PLATO'S THEORY OF NUMBER

Dissertation for the Degree of Ph. D.

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ABSTRACT

PLATO'S THEORY OF NUMBER

By

Bess Makris Stamatakos

How Plato's concept of number relates to his central metaphysical doctrine of the forms or ideas has been problematic, in that interpreters of Plato have not agreed whether Plato viewed numbers as forms, or as ontologically separate kinds of entities, which are neither forms nor physical things in this world. Chapter I of this dissertation presents a summary of the various positions taken by scholars regarding Plato's view of number relative to forms.

It is argued in this dissertation that, for Plato, numbers are forms. This investigation of number is separated into two parts. The first part of the investigation answers the question, "What sort of thing is number?" and is answered in Chapter II of this dissertation. The argument is developed by using Plato's argument from kinship (Phaedo) as a model to show that numbers and forms share the same properties or characteristics. These characteristics, which are all found mentioned in the dialogues and are either explicitly or implicitly associated with forms and numbers are listed in this chapter as : 1) intelligible or invisible, 2) causing or ruling, 3) immortal or eternal,

4) constant and invariable, 5) independently existing, 6) objects of knowledge, 7) indivisible or incomposite, and 8) unique and perfect.

Chapter III, the second part of the investigation responds to the question, "What is number?". First, Plato's criteria of an adequate definition are established. Then, it is suggested in this chapter that Plato defines number in a weak sense as 'the odd and the even', even though this definition does not satisfy his own criteria of an adequate definition. In addition, it is argued that although Plato was familiar with fractions and irrationals, he does not consider them to be numbers.

Chapter IV is a critical evaluation of an alternative position presented by A. Wedberg, which states that numbers are not only forms, for Plato, but that, ontologically, they are also separately existing entities called 'intermediates'. This chapter is polemical in nature and argues against this alternative view by examining the interpretation of various Platonic passages that Wedberg offers in his support. The conclusion of this chapter, after considerations of the plausibility of a theory of 'intermediates', is that there is nothing in the Plato text that demands an interpretation of numbers as 'intermediates.'

PLATO'S THEORY OF NUMBER

By

Bess Makris Stamatakos

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1975

DEDICATION

To my husband, Lou,

for his love and understanding

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CHAPTER I

INTRODUCTION

The importance of mathematics in Plato's works cannot be overestimated. The Dialogues abound with mathematical examples, both geometric and arithmetic in nature. And yet, in spite of the frequency of these examples, no dialogue is directly concerned with the question, "What is a number?". Because this question about the nature of number is never asked and only occassionally dealt with by Plato, subsequent philosopher-scholars have felt a need to give some kind of reconstruction of Plato's view of number, and interestingly, the accounts have varied greatly.

Three main lines of thought have emerged through the years in connection with Plato's view of number. One interpretation is the claim that numbers are neither forms, for Plato, nor sensible particular things, but that they are entities that hold a separate ontological position, intermediate between them. They are not forms because they are not unique and they are not sensible particular things because they are eternal and unchangeable. This is the position that one finds stated most often by Aristotle, who tells us:

> Further, besides sensible things and forms, he [Plato][1] says that there are objects of mathematics, which occupy

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[1] Plato is mentioned by name in the preceding sentence, Metaphysics 987b11.

> an intermediate position, differing from sensible things and being eternal and unchangeable, from Forms in that there are many alike, while the Form itself is in each case unique.[2]

When Aristotle compares Plato with the Pythagoreans he says:

> ...and so his view that numbers exist apart from sensible things, while they [Pythagoreans] say that the things themselves are Numbers, and do not place the objects of mathematics between Forms and sensible things.[3]

And again,

> ...those who believe both in forms and in mathematical objects intermediate between these and sensible things.[4]

Again, when Aristotle is to talk about substance, he says;

> Plato posited two kinds of substance - the Forms and the objects of mathematics - as well as a third kind, viz, the substance of sensible bodies.[5]

Along these lines, Wedberg[6] presents an interpretation of Plato's philosophy of mathematics which, in all main points, agrees with Aristotle's exposition. He argues that Plato posited two kinds of number, the 'ideal numbers' which are Ideas and the 'mathematical numbers' which are not the Ideas but which nevertheless share the mode of existence characteristic of Ideas. These mathematical numbers are

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[2] *Metaphysics* 987b14-31. This and other translations from *Metaphysics* are by Ross.

[3] *Metaphysics* 987b28.

[4] *Metaphysics* 995b16-18.

[5] *Metaphysics* 1028b19-21.

[6] Anders Wedberg, *Plato's Philosophy of Mathematics*, Almquist and Wiksell, Stockholm, 1955.

referred to, by Aristotle and those that hold this view, by the name of 'Intermediates'.[7] Wedberg admits that Plato never really explicitly asserts the existence of these intermediates, nor does he use the terminology Aristotle uses, but that in the Republic 525c-526c, along with a few other passages[8] Plato describes a kind of number which fits the description of Aristotle's mathematical numbers.

Along the same line Robin[9] says that although numbers are the models of the ideas, the mathematical numbers appear to be intermediate between the ideal and sensible realms, and their function is to introduce quantity into the sensible world. And Hardie[10] states that the doctrine of the intermediates is plainly presupposed by Plato and is an explicit doctrine of the Republic.

At the other end of the scale are those that believe that Plato held no view of number as 'intermediates', but that the specific numbers, two, three, and so on, are forms. Among those that hold this view is Wilson[11], who argues that Aristotle's notion of intermediates is due to his misunderstanding of Plato's theory of numbers, and that

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[7]The 'intermediates' include both mathematical numbers, and ideal geometrical figures, but this paper is concerned only with the status of number. Cf. Wedberg, p. 11.

[8]Notably, the Philebus 56d ff. and the Phaedo 74c ff.

[9]L. Robin, La theorie platoniscienne des idees et des nombres d'apres Aristotle, Paris, 1908.

[10]W.F.R. Hardie, A Study of Plato, Oxford University Press, 1936, p. 50.

[11]Cook Wilson, "On the Platonist Doctrine of ἀσύμβλητοι ἀριθμοί , Classical Review, 1904.

the doctrine of the intermediates cannot be found in the dialogues. Numbers are forms, i.e., universals of numbers, and as such, certain difficulties arise which might have caused Plato's disciples to come up with the notion of intermediates to explain these difficulties. Shorey[12] takes this line when he states that the objects of mathematics are explicitly stated in the Republic to be νοητά and, as such, are the objects of knowledge, i.e., forms. Raven[13] who, like Shorey, relates the question of the intermediates primarily to the Republic, argues that mathematical objects are not the sole objects of the state of mind intermediate between νόησις and πίστις.

Cherniss'[14] argument against the view of numbers as intermediates is most forceful in that he questions the reliability of Aristotle's report. He says that to include Aristotelean text in an interpretation of Plato's views is unwarranted for several reasons. First, he claims that it is rather doubtful that Plato expounded any theories about forms and numbers beyond what we possess in the dialogues, in spite of Aristotle's reference to Plato's 'unwritten doctrines'; secondly, that Aristotle is never perfectly clear to whom he is referring in his remarks about forms and numbers, as there were at least two other prominent philosopher-mathematicians at the Academy at the same time, namely Xenocrates and Speusippus, whose theories varied from each others' and from Plato's; third, because of Aristotle's

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[12] P. Shorey, The Unity of Plato's Thought, Archon Books, 1903, 1968, p. 83.

[13] J.E. Raven, Plato's Thought in the Making, Cambridge, 1965, p. 158.

[14] H. Cherniss, The Riddle of the Early Academy, Russell and Russell, New York, 1962, Chapter II.

polemical nature and because of his desire to present his own ideas relative to those views of previous philosophers, he tends to present a skewed picture of the views of earlier philosophers, by introducing his own terminology; and four, Aristotle himself gives inconsistent reports about Plato's view of numbers. For these reasons Cherniss rejects the view that Plato posited 'intermediates'.

The third line of thought is one taken by Ross. Ross argues that in the Republic Plato thought of Ideas falling into two divisions, a lower division consisting of Ideas of number or space, and a higher division not involving these.[15] Ross also claims that Cherniss goes too far in the opposite direction when he denies that Plato ever believed in intermediates at all, because "the distinction between 'intermediates' and ideas is below the surface in dialogue after dialogue, only waiting to be made explicit"[16], and "it is a doctrine which Plato seems from time to time to be on the verge of stating but never quite states."[17] Ross, who bases his views on both Platonic and Aristotelean texts, doubts that Aristotle, who distinguishes the views of Plato from Speusippus and Xenocrates in Books M and N of the Metaphysics, would without good reason have committed himself to a distinction that, if it were erroneous, could easily be repudiated.[18]

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[15] Sir David Ross, Plato's Theory of Ideas, Oxford, 1951, p.65.

[16] Ibid., p. 66.

[17] Ibid., p. 177.

[18] Ibid., p. 66.

Annas[19] is in agreement with Ross that Cherniss' conclusion that Plato did not expound any theories about Forms and numbers beyond what we possess in the dialogues is too sweeping, and that this conclusion should be narrowed.[20] Annas accepts the view that Plato held to a belief in intermediates, but acknowledges the fact that this view is based largely on Aristotelean texts, though "Plato's theory of number did unquestionably try to combine the thesis that numbers are forms with the concept of numbers as sets of units."[21]

The following thesis is based primarily upon Platonic text[22] and it is the view of the writer that all textual considerations point to the view that numbers, for Plato, are forms. In Chapters II and III we shall argue for this view in a positive fashion from the Platonic texts, and in Chapter IV we shall argue for it negatively by criticizing recent arguments for the view that Plato held a theory of intermediates. In asking, "What is number", it will be helpful to make use of a distinction Plato makes between a ti esti question and a poion question.

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[19] J.E. Annas, "Aristotle's Criticism of Plato's Theory of Numbers," Harvard Ph.D. Dissertation, 1973.

[20] Ibid., p. 303.

[21] Ibid., p. 233. Since forms are unitary (indivisible) to say that numbers are sets of units and forms simultaneously is contradictory.

[22] There must have been many discussions among the members of the Academy concerning the nature of number and many doctrines might have emerged from these discussions, but the fact that nothing has been preserved in written form concerning Plato's 'unwritten doctrines', makes the task of reconstructing these doctrines highly speculative. I have, for this reason, chosen to adhere primarily to Platonic text, in my interpretation of Plato's view of number.

B.

A ti esti question is one that asks the question, "What is x?" and the answer to this kind of question is one that tells of x's real nature, its essence, or what x actually is. In contrast, a poion question asks, "What sort of thing is x?" and might satisfactorily be answered by enumerating the characteristics of x, or listing the things that can be said _about_ x. This distinction is found throughout the dialogues. In the _Gorgias_ (448e6) Socrates distinguishes between what kind of art Gorgias is engaged in and what art it actually is. In the _Protagoras_ (360e7) Socrates says that there is a difference between learning _about_ virtue, and what virtue, in itself, is, and in the _Meno_ Socrates asks how can one know a property of something if he doesn't even know what it is (71b-c) and at 86d5, he says that the main question is not whether virtue can or cannot be taught, but what virtue is. Once again, in the _Theaetetus_ (146e8) the question is not what are the objects of knowledge, nor how many sorts of knowledge there are, but what the thing itself, knowledge, is.

The following chapters of this dissertation rest upon this distinction. Chapter II answers the poion question, or what sort of things numbers are. It is shown in this chapter that numbers exhibit basically the same characteristics that forms exhibit, and as such, numbers and forms are the same kinds of entities. Plato uses the same kind of argument in the _Phaedo_ when he argues that the soul is like the forms. The shortcomings of this sort of argument are acknowledged.

But to ask "What is number?" would, in accordance with Plato's distinction, be to ask a ti esti question. One is interested in something more fundamental than the kinds of things that can be said about number. What one asks the ti esti question of number, one is interested in determining what is the true nature of number, and when one has successfully answered this question, one has true knowledge of what number is. According to Plato, it is only with the apprehension of the real nature of anything that true knowledge of it is attained. (*Phaedo* 65d12).

Chapter III is an attempt to answer the ti esti question with regard to number. Since this calls for a definition of number, the criteria that Plato suggests in the dialogues for an adequate definition are established in the first part of the chapter. Then possible candidates for a definition of number are considered in light of these criteria. The ensuing discussion considers whether irrationals and/or fractions were considered to be numbers by Plato.

The concluding chapter refers back to the problem as stated in Chapter I with regard to the intermediates, and is negative in intent. Whereas Chapters II and III look for evidence in the dialogues to support the positive claim that numbers are forms, Chapter IV is to show that there is no evidence in the dialogues to support the claim that Plato viewed numbers as intermediates, and that it is highly improbable that Plato viewed numbers in this way.

CHAPTER II

A.

Among the many arguments that Plato gives in the Phaedo to prove the immortality of the soul, there is one that is peculiarly unique because it is explicitly a probabilistic argument and because it tells us as much about forms as it does about souls. The intent of the argument is to prove the soul's immortality; it attempts to do so by arguing that the soul is more akin to forms than it is to bodily things, and, as such, is 'less likely to dissolve or disintegrate'.

Briefly, the argument goes like this: Of things, there are two classes, those that are constant and invariable and those that are inconstant and variable. Of those that are constant and invariable, it is extremely probable that they are incomposite, and those that are inconstant and variable are composite (most likely to break up). The forms, such as absolute beauty, etc., are examples of the constant, invariable, and incomposite entities. The concrete instances of the forms, beautiful things, etc., are examples of those things that are inconstant, variable, and composite.

Of things, there are two more classes, those that are visible and those that are invisible. The forms, in addition to being constant, invariable, and incomposite, are invisible, and their instances, in addition to being inconstant, variable, and composite, are visible.

The soul, since it is invisible, is more like those things that are invisible, therefore it is constant, invariable, and incomposite, just as the forms are. Whereas, the human body, since it is visible, is more like those things that are visible; therefore, it is inconstant, variable, and composite.

The final two classes of things are those that are divine and those that are mortal. The soul is more like that which is divine since it rules and governs, and the body is more like that which is mortal, since it serves and is subject. The conclusion of the argument is:

> The soul is most like that which is divine, intelligible (invisible), uniform, indissoluble, and ever consistent and invariable; whereas the body is most like that which is human, mortal, multiform, unintelligible, dissoluble, and never self-consistent. (_Phaedo_ 80b1).

Socrates goes on to say that because the soul has all these characteristics, when a human dies, it is natural for the body to disintegrate rapidly, but for the soul to be quite or very nearly indissoluble. (_Phaedo_ 80b8).

An analysis of this argument shows that Plato works with four characteristics of things and their opposites:

All Entities		
	Souls and Forms	Bodily Things
a.	constant and invariable	inconstant and variable
b.	incomposite	composite
c.	invisible	visible
d.	divine	mortal

Forms are explicitly related to the first three characteristics on the left. Plato never declares forms divine, nor does he deny it, but since forms are causes (_Phaedo_ 100b), they may be viewed as ruling. Forms can be called divine because the divine-mortal dichotomy rests upon the ruling-to be ruled dichotomy. Thus, souls and forms both exhibit the same characteristics.

Two things are noted about this argument: 1) Plato says that because souls are c) invisible, they are more like those things that are invisible, namely, forms. And in being like forms, they exhibit characteristics a) constancy and b) incompositeness. Likewise, since the human body is c) visible, it is more like those that are visible, so it is a) inconstant and b) composite. 2) The conclusion is that souls are immortal because they are like forms. The underlying thought is that forms are immortal since they are constant, invariable, and incomposite. So:

Forms are a, b, c, d, and immortal.

Souls are a, b, c, d.

∴ Souls are like Forms (because of a, b, c, d).

∴ Souls are immortal.

The argument is a weak inductive for several reasons. The first reason is that it does not follow that because two entities share the same property that they will necessarily share any other properties. Thus, it does not necessarily follow that because the soul is invisible that it will be a) constant and b) incomposite. Plato's method of assigning the characteristics of constancy and incompositeness to the soul weakens the conclusion. The second reason is that Plato never explicitly says in this passage that forms are immortal. The conclusion would have been considerably strengthened if he had said:

Forms are a, b, c, d, and e.

Souls are a, b, c, and d.

∴ Souls are e.

But even if he had said this, it is not clear that the conclusion would follow, say, in case souls are not like forms.

In order to prove that the soul is, in fact, immortal, Plato would have had to state that souls are forms, and then argue deductively that since forms are immortal, souls will also be immortal. But to show that souls are forms might have involved him not only in something that he could not have shown, since it would entail that every characteristic of souls is identical to every characteristic of forms, but in something that he probably did not believe to be true.

With these reservations about the probabilistic nature of the argument from kinship, I will, nevertheless, use it as a model to determine the place of number in Plato's ontology of forms and sensible particular things. The couples of characteristics used in the following pages of this dissertation are listed as follows:

1.	Intelligible or Invisible	Sensible or Visible
2.	Causing or Ruling	Caused or Ruled
3.	Immortal or Eternal	Mortal or Perishable
4.	Constant or Invariable	Inconstant or Variable
5.	Independently Existing	Dependently Existing
6.	Objects of Knowledge	Objects of Opinion
7.	Indivisible or Incomposite	Divisible or Composite
8.	Unique and Perfect	Many and Imperfect

These characteristics and their opposites were chosen for several reasons. They are the characteristics that are explicitly and most often discussed in the dialogues in description of forms. They are the necessary conditions of being a form. Each of the above characteristics is also either explicitly or implicitly ascribed to numbers in the dialogues.

The argument in the following pages will be in the form:

Properties 1, 2, ...8 are explicitly assigned to forms in the dialogues.

Properties 1, 2, ...8 are either explicitly or implicitly assigned to numbers in the dialogues.

Therefore, numbers are the same sort of thing as forms.

The intent of the remainder of this chapter is to show that, for Plato, numbers are the same sort of thing as forms because they exhibit the same characteristics (those in the left column above) and it is probable that numbers are forms, rather than being a separate species of a higher genus of intelligible things. This argument is inductive in nature, but it is stronger than Plato's argument from kinship because no characteristic is assigned to either numbers or forms on the basis of their having another common characteristic.

It was suggested that the characteristics chosen are necessary conditions for being a form, but it should be added that it is not certain that together they are sufficient conditions. If one could be confident that the list of characteristics were complete, i.e., that forms exhibit no other characteristics, than one could conclude that numbers are forms. But Plato never tells us this in the dialogues. On the other hand, one cannot help but feel that if forms exhibited any additional characteristics Plato would have told us.

Because the model's limitations as a proof restrict the strength of the conclusion that can be drawn, as in the case of the argument from kinship, one can at most say that the conclusion that numbers are forms is probable. The probabilistic nature of the conclusion is no great cause for alarm, however, because it is quite in keeping with Plato's method to leave answers inconclusive.

B.

1. Intelligible or Invisible

It is true of both forms and numbers that they are apprehended only by thought. Plato tells us that the man who pursues the truth by applying his pure thought and cuts himself off from his eyes and ears and virtually all the rest of his body will reach the goal of reality. (Phaedo 66a1). That goal is the true perception of the nature of any given thing, absolute beauty, absolute uprightness, absolute goodness (Phaedo 65d1). The ideas, we are told, absolute beauty, absolute good, and so on, are a class of things that can be thought but not seen (Republic 507b13). In the Phaedo 71a1, Socrates says "...these constant entities (forms) you cannot possibly apprehend except by thinking are invisible to our sight."

Even in the later dialogues, the Stranger in the Statesman says, "...the existents which are the highest value and chief importance, are demonstrable only by reason and are not to be apprehended by any other means." (286a9). The forms are invisible and imperceptible by any sense, and the contemplation of them is granted to intelligence only. (Timeaus 52a2).

That number is apprehended only by thought is told several times in the dialogues.

> What numbers are these you are talking about?...they are speaking of units which can only be conceived by thought and which it is not possible to deal with in any other way. (Republic 526a2).

And when asked through which part of the body our mind perceives the "commons that apply to everything" (τὰ κοινά περὶ πάντων ἐπισκοπεῖν) such as numbers in general and the even and odd, Theaetetus answers: "...it is clear to me that the mind itself is its own instrument" for contemplating τὰ κοινά . (Theaetetus 185e1).

2. Causing or Ruling

Plato's approach to the problem of causality is to distinguish between the cause of a thing and the condition without which it could not be a cause. (Phaedo 99b3). As a young man, Socrates says, he had studied science so that might learn the cause for which each thing comes and ceases to be. (Phaedo 96a7). He had been content to think that the cause (αἰτία) of his sitting in a bent position was due to the fact that his body was composed of flexible joints, and that his bones moved freely in these joints by relaxing and contracting. (Phaedo 98d).

But he had come to realize that these are conditions without which there could be no cause, but they are not causes. His bodily composition had to be of a certain kind in order to be able to bend, which was the necessary condition of his bending, but it was not the cause of his bending. Man has the choice of whether to bend or not, so the cause had to be tied up to mind.

Plato is to introduce the Forms, absolute beauty, and goodness, and magnitude and all the rest of them, and "with their help is to explain causation." (Phaedo 100b9). Socrates says whatever is beautiful apart from absolute beauty is because it partakes of that absolute beauty and for no other reason (Phaedo 100c4, 100d5). An object's beauty is not due to its gorgeous shape or color, or any other such attribute (Phaedo 100c9). Likewise, whatever is taller than something else is simply so because it participates in Tallness, and similarly for Shortness (Phaedo 101a1). Plato makes it clear in the foregoing discussion that forms are causes and not conditions that make a cause possible. One must have have the conditions before one can have the cause, and as such, these conditions have ontological priority, but the cause has logical priority.

Numbers are causes in the way forms are causes. Socrates says that he cannot believe that:

> ....when you divide one, that this time the cause of its becoming two is division, because this cause of its becoming two is the opposite of the former one; then it was because they were brought close together and added one to the other, but now it is because they are taken apart and separated one from the other. (Phaedo 97a5)

The addition of one and one is not the cause of something becoming two; the thing's participation in 'duality' or 'twoness' is the cause of its being two. (Phaedo 101c1). Similarly, whatever is to become one must participate in unity (Phaedo 101c8).

> You would surely avoid saying that the cause of our getting two is the addition, or in the case of a divided unit, the division. You would loudly proclaim that you know of no other way in which any given object can come into being except by participation in the reality peculiar to its appropriate essence (οὐσία), and that in the cases which I have mentioned you recognize no other cause for the coming into being of two than participation in duality -- whatever is to become two must participate in this (Phaedo 101c1).

A little later in the Phaedo Socrates says once again, "...when the form of three (τῶν τριῶν ἰδέα) takes possession of any group of objects, it compels them (ἀνάγκη αὐτοῖς) to be odd as well as three." (Phaedo 104d7). Absolute beauty is the cause of a thing's beauty, just as 'twoness' is the cause of two things being two. Twoness, threeness, etc., are causes in the same sense as a form is a cause.

3 and 4. Immortal or Eternal and Constant or Invariable.

The third and fourth characteristics shared by forms and numbers is their immortality or eternality and their constancy or invariability. In the case of Forms, they are neither subject to generation nor destruction (Philebus 15b3), are eternal and unchanging (Republic 484b5), just as Beauty is always beautiful "neither more nor less" (Symposium 211a), and remain constant, invariable, and unchanging:

> Does that absolute reality which we define in our discussions remain always constant and invariable or not? Does absolute equality or beauty or any other independent entity which really exists ever admit change of any kind? Or does each one of these uniform and independent entities remain always constant and invariable, never admitting any alteration in any respect or in any sense? (Phaedo 78d1).

The answer is that they must be constant and invariable, and never admit of alteration. The idea of beauty itself always remains the same and unchanged (Republic 479a3) and such things as man, ox, the beautiful, and the good are each always one and the same. (Philebus 15b3). Again in the Philebus, Socrates says that that which exists in reality is ever unchanged (58a3) and that those who study the universe around us have "nothing to do with that which always is, but only with what is coming into being, or will come, or has come". (59a5-10). The stranger in the Sophist attributes to the friends of the forms the view that "real being is always in the same unchanging state" (248a14). And in the Timaeus:

> We must acknowledge that one kind of being is the form which is always the same, uncreated and undestructible, never receiving anything in itself from without, nor itself going out to any other...(51e8).[23]

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[23] G.E.L. Owen ("The Place of the Timaeus in Plato's Dialogues", p. 322-4 in Studies in Plato's Metaphysics, ed. Allen) argues that the distinction between 'οὐσία" and 'γένεσις' is jettisoned after the middle dialogues, but here are three dialogues, the Sophist, the Philebus, and the Timaeus which seem to retain it. H.F. Cherniss, "Relation of the Timaeus to Plato's Later Dialogues", p. 329-60 in Studies, ed. Allen, says that Owen's placement of the Timaeus in the middle Platonic period does not explain the occurrence of the distinction in the Sophist and the Philebus.

Numbers, like forms, are eternal, ungenerable, imperishable, constant, and invariable. With regard to the characteristics of constancy and invariability, it is said of a unit that every one is equal to every other without the slightest difference (Republic 526a3). And in the Philebus (56d12), Socrates says that every unit is precisely equal to every other unit.[24] Socrates tells us that "it is the very nature of three and five and all the alternate integers that every one of them is invariably odd, although it is not identical with oddness" (Phaedo 104a7). Similarly, two and four and all the rest of the other series are not identical with the even, but each one of them is always even. Since the oddness of three is its very nature, it cannot admit the form of even:

> ...five will not admit the form of even, nor will ten, which is double five admit the form of odd. Double has an opposite of its own, but at the same time will not admit the form of odd. Nor will one and a half, or others like these such as one-half and a third admit the form of whole." (Phaedo 105b1)[25]

The inadmissibility of a form opposite to a characteristic of a number guarantees us the number's invariability.

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[24]More will be said about both the Republic and Philebus pas-passages later in this chapter.

[25]This is my translation. Some translations insert the word 'fractions' in the last sentence of the quote. Notably, Hackforth's and Tredennick's translations read, respectively: "Again the fraction three-halves and all the other members of the series of halves will not admit the character of wholeness, and the same is true of one-third and all the terms of that series." Nor will one and a half, or other fractions such as one-half or three-quarters and so on, admit the form of whole." In Greek the last sentence reads: "οὐδέ δή τό ἡμιόλιον οὐδέ τἆλλα τά τοιαῦτα, τό ἥμισυ, τήν τοῦ ὅλου, καί τό τριτημόριον αὖ καί πάντα τά τοιαῦτα." There is no Greek equivalent for the word 'fraction' in this passage. ' τἆλλα τά τοιαῦτα ' is best translated 'others like these'.

Though a number, say, three, may not admit an opposite form, one might want to argue that it may still be variable in the way fire can be more or less hot, even though it will never admit of cold. Does Plato ever say that a number can be more or less that number, or must it be an invariable and constant quantity? The answer is that "numbers must be just what they are or not be at all. The number ten at once becomes other than ten if it be increased, and so of any other number..." (Cratylus 432a9).

In the Philebus, Socrates introduces us to "two constituents of things, the unlimited and the limit" (23c10). Of the unlimited he says that "when they are present in a thing, they never permit it to be a definite quantity but introduce into anything the character of being 'strongly' so and so as compared with 'mildly' so and so, or the other way around. They bring about a 'more' or a 'less' and obliterate definite quantity." (Philebus 24c2). "When we find things becoming 'more' or 'less' so-and-so, or admitting of terms like 'strongly', 'slightly', 'very' and so forth, we ought to reckon them all belonging to a single kind, namely, that of the unlimited." (Philebus 24e8).

On the other hand, things that do not admit of these terms come under the limit. Examples of such things are "equal" and "equality", "double" and "any term expressing a ratio of one number to another" or "one unit of measurement to another." (Philebus 25a8).[26]

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[26] It is not clear why Plato makes a distinction between a ratio of one number to another and of one unit of measurement to another. One possible interpretation is that it is his way of incorporating geometric entities such as line segments under the categorh of limit. Measurement is often associated with geometry in the dialogues (Philebus 56e7, Republic 526d2, Republic 534d7). M. Brown says that for Plato, arithmetic and geometry are two distinct things. "Plato Disapproves of the Slave Boy" in Plato's Meno, edited by M. Brown, p. 199.

Numbers are given as examples of those kinds of entities that do not admit any variability. The introduction of number "puts an end to the conflict of opposites with one another" and makes things "well proportioned and harmonious". (Philebus 25e7).

Plato does not explicitly say that numbers are eternal and ungenerable, but there are evidences in the dialogues that they are no different from forms in this respect. In the Republic, Socrates says, "Geometry is the knowledge of the eternally existent" (Republic 527b6). The reference is to geometry, but the whole section deals with numbers as well as geometric objects. The science of arithmetic, which is wholly concerned with number (529a9), leads to the apprehension of truth (525b1) just as the objects of geometry draw the soul to truth. (527b8).

The imperishability of numbers is shown in the Phaedo (106ff) during the proof for the imperishability of the soul. Socrates argues that if what is immortal (the soul) is also imperishable, then at the approach of death, it would not perish but retire and depart. Now in the case of that which is not even "we could not insist that the odd does not cease to exist -- because what is not even is not imperishable.. but we could easily insist that, at the approach of even, odd and three retire and depart." (Phaedo 106c1). Socrates is saying that 'what is not even' is not imperishable, but that odd and three may be imperishable, since they merely retire and depart. In order to make sense of this, the 'what is not even', must be different from three and odd. Because if we are to suppose that 'what is not even' refers to the number three, the argument ends in contradiction. The number three is not perishable and it is perishable. It must therefore mean three sheep, or five apples, or any group of material or substantial entities that can be called odd.

This interpretation fits with the earlier analogies of the passage. 'What is not hot' is later called snow, and 'what is not cold' is called fire. The outcome of the passage is that not only is the soul imperishable, because it retires and departs at the approach of death, but number is also imperishable, since it merely retires and departs.[27]

Number is not only imperishable, but existed prior to the objects of sense in this world. In the Timaeus, it is said that God fashioned the four kinds of elements, which make up the physical universe ("The objects which I have been describing are necessarily objects of sense." Timaeus 61c7) by form and number. (Timaeus 53a12).

The ontological priority of numbers to soul, as well as body, is also suggested. In the Phaedo Socrates argues that the soul is not an attunement, as Simmias suggests in the analogy of the soul to a lyre, because in the case of a lyre, the strings and untuned notes come first (Phaedo 92e1), i.e., before the harmony or attunement. The soul "directs or leads all the elements of which it said to consist." (Phaedo 94c10). So the soul cannot be linked with any physical parts, like parts of an instrument. But the soul partakes of harmony (Timaeus 37a2) and that harmony is linked to numbers.[28]

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[27] The argument is not at all convincing. If 'x' is imperishable, then it will merely retire and depart. But arguing that something merely retires and departs, does not show that it is imperishable. The invalidity of the argument applies to conclusions about both souls and numbers.

[28] W.K.C. Guthrie, History of Greek Philosophy, Cambridge University Press, Vol. I, 1962, argues that the view that the soul is a harmony does not conflict with its immortality, because the harmony is seen as a numerical, rather than a physical one. (That numbers are non-physical entities will be argued shortly. The non-materiality of the soul ceases to be threatened if numbers are non-physical entities.) p. 316.

In the Timaeus God fashioned the soul out of being, same, and different "divided and united in due proportion". (37a5). The proportions God used in designing the soul are numerical in nature. God takes the whole mixture of being, same, and different and takes away quantities from this mixture to form two series: one series being 1, 2, 4, 8..... and another series being 1, 3, 9, 27 ..... And in each interval of each series, he places two kinds of means, the harmonic and arithmetic means. So with the placement of the first kind of mean, the even series becomes 1, 4/3, 2, 7/3, 4, 16/3, 8, 32/3........ and the odd series becomes 1, 3/2, 3, 9/2, 9, 27/2, 27...and so on. The second means are then placed so that the interval between each of the numbers of the series are further decreased. This is the process of reducing the size of the interval until the last interval is expressed in the ratio 256:243[29], whence the whole mixture is exhausted. (Timaeus 35b4-36b4).

Archytas[30] uses the same procedure of finding intervals for the tetrachord for the harmonic, chromatic, and diatonic scales in music. Interestingly, the product of the intervals in the tetrachord for each of these scales is 4/3. The harmonic and arithmetic means are also discussed in the Epinomis (990c) and the value of these means to a double, (say between 1 and a) is 3/2 and 4/3. In the Epinomis, these ratios are called "a Gift from the blessed choir of the Muses." (991b3).

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[29] The value of this ratio is between 4/3 and 3/2.

[30] Diels and Kranz, Die Fragmente der Vorsokratiker, Weidman Dakota, Dublin, Ireland, 1968, according to Ptolemy, Harm. I13, p.30.0.

It is clear that, for Plato, numbers existed not only before the body, but even before the soul since they were both made according to numbers.

There is one passage in the *Parmenides* that has been traditionally viewed as one in which Plato explicitly states that numbers are generable by addition. The argument goes like this: From the hypothesis "ἓν εἰ ἔστιν "[31] Parmenides goes on to distinguish 'one' from 'being' and 'difference'. If we then select any pair from 'one', 'being', and 'difference', we have a pair which can be spoken of as 'both'. And if we have both, we have 'two', and each term must be 'one'. If any 'one' be added to any pair (which is two), we have 'three'. Then Parmenides proceeds to say:

> And three is odd, two even. Now if there are two, there must also be *twice times*, if three, *three times*, since two is twice times one, and three is three times one. And if there are two and twice times, three and three times, there must be *twice times two* and *three times three*. And if there are three which occur twice and two which occur three times, there must be *twice times three* and *three times two*. Thus, there will be even multiples of even sets, odd multiples of even sets, odd multiples of odd sets, and even multiples of odd sets. That being so, there is no number left, which must not necessarily be."[32]

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[31] This is what has been traditionally known as Hypothesis II, starting at *Parmenides* 142b3. Taylor, Hardie, and Cornford translate this hypothesis "if a one is...". Ryle in "Plato's *Parmenides*" in *Studies*, ed. Allen argues that this hypothesis is the same as Hypothesis I, namely " εἰ ἓν ἔστιν ", p. 113. W.G. Runciman in "Plato's *Parmenides*', *Studies*, ed. Allen agrees with Ryle "that the only feasable translation of 'ἓν ' is not 'the one', but 'unity'.

I have translated 'ἓν' as 'one', primarily because it more readily lends itself to the mathematical passage which follows, although it must be added that acceptance of this translation does not indicate my readiness to take sides on this controversy as to the meaning of 'ἓν ' in the whole of part II of the *Parmenides*.

[32] Cornford's translation.

The conclusions are that "if a one is", number exists (or there must also be number). And "if number is, there must be many things and indeed a plurality of things that are." (144a6).

Cornford[33] says of this passage:

> Thus, from the simple consideration of 'One Entity' with its two parts and the difference between them, we have derived the unlimited plurality of numbers. Each of the three terms is 'one entity' and can thus be treated as a unit: and by adding and multiplying these units we can reach any number (plurality of units) however great.

Thus, in Cornford's view, the method beginning at 143a by way of addition and multiplication, "explicitly deduces the existence of the number series."[34]

Allen[35] says that Cornford's view is mistaken because in order to derive a number, one must prove that there _are_ as many units as the number. One must make an existence assumption about the number of units available. So, for example, in order to derive four, it is required that there are four units, none of which is identical with the other.

> In short, if number is a plurality of units, it cannot be generated by the use of multiplication, since the use of multiplication assumes the existence of pluralities corresponding to its product. [36]

But let us suppose that Parmenides was unaware of the need for

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[33] F.M. Cornford, _Plato and the Parmenides_, Bobbs-Merrill Co., Inc., 1957, p. 141.

[34] _Ibid_., p. 141.

[35] R.D. Allen, "The Generation of Numbers in Plato's _Parmenides_" _Classical Philology_, 1970, p. 30.

[36] _Ibid_., p. 31.

an existence axiom; the question remains, is it possible that the passage is *intended* as a derivation or generation of number? Allen argues no. He views the whole passage as one in which arithmetic is used to conclude the plurality of things, and not to derive numbers as multiples of units. The argument is of a hypothetical nature. If the existence of pluralities having two or three members is granted, then the existence of all numbers follows.[37] In Allen's view, this kind of existence proof is not a generation of number:

> Parmenides has provided no indication that any number or numbers can be constructed or derived from simpler constituents. His argument is compatible with the view that numbers are timeless objects which no more admit of generation than they admit of destruction; it is also compatible with the view that numbers are simple essences incapable of analysis into ontologically (as distinct from numerically) prior and posterior elements. In short, Parmenides' account is compatible with the assumption that numbers are Forms or Ideas. [38]

Ross[39], too, argues that the proof in the *Parmenides* for the generation of numbers is an exercise in dialectic rather than an exhibition of doctrine. In particular, the four-fold classification of numbers makes no provision for prime numbers, other than 2 and 3, so it is incomplete[40], and secondly, and most importantly it seems for Ross, the account of generation of numbers does not square with the account Aris-

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[37] *Ibid.*, p. 31. If one adds the axiom: "If a and b are integers, the product and the sum of a and b are integers", which Allen claims Parmenides does not explicitly do.

[38] *Ibid.*, p. 31.

[39] Sir David Ross, *op. cit.*, p. 187-8.

[40] According to Allen, the primes can be accounted for by this method if 1 is considered an odd number. Then (5 = 5 times 1) is an odd multiple of odd sets. One is considered an odd number in *Hippias Major* 302a6.

totle gives us. It makes no use of the "principles answering to the One and the great and the small, but produces the numbers by the ordinary processes of addition and multiplication."

Both Allen and Ross agree in opposition to Cornford, but for different reasons, that the paragraph in the Parmenides ie not specifically concerned with the generation of numbers. For Allen, the purpose of the passage is not to generate numbers, but to generate a plurality of things, and the mathematical digression is of a hypothetical nature. For Ross, the purpose of the whole of the second part of the Parmenides is dialectical in nature, rather than an exposition of doctrine, and the particular paragraph under examination does not square with Aristotle's account of generation of numbers.

The suggestion of Ross' that the whole of the second part of the Parmenides is dialectical in nature, is, however, compatable with the view that it might also expose doctrine. One finds in the Phaedo, where the purpose of the dialogue is to all appearances a discussion of the immortality of the soul, that the Forms are accepted as hypotheses for one argument of the soul's immortality. And in the Philebus, where the discussion concerns itself with the 'good', one finds Plato introducing, again in an arbitrary and hypothetical manner, the four kinds of being. In other words, the Parmenides passage might tell us something concerning the nature of number, and it might even tell us that numbers can be generated by addition of units. Ross' suggestion that the Parmenides account does not square with Aristotle's account arbitrarily plays down the possibility that the Parmenides can be evaluated in its own right.

Allen's account is such an evaluation and interpretation, but even though he says that the mathematical digression is of a hypothetical nature, he states that Parmenides accepts certain mathematical truths, such as 3 = 2 + 1, and 3 = 3 x 1, which casts some doubt on the alleged non-generability of numbers.

One of the difficulties that obscures the interpretation of this passage seems to be that it is not clear at all times, if the terms 'two', 'three', etc., refer to two or three things in this world, or to the number two and the number three, etc. The paragraph begins with the referents of 'being', 'one', and 'difference', which we can call things, or entities, in this world. Parmenides makes it clear that any pair of these entities is two and each of them is one. He specifically says that one and two apply to the entities 'one', 'being', and 'different' (143d3). And there will be three when they are joined together in order, or in a union. (Εἰ δὲ ἓν ἕκαστον αὐτῶν ἐστι, συντεθέντος ἑνός ὁποιουοῦν ἡτινιοῦν συζυγία οὐ τρία γίγνεται τα πάντα.) There is no mention of adding three units to generate the number three. The referents are things and we have three of them when they are so joined together. This does not entail any claims about what numbers are, or about how numbers relate to each other, or are generated. Things in this world are the referents, once again, in the conclusion of the argument, when Parmenides says that "there must be many things and indeed an unlimited plurality of things." (144a6).

Beginning with 143d3, where Parmenides says "three is odd, two even", one is tempted to say that something is being said about numbers, since odd and even are typically kinds of numbers for Plato. But things in this world are also called even and odd. In the Phaedo Socrates says " ... when the form of three takes possession of any group of objects, it compels them to be odd as well as three." (104d7).

Parmenides goes on to say that now since we have 'two', we have 'twice' (δυοῖν ὄντοιν οὐκ ἀνάγκην εἶναι καὶ δίς) (143d8) and since we have 'three' we have 'thrice' (καὶ τριῶν ὄντων τρίς) (143e1).[41] We now have the entities: two, three, twice, thrice, one, being, and difference. Parmenides might have continued this procedure to arrive at an unlimited number of entities. But he does not do this. He makes combinations of four of these entities in order to make a four-fold classification: 'two-twice', 'three-thrice', 'three-twice', and 'two-thrice'. Then he states: "There will be even multiples of even sets (ἄρτια ἀρτιάκις), odd multiples of odd sets (περιττά περιττάκις), odd multiples of even sets (ἄρτια περιττάκις) and even multiples of odd sets [42] (περιττά ἀρτιάκις)"(144d1), to correspond with the four combinations above. If this four-fold classification exhausts all number, then there

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[41] It might be argued that 'twice' really means two times one or one plus one, in which case two is a multiple of units, and likewise for three. The Greek text makes no such implication. A common practice with Plato is to use incomplete predicates, such as 'δίς' and 'τρίς '.

[42] This classification corresponds to Definition 8 through Definition 10 of <u>Euclid's Elements</u>, Book VII. Definition 8 is of even-times even number (ἀρτιάκις ἄρτιος ἀριθμός). Definition 10 is of odd-times odd number (περισσάκις,περισσός ἀριθμός ,), and Definition 9 is of even-times odd number (ἀρτιάκις περισσός ἀριθμός). It is noted that Euclid does not include the odd-times even number. Nicomachus, Theon, and Iamblichus, according to Heath, are each to include it in their respective elements of geometry. <u>The Thirteen Books of Euclid's Elements</u>. Translation and commentary by Sir T.L. Heath. Second Edition. Dover Publications, Inc., 1956, Volume II, pp. 281-4.

must be an indefinite number of things in this world.[43]

The numerical digression is used, it seems, as a technique to aid in the argument that there are a plurality of things in this world. The current mathematical view at Plato's time of the division of even and odd numbers into a four-fold classification serves as a model for the claim that all numbers are thus exhausted or accounted for. Just as there is an indefinite plurality of numbers, there is an indefinite plurality of things in this world. This view supports Ross' and Allen's claim that the purpose of the passage is not intended to describe the generation of numbers by addition or multiplication, but that it is to establish the fact that there are an indefinite number of things in this world.

Numbers, then, like forms, are viewed by Plato as eternal, imperishable and ungenerable.[44]

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[43]Allen claims that it is not clear that the classification in the Elements is exhaustive. Heath says that the even numbers fall into three of the categories, the even-times even, the odd-times even and the even-times odd; and since the odd numbers are the odd-times odd (this includes primes if 1 is admitted as a number) then both even and odd numbers are accounted for in the Elements. Plato must have felt, at any rate, that the classification was exhaustive since Parmenides says no number is left out.

It must be further noted, that the classification in the Elements has nothing to do with generating new numbers by multiplying or adding. The whole procedure is one of the form: given a number, then in order to determine into which category it falls, one must halve it, until it can no longer be halved. An even-times even number, for instance, is the number which has its halves even, the halves of the halves even, and so on, until unity is reached. The even-timed odd number is such a number as when once halved leaves as quotient an odd number. In short, the definitions refer to a procedure which would be the opposite of generating a new number.

[44]The question of the generability of numbers is directly related to the question of whether a number is divisible into parts. The connection is that if the number four can be generated by adding two 'two's', then the number four is also divisible into two equal parts. The section on the indivisibility characteristic goes into this in more detail.

5. Independently Existing

The fifth characteristic which marks both forms and numbers is their independence of physico-sensible things in this world. In the Phaedo (78d3) Socrates calls the forms "independent entities which really exist."[45] In the Symposium, in his exposition of one's knowledge of the Beautiful (21031), Socrates describes Beauty itself as neither a face, or hands, or anything that is of flesh nor words.

> ...nor a something that exists in something else, such as a living creature, or the earth, or the heavens, or anything that is - but subsisting of itself and by itself in eternal oneness...

When Socrates distinguishes beautiful things from Beauty itself, he says beauty itself has "real existence" (Hippias Major 287c12). And in the Statesman (286a2) the stranger refers to the forms as 'existents'.[46] In consideration of the generation of the elements, prior to God's creation of the universe, the forms, which "enter into and go out of her (the receptacle) in a wonderful and mysterious way" (Timaeus 50c3), are likenesses of eternal realities.

When Plato speaks of true knowledge, he says it is attained only by apprehension of the forms. (Republic 511c2). Since true knowledge is apprehending the real nature of a given thing or what it actually is (Phaedo 65d12) and forms are seen as causes of a thing's being (Phaedo 192b2, 100d3), then true knowledge can be attained only by their apprehension. Forms are seen as objective realities, waiting to be apprehended. In Phaedrus (247c7) Socrates says that true being dwells beyond the heavens, without color or shape, and that it cannot be touched. Reason

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[45] In Greek: "αὐτὸ ἕκαστον ὁ ἔστιν, τὸ ὄν."

[46] In Greek: "κάλλιστα ὄντα καὶ μέγιστα."

alone can behold it, and all true knowledge is knowledge of it.

In the Theaetetus (194e4-198a2), Socrates pictures the mind as an aviary and birds as pieces of knowledge. At birth the aviary is empty, but whenever a person acquires a piece of knowledge, he shuts it up in his enclosure. The metaphor suggests that the knowledge itself may or may not be acquired by a mind and that having knowledge is the process of catching the bird, but the objects of knowledge themselves are the birds, which are independent of the acquisitor and are waiting to be acquired. Once again in the Theaetetus (191c10), the objects of knowledge are independent and real entities. The mind is viewed as a block of wax and "we hold this wax under the perceptions or ideas and imprint them on it as we might stamp the impression of a seal ring." (191d4). The knowledge itself may come and go, since it is viewed as an impression on the wax, but the thing that makes the impression is the object of knowledge which must exist before knowledge is possible. It has not been established in these passages of the Theaetetus that the objects of knowledge must be forms, but that the objects must be real and exist independently of their being known.

It is not without some doubts about their reality that Plato talks about forms in the Parmenides. Socrates agrees when Parmenides says that one is perplexed about forms and inclined to "either question their existence, or to contend that, if they do exist, they must certainly be unknowable by our human nature." (Parmenides 135a4). He concedes that "a man of exceptional gifts will be able to see that a form, or essence just by itself, does exist in each case" (Parmenides 135a9), and that "we ourselves are intellectual invalids" (Phaedo 90e3). In spite of some doubts, Plato is to reassure us time and time again as to their reality. Perhaps it is never said as strongly as in the following passage:

> ...do all those which we call self-existent exist, or are only those things which we see or in some way perceive through the bodily organs truly exist, and nothing whatever besides them? And are those intelligible forms, or which we are accustomed to speak, nothing at all, and only a name? Here is a question which we must not leave unexamined or undetermined, nor must we affirm too confidently that there can be no decision...
>
> Thus, I state my view. If mind and true opinion are two distinct classes, then I say that there are certainly these self-existent ideas unperceived by sense, and apprehended only by the mind; if, however, as some say, true opinion differs in no respect from mind, then everything we perceive through the body is to be regarded as most real and certain. But we must affirm them to be distinct, for they have a distinct origin and are of a different nature; the one is implanted in us by instruction, the other by persuasion; the one is always accompanied by true reason, the other is without reason; the one cannot be overcome by persuasion, but the other can; and lastly, every man may be said to share in true opinion, but mind is the attribute of the gods and of very few men. (<u>Timaeus</u> 51c2).

Plato never explicitly tells us that numbers are independently existing entities but there are several reasons to suppose that they are. Plato refers many times in the dialogues to the concept of 'pure numbers', and several dialogues suggest that 'pure numbers' are something other than numbers attached to material or to physical entities.

In the <u>Republic</u>, those who are to be in the highest positions in the state are to study calculation (λογιστική) until "they attain to the contemplation of the nature of number, by pure thought, not for the purposes of buying or selling, but for ... facilitating the conversion of the soul." (525c3). Here number is seen as the object of pure thought, not as related to the buying and selling of things. At 527d7, it is the study of calculation (περί τούς λογισμούς μαθήματος) that "directs the soul upward and compels it to discourse about pure numbers"; not "numbers attached to visible and tangible bodies." Thus, when one contemplates and attains the nature of pure number, it is as though the numbers are there, waiting to be contemplated.

Again, in the Republic (531b7), Socrates denounces the method of the Pythagoreans by saying of them:

> Their method corresponds to that of the astronomer, for the numbers they seek are those found in these heard concords, but they do not ascend to generalized problems and the consideration which numbers are inherently concordant and which not and why in each case.

The Pythagoreans associate their numbers with audible concords and sounds, and do not study the qualities of numbers, in themselves.

In his account of false judgment, Socrates asks Theaetetus if it is possible to make a mistake when one considers in his mind five and seven - "I don't mean five men and seven men or anything of the sort, but just five and seven themselves." (Theaetetus 196a1). The 'five and seven men' are different than 'the five and seven in themselves.'

In the Euthydemus, mathematicians (οἱ λογιστικοί) are a sort of hunter, who, even though they construct diagrams, they try to discover the real existents (τὰ ὄντα ἀνευρίσκουσιν) and hand over their discoveries. (290c1). Socrates goes on to reaffirm this in the Theaetetus. An arithmetician is a hunter (198a) and he "chases after knowledge about all numbers, even or odd." (Ταύτη δή ὑπόλαβε θήραν ἐπιστήμων ἀρτίου τε καιπεριττοῦ παντός.)When he has in his control knowledge of number, he can then hand over this knowledge to someone else. (198b1). So that the 'five and seven in themselves', or any other numbers, are viewed as independent of the subject, waiting to be discovered.

Several other passages in the Theaetetus suggest that numbers are independently existing entities. In Plato's metaphor of the mind as a block of wax, imprints are made on that wax by perceptions and ideas. And it is the misfitting of perceptions and thoughts in which false judgment resides. (Theaetetus 195d1 and 196c6). But people often, in their thoughts, mistake the sum of five and seven and say eleven, so the block of wax metaphor is a bad account of false knowledge. Here, numbers are seen as some kinds of entities that leave impressions on the block of wax called our mind. Again, in the Theaetetus, a finished arithmetician knows all numbers (198b9) and may sometimes count either the numbers themselves in his own head or some set of external things that have a number. (198c1). In this case, an arithmetician deals with pure as well as non-pure numbers (e.g., countable things are his material).

In the Philebus (56d), Socrates distinguishes between two kinds of arithmetic, that used by the ordinary man and that used by the philosopher. The ordinary man deals with two cows or two armies and his units are unequal, but the philosopher deals only with equal units.[47] The ordinary man's units are those concreted in bodies, while the philosopher's units, like the 'pure numbers' of the Republic, are not concreted in bodies.

In the Gorgias (451b1 - c4), Plato makes a distinction between the science, or art, of calculation (λογιστική) and arithmetic (ἀριθμητική) and continues to refer to these two sciences in a dichotomous

[47]This passage will be considered in more detail in Chapter III.

fashion in several of the dialogues. In the Gorgias, Socrates says:

> What is the art of arithmetic (ἀριθμητικὴ)? It is the one of the arts which secure their effect through speech. And if he should further inquire in what field, I should reply that of the odd and the even and how much either happens to be.[48] (451a9)
>
> What art do you call calculation (λογιστικὴ)?[49] I should say that this art too is one of those that secure their entire effect through words...the art of calculation (λογιστικὴ) resembles arithmetic (ἀριθμητικὴ) for its field is the same, the even and the odd, - but that calculation differs in this respect, that it investigates how the odd and the even are related with respect to the multitude, they make with themselves and with each other.[50] (451b8)

This distinction appears again in the Republic and the Laws. Those that share the highest function of the state should take those studies in their preparation that lead them to the apprehension of truth, the study of "number (ἀριθμόν τε) and calculation (λογισμόν)". (Republic 522c6). Should we not, asks Socrates, "set down as a study requisite for a soldier the ability to calculate (λογίζεσθαι τε) and number (ἀριθμεῖν)?" (Republic 522e1). And..."calculation (λογιστικὴ)

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[48]The Greek of the underlined phrase reads: "ὅτι τῶν περὶ τὸ ἀρτίον τὲ καὶ περιττόν (γνῶσις), ὅσα ἂν ἑκάτερος τυγχάνῃ ὄντα."

[49]It must be noted that the Greek word 'λογιστικὴ' has been handled by translators in various ways. A.E. Taylor translates it as 'ciphering' (Laws VII, 817e8). Shorey translates it as 'reckoning' (Republic 529a9), 522e1). Jowett translates it as 'calculation' (Charmides 166a5), as does Woodhead (Gorgias 451b-c). I have chosen to translate 'λογιστικὴ' as 'calculation' in the passages I quote from different dialogues in order to retain some uniformity.

[50]The Greek of the underlined phrase reads: "διαφέρει δε τοσοῦτον, ὅτι καὶ πρός αὐτὰ καὶ προς ἄλληλα πῶς ἔχει πλήθους ἐπισκοπεῖ τὸ περιττὸν καὶ τὸ ἄρτιον."

and the science of arithmetic (ἀριθμητική) are wholly concerned with number." (Republic 525a9). In the Laws, "calculation (λογιστική) and arithmetic (ἀριθμητική) make one subject" which all freeborn men must study. (817e8).

Now while it is clear that Plato is making a distinction between arithmetic and calculation, it is not as clear upon what grounds this distinction rests, or if the distinction carries with it any ontological import. What is suggested, particularly in the Gorgias passage cited, is that perhaps Plato's distinction between arithmetic and calculation rests upon the view that the science of arithmetic deals with pure numbers, unassociated with bodily or material things (what one might call today a branch of pure mathematics) and the science of calculation deals with the numbering or counting of physical things (what one might call applied mathematics). If this view can be textually supported, it would strenghen the case for Plato's belief in numbers as independently existing entities.[51]

This view finds strong support in Geminus' classification of the mathematical sciences[52] in which mathematics is divided into two parts. The one part is concerned with intelligibles only, and the

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[51] Jacob Klein (Greek Mathematical Thought and the Origin of Algebra, M.I.T. Press, Cambridge, Mass., 1968) traces the development of the concept of number and tries to show that Plato reduces practical logistics and practical arithmetic to theoretical logistics and theoretical arithmetic, wherein lie the true presuppositions of the practical activities. (p. 7.) Klein is also quoted by J.E. Annas ("Aristotle's Criticism of Plato's Theory of Numbers," Harvard Dissertation, 1973). Annas argues that, for Plato, numbers are independently existing entities.

[52] Diadochus Proclus, A Commentary on the First Book of Euclid's Elements, translated by G.E. Morrow, Princeton, New Jersey, Princeton University Press, 1970, p. 38-42. Geminus' classification is dated at about 73-67 B.C. This work has been lost.

other part is concerned with perceptibles. By intelligibles is meant "those objects that the soul arouses by herself and contemplates in separation from embodied forms"[53] and included in this part are the studies of geometry and arithmetic. The study of arithmetic "examines number as such and its various numbers as they proceed from the number one."[54] In the second part is included the study of calculation. The student of calculation:

> (does not) consider the properties of number as such, but of numbers as present in sensible objects; and hence he gives them names from the things being numbered, calling them sheep numbers or cup (bowl) numbers ... for when he is counting a group of men, one man is his unit.[55]

The same distinction appears in a Neoplatonic commentary to Plato's dialogues.[56] The scholium to Charmides reads:

> Logistic is the science that concerns itself with counted things, but not with numbers...for example, three things as 'three' and ten things as 'ten'. Thus it investigates...the 'sheep numbers' and the 'bowl-numbers' and also other class of bodies perceptible by the senses...All countable things are its material...[57]

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[53] Ibid., p. 31. [54] Ibid., p. 32.

[55] Ibid., p. 33. J. Klein suggests that the reference might be to Plato's Laws VII 819b - c, where teaching children how to calculate involves playing games with apples and saucers (cups). In Greek, 'μῆλον' means both 'sheep' and 'apple'.

[56] These sources are listed in J. Klein's Greek Mathematical Thought and the Origin of Algebra, p. 11. They are also found in Quellen und Studien zur Geschichte der Mathematik, Abt. B. III, 1934. Scholia quoted by Klein are to be found in Neue Jahrbucher für Philologie and Pädagogik, Jahns Jahrbucher, Suppl. 14, Leipzig, 1848, p. 131ff.

[57] Ibid., p. 12.

Yet, the Olympiodorus scholium to Gorgias reads:[58]

> But arithmetic concerns itself with their kind (the even and odd), while logistic concerns itself with their material, not only the even and odd as they are by themselves, but also their relation to one another in respect to their multitude."

And the claim is being made that arithmetic and calculation are not dichotomous. Examination of the relevant passages reveals that neither arithmetic nor calculation is exclusively associated with pure numbers in the dialogues by Plato. In the passages from the Republic, the study of calculation (λογιστική) leads and directs the soul to the contemplation of pure numbers, but it is not stated that one actually contemplates pure number in the study of arithmetic. In the Euthydemus, calculators (λογιστικοί) construct diagrams in order to discover the real existents. Here, the objects of study of calculation could be the material things (i.e., the diagrams drawn before the discoveries are made) or be the real existents (i.e., the pure numbers after the discovery has been made.) In the Theaetetus (198b9), an arithmetician (ἀριθμητικός) sometimes deals with pure numbers, and sometimes with "things that have a number." (198c1). And in the Philebus, two kinds of numbers are distinguished, one being pure number and the other being that which is attached to visible things, but Socrates refers to the sciences that deal with each of these as two kinds of arithmetic, and makes no distinction between arithmetic and calculation.

Even in the early dialogues, where the distinction between calculation and arithmetic first appears, calculation does not have to do solely with numbers concreted in physical things. In the Euthyphro,

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[58] Ibid., p. 14.

7b6, where it is said, "If you and I were to differ about numbers, on the question which of two was greater...should we not settle things by calculation (ἐπὶ λογισμὸν)?" And in the Hippias Minor (366c6) a skilled calculator (λογιστικῆς) is one who is able to give a true answer for the sum of three multiplied by seven hundred. In both these passages, calculation (λογιστική) is associated with pure numbers.

So, while it is true that Plato makes a distinction between 'pure numbers' and 'numerical units attached to material things' and that he holds to some kind of a distinction between arithmetic and calculation, he does not consistently say that arithmetic deals only with pure numbers and calculation only with numerical units attached to material things.

The Neo-Platonic commentaries on Plato come a good number of years after Geminus' classification and by that time the various branches of mathematics and their respective areas of study might have enjoyed a more stable position than they did at the time of Plato. In fact, Geminus' classification might have aided the Neo-Platonists in the formulation of their own ideas concerning mathematical studies. Indeed, Geminus' classification itself comes a good three hundred years after Plato's writing. Perhaps the distinction the Platonic Socrates makes between calculation and arithmetic is the kind of distinction that was in common use in Plato's day, and was not intended to carry with it any ontological overtones.

Plato's failure to classify consistently the mathematical sciences according to the kind of object studied does not weaken the thesis that he was advocating a kind of number that is 'pure' and not attached to physical things.[59] Passages in the Republic, in the Theaetetus, and in the Philebus support the claim that numbers are seen by Plato to be independently existing entities.

6. Objects of Knowledge

The sixth characteristic common to both forms and numbers is that they are, for Plato, objects of knowledge. The distinction between opinion (doxa) and knowledge (episteme) is first introduced in the Meno. Socrates says that a man that does not know has in himself true opinions on a subject without having knowledge (85c8), and that although true opinion is as good a guide as knowledge for the purpose of acting rightly (97b9), right opinion and knowledge are two different things:

> But it is not, I am sure, a mere guess to say that right opinion and knowledge are different. There are few things that I should claim to know, but that, at least, is among them, whatever else is. (98b2).

In the Meno very little is said about how true opinion and knowledge differ, except that like the statues of Daedalus, true opinions are fine things as long as they stay in their place and do not run away. On the other hand, when they are tied down, they become knowledge and are stable (98a4). So knowledge is more valuable than true opinion because knowledge has the characteristic stability that true opinion does not have.

Once again in Book V of the Republic, Socrates says that there is a difference between the lovers of sounds, sights, beautiful tones, and colors and shapes, and of everything that art fashions out of these,

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[59]According to F. Lasserre, The Birth of Mathematics in the Age of Plato, Hutchinson and Co., London, England, 1964, what distinguishes the mathematics of Plato from that of the Pythagoreans is Plato's view that "numbers are intelligible objects, realities inaccessible to the senses" and for this reason, he calls the mathematics of Plato 'ontological mathematics.' p. 28.

and the lovers of the nature of the beautiful in itself. The lover of sounds, sights, etc., in delighting in these beautiful things is mistaking resemblance for reality. But the lover of the beautiful itself is able to distinguish beauty itself from the things that participate in it. Plato calls the mental state of the former lover 'opining' (δοξάζοντος) or 'opinion' (δόξα) and the mental state of the latter lover as 'knowing' (γιγνώσκοντος) or 'knowledge' (γνώμην) (476d6).

The distinction between opinion[60] and knowledge[61] is retained even in the later dialogues. In the Theaetetus, the many unsuccessful attempts to define knowledge or episteme, end up in a negative claim, namely, that episteme is not true opinion (ἀληθής δόξα). (201a8). In the Timaeus knowledge (νοῦς) and opinion (δόξα) are distinct classes (γένη), (51d9) because knowledge is implanted by instruction, cannot be overcome by persuasion, is always accompanied by true reason, and is an attribute of the gods and very few men. Opinion, on the other hand, is implanted by persuasion, is overcome by persuasion, is without reason, and is shared by every man.

It is clear that there is a distinction between knowledge and opinion, but the most important distinguishing mark of the two seems

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[60]Sometimes 'δόξα' is translated 'belief', but Plato seems to want to distinguish belief (πίστις) from opinion (δόξα). At Republic 533e3, belief (πίστις) and image-making (εἰκασία) are collectively called opinion (δόξα), so belief is only a part of opinion.

[61]It must be noted that 'γνῶσις', 'νόησις ', 'νοῦς ', and 'ἐπιστήμη ' are interchangeably used by Plato for 'knowledge'. So that while he may use 'νοῦς ' in one dialogue to mean knowledge, he may use 'ἐπιστήμη ' in another dialogue to mean the same thing.

to be with regard to the objects with which each deals. Socrates argues for this in the following way: Faculties or powers are distinguished by the things to which they relate and by the affect they have on this things. Thus, the faculty of hearing relates to sounds, the faculty of seeing to sight (objects seen) and so on. Now, since the faculty of opinion is different than the faculty of knowledge, their objects also differ. (Republic 478a4)). And in the Timaeus, two kinds of being are distinguished, one that is apprehended by intelligence (νόησις) only, and the other of the same name with the first kind of being but different from it, which is apprehended by opinion (δόξα). (52a1). Clearly, for Plato, throughout the dialogues, knowledge and opinion are different. It is also clear, after textual analysis, that the objects of knowledge and opinion also differ.

That the objects of knowledge are the forms has strong textual verification. The theory of recollection appears in the Meno and later in the Phaedo, this theory is related to the forms. In the Meno, we are told that we can know what we don't know, because we knew everything that is (81c3) and that the truth about reality is always in our soul. (86b1). The slave boy will have knowledge when he knows what a diagonal is, but until then, only opinion. In the Phaedo, one has knowledge of absolute equality (and the other absolute ideas) prior to birth. (75d4). Coming to know what, say, beauty means, for Plato:

> Starting from individual beauties, the quest for beauty itself must find him ever mounting the heavenly ladder stepping from rung to rung -- that is, from one to two, and from two to every lovely body, from bodily beauty to the beauty of institutions from institutions to special lore that pertains to nothing but beautiful itself -- until at last he comes to know what beauty is. (Symposium 211c2).

When one knows beauty itself, his mental state is knowing (γνώμην).

(Republic 476d7), and knowledge (γνῶσις) pertains to that which is. (Republic 477a11). Since the forms, as argued earlier, are the true realities, i.e., independently existing entities, then they are the things that are knowable.

Just as the sun, in the Sun Analogy of the Republic, gives the objects of our experiences the power of visibility and our eyes the power of vision, so the Idea of the Good (ἀγαθοῦ ἰδέαν) gives truth to the objects of human knowledge and the power of knowledge to the knower. (Republic 508e1). In the Divided Line passage of the Republic the forms are once again treated as the objective correlates of intellection or reason (νόησις), which is the top section of the intelligible world. Socrates says of this section:

> While there is another section in which it (the soul) advances from its assumption to a beginning or principle that transcends assumption, and in which it makes no use of the images employed by the other section, relying on ideas only and progressing systematically through ideas. (Republic 510b9).

And in the same passage:

> ...by the other section of the intelligible I mean that which the reason itself lays hold of by the power of dialectic, treating its assumptions not as absolute beginnings but literally as hypotheses, underpinnings, footings, and springboards so to speak, to enable it to rise to that which requires no assumption and is the starting point of all... making no use whatever of any object of sense but only of pure ideas moving on through ideas to ideas and ending with ideas. (Republic 510b4).

The kinds of being that are apprehended by intelligence (νόησις) are only the forms. (Timaeus 51e9).

In the Seventh Letter, which is considered authentic by many scholars, Plato says that there are four classes of objects through which knowledge (ἐπιστήμη) must come. First is the name of the object, second is the description of the object, third is the image of the object, fourth is the knowledge itself and last is the actual object of knowledge (actually this makes five, but the fourth, since it is the knowledge itself cannot be considered a class of objects through which knowledge must come). The actual object, which can be known through the first three is the true reality. (342b1). The actual object, say a real circle, is not a verbal description, nor an image of a circle as one might draw on paper, nor is it a thought in minds. The first three fall short of the real object, because of the inadequacy of language (343a1) and "they confront the mind with unsought particulars whether in verbal or in bodily form" (343c1), whereas the real object, the circle, is an essential reality. (343b10).

> Every circle that is drawn or turned on a lathe in actual operations abounds in the opposite of the fifth entity, for it everywhere touches the straight, while the real circle, I maintain, contains in it neither much or little of the opposite character. (<u>Letters VII</u>, 343a6).

This passage eliminates the possibility of viewing the object of knowledge as one might view a concept, it also eliminates bodily or physical entities as objects of knowledge. The letter goes on to give testimony of the difficulty of "answering questions and giving proofs in regard to the actual object." (343d3). Plato never says in this letter that the actual object of knowledge is a form, but his reference to it as the essential reality, as one exemplifying stability, of not partaking of any opposite qualities, and of the difficulty of man's apprehending it, make the forms prime candidates for the actual object of knowledge.

One of Parmenides' criticisms of the theory of forms, namely, that they can't be known, since they are in another world, is based on the view that knowledge itself, for Plato, "is knowledge of that reality, the essentially real." (Parmenides 134a6). So "beauty itself or goodness itself, and all the things we take as forms in themselves are unknowable to us." (Parmenides 134c1).

In the Theaetetus, where Socrates talks about what he would call forms in other dialogues, he says: "If a man cannot reach the truth of a thing can he possibly know that thing?" (186c9). And he answers, no:

> Knowledge (ἐπιστήμη) does not reside in the impressions but in our reflection upon them. It is there, seemingly, and not in the impressions that it is possible to grasp existence (οὐσίας) and truth (ἀληθείας). (186d2).

Plato is grappling with the problem of what, if any, are the objective correlates of opinion and knowledge, and it is not entirely clear if he wants to appoint a separated object 'x' which is always an object of knowledge in the Theaetetus as he does in the Republic, but it is clear from this passage that true knowledge is attained by the mind and does not reside in our impressions or perceptions of thing.

When Theaetetus is asked through what organ are the common things (τὰ κοινά) (Theaetetus 185c5), such as existence and non-existence, likeness and unlikeness, sameness and difference, etc., perceived, Socrates is to answer that they are not perceived by any physical organ:

> It is clear to me that the mind in itself is its own instrument for contemplating the common terms (τὰ κοινά) that apply to everything. (185e1).

It is likely that these common terms are forms, even though the word 'εἶδος' is not used in the the Theaetetus, because in the Parmenides some of these same terms, namely, unity, sameness, unlikeness, existence, and nonexistence are called forms 'εἴδη'. (135d9, 135c, 135d1). And in the Sophist, existence, sameness, and difference are once again called 'εἴδη' or forms (253d1, 253d5).

In the Theaetetus, 'τά κοινά ' or the common things, as objects of knowledge invalidate the view that knowledge is perception (αἴσθησις), because at least some knowledge is not perception. And the view that there are some objects of knowledge that are grasped with the mind and not the senses is perfectly consistent with the earlier doctrine, as found in the Republic, that all knowledge is of what is intelligible and not of what is visible.

There is as much reason to believe that for Plato numbers are objects of knowledge as are forms. In the mathematical digression in the Meno, the slave boy gains true opinions about a diagonal (85c8), but knowledge of it will not come from teaching (85d3) but from a spontaneous recovery of knowledge (ἐπιστήμη) which is in him" (Meno 85d6). The truth about reality, in this case about the diagonal, is always in his soul. (Meno 85b1). In the Phaedo's argument from recollection, it is "not only absolute equality" which we knew prior to birth and now recall, but "of relative magnitudes and all absolute standards". (Phaedo 75d7). Knowledge (ἐπιστήμη) is correlated with these absolute entities which includes numbers. In Socrates' discussion of the studies appropriate for the guardians of the state, the studies of mathematics are recommended, because they lead the soul to the apprehension of the truth (Republic 525b1), because they deal with the eternally existent (527b6), and the real object of the entire study is pure knowledge (527a9). And in the Theaetetus, numbers are the counter-examples where opinion is regarded as knowledge.

> ...does a man ever consider in his own mind... five and seven themselves...among which there can be no false judgment...does anyone ever take into consideration and ask himself in his inward conversation how much they amount to, and does one man believe and state that they make eleven, another that they make twelve...? (196a1).

> Some people say eleven, and if larger numbers are involved, the more room there is for mistakes. (196a1).

Earlier, Socrates says that one cannot mistake a man for a horse (195d7), or the beautiful for the ugly (190d1), but one can mistake eleven for twelve.

The Divided Line passage in the Republic has been the cause of much controversy with regard to the status of mathematical entities in relationship to the forms.

On one side of the controversy, it has been argued that numbers are, indeed, objects of knowledge (νόησις) and that they do not differ in this respect from forms. To mention only a few who hold this view, Wilson[62] argues that the mathematical objects do not constitute the whole of the objects of dianoia[63] as some claim. All ideas are objects of dianoia and even though Aristotle speaks of 'intermediates', he never refers to the Divided Line passage. In agreement is Raven[64] who says that the classification of dianoia was not intended to refer uniquely to mathematical objects, and Shorey[65] who says that even though mathematical science as dianoia is midway between nous and doxa, because of its method, that "the objects of mathematics are explicitly stated to be pure noeta." Stocks[66] also argues that there are no 'mathematica' separate from ideas

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[62]Cook Wilson, op. cit., pp. 247-257.

[63]dianoia is transliterated from the Greek 'διάνοια ' and will be used in the remainder of this chapter.

[64]J.E. Raven, op. cit., p. 158.

[65]P. Shorey, op. cit., p. 83.

[66]J.L. Stocks, "The Divided Line of Plato Republic XI," Classical Quarterly, No. 2. 1911, pp. 84-85.

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because dianoia, analogous to the level of eikasia in Divided Line, represents an incomplete kind of knowledge, so that no one kind of object can be associated with it. And finally, Cherniss[67] claims that:

> ...although he (Plato) makes it abundantly clear that the objects of the mathematician's thought are ideas, still he asserts that in mathematics these objects are not treated as ideas and he calls this mental process 'something intermediate between opinion and reason'.

On the other hand, it is claimed by some scholars that numbers are not objects of knowledge as are forms. Wedberg,[68] for example, argues that in the Republic's Divided Line passage Plato describes a kind of number which fits the description of Aristotle's 'mathematical numbers,' and which does not conform with his definition of Ideas. He further claims that Plato never clearly expressed the doctrine of intermediates, in the Republic, but it is, so to speak, striving to come to the surface.[69] Hardie[70] states that the doctrine of intermediates is plainly presupposed and is an explicit doctrine of the Republic.

Intermediate between these is the line of thought taken uniquely by Ross[71] who argues that Plato meant to draw a distinction in the Divided Line passage between the objects of dianoia and those of noesis and that "he thought of ideas as falling into two divisions, a lower

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[67] H. Cherniss, op. cit., p. 78.

[68] A. Wedberg, op. cit., pp. 13-14.

[69] Ibid., p. 109.

[70] W.F.R. Hardie, op. cit., p. 50.

[71] Sir David Ross, op. cit., p. 64.

division consisting of Ideas of number or space, and a higher division not involving these." He goes on to say, "I conclude that the objects of dianoia are not the intermediates but are simply the mathematical ideas, and those of nous, the other ideas." Ross elsewhere says that "reflection on the logical requirements of the simile led Plato very soon to formulate the doctrine of the intermediates,"[72] so that although the doctrine of intermediates was never explicitly stated by Plato, it was on the verge of being stated.[73]

This controversy cannot be settled without examination of the Divided Line passage of the Republic. In the pages that follow it will be argued that the objects of dianoia are not just the mathematical objects, and that numbers, like forms, are the objects of knowledge.(νόησις).

The Divided Line passage is a natural continuation of the Sun Analogy. Socrates summarizes the analogy by saying that there are two entities, the Sun and the Idea of the Good that govern, respectively, the visible (τὸ δ' αὖ ὁρατοῦ) and the intelligible order and its place or locus (τὸ μὲν νοητοῦ γένους καὶ τόπου). The line that is introduced in the next sentence is divided initially into two unequal parts, each line segment representing each of the above designated orders, i.e., the intelligible and the visible. There is no break in the dialogue, nor any shift of emphasis. Glaucon impels Socrates to go on and not to omit a thing. Socrates agrees not to pass over much, and as far as presently practicable he promises not to leave out anything. Both the Sun Analogy and the Divided Line exhibit a division between the intelligible and

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[72]Sir David Ross, Aristotle's Metaphysics, Oxford University Press, Oxford, 1924, p. 52.

[73]Sir David Ross, op. cit., p. 177.

visible realms and exhibit a pronounced emphasis on the intelligible realm. The object of the analogy seems to have been to introduce the Idea of the Good, as the object of the Divided Line seems to have been to further illustrate the relation of forms to knowledge.

The most striking difference between the Sun Analogy and the Divided Line passage is the fact that Plato uses a line segment to represent whatever it is he is representing, whereas the representation of the sun is poetical and metaphorical, with not even a suggestion of a continuum of any kind. The fact that Plato chooses to use a line indicates that there is some common variable between any two sections of the line. To put it in other terms, there is a common unit of division, such as kinds of objects, states of mind, or some such thing that it is the line measures.

Socrates tells us to take a line and to divide it into two unequal parts, and then to divide the remaining segments again in the same ratio (κατὰ τὸν αὐτὸν λόγον)[74], thus dividing the original line into four proportional parts. One possible interpretation of how the line is divided is that the unit of division is based on the different kinds of objects that are associated with each level. Thus, it should be possible to associate distinct kinds of objects with each level of the line. At the first level, or "one of the sections of the visible world"[75] will be images. By images is meant "shadows, and then reflections in water and on surfaces of dense, smooth, and bright texture."[76] At the second level of the visible world, the objects are those of which

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[74] Republic 509d8. More will be said shortly about this ratio.

[75] Republic 509e1.

[76] Republic 510a2.

the objects of the first level are likenesses or images, "that is, the animals about us and all plants and the whole class of objects made by man."[77] At the fourth level of the intelligible realm, the objects are clearly the forms.[78]

If one remains consistent with the interpretation under consideration, namely, that each level is marked off by a different kind of object, then one must find some textual evidence for some kind of object different from the objects of this world and from the ideas or forms, that will go to make up the third level of the line. The following passage allegedly supports such a view:

> Well, I will try again, for you will better understand after this preamble. For I think you are aware that students of geometry and reckoning and such subjects first postulate the odd and the even and the various figures and three kinds of angles, and other things akin to these in each branch of science, regard them as known, and treating them as absolute assumptions, do not deign to render any further account of them to themselves or others, taking it for granted that they are obvious to everybody. They take their start from these, and pursuing the inquiry from this point on consistently, conclude with that for the investigation of which they set out." (Republic 510c1).

It is on this evidence, but not exclusively, that various Plato scholars have argued that the objects at level three, or the lower level of the intelligible world, are the objects of mathematics, such as angles, triangles, numbers, etc., as mentioned by Plato in the above quote, and that the doctrine that mathematical entities are neither sensible things nor forms is presupposed in the metaphysical scheme of the Republic.

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[77] Republic 510a6.

[78] See quotes p. 43.

There are several objections to dividing the line according to objects, and these will now be considered.

At 511d10, Socrates says:

> And now, answering to these four sections, assume these four affections occurring in the soul - intellection or reason (νόησιν) for the highest, understanding (διάνοιαν) for the second, belief (πίστιν) for the third, and the last, picture thinking or imagining (εἰκασίαν), and arrange them in a proportion, considering that they participate in clearness and precision in the same degree as their objects partake of truth and reality.

The shift in this passage is from a consideration of the line divided in terms of objects to the line's division in terms of states of mind (παθήματα). 511d10.

We are further told in this passage that the four states of mind are to be arranged in a proportion as they participate in clearness and precision in the same degree as their objects partake of truth and reality. Thus:

$$\frac{\text{intellection or reason (νόησις)}}{\text{understanding (διάνοια)}} = \frac{\text{belief (πίστις)}}{\text{imaging (εἰκασία)}}$$

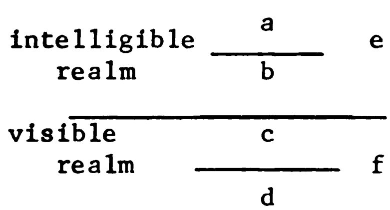
The proportion is set up in terms of clarity of thought, not in terms of the kinds of objects. Reason is related to the understanding in the same way seeing objects is related to seeing images of objects. The objects in the numerators of the proportion partake more of truth and reality than do the denominators, but they are not different kinds of objects.

The strongest objection to dividing the line according to objects is that there are two places in the Divided Line passage that explicitly state that the objects of the third level, dianoia, are the same objects as those of the second level, πίστις. At 510b2, Socrates says:

> Consider then again the way in which we are to make the division of the intelligible section. By the distinction that there is one section of the intelligible which the soul is compelled to investigate by treating as images the things imitated in the former division.

Later in the same passage, Socrates says that the soul "uses as images or likenesses the very objects that are themselves copied and adumbrated by the class below..."[79] At this third level the objects are the very objects that have been ascribed to the second level, but they are treated as images.

If we follow Socrates' instructions in the construction of the line, we get the following proportion:



$$\frac{e}{f} = \frac{a}{b} = \frac{c}{d}$$

where $a + b = e$ and $c + d = f$.

From this, we get :

$$\frac{a+b}{c+d} = \frac{a}{b} = \frac{c}{d}$$

But if $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$

Therefore: $\frac{a+b}{c+d} = \frac{b}{d}$ and $\frac{a+b}{c+d} = \frac{a}{c}$

But: $\frac{a+b}{c+d} = \frac{c}{d}$ and $\frac{a+b}{c+d} = \frac{a}{b}$

Therefore: $c = b$

It turns out that the second and third sections of the Divided Line are equal in length. And there is a sense in which they are importantly so. Socrates says that the objects in the second section are the same as those of the third section; therefore, the length of the line

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[79] Republic 511a6

representing these objects needs be the same in both cases.[80] But this is not to say that the line is divided according to the kinds of objects one is dealing with. For then, there would be no need for two separate line segments to represent the same objects. But if it is divided to represent states of mind, since we have two states of mind for the same objects, we need four divisions for the three objects.

The Cave Allegory, which immediately follows the Divided Line passage, seems to reenforce the claim that it is the clarity with which one apprehends objects that is Plato's main concern in the Divided Line passage. In the cave, the prisoner views the images on the wall, never having seen the originals nor the fire causing the images. In this state of mind, the prisoner either believes it is an original, or possibly conjectures it as an image but without conviction, not having seen the original. In the next state of mind, the prisoner views the originals, but it is not until he views the fire and understands how the images are caused that he is in the highest state of mind in the cave.

Analogously, when the prisoner is led out of the cave, first he sees shadows and reflections in water of men and things, then he sees the things themselves, but it is only when he sees the sun itself that he knows how shadows and reflections of things are generated from things.

If one is to take the Cave Allegory to be illustrative of the Divided Line, then a difficulty arises and challenges the interpretation given above, namely, that the objects of dianoia are the same as the ob-

[80] It has been argued at various times that when Plato divides the line in the aforesaid proportion, he is unaware of this consequence, or that the consequence is unintentional on his part. But it is hardly possible that someone as up to date with the mathematics of his time as Plato was, would not be aware of the result of this proportion. My contention is that Plato was aware of the consequence.

jects of πίστις , regarded in different ways. The difficulty some critics have raised is that the objects that are viewed as originals in the cave are not the same as the objects that are viewed as images out of the cave.

How the Cave Allegory relates to the Divided Line has been a point of some controversy. Hardie[81] argues that the Cave Allegory is analogous to the Divided Line, in that they both represent a progression in terms of clarity, and if so, then the objects of dianoia and in the Divided Line cannot be the same because the objects viewed as originals in the cave are different than the objects viewed as images out of the cave. Stocks[82] argues that the Divided Line is not a progression, but it is an analogy, since it follows the Sun Analogy. The lower half of both sections is incomplete. Conjecture and dianoia are the states of mind that exhibit only partial understanding with the help of the image.

The Cave Allegory might be viewed as an analogy in that it shows marked similarities to the Sun Analogy. There is a definite break inside the cave and outside the cave, as there is in the Sun Analogy between the visible world and the intelligible world. Also, in the Cave Allegory, the fire in the cave is the cause of the shadows on the wall, as the sun outside the cave is the cause of the visibility of the visible, as the Idea of the Good is the cause of our true knowledge.

But the Divided Line passage seems to differ from the Sun Analogy and Cave Allegory exactly in those two ways. There is not a different kind of cause associated with the lower level of the line and another for the upper level, but only one cause of true knowledge, name-

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[81] W.F.R. Hardie, op. cit., p. 50-1.

[82] J.L. Stocks, op. cit., p. 74.

ly, the forms. Secondly, the objects of the Divided Line seem to illustrate some kind of ontological progression, as more or less real. (Republic 515d2). First, are images of things, then the things in this world, and finally the forms.

But the Cave Allegory, though it might viewed as an analogy, might also be viewed as a progression, without changing the doctrine of the Divided Line. The originals in the cave (human images, shapes of animals made out of various kinds of material) (Republic 514c1), according to Socrates, are more real, (Republic 515d2), than the shadows cast on the wall, but they are also images or copies of the things outside, so they have the same ontological status in relation to the things outside as do the images outside. So while the originals in the cave are not identical to the images outside the cave, they both have the same relationship to the objects outside the cave in being copies of them. However, for purposes of converting the soul, the images outside the cave are better because they are brighter. So while the Cave Allegory preserves the feature of being an analogy from one point of view, it can be viewed as a progression from another point of view.

It might be argued that in Book V of the Republic, Socrates states that opinion and knowledge are related to different objects, so that analogously, dianoia should relate to an object different than those of knowledge and opinion. But this objection does not hold, because the first pair, namely, knowledge and opinion are clearly called 'powers' (δύναμις) of the mind (Republic 478a4) and dianoia is called an 'affection (πάθημα) occurring in the soul." (Republic 511d10).

Another important consideration in how one is to view dianoia is the passage 533e1 in the Republic, where Socrates says:

> ...call the first division knowledge (ἐπιστήμην) the second understanding (διάνοιαν), the third belief (πίστιν), and the fourth conjecture or picture thought (εἰκασίαν)---and the last two collectively opinion (δόξαν) and the first two intellection (νόησιν), and opinion (δόξαν) dealing with becoming (γένεσιν) and intellection dealing with being (οὐσίαν), and this relation being expressed in the proportion: as being is to becoming, so intellection to opinion, and as intellection to opinion, so is knowledge to belief and understanding to image thinking.

For the sake of clarity, we schematize the suggested proportion in the following way:

$$\frac{\text{being}}{\text{becoming}} = \frac{\text{intellection}}{\text{opinion}} = \frac{\text{knowledge}}{\text{belief}} = \frac{\text{dianoia}}{\text{conjecture}}$$

The proportion suggests that dianoia deals with the same kind of objects as does knowledge, since they are both in the numerators, and the objects of knowledge are clearly the forms themselves. But dianoia is importantly distinguished from knowledge, by Socrates, because of its dependence on hypotheses. In dianoia, "the soul is compelled to investigate by means of assumptions from which it proceeds not up to a first principle but down to a conclusion." (Republic 510b7). For example, a geometer must first hypothesize the odd and the even and the various and kinds of angles in his investigations. True knowledge, on the other hand, through the power of dialectic (Republic 511c6) advances to a beginning or principle that is unhypothetical. (Republic 510b9).

On the other hand, dianoia relates to conjecture, both the lower sections of the Divided Line, in that they both deal with images. But they, too, are importantly distinguished because dianoia deals with images of the forms (511d1, 510d7), while conjecture deals with the images of the things in this world. (Republic 510a1).

It has been established in Book VI that knowledge and opinion as different powers of the mind relate to different objects and that the objects of knowledge are forms and the objects of opinion are visible things. (Republic 476b4, 476d6). In Book VII, Socrates says that intellection and dianoia go to make up knowledge, (Republic 533c3), and belief and conjecture go to make up opinion. These four are called affections of the soul. These two passages together suggest that there are two affections of the soul related to each power of the mind, or two different ways of dealing with an object. The higher affection of the soul in each case deals directly with the object of the power, intellection with forms, belief with things. The lower affection of the soul deals indirectly with the object of the power by contemplating the image of that object.

Thus, conjecture, in so far as it is a power, is a power of conjecturing about a thing. Insofar as it is a state of mind, the object of that state of mind is the image of the thing.

Stocks and Ferguson[83] have argued that in the state of conjecture the subject is aware of the fact that it is an image and makes conjectures about the original. Hardie[84] says that this view of Stocks' and Ferguson's does not square with the Cave Analogy, because the prisoners in the cave, since they are compelled to hold their heads unmoved through life (Republic 515b1) are not aware that the images they are viewing are only images. If Hardie is correct and the analogy between the image of a thing and a thing is to hold at the two levels of the Divided Line, then one in the state of dianoia cannot possibly know that he is dealing with images, say, when he draws triangles.

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[83] J.L. Stocks, op. cit., p.86 and A.S. Ferguson, "Plato's Simile of Light," Classical Quarterly, Vol.XV, No. 2, April, 1921, p.145.

[84] W.F.R. Hardie, op. cit., p. 60-61.

But if our interpretation is correct, there is a sense in which both views are right. It is possible when one is in the state of conjecture to be aware that the image is an image, in which case one makes conjectures about the imaged object, or to be unaware, in which case the subject would be under the illusion that the image is real. When the prisoners of the cave have not seen the real objects, they are under the illusion that the shadows are real, but after seeing the real objects in the cave that cast the shadows, they make conjectures about those real objects by viewing their images. In both cases, the object is the same, namely, the reflections and shadows on the wall, but the intended object differs depending upon whether one believes it is an image or not. Thus, there are two different affections of the mind dealing with the same object each in its own way, when one is in the state of conjecture.

Analogously, dianoia functions like conjecture. In the state of dianoia, the intended object could be either the image of the form itself, i.e., a physical thing, which is the correlative object of belief, or a form, the correlative object of intellection. These two objects, since they are ontologically indistinguishable, are the same object, namely the physical thing. Yet, how one handles this object varies with ones intention. For instance, a geometer may claim upon viewing a square that the diagonal of a square bisect it. The object is the drawn square. Yet, when he makes this claim, he may be conjecturing about the square-itself on the basis of what looks likely in the case of the drawn square, or he may make this claim under the illusion that the claim is about physical things, namely all drawn squares.

Thus, while the objects of belief and dianoia are not ontologically distinguishable, they are distinguishable as objects of states of mind, since for one state of mind the object is an original and for the other it is an image.

It also follows from this reconstruction of the Divided Line that the objects of dianoia are not ontologically distinct from the objects of intellection, once again, being distinguishable only as objects of states of mind. Thus the claim that dianoia deals only with ontologically distinct mathematical entities cannot be supported by the Divided Line passage, alone.

Perhaps Plato chose mathematics to display the efficacy of dianoia because he saw it as "the common thing that all arts and forms of thought and all sciences employ, and which is among the first things that everybody must learn." (Republic 522c1). But there is no reason to suppose that he intentionally excluded other courses of study. In fact, the contrary claim can be textually supported, namely, that the distinction between studying an object, as an object, and studying an object for the sake of true knowledge about that object is made in the study of astronomy and harmonics, as well.

Socrates is to criticize contemporary astronomy because it regards the "sparks that paint the sky as the fairest and most exact of material things," (Republic 529c7), whereas, "we must use the blazonry of the heavens as patterns to aid in the study of their realities." (Republic 529d8). He says the same thing about those that study harmonics, that "their method exactly corresponds to that of the astronomer" (Republic 531b7), for, "they transfer it [the goal of knowledge][85] to hearing and measure audible concords and sounds against one another,

[85] My parenthesis.

expending much useless labor just as the astronomers do." (Republic 531a1).

Mathematics, astronomy, and harmonics, alike, are pursued as if their objects were sensible things, instead of using the sensible things as patterns. In short, the objects of mathematics, alone, do not go to make up the objects of dianoia, whether they be objects of true knowledge, or objects, as objects. The objects of astronomy and harmonics also serve as examples.

When Wedberg[86] argues that the Platonic writings offer evidence for the postulation of intermediate mathematical objects, but not for a class of intermediate astronomical entities, or a class of intermediate musical entities, he seems to overlook the Platonic textual evidence cited above. In view of this, Wedberg cannot convincingly argue that mathematical objects are somehow 'special', and that the objects of mathematics hold an ontologically distinct position between forms and sensible things in the Republic.[87]

It has been argued in this section, that numbers, for Plato, no differently than forms, are objects of knowledge.

7. Indivisible or Incomposite

The seventh characteristic shared by forms and numbers is their indivisibility or incomposite nature. It is a clear doctrine of the early and middle dialogues that the forms are unified wholes, indivisible into parts. In the Phaedo, the forms are constant and invariable (78d1) and that which is constant and invariable is probably incomposite. (78c7).

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[86] Wedberg, op. cit., p. 90.

[87] A fuller discussion of Wedberg's position is taken up in Chapter IV.

The oneness of Beauty in the Symposium is seen in final revelation as:

> ...subsisting of itself and by itself in an eternal oneness, while every lovely thing partakes of it in such sort, that, however much the parts may wax and wane, it will be neither more nor less, but still the same inviolable whole. (211b1)

The form's unitary nature is reiterated by Parmenides when he calls the form a whole, a single thing (131a9), and a one. (131b1).

It is true of numbers, as it is for forms, that each number is a unity (ἐν), indivisible, and incomposite. In the Phaedo, twoness (δυάδος) is the cause of whatever becomes two (101c7), exactly the same way that Beauty is the cause of whatever is beautiful (100d5). Even if two things participate in duality, this does not change the unitary nature of duality.

In the Republic, Socrates explicitly says:

> For you are doubtless aware that experts in this study, if anyone attempts to cut up the 'one' (τὸ ἕν) in argument, laugh at him and refuse to allow it, but if you mince it up, they multiply, always on guard lest the one should appear to be not one but a multiplicity of parts. (525e1).

While this passage is explicitly about the form of ἕν , Socrates says subsequent to this passage that "if it is true of the one (τὸ ἕν), the same holds of all number, does it not?" (Republic 525a6). So even if by ἕν , Plato means 'unity', it is clear that what is true of unity applies also to number. Thus, number is seen as being whole and indivisible, and if anyone tries to cut it up, the experts will multiply in number in order to save this unitary nature.[88]

In the Theaetetus, Socrates says:

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[88]This is a play on words. The experts that will multiply in number in order to retain the unitary nature of pure number, exemplify numbers "attached to visible and tangible bodies." (Republic 525d8).

> ...at any rate in the case of things that consist of a number, the words 'sum' and 'all the things' denote the same thing. (204d1)

And in explanation of this, Socrates gives the examples that an acre is a whole, when considered as an acre, but many parts when considered as the number of square feet in the acre. "The number of units in any collection of things cannot be anything but parts of that collection". (204e). Yet, the acre, considered as a whole, and the number of square feet considered as parts, denote exactly the same thing. (204d12). So, while a collection of things may be both one and many, this cannot be said of unity itself or number itself. Socrates is not saying that the multitude of units in a number are parts of that number, but that the number of things in a collection are parts of that collection.

In the Parmenides, however, Plato raises some questions concerning the unity of the forms. The doubts are voiced by Parmenides:

> Do you hold, then, that the form as a whole, a single thing, is in each of the many, or how?
>
> Why should it not be in each?
>
> If so, a form which is one and the same will be at the same time, as a whole, in a number of things, which are separate, and consequently will be separate from itself. (131a9).
>
> In that case, Socrates, the forms must be divisible into parts, and the things which have a share in them will have a part of their share. Only a part of any given form, and no longer the whole of it, will be in each thing. (131c4).

According to Parmenides, if many things participate in the form, then the form must be split up into many parts, and thus lose its unity. Again, in the Philebus, Socrates voices the same concern:

> ...this single unity comes to be in an infinite number of things that come into being -- an identical unity being thus found simultaneously in unity and in plurality. Is it torn into pieces or does the whole of it, and this would seem the extreme impossibility, get apart from itself. (15b5).

Aristotle is to make this difficulty with regard to the unity of the forms one of his key criticisms of Plato's theory. He says:

> If the 'animal' in the'horse' and in 'man' is one and the same, as you are with yourself, how will the one in things that exist apart be one, and how will this 'animal' escape being divided even from itself. (Metaphysics 1039a23-35).

In the dialogues after the Parmenides, Plato does not seem to hold the view that forms are unitary in nature with the same conviction that he did in the earlier dialogues.

One passage voiced by the Stranger in the Sophist states positively that unity itself is rightly defined as altogether without parts (245a8), yet only a few lines before this statement is made, the Stranger says:

> If a thing is divided into parts, there is nothing against its having the property of unity as applied to the aggregate of all the parts and being in that way one, as being a sum or whole. (244a1).
>
> On the other hand, the thing which has these properties cannot be just unity itself. (245a5).

Thus, while things have the properties of one and many (i.e., divisible into parts), the form of unity cannot have either property. And if the form of unity cannot have either the property of one, or the many, then what of the other forms?

Socrates says in the Parmenides that he would be surprised if unity itself is many or that plurality is one. One can say of things:

> ...that all things are one by having share in unity and at the same time many by sharing in plurality. But if anyone can prove that what is simply unity itself is many or that plurality itself is one, then I shall begin to be surprised. (Parmenides 129b4).

These two passages in the Parmenides are consistent with the statement in the Sophist (245a1-5). The Sophist passage tells us that the form of unity cannot be one or many (divided into parts). The two passages in the Parmenides tell us that the form of unity cannot be many and the form of plurality cannot be one.

No passages after the Parmenides give us any insight into what can be said about any of the forms, or of forms, in general, regarding the characteristics of unity or divisibility. Part II of the Parmenides seems to argue that the form of ἕν is both one and many (Parmenides 145a2) and neither one nor many. (Parmenides 155e4).

There are no passages after the Parmenides that explicitly state that number itself is unitary, but on the other hand, there are no passages that posit that number is divisible into parts, either. One statement in the Sophist is consistent with the claim that numbers are unitary. The stranger says:

> ...what is not a whole cannot have a definite number, for if a thing has a definite number, it must amount to that number, whatever it may be, as a whole. (245d9).

Any collection of things that is not a whole cannot participate in any definite number, just as it cannot participate in unity. Or stated in another way, if a whole is some collection of things which participate in unity and thus form a whole, they may then also participate in a definite number. But neither the unity itself, nor the number itself need have parts on this account.

Plato acknowledges through Socrates that there is a problem, namely, that the form cannot be one and divided at the same time into many, that "there is some weight to these objections"[89] (Parmenides 135a7).

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[89]The unity of the form is only one of many objections raised in the Parmenides to Plato's theory of forms.

and that it is "a problem that will assuredly never cease to exist" (Philebus 15d7). But even though some of the objections to the theory of forms that appear in Part I of the Parmenides seem to be formidable, it is doubtful that this particular objection concerning the alleged divisibility of the forms was taken very seriously by Plato.

In spite of the objection, Plato states in a passage which directly follows the introduction of the objections in the Parmenides, that without the forms "man will have nothing on which to fix his thought ... and in so doing he will completely destroy the significance of all discourse." (135b5).

Thus, even though Plato is to raise the question regarding the relationship between the form and its instances, i.e., how can the form be one, when many partake of it, he seems to want to retain the form's unity in the later dialogues. And it is clear that numbers, in the later dialogues, are seen by Plato in no different light than forms. And even if Plato had come to believe that forms are divisible, because of the objection in the Parmenides, it is clear that he could not have consistently assigned divisibility as a characteristic of forms, and not assigned it as a characteristic of numbers.

8. Unique and Perfect

The eighth and last characteristic that is common to both forms and numbers is their uniqueness and perfection. That there is only one form for the multiplicity of things in this world named after it is one of Plato's key principles, and it is stated many times throughout the dialogues.

In the Republic (476a1) Socrates says that in respect of all the ideas or forms, (e.g., the just, the unjust, the good, the bad), in itself each is one, but that by virtue of their communion with actions and bodies and with one another they present themselves everywhere, each as a multiplicity of aspects.

Later in the same dialogue, Socrates reminds us that there is a beautiful (or an idea of beauty) in itself which is one and just one (Republic 479a2, 470e2) and that "in the case of all the things we posit as many, we turn around and posit each as a single idea or aspect" (Republic 507b10).

The uniqueness of the forms is never brought out more strongly by Plato than in Book X of the Republic, in Socrates' discussion of a craftsman and his production. He begins by reiterating what he had said earlier:

> We are in the habit of positing a single idea or form in the case of the various multiplicities to which we give the same name.(596a7).

and goes on to say that the craftsman may make many couches, (or tables, etc.), but that "he does not make the idea or form which we say is the real couch" (597a1). God made the real couch and made only one:

> Now, God, whether because he so willed or because some compulsion was laid upon him not to make more than one couch in nature, so wrought and created one only, the couch which really and itself is. But two or more such were never created by God and never will come into being.

Plato follows this with a *reductio* proof which goes like this: Suppose God made only two real couches, then there would appear again one other couch of which they both would possess the form or idea, and that would be the couch that is in and of itself,[90] and not the other

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[90] The underlying premise is that there is only one form of couch which makes things couches, which is what Plato is trying to prove. This points to the invalidity of the argument.

two. Therefore, we would have reached a contradiction because we started by saying God made only two real couches. He concludes:

> God, then, I take it, knowing this and wishing to be the real author of the couch that has real being and not of some particular couch, nor yet a particular cabinet maker, produced it in nature unique." (597d1)

The uniqueness of forms is never again brought into such explicit light but it is referred to in several places. In the Philebus, a form is a single unity (15b5) and in the Parmenides reference is made to a form of rightness, a form of beauty, a form of goodness, etc. (130b10). And when Parmenides restates the Socratic view he says:

> I imagine your ground for believing in a single form for each case is this. When it seems to you that a number of things are large, there seems, I suppose, to be a certain single character which is the same when you look at them all; hence you think that largeness is a single thing." (Parmenides 131e10).

As regards the relationship between a form and its instance in a particular sensible thing in this world, it can be said of the form that it is not only unique, but that it is perfect; that is, the form of beauty is absolutely beautiful, the form of justice if absolutely just, and so on, whereas the instance of that form in a particular sensible thing (or the instances in particular sensible things) is (are) "always striving after that absolute anything, but falling short of it" (Phaedo 74b1). The instances of the perfect forms in sensible things are imperfect copies of these forms.[91]

The uniqueness of numbers is never explicitly mentioned by Plato as is the uniqueness of forms. But there are several very good reasons

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[91]The distinction between ousia and genesis was made earlier on p. 17 of this chapter. Those things that are, are the forms, and the things in this universe around us are always in the state of becoming. (cf. Phaedo 78d1, Republic 470a3, Philebus 15b3, 58a3, 59a5-10, Sophist 248a14, Timaeus 51e8-52e3).

to believe that numbers are unique.

First, there is no reason to believe that Plato's key principle mentioned above, namely, that there is one idea or form for the various multiplicity of things in this world to which we give the same name, would exclude number. In the Phaedo, Socrates tells us that a beautiful thing is beautiful because it participates in the form of beauty (100d5), and something becomes two by its participation in 'duality', or one, by its participation in 'unity'. 'Unity' and 'duality' are not seen in any different light than absolute beauty is seen. Later Socrates says "...when the form of three takes possession of any group of objects, it compels them to be odd as well as three" (104d7); here 'threeness' is explicitly stated as 'the form of three', and it is in the singular.

Second, in the Republic's finger example, Socrates says there are no doubts in one's mind that a finger is a finger. The soul of most men is never compelled to ask what in the world a finger is, since perception never signifies that a finger is the opposite of a finger. But the senses are defective in their reports as to the size, the hardness, the thickness, and so on. (Republic 524a1). In visual perception we see the same thing at once as one (ὡς ἕν) and as an indefinite plurality. (525a3). So the soul is summoned to contemplate the great and the small, in the opposite way from sensation (524c4). In this contemplation, the soul is drawn to the study of unity. The study of 'one' (τὸ ἕν) and the quality of number (since what is true of 'one', τὸ ἕν , holds for all number (525a6)), leads the soul to the apprehension of truth. It is implied in this passage that 'one' , τὸ ἕν , is unique. For if it were not, and say that there were two 'ones', then it would no longer serve as a guide or standard for judging between contradictory reports. (524e1).[92]

But as strong as the case is in the middle dialogues for the uniqueness of the forms, it does not remain unquestioned, as did the case for the indivisibility of the forms. Plato voices these doubts:

> But now take largeness itself and the other things which are large. Suppose you look at all these in the same way in your mind's eye, will not yet another unity make its appearance - a largeness by virtue of which they all appear large?
>
> So it would seem.
>
> If so, a second form of largeness will present itself over and above largeness itself and the things that share in it, and again, covering all these, yet another, which will make all of them large. So each of the forms will no longer be one, but an indefinite number. (Parmenides 132a6).

The forms become many, if the relationship between the form and its

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[92] It is Owen's position that Plato's view as to the kinds of things that require a form or idea as a standard by which to judge contradictory perceptual reports seems to have altered by the time Plato wrote the Parmenides, "Peri Ideon", p. 307-8, Studies, ed. Allen. He argues that Plato, at least in the early dialogues, includes only ideas of relative (πρός τι) or incomplete predicates, and that only in the later dialogues does he consider the non-relative (καθ' αὑτό) predicates. In the Parmenides, younger Socrates is asked if there is a form of objects such as dirt, and mud, and Socrates does not answer affirmatively nor negatively, but admits that he has been troubled by this consideration. Cherniss argues that there is no reason to believe that Plato did not mean to establish the existence of ideas or forms in the case of all predicates. (Aristotle's Criticism of Plato and the Academy, pp. 280-283.

The status of numbers is unaffected by this controversy, insofar as 'twoness' and 'threeness', etc., are, to use Owen's terminology are relative or πρός τί predicates in the early as well as the later dialogues.

instances is seen as one of sharing the common characteristic.

One of the issues that arises in determining whether the argument is valid or not, is whether Plato intended the forms to be self-predicative. If the form of beauty is absolutely beautiful then the form of beauty and beautiful things will share in the common characteristic, namely, beauty, in which case another form of beauty must be posited in which both the form of beauty and beautiful things partake, and so on, ad infinitum. Thus, the form of beauty is no longer unique.[93]

In the case of numbers, the self predicative nature of forms takes on an added dimension. For not only will there be an infinite number of 'twonesses' (by the third man argument), but if the form of two is actually two, then the form of two is divisible into two parts, and Plato seems possibly willing to accept this when Socrates says that the form of many cannot be one. (Parmenides 129b4).

The objection could be met if Plato had more fully developed in the dialogues a theory of predication. He might have argued that

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[93]If it can be argued that largeness is not itself large, then the regress cannot begin. R.E. Allen argues in "Participation, and Predication in Plato's Middle Dialogues", Studies, ed. Allen, p. 43, that the regress never begins. H. Cherniss agrees with Allen that the regress never gets started in "Relation of the Timaeus to Plato's Later Dialogues", Studies, ed. Allen, p. 367. P.T. Geach, in accordance with the above says in"The Third Man Again", Studies, p. 267, that "a form is not an attribute or characteristic, but a standard." G. Vlastos, "The Third Man Argument in the Parmenides", Studies, ed. Allen, p. 231, argues that there is an assumption of self-predication in the argument, but a valid objection might have been supplied by Plato, which he never does. On the other side of the controversy is Owen's view that the argument is valid and Plato never attempted to answer it, "The place of the Timaeus in Plato's Dialogues, Studies, ed. Allen, p. 318-322.

one predicates of forms in a different sense than one predicates of things that participate in forms. There are some hints of this in the Phaedo. The form of Beauty gets the predicate 'beautiful' in the preeminent sense, and beautiful things are called beautiful derivatively (called after them, Phaedo 102b-3c). If one does not assume a theory of preeminent predication, then one cannot account for Plato's reluctance to define number as a multiple of units. If twoness is two, then twoness might have been defined as a multiple of units. Yet, nowhere in the dialogues does Plato so define number.[94]

The form's uniqueness is questioned, once more, by Parmenides, who claims that even if the relationship between a form and its instances is not one of sharing a common characteristic, but one of participation (μέθεξις), so that things are made in the image (παραδείγματα) of forms and as likenesses (ὁμοιώματα), then the form must be like that which was made in its image.[95] In which case:

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[94]Chapter III deals specifically with this issue.

[95]This seems to imply that the relation of form and particular is symmetrical, but A.E. Taylor points out that it is asymmetrical. ("Parmenides, Zeno, and Socrates," Proceedings of the Aristotelean Society, XVI, 1916, p. 234-89) and that consequently this argument is invalid and Plato knew it. Cornford follows Taylor in this point, but later writers (Owen, "The Place of the Timaeus in Plato's Dialogues", Studies, ed. Allen) rightly remark that while the relation of pattern and copy is asymmetrical, this asymmetrical relation implies another symmetrical relation, namely, of likeness, and so the objection is a valid one. Geach says the copy is asymmetrically related to the standard ("The Third Man Again", Studies, ed. Allen, p. 272) but he does not reply to the rejoinder that this asymmetrical relation implies the symmetrical relation of "is like".

> ...a second form will always make its appearance over and above the first form, and if the second form is like anything, yet a third. And there will be no end to this emergence of fresh forms, if the form is to be like the thing that partakes of it. (Parmenides 132e7).

A large bulk of Platonic scholarship has been devoted to the question whether the criticisms of the theory of forms in the Parmenides Part II were weighty enough to cause Plato to reformulate or even abandon his early version of the theory of forms, and various points of view have emerged.

Some have argued that the weight of the objections raised in the Parmenides caused Plato to revise his view of the forms as patterns, particularly since the later dialogues never mention the forms again as paradigms.[96] Others[97] have argued that the objections in the Parmenides are not logically valid, that Plato knew this, and because of this, there was no reason for him to abandon his earlier view of the forms. Others[98] have maintained that the arguments of the Parmenides are invalid, but that Plato was honestly perplexed about how to handle the objections.

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[96] G.E.L. Owen, "The Place of the Timaeus in Plato's Dialogues", Studies, ed. R.E. Allen. The Timaeus, which has been traditionally placed as a late dialogue, states the relationship of forms to its instances as pattern to copy. But Owen argues that the placement of the Timaeus, as a later dialogue, is incorrect. According to Cherniss, "Relation of Timaeus to Plato's Later Dialogues", Studies, ed. R.E. Allen, p. 340, Owen's position is not unique. F. Tocco, Studi Italiani di Filologia Classica II, 1894, D. Peipers, Ontologia Platonica, 1883, Teichmuller, 1881-43, Susemihl, Woch für Class, Philologie I, 1884, to mention only a few, support Owen's position and argue that the Timaeus either follows the Republic, or was at least written before the critical period.

[97] A.E. Taylor, op. cit., F.M. Cornford, Plato and the Parmenides, and Sir D. Ross, Plato's Theory of Ideas.

[98] G. Vlastos, "The Third Man Argument in the Parmenides," Studies, ed. R.E. Allen, p. 231-261.

If Plato had felt that the objections raised in the Parmenides and in the Philebus concerning the uniqueness of the forms were valid and permanently damaging to his theory, he might have reformulated or abandoned his theory of forms. But the fact that Plato turns to epistemological considerations rather than ontological reconstructions suggests that he did not feel that the objections were permanently damaging.

He tells us, for instance, that "only a man of exceptional gifts will be able to see that a form or essence just by itself, does exist in each case" (Parmenides 135a9) and that one should not "undertake to define 'beautiful', 'just' and 'good', and other particular forms too soon before he has had a preliminary training" (Parmenides 135d1).

In the Sophist, Plato turns his attention to what can be said about existence and non-existence, and the more general kinds such as motion, rest, sameness and difference, rather than in establishing their existence. His concerns are 1) how can one combine these kinds in discourse in order to be meaningful and 2) how does one go about defining a kind. To the extent that Plato's concerns are linguistic, they are also epistemological.

He further suggests in the Parmenides:

> ...if, in view of all these difficulties and others like them, a man refuses to admit that forms of things exist or to distinguish a definite form in every case, he will have nothing on which to fix his thought, so long as he will not allow that each thing has a character which is always the same, and in so doing he will completely destroy the significance of all discourse. (135b5).

Socrates says that the method of approach that he suggests is "a gift of the gods -- that they let fall from their abode, and it was through Prometheus, or one like him, that it reached mankind, together with a fire exceeding bright." (Philebus 16c6).

In summary of this chapter, it has been shown that Plato explicitly holds that forms are 1) intelligible or invisible, 2) causing or ruling, 3) immortal or eternal, and 4) constant and invariable. It has been argued that there is enough evidence in the dialogues to show that Plato thought forms to be 5) independently existing entities and 6) objects of knowledge. It has been likewise argued that there is enough evidence in the dialogues to show that numbers are 5) independently existing entities and 6) objects of knowledge, for Plato.

Finally, it was explicitly part of Plato's earlier doctrine of the forms that forms are 7) indivisible or incomposite and 8) unique and perfect, but he questioned these characteristics following the middle dialogues. It was also explicitly Plato's view in the early doctrine that numbers are 7) indivisible and incomposite, and it has been argued that there is enough textual evidence to show that Plato thought them to be 8) unique.

The conclusion that is drawn in this chapter is that numbers are probably the same kind of thing as forms, since they share the same characteristics. And though the characteristics chosen may not be the <u>only</u> characteristics of forms and of numbers, they are the only characteristics mentioned in the dialogues. The fact that these characteristics are the <u>only</u> ones mentioned does not change the inductive nature of the argument, but it strengthens the conclusion.

CHAPTER III

A.

Many things have been said about numbers; that, like forms, they are imperishable, indivisible, incomposite, and so on. And because numbers exhibit these characteristics, it may be said of numbers that they are the same sort of thing, for Plato, as are forms. The concern of this chapter is to determine if Plato, anywhere in the dialogues, tells us, or even hints at, what a number really is, rather than what sort of thing it is.

Many of the early dialogues are attempts to answer a "What is it?" question. Socrates asks Charmides, "What is temperance?" and wants an indication of "her nature and qualities." (*Charmides* 158e8). And he asks Laches, "What is courage?" or that common quality which is "the same in all cases and which is called courage". (*Laches* 191e10, 192b6). And in the *Hippias Major* Socrates says, "Tell me what beauty itself is," in order to explain why we apply the word 'beautiful' to the various kinds of beauty (288a7). In the *Euthyphro*, Socrates, after his second attempt to elicit a response from Euthyphro to the question "What is piety?", says, "I wanted you to tell me what is the essential form of holiness (αὐτὸ τὸ εἶδος) which makes all pious actions pious." (6d10). And in the *Meno*, Socrates says to Meno,

> Suppose I asked you what a bee is, what is its essential nature (περὶ οὐσίας ὅτι ποτ' ἐστίν)and you replied that bees were of many different kinds. What would you say if I went on to ask, And is it being bees that they are many and various and different from one another?

> Or would you agree that it is not in this respect that they differ, but in something else, some other quality like size or beauty?" (72b1).

Meno answers that in so far as they are bees, they do not differ at all and Socrates goes on to ask for that character in respect of which they do not differ at all...the common character that makes them all bees. The same thing is asked by Socrates as regards virtue, namely, what is the common character in respect of which the various kinds of virtues are called virtue. (Meno 72c6, 74a5, 74d4).

It is clear in all these dialogues that Socrates is searching for the essential nature (οὐσία) of each of the things under discussion, and gaining knowledge of the essential nature of each comes from apprehending the appropriate form. In the Meno, the theory of recollection is introduced as a means to such knowledge; it is by looking at the idea or form that one knows what an entity is. (81c3, 72c8, 85d). The same view is more explicitly developed in the Phaedo (65d13) when Socrates tells us that what a thing really is, its real nature (οὐσία) is apprehended by man through contemplation of its form and that this is true of any existing thing.

The early dialogues tell us that the answer to a ti esti question must be one that gives an entity's essential nature or οὐσία, but they do not tell us if there is any answer, in the form of a judgment stating that entity's essential nature, that will satisfy the question. Evidently, apprehension of an entity's form is a condition for knowing what that entity really is, but whether one can beyond this visionary apprehension and express it in a λόγος or give an account of it, is not clear, at least in the early dialogues. The question is, is there any sort of λόγος that will adequately convey the οὐσία of an entity. The fact that all the attempts at definition of the key topics under discussion in these early dialogues end in failure seems to indicate that there can be no judgment satisfying the ti esti question.[99]

Even if this is true of Plato's position in the early dialogues, it is quite clear that in the later dialogues, Plato believes that some sort of judgment could adequately convey the essential nature of a thing, in view of the fact that he continues his search for definitions. It is unlikely when he asks "What is knowledge?" in the Theaetetus, and "What is a sophist?" in the Sophist, and "What is a statesman?" in the Statesman, that he felt that he was asking for the impossible.[100]

In the middle and later dialogues Plato relates definition to the essence of the thing one is defining. In the Phaedrus, Socrates says, "You must know the truth about the subject that you speak or write about,

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[99] W. Jaeger in Paideia: The Ideals of Greek Culture, Vol. II, Oxford University Press, New York, 1943 strongly endorses this view. He says, "Neither in the early dialogues nor here in the Meno is a real definition of ἀρετή ever given; and it is clear that when he asks for the nature of ἀρετή he does not want a definition for an answer."... "The answer to "What is virtue?" is not a definition, but an Idea." p. 163. "The question, "What is virtue?" points straight to the οὐσία , to the real being of virtue, and that is just the Idea of virtue." p. 164. This view is not shared by the majority of commentators. Bluck argues that Plato would have been satisfied with a definition similar to the one given for shape as the only thing that accompanies color, because although it may not state the οὐσία of shape, it may aid one in recollecting what the οὐσία is. p. 7. L. Grimm's argument in Definition in the Meno, Oslo University Press, Oslo, 1962, is based on the assumption that definition is what Plato is after, and the author makes some important distinctions as to the kind of definition Plato wants.

[100] R.S. Bluck, Plato's Meno, Liberal Arts Press, New York, 1954, argues that even in the Meno which is considered an early dialogue, Plato is not demanding the impossible. p. 6. The attempts at definition of virtue, even though they fail to state the ousia of virtue "might help both Socrates and Meno to recognize (or recollect) what that οὐσία was." p. 7. According to Bluck, "the vision resulting from recollection would transcend definition" but attempts at defining an entity would aid recollection of it.

that is to say, you must be able to isolate it in definition (κατ' αὐτὸ τε πᾶν ὁρίζεσθαι δυνατός γένηται)." (277b5). And knowing the truth of anything entails apprehension of its form. And in the Laws X, the Athenian says there are three points to be noted about anything:

> I mean, for one, the reality of a thing, what it is (τὴν οὐσίαν), for another, the definition of this reality, (τῆς οὐσίας τὸν λόγον), for another, its name (τὸ ὄνομα).(895d).

The phrase 'the definition of this reality ' (τῆς οὐσίας τὸν λόγον) reenforces the view that, at least in the later dialogues, a proper definition of an entity's οὐσία is possible. In the Republic Socrates tells us that at the top level of the intelligible world, mind itself lays hold of the power of dialectic (511b4) whereby it grasps the true nature of things through contemplation of the forms. That dialectic is viewed as the art of definition is told in several dialogues.

In the Phaedrus, Socrates advises Phaedrus that in order to to construct an effective speech, one must follow a certain pair of procedures. The first is:

> That in which we bring a dispersed plurality under a single form, seeing it all together - the purpose being to define so-and-so, and thus to make plain whatever may be chosen as the topic of exposition. (265d4).

The second procedure is the reverse of this:

> ...whereby we are enabled to divide into forms..the single general form which they postulated was irrationality; next, on the analogy of a single natural body with its pair of like-named members, right arm or leg, as we say and left, they conceived of madness a single objective form existing in human beings. Wherefore the first speech divided off a part on the left and continued to make divisions...The other speech conducted us to the forms of madness which lay on the right-handed side..discovering a type of love that shared its name with other, but was divine...(266a1).

In this exercise love is defined by postulating irrationality and making appropriate divisions of irrationality into forms of madness which are divine. The first procedure Socrates is to call collection (συναγωγή) (266b4) and the second, division (διαίρεσις) (266b4). The procedure of διαίρεσις , as dividing an entity into kinds until you reach the limit of division and have isolated the definiendum in a definition (277b8), Socrates calls 'dialectic' (266b9).

Once again in the Statesman (262b10), the Stranger tells us that the whole art of definition consists in finding the real cleavages among the forms and that this is the science of dialectic:

> Dividing according to kinds, (τὸ κατὰ γένη διαιρεῖσθαι), not taking the same form for a different one or a different one for the same -- is not that the business of the science of dialectic? (Sophist 253d1).

We know from other passages in the Statesman that there are real divisions among forms (262b, 262b8, 263a1, 285a5) and that the task of defining is to find those real divisions.[101] The stranger continues that the following would be the right method of this science:

> Whatever is the essential affinity between a given group of forms, which the philosopher perceives on first inspection, he ought not to forsake the task until he sees clearly as many differences as exist within the whole complex unity - the differences which exist in reality and constitute the several species. Conversely, when he begins by contemplating all the unlikenesses of one kind or another, which are to be found in the various group of forms... he must gather together all the forms which are in fact cognate and comprehend them all in their real general group. (285b1)

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[101]If what Socrates tells us is to be taken seriously, there is a form for all the multiplicities in the world to which we give the same name (Republic X 596a7), then we must take this to mean that there is a form for 'maleness' and a form for 'femaleness' as well as a form for 'humanity'. This means in the realm of forms, since 'maleness' and 'femaleness' make up the class called 'humanity', (Sophist 262e3) there is a hierarchy. And defining, as in the sense just discussed, is finding out that hierarchy.

Thus, it is through the dialectic that the art of definition is possible and this consists in finding the real divisions among the forms. The procedure suggested in the Phaedrus, the Sophist, and in the Statesman is that by starting with a general kind and making appropriate subdivisions, one reaches the specific kind which is to be defined.

To follow Plato's own example (Sophist 265a4), if one wants to define a sophist, he might begin by dividing art into productive and acquisitive, then divide productive art into divine and human. Once more, one can divide each of these two (the divine and human arts) into two parts: the one part will be the production of originals and the other part will be the production of images. The production of images can be further divided into two kinds, one producing likenesses and the other producing semblances. Once more, the kind that produces semblances might be divided into two kinds, that which produces semblance by means of tools, and that which produces semblances by mimicry. And mimicry might be further divided into the kind of mimicry which is guided by opinion ('conceit mimicry' (267e1)) and into that which is guided by knowledge ('mimicry by acquaintance' (267e2)). The art called 'conceit mimicry' may be further divided into the sincere mimic and the insincere mimic. The insincere mimic is of two kinds: the one keeps up his dissimulation publicly in long speeches to a large assembly and the other uses short arguments in private and forces others to contradict themselves in conversation. If one then "collects all of the elements of his description, from the end to the beginning" (268c3), one has defined sophistry as:

The art of contradiction making, descended from an insincere kind of 'conceited mimicry,' of the semblance-making breed, derived from image making, distinguished as a portion, not divine, but human, of production, that presents a shadow play of words..."(268c7).

The process of dividing according to kinds enables one to reach a definition of an entity.[102] One cannot define living things by dividing the entities of the world into vegetable and animal and so on. But one can define animal, or a specific animal by beginning with living things and making the appropriate divisions. This is in accordance with Plato's dictum in the Republic, Book VI, that when one has true knowledge of an entity's nature, one has already apprehended the form and stands at the top level of the Divided Line, looking at the various entities subsumed under that form. Whereas, when one does not have true knowledge, but wants to attain it, the suggested procedure is the reverse: namely, to view the various kinds of objects and then to subsume them under the proper form or forms. This is also in line with

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[102]Whether the process of collection and division alone is adequate for defining an entity is one raised by many of Plato's interpreters. J. Stenzel, Plato's Method of Dialectic, Oxford, 1940, says that in the later theory of forms, Plato sees forms as interrelated in a complex structure, p. 62, and "mere definition is adequate to reproduce the full content of the new universal' without the aid of recollection. pp. 55-72. R.S. Bluck, op. cit., p. 55, who disagees, says that division presupposes recollection. H. Cherniss in Aristotle's Criticism of Plato and the Early Academy, Russell and Russell, New York, 1944, says "The formal method alone may lead to a number of definitions of the same thing unless one has the additional power of recognizing the essential nature that is being sought. In short, diaeresis appears to be only an aid to reminiscence of the idea." p. 46-7. And again, he says, "Diaeresis is merely a practical expedient for recalling the essential nature of a given object." p. 60, note 50.

I will not take sides in this controversy because whether recollection is retained in the middle and later dialogues does not directly affect the primary concern in this chapter, namely, what does Plato say about definition itself.

the statements of the Philebus (18b1) that when you are compelled to start with the multitude:

> ...you must not immediately turn your eyes to the one, but must discern this or that number embracing the multitude, whatever it may be; reaching the one must be the last step.

For Plato, "it is the recognition of the intermediates that makes all the difference between a philosophical and contentious discussion." (Philebus 17a5).

In this discussion of Plato's view of definition, certain criteria of an adequate definition emerge. First, if one is defining x, it is clear that, for Plato, a definition of x is not a list of the different kinds or sorts of x's. As was seen earlier, the answer to a "What is it?" question must give a single characteristic or quality that gives an entity its essential nature.

In the Meno, Socrates pleads with Meno to give him the definition of "this single virtue which permeates each of the number of virtues" (74a5). "Even if they are many and various, at least they all have some common character which makes them virtues." (Meno 72c6). In this dialogue, Socrates gives several examples of what is not to count as a definition. In answer to "What is shape?" , roundness is not a correct answer, for though it is a shape, it is only one of many shapes, therefore not a definition. In answer to "What is color?", white is not a correct answer, because there are other colors which correctly answer the question.

Socrates says:

> We always arrive at a plurality, but this is not the kind of answer I want. Seeing that you call these many particulars by one and the same name, and say that every one of them is a shape, even though they are contrary to each other, tell me what this is which embraces round as well as straight and what you mean by shape when you say that straightness is a shape as much as roundness. (74d4).

In the Theaetetus, Socrates once again tells us that in the attempts to define knowledge, it is not the many sorts of knowledge that would satisfy his quest, but what knowledge itself is.

> ...the question you were asked, Theaetetus, was not, what are the objects of knowledge, nor yet how many sorts of knowledge there are. We do not want to count them, but to find out what the thing itself - knowledge - is. (146e7).

And he continues once again, as in the Meno to give examples of inadequate definitions: "Suppose you were asked about some obvious common thing, for instance, what clay is, it would be absurd to answer: potter's clay, and ovenmaker's clay, and brickmaker's clay." (147a1).

This view of definition as not being an enumeration of kinds is further supported by Plato's proposed procedure of collection and division. One does not define an entity, say humanity, by classifying it into kinds, say, male and female, but one can define either male or female by subsuming it under the appropriate form. So that a definition according to this procedure cannot be kinds or sorts of x's.

On the positive side of this view, a criterion of a good definition is that it is always true of all the things that go by that name. The final definition of shape that Socrates is to offer is that it is the limit of a solid. Then, regardless of the kind of boundary a solid has, it is always a shape.

A second criterion of an adequate definition is that the name of the thing defined and the definition of the thing denote the same thing. (Laws X 895e1-4). Using Plato's example, "it is the same thing we describe indifferently by the name 'even' and the definition 'number divided into two equal parts." (Laws X 895e8). In the Euthyphro, Socrates argues that 'what is pleasing to the Gods' cannot be the definition of 'piety' because they do not denote the same thing and that "the two are absolutely different from each other." (11a5). He concludes this because 'piety' and 'what is pleasing to the Gods' cannot be substituted for each other in sentences without changing the truth values of the sentences.[103]

If the name of the thing defined and the definition of the thing denote the same thing, this means that the definition must be true of only those things with that name, in order to retain interchangeability. So the second criterion places a further restriction on the first criterion. It is not only true that an adequate definition is one that is true of all the things that go by that name, but it is true only of those things. So that if y says something about x, then, in order for y to be an adequate definition of x, it must not only be true of all those things that are x, but only those that are x.

Socrates suggests to Meno that they define shape as the only thing which always accompanies color. So if we let x = shape, and y = the only thing which always accompanies color, then y is an adequate definition of x, because it is always true of all those things we call shapes, and because it is not true of anything else but shapes, since it is the only thing which always accompanies color. This definition is acceptable to Socrates and he says of it, "I should be con-

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[103]P.T. Geach, "Plato's Euthyphro", The Monist, July 1966, claims that the principle underlying the argument appears to be the Leibnizian principle that two expressions for the same thing must be mutually replaceable salva veritate - so that a change from truth to falsehood upon such replacement must mean that we have not two expressions for the same thing. p. 376.

tent if your definition of virtue were on similar lines" (75b12).[104] The final definition of shape as the limit of a solid satisfies both criteria: it is true of all those things called shapes and it is true only of those things called shapes.

B.

Plato defines many kinds of entities in the dialogues. For example, he defines a soul as a self-moving mover (Laws X 896a1, Phaedrus 245c8), clay as earth mixed with moisture (Theaetetus 147c7), round as that whose extremity is everywhere equidistant from its center (Parmenides 137c2), straight as that of which the middle is in front of both extremities (Parmenides 137e3), a square number as any number which is the product of a number multiplied by itself, and an oblong number as any number that cannot be obtained by multiplying a number by itself, but has one factor greater or less than the other (Theaetetus 147e4-7).

But nowhere in the dialogues does Plato explicitly define number in terms of its true nature.[105] Nor does he define any individual number, such as two, three, and so on.

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[104] The reason Plato goes on to give another definition of shape instead of settling with this one is that the truth of the statement that shape is the only thing which always accompanies color is not as easily established. One must know more about color in order for the definition to be effective and illuminating. (Meno 75c5).

[105] Aristotle claims that Plato never maintained the existence of an Idea embracing all numbers. (Nicomachean Ethics 1096a18). This claim will be discussed in more detail at the end of this chapter.

What might serve as a possible candidate for a definition of number is the phrase 'the odd and the even', because Plato uses it in many passages throughout the dialogues where 'number' might have been used instead. But examination of these passages shows that Plato uses the phrase 'the odd and the even' to refer to kinds of number. In the Euthyphro (12c6) Socrates says, "Reverence is a part of fear, as the uneven is a part of number, thus you do not have the odd wherever you have number, but where you have the odd you must have number." Here 'uneveness' (or oddness) is a kind of number. In the same passage Socrates says, "Suppose you had asked me what part of number is the even, and which the even number is. I would say that it is the one that corresponds to the isosceles and not to the scalene." (12d9). The 'even' relates to number, in general, just as isosceles, a kind of triangle relates to triangles, in general. Thus, the even number is a kind of number. In these Euthyphro passages, Plato tells us that even is one kind of number and odd is another kind of number.

In the Republic VI, the Divided Line passage, Socrates states, "the students of geometry and reckoning and such subjects first postulate the odd and the even and the various figures and three kinds of angles and other things akin to these." (510c2). One may infer here since he speaks of kinds of angles and kinds of figures, that he also means that the odd and the even are kinds of numbers.

In the Phaedo it is said that "it is the very nature of three and five and all alternate integers that every one of them is invariably odd. Similarly, two and four and all the rest of this series is not identical to even, but each of them is always even." (104a7). Odd and even are essential properties of certain numbers, but they are not the numbers themselves. As Socrates says, "...'odd' and 'three' are not the same thing." (Phaedo 104a6).

The relationship between 'the odd and the even' and number is never put as strongly elsewhere in the dialogues as in the Statesman during the Stranger's discussion of classification according to kinds.

> Surely, it would be better and closer to the real structure of the forms to make a central division of number into odd and even, or of humankind into male and female. (262e3).

In this passage, the odd and the even are explicitly kinds of number in exactly the same way male and female are kinds of humankind.

In all these passages, the odd and the even are kinds of numbers, and in so being, they cannot satisfy Plato's own criterion of an adequate definition. A definition of number, for Plato, cannot be an enumeration of the different kinds or sorts of numbers. Furthermore, on the basis of what Plato says about dividing according to kinds, one cannot define an entity by breaking down that same entity into kinds. For instance, humanity cannot be defined by classifying humanity into male and female. But female can be defined with this type of classification, by saying that a female is a human...and so on. Similarly, number can be classified into kinds in hopes of defining certain kinds of number, but not for the purpose of defining number itself. "An even number is a number which..." is the proper form of a definition, but "a number is anything which is either odd or even", according to Plato's own criteria cannot be a satisfactory definition, because it does not exhibit the essential feature or οὐσία of each and every number.

It is probable that Plato regarded the phrase 'the odd and

the even' as a classification of number, but the question of primary importance is whether that phrase was considered to be an exhaustive classification of number by Plato. For if the classification was considered to be exhaustive by him, he may have viewed it as a working definition of number, even though it was not arrived at by diaeresis. Thus, the phrase 'the odd and the even' might have been regarded by Plato as a working definition of number. But if Plato held it to be an exhaustive classification of number, he could not have consistently claimed that fractions and irrationals are numbers, since they are neither odd or even.[106]

In the Gorgias where Socrates says that the art of arithmetic is one of the arts which secures its effect through speech in the field of 'the odd and the even' (451b1), one is led to believe that, at least in the early dialogues, Plato thought that arithmetic deals only with the odd and the even. Yet in many of the dialogues written after the Gorgias (the Meno, the Republic, the Theaetetus) irrationals are mentioned, and in the Phaedo and in the Timaeus, reference is made to fractions.

These passages will now be considered in more detail in order to ascertain to what degree, if at all, Plato believed that fractions and irrationals are numbers. For if they are not numbers,

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[106]The proof for the irrationality of $\sqrt{2}$, which was according to Heath, familiar to the Pythagoreans, consists of showing that $\sqrt{2}$ is neither odd nor even. Euclid's Elements, Introduction to Book X, Vol. 3, p. 1. This proof is also found in Aristotle, Prior Analytics, 41a26-7.

In contemporary classifications of numbers, the real numbers are the rational and irrational. The rational numbers include natural numbers and fractions, whose decimal form is either termination or repeating. This distinguishes them from the irrational numbers, whose decimal form is non-termination and non-repeating.

then it is possible that Plato considered the phrase 'the odd and the even' as a working definition of number.

In the Phaedo, the fractions (what we call fractions) one-half (τὸ ἥμισυ) and a third (τὸ τριτομόριον) are mentioned, and all those respectively like these; namely, all multiples of halves and thirds. (Phaedo 105b1). Socrates says five will not admit of being even, and ten will not admit of being odd. (105a6). And one-half (ἥμισυ) and a third (τριτομόριον) will not admit to being whole (τοῦ ὅλου). The point of the passage is that if anything is always accompanied by a characteristic which has an opposite, then it will not admit of the opposite of that characteristic. While Plato does not say that these fractions are always accompanied by the form of parts, it is likely that this is what he meant. Because it is implied that five will not admit to being even, because it is always accompanied by odd, that ten will not admit to being odd because it is always accompanied by even, and one-half and one-third will not admit of whole, because it is always accompanied by part.

The question, of course, is if Plato viewed those entities which partake of 'parts', i.e., one third and one-half, as numbers. The concept of part (parts) is one that must have been used liberally among the mathematicians of Plato's day, since these concepts are defined in Euclid's Elements, which, according to Heath, appeared at about 300 B.C. as a summation of the mathematical works up to that time.

Definition 1 in Book V says: "A magnitude is a part of a magnitude, the less of the greater, when it measures the greater."

Definition 3 in Book VII[107] says: "A number is a part of a number, the less of the greater, when it measures the greater." And Definition 4 of Book VII says: "but parts of it when it does not measure it."[108] Thus a number is a part or parts of a larger number depending upon how it relates to it.

Heath says in his note to Definition 3 that "by a part Euclid means a submultiple,"[109] and goes on to say that we are to understand the meaning of submultiple as a number, such that when it is compared with a greater number it measures the greater number; that is, it goes into the greater number evenly. Heath's note to Definition 1 of Book V of the Elements gives the example of 1, 2, or 3, is each a part of six, because each goes into six evenly.[110] So a part denotes some whole number, 1, 2, 3, as it relates to another greater whole number, say, 6.

Definition 4 in the same book goes on to say: "but parts (μέρη , the plural of μέρος), when it does not measure it." The example given in the note to Definition I, Book V, of parts is "4 is parts of 6", because 4 does not go evenly into 6 (or does not measure it). Thus, while 1, 2, 3, are each a part of 6, 4 is parts of 6, the distinction between part and parts depends upon whether they measure a whole number or not. In both cases, part and parts refer to whole

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[107] Heath claims that Euclid might have saved himself a lot of repetition if he had consolidated these two books as one, because a number is a kind of magnitude. But he excuses him (Euclid) as being anxious to give credit where it was due. Book V, by general consensus, was written by Eudoxus. Book VII is Pythagorean.

[108] Definition 4 is a continuation of Definition 3, therefore, it is an incomplete sentence.

[109] Euclid's Elements, Vol. II, p. 280.

[110] Ibid., p. 115.

numbers and how they relate to a greater whole number.

The question raised is, can the number that is referred to as a _part_ or _parts_ be a fraction? The examples that Heath gives in his notes to Definition 1, Book V are whole numbers. But then he says in his note to Definition V, Book VII:

> ...by the expression _parts_, Euclid denotes what we should call a _proper fraction_. That is, a _part_ being a submultiple, the rather inconvenient term _parts_ means any number of such submultiples making up a fraction less than unity. I have not found the word used in this special sense elsewhere, e.g. in Nicomachus, Theon of Smyrna, or Iamblichus, except in one place (Theon, p. 79.26, where it is used as a proper fraction of which 2/3 is an illustration).[111]

It is curious that Heath, in his note to Definition 4 refers to _parts_ as proper fractions, and does not do this in his note to Definition 3 as well. There are some proper fractions, e.g., 2/3, which can be properly called a _part_ of some number, say 8, since it goes into it exactly twelve times. Just as there are some proper fractions, e.g., 2/3, which can be properly called _parts_ of 9, since it will not go into it evenly (13 1/2 times). In the first case, 2/3 is a submultiple of 8, and in the second it is not a submultiple of 9. Yet, in both cases, the proper fraction satisfies the definitions.

Heath's distinction between _part_ and _parts_ as it is stated in this last note imposes a further restriction on the conditions of the definitions, which are not stated in the definitions themselves, namely, that the _parts_ of a number are any number of submultiples that make up a fraction less than unity. But there is nothing in Euclid's definitions of _part_ or of _parts_ to indicate that the phrase 'greater number' must refer to the number 1. Under Heath's restriction, 2/3 could never

[111] _Ibid._, p. 280.

be a part or a submultiple, since it does not go into the number one evenly. Yet, we know, that according to Definition 3, it could be a submultiple of, say, 8.

There are several minor considerations that, though they give no conclusive evidence, tend to make one believe that the authors of Book V did not consider fractions to be numbers.

One consideration is that Definition 2 of Book V defines number as a multitude composed of units. And Definition 1 of the same book says that a unit is that by virtue of which each of the things that exist is called one. In a very unusual sense, a fraction might be said to be composed of units: one might say that 2/3 is composed of 1/3 and 1/2, and the 1/3rds are the units. Or that 1/3 is composed of 1/9 and 1/9 and 1/9, and the 1/9ths are the units. Then the 1/3rds and the 1/9ths in virtue of their being units, would be called one. Though this interpretation is possible, it is very unlikely. It is hardly likely that the Pythagorean mathematical thinking had reached this point of sophistication, when we know that they were still talking of numbers as even and odd,[112] that one was not a number, but a starting point of number,[113] that two was not a number, but the beginning of the even, and that by imitating figures of living things with pebbles, one can determine the living thing's number.[114]

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[112]Diels and Kranz, Die Fragmente der Vorsokratiker, Weidman Dakota, Dublin, Ireland, 1968, 58 B20, p. 451, as reported by Stobaeus. Also Euclid's Elements, Book VII, Definitions 6-10.

[113]Diels and Kranz, Ibid., p. 449, 58B4.

[114]Aristotle says this of Eurytus, a disciple of the Pythagorean Philolaus, Metaphysics 1092b10.

The other consideration is that by the time Book V was written by Eudoxus, several changes had been made. No reference is made to the even and the odd, number or unit is not defined, the general theory is presented in terms of magnitudes, rather in terms of numbers, and the distinction between part and parts is erased. Although fractions are never mentioned, they might have been with no threat to doctrine. In Book V, a ratio is defined as a sort of relation in respect of size between two magnitudes of the same kind. Since these magnitudes could have been both commensurable as well as incommensurable, fractions _might_ have been viewed as commensurable. Perhaps Heath's reference to fractions in the note to Definition 4, Book VII, would have been more appropriate under Definition 1, Book V.

Plato's familiarity with Pythagorean mathematics is unquestioned, but his familiarity with the Eudoxean general theory of ratio and proportion is controversial.[115] One thing is clear: that the _Phaedo_ passage was written with no knowledge of Eudoxus' theory, since the _Phaedo_ was written at least 30 years before Plato's acquaintance with Eudoxus. Since the _Phaedo_ exhibits many examples of Pythagorean influence[116] we might reasonably suppose that the Pythagorean view of number[117] was also influential in forming Plato's view at this time. It is likely that Plato

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[115]The dating of Plato's and Eudoxus' lives is discussed later in this chapter.

[116]_Phaedo_ passages that indicate Pythagorean influence: 86b-d, the Pythagorean theory of soul as an attunement as presented by Simmias, 61d-e, Pythagorean view of the illegitimacy of suicide, as related to Simmias and Cebes by Philolaos.

[117]Books I, II, III, IV, VII, VIII, and IX of _Euclid's Elements_ are Pythagorean, according to Heath and van der Waerden, _Science Awakening_, translated by Arnold Dresden, Groningen, P. Noordhoff, 1954.

did not view fractions as numbers at the time of the Phaedo. An interpretation of Definition 3 and Definition 4 of Book VII, more in line with the Pythagorean view is that 3 is a number and 6 is a number, and 3 is a part of 6. But the ratio 3/6 is not itself a number. The part-parts relationship is one that exists between numbers, but it is not a number itself. Accordingly, for Plato, two is a number, and six is a number, and two participates in the form of 'part' with respect to six. The 'part' is not itself a number. It is a relative form, such as equal, double[118] and so on, that relates two or more things, but the result of this participation does not amount to an introduction of new entities as numbers.

In the Timaeus fractions are once again used by Plato in God's making of the soul. After God establishes two series, 1, 3, 5, 7, and 1, 2, 4, 8, he fills the intervals in each series with two kinds of means. The one mean gives fractions in terms of halves, the other, fractions in terms of thirds. These fractions are viewed by Plato as relating in a specific way to the number at the beginning of an interval and to the number at the end of the interval. The fractions which result are relationships that exist between numbers, rather than new kinds of numbers. "4/3 is one third of 1 more than 1, and 1/3 of 2 (2/3) less than 2." (Timaeus 36a6). Thus, 4/3 expresses a relationship between 1 and 2. The ratio itself is expressed in terms of the numbers 3 and 4, but the 4/3 is not a number, but a ratio, as in the Phaedo. Here again, as in the Phaedo, 4/3 and 2/3 are equal parts of its extremes 1 and 2. (36a4). Thus 4/3 exceeds 1 by 1/3 and is exceeded by 2 by 2/3.

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[118]At Phaedo 105a7, Socrates says ten, which is double five, will not admit the form of odd, nor will double, which has an opposite of its own, admit the form of odd.

It seems fairly clear from the above two passages that Plato viewed fractions as parts of numbers, and that these parts can be expressed as a relationship between whole numbers in terms of a ratio. This means that they were probably not viewed by him as kinds of numbers.

The final evidence which supports this interpretation is that in neither the Phaedo passage nor in the Timaeus passage does Plato use a Greek equivalent for the word 'fraction'. The standard translations of these passages have obscured rather than facilitated Plato's meaning and have, perhaps unknowingly, made committments for Plato that Plato himself might not have made.

There is a sense in which he might have viewed fractions as rational in the broad sense: i.e., reasonable. He refers to irrationals as ἄλογος (meaning without a reason or an account) because in calculating for the value of, say, $\sqrt{2}$, the mathematicians of his day found that it could not be expressed in terms of a ratio between two natural numbers.[119] But even taken in this broader sense of acting more reasonably than irrationals, the case for fractions being viewed as real numbers is not strengthened. It is probably because fractions are more reasonable and expressible as a relationship between two whole numbers, that Plato may have felt no need to include them as separate real numbers.

If numbers are properly and exhaustively classified as 'the odd and the even,' then irrationals cannot have been regarded by Plato as real numbers, either. Historical evidence is conclusive that the

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[119] As noted in footnote 106, the Pythagoreans were familiar with the proof for the irrationality of $\sqrt{2}$. Also, it must have been familiar to Plato, since Theaetetus tells us that he had been working on the proofs for the irrationality of $\sqrt{3}$ ....$\sqrt{17}$, in the Theaetetus 147d following.

irrational was discovered well before Plato. Heath tells us that although the

> ...problem of determining how much of the Pythagorean discoveries in mathematics can be attributed to Pythagoras himself is not only difficult, it may be said to be insoluble[120] that it is not disputed that the Pythagoreans discovered the irrational.[121]

The Scholium No. 1 to Book X of the Elements,[122] which Heath uses as his strongest evidence, says that:

> They (the Pythagoreans) called all magnitudes measurable by the same commensurable, but those not subject to the same measure incommensurable.

Though Neugebauer [123] places the discovery of the irrational well before Pythagoras:

> ...the traditional stories of the discoveries made by Thales and Pythagoras must be discarded as totally unhistorical...We know today that all the factual mathematical knowledge which is ascribed to the early Greek philosophers was known centuries before.

he is in agreement with Heath to the degree that the discovery was assuredly pre-Platonic. Plato, himself, acknowledges the debt to the Egyptians when he says that Toth is the inventor of arithmetic, geometry, and astronomy.[124]

There is also no doubt about the fact that Plato was familiar with the concept of irrationality. In the Meno, the slave boy is asked

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[120] Euclid's Elements, Vol. I. p. 411.

[121] Ibid., p. 351.

[122] Heath lists the source as Heiberg's Om Scholierne til Euklid's Elementer, Kjohehaun, 1888.

[123] O. Neugebauer, The Exact Sciences of Antiquity, Princeton University Press, Princeton, New Jersey, 1952, p. 142.

[124] Phaedrus 274c

to state the length of the side of a square eight times as large as a given square, which length is an irrational number. In the Republic, Socrates refers to those without knowledge as being irrational (ἀλόγους) as the lines of geometry.[125] In the Parmenides, Plato voices through Parmenides the definition of equal in terms of both commensurables and incommensurables.[126] The strongest statement of Plato's familiarity with irrationals appears in the Theaetetus, when the discussion revolves around Theaetetus' new classification of numbers in terms of oblong or square, the square root of an oblong number being irrational, and of a square number, rational.[127]

But as in the case of fractions, it is not as easily shown that irrationals are, or are not considered by Plato to be real numbers. Several considerations lead one to believe that they are not.

Wedberg[128] argues that Plato viewed the irrational as a geometric rather than an arithmetic concept since he associates the term 'irrational' with lines. In the Meno the irrational under investigation refers to the length of line segments, in the Republic (546c5) in his explanation of how rulers will breed, and according to what season, the dimensions discussed are in terms of rational and irrational diameters. After Theaetetus has explained his new classification of square and oblong numbers, he goes on to relate these to geometric figures. All

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[125] Republic 534d5.

[126] Parmenides 140b7.

[127] Theaetetus 147d following.

[128] A. Wedberg, op. cit., p. 24.

the lines that form the sides of a square whose area is a square number are defined as length, while those lines which form the sides of a square whose area is an oblong number are called roots. Roots are incommensurable with others in length.[129] The irrational is illustrated by a certain kind of length rather in terms of numbers. 'Irrational' refers explicitly to lines, when Socrates says "...as irrational as the lines of geometry." (Republic 534d7). And in the Parmenides, equal is defined in terms of number of measures in commensurable or incommensurable lengths.

But the fact that Plato talks about irrationals in terms of lines, instead of numbers, does not exclude the possibility that irrationals are numbers. Even in Euclid's Elements, Book V, whose contents is the general theory of ratio and proportion and which applies to all kinds of magnitudes, i.e., geometrical as well as arithmetical, one finds that all these kinds of magnitudes are represented by straight line segments. This gives all the proofs of propositions a geometric appearance, yet the magnitudes referred to are numbers and lines, commensurable, as well as incommensurable. Book VII makes the same point in stronger terms. The proofs in Book VII which treats of the theory of proportion with reference to the particular case of numbers, are all demonstrated by line segments. This means that Plato's reference to 'irrational' lines does not mean that irrationals were only lines in a definitional sense, but that irrationals could be represented by lines, and were most often associated with lines.

The more important consideration is not whether irrationals were

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[129]For instance, if the area of a square is 16 (a square number), then the length of each side is 4. And if the area of a square is 15 (an oblong number), then its side is a root, or surd, namely, the square root of 15. The square root of 15 is incommensurable with the square root of 16, or four.

geometric concepts, but when, in the course of the mathematical discoveries flourishing in Plato's time, the irrational was incorporated into a general theory of number, as a bona fide member. What is difficult to determine is not that it was discovered well before Plato's lifetime, but how and when it was finally accepted into a system of numbers, and if Plato was aware of and acknowledged this incorporation.

We know from the Theaetetus that up to the writing of that particular dialogue, Theaetetus had not as yet formulated a general theory to show that irrationals are not capable of expression in terms of a ratio of two natural numbers. He says specifically that he had shown this to be true of "all the separate cases up to $\sqrt{17}$, the root of seventeen square feet." (147d8). We also that that it was Theaetetus who was responsible for generalizing the theory of roots,[130] and that this theory was incorporated in Leon's Elements,[131], in Theudius' Elements (a pupil of Eudoxus), and was systematically made into Book X of Euclid's Elements.[132] It has been said that Theaetetus' merit lies in:

> his being the first to understand that a mathematical theory develops from definitions which are broad enough to contain within them the solutions of all the problems posed in such a theory.[133]

Plato's acquaintance with Theaetetus, with his on-going work

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[130] Testimony for this is Proclus, op. cit., p. 54, 66.16; Heath in Euclid's Elements, Vol. 3, p. 1-4; and Pappas, Commentary on Euclid (Pappas of Alexandria, La Collection Mathematique, Paris, 1933) as sighted in E. Maziarz and T. Greenwood, Greek Mathematical Philosophy, Frederick Ungar Publishing Co., New York, 1968, p. 34.

[131] 365-360 B.C.

[132] Heath's commentary on Book X supports this, as well as B.L. Van der Waerden, op. cit., p. 172 and D. Proclus, op. cit., 66.16, p. 54.

[133] F. Lasserre, op. cit., p. 17.

on irrationals, and with the importance of broad definitions are supported by the dialogue, Theaetetus. That Theaetetus was, in fact, a member of the Academy, arriving in Athens about 375-369 B.C. is accepted by most Plato scholars.[134] The date of Theaetetus' arrival in Athens coincides with Plato's writing of the Republic (375-370 B.C.).

It was not until Eudoxus' doctrine of ratio and proportion of magnitudes,[135], however, that incommensurables are incorporated into a general theory of number.

> The word, λόγος , ratio, acquired in Euclid Book V, a wider sense covering the relative magnitude of incommensurables, as well as commensurables.[136]

It was during a twenty-five period of time, from 375-369 B.C. (when Theaetetus joined the Academy) until 350-347 B.C. (when Eudoxus appeared) that Greek mathematical activity was going through its most creative period. One can be certain that Plato knew the work of Theaetetus, but whether Plato was familiar with Eudoxus' general doctrine of ratio and proportion is uncertain.

Plato was familiar with proportionals in terms of numbers and the various kinds of means (Timaeus 32A-B, 35D) in which irrationals are involved. And the problem of duplicating a cube, which is connected with finding two mean proportionals, was of particular inter-

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[134] Proclus tells us that Theaetetus was a member of the Academy, p. 54, 66., and F. Lasserre, op. cit., dates his arrival in Athens.

[135] Proclus attributes this to Eudoxus, p. 55, 67.2. Also Heath, Euclid's Elements, Vol. 2, p. 122 and van der Waerden, op. cit., p. 184 attributes this doctrine to Eudoxus and as being the contents of Book V of the Elements.

[136] Heath's introduction to Book V of Euclid's Elements, Vol.2, p. 117.

est to him.[137] Plato is mentioned in the legends attached to the history of this problem, as recounted by Theon of Smyrna and Eutocius on the authority of Eratosthenes' Platonicus.[138] The same story is found in Plutarch [139] that Plato referred the Delians to Eudoxus or Helicon of Cyzicus for the solution of the problem.[140] He was also familiar with the contents of Book VII, VIII, and IX of Euclid's Elements[141] which deal with proportions of natural numbers only.

Reports on whether Plato was familiar with Eudoxus' general theory of ratio and proportion vary. Proclus tells us that Eudoxus of Cnidus was a member of Plato's group[142] and that:

> Eudoxus of Cnidus...was the first to increase the number of the so-called general theorems; to the three proportionals already known he added three more and multiplied the number of propositions concerning the section which had their origin in Plato, employing the method of analysis for their solution.[143]

Van Der Waerden claims that by 'general theorems' Proclus probably

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[137] E. Maziarz and T. Greenwood, op. cit.,refer us to Theon's Exposition Rerum Mathematicarum ii 7-12, p. 80. Hippocrates, born 430 B.C., is responsible for showing that the problem of duplicating a cube is reducible to the problem of finding the mean proportional. The problem had not been solved even half a century after Hippocrates' reformulation. Solutions were found by Archytas, Eudoxus, and Menaechmus, and later by Eratosthenes, Nicomedes, and Appolonius, as sighted by van der Waerden, op. cit., p. 139.

[138] B.L. van der Waerden, op. cit., p. 160-161, also E. Maziarz and T. Greenwood, op. cit., p. 80.

[139] (deDei apud Delphos) 386E and (de genio Socrates) 579CD.

[140] Eudoxus' solution to the duplication of the cube is lost.

[141] According to both Heath and van der Waerden, these books can be attributed to the Pythagoreans.

[142] Proclus, op. cit., p. 55, 67.2.

[143] Proclus, op. cit., p. 55, 67.

refers to the ones of Book V, which is Eudoxus' work, but that it might refer to the axioms of Book I, that he is known to have added more propositions to those already started by Theaetetus, and that by 'section' he refers to the 'Golden Section' which is treated three times in the Elements, Book II, early Pythagorean, Book VI, and in Book XIII, which is due to Theaetetus. So, according to van der Waerden, it cannot be said with certainty, on the basis of Proclus' report that Plato knew of his general theory in Book V.

Lasserre discounts Proclus' report by saying:

> The discoveries of Eudoxus which owed nothing to Plato's influence, prove that the need for generalization is not, in fact, peculiar to the Academy, and that Eudemus is one of Plato's admirers and gives him far too much credit.[144]

Lasserre also argues that Eudoxus was neither a pupil of Plato nor a member of the Academy.[145] According to Lasserre, Eudoxus was to have arrived at the Academy slightly before 350 B.C. where he met Plato and left shortly thereafter for Cnidus leaving behind him some pupils. Since Plato's death takes place about 348-7 B.C., his acquaintance with Eudoxus would have had to have been very short.

Cherniss also argues that "his chief works appear too late for the Philosopher to have been able to give them much attention."[146] And Owen agrees that at least Eudoxus' mathematical theory of astrono-

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[144]F. Lasserre, op. cit., p. 39. Eudemus' geometry, which is now lost, is the basis of Proclus' comments in his commentary of the Elements.

[145]F. Lasserre, op. cit., p. 17.

[146]H. Cherniss, op. cit., p. 86.

my has no effect on the astronomical theories presented in the *Timaeus*[147] though he dates this theory as being brought to Athens by Eudoxus by 368 B.C., at latest, which is somewhat earlier than the date quoted by Lasserre, i.e., 350 B.C. Van Der Waerden also dates Eudoxus' arrival to Athens at about 368-5 B.C.[148]

Even if the true date of Eudoxus' arrival to Athens was around 368-5 B.C., this would coincide roughly with the time of Plato's second and third trips to Sicily (367-6 B.C. and 361-0 B.C.) from which he returns to Athens at about 360 B.C. This would be more in line with Lasserre's dating of Plato's acquaintance with Eudoxus; and the period of Plato's acquaintance had to have been very short to be followed by Plato's death in 348-7 B.C.

In view of these chronological considerations, it is likely that in Plato's day, mathematicians were on the verge of relating irrationals to natural numbers in a general theory, but had not yet done so. In the *Parmenides* (140c), Plato talks about commensurables and incommensurables, but in this passage his terminology is devoid of any reference to ratio or proportions. He speaks in terms of 'fewer or more measures' and 'greater or less' which terminology is more in line with the propositions of Book VII of the *Elements*, which is Pythagorean, and not of Book V, which is Eudoxean.

Plato's introduction of triangles with irrational sides as the elementary forms out of which God fashioned the four elements in

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[147] G.E.L. Owen, "The Place of the *Timaeus* in Plato's Dialogues", *Studies*, ed. Allen, p. 325. This is one of Owen's reasons for the placement of the *Timaeus* as following the *Republic*.

[148] B.L. van der Waerden, *op. cit.*, p. 179.

the _Timaeus_, suggests that irrationals were important enough, for Plato, to be included in the primary stuff out of which things are made, but the proofs he gives for the five regular polyhedra, which start with these triangles and are consequently related to each of the elements, demands no more than an elementary knowledge of geometry, with which the Pythagoreans had adequately supplied him. The proofs, which are constructions and geometrical in nature, do not demonstrate a knowledge of a general theory of number.

It does not follow necessarily that Plato was unfamiliar with a general theory of number incorporating irrationals, but it does follow that either he knew about Eudoxus' theory and did not write about it, or that he favored a broader classification of numbers (_Theaetetus_), but he was in no position as a metaphysician to relate these different kinds of numbers to a general theory.

Thus, since it is likely that neither fractions nor irrationals were considered by Plato to be bona fide members of the number system, then the phrase 'the odd and the even' would be an exhaustive classification of number. And if 'the odd and the even' is an exhaustive classification of number, it would satisfy the first criterion of an adequate definition, namely, that it be true of all those things called numbers.

The final question that must be asked is if the phrase 'the odd and the even' denotes things other than number, because if it does it fails to satisfy the second criterion, namely, that it is true _only_ of numbers. One might argue that Plato calls things, e.g., physical objects, odd or even as in the _Phaedo_ when Socrates says that when the form of three takes possession of a group of objects, it compels them to be odd as well as three. (104d7). Now, while it is true that the definition of, say, beauty will describe a beautiful thing, as well as the form of beauty, the beautiful thing is less perfectly beautiful than beauty itself is beautiful. Similarly, the number three, like the form ob beauty, is odd in a more perfect way than three things are odd. Three things are both odd and even in that they may be odd in one respect and even in another. Socrates says in the Parmenides, that he is both one and many, since he is one man, but with many members and there is nothing "surprising in someone pointing out that I am one thing and also many." (129c6). But the number three itself cannot be both odd and even. So that while the phrase 'the odd and the even' may apply to physical things, it applies to them in a more imperfect way than it does to the series of natural number.

This being so, one finds that since the phrase 'the odd and the even' is an exhaustive classification of number, and since it does not apply to things other than numbers in the same way it applies to numbers, then the criteria for an adequate definition are satisfied and the denotata of 'the odd and the even' would be the same as the denotata of 'number'. But any statement that regards the phrase 'the odd and the even' as an adequate definition of number must overlook the strong doctrine of the dialogues that a definition of x is not an enumeration of the kinds or sorts of x's, but a statement of x's οὐσία or essential character. For just as one cannot properly define animal by listing all the kinds of animals, one cannot define number by listing all the kinds of numbers. Because, on the one hand, the 'odd and the even' satisfies all the criteria of an adequate definition, and because, on the other, it is a listing of the kinds of numbers, it is possible that Plato might have legitimately used the phrase whenever he meant number, and might have regarded it as a working definition of number.

Other than 'the odd and the even', there is not even a suggestion in the dialogues of what else might serve as a definition of number (even though Plato defines various kinds of numbers.) Even number, odd number, prime number, composite number, square number, and perfect number are all defined. Plato's definitions of these terms are surprisingly similar to the definitions of these same terms that are found in Euclid's Elements, Book VII, which is Pythagorean doctrine.

Plato's Definitions	Euclid's Elements
An even number is that which is divisible into two equal parts. (Laws X 895e3).	An even number is that which is divisible into two equal parts (Def. 6).
"If you ask what must be present in a number to make it odd, I shall not say oddness, but unity. (μονάς) (Phaedo 105c5).	An odd number is that which is not divisible into two equal parts or that which differs by an unit (μονάς) from an even number. (Def. 7).
"Thus there will be even multiples of even sets, and odd multiples of odd sets, odd multiples of even sets, and even multiples of odd sets". (Parmenides 144a1).	An even-times even number is that which is measured by an even number according to an even number. An even-times odd number is that which is measured by an even number according to an odd number. An odd-times odd number is that which is measured by an odd number according to an odd number. (Def. 8 - 10).
The numbers produced by multiplying two numbers represents the area of a plane figure, and the multiples its sides. This procedure is discussed in the Theaetetus 148ff.	And when two numbers having multiplied one another make some number, the number so produced is called plane, and its sides are one of the numbers which have multiplied one another. (Def. 16).
A square number is a number which is the product of a number multiplied by itself. (Theatetus 147e5).	A square number is equal multiplied by equal, or a number which is contained by two equal numbers. (Def. 18).
Plato never defines cube number, but mentions it at Timaeus 31c3).	A cube number is equal multiplied by equal and again by equal or a number which is contained by three equal numbers. (Def. 19).

Plato's Definitions (Cont'd.)	Euclid's Elements (Cont'd.)
A proportion is such as "Whenever in any three numbers, whether cube or square, there is a mean, which is to the last term what the first term is to it. (Timaeus 31c3).	Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second, that the third is of the fourth. (Def. 20).
Plato never defines perfect number, but mentions it at Republic 546b6).	A perfect number is that which is equal to its own parts. (Def. 22).

Plato seems to avoid one definition. That is the definition of number or Definition 2 in Euclid's Elements of Book VII.[149] Number is defined in this book as a "multitude composed of units". It is not as though Plato were not familiar with this definition. We have seen that he most assuredly was familiar with the remaining definitions of Book VII.

Heath tells us in his note to Definition 2 that "the definition of number is again only one out of many that are on record."[150] But the others that Heath gives are again all definitions that highly resemble Euclid's. Nicomachus says number is a 'defined plurality' (πλῆθος ὡρισμένον), or a 'collection of units' (μονάδων σύστημα). Theon, in almost identical words with those attributed to Moderatus,[151] a Pythagorea, says "a number is a collection of units, or a progression of multitude beginning from a unit and a retrogression closing at a unit."

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[149] The only exception is the first definition, namely, that of 'unit', which is "that by virtue of which each of the things that exist is called one."

[150] Euclid's Elements, Vol. 2, p. 280.

[151] Heath in Euclid's Elements, Vol. 2, p. 280 states that this is cited by Stobaeus in Eclogae, I. 1.8.

The definition 'collection of units' was also applied to number by Thales, according to Iamblichus.[152] Eudoxus also said number was 'a defined multitude' (πλῆθος ὡρισμένον), and Aristotle offers a number of definitions: number is 'limited plurality' (πλῆθος πεπερασμένον), 'plurality of units' (πλῆθος μονάδων), 'plurality measurable by one' (πλῆθος ἑνί μετριτού), 'a measured plurality' (πλῆθος μεμετρημένον), and 'a plurality of measures' (πλῆθος μέτρων).[153]

Even if we concede the possibility that Plato was not familiar with the definition of number as it appears in the Elements, which would be peculiar, indeed, since he was familiar with all the other definitions in this book and used them, he must have been familiar with the general trend of other mathematicians to define number as 'a multiple of units'.[154] And yet, in spite of his probable familiarity Plato never defines number in this way in the dialogues.

One explanation of his failure to do so is that the on-going definitions of Aristotle's, Euclid's, Eudoxus', etc., divided number into parts. But Plato's inclination to view numbers as forms, and as incomposite wholes, drew him away from such a definition. Perhaps he was not prepared to give up the characteristics of unity, indivisibility, and uniqueness of number in exchange for a definition of number in terms of parts or units.

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[152] Heath in Euclid's Elements, Vol. 2, p. 280.

[153] Metaphysics 1020a13, 1053a30, 1057a3, 1088a5. According to Heath, these all amount to the same thing.

[154] It is likewise argued by A. Wedberg, op. cit., that the definition of number current in Greek mathematics already before Plato's time is 'plurality of units', but he goes on to say that Plato accepted this by eventually forming mathematical numbers, which are separate from idea numbers. p. 71.

It has been suggested by some commentators on Plato that Plato did not believe that there was a single form for all numbers[155] and consequently, no definition could be given exhibiting the nature of all numbers. Aristotle tells us that:

> The men who introduced this doctrine [of the forms] did not posit Ideas of classes within which they recognized priority and posteriority (which is the reason why they did not maintain the existence of an Idea embracing all number).[156]

This quote tells us that Plato did not maintain the existence of an Idea embracing all number, and if this is true, then, Plato would not have defined number, in general, with a real or essential definition.

An understanding of Aristotle's statement depends upon what he means by the phrase 'priority and posteriority'. Ross[157] says that the underlying notion here is the Aristotelean view that "a truly generic nature must be one that is expressed equally, though differently, in a diversity of species." Aristotle says:

> But we must not forget that things of which the underlying principles differ in kind, one of them being first, another second, another third, have, when regarded in this relation, nothing, or hardly anything worth mentioning in common.[158]

Accordingly, for Aristotle, many general terms cannot be defined. In the case of a citizen, "we see that governments differ in kind, some of

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[155] A.E. Taylor, op. cit., p. 505ff.; H. Cherniss, op. cit., p.36; Sir David Ross, op. cit., p. 182.

[156] Nicomachean Ethics 1096a18.

[157] Sir David Ross, op. cit., p. 182.

[158] Politics 1275a35

them are prior and others are posterior"[159] and since the citizen of each kind of government of necessity differs, each must be defined separately.

Being, for Aristotle, is not defined, because there are as many kinds of being as there are categories,[160] and of the good, he says:

> Further, since 'good' has as many senses as 'being', clearly it cannot be something universally present in all cases and single; for then it could not have been predicated in all the categories, but in one only.[161]

In all these cases, where the diversity of species is related as prior and posterior, a specific definition of each of the kinds of species must be given, and no general definition can be given.

Ross attributes this same view to some Platonists[162] and says that because numbers exhibit plurality, only unequally, that number was not a genuine idea. Apparently, since four is made up of four units, and three is made of three units, a definition of number, in general, cannot be given that will give the true nature of all numbers, inasmuch as each number differs as to its true nature. This, according to Ross, is why Aristotle says numbers are prior and posterior.

Cherniss understands Aristotle's prior and posterior distinction in a similar way. He argues that the Aristotelean distinction has to do with ontological priority. An individual number is prior to one and posterior to another number, insofar as numbers are generable, and as such,

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[159] Politics 1275a38.

[160] Metaphysics 1017a24.

[161] Nicomachean Ethics 1096a23.

[162] Sir David Ross, op. cit., p. 182 says,"We do not know whether Plato was among them."

ontologically prior and posterior. So that three is generated from two by the addition of one, and is thus posterior to two, and so on for all the other numbers.[163] But this distinction, Cherniss argues, is an Aristotelean one, and not a Platonic one.[164] But he goes on to agree with Ross that Plato did not posit an idea of number, in general, because it would amount to a duplication of the series of ideal numbers. In explanation of this he says:

> As soon as the essence of each idea of number is seen to be just its unique position as a term in this ordered series, it is obvious that the essence of number in general can be nothing but this very order, the whole series of these unique positions. The idea of number, in general, then, is the series of ideal numbers itself, and to posit an idea of number apart from this would be merely to duplicate the series of ideal numbers. The proof that no such duplication of an idea is possible occurs in both the Republic and the Timaeus.[165]

Thus, according to Cherniss, it is Plato's view, that 1) the essence of each idea of number, say, twoness or threeness, is its unique position in an ordered series, and 2) if the essence of the idea of number in general is the very series of ideal numbers, then to posit an idea of number is to unnecessarily duplicate the series of ideal numbers.

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[163] H. Cherniss, op. cit., p. 35. According to Cherniss, ontological priority is given all ideas by Plato and posteriority to all particulars. This position is shared by C. Wilson, op. cit., who says, "The ideas of numbers, as being the Universals of number and therefore ἀσύμβλητοι , are entirely outside one another, in the sense that none is a part of another. Thus, they form a series of different terms, which have a definite order. They are nothing but what the mathematicians call 'the series of natural numbers' where the definite article is right because there is only one such series consisting of Universals, each of which is unique." p. 253.

[164] The notion of 'priority and posteriority' as regards numbers, is associated throughout Book XIII of the Metaphysics with the generation of number. cf. 1081a25-27, 1080a20-35, 1082a27-37, 1082a10, 1084a3, 1091a1, 1091a10.

[165] H. Cherniss, op. cit., p. 36.

There are some difficulties in both Ross' and Cherniss' interpretations of Aristotle's statement. According to Ross, if Plato viewed the essence of twoness as something that is made up of two units and so on for all the integers, then, Plato might have legitimately defined number, in general, as a multiple of units, which Ross claims Plato did not do. For example, Aristotle can consistently define number, in general, as a multiple of units and still maintain the view that general definitions cannot be given for differing species, because he does not view the individual numbers as differing in kind. Numbers are generated from one another by the addition of units. And the addition of units does not change the essential nature of each number. Thus, Ross' explanation of why numbers differ in kind, and subsequently, are prior and posterior, cannot be accepted on the basis of the fact that numbers exhibit plurality only unequally.

Cherniss' account that by 'prior and posterior' Aristotle meant that, for Plato, the essence of each number is its unique position in the number series is better, because it square with Aristotle's statement that since numbers, for Plato, differ in kind, and they would if each is characterized by its unique position, no general idea of number was posited by him.[166] But Cherniss' conclusion does not follow from his premises. For if the essence of each form of a number is its unique position <u>in</u> an ordered series, and one then forms this series by placing the individual numbers in their appropriate position, say like: twoness, threeness, fourness, etc., the total series itself is something different than the position of each number <u>in</u> that

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[166]Aristotle says in <u>Metaphysics</u> 1080b11 that one kind of number, the ideal number, has the prior and posterior character of the ideas. (τό πρότερον καί ὕστερον τάς ἰδέας).

series. So if there is a form of number in general, no duplication results. In other words, the statement that each individual number holds a unique position in a series is clearly different than the statement that the placement of all the individual numbers results in a series of ordered numbers. An idea of number, in general, would be an expression of the total series of natural numbers, whereas the idea of each specific number is not an expression of that total series.

Both Cherniss' and Ross' account of the indefinability of number, in general, is based on the view that numbers differ in kind, but for different reasons: Ross' is that numbers exhibit unequal multiplicity, and Cherniss' is that each number occupies a unique position in an ordered series. And it has been argued that, <u>if</u> numbers for Plato <u>do</u> differ in kind, that Ross' account does not explain this difference, but that Cherniss' does.

But even if numbers do differ in kind and it is likely that they do if each is viewed as a separate form, [167] it does not follow that, for Plato, this means there is no form of number, in general. It may be an Aristotelean doctrine that no general definition can be given for species that differ in priority and posteriority, but there is no indication that Plato subscribed to such a doctrine.

Throughout the dialogues, Plato asks for definitions of virtue, of justice, of knowledge, of good, and any answer in terms of kinds of virtue, and kinds of justice, etc., is unsatisfactory. This means that Plato is asking for a general definition that applies to all the kinds. His failure to find general definitions does not mean there is no general form of virtue or justice, but only that finding such a definition,

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[167] See pages 74-5.

if it is possible, is a very difficult task. Accordingly, it does not follow that because each of the individual numbers for Plato differ in kind, that there is no form of number in general which subsumes the various kinds under it, and to assert that something is undefineable or defineable is not to assert that it does not or does exist. Furthermore, in view of the fact that Plato posits forms of even the most general terms, such as existence, sameness, difference, motion, and rest,[168] which he does not define, it would have been peculiar if he had not posited a form of number, in general.

Several claims are made by Aristotle in the passage in the Ethics. Aristotle says that 1) the Platonists did not posit ideas of classes where there is priority and posteriority and 2) the Platonists did not posit an idea of number, and 3) (2) is true because of (1). But there is no evidence in the Platonic text that Plato held the philosophical positions mentioned in either (1) or (2).

Aristotle's view that Plato believed that there is no form of number in general is a conclusion that can be inferred from the application of Aristotelean principles.[169] Aristotle is saying that Plato

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[168] Sophist 254d-255e.

[169] In some cases Aristotle himself held (1), as in the Politics. But it was not a principle Aristotle slavishly adhered to, because even in the classes where there is priority and posteriority, there may be a single essence also, as in the case of the soul, in which case a general definition can be given. But in other cases, as in the case of a citizen in a good or a bad state, there may no common essence, only posteriority and priority, and so no general definition. This is also the case in 'good' and 'substance' for Aristotle. Whether Plato held (1) is not clear, although Aristotle says he did. In short, Aristotle seemed to hold (1) when the only connection between differing species is one of priority and posteriority.

cannot consistently maintain a form of number, in general, because individual numbers differ in priority and posteriority and on Aristotelean grounds, this is not possible. And again, on Aristotelean grounds, there is no general definition that can describe all the kinds.

Thus, it is unlikely that the reason Plato fails to define number is because he does not posit a form of number embracing all the individual numbers. In fact, there are several passages in the dialogues that indicate the contrary. In the Theaetetus, Socrates tells us that there are certain things that can be apprehended neither through hearing nor through sight, nor any of the other senses, but can be contemplated only by the mind itself. These things he calls 'κοινά', because they are common to all things and says:

> You mean existence and non-existence, likeness and unlikeness, sameness and difference, and also unity and number in general...and clearly your question covers 'even' and 'odd' and all that kind of notions.[170]

Some of these same 'κοινά', namely, existence, sameness, and difference are later referred to by the stranger in the Sophist as forms.[171] Thus, he is talking of forms in the Theaetetus passage, and number, in general, is one such form.

And in the Sophist, the stranger says:

> Well, among things that exist, we must include number in general.[172]

These passages strongly repudiate the claim that Plato did not posit

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[170] Theaetetus 185e12.

[171] Sophist 254-256.

[172] Sophist 238a11. In Greek: Ἀριθμὸν δὴ τὸν σύμπαντα τῶν ὄντων τίθεμεν.

a form of number, in general, and therefore, did not define it. It is clear, that for Plato, the form of number, in general, exists.

The question, of course, remains as to why he fails to define it and the answer to this question must be somewhat speculative because of the lack of textual evidence. First, there is nothing peculiar in Plato's not defining number, in general, because there are many general kinds that Plato does not define. While he defines a sophist and a statesman in terms of higher kinds, he does not define the higher kinds under which each is subsumed. Of the general kinds, sameness, other, motion, virtue, justice, there are few he defines. Analogously, while he defines different kinds of numbers, he does not define number, in general.

It is possible that Plato considered some of the more general terms undefineable. Moravcsik argues that in the Sophist Plato is telling us that Existence, one of the most general forms, cannot be defined at all, because it "is necessarily all-inclusive" and "one cannot withhold existence from any class of entities".[173] Existence, according to Moravcik, is not a definable attribute, because when it is predicated of an entity it does not separate this entity or any class of such entities from all other entities.[174]

There are some striking similarities between the general term of existence, and that of number. Number, like existence can be predicated of all things, and in this sense might be said to be all-inclusive. Any existent can be said to be made up of any number of parts,

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[173] J.M.E. Moravcsik, "Being and Meaning in the Sophist", Bobbs-Merrill Reprint Series in Philosophy, Reprinted from ACTA Philosophica Fennica, Fasc. XIV, 1962, p. 28.

[174] Ibid., p. 28.

depending upon what one takes as the unit. Like existence, number does not separate entities into those that number can be predicated of, and those that number cannot be predicated of. All entities are numerable and nothing is non-numerable. Thus, while the specific number forms, such as twoness or threeness, separate entities into groups, e.g., a group of three men is separated from all groups with fewer or more than three men, the form of number, in general, does not bring about this kind of separation upon predication.

If Moravcsik's view is correct, then one might want to say that since number is like Existence, one of 'τὰ κοινά', Plato does not define number because these common forms are not definable. The contradictions one encounters in attempting to define any form of great generality, such as number, is well illustrated in Part of the Parmenides.[175] The form of 'ἕν'[176] is found to be a whole and also to have parts (142d8, 145a3), to be a one and to be many (145a3, 143a2, 144e3, 144e6), and to be neither one nor many (155e4). It is, furthermore, both different

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[175]Ryle urges us to take Part II of the Parmenides seriously, and not as a 'jest' as suggested by Burnet and Taylor, in "Plato's Parmenides", Studies, ed. Allen, p. 97. He maintains that Part I of the Parmenides criticizes the relation of the form to its instances and Part II is concerned with the contradictions in the forms themselves: meeting the challenge in Part I where Socrates says you would never be able to show that the form of Like was unlike, or the form of on was many. I agree with Ryle that Part II is a discussion of the form of ἕν. This is substantiated by Socrates when he says he wants to extend his discussion to "those objects which are specially apprehended by discourse and can be regarded as forms."(Parmenides 135e4).

[176]As discussed on page 23, footnote 31, of Chapter II, ἕν may be translated 'unity' or 'one'. Which translation is used is irrelevant to the point that is being made, namely, that a form, no matter which one, is being discussed, as long as it is a general one, and both unity or one are general forms.

from the others and from itself, and the same with the others and itself. (147b11). It is both younger and older than itself and others and neither younger or older than itself or the others (151e4). It is both at rest and in motion (162e2). If one arrives at such contradictions in the search of the nature of ἕν , it is reasonable to suppose that Plato felt that any of ' τὰ κοινά ' mentioned in the <u>Theaetetus</u> and in the <u>Sophist</u> might be susceptible to the ensuing contradictions.

Even though Plato never discussed the general concept of number in any detail anywhere in the dialogues, one can assume that there are certain analogies to be drawn between the general forms that he does discuss and those that he does not discuss. On the basis of these considerations, one must surmise that number, like Existence and ἕν , is not definable.

Perhaps Plato felt that his exhaustive classification of number in terms of 'the odd and the even' might serve as a weak kind of definition. Thus, each number is an individual form and since there are the forms of the odd and the even which classify all numbers, every number shares either in the odd or the even. And though this kind of definition did not match up to what he himself demanded of a definition, perhaps he felt this was the best he could do.

CHAPTER IV

Any discussion of Plato's theory of number would be incomplete without a consideration of what some of Plato's interpreters have attributed to him concerning number. This chapter will deal with what Aristotle called, and what several Platonic scholars have since come to call, the 'intermediates'. Many passages in Aristotle's Metaphysics support the claim that besides sensible particulars and forms, Plato posited a distinct ontological position for numbers, such that they were like forms in some respects and like sensible particulars in others. One such passage is the following:

> Further, besides sensible things and Forms he[177] says there are the objects of mathematics, which occupy an intermediate position, differing from sensible things in being eternal and unchangeable, from Forms in that there are many alike, while the Form itself is in each case unique. (Metaphysics 987b14).

The objects of mathematics, according to Aristotle are like forms in their being eternal and unchangeable and unlike forms because there are many alike, thus not unique.

Elsewhere in the Metaphysics, Aristotle attributes to Plato the view that there are two kinds of numbers, the ideal numbers and the mathematical numbers:

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[177] Plato is specifically named as the referent in the context of the passage, 987a30ff.

> Some (Plato is meant)[178] say both kinds of number exist, that which has a before and after, being identical with the Ideas, and mathematical number being different from the Ideas and from sensible things, and both being separable from sensible things. (Metaphysics 1080b11).

The mathematical numbers in this second Aristotelean passage are the ones that occupy an intermediate position in the first passage. Thus, the mathematical numbers are numbers that are different from forms because there are many alike. Aristotle is to associate mathematical numbers with intermediates:

> Further, they must set up a second kind of number (with which arithmetic deals) and all the objects which are called 'intermediate' by some thinkers; (Metaphysics 991b29).[179]

According to Aristotle, the ideal numbers, which he calls 'ἀσύμβλητοι ἀριθμοί',[180] are counted thus: after 1, a distinct 2 which does not include the first 1, and a 3 which does not include 2, and so on with the rest of the number series. While the mathematical numbers, which he calls 'σύμβλητοι ἀριθμοί' are counted thus: after 1, 2, which

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[178] 'Plato is meant' is a footnote by the translator, Sir David Ross. In his commentary to the Metaphysics, Ross says that "οἱ μέν, who believed in both, means Plato and his orthodox followers." (p. 428). Ross refers us to his comments to 987b14-18, in which he says, "The ground of Plato's belief in mathematical objects as a distinct class of entities is indicated clearly enough in the present passage." Ross goes on to state that although Aristotle attributes this doctrine to Plato, it is not a doctrine that is explicitly found in the dialogues. (Commentary on Metaphysics, p. 167).

[179] Also, the same view is found at Metaphysics 995b16-18.

[180] 'σύμβλητοι' and 'ἀσύμβλητοι' have been most often translated 'comparable' and incomparable'.

consists of another 1 besides the former 1, and 3, which consists of another 1 besides the 2, and the other numbers similarly. (Metaphysics 1080a31). Thus, the mathematical is generated by the addition of one and might be defined as 'a multiple of units'.[181]

Aristotle criticizes the notion of ' ἀσύμβλητοι ἀριθμοί ' or ideal numbers:

> ...if the first 2 is an Idea, these 2's that generate 8, will also be Ideas of some kind ...so an Idea will be composed of Ideas. (Metaphysics 1082a32).

The criticism loses its force when one notes that it is based upon the assumption that numbers are generated, and this view is an Aristotelean one, and not a Platonic one. Numbers, as viewed by Plato, are forms and because they do not consist of units, are inadditive.[182] Twoness is the common characteristic of all pairs, and threeness the common characteristic of all triplets, and threeness cannot be generated from twoness.

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[181] Also, Metaphysics 990a29. It is Aristotle's own view that there are only mathematical numbers. "Number must be counted by addition, e.g., 2 by adding another 1 to the one, 3 by adding another 1 to the two, and 4 similarly." (Metaphysics 1081b12). Also, he says, "We, for our part, suppose that in general 1 and 1, whether the things are equal or unequal is 2...but those that hold these views say that not even two units are 2." (Metaphysics 1082b17).

[182] This view of Plato's theory is shared by R.E. Allen, op. cit., p. 32, Cook Wilson, op. cit., p. 247, and Sir David Ross, op. cit., p. 180. Cherniss, op. cit., says "...two plus three are five, for example, means that any two units plus any three units are five units, but the ideal two or twoness is not two units, but one, the ideal three or threeness is not three units but another one, and these ones are entirely different from one another." pp. 34-5.

That Plato did, in fact, believe in intermediates, as Aristotle claims he did, is not a case that can be convincingly made for several reasons. First, if Plato had posited mathematical numbers, as separately existing entities, it is indeed curious that he didn't say anything about them in the dialogues,[183] nor to anyone else.[184] Aristotle testifies that Plato did not even tell him about them:

> ...how do these exist or from what principles do they proceed? Or why must they be intermediate between the things in this sensible world and the things themselves? (Metaphysics 991b29).

And:

> And those who first posited two kinds of number; that of the Forms and that which is mathematical, neither said nor can say how mathematical number is to exist and of what it is to consist. (Metaphysics 1090b33).

It is clearly evident that Plato viewed numbers as forms, and passage after passage in the dialogues support this claim, as was seen in Chapter II.

Yet, in spite of this overwhelming evidence, some scholars have

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[183] H.F. Cherniss, op. cit., says, "This separate and intermediate existence of mathematicals -- of this there is not a word in the Platonic dialogues." p. 8. Also, P. Shorey, op. cit., says that "the doctrine of numbers as intermediate entities is not to be found in Plato." p. 82.

[184] Aristotle refers to Plato's "unwritten doctrine" (Physics 209b15, De Anima 404b20), but Cherniss, op. cit., argues convincingly that the De Anima passage is directed to Xenocrates, not Plato, and that in the passage in the Physics, Aristotle misquotes the Timaeus, and thus the remark about the 'unwritten lecture' loses its force.

Aristoxenus, in the Preface to the Harmonics refers to Plato's lecture on the Good, but his remarks about the contents of that lecture are extremely vague. There seems to be no unquestionable evidence that Plato had a doctrine other than that presented in the dialogues.

H.F. Cherniss, op. cit., says that Aristotle is inconsistent in his account of Plato's theory. First, he attributes a theory of intermediates to Plato, and then he says, that he does not give us any clues about them. p. 33.

felt the need to reconcile the apparent disparity between the Platonic text and the Aristotelean text and have reconstructed a Platonic theory of number to include the intermediates, or the mathematical number. These reconstructions have been based upon a few isolated passages in the dialogues.

A. Wedberg presents one such reconstruction and argues that Plato believed in both ideal numbers and mathematical numbers. It will be shown in the following pages that the passages in the dialogues that Wedberg leans on for his construction do not lend themselves to the kind of interpretation he suggests. There are no passages in the dialogues that demand mathematical numbers, and, in fact, some of the passages, which according to Wedberg are most highly suggestive of mathematical numbers, are to be seen as further supporting the claim that Plato sees numbers as forms, or as Wedberg calls them, ideal numbers.

According to Wedberg, the mathematical numbers and the ideal numbers are characterized by the following properties:[185]

<u>Mathematical Numbers</u>

1. They are made up of certain ideal units or "1's". The mathematical number N is a set of N such units.
2. Of such ideal units, or 1's, there exists an infinite supply.
3. There is no difference between the ideal units; two such units are completely indistinguishable.
4. An ideal unit does not contain any plurality of parts, or constituents, or characteristics: from whatever point of view we consider such a unit, it is One and One only.

<u>Ideal Numbers</u>

1. They are Ideas, viz. the Ideas of Oneness, Twoness, Threeness, and so on.
2. As Ideas, the Ideal Numbers are simple entities.
3. In particular, they are not sets of units like the Mathematical Numbers.
4. The notions of arithmetic, which are of a set-theoretical kind, are not defined for Ideal Numbers. Thus, the statements of arithmetic are not concerned with them. For Ideal Numbers the relation $<$ is not defined.

[185] A. Wedberg, <u>op. cit</u>., p. 65-7.

Mathematical Numbers (Cont'd.)	Ideal Numbers (Cont'd.)
5. Of each Mathematical Number there are infinitely many copies.	5. However, there is a relation of 'priority' among the Ideal Numbers, by which they are ordered in a series that runs parallel to the series of Mathematical Numbers, ordered according to size.
6. The elementary arithmetical notions are simple set-theoretical notions.	
7. Mathematical Numbers are the numbers studied by arithmetic. It is for them, and only for them, that the concepts of arithmetic are defined.	6. The study of Ideal Numbers belongs to the general theory of Ideas, the Dialectic.

Wedberg says that the Mathematical Numbers are 'intermediates' between the Ideal Numbers and sensible things or collections of sensible things in this world. Plato was convinced, according to Wedberg, that the statements of arithmetic are true, but they are not true of sensible things. Hence, they must be true of something else, and that of which they are true are Mathematical Numbers. The logic of the argument goes this way:

1. Arithmetic is true.

2. The truth of arithmetic presupposes the existence of objects which truly participate in the Ideas of Oneness, Twoness, and so on, i.e., in the Ideal Numbers.

3. In the world of senses, there are no perfect instances of the Ideal Numbers.

∴ 4. Perfect instances of the Ideal Numbers exist somewhere outside the world of the senses. These perfect instances are the Mathematical Numbers.[186]

Wedberg says that "the concept of Mathematical Numbers is apparently assumed by Plato in the *Republic*, the *Philebus*, and in the *Theaetetus*, and in the *Phaedo* numbers conceived as Ideas are contrasted with numbers

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[186] A. Wedberg, *op. cit.*, p. 67.

conceived as Mathematical Numbers."[187]

The _Phaedo_[188] passage is the one in which Socrates states that the addition of 1 plus 1 is not the cause of 2, but that the cause of 2 is its participation in Twoness. (101b-d). Wedberg's claim is that Socrates does not deny that 1 plus 1 equals 2, but only that adding 1 to 1 is not "the real reason"[189] for its being two. Here he distinguishes between a mathematical number, a 2 which is made up of 1 and 1, and an ideal number, twoness, which is the cause of something's being 2. And he says: "The true reason is that this Mathematical Number participates in the Idea of Duality or Twoness."[190]

The _Republic_ passage is the one in which Socrates says that with regard to numbers, "each one is such that each unit is equal to every other without the slightest difference and admitting no division into parts." (_Republic_ 525c-526b).[191] These numbers are those used by the arithmeticians, according to Wedberg, and are the Mathematical Numbers.[192]

The _Philebus_ passage is the one in which Socrates distinguishes between the arithmetic of the ordinary man and that of the philosopher. (_Philebus_ 56d1). The ordinary man operates with unequal units (two cows or two armies) whereas the mathematician operates with equal units. The

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[187] A. Wedberg, _op. cit._, p. 123.

[188] The passages here mentioned, i.e., the _Phaedo_, the _Republic_, the _Philebus_, and the _Theaetetus_ will be discussed in detail following this resume of Wedberg's interpretations of each.

[189] A. Wedberg, _op. cit._, p. 135.

[190] A. Wedberg, _op. cit._, p. 135.

[191] This is Wedberg's translation and it virtually corresponds to my translation in the discussion of this passage.

[192] A. Wedberg, _op. cit._, p. 124.

equal units, Wedberg takes to mean, the ones that go to make up a number. Thus:

> When the pure arithmetician calculates, e.g., the sum of 2 and 3, he forms the sum of a set of two units and another set of three units and counting the units in that sum he finds that they are five. The units occuring in this calculation are the same.[193]

These numbers are the Mathematical Numbers, according to Wedberg, and not the Ideal Numbers.

In the Theaetetus passage (198a-d) Socrates distinguishes numbers which the arithmetician has in his mind, five and seven, and such external objects as possess number, e.g. seven men and five men. Wedberg "safely concludes that he (Socrates) is thinking of the so-called Mathematical Numbers, and not the Ideal Numbers,"[194] because the Ideal Numbers do not figure in arithmetical computations and the Mathematical Numbers do. Further, counting is the same as considering how great a given number is.(Theaetetus 198c5). When we count a set of external objects, we assign 'one' to one object, 'two' to another, and so on. The arithmetician proceeds in the same manner when he calculates how much 5 and 7 is. 5 + 7 is 5 units plus 7 units, and one counts the units to determine how many 5 and 7 make. Since these are sets of ideal units, they are Mathematical Numbers, not Ideal Numbers, according to Wedberg.

These four passages in the Platonic text are the ones that Wedberg uses to support his claim that, for Plato, there are Mathematical Numbers, as well as Ideal Numbers.

What is most surprising about Wedberg's account is that even

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[193]A. Wedberg, op. cit., p. 127.

[194]A. Wedberg, op. cit., p. 130.

as early as the Phaedo, Plato had the distinction between mathematical and ideal numbers well in hand. And yet, Wedberg says of the Divided Line Passage in the Republic, which was written after the Phaedo:

> Plato seems to have been somewhat muddled in his views on the subject matter of mathematics, he was, I think, groping for a mathematical ontology essentially conforming with the Aristotelian scheme.[195]

and that:

> Plato had not quite made up his mind on the question whether or not there exists a class of ideal mathematical objects distinct from the mathematical ideas.[196]

Of the Phaedo passage (101b-d) Wedberg states that the Mathmatical Number, say two, which is made up of 1 + 1, participates in the idea of Duality or Twoness. So 2 is both the sum of 1 + 1 and the result of participation in Twoness. There seems to be nothing wrong with viewing a number from two different points of view. But Wedberg goes on to say that "the Mathematical Number participates in the Idea of Duality".[197] He makes the same claim in step 2 of his argument: "The truth of arithmetic presupposes the existence of objects which truly participate in the Ideas of Oneness, Twoness, and so on, i.e. in the Ideal Numbers."

But those things that participate in the Forms are never absolutely perfect, but striving for perfection (Phaedo 74a-75b). So it seems that the Mathematical Number, since it participates in the Ideal Number, must be imperfect, and striving for perfection. Yet, according

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[195] A. Wedberg, op. cit., p. 14.

[196] A. Wedberg, op. cit., p. 44.

[197] A. Wedberg, op. cit., p. 135.

to Wedberg, the perfection of the Mathematical Number is one its distinguishing properties. If we allow Wedberg's theory, for which we have no text, that something (namely, a Mathematical Number) can participate in something else (an Ideal Number) and still be perfect, then as Aristotle remarked about the theory of many perfect units (Metaphysics 991b25) "other absurdities follow". How will the mathematical, which is perfectly two, be distinguishable from twoness itself? Or one mathematical two from another mathematical two? Or, as Aristotle remarked, one of the units composing these numbers from another unit? Not with respect to twoness, evidently, but in manner of generation; being generated out of units. But if the mathematical is literally generated, then it is not ungenerable being, but belongs to the realm of becoming.

It would appear unusual that Plato would violate his own being-becoming dichotomy by accepting some entities, namely mathematical numbers that are not in the class of ungenerable being, and yet exemplify perfection. Plato's acceptance of mathematical numbers would have challenged the very dichotomy that he establishes in the Republic (479a) and reiterates again in the Philebus (15b, 58a3), in the Sophist (248a14), and in the Timaeus (51e).

Furthermore, it is not clear from the language of the Phaedo passage if Plato does accept mathematical numbers, as ontologically different kinds of numbers. Plato is interested in explaining the true cause of a thing's being, and in this conceptual framework, one can only conclude that adding one to one is not the true cause of two, but participation in Twoness is the true cause. It is not clear that Plato is making any ontological claims about mathematical numbers in this passage in the Phaedo.

The *Republic* passage that Wedberg claims describes a kind of number that fits the characteristics of a Mathematical Number is the one in which Socrates says:

> What numbers are these you are talking about, in which the one (τὸ ἕν) is such as you postulate, each equal to every other (ἴσον τε ἕκαστον) without the slightest difference and admitting no division into parts. (526a2).

Unfortunately, this passage is not as easily translatable as Wedberg would want us to believe. There is an ambiguity that clouds the answer to the question, "What kind of numbers are these?" The problem is to what does 'τὸ ἕν' refer? It is not clear if 'τὸ ἕν' refers to the individual numbers of the series of natural numbers, in which case 'τὸ ἕν' is being used to mean 'a unity', or if it refers to the units that go to make up an individual number, in which case 'τὸ ἕν' is being used to mean the number one.

Plato says of 'τὸ ἕν', that each is equal to every other without the slightest difference and that each admits of no division into parts. But this does not help decide the case. For if 'τὸ ἕν' refers to an individual number, considered as a unity, it strengthens the case that each number is indivisible and like every other number, therefore like a form (Wedberg's Ideal Number). On the other hand, if 'τὸ ἕν' refers to the units that go to make up the individual number, a number is divisible into parts and fits the description of what Wedberg calls a Mathematical Number.

There is textual evidence to support both interpretations. In the *Phaedo* (101b-c), 'ἕν' clearly means the number one. "Whatever is to become one 'ἕν' must participate in unity." This is said in the context of numbers. In the *Laws* (818c4) when the Athenian admonishes those that cannot distinguish "μήτε ἕν, μήτε δύο, μήτε τρία, " 'ἐν' is the number one. In some passages of the Parmenides, 'ἐν ' is used in the sense of a unity and not the number one. At 131c10, Parmenides asks if the single form will actually be divided or if it will be 'ἕν '. The 'ἕν ' is clearly not the number one, but one in the sense of unity, because Parmenides it not talking specifically about numbers, but forms, in general.[198] In the Phaedo (104a-b), Socrates omits one from the series of odd numbers by starting with three, which suggests the stronger claim that ' ἕν' was not considered to be a number, at all, by Plato.[199]

But in the Republic's finger example (524-525c) 'ἐν ' refers to the number one at times, and to unity, at others. The contradictory reports of the senses demand that a standard of some kind be used in judging their correctness. Plato suggests that the study of 'ἕν ' will convert the soul to the contemplation of true being (524e6). Obviously, 'ἕν ' cannot mean the number one. The number one cannot decide for us if an object is light or heavy, or hard. (524a2). Furthermore, Plato asks, "'Αριθμός τε καὶ τὸ ἐν ποτέρων δοκεῖ εἶναι." (524d7). Number and ' ἕν ' are differentiated. These considerations suggest

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[198] G. Ryle, op. cit., argues that in all of Part II, ' ἕν ' means unity. W. Runciman, "Plato's Parmenides", Studies, ed. Allen, says that the only feasable translation of ' ἕν ' in the Parmenides is unity, although Plato did not treat it unambiguously. ' ἕν ' at Republic 524d means both of two things to Plato: oneness, by which is meant the form or idea of the number one, and singleness or unity.

[199] We know that Aristotle did not consider one to be a number. (Physics 207b5, Metaphysics 1088a6). In Euclid's Elements, Book VII, Heath's note to Definition 11 says that a prime number is a number measured by no number but by a unit only. Aristotle, too, says that a prime number is not measured by a number, an unit not being a number. (Anal. Post. II 13, 96a36).

that ' ἕν ' means unity.

On the other hand, the point of the whole passage is that through the study of number, one can be led to the apprehension of the truth. If so, then it would follow that ' ἕν ' is the number one. Socrates says, "If it is true of ' ἕν ', the same holds for all number." (525a6).[200]

Wedberg's claim that ' ἕν ' means the number ones or units that go to make up a number seems to overlook the ambiguous status of ' ἕν ' in the dialogues.

The same kind of ambiguity is present in the Philebus passage, with regard to the Greek word ' μονάς ', but in this passage Wedberg acknowledges the various senses of unity that are possible. The Philebus passage reads:

> The ordinary arithmetician surely operates with unequal units (μονάδας ἀνίσους); his 'two' may be two armies, or two cows or two anythings from the smallest thing in the world to the biggest, while the philosophers will have nothing to do with him, unless he consents to make the unit of each unit (μονάδα μονάδος ἑκάστης) of the countless multitude not differ from one another in the slightest. (56d10).[201]

The unit concept (μονάς) is what is in question here. As in the case of ' ἕν ', ' μονάς ' might refer to an individual number, 2, 3, 4, each as a unity, or it might refer to the specific number one that goes to make up the individual number, e.g., 1 + 1 = 2.

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[200]Cornford translates ' το ἕν ' as unity in the Republic (Cornford translation of The Republic of Plato) and as the one in the Parmenides, in Plato and the Parmenides.

[201]This is my translation. The last part of Wedberg's translation, which is basically the same as mine reads: "...unless he consents to make every single one of his infinitely many units precisely the same as every other."

Unfortunately, Plato uses the Greek word 'μονάς' only twice in all the other dialogues and those two times both appear in the Phaedo. Each time 'μονάς' is used in a different sense. At Phaedo 105c5 Socrates says, "...what must be present in a number to make it odd, I shall say not oddness but unity (μονάς)." 'Μονάς' is the number one that is present in odd numbers to make them odd. At Phaedo 101c7, Socrates says,.."whatever is to become one (ἕν) must participate in unity (μο - νάς)." 'Μονάς' is the unity in which the number one must participate to be one, so it is not the number one, but oneness.

These passages are not instructive or helpful in deciding which way 'μονάδα' is to be translated in the Philebus passage, so the passage will have to stand on its own.

In the first part of the passage, Socrates says that the units (μονάδες) used in popular arithmetic are unequal, such as their two, which may, for instance, be two cows, two armies, etc. There are two senses of inequality that Socrates does not distinguish. a) The one sense of unequal units is one in which the same number two is applied to unequal units, cows, in the one case, armies in the other, in which the 'cows' and the 'armies' represent the unit concepts that are unequal. b) The other sense of unequal units is the sense in which the number two is applied to cows, or the number two is applied to armies, in which case the individual cows in the group of two cows and the individual armies in the group of two armies go to make up the unit concept. Since they are not the same cow or the same army they are unequal in some respects.

Plato does not distinguish between these two senses, but according to Wedberg, it is primarily sense (a) that Socrates is interested in pointing out, and his claim is rightly supported by the phrase that the unequal units may be anything "from the smallest thing in the world to the biggest."

In the philosopher's arithmetic, the passage tells us that the units (μονάδες) are equal, rather than unequal as in the case of popular arithmetic. But there are two senses of equality in the philosopher's arithmetic which correspond to the two senses of inequality in popular arithmetic. Corresponding to the first sense of inequality, a') the unit concept is the individual number of the series of natural numbers, and what the philosopher deals with are ideal units which are always the same and constant. The philosopher does not switch from one unit concept 'cow' to another unlike it, like 'army'. Corresponding to the second sense of inequality, b') the unit concepts are the individual units that go to make up a specific number, and these are equal in contradistinction to the cows which are called two, but are unequal.

It would appear that since Socrates intends inequality in sense a) for the arithmetic of the common man, that he would accordingly retain the corresponding sense of equality a') in the case of the philosopher's arithmetic.

But according to Wedberg, Socrates does not do this. He switches to sense b') of equality, where the unit concept is the many units subsumed under the unit concept.

> When the pure arithmetician calculates e.g. the sum of 2 and 3, he forms the sum of a set of two units and another set of three units and counting the units in the sum he finds that they are 5. The units occurring in this calculation are all the same.[202]

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[202] A. Wedberg, op. cit., p. 127.

Wedberg's rationale for this switch is that Socrates says of the philosopher's units that they do not vary in the slightest from one another. He concludes:

> Together, the passages from the Republic and the Philebus prove beyond doubt that the notion of Mathematical Number as described by Aristotle was familiar to Plato.[203]

Such a strong claim is unwarranted in view of the fact that both ' ἓν ' in the Republic passage, and ' μονάς' in the Philebus passage are used ambiguously, and there is no way to decide in which of two senses these Greek words are used. If Socrates intended in the Philebus passage equality in sense a'), then he is telling us nothing about the units that go to make up a number, but that each and every unit is an ideal unit, and not two cows here and two horses there. The conclusion is that each ideal unit is indistinguishable from the other.

This conclusion is not as surprising as it may seem. Aristotle took Plato's view to be that numbers do not differ and cannot be differentiated in the slightest and believed it to be an absurd consequence:

> In general, to differentiate the units (τὰς μονάδας) in any way is an absurdity and a fiction; and by a fiction I mean a forced statement made to suit a hypothesis. For neither in quantity nor in quality do we see unit differing from unit (διαφέρουσαν μονάδα μονάδος), and number must be either equal or unequal -- so that if one number is neither greater nor less than another, it is equal to it, but things that are equal and in no wise differentiated we take to be the same when we are speaking of numbers. (Metaphysics 1082b1).

The unit concept in Aristotle's quote is the individual number, as in

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[203] A. Wedberg, op. cit., p. 127.

sense a), or he could never conclude that Plato's numbers cannot be differentiated.[204] This is a possible outcome of interpreting the Philebus in a way Wedberg chooses not to interpret it.

The final passage Wedberg uses is the Theaetetus (198a-d) when Socrates argues that false opinion can be a confusion of two distinct objects that are merely thought, and need not be a union of thought and perception. To support this claim Socrates says that when we ask what the sum of the number 5 and the number 7 is, people often mistake the number 11, for the number 12, which are merely thought of. Wedberg says that these numbers in the abstract (which are merely thought of) "are clearly not the Ideal Numbers",[205] since these abstract numbers figure in arithmetical computations, and the Ideal Numbers do not.

What Wedberg seems to have overlooked is that each of these so-called abstract numbers, the 5, the 7, the 11, and the 12, are looked upon by Socrates in this passage as imprints on a block of wax (our minds). He says:

> I don't mean five men and seven men or anything of that sort, but just five and seven themselves, which we describe as records in that waxen block block of ours. (196a1).

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[204] H. Cherniss, op. cit., says that what distinguishes each of the numbers from all the rest is its position in the series and that for Plato numbers are not prior or posterior ontologically, i.e., they cannot be generated from one another by addition. p. 35. C. Wilson, op. cit., says, "The numbers...form a series of different terms, which have a definite order. They are nothing but what the mathematicians call 'the series of natural numbers, where the definite article is right, because there is only one such series consisting of Universals, each of which is unique." p. 253.

[205] A. Wedberg, op. cit., p. 130. The rest of the quote is, "...they figure in arithmetical computations which Ideal Numbers do not. Although Socrates does not go into any details as to the nature of these abstract numbers, we can, I believe, safely conclude that he is thinking of the so-called Mathematical Numbers."

And later:

> Is it not thinking that the twelve itself that is stamped on the waxen block is eleven? (196b5).

There is not a word about computing to get 12, or 11. The 11, he says, is mistaken for the 12, both of which are imprints on that block of wax.

Socrates does not say anything about the nature of these abstract numbers (how and if they are related, or how they become impressed in the wax), as Wedberg readily admits, but viewing them as impressions seems to suggest that each number is a single, unitary entity. If anything can be concluded it is that these are examples of Wedberg's Ideal Numbers, and not the Mathematical Numbers.

Consideration of these four passages, Phaedo 101b-d, Republic 525 - 526b, Philebus 56c-e, Theaetetus 198a-d,[206] has shown that as regards two of them, the Republic and the Philebus, the ambiguous status of 'ἕν ' and 'μονάς', respectively, weakens Wedberg's strong claim that Plato believed in Mathematical Numbers, to a weaker claim, i.e., that it is possible that Plato viewed numbers in these two ways. Of the remaining two passages, the Phaedo and the Theaetetus, the Phaedo says nothing of the ontology of a new kind of number which is additive, though it suggests that the process of adding should not be viewed as a true causal process, and the Theaetetus, perhaps the weakest evidence for Wedberg's view, might support the opposite view that number, for Plato is a unitary, unique entity.

There is one other passage in the dialogues that has suggested to Plato's interpreters that numbers are perfect like forms, yet many, like sensible particulars. This passage appears in the Phaedo 74e2.

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[206] A. Wedberg does not say this is the only evidence for Mathematical Numbers, but the only Platonic textual evidence. His work is heavily supported by Aristotelean text.

Following a brief resume of the Recollection Theory as it was first presented in the <u>Meno</u>, Socrates says that absolute equality is something quite distinct from equal things. In the first place, equal things or objects in this world may appear unequal to some people and equal to others, but this is not true of absolute equality. Socrates goes on to ask the question:

> ...αὐτα τὰ ἴσα ἔστιν ὅτι ἄνισα σοι ἐφάνη,
> ἢ ἡ ἰσότις ἀνισότης; (74)[207]

The answer by Simmias to Socrates' question is no, never. And Socrates concludes that equal things (sticks, stones, etc.) which seem to us to be equal are not equal in the sense of absolute equality, but always fall short of equality. The same point is made again at 74e, 75, 75g: sensible equals, those that we judge to be equal by use of our senses, are striving after absolute equality but fall short of it. "All equal objects of sense are desirous of being like (equality); but are only imperfect copies." (75b6).

With this, Plato establishes two kinds of entities, with respect to equality, those things in this world which are never absolutely equal, and absolute equality itself. The passage in question presents us with this problem, however: How does one translate the first part of the sentence 'αὐτα τὰ ἴσα ' without doing violence to the very dichotomy that is being established? If 'αὐτα τὰ ἴσα ' means 'equals themselves' or 'things that are really equal', as some of the translations

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[207]Using Trendennick's translation: "...have you ever thought that things that were absolutely equal were unequal, or that equality was inequality?" Bluck's translation: "Have the things that are really equal ever seemed to you to unequal? Or has Equality seemed the same as inequality?" Hackforth's translation: "What about equals themselves? Have they ever appeared to you to be unequal, or equality to be inequality?

have suggested, such a violation has occurred, by the admission that there are really equal objects or things in this world. This means that either the translation is wrong, or Plato has in mind some other kind of entities when he says 'αὐτα τὰ ἴσα '.

Hackforth, Bluck, and Ross take the latter alternative. Bluck states, "Plato probably has in mind mathematical 'equals'".[208] And Hackforth says, "...These can only be mathematical objects, for example, two triangles, or (as Burnet suggests) the angles at the base of an isosceles triangle."[209] Both Bluck and Hackforth go on to say in their footnotes, that if Plato ever developed a doctrine of mathematical entities distinct both from forms and sensible objects, it is unlikely that it had as yet been formulated.

Ross[210] says that "perfect particular instances of an Idea are distinguished both from imperfect sensible particulars and from the Idea itself." But at another point[211] Ross says "this passage does not necessarily imply in the existence of perfect equals; Plato may only mean that no pair of things known to be perfectly equal has ever appeared to be unequal." After the first quote, Ross says that this is the earliest hint of a belief in mathematical entities as something intermediate be-

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[208]R.S. Bluck, Plato's Phaedo, Cambridge University Press, 1961, p. 67, footnote 3.

[209]R. Hackforth, Plato's Phaedo, Cambridge University Press, 1955, p. 69, footnote 2.

[210]Sir David Ross, op. cit., p. 22.

[211]Sir David Ross, op. cit., p. 60.

tween Ideas and sensible particulars. After the second quote, Ross says if Plato had already to believe in intermediates, he could hardly have failed to stress their existence.

In spite of Ross' varying reports, one alternative to the problematic passage is to state that Plato, if he has not already come to believe in them, is at least hinting at the possibility of a class of mathematical entities intermediate between sensible particulars and Ideas, that like the Idea, of say, absolute equality, display absolute equality.

A second interpretation is to translate 'αὐτα τὰ ἴσα' in such a way that no doctrine is violated. This is the route that Geach[212] and Vlastos take.[213] 'Αὐτα τὰ ἴσα' is taken to mean the Form of Equality itself. They argue that it was common usage in Plato to use the plural in expressing forms. Plato speaks of <u>the</u> Bed, <u>the</u> Man, not Manhood and Bedness, as we do today. From this 'hypostatising' definite article, as Geach calls it, going to the plural form is a natural step. Plato often uses 'the equals', 'the equal', and 'equality' interchangeably.[214] Vlastos[215] supports this by giving even further examples, 'δίκαια', 'δικαιοσύνη', 'το δίκαιον' in <u>Gorgias</u> (454e-455a).

It is also suggested, according to this view that with regard to the statement in question, that the second part of the statement is merely a restatement of the first part, the 'or' indicating an alter-

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[212] P.T. Geach, "The Third Man Again", <u>Studies</u>, ed., Allen, p. 267-9.

[213] G. Vlastos, "Postscript to the Third Man: A Reply to Mr. Geach", <u>Studies</u>, ed., Allen, p. 289.

[214] P.T. Geach, <u>op. cit</u>., gives the following Platonic sources: <u>Phaedo</u> 74a11-12, c1, c4-5, p. 269.

[215] G. Vlastos, <u>op. cit</u>., p. 270.

native way of saying the same thing the first part of the statement says. Both 'αὐτὰ τὰ ἴσα ' and ' ἰσότης' refer to the form of equality. This interpretation makes the whole passage run smoothly. First Socrates points to the fact that sensible equals are sometimes unequal, then that the form of equality can never exemplify inequality, therefore, there is a distinction between sensible equals and the form of equality.

A third interpretation has been suggested by Brown.[216] According to Brown, one cannot understand what Plato means by this passage about equality unless one is familiar with the mathematical debate that was concurrently taking place. That debate centered around the problematic 'diagonal' and the related problem of squaring the circle. In effect, all these related problems were due to the lack of a definition of equality, such that what we call 'irrational numbers' today could be accounted for without resorting to unit measures, or as Brown puts it, a measure restriction. He argues that the early version of an equality definition is that two things are equal if and only if they are measured the same number of times by the unit, and that is is implied by Definition 20, Book VII of the Elements, generally held to date back to the fifth century Pythagoreans, so that Plato must have been familiar with it. That definition states that numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second, that the third is of the fourth.[217]

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[216]Malcolm Brown, "The Idea of Equality in the Phaedo," Archiv fur Geschichte der Philosophie, 51, Band, 1972.

[217]It might be added that there are in the Elements explicit definitions of equality to be found, but they apply to geometric figures, rather than to numbers. Common Notion 5, Book I, states that things which coincide with one another are equal to one another. In the later propositions of Book I, the definition of equality as congruence is expanded to include figures which are equal in area or in content, but need not be the same form.

The controversy begins when one wants to state an equality in arithmetic terms, such as $1:x = x:2$, where the number of the unit, say 1, cannot, in this case, be commensurate with x. The analogous situation is found on the geometric side. At the moment one wishes to get two figures equal in numerical area, say a square and a circle, one is up against 'non-measurable' numbers. The attempts to broaden the definition of equality were made by Bryson and Theon, with whose methods, Brown argues, Plato was familiar.

Bryson was interested in solving the problem of squaring the circle. His method consisted of constructing a series of polygons inside the circle and another series outside it. As each series of polygons increased in its number of sides, each approached the area of the circle, but the outside series always remained too large, and the inside series always remained too small. At this point Bryson's argument makes an appeal to a new idea of equality. His axiom was to read that those things which are greater and less than the same thing are equal to each other. In effect, equality is defined in terms of inequality.

Theon's method of approximating the value of the square root of 2 was worked out by the Pythagoreans before Plato, according to Heath,[218] and was called by the name of side-diagonal numbers. Each number that was arrived at, by use of this method, was either too small for the value of the square root of 2, or too large, but each calculation brought one within smaller limits to the exact value. What Theon was to state is that this method is an unqualified success for getting the value of the square root of 2, if one also presupposes a new definition

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[218] Euclid's Elements, Book II, Theorem 10.

of equality: namely, "the neither exceeding nor falling short."[219]

The upshot of Brown's discussion is that ' αὐτα τὰ ἴσα', turns out to mean 'things equal by definition.' [220] This is not to say that any two or more things that we decide by hypothesis or agreement to be equal are necessarily so, but rather, that two things are equal if they 'fit' the definition of equality in terms of inequality. The result of accepting Brown's interpretation of equality in terms of inequality is that one avoids any committment to a theory of mathematicals, or any other kinds of entities besides forms and sensible things in this world. 'Things' could mean sticks, stones, or objects in this world. There is no need to manufacture some third entity, other than sensible particulars and forms that will 'fit' the passage. Socrates is, after all, talking about sticks and stones. And by the Bryson definition of equality, two sticks can be equal in length, and if the definition of equality is satisfied, they cannot be unequal. This can become more clear with an example. Two sticks are equal if each, respectively, is greater than x and less than y. Let us suppose that stick A (12") is greater than stick C (10") and less than stick B (14"). Now, let us say that stick X (11") is also greater than stick C and less than stick B. We can, by definition, conclude that stick A (12") is equal to stick x (11").

Now, let us proceed to make stick C longer and stick B shorter. There will come a time in this process, that stick A will equal stick X in length. This will be only theoretically possible, in view of the

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[219] Theon of Smyrna's <u>Expositio</u>, as cited by M. Brown, <u>op. cit</u>.

[220] M. Brown, <u>op. cit</u>., p. 32.

fact that it will take an infinite number of steps to make stick A actually equal in length to stick X. The phrase 'αὐτα τὰ ἴσα ' seems to mean, for Brown, those things that can theoretically be gotten to be equal, but are not ever actually equal.

If it is true that Plato had this kind of notion of equality in mind[221], then the form of equality itself would have to be effected by this definition, which Brown readily admits. And then the idea of equality reduces to the paradoxical notion that equal is defined by unequal.[222] There is nothing wrong with this, in itself. But, if Plato's purpose is to mark a distinction between sensible equals and absolute equality, the distinction is erased by the new definition. Socrates' statements to the effect that sensible equals are 'striving' to be like equality itself, but are imperfect copies of it, become meaningless. According to the Phaedo passage sensible equals are equal in a different sense than absolute equality is equal, and yet, if Plato is writing this passage in accordance with Bryson's definition, no such distinction can be made.

It is likely that Plato knew of Bryson's definition and his acquaintance with Archytas[223] points to the likelihood that he knew of Archytas' interest in squaring a circle. But his acquaintance with him does not preclude omission of the definition from Plato's works.

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[221] There seems to be some indication of this. In one passage of the Parmenides Plato defines equality in terms of equal measures (140B) and in another passage, (161D) in terms of greatness and smallness.

[222] M. Brown, op. cit., p. 33.

[223] Plato's friendship with Archytas is mentioned in the Seventh Letter (338c7, 350a6). Also, the Ninth and Twelfth Letters are addressed to Archytas, and Plato praises him for his mathematical contributions, but these are more probably spurious than the Seventh Letter.

Because Bryson's definition of equality is heavily laden with the notion of process, it is likely that a definition in terms of process would have been antithetical to Plato's whole mode of thinking.

As regards the first alternative, that Hackforth and Bluck suggest, it is indeed curious that Plato should posit the existence of a class of entities that are absolutely equal that are not forms, in the very passage that he wants to distinguish a kind of equality that is true of things in this world such as sticks and stones, from the kind of absolute equality that is true only of forms. To introduce or even suggest, in the same breath, another kind of entity apart from forms that exemplifies perfect equality would have been counter to the logic of the whole passage.

Herein lies the strength of the interpretation offered by Geach and Vlastos. By saying that 'αὐτα τὰ ἴσα ' refers to the form of equality, no violence is done to Plato's dichotomy between equal things and absolute equality itself.[224] In addition, this interpretation is supported on firm grounds by the claim that the use of a plural form does not mean that Plato is thinking of a plural object. To mention still a few more examples where Plato seems to be talking about a form in the plural, in the Gorgias (454a-5a) Plato uses 'περὶ τῶν δικαίων τε ' and 'περὶ τὸ δικαιόν ' in the same paragraph, in Euthyphro 'τὸ ὅσιον ' is used at 6d1 and 'τὰ ὅσια ' at 7d11. In the Republic at 520c5, 'καλῶν τε καὶ δικαίων καὶ ἀγαθῶν πέρι ' and 539c7 'περὶ δικαίων καὶ καλῶν ' are again used as plurals. And in the Phaedo at 104e, three is some-

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[224]This is not to say that this interpretation raises no questions at all. One might ask, does the form of equality consist of two equals? In that case, the unity of the form is destroyed. Geach considers this and answers that there is no contradiction concerning the oneness of the form, as long as it is looked upon as a standard. op. cit., p. 270.

times ' τὰ τρία ' and at other times ' ἡ τριάς ', where the form of three is being discussed. So, to conclude that ' αὐτα τὰ ἴσα ' is the form of equality, is not made impossible by the use of the plural.

The foregoing has been a detailed consideration of the passages that some of Plato's interpreters have found most highly suggestive of a Platonic theory of intermediates.[225] Upon examination of these passages, one can conclude that there are no passages that demand a theory of intermediates. Of these passages there are a few (the *Phaedo* and the *Theaetetus*) that what is demanded is a kind of number that Wedberg calls Ideal, where numbers are viewed as forms.

Even Aristotle speaks against Wedberg in spite of his alleged support for Wedberg's thesis of a theory of intermediates. Aristotle says that if one is to accept Plato's view that mathematical objects are intermediate between forms and sensible objects, then it must be true of all kinds of other objects, as well. (*Metaphysics* 997b15). There will be a sun besides a sun-itself and a sensible sun (*Metaphysics* 1007b17), animals intermediate between animals themselves and perishable animals (*Metaphysics* 997b24), and so on. Similarly for the objects of astronomy, optics, and harmonics. (*Metaphysics* 1077a1-4).[226] Aristotle seems to be saying that the logic of the intermediates theory cannot be reserved for mathematical objects only, and that what Plato says about mathematical objects must be true of all kinds of other objects, as well. The objects of study of the mathematician, the astronomer, the musician, and so on, are all dealt with in the same way. To this extent

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[225] Of course, the theory of intermediates has also been involved in explaining the Divided Line passage in the *Republic*. See Chapter II, B.6, p. 47.

[226] Aristotle states that this conclusion is absurd.

Aristotle is correct: the real objects of knowledge, for Plato, cut across all the sciences and do not separate them.

One might want to argue that the introduction of mathematical numbers in Plato's later works would have been consistent with the difficulties voiced in the Parmenides concerning the divisibility and non-unique characteristics of the forms. Thus, the argument might go, once Plato saw that forms are divisible and non-unique, the mathematical number found its place as an entity, neither as a form, nor as a sensible thing, being divisible and many, and thus functioned as an answer to the objections in the Parmenides (at least with regard to the mathematical forms).

This claim cannot be substantiated, for several reasons. First, the problem of the form's divisibility as presented by Parmenides is not parallel to the divisibility of a number seen as an entity composed of many units. The Parmenides objection has to do with the relationship between the one form and the many instances of it in the sensible world. A form is divided only in the sense that many things partake of it. On the other hand, a number viewed as a mathematical, is made of parts, and thus divisible. The number is not broken up into parts because many things partake of it.

The second reason is that the arguments against the form's uniqueness that are presented in the Parmenides are not resolved by the introduction of intermediates. Again, the Parmenidean objection is concerned with the uniqueness of the form in the way that it relates to a thing that participates in the form. So that another form presents itself besides the original form, in which both the participating thing and the original form partake. This duplicates the forms, ad infinitum. The sense of the non-uniqueness of mathematical numbers is different than the sense in which non-uniqueness appears in the objection. The sense of many in the case of mathematicals is that there are an infinite number of 2's that a mathematician works with, and it has nothing to do with the relationship between 'twoness' and the things that partake of twoness.

Thus, Plato could not have been motivated to accept a theory of mathematicals on the basis of the objections to the forms in the *Parmenides*, because the theory of mathematicals does not answer to these objections.

If Plato had viewed numbers as 'multiples of units', as well as forms, he might have posited a theory of intermediates in order to accommodate this second kind of number, a 'multiple of units' called a 'mathematical'. But Plato never saw numbers as multiples of units. In fact, Plato avoided the on-going definition of number as a multiple of units.[227] Therefore, in the context of the Platonic theory of forms, as presented in the dialogues, a theory of intermediates had no useful job to do. It did not answer the *Parmenidean* objections to the forms, and it did not give a new ontological setting that would have been required if Plato had defined number as divisible, or as a multiple of units.

Finally, it is not always clear that Aristotle is always referring to Plato when he discusses the intermediates. In the passage quoted from the *Metaphysics* (1080b11), 'Plato is meant' is a footnote

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[227] See Chapter III, p. 109.

by the translator and editor, Sir David Ross. Aristotle merely says 'some'. In this same passage, Aristotle says:

1. Some say both kinds of numbers exist. (1080b11).
2. Others say mathematical number alone exists. (1080b15).
3. Another thinker says the first kind of number, that of the Forms alone exists. (1080b23).
4. Some say mathematical number is identical with this. (1080b24).

Aristotle does not specifically state who held each view, and the editor has taken the liberty to say that the first statement refers to Plato, the second statement to Speusippus, the third to some unknown Platonist, and the fourth to Xenocrates. According to Cherniss,[228] Speusippus would be the author of (2), Xenocrates would be the author of (4), and Plato, the author of (3). The disagreement indicates that Ross' editorial note cannot be taken unquestioned.

In the passage quoted at Metaphysics 991b29, 'they' refers to those that say that forms are numbers. This is a doctrine clearly not found in the Platonic dialogues.[229]

Doubtlessly, there were fervent discussions between the members of the Academy concerning the status of numbers and forms[230] and if any theory of intermediates emerged from these discussions, it is

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[228] H. Cherniss, op. cit., p. 33-4.

[229] The doctrine that forms are numbers is often mentioned and criticized by Aristotle. (Metaphysics 991b10, 1043b35, 1073a18, 1083a8, 1084a10, 1091b26). Whether it was a doctrine that Plato adhered to is a point of controversy, though most commentators will agree that the doctrine is not presented to us by Plato in the dialogues.

[230] P. Shorey, op. cit., says, "The problem...of the supposed mathematical numbers, and of ideal numbers offerred a rich feast for the quibblers and the 'ὀψιμαθεῖς' of the Academy". p. 84. H. Cherniss, op. cit., is in basic agreement with Shorey. p. 32.

likely that not Plato, but his disciples , came up with the theory.[231]

The relationship between the one form and the many instances of it in the world was a dilemma inherent in the very theory of forms. Thus, according to Plato's theory, every idea is one, yet reflected in the world of things, it appears as many. Likewise, the form of each individual number is not an aggregate of units, but a perfect and unique whole without parts, yet in the mathematician's imagination the numbers appear as aggregates of units. In the world of things, aggregates of three things, for example, are less perfect than the form of three, in appearing as a multitude of units instead of an indivisible, unique whole.

Apparently, members of the Academy either misunderstood Plato, or disagreed with him and deemed it necessary to give other grounds for the multiplicity of numbers that appear in arithmetic statements such as $2 + 2 = 4$, where '2' appears twice. What they perhaps failed to realize was that in so doing, in positing a theory of intermediates, the problem of the one in the many was further compounded, if taken in the context of the Platonic theory, for then, one would have to explain the relationship between the mathematical number and the ideal number, and distinguish one mathematical number from another of the same kind.

In summary, all considerations point to the view that the theory of intermediates was not a Platonic doctrine for the following reasons: there is no evidence in the dialogues that demands this theory, it does not solve any problems that Plato faced, specifically the objections raised concerning the relationship between the form and its instances, it is not clear that all of Aristotle's statements about

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[231] This view is suggested by C. Wilson, _op. cit_., p. 252, H. Cherniss, _op. cit_., p. 33-35, and P. Shorey, _op. cit_., p. 82.

intermediates refer to Plato, and finally, insertion of the theory of intermediates into the Platonic doctrine of forms would require further clarification as to the relationship between the intermediates and the forms.

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