ENGINEERING PARAMETERS RELATED TO THE HARDNESS OF CARROTS

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ABSTRACT

ENGINEERING PARAMETERS RELATED

TO THE HARDNESS OF CARROTS

by

Bassam Ahmed Snobar

Studies were conducted to indentify and evaluate objectively a textural parameter related to the long-term storage of carrots. Mechanical and rheological properties were used in this evaluation.

An equation was derived, using Hertz's Contact Theory, to calculate the modulus of elasticity of a cylindrical sample compressed in the radial direction. This equation was used to calculate the tangent modulus of a one-inch diameter carrot sample after it had been subjected to a 0.1-inch displacement in the radial direction. Relaxation tests were also conducted to study the relaxation behavior of cylindrical samples as related to the long term storage of carrots.

The tangent modulus varied significantly as the moisture content in an outer ring of the carrot decreased. The tangent modulus decreased from 869 psi to 27 psi as the moisture content decreased from 86.6 percent to 72.5 percent (wet basis). Similar variations were observed for the coefficients C_1 in the relaxation equations

$$F(t) = \sum_{L=1}^{3} C_1 e^{-\alpha_1 t}$$

used to fit the relaxation data (generalized Maxwell Model). No significant pattern was observed for the α_1.

The Texture Profile Analysis procedure was applied to carrot samples using an axial compression load. No significant changes were detected even though the physical appearance of the carrots changed significantly. This fact was accounted for by the fact that the center core of a carrot is stronger than the outer ring of material and thus biased the testing procedure.

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ENGINEERING PARAMETERS RELATED TO THE HARDNESS OF CARROTS

by

Bassam Ahmed Snobar

A DISSERTATION

submitted to

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The author dedicates this work to his father, Ahmed M. Snobar, his mother, Mrs. Yussra Nabhaan, and to his family. Their foresight, encouragement and personal sacrifice inspired him to undertake a career in agricultural engineering.

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I. INTRODUCTION AND OBJECTIVES

The carrot (Daucas carota) is a popular vegetable and is increasing in importance, owing to the fact that its value in the diet is better understood today than in past years. It is rich in carotine, a precursor of vitamin A, and contains appreciable quantities of thiamine and riboflavin. The carrot also has a high sugar content.

Concern has risen among growers and processors relative to the changes in the texture of carrots particularly during storage. In order to identify these changes, the texture parameters of carrots must be defined and objectively measurable. Sensory, or subjective, analysis has been used to measure the texture of carrots, but this requires a trained taste panel which is not readily available to many investigators.

Lord Kelvin (1891) said, "I often say that when you can measure what you are speaking about and express it in numbers you know something about it, but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind, it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of science, whatever the matter may be." There is a very limited number of reports in the literature related to the objective evaluation of the texture of carrots. An objective procedure is needed in order to bring consistency to the investigation of the effects of storage time on the texture characteristics of carrots.

The specific objectives of this investigation are:

1. To define the important textural parameters related to the long term storage of carrots.
2. To investigate methods of objectively measuring the textural parameter hardness.
3. To investigate the relationship between mechanical properties and moisture content.

II. REVIEW OF LITERATURE

2.1 Methods of texture evaluation

Texture is one of the three main attributes of foods that cause pleasure in eating; the other two being flavor, and appearance. There are many definitions for texture. Reidy (1970) reported a dictionary definition of texture as "an identifying quality; the disposition or manner of union of the particles of a body or substance". The Institute of Food Technologists offered another definition for texture of food (Kramer, 1959) as: "The mingled experience deriving from the sensation of the skin in the mouth after ingestion of a food or beverage. It relates to density, viscosity; surface tension and other physical properties of the material being sampled". A possible definition of texture in carrots may be stated as: "Texture of fresh carrots is the feel of hardness or crispness of the tissue in the mouth".

The present methods for evaluation of textural characteristics are classified into:

1. Subjective or sensory evaluation.
2. Objective or instrumental measurements.

Subjective estimations of the textural quality of foods have been performed since mankind began eating food and they continue to this day. This method of evaluation depends on human senses. Due to the fact that human senses are subject to the influence of various factors which lead to error, scientists began the search for objective or instrumental methods of texture measurements.

Scott Blair (1958) classified objective methods of texture measurement under the headings: fundamental, empirical and imitative. Szczesniak (1966) further defined Scott Blair's classification system as follows: fundamental methods measure the rheological properties, such as elastic modulus and viscosity, and relate the nature of the product to two basic rheological prototypes; a dashpot for a Newtonian liquid and a metal spring for a Hookean solid. The springs represent elastic moduli. The dashpots represent viscosities. Empirical tests measure characteristics related to textural quality using penetration force test, resistance to compression force test, and shearing force test. Imitative tests are performed under conditions simulating those to which the material is subjected in practice.

Bourne (1966a) classified the methods for objectively measuring the textural properties of food under the headings: force-measuring, distance-measuring, time-measuring, energy-measuring, ratio-measuring, multiple measuring, and multiple-variable instruments.

The first simple instruments used to objectively assess food texture came with the advent of scientific research into food quality. An examination of the literature of food texture measurement shows that these simple mechanical devices generally compressed, sheared or punctured the food in some way.

Experimental methods for measuring food texture date back at least to 1905, when Hankoczy in Hungary designed an apparatus for measuring the strength of gluten and in 1907 Lehmann described two instruments for testing the tenderness of meat. Morris (1917) constructed a simple device for measuring the resistance of fruits to penetration. In 1925, Magness and Taylor developed the Magness-Taylor fruit pressure tester. This instrument which is still widely used, consists of a plunger with either a 5/16-inch or 7/16-inch diameter tip attached to a calibrated spring. The round tip is pressed into the fruit to a depth of 5/16-inch, and the penetrating force is read on the scale. The skin is removed from some fruits before the measurement is made. Kramer et al. (1951) and Decker et al. (1957) developed the shear press which is one of the popular instruments for measuring textural qualities of both fresh and processed fruits and vegetables.

Kattan (1957) described an instrument for measuring firmness of tomatoes based upon compression of the fruit by a concentric chain which encircled the fruit.

Drake (1962) described an apparatus for automatic recording of mechanical resonance curves for test specimens of foodstuffs with the approximate size of 6x12x50 mm. The simple evaluation procedure described gave information on the modulus of elasticity (divided by the density) and the degree of dampening.

Schomer et al. (1963) developed an instrument called the "mechanical thumb" which operates on a principle similar to the Magness-Taylor tester. However, their test is nondestructive in that the fruit can be evaluated with the skin intact, and the depth of indentation (0.05 inch) causes no significant damage to the commodity. Parker et al. (1966) developed a simple, portable, inexpensive micrometer type device for evaluating cherry firmness.

Bourne (1965) evaluated the performance of pressure testers by making punch tests on apples with pressure tips mounted in a universal testing machine. His study showed that the yield point is reached when the pressure tip begins to penetrate the fruit tissue.

With the present realization of the importance of texture in consumer acceptance, an increasing amount of attention is being paid to correlating experimental measurements with sensory methods of texture evaluation. Friedman et al. (1963) studies the correlation between instrumental values using texturometer and subjective evaluation by a trained texture profile panel. This study was applied to measurement of the mechanical textural parameters: hardness, cohesiveness, viscosity, elasticity, adhesiveness, brittleness, chewiness, and gumminess. This study gave good correlation between objectively determined values and subjective evaluation.

Szczesniak et al. (1963) developed a standard rating scale for mechanical parameters of texture and correlation between the objective and the sensory methods of texture evaluation. Standard rating scales of hardness, brittleness, chewiness, gumminess, viscosity, and adhesiveness were established for quantitative evaluation of food texture. Hardness is judged organoleptically as the force required to penetrate a substance with molar teeth. The evaluation was restricted to solids and some semisolids because human perception of hardness is limited to samples that can be confined between the teeth. In their study, Szczesniak et al. avoided fresh fruits and vegetables whose texture varies greatly with variety, degree of maturity, and other factors, and items that required cooking, baking, etc.

Table 1 shows the nine points which were selected to represent the scale of hardness. Correlation was very good between taste panel and objective evaluation on the hardness scale (Fig. 1).

Numerous methods of objectively measuring texture of agricultural products have been developed, adapted, or studied by many scientists.

Szczesniak et al. (1963) developed a Texture Profile Analysis (TPA) technique by which textural parameters were derived from force vs. distance curves plotted on the General Food Texturometer. The (TPA) parameters are: hardness, brittleness, chewiness, gumminess, viscosity, cohesiveness, elasticity, and adhesiveness.

Brandt et al. (1963) developed a texture profile method that uses the A.D. Little flavor profile method as a model. They defined textural profile as the organoleptic analysis of the texture complex of a food in terms of its mechanical, geometrical, fat, and moisture characteristics, the degree of each present, and the order in which they appear from first bite through complete mastication. The procedure they followed to evaluate texture was mechanical, and geometrical evaluation. The mechanical parameters were evaluated with standard rating scales developed by (Szczesniak et al., 1963). The geometrical characteristics of texture were related to the size, shape, and arrangement of particles within a food. Table 2 shows the procedure used in evaluating the different textural characteristics with respect to their appearance.

Destructive and nondestructive techniques were developed to measure the texture of foods. Mohsenin et al. (1965) have suggested a "nondestructive" technique for evaluating firmness of apples based upon the appearance of a "yield point" within the fruit.

Bourne (1966b) designed a study to separate and measure the compression components (proportional to area) and the shear components (proportional to perimeter) of a simple puncture test. His puncture test measured the force required to push different types of punches into a food product. The test is characterized by: a) using a force-measuring instrument; b) penetration of the punch into the food; and c) a penetration distance usually held constant. Bourne used two sets of punches (one with a constant area and a variable perimeter, and the other with a constant perimeter and a variable area) to measure the compression and shear components in representative foods. The puncture force was

Table 1. Standard hardness scale (Szczesniak et al. 1963).

Panel rating	Product	Brand or type	Manufacturer	Sample size	Temp.
1	Cream cheese	Philadelphia	Kraft Foods	½"	45–55°F
2	Egg white	hard-cooked 5 min	.....	½" tip	room
3	Frankfurters	large, uncooked, skinless	Mogen David Kosher Meat Products Corp.	½"	50–65°F
4	Cheese	yellow, American, pasteurized process	Kraft Foods	½"	50–65°F
5	Olives	exquisite giant size, stuffed	Cresca Co.	1 olive	50–65°F
6	Peanuts	cocktail type in vacuum tin	Planters Peanuts	1 nut	room
7	Carrots	uncooked, fresh	......	½"	room
8	Peanut brittle	candy part	Kraft Foods	.....	room
9	Rock candy	.....	Dryden & Palmer	......	room

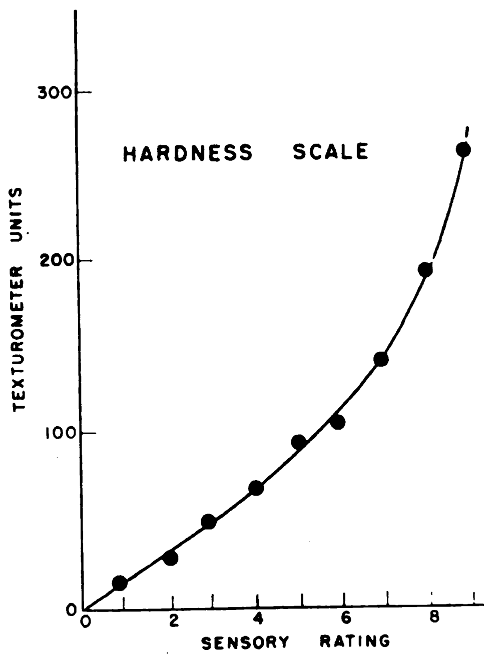


Fig. 1. Correlation between the panel and the texturometer on the hardness scale (Szczesniak et al. 1963).

expressed by the equation $F = K_s P + K_c A + C$, where K_s, K_c, and C are constants, P is the perimeter of the punch, and A is the area of the punch. K_s represents the shear coefficient and K_c the compression coefficient of the food being tested. Bourne tested the validity of the equation postulated above and found that the experimental data obtained fitted the equation. Table 3 gives the numerical values of the coefficients K_c, K_s, and C as measured by Bourne for various food commodities. Bourne stated that K_c and K_s can be a measure of the texture quality of foods. Although Bourne did not specify the direction of applying the compression force on the specimen, the work of Howard and Heinz (1970) seems to indicate that the force was perpendicular to the longitudinal axis.

Bourne was one of the first people to use the Instron Testing Machine for measuring the properties of food products. In one of his studies (Bourne 1967a), he used this machine to study the deformation rate of food under constant force. This test was used to determine the softness of the food as means of measuring food quality.

Bourne and Mondy (1967b) measured the deformation of: 1) standard cylinders of potato tissue and 2) whole potatoes under a metal punch, using a constant force, as an indication of the firmness of whole potatoes. They found that measuring deformation under a punch is preferred over measuring the deformation of a cylinder because a) it can be performed more quickly and easily; b) it is not destructive; c) the correlation with sensory evaluation is slightly improved. Both methods of measuring deformation was found to be a useful objective index of potato firmness.

Bourne (1967c) described a model system which closely represents the deformation of a food as it is squeezed in the hand. The model

Table 2. Procedure for evaluating texture (Brandt et al. 1963)

- *Initial* (perceived on first bite)
  - Mechanical
    - hardness
    - viscosity
    - brittleness
  - Geometrical
    - any, depending upon product structure
- *Masticatory* (perceived during chewing)
  - Mechanical
    - gumminess
    - chewiness
    - adhesiveness
  - Geometrical
    - any, depending upon product structure
- *Residual* (changes made during mastication)
  - rate of breakdown
  - type of breakdown
  - moisture absorption
  - mouthcoating

Table 3. Numerical values of coefficients for various commodities (Bourne 1966b)

Commodity	Compression coefficient K_c (Kg/cm²)	Shear coefficient K_s (Kg/cm)	Constant C (Kg)
Expanded polystyrene	4.86	0.34	−0.23
High-density polystyrene	13.14	2.20	−2.75
Polyurethane	3.57	0.29	−0.47
Apples (raw, Limbertwig variety)	7.52	0.16	0.03
Apples (raw, Fr. von Berl variety)	6.43	0.07	0.40
Banana (ripe, yellow)	0.43	0.06	−0.06
Creme filled wafers	1.06	0.14	0.64
Carrot (uncooked core tissue)	28.0	−0.03	2.18
Wiener (cold)	1.69	0.004	0.15
Potato (Irish, uncooked)	10.79	0.52	0.60
Rutabaga (uncooked)	29.58	0.86	−0.15
Sweet potato (uncooked)	19.8	0.90	0.35
1% agar gel	0.15	0.005	−0.01
2% agar gel	0.63	0.029	−0.02
3% agar gel	1.21	0.16	−0.33

consists of a set of true springs of differing heights and with differing Hooke's constants arranged in parallel. No dashpots were needed in this simple model. The model was restricted to represent the physical response of the food to a single compression. He described a graphical method for measuring the number, size, and Hooke's constants of the springs in the model. The spring model showed that with some foods at least, small compression forces measured differences in softness better than large forces.

Bourne (1968a) described a method to determine TPA parameters from force-distance curves produced upon twice compressing a specimen to a fixed deformation on the Instron machine. The curve produced for deriving the TPA parameters for pear parenchyma tissue from Instron force vs. distance is shown in Fig. 2. Brittleness, hardness and elasticity parameters are shown on the curve. Other TPA parameters can be calculated as follows:

$$\text{Cohesiveness} = \frac{A_2}{A_1}$$

$$\text{Gumminess} = \text{Hardness} \times \text{Cohesiveness}$$

$$\text{Chewiness} = \text{Gumminess} \times \text{Elasticity}$$

where

A_1 is the area under the first curve and

A_2 is the area under the second curve.

Ourecky and Bourne (1968) measured the texture of strawberry with an Instron machine. In this test, skin toughness and flesh firmness were determined by obtaining a puncture-force curve on the Instron. Many of the curves consisted of two or three distinct peaks. The first peak was defined as the puncture-force required to penetrate the skin. The second or middle peak was interpreted as the resistance of the flesh

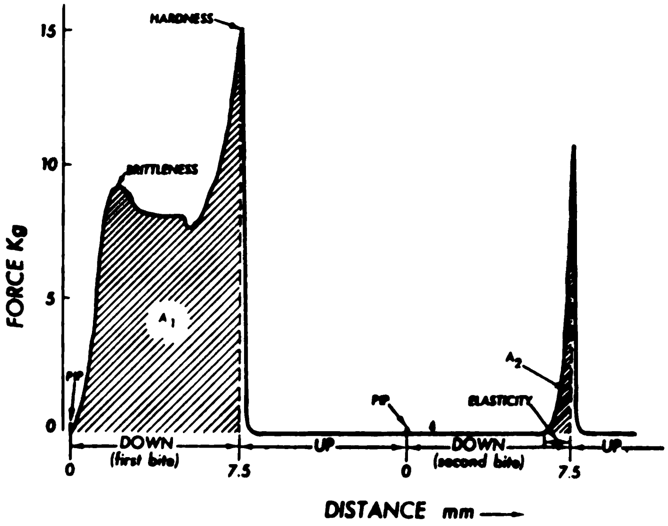


Fig. 2. Direct trace (heavy line) of force-distance curve obtained for a G. F. Texture Profile on a cylinder of pear tissue in the Instron machine. The test consists of two complete compression-decompression cycles. (Bourne, 1968a)

or cortex and vascula cylinder to the penetrating probe. The third peak was defined as the maximum force required to penetrate the fruit. Fruits with a uniform flesh and core area gave no second peak.

Bourne and Moyer (1968b) reported studies on an extrusion type of texture-measuring test cell as an attempt to measure the texture of fresh peas. The extrusion cell was mounted on the Instron. The studies included the effect of plunger speed, effect of annulus width, and effect of sample size on the extrusion force. The force required for this extrusion was measured using green peas as a test material. From their studies they found that this type of cell showed promise as a routine testing instrument in commercial use because of its simplicity in construction and operation, and comparative low cost.

Bourne (1969) determined the possible relationship between deformability, which is related to Young's modulus of elasticity, and the puncture test, which is related to the bioyield point. He measured the deformability and bioyield of eleven different apple varieties. The deformability was measured as the distance the whole apple deformed under a 5/16-inch diameter Magness-Taylor punch between 0.5 kg and 2.5 kg force using a universal testing machine. The bioyield point was measured using the same 5/16-inch diameter Magness-Taylor punch in the universal testing machine and measuring the force necessary to reach the bioyield point (Bourne, 1965). The resulting data showed that there is no apparent correlation between firmness as measured by deformability with the firmness as measured by the bioyield point. Both measurements are considered to be an index of apple firmness.

Finney (1969) gave a brief description of methods for measuring the texture of meats, dairy products, bakery foods, fresh fruits and vegetables, and processed commodities. He also gave new techniques for objective evaluation of texture of foods. These techniques were based upon analyses of sound, light transmission, and vibration phenomena.

Howard and Heinz (1970) stated that there were no reports in the literature on the correlation between objective methods and sensory analysis for determining the texture of carrots. These investigators studied the texture of carrots measuring the compression and shear strength of individual carrots with the Instron Universal Testing Machine. Based upon the values reported for uncooked carrots (Bourne, 1966b) as K_c = 28.0 kg/cm^2, K_s = -0.03 kg/cm, and C = 2.18 kg, they predicted that the compression measurements would be a better indication of texture than shear measurements.

A compression device was used with the Instron to deform a carrot in the radial direction (Fig. 3). A typical compression force-distance curve is illustrated in Fig. 4. The compression test was performed on carrots that were purchased locally and stored in plastic bags at 0 - 2°C. The carrots were removed from the bags prior to testing and stored on trays in a conditioning room at 21°C and 65 percent relative humidity for one to twenty-four hours. Carrots were evaluated manually for compressibility and flexibility by five well-trained judges, using a nine-point scale. Compressibility was determined by pressing the middle portion of the carrot with the fingers and evaluating the resistance. Flexibility was determined by gently bending the carrot at the middle with both hands. The results showed that compressibility measurements were highly correlated with sensory hardness as judged by the taste panel. Shear measurements had a low correlation with the sensory evaluation.

Breene et al. (1972) determined the TPA parameters of cucumbers using Bourne's (1968a) method of force vs. distance curves. A sliced

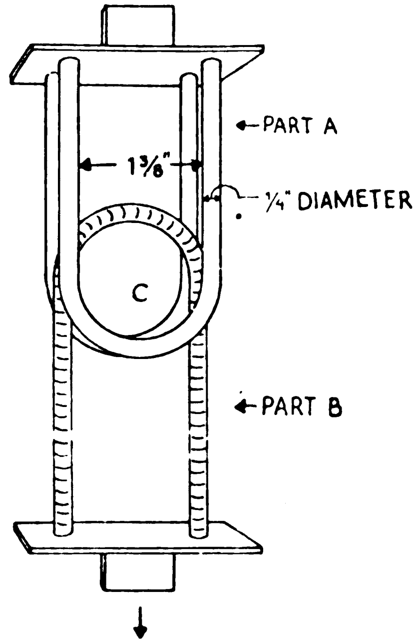


Fig. 3. Compression device used for compressing carrots. Part A is attached to the stationary crosshead of the Instron. Part B is attached to the moving crosshead. Space C is where the carrots are placed to be compressed (Howard and Heinz, 1970).

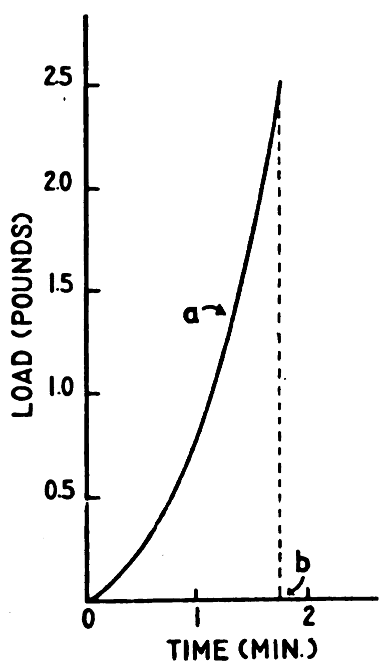


Fig. 4. Typical compression force-distance curve obtained for carrots. -(a) Load, with crosshead moving. -(b) Distance proportional to the change in diameter of the carrot compressed under a 2.5 pounds load (Howard and Heinz, 1970).

specimen of one-inch diameter and one-centimeter long was used for the test. The specimen was placed on the load cell of the Instron machine, and subjected to an axial load. The results of this test indicated variability in texture from one end of a cucumber to the other.

2.2 Rheological Models

Rheology is defined as "a science devoted to the study of deformation and flow", Mohsenin (1970). Therefore, when the action of forces result in deformation and flow in the material, the mechanical properties will be referred to as rheological properties. The rheological behavior of a material is expressed in terms of the three parameters; force, deformation, and time. Examples of rheological properties are time-dependent stress and strain behavior, creep, stress relaxation, and viscosity. The rheological behavior of linear viscoelastic materials can be explained and interpreted by use of mechanical models consisting of springs and dashpots. Based on experimental evidence, agricultural products are viscoelastic. From the very limited data available in this area, it appears, however, that the viscoelastic behavior is non-linear. Since solutions of non-linear problems are very difficult to obtain, the general procedure has been to make simplifying assumptions and apply the theories of linear viscoelasticity in an attempt to explain the rheological behavior of agricultural products.

The two basic mechanical elements used in mechanical models are a spring which obeys Hooke's law and a dashpot with the properties of a Newtonian liquid. The two elementary combinations of these elements, known as the Kelvin Model and Maxwell Model, are used as the rheological models. Maxwell's model is usually used to represent stress relaxation under constant strain. Kelvin's model is usually used to represent creep under constant stress. The two models, the stress relaxation and the creep are illustrated in Fig. 5.

Viscoelastic behavior of agricultural products has been studied by many investigators.

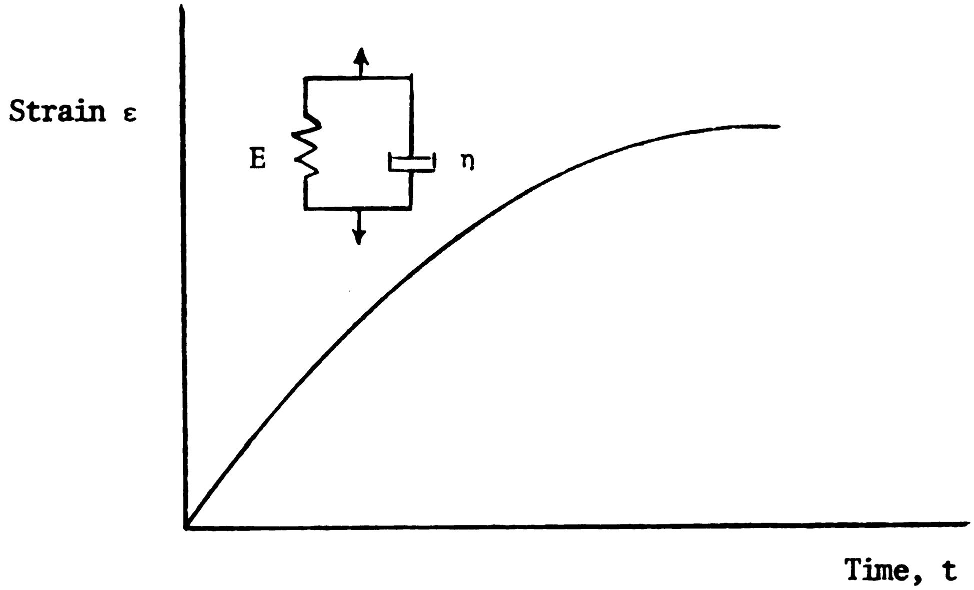
Barkas (1953) found that the resistance of organic materials to deformation is mainly dependent on the moisture held by molecular forces with the capillary water having little effect.

Zoerb (1958) studied small core samples of wheat kernels and found the response to be more non-linear approaching an elastic-plastic behavior with strain hardening tendencies.

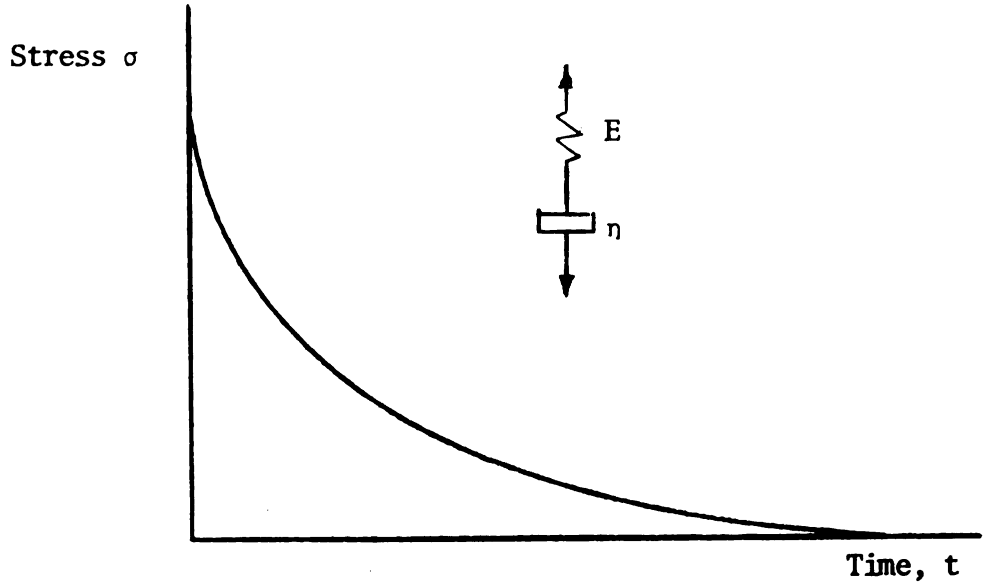
Stewart (1964) found that an inter-relationship existed between the wheat kernels moisture content and their viscoelastic properties.

Reidy (1970) in an attempt to find relationships between engineering and texture parameters of pre-cooked freeze-dried beef, developed two models: a) a four-element linear viscoelastic model of Kelvin and Maxwell bodies in series (Model 1); and b) an empirical constitutive equation (Model 2) which contained a probably non-linear term. He concluded that: a) Model 1 successfully predicted relaxation functions of the freeze-dried product; b) Model 2 predicted responses to relaxation, creep, and cyclic tests. This model predicted the texture indices of hardness and chewiness; c) water activity had an influence on the stress-strain behavior of pre-cooked freeze-dried beef. Resistance to deformation decreased at higher moisture contents; and d) relaxation stresses decreased with increased water activity.

Herum et al. (1973) studied the viscoelastic behavior of soybeans due to temperature and moisture content. They determined the time-dependent uni-axial moduli of intact soybeans by relaxation tests in parallel plate compression. Four temperatures and four levels of



Strain vs. time relationship corresponding to step function stress history.



Stress vs. time relationship corresponding to step function strain history.

Fig. 5. Kelvin and Maxwell models showing creep and stress relaxation characteristics (Sharma, 1964)

moisture content were used in the tests in an effort to determine if the techniques of time-temperature and time-moisture shift factors could be applied to describe a relaxation modulus for intact soybeans. Their goal was to identify the individual and joint contributions of temperature and moisture content upon the overall response. They concluded that soybeans may be described as thermo-rheologically and hydro-rheologically simple.

Bashford (1973a) studied creep and relaxation of meat related to tenderness. In his study Bashford found that rheological parameters obtained from creep and relaxation investigations correlated to taste panel evaluation. He concluded that these parameters are potentially good indicators of meat tenderness.

Chen et al. (1971) developed a computer program to determine the coefficient C_i and exponential α_i in the general relaxation equation:

$$F(t) = \sum_{i=1}^{n} C_i \, e^{-\alpha_i t'}$$

for the generalized Maxwell relaxation model,

where

$$C_i = 3/2 \, a^{3/2} \, KE_i \int_0^{t_1} e^{-\alpha_i (t_1 - \tau)} \, \tau^{1/2} \, d\tau$$

and

$$\alpha_i = E_i / \eta_i$$

Bashford et al. (1973b) developed a computer program to calculate the elastic and viscous parameters for the generalized Maxwell and Kelvin models. Maxwell's model is used as a relaxation model. They used the general form of relaxation equation $F(t) = \sum_{i=1}^{n} C_i \, e^{-t/T_i}$ to determine the coefficient C_i and exponential t/T_i.

2.3 Hertz Contact Problem

Hertz (1896) proposed a solution for contact stresses in two elastic isotropic bodies, such as the case of two spheres of the same material touching each other.

In his problem, Hertz attempted to find answers to such questions as the shape of the contact surface, the normal pressure distribution on the surface of contact, the magnitude of the maximum pressure, and the approach of the centers of the bodies. In developing his theory, Hertz made some fundamental assumptions.

These assumptions, given by Kosma and Cunningham (1962) are the following:

1. The material of the contacting bodies is homogeneous.
2. The loads applied are static.
3. Hooke's law holds.
4. Contacting stresses vanish at the opposite ends of the body (semi-infinite body).
5. The radii of curvature of the contacting solid are very large when compared with the radius of surface of contact.
6. The surfaces of the contacting bodies are sufficiently smooth so that tangential forces are eliminated.

The first known application of the Hertz solution for contact stresses in agricultural products was reported by Shpolyanskaya (1952), who applied it to evaluate modulus of elasticity of wheat kernels.

Morrow and Mohsenin (1966) have applied the results of Hertz's work to calculate the relaxation modulus and creep compliance of apples.

Fridley et al. (1968) obtained experimental force-deformation curves for peaches and pears and compared them to the theoretical curves as calculated using the Hertz equation for plate against sphere.

Horsfield et al. (1970) applied theory of elasticity to the design of fruit harvesting and handling equipment for minimum bruising. They extended Hertz's contact theory, given by Timoshenko and Goodier (1951), for the colliding of two spheres, to determine internal shear stresses generated under impact considering effects of the modulus of elasticity and radius of both the fruit and the impact surface. They discussed experimental techniques for measurement of modulus of elasticity under impact loading. They concluded that the modulus of elasticity and surface radius of both surfaces, in addition to the impact energy and flesh strength, permit meaningful prediction of bruising and serve as good criteria for design of machines to reduce bruising.

Mohsenin (1970) reported several applications of Hertz's contact theory on agricultural products. He reported the formulas to calculate the modulus of elasticity for a convex body of an agricultural material being tested under a steel flat plate or under a spherical indenter.

Hoki (1973) studied the mechanical strength and damage analysis of navy beans. In his study, Hoki used Hertz's contact theory given by Timoshenko and Goodier (1951) to determine the radius of the contact surface and the approach of two spheres when compressed together. He concluded that: a) using the contact theory to predict mechanical damage of navy beans showed promise; and b) it is appropriate to apply the contact theory to predict bean deformation under static loading for beans with low moisture content.

The solution of the viscoelastic counterpart of the Hertz problem in elasticity can be deduced from the elastic solution (Lee and Radok, 1960).

Föppl (1922) derived an equation to determine modulus of elasticity of a cylindrical specimen compressed in the direction parallel to the longitudinal axis. His assumption of the contact surface pressure differs from that used by Hertz. Föppl used a parabolic distribution while Hertz used the semi-ellipsoid distribution.

III. PRELIMINARY STUDIES

Due to the fact that a very limited amount of information was available on the texture evaluation of carrots, preliminary studies were conducted. The Texture Profile Analysis (TPA) parameters, modulus of elasticity of the two distinct areas of cross-sectional samples of carrots and Poisson's ratio were measured. The effect of storage time on these parameters was also investigated.

In looking at a cross-section of a cylindrical sample taken from a carrot, one can identify two areas, a center core, dark in color, and outer ring, light in color. In a cylindrical sample of one-inch in diameter, the center core is approximately one-half-inch in diameter.

Preliminary measurements of the modulus of elasticity for the center core and the outer ring were made using axial loading of cylindrical samples. The results showed that the modulus of elasticity, E_c for the center core is approximately twice the modulus of elasticity, E_o for the outer ring (Table 4).

Moisture content of the whole sample was measured for fresh carrots and for carrots stored in the Aminco chamber at 70°F and 50 percent relative humidity. Also, the moisture contents of the center core and the outer ring of samples, stored at 120°F and 50 percent relative humidity for 14 hours, were measured (Table 5). The results of the moisture content measurements showed no significant changes in the moisture content when measuring for the whole sample or the center core. But the moisture content changed significantly in the outer ring of the sample.

Table 4. Modulus of elasticity for the center core and the outer ring of six fresh carrot samples.

Sample No.	E_o psi	E_c psi
1	796	1475
2	847	1000
3	847	1750
4	593	1250
5	847	2000
6	847	1375
Ave.	796	1475

Table 5. Moisture content of fresh carrots and of carrots stored under different conditions and time periods.

Sample No.	M.C. % of fresh carrots	M.C. % of carrots stored at 70°F and 50% R.H. for 24 hours	M.C. % of center core and outer ring of carrots stored at 120°F and 50% R.H. for 14 hours	
			center core	outer ring
1	88.7	85.6	84.0	80.2
2	86.3	84.3	84.8	72.3
3	85.7	83.7	84.8	72.3
4	87.3	84.1	85.2	75.6
5	88.5	83.5	85.5	79.5
6	88.9	85.0	85.5	79.5
7	88.8	83.2	84.8	79.1
8	87.9	84.0	84.4	76.2
9	86.3	84.6	83.6	78.1
10	87.5	84.5	84.6	82.4
Ave.	87.6	84.5	84.7	77.5

A Texture Profile Analysis (TPA) parameters were determined by the method described by Bourne (1968a) from force-deformation curves produced upon twice compressing each sample 0.1-inch on the Instron Testing Machine.

Cylindrical samples of one-inch in diameter and 3/4-inch long, were compressed axially. Typical "first bit" and second bit" curves were as shown in Fig. 6.

The TPA parameters tested were:

1. Hardness
2. Cohesiveness
3. Gumminess
4. Brittleness

Although there was a noticeable change in the appearance of the carrots after seven hours of storage, the data showed no significant change in the values of the measured parameters, especially the hardness parameter. Hardness data for samples of fresh carrots and carrots stored at a temperature of 70°F and 60 percent relative humidity for 24 hours are given in Table 6.

It was concluded that axial loading of the cylindrical samples was not useful in detecting the change in the texture parameters such as hardness. The nonsignificant results in determining the TPA parameters is believed to be due to the difference in the values of modulus of elasticity of the center core and the outer ring of samples. It may also be due to the nonsignificant losses of moisture from the inside core compared to the outer ring. In deforming the samples axially, the center core dominates the reaction to the applied force at a given deformation, because it is stiffer and contains higher moisture than the outer ring, leaving the outer ring to follow the behavior of the

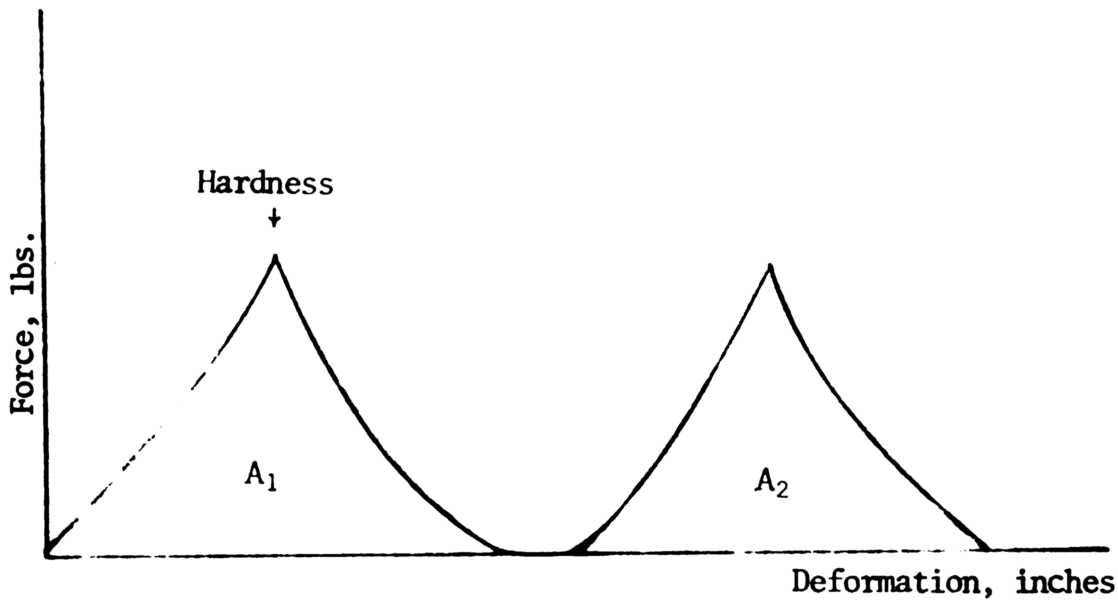


Fig. 6. Typical force-deformation curve for TPA parameters test.

Table 6. Hardness parameter measurements using the TPA method of axial deformation. The hardness is measured from force-deformation curves produced upon twice compressing cylindrical samples of carrots previously stored in perforated plastic bags at 32°F and 95% Relative Humidity for three weeks then removed from the bags prior to testing and stored in a room at 70°F and 60% Relative Humidity for 24 hours

Sample No.	Hardness, lbs. obtained at different storage times at room conditions of 70°F and 60% Relative Humidity					
	0 hours	4 hours	7 hours	16 hours	21 hours	24 hours
1	40	38	37	30	33	23
2	35	38	36	34	34	20
3	40	36	37	32	31	21
4	38	39	38	35	30	26
5	35	39	35	33	32	20

center core under the applied load. The outer ring from the previous experiment, appears to be controlling the decrease in the moisture content. It receives the first action when the carrots are bit or physically tested for consumer acceptance of fresh products.

This conclusion suggested that the samples be loaded in the radial (parallel to the longitudinal axis) direction between two flat steel plates and that the moisture content be measured in the outer portion of the cylindrical samples.

IV. THEORY

Mechanical properties have been defined as "those properties having to do with the behavior of the material under applied force".

Mechanical properties are widely used to study engineering materials or non-biological products. As of late, they have also been used to characterize agricultural products. The modulus of elasticity is one of the properties which is of considerable interest. The Hertz contact problem has been one approach to determining the elastic modulus in agricultural products.

Hertz's problem of contact stress, was originally developed to calculate the stresses resulting from the contact of common engineering materials. One result of the theory is an equation giving the modulus of elasticity, E, for a convex body compressed under a flat steel plate. This equation is

$$E = \frac{0.338\ k^{3/2}\ F\ (1 - \nu^2)}{\alpha^{3/2}} \left(\frac{1}{R} + \frac{1}{R'} \right)^{1/2} \qquad [1]$$

where

α = Total deformation of the body, in.

F = Compression force, lbs.

k = Constant taken from a table (A-1)

R = Minimum radii of curvature, in.

R' = Maximum radii of curvature, in.

The total deformation of the body along the axis of load at the point of contact is given by

$$\alpha = \frac{k}{2} \left[9F^2 \; K^2 \; \left(\frac{1}{R} + \frac{1}{R'}\right) \right]^{1/3} \qquad [2]$$

where

$$K = \frac{1 - \nu^2}{\Pi E}$$

The value of k depends on the principal curvatures of the bodies at the point of contact and the angle ϕ between the normal planes containing the principal curvatures. This value can be obtained from Table A-1 by first calculating cos T, for general case of two bodies, 1 and 2 pressed together, from

$$\cos T = \frac{\left[\left(\frac{1}{R_1} - \frac{1}{R_1'}\right)^2 + \left(\frac{1}{R_2} - \frac{1}{R_2'}\right)^2 + 2\left(\frac{1}{R_1} - \frac{1}{R_1'}\right)\left(\frac{1}{R_2} - \frac{1}{R_2'}\right) \cos 2\phi \right]^{1/2}}{\left(\frac{1}{R_1} + \frac{1}{R_1'} + \frac{1}{R_2} + \frac{1}{R_2'}\right)} \qquad [3]$$

where

R_1 and R_1' are minimum and maximum radii of curvature for body one, and

R_2 and R_2' for body two.

The preceeding equations are valid when the specimen is a sphere. When the specimen is a cylindrical (i.e. $R_1 = \infty$) cos T becomes one and the value of k is not defined.

To determine the elastic modulus for a cylindrical specimen compressed in the radial direction between two flat plates, an equation has to be derived based upon Hertz's contact theory.

4.1 Derivation of an equation to determine modulus of elasticity of a cylindrical specimen compressed radially.

Timoshenko and Goodier (1951) expressed the deformation at the surface of contact of two bodies as

$$\alpha = (K_1 + K_2)\iint \frac{q\, dA}{r'} + BY^2 + CX^2 \qquad [4]$$

In which B and C are constants depending on the magnitudes of the principal curvatures of the surfaces in contact and on the angle between the planes of principal curvatures of the two surfaces. If R_1 and R_1' denote the principal radii of curvature at the point of contact of one of the bodies, R_2 and R_2' those of the other body and Ψ the angle between the normal planes containing the curvatures $1/R_1$, and $1/R_2$, then the constants B and C are determined from the equations

$$B + C = \frac{1}{2}\left(\frac{1}{R_1} + \frac{1}{R_1'} + \frac{1}{R_2} + \frac{1}{R_2'}\right)$$

$$C - B = \frac{1}{2}\left[\left(\frac{1}{R_1} - \frac{1}{R_1'}\right)^2 + \left(\frac{1}{R_2} - \frac{1}{R_2'}\right)^2 + 2\left(\frac{1}{R_1} - \frac{1}{R_1'}\right)\left(\frac{1}{R_2} - \frac{1}{R_2'}\right)\cos 2\Psi\right]^{1/2}$$

where

α = total deformation of the cylinder (in.)

q = pressure at any point of load distribution (psi)

dA = element area on the area of contact (in^2)

r' = the distance from the center of the area of contact to the element area dA (in.)

$$K_1 = \frac{1 - \nu_1^2}{\Pi E}$$

$$K_2 = \frac{1 - \nu_2^2}{\Pi E}$$

ν = Poisson's Ratio

E = Modulus of Elasticity (psi)

In the case of contact of a cylinder with a plane surface, Figs. 7 and 8, B and C become

$$B + C = \frac{1}{2R_1}$$

$$C - B = \frac{1}{2R_1}$$

which means that $B = 0$ and $C = \frac{1}{2R_1}$.

Taking X to be equal to one-half the width of the contact area, b, at maximum deformation, equation [4] becomes

$$\alpha = (K_1 + K_2)\int\int \frac{q dA}{r'} + \frac{b^2}{2R} \qquad [5]$$

The problem now is to find a distribution of pressure to satisfy equation [5]. Hertz showed that this requirement is satisfied by assuming that the intensity of pressure q over the surface of contact is represented by the ordinates of a semi-ellipsoid constructed on the surface of contact as shown in Figs. 9 and 10.

$$q = \frac{q_o}{b}\sqrt{b^2 - x^2} \qquad [6]$$

The maximum pressure q_o is clearly along the center line of the surface of contact. The magnitude of the maximum pressure is obtained by summing forces in the y direction, yielding

$$P = 2\int_{-L/2}^{L/2}\int_o^b q dA = 2L\int_o^b q dx \qquad [7]$$

or

$$P' = \frac{P}{L} = \frac{2q_o}{b}\int_o^b \sqrt{b^2 - x^2}\, dx$$

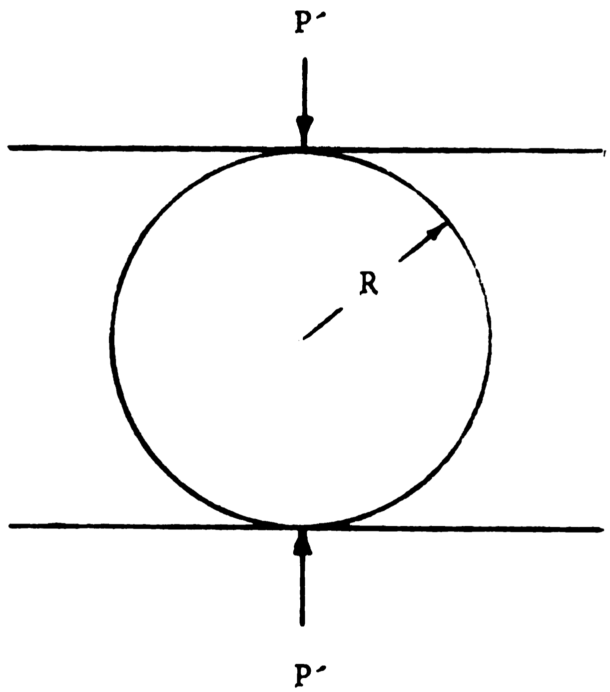


Fig. 7. Cylindrical sample before compression

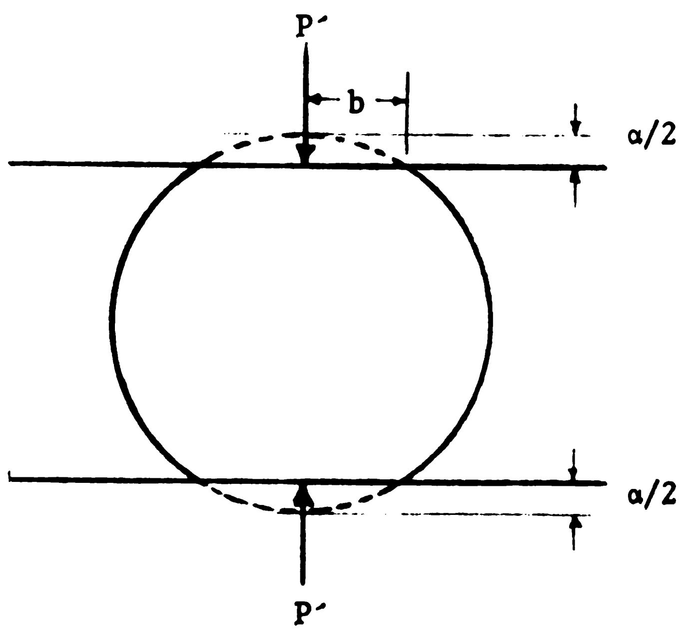


Fig. 8. Cylindrical sample after compression

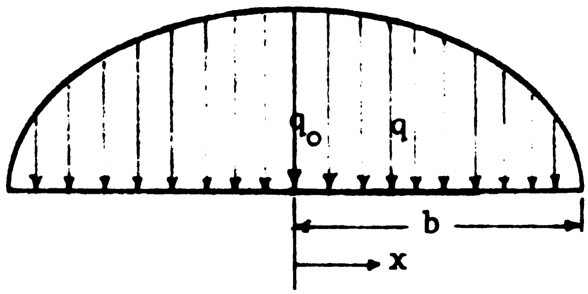


Fig. 9. Semi-ellipsoid pressure distribution

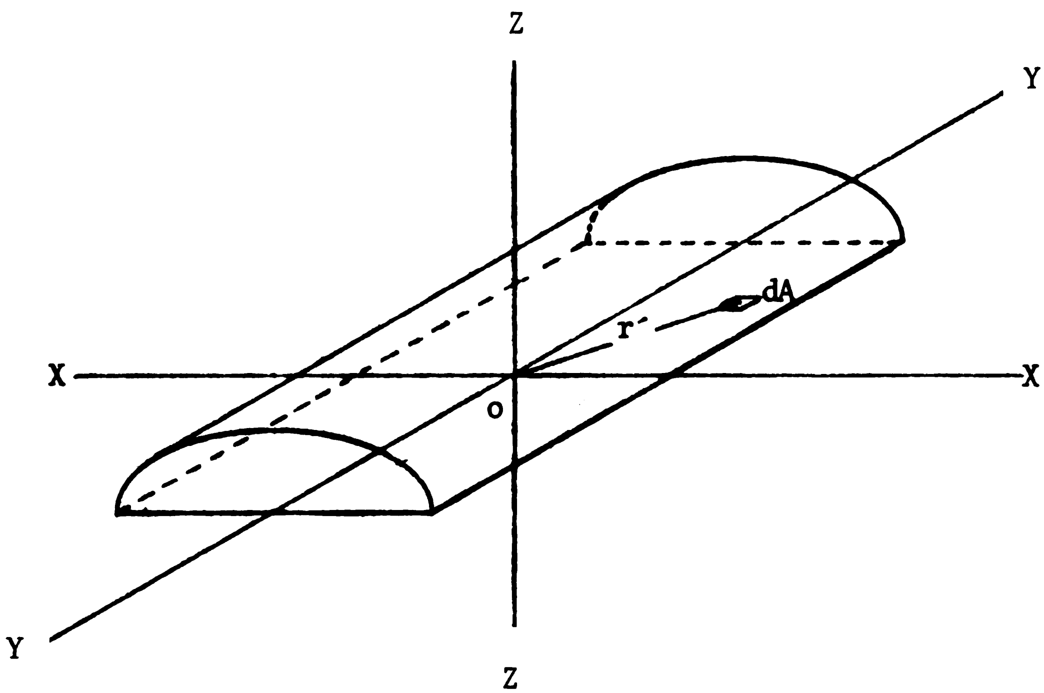


Fig. 10. Pressure distribution over the surface of contact

$$P' = \frac{q_o b\Pi}{2} \text{ and } q_o = \frac{2P'}{\Pi b} \qquad [8]$$

where P is total load and P′ is load per inch of length.

Timoshenko and Goodier give the expression

$$b = \sqrt{\frac{4P' (K_1 + K_2) R_1 R_2}{R_1 + R_2}} \qquad [9]$$

for the case of contact of two cylinders with parallel axes. R_1 and R_2 are the radii of the two cylinders.

When a cylinder contacts a flat surface, R_2 becomes ∞ and b becomes

$$b = \sqrt{4P' (K_1 + K_2) R_1}$$

If the cylinder is a biological material with small modulus of elasticity E_1, and the flat plate is made from steel, $E_2 = 30 \times 10^6$ psi, then $K_2 \to 0$ and b reduces to

$$b = \sqrt{4P' KR} \qquad [10]$$

where R is the cylinder radius.

Equation [5] now becomes

$$\alpha = K \iint \frac{qdA}{r'} + \frac{b^2}{2R} \qquad [11]$$

Utilizing equation [11] and the assumption of semi-elliptic pressure distribution for the case where the cylinder is loaded with a flat steel plate, (Fig. 11), an equation for the elastic modulus, E, of the material can be derived as outlined below.

Starting with [11] and substituting [6] gives

$$\alpha - \frac{b^2}{2R} = 4K \int_o^{L/2} \int_o^b \frac{q_o}{br'} \sqrt{b^2 - x^2}\, dx\, dy$$

where the integral is over one-fourth of the contact area. Two changes are now in order if evaluation of the integral is to be simplified. The first is a change in variables by letting

$$X = \frac{x}{b}, \quad Y = \frac{y}{b}, \quad \text{and } r = \frac{r'}{b}.$$

This change produces

$$\alpha - \frac{b^2}{2R} = \frac{4Kq_o}{b} \int_0^M \int_0^1 \frac{\sqrt{b^2 - b^2X^2}}{br} \, b^2 dXdY$$

or

$$\alpha - \frac{b^2}{2R} = 4Kq_o b \int_0^M \int_0^1 \frac{\sqrt{1 - X^2}}{r} \, dXdY$$

where

$$M = \frac{L}{2b}$$

The second involves dividing the contact area (Fig. 11) into two triangles with θ and ϕ defined as shown in Figure 12, and then chaning the equation into the Polor Coordinates. This transformation yields

$$\frac{1}{4Kq_o b}\left(\alpha - \frac{b^2}{2R}\right) = \int_0^{\tan^{-1} M} \int_0^{\sec\theta} \frac{\sqrt{1 - r^2\cos^2\theta}\, r\, dr\, d\theta}{r} + \int_0^{\tan^{-1} 1/M} \int_0^{M\sec\phi} \frac{\sqrt{1 - r^2\sin^2\phi}\, r\, dr\, d\phi}{r}$$

or

$$\frac{1}{4Kq_o b}\left(\alpha - \frac{b^2}{2R}\right) = I_1 + I_2.$$

Considering the individual components I_1 and I_2 gives

$$I_1 = \int_0^{\tan^{-1} M} \int_0^{\sec\theta} \frac{\sqrt{1 - r^2\cos^2\theta}}{r} dr\, d\theta$$

$$= \int_0^{\tan^{-1} M} \int_0^{1} \sqrt{1 - X^2}\, \frac{dX}{\cos\theta}\, d\theta, \quad X = r\cos\theta$$

$$= \int_0^{\tan^{-1} M} \frac{\Pi}{4} \sec\theta\, d\theta$$

$$= \frac{\Pi}{4} \log_e (M + \sqrt{1 + M^2})$$

and

$$I_2 = \int_0^{\tan^{-1} 1/M} \int_0^{M\sec\phi} \frac{\sqrt{1 - r^2\sin^2\phi}}{r}\, r\, dr\, d\phi$$

$$= \int_0^{\tan^{-1} 1/M} \int_0^{M\tan\phi} \sqrt{1 - Y^2}\, \frac{dY}{\sin\phi}\, d\phi, \quad Y = r\sin\phi$$

$$= \int_0^{\tan^{-1} 1/M} \left[\frac{1}{2}\sin^{-1} Y + \frac{1}{2} Y \sqrt{1 - Y^2}\right]_0^{M\tan\phi} \csc\phi\, d\phi$$

$$= \frac{1}{2}\int_0^{\tan^{-1} 1/M} [\sin^{-1} (M\tan\phi) + M\tan\phi \sqrt{1 - M^2\tan^2\phi}]\csc\phi\, d\phi$$

Substituting $\sin v = M\tan\phi$, $\sin\phi = \dfrac{\sin v}{\sqrt{M^2 + \sin^2 v}}$,

$$d\phi = \frac{M\cos v\, dv}{M^2 + \sin^2 v}$$

yields

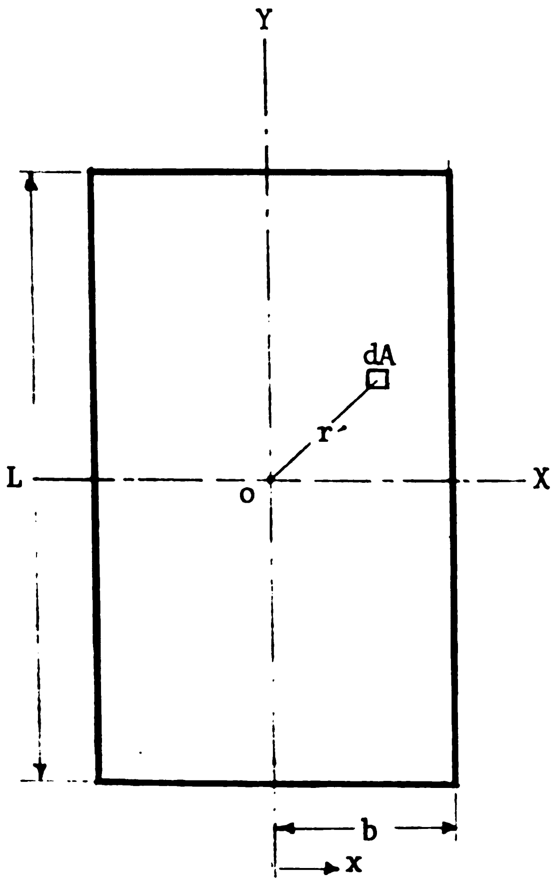


Fig. 11. Area of contact

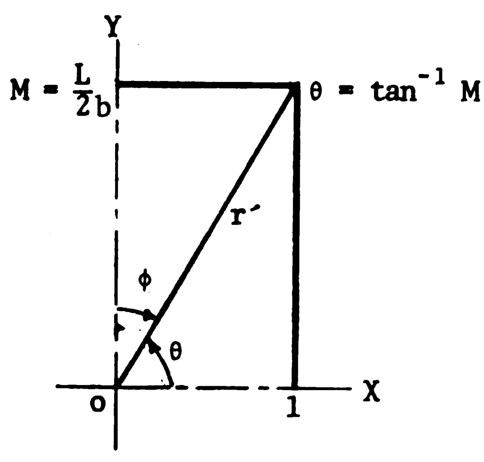


Fig. 12. One-fourth the area of contact divided into two triangles

$$I_2 = \frac{1}{2} \int_0^{\Pi/2} (v + \sin v \cos v) \frac{\cot v \, dv}{\sqrt{1 + (\sin^2 v) / M^2}}$$

since

$$M > 2, \ (\sin v) / M < \frac{1}{2},$$

we can expand

$$(1 + \frac{\sin^2 v}{M^2})^{-1/2}$$

by the binomial theorem as

$$(1 + \frac{\sin^2 v}{M^2})^{-1/2} = [1 - \frac{1}{2} \ \frac{\sin^2 v}{M^2} + \frac{1}{2} \cdot \frac{3}{4} \frac{\sin^4 v}{M^4} \ \ldots\ldots]$$

Then I_2 may be written as

$$I_2 = \frac{1}{2} \int_0^{\Pi/2} (v \cot v + \cos^2 v) \ (1 - \frac{1}{2} \frac{\sin^2 v}{M^2} + \frac{1}{2} \cdot \frac{3}{4} \frac{\sin^4 v}{M^4} \ldots) \ dv$$

Integration by parts gives

$$\int_0^{\Pi/2} v \cot v \, dv = - \int_0^{\Pi/2} \log_e \ \sin v \, dv = \frac{\Pi}{2} \log_e 2$$

and

$$\int_0^{\Pi/2} v \cot v \sin^{2n} v \, dv = \int_0^{\Pi/2} v \sin^{2n-1} v \cos v \, dv$$

$$= [\frac{v \sin^{2n} v}{2 n}]_0^{\Pi/2} - \frac{1}{2n} \int_0^{\Pi/2} \sin^{2n} v \, dv$$

$$= \frac{\Pi}{4n} \ [\ 1 - \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n - 1)}{2 \cdot 4 \cdot 6 \cdots 2n}]$$

From that I_2 becomes

$$I_2 = \frac{\Pi}{4} \left[(\log_e 2 + \frac{1}{2}) - \frac{1}{2} (\frac{1}{2} - \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{4}) \frac{1}{M^2} + \right.$$

$$\frac{1}{2} \frac{3}{4} (\frac{1}{4} - \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{3}{4} + \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{3}{6}) \frac{1}{M^4} +$$

$$\left. \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} (\frac{1}{6} - \frac{1}{6} \cdot \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} + \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{3}{6} \cdot \frac{5}{8}) \frac{1}{M^6} \right]$$

$$I_2 = \frac{\Pi}{4} [\log_e 2 + \frac{1}{2} - \frac{3}{16\ M^2} + \frac{21}{256\ M^4} - \frac{295}{6144\ M^6}]$$

$$I_2 = \frac{\Pi}{4} [\log_e 2 + \frac{1}{2} - \frac{3}{16\ M^2} (1 - \frac{7}{16\ M^2} + \frac{98}{384\ M^4})]$$

Since

$$\frac{1}{1 + X} = 1 - X + X^2 \ .....$$

$$(1 - \frac{7}{16\ M^2} + \frac{98}{394\ M^4}) \quad \text{is approximately} \quad \frac{1}{1 + \frac{7}{16\ M^2}} .$$

and I_2 becomes

$$I_2 = \frac{\Pi}{4} [\log_e 2 + \frac{1}{2} - \frac{3}{16\ M^2} (\frac{1}{1 + \frac{7}{16\ M^2}})$$

$$= \frac{\Pi}{4} [\log_e 2 + \frac{1}{2} - \frac{3}{16\ M^2 + 7}]$$

Then

$$\frac{1}{4Kq_o b}\left(\alpha - \frac{b^2}{2R}\right) = \frac{\Pi}{4}\left[\log_e \left(M + \sqrt{1 + M^2}\right) + \log_e 2 + \frac{1}{2} - \frac{3}{16M^2 + 7}\right]$$

$$= \frac{\Pi}{4}\left[\log_e 2\left(M + \sqrt{1 + M^2}\right) + \frac{1}{2} - \frac{3}{16\,M^2 + 7}\right]$$

But

$$q_o = \frac{2P'}{\Pi b}$$

$$b = \sqrt{4\,P'KR}\ , \quad K = \frac{b^2}{4P'R}$$

$$M = \frac{L}{2b}$$

Therefore

$$\alpha - \frac{b^2}{2R} = \Pi K q_o b\left[\log_e \left(M + \sqrt{1 + M^2}\right) + \frac{1}{2} - \frac{3}{16M^2 + 7}\right]$$

$$= \frac{b^2}{2R}\left[\log_e 2\left(M + \sqrt{1 + M^2}\right) + \frac{1}{2} - \frac{3}{16M^2 + 7}\right.$$

$$\alpha = \frac{b^2}{2R}\left[\log_e 2\left(M + \sqrt{1 + M^2}\right) + \frac{3}{2} - \frac{3}{16M^2 + 7}\right]$$

Letting $W = \frac{L}{b} = 2M$ yields the following equation

$$\frac{\alpha R}{L^2} = \frac{1}{2\,W^2}\left[\log_e \left(W + \sqrt{4 + W^2}\right) + \frac{3}{2} - \frac{3}{4\,W^2 + 7}\right] \qquad [12]$$

Table A-2 was prepared from equation [12] for values of W at different values of $\frac{\alpha R}{L^2}$. Once W is know, E can be calculated as follows

$$W = \frac{L}{b} = \frac{L}{\sqrt{4P'KR}}$$

$$K = \frac{L^2}{4P'W^2R}$$

$$\frac{1 - \nu^2}{\pi E} = \frac{L^2}{4\ P'W^2R}$$

$$E = \frac{4\ (1 - \nu^2)\ P'W^2R}{\pi L^2} \qquad [13]$$

4.2 Föppl's Equation

An equation which gives the modulus of elasticity for a cylinder compressed by a flat plate was found after [13] had been derived. This equation was developed by Föppl (1922). There is a major difference in the assumptions used by Föppl when compared to the work done by Hertz. Föppl used the second power of the semi-elliptic function for the pressure distirbution. His equation for q was

$$q = \frac{q_o}{b^2}\ (b^2 - x^2) \qquad [14]$$

which resulted in the following relationships

$$\alpha = 4P'\ (\frac{1 - \nu^2}{\pi E})\ (1/3 + \log_e \frac{2R}{b}) \qquad [15]$$

$$E = \frac{8\ (1 - \nu^2)\ \ P'\ Z^2}{\Pi\ D} \qquad [16]$$

where Z = R/b and values are given in Table (A-3).

4.3 Relaxation test

Stress relaxation experiments have been conducted on several agricultural products. In this rheological test, the specimen is suddenly brought to a given deformation (strain), and the stress required to hold the deformation constant is measured as a function of time. Since the deformation of the product under load is held constant, it is usually assumed that the loaded area of contact remains constant during the relaxation test and the recorded force-time is representative of the stress-time curve.

The generalized Maxwell model can be used to represent force or stress relaxation. A generalized Maxwell model is composed of n Maxwell elements with a spring in parallel with the nth element as illustrated in Fig. 13. When the model is subjected to a constant strain ε_o at time = 0, the stress can be represented by

$$\sigma\ (t) = \varepsilon_o\ (E_1 e^{-\alpha_1 t} + E_2 e^{-\alpha_2 t} \ldots + E_n e^{-\alpha_n t}) \qquad [17]$$

where $\alpha_n = E_n/\eta_n$, E is stiffness or modulus of the spring, η is viscosity coefficient of the liquid in the dashpot. Then the equation representing the portion of the curve where $t > t_1$, (Fig. 14) can be written as

$$F(t) = \sum_{i=1}^{n} C_1 \cdot e^{-\alpha_1 t'} \qquad [18]$$

where $C_1 = \varepsilon_o E_1$

$$\varepsilon_o = 3/2\, a^{3/2}\, K \int_o^{t_1} e^{-\alpha_1 (t_1 - \tau)} \, {}^{1/2}_{\tau}\, d\tau$$

$$t' = t - t_1$$

a = rate of deformation in./min.

Factor K is being a function of the geometric parameters of the specimen.

The approximate values of C_1 and α_1 can be determined by the numerical methods which used a computer program developed by Chen (1971).

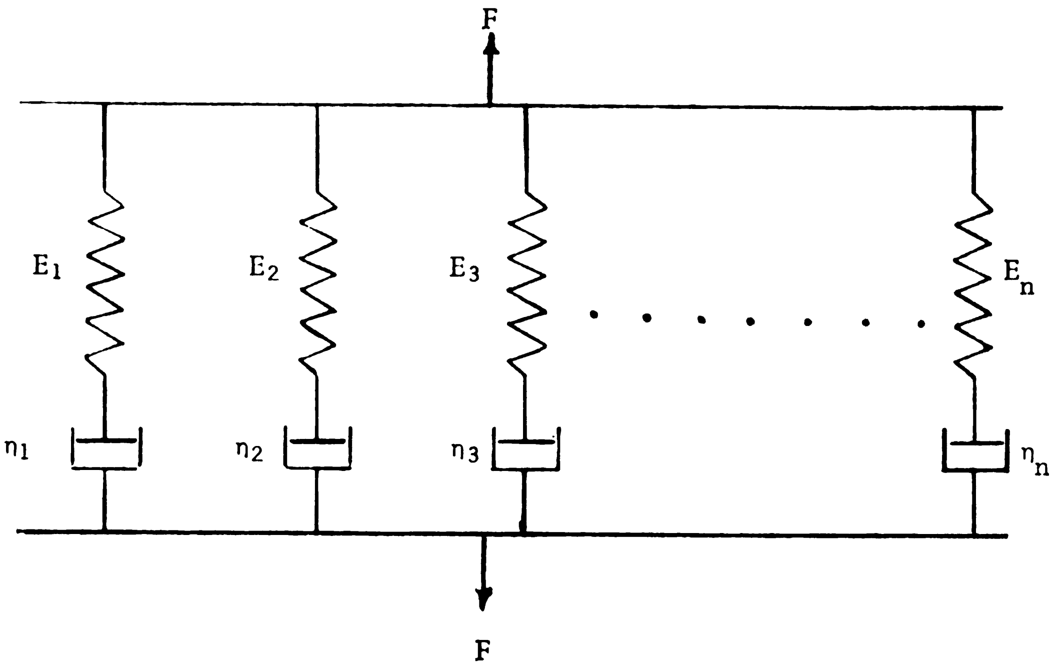


Fig. 13. Generalized Maxwell model

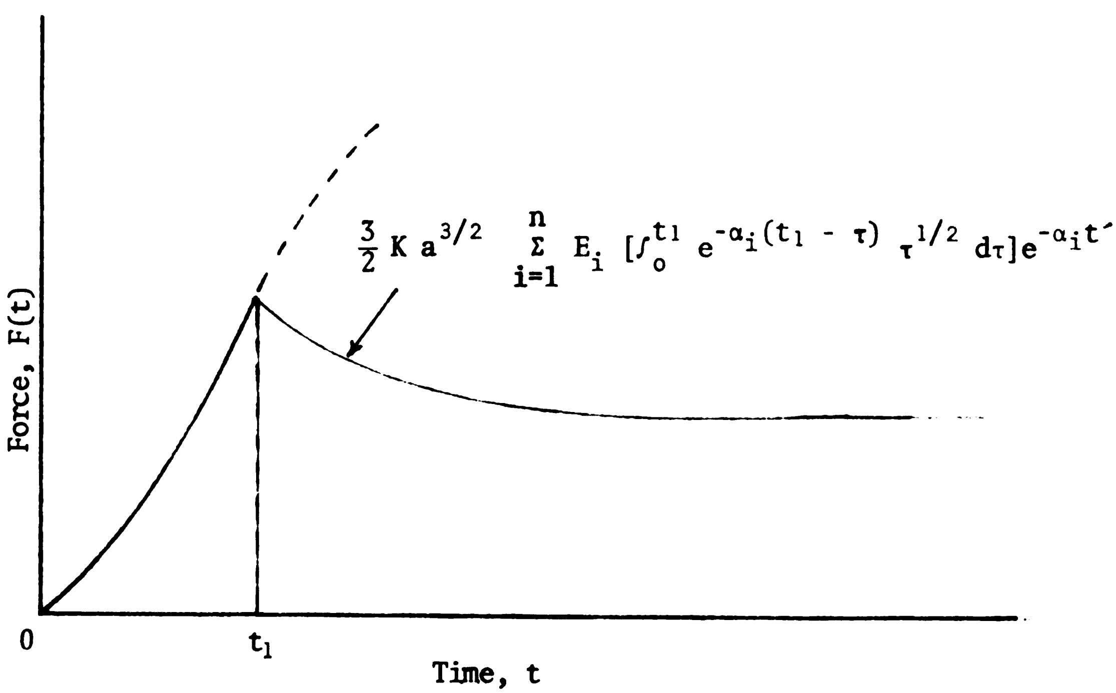


Fig. 14. Graphical representation of equation [18]

V. EXPERIMENTAL INVESTIGATION

5.1 Equipment

To obtain one-inch diameter samples, one-inch long, a corer (A), two inches long, was constructed with an internal diameter of one-inch and mounted on a base (B), (Fig. 15). A holding jug (C), one-inch long with internal diameter of one inch, and two-bladed knife (D), (Fig. 16), were used to trim the samples to a proper length of one inch for the radial and Poisson's ratio tests.

To measure the restrained modulus needed to determine Poisson's ratio, ν, a cylindrical die (E) with a one-inch inside diameter mounted on a base (B) with spacer (F) of outside diameter to exactly fit inside die (E), (Fig. 17), were used.

Another corer (G), (Fig. 18), with an outside diameter of 7/8-inch was used to separate a 1/8-inch thick outside ring from the one-inch diameter sample. This ring was removed after the radial test, for the purpose of measuring the sample moisture content.

An Aminco-Air unit was used to maintain a constant temperature and relative humidity in the chamber where the carrots were stored, (Fig. 19).

An Instron Universal Testing Machine, table model TM, 200 kilograms capacity, with standard crosshead speeds of 0.2 to 50 in/min. was used to load the samples, (Fig. 20).

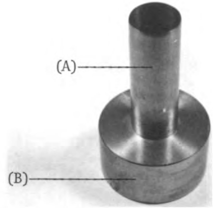


Fig. 15. Sampler, (A) is Corer and (B) is Base.

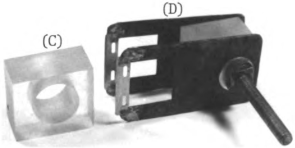


Fig. 16. Trimmer, (C) is holding jug (D) is two-bladed knife.

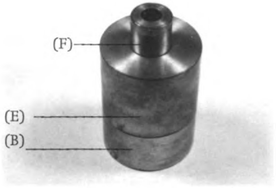


Fig. 17. Die to be used in measuring the Poisson's Ratio, (B) is base, (E) is sample holding die, and (F) is spacer.

Fig. 18. Corer with 7/8" outside diameter to be used to separate 1/8" thick outer ring from the 1" diameter sample.

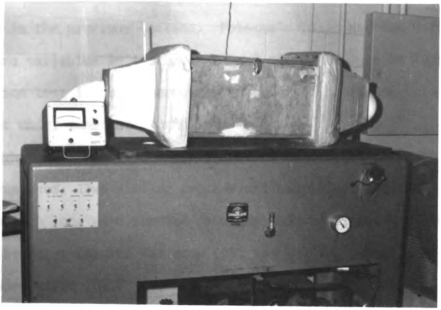


Fig. 19. Aminco unit with chamber to store carrots at given temperature and relative humidity.

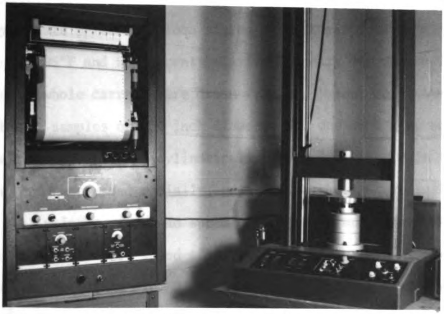


Fig. 20. Instron Universal Machine.

5.2 Experimental procedure

The modulus of elasticity is to be calculated using the equation derived in the previous section. Poisson's ratio and the deformation force are variables in the equation and therefore must be measured. The relaxation test is to be performed in order to study the relaxation behavior under constant strain.

Hughes and Segerlind's (1972) method of measuring the Poisson's ratio was used. Two similar cylindrical samples of one-inch in diameter and one-inch long were removed from the individual carrot (length-wise) using the corer (A) and the base (B), the holding jug (C), and the two-bladed knife (D).

One sample was axially compressed in the Instron Machine, (Fig. 21) for the unrestrained test. The other sample was placed in the die (E) as shown in Fig. 22 and then compressed. The two force-deformation curves were obtained (Fig. 23) and used to determine Poisson's ratio.

Carrots grown in the State of Arizona and purchased through the Michigan State University food stores were stored in the Aminco unit chamber at 85°F and 50 percent Relative Humidity for one to 72 hours. Samples of whole carrots were drawn from the Aminco unit chamber and cylindrical samples of one-inch diameter and one-inch long were prepared as described above. The cylindrical samples were placed in the Instron Machine and compressed radially, parallel to the longitudinal axis, (Fig. 24), using a crosshead speed of 5 in./min and a maximum deformation of 0.1 inches at chart speed of 50 in./min. A force-displacement curve similar to the one in Fig. 25 was obtained. The experiment was conducted using fresh carrot samples and samples drawn from the Aminco chamber at time intervals of 4, 6, and 8 hours over a period of 72 hours. Moisture

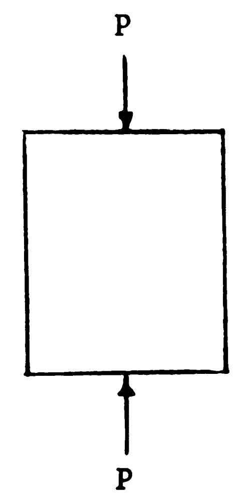


Fig. 21. Loading for the unrestrained test.

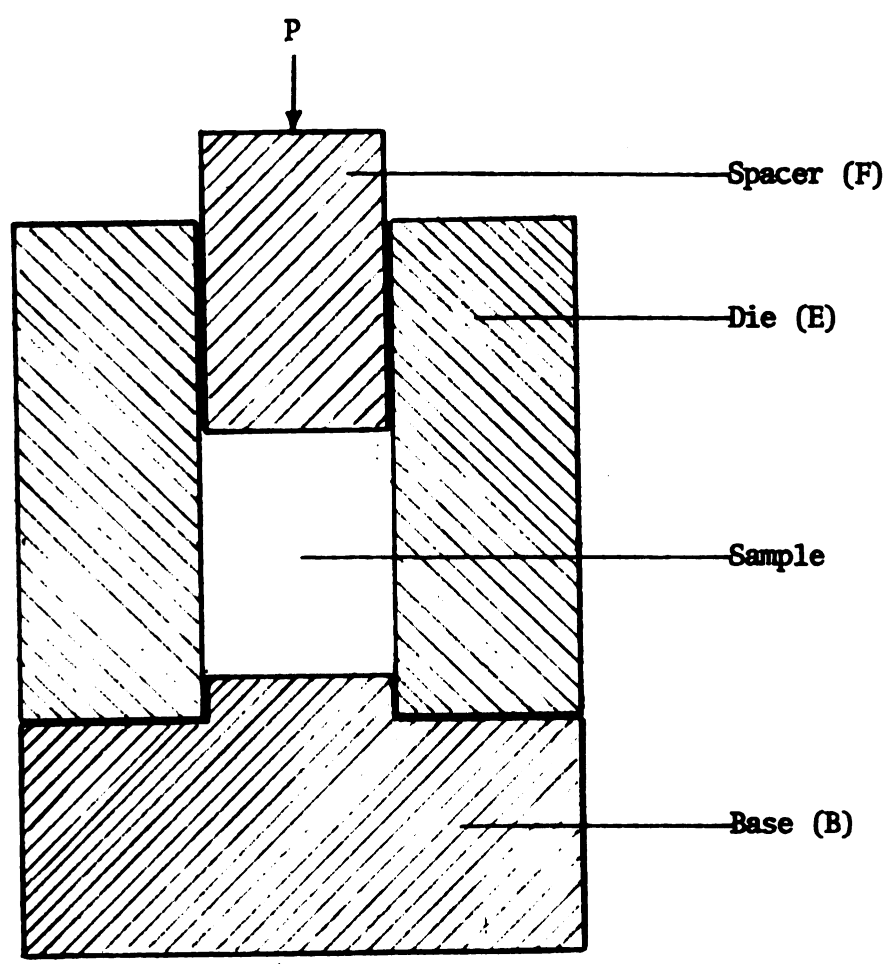


Fig. 22. Loading for the restrain test.

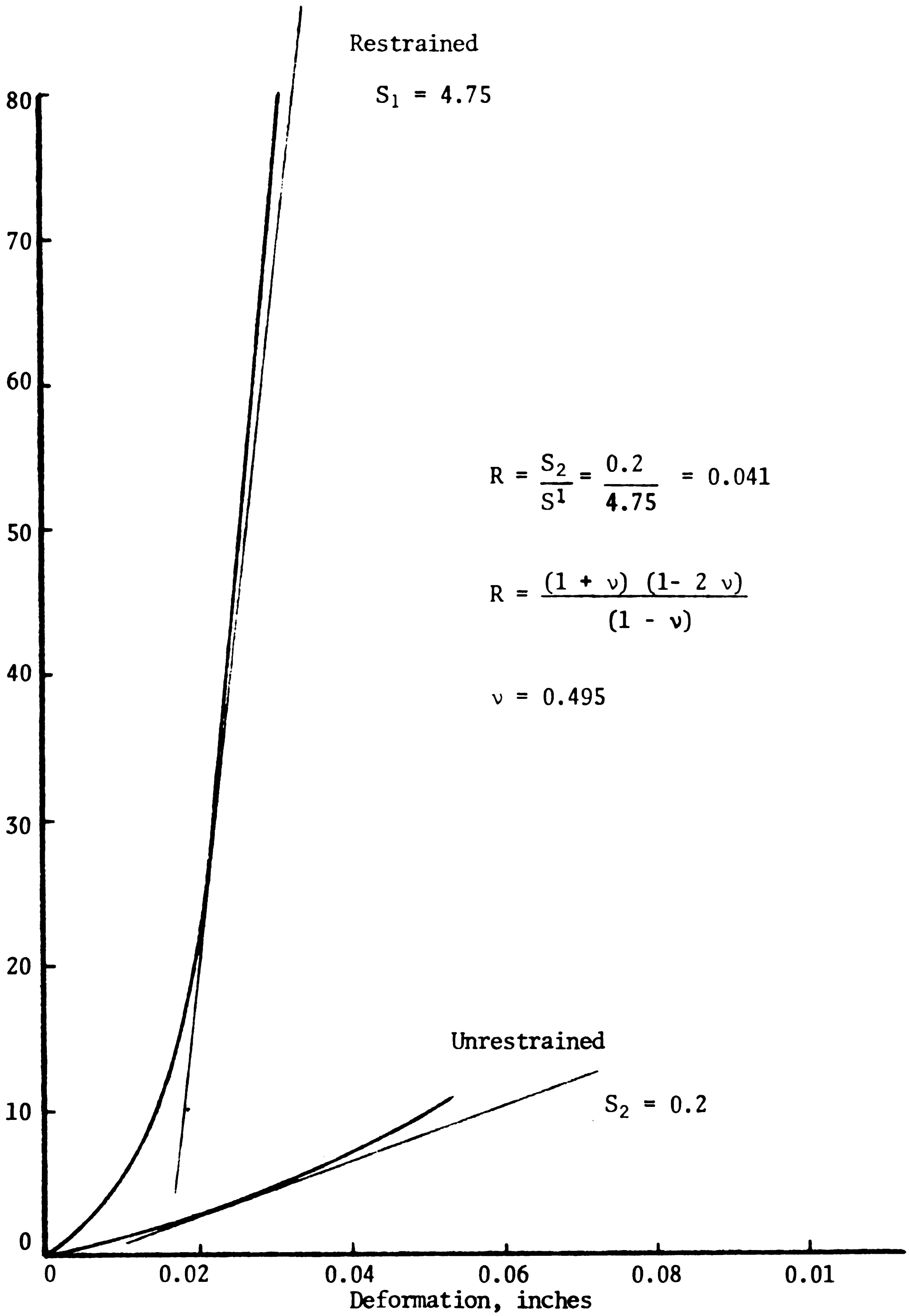


Fig. 23. Force-deformation curves for a cylindrical sample of carrots with one-inch in diameter and one-inch long.

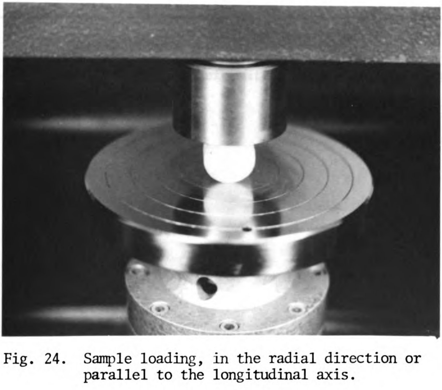


Fig. 24. Sample loading, in the radial direction or parallel to the longitudinal axis.

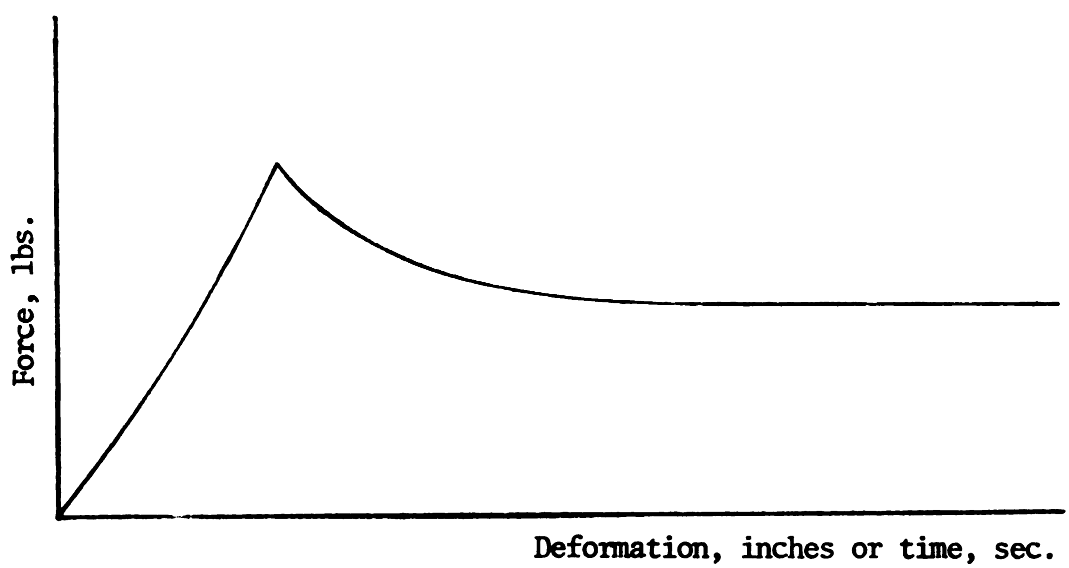


Fig. 25. Typical curve for deformation and relaxation tests.

content was determined by removing a 1/8-inch thick outside ring cut from the cylindrical samples and dried in an oven at 70°C for 20 hours.

Two sets of data were obtained, 138 samples in Set #1 (Table A-4) and 70 samples in Set #2 (Table A-5). The moisture content of the 1/8-inch thick outer ring was determined for each sample. The samples were divided according to the peak compression force into 11 groups and the average force and moisture content of the outer ring of each group were obtained (Set #1, Table A-6; Set #2, Table A-7). By grouping the data according to force, it was possible to select force-deformation curves with a peak force equal to the group average force and then use the selected curves to obtain the relation between deformation force, modulus of elasticity and moisture content.

The relaxation behavior was studied by maintaining a constant deformation (strain) of 0.1-inch on the sample for 30 seconds. A crosshead speed of 5 in./min and chart speed of 50 in./min were used. Twelve points on each curve were used to determine the coefficients and exponential of the generalized Maxwell model.

VI. RESULTS AND DISCUSSION

6.1 Accuracy of the derived equation

The equation derived for determining the modulus of elasticity was verified experimentally. The method of examining was a comparison between the modulus of elasticity values for a homogeneous and isotropic agricultural product as measured by applying an axial load and a radial load on separate cylindrical specimens. Homogeneous and isotropic materials should have the same modulus of elasticity value in both axial and radial directions. The potato was used for the specimens since it represents a homogeneous and an isotropic agricultural product. Specimens with various combinations of 0.5-inch and 1.0-inch diameters and lengths were prepared and subjected to a deformation in the axial or radial direction, of 0.025 inches, 0.05 inches or 0.1 inch. The Instron Testing Machine was used to deform the cylindrical samples. A cross-head speed of 5 in/min and a chart speed of 50 in/min were used. Two specimens from the same location of the potato were prepared. The modulus of elasticity was calculated using the axial equation, the derived equation, and Föppl's existing equation.

Assuming Poisson's Ratio, ν, for potatoes to be 0.5, the derived equation [13] and Föppl's existing equation [16] become:

Derived: $$E_1 = \frac{4(1 - 0.25)}{\pi L^2} P'W^2R = 0.955 \frac{P'W^2R}{L^2} \qquad [19]$$

Föppl's: $E_2 = \frac{8(1 - 0.25)}{\pi D} P'Z^2 = 1.91 \frac{P'Z^2}{D}$ [20]

The axial loading equation to determine the modulus of elasticity is:

$$E_3 = \frac{F}{A} \frac{L}{\alpha}$$ [21]

A summary of the calculated values of E for different specimen sizes and deformations is presented in Table 7. The averaged values of E were 415 psi, 496 psi, and 419 psi as calculated by the derived equation, Föppl's equation and the standard axial equation respectively.

The mean, variance, and standard deviation were calculated (Table A-8) and an F statistical test was used to determine whether the population variances are equal or not. The null hypothesis $\sigma_1^2 = \sigma_2^2$ was tested at the 0.05 level. The results showed that there is insufficient evidence to indicate a difference in the population variances. The conclusion is that $\sigma_1^2 = \sigma_2^2 = \sigma_3^2$ is not rejected.

6.2 Poisson's ratio

The Poisson's ratio for fresh carrots was found to be approximately 0.5. The value for carrots stored in a room temperature of 70°F and relative humidity of 60 percent for a time period of 24 hours was found to be essentially the same, 0.50. Twenty-one samples of carrots were studied in each case. The average values were 0.492 for fresh carrots and 0.496 for stored carrots (Table 8). The derived equation [13] reduces to [19] for carrots since Poisson's ratio is 0.5.

Table 7. Summary of calculated values of E for different potato specimen sizes and deformation using:

$E_1 = 0.955 \frac{P'W^2R}{L^2}$, $E_2 = 1.91 \frac{P'Z^2}{D}$, $E_3 = \frac{F}{A} \frac{L}{\alpha}$ where A is cross sectional area.

α inch	D inch	L inch	F lb	P lb/in	$\frac{\alpha R}{L^2}$	α/D	W	Z	E_1 psi	E_2 psi	E_3 psi
0.1	1.0	1.0	34.7	19.3	0.05	0.1	6.36	3.34	373	411	441
0.1	1.0	0.5	68.1	21.9	.20	0.1	2.86	3.34	343	467	434
0.05	1.0	1.0	15.8	9.1	0.025	0.05	9.42	5.17	385	464	405
0.05	1.0	0.5	36.4	11.2	0.10	0.05	4.28	5.17	392	572	463
0.05	0.5	1.0	4.6	11.7	0.0125	0.10	13.89	3.34	540	500	464
0.05	0.5	0.5	8.5	11.0	0.050	0.10	6.36	3.34	423	467	434
0.025	1.0	1.0	5.5	4.6	0.0125	0.025	13.89	7.85	420	536	278
0.025	1.0	0.5	13.8	4.5	0.050	0.025	6.36	7.85	345	526	350
0.025	0.5	1.0	2.8	4.8	0.00625	0.050	20.40	5.17	473	486	577
0.025	0.5	0.5	3.2	5.3	0.025	0.050	9.42	5.17	452	544	324
								mean	415	496	419

Table 8. Poisson's ratio, ν, measurements for fresh carrots and for carrots stored in a room at 70°F and 60% relative humidity for 24 hours.

Sample No.	Fresh carrots	24 hours stored carrots
1	0.493	0.496
2	0.493	0.497
3	0.495	0.496
4	0.492	0.497
5	0.494	0.498
6	0.493	0.496
7	0.494	0.497
8	0.492	0.498
9	0.493	0.494
10	0.489	0.494
11	0.493	0.498
12	0.488	0.495
13	0.483	0.495
14	0.494	0.498
15	0.494	0.497
16	0.485	0.497
17	0.490	0.497
18	0.490	0.495
19	0.486	0.495
20	0.497	0.493
21	0.495	0.496
Ave.	0.492	0.496

6.3 Deformation Test

The relationship between the peak force needed to deform the sample 0.1 inch in the radial direction and the moisture content in the outer ring (percent wet basis) was plotted from the results summarized in Table 9. A best fit-curve was drawn using a statistical sub-routine (Fig. 26). A second degree polynomial curve was found to be the best fit of the plotted data. This curve showed a significant relationship between the peak force and the moisture content. As the moisture content decreased, the force needed to deform the sample, also decreased.

A correlation coefficient of 0.937 was obtained for the curve.

The modulus of elasticity, E, was calculated using the derived equation, [19], and the results are plotted as related to the moisture content (Fig. 27). The second degree polynomial curve indicated that there is a significant relationship between the modulus of elasticity and the moisture content. As the moisture content decreased, the modulus of elasticity decreased. The correlation coefficient was 0.937. The modulus of elasticity at 85 percent moisture content was 725 psi which is almost ten times greater than the value of 70 psi at 76 percent moisture content. The sharpest drop in the modulus of elasticity occurs during the first five percent drop of the moisture content below that of fresh carrots. After this, the rate of decreases changes less rapidly.

The decrease in the modulus of elasticity is believed to be a measure of the hardness parameter of carrots. As the carrots lose moisture from the outer surface of the sample, it becomes less rigid causing the deformation force to drop.

The results of the deformation indicate that a slight drop in the moisture content of the outer ring creates a sharp drop in the hardness of carrots. Earlier results had shown no difference in the hardness of carrots as related to storage time when the measurements were in the axial direction of cylindrical samples.

6.4 Relaxation tests

The computer program by Chen (1971) was used to calculate the coefficients C_i and the exponentials α_i using a three element (i=3) generalized Maxwell model [18].

Table 9. Values of moisture content, modulus of elasticity as calculated from the derived equation, peak force as the selected experimental curves indicated, and the relaxation coefficients and exponential as calculated from the assumed generalized Maxwell model.

M.C.%	E lbs/in^2	F lbs	C_1	C_2	C_3	α_1	α_2	α_3
86.6	869	45.0	4.80	4.77	35.17	3.83	0.432	0.0030
84.7	684	35.4	3.34	3.71	28.38	4.54	0.475	0.0038
83.3	657	34.0	3.20	3.42	27.08	3.70	0.409	0.0028
83.0	527	27.3	2.95	3.13	21.05	3.23	0.309	0.0025
82.2	464	24.0	2.52	2.82	18.48	2.69	0.370	0.0039
81.5	317	16.4	2.30	2.10	11.90	3.85	0.435	0.0045
79.8	241	12.5	2.02	1.67	8.71	3.90	0.383	0.0037
79.3	164	8.5	1.32	1.43	5.58	3.91	0.465	0.0070
77.7	112	5.8	0.99	1.00	3.80	3.85	0.473	0.0050
75.1	54	2.8	0.55	0.55	1.65	5.63	0.587	0.0086
72.5	27	1.4	0.28	0.27	0.84	3.67	0.571	0.0088

The coefficients C_1, C_2, C_3 and the exponentials α_1, α_2, α_3, were determined by taking points from the force-time curve. Table 9 shows the values of C_1 and α_1 at various moisture contents.

Three relationships between C_1, C_2, and C_3 and the moisture content were plotted and the second degree polynomial curves were the best fit for the plotted data. These relationships are shown in Figures 28, 29 and 30 for C_1, C_2, and C_3, respectively. Each curve shows a significant relationship between C_1, C_2, and C_3 and the moisture content with correlation coefficients of 0.963, 0.965, and 0.927, respectively. The relaxation coefficients C_1, C_2, or C_3 decreased as the moisture content decreased.

The relaxation coefficients C_1, C_2, and C_3 and the exponentials α_1, α_2, and α_3 were used to calculate the force-time experimental curves (Table A-9). The experimental and calculated curves of the generalized Maxwell model, with three elements, are shown in Fig. 31, at sample moisture contents of 86.6 percent, 83.3 percent, 79.8 percent, and 77.7 percent.

The calculated relaxation curves are identical with the experimental curves, indicating that the force relaxation curve of a carrot could be represented by equation [18].

Table 9 shows that there is no significant relationship between the exponential α_1 and the moisture content since the data fluctuated with no behavior pattern.

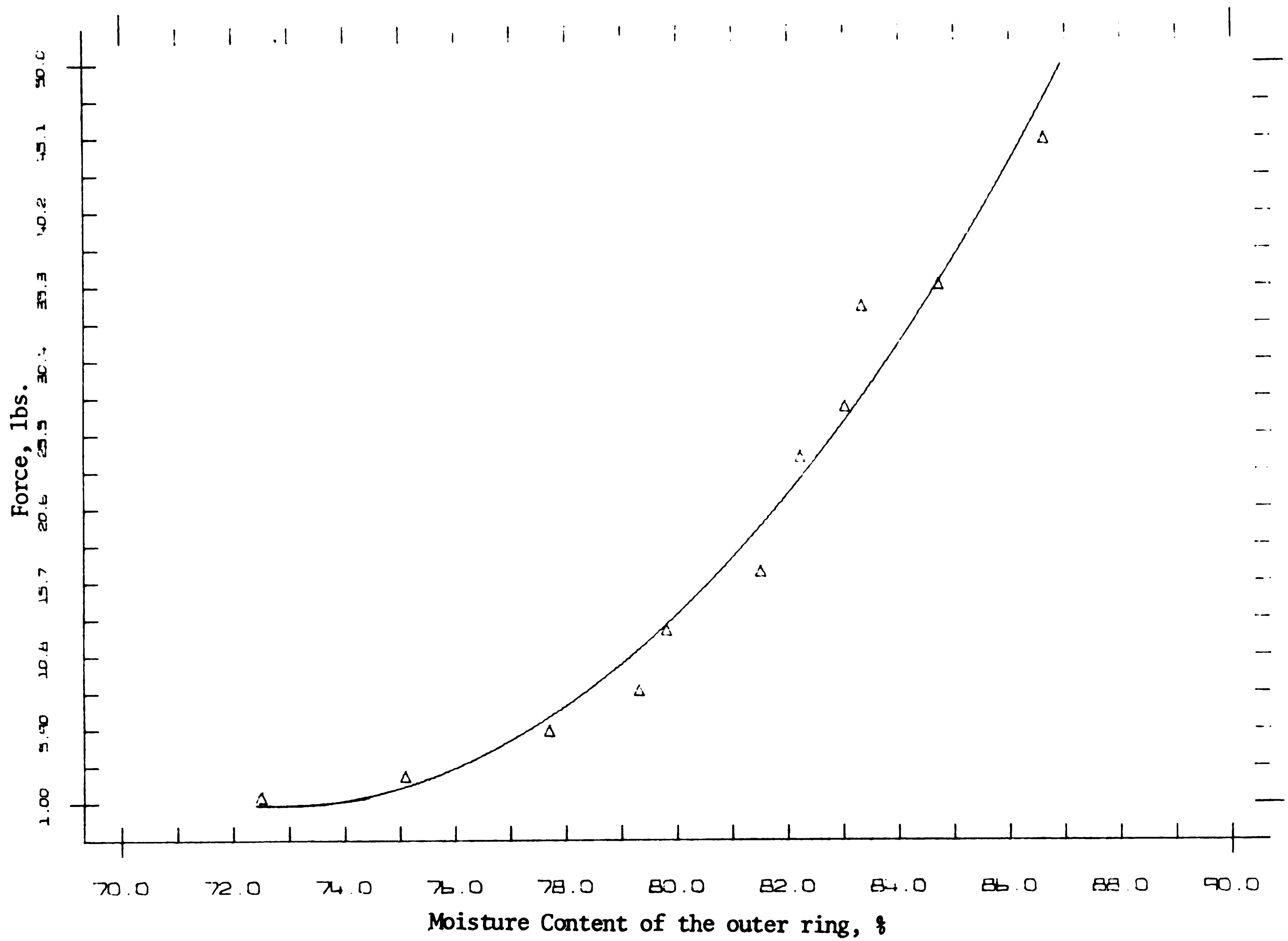


Fig. 26. Effect of moisture content on the applied radial force in deformation test.

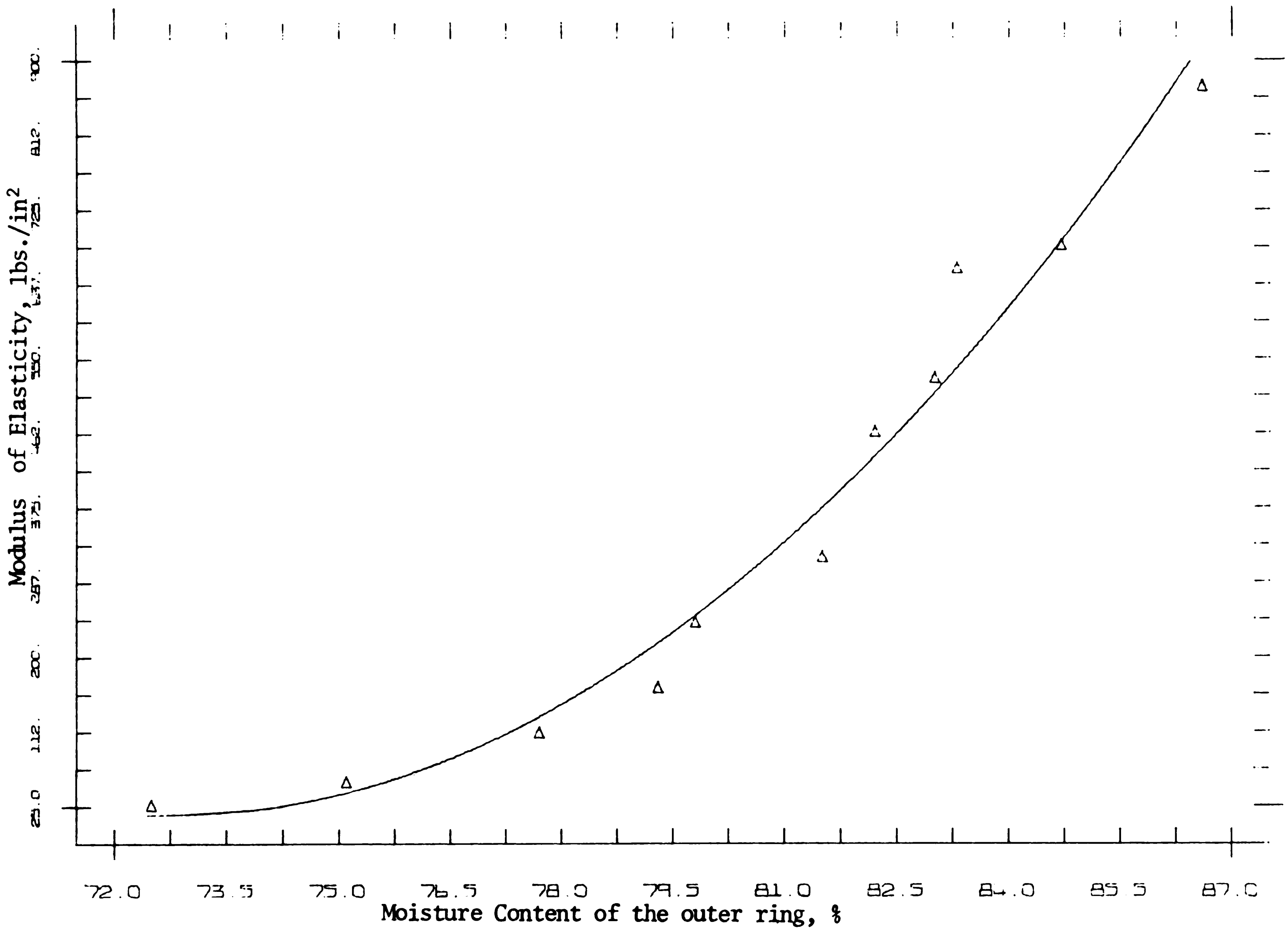


Fig. 27. Modulus of elasticity of carrots as calculated by the derived equation.

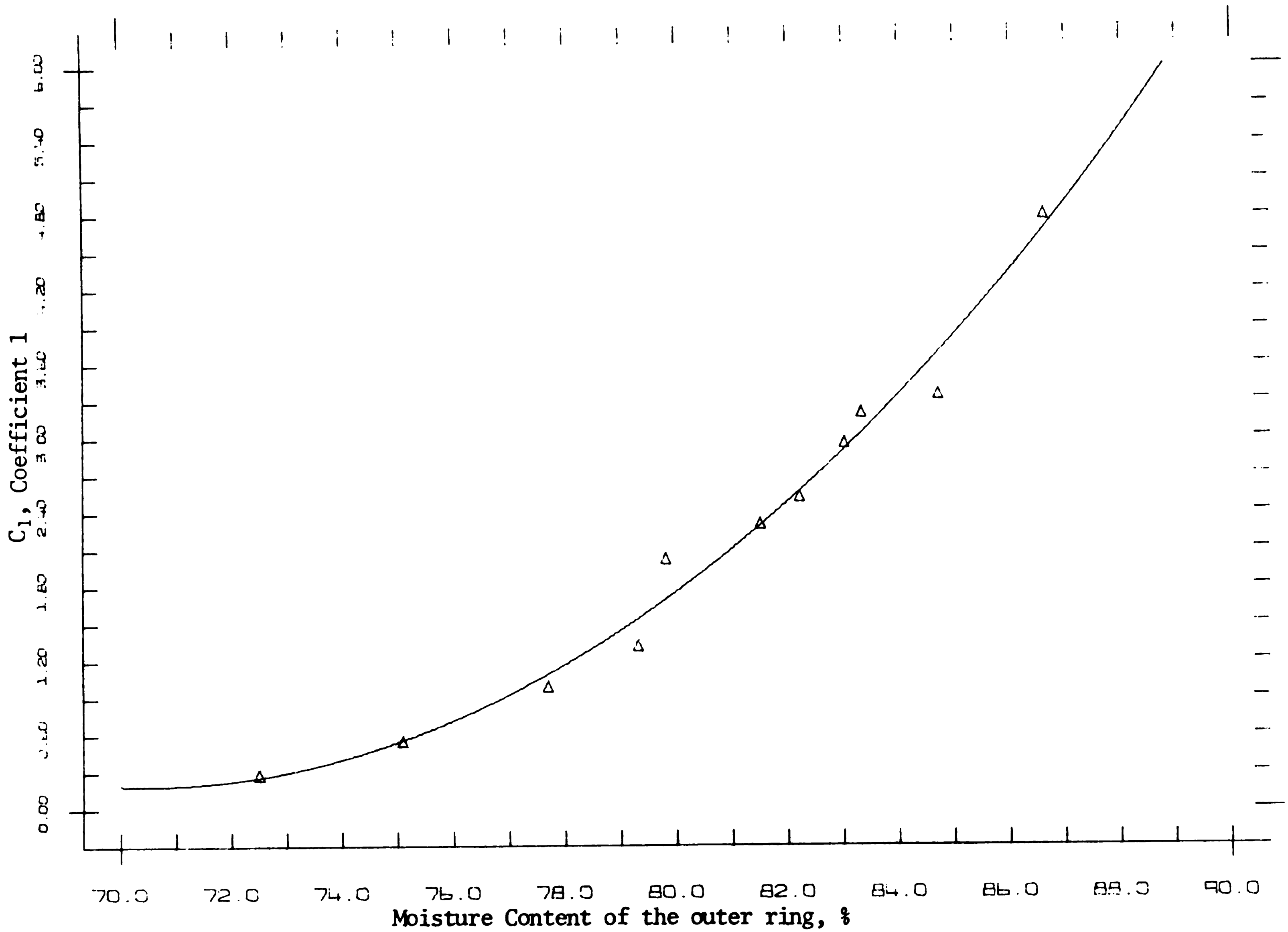


Fig. 28 . Effect of moisture content on the first coefficient in relaxation test.

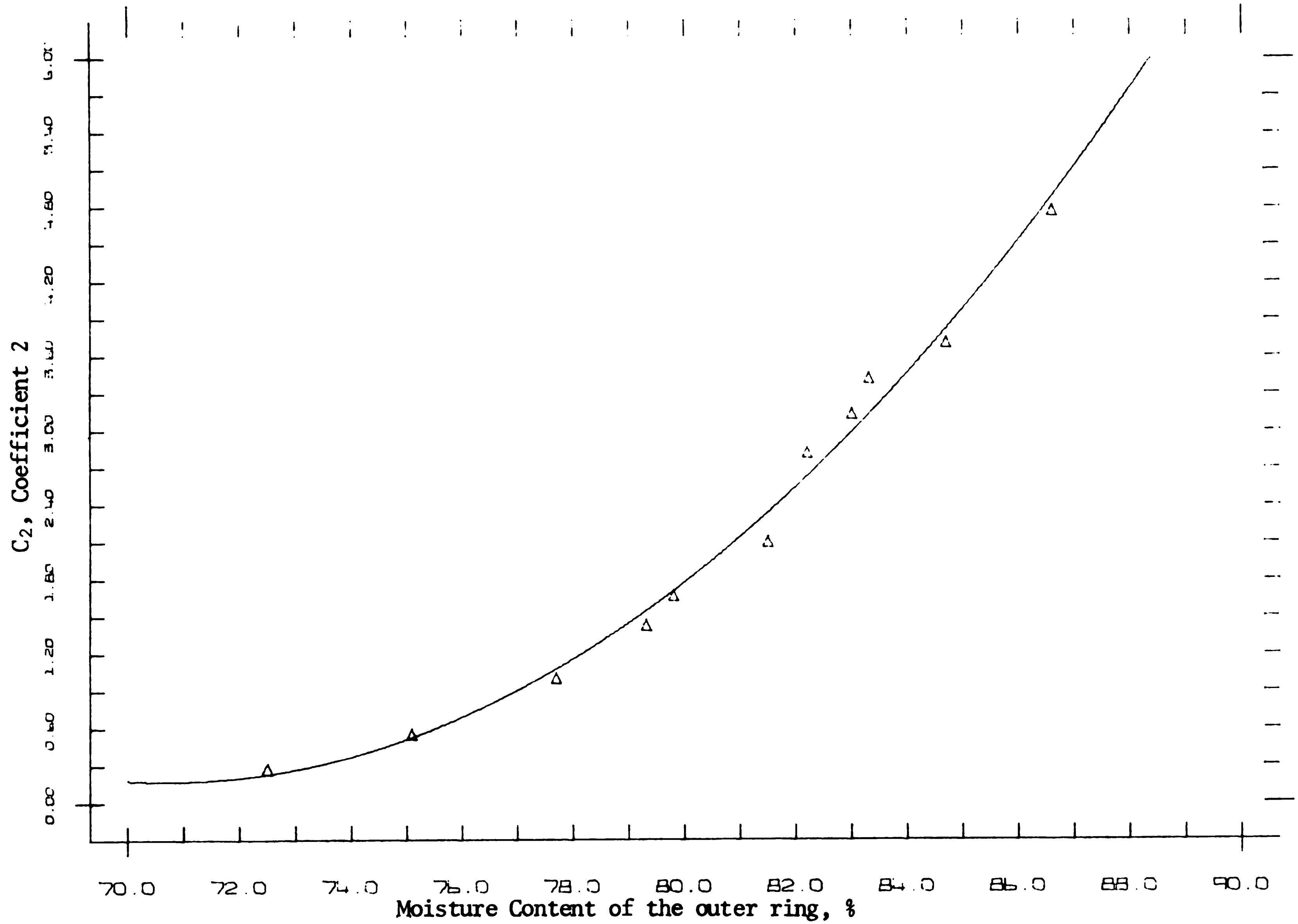


Fig. 29. Effect of moisture content on the second coefficient in relaxation test.

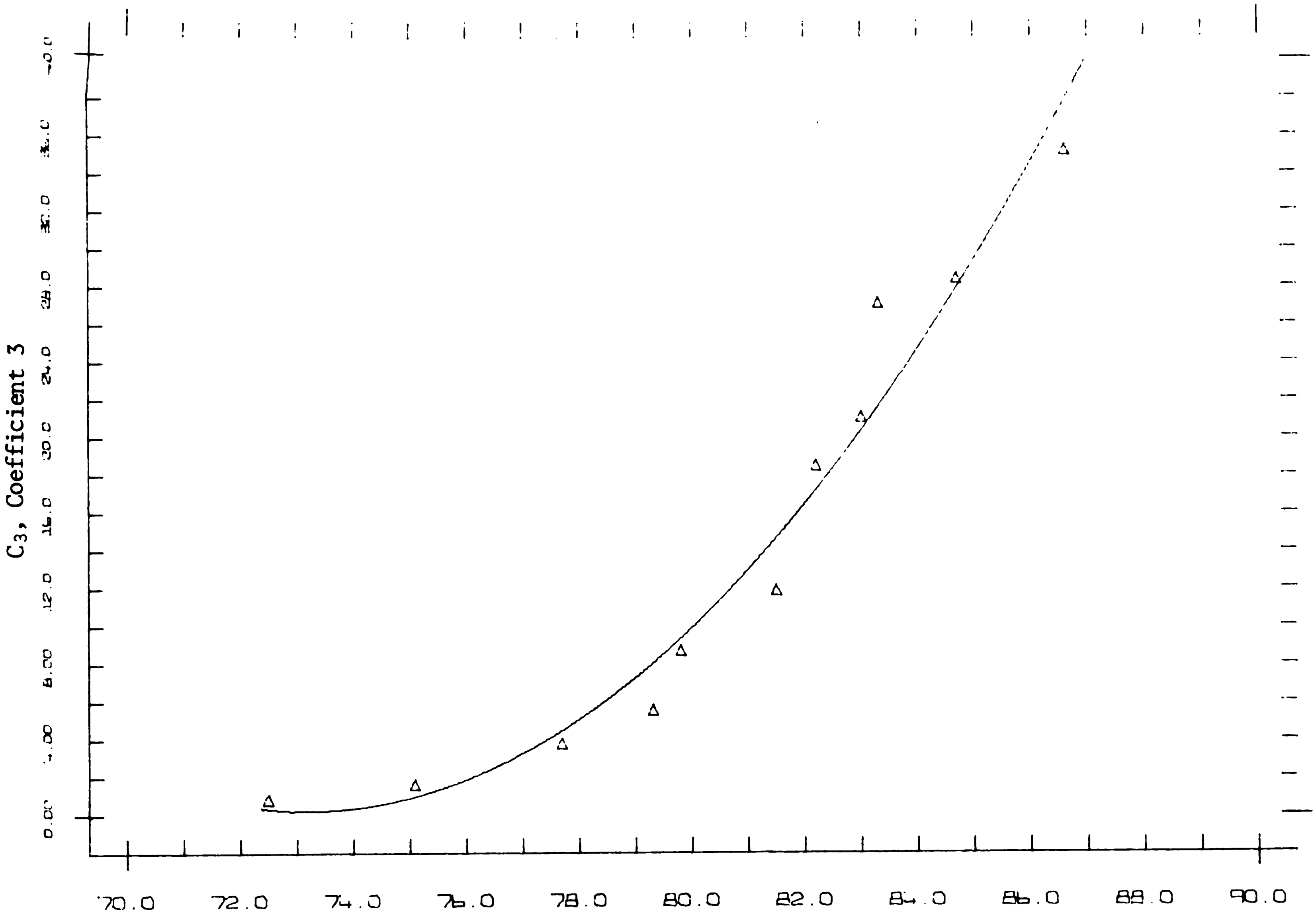


Fig. 30. Effect of moisture content on the third coefficient in relaxation test.

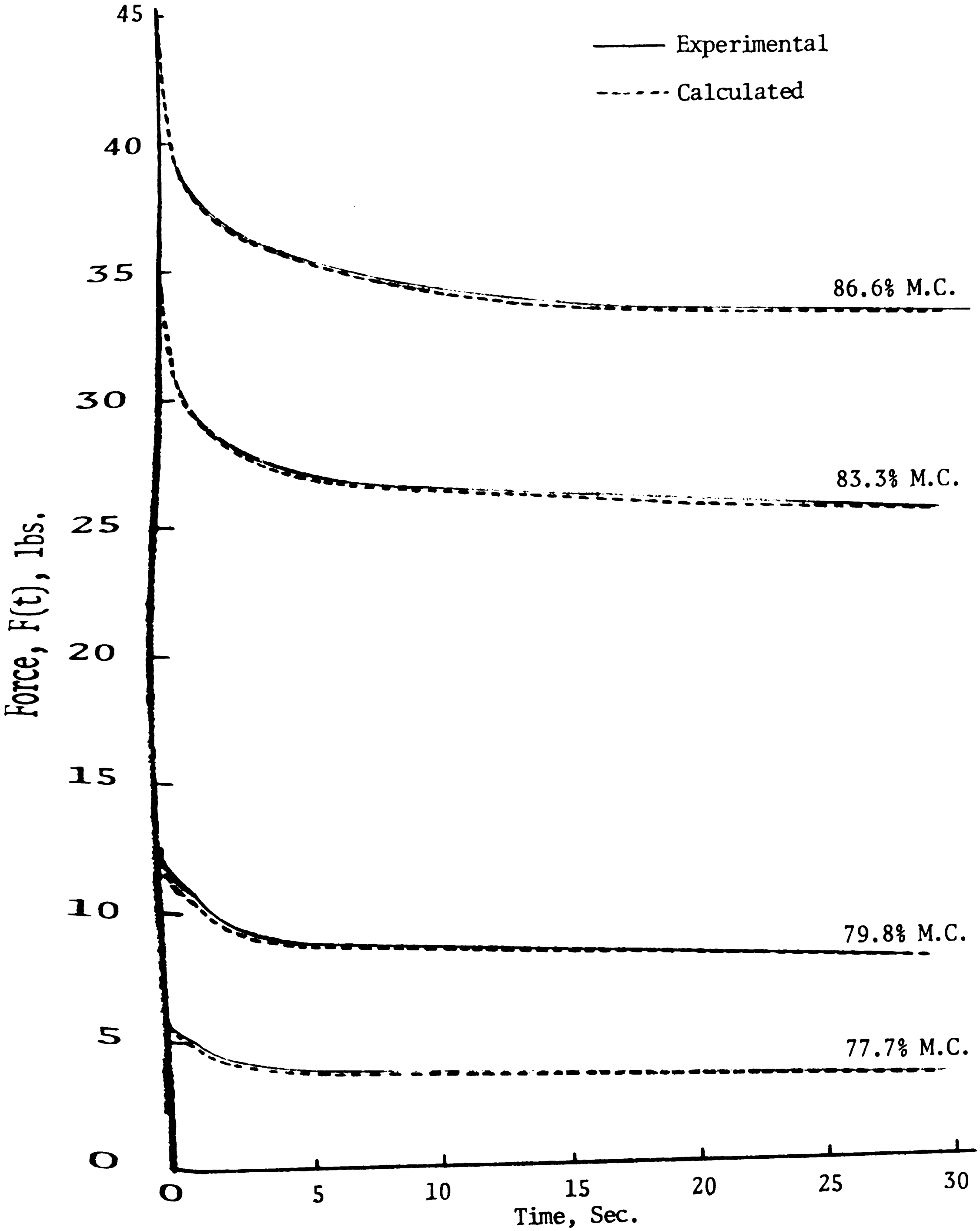


Fig. 31. Experimental and theoretical results for relaxation test at different moisture content. Table (A-9)

VII. CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

The conclusions derived from this study are as follows:

1. Measuring the change in TPA parameters, using an axial load on a sample of carrot, gave no significant results between hardness as a textural parameter and moisture content of the outer ring.
2. There are two distinct areas in a cross section of a cylindrical sample, a center core and an outer ring. The center core has a modulus of elasticity twice that of the outer ring when measured in the axial direction of a cylindrical sample.
3. The moisture content after a period of storing is not uniform throughout the cylindrical sample. It increases as the center core is approached.
4. The Poisson's Ratio of a carrot is approximately 0.5 and is independent of moisture content.
5. An equation for calculating the modulus of elasticity, E, of a cylindrical sample compressed in the radial direction was derived as

$$E = \frac{4(1 - \nu^2)}{\pi L^2} P' W^2 R$$

6. Comparison between the derived equation and Föppl's equation for calculating the modulus of elasticity of a radially compressed sample gave no significant variation in the modulus values, indicating that the assumption relative to the pressure

distribtution may not be a critical factor.

7. The hardness of texture parameter was found to be an important parameter as related to moisture content. As the moisture content decreased, the hardness, as measured to be the force to deform cylindrical sample compressed radially, decreased.

8. The force-time relaxation curve can be predicted by using three elements of the generalized Maxwell Model, i.e.

$$F\ (t) = C_1 e^{-\alpha_1 t} + C_2 e^{-\alpha_2 t} + C_3 e^{-\alpha_3 t}$$

9. The coefficients C_1, C_2, and C_3 of the relaxation equation showed a significant variation when plotted against moisture content.

10. The exponential coefficients α_1, α_2, α_3 showed no significant variation when plotted against moisture content.

11. As the moisture content in the outer ring decreases, the calculated modulus of elasticity decreases. The modulus of elasticity is an indication of stiffness.

12. Moisture content is the critical factor in carrot texture as indicated by the hardness parameter.

13. The hardness parameter should be measured as the peak force applied to deform cylindrical sample radially.

7.2 Recommendations

Further study should be made to determine the (TPA) parameters from force-deformation curves produced by twice compressing a cylindrical sample in the radial direction.

A study is recommended on measuring the modulus of elasticity of carrots stored under different temperatures and relative humidity to find the range of the modulus of elasticity value as related to the consumer acceptance of the product. This range could be used as a scale to measure carrot freshness objectively. The K value (Chen, 1972) in the relaxation equation should be determined so the modulus of elasticity can be calculated from the relaxation test and compared to the value calculated from the derived equation.

The study of the moisture content distribution from the center to the outer surface of a cross-section cylindrical sample of carrot is also recommended.

APPENDIX

Table A-1. Values of m, n, and k corresponding to Cos T (After Kosma and Cunningham, 1962).

Cos T	m	n	k	Cos T	m	n	k
0	1.000	1.000	1.3514	0.760	2.111	0.571	1.111
0.020	1.013	0.987	1.3512	0.780	2.195	0.558	1.091
0.040	1.027	0.974	1.3507	0.800	2.292	0.545	1.070
0.060	1.041	0.961	1.3502	0.820	2.401	0.530	1.047
0.080	1.056	0.948	1.3494	0.835	2.494	0.518	1.027
0.100	1.070	0.936	1.3484	0.845	2.564	0.511	1.013
0.120	1.085	0.924	1.3469	0,855	2.638	0.502	0.998
0.140	1.101	0.912	1.3453	0.865	2.722	0.494	0.983
0.160	1.117	0.901	1.343	0.875	2.813	0.485	0.966
0.180	1.133	0.889	1.341	0.885	2.915	0.476	0.948
0.200	1.150	0.878	1.339	0.895	3.029	0.466	0.929
0.220	1.167	0.866	1.336	0.905	3.160	0.455	0.908
0.240	1.185	0.855	1.334	0.912	3.262	0.448	0.892
0.260	1.203	0.844	1.330	0.916	3.326	0.443	0.882
0.280	1.222	0.833	1.327	0.920	3.395	0.438	0.872
0.300	1.242	0.822	1.323	0.924	3.468	0.433	0.862
0.320	1.262	0.812	1.319	0.928	3.547	0.428	0.851
0.340	1.283	0.801	1.315	0.932	3.631	0.423	0.839
0.360	1.305	0.790	1.310	0.936	3.723	0.418	0.828
0.380	1.327	0.780	1.305	0.940	3.825	0.412	0.815
0.400	1.351	0.769	1.300	0.944	3.935	0.406	0.801
0.420	1.375	0.759	1.294	0.948	4.053	0.399	0.787
0.440	1.401	0.748	1.288	0.952	4.187	0.393	0.772
0.460	1.428	0.738	1.281	0.956	4.339	0.386	0.756
0.480	1.456	0.728	1.274	0.960	4.509	0.378	0.739
0.500	1.484	0.717	1.267	0.964	4.700	0.370	0.719
0.520	1.515	0.707	1.259	0.968	4.94	0.361	0.699
0.540	1.548	0.696	1.251	0.972	5.20	0.351	0.676
0.560	1.583	0.685	1.242	0.976	5.52	0.341	0.650
0.580	1.620	0.675	1.232	0.980	5.94	0.328	0.621
0.600	1.660	0.664	1.222	0.984	6.47	0.314	0.586
0.620	1.702	0.653	1.211	0.988	7.25	0.298	0.545
0.640	1.748	0.642	1.200	0.991	8.10	0.281	0.504
0.660	1.796	0.631	1.187	0.993	8.90	0.268	0.472
0.680	1.848	0.619	1.174	*0.995	10.14	0.251	0.432
0.700	1.905	0.608	1.160	*0.997	12.26	0.228	0.376
0.720	1.966	0.596	1.145	*0.999	18.49	0.185	0.278
0.740	2.035	0.584	1.129				

*These intervals cannot be interpolated.

Table A-2. Values of W at different values of $\frac{\alpha R}{L^2}$ of the equation:

$$\frac{\alpha R}{L^2} = \frac{1}{2W^2} [\log_e (W + \sqrt{4 + W^2}) + \frac{3}{2} - \frac{3}{4 W^2 + 7}]$$

W	$\frac{\alpha R}{L^2}$	W	$\frac{\alpha R}{L^2}$	W	$\frac{\alpha R}{L^2}$
2.00	.36801	4.20	.10322	6.40	.04950
2.05	.35289	4.25	.10113	6.45	.04883
2.10	.33873	4.30	.09910	6.50	.04817
2.15	.32546	4.35	.09713	6.55	.04753
2.20	.31298	4.40	.09522	6.60	.04689
2.25	.30124	4.45	.09337	6.65	.04628
2.30	.29018	4.50	.09158	6.70	.04567
2.35	.27975	4.55	.08984	6.75	.04508
2.40	.26990	4.60	.08815	6.80	.04450
2.45	.26058	4.65	.08651	6.85	.04393
2.50	.25175	4.70	.08491	6.90	.04337
2.55	.24339	4.75	.08337	6.95	.04282
2.60	.23546	4.80	.08186	7.00	.04228
2.65	.22792	4.85	.08040	7.05	.04175
2.70	.22076	4.90	.07897	7.10	.04124
2.75	.21394	4.95	.07759	7.15	.04073
2.80	.20745	5.00	.07624	7.20	.04023
2.85	.20126	5.05	.07493	7.25	.03975
2.90	.19536	5.10	.07365	7.30	.03927
2.95	.18972	5.15	.07241	7.35	.03880
3.00	.18434	5.20	.07120	7.40	.03834
3.05	.17919	5.25	.07002	7.45	.03788
3.10	.17426	5.30	.06887	7.50	.03744
3.15	.16954	5.35	.06775	7.55	.03700
3.20	.16501	5.40	.06666	7.60	.03657
3.25	.16068	5.45	.06560	7.65	.03615
3.30	.15651	5.50	.06456	7.70	.03574
3.35	.15252	5.55	.06354	7.75	.03533
3.40	.14868	5.60	.06255	7.80	.03493
3.45	.14499	5.65	.06159	7.85	.03454
3.50	.14144	5.70	.06065	7.90	.03416
3.55	.13803	5.75	.05973	7.95	.03378
3.60	.13474	5.80	.05883	8.00	.03340
3.65	.13157	5.85	.05795	8.05	.03304
3.70	.12852	5.90	.05709	8.10	.03268
3.75	.12558	5.95	.05625	8.15	.03233
3.80	.12274	6.00	.05544	8.20	.03198
3.85	.12000	6.05	.05463	8.25	.03164
3.90	.11735	6.10	.05385	8.30	.03130
3.95	.11479	6.15	.05309	8.35	.03097
4.00	.11232	6.20	.05234	8.40	.03064
4.05	.10994	6.25	.05160	8.45	.03032
4.10	.10762	6.30	.05089	8.50	.03001
4.15	.10539	6.35	.05019	8.55	.02970

Table A-2 continued.

W	$\frac{\alpha R}{L^2}$	W	$\frac{\alpha R}{L^2}$	W	$\frac{\alpha R}{L^2}$
8.60	.02939	11.45	.01766	14.25	.01194
8.65	.02909	11.50	.01753	14.30	.01187
8.70	.02879	11.55	.01739	14.35	.01179
8.75	.02850	11.60	.01726	14.40	.01172
8.80	.02822	11.65	.01713	14.45	.01164
8.85	.02794	11.70	.01700	14.50	.01157
8.90	.02766	11.75	.01687	14.55	.01150
8.95	.02738	11.80	.01674	14.60	.01143
9.00	.02711	11.85	.01661	14.65	.01136
9.05	.02685	11.90	.01649	14.70	.01129
9.10	.02659	11.95	.01637	14.75	.01122
9.15	.02633	12.00	.01624	14.80	.01115
9.20	.02608	12.05	.01612	14.85	.01109
9.25	.02583	12.10	.01601	14.90	.01102
9.30	.02558	12.15	.01589	14.95	.01095
9.35	.02534	12.20	.01577	15.00	.01089
9.40	.02510	12.25	.01566	15.05	.01082
9.45	.02487	12.30	.01554	15.10	.01076
9.50	.02463	12.35	.01543	15.15	.01070
9.55	.02440	12.40	.01532	15.20	.01063
9.60	.02418	12.45	.01521	15.25	.01057
9.65	.02396	12.50	.01510	15.30	.01051
9.70	.02374	12.55	.01499	15.35	.01045
9.75	.02352	12.60	.01489	15.40	.01039
9.80	.02331	12.65	.01478	15.45	.01033
9.85	.02310	12.70	.01468	15.50	.01027
9.90	.02289	12.75	.01457	15.55	.01021
9.95	.02269	12.80	.01447	15.60	.01015
10.00	.02249	12.85	.01437	15.65	.01009
10.05	.02229	12.90	.01427	15.70	.01003
10.10	.02209	12.95	.01417	15.75	.00997
10.15	.02190	13.00	.01408	15.80	.00992
10.20	.02171	13.05	.01398	15.85	.00986
10.25	.02152	13.10	.01388	15.90	.00981
10.30	.02133	13.15	.01379	15.95	.00975
10.35	.02115	13.20	.01370	16.00	.00970
10.40	.02097	13.25	.01360	16.05	.00964
10.45	.02079	13.30	.01351	16.10	.00959
10.50	.02062	13.35	.01342	16.15	.00953
10.55	.02044	13.40	.01333	16.20	.00948
10.60	.02027	13.45	.01324	16.25	.00943
10.65	.02010	13.50	.01316	16.30	.00938
10.70	.01993	13.55	.01307	16.35	.00933
10.75	.01977	13.60	.01298	16.40	.00927
10.80	.01961	13.65	.01290	16.45	.00922
10.85	.01945	13.70	.01281	16.50	.00917
10.90	.01929	13.75	.01273	16.55	.00912
10.95	.01913	13.80	.01265	16.60	.00907
11.00	.01897	13.85	.01257	16.65	.00902
11.10	.01867	13.90	.01248	16.70	.00898
11.15	.01882	13.95	.01240	16.75	.00893
11.20	.01837	14.00	.01233	16.80	.00888
11.25	.01823	14.05	.01225	16.85	.00883
11.30	.01809	14.10	.01217	16.90	.00879
11.35	.01794	14.15	.01209	16.95	.00874
11.40	.01780	14.20	.01202	17.00	.00869

Table A-2 continued.

W	$\frac{\alpha R}{L^2}$	W	$\frac{\alpha R}{L^2}$	W	$\frac{\alpha R}{L^2}$
17.05	.00865	19.05	.00708	21.05	.00591
17.10	.00860	19.10	.00704	21.10	.00588
17.15	.00856	19.15	.00701	21.15	.00586
17.20	.00851	19.20	.00698	21.20	.00583
17.25	.00847	19.25	.00695	21.25	.00581
17.30	.00842	19.30	.00691	21.30	.00578
17.35	.00838	19.35	.00688	21.35	.00576
17.40	.00834	19.40	.00685	21.40	.00573
17.45	.00829	19.45	.00682	21.45	.00571
17.50	.00825	19.50	.00679	21.50	.00569
17.55	.00821	19.55	.00675	21.55	.00566
17.60	.00817	19.60	.00672	21.60	.00564
17.65	.00812	19.65	.00669	21.65	.00562
17.70	.00808	19.70	.00666	21.70	.00559
17.75	.00804	19.75	.00663	21.75	.00557
17.80	.00800	19.80	.00660	21.80	.00555
17.85	.00796	19.85	.00657	21.85	.00552
17.90	.00792	19.90	.00654	21.90	.00550
17.95	.00788	19.95	.00651	21.95	.00548
18.00	.00784	20.00	.00648	22.00	.00545
18.05	.00780	20.05	.00645	22.05	.00543
18.10	.00776	20.10	.00642	22.10	.00541
18.15	.00772	20.15	.00639	22.15	.00539
18.20	.00769	20.20	.00637	22.20	.00537
18.25	.00765	20.25	.00634	22.25	.00534
18.30	.00761	20.30	.00631	22.30	.00532
18.35	.00757	20.35	.00628	22.35	.00530
18.40	.00754	20.40	.00625	22.40	.00528
18.45	.00750	20.45	.00623	22.45	.00526
18.50	.00746	20.50	.00620	22.50	.00524
18.55	.00743	20.55	.00617	22.55	.00522
18.60	.00739	20.60	.00614	22.60	.00519
18.65	.00735	20.65	.00612	22.65	.00517
18.70	.00732	20.70	.00609	22.70	.00515
18.75	.00728	20.75	.00606	22.75	.00513
18.80	.00725	20.80	.00604	22.80	.00511
18.85	.00721	20.85	.00601	22.85	.00509
18.90	.00718	20.90	.00599	22.90	.00507
18.95	.00715	20.95	.00596	22.95	.00505
19.00	.00711	21.00	.00593	23.00	.00503

Table A-3. Values of Z at different values of $\frac{\alpha}{D}$ in equation

$$\frac{\alpha}{D} = \frac{1}{2Z^Z} (\log_e 2Z + \frac{1}{3})$$

Z	$\frac{\alpha}{D}$	Z	$\frac{\alpha}{D}$	Z	$\frac{\alpha}{D}$
1.00	.51324	3.55	.09099	6.10	.03809
1.05	.48765	3.60	.08902	6.15	.03758
1.10	.46354	3.65	.08711	6.20	.03708
1.15	.44092	3.70	.08527	6.25	.03659
1.20	.41972	3.75	.08349	6.30	.03611
1.25	.39987	3.80	.08176	6.35	.03564
1.30	.38131	3.85	.08009	6.40	.03519
1.35	.36394	3.90	.07848	6.45	.03474
1.40	.34769	3.95	.07691	6.50	.03429
1.45	.33247	4.00	.07539	6.55	.03386
1.50	.31821	4.05	.07392	6.60	.03344
1.55	.30483	4.10	.07250	6.65	.03302
1.60	.29228	4.15	.07111	6.70	.03261
1.65	.28048	4.20	.06977	6.75	.03221
1.70	.26939	4.25	.06846	6.80	.03182
1.75	.25895	4.30	.06720	6.85	.03144
1.80	.24911	4.35	.06597	6.90	.03106
1.85	.23983	4.40	.06477	6.95	.03069
1.90	.23107	4.45	.06361	7.00	.03033
1.95	.22278	4.50	.06248	7.05	.02997
2.00	.21495	4.55	.06138	7.10	.02962
2.05	.20753	4.60	.06031	7.15	.02927
2.10	.20050	4.65	.05927	7.20	.02894
2.15	.19382	4.70	.05826	7.25	.02860
2.20	.18749	4.75	.05727	7.30	.02828
2.25	.18147	4.80	.05631	7.35	.02796
2.30	.17574	4.85	.05538	7.40	.02764
2.35	.17029	4.90	.05447	7.45	.02733
2.40	.16509	4.95	.05358	7.50	.02703
2.45	.16014	5.00	.05271	7.55	.02673
2.50	.15542	5.05	.05187	7.60	.02644
2.55	.15090	5.10	.05105	7.65	.02615
2.60	.14659	5.15	.05024	7.70	.02587
2.65	.14247	5.20	.04946	7.75	.02559
2.70	.13852	5.25	.04870	7.80	.02531
2.75	.13474	5.30	.04795	7.85	.02504
2.80	.13112	5.35	.04722	7.90	.02478
2.85	.12765	5.40	.04651	7.95	.02452
2.90	.12432	5.45	.04582	8.00	.02426
2.95	.12113	5.50	.04514	8.05	.02401
3.00	.11806	5.55	.04448	8.10	.02376
3.05	.11511	5.60	.04383	8.15	.02351
3.10	.11227	5.65	.04320	8.20	.02327
3.15	.10954	5.70	.04258	8.25	.02304
3.20	.10691	5.75	.04197	8.30	.02280
3.25	.10438	5.80	.04138	8.35	.02258
3.30	.10194	5.85	.04080	8.40	.02235
3.35	.09959	5.90	.04023	8.45	.02213
3.40	.09732	5.95	.03968	8.50	.02191
3.45	.09514	6.00	.03914	8.55	.02169
3.50	.09303	6.05	.03861	8.60	.02148

Table A-3 continued.

Z	$\frac{\alpha}{D}$	Z	$\frac{\alpha}{D}$	Z	$\frac{\alpha}{D}$
8.65	.02127	10.30	.01582	11.95	.01227
8.70	.02107	10.35	.01569	12.00	.01219
8.75	.02086	10.40	.01557	12.05	.01210
8.80	.02066	10.45	.01544	12.10	.01201
8.85	.02047	10.50	.01531	12.15	.01193
8.90	.02027	10.55	.01519	12.20	.01185
8.95	.02008	10.60	.01507	12.25	.01176
9.00	.01989	10.65	.01495	12.30	.01168
9.05	.01971	10.70	.01483	12.35	.01160
9.10	.01953	10.75	.01471	12.40	.01152
9.15	.01935	10.80	.01460	12.45	.01144
9.20	.01917	10.85	.01448	12.50	.01136
9.25	.01899	10.90	.01437	12.55	.01128
9.30	.01882	10.95	.01426	12.60	.01121
9.35	.01865	11.00	.01415	12.65	.01113
9.40	.01848	11.05	.01404	12.70	.01106
9.45	.01832	11.10	.01393	12.75	.01098
9.50	.01815	11.15	.01382	12.80	.01091
9.55	.01799	11.20	.01372	12.85	.01083
9.60	.01783	11.25	.01361	12.90	.01076
9.65	.01768	11.30	.01351	12.95	.01069
9.70	.01752	11.35	.01341	13.00	.01062
9.75	.01737	11.40	.01331	13.05	.01055
9.80	.01722	11.45	.01321	13.10	.01048
9.85	.01707	11.50	.01311	13.15	.01041
9.90	.01693	11.55	.01301	13.20	.01034
9.95	.01678	11.60	.01292	13.25	.01028
10.00	.01664	11.65	.01282	13.30	.01021
10.05	.01650	11.70	.01273	13.35	.01015
10.10	.01636	11.75	.01264	13.40	.01008
10.15	.01622	11.80	.01254	13.45	.01002
10.20	.01609	11.85	.01245	13.50	.00995
10.25	.01596	11.90	.01236	13.55	.00989

Table A-4. Set #1, Experimental data of deformation force and moisture content.

Sample No.	Force at 0.1" def. lbs.	M.C.% W.B.	Sample No.	Force at 0.1" def. lbs.	M.C.% W.B.
1	50.4	88.3	47	25.0	83.9
2	49.2	87.5	48	25.0	86.4
3	49.0	87.5	49	25.0	87.2
4	47.5	87.5	50	24.2	83.9
5	47.2	88.0	51	23.8	85.4
6	46.3	88.7	52	23.5	86.2
7	44.5	89.1	53	21.4	84.4
8	44.4	89.1	54	21.0	84.7
9	43.0	88.3	55	21.0	86.2
10	43.0	88.3	56	19.4	85.7
11	42.0	87.0	57	19.0	85.4
12	42.0	86.0	58	17.0	85.7
13	41.5	87.9	59	16.0	84.7
14	41.4	89.9	60	15.7	85.5
15	41.0	85.8	61	15.5	84.2
16	40.7	87.9	62	15.4	83.7
17	40.0	89.4	63	15.0	85.7
18	39.0	86.7	64	14.4	85.2
19	38.3	86.6	65	13.0	84.2
20	38.0	87.7	66	13.0	85.2
21	38.0	87.7	67	12.5	85.5
22	38.0	89.6	68	12.5	85.7
23	38.0	86.6	69	11.6	83.7
24	37.5	86.7	70	11.5	82.3
25	37.0	86.7	71	11.3	85.1
26	37.0	86.6	72	11.0	83.4
27	37.0	87.7	73	10.7	85.5
28	36.0	89.2	74	10.3	82.7
29	36.0	86.0	75	10.0	84.7
30	35.0	87.7	76	10.0	85.1
31	34.4	85.3	77	10.0	84.8
32	34.2	87.2	78	9.5	81.1
33	32.0	87.2	79	9.5	82.7
34	32.0	85.3	80	8.5	81.8
35	32.0	86.6	81	8.2	84.0
36	30.0	85.6	82	8.0	82.5
37	30.0	83.9	83	7.5	85.1
38	28.7	85.6	84	7.5	84.3
39	28.6	86.2	85	7.4	79.3
40	28.0	86.2	86	6.6	82.3
41	27.0	87.2	87	6.7	81.8
42	26.4	86.4	88	6.3	81.0
43	26.0	86.4	89	6.0	84.8
44	25.8	86.2	90	5.9	80.4
45	25.3	84.0	91	5.7	81.1
46	25.0	85.3	92	5.4	81.0

Table A-4 continued.

Sample No.	Force at 0.1" def. lbs.	M.C.% W.B.	Sample No.	Force at 0.1" def. lbs.	M.C.% W.B.
93	5.0	81.1	116	2.9	74.5
94	4.5	81.1	117	2.8	76.6
95	4.5	73.9	118	2.8	79.6
96	4.2	82.4	119	2.5	79.6
97	4.0	80.4	120	2.5	72.3
98	4.0	66.7	121	2.5	78.4
99	4.0	75.4	122	2.4	73.4
100	4.0	79.1	123	2.4	74.1
101	4.0	78.9	124	2.3	75.2
102	4.2	76.1	125	2.3	71.0
103	3.8	84.0	126	2.2	77.8
104	3.7	84.8	127	2.0	73.9
105	3.6	81.1	128	1.8	77.4
106	3.7	75.5	129	1.8	69.8
107	3.5	81.0	130	1.8	78.1
108	3.5	79.0	131	1.6	75.2
109	3.5	71.1	132	1.6	64.7
110	3.3	77.9	133	1.5	75.2
111	3.2	72.4	134	1.5	76.9
112	3.1	75.0	135	1.5	73.9
113	3.0	82.4	136	1.5	77.8
114	3.0	75.0	137	1.5	75.7
115	3.0	79.2	138	1.0	73.0

Table A-5. Set #2, Experimental data of deformation force and moisture content.

Sample No.	Force at 0.1" def. lbs.	M.C.% W.B.	Sample No.	Force at 0.1" def. lbs.	M.C.% W.B.
1	52.0	87.1	36	14.5	79.4
2	47.0	87.9	37	14.0	79.8
3	46.0	86.0	38	13.0	82.5
4	45.0	87.1	39	13.0	80.0
5	45.0	87.6	40	11.5	78.7
6	42.0	85.6	41	11.0	76.2
7	42.0	85.7	42	11.0	81.4
8	40.0	86.1	43	10.0	80.4
9	36.0	84.3	44	9.5	79.8
10	35.0	83.7	45	9.5	81.1
11	35.0	86.2	46	9.0	79.5
12	34.0	83.6	47	9.0	78.1
13	33.0	82.4	48	8.5	78.6
14	32.5	81.9	49	7.0	78.7
15	32.0	85.2	50	6.5	79.5
16	32.0	83.3	51	6.5	79.2
17	28.0	80.7	52	6.5	79.4
18	26.0	85.3	53	6.0	77.9
19	24.0	84.0	54	5.8	79.6
20	23.0	80.2	55	5.6	70.6
21	23.5	81.0	56	5.6	75.3
22	20.0	83.5	57	5.0	79.7
23	19.5	82.1	58	4.5	78.0
24	19.5	84.7	59	4.3	75.4
25	19.0	79.7	60	4.1	79.3
26	18.5	84.6	61	3.7	74.6
27	18.0	82.4	62	3.6	76.6
28	18.0	82.2	63	3.5	75.7
29	17.0	81.0	64	2.8	75.8
30	16.0	80.3	65	2.5	74.0
31	16.0	80.9	66	2.2	74.0
32	16.0	76.8	67	2.1	75.2
33	16.0	81.8	68	1.7	72.9
34	15.5	80.5	69	1.7	70.7
35	15.0	82.5	70	1.4	73.2

Table A-6. Set #1, Average deformation force and moisture content of samples grouped according to force range.

Force Range	Ave. Force lbs.	Ave. M.C.% W.B.
50 - to 40	43.4	88.2
39.9 to 35	37.2	87.4
34.9 to 30	33.3	86.5
29.9 to 25	26.6	85.6
24.9 to 20	21.4	84.9
19.9 to 15	16.8	84.4
14.9 to 10	13.7	83.7
9.9 to 7	8.5	82.4
6.9 to 4	4.8	78.6
3.9 to 2	2.9	75.8
1.9 to 0	1.7	74.4

Table A-7. Set #2, Average deformation force and moisture content of samples grouped according to force range.

Force Range	Ave. Force lbs.	Ave. M.C.% W.B.
50 - to 40	45.0	86.6
39.9 to 35	35.3	84.7
34.9 to 30	32.7	83.3
29.9 to 25	27.0	83.0
24.9 to 20	22.6	82.2
19.9 to 15	17.2	81.5
14.9 to 10	12.3	79.8
9.9 to 7	8.8	79.3
6.9 to 4	5.5	77.7
3.9 to 2	2.9	75.1
1.9 to 0	1.5	72.5

Table A-8. The calculated values of E for different potato specimen sizes and deformation using the equations:

$$E_1 = 0.955 \frac{P'W^2R}{L^2},\ E_2 = 1.91 \frac{P'Z^2}{D},\ E_3 = \frac{F}{A}\frac{L}{\alpha},$$ A is cross sectional area

α inch	D inch	L inch	F lb	P´ lb/in	$\frac{\alpha R}{L^2}$	$\frac{\alpha}{D}$	W	Z	E_1 psi	E_2 psi	E_3 psi
0.1	1.0	1.0	39.0	21.5	0.05	0.1	6.36	3.34	415	458	497
			27.5	11.5					222	245	350
			43.2	27.2					525	580	550
			29.0	17.0					328	362	369
	1.0	0.5	68.1	22.2	0.20	0.1	2.86	3.34	347	473	434
			67.2	18.8					294	401	428
			56.0	24.8					387	528	357
			81.0	—							516
0.05	1.0	1.0	9.0	6.3	0.025	0.05	9.42	5.17	267	322	229
			16.6	10.7					453	546	423
			14.8	9.5					403	485	377
			17.0	7.2					305	368	433
			22.0	11.7					496	597	561
	1.0	0.5	31.0	11.4	0.10	0.05	4.28	5.17	399	582	395
			40.0	10.0					350	511	509

Table A-8 continued.

α inch	D inch	L inch	F lb	P′ lb/in	$\frac{\alpha R}{L^2}$	$\frac{\alpha}{D}$	W	Z	E_1 psi	E_2 psi	E_3 psi
0.05	1.0	0.5	35.0	12.2	0.10	0.05	4.28	5.17	427	623	446
			41.2	11.2					392	572	525
			34.6	—					—	—	441
	0.5	1.0	4.5	13.7	0.0125	0.1	13.89	3.34	631	584	458
			5.2	10.9					502	464	530
			4.4	11.4					525	486	448
			4.1	10.9					502	464	418
	0.5	0.5	8.5	10.6	0.050	0.1	6.36	3.34	409	452	433
			8.2	11.2					433	477	418
			8.8	11.0					425	469	448
			8.6	11.0					425	469	438
0.025	1.0	1.0	4.5	2.8	0.0125	0.025	13.89	7.85	258	330	229
			7.6	6.4					590	753	387
			4.3	4.0					369	471	219
			—	5.0					461	588	—

Table A-8 continued.

α inch	D inch	L inch	F lb	P´ lb/in	$\frac{\alpha R}{L^2}$	$\frac{\alpha}{D}$	W	Z	E_1 psi	E_2 psi	E_3 psi
0.025	1.0	0.5	11.8	3.2	0.050	0.025	6.36	7.85	247	377	300
			11.4	4.0					309	471	290
			18.1	6.2					479	730	461
	0.5	1.0	3.5	4.0	0.00625	0.050	20.40	5.17	397	408	713
			2.2	4.5					447	459	448
			2.8	5.8					576	592	570
	0.5	0.5	2.2	4.0	0.025	0.050	9.42	5.17	339	408	224
			4.3	6.6					559	674	438
			3.7	5.4					458	551	377
			2.5	—					—	—	255
								Mean =	415	496	419
								σ^2 =	9928	12089	11219
								σ =	99.64	109.95	105.9

$$F_1 = \frac{\sigma_3^2}{\sigma_1^2} = \frac{105.9}{99.64} = 1.06$$

$$F_2 = \frac{\sigma_2^2}{\sigma_3^2} = \frac{109.95}{105.9} = 1.04$$

from tables $F_{0.05} = 1.7$

since F_1 and $F_2 < F_{0.05}$, then $\sigma_1^2 = \sigma_2^2 = \sigma_3^2$ is not rejected.

Table A-9. Experimental and calculated points for force-time relaxation curves at different moisture contents.

Point	86.6% M.C. F(t), lbs Experimental	86.6% M.C. F(t), lbs Calculated	t´ sec	84.7% M.C. F(t), lbs Experimental	84.7% M.C. F(t), lbs Calculated	t´ sec
1	45.0	44.7	0.0	35.4	35.4	0.0
2	42.7	42.7	0.12	35.0	35.4	0.0
3	41.2	41.4	0.24	33.5	33.5	0.15
4	39.8	39.8	0.48	32.6	32.6	0.27
5	39.0	38.9	0.72	31.6	31.6	0.51
6	37.7	37.8	1.32	30.4	30.5	1.11
7	37.0	37.1	1.92	29.9	29.8	1.71
8	36.1	36.1	3.12	29.0	29.0	2.91
9	35.1	35.0	5.52	28.3	28.3	4.71
10	34.5	34.5	7.92	27.8	27.8	7.11
11	34.0	34.2	10.32	27.2	27.3	10.71
12	33.0	33.0	21.12	26.1	26.6	16.71

Point	83.3% M.C. F(t), lbs Experimental	83.3% M.C. F(t), lbs Calculated	t´ sec	83.9% M.C. F(t), lbs Experimental	83.9% M.C. F(t), lbs Calculated	t´ sec
1	34.0	33.7	0.0	27.4	27.1	0.0
2	33.0	33.2	0.04	26.2	26.2	0.10
3	32.0	32.0	0.16	25.4	25.4	0.22
4	30.7	30.7	0.40	24.5	24.4	0.46
5	30.0	30.0	0.64	23.7	23.6	0.82
6	29.1	29.1	1.24	23.0	23.0	1.42
7	28.1	28.2	2.44	22.6	22.6	2.02
8	27.6	27.6	3.64	22.1	22.0	3.22
9	27.0	26.9	6.04	21.5	21.6	4.42
10	26.4	26.4	9.64	21.2	21.1	6.82
11	25.8	25.9	15.64	20.5	20.5	11.62
12	25.1	25.1	27.64	19.9	19.9	22.47

Table A-9 continued.

Point	82.2% M.C. F(t), lbs Experimental	82.2% M.C. F(t), lbs Calculated	t´ sec	81.5% M.C. F(t), lbs Experimental	81.5% M.C. F(t), lbs Calculated	t´ sec
1	24.0	23.8	0.0	16.3	16.3	0.0
2	23.1	23.1	0.10	15.5	15.4	0.11
3	22.3	22.4	0.22	14.9	14.8	0.23
4	21.6	21.6	0.46	14.1	14.0	0.47
5	21.0	21.0	0.70	13.7	13.6	0.71
6	20.4	20.4	1.06	13.1	13.0	1.31
7	19.6	19.5	2.26	12.8	12.7	1.91
8	19.0	19.0	3.46	12.4	12.3	3.11
9	18.4	18.4	5.86	12.1	12.0	4.61
10	18.0	18.0	8.26	11.6	11.4	9.41
11	17.7	17.7	11.86	11.0	11.0	17.21
12	17.0	17.0	21.46	10.7	10.6	26.81

Point	79.8% M. C. F(t), lbs Experimental	79.8% M. C. F(t), lbs Calculated	t´ sec	79.3% M.C. F(t), lbs Experimental	79.3% M.C. F(t), lbs Calculated	t´ sec
1	12.5	12.4	0.0	8.5	8.3	0.0
2	11.5	11.5	0.12	7.9	8.0	0.06
3	11.0	11.0	0.24	7.5	7.5	0.18
4	10.4	10.4	0.48	7.0	7.0	0.42
5	10.0	10.0	0.84	6.5	6.5	0.90
6	9.7	9.7	1.32	6.3	6.3	1.38
7	9.4	9.4	1.92	6.0	6.0	2.22
8	9.0	9.0	3.72	5.8	5.8	3.06
9	8.7	8.7	5.52	5.5	5.5	5.46
10	8.5	8.5	9.12	5.3	5.3	7.86
11	8.3	8.3	13.92	5.1	5.1	12.66
12	8.0	8.0	22.32	4.7	4.7	24.66

Table A-9 continued.

Point	77.7% M.C. F(t), lbs Experimental	77.7% M.C. F(t), lbs Calculated	t′ sec	75.1% M.C. F(t), lbs Experimental	75.1% M.C. F(t), lbs Calculated	t′ sec
1	5.8	5.8	0.0	2.8	2.8	0.0
2	5.6	5.6	0.04	2.5	2.5	0.08
3	5.3	5.3	0.16	2.3	2.3	0.20
4	5.0	5.0	0.28	2.3	2.2	0.32
5	4.7	4.7	0.52	2.1	2.1	0.44
6	4.5	4.5	0.76	2.0	2.0	0.80
7	4.3	4.2	1.60	1.9	1.9	1.28
8	4.0	4.0	2.80	1.8	1.8	1.88
9	3.8	3.8	5.20	1.7	1.7	3.08
10	3.7	3.7	7.60	1.6	1.6	5.48
11	3.5	3.5	13.60	1.5	1.5	9.68
12	3.4	3.4	23.20	1.4	1.4	19.28

Point	72.5% M.C. F(t), lbs Experimental	72.5% M.C. F(t), lbs Calculated	t′ sec
1	1.40	1.4	0.0
2	1.30	1.3	0.10
3	1.20	1.2	0.22
4	1.15	1.14	0.34
5	1.10	1.10	0.46
6	1.05	1.04	0.70
7	1.00	1.00	0.94
8	0.98	0.97	1.18
9	0.88	0.87	2.98
10	0.80	0.81	5.98
11	0.78	0.77	9.58
12	0.70	0.70	21.58

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