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OPTIMAL DESIGN OF VIBRATION ISOLATORS BASED ON FREQUENCY RESPONSE

By

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ABSTRACT

OPTIMAL DESIGN OF VIBRATION ISOLATORS BASED ON FREQUENCY RESPONSE

By

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In this thesis, an approach to vibration isolator design is defined based on the frequency response at important points. The frequency response approach is incorporated into a numerical optimization scheme for vibration isolators. A planar test case is defined which shows the viability of the frequency response approach. Using the test case, this approach is compared to another approach based on natural frequencies. The test case shows that the frequency response approach may yield less conservative isolator designs than the natural frequency approach.

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Figure C.1 Subroutine Calling Tree for the PVIP Program C.2

NOMENCLATURE

a,b	-	weighting factors between size of design change and dynamic response
C	-	size of design change part of objective function
[C]	-	(6x6) viscous damping matrix for the test case
c_i	-	weighting factor on magnitude of displacement at ith important point
$\underline{e}$	-	(qx1) design parameter vector
Δe_j	-	change in jth design parameter
E_j	-	normalization factor for change in jth design parameter
$\underline{f}$	-	(6x1) excitation vector for the test case
$\underline{F}$	-	(6x1) time invariant force amplitudes for the test case
G	-	excitation frequency probability function
[K]	-	(6x6) stiffness matrix for the test case
$[k_{mi}]$	-	(3x3) stiffness matrix for the ith mount compliant element
$[k_{fi}]$	-	(3x3) stiffness matrix for the ith support structure compliant element
[M]	-	(6x6) mass matrix for the test case
P	-	objective function
q	-	number of design parameters
r	-	number of important points
R	-	dynamic response part of objective function (based on frequency response)

R' - dynamic response part of objective function (based on natural frequencies)

S_i - magnitude of displacement at ith important point

$\underline{s}_i^*$ - (3x1) complex frequency response at ith important point

$[T]$ - (3rx6) transformation matrix which maps general body co-ordinates of test case to design-significant co-ordinates

$\underline{x}$ - (6x1) displacement vector for test case

$\underline{x}^*$ - (6x1) complex frequency response vector for test case

$\underline{x}_b$ - (3x1) displacement vector at c.g. of excited body: $[\ x_b\ y_b\ \theta_b\ ]^T$

$\underline{x}_f$ - (3x1) displacement vector at c.g. of support structure: $[\ x_f\ y_f\ \theta_f\ ]^T$

$\underline{y}$ - (3rx1) design-significant co-ordinates vector

ω_c - critical excitation frequency

ω_l - lower limit of excitation frequency

ω_h - upper limit of excitation frequency

INTRODUCTION

Vibration isolation is a commonly occuring engineering problem which arises when a vibrating body is mounted to a supporting structure. The isolation design problem can take one of two forms. In the first of these, the vibrating body is excited by forces acting on it and the goal is to reduce vibrations transmitted to the supporting structure. Examples include the mounting of machines with rotating imbalances such as electric motors, gas compressors, pumps, turbines, and automotive engines. In the second type of isolation problem, support structure motions are the source of excitation and the goal is to reduce forces transmitted through the mounts to the vibrating body. The design of building foundations is an example of this type of isolation problem.

A common approach to vibration isolation problems is to reduce transmitted forces in the mounts by moving the system's natural frequencies away from the excitation frequency. This approach originates from the transmissibility concept applied to simple uniaxial systems. The advantage of this approach is that it reduces vibration throughout the system. A disadvantage of this approach is that it does not take full advantage of the mechanisms by which vibration in a system can be reduced. The mode shapes of a system also determine the degree to which it is excited. If the mode shape can be modified so that it is orthogonal to the excitation, the excitation frequency can match the natural frequency and the mode will not be excited. Also, if particular points in the system are more important than others in regard to vibration, the vibration at these points can be reduced by modifying mode shapes. Based on these comments, the transmissibility approach to vibration isolation problems could be conservative. In design cycles which limit mount design options, a conservative approach can be costly.

The approach to vibration isolation problems taken in this research is to find a design which reduces the vibration at points in the system where isolation is particularly important. The dynamics of the system for different mount designs can be simulated on a computer and the frequency response at important points can be calculated. Searching for a design which maximally reduces vibration can be done manually, but the large number of possible design changes often makes this impractical. A better approach is to numerically identify the mount design using an optimization technique similar to that used previously [4].

In this thesis, the viability of the frequency response approach will be demonstrated using a planar test case. An objective function based on the vibration at important points will be defined for the test case. This objective function will be incorporated into a computer program which simulates the test case dynamics and optimizes the mount design. A previously defined [4] objective function based on the transmissibility approach will also be presented. The mount design found using the transmissibility-based objective will be compared to the mount design found using the frequency-response-based objective.

SYSTEM RESPONSE ANALYSIS

There are several ways to characterize the vibration at a point in a system. The quantity which will be used in the proposed objective function is the magnitude of the translational displacement at a point. Small motions, linear stiffnesses, and a combination of structural and viscous damping are assumed for the complete system consisting of vibratory body, mounts, and support structure. These simplifying assumptions allow the equations of motion for the complete system to be written in the following form [3]:

$$[M]\ddot{\underline{x}} + [C]\dot{\underline{x}} + [\ [K] + [D]i\ ]\underline{x} = \underline{f} \qquad (1)$$

The excitation vector $\underline{f}$ is assumed to be harmonic:

$$\underline{f} = \underline{F}\, e^{i\omega t} \qquad (2)$$

The vector $\underline{F}$ represents the time invariant force amplitudes. To solve for $\underline{x}$, a harmonic solution of the same frequency but with a phase shift is assumed.

$$\underline{x} = \begin{bmatrix} X_1 e^{i\phi_1} \\ X_2 e^{i\phi_2} \\ \\ X_n e^{i\phi_n} \end{bmatrix} e^{i\omega t} \qquad (3)$$

Substituting the assumed solution into the equations of motion, the following equation is formed:

$$[A]\underline{x}^* = \underline{F} \tag{4}$$

where

$$[A] = [K] - \omega^2[M] + \omega[C]i + [D]i \tag{5}$$

$$\underline{x}^* = \begin{bmatrix} X_1 e^{i\phi_1} \\ X_2 e^{i\phi_2} \\ \\ X_n e^{i\phi_n} \end{bmatrix} \tag{6}$$

The complex frequency response of the system in body co-ordinates can be found by solving for $\underline{x}^*$ in Equation 4. The body co-ordinates can then be transformed to a set of design-significant co-ordinates which express the motion at points of importance:

$$\underline{y}^* = [T]\underline{x}^* \tag{7}$$

where

$\underline{y}^*$ = frequency response in design-significant co-ordinates

$[T]$ = appropriate transformation matrix

The magnitude of translational vibration at points of importance can then be computed from the components of $\underline{y}^*$.

NUMERICAL OPTIMIZATION OF THE MOUNT DESIGN

The best mount design can be found from the large number of possible designs by performing an unconstrained optimization. The design is first parameterized and then a parameter-dependent objective function is formulated which decreases as the design objectives are met. By finding the design parameters which minimize the objective function, the best mount design can be found. The search for the optimal design parameters and the subsequent evaluation of the objective function can be performed by a computer program.

The parameters for the mount design are the stiffness, location, and orientation of each of the mounts. The objective of the mount design is to isolate vibrations in the system at the least cost. The cost of a design is defined by the size of design changes from a nominal design. Large changes in the nominal design represent a great cost. The isolating capability of the design is measured by the response of the system to the excitation. An objective function which incorporates both system response and changes in design is as follows [5]:

$$P(\underline{e}) = aR(\underline{e}) + bC(\underline{e}) \qquad (8)$$

The vector $\underline{e}$ contains the mount design parameters. The scalar R measures the dynamic response of the system, while the scalar C measures changes in the nominal design. The scalars a and b are weighting factors which define the relative importance between dynamic response of the design and changes in the nominal design.

The size-of-change function, C, used in this work is positive definite and increases with increasing differences between the current and the nominal design [5].

$$C(\underline{e}) = \sum_{j=1}^{q} (\Delta e_j/E_j)^2 \tag{9}$$

The design change measure, Δe_j, is the difference between the current and nominal value of the jth design parameter. Because different design changes are not always comparable, a factor E_j is used to normalize each term.

The response function, R, used in this work is positive definite and increases with increasing vibration at points in the system defined as important.

$$R(\underline{e}) = \int_{\omega_1}^{\omega_h} G(\omega) \sum_{i=1}^{r} c_i S_i(\underline{e},\omega) \quad d\omega \tag{10}$$

$S_i(\underline{e},\omega)$ is the magnitude of the translational displacement at an important point. A weighted summation of the important points is taken so that the relative importance of each point is defined. The function $G(\omega)$ expresses the likelihood of excitation occurring at the frequency ω (Figure 1). The interpretation of the triangular form is that the excitation is most likely to occur at the frequency ω_c.

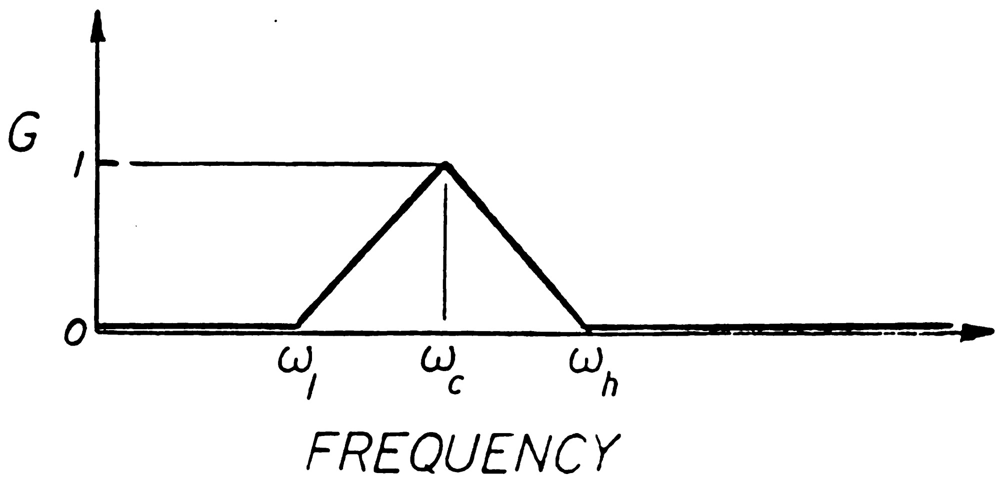


Figure 1 Example of Excitation Frequency Probability

Another type of response function based on system natural frequencies was proposed by Spiekermann, Radcliffe, and Goodman [4].

$$R'(\underline{e}) = \sum_{i=1}^{n} r_i(\underline{e}) \tag{11}$$

where

$$r_i(\underline{e}) = 0.0 \qquad \text{for} \qquad \omega_i < \omega_l \quad \text{or} \quad \omega_i > \omega_h \tag{12}$$

$$r_i(\underline{e}) = .5 + .5\mathrm{COS}(\ 2\pi(\omega_i - (\omega_h + \omega_l)/2)/(\omega_h - \omega_l)\ )$$
$$\text{for} \qquad \omega_l < \omega_i < \omega_h$$

In the above equation, $r_i(\underline{e})$ is the contribution of the ith natural frequency of the system to the response function, R'. The frequencies ω_l and ω_h are the lower and upper frequency bounds of the excitation. The frequency ω_i is the ith natural frequency of the system. The natural frequencies are dependent on the equations of motion of the system, and hence the design parameters $\underline{e}$.

TEST CASE

The test case consists of two rigid bodies whose motion is restricted to a plane. The test case is shown schematically in Figure 2. The upper rigid body is the excited vibratory system and the compliant elements M1 and M2 are the mounts. The lower rigid body represents the support structure and the compliant elements F1 and F2 represent the stiffness properties of the support structure.

Figures 3 and 4 show the model used for a compliant element. As shown in Figure 3, there is a local co-ordinate system associated with each compliant element. The position and orientation of each compliant element is defined by the position (q_{xi},q_{yi}) and orientation (ϕ_i) of its local co-ordinate system. It is assumed that a compliant element is small in relation to the rigid bodies, therefore its position can be defined by a point. Figure 4 gives the force/deformation relationship for a compliant element. Several simplifying assumptions are made in deriving this relationship. First, a compliant element is assumed to behave in a linear elastic manner. Second, the deformations of a compliant element are assumed small in relation to its dimensions; therefore, its stiffness matrix is diagonal. Third, the contribution of a compliant element's torsional stiffness to the total torque exerted on a rigid body is assumed to be small, therefore an element's torsional stiffness is neglected. Lastly, a compliant element is assumed to possess sub-critical viscous damping in proportion to its stiffness.

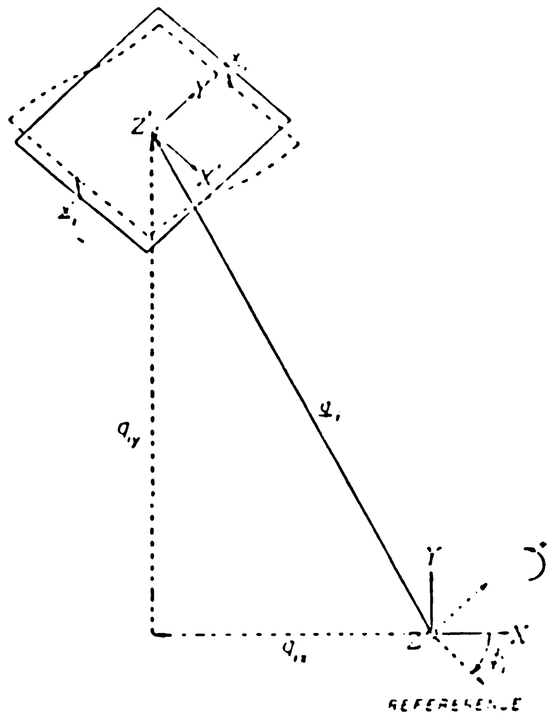
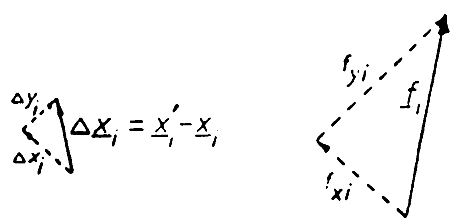


Figure 3 Position and Orientation of a Compliant Element



$$\begin{bmatrix} f_{xi} \\ f_{yi} \\ T_{zi} \end{bmatrix} = \begin{bmatrix} k_{xi} & 0 & 0 \\ 0 & k_{yi} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta \theta_i \end{bmatrix} + \begin{bmatrix} c_{xi} & 0 & 0 \\ 0 & c_{yi} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \dot{x}_i \\ \Delta \dot{y}_i \\ \Delta \dot{\theta}_i \end{bmatrix}$$

Figure 4 Force/Deformation Relationship for a Compliant Element

Table 1 Compliant Element Parameters for the Planar Test Case

	Axial Stiffness (N/M)	Shear Stiffness (N/M)	Horizontal Position (M)	Vertical Position (M)	Orientation (Degrees)
M1	5.0×10^5	6.5×10^5	-.400	.070	0.0
M2	5.0×10^5	6.5×10^5	-.250	.070	0.0
F1	1.2×10^7	1.9×10^6	-.560	0.00	0.0
F2	1.2×10^7	1.9×10^6	.560	0.00	0.0

Table 2 Rigid Body Parameters for the Planar Test Case

	Mass (kg)	Moment of Inertia (kg*m**2)	Horizontal Position (M)	Vertical Position (M)	Orientation (Degrees)
Support Structure	60.0	15.0	0.00	0.00	0.0
Excited Body	30.0	1.00	-.325	.270	0.0

Each compliant element is characterized by the five parameters q_{xi}, q_{yi}, ϕ_i, k_{xi} and k_{yi}. Each of the rigid bodies is also characterized by five parameters. A local co-ordinate system is placed at the c.g. of each rigid body and therefore three of the parameters define the position and orientation of the body's local system with respect to the reference system. The other two parameters define the mass and rotational inertia of the body.

The design parameters for the test case are the ten parameters which define the two mount compliant elements. The other twenty parameters which define the two support structure compliant elements and the two rigid bodies remain fixed. All thirty parameters used in the test case are given in Tables 1 and 2.

Using the assumed mount model, the equations of motion (eqn. 1) are developed in Appendix A using previously defined matrix methods [1]. The excitation for the test case is modelled by a force and torque applied to the c.g. of the upper rigid body. The displacement vector for the test case, $\underline{x} = [\ x_B\ y_B\ \theta_B\ x_F\ y_F\ \theta_F\ ]$, defines the rotation and translation at the c.g. of each body. The mass, stiffness, and damping matrices are as follows:

$$[M] = \begin{bmatrix} m_b & 0 & 0 & 0 & 0 & 0 \\ 0 & m_b & 0 & 0 & 0 & 0 \\ 0 & 0 & I_b & 0 & 0 & 0 \\ 0 & 0 & 0 & m_f & 0 & 0 \\ 0 & 0 & 0 & 0 & m_f & 0 \\ 0 & 0 & 0 & 0 & 0 & I_f \end{bmatrix} \tag{13}$$

$$[C] = \begin{bmatrix} [c_1] & -[c_2] \\ -[c_2] & [c_3] \end{bmatrix} \tag{14}$$

$$[K] = \begin{bmatrix} [k_1] & -[k_2] \\ -[k_2] & [k_3] \end{bmatrix} \tag{15}$$

where

$$[k_1] = \left[\sum_{i=1}^{2} [P_{bmi}][R_{bmi}][k_{mi}][R_{bmi}]^T[P_{bmi}]^T \right] \tag{16}$$

$$[k_2] = \left[\sum_{i=1}^{2} [P_{bmi}][R_{bmi}][k_{mi}][R_{fmi}]^T[P_{fmi}]^T \right] \tag{17}$$

$$[k_3] = \left[\sum_{i=1}^{2} [P_{ffi}][R_{ffi}][k_{fi}][R_{ffi}]^T[P_{ffi}]^T \right] + \left[\sum_{i=1}^{2} [P_{fmi}][R_{fmi}][k_{mi}][R_{fmi}]^T[P_{fmi}]^T \right] \tag{18}$$

The P and R matrices used in equations 16-18 are 3x3 translation and rotation matrices. The matrices $[k_{mi}]$ and $[k_{fi}]$ are the 3x3 stiffness matrices of the compliant elements. The submatrices $[c_1]$, $[c_2]$, and $[c_3]$ are proportional to $[k_1]$, $[k_2]$, and $[k_3]$ by a factor of .001 .

Once the equations of motion are formulated, the frequency response at points of importance can be calculated using equations 4 and 7. For the test case, the points defined as important are part of the support structure. The transformation from the general body co-ordinates to the design-significant co-ordinates is as follows:

$$\underline{y}^* = \begin{bmatrix} \underline{s}_1^* \\ \underline{s}_2^* \\ \\ \underline{s}_i^* \\ \\ \underline{s}_r^* \end{bmatrix} = \begin{bmatrix} [T_1] \\ [T_2] \\ \\ [T_i] \\ \\ [T_r] \end{bmatrix} \underline{x}^* \qquad (19)$$

where

$$\underline{s}_i^* = \begin{bmatrix} s_{ix}^* \\ s_{iy}^* \\ s_{i\theta}^* \end{bmatrix} \qquad (20)$$

$$[T_i] = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & -r_{yi} \\ 0 & 0 & 0 & 0 & 1 & r_{xi} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (21)$$

$$\underline{r}_i = [\ r_{xi} \quad r_{yi} \quad 0\ ]^T \qquad (22)$$

The vector $\underline{x}^*$ is the complex frequency response for the test case in body co-ordinates. The vector $\underline{s}_i^*$ is the complex frequency response at the ith important point. The vector $\underline{r}_i$ is the position of the ith important point relative to the c.g. of the support structure; it is expressed in local support structure co-ordinates. The magnitude of the translational vibration at the ith important point is given by the following equation:

$$s_i = \left(|s_{ix}^*|^2 + |s_{iy}^*|^2 \right)^{1/2} \tag{23}$$

Once the vibration at points of importance have been defined, the previously defined objective function P(e) can be used to optimize the mount parameters for the test case. The optimization is performed by a program called PVIP, an acronym for Planar Vibration Isolation Program. The program first reads in the various quantities necessary to calculate the objective function. These include the inertial and stiffness properties of the test case, the excitation, the design change normalizing factors, the important points, and the objective weighting factors. With this information, the minimization of the objective function is performed by the IMSL subroutine ZXMIN [2]. The objective function which is evaluated can use either of the two dynamic response objectives, the objective based on vibration at important points or the objective based on the system's natural frequencies. The resulting optimal design is output along with the initial design. The program also plots the vibration at important points versus excitation frequency. Other capabilities of the program include the plotting of two-dimensional design spaces and the calculation of undamped eigenvalues and eigenvectors.

For the test case, there are a total of ten design parameters which can be optimized. A partial optimization can be performed in which eight of the parameters are held constant and the remaining two parameters are optimized. Results from a partial optimization are useful to visualize the design space.

RESULTS

Two partial optimizations are performed with different excitations and design parameters. In the first optimization, the optimal axial stiffness for the two mounts is found. The important point in the support structure is defined to be .25 meters to the right of the center of gravity of the support structure. The system is excited by a 30.0 newton horizontal force acting on the upper body. The excitation is most likely to occur at 8.0 Hz, with a 2.0 Hz band of uncertainty. The form of $G(\omega)$ is shown in Figure 1 with ω_c, ω_l, and ω_h defined as 8.0, 7.0, and 9.0 Hz, respectively. The results of this optimization using both the frequency response objective and the transmissibility objective are shown in Table 3.

The values of the frequency response objective and the transmissibility objective over the design space are shown in Figures 5 and 6. The apex of the inverted pyramid represents the initial design, the apex of the upright pyramid the optimal design. The captions above the plots define the quantity plotted along each of axis. The numbers enclosed in parentheses define the range of values which appear on the plot for each quantity. The design space of the frequency response objective appears smooth, a condition necessary for convergence of the optimization. In both plots it appears that the optimization has converged to a local minimum. Both types of objectives reveal a ridge which arcs through the design space. This ridge represents those

Table 3 Optimal Axial Stiffness for the Mounts

	Initial Axial Stiffness (N/M)	Optimal Axial Stiffness (N/M)	Normalizing Factor, E	Normalized Change
Frequency Response Objective				
Mount #1	5.00×10^{5}	1.57×10^{5}	2.00×10^{6}	-1.71×10^{-1}
Mount #2	5.00×10^{5}	4.68×10^{5}	2.00×10^{6}	-1.61×10^{-2}
Transmissibility Objective				
Mount #1	5.00×10^{5}	3.90×10^{5}	2.00×10^{6}	-5.51×10^{-2}
Mount #2	5.00×10^{5}	3.94×10^{5}	2.00×10^{6}	-5.29×10^{-2}

X: MOUNT #1 AXIAL STIFFNESS, N/M (RANGES FROM 62500 TO 937500)
Y: MOUNT #2 AXIAL STIFFNESS, N/M (RANGES FROM 62500 TO 937500)
Z: FREQUENCY RESPONSE OBJECTIVE (RANGES FROM .0437 TO 1.08)

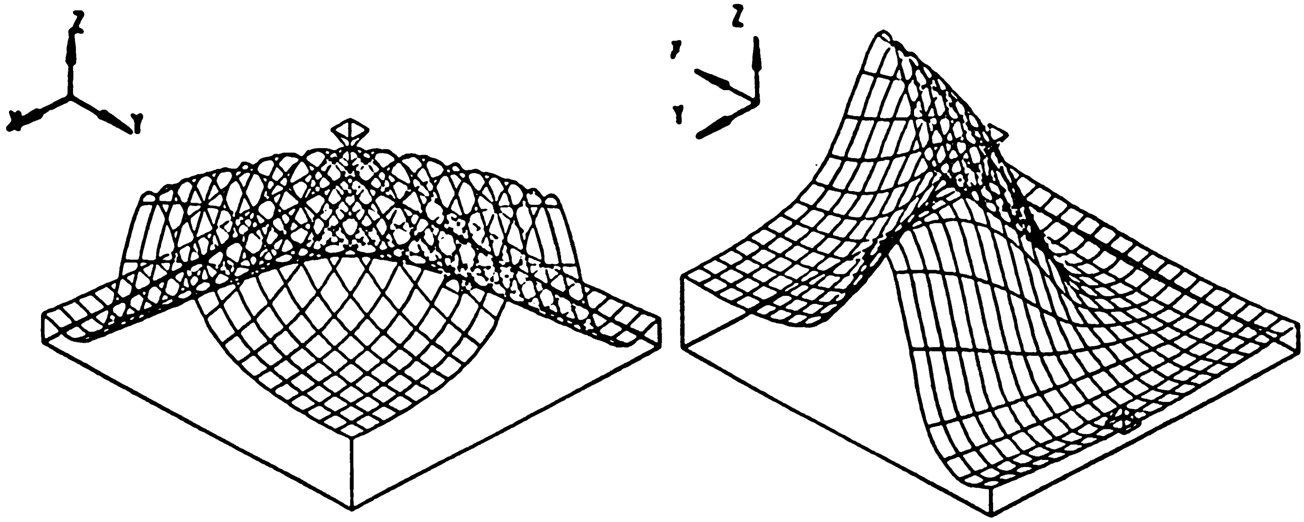


Figure 5 Design Space for Axial Stiffness Optimization Using the Frequency Response Objective

X: MOUNT #1 AXIAL STIFFNESS, N/M (RANGES FROM 62500 TO 937500)
Y: MOUNT #2 AXIAL STIFFNESS, N/M (RANGES FROM 62500 TO 937500)
Z: TRANSMISSIBILITY OBJECTIVE (RANGES FROM .00614 TO 1.05)

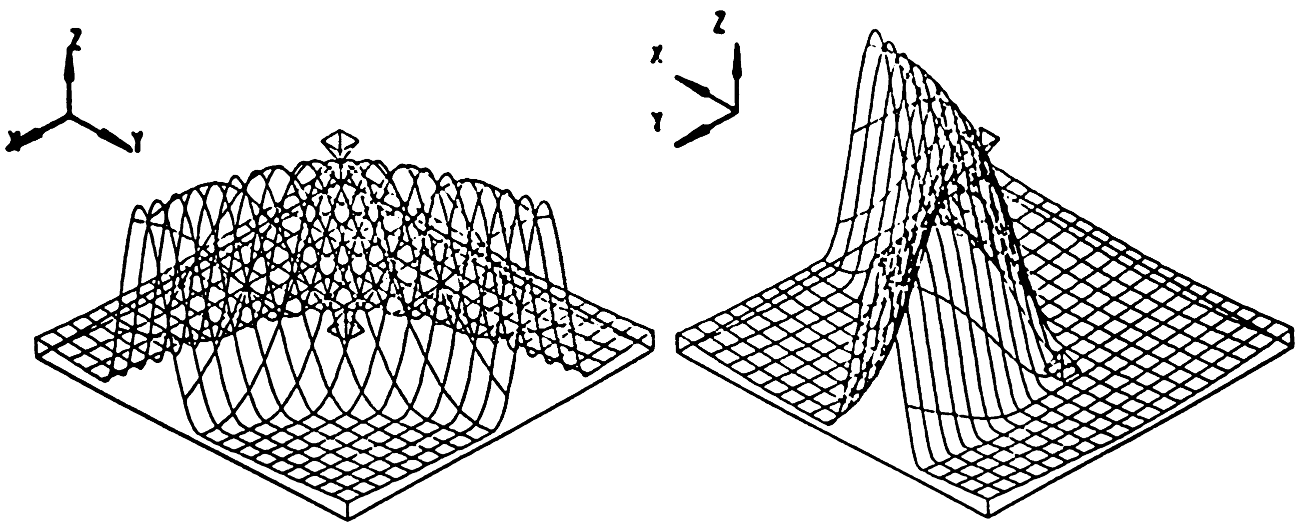


Figure 6 Design Space for Axial Stiffness Optimization Using the Transmissibility Objective

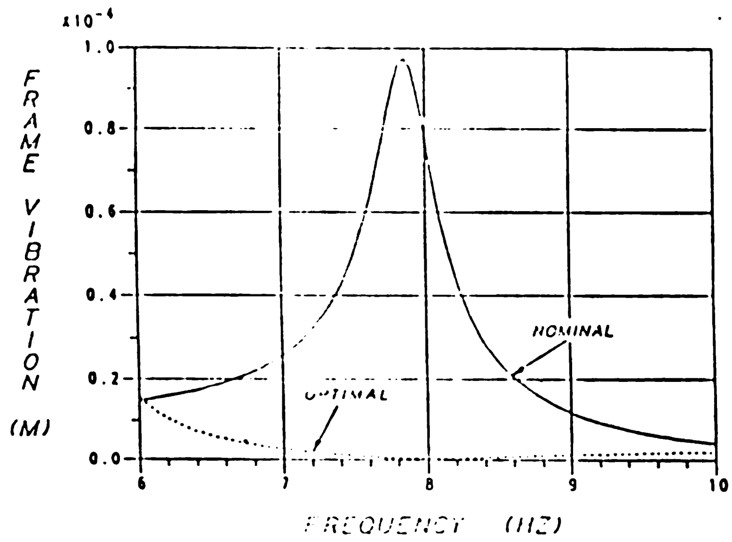


Figure 7 Support Structure Vibration for Optimal Axial Stiffness Found Using the Frequency Response Objective

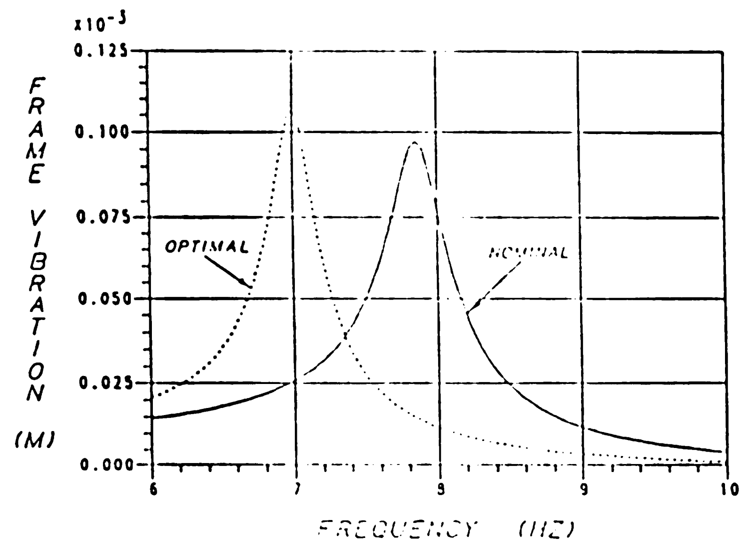


Figure 8 Support Structure Vibration for Optimal Axial Stiffness Found Using the Transmissibility Objective

designs which have a natural frequency at 8.0 Hz. Both objectives appear to be sensitive to shifts in the natural frequency. For the frequency response objective, this sensitivity to natural frequency extends over a wider range of design parameters, as evidenced by the wider, more gradual ridge. This characteristic of the frequency response objective is responsible for the differences between the two optimal designs under these conditions.

The definition of the frequency response objective accounts for the differences between the two optimal designs, as shown in Figures 7 and 8. These figures show the effects of the optimization on the vibration at the important point in the support structure. A resonant peak occurs at 7.8 Hz for the initial design. The optimal design as found by both types of objectives shifts the resonant peak to a lower frequency. The transmissibility objective moves the peak to 7.0 Hz, just outside the range of excitation. The frequency response objective moves the peak to 5.4 Hz, not only driving the resonant peak out of the range of excitation, but also minimizing the vibration of the support structure within this range. The cost in terms of design changes is higher for the frequency response objective (2.92×10^{-2}) than for the transmissibility objective ($.58 \times 10^{-2}$).

In the second optimization example, the optimal horizontal position for the two mounts is found. The excitation and the important point in the support structure are the same as in the axial stiffness optimization, except that the system is excited by 30.0 newton vertical force acting on the upper body. The results of this optimization for both types of objectives are shown in Table 4.

Table 4 Optimal Horizontal Position for the Mounts

	Initial Hor. Position (M)	Optimal Hor. Position (M)	Normalizing Factor, E	Normalized Change
Frequency Response Objective				
Mount #1	-4.00×10^{-1}	-4.00×10^{-1}	7.50×10^{-1}	-5.60×10^{-4}
Mount #2	-2.50×10^{-1}	-2.50×10^{-1}	7.50×10^{-1}	-5.34×10^{-4}
Transmissibility Objective				
Mount #1	-4.00×10^{-1}	-3.91×10^{-1}	7.50×10^{-1}	1.16×10^{-2}
Mount #2	-2.50×10^{-1}	-2.58×10^{-1}	7.50×10^{-1}	-1.13×10^{-2}

X: MOUNT #1 HORIZONTAL POSITION, M (RANGES FROM -.550 TO -.250)
Y: MOUNT #2 HORIZONTAL POSITION, M (RANGES FROM -.400 TO -.100)
Z: FREQUENCY RESPONSE OBJECTIVE (RANGES FROM .936 TO 30.9)

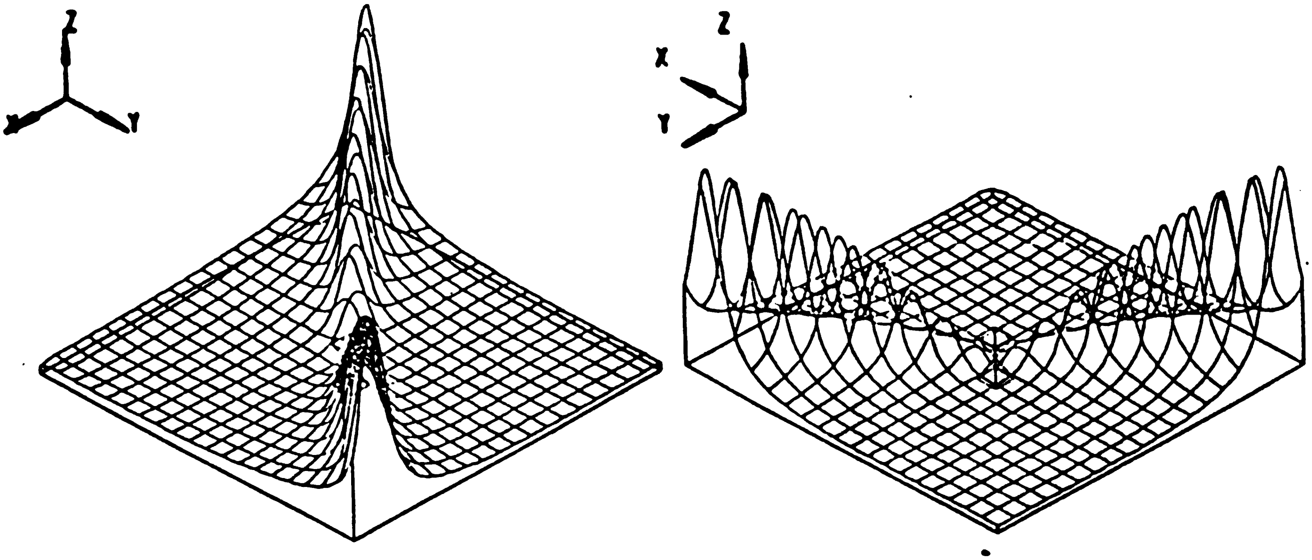


Figure 9 Design Space for Horizontal Position Optimization Using the Frequency Response Objective

X: MOUNT #1 HORIZONTAL POSITION, M (RANGES FROM -.550 TO -.250)
Y: MOUNT #2 HORIZONTAL POSITION, M (RANGES FROM -.400 TO -.100)
Z: TRANSMISSIBILITY OBJECTIVE (RANGES FROM .000328 TO 1.07)

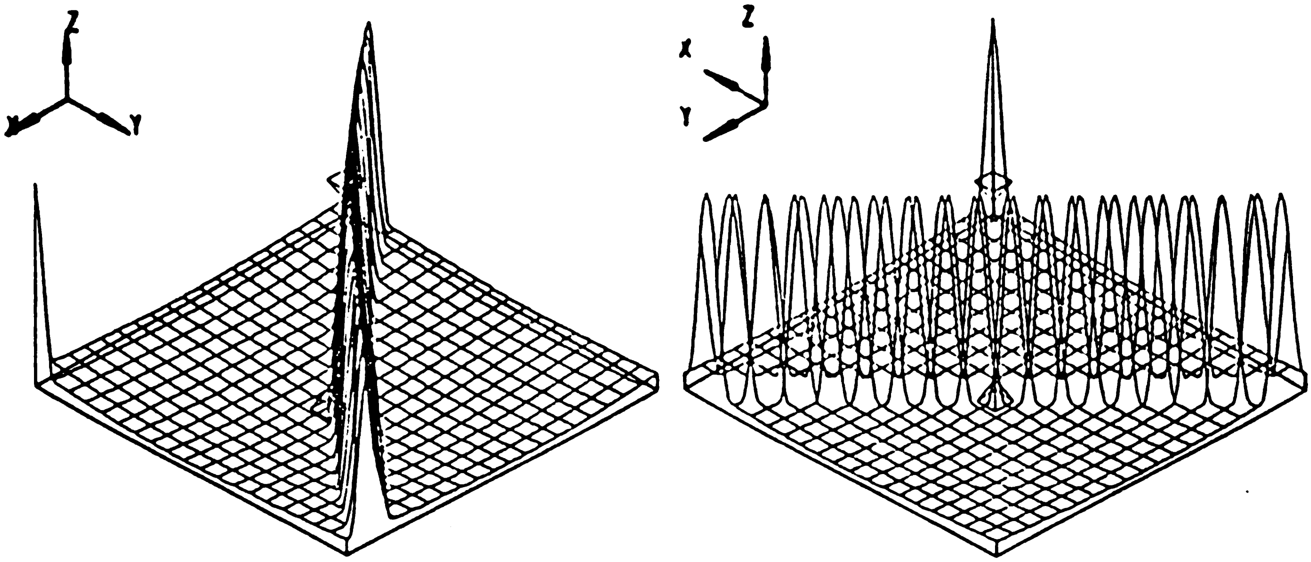


Figure 10 Design Space for Horizontal Position Optimization Using the Transmissibility Objective

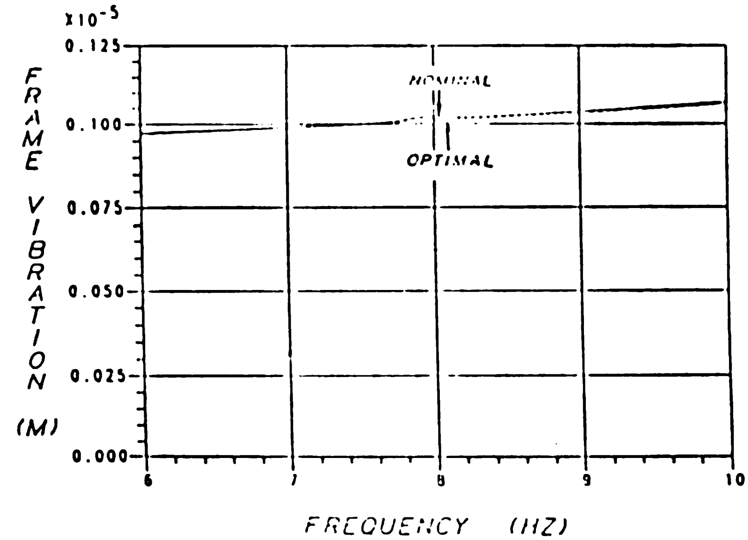


Figure 11 Support Structure Vibration for Optimal Horizontal Position Found Using the Frequency Response Objective

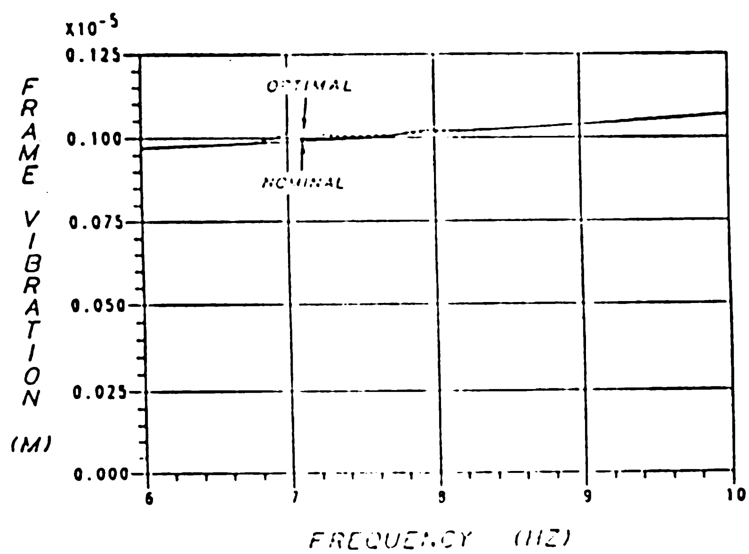


Figure 12 Support Structure Vibration for Optimal Horizontal Position Found Using the Transmissibility Objective

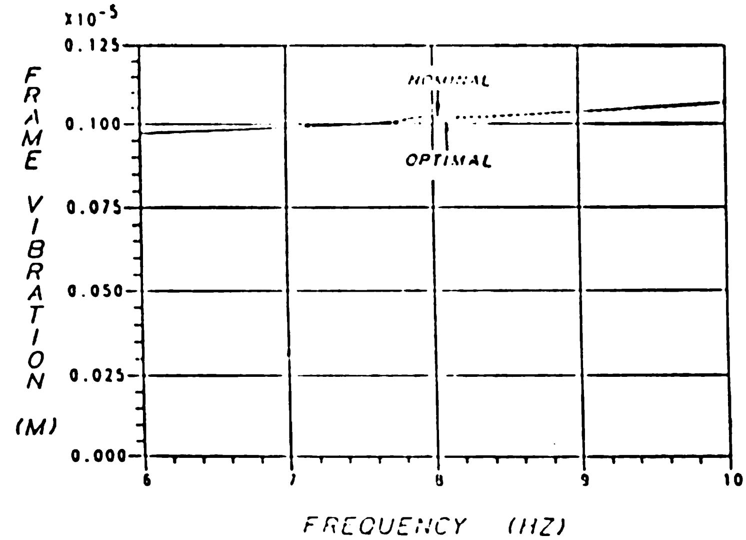


Figure 11 Support Structure Vibration for Optimal Horizontal Position Found Using the Frequency Response Objective

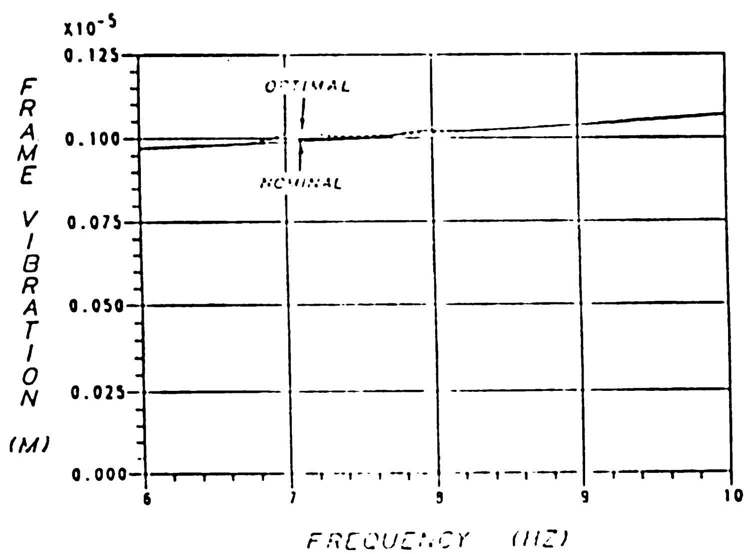


Figure 12 Support Structure Vibration for Optimal Horizontal Position Found Using the Transmissibility Objective

The design space for both types of objectives is shown in Figures 9 and 10. For both objectives, a ridge arcs through the design space. As in the stiffness optimization, this ridge represents those designs with a natural frequency of 8.0 Hz. Again, both objectives show a sensitivity to shifts in the natural frequency. However, the frequency response objective has a gap in the ridge that the transmissibility objective does not have. This is because the frequency response objective is not only sensitive to the frequency of the resonant peak,. but also to the mode shape. The gap presents a design possibility that the unbroken ridge of the transmissibility objective does not present.

Table 4 shows that the nominal and optimal mount horizontal position are the same for the frequency response objective. No design changes need to be made to get the optimal dynamic response. The excitation frequency and a natural frequency of the nominal design almost coincide, yet Figure 11 shows that a resonant peak does not occur. The mode is not excited because the excitation is orthogonal to the mode shape. The frequency response objective can detect this orthogonality condition and that is why no change is made to the design.

The transmissibility objective indicates that the nominal design is very undesirable from the standpoint of dynamic response. Because the transmissibility objective simply compares the excitation frequency to the system's natural frequencies, it cannot detect the ortogonality condition which exists between excitation and mode shape. Guided by the transmissibility objective, one would make unneccessary design changes and therefore incur additional costs.

CONCLUSIONS

The results indicate that the frequency response objective is valuable for finding the mount design which best isolates vibrations in a system. The objective function is smooth over the design space, allowing the optimization to converge to a local minimum. The test case showed that by minimizing the frequency response objective, the vibration at important points is minimized over a range of excitation frequencies.

The test case showed that, unlike the transmissibility objective, the frequency response objective is sensitive to changes in the mode shapes of a system. This was demonstrated in the test case when the frequency response objective made no changes to a design which had a natural frequency very close to the excitation frequency. The design was not altered because the corresponding mode shape was orthogonal to the excitation, and therefore a resonant peak did not occur. The transmissibility objective, on the other hand, made unneccessary changes to the design which resulted in increased costs.

It has been demonstrated that the frequency response objective is sensitive to mode shape changes and that the excitation of modes can be avoided by altering (or in the test case, leaving unaltered) the mode shapes. The test case should be generalized to the three-dimensional case and the degrees of freedom of the support structure model increased. This generalized case should then be used to study the altering of modes so that important points in the support structure are nodes. Thus even though the mode is excited, the vibration at important points is still kept to a minimum.

The frequency response objective required more computations than the transmissibility objective. Ways of decreasing the number of computations should be also be explored.

LIST OF REFERENCES

LIST OF REFERENCES

1. Fox, G.L., "Matrix Methods for the Analysis of Elastically Supported Isolation Systems", Shock and Vibration Bulletin, Bulletin 46, Part 5.

2. International Math and Science Library User's Manual, IMSL Inc., Edition 9.2, 1984.

3. Meirovitch, L., Analytical Methods in Vibrations, The Macmillan Company, 1967.

4. Spiekermann, C.E., Radcliffe, C.J., and Goodman, E.D., "Optimal Design and Simulation of Vibrational Isolation Systems", ASME Journal of Mechanisms, Transmissions, and Automation in Design, 1984.

5. Starkey, J.M., "Redesign Techniques for Improved Structural Dynamics", Ph.D. Thesis, Michigan State University, 1982.

APPENDICES

APPENDIX A

EQUATIONS OF MOTION

APPENDIX A

EQUATIONS OF MOTION

The equations of motion for the planar test case are derived by applying Newton's second law to a rigid body restricted to planar motion:

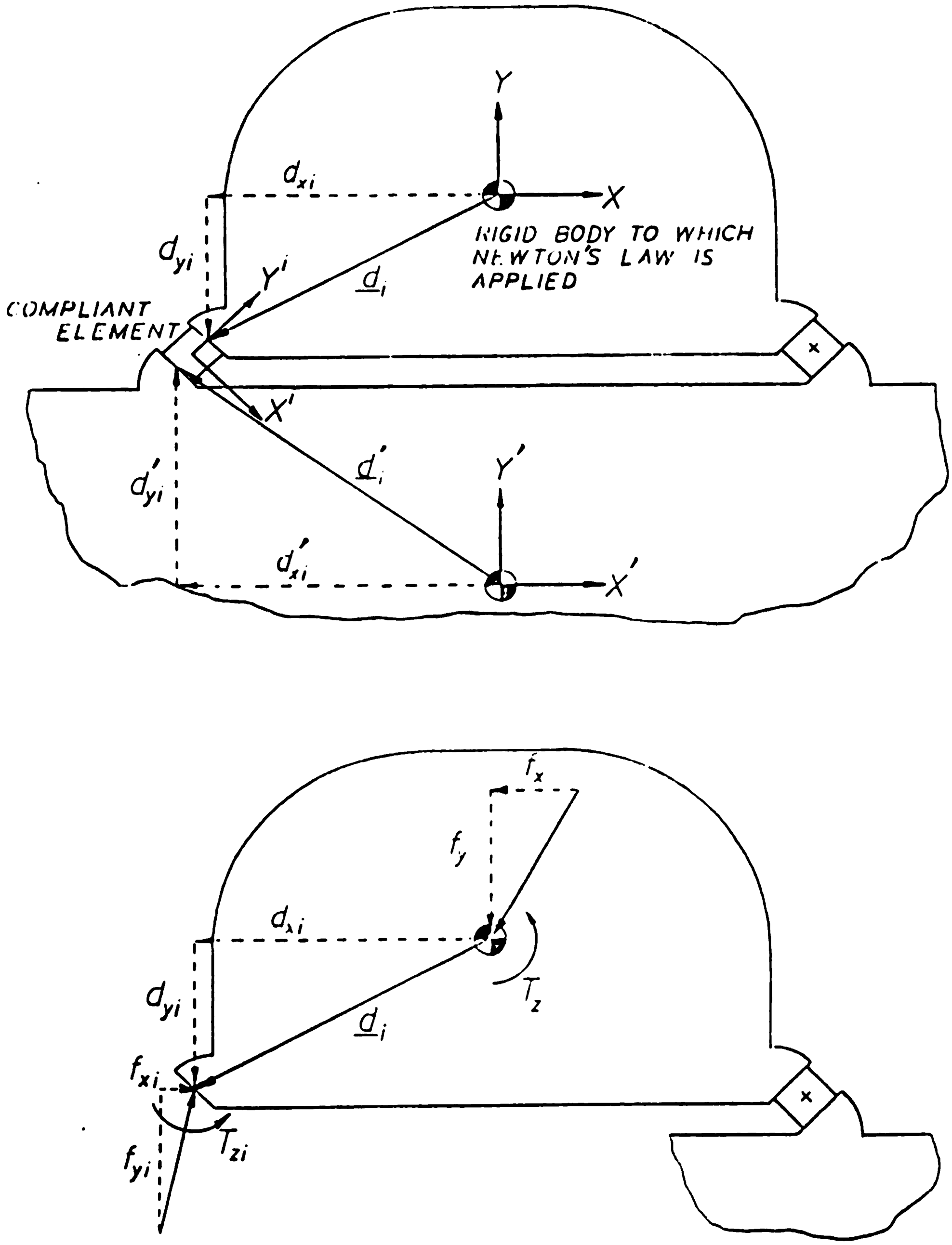
$$F_x = m\ddot{x} \tag{A.1}$$

$$F_y = m\ddot{y} \tag{A.2}$$

$$T_z = I\ddot{\theta} \tag{A.3}$$

The total force acting at the center of gravity of the body is given by F_x and F_y. The total torque acting on the body is T_z. The acceleration at the center of gravity of the body is given by $\ddot{x}$ and $\ddot{y}$. The angular acceleration of the body is $\ddot{\theta}$. The mass of the body is m and the mass moment of inertia of the body is I.

Figure A.1 shows two rigid bodies connected by compliant elements; it also shows the forces exerted on the upper body. In general, an external exciting force and torque act on a body at its center of gravity. In addition, compliant element forces also act on a body at the compliant element attachment points.



.Figure A.1 Forces Exerted On a Rigid Body

A compliant element force can be moved to the center of gravity by adding a compensating torque. For a force in the x direction, the compensating torque is $-d_{yi}f_{xi}$. For a force in the y direction, the compensating torque is $d_{xi}f_{yi}$. Newton's law can then be re-written as follows:

$$\underline{f} + \sum_{i=1}^{q} [P_i]\underline{f}_i = [m]\ddot{\underline{x}} \tag{A.4}$$

or

$$\underline{f} = [m]\ddot{\underline{x}} - \sum_{i=1}^{q} [P_i]\underline{f}_i \tag{A.5}$$

where

$$[P_i] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -d_{yi} & d_{xi} & 1 \end{bmatrix} \tag{A.6}$$

$$\underline{d}_i = \begin{bmatrix} d_{xi} \\ d_{yi} \\ 0 \end{bmatrix} \tag{A.7}$$

$$[m] = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{bmatrix} \tag{A.8}$$

$$\underline{x} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \tag{A.9}$$

$$\underline{f} = \begin{bmatrix} f_x \\ f_y \\ T_z \end{bmatrix} \tag{A.10}$$

$$\underline{f}_i = \begin{bmatrix} f_{xi} \\ f_{yi} \\ T_{zi} \end{bmatrix} \tag{A.11}$$

All vectors in equation A.5 are expressed in the local co-ordinates of the rigid body.

The compliant element model used for the test case is shown in Figures 3 and 4. The force/deformation relationship for a compliant element is as follows:

$$\underline{f}_i = [k_i]\Delta\underline{x}_i \tag{A.12}$$

where

$$[k_i] = \begin{bmatrix} k_{xi} & 0 & 0 \\ 0 & k_{yi} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{A.13}$$

$$\Delta \underline{x}_i = \underline{x}'_i - \underline{x}_i = \begin{bmatrix} x'_i \\ y'_i \\ \theta'_i \end{bmatrix} - \begin{bmatrix} x_i \\ y_i \\ \theta_i \end{bmatrix} \tag{A.14}$$

The vector $\underline{f}_i$ has the same meaning as in Equation A.10 except that it is expressed in compliant element co-ordinates instead of in rigid body co-ordinates. All vectors in equation A.12 are expressed in compliant element co-ordinates.

The displacement at any point in a rigid body can be found through a transformation of the displacement at the c.g. of the rigid body. The displacement at a compliant element attachment point is as follows:

$$\underline{x}_i = [P_i]^T \underline{x} \tag{A.15}$$

where

$$[P_i]^T = \begin{bmatrix} 1 & 0 & -d_{yi} \\ 0 & 1 & d_{xi} \\ 0 & 0 & 1 \end{bmatrix} \tag{A.16}$$

The vector $\underline{x}_i$ has the same meaning as in Equation A.14 except it is expressed in rigid body co-ordinates.

A vector expressed in one co-ordinate system can be expressed in another co-ordinate system by multiplying the vector by a rotation matrix as follows:

$$\underline{v}^{\alpha} = [R]\underline{v}^{\beta} \tag{A.17}$$

$$\Delta \underline{x}_i = \underline{x}_i' - \underline{x}_i = \begin{bmatrix} x_i' \\ y_i' \\ \theta_i' \end{bmatrix} - \begin{bmatrix} x_i \\ y_i \\ \theta_i \end{bmatrix} \tag{A.14}$$

The vector $\underline{f}_i$ has the same meaning as in Equation A.10 except that it is expressed in compliant element co-ordinates instead of in rigid body co-ordinates. All vectors in equation A.12 are expressed in compliant element co-ordinates.

The displacement at any point in a rigid body can be found through a transformation of the displacement at the c.g. of the rigid body. The displacement at a compliant element attachment point is as follows:

$$\underline{x}_i = [P_i]^T \underline{x} \tag{A.15}$$

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The vector $\underline{x}_i$ has the same meaning as in Equation A.14 except it is expressed in rigid body co-ordinates.

A vector expressed in one co-ordinate system can be expressed in another co-ordinate system by multiplying the vector by a rotation matrix as follows:

$$\underline{v}^{\alpha} = [R]\underline{v}^{\beta} \tag{A.17}$$

where

$$[R] = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A.18}$$

The vector $\underline{v}^{\alpha}$ is any general vector expressed in the α co-ordinate system and the vector $\underline{v}^{\beta}$ is the same vector expressed in the β co-ordinate system. The angle ϕ is the amount by which the β co-ordinate system is rotated from the α co-ordinate system. It can be easily shown that the inverse rotation is as follows:

$$\underline{v}^{\beta} = [R]^T \underline{v}^{\alpha} \tag{A.19}$$

The following result can be obtained by applying the appropriate rotation matrices and by combining Equations A.5, A.12, and A.15 .

$$\begin{aligned} \underline{f} = [m]\underline{x} &+ \sum_{i=1}^{q} [P_i][R_i][k_i][R_i]^T[P_i]^T \underline{x} \\ &- \sum_{i=1}^{q} [P_i][R_i][k_i][R_i']^T[P_i']^T \underline{x}' \end{aligned} \tag{A.20}$$

where

$$[R_i] = \begin{bmatrix} \cos\phi_i & -\sin\phi_i & 0 \\ \sin\phi_i & \cos\phi_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A.21}$$

$$[R_i'] = \begin{bmatrix} \cos\phi_i' & -\sin\phi_i' & 0 \\ \sin\phi_i' & \cos\phi_i' & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A.22}$$

$$[P_i'] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -d'_{yi} & d'_{xi} & 1 \end{bmatrix} \tag{A.23}$$

The rotation matrix $[R_i]$ transforms a vector expressed in the ith compliant element co-ordinates to a vector expressed in the body co-ordinates. The angle ϕ_i is the amount by which the co-ordinate system of the ith compliant element is rotated from the co-ordinate system of the body. The vector $\underline{x}'$ is the displacement of the secondary rigid body attached to the ith compliant element. A prime mark is used to indicate those quantities which are associated with the secondary body attached to the ith compliant element.

For the planar test case, two rigid bodies are present. One of the rigid bodies represents the support structure, the other represents the externally excited body. The two rigid bodies are connected by two compliant mounts. Additionally, the support structure is connected to a stationary reference frame through two compliant elements. Newton's second law in the form of equation A.20 can be applied to each of the rigid bodies:

$$\underline{f}_b = [m_b]\ddot{\underline{x}}_b + [k_1]\underline{x}_b - [k_2]\underline{x}_f \tag{A.24}$$

$$\underline{0} = [m_f]\ddot{\underline{x}}_f - [k_2]^T\underline{x}_b + [k_3]\underline{x}_f \tag{A.25}$$

where

$$[k_1] = \left[\sum_{i=1}^{2} [P_{bmi}][R_{bmi}][k_{mi}][R_{bmi}]^T[P_{bmi}]^T \right] \tag{A.26}$$

$$[k_2] = \left[\sum_{i=1}^{2} [P_{bmi}][R_{bmi}][k_{mi}][R_{fmi}]^T[P_{fmi}]^T \right] \tag{A.27}$$

$$[k_3] = \left[\sum_{i=1}^{2} [P_{ffi}][R_{ffi}][k_{fi}][R_{ffi}]^T[P_{ffi}]^T \right] + \left[\sum_{i=1}^{2} [P_{fmi}][R_{fmi}][k_{mi}][R_{fmi}]^T[P_{fmi}]^T \right] \tag{A.28}$$

The subcripts used in these equations come from Figure 1.

f = support structure body

b = excited body

fi = support structure compliant element

mi = mount compliant element

bmi = excited body / mount compliant element

fmi = support structure body / mount compliant element

ffi = support structure body / support structure compliant element

Analagous derivations are used to obtain structural and viscous damping matrices. When damping forces are included in equations A.24 and A.25, the following equation results:

$$[M]\ddot{\underline{x}} + [C]\dot{\underline{x}} + [K]\underline{x} + i[D]\underline{x} = \underline{f} \tag{A.29}$$

where

$$\underline{x} = \begin{bmatrix} \underline{x}_b \\ \underline{x}_f \end{bmatrix} \tag{A.30}$$

$$\underline{f} = \begin{bmatrix} \underline{f}_b \\ \underline{0} \end{bmatrix} \tag{A.31}$$

$$[M] = \begin{bmatrix} [m_b] & [0] \\ [0] & [m_f] \end{bmatrix} \tag{A.32}$$

$$[C] = \begin{bmatrix} [c_1] & -[c_2] \\ -[c_2] & [c_3] \end{bmatrix} \tag{A.33}$$

$$[K] = \begin{bmatrix} [k_1] & -[k_2] \\ -[k_2] & [k_3] \end{bmatrix} \tag{A.34}$$

$$[D] = \begin{bmatrix} [d_1] & -[d_2] \\ -[d_2] & [d_3] \end{bmatrix} \tag{A.35}$$

APPENDIX B

INSTRUCTIONS FOR RUNNING THE PVIP PROGRAM

APPPENDIX B

INSTRUCTIONS FOR RUNNING THE PVIP PROGRAM

The planar vibration isolation program finds the optimal mount parameters for the planar test case shown in Figure 2, page 10. A complete list of the program's capabilities is as follows:

1) Reads in the test case data and formulates the mass, damping, and stiffness matrices.
2) Calculates the undamped eigenvalues and eigenvectors.
3) Optimizes the mount design.
4) Searches the design space for the global minimum of the objective function.
5) Plots the weighted summation of the vibration at important points versus the excitation frequency.
6) Plots the objective function versus two design parameters.

The PVIP program first displays a primary menu. The four items on the primary menu are as follows:

1) Read in raw data
2) Optimize the mounts
3) Calculate the eigenvalues/eigenvectors
4) Exit

The first option in the primary menu, reading in raw data, must be selected before options 2 and 3 in the menu are selected. Upon selecting option 1, the user is prompted for the names of the files which define the test case data. Two data files are used to define the test case data. The first of these files defines the data for the excited upper body and the mount compliant elements. Figure B.1 shows the template which should be used to create this file. The second of these files defines the data for the support structure body and support structure compliant elements. Figure B.2 shows the template which should be used to create this file.

Once the raw data for the test case has been input, option 2 or 3 in the primary menu can be selected. If option 2 is selected, the global mass, stiffness, and damping matrices are calculated for the test case, as are the undamped eigenvalues and eigenvectors. The program creates an output file called T_EIGER which contains these results.

The third option in the primary menu, optimizing the mount design, serves as the starting point for the last four of the listed features. When this option is selected, the user is first prompted for data common to all four of the features:

1) Type of penalty?
2) Optimization data file name?
3) Number of significant digits to which design is to be found?
4) Maximum number of optimization iterations?

The first question refers to the type of dynamic response objective used in the optimization or plotted over the design space. The user can choose either the frequency response based objective function or the natural frequency based objective function. Next the name of the file which contains the information necessary to perform an optimization or to generate a plot is specified. The template which should be used to create this file is shown in Figure B.3 . Only mount parameters with non-zero normalization factors (E factors) will be optimized or plotted. If a plot is to be generated, exactly two of the mount parameters should have non-zero normalization factors. After the optimization data file name is specified, the convergence criteria for the optimization must be given. The optimization terminates when the specified number of significant digits is achieved for each of the optimized parameters or when the number of optimization iterations exceeds the specified number. These quantities do not enter into the plotting of the objective function over the design space. Once the convergence criteria has been specified, either of the following can be performed: a search for the global optimum over the design space, a plot of the objective function over the design space, or a simple optimization.

```
MOUNT AND UPPER BODY DATA FILE
MASS MATRIX
   30.0   0.0   0.0
   0.0   30.0   0.0
   0.0   0.0   1.0
POSITION OF RIGID BODY C.G.
   -.325   .27   0.0
ORIENTATION OF RIGID BODY
   0.0
NUMBER OF MOUNTS
   2
MOUNT NO. 1
   STIFFNESS
     650000.0   0.0   0.0
      0.0   500000.0   0.0
      0.0   0.0   0.0
   VISCOUS DAMPING
      650.0   0.0   0.0
      0.0   500.0   0.0
      0.0   0.0   0.0
   STRUCTURAL DAMPING
      0.0   0.0   0.0
      0.0   0.0   0.0
      0.0   0.0   0.0
   MOUNT POSITION
      -.40   .07   0.0
   MOUNT ORIENTATION
      0.0
MOUNT NO. 2
   STIFFNESS
     650000.0   0.0   0.0
      0.0   500000.0   0.0
      0.0   0.0   0.0
   VISCOUS DAMPING
      650.0   0.0   0.0
      0.0   500.0   0.0
      0.0   0.0   0.0
   STRUCTURAL DAMPING
      0.0   0.0   0.0
      0.0   0.0   0.0
      0.0   0.0   0.0
   MOUNT POSITION
      -.25   .07   0.0
   MOUNT ORIENTATION
      0.0
```

Figure B.1 Data File of Mount Compliant Element Parameters and Upper Body Parameters

```
SUPPORT STRUCTURE DATA FILE
MASS MATRIX
   60.0   0.0   0.0
   0.0   60.0   0.0
   0.0   0.0   15.0
NUMBER OF SUPPORTS
   2
SUPPORT NO. 1
   STIFFNESS
      1900000.0   0.0   0.0
      0.0   12000000.0   0.0
      0.0   0.0   0.0
   VISCOUS DAMPING
      1900.   0.0   0.0
      0.0   12000.   0.0
      0.0   0.0   0.0
   STRUCTURAL DAMPING
      0.0   0.0   0.0
      0.0   0.0   0.0
      0.0   0.0   0.0
   SUPPORT POSITION
      .56   0.0   0.0
SUPPORT NO. 2
   STIFFNESS
      1900000.0   0.0   0.0
      0.0   12000000.0   0.0
      0.0   0.0   0.0
   VISCOUS DAMPING
      1900.   0.0   0.0
      0.0   12000.   0.0
      0.0   0.0   0.0
   STRUCTURAL DAMPING
      0.0   0.0   0.0
      0.0   0.0   0.0
      0.0   0.0   0.0
   SUPPORT POSITION
      -.56   0.0   0.0
```

Figure B.2 Data File of Support Structure Compliant Element Parameters and Support Structure Rigid Body Parameters

```
OPTIMIZATION DATA FILE
FREQUENCY RESPONSE PENALTY
   WEIGHTING ON FREQUENCY RESPONSE
   1.0
   FORCING FUNCTION
      MAGNITUDE
      30.0   0.0   0.0
      FILTER DATA
      2
      7.0  8.0  9.0  0.0  1.0  0.0
   LOCATIONS AT WHICH FREQUENCY RESPONSE IS TO BE OPTIMIZED
      NO OF LOCATIONS
      1
         LOCATION #1
            POSITION
            .25    0.0   0.0
            WEIGHTING FACTOR
            1.0
SIZE OF CHANGE PENALTY
   WEIGHT ON SIZE OF CHANGE
   1.0
   NUMBER OF MOUNTS
   2
   MOUNT #1
      MOUNT STIFFNESS
      2.0E6   0.0   0.0
      0.0   2.0E6   0.0
      0.0   0.0   0.0
      MOUNT POSITION
       .75   .75   0.0
      MOUNT ORIENTATION
      300.0
   MOUNT #2
      MOUNT STIFFNESS
      2.0E6   0.0   0.0
      0.0   2.0E6   0.0
      0.0   0.0   0.0
      MOUNT POSITION
       .75   .75   0.0
      MOUNT ORIENTATION
      300.0
```

Figure B.3 Data File of the Optimization Parameters

```
SEARCH LIMIT DATA FILE
MOUNT #1
  STIFFNESS
  568750.0   0.0   0.0
  0.0   0.0   0.0
  0.0   0.0   0.0
  POSITION
  0.0   0.0   0.0
  ORIENTATION
  0.0
MOUNT #2
  STIFFNESS
  568750.0   0.0   0.0
  0.0   0.0   0.0
  0.0   0.0   0.0
  POSITION
  0.0   0.0   0.0
  ORIENTATION
  0.0
```

Figure B.4 Data File of Search Limits or Plotting Limits

SEARCHING FOR A GLOBAL MINIMUM

After the convergence data is input, the user is asked whether or not a search for the global minimum is to be performed. If a search is performed, the user is prompted for the name of the file containing the range of design parameters over which the search is to be conducted. The template which should be used to create this data file is shown in Figure B.4 . The values specified in the search data file are added to or subtracted from the nominal design parameter values to give the upper and lower limits for the search.

In addition to the range of design parameter values, the number of search starting points per design parameter must be specified. If r is the number of design parameters and p is the number of starting points per design parameter, p^r is the number of optimizations performed. Therefore, when performing a search, it is recommended that the maximum number of optimization iterations be kept low.

When a search is performed, two output files are generated. The first file, T_LOCMIN, lists the final value of the objective function for each of the p^r optimizations which are performed. The user should look for the lowest values in the list and note the corresponding design number. The other file, LMINDAT, contains the design parameters for each design number listed in T_LOCMIN. After the search is performed, the program returns to the primary menu.

PLOTTING THE OBJECTIVE FUNCTION OVER THE DESIGN SPACE

Initially, the procedure for generating plots of the objective function over the design space are the same as for conducting a search of the design space. After the convergence data is input, the user is asked whether or not a search for the global minimum is to be performed. For plotting of the design space, the user should reply "yes". The range of design parameters over which the plot is to be generated must then be defined. The user is prompted for the name of the file which defines the plotting limits. The template which should be used to create this file is shown in Figure B.4 . The values specified in this data file are added to or subtracted from the nominal design parameter values to give the upper and lower limits for the plot. At this point the program asks the user if he wants to abort the search and plot instead. After answering "yes", the following menu is displayed:

1) Draw
2) Save
3) Viewpoint
4) Optimum
5) Magnify
6) Title
7) Ensdat
8) Quit

A description of the menu items is given on the following page. Items 2-7 should be selected before the draw option is selected.

Draw: Draws a plot of the design space. The points to be plotted can either be read in from a previously generated file or can be generated using the defined objective function. If the points are read in from an existing file, the user will be prompted for the name of the file. The program will then ask the user if he wishes to tack on the currently defined size of change part of the objective function. The user would only want to do this if the size of change part of the objective function was not included when the data file was generated. If the points to be plotted are generated on the spot, the points can be saved in a data file after they are generated. The user is prompted for the name of the file in which he wishes to save the data.

Save: Saves the generated plot in a graphics file. The user is prompted for the name of the graphics file.

Viewpoint : Changes the point from which the generated plot is viewed. The user is prompted for the two points which define the new viewing vector.

Optimum : Displays the inital and optimal points if an optimization of the two design parameters has been performed. The initial and optimal points must be saved when the optimization is performed. The user will be promted for the name of the file containing the initial and optimal points.

Magnify : Magnifies the plot to be generated. User is prompted for the magnification factor (1=no mag).

Ensdat : Generates files that allow the design space to be displayed on the Evans and Sutherland computer. An in-house program called PLOTEM written by Steve Southward is used to display the file. The advantage of plotting the design space on the E & S is that different viewpoints are easily obtained. The PLOTEM co-ordinate file generated is called ENS.CORD while the PLOTEM sequence file is called ENS.SEQ .

Quit : Return to primary menu.

PERFORMING A SIMPLE OPTIMIZATION

After the convergence data is input, the user is asked whether or not a search for the global minimum is to be performed. The user should answer "no" to perform a simple optimization. The program will then ask the user if the starting point of the optimization is to be read in from a global minimum result file. If the user replies "yes", the program will ask the user for the design number. This is simply the line number which the user has selected from T_LOCMIN (see page B.6). If the user replies "no", the starting point of the optimization defaults to the nominal parameters. Once the optimization starting point is defined, the program will ask the user if the initial (starting) point and optimal point are to be saved for later plotting. If two parameters are being optimized, a file which saves the initial and optimal points can be generated. If the user wishes to generate such a file, he will be asked for the name of the file in which the points are to be saved. If more than two parameters are being optimized the user should answer "no". At that point, the program will perform the optimization. Two output files will be generated. One of the files, G_RESPONSE, will contain a plot of the weighted sum of the vibration at important points versus the excitation frequency. The other file, T_OPTIMIZ, will contain the initial design, the optimal design, and the changes in the design. After the optimization is performed, the program returns to the primary menu.

APPENDIX C

DOCUMENTATION FOR THE PVIP SOURCE CODE

APPENDIX C

DOCUMENTATION FOR THE PVIP SOURCE CODEE

The subroutine calling tree for the PVIP program is shown in Figure C.1 . Common block occurences for the program are shown in Table C.1 . Descriptions of the routines employed in the PVIP program are listed on pages C.3 through C.11 .

Table C.1 Common Block Occurences for the PVIP Program

Common Block	Subroutines
DSGN	DRAWIT, GRAPH
EANDS	DRAWIT, DR3, GRAPH, MV3
FRCE	FRESP, PENALTY, RESPNTS
MDS	EIGER, EQNS, FRESP, NODAMP
OPM1	OPARAM, OPTIMIZ
OPM2	FUNC1, NATFPEN, OPTIMIZ, PENALTY, RESPNTS
OPM3	OPARAM, OPTIMIZ
PARCOM	DRAWIT, GRAPH, OPTIMIZ
PENTYPE	OPTIMIZ, PENALTY
RBDATA1	EQNS, RAWDAT
RBDATA2	EQNS, OPARAM, OPTIMIZ, RAWDAT
SCALE	OPARAM, OPTIMIZ

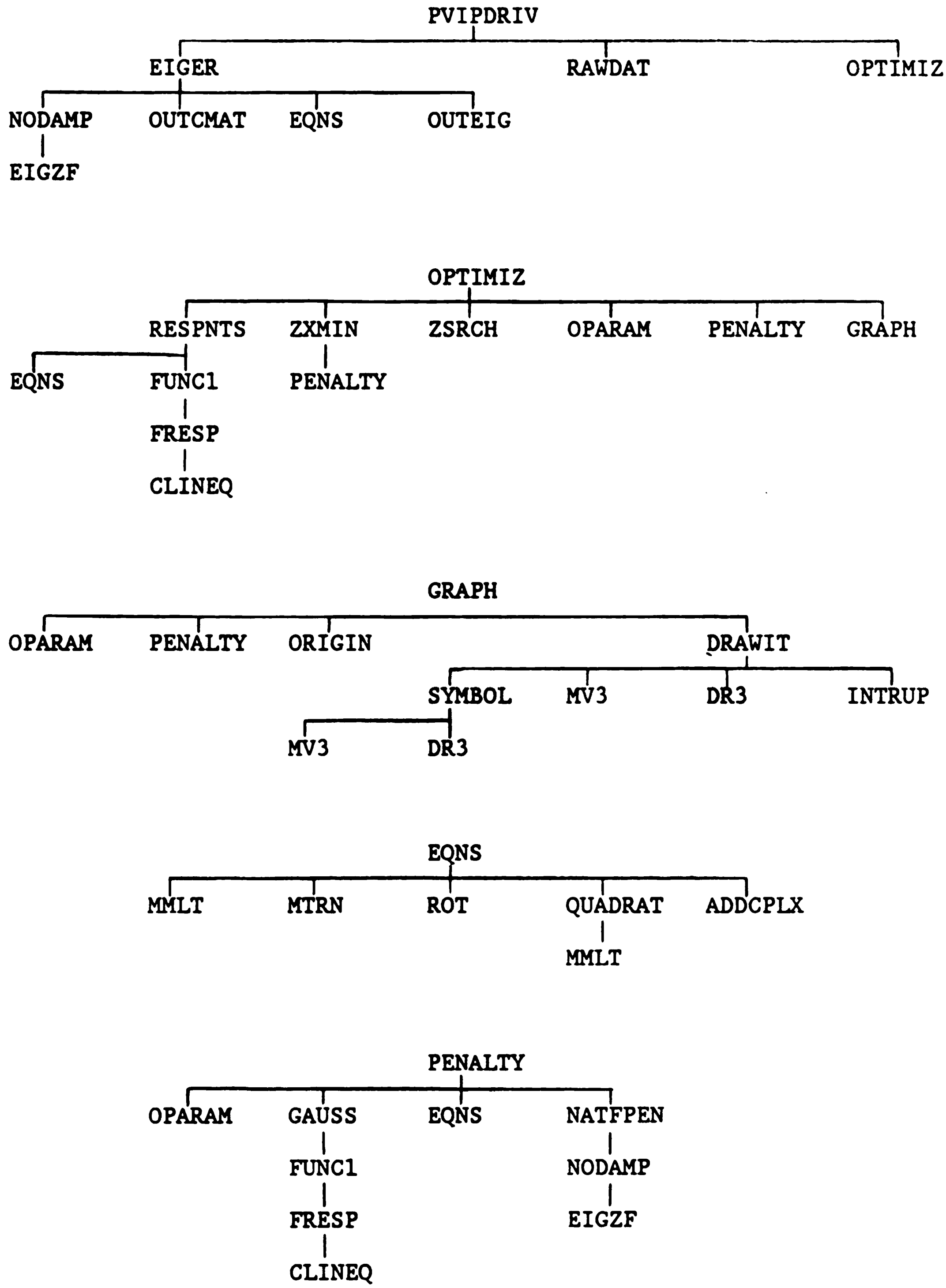

Figure C.1 Subroutine Calling Tree for the PVIP Program

DESCRIPTION OF THE ROUTINES USED IN THE PVIP PROGRAM

```
C-----------------------------------------------------------C
C                                                           C
C    SUBROUTINE ADDCPLX(SUM,REALMAT,IMAGMAT)                C
C                                                           C
C    SUBROUTINE 'ADDCPLX' CONVERTS 2 REAL MATRICES INTO      C
C    A COMPLEX MATRIX.  THIS COMPLEX MATRIX IS THEN ADDED   C
C    TO THE COMPLEX MATRIX 'SUM'.  THE RESULT IS RETURNED   C
C    IN 'SUM' .                                             C
C                                                           C
C    CALLING ROUTINES:                                      C
C                     EQNS                                  C
C                                                           C
C-----------------------------------------------------------C
C                                                           C
C    IN-HOUSE ROUTINE CMMLT                                 C
C                                                           C
C    SUBROUTINE 'CMMLT' CALCULATES THE PRODUCT OF 2 COMPLEX C
C    MATRICES.                                              C
C                                                           C
C    CALLING ROUTINES:                                      C
C                     FUNC1                                 C
C                                                           C
C-----------------------------------------------------------C
C                                                           C
C    SUBROUTINE DR3(X,Y,Z)                                  C
C                                                           C
C    SUBROUTINE 'DR3' PERFORMS A 3-D DRAW AND WRITES THE    C
C    NECESSARY LINES TO THE ENS DATA FILES IF THE ENS FLAG IS ON. C
C                                                           C
C    CALLING ROUTINES:                                      C
C                      DRAWIT                               C
C                      SYMBOL                               C
C                                                           C
C-----------------------------------------------------------C
C                                                           C
C    SUBROUTINE DRAWIT(ZMIN,ZMAX)                           C
C                                                           C
C    SUBROUTINE 'DRAWIT' DRAWS THE DESIGN SURFACE GENERATED C
C    IN THE ROUTINE 'GRAPH'.  IT ALSO GENERATES CO-ORDINATE AND C
C    SEQUENCE FILES FOR THE EVANS & SUTHERLAND PROGRAM 'PLOTEM'. C
C    THE ENS OPTION IN 'GRAPH' MUST BE SPECIFIED FOR THIS   C
C    FUNCTION TO BE PERFORMED.  THIS ROUTINE ALSO GENERATES C
C    SYMBOLS TO SHOW WHERE THE INITIAL AND OPTIMAL DESIGN ARE C
C    LOCATED.  THE OPTIMAL OPTION MUST BE SPECIFIED IN 'GRAPH' C
C    FOR THIS FUNCTION TO BE PERFORMED.                     C
C                                                           C
C    CALLING ROUTINES:                                      C
C                     GRAPH                                 C
C                                                           C
C-----------------------------------------------------------C
```

```
C-------------------------------------------------------------C
C                                                             C
C     SUBROUTINE EIGER                                        C
C                                                             C
C     SUBROUTINE 'EIGER' CALCULATES THE EIGENVALUES AND       C
C     EIGENVECTORS FOR THE RIGID BODY VIBRATION ISOLATION     C
C     PROGRAM.  THE RESULTS ARE WRITTEN TO A FILE CALLED      C
C     'T_EIGER'.                                              C
C                                                             C
C     CALLING ROUTINES:                                       C
C                       PVIPDRV                               C
C                                                             C
C-------------------------------------------------------------C
C                                                             C
C     IMSL ROUTINE EIGZF                                      C
C                                                             C
C     SUBROUTINE 'EIGZF' FINDS THE EIGENVALUES AND EIGENVECTORS C
C     FOR THE FOLLOWING EIGENVALUE PROBLEM:  [A]x - λ[B]x = 0 . C
C     [A] AND [B] ARE REAL NXN MATRICES.                      C
C                                                             C
C     CALLING ROUTINES:                                       C
C                       NODAMP                                C
C                                                             C
C-------------------------------------------------------------C
C                                                             C
C     SUBROUTINE EQNS                                         C
C                                                             C
C     SUBROUTINE 'EQNS' CALCULATES THE GLOBAL MASS, DAMPING   C
C     AND STIFFNESS MATRICES.                                 C
C                                                             C
C     CALLING ROUTINES:                                       C
C                       EIGER                                 C
C                       RESPNTS                               C
C                       PENALTY                               C
C                                                             C
C-------------------------------------------------------------C
C                                                             C
C     SUBROUTINE FRESP(W,Z)                                   C
C                                                             C
C     SUBROUTINE 'FRESP' CALCULATES THE FREQUENCY RESPONSE OF C
C     THE PLANAR 6 D.O.F. ISOLATION PROBLEM.                  C
C                                                             C
C     CALLING ROUTINES:                                       C
C                       FUNC1                                 C
C                                                             C
C-------------------------------------------------------------C
```

```
C-----------------------------------------------------------C
C                                                           C
C    FUNCTION FUNC1(IFILT,W)                                C
C                                                           C
C    FUNCTION 'FUNCT' FINDS THE RESPONSE AT THE SPECIFIED POINTS  C
C    OF CONCERN IN THE VIBRATIONAL SYSTEM.  IT THEN FINDS THE     C
C    NORM AT THESE POINTS AND WEIGHTS THEM AS SPECIFIED.  THE SUM C
C    OF THESE WEIGHTED NORMS IS RETURNED.                         C
C                                                           C
C    CALLING ROUTINES:                                      C
C                       RESPNTS                             C
C                       GAUSS                               C
C                                                           C
C-----------------------------------------------------------C
C                                                           C
C    SUBROUTINE GAUSS(XL,XH,NINC,IORDER,FUNCT,SUM,IERR)     C
C                                                           C
C    SUBROUTINE 'GAUSS' NUMERICALLY INTEGRATES A FUNCTION USING   C
C    GAUSSIAN QUADRATURE.  THE FUNCTION TO BE INTEGRATED IS       C
C    DEFINED BY THE USER IN 'FUNCT'.  THE ORDER OF THE POLYNOMIAL C
C    APPROXIMATING THE FUNCTION CAN BE SPECIFIED BY THE VALUE OF  C
C    'IORDER'.  ONLY 3RD AND 5TH ORDER POLYNOMIALS CAN BE         C
C    USED, OR ELSE THE ERROR FLAG (IERR) IS SET.                  C
C                                                           C
C    CALLING ROUTINES:                                      C
C                        PENALTY                            C
C                                                           C
C-----------------------------------------------------------C
C                                                           C
C    SUBROUTINE GRAPH                                       C
C                                                           C
C    SUBROUTINE 'GRAPH' DRAWS 3-D PLOTS.  VARIATIONS IN A PAIR    C
C    OF DESIGN PARAMETERS ARE PLOTTED ALONG TWO OF THE AXES, AND  C
C    THE CORRESPONDING VARIATION IN THE PENALTY FUNCTION IS       C
C    PLOTTED ALONG THE THIRD AXIS.  THE RANGE OF DESIGN PARAMETER C
C    VARIATIONS IS READ IN FROM A FILE WHILE IN THE ROUTINE       C
C    'OPTIMIZ'.  THE ROUTINE 'OPTIMIZ' PASSES THESE LIMITS TO     C
C    'GRAPH' THROUGH THE COMMON ARRAY 'PNT'.  THE POINTS          C
C    COMPRISING THE PLOT CAN EITHER BE GENERATED OR READ IN FROM  C
C    A FILE.  IF THE POINTS ARE GENERATED, AN OPTION TO SAVE THE  C
C    THE POINTS FOR LATER PLOTTING IS AVAILABLE.  THE ROUTINE     C
C    ALSO ALLOWS THE MAGNIFICATION AND THE VIEWPOINT OF THE PLOT  C
C    TO BE CHANGED.  THE INITIAL AND OPTIMAL DESIGN POINTS CAN    C
C    ALSO BE DISPLAYED, ASSUMING THAT AN OPTIMIZATION WAS         C
C    PERFORMED SOLELY WITH RESPECT TO THE PLOTTED DESIGN          C
C    PARAMETERS.  LASTLY, THE ROUTINE CAN GENERATE FILES          C
C    COMPATIBLE WITH THE EVANS AND SUTHERLAND PROGRAM 'PLOTEM'.   C
C                                                           C
C    CALLING ROUTINES:                                      C
C                        OPTIMIZ                            C
C                                                           C
C-----------------------------------------------------------C
```

```
C-------------------------------------------------------------C
C                                                             C
C    SUBROUTINE INTRUP(IVAR)                                  C
C                                                             C
C    SUBROUTINE 'INTRUP' SETS IVAR=1 IF TTY$IN FINDS THAT INPUT  C
C    BUFFER HAS HAD INPUT FROM KEYBOARD, OTHERWISE IVAR=0.    C
C    CISSUE CLEARS THE INPUT BUFFER.  MUST LOAD  DYNTLIB  FOR C
C    TTY$IN AND CISSUE.                                       C
C                                                             C
C    CALLING ROUTINES:                                        C
C                      INTRUP                                 C
C                                                             C
C-------------------------------------------------------------C
C                                                             C
C    IN-HOUSE ROUTINE MMLT                                    C
C                                                             C
C    SUBROUTINE 'MMLT' CALCULATES THE PRODUCT OF 2 REAL MATRICES  C
C                                                             C
C    CALLING ROUTINES:                                        C
C                      EQNS                                   C
C                                                             C
C-------------------------------------------------------------C
C                                                             C
C    IN-HOUSE ROUTINE MTRN                                    C
C                                                             C
C    SUBROUTINE 'MTRN' CALCULATES THE TRANSPOSE OF A REAL MATRIX  C
C                                                             C
C    CALLING ROUTINES:                                        C
C                      EQNS                                   C
C                                                             C
C-------------------------------------------------------------C
C                                                             C
C    SUBROUTINE MV3(X,Y,Z)                                    C
C                                                             C
C    SUBROUTINE 'MV3' PERFORMS A 3-D MOVE AND WRITES THE      C
C    NECESSARY LINES TO THE ENS DATA FILES IF THE ENS FLAG IS ON. C
C                                                             C
C    CALLING ROUTINES:                                        C
C                       DRAWIT                                C
C                       SYMBOL                                C
C                                                             C
C-------------------------------------------------------------C
C                                                             C
C    SUBROUTINE NATFPEN(FPEN)                                 C
C                                                             C
C    SUBROUTINE 'NATFPEN' CALCULATES THE PENALTY DUE TO A     C
C    NATURAL FREQUENCY OF THE SYSTEM LYING WITHIN THE         C
C    PENALIZED FREQUENCY BAND.                                C
C                                                             C
C    CALLING ROUTINES:                                        C
C                       PENALTY                               C
C                                                             C
C-------------------------------------------------------------C
```

```
C-------------------------------------------------------------C
C                                                             C
C    SUBROUTINE NODAMP(NF,RZ)                                 C
C                                                             C
C    SUBROUTINE 'NODAMP' CALCULATES THE UNDAMPED NATURAL      C
C    FREQUENCIES OF THE SYSTEM.                               C
C                                                             C
C    CALLING ROUTINES:                                        C
C                     NATFPEN                                 C
C                     MODRESP                                 C
C                                                             C
C-------------------------------------------------------------C
C                                                             C
C    SUBROUTINE OPARAM(IASSIGN,NPARAM,X,XA,XB,SPEN)           C
C                                                             C
C    SUBROUTINE 'OPARAM' DETERMINES THE NUMBER OF SYSTEM      C
C    PARAMETERS TO BE OPTIMIZED (NPARAM).  USING THE INITIAL  C
C    SYSTEM PARAMETER VALUES READ IN BY 'RAWDAT', IT INITIALIZES C
C    THE PARAMETER LIST PASSED TO THE OPTIMIZING ROUTINE.     C
C    CONVERSELY, IT CAN TAKE A PARAMETER LIST PASSED FROM THE C
C    OPTIMIZING ROUTINE AND CONVERT IT INTO CHANGES IN THE    C
C    SYSTEM PARAMETERS.  IN ADDITION, THE SIZE OF CHANGE PART OF C
C    THE PENALTY FUNCTION IS CALCULATED IN THIS ROUTINE.  THE C
C    FUNCTION PERFORMED BY THIS ROUTINE DEPENDS ON THE VALUE OF C
C    'IASSIGN'.                                               C
C                                                             C
C    CALLING ROUTINES:                                        C
C                     OPTIMIZ                                 C
C                     GRAPH                                   C
C                     PENALTY                                 C
C                                                             C
C-------------------------------------------------------------C
C                                                             C
C    SUBROUTINE OPTIMIZ                                       C
C                                                             C
C    SUBROUTINE 'OPTIM' OPTIMIZES THE MOUNT DESIGN WITH RESPECT C
C    TO THE FREQUENCY RESPONSE OF SPECIFIED POINTS IN THE     C
C    REFERENCE SYSTEM.  SYSTEM PARAMETERS MUST FIRST BE READ IN C
C    BEFORE THIS ROUTINE CAN BE EXECUTED.                     C
C                                                             C
C    CALLLING ROUTINES:                                       C
C                      PVIPDRV                                C
C                                                             C
C                                                             C
C-------------------------------------------------------------C
```

```
C-----------------------------------------------------------------C
C                                                                 C
C     SUBROUTINE ORIGIN(V,OX,OY,ORIG)                             C
C                                                                 C
C     SUBROUTINE 'ORIGIN' CALCULATES THE POSITION OF THE          C
C     CO-ORDINATE SYSTEM DRAWN ON 3-D PLOTS.  THE COORDINATE      C
C     SYSTEM ALWAYS APPEARS IN THE SAME AREA OF THE SCREEN,       C
C     REGARDLESS OF CHANGES IN THE VIEWPOINT.                     C
C                                                                 C
C     CALLING ROUTINES:                                           C
C                      GRAPH                                      C
C                                                                 C
C-----------------------------------------------------------------C
C                                                                 C
C     SUBROUTINE OUTCMAT(IUNIT,CMAT)                              C
C                                                                 C
C     SUBROUTINE 'OUTCMAT' PRINTS OUT THE SPECIFIED COMPLEX       C
C     6*6 MATRIX.                                                 C
C                                                                 C
C     CALLING ROUTINES:                                           C
C                       EIGER                                     C
C                                                                 C
C-----------------------------------------------------------------C
C                                                                 C
C     SUBROUTINE OUTEIG(IUNIT,NF,RZ)                              C
C                                                                 C
C     SUBROUTINE 'OUTEIG' PRINTS OUT EIGENVALUES AND THE          C
C     EIGENVECTORS ASSOCIATED WITH THE EIGENVALUES.               C
C                                                                 C
C     CALLING ROUTINES:                                           C
C                        EIGER                                    C
C                                                                 C
C-----------------------------------------------------------------C
C                                                                 C
C     SUBROUTINE PENALTY(N,X,PEN)                                 C
C                                                                 C
C     SUBROUTINE 'PENALTY' FINDS THE PENALTY FOR A GIVEN MOUNT    C
C     DESIGN FOR THE VIBRATIONAL SYSTEM.                          C
C                                                                 C
C     CALLING ROUTINES:                                           C
C                        ZXMIN                                    C
C                        GRAPH                                    C
C                                                                 C
C-----------------------------------------------------------------C
C                                                                 C
C     PROGRAM PVIPDRV                                             C
C                                                                 C
C     PVIPDRV IS THE DRIVER FOR THE PLANAR VIBRATION ISOLATION    C
C     PROGRAM                                                     C
C                                                                 C
C-----------------------------------------------------------------C
```

```
C-----------------------------------------------------------------C
C                                                                 C
C     SUBROUTINE QUADRAT(QUAD,LMAT,CMAT,RMAT,I)                   C
C                                                                 C
C     SUBROUTINE 'QUADRAT' TAKES 3 MATRICES LMAT, CMAT, RMAT,     C
C     AND COMPUTES THE PRODUCT LMAT*CMAT*RMAT.  IT STORES THE     C
C     PRODUCT IN 'QUAD'.                                          C
C                                                                 C
C     CALLING ROUTINES:                                           C
C                       EQNS                                      C
C                                                                 C
C-----------------------------------------------------------------C
C                                                                 C
C     SUBROUTINE RAWDAT(IERR)                                     C
C                                                                 C
C     SUBROUTINE 'RAWDAT' READS IN DATA TO BE USED IN THE PLANAR  C
C     VIBRATION ISOLATION PROBLEM                                 C
C                                                                 C
C     CALLING ROUTINES:                                           C
C                       PVIPDRV                                   C
C                                                                 C
C-----------------------------------------------------------------C
C                                                                 C
C     SUBROUTINE RESPNTS(IPNTS,X,Y,FMAX)                          C
C                                                                 C
C     SUBROUTINE 'RESPNTS' CALCULATES THE FREQUENCY RESPONSE      C
C     OF THE VIBRATIONAL SYSTEM.                                  C
C                                                                 C
C     CALLING ROUTINES:                                           C
C                       OPTIMIZ                                   C
C                                                                 C
C-----------------------------------------------------------------C
C                                                                 C
C     SUBROUTINE ROT(THETAZ,ROTMAT)                               C
C                                                                 C
C     SUBROUTINE 'ROT' CALCULATES THE ROTATION MATRIX FOR A       C
C     ROTATION THETAZ ABOUT THE Z AXIS.  THETAZ IS ASSUMED        C
C     TO BE IN DEGREES.                                           C
C                                                                 C
C     CALLING ROUTINES:                                           C
C                       EQNS                                      C
C                                                                 C
C-----------------------------------------------------------------C
C                                                                 C
C     SUBROUTINE SYMBOL(P1,P2)                                    C
C                                                                 C
C     SUBROUTINE 'SYMBOL' DRAWS THE SYMBOLS WHICH POINT TO THE    C
C     INITIAL AND OPTIMAL DESIGN POINTS.                          C
C                                                                 C
C     CALLING ROUTINES:                                           C
C                         DRAWIT                                  C
C                                                                 C
C-----------------------------------------------------------------C
```

```
C-------------------------------------------------------------C
C                                                             C
C    IMSL ROUTINE ZXMIN                                       C
C                                                             C
C    SUBROUTINE 'ZXMIN' FINDS THE MINIMUM OF A FUNCTION OF    C
C    SEVERAL PARAMETERS.  THE USER DEFINES THE FUNCTION AND   C
C    SUPPLIES THE STARTING POINT AND THE CONVERGENCE CRITERIA. C
C    THE ROUTINE RETURNS THE SET OF PARAMETERS WHICH GIVES THE C
C    MINIMUM VALUE OF THE FUNCTION.                           C
C                                                             C
C    CALLING ROUTINES:                                        C
C                      OPTIMIZ                                C
C                                                             C
C-------------------------------------------------------------C
C                                                             C
C    IMSL ROUTINE ZSRCH                                       C
C                                                             C
C    SUBROUTINE 'ZSRCH' GENERATES A SET OF STARTING POINTS USED C
C    IN SEARCHING FOR THE GLOBAL MINIMUM OF A MULTI-PARAMETER C
C    FUNCTION.  THE USER MUST SUPPLY THE LOWER AND UPPER LIMITS C
C    OF EACH OF THE PARAMETERS AND THE TOTAL NUMBER OF STARTING C
C    TO BE GENERATED.                                         C
C                                                             C
C    CALLING ROUTINES:                                        C
C                      OPTIMIZ                                C
C                                                             C
C-------------------------------------------------------------C
```