This is to certify that the

thesis entitled

THE EFFECT OF GRAVITY ON QUANTUM PHASE MEASUREMENT, WITH SPECIAL CONSIDERATION OF THE EQUIVALENCE PRINCIPLE

presented by

Eric Grant Young

has been accepted towards fulfillment of the requirements for

M.S. degree in Physics

Major professor

Date 5/15/87

THE EFFECT OF GRAVITY ON QUANTUM PHASE MEASUREMENT, WITH SPECIAL CONSIDERATION OF THE EQUIVALENCE PRINCIPLE.

by

Eric Grant Young

A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

MASTER OF SCIENCE

Department of Physics and Astronomy

1987

ABSTRACT

THE EFFECT OF GRAVITY ON QUANTUM PHASE MEASUREMENT, WITH SPECIAL CONSIDERATION OF THE EQUIVALENCE PRINCIPLE.

by

Eric Grant Young

The observed effect of gravity on quantum phase measurements is discussed, and analyzed, in particular, with the Feynman-Hibbs[1] method of quantum mechanics. Consideration is given to the problems these experiments pose of the equivalence principle of general relativity.

I. Introduction

That a gravitational field can produce measurable phase effects in quantum systems has been established by experiment. Collela, et al.[2], observed such effects in a neutron interferometer experiment. Specifically, they observed a phase difference between the two "paths" of the interferometer, which depends on the potential difference between the two "paths," and the neutron's mass. More recently, Kuhn and Schoelkopf[3] have observed phase effects in a superconducting loop. Jain, et al.[4], have considered such effects using Josephson-effect batteries.

I begin by introducing the Feynman-Hibbs[1] method of solving quantum mechanical problems. I show that the Feynman-Hibbs method is equivalent to solving the Schrodinger equation, and that low velocity problems can satisfactorily be treated with the Feynman-Hibbs method. Next, I analyze the Collela, et al.[2], experiment using the Feynman-Hibbs method; and demonstrate that this method correctly reproduces their results. As a theoretical aside, I show that, when kinetic energy is much greater than potential energy, the integral of the Lagrangian over time reproduces the integral of momentum over distance. I briefly mention the Jain, et al.[4], experiment; and show that the Schrodinger equation for Kuhn's[3] experiment leads to Airy[5] functions. Kuhn's experiment has a spatially invariant current, which poses difficulty for general relativity. This paradox is discussed, but not resolved. A covariant form of the Schrodinger equation, derived from the Klein-Gordon equation, is presented. The paper ends with a discussion of the local/non-local paradox of general relativity. The question of an accelerated charge, verses a charge at rest in a gravitational field, has significance for the equivalence principle, and has been discussed previously by Bondi, and Gold.[6] Greenberger[7] has considered the breakdown of the weak equivalence principle for quantum systems. This has been confirmed experimentally, in particular, by Collela, et al.[2] There is concern because the mass of a particle can be determined by measuring its phase produced by a gravitational field. Finally, I will show that weak gravitational fields are formally equivalent to accelerated reference frames in special relativity.

II. The Feynman-Hibbs method, and the Collela, et al., experiment

The Feynman-Hibbs[1] method is an integral method of solving quantum mechanical problems, which is formally equivalent to the Schrodinger equation. In practice, those problems which are tractable with the Feynman-Hibbs method may differ from those which are tractable with the Schrodinger equation. The method consists of two parts: An integral over the classical action which contributes only to the phase; and, a complex prefactor which provides the magnitude, and contributes to the phase. Together, these form a kernel which is integrated to obtain the probability of transition from one state to another. The kernel has the form

$$K(b,a) = \int_a^b \exp[i/h \int_{t_a}^{t_b} L(\dot{x},x,t)dt]\ \mathcal{D}x(t), \qquad (1)$$

where the differential, $\mathcal{D}x(t)$, implies integration over all possible paths between a and b. This is treated by letting $x = \bar{x} + y$, and using the action

$$S[x(t)] = S[\bar{x}(t) + y(t)]$$

$$= \int_a^b [a(t)(\dot{\bar{x}}^2 + 2\dot{\bar{x}}\dot{y} + \dot{y}^2)\ldots]dt$$

$$= S_{cl}[b,a] + \int_{t_a}^{t_b} [a(t)\dot{y}^2 + b(t)y\dot{y} + c(t)y^2]dt. \tag{2}$$

Substituting (2) into (1) gives

$$K(b,a) = \int_0^0 \exp\left\{i/h\int_{t_a}^{t_b} [a(t)\dot{y}^2 + b(t)\dot{y}y + c(t)y^2]dt\right\} \mathcal{D}x(t) \ X$$
$$e^{iS_{cl}[b,a]/h}. \tag{3}$$

The integral part of (3) is designated $F(t_b,t_a)$; since the end point times don't effect the integral directly, F must be of the form $F(t) \equiv F(t_b-t_a)$.

It is instructive to consider a particle traveling in a potential, for which the Lagrangian is

$$L = \frac{m}{2}\dot{x}^2 - V(x). \tag{4}$$

The classical action between two points for such a particle is

$$S_{cl}[b,a] = \frac{m}{2t}(x_b-x_a)^2 - V(x_b,x_a)t, \tag{5}$$

where $t \equiv t_b-t_a$, and $V(x_b,x_a) \equiv V(x_b) - V(x_a)$. The kernel K can be written

$$K(b,a) = \int_{x_c} K(b,c)K(c,a)dx_c$$

or, changing notation,

$$F(t+s)e^{iS_{cl}(t+s)/h} = \int_{x_c} F(t)e^{iS_{cl}(t)/h} F(s)e^{iS_{cl}(s)/h} dx_c$$

$$= F(t)F(s)\int_{x_c} dx_c \ \exp\left\{\frac{im}{2ht}(x_b-x_c)^2\right.$$

$$\left. +\frac{im}{2hs}(x_c-x_a)^2 - V(x_b,x_c)t - V(x_c,x_a)s\right\}$$

$$= F(t)F(s)e^{iS_{cl}(t+s)/h}\int_{x_c} dx_c e^{\frac{im}{2h}x_c^2\left(\frac{1}{t}+\frac{1}{s}\right)}$$

Thus, with the standard form for the gaussian integral,

$$F(t+s) = F(t)F(s)\left[\frac{2i\pi h}{m(1/t+1/s)}\right]^{1/2}.$$

Let $F(t) \equiv f(t)[2i\pi ht/m]^{-1/2}$, then

$$f(t+s)\left[\frac{m}{2i\pi h(t+s)}\right]^{1/2} = f(t)f(s)\frac{m}{2i\pi h}\left[\frac{2i\pi h}{tsm(1/t+1/s)}\right]^{1/2},$$

which yields

$$f(t+s) = f(t)f(s), \text{ or } f(t) = e^{at}.$$

Take $f(t)$ to be identically one. Then, for a particle in a potential,

$$K[b,a] = e^{iS_{cl}[b,a]/h}\left[\frac{m}{2i\pi h(t_b-t_a)}\right]^{1/2}. \tag{6}$$

The Feynman-Hibbs method is formally equivalent to the Schrodinger equation as follows: Begin with the probability of transition from one state to another

$$\psi(x_2,t_2-t_1) = \int_{-\infty}^{\infty}K(x_2,x_1;t_2,t_1)\psi(x_1,t_1)dx_1 \tag{7}$$

Restate this in infinitesimal form,

$$\psi(x,t+\varepsilon) = \int_{-\infty}^{\infty}dy\ \frac{1}{A}\exp\left[\varepsilon\frac{i}{\hbar}L\left(\frac{x-y}{\varepsilon},\varepsilon t\right)\right]\psi(y,t). \tag{8}$$

where A is to be determined. Using (5), the Lagrangian of a particle in a potential,

$$\psi(x,t+\varepsilon) = \frac{1}{A}\int_{-\infty}^{\infty}dy\ \exp\left[\frac{i}{\hbar}\ \frac{m(x-y)^2}{2\varepsilon}\right]\left[-\frac{i}{\hbar}\varepsilon V\left(\frac{x+y}{2},\varepsilon t\right)\right]\psi(y,t)$$

Let $y = x + \eta$; then

$$\psi(x,t+\varepsilon) = \frac{1}{A}\int_{-\infty}^{\infty}d\eta\ e^{im\eta^2/2h\varepsilon}e^{-i\varepsilon V(x+\eta/2,t)/h}\psi(y,t) \tag{9}$$

Expand (9) to first order in ε, and second order in η, replacing $\varepsilon V(x+\eta/2,t)$ with $\varepsilon V(x,t)$ (the error is $O(\varepsilon^2)$):

$$\psi+\varepsilon\frac{\partial\psi}{\partial t} = \frac{1}{A}\int_{-\infty}^{\infty}d\eta\ e^{im\eta^2/2h\varepsilon}\left[1-\frac{i\varepsilon}{\hbar}V(x,t)\right]\left[\psi+\eta\frac{\partial\psi}{\partial x}+\frac{1}{2}\eta^2\frac{\partial^2\psi}{\partial x^2}\right] \tag{10}$$

The leading terms give

$$\psi = \frac{\psi}{A}\int_{-\infty}^{\infty}d\eta\ e^{im\eta^2/2h\varepsilon} = \frac{\psi}{A}\left[\frac{2i\pi h\varepsilon}{m}\right]^{1/2}.$$

Therefore,

$$A = \left[\frac{2i\pi h\varepsilon}{m}\right]^{-1/2}. \tag{11}$$

The higher terms are treated with two integrals,

$$\frac{1}{A}\int_{-\infty}^{\infty}e^{im\eta^2/2h\varepsilon}\eta d\eta = 0,$$

and

$$\frac{1}{A}\int_{-\infty}^{\infty}e^{im\eta^2/2h\varepsilon}\eta^2 d\eta = \frac{ih\varepsilon}{m}.$$

Equation (8) is now

$$\psi + \varepsilon\frac{\partial\psi}{\partial t} = \psi - \frac{i\varepsilon}{m}V\psi + \frac{ih\varepsilon\partial^2\psi}{2m\partial x^2}$$

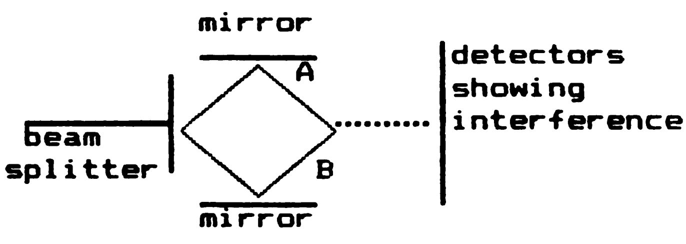
This can be manipulated to become the Schrodinger equation

$$-\frac{h}{i}\frac{\partial\psi}{\partial t} = -\frac{h^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi. \qquad (12)$$

Since the Schrodinger equation applies in the low velocity limit, and can be obtained from the Feynman-Hibbs integral, the Feynman-Hibbs method must be able to treat low velocity problems satisfactorily.

The Collela, et al.[2], experiment is a classic neutron interferometer experiment which firmly establishes the effect of gravitational fields on the quantum mechanical phase of a particle. The experiment is shown schematically in Figure 1. The potential difference between "paths" A and B is varied by rotating the

Figure 1



The Collela, et. al., experiment

interferometer about its axis. The effect of the gravitational potential on the phase, i.e. the phase difference between the two branches of the interferometer, is detected as shifting of the interference pattern. This difference is stated as

$$\beta = q_{grav}\sin\phi, \tag{13}$$

where ϕ is the angle of rotation of the interferometer. Ignoring ϕ , and other angles that parameterize β, Collela, et al., obtain for β

$$2\frac{Mg}{\hbar}\frac{M\lambda}{\hbar}A = 2MgA/hv, \tag{14}$$

where A is the area enclosed by the branches of the interferometer.

This experiment can be treated exactly by the Feynman-Hibbs method. The Lagrangian for this problem is

$$L_i = \frac{m}{2}\left(\dot{x}_i{}^2+\dot{y}_i{}^2\right) - mgx_i, \tag{15}$$

where i = 1,2 designates the two branches of the interferometer. Similarly, the Hamiltonian is

$$\begin{aligned} H_i &= \frac{1}{2m}\left(p_{xi}{}^2+p_{yi}{}^2\right) + mgx_i \\ &= \frac{m}{2}\left(\dot{x}_i{}^2+\dot{y}_i{}^2\right) + mgx_i \\ &= \text{constant.} \end{aligned} \tag{16}$$

Suppose that $x_{i0} = 0$, and that

$$H_i = \frac{m}{2}\left[\dot{x}_{i0}^2+\dot{y}_{i0}^2\right] .$$

Then,

$$mgx_i = \frac{m}{2}\left[\dot{x}_{i0}^2-\dot{x}_i^2+\dot{y}_{i0}^2-\dot{y}_i^2\right].$$

This gives

$$mgx_i = \frac{m}{2}\left[\dot{x}_{i0}^2-\dot{x}_i^2\right]; \ \dot{y}_i = \dot{y}_{i0}. \quad (17)$$

Let $\dot{X}^2 \equiv \dot{x}_i^2 - \dot{x}_{i0}^2 = -2gx_i$;

$$L_i = \frac{m}{2}\left[\dot{X}_i^2 + \dot{x}_{i0}^2 + \dot{y}_{i0}^2\right] - mgx_i$$

$$= \frac{m}{2}\left[\dot{x}_{i0}^2+\dot{y}_{i0}^2\right] - 2mgx_i . \quad (18)$$

Suppose $\dot{x}_{10}^2 = \dot{x}_{20}^2$; $\dot{y}_{10}^2 = \dot{y}_{20}^2$:

$$\beta = \frac{1}{\hbar}\left|\int dt\,(L_1 - L_2)\right|$$

$$= \frac{1}{\hbar}\left|\int dt\left[2mg\left(x_1(t)-x_2(t)\right)\right]\right|$$

$$= \frac{2mg}{\hbar}\left|X(t)t\right|_{t_a}^{t_b}, \ X(t) \equiv x_1(t) - x_2(t). \quad (19)$$

Define $\bar{X}(T)$ to be the average of $X(t)$ over the interval $T \equiv t_b - t_a$. Thus,

$$\beta = \frac{2mg}{h} \bar{X}(T)T. \tag{20}$$

Since $T = y/\dot{y}$, Equation (20) becomes

$$\beta = \frac{2mg}{h} \frac{y\bar{X}(y/\dot{y})}{y} = 2mgA/h\dot{y}, \tag{21}$$

the equation Collela, et al., obtained.

If kinetic energy is much greater than potential energy, then the integral of the Lagrangian over time reproduces the integral of momentum over distance. In this limit the Lagrangian is

$$L \cong 2T = m\dot{x}^2.$$

(Multiplying the Lagrangian by a constant in no way changes the underlying physics.) The integral of the Lagrangian over time is

$$\begin{aligned} \int L\,dt &= m\int \dot{x}^2\,dt \\ &= m\int \left(\frac{dx}{dt}\right)^2 dt \\ &= m\int \left(\frac{dx}{dt}\right) dx \\ &= \int p\,dx, \end{aligned}$$

the integral of momentum over distance.

III. The Kuhn, and Schoelkopf experiment, and the Jain, et al., experiment

Jain, et al.[4], have looked for gravitational effects on charged particles. In particular, they sought behavior in charged particles in analogy to the gravitational redshift. Their experiment was to perform a null measurement of the potential difference between two Josephson-effect batteries connected in opposition, and at different potentials. They obtained results which are consistent with their theoretical introduction. However, there is no apparent conclusion to draw from this experiment about gravitational phase effects.

Kuhn, and Schoelkopf[3], have examined gravitational phase effects in a superconducting loop suspended dynamically in a gravitational field. The loop was tightly coiled about itself so that its area would be negligible, and connected to a superconducting quantum interference device (SQUID) for detection. The dewar, which housed the coil, was suspended from a tunable pendulum so that the loop was subject to a time dependent acceleration. A null measurement was made of the flux modulation induced in the loop by the time dependent acceleration. The induced electric field was consistent with gravitational force balance to better than one part in 10^5, and to have a relaxation rate shorter than the acoustic time scale of the loop.

Kuhn's experiment can not be analyzed exactly; some approximation method must be employed to analyze the Schrodinger equation. A series expansion of the Schrodinger equation leads to Airy functions.[5] The Schrodinger equation is

$$ih\frac{\partial\psi}{\partial t} = \left[-\frac{h^2}{2m}\frac{\partial^2}{\partial x^2} \quad +mgz \right]\psi. \tag{22}$$

Letting $\psi = f(t)Z(z)$, Equation (22) becomes

$$\frac{ih}{f}\frac{\partial f}{\partial t} = \frac{1}{Z}\left[-\frac{h^2}{2m}\frac{\partial^2}{\partial z^2} \quad + mgz\right]Z = E. \tag{23}$$

Equation (23) can be solved immediately for f,

$$f(t) = e^{iEt/h}. \tag{24}$$

The equation to solve for Z is

$$0 = -\frac{\partial^2}{\partial x^2}Z + \frac{2m}{h}\left(mgz - E\right)Z.$$

Let $Z(z) = \sum_{k=o}^{\infty} a_k z^k$, then

$$0 = -\sum_{k=o}^{\infty} a_{k+1} z^k + \frac{2m}{h^2}\left\{\sum mga_{k-1} z^k - \sum Ea_k z^k\right\}. \tag{25}$$

Equation (25) gives the recursion relation

$$a_{n+3} = \frac{2m}{h^2}\left\{mga_n - Ea_{n+1}\right\},$$

which is an Airy function. This solves the eigenvalue problem exactly if carried out to all orders in n.

Using the WKB approximation, Kuhn and Schoelkopf obtain a current for their loop which is independent of position; in particular

$$i \propto \left[n^2\pi^2 + \frac{4m^2 l^2 Hg}{h^2} \right]^{1/2} \tag{26}$$

where H is the vertical height of the loop and 2l is its length. Like the weak equivalence violation paradox (see next section), Equation (26) presents another paradox, since general relativity predicts that the current should vary with position as follows: We begin with the continuity equation

$$j^{\alpha}{}_{;\alpha} = 0, \tag{27}$$

and ignore the 1 and 2 components (potential difference only along the z-axis). The nearly-Newtonian metric

$$g_{\alpha\beta} = \begin{vmatrix} -1-2mgx^3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \tag{28}$$

is used. For the relevant components of the connectivity this gives

$$\Gamma^{o}{}_{3o} = \Gamma^{o}{}_{o3} = mg.$$

Since the loop is closed, the number of charge carriers in the loop can not vary with time; i.e.

$$j^{o}{}_{,o} = 0.$$

The continuity equation (27) is now

$$j^{3}{}_{,3} + mgj^{3} = 0,$$

which has the solution

$$j^{3} = e^{-mgx^{3}} j^{3}\Big|_{t=o}. \qquad (29)$$

At this time, no satisfactory means of resolving the conflict between Equations (26), and (29) is known.

A covariant form of the Schrodinger equation can be obtained by expanding the d'Alembertian of the Klein-Gordon equation

$$(\Box - m^{2})\psi(x)e^{i(m+\varepsilon)t} = 0, \qquad (30)$$

where ε is small such that ε^{2} can be neglected. The d'Alembertian expands as

$$\begin{aligned} \Box\psi &\equiv g^{\mu\nu}\nabla_{\mu}\nabla_{\nu}\psi \\ &= g^{\mu\nu}\nabla_{\mu}\partial_{\nu}\psi \\ &= g^{\mu\nu}\partial_{\mu}\partial_{\nu}\psi - g^{\mu\nu}\Gamma^{\alpha}{}_{\mu\nu}\partial_{\alpha}\psi. \end{aligned} \qquad (31)$$

$g^{\mu\nu}$ is taken to have the nearly-Newtonian form

$$\begin{vmatrix} -(1+2\Phi)^{-1} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}.$$

The pertinent components of the connectivity are

$$\Gamma_{i00} = -\Gamma_{0i0} = -\Gamma_{00i} = \partial_i \Phi.$$

Equation (31) becomes

$$\Box\psi = (1+2\Phi)^{-1}(m+\varepsilon)^2 e^{i(m+\varepsilon)t}\psi(x) + e^{i(m+\varepsilon)t}\nabla^2\psi(x)$$

$$+ (1+2\Phi)^{-1}\nabla\Phi\cdot\nabla\psi(x)e^{i(m+\varepsilon)t}\psi(x). \qquad (32)$$

Substituting Equation (32) into equation (30) gives

$$\left[-m^2 + m^2 + 2m\varepsilon - 2\Phi m^2 - 4m\Phi\varepsilon + \nabla^2 + (1+2\Phi)^{-1}\nabla\Phi\cdot\nabla\right]\psi(x) = 0.$$

The term $4\Phi m\varepsilon$ can be neglected, since ε and Φ are both small, and $\Phi m = -V$. So that, dividing by 2m yields a modified Schrodinger equation

$$\varepsilon\psi(x) = \frac{-1}{2m}\nabla^2\psi(x) - V(x)\psi(x) - \frac{\nabla\Phi(x)\cdot\nabla\psi(x)}{2m(1+2\Phi(x))} \qquad (33)$$

IV. The local verses non-local paradox

The question of an accelerated charge, verses a charge at rest in a gravitational field has significance for the equivalence principle.[6] As is well known, an accelerated charge emits electromagnetic radiation. The strong equivalence principle (SEP) states that locally gravitational fields are indistinguishable from accelerated reference frames. Based on the SEP, one would conclude that a particle suspended statically in a gravitational field should radiate. That such a suspended charge does not radiate demands attention. Careful consideration of the problem shows that the SEP is not tested because the two situations are gravitationally distinguishable, except for uniform fields of large extent where the paradox remains unresolved. Accelerated reference frames are homogeneous by definition to all distances. Except for uniform fields of large extent, gravitational fields exhibit inhomogeneities when consideration is extended to a distance of the order of the inverse of the corresponding acceleration. That is inhomogeneities appear in the gravitational field, unless distances are kept small compared to c^2/g. Since one must extend observation to a distance of one over the acceleration (c^2/g) to study a radiation field, comparison of an accelerated charge to a static charge in a gravitational field is not a proper test of the SEP; the SEP escapes scrutiny.

One possible statement of the weak equivalence principle (WEP) is that a particle's mass can not be determined by its free motion in a gravitational field. The WEP breaks down for quantum systems.[7] The WEP is in conflict with quantization as follows: Classically, the WEP states that the motion of a particle may be parameterized by its velocity, which is independent of its mass. In the Bohr limit of quantum theory,

$$\int p \, dx = nh,$$

or,

$$\int v \, dx = nh/m. \tag{34}$$

Thus, the velocity is parameterized by (h/m), dependent on mass in contradiction with the WEP. The Schrodinger equation is similarly parameterized. Dividing the Schrodinger equation by mass gives

$$\left[-\frac{1}{2}\left(\frac{h}{m}\right)\frac{\partial^2}{\partial x^2} + \phi\right]\psi_n = \varepsilon_n \psi_n, \tag{35}$$

so that

$$\psi_n = f(h/m), \ \varepsilon_n = f(h/m), \ E_n = mf(h/m). \tag{36}$$

That the WEP breaks down for quantum systems has been established experimentally by Collela, et al.[2] This experiment establishes that the mass of a particle can be determined by gravitational induced phase effects. The SEP -- local equivalence of gravity to acceleration -- is not contradicted by quantum theory, nor by quantum experiment.

Is the ability to determine the mass of a particle from its gravitationally produced phase worrisome? While this is clearly a departure from classical thinking, as long as the mass dependence is confined to the phase, and so far it is, I claim that this is not problematic. The WEP is a classical concept, which does not survive in the quantum domain; while the phase is a strictly quantum mechanical concept. That the non-classical phase violates the WEP should be cause for thought but not worry. However, if the amplitude, from which classical variables, e.g. velocity, can be obtained by integration, should be shown to violate the WEP, there might be extreme concern.

Weak gravitational fields may be equivalent to accelerated reference frames in special relativity; i.e. the metrics for the two have the same form. This is important, from the geometric perspective of general relativity, since this perspective holds that, locally, gravitational fields are indistinguishable from the

Minkowskian space of special relativity, and the criterion for locality is the same criterion that gives a weak field. Begin with the condition distance << g^{-1}, local coordinates $\xi^{\mu'}$, and basis vectors $\underline{e}_{k'}$ for the locally defined hypersurface.[8] The observer is located at $P(\tau)$, which is displaced from the origin by $\underline{Z}(\tau)$. The typical point in this hyperplane is

$$\underline{x} = \xi^{k'} \underline{e}_{k'}(\tau) + \underline{Z}(\tau). \tag{37}$$

$\xi^{0'}$ is identified with τ, and the coordinates of a point are found by solving

$$x^{\mu} = \xi^{k'}\left[e_{k'}\left(\xi^{0'}\right)\right]^{\mu} + Z^{\mu}\left(\xi^{0'}\right), \tag{38}$$

for $\xi^{\mu'}$. The equations obtained are

$$x^{0} = \left(g^{-1} + \xi^{1'}\right) \mathrm{sh}\left(g\xi^{0'}\right),$$

$$x^{1} = \left(g^{-1} + \xi^{1'}\right) \mathrm{ch}\left(g\xi^{0'}\right),$$

$$x^{2} = \xi^{2'},$$

$$x^{3} = \xi^{3'}. \tag{39}$$

Thus, the metric for an accelerated reference frame is

$$ds^2 = \eta_{\mu\nu} dx^{\mu} dx^{\nu}$$

$$= -\left(1 + g\xi^{1'}\right)^2 \left(d\xi^{0'}\right)^2 + \left(d\xi^{1'}\right)^2 + \left(d\xi^{2'}\right)^2 \left(d\xi^{3'}\right)^2. \tag{40}$$

The coefficient of the first term is

$$1 + 2g\zeta^{1'} + \left[g\zeta^{1'}\right]^2.$$

Since we are restricted to $\zeta^{1'} << g^{-1}$, or $g\zeta^{1'} << 1$, $(g\zeta^{1'})^2$ can be neglected. Further, $g\zeta^{1'}$ is identifiable as the gravitational potential Φ. The metric (40) becomes

$$ds^2 = -\left[1 + 2\Phi\right]\left[d\zeta^{0'}\right]^2 + \left[d\zeta^{1'}\right]^2 + \left[d\zeta^{2'}\right]^2 + \left[d\zeta^{3'}\right]^2 \qquad (41)$$

The metric (41) is the standard metric for nearly-Newtonian gravitational fields.

V. Conclusion

Two conclusions can be drawn from this discussion: 1) Theory and experiment are in agreement. 2) The application of the equivalence principle leads to paradoxes which remain to be explained. The status of the strong equivalence principle for a charged particle suspended in a gravitational field which is uniform to large extent remains uncertain. The position independent current observed by Kuhn, and Schoelkopf is an unresolved paradox for general relativity. That gravitational phase effects are mass dependent in contradiction with the weak equivalence principle is not worrisome, but awaits an adequate explanation.

Acknowledgements

I have benefitted priciplely from J. Kuhn's supervison. W. Repko also provided helpful conversation.

References

[1] R.P. Feynman and A.R. Hibbs, Quantum Mechanics and Path Integrals (McGraw-Hill, New York, 1965)

[2] R. Collela, A.W. Overhauser, and S.A. Werner, Phys. Rev. Lett. 34, 1472 (1975)

[3] J.R. Kuhn and R. Schoelkopf, preprint

[4] A.K. Jain, J.E. Lukens, and J.-S. Tsai, Phys. Rev. Lett. 58, 1165 (1987)

[5] C.M. Bender and S.A. Orszag, Advanced Mathematical Methods for Scientists and Engineers (McGraw-Hill, New York, 1978), p. 51

[6] H. Bondi and T. Gold, source unknown, but believed to be Proc. Roy. Soc. (1955)

[7] D.M. Greenberger, Rev. Mod. Phys. 55, 887 (1983)

[8] C.W. Misner, K.S. Thorne, and J.A. Wheeler, Gravitation (W.H. Freeman, San Francisco, 1973), p. 172