### ABSTRACT

## HELIUM II FILM TRANSFER RATES FOR SOLID-ARGON BEAKERS

By

### Thomas O. Milbrodt

Helium II film transfer rates have been measured for filling and for emptying beakers of solid argon. Liquid argon was frozen slowly in a mold which was later removed, leaving free-standing, transparent solid-argon beakers. The beakers used were of 3 mm i.d., 5 mm o.d., and 3.5 cm in height. Experiments were carried out in the temperature range 1.45 - 2.0 K. The dependence of the transfer rate  $\sigma$  (cm<sup>3</sup>/ sec-cm of circumference) on the difference between the inner and outer helium levels Z, height of the beaker lip above the level of the liquid source H, and temperature, were studied. The transfer rates were lower than those observed with glass beakers, in agreement with theory. For beaker fillings with a level difference of 1 mm and at a temperature of 1.66 K, the measured transfer rates may be described by the relation  $\sigma = 4.8$  H<sup>-0.21</sup> x 10<sup>-5</sup> cm<sup>2</sup>/sec, where H is in cm. HELIUM II FILM TRANSFER RATES FOR SOLID\_ARGON BEAKERS

By  $\rho^{i} \rho^{i}$ . Thomas O. Milbrodt

A DISSERTATION

.

Submitted to

Michigan State University

in partial fulfillment of the requirements

for the degree of

DOCTOR OF PHILOSOPHY

Department of Physics



I would very much like to thank Professor G. L. Pollack, who suggested this experiment, for his guidance during the course of this research. I also wish to thank David Christen and Jon Opsal for many helpful discussions and suggestions, and my wife, Jane, for her patience and support. Finally, I would like to acknowledge the financial support of the United States Atomic Energy Commission and the National Science Foundation.

# TABLE OF CONTENTS

Chapte	r	Page
Ī.	INTRODUCTION	1
	A. Description of the Transfer Effect	1
	B. Motivation for the Experiment	4
	C. Some Properties of Solid Ar	7
11.	THEORY	8
	A. General	8
	B. The Film Thickness d	9
	C. The Superfluid Velocity v	14
	D. Theoretical Expression for the Transfer Rate	18
111.	EXPERIMENTAL PROCEDURE	20
	A. Description of the Apparatus	<b>2</b> 0
	B. Procedure Used To Make the Ar Beakers	27
	C. Data Collection	33
IV.	RESULTS OF THE EXPERIMENT	38
	A. Reproducibility of the Results	38
	B. Results of Beaker Filling Experiments	38
	C. Regults of Beaker Buntving Experiments	42
	D. Discussion of the Results	45
v.	SUMMARY AND CONCLUSIONS	48
<b>Appen</b> d	ices	
A.	DATA REDUCTION PROGRAM	<b>5</b> 0
В.	TABULATION OF EXPERIMENTAL DATA	5 <b>3</b>
	LIST OF REFERENCES	56

# LIST OF TABLES

Table	à	Page
1	Values of the He film thickness d on a vertical Ar surface as a function of height h above the bulk He, calculated using the results of Sabisky and Anderson. <sup>21</sup>	14
2	<b>Comparison</b> of the experimental results for beaker filling and emptying rates with the theoretical predictions. The transfer rates $\sigma_0$ correspond to lum level difference and 1 cm film height.	44
A1	List of the FORTRAN program SIGMAZ used to derive the transfer rates from the raw data.	50
<b>B1</b>	Results of beaker filling runs made with Beaker 3.	53
B2	Results of beaker filling runs made with Beaker 4.	54
B3	Results of beaker emptying runs made with Beaker 3.	<b>5</b> 5
B4	Results of beaker emptying runs made with Beaker 4.	<b>5</b> 5

# LIST OF FIGURES

Figur	e	Page
1	The results of the numerical calculations of Sabisky and Anderson for the potential energy between a He atom and a substrate a distance d away.	12
2	A sketch of the apparatus. The upper part of the mold may be raised and lowered from outside the cryostat.	21
3	Photograph of one of the Ar beakers. The beaker is filling, and the liquid He levels inside and outside it can be seen.	32
4	The He level difference as a function of time for three beaker fillings at three different heights H from the outer He level to the beaker rim.	35
5	The transfer rate as a function of level difference for the three beaker fillings of Figure 4.	37
6	The transfer rate at 1 mm level difference as a function of the height H from the outer level to the top of the beaker for beaker fillings at a temperature of 1,66 K.	41
7	The transfer rate as a function of the height H <sub>T</sub> from the inner He level to the beaker rim at three different heights H from the outer He level to the beaker rim.	43

## I. INTRODUCTION

## A. Description of the Transfer Effect

When liquid He<sup>4</sup> under its saturated wapor pressure is cooled through 2.17 K, the helium goes through a phase transition, called the lambda transition, and enters a new liquid phase called He II. Liquid helium at temperatures above the phase transition is correspondingly called He I. Through the efforts of London<sup>1</sup>, Landau<sup>2</sup>, and Tisza<sup>3</sup> the two-fluid model of He II was developed in the late thirties. With the model the behavior of He II may be described by supposing that a finite fraction of it is superfluid. This superfluid is mixed uniformly with the rest of the He II, which is called normal fluid. The superfluid fraction increases as the temperature of the He II is reduced and decreases continuously to zero as the temperature is raised to the temperature of the lambda transition. The superfluid carries no entropy, which suggests that it is to be associated with the ground state of bulk helium.

Kamerlingh Oanes was the first to succeed in the liquefaction of He<sup>4</sup>, which he did in Leiden in 1908.<sup>4</sup> At the time there was little reason to expect the liquid to have unusual properites, since the atoms are spherically symmetrical and chemically inert, and quantum mechanics, which is at the root of the properties of liquid helium, was not yet understood. There was a reluctance to consider unusual effects meaningful, and it was not until 1927, when Keesom

and Wolfke found a discontinuity in the dielectric constant of liquid helium in the vicinity of 2.2 K, that it was suggested that the liquid must undergo some sort of transition. A few years later Keesom and coworkers made careful measurements of the specific heat of He<sup>4</sup>. They discovered no measurable latent heat accompanying the transition, but they did find a specific heat curve with a discontimuity at the transition temperature and a shape similar to the Greek letter "lambda." For this reason Keesom called the transition temperature the "lambda point."<sup>5</sup>

The properties of He II have their origin in quantum mechanics. Because of the low mass of the He<sup>4</sup> atom, its quantum mechanical zero point motion is large, and the weak He-He interatomic forces cannot hold the atoms in a solid array at any temperature. Only under an external pressure greater than 26 atm will He<sup>4</sup> form a solid, even very mear 0 K. Nevertheless, thermodynamics requires that the entropy of a system approach zero as the temperature approaches 0 K. This is satisfied in the case of liquid He by order in mementum space rather than order in real space. Since the He<sup>4</sup> atoms obey Bose-Einstein statistics, a finite fraction of them may "condense" into a single ground state in momentum space. This can be shown to happen below some critical temperature for the ideal Bose-Einstein gas, and it is believed that an analogous thing happens with He<sup>4</sup>. The complete theoretical solution to the He<sup>4</sup> problem has not yet been worked out, however.

He II has many well-known unique properties. One is that the superfluid is able to flow without measurable viscosity as long as the magnitude of its flow velocity is less than some value, called

the critical velocity. (The existence of a well defined critical velocity is now believed to be only an approximation, as will be discussed later.) He II also has an extremely high thermal conductivity, typically 1000 times the conductivity of room temperature copper. Other phemomena that can be observed in He II are the propagation of entropy waves, called second sound, and the production of a pressure difference by a temperature difference, called the fountain effect. The properties of helium and the two-fluid model are discussed in detail in many books.<sup>4-8</sup>

The He II film transfer effect may be observed by partially submerging an empty beaker in a bath of He II. The beaker will fill until the He levels inside and outside it are equal. It will similarly empty until the levels are equalized if the level inside it is initially higher than the level outside. The effect, as first proposed by Rollin,<sup>9</sup> depends on the existence of a thin He film adsorbed on cold surfaces in contact with the He liquid or vapor. This film is typically only 100 to 300 Å thick, but the superfluid component of He II can nevertheless flow through it quite readily. The normal fluid component of the He film is locked in place by its viscosity, and therefore, for He I, where all of the fluid is normal, there is no transfer of the liquid He.

The magnitude of the parameters associated with the transfer effect are such that measurements of transfer rates can be made in a very straightforward manner in the laboratory. The velocity of the superfluid in the adsorbed film has been determined by other experiments to be of the order of 25 cm/sec.<sup>10</sup> The fraction of the He II that is superfluid varies from 60% to 80% at the temperatures at which

we conducted our experiments.<sup>7</sup> For a film thickness of 200 Å, one can calculate a typical transfer rate of  $4 \times 10^{-5}$  cm<sup>3</sup>/sec·cm of circumference of the beaker lip. This implies that for a beaker with an i.d. of 3 mm, such as we used, the inner level should change at a rate of the order of 0.3 mm/min when He is being transferred into or out of the beaker. It is possible, therefore, to make fairly good measurements with just a cathetometer and a stopwatch.

B. Metivation for the Experiment

The results of the first systematic study of the He II film transfer effect were reported by Daunt and Mandelssohn<sup>11</sup> in 1939. They observed that the transfer rate depended mostly on the temperature. By doing experiments with constricted beakers, they determined that the transfer rate was proportional to the smallest circumference of the beaker surface. They failed to observe a dependence of the transfer rate on level difference or film height. They proposed that the rate was limited by a maximum velocity at which the superfluid could flow without friction.

In 1941 Schiff<sup>12</sup> pointed out that it should be possible to calculate the thickness of a He II film on a vertical surface by equating the gravitational potential energy of a He atom on the surface of the film to the net van der Waals interaction emergy with the surface. Such an analysis implies that if the interaction potential varies as  $r^{-6}$ , where r is the separation distance between two interacting atoms, then the film thickness should vary as  $h^{-1/3}$ , where h is the height above the surface of the bulk liquid. This height dependence is not very strong, which explains why Daunt and Mendelssohn could not resolve any dependence at all. In 1950 Atkins<sup>13</sup> reported

seeing a film height dependence of the transfer rate in a more careful experiment.

The great importance of having a smooth and contamination free beaker surface to obtain reproducible results was shown in experiments reported by Bowers and Mendelssohn<sup>14</sup> in 1950. They observed the effects of coating a beaker with various thicknesses of frozen air, H, or Ne. The transfer rates were greatly increased, by over an order of magnitude for the thickest layers, relative to the rates for the uncoated beaker. This was true even for a Ne coating. Since the van der Waals interaction between Ne and He is weaker than it is between glass and He, there should have been no film thickness increase for the Ne coating. The increase, therefore, must have been due to roughness of the Ne surface causing the microscopic perimeter of the beaker to be greatly increased.

In the past several years He II film transfer experiments have been conducted with glass beakers by Allen and Armitage<sup>15</sup> and by Duthler and Pollack<sup>16-18</sup>. In these experiments great care was taken in order to ensure that the glass was clean. Their results are in fairly good agreement and may be used with independent direct measurments of the He film thickness on glass by Ham and Jackson<sup>19</sup> to determine what the superfluid velocity in the film must be.

A very interesting way to test out knowledge of the transfer effect and the van der Waals potential would be to change the film thickness by changing the material of which the beaker wall is made. A good candidate for a different wall material is a solid moble gas, since the solid structure is simple and the interaction, even among the solid atoms, is purely of the van der Waals type,<sup>20</sup> Also, the

van der Waals attraction energies between He and most common beaker materials, such as glass and metal, are very similar in magnitude, so that the He film thickness does not vary much among them.<sup>12</sup> Ar and Ne, on the other hand, are significantly less polarizable than common materials and should support thinner He films, thus reducing the film transfer rates.

Duthler and Pollack<sup>17,18</sup> conducted experiments in which they coated a glass beaker with a 1000 Å layer of Ne to make a beaker with an effective Ne surface. The problem of surface roughness is a serious one in such an experiment. If the glass is coated near the Ne triple point, the solid Ne will contract under further cooling and probably form microscopic cracks. The results indicate that the surfaces did tend to be somewhat rough, but not nearly so rough as those of Bowers and Mendelssohn. Transfer rates considerably lower than those for glass were observed on some runs, but none were as low as one would predict by using the most recent and complete calculations for the van der Waals potential.<sup>21</sup>

We decided to try the approach of making the beakers for our experiments from a solid noble gas, specifically from solid Ar. We chose Ar because it is relatively plentiful and cheap and because it has a vary convenient triple point temperature of 83.8 K, just above the liquid N<sub>2</sub> boiling temperature of 77.4 K. Recent calculations of the van der Waals potential between He and an Ar substrate<sup>21</sup> have shown that the He film on Ar should be about 35% thinner than it is of glass, resulting in measurably lower transfer rates.

## C. Some Properties of Solid Ar

Solid Ar is a difficult material to work with near its triplepoint because of its softness, low thermal conductivity, and high vapor pressure. Solid Ar near the triple-point may be fairly easily deformed or broken. It has been said that it has a consistency similar to wax.<sup>22</sup> At lower temperatures the solid becomes more brittle. The thermal conductivity of solid Ar near the triple-point is only half that of glass, which makes the removal of temperature gradients difficult. The sublimation pressure of Ar is 517 Torr at the triple-point temperature of 83.8 K and is 204 Torr at 77.5 K.<sup>23</sup> This very large dependence of sublimation pressure on temperature means that, just below the triple-point, relatively small temperature gradients in solid Ar can produce large pressure gradients that can push the soft solid apart.<sup>22</sup> The sublimation pressure is high enough at the liquid N boiling temperature that substantial amounts of the solid will migrate from one place to another under the influence of a small temperature gradient. It is therefore necessary to keep solid Ar under liquid He if one wants to preserve its shape. Solid Ar contracts 3% between 83 K and 4 K.<sup>20</sup>

Solid Ar also has some properties that made the experiment easier. It is completely transparent to visible light, so that one can measure the He level inside an Ar beaker optically. Also, the high sublimation pressure makes it possible to release solid Ar from a mold by just gently pumping on it.

#### II. THEORY

## A. General

We relate the transfer rate to the experimental parameters in the usual way with the equation<sup>24</sup>

$$\sigma = (\rho_{\rm g}/\rho) v_{\rm g} d \qquad (1)$$

In Equation 1 d is the thickness of the adsorbed helium film at the lip of the beaker,  $\mathbf{v}_{g}$  is the maximum superfluid velocity in the film, which occurs where the film is thinnest, mear the beaker lip, and  $\mathbf{P}_{g}/\mathbf{P}$  is the superfluid fraction, expressed in terms of the ratio of the superfluid density  $\mathbf{P}_{g}$  to the total helium density  $\mathbf{P}$ . The interpretation of Equation 1 is that only the superfluid component of the thin helium film is flowing, the normal fluid being locked in place on the beaker surface by its viscosity. Since the film is thinnest at the top of the beaker, the superfluid velocity at that point will be the highest and will limit the flow rate.

The superfluid fraction is taken to be the same in the film as it is in bulk He II, for which it has been carefully measured.<sup>6-8</sup> The temperature dependence of  $\rho_s/\rho$  is the principle source of the temperature dependence of  $\sigma$ . The actual superfluid fraction in the film is probably slightly less then it is in the bulk because the relatively strong attractive potential of the substrate destroys the superfluidity very near the surface. Recent experiments have indicated, however, that the superfluid fraction of a saturated film not

too near the lambda-point is very close to that of the bulk.<sup>25,26</sup>

The other factors in Equation 1 cannot be disposed as easily or quickly as  $\rho_s/\rho$ . The derivation of values for d and for  $v_s$  will be discussed at some length in the next three sections of this chapter.

## B. The Film Thickness d

Thirty years ago Schiff<sup>12</sup> and Frenkel<sup>27</sup> independently suggested that the He film thickness d on any vertical wall at a height h above the bulk liquid may be calculated by applying the condition that the potential energy of a He atom on the surface of the film must equal the potential energy of an atom on the surface of the bulk liquid. This may be expressed by the simple relation

$$V(d) - V_{H_0}(d) + mgh = 0,$$
 (2)

where m is the mass of a He atom, V(d) is the attractive van der Waals potential energy between a He atom and a substrate a distance d away, and  $V_{\text{He}}(d)$  is what V(d) would be if the substrate were He. The potential  $V_{\text{He}}(d)$  is usually much smaller than V(d) and is frequently neglected. This calculation, of course, does not depend on the superfluid properties of He II, but applies to any equilibrium case of a liquid and substrate where the liquid atoms are more strongly attracted to the substrate atoms than they are to each other.

The calculation of V(d) has usually been based on the London<sup>28</sup> theory of the van der Waals attraction. London showed the attraction to be due to the fact that a neutral molecule or atom possesses a fluctuating electric dipole moment which enables it to interact with neighboring molecules, through the induced moment in these molecules, to produce an attractive potential. Casimir and Poulder<sup>29</sup> have pointed out, however, the polarization of the neighboring molecules

does not instantaneously follow the first molecule because of the finite velocity of light. The London potential for a separation distance r between the interacting atoms varies as  $r^{-6}$ , but Casimir and Polder have shown that the potential varies as  $r^{-7}$  when retardation becomes important. The other factor that must be considered in calculations of V(d) is that condensed systems are involved, and, therefore, screening and perhaps other many body effects may be important.

Lifshitz<sup>30</sup> has published a comprehensive theory for the van der Waals force which treats the matter involved as a continuum with a well-defined frequency-dependent dielectric susceptibility. Dzyaloshinskii, Lifshitz, and Pitaevskii<sup>31</sup> have applied this theory to the problem of calculating the molecular forces between a thin film and a substrate. Their general equations require only information about the dielectric properties of the bodies, but they did not give numerical results because the integrals in their equations can only be done using digital computer techniques and because they felt that the frequency-dependence of the dielectric susceptibility of materials was not known in sufficient detail.

Parsegian and Ninham<sup>32</sup> have applied the Lifshitz theory to a study of the forces between biological membranes. They have shown that accurate results may be obtained from the theory with only a partial knowledge of the dielectric susceptibility of the materials involved. Sabisky and Anderson<sup>21</sup> have published the results of accurate measurements of He films on cleaved surfaces of alkalineearth fluoride crystals in which they used an acoustic interferometry technique. In their paper they have included the results of

calculations of V(d) for He on a number of substrates, including Ar, based on the equations of Dzyaloshinskii, Lifshitz and Pitaevskii. Their experimental results agree very well with the theoretical potential energies, providing a strong confirmation of the Lifshitz theory. Before discussing the results of the calculations of Sabisky and Anderson, we shall briefly describe the much simpler traditional calculation for V(d) based on the London theory.

The  $r^{-6}$  dependence of the London potential yields V(d)  $\propto d^{-3}$ if one assumes the forces are additive and just integrates to determine the potential between a single atom and a semi-infinite solid whose plane surface is a distance d away. If one plugs this into Equation 2 then the result is

$$d = k h^{-1/3}$$
, (3)

where k is a constant. We have used published potential parameters for He-He<sup>33</sup> and Ar-Ar<sup>20</sup> attractive forces, together with a combining rule consistent with the London theory<sup>34</sup>, to determine the He-Ar force. From this we calculate that for He on an Ar substrate  $k = 2.9 \times 10^{-6}$  cm <sup>4/3</sup>. This is about the same magnitude for k as has been measured for He on glass.<sup>19</sup>

The results of the numerical calculations of Sabisky and Anderson, as given in Figure 8 of Reference 21, are plotted in Figure 1. Notice that the quantity plotted is  $-d^3 V(d)$  vs. d, so that using the uncorrected London theory corresponds to a horizontal line on this graph. Furthermore, retardation and screening effects are important even for a He atom arbitrarily close to the substrate surface, since a substantial fraction of the attractive force there is still due to atoms more than 10 Å away. The magnitude of the potential is only





about 2/3 of that determined above, where retardation was neglected and additivity assumed. The curve in Figure 1 does appear to become level for d < 10 Å. The most important result of using this new, more correct potential is that the theoretically calculated transfer rates for Ar are considerably reduced. The effect of the fact that the potential is no longer proportional to  $d^{-3}$  is relatively small.

For sufficiently large d, V is proportional to  $d^{-1/4}$ , but for the range of d of interest to us, 120 to 250 Å, there is no simple relationship. To obtain a relationship that could be used easily in numerical calculations, we did the following thing: We determined an approximate expression for  $\alpha(d) = -V d^3$  by fitting the three parameter form

$$\alpha (\mathbf{d}) = \mathbf{A} + \mathbf{B} \, \mathbf{d} + \mathbf{C} \, \mathbf{d}^{\mathbf{Z}} \tag{4}$$

to three points extracted from the results of Sabisky and Anderson:  $\alpha$  (127 Å) = 0.60 x 10<sup>-13</sup> erg Å<sup>3</sup>,  $\alpha$  (178 Å) = 0.50 x 10<sup>-13</sup> erg Å<sup>3</sup>,  $\alpha$  (250 Å) = 0.41 x 10<sup>-13</sup> erg Å<sup>3</sup>. The resulting parameters are  $A = 0.98 \times 10^{-13}$  erg Å<sup>3</sup>,  $B = -3.72 \times 10^{-16}$  erg Å<sup>2</sup>, and C =5.78 x 10<sup>-19</sup> erg Å. We include this in our theory by replacing k in Equation 3 with k(d), a relatively slowly varying function of d that may be calculated for any d through the use of Equation 4 and the parameters we determined. As can be seen from Equation 2, we meed not only  $\alpha$  (d), but also  $\alpha_{\text{He}}(d)$ , in order to calculate k(d). Sabisky and Anderson do not give  $\alpha_{\text{He}}(d)$  for all d, but only for d mear zero, i.e., less than 10 Å. This value for  $\alpha_{\text{He}}$  is about 13% of the corresponding value for Ar. Based on this, we set

$$\alpha$$
 (d) -  $\alpha_{\mu}$  (d) = 0.87  $\alpha$  (d). (5)

The other equations are

$$d = k(d) h^{-1/3}$$
 (6)

\_ \_ ...

$$k(d) = [(\alpha (d) - \alpha_{He}(d)) / mg]^{1/3},$$
 (7)

where m is the mass of a He atom and g is the acceleration of gravity. Equation 6 can be easily solved for d(h) by an iterative procedure. Given an h, one chooses a trial value for d, say 150 Å. This walue of d is used to calculate k(d), and then Equation 6 is used to calculate a new value for d. This new d is used to calculate a new k(d), and so on. Because k(d) varies so slowly with d, the procedure converges after just a few cycles. The results of our calculations for d(h) are diplayed in Table 1.

> TABLE 1 Values of the He film thickness d on a vertical Ar surface as a function of height h above the bulk He, calculated using the results of Sabisky and Amderson,<sup>21</sup>

<b>h (cm)</b> 0 <b>,2</b> 5	d (Å) 276	h ( <b>cm</b> ) 1.50	d (Å) 167
0 <b>.50</b>	226	1.75	160
0.75	202	2.00	154
1.00	187	2.25	149
1.25	175	2.50	144

C. The Superfluid Velocity v.

The problem of the velocity of the superfluid in the transfer effect is not understood theoretically as well as the He film thickness. It is only in the past few years, in fact, that even a qualitative understanding has been developed. As shall be seen, the concept of a critical velocity is a rather fuzzy one in the most recent theories of He II. We shall continue to use the concept for the time being, however. We first consider the case in which  $v_s$  is less than  $v_{s,c}$ , the superfluid critical velocity. The usual two-fluid equations of motion for this case are<sup>35</sup>

$$\rho_{s} \frac{d\vec{v}_{s}}{dt} = -\frac{\rho_{s}}{\rho} \nabla P + \rho_{s} s \nabla T \qquad (8a)$$

$$\rho_{\mathbf{n}} \frac{d\vec{\mathbf{v}}_{\mathbf{n}}}{d\mathbf{t}} = -\frac{\rho_{\mathbf{n}}}{\rho} \nabla \mathbf{p} - \rho_{\mathbf{s}} \mathbf{s} \nabla \mathbf{T} + \eta \nabla^{2} \vec{\mathbf{v}}_{\mathbf{n}}, \qquad (8b)$$

where  $\rho$  is the total He density, P is the pressure, s is the specific entropy, and T is the temperature. The normal fluid need not be considered here because it is locked in place on the wall and does not move, i.e.  $v_n = 0$ . We consider experiments done isothermally, so that the factor  $\nabla T$  vanishes. If we include this, and also expand the L.H.S., Equation 8a becomes

$$(\partial \vec{v}_s / \partial t) + (\vec{v}_s \cdot \nabla) \vec{v}_s = - (1/\rho) \nabla P$$
 (9)

Next we integrate both sides of this equation along a line leading from the outer He level, up the beaker wall, over the lip, and down to the inner level. The result is

$$\int \frac{\partial \vec{v}_{s}}{\partial t} \cdot \vec{d1} + \frac{1}{2} |v_{s}|^{2} \qquad \text{inside} = -\frac{1}{\rho} \Delta P \quad . \tag{10}$$

Since  $\Delta P$  is just  $\rho gZ$ , where Z is the level difference, and the velocity  $v_g$  vanishes at the bulk surface, we have

$$\rho \int \frac{\partial \vec{v}_{e}}{\partial t} \cdot \vec{d1} = -\rho g Z \qquad (11)$$

We can relate  $v_g$  to the level difference with the equation

$$2 \operatorname{mrd} \frac{\rho_{\mathrm{s}}}{\rho} v_{\mathrm{s}} = \operatorname{mr}^2 \frac{\mathrm{d}Z}{\mathrm{d}t} , \qquad (12)$$

where r is the radius of the beaker. Combining Equations 11 and 12 yields

$$\left[\frac{\rho^2}{\rho_s}\frac{\mathbf{r}}{2}\int\frac{d\mathbf{l}}{d\mathbf{l}}\right]\frac{d^2\mathbf{Z}}{d\mathbf{t}^2} = -\rho_g\mathbf{Z} \qquad (13)$$

This equation predicts oscillation of the He level difference, as one would have to expect for flow without viscosity. Oscillations of this type have been observed, but they occur only as long as  $v_8$ remains less than  $v_{8,c}$ . In our experimental configuration this occurs for a maximum oscillation amplitude so small as to be unmeasurable with our cathetometer.

What is actually observed in our transfer experiments is that the transfer rate is almost constant. This means that at some superfluid velocity  $v_s$  a viscous effect sets in to dissipate the kinetic energy of the flowing He film, preventing an increase in  $v_s$ . The velocity at which this occurs may be called the critical velocity.

More careful experiments in the past few years have shown that the above description is only approximately correct. 16-18, 36-41Experimental results are better described by the relation 42

$$- \nabla \mu = G \exp \left[ b(T) \left( 1 - \frac{\mathbf{v}_{c}(T)}{\mathbf{v}_{s}} \right) \right]$$
(14)

where  $\mu$  is the chemical potential, G is the value of the chemical potential gradient at which  $v_s = v_c(T)$ , and b(T) is a relatively slowly varying function of the temperature. In our case the driving chemical potential difference is just the gravitational potential energy difference between the two He levels.

Because of these rather recent experimental results, the theory of He superflow has been revised, and, in some sense, the phenomenon is now probably better understood. The basic ideas behind the new theory are that any state of non-zero superflow is in principle unstable and that the decay of superflow is initiated through thermally activated fluctuations.<sup>43-47</sup> It is generally believed that the

fluctuations that cause the decay of the flow are vortex rings.

Chester and Ziff<sup>48</sup> have shown that, at any given flow velocity, a vortex ring with an initial energy greater than a certain minimum amount will "run away", growing until it reaches the physical bounds of the system and removing energy from the fluid flow as it grows. The rate at which vortices with sufficient energy are created by fluctuations is proportional to the usual Boltzmann factor,  $e^{-E/kT}$ , where E is the vortex ring energy. As the flow velocity increases, the minimum vortex ring energy required for growth decreases, and the dissipation increases very rapidly. Really good first principles calculations for the parameters of Equation 14 have not been done yet for the constricted geometries in which He superflow experiments usually take place. The calculations of Chester and Ziff ignored the effects of physical boundaries.

It is known from experiments that the parameters of Equation 14 are such that the critical velocity description of superflow is almost numerically right. That is,  $\mathbf{v}_{s}$  changes very little as  $\nabla \mu$  varies over several decades. Persistent currents are considered metastable in this theory.<sup>47</sup> For a flow velocity not much less than  $\mathbf{v}_{c}(T)$ , the fluctuations that cause the dissipation occur so rarely that the decay times of the persistent currents are inaccessibly long.

We use the above theory to analyze the level difference dependence of our transfer rates, but we cannot use it to predict the actual magnitude of  $v_g$  because the theory has not been developed in sufficient detail. To actually calculate expected values for  $v_g$  in the film, we use an empirical rule called the "Leiden Rule," so called because it was proposed by van Alphen, <u>et al.</u>, at Leiden. The rule

is a summary of the results of many experiments in which  $v_s$  was measured for flow in channels of different sizes. It is stated in terms of a critical velocity  $v_{s,c}$ . The empirical relationship is<sup>10</sup>

$$v_{s,c} = d^{-1/4}$$
, cgs units, (15)

where d is the channel width. The rule fairly accurately describes the results of many experiments with channel widths varying over several orders of magnitude, from  $10^{-7}$  cm to  $10^{-3}$  cm. The fact that the rule is expressed in terms of a critical velocity means that the measurements were made for experimentally small driving pressure differences, say, of the order of 1 mm He pressure head, which is equivalent to about 14 dynes/cm<sup>2</sup> pressure.

D. Theoretical Expression for the Transfer Rate If we combine Equations 1, 6, and 15 we obtain the result

$$\sigma = \frac{\rho_{\rm R}}{\rho} d^{3/4}({\rm H}) = \frac{\rho_{\rm R}}{\rho} k^{3/4} {\rm H}^{-1/4} .$$
(16)

This result has been experimentally verified to correctly describe the transfer rates for small level differences over clean glass.<sup>15-18</sup> In this case the value of k for glass was determined by an independent experiment<sup>19</sup> to be  $3 \times 10^{-8}$  cm<sup>4/3</sup>.

Equation 19 requires several comments. First, there is a question of what experimental parameter to use for H, the height that determines the film thickness at the top of the beaker, since there are really two different heights involved when there is a level difference. The usual choice in the literature is the smaller height, that is, the distance from the source He level to the top of the beaker. It has recently been suggested that the mean height may be a better choice,<sup>40</sup> and one might even argue that the larger height should be used for H, since it gives the thinnest film and therefore the highest velocity of superflow. This point will be discussed later in relation to our experimental results.

The empirical relationship of Equation 15 that is used in Equation 16 is intended to apply to He flowing at the critical velocity, as was discussed earlier. Since 2 = 1 mm is approximately the smallest level difference for which we have transfer rate data, we shall compare Equation 16 to our results for 1 mm level difference.

The final point to discuss with respect to Equation 16 is that all the dependence on H is not explicitly displayed. Since in the Lifshitz result for the van der Waals potential k varies slightly, k has effectively some dependence of H. Unfortunately, there is no simple analytic expression for the dependence. We derive an approximate simple relation in the following way. We take the values for d(h) from Table 1 and plug them into Equation 16, so that we obtain a table of  $\sigma/(\rho_g/\rho)$  vs. H. Then we do a two parameter least squares fit of the equation  $\sigma/(\rho_g/\rho) = a H^{-n}$  to the Table. The result is that to a very high degree of accuracy (better than 0.2%) we can represent the predicted transfer rates by the equation

$$\sigma = 5.05 \ (\rho_s / \rho_s) \ H^{-0.21} \times 10^{-5} \ cm^2/sec.$$
 (17)  
Since the van der Waals potential theory that ignores retardation  
predicts n = -0.25, and the Lifshitz theory in the long distance

limit predicts n=-0.1875, we can see that the best fit value for n is intermediate, as would be expected. Equation 17 is of course only valid for the range of interest, in which H is between 0.1 cm and 2.5 cm.

### III. EXPERIMENTAL PROCEDURE

## A. Description of the Apparatus

The procedure that we chose to follow was to make free-standing beakers of solid Ar and then to observe the He II film transfer visually with a cathetometer. Preliminary experiments indicated that it should be possible to make such a beaker by slowly freezing liquid Ar in a mold under its own vapor pressure. Because Ar has a high triple-point vapor pressure, 517 Torr, it is then fairly easy to free the solid Ar from the mold walls by just slowly pumping Ar vapor from the mold, causing some of the solid to sublime away. The solid tends to sublime fastest where it is in contact with the mold, thus freeing the beaker. The apparatus that we built enabled us to control the freezing of the liquid Ar in a mold and then to remove the mold, leaving the beaker standing in place. We could then carry out transfer experiments with the beaker.

A cross-section of the apparatus that we used is shown in Figure 2. The walls of the chamber in which the experiments were carried out are made of glass, with Kovar metal to glass graded seals at both ends. All of the Kovar seals used in the apparatus were obtained from the Kontes-Martin Company, Evanston, Illinois. They are used to match the thermal contraction properties of borosilicate (Pyrex) glass to metal. Kovar metal is an alloy of iron, nickel, and cobalt.<sup>49</sup> It is joined to a special kind of glass which matches its thermal



Figure 2: A sketch of the apparatus. The upper part of the mold may be raised and lowered from outside the cryostat.

contraction properties fairly well. This glass is joined to a band of a second special glass, and this second glass is finally joined to ordinary Pyrex glass. To make the glass wall of the experimental chamber, and the outermost glass wall of the exchange gas chamber, two graded seals were taken in each case and joined by the MSU Glass Shop. The inner wall has an o.d. of  $1\frac{1}{2}$  inches and the outer wall has an o.d. of 2 inches. Since there is distortion in the glass in the graded region, within about  $1\frac{1}{2}$  inches of the glass bead around the Kovar, and also in the joint made by the MSU Glass Shop. care was taken in designing the apparatus so that the line of sight to the beaker would be through distortion-free glass.

The inner glass wall forms the wall of the experimental chamber, in which the beakers are made. The Kovar sections at both ends of the inner wall are soft-soldered into brass flanges. Screws hold the bottom of the chamber, called the brass block in the drawing, to the lower flange and the joint is sealed with an indium O-ring. The brass block has its unusual shape in order to lift the bottom of the beaker, which rests on it, above the section of the glass wall that is uneven and causes distortion. The upper flange is soldered to a thin-walled (0.010 inch wall thickness) 3/4 inch o.d. stainless steel tube, which supports the chamber.

Between the inner glass wall and the outer glass wall is an annular exchange gas region. This annular space can be evacuated or filled with an appropriate amount of exchange gas as necessary to help in controlling the temperature of the experimental chamber. The Kovar sections at the ends of the outer glass wall are both soldered into demountable flanges made of brass. These flanges are

also held together with screws and sealed with lead O-rings made from Buss fuse wire.

A needle valve is mounted on top of the uppermost flange and is connected by an 1/8 inch o.d. stainless steel tube to the experimental chamber. The valve is used to admit liquid He to the chamber from the surrounding bath. The valve body is soldered to the upper brass flange and the stainless steel tube is soldered to the top of the experimental chamber to seal the hole through which it passes. The needle valve seat is part of the valve body, which is made of brass and was fabricated in the Physics Department Machine Shop. The meedle is made of stainless steel and has a  $15^{\circ}$  taper where it fits in the seat. The meedle and valve body are threaded so that the valve may be closed by turning the needle. A 1/4 inch thin-walled stainless steel tube is mechanically attached to the needle and extends through a rubber 0-ring on top of the cryostat to a knob that is turned to open and close the valve.

The upper part of the mold in which the Ar beakers are formed consists of two glass tubes. The outer tube has an o.d. of 3/8 inch and is attached at its top to another glass to Kovar graded seal. The Kovar is soldered into a brass collar, which serves as a spacer to keep the mold centered in the chamber and which slides in the stainless steel tube. The inner glass tube forms the bore of the beaker. It is made from standard 3 mm tubing obtained from the MSU Glass Shop. It is sealed at its lower end and there is a loop of heater wire of about 15  $\Omega$  resistance in it. The inner tube is supported at its top in the brass collar by an arrangement of two wires and also in the outer tube by a Teflon spacer. This spacer is

above the point to which the mold is filled with condensed Ar. A 1/4 inch o.d. thin-walled stainless steel tube is soldered to the top of the brass collar. It slides through a rubber O-ring at the top of the cryostat (at room temperature) and is used to move the upper part of the mold up and down as well as to admit Ar gas to the inside of the mold.

The base of the mold is made of Teflon. It has a conical seat with half angle 5° into which the upper part of the mold is pressed to form a seal. The lower, sealing edge of the mold tube must be ground flat to make the seal gas tight. The hollow spaces machined in the base fill with solid Ar, which helps to support the beaker and to hold the beaker down when the upper part of the mold is pulled away. A threaded post on the bottom of the Teflom base screws into a hole in the brass block beneath it. This holds the base in place. The base is slightly loose in the hole so that it can move and tilt a bit as the mold is fitted together, making it easier to obtain a good seal. The bottom of the base is coated with Apiezon N vacuum grease to improve thermal contact with the brass block.

The apparatus is supported by the stainless steel tubes in a fairly standard double dewar cryostat. The dewars are glass and have unsilvered viewing strips. The inner dewar may be sealed and evacuated to reduce the temperature of the liquid He bath in it. A Meraeus-Engelhard #E225, air-cooled, 147 cfm vacuum pump is used to evacuate the dewar and a Cryometics Mark II mechanical pressure regulator is used to regulate the bath temperature. Temperatures in the liquid He range are determined by measuring the vapor pressure of the He in the bath with a manometer.

The Ar gas pressure in the mold is controlled manually by admitting or removing Ar gas through an external needle valve. A Wallace and Tiernan Model FA 145 pressure gauge is used to monitor the pressure. We use Matheson ultra-high purity grade Ar gas (purity 99.999%) in making the beakers.

The temperature of the mold is also controlled manually. There are two electric resistance heaters, one on the brass block at the bottom of the experimental chamber and one on the brass collar at the top of the mold. Both heaters are made of turns of nichrome wire that are attached and insulated with cigarette paper and Glyptal varnish. Their resistances are 80  $\Omega$  for the lower heater and 100  $\Omega$ for the upper heater. Current for the heaters is conducted down #40 varnished copper wires from a vacuum feed-through at the top of the cryostat. The current source is a 12 volt storage battery, with control obtained by adjusting variable resistances in series with the heaters. The heater currents are monitored with a milliammeter. A typical current for the lower heater under operating conditions is 30 mA. The upper heater was rarely used because the heat leak down the stainless steel tubes is usually enough to keep the collar sufficiently warm. The heater control box is wired to permit a maximum current of 100 mA in either heater.

Two miniature platinum resistance thermometers are used to monitor the mold temperature. One is placed near each heater. Both thermometers were obtained from Artronix Instrumentation, the lower one being a Model PS-3 and the upper one a PS-1. A drop of vacuum grease is used on each thermometer to enhance the thermal contact between it and the brass beneath it. The thermometers have similar

resistance characteristics, with resistances at  $0^{\circ}$  C of about 100  $\Omega$ and at 77 K of about 20  $\Omega$ . To measure the thermometer resistances, a 4-wire potentiometric method was used. The thermometers are wired in series and a 1 mA current run through them. Two separate leads are also run from each thermometer to a Leeds and Northrup K-3 potentiometer to measure the voltage drop across each thermometer and thereby determine the resistance. The thermometers were calibrated to the vapor pressure of liquid N<sub>2</sub> in the range 66 K to 77 K and the calibration lines extrapolated for use to 90 K. This yielded more than enough precision for this experiment.

While conducting film transfer experiments with a given beaker, it is necessary to remove the liquid He from the experimental chamber before transferring additional liquid He into the cryostat. This is done by pumping the He from the chamber with a mechanical vacuum pump. To prevent unwanted vapors from traveling back up the pumping line and contaminating the beaker surface, a cold trap is installed in the line. The trap contains granular Molecular Sieve and is cooled with liquid N<sub>2</sub>. A mercury bubbler is also installed in the line to prevent the chamber pressure from rising more than a few Torr above atmospheric pressure. The same cold trap prevents any mercury vapor from the bubbler from contaminating the beaker.

The experimental chamber also contains a He II fountain, not shown in Figure 2. The fountain is made from a section of 3 mm o.d. glass tubing about 8 cm long and is lashed to the side of the brass block that forms the bottom of the experimental chamber. The top of the tube is slightly constricted and bent to direct the He stream into the beaker. At the bottom of the tube, below the level of the

bottom of the beaker, is the heater that developes the pressure head for the fountain. The heater is a 10  $\Omega$ , 1/8 watt carbon resistor with the insulating coating sanded off. Below the heater, at the bottom of the tube, is a superleak. The superleak is a few mm of jeweller's rouge packed tightly into the tube, and held in place with small wads of cotton. When the fountain is used, both the heater and the superleak are under the surface of the liquid He in the chamber. A current of from 20 to 60 mA is applied to the heater, warming slightly the He in the tube that is superfluid. To restore equilibrium, superfluid flows into the tube through the superleak. Because of its viscosity, the normal fluid cannot flow out through the superleak at a significant rate, and enough of a pressure head to squirt the He out of the end of the fountain develops.

B. Procedure Used To Make the Ar Beakers

Before cooling down for a run, the apparatus and the gas handling plumbing are evacuated by pumping with a Consolidated Vacuum Corporation oil diffusion pump through a liquid N<sub>2</sub> cooled cold trap for at least 24 hours. It was usually possible to reduce the pressure in the system to between 1 and 3 x  $10^{-6}$  Torr, as determined with a cold-cathode ionization gauge, in this way.

In order to make a gas tight seal at the bottom of the wold, the glass tube must be pressed into the Teflon seat while the apparatus is at room temperature. Upon cooling, the Teflon, which has a larger coefficient of thermal expansion than glass, contracts tightly around the glass mold tube, but not so tightly that the seal cannot be pulled apart with a bit of tugging. We were never able to obtain

a good seal when the mold was mated after cooling the apparatus with liquid N<sub>2</sub>. The Teflon apparently becomes too hard at cryogenic temperatures to conform sufficiently well to the shape of the glass. We obtain consistently good results with seals that pass a leak test at room temperature. We conduct the leak test by filling the mold with He gas to a pressure of 700 Torr and monitoring the pressure in the chamber with a Pirani gauge. If the chamber pressure rises by about 1 millitorr per minute or less when the chamber is isolated under these circumstances, the seal is judged sufficiently good. If the pressure rise is greater, the mold is separated and remated, the experimental chamber reevacuated, and the leak test is conducted again.

During the time that the Ar is condensed and fromen, the apparatus is cooled by liquid  $N_2$  and exchange gas. The outer dewar of the cryostat is kept filled with liquid  $N_2$ , the inner dewar contains about 500 Torr of He exchange gas, and the annular exchange gas space is filled with 20 Torr of He gas. In this kind of situation, the bottom of the apparatus cools eventually to about 78 K.

Before Ar gas is introduced into the mold, the lower heater is turned on and the current is adjusted to bring the temperature of the mold to about 0.5 K above the Ar triple-point temperature of 83.8 K. With the experimental chamber evacuated, the temperature of the top of the mold tends to 88 - 90 K with the upper heater turned off.

Ar gas is admitted to the mold until a sufficient amount of liquid has condensed in it, typically to where the liquid level is about 3.5 cm above the top of the Teflon base of the mold. The Ar gas pressure in the mold is maintained at about 700 Torr while the

Ar is condensing, and about 30 millitorr of He gas is admitted to the experimental chamber to hasten the condensation. It takes about an hour to collect enough liquid Ar.

After the Ar gas supply to the mold is shut off, the experimental chamber is evacuated with the diffusion pump to as low a pressure as can be obtained. This ensures that the heat of fusion of the Ar as it freezes is carried away through the already solid Ar and the mold. In this way the freezing rate can be controlled and formation of new crystal grains on the mold walls is discouraged. A small reduction in the electric current in the lower heater causes the temperature of the bottom of the mold to fall slightly below the triple-point temperature, and the Ar begins to freeze from the bottom. After a few mm of solid have formed, a further reduction of the temperature of the bottom of the mold is begun until, after several hours, it has fallen to 78 K. It takes about 12 hours for all of the liquid Ar to freeze under these conditions. Since the thermal conductivity of glass is about twice that of Ar, the solid has a slightly concave upper surface.

After the last of the Ar has frozen, the temperature of the top of the solid begins to fall below the triple-point temperature and solid Ar begins to condense on top of the beaker directly from the Ar gas remaining in the mold and the Ar handling lines. In order to prevent this and the associated unevenness of the beaker top, Ar must be slowly pumped from the mold as the sublimation pressure of Ar corresponding to the temperature of the top of the beaker falls from the triple-point value of 517 Torr to 220 Torr at 78 K. The rate at which to remove Ar gas is determined visually by

pumping through the external needle valve and opening the valve just enough to allow only a very small amount of Ar frost to condense on the mold wall above the beaker. A few millitorr of He exchange gas is added to the experimental chamber to help bring the solid Ar to a uniform temperature. This pressure is gradually increased to 100 millitorr as the top of the solid Ar cools. The temperature gradient is almost completely removed in about an hour, but the beaker is allowed to stand in the mold for about 12 more hours before an attempt is made to remove the mold. This ensures that the solid is at a uniform temperature and that the experimenter is rested when the next phase begins.

The next step is to begin again slowly pumping Ar gas from the mold, but this time, since the Ar pressure is being reduced to below the sublimation pressure of the solid, the solid begins to sublime. The sublimation of some of the solid Ar cools the remaining solid and frees the Ar beaker from the outer glass tube. The solid is released from the outer tube both because the tube acts as a heat source to enhance the sublimation and because the solid contracts as it cools. It is necessary to use the heater in the inner glass tube of the mold during this time. The current in the heater is gradually increased from 20 mA to 40 mA. The amount of current needed in this heater is quite critical and was determined by trial and error. If too little heat is added, the Ar presses very tightly about the inner tube as it cools and contracts, causing the formation of fissures in the Ar. If too much heat is added, or if the current is turned up too fast, the temperature gradient and resulting vapor pressure gradient that is formed in the Ar around the inner

tube also causes the solid Ar to fissure. The fissures appear as white, feathery lines in the solid, and they grow as the solid is cooled further.

By pumping, the Ar pressure in the mold is reduced to about 0.5 Torr over a period of about an hour. This is as low as we can reduce the pressure with our pumping system since the remaining solid Ar, sitting in a 78 K environment, is subliming away rapidly at this pressure. By this time, if there has been enough heat added to the inner tube, the beaker is loose in the mold. The upper part of the mold is pulled out of the Teflon and up clear of the solid Ar, leaving the beaker standing free on the Teflon base of the mold. Liquid Ne is promptly transferred into the eryostat to cool the beaker to 4.2 K. At 4.2 K the Ar has a megligible sublimation pressure and the beaker may be kept indefinitely. We have stored beakers for over a week under liquid Ne.

Our beakers are typically 3.5 cm in height (measured from the top of the Teflon base), with 0.55 cm o.d., 0.30 cm i.d., and 2.5 cm bore length. The Ar is transparent, but usually has a few visible defects. Quite often one of these defects, or fissures, will penetrate the wall of a beaker, allowing it to leak He I and rendering it useless for transfer experiments. Figure 3 is a photograph of one of our beakers. The beaker is filling, and the liquid He levels inside and outside it can be seen. Some defects are visible in the solid Ar, especially below the end of the bore.

It is necessary to protect the beaker at all times from contaminating vapors with a liquid  $N_2$  cooled cold trap. The effect of surface contamination can be quite dramatic. The first leak free



Figure 3: Photograph of one of the Ar beakers. The beaker is filling, and the liquid He levels inside and outside it can be seen. beaker that we produced had no such protection, and as contamination collected during two days of conducting transfer experiments with the beaker, transfer rates were observed to increase by an order of magnitude. After installation of the cold trap, described earlier in the thesis, transfer rates remained completely consistent throughout the time that we kept any beaker.

#### C. Data Collection

We collect the transfer rate data in a very straightforward manner with a cathetometer and a stopwatch. To initiate a beaker filling we admit He from the bath through the needle valve into the chamber until the liquid level in the chamber is about 1 cm higher than it is inside the beaker. We initiate a beaker emptying by using the stream from the He II fountain to fill the beaker. We observe the liquid level in the beaker with a Wild-Heerbrug model KM326 cathetometer and record the height of the level every 30 seconds. The scale of the cathetometer can be read to ±0.001 cm, but because the cross-hairs of the cathetometer telescope must be placed on a moving and sometimes dim target, the actual measurement error is probably at least twice that.

We have collected data at several temperatures below the lambda-point. The temperature is determined to 0.01 K accuracy by measuring the vapor pressure of the He bath with an octoil filled manometer. By comparing the vapor pressure of liquid He in the experimental chamber with the bath pressure in a separate run, it was determined that the chamber is 0.01 K warmer than the bath in the relevant temperature range, this because the two are connected only through the exchange gas jacket. A corresponding correction

is used to obtain the experimental temperature from the measured bath temperature. It was also determined in the same separate run that, while using the fountain heats the chamber slightly, the chamber returns to its original temperature within two minutes, a megligible part of the hour or more it usually takes for a beaker emptying run.

Figure 4 is a plot of the liquid level inside the beaker as a function of time for three filling runs. The ordinate Z is the difference between the inner and outer levels in cm. Since the volume of the chamber is much greater than that of the beaker, the outer level may be taken to remain constant during a run. The quantity  $\sigma$  (cm<sup>3</sup>/ sec - cm of circumference), the volume transfer rate per unit length of the circumference of the beaker lip, is proportional to the slope of the Z(t) curve.

Differentiating the 2(t) data directly results in a great deal of scatter in  $\sigma$  (Z) because of the random errors in the height measurements. The scatter is reduced by using the following smoothing procedure: A line Z(t) = a t + b is fit to each set of 8 adjacent data points. The slope of the line is used to calculate the transfer rate associated with the center of the smoothing interval. The smoothing interval size of 8 points was chosen somewhat arbitrarily, but is consistent with the requirements that the interval be large enough to give a reasonably smooth set of experimental rates, but small enough that the expected real variations in  $\sigma$  are not concealed. The smoothing is done by using the MSU 6500 computer operating under a FORTRAN program called SIGMAZ, which is a revision of a program written by C, J. Duthler. A list of the program appears in



The He level difference as a function of time for three beaker fillings at three different heights H from the outer He level to the beaker rim. Figure 4:

Appendix  $\Lambda$ .

The geometric relation between the transfer rate and Z is just

$$\sigma = \frac{r}{2} \frac{dZ}{dt} , \qquad (18)$$

where r is the inner radius of the beaker. There is a slight flare in the i.d. of our beakers near the lip, so that the i.d. at the lip is usually about 7% greater than r. The flare is never more than 2 or 3 mm long. This flare is caused by our technique of freeing the solid from the mold. When we pump Ar gas from the mold, solid Ar naturally sublimes first from the top of the beaker, with the result in the end that less solid remains there. Below the top few millimeters the i.d. of the beaker is actually remarkably constant. For purposes of our analysis of the experimental results, as given in the following section, we neglect the flare. The radius r is measured to within 3% with a horizontal cathetometer. Corrections are made for the index of refraction of the solid Ar, for which we used the value 1.27 determined by Marcoux<sup>50</sup>, and for the optical effects of the glass dewar system, which we measured independently.

Figure 5 is a plot of the transfer rates derived from the raw data of Figure 4 through the use of the method described above. The solid curves in Figure 5 will be discussed in the next chapter.





## IV. RESULTS OF THE EXPERIMENT

## A. Reproducibility of the Results

In the course of many attempts, we succeeded in making five Ar beakers that did not leak liquid He I at an observable rate, and which were kept free of surface contamination. Of the transfer rates observed with these five beakers, the transfer rates from two of the beakers are the lowest and are in reasonably good agreement with each other. The results of the experiments conducted with these two beakers will be described in this chapter.

The other three beakers showed transfer rates from 15% to 50% higher than the ones we will report here. We cannot determine the reason for these higher rates for certain, but they may have been due to an increased microscopic surface area of the beaker walls or possibly due to tiny leaks in the walls, much too small to be seen with He I. We believe that surface contamination was not a factor in the increased rates for these beakers since considerable care was taken to prevent it. An indication that we were successful in preventing contamination is that the transfer rates for any given beaker were reproducible from day to day over the week long period that we kept the beaker at 4.2 K.

B. Results of Beaker Filling Experiments

In our data analysis we shall adopt the conventional procedure of using the source film height as the height that determines the

flow limiting film thickness. Because of this, we can use two equations in our analysis of the data collected from beaker filling experiments. Since during each filling the source film height H is constant, only the level difference dependence is important, and we may fit the transfer rate data from each filling to the two parameter form

$$\sigma = 1 / (A - B \ln 2), \qquad (19)$$

where Z is in cm and  $\sigma$ , as usual, is in cm<sup>2</sup>/sec. Equation 19 is equivalent to Equation 14 as long as the total chemical potential difference between the levels,  $\Delta \mu = \rho$  g Z, is proportional to  $\nabla \mu$ in the region of the film at the beaker lip where the dissipation is taking place. This point is discussed in more detail below. The solid curves in Figure 5 are least squares fits of Equation 19 to the corresponding transfer rates. We observed a capillary rise of about 0.07 cm of the He in the beaker in equilibrium, and we corrected our level differences by this amount when making the fits. The values of A and B determined from the results of each filling run are tabulated in Appendix B.

The magnitude of B that we determine from an average of all of our results for beaker fillings is  $B = (3.2 \pm 1.7) \times 10^3 \text{ sec/cm}^2$ . Scatter in B masks any dependence of it on temperature. This value for B agrees in order of magnitude with the results of other experimenters. Duthler and Pollack<sup>16-18</sup> determined a value for B that is about 30% of our value. From our B one can calcuate that the corresponding b(T) of Equation 14 is about 15. Cannon, Chester, and Jones<sup>40</sup> have found for b(T) temperature dependent values ranging from 30 to 70 in the temperature range of our experiment. The range of 2 over which observations were made in our experiment is too small to make this experiment a very good one for determining B.

Since the transfer rates predicted by Equation 17 are appropriate to small level differences, when  $v_s$  may be expected to be given by  $v_{s,c}$  of Equation 15, we deduce from the fitted curve for each beaker filling the transfer rate corresponding to 1 mm level difference. This is just

$$\sigma (2=0.1cm) = 1 / (A - B \ln(0.1)) , \qquad (20)$$

We choose this level difference because it does not involve extrapolating the fitted curve for  $\sigma$  beyond the range of our data.

Figure 6 is a plot of our results for beaker filling experiments carried out at 1.66 K for  $\sigma$  at 1 mm level difference vs. the height H from the outer He level to the top of the beaker. The solid curve is a fit of the form

to the data points. This is the second equation that we use in our analysis of the beaker filling data. The values of the parameters for the curve in Figure 6 are  $\sigma_0 = 4.8 \times 10^{-5} \text{ cm}^2/\text{sec}$  and n = 0.21. The rms deviation of the data points from this curve is  $0.5 \times 10^{-5} \text{ cm}^2/\text{ sec}$ .

We have collected beaker filling data at several other temperatures and have fit the rates for 1 mm level difference at each temperature to Equation 21. The results in terms of  $\sigma_0$  and n are summarized in Table 2, which appears at the end of the next section. The other entries in the Table will be discussed later.





C. Results of Beaker Suptying Experiments

The transfer rates obtained from three typical beaker emptying experiments are plotted in Figure 7. The variable  $H_I$  is the height from the He level inside the beaker to the lip, which increases as the beaker empties. In the case of a beaker emptying, both the level difference and the source film height change simultaneously. Therefore, we cannot separate the two dependences into two equations as we did for beaker fillings. Instead, we fit each beaker emptying to the two- parameter form

$$\sigma = H_{T}^{-0.21} / (A - B \ln 2)$$
 (22)

This expression contains both the film height and the level difference dependence. As with the beaker fillings, we make an additive correction of 0.07 cm to the level difference Z to compensate for the observed capillary rise of the Ne inside the beaker. The solid curves in Figure 7 are fitted curves of Equation 22 to the data. The best fit values of A and B for the beaker emptying experiments are given in Appendix B.

In order to compare the beaker emptying rates with the beaker filling rates, we calculate, using the parameters A and B from each emptying, the transfer rate  $\sigma_0$  for 1 mm level difference and 1 cm film height. The mean values of  $\sigma_0$  for the beaker emptying experiments are displayed in column 6 of Table 2. All of the values of  $\sigma_0$  obtained are tabulated in Appendix B. The best fits result in a value of B for the beaker emptyings of (4.8 ± 2.0) x 10<sup>3</sup> sec/cm<sup>2</sup>, somewhat higher than, but overlapping with, the value determined from the beaker filling data.





		)				
	Theory (Equation 17)	F1111	g Experime	ents	Emptying E	xperiments
	σ <sub>0</sub> × 10 <sup>5</sup>	σ <sub>0</sub> × 10 <sup>5</sup>		Number	مي × 10 <sup>5</sup>	Number
<b>T(K)</b>	(cm <sup>2</sup> / sec)	(cm <sup>2</sup> / sec)	<b>[</b> ]	or Fillings	(cm <sup>2</sup> / sec)	or Emptyings
1.99	2.28	3.2	0.41	9	1	;
1.77	3.54	3.9	0.26	9	3.1	2
1.66	3.99	4.8	0.21	21	4.3	12
1.47	4.59	4.7	0.22	11	4.3	£

TABLE 2

Comparison of the experimental results for beaker filling and emptying rates with the theoretical predictions. The transfer rates  $\sigma_0$  correspond to 1 mm level difference and 1 cm film height.

#### D. Discussion of the Results

As can be seen from Figures 5 and 7, the level difference dependence of the transfer rates we have measured is described well by fits to the dependence given in Equation 19. This is especially apparent in the case of beaker emptyings, shown in Figure 7. In this case, as long as the source height determines the film thickness, without any level difference dependence, the emptying rates should be independent of the height H from the outer level to the beaker lip. A rather sharp drop in the transfer rate for the emptyings can be seen as the inner level approaches its equilibrium value, and this is also matched quite well by the fit.

We made some effort in analyzing the data to determine whether the source film height or the mean of the inner and outer heights was more appropriate to use in determining the film thickness at the beaker top. In the case of beaker fillings, there is generally not enough difference in the results of the two models to make a determination since, for small level differences, the source height is almost equal to the mean height. Near the start of a beaker emptying, however, the source height is much less than the mean height. In this case, using the source height rather than the mean height in the fitting procedure seemed to result in somewhat better fits to the experimental data. Because of this, and because the source height has generally been used by other researchers, we continue to use it in our analysis of the results.

If one compares column 6 of Table 2 with column 3 of the same Table, one sees that the transfer rates for beaker emptyings were consistently lower than the corresponding rates for beaker fillings.

1 i This may be a result of the geometry of the beaker in the sense that the flair in the i.d. at the top may cause an extension of the region in which the dissipation takes place. Keller and Hammel<sup>4,36</sup> have reported seeing a dependence of the transfer rates for beaker emptyings on the distance in the film in which dissipation was taking place. If the distance in the film over which the chemical potential changes is increased, then  $\nabla \mu$  in the region of dissipation is decreased for a given total  $\Delta \mu$ . By Equation 14, a smaller  $\nabla \mu$  implies a smaller v<sub>g</sub>, and therefore, a lower transfer rate. The flare may encourage the region of dissipation to be longer since the crosssectional area of the film is almost constant on the inner wall of the beaker for a few millimeters near the lip. This is because the increased circumference mearly compensates for the decreased film thickness with height near the top of the i.d.

From Table 2, the temperature dependence of the observed transfer rates is somewhat weaker than Equation 17 predicts, although the observed rates do decrease substantially at the higher temperatures. We do not have very much data at some of the temperatures, so the temperature dependence that we obtain cannot be taken too seriously. We did not have enough good samples and, because we ran out of liquid He on some occasions, were not able to retain the ones we had long enough to really concentrate on more than one temperature.

The He II film transfer rates that we observed using Ar beakers are, to the best of our knowledge, the lowest that have been observed by anyons for the same temperatures and film heights. They are about 10% lower than the rates of Allen and Armitage<sup>15</sup> and 40% lower than

Duthler and Pollack<sup>16-18</sup>, all of whom measured transfer rates for carefully cleaned glass. This is of course consistent with the fact that Ar has a weaker van der Waals attractive potential than glass.

Our transfer rates are generally higher than the theoretical results, but, as film transfer experiments go, are in fairly good agreement with theory. It is to be expected that our experiment should really give only an upper bound to the transfer rate, since the beaker surfaces probably have some microscopic roughness. It would be surprising if the process of subliming away a significant fraction of the solid surface did not produce some surface irregularities. The agreement with theory indicates that the surface must be fairly smooth.

## V. SUMMARY AND CONCLUSIONS

In this experiment we have studied the He II film transfer effect over a noble gas substrate. Since the transfer occurs as a flow of He through the He film adsorbed on the substrate surfaces, the experiment gave us an indirect measure of the adsorbed film thickness. The beakers used in performing the experiments were made entirely of the noble gas material, solid Ar, and it was possible to produce substrate surfaces that were fairly smooth in this way. Because the substrate surfaces were never exposed to contaminants, we expect that they were also very clean.

The results of our experiments yielded dependences of the transfer rate on level difference and film height in agreement with the results of previous researchers. The weak level difference dependence agrees in form with that of other experiments and recent theory. The magnitude of the level difference dependence is also in reasonable agreement when one considers the small size of the dependence under our experimental conditions. The form of the film height dependence is also in agreement with the usual theory of the Me film thickness.

The most important result of the experiment is that our data are in agreement with the Lifshits theory of the van der Waals attractive potential. This theory includes retardation and screening effects. Results of the Lifshits theory were used by us to calculate

theoretical values for the adsorbed He film thickness. To obtain accurate results, the theory requires only a rough knowledge of the frequency-dependent dielectric susceptibility of the materials involved and some computer time to do the necessary calculations.

It is our hope that this study of the substrate dependence of the transfer effect will make a contribution to a better understanding of the phenomenon, and to He II and surface physics in general. APPENDICES

### TABLE A1

List of the FORTRAN program SIGMAZ used to derive the transfer rates from the raw data.

```
PROGRAM SIGMAZ (INPUT, OUTPUT, TAPE60)
      DIMENSION T(150), H(150), TS(150), Z(150), SIGMA(150)
      DIMENSION AF(30), AE(30), BF(30), BE(30), RF(30), RE(30), NPTSF(30)
      DIMENSION RUF(30), RUB(30), TF(30), TE(30), HF(30), HE(30), NPTSE(30)
  100 FORMAT (5F10.3)
  101 FORMAT (2F10, 3, 215)
  102 FORMAT (12F6.3)
  103 FORMAT (110)
  104 FORMAT (1H1,6X,4HRUN ,F5,2.6X,3HH= ,F6,3,6X,6HTEMP= ,F5,2,6X,4HR0=
     1 , F6.3, 6X, 3HG= , F6.0, 6X, *NPTS= *, I3, 6X, *NFIT= *, I3 ///)
  105 FORMAT (14X, 1HN, 11X, 5HT MIN, 11X, 4HH CM, 10X, 5HDELTA, 12X, 1HZ, 12X, 5HS
     1IGMA,11X,4HLN 2,12X,2HHI //)
  106 FORMAT (116, F15, 2, 3F15, 3)
  107 FORMAT (116, F15, 2, 6F15, 3)
  108 FORMAT ( ///6X.43HLEAST SQUARES FIT TO SIGMA= 1./(A-B*LN(Z))
                                                                        1)
  109 FORMAT (*
                  A= *, B12,4,5X,*B= *, B12,4,5X,*R= *, B12,4)
  110 FORMAT (/4X.*1/A= *,F8.3)
  111 FORMAT (1H1,* DATA FROM RUN NUMBER*, P6, 2, * IS IN THE WRONG GROUP
     1 *)
  112 FORMAT (1H1,10X,* SUMMARY OF BEAKER FILLING RUNS *,///.9X,3HRUN
     1,13X,1HH,13X,4HTEMP,10X,3H1/A,12X,1HB,14X,1HR,11X,4HNPTS /)
  113 FORMAT (/6X, F7.2, F15.3, F15.2, F15.3, 2B15.4, I10)
                          SUMMARY OF BEAKER EMPTYING RUNS *,///,9X, 3HRUN
  114 FORMAT (1H1.10X.*
     1,13X,1HH,13X,4HTEMP,10X,3H1/A,12X,1HB,14X,1HR,11X,4HNPTS /)
  115 FORMAT (///6X, 51HLEAST SQUARES FIT TO SIGMA= (H**=0.25)/(A=B*LN(2)
     1)
            116 FORMAT (F5.2, F9.3, F4.2, F6.3, F6.0, 13, 12, 3F15, 12)
  117 FORMAT (*100, *)
      IFL=0 $ IB=0
   13 READ 100. RUNI, RUNF, HTOP, G, TEMP
С
   NFIT WILL BE AUTOMATICALLY CHOSEN IF NOT DEFINED
   11 READ 101, RUN, HO, NPTS, NFIT
      READ 102, (T(I), H(I), I=1, NPTS)
      READ 103. LAST
      IF (RUN.LT.RUNI) GO TO 14
      IF (RUN.GT.RUNF) GO TO 14
      IF (NFIT.GT.0) GO TO 15
      NFIT= NPTS/3+1
      IF (NFIT-8) 15,15,16
   16 NFIT=8
```

Table Al (cont'd.) 15 DO 1 I=1.NPTS TS(I) = 60, \*T(I) $1 Z(I) = HO_H(I)$ JEND= NPTS-NFIT+1 \$ N2= NFIT/2 2 DO 6 J= 1. JEND ST=0.\$ ST2=0.\$ SZ=0.\$ STZ=0. IEND= J+NFIT-1 DO 7 I=J.IEND ST= ST+TS(1) \$ ST2= ST2+TS(1)\*\*2 S2=SZ+Z(I)7 STZ= STZ+Z(I)\*TS(I)  $B = (ST2_(ST*S2)/NFIT)/(ST2_(ST**2)/NFIT))$ I = J + N26 SIGMA(I)= ABS(G\*B) 5 HDIF=HTOP-HO PRINT 104, RUN, HDIF, TEMP, HO, G, NPTS, NFIT **PRINT 105** FE=H(2)-H(1)DELTA=0. \$ I=1 PRINT 106. I.T(I), H(I), DELTA, Z(I) NLINS=1 IF (N2.LT.2) GO TO 26 DO 8 I=2.N2DELTA= H(I)-H(I-1)NLINS=NLINS+1 8 PRINT 106, I,T(I),H(I),DELTA,Z(I) 26 SX=0,\$ SY=0,\$ SX2=0,\$ SY2=0,\$ SXY=0. N= N2+1 \$ IEND= JEND+N2 DO 9 I=N,IEND NLINS=NLINS+1 IF (NLINS.LT.51) GO TO 28 PRINT 104. RUN, HDIF, TEMP, HO, G, NPTS, NFIT PRINT 105 NLINS=1 28 DELTA=H(I)-H(I-1)HI = HTOP - H(I)IF (FB.GT.0) GO TO 22 ASIG= 1./(SIGMA(I)\*(HI\*\*0.25)) GO TO 23 22 ASIG= 1./SIGMA(I) 23 ALZ= ALOG(ABS(Z(I)))SX= SX\_ALZ \$ SY= SY+ASIG SX2= SX2+ALZ\*\*2 \$ SY2= SY2+ASIG\*\*2 SXY= SXY\_ALZ\*ASIG 9 PRINT 107, I,T(I),H(I),DELTA,Z(I),SIGMA(I),ALZ,HI IF (NLINS.LT.43) GO TO 29 PRINT 104. RUN. HDIF. TEMP. HO.G. NPTS. NFIT PRINT 105 29 IEND=IEND+1 IF (NPTS.LT.IEND) GO TO 27 DO 10 I=IEND, NPTS

Table Al (cont'd.) DELTA= H(I)-H(I-1)10 PRINT 106, I,T(I),H(I),DELTA,2(I) 27 N= IEND-N  $\mathbf{B} = (\mathbf{S} \mathbf{X} \mathbf{Y}_{-} (\mathbf{S} \mathbf{Y}^{*} \mathbf{S} \mathbf{X}) / \mathbf{N}) / (\mathbf{S} \mathbf{X} \mathbf{2}_{-} (\mathbf{S} \mathbf{X}^{*} \mathbf{2}) / \mathbf{N})$  $A = (SY - B \times SX)/N$ **R= (SXY-(SX\*SY)/N)/SURT(SX2\*SY2-(SX\*\*2)\*SY2/N-SX2\*(SY\*\*2)/N** 1 +(SX\*SY/N)\*\*2) IF (FE.GT.O) GO TO 24 **PRINT 115** GO TO 25 24 PRINT 108 25 PRINT 109, A.B.R WRITE(60,116) RUN, HTOP, TEMP, HO, G, N, NFIT, A, B, R С N = THE NUMBER OF DATA POINTS WRITTEN ON 60 N=N2+1 \$ IEND=JBND+N2 WRITE(60,102) (H(I), SIGMA(I), I=N, IEND) A=1./A PRINT 110, A IF (FE) 17.17.18 17 IE=IE+1 RUB(IE)=RUN \$ TE(IE)=TEMP \$ HE(IE)=HDIF \$ AE(IE)=A BB(IE)=B \$ RE(IE)=R \$ NPTSE(IE)=NPTS GO TO 19 18 IFL= IFL+1 RUF(IFL)=RUN \$ TF(IFL)=TEMP \$ HF(IFL)=HDIF \$ AF(IFL)=A BF(IFL)=B \$ RF(IFL)=R \$ NPTSF(IFL)=NPTS GO TO 19 14 PRINT 111, RUN 19 IF(LAST) 12.11.13 12 PRINT 112 DO 20 I=1.IFL 20 PRINT 113, RUF(I), HF(I), TF(I), AF(I), BF(I), RF(I), NPTSF(I) PRINT 114 DO 21 I=1,IE 21 PRINT 113. RUE(I), HE(I), TE(I), AE(I), BE(I), RE(I), NPTSE(I) WRITE(60,117) END

# APPENDIX B: TABULATION OF EXPERIMENTAL DATA

# TABLE B1

H	Temperature	. <b>A</b>	В	$\sigma$ (Z=lmm)
(cm)	<u>(K)</u>	$(10^4 \text{ sec/cm}^2)$	$(10^4 \text{ sec/cm}^2)$	$(10^{-5} \text{ cm}^2/\text{sec})$
1.279	1.67	1.92	0,309	4.06
1.852	1.66	2.14	0 <b>.29</b> 5	3.75
0.581	1.66	1.59	0.232	4.99
1.856	1.66	2.13	0.224	3.95
1.024	1.66	1.91	0.242	4.28
0.247	1.66	1.41	0.144	5,99
2.120	1.66	2.09	0.298	3,82
1.219	1.66	1.86	0.373	3.97
0.364	1.66	1.44	0.341	4.89
2.163	1.77	2.13	0.645	3.06
0.951	1.77	1.92	0.371	3.88
0.251	1.77	1.37	0,248	5,53
1.598	1.77	2.11	0.404	3.54
0.804	1.77	1.75	0.305	4.37
0.260	1.77	1.33	0.258	5,58
2.021	1.47	2.14	0.367	3,59
0.953	1.47	1.72	0.339	4.31
0.227	1.47	1.33	0,231	5.75
1.698	1.47	1.93	0.282	4.11
0.608	1.47	1,58	0.245	4,98

Results of beaker filling runs made with Beaker 3.

# TABLE B2

Results of beaker filling runs made with Beaker 4.

R	Temperature	<b>A</b>	B	$\sigma$ (2=1mm)
(cm)	<u>(K)</u>	$(10^4 \text{ sec/cm}^2)$	$10^4 \text{ sec/cm}^2$	$\frac{(10^{-5} \text{ cm}^2/\text{sec})}{10^{-5} \text{ cm}^2/\text{sec}}$
1.840	1.66	1.88	0.227	4.38
0.810	1.66	1.49	0.114	5, <b>92</b>
0.307	1.66	1.16	0.253	6.20
2.156	1.66	2.03	0.215	4.15
1.260	1.66	1.55	0,191	5.29
0.395	1.66	1.33	0.148	6.30
1.700	1.66	1.80	0,157	4.80
0.853	1.66	1.54	0.584	6.10
0.177	1.66	1.08	0.171	7.22
1.929	1.66	1.92	0.232	4.30
1.084	1.66	1.50	0.152	5.66
0.325	1.66	1.22	0.155	6.69
1.872	1.99	2.76	0.652	2,55
1.003	1.99	2.36	0.325	3.41
0.283	1.99	1.62	0.163	5.24
2.094	1.99	2.97	0.814	2 <b>.2</b> 7
1.246	1.99	2.63	0.608	2.70
0.527	1.99	2.02	0.224	4.13
1.715	1.46	1.76	0.355	4.19
0.867	1.46	1.50	0.125	5.80
0.300	1.46	1.24	<b>0.2</b> 07	6.24
2.006	1.46	1.93	0.304	4.05
1.147	1.46	1,52	<b>0.28</b> 0	4.96
0.453	1.46	1.25	0,201	6.21

# TABLE B3

H (cm)	Temperature <u>(K)</u>	$\frac{A}{(10^4 \text{ sec/cm}^2)}$	$\frac{B}{(10^4 \text{ sec/cm}^2)}$	(10-5 cm <sup>2</sup> /sec)
1.278	1.67	1.92	0.571	3.41
1.850	1.66	1.96	0,412	3.72
1.849	1.66	1.85	0.592	3,45
2.753	1.66	1.85	0.300	<b>4.2</b> 0
2.076	1.66	1.95	0,566	3.39
1.202	1.66	1.90	0.673	3.23
2.159	1.77	1.99	0.861	2.85
1.590	1.77	2.09	0,509	3.34
2.007	1.47	1.74	0.368	4.18
2,601	1.47	1.71	0.279	4,54
1.696	1.47	1.72	0.423	4.04

Results of beaker emptying runs made with Beaker 3.

# TABLE B4

Results of beaker emptying runs made with Beaker 4.

H (cm)	Temperature (K)	$\frac{A}{(10^4 \text{ sec/cm}^2)}$	$\frac{B}{(10^4 \text{ sec/cm}^2)}$	$\frac{\sigma_0}{(10^{-5} \text{ cm}^2/\text{sec})}$
2.621	1.66	1.56	0.304	4.76
2,154	1.66	1.53	0.287	4.90
1.261	1.66	1.53	0.211	5.24
2.648	1.66	1.56	0,285	4.84
1.926	1.66	1,51	0.267	5.04
1.084	1.66	1.46	0 <b>.2</b> 89	5.07

LIST OF REFERENCES

•



### LIST OF REFERENCES

- 1. F. London, Superfluids, Vol. II (Wiley, New York, 1954).
- L. Landau, Phys. Rev. <u>60</u>, 357 (1941); J. Phys. (USSR) <u>5</u>, 71 (1941).
- 3. L. Tissa, Nature 141, 913 (1938).
- 4. W. E. Keller, <u>Helium-3 and Helium-4</u> (Plenum Press, New York, 1969).
- 5. C. T. Lane, Superfluid Physics (McGraw-Hill, New York, 1962).
- 6. K. R. Atkins, <u>Liquid Helium</u> (Cambridge University Press, Cambridge, England, 1959).
- 7. J. Wilks, Liquid and Solid Welium (Clarendon Press, Oxford, England, 1967).
- 8. R. J. Donnelly, <u>Experimental Superfluidity</u> (University of Chicago Press, Chicago, 1967).
- 9. B. V. Rollin and F. Simon, Physica 6, 219 (1939).
- W. M. van Alphen, G. J. van Haasteren, R. De Bruyn Ouboter, and K. W. Taconis, Phys. Letters 20, 474 (1966).
- 11. J. G. Daunt and K. Mendelssohn, Proc. Roy. Soc. A170, 423 (1939); A170, 439 (1939).
- 12. L. I. Schiff, Phys. Rev. 59, 839 (1941).
- 13. K. R. Atkins, Proc. Roy. Soc. A203, 240 (1950).
- 14. R. Bowers and K. Mendelssohn, Proc. Phys. Soc. A63, 1318 (1950).
- 15. J. F. Allen and J. G. M. Armitage, Phys. Letters 22, 121 (1966).
- 16. C. J. Duthler and G. L. Pollack, Phys. Letters 31A, 390 (1970).
- 17. C. J. Duthler and G. L. Pollack, Phys. Rev. A 3, 191 (1971).
- C. J. Duthler, <u>Dependence of the Helium II Film Transfer Rate</u> of Pressure Head, Film Height, and Substrate, Ph.D. Thesis, Michigan State University (1970).

- 19. A. C. Ham and L. C. Jackson, Proc. Roy. Soc. A240, 243 (1957).
- 20. G. L. Pollack, Rev. Mod. Phys. 36, 748 (1964).
- 21. E. S. Sabisky and C. H. Anderson, Phys. Rev. A 7, 790 (1973).
- 22. M. Gsänger, H. Egger, G. Fritsch, and E. Lüscher, Z. Angew. Phys. <u>26</u>, 334 (1969).
- 23. C. W. Leming and G. L. Pollack, Phys. Rev. B 2, 3329 (1970).
- 24. Ref. 4, Chapter 8.
- 25. M. Chester, L. C. Yang, and J. B. Stephens, Phys. Rev. Letters 29, 211 (1972).
- 26. E. S. Sabisky and C. H. Anderson, Phys. Rev. Letters <u>30</u>, 1122 (1973).
- 27. J. Frenkel, J. Phys. (USSR) 2, 365 (1940).
- 28. F. London, Z. Physik 63, 245 (1930).
- 29. H. B. G. Casimir and D. Poulder, Phys. Rev. 73, 360 (1948).
- 30. B. M. Lifshitz, Zh. Eksp. Teor. Fiz. 29, 94 (1955) (Sov. Phys.-JETP 2, 73 (1956)).
- 31. I. E. Dzyaloshinskii, E. M. Lifshitz, and L. P. Pitaevskii, Adv. Phys. <u>10</u>, 165 (1961).
- 32. V. A. Parsegian and B. W. Ninham, J. Chem. Phys. <u>52</u>, 4578 (1970); Biophys. J. 10, 646 (1970); Biophys. J. 10, 664 (1970).
- 33. Ref. 4, Chapter 3.
- 34. R. J. Good and C. J. Hope, J. Chem. Phys. 53, 540 (1970).
- 35. Ref. 5, Chapter 8.
- 36. M. E. Keller and E. F. Hammel, Physics 2, 221 (1966).
- 37. H. A. Notarys, Phys. Rev. Letters 22, 1240 (1969).
- 38. D. H. Liebenberg, J. Low Temp. Phys. 5, 267 (1971).
- 39. D. J. Martin and K. Mendelssohn, J. Low Temp. Phys. <u>5</u>, 211 (1971).
- 40. W. C. Cannon, M. Chester, and B. K. Jones, J. Low Temp. Phys. 9, 307 (1972).
- 41. J. R. Clow and J. D. Reppy, Phys. Rev. A 5, 424 (1972).

- 42. D. R. Williams and M. Chester, Phys. Rev. A 4, 707 (1971).
- 43. S. V. Iordanskii, Zh. Eksp. Teor. Fiz. <u>48</u>, 708 (1965) (Sov. Phys. JETP 21, 467 (1965)).
- 44. J. S. Langer and M. B. Fisher, Phys. Rev. Letters <u>19</u>, 560 (1967).
- 45. R. J. Donnelly and P. H. Roberts, Proc. Roy. Soc. <u>A312</u>, 519 (1969).
- 46. P. H. Roberts and R. J. Donnelly, Phys. Rev. Letters 24, 367 (1970).
- 47. J. S. Langer and J. D. Reppy, in <u>Progress in Low-Temperature</u> <u>Physics</u>, edited by C. J. Gorter (North-Holland, Amsterdam, 1970), Vol. 6, p. 1.
- 48. M. Chester and R. Ziff, J. Low Temp. Phys. 5, 285 (1971).
- 49. G. K. White, Experimental Techniques in Low-Temperature Physics, (Clarendon Press, Oxford, England, 1968), Chapter 10.
- 50. J. Marcoux, Can. J. Phys. 48, 1947 (1970).
- 51. T. O. Milbrodt and G. L. Pollack, Phys. Rev. A (to be published).

ł ł

