

AN EXPLICATION OF THE LOGICAL
MODEL OF ROLE SYSTEMS

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This is to certify that the

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An Explication of the Logical
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A handwritten signature in cursive script, reading "Sant F. Camilleri".

Major professor

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ABSTRACT

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by Francis Montgomery Sim

Concepts of status, position, role, etc., are fundamental in social sciences. Increasing emphasis has been given to defining role concepts in terms of relationships in a system, such relationships being of several sorts, e.g., norms, expectations, patterns of behavior, etc. Any position may have relationships with several other positions; a role system contains a set of positions connected in this way.

This work explicates the formal moments of the concept of a role system. It is asserted that positions and roles must be specified interdependently; roles are taken to be sets of relationships between positions, and each position is specified by its relationships with other positions. Neither position nor role is substantively prior. The logical structure of this conception is coordinated to the mathematical structures studied in the theory of multilinear directed graphs or "nets." In showing the coordination, it is convenient to take the concepts of position and relation as primitive;

then an "axiom of relation" is stated, and role and other related concepts (especially "role sector," and qualifiers "focal" and "counter") are defined in terms of the primitives and axiom. A binary matrix representation isomorphic to a net is also introduced, since it is more convenient for some purposes.

In concrete role systems, positions are occupied by actors, and this requires representation in the logical structure. The term "actor" is taken as an additional primitive (though it is intended to include any social object), and the necessary representation is entailed by an "axiom of incumbency." This axiom states that any actor in a position must have the relations of that position with some actor(s) in each of the other related positions. It allows any actor to occupy one or more positions in the system. Further, it guarantees some "mapping" of actors, and of relationships among them, into positions and roles. Such mappings depend on the number and kind of additional restrictions used to define any particular role system, and some possible restrictions and substantive interpretations of them are explored. In general, there are many possible mappings for any set of actors and relations.

There are several salient features of this reconstruction of the concepts. It emphasizes the selection of a set of relations, in terms of which

positions and roles -- and thereby the role system -- are to be defined, as a crucial analytic requirement for the investigator. Also, while the model allows public identification of positions, etc., it does not require it, and it specifically eliminates identities as defining characteristics of positions; a role system may be either latent or manifest (or mixed). Finally, the incumbency axiom appears to be a unique statement of a necessary concept. Suggestions are made for continuations of analysis of the formal characteristics of role systems.

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CONTENTS

Chapter	Page
I. EXPLICATION AND MODELS	1
Explication	1
Models	9
"Explicational Models"	15
Purpose and Overview	28
II. THE ROLE CONCEPTS	30
State of Agreement on the Concepts	31
Selective Emphases and Organiza-	
tion of the Review	34
The Elements of the System	40
The Variety of Concepts	40
Benchmarks: Behavior, Structure,	
and the Emphasis on Consensus	42
Norms	54
Expectations	56
Minority Views	62
Summary of Themes	65
Attributive and Relational Con-	
cepts of Role	68
Positions and Actors	73
The Role System	76
The Graphic Model of Role Systems	83
III. PRIMITIVES, DEFINITIONS, AND AXIOMS	89
Primitive and Defined Terms and	
the Axiom of Relation	89
The Incumbency Axiom	96
IV. THE FORMAL MODEL	.114
Linear Graphs and Matrices	.114
Coordination of the Systems	.120
The Incumbency of Actors	.126
Comparison With Other Formali-	
zations	.131

Chapter	Page
V. MAPPING ACTORS INTO ROLE SYSTEMS. . . .	.140
Mapping Procedures	.141
Assumption of Exhaustiveness	.141
Agreement	.143
Canonical Form.	.144
Maximality	.145
Logical Restrictions.	.147
Symmetry.	.147
Quantification	.151
Connectedness	.152
Iteration on G_p and $G(A)$	.153
Bookkeeping Matrices	.154
Equivalence	.155
Problems of Equivalence.	.157
Nesting	.161
Types of Structures	.164
Some Substantive Coordinations	.170
VI. ROLE CONCEPTS REVISITED	.179
Summary of the Model and Some	
Interpretations	.180
Evaluation of the Explication	.193
Suggestions for Further Research	.198
BIBLIOGRAPHY	.205

CHAPTER I

EXPLICATION AND MODELS

The work reported in this thesis undertakes an analysis and reconstruction of several related concepts -- particularly status or position, and role -- widely employed as elements of social systems, and it purports to show how we may conceive of the logical form of a role system. The discussion in this first chapter is presented as a sketch of some facets of the context in which the writer believes the work fits and should be seen. We will make a statement on the general procedure employed before developing the problem treated with it.

This introduction is directed primarily to explaining our meaning in regard to some terms which appear in the title, especially "explication" and "model." Received definitions require emendation in order to be applied here (and, we believe, elsewhere as well). Our brief critique of them is a convenient vehicle for the contextual sketch.

Explication

In contemporary usage, the term and concept "explication" was given its initial sense in the statements of the

philosopher Rudolf Carnap,¹ though it appears to have acquired further import in use. It refers to characteristic problems and forms of concept formation and definition, and generally indicates efforts toward redefinition which attempt to clarify and perhaps extend the meanings of concepts. Carnap proposed that the term be used in situations ". . . where a concept already in use is to be made more exact or, rather, is to be replaced by a more exact new concept."² In this case, ". . . we call the old concept, used in a more or less vague way either in every-day language or in an earlier stage of scientific language, the explicandum; the new, more exact concept which is proposed to take the place of the old one the explicatum."³ As these passages suggest, he lays particular emphasis on the vagueness of the explicandum and the precision required of the explicatum.

¹First, in his "The Two Concepts of Probability," in H. Feigl and M. Brodbeck, eds., Readings in the Philosophy of Science, New York: Appleton-Century-Crofts, 1953, pp. 438-455; and more fully in Logical Foundations of Probability, Chicago: The University of Chicago Press, 1950, Chapter 1.

²Carnap, "The Two Concepts of Probability," . . . , p. 455.

³Ibid. The philosophic reasons for this exact choice of terms are given in Logical Foundations of Probability, p. 3, but they are not required here.

Hempel, in his work on concept formation, places explication in the context of more usual types of definition. He notes that it is an especially important kind of "real" (as opposed to "nominal") definition.⁴ He suggests that explication goes beyond the "Meaning analysis, or analytic definition . . ." of real definition which aims at ". . . characterizations of approximately uniform patterns of usage."⁵ An explication ". . . combines essential aspects of meaning analysis and empirical analysis."⁶ He introduces the latter element not to indicate that an explication requires immediate examination of relevant evidence, but to denote the fact that it is concerned with the intended range of reference of the concept as well as with internal consistency of usage. Insofar as this extended purpose is involved in the analysis, Hempel suggests that explication transcends definition as such and becomes concerned with "scientific explanation"; the focus becomes concept formation more than terminological specification.

⁴C. G. Hempel, "Fundamentals of Concept Formation in Empirical Science," International Encyclopedia of Unified Science, Vol. II, no. 7, Chicago: The University of Chicago Press, 1952, especially pp. 2-14. A nominal definition introduces a (new) term to stand as a shorthand for an established usage. Hempel notes that explicational procedure is sometimes ". . . called logical analysis or rational reconstruction." (p. 11).

⁵Ibid., p. 10.

⁶Ibid., p. 12

Carnap notes ". . . one of the puzzling peculiarities of explication,"⁷ viz., that the explicandum is vague, thus the ". . . problem itself is not stated in exact terms; and yet we are asked to give an exact solution."⁸ He emphasizes that it is not possible to decide whether a solution is right or wrong, though it may be possible to say whether it is satisfactory, or more or less satisfactory than some alternative. Similarly, Hempel states that,

An explication sentence does not simply exhibit the commonly accepted meaning of the expression under study but rather proposes a specified new and precise meaning for it.

Explications, having the nature of proposals, cannot be qualified as being either true or false.⁹

In these circumstances, then, one does not have recourse to usual logical or material tests of validity; so that -- in lieu of other canons -- personal taste, arbitrary convention, or more likely "whatever works" would have to be taken as the guide. Both authors stress criteria for an explication, apparently in order to avoid a purely pragmatic test of adequacy. Carnap suggests

⁷Carnap, Logical Foundations . . . , p. 4.

⁸Ibid.

⁹Hempel, op. cit., p. 11.

that an adequate explicatum should be characterized by
 ". . . (1) similarity to the explicandum, (2) exact-
 ness, (3) fruitfulness, (4) simplicity."¹⁰ The last
 item is distinctly subsidiary to the others, and amounts
 to usual notions of parsimony. The first item is con-
 siderably qualified by the assumed vagueness and more
 importantly by conflict with the third.¹¹ The second
 and third criteria refer respectively to syntactic pre-
 cision and to empirical and/or conceptual relevance.
 Hempel redevelops them (and the first as well) in
 observing that explications,

. . . are by no means a matter of arbitrary
 convention for they have to satisfy two
 major requirements: First, the explicative
 reinterpretation of a term, or -- as is often
 the case -- of a set of related terms, must
 permit us to reformulate, in sentences of a
 syntactically precise form, at least a large
 part of what is customarily expressed by
 means of the terms under consideration.
 Second, it should be possible to develop, in
 terms of the reconstructed concepts, a com-
 prehensive, rigorous, and sound theoretical
 system.¹²

¹⁰Carnap, Logical Foundations . . . , p. 5.

¹¹Carnap does not state the matter in just this
 way, but his discussion clearly implies this priority;
ibid., p. 5f. In preparation for his discussion on
 these requirements he remarks on treatment of the expli-
 candum, and we return to this point below.

¹²Hempel, loc. cit.

However, these instructions to produce well-formed and effective constructs do not seem -- at least to the present writer -- to apply with unique force to explication as compared to other modes of concept formation. The flavor is more hortative than directive; the objectives of the procedure (e.g., precision, clarity, specificity, extension) are restated, but there are not additional guides for deciding when they have been reached, or, more important, when they are being approached. We stress that the authors have stated a normative ideal, not a method.

There are several points about this analysis which need consideration here. It is our opinion that the emphasis on the vagueness of the explicandum (especially by Carnap) is overdone. Sometimes the apparent lack of precision is due to unnoticed instances of what is technically better called ambiguity -- Carnap's own analysis of probability is an example. More than this, there does not seem to be any a priori reason for believing that all aspects of any given concepts would be equally problematic in respect to vagueness, or even that any would be, or be recognized as such.

Further, it is certain that Carnap does not believe that the explication proper (i.e., the solution) begins with the explicandum in its "natural" obscurity.

There is a temptation to think that, since the explicandum cannot be given in exact terms anyway, it does not matter much how we formulate the problem. But this would be quite wrong. On the contrary. . . we must, in order to prevent the discussion of the problem from becoming entirely futile, do all we can to make at least practically clear what is meant as the explicandum.¹³

This means that reasonably close attention must be given to the sense and use of the concepts to be explicated. For example, in our own analysis, it would be improper to begin by asking, in effect, "What is a role?" and straight-away to begin looking for a solution, ". . . without first examining the tacit assumption that the terms of the question are at least practically clear enough to serve as a basis for an investigation."¹⁴ The fact that we do not take these preliminaries to be part of an explication as such also reflects the freedom permitted in choosing a solution, which both writers stressed in different ways.

In the emphasis on vagueness there seem to be assumptions that what is vague is also wrong, and that what is right is also precise. Moreover, there appears to be a tendency to convert the latter proposition; i.e., what is precise is also right, or at least more right than what is not precise. To be sure, exactness

¹³Carnap, Logical Foundations . . . , p. 4.

¹⁴Ibid.

is a condition for assessing the correctness of an explicative proposal, but it certainly does not assure that the proposal is correct. Our own analysis will yield examples of proposals which are precise, and precisely wrong. The point we wish to emphasize is that, of the suggested criteria, "fruitfulness" is both the salient purpose and test of an explication; the other "internal" criteria are important in that they contribute to this and to our ability to judge it.

There is one final point of reinterpretation needed for our work. Although Hempel notes that often several terms are involved in an explication, both he and Carnap treat the subject as though the concepts being analyzed are all of a piece. The explication is to provide a whole new meaning. Our own view is that it is possible to undertake explication of certain facets of a set of concepts without attempting to adjudicate competing formulations of other aspects. In this event we should not expect improved precision beyond those facets treated, but we might anticipate that the analysis would shed some light on other aspects of the concepts. Also, a partial explication cannot be said to result in replacement of the explicandum in the way suggested above, for it explicitly recognizes that some of the pre-existing features have not been qualified directly.

To summarize: By explication we understand the reconstruction and extension of some aspect(s) of a set of related concepts which are already in established, though perhaps disparate, usage. The results are in some respects novel relative to that usage; they bring to attention what had gone unnoticed and propose appropriate incorporations. Thus, the first step is to assay the state of the concepts, but the proposal is not bound by them -- it may (perhaps must) transcend that state, consistent with the objective of improved understanding. It is this objective which places stress on precision; exactness is not the test of adequacy, but it is an important condition for allowing comparisons with alternatives and empirical referents.

One final note on this subject here. It is the author's opinion that the appeal of the idea of explication (under whatever name) is that it breaks out of the narrow and unreal confines of the traditional philosophic treatments of definition, and comes closer to formulating the conditions of disciplined reanalysis which is characteristic of cumulative concept formation in science.

Models

The concept of a model has excited considerable attention in recent scientific and philosophic discussions,

and it has been the object of explication as an essential of the scientific tool kit. Here we attempt to summarize only some important points and to say what sense is relevant to our analysis.

Particularly on the philosophic side, emphasis has been placed on the relationship between models and theories, especially in terms of formal or syntactic aspects. The key concept in diagnosing this relationship is isomorphy. The usual development is to say that two sets of propositions -- each of which may have some conceptual intentions, but not necessarily the same ones -- are isomorphic if they have the same logical form, i.e., there is one-to-one correspondence between their elements and the same kind of relations are observed among them.¹⁵ The relations are usually taken to be of the form of laws. An alternative way of stating the relationship is in terms of the underlying logical calculus or formal deductive apparatus of the systems, and while this leads to some difference of emphases the general result is equivalent.¹⁶ A model is

¹⁵This follows Brodbeck's statement most closely; see M. Brodbeck, "Models, Meaning, and Theories," in L. Gross, ed., Symposium on Sociological Theory, New York: Harper and Row, 1959, pp. 373-403.

¹⁶E.g., R. B. Braithwaite, Scientific Explanation, New York: Harper and Brothers, 1960, especially Chapter IV; or E. Nagel, The Structure of Science, New York: Harcourt, Brace & World, 1961, Chapter 5 (and also 6).

isomorphic to a theory for which it is a model, but they are not the same and the difference is denoted by saying that one or the other or both (depending on the author) is "interpreted"; while the elements of each are formally coordinate, they have different referents. This interpretation provides a vehicle by which the model becomes a convenient reasoning device for generating results relevant to the theory. The manner of emphasis on a formal deductive system is not uniform among all such discussions (the emphasis on a strict logical calculus reaches its nadir in Braithwaite's analysis), but it is a central theme in the philosophic treatment and the effect is in part pernicious. It results in a conceptual tendency to narrow the range of reference of "theory" -- and therefore of "model" -- to efficacious instances. Strictly speaking, deduction is not a defining requirement of theories, though it is generally supposed to be a requisite of effective ones. Any conjunction of propositions with the same range of reference constitutes a theory, even if not a powerful one. At least, it is only in this sense that we can understand how scientists use the concept of a model (and still save the form of the philosophic analysis) when the object is not a well-formed theory (i.e., an effectively deductive one). And scientists do this, and do it effectively, as we shall see in the next section.

There is another idea of some importance in our concept of a model which usually receives little, if any, notice in these discussions, but it is common both to the model-theory circumstance and to other instances of modelling as well. We refer to the fact that a model is always in some sense "less than" what it is a model of. In deductive systems, we refer to the interpretation of an axiom set as a model because it is not as rich as the interpretation. In other circumstances, a toy which is smaller and less detailed than its full-scale counterpart is called a model. Other examples requiring more subtle justification of the point could be given, but it is essential to our sense of the term that a model does not account for all aspects of its object; if it did, it would not be a model. All of this is quite clear in the discussion based on deductive systems, but the point is frequently lost in the tacit assumption that the acquisition of such systems is, anyway, the really important step in the development of powerful theory. It seems to us that the relative poverty of a model needs reemphasis when the characteristics of the object in view do not consist in a well-formed deductive apparatus. A model constitutes an attempt to develop by judicious selection a device for reasoning about the object, but it is not

necessarily a complete calculating system.¹⁷ We alluded to this point above in remarking that an explication may develop only a part of the conceptual equipment being analyzed.

In view of this discussion, it may seem either redundant or anomalous that we qualify our analysis as a "logical model." While we actually develop an axiomatic treatment of the role concepts, this is not taken to be a calculus, or a deductive system (or a set of laws) of importance as such, since the object is not a theory in the narrow sense above.¹⁸ Rather, this is an instrument for defining a set of concepts in terms of certain of their aspects, viz., the logical form of propositions about them. The term "logical" appears in the title to denote the character of the contents of the

¹⁷Brodbeck, op. cit., p. 381f., castigates social scientists for use of the term "model" to refer to theories which do not conceptualize all aspects of the phenomena considered. "All theories . . . omit some variables simply because they are not relevant to the phenomena to be explained Selfconsciousness . . . about such perfectly legitimate omissions seems to be peculiar to social science." Of course, the key term here is "relevant," and we feel that Brodbeck misses the fact that a theory may not be thought to organize all relevant aspects, i.e., there may be external grounds for believing that some omitted variables would be included in a proper theory of the empirical matters in view.

¹⁸That is, a theory as a deductively interrelated set of propositions; this seems to be the common sense of the unqualified term, but we do not adhere to it in the sequel.

inquiry, not the form of the results, even though it can be correctly applied to them. This facet of the role concepts seems to have gone unnoticed, or at least has not been properly apprehended before, and thus requires explication.

Some last remarks on the concept of a model at this point concern the question of interpretation mentioned above. In his discussion, Nagel -- who uses "model" to signify an interpreted calculus (much in the way of mathematicians) -- states that there is still a third component of a theory (besides the calculus and the model), viz., the "Rules of Correspondence." These are ". . . a set of rules that in effect assign an empirical content to the abstract calculus by relating it to the concrete materials of observation and experiment . . ." ¹⁹ The significance for present interests is that conceptual interpretation relevant to a model does not require "operationalizing the concepts," though it does not preclude it. Further, our analysis does not yield an interpretation of the model; it is developed from several somewhat different interpretations of the same concepts.

¹⁹ Nagel, op. cit., p. 90; and see also pp. 97-105.

"Explicational Models"

The heading above is entered in quotations because the phrase is taken directly from the analyses of Berger, Cohen, Snell, and Zelditch.²⁰ The fact that the expression combines two of our key orienting concepts would be warrant for at least considering the relevance of their conception here. But more than this, their work is of interest to us both as evidence regarding some of the directions of the preceding discussion and as a trenchant source of further considerations on the problems of scientific concept formation.

In their work this group has emphasized that -- at least at the present stage of analysis -- the most useful classification of formal models is by the functions they are intended to serve with respect to the theories they are intended to assist.²¹ They assert that these goals differ according to the stages of research and theory development, that characteristics and functions also vary, and that models are helpful at more than one

²⁰Types of Formalization in Small-Group Research, Boston: Houghton Mifflin, 1962.

²¹"By 'formalization' we refer to the general process of making explicit the logical structure of a set of assertions." Ibid., p. 3, N. 1. In this sense, our own analysis would be called a formal model.

stage. We would add (and they clearly imply) that models help in getting from one stage to the next; models are not passive residues of codification of earlier thought, they are important agents in cumulative reconstruction.

In addition to explicational models they also distinguish two other types which they call "representational" and "theoretical-construct" models. We will review all three of them in a brief way first and then return to a summary consideration of their example of the explicational case.

An explicational model deals with some key concept(s) though not with all of the propositions of a theory, and its work is to clarify the status and meaning of the concept in the way we have outlined above. They suggest that to be fruitful such models must be concerned with really central concepts, and that the formalization must be checked closely for its coordination with the performance of the concept in the theory. Expectably, explications may yield an additional reward in new concepts, as we shall see in their example.

A representational model is appropriate to circumstances in which a known social phenomenon (apparently usually a process) is "fitted" by a formalization. (The example which they analyze is Cohen's formalization of the Asch effect in conformity experiments.) They

stress as requirements both simplicity of the representation and that it should give a good fit. In fact, a representational model for a particular process may produce a better approximation to the empirical material than a general explanatory theory which intends to account for a variety of observed processes. The other side of this accuracy is that this intermediate stage is in general subject to the criticism of "curve-fitting." However, it is sometimes the best that can be done, and the authors seem to imply that it marks a step up in prediction from the reconstruction of the first stage.

A theoretical-construct model is the full step up to formalization of a ". . . general explanatory theory . . ." ²² (i.e., one which is well-formed and efficacious). This puts most constraint on the model builder, since simplicity and adequacy of empirical relevance are still required, but one incurs the additional necessity of embodying a set of propositions.

There are some observations to be made about this typology of models and about the model of the research process which we believe to be implied by it. These can be brought out by considering the terms they choose to designate the three types. We noted above that their typology is based on "analyzing" the goals

²²Ibid., p. 67.

(functions) of any particular formalization, and this is the source of the type names.²³ But it appears to the writer that there are other constant features of each situation, and that these could have been used to designate the types.

This can be seen by the conventions that we 1) explicate concepts, 2) represent phenomena, and 3) construct theories. Their types could be called 1) concept, 2) phenomena, and 3) theory models respectively, and better denote the central moment of each. This amounts to shifting attention from the function of the formalization to the structure of each type of situation, and the writer's reading of their work strongly suggests that it is in fact the character of the situation which determines the salient kind of formal assistance required, and not the other way around.

But, we would argue in addition that any particular kind of assistance (explication, representation, construction²⁴) is never more than salient; particularly, in terms of our present interests, it is believed that

²³"Analyzing" is set off here because in no case do they attempt to justify their imputation of goals in a direct way; but in each case the goals are, by any reasonable standard, self-evident.

²⁴This typology suggests a further line of analysis, but it would certainly take us even farther afield into philosophic and methodological questions than seems necessary to the writer.

explication can be used properly to denote certain aspects of the process of formalization in all three of their types. That is, freeing their terms from particular kinds of situations suggests a useful extension of the earlier summary of our understanding of explication. There we said that explication is performed upon "a set of related concepts," and here it is necessary to notice only that all representations of a phenomenal process are made in terms of such a set, and that "related concepts" similarly are fundamental constituents of any well-formed theory. In these terms, explication is relevant to all three of their types of formalization.²⁵ Similarly, while it involves some ambiguity in the use of the term, representation is a function that models perform for concepts as well as for phenomena, and we shall frequently use it in this sense later.

We now need to reemphasize that their specification of kinds of situations seems to us to be essentially correct, and that insofar as a particular instance of formalization approximates to being a "pure" type, the relevant kind of development will be most useful. In our own work the main aim is to render assistance to

²⁵We believe that our commentary also suggests that Berger and his associates have embodied a dialectic conception of the interplay between theory and research, but, again, this observation cannot be pursued here.

a set of concepts, and in this connection it is helpful to review the highlights of their analysis of the example of an explicational model. The example which they use as a type case is the development by Cartwright and Harary of the concept of "balance" in Heider's theory of the same name.²⁶

Heider's theory concerned the organization of relations among elements of the ". . . life-space of a fixed person . . . ,"²⁷ which we will loosely call cognitive structure. This structure is characterized by two kinds of elements, persons and non-persons, and two kinds of relations, which Heider denoted by L and U. L referred to a sentiment and U ". . . covered virtually every relation . . . which was not a sentiment."²⁸ Because of the "life-space" restriction, a relation between some other person and another element (of either kind, including the fixed person) was interpreted as a thought of the fixed person; his own sentiment or activity was expressed by designating him as the first element of the appropriate relation, with another element as second. The only other restriction on correct formation of relations

²⁶Citations to relevant works are in ibid.

²⁷Ibid., p. 11.

²⁸Ibid., p. 10.

between elements was that a non-person could not be the first element of an L relation, e.g., tables and baseball games do not love anything.

In his analysis, Heider considered a variety of examples in order to define a state of balance in such a structure, and we will not review them. He was concerned largely with the case of three elements (of which one was the fixed person), with one relation of either kind between each of the three pairs, and with defining balance in terms of the positive or negative character of each relation.

In general . . . Heider concluded that balance occurs when either all relations in a system are positive or any two are negative and the third positive. In this definition L and U relations were 'exchangeable' in the sense that, in classifying systems as balanced, it made no difference which relations were negative and which positive, so long as two were negative and one positive, or else all were positive.²⁹

He made several propositions about the consequence of imbalance for reorganization of the structure (or, if this was not possible, for the tension state of the person), and other matters as well, but these are not needed directly in our summary of the formalization and we omit them here. Similarly, Heider's conceptions

stimulated a number of interesting experiments which Berger, et al., review, but we may pass over these with their summary observations.

It may be said, then, that in general the body of experimentation stimulated by Heider's theory provided empirical confirmation for some of its principal assertions, but that it also pointed to the need to extend the theory beyond its original scope and provided evidence that the original theory was in need of conceptual clarification.³⁰

Such a clarification was made for the concept of balance (as defined in a quotation above) by Cartwright and Harary using the theory of linear graphs. This is of interest to us both as an explicational model and because we use the same general kind of formal apparatus in our own analysis. Since we review concepts of linear graphs at a later point (see Chapter IV), we will be brief here.

The theory of linear graphs deal with finite collections of points, and lines between pairs of these points. A set of points, all, some, or none of which is connected by these lines, is called a graph.³¹

In our review above we have used the terms "structure" and "element" and they may be equated to "systems" and "entity" respectively in the following.

To the formal definitions of graph theory . . . coordinate the concepts . . .

³⁰Ibid., p. 19.

³¹Ibid., p. 20.

according to the following rules of interpretation: each point in a graph is an 'entity' in Heider's theory; each line in a graph is a 'relation' . . .; a graph may then represent a system of such entities and relations For each kind of graph a definition of a balanced graph can be made that is consistent with Heider.³²

If positive and negative indications are attached to each line of a graph (thus constituting a "signed" graph), then the concept of balance may be defined in terms of path and cycle. A path is a connected sequence of lines which is not redundant -- the same line does not appear twice -- and a cycle is a closed path. "The sign of a cycle is the product of the signs on all lines of a cycle."³³ This product is negative if there are an uneven number of such signs, and it is positive otherwise. Finally, "A signed graph is balanced if all its cycles are positive."³⁴

Useful features of this explication are quite directly obvious even in the greatly attenuated form given here. For example, since Heider considered only one relation between each pair out of three entities,

³²Ibid., pp. 20-21.

³³Ibid., p. 22.

³⁴Ibid.

there could be only one cycle. In the graphic model, Cartwright and Harary did not limit themselves to systems of three entities, and consequently were able to consider the more general case of having many cycles, some of which might be balanced and others not.³⁵ This led to the concept of degree of balance; this development is well remarked by Berger, et al., in their evaluation of the explication, and we turn to this now.

The authors summarize the "Characteristics and Functions of an Explicational Model" in a series of five major points, which we reproduce with a selected condensation of the exposition.³⁶

1. The model is selective with respect to the original conceptualization. We have noted this feature in our earlier development. Here, the explication deals with the concept of balance, not with all of Heider's propositions. Another example is that sentiments of a person toward himself are not handled since the explication treats irreflexive graphs.

2. The model provides a means of clarifying the original conceptualization. Heider did not distinguish between the complement of a relation and its opposite, but this is easily accomplished in a graph by the distinction between omitting a line and attaching a negation to it. Clarifying nonexistent relations led to the concept of "vacuous" balance -- a graph with no cycles at all -- and this improved the fit of experimental results, the deductive completeness, and the interpretation of the theory.

³⁵The number of possible cycles is increasingly larger as entities are added and may be represented as a factorial function.

³⁶Ibid., pp. 24-36.

3. The model provides a means of refining the original conceptualization. We noted above the extension to more than three points; together with the ideas of partitioning the set of cycles according to 1) inclusion of a point, 2) the number of points (or lines) in a cycle, and 3) the balance of a cycle, this leads to concepts of "balance at a point," "local n-balance," and "degree of balance."

4. The model provides a means of generalizing the original conceptualization. Again, the extension to more than three entities is important, but perhaps more important (from the present writer's view) is the possibility of incorporating more than one relation between a pair of entities. This latter development is more programmatic than others (Cartwright and Harary actually did not extend the theory to higher order graphs), but it has served to locate gaps in the original formulation and its development.

5. The model provides a means for determining implications within the original conceptualization. An especially interesting example occurs in the Cartwright-Harary development of the "structure theorem" which shows that in a balanced graph there must be two mutually exclusive and exhaustive subsets of points such that signs of lines are always positive within a subset and always negative between subsets. This appears to be equivalent to a proposition about balance of segregated entities made by Heider, and it shows that it is actually deducible from others.

We have reproduced the example and discussion of an explicational model in some detail for two reasons. First, it seems useful to give the reader who is not familiar with the original work (or some comparable development) some orientation to the kinds of questions and tools employed through an example of a particularly efficacious explication. Second, it is thought that the explication (!) by Berger, et al., especially the five

points summarized immediately above, provides a very useful extension to our guidelines for evaluation of the results of our own work. However, there are some points of difference in the circumstance and we should take note of them.

There seem to be two salient and related points of difference between the example reviewed and the corpus of concepts to be attacked in our work. In the Cartwright-Harrary formalization of balance there was only one source (or tradition) for the concept and its uses. This has the beneficial effect (for the explicator) of limiting the variety of vagueness and ambiguity confronted in the original conceptions. On the other hand, in the analysis of role concepts carried out in the following chapters we are confronted with a much more diffuse array of meanings and uses, to the point that it is difficult to make classifications of them to show which ones "go together."³⁷ The second, and we believe more consequential, point is that the concept of balance was embedded in a fairly well developed and coherent theory. In the functions enumerated by Berger, et al., many of

³⁷This is relieved somewhat by the focus of our examination on the logical characteristics of the concepts, as we will indicate in the next chapter.

the most important results concern not just the concept of balance as such but the wider theory and its reconstruction.³⁸ In dealing with role concepts we must attend to questions as to what theory exists, and what the consequences of our explication are in this regard. In connection with this we must take notice here of a warning given by Berger and his associates.

It is probable that where the model-builder attempts to explicate a concept which is not part of a significant substantive theory, there will be few . . . constraints on the way in which he formalizes the concept. It is probably also true that in such a case the explication may be a relatively idle exercise, unless a theory is later developed which makes use of the model-builder's particular formulation.³⁹

There are some difficulties in interpreting this injunction on the perils of bootstrap efforts. It is uncertain whether one should emphasize "significant" or "theory" or both, though one can make an informed guess that the last option is intended. It is then necessary to inquire what they mean by "significant theory," and one need not guess here that they mean a well-formed one

³⁸We take this as confirming evidence of the points made earlier concerning our views ". . . that models help in getting from one stage to the next," and that one may significantly speak of explication of a theory as well as of concepts.

³⁹Ibid., p. 104.

which is effective vis-a-vis its intended range of phenomena. However, it is felt that there is an implied counsel of excessive caution; in the absence of a well-formed object for our investigation we had best begin with what is at hand lest one never be developed. The writer shares the concern for responsible and responsive theory construction which is surely behind their notice of the dangers of free thinking, but he does not share their pessimism concerning the cumulation of viable concepts. It is well known that "exercise" sometimes locates and develops ligature previously unknown.

Of course, all of these remarks are directed to clearing away objections to our undertaking herein, and they will be effectively otiose if it proves unsatisfactory. But they do serve to point up our concern with the frequently stated fact that "role theory" is more an aspiration than a reality. We aim to aid the realization of this ambition in the knowledge that such attempts should not be made or discarded lightly. And it seems best to make the statement of our analysis in as definite form as possible in the realization that others will accept and treat it as provisional.

Purpose and Overview

It is our intention to review and examine the concept of role and related concepts as they are defined

when being used as elements of a structure or system. The focus will be on the logical character of certain features of propositions about these concepts so used, and we try to show the consequences regarding formulation of these concepts and the system. After explicating our model of a role system, we further intend to explore the consequences of the model for our conceptions, or in one of the senses of the now familiar phrase "its feedback on the system." Naturally enough, this last task, is saved for the final chapter. The actual model and a solution for a problem it brings out are presented in Chapters IV and V respectively. In order to specify compactly the appropriate formal character of the model, in Chapter III we set out a verbal axiomatization (and some considerations of its justification) which entails the logical structure developed in the succeeding chapters. The first burden of Chapter II is to provide a background upon which the justification of the axioms can be based. It attempts ". . . to make at least practically clear what is meant as the explicandum,"⁴⁰ and we now proceed to this task.

⁴⁰Carnap, Logical Foundations . . . , p. 4.

CHAPTER II

THE ROLE CONCEPTS

The concepts of role¹ have a long and useful history in the literature of sociology, both in theory, either speculative or analytic, and more recently in empirical analyses. The history is also one of controversy, of clarification, specification, definition, and redefinition, of caveats against one or another of the associated terms or ideas, of great anticipations and sometimes small realizations, and of appeals for a common tongue. In spite of their parentage in speculative discourse, the concepts of role have not been relegated to the intellectual dustbin by modern empirical social science. The amount of recent effort toward developing the theoretical refinement and empirical relevance of the concepts is substantial

¹In the sequel, we will sometimes, as here, use the term "role" to stand for a battery of combining and related terms (especially "status," "position," "role behavior," and the like) in order to avoid repetitious locutions. We will also use it in more restricted ways, and the context should indicate our intention.

evidence of their felt importance in sociology, and in the neighboring disciplines of social psychology, social and cultural anthropology, political science, social work and education. There is no need in the present work to justify the importance of the concepts or the empirical referents they assay.

State of Agreement on the Concepts

The task which we have set for this analysis is to examine the concepts as they are used by social scientists with a view to determining their meanings with somewhat greater precision than has been achieved to date. What we hope to show is that there are common features in what sometimes have appeared to be disparate conceptualizations, and that they form the basis for a model of a structure or system of roles as a general conceptual tool. It is clear, however, that other observers do not share the belief that commonalities are noticeable in the definitional schemes that have been offered. Neiman and Hughes in their review of a decade and a half ago presented a pessimistic appraisal of the then current condition of the concepts.² They professed to find little convergence beyond several rather general

²L. J. Neiman and J. W. Hughes, "The Problem of the Concept of Role -- a Re-survey of the Literature," Social Forces, 30 (1951), pp. 141-149.

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assertions about the possibility and general character of the analysis of human behavior, which did not really entail the concept role or any of its relatives.³ They reached the conclusions that the concept was vague, nebulous, non-definitive, reified, little used in research, and used for ad hoc explanation of human behavior. After an intervening decade in which the concepts had come into wider usage in reports of empirical research, Biddle similarly inveighed against a babel of competing formulations and declared that each of the several summaries which had appeared in the interval ". . . has emphasized the lack of structure in the field and has called for rectification of . . . [the manifest dissensus]." ⁴ However, the present view that there is an important measure of consensus among social scientists on the role concept finds support in the recurrent efforts to give expression to the consensus through reconceptualization, through the observable continuity given by the adoption by investigators of the formulations offered by others, and in the conclusions of some of the above-mentioned summary evaluations. Particularly, Gross, Mason and McEachern, in their review of a rather

³Ibid., p. 147.

⁴Bruce J. Biddle, The Present Status of Role Theory, Columbia, Missouri: University of Missouri, Social Psychology Laboratory, 1960, p. 3.

wide range of sources, concluded that certain common themes did occur in almost all conceptualizations, whether or not they were explicitly incorporated in stated definitions.⁵

There is, however, an important difference in the line of argument to be pursued herein with respect to conceptual agreement. Almost all of the conceptual reconstructions alluded to above have taken as their focus the analysis of the substantial character of concepts and of the kinds and ranges of empirical phenomena to which they should apply. In the present analysis we shift the focus to the formal character of the concepts, or what might be termed their logical foundations as a conceptual system. It is tempting to say that what we intend is to "move to a higher level of abstraction," but this phrase carries some inappropriate connotations. As will be seen later, we do not intend to "abstract" in the sense of eliminating the substantive content of the concepts. This content must be used to determine the relevant logical forms, and these forms must be an adequate reflection of content.

⁵N. Gross, W. S. Mason, and A. W. McEachern, Explorations in Role Analysis, New York: John Wiley and Sons, 1958, Chapter 2.

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This shift of focus does make it possible we believe to specify certain dimensions of agreement which transcend particular substantive differences.

Selective Emphases and Organization of the Review

It is commonplace, as we observed above, to notice that different authors and even the same author have spoken with many tongues in the history of the development of role theory. This is true with respect both to the use of different terms or phrases which apparently have the same referent, and to the use of the same term with manifestly different meanings. At a later point, we will settle our own usage on two terms, position and role, and on some combining forms of them in a manner to be specified here. To do this we want to get something of the flavor and the content of these many uses in order to show that ours is not completely idiosyncratic. Fortunately, in view of the quite literally vast interest in the subject, a number of summaries and reviews have appeared, as was remarked above, and they will provide some assistance. However, we will not be able to rely on them entirely, since some of the facets of the concepts which are necessary to our purpose have not been adequately established, and we will want to be able to see at what point certain developments of the concepts enter which are important

to our formulation.⁶ Both the volume of the literature⁷ and our own interests direct us to employ several criteria of selection for the material to be reviewed.

The first restriction that we impose is that we will be interested in what may be termed modern or contemporary views of role concepts. It is our opinion that no useful purpose would be served for our explication by reaching back beyond the last generation of writers for sources.⁸ In saying that our object is an appraisal of current perspective, we are not thereby denying important antecedents; much of the conceptual development can be seen as a "working out" of earlier specifications.

A very salient element of selectivity in our interest relates to one of the major points of significance of the role concepts. It is universally accepted

⁶This last remark should not be taken to mean that we see a unilinear evolution of the concepts, only that certain changes to be noted have emerged, even though they are not uniformly accepted.

⁷In their forthcoming study of role theory, B. J. Biddle and E. Thomas include a bibliography of about fifteen hundred items which they believe to be incomplete even in the particular areas covered. The writer expresses his appreciation for their scholarly aid in making this compilation available.

⁸This office is well performed elsewhere; see footnote 13 on antecedents below, and especially the citation to W. B. Catton, Jr.

that roles are the intersection or articulation of the individual and the larger milieu, however defined, in which he is situated. It is this linking function which gives them their wide substantive and empirical relevance; on the one hand they are crucial to analyses of personality, attitudes, etc., and on the other they are the building blocks of social structure. In this study, we are interested in the latter aspect: the conceptualization of roles as elements of a social system. This does not mean that we may thereby eliminate social psychological views from our review; many, if not most, of the authors who have contributed materially to clarification of the concepts have been much concerned with the linking function. In recent years this interest has found major expression in a proliferation of studies of the sources and effects of role conflict. Our effort will be to attend primarily to structural aspects, introducing these parallel considerations insofar as they have immediate bearing on our concepts of a role system.

In limiting our attention to the structural relevance of role concepts we thus neglect problems of socialization, role learning, and development of selfhood, of role playing and its relationships to characteristics of the individual or personality, of the intra-personal experience and effects of multiple

roles, and in general most of the manifold ways in which role concepts have been employed in analyses of the internal dynamics of the personality system. In effect, we adopt the widely accepted assumption that it is perfectly feasible to ignore these questions, specifically as at present when dealing with roles as parts of a social system.⁹

The concept of a system of roles and related elements provides a general orientation for the present analysis. The concept of system draws our attention in the review of existing conceptualizations to a number of orienting themes and questions: What sort of system is postulated? What are its elements? And in what way are they taken to be related? Are sub-systems assumed? And what other systems (and their characteristics) are required? Such questions tend to imply a univocality of response, which we have already agreed does not exist in detail, and to suggest that simple, explicit answers can be given. This is not the case, but the

⁹This view has been put forth most insistently by Talcott Parsons in many writings, particularly in The Social System (Glencoe: The Free Press, 1951). It is a view which seems "natural" (at least to sociologists!) and is regarded as self-evidently valid. It is worth noting that the concept is of fundamental importance in physical science, where it is referred to as "The independence of components," and it is taken as a postulate of the discipline of physical systems analysis. See H. Koenig, et al., The Analysis of Discrete Physical Systems, East Lansing: Michigan State University, Department of Electrical Engineering, 1964. The present author's conception of a system has been substantially influenced by this work.

orientation does have the merit of forcing us to look for whatever general agreement there is to be found. For the present it is sufficient to say with Parsons that what we have in mind is ". . . a system in the scientific sense . . . ,"¹⁰ i.e., a set of interrelated, or interdependent (or interacting) elements, or parts, or components. The particular system we have in mind (or rather what we believe to be implied by the concepts as they have been used) will become clearer as we proceed, and it will be given explicitly in Chapter IV.

Our review of the concepts will be divided into two main parts, and these roughly correspond to examination of the elements of the system and examination of the composition of the system. We make this do double duty in dividing time periods of the selected writings as well. This serves to point up our view that it is only relatively recently that the idea of a role system has been worked out sufficiently to provide a basis for the kind of explication carried through in the sequel.

The selection of particular writings for our review is somewhat arbitrary. The primary purpose is to illustrate important themes which have been used in specifying the concepts rather than a detailed comparison

¹⁰Parsons, op. cit., p. 3.

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of particular terms. In doing this we have attempted to use the work of authors who have been thought important by succeeding commentators. Also, we have avoided using more than is deemed necessary and consequently we have not included all of those whom others have commended. In most cases, the work of a particular author is discussed at a single point under a topic heading which it seems to illuminate best, even though it reflects other concerns as well. This treatment seems more convenient here than such an alternative as collecting definitions of a single term together for comparison; this is a result of the ambiguities of substantive usage, their essential interdependence which we stress in our reconstruction, and the relevance of the substantive themes for determining the logical characteristics which are the main object of our attention.

While we have not organized the presentation around individual concepts, it will be an aid to the reader to point out that our concern in the first section of the review is with the general sense of role, status or position, and, in a subsidiary way, actor; and that we introduce the idea of relation as a fundamental logical element. The shift of attention to system organization in the second section introduces several qualifying terms for position and role which denote different system aspects. We will leave their enumeration to that point.

The Elements of the System

The Variety of Concepts

In their review of the literature, Neiman and Hughes classified the definitions of the concept role known to them in terms of the contexts and contents of these uses. It is interesting to reproduce the main headings of their classificatory scheme, since they reflect the variety of senses of the single term and emphasize the contextual relevance characteristic of the definitions. With appended names of writers cited as examples, the categories are as follows:¹¹

- A. Definitions in terms of the dynamics of personality development
 - 1. Role as a basic factor in the process of socialization - Park and Burgess, but preeminently G. H. Mead.
 - 2. Role as a cultural pattern - Linton, Znaniecki, Parsons.
- B. Functional definitions in terms of society as a whole
 - 1. Role as a social norm, no connection with status explicit, but implicit - Komarovsky, Sherif, Benedict, M. Mead, Stouffer.
 - 2. Role as a synonym for behavior, non-definitive - Neiman, Hughes.

¹¹ Op. cit. The appearance of the word "Resurvey"(see previous footnote) in the title of this article is somewhat puzzling, since there is no evidence of earlier comparable summarizations.

C. Functional definitions in terms of specific groups

1. Status-role continuity, definitive as activated status - Linton, Hughes, K. Young, Hiller, Znaniecki, Merton, G. Murphy.
2. Role defined as participation in a specific group - Moreno.

These category titles are interesting both for features that they emphasize and for ones that they overlook. Particularly, it is apparent that role has been defined both by what it "does" and by what it "is." For some writers it "functions" in the socialization process and for others in the activation of status. Other writers (and sometimes the same ones) see it as composed of patterns of behavior, perhaps culturally given, or as made up of or oriented by norms.¹² On the other hand, the categories seem to emphasize an idea of a role as a singular element rather than as a factor in relationships between persons, and these relationships are of considerable importance in the literature and in the sequel.

In a later review, Biddle compiled formidable evidence that a great variety of different terms have

¹²This difference, which is generally correlative to the function-structure distinction, points to another factor of selectivity in the formalization to be developed which was implicit in our earlier remarks. The model to be presented is a conception of the structure of a role system, and this directs our attention to consider what the concepts "are" as elements in the system.

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been used with meanings which are -- at least according to the definitions of his own array -- the same (as well as being used frequently with more than one meaning, again according to his distinctions). A few samples will suffice to indicate the extent of this proliferation. For his term "position" Biddle finds 16 other different words or phrases; for his "expectation" he finds 33 other expressions; and for "norm" he enumerates 28 others. Clearly, there is a lesson to be learned from this depressing display. Both Biddle's analysis and one's own experience suggests that more names than concepts are involved. In the following pages we will review some of the important examples from the material, and then attempt to summarize the aspects relevant for setting out our own definitions.

Benchmarks: Behavior, Structure, and the Emphasis on Consensus

An attempt to make appropriate selections presents a rather difficult task. A simple list of authors who have written on and contributed to our concepts would read like an honor roll of the social and behavioral sciences. Similarly, assessment of priority or antecedents for concepts leads to an impressive array of founders.¹³ We have said that we

¹³A few of the persons to whom credit for significant innovation is accorded by recent authors include Pareto, Weber, James, Baldwin, Dewey,

are mainly concerned with the contemporary condition of the concepts, but it is sometimes difficult to distinguish between history and current events. It seems fairly representative of the consensus on conceptual origins to say that two writers whose publications appeared in the decade of the 1930's (one on the side of social psychology and the other on that of social structure, though neither was much disposed to observe boundaries) stand as important intermediaries between the past and the present, and they are George Herbert Mead and Ralph Linton. Both appear to have served -- in an imperfect analogy -- as "funnels" through which ideas have passed into our hands.

W. I. Thomas, Burgess, Sumner, Cooley, G. H. Mead, R. E. Park, Moreno, and Linton. See S. F. Nadel, The Theory of Social Structure, Glencoe: The Free Press, 1957, p. 21; E. C. Hughes, Men and Their Work, Glencoe: The Free Press, 1958, p. 56; Neiman and Hughes, op. cit., p. 141; T. R. Sarbin, "Role Theory," in G. Lindzey, ed., Handbook of Social Psychology, Vol. I., Cambridge, Mass.: Addison-Wesley, 1954, p. 223; F. L. Bates, "Position, Role, and Status: A Reformulation of Concepts," Social Forces, 34 (1956), p. 313; Biddle, op. cit., p. 20.

W. R. Catton, Jr., retrenches further than most in tracing the concepts from Comte, Spencer, Main, Toennies, Durkheim, Weber, and Simmel. See W. R. Catton, Jr., "The Development of Sociological Thought," in R. E. L. Faris, ed., Handbook of Modern Sociology, Chicago: Rand McNally, 1964, Chapter 24. Pages 936-943 contain a compact summary of the historical and recent development of the concepts of status, role, position, and related terms, primarily as components of societal structure. This orientation leads to an emphasis on conceptions of ranking which the present account will not share.

Our deemphasis of the relevance of role concepts as used in analysis of the individual might suggest that Mead's work would have relatively little to offer for present purposes. It is important to our analysis because of his profound effect on others coming after. Two main features of this heritage are the markedly behavioral quality which he imparted to definitions of role and an emphasis on the consensual character of social structure. Substantively, we are interested in his conception of the "generalized other" as an aspect of the development of the organization of the self, and in his use of "games" and their rules to illustrate the formation of the individual's conception of the social structure. His explicit introduction of the main concept follows.

The organized community or social group which gives to the individual his unity of self may be called 'the generalized other.' The attitude of the generalized other is the attitude of the whole community. Thus, for example, in the case of such a social group as a ball team, the team is the generalized other insofar as it enters -- as an organized process or social activity -- into the experience of any one of the individual members of it.¹⁴

Mead was centrally concerned with analyzing the relationship of the individual to a total group, and with the way

¹⁴George H. Mead, Mind, Self and Society, Chicago: University of Chicago Press, 1934, p. 154.

in which this organized the behavior of the individual. In an abstract, speculative formulation it was not necessary to raise much question regarding veridicality or agreement. The use of illustrations in which explicit rules are common to all heightened the emphasis on the shared characteristics of outlook and performance in functional detail as in the remark,

" . . . in a game where a number of individuals are involved, . . . the child taking one role must be ready to take the role of everyone else. If he gets in a ball nine he must have the responses of each position involved in his own position.¹⁵

He did not depreciate individual differences, but their relatively low salience for his analysis seemed to lead by conversion to an emphasis on structural integration and a deemphasis of differentiation in parts of the structure.

Of course, we are not only what is common to all: each one of the selves is different from everyone else; but there has to be such a common structure as I have sketched in order that we may be members of a community at all. . . . We cannot have rights unless we have common attitudes. . . . Selves can exist only in definite relationships to other selves.¹⁶

In the original, these passages were relevant to the import of the structure of the social group for an

¹⁵Ibid., p. 151.

¹⁶Ibid., pp. 163-64.

understanding of the development and behavior of the individual. Here, they are of primary interest for the conception of that structure (and of the ideas of it held by participants) which they convey. The emphasis on the rules and logic of games presents a unitary picture of group structure. At least some of the credit for the acceptance of interpersonal consensus as a basis and mark of structure should be attributed to the concept of the generalized other. And it is this consensus which has become a point of some dispute in later formulations.

An emphasis on consensus also characterizes the formulation of the concepts of status and role presented by Ralph Linton in the now classic eighth chapter of The Study of Man. In contrast to Mead, Linton had a definite concern in this work with the nature of societal structuring of these elements independently of the way in which they are incorporated into the individual. His statement is a forerunner explicitly incorporated in many later works, or at least a landmark to be reckoned with. In part, this may stem from the unusually compact and straightforward statement and exposition of definitions.¹⁷

¹⁷ The first two pages of the chapter must be among the most frequently quoted passages in the literature of sociology.

Many, though not all, of the points to be found in later structural role theory are contained in them, as well as some which have not survived intact. The following extract contains the main analytic specifications.

. . . the functioning of societies depends upon the presence of patterns for reciprocal behavior between individuals or groups of individuals. The polar positions in such patterns of reciprocal behavior are technically known as statuses. The term . . . has come to be used with a double significance. A status in the abstract, is a position in a particular pattern. It is . . . correct to speak of each individual as having many statuses, since each individual participates in a number of patterns. However, unless the term is qualified in some way, the status of any individual means the sum total of all the statuses which he occupies. It represents his position with relation to the total society. . . .

A status, as distinct from the individual who may occupy it, is simply a collection of rights and duties. . . .

A role, represents the dynamic aspect of a status. The individual is socially assigned to a status and occupies it with relation to other statuses. When he puts the rights and duties . . . into effect, he is performing a role. Role and status are quite inseparable, and the distinction between them is of only academic interest. There are no roles without statuses or statuses without roles. Just as in the case of status, the term role is used with a double significance. . . .

Although all statuses and roles derive from social patterns and are integral parts of patterns, they have an independent function with relation to the individuals who occupy particular statuses and exercise their roles. To such individuals the combined status and role represent the minimum of attitudes and behavior which he must assume. . . . [They] . . . serve to reduce the ideal patterns of social life to individual terms. They become

models for organizing the attitudes and behavior of the individual so that these will be congruous with those of the other individuals participating in the expression of the pattern.¹⁸

The conception of status as a position, i.e., location in a pattern, with associated rights and duties, and of role as what is done in consequence of being in a status, has survived in substance if not always in name in much of the later discussion. Linton's own reformulation at a later date represents a shift in the content of the individual concepts and an extension (or at least development) of the total substance.

The place in a particular system which a certain individual occupies at a particular time will be referred to as his status with respect to that system. . . . The second term, role, will be used to designate the sum total of the culture patterns associated with a particular status. It thus includes the attitudes, values and behavior ascribed by the society to any and all persons occupying this status. It can even be extended to include the legitimate expectations of such persons with respect to the behavior toward them of persons in other statuses within the same system.¹⁹

Thus, "rights and duties" have been removed from status, it being reduced to position, and some extended counterpart

¹⁸R. Linton, The Study of Man, New York: D. Appleton-Century, 1936, pp. 113-114.

¹⁹R. Linton, The Cultural Background of Personality, New York: Appleton-Century-Crofts, 1945, pp. 76-77.

with a more explicitly normative ring in the form of ". . . attitudes, values, . . . and legitimate expectations . . ." has been added to role. It is worth emphasizing that in neither of the two statements is a role taken to be the actual behavior of an individual, since there has been some confusion on the point.²⁰

In so far as it represents overt behavior, a role is the dynamic aspect of a status: what the individual has to do [but not what he does] in order to validate his occupation of the status.²¹

There are several other points of importance about Linton's conception to be brought out here. One of these is the root of the concepts in "patterns for reciprocal behavior" and "polar positions," which implies that a status and role entail other related statuses and roles. Linton generally treats statuses as pairs or dyads, though this may be for simplicity of exposition (as some later observations in this chapter tend to show).²² Similarly, though he recognizes that individuals have a

²⁰See M. J. Levy, Jr., *The Structure of Society*, Princeton, New Jersey: Princeton University Press, 1952, p. 154; also, M. J. Daniels "Relational Status and the Role Concept," Pacific Sociological Review, 2 (1959), p. 44.

²¹Linton, op. cit., p. 77. Emphasis and brackets added.

²²Some other writers have an explicitly dyadic concept of statuses. See, e.g., E. T. Hiller, Social Relations and Structures, New York: Harper, Chapter 22, et passim.

multiplicity of roles (or statuses), there is little indication as to how these may be conjoined, except by being played by the same individuals and perhaps by the concrete situation.²³ In his explicit definitions there is a one-to-one relation between each status and its role and no more. Another feature, related to the reciprocity of roles and statuses is notable by its absence in Linton's concepts.²⁴ This is the factor of ordering or ranking which long has been (and still is) associated with the term status. Catton points to this divorcement of hierarchical evaluation from the concept of social location as one of the essential steps in the purification of the concepts.²⁵ Other authors (e.g., Gross, et al., and Biddle) have preferred to drop status in favor of a more neutral term, usually position, because it is thought still to carry the connotation of hierarchy even though defined without it.

Perhaps the most enduring elements of the specification are the idea of position as an abstract model,

²³See his example of a department store clerk, in Linton, op. cit., p. 78f.

²⁴This is not precisely true. "Status has long been used with reference to the position of an individual in the prestige system of his society. In the present usage this is extended to apply to his position in each of the other systems." Ibid., p. 77. However, the extension to other systems removes ranking as essential to the definition.

²⁵Catton, op. cit., p. 937, 939.

and the spatial analogy and the idea of occupancy which it entails. With one exception to be noted below, these elements have been retained to the present,²⁶ though not always uniformly interpreted. Over time, position has become a more or less defined term in its own right, rather than the relatively "primitive" (in the logical sense) concept which Linton used. This is a point to which we will return repeatedly and in the end quite extensively below.

The impact of Linton's comparatively brief statements can hardly be overemphasized.²⁷ The basic "static-dynamic" dichotomy has influenced (one is tempted to say plagued) much of the later thought,²⁸ though it does not appear to have been intended quite so literally by its

²⁶E.g., see the definition of "office" in R. L. Kahn, et al., Organizational Stress: Studies in Role Conflict and Ambiguity, New York: Wiley, 1964, p. 13.

²⁷Merton is especially emphatic on this point: "To say that Linton was not the 'first' to introduce these twin concepts into social science would be as true as it is irrelevant. For the fact is that it was only after his famous Chapter . . . that these concepts, and their implications, became systematically incorporated into a developing theory of social structure." R. K. Merton, Social Theory and Social Structure, rev. ed., Glencoe: The Free Press, 1957, p. 368, n. 112.

²⁸E.g., ibid., and R. K. Merton, "The Role-Set: Problems in Sociological Theory," The British Journal of Sociology, 8 (1957), p. 110; Parsons, op. cit., p. 25, and in other writings; C. P. Loomis, Social Systems: Essays on Their Persistence and Change, Princeton, New Jersey: D. VanNostrand, 1960, p. 19.

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author. In addition, the unqualified emphasis on the societally given character of the content of the concepts has only gradually yielded in the confrontation of empirical research.

Another formulation of related concepts made shortly after Linton's original statement appears in the work of E. C. Hughes. It is of interest here for comparative purposes, since it both agrees with the preceding in some respects and differs in certain emphases.

Status assigns individuals to various accepted social categories; each category has its own rights and duties. . . . in its active and conscious aspect, [it] is an elementary form of office. An office is a standardized group of duties and privileges developing upon a person in certain defined situations.

In current writings on the development of personality, a great deal is made of social role. What is generally meant is that the individual gets some consistent conception of himself in relation to other people. This conception, although identified with one's self as a unique being, is a social product; . . . But role, however individual and unique, does not remain free of status. Indeed, Linton says 'a role is the dynamic aspect of a status.' Role is dynamic, but it is also something more than status.²⁹

Disregarding the salient personalization of role (in a manner similar to Mead, which we did not emphasize above),

²⁹E. C. Hughes, "Institutional Office and the Person," The American Journal of Sociology, 43 (1937), p. 404.

the most important difference for our purposes is the absence of some idea of location and an associated spatial analogy as underlying status. This is also associated with an attributive quality of the concepts; the idea of social locations as mutually oriented is not explicit and the relational component appears most expressly in the function of a social role in the individual's conception of himself.

In another presentation which, to judge by the number of secondary citations, impressed others by its concise and incisive character, Cottrell combined the two earlier formulations of role in the following way:

. . . I shall . . . [use] . . . the term role to refer to an internally consistent series of conditioned responses by one member of a social situation which represents the stimulus pattern for a similarly internally consistent series of conditioned responses of the other(s) in that situation. Dealing with human behavior in terms of roles, therefore, requires that any item of behavior must always be placed in some specified self-other context.

By way of further clarification it is necessary to call attention to the distinction between the use of the term role to refer to a modal system of responses which constitutes the culturally expected behavior and the particular system of responses with which a specific individual operates. . . . We may refer then to cultural roles and deviant roles. The distinction is most obvious when we have a person equipped with both a cultural and a deviant pattern.³⁰

³⁰L.S. Cottrell, Jr., "The Adjustment of the Individual to His Age and Sex Roles," American Sociological Review, 7 (1942), p. 617.

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While he makes no explicit definition of status or position, Cottrell recognizes the assignment function of social categories in determining what roles an individual has. His emphasis on the non-isolability of roles and on how the ideal patterns of expectancy are met in actual behavior weaves together some strands which were present in the conceptions of Mead and Linton, and also presages a line of interest which flowered in the next two decades in empirical research on role conflict.

Norms

A later contribution to thinking on role conflict was made by Samuel A. Stouffer, in which he pointed out that multiple roles may well place an individual under the restraints of incompatible norms.³¹ This way of conceiving the question shifts attention from consideration of a role as a (usually coherent) bundle to the elements which determine or compose the bundle; i.e., Stouffer was not much concerned with roles as units, but rather with the consequences of their composition. One particularly influential consequence of this is noted in the statement,

It is the viewpoint of this paper that the range of approved or permissible behavior as perceived by a given individual is an important

³¹S. A. Stouffer, "An Analysis of Conflicting Social Norms," American Sociological Review, 14 (1949), pp. 707-17.

datum for the analysis of what constitutes a social norm in any group, . . .³²

And in his concluding observation,

From the theoretical standpoint, the most important implication of this paper may stem from its stress on variability. . . . it is common and convenient to think of a social norm as a point, or at least as a very narrow band on either side of a point. This probably is quite unrealistic as to most of our social behavior. And it may be precisely the ranges of permissible behavior which most need examination, . . .³³

The question of whether the constituent elements of roles are in fact unitary is of considerable importance in the sequel, and there is one observation that we ought to make on the matter at this point. While Stouffer emphasized variation, this does not preclude the possibility that norms may be taken to be essentially singular for some purposes. His idea of a permissive range seems also to indicate that this range has bounds and that what falls within them is permitted, i.e., the bounds act as cutting points to establish two areas, that which is permitted and that which is not. His use of norm is also terminologically interesting in that it and similar terms have been incorporated in subsequent uses to stand for rules in general, whether effected as rights or duties.

³²Ibid., p. 708.

³³Ibid., p. 717.

Expectations

We have digressed somewhat from our main line of structural emphasis and we need now to go further afield to explore the use of another term which has been used in a somewhat parallel way to norm, but usually with a pronounced interest in the behaving individual who is incumbent in the social location and performs the associated activities in context of the relevant rules. The reference is to the concept of "expectations" which has played an increasing part in the last 15-20 years. Of course, our attention is directed to the consequences these usages have for our idea of a role system. We will consider first three sources -- Parsons, Rommetveit, and Sarbin. A fourth will be delayed to a summarization point in later pages.

It may seem amiss to introduce our selections from Parsons' treatment of role concepts in connection with otherwise "social psychological" considerations since his work is intimately identified with the structural-functional disposition of contemporary sociological theory and the very concept of a social system. The reason is simply that we believe that he has been very effective in impressing the concept of expectation on role analysis with respect to the incorporation of standards into action via orientations of the actor.

We will pass over his early statements, since he later regarded their context as "completely obsolete."³⁴ A number of statements appearing in the early 1950's present similar views, and the following extracts from The Social System are representative. After noting the act (in interaction) as the most elementary unit of a social system, and status-role as a (most) convenient higher order unit, he says:

. . . it is the structure of the relations between actions . . . which is essentially the structure of the social system. The system is a network of such relationships.

Each individual actor is involved in a plurality of such interactive relationships each with one or more partners in the complementary role. Hence it is the participation of an actor in a . . . relationship which is for many purposes the most significant unit. . . .

This participation in turn has two principal aspects. . . . where the actor in question is 'located' in the social system relative to other actors. This is what we will call his status . . . On the other hand there is the processual aspect, that of what the actor does in his relations with others seen in the context of its functional significance for the social system. It is this which we shall call his role.³⁵

³⁴Specifically, a brief taxonomic paper (circa 1941) which was later reproduced in the first edition of Essays in Sociological Theory (1949). See T. Parsons, Essays in Sociological Theory, rev. ed., Glencoe: The Free Press, 1954, p. 12.

³⁵Parsons, The Social System, op. cit., p. 25. This is the first appearance of the term 'network' in any of the extracts cited here. Parsons did not introduce the idea nor did he work out its significance in the relevant way.

As we have seen, the locational and behavioral components were not novel. A few pages later in discussing "the integration of the motivation of actors with the normative cultural standards," Parsons recasts role from the actor's perspective. After remarking that value standards have both expressive and instrumental significance to an actor in his expectation system as "role-expectations" and "sanctions" respectively, he continues,

The relation between role-expectations and sanctions then is clearly reciprocal. What are sanctions to ego are role-expectations to alter and vice versa.

A role then is a sector of the total orientation system of an individual actor which is organized about expectations in relation to a particular interaction context, that is integrated with a particular set of value-standards which govern interaction with one or more alters in the appropriate complementary roles. These alters need not be a defined group of individuals, but can involve any alter if and when he comes into a particular complementary interaction relationship with ego which involves a reciprocity of expectations with reference to common standards of value-orientation.³⁶

The concept of expectation is a summary psychological conception of the characteristics of cognitions under repeated stimuli.

. . . action . . . does not consist only of ad hoc 'responses' . . . the actor develops a

³⁶ Ibid., pp. 38-39.

system of 'expectations' relative to the various objects of the situation. These may be structured only relative to his own need-dispositions. . . . But in the case of interaction with social objects a further dimension is added. Part of ego's expectation, in many cases the most crucial part, consists in the probable reaction of alter to ego's possible action, a reaction which comes to be anticipated in advance and thus to effect ego's own choices.³⁷

It is clear from his discussion of the employment of expectations that their essential substantive relevance to the social system is that they become shared. However, the main point of relevance for present purposes is that these expectations involve at least two actors (or social locations), one who expects and another of whom something is expected. Any explicit proposition concerning an expectation or expectational state would always require a predicate of second-order, i.e., the logical character requires a major operator of relational form. This point stands out again in the next two formulations.

Rommetveit's analysis is a highly technical exposition, and it is even less possible to do justice to it here than to many other statements. The main substantive foundation is an explication of the concept "norm," whose formal specification is then used to

³⁷Ibid., p. 5.

define role. (He makes residual use of status or position, accepting it as received.) He finds three main themes in the concept norm, viz., shared frame of reference (from Sherif), statistical behavioral uniformity, and social pressure or role obligation (from Festinger and Stouffer), and adopts the third in his formal definition.

A social norm is a pressure existing between a norm-sender (A) and a norm-receiver's (E's) behavior in a category of recurrent situations (S) manifesting itself as follows:

A expects E to behave in a specific way x_i or A wishes E to do x_i or A is satisfied (dis-satisfied) when E ⁱdoes (not) x_i or A applies overt sanctions when E does (not) x_i

or

E perceives that A expects him to do x_i or E perceives that A wishes him to do x_i or E anticipates A's (dis-)satisfaction ^jwhen x_j will (not) be done or E anticipates overt ^jsanctions from A upon performance (non-performance) of x_j , x_i and x_j referring to similar but not necessarily identical modes of behavior within the class of possible behaviors x in S.³⁸

Our reason for quoting this exact form is that the logically relational form of the constituent elements is suppressed in the verbal statement of his definition of social role.³⁹

³⁸R. Rommetveit, Social Norms and Roles, Minneapolis: University of Minnesota Press, 1954, p. 51.

³⁹He also uses extensive symbolic notation to provide explicit definitions of concepts (reproducing it here would require extensive explanation) which exhibits the relational form.

A social role is a system of social norms directed toward one and the same individual as member of a group or representative of a psychologically distinguishable category of individuals.⁴⁰

In his psychologically oriented review of role theory of a decade ago, Sarbin adopted definitions which represent a meld of several of the themes which had appeared (which in fact is the case with the greatest number of the statements in the literature). We cite his formulations here both for the character of their content and as an avowed stocktaking at a time when developments which are very important for our purpose were in the offing.

The writer would regard a position in a social structure as a set of **expectations** . . . That is to say, the person learns (a) to expect or anticipate certain actions from other persons and (b) that others have expectations of him. . . . In other words, a position is a cognitive organization of expectations . . . These expectations, organized as they are around roles, may justifiably be called role expectations. Thus, a position is a cognitive organization of role expectations.

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A role is a patterned sequence of learned actions or deeds performed by a person in an interaction situation.

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Two general kinds of expectations are found: rights and obligations. Rights are role

⁴⁰Ibid., p. 84.

expectations in which the actor of the role anticipates certain performances from the actor of the reciprocal role; . . . Obligations (or duties) are role expectations in which the actor of a role anticipates certain performances directed toward the actor of the reciprocal role; . . .⁴¹

It may appear that a new concept of position enters here, but we regard it as a rearrangement of denotata rather than any new substance. The locational idea has been suppressed and the term used to compass contents variously assigned to status or role by others. The phrase "cognitive organization" might seem to introduce some new moment, but the idea has been used extensively before and the emphasis is on the character of expectations. Again we note the explicit emphasis on the relational form.

Minority Views

We have followed a brief historical route through a few of the salient contributions to the role concepts and presently will make a provisional summary of the material. Before doing so let us examine some suggestions which have not been incorporated into general use or run against it. Selections from two authors will suffice.

In the material above, we have seen that most authors associate the rights, duties, obligations, norms, expectations, cognitions, behavior patterns or other non-locational elements of the role concepts with one or

⁴¹Sarbin, op. cit., pp. 225-26.

another of the variety of available terms, but that the elements of similar content are usually tied to only one of the terms for any given author. A somewhat different view which may appear to introduce a distinctive way of handling the point is represented in the following:

A natural distinction between role and status . . . suggests itself, i.e., that role is somehow associated with duties or obligations, and status with rights, privileges, and sanctions, and that there is no necessary logical relationship between them.⁴²

That is, in a role the "responsibility" is on ego, and in a status it is on alter. This way of putting the matter has a certain persuasiveness since we do frequently associate "activity" with role and the converse with status, and responsibility connotes doing (or giving) while privilege connotes passive reception. However, the point which is missed is that while there is no "necessary logical relationship" between rights and duties, neither is there any among rights or among duties. Again, the proposal seems to amount analytically to little more than a reshuffling of terms, as the author seems partially to recognize in the statement:

Recognition of the analytic separability of role and status in no wise denies of course that the relationship between the two is often institutionalized in such a manner as to give

⁴²A. Pierce, "On the Concepts of Role and Status," Sociologus, N.S. 6 (1956), p. 30.

an appearance of inseparability. It is not uncommon however for some of the elements comprising one to change with no related change in the other.⁴³

He fails to add that it is also common for such related changes to occur. The argument seems to proceed from observed statistical irregularity to logical independence. The point is however that dropping the archaic use of status as necessarily associated with privilege is precisely one of the purifications of contemporary role theory.

Another anomaly is represented by the commentary of Argyle. His discussion is intended to be a clarification of the concepts but leads to a view which is incompatible with main currents. He does not provide a concise statement of position but offers several examples, as social categories (age, sex), membership in formally constituted groups, and in institutionally established structures. In terms of these,

There may be said to be a role in the social structure sense if the behavior of occupants of a position (as defined above) [sic] is modally distributed . . . [for situations] . . . and if the mode differs significantly in adjacent positions.⁴⁴

Later, he utilizes the notion of position as follows:

⁴³Ibid., pp. 31-32.

⁴⁴M. Argyle, "The Concepts of Role Status," Sociological Review, 44 (1952), p. 41.

An informal group may be defined as a group without positions -- though of course members will occupy positions in the wider society.⁴⁵

It appears to us, however, that the notion of a human aggregate to which the term "social" or any equivalent locution would be applied and that was devoid of positions (and thereby of roles) is unusual in sociological thought. We do not refer to the "real" nature of human groups, but to how sociologists choose to think about them, and most do not choose this way. However, we will see later on that when the choice is less apparent another view is often accepted which has operational consequences essentially similar.

Summary of Themes

We come now to the point of intermediate summarization promised earlier. For this purpose we turn to the excellent review of role and related terms which Gross, Mason and McEachern provided as a preliminary to their own reformulation of the concepts. They distinguish three main categories of definitions of the term role, ". . . which, if not exhaustive, are at least representative of the major role formulations in the social

⁴⁵Ibid., p. 43.

science literature."⁴⁶ They are as follows:

Definitions of role which either equate it with or define it to include normative culture patterns . . . which includes . . . Linton's often-quoted definition.⁴⁷

. . . definitions . . . as an individual's definition of his situation with reference to his and others' social positions . . .⁴⁸

. . . definitions which deal with role as the behavior of actors occupying social positions . . . this . . . does not refer to normative patterns for what actors should do, not to an actor's orientation to his situation, but to what actors actually do as position occupants.⁴⁹

For brevity, let us refer to these as (1) normative, (2) cognitive, and (3) behavioral conceptions. We use the term "cognitive" for the second set to emphasize the salience of the actor as the definer; "expectational" would better suggest the content of such individual definitions, but the term is also associated with ideas of sharing or consensus, either among a plurality of actors or as socially or culturally given.

⁴⁶Gross, et al., op. cit., p. 11. A materially equivalent set of categories appears in D. J. Levinson, "Role, Personality and Social Structure in the Organizational Setting," Journal of Abnormal and Social Psychology, 58 (1959), p. 172.

⁴⁷Ibid.

⁴⁸Ibid., p. 13. Expectations are included here.

⁴⁹Ibid., p. 14.

It seems more accurate to regard these ways of approaching the definition of the concept of role (and of status as well) as themes or components of the conceptions which recur with frequency in the literature. Sometimes more than one of these ways of looking at the question will appear in the same explicit statements, or the same author will employ each in different places as he needs to for different purposes.⁵⁰

There are a number of points to be noticed about these different conceptual elements. First, the concept of role is not in itself taken to be logically primitive, i.e., there is a clear consensus that it is not to be viewed analytically as an undefined term or element, but that it should be defined as a collection, or in a more conventional formal term a "set," of other elements (which usually are taken as formally undefined, though some specifications of them may be made). Second, it is not possible in general to choose from among these kinds of elements; there is not consensus as to which ones should be included in the "real" definition of role, each being required for different purposes. Third, in these

⁵⁰E.g., see Parsons, Essays in Sociological Theory, rev. ed., op. cit., pp. 230, 337, 393f, The Social System, op. cit., pp. 25f, 38f, and Structure and Process in Modern Societies, Glencoe: The Free Press, 1960, pp. 171, 251.

terms, it might well seem that there are in reality a series of definitions of role ($\text{role}_1, \text{role}_2, \dots, \text{role}_n$) and that there is no point of connection among them (though there may be substantial connection in specified theoretical contexts). We intend to argue that this last point is incorrect.

Attributive and Relational Concepts of Role

The line of the argument was indicated above, and the particular point of attack was taken up in connection with the concept of expectations as elements of role definitions. It has been suggested that the properties of the logical character of the concepts taken as elements of role concepts provide a basis for agreement among the ways in which the otherwise disparate elements are used. What we are particularly concerned with is what we will call attributive and relational uses of the concepts.

Attributive concepts are ones which attend explicitly (or entail operations which require) only a single object (position, actor) in making specifications of a role. Consider the way in which we might describe an occupational role (with any of the "primitive" elements) such as machine operative in a factory: the occupant of the position must carry out the actual performances needed to run the machine, do "a fair day's

work" (though the actual specification would be much more precise), be prompt, and perhaps we should include that he display courtesy and friendliness. The accuracy or completeness of the role description does not concern us, but the form of the statements shows that it is possible and quite usual to specify characteristics of a role by stating attributes or properties of the incumbent of the position.

On the other hand, relational concepts are ones which attend to two or more objects in specifying a role. The same machine operative would take instructions from a foreman, receive stock to be processed from fellow workers and pass completed pieces to still others, and most particularly courtesy, friendliness, etc., would specifically imply his appropriate demeanor vis-a-vis both peers and superiors. (Obviously, attributive and relational concepts are of a much more general order than exhibited in this example. Logically, they correspond in syntactic analysis to first- and higher-order predicates respectively.)

It is our contention that any of the generally used kinds of elements of role definitions can be analyzed either attributively or relationally. In general on the relational side which is our focus of interest, this follows in almost all formulations from such assumptions or remarks as "reciprocal patterns of

behavior," "self-other context," "polar positions," etc. This seems to give little difficulty with respect to normative or cognitive components. The normative elements are most usually factored into rights and duties, and it is sufficient to note the frequent "one's rights are another's duties." Similarly, cognitive elements become role-relevant by virtue of their employment in orientation to alters, as noted above for expectations. It may be thought that behavioral components present some difficulty here, since the idea of a "pattern of behavior" of an actor does not require any notion of present or even relevant alters. But it is, exactly on this point that behavioral definitions of role should be challenged.⁵¹

Here we need to take note of a similar formulation by Nadel in order to make clear what is and what is not intended by our distinction between logically attributive and relational characteristics of role definitions. In his very important contribution to role theory, which we have not referred to explicitly up to now, Nadel defines two sub-classes of "logically related" roles in the following way.

⁵¹Cf. remarks by M. J. Levy and M. J. Daniels in the citations in footnote 20, and Cottrell's definition cited above.

One is constituted by roles in the case of which the logical relation also implies, with logical compulsion, an actual relationship between the respective actors. [i.e.] . . . a given role . . . requires to be enacted vis-a-vis another counterpart or correlative role. The second extreme is constituted by roles in the case of which the logical relation merely means that actual relationships between the actors are possible, not in any logical sense necessary. . . . I will call the first type of roles dependent, and the second, independent roles.⁵²

. . . dependent and independent roles represent only extremes, between which all concrete roles are ranged along a continuous scale or at least one of many degrees.⁵³

First, we need to notice what he means by "logically related" roles. He refers by this to sets of roles which have the same differentiae in the sense of belonging to the same class (e.g., ones that are all occupations). Nadel has reference to substantive categories, whereas our use of "logical properties" refers to the analytic character of role elements. Second, the distinction between dependent and independent roles, depends on whether or not an "actual relationship" is more or less required, and again this is not what we have in mind by the attributive-relational distinction. In our view, roles always entail "actual relationships," albeit

⁵²S. F. Nadel, The Theory of Social Structure, Glencoe: The Free Press, 1957, pp. 79-80.

⁵³Ibid., p. 84.

possibly not with others which only share the property of having been named (or thought of) in terms of the same differentiating concepts by the culture in question.⁵⁴ And when we speak of a relational definition of a role, we mean that relationships are to be considered as the analytic components of a role, and thus it would be formally incorrect to speak of the relationships as between roles.

It should be clear that we have not "taken sides" as to what are the contents of roles, but have only tried to show how the major kinds of such contents can be handled as to their logical form. Further, we have not argued that role elements can have only the form of a relation or only the form of an attribute. It seems quite clear that both are useful, but for different purposes. One would use attributes in order to describe a particular role, or the qualities or performance of an incumbent of the position, especially with regard to recruitment into it. That is, it is quite possible when dealing with a single actor to make many substantively

⁵⁴We should note that "relationships" does not have the meaning for Nadel which is connoted by the first passage quoted above. Elsewhere, he defines them as "determinate ways of acting towards," with the emphasis on a rule-determined form, and the discussion between the two passages quoted shows the same intention here.

significant statements about characteristics and behavior without taking explicit notice of any other actors; and it is possible to extend this analysis to a collection of actors, describing the properties and actions of each without reference to any of the others explicitly, and even to compile the "data" so as to show summary characteristics of the collectivity. However, if it is desired to treat such a collectivity as more than an aggregate, then some explicit notice must be taken of more than one of the members at a time, i.e., one must have recourse to statements of a relational form. What we have tried to show is that such statements are (or more precisely, may be) about roles.

Thus, from the point of view of logical form, we wish to extract a certain common moment from the concepts of roles, and to say, provisionally, that they are sets of relations among positions, and that the substantive content of these relations may be one or more of several kinds.

Positions and Actors

There are two other terms which have been much used in our review and discussion, and which we will require in the development of our model, viz., position and actor.

We have observed above that the concept of a position enters most discussions as an undefined term. It is sometimes given some suggestive specification as a "location in a set of social relationships" or some similar expressions. However, this is not usually accompanied by specific instructions on what is to be understood by "social relationships." Since we have already taken relations as the formal component elements of roles, one might begin to wonder whether one or the other of the terms "role" and "position" is redundant.⁵⁵ Our answer, which will be developed in the next section, is that while they can be analyzed in terms of the same contents and are connected, they are not redundant.

As a preparation for this and in order to complete our review of system elements, let us take notice at this point of the exceptional definition of position made by Biddle.

A position is a set of persons who exhibit similar characteristics, who are treated similarly by others, or for whom a cluster of unique cognitions are maintained either by themselves or others.⁵⁶

⁵⁵Nadel argues that the static-dynamic distinction is "not only redundant but misleading," since it amounts to knowledge-performance of the same rights and duties, but this is not the same as the question here. Ibid., p. 29.

⁵⁶Biddle, op. cit., p. 5.

The reference to "a set of persons" is unique, and replaces explicit mention of "location" or the like. Biddle remarks that definition by persons or by "location in a social system . . . are equivalent in denotation," but does not go on to show why or how this is true. The definition which we will develop in the formal model displays the correctness of his assertion regarding the two modes of the definition. A point of interest about Biddle's and all other definitions of position known to the present author is that the positions in a group, system, or structure are always assumed to be fixed. Moreover, in most uses they are assumed to be given, i.e., not open to question by the participants. As we shall see later, positions differ very markedly from roles in this respect, and we shall have reason to inquire why this should be the case.

We turn now to the term "actor." Most conceptualizations of the role concepts are not definite on this point, but it is usually clear in context that the authors intend the range of their terms to take in living humans in social groups. Whether other objects are allowable as referents is often uncertain, but they are not in general precluded. Part of the indecision here stems probably from the fact that most writers seem to have essentially attributive "pictures" of the concepts in mind and to attend primarily to the "ego" of the ego-alter pair

assumed as the fundamental context. This seems to lead naturally enough to semantic restrictions of the actor to single and potentially existent persons. On the other hand, there are frequent references to the possibility of "empty" (i.e., unoccupied) positions, or "mythical" ones (i.e., occupied by symbolically significant but unreal figures conceived by the participants). It seems to be necessary to pay attention to the possible incumbents of the "alter" in order to determine the range of permissible referents. We do not need to attempt to decide what kinds of objects can allowably be actors, since we take the term as formally undefined in our analysis. However, it is intended that it would include, operationally, any symbolic referents capable of being treated as alters.⁵⁷

The Role System

We turn now to the problem of how elements of a role system are related to each other. Definitions of

⁵⁷In a footnote, Mead makes an observation about the composition of the generalized other which is of interest here. "It is possible for inanimate objects . . . to form parts of the generalized . . . other for any human individual, insofar as he responds to such objects socially. . . ." G. H. Mead, op. cit., p. 154.

Imaginary persons as incumbents of positions are used not only by participants, but also by field workers. See M. J. Herskovits, "The Hypothetical Situation: A Technique of Field Research," Southwestern Journal of Anthropology, 6 (1950), pp. 32-40.

role concepts which were generally in use up to about the middle 1950's postulated a "primitive" positional element and associated with it in a one-to-one way a collection of contents which taken with the position constituted the status and/or role (or office, or other similar term). Further, it was frequently suggested that each such status was paired with another status similarly constituted of position and associated contents in a dyadic relationship. Usually, no other general specification was made and the matter was simply left at the assertion that each status and position was associated with one role, and vice versa. Sometimes, following Linton, it was said that a collection of all of a person's statuses constituted his concrete status.

A number of different authors have presented largely congruent reformulations of the relationship between status and position and role. Merton, who was one of these writers, remarked in a felicitous phrase regarding his own reconception that it involved ". . . initially a small . . . shift in the angle of vision . . . ," and he gave to the formulation what has come to be its distinctive title, "the role-set." The full quotation, which in context was preceded by a restatement of Linton's definitions, follows.

It is at this point that I find it useful to depart from Linton's conception. The difference is initially a small one, some might

say so small as not to deserve notice, but it involves a shift in the angle of vision which leads, I believe, to successively greater differences of a fundamental kind. Unlike Linton, I begin with the premise that each social status involves not a single associated role, but an array of roles. This basic feature of social structure can be registered by the distinctive but not formidable term, role-set. To repeat, then, by role-set I mean that complement of role-relationships in which persons are involved by virtue of occupying a particular social status.⁵⁸

Merton's succinct term has persisted, but his own and others further analysis and use is somewhat clouded by the fact that the term has been taken to refer to the set of persons incumbent in the reciprocal positions, rather than to the set of roles as such which seemed to be implied by the passage above.

In terms of priorities, it appears that honors for this "small shift" must be accorded separately 1) to an article by F. L. Bates⁵⁹ (which was known to Merton⁶⁰ and to Gross, et al.,⁶¹ whose ideas were also known to each other), and 2) to a less noticed paper by J. E. Haas.⁶²

⁵⁸Merton, "The Role-set . . . ," op. cit., p. 110.

⁵⁹Bates, op. cit.

⁶⁰R. K. Merton, Social Theory and Social Structure, op. cit., p. 368, n. 115.

⁶¹Gross, et al., op. cit., p. 69, n. 26.

⁶²Another development of the same line of argument, though in quite different terminology, which appears to be innocent of any knowledge of these is contained in

Bates way of developing the underlying idea has certain defects which we will consider in connection with a topic of the next chapter. On the other hand, Haas' presentation is very compatible with the views we wish to develop, and we will set out the essential points of it in the following extracts.

. . . in any group where the members interact repeatedly we can find sets of normative specifications for behavior (norms) which apply to distinct units of social interaction. Such units . . . refer to the interaction of the persons who occupy positions within the group. We may refer to a set of normative specifications for the behavior involved in a unit of social interaction as a role. . . . when role is defined in this manner, the concept 'position' is usually left undefined. . . . [but] . . . It is simply a label we give to a whole system of normative specifications which apply to a number of units of interaction.⁶³

He then introduces the position "doctor" to show how various roles (as doctor-patient, doctor-nurse, etc.) of the same position may differ in many respects, but are related by the fact, at least, that the incumbent "plays" all of them.

Ward H. Goodenough, "Rethinking 'Status' and 'Role'; Toward a General Model of the Cultural Organization of Social Relationships," Association of Social Anthropologists Monographs: 1, The Relevance of Models for Social Anthropology, London: Tavistock, 1965, pp. 1-24. "This is a revised and expanded version of a paper entitled 'Formal Properties of Status Relationships' read at the Annual Meeting of the American Anthropological Association, 16 November 1961." Ibid., n. 1.

⁶³J. E. Haas, "Role, Position, and Social Organization: A Conceptual Formulation," The Midwest Sociologist, 19 (1956), p. 34.

Thus it seems useful to define position as a cluster of roles that are conceived of as belonging together. . . . It refers to no more or no less than the normative specifications of which its roles are composed.

. . . A role may be viewed, then, as being a segment or part of each of two positions. In the course of describing a position by indicating the roles of which it is composed, there will be an explicating of at least some segments of the other positions involved.⁶⁴

Here, then are the main points of the "break-through" for defining roles and positions as common elements of a system. Roles are made up of elements (norms) and positions are made up of roles, and consequently positions are also made up of the same elements, though in larger aggregates. We have tried to explicate the logically relational character of roles by the requirement of always designating two positions when specifying the elements of -- and thereby defining -- a role. It is not clear that Haas believed that the same requirement applied as well to a position, i.e., that it could only be concretely specified in terms of statements which also specified other positions at the same time, though it necessarily follows from this way of formulation. Also, he does not take note of the fact that defining a role in terms of a pair of positions can be done, so to speak,

⁶⁴Ibid., pp. 34-35.

from either point of view (as doctor-nurse, and nurse-doctor). (These points are clear in Gross, et al., to which we will turn presently.) Thus it develops that position and role are to be defined interdependently, and in fact that this is to be done in the context of a system of them.⁶⁵ Before leaving our discussion of Haas' proposals, we must emphasize that we have not accepted his specification of norms as the elements but only his formulation of the connections between positions and roles. Further, contrary to his specifications,⁶⁶ we retain the notion that actors "occupy" positions (and one might say roles as well) since it is a useful locution that can be given operational interpretation.

The similar analysis carried out by Gross and his associates has certain terminologic advantages for us, though there are conceptual shortcomings in their treatment (which we shall analyze more fully in the next chapter) concerning positions. Foreshadowing

⁶⁵Biddle, and Coult after him, has made a point of insisting that position and role ought to be defined independently, though for reasons that have nothing to do with whether they are defined interdependently or not. In the sequel we will find that it is precisely through this interdependent definition that we are able to show that Biddle's definition of position cited before is equivalent to the locational definition. See Biddle, op. cit., p. 4, and A. D. Coult, "Role Allocation, Position Structuring, and Ambilineal Descent," American Anthropologist, 66 (1964), pp. 30-31.

⁶⁶Haas, op. cit., p. 35.

Biddle, they employ a definition of position as "the location of an actor or class of actors in a system of social relationships,"⁶⁷ but their definition is incomplete in not making clear the nature of the "social relationships." Their definition of role is "a set of expectations [which are "evaluative standards"] . . . applied to the incumbent of a particular position."⁶⁸ Formally, this is equivalent to Merton's "role-set" in its original sense and to Haas' "cluster of roles." They introduce the term "role-sector"⁶⁹ to correspond to what Haas and Merton (and Bates) called "role," and we will adopt the term in connection with specification of the model to avoid the misleading "role-set" appellation. In their analysis, the author's recognize that a set of related positions is formally constitutive of a system and that it is necessary to make "relational specifications of positions."⁷⁰ In their definitional development, however, this amounts only to the enumeration of other positions which are related to

⁶⁷Gross, et al., op. cit., p. 48.

⁶⁸Ibid., p. 60.

⁶⁹"A role sector is defined as a set of expectations applied to the relationship of a focal position to a single counter position." Ibid., p. 62. On focal and counter positions, see below.

⁷⁰Ibid., p. 50f.

a given position (again, see the remarks in the next chapter). Though they define a "system model," their main analysis is concerned with a single "focal position (school superintendent) and its relationships to any other "counter" position(s). Thus they refer to their model as "position-centric."⁷¹ While their definitions do not suit our present purposes precisely, we will adopt the terms "focal" and "counter" to denote the necessary ordering of positions, etc., noted in the summary of Haas' concepts. We will use these terms to modify "position," "role," and "role-sector" as needed, and when the qualifiers are not used we will intend that the term(s) be understood as referencing both aspects.

The Graphic Model of Role Systems

In order to complete our review of role concepts, we need to take notice again of a feature which characterizes most of the conceptualizations, and particularly those most directly concerned with the structure of the system, viz., the implied or explicit spatial analogy. We do this to underline the essential correctness of the redefinition of position and role as interdependent and to indicate how the commonly used spatial conception

⁷¹
Ibid., p. 51f.

is inadequate. We should emphasize that, even though our treatment in this section is informal, the shift in the focus turns our discussion away from the substantive concepts and toward major characteristics of their underlying calculus.

Many authors, especially recent ones (particularly Haas, Bates, and Gross, et al.), use graphic diagrams as an aid in their expositions. The use of a geometric conception of some sort seems to be required by the locational idea. This is well expressed by Linton in his discussion of the concept of a social system.⁷²

Perhaps the nature of a social system can best be understood if we compare it to a geometric figure, Actually, there is nothing else within the range of common experience which would be so closely comparable. A geometric figure consists of a series of spatial relationships which are delimited by points. These points are established by the relationships and can be defined only in terms of the relationships. They have no independent existence. Each of the patterns which together compose a social system is made up of hypothetical attitudes and forms of behavior, the sum total of these constituting a social relationship. The polar positions within such patterns, i.e., the statuses, derive from this relationship and can only be defined

⁷²"Social systems consist of the mutually adjusted ideal patterns according to which the attitudes and behavior of a society's members are organized. A society is an organization of individuals; a social system is an organization of ideas." Linton, The Study of Man, op. cit., p. 253.

in terms of it. They have not more independent existence than do the points of the geometric figure. Any status, as distinct from the individuals whom society may designate to occupy it, is simply a collection of rights and duties. Thus the status of employer derives from the relationship between employer and employee and can be defined only in terms of the attitudes and behavior which the total pattern for this relationship ascribes to this one of its two polar positions.⁷³

Linton's statement here corresponds closely to the "role-set" redefinition of the concepts, and it explicitly eliminates any idea of delimitation of a particular position outside the content of the relationships. Such a definition of a set of points suggests that the spatial concept is what is technically called a "linear graph."⁷⁴ We will deal with the concept of a linear graph more fully in Chapter IV in preparation for formally specifying our model, but a few points about it here should be

⁷³Ibid., pp. 256-57.

⁷⁴More familiar geometric conceptions have also been attempted, as in R. E. Park, "The Concept of Position in Sociology," Publication of the American Sociological Society, 20 (1925), pp. 1-14, which uses a more or less Euclidean notion; and in R. Linton, "A Neglected Aspect of Social Organization," American Journal of Sociology, 45 (1940), pp. 870-886, which employs a system of coordinates.

The context of an idealized patterning may also suggest why Linton did not utilize the graphic concept to define a total system. If he had been interested in the concrete "society" (see definition in footnote above), he might have used the analogy to represent the ramified pattern of connection between persons. In the abstract social system it is possible to conceive of isolated pairs as representing the structure, though there is

helpful. Roughly, it conceives of a space without independent referents of distance and direction, and so differs from intuitive notions of space and from Euclidean geometry which we usually think of as their formal representation. A drawing of such a graph shows a group of points with lines connecting some, though usually not all, of the possible pairs of such points. Sociographs are an example, and are, in fact, one of the stimuli for the study of the properties of such graphs. We have said that there are not measures of distance and direction assumed which are independent of the particular collection of point and lines, and this is seen in one of the "peculiarities" of such a geometry: There are many (in fact infinitely many) different possible drawings of a given collection of points and lines (i.e., just so many points and just such connections between them), and all of them are indifferently equivalent from the point of view of linear graph theory.⁷⁵

certainly warrant for considering more extensive structures; e.g., Mead's example of "the ball nine" suggests this. However, it should be noticed that the source work on graph theory by D. König was being published in Germany at about the same time (1936) that Linton's classic appeared in this country; Linton did not have a properly worked out geometry available for his purposes.

⁷⁵Indeed, this point quickly became clear in early sociography and considerable effort has been expended to develop ways of using more usual ideas of spatiality to produce ways of drawing sociographs which "intuitively" display relevant properties.

Another recent statement by Kahn, et al., exhibits essentially the same spatial analogy, but also brings out more clearly the point on which the analogy is inadequate.

If the organization is a network of inter-related roles (or, more precisely, role behaviors), it follows that the bonds which connect one role to another may be of many kinds, not of formal authority alone. Two roles may be related in terms of authority, to be sure, but they may also be related because of the sequence of work flow, or of information, or because of the liking which one person has for another, or because of expertise. . . . The organization is a complex network of roles connected by such bonds of expectation and influence, some of them reciprocal, some asymmetrical. Thus no role in an organization is intact or fully separable from others. . . .

We may wish to deal with a single role, and for that purpose we figuratively pluck it out of the network of other roles. . . . When we do so, we find that what we have in hand is an assortment of duties and obligations, expectations and rights which state relationships between this role and various others. These relational statements dangle from the role like strands from a knot which has been cut out of a larger net of which it was a part. And if we try to eliminate those bonds and define the role without mentioning its connections to any other, we make a startling discovery: there is virtually nothing left. The role is defined in terms of its relationships to others, just as the knot in a net is no more than the intersection of bonds and disappears if we try to trim too closely.⁷⁶

⁷⁶ Kahn, et al., op. cit., pp. 388-89. We would add that "virtually" is unnecessarily cautious -- for their purposes, and ours, there would be nothing left at all.

There is a point of ambiguity in the observation of plural bonds of connection between a single pair of roles (i.e., positions). The graphic and pictorial concepts that are used are of a sort suggesting a single connection between each pair of positions. (For example, our picture of a fish net or a spider web is of a structure such that each line segment runs between a pair of knots or intersections, but any given pair is joined directly by at most one such line.) But the substance declares that there are many connections of one position to another and that in general these are conceived as being of different kinds. In connection with this we shall need to reconstruct the spatial analogy and more formally the graph theoretic conception of it in the specification of our model.

We have remarked on the sociographic image of the spatial conception, and at this point it is also possible to say succinctly why we have passed over the work of sociometric analysis in our review of role concepts. On the one side, it has not made any unusual contributions to the substance of role concepts, and on the other side, while its emphasis on relational characteristics is essentially correct, the sociographic model is inadequate for a full definition of the logical structure of a role system.

CHAPTER III

PRIMITIVES, DEFINITIONS, AND AXIOMS

In the preceding chapter we introduced the general ideas involved in the redefinition of the concepts of position and role that have emerged in the work of several different authors in the past decade. We tried to show in an approximate way the points of consensus in these conceptualizations and some of the underlying inadequacies. The task of the present chapter is to restate the consensus in a more rigorous way and to modify and extend it to permit the statement of a definite model of the logical structure of a role system in the following chapter. In order to accomplish this we develop a somewhat informal and incomplete axiomatization.

Primitive and Defined Terms and the Axiom of Relation

In the foregoing discussion we listed a collection of terms which we will require in our analysis, viz., position, relation, role, role-sector, and the terms focal and counter which are to qualify the others as necessary.¹

¹Another term to be used as a primitive, actor, will be introduced later.

The first two terms will be selected as our primitives, then an axiom will be stated which serves to formulate the connection between these primitives, and this gives the basis for defining the remaining terms.

A few words seem to be required about our selection of the primitive terms (especially since the choice of one of these, position, may seem to contradict the results of the last chapter where we emphasized the interdependence of position and role). In general, there is no formal basis for our specific choice of primitives here, since others in the main set could be substituted, or even a single term employed as a primitive, and an equally rigorous and formally satisfactory development could be made which would provide a logically identical result. However, it is our judgment that the two selected provide the best intuitive basis for the exposition of the system. Moreover, they, they or their analogs of whatever name, appear most frequently in functionally equivalent uses in the substantive conceptualizations reviewed from the literature. In addition, these terms can be coordinated most easily to corresponding primitives in the standard treatment of the mathematical equipment which we shall introduce in the next chapter.

There does not seem to be any difficulty in accepting the intuitive validity of treating relations

as primitive when they are understood in the sense of their logical use in the discussion of role concepts which we sketched in the preceding chapter. That is, rights and duties, norms, obligations, expectations, etc., and even behavior patterns, when taken as the elements of status, position, role, office, etc., can always be interpreted as operationally requiring a predicate of second degree, i.e., one which needs two arguments to make a complete expression;² and such concepts are always used logically in this way. However, the situation is a bit different with respect to position.

As we have seen in the theoretical statements, the positional concept is indeed frequently taken as undefined, or rather it is effectively assumed that the reader will assign a meaning to it from his own intuition or experience and that there is a widely shared meaning (which is generally true since sociography). However, in our review we applauded the recognition (specifically by Haas, and by Kahn and associates, but also by Linton) that a position can be denoted only definitively by specifying all of its relations with other positions. Since the same relations also make up

²Again, it may well be asked why we should stop at second degree, when examples requiring higher orders can easily be given. Provisionally, the answer is that most conceptualizations do not make this extension. However, it is still a relevant topic and we return to it in the final chapter.

the roles, position and role are interdependent in their use. Now it would seem that either position is a more "basic" concept and role is defined in terms of it, or they are specified interdependently, but that we cannot have it both ways at the same time. The conflict is only apparent, however, and in fact arises from an ambiguity which shows up in the use of position.

The abstract or ideal concept of position refers to a location or point of some space, the location being capable of specification relative to other points. Put another way, the concept refers to the possibility of such a point, but not to the existence of any one in particular. It is in this sense that position is to be used as a primitive. This concept of position leaves open the question of where any given position is located. In any space, the particular location of a point is not given by the fact that it is there or even by naming it (unless the label implicitly or explicitly includes further information, as in the use of coordinates to designate the elements of a Cartesian system). "Locating" requires the use of rule(s), and in the kind of space which we need here, positions are "located" with reference to each other. That is, knowing where (or what) a singular position is requires knowing where it "stands" relative to the other positions in that system. And this means knowing what its relations are, or in general what

the relations are in the system; this is the same as knowing what (or where) the roles are in the system. It is in this sense that we say that positions and roles are specified interdependently. Empirical knowledge of roles in a system implies positions, and vice versa.

Turning now to some hitherto neglected considerations, we note that in any role system the number of positions and the number of kinds of relations by which positions are to be specified are both finite, though usually of different magnitude. It follows that the number of relations of whatever kinds which connect any pair of positions, and the total collection of all such connections, are also finite. Either this point is trivially obvious to most theorists, or they do not accept it, or it is of no consequence in their intended analyses, and this last alternative certainly seems to be the likely one. At any rate, the present author is aware of only one writer who makes any explicit remark on the matter. Bates says that, "Within any given culture there exists a limited number of roles which were combined in various ways to compose a limited number of different positions."³ A position ". . . is associated with a set of social norms," and a role is "A part of a

³ Bates, op. cit., p. 315.

social position consisting of a . . . sub-set of social norms . . .," i.e., of the ones for that position.⁴

Thus, positions and roles are taken by him to be finite (limited) in number in a given system; it is still possible that he intended to take the number of norms as unlimited, but this hardly seems likely.

A less obvious point is implied by our use of the phrase ". . . kinds of relations by which they are to be specified . . ." in the paragraph above. In general, we assume that any of the finite set of relations is a candidate for the connection between any given pair of positions, and that all of the positions (and roles, etc.) in a given system can be definitely and completely described in terms of some set of relations, properly enumerated. On one side, this point is not at all clear in most statements, since most only deal explicitly to any extent with a dyadic model; lacking an attempt to define a total system, there is little interest in considering whether the set (of whatevers) has a parent, or how it compares analytically to other such sets. On the other side, this specification seems to be no more than a sensible operational requirement. A related assumption which we shall use is that each relation is binary, i.e., that it either applies to a pair of positions, or it does

⁴Ibid., p. 314.

not. This is mostly for simplification, and we will see later that it can be relaxed somewhat.

With this background on our primitives (as a result of which one might say that they are not quite undefined), we may state a fundamental postulate of our model of a role system, which we will designate as the axiom of relation. Consider any relation, R_i , and any two positions, P_j and P_k (which are not necessarily distinct), each having been selected from its appropriate set. Then R_i either joins P_j and P_k or it does not, and R_i either joins P_k and P_j or it does not. Taken together these two conditions result in four possible combinations which we could symbolize as follows, using " $\overline{R}_i$ " to show that the relation does not join the pair.

- 1) $P_j R_i P_k$ and $P_k R_i P_j$,
- or 2) $P_j R_i P_k$ and $P_k \overline{R}_i P_j$,
- or 3) $P_j \overline{R}_i P_k$ and $P_k R_i P_j$,
- or 4) $P_j \overline{R}_i P_k$ and $P_k \overline{R}_i P_j$.

This display shows a feature mentioned in the preceding chapter but not yet recalled here, viz., that in general relations are taken to be "directed," which position comes first and which second must be specified. This feature is required for our formal definitions of role-sector and role. In the following we will refer to

an expression of the form " $P_j R_i P_k$ " as a "predicate" and one of the form " P_j " (or " P_k ") as an argument for ease of reading.

- 1) A role-sector is a collection of predicates whose first arguments are identical and whose second arguments are identical.
- 2) A focal role is a collection of predicates whose first arguments are identical.
- 3) A counter role is a collection of predicates whose second arguments are identical.
- 4) A role is a focal role and a counter role whose respective first and second arguments are identical.

We should notice that each role contains at least one role-sector, and that it may contain only one, in which case the denotata of all four terms are the same. Also, though " $\overline{R}_i$ " does not appear explicitly it is accounted for as the remainder when a relevant set of " R_i " has been collected.

The Incumbency Axiom

We now need to formulate explicitly some assumptions of considerable importance as a foundation for the subsequent developments of our analysis. We shall refer to these assumptions as "the incumbency axiom." It requires an additional primitive term, "actor," which we

introduce without comment at this point. The axiom may be stated in two parts. Assume that A_i represents an actor and P_j and P_k represent two positions.

- 1) Either A_i occupies P_j (or P_k or both) or he does not. (There is no such thing as a slight case of incumbency.)
- 2) If A_i occupies P_j , then for any role-sector of P_j , A_i has all of the relations which constitute the role-sector with some incumbent of P_k .

Insofar as the writer is aware, this precise statement of the assumptions does not appear in this form in the literature of the concepts with which we are concerned. In fact, some of the drift of theorizing in this area might well be said to run counter to our axiom. We must therefore consider its desirability with some care. Let us begin with some general considerations and then proceed to examine arguments against it, but hopefully also some for it, from the literature.

We have said previously that our task in this work is taken to be one of concept formation (or reformation). In the question at hand, we believe that a clue to desirable characteristics of our concepts is available in measurement theory. Even at the lowest level of measurement, i.e., what is usually called a

nominal scale, or otherwise may be thought of as a set of categories, we require that the categories be mutually exclusive. That is, that any object for which the particular scale of measurement is relevant may be assigned to one and only one of these categories. We have said that there is a clue quite deliberately in order to avoid giving the impression that measurement considerations are to take precedence over others, or that they are a directly adequate model of the empirical reality particularly as it is, so to speak, experienced.⁵

What we do want to suggest is that the criterion of exclusivity carries the kernel of the problem we confront when we have to decide whether two persons (or one person in two different situations) are involved in the same role-sector. The concept of exclusivity implies that with respect to the variable to be measured (which may be complex), the object in question has only that set of properties associated with a particular category of that measure.⁶ In our present case, exclusivity would suggest that the same role-sector exists for actors A and

⁵Nadel does hold a view of this sort in his concept of logical differentiae of role frames. Nadel, op. cit., p. 74.

⁶This way of stating the point may at first seem strange, since we most frequently think of a measure as unidimensional, as involving only one property. However, consider the well-known model of a Guttman scale,

B and for actors C and D, only if the same relations hold between A and B that also hold between C and D.

As an example of a possibly contrary view, let us consider the case which Bates puts forward in arguing that the content of a role may not be fixed. The proposition is stated as a "postulate" of his system, viz.,

Postulate 3. . . . When a given role is a part of two different positions, it tends to contain slightly different contents in each position.⁷

He argues that this is justified by two earlier postulates,⁸ which are stated as follows.

Postulate 1. Within any given culture there exists a limited number of roles which were [sic] combined in various ways to compose a limited number of positions.

Corollary: A. Some roles, though not all, are found as parts of a number of different positions.

Corollary: B. Within any given culture, a given rôle tends to contain basically the same norms regardless of the position of which it is a part. . . .

Postulate 2. Within any given position there tends to be a strain towards consistency or adjustment between the various roles composing a position.

Corollary: A. In the long run, roles which are part of the same position and

which is given as a model for extracting a unidimensional ordinal scale from a multiplicity of properties. Further, notice what alternatives exist when "the data doesn't scale." We may either seek a multidimensional account of the scale and non-scale (or quasi-scale) types, or we may treat them as categories of a nominal scale.

⁷Bates, op. cit., p. 316.

⁸In which case, of course, it might well be called a theorem.

also inconsistent . . . will either be changed . . . or at least one of the two inconsistent roles will be eliminated . . .
 Corollary: B. In the short run, various devices or mechanisms may be developed . . . which will allow two inconsistent . . . roles to be maintained as a part of the same position.⁹

There is a logical problem here that we must dispose of before turning to the empirical example with which Bates buttresses the postulate (or theorem) cited above. In his definitions of the concepts of position, role, and norms, he states that a position "is associated with a set of norms" and that a role is "A part of a position consisting of a more or less integrated or related subset of social norms . . . distinguishable from other sets . . . forming the same position."¹⁰ In this context, the "norm" is the element and the content of a "role" is a particular sub-set of such elements. The phrase ". . . more or less integrated . . ." refers to relationships between these norms, not to which ones are included. However, in the statement of the postulates, he has in effect given primacy as an element to the concept of role.

If a given role is combined with different other roles to form two different positions,

⁹Ibid., p. 315.

¹⁰Ibid., p. 314.

then, its content, (i.e., the norms of which it is made) must be slightly different in each case in order for it to be consistent with the unique contents of each position.¹¹

The difficulty in his formulation is that "role" has been defined in two different ways, once in the explicit definition, and then again in Postulate 1, but at what we usually call two different levels, or at least in two different ways, of abstraction. When a particular role is part of a position, for Bates it partakes of the "unique contents" of that position. There is no question raised here as to whether the same role in the same position may vary in its contents. But a question does arise when this "same" role appears in two different positions: In what sense are we to take this as the "same" role? In the context of the passage under consideration and elsewhere,¹² the reason for this assertion regarding the sameness of roles is that the term is being considered at two levels at once, culture and group structure. What is "known" about the culture is applied to the group structure in order to "see" that the two roles are the "same." Now we must make very clear that the present argument is not inveighing against

¹¹Ibid., p. 316.

¹²F. L. Bates, "A Conceptual Analysis of Group Structure," Social Forces, 36 (1957), pp. 103-111.

the obvious and considerable benefits for substantive theory and empirical analysis of such an application of knowledge. But it must also be recognized that a precise rendering of concepts does not permit their indiscriminate use, and in the present case the sameness of the roles as parts of positions in different group structures does not arise solely from those structures when defined as sets of related positions.¹³

In order to recapitulate these remarks, let us consider the concrete example used by Bates.

Let us suppose the 'A' represents the role of 'disciplinarian' and that position 1 is the position of father in a family while position 2 represents that of foreman in a factory. What Postulate 3 states is that the father's role as disciplinarian differs from the foreman's role as disciplinarian since it is combined with different other roles in each position. . . . let it be supposed that the other roles in the father's position include 'teacher,' 'playmate,' 'spouse of mother,' and 'provider'; while those of the foreman include 'representative of management,' 'technician,' and 'friend.' It is seen that the father's role of disciplinarian is conditioned by a totally different set of other roles than in the case of the foreman. In order for it to be integrated into each position and to be consistent with other roles contained within that position, it must vary in its contents.¹⁴

¹³At the same time we should note that there is no implication in Bates' analysis that a role as a culture element exhibits any intrinsic variability at that level. We can only assume that it is a particular sub-set of norms.

¹⁴Bates, "Position, Role, and Status: . . .," op. cit., p. 316.

In Bates' conception, each of the named roles shown in single quotes is one of the "limited number of roles" available from the culture, and these are put together in the position. We have raised the question whether "disciplinarian" is the same role in "father" as in "foreman," and we have said that at the level¹⁵ of group structure there are two different roles, which are referred to by the same identifying term and indeed bear considerable resemblance to each other because, so to speak, they are drawn from the same cultural model.¹⁶

At this point, it is somewhat anticlimactic to observe that Bates' postulation of role variability actually does not contradict our axiom of incumbency as it is stated above. The axiom only requires that any actor who occupies a particular position will have its roles as given, and we have seen that he does not assert that "the same role in the same position" will vary in its contents. So, in effect, we have argued for a bit more than we need for immediate purposes in suggesting that the "same" role in two distinguishable positions

¹⁵"Level" is an unfortunate term here, but it is hallowed by usage; a more accurate rendering might be "when considering" or "in reference to."

¹⁶These remarks should not be taken to mean that our explicated model does not apply "at the level of culture." Specifically, there seems little doubt that the elements of culture that Bates has in mind come in reciprocally connected pairs.

must have the same content. Later it will be seen that in its narrowest sense our graphic model defines the logical structure of a role only with respect to the specific positions with which it is associated. The argument has been carried through both to help bring out what the axiom does require and to show how this particular argument for role variability ought to be construed.

"Cultural" identification of positions and roles is also involved in the next set of views which we need to examine in connection with our axiom. We refer to the arguments put forward by Gross, Mason, and McEachern. The questions which their discussion raise flow most immediately from their denial of "the postulate of role consensus," but we will need to review other points of their presentation as well.

They argue (with good cause) against the idea that the expectations for and patterns of behavior of the members of a group are social or cultural givens, and/or that there is complete agreement among members on these matters. In their third chapter they have amassed evidence that a number of theoretical discussions are committed to such assumptions, and they cite relevant empirical evidence which confounds the position. We are concerned here with whether their criticisms argue against the advisability of the incumbency axiom.

There are a number of interrelated questions to be examined, and they stem generally from what Gross, et al., have called the "role definers problem."¹⁷ By this they mean that in any empirical research, the investigator must specify whose definition(s) of the expectations which are taken to constitute the roles in question are to be analyzed. Their denial of role consensus as a postulate is based on the empirical fact that a plurality of definers may disagree as to how these expectations apply.

One of the keys here is the term "plurality." It is quite clear that the authors do not consider the question of consensus to be relevant if only a single role definer is being considered as such, whether it be a person, or an actor more generally (as a collectivity), or the modalities of a social system in general.¹⁸ Ambiguity or vagueness in the cognitions (in Biddle's sense) of a specified definer are not viewed as theoretically relevant, and they would probably be treated as residual errors of measurement, though the authors are not explicit on this point. In effect, for a specified

¹⁷Gross, et al., op. cit., p. 70.

¹⁸They do not allow this generality explicitly, but the intention of their scheme requires it: ". . . a special point was made of defining concepts without specifying whether they applied to individual, social, or cultural phenomena." Ibid.

role definer and a specified position incumbent the expectations for that incumbent vis-a-vis a specified alter "are what they are," and this is true for all specified alters.

This much agrees with the second part of our axiom, but what of the first part? Here there seems to be little difficulty.¹⁹ Their views seem clearly and completely in support of the assertion that an actor either occupies a position or he does not. However, there is another element involved in this view of positions which lends further and substantial credence to the second part of our axiom, which deals with role(s) in positions.

We may express the authors' views on positions by saying that they have adopted a "postulate of position consensus." Particularly, if a person is a superintendent of schools or a member of a board of education, they assume that all role definers (or perhaps all rational ones or the like) will agree to the allocation of such positions. But what does it mean in their scheme for a person to occupy a position? It is not simply that certain "labels or identities"²⁰ are uniformly applied,

¹⁹The same remark applies to our discussion above of Bates' scheme, though we did not take note of it here.

²⁰Ibid., p. 40.

but that the incumbent is located in a system of relationships with others; this latter feature is, of course, their own formal definition of a position. The essential question here is how positions are located. We are not referring to the way in which an investigator decides which positions to study or what descriptions of them he provides; rather we are asking 1) what it means in operational terms for an investigator to locate a position in a system and 2) what additional consequences are assumed for the incumbents' behavior?

Gross and his associates provide only a partially explicit answer to the first part of this query in their theoretical discussion, but some suggestions about the answer to the second part may be made on the basis of their use of the theoretical concepts in their own research. They propose "two aspects of position specification, the relational and the situational."²¹

Relational specification in its fullest use amounts to the enumeration for a given position of all other positions to which it is related. We assume they mean this to be a dependent aspect which requires the situational specifications; it is difficult to conceive how the enumeration could be made without some knowledge of the situation. But the operational meaning of situational

²¹Ibid., p. 50.

specification is not made clear in their theoretical remarks, which consist mainly of examples of "situational context" but do not provide a hard definition.²² We believe that this is because, in practice, the concept of situational specification does double duty in 1) setting the limits of generalization, which is the salient facet in their theoretical analysis, and 2) providing constraints on the system which, in effect, explain and justify the relational specification. The latter function becomes apparent in their empirical analysis.

In considering the specifications of the object population, it is pertinent to make certain observations about the kind of system of social relationships in which the positions of school superintendent and school board member are involved.²³

Following this remark, they list five features of the board system and discuss their general consequences for the structure of the situation. That is, at a fairly general level and without mentioning the term explicitly, they enumerate features whose operational and behavioral consequences can only be that they set certain expectations for incumbents of the positions.

We have gone to some trouble, and perhaps the long way round, to say that the authors have turned the postulate

²²Ibid., p. 56f.

²³Ibid., p. 99.

of role consensus away from their front door, only to readmit it at the servants entrance. Gross and his associates have denied the "postulate of role consensus" and declared that it is a matter for empirical investigation, and one cannot take exception with this. But when confronted with evidence of consensus, they take no notice of its consequences for their definitional system. If we are correct in believing that their description of a school system as ". . . a form of organization that has most of the characteristics of . . . bureaucracy,"²⁴ entails 1) uniformity of 2) certain expectations regarding positions, and given their definition of role as ". . . a set of expectations applied to an incumbent. . .,"²⁵ then it seems that there is consensus and it is about roles.

It seems fair to say that the authors recognize that it is necessary to assume a "fixed" core in the structure of a system,²⁶ but they choose to conceptualize it in position and to assign elements of variation to role. In some respects this seems a very natural usage. The concept of position is usually taken as similar to

²⁴Ibid., pp. 99-100.

²⁵Ibid., p. 67.

²⁶"That the members of a social system, whether a dyad or a total society, must agree among themselves to some extent on values or expectations is a matter of definition." Ibid., p. 43.

"point," and the idea of unambiguous fixity is clearly contained in the common meaning.²⁷ If a role is thought of as "the dynamic aspect of status," it is easy to assimilate the idea of change and variability to it. However, the trend of conceptualization in recent decades has been toward taking the concept of role up more firmly into "structure." Even conceived as expectations there is no inherent reason for requiring it to "soak up the variance" unless it is tacitly assumed that there can be no variability about position and that it is necessary to locate variability in one or the other. The present analysis will try to suggest that there is no need to locate variation in either. Both position and role can be taken up interdependently in a model of a "fixed" structure, and variability can then be conceptualized in terms of the structure(s) as such.

These remarks have led us ahead of our immediate interest, and explanation of them must be delayed. There is yet one aspect of the usages of Gross, et al., to be noted here. We refer to the use of the "identities" of positions which were mentioned before. In actual practice

²⁷Cf. Bates remarks on the "imperfect spatial analogy" of having an actor occupy more than one position in a structure. Bates, "Position, Role, and Status: . . .," op. cit., p. 313. However, we do not accept his argument herein; it is a bad example of allowing representational techniques to decide form, rather than suiting them to content.

it appears that they believe that the names of positions are to be used to locate them, even though ". . . positions have been defined as locations of actors in systems of social relationships. . ."²⁸ It is of considerable interest to inquire whether this use of positional labels is necessary or even possible in all cases. The answers would seem to be that names are quite handy if they exist, but that it is quite possible to conceive of an actor or actors in a system of social relationships whose location is not manifestly identified by participants. It seems perfectly sensible to conceive of a latent structure of positions which are neither "intended nor recognized" by members of the system.²⁹ The only possible way then to locate the actors would be in terms of the "social relationships" themselves. Indeed, this is what is done in studies of "informal organization." That is, actors are "located" in the "group structure" in terms of one or a few relations of whatever content.³⁰

²⁸Gross, et al., op. cit., p. 49. Emphasis added.

²⁹We use "latent" here in the sense defined by R. K. Merton. The phrase "latent structure" may be infelicitous since it is used by P. F. Lazarsfeld in another sense.

³⁰Many examples could be cited but a few will suffice here: A. Zalesnik, et al., The Motivation, Productivity, and Satisfaction of Workers, Boston: Harvard University, Graduate School of Business Administration, 1958; R. S. Weiss and E. Jacobson, "A Method for the Analysis of the Structure of Complex Organizations," American Sociological Review, 20 (1955), pp. 661-68; H. White, "Management Conflict and Sociometric Structure," American Journal of Sociology, 67 (1961), pp. 185-199.

This is a question of much interest to our analysis in other respects and it is taken up elsewhere, but it is introduced here for its bearing on the advisability of our axiom of incumbency. What can it tell us on the matter?

An unequivocal answer cannot be given, but the general indications are positive. The term role is rarely used to refer to patterns of the relations analyzed in such studies, though the formal definitions of the concept which we have reviewed would not preclude this usage. The term position is used frequently, but in a way which parallels the absence of role, viz., as the location of an actor rather than as, so to speak, the location of a position. When a collection of actors are observed to have the same pattern of relations with others, they are usually designated by "group" or some similar locution, but not as occupying a common position, although, again, this would be permissible in the usual definition. However, the main point of interest here is what happens to disagreements concerning relations among actors, and the answer is that they are taken up as part of the structure. "Contradictions" may exist, but they are seen as reflecting different positions (and perhaps dissensus as well). In terms of our axiom, if an actor occupies a particular position, he does so in virtue of the unique pattern of relations for that position vis-a-vis others which actually defines the position.

We have reviewed a number of substantive considerations, including potential objections, regarding the axiom of incumbency, and we conclude that it survives these tests. In addition, these considerations have served to give further direction to our reinterpretation of the concepts of position and role and to suggest features which the conceptualization must represent adequately.

In an important formal sense, it is unnecessary to interpret our analysis in this chapter as justifying the terms and axioms. In the formal structure of a deductive system, such conceptions are established as satisfactory by leading to desirable propositional consequences, in the present case by assisting in the development of a structural model which properly reflects important concepts. It will be the burden of later analysis to show that this is the case.

CHAPTER IV

THE FORMAL MODEL

In the last chapter, we set out a verbal axiomatization of the concepts of a role system which we believe is adequate to compass the structure we have detected in the sociological literature. The work of this chapter is to coordinate these results with an appropriate mathematical representation which we can study to learn whether such a representation has properties satisfactory for our needs. We shall specify two analytically equivalent mathematical representations, but we will find that they are incomplete and require some extension to represent our concepts and that this leads to as yet unanalyzed problems.

Linear Graphs and Matrices

Our discussion of graphs and matrices in this section is mathematically informal, but it is still technical and quite compressed. There are some important points which we wish to emphasize at the beginning so we may ignore them when it is convenient to do so in the exposition. We have said that we will specify two equivalent representations. However, there is really only one system -- the same axioms apply to both

representations. In Nagel's sense, each of the representations, graphs and matrices, is an interpretation of the axiom system (i.e., the "calculus"), and each is thus a model.¹ It is useful to describe both since each aids our understanding in different ways. For example, our primitive terms (position and relation) seem to "come through" better in the graphic representation, and our defined terms (role and role-sector) and qualifiers (focal and counter) do best in the matrix display, though all of these terms can be discussed in either representation. By convention, the axiom system is associated with the graphic model. In view of this one might think of the matrix representation as only a convenient alternative to the "real" thing.² However, we use the matrix interpretation heavily in the sequel, and it seems best to the writer to acknowledge that it is equally as legitimate as the graphic.

The theory of linear graphs deals with a kind of geometric structure which is outside our ordinary intuitive conceptions of spatiality, though it is somewhat familiar to sociologists since a sociograph is a specie of the general kind. Linear graphs are pictorially displayed as

¹Nagel, op. cit., p. 90f.

²Also, the reader familiar with matrix theory will see that our matrix representation admits only a small part of the operations of that mathematical discipline. The possible confusion from our use of the term "matrix" is regrettable, but we have not been able to devise an alternative.

collections of points and of lines which join them. They differ from some other geometric structures in that there is no assumption of ancillary arbitrary referents by which distance and direction are determined.³ A point does not have a location in the sense of the familiar Cartesian or polar coordinate systems of analytic geometry. Neither does a line have any length, regardless of the way in which we might draw it in a diagram, though we might say that each line has length "1". However, a point does have a location relative to all the other points in the system in terms of the lines which join them.

The example of sociography may suggest to the reader characteristics of such collections of points and lines which were not strictly intended in the preceding paragraph, viz., that each line goes from one point to another point, so that we might say that it is "directed," and also that there is at most one line directed from one point to another and/or another line in the reverse direction. In general, we will assume that all lines are directed,⁴ but we will not assume that only one line may be directed from one point to another. The term "linear

³Concepts of distance and direction are used in analyzing linear graphs, but they are defined in the system itself.

⁴Actually, undirected graphs can be taken as a special case of directed graphs in which there are always lines in both directions between a pair of points, if there is one in either direction; i.e., they are point symmetric. However, they are of great interest in applications, and the case of graphs with single undirected lines has been intensively studied because of its importance in physical systems analysis.

(directed) graph" is usually restricted to ones in which there is only one line in a given direction between a pair of points. (The term "unilinear graph" seems to us to be better.) Also, the terms "edge" or "arc" are usually used for lines (and we may sometimes call them "arrows"), and a line is said to be "incident from" or ". . . to" a point to show its orientation. Finally, a line incident from and to the same point is sometimes called a "loop," and two lines incident from and to the same points are said to be "parallel."

As we have indicated, another mathematical structure which can be shown to be completely isomorphic with a unilinear directed graph is a matrix, or more specifically a square binary matrix.⁵ A matrix is a rectangular array of elements such that each element can be uniquely denoted by reference to the row and column in which it appears, that is, the element lies at their intersection, which is called a cell. A square matrix is one which has the same number of rows and columns; a chess board is a pictorial example of a square matrix with eight rows and columns. By calling such a matrix binary we mean that there are only two kinds of elements which can appear in it (one or the other in each cell), and we usually use the digits

⁵See L. Katz and J. H. Powell, "Probability Distributions of Random Variables Associated with a Structure of the Sample Space of Sociometric Investigations," Annals of Mathematical Statistics, 28 (1957), pp. 442-48.

"0" and "1" for representation. The rows and columns are denoted by assigning the integer digits from "1" to "n" (where n is the number of rows and of columns) to the members of each set, usually in order and starting from the upper left, so the first row is the top-most horizontal array, the first column is the left-most vertical array, and so on.

We can coordinate the elements of a unilinear directed graph and of a square binary matrix in the following way. In a graph of n points, we assign the digits 1, 2, . . . , n, to the points one-to-one in any arbitrary order. We associate a given point of the graph with the row and the column of the matrix which have the same digital referent, which serves to represent the originating (rows) and terminating (columns) aspects of the points with respect to the edges which join them. We then inspect each edge in the graph, noting the numbers of the points, which we use to determine the row ("from") and the column ("to"), i.e., the cell of the matrix in which we are to place a "1." Having inspected all edges and entered 1's in the appropriate cells, we enter 0's in all remaining cells. This is because we earlier had assumed that there either is or is not an edge of given orientation between each pair of points. Obviously, a similar procedure could be followed to "map" in the converse way, from a matrix to a graph. The following illustration shows a unilinear directed graph of three points, with

arbitrarily chosen index digits and edges, and the corresponding matrix representation.



In order to provide a corresponding matrix representation for a multilinear directed graph (i.e., one which possibly has more than one edge of given orientation between any given pair of points, or more precisely, one which has a plurality of different kinds of possible edges) we need to think of having a separate matrix for each of the kinds of edges represented in the graph. We might think of these as appearing like sheets of paper in a file drawer, each with the same square two-dimensional matrix form, but each referring to a different one of the possible kinds of lines used in the multi-graph. This suggests extending our matrix into three dimensions, each plane of the third dimension (besides rows and columns) representing an edge type. Mappings could then be constructed by an extension of the foregoing procedure, from the multilinear directed graph to the three-dimensional matrix and conversely.

Coordination of the Systems

We have described graphs and matrices in an informal way, and we now need to show that they are indeed mathematical representations or structures which are formally equivalent to our conception of the structure of a role system as stated in the axiomatization of the last chapter. For this purpose, we examine the primitives and axioms of multilinear directed graphs and show that they can be coordinated to our previous results. Our source is the recent treatment by Harary, Norman, and Cartwright. We should note that these authors use the term "net" to refer to what we have called a multilinear directed graph, "relation" to refer to our unilinear directed graph, and "digraph" to designate a "relation" with no "loops."⁶ Their terms are more economical, but we have avoided them in the preceding description for several reasons. First, the term "net" conjures an image of only single strands between knots, which seems to us to be misleading. Second, we have used the term "relation" in our earlier discussion and did not want to assign it prematurely to the corresponding mathematical structure. And finally, the restriction of graph theory to "no loops" is not completely standard (this may be their reason for coining the term "digraph," which is not used elsewhere). In a sense, all

⁶Such a "digraph" maps into a "hollow" matrix -- one with 0's on the principal diagonal, the set of cells each of which has its subscripts identical.

of this is by the way, since we will mainly use the matrix representation in our development. However, matrix theory includes a good deal more than our rather restricted representation, and the graphic isomorph is required for its axiomatization.

The four primitives of nets (and also of relations and of digraphs) are:

- P₁. A set V of elements called 'points.'
- P₂. A set X of elements called 'directed lines,' or more briefly, 'lines.'
- P₃. A function f whose domain is X and whose range is contained in V.
- P₄. A function s whose domain is X and whose range is contained in V.

The first two of these primitives are self-explanatory. The second two relate the lines to the points by means of two functions f and s which serve to identify the 'first' and the 'second' point of each line respectively. . . . In general, for any line x of X, the image fx of the function f is called the first point of x and the image sx of the function s is the second point of x. Thus every line of a net is directed from its first point to its second point.

The axioms for a net are:

- A₁. The set V is finite and not empty.
- A₂. The set X is finite.

The first axiom excludes consideration of an empty net with no points at all and of a net with an infinite number of points. Then the second axiom avoids nets with a finite number of points but an infinite number of lines. These two axioms impose no restrictions on the structure of a net other than the number of its points and lines.⁷

It will be seen that this is not manifestly identical with out earlier systematization, so we must examine the several items to see how the coordination may be made.

⁷F. Harary, et al., Structural Models: An Introduction to the Theory of Directed Graphs, N.Y.: John Wiley & Sons, 1964, p. 5.

First, the primitives P_1 for points and P_2 for directed lines may be taken as equivalent to our positions and relations respectively. Next, the primitives P_3 and P_4 , which specify the functions $\underline{f}$ and $\underline{s}$ for first and second points of edges respectively, serve the same purpose as our axiom of relation. For our purposes, the earlier way of stating the assumptions (N.B., primitives are assumed) is intuitively preferable in that it displays a catalog of possible decisions which can be taken as determining entries in the cell(s) of a matrix. Also, it explicitly denotes the kinds of relations (edges) which may appear, though this will later be shown to be a matter of formal indifference to the theory of "nets." However, the formulation by Harary, et al., is clearer in showing how the concept of direction is defined. Finally their axioms A_1 and A_2 appeared in our commentary, though not as explicit elements of the axiomatization, with the exception of the provision that there may be no edges in a graph. This provision is perfectly acceptable though only trivially interesting.⁸ Thus, we see that the formal concepts of a role system are mathematically represented in the theory of "nets" (or in our term, multilinear directed graphs).

⁸It does raise a point which we have not mentioned before, in that one usually thinks of a position in terms of its, so to speak, manifest relations with other positions. But when one shifts attention to the total set of

It is also interesting to consider the import of two additional axioms which we do not use. The first is required for the theory of relations, and both for the theory of digraphs, as well as all primitives and axiom A_1 above (A_2 can be derived from A_1 and A_3).

A_3 : No two distinct lines are parallel.

A_4 : There are no loops.⁹

Either of these would be mischievous in our system, except in special cases, and we are thus debarred in general considerations from the aid of the theories of unilinear graphs. Unfortunately, it is precisely these topics (particularly systems requiring axiom A_3) which have received most extensive treatment.¹⁰ It can fairly be said that the mathematical theory for role systems has yet to be written. However, the theories of relations and digraphs are still of no small comfort since they can

relations which may possibly exist between any pair of positions, it becomes clear that a position is also defined by the relations it does not have with others. And the idea of a null position is quite intelligible as a residual category which is not part of the connected system. These ideas will become somewhat clearer in the sequel.

⁹
Ibid., p. 9.

¹⁰We should note that the modern treatment of these topics as graphs is usually dated from the work of D. König (in 1936) who suggested the name and began a systematic study of their properties. See C. Berge, The Theory of Graphs and its Applications, London: Methuen & Co., 1962, p. ix.

be applied to single relations (which we shall not do to any extent in the present work) and they generate concepts which can be applied by extension to the multirelational case, and we shall note these as we go along.

We have coordinated our primitive terms and axioms to the mathematical representations, and we now turn to coordinating our defined terms which denote various forms of the concept of role. To do this it will be convenient to shift our attention to the matrix representation. We have described this as a three-dimensional array of cells, each of which contains either the element "1" or the element "0." The file drawer example will again be handy. On each page (which we will subsequently refer to as a table) in our file, we have a square matrix whose rows and columns represent positions in what we may now call their focal and counter aspects respectively. If we consider a particular cell (say the " i, j_{th} " one, where i and j are integers between 1 and n) in a table and its counterparts in all other tables we may call this a cell vector, and the pattern of relations reflected by the contents of these cells represent a role sector (one of two, the other being the " j, i_{th} " one, unless $i=j$) for the i_{th} and j_{th} positions. If we consider a collection of all cell vectors whose first position identifications (subscripts) are the same, then we shall call the collection of

patterns of relations a focal role; a counter role is defined similarly for second subscripts. Such a collection of cell vectors from the same row (or column) of our supermatrix is represented by a horizontal (or vertical) plane, and is itself, of course, a rectangular matrix. Finally, a role is coordinated to the collection of contents of a set of two such planes, a row plane and a column plane with same subscripts (only one subscript is required to identify a plane).

Thus, as anticipated, we have developed a representation of a role system which provides interdependent specification of positions and roles. That is, we may think of the concepts, as Linton suggested, as indicating different aspects of the same content; we might think of positions as denotative, and roles as definitive. To rephrase the original aphorism, a position is a locational aspect of a role, and a role is a substantial aspect of a position.

In order to have a compact way of referring to this three-dimensional array (or as we called it above, supermatrix), we shall adopt some notational conventions. We will call such a representation of a role system a position matrix (without prejudice to other concepts, but in anticipation of some other terms to be adopted later), or a **P**-matrix, or just **P** as the circumstance permits. On occasion, we shall have need for reference to sub-sets

of cells of the sorts used above to define concepts, and we will indicate them by appending appropriate subscripts, the first for rows, the second for columns, and the third for tables, with dots (.) to show a collection over an entire dimension (as $P_{ij.}$ for a cell vector, and $P_{..k}$ for a table).

The Incumbency of Actors

To complete the main outlines of our formal model, it remains to consider the representation entailed by our incumbency axiom developed in the last section of the preceding chapter. It will be recalled that it was stated in two parts; the first guaranteed that each actor could be determinately assigned or denied occupancy of each position in the system, and the second required an invariance of relations, that an incumbent of a position must have each role sector with at least one incumbent of the relevant other position (N.B., we do not use the terms focal and counter here, as the incumbent's position must be considered in both aspects). There are two features of this axiom, one regarding each part, that we have not mentioned before, and they are somewhat easier to conceive when a complete representational scheme is in hand.

The first point is that the axiom does not assume that there are any actors occupying the system.

We need to be able to conceive of systems abstractly (this was the concept Linton had in view in discussing social systems, and the statuses and roles which were to compose them), and the P-matrix does this. For this purpose, the incumbency axiom is supernumerary.

Secondly, while the axiom specifies invariant relations between the actor and some actors in other relevant positions, it is silent concerning what relations he has with the remaining actors in those other positions. This is quite reasonable since we cannot require, for example, that a given adult female be mother to all children of her community, nor that a foreman supervise workers in departments other than his own. In addition, we ought to allow the possibility, say, of friendship in either case. This exhibits the possibility that these residual alters may be the salient ones for our hero or heroine in some other position. However, further examination of these features would take us into a consideration of problems which will be delayed until the next chapter. We return to the present question of the representation of incumbency, with these observations as an aid to see what is implied.

The first part of the axiom, again, says that every actor is or is not incumbent in every position. It seems quite natural to display this in a binary rectangular matrix, in which we will take rows to refer to

actors and columns to refer to positions, with a "1" in a cell to show that the actor occupies the position and a "0" otherwise. We will call this the incumbency matrix, or the I-matrix, or I, in a way similar to our notation for the position matrix.

The second part of our axiom of incumbency needs a bit more attention as it yields an additional matrix representation which is not quite so obvious from its statement. The representation has the same form as **P**, and in particular the same number of tables. But it need not have the same number of rows and columns (they will be equal to the number of rows of I, say m), and its precise contents are in general not specified from a given **P** and I because of the qualifying "some other actor" in the statement of the axiom. An example of this would be a **P** representing the skeletal line of an organization (though most organization charts would not be so complete), and an I representing the roster of duty assignments of employees. However, the axiom does guarantee the existence of this matrix (which we shall call the actor matrix, or A-matrix, or A, as before), which would be constituted as follows. There will be one table for each table of **P**, and there will be one row and column for each row of I, and each plane will be coordinated appropriately.

We can demonstrate the existence of A with the following algorithm. For a given row (or column) of A , search the relevant row of I ; each "1" encountered resides in a column of I which designates a row (or column) of P ; search this row (or column) of P ; each non-empty cell vector of P designates a pattern of relations and the column (or row) designates a column of I ; search the column of I and note the row of each "1"; these rows designate columns (or rows) of A , and at least one of the cell vectors formed by their intersection with the originally given row (or column) of A must contain the pattern of relations secured from P . This completes the proof of the existence of A .

We should notice particularly that empty cell vectors of P are idle in this proceeding. This results from the requirement of "exhaustiveness" of the second part of the axiom, that each actor of a given position must be properly connected to some actor of the other position in question for the role sector to be affirmed (and this holds for both focal and counter aspects). It exhibits a fact of considerable importance, that the axiom of incumbency works a marked effect on the concept of a role system. This can be conceptualized by observing that, in general, A cannot be obtained as a set of single-valued functions of the I and P matrices (as our proof of the existence of A has shown in the indeterminacy of role

sectors between actors), i.e., symbolically $A \neq g(I, P)$; but on the other hand, I and P must jointly be determined by single-valued functions of A , i.e., $(I, P) = f(A)$. We say "functions of A ," because in general the " f " of our equation must be thought of as complex, using several such functions¹¹ (and iteration). (A proof of the existence of " f " is that one such set of mappings is developed in the next chapter.)

The more important side of the point is a substantive one. When we accept the incumbency axiom we thereby accept the consequence that the role systems for an (empirical) group are definitely determined by the relevant relations specified between the actors in the group. This emphasizes that role systems are analytic constructs of the sociologist, even though he will prefer to construct them in a way which he believes coincident with or representative of reality. There is more to be said on the subject and we return to it in the final chapter.

¹¹The mathematical concept of a single-valued function involves what is called a "mapping" of the elements of one set into the elements of another set, under the restriction that each element of the first must go to only one element of the second, but not conversely. The first set is called the "domain" and the second set the "range." The concept is used in the quotations from Harary, et al., above.

Comparison With Other Formalizations

We have specified a formal representation which we believe to be the correct logical model of the concepts of role systems. It is now of interest to inquire provisionally about its comparative virtues vis-a-vis other attempts in this direction. First, let us notice that every other formulation of which the present author is aware assumes, sometimes explicitly but usually not, the theory of graphs or in its most general form "nets." Since our development, not including the incumbency axiom, uses the theory of nets as a model, but no more, all other formalizations must be "special" cases thereof, i.e., properly explicated they are either isomorphic or utilize additional axioms. If they incorporate additional axioms, the resulting formal structure will be richer, but we must question whether it then properly represents our ideas of role systems in general.¹²

There are two other formalizations of which the author is aware that are intended for the same purpose in general and are sufficiently developed to warrant consideration.¹³ These are contained in 1) a series of articles by

¹² An excellent example of a richer axiom set is H. White, An Anatomy of Kinship, Englewood Cliffs: Prentice Hall, 1963. We should be clear that this author's purpose is restricted.

¹³ It is believed that the present analysis is an acceptable explication of the intentions of Hass, Gross,

Bates, beginning with the definitional one frequently cited above, and 2) two papers by Oeser and Harary. Though they are quite different, they share certain faults. Chief among these is a functionalist orientation which leads to a focus on goals (Bates) and tasks (Oeser and Harary), and in each case this vitiates the possibility of achieving a proper structural analysis. Equally unacceptable are the different restrictions made regarding assignment of actors to positions in a group, but in each case the restriction is a locution more than a reality since they subvert it at the first turn, each in their own way. We will consider each conceptualization as briefly as possible.

Bates' definitions of the role concepts have been cited in Chapter III and they do not need to be repeated here. His extension of these concepts into a definition of group structure is made in the next two papers in the series.¹⁴ The first of these specifies the concept of a

et al., Merton, Goodenough, and Kahn, et al., in the works cited in preceding chapters, or rather of what their intentions would have been if fully carried out.

¹⁴F. L. Bates, "A Conceptual Analysis of Group Structure," op. cit., and "Institutions, Organizations, and Communities, A General Theory of Complex Structures," Pacific Sociological Review, 3 (1960), pp. 59-70. Respectively, these are referred to as "Group Structure" and "Complex Structures" in the next footnotes.

group in terms of position and roles, but the intent and implications of this only become clear in the second. He states his definition of a group in terms of two conditions.

Condition 1. A group consists of at least two individuals who interact with each other as the occupants of two positions, each of which contains at least one role reciprocal to a role in the other position.

• • • • •
Condition 2. A group is composed of all individuals who occupy positions reciprocal to all other positions in the group structure and includes no individuals who do not meet this condition.¹⁵

We may interpret this in terms of our own model as follows. It imposes a restriction on the P-matrix such that it can be partitioned into sub-sets of positions which are non-overlapping and dense. By dense we mean that each sub-set has no empty off-diagonal cell vector,¹⁶ and by non-overlapping we mean that all cell vectors not contained in such sub-sets are empty. (It appears that he would also intend the principal diagonal to be empty, but the point is not clear.) These requirements may not be immediately clear from the conditions, but it may help to recognize their formal equivalence to the definition of a clique as a maximal sub-set of persons in a sociomatrix who enter into

¹⁵ "Group Structure," pp. 104-5.

¹⁶ "Off-diagonal cell vector" means P_{ij} , such that $i \neq j$.

reciprocal (friendship) choices.¹⁷ There may be many such cliques and they may overlap with regard to persons. However, in Bates' application the positions of each group are taken to be unique, the linkage between groups being performed by multiple incumbency of individuals but in positions of several different groups. Apparently, these specifications are motivated by a consideration introduced in the earliest of his papers, viz., "The concept of social position depends on an imperfect spatial analogy since it allows a given individual to occupy two positions in the same social space at the same time."¹⁸ Congruent with this, "the same social space" can compass only a single group, and other related groups simply are not in the same or even determinately relatable spaces, i.e., not in terms of the same operations used to define each separate space. However, his later usage suggests that the concept of a group only serves to "save the forms," to retain the idea of singular incumbency without serious

¹⁷L. Festinger, "The Analysis of Sociograms Using Matrix Algebra," Human Relations, 2 (1949), pp. 153-58; and R. D. Luce and A. D. Perry, "A Method of Matrix Analysis of Group Structures," Psychometrika, 14 (1949), pp. 95-116.

¹⁸F. L. Bates, "Position, Role, and Status; . . .," op. cit., p. 313, emphasis added.

prejudice to the definition of larger enclaves. In an example used to show group linkage he remarks of the individuals performing this function, "In other words, these two actors occupy two positions in the structure of a multigroup system."¹⁹ In connection with all of this, we should notice that in our own constructions of the A and P matrices, each actor and position do occupy unique locations in the corresponding multigraphs. Multiple incumbency is accounted for in the I matrix, and it employs a quite common spatial concept (in the mathematical sense) of a mapping of one set of elements into another. In part, Bates' construction stands on an imperfect understanding of spatial analogies and an attempt to fit his concepts to a predetermined form, rather than allowing them to generate the form they require.

A similar criticism applies to the formalization offered by Oeser and Harary.²⁰ In our view, they have stated a model in search of a theory, but not the one specified in the titles or context of their articles.²¹

¹⁹

"Complex Structures," p. 60. Emphasis added.

²⁰We do not mean that their spatial concepts are incorrect; Professor Harary has been one of the foremost contributors to the development of graph theory, and we would hardly challenge his mathematical credentials.

²¹O. A. Oeser and F. Harary, "A Mathematical Model for Structural Role Theory, I," Human Relations, 15 (1962), pp. 89-109, and ". . . , II," . . . , 17 (1964), pp. 3-17.

Their model does seem to codify some common conceptions about the structure of formal (complex, large-scale, purposive, bureaucratic) organizations, but it is not put forward as such. Fundamentally, the difficulty arises from their effective acceptance of the proposition that the structure of a role system is given by the formal (purposive) organization, which they take to exist in every social structure, and that this is only subject to ancillary modification by the informal organization. Since they take the formal organization to be representable by a collection of related digraphs (which is quite conventionally accepted), they are able to take the theory of digraphs as the representational system for their construction. It is in this sense that we say that they have fitted the substantive theory to the formal model in an unacceptable way; the substantive theory is wrong to begin with. However, it is always possible to get the right results for the wrong reasons, but a brief further consideration of their development will show that they have not represented role concepts properly as a general structural system.

The following quotation is a summary description of their concepts.

<u>elements:</u>	persons (h_i), positions (p_j), tasks (t_k^i).
<u>relations:</u>	R_0 , between persons (the M graph);

- R_1 , between positions (the P graph, or organization chart);
- R_2 , between tasks (the T graph, or work layout);
- R_3 , between persons and positions (the H-P graph, or personnel assignment);
- R_4 , between positions and tasks (the P-T graph, or task allocation);
- R_5 , between persons and tasks (the H-T graph, or induced personnel assignment).

roles:

- There are three kinds of role.
- (a) The informal role of a person, $R'(h_i)$;
 - (b) The formal role of a position, $R'(p_i)$;
 - (c) The actual, or operational, role of a position, which is the second as augmented by the first.²²

The general bent of their formulation which we noted above is rather well indicated by the content of the quotation. All graphs are unilinear except H, which represents the "informal social relationships" between persons, and it is not given extensive consideration. R_3 is a function which assigns each position to one person, but not conversely. However, R_4 allows overlap so that a task may be assigned to more than one position, and in consequence R_5 allows overlap of person-to-task assignments. R_3 is obviously not a satisfactory formulation of incumbency, but R_5 immediately recovers some of the necessary content.²³ Their formal

²²Ibid., p. 6.

²³ R_3 is not even a good statement of their own "intuitive" observations set out in the first article regarding positions. For an example of the strain to employ digraphs, see Ibid., p. 8, n. 4.

statement of the definitions of role shows the consequences of their assumptions.

D3. The informal role of a person, $R'(h_i)$, consists of all relationships entailing h_i , in the set R_0 .

D4. (a) The formal role of a position, $R'(p_j)$, consists of all relationships entailing p_j .

(b) The actual or operational role of a position is

$$AR'(p_i) = R'(p_j) \cup R'(h_i)$$

where person h_i is assigned to position p_j . In other words, the actual or operational role of a position is the augmentation of its formal role by the informal relationships of its office-holder.²⁴

This makes difficult reading, as there are several technical problems as well as substantive ones. The use of R' throughout is misleading since the relations referenced in D3 and D4(a) are different, being R_0 in the former and R_1 , R_3 , and R_4 , in the latter. The appearance of i as a subscript on the left side of D4(b) is either a misprint or wrong, in consequence of R_3 and their own verbal statement following the definition. With this correction, it will be seen that in addition to the uniqueness of every position we have a correspondingly unique actual role associated with that position, and only one. Further, while it might be possible to compare positions (in the P - T graph) for equivalence of task assignments, and thus arrive at some higher level of classification of them, there is no formal provision for comparison of different positions in terms of "informal

²⁴Ibid., p. 9.

social relationships" for equivalence of roles. And each of these features must be counted as a defect when compared to actual sociological usage.

There is one final point of comparison with other models (explicit or otherwise) with which we may close this chapter and provide a motivation for the next. It appears that the only other intentional use of any formal specification like the incumbency axiom is the one used by Oeser and Harary in the material we have just reviewed. This axiom seems to us to be an absolutely necessary conceptualization for representing the concepts of role systems in their fullest intention, and it leads to interesting and potentially rewarding problems which it is the business of the next chapter to explore.

CHAPTER V

MAPPING ACTORS INTO ROLE SYSTEMS

In the preceding chapter we noted that the incumbency axiom generates a relationship between the A matrix and the I and P matrices that we expressed as $(I,P) = f(A)$. That is, if it is assumed that a set of actors occupy a role system, then it must be possible to determine their incumbency and the positions, roles, etc., from the information contained in the A matrix. This is a consequence of the theoretical assumptions embodied in the incumbency axiom. As is the case with any formal postulates of a theory with empirical intentions, the incumbency axiom is subject to revision and rejection on evidence of inadequacy. The incumbency axiom, or some systemic equivalent, must serve as the first step in specifying the rules to coordinate the purely constructural concept of a role system to intended ranges of relevant phenomena. It is the purpose of this chapter to examine implications of these considerations.

We observed earlier that the displaying of a candidate mapping procedure in this chapter furnishes proof that the (multiple) function exists. This display

will not constitute proof of the uniqueness of the set of techniques. It is nevertheless possible that they are a unique solution. In fact, it is the author's belief that certain of the steps in the procedure to be outlined do exhaust the possibilities permitted by the axiom at those points, but they require a further assumption not stated and not implicit in the axiom.

Mapping Procedures

Assumption of Exhaustiveness

If we consider a single cell vector of A, which contains the information about the relevant relations between a pair of actors taken as focal and counter, then it seems natural to suppose that the second part of the incumbency axiom directs us to take this as a role sector to be mapped into P. However, the conditional form in which the proposition is given (which see) only permits this, it does not require it. "Permits" is perhaps an inadequate term here, since it does not forcefully exclude other alternatives. That is, what the second statement in the axiom is intended to mean is that any cell vector of A can be used only for a determinate assignment of the relations as a role sector, and of the actors each to positions in the I matrix. If the cell vector of A is not used for this purpose, then it has no other significance, i.e., it is ignored.

This possible ambiguity could have been circumvented in the statement of the axiom simply by changing the constituent propositions and their order in the conditional to read:

Assume A_h, A_i ;

If A_h is focally joined to A_i by a pattern of relations, then there is a P_j and a P_k such that A_h and A_i are incumbent in them respectively and the pattern constitutes

the role sector for P_j focal and P_k counter.

However, this amounts only to burying a necessary assumption of exhaustiveness in the axiom, and there appear to be good reasons against doing so. It is not altogether clear that the assumption is acceptable; the sociological literature is either silent or vague on the matter of when we are to take what as evidence of a role (sector).¹ It seems better to state exhaustiveness as a separate assumption which can be dealt with independently of incumbency. The issue is of considerable importance, and we return to it in the final chapter. Suffice it to say at present that we accept exhaustiveness essentially as an additional axiom in the development of our solution to the mapping problem. Thus, for any mapping of an A matrix, each cell vector of A must be mapped to P, and

¹The conceptual situation seems to be similar to the one summarized in Freud's famous comment on cigars.

the associated actors must be mapped to the associated positions in I.

The statement reveals another facet of the problem which has not been touched on before. In general there is always the possibility of more than one **P** and I into which A can be mapped, but only one of these mappings might have a significant interpretation. In the remarks below we consider the logical decisions which seem to be required in making a mapping.

Agreement

The cell vectors of A must always be taken as the elements of the domain (or "values of the variable") which the specific mapping functions carry into the object range, I and **P**. Incumbency requires that all relations of the set be considered, and a given pattern must be considered as a unit. The next question is how we shall decide when two actors are incumbents of the same position, and the natural answer is that they must have the same patterns of relations (the same "values" of the units) with incumbents of the other positions to which the position is joined. This does not mean that they must have exactly the same patterns of relations with all other members of the set of actors (and with each other), but only that each will have a pattern of relations with some incumbent of each other position to which their position is joined,

and that for each position taken separately the pattern of relations will be identical. This does not require that they be related to each other or even to related alters. On the other hand, it does not enjoin against any of these possibilities, and, in fact, we sometimes wish to use some of them in specifying a position in a particular role system. For example, for certain purposes we might wish to specify (among other things) that two assembly operatives occupy the same position only if they are supervised by the same foreman. But in such a case is not the statement of the incumbency axiom inadequate? The answer is yet and no. The statement is given in the most general form necessary, and it allows other restrictions as special cases. The burden of our discussion is to consider the logical alternatives which can be generated to specify the restrictions.

The proposal above is that we shall take cell vector identity, or as we shall say "agreement," as the basis for the first step in our mapping procedure. This requires that we develop a procedure for determining when or which cell vectors agree and how they (and their actors) will be aggregated for consideration as candidates to be mapped. We will do this in several parts.

Canonical Form

First, we define what might be called a "canonical" form for the A matrix (which applies to **P** as well). By

requiring patterns to agree, we mean that they must have the same order of 1's and 0's in cells taken in an arbitrary but fixed order, say from "front" to "back." If r is the number of relations in the set, then there are 2^r possible distinct patterns. This generates a geometric series of weights; so we may assign a unique integer from the series $0, 1, \dots, 2^r-1$, to each of the possible patterns by assigning integers $0, 1, \dots, r-1$, uniquely to the relations, raising 2 to the same power to be used as a weight for each relation, multiplying the contents of each cell by the weight of its relation, and summing over these products for each cell vector separately. These sums can be cast into a square matrix (which is not binary), and this matrix contains all of the information which was in the original three-way array. The two forms are completely equivalent under the weighting function, but each is an intuitively more convenient representation for certain further constructions. We call this new representation $G(A)$ (for "geometric weights over A ").

Maximality

We can locate agreeing cell vectors of A by finding cells of $G(A)$ which are equal. It seems natural to inquire which of an agreeing set of entries we shall associate together, and equally as natural to decide to take all of them, under whatever other restrictions we

might want to impose to specify "special cases." That is, it is proposed that we adopt a criterion of maximality. In order to define this it is convenient to employ the concept of partitioning of a matrix. (This is part of the practical motivation for defining $G(A)$.) Strictly speaking, the concept of partitioning in matrix theory involves the separating out of sub-sets of rows and of columns, but not necessarily the same ones; the rectangular arrays of cells of these sub-sets then being taken as the elements of a reduced matrix. However, we only require the idea of separating out sub-sets of rows and of columns, i.e., we wish to construct sub-matrices from $G(A)$. We need to think of constructing a separate sub-matrix for each of the 2^r possible entries in $G(A)$. We construct each under the restriction that each row and each column must contain at least one entry of the value for which the sub-matrix is being constructed. This satisfies the requirement that each focal actor must have the pattern of relations which defines the role sector with some counter, and vice versa. Our maximality criterion directs that all such rows (and columns) of $G(A)$ will be included in the sub-matrix. We should note particularly that each sub-matrix must be thought of as extracted separately (or at a separate round) from $G(A)$, since any given row or column may appear in a great many of them. Now since the defining characteristic of such

a sub-matrix is that some entries (properly distributed) are the defining value, we may conveniently think of another operation on it such that we replace all entries of that value by "1" and all other entries by "0." Let us denote this result as G_p (where p ranges from 0 to $2^r - 1$ corresponding to the defining value from $G(A)$). Our following development requires the idea that we retain $G(A)$ subscripts in G_p .

Logical Restrictions

With the idea of the rectangular binary matrix G_p in hand, we are now in a position to define the restrictions which can produce "special cases" as further substructions on G_p ; i.e., since G_p is binary, the only restrictions on it must deal with further partitioning to produce sub-matrices of G_p by selection of appropriate rows and columns. Let us denote any such sub-matrix of G_p by G_p^* , and note that, again, there may be many, and they may overlap, just as G_p 's may overlap "in" $G(A)$.

Symmetry. We first consider the import of three logical properties of relations, viz., reflexivity, symmetry, and transitivity. We are interested in them not for their reference to relations as such but for what they suggest regarding selections of actors as incumbents of focal and counter positions. Transitivity

is mentioned here only for completeness since logical usage associates it with the others, but in view of our limitation to consideration of pairs in the formal development, it does not appear to have any immediate application to the question. (It is a question to which we shall return in the final chapter.) Consequently, we assume that it is safe to treat all G_p as though the set of relations represented is essentially non-transitive (i.e., indifferently transitive or intransitive). Similar remarks apply to reflexivity, though for different reasons. It is not difficult to construct examples of actor-reflexive relationships between positions, e.g., a university departmental head who writes a letter to himself as chairman of a guidance committee, but such cases do not seem to be of such substantive importance as to warrant special consideration. We shall assume that non-reflexivity is the general case, which is interpreted operationally to mean that the content of a cell of G_p which has identical row and column subscripts is unimportant.

It appears that symmetry is a most decisive factor in determining desirable restrictions. Non-symmetry is already a characteristic of G_p as defined above, i.e., rows and columns (actors as focal and counter) with same subscripts may appear or not. However, it is easy to cite important examples of symmetric

and asymmetric relationships, e.g., friendship as a reciprocal set of expectations (and as operationalized in the definition of a clique by Festinger, and Luce and Perry mentioned in the previous chapter), or well-formed authority as a superior-subordinate relationship. Symmetry would require that any rows or columns of G_p which do not have matching subscripts in the other set must be dropped in forming a G_p^* , then empty vectors are dropped, and the procedure iterated. Asymmetry would require that in any case of matching the row or column must be dropped, but of itself does not indicate which.

It might seem that maximality could provide a solution, say that the product of the number of rows and the number of columns in G_p^* be as large as possible. This is the solution used in a technique called Multiple Agreement Analysis, and it has some application at a point to be noted below, but it has some defects for present purposes. First, it is clear that it does not provide a unique solution, as in the case when there is only one offending subscript and an equal number of rows and columns. Second, even if the product solution were unique for a particular G_p , it might be at the expense of the total number of 1's in G_p^* . (This objection disappears under another type of restriction to be noted below.) Another approach to the solution

also developed in Multiple Agreement Analysis, which does generate unique solutions is to apply maximality to rows or columns, i.e., to require either that only columns or only rows be dropped.

Thus, up to this point, non-symmetry leaves $Gp^* = Gp$, symmetry provides a unique solution, and asymmetry yields several competitors, two of which are unique. There might seem to be no intrinsic reason why any of these restrictions must provide unique solutions, i.e., neither our model nor the additional criterion of maximality assumed in this chapter require that a given cell vector of A be mapped to one and only one cell vector of P . However, our criterion of exhaustiveness might be interpreted as intending to require this so as to satisfy in a simple way the assertion $(I,P) = f(A)$, that "f" is constituted of well-defined functions. In this case one would prefer the maximization of rows or columns as equally acceptable alternatives. However, it should be made clear that a relaxation of these requirements would still yield only a finite number of solutions, and each of these could be treated as a separate function by denoting different P 's, barely saving the form. It would be equally admissible to choose product maximization as a first choice which, if it failed to yield uniqueness, could be replaced by row or column maximization, and this would yield a unique final result. In general, it seems

best to interpret the "f" in the expression as denoting a complex function and to admit all of the alternatives as possible "parameters" for the asymmetric case, on the grounds that the non-unique solutions do yield ones which are finite and determinate. That is, each G_p^* is equally definite, since though an A_{ij} may thus map to several P_{ij} , it will definitely go to each of them. (Indefiniteness is the reason for the general case of $A \neq g(I,P)$, as shown by our proof of the existence of A in the preceding chapter.)

Quantification. We now turn to a second general set of restrictions on G_p suggested by the logical quantification of propositions. The statement of the incumbency axiom says that an actor must have a role sector with some alter of a matching position. A restriction on this which can be represented as a sub-set of G_p is the requirement that an actor have a role sector with all incumbents of the reciprocal position. Of course, the restriction automatically guarantees that it applies equally to incumbents of both positions, so that it yields a rectangular matrix which we could describe as "off-diagonally dense," i.e., all cells other than those with identical subscripts have 1's as entries, remembering that subscripts come from $G(A)$.

In this case of "each to all" the problem of multiple solutions applies to symmetric, non-symmetric,

and asymmetric choices, though in different ways. The application to the symmetric case is easily seen from the definition of a sociometric clique by Festinger (and others), i.e., such cliques may overlap. However, there is not any problem of maximization since the Gp^* must always be square. Any non-symmetric Gp^* may in general be taken as made up of a symmetric portion and a remainder which is asymmetric. The maximization solution for such a non-symmetric case has the same alternatives as for asymmetry outlined above -- which also apply to asymmetry here -- though they may yield different results, dependent on the actual contents of matrices. While we have taken some effort to point out the maximization problems, we will not need to consider them extensively in the sequel since we assume that the existence of determinate solutions satisfies our needs for $(I,P) = f(A)$. Moreover, they appear to be of limited substantive relevance except as noted below.

Connectedness. The combinations of "some" and "all" and the various modes of symmetry yield a general six-fold classification of functions which can be applied to Gp to generate candidates for incumbencies. There is yet another logical property of Gp^* which may be used to create a further dichotomy of the "some" group. Any Gp^* derived under "all" and symmetry must be totally connected, each actor reciprocally to every other. Consider an

instance in which a G_p yields two G_p^* under these restrictions, such that they are disjoint and exhaustive of G_p . The same G_p under symmetry and "some" would yield a single G_p^* ($=G_p$), which contained both of the dense sub-matrices, but there would be no connection between them. This could be determined by a partitioning of G_p to reveal the null matrices which represent the disconnected character.² The concept of connectedness can also be extended to the asymmetric and non-symmetric cases, and it is implicit in the "all" restriction, so that it does not provide a way of further dividing those cases.³ The nine-fold classification which results has certain interesting substantive applications, but we delay consideration of these until completion of our mapping procedure and its ramifications.

Iteration on G_p and $G(A)$. To complete the criteria for application of this set of "parameters" for extraction of G_p^* , we need to provide that the selected

²A connected symmetric matrix has the interesting property that it must always contain at least one "cycle" as defined in graph theory. While we have not remarked the fact above, perhaps the reader has noted that G_p and G_p^* are always digraphic. For the definition of a cycle, see Harary, et al., op. cit., Chapter 2, and on connectedness, see Chapter 3.

³Connectedness goes beyond our restriction to pairs, but it can be determined in a digraph by the existence of "semi-paths" between pairs of actors. Ibid., p. 31.

modes are applied iteratively to G_p , with multiple mappings of equivalently maximal G_p^* as necessary, and that after each "level" of maximality has been exhausted all entries used for that level are "swept out," i.e., the l 's are set to 0, until G_p is null. This is required to satisfy our assumption of exhaustiveness. This procedure must be repeated for all G_p extracted from $G(A)$, with parameter values as appropriate for the given G_p . It should be noted that while in general there are 2^r possible G_p , there may not be more than $m(m-1)$ for a given A and set of relations, where m is the number of actors. Still, the problem of conceiving how this mass of information is to be used to construct I and P suggests the utility of some additional bookkeeping devices.

Bookkeeping Matrices

For this purpose, we introduce three new rectangular matrices which are not essential to the solution but are convenient as devices for isolating steps in the procedure. The first stores focal positions from G_p^* , the second stores counter positions, and the third stores role sectors, all by columns, with rows for actors in the first two and relations in the third. We shall denote them as F , C , and R , respectively. All of them have as many columns as there are G_p^* from all G_p . F and C have m rows, and R has r rows, and we employ binary entries in the now familiar way. (We could as well use three vectors with geometric weights

of the appropriate sort, but the binary representation seems more natural at this point, and is essential for F and C in connection with "nesting" below.)

We now add an additional row to each of the F and C matrices in which to record position numbers to be used in constructing I and P. There are two apparent candidates for the criterion required to generate these position numbers, and we shall designate them as "equivalence" and "nesting." We shall develop the procedure with respect to the first (which is more general), and then examine the consequences of the second. The purpose of the criterion is to determine which columns of F and C are to be assigned the same position number. In our discussion, we use subscripts to denote the columns of F, C, and R.

Equivalence

This criterion requires that if two or more columns of F and C are to be assigned the same position number, then they must display identical patterns, i.e., the actors incumbent must be exactly the same. We begin by choosing any arbitrary column, say F_1 , entering a 1 as its position number, scanning all remaining columns of F and C for equivalence with the starter, and entering 1's in those which are equivalent. Next, we choose a column not yet numbered, enter a 2, and scan unnumbered columns entering 2's in equivalent ones. The procedure

is repeated until all columns of F and C have been numbered, and the final number used is the number of positions in I and P . I may be generated during the numbering procedure by recording the appropriate patterns. P is generated by examining the columns of F , C , and R , in the sets in which they were given by Gp^* , using the new position numbers of F and C as row and column subscripts and the column of R as the cell vector of P .

We now examine the consequences of the use of this criterion to combine the provisional positions extracted by Gp^* . There are two salient points, both concerned with the parameter values used to define Gp^* . Since, in general, we do not assume that a unique set of parameters has been used for all Gp^* (from all Gp) it is possible for a given position to have role sectors with separate other positions which have been defined by disparate parameters. However, we do assume that a given type of role sector (a particular Gp) has been defined by only one unique set of values of quantification, symmetry, and connectivity.

Further, the question arises whether the new numbering of positions may not require combining role sectors from R in P , and the answer is that this can occur only if errors are made in forming the Gp . Consider the results of two Gp^* , as F_i , F_j , C_i , C_j , and R_i , R_j . If either $F_i \neq F_j$ or $C_i \neq C_j$, then separate

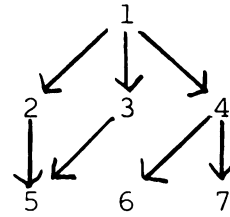
role sectors in P result. If $F_i = F_j$, and $C_i = C_j$, then if $R_i = R_j$, the G_p^* is redundant; or if $R_i \neq R_j$, then a type of element of $G(A)$ has been used in two different G_p , contrary to assumption. (This is not true for "complex" structures to be defined below, but as noted there it causes no problem.)

In the general case, the criterion of equivalence is the only rule for equating columns of F and C which insures satisfaction of the second part of the incumbency axiom. If any other rule less than complete identity were employed, an actor might be assigned as incumbent to a position in virtue of relations to an incumbent of a reciprocal position, but thus be attributed relations to still a third position in virtue of another incumbent of his own position. There is a special case of some interest because of its substantive applications, and it requires that P be what we shall call "locally digraphic" (i.e., defined by a single G_p), and "articulated through maximal positions." A much simplified example will assist in explaining these points.

Problems of Equivalence. Consider the following matrix of seven actors and one relation (which we might take to be something in the nature of authority or influence) as an A . (A digraphic representation is given as well for clarity.) Let us assume symmetry and "all"

A

	1	2	3	4	5	6	7
1		1	1	1			
2					1		
3					1		
4						1	1
5							
6							
7							



as the criteria for Gp^* . Gp consists of the first four rows and the last six columns of A (which is already conveniently binary). Gp^* yields three position pairs which are enumerated in F and C (it is not necessary to represent R for a digraph, since it is always a vector of 1's). Examination of F and C under the criterion of equivalence reveals six distinct positions.

F			C			P						I					
a	b	c	a	b	c	1	2	3	4	5	6	1	2	3	4	5	6
1	1					1			1			1	1				
2		1		1		2				1		2		1		1	
3		1		1		3					1	3		1		1	
4			1	1		4						4			1	1	
5					1	5						5					1
6						6						6					1
7												7					1
	1	2	3	4	5	6											

However, a glance at the digraph of A suggests that this does not reflect facts of considerable substantive interest. There are two "condensations"⁴ of the graph which it would be desirable to be able to extract in a mapping of A (into different P 's). The first would take actors 2 and 3 as incumbents of a position, and also 6 and 7 as mates, leaving 1, 4, and 5, as occupants of

⁴Ibid., p. 57f.

separate positions. The second would associate 2, 3, and 4, in one position, and 5, 6, and 7, in another, with 1 in a singular position. Examination of the columns of F and C suggests ways in which this could be accomplished. A comparison of the second and third columns of F and the first column of C shows that the logical sum of the first two yields the third. There are two alternatives for using this information, and we examine both.

First, the columns of F might be used to determine that the first column of C ought to be broken up, i.e., a further partitioning on the first Gp*. This could be done by adding a fourth Gp*, with 1 as focal and 4 as counter, and deleting 4 from the counter of the first Gp*. This would yield five distinct columns in F and C, and the resultant P and I as shown which is the first "condensation." At this point it might be

F					C					P					I				
a	b	c	d		a	b	c	d		1	2	3	4	5	1	2	3	4	5
1	1		1							1		1	1		1	1			
2		1			1					2			1		2		1		
3		1			1					3				1	3		1		
4			1					1		4					4			1	
5						1				5					5				1
6							1								6				1
7							1								7				1
1 2 3 1					2 4 5 3														

observed that entries of P show further association between the now positions 2 and 3 in their relation to 1. Perhaps, this could be utilized by taking P "as an A" (it should be recalled that A is of the same form as

P, and is in fact a special case of P in which each actor occupies one and only one position) and repeating the procedure. However, this would not combine positions 2 and 3 (actors 2,3 = position 2, and 4 = 3) since each appears separately with still another position. This reveals both a limitation and the character of this technique of further partitioning of Gp^* which we might call "de-nesting" by comparison with the technique to be outlined below. If the elements of A (or P if it is already a result of a mapping) which are associated in the F and C in a group of columns (which form a logical sum represented by one or more columns) appear among these columns as singular incumbents, then repetition will reproduce the same A. In a sense, it thus guarantees the lowest possible level of definition, i.e., the one closest to the original A and consistent with the several criteria used.

It will be seen above that we have implicitly used equivalence after the partitioning of Gp^* as the definition of position combination, and this suggests correctly that partitioning may be used on sets of columns of F and C matrices resulting from heterogeneous Gp^* . The partitioning would have the advantage of yielding P's which are as connected as possible since disconnectedness is always the result of non-equivalent positions and de-nesting guarantees a maximization of

equivalence. However, the restriction to very concrete representation may outweigh this advantage.

The iteration of the procedure is an important additional technique, but one which does not apply in general when equivalence is required, i.e., when more than a single Gp is involved and the criteria for a given Gp* are held constant. However, if the criteria are changed at each round from the most stringent ("all" and symmetry or asymmetry) to less stringent ones ("some," disconnected, and nonsymmetry), then iteration is significant under equivalence and is analogous to a shift in the "level" of definition from concrete groups to societal or cultural (or better, "system component") models. In the present example, such changes would yield a single position pair, with 1, 2, 3, and 4 as focal and 2, 3, 4, 5, 6, and 7 as counter, which represents the only general type of role in the system.

Nesting

The second alternative for using the information on the relation between the second and third columns of F and the first column of C in our example is to take the logical sum as evidence that only a single position is involved and thus assign the same number to all and the sum as a column of I. It is this procedure that we designate as "nesting" and it is strictly limited to local digraphs which are articulated to a larger multigraph

through positions which are maximal in being logical sums. In the example, this criterion would first yield four positions composed of 1; 2, 3, 4; 5; and 6, 7. An

F			C			P				I			
a	b	c	a	b	c	1	2	3	4	1	2	3	4
1	1					1		1		1	1		
2		1	1			2			1 1	2		1	
3		1	1			3				3		1	
4			1			4				4		1	
5				1						5			1
6					1					6			1
7					1					7			1
1 2 2			2 3 4										

iteration would then yield three positions by combining 5 and 6,7. It should be noticed that here the iteration

F		C		P			I		
a	b	a	b	1	2	3	1	2	3
1	1			1		1	1	1	
2		1		2		1	2		1
3			1	3			3		1
4			1				4		1
1 2		2 3					5		1
							6		1
							7		1

is absolutely essential, since the first result yields a position composed of 2,3,4, which is joined to separate positions of 5, and 6,7, and 4 is not connected to the first of these nor are 2,3, to the second, and this violates the "some" requirement of the second part of incumbency. The iteration combines 5,6,7, and joins it to 2,3,4, which satisfies the axiom.

The restrictions stated above are necessary to avoid violating the second part of the axiom. The reasoning is as follows: any appearance of a strict

sub-set of the logical sum will be assigned the same position number, and if this sub-set in F and C was generated by some Gp other than the one being used to permit the nesting, then the sum will be articulated to a second position such that some of the sum's incumbents do not have the second Gp with any incumbents of the second position.

Nesting is essentially a technique that takes into account the simplest order existent in a set of positions, and this entails effectively operating on more than a pair of positions, even though they are taken a pair at a time. That is, mapping still proceeds by pairs, but the effect of iteration is to link pairs. It is of interest because of the substantive significance of ordering in producing ranks. In connection with this, there are some restrictions on the effectiveness of nesting which are best stated in terms of digraphs. Any set of positions which are to be nested must be mapped from a set of actors for whom the associated digraph of Gp is no more than "strictly unilateral."⁵ This means that for any pair of actors there may be only one "path" between them: e.g., consider A_i and A_j , then $A_i(G_p)A_j$ or conversely (or neither) but not both; further, for any A_k , $A_i(G_p)A_k$ and $A_k(G_p)A_j$, or conversely (or neither),

⁵Ibid., p. 69.

but not both; and so on by extension. In digraphs, this means that there are no "cycles" (i.e., a closed path) on G_p in the set of actors. If cycles exist, nesting will produce ambiguous results.

There is an interesting substantive aside which is related to the restrictions on nesting. It is well known that the line and staff structure exhibited in an organization chart does not represent the "real" structure of a bureaucracy, because office holders form other relationships with fellow members of the organization than those intended by the chart. This results in erstwhile incumbents of the same position entering in disparate relations with others, which violates the articulation restriction of nesting. The chart becomes not merely incomplete but wrong. Of course, even though position mates were as much alike as tweedle-dum and tweedle-dee in their extra-formal relationships, these latter might still take precedence to the point that the chart while limitedly correct was trivial. Which underlines the point that our considerations herein are not thought to guarantee substantive significance.⁶

Types of Structures

Up to this point, we have taken as assumed that the procedures described are conceptually applicable to

⁶This example should not be taken to suggest that ordering is only relevant to formal structure.

the mapping of a set of actors into a role system on the basis of a set of relations considered as a whole. We now need to consider whether this assumption is required in all cases, and whether it adequately represents the operations we wish to conceive. As often, the answer is both yes and no. On the technical side, it appears best to assume that every effort should be made to retain the procedures, as described on the assumption that an entire set of relations are exhaustively used in a mapping, as the basis of any elaboration, since any relaxation would only ramify an already complex conceptual structure. However, on the substantive side it may seem that the procedures described are inadequate, and this revolves around our definition of a position under the incumbency assumption.

When incumbency is assumed, our definition of a position is that it is specified by its pattern of connections with other positions through the set of relations and that it is occupied by a set of actors. The mapping procedures outlined operate to insure that this set is unique; though it may overlap with other sets, it is not identical with any set involved in the specification of any other position. Further, for any given position, there may not be more than one role sector with any other position. And here an apparent difficulty arises. While most theoretical conceptions

do not require (and some do not permit) more than one sector between positions, they do not prohibit (and some require) that a unique set of actors may occupy more than one position in different contexts and thus jointly hold more than one role sector with another unique set of actors in a larger sense. In any concrete empirical situation, more than one context may be (and usually is) relevant to the definition of the situation.

How may we save the form which we have developed and still incorporate these considerations? There appear to be two alternatives. The first is to affirm that the form applies only to a definitive singular context and that there always may be multiple P's in any empirical situation, and this seems to be the preference of some theorists.⁷ The other alternative is to develop a way of extending our conception of the formal structure of a situation, using the ideas of mapping as a basis, and we will explore this briefly.

The question centers on what we conceive as the character of the relations in the set used to specify A and P. We began with the idea that they are contentless, or more properly that they represent the logical form of each of several kinds of contents which are frequently

⁷ E.g., W. Goodenough, op. cit.

used in substantive analyses of the role concepts, and this is still to be maintained. The problem we want to consider is how these contents of whatever kinds may be analytically related to each other. Let us use the following example.

Suppose that two actors as incumbents of related positions are related by both authority and friendship, as is not infrequently the case. As labels, authority and friendship are distinct, but we might wish to decompose them into some analytic constituents, and assume further that one of the constituents of each of these is communication. (We are not concerned here with how any of these implied measurements are carried out in practice, only with the analytic consequence of the overlap.) If we wished to map a P for authority, we would naturally take those relations conceived as constituent of it, and similarly for friendship. We could easily conceive of creating a revised and extended P (and I) which combined both structures in one representation, but in order to do this we would have to consider what to do with the analytic constituent communication. If it appeared only once in A and P (as we have up to now assumed), then the authority-friendship structure would be intermeshed through combined role sectors.

On the other hand, we easily think of a modification of our earlier design, such that the authority and

friendship structures are represented each by unique sets of role sectors and the two structures are linked through positions. We do this by supposing that we allow the set of relations (r in number) to be divided into sub-sets (by definition, sub-sets of a set may overlap), and that each of these be used to specify an A . Then each A may be used to form $G(A)$, and all G_p and G_p^* are extracted and mapped into F , C , and R , and the procedure iterated for all such A 's generated from the parent set of relations. In order to do this some modification of R (and P) is required in order to distinguish the results of distinct A 's. This is conveniently accomplished by supposing that we use geometric weights for this purpose, and that each successive A requires a single row in R and a single table in P . (In "complex" structures, role sectors may be combined in mapping from R to P , and this avoids redundancies. We noted above that in simple structures role sectors of R are never combined in mapping to P .) However, this is only a convenience, since the weights are completely equivalent to a binary representation, and in such a representation each overlapping relation would appear more than once, in R and P , and in an A expanded in the same way.

As a matter of nomenclature, we may designate our original development of the mapping procedure as the model of the simple structure of a role system and the extension suggested here as the model of the complex structure. Just

as there are many possible simple structures under various combinations of parameters for various Gp^* , there are also many possible complex structures. Since each of the sub-sets of the r relations might be used in specifying any of the constituent tables (using the weighting notation) of P (and A), there are $2^r - 1$ possible tables (excluding the case in which all relations are irrelevant) which might or might not appear in P . Each of these in itself generates a complete simple structure (including $G(A)$, all possible Gp , selected Gp^* , etc.). Excluding these, there are $2^{2^r - 1} - (2^r - 1)$ possible complex P 's, each of whose constituent tables may be defined in all of the ways of any simple structure (!) (Since each relation may or may not be used to specify the sub-set for the simple structures, and if used it may or may not appear in a role sector for a particular structure, there are $3^r - 1$ ways -- excluding the case when all are unused -- of specifying role sectors for simple structures.) Of course, these potentially rather large numbers represent an enumeration of logical possibilities but certainly not all of them could be given substantively significant interpretations in any empirical situation. What they do serve to illustrate is the (literally) exponentially increasing ramifications of even such a simple "set" definition of an extended structural concept which seems more nearly adequate to substantive requirements than our already rather complicated specification of the mapping of a "simple" structure.

Some Substantive Coordinations

We remarked above that the nine-fold classification of the main values of parameters for extracting Gp* may be given certain general interpretations in terms of their use in specifying types of role sectors. It should be emphasized that they apply only to role sectors and not to total systems, but in fact some systems are constructed mainly through the use of one set of values. The following table reproduces the nine-fold classification with numbered cells for ease of reference.

	"some"		"all"
	<u>disconnected</u>	<u>connected</u>	
symmetric	1	2	3
non-symmetric	4	5	6
asymmetric	7	8	9

We noted before that cell 4 represents the most general definition of a role sector under the incumbency axiom (with these parameters) and that all others are special cases of it. As such, it cannot have any specific application and is of residual interest here. The remaining cells in the row, 5 and 6, are of interest but are better examined with and after others.

The difference between the first and last rows of the table is essentially comparable to that between peer (or coordinate) relationships and non-peer (or ordinal) relationships. There can be little question that this

reflects a deep chord in sociological concern; the distinction between associative and differentiating connections between social elements is repeated again and again in human life (and in human sociology), and one or the other or both types of relationships seem to be fundamental building blocks of the structure of any social group. As such, it appears that a "parameter" of this sort is absolutely required in any conception of how such structures are built up.⁸ But it is not particularly surprising that there should be congruence between ways of conceiving relations abstractly and the ways in which we construct relationships in role systems.

The implementation of symmetry in empirical groups and in sociological analysis reveals an interesting facet of the other main dimension of the table. We have noted at several places above that cell 3 represents the definition of "clique" offered by Festinger (and others). It may seem curious that this definition has been used (to the best of the author's knowledge) mainly for analytic

⁸The parenthetical reference to the ordinality of the last row is only residually correct in view of our general limitation to consideration of pairs (an order can always be imposed on any pair), but any, usually hierarchical, order is fundamentally distinguished by asymmetry. And it is in this connection that we found it necessary to introduce the idea of "nesting" in order to represent a fundamental way in which combinations are formed in a positional structure.

purposes, and that most empirical sociography has used definitions that would fall either in cell 2 or in cell 5, the former for "cliques," and the latter for "crowds." In existent social groups it has proved difficult to find the consensus required by the "all" restriction. However, this is also a function of the kinds of problems, i.e., the kinds of relationships, which have been subjected to sociometric analysis. Most usually these are what is broadly designated "informal" (and usually associative, especially in the original formulation), and it turns out to be asking a bit much of a collection of persons, however solidary or well-informed, to agree upon and/or reciprocate all expressed connections. However, cell 3 actually is used and with great frequency when the context is "formal" or effectively so. Collegial relationships of all sorts fall here, but these are hardly the kinds of phenomena that one would think of subjecting to a sociometric analysis; the answers are already known and quite efficiently represented. The point is that the elementary architectural decisions are not of a completely different order, but that it requires broad social givens to establish the completeness of consensus demanded by such a definition. Even here the reference of such givens is to existent structures, since the higher "level" specifications slide over to lower numbered cells. However, the right-most column is used and with important effects (except as noted below regarding cell 6).

As noted above, the last row of the table is mainly associated with ordinally differentiated structures, though not always so, e.g., in some definitions of communication (when feedback is neglected). Authority (and perhaps influence) as usually pictured fall in cell 9, which is the basic component for any "line" structure of a formal organization. It is also customarily associated with notions of status⁹ or prestige, though cell 8 would appear to be closer to the concept of a pair of strata in a system, since it does not require that every member be immediately comparable to every other, only that the comparison can be made at some determinate remove. Cell 7 serves (as does cell 1 in the first row) only to indicate the existence of a mode of relationship in an aggregate.

In all instances, the most general application of any type of relationship in an existent system tends to lead to cell 5, since all ways of formulating connections between locations in groups lead to both symmetries and asymmetries through multi-relational incumbencies. But the importance of the requirements on relational reciprocity and quantification is rather well exhibited by the difficulty of discovering position pairs which may

⁹Cf. F. Harary, "Status and Contrastatus," Sociometry, 22 (1959), pp. 22-43, in which a canonical ordinal measure is developed from this assumption.

be cast into cell 6. It is possible that some examples may be drawn from deference behavior in kinship systems¹⁰ (which are notable instances of formal circumstances in which both symmetry and asymmetry are combined and required -- as in husband, wife, spouse -- though usually with contextual clarity). However, it is of interest that such deference patterns (in which some of a set defer to all of another, and others in the first set defer reciprocally with some of the second set) are defined "on top" or in terms of the kin positions as such, and not conversely.

The purpose of all of these remarks is to suggest that the parameters of our agreement substruction have fundamental substantive import, but not to suggest that they are the only possible or significant ones. Others might be outlined, but they would not confront a more basic question which may be raised concerning the concept of mapping as applied to analyses of role systems. Throughout the present chapter we have spoken "as if" the mapping procedure for assigning actors as incumbents in role systems were a conceptual and empirical reality in the measurements required by sociological analysis. The skeptical (or better, realistic) reader may well inquire whether the "as if" is, or could be, a reality. The formal model which was exposed in the preceding chapter

¹⁰ See W. Goodenough, op. cit.

may be all well enough, but does it logically or materially entail the analysis outlined here? Once more, our answer is equivocal.

At the outset of the chapter, it was remarked that the details of the mapping described may not constitute a unique solution, though some of them seem to be required. (Particularly, Gp as the fundamental starting point for agreement substruction, and equivalence as the general definition of congruent incumbency, since both are circumscribed directly by the second part of the incumbency axiom.) On the other hand, it is reasserted that the model, with the additional assumption of exhaustiveness, requires that some solutions exist if actors are to be said to be in role systems. And the assumption of exhaustiveness may be replaced with any other determinate directions for deciding when a cell vector of an A is significant, and the same consequence follows.

So much for the logical necessity, but what of its real application? And it is at this point that the "as if" enters. As a means of discourse, it is naturally preferable to be able to speak as though our statements designate empirical truths, but it does raise a question of their validity. It is rather obvious that the full array of armaments described herein have never been applied, even separately, to the problems of summary and descriptive measurement of an existent social system,

though certain rather limited cases are in evidence, mainly in sociography as a degenerate case dealing with a single relation. Nor is the author led to believe that the battery of techniques should or could in general be applied to any good purpose in specific empirical analyses at the present time. But then what material merit can such an analysis as the one carried through in this chapter have for sociological thought?

Our answer to this question has been given before, and we need to repeat it here for emphasis. The development of the details of the procedure for mapping of actors into role systems is not intended primarily as a contribution to the methods of measurement in structural analysis, but as a device to aid in clarifying our concepts of a role system. And it is asserted that it does serve this with merit.

First, the very elaborateness in terms of expectable detail in any empirical application which has issued from a few essentially very simple operational specifications serves to indicate why such a mapping could hardly be expected to be carried out, except in technically degenerate cases as noted above. The richness of the results would make them most difficult to use. However, we may observe that this embarrassment of riches is in useless currency just because it is not backed by substantive coinage. The manifold possible outcomes are all

about equally plausible in the absence of specifications which tell us which ones are of interest.¹¹ This circumstance underlines an observation that has been noted repeatedly elsewhere, and which amounts to questioning whether "structural role theory" ought to be called a theory or even a theory in an empirically corrigible and substantively extensive sense.

Second, we have, by our development of the construction, been led quite naturally to notice similarities between the operational categories devised and extant techniques of structural description, most notably in sociography. We have intended these operations as the working out of the logical form of role systems, but must this be taken as evidence that all sociography is properly seen as the analysis of positions and roles? Surely an individual's choice of luncheon partners is a bit more ephemeral than the kinds of social order that the role concepts intend to compass. The logical form is not decisive on such questions but it leads to their consideration and to some suggestions for answers.

¹¹There is a similarity to the rather common circumstance in 'survey' research in which the investigator (who perhaps began with rather immodest aspirations) confronted with the wealth of alternative possibilities in 'multivariate' designs falls back on the use of bivariate contingency tables, for lack of a theoretical basis for making the necessary specifications.

Other suggestions as to the conceptual consequences of our mapping analysis may occur, but all of these go somewhat beyond the originally rather technical intent of the present chapter. It is the purpose of the final section of this work to attempt an appraisal of some of the questions raised by the formalization of the role concepts which has been developed.

CHAPTER VI

ROLE CONCEPTS REVISITED

We arrive now at our final stock-taking and appraisal of the role concepts and the analysis carried out in the preceding chapters. There are several purposes and parts to this discussion. First, we will summarize the logical model of a role system which has been stated and attempt to specify some interpretations of this development for other aspects of the conceptual system which are not directly involved in the statement of the logical model. Some of these points have been outlined to a greater or lesser extent in previous commentary, but we must try to pull them together here to get an overview of the consequences of our explication. Briefly, the object is to find out what the model says about the role concepts. A second objective is to evaluate the explication qua explication along the guidelines set out in the first chapter. This, also, will be provisional for reasons we will note at that point. Finally, we will try to enumerate additional formal problems of the conceptualization of role which have not been incorporated in our model. The reader may agree with the

present writer that the model at least gives illumination to some of these topics. Our purpose in the final section is to indicate what the model does not say about the role concepts, and thus to suggest continuing directions for further analysis.

Summary of the Model and Some Interpretations

It is possible to think of the model which we have articulated as having two related representational objectives, corresponding to the two axioms stated in the third chapter above. The first axiom (of relation), together with the primitive and defined terms, sets up a representation of an "abstract" role system in terms of positions, roles, etc. While position and relation were taken as primitive terms for convenience in coordinating the system to its model in the theory of "nets," we noted that this was not formally necessary. Both terms were given contextual specification through the syntactic rules of the axiom itself. A relation involves two positions, and a position is known by its relations to other positions. A directed set of relations between two positions is called a role-sector, and a role is the set of role-sectors for a position.¹ Thus, both positions and roles may be thought of as specified

¹For brevity, we omit focal and counter qualifiers here.

by role sectors. Representation of the system is given by the three-dimensional array which we call the position (P) matrix.

The second axiom (of incumbency), together with the additional primitive term "actor," extends the apparatus to represent a "concrete" role system, one which is assumed to have occupants of its positions. Again, the new primitive term was specified contextually by the additional rules of syntax -- the axiom states the conditions an actor must satisfy to be an incumbent. Further, the stipulations of the axiom are such that the role system is derivable from relations among actors. The resulting representation is made in two new matrices, one like the P-matrix with actors rather than positions, which we call the actor (A) matrix, the other a two-dimensional array of actors in positions, which we call the incumbency (I) matrix. While the A- and I-matrices result from the incumbency axiom, the I-matrix stands on the other side of the relationship between A and P. The axiom entails two associated mappings, of actors to I and of cell vectors to P, coordinated by positions in each. We denote these mappings by the expression $(I,P) = f(A)$. Finally, in our development of a set of mapping procedures, we accepted the additional assumption of exhaustiveness -- each cell vector of A must be used in the mapping.

We will not try here to examine all of the conceptual problems which might be exposed or illuminated by our considerations of the logical model.

The writer's experience in attempting to formulate the problem and solution has led to a host of speculative asides of considerable interest. However, some of these resulted more from the cognitive exercise than from the formal considerations, and others remain as yet speculations; both classes are largely deleted from our comments here. Naturally, we attempt to summarize what seem to us the most important implications.

Some of the questions we wish to consider here appear to be so highly interrelated that it is quite difficult to separate them, if they are, indeed, separate questions. At various points in our analysis we have remarked on such problems as 1) whether it is useful to conceive of social groups without positions and roles, 2) the use of manifestly given names or labels to identify positions and roles in a system, 3) the possibility of latent positions and roles, 4) the importance of specifying role definers and the question of consensus of expectations which are central to the concepts of Gross and his associates, 5) whether any and all relationships between actors must be taken to be components of role sectors, and so on. Some of our earlier remarks

on these questions easily could be seen as talking on both sides of an issue, even though at different times.²

One line of exploration of the questions enumerated, and a proper reply to them, can be pursued by a "shift in the angle of vision" on the role definers problem stated by Gross, et al. Nadel has observed ". . . that the role concept is not an invention of anthropologists or sociologists but is employed by the very people they study."³ He goes on to remark that these scientists, as well as recognizing the public use of the concept, have made of it ". . . a special analytic tool." With most other observers, he appears to believe that this dual usage is a resource of great strength. But we must also recognize that it is a source of considerable peril.

When Gross and his associates speak of the problem of role definers, they refer to the use of concept by the "people they study." The authors have, of course, set down their own definition of the concept, and we have already asserted that our model adequately

²For example, we asserted that it seems unusual to conceive of role-less groups and that the idea of a latent structure of positions and roles is natural extension of sociological usage which is approached though not directly employed in studies of informal organizations. On the other hand, we have expressed doubt that sociographic relationships (or more specifically, the kinds of transitory choice behavior sometimes employed in such studies) ought to be seen as evidence of role systems.

³S. F. Nadel, op. cit., p. 20.

embodies the logical features of their definition. However, their construal of the actual specification of role elements as finally and definitively given by the participants introduces an additional restriction of their definition and works mischief on the concepts. If it be allowed it assuredly removes some degrees of freedom from the possibility of latent positions and roles. If the conceptualization of role systems is to be "a special analytic tool," it must both take account of the public use of role concepts and transcend them. In brief, the scientist must be admitted as the ultimate "role definer," though in a fundamentally different sense.

In our explication, our objective has been to develop the scientific rather than the common sense of the concepts. The former provides the substantive definition and the latter is included in the order of phenomena used for the specification of the relationships examined in empirical analyses. In this sense, the public, and particularly the participants', concepts of relations in the system being examined are part of the data, and as such they cannot "speak for themselves." Their employment requires the active conceptual intervention of the investigator.

Simply, it appears that the concept of participant role definition indicates that every actor has a

"cognitive mapping" of the relationships among participants. In general, such a "mapping" could be, but probably is not, of the same detailed form as our model and its constituents and mappings; especially, such cognitions are likely to be incomplete respecting enumeration both of participants and relationships. The congruence among the "mappings" of the actors in the system would then be seen as yielding measures of consensus on participant role definitions.

More important than the simple enumeration of participants' conceptions of roles is the question of the perceived content of relations, which is required for decisively defining the system. In the situation studied by Gross, et al., are the "expectations" held by superintendents and board members and elicited by the investigators the only and real content of the roles? In connection with our statement of the incumbency axiom we observed that they are not, and there is another source of evidence which is suggestive on this same point. In their conceptual reformulations, Gross and his associates state definitions of a battery of concepts. These include role and role-sector, which are defined in terms of expectations. However, one may observe that these key terms are used relatively infrequently in their subsequent discussion of the actual data analyzed in their research. There the preference is for expectations as such, though combining forms, such as "intrarole conflict" and "role congruency," which restrict

the range of empirical reference, are used liberally. Perhaps this happens only because the detailed analysis necessarily deals with a restricted range of phenomena. But the issue does arise as to just what order of extensiveness and significance of expectations for a pair of positions must be encompassed before it is appropriate to refer to the set as a role-sector. No answer to this question is given.

We have reviewed these questions regarding the usages of Gross and his associates as a vehicle for reintroducing the general question of the relations taken as primitive in our model, for this issue is also involved in our views on the "necessity" of positions and roles, and on manifest identities and latent systems. We have said that our aim is to represent the formal character of the concepts of sociologists rather than of those they study, and that logical relations are uniformly appropriate for the several main ways of defining these concepts. But our model goes beyond this. It introduces the idea that a full specification of a role system as a conceptual tool can be made only in terms of a definite set of relations of whatever kinds. In terms of our objective these relations must be specified by the investigator, on grounds of substantive import. Participants in a concrete system may have cognitions about the relations and these may be relevant data, but neither is required. The model is indifferent to whether a concrete system is latent or manifest.

However, since the model can represent a manifest system, is it not then permissible in such instances to use the public identities of positions to locate them, as most investigators have assumed? In our strict construal of the model, the answer is "No" -- the formulation of the incumbency axiom does not permit this. We noticed in our summary that A and I are the additional representation needed for a concrete role system, but that I "stands on the other side of the relationship between A and P." That is, the expression $(I,P) = f(A)$ is our notation for saying that incumbency and positions are functions of the chosen set of relations as observed in the object group. If public identities of positions were used, then this might be expressed in a similar way by $P = f(A,I)$, or $A = f(P,I)$, dependent on other assumptions. Further, all of these expressions might yield equivalent A's, P's, and I's, for a given group and set of relations, as, for example, in the authority structure of a formal organization. But this need not hold in general. The incumbency axiom is intended to represent what we detect as the sociologic definition of the concepts rather than their public specification. It should be clear that we view the investigator's selection of the set of relations as a central step in the full specification of a structure.

In terms of this we may see the reason for holding doubts about the appropriateness of "just any" content for

the set of relations used to specify a structure. We have observed before that investigators sometimes are reluctant to use role terms to designate detailed aspects of relational systems, for example in studies of informal organization. This seems to reflect sensitivity to the inadequacy of one or a few observed relationships to delimit "a person's role" or "the total structure." Moreover, it indicates a diffuse concern with the matter of substantive significance of the structures under analysis.⁴ We also have observed that the model does not preclude use of any set of relations whatsoever, but it might better be said that it is silent on the matter -- the model neither precludes nor requires treatment of any given relational structure as a role system. As a conceptual tool it may be used well or poorly, or it may not be used at all. If it is used, then the role concepts are applied to the chosen content. Whether it should be used must be decided in terms of substantive considerations of theoretical relevance, and these are not resolved by the logical form.

⁴By comparison, role terms and concepts are used quite freely in discussions of the structure of small laboratory groups, where there is no implicit idea of a transcendent social system whose significance overshadows that of the particular task relationships formed therein.

The final point we wish to make on the questions of public identities, latency, and relational specifications, concerns our assumption of exhaustiveness (that every pattern of the relevant relations between pairs of actors is a role-sector) which was introduced in connection with the mapping solutions developed in the last chapter. We remarked there that sociological literature is not determinate on the matter of "when we are to take what as evidence" of the existence of positions and roles. Perhaps we might say that in the broad context of definitional statements the matter is ambiguous, since both "public identity" and "relationship" criteria are advanced. Our rejection of the identity criterion and acceptance of exhaustiveness are related. If positions and roles must be recognized (and named) as such by participants, then it is quite conceivable that the recognition might both occur and be absent in separate instances having the same relational pattern. This is equivalent to ignoring a particular cell vector of A in the mapping. On the other hand, in the use of the relational criterion for positions and roles, there is no intrinsic reason for discriminating between patterns in one or another of their instances, and exhaustiveness is the sensible criterion on grounds of parsimony.

We now turn our attention to another question of the interpretation of the analysis which has been made.

In the beginning of our review of role concepts it was stated that the concept of a system would provide a general orientation for our work. Indeed, we have developed the logical model in terms of elements and their connections as displayed in several preceding chapters. We also activated the common assumption that it is permissible to attend to the social structural aspects of role concepts without attempting to incorporate social psychological considerations in any comprehensive way. There is another broad assumption of a similar sort which has been implicit in this development, but it is on the side of social structure itself. We have used the term "role system" to refer to the structure whose logical features have been analyzed, but we have not been explicit as to what relationship is assumed between role systems as conceived here and concepts of social system or social structure. Obviously, it is assumed that there is an intimate and important connection, but, as often, the received opinions in the literature are not decisive. There is general agreement that roles (etc.) are important parts of (social) structure, which in turn is an essential characteristic of a social system, but variations in the working out of this formulation are too well known to require more than brief comment. Consider, for example, the differences between Nadel and Parsons. For Nadel, what we have called the role

system is the social structure (assuming, we suppose, that the right relationships are chosen). His ". . . definition of social structure . . ." is given as follows:

We arrive at the structure of a society through abstracting from the concrete populations and its behaviour the pattern or network (or 'system') of relationships obtaining 'between actors in their capacity of playing roles relative to one another.'⁵

On the other hand, for Parsons what we assume to be equivalent to our role system is only one of two "levels" of analysis or ". . . points of view, both of which are essential to completeness. . ." in analyzing the structure of a social system.

The first is the 'cultural-institutional' point of view which uses the values of the system and their institutionalization in different functional contexts as its point of departure; the second is the 'group' or 'role' point of view which takes suborganizations and the roles of individuals in the functioning of the organization as its point of departure.⁶

However, such differences in formulation need not introduce any fundamental ambiguity concerning the status of the concept of a role system, since they agree that role systems can be demarcated as such, whether as the system or as a sub-system of some larger one, and this is all that is required here. The converse of this is also important and

⁵S. F. Nadel, op. cit., p. 12.

⁶T. Parsons, Structure and Process in Modern Societies, op. cit., p. 20.

should not go unnoticed; if we assume that role systems are capable of being treated in certain respects as independent components of larger social structures or systems, then we also have assumed that the larger system can accept or account for such a component. It is our belief that these assumptions are entailed by sociological use, and this is sufficient for our purpose.

To conclude our remarks on these interpretations of the model of role systems, we may observe that they are not the only way of construing the model. We have developed the model as a means of capturing the logical moments of the concepts of position and role, but we have been selective in admitting and reconstructing views of the concepts. Let us go back once more to the general question of variability of role contents (or consensus on them). We have argued that it is not necessary or parsimonious to assume that such variability should be incorporated in the concept of a role, that it is better to postulate a determinate structure and to conceptualize variability in reference to it. Another line of argument could be made which accepts the model as given in all formal particulars (A, P, I, etc.) but places a different interpretation on the elements. It may be recalled that the concept of a position is uniformly assumed to be an element of a fixed structure, i.e., as a location is a determinate matrix of relationships. Strictly speaking, this is sufficient to

entail the formal apparatus, in which case it might be called a "positional system," and the role terms used for other concepts. But it is our interpretation that sociological concepts of roles as structural elements imply that the sets of relations between positions should be designated by the role terms. The point of this example is to emphasize that the model is intended to represent important conceptual facts, but that they are selected aspects whose fit in the larger conceptual structure may be changed without requiring modification of the representation. In this sense, the model may be more enduring and fundamental than the particular interpretation which we have placed upon it.

Evaluation of the Explication

In the first chapter of this report we made a brief review of the concept of an explication and related topics in order to provide a frame of reference for understanding and evaluating our analysis of role concepts. Here we will attempt to apply the guidelines which were suggested to make an estimate of the results which have been obtained. In discussing explication we first reviewed some rather general considerations and later summarized points made by Berger, et al., which are more specific, and we will repeat that order of presentation here.

The overarching criterion of explication seems to be "fruitfulness," though we have questioned whether this is

more than an exhortation to do well. It is the writer's belief that he may reasonably be excused from any attempt to make a definitive declaration on the question, both on the grounds that it is an unnecessary jeopardy and that a litigant ought not be permitted to sit in his own judgment. However, it does seem reasonable to point out some main features of the work which ought to be considered in rendering such an evaluation. Briefly, the writer believes that the following characteristics are central.

First, the concepts of position and role are defined interdependently, and this is accomplished in terms of specific syntax; apparently conflicting or unrelated conceptions are shown to be compatible, and, in fact, to entail the same referents under comparable specifications. Second, models which have been used previously (and often intuitively) are shown to be inadequate; in the most general case, role concepts require multilinear directed graphs, or some isomorph, as a model. Third, to the best of the writer's knowledge, the axiom of incumbency is a unique statement of a necessary concept. Finally, the model states a general system which does not require the restricting use of public identities to specify the concepts.

Two other features which were stressed in our initial statement on explication are the likelihood of novelty relative to established usage and the value of

exactness or precision of presentation. It is felt that the development has exhibited both of these traits in good measure. Of course, the new features are not in the substance of the concepts but in correctly apprehending the common form of usage, and it is this form which makes possible relatively precise statement. But we need to emphasize once again that neither of these features is valuable in itself -- they are of interest because they are frequently associated with worthwhile results, and in respect to exactness because it facilitates appraisal.

Let us turn now to the five-point summary of Berger and his associates, as more concrete measures of the work. The first of their characteristics is not important for this purpose, though we have noticed repeatedly that it is certainly characteristic of the model, viz., that it is selective with respect to original formulations. The remaining points are the ones we require and they are that an explication 1) clarifies, 2) defines, 3) generalizes, and 4) determines internal implications of the original conceptualization.

As evidence of clarification of the concept of a role system, we may cite two developments in particular. First, the model makes clear (one might almost say that the techniques of representation require) that the specification of a given role system can only be made with respect to a definite set of relations; even if these must be

further substructured to produce a complex structure, the model draws attention to the fact that structures cannot be defined by simply enumerating their "elements," i.e., positions and some associated behaviors. Further, it emphasizes that, whatever its empirical intentions, a given role system is a construct of the investigator. Second, the incumbency axiom brings out a necessary assumption, and it casts the assumption in a form such that its consequences can be considered independently of other questions. If subsequent analysis and opinion should find the assumption untenable, it may be elided without prejudice to the abstract conception of a role system.

From the examples which are mentioned in our first chapter, it appears that the criterion of refinement refers essentially to developments in the direction of producing measurement techniques for the concepts under analysis. It is the writer's opinion that our analysis of role concepts fares less well on this point than on any of the others under review. Broadly understood, the analysis of $(I,P) = f(A)$ carried out in the previous chapter is a specification of procedures for measurement of the elements of a role system. But in our conclusion to that analysis we expressed our doubts regarding the usability of the mapping techniques which were outlined. We reiterate the opinion here and further emphasize that the techniques will neither be used or transcended in the

absence of theoretical grounds for specifying structures in terms of their component relations, of whatever kinds.

With respect to generalization of the concept of a role system, two key factors should be noticed. The first of these is the explicit postulation of a model of a multi-relational system in the theory of "nets," in place of the more usual and somewhat haphazard use of unilinear graphs. The second important direction is the definition of a system which eliminates the competing assumption of public identity from the specification of positions and roles; such identities imply relations, but relations do not imply identities. In consequence, the concept of a role system becomes a more general tool for the investigator.

Finally, the model of a role system provides a means of rigorously showing in what way positions and roles may be said to mutually imply each other without also thereby being redundant. This interdependence of the concepts has been remarked so frequently before in our discussion that it needs no further comment here.

With these points for consideration, and setting aside just for the moment our earlier reluctance to judge, what may we conclude about the explication that has been carried out? Is it a good, or at least satisfactory, one? Or if such an absolute appraisal cannot be made, how does it compare to others? Specifically, say, how does it fare

when set beside the Cartwright-Harary formalization of balance theory which we summarized in the first chapter? Simply, we have started out with less to work with and ended up with less striking results. We noted especially that the concept of balance was part of a relatively "tight" theory, and the role concepts which we have examined are not. On the other hand, we would repeat the assertion made in the concluding section of the fourth chapter where we argued that the model does the general concepts more justice with respect to its emphases than any alternative known to us.

There is further reason for suggesting that no final evaluation may be made, for the work is not done. The final pages of this report indicate only a few of the additional questions which seem to require exploration before the "fruitfulness" of our analysis could be determined.

Suggestions for Further Research

In these suggestions we attempt to maintain continuity with the kinds of concerns which have been evident throughout the report. Thus, the main questions concern relations, namely, what order and how many of them must a role system model confront, and what are the relations among the relations themselves? We will not ask after what kinds of relations should be included in terms of appropriate substantive content; we have

eliminated this order of inquiry up to now, and there is quite sufficient material yet in the line of thought being pursued to keep our attention.

In our review of role concepts a distinction was made between attributive and relational concepts of role, wherein we recognized that both modes of predication could be used in describing and analyzing the concepts and their applications. Henceforth we dropped further consideration of attributive forms in keeping with our judgment that relational forms provide a correct basis for the model of a role system. However, the question remains open whether it is possible to construct an attributive model, and whether this would not represent the concepts as well or better than the model we have devised. A salient candidate exists in the work of Nadel.⁷ He develops a notation system which mainly utilizes first-order predicates (i.e., attributes) to symbolize features of relations which we have detected in the literature. However, though he correctly notes that the ". . . system approximates to a calculus,"⁸ it does not yield a satisfactory representation of a role system. In fact, careful reading of the original definition and subsequent propositions reveals that it is

⁷S. F. Nadel, op. cit.

⁸Ibid., p. 57.

based on suppressed relational operators⁹ (i.e., second-order predicates), and this development is one of the sources of the writer's conviction that a role system model must be fundamentally relational. But persuasion is not proof, and the problem remains open.

Turning our attention from the reduction of the number of arguments permitted for the basic predicates, we may look in the other direction and inquire whether we need to consider increasing the number. In our development of the model we used the term "relation" to refer to dyadic connections, though its technical meaning does not require this. The use was dictated by the fact that almost all of the extant definitional materials assumes just this much complexity. Is there evidence that our model ought to be able to deal explicitly with, say, third-order predicates? The role definers problem which we have commented on above, as well as Biddle's conceptions of "Levels of Cognition,"¹⁰ might suggest this requirement, but in each case it appears that the authors do not see the actor's conception of the relationship between others as denoting incumbency and its consequences. However, Nadel, again, provides an example of the kind of

⁹Ibid., p. 10, et passim.

¹⁰B. J. Biddle, op. cit., p. 14.

consideration which might require such an extension in his concept of ". . . triadization . . .",¹¹ which denotes the concern or involvement of a "third party" in a dyadic relationship. This amounts analytically to a relation between a position and the role-sectors of another pair of positions. It should be observed immediately that the usual graphic representation cannot incorporate this feature without considerable complication, though the matrix representation could do so by the addition of a fourth dimension. In either case, the result is awe-inspiring. Perhaps it is fortunate that the makers of definitions of role concepts have not concerned themselves with third parties to any significant extent.

There is another numerical question of some interest concerning relations. In our development we have assumed that the model requires a definite set of relations, but we have not asked how many might be required in any concrete system. In general, of course, there is little but speculation to guide such inquiries, but the work of some investigators seems to suggest that the number might not be very large.¹² We have emphasized that the actual composition of a set of relations used to define a structure

¹¹S. F. Nadel, op. cit., p. 86.

¹²E.g., A. F. C. Wallace, "On Being Just Complicated Enough," Proceedings of the National Academy of Sciences, 47 (1961), pp. 458-464.

is a theoretically based decision which has to be made by the investigator, but there is nothing which need constrain him to superfluity. If a small number would suffice, the practical relevance of the kind of considerations outlined in our fifth chapter might increase sharply.

Practical restriction in the actual number of relations utilized might also be a heuristic circumstance for solutions to two connected problems concerning the relations among the relations themselves. We refer to the question of ranking which has long been associated with role systems through the concept of status, and to the property of transitivity of relations which is fundamental to the concept of an ordering of a set of elements. Transitivity is defined with respect to single relations and the representation is essentially "digraphic." However, it is sometimes possible to define a single common property of a set of different relations, e.g., "power" as a component of both "authority" and "influence," and thus satisfy the requirement. Or more complex relationships might be developed which induce transitivity. In the same example, a three position chain of "influence" followed by "authority" might be thought to entail "authority" between the first and third positions, while "authority" followed by "influence" might not do so. A still broader approach might be made by the use of multidimensional scaling techniques, and this would have much to recommend it. The problems of multiple

dimensions of ordering in social structures have excited much interest, for example in such relatively recent conceptualizations as status crystallization, congruence, or equilibration, though such interests have been directed to investigating ways in which multiple orders collapse into single dimensions.

Another kind of interest in relations among relations concerns their compatibility, and while this requires substantive considerations of the content of relations, there are interesting formal features which a model may isolate. The outstanding source of contemporary interest in this topic appears in the proliferation of studies of role conflict. The focus of these interests is often in the dynamic consequences for the participant, and on this such a model as ours has little to say. But the antecedent stress is assumed to be structural, and it would seem that a useful model ought to provide some representation for this. For example, in the P-matrix representation of the role system it is possible to form pairs of cells for comparisons regarding compatibility in eight different ways by enumerating ones with like and unlike subscripts in all three dimensions. In fact this might be taken to represent a basis for a more general typology of general structural conflict within a system since it allows cases which concern elements of role sectors between completely different position pairs.

But such observations are little more than suggestive, and their import goes beyond our purpose here of noting another possible extension of analysis.

To repeat, the intent of this final section is to underline the writer's belief that there are manifold possibilities for fruitful extension and reconstruction of the formal characteristics of our conceptualizations of role systems. We conclude in agreement with another recent observer of the field.

The final way of dissecting this conceptual complex has doubtless not yet occurred.¹³

¹³W. R. Catton, Jr., op. cit., p. 942.

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