This is to certify that the

thesis entitled

SIMULATING RIGID BODY ENGINE DYNAMICS

presented by

Charles E. Spiekermann

has been accepted towards fulfillment
of the requirements for

Master of Science degree in Mechanical Engineering

Clark Radcliffe
Major professor

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SIMULATING RIGID BODY ENGINE DYNAMICS

By

Charles E. Spiekermann

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ABSTRACT

SIMULATING RIGID BODY ENGINE DYNAMICS

By

Charles E. Spiekermann

The vibration response of a vehicle is dependent on the location and direction of the forces acting on it. Motions of the engine contribute a significant excitation transmitted through the engine mounting points. Currently, packaging constraints and mounting locations have already been decided by the time engine vibration modes can be experimentally determined. An advance knowledge of the dynamic characteristics of the engine/mounts system would allow better mount locations to be designed into the vehicle structure. This thesis develops the analysis for a computer simulation of the dynamics of an engine modeled as a rigid body on elastic mounts tied to ground.

The analysis is based on linearized equations of motion that are incorporated into an interactive computer program which utilizes computer graphics and animation. Results of the simulation are the natural frequencies, mode shapes, static deflection, mount forces, elastic axes, and torque axis of the engine/mounts system.

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with love to my wife, LINDA

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NOMENCLATURE

DOF	Degrees of freedom	
{H}	Angular Momentum vector,	3 by 1.
[I]	Inertia Matrix,	3 by 3.
{ω}	Angular velocity vector,	3 by 1.
{M}m	Externally applied moment vector,	3 by 1.
[M]	Mass/inertia coefficient matrix,	6 by 6.
[C]	Viscous damping coefficient matrix,	6 by 6.
[D]	Structural damping coefficient matrix,	6 by 6.
[K]	Stiffness coefficient matrix,	6 by 6.
{F}	Force/torque vector,	6 by 1.
{x}	Generalized coordinate vector,	6 by 1.
λ	Eigenvalue	
{A}	2nd order eigenvector,	6 by 1.
[U]	Modal matrix, columns are eigenvectors {A},	6 by 6.
{y}	2nd order modal coordinate vector,	6 by 1.
{z}	1st order transformation vector,	12 by 1.
[M]z	1st order mass matrix,	12 by 12.
[K]z	1st order stiffness matrix,	12 by 12.
{F}z	1st order force/torque function,	12 by 1.
{B}	1st order eigenvectors,	12 by 1.
[V]	1st order modal matrix,	12 by 12.
{p}	1st order modal coordinate vector,	12 by 1.
[A]	Flexibility matrix,	6 by 6.

CHAPTER 1

INTRODUCTION

Smaller and more fuel efficient vehicles must be designed throughout the automobile industry to satisfy consumer demand and federal laws. In the future, vehicle designs will be developed and put into production in much less time than was done in the past in order to meet this demand. This thesis will show how the use of computer aided design techniques can be used to help achieve this goal.

This thesis presents the analysis for a computer simulation of the dynamics of an engine on elastic mounts tied to ground. The simulation predicts the natural frequencies and mode shapes of this system and also finds the forces transmitted through the engine mounts to the vehicle structure. An accurate prediction of these forces before a vehicle is even built can be used to design an engine mounting system with reduced vehicle vibration response.

The motivation for this thesis was provided by the increased implementation of diesel engines and lighter weight vehicles to achieve better fuel economy. This new combination has resulted in increased noise and vibration control problems since the high compression ratios and sharp torque pulse rise experienced in diesel combustion cause higher force levels which are then transmitted through the diesel engine mounting locations. Vehicle structural vibration is then increased. The engine mounting system is the preferable component to change to control this problem, but the packaging constraints of smaller engine compartments allow less design flexibility.

As an example, consider the recent down-sizing which included converting from vehicles with longitudinal mounted engines and rear wheel drive to vehicles with transverse mounted engines and front wheel drive (TFWD) with diesel engines. This is being widely used. An extensive re-design of existing vehicles was required. The vibration characteristics of TFWD mounted engines are not well known because of the lack of practical experience with this design. The past design method of physically testing many possible component designs (e.g. engine mount locations, orientation, and spring rates) is too slow and inconclusive. A computer simulation can be used to evaluate the best designs.

By using a simulation, the engineer can change one parameter or many parameters, such as the engine mount locations, and see the results in a very short time. This eases the process of sorting out the effects of design changes. Ultimately, physical testing of designs is required to insure good correlation between the simulation and the actual design, but the amount of testing is reduced.

In the past, it was not clear whether inertia or stiffness effects were more important as engine/mounts system design considerations. Separate consideration of these effects was necessary without a computer to handle the large computations involved. If desired, both effects can be considered separately in a computer simulation. However, neither effect acts just by itself. The complete system response is really a combination of many effects.

Chapter two develops the equations of motion for the rigid body dynamics involved in the engine/mounts simulation. Chapter three formulates and solves the associated eigenvalue problem. Chapter four discusses coordinate system conversions. Chapter five describes special design calculations dependent only on the inertia and stiffness properties. Chapter six presents an example problem. Appendix A discusses an interactive computer program, ENGSIM, which incorporates the previous topics.

CHAPTER 2

RIGID BODY DYNAMICS

The goal here is to develop an analytical model of an automobile engine/mounts system which will predict its vibration characteristics. The approach to be taken will be to first construct a conceptual model of the system. In a rigid body model, all of the elements of the body are at fixed distances from each other. Since the flexural natural frequencies of the engine are much higher than any of the natural frequencies of the engine on its mounts, the engine/mounts system will be viewed as a rigid body.

The motion of all points on the engine can be described as translations and rotations about the center of mass of that engine. A rectangular coordinate system with its origin at the center of gravity will be placed so that the positive Y axis is along the crankshaft and facing the front of the engine. (Figure 1) The Z axis points up and the X axis points to the rear of a vehicle with a transverse mounted engine.

A rigid body model has six degrees of freedom (DOF) which means that there can be translations along each of the three coordinate axes and rotations around each of those axes.

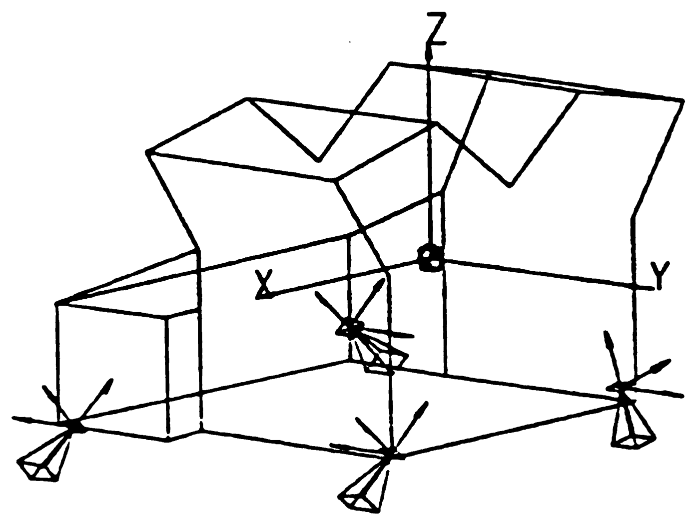


Figure 1 Engine/mounts coordinate system

Six independent coordinates are needed to specify completely the location and orientation of a general rigid body motion. For example, three cartesian coordinates may be used to specify the location of the center of mass of the body and three angles may be used to specify the orientation of the body about this point. For each of these six DOF, it is required that an equation of motion be written in order to completely describe the motion of the body.

Three of the equations equate the vector sum of the external forces acting on the engine rigid body with its rate of change of linear momentum. The other three equations equate the vector sum of the external torques acting on the engine rigid body with its rate of change of angular momentum. The three translational equations are represented in matrix form as

$$\{F\} = [M]\ \{a\} \qquad (1)$$

Each is a statement of Newton's Second Law and relates the external force, {F}, acting on the body to the acceleration, {a}, of the body's mass center, [M].

The three rotational equations relate the body's rotational motions to the moments created by torques or external forces. In general, these differential equations will be difficult to solve because the elements of the inertia matrix, [I], will produce non-zero terms when taking the time derivative of the angular momentum, {H},

$$\{H\} = [I]\ \{\omega\} \qquad (2)$$

where ω is the angular velocity. Choosing the reference coordinate system to be fixed in and to move with the body, as in Figure 1, will alleviate this problem. Then the moments and products of inertia relative to these axes will be constant during motion and their derivatives are zero. [1]* The general rotational equation of motion

* Numbers in brackets [ ] designate references at end of thesis.

is

$$\{M\}_m = \{dH/dt\} + \left[\{\omega\} \times \{H\}\right] \qquad (3)$$

where {M}m are the externally applied moments. This equation allows for both the time rate of change of the angular momentum itself and also the change in angular momentum caused by the rotation of the coordinate system. If only small motions are assumed to occur, the higher order non-linear terms can be assumed to be very small and disregarded to obtain

$$\{M\}_m = [I]\{d\omega/dt\} \qquad (4)$$

where $\{M\}_m$ are the external moments applied to the body, [I] is the inertia matrix, and dω/dt is the rotational acceleration. These three scalar rotational equations of motion, together with the three scalar translational equations of motion, can be solved to find the natural frequencies and mode shapes of the rigid body. The external forces applied to the body can be restoring forces from elastic elements such as springs, dissipative forces from dampers, or other externally applied forcing functions.

The general case to be solved is the damped forced vibration problem. The form of the general equation is

$$[M]\{d^2x/dt^2\} + [C]\{dx/dt\} + \frac{[D]}{\pi\omega}\{dx/dt\} + [K]\{x\} = \{F\} \qquad (5)$$

where [M] is the mass/inertia matrix, [C] is the viscous damping coefficient matrix, [D] is the structural damping coefficient matrix,

ω is the excitation frequency, [K] is the stiffness coefficient matrix, {F} is the externally applied harmonic force/torque, t is time, and {x} is a generalized coordinate vector.

$$\{x\}^T = \{X, Y, Z, \Theta x, \Theta y, \Theta z\} \tag{6}$$

Viscous damping takes the form of a force proportional in magnitude to the velocity and acting in the direction opposite to the direction of the velocity. Structural damping is associated with internal energy dissipation due to the hysteresis in cyclic stresses. The energy loss per cycle of stress is proportional to the amplitude squared. The structural damping has been treated in Equation 5 as an equivalent viscous damping term which is inversely proportional to the driving frequency, ω, of the harmonic excitation. [2] These damping models are the two used most frequently to represent damping effects.

CHAPTER 3

EIGENVALUE PROBLEM FORMULATION

The solution of Equation (5) begins by looking at the homogeneous case with no forcing functions, {F}, present. In many cases, the system damping is very small and can be neglected. This undamped free vibration problem is the easiest to solve and still results in much useful information. Equation (5) is then reduced to

$$[M]\{d^2x/dt^2\} + [K]\{x\} = \{0\} \tag{7}$$

where [M] and [K] are six by six matrices representing the inertia and stiffnesses of the rigid body. In this case, the only external forces applied to the body are from the elastic elements. The eigenvalue problem [3] formed from Equation (7) is

$$[K]\{A\} = \lambda[M]\{A\} \tag{8}$$

This eigenvalue problem is useful because it provides information about the natural frequencies and mode shapes of the system. The physical interpretation of this eigenvalue problem is straightforward. The constant, λ, is an eigenvalue representing the square of a natural frequency. Assume that six distinct eigenvalues, one for each of the six degrees of freedom (DOF), will be found for this problem. Once the six eigenvalues are known, each will have an eigenvector, $\{A\}$, associated with it. The eigenvectors are distinct, except for the possibility of a constant multiple of themselves.

Each eigenvector is a mode shape formed from the six DOF, $\{x\}$, in Equation 6. The six DOF locate any point on the rigid body with three translations of the center of mass along the reference coordinate axes and a rotation of the body around each of those axes. With no damping present, the eigenvectors will all be real values and each DOF will reach its maximum and minimum at the same time because there is no phase lag due to damping. [2]

A modal matrix, $[U]$, is formed by letting the columns of $[U]$ be the six eigenvectors, $\{A\}$. The modal matrix forms a basis for this system and can be used to decouple the system of differential equations by making the substitution

$$\{x\} = [U]\{y\} \tag{9}$$

into Equation (7) and then premultipling by $[U]^T$ to obtain

$$[U]^T[M][U]\{d^2y/dt^2\} + [U]^T[K][U]\{y\} = \{0\} \tag{10}$$

The matrices $[U]^T[M][U]$ and $[U]^T[K][U]$ are diagonal, as long as [M] and [K] are symmetric positive definite. [4] The system of equations is now six decoupled differential equations and can be solved easily. The response to a harmonic excitation, {F}, with a frequency equal to a system natural frequency will be infinite because there is no dissipation. [5] Damping is needed to get a meaningful frequency response.

A computer solution to this eigenvalue problem, Equation (8), can be obtained by standard eigenvalue solution techniques. One of these is the International Mathematics and Statistical Library (IMSL) Fortran subroutine EIGZF. [6]

The forced damped vibration problem, Equation (5), cannot in general be solved in the same manner as above. Unless [C] and [D] are some linear combination of [M] and [K], they will not be diagonalized by the modal matrix, [U]. Thus, a transformation must be made to facilitate finding a solution. The transformation to be used here is shown in Equation (11).

$$\{z\} = \{dx/dt, x\} \tag{11}$$

The structural damping will be included as a complex stiffness term [2] where {x} will be assumed to be of the same form as the harmonic excitation.

$$\{x\} = e^{i\omega t} \tag{12}$$

Substitute Equations (11) and (12) into Equation (5) to get

$$\begin{bmatrix} [0] & [M] \\ [M] & [C] \end{bmatrix} \{dz/dt\} + \begin{bmatrix} -[M] & [0] \\ [0] & [K]+\frac{i}{\pi}[D] \end{bmatrix} \{z\} = \begin{bmatrix} \{0\} \\ \{F\} \end{bmatrix} \quad (13)$$

This is simplified to

$$[M]z \{dz/dt\} + [K]z \{z\} = \{F\}z \quad (14)$$

where $[M]_z$ and $[K]_z$ are twelve by twelve matrices and {Z} and $\{F\}_z$ are twelve by one vectors. There are twice as many DOF in this first order problem as were in the second order problem above. The upper six DOF are three translational velocities and three angular velocities. The lower six DOF are three translations and three rotations.

To solve Equation (14), first look at the homogeneous equation with $\{F\}_z$ equal to {0}. Then assume a solution to be of the form

$$\{z\} = \{B\} e^{(\lambda t)} \quad (15)$$

Substitute this in and rearrange to obtain

$$[K]\{B\} = -\lambda[M]\{B\} \quad (16)$$

Equation (16) is in a form ready to be solved by the IMSL Fortran subroutine EIGZC. [6] The eigenvalues, λ, are complex. The imaginary part can be associated with the natural frequency and the real part can be associated with the damping present. The eigenvector, {B}, is a twelve by one complex vector representing the mode shape. [2]

When viscous damping is present but there is no structural damping, the twelve eigenvalues and eigenvectors will each appear in six complex conjugate pairs, $\lambda_i = (R \pm jI)$. The real and imaginary parts of the eigenvalue are represented by R and jI. If structural damping is the only damping present, the eigenvalues and eigenvectors appear as if pure imaginary complex conjugate pairs were rotated through the same angle, $\lambda_i = \pm(-R + jI)$. A combination of the above two effects will be observed if both viscous and structural damping are present. In all cases, for small damping the natural frequency values change only slightly from the undamped case.

To solve for the forced response, first form another modal matrix, [V], by letting the columns of [V] be the eigenvectors, {B}. Then make the transformation

$$\{z\} = [V]\{p\} \tag{17}$$

Substitute Equation (17) into Equation (14) and premultiply by $[V]^T$ to obtain

$$[M]p \; \{dp/dt\} + [K]p \; \{p\} = \{F\}p \tag{18}$$

The matrices [M]p and [K]p are diagonal and Equation (18) is a set of decoupled first order ordinary differential equations.

To solve for the forced response of one of these differential equations, choose a representation for the harmonic forcing function.

$$\{F\}p = \{f\}\ e^{(i\omega t)} \tag{19}$$

Then assume a solution to the differential equation of the same form.

$$\{p\} = \{P\}\ e^{(i\omega t)} \tag{20}$$

Substitute Equation (20) into Equation (18) and rearrange to obtain

$$P = \frac{f/m}{i\omega-\lambda}\ e^{(i\omega t)} \tag{21}$$

where P is the response for one of the twelve DOF and λ is defined by Equation (16). The response in the reference coordinate system, {x}, is obtained by using the previous transformations, Equations (12) and (17). This response is a complex value with the magnitude equal to the square root of the sum of the squares of the real and imaginary parts. The phase is the arctangent of the angle found from the imaginary divided by the real part of the response.

The magnitude of the modal response, P, is effected significantly by the method of normalizing or scaling the eigenvectors. Normalizing with the mass matrix means that the equation

$$[U]^T[M][U] = [I] \tag{22}$$

is solved to find the appropriate scale for the ratios between the DOF. This type of normalization gives an indication of the amount of kinetic energy in a mode. The kinetic energy in each mode is proportional to the natural frequency squared as seen by

$$\{U\}_i = \{x\}\ e^{\lambda t} \tag{23}$$

$$\text{Kinetic Energy} = \lambda^2 \{x\}^T [M] \{x\} \tag{24}$$

Normalizing with the stiffness matrix gives an indication of the amount of potential energy in a mode. This is done by solving the equation

$$[U]^T [K] [U] = [I] \tag{25}$$

Here each mode has the same potential energy.

Normalizing to the largest DOF in each mode is useful to graphically display the mode. Since the largest DOF after this normalization is equal to one, all modes have the same amount of deflection during animation.

CHAPTER 4

COORDINATE SYSTEM CONVERSIONS

In Chapter two, the equations of motion were derived for the engine/mounts system. The equations require inertia, stiffness, and damping parameters. These parameters can be defined by experimental testing. Mount stiffness and damping values are measured for the three local orthogonal principle mount directions; compression, lateral, and fore/aft. An engine mount is commonly installed in the reference coordinates (Figure 1) at some angle and location specified by the mount system designer. The stiffness values expressed in the reference coordinate system are needed for Equation 5. This chapter discusses the transformation between values expressed in the local mount coordinates to those expressed in the reference coordinates.

If the compression, lateral, and fore/aft directions of the mount local coordinate system are aligned with the X, Y, and Z directions of the reference coordinate system, no conversion has to be made. As the mount is rotated and orientated differently, the principle stiffness values in the local mount coordinate system do not change, but the stiffness values as viewed in the reference coordinate system do change.

Consider that the tip of the mount is free to move and that the mount can rotate in any direction around its base. Position the mount so that the compression (P), lateral (Q), and fore/aft (R) directions are aligned with the X, Y, and Z directions. (Figure 2)

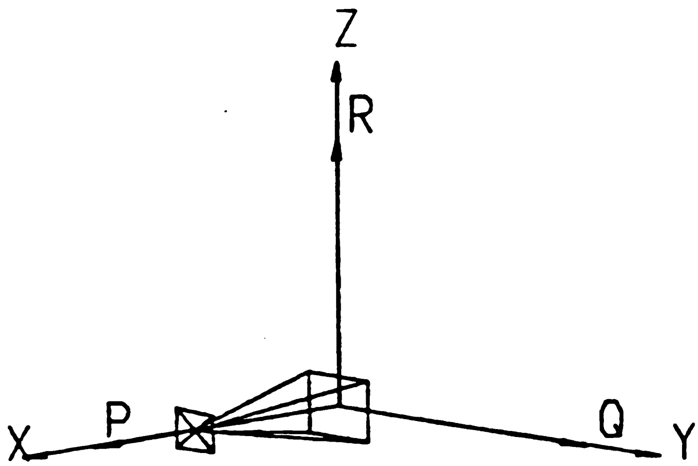


Figure 2 Mount coordinate system

The tip of the mount can be pointed in any desired direction by performing three rotations. Each rotation is additive to the previous one. Many combinations of rotations will put the mount in the same orientation. Once a combination of three rotations is selected, they must be performed in that specific order. Changing the order will generally orientate the mount in another direction.

Euler angles are commonly used for this purpose because they are a unique set for any specific orientation. But because the rotation axes move with each additional rotation, errors can easily occur when prescribing the rotations. For this reason, rotations around the fixed X, Y, and Z reference coordinates (Figure 2) are used here. The rotations are done in the following order: a rotation around the X axis, then a rotation around the Y axis, and finally a rotation around the Z axis. This method is easier to visualize and will orient the P, Q, and R axes in any way desired.

These rotations are accomplished by multiplying by the appropriate three by three direction cosine rotation matrix. The following matrices produce a rotation of the end point of a vector when post-multiplied by that three by one vector.

$$[Rx] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & \sin\theta_x \\ 0 & -\sin\theta_x & \cos\theta_x \end{bmatrix} \qquad (26)$$

Rotation about X

$$[Ry] = \begin{bmatrix} \cos\theta_y & 0 & -\sin\theta_y \\ 0 & 1 & 0 \\ \sin\theta_y & 0 & \cos\theta_y \end{bmatrix} \qquad (27)$$

Rotation about Y

$$[Rz]=\begin{bmatrix} \cos\theta_z & \sin\theta_z & 0 \\ -\sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (28)$$

Rotation about Z

The angles θ_x, θ_y, and θ_z are measured according to the right hand rule around the X, Y, and Z axes respectivly. Several vectors can be transformed at once by letting the vectors be the columns of a coordinates matrix. A combination of these rotations will move a mount local coordinate system from an orientation where the P, Q, and R axes are aligned with the X, Y, and Z axes to one where the P, Q, and R axes are in the correct design orientation. Doing the rotations in the reverse order will do the opposite transformation. Consecutive rotations are performed by consecutive multiplications of these matrices, Equations (26),(27),(28). The desired rotation matrices can be combined into one matrix.

$$[R] = [Rz]\ [Ry]\ [Rx] \qquad (29)$$

The conversion of stiffness values from one coordinate system to another is done as follows. [7]

$$\begin{bmatrix} Kxx & Kxy & Kxz \\ Kyx & Kyy & Kyz \\ Kzx & Kzy & Kzz \end{bmatrix} = [R] \begin{bmatrix} Kpp & 0 & 0 \\ 0 & Kqq & 0 \\ 0 & 0 & Krr \end{bmatrix} [R]^T \qquad (30)$$

The stiffness matrix on the right is evaluated in the local mount coordinate system and is diagonal because it contains only the principal mount stiffness values. The stiffness matrix on the left is evaluated in the reference coordinate system and all six values can be non-zero. For instance, if the term Kxz is non-zero there will be a force in the X direction due to a Z displacement. The same procedure is used for transforming damping values.

CHAPTER 5

INTRINSIC PROPERTIES OF INERTIA AND STIFFNESS

This chapter describes properties dependent upon the inertia and stiffness matrices that are useful during the engine mounting design process. Each depends solely on the inertia or stiffness matrix and describes something about the engine/mounts system dynamic or static characteristics. The properties to be discussed are the elastic axes, the torque axis, the static deflection, and the mode dependent mount forces. These are time consuming to calculate manually, but can easily be incorporated into a computer program which can perform the calculations quickly. More detailed derivations of the elastic axes and torque axis calculations are presented in the references cited.

ELASTIC AXES

Ideally, the elastic axes form an orthogonal coordinate system in which the only displacement or angular response due to an applied force or torque is in the same direction as the degree of freedom

(DOF) in which the input force or torque is directed. The center of elasticity is the origin of this coordinate system. With the elastic axes coordinate system, the response will be pure decoupled translational modes and pure decoupled rotational modes. The desired result is to achieve a design in which the elastic axes are aligned with the reference coordinate system about which the input torque acts so that a torque input does not cause a displacement response.

This ideal definition of elastic axes can only occur in planar analysis. In planar problems, motion is described by two translations in a plane and a rotation about an axis normal to that plane. The elastic axes are found so that if a force is applied along one of the axes in the plane, only a translation along that axis will result. If a torque is applied around the axis normal to the plane, only a rotation around that axis will result. Elastic axes can be easily found for planar problems by finding the coordinate system in which the flexbility matrix is diagonal.

The elastic axes are found by using the general flexibility matrix, [A], which is the inverse of the stiffness matrix, [K]. The flexibility matrix post-multiplied by a force vector, {F}, is equal to a shape vector, {x}, describing the location and orientation of the body using the six DOF.

$$\{x\} = [K]^{-1}\{F\} = [A]\{F\} \tag{31}$$

Planar problems have a flexibility matrix given by

$$[A] = \begin{matrix} * & * & * \\ * & * & * \\ * & * & * \end{matrix} \qquad * = \text{non-zero terms} \tag{32}$$

The DOF needed are two translations describing motion in the plane and a rotation about an axis normal to the plane. The planar flexibility matrix, [A], evaluated in the elastic axes coordinates would be diagonal. (Equation 33)

$$[A] = \begin{matrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{matrix} \qquad * = \text{non-zero terms} \tag{33}$$

There is not a clear analogy between the planar problem and the general three dimensional problem with six degrees of freedom. In the latter case, the flexibility matrix is a six by six symmetric matrix.

$$[A] = \begin{matrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{matrix} \qquad * = \text{non-zero terms} \tag{34}$$

Changing six DOF can introduce at most six symmetric zeroes, a total of twelve zeroes. [8] Because there are thirty off diagonal terms, coupling is always present. A decision must be made as to what kind of partial decoupling is acceptable or most useful.

One choice would be to maximize decoupling between the translational modes and the rotational modes by diagonalizing or possibly zeroing out rows or columns of the off diagonal sub-matrices. The diagonalization method [8] used to obtain Equation 35 will be discussed here. In the diagonalization method, if a force is applied along one of the coordinate axes, the only resultant rotation will be around that axis combined with a translation along an axis which in general is not one of the coordinate axes. If a torque is applied around one of the coordinate axes, the only resultant translation will be along that axis combined with a rotation about an axis which is in general not one of the coordinate axes.

$$[A] = \begin{matrix} * & * & * & * & 0 & 0 \\ * & * & * & 0 & * & 0 \\ * & * & * & 0 & 0 & * \\ * & 0 & 0 & * & * & * \\ 0 & * & 0 & * & * & * \\ 0 & 0 & * & * & * & * \end{matrix} \qquad (35)$$

$*$ = non-zero terms

This is not analogous to the planar analysis, which one could attempt to imitate in the three dimensional problem by assuming infinite stiffness normal to a plane containing two of the reference axes. Obviously, this does not represent the problem as accurately as the three dimensional elastic axes.

TORQUE AXIS

The torque axis is defined as that axis about which the engine will tend to rotate due to a torque T applied around the Y axis, which is the crankshaft axis in this problem. It is calculated [9] solely as a property of the principle inertia axes centered at the center of mass and their geometric relationship to the reference coordinate system. No effects of stiffness are considered.

The computations involved find the reactions around each of the principle inertia axes due to the components of the torque T in terms of the principle inertia values and the inertia direction cosines. An axis about which these reactions add up to a pure rotation with no translation is then found. This is the torque axis relative to the principle inertia axes. It is then transformed to the reference coordinate system for interpretation by the user.

STATIC DEFLECTION

An important question to the engineer long before any physical parts are made is whether the designed engine package will fit into the engine compartment. The engine mounts will be compressed because of the weight of the engine and will allow the engine to be displaced to a static deflection condition. The mount spring rates and mounting locations could be such that this static deflection collides with the engine compartment. Also, the maximum travel of the engine mounts could be reached.

To find the static deflection, the following problem is solved.

$$\{x\} = [A]\{F\} \tag{36}$$

$\{x\}$ is the six by one static displacement vector of the center of gravity. $[A]$ is the six by six flexibility matrix built up from all the engine mounts. $\{F\}$ is the six by one force vector and has as its only non-zero element the vertical weight of the engine. The displacement vector, $\{x\}$, locates the static position of the engine.

MOUNT FORCES

Each of the six mode shapes of the engine may be associated with forces transmitted through the engine mounts because of the displacement of the elastic elements involved. These forces are useful in themselves and can be used as input to other models, for example, a model of the vehicle structure.

The forces transmitted through a mount are found by building up the six by six stiffness matrix using only the stiffness values for that mount and solving

$$\{F\} = [K]\{x\} \tag{37}$$

This is done for each of the six eigenvectors, {x}. The result is a force vector containing three component forces and three component torques. In this case, the component torques at each engine mount location are zero because the mounts were assumed to have zero rotational stiffness. It is useful to normalize the component forces for each mode to the largest value. The absolute magnitudes may vary depending on how the eigenvectors are normalized, but the relative magnitudes will stay the same.

CHAPTER 6

EXAMPLE PROBLEM

The development of an engine/mounts simulation and the properties obtainable from the stiffness and inertia matrices have been discussed in this thesis. This chapter will present an example combining these ideas into an interactive computer program to solve a practical problem.

The Albert H. Case Center for Computer Aided Design, a part of the College of Engineering at Michigan State University, has recently finished a joint project with the Oldsmobile Division of General Motors Corporation. [10,11,12] A partial list of the goals of this project were to

1. Develop an interactive computer engine mounting design tool.
2. Predict forces transmitted from the engine to the vehicle structure.
3. Develop methods to explain why accepted engine mounting design concepts of the past were not working on current transverse front wheel drive designs.

These goals were accomplished by developing a computer program entitled ENGSIM, or rigid body engine dynamics simulation.

ENGSIM is an interactive, "user friendly" computer program. It asks questions which prompts the user to reply and in this way, leads the user through the program without requiring detailed knowledge of the program structure or the analysis performed. ENGSIM includes the various calculations discussed previously in this thesis into a single program allowing many design questions to be investigated with this one program package. The output relies heavily on computer graphics and animation. An example run of ENGSIM is presented in Appendix A.

The input data to the program describes the configuration of the engine and mounts being simulated. It includes the engine inertia data and the mounts spring rates, damping, location, orientation, and number. Both viscous and structural damping may be input. The values used in this example were measured experimentally by Oldsmobile.

The output of the simulation includes engine vibrational modes and natural frequencies, engine static deflection, elastic and torque axes, and the normalized mount forces for all modes. Mode shapes can be animated. The input data can be modified interactively and the resultant output viewed to determine the effects of the change.

A comparison of this analytical model with experimentally obtained results was conducted by Oldsmobile. A modal test was done on a V-6 diesel engine mounted in a cradle by four engine mounts. The cradle was attached directly to a bed plate. Other connections such as hoses, exhaust system, and the driveline were disconnected. A shaker located near the front transmission mount provided random excitation. The experimental modal data was analyzed with a Hewlett Packard Structural Analyzer (HP5423A) to determine the natural frequencies and mode shapes with the output stored on cassette tape. The results are shown in Table 1.

TABLE 1

Comparison of predicted and measured engine natural frequencies.

NATURAL FREQUENCIES

MODE	COMPUTER SIMULATION	EXPERIMENTAL TESTING	DIFFERENCE
1	4.47 Hz	4.17 Hz	7.2%
2	5.97 Hz	5.66 Hz	5.5%
3	7.48 Hz	6.47 Hz	15.6%
4	9.87 Hz	8.76 Hz	12.7%
5	12.26 Hz	12.47 Hz	-1.7%
6	16.46 Hz	----	----

The natural frequencies in Table 1 show that the computer prediction and the experimental values correlate well except for the sixth mode. The experiment did not find a sixth mode. The mode shape predicted for the sixth mode by the simulation has a node very near the point on the engine where the shaker was placed in the experimental tests. (Figure 3) This may have resulted in insufficient excitation of the sixth mode in the experimental tests.

Figure 4 compares the fifth mode shapes from the simulation and from the experiment. The same general shape is seen here and were also seen for the other modes. Differences in natural frequency and mode shape could be caused by the difficulty in accurately determining the stiffness and damping values of the engine mounts. Only the compression stiffness values could be easily obtained because of the test procedure used. Lateral and fore/aft values were arrived at by using empirical ratios for mounts of similar geometry and durometer. Differences could also be attributed to error in the assumption of small motion made in the simulation.

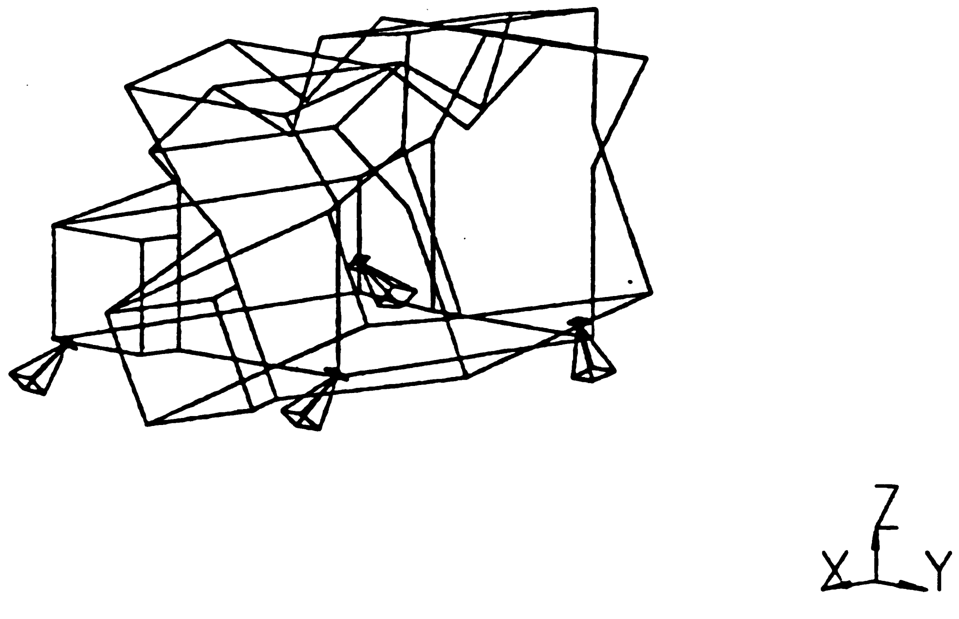


Figure 3 Predicted sixth mode of engine/mounts

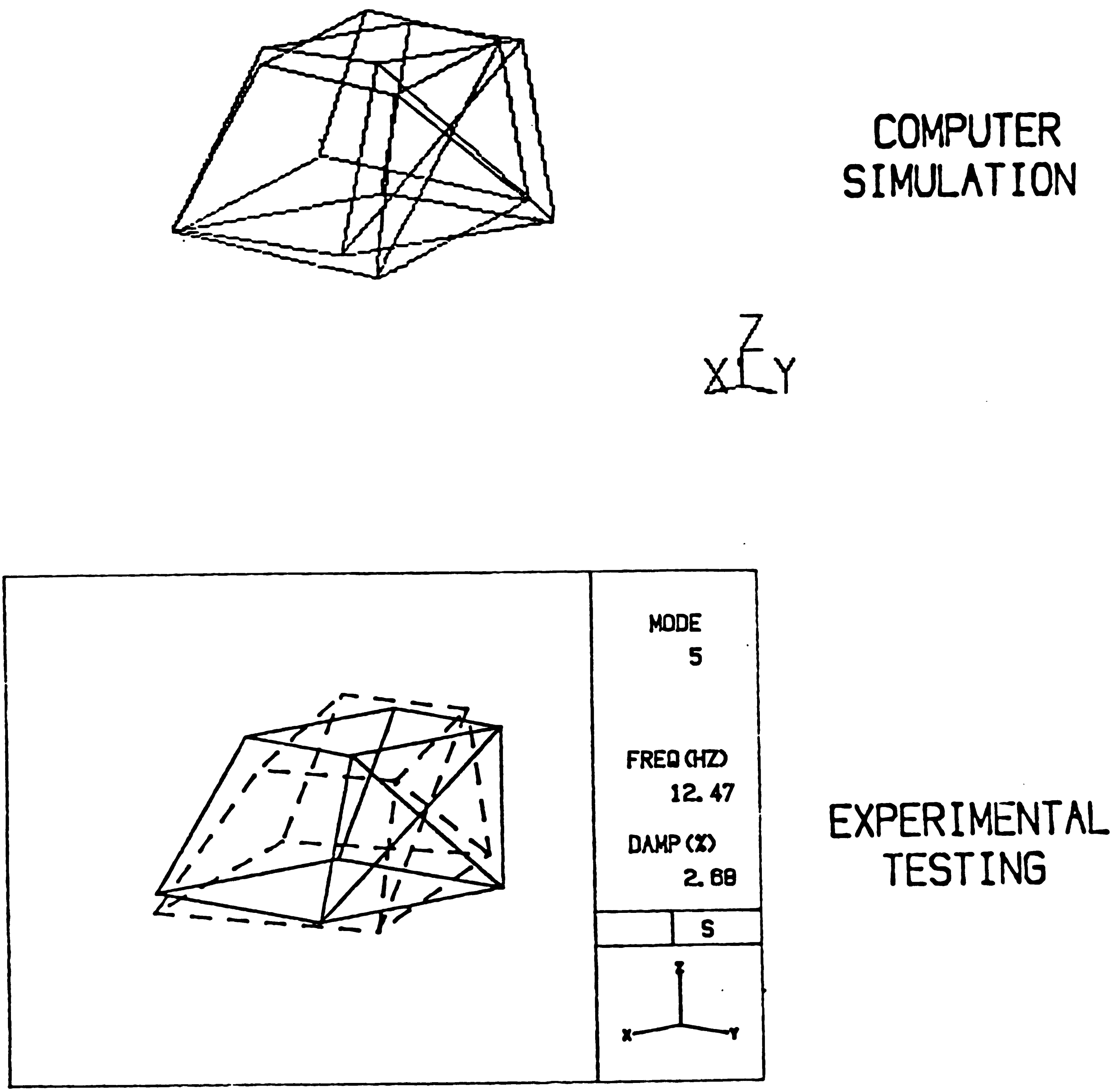


Figure 4   Predicted and measured fifth mode

CHAPTER 7

CONCLUSIONS

This thesis has developed the analysis for a computer simulation of the dynamics of a rigid body engine on elastic mounts tied to ground. The results of the simulation are the natural frequencies, mode shapes, static deflection, mount forces, elastic axes, and torque axis, of the engine/mounts system. The simulation predictions of natural frequencies and modes shapes for a specific V-6 diesel engine were then compared to experimentally measured data. The two results correlate within about ten percent.

The analysis is based on linearized equations of motion which are incorporated into a "user friendly" interactive computer program. The program asks questions of the user and leads one through without requiring detailed knowledge of the analysis being performed. Data input to the simulation include the engine inertia and the engine mount spring rates, damping, location, orientation, and number. Both viscous and structural damping may be entered.

The computer simulation provides the engine mount designer with direct visual information on the specific engine mounts which pass the largest idle shake forces to the structure. This is done with plots and animations of the engine/mounts system mode shapes and is useful when optimizing a mount design. Using the animation and natural frequencies provided by the simulation, the designer can minimize mount deflections likely to excite vehicle structural modes.

The three dimensional elastic axes analysis is a better approach than a two dimensional analysis. The two dimensional definition requires inaccurate assumptions which are not needed in the three dimensional problem.

The simulation is valuable when designing engine mounting packages and can be used separately from any vehicle structural model to aid in optimizing mounting designs. The early specification of mount locations is useful for improving the idle shake isolation of future vehicle models. This will allow better mount locations to be designed into the vehicle structure and reduce the physical testing required.

APPENDIX

APPENDIX A

USING THE ENGINE DYNAMICS SIMULATION ENGSIM

This appendix presents an example run of the interactive rigid body engine dynamics simulation, ENGSIM, which was developed to incorporate the analysis discussed in this thesis. An input file will be shown along with a complete run of ENGSIM showing the program prompts, the user answers, and the program output. Before trying to run the program, a user should have a run image of ENGSIM, an input file with the input data, and a geometry file used when drawing the pictures. The input and geometry files must be available before starting because ENGSIM asks for them.

The XYZ reference coordinate system with its origin at the center of mass of the engine is shown in Figure A1. This is the coordinate system which mount coordinates given in vehicle global coordinates are converted to. The user must be cautious to convert input data measured in a coordinate system oriented another way to this one in order for the results to make sense.

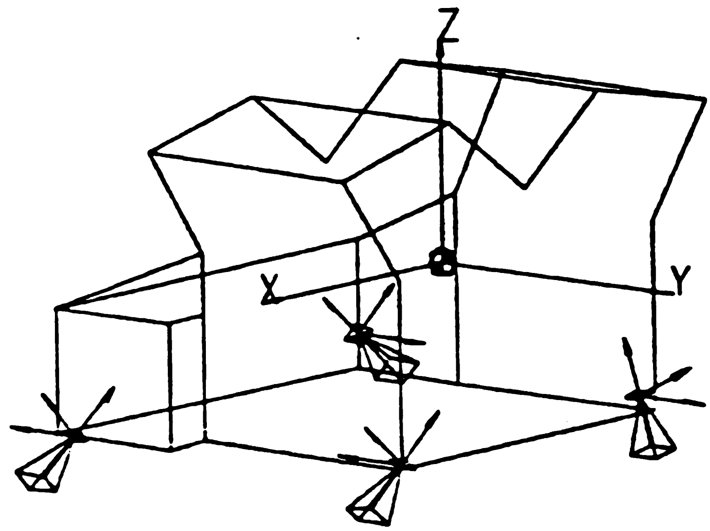


Figure A1 Engine/mounts coordinate system

The following is the form that the input file must be in. This input file name is INPUT.2 and will be asked for by ENGSIM.

```
---------- INPUT.2 ----------
****************************
Oldsmobile engine test stand configuration.
**
*************************
Mass of engine. Kg mass and Kg force are numerically equal.
**
276.7
*************************
Engine center of gravity coordinates.  (X Y Z  Meters)
**
1.5366 .0693 .661
*************************
Number of engine mounts.
**
4
```

```
************************
Engine mount coordinates.(Meters)(X Y Z mount  1, X Y Z mount  2 etc.)
**
1.312  -.24   .462
1.898  -.213  .41
1.342   .21   .432
1.83    .236  .416
************************
Mount Stiffness.
** Compression Lateral Fore/Aft (N/m) ThetaX ThetaY ThetaZ (Degrees)
223667. 44733.  44733.  0.  -45.  0.
170167. 126050.  48619.  0.  -39.  180.
217167. 434334. 108583.  0.  -75.  0.
232167. 464334. 116083.  0.  -45.  180.
******************
Engine mass moment of inertia matrix.  (N-M-SEC2)
**
15.80 -0.80   .9
-0.80 11.64 -3.2
  .90 -3.2  15.69
******************
Direction Cosine Angles to Principal Inertia Axis  (Degrees)
**
17.87  73.91  82.43
100.52 31.07 118.87
104.28 64.19  30.03
******************
Mount viscous damping.  Compression Lateral Fore/Aft   (N-sec/M)
**
45. 100. 150.
45. 100. 150.
45. 100. 150.
45. 100. 150.
******************
Mount structural damping.  Compression Lateral Fore/Aft   (N/M)
**
26000. 25000. 27000.
26000. 25000. 27000.
26000. 25000. 27000.
26000. 25000. 27000.
************************
Number of cradle mounts. (enter 0 for no cradle)
**
6
************************
Cradle mount coordinates.(Meters)(X Y Z mount  1, X Y Z mount  2 etc.)
**
1.125 -.541   .511
1.125  .541   .511
2.041 -.4265 .3495
2.041  .4265 .3495
2.168 -.584   .3495
2.168  .584   .3495
```

```
*************************
Cradle stiffness (N/M) (X Y Z mount  1,X Y Z mount  2 etc.)
**
144000 280000 400000 0 0 0
144000 280000 400000 0 0 0
250000 520000 950000 0 0 0
250000 520000 950000 0 0 0
250000 520000 950000 0 0 0
250000 520000 950000 0 0 0
***************************
EOF
```

The mass of the engine, mass moment of inertia matrix, and direction cosine angles to principal inertia axes were obtained from Oldsmobile. The inertia values, I, are entered as

$$\begin{matrix} IX'X & IX'Y & IX'Z \\ IY'X & IY'Y & IY'Z \\ IZ'X & IZ'Y & IZ'Z \end{matrix} \qquad (A1)$$

where XYZ is the reference coordinate system in Figure A1 and X'Y'Z' are the principal inertia axes of the engine. The inertia direction cosine matrix, L, locates the X'Y'Z' axes relative to the XYZ reference coordinate system. They are entered as

$$\begin{matrix} LX'X & LX'Y & LX'Z \\ LY'X & LY'Y & LY'Z \\ LZ'X & LZ'Y & LZ'Z \end{matrix} \qquad (A2)$$

The coordinates locating the mounts and the center of mass were obtained from drawings. Mount stiffness and damping values were obtained from an elastomer dynamic rubber test. Remember, the coordinate system used must conform to the reference coordinate system.

Mounts are orientated by performing three rotations. First, assume that the compression axis (P), lateral axis (Q), and fore/aft axis (R) are aligned with the XYZ axes respectively as in Figure A2.

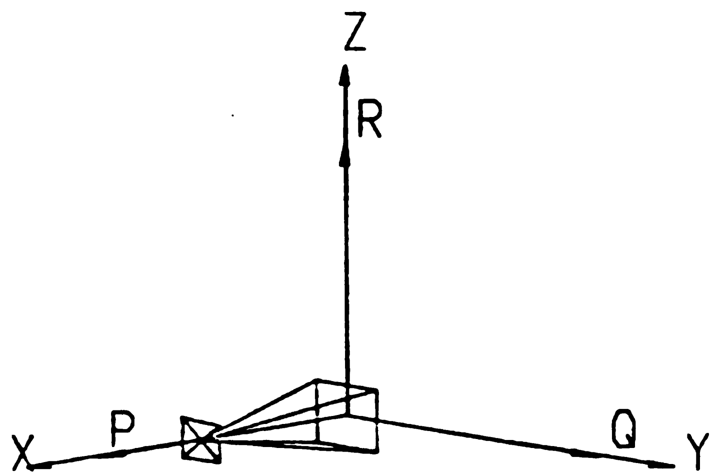


Figure A2 Mount coordinate system

The object is to orient the PQR axes in the design orientation of the mount. The rotations are done as follows: (1) rotate around the X axis (2) rotate around the Y axis and (3) rotate around the Z axis. They must be performed in this order. An example of rotating zero degrees around the X axis ($\Theta x=0$), then adding a rotation of minus sixty degrees around the Y axis ($\Theta y=-60$), and then adding a rotation of twenty degrees around the Z axis ($\Theta z=20$) is shown in Figures A2,A3,A4.

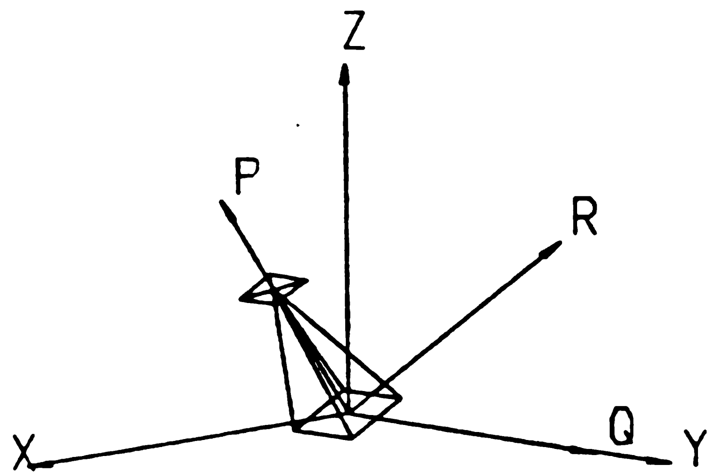


Figure A3 **Additional mount rotation of $\Theta y= -60$**

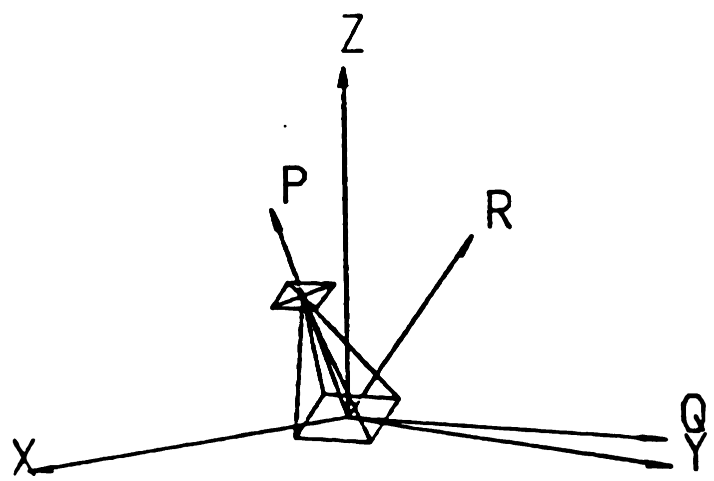


Figure A4 **Additional mount rotation of $\Theta z= 20$**

Positive rotations are determined according to the right hand rule. Three rotations such as this can orientate a mount in any way desired.

Another file used is the geometry file that contains the geometry for drawing pictures of the engine. The first value is the number of points, N, in the picture. The next N lines are the X, Y, and Z coordinates of the points measured from the C.G. of the engine. The next value is the number of entries, M, describing how to connect the points. The next M values are point numbers. If a point number is positive, a line will be drawn from the current cursor position to the coordinates of that point number. If a point number is negative, the cursor will move to those coordinates without drawing a line. The following file is called GEOENG and will be used to draw a V-shaped block resembling an engine. Distances here are measured in meters.

```
---------- GEOENG ----------
29
 .2934   .1667  -.251
-.1946   .1667  -.251
-.1946  -.137   -.251
-.2246  -.3093  -.251
     .3614  -.3093  -.251
     .3614  -.137   -.251
     .2934  -.137   -.251
     .2934   .1667   .251
-.1946   .1667   .251
-.1946  -.137    .05
 .2934  -.137    .05
-.2246  -.3093  -.051
     .3614  -.3093  -.051
     .3614  -.137   -.051
     .2934 -.1370   -.051
     .2934  .1667    .05
 .3934  .1667    .225
 .1934  .1667    .277
 .0494  .1667    .15
-.0946  .1667    .277
```

```
     -.2946  .1667     .225
     -.1946  .1667     .05
       .2934 -.137     .05
       .3934 -.137     .225
 .1934 -.137      .277
 .0494 -.137      .15
-.0946 -.137      .277
-.2946 -.137      .225
     -.1946 -.137      .05
46
 -1    2    3    4    5    6    7    1
 16   17   18   19   20   21   22    2
 -7   23   24   25   26   27   28   29
  3  -17   24  -18   25  -20   27  -21
 28  -10   12    4  -12   13    5  -13
 14    6  -14   15  -13   11
```

A sample run of ENGSIM is presented next. Program prompts and output are indented and user typed answers shown underlined.

```
SEG #ENGSIM
                         M   M SSSSS U   U
                         MM MM S     U   U
                         M M M SSSSS U   U
                         M   M     S U   U
                         M   M SSSSS UUUUU
                             ENGSIM

                   RIGID BODY ENGINE DYNAMICS SIMULATION

** Rev. 7.1  JUL 26,1982 **

  ENTER NAME OF FILE WITH MOUNT GEOMETRY,STIFFNESSES,and DAMPING
INPUT.2

 WEIGHT (NEWTONS)  2711.66        MASS (KILOGRAMS)   276.70
 COORDINATES (METERS) GLOBAL                          LOCAL
                  X        Y        Z         X        Y        Z
  C.G.         1.5366   0.0693   0.6610    0.0000   0.0000   0.0000
  MOUNT 1      1.3120  -0.2400   0.4620   -0.2246  -0.3093  -0.1990
  MOUNT 2      1.8980  -0.2130   0.4100    0.3614  -0.2823  -0.2510
  MOUNT 3      1.3420   0.2100   0.4320   -0.1946   0.1407  -0.2290
```

```
  MOUNT 4     1.8300   0.2360   0.4160      0.2934   0.1667  -0.2450
 MOUNT STIFFNESS  (NEWTONS/METER)
       COMPRESSION LATERAL FORE/AFT          THETAX   THETAY   THETAZ
  MOUNT 1 223667.   44733.   44733.           0.0     -45.0      0.0
  MOUNT 2 170167.  126050.   48619.           0.0     -39.0    180.0
  MOUNT 3 217167.  434334.  108583.           0.0     -75.0      0.0
  MOUNT 4 232167.  464334.  116083.           0.0     -45.0    180.0
 MOUNT DAMPING (N-sec/M)    VISCOUS                         STRUCTURAL
       COMPRESSION LATERAL FORE/AFT    COMPRESSION LATERAL FORE/AFT
  MOUNT 1      45.0    100.0    150.0        26000.0  25000.0  27000.0
  MOUNT 2      45.0    100.0    150.0        26000.0  25000.0  27000.0
  MOUNT 3      45.0    100.0    150.0        26000.0  25000.0  27000.0
  MOUNT 4      45.0    100.0    150.0        26000.0  25000.0  27000.0

  DO YOU WANT TO CHANGE ANY OF THESE VALUES ENTER Y OR N
N

  ENTER COMPREHENSIVE LEVEL OF OUTPUT     (MINIMUM= 1  MAXIMUM= 4)
1

  MASS MATRIX EQUALS...
  276.70      0.00      0.00      0.00      0.00      0.00
    0.00    276.70      0.00      0.00      0.00      0.00
    0.00      0.00    276.70      0.00      0.00      0.00
    0.00      0.00      0.00     15.80      0.80     -0.90
    0.00      0.00      0.00      0.80     11.64      3.20
    0.00      0.00      0.00     -0.90      3.20     15.69

  STIFFNESS MATRIX EQUALS...
546210.00        0.04     -875.04  -16746.68  -62636.55   30629.07
     0.04  1069450.50        0.02  253764.56      -0.02   87221.59
  -875.04        0.02  614975.25  -10264.08   -9948.91   16746.70
-16746.68   253764.56  -10264.08   89904.45     569.74   20711.11
-62636.55       -0.02   -9948.91     569.73   42328.55    2635.68
 30629.07    87221.59   16746.70   20711.11    2635.68  104834.89

  CENTER OF ELASTICITY (Meters)  (Measured from the C.G.)
       X          Y          Z
    0.034135  -0.023090  -0.182742

  ELASTIC CENTER ROTATION MATRIX
   -0.786903   0.615756  -0.040357
    0.565569   0.745833   0.351944
   -0.246812  -0.254121   0.935150

  X Y Z  ON TORQUE AXIS FROM C.G.
   -0.33      5.25     -1.09

   ENTER VALUE FOR (DYNAMIC/STATIC) STIFFNESS
1.4
```

```
STATIC DEFLECTIONS ARE ... (METERS)
              X         Y         Z       THETAX   THETAY   THETAZ
  C.G.     -0.00039  0.00046 -0.00628 -0.00235 -0.00211  0.00126
  MOUNT  1  0.00041 -0.00029 -0.00603
  MOUNT  2  0.00049  0.00032 -0.00486
  MOUNT  3 -0.00009 -0.00033 -0.00702
  MOUNT  4 -0.00009  0.00025 -0.00606

Choose the MODE SHAPE NORMALIZATION method
MASS---------(MA)      STIFFNESS---(ST)      LARGEST DOF--(LD)
LD

THE MODE SHAPES ARE ...    (normalized to largest DOF)
   0.080     0.414     -.137     -.093     0.078     0.018
   -.300     0.107     0.058     -.018     -.117     0.180
   0.029     0.050     1.000     -.004     0.029     0.005
   1.000     -.220     -.021     -.134     -.609     1.000
   0.163     1.000     0.015     1.000     -.708     -.491
   0.019     -.195     -.243     0.392     1.000     0.987
NATURAL FREQUENCIES ... (CYCLES/SEC)
   4.47      5.97      7.48      9.87      12.26     16.46

Choose one of the options by entering the two letters.
MODE SHAPES--(MS)  ELASTIC AXIS---(EA)  RESTART PROGRAM---(RS)
MOUNT FORCES-(MF)  FREQ RESPONSE--(FR)  RESTART W/NU DATA-(NU)
CHG NORM-----(NO)  NEW INPUT FILE-(IN)  QUIT--------------(QU)
```

SEG #ENGSIM was entered to run the program. The title header appeared and the user was asked for the name of the input file. The user entered INPUT.2 and ENGSIM read in that file. Then this information was displayed in tabular form. The coordinates were displayed in both the global vehicle coordinates and the local coordinates with origin at the center of mass. ENGSIM performs all calculations using the local coordinate system. Engine mount stiffness, damping, and orientation were also displayed. Any of these input design parameters may be changed within ENGSIM to see the effect on program output. Changing parameters at this point does not change the input file. The comprehensive level of output determines how much output the user will see at the terminal. Level 1 prints out what is considered the basic useful information. Higher levels print out more data.

At this point, ENGSIM begins forming matrices and computing output. The mass, stiffness, viscous damping, and structural damping matrices were formed and an eigenvalue problem was solved for engine natural frequencies and mode shapes. The location and orientation of the elastic axes and torque axis were found and printed out.

In order to calculate static deflection of the engine due to its weight on the mounts, the user was then asked to enter the ratio of dynamic to static stiffness rate for all mounts. The dynamic stiffness rate is entered in the input file above, but the static rate must be known to calculate static deflections. The deflection of the center of mass in terms of the six degrees of freedom and the deflection of the engine mounts in terms of the X, Y, and Z coordinates of the local coordinate system was printed out.

Next the user was asked to select the type of mode shape normalization to be used. The mode shapes can be normalized to the mass matrix, the stiffness matrix, or the largest degree of freedom. Normalizing to the mass matrix scales by the amount of kinetic energy in each mode. Normalizing to the stiffness matrix results in all modes having equal potential energy. Normalizing to the largest degree of freedom means that the largest entry in the mode shape will always be equal to one after normalization.

The mode shapes and natural frequencies for the six undamped free vibration modes are printed out next. Modes are shown from lowest frequency to highest frequency. The six entries in the mode shapes represent the six degrees of freedom. They are, from top to bottom, translations along the X, Y, and Z coordinate axes and rotations around the X, Y, and Z axes. Several options are available to the user at this point.

Two letter acronyms are entered by the user to indicate the option selected. Selecting the MODE SHAPES options allows the user to view any of the six mode shapes from any viewpoint and with the user defined input geometry representing the engine. The mode shape may be animated depending on the type of output terminal device being used.

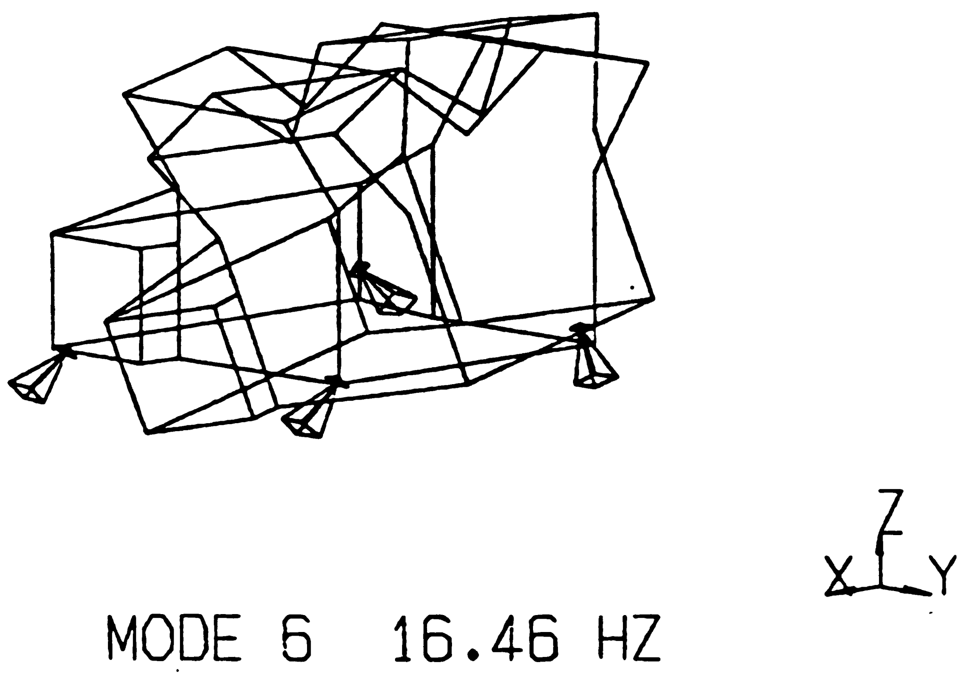
The MOUNT FORCES option will print out the forces at each engine mount caused by the maximum motion of a mode shape. The X, Y, and Z components and the resultant magnitude are shown for each mount force. For each mode, the values are normalized to the largest entry. Therefore, the user can find the largest values and know at which engine mount and in which direction the significant forces are expected.

The CHG NORM option simply allows the user to go back to the prompt asking what type of normalization procedure to use. The program procedes from that point then.

The NEW INPUT FILE command writes the input data currently being used in ENGSIM to a file with the proper form to be used as a new input file at a later time. RESTART PROGRAM begins ENGSIM again, but uses the same input data. It allows the user to go back and change a value for a coordinate or stiffness etc. RESTART W/NU DATA also begins ENGSIM again. This time though, the user can read in a new input file. QUIT will close all files and exit ENGSIM. Examples of some of these options are shown below.

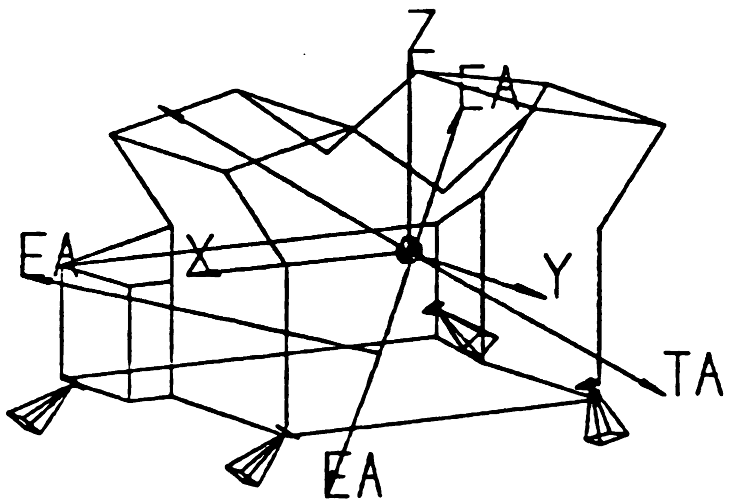
```
Choose one of the options by entering the two letters.
MODE SHAPES--(MS)  ELASTIC AXIS---(EA)  RESTART PROGRAM---(RS)
MOUNT FORCES-(MF)  FREQ RESPONSE--(FR)  RESTART W/NU DATA-(NU)
CHG NORM-----(NO)  NEW INPUT FILE-(IN)  QUIT--------------(QU)
MF

  NORMALIZED MOUNT FORCES ARE ...
                           FX      FY      FZ     F RESULT
    MODE  1   MOUNT  1   -0.26   -0.09   -0.50    0.57
    MODE  1   MOUNT  2    0.44   -0.10   -0.59    0.74
    MODE  1   MOUNT  3    0.18   -0.59    0.79    1.00
    MODE  1   MOUNT  4   -0.04   -0.42    0.43    0.60

    MODE  2   MOUNT  1    0.65    0.06    0.75    1.00
    MODE  2   MOUNT  2    0.35   -0.03   -0.38    0.52
    MODE  2   MOUNT  3    0.38    0.52    0.64    0.91
    MODE  2   MOUNT  4    0.65   -0.02   -0.76    1.00

    MODE  3   MOUNT  1    0.29    0.02    0.55    0.62
    MODE  3   MOUNT  2   -0.40   -0.02    0.52    0.65
    MODE  3   MOUNT  3    0.07    0.21    0.98    1.00
    MODE  3   MOUNT  4   -0.35   -0.04    0.84    0.91

    MODE  4   MOUNT  1    0.01   -0.08    0.28    0.29
    MODE  4   MOUNT  2   -0.13    0.16   -0.25    0.32
    MODE  4   MOUNT  3   -0.54   -0.76    0.36    1.00
    MODE  4   MOUNT  4   -0.72    0.41   -0.45    0.94

    MODE  5   MOUNT  1    0.38   -0.10    0.28    0.48
    MODE  5   MOUNT  2    0.19    0.06    0.06    0.21
    MODE  5   MOUNT  3    0.03   -0.98   -0.19    1.00
    MODE  5   MOUNT  4    0.03    0.06    0.09    0.12

    MODE  6   MOUNT  1    0.06    0.02   -0.05    0.08
    MODE  6   MOUNT  2    0.17    0.29   -0.10    0.35
    MODE  6   MOUNT  3    0.00    0.28    0.03    0.28
    MODE  6   MOUNT  4   -0.07    0.98    0.17    1.00

 Choose one of the options by entering the two letters.
 MODE SHAPES--(MS)  ELASTIC AXIS---(EA)  RESTART PROGRAM---(RS)
 MOUNT FORCES-(MF)  FREQ RESPONSE--(FR)  RESTART W/NU DATA-(NU)
 CHG NORM-----(NO)  NEW INPUT FILE-(IN)  QUIT--------------(QU)
MS
  IS THIS TO BE THE COLOR VERSION  ?
  ENTER YES, NO
NO
```

ENTER THE GEOMETRY DISPLAY FILE
GEOENG
ENTER THE MODE NUMBER TO BE SHOWN
6
ENTER VIEWPOINT
10 10 3
ENTER THE PLOT FILE NAME
PLOT1.CS
ENTER THE PLOT TITLE
MODE 6 16.46 HZ
DO YOU WANT TO DRAW IN THE MOUNTS YES OR NO ?
Y



DO AGAIN YES OR NO ?
N
CHANGE THE VIEW POINT YES OR NO ?
N
DO ANOTHER MODE YES OR NO ?
N

```
 Choose one of the options by entering the two letters.
 MODE SHAPES--(MS)  ELASTIC AXIS---(EA)  RESTART PROGRAM---(RS)
 MOUNT FORCES-(MF)  FREQ RESPONSE--(FR)  RESTART W/NU DATA-(NU)
 CHG NORM-----(NO)  NEW INPUT FILE-(IN)  QUIT--------------(QU)
EA
   IS THIS TO BE THE COLOR VERSION YES OR NO ?
N
   ENTER THE GEOMETRY DISPLAY FILE
GEOENG
   ENTER VIEWPOINT
10 12 5
   ENTER THE PLOT FILE NAME
PLOT2.CS
   ENTER THE PLOT TITLE
ELASTIC and TORQUE AXIS
   DO YOU WANT TO DRAW IN THE MOUNTS YES OR NO ?
Y
```



ELASTIC & TORQUE AXES

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