to the system (or an input to some other internally situated component or subsystem of the system) so that the appropriate control action may be formed as some function of the output and input.

INPUT, the stimulus or excitation applied to a system from an external energy source, usually in order to produce a specified response from the system.

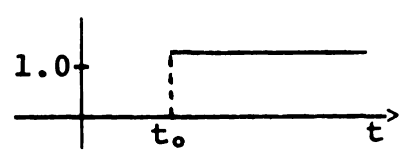
OUTPUT, the actual response obtained from a control system. It may or may not be equal to the specified response implied by the input.

SYSTEM STATE, the minimal set of numbers which specifies the status (State) of the system of a given instant in time. Knowledge of the system state at t_i plus knowledge of the inputs over time t_i to t_{i+1} suffices to completely determine the behavior of the system over this period of time (t_i to t_{i+1}).

SERVOMECHANISM, a power-amplifying feedback control system in which the output controlled variable is mechanical position, or a time derivation of position such as velocity or sales revenues of a commodity.

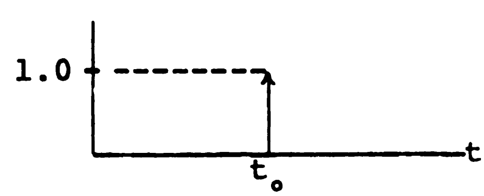
UNIT STEP FUNCTION, is a function of time denoted by $u(t-t_o)$ and defined by

$$u(t-t_o) = \begin{matrix} 1 & t>t_o \\ 0 & t<t_o \end{matrix}$$

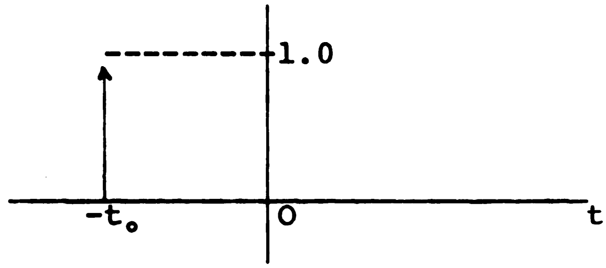


UNIT IMPULSE FUNCTION, denoted by $S(t-t_o)$ and defined by:

$$S(t-t_o) = \frac{du(t-t_o)}{dt}$$



<u>IMPULSE DELAY FUNCTION</u>, denoted by $S(t+t_0)$ and shifts the pulse to units of time to the past.



Appendix 2: The Supply Response Program, Based on the Hog Population Model

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PROGRAM      HOGDIS                                   CDC 6500 FTN V3.0-P336 OPT=1  09/20/73 .10.10.53.      PAGE     1

              PROGRAM HOGDIS(INPUT,OUTPUT)
C
C     ***** H O G   P O P U L A T I O N   M O D E L *****
C             ********   HOVAV  TALPAZ   ********
C
      DIMENSION  SFI(200),SF(200),HF(200),CF(200),TLWI(180),AVLWI(108)
      DIMENSION FES(108),BGES(108),BGENS(108)
      DIMENSION TMHGI(144),BGSI(144), SOWSI(144),X(80)
      DIMENSION  P(6,6)  , IMN(12)  , SFFL(12)
      DIMENSION ALWS(108),FAS(108),FES(108),FCS(108),FDS(108)
      DIMENSION BGAS(108),BGBS(108),BGCS(108),BGANS(108),BGBNS(108)
      DIMENSION  BGCNS(108),BGDNS(108) ,FFS(108) ,BGDS(108)
      DIMENSION  EBGS(108),SBGS(108)  ,  AWD(108,16)
      DIMENSION  HOGIN(4,4) , TSP(4,4),BSKT(5) , BGN(5),IBG(5),INT(5)
      DIMENSION RAT(108,5)
      CALL NOBLANK
C
C     ******   REC.  SYSTEM  SINCE  JAN.  1966    *****
C
      SD = 19.5
      JS = 24
C
      IT = 7 * 12
      W =  2.0 * 3.1415927 / 48.0
      DO  1 I = 1,1728
    1 AWD(I) = 0.0
C
C    *****  READ  INPUT  DATA  ****
C
    5 FORMAT (12F4.2)
      READ 10 ,(SFI(I),I=1,192)
      READ 10 ,( HF(I),I=1,192)
      READ 10 ,( CF(I),I=1,192)
      READ  10, (TMHGI(I),I=1,108)
      READ  10, (BGSI (I),I=1,108)
      READ  10, (SOWSI(I),I=1,108 )
      READ 10,(TLWI(I),I=1,108)
      READ 10, (AVLWI( I),I=1,108)
   10 FORMAT(8X,12F6.0)
C
C    *****  PERCENTAGE  ALLOCATION  INPUT DATA ****
C
      DO 20 J=1,3
      READ 19,(F(J,I),I=1,6)
   19 FORMAT(6F4.2)
   20 CONTINUE
      READ 16,(SFFL(I),I=1,12)
      READ 18,(IMN(I),I=1,12)
   18 FORMAT(12(A2,1X))
   16 FORMAT (12F6.3)
      READ 14,(INT(I),I=1,5)
   14 FORMAT (5(3X,I3))
      READ 17, (IBG(I),I=1,5)
   17 FORMAT (5(I3,3X))
C
```

```
C
C  *******PRINT  INPUT  DATA   *********
C
      PRINT 8
    8 FORMAT(*1*,*        ---  R E P O R T E D   D A T A  ---
     1 1964 - 1971*/)
      PRINT 6
    6 FORMAT (*0*, *   TIME          SF         HP         CP     H-C PRT*,
     1 *  ALL HOGS*,
     2  *  BG SOLD    SOWS SOLD  BOARSLD  AVELIVWT*,
     3  *                      PIGSPRLTR*)
      NR = 0
      DO  9     NYR=1,8
      DO  9     NM=1,12
      NR=NR+1
      NT= IT+ NR
      NL = NR + 12
      AK = 7.34          +NYR*1.0/100. + SPFL(NM)
      HCPR = HP(NT) / CP(NT)
      BGS = BGSI(NL) / 100.    *TMHGI(NL)
      SWS = SOWSI(NL) / 100.    *TMHGI(NL)
      BRS = TMHGI(NL) - BGS -SWS
      IYR = 1963 + NYR
      IF(NM.EQ.1) PRINT 170,IYR
      PRINT 7 ,IPN(NM) ,SFI(NT),HP(NT),CP(NT),HCPR,TMHGI(NL),BGS,SWS,
     1   BRS,AVLWI(NL),AK
    7 FORMAT(*0*,4X,A2,4X,9F10.3,20X,F10.3)
    9 CONTINUE
C
      PRINT 26
   26 FORMAT (*1*/*   MARKET  ALLOCATION SCHEME*//)
      DO 28 J=1,3
   28 PRINT 29,J,(P(J,I),I=1,6)
   29 FORMAT(*0*,10X,I2,6F10.2)
C
      PRINT 30
   30 FORMAT(//////* SEASONALITY  OF  PIGS  PER  LITTER*//)
      PRINT 31,(SPFL(I),I=1,12)
   31 FORMAT(* *,5X,12F8.2)
C
C
C ***** INITIALIZATION *****
C
      WPA= 6000.
      WPB = 4500.
      PIGSA = 4000.
      PIGSB = 6500.
      PIGSH = 6000.
      DO 505 I=1,4
      HOGIN(I,1) =  8000. / 4.
      HOGIN(I,2) =  3500. / 4.
      HOGIN(I,3) =  1200. / 4.
  505 HOGIN(I,4) =   500. / 4.
      PIGSF = 300.
      BOARS = 50.
```

```
      GILTS = 50.
      TFH = 6500.
      TMALES = TFH / 12.
      PRINT 11
   11 FORMAT ("1","     --- S I M U L A T I O N    R E S U L T S ---")
      PRINT  12
   12 FORMAT( // " T SF-NR-  WPA   WFE  PIGSA PIGSB PIGSC PIGSD PIGSE PI
     1GSF GILTS BCARS  TFH   SGS  SMALES TSPC  TSPC  TSPE  TSPF    Y  C
     2S  DY  JB")
      NR = 0
C
C**DYNAMIC   SECTION ***
C
      DO  999   NYR=1,8
      DO  999   NM=1,12
      NR=NR+1
      NT= IT+ NR
      T=NR*W
      AK = 7.34          +NYR*1.0/100. + SPFL(NM)
      DO 72 I=1,12
   72 X(55+I)=0.0
      X(55+NM)=1.0
      DO 640  I=1,26
  640 X(I+17)= HP(NT+ 1 - I)/  CP(NT+1-I)
      DO 680 I=1,12
       X(67 +I) = 0.0
      IF ( X(21   ) .LT. X(I+21)) X(67+I) = 1.0
  680 CONTINUE
C
C**ESTIMATE  SOWS  FARROWING ****
C
      HCPR5 =HP(NT-5 )/CP(NT-5 )
      HCPR21=HP(NT-21)/CP(NT-21)
      VSF6  = SFI(NT-6)
      IF(NR.LT.JS) GO TO 35
      VSF6 = SF(NR-6)
   35 SF(NF) = -231.32 + 0.3973*VSF6 + 12.981*HCPR5 +11.73*HCPR21
     1 +39.75*SIN(2.*T) + 1145.6*SIN(4.*T) -312.98*X(56) -507.8*X(57)
     2 -380.99*X(58)+ 529.45*X(61) + 948.3*X(62) + 1515.78*X(63)
     3 + 1855.26*X(64) + 1096.1*X(65) + 478.7*X(66) + 34.52*X(74)
C
C TRANSFER  POPULATION   AND  COUNT  FOR  DEATH  LOSS  AND  FARM  SLTR.
C
      GILTS = PIGSF * .920
      BCARS = PIGSF * .060
      PIGSF = 0.0
      DO 509  I=1,4
  509 PIGSF = PIGSF + HOGIN(I,4)
      DO 508 I=1,4
      HOGIN(I,4) =  .995 * HOGIN(I,3)
      HOGIN(I,3) =  .995 * HOGIN(I,2)
      HOGIN(I,2) =  .993 * HOGIN(I,1)
      HOGIN(I,1) =  .99  * PIGSH /  4.
  508 CONTINUE
      PIGSH = .99 * PIGSB
```

```
      PIGSB = .987*PIGSA
      PIGSA = .985*WP9
      WP9 = .975* WPA
      WPA = AK  * SF(NR)
      X(55+NM) = 0.0
      NK=NM+4
      IF(NK.GT.12) NK= NK-12
      X(55+NK)=1.0
C
C*** ESTIMATE  TOTAL  FEMALE  BRED  ****
C
      HCPR17=HP(NT-17)/CP(NT-17)
      HCPR1 =HP(NT-1 )/CP(NT-1 )
      VSF2 =        SFI(NT-2)
      IF(NR.LT.JS) GO TO  37
      VSF2 = SF(NR-2)
   37 SFT4 =-227.02+0.4126*VSF2+12.46*HCPR1+11.99*HCPR17
     1+40.8*SIN (2.*(NR+4)*W)+1163.4*SIN(4.*(NR+4)*W)-324.4*X(56)
     2-529.2*X(57)-406.2*X(58)+ 550.6*X(61)+980.02*X(62)
     3 +1534.02*X(63) +1859.2*X(64) +1098.01*X(65) +483.07*X(66)
      TFBD= 1.2987 * SFT4
      X(55+NK)=0.0
      X(55+NM)=1.0
C
C*** ESTIMATE  SOWS SOLD ****
C
      HCPR2=HP(NT-2)/CP(NT-2)
      VSF3 = SFI(NT-3)
      IF(NR.LT.JS) GO TO  38
      VSF3= SF(NR-3)
   38 SGS=620.2 +.05896*VSF3 - 10.367*HCPR2 - 75.82*X(56)-145.8*X(57)
     1 -127.8*X(58) - 77.2*X(59) + 59.3*X(62) + 92.3*X(63)
      TFH=  TFH+ GILTS
      IF((TFH-SGS).LT.TFBD) GO TO 40
      TFH = TFH - SGS  * 1.085
      GO TO 45
   40 CONTINUE
      SGS = TFH- TFBD
      IF( SGS) 42,44,44
   42 SGS =0.0
   44 TFH = TFH-SGS  * 1.085
   45 CONTINUE
C
C  MARKETING   OF  HOGS
C
   70 DMALES = TFH / 12.
      TMALES = TMALES + BOARS
      SMALES =  TMALES - DMALES
      IF(SMALES .LT.0.0) SMALES = 0.0
      TMALES = TMALES - SMALES  * 1.10
      TPIGS = PIGSA + PIGSB + PIGSC + PIGSC + PIGSE  + PIGSF
C
C*** ESTIMATE  TARGETED  BARROWS AND GILTS SOLD ****
C
      HCPR12=HP(NT-12)/CP(NT-12)
```

```
      VSF6 =SFI(NT-6)
      VSF8 = SFI(NT-8)
      VSF10= SFI(NT-10)
      IF(NR.LT.JS) GO TO 39
      VSF6 =SF(NR-6)
      VSF8  = SF(NR -8 )
      VSF10 = SF(NR -10)
   39 EGS=-2507.03 + 1.4099*VSF6 + 3.246*VSF8 +2.017*VSF10+43.73*HCPR12
     1+1896.3* SIN(4.*T) -1551.4*X(57) -437.1*X(60)+1203.4*X(61)
     2 +2242.9*X(62) +3915.05*X(63) +4388.9*X(64) +3530.5*X(65)
     3 + 1460.2*X(66)
C
C
C **** ESTIMATE  AVE.  LIVEWIGHT  ****
C
      VSF7 = SFI(NT-7)
      VSF9 = SFI(NT-9)
      VSF10= SFI(NT-10)
      IF(NR.LT.JS) GO TO 52
      VSF7 = SF(NR-7)
      VSF9 = SF(NR-9)
      VSF10= SF(NR-10)
   52 AVLW = 217.36475 + .0056209*VSF7 + .C1013765*VSF9 + .0048398*VSF10
     1 +.79961*HP(NT-2) - 1.16594*HP(NT-8) + .33968*HP(NT-25)
     2 -4.8118*COS(4.*T) -5.7801*SIN(T) + 2.1706*SIN(2.*T)-5.6542*X(61)
     3 -9.9588*X(62) - 8.55498*X(63) - 4.62288*X(64) - .9499*COS(2.*T)
     4 + .5942*COS(5.*T) + .3223*COS(6.*T)
C
C
C ****** TO  CALCULATE  THE  AREA  UNDER  THE  NORMAL  DISTRIBUTION  CURVE  **
C
      XBAR = AVLW
      Z18=  (180. - XBAR ) / SD
      Z20=  (200. - XBAR ) /SD
      Z22=  (220. - XBAR )/ SD
      Z24=  (240. - XBAR )/  SD
      Z27=  (270. - XBAR) /  SD
      Z30 = (300. - XBAR ) / SD
      CALL  NOTR (Z18,P18,T18)
      CALL  NOTR (Z20,P20,T20)
      CALL  NOTR (Z22,P22,T22)
      CALL  NOTR (Z24,P24,T24)
      CALL  NOTR (Z27,P27,T27)
      CALL  NOTR (Z30,P30,T30)
      FA = P20
      FB = P22 - P20
      FC = P24 - P22
      FD = P27 - P24
      FE = 1.0 - P27
      FA = 0.03554 + 0.571898*FA - 0.0136* X(56) - 0.018*X(58)-0.025*X(
     1 59) - 0.0144*X(60) -0.0146*X(64) -0.0306*X(65)-0.019*X(66)
      FB = 0.1532 + 0.56875*FB -0.0328*X(59)-0.0362*X(60) +0.0548*X(62)
     1 + 0.04457*X(63) - 0.027*X(66)
      FC = 0.1031 + 0.38682*FD + 0.05386*X(59) +0.0237*X(60)-0.0526*X(62
     1 ) -0.0414*X(63) - 0.04*X(64) + 0.0298*X(65) + 0.02*X(66)
```

```
      FE = 0.022893 + 0.8745*FE -0.0138*X(58)+0.0186*X(59)+0.016*X(61)
     1 - 0.0211*X(62) -0.0121*X(65)
      FC = 1.0 - ( FA + FB + FD + FE )
C
C  ***  CORRECT  THE  NORMAL  DISTRIBUTION  ****
C
   50 CONTINUE
C
C
C  *****    COMPUTE  NUNBER  OF  B+G  FOR  EACH  WEIGHT  GROUP     ******
C
      BGAN = FA * BGS
      BGBN = FB * BGS
      BGCN = FC * BGS
      BGDN = FD * BGS
      BGEN = FE * BGS
C
C
C   *****    COMPUTE  LIVEWEIGHT  FOR   EACH  WEIGHT   GROUP    *****
C
      BGA = BGAN * .192
      BGB = BGBN * .211
      BGC = BGCN * .230
      BGD = BGDN *.255
      BGE = BGEN *.280
C
C   ********SEARCH  MARKETING  LOOP  ******
C
      LOOP = 0
      YB = 0.0
      BGN(1) =  BGAN
      BGN(2) =  BGBN
      BGN(3) =  BGCN
      BGN(4) =  BGDN
      BGN(5) =  BGEN
      TSPA = TSPB = TSPC = TSPD = TSPE =   TSPF =  0.0
      BA = BB = BC = BD = BE = BF =  1.0
      DO 507  I=1,16
      TSP (I) = 0.0
  507 CONTINUE
   77 CONTINUE
      DO 560 JJ = 1,5
      J = 6 - JJ
      BSKT(J) =0.0
      LOOP = 0
      CT = BGN(J) / 3.0
      KEND = INT(J) + IBG(J) - 1
      KBG  = IBG(J)
  510 CONTINUE
      SM = 0.0
C
      DO 504 I=KBG,KEND
  504 SM = SM + HOGIN(I)
      IF(SM .LT. QT    ) QT = SM
C
```

```
      CC 550 I=KBG,KEND
      IF(SP .LE. 0.0) SM= .01
      SP = QT * ( HOGIN(I) / SM )
      IF( SP .GE. HOGIN(I)) SP = .99 * HOGIN(I)
      HCGIN(I) = HCGIN(I) - SP
      BSKT(J) = BSKT(J) + SP
      TSF(I)= TSF(I) +SP
      AWD(NR,I) = AWC(NR,I)  + SP
  550 CCNTINUE
C
      KEND = KENC + 1
      KBG = KBG - 1
      IF(    KBG .LT .  1)  KBG = 1
      IF(    KENC .GT . 16)  KEND = 16
      LCCP = LOOF + 1
      IF(LCCF .LT.3 ) GO TO 510
  559 RAT (NR,J) = BSKT(J) / BGN(J)
      IF(RAT(NR,J) . GT .  0.995)  GC TC 560
      QT = BGN(J) - BSKT(J)
      KEND = KENC +3
      KBG = KBG - 3
      IF(    KBG .LT .  1)  KBG = 1
      IF(    KENC .GT . 16)  KEND = 16
      IF ( LCOP .LT. 4 ) GO TO 510
C
  560 CCNTINUE
      YB=0.0
      CC 565 I=1,5
  565 YB =YB + BSKT(I)
      DC 570  I=1,4
      TSFC =  TSFC  + TSP(I,1)
      TSFD =  TSFC  + TSP(I,2)
      TSFE =  TSFE  + TSP(I,3)
      TSPF =  TSFF  + TSP(I,4)
  570 CCNTINUE
      PIGSC=FIGSC = FIGSE = FIGSF = C.0
      DC 580  I= 1,4
      FIGSC  = FIGSC  +  HOGIN(I,1)
      FIGSD  = FIGSD  +  HOGIN(I,2)
      FIGSE  = FIGSE  +  HOGIN(I,3)
  580 PIGSF  = FIGSF  +  HOGIN(I,4)
      Y = YB    + SGS   + SMALES
C
C*** CCMPUTE  DEVIATICN  FROM  REAL  WORLD  CATA ****
C
      CS= SF(NR)-SFI(NT)
      CS = CS* 100. /SFI(NT)
      CY = Y -TMHGI (NR+12)
      CY = DY * 100. / TMHGI(NR+12)
      BG = BGSI(NR+12)  * TMHGI(NR+12)  /  100.
      CB= YB - EG
      CB = DB *100. / BG
C
C  PRINT  CUTPUT
C
```

```
      IYR = 1963 + NYR
      IF(NM.EQ.1) PRINT 170,IYR
  170 FORMAT (*0--*,I4,*--*)
      PRINT 80,IMN(NM) ,SF(NR),WPA,WFB,PIGSA,PIGSB,PIGSC,PIGSD,PIGSE,
     1PIGSF,GILTS,ECARS,TFM,SGS,SMALES,TSPC,TSPD,TSPE,TSPF,Y,DS,DY,DB
   80 FORMAT (*0*,A2,   19F6.0,3F4.0)
C
C  ******  STOR  PORK  VARIABLES  FOR  PRINTING  AT  THE  END   ******
C
      FF =  YB /(BGAN + BGBN + BGCN + BGDN + BGEN)
      FFS(NR)=FF
      ALWS(NR)= AVLW
C
C  ***   COMPUTE  PERCENT  WIGHT  GROUP  OUT  OF  ALL  HOGS  SOLD    ****
C
      FAS(NR) = BGAN * FF * 100. /YB
      FBS(NR) = BGBN * FF * 100. /YB
      FCS(NR) = BGCN * FF * 100. /YB
      FDS(NR) = BGDN * FF * 100. /YB
      FES(NR) = BGEN * FF * 100. /YB
      BGAS(NR) =  BGA * FF
      BGBS(NR) =  BGB * FF
      BGCS(NR) =  BGC * FF
      BGDS(NR) =  BGD * FF
      BGES(NR) =  BGE * FF
      BGANS(NR) = BGAN * FF
      BGBNS(NR) = BGBN * FF
      BGCNS(NR) = BGCN * FF
      BGDNS(NR) = BGDN * FF
      BGENS(NR) = BGEN * FF
      BBGS(NR)= BGS
      SBGS(NR)= YB
C
C ***** AGE -- WEIGHT  DISTRIBUTION   ******
C
  999 CONTINUE
C
C  *****  PRINT  PORK  DATA   *****
C
      PRINT  179
  179 FORMAT (*1*,*                        -- P O R K   R E S U L T S --*//
     2* TIME RAT.  AVLW  F18-20 F20-22 F22-24 F24-27 F27+UP*,
     3*  P18-20  P20-22  P22-24  P24-27  P27+UP  N18-20   N20-22*,
     4*   N22-24   N24-27 N27+UP*)
      NR = 0
      DO 9999 NYR=1,8
      DO 9999 NM =1,12
      NR=NR+1
      IYR =1963 + NYR
      IF(NM.EQ.1) PRINT 170,IYR
      PRINT 180,IMN(NM),FFS(NR),ALWS(NR),FAS(NR),FBS(NR),FCS(NR),FDS(NR)
     1,FES(NR),(RAT(NR,I),I=1,5), BGANS(NR),
     2 BGBNS(NR), BGCNS(NR),BGDNS(NR), BGENS(NR)
  180 FORMAT(*0*,A2,F6.2,F8.2,5F7.2,6F8.2,3F9.2,F7.1 )
 9999 CONTINUE
```

```
C
      PRINT  340
  340 FORMAT(*1*,*                  A G E ---W E I G H T   D I S T R I B U
     1T I O N    O F   B+G*//
     2 *          C1      C2       C3*,
     3          *         C4         D1       D2       C3       D4          E1       E2*,
     4   *      E3      E4          F1       F2       F3       F4*/)
      NR=0
      DO 777 NYR=1,9
      DO 777 NM =1,12
      NR=NR+1
      IYR = 1963 + NYR
      IF(NM.EQ.1) PRINT 170,IYR
      PRINT 345,IMN(NM), (AWD(NR,I),I=1,16)
  345 FORMAT (*0*,A2,4(3X,4F7.0))
  777 CONTINUE
C *****     PRINT  B+G  PERFORMANCE   *******
C
      PRINT 190
  190 FORMAT (*1*,*                  ------  B + G    P E R F O R M A N C
     1E  ---------*//
     2   * TIME        REP. BGS  EST. BGS  SIM. BGS  DIF E-R  PERCDIFE-R
     3*, *  DIF S-R PERCDIFS-R  DIF S-E PERCDIFS-E*)
      NR=0
      DO 888 NYR=1,8
      DO 888 NM =1,12
      NR=NR+1
      IYR = 1963  + NYR
      IF(NM.EQ.1) PRINT 170,IYR
      PBG = BGSI(NR+12) * TMHGI(NR+12) / 100.
      DIFER =EBGS(NR)  - PBG
      PERER = DIFER * 100. / PBG
      DIFSR = SBGS(NR) - PBG
      PERSR = DIFSR * 100. / PBG
      DIFSE = SBGS(NR) - EBGS(NR)
      PERSE = DIFSE * 100. / EBGS(NR)
      PRINT 195,IMN(NM),PBG,EBGS(NR),SBGS(NR),DIFER,PERER,DIFSR,PERSR,
     1DIFSE, PERSE
  195 FORMAT(*0*,A2,8X,9F10.2)
  888 CONTINUE
      END
MORE MEMORY WOULD HAVE RESULTED IN BETTER OPTIMIZATION
```

```
      SUBROUTINE  NOTR(X,P,D)
C
C ****** TO  CALCULATE  THE  AREA  UNDER  THE  NORMAL  DISTRIBUTION  CURVE
C  ****    FOR   F(X) LESS THEN  X    ****
C
      AX = ABS(X)
      T= 1./(1.0+ .2316419 *AX)
      D=0.3989423* EXP(-X*X/2.0)
      F= 1.0 - D*T*((((1.330274*T - 1.821256)*T+1.781478)*T  -
     1 0.3565638)*T + 0.3193815)
      IF(X)1,2,2
    1 F=1.0 - F
    2 RETURN
      END
```

Appendix 3. Barrows and Gilts Weight Groups Percentage, 7 Market Survey*

		180 lbs. -200 lbs.	201 lbs. -220 lbs.	221 lbs. -240 lbs.	241 lbs. -270 lbs.	271 lbs. and up
Jan 70	1	2.39	23.51	35.03	28.06	8.31
	2	5.10	25.66	34.66	27.14	5.90
	3	3.22	25.71	39.33	25.77	4.87
	4	2.61	16.86	32.71	35.11	10.47
	5	2.94	17.08	30.86	33.83	11.51
	6	4.62	20.95	33.04	25.31	12.44
	7	5.55	33.40	40.71	15.63	3.35
	8	8.59	36.85	37.80	14.90	1.76
	9	5.57	28.85	43.43	17.61	3.79
	10	3.66	23.97	39.26	27.40	5.19
	11	3.58	22.48	38.90	27.13	6.22
	12	5.16	23.23	35.45	25.47	8.02
Jan 71	13	4.97	24.78	32.27	28.42	6.96
	14	6.42	27.54	38.57	21.04	4.97
	15	4.95	27.24	42.96	20.25	3.31
	16	2.19	23.08	33.65	29.40	8.65
	17	2.83	17.99	39.04	28.80	8.49
	18	4.98	21.90	34.28	28.17	7.52
	19	6.88	30.47	36.88	21.29	2.68
	20	6.82	32.21	38.99	17.66	3.77
	21	6.08	31.86	42.03	16.32	3.25
	22	1.97	27.33	41.61	26.01	2.64
	23	2.49	19.46	40.15	29.78	6.89
	24	4.62	26.14	35.67	25.01	7.01
Jan 72	25	4.18	25.43	35.33	26.10	6.93
	26	4.66	24.99	40.99	23.74	4.51
	27	3.33	22.06	40.63	27.14	5.48
	28	2.51	22.73	35.22	30.53	7.05
	29	2.56	19.02	38.87	29.46	7.49
	30	3.04	23.55	35.17	27.20	8.02

*Unpublished U.S.D.A.--Purdue University Survey Conducted at Indianapolis, Kansas City, Omaha, National Stock Yards, Sioux City, South St. Joseph and South St. Paul markets.

Appendix 4--- REPORTED DATA --- 1964 - 1971

TIME	SF	HP	CP	H-C PRT	ALL HOGS	BG SOLD	SOWS SOLD	BOARSLD	AVELIVWT	PIGSPRLTR
--1964--										
JA	785.000	14.700	1.200	12.250	8006.400	7421.933	520.416	64.051	238.000	7.305
FB	1045.000	14.700	1.190	12.353	6828.700	6425.807	361.921	40.972	234.000	7.320
MR	1557.000	14.480	1.220	11.869	7409.500	7001.977	355.656	51.867	234.000	7.330
AP	1605.000	14.160	1.240	11.419	7441.500	6920.595	461.373	59.532	237.000	7.390
MY	1068.000	14.840	1.250	11.872	6356.200	5809.567	508.496	38.137	239.000	7.370
JN	959.000	15.830	1.230	12.870	5932.800	5220.864	652.608	59.328	237.000	7.350
JL	861.000	17.110	1.200	14.258	5798.000	4963.088	765.336	69.576	232.000	7.340
AG	1083.000	17.050	1.220	13.975	5707.900	5000.120	662.116	45.663	226.000	7.330
SP	1379.000	16.760	1.220	13.738	6563.000	6005.145	518.477	39.378	228.000	7.320
OC	811.000	15.390	1.230	12.512	7797.200	7204.613	545.804	46.783	235.000	7.310
NV	432.000	14.430	1.230	11.732	7486.200	6842.387	606.382	37.431	236.000	7.305
DC	517.000	15.550	1.270	12.244	7690.800	6967.865	661.409	61.526	234.000	7.300
--1965--										
JA	708.000	16.060	1.290	12.450	6995.700	6512.997	440.729	41.974	237.000	7.315
FB	953.000	17.010	1.280	13.289	6161.900	5816.834	314.257	30.810	231.000	7.330
MR	1404.000	16.980	1.300	13.062	7526.100	7089.586	391.357	45.157	230.000	7.340
AP	1395.000	17.630	1.320	13.356	6690.800	6182.299	461.665	46.836	233.000	7.390
MY	913.000	20.290	1.300	15.608	5513.800	5017.558	463.159	33.083	234.000	7.380
JN	855.000	23.380	1.290	18.124	5479.000	4832.478	602.690	43.832	232.000	7.360
JL	749.000	24.270	1.280	18.961	5142.000	4586.664	529.626	25.710	229.000	7.350
AG	944.000	24.670	1.230	20.057	5529.100	4970.661	519.735	38.704	225.000	7.340
SP	1251.000	22.920	1.230	18.634	6340.800	5865.240	437.515	38.045	226.000	7.330
OC	768.000	23.360	1.190	19.630	6255.400	5805.011	412.856	37.532	232.000	7.320
NV	439.000	24.330	1.190	20.445	6334.500	5815.071	487.756	31.673	236.000	7.315
DC	526.000	28.070	1.260	22.278	5814.100	5343.158	430.243	40.699	236.000	7.310
--1966--										
JA	732.000	27.930	1.290	21.651	5533.000	5173.355	331.980	27.665	241.000	7.325

FB	963.000	27.800	1.290	21.550	5408.400	5110.938	275.828	21.634	237.000	7.340
MR	1448.000	24.410	1.210	20.174	6717.300	6334.414	349.300	33.587	237.000	7.350
AP	1531.000	22.260	1.250	17.808	6138.700	5739.684	374.461	24.555	241.000	7.400
MY	1001.000	23.160	1.240	18.677	5719.700	5187.768	509.053	22.879	243.000	7.390
JN	977.000	24.720	1.260	19.619	5481.000	4806.837	646.758	27.405	239.000	7.370
JL	951.000	25.090	1.330	18.865	4943.800	4271.443	642.694	29.663	228.000	7.360
AG	1081.000	25.750	1.380	18.659	5943.300	5224.161	695.366	23.773	224.000	7.350
SP	1423.000	23.160	1.350	17.156	6750.700	6163.389	540.056	47.255	227.000	7.340
OC	887.000	21.570	1.350	15.978	6944.300	6430.422	479.157	34.722	233.000	7.330
NV	492.000	19.870	1.340	14.828	7175.300	6622.802	530.972	21.526	239.000	7.325
DC	606.000	19.670	1.360	14.463	7255.100	6732.733	500.602	21.765	241.000	7.320
--1967--										
JA	793.000	19.460	1.330	14.632	7304.300	6822.216	445.562	36.522	243.000	7.335
FB	1051.000	19.380	1.310	14.794	6581.600	6193.286	361.988	26.326	236.000	7.350
MR	1517.000	18.430	1.370	13.453	7688.500	7250.255	407.490	30.754	235.000	7.360
AP	1545.000	17.620	1.270	13.874	6768.000	6328.080	412.848	27.072	238.000	7.410
MY	1058.000	21.830	1.300	16.792	6205.400	5640.709	533.664	31.027	240.000	7.400
JN	971.000	22.290	1.310	17.015	6010.200	5355.088	619.051	36.061	237.000	7.380
JL	907.000	22.580	1.280	17.641	5535.500	4948.737	553.550	33.213	234.000	7.370
AG	1091.000	21.040	1.180	17.831	6732.100	6072.354	626.085	33.661	229.000	7.360
SP	1459.000	19.460	1.160	16.776	7008.600	6447.912	525.645	35.843	231.000	7.350
OC	908.000	18.160	1.140	15.930	7675.800	7176.873	468.224	30.703	235.000	7.340
NV	558.000	17.360	1.110	15.640	7481.400	6935.258	523.698	22.444	240.000	7.335
DC	527.000	17.290	1.140	15.167	7132.200	6611.549	499.254	21.397	240.000	7.330
--1968--										
JA	815.000	18.310	1.150	15.922	7567.300	7060.291	469.173	37.836	239.000	7.345
FB	1215.000	19.410	1.150	16.879	6633.100	6241.747	358.187	33.166	235.000	7.360
MR	1584.000	19.070	1.170	16.299	7129.600	6765.990	327.962	35.648	235.000	7.370
AP	1557.000	19.000	1.190	15.966	7367.100	6888.238	442.026	36.836	238.000	7.420
MY	971.000	18.880	1.220	15.475	7263.900	6740.899	493.945	29.056	240.000	7.410
JN	963.000	20.430	1.190	17.168	5871.600	5354.899	487.343	29.358	238.000	7.390

JL	932.000	21.480	1.160	18.517	6209.800	5607.449	558.882	43.469	231.000	7.380
AG	1257.000	20.080	1.100	18.255	6924.400	6377.372	512.406	34.622	227.000	7.370
SP	1564.000	19.930	1.170	17.034	7123.100	6645.852	448.755	28.492	228.000	7.360
OC	919.000	18.290	1.180	15.500	8300.100	7818.694	448.205	33.200	234.000	7.350
NV	494.000	17.920	1.180	15.186	7422.800	6851.244	534.442	37.114	237.000	7.345
DC	530.000	18.760	1.170	16.034	7546.900	6950.695	543.377	52.828	235.000	7.340
--1969--										
JA	819.000	19.770	1.200	16.475	7704.100	7187.925	454.542	61.633	233.000	7.355
FB	1221.000	20.410	1.190	17.151	7004.300	6584.042	364.224	56.034	229.000	7.370
MR	1460.000	20.690	1.210	17.099	7525.600	7134.269	346.178	45.154	230.000	7.380
AP	1435.000	20.380	1.250	16.304	7550.800	7105.303	385.091	60.406	234.000	7.430
MY	895.000	23.140	1.290	17.938	6683.700	6215.841	414.389	53.470	237.000	7.420
JN	893.000	25.160	1.260	19.968	6184.700	5628.077	494.776	61.847	238.000	7.400
JL	865.000	26.050	1.230	21.179	6354.500	5744.468	540.132	69.900	232.000	7.390
AG	1166.000	26.910	1.220	22.057	6284.000	5737.292	509.004	37.704	226.000	7.388
SP	1472.000	25.940	1.170	22.171	7229.100	6636.314	527.724	65.062	229.000	7.370
OC	866.000	25.530	1.170	21.821	7772.200	7173.741	544.054	54.405	234.000	7.360
NV	465.000	25.770	1.170	22.026	6462.000	5977.350	432.954	51.696	239.000	7.355
DC	567.000	26.930	1.160	23.216	7083.500	6552.237	481.678	49.585	241.000	7.350
--1970--										
JA	876.000	27.400	1.200	22.833	6832.400	6442.953	341.620	47.827	242.000	7.365
FB	1307.000	28.230	1.200	23.525	6084.400	5749.758	285.967	48.675	236.000	7.380
MR	1703.000	25.940	1.190	21.798	7032.500	6701.972	288.332	42.195	238.000	7.390
AP	1674.000	24.020	1.220	19.689	7296.200	6902.205	350.218	43.777	243.000	7.440
MY	1044.000	23.530	1.240	18.976	6420.100	6009.214	365.946	44.941	246.000	7.430
JN	1067.000	24.040	1.260	19.079	6261.600	5741.887	469.620	50.093	243.000	7.410
JL	1033.000	25.130	1.280	19.633	6359.800	5711.100	604.181	44.519	234.000	7.400
AG	1393.000	22.120	1.360	16.265	6616.800	5981.587	588.895	46.318	227.000	7.390
SP	1792.000	20.350	1.400	14.536	7641.900	7038.190	550.217	53.493	230.000	7.380
OC	1054.000	17.910	1.340	13.366	8352.500	7759.472	542.912	50.115	234.000	7.370

NV	566.000	15.690	1.340	11.709	8093.800	7494.859	550.378	48.563	238.000	7.365
DC	680.000	15.670	1.430	10.958	8824.800	8207.064	555.962	61.774	237.000	7.360
--1971--										
JA	955.000	16.250	1.470	11.054	8250.600	7730.812	462.034	57.754	235.000	7.375
FB	1425.000	19.430	1.460	13.308	7017.500	6596.450	385.962	35.088	231.000	7.390
MR	1641.000	17.130	1.440	11.896	8988.300	8484.955	422.450	80.895	231.000	7.400
AP	1612.000	16.190	1.430	11.322	8457.600	7983.974	397.507	76.118	236.000	7.450
MY	1006.000	17.430	1.450	12.021	7548.600	7020.198	460.465	67.937	241.000	7.440
JN	978.000	18.380	1.480	12.419	7603.200	6934.118	593.050	76.032	239.000	7.420
JL	947.000	19.940	1.410	14.071	6803.700	6273.011	455.848	74.841	234.000	7.410
AG	1276.000	19.050	1.240	15.363	7512.800	6866.699	570.973	75.128	230.000	7.400
SP	1627.000	18.910	1.150	16.443	7991.200	7383.869	543.402	63.930	229.000	7.390
OC	956.000	19.800	1.140	17.368	7780.200	7251.146	466.812	62.242	234.000	7.380
NV	514.000	19.390	1.150	16.861	8217.000	7617.159	525.888	73.953	239.000	7.375
DC	550.000	20.980	1.240	16.919	8267.200	7630.626	570.437	66.138	238.000	7.370

Appendix 5:

--- S I M U L A T I O N   R E S U L T S ---

T	SF-NR-	WPA	WFB	PIGSA	PIGSB	PIGSC	PIGSD	PIGSE	PIGSF	GILTS	BOARS	TFH	SGS	SMALES	TSPC	TSPD	TSPE	TSPF	Y	DS	DY	DB
--1964--																						
JA	767.	5605.	5850.	4432.	3948.	5547.	4052.	958.	805.	276.	24.	6267.	469.	43.	393.	3892.	2525.	389.	7711.	-2.	-4.	-3.
FB	1089.	7972.	5465.	5762.	4375.	5742.	2509.	1311.	741.	740.	64.	6598.	378.	33.	628.	2999.	2721.	212.	6971.	4.	2.	2.
MR	1535.	11248.	7773.	5383.	5687.	3250.	1531.	324.	1014.	682.	59.	6849.	397.	35.	620.	4171.	2173.	290.	7686.	-1.	4.	4.
AP	1550.	11440.	10967.	7656.	5313.	2277.	212.	2.	33.	933.	81.	7281.	461.	42.	2011.	3015.	1521.	289.	7339.	-3.	-1.	-1.
MY	956.	7047.	11154.	10802.	7557.	2089.	2.	0.	0.	31.	3.	6705.	559.	46.	3485.	2259.	211.	2.	6563.	-10.	3.	3.
JN	947.	6960.	6871.	10987.	10662.	1819.	0.	0.	0.	0.	0.	6061.	594.	49.	3389.	2074.	2.	0.	6107.	-1.	3.	5.
JL	864.	6345.	6786.	6768.	10844.	4041.	126.	0.	0.	0.	0.	5355.	651.	54.	3365.	1680.	0.	0.	5751.	0.	-1.	2.
AG	1110.	8134.	6186.	6684.	6680.	7918.	1236.	0.	0.	0.	0.	4658.	642.	53.	2532.	2777.	125.	0.	6128.	2.	7.	9.
SP	1440.	10538.	7931.	6094.	6597.	9789.	3558.	154.	0.	0.	0.	4084.	529.	43.	839.	4305.	1075.	0.	6790.	4.	3.	4.
OC	840.	6140.	10274.	7812.	6014.	6173.	5739.	1182.	82.	0.	0.	3514.	526.	43.	374.	3982.	2358.	72.	7355.	4.	-6.	-6.
NV	425.	3108.	5986.	10120.	7711.	5988.	3360.	2506.	933.	76.	7.	3002.	542.	45.	478.	2769.	3204.	243.	7280.	-2.	-3.	-2.
DC	476.	3474.	3030.	5896.	9988.	5292.	2634.	1021.	2046.	858.	75.	3240.	572.	50.	603.	3313.	2322.	447.	7307.	-8.	-5.	-4.
--1965--																						
JA	682.	4991.	3387.	2985.	5820.	6790.	1803.	349.	709.	1883.	164.	4612.	471.	44.	767.	3452.	2271.	308.	7313.	-4.	5.	4.
FB	994.	7282.	4866.	3336.	2946.	9025.	3131.	314.	203.	652.	57.	4859.	373.	32.	765.	3611.	1481.	145.	6406.	4.	4.	3.
MR	1379.	10119.	7100.	4793.	3293.	5234.	4746.	1079.	232.	187.	16.	4618.	394.	33.	470.	4215.	2036.	80.	7228.	-2.	-4.	-4.
AP	1408.	10408.	9866.	6994.	4731.	2585.	2492.	1766.	811.	213.	19.	4347.	447.	38.	302.	2706.	2956.	264.	6713.	1.	0.	1.
MY	834.	6156.	10143.	9718.	6903.	2593.	443.	261.	1389.	746.	65.	4505.	541.	48.	634.	2124.	2218.	368.	5934.	-9.	8.	7.
JN	809.	5954.	6002.	9996.	9591.	2753.	329.	0.	25.	1278.	111.	5171.	565.	51.	1584.	2246.	441.	235.	5420.	-5.	-1.	-1.
JL	747.	5487.	5806.	5912.	9866.	4681.	551.	1.	0.	23.	2.	4543.	600.	49.	2085.	2183.	325.	0.	5242.	-0.	2.	0.
AG	952.	6991.	5350.	5718.	5836.	8234.	1327.	3.	0.	0.	0.	3915.	578.	47.	1167.	3321.	545.	1.	5660.	1.	2.	1.
SP	1321.	9686.	6816.	5270.	5644.	8999.	4159.	275.	0.	0.	0.	3401.	474.	38.	671.	4017.	1045.	3.	6247.	6.	-1.	-2.
OC	753.	5514.	9444.	6714.	5201.	5408.	5566.	1772.	190.	0.	0.	2906.	456.	37.	311.	3370.	2367.	84.	6625.	-2.	6.	6.
NV	407.	2979.	5376.	9303.	6626.	5145.	3081.	2695.	1492.	175.	15.	2557.	483.	41.	387.	2289.	2844.	271.	6314.	-7.	-0.	-0.
DC	475.	3476.	2904.	5296.	9182.	4618.	2378.	991.	2222.	1372.	119.	3392.	495.	46.	480.	2731.	2075.	459.	6285.	-10.	8.	8.
--1966--																						
JA	730.	5345.	3389.	2861.	5227.	5962.	1869.	399.	677.	2045.	178.	5028.	377.	37.	532.	2717.	1968.	310.	5940.	-0.	7.	7.

FB	1032.	7578.	5212.	3338.	2823.	8459.	3097.	411.	233.	622.	54.	5360.	267.	23.	540.	2824.	1448.	163.	5265.	7.	-3.	-3.
MR	1492.	10968.	7388.	5133.	3294.	4820.	4907.	1077.	280.	215.	19.	5254.	296.	25.	303.	3493.	2005.	130.	6251.	3.	-7.	-6.
AP	1527.	11301.	10694.	7278.	5067.	2580.	2725.	1946.	753.	257.	22.	5118.	363.	31.	187.	2062.	2937.	319.	5899.	-0.	-4.	-4.
MY	995.	7351.	11019.	10533.	7183.	2826.	740.	399.	1417.	692.	60.	5298.	472.	42.	403.	1821.	2312.	519.	5570.	-1.	-3.	-3.
JN	969.	7141.	7168.	10853.	10396.	3687.	601.	3.	88.	1304.	113.	6034.	524.	48.	1279.	2205.	734.	308.	5097.	-1.	-7.	-6.
JL	934.	6871.	6963.	7060.	10712.	5902.	766.	2.	0.	81.	7.	5490.	576.	48.	1138.	2896.	596.	3.	5256.	-2.	6.	8.
AG	1171.	8606.	6699.	6858.	6968.	9164.	2392.	96.	0.	0.	0.	4874.	568.	47.	1026.	3468.	665.	2.	5776.	8.	-3.	-1.
SP	1509.	11073.	8391.	6598.	6769.	9799.	5252.	876.	54.	0.	0.	4352.	482.	39.	700.	3847.	1505.	42.	6615.	6.	-2.	-1.
OC	922.	6760.	10796.	8265.	6513.	6459.	6161.	2521.	713.	49.	4.	3878.	482.	40.	370.	3569.	2705.	158.	7325.	4.	5.	6.
NV	525.	3847.	6591.	10634.	8158.	6225.	3795.	2921.	2129.	656.	57.	3979.	511.	45.	409.	2619.	3209.	380.	7173.	7.	-0.	-0.
DC	586.	4291.	3751.	6492.	10496.	5848.	3016.	1197.	2312.	1958.	170.	5348.	544.	52.	535.	3165.	2579.	594.	7468.	-3.	3.	2.
--1967--																						
JA	823.	6033.	4184.	3694.	6408.	7378.	2424.	488.	762.	2127.	185.	6993.	445.	43.	618.	3383.	2513.	429.	7431.	4.	2.	2.
FB	1139.	8369.	5882.	4121.	3646.	9607.	3695.	522.	285.	701.	61.	7308.	355.	30.	680.	3631.	1891.	201.	6789.	8.	3.	3.
MR	1559.	11475.	8160.	5794.	4068.	5851.	5192.	1177.	361.	262.	23.	7163.	375.	32.	429.	4348.	2500.	158.	7842.	3.	2.	3.
AP	1542.	11428.	11188.	8037.	5719.	3271.	2922.	1739.	821.	332.	29.	7019.	438.	38.	303.	2889.	3428.	350.	7445.	-0.	10.	10.
MY	969.	7173.	11142.	11020.	7933.	3388.	694.	233.	1214.	755.	66.	7180.	548.	49.	598.	2553.	2674.	515.	6937.	-8.	12.	12.
JN	934.	6895.	6993.	10975.	10877.	3587.	568.	0.	1.	1117.	97.	7681.	568.	51.	2018.	2797.	691.	231.	6355.	-4.	6.	7.
JL	889.	6555.	6723.	6888.	10832.	5621.	757.	3.	0.	1.	0.	7035.	596.	49.	2154.	2804.	562.	0.	6165.	-2.	11.	12.
AG	1125.	8278.	6391.	6622.	6799.	9470.	1574.	4.	0.	0.	0.	6392.	593.	49.	1190.	4008.	749.	3.	6592.	3.	-2.	-2.
SP	1511.	11106.	8071.	6295.	6536.	9928.	4719.	282.	0.	0.	0.	5857.	492.	40.	689.	4686.	1284.	4.	7195.	4.	3.	3.
OC	937.	6881.	10829.	7950.	6213.	6306.	5981.	1804.	174.	0.	0.	5328.	488.	40.	358.	3877.	2891.	106.	7761.	3.	1.	1.
NV	523.	3837.	6709.	10666.	7847.	6010.	3628.	2611.	1424.	160.	14.	4932.	513.	43.	396.	2634.	3340.	371.	7295.	-6.	-2.	-3.
DC	606.	4438.	3741.	6608.	10528.	5572.	2862.	1078.	2007.	1310.	114.	5652.	544.	50.	517.	3106.	2531.	591.	7339.	15.	3.	2.
--1968--																						
JA	832.	6115.	4328.	3685.	6522.	7053.	2252.	491.	723.	1846.	161.	7024.	438.	41.	638.	3281.	2357.	350.	7105.	2.	-6.	-6.
FB	1157.	8513.	5962.	4263.	3637.	9628.	3599.	547.	317.	666.	58.	7312.	348.	30.	690.	3404.	1694.	172.	6338.	-5.	-4.	-5.
MR	1633.	12037.	8300.	5872.	4207.	5968.	5433.	1317.	410.	291.	25.	7209.	363.	31.	424.	4128.	2264.	134.	7344.	3.	3.	3.
AP	1626.	12069.	11736.	8175.	5796.	3303.	3262.	2088.	955.	378.	33.	7134.	417.	36.	261.	2665.	3318.	355.	7052.	4.	-4.	-4.
MY	1038.	7691.	11767.	11560.	8069.	3595.	935.	476.	1545.	879.	76.	7449.	519.	47.	528.	2345.	2770.	532.	6741.	7.	-7.	-8.
JN	1021.	7544.	7499.	11590.	11409.	4182.	770.	3.	94.	1422.	124.	8273.	551.	50.	1499.	2800.	928.	379.	6207.	6.	6.	5.

JL	954.	7043.	7356.	7386.	11440.	6512.	838.	2.	0.	86.	8.	7692.	615.	51.	1397.	3315.	764.	3.	6144.	2.	-1.	-2.
AG	1206.	8891.	6867.	7245.	7290.	10093.	2319.	40.	0.	0.	0.	7046.	596.	49.	1090.	4147.	794.	2.	6677.	-4.	-4.	-5.
SP	1532.	11277.	8668.	6764.	7151.	10536.	5644.	618.	4.	0.	0.	6516.	488.	39.	676.	4378.	1690.	35.	7307.	-2.	3.	2.
OC	911.	6698.	10995.	8538.	6676.	6744.	6536.	2634.	478.	4.	0.	5991.	487.	40.	401.	3927.	2981.	137.	7974.	-1.	-4.	-5.
NV	494.	3626.	6530.	10830.	8427.	6515.	3807.	3117.	2257.	439.	38.	5872.	515.	44.	494.	2890.	3386.	364.	7692.	-0.	4.	4.
DC	572.	4202.	3535.	6432.	10690.	5873.	2923.	1267.	2632.	2077.	181.	7352.	550.	53.	670.	3547.	2521.	470.	7811.	8.	3.	4.
--1969--																						
JA	832.	6116.	4097.	3482.	6349.	7492.	2223.	485.	915.	2421.	211.	9296.	441.	43.	768.	3609.	2423.	346.	7630.	2.	-1.	-1.
FB	1165.	8586.	5964.	4035.	3437.	9628.	3545.	498.	322.	842.	73.	9771.	337.	29.	849.	3894.	1713.	161.	6984.	-5.	-0.	1.
MF	1515.	11179.	8371.	5874.	3983.	5706.	5024.	1237.	384.	296.	26.	9682.	355.	30.	516.	4536.	2290.	111.	7839.	4.	4.	4.
AP	1527.	11346.	10899.	8246.	5798.	3034.	2645.	1606.	888.	354.	31.	9596.	414.	36.	335.	3021.	3393.	343.	7543.	6.	-0.	-0.
MY	951.	7057.	11062.	10736.	8139.	3166.	427.	124.	1152.	817.	71.	9848.	512.	46.	737.	2585.	2507.	446.	6833.	6.	2.	1.
JN	931.	6889.	6880.	10896.	10596.	3361.	369.	0.	0.	1060.	92.	10321.	540.	48.	2321.	2775.	425.	124.	6234.	4.	1.	0.
JL	880.	6505.	6717.	6777.	10754.	5497.	721.	1.	0.	0.	0.	9638.	584.	48.	2480.	2616.	367.	0.	6094.	2.	-4.	-5.
AG	1126.	8311.	6342.	6616.	6689.	9134.	1521.	5.	0.	0.	0.	9079.	562.	46.	1251.	3938.	713.	1.	6510.	-3.	4.	3.
SP	1442.	10624.	8104.	6247.	6530.	9782.	4492.	293.	0.	0.	0.	8584.	456.	37.	759.	4578.	1220.	5.	7053.	-2.	-2.	-1.
OC	879.	6470.	10359.	7982.	6165.	6177.	5750.	1668.	186.	0.	0.	8103.	443.	36.	378.	3963.	2802.	106.	7728.	2.	-1.	1.
NV	499.	3673.	6308.	10203.	7878.	5986.	3499.	2496.	1331.	171.	15.	7779.	457.	38.	414.	2635.	3226.	329.	7099.	7.	10.	10.
DC	605.	4444.	3581.	6214.	10071.	5495.	2729.	987.	1927.	1225.	106.	8484.	479.	44.	548.	3215.	2494.	557.	7336.	7.	4.	4.
--1970--																						
JA	862.	6349.	4333.	3527.	6133.	7111.	2209.	397.	614.	1773.	154.	9857.	368.	35.	610.	3247.	2318.	368.	6948.	-2.	2.	2.
FB	1190.	8780.	6191.	4268.	3481.	9251.	3604.	439.	207.	564.	49.	10136.	263.	22.	619.	3457.	1759.	188.	6309.	-9.	4.	5.
MR	1619.	11963.	8560.	6098.	4213.	5653.	5221.	1129.	273.	190.	17.	10011.	291.	25.	358.	3966.	2457.	165.	7261.	-5.	3.	4.
AP	1648.	12258.	11664.	8432.	6019.	3184.	3083.	1746.	710.	251.	22.	9882.	350.	30.	228.	2531.	3448.	413.	7001.	-2.	-4.	-4.
MY	1094.	8127.	11952.	11489.	9322.	3680.	967.	370.	1131.	653.	57.	10031.	464.	41.	449.	2194.	2697.	607.	6453.	5.	1.	-1.
JN	1109.	8217.	7923.	11772.	11339.	4433.	869.	1.	34.	1040.	90.	10516.	512.	46.	1466.	2786.	961.	335.	6105.	4.	-3.	-3.
JL	1047.	7746.	8011.	7804.	11619.	7062.	902.	2.	0.	31.	3.	9918.	580.	48.	1094.	3500.	863.	1.	6086.	1.	-4.	-4.
AG	1295.	9570.	7553.	7891.	7703.	10060.	2748.	75.	0.	0.	0.	9290.	579.	48.	1054.	4265.	823.	2.	6770.	-7.	2.	3.
SF	1628.	12013.	9331.	7439.	7788.	10681.	5607.	827.	28.	0.	0.	8767.	482.	39.	708.	4383.	1907.	46.	7564.	-9.	-1.	0.
OC	1021.	7526.	11713.	9191.	7343.	7115.	6471.	2485.	647.	26.	2.	8236.	513.	43.	435.	4135.	3094.	176.	8395.	-3.	1.	1.

NV	603.	4440.	7338.	11537.	9072.	7145.	4044.	2959.	2058.	595.	52.	8239.	546.	47.	498.	3022.	3579.	415.	8097.	6.	0.	0.
DC	699.	5146.	4329.	7228.	11387.	6566.	3347.	1188.	2239.	1893.	165.	9505.	578.	54.	630.	3748.	2835.	606.	8451.	3.	-4.	-5.
--1971--																						
JA	871.	6424.	5017.	4264.	7134.	8206.	2729.	526.	733.	2060.	179.	11041.	483.	46.	685.	3792.	2804.	450.	8260.	-9.	0.	0.
FB	1181.	8730.	6263.	4942.	4208.	10402.	3981.	520.	284.	674.	59.	11285.	396.	34.	759.	4168.	2195.	240.	7791.	-17.	11.	12.
MR	1594.	11797.	8511.	6169.	4878.	6519.	5539.	1163.	336.	261.	23.	11032.	419.	35.	473.	4790.	2798.	181.	8696.	-3.	-3.	-3.
AP	1567.	11675.	11502.	8394.	6089.	3853.	3427.	1486.	625.	309.	27.	10906.	456.	39.	271.	3047.	4025.	533.	8371.	-3.	-1.	-1.
MY	978.	7277.	11383.	11329.	8275.	4241.	1058.	278.	840.	575.	50.	10866.	567.	49.	540.	2768.	3132.	639.	7695.	-3.	2.	1.
JN	974.	7229.	7095.	11212.	11182.	4143.	922.	0.	0.	773.	67.	10991.	597.	52.	1825.	3290.	1053.	276.	7092.	-0.	-7.	-7.
JL	915.	6781.	7048.	6988.	11066.	6210.	813.	4.	0.	0.	0.	10289.	647.	53.	1900.	3301.	913.	0.	6815.	-3.	0.	-3.
AG	1175.	8632.	6611.	6942.	6898.	9726.	1739.	2.	0.	0.	0.	9593.	641.	53.	1233.	4427.	807.	4.	7166.	-8.	-5.	-6.
SP	1503.	11106.	8474.	6512.	6852.	10216.	4830.	278.	0.	0.	0.	9016.	532.	43.	630.	4829.	1453.	2.	7486.	-8.	-6.	-6.
OC	899.	6636.	10829.	8347.	6427.	6398.	6115.	1790.	164.	0.	0.	8457.	515.	42.	362.	4029.	3016.	113.	8077.	-6.	4.	4.
NV	490.	3614.	6471.	10666.	8239.	6342.	3781.	2627.	1358.	151.	13.	8045.	519.	43.	374.	2572.	3458.	424.	7390.	-5.	-10.	-10.
DC	594.	4378.	3524.	6374.	10527.	5841.	3277.	1149.	1892.	1249.	109.	8720.	529.	48.	459.	3021.	2614.	722.	7392.	8.	-11.	-11.

Appendix 6: -- F O R K   R E S U L T S --

TIME	RAT.	AVLW	F18-20	F20-22	F22-24	F24-27	F27+UP	P18-20	P20-22	P22-24	P24-27	P27+UP	N18-20	N20-22	N22-24	N24-27	N27+UP
--1964--																	
JA	1.00	243.92	2.89	20.88	36.79	29.23	10.21	1.00	1.00	1.00	1.00	1.00	207.98	1503.39	2648.45	2104.05	734.6
FB	1.00	235.88	5.43	25.26	38.64	24.87	5.90	1.00	1.00	1.00	1.00	1.00	356.48	1657.23	2535.02	1631.24	380.2
MR	1.00	233.34	4.25	26.88	42.01	23.32	3.54	1.00	1.00	1.00	1.00	1.00	308.27	1949.84	3047.38	1691.66	256.7
AP	1.00	236.89	2.73	21.36	36.99	30.85	8.06	1.00	1.00	1.00	1.00	1.00	186.45	1460.57	2528.84	2109.41	551.3
MY	1.00	239.47	3.34	19.53	40.38	29.33	7.42	1.00	1.00	1.00	1.00	1.00	199.19	1163.34	2405.95	1747.33	442.2
JN	1.00	236.32	5.34	24.99	36.97	25.13	7.57	1.00	1.00	1.00	1.00	1.00	291.94	1365.79	2020.24	1373.08	413.5
JL	1.00	229.65	7.22	34.80	40.29	15.82	1.86	1.00	1.00	1.00	1.00	1.00	364.56	1755.93	2032.81	798.46	94.0
AG	1.00	225.00	9.27	36.78	36.44	14.31	3.21	1.00	1.00	1.00	1.00	1.00	503.55	1998.23	1979.88	777.39	174.3
SP	1.00	226.76	6.95	31.21	42.97	15.41	3.45	1.00	1.00	1.00	1.00	1.00	432.47	1941.04	2672.13	958.35	214.7
OC	1.00	234.05	2.80	26.42	40.11	26.73	3.93	1.00	1.00	1.00	1.00	1.00	190.30	1793.06	2721.70	1813.94	266.8
NV	1.00	234.74	3.79	23.28	41.38	26.18	5.38	1.00	1.00	1.00	1.00	1.00	253.87	1558.18	2769.62	1752.10	359.9
DC	1.00	235.65	5.49	25.41	38.68	24.72	5.70	1.00	1.00	1.00	1.00	1.00	366.69	1695.57	2585.51	1652.71	381.4
--1965--																	
JA	1.00	236.16	4.02	25.09	39.96	25.03	5.90	1.00	1.00	1.00	1.00	1.00	273.02	1705.84	2716.35	1701.50	401.2
FB	1.00	231.27	6.67	28.25	38.69	22.05	4.34	1.00	1.00	1.00	1.00	1.00	400.06	1695.13	2321.69	1323.54	260.7
MR	1.00	229.85	5.35	29.18	41.62	21.21	2.64	1.00	1.00	1.00	1.00	1.00	363.92	1985.00	2830.94	1442.29	179.4
AP	1.00	233.01	3.64	23.82	37.36	28.50	6.68	1.00	1.00	1.00	1.00	1.00	226.82	1483.40	2326.64	1775.00	415.9
MY	1.00	233.82	4.48	22.95	41.52	25.98	5.07	1.00	1.00	1.00	1.00	1.00	239.60	1226.68	2219.08	1388.86	270.9
JN	1.00	233.59	5.98	26.72	37.23	23.47	6.59	1.00	1.00	1.00	1.00	1.00	287.56	1284.02	1788.91	1127.69	316.8
JL	1.00	229.08	7.44	35.18	40.15	15.49	1.75	1.00	1.00	1.00	1.00	1.00	341.66	1615.67	1844.12	711.41	80.3
AG	1.00	225.28	9.13	36.61	36.57	14.46	3.24	1.00	1.00	1.00	1.00	1.00	459.53	1843.00	1840.97	727.75	163.3
SP	1.00	227.65	6.56	30.63	43.29	15.92	3.60	1.00	1.00	1.00	1.00	1.00	376.17	1756.97	2482.98	912.96	206.3
OC	1.00	234.26	2.75	26.29	40.10	26.86	4.00	1.00	1.00	1.00	1.00	1.00	168.62	1611.66	2458.64	1647.08	245.4
NV	1.00	235.43	3.63	22.84	41.31	26.59	5.62	1.00	1.00	1.00	1.00	1.00	210.37	1322.74	2391.87	1540.01	325.7
DC	1.00	237.84	5.05	24.08	38.22	26.03	6.62	1.00	1.00	1.00	1.00	1.00	290.09	1383.09	2195.72	1495.34	380.5
--1966--																	
JA	1.00	239.47	3.42	23.14	39.05	26.96	7.43	1.00	1.00	1.00	1.00	1.00	189.10	1278.85	2158.01	1490.11	410.4
FB	1.00	236.21	5.36	25.06	38.59	25.06	5.92	1.00	1.00	1.00	1.00	1.00	266.85	1246.56	1919.65	1246.85	294.7

MR	1.00	237.06	3.39	24.54	41.60	25.57	4.90	1.00	1.00	1.00	1.00	1.00	201.26	1455.20	2466.61	1516.21	290.3
AP	1.00	241.55	2.00	18.75	35.31	33.47	10.47	1.00	1.00	1.00	1.00	1.00	110.11	1032.09	1943.68	1842.33	576.5
MY	1.00	242.37	2.97	18.00	39.04	30.87	9.13	1.00	1.00	1.00	1.00	1.00	149.96	910.04	1973.73	1560.57	461.8
JN	1.00	239.10	4.84	23.35	36.23	26.75	8.83	1.00	1.00	1.00	1.00	1.00	219.01	1056.76	1639.41	1210.68	399.7
JL	1.00	231.04	6.74	33.88	40.55	16.66	2.18	1.00	1.00	1.00	1.00	1.00	312.27	1569.48	1878.47	771.75	100.9
AG	1.00	223.33	10.18	37.78	35.59	13.44	3.02	1.00	1.00	1.00	1.00	1.00	525.23	1949.70	1836.86	693.64	155.8
SP	1.00	225.85	7.38	31.79	42.60	14.91	3.32	1.00	1.00	1.00	1.00	1.00	449.92	1937.45	2596.10	908.48	202.3
OC	1.00	233.83	2.86	26.56	40.12	26.60	3.86	1.00	1.00	1.00	1.00	1.00	194.57	1807.11	2729.09	1809.57	262.7
NV	1.00	237.69	3.18	21.47	40.86	27.94	6.55	1.00	1.00	1.00	1.00	1.00	210.27	1420.57	2704.03	1848.73	433.6
DC	1.00	240.87	4.59	22.38	37.10	27.72	8.20	1.00	1.00	1.00	1.00	1.00	315.18	1538.39	2550.07	1905.51	563.8
--1967--																	
JA	1.00	241.74	3.12	21.94	38.04	28.18	9.73	1.00	1.00	1.00	1.00	1.00	216.52	1523.05	2641.33	1956.19	605.8
FB	1.00	235.94	5.42	25.23	38.63	24.90	5.82	1.00	1.00	1.00	1.00	1.00	347.17	1615.27	2473.63	1594.22	372.4
MR	1.00	235.92	3.63	25.24	41.82	24.89	4.43	1.00	1.00	1.00	1.00	1.00	269.55	1876.41	3109.10	1850.61	329.5
AP	1.00	237.87	2.55	20.78	36.75	31.43	8.49	1.00	1.00	1.00	1.00	1.00	177.41	1448.44	2561.27	2190.49	592.0
MY	1.00	239.71	3.31	19.39	40.29	29.47	7.55	1.00	1.00	1.00	1.00	1.00	209.66	1229.46	2554.65	1868.41	478.9
JN	1.00	237.76	5.06	24.12	36.64	25.98	8.19	1.00	1.00	1.00	1.00	1.00	290.47	1383.75	2102.00	1490.54	469.7
JL	1.00	234.11	5.85	31.86	40.71	18.53	3.05	1.00	1.00	1.00	1.00	1.00	322.86	1758.90	2246.94	1022.75	168.4
AG	1.00	228.46	7.68	34.56	37.77	16.24	3.74	1.00	1.00	1.00	1.00	1.00	457.27	2056.74	2247.44	966.49	222.5
SP	1.00	230.92	5.32	28.47	44.10	17.85	4.26	1.00	1.00	1.00	1.00	1.00	354.41	1897.28	2938.32	1189.16	283.9
OC	1.00	236.65	2.21	24.79	39.80	28.31	4.89	1.00	1.00	1.00	1.00	1.00	160.15	1792.88	2878.35	2047.46	354.0
NV	1.00	239.37	2.90	20.50	40.33	28.91	7.37	1.00	1.00	1.00	1.00	1.00	195.27	1381.52	2717.95	1948.16	496.9
DC	1.00	241.53	4.50	22.04	36.79	28.07	8.60	1.00	1.00	1.00	1.00	1.00	303.79	1486.95	2481.62	1893.27	579.8
--1968--																	
JA	1.00	239.18	3.47	23.30	39.16	26.80	7.27	1.00	1.00	1.00	1.00	1.00	229.68	1544.09	2594.40	1775.64	482.0
FB	1.00	233.49	6.01	26.78	38.83	23.41	4.96	1.00	1.00	1.00	1.00	1.00	358.18	1596.31	2314.32	1395.35	295.9
MR	1.00	233.80	4.13	26.58	42.01	23.60	3.68	1.00	1.00	1.00	1.00	1.00	286.87	1847.46	2919.44	1640.30	255.9
AP	1.00	238.76	2.39	20.26	36.48	31.95	9.92	1.00	1.00	1.00	1.00	1.00	157.94	1337.18	2407.07	2108.16	588.8
MY	1.00	240.14	3.24	19.15	40.11	29.70	7.79	1.00	1.00	1.00	1.00	1.00	200.32	1182.60	2476.81	1834.03	486.9
JN	1.00	239.77	4.74	22.37	35.97	27.13	9.19	1.00	1.00	1.00	1.00	1.00	265.56	1287.91	2016.56	1520.81	514.9

JL	1.00	232.95	6.15	32.61	40.71	17.82	2.69	1.00	1. [illegible]	1.00	1.00	1.00	337.33	1786.80	2230.51	976.40	147.5
AG	1.00	228.40	7.71	34.60	37.75	16.21	3.73	1.00	1.00	1.00	1.00	1.00	464.94	2087.35	2277.47	977.88	224.9
SP	1.00	230.13	5.59	29.00	43.96	17.37	4.08	1.00	1.00	1.00	1.00	1.00	379.05	1966.01	2980.03	1177.55	276.4
OC	1.00	233.70	2.89	26.65	40.12	26.52	3.82	1.00	1.00	1.00	1.00	1.00	215.55	1984.45	2987.49	1974.75	284.4
NV	1.00	235.01	3.73	23.11	41.35	26.33	5.47	1.00	1.00	1.00	1.00	1.00	266.11	1648.56	2949.64	1878.40	390.1
DC	1.00	234.61	5.72	26.06	38.79	24.09	5.33	1.00	1.00	1.00	1.00	1.00	412.64	1878.65	2795.78	1736.73	384.2
--1969--																	
JA	1.00	236.36	3.97	24.97	39.92	25.15	5.99	1.00	1.00	1.00	1.00	1.00	283.93	1784.14	2852.91	1797.56	427.7
FB	1.00	230.09	7.07	29.03	38.49	21.35	4.07	1.00	1.00	1.00	1.00	1.00	467.59	1920.89	2547.00	1412.45	269.2
MR	1.00	230.34	5.18	28.86	41.72	21.50	2.74	1.00	1.00	1.00	1.00	1.00	385.91	2151.25	3109.69	1602.29	204.6
AP	1.00	235.78	2.96	22.04	37.19	30.19	7.62	1.00	1.00	1.00	1.00	1.00	209.66	1563.64	2638.02	2141.46	540.2
MY	1.00	236.66	3.83	21.16	41.19	27.70	6.11	1.00	1.00	1.00	1.00	1.00	240.48	1327.98	2585.02	1738.49	383.3
JN	1.00	237.22	5.16	24.45	36.78	25.66	7.94	1.00	1.00	1.00	1.00	1.00	291.55	1380.12	2076.45	1448.74	448.5
JL	1.00	231.13	6.71	33.82	40.56	16.71	2.20	1.00	1.00	1.00	1.00	1.00	366.68	1847.56	2215.62	912.80	120.1
AG	1.00	226.17	8.69	36.05	36.96	14.94	3.36	1.00	1.00	1.00	1.00	1.00	512.97	2127.84	2181.42	881.95	198.6
SP	1.00	227.99	6.42	30.42	43.40	16.11	3.65	1.00	1.00	1.00	1.00	1.00	421.03	1995.59	2847.72	1057.06	239.7
OC	1.00	235.45	2.47	25.53	39.99	27.58	4.42	1.00	1.00	1.00	1.00	1.00	179.01	1850.75	2898.81	1999.25	320.4
NV	1.00	237.93	3.13	21.33	40.80	28.08	6.66	1.00	1.00	1.00	1.00	1.00	207.03	1408.44	2694.36	1854.33	440.0
DC	1.00	240.22	4.67	22.73	37.39	27.37	7.83	1.00	1.00	1.00	1.00	1.00	318.42	1548.73	2547.51	1865.06	533.5
--1970--																	
JA	1.00	240.90	3.22	22.37	38.45	27.74	8.22	1.00	1.00	1.00	1.00	1.00	210.91	1464.10	2516.60	1815.38	537.8
FB	1.00	236.96	5.21	24.60	38.44	25.51	6.23	1.00	1.00	1.00	1.00	1.00	314.04	1481.89	2315.49	1536.66	375.5
MR	1.00	238.94	3.06	23.44	41.06	26.66	5.77	1.00	1.00	1.00	1.00	1.00	212.83	1627.94	2851.62	1851.62	400.8
AP	1.00	243.46	1.79	17.81	34.25	34.41	11.74	1.00	1.00	1.00	1.00	1.00	118.66	1179.45	2267.51	2278.11	777.3
MY	1.00	244.83	2.73	16.86	37.52	32.00	10.90	1.00	1.00	1.00	1.00	1.00	162.29	1002.54	2231.32	1903.07	648.0
JN	1.00	241.93	4.46	21.84	34.99	28.27	10.45	1.00	1.00	1.00	1.00	1.00	247.20	1211.54	1940.68	1568.28	579.4
JL	1.00	233.80	5.93	32.06	40.72	18.34	2.95	1.00	1.00	1.00	1.00	1.00	323.53	1749.91	2222.23	1001.03	161.1
AG	1.00	227.88	7.92	34.34	37.59	15.91	3.64	1.00	1.00	1.00	1.00	1.00	486.65	2146.62	2309.43	977.42	223.3
SP	1.00	230.17	5.58	28.98	43.97	17.39	4.09	1.00	1.00	1.00	1.00	1.00	392.84	2040.73	3096.57	1225.02	287.8
OC	1.00	234.18	2.77	26.34	40.10	26.81	3.98	1.00	1.00	1.00	1.00	1.00	217.17	2064.68	3143.90	2102.12	311.7
NV	1.00	237.55	3.20	21.55	40.90	27.86	6.49	1.00	1.00	1.00	1.00	1.00	240.27	1616.85	3068.92	2090.50	487.1

DC	1.00	239.69	4.75	23.03	37.61	27.08	7.53	1.00	1.00	1.00	1.00	1.00	371.51	1800.54	2940.81	2117.17	589.1
--1971--																	
JA	1.00	241.67	3.13	21.97	38.08	29.14	8.69	1.00	1.00	1.00	1.00	1.00	241.68	1698.40	2943.60	2175.67	671.5
FB	1.00	237.02	5.20	24.57	38.43	25.54	6.26	1.00	1.00	1.00	1.00	1.00	382.99	1808.47	2828.70	1880.23	460.6
MR	1.00	237.06	3.39	24.54	41.60	25.57	4.90	1.00	1.00	1.00	1.00	1.00	279.76	2022.66	3428.34	2107.27	403.5
AP	1.00	245.89	1.59	16.75	32.61	35.45	13.61	1.00	1.00	1.00	1.00	1.00	124.90	1319.28	2568.31	2791.80	1071.9
MY	1.00	245.25	2.70	16.68	37.23	32.17	11.22	1.00	1.00	1.00	1.00	1.00	190.79	1180.75	2635.58	2277.37	794.3
JN	1.00	244.10	4.23	20.80	33.71	29.31	11.94	1.00	1.00	1.00	1.00	1.00	272.71	1340.35	2172.40	1888.65	769.3
JL	1.00	238.28	4.97	29.30	39.98	21.03	4.72	1.00	1.00	1.00	1.00	1.00	304.05	1791.32	2444.65	1285.53	288.6
AG	1.00	232.14	6.39	32.13	38.46	18.44	4.57	1.00	1.00	1.00	1.00	1.00	413.82	2079.22	2489.23	1193.71	295.8
SP	1.00	235.00	4.17	25.82	44.21	20.33	5.47	1.00	1.00	1.00	1.00	1.00	288.44	1784.55	3056.04	1405.10	377.8
OC	1.00	237.26	2.10	24.42	39.66	28.67	5.15	1.00	1.00	1.00	1.00	1.00	157.57	1836.21	2982.35	2155.95	387.6
NV	1.00	241.99	2.55	19.11	39.15	30.31	8.89	1.00	1.00	1.00	1.00	1.00	174.01	1304.56	2672.99	2069.08	606.8
DC	1.00	245.60	4.11	20.15	34.29	29.95	11.51	1.00	1.00	1.00	1.00	1.00	279.92	1373.16	2336.55	2040.86	784.5

Appendix 7: A G E ---W E I G H T D I S T R I B U T I O N O F R+G

	C1	C2	C3	C4	D1	D2	D3	D4	E1	E2	E3	E4	F1	F2	F3	F4
--1964--																
JA	0.	13.	126.	255.	514.	763.	1144.	1471.	680.	581.	602.	561.	133.	103.	77.	77.
FB	0.	28.	198.	402.	550.	715.	851.	883.	1065.	868.	512.	276.	65.	48.	46.	53.
MR	0.	25.	211.	383.	832.	1073.	1171.	1056.	855.	683.	405.	229.	143.	80.	39.	28.
AP	0.	456.	591.	863.	854.	846.	735.	580.	746.	479.	212.	85.	63.	57.	78.	92.
MY	0.	807.	1289.	1390.	1064.	611.	377.	207.	106.	89.	16.	0.	0.	0.	1.	1.
JN	131.	719.	1238.	1302.	1394.	583.	104.	4.	1.	0.	1.	0.	0.	0.	0.	0.
JL	0.	645.	1193.	1527.	1078.	543.	59.	0.	0.	0.	0.	0.	0.	0.	0.	0.
AG	0.	435.	959.	1237.	1122.	912.	520.	322.	84.	36.	5.	0.	0.	0.	0.	0.
SP	0.	35.	263.	541.	802.	1081.	1236.	1186.	636.	339.	99.	1.	0.	0.	0.	0.
OC	0.	10.	119.	245.	603.	863.	1178.	1337.	1207.	726.	308.	117.	39.	20.	13.	0.
NV	0.	17.	148.	312.	484.	542.	795.	847.	1247.	1036.	594.	327.	147.	65.	24.	8.
DC	0.	27.	193.	383.	600.	789.	943.	980.	872.	728.	445.	277.	216.	126.	61.	44.
--1965--																
JA	0.	25.	241.	501.	602.	863.	1012.	975.	910.	720.	418.	223.	117.	78.	57.	55.
FB	0.	33.	238.	494.	583.	844.	1069.	1114.	723.	482.	197.	78.	47.	32.	34.	32.
MR	0.	20.	152.	298.	726.	971.	1208.	1310.	896.	682.	322.	136.	43.	21.	11.	5.
AP	0.	12.	98.	192.	528.	656.	758.	764.	1153.	932.	554.	316.	127.	77.	39.	21.
MY	0.	28.	211.	396.	474.	579.	570.	501.	937.	690.	435.	256.	186.	107.	44.	32.
JN	0.	410.	641.	833.	657.	640.	541.	408.	242.	125.	49.	25.	46.	43.	60.	86.
JL	0.	417.	687.	981.	836.	568.	455.	324.	143.	133.	50.	0.	0.	0.	0.	0.
AG	0.	68.	364.	734.	806.	891.	919.	705.	314.	175.	56.	0.	0.	0.	1.	0.
SP	0.	28.	210.	432.	635.	964.	1192.	1227.	598.	295.	51.	0.	0.	0.	3.	0.
OC	0.	8.	99.	204.	526.	730.	986.	1127.	1018.	767.	396.	186.	58.	21.	5.	0.
NV	0.	14.	119.	254.	404.	529.	655.	701.	1074.	909.	543.	318.	143.	78.	34.	17.
DC	0.	21.	153.	306.	481.	541.	782.	827.	760.	646.	407.	262.	215.	128.	64.	51.
--1966--																
JA	0.	16.	166.	350.	434.	649.	812.	822.	775.	622.	367.	203.	113.	78.	60.	59.

FB	0.	21.	166.	353.	406.	619.	850.	949.	658.	475.	218.	97.	57.	39.	36.	31.
MR	0.	10.	97.	195.	509.	741.	1029.	1214.	813.	659.	351.	176.	65.	36.	20.	10.
AP	0.	5.	59.	123.	340.	457.	601.	664.	1087.	917.	578.	355.	144.	92.	53.	30.
MY	0.	14.	128.	261.	330.	459.	523.	510.	840.	719.	466.	288.	248.	150.	72.	49.
JN	0.	257.	418.	604.	502.	553.	607.	543.	356.	221.	103.	53.	79.	67.	75.	88.
JL	0.	103.	361.	674.	704.	772.	787.	633.	298.	233.	65.	0.	0.	0.	2.	1.
AG	0.	49.	320.	657.	683.	886.	983.	916.	463.	179.	23.	0.	0.	0.	2.	0.
SP	0.	31.	219.	450.	655.	301.	1109.	1182.	690.	491.	227.	97.	29.	11.	3.	0.
OC	0.	10.	117.	243.	574.	779.	1037.	1179.	1060.	869.	499.	277.	88.	44.	19.	7.
NV	0.	14.	126.	269.	434.	589.	759.	837.	1192.	1017.	627.	383.	179.	108.	55.	38.
DC	0.	23.	169.	343.	526.	722.	917.	1001.	937.	903.	510.	329.	261.	163.	93.	77.
--1967--																
JA	0.	18.	192.	407.	516.	787.	1019.	1060.	974.	793.	475.	271.	157.	109.	84.	79.
FB	0.	27.	210.	442.	537.	805.	1086.	1204.	852.	621.	289.	128.	72.	48.	44.	37.
MR	0.	15.	138.	276.	665.	953.	1277.	1452.	1034.	821.	432.	213.	79.	43.	24.	12.
AP	0.	10.	97.	195.	501.	566.	834.	888.	1323.	1083.	650.	373.	158.	100.	58.	34.
MY	0.	22.	194.	383.	492.	679.	720.	663.	1009.	933.	523.	308.	245.	149.	72.	50.
JN	0.	447.	680.	891.	739.	738.	710.	609.	393.	198.	70.	29.	44.	40.	61.	85.
JL	0.	427.	706.	1021.	940.	710.	647.	507.	250.	228.	84.	0.	0.	0.	0.	0.
AG	0.	48.	369.	773.	885.	1068.	1141.	914.	449.	235.	65.	0.	0.	0.	3.	0.
SF	0.	26.	216.	447.	685.	1108.	1413.	1479.	962.	358.	62.	2.	1.	0.	3.	0.
OC	0.	8.	114.	236.	580.	828.	1146.	1324.	1246.	939.	483.	224.	73.	26.	7.	0.
NV	0.	13.	122.	261.	425.	589.	768.	851.	1251.	1065.	641.	382.	191.	108.	50.	23.
DC	0.	22.	163.	332.	514.	708.	902.	982.	930.	790.	496.	316.	264.	165.	92.	70.
--1968--																
JA	0.	20.	199.	419.	522.	776.	978.	1005.	930.	749.	438.	240.	134.	90.	66.	60.
FB	0.	29.	213.	449.	523.	765.	1009.	1107.	758.	555.	256.	115.	63.	41.	36.	30.
MR	0.	15.	136.	273.	647.	908.	1203.	1371.	933.	745.	392.	194.	68.	36.	19.	10.
AP	0.	8.	83.	170.	450.	502.	773.	840.	1246.	1040.	644.	388.	158.	101.	59.	36.
MY	0.	19.	168.	340.	431.	595.	670.	648.	1026.	867.	549.	328.	253.	153.	74.	52.
JN	0.	304.	492.	703.	641.	704.	769.	686.	451.	280.	130.	67.	97.	82.	93.	107.

JL	0.	154.	446.	796.	830.	883.	890.	712.	331.	299.	85.	0.	0.	0.	2.	1.
AG	0.	44.	339.	707.	773.	1063.	1212.	1098.	552.	214.	27.	0.	0.	0.	2.	0.
SF	0.	26.	211.	439.	669.	393.	1296.	1420.	856.	546.	219.	69.	22.	9.	4.	0.
OC	0.	11.	127.	263.	632.	857.	1140.	1298.	1205.	968.	532.	277.	87.	38.	12.	1.
NV	0.	18.	152.	323.	507.	667.	828.	868.	1260.	1075.	656.	394.	182.	105.	48.	29.
DC	0.	31.	214.	425.	652.	946.	1004.	1045.	939.	791.	488.	303.	220.	130.	65.	55.
--1969--																
JA	0.	25.	241.	502.	616.	886.	1063.	1045.	955.	765.	452.	251.	133.	88.	63.	61.
FB	0.	38.	264.	546.	652.	911.	1137.	1194.	804.	562.	245.	102.	56.	37.	35.	33.
MR	0.	22.	167.	328.	785.	1049.	1300.	1402.	375.	758.	381.	177.	58.	30.	15.	8.
AP	0.	12.	109.	214.	560.	722.	860.	879.	1314.	1070.	642.	367.	155.	98.	56.	34.
MY	0.	29.	242.	466.	556.	717.	700.	612.	961.	780.	485.	281.	213.	128.	60.	44.
JN	0.	492.	806.	1024.	797.	801.	671.	506.	279.	107.	29.	11.	18.	18.	31.	57.
JL	0.	521.	817.	1142.	1004.	684.	533.	394.	171.	139.	56.	0.	0.	0.	0.	0.
AG	0.	54.	391.	807.	948.	1055.	1090.	845.	404.	237.	72.	0.	0.	0.	1.	0.
SF	0.	31.	238.	489.	725.	1108.	1360.	1385.	837.	327.	54.	1.	0.	0.	4.	0.
OC	0.	9.	121.	248.	610.	861.	1167.	1325.	1206.	911.	467.	217.	75.	25.	6.	0.
NV	0.	14.	128.	273.	435.	597.	766.	837.	1230.	1035.	610.	351.	168.	95.	45.	21.
DC	0.	24.	174.	350.	542.	742.	931.	1000.	923.	780.	485.	306.	254.	156.	84.	63.
--1970--																
JA	0.	18.	190.	403.	501.	767.	979.	1000.	924.	735.	426.	232.	134.	93.	73.	68.
FB	0.	25.	191.	403.	495.	757.	1041.	1164.	807.	578.	261.	113.	66.	45.	42.	35.
MR	0.	11.	114.	232.	568.	942.	1178.	1377.	1005.	806.	430.	216.	82.	46.	25.	12.
AP	0.	6.	72.	150.	396.	552.	747.	835.	1287.	1081.	674.	407.	185.	120.	71.	38.
MY	0.	15.	141.	292.	367.	546.	644.	636.	1002.	846.	531.	319.	279.	176.	92.	60.
JN	3.	396.	480.	691.	604.	668.	779.	735.	477.	293.	129.	61.	85.	72.	85.	92.
JL	0.	56.	342.	697.	790.	951.	970.	788.	419.	340.	104.	0.	0.	0.	1.	0.
AG	0.	44.	329.	690.	760.	1075.	1246.	1164.	617.	191.	15.	0.	0.	0.	2.	0.
SP	0.	27.	221.	459.	696.	1002.	1290.	1404.	885.	630.	282.	109.	33.	11.	2.	0.
OC	0.	12.	138.	285.	659.	902.	1204.	1369.	1240.	1000.	557.	297.	100.	50.	20.	5.

NV	0.	16.	151.	321.	497.	683.	878.	964.	1329.	1137.	694.	419.	199.	118.	59.	39.
DC	0.	27.	200.	403.	627.	860.	1086.	1176.	1050.	888.	551.	345.	267.	166.	95.	78.
--1971--																
JA	0.	20.	213.	451.	579.	886.	1144.	1183.	1105.	888.	522.	290.	163.	114.	90.	83.
FB	0.	30.	235.	493.	611.	925.	1251.	1380.	990.	721.	336.	148.	87.	58.	52.	42.
MR	0.	16.	152.	305.	719.	1047.	1416.	1608.	1172.	921.	476.	229.	89.	58.	28.	14.
AP	0.	7.	86.	178.	455.	659.	914.	1019.	1516.	1261.	782.	466.	228.	153.	97.	54.
MY	0.	18.	171.	351.	458.	699.	816.	796.	1206.	997.	595.	334.	292.	184.	101.	62.
JN	0.	422.	603.	799.	766.	796.	890.	838.	563.	317.	122.	51.	69.	58.	72.	77.
JL	0.	332.	617.	951.	942.	832.	839.	688.	419.	371.	123.	0.	0.	0.	0.	0.
AG	0.	72.	385.	777.	894.	1168.	1296.	1069.	537.	229.	42.	0.	0.	0.	4.	0.
SP	0.	21.	197.	412.	649.	1124.	1481.	1573.	946.	432.	74.	0.	0.	0.	2.	0.
OC	0.	8.	115.	239.	592.	852.	1193.	1392.	1331.	973.	490.	221.	74.	30.	9.	0.
NV	0.	11.	115.	248.	397.	564.	758.	854.	1277.	1095.	672.	414.	219.	123.	57.	24.
DC	0.	19.	144.	295.	469.	662.	886.	1004.	942.	811.	518.	341.	309.	200.	121.	92.

Appendix 8: ------ B + G P E R F O R M A N C E --------

TIME	REP. BGS	EST. BGS	SIM. BGS	DIF E-R	PERCDIFE-R	DIF S-R	PERCDIFS-R	DIF S-E	PERCDIFS-E
--1964--									
JA	7421.93	7198.48	7198.48	-223.45	-3.01	-223.45	-3.01	-.00	-.00
FB	6425.81	6560.16	6560.16	134.35	2.09	134.35	2.09	-.00	-.00
MR	7001.98	7253.83	7253.83	251.85	3.60	251.85	3.60	-.00	-.00
AP	6920.59	6836.55	6836.55	-84.04	-1.21	-84.04	-1.21	-.00	-.00
MY	5809.57	5957.98	5957.98	148.41	2.55	148.41	2.55	-.00	-.00
JN	5220.86	5464.54	5464.54	243.68	4.67	243.68	4.67	-.00	-.00
JL	4963.09	5045.81	5045.81	82.72	1.67	82.72	1.67	-.00	-.00
AG	5000.12	5433.37	5433.37	433.25	8.66	433.25	8.66	-.00	-.00
SP	6005.14	6218.66	6218.66	213.52	3.56	213.52	3.56	-.00	-.00
OC	7204.61	6785.79	6785.79	-418.83	-5.81	-418.83	-5.81	-.00	-.00
NV	6842.39	6693.65	6693.65	-148.73	-2.17	-148.73	-2.17	-.00	-.00
DC	6967.86	6684.85	6684.85	-283.02	-4.06	-283.02	-4.06	-.00	-.00
--1965--									
JA	6513.00	6797.93	6797.93	284.94	4.37	284.94	4.37	-.00	-.00
FB	5816.83	6001.13	6001.13	184.30	3.17	184.30	3.17	-.00	-.00
MR	7089.59	6801.51	6801.51	-288.08	-4.06	-288.08	-4.06	-.00	-.00
AP	6182.30	6227.73	6227.73	45.43	.73	45.43	.73	-.00	-.00
MY	5017.56	5345.13	5345.13	327.57	6.53	327.57	6.53	-.00	-.00
JN	4832.48	4805.01	4805.01	-27.47	-.57	-27.47	-.57	-.00	-.00
JL	4586.66	4593.16	4593.16	6.50	.14	6.50	.14	-.00	-.00
AG	4970.66	5034.52	5034.52	63.86	1.28	63.86	1.28	-.00	-.00
SP	5865.24	5735.33	5735.33	-129.91	-2.21	-129.91	-2.21	-.00	-.00
OC	5805.01	6131.42	6131.42	326.41	5.62	326.41	5.62	-.00	-.00
NV	5815.07	5790.71	5790.71	-24.36	-.42	-24.36	-.42	-.00	-.00
DC	5343.16	5744.74	5744.74	401.58	7.52	401.58	7.52	-.00	-.00
--1966--									
JA	5173.35	5526.50	5526.50	353.15	6.83	353.15	6.83	-.00	-.00
FB	5110.94	4974.66	4974.66	-136.28	-2.67	-136.28	-2.67	-.00	-.00

MR	6334.41	5929.61	5929.61	-404.80	-6.39	-404.80	-6.39	-.00	-.00
AP	5739.68	5504.72	5504.72	-234.96	-4.09	-234.96	-4.09	-.00	-.00
MY	5187.77	5056.06	5056.06	-131.71	-2.54	-131.71	-2.54	-.00	-.00
JN	4806.84	4525.57	4525.57	-281.27	-5.85	-281.27	-5.85	-.00	-.00
JL	4271.44	4632.90	4632.90	361.46	8.46	361.46	8.46	-.00	-.00
AG	5224.16	5161.25	5161.25	-62.91	-1.20	-62.91	-1.20	-.00	-.00
SP	6163.39	6094.28	6094.28	-69.11	-1.12	-69.11	-1.12	-.00	-.00
OC	6430.42	6803.04	6803.04	372.62	5.79	372.62	5.79	-.00	-.00
NV	6622.80	6617.17	6617.17	-5.63	-.08	-5.63	-.08	-.00	-.00
DC	6732.73	6872.91	6872.91	140.18	2.08	140.18	2.08	-.00	-.00
--1967--									
JA	6822.22	6942.92	6942.92	120.71	1.77	120.71	1.77	-.00	-.00
FB	6193.29	6402.73	6402.73	209.45	3.38	209.45	3.38	-.00	-.00
MR	7250.26	7435.13	7435.13	184.87	2.55	184.87	2.55	-.00	-.00
AP	6328.08	6969.62	6969.62	641.54	10.14	641.54	10.14	-.00	-.00
MY	5640.71	6341.09	6341.09	700.38	12.42	700.38	12.42	-.00	-.00
JN	5355.09	5736.44	5736.44	381.35	7.12	381.35	7.12	-.00	-.00
JL	4948.74	5519.88	5519.88	571.15	11.54	571.15	11.54	-.00	-.00
AG	6072.35	5950.39	5950.39	-121.96	-2.01	-121.96	-2.01	-.00	-.00
SP	6447.91	6663.04	6663.04	215.13	3.34	215.13	3.34	-.00	-.00
OC	7176.87	7232.83	7232.83	55.95	.78	55.95	.78	-.00	-.00
NV	6935.26	6739.79	6739.79	-195.47	-2.82	-195.47	-2.82	-.00	-.00
DC	6611.55	6745.43	6745.43	133.88	2.03	133.88	2.03	-.00	-.00
--1968--									
JA	7060.29	6625.81	6625.81	-434.48	-6.15	-434.48	-6.15	-.00	-.00
FB	6241.75	5960.04	5960.04	-281.71	-4.51	-281.71	-4.51	-.00	-.00
MR	6765.99	6949.98	6949.98	183.99	2.72	183.99	2.72	-.00	-.00
AP	6888.24	6599.14	6599.14	-289.10	-4.20	-289.10	-4.20	-.00	-.00
MY	6740.90	6174.62	6174.62	-566.28	-8.40	-566.28	-8.40	-.00	-.00
JN	5354.90	5605.78	5605.78	250.88	4.69	250.88	4.69	-.00	-.00

JL	5607.45	5478.50	5478.50	-128.95	-2.30	-128.95	-2.30	-.00	-.00
AG	6377.37	6032.55	6032.55	-344.82	-5.41	-344.82	-5.41	-.00	-.00
SP	6645.85	6779.04	6779.04	133.19	2.00	133.19	2.00	-.00	-.00
OC	7818.69	7446.66	7446.66	-372.03	-4.76	-372.03	-4.76	-.00	-.00
NV	6851.24	7132.79	7132.79	281.55	4.11	281.55	4.11	-.00	-.00
DC	6950.69	7208.01	7208.01	257.31	3.70	257.31	3.70	-.00	-.00
--1969--									
JA	7187.93	7146.26	7146.26	-41.67	-.58	-41.67	-.58	-.00	-.00
FB	6584.04	6617.13	6617.13	33.09	.50	33.09	.50	-.00	-.00
MR	7134.27	7453.72	7453.72	319.45	4.48	319.45	4.48	-.00	-.00
AP	7105.30	7093.03	7093.03	-12.27	-.17	-12.27	-.17	-.00	-.00
MY	6215.84	6275.28	6275.28	59.44	.96	59.44	.96	-.00	-.00
JN	5628.08	5645.31	5645.31	17.23	.31	17.23	.31	-.00	-.00
JL	5744.47	5462.80	5462.80	-281.66	-4.90	-281.66	-4.90	-.00	-.00
AG	5737.29	5902.77	5902.77	165.48	2.88	165.48	2.88	-.00	-.00
SP	6636.31	6561.15	6561.15	-75.17	-1.13	-75.17	-1.13	-.00	-.00
OC	7173.74	7248.20	7248.20	74.46	1.04	74.46	1.04	-.00	-.00
NV	5977.35	6604.12	6604.12	626.77	10.49	626.77	10.49	-.00	-.00
DC	6552.24	6813.18	6813.18	260.94	3.98	260.94	3.98	-.00	-.00
--1970--									
JA	6442.95	6544.78	6544.78	101.82	1.58	101.82	1.58	-.00	-.00
FB	5749.76	6023.56	6023.56	273.80	4.76	273.80	4.76	-.00	-.00
MR	6701.97	6944.83	6944.83	242.86	3.62	242.86	3.62	-.00	-.00
AP	6902.21	6621.00	6621.00	-281.21	-4.07	-281.21	-4.07	-.00	-.00
MY	6009.21	5947.17	5947.17	-62.04	-1.03	-62.04	-1.03	-.00	-.00
JN	5741.89	5547.12	5547.12	-194.77	-3.39	-194.77	-3.39	-.00	-.00
JL	5711.10	5457.82	5457.82	-253.28	-4.43	-253.28	-4.43	-.00	-.00
AG	5981.59	6143.47	6143.47	161.88	2.71	161.88	2.71	-.00	-.00
SP	7038.19	7042.93	7042.93	4.74	.07	4.74	.07	-.00	-.00
OC	7759.47	7839.57	7839.57	80.10	1.03	80.10	1.03	-.00	-.00
NV	7494.86	7503.66	7503.66	8.81	.12	8.81	.12	-.00	-.00

DC	8207.06	7819.16	7819.16	-387.90	-4.73	-387.90	-4.73	-.00	-.00
--1971--									
JA	7730.81	7730.86	7730.86	.04	.00	.04	.00	-.00	-.00
FB	6596.45	7360.95	7360.95	764.50	11.59	764.50	11.59	-.00	-.00
MR	8484.96	8241.49	8241.49	-243.47	-2.87	-243.47	-2.87	-.00	-.00
AP	7983.97	7876.22	7876.22	-107.75	-1.35	-107.75	-1.35	-.00	-.00
MY	7020.20	7078.82	7078.82	58.62	.84	58.62	.84	-.00	-.00
JN	6934.12	6443.44	6443.44	-490.67	-7.08	-490.67	-7.08	-.00	-.00
JL	6273.01	6114.13	6114.13	-158.88	-2.53	-158.88	-2.53	-.00	-.00
AG	6866.70	6471.81	6471.81	-394.89	-5.75	-394.89	-5.75	-.00	-.00
SP	7383.87	6911.90	6911.90	-471.97	-6.39	-471.97	-6.39	-.00	-.00
OC	7251.15	7519.69	7519.69	268.54	3.70	268.54	3.70	-.00	-.00
NV	7617.16	6827.43	6827.43	-789.73	-10.37	-789.73	-10.37	-.00	-.00
DC	7630.63	6814.99	6814.99	-815.63	-10.69	-815.63	-10.69	-.00	-.00

Appendix 9: The Impact of the Open-Loop Control Policy. (See Chapter VI)

Time	Hog-Corn Price Ratio			Sows Farrowing			Barrows and Gilts Sold		
	Uncontrolled	Controlled	Difference	Uncontrolled	Controlled	Difference	Uncontrolled	Controlled	Difference
64 1	14.97	16.46	-1.49	792.48	784.28	8.20	7214.59	7338.32	-112.73
64 2	14.77	16.45	-1.68	1095.51	1079.97	15.54	6609.68	6723.07	-113.39
64 3	14.15	15.96	-1.81	1595.24	1574.03	21.21	7328.68	7425.97	-97.29
64 4	13.77	15.65	-1.89	1503.66	1479.14	24.52	6918.54	6997.14	-78.60
64 5	14.23	16.13	-1.90	1075.47	1050.49	24.98	6037.17	6097.56	-60.39
64 6	15.38	17.26	-1.88	938.46	916.17	22.29	5566.34	5611.55	-45.21
64 7	16.39	18.23	-1.84	914.68	898.27	16.41	5162.93	5197.75	-34.83
64 8	16.44	18.24	-1.80	1166.42	1158.89	7.53	5581.95	5612.04	-30.10
64 9	15.53	17.30	-1.77	1424.12	1428.03	-3.91	6388.28	6419.21	-30.93
64 10	14.43	16.18	-1.75	930.19	947.46	-17.27	6888.34	6924.76	-36.41
64 11	13.98	15.70	-1.72	412.56	44.30	-31.74	6789.69	6834.78	-45.09
64 12	14.33	16.10	-1.67	551.37	597.77	-46.40	6806.93	6862.16	-55.23
65 1	14.90	16.46	-1.56	724.01	784.28	-60.26	6916.92	6982.06	-65.14
65 2	15.11	16.45	-1.34	1007.60	1079.97	-72.37	6106.75	6180.21	-73.46
65 3	14.98	15.96	-.99	1492.20	1574.03	-81.83	8901.18	6980.50	-79.33
65 4	15.18	15.65	-.47	1391.24	1479.14	-87.90	6330.47	6412.95	-82.49
65 5	16.34	16.13	.21	960.46	1050.49	-90.03	5448.17	5531.40	-83.23
65 6	18.30	17.26	1.03	828.26	916.17	-87.91	4914.91	4997.16	-82.25
65 7	20.20	18.23	1.97	816.80	898.27	-81.47	4686.26	4766.70	-80.44
65 8	21.20	18.24	2.96	1087.97	1158.89	-70.92	5142.37	5220.98	-78.60
65 9	21.22	17.30	3.91	1371.31	1428.03	-56.72	5813.78	5891.02	-77.24
65 10	20.94	16.18	4.76	907.90	947.46	-39.56	6215.38	6291.72	-76.34
65 11	21.12	15.70	5.42	423.98	444.30	-20.32	5888.97	5964.30	-75.33
65 12	21.80	16.00	5.80	597.77	597.77	.00	5835.46	6463.77	-628.31
66 1	22.34	16.46	5.87	804.59	784.28	20.32	6371.93	7812.13	-640.20
66 2	22.06	16.45	5.61	1119.53	1079.97	39.56	5873.14	5894.32	-621.19
66 3	20.99	15.96	5.02	1630.75	1574.03	56.72	6566.19	7135.30	-569.11
66 4	19.81	15.65	4.16	1550.06	1479.14	70.92	6281.34	6765.29	-483.95
66 5	19.21	16.13	3.08	1131.96	1050.49	81.47	5415.00	5783.06	-368.07
66 6	19.14	17.26	1.88	1004.08	916.17	87.91	5480.82	5707.06	-226.24
66 7	18.89	18.23	.66	988.30	898.27	90.03	4947.56	5013.06	-65.49
66 8	17.77	18.24	-.47	1246.79	1158.89	87.90	6042.76	5937.41	105.35
66 9	15.85	17.30	-1.44	1509.86	1428.03	81.83	6679.40	6403.15	276.25
66 10	13.99	16.18	-2.19	1019.83	947.46	72.37	7422.15	6985.40	426.75
66 11	13.04	15.70	-2.66	504.56	444.30	60.26	7440.73	6863.92	576.81
66 12	13.13	16.00	-2.87	644.17	597.77	46.40	7151.47	6463.77	687.70
67 1	13.63	16.46	-2.83	816.02	784.28	31.74	7774.83	7012.13	762.70
67 2	13.86	16.45	-2.59	1097.24	1079.97	17.27	6692.05	5894.32	797.72
67 3	13.74	15.96	-2.22	1577.94	1574.03	3.91	7926.95	7135.30	791.66
67 4	13.86	15.65	-1.80	1471.61	1479.14	-7.53	7511.72	6765.29	746.43
67 5	14.75	16.13	-1.38	1034.08	1050.49	-16.41	6449.86	5783.06	666.80
67 6	16.23	17.26	-1.03	893.88	916.17	-22.29	6266.96	5707.06	559.88
67 7	17.43	18.23	-.80	873.29	898.27	-24.98	5447.53	5013.06	434.48
67 8	17.55	18.24	-.69	1134.37	1158.89	-24.52	6237.68	5937.41	300.27
67 9	16.59	17.30	-.71	1406.82	1428.03	-21.21	6570.10	6403.15	166.95
67 10	15.35	16.18	-.83	931.92	947.46	-15.54	7028.83	6985.40	43.43
67 11	14.67	15.70	-1.03	436.10	444.30	-8.20	6801.05	6863.92	-62.87
67 12	14.73	16.00	-1.26	597.77	597.77	.00	6317.26	6463.77	-146.51
68 1	14.97	16.46	-1.49	792.48	784.28	8.20	6807.79	7012.13	-204.34
68 2	14.77	16.45	-1.68	1095.51	1079.97	15.54	5658.82	5894.32	-235.51
68 3	14.15	15.96	-1.81	1595.24	1574.03	21.21	6893.94	7135.30	-241.36
68 4	13.77	15.65	-1.89	1503.66	1479.14	24.52	6540.23	6765.29	-225.06
68 5	14.23	16.13	-1.90	1075.47	1050.49	24.98	5591.89	5783.06	-191.17
68 6	15.38	17.26	-1.88	938.46	916.17	22.29	5561.96	5707.06	-145.11
68 7	16.39	18.23	-1.84	914.68	898.27	16.41	4920.39	5013.06	-92.66
68 8	16.44	18.24	-1.80	1166.42	1158.89	7.53	5897.88	5937.41	-39.53
68 9	15.53	17.30	-1.77	1424.12	1428.03	-3.91	5412.21	6403.15	9.05
68 10	14.43	16.18	-1.75	930.19	947.46	-17.27	7034.00	6985.40	48.60
68 11	13.98	15.70	-1.72	412.56	444.30	-31.74	6939.42	6863.92	75.50
68 12	14.33	16.00	-1.67	551.37	597.77	-46.40	6550.90	6463.77	87.13
69 1	14.90	16.46	-1.56	724.01	784.28	-60.26	7093.97	7012.13	81.84
69 2	15.11	16.45	-1.34	1007.60	1079.97	-72.37	5953.30	5894.32	58.97
69 3	14.98	15.96	-.99	1492.20	1574.03	-81.83	7154.11	7135.30	18.81
69 4	15.18	15.65	-.47	1391.24	1479.14	-87.90	6727.99	6765.29	-37.42
69 5	16.34	16.13	.21	960.46	1050.49	-90.03	5675.50	5783.06	-107.56
69 6	18.30	17.26	1.83	828.26	916.17	-87-91	5518.53	5707.06	0188.53
69 7	20.20	18.23	1.97	816.80	898.27	-81.47	4736.74	5013.06	-276.32
69 8	21.20	18.24	2.96	1087.97	1158.89	-70.92	5571.33	5937.41	-366.08
69 9	21.22	17.30	3.92	1371.31	1428.03	-56.72	5950.90	6403.15	-452.25
69 10	20.94	16.18	4.76	907.90	947.46	-39.56	6456.63	6985.40	-528.78
69 11	21.12	15.70	5.42	423.98	444.430	-20.32	6274.48	6863.92	-589.44
69 12	21.80	16.00	5.80	597.77	597.77	.00	5835.46	6463.77	-628.31

Appendix 9: Continued.

Time	Hog-Corn Price Ratio			Sows Farrowing			Barrows and Gilts Sold		
	Uncontrolled	Controlled	Difference	Uncontrolled	Controlled	Difference	Uncontrolled	Controlled	Difference
70 1	22.34	16.46	5.87	804.59	784.28	20.32	6371.93	7812.13	-640.20
70 2	22.06	16.45	5.61	1119.53	1079.97	39.56	5273.14	5894.32	-621.19
70 3	20.99	15.96	5.02	1630.75	1574.03	56.72	6566.19	7135.30	-569.11
70 4	19.81	15.65	4.16	1550.06	1479.14	70.92	6281.36	6765.29	-483.95
70 5	19.21	16.13	3.08	1131.96	1050.49	81.47	5414.99	5783.06	-368.07
70 6	19.14	17.26	1.88	1004.08	916.17	87.91	5480.82	5787.06	-226.24
70 7	18.89	18.23	.66	988.30	898.27	90.03	4947.56	5013.06	-65.49
70 8	17.77	18.24	-.47	1246.79	1158.89	87.90	6042.76	5937.41	105.35
70 9	15.85	17.30	-1.44	1509.86	1428.03	81.83	6679.30	6403.15	276.25
70 10	13.99	16.18	-2.19	1019.83	947.46	72.37	7422.15	6985.40	436.75
70 11	13.04	15.70	-2.66	504.56	444.39	60.26	7440.73	6863.92	576.81
70 12	13.13	16.00	-2.87	644.17	597.77	46.40	7151.47	6463.77	687.70
71 1	13.63	16.46	-2.83	816.02	784.28	31.74	7774.83	7012.13	762.70
71 2	13.86	16.45	-2.59	1097.24	1079.97	17.27	6692.05	5894.32	797.72
71 3	13.74	15.96	-2.22	1577.94	1574.03	3.91	7926.95	7135.30	791.66
71 4	13.86	15.65	-1.80	1471.61	1479.14	-7.53	7511.72	6765.29	746.43
71 5	14.75	16.13	-1.38	1034.08	1050.49	-16.41	6449.86	5783.06	666.80
71 6	16.23	17.26	-1.03	893.88	916.17	-22.29	6266.94	5787.06	559.88
71 7	17.43	18.23	-.80	873.29	898.27	-24.98	5447.53	5013.06	434.48
71 8	17.55	18.24	-.69	1134.37	1158.89	-24.52	6237.68	5937.41	300.27
71 9	16.59	17.30	-.71	1406.82	1428.03	-21.21	6570.10	6483.15	166.95
71 10	15.35	16.18	-.83	931.92	947.46	-15.54	7028.83	6985.40	43.43
71 11	14.67	15.70	-1.83	436.10	444.30	-8.20	6801.05	6863.92	-62.87
71 12	14.73	16.01	-1.26	597.77	597.77	.00	6317.26	6463.77	-146.51

Appendix 10: Hog Production Simulator Code

Line Reference	Variable Name		Remarks
13	SD	Standard deviation	Constant
14	JS	Switch to convert program from nonrecursive system	
15	IT	Index	Deals with unequal data time series length
16	W	Fourier Series Constant	
18	SFI	Sows Farrowing	Array of Reported Data
19	HP	Hog Price	Reported data
20	CP	Cow Price	" "
21	TMHGI	Hogs slaughtered	Reported Total
22	BGSI	Barrows and Gilt Sold	" "
23	SOWSI	Sows Slaughtered	" "
24	TLWI	Total live weight	" "
25	AVLWF	Average live weight	Reported
28	P	Market allocation constant	
31	SPPL	Pigs per litter	Seasonal variation
33	IMN	Monthly name array	
39	NR	Time index	
40	NYR	Year index	
41	NM	Month index	
43	NT	Time = IT + NR	
44	NL	NR + 12	
45	AK	Pigs per litter	
46	HCPR	Hog corn price ratio	[HCPR - for time lag]
47	BGS	Barrows and Gilts sold	BGS = BGSI
148	BGS	Barrows and Gilts sold	Estimated
48	SWS	Sows sold	SWS = SOWSI
49	BRS	Boars sold	Reported
50	IYR	Year	
64	WPA	Pigs 1 to 30 days	Initial value
65	WPB	Pigs carryover from WPA	" "
66	PIGSA	Inventory adjustment on WPB	Initial value
67	PIGSB	" " " PIGSA	" "
68	PIGSC	" " " PIGSB	" "
69	PIGSD	" " " PIGSC	" "
70	PIGSE	Inventory Adjustment on PIGSD	" "
71	PIGSF	" " " PIGSE	" "
72	BUARS	Male hogs to Breeding Herd	From PIGS F
73	GILTS	Female hogs to breeding herd	From PIGS F
74	TFH	Total Females held in breeding herd	
75	TMALES	Total males held in breeding herd	
88	X	Monthly dummy variables	
115	SFT4	Sow farrowed at T + 4	Estimated
	TFBD	Bred females required to yield SFT4	
123	SGS	Sows sold	Estimated
134	DMALES	Boars required to service TFH	
136	SMALES	Boars sold	By deduction
156	AVLW	Average live weight on BG3	Estimated
175	FA	BGS under 200# (Pct of BGS)	Normal curve estimate
176	FB	" 200 - 220 # " " "	" " "
177	FD	" 240 - 270 # " " "	" " "
178	FE	" 270 and above" " "	" " "
179	FC	" 222 - 240	By deduction
180	BGAN	Barrows and Gilts, Number under 200#	
181	BGBN	Barrows and Gilts, Number 200-220	
182	BGCIV	" " " " 240-270	
183	BGDN	" " " " 270 +	
184	BGEN	" " " " 220 - 240	
191	YB	BGS Register Total	
192	RP	Fraction used in allocation scheme	
193	TSP__	BGS register by age group	
194	BP_	Zero - one variable	
199 - 206	SPG__	Adjusted BGS by weight group	
241	Y	Total hogs sold	
248 - 254	D	Simulated deviation from real world	Percent
261	FF	Ratio of BGS to targeted BGS	
332	__S	Stored variable for__	
356 - 362	DIF__	Difference between reported and simulated	