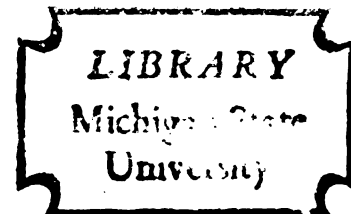


A DISCRETE STATE DETERMINISTIC SYSTEM MODEL
FOR ANALYSIS OF THE FIRM

Thesis for the Degree of Ph. D.
MICHIGAN STATE UNIVERSITY
B. LINN SOULE
1967



This is to certify that the
thesis entitled
A DISCRETE STATE DETERMINISTIC SYSTEM MODEL
FOR ANALYSIS OF THE FIRM
presented by
B. LINN SOULE

has been accepted towards fulfillment
of the requirements for
Ph.D. degree in MANAGEMENT

Date April

w-137



ABSTRACT

A DISCRETE STATE DETERMINISTIC SYSTEM MODEL FOR ANALYSIS OF THE FIRM

by B. Linn Soule

Techniques originally developed to model and analyze complex systems of interacting physical components have recently been adapted for use in analyzing economic systems. These techniques can be applied to any phenomenon which can be identified as a collection of components interacting at clearly defined interfaces, as long as the behavioral characteristics of these components can be described mathematically in terms of common flow and/or propensity variables. The vehicle for analysis is a system model composed of a set of state equations and an output vector.

The advantages of this methodology are twofold:

(1) precise procedural techniques are available for combining the individual component models into an equation set suitable for analysis, and (2) well developed analytical techniques can be applied to the resulting system model. The system model is formulated from two structural features of the system: (1) the equations describing the unconstrained characteristics of the components of the system, and (2) the constraints imposed on the system by the interconnection pattern of its

components as derived from a linear graph of the system. The state model is a set of difference equations characterizing the internal states of the system as a function of the inputs, and the output model is a set of algebraic equations giving the system response as a function of the internal states and the inputs.

Analysis of the system, as represented by the system model, provides knowledge of its dynamic characteristics. The limiting characteristics of the system can be examined for stability, strategies can be derived to achieve specified future objectives, and sensitivity analysis performed by means of computer simulations.

This research applies these physical systems techniques to a typical business firm. The resulting model depicts the firm as a system of interacting flows of funds driven by the sales of its products. The firm is perceived as a medium for accumulating the resources necessary to produce sufficient goods to satisfy this sales driver. The system is patterned after a real firm and parameter values are derived from its financial reports. The system as modeled is shown to be both stable and controllable. Operation of the firm is simulated on the computer and the sensitivity of profit and debt to selected parameter changes is studied.

The study shows that a business firm can be modeled and analyzed with these physical techniques. Controllability is

shown to be a viable concept for application to the business system. The validity and value of the analysis is determined by the precision and detail with which the system is modeled, which in turn depends on the descriptive detail of the component models. More research is needed in the area of component modeling; better techniques for modeling the components of economic systems are necessary for this system analysis methodology to become operational.

A DISCRETE STATE DETERMINISTIC SYSTEM MODEL
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By

B. Linn Soule

A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Management

1967

646147
9/27/67

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1967

ACKNOWLEDGMENTS

The author wishes to acknowledge the assistance of the many persons who have so generously contributed both substantively and implicitly to the program of study and research culminating in this dissertation. Both Professor Gonzalez, chairman of the research committee, and Professor Mossman, skillfully guided the research effort and provided the encouragement so essential to satisfactory completion of a program of this type. The contribution of Professor Koenig, both as a source of methodology and a continuing guide and evaluator, is gratefully acknowledged. The aid of the industrial firm which supported the research both financially and with business data is greatly appreciated. The continuing nature of the research and the proprietary nature of various data preclude public disclosure of the specific firm at this time. The continued encouragement of Clarence F. Hyde throughout the course of this program and the unstinting tolerance and devotion of my family have been vital to the completion of this work.

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CHAPTER I

INTRODUCTION

Throughout history attempts to describe and analyze the natural phenomena of the universe have resulted in comprehensive mathematical descriptions of the phenomenon under study. These scientific contributions to the general knowledge about nature have increased the efficiency with which man adapts his environment to his needs. The advent of high speed digital computers and the promulgation of viable quantitative techniques have provided scientists with many powerful tools for the analysis and design of various types of complex systems. This thesis applies some of these tools of analysis to a business system in order to project future effects of various changes in policy and system structure. This effort is consistent with the generally increased emphasis on scientific approaches to business problems.

In the last three decades, the use of scientific methods and quantitative techniques in business has accelerated noticeably. The trend of modern managers has been to abandon many rule-of-thumb practices in favor of more scientific methods. This increased effort to quantify business factors has led to a rather recent cleavage of management personnel. Minas succinctly summarized this cleavage between the

management scientist, whom he called the formalist, and the manager who uses heuristics as his dominant guide, in the following manner:

The formalist is usually very much oriented towards mathematics. . . . The realist is apt to rely more heavily on "raw" observations than on formal devices such as mathematics; he goes in for what might loosely be called "clinical" procedures.¹

The management scientist has borrowed heavily from other disciplines and sciences in a continuing effort to optimize business performance. McGuire attempts to validate this interdisciplinary approach in the following statement:

Theories originally applicable in alien situations, which were not meant specifically to be used to explain business activities, often have provided insights and explanations useful in business. If such "borrowed" theories--whatever their origin--prove useful in explaining or predicting business behavior, then, by all means, let us borrow them.²

Material borrowed from another discipline or science can be viewed as consisting of three classes: (1) content, (2) techniques, and (3) theory.³ The content of a science, consisting of specific observations and measurements, is the most specific of these three classes, and can be borrowed quite easily, provided that care is used in transferring it across the boundary separating the two disciplines.

¹J. Sayer Minas, "Formalism, Realism and Management Science," Management Science, III (October, 1956), p. 12.

²Joeseeph W. McGuire, Interdisciplinary Studies in Business Behavior (Cincinnati, Ohio: South-Western Publishing Company, 1962), p. 8.

³Michael Halbert, The Meaning and Sources of Marketing Theory (New York: McGraw-Hill Book Company, 1965), pp. 10-15.

Techniques include both measurement and analysis; borrowing from this class is often labeled adaptation of methodology.⁴ Techniques have been broadly adapted and extensively utilized by the management scientist in pushing back the frontiers of business knowledge. The main thrust of this thesis is adaptation of these two classes of material to the entity known as the business firm.

The sheer scope and complexity of the operations of a large business firm contribute significantly to sub-optimization in decision making. The concept of "system" is a viable one in the physical universe, and should provide the manager of an economic system with a powerful and comprehensive tool. The pervasive nature of the system concept is aptly presented by Boulding's comments on the emergence of a general theory of a system:

General Systems Theory is the skeleton of science in the sense that it aims to provide a framework or structure of systems on which to hang the flesh and blood of particular disciplines and particular subject matters in an orderly and coherent corpus of knowledge.⁵

Systems Concepts

Engineers and physical scientists have long been able to describe with mathematical precision the characteristics of

⁴Ibid.

⁵Kenneth E. Boulding, "General Systems Theory--The Skeleton of Science," Management Science, II (April, 1956), p. 208.

most of the components of the systems with which they deal. The hydraulic concepts of pressure and flow, and the electrical concepts of voltage and current have provided the physical scientist with the common denominator for study of their system components and a means for characterizing and classifying component behavior. Using the scientific knowledge of components thus generated, the engineer has been able to synthesize systems designed to meet specific objectives.

In the nineteenth century Kirchoff⁶ developed a methodology for analyzing electrical networks with known physical components by calculating currents and voltages in all parts of the network. Kirchoff's analytical methods provided a rigorous methodology for formulating, for any electrical network, a system of mathematical equations called a model. He introduced the basic concepts of linear graph theory for model formulation, which have since been formalized by mathematicians such as Whitney⁷ and electrical engineers such as Seshu and Reed.⁸ They have recently been generalized by

⁶G. Kirchoff, "Über die Auflösung der Gleichungen, auf welche man bei der Untersuchungen der Linearen Verteilung Galvanischer Ströme geführt wird," English translation, Transaction of the Institute of Radio Engineers, CT-5 (March, 1958), pp. 4-7.

⁷H. Whitney, "Non-separable and Planar Graphs," Transactions of the American Mathematical Society, XXXIV (1932), pp. 339-362.

⁸Sundaram Seshu and Myril B. Reed, Linear Graphs and Electrical Networks (Reading, Massachusetts: Addison Wesley Publishing Company, 1961).

Koenig, Blackwell,⁹ and others, in their application to essentially the entire class of physical systems. These concepts coupled with more recent developments form much of the foundation for a discipline known as Systems Science.

The system model utilized in analysis and control is formulated from two major structural features of the system under study: (1) the equations describing the unconstrained characteristics of the components of the system, and (2) the constraints imposed on the system by the interconnection pattern of its components as derived from a linear graph of the system. The sets of equations characterizing these two features of the system together constitute the system model and form the basis for analysis of the system.

A study of the system model can lead to considerable insight regarding the operational characteristics of the system. In particular, the stability of the system can be examined to determine its limiting characteristics and strategies can be derived to achieve a specified objective at a future point in time.

The applicability of this discipline to socio-economic systems has been advocated for several years by Professor Koenig through a series of graduate courses in Systems Analysis for Social Scientists offered by him in the College of

⁹Herman E. Koenig and William A. Blackwell, Electro-mechanical System Theory (New York: McGraw-Hill Book Company, 1961).

Engineering at Michigan State University. His class notes¹⁰ from these courses are the primary reference source for this thesis.

Research Design

The focus of this study is a large modern corporation manufacturing and selling a consumer product in the United States. This corporation serves as the pattern for the system model and as a source of data for analysis. The firm is viewed in its most basic form as an economic system which collects the capital assets required to produce and sell an economic good. The basic objectives of the study are threefold:

1. Develop a model of the firm as a system of interacting components or sectors.
2. Examine the system for stability and controllability.
3. Describe quantitatively the way in which the firm reacts to its environment, and trace the time path of its key variables by computer simulation.

The first step in developing a system model is to identify the system, that is, within the level of aggregation desired, the components of the system must be identified and their characteristics described mathematically. In this study the firm is perceived as a complex system of financial transactions which take place both internally and externally.

¹⁰Herman E. Koenig, Unbound class notes for the three course sequence entitled "Systems Analysis for Social Scientists" (College of Engineering, Michigan State University, 1965-66 school year). (Mimeographed.)

These resulting flows of money accumulate pools of assets at various places in the firm. The flows are evaluated at discrete points in time, and the dynamic characteristics of the process are presented by a set of difference equations called a state model.

Data for empirical determination of parameter values are obtained by analysis of the firm's financial reports. Sensitivity of the model to a selected set of parameters is investigated by simulating operation of the firm for twenty-five years and introducing various incremental changes in the coefficients of the component equations. The transition matrix of the state model is examined to determine system stability, and a general strategy for controlling the system state is calculated. A large scale digital computer is used to calculate the specific system values required for analysis.

Thesis Organization

This thesis follows the natural evolution of the research it relates; beginning with a brief description of the systems analysis methodology, proceeding to development of the model and its use for analysis, and ending with the conclusions drawn from the research.

A system model of the firm is developed in Chapter II using the methodology outlined in this chapter. Chapter II begins with a linear graph of the system conceptualized to represent the "anatomy" of the business operation. Component

behavior is modeled mathematically and system constraints applied, resulting in two sets of equations: (1) a set of difference equations characterizing the internal states of the system as a function of the inputs, and (2) a set of algebraic equations giving the output as a function of the internal states and the inputs.

Chapter III relates the effort to derive numerical values for the system model matrices, and the results of the stability and control analysis. Several computer simulations are described in Appendix B, and selected results are shown graphically.

The Bibliography represents a comprehensive and thorough search of the literature. Because of the uniqueness of the particular methodology used in this study there is a dearth of directly pertinent literature. However, much material felt to be of a generally related and constructively pertinent nature is included.

The complete computer program used for analysis, along with a brief description, is listed in Appendix A, and Appendix C contains a selected sample of computer output.

CHAPTER II

MODEL OF THE FIRM

The system being examined in this study is a typical firm operating in the United States. A macro view is taken of the firm and its money flows are aggregated into a minimum set of decision parameters.

Structure of the Firm

The firm as conceived consists of two basic sectors: (1) production, and (2) finance. The production sector is assumed to have three interfaces with its environment (the surrounding economy) and the finance sector has two. The production sector provides the salable goods that result in a flow of funds into the firm in the form of cash sales. The flow at one of the two remaining environmental interfaces represents the cost of operating the firm, and the other is the payment of federal income taxes. The production sector of the firm reduces the gross sales income by the sum of the cash cost of operation and federal taxes, and passes the remainder to the finance sector, in the form of profit and depreciation. It is assumed to possess exactly the proper quantity of fixed assets to satisfy production requirements.

The finance sector of the firm provides the assets with which the production sector manufactures the goods the firm sells, and maintains sufficient working capital to support current operations. It has two sources of funds from which to meet these requirements: (1) cash flow from the production sector, and (2) environmental sources. It is assumed to provide exactly the quantity of fixed assets and working capital required, and the funds available from the environmental sources are without limit; that is, they make available whatever amount of funds the finance sector demands. Let the structure of the firm as perceived above be represented by the graph shown in Figure 1, having ten edges as indicated.

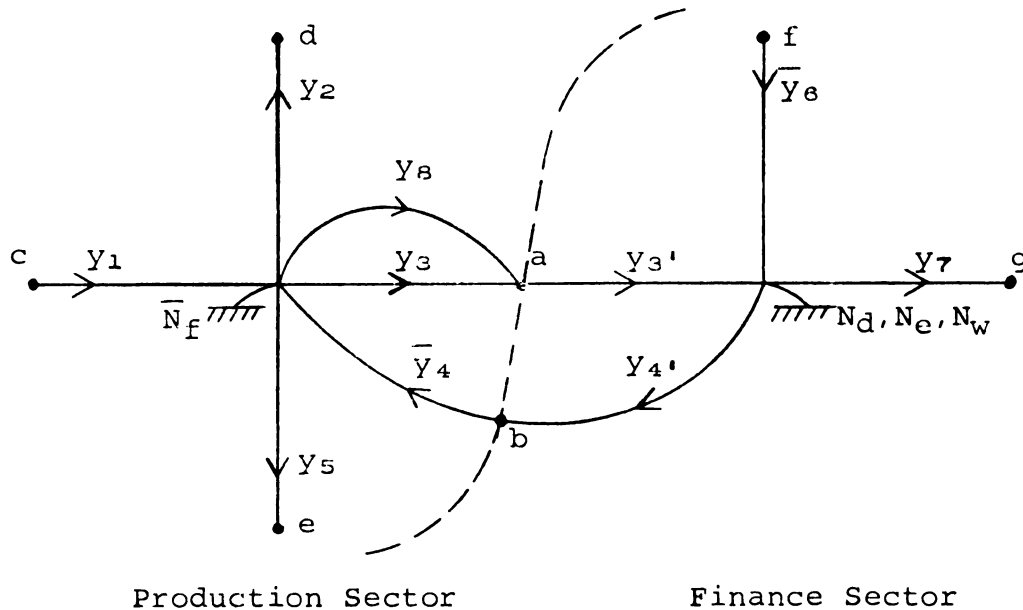


Figure 1.--System graph of the firm depicting input-output flows and the interconnection pattern of the firm's two sectors.

The arrows show the assumed directions of flow of goods or funds. Reversed flow will be indicated in the solution set by negative values. Each of the end edges 1, 2, 5, 6, and 7, represent flows to or from the economic environment, and the remainder are intra-firm flows.

All edge flows are measured in units of dollars per year. The flows y_i , $i = 1, 2, 3, 3', 4, 4', \dots, 8$, identified by the edges of the system graph are specifically

y_1 = Sales	$y_{4'}$ = Capital expenditure
y_2 = Federal income tax	y_5 = Cash cost of operation
y_3 = Net profit	$\bar{y}_6$ = Flow of outside funds
$y_{3'}$ = Cash flow	y_7 = Dividends
$\bar{y}_4$ = New capital items	y_8 = Depreciation

Each of these flows can, if desired, be regarded as a vector.

The fixed assets of the production sector are represented by the variable $\bar{N}_f$, which is called an internal state of the production sector. Similarly the variables N_d , N_e , and N_w representing respectively, debt, paid in capital, and working capital are internal states of the finance sector. All of these internal states are measured in dollar units. Specifically, these internal states, which also can be regarded as vectors if desired, are defined as

N_d = Total corporate debt
N_e = Total paid in capital
$\bar{N}_f$ = Net fixed assets
N_w = Net working capital

Standard accounting procedures are used for measuring financial flows and balances. Since the year is a commonly used time interval in accounting practice, and as a planning horizon, all flows refer to an aggregate flow over a time period of one year, and the variable n is used to represent points in time at regular twelve month intervals. The state variables are measured at the beginning of the interval n . The set of variables defined above represents the critical items found in a firm's Balance Sheet, Profit and Loss, and Source and Application of Funds statements, usually presented in the annual financial report.

In more generalized models representing actual flows of goods or physical units of resources, a complementary variable which reflects unit values or prices can be introduced. This variable, analogous to the physical concept of propensity, or pressure, must satisfy the generalized Kirchoff circuit postulate.¹ In applications which utilize this concept it is possible to relate policy parameters to prices as well as to flows. In such a development the dollar value is the product of the two variables. No attempt is made in this study to include these effects.

¹An excellent reference for explanation of this postulate and amplification of general systems analysis methodology is the book by Koenig, Tokad, and Kesavan entitled Analysis of Discrete Physical Systems (New York: McGraw-Hill, 1967). Chapters 4 and 5 are especially appropriate for economic systems.

Sector Models

A model of the system is developed by first considering each sector of the firm independent of any constraints imposed by the interconnections between the sectors; that is, strictly in terms of the variables associated with that sector. The model so constructed can also be used in the context of other constraints with equal validity. The mathematical equations characterizing the unconstrained behavior of the sectors of the system follow.

Edge 1. This is the edge in the system representing the annual sales income of the firm. It is considered to be the flow driver for the system and is therefore taken as the general function of time

$$y_1(n) = F(n) \quad (2-1)$$

Edge 2. This edge represents the annual federal income tax paid by the firm. The tax is based on the gross profit of the firm and is assessed at a rate T . Since gross profit is determined by subtracting total cost of operations from sales income, the tax component equation is

$$y_2(n) = T[y_1(n) - y_5(n) - y_8(n)] \quad (2-2)$$

Edge 3. Net profit is represented in the system by edge 3. Net profit equals gross profit minus tax and the resulting equation is

$$y_3(n) = y_1(n) - y_2(n) - y_5(n) - y_8(n) \quad (2-3)$$

Edge 4. The flow on this edge is the total annual expenditure for various capital items used in the production process. Due to the extreme range of types of assets and wide differences in their characteristics, it is worthwhile to consider this edge as a vector. The capital assets of the firm are considered to be of three basic types: (a) tools, (2) machinery, and (3) buildings. These are represented respectively as components y_{41} , y_{42} , y_{43} of the vector $\bar{y}_4$. This equation attempts to capture the management decision process regarding the procurement of capital goods, and appears as

$$\bar{y}_4(n) = \begin{bmatrix} y_{41}(n) \\ y_{42}(n) \\ y_{43}(n) \end{bmatrix} = 1/P \begin{bmatrix} R_1 \\ R_2/K_2 \\ R_3/K_2 \end{bmatrix} y_1(n+1) - \begin{bmatrix} 0 \\ N_{f2}(n) \\ N_{f3}(n) \end{bmatrix} \quad (2-4)$$

Product obsolescence requires the firm to continuously re-tool, based on predicted sales volume in the $n+1$ time period, regardless of present tooling capacity. The tooling component of the capital expenditure vector is therefore determined entirely by the sales forecast and the size of the production parameter R_1 . The coefficient P is the selling price of the product and converts the sales dollar volume into physical units. The coefficient R_i is the number of dollars per production unit spent for capital production goods of the i^{th} type; it converts the dollar value of the i^{th} type of fixed

asset into production units. The states N_{f2} and N_{f3} are respectively net fixed machinery assets and net fixed building assets, and K_2 is the percentage of maximum machine capacity utilized in normal operations.

The vector components of $\bar{N}_f$ represent the dollar value of different classes of capital assets which can be converted into units of production capacity by a representation such as N_{fi}/R_i ; this relates the dollar value of the i^{th} type of asset to the maximum number of units it can produce. When this maximum value is operated on by K_i it is converted to a normal, or desired utilization level. The capital expenditure needed for the i^{th} type of asset can be expressed basically as the difference between present capacity and forecasted sales volume. This difference, when operated on by R_i and K_i results in the dollar expenditure required to equate that asset's need with its present value. The basic form for the machinery component in this system is

$$y_{42}(n) = R_2/K_2 [y_1(n+1)/P - (K_2/R_2)N_{f2}(n)] \quad (2-4a)$$

The decision rule for adding to building capacity is based on the concept of slack between present capacity to contain machinery and anticipated machinery needs in the $n+1$ period. If present building capacity is greater than the predicted machinery need at $n+1$, there is no need for building addition at time n . The basic form for the building component is

$$y_{43}(n) = R_3 [y_1(n+1)/K_2P - N_{f3}(n)/R_3] \quad (2-4b)$$

Simplifying these equations and adding y_{41} yields the resulting component equation 2-4.

Edge 5. Cash cost includes all operating expenses except depreciation, and in this system is visualized as consisting of three basic components: (1) fixed costs, (2) variable costs, and (3) interest expense. Interest expense is kept as a separate item so that the impact on cost of various debt positions can be analyzed. The fixed cost portion is related to the scale of plant involved in the production process and the variable cost is a function of production volume. Applying an average interest rate to the outstanding debt yields the interest cost for the period. Since this model considers three types of fixed assets, these three separate types are included in the model as

$$y_5(n) = C_{11}N_{f1}(n) + C_{12}N_{f2}(n) + C_{13}N_{f3}(n) \\ + C_2y_1(n) + R_6N_d(n) \quad (2-5)$$

Edge 6. The flow of funds between the firm and the environment is a vector represented by edge number six. The two components of the vector are: (1) debt, represented by y_{61} , and (2) equity, represented by y_{62} . Outside funds are needed whenever those generated internally from operations are not sufficient. The funds generated internally during period n are called cash flow and are represented by edge 3' in the system graph. Cash flow enters into the working capital pool N_w , and the amount of N_w at time n is the sum

total of internal cash available to the firm. Basically there are two uses of funds within the firm: (1) capital expenditures of the three types, and (2) cash for operating expenses during the period. Operating expenses are a function of the sales level during the period and capital expenditures are determined by the flow on edge four. In this model outside tooling funds are selected entirely from debt sources while funds for machinery and buildings are divided between debt and equity. When additional working capital is required, this need is allocated between debt and equity also. It is only the change in N_w that is allocated in this manner, the final composition of the pool cannot be specified. The resulting equation for capital expenditure is therefore

$$\begin{aligned} \begin{bmatrix} \bar{y}_{61}(n) \\ \bar{y}_{62}(n) \end{bmatrix} &= \begin{bmatrix} 1 & F_1 & F_2 \\ 0 & 1-F_1 & 1-F_2 \end{bmatrix} \begin{bmatrix} y_{41}(n) \\ y_{42}(n) \\ y_{43}(n) \end{bmatrix} \\ &+ \begin{bmatrix} G \\ 1-G \end{bmatrix} W_{Y_1}(n) - \begin{bmatrix} G \\ 1-G \end{bmatrix} N_w(n) \end{aligned} \quad (2-6)$$

Edge 7. Dividend payments to the owners of the firm are based strictly on the amount of cash flow during the period. This relationship is formulated as

$$y_7(n) = D_5 y_3'(n) \quad (2-7)$$

Edge 8. Depreciation in the firm is based on the value of the fixed assets measured at the beginning of period n and

the capital expenditures during the period. The acquisition of assets throughout the year is assumed to be normally distributed with a mean depreciation period of six months; consequently, depreciation can be calculated on new assets at one-half the annual rate for its class. When the three classes of assets have depreciation rates D_1 , D_2 , and D_3 , and the declining balance method of depreciation is used, the depreciation equation is

$$\begin{aligned}
 y_8(n) = & \begin{bmatrix} D_1 & D_2 & D_3 \end{bmatrix} \begin{bmatrix} N_{f1}(n) \\ N_{f2}(n) \\ N_{f3}(n) \end{bmatrix} \\
 & + \begin{bmatrix} D_1/2 & D_2/2 & D_3/2 \end{bmatrix} \begin{bmatrix} y_{41}(n) \\ y_{42}(n) \\ y_{43}(n) \end{bmatrix} \quad (2-8)
 \end{aligned}$$

As this system is perceived there are six internal state variables: (1) net debt, (2) total paid in capital, (3) net working capital, (4) net tooling assets, (5) net machinery assets, and (6) net building assets. These internal states appear as variables in equations 2-4, 2-5, 2-6, 2-8, and can be expressed as a function of the flows as shown next.

Net debt. In determining the debt position of the firm it is assumed that some fixed percentage of the outstanding debt is repaid each period. Any new debt incurred is then added to give the resultant net debt in the $n+1$ time period.

The resulting difference equation is

$$N_d(n+1) = (1 - R_4)N_d(n) + y_{s1}(n) \quad (2-9)$$

Paid in capital. Total paid in capital at $n+1$ is represented simply as the sum of the amount at time n and any new capital paid in during the period, thus

$$N_e(n+1) = N_e(n) + y_{s2}(n) \quad (2-10)$$

Net working capital. If R_5 represents the imputed effective rate of interest earned on the total working capital pool resulting in interest income to the firm,² and R_4N_d is the annual debt installment, the working capital at the $n+1$ period is

$$\begin{aligned} N_w(n+1) = & (1+R_5)N_w(n) + y_{31}(n) + y_{s1}(n) \\ & + y_{s2}(n) - y_{41}(n) - y_{42}(n) - y_{43}(n) \\ & - y_7(n) - R_4N_d(n) \end{aligned} \quad (2-11)$$

The difference equation for this state variable is similar to the accounting statement "Source and Application of Funds." In addition to the entries of that statement, since the difference equation represents the cumulative working capital pool, it includes the value of the pool at the beginning of the time period.

²The total working capital pool and an imputed interest rate are used because the rapid and large changes in the actual amounts producing interest income at any point in time make it impossible to track exactly.

Net fixed assets. The net fixed assets can be represented for the $n+1$ time period as the sum of the depreciated values at the beginning of period n and the properly depreciated expenditures made during the period. The three-component vector of assets is

$$\begin{aligned} \bar{N}_f(n+1) &= \begin{bmatrix} N_{f1}(n+1) \\ N_{f2}(n+1) \\ N_{f3}(n+1) \end{bmatrix} = \begin{bmatrix} 1-D_1 & 0 & 0 \\ 0 & 1-D_2 & 0 \\ 0 & 0 & 1-D_3 \end{bmatrix} \begin{bmatrix} N_{f1}(n) \\ N_{f2}(n) \\ N_{f3}(n) \end{bmatrix} \\ &+ \begin{bmatrix} 1-D_1/2 & 0 & 0 \\ 0 & 1-D_2/2 & 0 \\ 0 & 0 & 1-D_3/2 \end{bmatrix} \begin{bmatrix} y_{41}(n) \\ y_{42}(n) \\ y_{43}(n) \end{bmatrix} \quad (2-12) \end{aligned}$$

System Constraints

When the components of the system are united at their interfaces, the terminal variables of the component models are no longer independent, but are constrained uniquely in a manner determined by the interconnection pattern depicted by the system graph shown in Figure 1. The constraint equations are developed from this graph by summing the flows of funds at the vertices of the graph. The seven interfaces, and the common reference point, in this system result in a total of eight vertices. Since edges 1, 2, 5, 6, and 7, are end edges their corresponding end vertices are eliminated from consideration in the constraint equations. At vertex b the constraint equation is

$$\bar{y}_4(n) - y_{4'}(n) = 0 \quad (2-13)$$

At vertex a the constraint equation is

$$y_3(n) + y_8(n) - y_{3'}(n) = 0 \quad (2-14)$$

These two constraint equations are linearly independent as is evident when they are written in matrix form

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} y_3(n) \\ \bar{y}_4(n) \\ \text{-----} \\ y_{3'}(n) \\ y_{4'}(n) \\ y_8(n) \end{bmatrix} = 0 \quad (2-15)$$

The System Model

When the set of linear constraints in equation 2-15 is substituted into the component models, a system model is obtained. The result after some algebraic reduction is

$$\begin{bmatrix} N_d(n+1) \\ N_e(n+1) \\ N_w(n+1) \\ N_{f1}(n+1) \\ N_{f2}(n+1) \\ N_{f3}(n+1) \end{bmatrix} = \begin{bmatrix} \alpha_1 & 0 & -G & 0 & -F_1 & -F_2 \\ 0 & 1 & \alpha_4 & 0 & \alpha_2 & \alpha_3 \\ \alpha_8 & 0 & R_5 & \alpha_9 & \alpha_{10} & \alpha_{11} \\ 0 & 0 & 0 & \alpha_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\alpha_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\alpha_7 \end{bmatrix} \begin{bmatrix} N_d(n) \\ N_e(n) \\ N_w(n) \\ N_{f1}(n) \\ N_{f2}(n) \\ N_{f3}(n) \end{bmatrix} \\ + \begin{bmatrix} \alpha_{12} \\ \alpha_{13} \\ \alpha_{14} \\ 0 \\ 0 \\ 0 \end{bmatrix} y_1(n) + \begin{bmatrix} \alpha_{15} \\ \alpha_{16} \\ \alpha_{20} \\ \alpha_{17} \\ \alpha_{18} \\ \alpha_{19} \end{bmatrix} y_1(n+1) \quad (2-16)$$

$$\begin{bmatrix} y_2(n) \\ y_3(n) \\ y_{41}(n) \\ y_{42}(n) \\ y_{43}(n) \\ y_5(n) \\ y_{61}(n) \\ y_{62}(n) \\ y_7(n) \\ y_8(n) \end{bmatrix} = \begin{bmatrix} -TR_6 & 0 & 0 & -\beta_1 & -\beta_2 & -\beta_3 \\ \beta_7 R_6 & 0 & 0 & \beta_4 & \beta_5 & \beta_6 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ R_6 & 0 & 0 & C_{11} & C_{12} & C_{13} \\ 0 & 0 & -G & 0 & -F_1 & -F_2 \\ 0 & 0 & \alpha_4 & 0 & \alpha_2 & \alpha_3 \\ D_5 \beta_7 R_6 & 0 & 0 & \alpha_9 D_5 & \alpha_{10} D_5 & \alpha_{11} D_5 \\ 0 & 0 & 0 & D_1 & \alpha_6 & \alpha_7 \end{bmatrix} \begin{bmatrix} N_d(n) \\ N_e(n) \\ N_w(n) \\ N_{f1}(n) \\ N_{f2}(n) \\ N_{f3}(n) \end{bmatrix} \\
 + \begin{bmatrix} T\beta_8 \\ -\beta_7 \beta_8 \\ 0 \\ 0 \\ 0 \\ C_2 \\ \alpha_{12} \\ \alpha_{13} \\ -D_5 \beta_7 \beta_8 \\ 0 \end{bmatrix} y_1(n) + \begin{bmatrix} -\beta_9 \\ \beta_{10} \\ R_1/P \\ R_2/K_2 P \\ R_3/K_2 P \\ 0 \\ \alpha_{15} \\ \alpha_{16} \\ \beta_{11} \\ \beta_{12} \end{bmatrix} y_1(n+1) \quad (2-17)$$

Where the coefficients in this model are determined in the development as

$$\begin{aligned}
\alpha_1 &= 1-R_4 \\
\alpha_2 &= F_1-1 \\
\alpha_3 &= F_2-1 \\
\alpha_4 &= G-1 \\
\alpha_5 &= 1-D_1 \\
\alpha_6 &= D_2/2 \\
\alpha_7 &= D_3/2 \\
\alpha_8 &= R_6(1-D_5)(T-1)-R_4 \\
\alpha_9 &= (1-D_5)(T(C_{11}+D_1)-C_{11}) \\
\alpha_{10} &= (1-D_5)(T(C_{12}+D_2/2)-C_{12}) \\
\alpha_{11} &= (1-D_5)(T(C_{13}+D_3/2)-C_{13}) \\
\alpha_{12} &= GW \\
\alpha_{13} &= (1-G)W \\
\alpha_{14} &= (1-D_5)(1-C_2)(1-T)+W \\
\alpha_{15} &= (1/P)(R_1+F_1R_2/K_2+F_2R_3/K_2) \\
\alpha_{16} &= (1/PK_2)(R_2(1-F_1)+R_3(1-F_2)) \\
\alpha_{17} &= (R_1/P)(1-D_1/2) \\
\alpha_{18} &= (R_2/PK_2)(1-D_2/2) \\
\alpha_{19} &= (R_3/PK_2)(1-D_3/2) \\
\alpha_{20} &= T(1-D_5)/(2PK_2)(D_1R_1K_2+D_2R_2+D_3R_3) \\
\beta_1 &= T(C_{11}+D_1) \\
\beta_2 &= T(C_{12}+D_2/2) \\
\beta_3 &= T(C_{13}+D_3/2) \\
\beta_4 &= (\alpha_9/1-D_5)-D_1 \\
\beta_5 &= (\alpha_{10}/1-D_5)-\alpha_6 \\
\beta_6 &= (\alpha_{11}/1-D_5)-\alpha_7
\end{aligned}$$

$$\beta_7 = T-1$$

$$\beta_8 = 1-C_2$$

$$\beta_9 = \alpha_{20}/1-D_5$$

$$\beta_{10} = \alpha_{20}(T-1)/(T(1-D_5))$$

$$\beta_{11} = D_5 \alpha_{20}/1-D_5$$

$$\beta_{12} = \alpha_{20}/(T(1-D_5))$$

Using matrix notation this model can be written in the form

$$\Psi(n+1) = P\Psi(n) + Q_1 Y_1(n) + Q_2 Y_1(n+1) \quad (2-18)$$

$$R(n) = M\Psi(n) + N_1 Y_1(n) + N_2 Y_1(n+1) \quad (2-19)$$

The matrix P is called the transition matrix, the matrices Q_1 and Q_2 are called excitation matrices, and $R(n)$ is called the output, or response, vector.

Note that the coefficients in the system model are dependent upon, and uniquely determined by, the coefficients in the sector models.

CHAPTER III

PROCESSING THE MODEL

The practical value of any model is determined by the accuracy with which it portrays the behavioral characteristics of the system it represents. The possible studies that can be made by use of the model range from simulation to control. Before any such analyses can be carried out, however, numerical values must be assigned to the coefficients in the final system model. These coefficients are calculated from the numerical values of the coefficients in the component models by the relations following equations 2-16 and 2-17. In this chapter coefficients are evaluated, the stability of the system is investigated, a control strategy is developed, and a simulation program is outlined.

Component Coefficients

The coefficients are evaluated from a data base corresponding to a typical large firm operating in the United States, as taken from its financial statements. The three primary statements used are: (1) Profit and Loss, (2) Balance Sheet, and (3) Source and Application of Funds. Those coefficients that cannot be calculated from these reports are imputed. Coefficients so obtained are given in Table 1.

Table 1.--Component Coefficients

Coeffi- cient	Base value	Definition
C ₁₁	1.978	Overhead cost factor of tooling
C ₁₂	1.978	Overhead cost factor of machinery
C ₁₃	1.978	Overhead cost factor of buildings
C ₂	0.540	Ratio of variable cost to sales
D ₁	0.500	Depreciation rate for tooling
D ₂	0.250	Depreciation rate for machinery
D ₃	0.100	Depreciation rate for buildings
D ₅	0.130	Ratio of annual dividends to cash flow
F ₁	0.400	Ratio of debt to equity in machinery expenditures
F ₂	0.100	Ratio of debt to equity in building expenditures
G	0.600	Ratio of debt to equity in working capital change
K ₁	$\$7.300 \times 10^6$	Slope of linear sales function
K ₂	0.476	Ratio of rated machine capacity to maximum
P	$\$2.500 \times 10^3$	Product unit sales price
R ₁	$\$6.000 \times 10$	Tooling expenditure per production unit
R ₂	$\$1.440 \times 10^2$	Machinery expenditure per production unit
R ₃	$\$3.600 \times 10$	Building expenditure per production unit
R ₄	0.100	Ratio of annual debt repayment to total debt
R ₅	0.005	Interest rate achieved on total working capital
R ₆	0.050	Interest rate on total debt
T	0.480	Federal tax rate on gross income
W	0.160	Ratio of working capital to sales

The initial values of the state variables derived from the financial statements are listed in Table 2.

Table 2.--State Variables

State Variable	Initial Value	Definition
N_d	\$ 4.25 x 10 ⁶	Corporate debt
N_e	\$11.98 x 10 ⁶	Paid in capital
N_w	\$ 9.80 x 10 ⁶	Working capital
N_{f1}	\$ 5.04 x 10 ⁶	Tooling
N_{f2}	\$12.09 x 10 ⁶	Machinery
N_{f3}	\$ 3.02 x 10 ⁶	Buildings

Development of the values for the coefficients in Table 1 is described in the order in which they appear in the eight component equations.

Sales (K_1). Both linear and non-linear sales functions are used in the analysis. The slope of the linear function, K_1 in this model, is determined by a simple linear regression using historical sales data. The number of years used in the regression was chosen to correspond to what is felt to be the "modern era," and would typify the near future characteristics of the firm and its industry.

Federal tax rate (T). The tax rate is given by federal statute. While it has some variation based on gross profit,

for a large firm only very slight error is introduced by considering it to be constant.

Capital expenditure coefficients (K_2, P, R_1, R_2, R_3). Since a machine can run an absolute maximum of one hundred sixty-eight hours per week, the value for K_2 is set by what the firm considers to be its standard work week. The rated capacity for the firm under study was determined by considering a standard work week of eighty hours. The product price P is established by dividing gross sales income by the number of units sold. The figure for capital assets per production unit can be approximated from data contained in the balance sheet, and summary production figures. By comparison of data from similar situations and consultation with business representatives, the total assets of the firm are allocated by type in the following way: sixty percent machinery, twenty-five percent tooling, and the remaining fifteen percent buildings.

Cash cost of operation coefficients ($C_{11}, C_{12}, C_{13}, C_2, R_6$). Depreciation is removed from the total cost of operation to determine actual cash cost. The average rate of interest on the total debt, designated R_6 , can be calculated from the total interest expense figure and total debt. After depreciation and interest expense are subtracted from the total cost of operation, the remaining cost can be allocated between fixed and variable costs. This allocation is forty percent fixed cost and sixty percent variable. Using this fixed cost

figure and the firm's fixed assets, the value of the vector $\bar{C}_1$ can be found. All three types of fixed assets contribute to fixed cost in the same manner, and thus all C_{1i} , $i = 1, 2, 3$, are taken as identical to $\bar{C}_1$. The remaining sixty percent of cost (considered variable) can be expressed as a function of sales, thereby yielding the value of C_2 .

Depreciation rates (D_1, D_2, D_3). This firm uses the declining balance method of depreciation at twice the straight line rate. Tool assets are taken to have an effective life of four years, resulting in a declining balance rate for D_1 of fifty percent per year. The rates for D_2 and D_3 are determined in a similar way, with their lives respectively calculated as eight years and twenty years.

Capital expenditure coefficients (F_1, F_2, W, G). The coefficients F_1 , F_2 and G are complex intra-firm decision parameters and can only be estimated. A value for W can be calculated from the gross sales and net working capital figures.

Dividend rate (D_5). The value for this parameter can be established by relating actual dividends paid with their corresponding cash flows.

Debt retirement rate and interest rate on working capital (R_4, R_5). Two additional coefficients are introduced in the state equations. The debt retirement rate R_4 can be

approximated by comparing the current portion of the long term debt with the total debt of the firm. Current and anticipated money market conditions must also be considered in establishing this coefficient value. It is difficult to arrive at an accurate value for R_5 , the effective rate of interest on total working capital, resulting in interest income. As used in this state model the value of R_5 must be imputed from some estimate of total interest income compared with net working capital.

The base values of the component coefficients as listed in Table 1 are used to calculate base values for the coefficients in the system model. The results are given in Table 3 on the following page.

General Solution

While the discrete form of the state model can be solved for any specific point in time by recursive substitution, this method is tedious for large values of n . It is helpful, both for ease of solution and for system analysis, to show the solution explicitly as a function of time. Applying the linear transformation

$$\Phi(n) = \Psi(n) - Q_2 Y_1(n) \quad (3-1)$$

the state model

$$\Psi(n+1) = P\Psi(n) + Q_1 Y_1(n) + Q_2 Y_1(n+1) \quad (3-2)$$

Table 3.--Definitions and Base Values of Coefficients in the System Model

Coefficient	Definition	Base Value
α_1	$1-R_4$	0.900
α_2	F_1-1	-0.600
α_3	F_2-1	-0.900
α_4	$G-1$	-0.400
α_5	$1-D_1$	0.500
α_6	$D_2/2$	0.125
α_7	$D_3/2$	0.050
α_8	$R_6(1-D_5)(T-1)-R_4$	-0.123
α_9	$(1-D_5)(T(C_{11}+D_1)-C_{11})$	-0.686
α_{10}	$(1-D_5)(T(C_{12}+D_2/2)-C_{12})$	-0.843
α_{11}	$(1-D_5)(T(C_{13}+D_3/2)-C_{13})$	-0.874
α_{12}	GW	0.096
α_{13}	$(1-G)W$	0.064
α_{14}	$(1-D_5)(1-C_2)(1-T)+W$	0.368
α_{15}	$(1/P)(R_1+F_1R_2/K_2+F_2R_3/K_2)$	0.075
α_{16}	$(1/PK_2)(R_2(1-F_1)+R_3(1-F_2))$	0.100
α_{17}	$(R_1/P)(1-D_1/2)$	0.018
α_{18}	$(R_2/PK_2)(1-D_2/2)$	0.106
α_{19}	$(R_3/PK_2)(1-D_3/2)$	0.029
α_{20}	$T(1-D_5)/(2PK_2)(D_1R_1K_2+D_2R_2+D_3R_3)$	0.009
β_1	$T(C_{11}+D_1)$	1.190
β_2	$T(C_{12}+D_2/2)$	1.012
β_3	$T(C_{13}+D_3/2)$	0.972
β_4	$(\alpha_9/1-D_5)-D_1$	-1.288
β_5	$(\alpha_{10}/1-D_5)-\alpha_6$	-1.083
β_6	$(\alpha_{11}/1-D_5)-\alpha_7$	-1.055
β_7	$T-1$	-0.520
β_8	$1-C_2$	0.460
β_9	$\alpha_{20}/1-D_5$	0.011
β_{10}	$\alpha_{20}(T-1)/(T(1-D_5))$	-0.033
β_{11}	$D_5\alpha_{20}/1-D_5$	0.001
β_{12}	$\alpha_{20}/(T(1-D_5))$	0.023

reduces to

$$\bar{\Phi}(n+1) = P\bar{\Phi}(n) + Q_3 y_1(n) \quad (3-3)$$

where the new matrix Q_3 is

$$Q_3 = PQ_2 + Q_1 \quad (3-4)$$

Substituting equation 3-3 recursively into itself, the general solution is

$$\bar{\Phi}(k) = P^k \bar{\Phi}(0) + \begin{bmatrix} P^{k-1} Q_3 & \dots & Q_3 \end{bmatrix} \begin{bmatrix} y_1(0) \\ y_1(1) \\ \vdots \\ y_1(k-1) \end{bmatrix} \quad (3-5)$$

Where $P^{k-1} Q_3 \dots Q_3$ is a matrix of order $j \times k$, j being the order of the state vector Ψ , and k the number of time periods involved. Substituting the linear transformation 3-1 into equation 3-5 gives the solution in terms of the original state vector, as

$$\begin{aligned} \Psi(k) = & P^k \Psi(0) - (P^k Q_2 - P^{k-1} Q_3) y_1(0) \\ & + Q_2 y_1(k) + \sum_{m=1}^{k-1} P^{k-m-1} Q_3 y_1(m) \end{aligned} \quad (3-6)$$

Using this general solution the state vector at any time k can be calculated if the initial state $\Psi(0)$, and the vector of inputs $y_1(n)$, $n=0,1,\dots,k$ are known.

Stability Analysis

There are several ways in which stability of linear time invariant systems can be defined mathematically. One definition is based on the concept that if a stable system is disturbed a small amount from an equilibrium state it should either return to that state or stay within some pre-assigned finite region near the equilibrium state. This definition, given originally by Lyapunov¹ applies to both linear and non-linear systems. The equilibrium states of the linear system $\Psi(n+1) = P\Psi(n)$ are stable in the sense of Lyapunov if:²

1. The magnitude of all the zeroes of the minimal polynomial $m(\lambda)$ of P are less than or equal to one, and
2. All zeroes of the minimal polynomial of magnitude one, if any, are simple.

If either of these conditions is violated the system has an unbounded solution. The solution is asymptotically stable if and only if the magnitudes of all the eigenvalues of P are less than one. With caution the characteristic polynomial can be used for the analysis since it has the same zeroes as the minimal polynomial, although perhaps of different multiplicity.

¹M. A. Lyapunov, "Problem General de las Stabilité du Mouvement," Annals of Mathematical Studies, XVII (Princeton, N. J.: Princeton University Press, 1947).

²Koenig, Tokad and Kesavan, Analysis of Discrete Physical Systems, p. 248.

Stability of the system in this study is investigated by examining the eigenvalues of the P matrix. In general the characteristic polynomial of any real matrix P of order n is of the form

$$D(\lambda) = \begin{vmatrix} \lambda U - P \end{vmatrix} = \lambda^n + p_1 \lambda^{n-1} + p_2 \lambda^{n-2} + \dots + p_n \quad (3-7)$$

Since the coefficients p_i are all real numbers, the zeroes of $D(\lambda)$ are either real or occur in complex conjugate pairs. For the system under study here the characteristic polynomial can be obtained by expansion of the determinant of the matrix

$$[\lambda U - P] = \begin{bmatrix} \lambda - \alpha_1 & 0 & G & 0 & F_1 & F_2 \\ 0 & \lambda - 1 & -\alpha_4 & 0 & -\alpha_2 & -\alpha_3 \\ -\alpha_8 & 0 & \lambda - R_5 & -\alpha_9 & -\alpha_{10} & -\alpha_{11} \\ 0 & 0 & 0 & \lambda - \alpha_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda + \alpha_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda + \alpha_7 \end{bmatrix} \quad (3-8)$$

In factored form the characteristic polynomial is

$$D(\lambda) = [(\lambda - 1)(\lambda - \alpha_5)(\lambda + \alpha_6)(\lambda + \alpha_7)] \\ [\lambda^2 - (R_5 + \alpha_1)\lambda + (\alpha_1 R_5 - \alpha_8 G)] \quad (3-9)$$

Substituting the base values for the parameters from Table 1 and Table 3 into equation 3-9 the eigenvalues are

$$\begin{aligned}
\lambda_1 &= 1 \\
\lambda_2 &= \alpha_5 = .500 \\
\lambda_3 &= -\alpha_6 = -.125 \\
\lambda_4 &= -\alpha_7 = -.050 \\
\lambda_5, \lambda_6 &= (\alpha_1 + R_5)/2 \\
&\quad \pm [\sqrt{(\alpha_1 + R_5)^2 - 4(\alpha_1 R_5 - \alpha_6 G)}] / 2 \\
&= .808, .097
\end{aligned} \tag{3-10}$$

Since the magnitude of all the eigenvalues is less than or equal to one, the system is stable, however it is not asymptotically stable since one eigenvalue is on the unit circle.

Control Strategy

Modern control theory provides a theoretical basis for determining the input controls required to take a system from one state to another in a finite number of time intervals. The controls determined to perform this function are called the control law, or control strategy. The control problem is the inverse of the problem of calculating the system state Ψ at time k as a function of the initial condition $\Psi(0)$ and the time vector of inputs Y_1 . When determining a control strategy the final state is specified along with the initial conditions, and the inputs required to achieve that final state are calculated. Specifically, the system under study is said to be state controllable if an arbitrary state $\Psi(0)$ can be reduced to zero (or any other

value) by a finite vector sequence of inputs $y_1(n)$, $n=1, 2, \dots, k$, in a finite number of time intervals k .

Let the general solution for the state model, equation 3-6, be re-written here in the condensed matrix form

$$\begin{aligned} \Psi(k) = & P^k \Psi(0) - Q^* y_1(0) + Q_2 y_1(k) \\ & + M y_1(i) ; i=1, \dots, k-1 \end{aligned} \quad (3-11)$$

where Q^* represents the six element column vector $P^k Q_2 - P^{k-1} Q_3$, M represents the product matrix $P^{k-2} Q_3 \dots Q_3$ of order six by $k-1$, and $y_1(i)$ is the time vector of inputs of order $k-1$. The matrix M can be converted to order six by k by including Q_2 as its k^{th} column. This adds $y_1(k)$ to Y_1 as the k^{th} row, increasing its order also, and maintaining the conformability of the two matrices. The resulting equation now appears as

$$\Psi(k) = P^k \Psi(0) - Q^* y_1(0) + P^* Y_{1k} \quad (3-12)$$

The matrix P^* is of order six by k and is defined as

$$P^* = [P^{k-2} Q_3 \quad P^{k-3} Q_3 \quad \dots \quad Q_3 \quad Q_2] \quad (3-13)$$

Given the boundary states $\Psi(0)$ and $\Psi(k)$, and the flow driver at time zero, equation 3-12 can be solved for the time sequence of scalar inputs Y_{1k} . The only requirement for solution is that the matrix P^* must be nonsingular. The minimum number of time intervals required for control is determined by the order of the state model. Since the system

model developed in this study is of order six, the state vector can be controlled in six intervals if P^* is non-singular.

To demonstrate the concept of control the strategy required to drive the initial state $\Psi(0)$ to zero in six time intervals is calculated. The computer program listed in Appendix A performs this function for the base coefficient values of Table 1 and initial state value of Table 2. The resulting control strategy is

$$\begin{aligned}
 y_1(1) &= 7.2713 \times 10^8 \\
 y_1(2) &= -1.4102 \times 10^9 \\
 y_1(3) &= 2.8178 \times 10^8 \\
 y_1(4) &= 9.3591 \times 10^7 \\
 y_1(5) &= 4.7499 \times 10^6 \\
 y_1(6) &= 4.7780 \times 10^4
 \end{aligned}
 \tag{3-14}$$

The negative sales in this strategy, forced by selection of a final state of zero, is equivalent to paying customers to take the product of the firm, and of course is not a feasible occurrence. The final state of zero is used only to demonstrate the control principle; in practice, any final state can be specified. The results of this control law are shown in Figures 2 and 3 which trace the system state variables during the six period control cycle. These graphs show that a minimum time control strategy of this type, when implemented, may subject the state variables to extreme variations. Other control techniques are available which minimize these

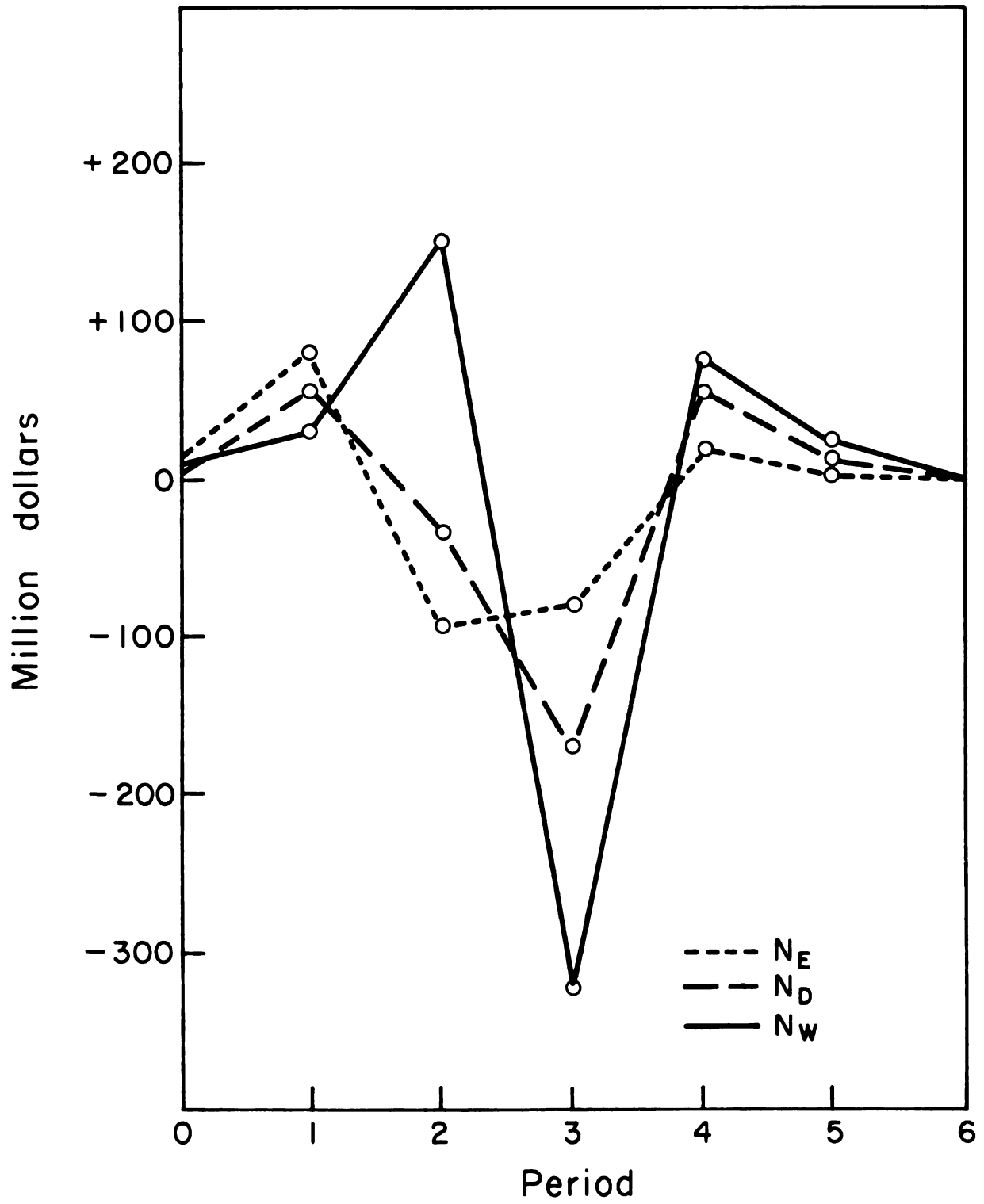


Figure 2.--Time paths of debt, equity and working capital under control law.

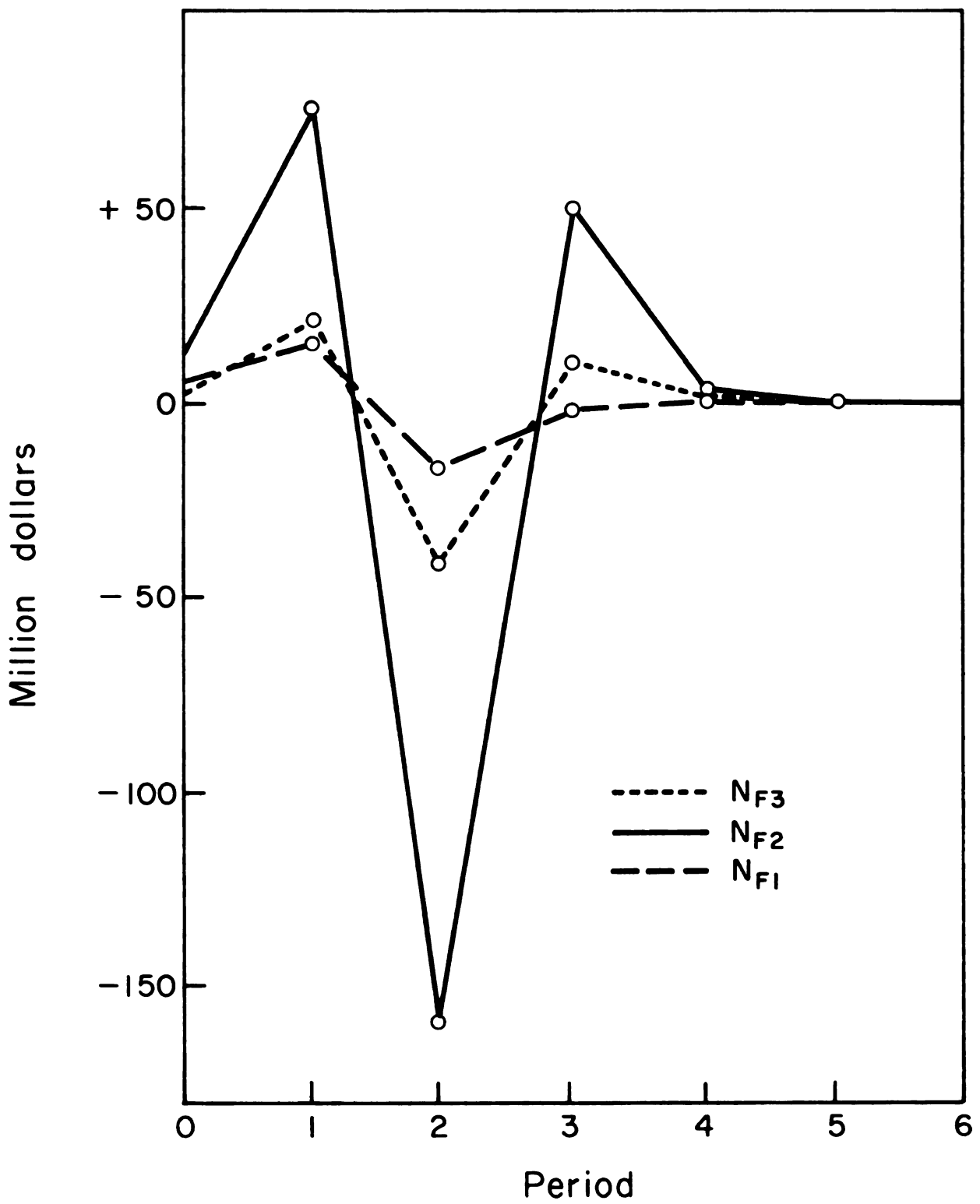


Figure 3.--Time paths of fixed assets under control law.

potentially harmful excursions by extending the control cycle over a longer time period. These optimal techniques which allow boundaries to be placed on the state variables during control are beyond the scope of this thesis.

Using the same base coefficient, and initial state and sales values, a general control strategy can be developed. This strategy is expressed as an explicit function of the desired final state at $k=6$. This strategy is

$$Y_{1k} = (P^*)^{-1} [\Psi(k)] - (P^*)^{-1} [P^k \Psi(0) + Q^* Y_1(0)] \quad (3-15)$$

or numerically

$$Y_{1k} = \begin{bmatrix} 682.5 & -1265.5 & -2.6 & 1332.2 & -437.0 & 3381.1 \\ -946.4 & 1628.2 & 100.3 & -2382.2 & 1564.5 & -7476.9 \\ 163.9 & -275.8 & -79.5 & 756.2 & -1603.4 & 5987.6 \\ 62.0 & -100.4 & 7.3 & 240.8 & 469.7 & -1697.4 \\ 3.6 & -5.2 & 3.6 & 15.6 & 0.5 & -4.1 \\ 0.1 & -0.1 & 0.1 & 0.3 & -1.4 & 39.6 \end{bmatrix} \Psi(k)$$

$$- \begin{bmatrix} 7.2713 \times 10^8 \\ -1.4102 \times 10^9 \\ 2.8178 \times 10^8 \\ 9.3591 \times 10^7 \\ 4.7499 \times 10^6 \\ 4.7780 \times 10^4 \end{bmatrix} \quad (3-16)$$

Simulation

McMillan and Gonzalez³ summarize simulation as an experimental problem solving technique applied to problems of system design or analysis. The simulation performed in this study is a portrayal of the operations of a firm as reflected by a numerical solution to the mathematical model of the firm. Operation of the firm is simulated for twenty-five periods, equivalent to twenty-five years of operation. The initial state conditions of Table 2 are used as the starting point along with sales of 100 million dollars at time zero. Values at discrete points in time are calculated by recursive substitution of the state model of equation 2-16 into itself. Response flows are calculated from the output model of equation 2-17.

Using the base coefficients in Table 1, identical simulations are run for both linear and nonlinear changes in sales with time. The specific functions used are

$$y_1(n) = K_1n + 100 \times 10^6 \quad (3-17)$$

and

$$y_1(n) = (K_1n + 10^8) + [10^7 \sin(2\pi n/5)] + (10^7x) \quad (3-18)$$

³Claude McMillan and Richard F. Gonzalez, Systems Analysis; A Computer Approach to Decision Models (Homewood, Illinois: Richard D. Irwin, Inc., 1965), p. 16.

where 10^7X is a random noise factor. The nonlinear sales function begins with the same zero values as does the linear function, and is a combination of a linear function, a sinusoidal function with a ten year period, and a random noise factor. The noise is normally distributed with unit variance, and both the periodic and random functions have maximum amplitudes of ten million dollars.

The effects on the system of changes in policy or environmental conditions can be simulated as described above. Sensitivity analyses of three system parameters is described in Appendix B and the results are shown graphically.

The computer program used for system analysis and simulation is listed in Appendix A, along with a brief description, and Appendix C contains a selected portion of the computer printout.

CHAPTER IV

SUMMARY AND CONCLUSIONS

The three primary objectives of the study as stated in Chapter I are:

1. Develop a model of the firm as a dynamic system.
2. Examine the system for stability and controllability.
3. Simulate the response of the firm to its environment and policy parameters.

A typical business entity, conceptualized as two interacting sectors characterized by flows of funds, is described in Chapter II. These two sectors are modeled by sets of algebraic and difference equations as though they were unconstrained. System constraints obtained from a graph of the system are applied to these models to derive the state and output equations of the system, referred to collectively as the system model.

In Chapter III numerical parameter values are established for the system model from analysis of the financial reports of a large production firm operating in the United States. The quantitative model resulting from the introduction of this real data is used for stability and control analysis, and computer simulations are prescribed for investigating the sensitivity of system response to parameter changes. The system as perceived is shown to be both

stable and controllable. Specific sensitivity analyses are reported in Appendix B.

The research has successfully demonstrated the application of systems techniques to a specific type of socio-economic system--a business firm. It has shown that a man-made system of this type can be viewed within the same conceptual framework as a natural system, and that meaningful results can be achieved by applying to these synthetic systems, procedural techniques originally developed for application to physical phenomena. Several conclusions result from the research.

This model, although gross, is a vehicle for analysis of the firm, and provides explicit knowledge of important decision parameters. A more detailed level of analysis can be approached if the system components are sufficiently magnified and refined; increased detail in the component models, achieved by a lower scale of aggregation, will increase the detail of the resulting analysis. Since the two sectors of the firm are modeled independently of constraints, each can be removed from the system and studied separately. This property of systems analysis methodology, when applied to the model developed in this study, should lead to considerable refinement in the component models.

While a manufacturing firm forms the basis of this research, the resulting model is not limited to such narrow applications. The generalized nature of the model allows its

use in any type of business situation characterized by money flows, however, to adapt it to any specific system, that system's parameter values must be evaluated by empirical observations.

Techniques for analysis are well developed and highly articulate within the physical sciences, and are directly applicable to business systems. The control concept has special appeal for the business manager; to know what system inputs would be required to reach a pre-assigned state would have considerable predictive value, and would greatly assist in performance of the planning function. If the control strategy does involve a feasible input set, then the manager might well gain competitive advantage by utilizing it.

Simulating the operation of the firm provides a valuable experimental device to test the results of both internal and external changes. If the sensitivity of the system to changes in various parameters is known, they can be given special consideration. A system model of this form is not an essential item for simulation; there are many other ways to model the firm which are well suited to simulation, however, using the system model in this way is a bonus which accrues to the system analyst after the model is developed.

While the research has demonstrated that many of the techniques used in analysis of physical systems have application in the realm of business systems, it has also shown that the resulting analysis can be no better than the system

model itself, which in turn depends on the quality of the component models. The weakest link in the chain of analysis of business systems is the ability to identify the system components and model them in a mathematically tractable form. If business components could be classified as neatly and described with the precision of physical components, the value of the discipline to the business environment would be greatly enhanced.

The first generation model resulting from this research is acknowledged to be crude; it deals only with first order effects and grossly aggregated system components. Systems analysis, when operational, has the potential for far greater analytical power than demonstrated here; however, this research shows the feasibility and merit of the concept and the need for continuing effort to develop an operational methodology of pragmatic value in the business world.

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APPENDICES

APPENDIX A

PROGRAM ANALYZE

The computer program used in this research, written in 3600 FORTRAN language, is listed in this appendix. It is not a general program, but is written specifically to satisfy the requirements of the research described by this thesis. All of the format statements are grouped at the beginning of the program, following the declarations. The initial state conditions of the system are listed next, followed by the statements which read coefficient values from data cards and calculate parameter values. The control strategy to drive the system state to zero at $n = 6$ is next calculated by calls to the subroutines for multiplying matrices, and computing a matrix inverse. After the control strategy is printed out, nineteen simulations of twenty-five years each are executed, the first using the base coefficient values and the next eighteen reading new values from the data deck. A twenty-five year simulation is executed for both a linear and a non-linear sales function by calling two subroutines; LINSALES for the linear values, and RANSALES for nonlinear. Subroutine RANSALES uses the same upward trend line as LINSALES but adds to this a sinusoid with maximum amplitude of ten million dollars and a period of ten years. The random number generator

in the computer is used to superimpose on this a random noise factor normally distributed between zero and ten million dollars with unit variance and a mean of five million dollars. This random sales function is repeatable since the statement `CALL RANFSET` specifies the point of beginning and the same random sequence is returned in each succeeding simulation.

Subroutine `MATMULT` used for matrix multiplication was adapted from the computer library, as was the subroutine `MATINV`, used by the main program to compute the inverse of the product matrix `PSTAR` required for determination of the control strategy.

The first eight pages of output from this program are shown in Appendix C.



```

PROGRAM ANALYZE
TYPE REAL K1,K2,K3,N3,NE,NF1,NF2,NF3,NW
DIMENSION ALPHA(30),ND(30),NE(30),NF1(30),NF2(30),NF3(30),NW(30)
DIMENSION SALES(30),Y2(30),Y3(30),Y4(30),Y42(30),Y43(30),Y5(30)
DIMENSION Y61(30),Y62(30),Y7(30),Y8(30)
DIMENSION Q1(6),Q2(6),Q3(6),Q4(6),QSTAR(6),QTEMP(6)
DIMENSION P1(6,6),P2(6,6),P3(6,6),P4(6,6),P5(6,6),P6(6,6)
DIMENSION PSTAR(6,6),PSI(6,7)
DIMENSION TEMP1(6),ITEMP1(6),ITEMP2(6,2)
COMMON SALES,LINN
1 FORMAT(1H1,46X,*CONTROL STRATEGY USING THE FOLLOWING PARAMETER VAL
  1UES*//)
2 FORMAT(8(F10.5))
3 FORMAT(4(E10.2))
4 FORMAT(7X,*C11*7X,*C12*7X,*C13*8X,*C2*8X,*D1*8X,*D2*8X,*D3*8X,*D5*
  18X,*F1*8X,*F2*9X,*G*8X,*K1*8X,*K2*/)
5 FORMAT(13(F10.5)///)
6 FORMAT(8X,*R4*8X,*H5*8X,*H6*9X,*T*9X,*W*19X,*P*18X,*R1*18X,*R2*18X
  1,*R3*/)
7 FORMAT(5F10.5,4E20.4///)
8 FORMAT(3X,*ALPHA*13,1X,* = *E12.5/)
9 FORMAT(1H1,62X,*LINEAR SALES*//**PERIOD*5X,**SALES*3X,*COST OP*4X,*P
  1ROFIT*7X,*Y61*6X,*DEBT*7X,*Y62*4X,*EQUITY*5X,*TOOLS*3X,*MACHINE*6X
  2,*BLDG*5X,*W CAP*7X,*Y42*7X,*Y43*)
10 FORMAT(1H0,15,13(E10.2))
11 FORMAT(1H1,39X,*SIMULATION NO.*13,2X,*USING THE FOLLOWING PARAMETE
  1R VALUES*//)
12 FORMAT(1H1,58X,*NON LINEAR SALES*//**PERIOD*5X,**SALES*3X,*COST OP*4
  1X,*PROFIT*7X,*Y61*6X,*DEBT*7X,*Y62*4X,*EQUITY*5X,*TOOLS*3X,*MACHIN
  2E*6X,*BLDG*5X,*W CAP*7X,*Y42*7X,*Y43*)
100 FORMAT(1H0,6F12.7,2(16X,F12.7))
101 FORMAT(1H1,*MATRIX P*80X,*MATRIX G1*19X,*MATRIX G2*/)
102 FORMAT(1H0,6F12.7)
103 FORMAT(1H0,*P SQUARED */)
104 FORMAT(1H0,*P CUBED */)
105 FORMAT(1H0,*P FOURTH */)
106 FORMAT(1H1,*P FIFTH */)
107 FORMAT(1H0,*P SIXTH */)
108 FORMAT(1H0,*MATRIX P STAR*/)
109 FORMAT(1H0,*PSTAR INVERSE*/)
110 FORMAT(1H0,/**P SIXTH TIMES PSI (0) =*E12.4,5(/24X,E12.4))
111 FORMAT(1H0,*QSTAR TIMES SALES (0) =*E12.4,5(/24X,E12.4))
112 FORMAT(1H0,*TRANSIENT STATE VECTOR AT N OF SIX =*E12.4,5(/37X,E12.
  14))
113 FORMAT(1H0,*STATE VECTOR ERROR AT N OF SIX =*E12.4,5(/33X,E12.4))
114 FORMAT(1H1,*THE CONTROL STRATEGY TO ACHIEVE PSI(6)=ZERO IS*/)
115 FORMAT(1H0,*SALES (*12,* ) =*E12.4/)
116 FORMAT(1H0,6F12.7)

```

```

1000 FORMAT(1H1,*PSTAR TIMES ITS INVERSE*/)
      NUMBER=LINN=0
      D=K1
      NJ(0)=4.25E6 $ NE(0)=11.98E6 $ NW(0)=9.80E6
      NF1(0)=5.04E6 $ NF2(0)=12.09E6 $ NF3(0)=3.02E6
      SALES(0)=100E6
      DO 15 I=1,6
15    PSI(I,7)=0
      PRINT 1
125  NUMBER=NUMBER+1
      JUMP=0
      READ BASE COEFFICIENT VALUES FROM DATA CARDS
      READ 2, C11,C12,C13,C2,D1,D2,D3,D5,F1,F2,G,K1,K2,R4,R5,R6,T,W
      READ 3,P,R1,R2,R3
      B1=K1*1E6
      C
      COMPUTE PARAMETER MATRICES
      ALPHA(1)=1-R4
      ALPHA(2)=F1-1
      ALPHA(3)=F2-1
      ALPHA(4)=G-1
      ALPHA(5)=1-D1
      ALPHA(6)=D2/2
      ALPHA(7)=D3/2
      ALPHA(8)=R6*(1-D5)*(T-1)-R4
      ALPHA(9)=(1-D5)*(T*(C11+D1)-C11)
      ALPHA(10)=(1-D5)*(T*(C12+D2/2)-C12)
      ALPHA(11)=(1-D5)*(T*(C13+D3/2)-C13)
      ALPHA(12)=G*W
      ALPHA(13)=(1-G)*W
      ALPHA(14)=(1-D5)*(1-C2)*(1-T)+W
      ALPHA(15)=(1/P)*(R1+F1*R2/N2+F2*R3/K2)
      ALPHA(16)=(1/(P*K2))*(1-F1)*R2+(1-F2)*R3)
      ALPHA(17)=(R1/P)*(1-D1/2)
      ALPHA(18)=(R2/(P*K2))*(1-D2/2)
      ALPHA(19)=(R3/(P*K2))*(1-D3/2)
      ALPHA(20)=(T*(1-D5))/(2*P*K2))*(D1*R1*K2+D2*R2+D3*R3)
      IF (NUMBER.EQ.1) GO TO 250
      PRINT 11,NUMBER
250  PRINT 4
      PRINT 5,C11,C12,C13,C2,D1,D2,D3,D5,F1,F2,G,K1,K2
      PRINT 6
      PRINT 7,R4,R5,R6,T,W,P,R1,R2,R3
      DO 20 I=1,20
20  PRINT 8,I,ALPHA(I)
      IF (NUMBER.GT.1) GO TO 206
      C
      INITIALIZE MATRICES
      DO 500 I=1,6
      TEMP1(I)=0

```



```

ITEMP1(I)=0
Q1(I)=Q2(I)=Q3(I)=Q4(I)=QSTAR(I)=UTEMP(I)=0
DC 400 K=1,2
400 ITEMP2(I,K)=J
DO 500 J=1,6
PSTAR(I,J)=P3(I,J)=0
500 P1(I,J)=P2(I,J)=P3(I,J)=P4(I,J)=P5(I,J)=P6(I,J)=0
C
ENTER VALUES IN PARAMETER MATRICES
P1(1,1)=ALPHA(1) + P1(3,1)=ALPHA(8) + P1(2,2)=1
P1(1,3)=-G + P1(2,3)=ALPHA(4) + P1(3,3)=R3
P1(3,4)=ALPHA(9) + P1(4,4)=ALPHA(5) + P1(1,5)=-F1
P1(2,5)=ALPHA(2) + P1(3,5)=ALPHA(10) + P1(5,5)=-ALPHA(6)
P1(1,6)=-F2 + P1(2,6)=ALPHA(3) + P1(3,6)=ALPHA(11)
P1(6,6)=-ALPHA(7)
P5(1,1)=ND(U) + P5(2,1)=NE(U) + P5(3,1)=NW(U)
P5(4,1)=NF(U) + P5(5,1)=NF2(U) + P5(6,1)=NF3(U)
Q1(1)=ALPHA(12) + Q1(2)=ALPHA(13) + Q1(3)=ALPHA(14)
Q2(1)=ALPHA(15) + Q2(2)=ALPHA(16) + Q2(3)=ALPHA(20)
Q2(4)=ALPHA(17) + Q2(5)=ALPHA(18) + Q2(6)=ALPHA(19)
PRINT 101
PRINT 100,((P1(I,J),J=1,6),Q1(I),Q2(I),I=1,6)
C
PERFORM MATRIX OPERATIONS
CALL MATMULT(P1,P1,6,6,8,P2,6,6,6)
PRINT 103
PRINT 102,((P2(I,J),J=1,6),I=1,6)
CALL MATMULT(P1,P2,6,6,6,P3,6,6,6)
PRINT 104
PRINT 102,((P3(I,J),J=1,6),I=1,6)
CALL MATMULT(P1,P3,6,6,6,P4,6,6,6)
PRINT 105
PRINT 102,((P4(I,J),J=1,6),I=1,6)
CALL MATMULT(P1,P4,6,6,6,P5,6,6,6)
PRINT 106
PRINT 102,((P5(I,J),J=1,6),I=1,6)
CALL MATMULT(P1,P5,6,6,6,P6,6,6,6)
PRINT 107
PRINT 102,((P6(I,J),J=1,6),I=1,6)
CALL MATMULT(P1,P6,6,6,6,Q3,6,6,6)
DO 150 I=1,6
150 Q3(I)=Q3(I)+Q1(I)
CALL MATMULT(P6,Q2,6,6,1,QSTAR,6,6,6)
CALL MATMULT(P5,Q3,6,6,1,Q4,6,6,6)
DO 151 I=1,6
151 QSTAR(I)=QSTAR(I)+Q4(I)
CALL MATMULT(P4,Q3,6,6,1,UTEMP,6,6,6)
N=0
152 DO 153 I=1,6
PSTAR(I,N+1)=UTEMP(I)

```

```

153 GTEMP(I)=0
    NEN+1
    GO TO (154,155,156,157)N
154 CALL MATMUL(P3,Q3,6,6,1,QTEMP,6,6,6) $ GO TO 152
155 CALL MATMUL(P2,Q3,6,6,1,QTEMP,6,6,6) $ GO TO 152
156 CALL MATMUL(P1,Q3,6,6,1,QTEMP,6,6,6) $ GO TO 152
157 DO 158 I=1,6
    PSTAR(I,5)=Q3(I)
158 PSTAR(I,6)=Q2(I)
    PRINT 108
    PRINT 102,((PSTAR(I,J),J=1,6),I=1,6)
    SAVE PSTAR MATRIX IN P1
    DO 159 I=1,6
    DO 159 J=1,6
159 P1(I,J)=PSTAR(I,J)
    COMPUTE INVERSE OF MATRIX PSTAR
    CALL MATINV(PSTAR,6,6,PSI,0,TEMP1,ITEMP1,ITEMP2,D)
    PRINT 109
    PRINT 116,((PSTAR(I,J),J=1,6),I=1,6)
    CHECK ACCURACY OF INVERSE
    CALL MATMUL(PSTAR,P1,6,6,6,P2,6,6,6)
    PRINT 1000
    PRINT 102,((P2(I,J),J=1,6),I=1,6)
    CALCULATE HOMOGENEOUS PORTION OF STATE VECTOR AT N=6
    CALL MATMUL(P6,PSI,6,6,1,QTEMP,6,6,6)
    PRINT 110,(QTEMP(I),I=1,6)
    CALCULATE RESULTS AT N=6 OF SALES (0)
    DO 199 I=1,6
199 QSTAR(I)=QSTAR(I)*SALES(0)
    PRINT 111,(QSTAR(I),I=1,6)
    CALCULATE TRANSIENT STATE AT N=6
    DO 201 I=1,6
201 GTEMP(I)=QTEMP(I)-QSTAR(I)
    PRINT 112,(GTEMP(I),I=1,6)
    CALCULATE ERROR SIGNAL AT N=6
    DO 202 I=1,6
202 PSI(I,7)=PSI(I,7)-GTEMP(I)
    PRINT 113,(PSI(I,7),I=1,6)
    CALCULATE CONTROL STRATEGY FOR PERIODS ONE THROUGH SIX
    DO 203 I=1,6
203 GTEMP(I)=PSI(I,7)
    CALL MATMUL(PSTAR,GTEMP,6,6,1,SALES,6,6,6)
    PRINT 114
    DO 204 I=1,6
204 PRINT 115,I,SALES(I)
    PRINT 11,NUMBER
    PRINT 4
    PRINT 3,C11,C12,C13,C2,D1,D2,D3,D5,F1,F2,G,K1,K2

```

```

PRINT 6
PRINT 7,R4,R5,R6,T,W,P,R1,R2,R3
DO 30 K=1,20
30 PRINT 8,K,ALPHA(K)
206 CALL LNSALES(J1)
GO TO 300
200 JUMP=1
CALL RANSALES(J1)
PRINT 12
GO TO 205
300 PRINT 9
205 DO 40 I=LINN,25
ND(I+1)=ALPHA(1)*ND(I)-F1*NF2(I)-F2*NF3(I)-G*NW(I)+ALPHA(12)*SALES
1(I)+ALPHA(15)*SALES(I+1)
NE(I+1)=NE(I)+ALPHA(2)*NF2(I)+ALPHA(3)*NF3(I)+ALPHA(4)*NW(I)+ALPHA
2(13)*SALES(I)+ALPHA(16)*SALES(I+1)
NF1(I+1)=ALPHA(5)*NF1(I)+ALPHA(17)*SALES(I+1)
NF2(I+1)=-ALPHA(6)*NF2(I)+ALPHA(18)*SALES(I+1)
NF3(I+1)=-ALPHA(7)*NF3(I)+ALPHA(19)*SALES(I+1)
NW(I+1)=ALPHA(8)*ND(I)+ALPHA(9)*NF1(I)+ALPHA(10)*NF2(I)+ALPHA(11)*
1NF3(I)+R5*NE(I)+ALPHA(14)*SALES(I)+ALPHA(20)*SALES(I+1)
Y2(I)=-T*R6*ND(I)-T*(C11+U1)*NF1(I)-T*(C12+U2/2)*NF2(I)-T*(C13+D3/
12)*NF3(I)+T*(1-C2)*SALES(I)-(ALPHA(20)/(1-D5))*SALES(I+1)
Y3(I)=R6*(T-1)*ND(I)+(ALPHA(9)/(1-D5)-U1)*NF1(I)+(ALPHA(10)/(1-D5)
1-ALPHA(6))*NF2(I)+(ALPHA(11)/(1-D5)-ALPHA(7))*NF3(I)+(1-T)*(1-C2)*
2SALES(I)+(ALPHA(20)*(T-1)/(T*(1-D5)))*SALES(I+1)
Y41(I)=(R1/P)*SALES(I+1)
Y42(I)=-NF2(I)+(R2/(K2*P))*SALES(I+1)
Y43(I)=-NF3(I)+(R3/(K2*P))*SALES(I+1)
Y5(I)=R6*ND(I)+C11*NF1(I)+C11*NF2(I)+C11*NF3(I)+C2*S1LES(I)
Y61(I)=-G*NW(I)-F1*NF2(I)-F2*NF3(I)+ALPHA(12)*SALES(I)+ALPHA(15)*S
1ALES(I+1)
Y62(I)=ALPHA(4)*NW(I)+ALPHA(2)*NF2(I)+ALPHA(3)*NF3(I)+ALPHA(13)*SA
1LES(I)+ALPHA(16)*SALES(I+1)
Y7(I)=D5*R6*(T-1)*ND(I)+ALPHA(9)*D5*NF1(I)+ALPHA(10)*D5*NF2(I)+ALP
1HA(11)*D5*NF3(I)+D5*(1-T)*(1-C2)*SALES(I)+(D5*ALPHA(20)/(1-D5))*SA
2LES(I+1)
Y8(I)=D1*NF1(I)+ALPHA(6)*NF2(I)+ALPHA(7)*NF3(I)+(ALPHA(20)/(T*(1-D
15)))*SALES(I+1)
40 PRINT 10,I,SALES(1),Y5(1),Y3(1),Y61(1),Y62(1),NE(1),NF1(1),N
1F2(1),NF3(1),NW(1),Y42(1),Y43(1)
IF(JUMP.EQ.0)GO TO 200
IF(NUMBER.LT.19)GO TO 125
END
SUBROUTINE LNSALES(B1)
DIMENSION SALES(30)
COMMON SALES,LINN
SALES(0)=100E6

```

```

DO 1 I=LINN,25
1 SALES(I+1)=B1+SALES(I)
END
SUBROUTINE RANSALES(B1)
DIMENSION SALES(30)
COMMON SALES,LINN
SALES(0)=100E6
JEN=59104 $ SINMAX=10E+6 $ XNOISE=10E+6
CALL RANFSET(JEN)
DO 2 I=LINN,25
X1=I+1
FORECAST=SALES(0)+X1*31+SINMAX*SINF(3.1416*X1/5)
X=0
DO 1 N=1,12
1 X=X+RANF(-1)
SIGNOISE=XNOISE*(X-6)
2 SALES(I+1)=FORECAST+SIGNOISE
END
SUBROUTINE MATMULT (A,B,L,M,N,C,MA,MB,MC)
DIMENSION A(MA,1),B(MB,1),C(MC,1)
DO 100 I=1,L
DO 100 J=1,N
SUM=0
DO 101 K=1,M
101 SUM=SUM+A(I,K)*B(K,J)
100 C(I,J)=SUM
END
SUBROUTINE MATINV(A,N,NMAX,B,M,PIVOT,INDEX,DETERM)
DIMENSION IPIVOT(N),A(NMAX,NMAX),B(NMAX,1),INDEX(N,2),PIVOT(N)
INITIALIZATION
10 DETERM=1.0
15 DO 20 J=1,N
20 IPIVOT(J)=0
30 DO 50 I=1,N
C SEARCH FOR PIVOT ELEMENT
40 AMAX=C.0
45 DO 105 J=1,N
50 IF (IPIVOT(J)-1) 60, 105, 60
60 DO 100 K=1,N
70 IF (IPIVOT(K)-1) 80, 100, 740
80 IF (ABSF(AMAX)-ABSF(A(J,K))) 85, 100, 100
85 IROW=J
90 ICOL=K
95 AMAX=A(J,K)
100 CONTINUE
105 IPIVOT(ICOL)=IPIVOT(ICOL)+1
C INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL

```

```

120 IF (IROW-ICOLU) 140, 260, 140
140 DETERM=DETERM
150 DO 200 L=1,N
150 SWAP=A(IROW,L)
170 A(IROW,L)=A(ICOLU,L)
200 A(ICOLU,L)=SWAP
205 IF(N) 260, 260, 210
210 DO 250 L=1, M
220 SWAP=B(IROW,L)
230 B(IROW,L)=B(ICOLU,L)
250 B(ICOLU,L)=SWAP
260 INDEX(1,1)=IROW
270 INDEX(1,2)=ICOLU
310 PIVOT(1)=A(ICOLU,ICOLU)
320 DETERM=DETERM*PIVOT(1)
C   DIVIDE PIVOT ROW BY PIVOT ELEMENT
330 A(ICOLU,ICOLU)=1.0
340 DO 350 L=1,N
350 A(ICOLU,L)=A(ICOLU,L)/PIVOT(1)
355 IF(M) 380, 380, 360
360 DO 370 L=1,M
370 B(ICOLU,L)=B(ICOLU,L)/PIVOT(1)
C   REDUCE NON-PIVOT ROWS
380 DO 550 LI=1,N
390 IF(LI-ICOLU) 400, 550, 400
400 T=A(LI,ICOLU)
420 A(LI,ICOLU)=0.0
430 DO 450 L=1,N
450 A(LI,L)=A(LI,L)-A(ICOLU,L)*T
455 IF(M) 550, 550, 460
460 DO 500 L=1,M
500 B(LI,L)=B(LI,L)-B(ICOLU,L)*T
550 CONTINUE
C   INTERCHANGE COLUYS
600 DO 710 I=1,N
610 L=N+1-I
620 IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
630 JRC=INDEX(L,1)
640 JCOLU=INDEX(L,2)
650 DO 705 K=1,N
660 SWAP=A(K,JROW)
670 A(K,JROW)=A(K,JCOLU)
700 A(K,JCOLU)=SWAP
705 CONTINUE
710 CONTINUE
740 RETURN
750 END

```

APPENDIX B

SENSITIVITY ANALYSIS

Effects of management decisions or changes in environmental conditions such as tax rates can be simulated by changing the magnitudes of the coefficients in the component equations. Three experiments with parameter sensitivity are described in this appendix and graphs of the resulting profit and debt positions are shown. The three experiments are conducted with the coefficients K_2 , R_1 , and W , which are respectively the factors dealing with the work week, tooling, and working capital. The system model used for these simulations allows unrestricted bidirectional flow in all components. Negative flow on the profit edge y_3 is equivalent to an operating loss, and negative debt, shown by the internal state variable N_d , is interpreted as a fund of money the firm has loaned to others, resulting in interest income to the firm rather than interest expense. Negative debt occurs only after all outstanding debt has been repaid and excess funds are generated from operations. Twenty-five years of operation are simulated for each parameter value, for both the linear and nonlinear sales functions, resulting in four separate graphs for each experiment.



The coefficient K_2 (the ratio of the firm's standard work week to the maximum hours possible) is varied around the base value in three steps of ten percent plus and minus. Increases in the work week correspond to management decisions to meet increased sales through increased use of present production assets. Since increasing the work week may require overtime or bonus pay to production workers, this increase in the variable cost of operation is reflected in the simulation by increasing the size of the coefficient C_2 in the cash cost of operation equation y_5 . If labor makes up about one-half of the variable cost, C_2 is increased in three related steps of five percent each. Decreasing the factor K_2 is the converse of the above management decision; however, if the firm has a guaranteed minimum wage, the variable cost will not be reduced below the base value, and C_2 will remain unchanged. The graphs of profit and debt for these parameter changes are shown in Figure 4 through Figure 7. These graphs show that maximum profit is achieved by continuing the present eighty hour work week and meeting increased production requirements through capital expenditures. The firm continues to earn a profit with all lower asset utilization rates except the lowest one considered in the experiment ($K_2=0.332$). For this lowest value of K_2 increasing losses are incurred and increased amounts must be borrowed, resulting in an undesirable operating condition.

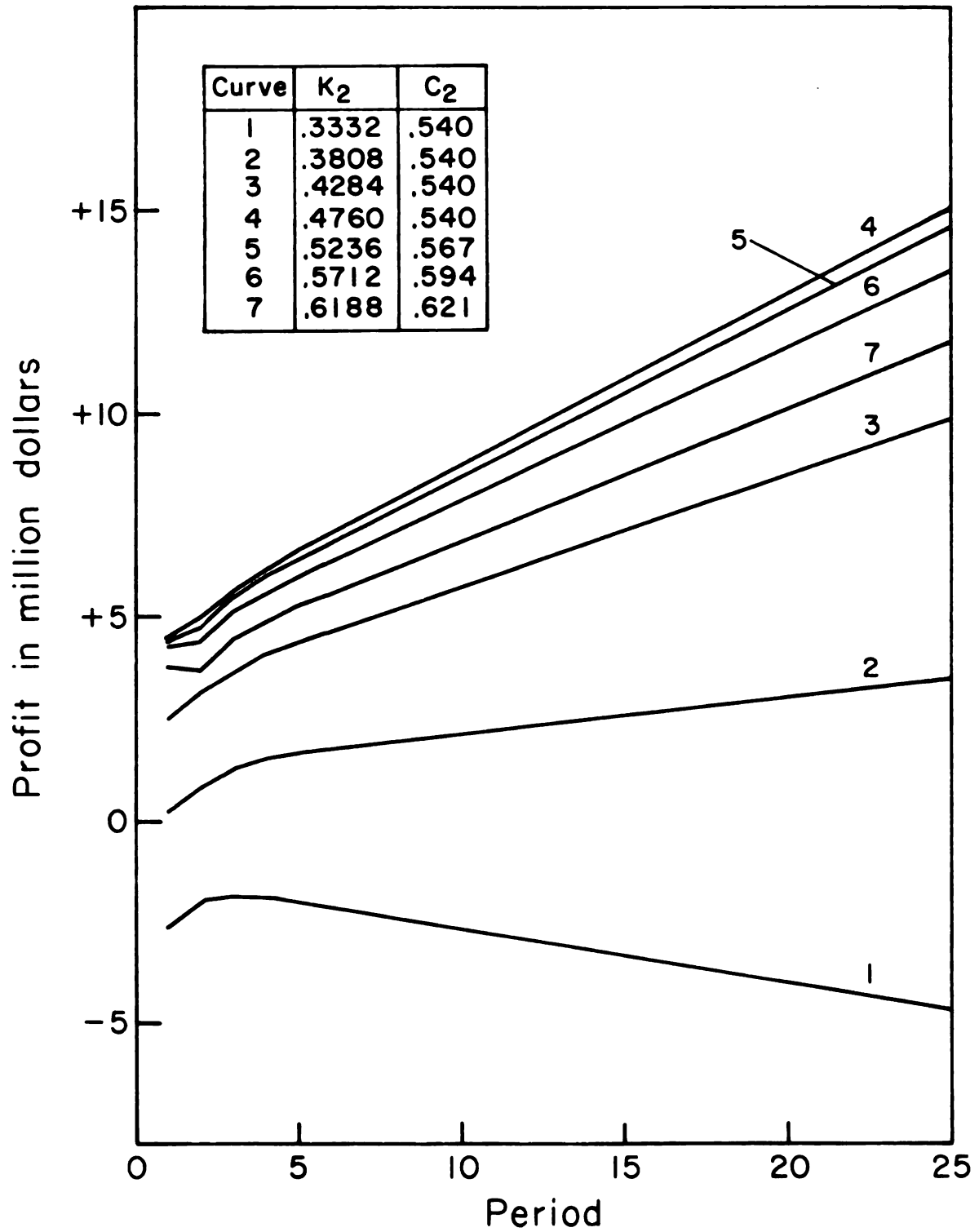


Figure 4.--Sensitivity of profit to changes in work week for linear sales.

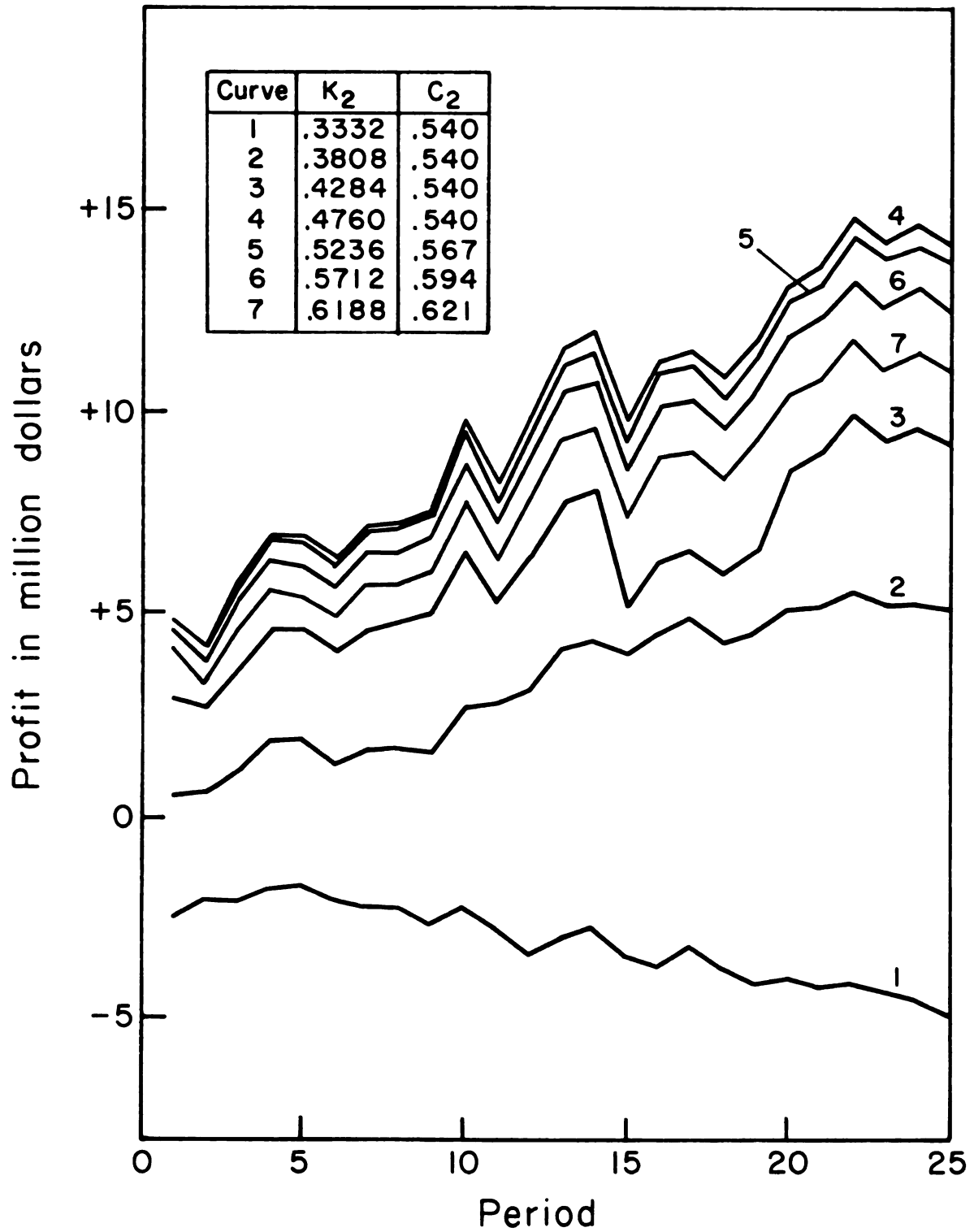


Figure 5.--Sensitivity of profit to changes in work week for nonlinear sales.

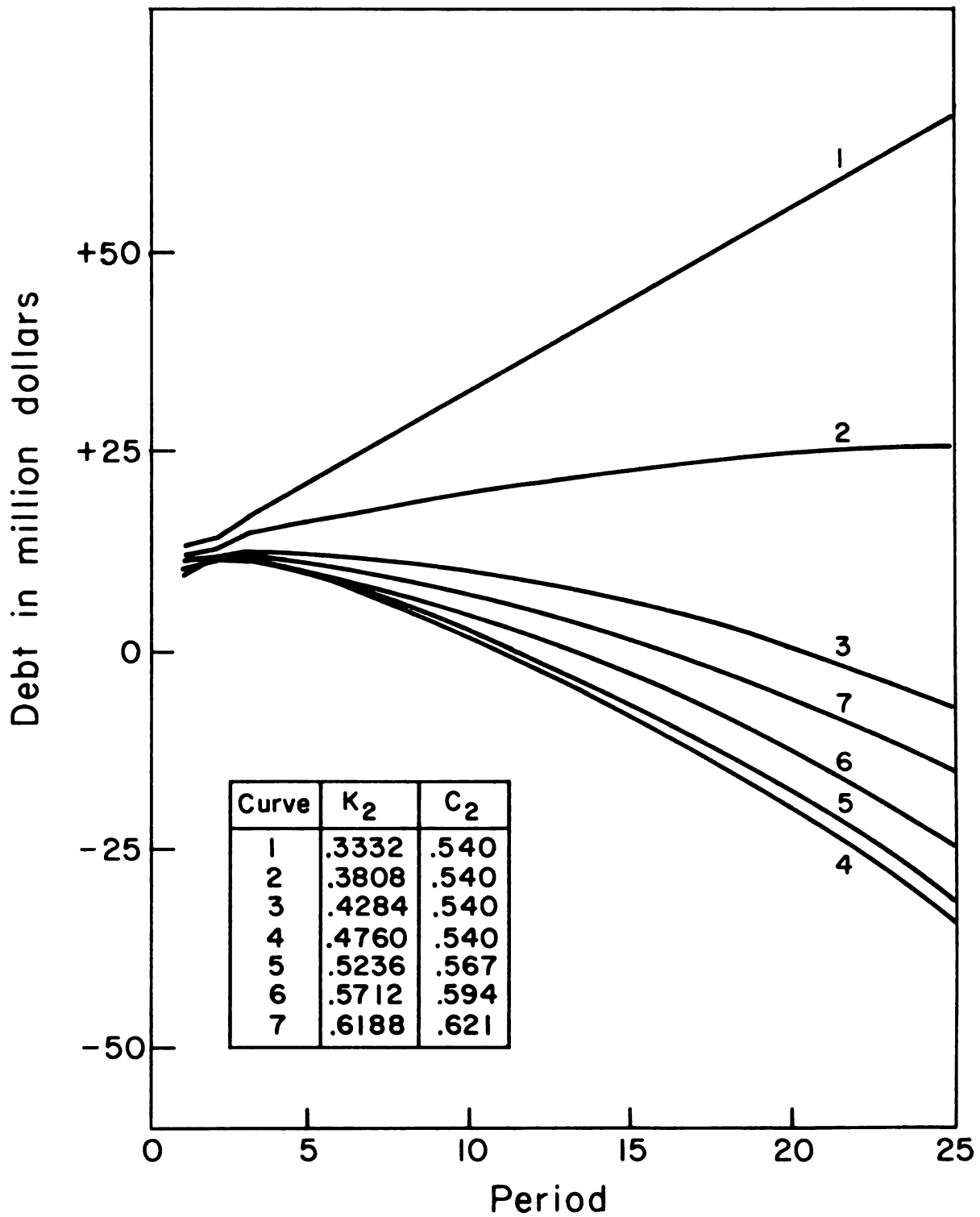


Figure 6.--Sensitivity of debt to changes in work week for linear sales.

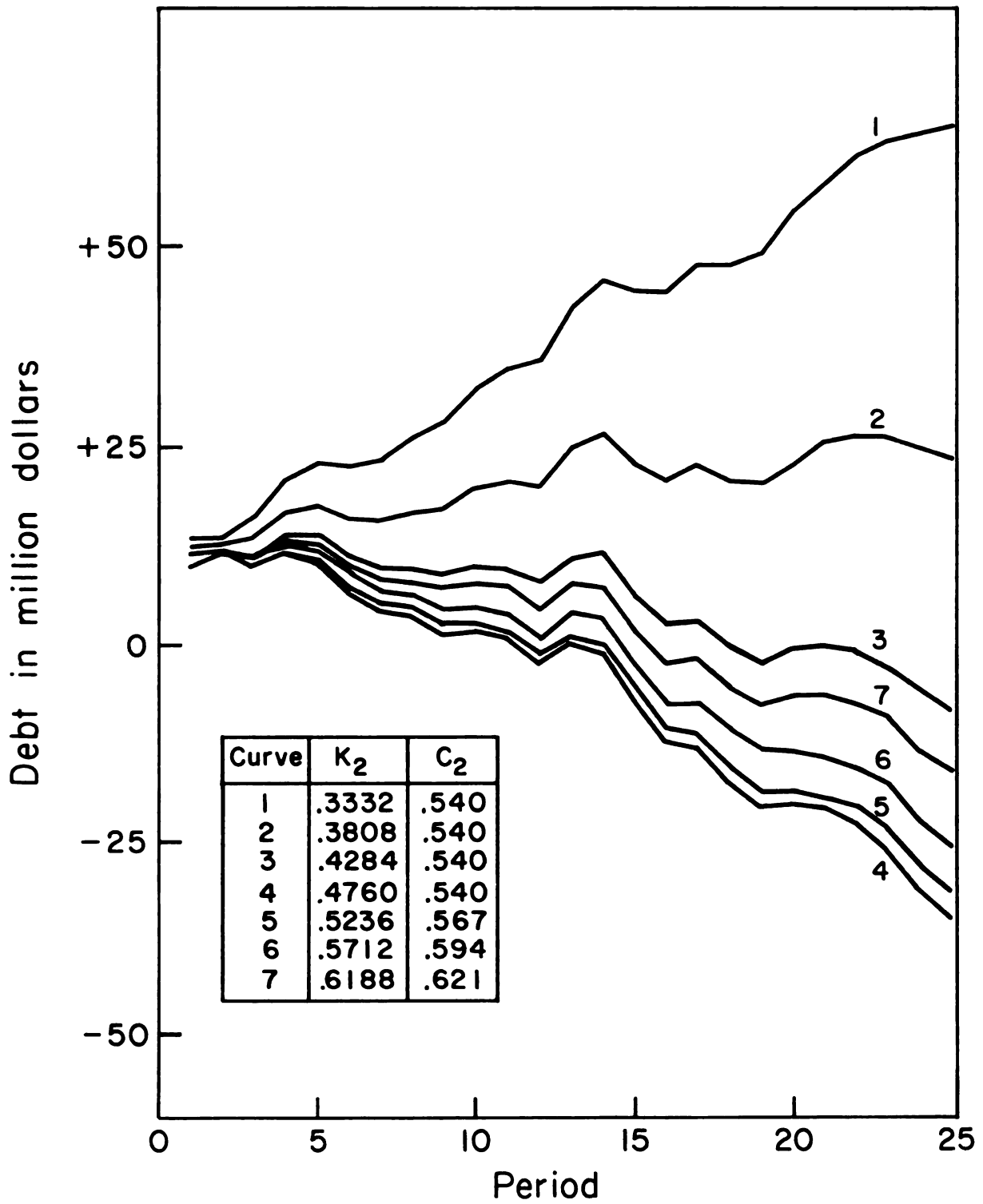


Figure 7.--Sensitivity of debt to changes in work week for nonlinear sales.

The second experiment is with the coefficient R_1 which represents the firm's annual tooling level, and directly influences sales through styling changes. This coefficient is varied around the base value of sixty dollars in steps of ten, twenty, and thirty percent, while holding the other assets per unit, R_2 and R_3 , constant. Each change in R_1 is assumed to result in an equal percentage change in the normal change in sales (K_1 in the sales function). The resulting graphs of profit and debt during the twenty-five years of the simulation are shown in Figure 8 through Figure 11. Although increased tooling expenditure does cause increased sales income, the maximum profit position is reached when only minimum expenditures are made. This condition also results in minimum borrowing requirements, and would be the recommended strategy for the firm to follow under these conditions.

Experiment number three deals with the ratio of working capital to sales (coefficient W in the system model). There are many things that might affect W , examples are changes in policies relating to current liabilities, or changes in accounts receivable. Policy changes in the credit offered to customers generally influence the sales volume in a direct manner; increased leniency in credit terms increases sales, and more stringent credit policies reduce sales. Policy changes of this type are simulated by varying W in steps of ten, twenty, and thirty percent above and below the base value. Customer reaction is reflected by introducing

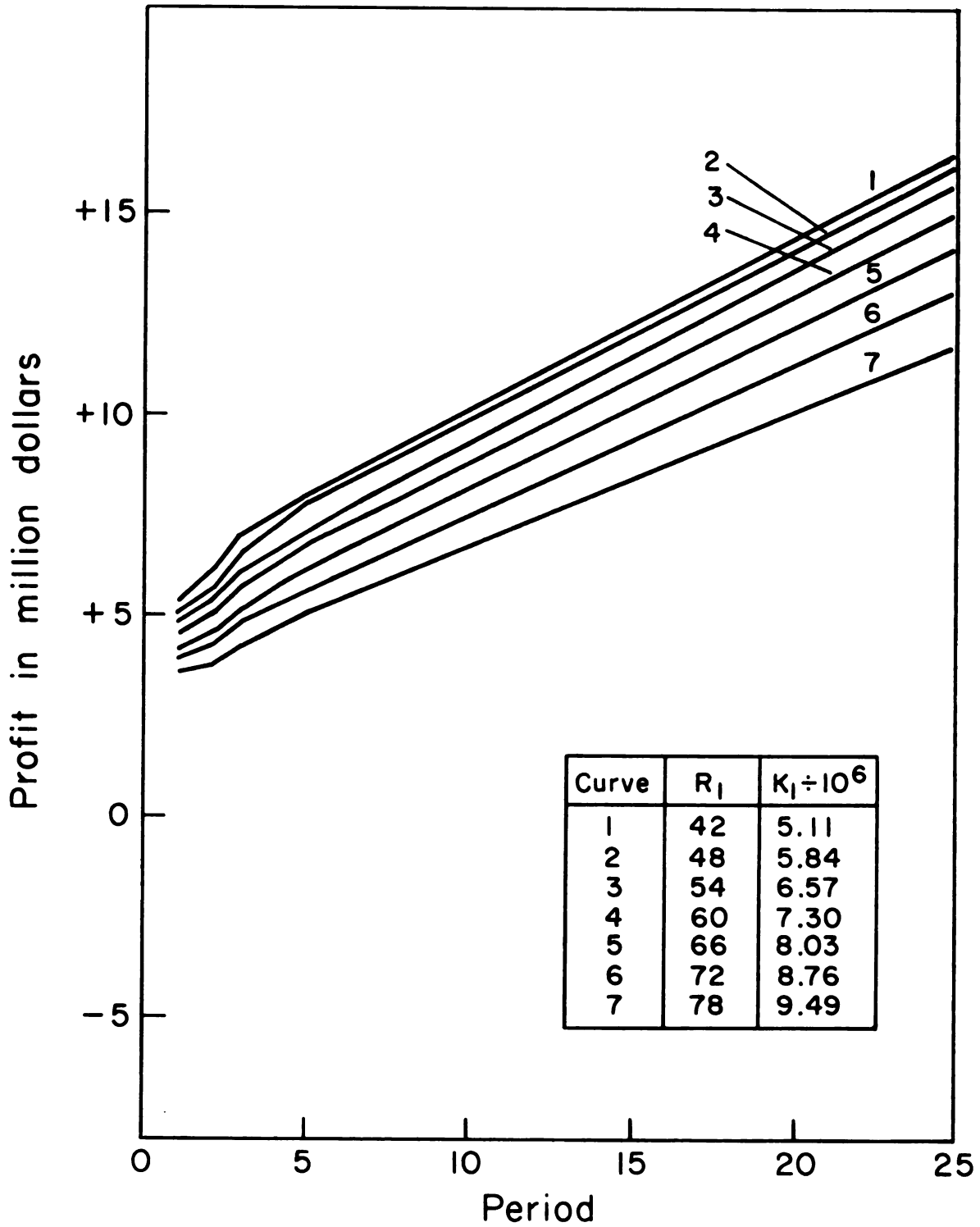


Figure 8.--Sensitivity of profit to changes in tooling for linear sales.

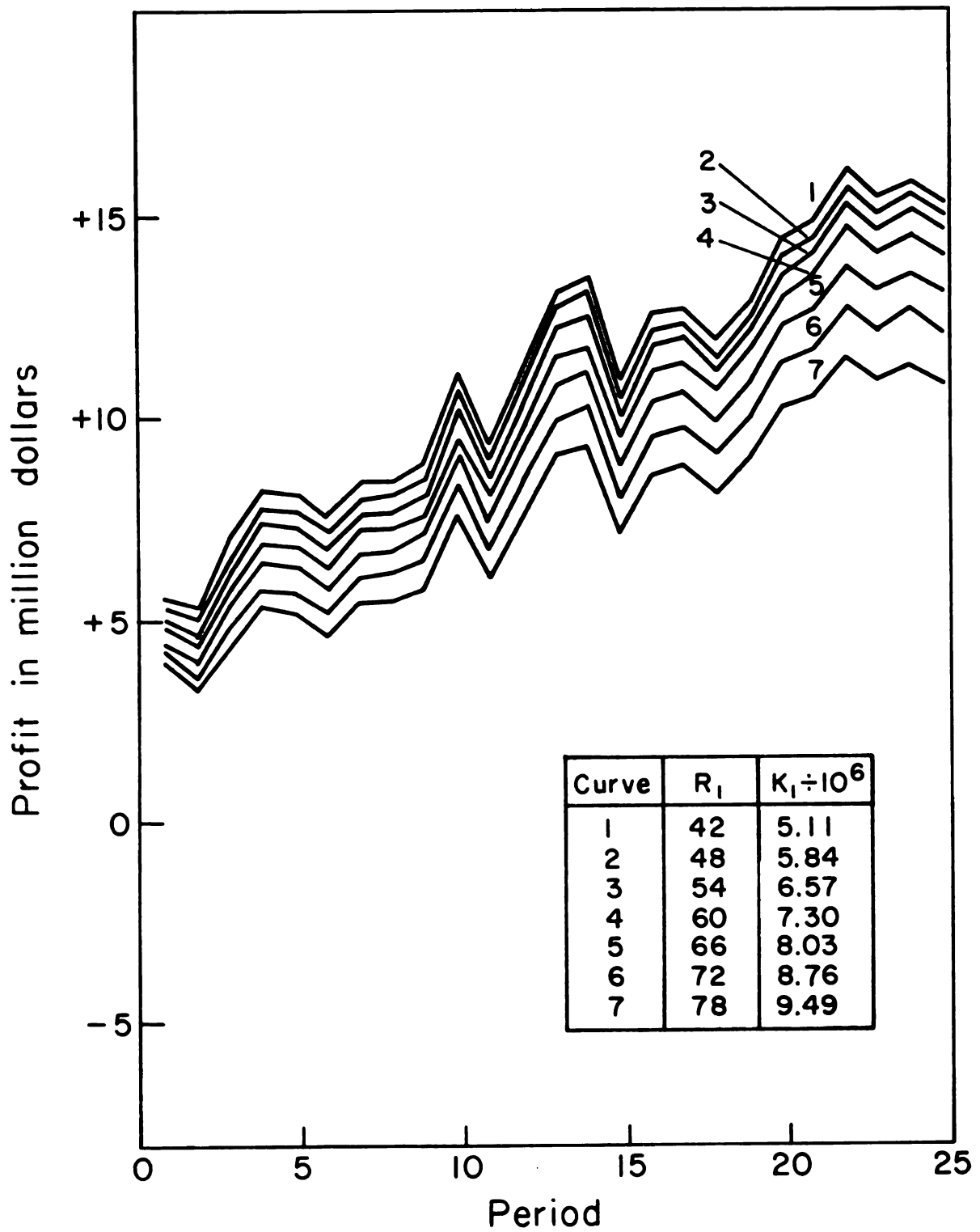


Figure 9.--Sensitivity of profit to changes in tooling for nonlinear sales.

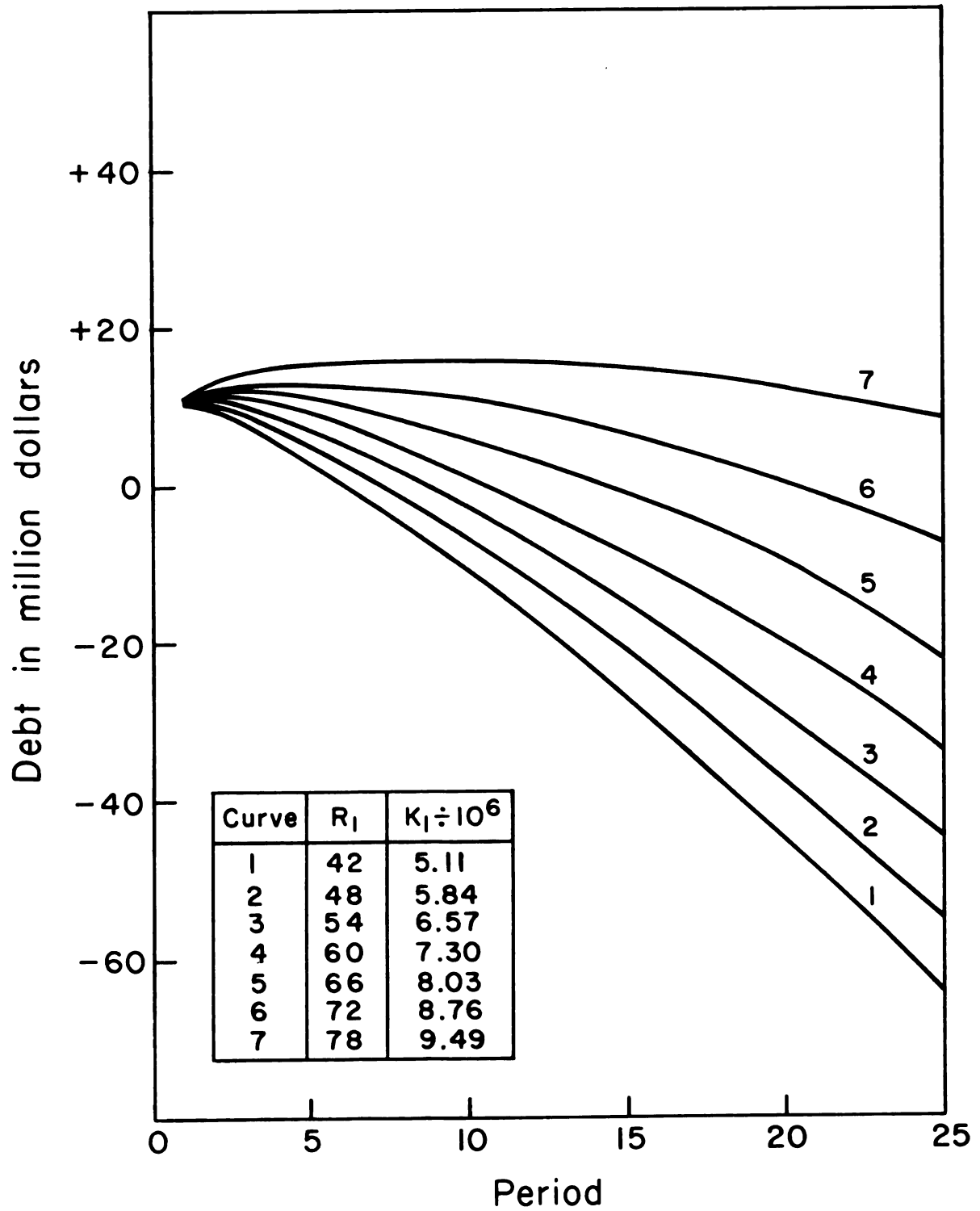


Figure 10.--Sensitivity of debt to changes in tooling for linear sales.

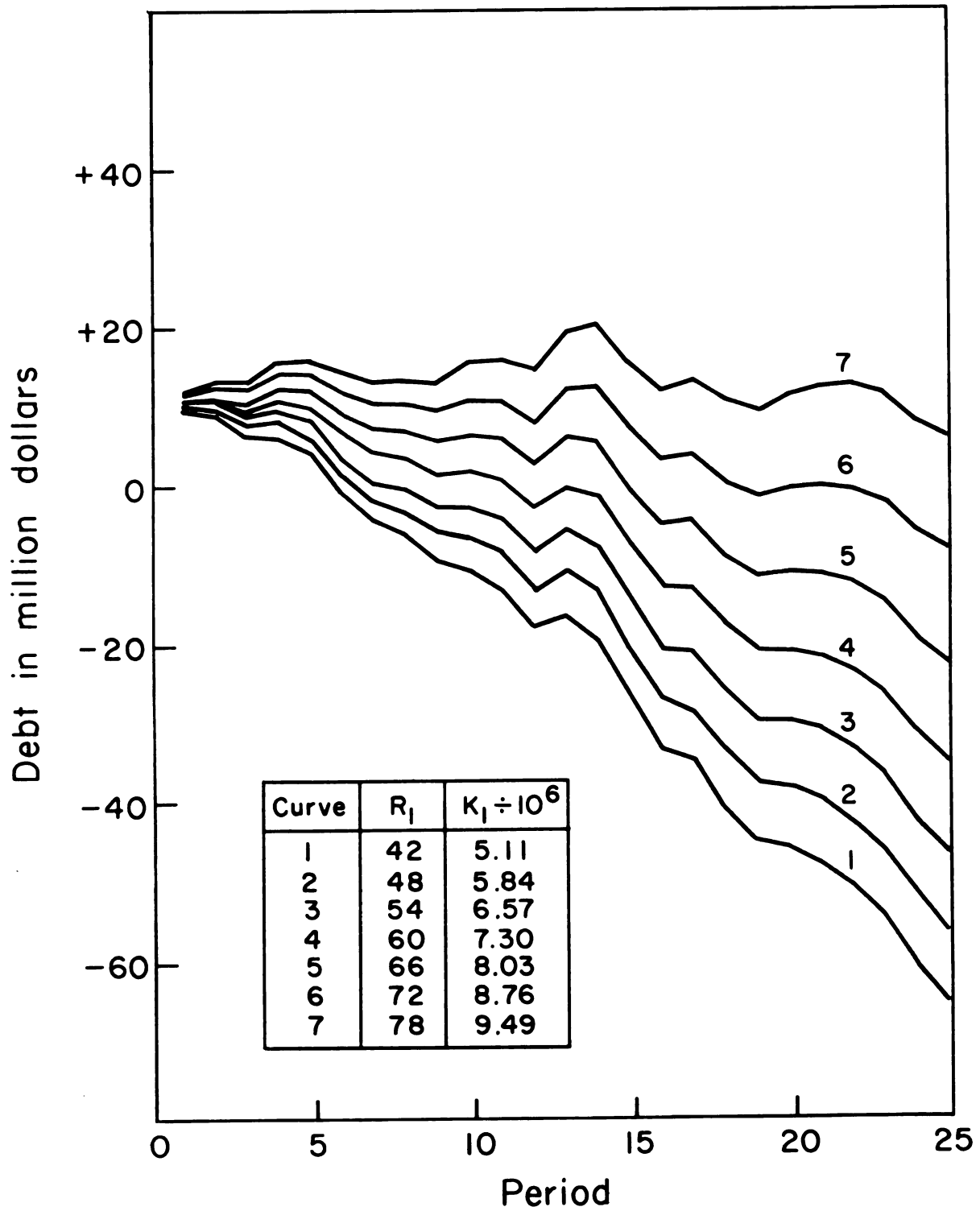


Figure 11.--Sensitivity of debt to changes in tooling for nonlinear sales.



corresponding changes in the normal change in sales (K_1 in the sales function). The graphs of Figure 12 through Figure 15 show the sensitivity of profit and debt to these policy changes. These graphs show that this firm can achieve greater profit by reducing the stringency of their credit policies, thus increasing sales volume and accepting the attendant increases in working capital requirements. This position, however, does increase the debt level and extend the point in time when the firm becomes debt-free.

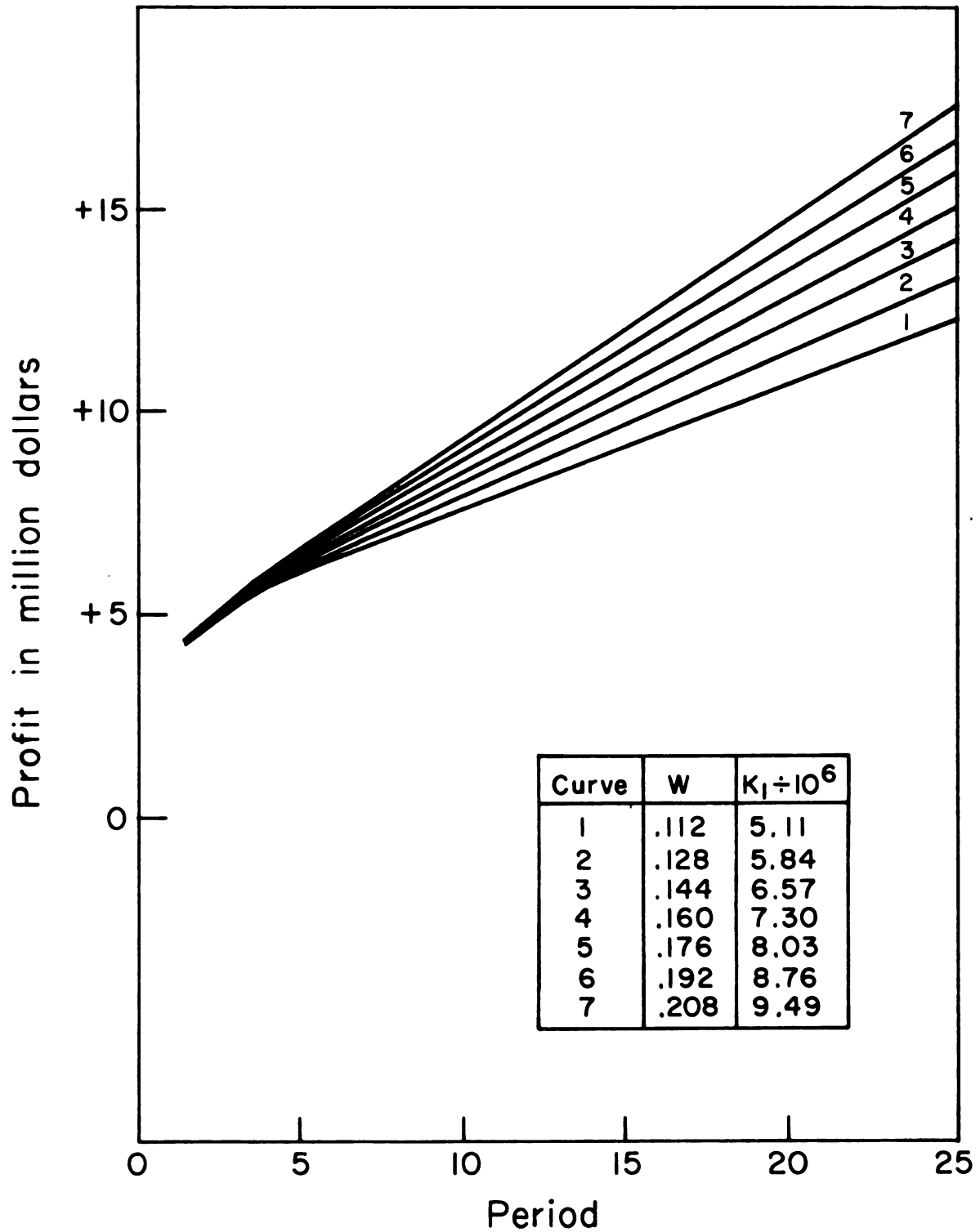


Figure 12.--Sensitivity of profit to changes in ratio of working capital to sales for linear sales.

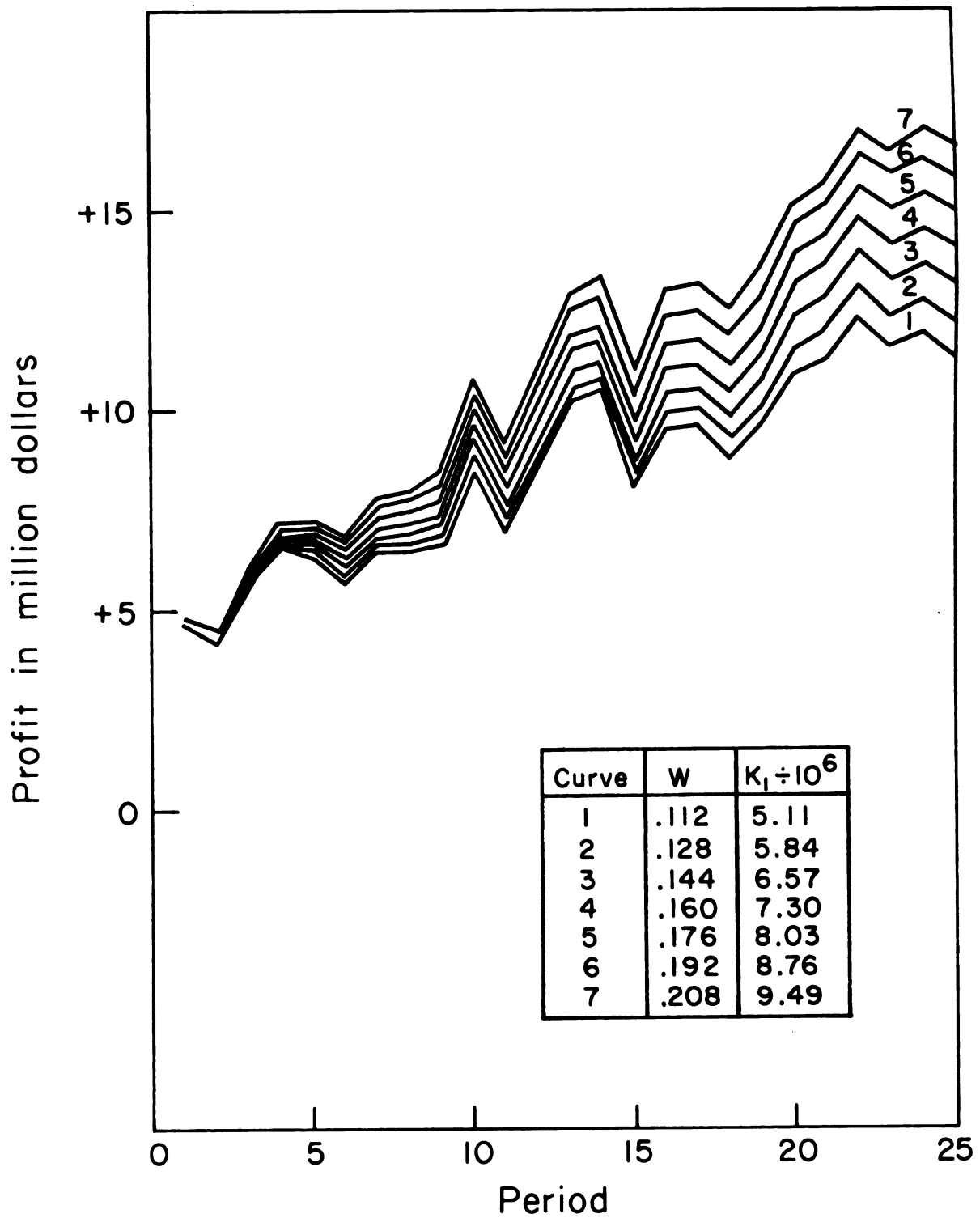


Figure 13.--Sensitivity of profit to changes in ratio of working capital to sales for nonlinear sales.

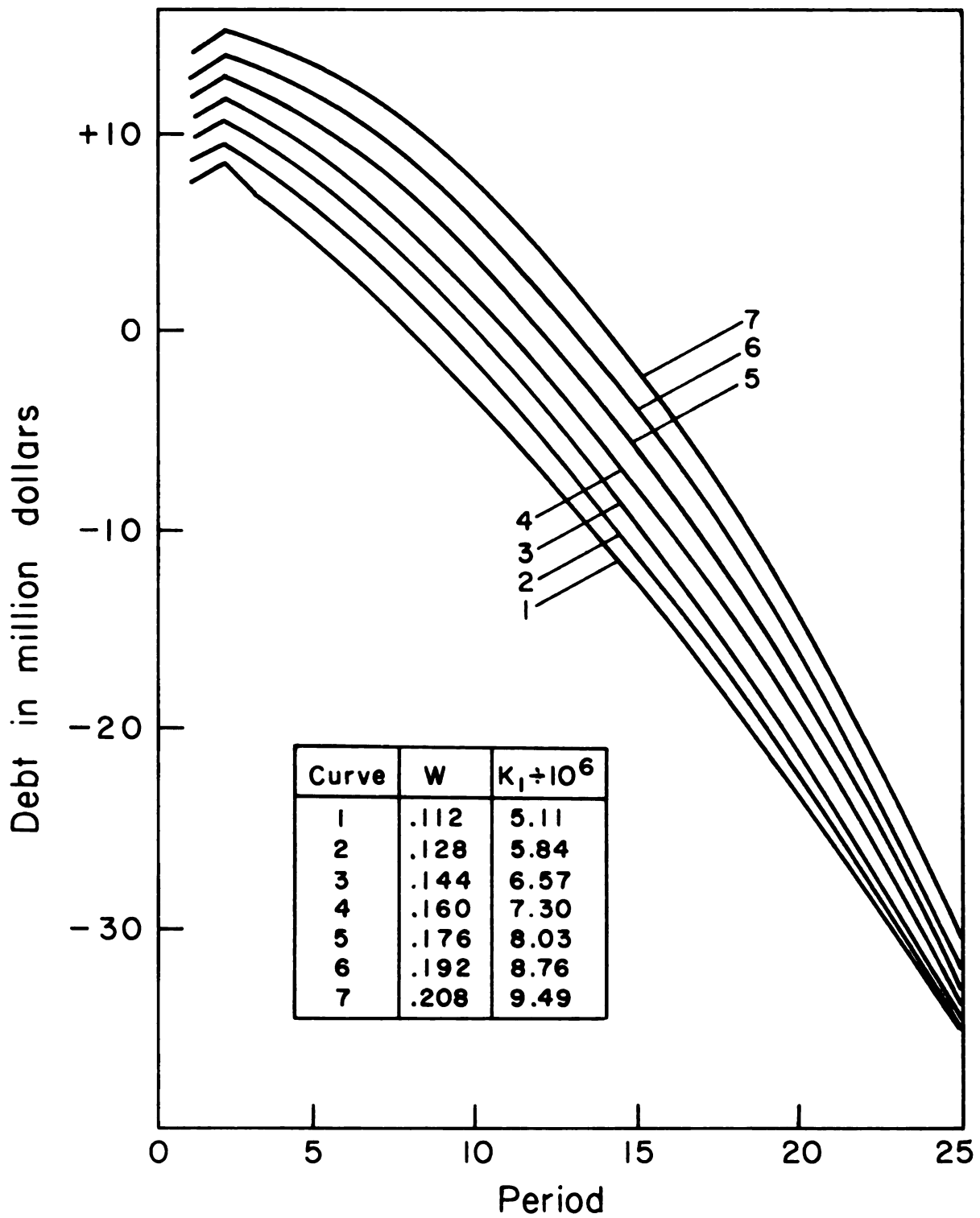


Figure 14.--Sensitivity of debt to changes in the ratio of working capital to sales for linear sales.



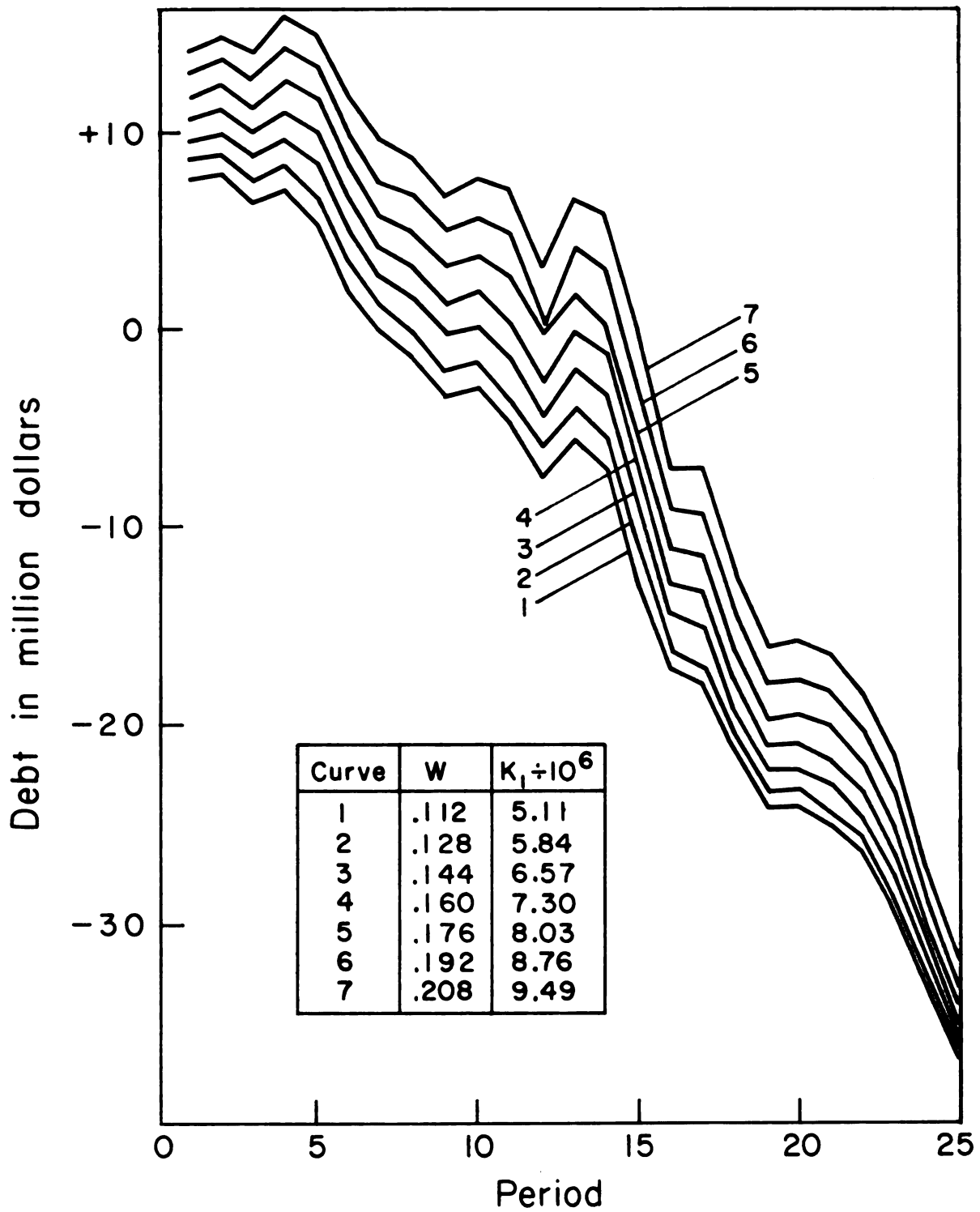


Figure 15.--Sensitivity of debt to changes in the ratio of working capital to sales for nonlinear sales.



APPENDIX C

SELECTED COMPUTER OUTPUT

This appendix contains the first eight pages of computer printout from program ANALYZE listed in Appendix A. The first page shows the coefficient values used in the control strategy computation and the resulting parameter values. Matrices P , Q_1 , and Q_2 in the system model are shown next along with the powers of P from two through six. The product matrix which operates on the input vector, called PSTAR, is shown next and its inverse is also shown. The result of the test of the accuracy of the inverse is shown to be a unit matrix with accuracy to seven significant figures. Intermediate steps in computation of the control strategy are printed out next, including the transient values of the state and sales boundary conditions, and the error signal in the sixth time period for the desired result of zero. The time vector of inputs to drive the state to zero at $n = 6$ is listed next. The last three pages of the appendix show the coefficient values and the results of the first simulations with base values and linear and nonlinear sales functions.



MATRIX P				MATRIX Q1				MATRIX Q2			
0.9000000	0.0000000	-0.4000000	0.0000000	0.0000000	-0.4000000	-0.1000000	-0.1000000	0.0960000	0.0754286		
0.0000000	1.0000000	-0.4000000	0.0000000	0.0000000	0.0000000	0.0000000	-0.4000000	0.0640000	0.0998319		
-0.1226200	0.0000000	0.0000000	-0.4000000	-0.4000000	-0.4000000	0.0000000	-0.4000000	0.3681040	0.0094539		
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0180000		
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.1058824		
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0287395		
P SQUARED											
0.0635720	0.0000000	-0.3400000	0.0000000	0.4116283	0.1955983	0.4393803					
0.0490480	1.0000000	-0.4000000	0.0000000	0.2744189	-0.1879411	-0.5054131					
-0.1109711	0.0000000	0.0700000	-0.4000000	-0.3444338	0.1501657	0.0515905					
0.0000000	0.0000000	0.0000000	0.0000000	0.2500000	0.0000000	0.0000000					
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0156250	0.0000000					
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0025000					
P CUBED											
0.0617975	0.0000000	-0.3300000	0.0000000	0.3783178	0.0768001	0.3642380					
0.0934364	1.0000000	-0.4300000	0.0000000	0.4130004	-0.2573824	-0.5282993					
-0.1088985	0.0000000	0.0649004	-0.4000000	-0.2237179	0.0343986	-0.0558038					
0.0000000	0.0000000	0.0000000	0.0000000	0.1250000	0.0000000	0.0000000					
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	-0.0019331	0.0000000					
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	-0.0001250					
P FOURTH											
0.0409568	0.0000000	-0.3197429	0.0000000	0.3547348	0.0943325	0.3613089					
0.1369958	1.0000000	-0.4582191	0.0000000	0.5024876	-0.2416511	-0.5058653					
-0.1062181	0.0000000	0.0646738	-0.4000000	-0.1577803	0.0083066	-0.0448326					
0.0000000	0.0000000	0.0000000	0.0000000	0.0625000	0.0000000	0.0000000					
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0012441	0.0000000					
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000062					

P FIFTH

0.4205920	0.0000000	-0.5071720	0.4030355	0.0007055	0.3520770
0.1794831	1.0000000	-0.4844804	0.5656037	-0.2304749	-0.4079379
-0.1036492	0.0000000	0.0640592	-0.1230905	-0.0110143	-0.0445333
0.0000000	0.0000000	0.0000000	0.0312500	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	-0.0000305	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	-0.0000003

P SIXTH

0.0007223	0.0000000	-0.4940910	0.4009122	0.0079070	0.3435693
0.2209427	1.0000000	-0.5101123	0.4151039	-0.2337309	-0.4701243
-0.1011592	0.0000000	0.0625090	-0.1059229	-0.0110429	-0.0433941
0.0000000	0.0000000	0.0000000	0.0156250	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000030	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000

MATRIX P STAR

-0.0215502	-0.0227300	-0.0257762	-0.0322031	0.1129065	0.0754206
-0.0110234	-0.0110912	-0.0114096	-0.0129959	0.0706554	0.0908319
0.0020035	0.0017013	-0.0006276	-0.0044592	0.2322145	0.0004539
0.0005625	0.0011250	0.0022500	0.0045000	0.0090000	0.0100000
-0.0000032	0.0000259	-0.0002060	0.0016544	-0.0132353	0.1050024
-0.0000000	0.0000002	-0.0000034	0.0000710	-0.0014370	0.0207395

PSTAR INVERSE

602.401	-1265.464	-2.646	1332.226	-437.007	3301.120
-946.396	1620.207	100.299	-2302.230	1564.491	-7476.003
163.070	-275.750	-79.477	756.170	-1603.400	5907.566
61.994	-100.424	7.325	240.703	469.609	-1697.430
3.570	-5.179	1.530	15.413	0.407	-4.130
0.051	-0.051	0.140	0.200	-1.360	59.620

PSTAR TIMES ITS INVERSE

1.0000000	-0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
-0.0000000	1.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000
-0.0000000	-0.0000000	1.0000000	-0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	1.0000000	-0.0000000	-0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	1.0000000	-0.0000000
-0.0000000	-0.0000000	-0.0000000	-0.0000000	0.0000000	1.0000000

P SIXTH TIMES PSI (0) = 4.1307+006
 6.7749+006
 -6.1566+005
 7.8750+004
 4.6120+001
 4.7188+002

QSTAR TIMES SALES (0) = 1.0792+007
 9.7112+006
 -1.3630+006
 0.0000+000
 0.0000+000
 0.0000+000

TRANSIENT STATE VECTOR AT N OF SIX = -6.6608+006
 -2.9363+006
 7.4736+005
 7.8750+004
 4.6120+001
 4.7188+002

STATE VECTOR ERROR AT N OF SIX = 6.6608+004
 2.9363+004
 -7.4736+005
 -7.8750+004
 -4.6120+001
 -4.7188+002

THE CONTROL STRATEGY TO ACHIEVE $PSI(6)=ZERO$ IS

$SALES(1) = 7.2713 \times 10^8$

$SALES(2) = -1.4102 \times 10^9$

$SALES(3) = 2.8178 \times 10^8$

$SALES(4) = 9.3591 \times 10^7$

$SALES(5) = 4.7499 \times 10^6$

$SALES(6) = 4.7780 \times 10^4$

SIMULATION NO. 1 USING THE FOLLOWING PARAMETER VALUES

C11	C12	C13	C2	D1	D2	D3	D5	F1	F2	G	K1	K2
1.97800	1.97800	1.97800	0.54000	0.50000	0.25000	0.10000	0.13000	0.40000	0.10000	0.160000	7.30000	0.47600

R4	R5	R6	T	W	P	R1	R2	R3
0.10000	0.00500	0.05000	0.48000	0.16000	2.50000003	6.00000001	1.44000002	3.60000001

ALPHA 1 = 9.000000001

ALPHA 2 = -6.000000001

ALPHA 3 = -9.000000001

ALPHA 4 = -4.000000001

ALPHA 5 = 5.000000001

ALPHA 6 = 1.250000001

ALPHA 7 = 5.000000002

ALPHA 8 = -1.226200001

ALPHA 9 = -6.860470001

ALPHA 10 = -8.426470001

ALPHA 11 = -8.739670001

ALPHA 12 = 9.600000002

ALPHA 13 = 6.400000002

ALPHA 14 = 3.681040001

ALPHA 15 = 7.542860002

ALPHA 16 = 9.983190002

ALPHA 17 = 1.800000002

ALPHA 18 = 1.058820001

ALPHA 19 = 2.873950002

ALPHA 20 = 9.453900003

LINEAR SALES

PERIOD	SALES	COST	MP	PROFIT	Y41	NET	Y42	EQUITY	TOOLS	MACHINE	BLDG	M CAP	Y42	Y43
0	1.00+008	9.41+007	-1.54+005	6.48+006	4.25+006	3.22+006	1.20+007	5.04+006	1.21+007	3.02+006	9.80+006	8.94+005	2.26+005	
1	1.07+008	9.26+007	4.44+006	2.07+006	1.05+007	1.33+006	1.52+007	4.45+006	9.85+006	2.93+006	2.11+007	4.02+006	5.34+005	
2	1.15+008	9.87+007	4.91+006	2.31+005	1.15+007	-6.27+004	1.65+007	4.29+006	1.09+007	3.15+006	2.55+007	3.95+006	5.41+005	
3	1.22+008	1.04+008	5.62+006	1.92+005	1.06+007	-1.07+005	1.65+007	4.14+006	1.15+007	3.35+006	2.72+007	4.09+006	5.63+005	
4	1.29+008	1.10+008	4.13+006	-1.11+005	9.73+006	-3.58+005	1.64+007	4.49+006	1.22+007	3.55+006	2.93+007	4.28+006	5.84+005	
5	1.36+008	1.16+008	4.59+006	-3.78+005	8.63+006	-5.57+005	1.60+007	4.70+006	1.29+007	3.75+006	3.13+007	4.48+006	6.05+005	
6	1.44+008	1.23+008	7.02+006	-4.25+005	7.39+006	-7.55+005	1.54+007	4.94+006	1.36+007	3.95+006	3.33+007	4.67+006	6.26+005	
7	1.51+008	1.29+008	7.43+006	-4.70+005	6.02+006	-9.52+005	1.47+007	5.19+006	1.43+007	4.15+006	3.53+007	4.87+006	6.47+005	
8	1.58+008	1.35+008	7.83+006	-1.12+006	4.55+006	-1.15+006	1.37+007	5.45+006	1.50+007	4.35+006	3.73+007	5.07+006	6.68+005	
9	1.66+008	1.41+008	8.24+006	-1.37+006	2.98+006	-1.35+006	1.26+007	5.71+006	1.57+007	4.54+006	3.93+007	5.26+006	6.89+005	
10	1.73+008	1.47+008	8.64+006	-1.63+006	1.31+006	-1.36+006	1.12+007	5.97+006	1.64+007	4.74+006	4.14+007	5.46+006	7.10+005	
11	1.80+008	1.53+008	9.05+006	-1.90+006	-4.54+005	-1.77+006	9.67+006	6.23+006	1.70+007	4.94+006	4.34+007	5.66+006	7.31+005	
12	1.88+008	1.59+008	9.46+006	-2.17+006	-2.31+006	-1.99+006	7.90+006	6.49+006	1.77+007	5.14+006	4.55+007	5.85+006	7.52+005	
13	1.95+008	1.65+008	9.87+006	-2.45+006	-4.25+006	-2.21+006	5.91+006	6.75+006	1.84+007	5.34+006	4.75+007	6.05+006	7.73+005	
14	2.02+008	1.72+008	1.03+007	-2.74+006	-6.27+006	-2.43+006	3.70+006	7.02+006	1.91+007	5.54+006	4.96+007	6.24+006	7.94+005	
15	2.09+008	1.78+008	1.07+007	-3.03+006	-8.33+006	-2.66+006	1.27+006	7.28+006	1.98+007	5.74+006	5.17+007	6.44+006	8.15+005	
16	2.17+008	1.84+008	1.11+007	-3.33+006	-1.06+007	-2.89+006	-1.39+006	7.54+006	2.05+007	5.94+006	5.38+007	6.64+006	8.36+005	
17	2.24+008	1.90+008	1.15+007	-3.63+006	-1.28+007	-3.13+006	-4.29+006	7.80+006	2.12+007	6.14+006	5.59+007	6.83+006	8.57+005	
18	2.31+008	1.96+008	1.20+007	-3.94+006	-1.52+007	-3.37+006	-7.42+006	8.07+006	2.19+007	6.34+006	5.80+007	7.03+006	8.78+005	
19	2.39+008	2.02+008	1.24+007	-4.26+006	-1.76+007	-3.62+006	-1.08+007	8.33+006	2.25+007	6.54+006	6.01+007	7.23+006	8.99+005	
20	2.46+008	2.08+008	1.28+007	-4.58+006	-2.01+007	-3.86+006	-1.44+007	8.59+006	2.32+007	6.74+006	6.22+007	7.42+006	9.20+005	
21	2.53+008	2.14+008	1.32+007	-4.90+006	-2.27+007	-4.12+006	-1.83+007	8.86+006	2.39+007	6.94+006	6.44+007	7.62+006	9.41+005	
22	2.61+008	2.20+008	1.37+007	-5.24+006	-2.53+007	-4.37+006	-2.24+007	9.12+006	2.46+007	7.14+006	6.65+007	7.81+006	9.62+005	
23	2.68+008	2.26+008	1.41+007	-5.57+006	-2.80+007	-4.63+006	-2.68+007	9.38+006	2.53+007	7.34+006	6.87+007	8.01+006	9.83+005	
24	2.75+008	2.32+008	1.45+007	-5.92+006	-3.08+007	-4.89+006	-3.14+007	9.64+006	2.60+007	7.54+006	7.08+007	8.21+006	1.00+006	
25	2.83+008	2.39+008	1.49+007	-6.26+006	-3.36+007	-5.16+006	-3.63+007	9.91+006	2.67+007	7.74+006	7.30+007	8.40+006	1.03+006	

NON LINEAR SALES

PERIOD	SALES	COST	PRFIT	Y41	DEBT	Y42	EQUITY	TOOLS	MACHINE	BLDG	W CAP	Y42	Y43
0	1.00+008	9.41+007	-1.42+005	4.45+006	4.25+006	3.46+006	1.20+007	5.04+006	1.21+007	3.02+006	9.80+006	1.18+006	2.98+005
1	1.10+008	9.46+007	4.71+006	1.51+006	1.07+007	3.78+005	1.54+007	4.49+006	1.01+007	3.00+006	2.11+007	2.70+006	1.98+005
2	1.06+008	9.12+007	4.27+006	-1.49+005	1.11+007	3.49+005	1.58+007	4.15+006	9.93+006	2.89+006	2.59+007	5.29+006	9.17+005
3	1.26+008	1.08+008	5.74+006	2.34+006	9.82+006	1.55+006	1.62+007	4.34+006	1.21+007	3.47+006	2.51+007	4.80+006	7.47+005
4	1.39+008	1.19+008	4.48+006	-1.35+005	1.12+007	-9.85+005	1.78+007	4.48+006	1.33+007	3.83+006	3.04+007	3.40+006	3.29+005
5	1.38+008	1.17+008	4.41+006	-2.58+006	9.92+006	-2.41+006	1.69+007	4.42+006	1.29+007	3.76+006	3.37+007	3.07+006	2.34+005
6	1.32+008	1.13+008	4.23+006	-1.74+006	6.35+006	-1.12+006	1.43+007	4.79+006	1.24+007	3.61+006	3.34+007	5.15+006	7.71+005
7	1.45+008	1.23+008	7.09+006	-4.73+005	3.98+006	-9.46+005	1.51+007	5.00+006	1.38+007	3.98+006	3.25+007	3.95+006	4.53+005
8	1.47+008	1.25+008	7.19+006	-1.51+006	3.11+006	-1.17+006	1.22+007	5.14+006	1.38+007	4.01+006	3.58+007	5.21+006	7.37+005
9	1.57+008	1.34+008	7.56+006	4.41+005	1.29+006	1.00+006	1.10+007	5.40+006	1.49+007	4.31+006	3.66+007	7.41+006	1.26+006
10	1.84+008	1.57+008	9.48+006	-1.09+006	1.91+006	-2.63+006	1.20+007	4.02+006	1.77+007	5.08+006	3.95+007	2.41+006	-6.48+004
11	1.66+008	1.41+008	9.05+006	-3.24+006	5.38+005	-1.19+006	9.38+006	5.99+006	1.54+007	4.51+006	4.60+007	8.71+006	1.50+006
12	1.99+008	1.69+008	2.44+006	2.34+006	-2.76+006	1.59+006	8.19+006	4.58+006	1.91+007	5.49+006	4.21+007	7.48+006	1.21+006
13	2.22+008	1.88+008	1.15+007	-1.15+006	-1.40+005	-1.79+006	9.78+006	7.28+006	2.11+007	6.09+006	5.04+007	5.99+006	6.70+005
14	2.24+008	1.90+008	1.19+007	-4.32+006	-1.27+006	-6.63+006	7.99+006	7.46+006	2.10+007	6.12+006	5.59+007	2.66+006	-1.97+005
15	1.96+008	1.66+008	9.56+006	-4.13+006	-7.47+006	-3.45+006	1.56+006	7.46+006	1.91+007	5.32+006	5.63+007	8.53+006	1.34+006
16	2.20+008	1.87+008	1.12+007	-1.48+006	-1.28+007	-2.17+006	-2.29+006	7.44+006	2.10+007	6.06+006	5.04+007	5.37+006	5.42+005
17	2.18+008	1.85+008	1.14+007	-5.90+006	-1.32+007	-5.27+006	-4.66+006	7.75+006	2.05+007	5.97+006	5.66+007	5.08+006	4.20+005
18	2.11+008	1.79+008	1.07+007	-4.06+006	-1.78+007	-3.33+006	-9.92+006	7.48+006	1.98+007	5.77+006	5.64+007	7.45+006	1.14+006
19	2.29+008	1.93+008	1.17+007	-1.46+006	-2.09+007	-1.41+006	-1.33+007	7.95+006	2.17+007	6.28+006	5.54+007	8.34+006	1.24+006
20	2.48+008	2.10+008	1.30+007	-2.83+006	-2.07+007	-2.44+006	-1.47+007	8.45+006	2.36+007	6.82+006	6.01+007	7.66+006	9.86+005
21	2.58+008	2.18+008	1.35+007	-3.97+006	-2.14+007	-3.04+006	-1.73+007	8.47+006	2.44+007	7.08+006	6.51+007	8.91+006	1.25+006
22	2.75+008	2.33+008	1.47+007	-5.25+006	-2.32+007	-5.23+006	-2.03+007	9.19+006	2.61+007	7.56+006	6.78+007	6.29+006	5.41+005
23	2.48+008	2.26+008	1.41+007	-7.52+006	-2.42+007	-5.84+006	-2.56+007	9.51+006	2.51+007	7.31+006	7.20+007	8.20+006	1.01+006
24	2.75+008	2.32+008	1.46+007	-4.89+006	-3.11+007	-6.24+006	-3.14+007	9.71+006	2.60+007	7.54+006	7.06+007	6.49+006	5.79+005
25	2.68+008	2.27+008	1.41+007	-7.44+006	-3.49+007	-5.75+006	-3.77+007	9.48+006	2.52+007	7.33+006	7.28+007	8.44+006	1.12+006



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