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**A STUDY OF THE EFFECT OF A TANGIBLE AND CONCEPTUALIZED  
PRESENTATION OF ARITHMETIC ON ACHIEVEMENT  
IN THE FIFTH AND SIXTH GRADES**

**BY**

**Patricia McHitt Spross**

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\* \*  
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**A DISSERTATION**

**Submitted to the School of Graduate Studies  
of Michigan State University in partial  
fulfillment for the degree of**

**DOCTOR OF EDUCATION**

**School of Education**

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1. The first part of the report is a general introduction to the subject of the study. It discusses the importance of the problem and the objectives of the research. It also mentions the scope of the study and the methods used.

2. The second part of the report is a detailed description of the experimental work. It includes a description of the apparatus used, the procedure followed, and the results obtained. It also discusses the errors and limitations of the experiment.

3. The third part of the report is a discussion of the results. It compares the results with the theoretical predictions and with the results of other experiments. It also discusses the implications of the results and the conclusions drawn from them.

4. The fourth part of the report is a summary of the work. It briefly reviews the main points of the report and states the conclusions. It also mentions the acknowledgments and the references.

Final examination

Dissertation

Outline of

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**Final examination: May 7, 1962, 1:00 P.M., 304 Education Building**

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Presentation of Arithmetic on Achievement in the Fifth  
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**BY**

**Patricia McNitt Spross**

**AN ABSTRACT**

**Submitted to the School of Graduate Studies  
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**School of Education**

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**Approved** *Irving L. Stearns*

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This study was concerned with the effect of a tangible and conceptualized presentation of arithmetic to children in the fifth and sixth grades as compared to a routine classroom presentation.

The regular arithmetic course of study of the school system was divided into concepts or mathematical ideas. To these were added enough additional concepts taken from pure mathematics theory to explain the arithmetic processes regularly taught in these grades. Tangible manipulative items that had cultural significance were used in developing the concepts whenever possible. The regular arithmetic text was used as a reference book only. In addition the children were supplied with other reference materials as they requested them.

Only one concept was presented each week in a discussion period in which the teacher acted as a resource person. Major points of interest relating to the topic were identified, and the means of developing these were planned by the students. The students reacted to the concept by producing some tangible item of their own choosing that they felt would represent their understanding of the particular mathematical idea for the week. The students were not given an assignment of problems to work in any text.

Class periods were limited to thirty-five minutes per day. There was no ability grouping, no individual help, no homework, or work at any other time. The lessons included no "drill".

Once a week students were given the opportunity to work as far as possible in a series of problems of gradual difficulty. These were corrected by the teacher and returned to them. This was done in order to keep the children and the patrons of the school informed of the class'

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progress. No student was ever required to complete these problems, only to do as many as possible in the class time.

The tangible and conceptualized method was compared to the routine self-contained classroom presentation in which the arithmetic text structured the program and was worked through as a method of completing the arithmetic course. The self-contained rooms were not prohibited from ability grouping, individual help, drill and rote learning methods, home work, direct correlation with other subjects, or any other method the teacher chose. These classrooms were informed that they were acting as control groups for the study.

There were 166 students in the study comprising 8 heterogeneous classes in two elementary schools. There were two experimental fifth grades and two fifth grade controls; likewise, two experimental sixth grades, and two sixth grade controls. The grades selected were deemed to be typical of the school population.

The relative effectiveness of the conceptualized and tangible presentation and the typical routine of the self-contained classroom was determined by students performance on standardized achievement tests. These were administered at the beginning, the mid-point and the end of the school year. Comparisons were made on the students' performance on achievement tests administered as part of the school routine. These were:

STEP Test - Form A

California Achievement Test - Elementary Arithmetic Form BB -  
Reasoning and Fundamentals

Tests given the students were treated by using the analysis of covariance model. F ratio showed that there existed a significant difference between experimental and control groups when measured by the STEP and California Achievement Test-Elementary Arithmetic-Form BB, reasoning. There was no significant difference found in California Achievement, fundamentals.



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Conclusions: It would appear as a result of the study that it would be possible for students to achieve essentially the same progress in arithmetic with an understanding of mathematical concepts and their use as they can in a routine classroom procedure in which rote methods of learning are employed. It would further appear that this can be accomplished in 35 minutes per day in a heterogeneous group in which there is no ability grouping, no individual help, and no additional work outside of the 35 minute class time when the burden of proof of learning rests with the student. It also would appear that it is possible for fifth and sixth grade children to understand sufficient mathematical theory as would give meaning to the arithmetic content at the fifth and sixth grade level.

CHAPTER

I. INTRODUCTION

II. REVIEW OF LITERATURE

III. DESCRIPTION OF THE STUDY

IV. ORGANIZATION OF THE STUDY

V. DATA ANALYSIS

VI. RESULTS AND DISCUSSION

VII. CONCLUSIONS

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## CHAPTER I

### INTRODUCTION

Improvement in the elementary curriculum has long been a part of educational research. Since 1900 much emphasis has been given to a consideration of methods for improving arithmetic programs.<sup>1</sup> Leaders in the field of education and in national mathematical organizations have given much thought to: (1) factors contributing to a need for an improved program; and, (2) the kind of program that will supply these needs. Price lists the factors which contribute to the need for change in mathematics curriculum as: (1) Research in mathematics; (2) Automation; and, (3) Automatic digital computing machines.<sup>2</sup> Advances in mathematics have created more profound mathematics, greater quantity of content, introduced new subject areas, and created additional and more extensive uses of mathematics. The sequential nature of mathematics makes it imperative that students and teachers possess a thorough knowledge of these continuing developments.<sup>3</sup>

Cultural implications for mathematics have arisen in the application of mathematics to the development of automation, which has in turn created the need for the production of high speed computers. In order to carry out the mechanization of automation, it has become

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<sup>1</sup>G. T. Buswell, "Arithmetic". Review of Educational Research, Chester W. Harris, Editor. New York: The Macmillan Company. 1960. P 62.

<sup>2</sup>G. Baily Price, "Progress in Mathematics and Its Implications for the Schools." The Revolution in School Mathematics. Washington: National Council of Teachers of Mathematics. 1961. Pp. 3-5.

<sup>3</sup>Ibid., p. 2.

... ..

necessary to design machines capable of computation at rates far beyond human abilities. The design, maintenance, and operation of such machines require the close association of mathematics with the fields of engineering and mechanics. Development of such processes have had a great influence on the way of life.<sup>4</sup>

Not only has content in mathematics changed, but areas of emphasis have changed. While it is still necessary to teach the traditional subject matter areas, they now receive less emphasis. Specific application of mathematics toward increased environmental control make a thorough understanding of the properties of functions just as necessary as a memorization of these processes. This is brought out by the Rockefeller report.

. . . we are moving with headlong speed into a new phase of man's long struggle to control his environment, a phase beside which the industrial revolution may appear a modest alteration of human affairs.<sup>5</sup>

Some idea of the scope and recency of the change may be obtained by considering a statement from the Fiftieth Yearbook of the National Society for the Study of Education which was published just nine years ago.

Arithmetic exhibits some marked contrasts when compared with some of the other content areas of education. Unlike chemistry, physics, and the social sciences, its content is not subject to radical changes due to discoveries. . . .<sup>6</sup>

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<sup>4</sup>Ibid., Pp. 4-5.

<sup>5</sup>Rockefeller Report. The Pursuit of Excellence, Panel Report V of the Special Studies Project. Garden City: Doubleday. 1958. P. 28.

<sup>6</sup>G. T. Bushnell, "Introduction." Fiftieth Yearbook of the National Society for the Study of Education, Part II. Chicago: University of Chicago Press. 1951 P. 1.



Further support indicating the importance of a changing mathematical program to the cultural development is offered by Schaaf.

The creative language of today is science, and mathematics is the alphabet of science. . . . Contemporary mathematics is to be distinguished from all previous mathematics in two vital respects: (1) the intentional study of abstractions, where the important considerations are not the things related, but the relations themselves; and, (2) the relentless experimentation of the very foundations - the foundations of ideas upon which the elaborate superstructure of mathematics is based. . . .<sup>7</sup>

A consideration of the arithmetic program needed in view of these changes implies the utilization of mathematics by the culture and, as a consequence, a change in methods of classroom presentation. Suelts in emphasizing the pressure to apply research results to classroom methodology lists three components of an arithmetic program that will tend to emphasize the social significance of mathematics:

1. The essence of mathematics.
2. Computations and procedures.
3. Application to the social, physical, cultural, and economic world.<sup>8</sup>

The importance of number systems (arithmetic) to the cultural is also emphasized by Suelts:

The fact (well accepted) that modern science and art could not well function without the number system in its present state of development tends to support the concept that the invention of it is, as it has been called, the greatest invention of the human mind.<sup>9</sup>

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<sup>7</sup>William L. Schaaf, "Mathematics as a Cultural Heritage." The Arithmetic Teacher, 8:3-5, January 1961.

<sup>8</sup>Ben A. Suelts, "A Time for Decision." The Arithmetic Teacher, 8:280, October, 1961.

<sup>9</sup>Ibid., P. 13.





The cautious planning that is needed to implement a mathematics program that will serve and support: (1) the abstractness of mathematical invention; (2) the rapid development of automation; and (3) the resulting construction of rapid mechanistic computation is brought out by Craig:

In the already crowded curriculum any subject should be most carefully delineated. Although arithmetic can claim a share of curriculum time as its cultural right, the delineation of that share of the curriculum should be most carefully considered. The question appears to be, "How can mathematics be treated in the curriculum to be of the most use?"

Although mathematics as such is a part of the cultural heritage, in the crowded curriculum of today, time spent on mathematics needs to be justified.<sup>10</sup>

It would seem that we need to make a choice as to what course the arithmetic should pursue: (1) Shall we continue in the way of abstraction only; (2) can abstraction have utility as a means of enhancing understanding; (3) is it possible for children in the elementary school to understand functions and processes as well as the memorization of facts; and (4) can this be accomplished in a reasonable amount of class time?

The arithmetic program of the future is pictured by Clark:

. . . arithmetic programs in the next decade will: (1) be directed by teachers of high scholarship; (2) be better supported by the lay public; (3) give greater emphasis to learning by thinking; (4) make better provision for the range of abilities in heterogeneous groups of pupils; (5) incorporate topics not generally found in school curricula; (6) include experiments with, and evaluation of, newer tools of learning.<sup>11</sup>

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<sup>10</sup>Gerald S. Craig, Science for the Elementary School Teacher. New York: Ginn and Company, 1958. P. 35.

<sup>11</sup>John R. Clark, "Looking Ahead at Instruction in Arithmetic." The Arithmetic Teacher, 8:394, December, 1961.



It is with these implications in mind that this study originated.

Need for the study. It has been pointed out that: (1) mathematics content is rapidly changing and developing; (2) mathematics content is rapidly increasing; (3) there is an urgent necessity for the utilization of mathematics in the control of a rapidly developing technological environment; and (4) these factors imply a change in the content presented to students and the methods by which this is done.

Studies involving mathematical research are numerous and attack many facets of the total mathematical problem situation.<sup>12</sup> However it would seem that some attempt might be made to inaugurate an arithmetic program in a typical classroom situation that would incorporate the implications of such studies.

The sequential nature of mathematics makes a thorough understanding of previous content necessary in order to master developing mathematical innovations.<sup>13</sup> It has been pointed out that the need to utilize mathematics in the control of the environment implies understanding on the part of the student rather than memorization. It would seem then that the elementary curriculum as the sequential antecedent to this developing mathematical program, could not advantageously be continued as a memorizing, or rote type, program.

It was the purpose of this study to attempt to design an elementary course of study in grades five and six that would: (1) be based on

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<sup>12</sup>Buswell, op. cit., p. 63.

<sup>13</sup>Price, op. cit., p. 2

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a thorough understanding of mathematical theory; as it relates to arithmetical processes; (2) be consistent as a sequential antecedent to the new content that is developing at the secondary levels; (3) provide academically talented students with an arithmetic program equal to their abilities; (4) provide the slow learners with arithmetic experiences compatible with their abilities; (5) provide experiences in the application of arithmetic to real-life situations; (7) be taught in a realistic manner consistent with principles of learning; and, (8) be accomplished in thirty-five minutes per day of class time.

In order to implement a program of this type an attempt was made to: (1) identify and analyze methods of improving the elementary mathematics program; (2) investigate methods of reducing the classroom teacher's total load as it relates to areas of specialization; (3) investigate a method of teaching elementary mathematics that would emphasize the understandings of concepts and their application to a social situation with tangible manipulative materials; and, (4) investigate the relationship of such teaching-learning procedures to the development of skill in the manipulation of abstract mathematical symbols.

Statement of the problem. The purpose of this study was to compare two methods of teaching arithmetic in the fifth and sixth grades. The two methods were designated as those practiced in: (1) the self-contained (or routine) classroom procedure; and, (2) a tangible and conceptualized presentation of arithmetic as mathematical theory to be applied in a social situation. In the self-contained classroom, arithmetic was taught in the routine manner prescribed by the curriculum of the school system in which the text structured the arithmetic program. In experimental



rooms the mathematics program was presented as a cultural tool. The two methods were compared as to the performance of students on selected standardized achievement tests. These were administered as a part of the arithmetic program.

In addition the study was concerned with incorporating into the content of the elementary arithmetic curriculum sufficient mathematical theory to serve as an adequate explanation of the arithmetical processes heretofore taught by rote - or memory.

It was the hypothesis of the experimental program that fifth and sixth grade mathematics presented as a mathematical concept to be used as a cultural tool will result in: (1) an improved understanding of mathematical principles; (2) improved ability to manipulate abstract mathematical symbols and to apply these in a social situation; and, (3) that these abilities would be exhibited on standardized achievement tests given as a part of the regular routine of the school.

Basic considerations. The basic considerations pertinent to the study were as follows:

1. Is it possible to increase understandings of mathematical concepts by the manipulation of tangible items?
2. Will an increased understanding of mathematical concepts result in greater mathematical achievement by students as measured by standardized achievement tests?
3. What is the extent which mathematics as a symbolic system may evolve skill in the manipulation of symbols?
4. What emphasis should be placed on the understanding of mathematical concepts as the manipulation of abstract symbols?





Definition of terms. Terms used are defined in appropriate places in the body of the thesis. However, in order to make clear the initial presentation, certain terms are defined here. The terms defined are: arithmetic, mathematical concepts, routine classroom procedure, and conceptualized presentation.

Arithmetic in this study was used interchangeably with mathematics and elementary mathematics. It also refers to the program used in the school to develop the part of the curriculum usually referred to as "arithmetic". The connotations given to arithmetic in the experimental program are defined in Chapter III.

Mathematical concepts was used to refer to the ideas and generalizations of mathematics which were taught in connection with the study of arithmetic. This term was also used to refer to the branches of arithmetic dealing with denominate numbers, and the areas of mathematics having to do with measurement. As used in the study, the term would include ideas, generalizations, principles and symbols referring to the various areas of mathematics.

Routine classroom procedure was the term applied to the process of arithmetic instruction carried on in the self-contained classroom. In this study it was referred to as "the way arithmetic was usually taught." This implies a presentation involving: (1) regular assignments to be completed in the text; (2) following the sequence of the text; (3) requiring the students to complete most of the practice and drill work in the text; (4) direct correlation with other classes; (5) "extra" help for slow learners; (6) additional time other than in class to complete assignments; (7) drill on "facts"; and, (8) homework when

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necessary. This is further developed in the chapter on the teaching procedures used in the study.

Conceptualized presentation was the term arbitrarily chosen to represent the presentation of a broad general idea to students as a teaching process. This was in contrast to a presentation involving a single operation. Conceptualized presentation included many aspects of an arithmetic concept and its application in a social situation.

Limitations of the study. The study was an evaluation of a method of teaching arithmetic to fifth and sixth grade students and was not an attempt to prove that one method was superior to another. The study was limited to the content, and the understanding of the content, of the mathematics contained in the course of study of the school system.

Specifically the study was not concerned with:

1. The evaluation of the existing routine classroom presentation.
2. The evaluation of textbooks.
3. The evaluation of the curriculum.
4. The evaluation of grade placement of mathematical concepts and/or materials.

The study was specifically concerned with ascertaining if it was possible to teach the mathematics contained in the curriculum, as appropriate to grades five and six, as an understanding of concepts rather than the memorization of facts and processes.

Structure of the thesis. Chapter I contained a review of some of the developments in mathematics and society that have given rise to the need for the study. Basic assumptions and terms have been described.

1. The first part of the document discusses the importance of maintaining accurate records of all transactions. It emphasizes that proper record-keeping is essential for the transparency and accountability of the organization. This section also outlines the various methods used to collect and analyze data, ensuring that the information is reliable and up-to-date.

2. The second part of the document focuses on the implementation of the proposed changes. It details the steps involved in the transition process, from the initial planning phase to the final execution. This section also addresses the potential challenges and risks associated with the changes, providing strategies to mitigate them.

3. The third part of the document discusses the impact of the changes on the organization's overall performance. It presents data and analysis showing the positive outcomes of the implementation, such as increased efficiency and cost savings. This section also highlights the areas where further improvements are needed and provides recommendations for future actions.

4. The fourth part of the document provides a summary of the key findings and conclusions. It reiterates the importance of the changes and the successful implementation of the proposed plan. This section also includes a final statement of commitment to the organization's mission and vision.

5. The fifth part of the document is a conclusion. It summarizes the main points of the document and provides a final statement of commitment to the organization's mission and vision. It also includes a statement of appreciation for the support and cooperation of all stakeholders throughout the process.

Chapter II will be concerned with a review of the literature that is pertinent to a conceptualized presentation of arithmetic, literature pertaining to learning theory that is particularly applicable to the study, and a review of current literature regarding present practices of teaching arithmetic in the elementary school. Chapter III will contain a detail of the two methods used in the study. Chapter IV will contain descriptions of: (1) the organization of the investigation; (2) the persons involved in the study; and, (3) the cultural and geographic areas participating in the study. Chapter V presents the data and results of the study. Chapter VI contains a summary and implications for further study.

## CHAPTER II

### REVIEW OF THE LITERATURE

It was indicated in Chapter I that the literature in regard to research in the methods and content of the arithmetic program is very extensive. It would have been beyond the scope of this paper to completely review all these productions. Consequently this chapter will concern itself with a review of literature pertinent to: (1) concepts in arithmetic curricula and methods of teaching them; (2) literature pertaining to theories of learning that are significant to this study; and, (3) literature regarding current practices of teaching arithmetic.

Review of literature pertaining to concepts of arithmetic curricula and methods of teaching them. It should be emphasized that the teaching of arithmetical concepts or ideas does not mean an incidental type of learning situation. This is brought out by Clark:

Changing conceptions and functions of education, of the philosophy of education, have been clearly reflected during the past several years both in curricula and teaching methods. . . . There is a middle ground between the two extremes of the now outmoded formalism of an over-burdened curriculum, and the established freedom of wholly incidental learning.<sup>14</sup>

The relation of teaching mathematical concepts with cultural application is further emphasized by Bredeoake and Groves:

. . . to supply children with those experiences which lead to real comprehension of number . . . children who failed to achieve success in "sum lessons" showed considerable grasp of

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<sup>14</sup>John R. Clark, Arthur S. Otis, and Caroline Hatton, Primary Arithmetic Through Experience. Yonkers-on-Hudson: World Book Company. 1939. P. 6.

Figure 1. The effect of the concentration of the *Agrobacterium* suspension on the transformation efficiency of *Agrobacterium* strains. The number of transformed cells was determined by the number of colonies obtained on the selective medium. The results are the mean of three independent experiments. Error bars represent the standard deviation.

[illegible]



the subject when shopping. . . a scheme which brought to the classroom some of the numerical activities found in daily life. . . .<sup>15</sup>

Many, if not most, references dealing with arithmetic instruction are chiefly concerned with mathematical content, with much less emphasis on method of teaching and learning. Banks specifically illustrates teaching individual and isolated content areas. He does indicate however that the choice of material in texts has made a real effort to "reflect interests, activities, and experiences of children."<sup>16</sup>

The need of structuring the arithmetic curriculum so that it contains mathematical concepts is emphasized by Hartung.

. . . more than a set of things it might be interesting to try, or a collection of enrichment materials, or drill materials . . . It must provide for a new and deeper understanding of arithmetic. . . . A few simple ideas provide the foundation stones upon which arithmetic is built. . . . The ultimate goal of a modern arithmetic program is that the child be able to solve problems involving quantitative ideas. To attain this goal, it is necessary for him to acquire certain essential concepts and techniques.<sup>17</sup>

The development of arithmetic programs has closely followed the needs of society. It has progressed from a primitive need for an expression of quantity, such as a one-to-one relationship, to the complicated processes demanded by current technology.<sup>18</sup> Evidence has been presented

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<sup>15</sup>E. Bradeoake and I.D. Groves, Arithmetic in Action. London: University of London Press. 1956. P. 128.

<sup>16</sup>J. Houston Banks, Learning and Teaching Arithmetic. Boston: Allyn and Bacon, Incorporated. 1959. P. 5.

<sup>17</sup>Maurice L. Hartung, et. al., Charting the Course for Arithmetic. Chicago: Scott, Foresman and Company. 1960. Pp. 4-19.

<sup>18</sup>David Eugene Smith, History of Mathematics. New York: Ginn and Company. 1951. P. 6.



in Chapter I to indicate this development. Of particular concern to this paper are the changes in content and methodology during the past century.

Mathematics during the colonial period showed little ingenuity.<sup>19</sup> Up to the year 1800 texts were of English origin and related largely to the needs of commerce.<sup>20</sup> Methodology consisted largely in the dictation of practical problems by the teacher which were copied by the students. Texts gave rules for the solution of problems, these were memorized by the students and applied with a minimum of emphasis on understanding. About the first significant change in arithmetic teaching came in 1821, when Colburn, under the influence of Pestalozzi, broke away from the deductive method of memorizing rules and applying them. As arithmetic content continued to incorporate additional mathematics (as it was needed by the culture) the bulk became so heavy it was deemed necessary that it be abridged and enriched. This was undertaken by the Committee of Ten (1893) and the Committee of Fifteen (1895).<sup>21</sup>

In order to reduce the amount of content the theory of "utility" was applied. In about 1920 arithmetic was organized as a succession of unit skills. This was followed by drill (the "law" of exercise) as a law of learning. In 1930 the National Society for the Study of Education

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<sup>19</sup>Walter S. Monroe, Development of Arithmetic as a School Subject, U.S. Bureau of Education Bulletin, No. 10. Washington: Government Printing Office. 1917. p. 8.

<sup>20</sup>Buswell, Op. cit., p. 63.

<sup>21</sup>Ibid.

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Yearbook paved the way to teaching for meaning.<sup>22</sup>

As the development of arithmetic programs followed a pattern of increasing content, it began to be opposed by arithmetic curricula which emphasized in turn: (1) mental discipline; (2) social utility; (3) real-life relationships; and, (4) social aims.<sup>23</sup> Wise made some attempt to show that much of the arithmetic taught during the early part of the century did not function in life.<sup>24</sup> Consideration of arithmetic as social utility helped to streamline much of the outmoded content. In addition to theories of social utility much emphasis has been placed on an attempt to make it meaningful.<sup>25</sup> It is now rather well accepted that both aims are necessary<sup>26</sup> and since content (up to the last few years) has been pretty well founded, research has dwelt on arrangement.<sup>27</sup>

This thesis would be incomplete without some reference to "activity" programs that have been inaugurated. Harap and Mapes attempted the learning of decimals in an activity program, and found that the upper and lower extremes of the class (according to I.Q.) acquired some mastery, and that the number of repetitions had nothing

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<sup>22</sup>Ibid., p. 64.

<sup>23</sup>Ibid., p. 65.

<sup>24</sup>Carl L. Wise, "A Survey of Arithmetic Problems Arising in Various Occupations", Elementary School Journal, 20:118-36. 1949.

<sup>25</sup>Buswell, op. cit., p. 65.

<sup>26</sup>Henry Van Engen, "An Analysis of Meaning in Arithmetic." Elementary School Journal. 49:321-29, 395-340; 1949.

<sup>27</sup>Herbert F. Spitzer and Robert L. Burch, "Methods and Materials in the Teaching of Mathematics." Review of Educational Research. 18:337-49; 1948.



to do with degree of mastery.<sup>28</sup> In as much as there was no control these findings might indicate further study. Willey reported a study of social expression intended to lead a child to understandings.<sup>29</sup> In a report of an experience curriculum, Williams showed satisfactory gains, but had no control group.<sup>30</sup>

Buswell reported in 1935.

. . . on the basis of evidence now available, the incidental, experience approach has not produced a superior substitute for the more systematic organization of content, but it may well provide some insight into supplements to the systematic program . . . (in an activity program) there are many uncontrollable variables . . . it is also practically impossible to duplicate a situation.<sup>31</sup>

Since the method of arithmetic instruction described in this study is concerned with "drill" teaching techniques, the work of Phillips,<sup>32</sup> Brown,<sup>33</sup> and Brownell<sup>34</sup> are significant in which there was some

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<sup>28</sup>Henry Harap and Charlotte E. Mapes, "The Learning of Decimals in an Arithmetic Activity Program". Journal of Educational Research, 29:686-93, 1936.

<sup>29</sup>Ray D. Willey, "A Study in the Use of Arithmetic in the Schools of Santa Clara County, California." Journal of Educational Research, 36:353-65, 1943.

<sup>30</sup>Catherine M. Williams, "Arithmetic Learning in an Experience Curriculum," Educational Research Bulletin, 28:154-62, 167-68; 1919.

<sup>31</sup>Buswell, op. cit., p. 67

<sup>32</sup>L. M. Phillips, "Value of Daily Drill in Arithmetic." Journal of Educational Psychology. 4:159-63, 1913.

<sup>33</sup>J. C. Brown, "An Investigation of the Value of Drill Work in the Fundamental Operation of Arithmetic," Journal of Educational Psychology, 3:485-92, 561-70; 1912.

<sup>34</sup>William A. Brownell and Charlotte B. Chezal, "The Effects of Premature Drill in Third Grade Arithmetic." Journal of Educational Research. 29:17-28, 1935.

[illegible]



indication that drill without understanding may result in poor learning and learning habits.

Review of learning theories of significance to this study. It is not our purpose to investigate all the literature published relative to theories of learning, but rather to confine this discussion to basic theories of learning, (as presented by accepted authorities) that are pertinent to this study. Neither is it the purpose of this dissertation to evaluate various learning theories, only to investigate such aspects as would tend to support the techniques which were used in the teaching of mathematics in this study. In other words, it is our intent to show that the teaching methods employed were based on some accepted theories of the learning process. It is further our purpose to point out that a consideration of some commonly accepted theories of learning would indicate that a sense-perception (tangible), and conceptualized presentation of arithmetic in a social situation would tend to enhance meaning.

The importance of the process of learning to the tremendous task of achievement in arithmetic is emphasized by Judd.

. . . children are not born with a number system as part of their physical inheritance . . . the school puts them in contact with a system of number symbols which is one of the most perfect creations of the human mind . . . in the short span of a few years, the child becomes expert in the use of a method of expressing ideas of quantity which cost the race centuries of effort to invent and perfect.

If arithmetic education is to accomplish such a task as this it would appear that every effort should be made to do it as effectually as

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<sup>35</sup>Charles H. Judd, Educational Psychology, New York: Houghton Mifflin. 1939. p. 270.

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possible. The fact that mathematics might be considered as an inherent or instinctive process should not be totally disregarded, however, there is some support to the theory that behavior in man is a learned process, or, learning is important to education. Instinctive and/or inherent determinants have been shown to have somewhat less significance, though not to be completely discounted.<sup>36</sup>

An investigation of much of the literature relating to the subject matter and methods of teaching, approaches the learning process and its accompanying problems in a symptomatic rather than a casual manner.<sup>37</sup> That is, reported research tends to analyze isolated problem areas and suggest methods of attack, but puts less emphasis on underlying principles of the thinking and learning activities of children. Basic then, to this paper is a consideration not only of learning theory, but the use to which it could be put in teaching and learning mathematics. It has been our assumption that learning theories, or generalisations, might be employed in predicting or anticipating desirable behavior on the part of the student in the area of mathematics.

Such a treatment of learning theory seems to involve three aspects: (1) the antecedents within the students background; (2) the results to be expected from the application of the "teaching process" on the student, and (3) resultant behavior. The learning theory employed in this study

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<sup>36</sup>N. Tenbergen, The Study of Instinct. London: Oxford University Press. 1951. As cited by Ernest R. Hilgard, Theories of Learning. New York: Appleton-Century-Crafts, Incorporated. 1958. p. 3.

<sup>37</sup>Buswell, op. cit., p. 63-67.



would need to deal with all of these areas. All three are difficult to describe in that there appears to be little scientific method which can be used to treat them.

The first of these has to do with what is already in the students mind when exposed to school learning situations. Children think not necessarily in relationship to the momentary classroom situation, but in relationship to this, plus the mental residue contained in their minds of what has gone before. This mental development "includes the sum total of memories, images, percepts, concepts, and attitudes built up over the years."<sup>38</sup>

The conclusions of Piaget are relevant to a methodology that considers the thinking process of children in the learning situation. This work analyses a situation in which children often "think" they understand when the particular understanding is the result of a "syncretistic perception."<sup>39</sup> In other words, children are apt to jump to conclusions when presented with instructional materials that are to be learned. The supposed understanding is no more than a quick relating of the new idea with the child's apperceptive mass. This leads to Piaget's conclusion:

Syncretistic understanding consists in . . . (a thought process) that the whole is understood before the parts are analyzed and that the understanding of the details takes place only as a function of the general scheme.<sup>40</sup>

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<sup>38</sup>David H. Russell, Children's Thinking. New York: Ginn and Company, 1956. p. 34.

<sup>39</sup>Jean Piaget, The Language and Thought of the Child. New York: Meridian Books, 1955. p. 140.

<sup>40</sup>Ibid., p. 162.



In view of this it would seem illogical to teach isolated "facts" in subject matter as a prerequisite to either understanding or assimilation of knowledge. This does not intimate that facts or mathematical ideas, in and of themselves, are unimportant, but rather that they should be viewed in their totality and their interrelationships.

Carpenter's work is significant in this regard:

We cannot always assume that the perception of the "whole" gestalt can somehow suddenly be grasped without making sure that the pupil knows the meaning of key concepts that compose the gestalt.<sup>41</sup>

This need for an understanding of parts as they relate to wholes is also brought out by Russell:

Not until eleven years can children understand how three objects are alike, and not until adolescence can they detect similarities in abstract words. This one example of mental development suggests that elementary children should have many concrete experiences.<sup>42</sup>

The importance of percepts and sensations contributing to a child's total concept of a situation are further emphasized by Russell who supports Piaget's conclusion and gives it application.

A child's thinking is based on his experience. His immediate interpretation of events in his external or internal environment are his percepts. . . . A percept, an image, or a memory seldom exist in isolation. . . . The older distinction that sensory impression is a lower form of activity than the higher "mental processes" of abstraction or problem solving is now considered to be largely a metaphysical one. Modern research suggests not a sharp break, but a continuum in the cognitive process from

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<sup>41</sup>Fanley Carpenter, "Conceptualization as a Function of Differential Reinforcement." Social Science and Mathematics. 38:284-294. 1954.

<sup>42</sup>Russell, op. cit., p. 59.

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relatively simple sensations and perceptions to elaborate aesthetic or creative experiment.<sup>43</sup>

To apply the foregoing to methodology in arithmetic it needs only to be remembered that the subject matter as it is presented to the child becomes a part of this environment, or the subject matter is interpreted by the child in his percepts.

Basic also to this discussion is the function of repetition or "drill" methods, commonly referred to as "rote" learning. A method by which content is presented to the students by the teacher, either orally or written, and which the students repeat a sufficient number of times until they can reproduce it. Thorndike's work is significant to this kind of methodology that involves the teacher telling the pupils "how" rather than building an understanding for them that tells "why". There is some indication in Thorndike's work to show that "response to a command situation will not result in a waning of the initially most frequent connection at the expense of the initially less frequent."<sup>44</sup>

If this were applied to a multiple choice problem situation in which a student may make either a correct or several incorrect answers, continued repetition of incorrect answers would not necessarily enhance the correct responses at the expenses of the incorrect answers.<sup>45</sup>

Further support for a conceptualized, learning and the use of broad ideas is offered by Carpenter:

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<sup>44</sup>Edward L. Thorndike, The Fundamentals of Learning. New York: Bureau of Publications, Teachers College, Columbia University. 1937. p. 18

<sup>45</sup>Ibid., p. 18



Functional learning of concepts is more efficient than rote learning when measured by retention and ability to verbalize meanings of learned concepts.<sup>46</sup>

In view of the evidence presented in Chapter I indicating that elementary mathematics education needs now to teach structure and concepts rather than isolated facts, the work of Hargrove which indicates a structured, rather than an incidental learning program is significant.<sup>47</sup>

The importance of understanding a number system and its application to social situations is a concern of teachers in instructing children in ways of solving story problems. The use of a thorough understanding of a number system, as it applies to the culture is brought out by Hamilton:

. . . A problem seldom if ever has any numbers associated with it until we recognize the situation and apply numbers to it. . . . We have to be able to think well enough to be able to arrange the elements of a situation and adapt a strategy using models from reality, concepts, mental imagery, . . . also know enough about numbers to recognize one or several properties that fit the situation, thus choosing an abstract model in the best mathematical sense.<sup>48</sup>

Support for learning in a social situation rather than in isolation is also offered by McHugh.<sup>49</sup>

The learning or behavior to be expected as a result of arithmetic instruction is basic to this paper. This consideration involves the

<sup>46</sup>Fenley Carpenter, "The Effect of Different Learning Methods on Concept Formation." School Science and Mathematics, 40:282-285, 1956.

<sup>47</sup>W. Richard Hargrove, "Proper Emphasis on Science and Mathematics in the Elementary School." School Science and Mathematics, 44:89-91, 1960.

<sup>48</sup>E. W. Hamilton, "About the Articles", The Arithmetic Teacher, 8:49, 1961.

<sup>49</sup>Walter Joseph McHugh, "Pupil Team Learning in Skill Subjects in Intermediate Grades." Dissertation Abstracts, 21:1461, Dec. 1960.

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relationship of mind to the learning process; or, as suggested on page 17, the use to which the learning (resulting from the instructional process) shall be put. Guthrie makes such a distinction:

Growth, reproduction, and defense mechanizations are life but they are not mind. Mind is something more; it is growth and reproductions and reactions serving these ends plus something else that common sense might call profiting by experience. . . . The ability to learn, that is to respond differently to a situation because of past response to the situation is what distinguishes those living creatures which common sense endows with minds . . . training leaves no observable changes . . . mind must be a mode of behavior which changes with use or practice.<sup>50</sup>

While it might be desirable to find a teaching method that would assure all the best possible results, such a possibility would seem improbable. In this light Guthrie continues:

So far as I am aware the only suggestions toward the description or explanation of the circumstances under which specific changes in behavior will or will not occur have been made in the form of association or conditioning . . . when the past of the individual is used for predicting behavior, we find our predictions are always in terms of associative learning . . . we can never understand him (man) but we can understand something of him and know something of him in terms of what we know of human nature in general and in terms of what we know of his past history and the nature of associative learning.

In contrast to this concept of learning as it relates to the total past and present experiences of the individual DeMay offers a brief summary of some early aspects of arithmetic instruction:

When number work was first thought to be a necessary part of instruction of the first two grades, teachers did not know what to do except to drill their pupils in the abstract combinations. Investigation later showed most of the time spent this way had been wasted . . . (the need is) to provide teachers with

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<sup>50</sup>E. R. Guthrie, The Psychology of Learning. New York: Harper and Brothers. 1935. Pp. 2-4.

<sup>51</sup>Ibid., Pp. 243-245.



sequential materials that will guide the development of exercises through which children will learn the significance of numbers rather than the manipulation of abstract symbols.<sup>52</sup>

This importance of learning theory to the teaching of arithmetic is emphasized by Lankford:

. . . the effective teacher of mathematics encourages creativity by helping pupils discover the basic laws, or principles of mathematics; he aims for understanding ahead of skills of operation; and he seeks to give students the stimulation that comes from accepting and realizing worthwhile goals.<sup>53</sup>

In setting forth the important elements in an effective arithmetic program Grossnickle lists:

1. The nature of the subject is such that it has a cultural value; it is structured; properly taught, it leads to unique quantitative ways of thinking; and it is basic to the further study of mathematics.

2. A program for the learning of arithmetic should recognize such factors as the mental hygiene of the classroom, adequate records for guidance, provision for optimum individual growth, use of materials, and ability to read quantitative statements.

3. A specific course in the background of mathematics is recommended as essential in the training of teachers of arithmetic.

The basic understandings needed, opportunities to create, and emphasis on meanings rather than drill are also advocated by Jones:

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<sup>52</sup>Amy J. DeMay, Guiding Beginners in Arithmetic. Evanston: Row, Peterson and Company. 1957. Pp. IX-X.

<sup>53</sup>Francis G. Lankford, Jr., "Implications of the Psychology of Learning for the Teaching of Mathematics," The Growth of Mathematical Ideas, Twenty-Fourth Yearbook. National Council of Teachers of Mathematics. Washington: The Council. 1959. p. 405.

<sup>54</sup>Foster E. Grossnickle, "Introduction." Instruction in Arithmetic, Twenty-Fifth Yearbook of the National Council of Teachers of Mathematics. Washington: The Council. 1960. p. 3.

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1. The best learning is that in which the learned facts, concepts, and processes are meaningful to and understood by the learner.

2. Understanding and meaningfulness are rarely if "all or none" insights in either the sense of being achieved instantaneously or in the sense of embracing the whole concept and its implications at any one time.<sup>55</sup>

The foregoing has been presented in support of a method of teaching arithmetic which would emphasize understandings, insights, creativity, and cultural utility; and which would give no emphasis to drill methods, repetitious problem solving, and meaningless memorization.

Current literature regarding present practices in teaching arithmetic. To review all the studies regarding present practices in arithmetic instruction would be far beyond the scope of this study. We have selected those that appear to be of significance and that are representative of the general trends in teaching for meaning and understanding. Glennon has stated the present condition of arithmetic research rather well as he points out that there is much research about what can be taught and little concern about what should be taught.<sup>56</sup> He states:

Prior to 1900 the curriculum was determined by:

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<sup>55</sup>Phillip S. Jones, "The Growth and Development of Mathematical Ideas in Children," The Growth of Mathematical Ideas, Twenty-Fourth Yearbook of the National Council of Teachers of Mathematics. Washington: The Council, 1959. p. 1.

<sup>56</sup>Vincent J. Glennon, "Editorial," Educational Leadership. 19:354-56, 1962.



1. The need for mathematical training by society.
2. The need for the subject to be taught as a system of related ideas.
3. The present emphasis is on the needs of the child.<sup>57</sup>

The author continues that all these emphases have drawbacks, there is a need for balance in the curriculum and that " . . . modernizing the school program is one way of changing the method as well as the content."<sup>58</sup>

Further support as to the disorganized state of arithmetic research is offered by Hartung:

Instructional programs in arithmetic include a layer of ideas and practices accumulated through the years. Examined critically . . . from the point of view of learning theory . . . they are seen to be questionable either on mathematical grounds or on psychological grounds or both . . . much confusion arises between means and ends. Computation for its own sake is fruitless . . . but essential to problem solving. . . . I see great possibilities for improvement in arithmetic instruction if we are willing to acknowledge that some of our teaching has been superficial at best. The remedy is not more drill, but deeper insight as to what is really basic arithmetic.<sup>59</sup>

The need for a culturally significant and useful arithmetic curriculum is brought out by Cook:

Arithmetic is undoubtedly one of the more poorly taught subjects in the elementary school. Educators may question whether present demands in mathematics are in focus with other areas of the curriculum, but we must recognize the needs of our youth in the world in which they will live. A reappraisal of our approach to arithmetic in the elementary schools, of the coordination with the mathematics program in the secondary schools, of the preparation of teachers, and

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<sup>57</sup> Ibid., p. 54.

<sup>58</sup> Ibid., p. 55.

<sup>59</sup> Maurice L. Hartung, "Distinguishing Between Basic and Superficial Ideas in Arithmetic Instruction." The Arithmetic Teacher, 6:65-7, 1959.

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of the place of inservice programs in the school system is imperative if we are to meet the demands facing us at the present time.<sup>60</sup>

The need of a program designed to build meaningful mathematical concepts of cultural and social significance is emphasized by Sueltz,<sup>61</sup> Madden,<sup>62</sup> Flournay,<sup>63</sup> Breuckner,<sup>64</sup> and Brownell.<sup>65</sup>

Sueltz offers a fine "time chart" of the development of arithmetic instruction and the factors influencing the development of arithmetic texts:

- 1850 - 1890 "Faculty" psychology.  
"Training" of the mind.
- 1900 - 1920 Principles of learning.  
Motivation and readiness.  
Exercise, effect, threshold of learning,  
over-learning.
- 1920 - 1935 Readiness.  
Exercise.  
Drill.
- 1930 - 1950 Progressive education.  
Satisfyingness, peer group.  
Whole child.

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<sup>60</sup>Raymond F. Cook, "Improving Arithmetic Instruction," National Elementary Principal, 38:37-39; 1959.

<sup>61</sup>Ben A. Sueltz, "Arithmetic in Historical Perspective," National Elementary Principal, 39:12-16; 1959.

<sup>62</sup>Richard Madden, "Major Issues in Teaching Arithmetic," National Elementary School Principal, 39:174; 1959.

<sup>63</sup>Frances Flournay, "Relating Arithmetic to Everyday Life," National Elementary Principal, 39:294; 1959.

<sup>64</sup>Leo J. Breuckner, "Testing, Diagnosis, and Follow-Up in Arithmetic," National Elementary Principal, 39:334; 1959.

<sup>65</sup>William A. Brownell, "Arithmetic in 1970," National Elementary Principal, 39:42-44; 1959.

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1950            Child development understandings.  
                  Meaning, action research.  
                  Discovery learning.  
                  Multisensory learning.<sup>66</sup>

Suelts continues:

Arithmetic will advance as teachers develop an insight into the subject matter and its significance and as they learn more and more about the boys and girls they teach and how the human minds, bodies, and emotions combine in the behavioral learning situation.<sup>67</sup>

In considering the characteristics of a good arithmetic program

Suelts advises:

A teaching process through an inductive-deductive cycle (in which) . . . new concepts are introduced in a socially significant situation, developed in their mathematical sequence, and returned to socially significant application.<sup>68</sup>

Methods which would teach arithmetic for understanding make it necessary to present the structure of the number system. This is emphasized by Brueckner.

It is generally agreed that children must understand the number system. They must also understand how the number system operates in the performance of number operations.<sup>69</sup>

The emphasis on sound mathematics as basic to arithmetic programs is supported by Lundberg<sup>70</sup> and Wilsberg.<sup>71</sup>

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<sup>66</sup>Suelts, op. cit., Pp. 15-16.

<sup>67</sup>Ibid.

<sup>68</sup>Ibid.

<sup>69</sup>Brueckner, loc. cit.

<sup>70</sup>Hazel Lundberg, "Mathematics in Elementary School." Educational Leadership, 19:364-68; 1962.

<sup>71</sup>Mary E. Wilsberg, "Freeing Children in Primary Arithmetic." Educational Leadership, 19:352-6; 1962.

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There are many isolated studies which support teaching methods employing meaningful application of mathematics, as pointed out at the beginning of this section. Some of the more outstanding are those of Stone,<sup>72</sup> DeLong,<sup>73</sup> Langer,<sup>74</sup> and Miller,<sup>75</sup> in which meaningful methods generally gave favorable results. Langer emphasized the dependence of significant technological advances on mathematics.

There have been a few studies recently reported showing the relationship of time in class to achievement. Daugherty, in a study conducted in the Des Moines Public Schools, found a fifty minute class favorable.<sup>76</sup> Denny<sup>77</sup> also showed an increased time allotment as contributing to greater achievement.

Changing methods of teaching arithmetic have prompted some studies in regard to evaluation procedures. Breuckner offers six

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<sup>72</sup>Marshall H. Stone, "Fundamental Issues in the Teaching of Elementary School Mathematics." The Arithmetic Teacher, 6:177-79; 1959.

<sup>73</sup>Arthur R. DeLong and Richard M. Clark, "Developing Creativity in Arithmetic." The Arithmetic Teacher, 6:208; 1959.

<sup>74</sup>Rudolph E. Langer, "To Hold As't Were the Mirror up to Nature; to Show the Very Age and Body of the Time." The Arithmetic Teacher, 6:289-94; 1959.

<sup>75</sup>G.H. Miller, "How Effective Is the Meaning Method?" The Arithmetic Teacher, 4:45-49; 1957.

<sup>76</sup>James Lewis Daugherty, "A Study of Achievements in Sixth Grade Arithmetic in Des Moines Public Schools," doctoral dissertation as reported in Analysis of Research in the Teaching of Mathematics. United States Department of Health Education and Welfare, Bulletin Number 8. Washington: U.S. Office of Education, 1900.

<sup>77</sup>Robert Ray Denny, "A Two-Year Study of an Increased Time Allotment Upon Achievement in Arithmetic in the Intermediate Grades, doctoral dissertation as reported in Analysis of Research in the Teaching of Mathematics, United States Department of Health Education and Welfare, Bulletin No. 8. Washington: U.S. Office of Education, 1900.

1. The first part of the document is a letter from the President of the United States to the Congress, dated January 3, 1862. It is a very important document, as it contains the President's annual message to Congress. The President discusses the state of the Union, the progress of the war, and the administration's policies. He also mentions the recent discovery of gold in California, which has led to a large influx of people into the state.

2. The second part of the document is a report from the Secretary of the Interior, dated January 10, 1862. It discusses the progress of the survey of the public lands in California. The report mentions that the survey has been completed for a large area of land, and that the results show that there is a large amount of land available for settlement. The Secretary also mentions that the survey has been completed for a large area of land, and that the results show that there is a large amount of land available for settlement.

3. The third part of the document is a report from the Secretary of the Interior, dated January 10, 1862. It discusses the progress of the survey of the public lands in California. The report mentions that the survey has been completed for a large area of land, and that the results show that there is a large amount of land available for settlement. The Secretary also mentions that the survey has been completed for a large area of land, and that the results show that there is a large amount of land available for settlement.

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criteria to be considered in a testing program to be used in connection with meaningful teaching procedures:

1. The selection and clarification of objectives.
2. The determination of the rate of growth.
3. Provision of a basis by which teachers can set up educational experiences adopted to the needs and ability of the learners.
4. Motivation and guidance of learning, especially by helping children to evaluate their own responses and behavior.
5. The location diagnosis and treatment of learning difficulties.
6. Bases for coordinating improvement programs in related fields such as arithmetic, reading, science, and social studies.<sup>78</sup>

He adds five basic steps to aid in evaluation:

1. Formulating general and specific objectives.
2. Defining objectives in terms of pupil behavior.
3. Designing and selecting suitable means of appraisal.
4. Securing a record of behavior and performance.
5. Interpreting and evaluating the information secured.<sup>79</sup>

Grouping has received some attention as offering a possible solution to differences of ability. Lorch reports:

Ability grouping as it has ordinarily been used does not solve the problem of providing for individual differences in arithmetic. The mere grouping of pupils with similar characteristics does little toward improving the teaching-learning situation.<sup>80</sup>

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<sup>78</sup>Breuckner, op. cit., p. 334.

<sup>79</sup>Ibid.

<sup>80</sup>Harold Lorch, "Inter-Class Grouping for Arithmetic Instruction-Critique and Criteria." The Arithmetic Teacher, 8:406; 1961.

1. The first of these is the fact that the system is not a closed system.

2. The second is the fact that the system is not a closed system.

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27. The twenty-seventh is the fact that the system is not a closed system.

28. The twenty-eighth is the fact that the system is not a closed system.

29. The twenty-ninth is the fact that the system is not a closed system.

A program of grouping based on individualized textbooks in which students corrected their own work, progressed at their own speed, and were instructed during individual teacher-pupil conference periods was tried by Whitaker. Success seems to have been measured by the amount of text covered.<sup>81</sup>

This review of present practices would be incomplete without references to the studies being conducted on content, or what it is possible to teach rather than why it should be taught, as pointed out previously. The School Mathematics Study Group has prepared materials for grades four to twelve relating to pure mathematics under the support of the National Science Foundation. These materials are being tried in some areas on an experimental basis. Preliminary reports seem to indicate that students accomplish about the same achievement in S.M.S.G. classes as regular classes. There was no indication given as to time students spent in class, teaching methodology, or as to whether these materials should comprise the total arithmetic program of the curriculum.<sup>82</sup>

Other current experiments include that of Suppes<sup>83</sup> in teaching "Sets and Numbers", Stanford University, California, "Geometry in the

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<sup>81</sup> Walter L. Whitaker, "Why Not Individualize Arithmetic?" The Arithmetic Teacher, 8:402; 1960.

<sup>82</sup> "School Mathematics Study Group," Newsletter No. 10. Iland Stanford University, 1961. See also, Fred J. Weaver, "The School Mathematics Study Group on Elementary School Mathematics," The Arithmetic Teacher, 8:32-35; 1959.

<sup>83</sup> "Improvement Projects Related to Elementary School Mathematics--A Selected Listing." Unpublished mimeographed material. Lansing: Michigan Department of Public Instruction, 1962.



Primary Grades" being conducted by Hawley also at Stanford University,<sup>84</sup> the University of Illinois Arithmetic Project<sup>85</sup> and the Syracuse University "Madison Project."<sup>86</sup> These studies are concerned with teaching pure mathematical content and not with a total arithmetic curriculum. They are examples of the "what-can-be-taught" studies pointed out by Glennon.

In contrast to the studies listed above, it was the purpose of this investigation to design an arithmetic program that would take into consideration the problems reviewed in this section: (1) offering a meaningful presentation of mathematics; (2) providing for individual differences; (3) structuring so that it could be accomplished in a minimum of class time; (4) teaching only the mathematical theory that was needed to adequately explain the arithmetic content of grades five and six; and, (5) enabling students to show significant achievement on standardized achievement tests at these grade levels.

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<sup>84</sup>Ibid.

<sup>85</sup>Ibid.

<sup>86</sup>Ibid.

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## CHAPTER III

### DESCRIPTION OF THE TWO METHODS USED IN THE STUDY

It is not the purpose of this ~~thesis~~ to present in detail all the materials used in the teaching of the arithmetic program during the year of this study. Representative lesson plans appear in the Appendix. This chapter will be concerned with a descriptive comparison of the experimental and routine classroom procedures. The experimental description will contain: (1) the method of presentation of the mathematical concepts taught; and (2) the developmental processes used. The routine classroom procedure will contain an analysis of interviews with the teachers of the self-contained, or routine, classroom situations.

The experimental method was concerned with concepts to be presented to the students as important mathematical ideas of social significance. These were selected arbitrarily by the experimental teacher as representing the arithmetical materials which the students were required to cover during the course of the year as prescribed by the existing curriculum of the school system. Topics from pure mathematics that would offer sound mathematical explanation for this material were added. One concept was presented to the students each week in a discussion period. Students were not told how to solve problems, but were encouraged to create and investigate ways of doing this. A list of the concepts presented and the approximate time spent on each appears in Table I. The same concepts were presented to both fifth and sixth grades at the same time during the year in the sequence indicated on the table.



TABLE I

CONCEPTS TAUGHT IN GRADES 5 AND 6

TOPIC	APPROXIMATE TIME
My Own Number System	1 week
Egyptian Number System	1 week
Greek Number System	2 days
Roman Number System	3 days
Hindu Arabic Number System	1 week
Linear Measurement	1 week
Square Measurement (area)	1 week
Dry Measurement (volume)	1 week
Weight	1 week
How We Can Add	1 week
How We Can Subtract	1 week
How We Can Multiply	1 week
How We Can Divide	1 week
Place Value as Powers of a Base	1 week
Other Bases (7 and 12)	1 to 2 weeks
Time (and Base 60)	1 week
Modular Arithmetic (calendar)	1 week
Centuries	1 week
Temperature	1 week
Averages	1 week
Data	1 week
Graphs (line)	1 week
Graphs (bar )	1 week
Graphs (area)	1 week
Decimals (as extension of place value)	1 week
Rounding and Estimating	1 week
Big Numbers (as powers of 10)	1 week
How to Add, Subtract, Multiply and Divide Fractions)	About 2 to 4 weeks. Taught at one time all four processes.
Cancellation	1 week
Accounting	1 week
Finding Wholes from Parts	1 week



Table II indicates the general structure of each weekly lesson. The teacher acted as a resource person during the class. The text was used as a reference book. In addition, other reference materials were supplied as students requested them. When children asked for suggestions about locating materials, the teacher helped students to find these. Since much of the pure mathematics needed by the students was not found in published materials, these were written for the students by the teacher. These appear in the Appendix to this study.

At all times the student chose the method he would use in presenting evidence of his understanding of the concept for the week. Students were not given assignments to complete. The only requirement was that each would demonstrate his understanding of the concept under consideration.

TABLE II  
WEEKLY ROUTINE OF EXPERIMENTAL CLASS

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MONDAY:	Presentation of Concept by teacher in discussion period. Children discuss: (1) what they already know regarding concept; (2) how to find additional materials; (3) what each individual can do to produce tangible evidence of his understanding of concept.
TUESDAY)	Children are free to work in groups or individually in the area under investigation for the week. The experimental teacher acts as resource person.
WEDNESDAY)	
THURSDAY)	
FRIDAY:	Children are presented problems in a series of increasing difficulty. These contain selections from the four arithmetical processes. Children are requested to work as many as they can during the class time. These are taken up, corrected by the teacher and generalizations discussed with the group as a whole.

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The tangible and conceptualized presentation was intentionally designed as a methodology which would attempt to accomplish the existing arithmetic curriculum in a minimum of time without the use of rote learning techniques. The intent was to increase understanding and to decrease the amount of paper-and-pencil work by both students and teachers. This seemed desirable because of the recent additions of content courses of elementary curricula. Chapter II pointed out the relationships between such a methodology and learning theory.



It was also felt that the elementary arithmetic curriculum should meet the needs of: (1) the student who may leave school at age sixteen, and; (2) the student who will pursue a career in mathematics. The former needs a mathematics program to provide him with a cultural tool for daily living; the later, needs a sound theoretical foundation that will furnish a sequential background for secondary mathematics. The tangible and conceptual presentation was designed to present mathematical theory in a simple manner to the whole group. Each child then took the concept and produced tangible evidence of his understanding. In this manner, slow children were permitted to produce tangible evidence at their own level; academically talented students were permitted to produce evidences representative of their intellectual level. The burden of proof of understanding rested at all times with the student. Details of the mathematical theory taught are presented in the Appendix.

The procedure for teaching the experimental class followed a very definite pattern. Since the classes were conducted in two buildings, a careful time schedule had to be followed. The experimental teacher entered the building, taught the class, or classes, and left. The regular classroom teacher stayed in the room and observed the experimental class. This teacher was prohibited from teaching any additional arithmetic at any other time, and from assigning arithmetic as "seat work" or for "discipline".

At the end of the thirty-five minute experimental class period, the experimental teacher left the room and travelled to the next school. This made it impossible to give any extra help to slow learners. It also prohibited correlation of the arithmetic program with other subject matter areas. Students were requested to stop working at the end of the thirty-five minute class. Table III contains a sample lesson.





TABLE III  
DESIGN OF SAMPLE LESSON

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YOUR OWN NUMBER SYSTEM

This lesson is designed to help children understand that a number system is an orderly sequence of symbols used to express quantity. Often students become so accustomed to counting and doing arithmetic by rote that they do not realize how a number system functions. Say to the children:

"Forget everything that you ever knew about arithmetic. Forget how to make numbers and how to do problems. Now pretend you are living many thousands of years ago and that you are in need of keeping track of something. This is really one of the most important uses of arithmetic. . . writing down amounts of things by using symbols. We call these numbers and numerals."

(Do not go into the "number and numeral" question at this point. Use terms the children already understand. This may be introduced as an outcome of the lesson, however. Point out that it is easier to manipulate numerals rather than things to figure something. For instance, if a farmer has a large flock of sheep and buys and sells some, he can simply use a numeral to represent the total flock and add and subtract other numerals to represent the amount he buys and sells. This is easier than bringing them all together everytime and counting them.) After developing these or similar ideas continue:

"After you have forgotten all the arithmetic you ever knew, pretend that you want to figure something out or keep track of something. Make up a number system and do this in your system."

The children will probably make rather simple symbols such as:

1.    |        ||        |||

Just marks are typical of a 1-1 relationship.

2.    L        E        E

This type of symbol is really 1-1 relationship also.

3.            

Sometimes children will use ideas like those in the text.

4.            

Symbols like these are vague.

Children need help in seeing that symbols, or a system of numeration should be:

1. simple
2. convenient
3. useful

After they have invented a few symbols, let them work some problems in their own system. Help them to simplify and improve their system.

[illegible]

1. The first step is to identify the problem. This involves understanding the current situation and what needs to be changed.

1. The first step in the process of the investigation is the identification of the problem. This is done by the investigator who is responsible for the study. The investigator must first identify the problem that is being studied. This is done by the investigator who is responsible for the study.

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TABLE III - (Continued)  
YOUR OWN NUMBER SYSTEM - RESUME OF LESSON

1. Discuss arithmetic as a way of keeping track of things.
2. Introduce the idea that our numbers are a system of symbols that express amounts.
3. Have children make up their own number system.
4. Let each child work problems in his number system.
5. Discuss how the number systems of the children work. Lead them to conclude that a number system should be:
  1. simple
  2. useful
  3. convenient

(It is best to have children "forget everything that they ever knew about how to do arithmetic" in order to do this)

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— *Journal of the American Medical Association*, 1997, 278: 1001-1002

1. The first group of variables is the *demographic* group, which includes age, sex, and marital status. The second group is the *education* group, which includes years of schooling and highest level of education. The third group is the *employment* group, which includes occupation, industry, and tenure. The fourth group is the *income* group, which includes household income and personal income. The fifth group is the *housing* group, which includes type of housing, tenure, and location. The sixth group is the *transportation* group, which includes mode of transportation and vehicle ownership. The seventh group is the *leisure* group, which includes leisure activities and participation in sports. The eighth group is the *health* group, which includes self-rated health and chronic conditions. The ninth group is the *social* group, which includes social network and social support. The tenth group is the *psychological* group, which includes life satisfaction and mental health. The eleventh group is the *environmental* group, which includes air quality and noise. The twelfth group is the *community* group, which includes community participation and community satisfaction. The thirteenth group is the *policy* group, which includes knowledge of public policy and participation in public policy. The fourteenth group is the *research* group, which includes research participation and research satisfaction. The fifteenth group is the *other* group, which includes other variables not covered by the other groups.

— *Journal of the American Medical Association*, 1997, 278: 1003-1004

Figure 1. The effect of the concentration of the *Agrobacterium* suspension on the transformation efficiency of *Agrobacterium* strains.

3. The  $\beta$  parameter is the probability that a node will be infected by a single contact with an infected node. The  $\beta$  parameter is the probability that a node will be infected by a single contact with an infected node. The  $\beta$  parameter is the probability that a node will be infected by a single contact with an infected node.

• *Staphylococcus aureus* is the most common cause of skin infections. It is a gram-positive, spherical bacterium that can form clusters. It is often found on the skin and in the nose. It can cause a variety of infections, including skin abscesses, impetigo, and cellulitis.

Specific procedures of the experimental class are listed in Table IV.

TABLE IV

PROCEDURES USED IN EXPERIMENTAL CLASSES

- 
- 
1. Class time limited to 35 minutes per day.
  2. No ability grouping.
  3. No home work.
  4. No individual help for "slow learners."
  5. No assignments were given that required students to finish.
  6. No direct coordination with other subjects.
  7. No time spent outside of class on arithmetic such as recess or after school.
  8. No arithmetic given as "discipline" or "busy work".
  9. No drill.
  10. No "story problems" from text worked.
  11. No attempt was made to work any problems in the text; other than those used as reference and resource.
  12. Students worked problems and handed them in once a week only.
- 
-

1. The first part of the report is a general introduction to the subject of the study. It discusses the importance of the study and the objectives of the research.

2. The second part of the report is a detailed description of the methodology used in the study. It includes information about the sample size, the data collection methods, and the statistical analysis techniques.

3. The third part of the report is a presentation of the results of the study. It includes tables and graphs showing the data and the findings of the research.

4. The fourth part of the report is a discussion of the results and their implications. It discusses the strengths and limitations of the study and the implications for future research.

5. The fifth part of the report is a conclusion and a summary of the findings. It provides a final statement on the results of the study and the overall conclusions.

6. The sixth part of the report is a list of references. It includes a list of all the sources used in the study, including books, articles, and other documents.

7. The seventh part of the report is an appendix. It includes any additional information that is relevant to the study, such as raw data, additional tables, or figures.

8. The eighth part of the report is a final statement. It provides a final summary of the study and its findings, and it includes a statement of the author's responsibility for the results.

In general the class routine was one which: (1) presented an idea or concept to the students; (2) gave them opportunity to investigate it; (3) gave them opportunity to react to it by producing some tangible evidence of their understanding of the concept; and, (4) gave them an idea of the progress that they would be making were they in a routine text. This was done by giving them a set of problems once a week in the four arithmetical processes which were structured in a series of increasing difficulty. This was felt necessary for the following reasons.

1. Patrons of the school wished to know "what page in the text their children were on." It was felt necessary to show them that the students were able to do the routine text materials although they were not using the text as such.

2. The children wanted to know if the conceptualized program would retard their regular progress through the grade. They seemed to wish some evidence of their progress. Doing some problems in the routine fashion seemed to help them feel they were making progress in the grade.

3. The problems presented once a week differed from the type contained in the text in that: (1) the four arithmetical processes were represented; (2) the problems progressed from very simple to very difficult ones. Each child did only the problems of which he felt capable; (3) these problems always contained representative examples from all preceeding lessons. Samples of these weekly lessons appear in the Appendix.

As each concept was presented, every effort was made to correlate and illustrate it with culturally significant items. Fractions were taught with measures. All four arithmetical processes involving

fractions were taught at the same time. These were presented as experiences with fractions within the culture rather than as the manipulation of abstract symbols. After the understanding was built by using tangible materials, abstract symbols were manipulated in problem situations. Pages 41 to 45 show a typical lesson in fractions.





## LINEAR MEASUREMENT

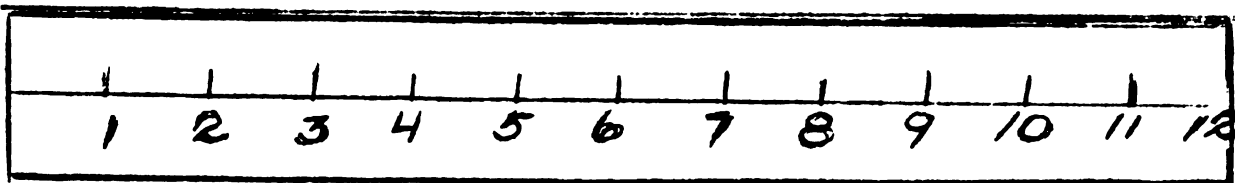
41

### GRADE 5

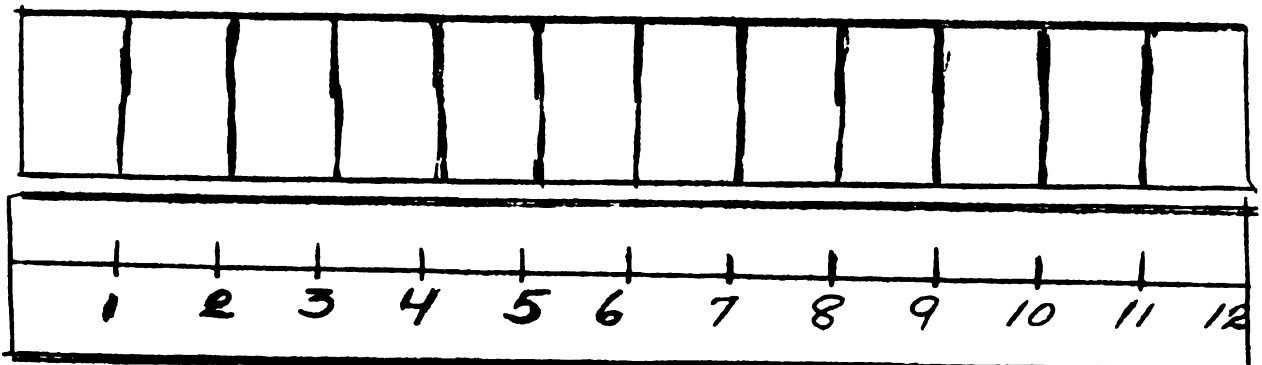
Linear (līn ē er) measurement is a measurement of line. We say a line is 6 inches long. We also measure line, or length, in feet, miles, rods, and yards. See how many different kinds of linear measurement you can find and list.

We can use linear measurement to help us understand fractions. This is a good thing to do because very few of the things that we measure come in whole inches, whole feet, whole yards, etc.

Take one of the strips of paper that have been given you, place your ruler on it so that you can draw a line down the middle of the strip. On the line mark off the inch divisions so that you have a little oak tag ruler of your own. Like this:



Do the same thing with the other strip, but cut this one up into one-inch sections. This will give you a ruler that you can divide up into fractional parts. Place the whole ruler beside the one that is cut up into inches on your desk like this:





Do the following things and answer the questions.

1. Divide the cut up inches into two groups.  $1/2$  foot = ? inches?
2. Divide the cut up inches into four groups.  $1/4$  foot = ? inches?
3. Divide the cut up inches into three groups. Each of these is a third of a foot.  $1/3$  foot = ? inches?

\*\*\*\*\*

There are 12 inches in a foot. Each inch is  $1/12$  of the whole foot. By moving the inch pieces about, answer the following questions.

A

1.  $1/4$  ft. = ?/12 ft.
2.  $1/2$  ft. = ?/12 ft.
3.  $1/2$  ft. = ?/4 ft.

B

- $3/12$  ft. = ?/4 ft.
- $3/4$  ft. = ?/2 ft.
- $2/4$  ft. = ?/2 ft.

Can you make a list of other statements that are true about the fractional parts of a foot?

\*\*\*\*\*

Answer these questions using your cut up inch pieces to help you.

A

1.  $1/2 = ?/4$

B

$1/3 = ?/6$

C

$3/4 = ?/12$

D

$2/2 = ?/12$

2.  $1/4 = ?/12$

$1/2 = ?/4$

$1/6 = ?/12$

$2/3 = ?/12$

Make as many additional true statements as you can.

\*\*\*\*\*

You have found that there are three-twelfths of a foot in one-fourth of a foot. We can write it this way:  $3/12$  foot =  $1/4$  foot, or just  $3/12 = 1/4$ . Use your inches and see if you can complete these statements so that they are true.

1. If  $1/4 = 3/12$ , then  $3/4 = ?/12$
2. If  $2/3 = 8/12$ , then  $1/3 = ?/12$
3. If  $1/2 = 6/12$ , then  $2/2 = ?/12$



These are more difficult. See if you can do them by thinking very carefully.

1. If  $1/2 = 2/4$ , then  $1 \frac{1}{2} = ?/4$ . We would read this this way: "If one half equals two fourths, then one and one half halves equal how many fourths?" The answer would be three fourths, or  $3/4$ . Do this one.

If  $1/2 = 2/4$ , then  $2 \frac{1}{2} = ?/4$ .

2. If  $1/3 = 4/12$ , then  $1 \frac{1}{2} = ?/12$ .
3. If  $6/8 = 3/4$ , then  $1/8 = ?/4$ .
4. If  $4/12 = 1/3$ , then  $5/12$  is how many twelfths more than  $1/3$ ?

We have a sign in mathematics that means "more than." It is: .

"Less than" is this sign: . You can remember them by thinking that more than is pointed in the direction that you write. Less than is pointed backwards to the way you write. Now write the answer to your problem like this:

If  $4/12 = 1/3$ , then  $5/12$  is  $1/12$   $1/3$ .

5. If  $1/6 = 2/12$ , then  $3/12$  is how many twelfths  $1/6$ ?
6. Is  $3/12$  or  $1/6$ ?

\*\*\*\*\*

You will need to think very carefully to do these. Use your cut up inches and see if they are true.

1.  $3/4$  foot =  $2 \frac{1}{2}$  thirds of a foot. (Hint: change both fractions to twelfths of a foot).
2.  $1/2$  foot =  $1 \frac{1}{2}$  thirds of a foot.
3.  $2/3$  foot =  $2 \frac{1}{4}$  fourths of a foot.

\*\*\*\*\*

1. The first part of the document discusses the importance of maintaining accurate records of all transactions. It emphasizes that proper record-keeping is essential for the transparency and accountability of the organization. This section also outlines the various methods used to collect and analyze data, ensuring that the information is reliable and up-to-date.

2. The second part of the document focuses on the financial aspects of the organization. It provides a detailed breakdown of the budget, including income, expenses, and the resulting surplus or deficit. This section also discusses the various financial risks and the strategies used to mitigate them, ensuring the financial stability of the organization.

3. The third part of the document addresses the operational aspects of the organization. It describes the various departments and their functions, as well as the processes used to manage the organization's resources. This section also discusses the various challenges faced by the organization and the strategies used to overcome them, ensuring the efficient and effective operation of the organization.

4. The fourth part of the document discusses the future of the organization. It outlines the various goals and objectives for the coming year, as well as the strategies used to achieve them. This section also discusses the various risks and challenges that the organization may face in the future and the strategies used to mitigate them, ensuring the long-term success of the organization.

5. The fifth part of the document discusses the various stakeholders of the organization. It identifies the various groups and individuals who have an interest in the organization and their respective roles. This section also discusses the various ways in which the organization interacts with its stakeholders and the strategies used to manage these relationships, ensuring the overall success of the organization.

You have seen that you can write  $2 \frac{1}{2}$  thirds like this:  $\frac{2 \frac{1}{2}}{3}$ .

Write  $1 \frac{1}{2}$  fourths. If you add like this, then is it true?

$$\begin{array}{r} \frac{1 \frac{1}{2}}{4} \\ \frac{1 \frac{1}{2}}{4} \\ \hline \frac{3}{4} \end{array}$$

Add:

|                            |                            |                            |
|----------------------------|----------------------------|----------------------------|
| $\frac{1 \frac{1}{2}}{3}$  | $\frac{2 \frac{1}{4}}{2}$  | $\frac{3 \frac{1}{2}}{4}$  |
| $\frac{1 \frac{1}{2}}{3}$  | $\frac{2 \frac{1}{4}}{2}$  | $\frac{3 \frac{1}{2}}{4}$  |
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[illegible]

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1. *Chlorophyll a* (Chl *a*)

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1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100. 101. 102. 103. 104. 105. 106. 107. 108. 109. 110. 111. 112. 113. 114. 115. 116. 117. 118. 119. 120. 121. 122. 123. 124. 125. 126. 127. 128. 129. 130. 131. 132. 133. 134. 135. 136. 137. 138. 139. 140. 141. 142. 143. 144. 145. 146. 147. 148. 149. 150. 151. 152. 153. 154. 155. 156. 157. 158. 159. 160. 161. 162. 163. 164. 165. 166. 167. 168. 169. 170. 171. 172. 173. 174. 175. 176. 177. 178. 179. 180. 181. 182. 183. 184. 185. 186. 187. 188. 189. 190. 191. 192. 193. 194. 195. 196. 197. 198. 199. 200. 201. 202. 203. 204. 205. 206. 207. 208. 209. 210. 211. 212. 213. 214. 215. 216. 217. 218. 219. 220. 221. 222. 223. 224. 225. 226. 227. 228. 229. 230. 231. 232. 233. 234. 235. 236. 237. 238. 239. 240. 241. 242. 243. 244. 245. 246. 247. 248. 249. 250. 251. 252. 253. 254. 255. 256. 257. 258. 259. 260. 261. 262. 263. 264. 265. 266. 267. 268. 269. 270. 271. 272. 273. 274. 275. 276. 277. 278. 279. 280. 281. 282. 283. 284. 285. 286. 287. 288. 289. 290. 291. 292. 293. 294. 295. 296. 297. 298. 299. 300. 301. 302. 303. 304. 305. 306. 307. 308. 309. 310. 311. 312. 313. 314. 315. 316. 317. 318. 319. 320. 321. 322. 323. 324. 325. 326. 327. 328. 329. 330. 331. 332. 333. 334. 335. 336. 337. 338. 339. 340. 341. 342. 343. 344. 345. 346. 347. 348. 349. 350. 351. 352. 353. 354. 355. 356. 357. 358. 359. 360. 361. 362. 363. 364. 365. 366. 367. 368. 369. 370. 371. 372. 373. 374. 375. 376. 377. 378. 379. 380. 381. 382. 383. 384. 385. 386. 387. 388. 389. 390. 391. 392. 393. 394. 395. 396. 397. 398. 399. 400. 401. 402. 403. 404. 405. 406. 407. 408. 409. 410. 411. 412. 413. 414. 415. 416. 417. 418. 419. 420. 421. 422. 423. 424. 425. 426. 427. 428. 429. 430. 431. 432. 433. 434. 435. 436. 437. 438. 439. 440. 441. 442. 443. 444. 445. 446. 447. 448. 449. 450. 451. 452. 453. 454. 455. 456. 457. 458. 459. 460. 461. 462. 463. 464. 465. 466. 467. 468. 469. 470. 471. 472. 473. 474. 475. 476. 477. 478. 479. 480. 481. 482. 483. 484. 485. 486. 487. 488. 489. 490. 491. 492. 493. 494. 495. 496. 497. 498. 499. 500. 501. 502. 503. 504. 505. 506. 507. 508. 509. 510. 511. 512. 513. 514. 515. 516. 517. 518. 519. 520. 521. 522. 523. 524. 525. 526. 527. 528. 529. 530. 531. 532. 533. 534. 535. 536. 537. 538. 539. 540. 541. 542. 543. 544. 545. 546. 547. 548. 549. 550. 551. 552. 553. 554. 555. 556. 557. 558. 559. 560. 561. 562. 563. 564. 565. 566. 567. 568. 569. 570. 571. 572. 573. 574. 575. 576. 577. 578. 579. 580. 581. 582. 583. 584. 585. 586. 587. 588. 589. 590. 591. 592. 593. 594. 595. 596. 597. 598. 599. 600. 601. 602. 603. 604. 605. 606. 607. 608. 609. 610. 611. 612. 613. 614. 615. 616. 617. 618. 619. 620. 621. 622. 623. 624. 625. 626. 627. 628. 629. 630. 631. 632. 633. 634. 635. 636. 637. 638. 639. 640. 641. 642. 643. 644. 645. 646. 647. 648. 649. 650. 651. 652. 653. 654. 655. 656. 657. 658. 659. 660. 661. 662. 663. 664. 665. 666. 667. 668. 669. 670. 671. 672. 673. 674. 675. 676. 677. 678. 679. 680. 681. 682. 683. 684. 685. 686. 687. 688. 689. 690. 691. 692. 693. 694. 695. 696. 697. 698. 699. 700. 701. 702. 703. 704. 705. 706. 707. 708. 709. 710. 711. 712. 713. 714. 715. 716. 717. 718. 719. 720. 721. 722. 723. 724. 725. 726. 727. 728. 729. 730. 731. 732. 733. 734. 735. 736. 737. 738. 739. 740. 741. 742. 743. 744. 745. 746. 747. 748. 749. 750. 751. 752. 753. 754. 755. 756. 757. 758. 759. 760. 761. 762. 763. 764. 765. 766. 767. 768. 769. 770. 771. 772. 773. 774. 775. 776. 777. 778. 779. 780. 781. 782. 783. 784. 785. 786. 787. 788. 789. 790. 791. 792. 793. 794. 795. 796. 797. 798. 799. 800. 801. 802. 803. 804. 805. 806. 807. 808. 809. 810. 811. 812. 813. 814. 815. 816. 817. 818. 819. 820. 821. 822. 823. 824. 825. 826. 827. 828. 829. 830. 831. 832. 833. 834. 835. 836. 837. 838. 839. 840. 84

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**Divide:**

Label your answer and your remainder if you have one.

**A**

$$2 \text{ in. } \overline{) 12 \text{ in.}}$$

**B**

$$3 \text{ in. } \overline{) 18 \text{ in.}}$$

**C**

$$2 \text{ in. } \overline{) 24 \text{ in.}}$$

**D**

$$6 \text{ in. } \overline{) 54 \text{ in.}}$$

**E**

$$7 \text{ in. } \overline{) 1 \text{ ft. } 4 \text{ in.}}$$

**F**

$$3 \text{ in. } \overline{) 2 \text{ ft.}}$$

**G**

$$6 \text{ in. } \overline{) 3 \text{ ft. } 6 \text{ in.}}$$

The routine or control classroom procedures were established by interviews with the teachers of these rooms. Pages 46 to 50 contain a summary of these interviews. The replies of the teachers of the routine (control) classrooms have been generalized and listed. A summary appears in the conclusion.

1. How were assignments made?

Control teacher No. 1

Assignments in texts at several levels.

Students required to finish assignment.

Memorizing tables, mandatory - drilled.

Homework about 3 times a week.

Text sequence covered.

Talented pupils worked extra problems in advanced texts.

Repetitive problem work for slow learners.

Control teacher No. 2

Assignments in text every day.

Some children "worked ahead".

Assigned extra work as "discipline".

Assigned homework.

Generally worked through text.

Control teacher No. 3

Assignments in text every day.

Generally covered text.

Control Teacher No. 3 - continued

Students required to finish given amount determined by teacher.

Tables memorized.

Control teacher No. 4

Assignments in text every day.

Teacher determined amount to be completed.

Generally covered text.

Followed text sequence.

2. Were story problems in text covered?

Control teacher No. 1

Generally covered except for slow learners.

Control teacher No. 2

Covered at least 60% of those in text and made up others.

Control teacher No. 3

Did most of the story problems in the text.

Control teacher No. 4

Worked most of the story problems in the text.

3. Did you give individual help?

Control teacher No. 1

During odd times all during the day.

Set aside 15 minutes twice a week for "extra" help.

Worked with slow learners.

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Control teacher No. 2

Gave extra help to talented and slow students.

Control teacher No. 3

Helped individuals all day through.

Control teacher No. 4

Gave individual help during the day and set aside 15 minutes per day extra for help.

4. How often were papers sent home?

Control teacher No. 1

At marking periods and also other times.

Sent homework assignments home.

Control teacher No. 2

Two or three times a year.

Control teacher No. 3

Twice per week.

Control teacher No. 4

Each week.

5. How were students grouped as to ability?

Control teacher No. 1

Students grouped themselves: below grade level, at grade level, and above grade level.

Control teacher No. 2

Some individual cases.



Control teacher No. 3

Grouped according to amount of text covered.

Control teacher No. 4

All stayed in same text but progressed at different levels.

6. What materials did students cover?

Control teacher No. 1

Used texts at various ability levels.

Control teacher No. 2

Covered text sequence.

Control teacher No. 3

Covered texts at various ability levels.

Control teacher No. 4

Followed text sequence - 2/3 to all of it.

7. How often did you correct papers?

Control teacher No. 1

At least 3 times a week.

Control teacher No. 2

More or less every day.

Control teacher No. 3

Children corrected own papers every day - teacher once a week.



2.  $\frac{1}{2} \log 2$

•  $\frac{1}{2} \log 2 = \frac{1}{2} \log 2$

3.  $\frac{1}{2} \log 2$

•  $\frac{1}{2} \log 2 = \frac{1}{2} \log 2$

4.  $\frac{1}{2} \log 2$

•  $\frac{1}{2} \log 2 = \frac{1}{2} \log 2$

•  $\frac{1}{2} \log 2 = \frac{1}{2} \log 2$

5.  $\frac{1}{2} \log 2$

•  $\frac{1}{2} \log 2 = \frac{1}{2} \log 2$

6.  $\frac{1}{2} \log 2$

•  $\frac{1}{2} \log 2 = \frac{1}{2} \log 2$

7.  $\frac{1}{2} \log 2$

•  $\frac{1}{2} \log 2 = \frac{1}{2} \log 2$

8.  $\frac{1}{2} \log 2$

•  $\frac{1}{2} \log 2 = \frac{1}{2} \log 2$

•  $\frac{1}{2} \log 2 = \frac{1}{2} \log 2$

9.  $\frac{1}{2} \log 2$

•  $\frac{1}{2} \log 2 = \frac{1}{2} \log 2$

10.  $\frac{1}{2} \log 2$

•  $\frac{1}{2} \log 2 = \frac{1}{2} \log 2$

Control teacher No. 4

Two times a week, but had children rework problems missed and then corrected these.

## 8. What other learning activities did you use?

Control teacher No. 1

Abacus, flash cards, games, extra papers, teamwork, outside of class productions, pictures.

Control teacher No. 2

Memorized tables by repetitious problems.

Control teacher No. 3

Correlated with science.

Had drill problems.

Control teacher No. 4

Some games, tables (drill)

## 9. How much instructional time was used per day for arithmetic?

Control teacher No. 1

45 minutes, plus extra help during the day and homework.

Control teacher No. 2

35 minutes, plus some homework and "discipline" problems.

Control teacher No. 3

35 minutes, plus extra help at odd times.

Control teacher No. 4

Given extra time to finish.

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100. 101. 102. 103. 104. 105. 106. 107. 108. 109. 110. 111. 112. 113. 114. 115. 116. 117. 118. 119. 120. 121. 122. 123. 124. 125. 126. 127. 128. 129. 130. 131. 132. 133. 134. 135. 136. 137. 138. 139. 140. 141. 142. 143. 144. 145. 146. 147. 148. 149. 150. 151. 152. 153. 154. 155. 156. 157. 158. 159. 160. 161. 162. 163. 164. 165. 166. 167. 168. 169. 170. 171. 172. 173. 174. 175. 176. 177. 178. 179. 180. 181. 182. 183. 184. 185. 186. 187. 188. 189. 190. 191. 192. 193. 194. 195. 196. 197. 198. 199. 200. 201. 202. 203. 204. 205. 206. 207. 208. 209. 210. 211. 212. 213. 214. 215. 216. 217. 218. 219. 220. 221. 222. 223. 224. 225. 226. 227. 228. 229. 230. 231. 232. 233. 234. 235. 236. 237. 238. 239. 240. 241. 242. 243. 244. 245. 246. 247. 248. 249. 250. 251. 252. 253. 254. 255. 256. 257. 258. 259. 260. 261. 262. 263. 264. 265. 266. 267. 268. 269. 270. 271. 272. 273. 274. 275. 276. 277. 278. 279. 280. 281. 282. 283. 284. 285. 286. 287. 288. 289. 290. 291. 292. 293. 294. 295. 296. 297. 298. 299. 300. 301. 302. 303. 304. 305. 306. 307. 308. 309. 310. 311. 312. 313. 314. 315. 316. 317. 318. 319. 320. 321. 322. 323. 324. 325. 326. 327. 328. 329. 330. 331. 332. 333. 334. 335. 336. 337. 338. 339. 340. 341. 342. 343. 344. 345. 346. 347. 348. 349. 350. 351. 352. 353. 354. 355. 356. 357. 358. 359. 360. 361. 362. 363. 364. 365. 366. 367. 368. 369. 370. 371. 372. 373. 374. 375. 376. 377. 378. 379. 380. 381. 382. 383. 384. 385. 386. 387. 388. 389. 390. 391. 392. 393. 394. 395. 396. 397. 398. 399. 400. 401. 402. 403. 404. 405. 406. 407. 408. 409. 410. 411. 412. 413. 414. 415. 416. 417. 418. 419. 420. 421. 422. 423. 424. 425. 426. 427. 428. 429. 430. 431. 432. 433. 434. 435. 436. 437. 438. 439. 440. 441. 442. 443. 444. 445. 446. 447. 448. 449. 450. 451. 452. 453. 454. 455. 456. 457. 458. 459. 460. 461. 462. 463. 464. 465. 466. 467. 468. 469. 470. 471. 472. 473. 474. 475. 476. 477. 478. 479. 480. 481. 482. 483. 484. 485. 486. 487. 488. 489. 490. 491. 492. 493. 494. 495. 496. 497. 498. 499. 500. 501. 502. 503. 504. 505. 506. 507. 508. 509. 510. 511. 512. 513. 514. 515. 516. 517. 518. 519. 520. 521. 522. 523. 524. 525. 526. 527. 528. 529. 530. 531. 532. 533. 534. 535. 536. 537. 538. 539. 540. 541. 542. 543. 544. 545. 546. 547. 548. 549. 550. 551. 552. 553. 554. 555. 556. 557. 558. 559. 560. 561. 562. 563. 564. 565. 566. 567. 568. 569. 570. 571. 572. 573. 574. 575. 576. 577. 578. 579. 580. 581. 582. 583. 584. 585. 586. 587. 588. 589. 590. 591. 592. 593. 594. 595. 596. 597. 598. 599. 600. 601. 602. 603. 604. 605. 606. 607. 608. 609. 610. 611. 612. 613. 614. 615. 616. 617. 618. 619. 620. 621. 622. 623. 624. 625. 626. 627. 628. 629. 630. 631. 632. 633. 634. 635. 636. 637. 638. 639. 640. 641. 642. 643. 644. 645. 646. 647. 648. 649. 650. 651. 652. 653. 654. 655. 656. 657. 658. 659. 660. 661. 662. 663. 664. 665. 666. 667. 668. 669. 670. 671. 672. 673. 674. 675. 676. 677. 678. 679. 680. 681. 682. 683. 684. 685. 686. 687. 688. 689. 690. 691. 692. 693. 694. 695. 696. 697. 698. 699. 700. 701. 702. 703. 704. 705. 706. 707. 708. 709. 710. 711. 712. 713. 714. 715. 716. 717. 718. 719. 720. 721. 722. 723. 724. 725. 726. 727. 728. 729. 730. 731. 732. 733. 734. 735. 736. 737. 738. 739. 740. 741. 742. 743. 744. 745. 746. 747. 748. 749. 750. 751. 752. 753. 754. 755. 756. 757. 758. 759. 760. 761. 762. 763. 764. 765. 766. 767. 768. 769. 770. 771. 772. 773. 774. 775. 776. 777. 778. 779. 780. 781. 782. 783. 784. 785. 786. 787. 788. 789. 790. 791. 792. 793. 794. 795. 796. 797. 798. 799. 800. 801. 802. 803. 804. 805. 806. 807. 808. 809. 810. 811. 812. 813. 814. 815. 816. 817. 818. 819. 820. 821. 822. 823. 824. 825. 826. 827. 828. 829. 830. 831. 832. 833. 834. 835. 836. 837. 838. 839. 840. 84

[illegible]

1. *Chlorophyll a* (Chl *a*)

THE UNIVERSITY OF CHICAGO

— *Journal of the American Medical Association*, 1997, 277: 1009-1010

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• *Lauren* is a 25-year-old female who is a graduate of the University of California, Berkeley, and is currently employed as a research assistant at a local university. She is a member of the Phi Kappa Phi Honor Society and is a member of the Phi Kappa Phi Honor Society. She is a member of the Phi Kappa Phi Honor Society.

1990

• 1980 - 1981 •

• 2 •

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1. *Chlorophyll a* and *Chlorophyll b* were determined by the method of Arar and Collins (1971).

[illegible]

1000

• What is the purpose of the study? (1 point)

— 2 —

[illegible]

2. *Phragmites* (Common Reed)

• *Chlorophyll a* (Chl *a*)

1997, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 26

TABLE V

SUMMARY OF TEACHING TECHNIQUES USED  
IN EXPERIMENTAL AND ROUTINE CLASSES

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| <u>EXPERIMENTAL</u>                    | <u>ROUTINE</u>                                |
|--|---|
| Assignments were student determined    | Assignments teacher determined                |
| Did not work story problems in text    | Worked most of story problems in text         |
| Received no individual help            | Received individual help                      |
| Worked problems to hand in once a week | Worked problems to hand in 2 - 5 times a week |
| Were not grouped                       | Were partially ability grouped                |
| Text did not structure program         | Text structured program                       |
| Had many other learning activities     | Learning activities limited to "drill" games  |
| Spent 35 minutes per day in class      | Spent over 35 minutes per day in class        |

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## CHAPTER IV

### ORGANIZATION OF INVESTIGATION

Design of this investigation. This dissertation was carried out during the course of the "Study in the Use of Special Teachers in Science and Mathematics in Grades Five and Six a part of the Science Teaching Program of the American Association for the Advancement of Science". Cooperating school systems were: Lansing, Michigan; Cedar Rapids, Iowa; Washington, D.C.; and Woodford County, Kentucky. Each school system hired a teacher who would teach only science or only mathematics. The purpose of the over-all study was to compare the two methods: (1) special teachers; or, (2) self-contained classroom teachers.

The use of a conceptualized and tangible presentation of arithmetic was devised as the special method of teaching which would be used in the Lansing, Michigan experimental center. As each of the four experimental centers were free to specialize in any way they chose, it was felt that this in no way either detract from or enhance the study of the American Association for the Advancement of Science.

Students in the study. The students chosen to participate in the study were considered as typical of those in the city. Two schools, each of which contained two fifth and two sixth grades were selected. One fifth grade and one sixth grade were taught by the special, or experimental teacher, and the other fifth and sixth grades were taught by the self-contained, or control room teacher in the routine manner of the school system.



There were, therefore, eight groups in the study. They were arbitrarily numbered as groups I to VIII, inclusive. Odd numbered groups were experimental, even numbers designated the control (self-contained) groups. No effort was made to match the members of the experimental and control classes. They were selected just as they existed in the school during the regular routine of grade placement. In the four fifth and four sixth grades, there were a total of 89 students in the experimental groups and 77 students in the control groups that completed the study. They have been treated as a total universe of experimental and control groups rather than separately.

No effort was made to match the members of the control and experimental classes. The classes were left as they existed in the normal routine of the school. Table VI compares the mean scores of the groups on the Otis Quick Scoring Test of Mental Ability given at the beginning of the study.

TABLE VI

COMPARISON OF MEAN TEST SCORES ON OTIS QUICK SCORING  
TEST OF MENTAL ABILITY GIVEN AT BEGINNING OF STUDY

| <u>EXPERIMENTAL</u>                                    |       | <u>CONTROL</u> |       |
|--|-------|----------------|-------|
| Group I  | 101.1 | Group II       | 103.1 |
| Group III  | 107.9 | Group IV       | 106.4 |
| Group V  | 104.4 | Group VI       | 103.9 |
| Group VII  | 103.9 | Group VIII     | 103.2 |
| MEAN OF THE MEAN SCORES ON OTIS OF EXPERIMENTAL GROUPS |       |                | 104.3 |
| MEAN OF THE MEAN SCORES ON OTIS OF CONTROL GROUPS      |       |                | 104.1 |



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Table VII shows the composition of the experimental and control groups.

TABLE VII  
COMPOSITION OF GROUPS PARTICIPATING IN STUDY

| <u>EXPERIMENTAL</u>             |                |                   |                    |                        |                   |
|---------------------------------|----------------|-------------------|--------------------|------------------------|-------------------|
| GROUP                           | GRADE<br>LEVEL | NUMBER<br>OF BOYS | NUMBER<br>OF GIRLS | AGE RANGE<br>IN MONTHS | TOTAL<br>IN GROUP |
| I                               | 6              | 14                | 9                  | 130 - 142              | 23                |
| III                             | 5              | 11                | 11                 | 119 - 136              | 22                |
| V                               | 6              | 11                | 13                 | 130 - 143              | 24                |
| VII                             | 5              | <u>8</u>          | <u>12</u>          | 118 - 137              | <u>20</u>         |
| TOTAL EXPERIMENTAL<br>GROUPS    |                | 44                | 45                 |                        | 89                |
| <u>CONTROL (SELF-CONTAINED)</u> |                |                   |                    |                        |                   |
| II                              | 6              | 8                 | 10                 | 131 - 149              | 18                |
| IV                              | 5              | 5                 | 12                 | 118 - 129              | 17                |
| VI                              | 6              | 10                | 13                 | 128 - 142              | 23                |
| VIII                            | 5              | <u>8</u>          | <u>11</u>          | 113 - 130              | <u>19</u>         |
| TOTAL CONTROL GROUPS            |                | 31                | 46                 |                        | 77                |

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Other personnel in the study. The regular classroom teacher remained in the room and observed the experimental lesson at most of the sessions. It was felt that this would have a certain inservice training effect. Although no study was made of the attitudes of these teachers, their feelings of anxiety were often expressed to the author. They indicated that they felt that the conceptualized presentation, the freedom which the students enjoyed in planning their own lessons, the fact that the sequence of the text was not followed, and that no regular assignments of problems to be completed and handed in would cause the children to fall behind in their grade level of achievement. It seemed difficult for them to disengage themselves from the arithmetic program as taught by the special teacher in their rooms. Pressure and anxiety became so great in some cases that the regular classroom teacher was requested to leave the room during the experimental class.

At the beginning of the study, the patrons of the school exhibited great anxiety about the experimental program. They frequently asked: (1) what page of the text the class was on; (2) why there were no papers worked and brought home by the children; (3) if their children could not be removed from the experimental class because "they were learning nothing". In order to answer such questions, group meetings were held in which the total experimental program was explained to the parents. They were invited to visit the classes and often did so. Frequently they asked to come and learn some of the "new" mathematics with the children. As the study progressed, the support of the parents appeared to increase. In many cases they expressed a desire to have the program continue as part of the regular curriculum. As an outgrowth of the experimental

• 1947年10月1日，中华人民共和国成立，这是中国历史上一个伟大的转折点。从此，中国结束了长达一个多世纪的半殖民地半封建社会，真正实现了民族独立和人民解放。在党的领导下，中国人民开始了建设社会主义的新征程。这一时期，国家在政治、经济、文化各方面都取得了长足的进步，为后来的发展奠定了坚实的基础。

• 1956年，社会主义改造基本完成，中国正式进入社会主义社会。这一时期，国家在工业、农业、交通运输业等方面取得了显著成就，人民生活水平得到了初步改善。同时，国家也面临着许多困难和挑战，但正是在党的领导下，中国人民克服了重重困难，实现了国家的统一和民族的团结。

• 1978年，改革开放政策的实施，标志着中国进入了社会主义现代化建设的新时期。这一时期，国家在经济、科技、教育、文化等方面取得了举世瞩目的成就，人民生活水平得到了显著提高。同时，国家也面临着许多新的挑战，但正是在党的领导下，中国人民克服了重重困难，实现了国家的繁荣和民族的振兴。

• 1992年，社会主义市场经济体制的建立，标志着中国进入了改革开放和社会主义现代化建设的新阶段。这一时期，国家在经济、科技、教育、文化等方面取得了举世瞩目的成就，人民生活水平得到了显著提高。同时，国家也面临着许多新的挑战，但正是在党的领导下，中国人民克服了重重困难，实现了国家的繁荣和民族的振兴。

• 2001年，中国加入世界贸易组织，标志着中国正式成为世界经济的重要组成部分。这一时期，国家在经济、科技、教育、文化等方面取得了举世瞩目的成就，人民生活水平得到了显著提高。同时，国家也面临着许多新的挑战，但正是在党的领导下，中国人民克服了重重困难，实现了国家的繁荣和民族的振兴。

(1) 1949年10月1日，中华人民共和国成立，这是中国历史上一个伟大的转折点。从此，中国结束了长达一个多世纪的半殖民地半封建社会，真正实现了民族独立和人民解放。在党的领导下，中国人民开始了建设社会主义的新征程。这一时期，国家在政治、经济、文化各方面都取得了长足的进步，为后来的发展奠定了坚实的基础。

• 1956年，社会主义改造基本完成，中国正式进入社会主义社会。这一时期，国家在工业、农业、交通运输业等方面取得了显著成就，人民生活水平得到了初步改善。同时，国家也面临着许多困难和挑战，但正是在党的领导下，中国人民克服了重重困难，实现了国家的统一和民族的团结。

• 1978年，改革开放政策的实施，标志着中国进入了社会主义现代化建设的新时期。这一时期，国家在经济、科技、教育、文化等方面取得了举世瞩目的成就，人民生活水平得到了显著提高。同时，国家也面临着许多新的挑战，但正是在党的领导下，中国人民克服了重重困难，实现了国家的繁荣和民族的振兴。

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• 2001年，中国加入世界贸易组织，标志着中国正式成为世界经济的重要组成部分。这一时期，国家在经济、科技、教育、文化等方面取得了举世瞩目的成就，人民生活水平得到了显著提高。同时，国家也面临着许多新的挑战，但正是在党的领导下，中国人民克服了重重困难，实现了国家的繁荣和民族的振兴。

arithmetic, an adult education class in "Arithmetic for Parents" was inaugurated. This class was taught by the author. The parents showed keen interest and often expressed the feeling that they could now "understand what their children were doing."

The geographic and cultural areas involved in the study were chosen mainly because of their accessibility, since it was necessary for the experimental teacher to travel from one school to the next in less than thirty minutes. One of the schools was in an area of the city which contained factories for manufacturing heavy machinery, the other participating school was in a residential district containing only local stores and offices. Table VIII indicates the types of employment listed on the Form CA 39's in the schools. These should be regarded as an indication only as these records are not detailed. When both parents were listed as employed, both occupations were counted.



TABLE VIII

OCCUPATIONS OF PARENTS OF STUDENTS IN STUDY AS INDICATED ON  
FORM CA 39 FILED IN THE TWO PARTICIPATING SCHOOLS

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| <u>"FACTORY" DISTRICT - GROUPS I, II, III, IV</u>       |                  |          |                               |          |         |
|---|------------------|----------|-------------------------------|----------|---------|
|   | SELF<br>EMPLOYED | LABORERS | SKILLED AND<br>ADMINISTRATIVE | CLERICAL | UNKNOWN |
| Experimental  | 2                | 33       | 6                             | 1        | 2       |
| Control   | 2                | 21       | 11                            | 3        | 1       |
| <u>"RESIDENTIAL" DISTRICT - GROUPS V, VI, VII, VIII</u> |                  |          |                               |          |         |
| Experimental  | 6                | 11       | 23                            | 0        | 1       |
| Control   | 4                | 4        | 31                            | 2        | 1       |

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Table IX indicates the general academic background of the parents in the two geographic areas. These listings were taken from the CA 39 records in the participating schools and should be regarded as an indication only. They do however offer some idea as to the composition of the area.

TABLE IX

ACADEMIC BACKGROUND OF PARENTS OF STUDENTS  
PARTICIPATING IN THE STUDY

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| "FACTORY" DISTRICT                                    |              |               |                |                |
|---|--------------|---------------|----------------|----------------|
| EDUCATION OF PARENTS IN YEARS - GROUPS I, II, III, IV |              |               |                |                |
|   | <u>0 - 8</u> | <u>9 - 12</u> | <u>12 - 16</u> | <u>OVER 16</u> |
| Experimental  | 10%          | 82%           | 5%             | 1%             |
| Control   | 8%           | 81%           | 7%             | 1%             |

| "RESIDENTIAL" DISTRICT                                  |    |     |     |    |
|---|----|-----|-----|----|
| EDUCATION OF PARENTS IN YEARS - GROUPS V, VI, VII, VIII |    |     |     |    |
| Experimental  | 3% | 66% | 29% | 1% |
| Control   | 7% | 61% | 24% | 3% |

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## CHAPTER V

### DATA AND RESULTS

The achievement of the students in this study was evaluated on the basis of the regular testing program of the school. It was felt that this would offer some indication as to whether or not the children had progressed satisfactorily in the regular school program. This was due to the fact that it was an integral part of the study to ascertain if the arithmetic content required by the curriculum of the school system could be accomplished: (1) in a decreased class time; (2) without drill or rote learning processes; and, (3) with a decreased quantity of "paper and pencil" problem solving.

In addition, the achievement of the students in this study as measured by the regular testing program of the school, could be used to test the assumption that the "experimental" teaching method was equal to or better than the routine (control) teaching method.

As pointed out above, all students were, at the inception of the study, given the STEP Test Form A. The California Achievement Test - Arithmetic Form BB - Reasoning and Fundamentals had been given in the school routine testing program. They were likewise tested at the conclusion (the end of the academic year) of the study. These standard tests were used to measure the relative gain in achievement of students after the treatment (tangible and conceptualized method) as opposed to routine teaching methods (control group).

Analysis of the Data. The "before-after" comparison was made by the use of analysis of Covariance. Lindquist, Taves, and others have



demonstrated the superiority of the analysis of variance model over most other statistical techniques. This model is especially superior to a standard "differences in the means" model which is based on the (t) distribution and hence "breaks down for small samples primarily because the standard deviations of small samples are not normally distributed."<sup>86</sup> When testing the hypothesis that the samples were drawn from equally variable populations, rather than dealing with the difference between the observed standard deviations (t) we deal with the ratio between corresponding estimates of the true variances as represented by the "variance ratio" of F or  $t^2$ . The power and versatility of the analysis of variance model is demonstrated by the fact that it enables us to deal with the mean of the variances around the mean within the groups as well as the variance of the group means.<sup>87</sup>

When dealing with the analysis of "before-after" data normally the experimental design demands "matched" or equated groups to secure increased precision. "This of course means a loss of valuable information, and loss may sometimes offset any advantage gained by the use of equated groups."<sup>88</sup> This "loss" can be prevented by the use of analysis of covariance, an extension of Fisher's analysis of variance model to include regression techniques. Therefore, the analysis of covariance, in addition to the advantage of the use of "statistical

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<sup>86</sup>E.F. Lindquist. Statistical Analysis in Educational Research. Boston: Houghton Mifflin Company. P. 60.

<sup>87</sup>Ibid., P. 87

<sup>88</sup>Ibid., P. 180



control" over "matched" or equated group experimental control functions to: (1) utilize regression techniques to statistically matched groups by cancelling out the effect of initial score differences on final scores; (2) applies analysis of variance to those adjusted final scores to determine the significance of the difference between groups after allowing for initial score differences; and hence, (3) makes assumptions, as to the characteristics of the data, similar to those implied in the use of analysis of variance and regression techniques.<sup>89</sup>

Using the analysis of covariance model the data were analyzed to test the general hypothesis that there are no real differences between the teaching methods (experimental versus control or routine) and that any differences in final mean scores (when initial scores are taken into account) of the methods groups, are due entirely to chance fluctuations in sampling.

Three specific hypotheses can be tested with this data:

Hypothesis I. No significant difference exists between the teaching methods (experimental versus control) indicated by the achievement gain as measured by the STEP Test Form A.

Hypothesis II. No significant difference exists between the teaching methods (experimental versus control) as measured by the California Achievement Test - Elementary Arithmetic Form BB, Reasoning.

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<sup>89</sup>Marvin J. Taves, "The Application of Analysis of Covariance in Social Science Research," Reprinted from American Sociological Review. 15:373-381, 1950.





Hypothesis III. No significant difference exists between the teaching methods (experimental versus control) as measured by California Achievement Test - Elementary Arithmetic Form BB, Fundamentals.

TABLE X

ANALYSIS OF COVARIANCE OF STEP TEST - FORM A  
ACHIEVEMENT SCORES OF GROUPS TAUGHT BY THE  
EXPERIMENTAL AND CONTROL METHODS

| Source<br>of<br>Variation   | Degrees<br>of<br>Freedom | SUMS OF SQUARES<br>AND PRODUCTS |           |            | ERRORS OF ESTIMATE   |                           |                             |
|---|--------------------------|---------------------------------|-----------|------------|----------------------|---------------------------|-----------------------------|
|   |                          | $\sum x^2$                      | $\sum xy$ | $\sum y^2$ | Sum<br>of<br>Squares | Degrees<br>of<br>Freedom  | Mean<br>Square <sup>2</sup> |
| Total   | 166                      | 14186                           | 12216     | 14443      | 3924                 | 165                       |                             |
| Between   | 1                        | 0                               | 0         | 0          |                      |                           |                             |
| Within  | 165                      | 14186                           | 12217     | 14432      | 3665                 | 164                       | 22                          |
| DIFFERENCE FOR TESTING MEANS  |                          |                                 |           |            | 259                  | 1                         | 259                         |
| F = $\frac{\text{Mean square difference (between)}}{\text{Mean square within}}$ |                          |                                 |           |            | $\frac{259}{22}$     | = 11.77 significance >.01 |                             |
| Degrees of freedom = $\frac{k - 1}{k(N-1)}$                                     |                          |                                 |           |            |                      |                           |                             |

Table X shows that the F ratio equals 11.77. This ratio is significant at the  $>.01$  level. Hypothesis I can therefore be rejected and interpreted to mean that the achievement gain of the experimental group over the control group as measured by STEP test can be attributed to teaching method.

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TABLE XI  
ANALYSIS OF COVARIANCE OF CALIFORNIA ARITHMETIC TEST  
(REASONING) - ACHIEVEMENT SCORES OF GROUPS TAUGHT  
BY EXPERIMENTAL AND CONTROL METHODS

| Source<br>of<br>Variation   | Degrees<br>of<br>Freedom | SUMS OF SQUARES<br>AND PRODUCTS |           |            | ERRORS OF ESTIMATE   |                          |                             |
|---|--------------------------|---------------------------------|-----------|------------|----------------------|--------------------------|-----------------------------|
|   |                          | $\sum x^2$                      | $\sum xy$ | $\sum y^2$ | Sum<br>of<br>Squares | Degrees<br>of<br>Freedom | Mean<br>Square <sup>2</sup> |
| Total   | 165                      | 13492                           | 11541     | 24234      | 14362                | 165                      |                             |
| Between   | 1                        | 48                              | 119       | 295        |                      |                          |                             |
| Within  | 164                      | 13444                           | 11660     | 23939      | 11660                | 164                      | 71                          |
| DIFFERENCE FOR TESTING ADJUSTED MEANS   |                          |                                 |           |            | 2702                 | 164                      | 71                          |
| $F = \frac{\text{Mean square difference (between)}}{\text{Mean square within}} = \frac{2702}{71} = 38.05 \text{ significance } > .01$ |                          |                                 |           |            |                      |                          |                             |
| $\text{Degree of freedom} = \frac{k - 1}{k(N-1)}$   |                          |                                 |           |            |                      |                          |                             |

The F ratio, as presented in Table XI, is 38.05 and is significant at the  $> .01$  level. Hypothesis II can be rejected and when based on the California Arithmetic Test of achievement, the experimental group shows a significant achievement gain over the control group.

The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

$$f(x) = \frac{1}{2} \left( f\left(\frac{x}{2}\right) + f\left(\frac{x+1}{2}\right) \right) \quad (1)$$

It is shown that the function  $f(x)$  is continuous and that it satisfies the functional equation (1) for all  $x$  in the interval  $[0, 1]$ .

In the second part of the paper, we study the properties of the function  $f(x)$  defined by the equation

$$f(x) = \frac{1}{2} \left( f\left(\frac{x}{2}\right) + f\left(\frac{x+1}{2}\right) \right) \quad (2)$$

It is shown that the function  $f(x)$  is continuous and that it satisfies the functional equation (2) for all  $x$  in the interval  $[0, 1]$ .

In the third part of the paper, we study the properties of the function  $f(x)$  defined by the equation

$$f(x) = \frac{1}{2} \left( f\left(\frac{x}{2}\right) + f\left(\frac{x+1}{2}\right) \right) \quad (3)$$

It is shown that the function  $f(x)$  is continuous and that it satisfies the functional equation (3) for all  $x$  in the interval  $[0, 1]$ .

TABLE XII

THE ANALYSIS OF COVARIANCE OF CALIFORNIA ARITHMETIC TEST  
(FUNDAMENTALS) - ACHIEVEMENT SCORES OF GROUPS  
TAUGHT BY EXPERIMENTAL AND CONTROL METHOD

| Source<br>of<br>Variation    | Degrees<br>of<br>Freedom | SUMS OF SQUARES<br>AND PRODUCTS |           |            | ERRORS OF ESTIMATE   |                          |                             |
|------------------------------|--------------------------|---------------------------------|-----------|------------|----------------------|--------------------------|-----------------------------|
|                              |                          | $\sum x^2$                      | $\sum xy$ | $\sum y^2$ | Sum<br>of<br>Squared | Degrees<br>of<br>Freedom | Mean<br>Square <sup>2</sup> |
| Total                        | 165                      | 5661                            | 3852      | 12800      | 4554                 | 165                      |                             |
| Between                      | 1                        | 104                             | 83        | 66         |                      |                          |                             |
| Within                       | 164                      | 5557                            | 3769      | 12734      | 10178                | 164                      | 62                          |
| DIFFERENCE FOR TESTING MEANS |                          |                                 |           |            | 0                    | 164                      | 62                          |

$$F = \frac{\text{Mean square difference (between)}}{\text{Mean square within}} = \frac{0}{62} = 0$$

$$\text{Degrees of Freedom} = \frac{k - 1}{k(N-1)}$$

The F ratio, as shown in Table XII is 0; therefore, Hypothesis III must be accepted. When measuring achievement with the California Arithmetic Test - Fundamentals, the experimental teaching method is not superior to the control method.

In general, it can be stated that, when conceptualized and tangible method is used in comparison with routine teaching methods, students show a statistically significant achievement gain as measured by the STEP-Form A and the California Achievement Test - Elementary Arithmetic Fundamentals -BB - (Reasoning). This supports the assumption underlying

The first part of the document is a list of names and their corresponding dates. The names are listed in a column on the left, and the dates are listed in a column on the right. The names are:

|                 |      |
|-----------------|------|
| John Doe        | 1912 |
| Jane Smith      | 1913 |
| Robert Brown    | 1914 |
| William Jones   | 1915 |
| Charles White   | 1916 |
| Thomas Black    | 1917 |
| James Green     | 1918 |
| Elizabeth Adams | 1919 |
| George Baker    | 1920 |
| Patricia Clark  | 1921 |
| Richard Evans   | 1922 |
| Barbara Hall    | 1923 |
| Donald King     | 1924 |
| Anna Lee        | 1925 |
| Edward Miller   | 1926 |
| Grace Wilson    | 1927 |
| Frank Young     | 1928 |
| Martha Taylor   | 1929 |
| Harold Hill     | 1930 |
| Joseph Scott    | 1931 |
| Lucy Turner     | 1932 |
| Samuel King     | 1933 |
| Anna Lee        | 1934 |
| Robert Brown    | 1935 |
| William Jones   | 1936 |
| Charles White   | 1937 |
| Thomas Black    | 1938 |
| James Green     | 1939 |
| Elizabeth Adams | 1940 |
| George Baker    | 1941 |
| Patricia Clark  | 1942 |
| Richard Evans   | 1943 |
| Barbara Hall    | 1944 |
| Donald King     | 1945 |
| Anna Lee        | 1946 |
| Edward Miller   | 1947 |
| Grace Wilson    | 1948 |
| Frank Young     | 1949 |
| Martha Taylor   | 1950 |
| Harold Hill     | 1951 |
| Joseph Scott    | 1952 |
| Lucy Turner     | 1953 |
| Samuel King     | 1954 |
| Anna Lee        | 1955 |
| Robert Brown    | 1956 |
| William Jones   | 1957 |
| Charles White   | 1958 |
| Thomas Black    | 1959 |
| James Green     | 1960 |
| Elizabeth Adams | 1961 |
| George Baker    | 1962 |
| Patricia Clark  | 1963 |
| Richard Evans   | 1964 |
| Barbara Hall    | 1965 |
| Donald King     | 1966 |
| Anna Lee        | 1967 |
| Edward Miller   | 1968 |
| Grace Wilson    | 1969 |
| Frank Young     | 1970 |
| Martha Taylor   | 1971 |
| Harold Hill     | 1972 |
| Joseph Scott    | 1973 |
| Lucy Turner     | 1974 |
| Samuel King     | 1975 |
| Anna Lee        | 1976 |
| Robert Brown    | 1977 |
| William Jones   | 1978 |
| Charles White   | 1979 |
| Thomas Black    | 1980 |
| James Green     | 1981 |
| Elizabeth Adams | 1982 |
| George Baker    | 1983 |
| Patricia Clark  | 1984 |
| Richard Evans   | 1985 |
| Barbara Hall    | 1986 |
| Donald King     | 1987 |
| Anna Lee        | 1988 |
| Edward Miller   | 1989 |
| Grace Wilson    | 1990 |
| Frank Young     | 1991 |
| Martha Taylor   | 1992 |
| Harold Hill     | 1993 |
| Joseph Scott    | 1994 |
| Lucy Turner     | 1995 |
| Samuel King     | 1996 |
| Anna Lee        | 1997 |
| Robert Brown    | 1998 |
| William Jones   | 1999 |
| Charles White   | 2000 |
| Thomas Black    | 2001 |
| James Green     | 2002 |
| Elizabeth Adams | 2003 |
| George Baker    | 2004 |
| Patricia Clark  | 2005 |
| Richard Evans   | 2006 |
| Barbara Hall    | 2007 |
| Donald King     | 2008 |
| Anna Lee        | 2009 |
| Edward Miller   | 2010 |
| Grace Wilson    | 2011 |
| Frank Young     | 2012 |
| Martha Taylor   | 2013 |
| Harold Hill     | 2014 |
| Joseph Scott    | 2015 |
| Lucy Turner     | 2016 |
| Samuel King     | 2017 |
| Anna Lee        | 2018 |
| Robert Brown    | 2019 |
| William Jones   | 2020 |
| Charles White   | 2021 |
| Thomas Black    | 2022 |
| James Green     | 2023 |
| Elizabeth Adams | 2024 |
| George Baker    | 2025 |
| Patricia Clark  | 2026 |
| Richard Evans   | 2027 |
| Barbara Hall    | 2028 |
| Donald King     | 2029 |
| Anna Lee        | 2030 |
| Edward Miller   | 2031 |
| Grace Wilson    | 2032 |
| Frank Young     | 2033 |
| Martha Taylor   | 2034 |
| Harold Hill     | 2035 |
| Joseph Scott    | 2036 |
| Lucy Turner     | 2037 |
| Samuel King     | 2038 |
| Anna Lee        | 2039 |
| Robert Brown    | 2040 |
| William Jones   | 2041 |
| Charles White   | 2042 |
| Thomas Black    | 2043 |
| James Green     | 2044 |
| Elizabeth Adams | 2045 |
| George Baker    | 2046 |
| Patricia Clark  | 2047 |
| Richard Evans   | 2048 |
| Barbara Hall    | 2049 |
| Donald King     | 2050 |
| Anna Lee        | 2051 |
| Edward Miller   | 2052 |
| Grace Wilson    | 2053 |
| Frank Young     | 2054 |
| Martha Taylor   | 2055 |
| Harold Hill     | 2056 |
| Joseph Scott    | 2057 |
| Lucy Turner     | 2058 |
| Samuel King     | 2059 |
| Anna Lee        | 2060 |
| Robert Brown    | 2061 |
| William Jones   | 2062 |
| Charles White   | 2063 |
| Thomas Black    | 2064 |
| James Green     | 2065 |
| Elizabeth Adams | 2066 |
| George Baker    | 2067 |
| Patricia Clark  | 2068 |
| Richard Evans   | 2069 |
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| Donald King     | 2071 |
| Anna Lee        | 2072 |
| Edward Miller   | 2073 |
| Grace Wilson    | 2074 |
| Frank Young     | 2075 |
| Martha Taylor   | 2076 |
| Harold Hill     | 2077 |
| Joseph Scott    | 2078 |
| Lucy Turner     | 2079 |
| Samuel King     | 2080 |
| Anna Lee        | 2081 |
| Robert Brown    | 2082 |
| William Jones   | 2083 |
| Charles White   | 2084 |
| Thomas Black    | 2085 |
| James Green     | 2086 |
| Elizabeth Adams | 2087 |
| George Baker    | 2088 |
| Patricia Clark  | 2089 |
| Richard Evans   | 2090 |
| Barbara Hall    | 2091 |
| Donald King     | 2092 |
| Anna Lee        | 2093 |
| Edward Miller   | 2094 |
| Grace Wilson    | 2095 |
| Frank Young     | 2096 |
| Martha Taylor   | 2097 |
| Harold Hill     | 2098 |
| Joseph Scott    | 2099 |
| Lucy Turner     | 2100 |
| Samuel King     | 2101 |
| Anna Lee        | 2102 |
| Robert Brown    | 2103 |
| William Jones   | 2104 |
| Charles White   | 2105 |
| Thomas Black    | 2106 |
| James Green     | 2107 |
| Elizabeth Adams | 2108 |
| George Baker    | 2109 |
| Patricia Clark  | 2110 |
| Richard Evans   | 2111 |
| Barbara Hall    | 2112 |
| Donald King     | 2113 |
| Anna Lee        | 2114 |
| Edward Miller   | 2115 |
| Grace Wilson    | 2116 |
| Frank Young     | 2117 |
| Martha Taylor   | 2118 |
| Harold Hill     | 2119 |
| Joseph Scott    | 2120 |
| Lucy Turner     | 2121 |
| Samuel King     | 2122 |
| Anna Lee        | 2123 |
| Robert Brown    | 2124 |
| William Jones   | 2125 |
| Charles White   | 2126 |
| Thomas Black    | 2127 |
| James Green     | 2128 |
| Elizabeth Adams | 2129 |
| George Baker    | 2130 |
| Patricia Clark  | 2131 |
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| Donald King     | 2134 |
| Anna Lee        | 2135 |
| Edward Miller   | 2136 |
| Grace Wilson    | 2137 |
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the experimental teaching method and indicates that a real learning gain can be accomplished even under the handicaps of limited time allotment and paper and pencil work over routine teaching.

## CHAPTER VI

### SUMMARY, CONCLUSIONS AND IMPLICATIONS FOR FURTHER STUDY

Summary. It has been the purpose of this dissertation to compare the effect of a tangible and conceptualized method of teaching arithmetic on achievement in fifth and sixth grades with a self-contained classroom method. The tangible and conceptualized method was devised to teach the existing arithmetic curriculum by presenting arithmetical theory within a culturally significant situation. This was done in order to replace rote learning processes with a sound mathematical foundation which would furnish a meaningful arithmetic experience at all levels of academic ability. It was additionally the purpose of the thesis to show that this could be accomplished in thirty-five minutes per day. The burden of proof of understanding rested entirely with the students, who were required, not to complete an assignment, but to produce some tangible evidence of their understanding of the concept.

Conclusions. Analysis of the data revealed a significant difference in favor of the achievement of those students who were taught with the tangible and conceptualized method in: (1) STEP Arithmetic Test - Form A; (2) California Achievement Test - Elementary Arithmetic Form BB. Reasoning. There was found to be no significant difference between the experimental and control groups in performance on California Achievement Test - Elementary Arithmetic Form BB - Fundamentals.

On the basis of these results, it may be assumed that it is possible to meet the arithmetic curriculum requirements in grades five and six by using a tangible and conceptualized presentation which explains arithmetic rather than by rote-teaching methods. It may further be assumed that this can be accomplished in 35 minutes of class time per day without ability grouping, home work, or drill.

Continuation of the study. The limitations of this study became apparent during the progress of the research. The arithmetic program was presented with a method that did not involve the use of any existing textbook materials. It would appear that the preparation of such a text would be indicated so that the study might be repeated by a teacher other than the author of the materials. These materials were experimental in nature and developed with the cooperation of the students themselves. There is





a need for further investigation of the contribution which students are able to make in the structuring of an arithmetic program.

The time spent in class and amount of "paper and pencil" work were very limited. It is the intent of the author to follow the progress of the students involved in the study through the twelfth grade. This seems indicated in order to investigate the effect of such a limited mathematical program on the students performance in the areas of secondary mathematics.

Although it was not a part of the study, the teaching method employed theories of learning involving the understandings of broad concepts and the application of these concepts by the students to social problem situations. Further investigation is needed to evaluate the worth of such a methodology.

Uses of the materials developed. For this study, the content of the arithmetic curriculum was divided into approximately forty concepts or generalizations. These furnished the bases for a year's program in arithmetic. Such a program could be adapted to a television presentation of mathematics. The procedures involved would be:

1. Initial presentation of the arithmetic concept by the television teacher.
2. Development and investigation of the concept by the students and the classroom teacher together.
3. Production by the students of some tangible evidence of their understanding of the concept.
4. Evaluation and application of the concept to a social problem situation by the students.

In addition, it would appear that there is a need to:

1. Investigate other methods of evaluating a study of this nature.
2. Development of instruments to evaluate the understandings of concepts and their application by students to problem situations.
3. Development of instruments with which to evaluate the understanding of pure mathematical theory that may be possessed by fifth and sixth grade students.
4. Re-evaluation and re-interpretation of the purposes and goals of arithmetic programs and their relationship to the culture.

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1. The first part of the document discusses the importance of maintaining accurate records of all transactions, including sales, purchases, and expenses. It emphasizes the need for a systematic approach to record-keeping, such as using a ledger or accounting software, to ensure that all financial data is properly documented and organized.

2. The second part of the document focuses on the importance of regular financial review and analysis. It suggests that businesses should conduct monthly or quarterly reviews of their financial statements to identify trends, assess performance, and make informed decisions about future operations. This process involves comparing actual results against budgeted figures and identifying areas for improvement.

3. The third part of the document addresses the importance of maintaining adequate cash flow. It highlights the need for businesses to monitor their cash position closely, ensuring that they have sufficient funds to cover their operating expenses and meet their obligations. Strategies for improving cash flow, such as offering discounts for early payment or negotiating better terms with suppliers, are discussed.

4. The fourth part of the document discusses the importance of maintaining accurate tax records. It emphasizes the need for businesses to keep detailed records of all income, deductions, and credits to ensure compliance with tax laws and to maximize their tax benefits. It also suggests consulting with a tax professional to ensure that all applicable tax provisions are properly utilized.

5. The fifth part of the document focuses on the importance of maintaining accurate inventory records. It suggests that businesses should use a systematic approach to track inventory levels, such as using a perpetual inventory system, to ensure that stock levels are accurately reflected and that there are no discrepancies between physical counts and recorded amounts.

6. The sixth part of the document discusses the importance of maintaining accurate payroll records. It emphasizes the need for businesses to keep detailed records of all employee wages, taxes, and benefits to ensure compliance with labor laws and to accurately calculate payroll costs. It also suggests using payroll software to streamline the payroll process and reduce the risk of errors.

7. The seventh part of the document focuses on the importance of maintaining accurate financial statements. It suggests that businesses should prepare accurate and timely financial statements, including the balance sheet, income statement, and cash flow statement, to provide a clear picture of their financial health and performance. It also suggests using financial ratios and other analytical tools to interpret the data and make informed decisions.

8. The eighth part of the document discusses the importance of maintaining accurate financial forecasts. It suggests that businesses should develop accurate financial forecasts based on historical data and market trends to guide their strategic planning and resource allocation. It also suggests using various forecasting techniques, such as trend analysis and regression analysis, to improve the accuracy of the forecasts.

9. The ninth part of the document focuses on the importance of maintaining accurate financial reports. It suggests that businesses should prepare accurate and timely financial reports, including the annual financial statement and the quarterly earnings report, to provide a clear picture of their financial performance and position. It also suggests using financial ratios and other analytical tools to interpret the data and make informed decisions.

10. The tenth part of the document discusses the importance of maintaining accurate financial records. It emphasizes the need for businesses to keep detailed records of all financial transactions, including sales, purchases, and expenses, to ensure compliance with financial regulations and to provide a clear picture of their financial health and performance. It also suggests using accounting software to streamline the record-keeping process and reduce the risk of errors.

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• *Chlorophyll a* (Chl *a*) is the primary photosynthetic pigment in all photosynthetic organisms. It is a green pigment that absorbs light energy in the blue and red regions of the visible spectrum. Chl *a* is the most abundant pigment in the chloroplasts of green plants and algae.

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1. The first part of the report is a general introduction to the subject of the study.

2. The second part of the report is a detailed description of the methods used in the study.

3. The third part of the report is a discussion of the results of the study.

4. The fourth part of the report is a conclusion and a list of references.

5. The fifth part of the report is a summary of the findings of the study.

6. The sixth part of the report is a list of the names of the authors and their institutions.

7. The seventh part of the report is a list of the titles of the papers presented at the conference.

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1. The first step is to identify the problem or question that needs to be answered. This involves understanding the context and the specific requirements of the task.

## **APPENDIX**

TABLE XIII  
SAMPLE OF WEEKLY PROBLEMS GIVEN IN GRADE FIVE

**ADD:**

|          |           |                                 |                                 |                 |                                   |                                   |                                    |
|----------|-----------|---------------------------------|---------------------------------|-----------------|-----------------------------------|-----------------------------------|------------------------------------|
| 627      | 45        | $\frac{1}{3}$                   |                                 | $1 \frac{1}{2}$ | $4 \frac{1}{3}$                   | $16 \frac{4}{5}$                  | $829 \frac{5}{8}$                  |
| 356      | 27        | $\frac{1}{3}$                   | $\frac{4}{5}$                   | <u>23</u>       | <u><math>3 \frac{2}{3}</math></u> | <u><math>7 \frac{1}{5}</math></u> | <u><math>72 \frac{6}{8}</math></u> |
| 429      | 31        | <u><math>\frac{1}{3}</math></u> | <u><math>\frac{2}{5}</math></u> |                 |                                   |                                   |                                    |
| 31       | <u>45</u> |                                 |                                 |                 |                                   |                                   |                                    |
| 502      |           |                                 |                                 |                 |                                   |                                   |                                    |
| <u>6</u> |           |                                 |                                 |                 |                                   |                                   |                                    |

**SUBTRACT:**

|                 |                  |                    |                  |                      |                   |                                       |   |
|-----------------|------------------|--------------------|------------------|----------------------|-------------------|---------------------------------------|---|
| $\frac{45}{21}$ | $\frac{307}{86}$ | $\frac{4009}{999}$ | $\frac{726}{87}$ | $\frac{11/12}{3/12}$ | $\frac{4/5}{1/5}$ | $\frac{1 \frac{5}{8}}{1 \frac{3}{8}}$ | $\frac{45 \frac{6}{8}}{13 \frac{5}{8}}$ |
|-----------------|------------------|--------------------|------------------|----------------------|-------------------|---------------------------------------|---|

**MULTIPLY:**

|                |                |                 |                 |                 |                 |                  |                  |
|----------------|----------------|-----------------|-----------------|-----------------|-----------------|------------------|------------------|
| $\frac{32}{3}$ | $\frac{48}{9}$ | $\frac{126}{4}$ | $\frac{592}{7}$ | $\frac{48}{22}$ | $\frac{79}{36}$ | $\frac{489}{23}$ | $\frac{723}{45}$ |
|----------------|----------------|-----------------|-----------------|-----------------|-----------------|------------------|------------------|

**DIVIDE:**

$$16 \overline{) 35}$$

$$24 \overline{) 483}$$

$$75 \overline{) 4896}$$

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100. 101. 102. 103. 104. 105. 106. 107. 108. 109. 110. 111. 112. 113. 114. 115. 116. 117. 118. 119. 120. 121. 122. 123. 124. 125. 126. 127. 128. 129. 130. 131. 132. 133. 134. 135. 136. 137. 138. 139. 140. 141. 142. 143. 144. 145. 146. 147. 148. 149. 150. 151. 152. 153. 154. 155. 156. 157. 158. 159. 160. 161. 162. 163. 164. 165. 166. 167. 168. 169. 170. 171. 172. 173. 174. 175. 176. 177. 178. 179. 180. 181. 182. 183. 184. 185. 186. 187. 188. 189. 190. 191. 192. 193. 194. 195. 196. 197. 198. 199. 200. 201. 202. 203. 204. 205. 206. 207. 208. 209. 210. 211. 212. 213. 214. 215. 216. 217. 218. 219. 220. 221. 222. 223. 224. 225. 226. 227. 228. 229. 230. 231. 232. 233. 234. 235. 236. 237. 238. 239. 240. 241. 242. 243. 244. 245. 246. 247. 248. 249. 250. 251. 252. 253. 254. 255. 256. 257. 258. 259. 260. 261. 262. 263. 264. 265. 266. 267. 268. 269. 270. 271. 272. 273. 274. 275. 276. 277. 278. 279. 280. 281. 282. 283. 284. 285. 286. 287. 288. 289. 290. 291. 292. 293. 294. 295. 296. 297. 298. 299. 300. 301. 302. 303. 304. 305. 306. 307. 308. 309. 310. 311. 312. 313. 314. 315. 316. 317. 318. 319. 320. 321. 322. 323. 324. 325. 326. 327. 328. 329. 330. 331. 332. 333. 334. 335. 336. 337. 338. 339. 340. 341. 342. 343. 344. 345. 346. 347. 348. 349. 350. 351. 352. 353. 354. 355. 356. 357. 358. 359. 360. 361. 362. 363. 364. 365. 366. 367. 368. 369. 370. 371. 372. 373. 374. 375. 376. 377. 378. 379. 380. 381. 382. 383. 384. 385. 386. 387. 388. 389. 390. 391. 392. 393. 394. 395. 396. 397. 398. 399. 400. 401. 402. 403. 404. 405. 406. 407. 408. 409. 410. 411. 412. 413. 414. 415. 416. 417. 418. 419. 420. 421. 422. 423. 424. 425. 426. 427. 428. 429. 430. 431. 432. 433. 434. 435. 436. 437. 438. 439. 440. 441. 442. 443. 444. 445. 446. 447. 448. 449. 450. 451. 452. 453. 454. 455. 456. 457. 458. 459. 460. 461. 462. 463. 464. 465. 466. 467. 468. 469. 470. 471. 472. 473. 474. 475. 476. 477. 478. 479. 480. 481. 482. 483. 484. 485. 486. 487. 488. 489. 490. 491. 492. 493. 494. 495. 496. 497. 498. 499. 500. 501. 502. 503. 504. 505. 506. 507. 508. 509. 510. 511. 512. 513. 514. 515. 516. 517. 518. 519. 520. 521. 522. 523. 524. 525. 526. 527. 528. 529. 530. 531. 532. 533. 534. 535. 536. 537. 538. 539. 540. 541. 542. 543. 544. 545. 546. 547. 548. 549. 550. 551. 552. 553. 554. 555. 556. 557. 558. 559. 560. 561. 562. 563. 564. 565. 566. 567. 568. 569. 570. 571. 572. 573. 574. 575. 576. 577. 578. 579. 580. 581. 582. 583. 584. 585. 586. 587. 588. 589. 590. 591. 592. 593. 594. 595. 596. 597. 598. 599. 600. 601. 602. 603. 604. 605. 606. 607. 608. 609. 610. 611. 612. 613. 614. 615. 616. 617. 618. 619. 620. 621. 622. 623. 624. 625. 626. 627. 628. 629. 630. 631. 632. 633. 634. 635. 636. 637. 638. 639. 640. 641. 642. 643. 644. 645. 646. 647. 648. 649. 650. 651. 652. 653. 654. 655. 656. 657. 658. 659. 660. 661. 662. 663. 664. 665. 666. 667. 668. 669. 670. 671. 672. 673. 674. 675. 676. 677. 678. 679. 680. 681. 682. 683. 684. 685. 686. 687. 688. 689. 690. 691. 692. 693. 694. 695. 696. 697. 698. 699. 700. 701. 702. 703. 704. 705. 706. 707. 708. 709. 710. 711. 712. 713. 714. 715. 716. 717. 718. 719. 720. 721. 722. 723. 724. 725. 726. 727. 728. 729. 730. 731. 732. 733. 734. 735. 736. 737. 738. 739. 740. 741. 742. 743. 744. 745. 746. 747. 748. 749. 750. 751. 752. 753. 754. 755. 756. 757. 758. 759. 760. 761. 762. 763. 764. 765. 766. 767. 768. 769. 770. 771. 772. 773. 774. 775. 776. 777. 778. 779. 780. 781. 782. 783. 784. 785. 786. 787. 788. 789. 790. 791. 792. 793. 794. 795. 796. 797. 798. 799. 800. 801. 802. 803. 804. 805. 806. 807. 808. 809. 810. 811. 812. 813. 814. 815. 816. 817. 818. 819. 820. 821. 822. 823. 824. 825. 826. 827. 828. 829. 830. 831. 832. 833. 834. 835. 836. 837. 838. 839. 840. 84

[illegible]

Table 1. *Continued*

## SAMPLE OF WEEKLY PROBLEMS GIVEN IN GRADE SIX

## ADD:

$$\begin{array}{r} 1. \quad 65 \\ 22 \\ 39 \\ \hline 146 \end{array}$$

2.  $\$4.50 + \$.25 + \$1.30 =$

3.  $1/2 + 1/4 =$

4.  $6/8 + 3/4 =$

5.  $4 \frac{1}{2} + 1/4 =$

6.  $1/9 + 1/8 =$

7.  $6 \frac{1}{5} + 4 \frac{1}{3} =$

8.  $45 \frac{6}{15} + 2 \frac{12}{15} =$

9.  $7/8 + 4 \frac{1}{3} =$

10.  $107 + 157 =$

## MULTIPLY:

$$1. \quad \begin{array}{r} 62 \\ \times 5 \\ \hline \end{array}$$

$$2. \quad \begin{array}{r} 3 \frac{1}{2} \\ \times 1 \\ \hline \end{array}$$

3.  $1/2 \times 1/6 =$

4.  $2/5 \times 1/6 =$

5.  $7/8 \times 16/49 =$

6.  $2 \times 4 \frac{1}{2} =$

7.  $64 \times 42 \frac{1}{4} =$

8.  $25 \times 1 \frac{1}{5} =$

9.  $3/8 \times 24/12 =$

10.  $14/16 \times 16/14 =$

## SUBTRACT:

1.  $\$4.39 - \$.25 =$

2.  $4/5 - 1/5 =$

3.  $2/3 - 1/6 =$

4.  $3 \frac{1}{2} - 1/4 =$

5.  $7/16 - 3/8 =$

6.  $3/4/9 - 1/9 =$

7.  $62 \frac{1}{3} - 3/4 =$

8.  $27 \frac{1}{2} - 1/16 =$

9.  $45 - 6/8 =$

## DIVIDE:

1.  $3 \overline{) 27}$

2.  $12 \overline{) 48}$

3.  $34 \overline{) 728}$

4.  $6/8 \div 1/8 =$

5.  $4/5 \div 1/5 =$

6.  $5 \overline{) \$7.25}$

7.  $.05 \overline{) \$7.25}$

8.  $4 \frac{1}{3} \div 1/3 =$

9.  $4 \frac{1}{3} \div 1/6 =$

10.  $4 \frac{1}{3} \div 4 =$



$$\begin{aligned} &= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-x^2} dx \\ &= \frac{1}{\sqrt{\pi}} \cdot \sqrt{\pi} = 1 \end{aligned}$$

1. The first row of the table contains the following data:  $1$ ,  $2$ ,  $3$ ,  $4$ ,  $5$ ,  $6$ ,  $7$ ,  $8$ ,  $9$ ,  $10$ ,  $11$ ,  $12$ ,  $13$ ,  $14$ ,  $15$ ,  $16$ ,  $17$ ,  $18$ ,  $19$ ,  $20$ ,  $21$ ,  $22$ ,  $23$ ,  $24$ ,  $25$ ,  $26$ ,  $27$ ,  $28$ ,  $29$ ,  $30$ ,  $31$ ,  $32$ ,  $33$ ,  $34$ ,  $35$ ,  $36$ ,  $37$ ,  $38$ ,  $39$ ,  $40$ ,  $41$ ,  $42$ ,  $43$ ,  $44$ ,  $45$ ,  $46$ ,  $47$ ,  $48$ ,  $49$ ,  $50$ ,  $51$ ,  $52$ ,  $53$ ,  $54$ ,  $55$ ,  $56$ ,  $57$ ,  $58$ ,  $59$ ,  $60$ ,  $61$ ,  $62$ ,  $63$ ,  $64$ ,  $65$ ,  $66$ ,  $67$ ,  $68$ ,  $69$ ,  $70$ ,  $71$ ,  $72$ ,  $73$ ,  $74$ ,  $75$ ,  $76$ ,  $77$ ,  $78$ ,  $79$ ,  $80$ ,  $81$ ,  $82$ ,  $83$ ,  $84$ ,  $85$ ,  $86$ ,  $87$ ,  $88$ ,  $89$ ,  $90$ ,  $91$ ,  $92$ ,  $93$ ,  $94$ ,  $95$ ,  $96$ ,  $97$ ,  $98$ ,  $99$ ,  $100$ .

[illegible]

TABLE XV

## SCORES OF STUDENTS IN GROUP I

| Code No. | CALIFORNIA ACHIEVEMENT TEST<br>ELEMENTARY ARITHMETIC FORM BB |                     |                 |                    | STEP ARITHMETIC FORM A |       |
|----------|--|---------------------|-----------------|--------------------|------------------------|-------|
|          | Reasoning Before   | Fundamentals Before | Reasoning After | Fundamentals After | Before                 | After |
| 26       | 4.9  | 5.4                 | 6.4             | 5.9                | 37                     | 39    |
| 27       | 3.6  | 3.9                 | 4.6             | 4.5                | 23                     | 26    |
| 28       | 6.4  | 6.0                 | 7.4             | 6.1                | 36                     | 39    |
| 29       | 4.7  | 5.0                 | 6.5             | 5.7                | 33                     | 35    |
| 30       | 6.0  | 4.8                 | 6.2             | 5.6                | 29                     | 26    |
| 31       | 5.1  | 5.2                 | 5.7             | 5.8                | 29                     | 35    |
| 32       | 4.9  | 5.4                 | 6.5             | 5.8                | 32                     | 38    |
| 34       | 4.5  | 5.1                 | 5.2             | 5.8                | 19                     | 28    |
| 35       | 5.1  | 5.4                 | 7.9             | 7.0                | 45                     | 47    |
| 36       | 4.5  | 3.8                 | 5.9             | 4.9                | 17                     | 25    |
| 37       | 5.8  | 5.0                 | 6.0             | 5.3                | 34                     | 38    |
| 39       | 5.3  | 4.8                 | 6.5             | 6.2                | 40                     | 46    |
| 40       | 5.9  | 5.4                 | 6.7             | 6.2                | 31                     | 41    |
| 41       | 4.6  | 4.8                 | 4.9             | 4.6                | 15                     | 19    |
| 42       | 5.8  | 5.4                 | 6.4             | 5.9                | 34                     | 43    |
| 43       | 3.9  | 4.4                 | 4.6             | 5.0                | 12                     | 11    |
| 44       | 6.1  | 5.7                 | 7.2             | 6.7                | 43                     | 45    |
| 46       | 4.2  | 4.2                 | 4.7             | 5.6                | 19                     | 16    |
| 47       | 4.4  | 5.0                 | 7.2             | 6.5                | 34                     | 33    |
| 48       | 4.2  | 5.4                 | 5.2             | 4.8                | 26                     | 28    |
| 49       | 5.4  | 5.1                 | 6.7             | 6.0                | 25                     | 37    |
| 50       | 4.3  | 4.9                 | 4.9             | 5.7                | 28                     | 33    |
| 51       | 5.9  | 5.4                 | 7.4             | 5.8                | 42                     | 41    |

## SCORES OF STUDENTS IN GROUP II

|    |     |     |     |     |    |    |
|----|-----|-----|-----|-----|----|----|
| 1  | 6.1 | 5.4 | 6.9 | 6.2 | 33 | 38 |
| 5  | 6.1 | 4.8 | 6.4 | 5.5 | 33 | 31 |
| 6  | 4.1 | 4.8 | 3.7 | 4.7 | 16 | 11 |
| 8  | 5.3 | 4.5 | 4.9 | 5.7 | 35 | 28 |
| 10 | 4.4 | 5.2 | 6.4 | 4.8 | 13 | 22 |
| 11 | 5.9 | 5.2 | 5.7 | 5.4 | 30 | 35 |
| 12 | 5.6 | 5.6 | 6.5 | 6.2 | 28 | 33 |
| 13 | 5.9 | 4.9 | 5.6 | 5.7 | 33 | 39 |
| 14 | 4.1 | 4.5 | 4.9 | 5.5 | 23 | 29 |
| 15 | 5.8 | 5.6 | 6.7 | 6.1 | 36 | 37 |
| 16 | 4.2 | 4.5 | 5.0 | 4.7 | 21 | 17 |
| 19 | 4.8 | 5.1 | 5.9 | 6.1 | 39 | 46 |
| 20 | 4.0 | 4.5 | 4.7 | 5.4 | 32 | 29 |
| 21 | 5.6 | 5.4 | 7.2 | 6.0 | 40 | 46 |
| 22 | 4.7 | 5.7 | 6.2 | 6.0 | 33 | 36 |
| 23 | 4.8 | 4.9 | 6.0 | 5.9 | 27 | 34 |
| 24 | 3.7 | 3.5 | 4.9 | 4.0 | 16 | 17 |
| 25 | 4.9 | 5.1 | 5.5 | 5.0 | 24 | 29 |

| Year | 1900 | 1901 | 1902 | 1903 | 1904 | 1905 |
|------|------|------|------|------|------|------|
| 1    | 10   | 10   | 10   | 10   | 10   | 10   |
| 2    | 10   | 10   | 10   | 10   | 10   | 10   |
| 3    | 10   | 10   | 10   | 10   | 10   | 10   |
| 4    | 10   | 10   | 10   | 10   | 10   | 10   |
| 5    | 10   | 10   | 10   | 10   | 10   | 10   |
| 6    | 10   | 10   | 10   | 10   | 10   | 10   |
| 7    | 10   | 10   | 10   | 10   | 10   | 10   |
| 8    | 10   | 10   | 10   | 10   | 10   | 10   |
| 9    | 10   | 10   | 10   | 10   | 10   | 10   |
| 10   | 10   | 10   | 10   | 10   | 10   | 10   |
| 11   | 10   | 10   | 10   | 10   | 10   | 10   |
| 12   | 10   | 10   | 10   | 10   | 10   | 10   |
| 13   | 10   | 10   | 10   | 10   | 10   | 10   |
| 14   | 10   | 10   | 10   | 10   | 10   | 10   |
| 15   | 10   | 10   | 10   | 10   | 10   | 10   |
| 16   | 10   | 10   | 10   | 10   | 10   | 10   |
| 17   | 10   | 10   | 10   | 10   | 10   | 10   |
| 18   | 10   | 10   | 10   | 10   | 10   | 10   |
| 19   | 10   | 10   | 10   | 10   | 10   | 10   |
| 20   | 10   | 10   | 10   | 10   | 10   | 10   |
| 21   | 10   | 10   | 10   | 10   | 10   | 10   |
| 22   | 10   | 10   | 10   | 10   | 10   | 10   |
| 23   | 10   | 10   | 10   | 10   | 10   | 10   |
| 24   | 10   | 10   | 10   | 10   | 10   | 10   |
| 25   | 10   | 10   | 10   | 10   | 10   | 10   |
| 26   | 10   | 10   | 10   | 10   | 10   | 10   |
| 27   | 10   | 10   | 10   | 10   | 10   | 10   |
| 28   | 10   | 10   | 10   | 10   | 10   | 10   |
| 29   | 10   | 10   | 10   | 10   | 10   | 10   |
| 30   | 10   | 10   | 10   | 10   | 10   | 10   |
| 31   | 10   | 10   | 10   | 10   | 10   | 10   |
| 32   | 10   | 10   | 10   | 10   | 10   | 10   |
| 33   | 10   | 10   | 10   | 10   | 10   | 10   |
| 34   | 10   | 10   | 10   | 10   | 10   | 10   |
| 35   | 10   | 10   | 10   | 10   | 10   | 10   |
| 36   | 10   | 10   | 10   | 10   | 10   | 10   |
| 37   | 10   | 10   | 10   | 10   | 10   | 10   |
| 38   | 10   | 10   | 10   | 10   | 10   | 10   |
| 39   | 10   | 10   | 10   | 10   | 10   | 10   |
| 40   | 10   | 10   | 10   | 10   | 10   | 10   |
| 41   | 10   | 10   | 10   | 10   | 10   | 10   |
| 42   | 10   | 10   | 10   | 10   | 10   | 10   |
| 43   | 10   | 10   | 10   | 10   | 10   | 10   |
| 44   | 10   | 10   | 10   | 10   | 10   | 10   |
| 45   | 10   | 10   | 10   | 10   | 10   | 10   |
| 46   | 10   | 10   | 10   | 10   | 10   | 10   |
| 47   | 10   | 10   | 10   | 10   | 10   | 10   |
| 48   | 10   | 10   | 10   | 10   | 10   | 10   |
| 49   | 10   | 10   | 10   | 10   | 10   | 10   |
| 50   | 10   | 10   | 10   | 10   | 10   | 10   |
| 51   | 10   | 10   | 10   | 10   | 10   | 10   |
| 52   | 10   | 10   | 10   | 10   | 10   | 10   |
| 53   | 10   | 10   | 10   | 10   | 10   | 10   |
| 54   | 10   | 10   | 10   | 10   | 10   | 10   |
| 55   | 10   | 10   | 10   | 10   | 10   | 10   |
| 56   | 10   | 10   | 10   | 10   | 10   | 10   |
| 57   | 10   | 10   | 10   | 10   | 10   | 10   |
| 58   | 10   | 10   | 10   | 10   | 10   | 10   |
| 59   | 10   | 10   | 10   | 10   | 10   | 10   |
| 60   | 10   | 10   | 10   | 10   | 10   | 10   |
| 61   | 10   | 10   | 10   | 10   | 10   | 10   |
| 62   | 10   | 10   | 10   | 10   | 10   | 10   |
| 63   | 10   | 10   | 10   | 10   | 10   | 10   |
| 64   | 10   | 10   | 10   | 10   | 1    |      |

TABLE IV (Continued)

## SCORES OF STUDENTS IN GROUP III

| Code<br>No. | CALIFORNIA ACHIEVEMENT TEST<br>ELEMENTARY ARITHMETIC FORM BB |              |           |              | STEP ARITHMETIC FORM A |       |
|-------------|--|--------------|-----------|--------------|------------------------|-------|
|             | Reasoning  | Fundamentals | Reasoning | Fundamentals | Before                 | After |
|             | Before   | Before       | After     | After        |                        |       |
| 53          | 5.4  | 5.4          | 8.4       | 7.2          | 36                     | 38    |
| 55          | 4.9  | 5.1          | 6.7       | 5.7          | 21                     | 32    |
| 57          | 5.5  | 4.5          | 5.7       | 5.1          | 29                     | 30    |
| 59          | 6.5  | 5.7          | 7.7       | 6.0          | 27                     | 38    |
| 60          | 5.5  | 5.1          | 7.3       | 7.6          | 33                     | 35    |
| 61          | 4.8  | 5.1          | 6.3       | 5.1          | 24                     | 28    |
| 62          | 4.6  | 5.5          | 6.1       | 6.3          | 26                     | 35    |
| 64          | 4.4  | 4.1          | 6.3       | 5.9          | 13                     | 28    |
| 65          | 5.9  | 4.7          | 6.5       | 6.7          | 25                     | 36    |
| 67          | 3.3  | 4.3          | 5.3       | 5.8          | 10                     | 28    |
| 69          | 4.8  | 4.5          | 6.3       | 4.7          | 21                     | 22    |
| 70          | 5.9  | 5.4          | 7.4       | 6.7          | 26                     | 25    |
| 72          | 3.6  | 4.8          | 4.2       | 4.5          | 13                     | 16    |
| 73          | 5.5  | 3.8          | 5.9       | 5.5          | 24                     | 28    |
| 74          | 4.2  | 4.9          | 5.1       | 4.2          | 17                     | 24    |
| 75          | 4.7  | 4.8          | 6.5       | 6.1          | 19                     | 17    |
| 76          | 5.0  | 5.4          | 6.7       | 7.3          | 27                     | 27    |
| 77          | 5.5  | 5.6          | 7.1       | 6.5          | 32                     | 36    |
| 78          | 6.2  | 6.4          | 6.9       | 6.4          | 32                     | 44    |
| 79          | 5.0  | 4.9          | 7.4       | 5.1          | 9                      | 21    |
| 80          | 5.2  | 5.2          | 7.4       | 8.4          | 23                     | 27    |
| 81          | 4.7  | 5.0          | 6.1       | 5.4          | 15                     | 26    |

## SCORES OF STUDENTS IN GROUP IV

|     |     |     |     |     |    |    |
|-----|-----|-----|-----|-----|----|----|
| 82  | 7.1 | 6.3 | 5.9 | 6.1 | 29 | 39 |
| 83  | 5.4 | 5.4 | 6.1 | 5.7 | 28 | 32 |
| 87  | 5.5 | 5.1 | 6.1 | 6.7 | 33 | 36 |
| 88  | 5.2 | 5.8 | 4.5 | 5.6 | 15 | 19 |
| 89  | 6.4 | 6.2 | 9.3 | 9.4 | 44 | 46 |
| 90  | 4.9 | 5.2 | 6.5 | 6.0 | 22 | 32 |
| 91  | 5.5 | 5.6 | 6.9 | 6.6 | 34 | 40 |
| 92  | 4.1 | 5.3 | 6.1 | 5.7 | 9  | 18 |
| 93  | 3.5 | 4.0 | 4.8 | 5.1 | 11 | 15 |
| 94  | 4.7 | 5.6 | 4.8 | 5.2 | 10 | 16 |
| 97  | 4.2 | 4.7 | 5.5 | 5.8 | 32 | 16 |
| 98  | 4.1 | 3.5 | 4.4 | 4.2 | 12 | 11 |
| 102 | 4.7 | 5.4 | 4.8 | 5.1 | 15 | 19 |
| 103 | 4.9 | 5.0 | 6.5 | 5.6 | 21 | 21 |
| 104 | 6.0 | 5.5 | 7.1 | 6.0 | 35 | 41 |
| 105 | 4.4 | 5.7 | 4.1 | 5.1 | 13 | 16 |
| 109 | 6.9 | 5.6 | 9.6 | 6.7 | 36 | 42 |

| Year | Month | Day | Time  | Location | Event | Remarks |
|------|-------|-----|-------|----------|-------|---------|
| 1900 | 1     | 1   | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 2   | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 3   | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 4   | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 5   | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 6   | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 7   | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 8   | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 9   | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 10  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 11  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 12  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 13  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 14  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 15  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 16  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 17  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 18  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 19  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 20  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 21  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 22  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 23  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 24  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 25  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 26  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 27  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 28  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 29  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 30  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 1     | 31  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 1   | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 2   | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 3   | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 4   | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 5   | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 6   | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 7   | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 8   | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 9   | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 10  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 11  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 12  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 13  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 14  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 15  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 16  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 17  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 18  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 19  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 20  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 21  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 | 2     | 22  | 10.00 | 10.00    | 10.00 | 10.00   |
| 1900 |       |     |       |          |       |         |

TABLE XV (Continued)

## SCORES OF STUDENTS IN GROUP V

| Code No. | CALIFORNIA ACHIEVEMENT TEST<br>ELEMENTARY ARITHMETIC FORM BB |                     |                 |                    | STEP ARITHMETIC FORM A |       |
|----------|--|---------------------|-----------------|--------------------|------------------------|-------|
|          | Reasoning Before   | Fundamentals Before | Reasoning After | Fundamentals After | Before                 | After |
| 111      | 3.0  | 4.8                 | 5.7             | 6.4                | 19                     | 14    |
| 112      | 4.7  | 4.6                 | 6.5             | 5.8                | 35                     | 45    |
| 113      | 4.7  | 4.6                 | 6.5             | 5.8                | 25                     | 33    |
| 114      | 4.6  | 4.3                 | 6.3             | 6.7                | 20                     | 34    |
| 115      | 5.6  | 5.0                 | 7.1             | 6.5                | 30                     | 37    |
| 116      | 6.1  | 5.1                 | 8.7             | 6.9                | 44                     | 46    |
| 117      | 5.9  | 5.1                 | 8.4             | 6.7                | 44                     | 40    |
| 118      | 5.9  | 5.4                 | 7.6             | 6.3                | 32                     | 38    |
| 120      | 4.4  | 4.3                 | 5.5             | 5.6                | 17                     | 24    |
| 122      | 5.1  | 4.5                 | 8.1             | 6.3                | 36                     | 41    |
| 123      | 5.6  | 5.5                 | 8.4             | 6.7                | 38                     | 41    |
| 124      | 4.9  | 4.9                 | 7.7             | 6.3                | 24                     | 33    |
| 125      | 2.7  | 3.5                 | 5.3             | 5.3                | 15                     | 18    |
| 126      | 4.2  | 3.9                 | 6.5             | 6.4                | 22                     | 29    |
| 127      | 4.8  | 5.1                 | 6.5             | 5.9                | 19                     | 22    |
| 128      | 6.1  | 5.1                 | 9.3             | 7.7                | 44                     | 43    |
| 129      | 5.3  | 4.3                 | 6.9             | 6.6                | 35                     | 42    |
| 132      | 4.8  | 4.7                 | 6.9             | 6.9                | 34                     | 36    |
| 133      | 4.8  | 4.3                 | 7.4             | 6.8                | 32                     | 37    |
| 134      | 4.4  | 4.9                 | 6.5             | 5.9                | 26                     | 33    |
| 135      | 4.9  | 4.7                 | 6.7             | 6.1                | 30                     | 38    |
| 136      | 5.4  | 5.4                 | 7.1             | 7.2                | 30                     | 43    |
| 138      | 4.5  | 4.5                 | 6.3             | 6.5                | 34                     | 33    |
| 139      | 5.9  | 5.0                 | 5.1             | 7.1                | 38                     | 46    |

## SCORES OF STUDENTS IN GROUP VI

|     |     |     |     |     |    |    |
|-----|-----|-----|-----|-----|----|----|
| 141 | 5.7 | 5.0 | 8.1 | 6.4 | 37 | 35 |
| 145 | 5.4 | 4.9 | 7.7 | 6.9 | 37 | 45 |
| 146 | 4.7 | 5.4 | 6.9 | 7.6 | 39 | 42 |
| 147 | 4.3 | 4.5 | 6.5 | 6.6 | 21 | 31 |
| 148 | 5.3 | 5.4 | 8.1 | 6.9 | 41 | 38 |
| 149 | 5.9 | 5.6 | 7.7 | 7.3 | 41 | 44 |
| 150 | 6.4 | 5.8 | 8.1 | 7.4 | 38 | 45 |
| 151 | 5.8 | 4.9 | 7.1 | 7.4 | 32 | 38 |
| 152 | 3.9 | 3.4 | 4.5 | 5.2 | 11 | 10 |
| 153 | 6.6 | 5.8 | 8.7 | 8.1 | 39 | 41 |
| 154 | 6.6 | 5.3 | 8.4 | 7.4 | 40 | 37 |
| 155 | 6.1 | 5.6 | 6.3 | 6.5 | 38 | 39 |
| 156 | 4.6 | 4.7 | 7.1 | 6.6 | 25 | 28 |
| 158 | 5.3 | 4.7 | 6.7 | 6.6 | 24 | 24 |
| 159 | 5.1 | 5.5 | 8.7 | 7.2 | 39 | 47 |
| 160 | 5.9 | 5.8 | 7.7 | 7.4 | 41 | 47 |
| 161 | 4.5 | 4.8 | 5.0 | 5.9 | 21 | 26 |
| 162 | 4.4 | 4.9 | 6.9 | 6.2 | 29 | 39 |
| 163 | 5.1 | 4.4 | 5.5 | 6.5 | 29 | 35 |
| 165 | 5.9 | 5.1 | 6.9 | 7.9 | 41 | 41 |
| 166 | 4.8 | 5.1 | 6.9 | 6.4 | 27 | 33 |
| 167 | 5.9 | 5.7 | 8.7 | 7.8 | 37 | 42 |
| 169 | 5.6 | 6.2 | 7.4 | 6.3 | 31 | 36 |

1. The first part of the document is a list of names and addresses.

2. The second part of the document is a list of names and addresses.

3. The third part of the document is a list of names and addresses.

4. The fourth part of the document is a list of names and addresses.

5. The fifth part of the document is a list of names and addresses.

6. The sixth part of the document is a list of names and addresses.

7. The seventh part of the document is a list of names and addresses.

TABLE XV (Continued)

## SCORES OF STUDENTS IN GROUP VII

| Code<br>No. | CALIFORNIA ACHIEVEMENT TEST<br>ELEMENTARY ARITHMETIC FORM BB |                        |                    |                       | STEP ARITHMETIC FORM A |       |
|-------------|--|------------------------|--------------------|-----------------------|------------------------|-------|
|             | Reasoning<br>Before  | Fundamentals<br>Before | Reasoning<br>After | Fundamentals<br>After | Before                 | After |
| 170         | 3.6  | 4.6                    | 5.5                | 5.4                   | 7                      | 15    |
| 171         | 5.7  | 5.3                    | 7.4                | 6.1                   | 25                     | 34    |
| 174         | 7.2  | 5.6                    | 7.4                | 5.8                   | 26                     | 29    |
| 177         | 4.4  | 4.4                    | 6.1                | 5.7                   | 20                     | 28    |
| 179         | 5.2  | 5.6                    | 6.1                | 5.5                   | 17                     | 17    |
| 180         | 7.0  | 4.7                    | 7.7                | 6.9                   | 33                     | 35    |
| 181         | 5.0  | 4.6                    | 6.2                | 5.2                   | 26                     | 27    |
| 183         | 4.9  | 5.7                    | 6.5                | 6.3                   | 24                     | 23    |
| 185         | 5.9  | 4.1                    | 6.7                | 5.6                   | 24                     | 24    |
| 187         | 5.4  | 5.0                    | 7.4                | 6.2                   | 29                     | 36    |
| 189         | 6.6  | 5.6                    | 8.4                | 6.8                   | 36                     | 41    |
| 190         | 6.1  | 5.5                    | 5.9                | 5.6                   | 23                     | 27    |
| 192         | 5.0  | 5.1                    | 7.4                | 6.0                   | 25                     | 27    |
| 193         | 6.8  | 5.8                    | 9.3                | 6.8                   | 35                     | 38    |
| 194         | 5.0  | 4.8                    | 6.5                | 5.8                   | 25                     | 31    |
| 195         | 5.7  | 4.0                    | 6.7                | 5.8                   | 25                     | 23    |
| 196         | 6.2  | 5.3                    | 5.3                | 5.9                   | 28                     | 36    |
| 197         | 5.2  | 5.6                    | 6.7                | 5.6                   | 20                     | 28    |
| 198         | 3.8  | 4.4                    | 4.4                | 5.2                   | 14                     | 12    |
| 199         | 6.6  | 5.0                    | 6.7                | 6.4                   | 27                     | 33    |

## SCORES OF STUDENTS IN GROUP VIII

|     |     |     |     |     |    |    |
|-----|-----|-----|-----|-----|----|----|
| 202 | 6.6 | 5.5 | 8.1 | 8.1 | 30 | 37 |
| 203 | 6.2 | 4.6 | 5.5 | 6.4 | 18 | 28 |
| 205 | 4.5 | 4.2 | 7.1 | 5.5 | 22 | 37 |
| 206 | 4.7 | 3.9 | 4.7 | 3.9 | 17 | 30 |
| 207 | 6.1 | 5.4 | 6.9 | 6.0 | 16 | 37 |
| 209 | 5.9 | 5.3 | 6.9 | 6.2 | 27 | 28 |
| 210 | 7.8 | 5.6 | 8.7 | 7.6 | 38 | 41 |
| 211 | 5.6 | 4.8 | 5.0 | 5.0 | 11 | 24 |
| 213 | 4.2 | 4.4 | 5.0 | 6.1 | 10 | 18 |
| 214 | 3.3 | 4.0 | 5.3 | 5.8 | 14 | 21 |
| 216 | 4.9 | 5.7 | 7.7 | 6.5 | 24 | 33 |
| 218 | 4.7 | 4.7 | 6.3 | 6.1 | 20 | 17 |
| 219 | 4.1 | 5.1 | 4.7 | 5.6 | 17 | 12 |
| 220 | 6.8 | 5.5 | 5.9 | 5.8 | 28 | 30 |
| 221 | 5.0 | 5.3 | 6.7 | 6.0 | 18 | 26 |
| 222 | 5.6 | 4.8 | 5.7 | 5.6 | 13 | 27 |
| 223 | 6.0 | 6.0 | 6.1 | 6.3 | 27 | 30 |
| 227 | 6.2 | 5.8 | 7.1 | 6.7 | 39 | 41 |
| 229 | 3.5 | 4.6 | 3.3 | 4.7 | 13 | 24 |





**ARITHMETIC THEORY TAUGHT TO GRADES  
FIVE AND SIX AS EXPLANATION FOR  
CONTENT AREAS FOUND IN  
CURRICULUM AT THESE LEVELS**

## **I.      NUMERATION**

**Arithmetic as a Symbolic Language**

**Our Own Number System**

**Suggested Activities**

## ARITHMETIC AS A SYMBOLIC LANGUAGE

Numeration is a way of expressing numbers in words. It is difficult to figure in written words alone; so over the years, different peoples have invented different ways of expressing numbers (or quantities) with symbols. Many types of symbols have been used throughout the history of mathematics. It would appear that man has always tried to make whatever number system he used more simple and convenient. In this sense, one could say that arithmetic just "grew". It has grown for some six or eight thousand years. Many number systems might not be recognized as such today.

Counting is the foundation of arithmetic. People count in many ways. For the most part, one can count to the answer in an arithmetic problem - if one has time enough. Since our mathematics should be simple and convenient in order to be of the most use, we try to count in bunches (or groups) rather than by naming every different quantity that we wish to express. We count in bunches (or groups) or tens. We call this a base 10 number system. There are other people who count differently. Some systems are very simple. There are records of one like this:

|      |                             |
|------|-----------------------------|
| one  | (1)                         |
| two  | (2)                         |
| some | (3)                         |
| many | (any amount of more than 3) |

It makes little difference what these symbols may be or how they are named. We can see that in this kind of a system four could be either many, or some plus one, or one plus some, or perhaps just onesome as one word. This would be something like our number twenty-one. When we say "twenty-one" we think of an amount, and seldom stop to think that it is equal to 2 tens plus 1 one. Other people use other words to express amount.



There are said to be some Eskimos who use a system of naming their fingers and toes and combining these names into a counting system. In this way, they count in groups of five instead of ten as we do. They can do this until they get to twenty and (having run out of fingers and toes) they are forced to count in groups of twenty. They might figure their number system something like this:

|    |                       |                                |
|----|-----------------------|--------------------------------|
| 1  | atauseq               | 1 finger                       |
| 2  | machdlug              | 2 fingers                      |
| 3  | pinasut               | 3 fingers                      |
| 4  | sisamat               | 4 fingers (hand less 1 finger) |
| 5  | tadlimat              | 5 fingers (hand)               |
| 6  | achfineq-atauseq      | 1 hand and 1 finger            |
| 7  | achfineq-machdlug     | 1 hand and 2 fingers           |
| 8  | achfineq-pinassut     | 1 hand and 3 fingers           |
| 9  | achfineq-sisamat      | 1 hand and 4 fingers           |
| 10 | qulit                 | 2 hands                        |
| 11 | achqaneq-atauseq      | 2 hands and 1 toe              |
| 12 | achquaneq-machdlug    | 2 hands and 2 toes             |
| 13 | achqaneq-pinassut     | 2 hands and 3 toes             |
| 14 | achqaneq-sisamat      | 2 hands and 4 toes             |
| 15 | achfechsaneq          | 2 hands and 1 foot             |
| 16 | achfechsaneq-atauseq  | 2 hands, 1 foot, and 1 toe     |
| 17 | achfechsaneq-machdlug | 2 hands, 1 foot, and 2 toes    |
| 18 | achfechsaneq-pinassut | 2 hands, 1 foot, and 3 toes    |
| 19 | achfechsaneq-sisamat  | 2 hands, 1 foot, and 4 toes    |
| 20 | inuk navdlucho        | 2 hands and 2 feet             |

You could make your own number system and do problems in it. You would need to make up some symbols to stand for amounts, and then show how they could be manipulated in order to keep track of things. The way the Eskimos have done should give you some ideas.

In the history of our number system, many things have been used to stand for different quantities. People have used letters like the Romans and the Greeks, and pictures like the Egyptians. Usually, the symbols had some meaning or relationship to the quantity that it was to represent, as in the Eskimo system above.

SECRET

For instance: a nose could stand for one, etc. We might have a system like this:

Nose stands for 1. (Almost everyone has just one of these.)

Eyes come in pairs. So, eyes have stood for two.

Sometimes wings were used.

Stopping to draw 2 eyes every time is a bother,

so one eye might come to mean 2 all by itself.

Nose-eye combined could stand for 3.

Since this is rather a nuisance to have to stop

to make these two separately . . . . .

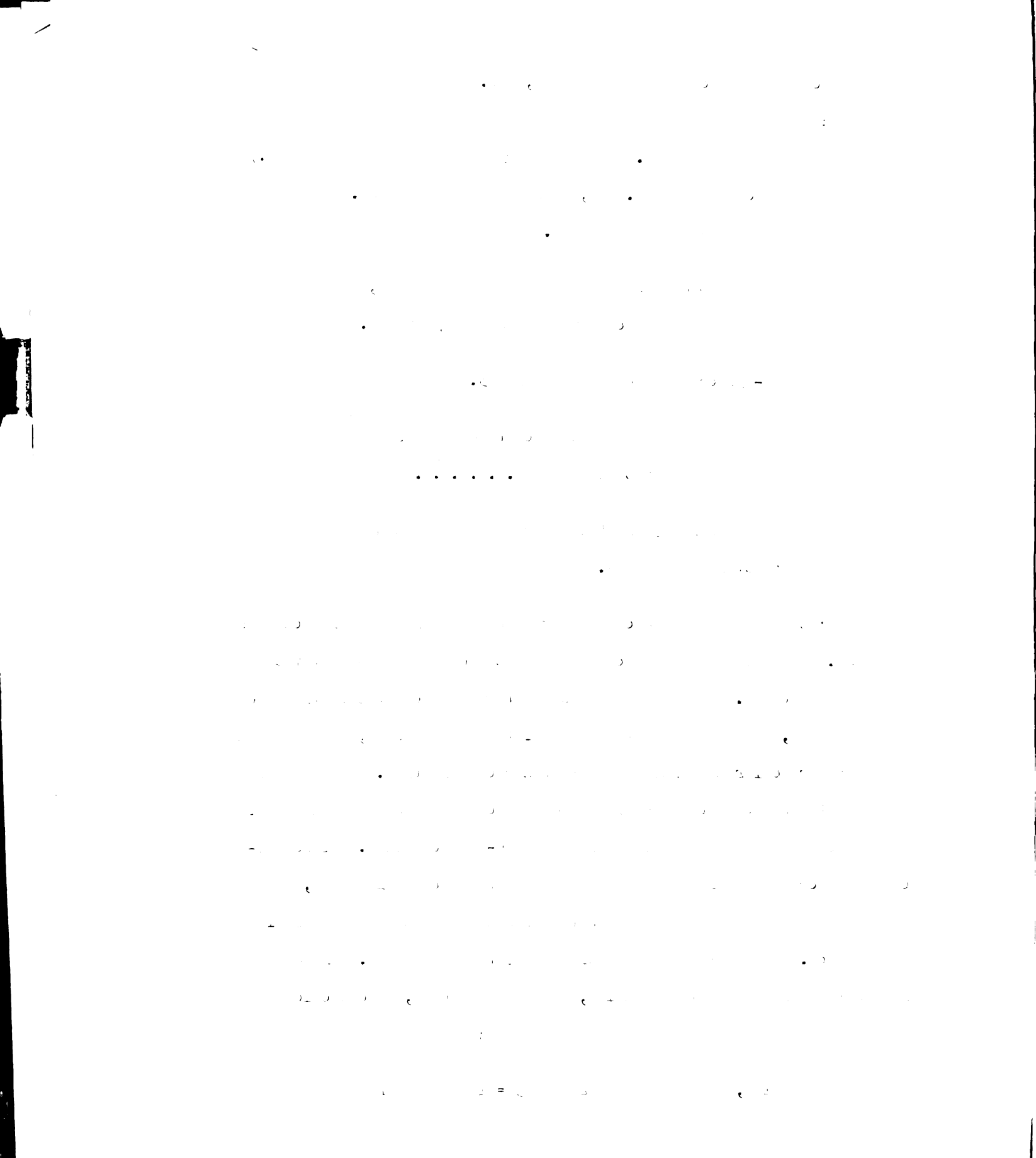
The symbol for 3 might come to be no more than

something like this. 

Symbols have grown and changed over hundreds of years in much this same way. They have grown and changed over the centuries to meet the needs of a society. As one studies the history of ancient societies and civilizations, we find a vast variation of the symbols used; but they all served a particular need for some particular civilization. Many of these were adapted by other civilizations and were changed and fitted to their needs much as we have done with our own Hinku-Arabic system. As we discover and create new forms of energy along with space exploration, we will probably be adding new symbols to our system to meet the needs of mathematics. A good illustration of this is the light year. If we figure that light travels about 186,000 miles a second, we can calculate the speed that light would travel in a year by:

$$186,000 \times 60 \times 60 \times 24 \times 365 = 1 \text{ light year}$$





## OUR OWN NUMBER SYSTEM

The exact beginnings of our own number system are vague. It is believed by some historians that it has been in use since the fifth or sixth century, B.C. It appears to have been in use for some time by the nomadic or "heathen" tribes of Arabia and India before it was recognized as being superior over the old Greek and Roman systems.

Historians disagree as to the exact time and place of its origin. Certain it is that the system has changed a great deal over the centuries. The symbols are of Hindu origin. The nomadic Arabians may have picked them up in India and carried them to Europe without using them much. It probably appeared first without the zero. This would have given it little advantage over the other older systems. It is thought that the zero may have come into use about the ninth century. The Hindus hit upon the ingenious idea of place value, or grouping of numbers. In this way, any amount can be expressed with only 9 digits and the zero to hold an empty place. It should be kept in mind that this so-called zero could be any symbol that would hold the place.

This Hindu-Arabic system was known in Europe as early as the thirteenth century. It was not much used until developments in science and trade made its computational advantages preferred over the old existing systems. This was possibly about the sixteenth century.

It is important to remember that over the past 2,000 years or so our number system has changed. These changes have been influenced by the changing needs of science and society. The Arabic and Hindu influences had much to do with its early formation. Our system has grown and changed with the centuries and it might be expected to continue to do so.

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## SUGGESTED ACTIVITIES

## I

Have the children imagine that they have need to "keep track of something"; but that they know of no number system to use. Ask them to make up symbols to stand for amounts and then do a problem or two in their system.

## II

Make a panarama to show how a shepherd might keep track of a flock of sheep by using 1 pebble to stand for each sheep.

Do the same thing, but show a 1 to 5 relationship or a 1 to 10. (One pebble for 5 sheep, or 1 pebble for 10 sheep.)

## III

Dramatize a situation of buying and selling in which characters use primitive ways of counting.

## IV

Look up measures used during Biblical times and produce charts, booklets, or posters describing them.

## V

Make a sign language for quantities and dramatize a situation involving a sale or exchange in which bargaining takes place.

## VI

Look up the signs used by auctioneers to indicate amounts.

## VII

Make a time line showing the development of different number systems. Use white wrapping paper and illustrate.

## VIII

Make a simple abacus or counter and demonstrate it to the class. A good one can be made with a shirt box and buttons on strings for counters. Others can be made with clay. Use the clay for grooves and small stones for counters. Wonderful World of Mathematics by Hogaben has some good material in it. This book should be in your library or office

1. The first part of the report is a general introduction to the subject.

2. The second part of the report is a detailed description of the methods used.

3. The third part of the report is a discussion of the results obtained.

4. The fourth part of the report is a conclusion and summary of the findings.

5. The fifth part of the report is a list of references.

6. The sixth part of the report is a list of figures and tables.

7. The seventh part of the report is a list of appendices.

8. The eighth part of the report is a list of footnotes.

9. The ninth part of the report is a list of acknowledgments.

10. The tenth part of the report is a list of the author's address and contact information.

11. The eleventh part of the report is a list of the author's other publications.

12. The twelfth part of the report is a list of the author's awards and honors.

13. The thirteenth part of the report is a list of the author's affiliations.

14. The fourteenth part of the report is a list of the author's dates of birth and death.

15. The fifteenth part of the report is a list of the author's dates of residence.

16. The sixteenth part of the report is a list of the author's dates of travel.

17. The seventeenth part of the report is a list of the author's dates of employment.

18. The eighteenth part of the report is a list of the author's dates of marriage and divorce.

19. The nineteenth part of the report is a list of the author's dates of death.

20. The twentieth part of the report is a list of the author's dates of burial.

21. The twenty-first part of the report is a list of the author's dates of cremation.

22. The twenty-second part of the report is a list of the author's dates of interment.

23. The twenty-third part of the report is a list of the author's dates of exhumation.

24. The twenty-fourth part of the report is a list of the author's dates of reinterment.

25. The twenty-fifth part of the report is a list of the author's dates of reburial.

26. The twenty-sixth part of the report is a list of the author's dates of reinterment.

27. The twenty-seventh part of the report is a list of the author's dates of reburial.

28. The twenty-eighth part of the report is a list of the author's dates of reinterment.

29. The twenty-ninth part of the report is a list of the author's dates of reburial.

30. The thirtieth part of the report is a list of the author's dates of reinterment.

## **II. PLACE VALUE**

**Pattern of Expressing a Numeral**

**Use of Abacus in Place Value**

**The Powers of a Base (Exponents)**



## PLACE VALUE

We count in bunches or groups of numbers. This makes it possible to do mathematics with only a few symbols. In our number system we count in groups of tens. We call this a BASE ten, or we say we have a number system in Base Ten. We have nine symbols to stand for the quantities from one through nine: 1, 2, 3, 4, 5, 6, 7, 8, 9, and a zero, 0. We need the zero because we give numbers a value according to the place in which they stand: 3 is equal to 3 ones. If we put the 3 in another place and make 30, we use the zero to literally push the 3 into the second place. It now stands for 3 tens, or thirty. We can hold the ten's place and put the three in hundred's place: 300. In each case, we have used the same symbol to stand for three different amounts. We could go on doing this without end.

We do not really have a symbol to stand for ten. We simply use a 1 and put it in ten's place, using the zero to hold the empty one's place. We read this number: 10, as "ten". This stands for one group of ten. Ten is our base. The 1 in the numeral 10 stands for 1 times the base of ten. If the base were something else, this one would stand for that amount. This is also called the model group of a number system.

If we think about counting it helps us to understand about how the base, or model group functions in our number system. It is really unfortunate that we learn to count before we understand what it means, because then we are so familiar with the process that we may fail to see the importance of understanding it. Counting is the very foundation of arithmetic. Counting with understanding can be a great help in learning arithmetical processes. It is absolutely necessary to understand it in order to master arithmetic processes.



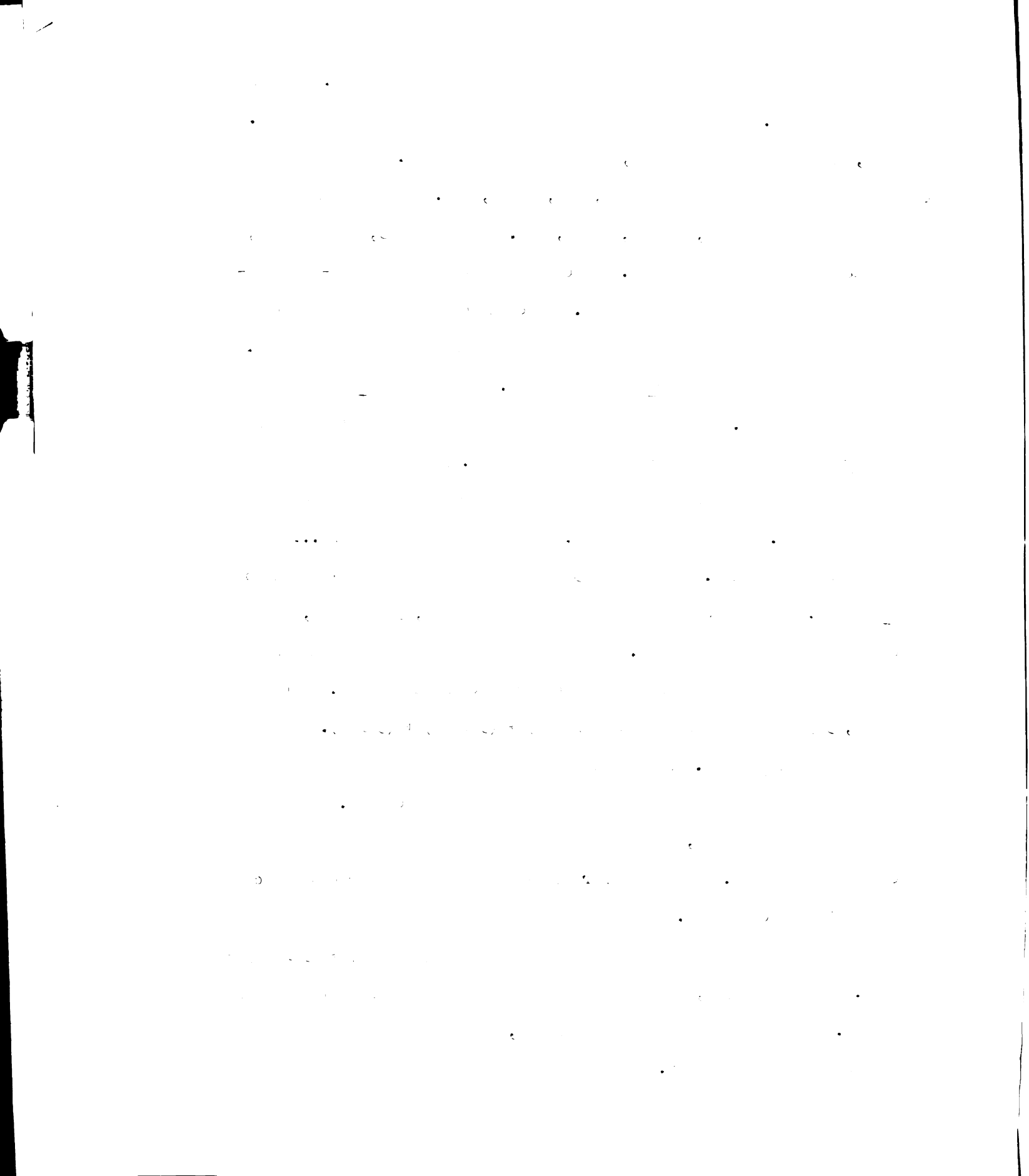
1. The first part of the report is a general  
introduction to the subject of the study.  
2. The second part is a description of the  
methodology used in the study.  
3. The third part is a description of the  
results of the study.  
4. The fourth part is a discussion of the  
results of the study.  
5. The fifth part is a conclusion of the study.

6. The sixth part is a list of references.  
7. The seventh part is a list of appendices.  
8. The eighth part is a list of figures.  
9. The ninth part is a list of tables.  
10. The tenth part is a list of footnotes.  
11. The eleventh part is a list of glossary.  
12. The twelfth part is a list of abbreviations.  
13. The thirteenth part is a list of symbols.  
14. The fourteenth part is a list of units.  
15. The fifteenth part is a list of definitions.

We use counting numbers to stand for different quantities. Counting is adding by ones. Each number has a position in relation to the others. That is, six always follows five, and comes before seven. We think of cardinal numbers as the numbers one, two, three, etc. We think of the ordinal numbers as first, second, third, etc. The number 3, for example, has a face value of three things. We could put it into a one-to-one relationship with a group of X X X things. We could count these and the last number we name "three" is the name of the number of members of the group. The number 3 is also the third number we name. This is the ordinal way of thinking of three. One good way to remember is that "third" expresses the order or arrangement of the series of numbers.

Counting numbers can be expressed as they relate to the place in which they stand. This is place value. A number has two values ... a face value and a place value. The number 3 has a value that makes it equal to X X X things. It also can be used to stand for 3 tens, 3 hundreds, or any place that we may choose to use. We express this as though the number were multiplying the value of the particular place it stands in. For instance, 333 can be expressed as  $(3 \times 100) + (3 \times 10) + (3 \times 1)$ . This is what 333 really means. It is necessary to think about counting numbers in this way to understand how a number system really functions. When it is thought of in this way, it is possible to understand arithmetical functions in any base. This also furnishes a basis for the process we call "borrowing" and "carrying".

The number 333 is simply easier to write than  $(3 \times 100) + (3 \times 10) + (3 \times 1)$ . In writing 333, it is also easy to lose sight of what the number really is. In order to see this more easily, it is well to indicate how numbers could be taken apart.



We will use parentheses to indicate the multiplication,  $3(1) = 3 \times 1$ , etc.

$$\begin{aligned} 1 &= 1(1) \\ 2 &= 2(1) \\ 3 &= 3(1) \\ 4 &= 4(1) \\ 5 &= 5(1) \\ 6 &= 6(1) \\ 7 &= 7(1) \\ 8 &= 8(1) \\ 9 &= 9(1) \end{aligned}$$

$$\begin{aligned} 10 &= 1(10) + 0(1) \\ 11 &= 1(10) + 1(1) \end{aligned}$$

$$\begin{aligned} 20 &= 2(10) + 0(1) \\ 26 &= 2(10) + 6(1) \end{aligned}$$

$$\begin{aligned} 45 &= 4(10) + 5(1) \\ 46 &= 4(10) + 6(1) \end{aligned}$$

$$\begin{aligned} 99 &= 9(10) + 9(1) \\ 100 &= 1(100) + 0(10) + 0(1) \\ 150 &= 1(100) + 5(10) + 0(1) \\ 268 &= 2(100) + 6(10) + 8(1) \end{aligned}$$

$$\begin{aligned} 1000 &= 1(1000) + 0(100) + 0(10) + 0(1) \\ 1004 &= 1(1000) + 0(100) + 0(10) + 4(1) \end{aligned}$$

$$\begin{aligned} 4579 &= 4(1000) + 5(100) + 7(10) + 9(1) \\ &\quad \dots \text{etc.} \end{aligned}$$

It is easy to see when we look at numbers in this fashion that:

(1) the use of place value makes our number system much more convenient;  
and, (2) we can change the model group or base very easily if we change  
the value of the places. This will become more apparent as we investigate  
the structure of other number systems.

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$$

二  
三  
四  
五  
六  
七  
八  
九  
十

Figure 1 is a schematic representation of the experimental design. It shows a sequence of events: 'Stimulus presentation', 'Response', 'Feedback', and 'Inter-trial interval'. Arrows indicate the flow from one stage to the next. A 'Start' box points to the first 'Stimulus presentation' box. A 'End' box points to the last 'Inter-trial interval' box. The sequence is repeated for multiple trials.

$\frac{1}{2}$

1.  $\frac{1}{2}$       2.  $\frac{1}{3}$       3.  $\frac{1}{4}$       4.  $\frac{1}{5}$       5.  $\frac{1}{6}$       6.  $\frac{1}{7}$       7.  $\frac{1}{8}$       8.  $\frac{1}{9}$       9.  $\frac{1}{10}$       10.  $\frac{1}{11}$       11.  $\frac{1}{12}$       12.  $\frac{1}{13}$       13.  $\frac{1}{14}$       14.  $\frac{1}{15}$       15.  $\frac{1}{16}$       16.  $\frac{1}{17}$       17.  $\frac{1}{18}$       18.  $\frac{1}{19}$       19.  $\frac{1}{20}$       20.  $\frac{1}{21}$       21.  $\frac{1}{22}$       22.  $\frac{1}{23}$       23.  $\frac{1}{24}$       24.  $\frac{1}{25}$       25.  $\frac{1}{26}$       26.  $\frac{1}{27}$       27.  $\frac{1}{28}$       28.  $\frac{1}{29}$       29.  $\frac{1}{30}$       30.  $\frac{1}{31}$       31.  $\frac{1}{32}$       32.  $\frac{1}{33}$       33.  $\frac{1}{34}$       34.  $\frac{1}{35}$       35.  $\frac{1}{36}$       36.  $\frac{1}{37}$       37.  $\frac{1}{38}$       38.  $\frac{1}{39}$       39.  $\frac{1}{40}$       40.  $\frac{1}{41}$       41.  $\frac{1}{42}$       42.  $\frac{1}{43}$       43.  $\frac{1}{44}$       44.  $\frac{1}{45}$       45.  $\frac{1}{46}$       46.  $\frac{1}{47}$       47.  $\frac{1}{48}$       48.  $\frac{1}{49}$       49.  $\frac{1}{50}$       50.  $\frac{1}{51}$       51.  $\frac{1}{52}$       52.  $\frac{1}{53}$       53.  $\frac{1}{54}$       54.  $\frac{1}{55}$       55.  $\frac{1}{56}$       56.  $\frac{1}{57}$       57.  $\frac{1}{58}$       58.  $\frac{1}{59}$       59.  $\frac{1}{60}$       60.  $\frac{1}{61}$       61.  $\frac{1}{62}$       62.  $\frac{1}{63}$       63.  $\frac{1}{64}$       64.  $\frac{1}{65}$       65.  $\frac{1}{66}$       66.  $\frac{1}{67}$       67.  $\frac{1}{68}$       68.  $\frac{1}{69}$       69.  $\frac{1}{70}$       70.  $\frac{1}{71}$       71.  $\frac{1}{72}$       72.  $\frac{1}{73}$       73.  $\frac{1}{74}$       74.  $\frac{1}{75}$       75.  $\frac{1}{76}$       76.  $\frac{1}{77}$       77.  $\frac{1}{78}$       78.  $\frac{1}{79}$       79.  $\frac{1}{80}$       80.  $\frac{1}{81}$       81.  $\frac{1}{82}$       82.  $\frac{1}{83}$       83.  $\frac{1}{84}$       84.  $\frac{1}{85}$       85.  $\frac{1}{86}$       86.  $\frac{1}{87}$       87.  $\frac{1}{88}$       88.  $\frac{1}{89}$       89.  $\frac{1}{90}$       90.  $\frac{1}{91}$       91.  $\frac{1}{92}$       92.  $\frac{1}{93}$       93.  $\frac{1}{94}$       94.  $\frac{1}{95}$       95.  $\frac{1}{96}$       96.  $\frac{1}{97}$       97.  $\frac{1}{98}$       98.  $\frac{1}{99}$       99.  $\frac{1}{100}$       100.  $\frac{1}{101}$       101.  $\frac{1}{102}$       102.  $\frac{1}{103}$       103.  $\frac{1}{104}$       104.  $\frac{1}{105}$       105.  $\frac{1}{106}$       106.  $\frac{1}{107}$       107.  $\frac{1}{108}$       108.  $\frac{1}{109}$       109.  $\frac{1}{110}$       110.  $\frac{1}{111}$       111.  $\frac{1}{112}$       112.  $\frac{1}{113}$       113.  $\frac{1}{114}$       114.  $\frac{1}{115}$       115.  $\frac{1}{116}$       116.  $\frac{1}{117}$       117.  $\frac{1}{118}$       118.  $\frac{1}{119}$       119.  $\frac{1}{120}$       120.  $\frac{1}{121}$       121.  $\frac{1}{122}$       122.  $\frac{1}{123}$       123.  $\frac{1}{124}$       124.  $\frac{1}{125}$       125.  $\frac{1}{126}$       126.  $\frac{1}{127}$       127.  $\frac{1}{128}$       128.  $\frac{1}{129}$       129.  $\frac{1}{130}$       130.  $\frac{1}{131}$       131.  $\frac{1}{132}$       132.  $\frac{1}{133}$       133.  $\frac{1}{134}$       134.  $\frac{1}{135}$       135.  $\frac{1}{136}$       136.  $\frac{1}{137}$       137.  $\frac{1}{138}$       138.  $\frac{1}{139}$       139.  $\frac{1}{140}$       140.  $\frac{1}{141}$       141.  $\frac{1}{142}$       142.  $\frac{1}{143}$       143.  $\frac{1}{144}$       144.  $\frac{1}{145}$       145.  $\frac{1}{146}$       146.  $\frac{1}{147}$       147.  $\frac{1}{148}$       148.  $\frac{1}{149}$       149.  $\frac{1}{150}$       150.  $\frac{1}{151}$       151.  $\frac{1}{152}$       152.  $\frac{1}{153}$       153.  $\frac{1}{154}$       154.  $\frac{1}{155}$       155.  $\frac{1}{156}$       156.  $\frac{1}{157}$       157.  $\frac{1}{158}$       158.  $\frac{1}{159}$       159.  $\frac{1}{160}$       160.  $\frac{1}{161}$       161.  $\frac{1}{162}$       162.  $\frac{1}{163}$       163.  $\frac{1}{164}$       164.  $\frac{1}{165}$       165.  $\frac{1}{166}$       166.  $\frac{1}{167}$       167.  $\frac{1}{168}$       168.  $\frac{1}{169}$       169.  $\frac{1}{170}$       170.  $\frac{1}{171}$       171.  $\frac{1}{172}$       172.  $\frac{1}{173}$       173.  $\frac{1}{174}$       174.  $\frac{1}{175}$       175.  $\frac{1}{176}$       176.  $\frac{1}{177}$       177.  $\frac{1}{178}$       178.  $\frac{1}{179}$       179.  $\frac{1}{180}$       180.  $\frac{1}{181}$       181.  $\frac{1}{182}$       182.  $\frac{1}{183}$       183.  $\frac{1}{184}$       184.  $\frac{1}{185}$       185.  $\frac{1}{186}$       186.  $\frac{1}{187}$       187.  $\frac{1}{188}$       188.  $\frac{1}{189}$       189.  $\frac{1}{190}$       190.  $\frac{1}{191}$       191.  $\frac{1}{192}$       192.  $\frac{1}{193}$       193.  $\frac{1}{194}$       194.  $\frac{1}{195}$       195.  $\frac{1}{196}$       196.  $\frac{1}{197}$       197.  $\frac{1}{198}$       198.  $\frac{1}{199}$       199.  $\frac{1}{200}$       200.  $\frac{1}{201}$       201.  $\frac{1}{202}$       202.  $\frac{1}{203}$       203.  $\frac{1}{204}$       204.  $\frac{1}{205}$       205.  $\frac{1}{206}$       206.  $\frac{1}{207}$       207.  $\frac{1}{208}$       208.  $\frac{1}{209}$       209.  $\frac{1}{210}$       210.  $\frac{1}{211}$       211.  $\frac{1}{212}$       212.  $\frac{1}{213}$       213.  $\frac{1}{214}$       214.  $\frac{1}{215}$       215.  $\frac{1}{216}$       216. <

1.  $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$   
 2.  $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$   
 3.  $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$   
 4.  $\frac{1}{4} \times \frac{1}{8} = \frac{1}{32}$   
 5.  $\frac{1}{8} \times \frac{1}{8} = \frac{1}{64}$   
 6.  $\frac{1}{8} \times \frac{1}{16} = \frac{1}{128}$   
 7.  $\frac{1}{16} \times \frac{1}{16} = \frac{1}{256}$   
 8.  $\frac{1}{16} \times \frac{1}{32} = \frac{1}{512}$   
 9.  $\frac{1}{32} \times \frac{1}{32} = \frac{1}{1024}$   
 10.  $\frac{1}{32} \times \frac{1}{64} = \frac{1}{2048}$   
 11.  $\frac{1}{64} \times \frac{1}{64} = \frac{1}{4096}$   
 12.  $\frac{1}{64} \times \frac{1}{128} = \frac{1}{8192}$   
 13.  $\frac{1}{128} \times \frac{1}{128} = \frac{1}{16384}$   
 14.  $\frac{1}{128} \times \frac{1}{256} = \frac{1}{32768}$   
 15.  $\frac{1}{256} \times \frac{1}{256} = \frac{1}{65536}$   
 16.  $\frac{1}{256} \times \frac{1}{512} = \frac{1}{131072}$   
 17.  $\frac{1}{512} \times \frac{1}{512} = \frac{1}{262144}$   
 18.  $\frac{1}{512} \times \frac{1}{1024} = \frac{1}{524288}$   
 19.  $\frac{1}{1024} \times \frac{1}{1024} = \frac{1}{1048576}$   
 20.  $\frac{1}{1024} \times \frac{1}{2048} = \frac{1}{2097152}$   
 21.  $\frac{1}{2048} \times \frac{1}{2048} = \frac{1}{4194304}$   
 22.  $\frac{1}{2048} \times \frac{1}{4096} = \frac{1}{8388608}$   
 23.  $\frac{1}{4096} \times \frac{1}{4096} = \frac{1}{16777216}$   
 24.  $\frac{1}{4096} \times \frac{1}{8192} = \frac{1}{33554432}$   
 25.  $\frac{1}{8192} \times \frac{1}{8192} = \frac{1}{67108864}$   
 26.  $\frac{1}{8192} \times \frac{1}{16384} = \frac{1}{134217728}$   
 27.  $\frac{1}{16384} \times \frac{1}{16384} = \frac{1}{268435456}$   
 28.  $\frac{1}{16384} \times \frac{1}{32768} = \frac{1}{536870912}$   
 29.  $\frac{1}{32768} \times \frac{1}{32768} = \frac{1}{1073741824}$   
 30.  $\frac{1}{32768} \times \frac{1}{65536} = \frac{1}{2147483648}$   
 31.  $\frac{1}{65536} \times \frac{1}{65536} = \frac{1}{4294967296}$   
 32.  $\frac{1}{65536} \times \frac{1}{131072} = \frac{1}{8589934592}$   
 33.  $\frac{1}{131072} \times \frac{1}{131072} = \frac{1}{17179869184}$   
 34.  $\frac{1}{131072} \times \frac{1}{262144} = \frac{1}{34359738368}$   
 35.  $\frac{1}{262144} \times \frac{1}{262144} = \frac{1}{68719476736}$   
 36.  $\frac{1}{262144} \times \frac{1}{524288} = \frac{1}{137438953472}$   
 37.  $\frac{1}{524288} \times \frac{1}{524288} = \frac{1}{274877906944}$   
 38.  $\frac{1}{524288} \times \frac{1}{1048576} = \frac{1}{549755813888}$   
 39.  $\frac{1}{1048576} \times \frac{1}{1048576} = \frac{1}{1099511627776}$   
 40.  $\frac{1}{1048576} \times \frac{1}{2097152} = \frac{1}{2199023255552}$   
 41.  $\frac{1}{2097152} \times \frac{1}{2097152} = \frac{1}{4398046511104}$   
 42.  $\frac{1}{2097152} \times \frac{1}{4194304} = \frac{1}{8796093022208}$   
 43.  $\frac{1}{4194304} \times \frac{1}{4194304} = \frac{1}{17592186044416}$   
 44.  $\frac{1}{4194304} \times \frac{1}{8388608} = \frac{1}{35184372088832}$   
 45.  $\frac{1}{8388608} \times \frac{1}{8388608} = \frac{1}{70368744177664}$   
 46.  $\frac{1}{8388608} \times \frac{1}{16777216} = \frac{1}{140737488355328}$   
 47.  $\frac{1}{16777216} \times \frac{1}{16777216} = \frac{1}{281474976710656}$   
 48.  $\frac{1}{16777216} \times \frac{1}{33554432} = \frac{1}{562949953421312}$   
 49.  $\frac{1}{33554432} \times \frac{1}{33554432} = \frac{1}{1125899906842624}$   
 50.  $\frac{1}{33554432} \times \frac{1}{67108864} = \frac{1}{2251799813685248}$   
 51.  $\frac{1}{67108864} \times \frac{1}{67108864} = \frac{1}{4503599627370496}$   
 52.  $\frac{1}{67108864} \times \frac{1}{134217728} = \frac{1}{9007199254740992}$   
 53.  $\frac{1}{134217728} \times \frac{1}{134217728} = \frac{1}{18014398509481984}$   
 54.  $\frac{1}{134217728} \times \frac{1}{268435456} = \frac{1}{36028797018963968}$   
 55.  $\frac{1}{268435456} \times \frac{1}{268435456} = \frac{1}{72057594037927936}$   
 56.  $\frac{1}{268435456} \times \frac{1}{536870912} = \frac{1}{144115188075855872}$   
 57.  $\frac{1}{536870912} \times \frac{1}{536870912} = \frac{1}{288230376151711744}$   
 58.  $\frac{1}{536870912} \times \frac{1}{1073741824} = \frac{1}{576460752303423488}$   
 59.  $\frac{1}{1073741824} \times \frac{1}{1073741824} = \frac{1}{1152921504606846976}$   
 60.  $\frac{1}{1073741824} \times \frac{1}{2147483648} = \frac{1}{2305843009213693952}$   
 61.  $\frac{1}{2147483648} \times \frac{1}{2147483648} = \frac{1}{4611686018427387904}$   
 62.  $\frac{1}{2147483648} \times \frac{1}{4294967296} = \frac{1}{9223372036854775808}$   
 63.  $\frac{1}{4294967296} \times \frac{1}{4294967296} = \frac{1}{18446744073709551616}$   
 64.  $\frac{1}{4294967296} \times \frac{1}{8589934592} = \frac{1}{36893488147419103232}$   
 65.  $\frac{1}{8589934592} \times \frac{1}{8589934592} = \frac{1}{73786976294838206464}$   
 66.  $\frac{1}{8589934592} \times \frac{1}{16777216} = \frac{1}{147573952589676412928}$   
 67.  $\frac{1}{16777216} \times \frac{1}{16777216} = \frac{1}{295147905179352825856}$   
 68.  $\frac{1}{16777216} \times \frac{1}{33554432} = \frac{1}{590295810358705651712}$   
 69.  $\frac{1}{33554432} \times \frac{1}{33554432} = \frac{1}{1180591620717411303424}$   
 70.  $\frac{1}{33554432} \times \frac{1}{67108864} = \frac{1}{2361183241434822606848}$   
 71.  $\frac{1}{67108864} \times \frac{1}{67108864} = \frac{1}{4722366482869645213696}$   
 72.  $\frac{1}{67108864} \times \frac{1}{134217728} = \frac{1}{9444732965739290427392}$   
 73.  $\frac{1}{134217728} \times \frac{1}{$

1. *Chlorophyll *a** was determined in 100% methanol extracts of the leaves and stems of the plants. The concentration of chlorophyll *a* was determined by the method of Arar and Johnson (1977).

[illegible][illegible]

1000

## USING AN ABACUS TO SHOW PLACE VALUE

Since 10 is our model group, we need to have only nine digits if we use place value. On the other hand, if we have only nine digits, we are forced to use some other way to express quantities over ten. An abacus can be used to demonstrate this very well.

Push up ten ones on the ones column,  
counting and writing the numbers as you do:  
1, 2, 3, 4, 5, 6, 7, 8, 9, ? .....

Now, after we get to 9, we have no more  
symbols to express additional quantities, so  
we use place value. Push down the 10 ones  
beads and push up 1 ten bead on the tens  
column. Now write 10. This is:  $1(10) +$   
 $0(1)$ . The zero indicates an empty column.

Do this again: push up ten ones. We  
have no symbol for tens so push down the ten ones  
and push up another ten bead. We now have 20,  
or  $2(10) + 0(1)$ .

Continue doing this until you have pushed  
up all the tens beads. Now push these down and  
use 1 one hundred bead to stand for  $1(100) +$   
 $0(10) + 0(1)$ . Again, the zeros stand for empty  
columns.

• • • • •

• • • • •

• • •

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•

You could continue until you have used up all of the hundred beads. After using 9 hundred beads, it is easy to see that we have no symbol for 10 hundred unless we go to the next place and, using place value, write 1,000. This is:  $1(1000) + 0(10) + 0(1)$ .

If we think carefully about place value, and remember that our model group, or BASE, is ten, we can make a place value chart that shows this mathematically rather than by using an abacus. Think first about place value as we have learned the names for these places. We will express these by using numbers that show how many times ten is multiplied by itself in order to equal each place.

#### THE POWERS OF THE BASE

|              |          |                   |               |           |          |      |      |
|--------------|----------|-------------------|---------------|-----------|----------|------|------|
| ten-millions | millions | hundred-thousands | ten-thousands | thousands | hundreds | tens | ones |
|--------------|----------|-------------------|---------------|-----------|----------|------|------|

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Each place is ten times the one to the right of it. For example, tens are equal to  $10 \times 1$ , and hundreds are equal to  $10 \times 10$ , etc.

Each place is also  $1/10$  of the place to the left of it. For example, tens are  $1/10$  of 100, and hundreds are  $1/10$  of thousands.

Another way to understand this is to make a chart like the following one in which the value for each place is expressed as the number of times ten is multiplied by itself.





|            |                     |  |
|------------|---------------------|--|
| 1          | (ones)              |  |
| 10         | (tens)              | $10 \times 1$  |
| 100        | (hundreds)          | $10 \times 10$   |
| 1,000      | (thousands)         | $10 \times 10 \times 10$   |
| 10,000     | (ten-thousands)     | $10 \times 10 \times 10 \times 10$                               |
| 100,000    | (hundred-thousands) | $10 \times 10 \times 10 \times 10 \times 10$                     |
| 1,000,000  | (millions)          | $10 \times 10 \times 10 \times 10 \times 10 \times 10$           |
| 10,000,000 | (ten-millions)      | $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ |

We have a more simple way of showing this by using small numbers written up to the right of the tens to show how many times ten is multiplied by itself. We call these exponents. These small figures show the powers of ten. We would read  $10^2$  as "ten squared" or as "ten to the second power." This means  $10 \times 10$  or 100. Expressed in another way  $10^1 \times 10^1 = 10^2$ . Here you will see that we can add exponents in order to multiply numbers that are expressed exponentially. This is not particularly time saving in the multiplication of small numbers, but it is very useful in dealing with large ones. It also forms the basis for some very necessary understandings in regard to place value and the base of a number system. These ideas are also used in developing the concept of logarithms.

It would be well at this point to consider a chart showing how the powers of ten can be used to indicate place value.

|            |                     |  |        |
|------------|---------------------|--|--------|
| 1          | (ones)              |  | $10^0$ |
| 10         | (tens)              | $10 \times 1$  | $10^1$ |
| 100        | (hundreds)          | $10 \times 10$   | $10^2$ |
| 1,000      | (thousands)         | $10 \times 10 \times 10$   | $10^3$ |
| 10,000     | (ten-thousands)     | $10 \times 10 \times 10 \times 10$                               | $10^4$ |
| 100,000    | (hundred-thousands) | $10 \times 10 \times 10 \times 10 \times 10$                     | $10^5$ |
| 1,000,000  | (millions)          | $10 \times 10 \times 10 \times 10 \times 10 \times 10$           | $10^6$ |
| 10,000,000 | (ten-millions)      | $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ | $10^7$ |

6

We can see why ten to the zero ( $10^0$ ) is equal to one if we do a problem using the powers of ten. We will start with an example to which we know the answer:

$$\begin{array}{r} 10 \\ \times 10 \\ \hline 100 \end{array}$$

or

$$\begin{array}{r} 10^1 \\ \times 10^1 \\ \hline 10^2 \end{array}$$

We have to get  $10^2$  for the answer since  $10 \times 10 = 100$ , and 100 is written with an exponent as  $10^2$ . Now, if we do a division example, we will see why  $10^0$  is equal to one.

$$10 \overline{) 100}$$

or

$$10^1 \overline{) 10^2}$$

This may also be expressed as a fraction:

$$\frac{10^2}{10^1} = 10^1$$

Since  $100 \div 10 = 10$ , we have to show the answer as ten. (Ten expressed with an exponent is  $10^1$ ; therefore, we can assume that we would subtract exponents to divide numbers; or  $10^2 \div 10^1 = 10^{2-1}$  or  $10^1$ ).

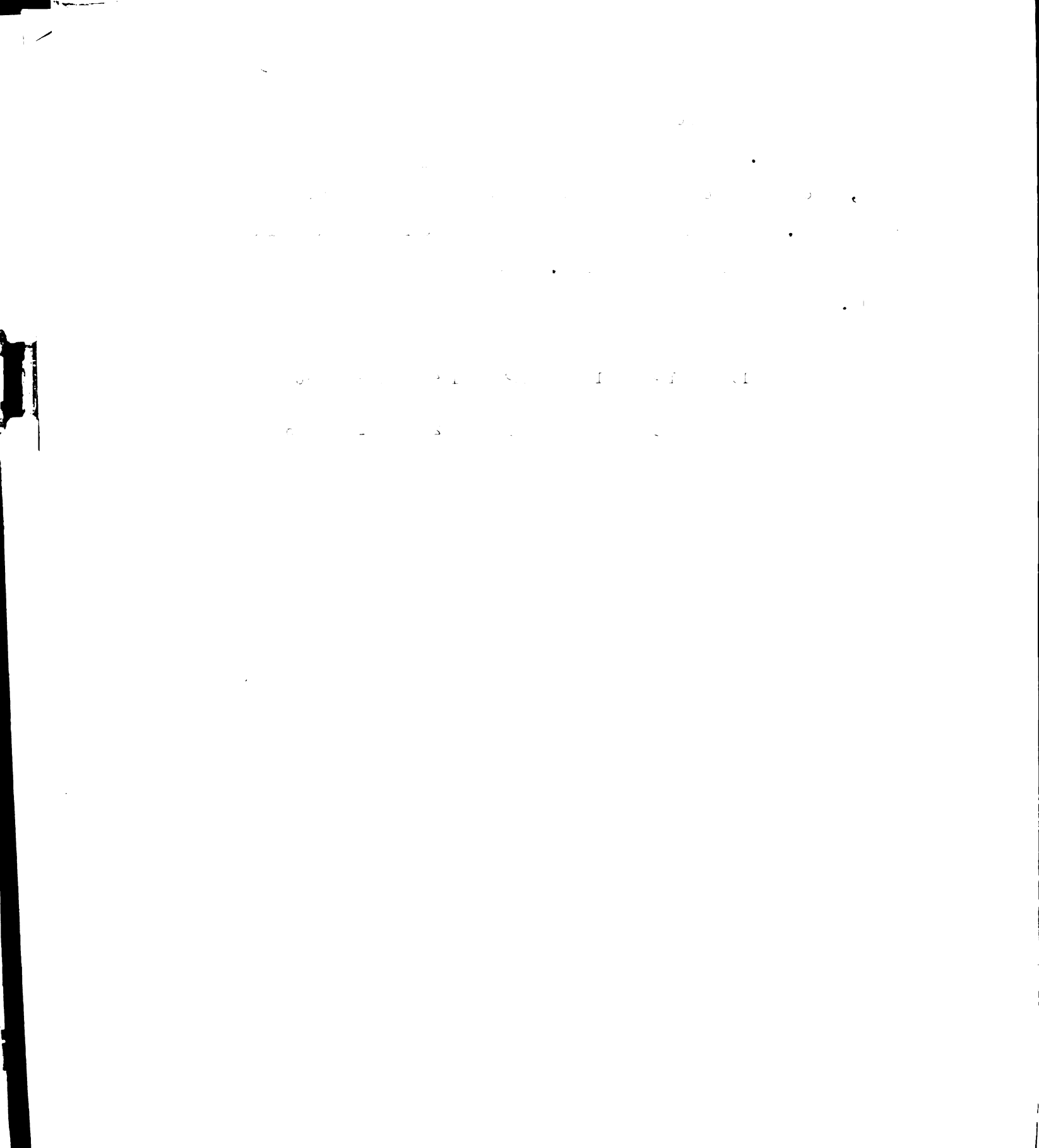
Since  $10 \div 10 = 1$ , we can now show that  $10^1 \div 10^1 = 10^0$ , because we subtract exponents to divide numbers. In this case, we could show it as  $10^1 \div 10^1 = 10^{1-1}$  or  $10^0$ . Or, in other words, ones can be expressed as  $10^0$ . From this, we might conclude that: any number (not a zero) whose exponent is zero ( $^0$ ) is equal to one, or  $N^0 = 1$ .

We should now remember that our number system is based on a model group of 10. Each position in place value can be expressed as a power of ten. These powers of ten are written as exponents. Knowing these things we can: (1) write any number of quantities that we wish to; and, (2) we can set up a number system in any base other than ten that we may choose. It is necessary to understand this as a foundation to the mathematics that is basic to arithmetic.



Often in mathematics when we wish to generalize we let letters stand for numbers. If we let the letter B stand for any base that we may choose, we can make a chart that will show place value for any model group or base. This will help to understand just exactly how place value operates in our familiar base 10 system. We could compare this with base 10.

|          |        |        |        |        |        |        |        |
|----------|--------|--------|--------|--------|--------|--------|--------|
| Base 10  | $10^6$ | $10^5$ | $10^4$ | $10^3$ | $10^2$ | $10^1$ | $10^0$ |
| Any Base | $B^6$  | $B^5$  | $B^4$  | $B^3$  | $B^2$  | $B^1$  | $B^0$  |



### III. OTHER SYSTEMS AND BASES

Egyptian Number System

Babylonian Number System

Greek Number System

Roman Number System

Base 7

Suggestions for Teaching



## THE EGYPTIAN NUMBER SYSTEM

Egypt is one of the oldest civilizations about which we have a record. From their great temples and tombs, which they constructed some four thousand years ago, we have been able to learn a great deal about their lives. Surprisingly enough, we know quite a bit about their mathematics. They seem to have been a clever folk, quick to invent mathematical ideas and methods that would help them in their daily lives. An arithmetic book has been found dating from about four thousand years ago.

The Egyptians and the people in the countries around this area appear to have used a kind of picture writing for their words and for the figures that they made. The Samaritans, Babylonians, Arabians, and Hindus, later the Greeks and Romans, have all contributed to our mathematical system. The earliest of these used hieroglyphics, or pictures, to make their words and numerals. Man appears first to have done his figuring only by drawing crude pictures to represent numbers. For instance:



It is easy to see how something like this could lead to a hieroglyphic writing and number system. There is some indication that men also wrote out words for each number. Instead of writing a symbol like 6 or 7, they would write out the word, six, and seven. It would be very difficult to do much figuring with this kind of number system. So, symbols probably were invented as a matter of convenience. One of the oldest of all these systems is the Egyptian. It is quite remarkable for its simplicity and usefulness. Their figures, like their written words, were hieroglyphics.

1. The first part of the report deals with the general situation of the country and the progress of the work during the year. It is divided into two main sections: the first section deals with the general situation and the second section deals with the progress of the work.

2. The second part of the report deals with the results of the work during the year. It is divided into two main sections: the first section deals with the results of the work in the field of research and the second section deals with the results of the work in the field of education.

3. The third part of the report deals with the financial situation of the institution during the year. It is divided into two main sections: the first section deals with the income and the second section deals with the expenditure.









4. The fourth part of the report deals with the personnel of the institution during the year. It is divided into two main sections: the first section deals with the staff and the second section deals with the students.

5. The fifth part of the report deals with the general conclusion of the year's work. It is divided into two main sections: the first section deals with the achievements and the second section deals with the problems.

### CONCLUSION

The year's work has been successful in many respects. The results of the work in the field of research and education have been satisfactory. The financial situation of the institution has been stable. The personnel of the institution have been efficient and dedicated. The general conclusion of the year's work is that the institution has made significant progress in its work.







We have words that illustrate this idea. Water wheel, water shed, water snake, and the name Waterman might be written in hieroglyphics like this:

|             |   |   |
|-------------|---|---|
| water shed  |  |  |
| water wheel |  |  |
| water snake |  |  |
| waterman    |  |  |

The Egyptians just made marks for the number from 1 to 9. It is easier to read these if the marks are grouped, and this is how they look:

| 1 | 2  | 3   | 4 | 5 | 6 | 7 | 8 | 9 |
|---|----|-----|---|---|---|---|---|---|
|   |    |     |   |   |   |   |   |   |
| , | '' | ''' |   |   |   |   |   |   |
|   |    |     |   |   |   |   |   |   |

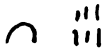

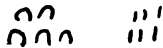
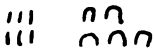


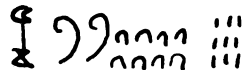
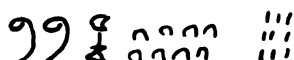
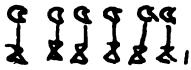
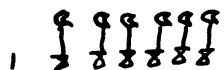


In addition to these, they had symbols for a sort of place value much like ours. It was based on 10.


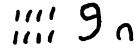


|           |   |                     |
|-----------|---|---------------------|
| 10        |  | heel bone           |
| 100       |  | coil of rope        |
| 1,000     |  | lotus flower        |
| 10,000    |  | bent line           |
| 100,000   |  | burbot              |
| 1,000,000 |  | man in astonishment |



We do not know why they picked these particular symbols. It is easy to imagine that a number like a million would be represented as a "man in astonishment". He probably was just overcome by such a large number. This reminds us of the very primitive number system of: "one, two, some, and a whole lot."

The Egyptians had no zero. They had no need of it in this system. The zero was to come many years later. If you write a few numbers, you can see that they would not miss a zero.

|           |   |    |   |
|-----------|---|----|---|
| 16        |    | or |    |
| 56        |    | or |    |
| 101       |   | or |   |
| 1,289     |  | or |  |
| 6,001     |  | or |  |
| 1,000,001 |  | or |  |

The Egyptians had a place value of sorts. Yet, it would make no difference to them if 128,  , was written as  or  or  , etc. We can see that they used the principle of addition in their system. It would make no difference if they added 10 + 6 to make 16, or 6 + 10 to make 16. This makes it rather easy to do problems in Egyptian hieroglyphics because all you have to do is to count the number of times a symbol appears and write it in tens. We will see how this system operated in the Roman number system later on.



## ROMAN NUMBERS

In order to write Roman numbers, we use addition, subtraction, multiplication, and repetition. When these are understood, reading and writing these numerals becomes a simple process.

Roman numerals are written with the use of seven (and sometimes eight) symbols: I V X L C D M and sometimes  $\bar{M}$ . This is a very old number system. Over the centuries there have been changes in the symbols as we know them today. At the time Roman numerals were first used, people did not need very large numbers or the symbols to write them. As the culture of the people developed and their possessions increased, they had need of a better number system; so they changed the symbols that they used to fit their particular needs.

Very early Roman numerals were quite different from those of today. There were several ways to write the same number. Examples of such numerals have been found in old ruins.



EARLY ROMAN NUMERALS

There are many interesting books about the development of our number system. See:

Smith, David Eugene, and Jekuthiel Ginsburg, Numbers and Numerals. The National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W. Washington, D.C. (35 cents postpaid)

Smith, David Eugene, Number Stories of Long Ago. The National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington, D.C.

The National Council also has fine lists of books on mathematics in many areas.

1. The first part of the document is a list of names and addresses of the members of the committee.

2. The second part of the document is a list of the names and addresses of the members of the committee who have been elected to the office of the chairperson.

3. The third part of the document is a list of the names and addresses of the members of the committee who have been elected to the office of the secretary.

4. The fourth part of the document is a list of the names and addresses of the members of the committee who have been elected to the office of the treasurer.

5. The fifth part of the document is a list of the names and addresses of the members of the committee who have been elected to the office of the clerk.

6. The sixth part of the document is a list of the names and addresses of the members of the committee who have been elected to the office of the assistant clerk.

7. The seventh part of the document is a list of the names and addresses of the members of the committee who have been elected to the office of the assistant treasurer.

8. The eighth part of the document is a list of the names and addresses of the members of the committee who have been elected to the office of the assistant secretary.



## THE USE OF ADDITION

|              |              |                |
|--------------|--------------|----------------|
| V equals 5   | X equals 10  | L equals 50    |
| C equals 100 | D equals 500 | M equals 1,000 |

A line over a numeral is sometimes used to multiply a number by 1,000;  $\overline{M}$  would equal one thousand thousand or a million.

|               |  |
|---------------|--|
| VI equals 6   | MDCXX equals 1,000 plus 500 plus 100 plus 10 plus 10 (1,620) |
| VIII equals 8 | LXV equals 50 plus 10 plus 5 (65)                            |
| XII equals 12 | DCCC equals 500 plus 100 plus 100 plus 100 or (800)          |

With the exception of M (1,000), no symbol is repeated more than three times. When it becomes necessary to repeat a symbol four times, the next higher one is used and we subtract.

|    |      |     |     |     |     |     |      |       |                |     |    |
|----|------|-----|-----|-----|-----|-----|------|-------|----------------|-----|----|
| 1  | 2    | 3   | 4   | 5   | 6   | 7   | 8    | 9     | 10             | 30  | 40 |
| I  | II   | III | IV  | V   | VI  | VII | VIII | IX    | X              | XXX | XL |
| 50 | 80   | 90  | 100 | 200 | 300 | 400 | 500  | 1,000 | 1,000,000      |     |    |
| L  | LXXX | XC  | C   | CC  | CCC | CD  | D    | M     | $\overline{M}$ |     |    |

In order to write several thousands, the symbol M is usually repeated.

\* \* \* \* \*

## THE USE OF SUBTRACTION

Only one symbol is subtracted in writing Roman numerals. We would not write 80 as ~~XXC~~ because we would be subtracting more than one ten.

|             |              |               |
|-------------|--------------|---------------|
| IV equals 4 | XL equals 40 | CD equals 400 |
| IX equals 9 | XC equals 90 | CM equals 900 |

As we write Roman numerals now, these are the only ones that we use subtraction for. Remember that we subtract to write the numbers with fours, (4, 40, 400) and the ones with nines (9, 90, 9000).

the first of these is the fact that the  
 the second is the fact that the  
 the third is the fact that the  
 the fourth is the fact that the  
 the fifth is the fact that the  
 the sixth is the fact that the  
 the seventh is the fact that the  
 the eighth is the fact that the  
 the ninth is the fact that the  
 the tenth is the fact that the

the eleventh is the fact that the  
 the twelfth is the fact that the  
 the thirteenth is the fact that the  
 the fourteenth is the fact that the  
 the fifteenth is the fact that the  
 the sixteenth is the fact that the  
 the seventeenth is the fact that the  
 the eighteenth is the fact that the  
 the nineteenth is the fact that the  
 the twentieth is the fact that the

the twenty-first is the fact that the  
 the twenty-second is the fact that the  
 the twenty-third is the fact that the  
 the twenty-fourth is the fact that the  
 the twenty-fifth is the fact that the  
 the twenty-sixth is the fact that the  
 the twenty-seventh is the fact that the  
 the twenty-eighth is the fact that the  
 the twenty-ninth is the fact that the  
 the thirtieth is the fact that the

the thirty-first is the fact that the  
 the thirty-second is the fact that the  
 the thirty-third is the fact that the  
 the thirty-fourth is the fact that the  
 the thirty-fifth is the fact that the  
 the thirty-sixth is the fact that the  
 the thirty-seventh is the fact that the  
 the thirty-eighth is the fact that the  
 the thirty-ninth is the fact that the  
 the fortieth is the fact that the

the forty-first is the fact that the  
 the forty-second is the fact that the  
 the forty-third is the fact that the  
 the forty-fourth is the fact that the  
 the forty-fifth is the fact that the  
 the forty-sixth is the fact that the  
 the forty-seventh is the fact that the  
 the forty-eighth is the fact that the  
 the forty-ninth is the fact that the  
 the fiftieth is the fact that the

the fifty-first is the fact that the  
 the fifty-second is the fact that the  
 the fifty-third is the fact that the  
 the fifty-fourth is the fact that the  
 the fifty-fifth is the fact that the  
 the fifty-sixth is the fact that the  
 the fifty-seventh is the fact that the  
 the fifty-eighth is the fact that the  
 the fifty-ninth is the fact that the  
 the sixtieth is the fact that the

the sixty-first is the fact that the  
 the sixty-second is the fact that the  
 the sixty-third is the fact that the  
 the sixty-fourth is the fact that the  
 the sixty-fifth is the fact that the  
 the sixty-sixth is the fact that the  
 the sixty-seventh is the fact that the  
 the sixty-eighth is the fact that the  
 the sixty-ninth is the fact that the  
 the seventieth is the fact that the

the seventy-first is the fact that the  
 the seventy-second is the fact that the  
 the seventy-third is the fact that the  
 the seventy-fourth is the fact that the  
 the seventy-fifth is the fact that the  
 the seventy-sixth is the fact that the  
 the seventy-seventh is the fact that the  
 the seventy-eighth is the fact that the  
 the seventy-ninth is the fact that the  
 the eightieth is the fact that the

## THE USE OF REPETITION

The only symbols that are repeated are:

I equal to 1

C equal to 100

X equal to 10

M equal to 1,000

Remember that with the exception of M, which stands for 1,000, these are only repeated one, two, or three times, never four.

II equals 2

CC equals 200

III equals 3

CCC equals 300

XX equals 20

MMMMM equals 6,000

\* \* \* \* \*

## THE USE OF MULTIPLICATION

In very early times, large numbers were not used a great deal. The Roman numerals that were written about the beginning of the Christian era were quite different than ours and very irregular. When it became necessary to express larger numbers, a bar,  $\overline{\quad}$ , was drawn over the number to multiply it by a thousand. This was similar to the Greek M for expressing larger numbers.

$\overline{\text{XXXIV}}$  equals 30 thousand plus 14, or 30,014

$\overline{\text{XX}}$  equals 20 thousand or, 20,000

$\overline{\text{MMD}}$  equals 2 million plus 500 thousand or 2,500,000. The bar over the two M's makes each of them one thousand thousand.

\* \* \* \* \*

## TWO HANDY RULES

- I. A smaller number after a larger number is added to the larger number.
- II. A smaller number before a larger number is subtracted from the larger number.

1.  $\mathcal{H}^1$  is a linear space.

2.  $\mathcal{H}^1$  is a linear space.

3.  $\mathcal{H}^1$  is a linear space.

4.  $\mathcal{H}^1$  is a linear space.

5.  $\mathcal{H}^1$  is a linear space.

6.  $\mathcal{H}^1$  is a linear space.

7.  $\mathcal{H}^1$  is a linear space.

8.  $\mathcal{H}^1$  is a linear space.

9.  $\mathcal{H}^1$  is a linear space.

10.  $\mathcal{H}^1$  is a linear space.

11.  $\mathcal{H}^1$  is a linear space.

12.  $\mathcal{H}^1$  is a linear space.

13.  $\mathcal{H}^1$  is a linear space.

14.  $\mathcal{H}^1$  is a linear space.

15.  $\mathcal{H}^1$  is a linear space.

16.  $\mathcal{H}^1$  is a linear space.

17.  $\mathcal{H}^1$  is a linear space.

18.  $\mathcal{H}^1$  is a linear space.

19.  $\mathcal{H}^1$  is a linear space.

20.  $\mathcal{H}^1$  is a linear space.

21.  $\mathcal{H}^1$  is a linear space.

22.  $\mathcal{H}^1$  is a linear space.

23.  $\mathcal{H}^1$  is a linear space.

24.  $\mathcal{H}^1$  is a linear space.

## BASE SEVEN

We do not give every separate number a separate name. Instead, we count in groups of numbers. This makes it possible to count to an infinite number with the use of only ten symbols. We call this group of numbers a model group or the base of the number system. This may be any size.

For the most part, we use a model group, or base, of ten. Many times we need to have a mathematical system based on some other model group, or base, such as 12, 7, 60, etc. We have used some of these model groups or bases for a long time without realizing it.

We can use a base of 12 in figuring linear measurements. If we think of these as a base of 12, it makes the addition and subtraction much more simple.

$$\begin{array}{r} 2 \text{ feet} \quad 8 \text{ inches} \\ +3 \text{ feet} \quad 6 \text{ inches} \\ \hline 5 \text{ feet} \quad 14 \text{ inches} \\ +1 \text{ foot} \quad -12 \text{ inches} \\ \hline 6 \text{ feet} \quad 2 \text{ inches} \end{array}$$

(removing 1 model group of 12  
and "carrying" it)

or

$$\begin{array}{r} 5 \text{ feet} \quad 2 \text{ inches} \\ -2 \text{ feet} \quad 6 \text{ inches} \\ \hline 4 \text{ feet} \quad 12+2 \text{ inches} \\ -2 \text{ feet} \quad 6 \text{ inches} \\ \hline 2 \text{ feet} \quad 8 \text{ inches} \end{array}$$

("borrow", or regroup, 5 feet  
2 inches into 4 feet 14 inches)  
(subtraction can now be completed)

In figuring time, we really are using a base of 60.

$$\begin{array}{r} 4 \text{ hours} \quad 50 \text{ minutes} \\ +2 \text{ hours} \quad 20 \text{ minutes} \\ \hline 6 \text{ hours} \quad 70 \text{ minutes} \\ +1 \text{ hour} \quad -60 \text{ minutes} \\ \hline 7 \text{ hours} \quad 10 \text{ minutes} \end{array}$$

(removing 1 model group of 60  
minutes and "carrying" it as  
1 hour)



The same operation may be performed for subtraction. We would regroup or "borrow" 1 model group of 60.

|                |                   |  |
|----------------|-------------------|--|
| 4 hours        | 20 minutes        |  |
| <u>-1 hour</u> | <u>40 minutes</u> |  |
| 3 hours        | 60+20 minutes     |  |
| <u>-1 hour</u> | <u>40 minutes</u> |  |
| 2 hours        | 40 minutes        |  |

(regroup 4 hours 20 minutes  
to 3 hours 60+20 minutes)

Science and industry find new uses for bases other than 10. It is sometimes desirable to organize a system of arithmetic to fit a particular situation. In order to be able to symbolize and to understand another number system using a different base or model group, it is often well to first learn to count in the system. It is extremely important to learn to count, and to understand counting, in any system because counting is the basis for the processes of multiplication, division, addition, and subtraction. A comparison with our familiar Base Ten will make this evident.

First, consider the number of symbols needed to write the system. The model group, or base, to be used dictates how many symbols will compose the number system. We have nine digits and a zero to write all our numbers in base 10: 1, 2, 3, 4, 5, 6, 7, 8, 9, and a zero, 0. We can do this by using place value. It would be almost impossible to use a different number of symbols with a base of 10 and place value as we use it. We give numbers two values to do this: (1) an amount which the number symbolizes (how many), and (2) an amount which it has because of its place.

If we count, we can see how this functions.

Count: "1, 2, 3, 4, 5, 6, 7, 8, 9 . . . ?????"

When we get to nine, we are forced to use place value simply because we have no more symbols to use. Our only alternative would be to invent more symbols. Therefore, we write one, zero (10) and call it ten. Ten is the name of this place. It is the size of the model group.





The 1 now, because of its place in the numeral, has a value of 1 ten, rather than just 1. Or, expressed in another way, it is equal to  $1 \times 10$  plus  $0 \times 1$ . We would write this more conveniently as  $1(10) + 0(1)$ , with the understanding that the parenthesis indicates that ten is multiplied by 1. Were there a 2 in this place, as in the numeral 20, we would show this as  $2(10) + 0(1)$ . This indicates that the second place, or tens place, is now multiplied by 2 and that 0 ones are added. We simply call this number "twenty" and, because of learning and habit, it means two tens to us. It is unfortunate that we have this habit, because it tends to keep us from understanding what the number really is, and how the system actually functions.

Now, if instead of counting in groups of 10, we change the size of the group and count in groups of 7, we have changed the base, or model group, of the system. We now have, and need, only six digits and a zero. That is all that we can use with a base of 7. To understand counting we need to recall that the values of each place in place value are expressed as powers of the base. That is, we have ones, tens, hundreds ( $10 \times 10$ ), thousands ( $10 \times 10 \times 10$ ) in Base Ten. Now we have: ones, sevens ( $7 \times 1$ ), forty-nines ( $7 \times 7$ ), three-hundred-forty-threes ( $7 \times 7 \times 7$ ), etc. The only difference is that we are accustomed to the terms for place value in a base of 10 and have special names for them. In a base of seven, we can either make up new names for these places or use terms that indicate their value in base 10. Making up new names necessitates extra memory work, so it is easier simply to think of them in the terms we know.

Again, counting in the base 7 system will show how the system functions.

Count: "1, 2, 3, 4, 5, 6, . . . ????"



We cannot use 7 because seven is our model group and is now written 1 group of sevens and no ones, or 10. In order to avoid mixing up this system with base 10, in which we call this place "ten", we will call this "one-oh". We might also call it 1 seven, but this would tend to be confusing. We have now changed the value of the second place to 7 instead of ten. We could show this by indicating the value for the places as we did before: 10 in base 7 is equal to  $1(7) + 3(1)$ ; in base 10 it is equal to  $1(10) + 0(1)$ .

It helps to understand a number system if you count in the system and write out the value for each place. This is true of base ten also, and helps for real understanding of the system and how it functions. First, however, we need to be sure of the place values. A comparison with base ten will help to understand base 7. If need be, refer back to the section on place value to review how the value for each place is established by the powers of the base. A comparison of the two bases is of some help in understanding this.

|  |                                 |                             |                       |      |
|--|---------------------------------|-----------------------------|-----------------------|------|
| ten-thousands<br>(10,000 = 10 x 1,000)       | thousands<br>(1,000 = 10 x 100) | hundreds<br>(100 = 10 x 10) | tens<br>(10 = 10 x 1) | ones |
| <hr/>  |                                 |                             |                       |      |
| Base Ten                                     |                                 |                             |                       |      |
| three-hundred-forty-threes<br>(343 = 7 x 49) | forty-nines<br>(49 = 7 x 7)     | sevens<br>(7 = 7 x 1)       | ones                  |      |
| <hr/>  |                                 |                             |                       |      |
| Base Seven                                   |                                 |                             |                       |      |

The first part of the document is a list of names and dates, which appears to be a record of some kind. The names are written in a cursive script, and the dates are in a more formal, printed style. The list is organized into columns, with names in the first column and dates in the second.

The second part of the document is a series of paragraphs of text, also written in cursive. The text is somewhat difficult to read due to the handwriting, but it appears to be a narrative or a report of some kind. The paragraphs are separated by small gaps, and the text is written in a consistent style throughout.

The third part of the document is a list of names and dates, similar to the first part. This list is also organized into columns, with names in the first column and dates in the second. The handwriting is consistent with the rest of the document.

The fourth part of the document is a series of paragraphs of text, similar to the second part. The text is written in cursive and appears to be a continuation of the narrative or report from the second part. The paragraphs are separated by small gaps, and the text is written in a consistent style throughout.

The fifth part of the document is a list of names and dates, similar to the first and third parts. This list is also organized into columns, with names in the first column and dates in the second. The handwriting is consistent with the rest of the document.

The bottom part of the document contains a series of lines of text, which appear to be a list or a table of some kind. The text is written in a cursive script, and the lines are organized into columns. The handwriting is consistent with the rest of the document.

The value for each place is established exactly the same in Base Ten as in Base Seven. If we think of the value for each place as a power of the base, it is much simpler. This means the number of times the base is multiplied by itself.  $10^2$  means  $10 \times 10$ ;  $7^2$  would mean  $7 \times 7$ . Comparing this to base ten, we would have a chart like this:

---

| BASE 10  |        | BASE 7   |       |
|--|--------|--|-------|
| ones   | $10^0$ | ones   | $7^0$ |
| tens<br>$10 \times 1$ , or (ten ones)  | $10^1$ | sevens<br>$7 \times 1$ , or (seven ones)   | $7^1$ |
| hundreds<br>$10 \times 10$ , or (ten tens)   | $10^2$ | forty-nines<br>$7 \times 7$ , or (seven sevens)  | $7^2$ |
| thousands<br>$10 \times 10 \times 10$ , or (ten hundreds)                                  | $10^3$ | three-hundred-forty-threes<br>$7 \times 7 \times 7$ , or (seven forty-nines)   | $7^3$ |
| ten-thousands<br>$10 \times 10 \times 10 \times 10$ , or (ten thousands)                   | $10^4$ | two-thousand-four-hundred-ones<br>$7 \times 7 \times 7 \times 7$ , or (seven three-hundred-forty-threes)                     | $7^4$ |
| hundred-thousands<br>$10 \times 10 \times 10 \times 10 \times 10$ , or (ten ten0thousands) | $10^5$ | sixteen-thousand-eight-hundred-sevens<br>$7 \times 7 \times 7 \times 7 \times 7$ , or (seven two-thousand-four-hundred-ones) | $7^5$ |

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21

5

1

11

1000

$P = \text{significance level} = .05$



1. THEORY

9161 512 1000

1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 2679, 2680, 26

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2019年12月31日 12:00 截止

A COMPARISON OF PLACE VALUE IN BASE  
TEN AND SEVEN

|             |           |          |          |          |          |          |
|-------------|-----------|----------|----------|----------|----------|----------|
| $10^6$      | $10^5$    | $10^4$   | $10^3$   | $10^2$   | $10^1$   | $10^0$   |
| 1,000,000's | 100,000's | 10,000's | 1,000's  | 100's    | 10's     | 1's      |
| $7^6$       | $7^5$     | $7^4$    | $7^3$    | $7^2$    | $7^1$    | $7^0$    |
| 117,649's   | 16,807's  | 2,401's  | 343's    | 49's     | 7's      | 1's      |
| <hr/>       |           |          |          |          |          |          |
|             | <u>3</u>  | <u>2</u> | <u>6</u> | <u>0</u> | <u>2</u> | <u>2</u> |

Imagine that we have the numeral 326,022. Each digit of this numeral has a value for the place in which it stands. The places are indicated in the chart above the numeral. For instance, hundred's place in a base of ten is only equal to forty-nine's place in a base of seven. This is reasonable when we think that the base is smaller. We would show the value of the above number in this way:

For base ten:

$$326,022 = 3(100,000) + 2(10,000) + 6(1,000) + 0(100) + 2(10) + 2(1)$$

We read this number "three hundred, twenty-six thousand, twenty-two". We do not have such names as thousand, etc., for the base of seven. That is we do not have a name that means to us: "thre-hundred-forty-threes", like "thousand" means ten tens. Therefore, we will use base ten number names for the places in Base Seven. This will help to understand the value for these places.

For base seven:

$$326,022 = 3(16,807) + 2(2,401) + 6(343) + 0(49) + 2(7) + 2(1)$$

We can read this number simply "three, two, six, zero, two, two," since we do not have place value names for base seven. We would write it with a little subscript indicating the base like this: 326,022<sub>7</sub>.

The value for this numeral in base seven is only 57,297 in Base ten, since the worth of each place is less in Base Seven. This is due to the fact that the model group is smaller.

Since counting may be considered as the basis of arithmetic, it would help to understand another model group if you write out a chart like the one following. Fill in the missing numbers indicated by . . ., and show the value for each place. Go as far as 100 at least in Base 10. This will make a counting chart that can be used later to construct addition and multiplication tables, with which it is possible to work problems in the new base.

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| BASE 10                          | BASE 7                                |
|----------------------------------|---------------------------------------|
| 1                                | 1                                     |
| 2                                | 2                                     |
| 3                                | 3                                     |
| 4                                | 4                                     |
| 5                                | 5                                     |
| 6                                | 6                                     |
| 7                                | 10 1 seven + 0 ones                   |
| 8                                | 11 1 seven + 1 one                    |
| 9                                | 12 1 seven + 2 ones                   |
| 10 1 ten + 0 ones                | 13 1 seven + 3 ones                   |
| .. ..                            | .. ..                                 |
| 14 1 ten + 4 ones                | 20 2 sevens + 0 ones                  |
| 15 1 ten + 5 ones                | 21 2 sevens + 1 one                   |
| .. ..                            | .. ..                                 |
| 21 2 tens + 1 one                | 30 3 sevens + 0 ones                  |
| 22 2 tens + 2 ones               | 31 3 sevens + 1 one                   |
| .. ..                            | .. ..                                 |
| 41 4 tens + 1 one                | 56 5 sevens + 6 ones                  |
| 42 4 tens + 2 ones               | 60 6 sevens + 0 ones                  |
| 43 4 tens + 3 ones               | 61 6 sevens + 1 one                   |
| .. ..                            | .. ..                                 |
| 48 4 tens + 8 ones               | 66 6 sevens + 6 ones                  |
| 49 4 tens + 9 ones               | 100 1 forty-nine + 0 sevens + 0 ones  |
| 50 5 tens + 0 ones               | 101 1 forth-nine + 0 sevens + 1 one   |
| .. ..                            | .. ..                                 |
| 101 1 hundred + 0 tens + 1 one   | 203 2 forty-nines + 0 sevens + 3 ones |
| 342 3 hundreds + 4 tens + 2 ones | 666 6 forty-nines + 6 sevens + 6 ones |
| 343 3 hundreds + 4 tens + 3 ones | 1000 1 three-hundred-forty-three +    |
| 344 3 hundreds + 4 tens + 4 ones | 0 forty-nines + 0 sevens + 0 ones     |
| .. ..                            | .. ..                                 |
| 2400 2 thousands + 4 hundreds +  | 6666 6 three-hundred-forty-threes +   |
| 0 tens + 0 ones                  | 6 forty-nines + 6 sevens + 6 ones     |
| 2401 2 thousands + 4 hundreds +  | 10000 1 two-thousand-four-hundred-one |
| 0 tens + 1 one                   | + 0 three-hundred-forty-three +       |
|                                  | 0 forty-nines + 0 ones                |

Reading across on this chart it is possible to see what digits would represent the same number in each base. For example: 42 in Base Ten (4 tens + 2 ones) is written 60 in Base Seven (6 sevens and 0 ones); 43 in Base Ten is written 61 in Base Seven. (6 sevens would give you 42 in Base Ten plus one more would make 43.)



The tables on the next page are constructed to show multiplication and addition in Base Seven. It would help to make a table of your own and then to compare it with these. Refer to the counting chart that you made previously.

For example, in constructing the chart, think:  $6 \times 6 = 36$  in Base Ten. Refer to the counting chart, 36 is written 51 in Base Seven, so write 51 in the space for the answer to  $6 \times 6$ . Do this for all the spaces. Remember that since you are working in a Base of Seven, you will be able to use only 6 digits on your chart. The seventh digit would become 10 in Base Seven, or 1 group of 7 ones plus 0 ones.

It is difficult to start thinking in another base when you first start working in it; although, if we thought as carefully in Base Ten as we are forced to think in Base Seven, we would probably understand arithmetic much better. For beginning, think the answer to problems in Base Ten and convert to Base Seven. Do this for a while, and you will probably find yourself thinking in Base Seven.

I.  $4_7 \times 5_7 = 26_7$

Think:  $4 \times 5 = 20$  in Base Ten. (Four and five are written the same in Base Ten and Base Seven.) But twenty in Base Ten is written 20, in Base Seven 26. This is because two tens are the same as two sevens plus six ones. You are working in groups of sevens instead of tens. Refer to your counting chart for the answers.

II.  $4_7 \times 6_7 = 33_7$

Think:  $4 \times 6 = 24$  in Base Ten, but this is written 33 in Base Seven. Write 33 in the space for the answer to  $4 \times 6$ . (This gives you 3 sevens plus 3 ones which equals 24 in Base Ten.)

III.  $4_7 \times 10_7 = 40_7$

Think: This is the same as  $4 \times 7$  in Base Ten, but seven in Base Seven is written 10 (or 1 group of seven ones). The problem is done exactly as in

1. The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation  $f(x) = \int_0^x f(t) dt$ . It is shown that  $f(x)$  is a constant function, i.e.,  $f(x) = C$  for all  $x$ .

2. The second part of the paper is devoted to the study of the properties of the function  $g(x)$  defined by the equation  $g(x) = \int_0^x g(t) dt$ . It is shown that  $g(x)$  is a constant function, i.e.,  $g(x) = C$  for all  $x$ .

3. The third part of the paper is devoted to the study of the properties of the function  $h(x)$  defined by the equation  $h(x) = \int_0^x h(t) dt$ . It is shown that  $h(x)$  is a constant function, i.e.,  $h(x) = C$  for all  $x$ .

4. The fourth part of the paper is devoted to the study of the properties of the function  $k(x)$  defined by the equation  $k(x) = \int_0^x k(t) dt$ . It is shown that  $k(x)$  is a constant function, i.e.,  $k(x) = C$  for all  $x$ .

5. The fifth part of the paper is devoted to the study of the properties of the function  $l(x)$  defined by the equation  $l(x) = \int_0^x l(t) dt$ . It is shown that  $l(x)$  is a constant function, i.e.,  $l(x) = C$  for all  $x$ .

6. The sixth part of the paper is devoted to the study of the properties of the function  $m(x)$  defined by the equation  $m(x) = \int_0^x m(t) dt$ . It is shown that  $m(x)$  is a constant function, i.e.,  $m(x) = C$  for all  $x$ .

7. The seventh part of the paper is devoted to the study of the properties of the function  $n(x)$  defined by the equation  $n(x) = \int_0^x n(t) dt$ . It is shown that  $n(x)$  is a constant function, i.e.,  $n(x) = C$  for all  $x$ .

8. The eighth part of the paper is devoted to the study of the properties of the function  $o(x)$  defined by the equation  $o(x) = \int_0^x o(t) dt$ . It is shown that  $o(x)$  is a constant function, i.e.,  $o(x) = C$  for all  $x$ .

Base Ten, but the 1's now stand for groups of sevens instead of tens.

The answer  $40_7$  is equal to 4 sevens or 28 in Base 10.

I.  $10 \times 10 = 100$  in Base Ten, and  $10_7 \times 10_7 = 100_7$  in Base Seven, BUT the ones in the second example stand for groups of sevens and forty-nines instead of tens and hundreds as in the first one.

ADDITION TABLE IN BASE SEVEN

| + | 0 | 1  | 2  | 3  | 4  | 5  | 6  |
|---|---|----|----|----|----|----|----|
| 0 | 0 | 1  | 2  | 3  | 4  | 5  | 6  |
| 1 | 1 | 2  | 3  | 4  | 5  | 6  | 10 |
| 2 | 2 | 3  | 4  | 5  | 6  | 10 | 11 |
| 3 | 3 | 4  | 5  | 6  | 10 | 11 | 12 |
| 4 | 4 | 5  | 6  | 10 | 11 | 12 | 13 |
| 5 | 5 | 6  | 10 | 11 | 12 | 13 | 14 |
| 6 | 6 | 10 | 11 | 12 | 13 | 14 | 15 |

MULTIPLICATION TABLE IN BASE SEVEN

| x | 0 | 1 | 2  | 3  | 4  | 5  | 6  |
|---|---|---|----|----|----|----|----|
| 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  |
| 1 | 0 | 1 | 2  | 3  | 4  | 5  | 6  |
| 2 | 0 | 2 | 4  | 6  | 11 | 13 | 15 |
| 3 | 0 | 3 | 6  | 12 | 15 | 21 | 24 |
| 4 | 0 | 4 | 11 | 15 | 22 | 26 | 33 |
| 5 | 0 | 5 | 13 | 21 | 26 | 34 | 42 |
| 6 | 0 | 6 | 15 | 24 | 33 | 42 | 51 |



It is unfortunate that it is often possible to learn to do arithmetic problems by rote with little or no understanding of the processes involved. Doing some simple problems in another number system helps to understand how a system really functions, since it is impossible to do them without this knowledge. The following problems are worked in both Base Ten and Base Seven so that you can see how a 7 would be carried as a model group in the same way that a 10 is. Learning to manipulate a model group of 7 will make plain how the base of a system functions.

BASE TEN

$$\begin{array}{r} 1. \quad 4 \quad 4(1) \\ +2 \quad 2(1) \\ \hline 6 \quad 6(1) \end{array}$$

BASE SEVEN

$$\begin{array}{r} 4_7 \quad 4(1) \\ +2_7 \quad 2(1) \\ \hline 6_7 \quad 6(1) \end{array}$$

Since there is no digit with a value over 7, the problem is the same as in Base Ten.

$$\begin{array}{r} 2. \quad 4 \quad 4(1) \\ +3 \quad 3(1) \\ \hline 7 \quad 7(1) \end{array}$$

$$\begin{array}{r} 4_7 \quad 4(1) \\ +3_7 \quad 3(1) \\ \hline 10_7 \quad 1(7) + 0(1) \end{array}$$

Now we are forced to "carry" in the Base Seven problem because 7 is the model group. The two answers have the same value, but they are in different bases.

$$\begin{array}{r} 3. \quad 3 \quad 3(1) \\ +8 \quad 8(1) \\ \hline 11 \quad 1(10) + 1(1) \end{array}$$

$$\begin{array}{r} 3_7 \quad 3(1) \\ +11_7 \quad 1(7) + 1(1) \\ \hline 14_7 \quad 1(7) + 4(1) \end{array}$$

In the problem  $3 + 8 = 11$ , we are forced to "carry" in the Base Ten situation, but not in Base Seven, we do not have a group of 7 to carry. The actual value of the two answers is, however, the same.

$$\begin{array}{r} 4. \quad 23 \quad 2(10) + 3(1) \\ +23 \quad 2(10) + 3(1) \\ \hline 46 \quad 4(10) + 6(1) \end{array}$$

$$\begin{array}{r} 32_7 \quad 3(7) + 2(1) \\ +32_7 \quad 3(7) + 2(1) \\ \hline 64_7 \quad 6(7) + 4(1) \end{array}$$

The process of addition is the same, except that the model group in one is ten and the other is seven.

$$\begin{array}{r} 5. \quad 19 \quad 1(10) + 9(1) \\ +24 \quad 2(10) + 4(1) \\ \hline 43 \quad 4(10) + 3(1) \end{array}$$

$$\begin{array}{r} 25_7 \quad 2(7) + 5(1) \\ +33_7 \quad 3(7) + 3(1) \\ \hline 61_7 \quad 6(7) + 1(1) \end{array}$$

In the case of the problem:  $19 + 24 = 43$  in Base ten, we have to "carry" 1 ten. Then the problem is converted into Base Seven, we still have to "carry" but in this case, we are working with the new base of seven, so this is our model group. The answers have different notation, but the same value.

• 1. The first part of the paper is devoted to a study of the properties of the function  $f(x)$  defined by the equation

$$f(x) = \int_0^x \frac{1}{1+t^2} dt$$

• 2. It is shown that the function  $f(x)$  is continuous and differentiable for all values of  $x$ . The derivative of  $f(x)$  is given by the formula

$$f'(x) = \frac{1}{1+x^2}$$

• 3. The function  $f(x)$  is also shown to be bounded for all values of  $x$ . The maximum value of  $f(x)$  is found to be  $\frac{\pi}{2}$ .

• 4. The function  $f(x)$  is also shown to be concave down for all values of  $x$ . The maximum value of  $f(x)$  is found to be  $\frac{\pi}{2}$ .

• 5. The function  $f(x)$  is also shown to be concave up for all values of  $x$ . The maximum value of  $f(x)$  is found to be  $\frac{\pi}{2}$ .

• 6. The function  $f(x)$  is also shown to be concave down for all values of  $x$ . The maximum value of  $f(x)$  is found to be  $\frac{\pi}{2}$ .

• 7. The function  $f(x)$  is also shown to be concave up for all values of  $x$ . The maximum value of  $f(x)$  is found to be  $\frac{\pi}{2}$ .

• 8. The function  $f(x)$  is also shown to be concave down for all values of  $x$ . The maximum value of  $f(x)$  is found to be  $\frac{\pi}{2}$ .

• 9. The function  $f(x)$  is also shown to be concave up for all values of  $x$ . The maximum value of  $f(x)$  is found to be  $\frac{\pi}{2}$ .

• 10. The function  $f(x)$  is also shown to be concave down for all values of  $x$ . The maximum value of  $f(x)$  is found to be  $\frac{\pi}{2}$ .

• 11. The function  $f(x)$  is also shown to be concave up for all values of  $x$ . The maximum value of  $f(x)$  is found to be  $\frac{\pi}{2}$ .



## HOW TO WORK A PROBLEM IN BASE SEVEN

ADDITIONBASE TEN

$$\begin{array}{r} 34 \\ +25 \\ \hline \end{array}$$

$$\begin{array}{r} 59 \\ \hline \end{array}$$

$$\begin{array}{l} 3(10) + 4(1) \\ 2(10) + 5(1) \end{array}$$

$$5(10) + 9(1)$$

BASE SEVEN

$$\begin{array}{r} 46_7 \\ +34_7 \\ \hline \end{array}$$

$$\begin{array}{r} 113_7 \\ \hline \end{array}$$

$$\begin{array}{l} 4(7) + 6(1) \\ 3(7) + 4(1) \end{array}$$

$$1(49) + 1(7) + 3(1)$$

Think:  $6 + 4 = 10$ . Ten in Base Seven is written on a counting chart as 13. (Call this "one three" so that you do not mix it up with Base Ten "thirteen"). Put down 3 and "carry" 1 seven,  $4 + 3 = 7$ . This however, really is 4 sevens plus 3 sevens are 7 sevens, plus the one that you are "carrying" makes 8 sevens. Eight on a counting chart is written 11 (call this "one one" so as not to mix it up with Base Ten "eleven".) Put down 11. This gives you the answer, using place value of:  $1(49) + 1(7) + 3(1)$ . It is the same process you would use in base ten, but the model group is 7.

$$\begin{array}{r} 342 \\ + 66 \\ \hline 408 \end{array}$$

$$\begin{array}{l} 3(100) + 4(10) + 2(1) \\ 6(10) + 6(1) \\ 4(100) + 0(10) + 8(1) \end{array}$$

$$\begin{array}{r} 666_7 \\ +123_7 \\ \hline 1122_7 \end{array}$$

$$\begin{array}{l} 6(49) + 6(7) + 6(1) \\ 1(49) + 2(7) + 3(1) \\ 1(343) + 1(49) + 2(7) + 2(1) \end{array}$$

THINK:  $6 + 3 = 9$ , but 9 in Base Seven is written one two (12). Put down 2 and "carry" 1 seven.  $6 + 2 = 8$ . Eight is written one one (11), add the 1 seven that you are "carrying". This makes one two (12). Put down 2 and "carry" 1. In this case, it is 1 forty-nine.  $6 + 1 = 7$ . Seven in Base Seven is written one oh (10). Add the one forty-nine that you are "carrying". This makes one one (11). The answer is 1,122<sub>7</sub> or  $1(343) + 1(49) + 2(7) + 2(1)$  which would equal 408.

$$\begin{aligned} & \frac{1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}} \cdot \frac{1}{\sqrt{1-x^2}} = \frac{1}{1-x^2} \\ & \frac{1}{1-x^2} = \frac{1}{(1-x)(1+x)} = \frac{1}{1-x} \cdot \frac{1}{1+x} \end{aligned}$$

$$\begin{aligned} & \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{for } |x| < 1 \\ & \frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n \quad \text{for } |x| < 1 \\ & \frac{1}{1-x^2} = \left( \sum_{n=0}^{\infty} x^n \right) \left( \sum_{n=0}^{\infty} (-1)^n x^n \right) \\ & = \sum_{n=0}^{\infty} \left( \sum_{k=0}^n (-1)^k \right) x^n \\ & = \sum_{n=0}^{\infty} (-1)^n x^n \quad \text{for } |x| < 1 \\ & \frac{1}{\sqrt{1-x^2}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} \binom{2n}{n} x^n \quad \text{for } |x| < 1 \end{aligned}$$

$$\begin{aligned} & \frac{1}{\sqrt{1-x^2}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} \binom{2n}{n} x^n \\ & \frac{1}{\sqrt{1-x^2}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} \frac{(2n)!}{n!n!} x^n \end{aligned}$$

$$\begin{aligned} & \frac{1}{\sqrt{1-x^2}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} \frac{(2n)!}{n!n!} x^n \\ & \frac{1}{\sqrt{1-x^2}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} \frac{(2n)!}{n!n!} x^n \\ & \frac{1}{\sqrt{1-x^2}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} \frac{(2n)!}{n!n!} x^n \\ & \frac{1}{\sqrt{1-x^2}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} \frac{(2n)!}{n!n!} x^n \end{aligned}$$

## ADDITION IN BASE SEVEN

These are a series of problems of increasing difficulty in Base Seven. They could be mimeographed and used with students either for enrichment or understanding. They might be presented in such a way that each student could go as far as possible in the series. The problems have been worked in Base Ten first and the answers indicated. This should make for more meaning than working in Base Seven alone with no way of comparing with Base Ten. It is not necessary to indicate the base with a subscript when it is understood, therefore, these are omitted. Encourage students to look for the pattern of counting.

| <u>BASE TEN</u>   | <u>BASE SEVEN</u>   |
|---|---|
| 1. $\begin{array}{r} 5 \\ 5 \\ \hline 10 \end{array}$ 1(10) + 0(1)    | $\begin{array}{r} 5 \\ 5 \\ \hline 13 \end{array}$ 1(7) + 3(1)            |
| 2. $\begin{array}{r} 6 \\ 6 \\ \hline 12 \end{array}$ 1(10) + 2(1)    | $\begin{array}{r} 6 \\ 6 \\ \hline 15 \end{array}$ 1(7) + 5(1)            |
| 3. $\begin{array}{r} 7 \\ 7 \\ \hline 14 \end{array}$ 1(10) + 4(1)    | $\begin{array}{r} 10 \\ 10 \\ \hline 20 \end{array}$ 2(7) + 0(1)          |
| 4. $\begin{array}{r} 8 \\ 8 \\ \hline 16 \end{array}$ 1(10) + 6(1)    | $\begin{array}{r} 11 \\ 11 \\ \hline 22 \end{array}$ 2(7) + 2(1)          |
| 5. $\begin{array}{r} 9 \\ 9 \\ \hline 18 \end{array}$ 1(10) + 8(1)    | $\begin{array}{r} 12 \\ 12 \\ \hline 24 \end{array}$ 2(7) + 4(1)          |
| 6. $\begin{array}{r} 10 \\ 10 \\ \hline 20 \end{array}$ 2(10) + 0(1)  | $\begin{array}{r} 13 \\ 13 \\ \hline 26 \end{array}$ 2(7) + 6(1)          |
| 7. $\begin{array}{r} 11 \\ 11 \\ \hline 22 \end{array}$ 2(10) + 2(1)  | $\begin{array}{r} 14 \\ 14 \\ \hline 31 \end{array}$ 3(7) + 1(1)          |
| 8. $\begin{array}{r} 12 \\ 12 \\ \hline 24 \end{array}$ 2(10) + 4(1)  | $\begin{array}{r} 15 \\ 15 \\ \hline 33 \end{array}$ 3(7) + 3(1)          |
| 9. $\begin{array}{r} 13 \\ 13 \\ \hline 26 \end{array}$ 2(10) + 6(1)  | $\begin{array}{r} 16 \\ 16 \\ \hline 35 \end{array}$ 3(7) + 5(1)          |
| 10. $\begin{array}{r} 14 \\ 14 \\ \hline 28 \end{array}$ 2(10) + 8(1) | $\begin{array}{r} 20 \\ 20 \\ \hline 40 \end{array}$ 4(7) + 0(1)          |
| 11. $\begin{array}{r} 35 \\ 35 \\ \hline 70 \end{array}$ 7(10) + 0(1) | $\begin{array}{r} 50 \\ 50 \\ \hline 130 \end{array}$ 1(49) + 3(7) + 0(1) |

The following table shows the results of the experiments conducted on the 10th of May 1900. The first column gives the number of the experiment, the second column the number of the subject, the third column the number of the trial, the fourth column the number of the error, the fifth column the number of the correct answer, and the sixth column the number of the total number of trials.

|     |   |   |   |   |   |
|-----|---|---|---|---|---|
| 1   | 1 | 1 | 1 | 1 | 1 |
| 2   | 1 | 1 | 1 | 1 | 1 |
| 3   | 1 | 1 | 1 | 1 | 1 |
| 4   | 1 | 1 | 1 | 1 | 1 |
| 5   | 1 | 1 | 1 | 1 | 1 |
| 6   | 1 | 1 | 1 | 1 | 1 |
| 7   | 1 | 1 | 1 | 1 | 1 |
| 8   | 1 | 1 | 1 | 1 | 1 |
| 9   | 1 | 1 | 1 | 1 | 1 |
| 10  | 1 | 1 | 1 | 1 | 1 |
| 11  | 1 | 1 | 1 | 1 | 1 |
| 12  | 1 | 1 | 1 | 1 | 1 |
| 13  | 1 | 1 | 1 | 1 | 1 |
| 14  | 1 | 1 | 1 | 1 | 1 |
| 15  | 1 | 1 | 1 | 1 | 1 |
| 16  | 1 | 1 | 1 | 1 | 1 |
| 17  | 1 | 1 | 1 | 1 | 1 |
| 18  | 1 | 1 | 1 | 1 | 1 |
| 19  | 1 | 1 | 1 | 1 | 1 |
| 20  | 1 | 1 | 1 | 1 | 1 |
| 21  | 1 | 1 | 1 | 1 | 1 |
| 22  | 1 | 1 | 1 | 1 | 1 |
| 23  | 1 | 1 | 1 | 1 | 1 |
| 24  | 1 | 1 | 1 | 1 | 1 |
| 25  | 1 | 1 | 1 | 1 | 1 |
| 26  | 1 | 1 | 1 | 1 | 1 |
| 27  | 1 | 1 | 1 | 1 | 1 |
| 28  | 1 | 1 | 1 | 1 | 1 |
| 29  | 1 | 1 | 1 | 1 | 1 |
| 30  | 1 | 1 | 1 | 1 | 1 |
| 31  | 1 | 1 | 1 | 1 | 1 |
| 32  | 1 | 1 | 1 | 1 | 1 |
| 33  | 1 | 1 | 1 | 1 | 1 |
| 34  | 1 | 1 | 1 | 1 | 1 |
| 35  | 1 | 1 | 1 | 1 | 1 |
| 36  | 1 | 1 | 1 | 1 | 1 |
| 37  | 1 | 1 | 1 | 1 | 1 |
| 38  | 1 | 1 | 1 | 1 | 1 |
| 39  | 1 | 1 | 1 | 1 | 1 |
| 40  | 1 | 1 | 1 | 1 | 1 |
| 41  | 1 | 1 | 1 | 1 | 1 |
| 42  | 1 | 1 | 1 | 1 | 1 |
| 43  | 1 | 1 | 1 | 1 | 1 |
| 44  | 1 | 1 | 1 | 1 | 1 |
| 45  | 1 | 1 | 1 | 1 | 1 |
| 46  | 1 | 1 | 1 | 1 | 1 |
| 47  | 1 | 1 | 1 | 1 | 1 |
| 48  | 1 | 1 | 1 | 1 | 1 |
| 49  | 1 | 1 | 1 | 1 | 1 |
| 50  | 1 | 1 | 1 | 1 | 1 |
| 51  | 1 | 1 | 1 | 1 | 1 |
| 52  | 1 | 1 | 1 | 1 | 1 |
| 53  | 1 | 1 | 1 | 1 | 1 |
| 54  | 1 | 1 | 1 | 1 | 1 |
| 55  | 1 | 1 | 1 | 1 | 1 |
| 56  | 1 | 1 | 1 | 1 | 1 |
| 57  | 1 | 1 | 1 | 1 | 1 |
| 58  | 1 | 1 | 1 | 1 | 1 |
| 59  | 1 | 1 | 1 | 1 | 1 |
| 60  | 1 | 1 | 1 | 1 | 1 |
| 61  | 1 | 1 | 1 | 1 | 1 |
| 62  | 1 | 1 | 1 | 1 | 1 |
| 63  | 1 | 1 | 1 | 1 | 1 |
| 64  | 1 | 1 | 1 | 1 | 1 |
| 65  | 1 | 1 | 1 | 1 | 1 |
| 66  | 1 | 1 | 1 | 1 | 1 |
| 67  | 1 | 1 | 1 | 1 | 1 |
| 68  | 1 | 1 | 1 | 1 | 1 |
| 69  | 1 | 1 | 1 | 1 | 1 |
| 70  | 1 | 1 | 1 | 1 | 1 |
| 71  | 1 | 1 | 1 | 1 | 1 |
| 72  | 1 | 1 | 1 | 1 | 1 |
| 73  | 1 | 1 | 1 | 1 | 1 |
| 74  | 1 | 1 | 1 | 1 | 1 |
| 75  | 1 | 1 | 1 | 1 | 1 |
| 76  | 1 | 1 | 1 | 1 | 1 |
| 77  | 1 | 1 | 1 | 1 | 1 |
| 78  | 1 | 1 | 1 | 1 | 1 |
| 79  | 1 | 1 | 1 | 1 | 1 |
| 80  | 1 | 1 | 1 | 1 | 1 |
| 81  | 1 | 1 | 1 | 1 | 1 |
| 82  | 1 | 1 | 1 | 1 | 1 |
| 83  | 1 | 1 | 1 | 1 | 1 |
| 84  | 1 | 1 | 1 | 1 | 1 |
| 85  | 1 | 1 | 1 | 1 | 1 |
| 86  | 1 | 1 | 1 | 1 | 1 |
| 87  | 1 | 1 | 1 | 1 | 1 |
| 88  | 1 | 1 | 1 | 1 | 1 |
| 89  | 1 | 1 | 1 | 1 | 1 |
| 90  | 1 | 1 | 1 | 1 | 1 |
| 91  | 1 | 1 | 1 | 1 | 1 |
| 92  | 1 | 1 | 1 | 1 | 1 |
| 93  | 1 | 1 | 1 | 1 | 1 |
| 94  | 1 | 1 | 1 | 1 | 1 |
| 95  | 1 | 1 | 1 | 1 | 1 |
| 96  | 1 | 1 | 1 | 1 | 1 |
| 97  | 1 | 1 | 1 | 1 | 1 |
| 98  | 1 | 1 | 1 | 1 | 1 |
| 99  | 1 | 1 | 1 | 1 | 1 |
| 100 | 1 | 1 | 1 | 1 | 1 |

## HOW TO WORK MULTIPLICATION PROBLEMS IN BASE SEVEN

BASE TEN

$$1. \quad \begin{array}{r} 8 \\ \times 3 \\ \hline 24 \end{array} \quad 2(10) + 4(1) = 24$$

$$\begin{array}{r} 11 \\ \times 3 \\ \hline 33 \end{array} \quad 3(7) + 3(1) = 24$$

THINK: To convert 8 to Base Seven, eight equals 1 seven plus 1 one. Write one one (11), read it "one one". There is no regrouping necessary because the ones do not combine to an amount over the base of 7.

$$2. \quad \begin{array}{r} 15 \\ \times 4 \\ \hline 60 \end{array} \quad 6(10) + 0(1) = 60$$

$$\begin{array}{r} 21 \\ \times 4 \\ \hline 114 \end{array} \quad 1(49) + 1(7) + 4(1) = 60$$

THINK: Fifteen in Base Seven is equal to 2 sevens plus 1 one. Write 21. Read it "two one". To multiply, think:  $4 \times 1 = 4$ . Four in Base Seven is simply written 4 because it is not over the model group of 7. Write down 4.  $4 \times 2 = 8$ . Eight in Base Seven is equal to 1 seven plus 1 one. Write 11. Read it "one one". The answer is the same amount as in Base Ten. The notation is different.

$$3. \quad \begin{array}{r} 53 \\ \times 24 \\ \hline 106 \\ 1272 \\ \hline \end{array} \quad 1(1000) + 2(100) + 7(10) + 2(1) = 1272$$

$$\begin{array}{r} 104 \\ \times 33 \\ \hline 315 \\ 315 \\ \hline 3465 \end{array} \quad 3(343) + 4(49) + 6(7) + 5(1) = 1272$$

THINK: Base Ten 53 is written 104 in Base Seven because 53 has 1 forty-nine + 0 sevens + 4 ones. To multiply, think:  $3 \times 4 = 12$  in Base Ten. Twelve in Base Seven is written 15. Put down 5 and carry 1 seven.  $3 \times 0 = 0$ . Add the 1 seven. Write 1.  $3 \times 1 = 3$ . Put down 3. Do the same for the next line, and add.

$$4. \quad \begin{array}{r} 79 \\ \times 7 \\ \hline 553 \end{array} \quad 5(100) + 5(10) + 3(1) = 553$$

$$\begin{array}{r} 142 \\ \times 10 \\ \hline 1420 \end{array} \quad 1(343) + 4(49) + 2(7) + 0(1) = 553$$

THINK: In the base seven problem the model group is 7, so one zero stands for 1 seven. The process is the same as base ten, but the notation is different. The answer will have the same value.

$$5. \quad \begin{array}{r} 17 \\ \times 3 \\ \hline 51 \end{array} \quad 5(10) + 1(1) = 51$$

$$\begin{array}{r} 23 \\ \times 3 \\ \hline 102 \end{array} \quad 1(49) + 0(7) + 2(1) = 51$$

THINK:  $3 \times 3 = 9$ . (Written in Base Seven as 12.) Write 2, carry 1 seven.  $3 \times 2 = 6$ . (Written in Base Seven as 6.) Add the 1 seven making 7 sevens. In Base seven this is written 10. Put down 10. Place value operates here the same as in Base Ten.

1

## MULTIPLICATION BASE SEVEN

It might be best to do problems in Base Ten first and then convert them to Base Seven, and check as indicated.

|     | <u>BASE TEN</u>   | <u>BASE SEVEN</u>  |
|-----|---|--|
| 1.  | $\begin{array}{r} 49 \\ 3 \\ \hline 147 \end{array}$                              | $\begin{array}{r} 100 \\ 3 \\ \hline 300 \end{array} 3(49) + 0(7) + 0(1) = 147$  |
| 2.  | $\begin{array}{r} 24 \\ 2 \\ \hline 48 \end{array}$                               | $\begin{array}{r} 33 \\ 2 \\ \hline 66 \end{array} 6(7) + 6(1) = 48$   |
| 3.  | $\begin{array}{r} 14 \\ 4 \\ \hline 56 \end{array}$                               | $\begin{array}{r} 20 \\ 4 \\ \hline 110 \end{array} 1(49) + 1(7) + 0(1) = 56$  |
| 4.  | $\begin{array}{r} 21 \\ 6 \\ \hline 126 \end{array}$                              | $\begin{array}{r} 30 \\ 6 \\ \hline 240 \end{array} 2(49) + 4(7) + 0(1) = 126$   |
| 5.  | $\begin{array}{r} 35 \\ 7 \\ \hline 245 \end{array}$                              | $\begin{array}{r} 50 \\ 10 \\ \hline 500 \end{array} 5(49) + 0(7) + 0(1) = 245$  |
| 6.  | $\begin{array}{r} 42 \\ 8 \\ \hline 336 \end{array}$                              | $\begin{array}{r} 60 \\ 11 \\ \hline 60 \\ 60 \\ \hline 660 \end{array} 6(49) + 6(7) + 0(1) = 336$                             |
| 7.  | $\begin{array}{r} 349 \\ 8 \\ \hline 2792 \end{array}$                            | $\begin{array}{r} 1006 \\ 11 \\ \hline 1006 \\ 1006 \\ \hline 11066 \end{array} 1(2401) + 1(343) + 0(49) + 6(7) + 6(1) = 2792$ |
| 8.  | $\begin{array}{r} 1072 \\ 12 \\ \hline 21144 \\ 1072 \\ \hline 12864 \end{array}$ | $\begin{array}{r} 3061 \\ 15 \\ \hline 21125 \\ 3061 \\ \hline 52335 \end{array} 5(2401) + 2(343) + 3(49) + 5(1) = 12864$      |
| 9.  | $\begin{array}{r} 364 \\ 14 \\ \hline 1156 \\ 364 \\ \hline 5096 \end{array}$     | $\begin{array}{r} 1030 \\ 20 \\ \hline 20600 \end{array} 2(2401) + 0(343) + 6(49) + 0(7) + 0(1) = 5096$                        |
| 10. | $\begin{array}{r} 98 \\ 4 \\ \hline 392 \end{array}$                              | $\begin{array}{r} 200 \\ 4 \\ \hline 1100 \end{array} 1(343) + 1(49) + 0(7) + 0(1) = 392$                                      |

1.  $\frac{1}{x^2} = x^{-2}$

2.  $\frac{1}{x^3} = x^{-3}$

3.  $\frac{1}{x^4} = x^{-4}$

4.  $\frac{1}{x^5} = x^{-5}$

5.  $\frac{1}{x^6} = x^{-6}$

6.  $\frac{1}{x^7} = x^{-7}$

7.  $\frac{1}{x^8} = x^{-8}$

8.  $\frac{1}{x^9} = x^{-9}$

9.  $\frac{1}{x^{10}} = x^{-10}$

10.  $\frac{1}{x^{11}} = x^{-11}$

11.  $\frac{1}{x^{12}} = x^{-12}$

12.  $\frac{1}{x^{13}} = x^{-13}$

13.  $\frac{1}{x^{14}} = x^{-14}$

14.  $\frac{1}{x^{15}} = x^{-15}$

15.  $\frac{1}{x^{16}} = x^{-16}$



## SUBTRACTION PROBLEMS IN BASE SEVEN

Subtraction in Base Seven is exactly the same as in Base Ten, except that we would "borrow", or regroup, sevens instead of tens. If we think about some problems, this is clear.

$$\begin{array}{r}
 1. \quad 20 \\
 \quad -6 \\
 \hline
 \quad 14
 \end{array}
 \qquad \text{or} \qquad
 \begin{array}{r}
 1 \text{ ten} \quad 0+10 \text{ ones} \\
 7 \text{ tens} + 0 \text{ ones} \\
 - \quad \quad 6 \text{ ones} \\
 \hline
 1 \text{ ten} + 4 \text{ ones}
 \end{array}$$

It is a mistake if we do not think of the 1 ten that we "borrow" as being added to the ones in the one's place. We are inclined to write the one beside the zero, in this case, and with little thought simply call it ten.

$$\begin{array}{r}
 2. \quad 22 \\
 \quad -6 \\
 \hline
 \quad 16
 \end{array}
 \qquad \text{or} \qquad
 \begin{array}{r}
 1 \text{ ten} \quad 2+10 \text{ ones} \\
 7 \text{ tens} + 2 \text{ ones} \\
 - \quad \quad 6 \text{ ones} \\
 \hline
 1 \text{ ten} + 6 \text{ ones}
 \end{array}$$

The ten ones that we "borrow" are added to the 2 ones that are already in one's place and this makes 12. If we do a problem in Base Seven, we can see this more clearly. We must remember that we are regrouping in terms of the base or model group.

$$\begin{array}{r}
 3. \qquad \qquad \text{BASE TEN} \qquad \qquad \qquad \text{BASE SEVEN} \\
 \qquad \qquad \quad 22 \qquad \qquad \quad 31 \text{ or } 2 \text{ sevens } 1+7 \text{ ones (or 8 ones)} \\
 \qquad \qquad \quad -6 \qquad \qquad \quad -6 \text{ or } 7 \text{ sevens } + 7 \text{ ones} \\
 \qquad \qquad \quad \hline \quad 16 \qquad \qquad \quad \hline \qquad \quad 6 \text{ ones} \\
 \qquad \qquad \qquad \qquad \qquad \quad 2 \text{ sevens } + 2 \text{ ones} = 16
 \end{array}$$

It is easy to see that it is NECESSARY TO ADD the model group (7) to the ones that are already in the one's place and not to simply write the 1 beside the 1 in the one's place and call it eleven. We may do this, if we have been able to think in the system that  $11-6=2$ . But, until one thinks in the system, or had memorized the facts, it is almost impossible to do this. Thinking of the model group as being added when it is "borrowed" makes it plain that 11 in Base Seven is composed of  $1(7) + 1(1)$  or is equal to 8 in Base Ten.

Students can gain an understanding of subtraction by making up problems, converting them to Base Seven, and then working them in both systems. The problems always should be proved according to the above pattern until the students are very sure of themselves in the new system.

Long division in Base Seven is easily done as multiple subtraction; therefore, the subtraction is not developed further here.

1. The first part of the document is a list of names and addresses of the members of the committee.

2. The second part of the document is a list of names and addresses of the members of the committee.

3. The third part of the document is a list of names and addresses of the members of the committee.

4. The fourth part of the document is a list of names and addresses of the members of the committee.

5. The fifth part of the document is a list of names and addresses of the members of the committee.

6. The sixth part of the document is a list of names and addresses of the members of the committee.

7. The seventh part of the document is a list of names and addresses of the members of the committee.

8. The eighth part of the document is a list of names and addresses of the members of the committee.

9. The ninth part of the document is a list of names and addresses of the members of the committee.

10. The tenth part of the document is a list of names and addresses of the members of the committee.

## LONG DIVISION IN BASE SEVEN

It is possible to do division in Base Seven either as multiple subtraction or in the form,  $2/\overline{8}$ , sometimes called the "divided-by" form. For beginners, it is probably easier to do it as multiple subtraction. Following are some examples. Note that this method carries on the ideas presented under subtraction.

| <u>BASE TEN</u>  | <u>BASE SEVEN</u>   |
|--|---|
| 1. $\begin{array}{r} 7 \overline{) 52} \text{ r.3} \\ \underline{49} \\ 3 \end{array}$ | $\begin{array}{r} 10 \overline{) 103} \text{ r.3} \\ \underline{-10} \quad 1 \\ 63 \\ \underline{-10} \quad 1 \\ 53 \\ \underline{-10} \quad 1 \\ 43 \\ \underline{-10} \quad 1 \\ 33 \\ \underline{-10} \quad 1 \\ 23 \\ \underline{-10} \quad 1 \\ 13 \\ \underline{-10} \quad 1 \\ 3 \end{array}$ <div style="display: flex; justify-content: space-between; margin-top: 10px;"> <div>10 1(7)=0(1)</div> <div> <math display="block">\begin{array}{r} 1 \text{ forty-nine} + 0 \text{ sevens} + 3 \text{ ones} \\ \underline{- 1 \text{ seven} + 0 \text{ ones}} \\ 6 \text{ sevens} + 3 \text{ ones} \\ \underline{- 1 \text{ seven} + 0 \text{ ones}} \\ 5 \text{ sevens} + 3 \text{ ones} \\ \underline{- 1 \text{ seven} + 0 \text{ ones}} \\ 4 \text{ sevens} + 3 \text{ ones} \\ \underline{- 1 \text{ seven} + 0 \text{ ones}} \\ 3 \text{ sevens} + 3 \text{ ones} \\ \underline{- 1 \text{ seven} + 0 \text{ ones}} \\ 2 \text{ sevens} + 3 \text{ ones} \\ \underline{- 1 \text{ seven} + 0 \text{ ones}} \\ 1 \text{ seven} + 3 \text{ ones} \\ \underline{- 1 \text{ seven} + 0 \text{ ones}} \\ 3 \text{ ones} \end{array}</math> </div> </div> |

Proof: 7

$$\begin{array}{r} \times 7 \\ \underline{49} \\ +3 \\ \hline 52 \end{array}$$

$$\begin{array}{r} 10 \quad 1(7)+0(1) \\ \times 10 \quad 1(7)+0(1) \\ \hline 100 \quad 1(49)+0(7)+0(1) \\ +3 \\ \hline 103 \quad 1(49)+0(7)+3(1) = 52 \text{ in base ten.} \end{array}$$

2. 
$$8 \overline{) 12} \text{ r.2}$$

$$\begin{array}{r} 15 \text{ r.2} \\ 11 \overline{) 200} \\ \underline{-66} \quad 6 \\ 101 \\ \underline{-66} \quad 6 \\ 2 \quad 15 \end{array}$$

$1(7) + 5(1) = 12$

THINK: Take 6 eights away from 200. You have to decompose 200 to 6 sevens + 7 ones. Complete the subtraction thinking in groups of 7's.

Proof: 12

$$\begin{array}{r} \times 8 \\ \underline{96} \\ +2 \\ \hline 98 \end{array}$$

$$\begin{array}{r} 15 \\ \times 11 \\ \hline 15 \\ 165 \\ +2 \\ \hline 200 \end{array}$$

THINK:  $1 \times 5 = 5$ ,  $1 \times 1 = 1$ , and repeat. Add the remainder. Remember  $5 = 2 = 10$ , or one group of seven. Carry 1 seven. This makes 6 sevens plus the one you are carrying. Put down the zero and carry again. You now have  $1 + 1 = 2$ . Your answer is  $2(49) + 0(7) + 0(1)$ .

1. The first part of the document is a list of names and addresses, which are arranged in a columnar format. The names are written in a cursive script, and the addresses are written in a more formal, printed style.

2. The second part of the document is a list of names and addresses, which are arranged in a columnar format. The names are written in a cursive script, and the addresses are written in a more formal, printed style.

3. The third part of the document is a list of names and addresses, which are arranged in a columnar format. The names are written in a cursive script, and the addresses are written in a more formal, printed style.

4. The fourth part of the document is a list of names and addresses, which are arranged in a columnar format. The names are written in a cursive script, and the addresses are written in a more formal, printed style.

5. The fifth part of the document is a list of names and addresses, which are arranged in a columnar format. The names are written in a cursive script, and the addresses are written in a more formal, printed style.

6. The sixth part of the document is a list of names and addresses, which are arranged in a columnar format. The names are written in a cursive script, and the addresses are written in a more formal, printed style.

7. The seventh part of the document is a list of names and addresses, which are arranged in a columnar format. The names are written in a cursive script, and the addresses are written in a more formal, printed style.

8. The eighth part of the document is a list of names and addresses, which are arranged in a columnar format. The names are written in a cursive script, and the addresses are written in a more formal, printed style.

## BASE TEN

$$\begin{array}{r}
 1. \quad 11 \text{ r. } 3 \\
 9 \overline{) 102} \\
 \underline{-63} \quad 7 \\
 39 \\
 \underline{-36} \quad 4 \\
 3 \quad 11
 \end{array}$$

$$\begin{array}{r}
 \text{Proof: } 11 \\
 \times 9 \\
 \hline
 99 \\
 + 3 \phantom{0} \\
 \hline
 102
 \end{array}$$

## BASE SEVEN

$$\begin{array}{r}
 11 \text{ r. } 3 \\
 12 \overline{) 204} \\
 \underline{-120} \quad 10 \\
 54 \\
 \underline{-51} \quad 4 \\
 3 \quad 11(7) + 4(1) = 11
 \end{array}$$

$$\begin{array}{r}
 14 \\
 \times 12 \\
 \hline
 31 \\
 14 \phantom{0} \\
 \hline
 201 \\
 + 3 \phantom{0} \\
 \hline
 204 \quad 2(49) + 0(7) + 4(1) = 102
 \end{array}$$

Remember as you work in base seven, you must either convert each number to base seven as you go along, or else think in the base.

$$\begin{array}{r}
 2. \quad 29 \text{ r. } 3 \\
 7 \overline{) 206} \\
 \underline{-49} \quad 7 \\
 157 \\
 \underline{-49} \quad 7 \\
 108 \\
 \underline{-49} \quad 7 \\
 59 \\
 \underline{-49} \quad 7 \\
 10 \\
 \underline{-7} \quad 1 \\
 3
 \end{array}$$

$$\begin{array}{r}
 \text{Proof: } 29 \\
 \times 7 \\
 \hline
 203 \\
 + 3 \phantom{0} \\
 \hline
 206
 \end{array}$$

$$\begin{array}{r}
 41 \text{ r. } 3 \\
 10 \overline{) 413} \\
 \underline{-100} \quad 10 \\
 313 \\
 \underline{-100} \quad 10 \\
 213 \\
 \underline{-100} \quad 10 \\
 113 \\
 \underline{-100} \quad 10 \\
 13 \\
 \underline{-10} \quad 1 \\
 3 \quad 4(7) + 1(1) = 29
 \end{array}$$

$$\begin{array}{r}
 41 \\
 \times 10 \\
 \hline
 410 \\
 + 3 \phantom{0} \\
 \hline
 413 \quad 4(49) + 1(7) + 3(1) = 206
 \end{array}$$

In this base seven problem, 10 stands for 1 group of 7 ones instead of ten as in base ten. Then  $10 \times 10 = 100$ , however, 100 in base seven is equal to 1 forty-nine instead of one hundred.

$$\begin{array}{r}
 4. \quad 40 \text{ r. } 10 \\
 12 \overline{) 490} \\
 \underline{-48} \\
 10
 \end{array}$$

$$\begin{array}{r}
 \text{Proof: } 40 \\
 \times 12 \\
 \hline
 80 \\
 40 \phantom{0} \\
 \hline
 480 \\
 + 10 \phantom{0} \\
 \hline
 490
 \end{array}$$

$$\begin{array}{r}
 55 \text{ r. } 13 \\
 15 \overline{) 1300} \\
 \underline{-114} \\
 130 \\
 \underline{-114} \\
 13
 \end{array}$$

$$\begin{array}{r}
 55 \\
 \times 15 \\
 \hline
 404 \\
 55 \phantom{0} \\
 \hline
 1254 \\
 + 13 \phantom{0} \\
 \hline
 1300 \quad 1(343) + 3(49) + 0(7) = 490
 \end{array}$$

This problem is done with the "dividedby" method. It is more difficult because you have to think in the base.

## PLACE VALUE CHART OF OTHER BASES

| BASE | $B^6$       | $B^5$     | $B^4$    | $B^3$   | $B^2$ | $B^1$ | $B^0$ |
|------|-------------|-----------|----------|---------|-------|-------|-------|
| 10   | 1,000,000's | 100,000's | 10,000's | 1,000's | 100's | 10's  | 1's   |
| 7    | 117,649's   | 16,807's  | 2,401's  | 343's   | 49's  | 7's   | 1's   |
| 2    | 64's        | 32's      | 16's     | 8's     | 4's   | 2's   | 1's   |
| 3    | 729's       | 243's     | 81's     | 27's    | 9's   | 3's   | 1's   |
| 5    | 15,625's    | 3,125's   | 625's    | 125's   | 25's  | 5's   | 1's   |

By using place value, we can make a number system in any base. This helps us to understand the structure of our own Base Ten, because we are forced to think of it as it functions as a base or model group, not as something we have learned by rote. This also helps to give us an insight into pure mathematics.

We have given no bases with a model group larger than ten, because this would make it necessary to invent extra symbols. However, a base, such as base twelve would function in the same way as any that used place value and a model group.

To convert to any base, follow this procedure using the appropriate place values. To convert 469 in base ten to base seven:

$$\begin{array}{r}
 469 \\
 -343 \\
 \hline
 126 \\
 -98 \\
 \hline
 28 \\
 -28 \\
 \hline
 0
 \end{array}
 \begin{array}{l}
 1 \text{ three-hundred-forty-three} \\
 2 \text{ forty-nines} \\
 4 \text{ sevens} \\
 0 \text{ ones}
 \end{array}$$

THINK: Take the largest place value out of 469 that you can. This is 343. You will have 1 three-hundred-forty-three. Then take out all you can of the next largest place value. This is 2 forty-nines. The next digit in the base seven number will be 2. Take out 4 sevens. You have used up all the 469 but you must hold the one's place with a zero, since you have no ones in this numeral. If you did not put a zero on the end to hold one's place, this would be only a three place number and the values would be wrong.

After you have subtracted as indicated above, it is possible to simply read down the column. This will be the new numeral in the new base. In this case, 1,240:  $1(343) + 2(49) + 4(7) + 0(1) = 469$ .



## SUGGESTIONS FOR TEACHING

1. Teach counting in the base first. Have students write out the values for the place value according to the pattern below. Do this for as far as the base squared at least. In Base Seven, this would be 49 in Base Ten.

| <u>BASE TEN</u> |                         | <u>BASE SEVEN</u> |                        |
|-----------------|-------------------------|-------------------|------------------------|
| 1               | 1(1)                    | 1                 | 1(1)                   |
| 2               | 2(1)                    | 2                 | 2(1)                   |
| 3               | 3(1)                    | 3                 | 3(1)                   |
| 4               | 4(1)                    | 4                 | 4(1)                   |
| 5               | 5(1)                    | 5                 | 5(1)                   |
| 6               | 6(1)                    | 6                 | 6(1)                   |
| 7               | 7(1)                    | 10                | 1(7) + 0(1)            |
| 8               | 8(1)                    | 11                | 1(7) + 1(1)            |
| 9               | 9(1)                    | 12                | 1(7) + 2(1)            |
| 10              | 1(10) + 0(1)            | 13                | 1(7) + 3(1)            |
| 11              | 1(10) + 1(1)            | 14                | 1(7) + 4(1)            |
| 12              | 1(10) + 2(1) . . . etc. | 15                | 1(7) + 5(1) . . . etc. |

2. After a counting chart has been made, use it to make addition and multiplication tables in the base. This helps the student to begin to THINK in the base.
3. Use these tables to work simple problems. Probably the problems that students make up themselves would be best to use. Work them in Base Ten first, and then convert, compare, and prove the problems back to base ten as suggested.
4. Have children produce a chart or booklet explaining what they understand about another number system other than Base Ten.
5. Give oral demonstrations of place value. Take place value pockets and change the base from ten to something else, and show how the values for the places would have to change.
6. Let slow children make simple counting charts in another base.
7. Write a story or develop the idea that a people with twelve fingers might have a number system based on twelve. This would require the invention of two additional symbols.



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**IV.     ARITHMETIC PROCESSES**

**Addition**

**Subtraction**

**Multiplication**

**Division**

**Review and Reinforcement Suggestions**

1. 1000  
2. 1000  
3. 1000  
4. 1000  
5. 1000  
6. 1000  
7. 1000  
8. 1000  
9. 1000  
10. 1000

## ADDITION

The process of addition for the most part is usually well established prior to the grades for which this material is written; however, some attention will be given to it here. Unfortunately, some students have learned their addition facts by drill and have little or no real understanding of what they are actually doing.

In addition, we are combining groups of irregular counting. Actually, addition is a matter of regrouping in whatever base you may be working. In our system of Base Ten, we group and regroup numbers based in this system. For example:  $5 + 7 = 12$ . We are saying a subgroup of 5 and a subgroup of 7 are regrouped to form a subgroup of 1 ten and 2 ones. In addition, we are regrouping numbers based on ten. In another base, such as Base Seven, we would be regrouping numbers based on 7. While it is not advisable to present addition to primary children in such terms as above, it is possible to help them to understand that they are dealing with groups of numbers rather than mere facts.

$$\begin{array}{r} 36 \\ 54 \\ \hline 90 \end{array} \qquad \begin{array}{r} 3 \text{ tens} \quad 6 \text{ ones} \\ 5 \text{ tens} \quad 4 \text{ ones} \\ \hline 8 \text{ tens} \quad 10 \text{ ones} \\ 1 \text{ ten} \quad -10 \text{ ones} \\ \hline 9 \text{ tens} \quad 0 \text{ ones} \end{array}$$

Think: In our number system, we have no digit or symbol higher than 9. So we say  $6+4$  gives us 1 ten and 0 ones. We "carry" 1 group of ones to the ten's column, or place, and this gives us 9 groups of tens plus no ones. It is easy to see that 8 tens + 10 ones would give us the same face amount; however, we could not write the numeral as 810 for this would be read "eight-hundred-ten". So, we are forced to combine the 8 tens and the 10 ones to read 9 tens or ninety.

Addition is still counting, but it is much easier and faster to count by groups rather than by ones. Instead of the student just saying  $8 + 7 = 15$ , he should understand and think of this as 1 group of tens and 1 group of 5 ones. Writing out these problems as we have indicated helps this process.

1 group of hundreds

|            |                 |    |            |                   |
|------------|-----------------|----|------------|-------------------|
| 11         | 1 group of tens | OR | 367        |                   |
| 367        |                 |    | 294        |                   |
| 294        |                 |    | <u>11</u>  | (7 + 4 = 11)      |
| <u>661</u> |                 |    | 150        | (60 + 90 = 150)   |
|            |                 |    | 500        | (300 + 200 = 500) |
|            |                 |    | <u>661</u> |                   |

Here, we are regrouping, or "carrying" ones, tens, and hundreds. THINK, 11 ones regroup to 1 ten and 1 one. 16 tens regroup to 1 hundred and 6 tens.

The second illustration shows how the problem could be done by simply adding the value for each place. This would Avoid "carrying".

The first of these is the fact that the  
 system is not a simple one. It is a  
 complex one, and it is not possible to  
 describe it in a simple way. It is a  
 system of many parts, and it is not  
 possible to describe it in a simple way.  
 It is a system of many parts, and it is  
 not possible to describe it in a simple  
 way. It is a system of many parts, and  
 it is not possible to describe it in a  
 simple way. It is a system of many  
 parts, and it is not possible to describe  
 it in a simple way. It is a system of  
 many parts, and it is not possible to  
 describe it in a simple way. It is a  
 system of many parts, and it is not  
 possible to describe it in a simple way.

The second of these is the fact that the  
 system is not a simple one. It is a  
 complex one, and it is not possible to  
 describe it in a simple way. It is a  
 system of many parts, and it is not  
 possible to describe it in a simple way.  
 It is a system of many parts, and it is  
 not possible to describe it in a simple  
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 system of many parts, and it is not  
 possible to describe it in a simple way.

The third of these is the fact that the  
 system is not a simple one. It is a  
 complex one, and it is not possible to  
 describe it in a simple way. It is a  
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 It is a system of many parts, and it is  
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 possible to describe it in a simple way.

## SUBTRACTION

Subtraction can be taught as the inverse process of addition. Instead of combining subgroups as we do in addition, we subtract subgroups. For example:  $7 - 5 = 2$ . We are saying: "A group of 7 less a subgroup of 5 equals a subgroup of 2". We can apply this to any combination except where the digit of the subtrahend is greater than the corresponding digit of the minuend. Then we must use decomposition, or "borrowing", before we can subtract.

$$\begin{array}{r} 53 \\ -36 \\ \hline 17 \end{array} \quad \text{or} \quad \begin{array}{l} (4 \text{ tens}) (3+10 \text{ ones}) \\ 5 \text{ tens} + 3 \text{ ones} \\ -3 \text{ tens} + 6 \text{ ones} \\ \hline 1 \text{ ten} + 7 \text{ ones} \end{array}$$

THINK: We cannot subtract a group of 6 from a group of 3, so we regroup by decomposing (rearranging) the 5 tens into 4 tens and 10 ones. Now we regroup the ones by adding the subgroup of 3 ones, to the ten ones we "borrowed" to make 13 ones from which to subtract the 6 ones of the subtrahend. The student should understand that this is what he is doing when he "borrows" from the next place and puts a 1 beside the digit from which he will subtract. He is increasing the minuend digit by the amount of the model group of the system.

$$\begin{array}{r} 8323 \\ -2756 \\ \hline 5567 \end{array} \quad \begin{array}{l} (2+10 \text{ hundreds})(1+10 \text{ tens}) \\ 7 \text{ thousands} \quad 2 \text{ hundred} \quad 1 \text{ ten} \quad 3+10 \text{ ones} \\ 8 \text{ thousands} \quad 3 \text{ hundreds} + 2 \text{ tens} + 3 \text{ ones} \\ -2 \text{ thousands} + 7 \text{ hundreds} + 5 \text{ tens} + 6 \text{ ones} \\ \hline 5 \text{ thousands} + 5 \text{ hundreds} + 6 \text{ tens} + 7 \text{ ones} \end{array}$$

THINK: We decompose the 2 tens to read 1 ten and 10 ones . . . regrouped, this gives us 1 ten plus 3 ones or 13 ones. The 3 hundred decomposes to 2 hundred and 10 tens . . . regrouped to give us 10 tens plus 1 ten, or 11 tens. The 8 thousand regroups to read 7 thousand and 10 hundreds, which regroups to give us 10 hundreds plus 2 hundreds or 12 hundreds.

This process might be explained by showing that the rearrangement will still equal the original amount:

$$7,000 + 1,200 + 110 + 13 = 8,323$$

Subtraction can also be done by the additive method. This is considered by some to be a better one since it employs the use of the addition facts with no new combinations to learn. However, the "take away" method is most commonly used. Essentially it is only a different way of thinking of the process.

| <u>TAKE AWAY</u>   |   | <u>ADDITIVE</u>  |   |
|--|---|--|---|
| $\begin{array}{r} 538 \\ -426 \\ \hline 112 \end{array}$ | We think: $8-6=?$<br>$3-2=?$<br>$5-4=?$ | $\begin{array}{r} 538 \\ -426 \\ \hline 112 \end{array}$ | We think: $6+7=13$<br>$2+7=9$<br>$4+7=11$ |



## MULTIPLICATION

An approach to multiplication through a thorough understanding of place value opens many interesting and worthwhile activities that can be used to motivate students and increase understanding. Following are given some different forms of working multiplication problems. Students, investigating these, may find it worthwhile to invent others.

$$1. \quad \begin{array}{r} 12 \\ \times 3 \\ \hline 36 \end{array} \quad \begin{array}{l} 1 \text{ ten} + 2 \text{ ones} \\ \quad \times 3 \text{ ones} \\ \hline 3 \text{ tens} + 6 \text{ ones} \end{array}$$

$$\begin{array}{r} 12 \\ \times 3 \\ \hline 30 \\ +6 \\ \hline 36 \end{array} \quad \begin{array}{l} (3 \times 10) \\ (3 \times 2) \end{array}$$

$$2. \quad \begin{array}{r} 14 \\ \times 3 \\ \hline 42 \end{array} \quad \begin{array}{l} 1 \text{ ten} + 4 \text{ ones} \\ \quad \times 3 \text{ ones} \\ \hline 3 \text{ tens} + 12 \text{ ones} \\ +1 \text{ ten} = 10 \text{ ones} \\ \hline 4 \text{ tens} + 2 \text{ ones} \end{array} \quad (\text{remove 1 ten and "carry" it})$$

$$3. \quad \begin{array}{r} 14 \\ \times 3 \\ \hline 12 \\ +30 \\ \hline 42 \end{array} \quad \begin{array}{l} (3 \times 4) \\ (3 \times 10) \end{array}$$

$$\begin{array}{r} 14 \\ \times 3 \\ \hline 30 \\ 12 \\ \hline 42 \end{array} \quad \begin{array}{l} (3 \times 10) \\ (3 \times 4) \end{array}$$

$$\begin{array}{r} 14 \\ 14 \\ 14 \\ \hline 12 \\ 30 \\ \hline 42 \end{array} \quad \begin{array}{l} (4+4+4) \\ (10+10+10) \end{array}$$

$$4. \quad \begin{array}{r} 243 \\ \times 24 \\ \hline 12 \\ 160 \\ 800 \\ 60 \\ 800 \\ 4000 \\ \hline 5832 \end{array} \quad \begin{array}{l} (4 \times 3) \\ (4 \times 40) \\ (4 \times 200) \\ (20 \times 3) \\ (20 \times 40) \\ (20 \times 200) \end{array}$$

$$\begin{array}{r} 243 \\ \times 24 \\ \hline 4860 \\ 972 \\ \hline 5832 \end{array} \quad \begin{array}{l} (20 \times 243) \\ (4 \times 243) \end{array}$$

$$\begin{array}{r} 243 \\ \times 24 \\ \hline 4860 \\ 12 \\ 160 \\ 800 \\ \hline 5832 \end{array} \quad \begin{array}{l} (20 \times 243) \\ (4 \times 3) \\ (4 \times 40) \\ (4 \times 200) \end{array}$$

Such different ways as these can make multiplication have a "new" look and furnish an excellent means of checking problems. They may not be used very successfully if a student does not have a good idea of place value. These methods should not be presented to students as something that they are required to memorize. They should be the result and natural outcome of understanding acquired in studying place value and the function of the number system.

When students are given freedom to invent methods of doing some process they should be required, however, to prove their invention is sound and will result in the correct answer.





## DIVISION

Division, or dividing numbers, can be thought of as subtracting in a special way. If we take the problem:

$$3 \overline{)12}$$

We may want to know how many threes are in 12, how much  $1/3$  of 12 would be, or how many times we can take 3 away from 12.

Putting the problem down like this:  $3 \overline{)12}$  is just a convenient way of arranging it so that we can find out what we want to know. The well-known "box" has absolutely nothing to do with this.

We could also put the problem down like this and find the answer:

$$\begin{array}{r} 12 \\ -3 \quad 1 \text{ (three taken away)} \\ \hline 9 \\ -3 \quad 1 \text{ (another three taken away)} \\ \hline 6 \\ -3 \quad 1 \text{ (another three taken away)} \\ \hline 3 \\ -3 \quad 1 \text{ (the last three taken away)} \\ \hline 0 \end{array} \quad \frac{1}{4}$$

We have taken away 4 threes, or  $4 \times 3 = 12$ . It took 4 threes to use up all of the 12. We could find out how many 3 inch pieces there were in twelve inches in the same way, simply by taking a 12 inch strip of paper and cutting it up, then count the pieces.

$$\begin{array}{r} 54 \text{ inches (the length of the snake in inches)} \\ -12 \quad 1 \text{ (take away one foot of 12 inches)} \\ \hline 42 \\ -12 \quad 1 \text{ (take away another foot of 12 inches)} \\ \hline 30 \\ -12 \quad 1 \text{ (take away another foot of 12 inches)} \\ \hline 18 \\ -12 \quad 1 \text{ (take away the last whole foot)} \\ \hline 6 \end{array} \quad \frac{1}{4} \text{ (there are 6 inches left)}$$

The snake was 4 twelve inches, or 4 feet, long. There were six inches left over.

A better way to do this problem would be like this:

$$\begin{array}{r} 54 \text{ inches} \\ -24 \quad 2 \text{ (take away 2 twelves)} \\ \hline 30 \\ -24 \quad 2 \text{ (take away 2 more twelves)} \\ \hline 6 \end{array} \quad \frac{1}{4}$$

2. The first of these is the fact that the

the first of these is the fact that the

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The snake was 4 feet 6 inches long. The children have divided 54 by 12: or we could say  $54 \div 12 = 4$ , remainder 6; or  $12/54$ , etc. It does not make any difference how we put it down to work it, if we are careful, we will get the right answer.

It is difficult to do very complicated long division problems with this kind of a subtractive process unless students have a very good background in place value. After this has been built up, division done with any of the processes can become meaningful. Following are some problems of increasing difficulty. These should demonstrate the process.

$$\begin{array}{r}
 15 \overline{) 65} \\
 \underline{-15} \quad 1 \\
 50 \\
 \underline{-15} \quad 1 \\
 35 \\
 \underline{-15} \quad 1 \\
 20 \\
 \underline{-15} \quad \frac{1}{4} \\
 5
 \end{array}
 \quad \text{or} \quad
 \begin{array}{r}
 1+1+1+1=4 \text{ r. } 5 \\
 15 \overline{) 65} \\
 \underline{-15} \\
 50 \\
 \underline{-15} \\
 35 \\
 \underline{-15} \\
 20 \\
 \underline{-15} \\
 5
 \end{array}$$

A slow student might do the problem this way. One with more talent might speed the process up in the following manner.

$$\begin{array}{r}
 15 \overline{) 65} \\
 \underline{-30} \quad 2 \\
 35 \\
 \underline{-30} \quad \frac{2}{4} \\
 5
 \end{array}$$

Some students would see that  $4 \times 15 = 60$  and know the answer. With this method, however, there is offered to students of many abilities an understandable way of doing the division process.

$$\begin{array}{r}
 6 \overline{) 90} \\
 \underline{-60} \quad 10 \\
 30 \\
 \underline{-30} \quad \frac{5}{15}
 \end{array}
 \quad
 \begin{array}{r}
 46 \overline{) 489} \\
 \underline{460} \quad \frac{10}{10} \\
 29
 \end{array}$$

Stop before doing a problem and think how many times the divisor should be taken out ... 1 time, 10 times, 100 times, etc. Do not try to take out so many times the divisor the multiplication becomes involved. For instance, it would be foolish to take out 15 times the divisor. This would make the problem more difficult and invite error.

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$$\begin{array}{r}
 15 \overline{) 650} \\
 \underline{150} \quad 10 \text{ (start with 10 times the divisor)} \\
 500 \\
 \underline{300} \quad 20 \text{ (it is easy to see 20 times the divisor now)} \\
 200 \\
 \underline{150} \quad 10 \text{ (it would be foolish to use 13 here)} \\
 50 \\
 \underline{45} \quad 3 \text{ (this is easier to multiply)} \\
 5
 \end{array}$$

$$\begin{array}{r}
 100+200+100+30+5+4=439 \text{ r. } 4 \quad (\text{the answer may be put in any convenient and meaningful way}) \\
 15 \overline{) 6589} \\
 \underline{1500} \quad 100 \\
 5089 \\
 \underline{3000} \quad 200 \\
 2089 \\
 \underline{1500} \quad 100 \\
 589 \\
 \underline{450} \quad 30 \\
 139 \\
 \underline{75} \quad 5 \\
 64 \\
 \underline{60} \quad 4 \\
 4 \quad 439
 \end{array}$$

THINK: There are 439 fifteens in 650. It is possible to find out the answer simply by taking them out until you have used up all the 650. It would be foolish to take them out 1 at a time. Take them out in groups.

The form in which the division problem is placed often can obscure the meaning. Such an approach as multiple subtraction will tend to: (1) increase understanding; (2) make the division process easier; and (3) relate back to the place value and subtraction concepts that the child already has mastered.

It should be understood that some problems are done more conveniently one way, some another. It would be well to help children understand many ways and then use whatever way best suits the situation. When a child can do this, he is using mathematics intelligently and not just repeating a rote, a meaningless process.



#### **IV. FRACTIONS**

**History and Development**

**Understandings Needed**

**Methods**



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## HISTORY AND DEVELOPMENT

People think of the amounts of things in two ways: as whole things, and as parts of things. It would appear that for some time during history, man did not think much about parts of things. Whatever accounts he kept and dealings that he had, were done, for the most part, in whole amounts. A way to "put down" or express fractions may have been difficult to invent. Ways to add, subtract, multiply and divide fractions were probably very confusing. We have records of very early fractions that were quite different from ours.

The word fraction comes from a Roman, or Latin, word, frangere which means to break. We have words like fracture that have much the same meaning. At first, people seemed to just express parts of things in any convenient way. Then, they tried to write fractions in such a way that all the numerators were 1. The Egyptians did this in the following manner.

Since  $\overline{|||}$  meant 3, they invented a symbol that means 1 part of three  $\overline{\bigcirc}$ . So  $\overline{\bigcirc}$  stood for  $1/3$ . They had a special symbol for  $2/3$ ,  $\overline{\bigcirc}$ ; and for  $1/2$   $\overline{\sqcap}$ . Other than these two, all fractions were unit fractions, or fractions with numerators of 1. This resulted in some unusual ways of figuring.

$$\frac{7}{8} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \overline{\sqcap \bigcirc \bigcirc}$$

$$\frac{4}{15} = \frac{1}{5} + \frac{1}{15} = \overline{\bigcirc \bigcirc}$$

$$\frac{8}{12} = \frac{1}{2} + \frac{1}{6} = \overline{\sqcap \bigcirc}$$

There was no set way of expressing fractions. We believe that some merchants "in the know" who had invented one used it to their own advantage. It is interesting to try to do this. You must start with the largest fraction that you can use, subtract this from the one that you want to express, use the next largest, etc.

We do much the same thing with money. So, you see, we are not so advanced after all. Suppose that you buy something for 26¢. You give the clerk \$1.00. You will get 74¢ change. The clerk will say: "26¢, 27¢, 28¢, 29¢, 30¢, 40¢, 50¢, \$1.00. She has given you:

$$\frac{1}{100} + \frac{1}{100} + \frac{1}{100} + \frac{1}{100} + \frac{1}{10} + \frac{1}{10} + \frac{1}{2}$$

Your change is equal to 74/100 or a dollar. Therefore:

$$\frac{74}{100} = \frac{1}{100} + \frac{1}{100} + \frac{1}{100} + \frac{1}{100} + \frac{1}{10} + \frac{1}{10} + \frac{1}{2}$$

1. The first part of the document is a list of names and addresses, which are arranged in a columnar format. The names are written in a cursive script, and the addresses are written in a more formal, printed style. The list appears to be a directory or a roster of some kind.

2. The second part of the document is a list of names and addresses, which are arranged in a columnar format. The names are written in a cursive script, and the addresses are written in a more formal, printed style. The list appears to be a directory or a roster of some kind.

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5. The fifth part of the document is a list of names and addresses, which are arranged in a columnar format. The names are written in a cursive script, and the addresses are written in a more formal, printed style. The list appears to be a directory or a roster of some kind.

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7. The seventh part of the document is a list of names and addresses, which are arranged in a columnar format. The names are written in a cursive script, and the addresses are written in a more formal, printed style. The list appears to be a directory or a roster of some kind.

8. The eighth part of the document is a list of names and addresses, which are arranged in a columnar format. The names are written in a cursive script, and the addresses are written in a more formal, printed style. The list appears to be a directory or a roster of some kind.

9. The ninth part of the document is a list of names and addresses, which are arranged in a columnar format. The names are written in a cursive script, and the addresses are written in a more formal, printed style. The list appears to be a directory or a roster of some kind.

10. The tenth part of the document is a list of names and addresses, which are arranged in a columnar format. The names are written in a cursive script, and the addresses are written in a more formal, printed style. The list appears to be a directory or a roster of some kind.

11. The eleventh part of the document is a list of names and addresses, which are arranged in a columnar format. The names are written in a cursive script, and the addresses are written in a more formal, printed style. The list appears to be a directory or a roster of some kind.

# UNDERSTANDINGS NEEDED

Fractions may be confusing to children because we tell them that they mean so many things. We say:

- $\frac{1}{2}$     number you have  
  name of parts
- or             $\frac{1}{2}$     shows 1 part of two equal parts
- or             $\frac{1}{2}$     shows the relationship between 1 and 2
- or             $\frac{1}{2}$     means 1 divided by 2

By the time that a child has been told all of this, it is probable that he does not know what to think and just gives up and memorizes everything that he needs to know about fractions in order to "pass". It would seem well at this point to make clear some of the basic understandings that a child needs, and to show some of the ways that these may be demonstrated.

Few of the things that we deal with in our daily lives come in wholes. We are always using parts of things. This is also true for children. Fractions can be put in the tangible context of a child's life, using the things that he knows. These can be used to help him understand the reality and usefulness of fractions. What he sees and can touch, he will remember better. There are many things that can be used in this way to help with fractions.

When fractions are approached through their historical development, we see that the manner of writing them is just one of convenience. Fractions can be just a logical, convenient way of figuring with parts of things. This brings us to the first understanding.

## 1. Fractions are Parts of Things

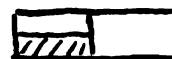
This can be shown graphically or with things drawn from children's environment.



$\frac{1}{2}$  of 1



$\frac{1}{2}$  of 4



$\frac{1}{2}$  of  $\frac{1}{2}$

This also can be demonstrated. Use liquid measures and show the following facts. Demonstrate them, list them, make graphs of them, and then draw conclusions.

100-100000

## FACT

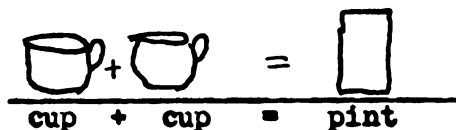
2 cups equal 1 pint

Conclusion: 1 cup =  $\frac{1}{2}$  pt.

$\frac{1}{2}$  cup =  $\frac{1}{4}$  pt.

..... etc.

## GRAPHIC



## FACT

2 cups equal 1 pint

2 pints equal 1 quart

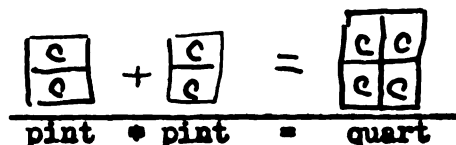
Conclusion: 1 c =  $\frac{1}{4}$  qt.

2 c =  $\frac{2}{4}$  qt. (or  $\frac{1}{2}$  qt.)

3 c =  $\frac{3}{4}$  qt.

or 3 c =  $\frac{1-\frac{1}{2}}{2}$  of a qt.

## GRAPHIC



These can be carried as far as the talents of students permit. Slow children may not go very far. More gifted ones will be able to draw quite advanced conclusions. Introducing fractions in this manner, at any grade level, puts the symbolization and the concrete illustration on the child's level of thinking. The next step would be to do simple problems, and then more difficult ones with denominate numbers. Below are some illustrations.

$$\begin{array}{r} 1 \text{ pt. } 2 \text{ c.} \\ +1 \text{ pt. } 1 \text{ c.} \\ \hline 2 \text{ pt. } 3 \text{ c.} \\ +1 \text{ pt. } -2 \text{ c.} \\ \hline 3 \text{ pt. } 1 \text{ c.} \end{array}$$

$$\begin{array}{r} \frac{1}{2} \text{ qt.} \\ \frac{1}{2} \text{ qt.} \\ \hline 2/2 \text{ qt.} = 1 \text{ qt.} \end{array}$$

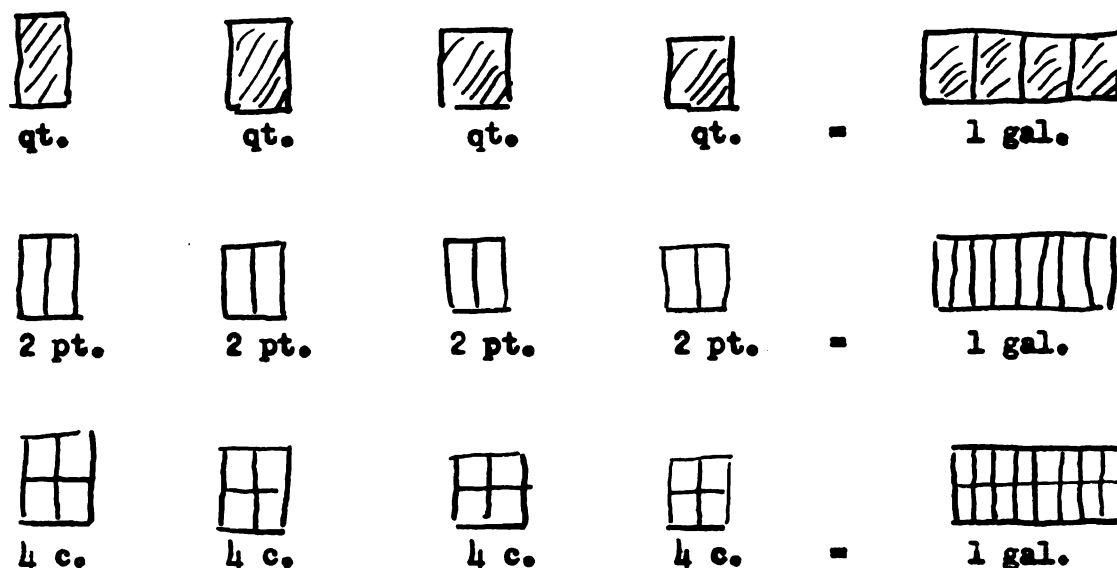
$$\begin{array}{r} 1 \frac{1}{2} \text{ gal.} \\ 1 \frac{1}{2} \text{ gal.} \\ \hline 2 \frac{2}{2} \text{ gal.} = 3 \text{ gal.} \end{array}$$

After children have demonstrated these facts, drawn conclusions from the facts, made graphics of the facts, and worked problems regarding them, they may see reasons for writing fractions as we do, the logic behind reducing fractions, and ways of combining them.



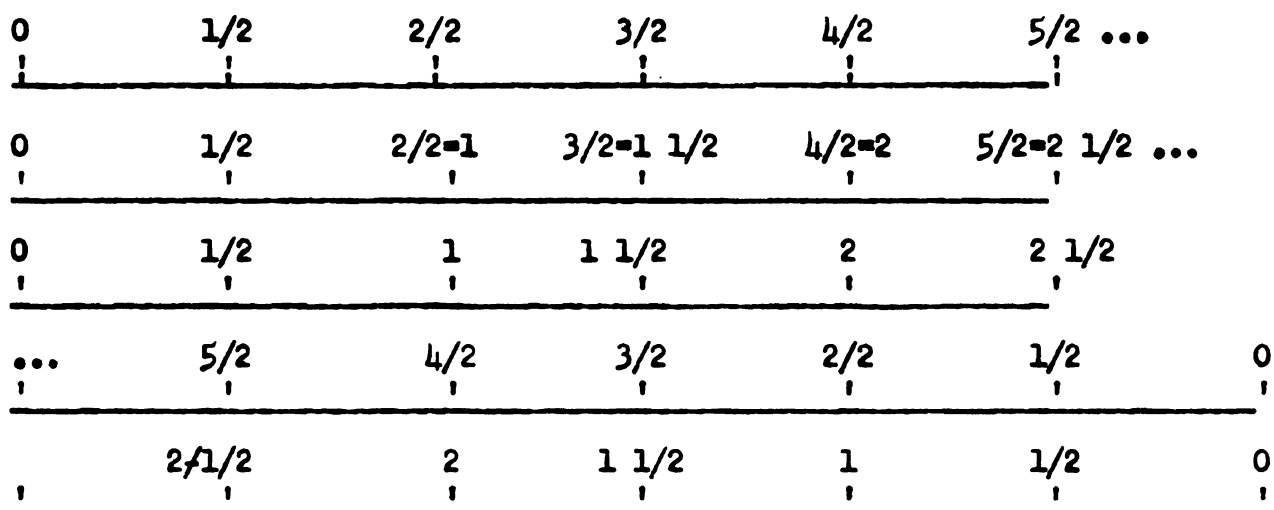
It is possible to help children make up very interesting displays or graphics developing any of the facts that they use. They could demonstrate that:

$$\begin{aligned} 4 \text{ quarts} &= 1 \text{ gallon} \\ 8 \text{ pints} &= 1 \text{ gallon} \\ 16 \text{ cups} &= 1 \text{ gallon} \end{aligned}$$



Conclusion from the above:  $1 \text{ pint} = 1/8 \text{ gallon}$   
 $1 \text{ cup} = 1/16 \text{ gallon}$   
 $2 \text{ pints} = 2/8 \text{ gallon} = 1/4 \text{ gallon, etc.}$

We can also use a number line to show the sequence of fractions. It is possible to count in fractions. This helps to put them in a logical arrangement for children. Children could make any kind of a number line for demonstrating fractions that was at their own particular level of thinking. Below are suggestions.



Any logical sequence makes a good number line for fractions. Counting forwards, backwards, and by reducing helps children to see how fractions relate. This is also excellent to help them see reduction of fractions.



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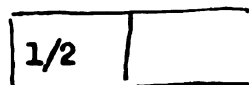
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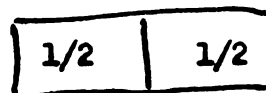
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A number line is also a fine help in teaching children that there are proper and improper fractions, while numbers and mixed numbers. This can be shown with things drawn from the child's environment and with graphics also. Below are some illustrations.

proper

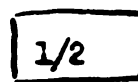


improper or  
mixed number



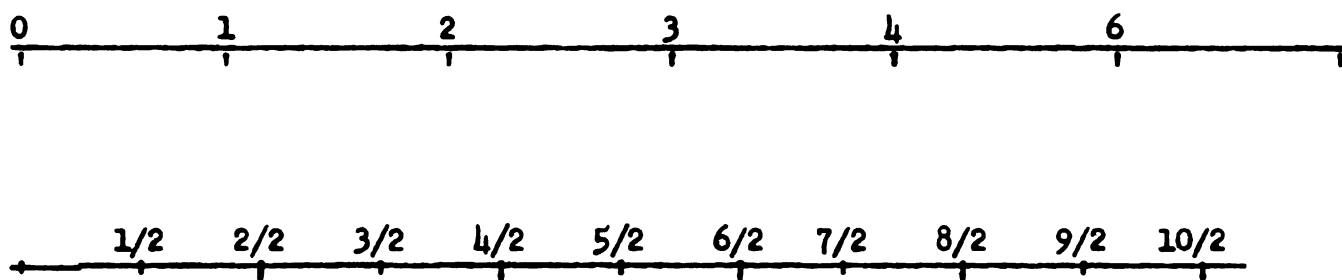
1

+



$1/2 = 3/2$

To show that all fractions can be changed to higher terms use measures and demonstrate or use a number line like the one below. These can be made of any convenient material. Oak tag is good and can be kept in desks. Wrapping paper makes a good one to put up for the whole class to use. This also makes a fine project for a group of students to do.



## 2. A Fraction is a Ratio

This is difficult for some children to understand. Perhaps a problem will help to demonstrate this idea. Again, this is putting the fraction concept into a concrete situation drawn from the child's experience.

John has 5 marbles, 2 are blue. What part are blue?

Solution: Let each marble represent  $1/5$  of the total. Therefore, 2 marbles, or  $2/5$  are blue.

THINK:  $1 \text{ marble} = 1/5$   
 $+1 \text{ marble} = 1/5$   


---

 $2 \text{ marbles} = 2/5$

100-100000

100-100000

100-100000

100-100000

100-100000

100-100000

100-100000

### 3. Fractions are Composed of Other Numbers

Just like whole numbers, fractions are composed of other numbers. The number 6 is made of:  $1+1+1+1+1+1$  and  $2+2+2$  and  $3+3$ , etc.  $4/5$  is made of  $2/5 + 2/5$ , or  $1/5 + 1/5 + 1/5 + 1/5$ , etc.

Also,  $4/5$  has the same value as any of these combinations. This is a fine way to help children see ideas that relate to lowest common denominators and to reducing. Opportunities to express fractions in as many different ways as one can are good. Another method that might lead to understanding would be to do problems in fractions as the Egyptians did. Below are some illustrations. Remember that it makes it easier to start with the largest fraction that you can.

Directions:

1. Express the following as unit fractions.

$$6/8 = 1/2 + 1/8 + 1/8$$

$$4/9 = 1/3 + 1/9$$

2. Express the following as unit fractions starting with the first fraction given.

$$1 \frac{1}{2} = 1/16 + \underline{\hspace{2cm}}$$

$$3/10 = 1/5 + \underline{\hspace{2cm}}$$

Many such activities can be made up by students and teachers to give meaning to lowest common denominators and reducing.

### 4. A Fraction is a Division Example

This is further developed in the section on division of fractions. It should be emphasized that division can be expressed in any convenient way, such as:

$$3/\overline{4}$$

$$4 \div 3 = 4/3$$

### 4. Fractions can be Added, Subtracted, Multiplied and Divided

This needs to be made plain with concrete materials and with symbols. It needs to be illustrated that unless this were true fractions would be of no use to the culture. It would be interesting for children to see if they could get along without fractions for a period of time, and then give a report on how they made out.

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## 5. Fractions can be Estimated

Just like whole numbers, it is possible to estimate fractions. Children need practice in this. They should have opportunities to practice, such as doing demonstrations, making charts and comparisons, etc.

Such expressions as:  $6/3 = ? > 1$

$3/5 = ? > 1/10$

$1/2 = ? < 6/8$ , etc.

should be encouraged and explored. These can be suited to each child's talent. Children should be encouraged to demonstrate their reasons and the method they use to arrive at an answer.

## METHODS WITH FRACTIONS

While it is fairly easy to visualize and to demonstrate addition and subtraction of fractions, this is more difficult to do with multiplication and division. Sometimes the method for these processes is emphasized so much that the meaning is obscured. Concentrating on the method until it is memorized may make it possible to "do the problems" but it helps very little with understandings and lasting learning.

Division and multiplication of fractions are closely related. They will be presented together here as a way of "thinking about fractions" rather than a process. There are several "methods" popular for teaching these. If the understandings behind these are mastered, the method as such, should develop of itself as a convenient way of doing the manipulation. This section will be divided into: (1) Understandings Basic to Division of Fractions, and (2) The Methods of Doing Division of Fractions. These will be accompanied with suggestions for classroom activities.

### 1. Basic Understandings

1. Children need to understand the relationship between multiplication and division. They should know that:

$3/\overline{12}$  means also: ?  $\times 3 = 12$

When a child does the first problem, or ones like it, he should also understand the second one and the way the two relate. Then

$3/4 \div 1 = 3/4$ , and ?  $\times 1 = 3/4$

would be the next step in understanding the process with fractions. This helps to make answers more meaningful.

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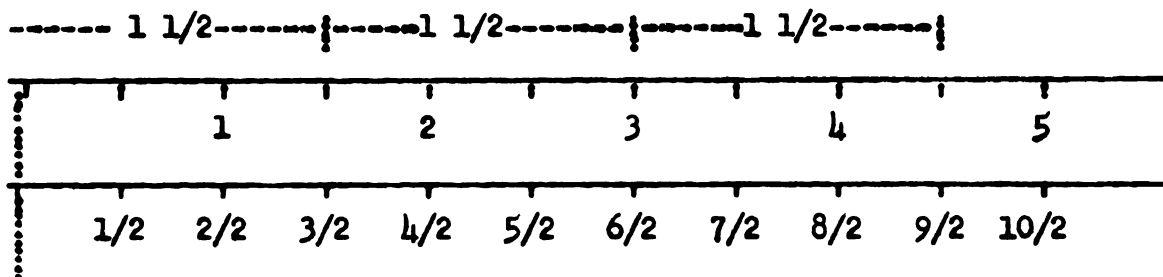
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2. Children need to be able to change whole numbers and mixed numbers to improper fractions. If they have acquired the basic understanding that fractions are parts of things, this can be demonstrated with tangible items and then carried to symbols. They should be led to see that:

$$1 \frac{1}{2} \times 4 = ?$$

$$1 \frac{1}{2} + 1 \frac{1}{2} + 1 \frac{1}{2} = \underline{\hspace{2cm}}$$

or with a number line or ruler:



or they can see that the problem could be done just like multiplication of whole numbers.

$$\begin{array}{r} 3 \\ 1 \frac{1}{2} \\ \hline 3 \quad (1 \times 3) \\ 1 \frac{1}{2} \quad (1/2 \text{ of } 3) \\ \hline 4 \frac{1}{2} \quad (\text{total of } (1 \times 3) + (1/2 \times 3)) \end{array}$$

or  $1 \frac{1}{2} = 3/2$ , so we have, finally:

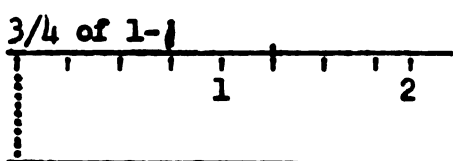
$$3/2 \times 3/1 = 9/2 = 4 \frac{1}{2}$$

Children need many experiences with each problem. They need to see it in the abstract and the concrete. The need to be able to find several ways of doing the same problem. Time might better be spent doing the same problem many ways and understanding it, than by doing many problems without understanding.

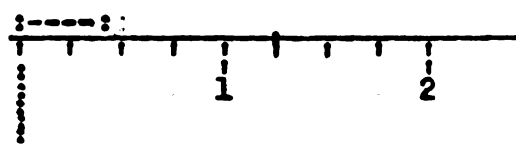
Children should be encouraged to invent reasonable, logical, and convenient ways of doing problems like this. They may make reports or do demonstrations, or make charts and graphs to explain their thinking.

3. How to multiply common fractions and mixed numbers may be approached in much the same manner as the above. Helping children to see the problem in many meaningful ways is better than dwelling on one method as such.

$3/4 \times 1 \frac{1}{2} = 3/4$  of 1 plus  $3/4$  of  $1/2$  would be easy to see if inches were used.



$3/4$  of  $1/2$





—

[illegible]

[illegible]

$$P_{\text{max}} = \frac{\rho}{2} \left( \frac{V_0}{A} \right)^2$$

— — — — —

[illegible]

1

Children can count the eighths and see that they will have  $9/8$  or  $1\frac{1}{2}$ . They can see that  $3/4 = 6/8$ . Half of  $3/4$  or  $6/8$  is then  $3/8$ . So, they will have  $6/8 + 3/8$  or  $9/8$  for the answer. Children need to be encouraged to think problems through logically.

To help in understanding that multiplication by fractions results in a smaller number than the multiplicand, introduce the expression taken so many times. Use this in the place of multiplied by, or times. Help children to understand that:

$$1\frac{1}{2} \times 3/4 = \text{means } 3/4 \text{ taken one and one-half } (1\frac{1}{2}) \text{ times.}$$

This may be shown in a sequence or number line relationship.

$$1 \times 3/4 = \text{means } 3/4 \text{ taken 1 time}$$

$$2 \times 3/4 = \text{means } 3/4 \text{ taken 2 times}$$

$$3 \times 3/4 = \text{means } 3/4 \text{ taken 3 times, etc.}$$

Now the half way points may be shown:

$$1 \times 3/4 = \text{means } 3/4 \text{ taken 1 time}$$

$$1\frac{1}{2} \times 3/4 = \text{means } 3/4 \text{ taken 1 and } 1/2 \text{ times}$$

$$2 \times 3/4 = \text{means } 3/4 \text{ taken 2 times}$$

$$2\frac{1}{2} \times 3/4 = \text{means } 3/4 \text{ taken 2 and } 1/2 \text{ times, etc.}$$

The problem may be done just as they have done multiplication with whole numbers in the past.

$$\begin{array}{r} 3/4 \\ \times 1\frac{1}{2} \\ \hline 3/4 \end{array} \quad (1 \times 3/4)$$

$$\begin{array}{r} 3/8 \quad (1/2 \times 3/4) \\ 3/4 + 3/8 \\ \hline \end{array} \quad \begin{array}{l} \text{(total of the two partial products)} \\ \end{array}$$

or

$$9/8 \quad \text{(combining with a common denominator)}$$

Such investigations as these, lead to understandings and promote insights that are often missed in rote situations. They are meaningful because they are based on things that the child already knows.

4. Children need to understand division as a process in order to understand division and multiplication of fractions.

$$\begin{array}{r} 4 \\ 3 \overline{) 12} \end{array} \quad \text{means} \quad \begin{array}{l} \text{there are 4 threes in 12} \\ \text{there are 4 groups of three in 12} \\ \text{3 can be subtracted from 12 four times} \\ 4 \times 3 = 12 \end{array}$$

1. The first part of the document is a list of names and addresses. The names are written in a cursive script, and the addresses are written in a more formal, printed style. The list is organized into columns, with names in the first column and addresses in the second column.

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In the same way:

$6/8 \div 3/8 = 2$  means there are 2 three-eighths in  $6/8$   
there are 2 groups of  $3/8$ 's in  $6/8$

$$\begin{array}{c} \text{or} \\ 6/8 = (1/8) (1/8) (1/8) \\ \quad (1/8) (1/8) (1/8) \end{array}$$

$3/8$  can be subtracted twice from  $6/8$   
 $2 \times 3/8 = 6/8$

Finding relationships and ways to express them such as these helps children to see that the answer 2 is not wrong or foolish, although it may first seem that way to them.

5. The fact that fractions can be written in different forms, all having the same value should be plain to children. This comes easily from the basic understanding that "fractions are parts of things". As illustrated on page 131, 1 cup is equal to  $1/16$  of a gallon. Taking this as a starting point, children can make true statements, such as:

1 cup =  $1/16$  gallon  
1 pint = 2 cups

therefore, 2 cups =  $2/16$  of a gallon

1 pint equals  $1/8$  of a gallon  
2 cups equal 1 pint

therefore,

1 pint equals  $2/16$  of a gallon

Children should be encouraged to make their own observations, record them and draw conclusions. The lesson in the appendix on the fractional or combo ruler helps children to see these relationships.

Making tables of equal fractions helps children to see this. Start with something that they know.

$$1/2 = 2/4, 4/8, 8/16, \text{ etc.}$$

$$1/3 = 2/6, 4/12, \text{ etc.}$$

Encourage the children to see that they can do this with tangible objects and with symbols. Then encourage them to draw the conclusion that they are multiplying the fraction, top and bottom, by the same number. After they have discovered this, help them see that the reason this multiplication does not change the value of the fractions is that  $n/n$  is always equal to one.

$2 \times \frac{1}{2} = \frac{2}{2}$   $\frac{2}{2}$  equals 1, therefore  $\frac{2}{4} = \frac{1}{2}$  because the fraction has been multiplied by the equivalent of 1. This leads directly to the discovery of lowest common denominators.

The following information was obtained from the records of the  
 Department of the Interior, Bureau of Land Management, regarding the  
 land owned by the United States in the State of California.  
 The total area of land owned by the United States in California is  
 approximately 100,000,000 acres. This land is divided into  
 several categories, including National Forest Land, National Monument  
 Land, National Park Land, and other public lands. The National Forest  
 Land is the largest category, covering approximately 60,000,000 acres.  
 The National Monument Land covers approximately 20,000,000 acres, and  
 the National Park Land covers approximately 10,000,000 acres. The  
 remaining 10,000,000 acres are classified as other public lands.  
 The land is managed by the Bureau of Land Management, which is  
 responsible for the conservation and management of the public lands.  
 The Bureau of Land Management is a part of the Department of the  
 Interior, and it is located in Washington, D.C. The Bureau of Land  
 Management has several offices throughout the State of California, and  
 it is responsible for the management of the public lands in each of  
 these offices. The Bureau of Land Management is also responsible for  
 the management of the public lands in the other States of the United  
 States.

6. Children need to have help in finding a lowest common denominator and help in a way to put it down so that it makes sense to them and can be read. Many children in the fifth and sixth grades have difficulty with fractions because their small muscle development has not reached a point where they can write them. If children have been taught that  $n/n = 1$ , they might be encouraged to write a problem involving lowest common denominators like the following:

$$\begin{array}{r} \frac{2}{2} \times \frac{1}{2} = \frac{2}{4} \\ \frac{1}{1} \times \frac{3}{4} = \frac{3}{4} \\ \hline \frac{5}{4} = 1 \frac{1}{4} \end{array}$$

It should be emphasized that in every case they are multiplying by the equivalent of 1.

7. In line with the above, they should see how to change a fraction to higher terms. They should see this graphically, tangibly, and then with symbols.

$$\begin{array}{r} \frac{3}{3} \times \frac{2}{5} = \frac{6}{15} \end{array}$$

THINK;  $2/5$  is equal to how many fifteenths? If I multiply the denominator by 3, that will give me fifteenths. I can multiply the numerator then by 3 and I will have 6 fifteenths. This is all right to do because  $3/3$  is equal to 1. Multiplying a number by 1 does not change its value.

8. In order to shorten difficult multiplication processes, children should know how to cancel. However, they should know that cancelling is the same as dividing the fraction, or fractions, by the equivalent of 1 or  $n/n$ .

$$\begin{array}{r} \frac{2}{3} \times \frac{3}{4} \times \frac{1}{2} = \\ \frac{2}{\cancel{3}} \times \frac{\cancel{3}}{4} \times \frac{1}{2} = \text{(dividing by } \frac{3}{3}) \\ \frac{2}{1} \times \frac{1}{4} \times \frac{1}{2} = \\ \frac{2}{\cancel{2}} \times \frac{\cancel{2}}{4} \times \frac{1}{2} = \text{(dividing by } \frac{2}{2}) \\ \frac{1}{1} \times \frac{1}{2} \times \frac{1}{2} = \end{array}$$

11-11-11

# UNDERSTANDINGS BASIC TO DIVISION OF FRACTIONS

1. Dividing by a number less than one will yield an answer greater than ONE. This should be thoroughly understood or the quotient in a division example is quite meaningless. Following are some suggestions as to how it may be approached.

Show division in a sequence of examples.

$$\begin{aligned} 3 \div 3 &= 1 \\ 3 \div 2 &= 1 \frac{1}{2} \\ 3 \div 1 &= 3 \\ 3 \div \frac{1}{2} &= 6 \\ 3 \div \frac{1}{4} &= 12, \text{ etc.} \end{aligned}$$

Another way to show this would be to indicate the number of halves or fourths in three. This can be done with symbols or a number line.

$$\begin{aligned} 3 \div \frac{1}{2} &= & 1 &= \frac{2}{2} \\ & & 1 &= \frac{2}{2} \\ & & 1 &= \frac{2}{2} \\ & & \hline & 3 &= \frac{6}{2} \end{aligned}$$

If a youngster knows "division as a process" as listed in the basic requirements, he can now see that  $3 \div \frac{1}{2}$  can mean how many halves in in three, etc.

$$\begin{array}{ccccccc} 3 \div \frac{1}{2} = & 0 & 1 & 2 & 3 & & \\ & 0 & \frac{1}{2} & \frac{2}{2} & \frac{3}{2} & \frac{4}{2} & \frac{5}{2} & \frac{6}{2} \end{array}$$

If the problem is one of the type:  $3 \div \frac{2}{4} =$ , we have much the same situation, except that the fourths are taken two at a time. If the youngsters have been introduced to this phase, they can see that now the question asks how many groups of two-fourths are there in three? This can be shown graphically and symbolically also:



$$3 \div \frac{2}{4} =$$

$$1 = \frac{4}{4} = \left(\frac{2}{4} + \frac{2}{4}\right)$$

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$$1 = \frac{4}{4} = \left(\frac{2}{4} + \frac{2}{4}\right)$$

$$3 = \frac{2}{4} + \frac{2}{4} + \frac{2}{4} + \frac{2}{4} + \frac{2}{4} + \frac{2}{4} = (6 \text{ two-fourths})$$

or there are 6 two-fourths in 3. The question is one of putting a label on the answer. It might be well for a time to have children label their answers for fraction problems in division.

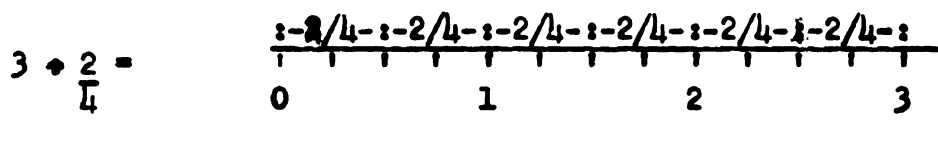
$$3 \div \frac{2}{4} = 6 \text{ two-fourths}$$

Logically, now students can be lead to see that we have to find out how many fourths there are in 3 and then take them 2 at a time. This means we have to multiply 3 by 4 and divide by 2, or INVERT and MULTIPLY.

$$3 \div \frac{2}{4} =$$

$$3 \times \frac{4}{2} = \frac{12}{2} \quad \text{the number of fourths in 3} = 6(2/4\text{'s}) \text{ taken two at a time.}$$

The same thing can be done with a number line and the problem shown graphically.



This is a logical approach to division of fractions. We simply have a convenient way of putting down the process. Children should have many experiences in "thinking about" what the process means. They should be encouraged to "find out for themselves and to experiment".

2. Another method for doing division of fractions is called the common denominator method. Children frequently discover this for themselves. Simply change the fractions to a lowest common denominator and divide as with whole numbers.

$$3 \div \frac{2}{4} =$$

$$\frac{12}{4} \div \frac{2}{4} = 6$$

or

$$3 \text{ fourths} \overline{) 12 \text{ fourths}} \quad \begin{array}{r} 6 \text{ times} \\ \hline \end{array}$$

This is a reasonable and understandable method for children that has an explanation lying within their previous experiences with fractions.

$$= \frac{1}{2} + \frac{1}{2} = 1$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

• The first part of the proof is to show that the function  $f$  is continuous at  $a$ . To do this, we need to show that for any  $\epsilon > 0$ , there exists a  $\delta > 0$  such that if  $|x - a| < \delta$ , then  $|f(x) - f(a)| < \epsilon$ .

$$= \frac{1}{2} + \frac{1}{2} = 1$$

• The second part of the proof is to show that the function  $f$  is differentiable at  $a$ . To do this, we need to show that the limit  $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$  exists and is equal to  $f'(a)$ .

$$= \frac{1}{2} + \frac{1}{2} = 1$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

• The third part of the proof is to show that the function  $f$  is differentiable at  $a$ . To do this, we need to show that the limit  $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$  exists and is equal to  $f'(a)$ .

• The fourth part of the proof is to show that the function  $f$  is differentiable at  $a$ . To do this, we need to show that the limit  $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$  exists and is equal to  $f'(a)$ .

$$= \frac{1}{2} + \frac{1}{2} = 1$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

3. A method that might be called the reciprocal method is possible to teach if children have mastered the idea that  $n/n = 1$ ; and that, for this reason, you can multiply both terms of a fraction by the same number without changing its value.

The difficulty in understanding this method is sometimes the difficulty of understanding the term reciprocal. Reciprocal refers to TWO numbers. OR TWO NUMBERS WHICH ARE MULTIPLIED TOGETHER SO THAT A PRODUCT OF 1 IS OBTAINED ARE RECIPROCALLS.

$$3 \times ? = 1 \quad 3 \times 1/3 = 1 \quad (3 \text{ and } 1/3 \text{ are reciprocals})$$

$$1/2 \times ? = 1 \quad 1/2 \times 2/1 = 1, \text{ etc.} (1/2 \text{ and } 2/1 \text{ are reciprocals})$$

We can state our division example as a fraction (complex fraction) and show this in a manner that will still be within the context of the previous experiences the children have had with fractions.

$$3 \div \frac{1}{4} =$$

$$\frac{3}{\frac{1}{4}}$$

If we multiply both terms by the same number, we are multiplying by 1 and this will not change the value of the fraction.

If we can make the denominator of this complex fraction 1, we will have the answer.

Therefore, multiply both terms by the reciprocal of the denominator and the denominator will then be 1.

$$\frac{\frac{4}{1} \times \frac{3}{1}}{\frac{1}{1} \times \frac{1}{4}} = \frac{\frac{4}{1} \times \frac{3}{1}}{1} = \frac{4}{1} \times \frac{3}{1} = \frac{12}{1} = 12$$

We have shown this in a round-about method. Now we can conclude, though, that the reciprocal of the divisor will be the divisor inverted, this will make the divisor 1 if we multiply both terms of the fraction by this reciprocal. So, all we have to do is to invert the divisor in the first place and multiply. This method gives a reason for inversion. Sometimes the reason is too difficult for all but a few youngsters. The important thing to remember is to give them a great many experiences, and many ways of thinking logically about fraction division. Without doubt, students and teachers can think of many additional ways of demonstrating the process. Time should be taken to investigate these. It will be well spent in developing interest in mathematics and real understanding.

## **V.      DECIMAL FRACTIONS**

**Decimal and Common Fractions**

**An Extension of Our Number System**

**Changing Common Fractions to Decimal Fractions**

Because decimal fractions all have denominators with powers of ten, we can leave the denominators off and write them as whole numbers if we can find a way to do it.

In order to do this, we put a . after the whole number and call all the numbers after the decimal point decimal fractions. The first place after the decimal point is tenths' place, the next, hundredths', etc., just as the places for the whole numbers.

WE CALL THIS WAY OF WRITING DECIMAL FRACTIONS AS AN EXTENSION OF OUR NUMBER SYSTEM. WE HAVE JUST EXTENDED THE SYSTEM IN ORDER TO WRITE THE DECIMAL FRACTIONS.

|                                 |                   |
|---------------------------------|-------------------|
| $10 \times 100,000 = 1,000,000$ | millions          |
| $10 \times 10,000 = 100,000$    | hundred-thousands |
| $10 \times 1,000 = 10,000$      | ten-thousands     |
| $10 \times 100 = 1,000$         | thousands         |
| $10 \times 10 = 100$            | hundreds          |
| $10 \times 1 = 10$              | tens              |
|                                 | ones              |
|                                 | DECIMAL POINT .   |
| $1/10$ of 1 =                   | tenths            |
| $1/10$ of $1/10$ =              | hundredths        |
| $1/10$ of $1/100$ =             | thousandths       |
| $1/10$ of $1/1000$ =            | ten-thousandths   |

|   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |    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185 | 186 | 187 | 188 | 189 | 190 | 191 | 192 | 193 | 194 | 195 | 196 | 197 | 198 | 199 | 200 | 201 | 202 | 203 | 204 | 205 | 206 | 207 | 208 | 209 | 210 | 211 | 212 | 213 | 214 | 215 | 216 | 217 | 218 | 219 | 220 | 221 | 222 | 223 | 224 | 225 | 226 | 227 | 228 | 229 | 230 | 231 | 232 | 233 | 234 | 235 | 236 | 237 | 238 | 239 | 240 | 241 | 242 | 243 | 244 | 245 | 246 | 247 | 248 | 249 | 250 | 251 | 252 | 253 | 254 | 255 | 256 | 257 | 258 | 259 | 260 | 261 | 262 | 263 | 264 | 265 | 266 | 267 | 268 | 269 | 270 | 271 | 272 | 273 | 274 | 275 | 276 | 277 | 278 | 279 | 280 | 281 | 282 | 283 | 284 | 285 | 286 | 287 | 288 | 289 | 290 | 291 | 292 | 293 | 294 | 295 | 296 | 297 | 298 | 299 | 300 | 301 | 302 | 303 | 304 | 305 | 306 | 307 | 308 | 309 | 310 | 311 | 312 | 313 | 314 | 315 | 316 | 317 | 318 | 319 | 320 | 321 | 322 | 323 | 324 | 325 | 326 | 327 | 328 | 329 | 330 | 331 | 332 | 333 | 334 | 335 | 336 | 337 | 338 | 339 | 340 | 341 | 342 | 343 | 344 | 345 | 346 | 347 | 348 | 349 | 350 | 351 | 352 | 353 | 354 | 355 | 356 | 357 | 358 | 359 | 360 | 361 | 362 | 363 | 364 | 365 | 366 | 367 | 368 | 369 | 370 | 371 | 372 | 373 | 374 | 375 | 376 | 377 | 378 | 379 | 380 | 381 | 382 | 383 | 384 | 385 | 386 | 387 | 388 | 389 | 390 | 391 | 392 | 393 | 394 | 395 | 396 | 397 | 398 | 399 | 400 | 401 | 402 | 403 | 404 | 405 | 406 | 407 | 408 | 409 | 410 | 411 | 412 | 413 | 414 | 415 | 416 | 417 | 418 | 419 | 420 | 421 | 422 | 423 | 424 | 425 | 426 | 427 | 428 | 429 | 430 | 431 | 432 | 433 | 434 | 435 | 436 | 437 | 438 | 439 | 440 | 441 | 442 | 443 | 444 | 445 | 446 | 447 | 448 | 449 | 450 | 451 | 452 | 453 | 454 | 455 | 456 | 457 | 458 | 459 | 460 | 461 | 462 | 463 | 464 | 465 | 466 | 467 | 468 | 469 | 470 | 471 | 472 | 473 | 474 | 475 | 476 | 477 | 478 | 479 | 480 | 481 | 482 | 483 | 484 | 485 | 486 | 487 | 488 | 489 | 490 | 491 | 492 | 493 | 494 | 495 | 496 | 497 | 498 | 499 | 500 | 501 | 502 | 503 | 504 | 505 | 506 | 507 | 508 | 509 | 510 | 511 | 512 | 513 | 514 | 515 | 516 | 517 | 518 | 519 | 520 | 521 | 522 | 523 | 524 | 525 | 526 | 527 | 528 | 529 | 530 | 531 | 532 | 533 | 534 | 535 | 536 | 537 | 538 | 539 | 540 | 541 | 542 | 543 | 544 | 545 | 546 | 547 | 548 | 549 | 550 | 551 | 552 | 553 | 554 | 555 | 556 | 557 | 558 | 559 | 560 | 561 | 562 | 563 | 564 | 565 | 566 | 567 | 568 | 569 | 570 | 571 | 572 | 573 | 574 | 575 | 576 | 577 | 578 | 579 | 580 | 581 | 582 | 583 | 584 | 585 | 586 | 587 | 588 | 589 | 590 | 591 | 592 | 593 | 594 | 595 | 596 | 597 | 598 | 599 | 600 | 601 | 602 | 603 | 604 | 605 | 606 | 607 | 608 | 609 | 610 | 611 | 612 | 613 | 614 | 615 | 616 | 617 | 618 | 619 | 620 | 621 | 622 | 623 | 624 | 625 | 626 | 627 | 628 | 629 | 630 | 631 | 632 | 633 | 634 | 635 | 636 | 637 | 638 | 639 | 640 | 641 | 642 | 643 | 644 | 645 | 646 | 647 | 648 | 649 | 650 | 651 | 652 | 653 | 654 | 655 | 656 | 657 | 658 | 659 | 660 | 661 | 662 | 663 | 664 | 665 | 666 | 667 | 668 | 669 | 670 | 671 | 672 | 673 | 674 | 675 | 676 | 677 | 678 | 679 | 680 | 681 | 682 | 683 | 684 | 685 | 686 | 687 | 688 | 689 | 690 | 691 | 692 | 693 | 694 | 695 | 696 | 697 | 698 | 699 | 700 | 701 | 702 | 703 | 704 | 705 | 706 | 707 | 708 | 709 | 710 | 711 | 712 | 713 | 714 | 715 | 716 | 717 | 718 | 719 | 720 | 721 | 722 | 723 | 724 | 725 | 726 | 727 | 728 | 729 | 730 | 731 | 732 | 733 | 734 | 735 | 736 | 737 | 738 | 739 | 740 | 741 | 742 | 743 | 744 | 745 | 746 | 747 | 748 | 749 | 750 | 751 | 752 | 753 | 754 | 755 | 756 | 757 | 758 | 759 | 760 | 761 | 762 | 763 | 764 | 765 | 766 | 767 | 768 | 769 | 770 | 771 | 772 | 773 | 774 | 775 | 776 | 777 | 778 | 779 | 780 | 781 | 782 | 783 | 784 | 785 | 786 | 787 | 788 | 789 | 790 | 791 | 792 | 793 | 794 | 795 | 796 | 797 | 798 | 799 | 800 | 801 | 802 | 803 | 804 | 805 | 806 | 807 | 808 | 809 | 810 | 811 | 812 | 813 | 814 | 815 | 816 | 817 | 818 | 819 | 820 | 821 | 822 | 823 | 824 | 825 | 826 | 827 | 828 | 829 | 830 | 831 | 832 | 833 | 834 | 835 | 836 | 837 | 838 | 839 | 840 | 841 | 842 | 843 | 844 | 845 | 846 | 847 | 848 | 849 | 850 | 851 | 852 | 853 | 854 | 855 | 856 | 857 | 858 | 859 | 860 | 861 | 862 | 863 | 864 | 865 | 866 | 867 | 868 | 869 | 870 | 871 | 872 | 873 | 874 | 875 | 876 | 877 | 878 | 879 | 880 | 881 | 882 | 883 | 884 | 885 | 886 | 887 | 888 | 889 | 890 | 891 | 892 | 893 | 894 | 895 | 896 | 897 | 898 | 899 | 900 | 901 | 902 | 903 | 904 | 905 | 906 | 907 | 908 | 909 | 910 | 911 | 912 | 913 | 914 | 915 | 916 | 917 | 918 | 919 | 920 | 921 | 922 | 923 | 924 | 925 | 926 | 927 | 928 | 929 | 930 | 931 | 932 | 933 | 934 | 935 | 936 | 937 | 938 | 939 | 940 | 941 | 942 | 943 | 944 | 945 | 946 | 947 | 948 | 949 | 950 | 951 | 952 | 953 | 954 | 955 | 956 | 957 | 958 | 959 | 960 | 961 | 962 | 963 | 964 | 965 | 966 | 967 | 968 | 969 | 970 | 971 | 972 | 973 | 974 | 975 | 976 | 977 | 978 | 979 | 980 | 981 | 982 | 983 | 984 | 985 | 986 | 987 | 988 | 989 | 990 | 991 | 992 | 993 | 994 | 995 | 996 | 997 | 998 | 999 | 1000 | 1001 | 1002 | 1003 | 1004 | 1005 | 1006 | 1007 | 1008 | 1009 | 1010 | 1011 | 1012 | 1013 | 1014 | 1015 | 1016 | 1017 | 1018 | 1019 | 1020 | 1021 | 1022 | 1023 | 1024 | 1025 | 1026 | 1027 | 1028 | 1029 | 1030 | 1031 | 1032 | 1033 | 1034 | 1035 | 1036 | 1037 | 1038 | 1039 | 1040 | 1041 | 1042 | 1043 | 1044 | 1045 | 1046 | 1047 | 1048 | 1049 | 1050 | 1051 | 1052 | 1053 | 1054 | 1055 | 1056 | 1057 | 1058 | 1059 | 1060 | 1061 | 1062 | 1063 | 1064 | 1065 | 1066 | 1067 | 1068 | 1069 | 1070 | 1071 | 1072 | 1073 | 1074 | 1075 | 1076 | 1077 | 1078 | 1079 | 1080 | 1081 | 1082 | 1083 | 1084 | 1085 | 1086 | 1087 | 1088 | 1089 | 1090 | 1091 | 1092 | 1093 | 1094 | 1095 | 1096 | 1097 | 1098 | 1099 | 1100 | 1101 | 1102 | 1103 | 1104 | 1105 | 1106 | 1107 | 1108 | 1109 | 1110 | 1111 | 1112 | 1113 | 1114 | 1115 | 1116 | 1117 | 1118 | 1119 | 1120 | 1121 | 1122 | 1123 | 1124 | 1125 | 1126 | 1127 | 1128 | 1129 | 1130 | 1131 | 1132 | 1133 | 1134 | 1135 | 1136 | 1137 | 1138 | 1139 | 1140 | 1141 | 1142 | 1143 | 1144 | 1145 | 1146 | 1147 | 1148 | 1149 | 1150 | 1151 | 1152 | 1153 | 1154 | 1155 | 1156 | 1157 | 1158 | 1159 | 1160 | 1161 | 1162 | 1163 | 1164 | 1165 | 1166 | 1167 | 1168 | 1169 | 1170 | 1171 | 1172 | 1173 | 1174 | 1175 | 1176 | 1177 | 1178 | 1179 | 1180 | 1181 | 1182 | 1183 | 1184 | 1185 | 1186 | 1187 | 1188 | 1189 | 1190 | 1191 | 1192 | 1193 | 1194 | 1195 | 1196 | 1197 | 1198 | 1199 | 1200 | 1201 | 1202 | 1203 | 1204 | 1205 | 1206 | 1207 | 1208 | 1209 | 1210 | 1211 | 1212 | 1213 | 1214 | 1215 | 1216 | 1217 | 1218 | 1219 | 1220 | 1221 | 1222 | 1223 | 1224 | 1225 | 1226 | 1227 | 1228 | 1229 | 1230 | 1231 | 1232 | 1233 | 1234 | 1235 | 1236 | 1237 | 1238 | 1239 | 1240 | 1241 | 1242 | 1243 | 1244 | 1245 | 1246 | 1247 | 1248 | 1249 | 1250 | 1251 | 1252 | 1253 | 1254 | 1255 | 1256 | 1257 | 1258 | 1259 | 1260 | 1261 | 1262 | 1263 | 1264 | 1265 | 1266 | 1267 | 1268 | 1269 | 1270 | 1271 | 1272 | 1273 | 1274 | 1275 | 1276 | 1277 | 1278 | 1279 | 1280 | 1281 | 1282 | 1283 | 1284 | 1285 | 1286 | 1287 | 1288 | 1289 | 1290 | 1291 | 1292 | 1293 | 1294 | 1295 | 1296 | 1297 | 1298 | 1299 | 1300 | 1301 | 1302 | 1303 | 1304 | 1305 | 1306 | 1307 | 1308 | 1309 | 1310 | 1311 | 1312 | 1313 | 1314 | 1315 | 1316 | 1317 | 1318 | 1319 | 1320 | 1321 | 1322 | 1323 | 1324 | 1325 | 1326 | 1327 | 1328 | 1329 | 1330 | 1331 | 1332 | 1333 | 1334 | 1335 | 1336 | 1337 | 1338 | 1339 | 1340 | 1341 | 1342 | 1343 | 1344 | 1345 | 1346 | 1347 | 1348 | 1349 | 1350 | 1351 | 1352 | 1353 | 1354 | 1355 | 1356 | 1357 | 1358 | 1359 | 1360 | 1361 | 1362 | 1363 | 1364 | 1365 | 1366 | 1367 | 1368 | 1369 | 1370 | 1371 | 1372 | 1373 | 1374 | 1375 | 1376 | 1377 | 1378 | 1379 | 1380 | 1381 | 1382 | 1383 | 1384 | 1385 | 1386 | 1387 | 1388 | 1389 | 1390 | 1391 | 1392 | 1393 | 1394 | 1395 | 1396 | 1397 | 1398 | 1399 | 1400 | 1401 | 1402 | 1403 | 1404 | 1405 | 1406 | 1407 | 1408 | 1409 | 1410 | 1411 | 1412 | 1413 | 1414 | 1415 | 1416 | 1417 | 1418 | 1419 | 1420 | 1421 | 1422 | 1423 | 1424 | 1425 | 1426 | 1427 | 1428 | 1429 | 1430 | 1431 | 1432 | 1433 | 1434 | 1435 | 1436 | 1437 | 1438 | 1439 | 1440 | 1441 | 1442 | 1443 | 1444 | 1445 | 1446 | 1447 | 1448 | 1449 | 1450 | 1451 | 1452 | 1453 | 1454 | 1455 | 1456 | 1457 | 1458 | 1459 | 1460 | 1461 | 1462 | 1463 | 1464 | 1465 | 1466 | 1467 | 1468 | 1469 | 1470 | 1471 | 1472 | 1473 | 1474 | 1475 | 1476 | 1477 | 1478 | 1479 | 1480 | 1481 | 1482 | 1483 | 1484 | 1485 | 1486 | 1487 | 1488 | 1489 | 1490 | 1491 | 1492 | 1493 | 1494 | 1495 | 1 |
|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-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Reading decimal fractions is comparatively simple if we refer to the chart. The next problem is how to change all fractions to decimal fractions. To approach this logically and within the context of what the children know about fractions and lowest common denominators, is better than presenting a rule to be memorized.

Some common fractions can be changed to decimal fractions with little or no margin of error. Some examples will demonstrate this.

$$\begin{array}{ll} \frac{1}{2} = \frac{?}{10} & \frac{1}{2} = \frac{?}{100} \\ \frac{5}{5} \times \frac{1}{2} = \frac{5}{10} & \frac{50}{50} \times \frac{1}{2} = \frac{50}{100} \end{array}$$

However:

$$\begin{array}{ll} \frac{1}{4} = \frac{?}{10} & \frac{1}{4} = \frac{?}{100} \\ \frac{2}{2} \times \frac{1}{4} = \frac{2}{10} & \frac{25}{25} \times \frac{1}{4} = \frac{25}{100} \end{array}$$

In the first example, we have a choice of multiplying 4 by 2 1/2 in order to make the decimal come out even. This is foolish, since the purpose of decimals is to make them easy to use. So, we carry the fraction one more place and it comes out even. Our error would be less if it came out 1/2 of a hundredth than if it came out 1/2 of a tenth.

WE ARE MULTIPLYING BOTH TERMS OF THESE FRACTIONS BY THE EQUIVALENT OF 1, JUST AS WE DID WITH LOWEST COMMON DENOMINATORS BEFORE.

In order to find out what to multiply the common fraction denominator by, THINK: "What do I multiply 4 by to get ten?" Or, "What do I multiply 4 by to get 100?", etc. A problem like the following, is more difficult. Students may actually need to perform the division in order to find out what number to use as a multiplier.

$$\frac{3}{16} = \frac{?}{100} \qquad 16 \overline{) 100} \qquad \frac{6}{6} \times \frac{3}{16} = \frac{18}{100}$$

It may be necessary for children to divide 100 by 16 in this case before they can complete the equation. 18/100 is as close as we can come with whole numbers.





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~~JUN 17 1966~~

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