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THE ANALYSIS OF THE OPERATIONS UNDERLYING
CONSERVATION OF NUMBER AND THEIR
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Paulette Margaret Valliere

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THE ANALYSIS OF THE OPERATIONS UNDERLYING

CONSERVATION OF NUMBER AND THEIR

ORDER OF ACQUISITION

By

Paulette Margaret Valliere

A THESIS

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ABSTRACT

THE ANALYSIS OF THE OPERATIONS UNDERLYING CONSERVATION OF NUMBER AND THEIR ORDER OF ACQUISITION

By

Paulette Margaret Valliere

Researchers have assumed that certain number concepts are necessary before a child is able to conserve number and that training in these concepts can induce number conservation.

The present study investigated eight number concepts assumed by previous researchers to underlie number conservation, testing for their presence in conserving and non-conserving children. They were given to 75 children along with the standard number conservation task. The data were analyzed by Guttman scalogram and linear multiple regression analyses to provide information on the developmental ordering of the tasks and their predictability of number conservation.

Results showed that the obtained sequence significantly correlated with the predicted outcome. The reliability of the number conservation task itself was found to be high. However, the eight tasks were poor

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predictors of number conservation when added to the regression equation, indicating the presence of other factors not tested in the present study.

DEDICATION

To my parents, Ray and Flo. Without your faith and love, I could never have accomplished so much. Keep telling me that I can do it, because you know I can.

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INTRODUCTION

Initially, investigators in the field of number development in children were interested mainly in the child's mastery of number concepts and their development. Much of the early work looked at how children count, perceive, and discriminate number. They also assessed children's knowledge of number combinations which provided them with normative charts showing which of these number facts children knew. These earlier studies (e.g., Brownwell, 1941) simply catalogued number facts which children of various ages knew, rather than question what children understood about addition and subtraction. They were not designed to determine the operations which children believe to change number.

Piaget, in the formulation of his theory of cognitive development, began the shift in methodology and reasoning to the investigation of these underlying operations in looking at number development in young children. Piaget (1952, 1953, 1968) defined the term conservation as the invariance of a characteristic in spite of the fact that transformations have been performed on the objects (or a collection of the objects) which possess this characteristic. More specifically, conse

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conservation of number is the concept where a collection of objects "conserves" its number when the shape or distribution of a collection is changed or the collection is separated into smaller subsets. With the onset of this newer approach, the framework of the research shifted from one where a child's responses to number were thought to simply indicate number mastery to one where responses were believed to reveal the functioning of the child's logical, or cognitive, operators.

Another theoretical viewpoint considers the origins of number concepts and their relation to the ordinal, cardinal, and natural theories of number. This research bases itself on the mathematical theories of number concepts put forth by Peano, Russell, Frege, and other logicians and attempts to link these to the behavioral foundations of these concepts. Researchers have attempted to determine the order of emergence of the concepts of cardination, ordination, and natural number in the young child.

The literature review suggests that children's ability to conserve number is preceded by the attainment of several concepts generally thought to underlie number conservation. Researchers have variously found, through training studies, that sometimes only one of these concepts or, at other times, several of them, is necessary for the child to become a conserver. The prese

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present study examined number conservation and the number concepts that have been assumed to be necessary for its attainment. The purpose of the present study was to determine whether or not certain mathematical concepts are present in young children before or after conservation of number has been attained. If these concepts are present before number conservation is achieved, it should be possible to place them in a developmental ordering in hopes of establishing a sequence of logical prerequisites to number conservation. If these concepts can be placed in developmental sequence, it then might be possible to analyze children's correct or incorrect responses to the number conservation task in terms of prerequisite concepts that have or have not been obtained. These concepts might also be analyzed in terms of their predictive power. That is, if a child has mastered a certain number concept, is it possible to predict his conservation ability from the knowledge that we have about his numerical abilities?

Piaget's Theory

Ginsburg and Opper (1969), in their explanatory text on Piaget's theory, indicate that the child in the preoperational period, that is between approximately ages 2 to 7, supposedly is not able to conserve number until towards the end of the stage when the child is about to becom

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become concrete operational. This is to say that when a child is presented two rows of equal number but unequal length and spacing, the child under 7 will most often say the rows are unequal in number. The child may be able to use the concept of one-one correspondence when the rows are lined up exactly but tends to concentrate on the row length when they are uneven. In doing so, the child fails to coordinate the two dimensions of length and density simultaneously and therefore is unable to construct sets equivalent in number.

Piaget described three distinct stages through which children pass on their way to attaining total conservation ability. Children who are consistent nonconservers are labeled as being in Stage I. They unfailingly make incorrect judgments and give inconsistent explanations for their answers. To be more exact, the children neither predict conservation preceding the transformation of one of the stimulus arrays nor do they maintain conservation after the transformation.

Children in Stage II can easily notice the equivalence of two sets. They are able to construct a set equal in number to another or can show one-one correspondence between two sets. They also can predict conservation prior to deformation of a set but fail to maintain conservation after the deformation has taken place

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place. This indicates that the child is somewhat unsure of the reasoning or criteria used to establish equality, or inequality, of number. At times, as in Stage I, the child centers on the length of the set, and, at other times, centers on the density of the sets. This focusing on only one of the relevant dimensions is referred to as the principle of centration.

The final stage, Stage III, is that during which true conservation emerges. Children are able to predict conservation prior to deformation and maintain it afterwards. They now have become concrete operational. With the emergence of this stage, the children need not rely on the perception of spatial proximity between the elements of each set. They either count them or use a sophisticated method of one-one correspondence. In the nonconserving of number, younger children center only on a limited amount of information available. They know that the compressed row will be equal to the unchanged row if spread out but perceptually, they feel that the number of a set changes when its appearance is altered.

Concrete operational children decenter their attention and begin to attend to the relevant dimensions of the stimuli. They recognize length and density as two separate dimensions of the stimulus array and associate the two. They then begin to understand the fact

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fact that as length of one line increases, the perceived density of the other line will also increase. This is the principle of reciprocity, which is a form of reversibility. The increase in length counteracts the density increase which results in a reversal and an equal number.

Conserving children are also capable of using the principle of negation. They realize that the changes to the rows can be annulled or negated. By spreading out the row again, the contraction is negated. This is opposed to Stage II children who are capable of empirical reversibility but can not focus on the reverse act of rearrangement. They attend to states, not transformations. They sometimes use an argument of identity, reasoning that the numbers must be the same since the same objects are involved; nothing has been added or taken away.

Number Development Studies

There have been attempts to provide a means by which the early development of number concepts and the child's conception of number can be studied. Some have begun by defining the basic concepts of operators and estimators. Gelman (1972b) defines operators as the cognitive processes by which children determine the consequences of transforming a quantity in various ways.

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Estimators are then defined as the cognitive processes by which children determine some quantity, such as the numerosity of an array.

It is important to differentiate between operators and estimators. Estimators are linked to perceptual input, whereas operators are not, the former involving a "lower" level of processing. An outcome of the application of a particular operation on a given amount can be specified by an operator whether or not the amount is actually present. Operators are also considered "more cognitive" than estimators, providing integrative connections between successive estimates. They are also more central to mature conceptions of number. Conservation is considered an operator task. Children are said not to have a concept of number until they have attained conservation. In previous scalogram analyses of number concepts, operational tasks were assigned a higher developmental ranking than estimation tasks (Siegel, 1971b; Wohlwill, 1960).

In summarizing the literature of estimators and counting in young children, Gelman (1972b) felt that a child's ability to abstract number appears to be related to counting abilities - a young child will count even when estimating small numbers. In her own research, she hypothesized that young children think about number in a multidimensional way where there exists a hierarchy of cues

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cues such as length, density, and numerosity that controls their attention to an array's numerosity. Experiences that set children to attend to the number of an array and not to the other features, such as length and density, increase the expectation of number abstraction. Children will develop better number skills as the use of number estimators decreases and number itself becomes the dominant factor.

In an experiment designed to test this hypothesis, Gelman (1972b) found that as children get older, they are more likely to estimate linear arrays on the basis of number. Younger children (4 to 7) are more likely to estimate on the basis of length, but they do not estimate on the basis of density. This finding suggests that the young child's concept of number is based on length and number first with the understanding of the relevance of density to numerosity developing later.

Operators mediate the ability to classify the relevancy of a set's transformation, anticipate the effect of a manipulation of numerical arrays, and integrate perceptions from two or more successive presentations of a set through inferences about manipulations that change or do not change number. Gelman (1972b) considers Piaget's conservation task and its relation to operators. If a correct answer is given (clas

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(classifed as a conserver), the child could be said to understand operators. However, a child who responds to the equivalent number array (time 1) and then to the transformed array (time 2) might be making two independent responses which could therefore also indicate the use of estimators. The presence of operators cannot always be inferred in such cases.

Gelman (1972b) devised an experiment using young children that would test for the presence of operators by requiring them to keep track of quantitative relationships. This was done by the use of the surprise reactions as an index of cognitive capacity, in which the surprise reaction occurred when the subject's perception failed to conform with his expectation. By using a magic show, Gelman changed the number of mice in an array previously seen by the subject and designated as the winner (the larger of two arrays). The subject was instructed to point out the winner after the two non-transformed arrays had been shown and then hidden. The experimenter transformed the larger row covertly and the reaction of the subject to the transformed "winner" row was noted when it was revealed.

The results of the study provided evidence that children as young as 3 years of age have number invariance operators, and can use the operators on small numbers (1, 2, 3) which they can estimate. No evidence

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was found of these operators in younger children (2 1/2 years). With the use of larger arrays, it was found that the hypothesis that whether or not children use number estimation predicts the use of number operators was supported. There was a tendency for those children who treated number as invariant to count out loud spontaneously. This was attributed to the possibility that children will count an array when a transformation has rendered the array discrepant to their expectation.

Similar findings showed that there are conditions which enhance young children's tendency to count (Gelman & Tucker, 1975). They found that the younger the child, the greater the tendency to count overtly. The decrease in overt counting with age may be attributed to covert counting being more prevalent in older children or the older children may rely more on estimation.

Using the magic show method again, Gelman (1972a) looked at the possibility of young children (mean age, 3-6) revealing a capacity to correctly classify the operations that are irrelevant to number. This magic paradigm showed the young children treating number as invariant under displacement and correctly explaining decreases or increases in number as the effects of subtraction or addition. This is in direct contradiction of the conclusions derived by the conservation paradigm. It is assumed that the conservation task tests only a chil

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child's logical capacity to deal with number invariance. The results of this study, however, indicate that it evaluates other skills as well, such as attention control, correct semantics, and estimation skills. Conservation ability is therefore a more sophisticated level of cognitive development in which many separate abilities are coordinated.

Another viewpoint concerning the child's concept of number is that of cardinal, ordinal, and natural numbers and their developmental sequence. Piaget holds that conservation of number is the criterion of natural number competence, whereas mathematicians claim that addition and subtraction of integers is the criterion. Piaget also claims that ordination and cardination are necessary precursors to natural number competence. Experimenters have therefore attempted to establish a sequence among the emergences of natural, ordinal, and cardinal numbers.

Brainerd (1973b) has found indications that (1) criterial performance on ordination tasks is the developmental antecedent of natural number competence, (2) criterial performance on ordination tasks is the developmental antecedent of criterial performance on cardination tasks, and (3) criterial performance on cardination tasks is not the developmental antecedent of natural number competence. This therefore gives us the sequence ordi

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ordination → natural number → cardination. This is supportive of the ordinal theory which states that the concept of natural number originates primarily in a prior understanding of ordination, where ordination is defined as the quantification of series of three terms united by any transitive-asymmetric relation (Brainerd & Fraser, 1975). Piagetian predictions do not receive any support from this viewpoint. Piaget's (1952, 1953) claim of cardination preceding natural number competence was proven to be unsupported by the results. Only in the ordination → natural number was it confirmed.

The finding of this specific sequence has been reported by Brainerd (1973c, Note 1) and Brainerd and Fraser (1975). It has also been reported that these concepts can be trained in children (Brainerd, 1973c). Ordination and cardination were shown to improve as a result of feedback training, but the average improvement in ordination performance was much greater than the average improvement of cardination principles. It was also found that children trained in ordination were superior to those not trained in number performance, but that there was no significant difference between the cardination-trained group and the untrained control group's number performance. This confirms the suggestion that natural number competence is preceded by a prior understanding of ordination but not by an understanding of cardination.

Brainerd and Fraser (1975), in comparing Piaget's number conservation as natural number criterion versus the mathematicians' addition and subtraction of integers criterion, showed number conservation to be an adequate estimate of natural number competence. This substitution for integer manipulation makes no significant difference. Brainerd and Fraser did find that children can acquire facility with the integers without prior knowledge of number conservation which is consistent with Piaget.

Schaeffer, Eggleston, and Scott (1974) considered number development in terms of skill acquisition. They postulated a model of number development consisting of six skills: (1) acquisition of more x's (where x is a number of objects), (2) judgments of relative numerosity, (3) pattern recognition of small numbers, (4) the counting procedure, (5) the cardinality rule, and, (6) one-one correspondences. These are listed in the order in which they were found to emerge. An emphasis was placed on counting skills. They hypothesized that, with the learning of the last two skills, the cardinality rule and one-one correspondences, which occur simultaneously, children then learn number conservation. This model assumes that number development is determined more by the application of number skills to object arrays than by thoughts about numbers in the absence of arrays or by "spontaneous cognitive reorganizations, i.e., equilibration." It is interesting to note that Schaeffer et al. (1974) used counting as an indication of the presence of the cardinality rule, which states that the last number named during counting denotes the number of objects in the array. Dodwell (1960) found that even though children counted two arrays out loud (equal number, unequal length), they then judged that arrays to be of differing number. He concluded that counting is not a guarantee of knowledge about cardinality.

Training Studies

In view of Piaget's developmental stage hypothesis which states that children should progress to the next stage or level only after they have acquired all concepts necessary and in the prescribed order, experimenters went about trying to disprove Piaget. They began by theorizing that these concepts could indeed be induced in children who did not possess them by intensive training in the concept or concepts deemed necessary to induce number conservation in those children who were still at the preoperational stage of cognitive development by training in the number conservation task itself, or by training of concepts thought by the experimenter to be necessary for conservation attainment.

As Brainerd (1978) states: "A subject can be taught a concept belonging to some given stage only if he is already at that stage or in a transition phase between that stage and the preceding stage. This is because learning a Piagetian concept is supposed to consist not of learning the concept per se but learning to generalize the mental structures appropriate to a given stage." He states that, according to Piaget, learning occurs only when two conditions are met. First, the training treatment must incorporate laws of spontaneous development. Second, the to-be-trained subjects must already possess the to-be-learned concept to some measurable extent. Brainerd feels that, at most, concept learning experiments consist of teaching children to generalize spontaneously acquired mental structures rather than to acquire concepts per se.

It also follows that, given an effective training procedure, Stage II children should learn more than Stage I children (Brainerd, 1977a). Concept learning is supposed to be a process where existing structures are generalized to new content. Stage I children are not capable of learning because they lack the grouping structures necessary for conservation. Stage II children possess them so therefore can be trained. Stage II is an unstable state during which learning experiences are maximally effective. The children are in a state called "cognitive disequilibrium." This proposal was found to be consistent with the findings of Brainerd (1972). He discovered that not only was trainability dependent on the present stage of the child but that it was independent of the subjects' age.

It is interesting to note that Mehler and Bever (1967) found children as young as 2-4 showed a type of number conservation. When showed two unequal rows of M&M's, 81% of the children under 2-8 correctly chose the bigger row. The appearance of conservation was found to be less frequent in the older children with a minimum in the group between 3-8 and 3-11. Conservation began to appear again at 4-6.

According to Mehler and Bever, the fact that very young children showed this conserving ability is an indication that they do have capacities which depend on the logical structure of the cognitive operations. Eventually they then develop an explicit understanding of these operations. They theorized that this inability to conserve is due to an overdependence on perceptual strategies. These perceptual strategies can be overcome given sufficient motivation. Eventually they develop more sophisticated integration of logical operations of their perceptual strategies, allowing them to count members of an array.

Piaget (1968) believes that the data of Mehler and Bever (1967) were inadequately analyzed with regards to the child's perception of length as a sufficient suppressor of a correct notion of number. When children evaluate number with regards to length, this process is actually based on the concept of ordinal quantification which is more complex than is shown in a number experiment. Piaget feels that Mehler and Bever's results actually show a simpler more primitive type of ordinal evaluation based on topological structures (proximity, separation, etc.) that the young child possesses before considering length. This topological evaluation is one based on the notions of "crowding" or "heaping." The child gets the impression that the short row is more crowded or bunched and therefore contains more than the longer one. In this way, children as young as 2 1/2 years gave correct answers almost 100% of the time in Mehler and Bever's (1967) study without really understanding the numerical relations with which they dealt.

Mehler and Bever (1968) redid their original study, this time using equal rows and found that there was no tendency for children at any age to choose the denser row and again owed this result to correct quantitative decisions on the part of the children without referring to relative densities of the rows. They felt that 2-year-olds have certain cognitive capacities that are limited, especially by memory and attention, and in overcoming these limitations, the children form intuitive, perceptual generalizations that extend their capacities beyond behavioral limits. When these generalizations fail, they are integrated into a system, formed by children through experience, that includes both the perceptual generalizations and their basic logical capacities. In this way, the children are able to deal with external reality, i.e., the array of counters in this case.

There have been several attempts at replicating Mehler and Bever (1967) with differing results. Rothenberg and Courtney (1968), in repeating Mehler and Bever's technique, found no indication of the U-shaped trend they reported but did find the high percentages of conserving responses among 2 1/2 to 4 1/2 year olds consistently. When Rothenberg and Courtney compared these results with those found in the same subjects by another more stringent method (Rothenberg, 1969), it was found that this method classified fewer conservers and a more consistent age increase was found. Rothenberg and Courtney attributed these differences to methodological flaws, especially language difficulties, in the Mehler and Bever study. Higgins-Trenk and Looft (1971) found virtually the same results as Rothenberg and Courtney (1968) in their replications and similarly concluded that the problems were due to limited language ability.

Willoughby and Trachy (1971) found similar results. They concluded that their failure to replicate was due to only a partial understanding of the term "more" in their subjects. They also felt that the study was not actually measuring number conservation but some partial concept of the stimuli being used. It was shown that the type of stimuli used, either M&M's or clay pellets, affected the subjects' responses.

Calhoun (1971), on the other hand, basically replicated Mehler and Bever (1967) successfully. However, positive results could not be obtained with the youngest group (2-4 to 2-7) due to problems in their understanding of the procedural instructions. They were found to conserve however when the method employed did not require verbal encoding by the youngest subjects. Calhoun did find a U-shaped age trend similar to the one found by Mehler and Bever and generally concluded that younger children could indeed conserve number.

Pufall and Shaw (1972) compared the positions of Mehler and Bever (1967) and Piaget (1952) on the structures of children's thought and their perception of number. Children (ages 3 to 6) were presented with six number problems in which length and density varied. In analyzing the performance of the children, it was found that only 4% made correct judgments on all problems. No child judged all problems with a relative density hypothesis, that is, judging that the rows are numerically equal because densities are equal or that the more dense row is numerically larger than the less dense row. A curvilinear relationship between number judgment and age on number equivalence tasks was found. This result questions Mehler and Bever's conclusion that young children's performances reflect cognitive intuitions analogous to operational structures in older children.

Early studies in the area of concept training were thought to be sufficient given what was known about the trainability of number conservation at that time (Flavell, 1963). In a later review of the existing training studies, Brainerd and Allen (1971) noticed that in all the successful studies, there was a common feature. All exposed the subjects to situations which specified the object bound form of operational reversibility, namely reversibility by inversion-negation. This fact is noteworthy in that Piaget attributes the onset of first-order thought and conservations to the acquisition of inversion-negation and reciprocity which are the two modes of cognitive reversibility.

Wohlwill (1959) was one of the first researchers who attempted to induce number conservation in nonconservers. Using subjects with a mean age of 5-8, he trained them in addition and subtraction facts with only three brief training sessions. Wohlwill theorized that the comprehension of addition and subtraction in the young child might be sufficient to induce conservation. However, the training proved to be unsuccessful and its lack of success was attributed to the brevity of the training sessions. It was concluded that even though addition and subtraction comprehension was not sufficient for number conservation, it did appear to be necessary.

Wohlwill and Lowe (1962) considered reinforcement, differentiation, and inference as possible alternative interpretations of the acquisition of number conservation. Three hypotheses were formulated and independent groups were trained in accordance with each hypothesis (mean age, 5-10) along with a control group. The <u>reinforcement</u> hypothesis proposed that, as children learn to correctly count and have experience with different types of arrays, they gradually learn that perceptual transformations of the arrays do not change the number of items in the array. So, systematic reinforced practice in counting rows of objects prior to and after transformation should induce conservation.

The second _differentiation_ hypothesis interprets inability to conserve in young children as a response to an irrelevant but highly visible cue, length, which tends to correlate with number. That is, when a row is longer, it usually indicates an increase of number. Repeated experience designed to neutralize the cue of length and weaken any association between length and number should therefore be expected to facilitate conservation acquisition.

The _inference_ hypothesis is based partly on Piaget's own analysis on the role of learning in the development of logical operations. He maintained that experience can be effective only in that it builds upon previously developed structure of thought. In the case of conservation, children might be led to infer it as a result of a change involving neither addition nor subtraction _per se_ but rather the cumulative effects of watching the adding or subtracting of an element to a collection.

The results revealed that the reinforced practice trials, which were expected to produce the best results, in fact showed poor results. This showed the ineffectiveness of continued reinforced practice in eliciting conservation responses. The greatest amount of improvement took place in the Addition and Subtraction group.

This result is consistent with the possible role of a process of inference, that is, conservation as the end-product of changes involving neither addition nor subtraction. No learning was found in the dissociation group suggesting that the act of counting interfered with the children's attention to the cue of length. The fact that the Addition and Subtraction group yielded the most improvement in conservation appears to suggest that a process of inference may be operative in the development of number conservation.

Reversibility was recognized to be a factor in conservation attainment. Piaget (1952) stressed the necessity of reversibility on number conservation acquisition. With this in mind, Wallach and Sprott (1964) attempted to induce conservation of number by showing children (mean age, 6-11) the reversibility of rearrangements, which they regarded as implying changes in number prior to conservation.

Dolls and beds were used with nonconserving subjects (classified by a pretest) in situations where a row of dolls and a row of beds were shown to each child. The eight situations showed equal numbers of beds and dolls with the rows unequal or unequal numbers of dolls and beds with equal rows. The child was asked to predict if there were enough beds for the dolls. This procedure was repeated until the child correctly

responded to each of the eight situations. The child was then given two posttests, one immediately after training and the other 14 to 23 days later. The results showed the training procedure to be effective in inducing number conservation in nonconservers. It was suggested that the training in reversibility contributed to conservation by providing subjects with the information that rearrangements are reversible, and by inducing subjects to think of reversal's possibility.

Cognitive conflict was also thought to be an essential condition for the development of conservation in children. It supposedly induces a reorganization of children's intellectual actions, which proceeds along the lines postulated by Piaget's equilibration theory. This reorganization then leads to the conservation strategy. Gruen (1965) compared the effectiveness of two types of training procedures: cognitive conflict and learning through reinforced practice. The former induces internal cognitive conflict which brings about number conservation. The latter provides external feedback through counting with the reinforcement being a "knowledge of results." The intention of this study was to isolate some of the variables that play a role in the acquisition of conservation. However, the results showed that neither training methods was successful in inducing number conservation in over half of the subjects. There were a number of subjects who did acquire conservation and when compared to some of the earlier studies, this was impressive. There was a better performance by the cognitive conflict group when compared to the reinforcement group, but this difference did not reach statistical significance.

A similar study was done by Winer (1968) where he induced addition and subtraction sets and investigated whether these sets could determine responses to conflict trials and to tests of conservation. It was hypothesized that practice in responding to either additions and subtractions or to changes in length would induce in subjects a set to respond to either of these manipulations. This set would be brought forward on conflict trials where changes in length opposed additions and subtractions. Those children (mean age, 5-11) who received addition-subtraction training would continue to respond to them in conflict trials while children who received practice in responding to length would continue to respond to changes in length. Winer also predicted conservation inducement through addition-subtraction practice.

The results showed that continued response to simple additions and subtractions can induce subjects to respond to these manipulations when they are made to contrast with changes in the length of a line and subsequently to acquire conservation. Although conflict trials can probably contribute somewhat to the acquisition of conservation, the effect is minimal and accomplished without the presence of overt conflict.

Mermelstein and Meyer (1969) compared four types of training conditions: cognitive conflict, multiple classification, verbal rule instruction, and language activation. In the first condition, subjects were shown arrays in one-one correspondence several times for 5 sessions. In the final three additional sessions, one-one correspondence was established, and then, one row was collapsed into a pile. The subjects were then questioned about the numerical equality of the two rows.

Multiple classification training established the possibilities of the multiple labeling, classification, and relations of the poker chips used. Each child was then trained in reversibility. They were shown two rows of chips in one-one correspondence and questioned about the row equality. One row was then collapsed into a pile and the subjects were again asked if there was the same number in each row. The verbal rule instruction group received the same reversibility training as this but one row was immediatley collapsed and the subject was questioned as to the equality of the two rows. The difference between the two methods stemmed from the fact that the experimenter said he was moving the row as he did so and commented that the number was not being changed, only its appearance. The language activation training was a technique based on the standard Piagetian conservation of discontinuous quantity task. Two equal boxes were filled with equal amounts of poker chips in a one-one correspondence technique. One box's contents were poured into a third taller box. The subject was asked to verify their number equality.

Results of the posttesting with various conservation tasks showed that none of the four training conditions were successful. All had been previously found to have been successful in number conservation training. It was felt that a point should be made that specific training differs from cumulative life experiences which Piaget suggested are partly the determining factors for acquiring number conservation.

Cognitive dissonance has been thought to be quite similar to Piaget's idea of equilibration and has been used to induce conservation in transitional conservers and nonconservers (Murray, Ames, & Botvin, 1977). It was felt that the need for establishing cognitive consistency was similar to the process that Piaget feels children undergo in cognitive development. Murray et al. (1977) found that conservation could in fact be induced permanently in nonconservers and transitional conservers by the use of a technique based on cognitive dissonance theory. They also found that conservers were not at all affected by the experimental treatment received. This study is quite important due to the fact that it ignored the training of concepts thought to be necessary for conservation and concentrated on the conflict that is thought to go on during cognitive development, namely disequilibrium.

Reversibility was again used as a training procedure. Wallach, Wall, and Anderson (1967) hypothesized that number conservation could be brought about by reversibility training, basically a replication of an earlier study (Wallach & Sprott, 1964) and also be induced by training in addition and subtraction, something that Wohlwill (1959) had suggested. It was thought that number conservation could be brought about by reversibility training without addition-subtraction experience. This was found to be true which suggests that number conservation is not affected by training in addition and subtraction. Training in addition and subtraction without reversibility experience was not found to be successful in inducing number conservation. The main effect of the reversibility training procedure may have been to lead the subjects (mean age, 6-11) to stop using a misleading perceptual cue. They felt that with this interpretation, reversibility training success cannot be regarded as providing evidence for the role of the recognition of reversibility in conservation. This procedure, while appearing to induce conservation, probably did so by leading the children to stop using misleading cues. However, reversibility was still looked upon as necessary for conservation.

Gelman (1969) hypothesized that young children fail to conserve because of inattention to relative quantitative relationships and attention to irrelevant features in classical conservation tests. Children (mean age, 5-4) were assigned to learning set training groups, with or without feedback. It was shown that, given appropriate training, one can elicit conservation in children who initially fail to conserve. Appropriate training seemed to involve two factors: (1) interaction with many different instances of quantitative equalities and differences, and (2) feedback, telling the subject what is and what is not relevant to the definition of quantity.

Whereas most of the previous studies focused on the training of one or two concepts to induce number conservation, Rothenberg and Orost (1969) trained their subjects in a sequence of concepts. They presented their subjects (mean age, 5-5) with a sequence of concepts derived from analysis of the components

assumed to underlie number conservation attainment. It was not stated how these concepts were chosen or analyzed. They were: (1) rote counting, (2) counting attached to objects, (3) "same" number, (4) the "same" versus "more" distinction in terms of number, (5) addition and subtraction representing a change in number, (6) one-one correspondence, (7) reversibility, and (8) the distinction of "more" referring to the actual number of objects versus "longer" referring to their arrangement in space. Each child was given three training sessions in all concepts. The above sequence of concepts was used, implying a rough estimation of their developmental occurence.

Results of this study implied that number conservation could be induced (or taught) to nonconserving children. The sequence of concepts provided a reasonable working effort for teaching number conservation. It was not proved that this sequence was the only sequence acceptable to the hypothesis. There was no way of noting if one or several of the concepts was excessive. Perhaps a combination of only a few would have been as successful as the use of all eight.

Murray (1972) theorized the number conservation could be taught to nonconservers through social interaction. Basing his hypothesis on Piaget's theory that the occurence of repeated communication conflicts between children was a necessary condition for the movement from preoperational thinking to concrete operations, Murray placed a nonconserver along with two conservers (mean age, 6-7). Each group had to come to an agreement on answers to various conservation tasks. Afterwards, each nonconserver was posttested and a significant number were found to have attained number conservation. It was most successful with Stage II children. The results indicate that social conflict or interaction is an important mediator of cognitive growth.

These findings were replicated by Botvin and Murray (1975) who also found that modeling was also effective in eliciting conservation responses. Those nonconservers who watched the social interaction groups give their correct conserving responses were found to conserve in a posttest and were able to justify their answers with reasons other than those given by the group they observed. This indicates that modeling is as effective as social conflict in eliciting conservation responses in nonconservers.

The effects of training children (ages 6 to 8) in the conservation task itself were examined by Figurelli and Keller (1972). Social class was also investigated as a variable. It was found that this type of training (repeated demonstrations with feedback) was successful, but differences across socioeconomic class were not significant.

Mpiangu and Gentile (1975) trained nonconservers, 4- to 6-years-old in rote and rationale counting (number recitation), number recognition (finding a missing number in a given sequence), relations (before, just before, after, just after, between), and number synthesis and analysis (giving correct answers and justifications to simple mathematics problems). These concepts include some of those singled out by Rothenberg and Orost (1969), but also include different mathematical concepts. The results showed dramatic increases on arithmetic performance and a significant effect on number conservation attainment. This result, while showing that training did facilitate the acquisition of number conservation, was interpreted by the authors to mean that conservation was not a crucial concept for mathematical understanding. Number conservation and mathematics were considered as concepts that develop simultaneously.

Emphasis was placed on discrimination by Halford and Fullerton (1970). To conserve number in the classical type of task, children must discover that number is determined by correspondence, potential or actual, to the standard array and not by length or spacing, unless these two compensate each other. They must discover that if number is constant, an increase or decrease in length is compensated by the opposite change in spacing, so length and spacing cannot be ignored as cues.

With this in mind, Halford and Fullerton trained children (mean age, 6-3) in a discrimination task. A row of beds was shown along with several differently arranged rows of dolls. The child was asked which row of dolls would fit the beds. Five individual training sessions were held. The principle of one-one correspondence was also shown as each child was shown the row of beds with a doll placed in each one. Results showed that two-thirds of the children had acquired a stable type of number conservation through this manner of training.

A similar line of research in the area of perceptual cues of number and length in number judgments has also been investigated by several researchers (LaPointe & O'Donnell, 1974; Lawson, Baron, & Siegel, 1974; Smither, Smiley, & Rees, 1974). Children 2- to 5-years-old were trained in the concept of a "bunch" by LaPointe and O'Donnell (1974). They were shown a small group of houses and told this was a bunch and then were asked to identify similar "bunches" of houses, referring to the concepts of "more" and "same." Each subject was then tested on the standard number conservation task and on a variety of conservation-like tasks designed to show changes in density, length, and number.

Results showed that children between the ages of 2 and 5 do not employ a consistent hypothesis in judging number. It appears that they attend to differences between the rows and give a response based on the most salient cue. The child understands number in terms of perceived correspondence and differences in arrays. The length cue appears to be the most salient.

Counting also was found to be related to conserving number. Four-year-old children who counted and conserved counted very carefully before and after transformation. It was suggested that this helps children to direct their attention to numerical correspondence between the two rows. In this way, a conserving response would be facilitated. It was also proposed that counting can increase the number of conservation responses without implying the lasting equivalence required by Piaget, indicating operational thought.

Lawson et al. (1974) on the other hand, found that number, given small arrays, was the salient cue rather than length as LaPointe and O'Donnell (1974) and Pufall and Shaw (1972) had found. Pufall and Shaw used larger arrays that perhaps might have been beyond the children's estimating abilities, making number less salient. Thus the rule might be: if the number arrays are beyond estimating, length is the cue and when the array is within estimating range, number is the salient cue.

Similar results were obtained by Smither et al. (1974). Children, ages 3, 4, and 6, were tested on a variety of number conservation-like tasks on which number, length, and density were varied. Results showed no support for three stages of development of number judgments. It was suggested that accurate number judgments develop in a continuous fashion and that the use of length and density cues depends on age, salience of the dimensions and also the magnitude of number and number differences. When the length differences and number were small, and the density and number difference large, the children tended to make accurate number judgments. However, when the numbers are large and the number difference small, the judgments were based on length. This seems to agree with the hypothesis and rule put forth by Lawson et al. (1974).

There have been several attempts at trying to establish the developmental sequence of number concepts. Wohlwill (1960) had one of the first studies attempting to establish a sequential pattern of these concepts with the use of scalogram analysis. After training children to match numbers to stimulus arrays of dots, they were tested on 7 tasks designed to assess certain number concepts. These concepts were found to occur in this order: (1) the ability to match two arrays on the basis of numerosity, (2) the ability to abstract number from irrelevant cues, (3) memory for numbers, (4) understanding addition and subtraction changes, (5) matching larger arrays of objects by number, (6) number conservation attainment, and, (7) ordinal correspondence.

Siegel (1971a, 1971b) administered 7 tasks to 3- to 5-year-old children. They were tested on continuous and discrete magnitude discrimination, equivalence, conservation, ordination, seriation, and addition. She found that equivalence and magnitude discriminations can both be understood as concepts at approximately the same time. Number conservation develops at a slightly later time with ordination, seriation, and addition developing later. Addition concepts came quite a bit later in Siegel's study than in Wohlwill's (1960) but the difference appears to be in the relative difficulties of the task type each used.

Hypotheses

From the review of the literature, eight concepts, or tasks, have been chosen as best representing the viewpoints of the current research as to what is necessary to induce number conservation in nonconserving children. All eight concepts were found either to be present in conservers or were able to induce conservation through training in them. However, not all were examined at the same time in any given subject in any of the reviewed studies. The present study differed from the others in that it combined those concepts which were individually thought to be necessary precursors to number conservation, and tested for their presence in both conserving and nonconserving children.

The questions raised in the present study were whether these concepts are all present in number conservers and whether or not all, or only several, are also present in nonconservers. The present study also examined the possibility of establishing the presence of a developmental sequence of these concepts. In this way, it was possible to judge whether the concept was a logical prerequisite for number conservation as thought by previous researchers.

With the selection of these eight concepts, it was then possible to make predictions as to the possible developmental sequence of emergence prior to number conservation. They are listed in order of predicted outcome, from earliest to the last emerging before number conservation.

1. Rote counting: Rote counting has been found by many to be present in conservers and in nonconservers

alike (Piaget, 1952; Pufall, Shaw, & Syrdal-Lasky, 1973; Rothenberg & Orost, 1969; Schaeffer et al., 1974; Wohlwill & Lowe, 1962). Rote counting, or reciting the number names, is in no way an indicator of true number understanding. It simply means that children are able to recite the number names in their proper order. Even so, counting has been hypothesized to be necessary for number conservation (Halford & Fullerton, 1970; LaPointe & O'Donnell, 1974; Mpiangu & Gentile, 1975).

2. Object counting: The next stage should be the ability of children to attach these memorized number names to objects and count an array (Rothenberg & Orost, 1969; Schaeffer et al., 1974). This ability has been found experimentally to come after rote counting (Pufall et al., 1973) and be necessary for number conservation (Halford & Fullerton, 1970). It was also found to precede one-one correspondence (Pufall et al., 1973).

3. Number judgments of two equal groups: The ability of children to count groups and make judgments on their equality was hypothesized to occur next. After the children are able to count and connect the counting with object arrays, they should next be able to make numerosity judgments. Schaeffer et al. (1974) theorized that in number skill development, equality judgments will emerge before the concept of inequality. This also

has been thought to occur in other training studies (LaPointe & O'Donnell, 1974; Rothenberg & Orost, 1969).

4. Number judgments of two unequal groups: Rothenberg and Orost (1969) thought that the training in this type of number concept would help to bring out number conservation in young children. Once children can count arrays and make judgments on the numerical equality, they should be able to make judgments on inequality. It was theorized that the knowledge that x + 1 is greater than x is developmentally later than equality recognition but occurring before one-one correspondence and number conservation (Schaeffer et al., 1974).

5. Addition/subtraction changes in a row: Many researchers have hypothesized as to the necessity of addition and subtraction skills in the acquisition of number conservation. It has been found to be necessary but not sufficient for conservation (Wohlwill, 1959), unsuccessful in bringing it about through training (Wallach et al, 1967), but more often its training has been successful in inducing number conservation (Gruen, 1965; Mpiangu & Gentile, 1975; Rothenberg & Orost, 1969; Winer, 1965; Wohlwill & Lowe, 1962). Gruen (1965) also found his experimental design showed addition and subtraction concepts to logically and developmentally

precede number conservation as also found by Rothenberg and Orost (1969).

6. <u>One-one correspondence</u>: The concept of one-one correspondence is a crucial one in children's understanding of number concepts. The understanding that if the density and length of two rows are both equal, then the number of the two rows is equal is a necessary concept in number conservation (Inhelder et al, 1974; Lawson et al., 1974). It has been found to be one of the later concepts to develop in children (Wohlwill, 1960) and theorized to be developmentally occurring just before conservation (Schaeffer et al., 1974). Training in one-one correspondence has both been found to be successful (Halford & Fullerton, 1970; LaPointe & O'Donnell, 1974; Rothenberg & Orost, 1969) and unsuccessful (Mermelstein & Meyer, 1969).

7. <u>Reversibility</u>: Reversibility is a concept that Piaget (1952) feels is a necessary part in the logical operations necessary to become concrete operational and a number conserver. Children must be able to understand that an operation performed on a numerical array can be reversed or undone to conserve number. Training in reversibility problems to induce number conservation has been attempted by Mermelstein and Meyer (1969), and Wallach et al. (1967) unsuccessfully, and by Wallach and Sprott (1964) and Rothenberg and Orost (1969)

successfully. Rothenberg and Orost (1969) placed reversibility developmentally and logically after one-one correspondence as Piaget (1952) and Inhelder et al. (1974) do.

8. Number versus length of arrays (Spatial arrangement): Overcoming perceptual cues to focus on the numerosity of an array is one of the last and possibly one of the more important concepts that children must attain. Children must learn to respond according to numerosity rather than by length or density. Pufall et al. (1973) showed that judging number using relative length increased with age. Wallach et al. (1967) identified reversibility training as the factor which permitted their subjects to overcome misleading cues, recognizing reversibility as well as not using misleading perceptual cues as necessary for conservation. These misleading perceptual cues (length, density) have been found to interfere with correct number judgments at all ages and also have been shown to be developmentally occurring very late, before the acquisition of number conservation (LaPointe & O'Donnell, 1974; Lawson et al., 1974; Rothenberg & Orost, 1969; Smither et al., 1974; Wohlwill & Lowe, 1962).

To summarize, the predicted developmental ordering of the eight tasks and number conservation is:

1. Rote counting

2. Object counting

3. Number judgments of two equal groups

4. Number judgments of two unequal groups

5. Addition/subtraction changes in a row

6. One-one correspondence

7. Reversibility

8. Number versus length of arrays (spatial arrangement

9. Number conservation

METHOD

Subjects

The subjects were 75 kindergarten students who were enrolled at the Williamston Memorial School in Williamston, Michigan, a predominantly lower-middle-class area. There were 42 males and 33 females, ranging in ages from 4-11 to 6-4, with a mean age of 5-6.

The testing was conducted very early in the school year, approximately four weeks after it began. Very little instruction in number concepts had been given to the children by this time, mainly counting numbers had been taught in each class. Each of the eight number concept tasks was administered to every child as was the standard number conservation assessment task.

Apparatus

Red and black wooden checkers and 13 white index cards with blue dots were used as stimuli. Each checker measured approximately 3.2 cm. x 0.8 cm. The index cards (12.8 cm. x 20.4 cm.) each had a varying number of dots (1.5 cm. in diameter) drawn on them (see Appendix A).

Procedure

Subjects were met individually at their classroom and escorted to the testing area by the experimenter to be individually tested. During this time, an effort was made to establish rapport to make the child feel more at ease with the experimenter. The experimenter also answered any questions the children had about what they were expected to do during the experiment. The child was then seated directly opposite the experimenter at a small table in a study area of the school.

Each child was first given the number conservation assessment task followed by rote counting. The following 7 tasks were given in a random order to each child to control for learning. Randomization was achieved through the use of a calculator with a random number generator. This series of tasks were then followed by a readministration of the number conservation assessment task.

Criterion for a subject to pass a given task was two correct responses in three trials, with a score of 1 for a pass, and 0 for a fail. This was the scoring procedure for all tasks except for Task 7, where 3 of 4 correct responses served as criterion. No feedback as to correct or incorrect responses was given to subjects and all responses given were recorded (see Appendix B).

<u>Task 1</u>: Number conservation assessment

The subject was shown two equal rows of 5 checkers each in one-one correspondence. The experimenter then asked, "Does your (my) row have the same number of checkers as my (your) row?" After the subject responded, the experimenter then transformed one of the rows, either by changing the density or length and repeated the question previously asked. This task was given a maximum of three times with the row to be pointed out first randomly changed each time. The row on which the transformation was to occur and the type of transformation to be performed were also randomly assigned.

This procedure is similar to that which Piaget (1952) designed. Two main differences did exist, however. Piaget required both correct judgment <u>and</u> explanation as criteria for passing the number conservation task. The present study required a judgment-only criterion in assessing number conservation. Brainerd (1973a, 1977b) stated that, in the current literature, the chances of an error in using a judgment-only criterion were fewer than if using a judgment plus explanation criterion. Errors can be made in two ways: (1) classifying a child as conserver when in fact she is not; and, (2) classifying a child as a nonconserver when in fact she is. Therefore, it was decided to use a judgment-only criterion based on these findings. Also, Piaget labeled his subjects as Stage I, II, or III conservers. Since we are using judments-only, it is not possible to classify subjects in this way, according to their responses, and will not be done in the present study.

Task 2: Rote counting

To ascertain whether the subject was able to recite the natural or counting numbers, the experimenter said, "I would like you to count from one to ten for me." The subjects were given additional trials if they stumbled over certain numbers or omitted any of them; otherwise they were scored 1 after one correct trial. Rothenberg and Orost (1969), Wohlwill and Lowe (1962), and Schaeffer et al. (1974) all used a task similar to this one in previous studies.

Task 3: Object counting

Schaeffer et al. (1974) and Rothenberg and Orost (1969) both used similar tasks to train or assess object counting abilities. Their procedures were used to devise the present task. The subjects were shown a group of checkers and were asked, "How many checkers are there in this group?" The task was repeated, each time with a randomly assigned number of checkers, from 4 to 7.

Task 4: Number judgments of two equal groups

The subjects were shown an index card with two rows of dots with equal length and number on it. They were asked, "Does each row have the same number of dots?" With each trial, the number of dots on the card that was shown was randomly assigned with either 4, 5, or 6 dots per row. A similar task has been used in previous studies (LaPointe & O'Donnell, 1974; Rothenberg & Orost, 1969).

Task 5: Number judgments of two unequal groups

Schaeffer et al. (1974) and Rothenberg and Orost (1969) used similar procedures to those described here to train or test their subjects on their ability to perceive number inequality. The subjects were shown two index cards, each with a row of dots, the two being unequal in number. They were then asked, "Does each row have the same number of dots in it?" The two cards used for each trial were randomly chosen, with the numbers varying from 4 to 7 dots.

Task 6: One-one correspondence

The present study used a one-one correspondence task quite similar to those in the studies by Halford and Fullerton (1970), Inhelder et al. (1974), LaPointe and O'Donnell (1974), and Rothenberg and Orost (1969). A row of checkers (5, 6, or 7 randomly ordered) was constructed before the subjects. They were then given a pile of chekcers and were told, "I would like you to make a row just like mine with the same number of checkers." Each trial was repeated with a different number of checkers.

Task 7: Addition/subtraction changes in a row

Two rows of 5 checkers were constructed in one-one correspondence in front of the subjects. The experimenter ascertained that the subjects understood that the two rows were numerically equal by asking, "Do these two rows have the same number of checkers in each?" If the child said yes, the experimenter then began the task. If no, the experimenter then counted each row out loud to show their equality.

This task differed from others in that there were 4 trials rather than 3. The experimenter then manipulated either her row or the subject's, twice on each in a random order. A checker was then added or subtracted (twice each in a random order) from or to one of the rows while the subject watched, with the manipulated row's length adjusted to match that of the non-manipulated row. The subject was then asked, "Who has more checkers, you or I, or do we have the same number?"

Pufall et al. (1973), Wallach et al. (1967), Rothenberg and Orost (1969) and others used a technique similar to the one developed for use in the present study.

<u>Task 8</u>: Number versus length of arrays (spatial arrangement)

Pufall et al. (1973) used several tasks to test judgments of numerosity in spite of misleading cues. These tasks were incorporated into one for the present study. The subjects were shown a card with two rows of dots on it. The rows were either numerically equal but unequal in length or equal in length but numerically unequal. The order of presentation was randomly assigned with no card presented twice to a subject. The experimenter then asked, "Do these two rows have the same number of dots in them?"

<u>Task 9</u>: Reversibility

The experimenter constructed two equal rows of checkers (6, 7, or 8). She then said to the subject, "These two rows of checkers have the same number in them. Now watch me." The experimenter then proceded to collapse, or "bunch up" one of the rows while the subject watched. The subject was then asked, "Does this row have the same number of checkers as the other row?" The number of checkers in the two constructed rows and the row to be manipulated were randomly chosen for each trial. This technique was similar to that used by Inhelder et al. (1974), Wallach et al. (1967), Rothenberg and Orost (1969), and Wallach and Sprott (1964).

RESULTS

Analyses

The data were analyzed in several different ways. The major analyses performed were a Guttman scalogram analysis and a linear multiple regression analysis. The Guttman scalogram was performed to give a developmental ordering of the eight number concept tasks and also of the number conservation task. This analysis also gives the number of errors committed by the subjects and the reproducibility coefficient of the scale. Multiple regression analysis was used to establish the ability of each number concept task to predict number conservation. This method also gives the predictability when these variables are combined in a forward stepwise fashion. In this manner, we are able to determine the percentage of variance accounted for by each variable. The .05 level of statistical significance was selected for all statistical analyses in the present study.

Sample Analyses

Table 1 lists the distribution of total scores (total passed) of the 75 subjects. The sample was divided into three age groups: (1) 4-11 to 5-4, (2) 5-5 to 5-9, and (3) 5-10 to 6-4. The mean total score and variances for each group are also listed.

Table 1

Total Scores, Means, and Variances of Age Groups

Total Score (Passed)	Age Group		
	4-11 to 5-4	5-5 to 5-9	5-10 to 6-4
2	1	0	0
3	0	2	0
4	3	5	1
5	8	3	2
6	6	5	3
7	6	10	5
8	2	3	6
9	1	2	1
Mean Total Score	5.8	6.1	6.9
σ^2	2.15	2.69	1.65

A chi-square analysis was computed to assess any differences in scores by age group. The analysis showed that there were no differences in the way the total scores were distributed across the three age groups (χ^2 (14) = 16.49, $p < .28$). It was not expected that the distributions would vary owing to the small age range of the sample.

A Levene test for homogeneity of variance (Keppel, 1973, p. 81) was performed on the data to assess sample homogeneity with regards to within-group variance. It was found that the sample variance was indeed homogeneous (F(2, 72) = .105, n.s.).

The data was not analyzed to check for sex differences. In general, it has been found that sex differences do not occur in studies of number conservation in children (Maccoby & Jacklin, 1975).

Correlational Analysis

The phi correlation coefficients between the 8 number concept tasks, number conservation and age are listed in Table 2, along with the phi max values. As can be seen, there was very little relationship between the variables themselves or between number conservation or age. The correlation coefficients were quite small, most very close to zero. However, a few were significantly correlated. Reversibility was found to be

Table 2

Intercorrelations Between Independent, Dependent, and Age Variables

Task	Phi Max	1	2	3	4	5	6	7	8	9
1. Number Conservation	-	-								
2. Rote counting	.53	.08								
3. Object counting	1.00	.15	.32**							
4. Equal number Judgments	.71	.10	-.05	.05						
5. Unequal number Judgments	.53	.12	.14	.29**	.14					
6. One-one Correspondence	.86	-.10	-.08	.21	-.10	-.01				
7. Addition/ Subtraction	.80	.19	.25*	.32**	-.13	.08	.24*			
8. Spatial Arrangements	.55	.22*	.14	.19	.08	.09	.17	.19		
9. Reversibility	.48	.23*	-.02	-.07	.12	.05	.04	.25*	.10	
10. Age	-	.10	.03	.01	.07	.04	.07	.15	.17	.27*

*$p < .05$ for the first statistical test

**$p < .01$

related significantly with age ($r = .27$, $p < .05$), number conservation ($r = .23$, $p < .05$), and addition/ subtraction in groups ($r = .25$, $p < .05$). Spatial arrangement was correlated with number conservation ($r = .22$, $p < .05$). These number concept tasks are those which are usually thought to occur later than other number concepts. Therefore, they appear relatively close to the appearance of number conservation with the probability of their appearance increasing with age.

The correlation between rote counting and object counting was highly significant ($r = .32$, $p < .01$). This is understandable owing that rote counting ability is a skill a child needs to count an array of objects effectively, and the teaching of this skill often involves arrays of objects. Object counting was also correlated with the counting of unequal groups ($r = .29$, $p < .01$), and the addition/subtraction changes in arrays ($r = .32$, $p < .01$). Both are tasks which are also dependent on counting ability.

The maximum phi was calculated for each correlation between criterion, number conservation, and each of the eight number task. Phi max indicates the maximum level each phi coefficient can attain given the marginal distributions (Guilford, 1954). Only when the two variables being correlated are of equal difficulty can phi coefficients be at their positive maximum. As can be seen on Table 2, most of the phi coefficient were less than 1.0, indicating that the difficulty levels between number conservation and the eight number tasks varied.

The number conservation task was given at the beginning and at the end of the session to each child to check for the test-retest reliability of the conservation task. Of the 65 children who were initially classified as nonconservers, only six conserved on the second trial and of the ten children who initially conserved, none failed the second time. The phi coefficient between conservation scores before and after the testing was .753 ($\underline{\phi}$ max = .753), showing that the number conservation task is reliable.

Developmental Sequence

To determine whether there was a certain sequence in which the subjects responded to the tasks, the Guttman scalogram analysis technique (Green, 1954, 1956; Torgerson, 1958) was employed. This technique is one commonly used in studies looking at developmental sequences (Kofsky, 1966; Siegel, 1971b; Wohlwill, 1960). The tasks were ranked in order of difficulty on the basis of the number of subjects who passed each task. This will give us an ordering of the logical prerequisites of number conservation and determine whether a subject who has passed a given task has also passed the logical prerequisites to that task.

Scalogram analysis assumes that, to achieve a perfect scale, subjects who pass a given task must pass all the logical prerequisites and fail all those tasks above the subjects' maximum level of achievement (Green, 1954). An error was made if a subject passed a task but failed a logical prerequisite to the task. A coefficient of reproducibility, Rep, can be calculated to determine the proportion of responses which fit this "perfect" sequence by each subject. The formula for Rep is:

$$\text{Rep} = 1 - \frac{E}{NK}$$

where N is the number of subjects, K, the number of tasks, and E, the number of errors made. A total of 88 errors were made in 675 responses by the subjects. The value of the coefficient for the present study was .87 which is high and indicates that the scale was fairly good and could be reproduced.

Table 3 shows the percentage of those subjects who passed each task. The order of difficulty of the tasks as given by the scalogram analysis from easiest to most difficult was: (1) rote counting, (2) number judgments of equal groups, (3) one-one correspondence, (4) object counting, (5) addition/subtraction changes in a row, (6) number judgments of unequal groups, (7)

Table 3

Percentage and Number of Subjects Passing Each Task and Number of Errors Made on Each Task

Predicted Order	Task	Passed	Percent	Guttman errors
1.	Rote counting	72	96.0	3
3.	Equal number judgments	70	93.3	5
6.	One-one correspondence	66	88.0	8
2.	Object counting	65	86.7	7
5.	Addition/subtraction changes	61	81.3	8
4.	Unequal number judgments	49	65.3	17
7.	Reversibility	46	61.3	20
8.	Spatial arrangement	25	33.0	14
9.	Number conservation	10	13.3	6

Note: N = 75

Total errors = 88

reversibility, (8) spatial arrangements, and (9) number conservation. The obtained results were somewhat different from the predicted order but were highly correlated with it. The Spearman rank-order correlation (Siegel, 1956) of the predicted logical sequence with the obtained sequence of tasks was .85 ($p < .01$).

Predictability of Tasks

Predictability of number conservation by the eight number concept tasks was analyzed by the use of linear multiple regression. In this way, we were able to see to what extent each variable predicted number conservation both alone and in combination with other variables. The eight variables were entered into a forward stepwise multiple regression procedure with number conservationas the predicted variable. A summary of the results of the analysis is presented in Table 4.

As can be seen, only the first 5 variables, or steps, added significantly to the regression equation ($F(5,69) = 2.41$, $p < .05$). Reversibility, spatial arrangement, one-one correspondence, object counting, and addition/subtraction changes were found to be significant predictors of number conservation when added in a stepwise procedure to the multiple regression equation. One-one correspondence received a negative beta-weight which could indicate that it is acting as

Table 4

Results of Multiple Regression Analysis Predicting Number Conservation from Eight Variables

Step	Variable entered	R	R^2 change	Beta	Standard error Beta	Significance of F
1	Reversibility	.23	.05	.191	.12	.046*
2	Spatial arrangement	.31	.04	.185	.12	.029*
3	One-one correspondence	.34	.02	-.192	.12	.034*
4	Object counting	.38	.03	.131	.13	.029*
5	Addition/Subtraction Changes	.39	.01	.121	.13	.045*
6	Equal number Judgments	.39	.00	.508	.12	.073
7	Unequal number Judgments	.39	.00	.392	.12	.116
8	Rote counting	.39	.00	-.318	.12	.173

*$p < .05$

a suppressor variable. However, it is relatively small (-.192) and therefore most likely is not contributing as a suppressor (Darlington, 1968). The three remaining variables did not improve predictive ability a significant amount. However, the 5 prediction variables only accounted for 15% of the variance with the remaining 3 variables contributing no variance. It appears that the tasks themselves are not especially good predictors of number conservation in spite of their significance. This fact will be discussed further in the following section.

DISCUSSION

The present study's major purpose was the determination of a developmental ordering of certain number concepts thought to be logical prerequisites of number conservation. These number concepts were among those proposed by numerous researchers as necessary for children's attainment of number conservation. In general, an ordering of these eight concepts was established, but evidence for their necessity was not found.

Rote counting was found to be the easiest, or first, concept that children acquired before number conservation. This finding upheld Prediction 1 and concurred with similar findings (Rothenberg & Orost, 1969; Schaeffer et al., 1974; Wohlwill & Lowe, 1962).

The next concept predicted to develop in children was the ability to count objects in an array. It was thought that once children possessed rote counting skills, the next skill to develop would be the application of the rote counting in determining number in arrays. This, however, was not found to be the case. Number judgments of two equal groups was shown to be the second concept to develop. Basically, counting skills were probably necessary to respond to this task, so it might have indirectly assessed this ability besides the recognition of equality. It is also possible that this task was easier than thought by other researchers (LaPointe & O'Donnell, 1974; Rothenberg & Orost, 1969).

One-one correspondence emerged as the third number concept acquired by the children. This concept had been predicted to emerge as the sixth concept, which would have made it a much more difficult one than has been shown here. With the prior emergence of correct number judgments of equal groups as a concept, it might be that the next logical concept to develop would be the ability to construct two such equal groups. This concept seemed to develop without the understanding of inequality as was thought necessary for one-one correspondence by Inhelder et al. (1974), Wohlwill (1960), and Schaeffer et al. (1974).

The fourth concept, object counting, had been predicted to develop in second position, immediately after rote counting. This indication of increased difficulty than predicted could be attributed to the manner in which the objects were displayed. The checkers were not presented to each subject in a straight line, but usually in a bunched group. The children might have had difficulty remembering if they had already counted a checker. Several subjects did show signs of confusion when counting the group of checkers. Perhaps, if the checkers had been arranged in a straight line, a higher percentage of subjects would have passed this task, placing it closer to its predicted position. This is also an indication of how perceptual factors and memory play an important part of children's number judgments.

Addition and subtraction changes in a row was shown to emerge as the fifth number concept to develop before number conservation. This confirmed the predicted order of its emergence.

Number judgments of unequal groups appeared in most children as the sixth concept to develop. This concept had been predicted to appear earlier (fourth) than the results showed. It is possible that children must understand the effects of addition and subtraction changes in equal rows before they are able to recognize that two groups of objects are unequal in number.

The last three concepts, reversibility, spatial arrangements, and, finally, number conservation, were found to appear in this order. Their order of emergence was in agreement with the predicted order where the first two were assumed to be logical prerequisites of number conservation; the observed order also agreed with the findings of other researchers (Inhelder et al., 1974; Pufall et al., 1973; Wallach et al., 1967).

It is interesting to note that the order of the eight concepts could have been changed very easily if a small number of subjects had responded differently. In looking at Table 3, it can be seen that there is only a spread of 11 correct responses among the first five tasks. They could very easily have come out in a different ordering, but not necessarily in the predicted order.

Apparently, one must be careful when interpreting the results of a Guttman scalogram analysis. This type of analysis is relatively sensitive to the feasibility of the chosen tasks as a scale in themselves. However, our data for the present study indicate that this analysis is insensitive to individual differences and leaves these differences unaccounted for. Use of the Guttman scale technique has often served to reinforce findings of literature reviews. That is, the researcher predicts a sequence of tasks and uses the scalogram analysis merely to confirm or reject the hypothesized ordering. No information is available to the researcher other than the "goodness," or unidimensionality, of the tasks as a scale. It is therefore usually necessary to use follow-up analyses in hopes of gaining more information about the tasks.

Each concept's ability to predict number conservation was also analyzed by a stepwise linear multiple regression procedure. The results of the analysis showed that reversibility, spatial arrangements, one-one correspondence, object counting, and addition/subtraction changes in a row were significant predictors of number conservation and accounted for 15 percent of the variance with the remaining variables contributing none of the variance.

How might the 85 percent of the variance missing be accounted for? It appears that there are other factors behind number conservation acquisition which are not taken into account when speaking of logical prerequisites. Factors such as memory, attention span, linguistic abilities, and also general reasoning or problem-solving skills might play an important part in the acquisition of number conservation. Results of the object counting task in the present study point to the part memory might play in conservation acquisition. No study has ever trained subjects in these skills together with number skills to determine their impact on conservation acquisition.

The number conservation task itself might also be responsible for the variance differences. It is assumed that the task tests for children's logical capacities in dealing with number invariance. It is possible that this task, while indeed testing number judgments, also taps other skills as well. In fact, Gelman (1972a) showed that this task possibly evaluated other skills such as attention control, correct semantics, and estimation skills. These results point to number conservation as a more sophisticated type of cognitive functioning where numerous skills are integrated into a holistic cognitive system. As far as the test-retest reliability of the task, the phi coefficient was quite high (r = .753). Therefore, unreliability of the task can be ruled out as a factor.

Another question which should be considered is of each number concept's necessity for conservation acquisition. The previous research naturally assumed that if training in a number concept induced number conservation, the concept was a necessary one for its attainment. There are problems in this reasoning. It is not at all possible to determine what skills each individual subject possessed before this training and how these skills affected the training and conservation acquisition. Are we to assume that the concept training itself was responsible for the acquisition of number conservation or did this training just add to the skill already possessed by the child creating adequate cognitive structures for conservation? It has been suggested that most concept learning experiments merely instruct subjects in the generalization of mental structures which are spontaneously acquired instead of acquiring the concepts themselves (Brainerd, 1978). The children learn to use spontaneously the trained concepts without true understanding of them as is assumed by the researchers when conservation is induced.

It cannot be assumed that the eight number concepts assessed in the present study are either necessary <u>or</u> sufficient for number conservation. It is obvious that several children conserved number without passing all eight tasks and that others who passed all eight tasks did not conserve. It is probable that those children who conserved but failed several logical prerequisites possessed other cognitive skills which were adequate for number conservation but were not assessed. The same reasoning may be applied to those who passed all tasks but who failed the conservation task. They perhaps lacked other necessary skills not tested by the present study.

In the present study, age was not a significant factor in the response patterns of the subjects. The age range used was not wide enough to show different patterns of responses or in the total correct scores in the different age groups. It could be expected that there would be a great deal more variability between age groups had the age range been extended in both directions (older and younger).

It was perhaps a failure on the part of the present study in relying on past findings as to the logical prerequisites of number conservation. Other aspects should have been investigated and included in the series of tasks used. Future research should investigate this area more fully. There are undoubtably other skills necessary for number conservation. In fact, by looking solely at number concepts, researchers have been ignoring vital information. Mpiangu and Gentile (1975) feel that number conservation and mathematical concepts develop simultaneously but independently of one another. This fact lends to the hypothesis that other factors play an important role in number conservation acquisition.

Some attention should be paid to memory and attentional factors and the part they play in cognitive skill acquisition. Researchers should investigate this area more closely, focusing less on the individual number concepts as necessary prerequisites. It is obvious that we are dealing with a concept which is more complex than that of a series of number concepts to be acquired before number conservation can be attained.

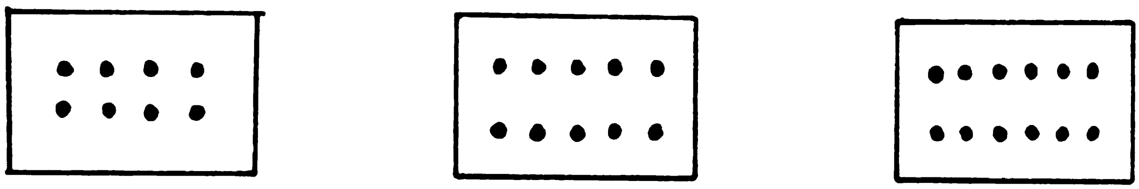
APPENDIX A

STIMULUS CARDS FOR TASKS 4, 5, AND 8

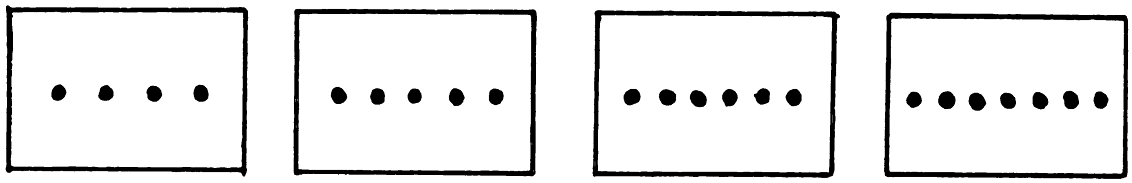
APPENDIX A

STIMULUS CARDS FOR TASKS FOR 4,5, AND 8

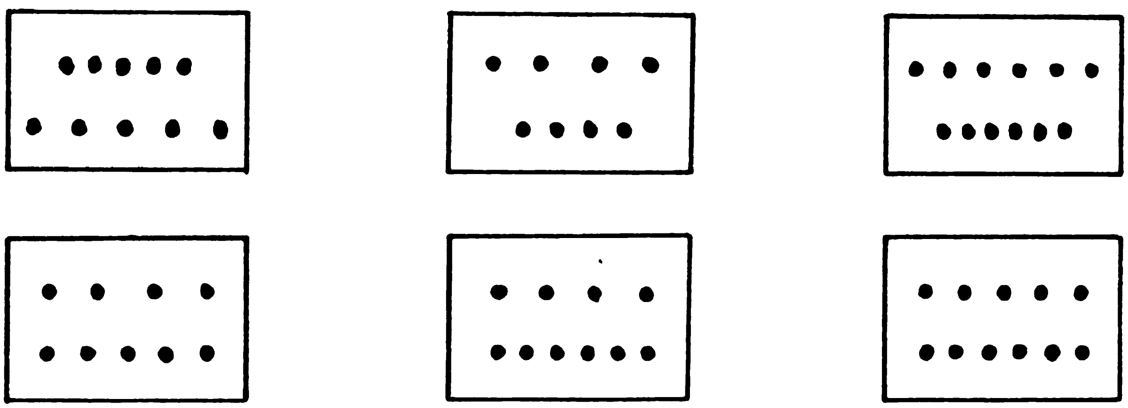
Stimulus cards for Task 4.



Stimulus cards for Task 5.



Stimulus cards for Task 8.



APPENDIX B

DATA SHEET

APPENDIX B

DATA SHEET

Name of child ____________________

Date of birth ____________________

Score 1 for pass, 0 for fail.

Task 1 Conservation of number assessment.

(1) Trial 1____ (2) Trial 1____ (1) Score ____

Trial 2____ Trial 2____ (2) Score ____

Trial 3____ Trial 3____

Task 2 Rote counting.

Trial 1____

Trial 2____

Trial 3____ Score ____

Task 3 Object counting.

Trial 1____ No. used____

Trial 2____ No. used____

Trial 3____ No. used____ Score ____

Task 4 Number judgments of two equal groups.

Trial 1____ 4 5 6

Trial 2____ 4 5 6

Trial 3____ 4 5 6 Score ____

Task 5 Number judgments of two unequal groups.

Trial 1____ 4 5 6 7

Trial 2____ 4 5 6 7

Trial 3____ 4 5 6 7 Score ____

Task 6 One-one correspondence.

Trial 1____ 5 6 7

Trial 2____ 5 6 7

Trial 3____ 5 6 7 Score ____

Task 7 Addition/subtraction changes in a row.

Trial 1____ A S

Trial 2____ A S

Trial 3____ A S

Trial 4____ A S Score ____

Task 8 Number vs. length of rows (spatial arrangement).

Trial 1____ 4 5 6 = ≠

Trial 2____ 4 5 6 = ≠

Trial 3____ 4 5 6 = ≠ Score ____

Task 9 Reversibility.

Trial 1____ 6 7 8

Trial 2____ 6 7 8

Trial 3____ 6 7 8 Score ____

REFERENCE NOTES

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