DETERMINATION OF CAPILLARY CONDUCTIVITY OF UNSATURATED POROUS MEDIA FROM MOISTURE CHARACTERISTICS

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This is to certify that the

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ABSTRACT

DETERMINATION OF CAPILLARY CONDUCTIVITY OF UNSATURATED POROUS MEDIA FROM MOISTURE CHARACTERISTICS

By

Abdul Raziq Qazi

Conductivity function is very essential for prediction of moisture movement in unsaturated porous materials. Some mathematical models have been proposed in previous studies for analytical determination of unsaturated capillary conductivity for porous materials from their moisture characteristics.

Moisture characteristics used in these models were obtained by static method, whereas, in nature the variation of moisture and capillary pressure in soil are gradual and dynamic. Therefore, in this study a dynamic method was developed and used to determine moisture characteristics of three natural soils. Gamma ray attenuation technique has been studied and used to determine the varying moisture content of the soils. The mass attenuation coefficient of water has been found to vary with thickness of water.

The curves of static and dynamic moisture characteristics have been found to differ from each other appreciably.

Capillary conductivities calculated by various models from both dynamic and static moisture characteristic are compared with the experimentally determined capillary conductivities. It is observed that no model can accurately predict the conductivity for adsorption Brooks and Corey's model does not seem to work for fine textured soils and for other soils at higher moisture range without modification. A modification has been suggested. Kunze's equation seems to have the tendency of giving lower conductivity values at low moisture contents. For glass beads of Topp and Miller, it tends to predict higher values of conductivity at all moisture contents using dynamic data. It gives fairly accurate results for static data for the soils studied. Direct numerical integration of the original Burdine equation using dynamic capillary pressure and saturation data gives conductivity values that approximate the experimental results better than other mathematical models.

Approved Approved

DETERMINATION OF CAPILLARY CONDUCTIVITY OF UNSATURATED POROUS MEDIA FROM

MOISTURE CHARACTERISTICS

Ву

Abdul Raziq Qazi

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INTRODUCTION

Need for Study

In order to understand, control and accurately predict the water movement in soils due to drainage, evaporation and infiltration after rainfall or irrigation one must understand the factors which control this movement. In agriculture this is important from the point of view of plant growth. In construction work it is important from the point of view of drainage of highways and seepage through walls, foundations and dams. To the petroleum industry it is important for removing oil from porous materials by displacement with either wetting or non-wetting fluids.

This complex process of water movement in porous materials needs more and more attention in order to increase agricultural production for the growing population of the world. The increasing loss of agricultural land to buildings and roads makes it more crucial to make use of these unused lands. Deserts or marshlands both offer problems of water movement in soil--in one case the supplemental irrigation is needed and in the other case the drainage is important.

Since D'Arcy discovered the relationship, over one hundred years ago, between flow velocity, hydraulic gradient and hydraulic conductivity of porous material, many investigators have been attempting to develop mathematical models describing flows in saturated soils. They attempted first to describe the one dimensional flow and then the two dimensional flow and finally the unsaturated flow with limited boundary conditions. D'Arcy's equation is basically for steady state laminar incompressible flow. Flow of liquids in soils is laminar and incompressible but not always steady. Richards in 1931 suggested a second order non-linear differential equation for general case by combining the D'Arcy law (V=KI) and the continuity equation $\nabla . V = d\theta/dt$.

These kind of equations are converted into a system of linear equations in the finite difference form. Knowing soil moisture characteristics, the conductivity or diffusivity function and the initial and boundary conditions, the solution of these equations through a process of iteration with the help of a computer gives a relationship between water content, tension, space and time. Thus the moisture conditions can be predicted at a given point at a certain time for certain problems.

When the medium is unsaturated and conductivity K becomes a function of moisture content, θ , it is no

longer a constant. The same is true for diffusivity $D(\theta)$ and water capacity $C(\theta)$ which are related as $K(\theta) =$ $C(\theta) \cdot D(\theta)$. Attempts have been made to determine $K(\theta)$ from soil moisture characteristics but soil moisture characteristics exhibit hysteresis when the medium goes through the process of desorption and then adsorption. Thus the conductivity function computed from soil moisture characteristics will be affected by this phenomenon. Moisture characteristics which characterize the pore size distribution of the porous material depend upon the process used to empty or fill the pores. Usually pressure steps are applied successively to drain or fill the pores, achieving equilibrium state after each step at which point the tension or the capillary pressure corresponds to the air pressure. Saturation of the media is accomplished by releasing the pressure in steps. In nature, however, the porous material is not saturated or desaturated by successive pressure steps but is in fact a gradual process which seems to differ from the step method.

Scope of Research

In this study, therefore, a gradual and dynamic method has been developed to vary the capillary pressure and moisture contents continuously, but gradually. The moisture contents are obtained by nondestructive method

of gamma ray attenuation. The unsteady state of flow was maintained and capillary conductivity values corresponding to measured values of moisture contents and capillary pressure were determined. Results of the dynamic data compared with the standard step method data are presented. Two mathematical models for determining the K(θ) from moisture characteristics were investigated to see if the K(θ) values predicted by these models fit the experimental K(θ) values. Effects of the hysteresis on these models was analyzed.

Effect of moisture content variations on the mass absorption coefficient of the water was studied. The exponential law for the attenuation of gamma radiation from C_s^{137} was verified for a fixed geometry. A regression technique to correlate moisture contents with the count rate is developed and analyzed. The results from the study of effects of external hydraulic gradients and air pressure application rates on the wetting phase pressure and capillary pressure are presented. The effects of the head loss in the barriers as related to the suction gradient between two points in the soils are discussed.

REVIEW OF LITERATURE

Use of Functions $K(\theta)$, $D(\theta)$, and $C(\theta)$ in the

Diffusion Equation and its Application

The heat diffusion equation and its application to unsaturated flow problems appear in early literature. The flow parameters--capillary conductivity $K(\theta)$, diffusivity $D(\theta)$, and water capacity $C(\theta)$ have been defined by the following relationships: Volume rate of flux per unit area is

$$-K(\theta) \quad \frac{\partial F}{\partial H} = -D(\theta) \quad \frac{\partial F}{\partial \theta}$$
(1)

where $D(\theta) = K(\theta)/C(\theta)$, $C(\theta) = \partial \theta/\partial H$, H is hydraulic head (cm), L is length of soil column (cm) and θ is volumetric moisture contents of the soil (cc/cc).

By combining the continuity equation

$$\nabla \cdot \mathbf{V} = \frac{\partial \theta}{\partial t}$$

with D'Arcy's equation

$$V = K(\theta)I$$

Richard (34) in 1939 obtained the following second order non-linear differential equation:

$$\nabla \cdot [K(\theta) \nabla H] = \frac{\partial \theta}{\partial t}$$
 (2)

where $(\nabla. V)$ is the divergence of velocity vector V, t is time in seconds and I is the hydraulic gradient (cm/cm). The hydraulic gradient can be expressed as

$$I = \nabla H = \frac{\nabla \phi}{g}$$
 (3)

where ϕ is the total potential (cm^2/sec^2) , and $\phi = \psi + gz$, ψ is the capillary potential (cm^2/sec^2) , gz is the gravitational potential (cm^2/sec^2) , g being the acceleration due to gravity and z the distance in the soil column along the vertical axis (cm). The intrinsic permeability $k(\theta)$ (cm²) is defined as the property of the media for transmitting fluid irrespective of the nature of the fluid passing and is related to capillary hydraulic conductivity $K(\theta)$ as

$$K(\theta) = (\rho g/\mu) k(\theta)$$
(4)

where ρ is the density and μ is the absolute viscosity of the fluid respectively. Substituting the relationships (3) and (4) in equation (2), results in the equation for vertical flow as

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[\frac{\rho}{\mu} \mathbf{k}(\theta) \frac{\partial \psi}{\partial z} + \frac{\rho g}{\mu} \mathbf{k}(\theta) \right].$$
(5)

If the change in the ψ is small and hence the change in moisture content is small, for horizontal flow the above expression reduces to

$$\frac{\partial \theta}{\partial t} = \left[\frac{\rho}{\mu} \mathbf{k} (\theta) \frac{\partial^2 \psi}{\partial \mathbf{x}^2} \right] . \tag{6}$$

Equation (6) implies one dimensional horizontal flow in isotropic material with $k(\theta)$ constant for a small change in moisture content. For vertical flow $H = \phi/g$, for horizontal flow $H = \psi/g$,

$$\frac{\rho \mathbf{g}}{\mu} \mathbf{k}(\theta) \frac{\mathbf{1}}{\mathbf{g}} \frac{\partial \psi}{\partial \mathbf{x}} = \mathbf{D}(\theta) \frac{\partial \theta}{\partial \mathbf{x}}$$
(7)

and

$$\frac{\partial \theta}{\partial t} = D(\theta) \frac{\partial^2 \theta}{\partial x^2}$$
 (8)

which is similar to the diffusion equation for one dimensional heat flow.

In order to predict moisture distribution after infiltration into the soil, Ashcroft <u>et al</u>. (1) developed a technique for solving an implicit difference analogue of the diffusion equation. The diffusivity values were chosen to convert the nonlinear diffusion equation into a system of linear equations which were finally solved by Gaussian elimination technique. The equation used in this method was

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} (D(\theta)) \frac{\partial \theta}{\partial x}$$

which was converted to finite difference form as

$$\frac{\theta_{j}^{n+1} - \theta_{j}^{n}}{\Delta t} =$$

 $\frac{\mathsf{D}(\boldsymbol{\theta}_{j+1/2}^{n+1/2}) \quad (\boldsymbol{\theta}_{j+1}^{n+1} - \boldsymbol{\theta}_{j}^{n+1}) \quad /\Delta \mathbf{x} - \mathsf{D}(\boldsymbol{\theta}_{j-1/2}^{n+1/2}) \quad (\boldsymbol{\theta}_{j}^{n+1} - \boldsymbol{\theta}_{j-1}^{n+1}) / \Delta \mathbf{x}}{\Delta \mathbf{x}}$

where superscripts and subscripts denote time and space respectively. Further, substituting the initial and boundary conditions and approximating

$$\theta_{j\pm 1/2}^{n=1/2}$$
 by $\theta_{j\pm 1/2}^{\star}$ and $D(\theta_{j\pm 1/2}^{n+1/2}$ by $D(\theta_{j\pm 1/2}^{\star}) = D(\theta_{j\pm 1/2}^{n})$

where a is an arbitrarily chosen small constant and $\theta_{j\pm1/2}^{n}$ is an average of two successive moisture contents at x and x + Δx , the following equation results:

$$-r D(\theta_{j=1/2}^{\star}) \theta_{j-1}^{n+1} + (1+r D(\theta_{j-1/2})+rD(\theta_{j+1/2}^{\star})) \theta_{j}^{n+1}$$

$$-r D(\theta_{j+1/2}^{\star}) \theta_{j+1}^{n+1} = \theta_{j}^{n}$$

where $r = (\Delta t)/(Lx)^2$. The above system of linear equations forms a coefficient matrix of the form

$$\begin{bmatrix} b_1 & c_1 \\ a_2 & b_2 & c_2 \\ & - & - & - \\ & & & & a_m & b_m \end{bmatrix}$$

where $a_1 = -rD(\theta_{i+1/2}^{\star})$, $b_1 = 1 + rD(\theta_{i-1/2}) + rD(\theta_{i+1/2}^{\star})$ and $c_i = -rD(\theta_{i+1/2}^{\star})$. The above tri-diagonal system of equations was solved by Gaussian elimination technique for θ_j^n . The solution so obtained was compared by them to the solution obtained by using Boltzmann transformation. They reported that the two solutions gave similar results for infiltration into horizontal columns of semiinfinite uniform media as obtained experimentally.

Hanks and Bowers (16) used similar techniques for flow upwards and downwards in vertical columns. They compared their results with other investigators and claim excellent agreement. However, the results were not verified experimentally. It was pointed out by them that the soil moisture characteristics and a relationship between soil moisture content and diffusivity must be known to use their method.

Whisler and Klute (44) used the equation of the type

$$C(h,z) \quad \frac{\partial h}{\partial t} = \frac{\partial}{\partial z} \quad K(h,z) \quad \frac{\partial h}{\partial z} + \frac{\partial K(h,z)}{\partial z}$$

to develop a numerical solution to determine time and depth distribution of water content and pressure head during the infiltration process in vertical columns when both wetting and drying is taking place in some parts of the soil column. Thus hysteresis effects had to be considered. However, moisture characteristics used by them were arbitrary and C(h,z) values were obtained by differentiating the moisture characteristic curve; $K(\theta)$ values were obtained by use of Millington and Quirk (29) formula and moisture characteristic curves. Whisler and Klute concluded that the position of wetting front was over estimated or under estimated if hysteresis was ignored depending upon whether the drainage or wetting curve was used.

Staple (37) conducted a similar experiment and used a numerical technique to compute infiltration and redistribution of water in vertical soil columns. Hysteresis was incorporated by using conductivity and soil moisture tension data obtained from the desorption curve for the upper portion of the column which was desaturating; at the lower end of the column the adsorption curve filled this need. Staple points out that use of diffusion equation involving D in the finite difference form predicted moisture profile for wetting which agreed better with experimental data than the equation using K. However, when accompanied by the hysteretic effects, the

error in the use of an equation involving K was small for drying part of the profile.

Methods for Determination of $K(\theta)$ and $D(\theta)$ Functions

It has been the goal of physicists, soil physicists and engineers to develop some efficient method for determination of the flow parameters used in unsaturated flow studies. Experimental methods which have been developed to determine these parameters indirectly are less time consuming than direct methods. Some of the indirect methods involve determining the advance of a wetting front in a soil column and the moisture contents at various points along the axis of flow. Other methods involve determination of outflow data or moisture characteristics. Some methods involve both. These methods are briefly discussed.

Transient methods

By using Boltzmann transformation $\lambda = xt^{-1/2}$ Bruce and Klute (4) avoided the assumption of constant K(θ) or D(θ) and equation (8) becomes

$$\frac{\partial \mathbf{f}}{\partial \theta} = \frac{\partial \mathbf{x}}{\partial \theta} \left(D(\theta) \frac{\partial \mathbf{x}}{\partial \theta} \right)$$

with the initial and boundary conditions as

$$\theta = \theta_{i} \quad x > 0 \quad t = 0$$
$$\theta = \theta_{i} \quad x = 0 \quad t \ge 0$$

where θ_i and θ_s are initial and saturation moisture contents of the soil. Solution of this expression yields

$$D(\theta_{\mathbf{x}}) = -\frac{1}{2t} \left(\frac{d\mathbf{x}}{d\theta}\right)_{\theta_{\mathbf{x}}} \qquad \theta_{\mathbf{i}} \quad \mathbf{x}d\theta \qquad (9)$$

which is evaluated from the plot of θ versus x graphically.

Their experiments indicated that $D(\theta)$ increases with moisture contents with a maximum value before saturation.

Gardner (12) used a pressure plate outflow method to determine capillary conductivity $K(\theta)$. He assumed that $K(\theta)$ was constant for small changes in soil moisture tension and hence also for small changes in θ . It was also assumed that θ is a linear function of soil moisture tension for a small increment in soil moisture tension -P i.e.

$$\theta = d + bP \tag{10}$$

Neglecting gravity, the one-dimensional flow equation (2) results in

$$\frac{\partial \theta}{\partial t} = \frac{1}{\rho g} \frac{\partial}{\partial z} (K(\theta) \frac{\partial P}{\partial z})$$

which by equation (10) reduces to

$$\frac{\partial P}{\partial t} = \frac{K}{b \rho g} \frac{\partial^2 P}{\partial z^2}$$
(11)

with boundary conditions, P(0,t) = 0, $(\partial P/\partial z)_{z=L} = 0$, P(2L,t) = 0, $P(z,0) = \Delta P$. Equation (11) was solved by method of separation of variables to yield

$$P(z,t) = \frac{4\Delta P}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin(\frac{n\pi z}{2L}) \exp(-a^2 Dt)$$

where a = $n\pi/2L$. Substituting θ from equation (10) and defining

$$b = \frac{\partial \theta}{\partial P} = \frac{\partial \theta}{\rho g \partial H} \quad \text{then } b = \frac{C}{\rho g} \& K = \rho g b D$$

$$\theta(z,t) = d + \frac{4b\Delta P}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi z}{2L}\right) \exp\left(-a^2 D t\right)$$

total moisture content is obtained by integrating the above expression which yields

W(t) =
$$\int_{0}^{L} A. \theta(z,t) dz = dV \frac{8b\Delta PV}{\pi^{2}} \sum_{n=1}^{\infty} \frac{1}{n^{2}} \exp(-a^{2}Dt)$$
 (12)

where A is cross sectional area and V is sample volume.

The series converge rapidly with time. The initial water content of the soil becomes $W_i = dV + bV\Delta P$, and final moisture content at $t = \infty$ is $W_f = dV$. Total outflow for the process is

$$Q_{O} = (W_{i} - W_{f}) = bV\Delta P$$

$$b = \frac{Q_{O}}{V\Delta P}$$
(13)

thus b can be calculated from experimental data. The equation for cumulative outflow at time t is

$$Q(t) = Q_0 \left(1 - \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp(-a^2 Dt) \right)$$

neglecting all but the first term of the series and taking the logarithm yields

$$\ln(Q_{O}-Q_{t}) = \ln(\frac{8Q_{O}}{\pi^{2}}) - a^{2}Dt.$$

Thus diffusivity D and the capillary conductivity K can be obtained by plotting the experimental data. Gardner (12) reported that the lower boundary condition $P(\theta,t)$ was not always met since the resistance to flow between the sample and the porous plate was not negligible especially at higher moisture contents when large amounts of water were released. A logarithmic relationship between conductivity and tension was suggested by Gardner as

$$Log K = Log (a') - b' Log (P).$$
(14)

Kunze and Kirkham (23) gave a method of determining $D(\theta)$ and $K(\theta)$ from experimental data without complicated computations. The technique takes into account the impedance of the porous plate and contact between the sample and the plate, and uses the initial outflow data for each pressure step thus reducing errors which may arise with time (due to air bubbles diffusing into the plate). The technique makes use of the theoretical curves $(Q/Q_0 \text{ versus } \alpha_1^2 \text{Dt/L}^2)$ plotted by using the equation

$$\frac{Q}{Q_0} = 1 - \sum_{n=1}^{\infty} \frac{2 \exp(-\frac{\alpha_n^2 \text{ Dt}/L^2}{\alpha_n^2 (1 + a + a_n^2 \alpha_n^2)}}{(1 + a + a_n^2 \alpha_n^2)}$$
(15)

where

- Q = volumetric cumulative outflow at time t (Cm³) Q_o = final cumulated outflow at equilibrium (Cm³) α_n = is obtained by solution of equation $a\alpha_n$ = $\cot \alpha_n$ D = diffusivity (Cm²/min) t = time (min)
- L = length of soil sample (Cm)

a = ratio of plate impedance to soil slab impedance. Appropriate values of t when $\alpha_1^2 Dt/L^2 = 1$, give $D = L^2/\alpha_1^2$ t and K = $(D \Delta \theta/P_2 - P_1)$, where change in moisture content corresponds to pressure change of $(P_2 - P_1)$. By these investigations some doubts were cast on the application of diffusion theory, firstly, because piston action was anticipated and remedied by placing a capillary tube in the sample to convey high pressure from upper surface of sample to the porous plate to remove moisture from the lower end first instead of the upper end and, secondly, because at low pressure the same values of K could not be obtained by using millipore filter and ceramic plates in spite of accounting for the impedance of the plates.

Nielsen et al. (30) experimentally investigated the applicability of diffusion equation by using oil or water as fluid entering the horizontal soil columns at different negative pressures. He assumed: (1) that D'Arcy's law is valid, i.e., the flux is proportional to water content gradient or pressure gradient for isothermal condition and (2) that there exists $\lambda(\theta)$ such that $\lambda(\theta) = xt^{-1/2}$, where λ is a single valued function of θ , and x is the distance of the wetting front at time t from the starting end. The results of the above study question the validity of one of the above assumptions because the relation between x and $t^{1/2}$ was curvilinear for negative entry pressures larger than -2mb. Also, the predicted relationship between θ and x was different from the measured values for higher negative pressures. They concluded that values of diffusivity calculated depend upon the boundary conditions at which the water enters the soil column and therefore those values cannot be used for the solution of diffusion equation for other boundary conditions.

Gardner (13) and Doering (10) simplified the calculation of diffusivity by using one large pressure step instead of several small steps thus eliminating the need of the assumption of constant diffusivity by using instantaneous outflow from the pressure cell apparatus. The relationship used by them has the same theoretical foundation, equation (12), and is given as:

$$D = \frac{-4L^2}{\pi^2 (W - W_f)} \quad \frac{dW}{dt}$$
(16)

where dW/dt is instantaneous outflow rate at time t, W is volume water content and W_f is the final equilibrium moisture content. Their results compared well with other methods. They found that the membrane impedance was not a significant factor except at saturation moisture content but boundary impedance had some effect when small pressure steps were used. For the multistep method the diffusivity values for negligible and non-negligible boundary impedance were not the same. Scatter of the points for diffusivity and conductivity versus moisture content was noticed in the case of multiple step method whereas in the one step method a smooth curve was obtained thus pointing out that the flow properties are affected by the applied pressure gradient.

Many methods for determination of capillary diffusivity have been suggested as mentioned earlier but this parameter is still not completely understood. Kunze (21) believed that besides the loss of moisture content there are some other factors such as size and rate of the applied pressure which are time dependent and affect this parameter. He observed that in order to determine moisture status in a draining soil, moisture history and relationships of these flow parameters with soil moisture tension must be known.

Skaggs <u>et al</u>. (35) used the modified form of equation (14) of Gardner to relate conductivity K(h) with tension -h as

$$K(h) = [(h/h_1)^a + b]^{-1}$$
(17)

with boundary and initial conditions

$$h = 0 \quad \theta = \theta_{s} \quad x = 0 \quad t \ge 0$$

$$h = h_{i} \quad \theta = \theta_{i} \quad x \ge 0 \quad t = 0$$

$$h = h_{i} \quad \theta = \theta_{i} \quad x = L \quad t \le t_{e}$$

where a, h_1 , and b are constants to be determined. θ_s is saturated moisture contents, θ_i and h_i are initial moisture contents and tension respectively in the soil column, and t_e is time at which the wetting front arrives at the bottom of the column. The constant b is determined at the end of the experiment when steady flow is reached at which time h is assumed to be zero whereby equation (17) gives $b = 1/K_o$, and a and h_1 are determined by trial and error. Value of K(h) so determined is used to solve the equation

$$\frac{\partial h}{\partial t} C(h) = \frac{\partial}{\partial x} \left[K(h) \frac{\partial h}{\partial x} \right] - \frac{\partial K}{\partial x} . \qquad (18)$$

C(h) in equation (18) is water capacity determined from the moisture characteristics for absorption. The results of equation (18) are compared with the experimental results from flow into the vertical soil column. The difference in the experimental results and that of the equation (18) is reduced by choosing another set of values for h_1 and a, until the difference is minimum. As a first approximation the value of a is taken as a = n, where n is the slope of a plot of log K(h) versus log (-h). The basis for this choice of value of a is the equation

$$K(h) = K_{s} \left(\frac{P_{b}}{-h}\right)^{\eta} \qquad \text{for } -h > P_{b} \qquad (19)$$

given by Brooks and Corey (3) which will be discussed later in this chapter. K_s is saturated conductivity, P_b is bubbling pressure and n is defined as pore size distribution index. According to equation (19) the slope of the plot of log K(h) versus log (-h) is a straight line with slope = n; equation (17) gives a straight line also with slope = a whose first value is thus taken as n.

This method is described by the authors as approximate since it uses equation (17) which is not universal in the sense that it does not hold true for all the soils. Secondly, two separate experiments are needed. The errors from both these experiments may accumulate as compared to some of the methods discribed later which only use the moisture characteristics.

Computational techniques for $K(\theta)$ measurements

Parallel model

Navier Stokes equations are applied to the flow through the unsaturated porous media. The flow is assumed to be laminar and incompressible for Newtonian fluid. The Navier-Stokes equation for creeping fluid is $\nabla P = \mu \nabla^2 \underline{V}$. For flow in a capillary tube the equation gives

$$\overline{u} = \frac{-r}{8\mu} \quad \nabla P \tag{20}$$

and

$$\overline{u} = \frac{-b^2}{12\mu} \quad \nabla P \tag{21}$$

for thin film flow on a flat plate and

$$\overline{u} = \frac{-b^2}{12\mu} \nabla P$$
 (22)

for flow between parallel plates, where \overline{u} is the one dimensional average velocity, r is the radius of the capillary tube, μ is the coefficient of absolute viscosity, ΔP is the drop in piezometric pressure, d is the thickness of the film and b is the distance between two plates.

When the above equations are applied to flow in porous materials they can be written as

$$u = \frac{R^2}{k_f u} \quad \nabla P \tag{23}$$

where R is the hydraulic radius of the pore and is defined as cross sectional area divided by wetted perimeter. If in equations (20), (21), and (22) the lengths r, d, and b are replaced by R, then values of k_f become 2 for equation (20) and 3 for equations (21) and (22). Carman (7), from this observation, concluded that the value of the shape factor k_f must lie between 2 and 3. However, the value of hydraulic radius R would differ considerably for the two cases, hence a mean value of $\overline{R^2}$ was suggested instead of R^2 . Another parameter called tortuosity $T = (Le/L)^2$ was incorporated in the equation (23); Le is the distance of the tortuous path and L is the direct distance. Thus the equation (23) becomes

$$\bar{u} = \frac{\overline{R^2}}{k_f^{\mu}T} \frac{\nabla P}{L}$$

The volume flux is

$$q = \overline{u}\phi S = \frac{\phi S \overline{R^2}}{k_f \mu T} \frac{\nabla P}{L}$$
(24)

where ϕ is the porosity and s is saturation defined by the relation S = θ/ϕ where θ is volumetric moisture content. According to the theories advanced by Purcell (33) and later used by Burdine (6) the mean of R² over the entire range of saturation can, when applicable, be expressed in terms of the capillary rise equation

$$r = \frac{2\gamma \cos}{P_c}$$

where P_{c} is the capillary pressure at saturation S, θ is contact angle, γ is surface tension and r is pore radius. Then R the hydraulic radius is

$$\frac{\text{Area}}{\text{Wetted perimeter}} = \frac{\pi r^2}{2\pi r} = \frac{r}{2}$$

$$R = \frac{\gamma \cos \theta}{P_{c}}, \quad S = \int_{0}^{S} ds$$
$$R^{2}S = (\frac{\gamma \cos \theta}{P_{c}})^{2} \int_{0}^{S} ds$$

then

therefore
$$R^2 = \frac{\gamma^2 \cos^2 \theta}{S} \int_0^S \frac{ds}{P_c^2}$$
 (25)

Equation (24) resembled D'Arcy's Law V = Ki, therefore, the intrinsic permeability is given by

$$k = \frac{\phi SR^{2}(s)}{\mu k_{f}T(s)}$$
(26)

T and R are functions of saturation. Burdine (6) obtained an empirical expression relating tortuosity and saturation

$$\left(\frac{T(1.0)}{T(s)}\right) = \left(\frac{S-S_{r}}{1-S}\right)^{2}$$
(27)

 $T_{1.0}$ is tortuosity at S = 1 and S_r is the residual saturation, i.e., saturation point on the moisture characteristic curve after which no appreciable amounts of liquid will drain. $(L/Le)^2 = 1/T_s$ varies from zero at
$S = S_r$ to 1.0 at S = 1.0, because Le becomes infinitely long at low saturation, S_r . This was verified by Corey (9). Equation (27) gives

$$T(s) = T(1.0) \left(\frac{1-S_r}{S-S_r}\right)^2$$

substituting T(s) and R^2 in equation (26) Burdine (6) obtained the expression for intrinsic permeability as

$$k(s) = \frac{\phi(\frac{S-s_r}{1-s_r})^2}{k_f^T(1.0)} \gamma^2 \cos^2\theta \int_0^s \frac{ds}{P_c^2}$$
(28)

for unsaturated flow, and

$$k = \frac{\phi}{k_{f}T(1.0)} \gamma^{2}\cos^{2}\theta \int_{0}^{1} \frac{ds}{P_{c}^{2}}$$
(29)

for saturated flow in isotropic media. The relationship

$$Se = \frac{S - S_r}{1 - S_r}$$
(30)

is called effective saturation. All terms including tortuosity can be eliminated if relative conductivity k_r is considered as the ratio $k_s/k = k_r$ resulting in

$$k_{r} = (Se)^{2} \int_{0}^{s} \frac{ds}{P_{c}^{2}} / \int_{0}^{1} \frac{ds}{P_{c}^{2}}$$
 (31)

This equation can be integrated graphically or numberically. However, Brook and Corey (3) obtained the following results

Se =
$$\left(\frac{P_b}{P_c}\right)^{\lambda}$$
 for $P_c > P_b$ (32)

where λ is the slope of the plot of log (Pc/pg) versus log (Se) and is termed the pore size distribution index of the medium, and P_b is defined as bubbling pressure of the medium at which the desaturation starts. Substituting equation (32) expressed as P_c = P_b(Se) $\frac{1}{\lambda}$ and the relation

$$ds = (1-S_r)ds_e$$

(obtained from equation (30)) in equation (28) and (29) we get

$$k(s) = \frac{\phi \gamma^2 \cos^2 \theta (Se)^2}{k_f T(1.0)} \quad \frac{(1-S_r)}{P_b} \int_0^{Se} \frac{2/\lambda}{S_e} dS_e$$
$$= \frac{\phi \gamma^2 \cos^2 \theta (1-S_r)}{k_f T(1.0) P_b^2} \quad (\frac{\lambda}{\lambda+2}) (Se) \frac{2+3\lambda}{\lambda} \quad (33)$$

and

$$k = \frac{\phi \gamma^2 \cos^2 \theta}{k_f^T (1.0)} \frac{(1-s_r)}{P_b} \frac{(\lambda)}{\lambda+2}$$
(34)

$$k_r = (s_e) \frac{2+3\lambda}{\lambda}$$
 (35)

or

$$k_{r} = \left(\frac{P_{b}}{P_{c}}\right)^{\eta}$$
(36)

where $n = (2+3\lambda/\lambda)$ is the slope of the plot of log (k_r) versus log $(Pc/\rho g)$. They observed good prediction of permeability by the above relationship except for unconsolidated fine sand, GE NO.13, and Touchet silt, GE NO. 3, and consolidated Berea Sandstone. They used the step method for determination of the soil moisture characteristics for desorption only. For nonconsolidated porous material they had to perform two separate experiments using different samples of the same materials, one to determine the moisture characteristic and the other for conductivity. They used hydrocarbons instead of water to avoid the swelling effects of water on porous media.

Laliberte (25), (26) modified equation (38) by substituting

$$T(1.0) = T(s) \left(\frac{S-S_r}{1-S_r}\right)^2$$

to obtain

$$k(s) = \frac{\phi \gamma^2 \cos^2 \theta}{k_f^T(s)} \quad \frac{(1-S_r)}{p_b} \quad (\frac{\lambda}{\lambda+2}) \quad (Se)^{\frac{\lambda+2}{\lambda}} \quad (37)$$

Where ϕ (1-Sr) may be denoted by ϕ_{e} and equation (37) may be written as

$$k_{(s)} = \frac{\gamma^2 \cos^2 \theta}{k_f T(s)} \qquad \frac{\phi e}{P_b^2} \qquad (\frac{\lambda}{\lambda+2}) \quad (Se)^{\frac{\lambda+2}{\lambda}} \qquad (38)$$

for saturated condition equation (38) becomes

$$k = \frac{\gamma^2 \cos^2 \theta \, \phi e}{k_f T(s) P_b^2} \qquad (\frac{\lambda}{\lambda+2})$$

Laliberte assumed $\cos \theta = 1$ and used values of Carman (7) for $k_f = 2.5$ and T = 2.0 to attain a dimensionless form of equation (38) as

$$\frac{\frac{\phi e \gamma}{k_f P_b^2}}{k_f P_b^2} \quad (\frac{\lambda}{\lambda+2}) = 5.0$$

He used this equation to determine the effects of changes in ϕ_e or bulk density and noted that the decrease in bulk density or increase in ϕ_e was followed by an increase in permeability and decrease in bubbling pressure and the pore size distribution index.

Series parallel model

Child <u>et al</u>. (8), Marshall (28) and Millington (29) have given methods of calculating the capillary conductivity from soil moisture characteristics. Child **and** Collis - George (8) used the equation for intrinsic permeability

$$k = m \sum_{\rho=0}^{R} \sum_{\sigma=0}^{R} \sigma f(\rho) \delta r f(\sigma) \delta r$$
(39)

which is based upon the probability of continuity of **pores.** In this equation $f(\rho)$ for is the cross sectional **area corresponding to the range of pore** ρ to ρ + for and $f(\sigma)$ for is the area corresponding to the range of pore σ to σ + δ r. R is the largest pore size of interest that remains full of water, m is a matching factor obtained by matching calculated and experimental values of the permeability at a certain point. The computations are based on Table 1 of reference (8).

Marshall (28) uses equation of the type

$$k = \frac{\phi^2 n^{-2}}{8} (r_1^2 + 3r_2^2 + 5r_3^2 + \dots + (2n-1)r_n^2)$$
(40)

where r_1 , r_2 --- r_n are radii of n equal classes of porosity of interest in decreasing order and ϕ is the porosity of the porous material. If in equation (40) r = $2\gamma/\rho gh$ is substituted in terms of h, where γ is surface tension and r_1 corresponds to h_1 , $h_1 < h_2$, etc., then equation (40) becomes

$$k = \left(\frac{2\gamma}{\rho g}\right)^{2} \quad \frac{\phi^{2} n^{-2} (h_{1}^{-2} + 3h_{2}^{-2} + 5h_{3}^{-2} + \dots + 2(n-1)h_{n}^{-2}}{(41)}$$

The above equations give intrinsic conductivity $(cm^2,)$ for various suction values. The value of capillary conductivity K(θ) is obtained as K(θ) = $\rho gk/\mu$ (cm/sec). Thus K becomes

$$K = \left(\frac{\rho g}{\mu}\right) \left(\frac{2\gamma}{\rho g}\right)^2 \frac{\phi^2 n^{-2}}{8} \left(h_1^{-2} + 3h_2^{-2} + \dots + 2(n-1)h_n^{-2}\right).$$
(42)

Marshall (28) pointed out that accuracy of these calculations depends upon the accuracy of measuring pore size distribution and will be affected in case of swelling materials.

Millington and Quirk (29) further improved the computational equation by taking $\phi^{4/3}$ instead of ϕ^2 in equation (41) and n as total number of porosity classes.

Jackson <u>et al</u>. (18) compared the results of the above three computational equations with the experimental results for capillary conductivity for graded sand. He concluded that the Child and Collis - George (8) method matched at saturated conductivity but did not predict the general shape of experimental curve. Marshall's (28) method predicted only the saturated conductivity and Millington and Quirk (29) method reasonably predicted the measured values for adsorption as well as desorption when calculated values were multiplied by a matching factor.

Kunze <u>et al</u>. (24) modified the equation (42) and used ϕ (instead of ϕ^2 or $\phi^{4/3}$) as water filled porosity and not the total porosity, and n as the total number of pore classes. This modified form of equation is written as:

$$K(\theta)_{i} = \frac{K_{s}}{K_{sc}} \frac{30 n^{-2} \gamma^{2} \phi}{\rho g \mu} \sum_{j=i}^{n} (2j+1-2i) \frac{1}{h^{2} j}$$
(43)

 $i = 1_{12} - - - n$.

where $K(\theta)_i$ is calculated conductivity for a specified moisture content or pressure class (cm/min), $K_s/K_{sc} = m$ is the matching factor which is the ratio of measured saturated conductivity to calculated saturated conductivity, γ is surface tension of water (dynes/cm), ρ is density of water (gm/cm³), g the acceleration due to gravity (cm/sec²), μ is fluid viscosity (gm/cm sec) and h is the pressure or suction head (cm).

By not raising the ϕ to any power, Kunze observed a better fit between experimental and calculated conductivities at lower moisture contents. He investigated the effects of a number of pore classes n and concluded that larger n reduced calculated conductivity and a larger m was required. But after m > 32 this effect was negligible. He also observed that the accuracy of the conductivity function depended upon the range of the moisture characteristic. A more complete moisture characteristic curve resulted in larger m values; m was smaller for fine textured soils as compared to coarse textured soils. The conductivities obtained from adsorption and desorption did not agree in all cases.

Kunze (22) also suggested a more economical computer method of solving the equations of the type (43), by adding the terms of the summation series backwards from r_n^2 the smallest radius to r_1^2 the largest radius. The calculation of K_n , the largest value of calculated

conductivity, by conventional computer programs requires (n-1) multiplications and additions; for K(n-1), (n-2) multiplications and additions are required, and so on. But if addition is started from the smallest radius going to the next higher radius, only four additions are needed for each K value. Thus the total number of additions and multiplications is reduced from n(n-1) to 4n.

Brutsaert (5) gives a theoretical discussion of various models advanced for permeability calculations. Some of them have been discussed earlier in this chapter. Some of his views are quoted below:

From the point of view of probability theory there is an interesting difference between the parallel and the series parallel model. In the later it is assumed by cutting and random rejoining, that the sizes of the pores in sequence are completely independent of one another. On the other hand, in the simple parallel model it is assumed that the sizes of pores in sequence are completely dependent on one another; as a matter of fact each flow channel is assumed to have a uniform cross section over its whole length.

Therefore the simple parallel model tends to over estimate the flow rate and the concept of tortuosity had to be introduced. In the parallel model the nonuniformity of the pores in the direction of the flow is taken care of by "tortuosity" while in series parallel model it is done by "cutting and rejoining."

Intuitively it would appear that the sizes of the pore sequences are not completely independent either. Therefore, the cutting and rejoining may yield an underestimation of the flow rate. Moreover, it is assumed that there is no bypassing of sequences of several pores, that the smaller pores in the sequence governs the flow rate, and that it remains uniformly narrow over its whole length.

However, there are also several assumptions which tend to yield an overestimate of permeability and which may thus cancel the effect of above. The tubes are assumed perfectly fitting, except in the model of Millington and Quirk, and straight without tortuosity. They are assumed to have a regular or even circular cross section while they are in fact highly irregular. There are also many dead-end pores which do not conduct water even though they are filled. In some cases the permeability is further over predicted because the porous medium has strong secondary structure. Often there is the possibility of non-Newtonian flow especially in small pores when the medium has a high clay content.

Another fact is that, while the size of the large pores in the sequence is assumed to govern the emptying suction, the size of the smallest is assumed to govern the flow rate. As mentioned, this may yield an underestimation of permeability at saturation. But when sequences consisting of large pores are emptied, the result is a large decrease in saturation and a disproportionately small decrease in permeability. The sequence of small pores with small pores have a permeability which is relatively higher, at least when one takes the average pore size of a sequence as reference.

The net effect of all these assumptions seems to be that, as was shown in most experiments, the relative permeability is overestimated at lower moisture contents by models of Childs and Collis-George and Marshall. Because Millington and Quirk assumed that the flow area in the individual pores decreases as the moisture content decreases, their method produces better results.

He further indicated that for most of the models the relationship between the effective saturation Se and relative permeability k_r can be simplified in the form

$$k_r = (Se)^{a+} \frac{c}{B}$$

where the parameters a, c, B vary for different models. For Brooks and Corey (3) model a = 3, c = 2 and B = λ , which is the pore size distribution index. Thus the equation becomes

$$k_r = (S_e)^{3+} \frac{2}{B}$$
 (44)

For Child and Collis-George's model

$$k_r = (s_e)^{2+} \frac{2}{B}$$
 (45)

the exponent is one less than the exponent in Brooks and Corey's equation. For a uniform material $B = \lambda = \infty$, thus Brooks and Corey's model reduces to

$$K_r = (S_e)^3$$

From this and from other models Brutsaert (5) concluded that the exponent depends on the pore size range and tortuosity. He also indicated that the two equations, (44) and (45), give different results because they represent different models.

Gamma Ray Attenuation Technique for Moisture Measurements

The attenuation of gamma rays has proved to be a very useful discovery in the soil moisture studies especially for rapidly varying moisture contents. Its major advantages are that it is nondestructive to the soil system, is very fast as compared to gravimetric methods, is very accurate and has high resolution.

A beam of gamma ray energy is directed towards a **Spot** in the soil column, part of which is attenuated

depending upon the density of the soil-water system. Some of the photons are scattered after collisions and the rest of the photons go through the soil. A detector picks up these photons and after amplification the resulting signal pulses are counted on a scaler.

The source of gamma photons usually used is C_s^{137} , which has a peak value of .661 mev.¹ The source is contained in a lead shield centered against a collimating hole in the shield. The detector consists of a NaI crystal photoelectrically connected to a photomultiplier and a preamplifier. The detector is also collimated and

 $¹_{C_s}$ has an half life of 30 years and emits both gamma and beta radiation. The specific gamma radiation constant in Roentgen/millicuries-hour at 1 cm. is 3.0. Exposure rate mR/hr at 1m=.003. One Roentgen = 2.58 x 10⁻⁴ Coulomhs/K gm of dry air, and is unit of available radiation concentration at certain distance from the source. Rad. is unit of absorbed radiations, i.e. energy absorbed/ gm. of a material. One Rad. is equivalent of 100 ergs absorbed / gm., or 0.01 Jouls per K gm. Rem. (radiation equivalent man), represents the biological effectiveness of different kinds of radiation and includes the quality factor of the radiation. The quality factor for $\mathbf{x}, \beta \leq \gamma$ rays is 1.0 and that of fast neutrons and protons up to 10 mev. and α particles is 10.0. Larger quality factor decreases the biological effectiveness. A millicurie_represents the rate of atom disintegration- $1 \text{ mc}_{i} = 10^7 \text{ x} 3.7/\text{sec.}$ Allowable safe dosage for the human body should not exceed 100 m Rem/week or 5 Rem/ year. For gamma radiation for persons above 18 years 1 Rem is equivalent to 1 Rad. .03 m Rad/hour is well within the A.E.C. background radiation regulation. This amount of radiation is absorbed by a man facing a 100 mc source in a 10 cm. lead shielding and standing at a distance of 10 cm. from the surface of the lead shielding.

shielded to prevent the counting of scattered photons. The scaler is set to measure a certain section of the energy spectrum usually very near the peak energy value (.611 to .711 mev.). The purpose is to discriminate against all lower energy photons outside of desired range which might have lost their energy slightly by collision with H atoms in the soil but were not absorbed completely.

The photomultiplier is energized by a high voltage; this of course varies from detector to detector. The high voltage is supplied to the detector through the scaler.

The equation

$$I = I_{oe}^{-\rho\mu x}$$
(46)

is used to determine the moisture contents. I is the signal pulse count number measured in a certain time. I_0 is the count number if an attenuating medium is absent, ρ is the density of the medium, x the thickness of the medium and μ is called the mass absorption coefficient of the medium.

Equation (46) can be adopted to the geometry and the procedure followed in the experiment to be presented later.

Topp (38), (39) used a somewhat different technique with 200 mc (millicure) source in hysteresis studies of

glass bead media. He used ionization chambers one of which acted as a detector chamber and the other a monitor chamber. Both these chambers converted the energy from the gamma rays into current. The difference of the two currents was amplified and recorded. The absorption by the sample was directly compared with a standard absorber. The correction for any drift was applied to the recorder reading. This standard absorber was frequently used to check the drift and make corrections.

Gardner (14) gives a description of the gamma radiation attenuation method. The relationship

$$\theta = \frac{\ln (\text{Nm/Nd})}{-\mu_w S}$$
(47)

is used to determine moisture contents θ , where N_m are counts in a certain time coming out of the sample at any moisture contents, N_d are counts coming out of a dry sample in the same time, μ_w is mass attenuation coefficient of the water for gamma rays and S is the thickness of the soil column. He also gives general specification for setup and design of the geometry of the system.

Considering a normal distribution for the gamma ray emission, randomness of the emission can be checked by the fact that the area under a normal curve covering One standard deviation on either side of the mean value

is 67.8%. One standard deviation as estimated by Gardner (14) is approximately equal to square root of the counts.

$$\sigma_{I_{m}} = \sqrt{I_{m}}$$
(48)

therefore the value of the counts measured in a certain time should be within $\pm \sqrt{I_m}$ of the mean value 68% of the time or roughly 7 out of 10 readings on the scaler should be $I_a \pm \sqrt{I_m}$ where I_a is mean value. Gardner (14) also gives a rough estimate of σ_{θ} , the standard deviation of moisture contents measured by this method as

$$\sigma = \frac{1}{\mu_{w} S / I_{m}}$$
(49)

which can be reduced by increasing either I_m or counting time.

Nutter (31) and Smith (36) used similar methods for moisture measurements. Smith mentions some investigators who achieved a precision of 0.006 gm. per cu. cm. for water contents from 0.05 to 0.40 gm. per cc. at dry density of 1.3 gm. per cc.

Ligon (27) applied the same technique for field measurements of moisture contents variation with percipitation and drainage. He reports some drift in the system indicated by the check on standard Mg and plexiglass absorbers. These checks were made before and after series of counts were taken. A mass absorption coefficient of $\mu = 0.0775 \text{ cm}^2/\text{gm}$ was used for both soil and water; individual values of μ_w were not determined. The equation

$$\rho = 103.81 - 32.21 \frac{\ln \frac{Isoil}{Img}}{\ln \frac{I}{Img}}$$
(50)

for soil density measurement was used which required three readings, one for soil and two for standard absorbers. If the two standard absorber readings, taken before and after the soil reading at a particular depth, differed from each other by more than 5%, the scaler was recalibrated and the readings were taken again. The change in the moisture contents of the soil was calculated by the equation

$$w = .4734 \begin{bmatrix} \frac{\ln \frac{I_{soil}}{Img}}{\frac{I_{pl}}{I_{mg}}} \end{bmatrix}_{1} = \begin{bmatrix} \ln \frac{I_{soil}}{I_{mg}} \\ \frac{I_{pl}}{I_{mg}} \\ \ln \frac{I_{pl}}{I_{mg}} \end{bmatrix}_{1}$$
(51)

Klock (19) used the gamma ray attenuation method for moisture determinations in his conductivity studies. The theory and general technique was the same as used by other investigators except that he used 229 mc. of Am²⁴¹ for the reason that 60% of its radiation has 0.061 mev. energy which required less shielding and less soil thickness for proper attenuation. Holland (17) investigated the possibilities of constructing calibration curves for moisture contents and radiation counts by use of regression technique. A beta guage was used for moisture measurements in leaves and a neutron guage was used in the soil moisture measurements. It was found that for fitting the regression equation

$$y = a + bx$$

by least squares method to the beta gauge data the dependent variate y (radiation counts) needed logrithmic transformation to fit the experimental data while neutron gauge data fitted better for a limited moisture range in soil with no transformations required.

Hydraulic Gradient and Its Application to D'Arcy's Equation in Unsaturated Flow

Vachaud (41) and Olsen <u>et al</u>. (32) studied the relationship between the flow velocities and suction gradients. Even though the techniques followed by both of these investigators are quite different from this study, their results are interesting to note. Vachaud used a transient flow technique. Initially dry soil was wetted in a horizontal position, the moisture contents at various points of the column were determined by the gamma ray attenuation technique. The distance and the time of advance of the wetting front were noted. However, the moisture contents versus suction relationship for wetting were determined in a separate experiment on the same soil by a suction plate method (step procedure). Moisture content θ versus time t relationship was plotted for each distance x, thus obtaining a family of curves. Equation of continuity for horizontal direction is

$$\frac{\partial \mathbf{q}}{\partial \mathbf{x}} = \frac{-\partial \theta}{\partial t}$$
(52)

and for a fixed moisture content θ_n , $d\theta_n = 0$, i.e.

$$d\theta_n = \frac{\partial \theta n}{\partial x} dt + \frac{\partial \theta n}{\partial t} dt = 0.$$
 (53)

Here q is the flow velocity; other variables are the same as used earlier. Combining equations (52) and (53) gives

$$\frac{\partial \mathbf{q}}{\partial \mathbf{x}} = \frac{\partial \theta}{\partial \mathbf{x}} \cdot \frac{\mathbf{d} \mathbf{x}}{\mathbf{d} \mathbf{t}}$$
(54)

where dx/dt = V is defined as the advancing velocity of a water content θ_n and is given by the slope of the relationship $x = f(\theta_n, t)$.

By integrating (54) one gets

$$q_{k+1} - q_k = \Delta V \left(\theta_{k+1} - \theta_k\right)$$
(55)

where $q_{k=1}$ and q_k correspond to the water contents θ_{k+1} and θ_k . ΔV is the small change assumed in V. If θ_i is the initial water contents of the soil and θ_0 the final, then θ_0 is calculated by the total intake of water by the soil volume and θ_i is determined before starting the flow. At t = 0, $q_i = 0$; q_0 corresponds to θ_0 and is determined at the end of the experiment by achieving a steady saturated flow.

At a fixed point x, an increment $\Delta \theta$ was assumed and corresponding Δt and Δx were calculated from the θ versus t curves with x as parameter (figure 4, reference (41).) Thus Δq was calculated from equation (55). The data of the family of curves (figure 4 of reference) of θ versus t with x as parameter was replotted as θ versus x with time as parameter (figure 5 of reference). Corresponding values of suctions obtained from a separate experiment were inserted along the θ axis. Suction gradients were thus obtained from this family of curves for each t_n at different values of θ . Lastly the suction gradients were plotted against the flow velocity for each value of θ (θ = 40% to 25%). At each θ the relationship was indicated as a straight line proving that for K = constant, V I and hence D'Arcy's law is valid for unsaturated flow.

Olson <u>et al</u>. (32) carried similar studies for steady state flow conditions for desaturation only. They also give a graphical solution for determining

flow velocity versus suction relationship. Their results are very similar to the transient flow results discussed above. However, no moisture characteristics are reported in this report.

Both the above mentioned studies, however, indicate that with decreasing moisture contents and flow velocity, the suction gradient increases but for fixed (θ) or (-h) V is directly proportional to hydraulic gradient.

Measurements of Moisture Characteristics and Conductivity

As mentioned earlier, most methods employed in the measurement of moisture characteristic use pressure steps. Topp (39) in his hysteresis study of glass beads media designed two water pressure regulators to keep the water pressure atmospheric at the level of these regulators. Since no air pressure was used, the level of the above regulators was changed to change the capillary pressure and moisture contents of the soil samples. The whole procedure and equipment was quite elaborate and several trial runs were necessary to acquaint himself with some of the unpredictable occurrences. Capillary pressure and pressure gradients were directly recorded by two transducers and flow measurement was measured by measuring the pressure drop in a capillary tube for both inflow and outflow and then applying Poiseuille's law to the average drop. In fact, the average drop was calibrated for flow velocity and recorded directly.

Moisture contents were measured by gamma radiation method as mentioned earlier. Thus θ , V, I, P_c and K were computed. His results and the results of other investigators have shown hysteresis in the θ versus P_c and P_c versus K relationship but only a small hysteresis was encountered in θ versus K relationship. The independent domain theory when applied did not predict the desirable results.

Topp et al. (39) investigated the difference in shapes of moisture characteristics of a sand for desaturation by several methods of obtaining the desaturation curve. The results reported show that the curve obtained by applying static pressure steps and achieving equilibrium after every stop, crosses the curves obtained by applying small pressure steps (of the order of a cm) while not achieving any equilibrium (semi dynamic method). These curves differed from each other considerably. The slope of static equilibrium method curve became steeper than the curve of the other method at tension of -30 cm. It flattened at about -50 cm. of tension and appeared to cross the other curve again at tension of -57 cm. This indicated that the desaturation curves of semidynamic method seem to retain more moisture initially

than the curves of the static equilibrium method. Finally, however, the dynamic curve becomes steeper and tends to join the static equilibrium curve. Topp <u>et al</u>. also reported the results of the rate of the pressure application on the shape of the curves. Three rates were compared, two of the curves were similar but the third was slightly different.

Another curve was obtained by achieving steady state of flow in the soil after each large pressure step. This curve lies near the static equilibrium curve. Apparently large steps in both cases gave similar shape of curve.

EXPERIMENTAL DESIGN AND PROCEDURE

Materials, Equipment and Methods

Materials

Three types of soils were used. The grain size distribution of these soils is given in Figure (1). Soil one is medium sand (Sphinx series), soil two is a fine sand (Ottawa series) and soil three is a sandy loam (Hillsdale series). The moisture characteristics of these materials as obtained by the static equilibrium method (pressure steps) and dynamic method (gradual pressure application and release) are given in Figures (17), (21) and (22).

The aluminum pressure cell (H), Figure (2b) used in the experimental work had an inside diameter of 12.8 cm. and a height of 11.65 cm. with an air inlet (E) and two tensiometers (C) and (D) set 4 cm. apart.

The cell had two end sections (G); each section Consists of a NRWP millpore 1 micron filter (K)¹

¹Millpore filter NRWP 142 is made of nylon, has a pore size of 1 micron, porosity of 63%, suggested bubbling pressure of 12 psi., thickness of 150 + 10 microns. Filters were obtained from Millpore Corporation, Bedford, Massachusetts.



Figure 2. (a) Cross sectional view of the tensiometer assembly.

- (b) Cross sectional view of one end section and one tensiometer assembled in the pressure cell.
- A,B Outlets for end sections
- C,D Tensiometers' outlets
- BR Top and bottom ring of tensiometer
- E Air pressure inlet
- F Top ring of end section
- G End sections
- G₁ Bottom plate of end section
- GS Rubber gaskets
- H Aluminum cylinder
- J Bolts
- K Millipore filter
- L Porous plate
- M Space to receive water
- N,O Screws
- P O-ring seals
- Q Porcelain ring
- R Center of soil solumn
- T₁,T₂ Tensiometers





supported on a very porous polyethylene disc (pore size 120 micron) 0.5 cm. thick.¹ This combination of filter and porous disc rests against a collar in a corrosion resistant brass bottom plate (G1). In the back of the porous plate is a small space (M) with two outlets (A,A). The plate and filter combination are secured between the top brass ring (F) and the bottom brass plate (G) by six metal screws (N). A rubber gasket (GS) serves as an air seal. The two end sections (G) of the cell can be easily assembled together by four bolts and wing nuts (J). The saturated conductivity of the filter and porous disc combination varies between 0.01 to 0.04 cm./min. The air entry value of this combination was ll cm. of Hg. It was observed that the saturated conductivity of porous discfilter combination decreased with time as fine soil particles settled on the surface of the filter. However, this combination proved much better than a plain porcelain plate of the same air entry value, but a much lower conductivity, or a porvic filter and porcelain combination that were used in earlier trial runs.

Details of tensiometers (Tl and T2) are shown in Figures 2a and b. The brass holder, which holds the

¹Polyethylene porous discs, Cat. No. F-1255, 120 mm. diameter, 5/10" thickness, pore size of 120 microns, were ground to smooth surface with 0.5 cm. thickness. These were obtained from Bel Art Products, Pequannock, New Jersey.

porcelain ring (Q), $\begin{pmatrix} 1 \\ \end{pmatrix}$ has an upper (BR_1) and a lower part (BR_2) . O-ring (P) is wedged between the porcelain ring and BR_1 . BR_2 pushes against two O-rings, one in contact with the porcelain ring and the other in contact with the lower part. The brass holder with porcelain and O-rings in place is held together with six screws. Water moving through the porcelain ring may be withdrawn from the tensiometers through copper tubing (C and D). The tubing (C and D) goes out of the aluminum cylinder wall and is fastened to the wall with a nut and a rubber gasket.

Equipment

The experimental equipment is shown in the schematic diagram Figure (3). The tensiometer outlet tubes are connected to a transducer (T) through a 3-way valve (U). The transducer used is a strain gage type Dynisco Model PT 14-01 with a maximum range of 15 psi. The liquid pressure and electrical connections of the transducer are shown in Figures (3) and (4). Four 1.5v. long life telephone batteries (Y) in series provide a six volt excitation voltage to the bridge of the strain gage.

¹The porcelain rings were cut from porcelain plates five inches in diameter having air entry value (factory suggested) of 3.5-4.6 psi. Actual tested value was 8-9 cm. of Hg, pore diameter of 9.2-12 microns, porosity of 39.4% and flow rate for water of 3-5 cc./sec/in² under 20psi. head. These plates were obtained from Coors Porcelain Co., 600 9th St., Golden, Colo.

Figure	3. (a) Schematic diagram of the equipment.(b) Side view of the pressure cell.
A ₁	Inflow Mariotte burrete
^B 1	Outflow Mariotte burrete
C,D,	Tensiometer connections to 3-way valve
Е	Air pressure inlet
I	Constant temperature and humidity chamber
S	Jacks
т	Pressure transducer
U	3-way valve
v	Detector
W	Recorder
x	Scaler/analyzer
Y	Batteries
Z	Lead shielding
l	Source holder
2	Source
3	Pressure regulator
4	Air inlet
5	Multispeed transmission
6	Driving motor
7	Manometer
8	Collimation plug







The strain gage is attached to the dry side of the transducer diaphram, i.e., the side which is open to atmospheric pressure. Full range sensitivity of the transducer is .918 uv/v. Recorder (W) used was Sargent recorder model S-72150. It is a versatile recorder as it has many ranges (1.25 millivolts or microamperes to 2500 volts or milliamperes.) Accuracy is .1% or 20 μ v. whichever is smaller. Chart speed can be varied from 1/3 inch to 12 inches/min. Pen speed is 1.8 seconds full scale. D.C. power is supplied by duracell mercury batteries supported by two dry cells placed inside the recorder. The useful features of this recorder are: (1) easy standardization at any time, (2) quick zero setting at any point of the chart by a displacement knob and (3) provision for rolling the chart in a backward direction for guick scanning of the data.

Calibration and response of tensiometer-transducer and recorder system.--This system had to be calibrated quite often by directly connecting a column of water to the transducer and raising or lowering it and observing the change in signals on the recorder. It was seen that although the results were exactly alike when positive or negative pressure was applied, yet after some time because of the weakening of batteries, the signal voltage decreased; therefore, the variation had to be checked occasionally and the system recalibrated. The variation was 7 mv/10 cm.

of water pressure for new batteries to 6.2 mv/10 cm. of water pressure when batteries were replaced after approximately six months.

The time constant and response of the system was determined by applying a sudden air pressure step to the pressure cell with saturated soil inside while inflow and outflow were closed. A pressure of 29.3 cm. of water was applied and was recorded as 27.5 cm. of water on the chart in 1.2 minutes giving a time constant of 0.43 minutes.¹ Experimentally t_c corresponding to $\Delta P = 63.2$ % of P was .4 minute.

The volume of water required for a displacement of the transducer diaphram was measured as lcc/250 cm. of water pressure.

Moisture content determination equipment.--One hundred mc. of C_s^{137} was used as gamma ray source. It was shielded in a cylindrical lead bucket 8 inches in diameter and 8 inches high. It was designed according to the equation

$$D(R,t) = 2.134 B(\mu, t) \frac{\mu a}{\rho} E_0 \frac{A \exp(-\mu t)}{4\pi R^2}$$

¹This follows the equation $\Delta P/P = (1 - e\frac{-t}{t_c})$ where ΔP is the recorded change in pressure in time t, t_c tc is time constant and P is the applied pressure step. If t = tc is put in the above equation the following equation results: $\Delta p = P(1 - \frac{1}{e}) = .632 P$. Similarly for t equal to 2, 3, and 4 time constants the recorded pressure will be 86.5%, 95% and 98.2% of the applied pressure respectively. However, the response becomes much slower at lower moisture contents.

given by Gardner (14).¹ In this equation D(R,t) is dose rate in millirad per hour at a distance R in cm. from the source of strength A in mc. and energy E_0 in mev; t is the thickness of shielding in cm. and μ the attenuation coefficient in cm⁻¹. B(μ ,t) is buildup factor; μ_a is energy absorption mass-attenuation coefficient for the source energy and the material which is to absorb the dosage; and ρ is density of the shielding material.

Source (No. 2 in Figure 3) is held by a bolt (No. 1 in Figure 3) screwed in a sleeve in the shielding. Thus it can be raised or lowered for minor adjustments of the source against the collimation plug (No. 8) which is 4 1/2 inches long with a 6 mm. circular hole drilled through it. The plug was first molded then machined. This size of collimation gave better results than other collimation sizes. The collimated gamma rays were directed at a spot (R) on the pressure cell in between the tensiometers. The detector (V) was shielded against scattered radiation by a 4 1/2-inch thickness of lead in all directions. A 3/16-inch hole was drilled in the lead for collimation in front of the detector and the two

¹For 0.661 mev. gamma radiation from C_s^{137} , μ_a/ρ as measured in tissues is 0.317 cm²/gm., μ is 1.134 cm⁻¹, B(μ ,t) for lead thickness of 3 to 20 cm. is 1.2 + 0.13t. When A is 100 and R = 10, D(R,t) = 0.1 mrad. at the surface of the lead and 0.03 mrad at 10 cm. from the surface of the lead. Four inches radius and 8 inches height for the bucket was thus a safe value.

collimation holes were aligned so that a maximum count was obtained. This was achieved by rotating (Z), lowering or raising jack (S) and bolt (No. 1).

The detector used was a modified Nuclear Chicago model DS 100 with a 1 1/2 inch NaI crystal connected to a photomultiplier and preamplifier. The detector was connected to a Nuclear Chicago model 8725 scaler-analyser.

The scaler was calibrated to represent one mev. for full scale of 10 volts baseline. The base was set at 0.611 mev. A narrow differential counting mode was used in which case the full scale setting of 10 on the window scans only 10% of the base. Since the energy scale was calibrated to represent 1 mev. for full scale baseline of 10 volts, the window width corresponded to 0.1 mev. Thus, a window setting of 10 scanned the energy spectrum from .611 to .711 mev. and rejected all other energies. The high voltage required to excite the scintillation NaI crystal was of the order of 910 volts. This voltage had to be readjusted occasionally in order to maintain a peak value of counts. The readjustment was necessitated by the drift in the baseline and/or amplifier or window. Need of this readjustment was determined by obtaining counts at several voltages higher and lower than the voltage in use previously. If the counts thus obtained were higher at a different voltage, this voltage was used in subsequent moisture measurements.

In order to account for the randomness of the gamma ray emission, ten one-minute counts were taken for slow runs of conductivity measurements and 3 to 4 oneminute counts were taken for fast runs of moisture characteristic determinations. Maximum c/m (counts per minute) for the fixed geometry were 351,000. The fixed geometry in this case was 4 1/2-inch thickness of lead shielding around the source with a 6-mm. collimation hole followed by a 6-inch air space and then another 4 1/2-inch lead shield for the detector with a 3/16-inch collimation hole. When the cell replaced the air space the c/m were of the order of 60,000/minute. However, the last digit is dropped in calculations as it varied considerably for constant moisture content. This does not affect the accuracy of results appreciably. The values of θ are calculated from the equation (64), Chapter IV:

$$\theta_{\rm m} = \theta_{\rm s} - \frac{1}{\mu_{\rm w} s} \ln \left(\frac{1}{{\rm I}_{\rm s}}\right)$$

where $\theta_{\rm m}$ is moisture content corresponding to $I_{\rm m}$ and $\theta_{\rm s}$ is saturated moisture content corresponding to $I_{\rm s}$, $\mu_{\rm w}$ and S are mass attenuation coefficient of water and thickness of soil, respectively. This equation involves the ratio $(I_{\rm m}/I_{\rm s})$ which will give correct results up to 4 decimal places even if the last digit in c/m is dropped. Plotting the graphs for more than 3 decimal places for θ is not very practical anyway. Theoretical standard deviation for counts of the order of 59,000 to 60,000 at saturation is of the order of ± 243 ($\sigma_{\theta} = \sqrt{I}_s$). Actual variation was well within this range. Theoretical standard deviation in moisture content at saturation is

$$\sigma_{\theta} = \frac{1}{\mu_{w} S / I_{s}}$$

or

$$\sigma_{\theta} = \frac{1}{.078 \times 12.8 \times 244} = \pm .00444$$

or \pm 0.44% which is quite reasonable.

Determination of mass absorption coefficient μ_W .--In literature different values of μ_W are reported; it is therefore necessary to determine the values of μ_W for the fixed geometry to be used. Four cubical cells of dimensions 2.85 x 4.80 x 5 cm. each were constructed from a 1/8 inch thick plexiglass plate. The cells were placed in between the two collimators at fixed positions. Seven one-minute counts were taken with the empty cells in the normal position. Then the first cell was filled with water and seven more one-minute counts were taken. Similarly, counts per minute for the remaining cells were taken after filling each. Using equation

$$\mu_{w} = \frac{1}{s\rho_{w}} \ln \frac{I_{o}}{I_{1}}$$
the mass attenuation coefficient of water was determined. In this equation s is the dimension 2.85 cm. through which the gamma rays pass. I_o and I_1 are the average of seven one-minute counts taken before and after filling a cell. The effect of the order in which the cells were filled did not seem to effect the μ_w value. But higher values of μ_w resulted when the number of cells filled was increased, (μ_w varied from 0.072 to .075), thus indicating that μ_w increases with thickness of water. The value of μ_w to be used in equation (56) was therefore obtained by using the aluminum cell (H, Figure 3), and taking counts before and after filling the cell with water. Here s is 12.8 cm.; μ_w obtained was 0.078 cm.²/gm.

The counter of the scaler/analyser is insensitive during the short interval in which the pulse builds up and this time is called dead time. A correction may be applied to obtain the true counts by equation

$$I_{m} = \frac{I_{mo}t}{(t-I_{mo}d_{t})}$$
(57)

where d_t is the dead time per pulse, I_{mo} are the observed counts in time t and I_m is the corrected value. The analyser used had a 1.5 μ sec. pulse pair resultation time and total dead time for minimum moisture contents (maximum c/m)

never exceeded I dt = 0.111 sec. Therefore, corrections were not necessary.

Methods

The pressure cell was packed with dry soil in layers of one inch. It was then tapped to settle the soil. When the cell became full it was wetted by imbibition with deaerated water containing .1% phenol for several hours after which some additional soil was needed to fill the cell. The soil level in the cell was kept slightly above the rim of the cell before assembling the three parts of the cell together to preclude the possibility of further settlement or consolidation of the soil later. After clamping all outlets, counts were taken at different levels of the cell to check packing irregularities. No appreciable difference was observed in count rates. Both end sections were flushed and then positive water pressure was applied to the lower end of the pressure cell. Outflows from the end sections were shut off while maintaining positive water pressure at the lower end section. When water started flowing out of the tensiometers, the cell was laid on its side with tensiometer outlets pointing upwards, then the tensiometers were connected to a suction device to remove air. The tensiometers were clamped and the cell was connected as shown in Figure (3).

All other connections were made as shown in Figure (3). The equipment enclosed in the rectangle (I) was set inside a constant humidity and temperature chamber¹ and kept at a dry bulb temperature of 72°F. and wet bulb temperature of 66°F. resulting in a relative humidity of 77 per cent.

Initially saturated conductivities were determined in a stainless steel permeameter. A sample from this permeameter was used to determine the saturated moisture content by oven drying. Saturated moisture content of the same soil used in the pressure cell were also determined at the end of the series of experiments by taking samples from the middle of the cylinder and oven drying them. This moisture content was lower than the saturated moisture content obtained from the permeameter sample. However, the saturated moisture content θ_s obtained from the actual pressure cell sample was used in equation (56) to determine θ_m , since I_s corresponding to θ_s was available. Saturated conductivities obtained in the pressure cell were also lower than those obtained by permeameter.

<u>Moisture characteristics determination</u>.--At first a water column was connected to one of the two outlets (C)

¹Aminco Air, Cat. No. 4-5478D and 4-5479D. Supplied by American Instrument Co., Inc., 8030 Georgia Ave., Silversprings, Md.

of the transducer. The other end of this water column was placed in level with the center of the cell corresponding to spot (R) in Figure (3). At this point the recorder pen was centered on the recorder chart establishing a zero gage pressure or a reference line on the chart. The tensiometer (C) was reconnected, and a marriot burrette (A1) was brought to the level of (R) and inflow was allowed. All other outlets were clamped. Valve (U) was so positioned that both tensiometers were connected to the transducer giving average pressure between two tensiometers. The air pressure inlet E was left open to let any air in the cell escape. These conditions were maintained until the pen reached the center of the chart, a position originally established to indicate zero gage pressure or zero suction. After achieving equilibrium state, inflow from (A_1) was clamped and the dripping point of (B_1) was raised to the level (R) and outlet (B_1) was unclamped. Then the air pressure hose was connected at (E). Now the air pressure could be applied to the soil either in steps (static method) or gradually (dynamic method) to change the moisture content. For static equilibrium, steps of approximately 10 cm. of water pressure were applied with an air pressure regulator and were noted on the manometer (7). When equilibrium was reached, as indicated by the fact that the recorder pen came back to the center of the chart and

outflow stopped, another step was applied. For the first few steps the equilibrium could be achieved in a few hours but for higher pressure, equilibrium state was reached after several days depending upon the kind of soil and range of pressures. The same process was followed for rewetting the soil. Several gamma counts were taken after each equilibrium state to calculate θ_m . Capillary pressure values were assumed at the corresponding values at equilibrium.

For the dynamic experiments the regulator stem was rotated at a controlled rate by the device shown by numbers (3), (5) and (6) Figure (3). Number (6) is a D.C. motor¹ controlled by a speed control.² The motor has two shafts. The main shaft is geared to the second shaft reducing its speed from 10 to one. This speed can be further controlled by the minarik speed control which can reduce the speed of the second shaft from a maximum of 173 RPM to zero RPM continuously. Through this control the direction of rotation of the motor can also be reversed to reduce air pressure in the cell. The motor is

¹The motor is type NSH-12 R, 115 v. D.C., 0.36 amps., 1726 RPM max. 1/50 H.P. reducer motor output shaft 173 RPM speed reduction 10:1, torque 49 in lbs. Supplied by Bodine Electric Co., Chicago.

²Speed control and converter from A.C. 60 C. 115 v, to D.C. 115 v. Model SH-12. Supplied by Minarik Electric Co., Los Angeles, California 90013.

further geared to a multispeed transmission¹ (No. 5, Figure 3) with a choice of 12 speeds, the fastest equaling the speed of the motor shaft. It can be reduced to 1/5000 by choosing the slowest gear speed. In between each step of gear speed any other speed can be chosen with the help of the Minarik speed control. Theoretically it is possible to obtain almost any RPM desired. Each revolution of the regulator shaft corresponds to 51.4 cm. of water pressure.

Once a speed is chosen, it is kept constant for that entire saturation and desaturation process. Several such speeds were selected by trial and error for the three soils. The equation, $P_c = P_w - P_{nw}$, was used to compute capillary pressure P_c from the air pressure P_A or non wetting phase pressure, P_{nw} , given by a manometer and P_w , absolute pore water or wetting phase pressure recorded by the tensiometer-transducer-recorder system. P_A and P_{nw} are used interchangeably. Values of θ were computed from the average of several gamma-ray counts, half of them taken before and half of them taken after P_w and P_A were recorded.

<u>Conductivity measurements</u>.--Experiments were also run for three soils to determine the experimental values

¹Multispeed transmission has 12 gear speeds and a nuetral. Neutral is used by applying pressure steps by manually rotating the regulator stem to get the desired pressure step indicated by the manometer. Supplied by Harvard Apparatus Co., Dover, Mass.

of K(P_c) along with θ and P_c for each soil. The values for the conductivity functions were determined as follows: initial saturated conductivity was determined for zero P_{c} and θ_{c} by keeping both inflow and outflow levels above the soil level R and establishing a saturated steady flow in the soil. Steady state flow was achieved when inflow and outflow were equal. Conductivity could be calculated by noting the head loss between tensiometers. A certain RPM value was chosen to affect an air pressure change. As the pressure started increasing, the inflow decreased and the outflow increased depending upon the pressure rate and the external levels of inflow and outflow points or the external hydraulic gradient. If this external gradient was too low, outflow could occur from both inflow and outflow outlets. If the gradient was too large, $K(P_{c})$ could not be measured accurately because of the dependency of K on P. Thus the external hydraulic gradient was chosen so that it took into account the above considerations and gave inflow and outflow values relatively close to each other (within lcc/min. in this study). The average flux was used in D'Arcy's equation to determine K(P_c). The choice of the pressure rates was governed by: (1) maximum time needed for each set of readings, (2) maximum desirable P_{c} and (3) difference between inflow and outflow values. If the pressure rate was too fast, Pw became larger and eventually would

go out of range of the recorder. The maximum value of P_A itself was limited by the air entry value of the filter disc combination and the air entry value of the tensiometers. If either of these air entry values was exceeded, air bubbles were forced into the water systems resulting in erroneous measurements. Rapid pressure changes caused larger differences in inflow and outflow quantities. Therefore, pressure rates for conductivity experiments were kept much lower than in other experiments in which only the moisture characteristic determination was the objective.

 P_w , P_A , inflow, outflow and gamma counts were recorded for an interval Δt . The internal head loss in soil ΔP_c was determined by positioning valve (U) to first connect one and then the other tensiometer. These values were averaged over the period Δt to correspond to a time $t_1 + \Delta t/2$, where t_1 was time of previous averaged readings. $K(\theta)$ was calculated from $K(\theta) = V/I$, $I = \Delta P_c/C\Delta l$ where ΔP_c is the head loss in terms of chart reading and C is calibration constant. Δl is the distance between the tensiometers and V is the average of inflow and outflow velocities. P_w was given by the average of the two tensiometer readings divided by calibration constant. Short programs were written to solve the equations

$$K(\theta) = \frac{Q}{\Delta t} \cdot \frac{C\Delta l}{\Delta h} (cm/min)$$

$$\theta_{\rm m} = \theta_{\rm s} - \frac{1}{\mu_{\rm w} {\rm s}} \ln \frac{{\rm I}_{\rm m}}{{\rm I}_{\rm s}} \, {\rm cc/cc}$$

and

$$P_{C} = \frac{CR}{C} - 2(MR-50)$$
 cm of water

on an electronic computer. Here Q is average inflow and outflow in time t (min.). A is crossectional area of flow. CR is chart reading $CR/C = P_w$, MR is manometer reading. Initially both legs of a manometer were set at 50.

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PRESENTATION AND ANALYSIS OF DATA

Theory

Theory of gamma radiation attenuation technique.

When count rate per minute was plotted against experimentally determined values of moisture content of a soil (contained in similar containers and saturated to different degrees of moisture content) on a semi-log paper, a straight line resulted as shown in Figure (5). This indicates that a logarithmic relationship of the form

$$\theta = -K \ln I \tag{58}$$

exists between θ , the moisture content, and I, the count rate. Since the value of I varies with geometry, source strength, type of soil and collimation, a reference moisture content θ_s of known value corresponding to I_s for a fixed geometry is necessary. Saturated conditions can be used for this purpose when

$$\theta_{\rm s} = -K \ln I_{\rm s}. \tag{59}$$

By subtracting equation (59) from (58) the equation

$$\theta = \theta_{s} - K(\ln I - \ln I_{s})$$
 (60)

or

$$\theta = \theta_{s} - K \ln \frac{I}{I_{s}}$$

results, where K is the slope of the plot.

Examination of the exponential law

$$I = I_{o} e^{-\rho \mu x}$$
(61)

for attenuation of gamma rays shows that K is the reciprocal of the product of S, the thickness of the soil, and μ_w , the mass attenuation coefficient, as follows:

For wet soil contained in an aluminum cylinder

$$I = I_{o} \exp -s \left(\rho_{soil} + \theta \mu_{w} \right) - 2S_{A} \mu_{A} \rho_{A}$$
(62)

where ρ , μ , s are density, mass attenuation coefficient and thickness of the respective material. I is the count rate recorded with attenuation and I₀ is the count rate recorded without any attenuation.

When the soil is saturated, equation (61) can be written as

$$I_{s} = I_{o} \exp - s \left(\rho_{soil} + \theta_{s} + \theta_{w}\right) - 2S_{A} \mu_{A} \rho_{A}$$
(63)

Dividing equation (62) by (63) and taking logarithm gives

$$\ln \frac{I}{I_{s}} = \ln \exp - s(\theta \mu_{w} + \theta_{s} \mu_{w})$$

$$= -s \mu_{w} (\theta - \theta_{s})$$
(64)

or

$$\theta = \theta_s - \frac{1}{s\mu_w} \ln \frac{I}{I_s}$$

Therefore, if $\frac{1}{s \mu_W} = K$, the exponential law can be used for moisture content determination. It was further verified by plotting the experimentally determined values of θ against the calculated values as shown in Figure (6). Disregarding possible experimental errors, the plot is practically a straight line.

The mass attenuation coefficient for water can be determined for the same fixed geometry by taking count rates before and after filling the experimental cylinder with water. For the empty cylinder the exponential law can be written as

$$I_{1} = I_{o} \exp^{-2t\rho} A^{\mu} A \cdot$$
 (65)

For the cylinder full of water the same law is expressed as

$$I_{2} = I_{0} \exp^{-2t\rho} A^{\mu} A^{-S\rho} w^{\mu} w .$$
 (66)

Dividing equation (65) by (66) and taking logarithm results in:

 $\ln \frac{I_1}{I_2} = s \rho_w \mu_w$ $\therefore \quad \mu_w = \frac{1}{\rho_w s} \ln \frac{I_1}{I_2}.$



θ - % Volumetric moisture contents

Figure 5.--Relationship between moisture contents and count rate.

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 $\theta_e^{-\$}$ Volumetric experimental moisture contents

Figure 6.--Verification of exponential law by plotting experimental values of moisture contents against calculated values.

μ_w - maga uttenuation coefficient of water



Figure 7.--Variation of mass attenuation coefficient of water with increase in thickness of water.

The fact that a change in the sample thickness results in the change of the mass attenuation coefficient is demonstrated by Figure (7). The mass attenuation coefficient was determined by increasing the thickness of water by successively filling four plastic cells placed between the source and detector and computing μ_w after each additional cell was filled.

Mathematical models used in computation of conductivity function

Three mathematical equations were used for conductivity function determination. Burdine's equation

$$K_{r} = (S_{e})^{2} \int_{o}^{s} \frac{ds}{P_{c}^{2}} / \int_{o}^{1} \frac{ds}{P_{c}^{2}} \text{ for } P_{c} > P_{b} \quad (67)$$

was solved by numerical integration (Appendix 1) for both adsorption and desorption. In this equation $\int_0^1 ds/P_c^2$ represents the total area under curves in Figures (44), (45), (46), (47), (48) and $\int_0^S ds/P_c^2$ is the fractional area under the curve corresponding to different values of Pc and S. The purpose of summing the areas directly is to avoid the use of relationship $\frac{1}{P_c^2} = \frac{1}{P_b^2} (S_e)^{2/\lambda}$ involving λ and P_b because λ and P_b cannot be correctly estimated for adsorption curves.

In a different approach instead of using the experimental moisture characteristic data, the functions

$$\frac{1}{P_{c}^{2}} = \frac{1}{P_{b}^{2}} \left(\frac{S-S_{r}}{1-S_{r}} \right)^{2/\lambda} \text{ for } P_{c} > P_{I}$$
 (68)

and

$$\frac{1}{P_{c}^{2}} = \frac{1}{P_{s}^{2}} \exp - c \left(\frac{1-s}{1-s_{i}}\right)^{d} \text{ for } P_{c} < P_{I}$$
(69)

generated the relationship between the saturation and the capillary pressure. Brooks and Corey's equation (68) generates a curve shown dotted in Figure (44) and there-fore, does not represent the experimental data completely. Function (69) is, therefore, a necessity because at $s = s_I$ and $P = P_I$ the curve changes its curvature from concave downwards to concave upwards.

At
$$s = s_{I}$$

 $\frac{1}{P_{C}^{2}} = \frac{1}{P_{I}^{2}} = \frac{1}{P_{S}^{2}} e^{-C}$
 $c = -\ln (\frac{P_{S}^{2}}{P_{I}^{2}}).$

The value of d = 0.4 was determined by trial and error. Theoretically P_s is capillary pressure at saturation when s = 1 and

$$\frac{1}{P_c^2} = \frac{1}{P_s^2}$$

For adsorption the value of P_s is 0 for the three soils investigated in this study. Therefore, in order to make the program (Appendix II) function properly, a finite value of P_s greater than zero has to be used. The relative conductivity is computed by substituting the function (68) for $P_c > P_I$ and function (69) for $P_c < P_I$ in equation (67). Capillary conductivity is calculated as

$$K = CSTD. x K_r$$

by numerical integration, where CSTD is the experimental value of conductivity corresponding to θ_s .

Equation (43) of Kunze

$$K(\theta)_{i} = \frac{K_{s}}{K_{sc}} \frac{30 n^{-2} \gamma^{2} \phi}{\rho g \mu} (h_{1}^{-2} + 3h_{2}^{-2} + --(2n-1)h_{n}^{-2})$$
(70)

was also used and the results are compared in Figures (49) to (56).

Presentation of Data and Analysis

Dynamic Method

Figures (8) to (16) represent the dynamic processes of desorption and adsorption. The air pressure (P_A) or nonwetting phase pressure and the wetting or absolute pore water pressure (P_w) are plotted as functions of time; the difference of these two pressures, i.e., the capillary pressure is also shown. Pore water pressure can be either positive or negative and is affected both by the externally applied hydraulic head and also by the time rate of air pressure application R_{AP} . As R_{AP} increases from 1.65 min./cm. in Figure (8) to $R_{AP} = 25.5$ min./cm. in Figure (9) for the same soil , maximum value of P_w decreases from 34 cm. H_2O to 11.0 cm. H_2O . Therefore, to achieve the same maximum value of P_c , the maximum value of P_{Nw} is larger for faster runs than slower runs. This can be observed by comparing Figures (9) and (11) where ΔH is the same but R_{AP} differs. Figure (9) and (10) show the effect of increasing ΔH . As a result of increase in ΔH , P_w is higher in Figure (10) as compared to Figure (9).

The same general behavior was observed in the other two soils. In fine sand (Figures (12), (13) and (14)) P_w decreased as R_{AP} increased from 0.625 min./cm. to 19.6 min./cm.

It was also observed that P_w does not increase as much in fine sand as in medium sand and increases the least in sandy loam (Figure (15)). Figures (16), (17a), (18), (19) and (20) show the lag between the air pressure and the capillary pressure. In fast runs it was noted that P_{nw} became zero before P_c became zero (Figure (8)) and that it took a long time afterwards for the soil to saturate suggesting that it took a long time to replace the entrapped air from the pores. The lag between P_{nw} and P_c for sandy loam is negligible and is not shown in Figure (22).

Dynamic method compared with static method

In Figure (17b), (21) and (22) the dynamic data are compared with the static data. In all cases it is noted that the two moisture characteristics do not follow the same path. These results agree with the results of Topp <u>et al</u>. (40). It is significant that when different R_{AP} are compared, as in Figures (17b) and (21), the desorption process follows approximately the same path. In Figure (17b) the maximum P_c achieved was the same for all R_{AP} values; hence, it is not unusual that the adsorption process also follows the same path. Since maximum P_c values in Figure (21) were different, adsorption curves were also different.

Head loss between tensiometers and the boundary

Figures (23), (24), and (25) show the variation of head loss between the two tensiometers installed in the soil with flow velocity in the three soils studied. Even though the curves for adsorption and desorption are not the same, they do follow a similar pattern. The flow velocity in the soil can be expressed as













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Figure 17. (a) Moisture characteristics by dynamic method and lag between P_C and P_{NW} for $R_{AP} = 1.65$ min./cm. (b) Comparison of moisture characteristics obtained by dynamic and static equilibrium method for medium sand.



Figure 18.--Moisture characteristics by dynamic method and lag between P_c and P_{nw} in fine sand for R_{AP} = 19.6 min./cm. H₂O and ΔH = 16 cm. H₂O.
















or

$$= K(\theta) \frac{\Delta h_s}{\Delta L}$$

Κ(θ)Ι

v =

and
$$\Delta H = \Delta h_s + 2\Delta h_b + 2\Delta h_f + 2\Delta h_p + \Delta h_t$$
 (71)

where ΔH is the total external applied head, and Δh_s , Δh_b , Δh_f , Δh_p , Δh_t are losses in soil, boundary between soil and supporting filter-plate combination, filter, plate and tubing. Head loss in the tubing can be calculated from

$$\Delta h_{t} = f \frac{L}{D} \frac{V^{2}}{2g}$$
 (72)

where f is the friction factor for laminar flow. D is diameter of tubing in ft., v is kinematic viscosity and L is the length of the tubing. h_f in the tubing was calculated as 0.000032 ft. or .000974 cm. of water which is negligible; therefore, the main causes of head loss are the soil, supports and boundary. The head loss determined experimentally in 4 cm. of soil ranged between 0.2 to 3.2 cm. of water, or 0.6 to 9.6 cm. of water in the full length of soil, the head loss being greater in finer soils. The total applied head ΔH ranged from 16.0 to 31.0 cm. of water for different soils.

From the Figures (23), (24) and (25) it is observed that the head loss in the soil between the



Figure 23.--Variation of hydraulic gradient between tensiometers with flow velocity in medium sand.



Figure 24.--Variation of hydraulic gradient between tensiometers with flow velocity in fine sand.



Figure 25.--Variation of hydraulic gradient between tensiometers with flow velocity in sandy loam.

tensiometers, and thus in the whole length of the soil, first increases as the velocity decreases but then decreases with a continuing decrease in velocity. The pattern is similar in adsorption and desorption, if the direction of the pathway is ignored.

Equation (72) suggests that initially when velocity is high, the head loss in the filter-plate combination is very high as compared to head loss in the soil. This is attributed to the very small pore diameters in the filter-plate. Head loss being proportional to the square of the velocity, it decreases considerably in the filterplate combination as the velocity decreases. The total head loss ΔH being constant throughout the run, Δh_{c} increases as large pores are emptied. After the moisture content is decreased, the boundary between the soil and the filter-plate combination may also contribute to head loss. With further decreases in moisture content, it appears that the head loss in the boundary becomes very large. Perhaps this is the result of separation of soil and filter or a loss of water film continuity.

K, θ and P_c data

Figures (26) to (28) present the experimental data for $K(\theta) - \theta$ relationships and Figures (29) to (33) present data for $K(P_c) - P_c$ relationships.



Figure 26.--Experimental capillary conductivity versus moisture contents for medium sand.



Figure 27.--Experimental capillary conductivity versus moisture contents for fine sand.



Figure 28.--Experimental capillary conductivity versus moisture content for sandy loam.



Figure 29.--Experimental capillary conductivity versus capillary pressure for medium sand.



Figure 30.--Experimental capillary conductivity versus capillary pressure for fine sand.

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Figure 31.--Experimental capillary conductivity versus capillary pressure for sandy loam.



Figure 32.--Variation of hydraulic conductivity of monodispersed glass beads with capillary pressure in rewet and redry loop.



Figure 33.--Variation of capillary conductivity with capillary pressure for aggregated glass beads for redry and rewet loop.

Figures (32) and (33) represent Topp's (38) experiments on mono-dispersed and aggregated glass beads. Topp's data was obtained by a dynamic method using tension supplied by a water column instead of air pressure. One point of difference between the data obtained from three natural soils in this study and Topp's glass bead data is that his adsorption curves for glass beads approach a horizontal asymptote as the capillary pressure becomes zero. But adsorption curves of the soils used in this investigation approach zero pressure with a sharp slope. Topp's data is included in this investigation to broaden the scope of testing the mathematical models for conductivity.

From these figures it is observed that hysteresis exists when curves are plotted for K versus P_c but little hysteresis was exhibited in plots of θ versus K. In the case of fine sand the adsorption and desorption curves even cross each other.

Figures (34) to (38) are plots of moisture characteristics using saturation $S = \theta/\phi$ instead of θ . These plots are needed to make the first estimate of S_r , the residual saturation by extrapolation as shown. S_r is to be used later for plotting S_e versus P_c on log-log scales (Figures (39) to (43). Here S_e , called the effective saturation, is equal to $(\frac{S-Sr}{1-Sr})$. From these plots the values of P_b and λ are estimated and used in equation (68) for computation of conductivity function.



Figure 34.--Saturation versus capillary pressure for medium sand for desorption and adsorption.



Figure 35.--Saturation versus capillary pressure data for fine sand for desorption and adsorption.



Figure 36.--Saturation versus capillary pressure for sandy loam desorption and adsorption.



Figure 37.--Saturation versus capillary pressure for mono-dispersed glass beads rewet-redry loop.



Figure 38.--Saturation versus capillary pressure for aggregated glass beads rewet-redry loop.







Figure 40.--Relationship between effective saturation and capillary pressure for fine sand.

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Desorption		Dynamic	Static
	P _b	89	122
	$\frac{1}{P_b^2}$.000127	.000067
	λ	2.43	8.5
Adsorption	P _b	5.8	49
	$\frac{1}{P_b^2}$.0297	.000417
	λ	.317	1.63

Figure 41.--Relationship between effective saturation and capillary pressure for sandy loam.



Figure 42.--Relationship between effective saturation and capillary pressure for mono-dispersed glass beads.



Figure 43.--Relationship between effective saturation and capillary pressure for aggregated glass beads.

Comparison of theoretical and experimental conductivity results

The theoretical results for conductivity function obtained by three different methods are presented separately for adsorption and desorption to avoid confusion because for each experimental desorption or adsorption curve six calculated curves are obtained, three for static and three for dynamic data. The plots of only K versus θ are compared. These plots are shown in Figures (49) to (56).

Figures (44) to (46) are plots of $\frac{1}{P_C^2}$ versus S for the three natural soils, for both adsorption and desorption. Figure (47) and (48) are plots for Topp's glass beads experiments. In the adsorption process (arrows upwards) it may be noted that the area under the adsorption curves is greater than the area under desorption curves.

In Figures (49) to (56) CSTD. is the measured value of capillary conductivity near saturation. $(FMT)_D$ and $(FMT)_S$ are the matching factors for dynamic and static processes respectively. The matching factor is the ratio of the measured conductivity to calculated conductivity near saturation. (FMB) is the measure of the deviation of the matched calculated conductivity from the measured conductivity at minimum moisture content, that is:

(FMB) = (Measured conductivity)

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(calculated conductivity).(FMT) Therefore (FMB) χ (FMT) gives matching factor at \mathfrak{S}_{\min} .



Figure 44.--Area under the curve for relative conductivity computation for medium sand.





Figure 45.--Area under the curve for relative conductivity computation for fine sand.



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Figure 46.--Area under the curve for sandy loam for relative conductivity computations.



Figure 47.--Area under curve for mono-dispersed glass beads for relative conductivity computations.



Figure 48.--Area under curve for aggregated glass beads for relative conductivity computations.

Legend for Figures (49) to (55) for the comparison of experimental conductivity with theoretical conductivity.



Figure 49.--Conductivity comparison for medium sand desorption.



Figure 50.--Conductivity comparison for medium sand adsorption.






Figure 52.--Conductivity comparison for sandy loam desorption.











Figure 55.--Conductivity comparison for aggregated glass beads desorption.



Figure 56.--Conductivity comparison for aggregated glass beads adsorption.

DISCUSSION

Gamma Radiation Attenuation Technique

Although this technique is very useful for the determination of varying moisture contents, there are some factors which must be considered for its successful application.

The detector must be shielded with lead as much as the source itself. Whereas the shielding of the source is necessary for personal safety, the shielding of the detector is necessary in order to prevent the counting of scattered radiation.

When a strong C_s^{137} source was used (100 mci) and the shielding was not adequate around the detector, it was noticed that even though the moisture content of the soil changed, the change in the count rate was not appreciable; therefore, accurate calculation of the moisture content was not possible. Additional shielding of the detector solved this problem. This factor must be considered in using this technique in the field. Another factor is proper collimation of the gamma beam both at the source and at the detector.

Static and Dynamic Methods

As in the studies of Topp et al. (40) it has been observed in these investigations also that moisture characteristics obtained by dynamic and static methods (Figures (17b), (21), (22)) differ from one another. It seems that more water drains in the beginning of the desorption process by the static method than by the dynamic method. The possible reason for this could be that when a certain pressure step P_2-P_1 is applied, all pores of radii r_2 to r_1 corresponding P_2 to P_1 tend to drain simultaneously, whereas when pressure is changed gradually, only the largest pore starts draining and in doing so creates discontinuities in the porous material making it difficult for the next smaller pore to drain. Therefore, the amount of water drained is smaller in the dynamic process initially. After a few pressure steps the same kind of discontinuities caused a slow down of drainage by the static method and therefore, the slope of the moisture characteristic curve tends to decrease.

Different rates of pressure application did not seem to affect the draining process and give approximately the same desorption curve for a particular soil. It was also observed that the adsorption curves did not become horizontal near saturation as reported by Topp (38). It approached saturation very steeply.

The plots in Figures (9), (12) and (15) for medium sand, fine sand and sandy loam show that a higher value of maximum P is achieved for sandy loam than for fine sand and similarly maximum P is higher in the case of fine sand than in medium sand. The reason for this is that P_w did not build up as fast in sandy loam as in medium sand. P, built up fastest in medium sand. This indicates that P_w builds up faster in coarse material than in fine material. In the case of medium sand (Figure (9)), it was observed that after $P_{nw} = 42 \text{ cm} \cdot \text{H}_2^0$, the increase in $P_{_{\rm U}}$ was approximately the same as the increase in P_{nw}. The reason for this appears to be that in fine textured soils the increase in P_w is alleviated by drainage of the excess water. Apparently in the coarser textured soils the film contact is lost, and hence there is no pathway out of the tensiometer in order to relieve this pressure.

Higher values of the external hydraulic gradient ΔH , are necessary in coarse material than in fine material to be able to measure Δh , the internal head loss in soil.

Experimental and Calculated Conductivity

The experimental values of conductivity plotted against moisture content (Figures (26)-(28)) show some scatter of data for adsorption and desorption processes. This scatter has been observed by Topp (38) but was not attributed to hysteresis. This observation is further strengthened in this study. In the case of medium sand the adsorption curve is below the desorption curve, in fine sand the two curves cross each other and in sandy loam the adsorption curve is above the desorption curve. The scatter seems to be due to experimental errors. Dotted lines in these figures show the average of the adsorption and desorption conductivity values. Plots in Figures (26), (27) and (28) are reproducible with the same degree of accuracy. Three runs for medium and fine sands produced similar results. For sandy loam (Figure (28)), only one run was made.

The experimental conductivity data when reproduced for comparison with the calculated values in Figures (49) to (56) are shown by open circles for both desorption and adsorption. Figures (29) to (33) show the plots of conductivity versus the capillary pressure. In these plots hysteresis is well marked. This has been shown by several investigators previously. Conductivity K(θ) may be assumed as a unique function of moisture content but not of capillary pressure, because for each value of P_c there are two values of θ , one for desorption and another

for adsorption; therefore, for each value of P_c there are two values of conductivity K.

Figures (49) to (56) give a comparison of experimental and calculated conductivities of five materials. Comparisons for the adsorption process of fine sand and sandy loam are not included because the calculated values compared very poorly with the experimental values. CSTD, the saturated conductivity, and matching factors for dynamic and static data are given on these plots. $(FMT)_D$ is the matching factor for the dynamic process and is the ratio of the experimental to calculated conductivities near saturation; (FMB) is a similar ratio at Θ_{min} after multiplying the calculated conductivity values with (FMT).

For the adsorption process none of the equations gave accurate results. By using Burdine or Brooks and Corey models, the calculated values are much smaller than the experimental values. The total area $\int_{0}^{1} ds/P_{c}^{2}$ shown in Figures (44) to (48) is much greater for adsorption than desorption; therefore, the ratio $\int_{0}^{s} ds/P_{c}^{2}/\int_{0}^{1} ds/P_{c}^{2}$ is smaller for adsorption than desorption and the resulting calculated conductivity K is smaller. The Brooks and Corey equation $S_{e} = (\frac{P_{b}}{P_{c}})^{\lambda}$ or $\frac{1}{P_{c}^{2}} = \frac{1}{P_{c}^{2}} (S_{e})^{2/\lambda}$ assumes a unique relationship between S_{e} and P_{c} but actually the same equation processes. If the above relationship is to be used for the adsorption process, a

different value of λ will have to be used. The use of Brooks and Corey's technique for the determination of λ for the adsorption process is not possible because the plot of log (P_c) versus log (S_e) is not a straight line but is a curve (Figures (39) to (43)). Even if a straight line is fitted through the adsorption data, it gives a different value of P_b. Theoretically, there should be only one value of bubbling pressure for a particular soil. Therefore, this approach does not hold true for the adsorption process.

Millington and Quirk's equation predicted higher values of conductivities for adsorption than the equations discussed above; however, the predicted values still differed very much from the experimental values. It appears, therefore, that the theory behind all of these approaches is not applicable to the adsorption process because the mechanics of the adsorption process differ from that of the desorption process. Adsorption of liquids by porous media is affected by air entrapped in the pores but this is not a significant factor in desorption.

Use of Millington and Quirk's equation with matching factors (as shown in Figures (49) to (56)) resulted in different matching factors for the upper and lower parts of the conductivity curves which indicate that the curves obtained from calculated values are not parallel

to the experimental curves. Generally, the curves matched better at low moisture content with the exception of the desorption curves for Topp's glass beads. The calculated values of conductivity are larger than the experimental values at saturation and when multiplied by the matching factor the calculated values became smaller than the measured values at low moisture contents.

For the desorption process, the difference between the experimental values and the calculated values of conductivity, even though not negligible, were not as much as for adsorption.

Brook's and Corey's equation $\frac{1}{P_C^2} = \frac{1}{P_b^2} (S_e)^{2/\lambda}$ was used to estimate $\frac{1}{P_C^2}$ and this was substituted in Burdine's equation (67) to compute conductivity. The approximation $\frac{1}{P_C^2} = \frac{1}{P_b^2} (S_e)^{2/\lambda}$ is shown by a dotted line in Figure (44). It does not exactly represent the experimental data. This approximation may give good results for those soils which have plots of $\frac{1}{P_C^2}$ versus S concave downwards. For soils having these plots either partially or completely concave upwards as shown in Figures (44) to (48) this approximation did not yield good results. For such soils approximation by equation (69) yielded better results (See Figure (44) open squares) for dynamic process.

For the above reasons conductivity values calculated by use of the original Burdine equation (67) did not match the values calculated by substituting equation

(68) and (69) in equation (67) (See Figure (49), open triangles and crossed circles). However, when the use of equation (67) is reduced, i.e., in plots of $\frac{1}{P_o^2}$ versus S, when the curves became concave upwards (See dark circles for static desorption (Figure (44), triangles and circles for desorption in Figures (45) to (48)), results of Burdine equation and that of equation (69) substituted in equation (67) gave similar results. These results are shown in Figure (49) dark triangles and crosses for static desorption and open triangle and crossed circles in Figures (52), (53) and (55) for dynamic desorption. It, therefore, seems more desireable either to use equation (67) directly or when the curves of Figures (44) to (49) are concave upwards, equation (69) may be substituted in equation (67) and then integrated for the conductivity calculation.

From the above analysis it may be concluded that the direct numerical integration of Burdine's equation, which is now possible with the aid of a computer, gave better results for dynamic desorption process. Equation (70) gave better conductivity values from static desorption data than from the dynamic desorption data (see closed squares compared with open squares in Figures (49) to (52)). In the case of glass beads it overestimated the conductivity values at all moisture contents. This probably was due to assumptions on which the development

of Millington and Quirk's equation is based, i.e., cutting the physical flow model at planes perpendicular to the direction of flow and then random rejoining, thus, overestimating the flow area and hence the conductivity. This is contrary to what had been anticipated since Millington and Quirk assumed that the flow area decreased with a decrease in moisture content.

In spite of these limitations, equation (70) involves no such parameters as S_r , λ , P_b , P_s etc. and, therefore, is easy to use. S_r is important in the use of Burdine's equation and in equation (68). Values of S_r cannot be correctly estimated unless complete moisture characteristics are available. Lambda (λ) can be estimated only if the relationship log P_c versus log (S_e) is a straight line.

From Figures (34) to (38) it may be observed that the moisture characteristic curves are extended beyond the experimental values to properly estimate the values of S_r .

CONCLUSIONS

 Dynamic moisture characteristics can be satisfactorily obtained by gamma radiation attenuation technique with proper shielding and collimation.

Mass attenuation coefficient of water varies with the thickness of water interposed between the source and detector.

Logarithmic relationship between the count rate and moisture content of the porous material is valid for gamma radiation attenuation technique.

2. The moisture characteristics obtained by the dynamic method differ from those obtained by the static method. The use of different pressure application rates in the dynamic method gave very similar results.

Moisture characteristics were obtained for medium sand by raising air pressure as fast as 1 cm. of water in 1.65 min. For finer soils such as fine sand and sandy loam this rate can be higher. In coarse soils such as medium sand the drainage of water from the soil is considerably impeded after a capillary pressure of 30 cm. H_20 ; whereas in sandy loam higher capillary pressure values can be reached before the drainage slows down.

3. The existing theories for the determination of capillary conductivity from moisture characteristics cannot predict the capillary conductivity values for the adsorption process. For the desorption process Burdine's equation gives better results for the dynamic process. Brooks and Corey's approximation for determination of P_c from S_e does not match experimental data; therefore, its use does not produce the same results as direct integration of Burdine's equation. However, when this approximation is replaceable by equation (69) (when $\frac{1}{P_c^2}$ versus S curves are concave upwards for desorption), the calculated values of conductivity by Burdine's equation and by using equation (69) along with equation (67) give similar results.

Equation (70) is more convenient to use, but the results obtained by employing dynamic data do not match experimental conductivity values. Using static data in equation (70) gives results which fit better with the experimental conductivity data. Generally, the conductivity values predicted for low moisture content by this equation require a smaller matching factor, showing that the calculated conductivities are higher than the experimental conductivity values.

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APPENDIX I

Computer Program and Data for Numerical Integration of Burdine Equation

$$K = CSTD \cdot (S_e)^2 \int_0^s \frac{ds}{P_c^2} / \int_0^1 \frac{ds}{P_c^2} .$$



FORMAT(*0*•2X•1HN•3X•6HTHETHA•7X•2HPC•12X•1HS•14X•2HSE•10X•6HDS/PC FORMAT (*0* • *N= * • 1 3 • 3X • *SR= * • F7 • 5 • 3X • *PH1 = * • F5 • 4 • 3X • *CSTD= * • F6 • 4 PRINT32+I+TINC([)+DPI([)+S([)+SE([)+TEMP([)+PROD([)+CR([)+CA(I) S(I)=TINC(I)/PHI \$ SE(I)=(S(I)-SR)/(1.0-SR) F0RMAT(*0***ST0INC=**F10*6*3X**PH1=**F10*6) Cond(1)=PROD(1-1)+STDINC/(PH1*DP1(1)**p) (/*4) 11 HSUM (DS/PC2) * 9X*2HKP * 13X*2HKA*/) TINC(M-1)=TINC(M)-STDINC\$J=J-1\$M=M-1 Cq(1)=SF(1)**2*(PROD(1)/PROD(L1)) FORMAT(* *.13.F10.7.F10.1.6F15.8) PRINT 34.N.SR.PHI.CSTD.PROD(L1) (W) Id0*(I+W) Id0/(W) Id0=(I-W) Id0 LEwp(1)=STD1NC/(DH1*DD1(1)**2) 1 • 3X • * PROD([]) = * • F10 • 7) 80) GO TO 88 [F(J.NE.0) GO TO PPINT7.STDINC.PHI CA(I)=CP(I)*CSTD 001 10 1=1 01 00 DO 37 1=M+L1 70 38 I=M+L1 DO 31 1=M.L1 DO 36 I=M+L1 DO 30 1=M+L1 1F(L].LT. p=(1)00ad GO TO R6 CONTINUE CONTINUE CONTINUE CONTINUE CONTINUE CONT INUF PPINT 6 L1=L1*2 U=3/N1 **CN**E ሳ የ С С ኒ r n r ٩. ř a 7 C ¢ 1

-25.0 0.011 -0 • C • 304 -1.0 • 302 .287 •29 •58 -30.0 -20.0 -30.0 -3•0 .303 ເ ເ ເ ເ ເ ເ .285 -1.5 •286 •29 •28 -15.0 -35-0 .297 •282 -2.0 •278 •312 -J. 0 -0-1 •28 •28 C•04--18.2 -15.0 -40.0 •306 -7.4 .272 •276 ເ ເ ເ ເ ເ ی • •26 62. .27 •31 -123.0-120.0-110.0-100.0-ar.0 -80.0 -70.0 -60.0 -50.0 -22.5 -8.78 -17.5 -43.0 -0°. •305 •304 -1.0 •26 •20 •24 •26 •27 PS OAZI SANDY LOAM (HILSDALE) DYN-MIC DESORPTION -28.5 -25.5 -47.5 -46.5 -19.0 .2915 -67.0 -57.7 -46.5 -36.7 -17.5 -17.5 -7.0 ADSORPTION -2.0 -11.0 -9.5 •262 A AZI MEDIUM SAND(SPINIX)DYNAMIC DESORPTION 25 QAZI FINE SAND (OTTAWA) DYNAMIG DESORPTION -15.5 -12. ADSORPTION • 29 د د. • • 25 • 22 • 25 25 0AZI MEDIUM SAND (SPINX)STATIC ADSORPTION 25 0AZI FINE SAND (OTTAW) STATIC DESORPTION e. -21•0 • 291 -4.2 .252 24 0AZI MEDIUM SAND (SPINIX) DESORPTION •24 •26 • 29 •16 •24 2 25 GAZI MEDIUM SAND(SPINIX)DYNAMIC -40°0 -44.0 -39.5 -37.0 -33.0 -30.0 -56.0 -55.5 -54.0 -53.0 -51.0 -49.2 -56.n -49.0 -38.5 -31.7 -26.0 -20.0 -29.5 -26.5 -24.5 -23.5 -23.1 -22.5 • 266 -49.n -40.0 -30.8 -24.0 -15.0 -13.0 -8-0 25 QAZI FINE SAND (OTTAWA) DYNAMI • 242 • 26 •14 • 18 • 23 • 23 • 23 -20.5 -22.5 -18.5 -15.0 -11.2 0.32 0.32 0.32 •267 **4**ۥ0 •228 • 1 1 •11 • 1 1 •228 •22 • • • •16 • 22 **N** •239 •159 •186 .215 •21 •12 .154 .17 • •21 0.06 0.06 0•06 0•06 • . . 80 °, •212 °, •00 といく・ •198 -20.0 -10.0 0.0 ŝ **∾** .125 .143 .125 .137 •182 •081 6 C • •19 .178 •0B •19 .312 0 **€ •** Ç 0.29 .312 .312 • 31 V 0.29 •28 - 82.7 .072 •188 •072 .188 **..** •172 ç 000. 11 **1**3 2 2 œ 4 •17 С. С.

-30.0 -20.0 5 0 1 •278 .237 .218 •276 -1•0 •206 -2.0 •27 -123.0-120.0-110.0-100.0-40.0 -80.0 -70.0 -60.0 -50.0 -40.0 •196 •266 0.01 -0• S -5.0 •189 •259 • 28 -154.0-140.0-104.0 -81.0 -63.0 -40.3 -10.7 -5.0 25 QAZI SANDY LOAM (HILDALES) STATIC DESORPTION 9 •28 0.06 0•34 STATIC ADSORPTION -10.3 -20.0 -10.0 SANDY LOAM(HILSDALF)DYNAMIC ADSORPTION •2314 •2771 •2771 •2314 0.01-**5481 5481** •278 •184 • 242 25 TOPP AND MILLER AGGRI BEADS DESORPTION 25 TOPP AND MILLER AGGRI-BEADS ADSORPTION 0.001 -154.0-111.0-89.0 -63.4 -45.2 -25.0 -12.0 25 TOPP AND MILLER MOND READS DESORPTION 25 TOPP AND MILLER MONO BEADS ADSORPTION •54B1 -10.0 .273 •238 .179 -65.0 -63.5 -55.0 -48.0 -42.0 -35.0 -45.0'-30.0 -28.0 36545481 .2396 .4263 -29.0 -23.5 • 249 -30.2 -23.0 -12.8 .215 .2428 .2727 .176 е О Ф α0. 25 AZI SANDY LOAM (HILSDALE) 0.34 0.34 .173 •03 С: С. •24 ۰ • .A358 .0391 .0652 .0978 .1952 • 0358 • 0414 • 0818 • 1434 -2192 •144 •1509 •1627 C• 04-10 5 10 •214 .171 • 774 1•0 •• 0.06 0 **6** 0.06 -65.0 -52.5 -36.0 -R4.5 -64.0 -58.5 .1077 -64.0 •0761 •1126 •1291 •199 -3.5 -1.0 •17 • 77 •326 •326 .609 009. -74.A .0913 .159 • 20 •17 25 OAZI •28 α Ω - 220 - R4 . R •124 .124 - NU-C • U <u>۳</u>: α • 17 v

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APPENDIX II

Computer Program and Data for the

Solution of Equations

$$K = \frac{\left(\text{CSTD x } (\text{S}_{e})^{2} \int_{\text{S}_{r}}^{\text{S}} \left(\frac{\text{S}-\text{S}_{r}}{1-\text{S}_{r}} \right)^{2/\lambda} \text{ds}}{\int_{\text{S}_{r}}^{1} \left(\frac{\text{S}-\text{S}_{r}}{1-\text{S}_{r}} \right)^{2/\lambda} \text{ds}} \text{for } P_{\text{C}} > P_{\text{I}}$$

and

$$K = \frac{CSTD \times (S_e)^2 \int_{S_r}^{s} exp(-c) \left(\frac{1-s}{1-s_I}\right)^{0.4} ds}{\int_{S_r}^{1} exp(-c) \left(\frac{1-s}{1-s_I}\right)^{0.4} ds} for P_c < P_I$$

 $CL_{1}C = C$ $CL_{2}C = 2/\lambda$ $PB2 = \frac{1}{P_{b}^{2}}$ $DTHETA = \theta_{s}/10,000$

AKS is experimental value of saturated conductivity corresponding to Saturation ST

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DIMENSION SP(100)+PROD(100)+TEMP(100)+ACPRO(100)+RAT(100)+SE2
                                                                                                                                                                        COMMON SP+PROD+TEMP+ACPRO+RAT+SE2+AKE+PC+DP+DIF+THETA+LOC+
                                                                                                                               1(100) AKE(100) PC(100) + PP(100) + DIF(100) + THETA(100) + LOC(9) +
                                                                                     PROGRAM COND (INPUT + TAPE60= INPUT + OUTPUT + TAPE61=OUTPUT)
                                                                                                                                                                                                                                                                                READIOI + PI + PS+ PB2+ CL2+ DTHETA+ SR+ SI + AKS+ ST
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         PROD(1)=PROD(1)+FUN(S+SI+CL1C+PS)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       PROD(1)=PROD(1)+FN(C,VCL2C,PB2)
                                                                                                                                                                                                                                                                                                                       CL1C=-ALOG(PS**2/P1**2)
                                                                                                                                                                                                                                                                                                                                                                                                                              I= (SP (M) -SR-.01) 2.2.1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            PROD(1)=PROD(1)*05
                                                                                                                                                                                                                                                                                                                                                                                                            10 • - (1 - W) as = ( W) as
0A71.T60.CM45000.PB.
                                                                                                                                                                                                                  READ 100 NN . LOC
                                                                                                                                                                                                                                      PRINT POOLOC
                                                                                                                                                                                                                                                                                                                                                                   SP(1)=ST-.005
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     IF (S-S1)7.7.4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  DO 9 J=1.100
                                                                                                                                                                                                                                                                                                                                                                                        DO 1 M=2.100
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          DO 4 1=1.100
                                                                                                                                                                                                                                                           IF (NN)5+3+3
                                                                                                                                                                                                                                                                                                                                              CL2C=2•/CL2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    S=ST+.00005
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     PROD(1)=0.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     S=ST-+0095
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              GO TO 31.
                                                                                                                                                   24KR(100)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              05=.0001
                                                                                                                                                                                                                                                                                                                                                                                                                                                    CONTINUE
                                                                                                                                                                                                                                                                                                     Y=1.-SP
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                DS=•001
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               S=S-DS
                                                                                                                                                                                                                                                                                                                                                                                                                                                                           M=100
                                                                                                                                                                                               A X A
                     FTN (LR)
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I6HDTHETA+8X+2HSR+8X+2HSI+8X+2HK5+8X+2HSI+6A+1HC+/+3+F1U+6+F1U+4)+ FORMAT(1H0+2X+1HN+5X+1HS +7X+5H1/PC2+7X+5HACCUM+7X+5HRA+10+9A+ 203+(1+SP(1)+TEMP(1)+ACPRO(1)+RAT(1)+SE2(1)+AKR(1)+ FORMAT(1H0+3X+2HP1+9X+2HPS+6X+8H1/PB(2)2+6X+4HL(2)+4X+ 13HSE2+10X+2HKR+2X+5HTHETA+8X+2HPC+10X+2HKE+11X+1HD+/) PPINT 201+PI +PS +PB2+CL2+DTHETA+SR+SI+AKS+ST+CL1C .929 993 . 463 .97 FORMAT(14+F6+3+5E12+4+F7+2+F10+2+2E12+4) • 80 • 80 • 80 • 80 THETA(I).PC(I).AKF(I).DIF(I).I=1.M) .684 .400 • 78 • 23 •11 • 1 1 THETA(L)=DTHETA*SP(L)*10000. MEDIUM SAND DYNAMIC DESATURATION • 1 1 • 1 1 MEDIUM SAND DYNAMIC SATURATUON D1F(L)=AKE(L)*DP(L)/DTHETA FORMAT(3(F7.0.F5.0).3F5.0) MEDIUM SAND STATIC DESORPTION MEDIUM SAND STATIC ADSORPTION FUNCTION FN(SS+SSR+CL+PB) Z=-C*((]-2)/(]-2])/**0-4 ป 3.71 .0031 0.853.0031 2.30 .0031 •01856 1.065.0031 FUNCTION FUN(S+SI+C+PS) FUN=(1/PS**2)*EXP(2) FORMAT(1H1.20X.9AB) Ñ ۱ FORMAT(12.9A8) FN = PB * (1. Z=(1.-SS)/SSR •00200 15.0 .0026 •0185 M.1=1 15 00 202 24F10.4) Q GO T O PRINT PD INT 2•0 8•5 0°0 STOP END END DND ND 23.0 C.052 24.0 44 0 201 202 203 100 101 õ 2 ហ

.975 .835 **992** -992 .992 566 666. -964 • 85 σ • 0 5 00 •05 • 0 0 •05 ດ ເ • 05 2.0 . . 0 TOPP AND MILLER AGGRI.BEADS ADSORPTION TOPP AUD MILLER AGGRI BEADS DESORPIION DESORPTION ADSORPTION •60 •69 • 75 .72 - -.82 .82 •58 •64 **9** 0.0 •03 3.17.00609 .08 • **0** 0 •34 80**.** •32 .32 •34 - 3 2 2 3 • 34 **4**C• SANDY LOAM STATIC DESORPTION SANDY LOAM DYNAMIC ADSORPTUON SANDY LOAM DYNAMIC DESORPTUON SANDY LOAM STATIC ADSORPTION 10.0 .002669 2.12.00609 TOPP AND MILLER MOND BEADS 28.0 12.0 00245 2.52 00326 •00058 2.92 •00292 •00905 0.545.00292 TOPP AND MILLER MONO BEADS 5.04.00326 DYNAMIC DESORPTION DYNAMIC ADSORPTION • COD6421 • 51 • 00291 89.0 30.0 0001272.43 0028 126.0 .5 0 .0397 0.317.0028 8.5 .0028 •000417 1•63•0028 STATIC DESORPTION END OF PROBLEM •000067 42.0 10.0 .000657 39.n 10.0 .00017 SAND SAND SAND 10.0 1.5 0.1 •52 108.0 5.0 FINE FINE FINE F INE 36en 56 o n 51.5 47.0 22.0 ī

APPENDIX III

1

Computer Program and Data for Solution of Equation

$$K = \frac{K_{s}}{K_{sc}} \frac{30n^{-2}\gamma^{2}\phi}{\rho g \mu} (h_{1}^{-2} + 3h_{2}^{-2} + \dots - (2n-1)h_{n}^{-2}).$$



6 FORMAT(*0*+*ACF=*+F6+5+3X+*V1S=*+F6+5+3X+*SURI=*+F5+2+3X+*M1N COND 1=*+F13+11+3X+*TEMP(C+)=*+F5+2+3X+*SMC=*+F5+4+3X+*SA1 COND=*+F10+6+ =*+F7+4+3X+*STD1NC=*+F5+4X (w)ld0*(i+w)ld0/(w)ld0=(i-w)ld0 \$ CO= (IWI-LINC(()-i)/210INC*(CBB())-CBB()-i))+CBB()-i) CL = (TMB-TINC(J-1))/STDINC*(CBB(J)-CBB(J-1))+CBB(J-1) VER=HOR & HOR=HOR+G(I) & CBB(I)=VER+HOR+CBB(I-1) T(N).T(1) *N1= *•12•3X•*TMB=*•F6•3•3X•*TMT=*•F6•3•3X• PRINT 6.ACF.VIS.SURT.TBI.TEMP.SMC.TBN.COEF PPINT 164.1.TINC(1).DPI(1).CBB(1).6(1) PDINT 8.FMB.FMT.STDINC .NI.TMB.TMT. FORMAT(* *.13. FR.4. F12.1.2F16.11) 2 *T(N)=*+F6+4+3X+*T(1)=*+F6+4) R FORMAT(* FMB=*+F7.4.3X+*FMT IF (TINC(J).GT.TMT) GO TO 166 IF(TINC(J).GT.TMR) GO TO 162 TINC(M-1)=TINC(M)-STDINC @(w-1)=@(w)/@(w+1)*@(w) **0**0 **0**0 80) GO TO IF(I.EQ.M) GO TO 12 CGR(1)=CBR(1)*FMT 2 3X ** COFF=* • F9 • 5) IF (J.NE.O) GO TO CPR(M) = HOP=G(M)DO 161 J=M+L1 DO 165 J=M•L1 DO 169 I=M.L1 DO 12 I=M.L1 FMB=TB1/CL FMT=TRN/CU IF (L1 .LT. CONTINUE CONTINUE CONTINUE GO TO 86 CONTINUE CONTINUE L1=L1+2 J=3/NL n 169 166 r v -154 2 50 161 100

•450 •046 •045 • 445 -25.0 •046 •044 • 304 -0 • 5 • 45 •45 .302 •287 •29 •28 0 0 -8-5 -43.0 -40.0 -35.0 -30.0 •286 • 303 -3.0 .285 -1.5 •29 • 28 22.22 22.22 22.22 22.22 22.22 22.22 22.2 22.22 -226 •29 •297 -22•5 -18•2 -15•0 •278 •272 •282 .312 -5.4 -2.5 •28 •28 -17-5 -15-0 0.0 •287 .287 • 285 •276 .305 •0725 •297 •072 •29 72.0 72.0 72.0 ņ 72.0 72.0 72.0 72.0 72.0 -7.4 -8.78 -5.5 •276 •29 .306 •31 •26 •27 ເດ ເ • 188 .125 •170 •188 .125 .172 -305 C•1--8.5 •304 •26 •26 •24 .27 25 0AZI SANDY LOAM (HILSDALE) DYNAMIC DESORPTION -49.0 -44.0 -39.5 -37.0 -33.0 -30.0 -29.5 -25.5 -54.0 -53.0 -51.0 -49.2 -47.5 -46.5 -20.55 -26.55 -23.55 -23.51 -22.55 -21.0 -19.0 .2915 -17.5 -17.5 -7.0 •000 25 CAZI MEDIUM SAND(SPINIX)DYNAMIC ADSORPTION •03064 •0099 -40.0 -30.8 -24.0 -15.0 -13.0 -11.0 -9.5 •03064 •nn99 -2.0 25 0AZI FINE SAND (OTTAWA) DYNAMIC DESORPTION •03064 •0099 •03064 •0099 •262 25 AZI MEDIUM SAND(SPINIX)DYNAMIC DESORPTION 6600. •03064 •0099 25 OAZI FINE SAND (OTTAWA) DYNAMIC ADSORPTION •03064 •0099 -56.n -49.0 -38.5 -31.7 -26.0 -20.0 -15.5 -12. • 29 • 22 25 QAZI MEDIUM SAND (SPINX)STATIC DESORPTION 25 0AZI MEDIUM SAND (SPINX)STATIC ADSORPTION • 22 • 25 • 25 24 CAZI FINE SAND (OTTAW) STATIC DESORPTION **ო** • • 03064 • 03064 •291 2.4-•24 .252 •16 • 29 •26 •24 N • • 266 -29.5 -22.5 -18.5 -15.0 -11.2 -8.0 • 242 •14 • 23 • 26 •13 • 23 • 23 -67.0 -57.7 -46.5 -36.7 •267 •228 • "J •228 • 35 •16 ເກ ຕ ر. ۳. •12 4 • 22 • 22 • () 4 4 4 •186 •239 .215 •17 •12 •21 •21 • 1 10 1.40F-02 9.00E-04 13 1.00F-03 1 • 00E-03 16 1.45F-03 13 1.45E-03 R.00F-04 1.40F-02 •212 •154 •159 •00 • •193 - 202 · • 1 -20.0 -10.0 0.0 د. • <u>م:</u> -55.5 11 •172 •182 **1**3 .125 .137 •081 .125 .143 4 α •178 •20 •072 •08 .183 .19 •19 CNU -92.7 - 20.0 010. 891. 0 d • 00-•17 α α

•044 1.75 110.0 •040 •039 C•C[] -0 • J •278 537 • 85 1.7 0 -123.n-120.0-110.n-100.0-90.0 -80.0 -70.n -60.0 -50.0 -40.0 -30.n -20.n -20.0 •218 •276 0.11 22.22 22.22 22.22 22.22 22.22 22.22 22.22 -153°u-150°0-110°0-100°0-00°0 -80°0 -20°0 -20°0 -20°0 -80°0 -30°0 •206 0.01 •27 •276 .270 •124 •273 • 4 4 •04 •0326 •27 72.0 72.0 72.0 72.0 72.0 72.2 72.2 •0326 •27 •196 •266 0.61 •0627 •0627 •124 • 17 5 • 0 -•259 -5.0 •189 •23 25 0AZI SANDY LOAM (HILDALES) STATIC DESORPTION C•U 25 0AZI SANDY LOAM (HILSDALE) STATIZ ADSORPTION -10.3 C • C [| 3654
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