MODELLING CONSUMER'S PURCHASE BEHAVIOR AS A STOCHASTIC PROCESS

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ABSTRACT

MODELLING CONSUMER'S PURCHASE BEHAVIOR AS A STOCHASTIC PROCESS

by Tanniru R. Rao

The behavior of the consumer and an understanding of the why's and ways of her purchase decisions are topics of growing interest in marketing. However, not much progress has been made in building realistic models of purchase behavior as a function of the dynamic environment in which she makes her buying decisions. Descriptive models of market behavior are still the focus of much research activity since they are a prerequisite for building causal models of a market; a market is treated as the composite behavior of all individual consumers.

Published evidence of brand choice models incorporating marketing variables, such as price, advertising, distribution, et cetera, is sparse. On the other hand, Bernoulli, Markov, and learning models, the stochastic models widely used in the literature, describe the consumer's brand choice as a function of her past purchases.

Purchase data of three consumer goods (a paper product, a drug product, and a food product), made available by the Chicago Tribune form the data base for this investigation. Probabilistic analyses of data reveal that housewives exhibit strong bias in the selection of a store for the purchase of any product and that a housewife's brand choice is different in different stores, suggesting that the choice of the store is a major intervening variable in executing the consumer's brand preferences. Accordingly, existing models of brand purchase have been extended by incorporating the place of purchase (store) as a variable.

The housewife's decisions regarding the size of purchase, the aggregate effect of store-brand interaction, and the effect of price on the market share of a brand are some of the issues considered in this investigation. Also, the increased number of private label purchases among store loyal customers and the lack of differentiation among private labels suggest the existence of certain market segments that are more prone to purchase private labels than others. Properly identified, these would help both the retailer and the manufacturer in planning and allocation of their marketing efforts.

This study is a first step in identifying the major components of the marketing system that have considerable effect (individual or interactional) on a consumer's purchase decision. More research is needed to include components such as advertising exposure, rate of consumption, time lapse between purchases, etc. This would enable the researcher not only to effectively simulate market behavior and pre-test marketing strategies or policy alternatives, but also to simultaneously study all the variables of the purchase decision for building dynamic aggregate models of the consumer's decision-making process.

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By

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In memory of my father, Tanniru Krishnamoorthy, a model
of the ideal parent, international citizen
and compassionate human being ....

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CHAPTER I

INTRODUCTION

Background

Market experimentation is a rapidly expanding area of knowledge in the marketing discipline today. Scientifically planned experimentation can be viewed as a means to observe the effectiveness of small or large changes in the value of the controllable inputs.[1] With the increasing complexity of marketing management, demands are placed more on scientific research than on limited fact finding activity. At the present stage of development, the operations research approach is the increasingly popular methodology in marketing. The operations research approach, systems approach and model building approach are treated as synonymous terms in describing certain phases of research in the literature.

The need for understanding market behavior in solving marketing problems and in developing an effective marketing program is obvious. This has given impetus for the develop-

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[1] William C. Hoofnagle, "Experimental Designs in Measuring Effectiveness of Promotion," Journal of Marketing Research, (May, 1965), pp. 154-162.

ment of various theories of market behavior. Formal statements of such theories are generally referred to as models.[2]

> Day:[3] In most business situations there are a few central factors which are of overriding importance in determining the outcome. A model can be confined to these few major factors and it specifies the nature of the important relationships among these variable factors. In other words, a model is a theory of the business system or process.

My major concern here is the study of "models of consumer's buying behavior." Such models of buying behavior are in great demand today as manufacturers are trying to out-guess their competitors by developing sophisticated customer-oriented marketing programs. The complexity of the marketing task makes model building extremely difficult, while the very nature of such complexity makes the model highly desirable.[4]

Scope of the problem

Identification of the central variables in any business situation constitutes an important step in developing models. There is often a great tendency on the part of the researcher to restrict himself to a few variables to make the models manageable. The very purpose of model building may become secondary in importance in this urge toward simplification. From this standpoint, this study takes a critical look at

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[2] Ralph L. Day, Marketing Models (International Text Book Company, 1964) p. 4.

[3] Ibid., p. 4.

[4] Ibid., p. 5.

some of the existing models of consumer buying behavior; major attention is focused on micro models - models based on an individual consumer's buying behavior.

The central thrust of the investigation is the study of the inter-relationships among the various selected elements of the consumer's purchase decision (store visited, brand purchased, size of purchase, price of purchase) and their implications for building realistic models of buying behavior. The study also highlights some hypotheses of special interest such as consumer's preference for private labels and its relation to store loyalty.

The approach to the study is descriptive; the consumer has been studied in terms of what she did and not in terms of what caused her to act the way she did. At times the cause and effect relationship has been studied on an inferential basis. Even with this limitation, the study should be helpful (1) in developing more realistic models of consumer's brand choice as a function of her environment, and (2) in developing an effective simulation of the consumer's decision-making process.

Statement of the problem

The housewife's purchase decision can be viewed as the outcome of a set of mutually related alternatives available to her in the market. In any purchase decision, the housewife, implicitly or explicitly, has selected a particular combination of store, brand, size and price. The decisions on these various aspects of the purchase are not completely independent due to the limitations imposed by dissimilar distribution, in-store services, advertising and promotion, and the dealing activities of competing brands and stores.

Due to the complexity of the problem, a considerable amount of research effort has culminated in descriptions of purchasing behavior in terms of the brands selected by the housewife. Accordingly, models published in the literature have incorporated only the information on the past history of brands purchased to predict the probability of subsequent brand purchase. Even here, empirical evidence is sparse in the literature. Where the models have incorporated other marketing variables such as price, store, advertising and so on, empirical evidence is even sparser.

In this study, brand choice will be described as a function of the consumer's past history of brand purchases and stores visited. The aggregate effect of a price differential on the market share of a brand, the store selection patterns of the consumer, and the consumer's decision on the size of purchase as affected by the brand or store change are studied in detail. In addition, the loyalty of the consumer to private brands and the frequent visiting of a store as it affects her preference for the store's private labels are studied. The specific focus of the study, in terms of hypotheses, are given in the following section.

Statement of Hypotheses

(1) A consumer's selection of a store is not completely random (i.e., has an equal likelihood of selecting each store); she exhibits bias in her choice. a) The more recent her purchase experience in a particular store, and b) the more frequent her

visits to the store, the more likely she is to repurchase the product in that store.

(2) As a corollary, consumers exhibit bias in the selection of the type of retail outlet (drugstore, food store, etc.) in which they would shop for a particular product.

(3) Store switching increases brand switching; the more a housewife changes stores, the more she changes the brand she purchases.

(4) Consumers change the size of their purchase when they change the store or brand; in general, they decrease rather than increase the size of their purchase with a change in store or brand.

(5) Loyalty (measured by the repurchase rate) to a particular store increases the preference for the private brands sponsored by the store. As a corollary, a housewife loyal to a private brand sponsored by a particular store is loyal to private labels regardless of store.

(6) Store-brand interaction is statistically significant after eliminating the effect of price. As a corollary, the effect of price on market share is significant and the interaction of brand and store cannot be explained by any linear function of the corresponding price difference.

Method of the Study

In the probabilistic approach to the purchase decision process, the housewife's purchase decision is treated as the result of the underlying parameters (probabilities of choosing different alternatives). Following the axioms of probability theory, the probability of a consumer purchasing any brand is measured by a number in the closed interval (0, 1). Since the housewife has to select one or another of the available brands, the probabilities of purchasing different brands of the product add up to one. This type of probabilistic approach can be used in any situation where one has to select from a set of mutually exclusive and collectively exhaustive alternatives.

> Kuehn:[5] The probabilistic approach provides a useful conceptual framework for considering the expected behavior of consumers. When the expected behavior of an individual is viewed as a set of probabilities related to the available brands, a richer and more flexible concept of brand loyalty is provided. Simple probability models are useful as building blocks in the construction of dynamic, aggregate models of consumer brand choice behavior.

The probabilistic approach has some definite advantages. One can measure the effectiveness of advertising, promotion, and distribution strategies by comparing the probabilities of purchasing different brands before, during, and after experimentation. In each period a simple weighted average of these probabilities over various individual consumers in the market (weighted by the frequency of purchases or rate of consumption) determines the market share distribution of brands. In this study, purchase histories of consumers over a period of three years form the data base for the analysis.

Sequences of two, three, and four purchases are aggregated

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[5] Alfred A. Kuehn, "Probabilistic Modelsof Consumer Buying Behavior," <u>Journal of Marketing</u>, 28 (October, 1964), pp. 27-31.

over individual consumers to describe purchasing behavior at the micro level in terms of conditional probabilities for three different products (paper product, drug product, and food product). At the macro level, the statistical technique of Analysis of Covariance has been used to study the interaction of brand and store. The detailed aspects of the research design and techniques are presented at a later stage.

Discussing the present stage of development of the probabilistic models of consumer behavior, Kuehn said that the emphasis has been on understanding the influence of marketing variables on the consumer's decision process.[6] If a particular probabilistic model gives good predictions of market behavior, the reasons the model works are of more importance than the fact that it does work.

Limitations of the Study

> Kuehn:[7] The more fundamental problem facing marketing science is the development of a model which will allow the simultaneous study of all the major marketing variables. Such a model would provide an extremely powerful tool, for the study of the effects of past purchase policies could serve as a basis for pre-testing proposed policies, and would provide an extremely sophisticated sales forecasting model.

Needless to say, the mathematical formulation of such a model is a formidable task. With the increasing uncertainties of such a task, simulation offers some immediate solutions; however, knowledge of the inter-relationships among the major

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6 Ibid.

7 Ibid.

variables of the consumer's marketing environment would be a prerequisite for purposes of constructive simulation. With these broad goals in mind, this study has been limited to the descriptive aspects of the consumer's interaction with the marketing environment as a first step.

In order to model the consumer's purchasing behavior as a dynamic stochastic process, the requirement of data on the consumer covering a period of time is essential. Panels have been the major source of such information. In general, panels record the information on brand, store, size, price, and time of each purchase for a selected sample of families over a period of time. However, panel data do not give any information regarding the advertising and promotions of stores or brands and only limited information on the presence of deals and price alternatives available to the consumer in the store on any purchase occasion. The presence of deals on a particular brand in a store can be inferred from data only if the actual purchase is made against that deal. Similarly, the panel data supply the price paid by the housewife but do not shed any light on the prices of competing brands available to her at the time of purchase. Added to these, the availability of brands, possible stock-outs, and the location and size of displays are variables that enter into the consumer's decision which, at best, can only be brought into focus with specially designed experiments. These limitations of data often leave no way for an investigator to tackle some of the detailed aspects of the consumer's purchasing behavior except on an inferrential basis.

To offset some of the said limitations of the data for the study of store-brand interaction, a portion of this investigation has been limited to the major national brands of the product-lines, as well as to the major retail outlets in the Chicago area where the specific brands have near 100 percent distribution. The structure of the competitition in the product lines, along with the market shares of the specific brands and stores selected, are presented in detail at a later stage.

Organization

The second chapter reviews the published literature on micro models of consumer buying behavior. Major importance is given to the models developed on the basis of panel data; the limitations and the underlying assumptions of the models are discussed in detail. For the purpose of convenience in presentation, the models have been grouped into broad categories such as: (1) Conceptual Models; (2) Bernoulli Models; (3) Markov Models; (4) Learning Models.

The third chapter deals with research design, the nature of panel data, and product descriptions. Research design includes the purpose of selecting the three specific products for this study and a description of differences among them in terms of consumer use, structure of competition, etc. A detailed account of the published literature, critical and supportive, of the panel data and its validity as a representative sample of the total consumer population, is presented in Appendix 1.

The fourth chapter deals with the analysis and the findings in regard to the varification of the hypotheses proposed in the earlier section. The empirical verification of each of the hypotheses has been presented in appropriate tables. Some of the statistical techniques used in the analysis are described in detail in Appendix V.

The presentation of the study is concluded with the author's remarks, where the findings have been discussed in terms of their implications to the task of building realistic models of consumer's buying behavior.

CHAPTER II

REVIEW OF THE LITERATURE ON MODELS OF BUYING BEHAVIOR

Models are of two types, descriptive and causal. A descriptive model describes the observed phenomena and, at best, the causal relationships involved can only be drawn on an inferrential basis. In a causal model, the experimental design facilitates the understanding of the cause and effect relationships involved in the behavior.

> Herniter:[1] Of primary importance to the marketing executive is a knowledge of how his company's promotional and advertising activities causally affect the sales of its products in a competitive environment. Unfortunately, the marketing process is so complex and our knowledge of it so rudimentary that the achievement of this goal is a problem in research rather than in application. As a step toward creating a causal model of the market, it is necessary to construct a descriptive model that reveals how the market is behaving. That is, before we can offer the reasons for the markets' behavior, we must be able to describe the behavior itself.

Considerable research effort is being directed toward developing various descriptive models of the consumer's buying behavior under different sets of assumptions.

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[1] Jerome D. Herniter and Ronald A. Howard, "Stochastic Marketing Models," working memorandum of Arthur D. Little, Inc. (Not dated).

Descriptive models can be purely conceptual or mathematical formulations. A conceptual model of the consumer and her interaction with the marketing environment is constructed using the existing knowledge of the marketing system and its components. Models of economic, affluent, limited rational, and social class consumers are examples of conceptual models based on different sets of assumptions and are widely published in the literature. One specific formulation presented by Breyer is discussed in detail at a later stage in view of its significance to the present study.

Mathematical models are conceptual. In addition, in a mathematical model all the variables are quantified for measurement and more often the mathematical models are based on empirical verification. Mathematical models of consumer behavior can be further classified into Micro and Macro models. A micro model describes the individual consumer's purchase behavior, and the market is treated as a composite behavior of many individual consumers. As panels are a major source of information on individual consumer purchases, most of the existing micro models make use of panel data. A macro model describes the over-all behavior of the market without any reference to the individual consumer. Macro models are also known as flow or gross models.

Conceptual Model

A discussion of the conceptual framework suggested by Breyer follows.[2] The model basically describes the consumer

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[2]Ralph F. Breyer, "Proposal Formation and Programmed Marketing," unpublished article, Pennsylvania State University.

and her interaction with the marketing environment. According to Breyer, the two elements of the consumer's purchasing environment are: (1) consumer make up; and (2) the total situation. Consumer make up includes the conditioning of the housewife's buying behavior by the past history of brand purchases, advertising exposure, and the exposure of the consumer to the promotional mix of all the competing brands in the market prior to her entering the store to make her purchase. In other words, Breyer coins the words "consumer make up" to include the psychological attitudes of the housewife toward the brand before she enters the store and "total situation" to signify the promotional environment of the brand that the housewife is exposed to in the store at the time of the purchase. Total situation is furthur classified as the physical setting, the proposition of the brand, and the broad environmental factors in the store. The proposition consists of the physical product, the price, and the accompanying services after the sale. The physical setting refers, among other things, to the quality and size of the display, sales talk, and in-store promotional services of the brand that would enhance the sales environment of the brand. In every purchase decision of the housewife, all the sub-elements and elements of this framework play their own important part. Assuming that the basic objective of the firm is to secure the purchase decision of the housewife in favor of the firm's brand, the manufacturer should carefully structure the marketing environment of its brand in each and every retail outlet. The wider implication of the statement is to argue for efficient channel management in addition to the management of advertising and promotion. For example, consumer make up is influenced by media advertising and promotional campaigns, proposition influenced by product development and pricing, and physical setting influenced by the distribution and good working relations at the retail level. Breyer pursues this framework further to argue for the optimal allocation of marketing effort among the various functions and suggests some alternatives for an efficient "programmed marketing."

In the case of frequently purchased consumer goods, price is the only important element of the proposition, as the physical product and accompanying services are more or less same among different brands of the product. However, consumer's relative preferences for different brands are widely varying as evidenced by the frequently observed phenomena of loyalty to brands. Thus, the consumer make up can be described by the past history of the consumer's brand purchases of the product. The physical setting of the brand is likely to be different from store to store. As a first step, let us assume that the brand's relative physical setting within a store remains the same over a period of time. Accordingly, the past history of brand purchases, the price of the product, and the store visited are the three major aspects of the consumer's purchase decision process. It is to be noted that these three elements of purchase decision occupy a central part of my investigation. This conceptual model has a close relationship to this investigation in that housewives with the same history of brand purchases visiting two different stores are likely to choose different brands due to the differences in price and physical setting of the brand between stores. In other words, store-brand interaction should be a major consideration in planning promotion and distribution strategies.

Macro Models

I shall not discuss Macro models in detail as they are not of immediate relevance to the purpose of my study. However, a brief mention of them is in order. Macro models depict the over-all behavior of the market with no consideration of the individual consumer. Frank and Massey[3] analyzed the time pattern of the market shares due to the changes in prices and the deal magnitudes, by fitting a multiple regression of current and lagged price and lagged market share on the current share of the brand in the market. Herniter and Magee[4] discussed some of the advantages of describing the over-all market behavior as a Markov process. Farley[5] hypothesized that the differences in the degrees of loyalty among consumers in different product lines can be explained by the structural variables in the market in which products are sold, such as: (1) the number of brands in

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[3] Ronald E. Frank and William F. Massey, "Short Term Price and Dealing Effects in Selected Market Segments," Journal of Marketing Research, 2 (May, 1965), pp. 171-185.

[4] Jerome D. Herniter and John F. Magee, "Customer Behavior as a Markov Process," Operations Research, 9 (January-February, 1961), pp. 105-122.

[5] John U. Farley, "Why Does Brand Loyalty Vary Over Products?" Journal of Marketing Research, 1 (November, 1964), pp. 9-14.

the product line; (2) the average rate of consumption by the consumer; (3) the degree of the price activity in the product line; (4) the multiple usages of the product; (5) the intensity of the distribution; and (6) the domination of the market by the leading brands.

Micro models

Herniter:[6] There is a growing body of literature on the analysis of panel data for frequently purchased, low cost consumer items. One of the methods of analysis has been the development of market models. The detailed structure of a model of a process is dependent upon the data that is available. Since panel data yield detailed consumer purchase histories, market models have been developed which describe the purchase behavior of individual consumers.

As stated earlier, micro models of buying behavior deal with the individual consumer's purchase decision process. The marketing manager's objective is to achieve the short and long run profit goals of the company. In achieving these goals, he uses different strategies, such as dealer promotions, price promotions, media advertising, et cetera, primarily to change the consumer's purchase decision in favor of his brand. Hence, a model that describes individual purchase behavior should help a marketing executive in allocation of his marketing effort.

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[6] Jerome D. Herniter, "Stochastic market models and the analysis of Consumer Panel Data," presented at the twenty-seventh National Meeting of the Operations Research Society of America, Boston, Massachusetts, May 6-7, 1965.

Herniter:[7] The behavior of the market is the composite behavior of many individual consumers. As a result, the market is a complex probabilistic system that is complicated in its interactions and difficult to observe. Yet, if there is to be any progress in controlling the market process as a whole, we have no alternative but to attempt to analyze it at its most fundamental level -- the activities of the individual consumer.

One particular method of describing individual consumer's purchase behavior is by means of a set of purchase probabilities. It would be very useful if the model could provide a method of revising the set of probabilities due to the passage of time, new purchase experiences, and other marketing influences.[8] With this brief introduction, I shall review some of the micro models.

Bernoulli Models

The simplest postulated brand switching model is the Bernoulli trial model based on the following assumptions:

"(1) In a K brand market each customer has a set of probabilities, p_j, j = 1, 2, . . , K, which define her probabilities of purchasing each brand. The customer maintains this set of probabilities indefinitely.

(2) Each purchase is independent of the customer's previous purchases.

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[7] Jerome D. Herniter and Ronald A. Howard, "Probabilistic Consumer Models," unpublished paper, (Not dated).

[8] Alfred A. Kuehn and Ralph L. Day, "Probabilistic Models of Consumer Buying Behavior," Journal of Marketing, 28 (October, 1964), pp. 27-31.

(3) For each p_j there is a distribution of p_j over the population."[9]

Assuming a beta distribution of p_j over the population, Herniter[10] tested the model with purchase data of different products. The results are not encouraging in terms of the model describing the observed brand switching behavior. The basic disadvantage of the model is that the probability of purchasing a particular brand depends on the number of purchases made by the individual consumer in her purchase history and independent of when they occurred in the sequence. This does not conform with the observed behavior where the recent purchases have more effect on brand choice than the earlier ones. However, if all the previous purchases had been of one particular brand, the model predicts that the probability of subsequently purchasing the same brand increases, and this conforms with the observed behavior.

Let $b_j(n)$ = 1 if brand j is purchased by the consumer n^{th} time,
= 0 otherwise.

According to the Bernoulli model, if the customer has a probability of purchasing brand j (p_j), then according to the model

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[9] Jerome D. Herniter, "Stochastic Market Models and the Analysis of Consumer Panel Data," working memorandum, Arthur D. Little, Inc., (Not dated).

[10] Ibid.

$$\left\{ b_j(n) = 1 \;\middle|\; b_j(n-1) = q_{n-1},\ b_j(n-2) = q_{n-2},\ \ldots\ ,\ b_j(1) = q_1 \right\} = p_j$$

$$j = 1, 2, \ldots, K$$

$$n = 2, 3, 4, \ldots$$

$$q_1, q_2, \ldots, q_{n-1} = 0,1$$

where $\{U\}$ indicates the probability of the event U, and K is the number of brands in the market.

Using the definition of conditional probability, we obtain

$$\left\{ b_j(n) = 1 \;\middle|\; b_j(n-1) = q_{n-1},\ b_j(n-2) = q_{n-2},\ \ldots,\ b_j(1) = q_1 \right\}$$

$$= \frac{\left\{ b_j(n) = 1,\ b_j(n-1) = q_{n-1},\ b_j(n-2) = q_{n-2},\ \ldots,\ b_j(1) = q_1 \right\}}{\left\{ b_j(n-1 = q_{n-1},\ b_j(n-2) = q_{n-2},\ \ldots,\ b_j(1) = q_1 \right\}}.$$

Suppose p_j has a beta distribution with parameters r' and n' over the population in the market.

Then,

$$\left\{ b_j(n) = 1 \;\middle|\; b_j(n-1) = q_{n-1},\ b_j(n-2) = q_{n-2},\ \ldots,\ b_j(1) = q_1 \right\}$$

$$= \frac{\int_{p_j} \left\{ b_j(n) = 1,\ b_j(n-1) = q_{n-1},\ b_j(n-2) = q_{n-2}, \ldots, b_j(1) = q_1 \;\middle|\; p_j \right\} \{p_j\}}{\int_{p_j} \left\{ b_j(n-1) = q_{n-1},\ b_j(n-2) = q_{n-2},\ \ldots,\ b_j(1) = q_1 \;\middle|\; p_j \right\} \{p_j\}}$$

$$= \frac{\langle p_j^{\,c+1}(1-p_j)^{n-1-c} \rangle}{\langle p_j^{\,c}(1-p_j)^{n-1-c} \rangle}$$

where $\langle\ \rangle$ is used to indicate expected values

$$= \frac{r'+c}{n'+n-1}$$

where c is the number of purchases of brand j in the past (n-1)

purchases of the consumer.

Note that the ratio (r'+c)|(n'+n-1) is affected only by the number of the purchases made by the consumer and not when they occurred in the sequence.

Frank[11] used Bernoulli Trial model (although he did not use the name) to analyze panel data for coffee purchases. A run in a purchase sequence is defined as the number of successive purchases of a particular brand by an individual consumer. Frank tested his hypothesis by using the distribution of runs in the purchase sequences of individual consumers. He found a considerable number of families with varying probabilities of brand purchase over time, thus rejecting his hypothesis.

Howard,[12] expanding the simple Bernoulli model by incorporating another stochastic process, suggested an interesting concept for describing the consumer's brand choice. The model is described in detail here in view of its implications to my study, though empirical verification of the model is lacking in the literature. A problem constantly facing the analysis of business systems is that of modeling situations where the underlying statistical parameters of the process may change from time to time. In other words, the parameters of one process are affected by another stochastic process. A method of analysing such a situation is to assume probability distributions of the

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[11] Ronald E. Frank, "Brand Choice as a Probability Process," Journal of Business, 35, (January, 1962), pp. 43-56.

[12] Ronald A. Howard, "Dynamic Inference," Operations Research, 13 (September-October, 1965) pp. 712-733.

underlying process parameters and approximate the behavior predicted by the model to the observed data. Howard coined this process "Dynamic Inference."

Suppose p_1 is the measure of preference of an individual consumer for a particular brand and that a change in p_1 is caused by a stochastic process. Howard assumed that the probability of changing p_1 on each purchase occasion is a constant equal to p_2; whenever a change occurs, a new p_1 is selected from a beta distribution. With these assumptions, Howard published tables depicting the probability of purchasing the given brand given the history of the previous four purchases.

Underlying the assumption of Howard's model is that there are two stochastic processes acting in consumer's brand choice, the latter affecting the parameters of the former. A consumer's brand choice may be inconsistent with her inherent preference for the brand on any particular purchase occasion because of a price cut or a change in the promotional environment of the brand in the store. The consumer's exposure to different advertising slogans between consecutive purchases may also be constantly changing her inherent preferences for different brands. Thus, the brand preferences of the consumer are changed by the external marketing influences. However, this change is not directly measurable and can only be inferred by the observed sequence of brand purchases. Further empirical verification of the model seems to be very promising.

Markov Models

"One approach is to construct a model and then assume that each individual in the market behaves according to the model; the behavior of each individual is the same as the behavior of the aggregate market."[13] Much of the published work describing the consumer's purchase behavior as a Markov process makes this assumption. A basic element of the consumer's purchase decision is the selection of the brand, and it is observed that the selection of the brand by the housewife is not completely random. In other words, the brand choices on consecutive purchase occasions are observed to be not independent. In a first order Markov process, the current purchase decision is described as a function of the most recent purchase. Defining the state of the system by the brand purchased within a time period, the brand switching behavior has been approximated by a first order Markov process. A brief description of the model follows.

Let the different brands of a product in the market be denoted by i = 1, 2, .. K, where K is the total number of brands in the market. The sequence of brand purchases made by a housewife can be represented by a vector whose n^{th} element is b(n).

$$[b(1), b(2), \ldots, b(n), b(n+1), \ldots]$$

where b(n) = i if i^{th} brand is purchased on n^{th} purchase occasion.

In a first-order Markov process, we assume that the probability of the housewife purchasing brand j for her n^{th} purchase depends only on her $n\text{-}1^{th}$ purchase and is independent of all the previous purchases. Let p_{ij} denote the probability that the

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[13] Donald G. Morrison, "New Models of Consumer Loyalty Behavior," Proceedings of the Fall Conference of the American Marketing Association, (1965), pp. 323-337.

purchase of j^{th} brand follows the purchase of i^{th} brand, on any two consecutive purchases.

$$p_{ij} = \{b(n) = j \mid b(n-1) = i\}$$

$$i, j = 1, 2, \ldots, K$$

$$n = 2, 3, \ldots$$

Transition-Probability Matrix (P)

Initial state \ Final state	1	2	·	j	·	K
1	p_{11}	p_{12}	·	·	·	p_{1K}
2	p_{21}	p_{22}	·	·	·	p_{2K}
·	·	·	·	·	·	·
i	·	·	·	p_{ij}	·	·
·	·	·	·	·	·	·
K	p_{K1}	p_{K2}	·	·	·	p_{KK}

The estimates of the transition probabilities can be obtained by observing the proportionate number of times, purchase of brand 'j' followed the purchase of brand 'i' in every pair of consecutive purchases for a consumer and aggregating over all the consumers. Denoting the purchase probability vector of a housewife after n time periods by $\Pi(n)$ and the transition probability matrix by P, we have the relation according to the model $\Pi(n + 1) = P\,\Pi(n)$. Successive substitutions will give $\Pi(n) = P^n\ \Pi(0)$ where $\Pi(0)$ denotes the initial vector of the probabilities of the housewife purchasing the different brands.

Knowledge of P and $\Pi(0)$ will enable us to predict through the model the probabilities of the housewife purchasing different brands after n time periods.

The model can be interpreted by tracing the individual probabilities of the housewife purchasing brands 1, 2, . . . , K in different time periods. This is based on the assumption that each individual in the market behaves in the same manner. For empirical evaluation of the model, the initial probability vector of the housewife is necessary. It is easily seen that the sum of the elements in every row of the transition matrix adds up to one because a purchase of the product must necessarily mean that one or another of the k brands has been purchased.

Herniter:[14]

> The Markov process is a very rich model for random phenomena. Many interesting parameters of the process can be analytically calculated and many of these parameters have important marketing interpretations. In fact, it might be said that the richness of the process has led to its abuse in marketing applications. There are very few cases indeed where the Markov process is an appropriate model in its present form.

While describing the random phenomena in the market, the Markov model provides a better description of certain changes in the market which traditional brand share data fails to provide. Three interesting properties of the Markov model are: (1) steady state equilibrium; (2) average staying time; and (3) average return time.[15]

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[14] Jerome D. Herniter and Ronald A. Howard, "Stochastic Marketing Models," working memorandum of Arthur D. Little, Inc., p. 44.

[15] Jane E. Draper and Larry H. Nolin, "A Markov Chain Analysis of Brand Preferences," Journal of Advertising Research, 4 (September, 1964), p. 35.

As the number of transitions tend to infinity, the Markov chain approaches a steady or equilibrium state. In this steady state the probability p_i that the Markov process is in any particular state i, is constant from trial to trial. The steady state values for brand purchase data would signify the long-run market shares of the brands.

We have the relation

$$\Pi(n + 1) = P\Pi(n)$$

$$\text{i.e.,} \qquad \Pi(n) = P^n\Pi(0)$$

Whatever brand purchase we start with in the initial state and if there are no trapping states in the system, $\Pi(n)$ approaches a steady state solution as n tends to infinity. (State i would be a trapping state if $p_{ii} = 1$; it follows then $p_{ji} = 0$ for $j \neq i$). We assume here that the transition matrix (P) remains the same over time. This is called the stationarity assumption of the Markov process. In a non-stationary Markov process, transition probabilities (p_{ij}) change with time and the mathematics of such a process becomes complicated.

Assuming that the Markov process is in a steady state, the expected number of periods the process stays in state 'i' is equal to $\frac{1}{1-p_{ii}}$. In marketing terminology, this is the expected number of times the consumer will purchase brand i in successive periods. Again, assuming the steady state, the expected number of time periods before the consumer returns to brand i after switching from brand i is given by $1/p_{ii}$.

Though these properties inherent in the model are of some interest, they are of little practical value as it is difficult to encounter any market situations operating under the steady state conditions or a true discrete time Markov process. The continuous nature of the competition among the brands not only makes it impossible to attain the steady state situation, but also changes the transition probabilities over time. Thus, the stationarity assumption of the Markov process is mere abstraction of the simplified reality.

In addition to the limitations of these assumptions, an operational problem often faced in fitting the model to the brand purchase data has been the division of the time into discrete time periods. In the model the state of the system is defined by the brand purchased during a certain period of time (a week, a month, etc.). The state is not well defined if two or more purchases, or no purchases are made during the time period. By appropriately selecting the time period, one can reduce the frequency of having two or more purchases. This would, however, increase the number of time periods when no purchase is made by the housewife. By creating a fictitious "no purchase" brand, the state can be defined as purchasing this fictitious brand of the product when, in fact, no purchase is made. Before we discuss some of these limitations in the conventional Markov chain analysis, let us briefly review the work of a few authors who put these models to use.

A manufacturer is not only interested in the dynamics of his market share, but also in the nature of his purchasers. It would be useful for him to know whether his brand's share depends on stable purchasers or brand switchers. In order to bring out the inherent dynamics of the market, Lipstein[16] divided the market into hard core and switcher components, and separately calculated the transition matrix (P) for the two groups. The hard core consumers are those who devoted 75 percent or more of their purchases to a particular brand. He finds the transition matrix for the switcher category closer to the steady state. Lipstein found this division of the market useful for the purposes of advertising allocation and evaluation of the new brand introductions.

However, the nature of Lipstein's division of market is self-defeating. By his very definition the hard core consumer is one who has higher repeat purchase rates (p_{ii}) of a particular brand. By removing these people, one would always end up with consumers who do not entertain any major preference for a brand and this market would be equally divided among all brands. Thus, the transition probabilities aggregated over the consumers are more or less the same. As such, the division into hard core and switcher groups is more meaningful if it is based on characteristics such as family size, income group, education, or a measure of advertising exposure, etc.

In another article, Draper and Nolin[17] approximated a first

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16 Benjamin Lipstein, "The Dynamics of Brand Loyalty and Brand Switching," Proceedings of the Fifth Annual Conference of the Advertising Research Foundation, (1959), pp. 101-108; by the same author and Harry Frank, "Dynamics of Brand Loyalty: A Markovian Approach," Operations Research, 10 (January-February, 1962) pp. 19-42.

17 Jean E. Draper and Larry H. Nolin, "A Markov Chain Analysis of Brand Preferences," Journal of Advertising Research, 4 (September, 1964), pp. 33-38.

order Markov process to brand switching behavior of housewives for cake-mix purchases. The three-year data has been divided into twelve time periods of three months each. Assuming a stationary Markov process, the authors separately calculated the transition probabilities for periods one to six and seven to twelve. The state of the system for a consumer in a particular quarter is labeled by the brand that received her largest expenditure. Defining the core customers as those who successively purchased the same brand in two time periods, the authors found that core users account for a relatively small portion of the market. Roughly 95 percent of the original users switched to other brands within a period of six months. The study also high-lighted some of the changes in the transition probabilities from one period to another due to the introduction of a new brand in the market. The way the authors defined the state of the system, the model depicts the phenomena of one mix of purchases to another rather than brand switching. This can be treated more as a flow model of consumer expenditures among the brands rather than a probabilistic model describing the underlying phenomena of the brand choice.

Thus far, we have been trying to describe the brand switching behavior by a first order Markov process. In an article by Styan and Smith,[18] the authors approximated product switching behavior between two substitutes by a Markov process. Each

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[18] George P. H. Styan and Harry Smith, Jr., "Markov Chains Applied to Marketing," Journal of Marketing, 1 (February 1964), pp. 50-55.

housewife has been characterized by one of the following four mutually exclusive and collectively exhaustive categories for each of the weeks of the study:

(1) Bought detergent only

(2) Bought soap powder only

(3) Bought both detergent and soap powder

(4) Bought no laundry powder.

Using this four-fold classification and the data collected for twenty-six weeks from a British panel of 100 housewives, the authors computed the twenty-five possible transition probability matrices for every pair of consecutive weeks. They tested for the stationarity assumption of the Markov process and established a first order stationary Markov process closely depicting the switching behavior of the market between the two products and used the model for predicting future market shares. This is an interesting extension of the Markov Model to depict the switching behavior between two substitute products.

Howard[19] and Herniter[20] raise some of the objections to conventional Markov analysis to business problems. The discussions of both the authors center around three points: (1) problem of aggregation; (2) arbitrary division of time into discrete time periods; and (3) problems of estimating the transition matrix. The problem of aggregation deals with how to generalize the market behavior from that of an individual, when

[19]Ronald A. Howard, "Stochastic Process Models of Consumer Behavior," Journal of Advertising Research, 3 (September, 1963), pp. 35-42.

[20] Jerome D. Herniter and Ronald A. Howard, "Stochastic Marketing Models," working memorandum, Arthur D. Little, Inc., (not dated).

the stochastic model describes the individual purchase behavior. In our present discussion, the individual purchase behavior is described by a discrete-time, first-order Markov process. The authors contend that the Markov process model is often treated as a flow model to predict the number of customers who would purchase a particular brand in successive periods of time. The model is not describing the flow of consumers but the change in the individual consumer's probability of purchasing a particular brand from purchase to purchase, and should be interpreted accordingly. This problem of aggregating the individual behavior to model the over-all market needs more attention.

Dealing with the problem of arbitrary division of time into discrete intervals, the authors suggest a modification that would incorporate the time intervals between purchases into the model. This would eliminate the need for the introduction of the fictitious "no purchase" brand in the model. In the generalized Markov process, known as the semi-Markov process, the time required for a transition (t) from state 'i' to 'j' is not fixed, but a random variable. Let the density function of this random variable, t, be denoted by $h_{ij}(t)$. In other words, the duration of the system in state i is selected from the density function depending on the transition to be made. The transition probabilities are estimated in the same manner as described earlier and the density function $h_{ij}(t)$ can be obtained by constructing a frequency distribution of the time between purchases whenever the purchase of brand j follows the purchase of brand i. The semi-Markov process is completely specified by the knowledge of p_{ij} and $h_{ij}(t)$ for all pairs of states i and j. Semi-Markov processes have limiting behaviors similar to that of Markov processes as the number of transitions increase and we do not go into the details here.

The generality provided by the semi-Markov process can incorporate the package size or frequency of purchases into the model. However, the problem of aggregation, as described in the earlier pages, remains unsolved for the semi-Markov process as well. Though semi-Markov process has conceptually solved some of the disadvantages existing in the simple Markov process, no empirical verification of this model describing the consumer's buying behavior has been published in the literature.

The third problem is the estimation of transition probabilities. Limited available data on the history of individual consumer purchases poses the problem of accurately estimating the transition probabilities. Also, the non-stationary nature of the market makes it difficult to incorporate new test data with the apriori knowledge of the transition probabilities. The solution is still a problem of academic research.

Even assuming that all these limitations of the Markov process models can be overcome, we still have the basic assumption that the individual's brand choice is independent of all the purchases she made prior to her last purchase which, by itself, is debatable. An extension of the first-order Markov process has been the n^{th} order Markov process.

In an n^{th} order Markov process, the probability of an individual purchasing a particular brand on her $n + 1^{th}$ purchase depends on her last n purchases. Accordingly, the transition matrix consists of conditional probabilities like

$$\left\{ b(n+1) = 1 \,\middle|\, b(1) = i_1,\ b(2) = i_2,\ b(3) = i_3,\ \ldots, b(n) = i_n \right\}$$

$$i,\ i_1,\ i_2,\ \ldots,\ i_n = 1,\ 2,\ \ldots,\ K.$$

In estimating the transition probabilities for approximating an n^{th} order Markov process, we have to consider sequences of n + 1 ordered purchases and calculate the proportionate number of times purchase of brand i follows the sequence of n brand purchases $i_1, i_2, \ldots, i_n$. I do not go into the details of higher order Markov processes here as they have not been extensively used in describing the consumer's buying behavior. However, it is to be noted that some authors[21] have worked with the second order Markov process in analyzing purchase data.

At this point, the presentation of two types of first order Markov process models described by Morrison[22] is in order. These are called brand loyal and last purchase loyal models. Let the original K-brand market be reduced to a two-brand market, where the brands are denoted by '0' and '1'; '1' indicating a particular brand and '0' any other brand. The brand purchase vector of any individual consumer can be represented by a sequence of 0's and 1's. The transition matrix of brand loyal model is given below.

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[21] Jerome D. Herniter, "Transition Matrices and Purchase Sequences," working memorandum, Arthur D. Little, Inc., (1964).

[22] Donald G. Morrison, "New Models of Consumer Loyalty Behavior," Proceedings of the Fall Conference of American Marketing Association, (1965), pp. 323-337.

		Brand purchased at time t	
		0	1
Brand purchased at time t-1	0	1-kp	kp
	1	1-p	p

"According to the model, an individual with a high probability of re-purchasing brand '1' (a high p) will also have a high probability of leaving brand '0' to buy brand '1'."[23] According to this model, the loyalty of the consumer is directed toward a particular brand. The assumptions of the model are:

(1) Each individual's purchase behavior in the market is described by the Markov process with the above transition matrix.

(2) 'p' is distributed beta (a,b) among the families of the population.

(3) 'k' is a constant, same for all families ($0 \leq k \leq 1$).

Instead of assuming the same transition matrix for all the consumers in the market, this model assumes a distribution of transition matrices in the population. This seems to be a realistic extension of the Markov Models discussed earlier.

The last purchase loyal model is based on the same assumptions as the brand loyal model, with a different structure of the transition matrix as given below:

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[23] Ibid.

Brand purchased at time 't'

Brand purchased at time t - 1	0	1
0	kp	1-kp
1	1-p	p

In this model, loyalty of the consumer in purchase choice is directed to the last brand purchased, in an opposite direction to that of the brand loyal population. "With the same notations of '0' and '1', an individual with a high probability of remaining with brand '1' (a high p) will also have a high probability of remaining with brand '0'."[24] Morrison claims that the brand loyal model gave a better description of the consumer's brand choice than the last purchase loyal model. In other words, he finds that consumers are more likely to have a favorite brand of a product rather than be affected by their most recent purchase.

All the Markov process models we have discussed so far use the information on the past history of brand purchases in order to describe the consumer's subsequent brand choice. No attempt has been made to incorporate any other marketing variables such as distribution of the brand, size of purchase, price, et cetera.

Learning Models

Markov process models describe the brand switching behavior by means of a set of transition probabilities. Identifying

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[24] Ibid.

the state of the system by the brand purchased, the model predicts the probability of a consumer purchasing a particular brand in a subsequent state. Bernoulli models describe the brand choice by the consumer's probability of purchasing a particular brand. No transition is involved here, but the inherent probabilities are treated either as independent of the past brand choices, or as transformed by the external marketing influences.

Kuehn[25] hypothesized that consumers change their purchase probabilities of a brand every time they make a purchase. In other words, the probability of consumer's purchasing a particular brand changes continuously depending upon the choice of the brand on her previous purchase occasion. The linear learning model presented below describes the postulated learning process underlying the consumer's brand choice in terms of her purchase probabilities.

The model was originally used by Bush and Mosteller to describe the learning behavior of rats. Kuehn used the model to describe the purchase data from the consumer panel. The assumptions of the model are:

(1) Every consumer has a probability of purchasing each brand in the market on each purchase occasion.

(2) If the consumer purchases a particular brand on one purchase occasion, her probability of purchasing the same brand for her next purchase

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[25] Alfred A. Kuehn, "Consumer Brand Choice as a Learning Process," Journal of Advertising Research, 2 (December, 1962), pp. 10-17.

increases and her probability of purchasing each of the other brands decreases.

Let $\Pi_j(n)$ denote the apriori probability of purchasing brand j on the n^{th} purchase occasion and assume a K-brand market for the product.

We use the notation b(n) to denote the brand purchased on the consumer's n^{th} purchase occasion.

Thus $\quad \Pi_j(n) = \{b(n) = j\}$

The brand purchase sequence of a consumer can be represented by a sequence of 0's and 1's by defining the variable $b_j(n)$ as follows:

$$b_j(n) = \begin{cases} 1 \text{ if } b(n) = j \\ 0 \text{ if } b(n) \neq j \end{cases} \qquad \begin{matrix} j = 1, 2, \ldots, K \\ n = 1, 2, \ldots \end{matrix}$$

Let $p_{ij}(n)$ denote the conditional probability of purchasing brand j for her n^{th} purchase given that brand i was purchased on the $n\text{-}1^{th}$ purchase occasion.

$$p_{ij}(n) = \{b(n) = j \mid b(n-1) = i\} \qquad 1 \leq i,\ j \leq K$$

In the linear learning model it is assumed that if the consumer purchases a particular brand on her $n\text{-}1^{th}$ purchase occasion, her probability of purchasing each of the brands for her n^{th} purchase are linear transformations of the purchase probabilities she had prior to her $n\text{-}1^{th}$ purchase. Thus, the equations of the linear learning model are:

$$p_{jj}(n) = gU_j + (1-g)\,\Pi_j(n-1) \qquad \begin{matrix} j = 1, 2, \ldots, K \\ n = 1, 2, \ldots \end{matrix}$$

$$p_{ij}(n) = gL_j + (1-g)\,\Pi_j(n-1) \qquad \begin{matrix} 1 \leq i \neq j \leq K \\ n = 1, 2, \ldots \end{matrix}$$

We have

$$\left\{b(n) = j\right\} = \Pi_j(n) = \begin{cases} p_{jj} & \text{if } b_j(n-1) = 1 \\ p_{ij} & \text{if } b_j(n-1) = 0 \end{cases}$$

So

$$\begin{aligned} \Pi_j(n) &= p_{jj}[b_j(n-1)] + p_{ij}\ [1-b_j(n-1)] \\ &= [gU_j + (1-g)\Pi_j(n-1)]\ b_j(n-1) \\ &\qquad + [gL_j + (1-g)\Pi_j(n-1)]\ [1-b_j(n-1)] \\ &= gL_j + (1-g)\Pi_j(n-1) + g(U_j-L_j)\ b_j(n-1) \\ &= L_j + (1-g)\ [\Pi_j(n-1) - L_j] + g(U_j-L_j)\ b_j(n-1) \end{aligned}$$

Successive substitutions will give

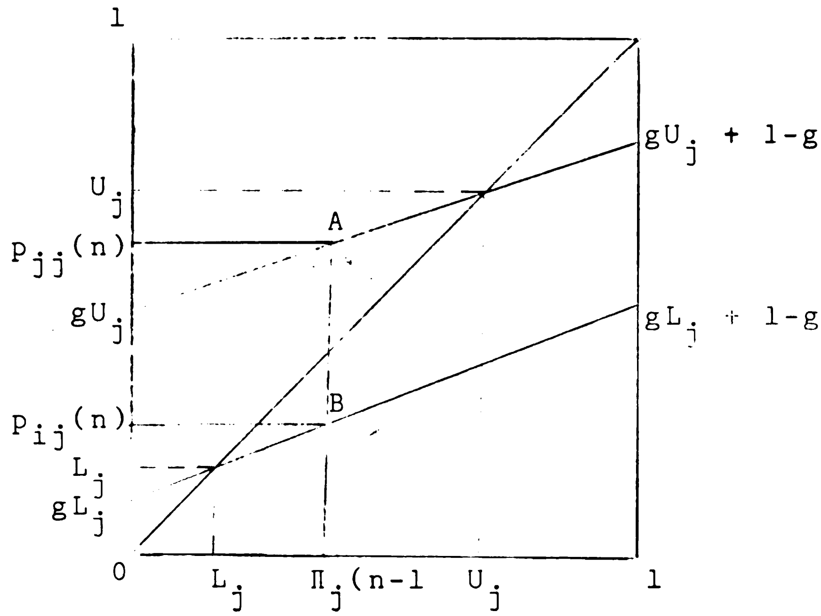
$$\Pi_j(n) = L_j + (1-g)^{n-1}\ [\Pi_j(1) - L_j] + g(U_j-L_j) \sum_{k=0}^{n-2} (1-g)^k b_j(n-1-k)$$

The above equation can be written as

$$\Pi_j(n) = A_j + B_j \sum_{k=0}^{n-2} c^k\ b_j(n-1-k) \qquad \begin{matrix} j = 1, 2, \ldots, K \\ n = 2, 3, 4 \ldots \end{matrix}$$

So the probability of purchasing brand j on the n^{th} purchase, $\Pi_j(n)$, is expressed as a linear combination of the previous n-1 purchases expressed in terms of the purchases of brand j ('1' if brand j is purchased, '0' otherwise). The ratio of the coefficients of successive purchases is constant.

The graphical presentation of the model is given below.



The parameters L_j and U_j are called the steady state probabilities of the consumer purchasing brand j (1) if she never purchased brand j, (L_j), and (2) if she always purchased brand j (U_j) respectively. According to the model, a housewife starting with an initial probability zero of purchasing brand j and continues to purchase some other brand, will have her probability of purchasing brand j, $\Pi_j(n)$, increase until it is equal to L_j. Similarly, a housewife starting with a probability one of purchasing brand j and continues to purchase brand j, will have her probability of purchasing brand j, $\Pi_j(n)$, decrease until it is equal to U_j. Thus, the long run values of $\Pi_j(n)$ for any housewife should belong to the closed interval (L_j, U_j). Extending the same reasoning for all the brands in the market, the probability that the consumer will purchase any particular brand is never zero. It follows then that the probability of purchasing a particular brand is never equal to one.

The arbitrary constraints imposed by the learning model on the consumer's probability of purchasing the brand is a strong limitation of the model. Whatever the historical purchase sequence may be, the probability of a consumer purchasing brand j can never be greater than U_j and can never be less than L_j. This limitation is a result of the assumption in the model that the incremental learning from purchase to purchase is a linear process.

By treating the last four purchases of the consumer as independent variables and the fraction of consumers purchasing a particular brand on the fifth purchase occasion as the dependent variable, Kuehn fitted a linear regression on purchase sequences of frozen orange juice product to check the validity of the model. Evidence of similar analysis done on a few other products shows the general tendency of the hypothesized learning behavior in the consumer's brand choice. The problem of aggregation that we discussed in the case of the Markov process model applies here with equal force. Frank[26] felt that Kuehn's finding of associate learning might have been a spurious relationship due to the aggregation over different families with constant but different probabilities of purchase. However, proof was to the contrary.

Though the model seems to have given a good description of the purchase data, it has many disadvantages. The model,

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[26] Ronald E. Frank, "Brand Choice as a Probability Process," Journal of Business, 35 (January, 1962), pp. 43-56.

as presented by Kuehn, ignores the influence of other marketing variables on consumer's brand choice. Herniter[27] pointed out that the basic problem with the formulation of the learning model is that there is no provision for incorporating the effect of promotion. He further suggests that the model, by taking into consideration the continuous nature of time, can study the effects of the time-interval between purchases; thus, the effect of advertising during inter-purchase time can be incorporated.

Kuehn[28] mentioned that the interval between purchases has a decreasingeffect on the re-purchase probability of the brand, though empirical verification of his claim is lacking in the published literature. However, if this were true, learning would be more important for frequent purchasers than for occasional buyers of the product. Also, according to the model, the purchase probabilities fluctuate widely from purchase to purchase depending upon the purchase sequence. It is difficult to understand why the probabilities of purchasing a brand should fluctuate as widely as suggested by the model. Non-availability of a particular brand or a price cut might have induced the housewife to purchase some other brand without any change in her inherent preference for her favorite brand. But the model predicts a considerable change in the

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[27] Jerome D. Herniter and Ronald A. Howard, "Stochastic Marketing Models," working memorandum of Arthur D. Little, Inc., (not dated).

[28] Alfred A. Kuehn, "Consumer Brand Choice as a Learning Process," Journal of Advertising Research, 2 (December, 1962), pp. 10-17.

probability of purchasing the favorite brand. In this sense the linear learning model seems to be exaggerating minor differences in the individual's probabilities of purchasing a particular brand, though on an aggregate the model seems to be giving a good description of the purchase data.

Relaxing the assumption on the linearity of the learning process, Howard[29] discusses some of the generalized learning models about which I will not go into the details here. Above all these problems, as yet, Kuehn has not published any method of estimating the parameters of the model. A suitable method of estimating the parameters may lead us into future research in expressing these parameters as functions of external marketing influences.

Closely related to the linear learning model is the gain-loss analysis suggested by Rohloff[30] to study market dynamics. Gain-loss analysis differs from purchase to purchase probabilistic analysis in that brand switching of a household is studied from one time period to the next. This analysis is based on the change in the mix of purchases. This method has specific application when the products have multiple usages.

"A common rule of thumb is that the gains of a brand come from each brand in the market in proportion to the market share

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[29] Ronald A. Howard, "Learning Models," working memorandum of Arthur D. Little, Inc., (1965).

[30] A. C. Rohloff, "New Ways to Analyze Brand to Brand Competition," in S. Greyser (ed.) Toward Scientific Marketing: Proceedings of the American Marketing Association Convention, Chicago (1963).

of the contributing brand:

$$g_{ij} = \lambda\beta_i \, S_{j,n-1} \qquad i \neq J$$

Where g_{ij} = share point gain of brand i from brand j

$S_{j,n-1}$ = share of market of brand j in the previous period

$\lambda\beta_i$ = a constant for brand i."[31]

Rohloff claims that the above rule of thumb is sufficiently accurate. Gain-loss analysis is useful in studying the dynamics of aggregate sales volume among the brands in the market, but not to our goal of modeling the individual brand switching behavior.

Brand and store loyalties

I shall discuss here one author's work which can be singled out as the only piece of published research on the consumer's store loyalty behavior and its relation to her brand loyalty. Brand-store interaction constitutes a major part of my study.

Cunningham[32] described the shopping habits of the consumers in terms of their brand and store loyalties. He defined

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[31] Ibid.

[32] Ross M. Cunningham, "Brand Loyalty and Store Loyalty Inter-relationships," Proceedings of the National Conference of the American Marketing Association, (June, 1959), pp. 210-214; by the same author, "Brand Loyalty, What, Where, and How Much," Harvard Business Review, 34 (January-February, 1956), pp. 116-128; and "Customer Loyalty to Store and Brand," Harvard Business Review, 39 (November-December, 1961), pp. 127-137.

the first and second brand loyalties in the following manner. The first loyal brand of a housewife is the one which receives the maximum amount of her product purchases. If housewife A purchases 80 units of brand A and 40 units of brand B and no other brand during a certain period, she is recorded as 67% loyal to the brand A, 33% loyal to the brand B, and not loyal to the other brands; thus, A is her first loyal brand and B her second loyal brand. In a similar fashion, the first loyal store of a housewife is defined as the one that receives the highest proportion of the family's total food purchases; in all, 18 food products are considered. Using these definitions, the author concluded the following: (Unless mentioned otherwise, loyalty refers to the first loyal brand or store)

(1) Families vary widely in their first store loyalties; store loyalty is independent of the total amount of food purchases made by the housewife.

(2) Families are more loyal to the chain stores than to the independent and the specialty stores.

(3) High store loyal families are more loyal to the private brands they purchase than are families with low store loyalty.

(4) Consumption characteristics and socio-economic characteristics appear to have no relationship to brand loyalty.

(5) No significant relationship exists between the brand purchasing and the store purchasing behavior of a housewife.

(6) There is little or no relationship between the size of purchase and the brand loyalties in the product groups.

Though defining the store and brand loyalties by the market share concept (as defined by Cunningham, oriented toward the distribution of the total purchases of a housewife) has some conceptual advantages, the results could be misleading. Let two housewives, A and B, purchase 200 units of a product, where A has shopped twice and B ten times. If both purchased 100 units of a particular brand, they are treated on an equal basis regarding the brand loyalty. However, the two consumers are different in terms of their having to make purchase decisions, A twice and B ten times. The significance of a housewife making her purchase decision in each of her shopping trips is lost by lumping all the purchases together. Some of the hypotheses tested by Cunningham are related to those proposed in my study, but the approach followed to analyze the data is different; my approach to the study is on a probabilistic basis.

During the review of the published research, detailed discussion has been limited to the models of immediate interest to the study. A brief summary at this point appears in order. Bernoulli, Markov and learning models are the three types of micro models extensively published in the literature describing the consumer's brand choice as a stochastic process. However, all these models have one thing in common. They used only the information on consumer's past history of brands purchased to predict the probability of her subsequent brand purchase. Empirical evidence is sparse in the literature where these models have incorporated any other marketing variables such as price, advertising, distribution, et cetera. The basic assumptions underlying these three types of models and their limitations are discussed in detail.

Published research by Cunningham is the single piece of evidence where the relationship between the housewife's loyalty to a store and to a brand is investigated. The study of this interaction between brand choice and store choice as well as its incorporation into the Bernoulli, Markov and learning models of brand purchasing behavior is the central focus of the remaining document.

CHAPTER III

DESIGN OF THE STUDY

Introduction to Data Base

Consumer panels have been the major source of information on purchase data in developing the micro models of consumer buying behavior. Barton defines a panel as "a controlled array of original data sources which permit current and repetitive examination of certain phenomena through a time series."[1] More specifically, "the consumer panel is a group of consumers so selected that it constitutes a representative sample of the market to be appraised. Such a representative panel is composed of a group of consumers properly weighted as to their income, age, sex, education, occupation and such other characteristics so as to conform to the national or regional pattern of the market under consideration."[2]

The Chicago Tribune panel consists of 750 families "who keep a chronological record of their purchases of food and household items. For each purchase in a given product class,

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[1] Samuel G. Barton, "The Consumer Pattern of Different Economic Groups," Journal of Marketing, 8 (July, 1943), pp. 51-53.

[2] Archibald S. Bennett, "Consumer Panels: Radar of the Sales Department," Sales Management, 55 (October 15, 1945), pp. 155-156

information is available as to the family's code number, selected demographic characteristics of the family, brand purchased, date, quantity, price, the type of outlet, and whether or not a deal was used in making the purchase."[3] As panel data provides a continuous record of the purchase information for an extended period of time, it is most suitable for developing the stochastic process models of consumer buying behavior.

Since the findings of any study based on the panel data are generalized from the observed behavior of the sample of housewives selected in the panel, a considerable amount of research effort has been directed in the past to assess the accuracy of the panel data and its representativeness of the general population. The detailed discussion of the problems involved in the panel administration, reliability and accuracy of panel data are deferred to a later stage, in Appendix 1. However, from the published evidence, it is reasonable to assume that the conclusions drawn from the data are not typical of any particular sample but depict the general tendencies of the consumers in the over-all market.

Research Design

Three different consumer products have been selected for the purpose of the study. The criterion for selecting the

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[3] Ronald E. Frank, "Brand Choice as a Probability Process," Journal of Business, 35 (January, 1962), p. 43.

three is rather arbitrary. The study of three products is better than one in terms of generalizing the results. On the other hand, costs of the computer runs and the time factor in analyzing the data were major constraints in extending the analysis to a few more products. At any rate, the three products selected are so distinct in terms of their market structures, distribution methods, rate of consumption by the consumer, and their importance to the consumer that the generality of the findings can be established.

The purchase data of these three products, covering the period of three years, 1960-61-62, is obtained from the Chicago Tribune. The Chicago Tribune restricts its sample of housewives to the Chicago metropolitan area. Some other panels, like MRCA, select the samples to represent the national market. As this study concentrates on the interaction of the housewife's brand choice with her choice of the store and the size of purchase, it is deemed suitable to restrict the data to a metropolitan area rather than the national market. The advantage of this restriction is that one can reasonably assume that all consumers are equally exposed to the different promotional influences of the brands and the stores within the area. In addition, the differences in the factors of availability of the brands and general price levels among different stores of the same chain are less likely to be prominent within the same metropolitan area; thus, we are able to study the behavioral patterns of the consumers on an equal basis.

The three products of the study are: (1) a paper product;

(2) a drug product (toothpaste); and (3) a food product (coffee), denoted for the purposes of convenience by A, B, and C respectively. Variables of the purchase which are of major concern to us are: (1) the brand of purchase; (2) the store of purchase; (3) the size of purchase; and (4) the price of purchase. The detailed description of the product data is presented in the later sections of the chapter.

Though the original data supplied by the Tribune includes the purchase histories of the families who either have dropped from the panel at some intermediate stage or missed a few diaries in reporting their purchases, care is taken to include only those families who have reported continuously for the period of three years (1960-62) without missing any single weekly diary. However, in order to be able to measure any over-all effects on the findings due to the inclusion of the families with missing diaries, the data on product B has been analyzed twice, once as it is supplied by the Tribune, and secondly after eliminating the families with missing diaries. Denoting the second part of the analysis by B', Appendix IV presents the findings of the two analyses side by side for comparison purposes. No significant differences in the over-all findings are observed. For each of the families the Tribune maintains activity cards which list receipt of the weekly diaries during their stay as panel members. This enables us to determine the number of missing weekly diaries during any particular period.

Method of Analysis

Individual consumer purchase decisions regarding the brand, the store, and the size of purchase are treated on a probabilistic basis. Sequences of two, three, and four successive purchases by a housewife are aggregated over different individuals and the period of time to describe the housewife's subsequent brand choice as a function of her past history of brands purchased and stores visited. The brand choice analysis often describes the individual's probability of purchasing a particular brand, say M, given the past history of her brands selected in terms of purchasing brand M (denoted by '1') or purchasing some other brand (denoted by '0'). The brand choice analysis in the study has been done separately for each of the major brands of the three products.

Analysis of the consumer's choice of the store is done separately for the purchase of each of the three products. An attempt has been made to include the different types of retail outlets. For studying store loyalty, only two types of stores could be incorporated, drug chains and food chains. However, for the analysis of loyalty to the type of outlet (by grouping all stores of the same type) three types of outlets could be studied; discount stores, independent food stores, and independent drugstores. Small drugstores could not be studied for individual store loyalty as fewer purchases are made in these stores by the panel sample.

Three major food chains in the Chicago area have an established market position for their private labels of coffee. Individual consumer's proneness to purchase private labels is obtained by estimating the conditional probabilities of purchasing private labels given the history of her past one or two purchases. In addition, frequent visiting of a store by a housewife is related to her proportionate number of private label purchases. For this purpose, the store loyalty index of a housewife is computed by defining it as the proportionate number of times the housewife visited the same store for the purchase of coffee in two consecutive purchases.

Statistical tests (Chi -square test, test of equality of proportions) are used wherever applicable to test the significance of null hypotheses. All the statistical null hypotheses are tested both at 5% and 1% levels of significance. Analysis of covariance is used to study the interaction of brand and store at an aggregate level, as well as to study the over-all effect of the price on the market share of the brand. The monthly market share of a brand in a store is treated as the dependent variable and the corresponding monthly average price of the brand in the store is taken as the concomitant variable. Detailed description of the technique is presented in Appendix V. In order to make it manageable, the analysis has been restricted to four major brands of the product and ten top outlets that contribute to nearly 80% of the market sales. In order to increase the readability of the document, definitions of the technical words and general notation followed in this study are presented in detail in Appendix II.

The computer programs for the analysis of the data have been developed by the author, since no standard routines suitable to the CDC 3600 computer are available to the specific nature of the problems under study. The detailed listings of the programs are not included here as a part of the presentation, but can be obtained from the author through a written request. The language of the programs has been the regular Fortran written for the Computer CDC 3600. The Chicago Tribune supplied the purchase data in the form of punched cards and the data has been reproduced on the magnetic tape after the necessary sorting of the cards.

Product Descriptions

The three products are similar in the sense that they all belong to the category of frequently purchased consumer goods. Promotional campaigns and price deals are frequent in the market for the three products both by the manufacturers and the retailers. Manufacturers' advertising on television and other media is heavy for promoting their brands of the products. However, the differences among the products are more important here than the similarities in terms of generalizing the findings.

Among the three products, Product C (coffee) is purchased more frequently, followed by Product B (toothpaste) and Product A (paper product). Frequency of purchasing a product is likely to contribute to better knowledge of the brands available in the market as well as to enable a housewife to make better price comparisons at the time of the purchase. Tables 1 and 2 present the characteristics of the distribution of families according to their rate of consumption of the product during the period of study.

Tables 3 and 4 describe the structure of the market com-

TABLE 1

PRODUCT, NUMBER OF FAMILIES, AND PURCHASES "PER FAMILY"

Product Code	Number of Families	Number of Purchases	Average Purchase per Family per Year
A	356	4297	4.02
B	777	10423	4.47
B'	356	5382	5.04
C	387	22888	19.71

TABLE 2

FREQUENCY DISTRIBUTION OF PURCHASES AMONG FAMILIES

No. of Purchases in 3 Yrs. (Frequency group)	Number of Families					
	Product A		Product B'		Product C	
	Units	Percentage	Units	Percentage	Units	Percentage
1 to 5	104	29.2	98	27.5	27	7.0
6 to 10	83	23.3	69	19.4	15	3.9
11 to 15	65	18.3	55	15.4	25	6.5
16 to 20	41	11.5	37	10.4	16	4.1
21 to 25	26	7.3	35	9.8	11	2.8
26 to 30	19	5.3	24	6.7	13	3.4
31 to 35	8	2.2	12	3.4	10	2.6
36 to 40	5	1.4	9	2.5	22	5.7
41 to 45	1	0.3	5	1.4	21	5.4
46 to 50	1	0.3	4	1.1	23	5.9
51 to 55	2	0.6	2	0.6	18	4.7
56 to 60	1	0.3	1	0.3	23	5.9
61 to 75	-	-	-	-	49	12.7
76 to 100	-	-	4	1.1	51	13.2
101 to 125	-	-	-	-	33	8.5
126 to 150	-	-	1	0.3	18	4.7
151 to 175	-	-	-	-	10	2.6
176 to 200	-	-	-	-	2	0.5
Total	356	100.0	356	99.9	387	100.1

TABLE 3

PRODUCT VS. NUMBER OF COMPETING BRANDS

Product Code	Number of Brands (Listed)	Number of Brands with at Least One Purchase Recorded in the Panel Data
A	33	25
B	126	23
C	282	36

TABLE 4

PRODUCT VS. DISTRIBUTION OF MARKET SHARE AMONG ITS BRANDS

Market Share	Number of Brands in Product		
	A	B	C
0.00	8	103	246
0.01-1.00	19	16	24
1.01-2.00	1	-	-
2.01-5.00	1	2	6
5.01-10.00	2	1	5
10.01-20.00	-	2	-
20.01-40.00	1	2	1
40.01 & Above	1	0	0
Total	33	126	282

petition among the brands of each of the product lines. The nature of brand competition is depicted by: (1) the number of brands in the market; and (2) the distribution of market shares of the brands for the period of study. In the paper product line, the two top brands account for nearly three-fourths of the market sales in the Chicago area, one of them accounting for nearly half of the market. Among the four leading brands of toothpaste, two brands compete for the dominant position in the market with each having around 25%, followed by the remaining two each having a share of above around 10%. Among the three products, coffee has the maximum number of brands in the market (including the private labels), and the leading brand has about 28% of the sales, followed by 6 to 7 brands each sharing the market more or less equally around 7%. Note that the degree of domination by the leading brands is clearly distinct in the three product lines.

Market share of the brand can be calculated in either of the following ways: (1) proportionate number of purchases; (2) proportionate volume of sales measured in product units or in dollar amounts. As we are interested in the process of brand selection by the housewife on different purchase occasions, market share determined by the proportionate number of purchases is preferred over others. Occasionally, other methods of market share computations are presented for the purpose of comparison.

The distribution structure of Product C (coffee), at the retail level, is different from that of A and B. Coffee is primarily sold in food stores in the Chicago area, while toothpaste and the paper product are distributed in all types of outlets; drugstores, food stores, and discount stores. Among the three products, the paper product is bulkiest and occupies a considerable amount of shelf space in the store, followed by coffee and then toothpaste. However, in each product the top ten outlets account for nearly 80% of the market sales. It should be noted that the retail outlets are coded by their ownership and not by their location. In other words, if the Chicago area has ten A & P stores, all the stores will be coded by a single number. Table 5 presents the distribution of product sales by the type of retail outlet, whereas Table 6 deals with the distribution of the market shares of each of the products among different outlets.

As mentioned in the earlier section, the brand choice analysis has been repeated for all the leading brands of the three products and, for the purposes of Analysis of Covariance, ten major stores have been selected to study the store-brand interaction on an aggregate level. Tables 7 and 8 present the market position of the leading brands and the share of the ten stores selected for each of the three products.

In addition to these structural differences among the three product markets, there could be inherent attitudinal differences toward their purchase by the housewife. For example, a housewife may be more careful in the selection of a brand of toothpaste in view of the cavity implications. For coffee, taste is likely to be more of an important factor in the brand

TABLE 5

TYPE OF RETAIL OUTLET VS. PRODUCT SALES

Type of Stores	Percentage of Sales (Market Share) in		
	Product A	Product B	Product C
Independent Food Stores	6.2	7.3	14.2
Food Chains	35.2	36.2	64.6
Independent Drugstores	8.4	14.9	0.3
Drug Chains	21.1	15.8	0.4
Discount Stores	7.2	7.5	0.0
All Others	21.5	18.3	20.5

TABLE 6

PRODUCT VS. DISTRIBUTION OF MARKET SHARE AMONG THE STORES

Market Share	Number of Stores for Product		
	A	B	C
0.00	45	52	65
0.01-1.00	34	30	23
1.01-2.00	9	6	3
2.01-5.00	3	4	3
5.01-10.00	6	4	1
10.01-20.00	2	3	3
20.01 & Above	0	0	1
Total	99	99	99

TABLE 7

PRODUCT VS. MARKET SHARE OF 10 MAJOR STORES

Product Code	Market Share of 10 Stores
A	73.53
B	77.10
C	87.60

TABLE 8

PRODUCT, LEADING BRANDS AND THEIR MARKET SHARE

Product Code	Brand Code	Market Share of Brand (estimated by data) as determined by		
		Number of Purchases	Volume of Sales (Units)	Dollar Volume of Sales
A	A_1	48.1	44.4	45.7
	A_2	29.1	31.2	31.5
B	B_1	26.0	23.8	24.5
	B_2	25.0	27.7	26.9
	B_3	17.0	17.1	17.7
	B_4	10.3	9.4	9.4
C	C_1	28.2	28.1	28.8
	C_2	9.3	10.5	10.1
	C_3	8.5	9.0	9.4
	C_4	8.2	9.2	8.7

selection. As such, consumers are likely to be disposed in a different manner in switching to a new brand of toothpaste than in switching to a new brand of coffee.

To summarize our discussion in this Chapter, the study uses the panel data supplied by the Chicago Tribune. The data base covers the purchase histories of consumers for three products that are distinct in terms of their importance to the consumer, rate of consumption by the consumer, distribution methods, structure of market competition, et cetera. This would ensure us with some confidence that the observed results are not typical of any particular product. Probabilistic approach is the central methodology of the study for analyzing the hypotheses at micro level and Analysis of Covariance is used at macro level

CHAPTER IV

ANALYSIS AND FINDINGS

Since the same type of analysis has been repeated for each of the three products, the problem of presenting all the relevant tables in this chapter has been magnified. Accordingly, for the obvious reasons of ease and convenience to the reader, only illustrative tables of the analyses and summary tables of the findings are presented in this chapter. The complete set of tables is presented separately in Appendix III. As previously mentioned, the product names and their leading brands are identified by their corresponding codes in the table descriptions.

HYPOTHESIS 1: A consumer's selection of a store is not completely random; she exhibits bias in her choice. a) The more recent her purchase experience in a particular store, and b) the more frequent her visits to the store, the more likely she is to repurchase the product in that store.

The existence of bias toward one particular store on the part of the consumer might be due to certain physical or service factors prevailing in the store. On the other hand, the bias could have been motivated by economical (price of product) or locational (nearness of the store) factors. Whatever the reason may be, according to the hypothesis, each store creates a favorable image on a certain segment of the market to draw it frequently.

The hypothesis has been verified using the probabilistic approach. Sequences of four purchases by a housewife have been aggregated over the individual consumers and the time to estimate the probability of a housewife purchasing a given product in a particular store given the history of the last three stores she visited for the purchase of the same product. If the selection of the store is completely at random, all the estimates of the probabilities must be equal to the market share of the store for the particular product. Looking at the pattern of the sequences and their corresponding probability estimates, the historical weighting of past brand purchases observed in consumer's brand choice seems to stand as well for store choice. The tables presented separately for each product describe the probability of a housewife purchasing in a given store given the history of her last three purchases. The columns of the tables stand for three different stores in which the analysis has been repeated. For the purposes of discussion, analysis of paper product is presented in Table 9 and the complete set of tables is given in Appendix III. (Refer to Tables 3.1, 3.2, and 3.3).

Each column of the table refers to a particular store to which the sets of the corresponding probabilities and purchase sequences are related. The figures in the parentheses indicate the sample sizes on which the corresponding probability estimates are based. Each purchase sequence has similar but differing

TABLE 9

PROPORTION OF HOUSEWIVES PURCHASING PRODUCT A IN THE STORE GIVEN THE PAST HISTORY OF THREE PURCHASES

PRODUCT A (Paper Product)

1-Purchased in the Store
0-Not Purchased in the Store

Past Purchase Sequence	Fraction Purchasing in Store No.		
	1	2	3
000	0.039 (2676)	0.047 (2328)	0.021 (2976)
010	0.192 (114)	0.255 (137)	0.181 (77)
001	0.298 (134)	0.368 (152)	0.253 (79)
011	0.523 (65)	0.625 (104)	0.375 (32)
100	0.219 (123)	0.264 (170)	0.189 (79)
110	0.430 (65)	0.413 (116)	0.500 (32)
101	0.562 (48)	0.546 (86)	0.433 (30)
111	0.717 (117)	0.710 (249)	0.621 (37)

Notes

1: Food Chain
2: Drug Chain
3: Food Chain

interpretations when referring to the different columns. Past store purchases are identified by sequences of 0's and 1's where the entries '0' and '1' stand for 'not purchased' or 'purchased' in the particular store identified by the column. The three positions in the sequence stand for the three most recent store visits for the purchase of the same product by the housewife. The sequence of purchases is in the time dimension of their actual occurrence, so that the first digit of the sequence refers to the third most recent purchase of the product, the second digit for the second most recent purchase, while the last digit refers to the most recent purchase of the product. As an example, suppose a housewife has bought product A on three previous occasions in stores denoted by the sequence given below.

Store 3 Store 6 Store 3

According to our notation, the sequence is written as '1 0 1' if we are referring her past history with respect to store 3, or '0 1 0' if our reference store is 6. So any purchase sequence in reference to a column, identified by a particular store, has the corresponding meaning.

Given the nature of a housewife's three past store visits for the purchase of a product, each column gives the set of conditional probabilities of her selecting the store for her subsequent purchase of the same product. Note that in the majority of cases, the conditional probabilities of purchasing in a store form an increasing order of magnitude as we go from left to right in the three sets of the sequences: 000, 100, 010, 001; 110,101, 011, 111; and 000, 001, 011, 111. Thus a housewife is showing stronger bias to a particular store for her purchase: (1) the more frequently she visited the store in the past, and (2) the more recently she has purchased in that store. Similar types of observations can be traced in the two sets of sequences 000, 010, 001, 011, and 100, 110, 101, 111. For example, the estimates of the probabilities in Table 9 for store No. 2 are as follows:

000	0.047	110	0.413
100	0.264	101	0.546
010	0.255	011	0.625
001	0.368	111	0.710

000	0.047
001	0.368
011	0.625
111	0.710

000	0.047	100	0.264
010	0.255	110	0.413
001	0.368	101	0.546
011	0.523	111	0.710

Though the estimates of the probabilities follow a general pattern as described above, there are occasionally a few reversals. For example, for store No. 2 in Table 9, the observed pattern has a reversal. A reversal is a deviation from an expected pattern, as shown below:

000	0.047
100	0.264
010	0.255
001	0.368

Acknowledging the fact of a few reversals, the over-all pattern of the estimates is consistent with the tendency to exhibit bias in the selection of a store for the purchase of any product.

A housewife has the maximum probability of purchasing a product in a particular store if she has visited it on all three previous purchase occasions, and the minimum probability of purchasing in that store if she has not visited it on all the three previous purchase occasions. In other words, the bias (or preference) of the housewife in selecting a particular store is strongly related to the uninterrupted most recent sequence of favorable choices of that store for purchase of the product. This phenomenon of bias is independent of the type of chain. For example, store No. 3 in the analysis is a food chain; store No. 2 is a drug chain. The customers' images of these stores may be due to various causes but, regardless of the reasons, both types of retail outlets enjoy a favorable image in a certain segment of the market.

These phenomena have been observed in the purchase of each of the products. However, it does not necessarily mean that consumers have a separate favorite store for the purchase of each of the above products. For some families, it could very well be the same store for all three products, since most of the purchases of this type are made in one shopping trip. This would increase the importance of store preference, since the store wants to attract a considerable amount of the consumer's dollar budget.

<u>HYPOTHESIS 2</u>: As a corollary, consumers exhibit bias in the selection of the type of retail outlet (drugstore, food store, discount store, etc.) in which they would like to shop for a product.

The same general notation described earlier is used for the purchase sequences in the following table (Refer to Table 10) except that the individual store has been replaced by drugstore, food store or discount store. For example, the sequence 111 in reference to Column 2 gives the conditional

TABLE 10

PROPORTION OF HOUSEWIVES PURCHASING PRODUCT A IN THE TYPE OF OUTLET GIVEN THE PAST HISTORY OF THREE PURCHASES

PRODUCT A (Paper Product) — 1-Purchased in Store Type; 0-Not Purchased in Store Type

Past Purchase Sequence	Fraction Purchasing in Store Type 1	Fraction Purchasing in Store Type 2
000	0.039 (2823)	0.040 (2755)
010	0.198 (116)	0.157 (127)
001	0.248 (37)	0.205 (136)
011	0.487 (41)	0.468 (47)
100	0.184 (114)	0.126 (142)
110	0.263 (38)	0.265 (49)
101	0.366 (137)	0.529 (34)
111	0.627 (43)	0.538 (52)

1: Discount stores
2: Independent Drugstores

probability of purchasing product A in an independent drugstore, given that she has purchased the product all three previous times in drugstores only. The only difference between this and the previous hypothesis is that the consumer's purchase sequences are analyzed not with respect to any particular store, but with respect to the type of retail outlet. As the chain stores are studied for individual store loyalty in the earlier hypothesis, only independent drug and food stores and discount stores are incorporated to analyze the loyalty to the type of outlet.

The figures in parentheses in the table indicate the sample sizes on which the corresponding probability estimates are based. The same general observations of the earlier hypothesis are repeated. The probability of a housewife purchasing a product in an outlet (drugstore, food store, or discount store) is higher: (1) the more frequently she purchased in that type of outlet in the past, and (2) the more recently she purchased there. The analysis of paper product is illustrated in Table 10. The magnitude of bias seems to be stronger toward a type of retail outlet than to an individual store. The complete set of tables is presented in Appendix III. (Refer to Tables 3.4, 3.5, and 3.6).

The above two hypotheses demonstrate that a housewife's selection of a store or a type of retail outlet for the purchase of any particular product is not completely random and strongly depends on her recent experience with that store or type of outlet respectively. Kuehn has demonstrated that a housewife's probability of purchasing a particular brand depends strongly on her recent purchase experience with that brand. In other words, the probability of a housewife purchasing a particular brand increases if she has purchased the same brand the previous time or decreases if she has purchased some other brand the previous time. The next obvious question is how these two (store preference and brand preference) are related to each other.

> Cunningham:[1] Intuitively it seemed possible that the forces governing store loyalty might be more compelling than the forces governing brand loyalty purchasing behavior. If so, then the environmental conditions within the stores would have some degree of restrictive effect upon complete freedom of brand choice. The assortments of brands available in each store and the policies followed in promoting manufacturer's brands versus their own private brands could be important factors.

As the manufacturer's marketing strategies are developed to create a favorable image of his brand, and to develop strong purchase loyalties, it would be equally important for him to know how the consumer's bias in the selection of a store is affecting her brand preference. The next two hypotheses deal extensively with this aspect of how store switching patterns of consumers affect brand selection and the size of purchase.

<u>HYPOTHESIS 3</u>: Store switching increases brand switching; the more a housewife changes stores, the more she changes the brand she purchases.

According to the hypothesis, a housewife who consistently

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[1] Ross M. Cunningham, "Brand Loyalty and Store Loyalty Interrelationships," <u>American Marketing Association: Proceedings of the National Conference</u>, (1959), pp. 201-215.

goes to the same store exhibits stronger brand preference than one who constantly changes stores. Such a behavior becomes obvious if 100 stores in the Chicago area display 100 completely different brands of the same product, each store carrying not more than one brand. But in reality, the situation is not that simple and the differences in the availability of brands in different stores would force the consumers to switch brands while shopping in different stores. For the purpose of demonstration, the major national brands of the products, each having better than 90% of the market distribution and available in practically all the major chains of the Chicago area, have been selected. Only the changes in the fraction of the consumers that switch these major brands as they switch stores are tabulated below, so that the differences can't be completely explained by distribution differences.

Two consecutive purchases of the same product have been aggregated over the different individuals and the period of three years to see whether a change of store from the previous purchase has any significant effect on the brand selection. The magnitude of this effect may vary from brand to brand and product to product. However, as a first step, the number of brand changes that follow a store change and the number that do not follow a store change are investigated. In other words, if a housewife purchases brand 1 in store 1 and subsequently purchases brand 3 in store 2, her change of brand followed a store change. If she has purchased the same brand in the two stores, the store change is not followed by a brand change. So, in this analysis, no specific brand or store has been identified and the aggregate effect of store switching on brand switching in two consecutive purchases is analyzed. Table 11 presents the proportionate number of times housewives purchased the same brand while visiting the same store or different stores for two consecutive purchases.

The null hypothesis that store change has no effect on the brand selection has been tested by using the statistical χ^2 test of significance. The value of χ^2 and the results of the tests are presented in the last two columns of Table 11. The null hypothesis has been rejected in all three cases; store change has significant effect on brand change. Going a step farther, sequences of two, three, and four brand purchases are analyzed to describe the conditional probability of a housewife purchasing a particular brand given the past history of brands purchased and the corresponding stores visited. Before presenting the detailed tables, a brief description of the general notation followed for denoting the past history of brands purchased and stores visited is in order. We assume here a K-brand and R-store market for the product.

The random variable $b_j(n)$ is defined as follows:

$b_j(n) = 1$, if brand j is purchased on her n'th purchase occasion.

0, if some other brand is purchased on her n'th purchase occasion.

$j = 1, 2, \ldots, K \quad n = 1, 2, 3, \ldots$

With this notation, the housewife's purchase history can

TABLE 11

STORE CHANGE VS. BRAND CHANGE
IN TWO CONSECUTIVE PURCHASES

Product	Brand Change	Store Change: Same Store	Store Change: Different Store	Marginals	X^2 Value	Test of Significance
A	Same Brand	0.700 (1229)	0.596 (1303)	0.643 (2532)	44.43	**
	Different Brand	0.300 (528)	0.404 (882)	0.357 (1410)		
	Marginals	0.446 (1757)	0.554 (2185)	1.00 (3942)		
B	Same Brand	0.662 (3078)	0.557 (2780)	0.607 (5858)	62.02	**
	Different Brand	0.338 (1572)	0.443 (2212)	0.393 (3784)		
	Marginals	0.482 (4650)	0.518 (4992)	1.00 (9642)		
C	Same Brand	0.683 (9645)	0.350 (2938)	0.553 (12583)	2365.42	**
	Different Brand	0.317 (4468)	0.650 (5445)	0.447 (9913)		
	Marginals	0.632 (14113)	0.368 (8383)	1.00 (22496)		

** Significant at 1% level

be described by a Vector of 1's and 0's, with respect to any particular brand 'j'. The ordering of the 1's and 0's depends upon the sequence of the customer's purchases of brand j and other brands.

Let us define the random variable $S_h(n)$ as follows:

$S_h(n) = 1$, if the consumer made her n^{th} purchase in store h

$= 0$, if the consumer made her n^{th} purchase in some other store.

$h = 1, 2, \ldots, R \quad n = 1, 2, 3, \ldots$

Let us also indicate by $S(t)$ the store visited by the consumer for her t^{th} purchase. Thus the past history of store visits prior to her n^{th} purchase is given by the vector

$[S(1), S(2), \ldots, S(n-1)]$

Given the information on consumer's past store visits and the store selected for her subsequent purchase, the purchase history of store visits can be described by a vector of S's and D's as defined below. Suppose the consumer has selected store h for her n^{th} purchase.

Then,

$S(t) = S$, if $S_h(t) = 1$, $S_h(n) = 1$ (the store selected for her t^{th} and n^{th} purchases is same)

$= D$, if $S_h(t) = 0$, $S_h(n) = 1$ (the store selected for her t^{th} purchase is different from the store selected for her n^{th} purchase)

$h = 1, 2, \ldots, R$

$t = 1, 2, \ldots, n-1.$

Accordingly, in the case of two purchases, the store background is indicated by "S" or "D" depending on whether she has visited the same store or different stores for purchase of the product. In a sequence of three purchases, if the individual has gone to A & P and then to Kroger for the purchase of a product and decided to make her subsequent purchase in A & P, her store background for the third purchase will be indicated by SD. If her two previous purchases were made in stores different from the subsequent store (DD), two sub-cases are identified depending on whether her two previous store visits are to the same store, indicated by D_1D_1, or different stores indicated by D_1D_2. First, two consecutive purchases of a housewife are analyzed with respect to visiting the same store (S) or different stores (D). The brand purchases are coded, as described earlier, by the sequences of '0' and '1'.

Table 12 presents the estimates of the probabilities of a housewife purchasing a particular brand (coded by 1), given the information on the past brand purchase and the corresponding store visited. The analysis has been repeated for all the leading brands of the three products. In each case, the null hypothesis that the re-purchase rate of a brand is independent of the store change has been tested by means of the statistical χ^2 test. The value of χ^2, and the test of significance are shown in the last two columns of Table 12. The first column of the table identifies the particular brand of the product against which the corresponding estimates of re-purchase probabilities are obtained. The figures in the parentheses in columns 3 and 4 indicate the sample sizes on which the relevant probability

TABLE 12

RE-PURCHASE RATE OF THE BRAND VS. STORE CHANGE

Brand	Previous Purchase Sequence	Proportion Purchasing Brand in Same Store (S)	Proportion Purchasing Brand in Different Store (D)	χ^2 Value	Test of Significance
A_1	0	0.191(843)	0.231(1190)		
	1	0.801(913)	0.721(995)	25.98	* *
A_2	0	0.151(1306)	0.177(1501)		
	1	0.611(450)	0.585(684)	8.68	* *
B_1	0	0.082(3359)	0.111(3767)		
	1	0.788(1290)	0.643(1228)	12.31	* *
B_2	0	0.094(3551)	0.116(3704)		
	1	0.740(1098)	0.701(1291)	6.29	*
B_3	0	0.071(3882)	0.094(4124)		
	1	0.643(767)	0.537(871)	1.44	- -
B_4	0	0.049(4153)	0.065(4494)		
	1	0.526(496)	0.435(501)	0.93	- -
C_1	0	0.098(10245)	0.187(5909)		
	1	0.734(3872)	0.567(2475)	11.27	* *
C_2	0	0.046(12915)	0.078(7501)		
	1	0.522(1202)	0.313(883)	13.50	* *
C_3	0	0.033(12985)	0.061(7622)		
	1	0.617(1132)	0.403(762)	7.67	* *
C_4	0	0.043(13011)	0.067(7653)		
	1	0.467(1106)	0.328(731)	5.37	*

* * Significant at 1% level

* Significant

\- - Not significant at 5% level

estimates have been based.

According to the hypothesis, a housewife who did not purchase the brand the last time has a higher probability of purchasing the brand if she changes the store than if she visits the same store. Similarly, a housewife who purchased the brand the last time has a lower probability of purchasing the brand if she changes the store than if she visits the same store. In all twenty cases of the ten brands tested, the tendency of the observed estimates are consistent with the hypothesis. In eight of the ten cases the differences in probability estimates due to store change are found statistically significant at the 5% level of significance and, thus, the hypothesis that store change is independent of the re-purchase rate of a brand is rejected.

Similarly, the three purchase sequences of a housewife are aggregated over the individual consumers and time to estimate the probabilities of a housewife's purchasing a given brand, as a function of her past two brand purchases and the corresponding stores visited. With our notation of "S's" and "D's" for store choices, there are five possible ways of classifying the housewife's past two store visits, as shown below.

The housewife can be uniquely classified in terms of her two previous brand choices (00, 01, 10, 11). Given the history of a housewife's two past brand choices and stores visited, along with the store of her subsequent purchase, she occupies a unique position in the 4 x 5 matrix of Table 13. The conditional probability in each cell of this 4 x 5 matrix is calculated by observing the actual fraction of housewives belonging to that cell and purchasing brand "1" on their subsequent pur-

Hypothetical Store Visits in Sequence

2nd Recent Purchase	Recent Purchase	Subsequent Purchase	Background of Stores Visited Indicated by
A & P	A & P	A & P	SS
A & P	Kroger	A & P	SD
Kroger	A & P	A & P	DS
Kroger	Kroger	A & P	D_1D_1
Kroger	Jewel	A & P	D_1D_2

chase. The last column of the table is a weighted summation of the first five columns, which gives the conditional probability of a housewife purchasing brand "1" given the history of her two previous brand purchases, without taking into consideration the pattern of store visits. The figures in the parentheses indicate the sample sizes on which the corresponding probability estimates are based.

According to the hypothesis, people switch brands when they switch stores. Accordingly, a housewife who has not purchased brand '1' the last two times (sequence 00) is more likely to purchase brand '1' for her subsequent purchase if she visits a store different from the earlier two rather than if she visits the same store all three times. It is observed that in the cases of store sequences D_1D_1 and D_1D_2 corresponding to the brand sequence '00', the probabilities of a housewife purchasing brand '1' are .163 and .172 respectively, compared to

TABLE 13

PROBABILITY OF A HOUSEWIFE PURCHASING BRAND A_1 GIVEN THE HISTORY OF HER PAST TWO BRAND PURCHASES AND THE CORRESPONDING STORES VISITED

PRODUCT A
(Paper Product)
BRAND A_1

1-Purchased Brand A_1
0-Not Purchased Brand A_1

Historical Sequence of Brands Purchased	Past History of Stores Visited					Irrespective of the Store Choice (over-all)
	SS	SD	DS	DD: D_1D_1	DD: D_1D_2	
00	0.116 (336)	0.119 (210)	0.109 (256)	0.163 (276)	0.172 (377)	0.139 (1455)
10	0.481 (104)	0.458 (59)	0.372 (78)	0.477 (65)	0.675 (118)	0.455 (424)
01	0.567 (90)	0.339 (59)	0.673 (92)	0.583 (60)	0.490 (100)	0.541 (401)
11	0.874 (435)	0.867 (188)	0.842 (214)	0.790 (248)	0.771 (253)	0.833 (1338)

Footnotes: Indicating the past two purchases by 1 and 2, and the subsequent purchase by 3 the notation stands for the following events:

Store Background	Event
SS	$S(1) = S(2) = S(3)$ or $S_h(1)=S_h(2)=S_h(3)=1$ for some h
SD	$S(2) \neq S(1) = S(3)$ or $S_h(2)\neq S_h(1)=S_h(3)=1$ for some h
DS	$S(1) \neq S(2) = S(3)$ or $S_h(1)\neq S_h(2)=S_h(3)=1$ for some h
D_1D_1	$S(1) = S(2) \neq S(3)$ or $S_h(1)=S_h(2)\neq S_h(3)=1$ for some h & $S_t(1)=S_t(2)=1$ for a $t \neq h$
D_1D_2	$S(1) \neq S(2) \neq S(3)$ or $S_h(1)=S_h(2)=S_h(3)=1$ for some h & $S_{t_2}(1)=1$, $S_{t_1}(1)=1$ for $t_1\neq t_2$

$h, t, t_1, t_2 = 1, 2, \ldots, R.$

the probability of .116 while visiting the same store (SS). (Refer to Table 13).

As another example, suppose a housewife has her brand history given by 01. Under the hypothesis, if the pattern of her store visits is given by SD, the probability of her purchasing brand '1' for her subsequent purchase is lower than if she visits the same store all along (SS), because the consumer's selection of brand '1' on her last purchase might have been prompted by her store change at that time rather than by any higher preference for brand '1'. If the same housewife has the pattern of store visits given by DS, her probability of purchasing brand '1' subsequently increases for the same reason as explained above: her purchase of some other brand on her second most recent purchase might have been due to her visiting a different store at that time rather than due to any higher preference for some other brand. The probabilities, as estimated by data in Table 13, are given below:

Brand Sequence	Pattern of Store Visits	Probability of Purchasing Brand '1
01	SS	0.567
01	SD	0.339
01	DS	0.673

The observed trend is consistent with the hypothesis. By looking at the observed magnitudes of the estimated probabilities, the numbers of comparisons that are consistent and inconsistent with the hypothesis are listed in Table 14. In all, eight comparisons are possible in this 4 x 5 matrix to test the validity of the hypothesis. The expected trends in the probability estimates under the hypothesis are shown below:

Historical Sequence of Brands Purchased	Past History of Stores Visited				
	SS	SD	DS	D_1D_1	D_1D_2
00	X_1			Greater than X_1	Greater than X_1
01	X_2	Less than X_2	Greater than X_2		
10	X_3	Greater than X_3	Less than X_3		
11	X_4			Less than X_4	Less than X_4

Since the analysis has been repeated over all the ten leading brands of the products, there are a total of eighty such comparisons possible for the verification of the hypothesis. The individual analysis of the ten brands, describing the housewife's probability of purchasing a brand given the history of her past two brand purchases and the corresponding store visits, are presented in Appendix 3. (Refer to Tables 3.7 to 3.16). The null hypothesis that the observed pattern in probability estimates can be attributed to purely chance factors is tested. In other words, the hypothesis implies that store change has no effect on the probability of a brand purchase by a housewife. Under the null hypothesis, the probability estimates will not have any specific pattern related to the stores visited, and as such, it is expected that in only 50% of the comparisons store change reduces the probability of purchasing a brand.

TABLE 14

LISTING OF THE COMPARISONS CONSISTENT WITH THE HYPOTHESIS

Reference No. (Appendix III)	Number of Comparisons Consistent with the Hypothesis	Number of Comparisons Inconsistent with the Hypothesis
3.7	7	1
3.8	7	1
3.9	7	1
3.10	6	2
3.11	7	1
3.12	7	1
3.13	8	0
3.14	8	0
3.15	8	0
3.16	8	0
Total	73	7
Percentage of Comparisons Consistent with the Hypothesis		91.2

Null Hypothesis: $\Pi = 0.50$ $n = 80$
Alternate Hypothesis: $\Pi > 0.50$ Observed value of $p = 0.91$

Z = Normal deviate under the hypotheses $= \frac{0.91-0.50}{\sqrt{\left(\frac{0.5 \times 0.5}{80}\right)}} = 0.41\sqrt{320} \simeq 7.38$

Z being greater than 2.33 (from Normal curve tables), the null hypothesis is rejected at 1% level of significance.

The observed value of 91% of the comparisons where store change has reduced the probability of purchasing a brand falls well in the critical region of the test, thus rejecting the null hypothesis. Accordingly, store switching increases brand switching. Details of the test of binomial proportions are presented following Table 14.

In sequences of four purchases a similar type of analysis has been repeated and presented in detail below. As earlier, sequences of 1's and 0's represent the past brand purchases of the consumer, and sequences of S's and D's represent the stores visited. Instead of presenting the 8 x 8 possible matrix of the conditional probabilities, only estimates that are relevant for demonstrating the hypothesis are presented. The complete set of tables describing the conditional probability of a housewife purchasing brand '1', given the history of her three past brand purchases and the corresponding stores visited, are presented in Appendix III (Refer to Tables 3.17 to 3.26). The ten tables refer to the analyses done separately for each of the ten major brands. For the purposes of illustration, the analysis of brand A_1 is given on the following page. (Refer to Table 15). The last column of the table gives the conditional probability of a housewife's purchasing brand '1' given the history of her three previous brand purchases irrespective of the pattern of store visits. With our notation, brand code '1' stands for the particular brand in the analysis, and the figures in parentheses refer to the sample sizes on which the corresponding estimates of probabilities are based.

TABLE 15

PROBABILITY OF A HOUSEWIFE PURCHASING BRAND A_1
GIVEN THE HISTORY OF HER PAST THREE BRAND PURCHASES
AND THE CORRESPONDING STORES VISITED

PRODUCT A
(Paper Product)
BRAND A_1

1-Purchased Brand A_1
0-Not Purchased Brand A_1

Historical Sequence of Brands Purchased	Past History of Stores Visited SSS			DDD	Irrespective of the Store Choice (over-all)
000	0.067 (165)			0.132 (416)	0.101 (1143)
010	0.478 (23)	DSD 0.412 (17)	SDS 0.125 (16)	0.377 (69)	0.383 (167)
001	0.393 (28)	DDS 0.655 (29)	SSD 0.077 (13)	0.459 (61)	0.462 (182)
011	0.562 (32)	DSS 0.750 (28)	SDD 0.389 (18)	0.532 (62)	0.614 (197)
100	0.378 (37)	SDD 0.389 (18)	DSS 0.375 (16)	0.306 (85)	0.335 (209)
110	0.595 (42)	SSD 0.524 (21)	DDS 0.414 (29)	0.508 (59)	0.522 (207)
101	0.667 (36)	SDS 0.800 (15)	DSD 0.417 (12)	0.558 (52)	0.622 (180)
111	0.921 (241)			0.825 (314)	0.877 (1033)

Under the hypothesis, a housewife with the brand history given by the sequence of 001 has a higher probability of purchasing brand '1' subsequently if her pattern of past store visits is DDS rather than SSS, because her purchasing of other brands the first two times might have been induced by her visiting a different store rather than by her higher preference for some other brand. The same housewife has a lower probability of purchasing brand '1' if her pattern of past store visits is SSD rather than SSS for the same reason. Her recent purchase of brand '1' might have been caused by her visiting a different store at that time.

The observed estimates in regard to the purchase of brand A_1 are (Refer to Table 15) given below:

Brand Purchase Sequence	Pattern of Store Visits	Probability of Purchasing Brand '1'
001	SSS	0.393
001	DDS	0.655
001	SSD	0.077

There are in all fourteen such comparisons possible, and the expected magnitudes of these probabilities under the hypothesis are given in the following page.

The comparisons have been made against the columns of S or SS or SSS (visiting the same store), since the hypothesis points to changing the store as a major factor in explaining a part of the variation in brand choice. Thus, the hypothesis

Historical Sequence of Brands Purchased	Past History of Stores Visited			
	SSS			DDD
000	X_1			Greater than X_1
001	X_2	DDS Greater than X_2	SSD Less than X_2	
010	X_3	DSD Greater than X_3	SDS Less than X_3	
011	X_4	DSS Greater than X_4	SDD Less than X_4	
100	X_5	SDD Greater than X_5	DSS Less than X_5	
101	X_6	SDS Greater than X_6	DSD Less than X_6	
110	X_7	SSD Greater than X_7	DDS Less than X_7	
111	X_8			Less than X_8

is checked by comparing brand purchase probabilities while switching stores as opposed to visiting the same store. Table 16 evaluates the validity of the hypothesis by listing the proportionate number of comparisons that are consistent with the hypothesis, considering the analysis of data on sequences of four purchases.

TABLE 16

LISTING OF COMPARISONS CONSISTENT WITH THE HYPOTHESIS

Table Reference No. (Appendix III)	Number of Comparisons Consistent with the Hypothesis	Number of Comparisons Inconsistent with the Hypothesis
3.17	12	2
3.18	11	3
3.19	11	3
3.20	11	3
3.21	12	2
3.22	12	2
3.23	14	0
3.24	14	0
3.25	14	0
3.26	13	1
Total	124	16
Percentage of Comparisons Consistent with the Hypothesis		88.6

Null Hypothesis: $\Pi = 0.50$ $n = 124$
Alternate Hypothesis: $\Pi > 0.50$ Observed value of $p = 0.89$

Z = Normal deviate
Under the hypothesis $= \dfrac{0.89 - 0.50}{\sqrt{\left(\dfrac{0.5 \times 0.5}{140}\right)}} = 0.39\sqrt{560} \doteq 9.24$

Z being greater than 2.33, (from Normal curve tables) the null hypothesis is rejected at 1% level of significance.

The null hypothesis that the observed pattern of probability estimates could have arisen because of chance factors have been tested on the same lines as explained earlier. The observed value of 89% of the comparisons is well above the expected value of 50%; thus, store switching increases brand switching. Details of the statistical test of binomial proportions are presented following Table 16.

In analyzing the sequences of two, three, and four purchases, this study arrived at the consistent finding that store change has increased the probability of brand change. This general finding has been consistent over all the national brands of the three products. Brand switching due to store switching is of extreme importance to manufacturers in: (1) developing effective distribution strategies, and (2) identifying the stores where the firm's brand has been losing or gaining customers.

So far the study has been concerned with the consumer's brand choices as she shops in different stores. However, from the manufacturer's point of view, another closely related and important characteristic of the consumer's choice is the size of her purchase. The market share of any brand (by dollar volume) is a simple multiplicative function of the unit price, the size of the purchase, and the probability of purchasing the particular brand. The size of the purchase, as used here, is the total number of units of a product purchased by a housewife on any one particular purchase occasion and, as such, it should not be confused with the package size. In the next hypothesis, we look into changes in the size of purchase as a housewife changes her store or brand or both.

HYPOTHESIS 4: Consumers change the size of their purchases as they change the store or brand; in general, they decrease rather than increase the size of their purchase with a change in store or brand.

Changes in purchase size may be prompted by many factors, such as: (1) the availability of different package sizes in different stores; (2) the lack of a uniform package size among different brands; (3) the unit price differential on higher package sizes; and (4) the customer's inherent demand variation in the use of the product. In addition to these, a housewife might be decreasing the size of her purchase as she changes brand because of her lack of familiarity with the new brand. Also, a housewife who visits a different store because of some advertised price promotion in the store is likely to increase the size of her purchase over the usual. Not much research has been published on the factors contributing to a housewife's decisions on purchase size. In this study, the proposition that a housewife's decision on purchase size is random is rejected, and some tendencies influencing change in her size of purchase are observed.

One can visualize the range of variation in the size of a purchase on a continuum from zero to infinity, and minor changes inevitably caused by package differences are quite likely to exaggerate the magnitude of the observed variation in the size of purchase. As such, using the distribution of sales over different sizes, the observed range in size of purchase has been grouped into low, medium and high volume categories. Only housewives changing from one category to another for two consecutive purchases are counted as changing their size of purchase. Table 17 presents the proportion of housewives who increased, decreased, or retained their purchase size as they visited different stores or purchased different brands while buying product A. The complete set of tables are given in Appendix III. (Refer to tables 3.27, 3.28, and 3.29).

The null hypothesis that size of purchase is independent of brand or of store change has been tested by computing the value of statistical χ^2. Table 18 presents the value of χ^2, which is significant in all three cases; thus, rejecting the null hypothesis. Hence, choice of brand and store has significant effect on the purchase size decision. Assuming that the effects of store and/or brand change on purchase size are independent of the product type, Table 19 summarizes the findings.

Reading from Table 19, among the housewives visiting the same store and purchasing the same brand in two consecutive purchases, only 13% decreased their purchase size as opposed to 10% who increased their purchase size. But with a change in the brand of purchase, an additional 11% of the housewives decreased their purchase size while only an additional 6% increased their purchase size. With a simultaneous change of brand and store, an additional 15% of the housewives decreased their purchase size as opposed to 11% who increased their purchase size. The differences between all these estimates of proportions of housewives increasing or decreasing the size of purchase caused by a brand and/or store change are found to

TABLE 17

PROPORTION OF HOUSEWIVES CHANGING THE SIZE OF PURCHASE OF PRODUCT A VS. THE PURCHASE PATTERN

PRODUCT A
(Paper Product)

Pattern of Two Consecutive Purchases	Increased the Size of Purchase	No Change in the Size of Purchase	Decreased the Size of Purchase	Marginal Total
Same Store & Same Brand	0.124 (152)	0.784 (963)	0.092 (113)	0.312 (1228)
Same Store & Different Brand	0.212 (112)	0.574 (303)	0.214 (113)	0.131 (528)
Different Store & Same Brand	0.184 (240)	0.630 (820)	0.186 (243)	0.333 (1303)
Different Store & Different Brand	0.236 (208)	0.521 (460)	0.243 (214)	0.224 (882)
Marginal Total	0.181 (712)	0.646 (2546)	0.173 (683)	1.000 (3941)

$$P_I = 13.30 + BC \times 7.00 + SC \times 4.20$$

$$P_D = 10.80 + BC \times 8.95 + SC \times 6.15$$

TABLE 18

TESTING THE EFFECT OF STORE CHANGE AND BRAND CHANGE ON THE SIZE OF PURCHASE

Product	Reference Table No.	Value of χ^2	Test of Significance
A	3.27	180.2	* *
B	3.28	276.3	* *
C	3.29	1606.7	* *

* * Significant at 1% level.

TABLE 19

AN OVER-ALL MEASURE OF STORE AND/OR BRAND CHANGE ON THE SIZE OF PURCHASE

Source of Variation	Percentage of housewives		
	Increased the size of purchase	No change in the size of purchase	Decreased the size of purchase
Same Store & Same Brand	13.10 (1503)	77.07 (11220)	9.83 (1263)
Same Store & Different Brand	19.27 (1115)	60.36 (4225)	20.37 (1180)
Different Store & Same Brand	17.17 (1191)	65.60 (4680)	17.23 (1194)
Different Store & Different Brand	24.03 (2021)	51.14 (4422)	24.83 (2062)

NOTE: Figure in the parentheses indicate the sample sizes on which the corresponding probability estimates are based.

$$P_D = 10.56 + BC \times 9.07 + SC \times 5.93$$
$$P_I = 12.92 + BC \times 6.52 + SC \times 4.42$$

be statistically significant at the 5% level. A brief explanation of the test is in order:

With a pair of proportional estimates, p_1 and p_2, based on two independent samples of sizes n_1 and n_2 respectively, the standard error of the difference $(p_1 - p_2)$ is given by

$$\sigma = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

The maximum value of $p(1-p)$ is equal to 1/4 when $p = 1/2$.

In Table 19, the minimum sample size for an estimate is 1100. Hence, the maximum value of the standard error of the difference between any two estimates is given by

$$\sqrt{\frac{1}{4}\left(\frac{1}{1100} + \frac{1}{1100}\right)} = 0.0213$$

In testing the equality of the binomial proportions, the observed difference is compared with the critical value (2σ limit) at 5% level of significance. Maximum value of the 2σ limit in our case is .0426. In Table 19, all the differences between observed estimates corresponding to the effect of store and/or brand change on the increase or the decrease in purchase size exceed the critical value of 4.3%. As such, all the effects are statistically significant.

Note that there is a greater tendency for consumers to decrease rather than to increase their size of purchase following a change in the brand purchased, or the store visited, or both. In Table 19, if 100 consumers visit the same store and purchase the same brand, we expect an average of ten consumers to decrease their size of purchase and thirteen to increase

their size of purchase. But if 100 consumers change their brand or store, or both, we expect an average of twenty-one consumers to decrease their size of purchase. This is obtained by taking a simple average of the three estimates for decreasing purchase size with a brand and/or store change ((20.37 + 17.23 + 24.83)/3 = 20.81); similarly, we expect twenty consumers to increase their size of purchase.

Comparing the above estimates, the change in brand and/or store has caused, on the average, an additional eleven housewives to decrease the size of their purchase and an additional seven to increase the size of their purchase. Accordingly, for every 100 housewives who increase purchase size with a change in brand and/or store, there are, on the average, 157 (= 11 x 100/7) who decrease purchase size.

Treating the data resulting from a simple designed experiment, a linear regression model is fitted between the percentage of housewives increasing or decreasing their purchase size and the factors of brand change and store change.

The linear additive models are:

$$P_I = \alpha + \beta \times BC + \gamma \times SC$$

$$P_D = \alpha^1 + \beta^1 \times BC + \gamma^1 \times SC$$

Where P_I = Percentage of people increasing the size of their purchase

P_D = Percentage of people decreasing the size of their purchase

α and α^1 are constants

β and β^1 are brand change effects

γ and γ^1 are store change effects

BC and SC are the variables that take the values of '0' and '1' in the following manner:

$$BC = \begin{cases} 1 \text{ if there is a change in the brand} \\ 0 \text{ if there is no change in the brand} \end{cases}$$

$$SC = \begin{cases} 1 \text{ if there is a change in the store} \\ 0 \text{ if there is no change in the store} \end{cases}$$

The regression equations are presented following each of the product data tables as well as the summary table. (Refer to Tables 17, 19 and 3.27 to 3.29). Note that from the estimates of β, γ, β^1 and γ^1 the following relations are consistently observed:

$$\beta < \beta^1; \ \gamma < \gamma^1$$

$$\beta > \gamma \ ; \ \beta^1 > \gamma^1$$

This shows that brand change has a more pronounced effect on the decrease or increase of purchase size than store change. However, both brand change and store change cause more people to decrease their purchase size than to increase their purchase size.

The next topic of my discussion is how store switching affects the store's private lable purchases. Private labels have limited distribution compared to national brands since they are available only in their sponsor stores. But private labels have one thing in common; they have, in general, a price advantage over national brands, and stores usually allocate more shelf space and better displays to their own private labels than to national brands. With these advantages, it is likely that a housewife loyal to a store may tend to purchase the store's private labels, and that a housewife loyal to a private label may tend to treat all private labels as substitutes regardless of the store she visits. The next hypothesis deals in detail with these two propositions.

HYPOTHESIS 5: Loyalty (measured by the re-purchase rate) to a particular store increases the preference for the private brands sponsored by the store. As a corollary, a housewife loyal to a private brand sponsored by a particular store is loyal to private labels regardless of store.

Among the three products of my study, only coffee has a considerable number of strong private labels in the market; therefore, this hypothesis has been tested with coffee data only. A store loyalty index has been calculated for each housewife by computing the fraction of times she has visited the same store for every pair of consecutive purchases. The families have been grouped into eleven categories depending on their store loyalty index, as shown in Table 20.

Three major food chains marketing private labels of coffee have been selected, and the fraction of coffee purchases made by each of the family groups in these three stores, as well as the fraction of their purchases in favor of the private labels of the three stores, are presented in columns (2) and (3) of the table respectively. However, only families who purchased the product on at least ten different occasions over the period of three years are included in this analysis. Column (1) of the table gives the total number of purchases made by each

TABLE 20

STORE LOYALTY VS. FRACTION OF PRIVATE LABEL PURCHASES OF COFFEE

PRODUCT: C
(Coffee)

Store group: Three major food chains

Store Loyalty Index	Total Number of Purchases (1)	Proportion of Purchases Made in the Store Group (2)		Proportion of Private Label Purchases (3)		Proportion of Private Label Purchases Relative to the Total Number of Purchases Made in the Store Group Col. (3) ÷ Col. (2)	
0.00-0.09	12	0.333		0.000		0.000	
0.10-0.19	289	0.356		0.183		0.515	
0.20-0.29	666	0.407	0.370	0.146	0.135	0.358	0.365
0.30-0.39	2460	0.361		0.127		0.351	
0.40-0.49	3975	0.530		0.234		0.441	
0.50-0.59	3909	0.515	0.509	0.201	0.234	0.389	0.459
0.60-0.69	2835	0.471		0.279		0.592	
0.70-0.79	2651	0.559		0.247		0.442	
0.80-0.89	3176	0.481	0.497	0.312	0.248	0.647	0.499
0.90-0.99	2457	0.451		0.165		0.365	
1.00	306	0.549		0.409		0.745	

family group. Grouping into sets of three, as shown in the table, it is evident that the proportion of private label purchases increased from .135 to .409 as we move in the ascending order of the loyalty index. Simultaneously, the proportion of purchases made in the store group (the three food chains) increased from 37% to 55% approximately.

To adjust for the obvious positive association between store traffic in terms of product purchases made in the store group, and coffee purchases in favor of their private labels, the last column indicates the proportion of private label purchases made by the family group relative to the number of total purchases made in the store group. With the same grouping, as done earlier, the proportion of private label purchases increased from .365 to .745 as we go from low to high store loyalty groups. This clearly states that after adjusting for store traffic figures the proportion of private label purchases in the stores has a high degree of association with the store loyalty index; the higher the store loyalty index of a housewife, the greater is the chance of her purchasing private labels. Looking at the estimates, private labels enjoy almost twice the proportion of sales from a completely loyal customer than from her counterpart, after adjusting for differences in the frequency of store visits. Accordingly, stores have more to gain in their sale of private labels by promoting the habit of store patronage. Thus loyalty of a housewife to a store is positively associated with her purchase of private labels in the store.

In the second part of the hypothesis, I am questioning the existence of consumer loyalty for private labels; do consumers differentiate the private labels of a product or treat them on an equal basis? At the outset, it is to be noted that this type of behavior may be observed by combining any set of national or regional brands, which is known as a brand-mix loyalty or loyalty to a group of brands. But one clear distinction in terms of the distribution is to be kept in mind. National or regional brands are available in many stores, but no two private labels are marketed in the same store.

Loyalty to private labels has been studied by calculating the proportion of housewives who purchased private labels while visiting the same or different stores in sequences of two and three purchases; Table 21 presents the probability estimates for sequences of two purchases and Table 22 for sequences of three purchases. Stores 1, 2, and 3 in both tables are food chains in the Chicago area marketing their own private labels of coffee, along with national and regional brands. A housewife's purchase of the store's private label of coffee in store 1, 2, or 3 is denoted by '1', whereas her purchase of another brand in that store obviously a national or regional brand, is denoted by '0'. Aggregating over all the housewives who made two consecutive purchases in one of these stores, the proportion of housewives who purchased private labels are compared in reference to the background of their store visits and brand purchases. If the phenomenon of loyalty to private labels does not exist in a consumer's mind, the past selection of a private label should not in any way influence her subsequent selection of a different store's private label.

TABLE 21

RE-PURCHASE RATE OF THE PRIVATE LABELS OF COFFEE VS. STORE CHANGE

PRODUCT: C (Coffee)   1-Purchased the store's private labels
0-Not purchased the store's private labels

Store Purchase Sequence	Brand Purchase Sequence	Probability of a Housewife Purchasing Private Labels of Coffee in the Store		
		1	2	3
1	0	0.152 (768)	0.106 (85)	0.221 (95)
	1	0.926 (1756)	0.424 (59)	0.577 (222)
2	0	0.432 (102)	0.092 (892)	0.197 (198)
	1	0.710 (31)	0.727 (275)	0.569 (51)
3	0	0.534 (188)	0.153 (190)	0.128 (1951)
	1	0.895 (153)	0.440 (59)	0.840 (1452)

TABLE 22

PROBABILITY OF A HOUSEWIFE PURCHASING PRIVATE LABELS OF COFFEE GIVEN THE HISTORY OF HER PAST TWO BRAND PURCHASES AND THE CORRESPONDING STORES VISITED

PRODUCT: C (Coffee)

1-Purchased the store's private labels
0-Not purchased the store's private labels

Past History of Stores Visited	Historical Sequence of Brands Purchased	Probability of a Housewife Purchasing Private Labels of Coffee in store		
		1	2	3
(1, 1)	00	0.049 (510)	0.100 (20)	0.136 (22)
	11	0.952 (1322)	0.263 (38)	0.603 (73)
(2, 2)	00	0.306 (36)	0.054 (552)	0.067 (60)
	11	0.750 (4)	0.825 (149)	0.455 (11)
(3, 3)	00	0.455 (44)	0.152 (66)	0.065 (1347)
	11	0.998 (55)	0.550 (20)	0.900 (972)
(1,2), (2,1) (2,3), (3,2) (1,3), (3,1)	00	0.231 (121)	0.043 (185)	0.128 (172)
	11	0.920 (138)	0.714 (35)	0.791 (153)

In other words, a housewife's selection of the private label in store 2 or 3 should be independent of her earlier brand selection in store 1. However, as shown in Table 20, about 42.4% and 57.7% of the housewives elected to continue with the purchase of private labels in stores 2 and 3 respectively, following their purchase of the private label in store 1. Only 10.6% and 22.1% of the housewives chose to purchase private labels in stores 2 and 3 respectively, following their purchase of some other brand in store 1. On the average, the carry-over effect of purchase preference from one private label to another is around 36%. This figure is obtained by taking the weighted average of the six estimates for the six possible cross-store traffic combinations. ((1,2),(1,3),(2,1),(2,3),(3,1),(3,2)). From the third column of the table, it is seen that store 1's private label is relatively stronger than others. Considerable proportion of consumers purchased store 1's private label even though they didn't choose to purchase the private labels in store 2 or 3 on their previous purchase.

Looking into the sequences of two consecutive purchases, the existence of carry-over effect in loyalty from one private label to another is evident. Table 21 presents findings of similar effect in sequences of three successive purchases. The first column of the table refers to the stores visited by the housewife for her past two purchases. Though the past two store visits can be classified in nine different combinations, only repeated purchase visits to the same store have been separately identified, leaving the six other combinations grouped together because of small sample sizes. The table presents the proportion of housewives purchasing private labels in a store, given the history of their last two brand purchases and the corresponding stores visited. Although the brand history of the last two purchases can be classified in four different ways (00, 01, 10, 11) only proportional estimates corresponding to 00 (private labels not selected in the last two purchases) and 11 (private labels selected the last two times) are shown in the table Surprisingly enough, the estimates in Tables 21 and 22 are based on relatively small sample sizes, although the three stores account for nearly 49% of coffee sales in the Chicago area, and the number of coffee purchases in the data base are around 22,000. However, it is to be noted that only consumers who purchased two or three successive times in the three stores can be included in the analysis

Let two housewives, A and B, buy coffee in store 1 on two consecutive purchase occasions and then switch to store 3 for their subsequent purchase of coffee. In addition, assume that housewife A purchased the private labels of store 1 during her past two visits, whereas housewife B purchased the national or regional brands. Reading from Table 22, housewife A will purchase the private label in store 3 with the probability of 0.60, whereas housewife B will purchase the private label with a probability of 0.14. In other words, due to the carry-over effect, the probability of purchasing the private label has been increased four times Similar observations on the differences for other stores show that the probability of purchasing has been increased approximately four times. The average increase in the probability (weighted by the sample size) due to the carry-over effect has been around 44%. The estimates of the carry-over effect, as obtained from Tables 21 and 22, are summarized in Table 23.

This strong carry-over effect suggests that in certain segments of the market, housewives feel that one private label can be substituted for another. In general, private labels have an advantage of price differential over national brands. Thus, it is likely that the observed purchasing behavior could be due to the housewife's price-consciousness. This inference has not been pursued further since the data does not supply the price alternatives available to the housewife in the store. It should also be kept in mind that private labels enjoy better in-store promotional services than national or regional brands, and this could be another causal factor in the observed carry-over effect. Further research by designed experimentation is necessary to test these cause and effect relationships. Thus far, we have shown that a housewife's loyalty to a store increases the probability of her purchasing the store's private labels, and that a housewife's loyalty to one private label positively influences her decision to a substantial degree to purchase private labels in a different store.

At the micro level we have been considering the consumer's many available alternatives in deciding among different brands, stores, and product sizes. Thus far, the discussion has centered around topics such as: (1) the effect of store switching

TABLE 23

MEASURE OF THE CARRY-OVER EFFECT IN PURCHASING THE PRIVATE LABELS OF COFFEE

Purchase History	Probability of purchasing private labels while visiting a store other than S_1 (Figures in parentheses indicate the sample size for corresponding probability estimates)	
Purchased the national or regional brand in Store S_1 on the previous purchase occasion	0.28	(858)
Purchased the private label in Store S_1 on the previous purchase occasion	0.64	(575)
Purchased national or regional brands in Store S_1 on the two previous purchase occasions	0.20	(248)
Purchased the private labels in Store S_1 on the two previous purchase occasions	0.64	(201)

on brand choice; (2) the loyalty of a housewife to a particular store and its effects on her purchase of private labels; and (3) the effect of store or brand change on the size of her purchase, et cetera. Although these findings are of considerable importance to the manufacturer, helping him to understand consumer buying behavior and to develop suitable marketing strategies, often the manufacturer initiates price promotions on his brand, either to meet the competitor's actions or to encourage an increase in the sales of his brand. A considerable amount of price activity is common in the market for frequently purchased consumer goods, initiated either by the manufacturer or the retailer. The assumption underlying price activity is that a housewife knows the price of a product because of her frequent purchases, and so any reduction in price should attract a greater volume of sales. Since this seems logical, it is interesting to explore the over-all market share variation as affected by prices over time. We shall also study the interaction between brand and store at an aggregate market share level, after suitably adjusting for price variation. This is an extension of our second hypothesis that store switching of housewives increases brand switching.

<u>HYPOTHESIS 6</u>: Store-brand interaction is statistically significant after eliminating the effect of price. As a corollary, the effect of price on the market share is significant and the interaction between brand and store cannot be explained by any linear function of the corresponding price difference.

In the ideal situation of a housewife purchasing the same brand in whatever store she visits, there would be no interaction between brand and store. It is assumed here that the over-all distribution of consumers shopping in different stores remains more or less the same in terms of their preferences for brands. Under this assumption and the ideal situations, a brand will maintain the same market share in each of the stores, and so the differences between two brand shares over all the stores remains the same; hence, no interaction between the brand and the store. Testing the interaction between brand and store would be difficult in real situations due to existing distribution differences; all brands are not available in all the stores. Therefore, four major brands of each product and ten stores that carry all these brands (as evidenced by actual purchases) were selected for the study.

For each month, the market share and the average price of each of the four brands in each of the ten stores are calculated. The price differential of the brands is one of the important external variables that affects the market share of the brand. In many situations it may be impracticable or uneconomical to keep constant all the other variables that affect the market share of a brand in a store; for example, the effect of variation in price levels is confounded in the observed differences in market shares. By measuring these extraneous variables or concomitant variables in statistical analysis, it is possible to adjust for their variations by the technique of Analysis of Covariance.

The price of the brand has been treated as the concomitant variable in performing the Analysis of Covariance. A detailed description of the technique is presented in Appendix V. Analysis of Covariance is done twice for each product, by calculating the market share of the brand first by the proportionate number of purchases and second by the proportionate volume of sales. In all, the analysis has been repeated six times for the three products. Each analysis is presented in sets of four tables. For the purposes of illustration, one set of tables in connection with the analysis of product A is presented below. (Refer to Tables 24 to 27). The first table (Table 24) tabulates the sums of products and the sums of squares (estimates of variation and the corresponding source of variation) for the market share variable (Y) and the concomitant variable, price (X). The second table (Table 25) is a part of the Analysis of Covariance for testing the null hypothesis that there is no interaction between the brand and the store after adjusting for the effect of price variation. The third table (Table 26) similarly tests the null hypothesis that the observed interaction (store x brand) is explained by a linear function of the corresponding price difference. The last table (Table 27) tests the null hypothesis that price has no over-all effect on the market share of the brand. All these hypotheses have been tested for their significance at both 5% and 1% levels. Two asterisks in the F-ratio column of a table indicates that the corresponding hypothesis has been rejected at the 1% level. A single asterisk indicates the rejection of the hypothesis at the 5% level. No asterisk means that the corresponding hypothesis has been accepted. The complete set

TABLE 24

SUMS OF PRODUCTS AND SUMS OF SQUARES

Product A
(Paper Product)

No. of Brands 4
No. of Stores 10
No. of Periods 36

Y_{ijk}: Market Share of Brand i in Store j and Period k as determined by number of purchases.

X_{ijk}: Average Price of Brand i in Store j and Period k.

Source of Variation	Degrees of Freedom	Sums of Squares (y^2)	Sums of Products (xy)	Sums of Squares (x^2)	Regression Coefficient
Between Brands	3	91.353	-0.261	0.033	-7.903
Between Stores	9	2.785	0.901	0.945	0.954
Between Periods	35	1.309	0.562	0.573	0.981
Brand x Store Inter-action	27	9.135	0.073	0.107	0.683
Brand x Perod Inter-action	105	11.453	-0.125	0.115	-1.086
Store x Period Inter-action	315	7.879	3.333	5.156	0.646
Error	945	82.518	0.252	1.082	0.233
Total	1439	206.433	4.736	8.012	0.591

TABLE 25

ANALYSIS OF COVARIANCE FOR PRODUCT A (Number of Purchases) (Paper Product)

Source of Variation	Degrees of Freedom	Sums of Squares S_{yy}	Sums of Products S_{xy}	Sums of Squares S_{xx}	Regression Coefficient β	Adjusted S_{yy}	Degrees of Freedom	Adjusted Mean Squares	F-Ratio
Brand x Store Interaction	27	9.135	0.073	0.107		9.105	27	0.337	3.83**
Error	945	82.518	0.252	1.082	0.233	82.460	944	0.088	
Brand x Store Interaction + Error	972	91.653	0.325	1.189	0.273	91.565	971		

TABLE 26

TESTING THE LINEARITY OF BRAND x STORE INTERACTION

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F-Ratio
Brand x Store Interaction	26	9.085	0.349	3.96**
Error	944	82.460	0.088	

TABLE 27

TESTING THE REGRESSION

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F-Ratio
Due to Regression	1	0.058	0.058	0.667
Error	944	82.460	0.087	

of tables are presented in Appendix III. (Refer to Tables 3.30 to 3.53). The summary of the results is given in Table 28. For the sake of convenience, let the hypotheses be identified by their numerical numbers.

Hypothesis 1: There is no interaction between the brand and the store after eliminating the effect of price variation.

Hypothesis 2: Brand-store interaction can be explained by a linear function of the corresponding price difference.

Hypothesis 3: Price has no over-all effect on the market share of the brand.

In all six cases, the three hypotheses have been rejected at the 1% level, except in the case of product A where the third hypothesis has been accepted; price does not seem to have any significant effect on the market share of the brand in the paper product market.

The rejection of the first two hypotheses in all the cases implies the existence of significant interaction between store and brand; also, this interaction is not explainable by any linear function of the price. In marketing terminology, brand-store interaction can be regarded as sales vs. promotion and distribution interaction. The differences in the market shares of the brands in different stores could be due to the differences in price and promotional environment. Promotional environment includes display, in-store promotional services, number of faces on the shelf, and special displays. From the rejection of the second hypothesis, brand-store interaction is not explainable by any linear function of the price differences.

TABLE 28

RESULTS OF THE ANALYSIS OF COVARIANCE

Product	Market share determined by	Ref. Tables	Hypothesis 1	Hypothesis 2	Hypothesis 3
A	Number of purchases	3.30-3.33	Rejected(**)	Rejected(**)	Accepted
	Volume of purchases	3.34-3.37	Rejected(**)	Rejected(**)	Accepted
B	Number of purchases	3.38-3.41	Rejected(**)	Rejected(**)	Rejected(**)
	Volume of purchases	3.42-3.45	Rejected(**)	Rejected(**)	Rejected(**)
C	Number of purchases	3.46-3.49	Rejected(**)	Rejected(**)	Rejected(**)
	Volume of purchases	3.50-3.53	Rejected(**)	Rejected(**)	Rejected(**)

As such, much of the interaction is caused by the differences in promotional environments of the brands over stores. Such interaction implies that the manufacturer is losing more sales in one store than in another. Though this is expected of any brand, consistent loss of sales in one store over another is hardly a matter to be overlooked. Further research is necessary to establish the causal factors behind this phenomena.

The significance of the third hypothesis once again reinforces the importance of price in the purchase of consumer goods. However, price seems to be an insignificant factor (within the observed range) in the purchase of product A. For the other two products, price variation significantly affected market share distribution.

Extending the hypothesis of store-brand interaction further, the significance of the interaction means that differences in market shares of two brands over different stores could not have arisen due to sampling disturbances. In other words, different stores exhibit different propensities for the sale of brands. Propensity signifies that each store has a certain sales parameter for each brand (a number between zero and one) depending on factors such as the availability of the brand in the store and the relative effectiveness of the promotional environment of the brand in the store. For example, the parameter of brand A in a store is '0' if brand A is not available in the store and '1' if the store carries only brand A. It is very likely that the magnitude of these parameters of brands in a store are altering the probability of a housewife purchasing a given brand while visiting the store.

Suppose a housewife has an apriori probability 'p' of purchasing brand A. Let there be two stores S_1 and S_2 with sales parameters for brand A as q_1 and q_2. As yet, there is no mention of any procedural outline of how to measure q_1 and q_2. Assuming q_1 to be greater than q_2, the housewife may have a higher probability of purchasing brand A while visiting store S_1 than she has of purchasing brand A in store S_2. On the average, store S_1 will have a higher percentage of sales of brand A than store S_2. This is precisely what has been observed through the significance of store and brand share interaction. The theoretical construction developed above resembles the suggested dynamic inference model by Howard. The change in the probability of a housewife purchasing a particular brand may be caused by another stochastic process, her store shopping behavior. The magnitude of the change in probability (positive or negative) is likely to be a function of the sales parameter of the brand in the store and her probabilities of visiting different stores. However, more research is needed to relate the physical and promotional aspects of the brand in the store to the change in the probability of purchasing the brand in the store.

Summary

Thus far, various hypotheses pertaining to the buying behavior of a housewife both at Micro and Macro levels are tested and at this stage a brief summary of my findings appears to be in order.

Concerning the store selection process of a housewife, I observed significant bias on the part of the consumer in the selection of a store for purchase of a product. The existence of bias toward store is not typical of any major food chain or drug chain, but observed for various types of outlet: independent drug stores; independent food stores; and discount stores. Accordingly, the following two hypotheses are accepted.

(1) A housewife exhibits strong bias in the selection of a store for purchase of any product. (a) The more recent her purchase experience in a particular store, and (b) the more frequent her visits to the store, the more likely she is to repurchase the product in that store.

(2) As a corollary, consumers exhibit bias in the selection of the typeof retail outlet (drug store, food store, discount store, etc.) in which they would like to shop for a product.

In analyzing the purchase sequences of different products with respect to store shopping, I observed the historical weighting of past purchases in a store or type of outlet affecting the probability of repurchasing in the store or the type of outlet respectively. This is similar to what has been observed by Kuehn in consumer's brand choice, which led to my next two hypotheses, the effect of bias in store selection on brand purchase and the size of purchase.

The analysis of the conditional probabilities of purchasing a particular brand given the past history of brand purchases and the corresponding stores visited, repeated for different brands of products consistently suggested that store change increases the probability of brand change. The probability estimates are obtained by aggregating the sequences of two, three and four purchases over individual consumers and time.

At a lesser detail, sequences of two purchases are analyzed to note that housewife's decision regarding the size of purchase is significantly affected by the change in her brand choice and/or store choice. A simple linear additive model is fitted to predict the effect of brand or store change on the percentage of consumers changing the size of their purchase. Thus, the two hypotheses about interaction of brand and store are accepted.

(3) Store switching increases brand switching; the more a housewife changes stores, the more she changes the brand she purchases.

(4) Consumers change the size of their purchase as they change the store or brand; in general, they decrease rather than increase the size of their purchase with a change in store or brand.

At this point, the study took a special look at the problem of consumer's preference for private labels and how the loyalty to store affects her preference. Defining the store loyalty index of a housewife by the repurchase rate, it is observed that the loyalty of a housewife to a store is positively associated with the proportion of her private label purchases, and that consumers show a substantial degree of carry-over effect in their purchase of private labels, which led to the acceptance of the following hypothesis.

(5) Loyalty to a particular store increases the preference for the private brands sponsored by a store. Also a housewife loyal to a private brand sponsored by a particular store is loyal to private labels regardless of store.

The hypothesis of store-brand interaction is extended to macro level by performing the analysis of covariance on monthly market share data and the significance of the interaction after adjusting for the effect of price variation is established. In addition, the store-brand interaction is not explainable by any linear function of the corresponding price difference, though price variation has significant effect on the market share variation of a brand. The results support the fact that the place of purchase (store) is a major factor acting as an intervening variable in executing consumer's brand preferences. Accordingly, the hypothesis given below is accepted.

(6) Store-brand interaction is statistically significant after eliminating the effect of price. As a corollary, the effect of price on the market share is significant and the interaction between brand and store cannot be explained by any linear function of the corresponding price difference.

CHAPTER V

CONCLUSIONS AND MODELLING IMPLICATIONS

The material in the following pages focuses on three points: (1) critical review of the basic assumptions underlying existing models of brand choice; (2) the significance of the study in building realistic models of brand choice; and (3) some tentative formulations for extending the existing models. The author intends to pursue these suggestions in greater detail in his future research.

Conclusions

The exhibition of strong bias by a housewife in the selection of a store, and the significant effect of store switching on the repurchase rate of a brand suggest that her brand purchase behavior is different in different stores. Accordingly, consumer's probability of purchasing a brand is not only affected by her past experience with the brand, but also by her selection of the store at the time of purchase. However, the existing models of brand choice ignored the effect of the store by describing a consumer's brand choice as a function of her past history of brand purchases. Evidence is sparse in the literature, where these models have incorporated any other marketing variables. In view of the above findings, it is deemed necessary that realistic models of brand purchase should incorporate the place of purchase (store) as a variable.

The two major components of store effect on brand purchase are: (1) the availability of a brand in the store; and (2) the promotional environment of the brand in the store. These components are reflections of the promotion and distribution policies of both the manufacturer and retailer. The effects of these factors can be partly controlled by both the channel members. These two factors along with the store patronage habits of a consumer restrict the exercise of complete freedom in brand choice by a housewife. Specifically, the nature of these constraints differ from housewife to housewife depending on the nature of stores she visits, distribution of the product, et cetera. The assumptions of any brand choice model should reflect the nature of these variations in consumer behavior, rather than act under a simplified assumption that all consumers behave in the same manner.

The extension of the store-brand interaction hypothesis at macro level once again reinforces the finding that store is a major intervening variable in executing consumer's preference for a brand. The collateral finding that price does not explain the observed interaction suggests that much of the interaction is due to the differences in the promotional environment of the brand among the stores. Non-availability of a brand is not a factor in this study as the analysis is restricted to the major national brands of a product that are available in all the stores. The identification of the stores where the brand is losing the sales may help the manufacturer in understanding the causal phenomena of store-brand interaction.

An important variable of consumer's purchase decision, the size of purchase, is found to be significantly ffected by the consumer's store selection and brand choice. The size of purchase has implications to building models of brand choice as it could affect the time lapse between purchases which is claimed to be related to the repurchase rate of a brand (Kuehn, 1964). The next step in extending the models of brand choice is to incorporate the continuous nature of time. However, in this study, the size of purchase is studied in lesser detail, and time lapse between purchases is not considered.

Another significant aspect of the study is regarding the consumer's purchase preference for private labels. The discussion of this subject is deferred to a later section of the chapter.

Modelling implications

Bernoulli models

In a Bernoulli model the assumption is that a consumer has a constant probability of purchasing a brand. Frank has observed in his findings that a housewife's probability of purchasing a brand is not constant and changes over time. A number of factors can be listed that are likely to contribute to this change in probability, such as new purchase experiences, distribution differences, and constant exposure to the competing promotional influences of the brands. It would be difficult to find a market situation in which these external influences are not present in order to measure the effect of these variables on the change in the probability of consumer's brand purchase. However, it can be reasonably assumed that a housewife who successively visits the same store for purchase of a product encounters the same promotional environment of the brands in the store. Also distribution differences will not affect her except for possible stock-outs. Let us see under these conditions, whether a simple Bernoulli model can describe the purchase behavior of a housewife who visits successively the same store. In other words, the null hypothesis states that a housewife has a constant probability of purchasing a brand as long as she visits the same store. In a Bernoulli model the probabilities of purchasing brand 1 depend only on the number of purchases of brand 1 in the past purchase history, but not on when they occurred in the sequence. In other words, there should not be any historical weighting of past purchases affecting the probability of purchasing the brand subsequently, a lack of recency effect in brand purchases. Under the null hypothesis, a simple Bernoulli model should describe the purchase data of consumers visiting the same store successively. The part of the independent brand analysis performed earlier (sequences of three and four purchases) corresponding to the store backgrounds SS or SSS form the data base to test the null hypothesis. Table 29 lists the conditional probability estimates of the ten different brands given the

TABLE 29

PROBABILITY OF PURCHASING THE BRAND VS. THE HISTORICAL SEQUENCE OF PAST TWO BRAND PURCHASES

Purchase Sequence \ Brand	A_1	A_2	B_1	B_2	B_3	B_4
10	0.481 (104)	0.381 (102)	0.352 (165)	0.397 (179)	0.358 (171)	0.305 (128)
01	0.567 (90)	0.390 (95)	0.409 (159)	0.436 (202)	0.413 (167)	0.338 (133)

TABLE 29 (Continued)

Purchase Sequence \ Brand	C_1	C_2	C_3	C_4	Irrespective of the Brand (Over-all)
10	0.357 (760)	0.312 (442)	0.217 (300)	0.236 (415)	0.322 (2766)
01	0.375 (738)	0.323 (434)	0.307 (306)	0.224 (428)	0.350 (2752)

information on the historical sequence of the past two purchases, corresponding to the store background SS (visiting the same store three times). Similar figures are given in Table 30 for sequences of four purchases, corresponding to the store background SSS.

Under the null hypothesis, the probability estimates do not show any effect of the recency of brand purchase.

Thus: $$\Pi(01) = \Pi(10) \qquad (1)$$

$$\left.\begin{array}{l}\Pi(001) = \Pi(010) = \Pi(100)\\ \Pi(011) = \Pi(101) = \Pi(110)\end{array}\right\} \qquad (2)$$

where $\Pi(abc)$ indicates the probability of purchasing brand 1 corresponding to the past purchase sequence given by abc.

However, if the recency effect of the brand purchase (historical weighting of past purchases) is present, we expect

$$\Pi(01) > \Pi(10) \qquad (3)$$

$$\left.\begin{array}{l}\Pi(001) > \Pi(010) > \Pi(100)\\ \Pi(011) > \Pi(101) > \Pi(110)\end{array}\right\} \qquad (4)$$

In nine out of the ten cases (refer to Table 29), the observed estimates are consistent with the relation given in (3), $\Pi(01) > \Pi(10)$, suggesting the effect of the recency of brand purchase in sequences of three purchases. In seven out of the twenty cases corresponding to sequences of four purchases (Refer to Table 30), the observed estimates are consistent with the set of equations given in (4). The significance of the hypothesis in this case is tested on the following lines.

Even under the null hypothesis, the conditional proba-

TABLE 30

PROBABILITY OF PURCHASING THE BRAND VS. THE HISTORICAL SEQUENCE OF PAST THREE BRAND PURCHASES

Purchase Sequence \ Brand	A_1	A_2	B_1	B_2	B_3	B_4
100	0.378 (37)	0.372 (43	0.164 (173)	0.324 (74)	0.182 (77)	0.119 (59)
010	0.478 (23)	0.308 (39)	0.219 (64)	0.338 (80)	0.187 (64)	0.269 (56)
001	0.393 (28)	0.309 (42)	0.295 (61)	0.361 (97)	0.365 (74)	0.328 (64)
110	0.595 (42)	0.464 (28)	0.511 (45)	0.490 (51)	0.510 (49)	0.387 (31)
101	0.667 (36)	0.438 (16)	0.683 (41)	0.652 (46)	0.500 (42)	0.365 (23)
011	0.562 (32)	0.372 (24)	0.468 (47)	0.750 (60)	0.478 (46)	0.455 (33)

TABLE 30 (Continued)

Purchase Sequence \ Brand	C_1	C_2	C_3	C_4	Irrespective of the brand (Over-all)
100	0.269 (390)	0.243 (251)	0.124 (185)	0.170 (247)	0.232 (1436)
010	0.271 (365)	0.273 (242)	0.145 (165)	0.244 (246)	0.253 (1344)
001	0.270 (397)	0.270 (237)	0.258 (186)	0.203 (246)	0.275 (1432)
110	0.523 (218)	0.459 (122)	0.370 (81)	0.293 (82)	0.463 (749)
101	0.560 (209)	0.481 (108)	0.519 (52)	0.238 (84)	0.509 (657)
011	0.553 (217)	0.482 (114)	0.449 (78)	0.419 (74)	0.513 (725)

bility estimates may follow a pattern consistent with the set of relations given in (4) due to sampling fluctuations, suggesting an effect of the recency of brand purchase. The probability that the observed set of estimates corresponding to a brand may form a pattern consistent with either one of the relations given in (4) is 1/6 because the six possible permutations are equally likely under the null hypothesis. But in seven out of the twenty triplets, the observed estimates follow a pattern consistent with either of the relations given in (4). Accordingly, a simple binomial test of proportions with $r = 7$, $n = 20$, and $\Pi = 1/6$ is valid here.

From the tables of the binomial probabilities:

$$P(r \geq 7 / n = 20, \Pi = 1/6) = 0.04$$

Thus the hypothesis is rejected at the 5% level, suggesting an effect of the recency of brand purchase. However, the magnitude of this effect appears to be very small. The aggregation of the consumer's purchasing in different stores may contribute to the over-estimation of the recency effect of brand purchase.

Though the purchase data confirm the tendency consistant with the following relations, the differences are statistically insignificant.

$$\Pi(01) > \Pi(10)$$

$$\Pi(001) > \Pi(010) > \Pi(100)$$
$$\Pi(011) > \Pi(101) > \Pi(110)$$

The maximum value of the standard error of the difference between any two proportional estimates p_1 and p_2 is given below for various sample sizes.

$$\text{Standard error of } (p_1 - p_2) = \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}$$

$$\leq \frac{1}{4}\left(\frac{1}{n_1} + \frac{1}{n_2}\right) = \frac{1}{2n} \text{ if } n_1 = n_2 = n$$

Sample Size	Standard error of $p_1 - p_2 (\sigma)$	2σ limit (5% level)
50	0.100	0.200
75	0.082	0.164
100	0.071	0.142
200	0.050	0.100
300	0.041	0.082
400	0.035	0.070
500	0.031	0.062

Testing for the significance of the differences in the probability estimates, only in seven out of the seventy possible comparisons the findings show a substantial effect of the recency of brand purchase. The seven cases are:

$\Pi(01) > \Pi(10)$ —— C_3

$\Pi(001) > \Pi(100)$ —— C_3

$\Pi(001) > \Pi(010)$ —— C_3

$\Pi(011) > \Pi(101)$ —— C_4

$\Pi(011) > \Pi(110)$ —— B_2

$\Pi(001) > \Pi(010)$ —— B_3

$\Pi(001) > \Pi(100)$ —— B_4

Thus, the recency effect of brand purchase in the purchase history is very small, when the consumer visits the same store successively for the purchase of a product. Accordingly, a simple Bernoulli model reasonably approximates the purchase behavior of a housewife in a particular store. This throws some serious doubts on what causes the apparent learning effect observed by Kuehn in brand choice. If the increase in the probability of buying a brand is completely due to past purchases in favor of the brand, then such learning should be more pronounced while visiting the same store since the external influences would be relatively less time-variant. However, it is to be noted that the linear learning model has the recency effect built into the model. Detailed analysis of purchase data by the store may highlight some of the underlying aspects of the consumer's brand choice, and the interaction with the choice of the store and external environment.

Markov models

Defining the state of the system by the brand purchased, the consumer's brand choice has often been described by a Markov process. In a K-brand market, a first order stationary Markov process is described by the transition matrix given below:

Initial State \ Final State	1	2	.	j	.	K
1	p_{11}	p_{12}	.	.	.	p_{1K}
2	p_{21}	p_{22}	.	.	.	p_{2K}
.	.	.	.	.	.	.
i	.	.	.	p_{ij}	.	.
.	.	.	.	.	.	.
K	p_{K1}	p_{K2}	.	.	.	p_{KK}

$p_{ij} = \{b(n)=j \mid b(n-1)=i\}$

b(n) indicating the brand purchased in the n^{th} state.

In a stationary Markov process, transition probabilities (p_{ij}) are independent of time. Much of the published work made this assumption and the additional assumption that the that the transition matrix (P) is the same for all individuals. However, the bias of consumer toward a store, the interaction between store choice and brand purchase, and the variation in the distribution of brands make these assumptions untenable. So, a realistic attempt to describe the brand choice as a Markov process should not only incorporate the place of purchase (store), but should also assume a distribution of transition probabilities in the market.

A simple way of incorporating the effect of the place of purchase into the model is through conditional probabilities.

Let $q_{ij.ah}$ indicate the probability that the purchase of brand j in store h follows the purchase of brand i in store a, in a K-brand and R-store market for a product.

With my earlier notation of b(n) and s(n) indicating brand and store selected for n^{th} purchase respectively, we have

$$q_{ij.ah} = \left\{ s(n)=h,\ b(n)=j \,\middle|\, s(n-1)=a,\ b(n-1)=i \right\}.$$

$$i, j = 1, 2, \ldots, K$$

$$a, h = 1, 2, \ldots, R$$

Defining the state of the system by the combination of brand purchased and store visited, we can describe the brand choice by a Markov process whose transition matrix 'Q' is of the order of KR. The transition probabilities ($q_{ij.ah}$) can be estimated by observing the fraction of times a housewife has purchased brand j in store h following her purchase of brand i in store a. The general form of the transition matrix (Q) is given in the following page.

Transition Matrix (Q)

Initial state \ Final state	(1,1)	.	(1,R)	(2,1)	.	(2,R)	.	(K,1)	.	(K,R)
(1,1)	$q_{11.11}$	.	$q_{11.1R}$	$q_{12.11}$	.	$q_{12.1R}$	.	$q_{1K.11}$	.	$q_{1K.1R}$
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
(1,R)	$q_{11.R1}$	.	$q_{11.RR}$	$q_{12.R1}$	.	$q_{12.RR}$	.	$q_{1K.R1}$	.	$q_{1K.RR}$
(2,1)	$q_{21.11}$	.	$q_{21.1R}$	$q_{22.11}$	.	$q_{22.1R}$	.	$q_{2K.11}$	.	$q_{2K.1R}$
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
(2,R)	$q_{21.R1}$	.	$q_{21.RR}$	$q_{22.R1}$	.	$q_{22.RR}$	.	$q_{2K.R1}$	.	$q_{2K.RR}$
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
(K,1)	$q_{K1.11}$	.	$q_{K1.1R}$	$q_{K2.11}$	.	$q_{K2.1R}$	.	$q_{KK.11}$	.	$q_{KK.1R}$
.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.
(K,R)	$q_{K1.R1}$	.	$q_{K1.RR}$	$q_{K2.R1}$	.	$q_{K2.RR}$	.	$q_{KK.R1}$	.	$q_{KK.RR}$

Knowledge of consumer's brand purchases and the corresponding store visits enable us to estimate the transition probabilities. The large number of transitions can be reduced by grouping in any particular fashion without doing undue violence to the model and can be used to pre-test different distribution strategies.

Aggregating over all the stores, we have:

$$\{b(n)=j \mid b(n-1)=i\}$$

$$= \sum_{a=1}^{R} \sum_{h=1}^{R} \{b(n)=j, s(n)=h \mid b(n-1)=i, s(n-1)=a\} \{s(n)=h, s(n-1)=a\}$$

i.e.

$$p_{ij} = \sum_{a=1}^{R} \sum_{h=1}^{R} q_{ij.ah} \{s(n)=h, \; s(n-1)=a\}$$

We shall assume that the transition probabilities ($q_{ij.ah}$) while switching stores ($h \neq a$) are independent of the store selected.

Then, $q_{ij.ah}=q_{ij.D}$ for all $a \neq h$, $1 \leq a, h \leq R$

Let us make another simplification by assuming that the transition probabilities ($q_{ij.ah}$) while visiting the same store ($a = h$) are independent of the store.

Then, $q_{ij.ah}=q_{ij.S}$ for all $a = h$, $1 \leq a, h \leq R$.

Accordingly, we have:

$$p_{ij} = \sum_{h=1}^{R} q_{ij.ah} \{s(n)=h, \; s(n-1) = h\} + \sum_{a=1}^{R} \sum_{\substack{h=1 \\ h\neq a}}^{R} q_{ij.ah} \{s(n)=h, \; s(n-1)=a\}$$

$$= q_{ij.S} \sum_{h=1}^{R} \{s(n)=h, \; s(n-1)=h\} + q_{ij.D} \sum_{a=1}^{R} \sum_{\substack{h=1 \\ h\neq a}}^{R} \{s(n)=h, \; s(n-1)=a\}$$

$$= q_{ij.S}\{S\} + q_{ij.D}\{D\}$$

With our earlier notation of 'S' and 'D' for store shopping

i.e., $P = Q_s\{S\} + Q_d\{D\}$, where Q_s and Q_d are the transition matrices corresponding to visiting the same store (S) or different store (D) respectively.

For example, we have in the case of brand C_1:

$$Q_s = \begin{bmatrix} 0.90 & 0.10 \\ 0.27 & 0.73 \end{bmatrix} \qquad Q_d = \begin{bmatrix} 0.81 & 0.19 \\ 0.43 & 0.57 \end{bmatrix}$$

$$\{S\} = 0.63 \qquad \{D\} = 0.37$$

where the two states of the system 1 and 2 are defined by the purchase of some other brand and brand C_1 respectively.

Multiplying the corresponding probabilities, the first order transition matrix without taking into consideration the store of purchase will be:

$$P = Q_s \{S\} + Q_d \{D\}$$

$$= 0.63 \begin{bmatrix} 0.90 & 0.10 \\ 0.27 & 0.73 \end{bmatrix} + 0.37 \begin{bmatrix} 0.87 & 0.13 \\ 0.43 & 0.57 \end{bmatrix}$$

$$= \begin{bmatrix} 0.87 & 0.13 \\ 0.33 & 0.67 \end{bmatrix}$$

The model suggested earlier can be further extended by assuming a distribution of Q_s and Q_d over the individuals in the market or even further by assuming a joint distribution of Q_s and $\{S\}$ as well as Q_d and $\{D\}$. Such a model would have taken into consideration the strong bias exhibited by the consumer in the selection of a store as well as the effect of her store switching on the brand purchase. This would allow us to bring into the model explicitly the effect of distribution.

<u>Learning models</u>

With the passage of time, a housewife purchases and uses different brands of the same product and according to the learning model the probability of her purchasing a brand is changed every time she makes a purchase decision. In the model, the change in the probability of purchasing a brand either by purchasing or rejecting the brand on a particular occasion, depends on the apriori probability of purchasing the brand and the slope (1-g) of the purchase and rejection operators.

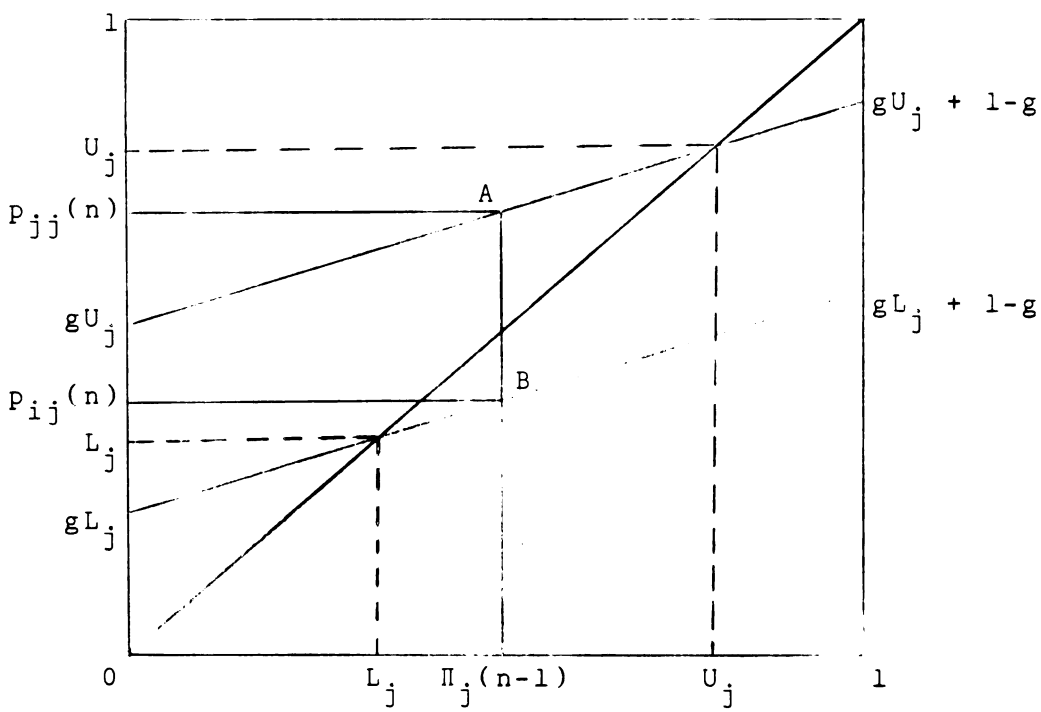
With our notation, the equations of the model are:

$$\left\{ b(n)=j \mid b(n-1)=i \right\} = p_{ij}$$

$$p_{ij} = \begin{cases} gU_j + (1-g)\ \Pi_j(n-1) & \text{if } i = j \\ gL_j + (1-g)\ \Pi_j(n-1) & \text{if } i \neq j \end{cases}$$

Where $\Pi_j(n-1)$ is the probability of purchasing brand j prior to consumer's n-1[th] purchase.

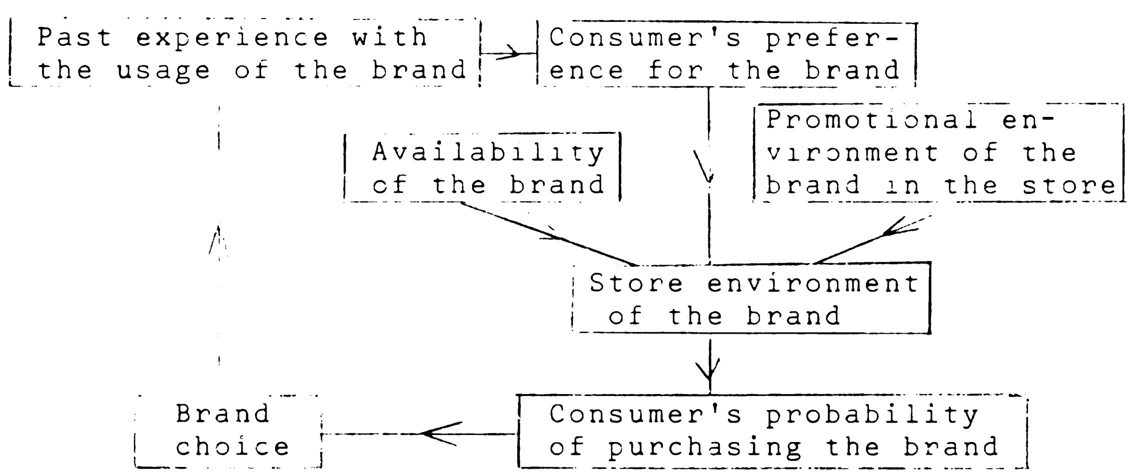
The graphical presentation of the model is given below:



The formulation of the learning model ignores the effect of the differences in the promotional policies of the retail outlets and the differences in the availability of brands.

Thus, the change in the probability caused by purchasing or rejecting the brand on a particular occasion is assumed to be independent of the store selected. A housewife with a positive probability of purchasing a particular brand (according to the model) obviously cannot purchase the brand if she makes her purchase in a store where the brand is not available. Similarly, visiting a store that de-emphasizes a particular brand will have some negative effect on the consumer's probability of purchasing that brand.

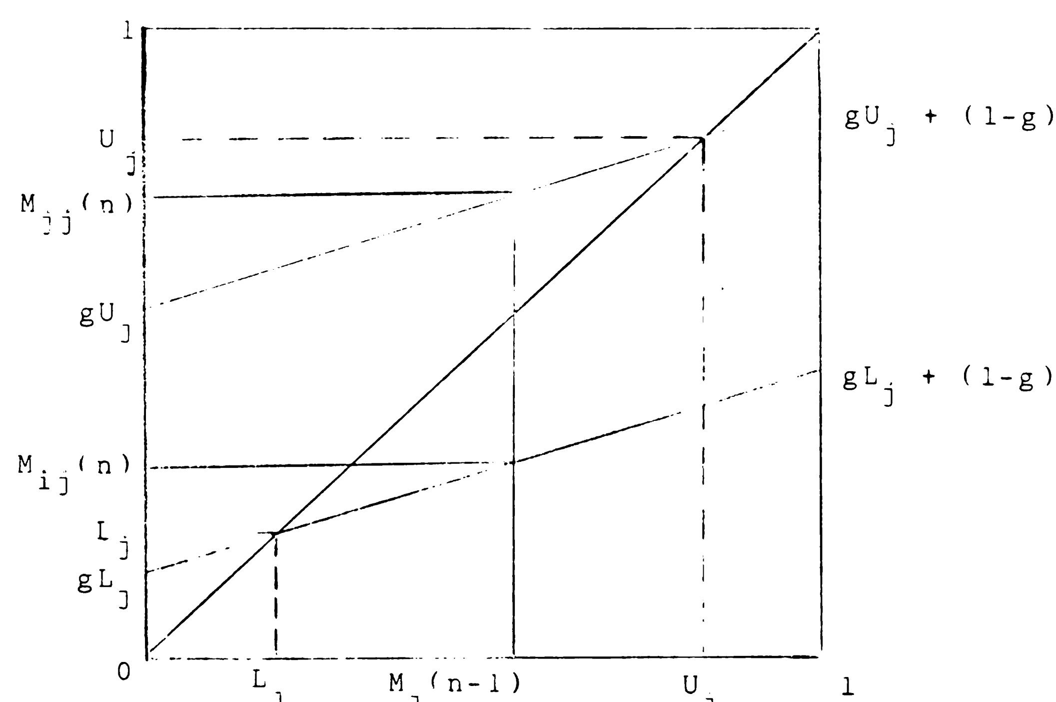
In other words, the preference developed by a housewife to a particular brand due to her past usage of the brand is likely to be modified by the store environment of the brand in her subsequent purchase. This can be conceived as a two-step process in the brand choice as shown below:



The store of purchase acts as an intermediate variable between the consumer's preference for a brand and the execution of her preference in terms of a purchase probability. In the ideal case where all stores are identical in terms of their distribution and promotion of the brands of a certain product we assume that the preferences of a consumer and her purchase probabilities will be the same.

We shall also assume that changes in the preference of a consumer for a particular brand are described by a linear learning model. The preference for a brand is measured by a number in the closed interval (0,1) and it is assumed that preference for a brand increases with the purchase of the brand and decreases with the purchase of some other brand on any purchase occasion.

Let $M_j(n)$ denote the consumer's preference for brand j prior to her n^{th} purchase and $\Pi_j(n)$ the probability of her purchasing brand j for her n^{th} purchase. Assume a K-brand and R-store market for the product.



The distinction between a housewife's preference for a brand, and her probability of purchasing the brand should be noted. We make an assumption here that preference for a brand is affected only by her usage of the brand. However, probability of purchasing a brand is affected by her preference for the brand as well as the marketing environment of the brand in the store.

If the consumer has purchased brand j on her n^{th} purchase, her preference for brand j on her subsequent purchase is given by:

$$M_{jj}(n+1) = gU_j + (1-g)M_j(n) \quad 1 \leq j \leq K$$

If the consumer has purchased brand 'i' on her n^{th} purchase, her preference for brand j on her subsequent purchase is

$$M_{ij}(n+1) = gL_j + (1-g)M_j(n) \quad 1 \leq j \neq i \leq K$$

Let $\Pi_{jh}(n)$ indicate the probability of a housewife purchasing brand j on her n^{th} purchase if she visits store h.

Assuming a linear relationship between $\Pi_{jh}(n)$ and $M_j(n)$, we can write:

$$\Pi_{jh}(n) = \alpha_{jh} + \beta_{jh} M_j(n)$$

where α_{jh} and β_{jh} are the parameters of brand j in store h. They reflect the distribution and the relative effectiveness of the promotion of brand j in store h. Indicating by $s(n)$ the store visited by the consumer for her n^{th} purchase, we have the relation:

$$\Pi_j(n) = \sum_{h=1}^{R} [\alpha_{jh} + \beta_{jh} M_j(n)] \{s(n) = h\} \qquad (1)$$

Assuming that the product (one or another of the K brands) is available in each of the R stores, we have the following constraints in the system of equations:

$$\sum_{j=1}^{K} \Pi_{jh}(n) = 1 \qquad h = 1, 2, \ldots, R$$
$$n = 1, 2, \ldots$$

$$\sum_{j=1}^{K} M_j(n) = 1 \qquad n = 1, 2, \ldots$$

$0 \leq \Pi_{jh}(n) \leq 1$ for all j, h and n

Thus, $0 \leq \alpha_{jh} + \beta_{jh} M_j(n) \leq 1$ for all $M_j(n)$ ———————— (2)

and $1 \leq j \leq K$; $1 \leq h \leq R$

and $0 \leq M_j(n) \leq 1$ for all j and n

Since equation (2) holds for all values of $M_j(n)$, it should hold for $M_j(n) = 0$ and $M_j(n) = 1$

Thus, we get: $0 \leq \alpha_{jh} \leq 1$, and $0 \leq \alpha_{jh} + \beta_{jh} \leq 1$

$j = 1, 2, \ldots, K$

$h = 1, 2, \ldots, R$

It follows then,

$0 \leq \alpha_{jh} \leq 1$; $0 \leq \beta_{jh} \leq 1$ $\qquad j = 1, 2, \ldots, K$

$h = 1, 2, \ldots, R$

We have,

$$1 = \sum_{j=1}^{K} \Pi_{jh}(n) = \sum_{j=1}^{K} [\alpha_{jh} + \beta_{jh} M_j(n)]$$
$$= \sum_{j=1}^{K} \alpha_{jh} + \sum_{j=1}^{K} \beta_{jh} M_j(n) \text{ ———— } (3)$$

Equation (3) holds for all values of $M_j(n)$. Let us take two arbitrary vectors given below:

$[M_1(n) = 1, M_j(n) = 0 \text{ for } j \neq 1]$

$[M_2(n) = 1, M_j(n) = 0 \text{ for } j \neq 2]$

Substituting these two in (3) we get:

$$\sum_{j=1}^{K} \alpha_{jh} + \beta_{1h} = \sum_{j=1}^{K} \alpha_{jh} + \beta_{2h}$$

$$\text{i.e.} \quad \beta_{1h} = \beta_{2h}$$

Extending the same argument with other arbitrary vectors, it can be verified that:

$$\beta_{1h} = \beta_{2h} = \ldots = \beta_{Kh} = \beta_h \qquad h = 1, 2, \ldots, R$$

It follows then

$$1 = \sum_{j=1}^{K} \Pi_{jh}(n) = \sum_{j=1}^{K} [\alpha_{jh} + \beta_h M_j(n)]$$

$$= \sum_{j=1}^{K} \alpha_{jh} + \beta_h \sum_{j=1}^{K} M_j(n)$$

$$= \sum_{j=1}^{K} \alpha_{jh} + \beta_h = 1$$

Therefore, $\sum_{j=1}^{K} \alpha_{jh} = 1 - \beta_h$

The linear transformation can be written as:

$$\Pi_{jh}(n) = \alpha_{jh}\beta_h + (1-\beta_h)M_j(n) \qquad \begin{array}{l} j = 1, 2, \ldots, K \\ h = 1, 2, \ldots, R \\ n = 1, 2, \ldots \end{array}$$

where $\sum_{j=1}^{K} \alpha_{jh} = 1$ for all h.

In a K-brand market there are K parameters to be estimated in each store. A hypothetical example of a two-brand market is given below.

Let the values of $M_1(n)$ be 0.65 and 0.90 respectively for two groups of consumers and the corresponding values of $\Pi_1(n)$

in store 1 be 0.55 and 0.70.

We have:

$$0.70 = \alpha_{11}\beta_1 + (1-\beta_1)\ 0.90$$

$$0.55 = \alpha_{11}\beta_1 + (1-\beta_1)\ 0.65$$

Solving the equations: $\alpha_{11} = 0.40;\ \beta_1 = 0.40$

Writing the equations for brand 2, we get

$$\alpha_{21} = 0.60;\ \beta_1 = 0.40$$

The parameters of the system in store 1 are:

$\alpha_{11} = 0.40;\ \alpha_{21} = 0.60;\ \beta_1 = 0.40.$

Note that $\alpha_{11} + \alpha_{21} = 1.00$

The principle of least squares can be used to find the best possible estimates of β_h and $(\alpha_{1h}, \alpha_{2h}, \ldots, \alpha_{kh})$ subject to the probability constraints in the model.

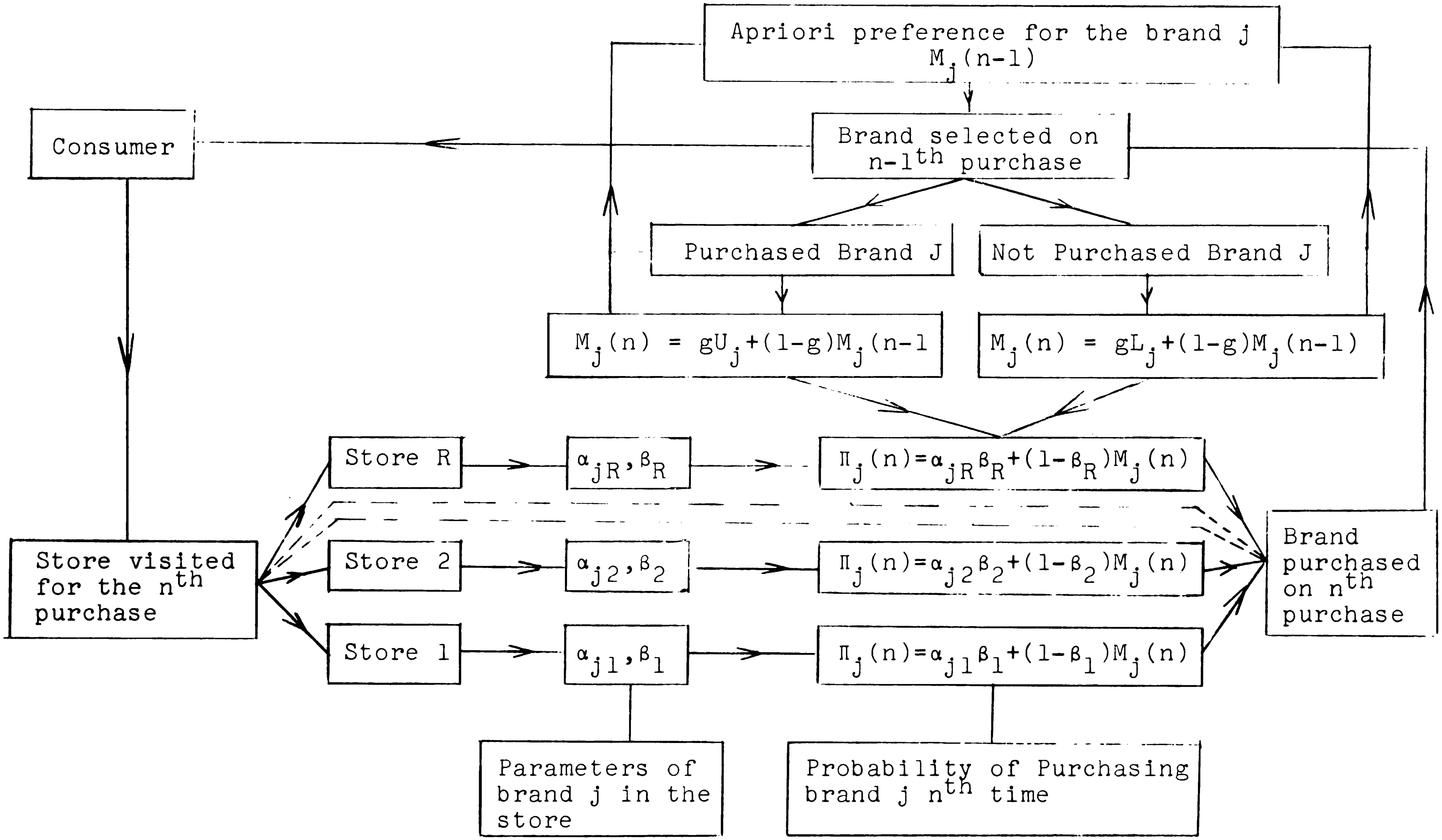
A flow diagram of the model on the following page is discussed here.

The diagram lists the inter-relationships between two consecutive purchases. A consumer starts with an apriori preference of $M_j(n-1)$ for brand j. The apriori preference for her n^{th} purchase will be $M_{jj}(n)$ if she purchases brand j or $M_{ij}(n)$ if she purchases some other brand in her $n-1^{th}$ purchase. This will become the apriori preference for her n^{th} purchase, $M_j(n)$. The consumer has to visit one of the available stores 1, 2, ... , R for her n^{th} purchase. If she purchases in store h, where α_{jh} and β_h are the parameters, her probability of purchasing brand j is given by:

$$\Pi_{jh}(n) = \alpha_{jh}\beta_h + (1-\beta_h)M_j(n)$$

The housewife makes the purchase of brand j on her n^{th}

FLOW DIAGRAM



purchase with a probability of $\Pi_{jh}(n)$, and some other brand with the probability of $(1-\Pi_{jh}(n))$. The final brand selection along with $M_j(n)$ will affect her apriori preference for brand j on her n + 1st purchase.

Equation (1) gives

$$\Pi_j(n) = \sum_{h=1}^{R} [\alpha_{jh}\beta_h+(1-\beta_h)M_j(n)]\{s(n)=h\}$$

$$= \sum_{h=1}^{R}\sum_{a=1}^{R} [\alpha_{jh}\beta_h+(1-\beta_h)M_j(n)]\{s(n)=h,\ s(n-1) = a\}$$

$$\sum_{h=1}^{R}\sum_{a=1}^{R} [\alpha_{jh}\beta_h+(1-\beta_h)M_j(n)]\{s(n)=h \mid s(n-1) = a\} X \{s(n-1) = a\} \qquad (5)$$

It is observed in this study that a housewife does not have a constant probability of purchasing in a store and the pattern of store choice is similar to the recency effect of brand purchases observed by Kuehn in brand choice. Now we shall incorporate the recency effect in store choice into the above model.

Let $\delta_h(n)$ denote the apriori probability of a housewife purchasing in store h for her nth purchase. Describing the change in the probabilities of visiting different stores by a linear learning model, the set of conditional probabilities are given by the following equations:

$$\{s(n) = h \mid s(n) = a\} = \begin{cases} \gamma_h\theta+(1-\theta)\ \delta_h(n-1) & \text{if } a=h \\ \varepsilon_h\theta+(1-\theta)\delta_h(n-1) & \text{if } a\neq h \end{cases} \qquad (6)$$

$$h = 1, 2, \ldots, R$$

θ is same for all stores because of our assumption regarding the linearity of the model.

Substituting equation (6) in (5) we get,

$$\Pi_j(n) = \sum_{h=1}^{R} [\alpha_{jh}\beta_h+(1-\beta_h)M_j(n)][\gamma_h\theta+(1-\theta)\delta_h(n-1)]\delta_h(n-1)$$
$$+\sum_{h=1}^{R}\sum_{\substack{a=1\\ a\neq h}}^{R}[\alpha_{jh}\beta_h+(1-\beta_h)M_j(n)][\varepsilon_h\theta+(1-\theta)\delta_h(n-1)]\delta_a(n-1)$$
$$= \sum_{h=1}^{R} [\alpha_{jh}\beta_h+(1-\beta_h)M_j(n)][\gamma_h\theta+(1-\theta)\delta_h(n-1)]\delta_h(n-1)$$
$$+ [\alpha_{jh}\beta_h+(1-\beta_h)M_j(n)][\varepsilon_h\theta+(1-\theta)\delta_h(n-1)][1-\delta_h(n-1)] \qquad (7)$$

<u>Case i</u>: $\beta_1 = \beta_2 = \ldots = \beta_R = 0$

Then we get,

$$\Pi_j(n) = M_j(n) \qquad j = 1, 2, \ldots, K$$

Since $\sum_{h=1}^{R}\sum_{a=1}^{R}\left\{s(n) = h,\ s(n-1) = a\right\} = 1$

In this case, consumers execute their brand preferences independent of the store visited.

<u>Case ii</u>: $\theta = 0$

In this case, consumers have constant probability of visiting a store.

Then we get:

$$\Pi_j(n) = \sum_{h-1}^{R} [\alpha_{jh}\beta_h+(1-\beta_h)M_j(n)]\delta_h \qquad j = 1, 2, \ldots, K$$

where δ_h is the probability of visiting store h.

<u>Case iii</u>: $g = 0$

In this case, consumers have constant preference for a brand. i.e. $M_j(n)$ is same for all n, (say) M_j $\quad j = 1, 2, \ldots, K$

Then the above formulation becomes a simple Bernoulli model within a store where the constant probability of purchasing brand j in store h is given by $\alpha_{jh}\beta_h+(1-\beta_h)M_j$ $\quad j = 1,2,\ldots,K$

$h = 1,2,\ldots,R$

If a housewife has gone to store a for her n-1[th] purchase, the probability of her purchasing brand j subsequently is given by:

$$\Pi_j(n) = [\alpha_{ja}\beta_a+(1-\beta_a)M_j(n)][\gamma_a\theta+(1-\theta)\delta_a(n-1)]$$
$$+ \sum_{\substack{h=1\\h\neq a}}^{R}[\alpha_{jk}\beta_h+(1-\beta_h)M_j(n)][\varepsilon_h\theta+(1-\theta)\delta_h(n-1]$$

The complete set of equations for this generalized model are:

$$M_j(n+1 = \begin{cases} gU_j+(1-g)M_j(n) & \text{if } b(n)=j \\ gL_j+(1-g)M_j(n) & \text{if } b(n)\neq j \end{cases} \qquad j = 1, 2, \ldots, K \quad n = 1, 2, \ldots$$

$$\Pi_{jh}(n+1) = \alpha_{jh}\beta_h+(1-\beta_h)M_j(n+1) \qquad h = 1, 2, \ldots, R \quad j = 1, 2, \ldots, K \quad n = 1, 2, \ldots$$

$$\Pi_j(n+1) = \sum_{h=1}^{R}[\alpha_{jh}\beta_h+(1-\beta_h)M_j(n+1)]\Big[[\gamma_h\theta+(1-\theta)\delta_h(n-1)]\delta_h(n-1)$$
$$+ [1-\delta_h(n-1)][\varepsilon_h\theta+(1-\theta)\delta_h(n-1)]\Big]$$

$$j = 1, 2, \ldots, K \quad n = 1, 2, \ldots$$

There is no need to assume that the relationship between $\Pi_j(n)$ and $M_j(n)$ is linear. The above model can be extended on these generalized lines:

$$\Pi_{jh}(n) = f_h[M_j(n)]$$

Then the equations of the model are:

$$M_j(n+1) = \begin{cases} gU_j+(1-g)M_j(n) & \text{if } b(n) = j \\ gL_j+1-g)M_j(n-1) & \text{if } b(n) \neq j \end{cases} \qquad \begin{array}{l} j = 1, 2, \ldots, K \\ n = 1, 2, \ldots \end{array}$$

$$\Pi_{jh}(n+1) = f_h[M_j(n+1)] \qquad \begin{array}{l} j = 1, 2, \ldots, K \\ h = 1, 2, \ldots, R \\ n = 1, 2, \ldots. \end{array}$$

$$\Pi_j(n+1) = \sum_{h=1}^{R} f_h[M_j(n+1)] \Big[\delta_h(n)[\gamma_h\theta+(1-\theta)\delta_h(n)] + [1-\delta_h(n)][\epsilon_h\theta+(1-\theta)\delta_h(n)]\Big] \qquad \begin{array}{l} j = 1, 2, \ldots, K \\ n = 1, 2, \ldots. \end{array}$$

If a consumer has visited store a for her n^{th} purchase, then:

$$\Pi_j(n+1) = f_a[M_j(n+1)][\gamma_a\theta+(1-\theta)\delta_a(n)] + \sum_{\substack{h=1 \\ h\neq a}}^{R} [\epsilon_h\theta+(1-\theta)\delta_h(n))]f_h[M_j(n+1)]$$

In addition to the task of developing a generalized relationship between $M_j(n)$ and $\Pi_{jh}(n)$, more research is needed to give a physical interpretation of the parameters in the model. In some sense, the parameters reflect the retail store policies as well as the distribution and promotion policies of the manufacturer. Such knowledge would be of immense use to the marketing practitioner and would also justify the academic task of building marketing models. The parameters not only help us in quantifying the effects of store-brand interaction, but also provide us with some concrete criteria for discriminating among the various types of outlets. The criteria can also be used to segment the retail market for effective and optimal allocation of marketing effort on the part of the manufacturer.

Thus far, we concentrated on the problem of incorporating store effect in brand choice models. The other important variables such as the size of purchase, the time-lag between purchases, and the price of purchase also deserve special attention. This investigation has not dealt at great length with these variables and, therefore, it would be premature to make any specific suggestions regarding their incorporation into models of brand choice, except that they should be incorporated.

Simulation

In the face of the complex task of building a mathematical model that incorporates all the elements of a consumer's purchase decision, simulation offers some immediate solutions to the researcher. However, constructive simulation needs the knowledge of the relationships among the various factors of buying behavior. This study highlighted some of these factors such as: store selection patterns of a consumer; the interaction between brand choice and store choice; the private label proneness of a store-loyal consumer; and the aggregate effect of price on the market share of a brand. Any of the suggested models in earlier pages can be used to simulate consumer's buying behavior. As an example, let us discuss the simulation of brand choice based upon the generalized learning model.

Assume a K-brand and R-store market for the product. We must start with a set of apriori probabilities of a consumer visiting each store and apriori preferences for each brand. Let these be, with our earlier notation:

$$\delta_h(1) \quad h = 1, 2, \ldots, R$$
$$M_j(1) \quad j = 1, 2, \ldots, K$$

Let the parameters of brand j in store h be (α_{jh}, β_h).
$h = 1, 2, \ldots, R$

If a consumer selects store h for his first purchase, the δ_h's for his second purchase are given by:

$$\delta_h(2) = \gamma_h\theta + (1-\theta)\delta_h(1)$$

$$\delta_a(2) = \varepsilon_a\theta + (1-\theta)\delta_a(1) \qquad a \neq h;\ a = 1, 2, \ldots, R$$

The probability of purchasing brand j in store h is given by:

$$\Pi_{jh}(1) = \alpha_{jh}\beta_h + (1-\beta_h)M_j(1)$$ (Assuming a linear effect of store x brand interaction)

Purchase of brand j or some other brand will affect her apriori preference for his second purchase as follows:

$$M_j(2) = \begin{cases} gU_j + (1-g)M_j(1) & \text{if } b(1) = j \\ gL_j + (1-g)M_j(1) & \text{if } b(1) \neq j \end{cases}$$

Using the random number generating function and the values of $\delta_h(n)$, $(h = 1, 2, \ldots, R)$ and $\Pi_{jh}(n)$ at each stage $(n = 1, 2, \ldots)$ the purchase data of a consumer in terms of her brand purchases and store visits can be simulated. The brand choice and store choice of n^{th} purchase will modify the apriori probabilities of $n + 1^{st}$ purchase through the intermediate variable $M_j(n+1)$. Starting with an initial distribution of the apriori preferences for brand j in the market, and distributions of apriori probabilities of visiting R stores, the performance of brand j can be simulated. As a first step, the model can be tested for its accuracy by comparing the simulated data with the panel data. Also, the simulated data can be used for pre-testing the policies of the manufacturer or for evaluating marketing alternatives.

Private label proneness of a consumer

Though the hypotheses regarding the private label purchases do not belong to the central theme of our discussion, they deserve a special mention in view of their importance to the retailer. The positive association between the store loyalty of a consumer and her proportion of private label purchases increases the importance of store patronage by a housewife to both the manufacturer and the retailer. Store patronage of a consumer enables the retailer not only to attract a considerable portion of the consumer's budget, but also to increase the preference of the consumer for the store's private labels. A consumer's store preference and her private label preference interact, positively reinforcing each other in favor of the retailer.

The substantial degree of substitutability of one private label for another suggests that in certain segments of the market, consumers do not differentiate between the individual private labels of stores. Identification of these segments through further research will help retailers to develop suitable promotional strategies. Except for a subtle difference in distribution, the observed behavior is similar to brand-mix loyalty (loyalty to a group of brands). The different brands of a brand-mix (normally referred to in the published literature as a set of national or regional brands) are available simultaneously in various stores, whereas no two private labels are available in the same store.

An interesting side light of the issue is that it has been the customary practice to group all the private labels into one category while analyzing the market data. This has been especially useful while fitting a Markov chain model to the purchase data, since the number of brands in the market are considerably reduced to make the analysis manageable. The evidence in this study supports the practice.

Future Research

In order to streamline future research efforts, a major goal should be to study simultaneously the major factors of the consumer's purchase decision process and incorporate them into models of buying behavior. It is a complex task, but not impossible to achieve in the author's view. One way of working toward this goal is to expand the existing models by expressing the parameters of the model as functions of the external marketing influences. The incorporation of store effect into the models of brand choice has been studied here.

Store-brand interaction observed at both micro and macro levels is only a partial reflection of the interactions among the various elements of the marketing mix: Advertising and distribution; price and distribution; product and promotion; retail store policies and manufacturers' promotions, etc. Further research, through designed experimentation, is necessary to understand the causal phenomena behind the observed interactions between the consumer and her marketing environment.

Such a diagnostic study will help the manufacturer look at the market situations in a more realistic perspective and plan and execute remedial strategies.

The size of the purchase, the time-lag between purchases, the price of the purchase, and the consumer's advertising exposure, are only a few of the important factors of purchase environment that should be explicitly brought into models of brand choice. These variables have not been studied in great detail in relation to the existing models of brand choice.

Constantly facing the studies of buying behavior is the problem of aggregation, to describe the behavior of the market when the stochastic model describes the behavior of an individual. How accurately and usefully one can interpret the market situation from the micro models of buying behavior is still a problem in research. The present method of grouping the successive purchases, giving rise to the duplication of individual brand purchases in estimating the conditional probabilities of brand purchase did not seem to draw universal approval among the academicians. A sincere attempt should be made to test any possible alternate methods of grouping purchase data to describe the consumer's buying behavior.

The study has achieved its purpose if it has raised some issues regarding our understanding of consumer's brand choice and the reliability of the existing models.

APPENDIX I

CONSUMER PANEL DATA: AN OVERVIEW

Introduction to Panel Data

The use of the panel technique in market research can be traced back to the 1940's. In 1940 the Farm Journal's cross country survey used the continuous reporting of family expenditure records to study their purchase behavior. In May, 1941, the Industrial Surveys Company (at present the Market Research Corporation of America) operated a continuous panel of more than one hundred families in Indiana.[1] In the last twenty years the panel technique has gained widespread attention and has become an integral part of marketing research. The consumer panel, "a unique market research tool," can be used to "consistently penetrate the mind of Mr. Consumer and discover his definite views on a host of subjects affecting product acceptance or merchandising."[2] The panel, in effect, "enables the marketer to take a motion picture of consumer purchase behavior, to classify this behavior by all sorts of demographic, psychological and geographic detail, and to watch these phenomena

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[1] H. L. Churchill, "How to Measure Brand Loyalty," Advertising and Selling, 35 (August, 1942), p. 24.

[2] ______________, "Consumer Panel as a Marketing Tool," Printer's Ink, 213 (November 9, 1945), p. 25.

change through time. By use of modern mathematical approaches, the panel technique may also provide the most powerful tool for predicting future behavior and determining strategy to affect this behavior."[3] The panel technique has wide application outside the marketing world; for example, it can be used to study the dynamics of people's opinions and attitudes. I shall restrict myself in the following pages to a discussion of panel research in the marketing sense.

A panel, in a general sense, "is a group of consumers organized to serve with some continuity in an advisory, a judiciary, or fact-finding capacity."[4] A panel in the marketing sense is "a controlled array of original data sources which permit current and repetitive examination of the phenomena through a finite time series."[5] In other words, a panel study enables us to study the purchase behavior of a group of consumers as a function of time.[6] The three functions of panels are: (1) to minimize memory loss by obtaining a record of the purchase on the day that the purchase is made; (2) to compile a mass of essential data

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[3] Seymour Sudman, "Maintaining a Consumer Panel," Marketing Keys to Profits in the 1960's: Proceedings of the American Marketing Association Convention, Cleveland (June, 1959), pp. 322-326.

[4] ______________, "Consumer Panel as a Marketing Tool," Printer's Ink, 213 (November 9, 1945), p. 25.

[5] Samuel G. Barton, "The Consumer Pattern of Different Economic Groups," Journal of Marketing, 8 (July, 1943), pp. 51-53.

[6] Franklin R. Cawl, "The Continuing Panel Technique," Journal of Marketing, 8 (July, 1943), pp. 45-50.

about each family on a basis convenient to the researcher and to the respondent family; and (3) to eliminate the bias of the respondent's present view in reporting past events.[7]

Panel Description and Administration

The *Chicago Tribune* panel consists of 750 families "who keep chronological records of their purchases of food and household items. For each purchase in a given product class, information is available as to the family's code number, selected demographic characteristics of the family, brand purchased, date, quantity, price, type of outlet, [some major outlets are coded by their ownership] and whether or not a deal was used in making a purchase."[8] Since panel data provides a continuous record of brand choice for an extended period of time, it is suitable for a study of the consumer's behavior patterns over time.

Panel designs are of two types: (1) natural; and (2) quasi-experimental.

Nicosia:[9] Natural designs (*Chicago Tribune*, *MRCA*) produce the data for descriptions of gross and net change and for the prediction of change. These designs generally are not intended to yield data for the study of evaluative, prescriptive or explanatory questions. The main feature of quasi-experimental designs is the purposeful manipulation or introduction of one or more stimuli into a system of variables interacting in real life rather than

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[7] Samuel G. Barton, "The Consumer Pattern of Different Economic Groups," *Journal of Marketing*, 8 (July, 1943), pp. 51-53.

[8] Ronald E. Frank, "Brand Choice as a Probability Process," *Journal of Business*, 35 (January, 1962), p. 43.

[9] Francesco M. Nicosia, "Panel Designs and Analysis in Marketing," *Proceedings of the Fall Conference of American Marketing Association*, (September, 1965), pp. 228-243.

laboratory settings. These are best suited to answer evaluative and prescriptive questions. Quasi-experimental designs are also a must when we want to explain changes in a variable in terms of its causal structure.

So far we have been talking about the conceptual aspects of the panel technique in marketing research. Some of the problems of administering a panel and the advantages and disadvantages of panel data for research purposes are discussed in detail in the following pages. Though a researcher is not directly confronted with the problem of panel administration, it is essential that he understand some of the aspects of the maintenance of a panel as they might reflect on the nature of the data.

The first problem of panel administration is to select the panel sample. As previously mentioned, a consumer panel is a group of consumers so selected that it constitutes a representative sample of the market to be appraised. The consumers are chosen in terms of income, age, sex, education, occupation, size of family, and ownership or rental of homes, in order to conform to the national, sectional, or regional patterns under observation.[10] Such a selected group of consumers is submitted "to a series of intermittent interviews or is required to make a series of reports over a period of time."[11]

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[10] Archibald S. Bennett, "Consumer Panels: Radar of the Sales Department," Sales Management, 55 (October 15, 1945), pp. 155-156.

[11] Robert N. Wadsworth, "The Experience of User of Consumer Panel," Applied Statistics, 1 (November, 1952), pp. 169-178.

Once the panel is established, the major problem is to enlist the cooperation of the panel members. Various incentive schemes are instigated as a part of the panel administration to ensure the continuity of the panel membership for a required time period. Sudman[12] presents an extensive discussion of these incentive schemes in a publication of the Journal of Marketing Research. Shaffer[13] discusses some of the operational problems involved in the organization of a consumer panel, such as: "(1) how long should the reporting period be; (2) on what days should the reporting period begin and end; and (3) should reporting be continuous or discontinuous." From the published evidence, it appears that weekly reporting is preferable to monthly reporting. Lewis[14] reports that families prefer weekly purchase panels and that more accurate and more complete information is obtained through the weekly reporting system. For brand loyalty studies, weekly panels have an additional advantage since they impose a more stringent scrutiny on the data studied.

The most important single advantage of the panel design is its analytical nature. "Since data are collected

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[12] Seymour Sudman, "Maintaining a Consumer Panel," Marketing Keys to Profits in the 1960's: Proceedings of the American Marketing Association Convention, Cleveland (June, 1959), pp. 322-326.

[13] James D. Shaffer, "The Reporting Period For a Consumer Purchase Panel," Journal of Marketing, 19 (January, 1955), pp. 252-257.

[14] Harrie F. Lewis, "A Comparison of Consumer Responses to Weekly and Monthly Purchase Panels," Journal of Marketing, 12 (April, 1948), pp. 449-454.

from the same individuals over time, the specific individuals who change or do not change (for example, those who switch to different brands and those who are loyal to one brand) can be studied. Analysis of timing of such changes may enable the researcher to develop hypotheses as to reasons for change. Since the same individuals are involved in all the 'before' and 'after' measurements, small changes can be identified more easily than if separate studies were made using two independent, but comparable, samples."[15] In addition, panel data are more accurate because purchases are immediately recorded rather than recalled later, as in questionnaire surveys. This claim of accuracy in panel data has been put to empirical test in several studies, and no evidence to the contrary has been found.[16] Besides these advantages of the panel technique over an ad hoc questionnaire survey, its utilitarian advantage is in top management decisions, since it enables management to understand how individual consumers behave from time to time. Today's business decisions require astute prejudgment of consumer acceptance for every competitive feature of a product.

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[15] Harper W. Boyd, Jr. and Ralph L. Westfall, Marketing Research (Text and Cases), (1964, Richard D. Irwin, Inc.,) pp. 116-117.

[16] Warren N. Gordall, "Are Nielsen Ratings Affected by Non-Cooperation, Conditioning, or Response Error?," Journal of Advertising Research, 2 (September, 1962), pp. 45-49, and "Response Variation Encountered With Different Questionnaire Forms," Marketing Research Report No. 163, U. S. Dept. of Agriculture.

For example, a panel can tell the manufacturer not only how he is meeting his competition, but whether he is gaining or losing ground in the market. Also, a panel can ascertain for a company the methods of distribution its competitors are employing. By reporting where they bought their goods, the consumers on the panel will reveal the stores and types of stores through which the competition is marketing the product. In other words, a consumer panel is "a barometer of recent and current purchases", as well as of anticipated demand.[17]

Panel Accuracy

The panel inherits a group of problems common to all survey methods; for example, "the attainment of optimum reliability, validity, and precision of measurement for a minimum cost."[18] The selection of a sample, the interviewing techniques, et cetera, are all more important in a panel operation than in any other type of ad hoc survey. "Respondents cannot be representative in their behavior if questions and reporting techniques focus their attention on certain things that they ordinarily would not notice" (e.g., price of a brand).[19]

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[17] Archibald S. Bennett, "Consumer Panels: Radar of the Sales Department," Sales Management, 55 (October 15, 1945), pp. 155-156.

[18] Francesco M. Nicosia, "Panel Designs and Analysis in Marketing," Proceedings of the Fall Conference of American Marketing Association, (September, 1965), pp. 228-243.

[19] __________, "Consumer Panel as a Marketing Tool," Printer's Ink, 213 (November 9, 1945), p. 25.

A problem unique to panel methodology is that panels suffer "a higher rate of refusal than cross-sectional surveys." Even with the universal "use of incentives to encourage both initial and continuing participation" of members on the panel, the mortality rate is high during the operation, and often the causal factors are not under the researcher's control.[20]

A considerable amount of literature has been published on various possible sources of bias in panel information and on the question of whether the panel sample truly represents the general market behavior. Some issues concerning panel accuracy are:

(1) Bias and other effects that may be introduced by re-interviewing

(2) Representativeness of the sample, since a certain proportion of people are not interested in panel participation

(3) Errors due to memory loss and mistakes in recording

(4) Possible sources of bias in recording due to the length of panel membership

(5) The large number of commodities for which purchases are to be recorded affecting the quality of reporting

(6) Possible over-statement of purchases from new panel recruits

(7) Response errors from the panel members because of their self-consciousness and "expertise" attempts to look good

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[20] Francesco M. Nicosia, "Panel Designs and Analysis in Marketing," Proceedings of the Fall Conference of American Marketing Association, (September, 1965), pp. 228-243.

(8) Possible conditioning of purchase behavior because of the continuous reporting of brands and the prices of past purchases

(9) Representativeness of national markets limited by the geographical concentration of panel members.

"Plausible as these arguments seem, the limited data available indicate that these effects do not occur, or occur in such a manner as to offset each other."[21] Consumer panels are continuously conducting experiments to study the possible validity of these arguments. Sudman[22] reports empirical evidence that panel membership does not condition future behavior in regard to purchases. Nicosia claims that bias due to the re-interviewing effect is not detrimental since "it makes the respondent a better reporter of her own actions and thoughts; indeed, repreated interviewing may be the only way to get at routine behavior patterns and unconsciously enacted psychological processes."[23] Ehrenberg[24] finds that the length of panel membership and the increase

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[21] James D. Shaffer, "The Reporting Period For a Consumer Purchase Panel," Journal of Marketing, 19 (January, 1955), pp. 252-257.

[22] Seymour Sudman, "Accuracy of Recording of Consumer Panels," Journal of Marketing Research, 1 (May, 1964 and August, 1964), pp. 14-20, pp. 69-80.

[23] Francesco M. Nicosia, "Panel Designs and Analysis in Marketing," Proceedings of the Fall Conference of American Marketing Association, (September, 1965), pp. 228-243.

[24] A. S. C. Ehrenberg, "A Study of Potential Biases in the Operation of Consumer Panel," Applied Statistics, 9 (March, 1960), pp. 20-27.

in the number of products to be reported do not affect behavior. Regarding the length of panel membership, Sandage[25] supplies evidence that people on the panel as long as eight to ten years did not exhibit any bias in their attitudes.

Relatively little evidence is available on the magnitudes of response errors because of self-consciousness and 'expertise' attempts to look good. However, Ehrenberg[26] indicates that new panel members tend to overstate their purchases. To eliminate the effect of this bias, panel operators exclude the data from new members from the final results. When members have belonged to the panel for a period of four to six weeks, they are included in the tabulations of the entire panel.[27] On the whole, evidence supports the position that a panel sample does not behave significantly different from the over-all market.

Some widespread discussions about panel limitations and panel representativeness are outlined here. However, for our research purposes, we can reasonably assume that the conclusions drawn from panel data are not typical of any particular sample, but only depict the general tendencies of consumers in the over-all market.

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[25] C. H. Sandage, "Do Research Panels Wear Out?," Journal of Marketing, 20 (April, 1956), pp. 397-401.

[26] A. S. C. Ehrenberg, "A Study of Potential Biases in the Operation of Consumer Panel," Applied Statistics, 9 (March, 1960), pp. 20-27.

[27] Seymour Sudman, "Maintaining a Consumer Panel," Marketing Keys to Profits in the 1960's: Proceedings of the American Marketing Association Convention, Cleveland (June, 1959), pp. 322-326.

APPENDIX II

DEFINITIONS AND NOTATION

For the sake of convenience to the reader, the terminology, the symbolism, and the notation used in this document are explained in the following pages:

Terminology

Matrix: An array of elements arranged in rows and columns.

Order of a matrix: If a matrix has m rows and m columns, then the order of the matrix is m.

Closed interval (a,b): The set of elements between a and b with the inclusion of a and b.

Probability: In simple terms, probability of an event E is a number in the closed interval (0,1) assigned to the event E and denoted by $\{E\}$ or P(E). An impossible event has probability zero and a certain event has probability one.

State: The state is the description of the system at a particular time. The state of the system in the Markov model is defined by the last brand purchased or a combination of the last brand purchased and the last store visited.

Transition probability: Probability that the system passes through state i to state j.

Transition matrix: A matrix whose elements are transition probabilities.

Stationary Markov process: A Markov process in which the transition probabilities are independent of time.

First order Markov process: In a first order Markov process, the present state is dependent on only the immediately preceding state. An extension of this is an n^{th} order Markov process in which the present state is dependent on the immediately preceding n states.

Model: An abstraction of the reality. A model formally states the relationships among various factors of a business situation or process.

Stochastic process: A situation in which the relationships among factors are probabilistic rather than deterministic.

Null hypothesis: In statistics, the hypothesis that is being tested is called the null hypothesis.

Level of significance: In statistical inference, this is known as Type I error. In any type of statistical testing, the researcher takes a risk of accepting wrong hypothesis or rejecting a correct hypothesis due to sampling fluctuations. The rejection of a null hypothesis at 5% level of significance means that the researcher is taking a one in twenty chance of rejecting the null hypothesis when in fact it is true. In other words Prob (Rejecting Null Hypothesis | Null Hypothesis is true) = 0.05.

Symbols

$\{\ \}$	Indicates the probability of an event in discrete case or the probability density of random variable in a continuous case.
$A\|B$	Indicates the event A given the occurrence of event B
$\{A\|B\}$	Indicates the conditional probability of A given the occurrence of B.
$\langle p \rangle$	Indicates the expected value of the random variable p.

Symbols like $\int$ (intergral), $\sum$ (summation), and [ ], (parentheses), convey the standard meanings used in mathematics

Notation As far as possible a uniform notation has been used throughout the document for representing the market situation.

$b(n)$	Brand purchased by the consumer for her n^{th} purchase.
$s(n)$	Store visited by the consumer for her n^{th} purchase.
$M_j(n)$	Preference for brand j prior to consumer's n^{th} purchase.
$\Pi_j(n)$	Probability of purchasing brand j prior to consumer's n^{th} purchase.
K	Number of brands of a product in the market.
R	Number of stores where the product is sold in the market.
j	A typical brand.
h	A typical store.

$\Pi_{jh}(n)$ — Probability of purchasing brand j in store h for n^{th} purchase.

α_{jh}, β_h — Parameters of brand j in store h.

$\delta_h(n)$ — Apriori probability of a consumer vising store h for her n^{th} purchase.

p_{ij} — Probability of purchasing brand j following the purchase of brand i.

$q_{ij.ah}$ — Probability of purchasing brand j in store h following the purchase of brand i in store a.

g, U_j, L_j — Parameters of the linear learning model for describing the change in the preference for brand j.

$\theta, \gamma_h, \epsilon_h$ — Parameters of the linear learning model for describing the change in the probability of visiting store h.

S — The event of a housewife visiting the same store in two consecutive purchases [(s(n) = h, s(n-1) = h), h - 1, 2, ..., R].

D — The event of a housewife visiting different stores in two consecutive purchases

$$[(s(n) = h,\ s(n-1) = a \neq h),\ 1 \leq a,\ h \leq R].$$

Brand and store backgrounds: The notation used in describing store and brand backgrounds is explained in the beginning of the fourth chapter (Refer to Pages 70 and 72).

$$b_j(n) = \begin{cases} 1 & \text{If brand j is purchased on } n^{th} \text{ purchase occasion} \\ 0 & \text{If brand j is not purchased on } n^{th} \text{ purchase occasion.} \end{cases}$$

$$s_h(n) = \begin{cases} 1 & \text{If the } n^{th} \text{ purchase is made in store h} \\ 0 & \text{If the } n^{th} \text{ purchase is not made in store h} \end{cases}$$

$$BC = \begin{cases} 1 & \text{If } b(n) \neq b(n-1) \\ 0 & \text{If } b(n) = b(n-1) \end{cases}$$

$$SC = \begin{cases} 1 & \text{If } s(n) \neq s(n-1) \\ 0 & \text{If } s(n) = s(n-1) \end{cases}$$

APPENDIX III

INDIVIDUAL ANALYSIS OF THE THREE PRODUCTS:

COMPLETE SET OF TABLES

List of Tables

Serial No.	Title of the Table	Product/Brand	Reference No.
33	Analysis of Covariance for Product (Market Share Determined by Number of Purchases)	A	3.31
34	"	B	3.39
35	"	C	3.47
36	Testing the Linearity of Store x Brand Interaction (Market Share Determined by Number of Purchases)	A	3.32
37	"	B	3.40
38	"	C	3.48
39	Testing the Regression (Market Share Determined by Number of Purchases)	A	3.33
40	"	B	3.41
41	"	C	3.49
42	Sums of Products and Sums of Squares (Market Share Determined by Volume of Sales)	A	3.34
43	"	B	3.42
44	"	C	3.50
45	Analysis of Covariance for Product (Market Share Determined by Volume of Sales)	A	3.35
46	"	B	3.43
47	"	C	3.51
48	Testing the Linearity of Store x Brand Interaction (Market Share Determined by Volume of Sales)	A	3.36
49	"	B	3.44
50	"	C	3.52

Serial No.	Title of the Table	Product/Brand	Reference No.
51	Testing the Regression (Market Share Determined by Volume of Sales)	A	3.37
52	"	B	3.45
53	"	C	3.53

TABLE 3.1

PROPORTION OF HOUSEWIVES PURCHASING PRODUCT A IN THE STORE GIVEN THE PAST HISTORY OF THREE PURCHASES

PRODUCT A
(Paper Product)

1-Purchased in the Store
0-Not Purchased in the Store

Past Purchase Sequence	Fraction Purchasing in Store No. 1	2	3
000	0.039 (2676)	0.047 (2328)	0.021 (2976)
010	0.192 (114)	0.255 (137)	0.181 (77)
001	0.298 (134)	0.368 (152)	0.253 (79)
011	0.523 (65)	0.625 (104)	0.375 (32)
100	0.219 (123)	0.264 (170)	0.189 (79)
110	0.430 (65)	0.413 (116)	0.500 (32)
101	0.562 (48)	0.546 (86)	0.433 (30)
111	0.717 (117)	0.710 (249)	0.621 (37)

Notes

1: Food Chain
2: Drug Chain
3: Food Chain

TABLE 3.2

PROPORTION OF HOUSEWIVES PURCHASING PRODUCT B IN THE STORE GIVEN THE PAST HISTORY OF THREE PURCHASES

PRODUCT B (Toothpaste)

1-Purchased in the Store
0-Not Purchased in the Store

Past Purchase Sequence	Fraction Purchasing in Store No. 1	2	3
000	0.032 (6777)	0.047 (6400)	0.006 (7791)
010	0.200 (249)	0.210 (346)	0.178 (56)
001	0.276 (286)	0.309 (381)	0.344 (61)
011	0.510 (145)	0.510 (186)	0.558 (43)
100	0.210 (299)	0.241 (377)	0.164 (73)
110	0.386 (150)	0.398 (173)	0.510 (47)
101	0.526 (114)	0.486 (144)	0.571 (35)
111	0.729 (292)	0.718 (305)	0.859 (206)

Notes

1: Food Chain
2: Drug Chain
3: Food Chain

TABLE 3.3

PROPORTION OF HOUSEWIVES PURCHASING PRODUCT C IN THE STORE GIVEN THE PAST HISTORY OF THREE PURCHASES

PRODUCT C (Coffee)

1-Purchased in the Store
0-Not Purchased in the Store

Past Purchase Sequence	Fraction Purchasing in Store No.		
	1	2	3
000	0.027 (16389)	0.027 (18123)	0.045 (14266)
010	0.312 (608)	0.244 (601)	0.300 (918)
001	0.334 (630)	0.301 (643)	0.322 (920)
011	0.622 (490)	0.550 (325)	0.577 (675)
100	0.275 (668)	0.224 (636)	0.281 (938)
110	0.527 (506)	0.425 (324)	0.564 (684)
101	0.598 (463)	0.471 (282)	0.570 (663)
111	0.840 (1999)	0.772 (819)	0.851 (2689)

Notes
1: Food Chain
2: Food Chain
3: Food Chain

TABLE 3.4

PROPORTION OF HOUSEWIVES PURCHASING PRODUCT A IN THE TYPE OF OUTLET GIVEN THE PAST HISTORY OF THREE PURCHASES

PRODUCT A (Paper Product)

1-Purchased in Store Type
0-Not Purchased in Store Type

Past Purchase Sequence	Fraction Purchasing in Store Type 1	Fraction Purchasing in Store Type 2
000	0.039 (2823)	0.040 (2755)
010	0.198 (116)	0.157 (127)
001	0.248 (37)	0.205 (136)
011	0.487 (41)	0.468 (47)
100	0.184 (114)	0.126 (142)
110	0.263 (38)	0.265 (49)
101	0.366 (137)	0.529 (34)
111	0.627 (43)	0.538 (52)

Notes

1: Discount stores
2: Independent Drugstores

TABLE 3.5

PROPORTION OF HOUSEWIVES PURCHASING PRODUCT B IN THE TYPE OF OUTLET GIVEN THE PAST HISTORY OF THREE PURCHASES

PRODUCT B (Toothpaste) — 1-Purchased in Store Type; 0-Not Purchased in Store Type

Past Purchase Sequence	Fraction Purchasing in Store Type 1	2
000	0.025 (7140)	0.028 (7255)
010	0.152 (216)	0.206 (174)
001	0.206 (223)	0.357 (246)
011	0.482 (85)	0.543 (114)
100	0.116 (265)	0.296 (182)
110	0.288 (90)	0.459 (87)
101	0.442 (61)	0.597 (67)
111	0.814 (232)	0.780 (187)

Notes:
1: Independent Food Stores
2: Discount Stores

TABLE 3.6

PROPORTION OF HOUSEWIVES PURCHASING PRODUCT C IN THE TYPE OF OUTLET GIVEN THE PAST HISTORY OF THREE PURCHASES

PRODUCT C (Coffee)

1-Purchased in Store Type
0-Not Purchased in Store Type

Past Purchase Sequence	Fraction Purchasing in Store Type	
	1	2
000	0.002 (21440)	0.036 (16773)
010	0.122 (57)	0.218 (738)
001	0.271 (70)	0.240 (756)
011	0.629 (27)	0.561 (365)
100	0.111 (63)	0.179 (767)
110	0.480 (25)	0.486 (374)
101	0.473 (19)	0.545 (343)
111	0.692 (52)	0.871 (1633)

Notes:
1: Drug Chains and Independent Drugstores
2: Independent Food Stores

TABLE 3.7

PROBABILITY OF A HOUSEWIFE PURCHASING BRAND
GIVEN THE HISTORY OF HER PAST TWO BRAND PURCHASES
AND THE CORRESPONDING STORES VISITED

PRODUCT A
(Paper Product)
BRAND A_1

1-Purchased Brand A_1
0-Not Purchased Brand A_1

Historical Sequence of Brands Purchased	Past History of Stores Visited					Irrespective of the Store Choice (over-all)
	SS	SD	DS	DD: D_1D_1	DD: D_1D_2	
00	0.116 (336)	0.119 (210)	0.109 (256)	0.163 (276)	0.172 (377)	0.139 (1455)
10	0.481 (104)	0.458 (59)	0.372 (78)	0.477 (65)	0.675 (118)	0.455 (424)
01	0.567 (90)	0.339 (59)	0.673 (92)	0.583 (60)	0.490 (100)	0.541 (401)
11	0.874 (435)	0.867 (188)	0.842 (214)	0.790 (248)	0.771 (253)	0.833 (1338)

TABLE 3.8

PRODUCT A
(Paper Product)
BRAND A_2

1-Purchased Brand A_2
0-Not Purchased Brand A_2

Historical Sequence of Brands Purchased	Past History of Stores Visited					Irrespective of the Store Choice (over-all)
	SS	SD	DS	DD: D_1D_1	DD: D_1D_2	
00	0.107 (633)	0.102 (313)	0.109 (375)	0.147 (401)	0.169 (443)	0.127 (2165)
10	0.381 (102)	0.518 (56)	0.330 (88)	0.296 (54)	0.303 (119)	0.356 (419)
01	0.390 (95)	0.370 (54)	0.535 (71)	0.494 (87)	0.350 (120)	0.422 (427)
11	0.719 (135)	0.753 (93)	0.726 (106)	0.720 (107)	0.705 (166)	0.722 (607)

TABLE 3.9

PROBABILITY OF A HOUSEWIFE PURCHASING BRAND
GIVEN THE HISTORY OF HER PAST TWO BRAND PURCHASES
AND THE CORRESPONDING STORES VISITED

PRODUCT B
(Toothpaste)
BRAND B_1

1-Purchased Brand B_1
0-Not Purchased Brand B_1

Historical Sequence of Brands Purchased	Past History of Stores Visited					Irrespective of the Store Choice (over-all)
				DD		
	SS	SD	DS	D_1D_1	D_1D_2	
00	0.050 (1858)	0.060 (765)	0.082 (1012)	0.083 (1008)	0.097 (1310)	0.072 (5952)
10	0.352 (165)	0.279 (86)	0.254 (114)	0.266 (94)	0.321 (196)	0.304 (654)
01	0.409 (159)	0.333 (78)	0.558 (111)	0.359 (92)	0.382 (202)	0.398 (641)
11	0.899 (711)	0.843 (191)	0.864 (242)	0.765 (260)	0.798 (302)	0.850 (1705)

TABLE 3.10

PRODUCT B
(Toothpaste)
BRAND B_2

1-Purchased Brand B_2
0-Not Purchased Brand B_2

Historical Sequence of Brands Purchased	Past History of Stores Visited					Irrespective of the Store Choice (over-all)
				DD		
	SS	SD	DS	D_1D_1	D_1D_2	
00	0.075 (2023)	0.072 (759)	0.058 (1005)	0.098 (1009)	0.101 (1276)	0.081 (6070)
10	0.397 (179)	0.442 (77)	0.292 (113)	0.329 (79)	0.376 (157)	0.368 (604)
01	0.436 (202)	0.488 (84)	0.596 (114)	0.548 (104)	0.487 (193)	0.499 (697)
11	0.857 (489)	0.825 (200)	0.830 (247)	0.763 (262)	0.818 (384)	0.824 (1581)

TABLE 3.9

PROBABILITY OF A HOUSEWIFE PURCHASING BRAND
GIVEN THE HISTORY OF HER PAST TWO BRAND PURCHASES
AND THE CORRESPONDING STORES VISITED

PRODUCT B
(Toothpaste)
BRAND B_1

1-Purchased Brand B_1
0-Not Purchased Brand B_1

Historical Sequence of Brands Purchased	Past History of Stores Visited					Irrespective of the Store Choice (over-all)
				DD		
	SS	SD	DS	D_1D_1	D_1D_2	
00	0.050 (1858)	0.060 (765)	0.082 (1012)	0.083 (1008)	0.097 (1310)	0.072 (5952)
10	0.352 (165)	0.279 (86)	0.254 (114)	0.266 (94)	0.321 (196)	0.304 (654)
01	0.409 (159)	0.333 (78)	0.558 (111)	0.359 (92)	0.382 (202)	0.398 (641)
11	0.899 (711)	0.843 (191)	0.864 (242)	0.765 (260)	0.798 (302)	0.850 (1705)

TABLE 3.10

PRODUCT B
(Toothpaste)
BRAND B_2

1-Purchased Brand B_2
0-Not Purchased Brand B_2

Historical Sequence of Brands Purchased	Past History of Stores Visited					Irrespective of the Store Choice (over-all)
				DD		
	SS	SD	DS	D_1D_1	D_1D_2	
00	0.075 (2023)	0.072 (759)	0.058 (1005)	0.098 (1009)	0.101 (1276)	0.081 (6070)
10	0.397 (179)	0.442 (77)	0.292 (113)	0.329 (79)	0.376 (157)	0.368 (604)
01	0.436 (202)	0.488 (84)	0.596 (114)	0.548 (104)	0.487 (193)	0.499 (697)
11	0.857 (489)	0.825 (200)	0.830 (247)	0.763 (262)	0.818 (384)	0.824 (1581)

TABLE 3.11

PROBABILITY OF A HOUSEWIFE PURCHASING BRAND
GIVEN THE HISTORY OF HER PAST TWO BRAND PURCHASES
AND THE CORRESPONDING STORES VISITED

PRODUCT B
(Toothpaste)
BRAND B_3

1-Purchased Brand B_3
0-Not Purchased Brand B_3

Historical Sequence of Brands Purchased	Past History of Stores Visited					Irrespective of the Store Choice (over-all)
	SS	SD	DS	DD D_1D_1	DD D_1D_2	
00	0 051 (2239)	0.062 (820)	0 048 (119)	0 094 (1140)	0 074 (1517)	0.064 (6831)
10	0.358 (171)	0.291 (79)	0 160 (125)	0.162 (80)	0 270 (163)	0.261 (618)
01	0 413 (167)	0.255 (96)	0.439 (98)	0.341 (85)	0.269 (160)	0.345 (606)
11	0 785 (316)	0 840 (125)	0 759 (137)	0.685 (149)	0 653 (170)	0.747 (897)

TABLE 3.12

PRODUCT B
(Toothpaste)
BRAND B_4

1-Purchased Brand B_4
0-Not Purchased Brand B_4

Historical Sequence of Brands Purchased	Past History of Stores Visited					Irrespective of the Store Choice (over-all)
	SS	SD	DS	DD D_1D_1	DD D_1D_2	
00	0.036 (2486)	0.042 (961)	0.033 (1215)	0.044 (1215)	0.050 (1696)	0.042 (7579)
10	0.305 (128)	0.295 (61)	0 239 (71)	0.190 (84)	0 287 (129)	0.268 (473)
01	0.338 (133)	0.383 (60)	0.477 (109)	0.362 (58)	0.266 (94)	0.366 (454)
11	0.630 (146)	0.576 (38)	0.770 (74)	0.546 (97)	0.527 (91)	0.610 (446)

TABLE 3.13

PROBABILITY OF A HOUSEWIFE PURCHASING BRAND GIVEN THE HISTORY OF HER PAST TWO BRAND PURCHASES AND THE CORRESPONDING STORES VISITED

PRODUCT C
(Coffee)
BRAND C_1

1-Purchased Brand C_1
0-Not Purchased Brand C_1

Historical Sequence of Brands Purchased	Past History of Stores Visited					Irrespective of the Store Choice (over-all)
	SS	SD	DS	DD: D_1D_1	DD: D_1D_2	
00	0.069 (7114)	0.098 (1658)	0.083 (1741)	0.190 (1983)	0.178 (1328)	0.100 (13824)
10	0.357 (760)	0.453 (300)	0.156 (456)	0.371 (248)	0.372 (288)	0.330 (2052)
01	0.375 (738)	0.229 (393)	0.396 (357)	0.325 (243)	0.368 (339)	0.361 (2070)
11	0.879 (2225)	0.786 (477)	0.798 (500)	0.732 (572)	0.714 (406)	0.822 (4180)

TABLE 3.14

PRODUCT C
(Coffee)
BRAND C_2

1-Purchased Brand C_2
0-Not Purchased Brand C_2

Historical Sequence of Brands Purchased	Past History of Stores Visited					Irrespective of the Store Choice (over-all)
	SS	SD	DS	DD: D_1D_1	DD: D_1D_2	
00	0.032 (9484)	0.043 (2363)	0.044 (2553)	0.081 (2620)	0.063 (1895)	0.044 (18915)
10	0.312 (442)	0.366 (172)	0.147 (224)	0.325 (123)	0.240 (196)	0.279 (1157)
01	0.323 (434)	0.152 (210)	0.432 (183)	0.193 (161)	0.229 (179)	0.277 (1167)
11	0.709 (477)	0.598 (83)	0.681 (94)	0.464 (140)	0.571 (91)	0.642 (885)

TABLE 3 15

PROBABILITY OF A HOUSEWIFE PURCHASING BRAND
GIVEN THE HISTORY OF HER PAST TWO BRAND PURCHASES
AND THE CORRESPONDING STORES VISITED

PRODUCT C
(Coffee)
BRAND C_3

1-Purchased Brand C_3
0-Not Purchased Brand C_3

Historical Sequence of Brands Purchased	Past History of Stores Visited: SS	SD	DS	DD: D_1D_1	DD: D_1D_2	Irrespective of the Store Choice (over-all)
00	0 025 (9692)	0 044 (2435)	0.040 (2646)	0.048 (2652)	0 060 (1959)	0 036 (19384)
10	0 217 (300)	0.304 (161)	0 108 (139)	0 171 (129)	0.218 (147)	0 209 (876)
01	0 307 (306)	0 119 (134)	0 441 (161)	0 243 (115)	0 256 (160)	0.285 (876)
11	0 807 (539)	0 796 (98)	0.787 (108)	0.527 (148)	0 642 (95)	0.746 (988)

TABLE 3 16

PRODUCT C
(Coffee)
BRAND C_4

1-Purchased Brand C_4
0-Not Purchased Brand C_4

Historical Sequence of Brands Purchased	Past History of Stores Visited: SS	SD	DS	DD: D_1D_1	DD: D_1D_2	Irrespective of the Store Choice (over-all)
00	0.032 (9611)	0.049 (2427)	0.049 (2597)	0 063 (2629)	0 060 (1990)	0.044 (19254)
10	0.236 (415)	0 293 (150)	0 115 (183)	0.191 (162)	0.147 (150)	0 204 (1060)
01	0.224 (428)	0 183 (169)	0.402 (189)	0.230 (126)	0.215 (149)	0 249 (1061)
11	0 723 (383)	0 622 (83)	0 682 (85)	0.480 (127)	0.542 (72)	0 649 (749)

TABLE 3.17

PROBABILITY OF A HOUSEWIFE PURCHASING BRAND A_1
GIVEN THE HISTORY OF HER PAST THREE BRAND PURCHASES
AND THE CORRESPONDING STORES VISITED

PRODUCT A
(Paper Product)
BRAND A_1

1-Purchased Brand A_1
0-Not Purchased Brand A_1

Historical Sequence of Brands Purchased	Past History of Stores Visited				Irrespective of the Store Choice (over-all)
	SSS			DDD	
000	0.067 (165)			0.132 (416)	0.101 (1143)
010	0.478 (23)	DSD 0.412 (17)	SDS 0.125 (16)	0.377 (69)	0.383 (167)
001	0.393 (28)	DDS 0.655 (29)	SSD 0.077 (13)	0.459 (61)	0.462 (182)
011	0.562 (32)	DSS 0.750 (28)	SDD 0.389 (18)	0.532 (62)	0.614 (197)
100	0.378 (37)	SDD 0.389 (18)	DSS 0.375 (16)	0.306 (85)	0.335 (209)
110	0.595 (42)	SSD 0.524 (21)	DDS 0.414 (29)	0.508 (59)	0.522 (207)
101	0.667 (36)	SDS 0.800 (15)	DSD 0.417 (12)	0.558 (52)	0.622 (180)
111	0.921 (241)			0.825 (314)	0.877 (1033)

TABLE 3.18

PROBABILITY OF A HOUSEWIFE PURCHASING BRAND A_2
GIVEN THE HISTORY OF HER PAST THREE BRAND PURCHASES
AND THE CORRESPONDING STORES VISITED

PRODUCT A
(Paper Product)
BRAND A_2

1-Purchased Brand A_2
0-Not Purchased Brand A_2

Historical Sequence of Brands Purchased	Past History of Stores Visited				Irrespective of the Store Choice (over-all)
	SSS			DDD	
000	0.079 (355)			0.159 (568)	0.105 (1752)
010	0.308 (39)	DSD 0.500 (16)	SDS 0.277 (13)	0.267 (75)	0.311 (225)
001	0.309 (42)	DDS 0.464 (28)	SSD 0.237 (21)	0.315 (92)	0.328 (256)
011	0.372 (24)	DSS 0.533 (15)	SDD 0.400 (15)	0.539 (54)	0.519 (154)
100	0.372 (43)	SDD 0.348 (23)	DSS 0.174 (23)	0.300 (70)	0.270 (241)
110	0.464 (28)	SSD 0.727 (11)	DDS 0.364 (22)	0.311 (45)	0.406 (155)
101	0.438 (16)	SDS 0.643 (14)	DSD 0.333 (9)	0.571 (63)	0.593 (135)
111	0.775 (57)			0.801 (151)	0.792 (400)

TABLE 3.19

PROBABILITY OF A HOUSEWIFE PURCHASING BRAND B_1
GIVEN THE HISTORY OF HER PAST THREE BRAND PURCHASES
AND THE CORRESPONDING STORES VISITED

PRODUCT B
(Toothpaste)
BRAND B_1

1-Purchased Brand B_1
0-Not Purchased Brand B_1

Historical Sequence of Brands Purchased	Past History of Stores Visited				Irrespective of the Store Choice (over-all)
	SSS			DDD	
000	0.041 (1184)			0.087 (1621)	0.060 (5104)
010	0.219 (64)	DSD 0.212 (33)	SDS 0.158 (19)	0.270 (126)	0.227 (348)
001	0.295 (61)	DDS 0.580 (50)	SSD 0.250 (20)	0.292 (154)	0.329 (398)
011	0.468 (47)	DSS 0.692 (26)	SDD 0.619 (23)	0.524 (82)	0.536 (233)
100	0.164 (73)	SDD 0.219 (32)	DSS 0.175 (40)	0.204 (147)	0.186 (424)
110	0.511 (45)	SSD 0.615 (13)	DDS 0.314 (35)	0.430 (86)	0.421 (240)
101	0.683 (41)	SDS 0.727 (11)	DSD 0.333 (18)	0.418 (55)	0.552 (183)
111	0.952 (505			0.842 (355)	0.903 (1376)

TABLE 3.20

PROBABILITY OF A HOUSEWIFE PURCHASING BRAND B_2
GIVEN THE HISTORY OF HER PAST THREE BRAND PURCHASES
AND THE CORRESPONDING STORES VISITED

PRODUCT B (Toothpaste)
BRAND B_2

1-Purchased Brand B_2
0-Not Purchased Brand B_2

Historical Sequence of Brands Purchased	Past History of Stores Visited				Irrespective of the Store Choice (over-all)
	SSS			DDD	
000	0.059 (1338)			0.078 (1589)	0.067 (5223)
010	0.338 (80)	DSD 0.462 (13)	SDS 0.261 (23)	0.302 (96)	0.295 (308)
001	0.361 (97)	DDS 0.569 (51)	SSD 0.324 (34)	0.432 (158)	0.425 (447)
011	0.750 (60)	DSS 0.806 (36)	SDD 0.810 (21)	0.664 (116)	0.715 (326)
100	0.324 (74)	SDD 0.290 (31)	DSS 0.281 (32)	0.262 (126)	0.261 (353)
110	0.490 (51)	SSD 0.500 (22)	DDS 0.300 (30)	0.353 (85)	0.480 (254)
101	0.652 (46)	SDS 0.865 (15)	DSD 0.636 (11)	0.620 (71)	0.667 (201)
111	0.887 (274)			0.844 (385)	0.860 (1194)

TABLE 3.21

PROBABILITY OF A HOUSEWIFE PURCHASING BRAND B_3
GIVEN THE HISTORY OF HER PAST THREE BRAND PURCHASES
AND THE CORRESPONDING STORES VISITED

PRODUCT B
(Toothpaste)
BRAND B_3

1-Purchased Brand B_3
0-Not Purchased Brand B_3

Historical Sequence of Brands Purchased	Past History of Stores Visited				Irrespective of the Store Choice (over-all)
	SSS			DDD	
000	0.045 (1485)			0.076 (1887)	0.056 (5944)
010	0.187 (64)	DSD 0.185 (27)	SDS 0.054 (37)	0.200 (115)	0.174 (368)
001	0.365 (74)	DDS 0.355 (45)	SSD 0.283 (46)	0.225 (138)	0.289 (402)
011	0.478 (46)	DSS 0.625 (16)	SDD 0.353 (17)	0.400 (65)	0.475 (198)
100	0.182 (77)	SDD 0.294 (34)	DSS 0.093 (54)	0.149 (141)	0.157 (421)
110	0.510 (49)	SSD 0.500 (14)	DDS 0.237 (21)	0.318 (66)	0.387 (204)
101	0.500 (42)	SDS 0.643 (14)	DSD 0.273 (11)	0.395 (43)	0.476 (145)
111	0.841 (183)			0.766 (171)	0.830 (624)

TABLE 3.22

PROBABILITY OF A HOUSEWIFE PURCHASING BRAND B_4
GIVEN THE HISTORY OF HER PAST THREE BRAND PURCHASES
AND THE CORRESPONDING STORES VISITED

PRODUCT B
(Toothpaste)
BRAND B_4

1-Purchased Brand B_4
0-Not Purchased Brand B_4

Historical Sequence of Brands Purchased	Past History of Stores Visited				Irrespective of the Store Choice (over-all)
	SSS			DDD	
000	0.033 (1693)			0.047 (2099)	0.037 (6739)
010	0.269 (56)	DSD 0.318 (22)	SDS 0.133 (15)	0.247 (89)	0.240 (263)
001	0.328 (64)	DDS 0.469 (49)	SSD 0.231 (13)	0.250 (80)	0.339 (292)
011	0.455 (33)	DSS 0.565 (23)	SDD 0.400 (10)	0.472 (53)	0.494 (154)
100	0.119 (59)	SDD 0.261 (23)	DSS 0.200 (25)	0.121 (108)	0.141 (326)
110	0.387 (31)	SSD 0.571 (7)	DDS 0.348 (23)	0.299 (67)	0.348 (158)
101	0.365 (23)	SDS 0.429 (7)	DSD 0.667 (9)	0.450 (40)	0.462 (119)
111	0.721 (61)			0.611 (90)	0.694 (255)

TABLE 3.23

PROBABILITY OF A HOUSEWIFE PURCHASING BRAND C_1 GIVEN THE HISTORY OF HER PAST THREE BRAND PURCHASES AND THE CORRESPONDING STORES VISITED

PRODUCT C
(Coffee)
BRAND C_1

1-Purchased Brand C_1
0-Not Purchased Brand C_1

Historical Sequence of Brands Purchased	Past History of Stores Visited				Irrespective of the Store Choice (over-all)
	SSS			DDD	
000	0.054 (5490)			0.173 (2265)	0.085 (12251)
010	0.271 (365)	DSD 0.355 (93)	SDS 0.111 (199)	0.312 (256)	0.235 (1292)
001	0.270 (397)	DDS 0.400 (140)	SSD 0.160 (187)	0.292 (295)	0.269 (1359)
011	0.553 (217)	DSS 0.713 (101)	SDD 0.343 (70)	0.471 (170)	0.525 (737)
100	0.269 (390)	SDD 0.360 (86)	DSS 0.098 (234)	0.316 (291)	0.243 (1344)
110	0.523 (218)	SSD 0.645 (62)	DDS 0.234 (94)	0.497 (149)	0.492 (725)
101	0.560 (209)	SDS 0.676 (74)	DSD 0.290 (62)	0.484 (151)	0.536 (662)
111	0.927 (1666)			0.794 (564)	0.886 (3383)

TABLE 3.24

PROBABILITY OF A HOUSEWIFE PURCHASING BRAND C_2
GIVEN THE HISTORY OF HER PAST THREE BRAND PURCHASES
AND THE CORRESPONDING STORES VISITED

PRODUCT C
(Coffee)
BRAND C_2

1-Purchased Brand C_2
0-Not Purchased Brand C_2

Historical Sequence of Brands Purchased	Past History of Stores Visited				Irrespective of the Store Choice (over-all)
	SSS			DDD	
000	0.023 (7596)			0.072 (3288)	0.038 (17783)
010	0.273 (242)	DSD 0.328 (61)	SDS 0.081 (99)	0.258 (182)	0.226 (826)
001	0.270 (237)	DDS 0.377 (77)	SSD 0.098 (102)	0.183 (197)	0.231 (828)
011	0.482 (114)	DSS 0.605 (38)	SDD 0.185 (27)	0.268 (71)	0.413 (315)
100	0.243 (251)	SDD 0.328 (58)	DSS 0.134 (112)	0.118 (169)	0.186 (818)
110	0.459 (122)	SSD 0.594 (32)	DDS 0.206 (34)	0.355 (62)	0.416 (310)
101	0.481 (108)	SDS 0.567 (30)	DSD 0.154 (26)	0.257 (70)	0.410 (315)
111	0.809 (282)			0.667 (102)	0.769 (558)

TABLE 3.25

PROBABILITY OF A HOUSEWIFE PURCHASING BRAND C_3 GIVEN THE HISTORY OF HER PAST THREE BRAND PURCHASES AND THE CORRESPONDING STORES VISITED

PRODUCT C
(Coffee)
BRAND C_3

1-Purchased Brand C_3
0-Not Purchased Brand C_3

Historical Sequence of Brands Purchased	Past History of Stores Visited				Irrespective of the Store Choice (over-all)
	SSS			DDD	
000	0.022 (7829)			0.052 (3378)	0.033 (18369)
010	0.145 (165)	DSD 0.242 (62)	SDS 0.033 (60)	0.136 (154)	0.136 (617)
001	0.258 (186)	DDS 0.348 (69)	SSD 0.071 (56)	0.230 (165)	0.227 (679)
011	0.449 (78)	DSS 0.559 (34)	SDD 0.214 (28)	0.306 (49)	0.440 (243)
100	0.124 (185)	SDD 0.244 (45)	DSS 0.043 (70)	0.129 (163)	0.118 (688)
110	0.370 (81)	SSD 0.618 (34)	DDS 0.231 (26)	0.410 (61)	0.395 (248)
101	0.519 (52)	SDS 0.741 (27)	DSD 0.083 (12)	0.405 (42)	0.497 (181)
111	0.894 (376)			0.690 (129)	0.850 (728)

TABLE 3.26

PROBABILITY OF A HOUSEWIFE PURCHASING BRAND C_4
GIVEN THE HISTORY OF HER PAST THREE BRAND PURCHASES
AND THE CORRESPONDING STORES VISITED

PRODUCT C
(Coffee)
BRAND C_4

1-Purchased Brand C_4
0-Not Purchased Brand C_4

Historical Sequence of Brands Purchased	Past History of Stores Visited				Irrespective of the Store Choice (over-all)
	SSS			DDD	
000	0.025 (7755)			0.059 (3365)	0.038 (17783)
010	0.244 (246)	DSD 0.234 (47)	SDS 0.143 (21)	0.383 (65)	0.226 (826)
001	0.203 (246)	DDS 0.455 (88)	SSD 0.108 (74)	0.235 (179)	0.231 (828)
011	0.419 (74)	DSS 0.628 (43)	SDD 0.143 (21)	0.383 (65)	0.413 (315)
100	0.170 (247)	SDD 0.315 (54)	DSS 0.112 (89)	0.140 (172)	0.189 (818)
110	0.293 (82)	SSD 0.500 (32)	DDS 0.069 (29)	0.210 (62)	0.416 (310)
101	0.238 (84)	SDS 0.500 (24)	DSD 0.222 (18)	0.200 (35)	0.410 (315)
111	0.821 (218)			0.578 (90)	0.769 (558)

TABLE 3.27

PROPORTION OF HOUSEWIVES CHANGING THE SIZE OF PURCHASE OF PRODUCT A VS. THE PURCHASE PATTERN

PRODUCT A
(Paper Product)

Patern of Two Consecutive Purchases	Increased the Size of Purchase	No Change in the Size of Purchase	Decreased the Size of Purchase	Marginal Total
Same Store & Same Brand	0.124 (152)	0.784 (963)	0.092 (113)	0.312 (1228)
Same Store & Different Brand	0.212 (112)	0.574 (303)	0.214 (113)	0.131 (528)
Different Store & Same Brand	0.184 (240)	0.630 (820)	0.186 (243)	0.333 (1303)
Different Store & Different Brand	0.236 (208)	0.521 (460)	0.243 (214)	0.224 (882)
Marginal Total	0.181 (712)	0.646 (2546)	0.173 (683)	3941

P_I = 13.30 + BC x 7.00 + SC x 4.20

P_D = 10.80 + BC x 8.95 + SC x 6.15

TABLE 3.28

PROPORTION OF HOUSEWIVES CHANGING THE SIZE OF PURCHASE OF PRODUCT B VS. THE PURCHASE PATTERN

PRODUCT B
(Toothpaste)

Pattern of Two Consecutive Purchases	Increased the Size of Purchase	No Change in the Size of Purchase	Decreased the Size of Purchase	Marginal Total
Same Store & Same Brand	0.188 (575)	0.687 (2143)	0.125 (400)	0.324 (3118)
Same Store & Different Brand	0.216 (331)	0.546 (835)	0.238 (364)	0.159 (1530)
Different Store & Same Brand	0.187 (527)	0.622 (1757)	0.191 (540)	0.292 (2824)
Different Store & Different Brand	0.253 (549)	0.473 (1025)	0.274 (594)	0.225 (2168)
Marginal Total	0.206 (1982)	0.597 (5760)	0.197 (1898)	9640

$$P_I = 17.85 + BC \times 4.7 + SC \times 1.8$$

$$P_D = 13.25 + BC \times 9.8 + SC \times 5.1$$

TABLE 3.29

PROPORTION OF HOUSEWIVES CHANGING THE SIZE OF PURCHASE OF PRODUCT C VS. THE PURCHASE PATTERN

PRODUCT C
(Coffee)

Pattern of Two Consecutive Purchases	Increased the Size of Purchase	No Change in the Size of Purchase	Decreased the Size of Purchase	Marginal Total
Same Store & Same Brand	0.081 (776)	0.841 (8114)	0.078 (755)	0.429 (9645)
Same Store & Different Brand	0.150 (672)	0.691 (3087)	0.159 (709)	0.199 (4468)
Different Store & Same Brand	0.144 (424)	0.716 (2103)	0.140 (411)	0.136 (2938)
Different Store & Different Brand	0.232 (1264)	0.540 (2937)	0.228 (1244)	0.236 (5445)
Marginal Total	0.139 (3136)	0.722 (16241)	0.139 (3119)	22496

$$P_I = 7.62 + BS \times 7.85 + SC \times 7.25$$

$$P_D = 7.62 + BS \times 8.45 + SC \times 6.55$$

TABLE 3.30

SUMS OF PRODUCTS AND SUMS OF SQUARES

Product A
(Paper Product)

No. of Brands 4
No. of Stores 10
No. of Periods 36

Y_{ijk}: Market Share of Brand i in Store j and Period k as determined by number of purchases.

X_{ijk}: Average Price of Brand i in Store j and Period k.

Source of Variation	Degrees of Freedom	Sums of Squares (y^2)	Sums of Products (xy)	Sums of Squares (x^2)	Regression Coefficient
Between Brands	3	91.353	-0.261	0.033	-7.903
Between Stores	9	2.785	0.901	0.945	0.954
Between Periods	35	1.309	0.562	0.573	0.981
Brand x Store Interaction	27	9.135	0.073	0.107	0.683
Brand x Period Interaction	105	11.453	-0.1250	0.115	-1.086
Store x Period Interaction	315	7.879	3.333	5.156	0.646
Error	945	82.518	0.252	1.082	0.233
Total	1439	206.433	4.736	8.012	0.591

TABLE 3.31

ANALYSIS OF COVARIANCE FOR PRODUCT A (Number of Purchases)

(Paper Product)

Source of Variation	Degrees of Freedom	Sums of Squares S_{yy}	Sums of Products S_{xy}	Sums of Squares S_{xx}	Regression Coefficient β	Adjusted S_{yy}	Degrees of Freedom	Adjusted Mean Squares	F-Ratio
Brand x Store Interaction	27	9.135	0.073	0.107		9.105	27	0.337	3.83**
Error	945	82.518	0.252	1.082	0.233	82.460	944	0.088	
Brand x Store Interaction + Error	972	91.653	0.325	1.189	0.273	91.565	971		

TABLE 3.32

TESTING THE LINEARITY OF BRAND x STORE INTERACTION

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F-Ratio
Brand x Store Interaction	26	9.085	0.349	3.96**
Error	944	82.460	0.088	

TABLE 3.33

TESTING THE REGRESSION

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F-Ratio
Due to Regression	1	0.058	0.058	0.667
Error	944	82.460	0.087	

TABLE 3.34

SUMS OF PRODUCTS AND SUMS OF SQUARES

Product A
(Paper Product)

No. of Brands 4
No. of Stores 10
No. of Periods 36

Y_{ijk}: Market Share of Brand i in Store j and Period k as determined by volume of sales.

X_{ijk}: Average Price of Brand i in Store j and Period k.

Source of Variation	Degrees of Freedom	Sums of Squares (y^2)	Sums of Products (xy)	Sums of Squares (x^2)	Regression Coefficient
Between Brands	3	98.391	-0.289	0.033	-8.759
Between Stores	9	2.569	0.872	0.945	0.923
Between Periods	35	1.154	0.524	0.573	0.914
Brand x Store Interaction	27	7.935	0.088	0.107	0.819
Brand x Period Interaction	105	11.773	-0.193	0.115	-1.680
Store x Period Interaction	315	7.647	3.364	5.156	0.652
Error	945	90.399	-0.438	1.082	-0.405
Total	1439	219.868	3.927	8.012	0.490

TABLE 3.35

ANALYSIS OF COVARIANCE FOR PRODUCT A (Volume of Sales)

(Paper Product)

Source of Variation	Degrees of Freedom	Sums of Squares S_{yy}	Sums of Products S_{xy}	Sums of Squares S_{xx}	Regression Coefficient β	Adjusted S_{yy}	Degrees of Freedom	Adjusted Mean Squares	F-Ratio
Brand x Store Interaction	27	7.935	0.088	0.107		8.009	27	0.297	3.16**
Error	945	90.399	-0.438	1.082	-0.405	90.222	944	0.094	
Brand x Store Interaction + Error	972	98.334	-0.350	1.189	-0.294	98.231	971		

TABLE 3.36

TESTING THE LINEARITY OF BRAND x STORE INTERACTION

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F-Ratio
Brand x Store Interaction	26	7.863	0.302	3.18**
Error	944	90.222	0.094	

TABLE 3.37

TESTING THE REGRESSION

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F-Ratio
Due to Regression	1	0.177	0.177	1.88**
Error	944	90.222	0.094	

TABLE 3.38

SUMS OF PRODUCTS AND SUMS OF SQUARES

Product B
(Toothpaste)

No. of Brands 4
No. of Stores 10
No. of Periods 36

Y_{ijk}: Market Share of Brand i in Store j and Period k as determined by number of purchases

X_{ijk}: Average Price of Brand i in Store j and Period k.

Source of Variation	Degrees of Freedom	Sums of Squares (y^2)	Sums of Products (xy)	Sums of Squares (x^2)	Regression Coefficient
Between Brands	3	13.379	0.352	0.030	11.646
Between Stores	9	0.847	-0.020	2.199	-0.009
Between Periods	35	1.031	-0.140	0.156	-0.902
Brand x Store Interaction	27	4.395	0.081	0.154	0.528
Brand x Period Interaction	105	9.050	-0.388	0.330	-1.176
Store x Period Interaction	315	3.543	-0.617	1.189	-0.519
Error	945	27.977	-0.778	1.932	-0.403
Total	1439	60.221	-1.510	5.989	-0.252

TABLE 3.39

ANALYSIS OF COVARIANCE FOR PRODUCT B (Number of Purchases)

(Toothpaste)

Source of Variation	Degrees of Freedom	Sums of Squares Syy	Sums of Products Sxx	Sums of Squares Syy	Regression Coefficient β	Adjusted S_{yy}	Degrees of Freedom	Adjusted Mean Squares	F-Ratio
Brand x Store Inter-action	27	4.395	0.081	0.154		4 476	27	0.166	5.72**
Error	945	27 977	-0.778	1.932	-0.403	27 663	944	0.029	
Brand x Store Inter-action + Error	972	32.372	-0.697	2.086	-0.334	32.139	971		

TABLE 3.40

TESTING THE LINEARITY OF BRAND x STORE INTERACTION

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F-Ratio
Brand x Store Inter-action	26	4.352	0.167	5.71**
Error	944	27.663	0.029	

TABLE 3.41

TESTING THE REGRESSION

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F-Ratio
Due to Regres-sion	1	0.314	0.314	10.83**
Error	944	27.663	0.029	

TABLE 3.42

SUMS OF PRODUCTS AND SUMS OF SQUARES

Product B (Toothpaste)

No. of Brands 4
No. of Stores 10
No. of Periods 36

Y_{ijk}: Market Share of Brand i in Store j and Period k as determined by Volume of sales.

X_{ijk}: Average Price of Brand i in Store j and Period k.

Source of Variation	Degrees of Freedom	Sums of Squares (y^2)	Sums of Products (xy)	Sums of Squares (x^2)	Regression Coefficient
Between Brands	3	13.902	0.300	0.030	9.924
Between Stores	9	0.817	-0.246	2.199	-0.122
Between Periods	35	1.062	-0.142	0.156	-0.910
Brand x Store Interaction	27	3.845	-0.098	0.154	-0.635
Brand x Period Interaction	105	10.391	-0.612	0.330	-1.857
Store x Period Interaction	315	3.938	-0.762	1.189	-0.641
Error	945	31.917	-2.429	1.932	-1.257
Total	1439	65.873	-3.989	5.989	-0.666

TABLE 3.43

ANALYSIS OF COVARIANCE FOR PRODUCT B (Volume of Sales) (Toothpaste)

Source of Variation	Degrees of Freedom	Sums of Squares S_{yy}	Sums of Products S_{xy}	Sums of Squares S_{xx}	Regression Coefficient β	Adjusted S_{yy}	Degrees of Freedom	Adjusted Mean Squares	F-Ratio
Brand x Store Interaction	27	3.845	-0.098	0.154		3.839	27	0.142	4.58**
Error	945	31.917	-2.429	1.932	-1.257	28.863	944	0.031	
Brand x Store Interaction + Error	972	35.762	-2.527	2.086	-1.211	32.702	971		

TABLE 3.44

TESTING THE LINEARITY OF BRAND x STORE INTERACTION

Source of Variance	Degrees of Freedom	Sums of Squares	Mean Squares	F-Ratio
Brand x Store Interaction	26	3.783	0.145	4.68**
Error	944	28.863	0.031	

TABLE 3.45

TESTING THE REGRESSION

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F-Ratio
Due to Regression	1	3.053	3.053	99.19**
Error	944	28.863	0.031	

TABLE 3.46

SUMS OF PRODUCTS AND SUMS OF SQUARES

Product C (Coffee)

No. of Brands 4
No. of Stores 10
No. of Periods 36

Y_{ijk}: Market Share of Brand i in Store j and Period k as determined by number of purchases.

X_{ijk}: Average Price of Brand i in Store j and Period k.

Source of Variation	Degrees of Freedom	Sums of Squares (y^2)	Sums of Products (xy)	Sums of Squares (x^2)	Regression Coefficient
Between Brands	3	24.957	2.054	0.339	6.065
Between Stores	9	8.002	2.312	5.135	-0.450
Between Periods	35	0.517	0.116	0.829	0.140
Brand x Store Interaction	27	6.834	-0.259	0.399	-0.648
Brand x Period Interaction	105	3.540	-0.587	0.341	-1.724
Store x Period Interaction	315	5.594	2.382	5.501	0.433
Error	945	21.608	-1.406	2.988	-0.471
Total	1439	71.053	-0.013	15.532	-0.001

TABLE 3.47

ANALYSIS OF COVARIANCE FOR PRODUCT C (Number of Purchases)

(Coffee)

Source of Variation	Degrees of Freedom	Sums of Squares S_{yy}	Sums of Products S_{xy}	Sums of Squares S_{xx}	Regression Coefficient β	Adjusted S_{yy}	Degrees of Freedom	Adjusted Mean Squares	F-Ratio
Brand x Store Inter-action	27	6.834	-0.259	0.399		6.677	27	0.247	11.23**
Error	945	21.608	-1.406	2.988	-0.471	20.946	944	0.022	
Brand x Store Inter-action + Error	972	28.442	-1.665	3.387	-0.492	27.623	971		

TABLE 3.48

TESTING THE LINEARITY OF BRAND x STORE INTERACTION

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F-Ratio
Brand x Store Inter-ation	26	6.666	0.256	11.64**
Error	944	20.946	0.022	

TABLE 3.49

TESTING THE REGRESSION

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F-Ratio
Due to Regres-sion	1	0.662	0.662	30.09**
Error	944	20.946	0.022	

TABLE 3.50

SUMS OF PRODUCTS AND SUMS OF SQUARES

Product C (Coffee)

No. of Brands 4
No. of Stores 10
No. of Periods 36

Y_{ijk}: Market Share of Brand i in Store j and Period k as determined by volume of sales.

X_{ijk}: Average Price of Brand i in Store j and Period k.

Source of Variation	Degrees of Freedom	Sums of Squares (y^2)	Sums of Products (xy)	Sums of Squares (x^2)	Regression Coefficient
Between Brands	3	21.898	1.872	0.339	5.528
Between Stores	9	7.947	-2.306	5.135	-0.449
Between Periods	35	0.610	0.108	0.829	0.130
Brand x Store Interaction	27	6.365	-0.531	0.399	-1.331
Brand x Period Interaction	105	4.604	-0.707	0.341	-2.077
Store x Period Interatiion	115	5.990	2.235	5.501	0.406
Error	945	25.858	-2.130	2.988	-0.713
Total	1439	73.273	-1.460	15.532	-0.094

TABLE 3.51

ANALYSIS OF COVARIANCE FOR PRODUCT C (Volume of Sales)

(Coffee)

Source of Variation	Degrees of Freedom	Sums of Squares S_{yy}	Sums of Products S_{xy}	Sums of Squares S_{xx}	Regression Coefficient β	Adjusted S_{yy}	Degrees of Freedom	Adjusted Mean Squares	F-Ratio
Brand x Store Interaction	27	6.365	-0.531	0.399		5.792	27	0.215	8.27**
Error	945	25.858	-2.130	2.988	-0.713	24.339	944	0.026	
Brand x Store Interaction + Error	972	32.223	-2.661	3.387	-0.786	30.131	971		

TABLE 3.52

TESTING THE LINEARITY OF BRAND x STORE INTERACTION

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F-Ratio
Brand x Store Interaction	26	5.658	0.218	8.35**
Error	944	24.339	0.026	

TABLE 3.53

TESTING THE REGRESSION

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F-Ratio
Due to Regression	1	1.519	1.519	58.42**
Error	944	24.339	0.026	

APPENDIX IV

ANALYSIS OF TOOTHPASTE DATA

As mentioned earlier, only the data for product B, D_1, include families with one or more weekly diaries missing during the period of study. For products A and C, only families who reported continuously without missing any single diary have been included for the purpose of analysis. In order to insure that the general results are not significantly affected by the inclusion of data on families with missing diaries, the screened data of product B, D_2, consisting only of families who reported continuously for three years have been re-analyzed. The results of the hypotheses tested against both the samples of D_1 and D_2 are tabulated side by side for comparison. Only hypotheses at micro level have implications in terms of a disconnected brand and or store sequence due to a missing observation. Some of these hypotheses are tested for both the samples and presented in the following tables. No significant differences in the estimates of conditional probabilities of brand choice are observed between the two samples D_1 and D_2.

However, this does not assure the reader that any type of brand and store analysis can be done on panel data with sets of families having missing diaries, though in my analysis the results are not significantly affected. A possible explanation could be that a missing weekly diary for a family in the panel does not always imply a missing purchase observation, especially when the time between two consecutive purchases is longer than a week, as in the case of product B (toothpaste).

TABLE 4.1

STORE CHANGE VS. BRAND CHANGE
IN TWO CONSECUTIVE PURCHASES OF PRODUCT B
COMPARED BETWEEN THE TWO SAMPLES D_1 AND D_2.

Product: B
(Toothpaste)

Brand Change \ Store Change	Data Base	Same Store	Different Store	Marginal Total
Same Brand	D_1	0.662(3078)	0.557(2780)	0.607(5858)
	D_2	0.684(1670)	0.637(1405)	0.612(3075)
Different Brand	D_1	0.338(1572)	0.443(2212)	0.393(3784)
	D_2	0.316(771)	0.363(1177)	0.388(1948)
Marginal Total	D_1	0.482(4650)	0.518(4992)	9642
	D_2	0.486(2441)	0.514(2582)	5023

TABLE 4.2

PROBABILITIES OF A HOUSEWIFE PURCHASING BRAND B_1 GIVEN THE HISTORY OF HER PAST TWO BRAND PURCHASES AND THE CORRESPONDING STORES VISITED COMPARED BETWEEN THE SAMPLES D_1 AND D_2.

PRODUCT: B (Toothpaste)
BRAND: B_1

1-Purchased Brand B_1
0-Not Purchased Brand B_1

Historical Sequence of Brands Purchased	Data Base	Past History of Stores Visited				
		SS	SD	DS	DD	
					D_1D_1	D_1D_2
00	D_1	0.050 (1858)	0.060 (765)	0.082 (1012)	0.083 (1008)	0.097 (1310)
	D_2	0.040 (985)	0.061 (411)	0.075 (535)	0.077 (534)	0.104 (721)
10	D_1	0.352 (165)	0.279 (86)	0.254 (114)	0.266 (94)	0.321 (196)
	D_2	0.354 (82)	0.256 (43)	0.200 (55)	0.213 (47)	0.287 (101)
01	D_1	0.409 (159)	0.333 (78)	0.558 (111)	0.359 (92)	0.382 (202)
	D_2	0.481 (79)	0.310 (42)	0.581 (62)	0.375 (40)	0.298 (104)
11	D_1	0.899 (711)	0.843 (191)	0.864 (242)	0.765 (260)	0.798 (302)
	D_2	0.887 (389)	0.859 (85)	0.855 (124)	0.773 (141)	0.777 (121)

TABLE 4.3

PROBABILITIES OF A HOUSEWIFE PURCHASING BRAND B_4 GIVEN THE HISTORY OF HER PAST TWO BRAND PURCHASES AND THE CORRESPONDING STORES VISITED COMPARED BETWEEN THE SAMPLES D_1 AND D_2.

PRODUCT: B
(Toothpaste)
BRAND: B_4

1-Purchased Brand B_4
0-Not Purchased Brand B_4

Historical Sequence of Brands Purchased	Data Base	Past History of Stores Visited				
		SS	SD	DS	DD	
					D_1D_1	D_1D_2
00	D_1	0.075 (2023)	0.072 (759)	0.058 (1005)	0.098 (1009)	0.101 (1276)
	D_2	0.080 (1023)	0.058 (520)	0.082 (390)	0.097 (538)	0.116 (666)
10	D_1	0.397 (179)	0.442 (77)	0.292 (113)	0.329 (79)	0.376 (157)
	D_2	0.365 (104)	0.232 (69)	0.386 (44)	0.405 (42)	0.368 (77)
01	D_1	0.436 (202)	0.488 (84)	0.596 (114)	0.548 (104)	0.487 (193)
	D_2	0.414 (116)	0.619 (63)	0.449 (49)	0.531 (49)	0.459 (111)
11	D_1	0.857 (489)	0.825 (200)	0.830 (247)	0.763 (262)	0.818 (384)
	D_2	0.887 (292)	0.791 (134)	0.767 (93)	0.797 (133)	0.825 (183)

TABLE 4.4

PROBABILITIES OF A HOUSEWIFE PURCHASING BRAND B_1 GIVEN THE HISTORY OF HER PAST TWO BRAND PURCHASES AND THE CORRESPONDING STORES VISITED COMPARED BETWEEN THE TWO SAMPLES D_1 AND D_2.

PRODUCT: B (Toothpaste)
BRAND: B_1

1-Purchased Brand B_1
0-Not Purchased Brand B_1

Historical Sequence of Brands Purchased	Data Base	Past History of Stores Visited			
		SSS			DDD
000	D_1	0.041(1184)			0.087(1621)
	D_2	0.036(635)			0.087(888)
			DSD	SDS	
010	D_1	0.219(64)	0.212(33)	0.158(19)	0.270(126)
	D_2	0.222(27)	0.056(18)	0.100(10)	0.217(69)
			DDS	SSD	
001	D_1	0.295(61)	0.580(50)	0.250(20)	0.292(154)
	D_2	0.346(26)	0.680(25)	0.286(14)	0.286(77)
			DSS	SDD	
011	D_1	0.468(47)	0.692(26)	0.619(23)	0.524(82)
	D_2	0.469(32)	0.706(17)	0.667(9)	0.460(37)
			SDD	DSS	
100	D_1	0.164(73)	0.219(32)	0.175(40)	0.204(147)
	D_2	0.143(35)	0.182(22)	0.118(17)	0.158(82)
			SSD	DDS	
110	D_1	0.511(45)	0.615(13)	0.314(35)	0.430(86)
	D_2	0.555(27)	0.667(9)	0.353(17)	0.391(46)
			SDS	DSD	
101	D_1	0.683(41)	0.727(11)	0.333(18)	0.418(55)
	D_2	0.783(23)	0.571(7)	0.167(6)	0.440(25)
111	D_1	0.952(505)			0.842(355)
	D_2	0.945(274)			0.842(177)

TABLE 4.5

PROBABILITIES OF A HOUSEWIFE PURCHASING BRAND B_4 GIVEN THE HISTORY OF HER PAST THREE BRAND PURCHASES AND THE CORRESPONDING STORES VISITED COMPARED BETWEEN THE SAMPLES D_1 AND D_2.

PRODUCT: B
(Toothpaste)
BRAND: B_4

1-Purchased Brand B_4
0-Not Purchased Brand B_4

Historical Sequence of Brands Purchased	Data Base	Past History of Stores Visited			
000	D_1	0.059(1338)			0.078(1589)
	D_2	0.064(669)			0.087(849)
			DSD	SDS	
010	D_1	0.338(80)	0.462(13)	0.261(23)	0.302(96)
	D_2	0.292(48)	0.333(9)	0.267(15)	0.345(55)
			DDS	SSD	
001	D_1	0.361(97)	0.569(51)	0.324(34)	0.43 (158)
	D_2	0.345(55)	0.607(28)	0.211(19)	0.417(84)
			DSS	SDD	
011	D_1	0.750(60)	0.806(36)	0.810(21)	0.664(116)
	D_2	0.781(32)	0.818(11)	0.864(21)	0.646(65)
			SDD	DSS	
100	D_1	0.324(74)	0.290(31)	0.281(32)	0.262(126)
	D_2	0.341(47)	0.286(14)	0.294(17)	0.260(77)
			SSD	DDS	
110	D_1	0.490(51)	0.500(22)	0.300(30)	0.353(85)
	D_2	0.500(22)	0.636(11)	0.363(11)	0.467(45)
			SDS	DSD	
101	D_1	0.652(46)	0.865(15)	0.636(11)	0.620(71)
	D_2	0.545(22)	0.889(9)	0.571(7)	0.600(40)
111	D_1	0.887(274)			0.844(385)
	D_2	0.908(184)			0.876(186)

TABLE 4.6

FRACTION OF HOUSEWIVES CHANGING THE SIZE OF PURCHASE OF PRODUCT B VS. THE PURCHASE PATTERN COMPARED BETWEEN THE TWO SAMPLES D_1 AND D_2.

PRODUCT B
(Toothpaste)

Pattern of Two Consecutive Purchases	Data Base	Increased the Size of Purchase	No Change in the Size of Purchase	Decreased the Size of Purchase	Marginal Total
Same Store & Same Brand	D_1 D_2	0.188 (575) 0.193 (322)	0.687 (2143) 0.697 (1134)	0.125 (400) 0.128 (214)	0.324 (3118) 0.332 (1670)
Same Store & Different Brand	D_1 D_2	0.216 (331) 0.215 (166)	0.546 (835) 0.546 (412)	0.238 (364) 0.239 (184)	0.159 (1530) 0.153 (771)
Different Store & Same Brand	D_1 D_2	0.187 (527) 0.179 (252)	0.622 (1757) 0.637 (895)	0.191 (540) 0.184 (258)	0.292 (2824) 0.279 (1405)
Different Store & Different Brand	D_1 D_2	0.253 (549) 0.253 (298)	0.473 (1025) 0.466 (548)	0.274 (549) 0.281 (331)	0.225 (2168) 0.235 (1177)
Marginal	D_1	0.206 (1982)	0.597 (5760)	0.197 (1898)	1.000 (9640)
Total	D_2	0.206 (1038)	0.596 (2998)	0.198 (987)	1.000 (5032)

Data Base D_1

$$P_I = 17.85 + 4.7 \times BC + 1.8 \times SC$$
$$P_D = 13.25 + 9.8 \times BC + 5.1 \times SC$$

Data Base D_2

$$P_I = 18.00 + 4.8 \times BC + 1.2 \times SC$$
$$P_D = 13.15 + 10.4 \times BC + 4.9 \times SC$$

APPENDIX V

ANALYSIS OF COVARIANCE

The technique of Analysis of Covariance is the combination of the methods of regression and Analysis of Variance. While making observations on a variable "Y", if some other additional factor 'X' varies, any dependence of "Y" on 'X' will tend to obscure and possibly invalidate the results of Analysis of Variance performed on 'Y'. A brief explanation of the technique of Analysis of Variance follows: If a set of observations can be classified according to one or more criteria, then the total variation between the members of the set can be broken up into components which can be attributed to the different criteria of classification. With the knowledge of this break-down, the investigator is able to identify the criteria and their contributions to the over-all variation.

In our present study of analyzing the market shares of brands, in addition to the variation due to the factors under investigation, the results are affected by the relative price

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Excerpts of this material are taken in part from the following books:

1. Jogabratha Roy, I. Chakravorti, and R. Laha, Handbook for Practical Work in Statistics, (Calcutta: Indian Statistical Institute, 1959), pp. 278-283.
2. Bennett C. Allan and Norman L. Franklin, Statistical Analysis in Chemistry and the Chemical Industry, (New York: Wiley Publications, 1959), pp. 441-461.

levels of the brands in the market. The factors under investigation are brand, store and the period of purchase denoted by A, B and C respectively. In performing the Analysis of Covariance, we assume a relationship between the average price of the brand and its market share, and the analysis of variance on market share data is performed by adjusting the market shares to some standard condition of price level. The regression relationship for adjustment of 'Y' due to variation in 'X' is normally sought within the data by a suitable analysis, unless a relationship based on previous experience is available.

It is assumed in the model that the distribution of 'Y' is approximately Normal, though moderate departures from Normality are unimportant in the Analysis of Variance. As the variable of proportional market share of a brand follows binomial distribution, the following transformation has been used to approximate the distribution to normality.

$$Y = \phi(m) \quad \text{Arc Sin}(\sqrt{m})$$

Where m = Proportional market share of brand
y = Transformed value of m

These transformations do not result in any substantial loss of efficiency for the estimates.

The model of our study is:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \delta_{ij} + \varepsilon_{jk} + \theta_{ik} + \beta X_{ijk} + \eta_{ijk}$$

Where

Y_{ijk} = Market share of Brand 'i', in store 'j', in Period 'k'

X_{ijk} = Average price of Brand 'i', in store 'j', in period 'k'

μ = constant

β = Regression coefficient of price on market share (assumed to be the same for all brands and stores)

α_i = Brand effect

β_j = Store effect

γ_k = Period effect

δ_{ij} - Brand x Store interaction effect

ε_{jk} = Store x Period interaction effect

θ_{ik} = Brand x Period interaction effect

η_{ijk} = Random effect (distributed normally with mean zero and variance σ^2)

The components of the variance are estimated by using the method of least squares, and Table 4.1 presents the corresponding algebraic equations. The subscripts 'i', 'j', 'k' run from 1 to r, 1 to s, and 1 to t respectively in the corresponding summations. In our investigation, the data has been classified according to 4 brands, 10 stores, and 36 (monthly) time periods.

Thus: $r = 4$; $s = 10$; and $t = 36$.

The sums of squares and the sums of products are computed as follows:

$$M_{ijo} = \sum_{k=1}^{t} y_{ijk} \qquad N_{ijo} = \sum_{k=1}^{t} x_{ijk}$$

$$M_{iok} = \sum_{j=1}^{s} y_{ijk} \qquad N_{iok} = \sum_{j=1}^{s} x_{ijk}$$

$$M_{ojk} = \sum_{i=1}^{r} y_{ijk} \qquad N_{ojk} = \sum_{i=1}^{r} x_{ijk}$$

$$M_{ioo} = \sum_{j=1}^{s} \sum_{k=1}^{t} y_{ijk} \qquad N_{ioo} = \sum_{j=1}^{s} \sum_{k=1}^{t} x_{ijk}$$

$$M_{ojo} = \sum_{i=1}^{r} \sum_{k=1}^{t} y_{ijk} \qquad N_{ojo} = \sum_{i=1}^{r} \sum_{k=1}^{t} x_{ijk}$$

$$M_{ook} = \sum_{i=1}^{r} \sum_{j=1}^{s} y_{ijk} \qquad N_{ook} = \sum_{i=1}^{r} \sum_{j=1}^{s} x_{ijk}$$

$$M_{ooo} = \sum_{i=1}^{r} \sum_{j=1}^{s} \sum_{k=1}^{t} y_{ijk} \qquad N_{ooo} = \sum_{i=1}^{r} \sum_{j=1}^{s} \sum_{k=1}^{t} x_{ijk}$$

$$N = rst \; ; \quad CF_{yy} = \frac{M^2_{ooo}}{N} \; ; \quad CF_{xt} = \frac{M_{ooo} \; N_{ooo}}{N} \; ; \quad CF_{xx} = \frac{N^2_{ooo}}{N}$$

$$S_{A.yy} = \frac{1}{st} \sum_{i=1}^{r} M^2_{ioo} - CF_{yy} \qquad S_{A.xy} = \frac{1}{st} \sum_{i=1}^{r} M_{ioo} \; N_{ioo} - CF_{xy}$$

$$S_{A.xx} = \frac{1}{st} \sum_{i=1}^{r} N^2_{ioo} - CF_{xx}$$

$$S_{B.yy} = \frac{1}{rt} \sum_{j=1}^{s} M^2_{ojo} - CF_{yy} \qquad S_{B.xy} = \frac{1}{rt} \sum_{j=1}^{s} M_{ojo} \; N_{ojo} - CF_{xy}$$

$$S_{B.xx} = \frac{1}{rt} \sum_{j=1}^{s} N^2_{ojo} - CF_{xx}$$

$$S_{C.yy} = \frac{1}{rs} \sum_{k=1}^{t} M^2_{ook} - CF_{yy} \qquad S_{C.xy} = \frac{1}{rs} \sum_{k=1}^{t} M_{ook} \; N_{ook} - CF_{xy}$$

$$S_{C.xx} = \frac{1}{rs} \sum_{k=1}^{t} N^2_{ook} - CF_{xx}$$

$$S_{AB.yy} = \frac{1}{t} \sum_{i=1}^{r} \sum_{j=1}^{s} M^2_{ijo} - S_{A.yy} - S_{B.yy} - CF_{yy}$$

$$S_{AB.xy} = \frac{1}{t} \sum_{i=1}^{r} \sum_{j=1}^{s} M_{ijo} \; N_{ijo} - S_{A.xy} - S_{B.xy} - CF_{xy}$$

$$S_{AB.xx} = \frac{1}{t} \sum_{i=1}^{r} \sum_{j=1}^{s} N^2_{ijo} - S_{A.xx} - S_{B.xx} - CF_{xx}$$

$$S_{BC.yy} = \frac{1}{r} \sum_{j=1}^{s} \sum_{k=1}^{t} M^2_{ojk} - S_{B.yy} - S_{C.yy} - CF_{yy}$$

$$S_{BC.xy} = \frac{1}{r} \sum_{j=1}^{s} \sum_{k=1}^{t} M_{ojk} N_{ojk} - S_{B.xy} - S_{C.xy} - CF_{xy}$$

$$S_{BC.xx} = \frac{1}{r} \sum_{j=1}^{s} \sum_{k=1}^{t} N^2_{ojk} - S_{B.xx} - S_{C.xx} - CF_{xx}$$

$$S_{AC.yy} = \frac{1}{s} \sum_{i=1}^{r} \sum_{k=1}^{t} M^2_{iok} - S_{A.yy} - S_{C.yy} - CF_{yy}$$

$$S_{AC.xy} = \frac{1}{s} \sum_{i=1}^{r} \sum_{k=1}^{t} M_{iok} N_{iok} - S_{A.xy} - S_{C.xy} - CF_{xy}$$

$$S_{AC.xx} = \frac{1}{s} \sum_{i=1}^{r} \sum_{k=1}^{t} N^2_{iok} - S_{A.xx} - S_{C.xx} - CF_{xx}$$

$$S_{T.yy} = \sum_{i=1}^{r} \sum_{j=1}^{s} \sum_{k=1}^{t} y^2_{ijk} - CF_{yy}$$

$$S_{T.xy} = \sum_{i=1}^{r} \sum_{j=1}^{s} \sum_{k=1}^{t} y_{ijk}\, x_{ijk} - CF_{xy}$$

$$S_{T.xx} = \sum_{i=1}^{r} \sum_{j=1}^{s} \sum_{k=1}^{t} x^2_{ijk} - CF_{xx}$$

$$S_{E.yy} = S_{T.yy} - S_{A.yy} - S_{B.yy} - S_{C.yy} - S_{AB.yy} - S_{AC.yy} - S_{BC.yy}$$

$$S_{E.xy} = S_{T.xy} - S_{A.xy} - S_{B.xy} - S_{C.xy} - S_{AB.xy} - S_{AC.xy} - S_{BC.xy}$$

$$S_{E.xx} = S_{T.xx} - S_{A.xx} - S_{B.xx} - S_{C.xx} - S_{AB.xx} - S_{AC.xx} - S_{BC.xx}$$

TABLE 4.1

SUMS OF PRODUCTS AND SUMS OF SQUARES

No. of Brands 4(r)
No. of Stores 10(s)
No. of Periods 36(t)

Y_{ijk}: Market Share of Brand i in Store j and Period k.

Y_{ijk}: Average Price of Brand i in Store j and Period k.

Source of Variation	Degrees of Freedom	Sums of Squares (y^2)	Sums of Products (xy)	Sums of Squares (x^2)	Regression Coefficient
Between Brands	(r-1) 3	$S_{A \cdot yy}$	$S_{A \cdot xy}$	$S_{A \cdot xx}$	$b_{ioo} = S_{A \cdot xy}/S_{A \cdot xx}$
Between Stores	(s-1) 9	S_{B} yy	$S_{B \cdot xy}$	$S_{B \cdot xx}$	$b_{ojo} = S_{B \cdot xy}/S_{B \cdot xx}$
Between Periods	(t-1) 35	$S_{C \cdot yy}$	$S_{C \cdot xy}$	S_{C} xx	$b_{ook} = S_{C \cdot xy}/S_{c \cdot xx}$
Brand x Store Interaction	(r-1)x (s-1) 27	$S_{AB \cdot yy}$	$S_{AB \cdot xy}$	$S_{AB \cdot xx}$	$b_{ijo} = S_{AB \cdot xy}/S_{AB \cdot xx}$
Brand x Period Interaction	(r-1)x (t-1) 105	S_{AC} yy	S_{AC} xy	$S_{AC \cdot xx}$	$b_{iok} = S_{AC \cdot xy}/S_{AC \cdot xx}$
Store x Period Interaction	(s-1)x (t-1) 315	$S_{BC \cdot yy}$	$S_{BC \cdot xy}$	$S_{BC \cdot xx}$	$b_{ojk} = S_{BC \cdot xy}/S_{BC \cdot xx}$
Error	(r-1)x (s-1)x (t-1) 945	$S_{E \cdot yy}$	$S_{E \cdot xy}$	$S_{E \cdot xx}$	$\beta = S_{E \cdot xy}/S_{E \cdot xx}$
Total	(N-1) 1439	S_{T} yy	$S_{T \cdot xy}$	$S_{T \cdot xx}$	$b = S_{T \cdot xy}/S_{T \cdot xx}$

The covariance correction for the regression of brand sales on price level increased the precision of the findings because the results are then based on a standard condition of price level. A brief description of the mathematical computations follow.

The computational procedure for obtaining sums of products and sums of squares (Refer to Table 4.1) is given in conventional text books (Bennett & Franklin, 1954; Lucas, 1957). In computing the sums of products, corresponding values of prices and market shares are multiplied instead of squared at each stage of the computation.

In Table 4.2 the effect of the regression of price (X) on brand sales (Y) is removed from the sums of squares for error, and sums of squares for store x brand interaction + error, by using the corresponding sums of squares for x and y and sums of products (xy) and the formula:

$$S_{yy} - (S_{xy})^2 / S_{xx}$$

One degree of freedom associated with the regression is subtracted from degrees of freedom for error, and interaction + error. The sums of squares due to the interaction effect is obtained by subtracting the adjusted sum of squares for error from the adjusted sum of squares for interaction + error. Degrees of freedom for interaction are obtained in a similar manner by subtraction. Detailed descriptions are given in Biometrics (Cochran, 1957). The F-test was used taking the ratio of mean squares (the sums of squares divided by the

TABLE 4.2

ANALYSIS OF COVARIANCE

Source of Variation	Degrees of Freedom	Sums of Squares S_{yy}	Sums of Products S_{xy}	Sums of Squares S_{xx}	Regression Coefficient β	Adjusted S_{yy}	Degrees of Freedom	Adjusted Mean Squares	F-Ratio
Brand x Store Inter-action	(r-1) x(s-1) = 27	$S_{AB \cdot yy}$	$S_{AB \cdot xy}$	$S_{AB \cdot xx}$		$AS_{yy} = AT_{yy} - AE_{yy}$	27	$AMS_{yy} = \frac{AS_{yy}}{27}$	$F = \frac{AMS_{yy}}{AME_{yy}}$
Error	(r-1) x(s-1) x(t-1) = 945	$S_{E \cdot yy}$	$S_{E \cdot xy}$	$S_{E \cdot xx}$	$\beta = \frac{S_{E \cdot xy}}{S_{E \cdot xx}}$	$AE_{yy} = S_{E \cdot yy} - \beta S_{E \cdot xy}$	(r-1)x (s-1)x (t-1)-1 = 944	$AME_{yy} = \frac{AE_{yy}}{944}$	
Brand x Store Inter-action + Error	N'=972	$T_{yy} = S_{AB \cdot yy} + S_{E \cdot yy}$	$T_{xy} = S_{AB \cdot xy} + S_{E \cdot xy}$	$T_{xx} = S_{AB \cdot xx} + S_{E \cdot xx}$	$\beta' = \frac{S_{T \cdot xy}}{S_{T \cdot xx}}$	$AT_{yy} = T_{yy} - \beta^1 T_{xy}$	(N'-1) = 971		

TABLE 4.3

TESTING THE LINEARITY OF BRAND x STORE INTERACTION

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F-Ratio
Brand x Store Inter-action	(r-1) x(s-1) -1 = 26	$BS_{yy} = S_{AB \cdot yy} - b_{ijo} \times S_{AB \cdot xy}$	$BMS_{yy} = BS_{yy}/26$	$F = \frac{BMS_{yy}}{AME_{yy}}$
Error	944	AE_{yy}	AME_{yy}	

TABLE 4.4

TESTING THE REGRESSION

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F-Ratio
Due to Regression	1	$\beta S_{E \cdot xy}$	$\beta S_{E \cdot xy}$	$F = \frac{\beta S_{Exy}}{AME_{yy}}$
Error	944	AE_{yy}	AME_{yy}	

corresponding degrees of freedom) of interaction and error to test the significance of store-brand interaction at both 5% and 1% levels.

In Table 4.3, the sums of squares corresponding to error and interaction are adjusted for regression of price in a similar manner as described above, this time by using the corresponding class regression coefficients obtained in the last column of Table 4.1. The ratio of mean squares for interaction and error is compared with percentage values of the F-distribution to test the null hypothesis that the observed interaction between brand and store can be explained by the class effect on regression coefficients. The estimates of regression coefficients in Table 4.1 are obtained by dividing the respective sums of squares of price of brand (χ), the independent variable, into the sums of products of prices and market shares.

In Table 4.4, the regression sums of squares with one degree of freedom, the part of total variation in market shares explained by the price levels (regression) is computed by the formula βE_{xy}, where β is the regression coefficient and E_{xy} is the error sum of products. The ratio of mean squares corresponding to regression and adjusted error, is compared with percentage points of the F-distribution to test the null hypothesis that price has no significant effect on the market share of a brand.

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