THESIS

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Martin John Vanderploeg

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THE LATERAL STABILITY OF THE MICHIGAN DOUBLE TANKER

By

Martin John Vanderploeg

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ABSTRACT

THE LATERAL STABILITY OF THE MICHIGAN DOUBLE TANKER

By

Martin John Vanderploeg

This thesis uses linear mathematics to analyze the Michigan tanker. Several mathematical models are used, from a two degree of freedom model of the dolly-pup trailer combination to higher degree of freedom models of entire vehicles.

The results of the modeling and analysis show that the Michigan double tanker cannot be improved by simple design changes involving easily changed items such as tongue length, axle spread or tire stiffness. More basic changes are therefore analyzed, including the so-called Canadian double, the four point hitch, and a constraint linkage between the semitrailer and the pup trailer. The Canadian double is shown to be the best idea, offering significantly improved dynamic properties over the Michigan double tanker.

ACKNOWLEDGEMENTS

I would like to thank my major professor and good friend, Dr. James E. Bernard, for his assistance and constant encouragement during my graduate career.

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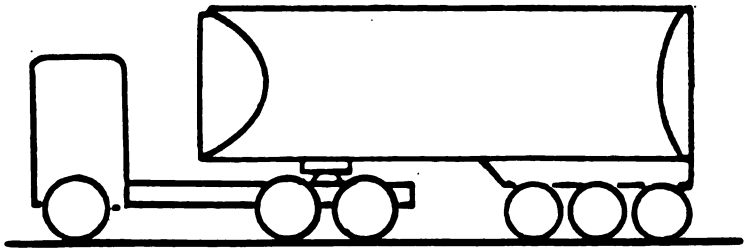
1.0 INTRODUCTION

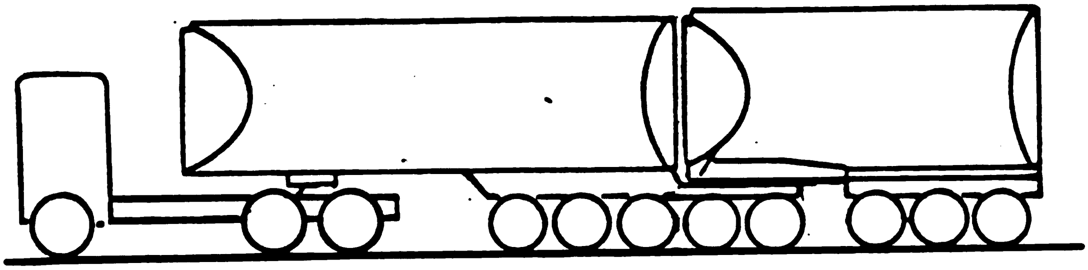
The transport of commercial cargo via trucks takes various forms in the United States and elsewhere depending on local rules. For example, it is clearly economically advantageous to use vehicles which are as long as local length limits will allow, thus carrying as much cargo as possible with each commercial vehicle.

Local length limits are usually lenient enough to lead to the desire to articulate the vehicle at least once and often more than once so that good low speed maneuvering is provided. Thus in various localities it is common to see simply articulated tractor semitrailers; doubly articulated Canadian doubles, or so-called B-trains; triply articulated vehicles such as the Michigan double tanker. A sketch of each of these vehicles is shown in Figure 1, which is reproduced from Reference 2.

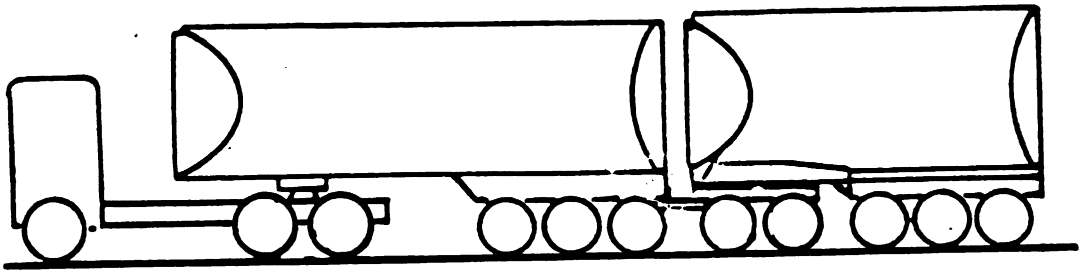
These vehicles have been the subject of analysis by various researchers over the years, starting as early as 1951 when Williams considered "Snaking of Commercial Vehicles" [13]. Williams was followed by several others, as will be indicated in the literature survey in this thesis. This survey clearly indicates an escalation in the complexity of the mathematical models as digital computers become more powerful and more convenient to use, culminating in 1979 with [5], in which a 128 degree of freedom model is presented to simulate the directional response of triples.



1. Tractor Semitrailer



2. Canadian Double



3. Michigan Double Bottom Tanker

Figure 1. Diagram of Various Vehicle Configurations

The main subject of this thesis is the so-called Michigan double tanker shown in Figure 1. Since this vehicle was often used to haul volatile fuel, and since it received significant notoriety due to several spectacular accidents during the middle and late 1970's, it is an appropriate subject for thoughtful analysis. This thesis will show that significant conclusions concerning the stability of the Michigan double can be reached using straightforward mathematics and simple models.

The thesis is divided into several sections. Section 2 presents a survey of the literature concerning the directional performance of multiply articulated vehicles. Section 3 presents the standard form of the linear model for the Michigan double and shows that the model can conveniently be broken into two simple models. Section 4 shows that these simple models yield the information that the inherent problems of the Michigan double are not amenable to simple solution via changes in pup trailer parameters. Section 5 introduces the design changes now commonly used to improve the performance of the Michigan double and Section 6 considers the penalty for these changes, degraded low speed offtracking performance.

Conclusions are presented in Section 7.

2.0 LITERATURE SURVEY

Analysis of the dynamic stability of multiply articulated vehicles first appeared in the literature in 1951. These first efforts were concerned with linear analysis, a procedure based on the assumption of constant speed and small angles. Later, as available computing capacity increased, researchers used non-linear models as well.

In 1951, Williams [13] used a linear model to determine stability criteria for a tractor-semitrailer. His results delt primarily with trailer mass center location. These results were extended to a tractor-dolly-pup trailer. His results indicated that snaking was inherent in the dolly-pup trailer configuration.

The first well known linear analysis was done by Jindra [8] in 1965. He obtained numerical solutions for the 6th order characteristic equation of a tractor-dolly-pup trailer. Extension of this to a tractor-semitrailer-pup trailer configuration, with an 8th order characteristic equation, was considered too large a problem for existing digital capabilities.

Hazemoto [6] numerically solved the 8th order equation in 1973, and showed that at normal highway operating speeds, the tractor-semitrailer-pup trailer configuration has a natural mode at around 0.8 hz with less than 20% critical damping. Frequency response calculations also showed that, at frequencies near 0.8 hz, the peak yaw rate of the pup trailer is significantly higher than the peak yaw rate of the tractor. Although many parameter variations were studied, no practical design change was suggested to significantly reduce the gain of the pup vehicle.

Hales [7], using the same techniques in 1975, also found a lightly damped mode at normal highway speeds. Again, no practical solution was offered.

Eshleman [2] developed AVDS, the first nonlinear digital simulation of multiply articulated vehicles, in 1973. AVDS was noteworthy because of its inverse formulation, i.e., the trajectory of the tractor is input and the steer angle and braking, as well as tractor sideslip and all the articulation angles, are output. The model was used in an attempt to judge multiply articulated vehicle stability. Eshleman concluded that "the double articulated vehicle is almost as stable as the single" [2], [3]. This conclusion, which is now generally regarded as incorrect, was based on steady turn results of a vehicle with a low center of gravity.

In 1974, Standberg and Nordström [11] presented an inverse model similar to the model developed by Eshleman. In 1973, a roll degree of freedom was added for each vehicle [12]. The dynamic behavior of the model was studied through simulation of a double lane change maneuver. Vehicle stability was determined by comparing mass center accelerations up to the rollover limit for each vehicle. They concluded that, in a lane change, the rearmost vehicle experienced the highest accelerations. In addition, they showed that an increased number of articulations among comparable vehicle combinations led to higher accelerations.

In 1978, Mallikarjunarao and Fancher [9] developed and used a linear model to study one particular vehicle, the Michigan double tanker. They computed eigenvalues, finding a lightly damped mode near 0.75 hz. No practical parameter changes were found that significantly added to the damping of this mode. They also noted that certain eigenvalues of the doubles combination closely matched eigenvalues of the tractor semitrailer without the pup trailer. This supported the idea that the dynamic coupling between the tractor semitrailer and the dolly pup trailer was weak.

Mallikarjunarao and Fancher also studied a transient lane change, paying close attention to the lateral accelerations of the mass centers of each vehicle. A large acceleration gain between the tractor and the pup trailer was found at frequencies near 0.75 hertz. Their suggested solution to this problem reconfigured the coupling between the semitrailer and the pup, effectively changing the vehicle to a tractor-semitrailer-semitrailer. This configuration offers better dynamic characteristics than the traditional tractor-semitrailer-dolly-pup trailer, with the penalty of degraded low speed maneuvering and tire wear.

In 1979, Gillespie et. al. [5] developed a multidegree of freedom simulation for vehicle configurations with a tractor, semitrailer and up to two dollies and full trailers. The model includes tandem axles for each vehicle and antiskid brakes.

The trend of the literature to date indicates escalation in the complexity of the models with time, an apparent indication of increasing availability of computational power. In the next section, an attempt will be made to reverse this trend by considering the information available from very simple models.

3.0 MODELS FOR MULTIPLY ARTICULATED VEHICLES

The previous section showed that research in the area of multiply articulated vehicles has been making use of increasingly complex models. As an example, consider Figure 2, a yaw plane model of the Michigan double tanker reproduced from Reference 9.

The linearized differential equations which describe the motion of this vehicle are based on four assumptions, namely, 1) lateral forces are a linear function of tire slip angles, 2) articulation angles are small, 3) forward speed is a constant and 4) all motion takes place in the yaw plane.

This section will show that additional information may be gained using much simpler models. For example, consider Figure 3, which presents a model of a tractor-semitrailer and a dolly pup trailer (a dolly pup trailer is often referred to as a full trailer).

The rationale for the use of simplified models in place of the more complete model of Figure 2 rests on the following observations: 1) Since the pup trailer kingpin is approximately vertically over the dolly's suspension center, only small lateral forces can be applied to the semitrailer by the dolly, and 2) since there is only lateral force coupling between pup trailer and semitrailer (linear analysis assumes constant speed), the pup trailer has little influence on the trajectory of the semitrailer. Thus, tractor-semitrailer calculations, which involve

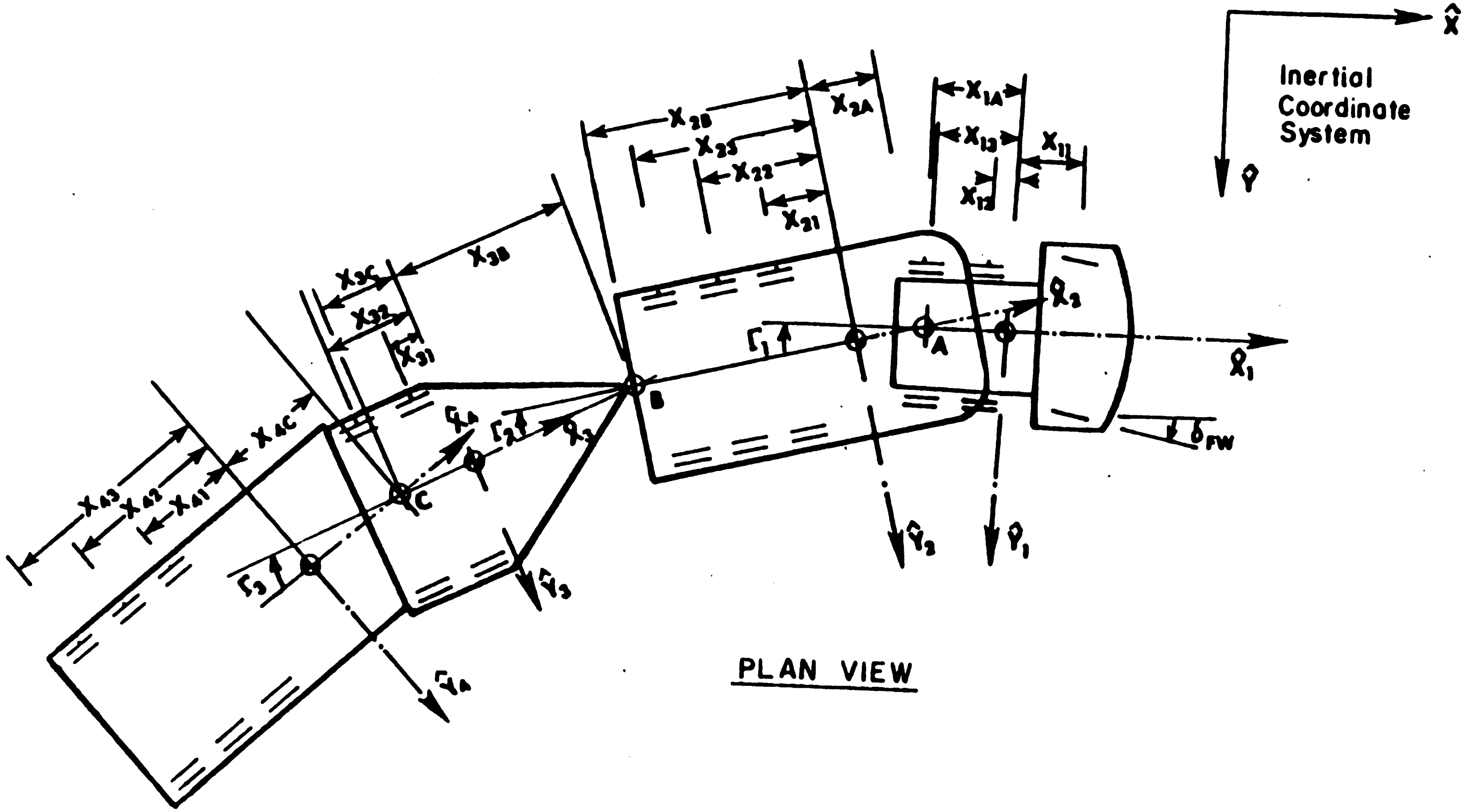
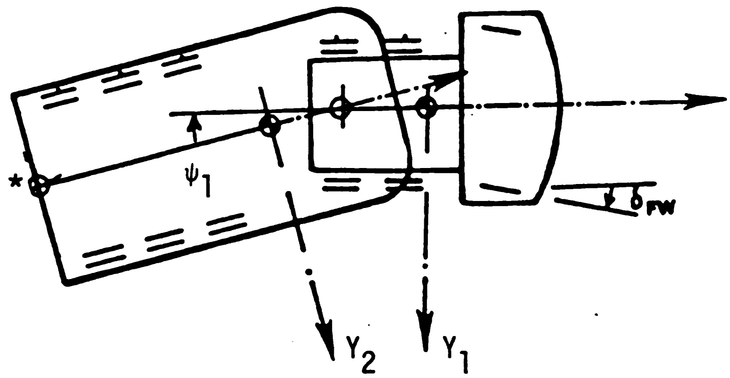
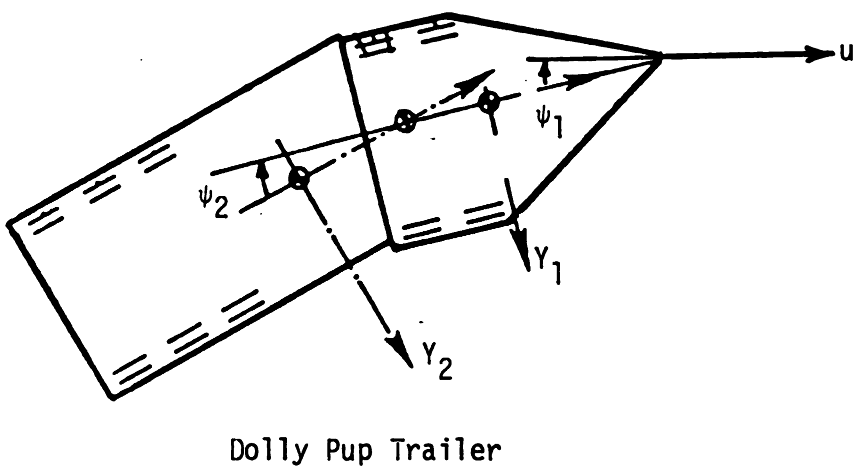


Figure 2. Yaw Plane Model of Michigan Double Bottom Tanker



Tractor Semitrailer



Dolly Pup Trailer
(Full Trailer)

Figure 3. Diagram of Tractor-Semitrailer and Dolly Pup Trailer Models

only three degrees of freedom, can be made without regard to the pup trailer, and the computed time-varying position of the pintle hook, point * in Figure 3, can be used as input to the pup trailer model. And the separate eigenvalues of each model should match the eigenvalues of the complete system. The advantage of this procedure is the relative simplicity of the analysis which will lead to clear and concise conclusions concerning pup trailer design.

The pup trailer model has two degrees of freedom, ψ_1 and ψ_2, which are the dolly and pup trailer articulation angles. Following methodology developed in [1], the dolly articulation angle is measured from the velocity vector of the hitch point at the rear of the semitrailer. It is assumed that this point has a constant speed U with variable direction. The rotation rate of the velocity vector is given by r_u. The pup trailer articulation angle is measured with respect to the center line of the dolly.

There was not an opportunity in the course of this thesis to design a test program for the purpose of testing the validity of the pup trailer model. Thus, results computed using the simple models were compared to the calculations and test results presented in Reference 3, which considered the full five degree of freedom model.

Parameters which correspond to the Figures and the nomenclature in Table 1 are listed in Table 2. This vehicle configuration, which will be referred to as the baseline vehicle, models the Michigan double tanker [9].

TABLE 1

LIST OF NOMENCLATURE USED IN DERIVATION OF THE EQUATIONS OF MOTION

A_{yi}	lateral acceleration of the ith vehicle.
C_{ij}	combined cornering stiffness at the jth axle of the ith vehicle.
F_{ij}	combined lateral tire force of the jth axle on the ith vehicle.
F_{1x}	longitudinal force at the hitch.
F_{1y}	lateral force at the hitch.
F_{2x}	longitudinal force at dolly kingpin.
F_{2y}	lateral force at dolly kingpin.
I_i	yaw inertia of the ith vehicle.
ℓ_1	distance from the dolly c.g. to the hitch.
ℓ_2	distance from the dolly c.g. to the kingpin.
ℓ_3	distance from the puptrailer c.g. to the kingpin.
M_i	mass of the ith vehicle.
M_{ij}	combined aligning moment of the jth axle on the ith vehicle.
P_{ij}	combined aligning moment coefficient of the jth axle of the ith vehicle.
r_i	yaw rate of the ith vehicle.
r_u	rotation rate of pintle hook point velocity vector.
u	magnitude of the pintle hook velocity vector.
u_i	forward velocity of the ith vehicle c.g.
v_i	lateral velocity of the ith vehicle, c.g.
x_{ij}	distance of axle ij from the mass center of the ith vehicle.
α_{ij}	tire slip angle at the jth axle of the ith vehicle.
ψ_i	articulation angle of the ith vehicle.

TABLE 2

LIST OF PARAMETERS FOR THE BASELINE DOLLY PUP TRAILER

M_1 = 4525.0 lb.	x_{12} = 21.0 in.
M_2 = 59975.0 lb.	x_{21} = 2.0 in.
I_1 = 21627. lb.*in.*sec.*sec	x_{22} = 44.0 in.
I_2 = 782079. lb.*in.*sec.*sec	x_{23} = 86.0 in.
C_{ij} = 1673 lb./deg.	ℓ_1 = 70.0 in.
P_{ij} = 248 ft. lb./deg.	ℓ_2 = 0.0 in.
x_{11} = 21.0 in.	ℓ_3 = 81.0 in.

Verification that the eigenvalues of the simpler models are close to the eigenvalues of the tractor-semitrailer-dolly-pup trailer [9] is presented in Figure 4. The figure indicates that the eigenvalues of the full trailer model closely match two of the eigenvalues of the more complete model. Figure 4 also indicates that two modes of the 8th order system can be predicted very accurately by a tractor-semitrailer model, thus illustrating the weak dynamic coupling between the tractor-semitrailer and the full trailer.

Transient response of the pup trailer model was compared to the lane change test and calculations presented in Reference 9. In this case, the input to the pup trailer simulation was the velocity vector of the pintle hook calculated using the tractor-semitrailer model. Figure 5 shows that the calculated acceleration of the pup mass center matches very closely the results from Reference 9.

The results of this section indicate that uncoupling the semitrailer from the full trailer for purposes of eigenvalue or transient analysis is a useful simplification. This point of view will be used in the next section wherein the pup trailer model will be utilized to illustrate dynamical phenomena which are peculiar to this configuration and to study potential pup trailer design changes.

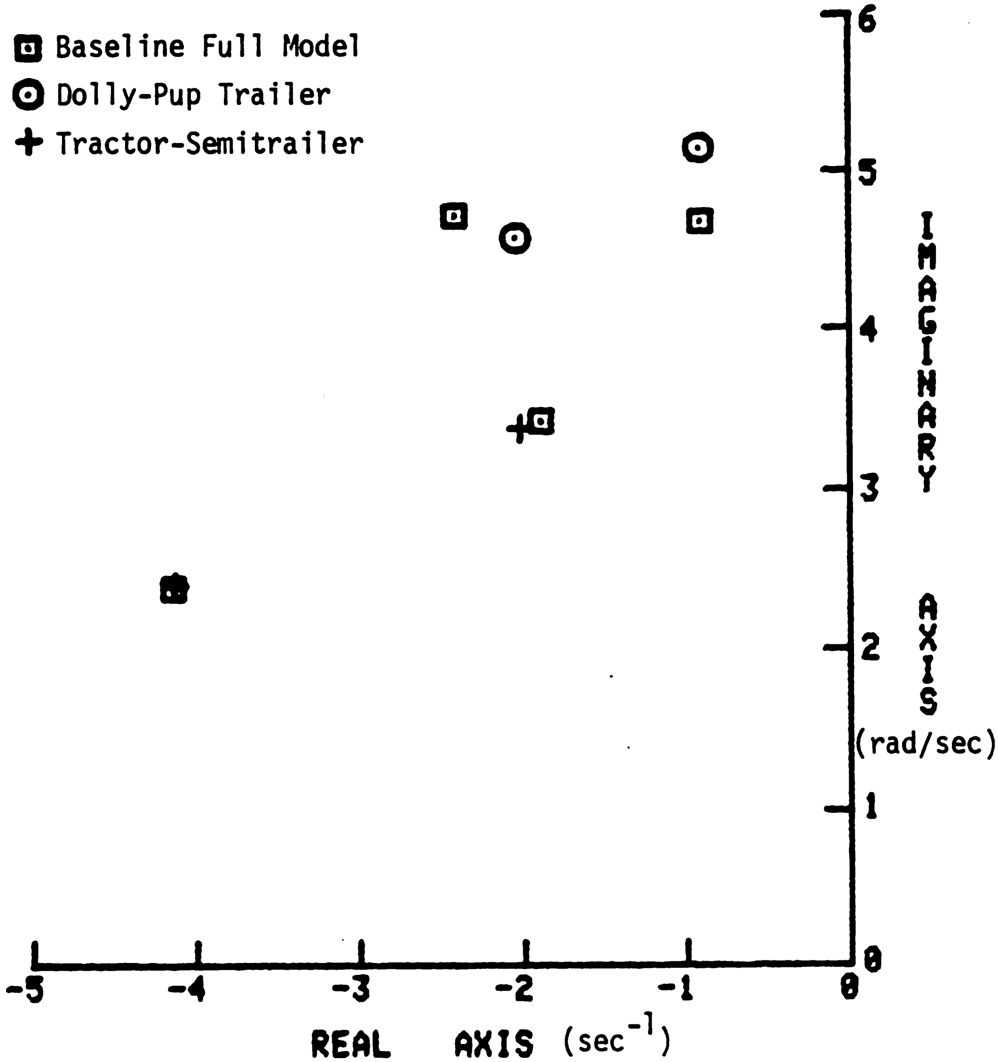


Figure 4. Eigenvalues of the Full Model and the Simplified Models

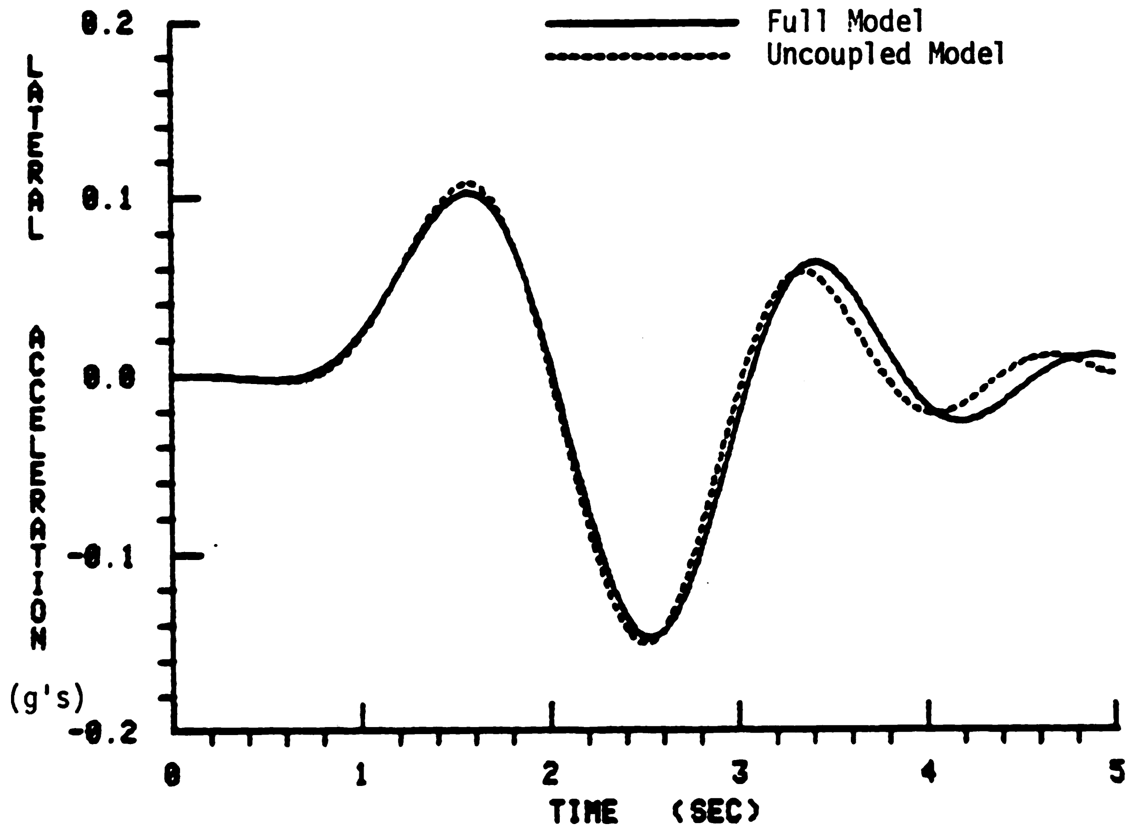


Figure 5. Lateral Acceleration of the Pup Trailer Mass Center in a Lane Change Maneuver

4.0 INFORMATION FROM THE PUP TRAILER MODEL

This section will be devoted to the study of the baseline full trailer model to determine feasible design improvements. This will be done via eigenvalue analysis and via examination of vehicle performance in lane change maneuvers.

4.1 Eigenvalue Analysis

Previous studies of articulated vehicles have used eigenvalues to study vehicle stability [9], [11]. Consider the eigenvalue $\lambda = a + ib$. The value of a is, for most vehicles, always negative. Researchers in vehicle dynamics commonly consider the damping ratio, $\cos\{\tan^{-1}(\frac{-a}{b})\}$, as an indicator of the amount of time for a disturbance to damp out. The worst case, positive a, occurs only for oversteer vehicles driven above their critical speed. The b term has often been considered to be a benign indicator of characteristic frequencies. However, since emergency lane change maneuvers of commercial vehicles typically entail frequencies which closely match the imaginary parts of one of the eigenvalues of tractor-semitrailer-full trailers, it's important to keep in mind both the imaginary and real parts of the eigenvalue.

The eigenvalues of the full trailer model change with design parameters. Since the frequency of an emergency lane change is near 0.5 hz [9] and the natural frequency of the baseline dolly-pup trailer is near 0.75 hz, it is desirable to change the design to increase the frequency of the full trailer roots, moving them further from the input frequencies. In addition, any resulting increase in the damping ratio is desirable because it decreases the peak response.

The full trailer model was used to compute eigenvalues corresponding to several feasible changes from the baseline configuration, including changes in dolly tongue length, the cornering stiffness of the tires, and the wheelbase. Special attention was given to the affect of these changes on the lightly damped mode.

Figure 6 compares the eigenvalues for several different tongue lengths. Lengthening the tongue from the baseline configuration leads to degraded performance as indicated by lower frequencies and lower damping. Shortening of the tongue led to favorable results, but this has limited applicability due to interference of the leading pup trailer axle with the trailing semitrailer axle in sharp turns.

Figures 7 through 9 present the eigenvalues for several different tire stiffness configurations. Small increases in damping ratio and frequency occur for some configurations. Simultaneous stiffening of dolly and pup tires gave the best results of the combinations modeled. A 20% increase in the stiffness of all tires resulted in a 10% increase in damping ratio and frequency of the lightly damped mode. Decreasing the tire stiffnesses reduces damping ratio and frequency in all cases modeled.

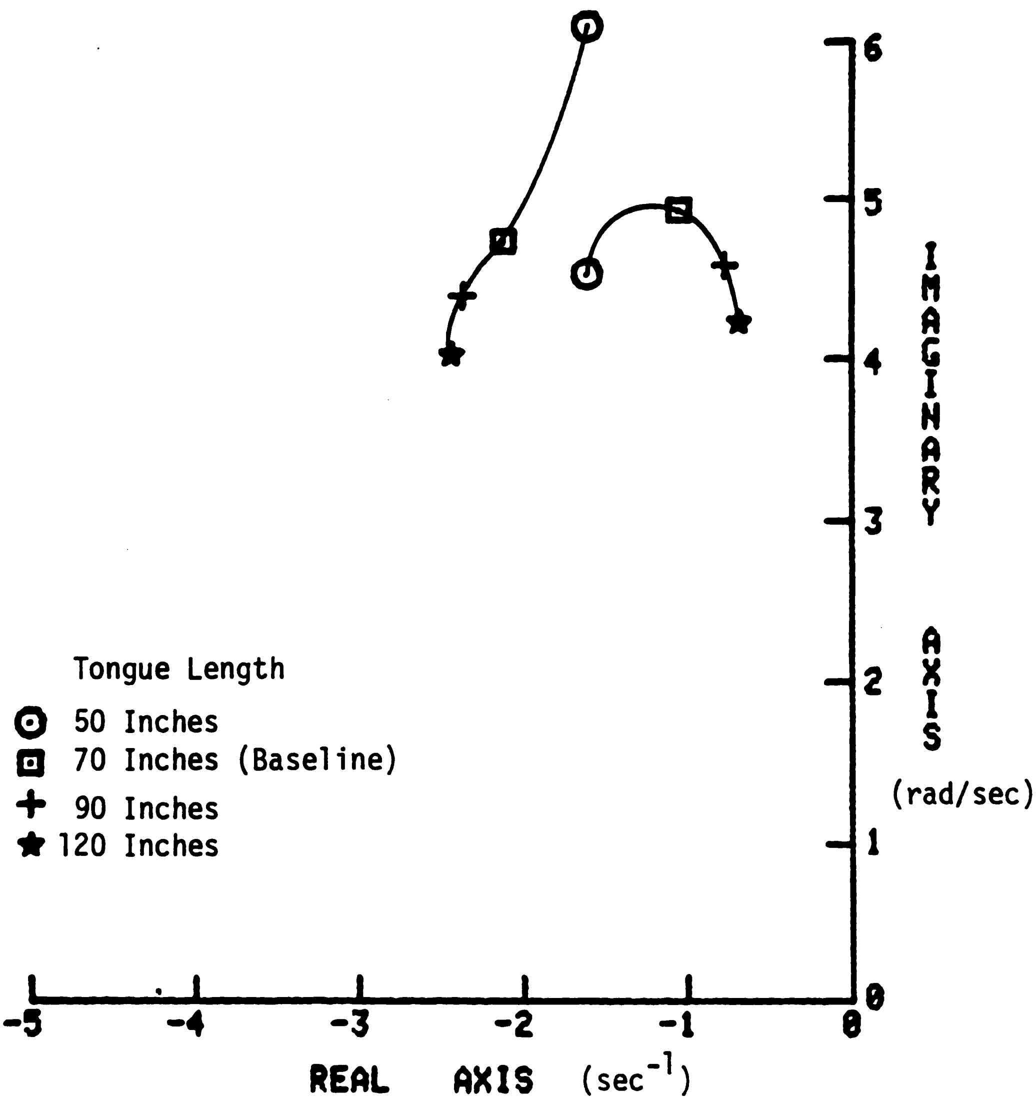


Figure 6. Effect of Tongue Length on Eigenvalues

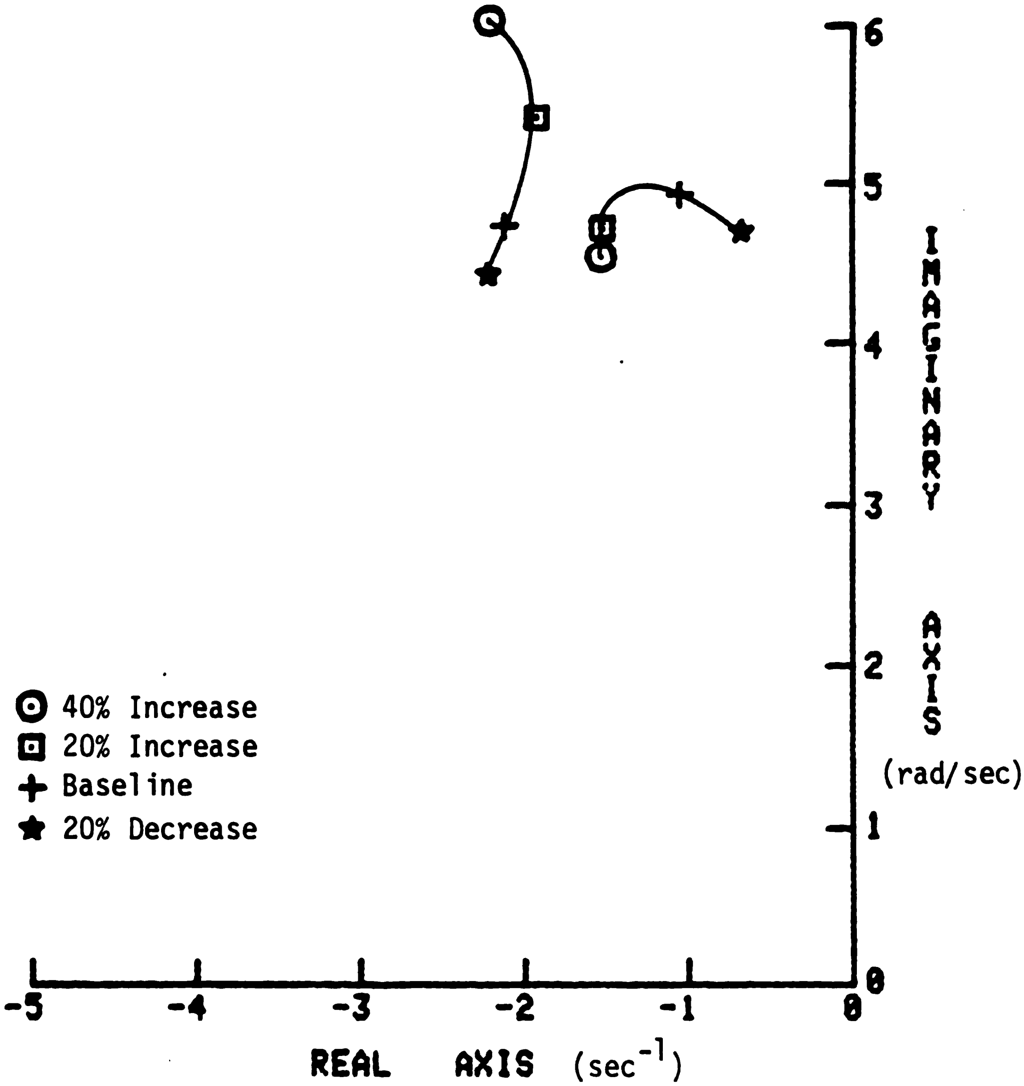


Figure 7. Effect of Stiffening Dolly Tires on Eigenvalues

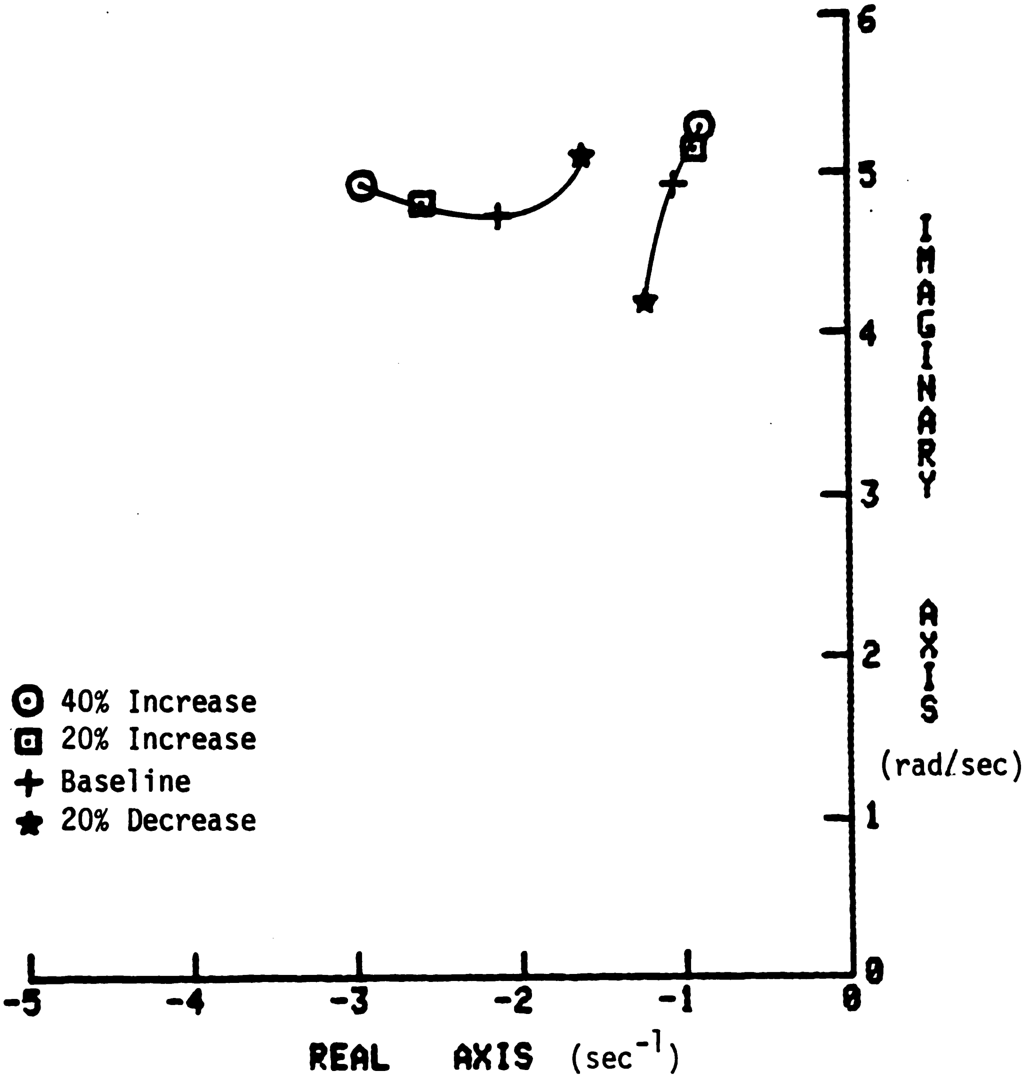


Figure 8. Effect of Stiffening Pup Trailer Tires on Eigenvalues

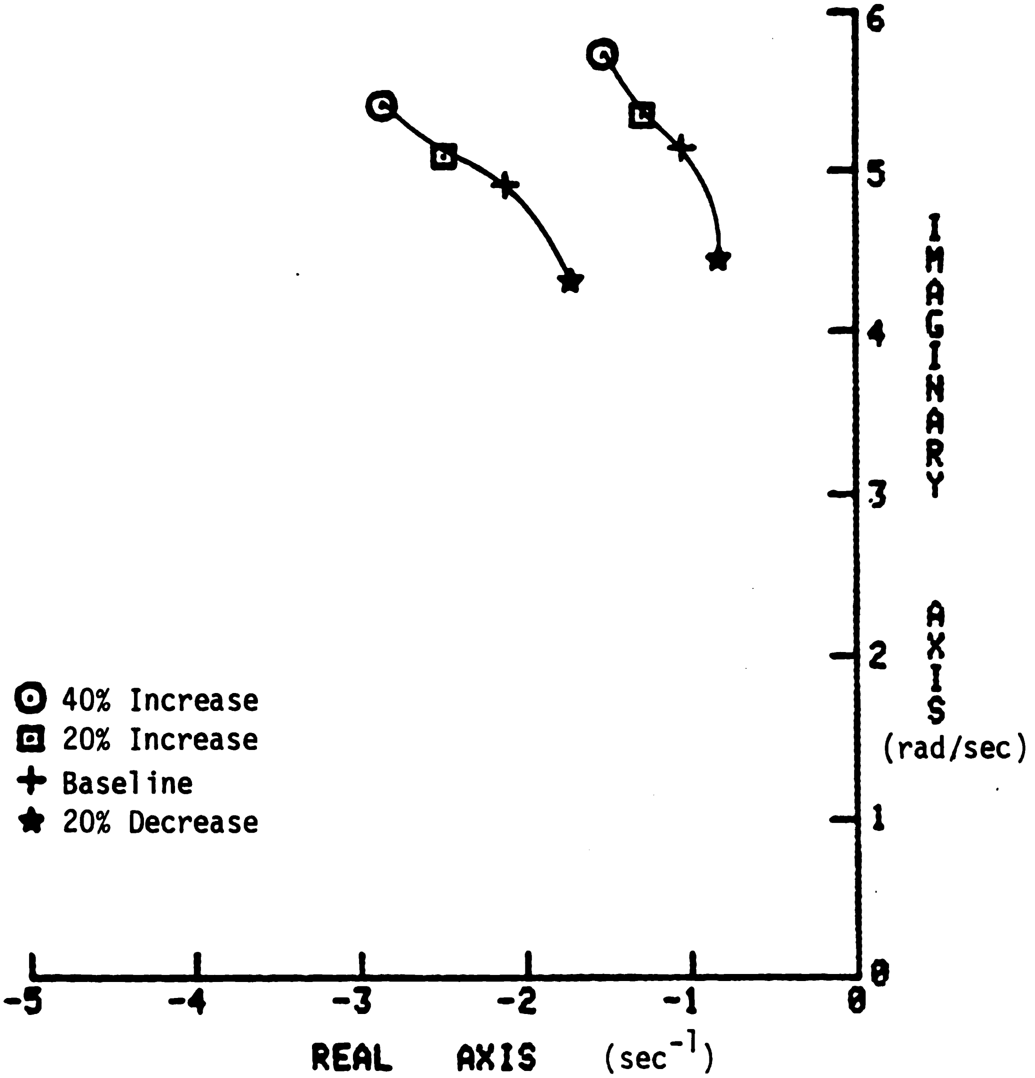


Figure 9. Effect of Stiffening Dolly and Pup Trailer Tires on Eigenvalues

Figure 10 shows the eigenvalues for different pup trailer wheel bases. Again, only small improvements are apparent.

Figure 11 presents the eigenvalues for different pup trailer tandem axle spreads. The figure indicates that frequency slightly increases but the damping ratio is reduced with increasing axle spread.

In summary, practical design changes of the full trailer show little potential for substantially improving the eigenvalues of the system.

4.2 Transient Analysis

The complete story of the linear range performance of multiply articulated vehicles depends on both the inherent properties of the vehicle as indicated by the eigenvalues and associated measures and on the time varying input to the vehicle system. Reference 9 indicates that a lane change is a particularly difficult manuever for these vehicles. This section explains how this manuever was used as an aid in understanding the Michigan double tanker and similar combinations.

Simulation of the full trailer alone was used to study the acceleration gain between vehicle components. To obtain input for the full trailer simulation, the tractor-semitrailer was simulated in a lane change. The lane change was run with the baseline configuration at 50 mph. The calculated velocity of the pintle hook was then used as input to the full trailer simulation.

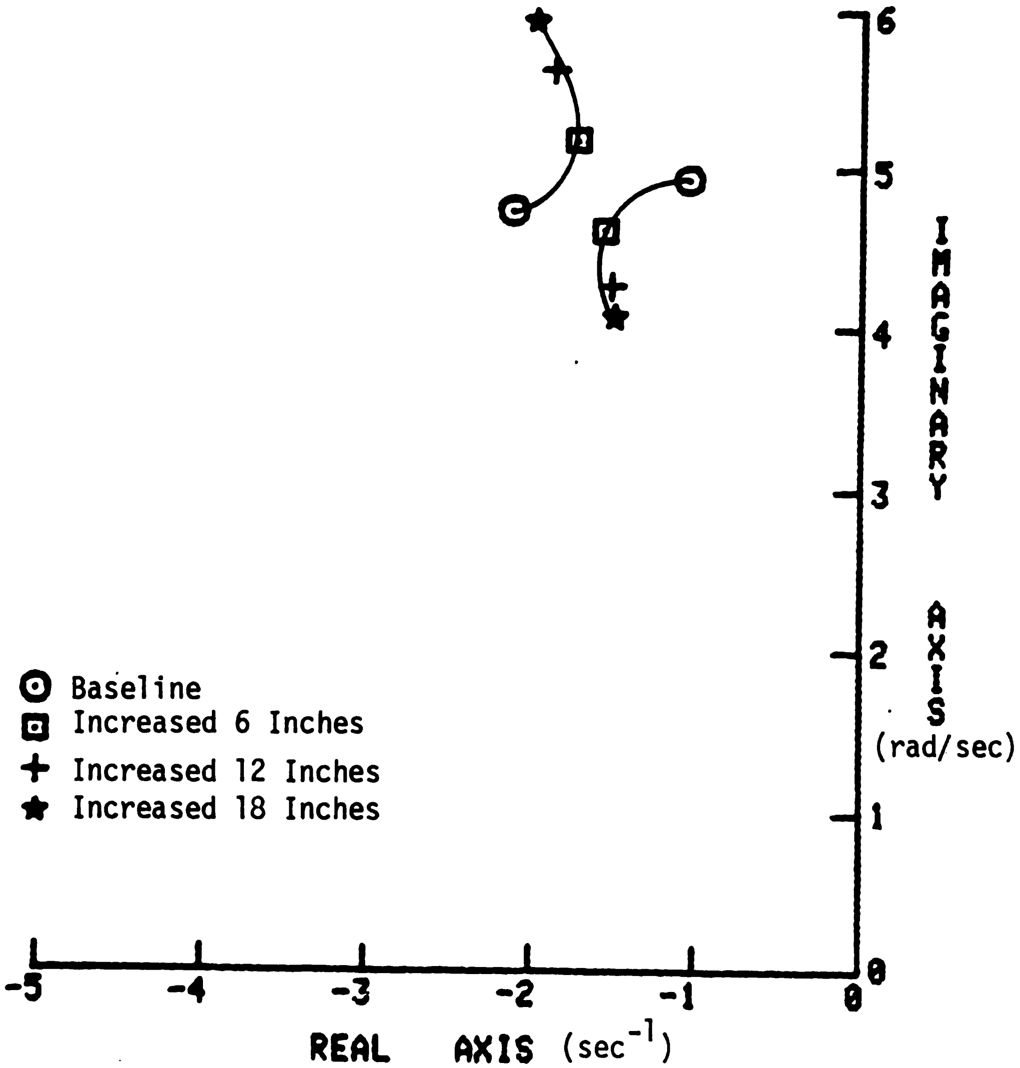


Figure 10. Effect of Changing Pup Trailer Wheelbase on Eigenvalues

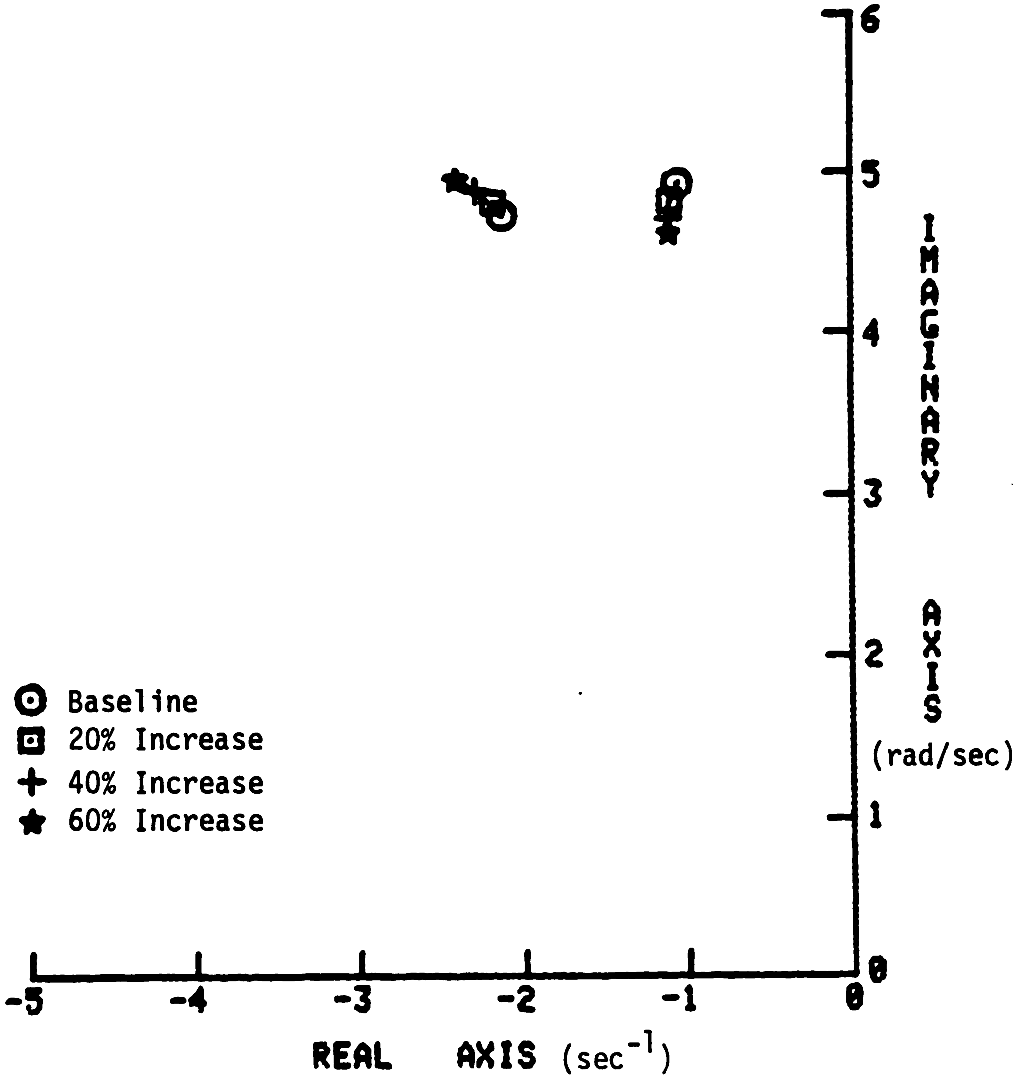


Figure 11. Effect of Changing Pup Trailer Axle Spread on Eigenvalues

Figure 12 presents accelerations of the tractor mass center, the pintle hook, and the pup trailer mass center for the baseline vehicle. It is of interest to note the acceleration gains between each component. The acceleration gain between the pintle hook and the pup trailer is 1.5. The acceleration gain between the tractor and the pintle hook is 1.6. This indicates the acceleration gain between the tractor and the pup trailer is only partially due to full trailer dynamics.

The lane change manuever can be used to illustrate the findings of the eigenvalue analysis. Table 3 presents the acceleration gain for various design changes. In each case, the tractor to pup trailer acceleration gain is only marginally improved.

The cummulative result of the changes is indicated by Figure 13 in which the baseline vehicle in the lane change is compared to a modified vehicle with shortened tongue length, increased cornering stiffness, increased wheelbase, and increased axle spread. The figure indicates no significant improvements in either peak response or settling time. The futility of significant design changes in the traditional pup trailer configuration suggests that more basic changes are in order. The next section will discuss various methods of restoring the dynamic coupling between the full trailer and the semitrailer, a procedure that offers the possibility of significant overall design improvements.

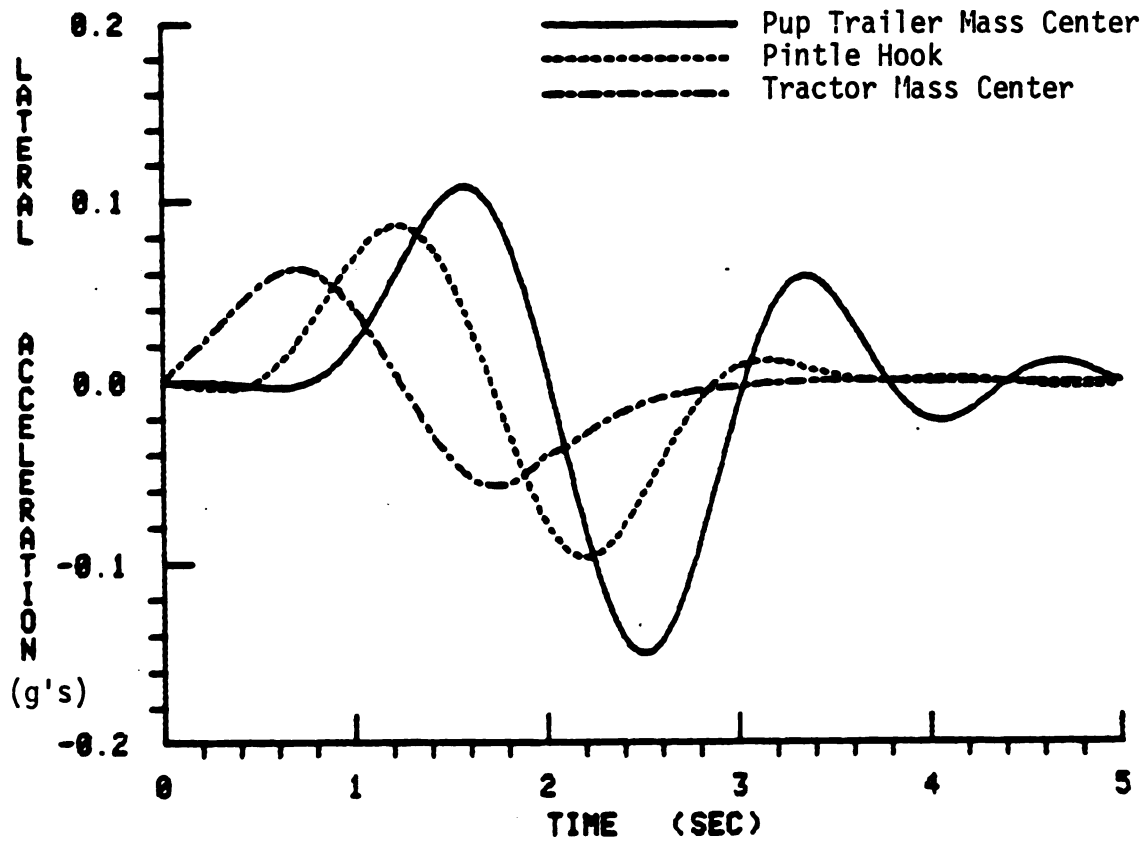


Figure 12. Lateral Accelerations of Vehicle Components of the Simplified Model

TABLE 3

PEAK LATERAL ACCELERATION GAIN FOR SEVERAL PARAMETER VARIATIONS

Parameter Modifications	Gain
Baseline vehicle	2.60
Tongue length shortened 20 inches	2.42
Tire lateral stiffness increased 20%	2.38
Wheelbase lengthened 12 inches	2.46
Rear axle spread increased 40%	2.63

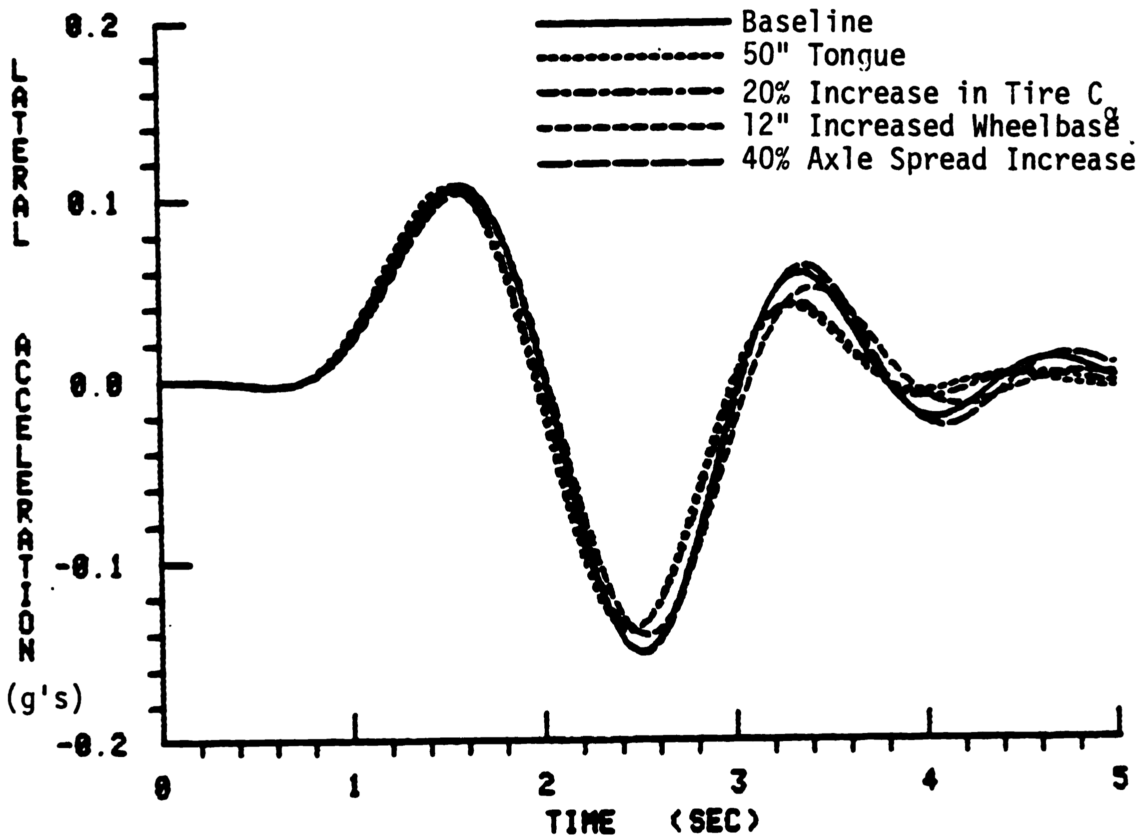


Figure 13. Effect of Parameter Changes on Lateral Acceleration of the Pup Trailer

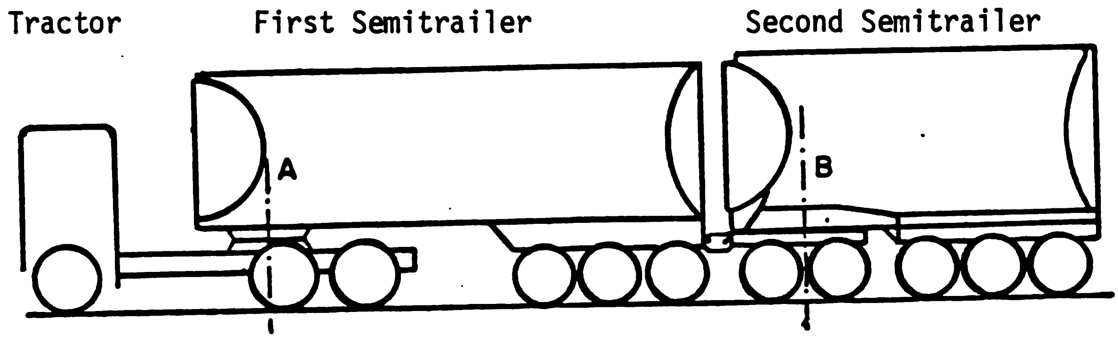
5.0 METHODS OF INTRODUCING DYNAMIC COUPLING

This section presents three different designs which produce dynamic coupling between the tractor-semitrailer and the dolly pup trailer. Each of these designs will be analyzed via eigenvalue and transient analysis to determine the improvement over the traditional setup.

The best known of these designs is the so-called Canadian double. (The Canadian double, as it is run in Canada, does not meet Michigan width limitations. Here we use the time to describe the hitch only.) This design rigidly attaches the dolly to the semitrailer, by the use of a rigid hitch, thus eliminating the yaw degree of freedom of the dolly. Figure 14 presents a schematic diagram of the Canadian double [2].

Another method uses a four point hitch between the dolly and the semitrailer. Figure 15 presents a schematic diagram of the hitch.

A third design uses a constraint linkage between the semitrailer and the pup trailer. The constraint is rigid in translation but the pup trailer is free to yaw at the point it is attached. This method also eliminates a degree of freedom because dolly yaw is dependent on pup trailer yaw. Figure 16 presents a schematic diagram of the linkage.



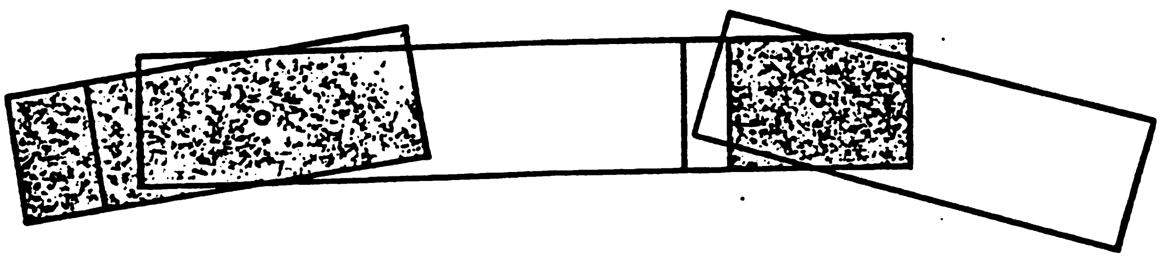


Figure 14. Diagram of the Canadian Double

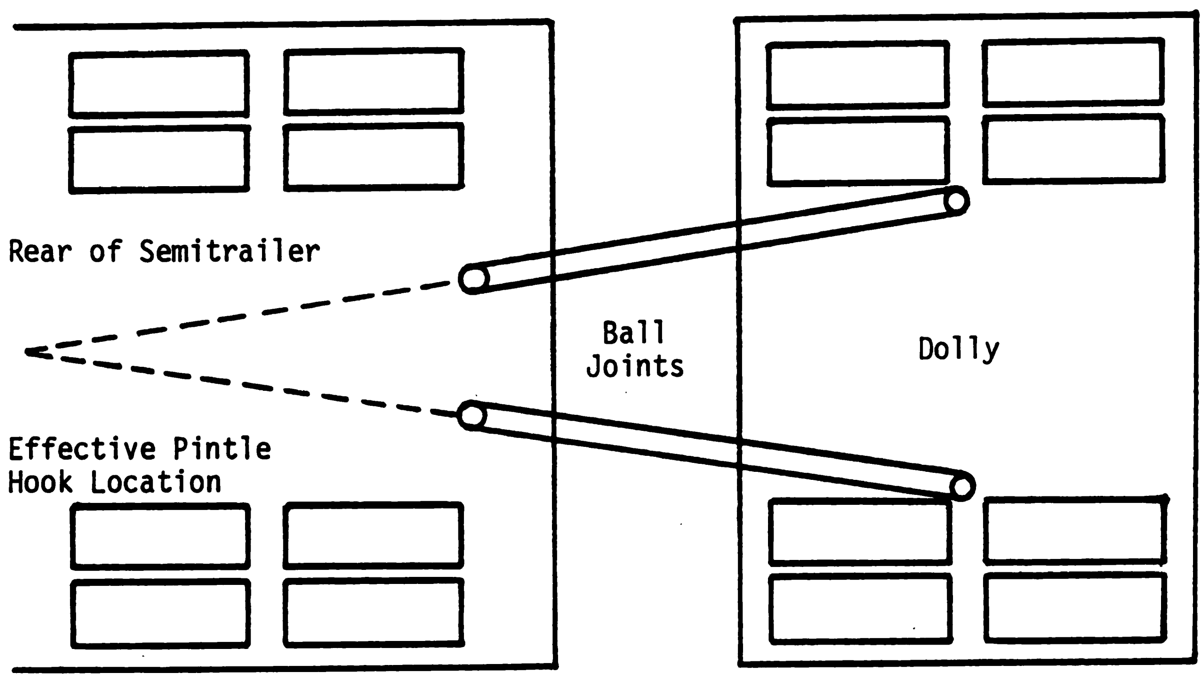


Figure 15. Diagram of the Four Point Hitch. (Effective pintle hook location at semitrailer mass center.)

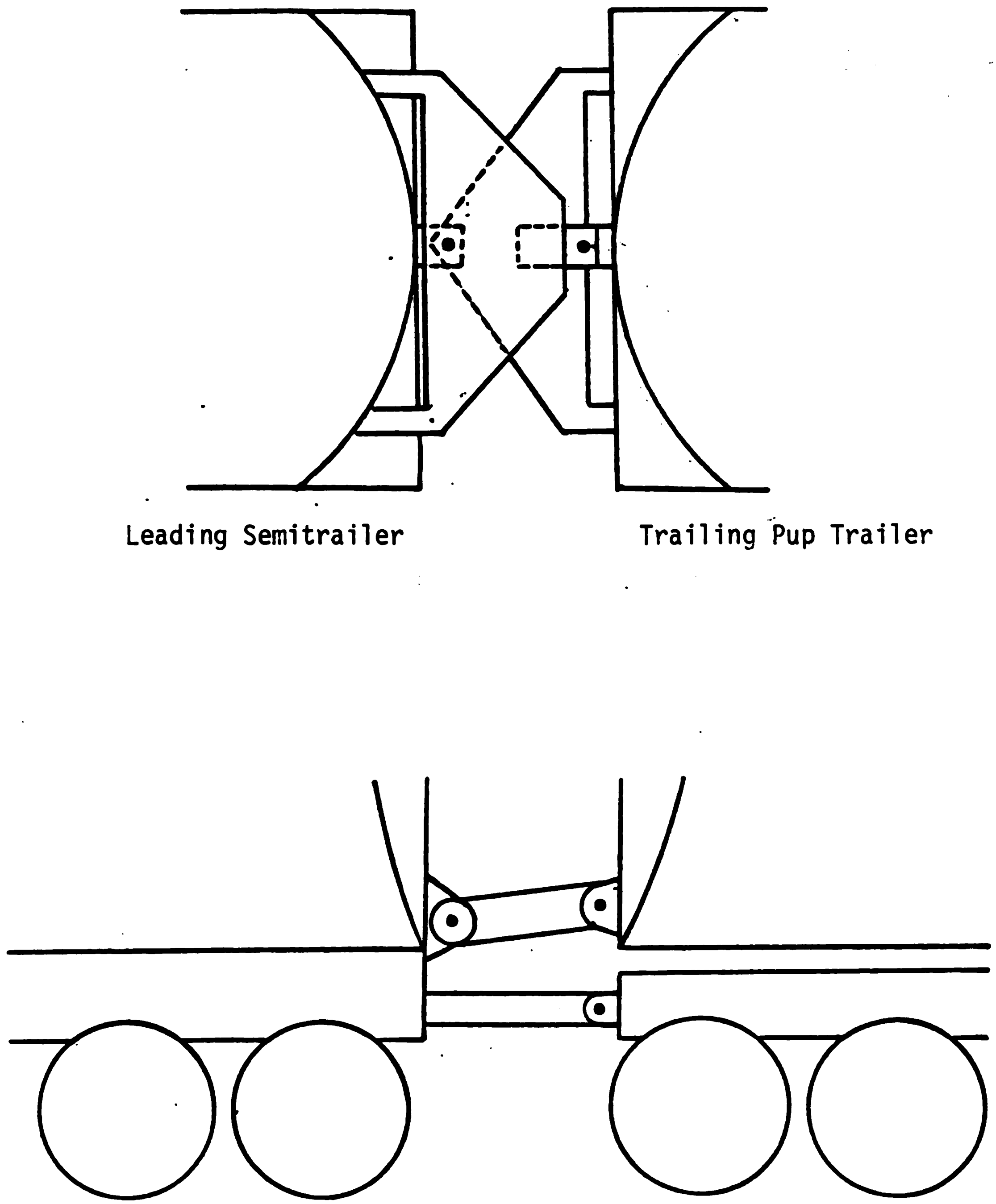


Figure 16. Diagram of the Constraint Linkage

5.1 Eigenvalue Analysis

This section presents an eigenvalue analysis of the three alternative designs. The source of the equations of motion and parameters describing each vehicle are given in Table 4. In each case, the coupled equations of motion were used to compute the eigenvalues because the dynamic coupling inherent in the alternative designs precludes consideration of the vehicle in a component by component fashion.

The eigenvalues for the modified designs are presented in Figure 17, along with eigenvalues of the baseline Michigan double bottom tanker model. Table 5 presents the damping ratio and frequency of the most lightly damped eigenvalue for each vehicle. The table indicates that both the Canadian double and the four point hitch increase the damping ratio of this eigenvalue by 80% over the baseline double tanker, and the frequency is reduced by 20%. Although the frequency of the eigenvalue is closer to the input frequency of an emergency lane change, transient analysis will indicate that the large increase in the damping ratio is dominant and reduces the peak response of the system. The constraint linkage design, on the other hand, increases the damping ratio by less than 10% over the baseline, and reduces the frequency by 35%.

The next section will present transient analyses of these configurations.

TABLE 4

SOURCE OF EQUATIONS AND PARAMETERS FOR DESIGN MODIFICATIONS

Vehicle	Source of Parameters	Source of Equations of Motion	Remarks
Baseline double bottom tanker	Reference 1	Reference 1	Figure 2
Canadian double bottom tanker	Reference 1	Reference 1	Figure 11
Baseline double bottom tanker with four point hitch	Measured	Appendix B	Figure 12
Baseline double bottom tanker with constraint linkage	Measured	Reference 10	Figure 13

TABLE 5

FREQUENCY AND DAMPING RATIO FOR SEVERAL DESIGN MODIFICATIONS

Vehicle	Damping Ratio	Frequency (hz)
Baseline double bottom tanker	.19	.74
Canadian double bottom tanker	.31	.59
Baseline double bottom tanker with four point hitch	.28	.58
Baseline double bottom tanker with constraint linkage	.19	.49

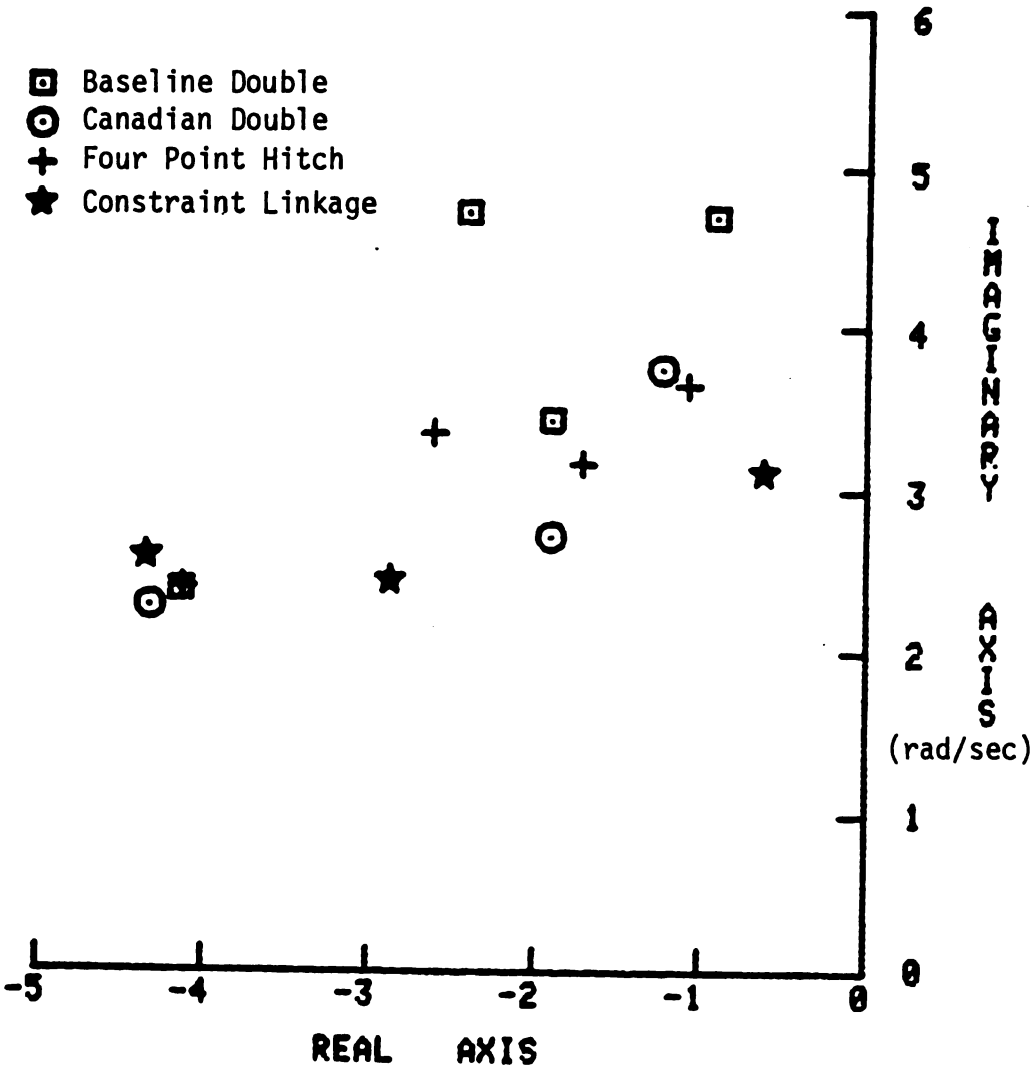


Figure 17. Eigenvalues of the Modified Designs

5.2 Transient Analysis

A lane change was simulated for each vehicle. The steer input for the lane change was one sine wave with a period of 2 sec. The amplitude of the steer input was varied to produce the same peak lateral acceleration at the tractor for each vehicle configuration. Figure 18 presents the acceleration of the pup trailer mass center for each of the vehicles including the baseline model.

Table 6 presents the peak acceleration gain and the ratio of amplitudes of the largest peak to the following peak in pup trailer acceleration for each vehicle configuration. The table indicates that the baseline double tanker had the highest peak lateral acceleration for the pup trailer. The Canadian double and the four point hitch both showed significant improvement over the baseline vehicle, a decrease in pup trailer peak acceleration of over 30%. The constraint linkage model reduced the peak lateral acceleration by 25%.

The ratio of the amplitudes of succeeding peaks for the baseline vehicle was also substantially higher than the ratio for the Canadian double model and the four point hitch model. But the constraint linkage showed less improvement. This is not surprising in view of the eigenvalue analysis presented in the previous section.

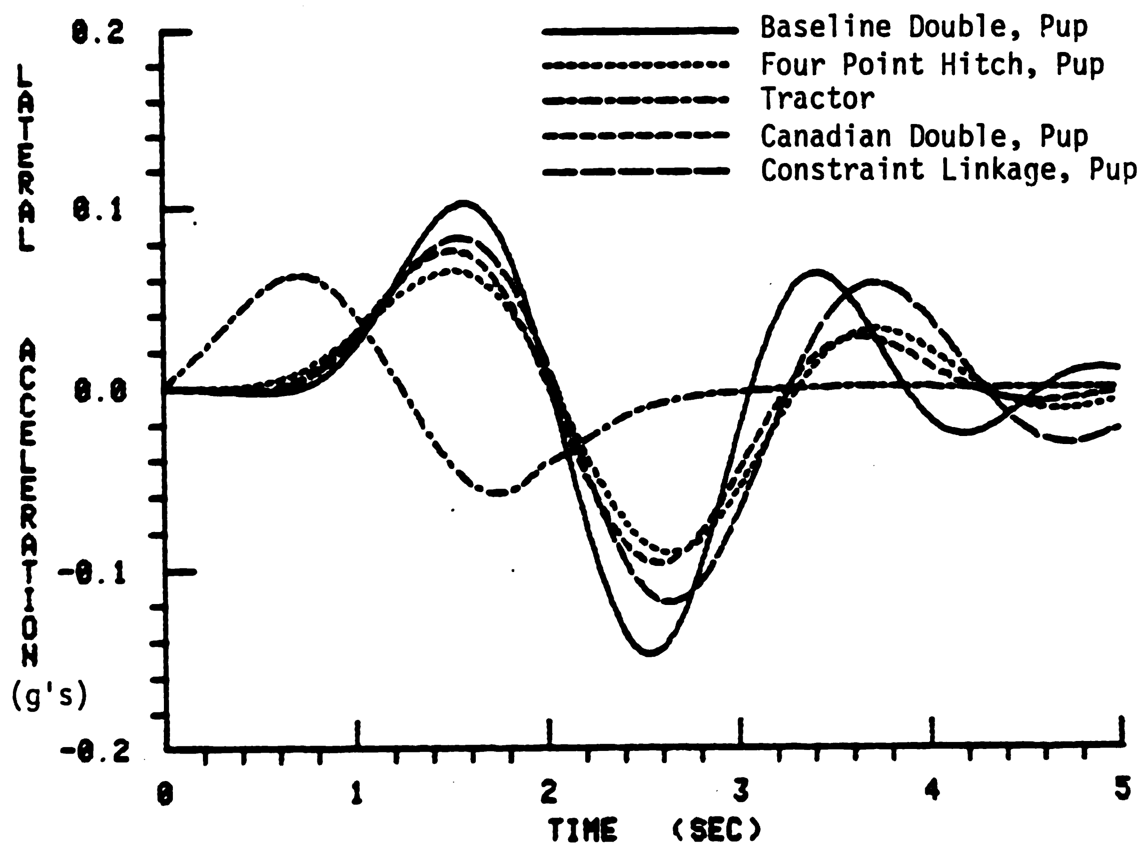


Figure 18. Lateral Acceleration in a Lane Change for the Modified Designs

TABLE 6

ACCELERATION GAIN FOR SEVERAL DESIGN MODIFICATIONS

Vehicle	Gain	Ratio of Successive Peaks
Baseline double bottom tanker	2.54	.61
Canadian double bottom tanker	1.76	.36
Baseline double bottom tanker with four point hitch	1.60	.50
Baseline double bottom tanker with constraint linkage	2.00	.69

6.0 OFFTRACKING

Offtracking of articulated vehicles is important at both low and high speeds. Excessive low speed offtracking can cause manueverability problems in tight access areas. Significant high speed offtracking can cause a trailer to hit objects outside the path of the tractor.

For use in this discussion, the offtracking of any tire will be defined as the distance it offtracks with relation to the corresponding tractor front tire. In a steady turn a positive sign convention will be given to offtracking outside of the trajectory of the tractor front tire. The high speed offtracking is computed using the steady turn solution of the equations of motion of each vehicle. The low speed offtracking is computed using equations developed in Reference 1.

Table 7 presents the offtracking of each vehicle at low speed (less than 1 mph) in a steady turn with a radius of 50 feet. Table 8 presents the offtracking of each vehicle in a steady turn with a radius of 600 feet at a velocity of 50 mph. (This corresponds to a lateral acceleration of .29 g's). The offtracking of several different axles on each vehicle is tabulated so that the contribution from each vehicle can be seen for both high and low speed offtracking.

The baseline double bottom tanker model had the least amount of low speed offtracking and the greatest amount of high speed offtracking. The Canadian double and the four point hitch both showed higher low speed offtracking than the baseline vehicle, but decreased high speed offtracking. Comparison of offtracking results with lateral acceleration results indicates a tradeoff between lateral acceleration gains and offtracking. The dynamically coupled vehicles in every case yielded greater low speed offtracking and less high speed offtracking than the baseline vehicle. And the high speed offtracking would seem to be somehow related to the real part of the eigenvalue, with the least offtracking corresponding to the vehicles with the highest damping.

TABLE 7

VEHICLE COMPONENT OFFTRACKING (INCHES) AT LOW SPEED IN A 50 FT. RADIUS TURN

Vehicle	Offtracking at Tractor Rear Tire	Offtracking at Semi-trailer Rear Tire	Offtracking at Dolly Rear Tire	Offtracking at Pup Trailer Rear Tire
Baseline double bottom tanker	-21.	-61.	-77.	-101.
Canadian double bottom tanker	-21.	-116.	--	-131.
Baseline double bottom tanker with four point hitch	-21.	-61.	-109.	-137.
Baseline double bottom tanker with linkage constraint	-21.	-61.	--	-120.

TABLE 8

VEHICLE COMPONENTS OFFTRACKING (INCHES) AT 50 MPH
IN A 600 FT. RADIUS TURN

Vehicle	Offtracking at Tractor Rear Tire	Offtracking at Semi-trailer Rear Tire	Offtracking at Dolly Rear Tire	Offtracking at Pup Trailer Rear Tire
Baseline double bottom tanker	5.4	9.5	13.3	18.9
Canadian double bottom tanker	6.0	12.0		15.8
Baseline double bottom tanker with four point hitch	5.4	9.5	9.6	17.2
Baseline double bottom tanker with linkage constraint	5.6	9.2		17.6

7.0 CONCLUSIONS

A study of the simplified model of a full trailer led to the conclusion that the lateral stability of the Michigan double bottom tanker cannot be improved significantly with practical parameter changes on the existing design.

This observation led to the study of three proposed design changes, each of which dynamically coupled the pup trailer to the semitrailer. Of these, the Canadian double has the best combination of qualities. Both the Canadian double and the full baseline model with a four point hitch equally reduced the lateral acceleration gain between the pup trailer and the tractor. The Canadian double also had a higher damping ratio than any other combination. The constraint linkage used on a full baseline model was inferior to the other designs in reducing the acceleration gain.

Both the Canadian double type hitch [2] and the constraint linkage on a baseline double bottom tanker are in use in Michigan. The constraint linkage offers slight improvement in directional performance over the Michigan double bottom tanker, but the constraint linkage is inferior to the Canadian double which offers marked improvements in directional response.

Baseline double bottom tankers can easily be converted to a Canadian double type vehicle by use of a rigid hitch as presented in [2]. Although there is a related penalty in low speed off-tracking and tire wear, the transient response well warrants the change.

APPENDICES

APPENDIX A

Derivation of Full Trailer Equations of Motion

A free body diagram of the dolly and pup trailer is shown in Figure A1. A nomenclature list is presented in Table 1. Subscripts are 1) vehicle number, 2) axle number. The slip angle of the jth axle on the ith vehicle is α_{ij}.

The dependent variables are dolly articulation angle and yaw rate, and pup trailer articulation angle and yaw rate (ψ_1, r_1, ψ_2, r_2). The dolly articulation angle is measured from the velocity vector of the pintle hook.

Application of Newton's second law to the dolly and pup trailer in Figure A1 yields

$$M_1 \cdot A_{1y} = F_{1y} - F_{2y} + \sum_j F_{1j} \tag{A1}$$

$$M_2 \cdot A_{2y} = F_{2y} + \sum_j F_{2j} \tag{A2}$$

Summing moments about each mass center yields

$$I_1(\dot{r}_1 + \dot{r}_u) = F_{1y} \cdot \ell_1 + F_{2y} \cdot \ell_2 - \sum_j F_{1j} \cdot x_{1j} + \sum_j M_{ij} \tag{A3}$$

$$I_2(\dot{r}_2 + \dot{r}_u) = F_{2y} \cdot \ell_3 - \sum_j F_{2j} \cdot x_{2j} + \sum_j M_{2j} \tag{A4}$$

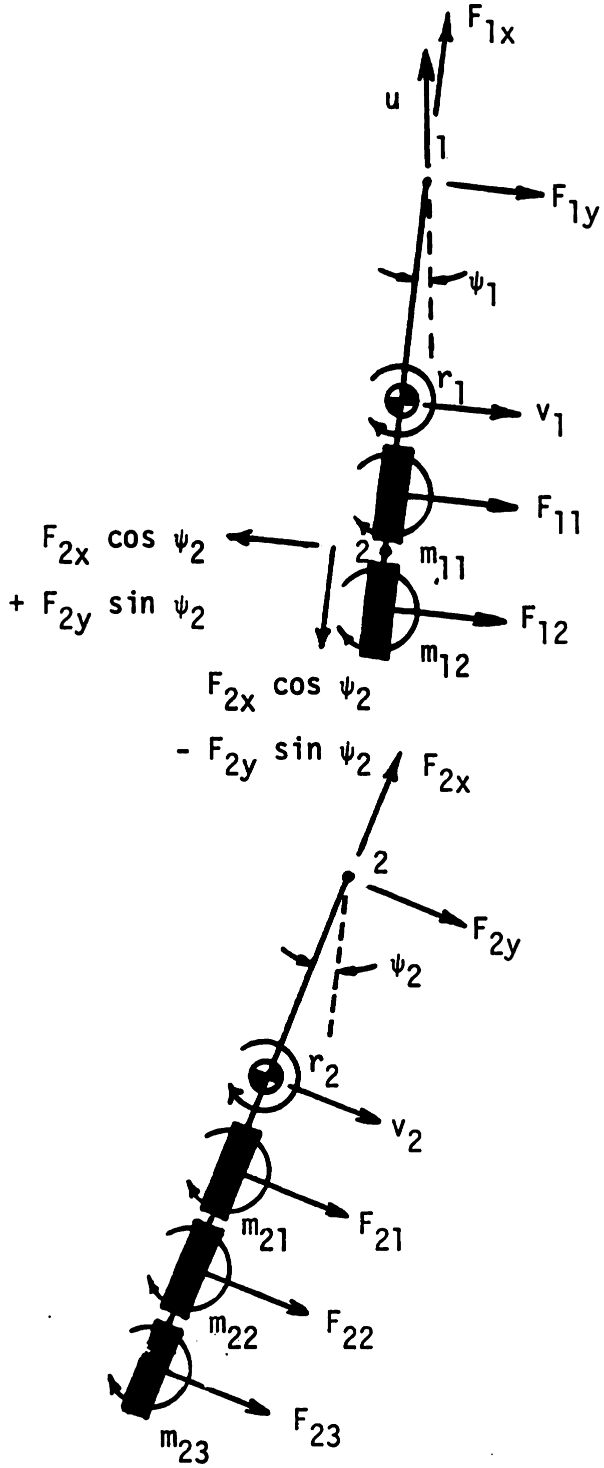


Figure A1. Free Body Diagram of Dolly Pup Trailer Model

The lateral forces F_{ij} are given by a linear slip law of the form

$$F_{ij} = -C_{ij} \cdot \alpha_{ij} \tag{A5}$$

where α_{ij} is the slip angle of the jth axle on the ith vehicle. The aligning moments are given by

$$M_{ij} = P_{ij} \cdot \alpha_{ij} \tag{A6}$$

The slip angle α_{ij} is defined as

$$\alpha_{ij} = \frac{v_i - x_{ij} \cdot r_i}{u_i} \tag{A7}$$

The numerator of equation gives the lateral velocity of the axle, and the denominator gives the longitudinal velocity of the axle.

The lateral velocity of each mass center can be expressed in terms of the four dependent variables ψ_1, r_1, ψ_2, r_2 as follows:

$$v_1 = u_1 \cdot \psi_1 - \ell_1 \cdot (r_1 + r_u) \tag{A9}$$

$$v_2 = u_1 \cdot (\psi_1 + \psi_2) - (\ell_1 + \ell_2) * (r_1 + r_u) - \ell_3 \cdot (r_2 + r_u)$$

(A10)

Substitution for all forces and moments in Equation A1 and A2 in terms of the four dependent variables yields two differential equations

$$\{I_1 + \ell_1 M_1 + (\ell_1 + \ell_2)^2 M_2\} \cdot \dot{r}_1 + \{\ell_3(\ell_1 + \ell_2)M_2\} \cdot \dot{r}_2$$

$$= \{\sum_j(\ell_1 + x_{1j})C_{1j} + (\ell_1 + \ell_2)\sum_j C_{2j} + \sum_j P_{ij}\} \cdot \psi_1$$

$$- \{\sum_j(\ell_1 + x_{1j})^2 C_{1j} + (\ell_1 + \ell_2)^2 \sum_j C_{2j} + \sum_j (\ell_1 + x_{1j})P_{1j}\} \cdot \frac{r_1}{u}$$

$$+ \{(\ell_1 + \ell_2) \sum_j C_{2j}\} \cdot \psi_2 - \{(\ell_1 + \ell_2)\sum_j((\ell_3 + x_{2j})C_{2j} + P_{2j})\} \cdot \frac{r_2}{u}$$

$$\text{(A11)}$$

$$\{\ell_3(\ell_1 + \ell_2)M_2\} \cdot \dot{r}_1 + \{I_2 + \ell_3^2 M_2\} \cdot \dot{r}_2$$

$$= \{\sum_j(\ell_3 + x_{2j})C_{2j} + \sum_j P_{2j}\} \psi_1$$

$$- \{(\ell_1 + \ell_2) \sum_j((\ell_3 + x_{2j})C_{2j} + P_{2j})\} \cdot \frac{r_1}{u}$$

$$+ \{\sum_j((\ell_3 + x_{2j})C_{2j} + P_{2j})\} \cdot \psi_2$$

$$+ \{\sum_j(\ell_3 + x_{2j})^2 C_{2j} + \sum_j(\ell_3 + x_{2j})P_{2j}\} \cdot \frac{r_2}{u} \qquad \text{(A12)}$$

Two additional differential equations relate the rate of change of the articulation angles to the yaw rates.

$$\dot{\psi}_1 = r_1 \tag{A13}$$

$$\dot{\psi}_2 = r_2 - r_1 \tag{A14}$$

These complete the set of four differential equations which determine the motion of the full trailer system as a function of r_u and $\dot{r}_u$.

APPENDIX B

Derivation of Equations of Motion for the Constraint Linkage Design

Figure B1 presents an articulated configuration for the constraint linkage. The articulation angle of the dolly is ψ_2, and the articulation angle of the pup trailer is ψ_3. Both are measured from the centerline of the semitrailer.

The addition of a linkage between the semitrailer and the pup trailer creates a kinematic relationship between ψ_2 and ψ_3. From Figure B1:

$$\ell_1 \sin \psi_2 = d \tag{B1}$$

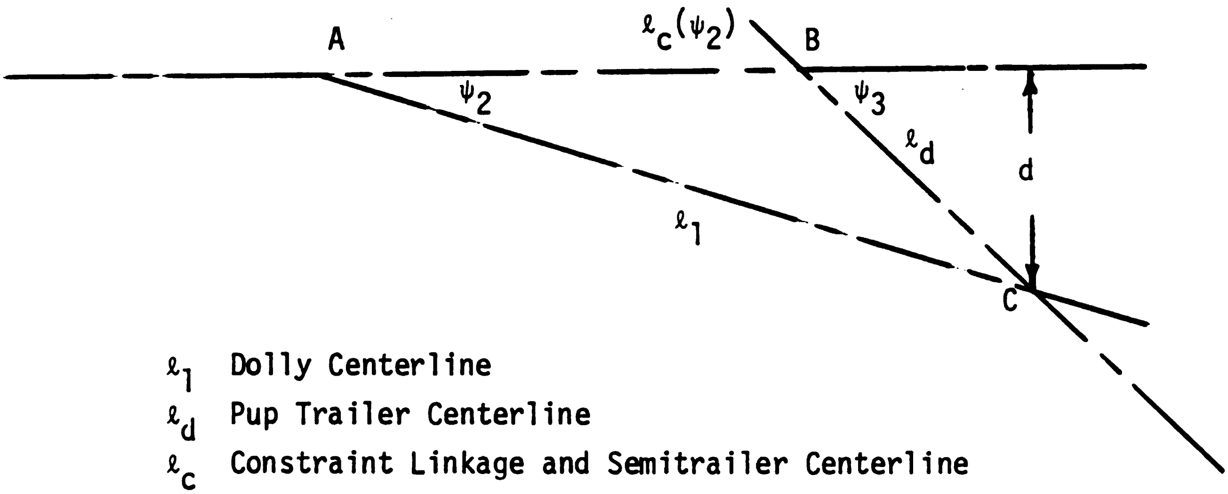
$$\ell_d \sin \psi_3 = d \tag{B2}$$

Combining Equations B1 and B2 yields

$$\sin \psi_2 = \frac{\ell_d}{\ell_1} \sin \psi_3 \tag{B3}$$

Assuming small angles

$$\psi_2 = \frac{\ell_d}{\ell_1} \psi_3 \tag{B4}$$



ℓ_1 Dolly Centerline

ℓ_d Pup Trailer Centerline

ℓ_c Constraint Linkage and Semitrailer Centerline

A Pintle Hook

B Linkage Attachment to Pup Trailer

C Dolly Kingpin

Figure B1. Constraint Linkage in a Deformed Configuration

This relationship eliminates a degree of freedom from the baseline double bottom tanker model.

Equations of motion for a tractor semitrailer-semitrailer obtained from Reference 9 were used to model the double bottom with the constraint linkage. The dolly was included in the model by adding forces from the dolly transmitted through the kingpin to the pup trailer. The mass and yaw inertia of the dolly were also added to those of the pup trailer. The dolly hitch force acting on the front semitrailer is assumed small and ignored.

Because the mass and yaw inertia of the dolly is small compared to the other vehicles, the dolly forces are computed quasi statically. It is assumed all lateral force from dolly tires is transmitted through the dolly kingpin to the pup trailer spring mass.

LIST OF REFERENCES

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1. Bernard, J. and Vanderploeg, M., "Static and Dynamic Offtracking of Articulated Vehicles." SAE Paper No. 800151, Feb. 1980.

2. Ervin, R.E., et al., "Ad Hoc Study of Certain Safety-Related Aspects of Double-Bottom Tankers." Final Report prepared for Office of Highway Safety Planning, Michigan Department of State Police, Contract No. MPA-78-002A, Highway Safety Research Institute, University of Michigan, Report No. UM-HSRI-78-18, May 7, 1978.

3. Eshleman, R.L. and Desai, S.D., "Articulated Vehicle Handling, Summary," Final Rept., DOT Contract No. DOT-HS-105-1-151 (Apr 1972).

4. Eshleman, R.L., Desai, S.D., and Sauza, "Stability and Handling Criteria of Articulated Vehicles," NHTSA, Aug. 1973.

5. Gillespie, T.D., "Simulation of the Effects of Increased Truck Size and Weight." Final Technical Report, DOT-FH-119330, Nov. 1979.

6. Hazemoto, T., "Analysis of Lateral Stability for Doubles." SAE Paper No. 730688, June 1973.

7. Hales, F.D., "The Rigid Body Dynamics of Road Vehicle Trains." The Dynamics of Vehicles on Roads and Tracks, Proceedings of IUTAM Symposium, Delft, August 1975.

8. Jindra, F., "Handling Characteristics of Tractor-Trailer Combinations." SAE Transactions, Vol. 74, Paper No. 650720, 1965.

9. Mallikarjunarao, C. and Fancher, P.S., "Analysis of the Directional Response Characteristics of Double Tankers," SAE 781064, December 1978.

10. Sharp, R.S., "The Steering Responses of Doubles." Veh. Syst. Dyn., 8, September 1979.

11. Nordström, O. and Strandberg, L., "The Dynamic Stability of Heavy Vehicle Combinations," National Swedish Road and Traffic Research Institute, Publication, 1974.

12. Strandberg, L., Norström, O. and Nordmark, S. "Safety Problems in Commerical Vehicle Handling." Commercial Vehicle Braking and Handling Proceedings, May 1975.

13. Williams, D., "The Mathematical Theory of the Snaking of Two-Wheeled Trailers." Proc., Automobile Division, Inst. Mech. Engrs. (1951-1952), pp. 175-187.