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Roger V. Varadarajan

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APPLICATIONS OF MODERN CONTROL THEORY TO
THE MANAGEMENT OF PEST ECOSYSTEMS

By

Roger V. Varadarajan

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ABSTRACT

As of the present, very little has been accomplished in terms of utilizing the available information on pest ecosystems to arrive at an optimum combination of control strategies that can be implemented in the field. Conventional pest management strategies based on field experience tend to be ad hoc and do not necessarily lead to satisfactory results. From the systems theoretic point of view, this problem can be interpreted as the determination of the "optimal control" strategies for the management of the ecosystem.

The present research focuses on the management of the cereal leaf beetle (CLB), Oulema melanopus, a key economic pest of cereal grains in the United States and Canada. A comprehensive state space model consisting of 33 state variables is developed for the CLB ecosystem, which includes the CLB, its larval parasite, T. julis, and a host plant component represented by oats. Both chemical and biological control aspects are incorporated into the model so that the model can be tested within the framework of Integrated Pest Management (IPM).

An economic optimization problem is formulated in which we seek to maximize the profit earned by the farmer. The optimal control problem is solved for both single season and multiple seasons. Optimal control strategies are characterized by emphasizing biological control and reducing chemical control usage, and are compared with conventional spraying schemes currently used. A sensitivity analysis is carried out

with reference to the timing and amount of pesticide sprayed. In general, the optimal strategies are at least as good as, and often times better than, the conventional schemes. It is found that the conventional spray is timed earlier in the growing season and is aimed at the CLB spring adults and eggs, while the optimal spray occurs a little later in the season and is targeted on the early larval instars of the CLB.

"Externality costs" are included to reflect the penalty imposed for environmental pollution caused by pesticide usage. Two different approaches are analyzed with reference to the externality problem. One is a regulatory approach, in which pesticide use is limited by absolute restrictions on the amount that can be used. Taxation is the other approach considered, in which case the performance measure is augmented with a tax. The discrete-time optimal control problems are solved using a first order successive approximation technique.

The necessity for stochastic estimation schemes in connection with pest management problems is pointed out. The Linear-Quadratic-Gaussian (LQG) approach is proposed for the combined stochastic estimation and control problem, leading to On-Line Pest Management (OLPM) systems. The overall approach to the pest management problem adopted in this work is general enough to be extended to a wide range of problems in biological resource management.

To My Wonderful Parents

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INTRODUCTION

The survival of human society requires that it exert some form of control over some of the other existing systems. In the case of pest-crop ecosystems there exists a competition between human and insect communities for resources like vegetation. Thus, exerting control on these systems is dictated more out of necessity than choice. However, control of natural systems is by no means trivial, in spite of our so-called technological and scientific progress. Insect populations no longer appear to be inert masses passively responding to changing environmental pressures (Wellington 1977). Oftentimes the consequences of our control actions have been counter-productive. Heavy crop losses in spite of tremendous application of pesticides, resistance to pesticides developed by pests, the adverse environmental effects of pesticides, and a low success rate with biocontrol attempts all provide corroborating evidence. It appears that we have been in search of a panacea, and somewhere along the way, have grossly underestimated the intricacies that typify biological systems. A logical outgrowth of these turns of events are the increasing demands for pest management programs that are not only economically feasible and profitable, but also ecologically compatible and acceptable.

Toward this end two significant ideologies have emerged: (1) the concept of "Integrated Pest Management" (Stern 1959, Smith 1962), incorporating strategies that attempt to utilize an optimal combination of all known pest control techniques including biological, cultural, and chemical approaches, and (2) the important concept of "On-Line Pest Management" (Tummala and Haynes 1977) which, in addition to being integrated in scope, provides for periodic updating of control strategies in light of changing meteorological conditions (hence changing ecological states) and the relative effectiveness of previous control strategies.

However, very little has been accomplished in terms of utilizing the available information on the pest ecosystems to arrive at an optimum combination of control strategies that can be implemented in the field. From the systems theoretic point of view this can be interpreted as the determination of "optimal control" strategies for the management of the ecosystem.

The major objective of this research is to provide the theoretical foundations for the design of control systems that will lead to optimal control strategies for on-line pest management. In general, the optimal control strategies will be characterized by efforts to emphasize biotic control while minimizing the use of pesticides. Economic and environmental trade-offs that are inherent in pest management problems will be discussed. Our research efforts will be directed toward the Cereal Leaf Beetle (CLB) (Oulema melanopus (L.)) ecosystem. However, the overall problem-solving methodology developed in this research will be general enough to be extended to a wide range of problems in biological resource management.

MODELING AND OPTIMIZATION IN THE CONTEXT OF ECOSYSTEM MANAGEMENT

Most biological systems (and many other real-world systems as well) are far too complex to be understood in all their details at any level, and far too intricate to be broken up into components without destroying the integrity of the system. Hence, models are used extensively in the representation of these systems. Modeling allows us, in principle, to isolate the components of a system and to study their interactions. This helps us recognize some important relationships that exist in the real-world system but are normally masked by complexities and interactions. More importantly models provide a framework for analyzing the system under various hypothetical situations. Especially for ecological problems these analyses can proceed well beyond what is experimentally possible to demonstrate under field conditions.

For our purposes, the models of ecosystems are classified under the broad categories of simulation models and mathematical models (Figure 1).

SIMULATION MODELS

Simulation models (sometimes referred to as descriptive models) describe ecosystem interactions usually in terms of a set of computer instructions. With the advent of modern digital computers, simulation models have found widespread use in the ecosystem analysis (see Patten 1970-1976). Simulation models can handle a great deal of detail and therefore tend to be very large. By their very nature, these simulation

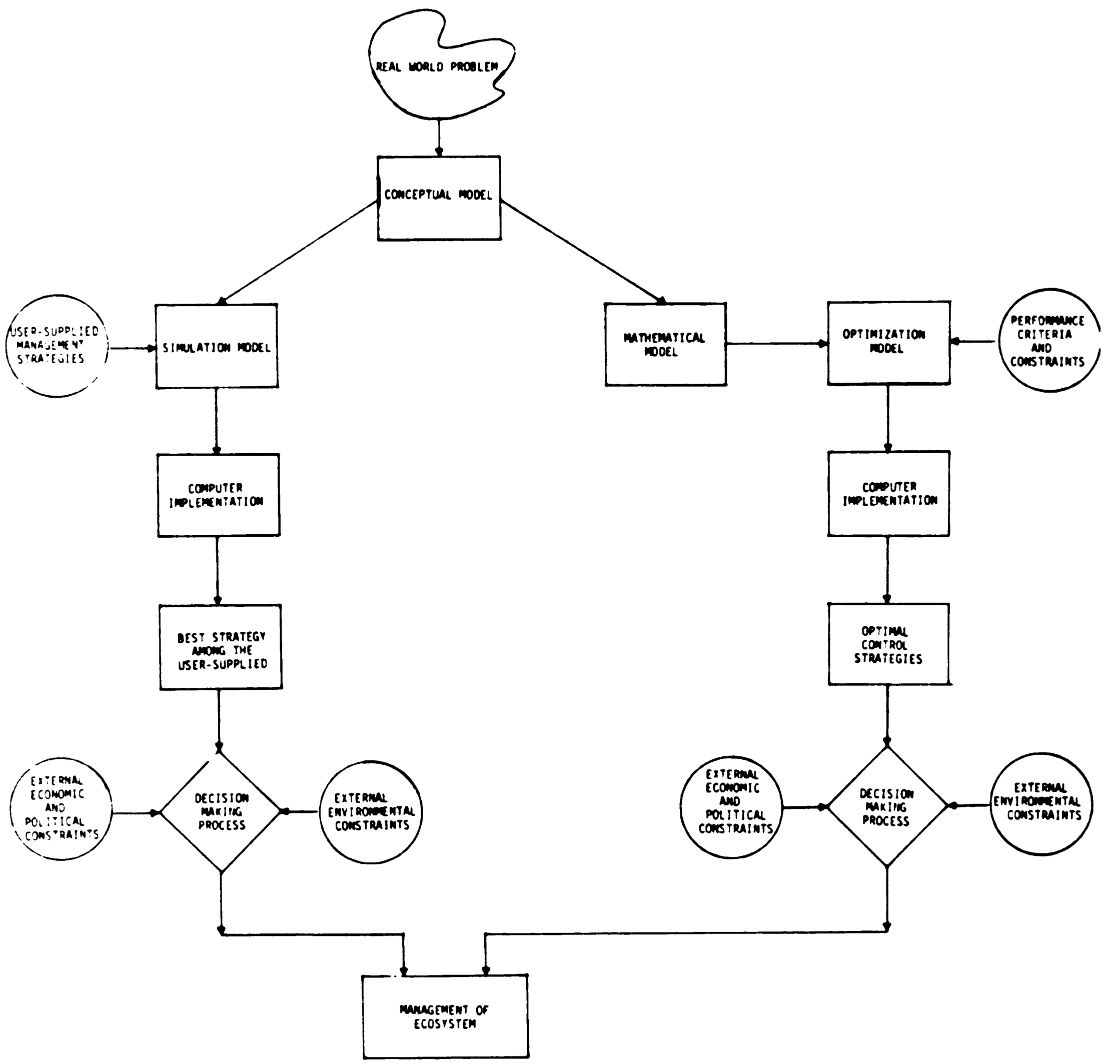

FIGURE 1. Flow diagram illustrating the methodology for ecosystem management using quantitative models.

models do not lend themselves to the analysis of alternative management options. Each one of the management options requires a large scale simulation, and the cost associated with the analysis of a large number of options is prohibitive. The main drawback is that the simulation approach does not provide the means of eliminating options that are not "optimal" in an efficient and systematic manner. *Thus, in the case of simulation models, the search for optimal policy is usually restricted to management strategies supplied apriori (i.e. user suppled policies).*

MATHEMATICAL MODELS

The shortcomings of the simulation models lead us to the second class of models, namely, "mathematical models." These models reflect the dominant features of the system, and their concise representation provides us with an alternative to the detailed (and large) simulation models.

Mathematical models are generally expressed in "state-space" form (Ogata 1967). The state equations can be expressed in differential, difference, or partial differential forms. The major advantage of using state-space models is that it is a very effective approach to mathematical representation of systems. Also, several important analytical tools like control theory, optimization theory, estimation theory, etc. are almost exclusively based on state-space models. Hence, for decision analysis involving optimization, mathematical models are generally preferred over simulation models. An excellent background on mathematical models for pest ecosystems can be found in Tummala et al (1975, 1976), Tummala (1974), Barr et al (1973), Shoemaker (1973, 1974), Kowal (1971) and Ruesink (1975) among others.

THE NEED FOR OPTIMIZATION SCHEMES

The need for optimization schemes arises in the context of goal-seeking in ecosystems or the so-called teleological approach to ecosystems. The controversy over the role and acceptability of the teleological approach to biological systems is both very old and still outstanding (Davis 1961). However, for many important classes of biological situations, only by using a goal-seeking description (Mesarovic 1968) an effective and constructive specification of the system can be developed. Indeed, the whole area of pest management, or in general, biological resource management, is basically a goal-seeking endeavor.

Essential to the management of any system is the inherent assumption that certain performance goals be defined--we would like to manage the system in such a manner that our performance goals are achieved. From the system theoretic point of view this can be interpreted as the determination of "optimal control" strategies for the management of the system. Though optimization techniques have found widespread use and success in engineering and physical systems there is only a limited amount of literature on the use of optimal control theory for ecosystem management. However, the value of optimization has been recognized by biologists. Patten (1971) stated: "The whole area of optimization theory is certainly pertinent to renewable resource management and could be used for...management schemes." Watt (1963) pointed out that "...many problems in the management of renewable natural resources are extremal problems: we would like to maximize fish yield* from a lake, lumber yield

*However, in the real world, one strives for profit maximization, not yield, because maximization of yield does not necessarily lead to maximum profit due to the existence of price elasticity.

from a farm, or minimize survival of a pest." Optimization schemes are most often used in conjunction with mathematical models in state-space form and are generally referred to as "optimization models" (refer to Figure 1).

LITERATURE REVIEW OF OPTIMIZATION MODELS IN PEST MANAGEMENT AND RELATED AREAS

In recent years several studies (Tables 1 & 2) have appeared in the literature related to optimization schemes for pest management. Watt (1963) was among the first to point out the potential use of optimization procedures like dynamic programming in pest management problems. The approach of Watt (1964) was based on an essentially brute-force technique. Several predetermined policies were tried on a spruce budworm model, and optimal policies were chosen on the basis of minimum total cost that included timber loss and control costs. Such an approach is limited by the number of policies to be considered--it only searches over a set of predetermined (i.e., user supplied) control policies and not over the entire policy space. This approach, though useful when used with simulation models, is clearly inefficient when used with mathematical models.

Jacquette (1970), Mann (1971), Becker (1970) have developed simple mathematical models in which pest populations are described by Markov processes, either continuous-time or discrete-time birth and death processes. They used dynamic inventory theory (similar to the principle of optimality) and calculus of variations to derive some necessary conditions. Jacquette (1972) pointed out that these are elementary models and have little practical use. Goh (1970, 1972) and Vincent (1975) have discussed the application of the maximum principle to population models described by simple Lotka-Volterra type equations. Goh et al (1975)

TABLE 1. Review of optimization models in pest management.

YEAR	AUTHOR(S)	SPECIFIC PEST MANAGEMENT PROBLEM/ HYPOTHETICAL PROBLEM	COMPONENTS OF THE MODEL	INCLUSION OF AGE-CLASSES	NATURE OF THE MATHEMATICAL MODEL	NUMBER OF STATE VARIABLES	OPTIMIZATION TECHNIQUE USED	TYPE OF PERFORMANCE MEASURE	INCLUSION OF STATE ESTIMATOR (FILTER)	GENERAL COMMENTS/ SPECIFIC RESULTS
1963	Watt	Spruce Budworm	Pest	NO	Deterministic	-	Dynamic programming	Economic optimization	NO	Introduced the use of dynamic programming for pest management problems.
1964	Watt	Spruce Budworm	Pest Parasite	YES	Deterministic	-	Exhaustive search using simulation	Minimize tree damage & control costs	NO	Does not use any optimization scheme. Simulations are made for each one of the user-supplied control policies & the best among them is taken as optimal.
1970	Becker	Hypothetical	Pest	NO	Deterministic continuous time	1	Maximum principle	Minimize pest damage & control costs	NO	Considers birth and death process in which state-transition probability is incorporated
1970	Jacquette	Hypothetical	Pest	NO	Deterministic discrete-time continuous time	1	Dynamic inventory theory	Minimize cost of pest damage and control costs	NO	Optimization procedure is similar to dynamic programming.
1971	Mann	Hypothetical	Pest	NO	Deterministic continuous time	1	Dynamic programming	Minimize control costs	NO	Derives basic decision rules for optimality.
1971	Headley	Hypothetical	Pest	NO	Deterministic	1	Ordinary calculus	Maximize profit	NO	Develops useful interpretation of "economic threshold".
1972 1974	Jacquette Swan									Survey paper on mathematical models for controlling growing biological population.

TABLE 1. (Continued)

YEAR	AUTHOR(S)	SPECIFIC PEST MANAGEMENT PROBLEM/ HYPOTHETICAL PROBLEM	COMPONENTS OF THIS MODEL	INCLUSION OF AGE-CLASSES	NATURE OF THE MATHEMATICAL MODEL	NUMBER OF STATE VARIABLES	OPTIMIZATION TECHNIQUE USED	TYPE OF PERFORMANCE MEASURE	INCLUSION OF STATE ESTIMATOR (FILTER)	GENERAL COMMENTS/ SPECIFIC RESULTS
1972	Carlos Ford-Le-vine	Hypothetical	Pest Parasite Plant	NO	Deterministic & stochastic discrete-time	3	Dynamic programming	Minimize crop loss and control cost	YES	Introduces estimators and propose a stochastic control problem to be solved using stochastic dynamic programming.
1973	Carlos Ford-Levine	Hypothetical	Pest Parasite Plant	NO	Deterministic discrete-time	3	Dynamic programming	Minimize crop loss and control cost	NO	Derives optimal control policies
1973	Hall & Norgaard	Hypothetical	Pest	NO	Deterministic continuous time	1	Calculus	Economic optimization	NO	Derives necessary conditions for optimality. Assumes a single application of control per season.
1973	Chatter-jee	Hypothetical	Pest	NO	Deterministic continuous time.	1	Calculus of variations	Minimize crop loss and control cost	NO	Considers a birth and death process with immigration.
1973	Shoemaker (3 part paper)	Mediterranean flour moth	Pest Parasite	NO	Deterministic discrete-time	2	Dynamic programming	Minimize pest damage & control costs	NO	Determines optimal policies in the form of a spray chart. Insecticide treatments are given as a function of pest, parasite densities.
1974	Goh et al	Hypothetical	Pest Parasite	NO	Deterministic continuous time	2	Maximum principle	Economic optimization	NO	Derives necessary conditions for maximum profits.

TABLE 1. (Continued)

YEAR	AUTHOR(S)	SPECIFIC PEST MANAGEMENT PROBLEM/ HYPOTHETICAL PROBLEM	COMPONENTS OF THE MODEL	INCLUSION OF AGE-CLASSES	NATURE OF THE MATHEMATICAL MODEL	NUMBER OF STATE VARIABLES	OPTIMIZATION TECHNIQUE USED	TYPE OF PERFORMANCE MEASURE	INCLUSION OF STATE ESTIMATOR (FILTER)	GENERAL COMMENTS/ SPECIFIC RESULTS
1974	Heuth & Regev	Hypothetical	Pest, Plant, susceptibility to insecticide	NO	Deterministic discrete-time	3	Maximum principle	Economic optimization	NO	Derives necessary conditions for optimality. Provides economic interpretation.
1974 1975	Dantzing Winkler	Spruce budworm	Pest, tree	No	Deterministic discrete-time	3	Modified dynamic programming	Economic optimization	NO	Provides Markov-Chain interpretation of dynamic programming. Optimal logging policies are derived.
1975	Talpaz & Borosh	Semi-realistic cotton crop ecosystem	Pest	NO	Deterministic continuous time	1	Calculus	Profit maximization	NO	Derives optimal frequency and application of pesticide.
1975	Vincent	Hypothetical	Pest Predator	No	Deterministic continuous time	2	Maximum principle	Minimize damage and control cost	NO	Derives necessary conditions for optimal prey-predator densities.
1975	Goh et al	Hypothetical	Plant	NO	Deterministic continuous time	1	Calculus	Maximize crop yield	NO	Derives optimal cropping rules for greenhouse vegetables.
1975	Taylor & Headley	Hypothetical	Pest	Includes genetic classes	Deterministic discrete-time also, stochastic case	3	Dynamic programming Monte-Carlo techniques.	Maximize genetic most susceptible to insecticides	NO	Incorporates pesticide resistance and its effect on genetic classes of pest populations.

TABLE 1. (Continued)

YEAR	AUTHOR(S)	SPECIFIC PEST MANAGEMENT PROBLEM/ HYPOTHETICAL PROBLEM	COMPONENTS OF THE MODEL	INCLUSION OF AGE-CLASSES	NATURE OF THE MATHEMATICAL MODEL	NUMBER OF STATE VARIABLES	OPTIMIZATION TECHNIQUE USED	TYPE OF PEFORMANCE MEASURE	INCLUSION OF STATE ESTIMATOR (FILTER)	GENERAL COMMENTS/ SPECIFIC RESULTS
1975	Rorres & Fair	Hypothetical	Pest	YES	Deterministic discrete-time	-	Calculus	Maximize the yield of specific age-class.	NO	Derives optimality conditions such that the pest population returns to identical age configuration after each harvest.
1975	Feder & Regev	Hypothetical	Pest Parasite	NO	Deterministic discrete-time	2	Calculus	Economic optimization	NO	Describes the effect of user-cost on optimal policies for regional pest management.
1975	Mitchiner et al	Hypothetical	Pest Parasite Plant	NO	Deterministic continuous time	4	Maximum principle	Profit maximization	NO	Optimal control problem is cast as a linear regular problem.
1976	Taylor	Hypothetical	Pest	NO	Deterministic & stochastic discrete-time	1	Dynamic programming	Profit maximization	NO	Derives optimal sterile-male release strategies.
1976	Regev et al	Alfalfa Weevil	Pest Plant	YES	Deterministic discrete-time	4	Nonlinear programming (reduced gradient technique)	Profit maximization	NO	Transforms the dynamic problem into a static optimization problem and then uses N/L programming. Optimal spraying rules are obtained.
1976	Marsolan & Rudd	Southern Green Stink Bug	Pest	YES	Deterministic distributed-parameter model	-	Maximum principle	Economic optimization	NO	Derives optimal control policies for a distributed-parameter pest management model.

TABLE 1. (Continued)

YEAR	AUTHOR(S)	SPECIFIC PEST MANAGEMENT PROBLEM/ HYPOTHETICAL PROBLEM	COMPONENTS OF THE MODEL	INCLUSION OF AGE-CLASSES	NATURE OF THE MATHEMATICAL MODEL	NUMBER OF STATE VARIABLES	OPTIMIZATION TECHNIQUE USED	TYPE OF PERFORMANCE MEASURE	INCLUSION OF STATE ESTIMATOR (FILTER)	GENERAL COMMENTS/ SPECIFIC RESULTS
1977	Vincent et al	Hypothetical	Prey Predator	NO	Deterministic continuous time	2	Maximum principle	Minimize pest damage and control costs	NO	Derives necessary conditions for a periodic optimization problem.
1977	Shoemaker	Alfalfa Weevil	Pest Parasite Plant	YES	Deterministic discrete-time	16	Dynamic programming	Profit maximization	NO	Derives multi-dimensional economic thresholds. Uses model decompositions to circumvent dimensionality problem.
1977	Regev et al	Alfalfa Weevil	Pest Plant	NO	Deterministic	2	Dynamic programming	Profit maximization	NO	
1977	Birley	Sugar cane froghopper	Pest	YES	Deterministic	-	Modified dynamic programming	Minimize damage and cost control	NO	Derive optimal policies by assuming binary-valued controls (spray no spray). Model does not take into account temperature effects on insects.
1978	Talpaz et al	Boll Weevil	Pest	NO	Deterministic	1	Nonlinear programming	Profit maximization	NO	Develops optimal spraying policies. Provides sensitivity analysis with respect to price changes in crop price.
1979	Gutierrez et al	Alfalfa Weevil	Pest Plant	NO	Deterministic	4	Nonlinear programming	Maximize Profit	NO	Optimal spraying rules are developed with pesticide resistance taken into account.

TABLE 2. Optimization problems in related areas.

YEAR	AUTHOR(S)	SPECIFIC PEST MANAGEMENT PROBLEM/ HYPOTHETICAL PROBLEM	COMPONENTS OF THE MODEL	INCLUSION OF AGE-CLASSES	NATURE OF THE MATHEMATICAL MODEL	NUMBER OF STATE VARIABLES	OPTIMIZATION TECHNIQUE USED	TYPE OF PERFORMANCE MEASURE	INCLUSION OF STATE ESTIMATOR (FILTER)	GENERAL COMMENTS/ SPECIFIC RESULTS
1967	Davis	Deer management	Deer forest (tree)	NO	Deterministic	-	Dynamic linear programming (DLP)	Maximize deer harvest and timber production	NO	Derives optimal harvesting for deer management.
1970	Swartzman	Hypothetical game management			Deterministic static	-	Non-linear programming		NO	Explores the use of non-linear programming for game management.
1970	Goh	Hypothetical fisheries problem	Fish	NO	Deterministic	1	Maximum principle	Maximize fish harvest	NO	
1975	Bahrami and Kim	Hypothetical cancer treatment	Cancer cells	YES	Deterministic	7	Maximum principle & gradient projection	Minimize final population of cancer cells and drug usage	NO	Uses a bilinear model with binary valued controls. The optimization technique is based on control vector iteration.
1975	Sancho and Mitchell	Hypothetical fisheries problem	Fish	NO	Deterministic	1	Dynamic programming	Profit maximization in fisheries	NO	
1975	Rauch et al	Semi-realistic lobster-growth facility in controlled environment	Lobster	NO	Deterministic	3	Maximum principle	Maximize profit by maximizing lobster weight	NO	Derives optimal rules for the control variables that include temperature, space, recirculation, and food in the controlled environment.
1975 1976	Walters	Salmon fisheries	Fish	NO	Stochastic	1	Stochastic dynamic programming	Maximize fish yield	YES parameter estimator	Derives optimal catching policies for salmon.
1976	Katz	African weaver birds	Bird (with 3 submodels)	NO	Deterministic	1	Calculus of variations	Maximize bird weight maximize survival of the bird, etc.	NO	Derives optimal feeding strategies for the bird. Uses sufficiency conditions for optimality.
1979	Hutchinson and Fisher	Atlantic sea scallop fishery	Fish	NO	Stochastic with random parameters	1	Stochastic dynamic programming	Maximize profit	NO	Derives optimal harvesting rules for the fishery.

have derived optimal cropping rules for greenhouse crops based on ordinary calculus. Vincent et al (1977) introduced the concept of isochronal systems and derived necessary conditions for periodic optimization of a scalar system representing biological populations.

Hueth and Regev (1974) proposed a hypothetical model with pest, plant, and pesticide-resistance components and derived necessary conditions for optimality using the maximum principle for a profit maximization problem. Marsolan and Rudd (1976) developed a distributed parameter model for the Southern Green Stink Bug, a major pest of soybeans, and used the maximum principle to derive optimal control strategies. Rorres and Fair (1975) considered an age-specific population and derived conditions for optimal harvesting subject to linear ecological and economic constraints. Feder and Regev (1975) pointed out the importance of externality costs and analyzed the effects of user-cost on optimal policy. Mitchiner et al (1975) discussed the application of optimal linear regulator theory to the pest management problem, and used the maximum principle to solve a hypothetical problem. Taylor and Headley (1975) presented a model with genotype classes in which physiological resistance to insecticides is incorporated, and suggested the use of dynamic programming to solve a simplified version of the problem. Headley (1971) in his elegant, yet simple, work reintroduced the concept of "economic threshold" that provided significant insights into the economic considerations that are inherent in pest management decision-making.

Shoemaker (1973) demonstrated the application of dynamic programming to arrive at optimal pest management strategies by considering a semi-realistic pest-parasite model for the Mediterranean Flour Moth. Hall and Norgaard (1973) derived an optimal quantity of pesticide spray under the assumption that there exists a single optimal time for the application of pesticides. Talpaz and Borosh (1974) derived optimal frequencies and quantities per application of pesticide spray for a cotton crop ecosystem consisting of a single pest population. Regev et al (1976) utilized non-linear programming to solve an economic optimization problem for the alfalfa weevil. Use of dynamic programming for the determination of optimal sterile male release strategies for a hypothetical pest population is discussed by Taylor (1976). Carlos Ford-Livene (1972, 1973) used dynamic programming to solve a hypothetical pest management problem described by a linear model and suggests the use of stochastic dynamic programming to solve the stochastic optimal control problem.

Birley (1977) proposed a transfer function approach to modeling pest ecosystems and used a modified dynamic programming technique to solve a linear optimal control problem with binary valued controls (i.e., spray/no spray scheme). Dantzig (1974) provided a Markov-chain interpretation to the dynamic programming approach by incorporating state-transitional probabilities. Winkler (1977) utilized a modified dynamic programming technique based on Dantzig's approach to solve a fairly realistic pest management problem for the spruce budworm. Talpaz et al (1978) discussed a simulation model for the boll weevil-cotton ecosystem, and used a modified version of Fletcher-Powell-Davidson's non-linear programming algorithm to derive optimal pesticide spraying schemes. Gutierrez et al (1979) presented a simplified model of the alfalfa weevil ecosystem with four components: (1) population dynamics of the alfalfa weevil, (2) dynamics of the alfalfa crop, (3) mortality induced by pesticides, and (4) evolution of pesticide resistance in the weevil population, and used non-linear programming techniques to derive optimal spraying strategies for two cases--with and without information on the development of resistance by the weevil population.

Shoemaker (1977) employed dynamic programming to successfully solve a comprehensive model for the alfalfa weevil. The approach was based on decomposing the original model into two coupled models in order to reduce the dimensionality. However, the model has some simplifying assumptions: for example, it was assumed that pest control measures are applied only once per generation, and the pest population has discrete (non-overlapping) generations. Further, the approach lacks generality because the assumptions made toward simplification of the problem are very specific to the pest ecosystem under consideration. Such inadequacies are likely to preclude its applications in many problems of practical interest. Nevertheless, the work represents one of the few notable exceptions that are oriented toward the solution of a realistic (and invariably complex) pest management problem.

In related areas, Walter (1975, 1976) utilized stochastic dynamic programming in conjunction with a scalar model to arrive at optimal catching policies for salmon. Goh (1970), Sancho and Mitchell (1975) proposed optimal management schemes for fisheries based on rudimentary models. Rauch et al (1975) used the maximum principle to determine temperature control schemes for lobster growth in a controlled environment. Katz (1978) explored the use of the maximum principle to gain insight into optimal feeding strategies for African weaver birds.

Hutchinson and Fischer (1979) discussed the application of stochastic control theory to fishery management, and solved a simple logistic model of the Atlantic sea scallop fishery using stochastic dynamic programming. Dynamic programming for deer management (Davis 1967) and non-linear programming to game management (Swartzman 1970) have been attempted in the area of wild life management. Optimal control approaches are also being explored for improving existing control schemes in medical and biomedical problems related to population control, health care, nourishment, etc. For example, Baharami and Kim (1975) presented optimal schemes for chemotherapy. They used a control vector interaction scheme based on the gradient projection method to solve a bilinear model with a binary valued {0,1} control. Detailed surveys of such problems in medical, biomedical, and related areas are presented by Jacquette (1972) and Swan (1973). Generally speaking, these models, like their counterparts in pest management, tend to be hypothetical and highly simplified, while dynamic programming remains the tool that is widely used with these simplified models.

DRAWBACKS IN PAST EFFORTS

The past works contributed immensely to the understanding of optimal control schemes for pest management and related problems. Nevertheless, several drawbacks exist.

Generally speaking, the shortcomings associated with past works are broadly classified into three categories: (1) drawbacks related to the model-representation of the biological system, (2) inadequacies in dealing with economic considerations, and (3) drawbacks related to the choice and use of mathematical representation and optimization schemes.

<u>Drawbacks related to the biological model representation</u>

The major shortcoming of most of these models is that they are over simplified from a biological point of view. Generally, the dimensions of the models were 3 or 4, with most of them scalar. Such highly simplified models could not capture most of the biological details. Ideally, the models should include the three major biological components of the ecosystem: pest, plant, and parasite. Several state variables may be needed to represent each one of these three components. Tummala (1977) pointed out that the advisability of implementing any control measure and the effectiveness of the control scheme depends on many factors, such as the age distribution of the pest population, the maturity and the vigor of the plant, the size of beneficial insect (parasite) populations, and weather.

The age distribution of the pest populations is very important because the damage an insect inflicts, and its susceptibility to insecticides, predators, and parasites depends on its stage of development. For example, some parasites attack only eggs, others attack only larvae of a specific size. The damage inflicted on crops also varies as an insect develops. For example, fourth instar larvae of the cereal leaf beetle (hereafter referred to as CLB) cause about 24 times the damage caused by the first instar larvae. Another related factor that should be taken into consideration in determining the effectiveness of control measures is plant vigor and maturity. Vigorous plants can often compensate for moderate damage so that pest infestations have little effect on yield. A plant's susceptibility to damage also varies as it matures.

Susceptibility also depends on the temporal relationship between the susceptible stages of the plant and the damaging stages of the insect population. The rates at which pest populations develop from one life stage to the next depend on temperature, humidity, and other climatic factors. Finally, the potential damage to the crop can be reduced if the pests are controlled by beneficial insects. However, the size of this reduction depends on the sizes of the pest population and the beneficial insects, the time synchrony between the two populations, and the age distribution of the pest when it is killed by its natural enemies.

It is obvious that most, if not all, of these factors have to be included in any real-world pest management problem, whereas the effect of including them will be that of tremendously increasing the number of state variables used in the model. It should be emphasized here that we do not advocate unnecessarily increasing the complexity of the models. As Holling (1977) points out,

> ...a major effort in modeling should be directed toward achieving a minimal representation of the system and one has to be ruthlessly parsimonious in selecting the state variables.

However, we do wish to emphasize the fact that most of the optimization models found in ecological literature are so simplified that they cease to represent even the dominant features of the ecosystem, and can no longer be identified with the real-world problem. As one would suspect, this is one of the prime reasons why biologists have chosen to largely ignore optimization models while opting for complex simulation models. Clearly there exists trade-offs between the simplicity desired for the purposes of optimization and mathematical analysis, and the complexity needed to capture the biological details--a meaningful and acceptable (to biologists) compromise must be found.

In addition to being simplified, most of the optimization models found in pest management literature are entirely hypothetical and are not based on biological studies. This is because functional forms for the pest ecosystem models are either not available, or (for the most part) are not sought after. Incidentally, this points out the significance and the need for an interdisciplinary approach to pest management and related problems.

Furthermore, past efforts have mostly ignored the parasite or the beneficial insect component of the ecosystem. This is a major drawback since the parasite component represents the biotic control of the pest population. Thus, optimal control schemes that have ignored the biotic control aspect of pest management and have concentrated only on chemical control do not represent an "integrated" control approach to pest management.

Inadequacies related to economic considerations

Another area which has not been adequately explored is that of the economic considerations involved in the pest management decision-making process. The science of economics enters into the design of pest management strategies primarily because the goals of pest management are mostly economic in nature.

Pest management problems are often posed as economic optimization problems (i.e., profit maximization or cost minimization). Economic theory, as it has been applied to pest management problems, is usually presented in the form of a threshold analysis. Such an analysis is popularly termed as the "economic threshold" in pest management problems (See Edens (1977) for an excellent critique of economic threshold, and Tummala and Varadarajan (1976) for an extensive bibliography).

The concept of economic threshold evolved as a direct application of microeconomic optimization techniques to agricultural management. In its most simplified form it is merely a restatement of the economic cost minimization criterion--undertake an additional expenditure only when the incremental increase in revenue which occurs as a result of the effort is greater than (or equal) to the incremental cost (Edens 1977).

In the context of pest management, the lowest pest density that can cause economic damage to the crop is often referred to as the Economic Injury Level. Based on this notion, the economic threshold is defined as the pest density at which control measures should be determined in order to prevent the pest population from reaching the economic injury level. It is obvious that the economic damage to any crop is dependent on a variety of factors, including the specific crop, the particular growing season, the prevailing crop price, the pesticide cost, etc. Hence, pest density represented by the economic threshold may vary through time, or vary from crop to crop, region to region, and season to season with society's changing scale of economic values. However, for the present, economic thresholds specified by entomologists tend to remain as static levels. For example, the economic threshold for the CLB is currently specified as 3 eggs and larva/stem (Ruppel 1974).

From the control theoretic point of view, the economic threshold can be interpreted as the optimal state (usually representing a specific lifestage with an associated age class) trajectory for the pest density obtained as a solution to the economic optimization problem. The more realistic the model of the pest ecosystem the more meaningful will be the resulting economic threshold.

Several agricultural economists (Headley 1971 & 1975, Hall and Norgaard 1973, Hilderbrant 1960, and Hueth and Regev 1974) considered economic optimization problems in pest management leading to useful theoretical interpretations of economic threshold. However, these works have serious drawbacks in terms of the ecosystem model considered. Generally speaking, the models were purely hypothetical and highly simplified, and lacked the biological control component. Except for a few isolated cases, e.g. Shoemaker (1977) attempts to solve meaningful economic optimization problems related to pest management are clearly lacking. As Edens (1977) rightly points out:

> The main impediment to the more generalized utilization of the threshold principle is the level of abstraction at which the concept is generally presented...Even though conventional optimization techniques are not all applicable to all pest management problems, they are currently underutilized, largely because of the difficulty involved in operationalizing them. As the increased cost of the chemical control becomes more apparent, it will be clearly recognized that complex optimization techniques based on the dynamic interactions of the agroecosystem have to be used in order to arrive at viable management strategies.

A closely related topic of interest that is often overlooked is that of "externality costs" associated with the chemical controls. Externalities arise due to the tacit assumption--that there is no difference between private and social benefits or between private and social costs--does not hold good in several instances. In the economic jargon, externality due to the environmental pollution caused by chemical controls will be classified as the "external diseconomy of production" (Mansfield 1975). An external diseconomy occurs when an action taken by an economic unit results in uncompensated costs to others. When such costs are due to increases in a firm's production, they are termed external diseconomies of production. Most of the environmental pollution problems fall into this category of external diseconomies of production. In such cases, the private costs do not reflect the full social costs since the firms responsible for the pollution are not charged for their actions that lead to environmental degradation.

In recent years, however, consumer groups have become increasingly vocal in protesting against environmental pollution--and rightly so. It is conceivable that legal enforcements will become widespread in the years to come. Therefore, it is worth our efforts to consider the problem of externality as it is present in pest management problems. This has to be carried out within the framework of hypothetical enforcement criteria, leading to potentially useful policies. Unfortunately, a very limited amount of literature exists in this area, especially with reference to quantitative analysis. Feder and Regev (1975), Regev et al (1976), Brook (1972, 1973) have made some initial attempts in this direction by considering hypothetical pest ecosystems. Obviously more research is needed. It is the belief of this author that it has to come from economists. Nevertheless, an attempt will be made in this research to gain insight into potential enforcement policies that take into account externalities, and more importantly, to provide control theoretic interpretations for such policies.

Drawbacks related to optimization schemes

One of the major drawbacks in past efforts is associated with the choice, and consequently, the limitations of the specific optimization schemes used. We are concerned here with discrete-time optimization techniques since most of the pest ecosystem and other biological systems are conceptually modeled as discrete approximation to continuous-time systems. Also, as Innis (1974) points out in his excellent paper "Dynamic Analysis of Soft-Science Studies: In Defense of Difference Equations," the use of difference equations is more appropriate in modeling biological systems. This is because insects have several distinct life stages, with time delays associated with maturation in each stage, which are modeled easily with difference equations. Further, it is more meaningful to model the control variables, such as pesticides, as discrete variables, since they are usually applied at certain discrete levels and not at a continuously varying level. Alternative modeling schemes for pest ecosystems include differential equations involving time delays, and partial differential equations (Barr et al 1973) that treat each individual's maturity or physiological age as a point in a continuum. In general, such modeling approaches are cumbersome (when compared with discrete-time models) from the standpoint of optimization. Thus, we are interested in optimization schemes that can handle discrete-time problems as well as rather large dimensional problems (since most of the real world pest management problems are large scale).

Basically there are three different optimization approaches that have been tried in the past: (1) dynamic programming, (2) maximum principle, and (3) non-linear programming.

Among the optimization techniques that were used, "dynamic programming" due to Bellman, is very appealing because a feedback solution is obtained. Further, hard constraints on state and control variables (which are very difficult to incorporate in most of the optimal control schemes) are very easy to handle with the dynamic programming approach. In fact, the presence of constraints on admissable state and/or control variables actually simplify the dynamic programming procedure by reducing the size of the region over which the search for optimal solution is made. Furthermore, extension of dynamic programming to stochastic areas is fairly straightforward. However, the straightforward dynamic programming technique is hampered by the "curse of dimensionality" (Kirk 1970). Thus, for a system with just three state variables and 100 quantization levels for each state, we will require $(100)^3 = 10^6$ storage locations which is enormous even for modern day computers to handle. As a result, several techniques have been developed that attempt to reduce the amount of storage locations required to implement the dynamic programming algorithm. State increment dynamic programming of Larson (1968) provides considerable savings in high speed storage requirements. Bellman and others have suggested polynomial approximations for the return function in order to reduce dimensionality requirements. Nevertheless, the computer solution of a dynamic programming problem still remains a formidable task when the dimension of the problem is greater than, say, 3 or 4 (Jacobson and Mayne 1970).

Yet another approach that has been utilized is that of transforming the dynamic optimization problem into an equivalent static optimization (i.e., mathematical programming) problem, and then employing mathematical programming techniques to arrive at optimal solutions (Pearson and Sridhar 1966, Cannon, Cullum and Polak 1969, and Tabak and Kuo 1971). This method has the advantage that constraints are easily handled and aperiodic problems can be considered. However, it is unwieldy in most cases, especially when the grid points in the discretized time horizon are large, resulting in an extremely large number of variables. In general, a dynamic optimization problem with N state variables, M control variables, and K grid points in the discretized time horizon will be transformed into an equivalent static optimization problem with (N + M) * K variables. Thus, a problem of the size considered in this research will result in (33 + 1) * 30 variables--over 1000 variables. Also, some computational simplifications that are possible with this approach, when the system equation is linear and time-invariant, cannot be extended to the case of biological systems that are generally non-linear and time-varying.

In recent years, the maximum principle has been applied to discrete-time problems. In reality, the (continuous) maximum principle is not universally valid for the case of discrete systems. Due to restrictions on possible variations of the control signal, the continuous maximum principle must be modified for the general discrete case (Sage and White 1977). Athans (1966, 1972) discusses the restrictions of the discrete maximum principle with reference to the convexity/directional convexity requirement on the reachable sets. Even though the discrete maximum principle has been used in deriving necessary conditions for optimality, very little computing experience with this approach is reported in literature (Kleinman and Athans 1966, Athans 1972, and Jacobson and Mayne 1970) in contrast to the continuous case.

The dimensions of real-world pest management thus dictate the use of a more suitable discrete-time optimization algorithm. However, it should be emphasized that the choice of the optimization algorithm depends on a variety of factors, including the specific problem on hand, the computational aspects of the algorithm, and the individual's own preference for any particular algorithm. In short, there are no general rules for choosing between optimization schemes.

Another major drawback associated with the currently available optimization models in pest management is the fact that they have completely ignored the stochastic aspects of the control problem and concentrated instead on the deterministic problem. In the deterministic optimal controller design, one assumes that <u>exact</u> measurement of <u>all</u> state variables are available. This is seldom the case in practical applications and especially so in pest management problems. For example, while it is generally easier to take measurement of larval stages of an insect, it is difficult to measure densities of pupa and adult. The problem is further compounded by the fact that certain age-classes (within life stages of an insect) introduced for modeling purposes, cannot be distinguished in the field, and therefore, cannot be measured. Even if one could measure all the state variables, there would be measurement errors introduced by physical sensors (or human errors) in carrying out the measurements. This measurement uncertainty should be taken into account in the design of the optimal controller. Also, in real world situations there is likely to be disturbance inputs acting on the physical process described by the system model, e.g. climatological changes affecting an ecosystem. It is obvious that a deterministic optimal controller will not be optimal in a real world stochastic situation. In order that we may take into account the stochastic aspects of the problem, the design of the optimal controller should include a stochastic estimator/filter and a scheme for stochastic feedback control (Athans 1971).

Very few people have discussed or attempted the stochastic aspects of pest management problems. Logan (1977) came up with an elementary form of filter based on regression equations to provide improved density estimations for the larvae of the CLB. Hildebrand and Haddad (1977) considered the estimation problem for insect populations and derived a parametric filter based on a distributed parameter model for the alfalfa weevil. These two approaches, however, are confined to the filtering problem and do not deal with the control problem.

Ford-Livene (1972) is the only one to have touched on the topic of stochastic estimation and optimal control for pest management. However, his approach appears short-sighted: he assumed a linear system dynamics and suggested the use of stochastic dynamic programming. As mentioned earlier, it will be a mistake to assume linear system dynamics for the generally non-linear pest ecosystem problems. Starting with a linear dynamics for the system (as Ford-Livene did) is markedly different from considering a linearized version of a non-linear system about the optimal trajectory. Further, Ford-Livene suggests that the use of stochastic dynamic programming for solving the stochastic optimal control problem. As Athans (1972) rightly points out, this approach is entirely impractical for most of the real world problems since the curse of dimensionality associated with dynamic programming is far more severe in the stochastic case as compared to the deterministic case. It is also worth noting in passing that while Ford-Livene outlined the stochastic optimal control problem, he has not provided the design of such a controller, nor extended it to a realistic pest management problem. In any case, the stochastic dynamic programming approach will be unsuitable to large scale pest management problems due to the curse of dimensionality.

Another approach, popular among statisticians, for handling some stochastic aspects is the use of stochastic models incorporating "birth and death processes" (Jacquette 1970, Mann 1971, and Chatterjee 1973). However, this approach is restricted to very simple applications, and caters mostly to theoretical interest. Also, this approach lacks the generality, usefulness, and the computational advantages of the state-space approach traditionally used in the engineering disciplines.

Another important aspect in the design of the optimal controller for pest management systems that has not been explored in the past works is the on-line capability of the controllers. Tummala and Haynes (1977) in their paper "On-Line Pest Management Systems" give a lucid account of the need and desirability of on-line features in pest management systems. They point out that pest management systems should have provisions for periodic updating of control strategies in light of changing meteorological conditions, ecological states of the ecosystem, and effectiveness of previous control strategies. This important feature of on-line capability has not been addressed in past works.

A complete survey of the algorithm and computational techniques for optimal control and estimation problems is beyond the scope of this writing due to space limitations. Besides, several excellent survey papers are available on these topics.

For example:

Survey papers on optimal control--Fuller (1962), Paiewonsky (1965), Athans (1966), Bryson (1967), Larson (1967), Athans (1971), Mendel and Gieseking (1971), Athans (1972), and Polak (1973).

Survey papers on estimation techniques--Rhodes (1971), Athans (1971), Mendel and Gieseking (1971), Athans (1972), and Leondes (1970).

In addition, there are several well-written texts in these areas--Bryson and Ho (1975), Meditch (1969), Jacobson and Mayne (1970), Schweppe (1973), Dyer and McReynolds (1970), Saridis (1970)--to name just a few.

Recapitulating, the drawbacks in past approaches (with a few exceptions in each case) are summarized as follows:

1. Most of the models are based on bypothetical ecosystems.
2. Most are overly simplified from a biological point of view.
3. The parasite component (which represents the biotic control component) has been largely ignored.
4. Age-distribution of the biological populations has not been taken into account in many cases.
5. Economic considerations have not been adequately addressed.
6. Most of the optimization schemes employed cannot handle more than 3 or 4 state variables.

7. Most of the approaches were deterministic. The need for stochastic control and estimation schemes has been mostly ignored.
8. On-line capability for the optimal controller has not been attempted.

PROBLEM DESCRIPTION

Briefly, the research problem can be stated as the determination of optimal decision rules for the "integrated control" of the CLB ecosystem. This involves the determination of both the timing and the amount of pesticide spray to be used in the field. In addition, the optimal decision rules to manage the CLB will be spearheaded by efforts to take maximum advantage of the beneficial effect of T. julis, a larval parasite of the CLB. Obviously there exists trade-offs between the use of biocontrol and chemical control approaches--especially with reference to revenue from the crop which is of great economic importance to the farmer. For example, chemical controls lead to short-term economic benefits. On the other hand, biocontrol attempts do not give instant pay-offs, but, over a long run, are likely to provide stable economic gains. As such the optimization attempts will be aimed at striking a reasonable balance between biocontrol and chemical control with minimal sacrifices in profit.

Within the framework of our model, optimal decision rules will be evaluated against current control practices (which are based on the recommendations of economic entomologists) in order to gain insight into potentially useful, and conceivably better, control strategies. Incidentally , this will also allow us to view in proper perspective the current spraying recommendations that are based on valuable field experience of entomologists but have never been quantitatively evaluated for either the timing or the amount of spray.

The individual farmer, who is mainly concerned about the ultimate revenue from the crop, has a tendency to emphasize chemical control and make some immediate monetary gains. This frequently leads to excessive spraying, and consequently to environmental pollution. In this context, there exist economic and environmental trade-offs in all pest management problems. In this work these trade-offs will be discussed within the framework of optimal decision rules for pest management.

For a single growing season, the larval parasite T. julis virtually plays no role, but its effect will be felt in the subsequent growing season. Thus, control policies have to be evaluated over multiple seasons in order to determine the effect of biocontrol. Repeated application of conventional control policies season after season, as well as repeated use of optimal policies (on a season by season basis) will be evaluated within the framework of the CLB ecosystem model.

As part of our approach to determine the optimal control strategies for the CLB problem, we will develop a discrete-time optimal control technique based on the successive approximation algorithm of Dyer and McReynolds (1970) that is similar in scope to the differential dynamic programming approach of Jacobson and Mayne (1970). The algorithm is utilized within a deterministic framework to solve several types of optimal problems associated with the integrated control approach to the CLB problem. Extension of the deterministic approach to the stochastic case is discussed, and the Linear-Quadratic-Gaussion (L-Q-G) methodology is proposed for the design of an on-line control system for pest management.

In the following sections, we will discuss at length, all of the aforementioned aspects of our approach to the research problem.

DESCRIPTION OF THE CLB ECOSYSTEM

The pest management problem considered in the present work is that of the CLB with its larval parasite, T. julis (TJ) and a crop component represented by oats.

The rationale behind the choice of the CLB ecosystem is two-fold:

1. The CLB is a key economic pest of the cereal grains in Michigan and several other states in the United States and Canada.
2. A large amount of data is available on a number of aspects of the CLB ecosystem from the research studies conducted at the Michigan State University over the years (Castro 1964, Yun 1967, Helgesen 1969, Ruesink 1972, Gage 1972 & 1974, Casagrande 1975, Jackman 1976, Logan 1977, Fulton 1975 & 1978, and Sawyer 1978).

The CLB, Oulema melanopus (L.) is native to Europe and Central Asia. The first reliable indentification of this pest was made in southwestern Michigan in 1962. Since then it has rapidly spread and has established itself as an economic pest in an area ranging from Pennsylvania to Wisconsin and from Kentucky to Michigan and Ontario. The CLB attacks small grains, mainly wheat, oats and barley. The annual combined acreage of these crops in the United States is close to 100 million acres (Cooper and Edens 1974). Radiation methods for sterilization of the CLB have proved to be ineffective because the dosage required to sterilize an adult is almost lethal. Studies of plant resistance have not provided a readily available method of control. Chemical control is the only viable control that is used extensively. There is a general acceptance among entomologists that satisfactory control of the CLB can be achieved only by a pest management program based on thorough ecological research (Haynes 1973).

The CLB overwinters as an adult in forest litter, grass, tree bark, or in small crevices protected from heat and cold. In Michigan, adults become active in April and feed on grasses and winter wheat prior to oviposition. The oviposition activity continues for 45-60 days; during this period each female lays an average of 50-150 eggs. The spring adult population declines to a negligible level due to natural mortality in about 60 days. The eggs hatch in a few days and the larval instars feed extensively on succulent leaves of wheat and (preferably) oats. There are 4 larval instars. New adults emerge in a few days, feed intensively on any available green grass, and disperse to overwintering sites. These adults diapause and do not lay eggs until the following spring. Most of the damage to the crop is caused by larval feeding during the early stages of plant development.

The CLB was introduced into North American with few, if any, of its natural control agents. Important biocontrol agents include the imported parasites: *Anaphes flavipes*, and egg parasite, and the three larval parasites--*Tetrastichus julis*, *Diaparsis carinifer*, and *Lemophagus curtus*. It appears that the egg parasite, *A. flavipes* will not have a major influence on the CLB population since the mortalities of the first and fourth instar CLB are density dependent; thus, as egg density is reduced, survival of larval instars will increase (Helgesen and Haynes 1972). Preliminary studies indicate that the larval parasite T. julis is better synchronized than the other larval parasites. Moreover, it has two generations per year, very high reproductive potential and a relatively low dispersal quality. The mathematical model considered focuses on this parasite. More detailed descriptions concerning the biology of the CLB can be found in Haynes (1973), Barr et al (1973), Tummala et al (1975), Lee et al (1976) and several theses cited in the literature.

MODELING ASPECTS

Considerable modeling work has been done on the CLB ecosystem. Presently four models are available on various aspects of the CLB ecosystem. Gutierrez et al (1974) provided a simulation model for the within field dynamics of the CLB in wheat and oats. Fulton (1978) developed a detailed simulation model for the CLB that can be used in an on-line fashion. Both models are aimed at providing detailed descriptions of the pest population dynamics through time. However, they did not include the parasite component represented by T. julis. Therefore these models are not suited for analyzing the biolgical control of the CLB. Furthermore, both models lack a dynamic host-crop component; hence the economic impact of the CLB feeding on the host plant cannot be evaluated. In addition, the models are based on extensive simulations, and are prone to all the drawbacks associated with the simulation approach to pest management (refer to earlier discussion). In short, these models are best suited for analyzing population dynamics of the CLB, but are not useful in evaluating a large number of management strategies.

Lee et al (1976) presented a comprehensive model of the CLB-T. julis ecosystem based on partial differential equations and an ordinary differential equation model for the host plant. The model, however, did not include the chemical control component. The model is used in a simulation mode to describe the maturity distribution of the CLB and its effect on the host plant through time. Since partial differential equations are rather cumbersome when used in conjunction with optimization schemes, the model of Lee et al is not particularly attractive for management purposes.

Tummala et al (1975) developed a detailed model of the CLB-T. julis dynamics based on a discrete component approach. The discrete-time state-space model was utilized to illustrate the effect of T. julis on the CLB under varying densities. Since the major objective of their work was to highlight the beneficial effect of biological control, they did not incorporate in their model the host plant component and the impact of chemical control on the pest-parasite complex.

In order to be useful in analyzing integrated control strategies, the models should include both biological and chemical control, and dynamic descriptions of the economic yield from the crop, and lend themselves suitable for use with optimization schemes. The state-space model of Tummala et al (1975) is particularly attractive for optimization purposes. In the present research, we will develop an updated version of the model of Tummala et al (1975) in such a manner that the final version of the model encompasses all the features necessary for analyzing integrated pest management options.

A chemical control component is added to the earlier version of the model; thus, mortality functions that account for the mortality caused by the application of pesticides are introduced. The pesticides used have impact on both the pest and the parasite. In addition, the pesticides used have different impacts on different life-stages of the insect. These factors should be incorporated into the model. Generally, insect mortality is described in terms of dosage response characteristics that give the relationship between the amount of pesticide applied and the corresponding mortality (expressed as a percentage) induced.

Sevin (carbaryl) and malathion are the pesticides that are extensively used in controlling the CLB. Both carbaryl and malathion are effective against the CLB larvae and adults, the larvae being more susceptible than the adults. In addition, carbaryl is a powerful ovicide (i.e., kills eggs), has a prolonged residual effect (compared to malathion), and is known to cause adverse side effects. The dosage response characteristics of the CLB to these pesticides are discussed in the literature (Yun and Ruppel 1965, Ruppel 1977, and Casagrande 1975).

Our model will focus on the pesticide malathion for the following reasons:

1. Malathion is widely used (states like New York recommend only malathion for CLB control).
2. More data is available on this pesticide.

3. Malathion has very little residual effectiveness, while Sevin has a prolonged residual effectiveness that is rather difficult to model and is likely to add quite a bit of complexity to the present model.

The dosage response curves used in the model are illustrated in Figure 2. These are based on published data and discussions with Dr. Ruppel. Currently there is no data available on the impact of malathion on the larval parasite T. julis. However, according to entomologists (Dr. Ruppel personal communication, Michigan State University) the effect of malathion on T. julis is likely to be very similar to that of malathion on the CLB larvae.

Due to the lack of availability of data, Tummala et al (1975) assumed a hypothetical function for the T. julis diapause. In the present model it is replaced by one that is based on field studies conducted by Gage (1974). The field data and the functional approximation (an eighth degree polynomial fit) used in the present model are illustrated in Figure 3.

The development of a model for the oats plant component is discussed in detail in the following section. (Note: The threshold temperature for oats is 42°F, whereas it is 48°F for the CLB and TJ. Hence, a transformation from 42°F to 48°F is used in calculating the cumulative degree days for the plant model. This transformation is required for optimization purposes. The error associated with the transformation is minimal because of the proximity of the thresholds.)

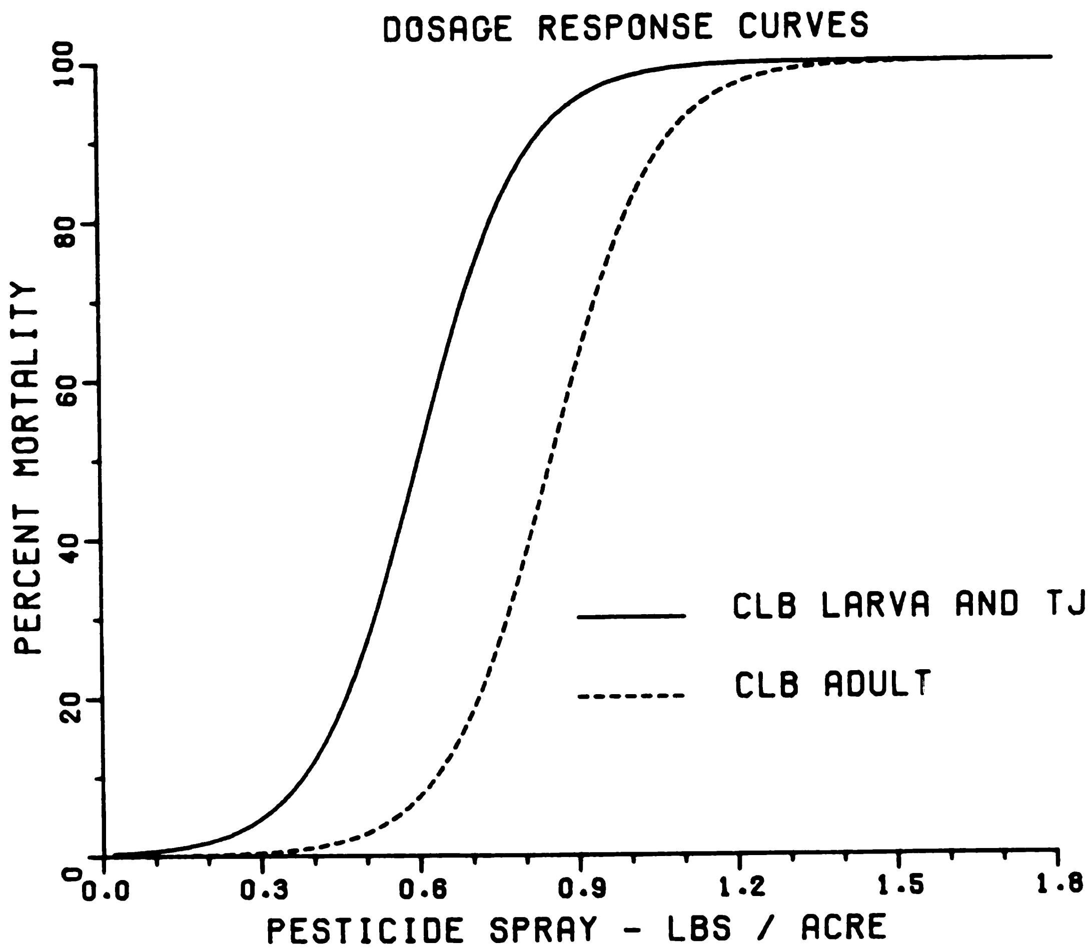

FIGURE 2. Dosage response characteristics of CLB larva, CLB adult, and *T. julis* to a pesticide spray of malathion.

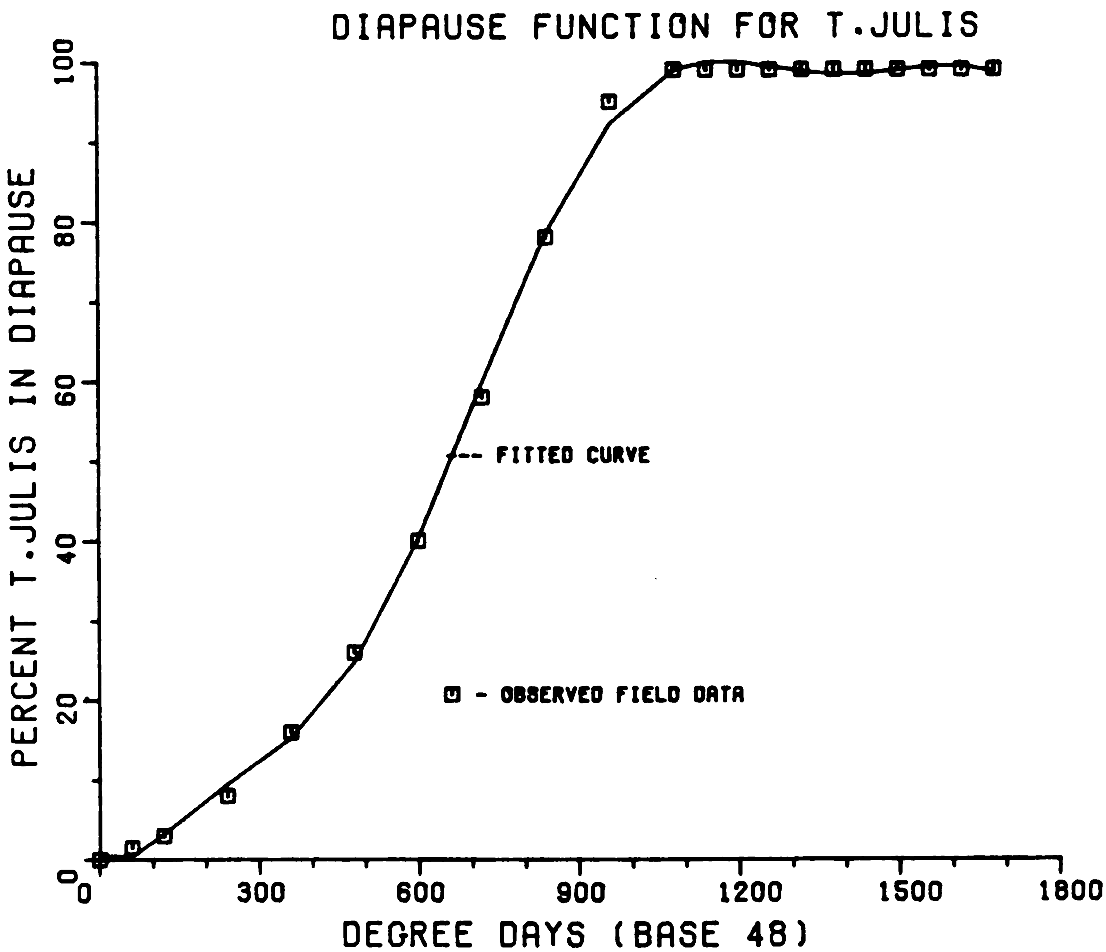

FIGURE 3. Diapause functions for T. julis--observed field data and fitted curve.

OATS PLANT MODEL

Since the goals of pest management are, to a large extent, economic in nature, the economic yield from the host plant is of utmost significance. For a realistic characterization of the oats plant as a component of the CLB ecosystem, it is necessary to identify the interactions between the CLB and the plant (for more details see Barr et al 1973, Lee et al 1976, and Gage 1972).

The feeding caused by the CLB population results in a reduced leaf surface area of the plant. The reduced photosynthetic capability in turn affects the final yield. Most of the CLB feeding occurs on the top three leaves, which are responsible for over 85 percent of the net photosynthetic activity (Gage 1972).

Plant growth is dependent on a variety of factors, including moisture, soil chemicals, light exposure, etc. However, in our model it is assumed that all these factors are prevalent in a non-stressed or "standard" condition. The key variables that are chosen to represent plant growth are the total weight of the plant W, the leaf surface area S, and the weight W_H and surface area S_H of the grain seeds as functions of degree days. The selection of these quantities as state variables is based on the following characteristic mechanism. The biomass generated through photosynthesis by the leaves is accumulated as plant biomass and is converted into seed when the plant matures. The weight of the heads directly reflects the quantity of seed produced. Moreover, as the heads develop, their surface area should also be considered as an active photosynthetic component.

The metabolic processes of plants are determined by the relationship between the active mass which undergoes catabolism (respiration) and the necessary surface to support anabolism (photosynthesis). The anabolism is expressed as the product of the rate k_1 at which mass is produced per unit area and the effective surface S through which exchanges take place. Similarly, catabolism is proportional to the entire bulk W of living material. Thus:

$$\frac{dW}{dt} = k_1 S + k_2 W$$

where: $k_1 > 0$

$k_2 < 0$

t = physiological time for the plant.

This equation was first proposed by von Bertalanffy (1957). The growth of surface area S also can be represented with a similar equation. Thus, a linear approximation for the plant dynamics can, in general, be expected to have the form:

$$\frac{d}{dt}\begin{bmatrix} W \\ S \\ W_H \\ S_H \end{bmatrix} = K \begin{bmatrix} W \\ S \\ W_H \\ S_H \end{bmatrix}$$

where: K = 4 x 4 matrix.

The above equation can be written in the difference equation form as follows:

$$\begin{bmatrix} W & (n+1) \\ S & (n+1) \\ W_H & (n+1) \\ S_H & (n+1) \end{bmatrix} = P \begin{bmatrix} W & (n) \\ S & (n) \\ W_H & (n) \\ S_H & (n) \end{bmatrix}$$

where: P = 4 x 4 matrix

n = discretized physiological time steps.

The elements of the matrix P are in general functions of soil fertility, moisture, light intensity, etc. The form of these functions is unknown at present. Evaluation of possible control strategies such as fertilization, irrigation, etc., so as to minimize crop damage from the CLB by manipulating plant growth would require extensive study to determine these functional forms. Nevertheless, development of management policies focusing on the manipulation of CLB and parasite densities under "standard" cultural conditions and practices for the crop component can proceed using the P matrix with constant parameters.

ESTIMATION OF PARAMETERS FOR THE OATS PLANT MODEL USING TIME-SERIES ANALYSIS

The essential features of our approach to parameter estimation for the oats plant model is illustrated in Figure 4. As discussed earlier, the structure of the plant model can be expressed as follows:

$$\underline{Y}\ (k + 1) = \underline{P}\ \underline{Y}(k)$$

or:

$$\begin{bmatrix} y_1\ (k+1) \\ y_2\ (k+1) \\ y_3\ (k+1) \\ y_4\ (k+1) \end{bmatrix} = \begin{bmatrix} P(1,1) & \dots & P(1,4) \\ \cdot & & \cdot \\ \cdot & & \cdot \\ P(4,1) & \dots & P(4,4) \end{bmatrix} \begin{bmatrix} y_1(k) \\ y_2(k) \\ y_3(k) \\ y_4(k) \end{bmatrix}$$

where: $y_1(k)$ = weight of oats plant/sq ft

$y_2(k)$ = leaf surface area/sq ft

$y_3(k)$ = weight of head/sq ft

$y_4(k)$ = surface area of head/sq ft

$\underline{Y}\ (o)$ = $\underline{Y}$ initial = initial conditions for the state variables $\underline{Y}$.

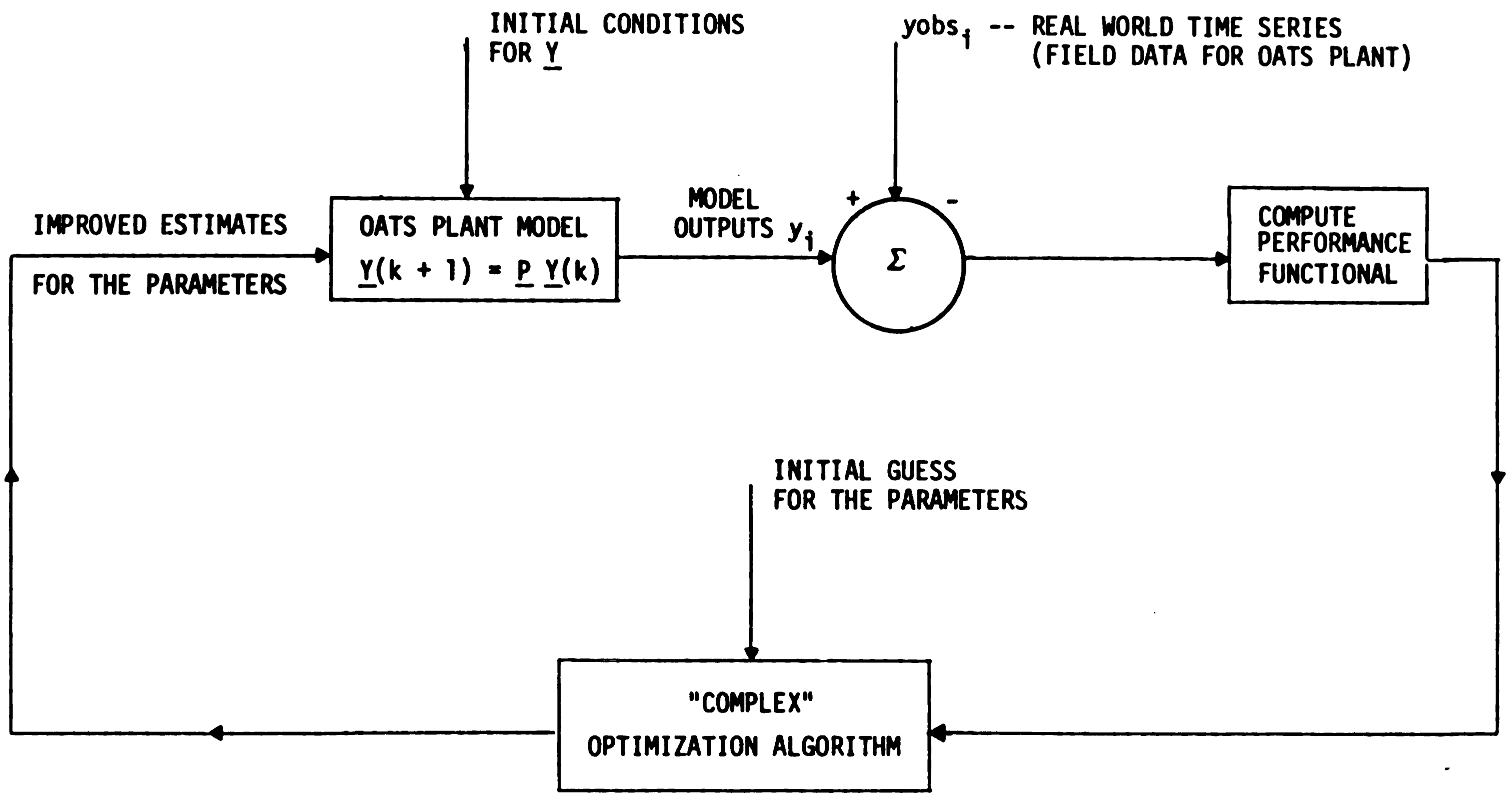

FIGURE 4. Block diagram illustrating the parameter estimation for oats plant model using time-series analysis.

The parameters of the P matrix remain to be estimated. These parameters have to be chosen in such a manner that the outputs of the plant model match closely the field data for the corresponding variables. The optimization algorithm of Box* (also known as the COMPLEX algorithm) is used in conjunction with the model as shown in Figure 4. An initial guess, as well as a lower and upper bound, are supplied for each one of the parameters. Based on the initial guesses for the parameters and the given initial condition, $\underline{Y}$ initial, for the state variables, the model generates the state variables $\underline{Y}(k)$ through time steps k, from initial time t_o to final time t_f. The outputs of the plant model are compared with the corresponding real world time-series (i.e., field data) at selected points in time. A weighted least squares criterion is used as the performance index (PI) in the optimization algorithm:

$$PI = \sum_{i=1}^{4} \sum_{k=1}^{n} \left[\frac{y_i(k) - yobs_i(k)}{yobsavg_i} \right]$$

where: $y_i(k)$ = data generated by the plant model

$yobs_i(k)$ = field data

$yobsavg_i$ = average values of the observed field data

n = final time step.

The COMPLEX optimization algorithm finds the values of the unknown parameters of the P matrix so that the performance index is minimized. The parameter values that are generated by the optimization procedure are fed back to the plant model, and the model is run with these new parameter values. When this process is repeated, a specified convergence is obtained with the optimization procedure. The aforementioned

*See Kuester and Mize (1973) for details of the algorithm.

procedure is repeated for different initial guesses of the parameters in order to make sure the global optimum is obtained with the optimization algorithm.

The unknown parameters of the P matrix estimated using the approach discussed here are given below.

P MATRIX:

$$\begin{bmatrix} 0.803 & 0.517 & -0.056 & 0.000 \\ -0.112 & 1.240 & -0.018 & 0.065 \\ 0.000 & 0.036 & 0.765 & 0.614 \\ 0.000 & 0.047 & -0.172 & 1.303 \end{bmatrix}$$

Convergence criterion employed:

$$|(PI/PILAST) - 1| \leq 1E - 5 \quad \text{for 3 consecutive iterations.}$$

The trajectories of the plant variables generated by the "best-fit" model, and the corresponding real world time-series form field data are illustrated in Figures 5 and 6.

CLB-OATS PLANT INTERACTIONS

The CLB-plant interactions are caused by the CLB feeding on the leaves of the host plant. The CLB-plant surface area coupling may be described by a simple form in view of the work of Gage (1972). He observed that leaf consumption by the four CLB larval instars is in the ratio 1.00, 2.87, 5.97, and 24.23, and that feeding by the adult CLB is negligible when compared to larval feeding.

Thus, the total larval feeding in terms of the first instar feeding equivalents can be written as:

$$L1EQ(n) = FEEDQ[L1(n) + 2.87\ L2(n) + 5.97\ L3(n) + 24.23\ L4(n)]$$

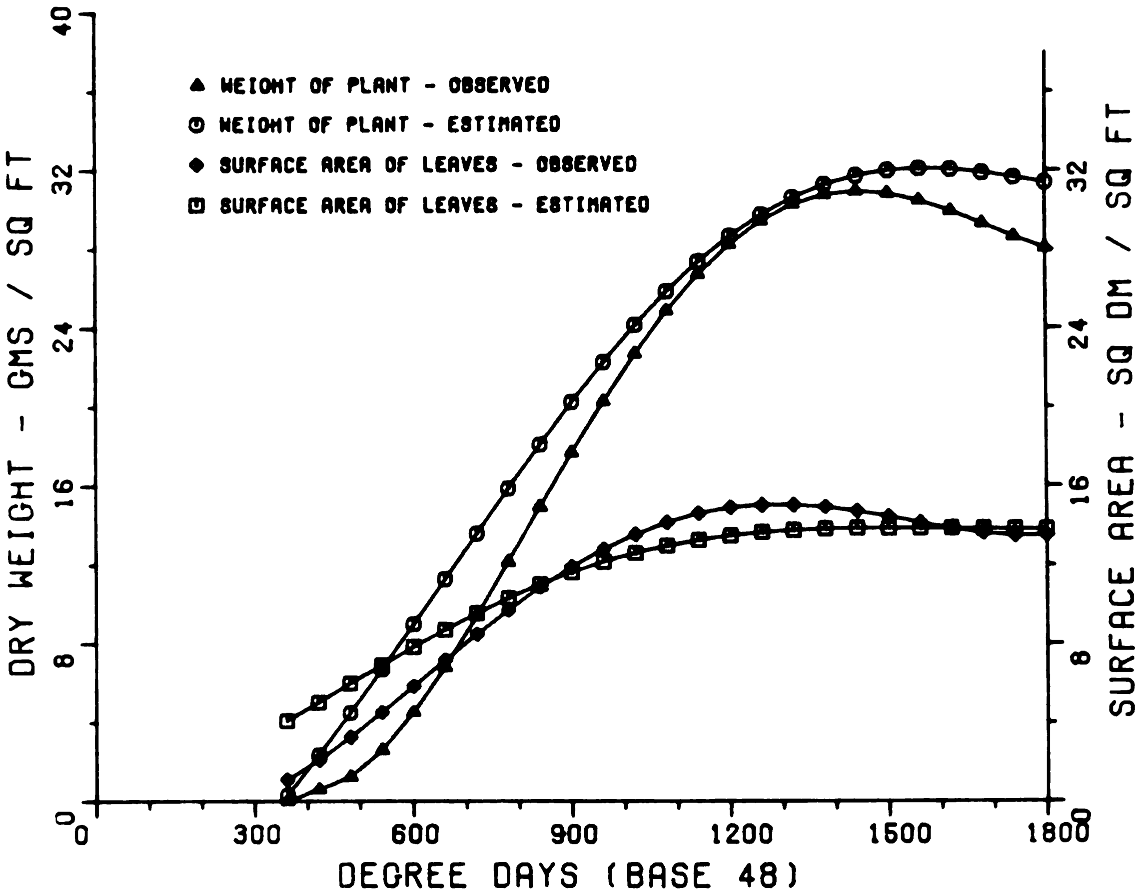

FIGURE 5. Weight of plant and surface area of leaves of the oats plants--observed and estimated.

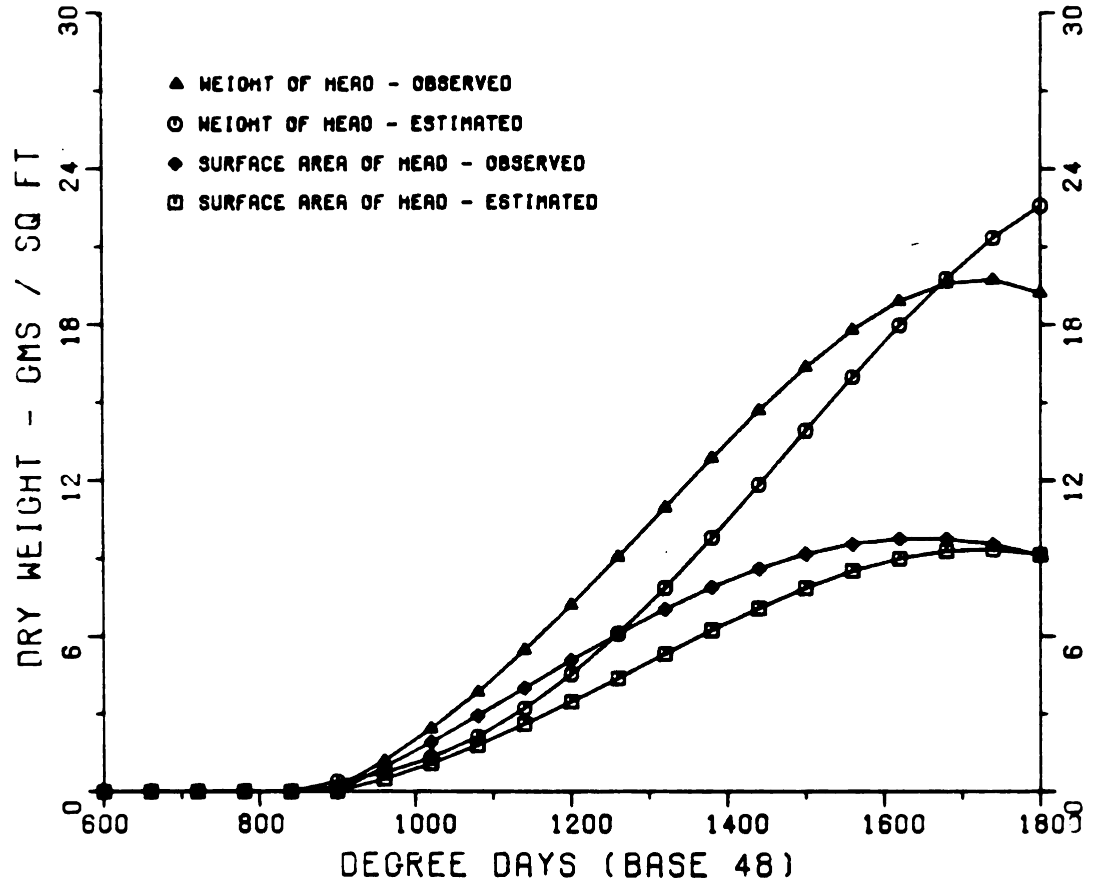

FIGURE 6. Weight of grainhead and surface area of grainhead of the oats plants--observed and estimated.

where: L1(n) . . L4(n) } CLB first instar through fourth instar larval density at time n

L1EQ(n) = first instar feeding equivalents at time n

FEEDQ = feeding coefficient (0.002029) that represents the leaf surface area (in sq dm) consumed by a first instar CLB larva in a time duration $60DD_{48}$

n = physiological time (in units of $60DD_{48}$).

Further, it is known that the biomass corresponding to 1 dm^2 of leaf surface area is 0.25 gm in oven dry weight (Gage 1972).

Hence, the CLB-plant coupling can be expressed as:

$$\begin{bmatrix} W\ (n+1) \\ S\ (n+1) \\ W_H(n+1) \\ S_H(n+1) \end{bmatrix} = \underline{P} \begin{bmatrix} W\ (n) \\ S\ (n) \\ W_H(n) \\ S_H(n) \end{bmatrix} - \begin{bmatrix} 0.25 \\ 1.00 \\ 0.00 \\ 0.00 \end{bmatrix} \text{L1EQ}(n)$$

The ultimate yield (i.e., seed weight) from the oats plant is directly related to the weight of the grain head at harvest time, $W_H(N)$, through the following equation (Reference: Lampert, unpublished data):

$$\text{YIELD gms/sq ft} = W_H(N) * 0.9119 - 0.003035$$

$$\text{YIELD bushels/acre} = \text{YIELD gms/sq ft} * 3.000992.$$

It is worth emphasizing here that unlike most of the pest management models of the past, the model developed in this research work is comprehensive (over 30 state variables), includes all the three major components of the CLB ecosystem (the CLB, *T. julis*, and oats plant), and provides dynamic relationships for calculating the crop yield. It has both biotic and chemical control components incorporated into it so that integrated control strategies can be evaluated. Most importantly, the model, to a large extent, is based on actual field studies, and not on hypothetical relationships. It should be obvious that models are the critical links in developing management strategies for complex pest ecosystems. The more realistic the model, the more meaningful will be the resulting management strategies.

A complete mathematical description of a system model for the CLB ecosystem is presented in the following pages.

SYSTEM MODEL FOR THE CLB ECOSYSTEM

DICTIONARY OF STATE VARIABLES

NAME	DESCRIPTION
X1	Spring Adult CLB Density
X2, X3, X4	CLB Egg Density
X5	First Instar CLB Density
X6	Second Instar CLB Density
X7	Third Instar CLB Density
X8	Unparasitized Fourth Instar CLB Density
X9, X10, X11, X12, X13, X14, X15	Unparasitized CLB Pupa Density
X16	Summer Adult CLB Dinsity
X17	Diapausing TJ Density
X18	Adult TJ Density
X19	Parasitized Fourth Instar CLB Density
X21, X22, X23, X24	Parasitized CLB Pupa Density
X20, X25, X26, X27, X28	Parasite Per Pest Individual in Different Stages

DICTIONARY OF STATE VARIABLES (continued)

NAME	DESCRIPTION
X29	CLB Larval Feeding
X30	Weight of Oats Plant
X31	Leaf Surface Area of Oats Plant
X32	Weight of Grain Head
X33	Surface Area of Grain Head

SYSTEM PARAMETERS

NAME	DESCRIPTION	VALUE
a	Spring Adult Survival	0.70
b	CLB Eggs/CLB Female/60DD	22.00
c	Summer Adult Survival	1.00
d	*T. julis* Adult Survival	0.60
e1	Max Eggs/TJ Adult/60DD	20.00
e2	Max TJ Eggs/CLB Larva/60DD	5.00
e3	TJ Searching Constant	100.00
rp	TJ Mortality Inside CLB	0.00
K1	Mortality of CLB Eggs	0.10
K2	Mortality of CLB L1	Variable
K3	Mortality of CLB L2	0.30
K4	Mortality of CLB L3	0.45
K5	Mortality of CLB L4	Variable
K6	Mortality of CLB Pupae	0.40
K7	Mortality of Overwintering CLB	0.77
K8	Mortality of Overwintering TJ	0.50
Cp	Exponent in Parasitism Equation	0.75
FEEDQ	Feeding Function Coefficient	0.002029
T_{sy}	Time When TJ First Shows	6.00 (i.e. 360DD Base 48)
T_e	Time When CLB Leaves Oats	16.00 (i.e. 960DD Base 48)
DF(n)	Diapause Function for TJ --an eighth degree polynomial fitted to field data from Gage (1974)	
P(1,1) . . P(4,4)	Parameters for the Oats Plant Model Obtained Through Time-Series Analysis	Refer to pg. 48

SYSTEM MODEL

$$X1(n+1) = a\,X1(n)\,SA$$

$$X2(n+1) = b\,X1(n)$$

$$X3(n+1) = X2(n)$$

$$X4(n+1) = X3(n)$$

$$X5(n+1) = (1 - K1)\,X4(n)\,SL$$

$$X6(n+1) = (1 - K2)\,X5(n)\,SL$$

$$X7(n+1) = (1 - K3)\,X6(n)\,SL$$

$$X8(n+1) = (1 - K4)\,X7(n)\,[1 - f2]\,SL$$

$$X9(n+1) = (1 - K5)\,X8(n)$$

$$X10(n+1) = X9(n)$$

$$X11(n+1) = X10(n)$$

$$X12(n+1) = X11(n)$$

$$X13(n+1) = X12(n)$$

$$X14(n+1) = X13(n)$$

$$X15(n+1) = X14(n)$$

$$X16(n+1) = (c\,X16(n) + (1 - K6)\,X15(n))\,SA$$

$$X17(n+1) = X17(n) + DF(n)\,X24(n)\,X28(n)$$

$$X18(n+1) = (d\,X18(n) + (1 - DF(n))\,X24(n)\,X28(n))\,SL$$

$$X19(n+1) = (1 - K4)\,f2\,X7(n)\,SL$$

$$X20(n+1) = f1/(f2\,X7(n))$$

$$X21(n+1) = (1 - K5)\,X19(n)$$

$$X22(n+1) = X21(n)$$

$$X23(n+1) = X22(n)$$

$$X24(n+1) = X23(n)$$

$$X25(n+1) = (1 - rp)\ X20(n)$$

$$X26(n+1) = X25(n)$$

$$X27(n+1) = X26(n)$$

$$X28(n+1) = X27(n)$$

$$X29(n+1) = FEEDQ\ [X5(n) + 2.87\ X6(n) + 5.97\ (X7(n) + 24.53\ (X8(n) + X19(n))]$$

$$X30(n+1) = P(1,1)\ X30(n) + P(1,2)\ X31(n) + P(1,3)\ X32(n) + P(1,4)\ X33(n) - 0.25\ X29(n)$$

$$X31(n+1) = P(2,1)\ X30(n) + P(2,2)\ X31(n) + P(2,3)\ X32(n) + P(2,4)\ X33(n) - X29(n)$$

$$X32(n+1) = P(3,1)\ X30(n) + P(3,2)\ X31(n) + P(3,3)\ X32(n) + P(3,4)\ X33(n)$$

$$X33(n+1) = P(4,1)\ X30(n) + P(4,2)\ X31(n) + P(4,3)\ X32(n) + P(4,4)\ X33(n)$$

ATTACK EQUATION

$$f1 = \left[\frac{X7(n)\ X18(n)}{\frac{X7(n)}{e1} + \frac{X18(n)}{e2} + \frac{1}{e3}}\right]$$

$$f2 = \left[\frac{f1}{X7(n)\,e2}\right]$$

DENSITY DEPENDENT MORTALITIES OF I AND IV INSTARS*

$$K2 = 0.46\ \log E - 0.85 \qquad 0 \leq K2 \leq 0.99$$

$$K5 = 0.28\ \log E - 0.18 \qquad 0 \leq K5 \leq 0.99$$

where: K2, K5 = 1st and 4th instar mortalities, respectively.

E = total number of eggs laid per sq ft for the entire season.

MORTALITIES INDUCED BY PESTICIDE**

Dosage response for CLB larva and TJ = (1 - SL)

$$= \frac{1}{1 + e^{-(10*u\ -\ 6.0)}}$$

Dosage response for CLB adult = (1 - SA)

$$= \frac{1}{1 + e^{-(10*u\ -\ 8.5)}}$$

where: SL = survival based on pesticide spray for CLB larva and TJ

SA = survival based on pesticide spray for CLB adults

u = pesticide spray of Malathion lb/acre

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*See Helgesen and Haynes 1972.

**Dr. Ruppel, Michigan State University--personal communication.

OPTIMIZATION SCHEME

The optimization procedure utilized (Dyer and McReynolds 1970) in the present work is derived from dynamic programming. It is a successive approximation technique, based on dynamic programming instead of the calculus of variations, for determining optimal controls of nonlinear dynamic (or static) systems. The method is motivated from a consideration of the first and second order expansion of the return function about some nominal control variable sequence. In each iteration, the system equations are integrated forward using the current nominal control, and the accessory equations (which yield the coefficients of a first or second order expansion of the cost function in the neighborhood of the nominal state trajectory) are integrated backward, thus yielding an improved control sequence. Iteratively, this method results in control sequences that successively approximate the optimal control sequence.

The first order technique of Dyer and McReynolds (1970) is known as the successive approximation by gradient method. The first order methods are characterized by slow convergence near optimum, but are simpler to compute and are very useful in getting starting solutions. The second order approach of Dyer and McReynolds (1970) is called the successive sweep method, while a similar second order approach of Jacobson and Mayne (1970) is popularly known as the differential dynamic programming. The second order methods have faster convergence than the first order schemes, but are computationally more involved and are susceptible to nominal controls. Thus, hybrid schemes, in which first order methods are used to start the optimization procedure and get an improved nominal control sequence, while second order methods are utilized later on to improve convergence, are more appealing. Since these optimization techniques are based on successive approximation schemes, the storage and computational time requirements are very small, compared to dynamic programming and possibly several other optimization schemes as well. However, it is to be noted that, unlike dynamic programming, a true feedback solution is not obtained with these approaches, although it is possible to compute optimal feedback control in the neighborhood of the optimal trajectory.

In the present work, the first order successive approximation algorithm is used in conjunction with the optimization model of the CLB ecosystem consisting of 33 state variables. This is a marked improvement considering the fact that almost all the optimization approaches employed in the past (in connection with pest management and related problems) are confined to dimensions of 3 or 4. Furthermore, the method is general enough to be extended to other problems in pest management and many other areas as well: the major requirements being an available state-space model of the system under consideration and a properly formulated optimization problem. In addition, the optimization approach fits in nicely with the overall methodology of Linear-Quadratic-Gaussian (L-Q-G) design (refer to Appendix B) proposed for on-line pest management.

We would like to point out here that the successive approximation algorithm and differential dynamic programming are increasingly used in solving non-linear optimal control problems, especially of the discrete-time type (Gershwin and Jacobson 1970, Iyer and Cory 1972, Jamshidi and Heidari 1977, to name just a few). Most recently, Professor Ohno of Kyoto University, Japan, has come up with a new approach to differential dynamic programming that can directly solve optimal control problems with hard constraints on state and/or control variables without adjoining them (Ohno 1978, Ohno--personal communication 1978). As of the moment, Ohno's approach is restricted to rather small dimensional problems. However, it is worth noting that efficient ways of handling constraints is one of the most difficult problems encountered in the computation of optimal controls. We envisage more widespread use of differential dynamic programming (and variations thereof) in the future, especially in large dimensional, discrete-time optimal control problems arising in pest management and related areas.

A detailed derivation of the first order successive approximation technique used in our work is given in the following pages. In addition, a flow chart for computer implementation of the technique and a listing of the FORTRAN program are included in Appendix A.

A FIRST ORDER SUCCESSIVE APPROXIMATION TECHNIQUE: THE GRADIENT METHOD

A brief description leading to the gradient algorithm is given here. For a detailed description of the gradient (first order) algorithm, the successive sweep (second order) algorithm, and other related second order algorithms, the interested reader can refer to Dyer and McReynolds (1970), and Jacobson and Mayne (1970).

System dynamics: The dynamics of the system are expressed in terms of a set of discrete equations. The process is assumed to have N stages and the state of the system through these stages is governed by a difference equation of the form:

$$x(i + 1) = F(x(i),u(i),\alpha), \quad i = 0,1, \ldots , N - 1 \qquad (1)$$

where,

$$F = (F_1,F_2, \ldots , F_n)^T$$

is an n - dimensional vector of functions, that are in general non-linear.

$$x(i) = \begin{bmatrix} x_1(i) \\ x_2(i) \\ \cdot \\ \cdot \\ \cdot \\ x_n(i) \end{bmatrix}$$

is an n - dimensional column vector of the state variables.

$$u(i) = \begin{bmatrix} u_1(i) \\ u_2(i) \\ \cdot \\ \cdot \\ \cdot \\ u_m(i) \end{bmatrix}$$

is an m - dimensional column vector of control variables which normally vary from stage to stage.

$$\alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \cdot \\ \cdot \\ \cdot \\ \alpha_p \end{bmatrix}$$

is a p - dimensional vector of control parameters that are constant and thus do not vary from stage to stage.

The performance index considered is of the form:

$$J = \phi(x(N),\alpha) + \sum_{i=0}^{N-1} L(x(i),u(i),\alpha) \tag{2}$$

where L and ϕ are scalar, single-valued functions of their respective arguments, and ϕ represents the performance attached to the final state of the system. The second term represents the summation (discrete-time counterpart of integration) of performance over the stages.

Further, constraints can be imposed on the system. The most general form of constraint is given by:

$$0 = \Psi(x(N),\alpha) + \sum_{i=0}^{N-1} M(x(i),u(i),\alpha) \tag{3}$$

where Ψ and M are single-valued, scalar or vector functions. When M=0, this constraint will be reduced to the so-called terminal constraint.

In addition to these equality constraints, inequality constraints on state and/or control variables,

$$C(x(i),u(i)) \leq 0 \qquad (4)$$

can also be imposed on the system.

The optimization problem, based on the foregoing definitions, is to find the sequence of controls $u(i)$, $i = 0, \ldots, n - 1$ and the control parameters α that maximize (minimize) the performance index of equation 2, subject to the system equation 1, and the constraints of equations 3 and 4.

The dynamic programming approach to this optimization problem is based on principles that are a direct consequence of the structure of the problem, namely the principle of causality, the principle of optimality, and the principle of optimal feedback control.

The principle of causality is a fundamental property of deterministic multistage systems, which says that the state $x(k)$ and control parameter α at the kth stage, together with the sequence of controls $u[k,r - 1] \triangleq [u(k), u(k + 1), -- u(r - 1)]$ uniquely determine the state of the rth stage, namely, $x(r)$.

The principle of optimality due to Bellman, can be stated as: An optimal sequence of controls in a multistage optimization problem has the property that whatever the initial stage, state, and controls are, the remaining controls must constitute an optimal sequence of decisions for the remaining problem with stage and state resulting from the previous controls considered as initial conditions.

An important consequence of the principle of optimality is that of optimal feedback control (i.e. the choice of the optimal control at some stage may be expressed as a function of the state at that stage). This is known as the principle of optimal feedback control which states that the optimal control at the kth stage, u(k), provided it exists and is unique, may be expressed as a function of the state at the kth stage, x(k), and the control parameter α. Thus there exists a function:

$$u^{opt}(x(k),\alpha,k)$$

such that,

$$u^{opt}(k) = u^{opt}(x(k),\alpha,k).$$

This function, generally referred to as the optimal control law, yields the closed-loop solution to the optimization problem.

From the principle of causality it follows that there exists a function,

$$V(x(k),\alpha,u[k,N-1],k)$$

such that,

$$V(x(k),\alpha,u[k,N-1],k) = \phi(x(N),\alpha) + \sum_{i=k}^{N-1} \{L(x(i),u(i),\alpha)\} \qquad (5)$$

This function, V, is referred to as the "return function", corresponding to the control sequence u[k,N - 1]. Now from the principle of optimality u[k,n - 1] must be chosen to maximize V(x(k),α,u[k,N - 1],k). If the control sequence u[k,N - 1] is replaced by the optimal control sequence $u^{opt}[k,N-1]$, V becomes the "optimal return function", $V^{opt}(x(k),\alpha,k)$ where:

$$V^{opt}(x(k),\alpha,k) = V(x(k),\alpha,u^{opt}[k,N-1],k). \qquad (6)$$

From the definition of the return function, equation 5, $V(x(k),\alpha,k)$ must satisfy the backward transition equation,

$$V(x(k),\alpha,k) = L(x(k),u(x(k),\alpha,k),\alpha) + V(x(k+1),\alpha,k+1) \qquad (7)$$

where $x(k+1)$ is given by the state equations,

$$x(i+1) = F(x(i),\alpha,u(x(i),\alpha,i)), \quad i = k,k+1, \ldots ,N-1$$

with $i = k$. At the final stage,

$$V(x(N),\alpha,N) = \phi(x(N),\alpha). \qquad (8)$$

The dynamic programming solution to the optimization problem will require the construction of this optimal return function.

The "straight forward" dynamic programming approach to this construction of the optimal return function (i.e., dynamic programming solution to the optimization problem) which will lead to the optimal control law is not possible except for simple systems with low dimensions due to the "curse of dimensionality".

An entirely different approach to the optimal control problem is to get an open loop solution (i.e., to arrive at the optimal control solution starting from a single initial state). The basic idea behind this approach is to utilize a successive approximation technique that will update some nominal control sequence so that it will converge, eventually, to the optimum control sequence. The advantage of this approach is that storage and time requirements are relatively small. As mentioned earlier, however, a true feedback solution is not obtained, although neighborhood extremal techniques can be used in some cases to get an optimal feedback solution in the neighborhood of the optimal solution.

The gradient method is one such approach. It is based on the consideration of a first-order expansion of the return function,

$$V(x(0),\alpha,u[0,N-1])$$

about some nominal control variable sequence,

$$u^j[0,N-1]$$

and a nominal parameter, α^j, viz,

$$V(x(0),\alpha^{j+1}, u^{j+1}[0,N-1]) = V(x(0),\alpha^j,u^j[0,N-1]) + \frac{\partial V(x(0),\alpha^j,u^j[0,N-1])}{\partial \alpha}\delta\alpha + \frac{\partial V(x(0),\alpha^j,u^j[0,N-1])}{\partial u[0,N-1]}\delta u[0,N-1]. \quad (9)$$

The variations in the control variables $\delta\alpha = \alpha^{j+1} - \alpha^j$ and $\delta u = u^{j+1} - u^j$ must be small enough to insure the validity of the expansion. Clearly, if $\delta u[0,N-1]$ and $\delta\alpha$ are chosen by,

$$\delta u[0,N-1] = \varepsilon\left[\frac{\partial V(x(0),\alpha^j,u^j[0,N-1])}{\partial u[0,N-1]}\right]^T \quad (10)$$

$$\delta\alpha = \varepsilon\left[\frac{\partial V(x(0),\alpha^j,u^j[0,N-1])}{\partial \alpha}\right]^T \quad (11)$$

where ε is some positive parameter, the return,

$$V(x(0),\alpha^{j+1},u^{j+1}[0,N-1])$$

will be greater than,

$$V(x(0),\alpha^j,u^j[0,N-1]).$$

Instead of forming,

$$V(x(0),\alpha,u[0,N-1])$$

explicitly and then differentiating, the gradients are computed more easily by means of a backward sequence of equations.

It is clear that a change in $u(k)$ will not affect $L(x(i),\alpha,u(k))$ for $i < k$. Hence, the gradient of the return function

$$V(x(0),\alpha,u[0,N-1])$$

with respect to the control function $u(k)$ is the same as the gradient of $V(x(k),\alpha,u[k,N-1])$, i.e.,

$$\frac{\partial V(x(0),\alpha,u[0,N-1])}{\partial u(k)} = \frac{\partial V(x(k),\alpha,u[k,N-1])}{\partial u(k)} . \tag{12}$$

Now the return function $V(x(k),\alpha,u[k,N-1])$ from its definition may be written,

$$V(x(k),\alpha,u[k,N-1]) = L(x(k),\alpha,u(k)) + V(x(k+1),\alpha,u[k+1,N-1]).$$

Hence, differentiating with respect to $u(k)$, we obtain,

$$\begin{aligned} \frac{\partial V(x(k),\alpha,u[k,N-1])}{\partial u(k)} & \\ &= \frac{\partial L(x(k),\alpha,u(k))}{\partial u(k)} + \frac{\partial V(x(k+1),\alpha,u[k+1,N-1])}{\partial x(k+1)} \\ &* \frac{\partial F(x(k),\alpha,u(k))}{\partial u(k)} . \end{aligned} \tag{13}$$

In the following analysis the arguments $x(i),\alpha,u[i,N-1]$ or $x(i),\alpha,u(i)$ will be replaced by the single argument i. Subscripts, unless specifically defined otherwise, denote partial derivatives. For example, equation 13 is written,

$$V_u(k) = L_u(k) + V_x(k+1)F_u(k).$$

In order to evaluate this expression it is necessary to obtain $V_x(k+1)$ for $k = 0,1, \ldots ,n-1$. A sequential set of relations for V_x evaluted along a trajectory may be obtained by taking partial derivatives of equations 7 and 8, i.e.,

$$V_x(N) = \phi_x(x(N),\alpha) \tag{14}$$

$$V_x(k) = V_x(k + 1)F_x(k) + L_x(k). \tag{15}$$

The partial derivative of V(0) with respect to α is obtained in a similar fashion by partially differentiating equation 7, 8, viz,

$$V_\alpha(N) = \phi_\alpha(x(N),\alpha) \tag{16}$$

$$V_\alpha(k) = V_\alpha(k + 1) + L_\alpha(k) + V_x(k + 1)F_\alpha(k) \tag{17}$$

where $V_x(k + 1)$ is given by equations 14 and 15. Thus, using equations 14-17 the gradients of the return $V(x(0),\alpha,u[0,N - 1])$ with respect to α, and $u[0,N - 1]$ may be formed as a backward sequence.

The gradient algorithm may now be summarized as follows:

1. Choose a nominal control sequence $u^j[0,N - 1]$ and control parameter α^j. Construct and store the trajectory $x[0,N]$ from the system equation 1 and the nominal control variables. Also compute the cost J where:

$$J = \phi(x(N),\alpha) + \sum_{i=0}^{N-1} \{L(x(i),\alpha,u(i))\}.$$

2. Compute the partial derivatives, $V_x(k) = V_x(x(k),\alpha^j,u^j[k,N - 1])$ and $V_\alpha^j(k) = V_\alpha(x(k),\alpha^j,u^j[k,N - 1])$ for $k = N,N - 1, \ldots, 0$ from

$$V_x^j(N) = \phi_x(N), \quad V_\alpha^j(N) = \phi_\alpha(N)$$

$$V_x^j(k) = V_x^j(k + 1)F_x^j(k) + L_x^j(k)$$

$$V_\alpha^j(k) = V_x^j(k + 1)F_\alpha^j(k) + V_\alpha^j(k + 1) + L_\alpha^j(k)$$

where:

$$F_\alpha^j(k) + F_\alpha(x(k),\alpha^j,u^j(k)) \text{ and } L_\alpha^j(k) = L_\alpha(x(k),\alpha^j,u^j(k)).$$

3. Compute and store the gradient with respect to the control at each stage from,

$$\frac{\partial V(x(0),\alpha^j,u^j[0,N-1])}{\partial u^j(k)} = V_x{}^j(k+1)F_u{}^j(k) + L_u{}^j(k),$$

$$k = N-1, \ldots, 0.$$

4. For some nominal parameter $\varepsilon > 0$, compute the new control from

$$u^{j+1}(k) = u^j(k) + \varepsilon[\partial V|\partial u^j(k)]^T.$$

5. At the initial stage compute the new control parameter α^{j+1} from $\alpha^{j+1} = \alpha^j + \bar{\varepsilon}V_\alpha{}^T(0)$, where $\bar{\varepsilon} > 0$.

6. Use the new control variables $u^{j+1}[0,N-1]$, α^{j+1}, and the system equations to construct a new trajectory $x^{j+1}[0,N]$ and compute the new cost J^{j+1}.

7. If $J^{j+1} > J^j$, continue with steps 2 and 3. If $J^{j+1} < J^j$, reduce the step size parameters ε, $\bar{\varepsilon}$; for example, set $\varepsilon = \varepsilon/2$, $\bar{\varepsilon} = \bar{\varepsilon}/2$, and repeat steps 4, 5 and 6 etc.

8. The iterations are continued until either no further increase in the cost is possible, or until the desired accuracy is attained.

Side constraints can be handled by appropriate modifications to this basic algorithm (Dyer and McReynolds 1970). Another alternative to a certain class of problems with side constraints, is the well-known penalty function technique. However, this technique can lead to very slow convergence in certain cases.

RESULTS AND DISCUSSIONS

As discussed in earlier sections, the major objective of this research is the determination of optimal control strategies for the integrated control of the CLB. In the systems terminology, the aforementioned objective can be transformed into an optimal control problem --derive the optimal timing and quantity of pesticide spray, given the state-space model of the CLB ecosystem, a desired performance measure to be optimized, and other constraints imposed on the problem to reflect the real-world situation.

We will compare the optimal decision rules determined through the use of optimal control techniques with conventional spraying schemes currently used in practice, and with the strategy of spraying no pesticide at all (which may be a viable option under certain circumstances). All three of the aforementioned strategies will be evaluated within a certain framework of the CLB ecosystem model under identical initial conditions.

Reasonable starting densities for the CLB and _T. julis_ have to be chosen for the problem as the optimal policies will be strongly dependent on the initial conditions for the biological variables. Figure 7 describes the relationship between the spring adult CLB and the yield from oats plant under no-spray conditions. This relationship is obtained through computer runs of the CLB ecosystem model under no-spray conditions. It can be observed from the figure that spring-adult-CLB

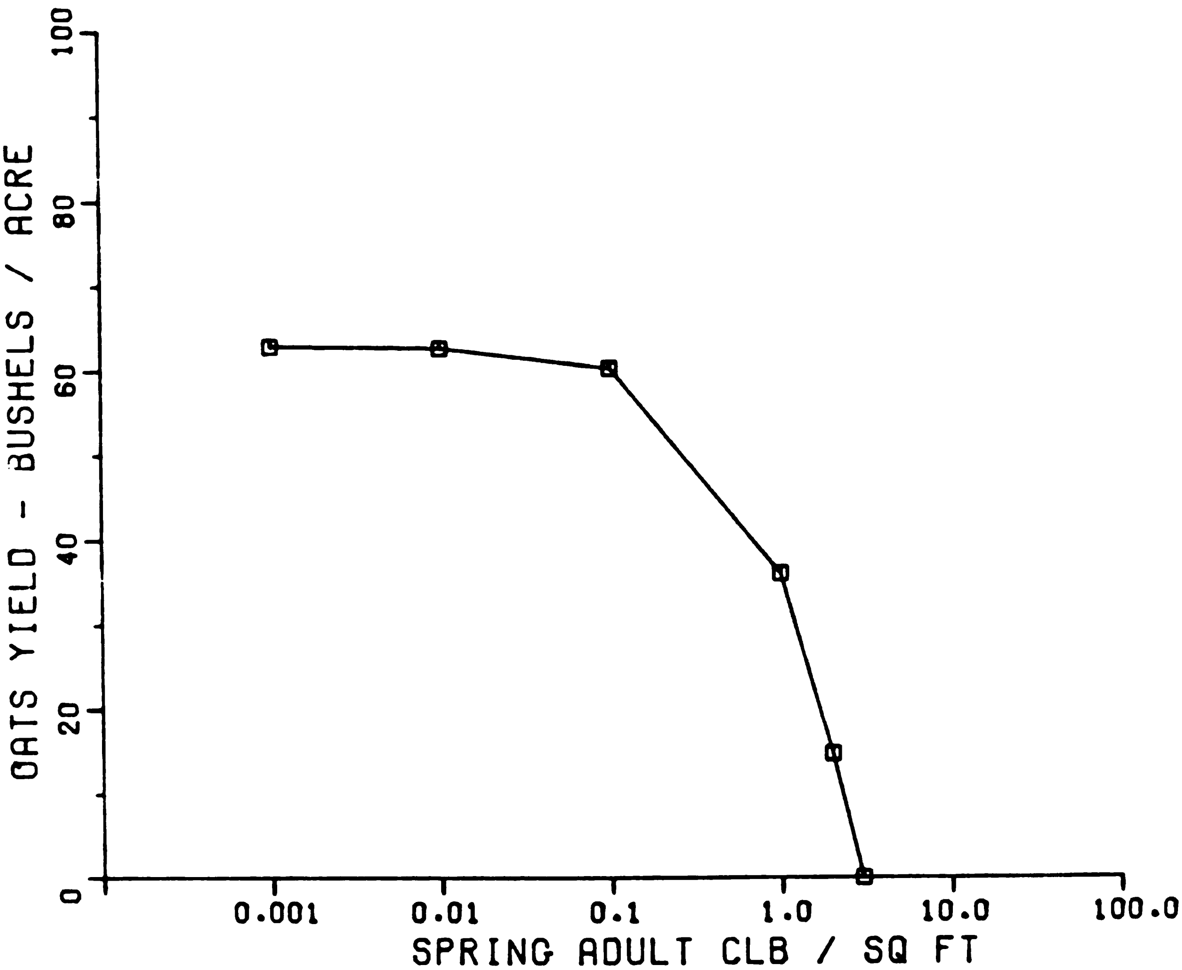


FIGURE 7. Relationship between spring-adult CLB density and yield from oats plant under no-spray conditions (plotted on a semi-logarithmic scale).

densities up to 0.1/sq ft make little impact on the ultimate yield from the oats plant which remains around 60 bushels/acre. At a CLB-spring-adult density of 1.0/sq ft the yield is fairly low--down to about 36 bushels/acre, and at a CLB density of 2.0/sq ft the yield is very low --about 14.7 bushels/acre. At spring adult densities of 3.0/sq ft and above the CLB almost completely destroys the oats crop. It is obvious that CLB densities below 0.1/sq ft require no chemical control at all. On the other hand, CLB spring adult densities of, say, 1.0/sq ft and above warrant chemical control measures so as to prevent economic damage to the crop. Based on the above information, and in order to illustrate vividly the differences between the optimal control strategies and conventional spraying schemes currently in use, the CLB starting density of 2.0/sq ft is chosen for our example, together with an initial density of 0.001/sq ft for the *T. julis*.

The conventional control scheme currently in use is to spray 1 lb/acre of the pesticide malathion when there are more than three eggs and larvae per stem of the oats plant (Michigan State University Cooperative Extension Service Bulletin E-829 by Dr. Ruppel, February 1977; Dr. Ruppel personal communication 1978). This criterion for conventional spraying was incorporated into the CLB ecosystem model and it was found that the spray of 1 lb/acre occurs at the third time step of the model (i.e., 180 DD_{48}) and results in 23.85 bushels/acre of oats yield along with a corresponding profit of $26.20/acre. In contrast, the no-spray scheme leads to 14.70 bushels/acre of yield from the oats plant and a profit of $16.85/acre (Table 3).

TABLE 3. Comparison of the optimal control policy with conventional spray and no-spray schemes for a single season problem.

Initial densities: CLB = 2.000/sq ft

TJ = 0.001/sq ft

Price of oats = \$1.35/bushel

Cost of malathion = \$3.00/lb

Cost of pesticide application = \$3.00/acre

TYPE OF DECISION RULE	TIME OF SPRAY IN $60DD_{48}$ UNITS	AMOUNT OF SPRAY lb/acre	OATS YIELD bu/acre	PROFIT #/acre	OVER WINTERING TJ/sq ft
no-spray	-	0.00	14.70	16.85	0.040
conventional spray	3	1.00	23.85	26.20	0.032
optimal spray	6	0.88	43.56	53.11	0.031

We will now focus on the determination of the optimal control strategy using optimization techniques. Optimization problems are characterized by the performance measure utilized in the formulation problem. Within the framework of pest management, the most important optimization problem that will be considered is the so-called profit maximization problem:

Max[revenue - control cost].

This is probably the most realistic type of problem with regard to the prevailing real world situation: the individual farmer's choice of pest control schemes is generally motivated by the profit maximization criterion, and, as of the present, the farmers are not required to bear the externality costs associated with the pesticide usage. In view of the aforementioned reason, the profit maximization problem is chosen as a typical example for the comparison of optimal strategies with control strategies currently in use.

The key state variable that is directly related to the yield from the oats plant is the weight of the grain head at harvest time ($X_{32}(N)$ in the system model, where N is the final time). The control costs consist of the cost of pesticides and the application costs. Further, the performance measure utilized in the profit maximization problem must reflect the integrated approach to pest management. In other words, the performance measure should be aimed at reducing pesticide use while at the same time, enhancing biological control of the CLB population. With reference to the optimal control problem, the pesticide spray is the only control variable that can be directly manipulated for timing and amount. The biological control manifested by the parasite population is only an indirect form of control. Further, there is a dynamic interaction between the CLB and *T. julis* throughout the season. Thus, the parasite populations are represented by state variables instead of control variables. Furthermore, for a single season optimization problem, the parasite, *T. julis*, virtually plays no role, but its effect will be felt in the subsequent growing seasons. The key entomological variable that captures the essence of this situation is the overwintering *T. julis* density at the end of the season. This can be directly transformed into a constraint on the terminal state (of the appropriate state variable) in the optimization problem.

In light of the above-mentioned attributes of the pest management problem, the following performance measure (referred to as the minimum control effort problem in the control literature) is found to be the most appropriate for characterizing the economic optimization problem.

$$\text{Maximize:} \quad J \triangleq X_{32}^2(N)P_1Q + \sum_{i=1}^{n-1} X_{32}^2(i)P_1Q - U^2(i)P_2R$$

subject to the terminal constraint on diapausing *T. julis*:

$$X_{18}(N) = \text{TJLAST}$$

where: P_1 = price of oats

P_2 = costs associated with chemical control

Q,R = weighting factors

N = terminal time

TJLAST = desired density of overwintering *T. julis* at the terminal time.

Obviously there exists trade-offs between minimizing pesticide use and maximizing revenue with reference to the profit maximization problem. Thus, the weighting factors can be adjusted so as to modify the relative emphasis on pesticide use and revenue. The terminal constraint for the overwintering T. julis is set so that the density of diapausing T. julis will be the same as that obtained using the conventional spraying scheme.

The profit maximization problem discussed above is solved using the optimization technique based on the successive approximation algorithm (refer to earlier discussions). The optimal control strategy is found to be a single spray of 0.88 lb/acre timed at $360DD_{48}$ (i.e. the sixth time step in the model) (Figure 8). Strictly speaking, the optimal strategy consists of a spray of 0.88 lb/acre at the sixth time step, along with sprays of 0.0001 lb/acre, or less, at several other time steps from 1 through 18. Since these sprays are totally insignificant when compared to the spray of 0.88 lb/acre, they are ignored. In this sense, the single spray of 0.88 lb/acre is suboptimal. However, the computer runs made with the optimal and suboptimal strategies lead to essentially identical results with an accuracy of 4 decimal places. In view of this situation, the suboptimal strategy is substituted in place of the optimal strategy.

Table 3 illustrates a comparison of the optimal control policy, with conventional spray and no-spray schemes for the single season of optimization problems considered above. It can be observed that the optimal strategy results in a 12% reduction in pesticide use--0.88 lb/acre as

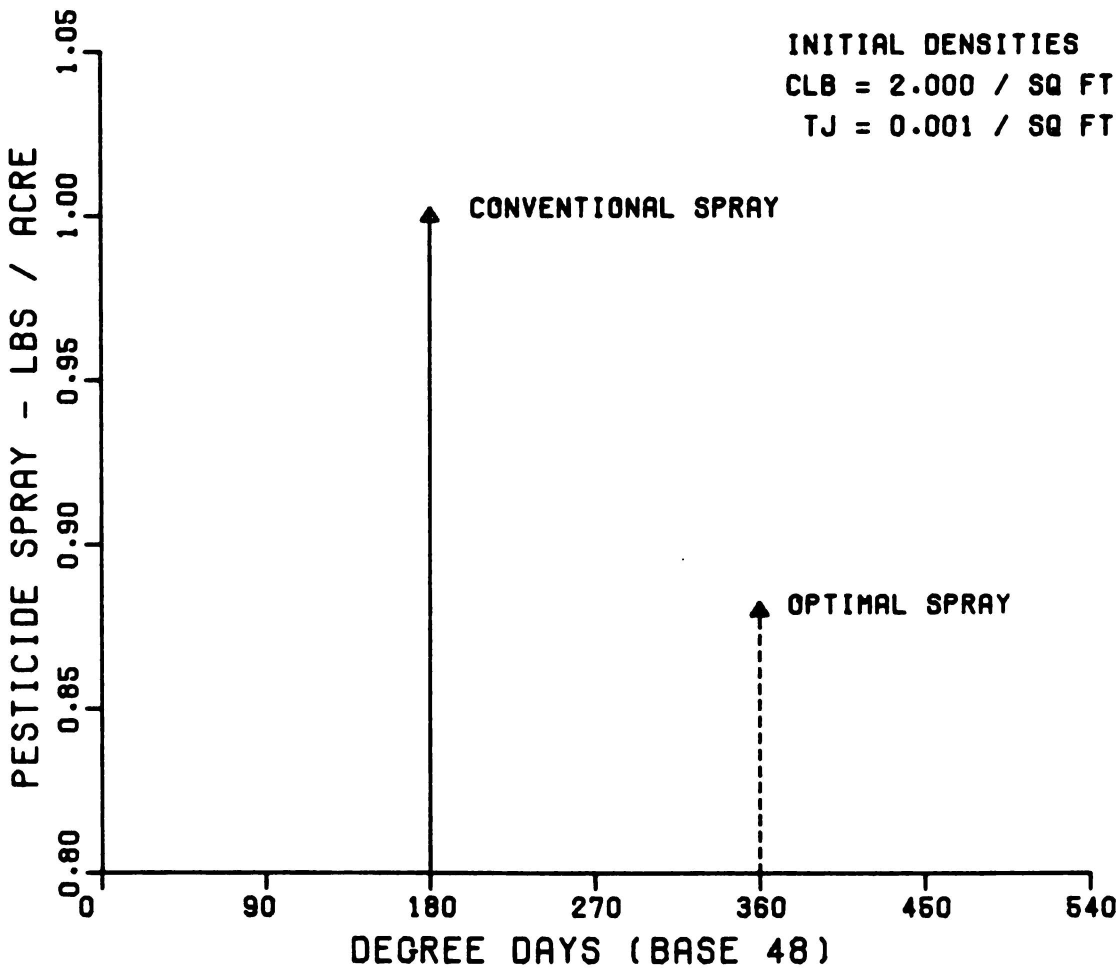


FIGURE 8. Comparison of the timing and amount of pesticide spray under optimal and conventional spraying strategies.

compared to 1.00 lb/acre with the conventional spray. More importantly, the density of overwintering T. julis is almost identical in both cases (0.031/sq ft in the optimal case as compared to 0.032/sq ft with the conventional spray) as required by the terminal constraint (on the overwintering T. julis) included in the optimal control problem. It is also worth noting that in comparison to the conventional strategy the optimal strategy leads to significant gains in oats yield (43.56 bushels/acre as compared to 23.85 bushels/acre with the standard policy) and almost doubles the profit ($53.11 as compared to $26.20 with the conventional spray).

From a biological point of view, there is a marked difference between the conventional spraying scheme and the optimal strategy--the conventional spray is carried out early in the season ($180DD_{48}$) and is aimed at CLB spring adults and eggs; whereas the optimal spray is timed later in the season ($360DD_{48}$) and is aimed at early larval instars of the CLB. It is also worth noting here that the CLB larvae are more susceptible to malathion, as compared to CLB adults.

Figures 9-21 illustrate the evolution of several important variables of the CLB ecosystem when subjected to different control strategies, namely, the conventional strategy, the optimal strategy, and the strategy of no spray at all. Figures 9 through 12 illustrate the state variables related to the oats plant, namely the weight of the plant, the leaf surface area, the weight of the grain head, and the surface area of the heads. It can be easily observed that, for the problem under consideration, the optimal strategy is far superior to the conventional strategy,

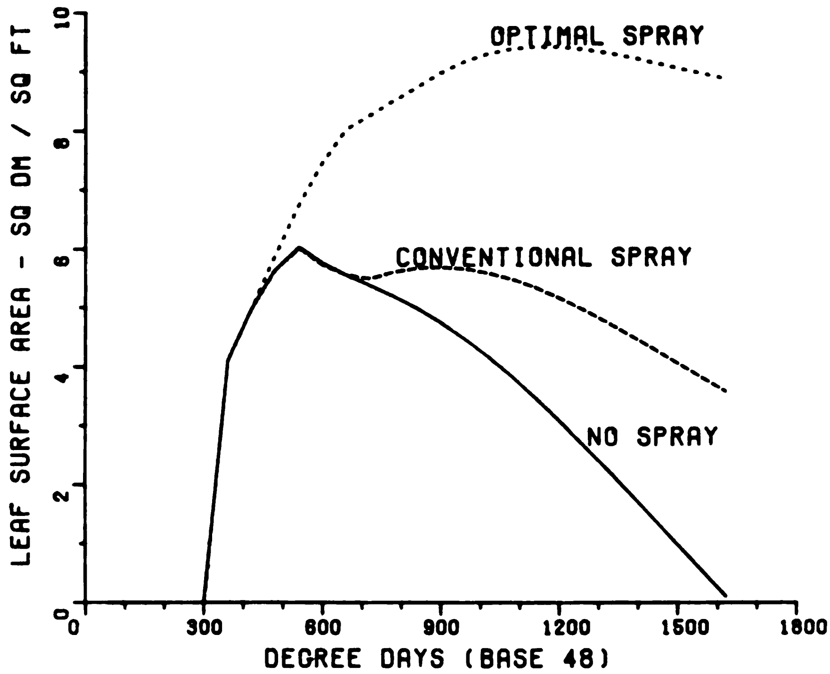


FIGURE 9. Leaf surface area of oats plant under optimal, conventional and no-spray schemes.

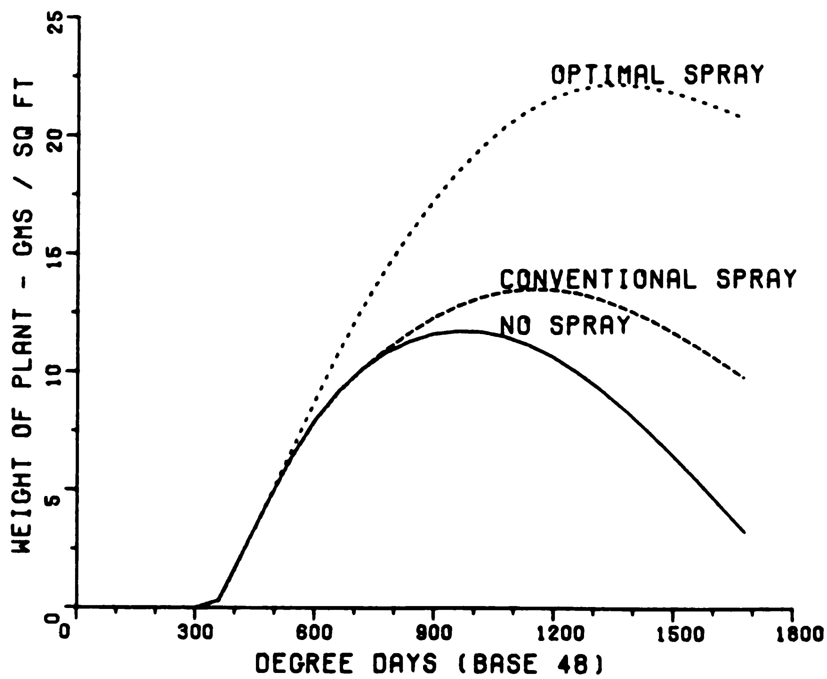


FIGURE 10. Weight of oats plant under optimal, conventional and no-spray schemes.

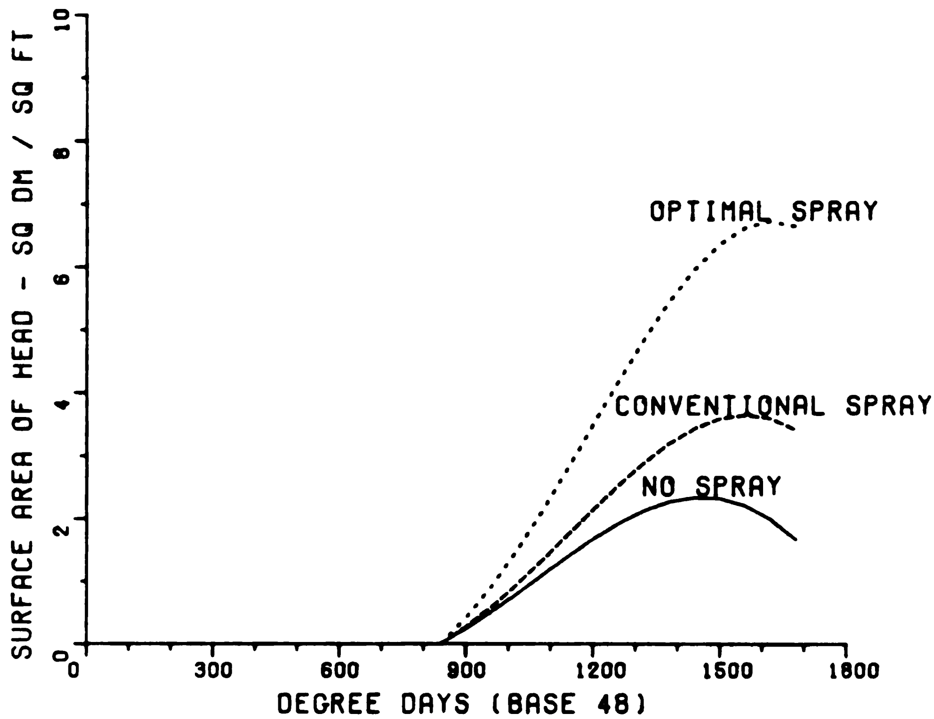


FIGURE 11. Surface area of grainhead of the oats plant under optimal, conventional and no-spray schemes.

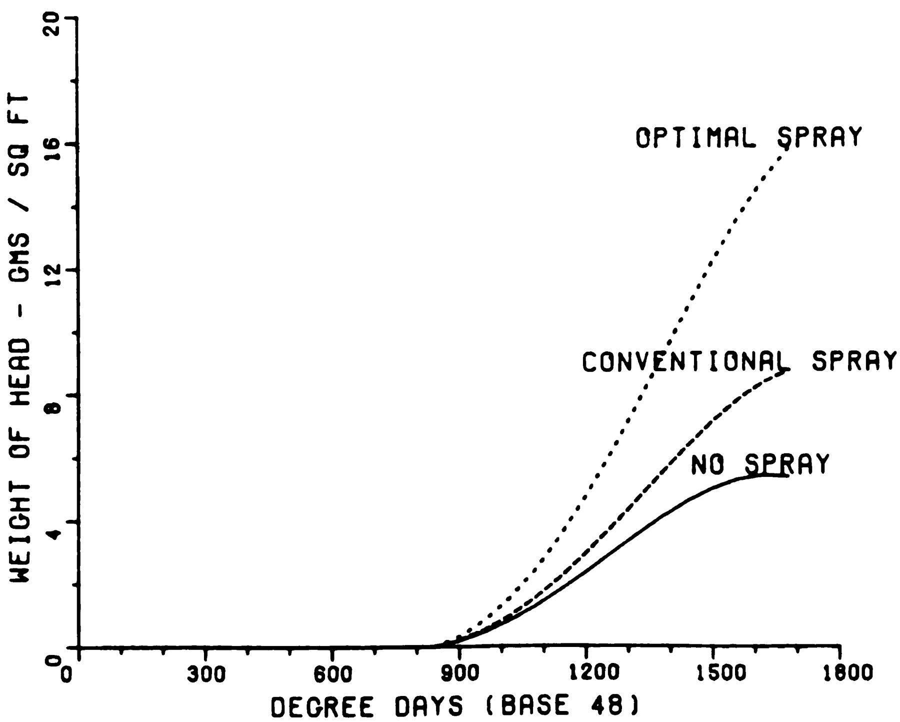


FIGURE 12. Weight of grainhead of the oats plant under optimal, conventional and no-spray schemes.

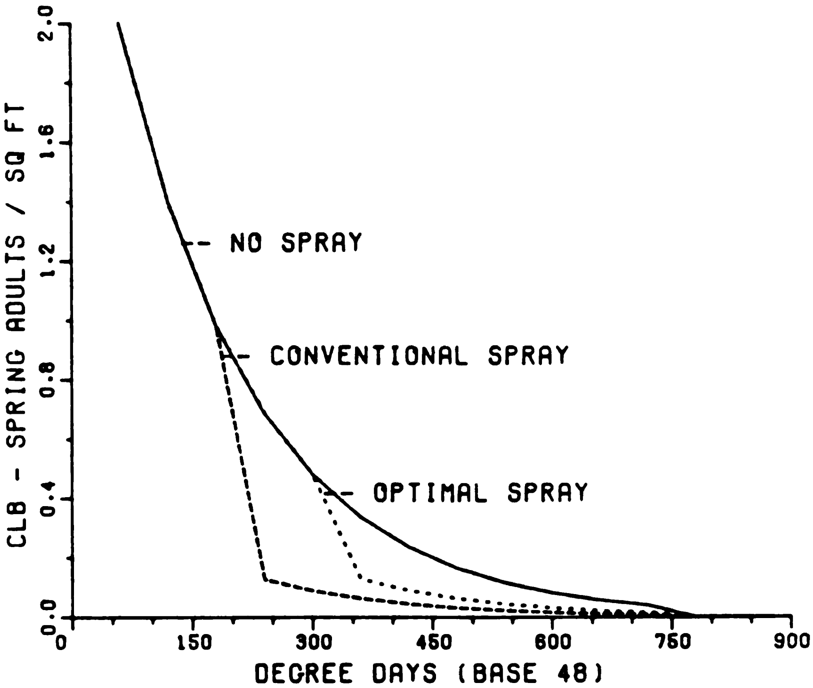


FIGURE 13. CLB-spring adult density under optimal, conventional and no-spray schemes.

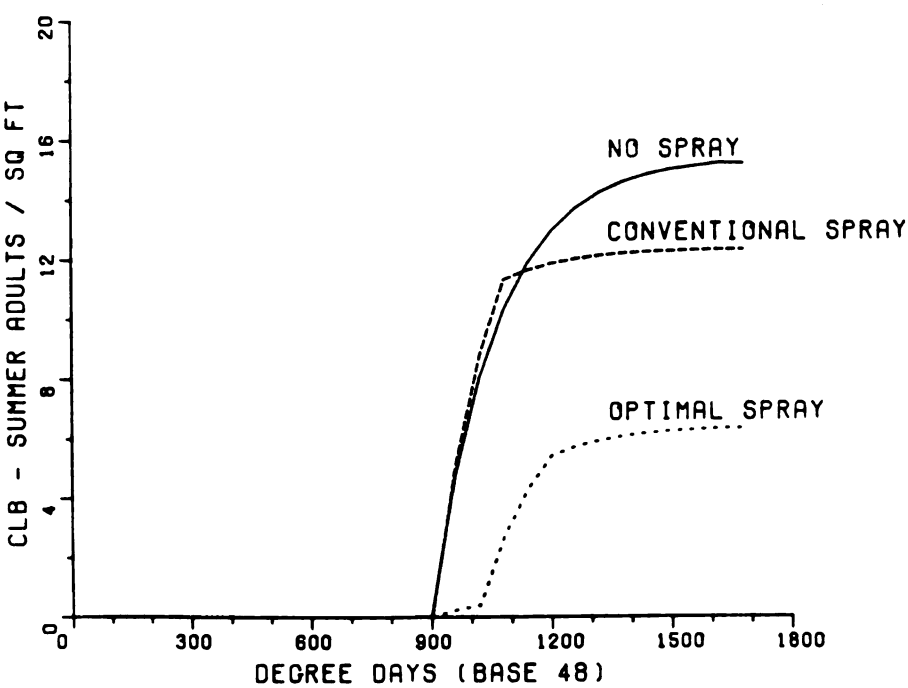


FIGURE 14. CLB-summer adult density under optimal, conventional and no-spray schemes.

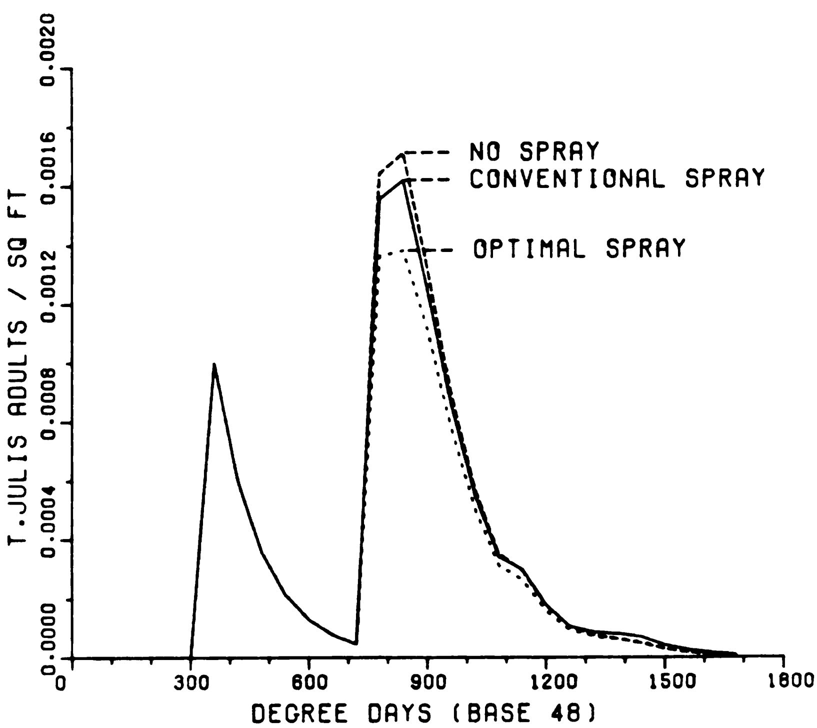


FIGURE 15. Density of adult T. julis under optimal, conventional and no-spray schemes.

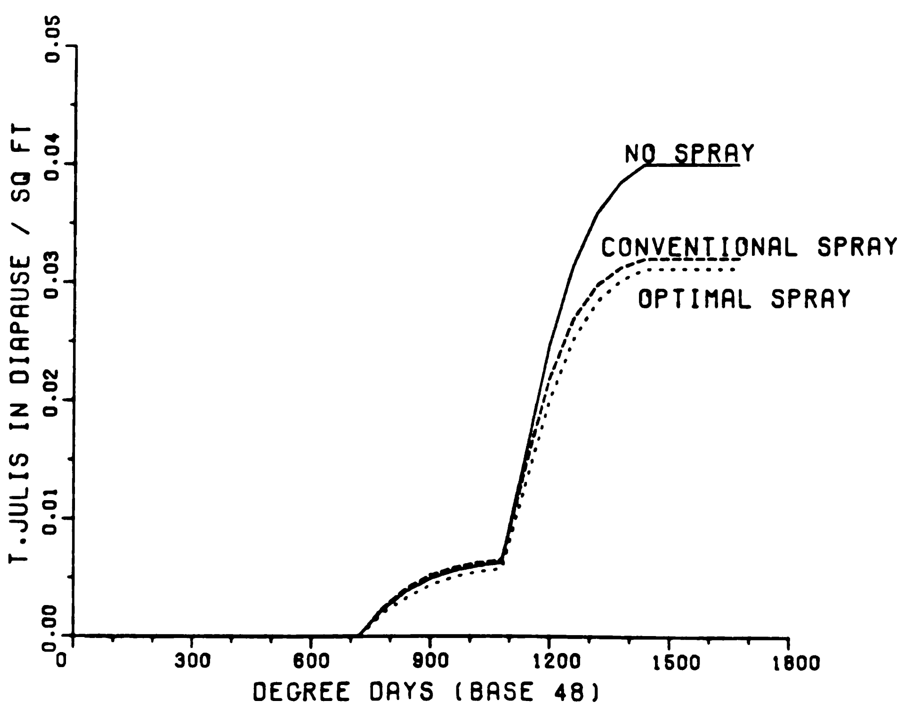


FIGURE 16. Density of diapausing T. julis under optimal, conventional and no-spray schemes.

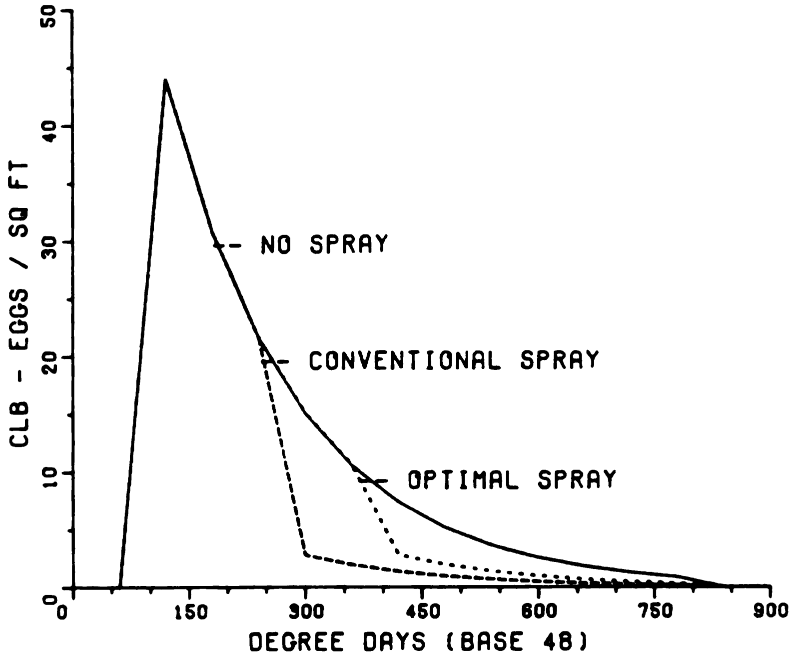


FIGURE 17. CLB-egg density under optimal, conventional and no-spray schemes.

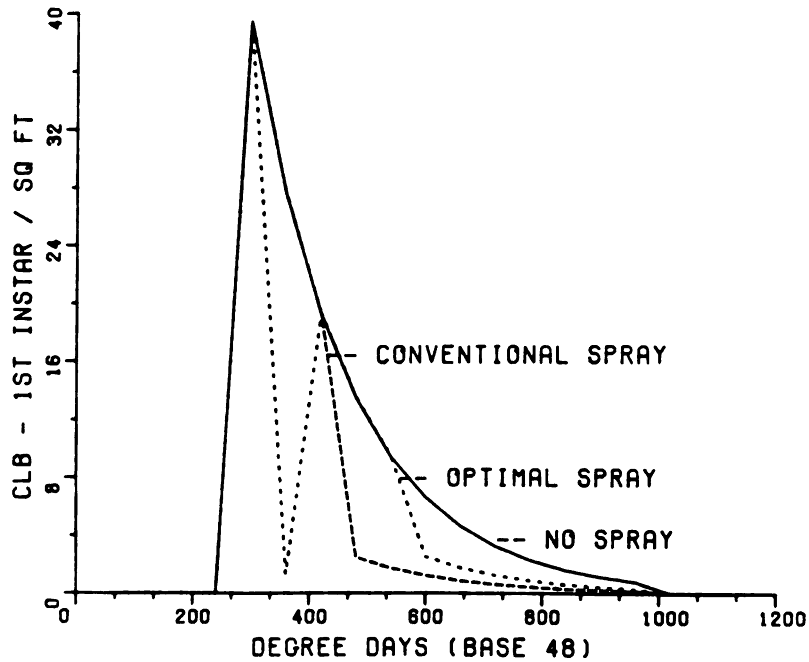


FIGURE 18. CLB-first instar density under optimal, conventional and no-spray schemes.

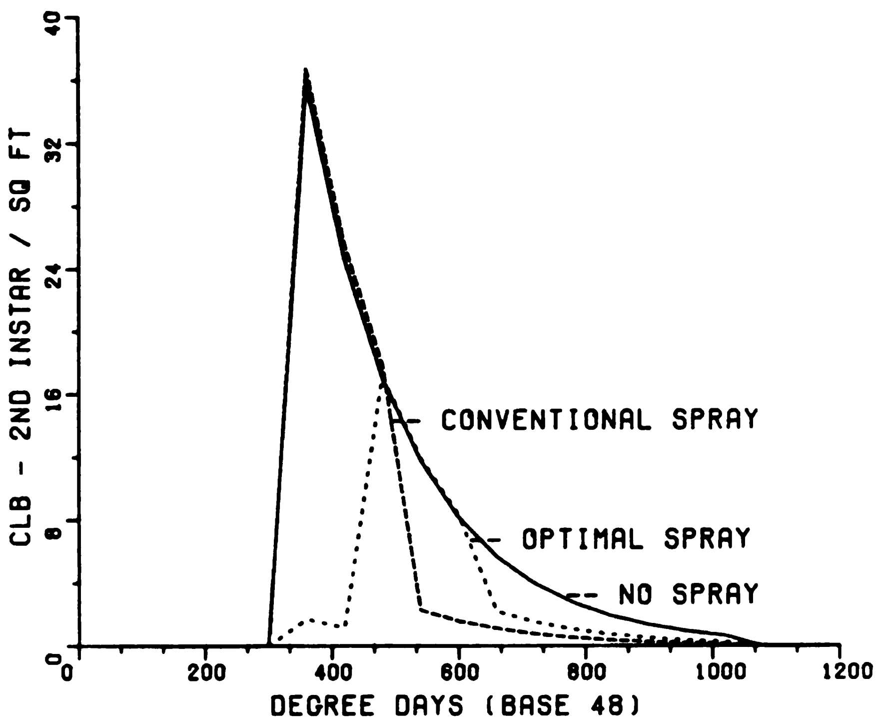


FIGURE 19. CLB-second instar density under optimal, conventional and no-spray schemes.

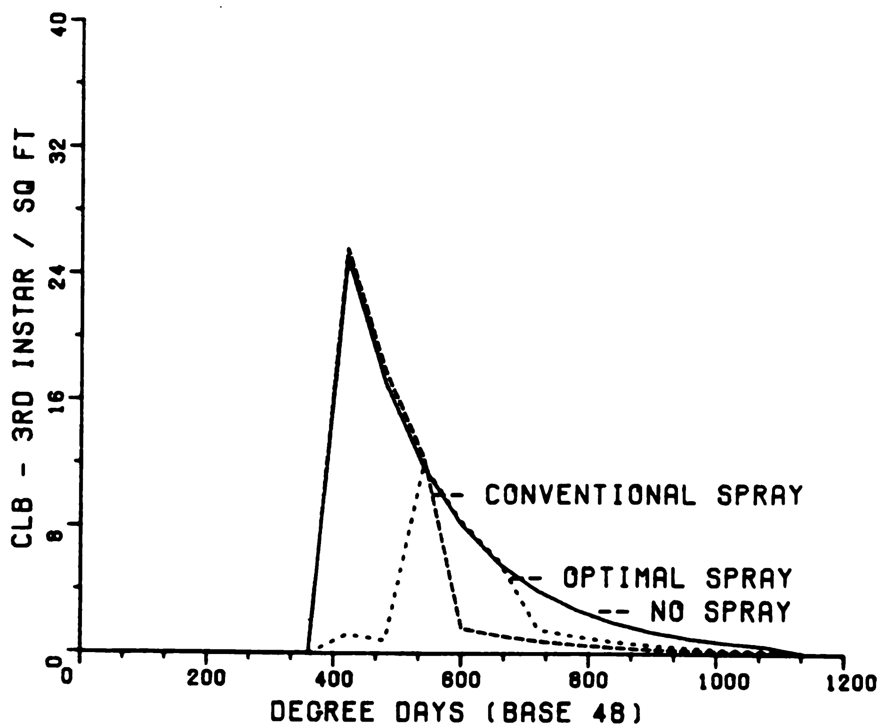


FIGURE 20. CLB-third instar density under optimal, conventional and no-spray schemes.

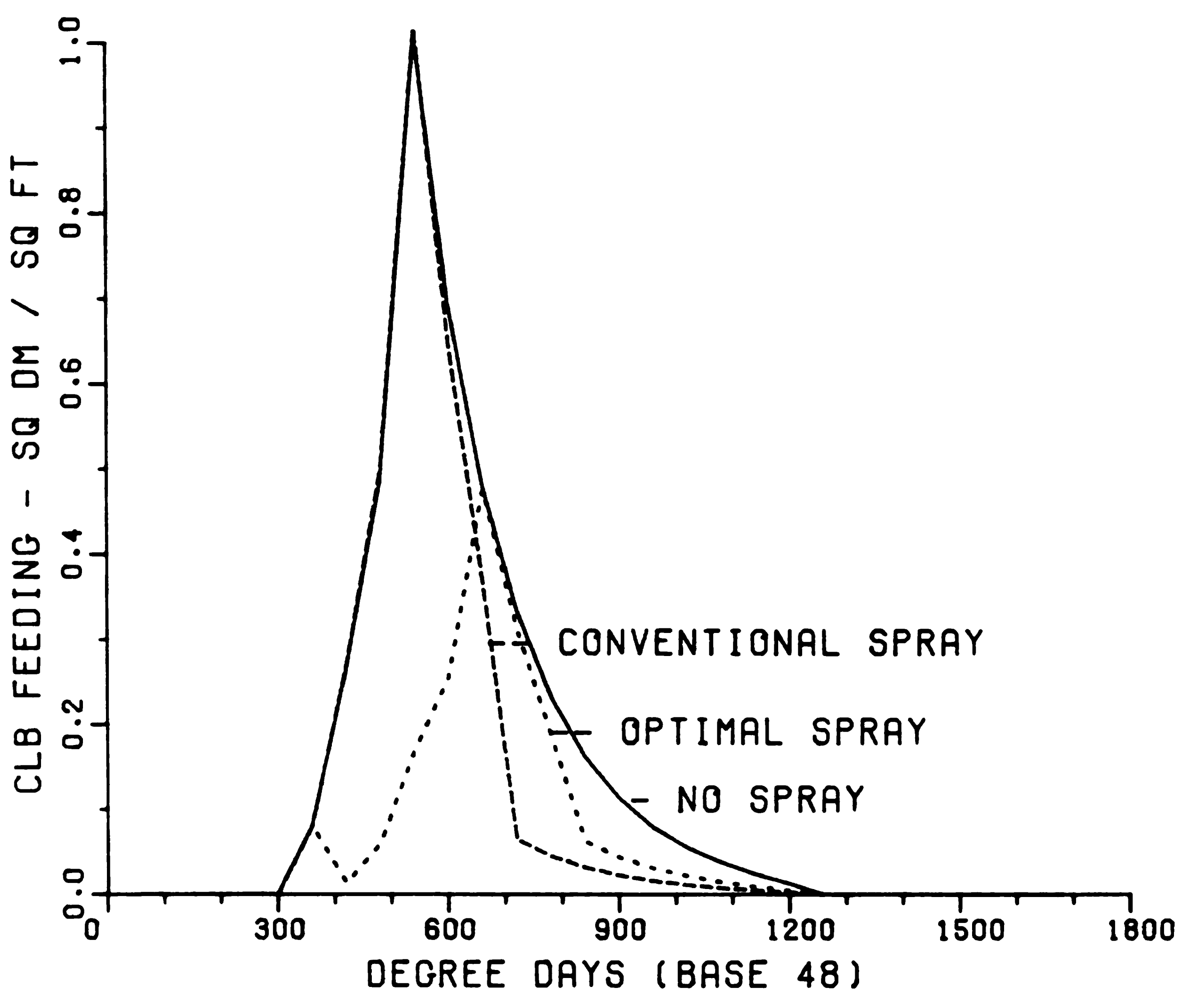


FIGURE 21. CLB feeding under optimal, conventional and no-spray schemes.

which as can be expected, is better than no control at all. It can be seen from Figure 13 that the conventional strategy is more effective in controlling the CLB spring adults as compared to the optimal strategy; whereas the optimal strategy is more effective against CLB summer adults (Figure 14). The effect of these different control strategies on the parasite population is depicted in Figures 15 and 16. It is interesting to note the density of *T. julis* in diapause at the end of the season is approximately the same for both the optimal and conventional strategies as necessitated by the terminal constraint imposed on the overwintering *T. julis*. The small difference between the two can be attributed to the fact that constraints are only approximately (and not exactly) satisfied in the computational implementation of the optimization algorithm. The major difference between the optimal and conventional strategies is highlighted in Figures 17 and 18, which illustrate the CLB egg and first instar densities under different control strategies; the conventional strategy is aimed at the CLB spring adults (Figure 13) and CLB eggs, while the optimal control strategy is directed toward the early larval instars of the CLB. The impact of the three different control strategies on the CLB second and third larval instars are portrayed in Figure 19 and 20. Figure 21 illustrates the CLB feeding on the oats plant for the three different control schemes, and clearly brings forth the effectiveness of the optimal spray in reducing the CLB feeding.

Optimal strategies may not always lead to such spectacular gains over conventional policies; however, optimal strategies will *always* be as good as, and often times better than, the conventional schemes. This is due to the fact that the optimization algorithm routinely searches numerous policy options, with reference to the timing and amount of pesticide spray, and chooses the one that optimizes the desired performance measure specified for the problem. In the event the conventional spraying scheme happens to be the optimal, the optimization scheme will automatically choose it.

SENSITIVITY ANALYSIS

In order that we may fully appreciate the effect of the timing and amount of pesticide spray on the economic yield from the crop and several other important variables characterizing the CLB ecosystem, a sensitivity analysis is carried out with reference to the timing and amount of pesticide spray. Thus, several computer runs of the CLB eocsystem model (initial conditions remaining as before at CLB = 2.000/sq ft and TJ = 0.001/sq ft) are carried out with the timing of pesticide spray kept fixed at $180DD_{48}$, while the amount of spray is varied from 0.1 to 2.0 lbs/acre. This process is repeated for several other spray times-- $240DD_{48}$, $300DD_{48}$, $360DD_{48}$, and $420DD_{48}$. The results are illustrated in Figures 22 through 27.

It can be seen from Figure 22 that the conventional spray timed at $180DD_{48}$ leads to very poor yield from the oats plant; whereas the optimal spray timed at $360DD_{48}$ results in the maximum yield. A similar situation exists in the case of profit (Figure 23). It can also be seen from Figures 22 and 23 that a pesticide spray of slightly over 1 lb/acre

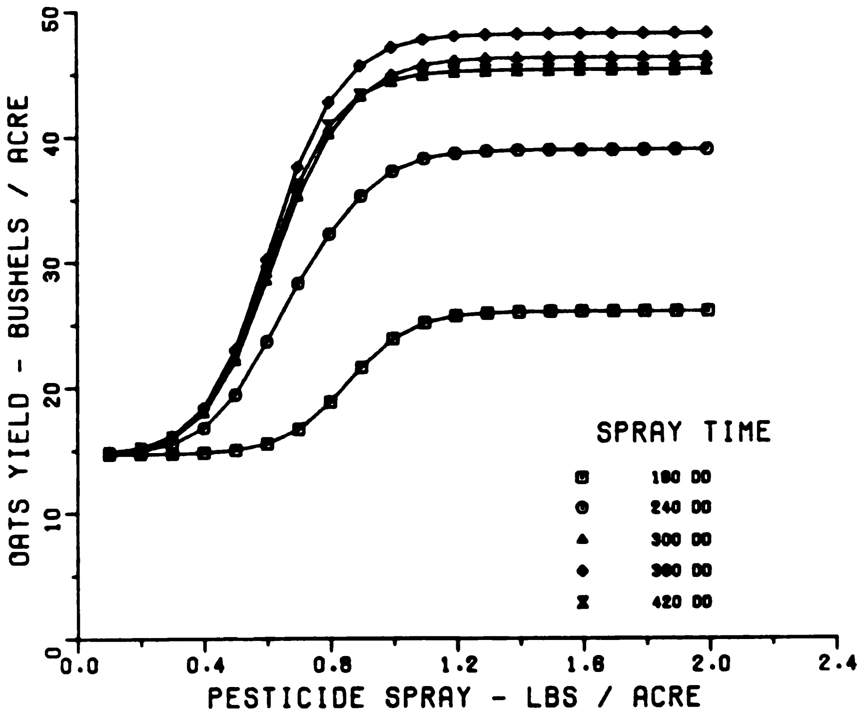


FIGURE 22. Sensitivity of oats yield (bushels/acre) with reference to changes in the timing and quantity of pesticide spray.

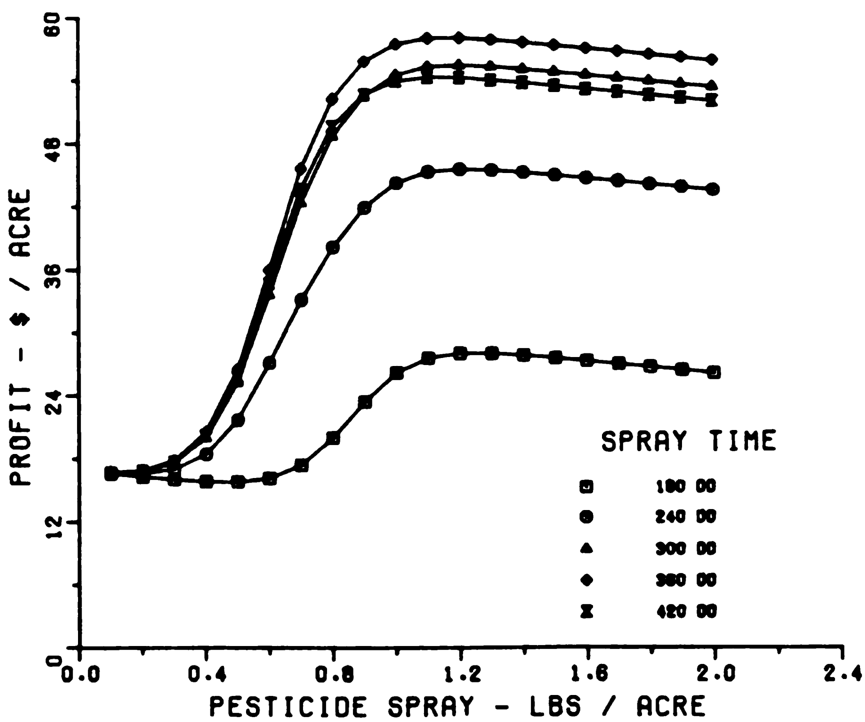


FIGURE 23. Sensitivity of profit (dollars/acre) with reference to changes in the timing and quantity of pesticide spray.

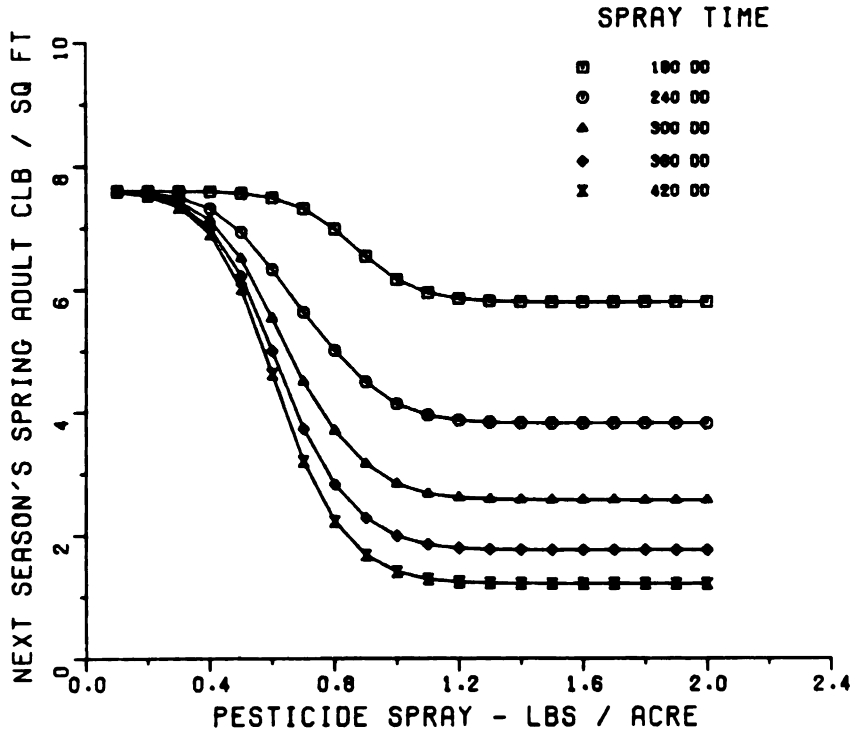


FIGURE 24. Sensitivity of the density of spring adult CLB (of the next season) with reference to changes in the timing and quantity of pesticide spray.

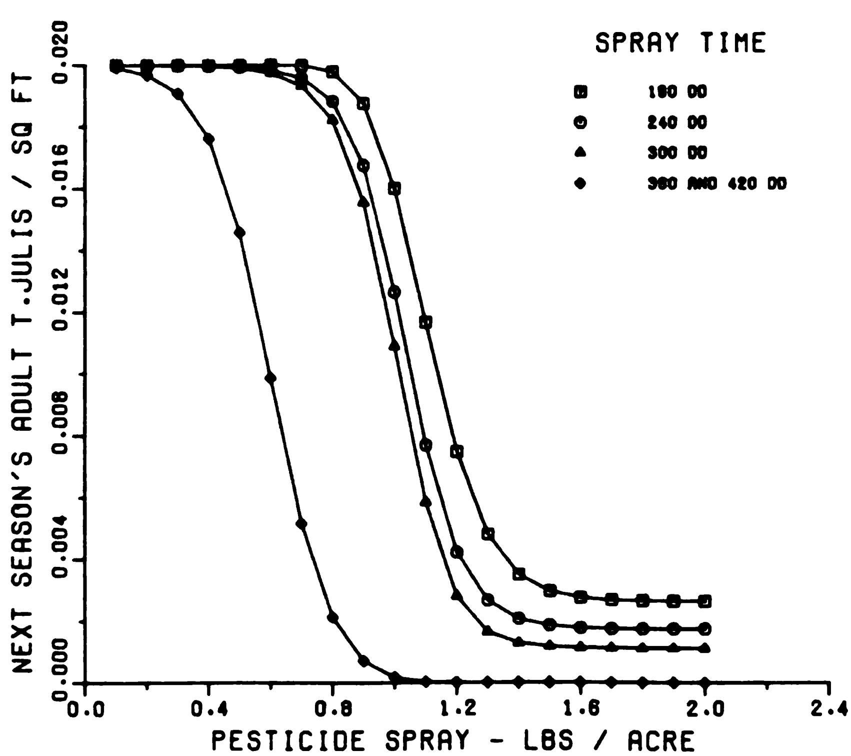


FIGURE 25. Sensitivity of the density of adult *T. julis* (of the next season) with reference to changes in the timing and quantity of pesticide spray.

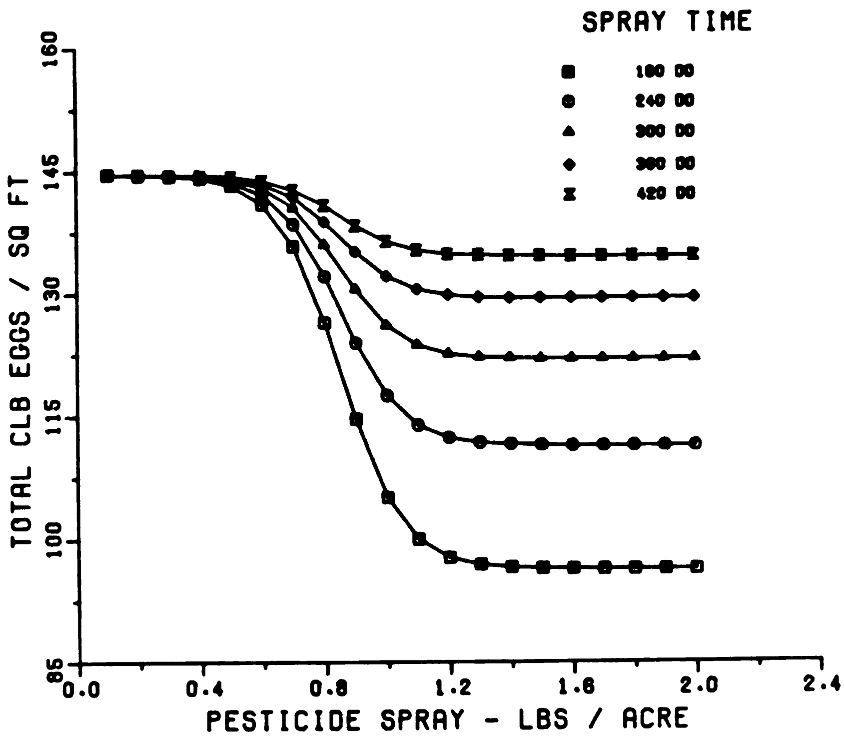


FIGURE 26. Sensitivity of the CLB egg density with reference to changes in the timing and quantity of pesticide spray.

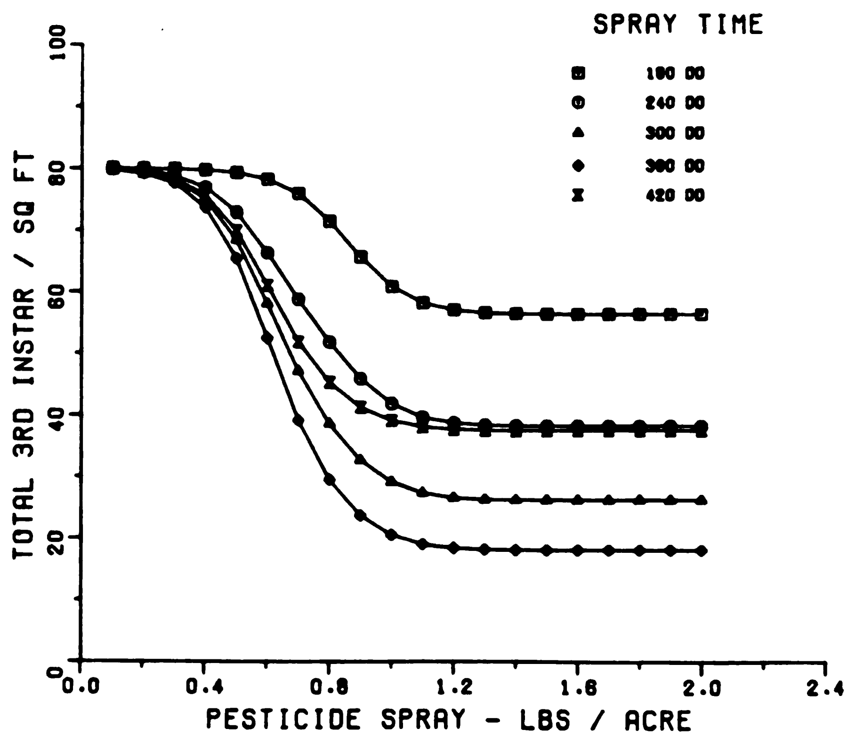


FIGURE 27. Sensitivity of the total CLB third instar density with reference to changes in the timing and quantity of pesticide spray

is the probable upper limit on the amount of a single spray--in other words, at any one particular time period, any amount of spray over and above this upper limit of 1 lb/acre will not improve the yield, but will only lead to a reduction in profits due to unnecessary expenses incurred on pesticide-related costs. It is also interesting to note that the amount of spray currently recommended, namely 1 lb/acre, also happens to be the upper limit on the amount of spray.

Figures 24 and 25 illustrate the effect of timing and the amount of the pesticide spray on the spring adult CLB and the adult *T. julis* of the subsequent season (i.e., the impact of spraying in the current season on the starting densities of CLB and TJ for the next season). With regard to the timing of the spray, early applications of pesticide are less effective in reducing the CLB population. However, early applications of pesticide are more beneficial to *T. julis* as compared to the sprays at a later point in the season.

The optimal policy chose a spraying amount of 0.88 lb/acre (in contrast to 1.0 lb/acre of the conventional scheme) such that the densities of overwintering *T. julis* at the end of the season are the same under the conventional and the optimal schemes, as required by the constraints we imposed on the optimization problem. Thus, the optimization algorithm simultaneously chooses the timing and the amount of spray so that the desired performance measure is optimized.

Figures 26 and 27 illustrate the results of the sensitivity analysis in the total CLB eggs and the total third larval instar for the entire season. It is obvious that the conventional spray is most effective against the CLB eggs while the optimal spray is most effective against the CLB larvae. Incidentally, this highlights the biological implications of the control schemes--conventional policy being aimed at CLB spring adults and eggs, while the optimal spray is aimed at the early larval instars of the CLB as described earlier.

Sensitivity analysis with reference to the timing and the amount of pesticide spray can be very useful in analyzing various control options and quickly narrowing the options to a few good (not necessarily optimal) strategies. Optimization techniques will still be needed to determine the best control strategy. Further, sensitivity analyses with reference to timing and amount of pesticide spray will be more cumbersome when there is a need to spray several times during the growing season as opposed to the single spray strategy that we have considered for the CLB ecosystem.

ANALYSIS OF CONTROL STRATEGIES FOR THE MULTISEASON PROBLEM

As discussed earlier, the single season optimization problem could not vividly illustrate the beneficial effect of the parasite population, because for any given season the impact of *T. julis* cannot be perceived during the current growing season, but the beneficial effect of the parasite manifests itself in subsequent growing seasons. In the single season optimization problem, this situation is implicitly taken into account by imposing a terminal constraint on the density of the diapausing *T. julis* at the end of the season. Nevertheless, in order to fully capture the beneficial impact of *T. julis* on the CLB ecosystem, it is necessary to consider multiple-season problems.

The multiple season optimization problem is solved as a series of single season optimization problems. In this sense, the optimal policy will be only suboptimal over the time horizon comprising the multiple season as a whole. Such multiple season problems extending over a four year period are solved for several different combinations of starting densities for the spring adult CLB and the adult *T. julis*:

1. Initial densities: CLB = 2.000/sq ft
   TJ = 0.001/sq ft
2. Initial densities: CLB = 2.000/sq ft
   TJ = 0.100/sq ft
3. Initial densities: CLB = 1.000/sq ft
   TJ = 0.001/sq ft

Furthermore, these initial densities are chosen in such a manner that comparison of cases 1 and 2 will illustrate the effect of a change in the initial density of *T. julis* (for the same initial density of CLB) on the evolution of the CLB ecosystem. Likewise, the comparison of cases 1 and 3 will exemplify the impact on the CLB ecosystem due to a change in the starting density of CLB spring adults.

In the optimization problem used with the multiseason analysis, the terminal constraint imposed on the overwintering *T. julis* is set in such a manner that the overwintering *T. julis* density under the optimal policy will be about 80% of that obtained under the no-spray scheme. Briefly, this implies a further increase of *T. julis* overwintering density as compared to the optimization problem considered before in which the constraint level of overwintering *T. julis* is set to be the same as that obtained with the conventional spraying scheme. Such an increase in the level of the terminal constraint on overwintering T. julis is incorporated in the optimization problem in order to fully capture the beneficial effect of the parasite population.

The results of the multiseason analysis of the repeated application of conventional and optimal policies over a four year period are given in Tables 4, 5 and 6 and are graphically illustrated in Figures 28 through 32. Figures 28 through 32 clearly illustrate the enormous advantages associated with the optimal strategy--of great significance is the amount of pesticide used, which is much less with the optimal scheme as compared to the conventional practice, for all of the three cases considered (see Figure 28). Furthermore, in comparison to the conventional policy, the optimal strategy always leads to higher yield from the oats plant (see Figure 29), correspondingly higher profits (Figure 30) and is superior in terms of controlling the CLB (Figure 31). In addition, the optimal strategy is more conducive to the parasite T. julis as compared to the control policy currently in use (Figure 32).

The beneficial effect of T. julis in controlling the CLB population and the manner in which the optimal strategy exploits this beneficial aspect to reduce the use of pesticides are clearly brought forth in the multiseason analysis. Thus, it can be seen that, for the same density (2.0/sq ft) of spring adult CLB, case 2 with a higher (as compared to case 1) T. julis population not only uses much less pesticide (Figure 28 a and b) but also results in higher yield (Figures 29 a and b) and correspondingly higher profit (Figures 30 a and b). This beneficial effect of T. julis can be observed in both the conventional and optimal

TABLE 4. Comparison of optimal and conventional spraying schemes for a multiseason problem with initial densities of CLB = 2.000/ sq ft and TJ = 0.001/sq ft.

Table 4a. Repeated application of conventional policy over a 4 year period.

YEAR	PESTICIDE SPRAY lb/acre	TIME OF SPRAY IN 60DD UNITS	OATS YIELD bu/acre	PROFIT $/acre	SPRING ADULT CLB/sq ft	ADULT TJ/sq ft
1	1.0	3	23.85	26.20	2.00	0.001
2	1.0	3	11.45	9.45	2.83	0.016
3	1.0	3	3.24	-	3.42	0.224
4	1.0	3	0.55	-	3.62	1.708

Table 4b. Repeated application of optimal policy over a 4 year period.

YEAR	PESTICIDE SPRAY lb/acre	TIME OF SPRAY IN 60DD UNITS	OATS YIELD bu/acre	PROFIT $/acre	SPRING ADULT CLB/sq ft	ADULT TJ/sq ft
1	0.96	5	42.58	53.08	2.00	0.001
2	0.90	5	39.39	48.92	2.18	0.029
3	0.95	5	37.41	46.20	2.46	0.902
4	0.92	5	39.08	48.49	2.25	5.288

TABLE 5. Comparison of optimal and conventional spraying schemes for a multiseason problem with initial densities of CLB = 2.0/sq ft and TJ = 0.1/sq ft.

Table 5a. Repeated application of conventional policy over a 4 year period.

YEAR	PESTICIDE SPRAY lb/acre	TIME OF SPRAY IN 60DD UNITS	OATS YIELD bu/acre	PROFIT $/acre	SPRING ADULT CLB/sq ft	ADULT TJ/sq ft
1	1.0	3	23.85	26.20	2.00	0.10
2	1.0	3	13.15	11.76	2.71	0.96
3	1.0	3	10.79	8.56	2.87	4.11
4	1.0	3	18.86	19.47	2.32	10.88

Table 5b. Repeated application of optimal policy over a 4 year period.

YEAR	PESTICIDE SPRAY lb/acre	TIME OF SPRAY IN 60DD UNITS	OATS YIELD bu/acre	PROFIT $/acre	SPRING ADULT CLB/sq ft	ADULT TJ/sq ft
1	0.96	5	42.58	53.08	2.00	0.10
2	0.91	5	40.97	51.05	2.04	1.56
3	0.91	5	44.60	55.85	1.80	7.15
4	0.90	5	51.36	65.08	1.08	11.78

TABLE 6. Comparison of optimal and conventional spraying schemes for a multiseason problem with initial densities of CLB = 1.000/ sq ft and TJ = 0.001/sq ft.

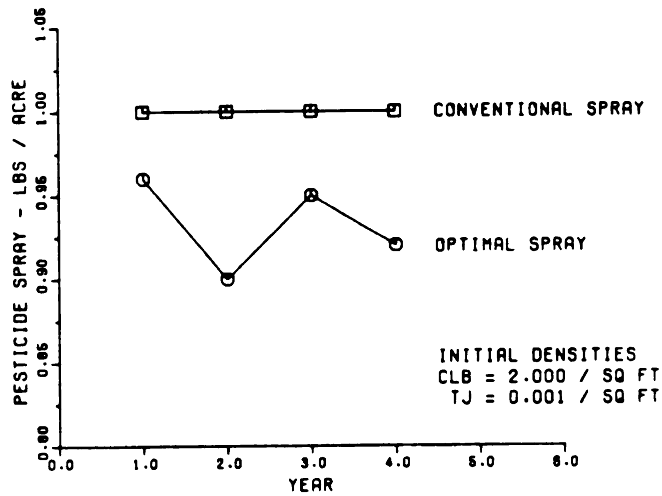
Table 6a. Repeated application of conventional policy over a 4 year period.

YEAR	PESTICIDE SPRAY lb/acre	TIME OF SPRAY IN 60DD UNITS	OATS YIELD bu/acre	PROFIT $/acre	SPRING ADULT CLB/sq ft	ADULT TJ/sq ft
1	1.0	4	48.79	59.86	1.00	0.001
2	1.0	4	45.97	56.06	1.19	0.011
3	1.0	4	43.88	53.24	1.37	0.127
4	1.0	4	43.27	52.42	1.43	0.818

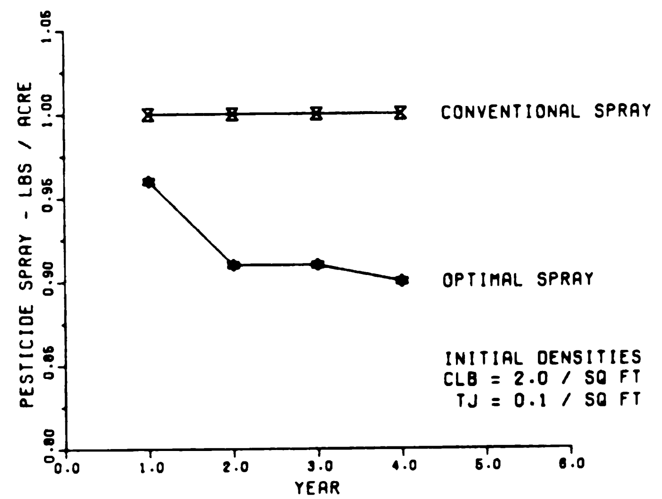
Table 6b. Repeated application of optimal policy over a 4 year period.

YEAR	PESTICIDE SPRAY lb/acre	TIME OF SPRAY IN 60DD UNITS	OATS YIELD bu/acre	PROFIT $/acre	SPRING ADULT CLB/sq ft	ADULT TJ/sq ft
1	0.88	5	51.94	65.86	1.00	0.001
2	0.85	5	48.42	61.20	1.25	0.028
3	0.91	5	46.83	58.89	1.57	0.822
4	0.86	5	46.70	58.84	1.43	3.664

a.



b.



c.

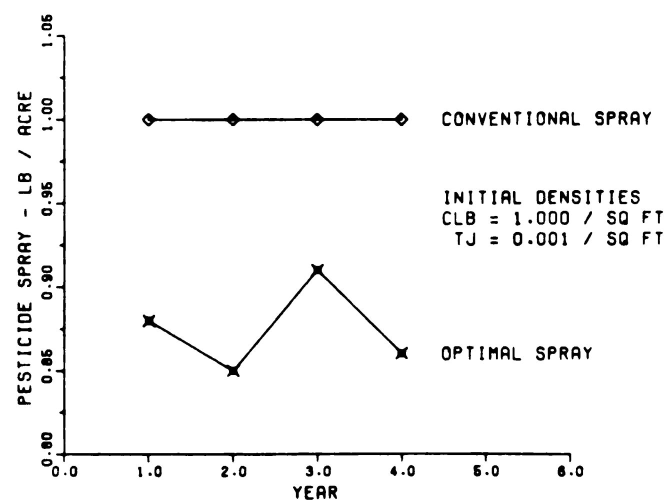


FIGURE 28. **Quantity of pesticide sprayed for the multi-season problem under different initial densities for the CLB and TJ.**

a.
b.
c.

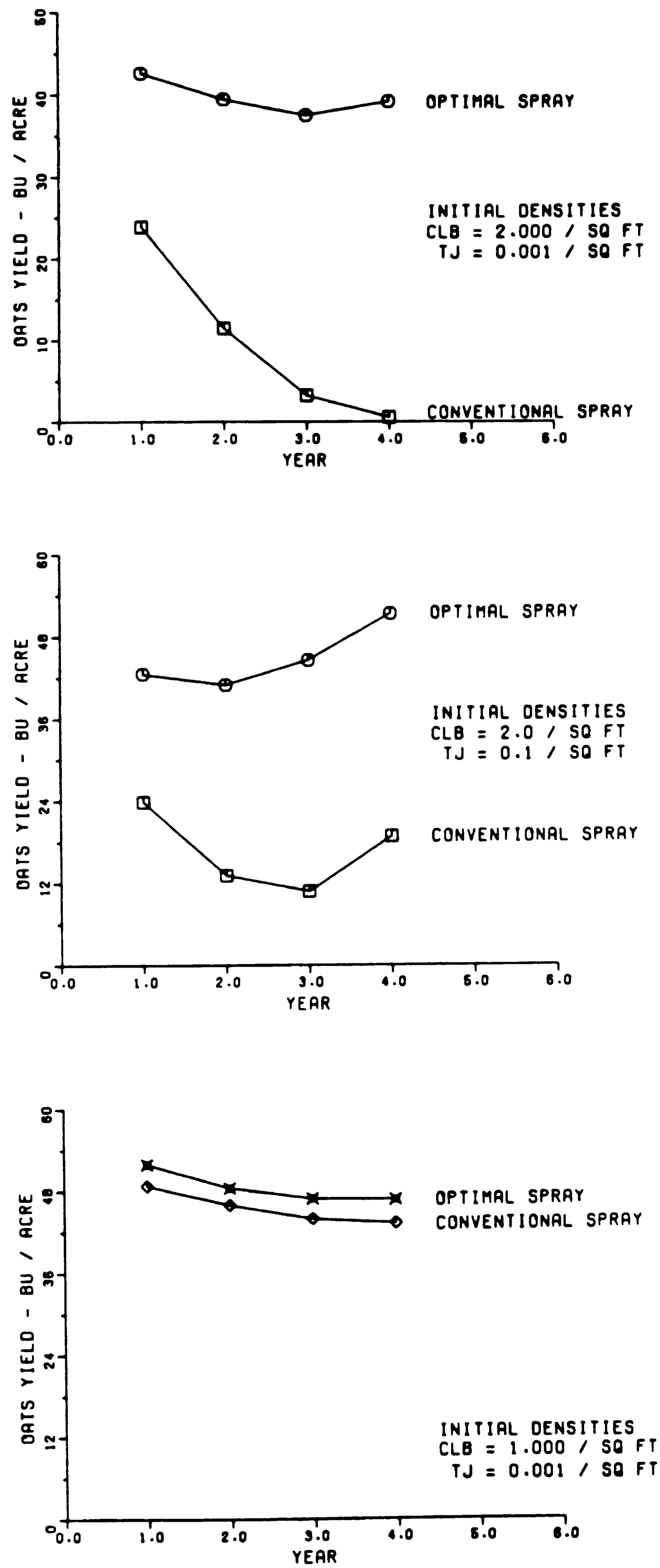


FIGURE 29. Yield from the oats plant for the multi-season problem under different initial densities for the CLB and TJ.

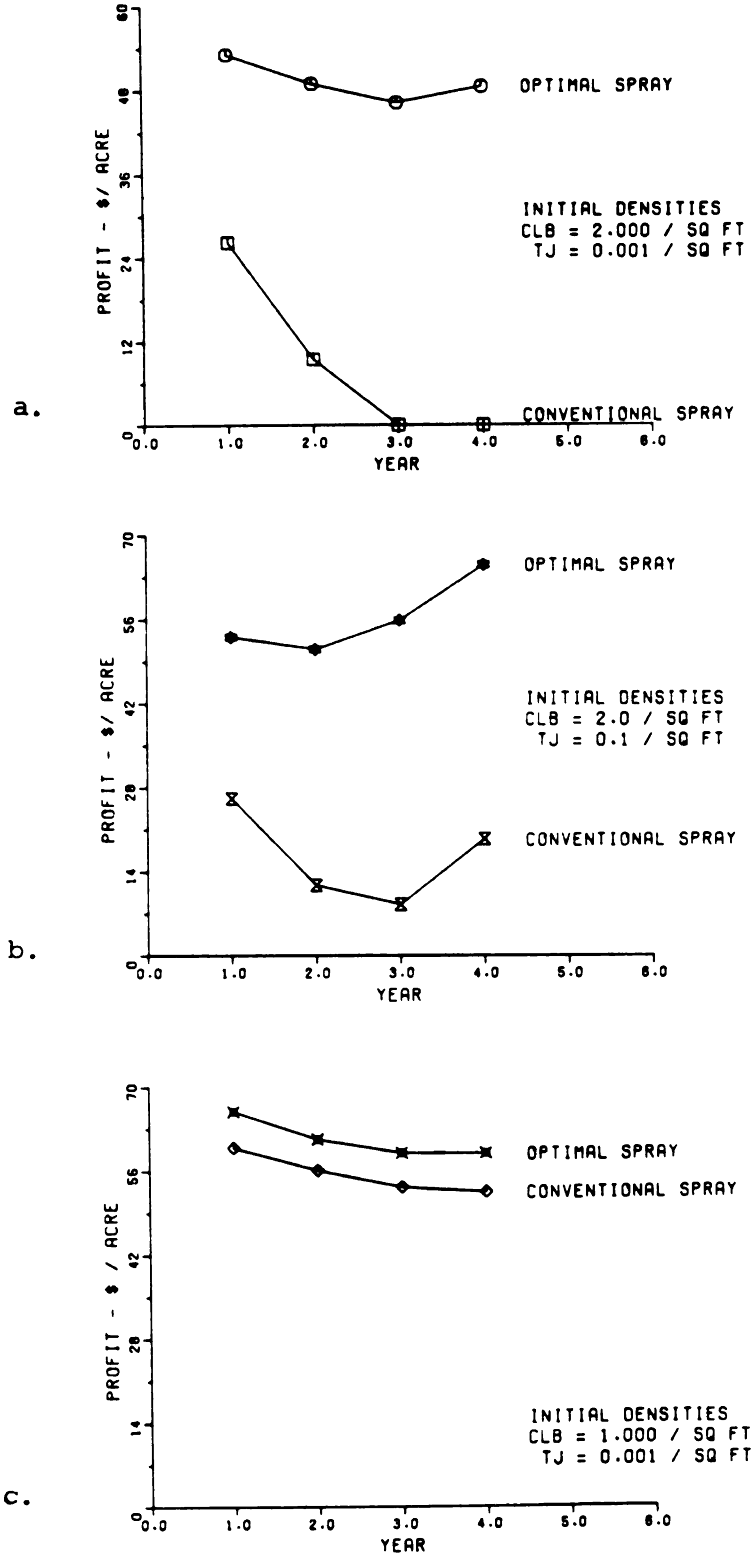
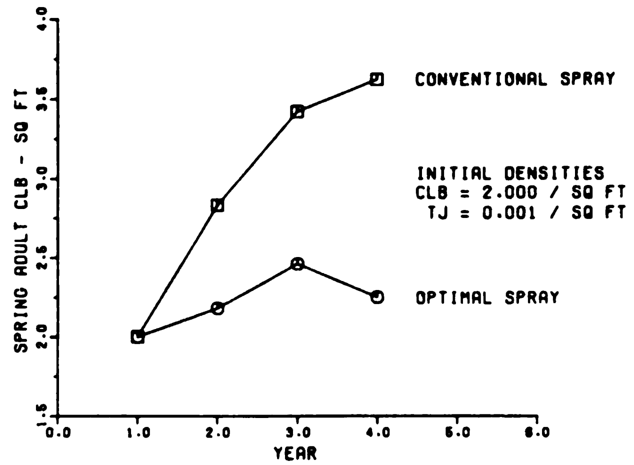
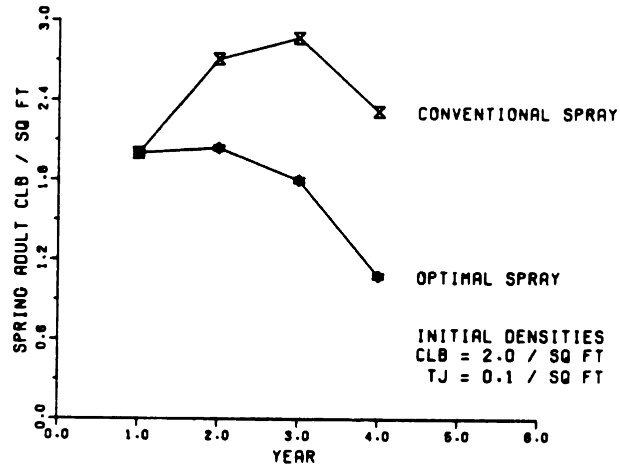


FIGURE 30. Profit obtained for the multi-season problem under different initial densities for the CLB and TJ.

a.



b.



c.

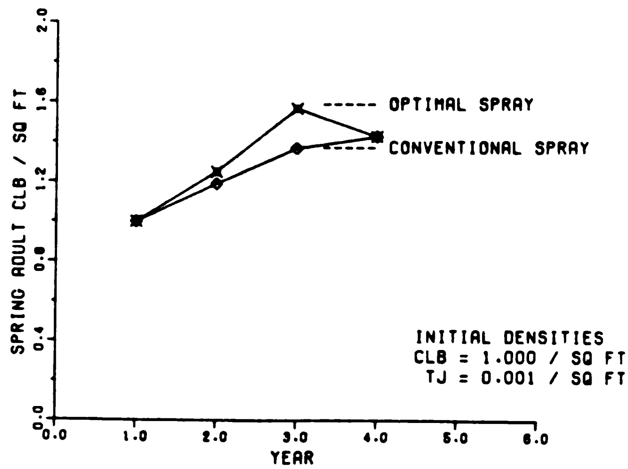


FIGURE 31. Density of spring adult CLB for the multi-season problem under different initial densities for the CLB and TJ.

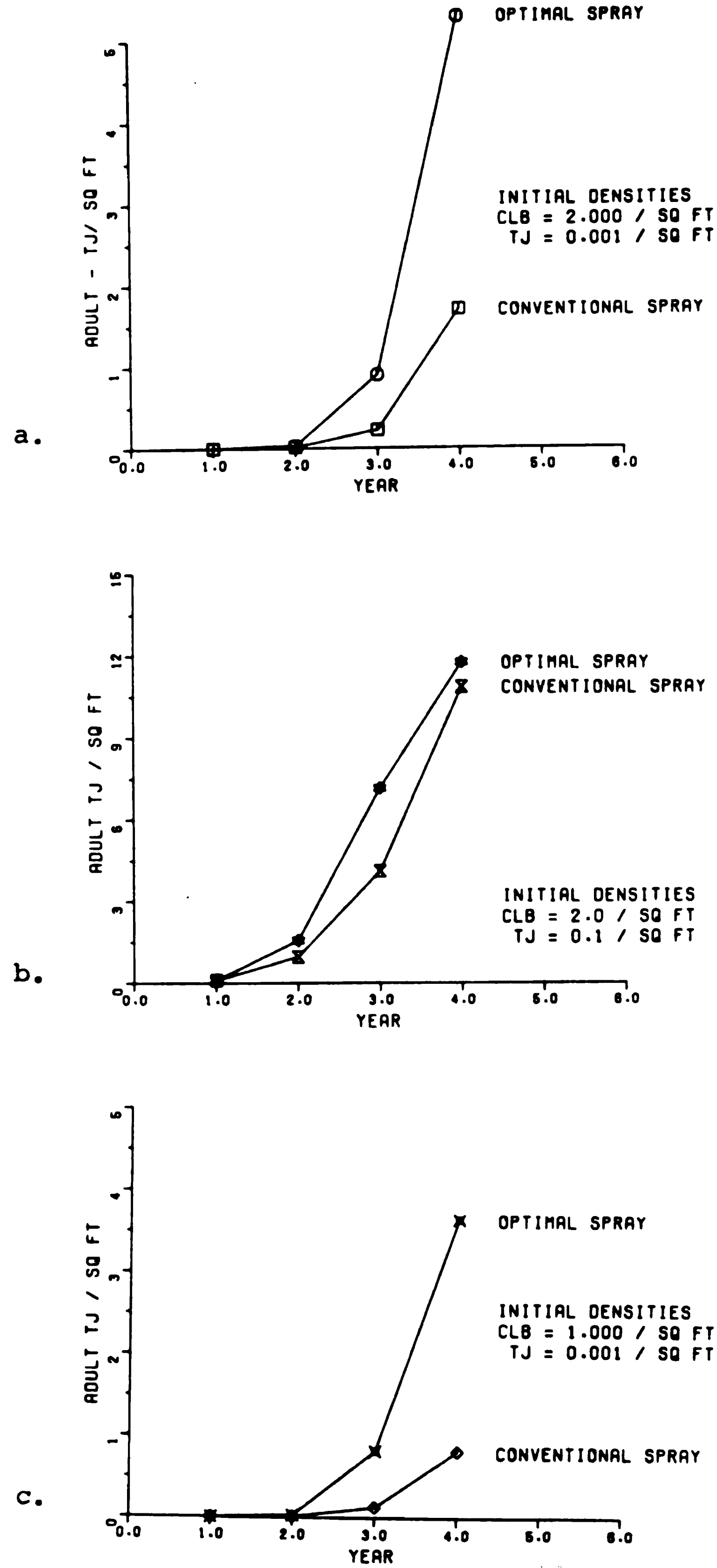


FIGURE 32. Density of adult T. julis for the multi-season problem under different initial densities for the CLB and TJ.

schemes; however, it is more pronounced in the optimal case. An analogous situation exists between case 1 and case 3, both of which have the same starting density for the *T. julis* (0.001/sq ft) but different starting densities for the CLB (2.0/sq ft in case 1 as opposed to 1.0/sq ft in case 3) (Figures 28-32).

It can also be observed (Figures 29c and 30c) that for case 2 with a low CLB density (1.0/sq ft) the conventional spray leads to high crop yield and large profit only slightly less than the corresponding ones obtained through the optimal strategy. However, bear in mind that these spectacular gains with the conventional spraying scheme have been achieved at the expense of using a much larger quantity of pesticide (Figure 28 c) as compared to the optimal strategy. Further, with the conventional spray, *T. julis* density through the years is much lower than the optimal case (Figure 32c) even though the spring adult CLB density is just about the same in both cases (Figure 31c).

It is remarkable that the optimal scheme is superior to the conventional scheme in all 3 cases, representing different combinations of CLB-TJ densities. The optimal policy leads to higher yields and greater profits, is more effective in suppressing CLB densities, and reinforces the increase of *T. julis* populations. More importantly, all of these gains are obtained with a much smaller pesticide use when compared to the standard practice.

ENVIRONMENTAL CONSIDERATIONS

The pest management problems discussed thus far have focused on the profit maximization criterion without due consideration to the externality costs associated with environmental pollution. More often than not, farmers are not held liable for most of the environmental damage caused by pesticide use. In the case of agricultural pest management problems we are concerned with negative externalities (i.e., external diseconomies of production) that result in uncompensated costs to the society. In this sense, there exists a divergence between private profits and social benefits. The major part of these externalities falls outside the scope of the market system and is not reflected in relative market prices (Kneese 1971, Kneese and Schultze 1975).

Externality problems have two important characteristics: (1) there exists an element of interdependency--interactions between the decisions of economic agents (e.g. the decisions of the individual farmer and those of neighboring farmers concerned with market prices for the crop, pesticide costs determined by the chemical companies, etc.) and (2) there exists no compensation; therefore, the one creating the externality costs (e.g. the farmer) is not legally or socially liable to pay for it. Another, but less important property of externalities, is emphasized by Mishan (1976) who points out that the environmental spill-over should be unintentional or an incidental by-product of some otherwise legitimate activity--which in our case is agricultural production. These characteristics make the externality problem very complex.

It is important to emphasize that an operational analysis of externalities is an extremely difficult task. Our approach to the externality problems arising in pest management will be confined to a rather simple analysis in terms of the economic considerations.

Generally speaking, there are two basic approaches to regulating pesticide use and associated pollution in pest management problems: direct regulation, and taxes and subsidies (see Judy 1970 for a detailed discussion on the instruments of environmental control).

The direct regulation approach involves directly regulating the amount of pesticide used in crop production. In terms of the optimization problem, the direct regulation criterion can be transformed into a constraint on the control variable, namely the amount of pesticide. In the present analysis, we impose an integral (isoperimetric) constraint on the profit maximization problem discussed earlier. Such a constraint implies that the total amount of pesticide used through the entire season will be set at certain prescribed levels by a regulatory agency. The regulatory agency will also have the task of overseeing the implementation of such a policy to insure that the prescribed levels of pesticide use are not exceeded. The profit maximization problem can now be restated as follows:

$$\text{Max} \sum_{i=1}^{N} [\text{revenue from oats}] - \text{control costs}$$

subject to integral constraint on pesticide use:

$$\sum_{i=1}^{N-1} u_i = \text{UMAX}$$

where UMAX is the prescribed level of pesticide use.

This optimization problem (together with a terminal constraint on overwintering _T. julis_) is solved using the same initial conditions as for the single season profit maximization discussed earlier for different constraint levels on the control variable (i.e., different amounts of pesticide used).

Figure 33 illustrates the yield from the oats plant and the corresponding profit in relation to the constraint level imposed on pesticide usage. It can be seen that a pesticide spray of 0.88 lb/acre results in maximum yield and correspondingly maximum profit. Any further increase in pesticide use does not increase the profit. In this particular case, the limit on the pesticide usage can be set at 0.88 lb/acre, which incidentally is less than the 1.00 lb/acre currently used in practice. However, the regulatory agency can choose to impose even a lower limit, say, 0.7 lb/acre, which will result in reduced yield and profit as compared to the pesticide use of 0.88 lb/acre.

It may be necessary to impose such a regulatory policy in order to meet certain guidelines of environmental toxicity (at the expense of lower profits to the private grower). Of course, in extreme cases the regulatory agency can ban the use of a pesticide.

There are several drawbacks with the direct regulation approach. First, pesticide use is strongly dependent on the degree of infestation in any given field, the presence or absence of biological control, etc. Ideally, there should be different levels of pesticide for different farmers growing the same crop, to allow them to make approximately the same monetary gains. These factors make direct regulation very difficult to implement. Second, it is very difficult to oversee the compliance of direct regulation. An attractive alternative is voluntary regulation under which the farmers are advised to follow certain recommendations as to the use of pesticides. As a matter of fact, the recommendations of extension service personnel can be categorized under the voluntary regulation approach.

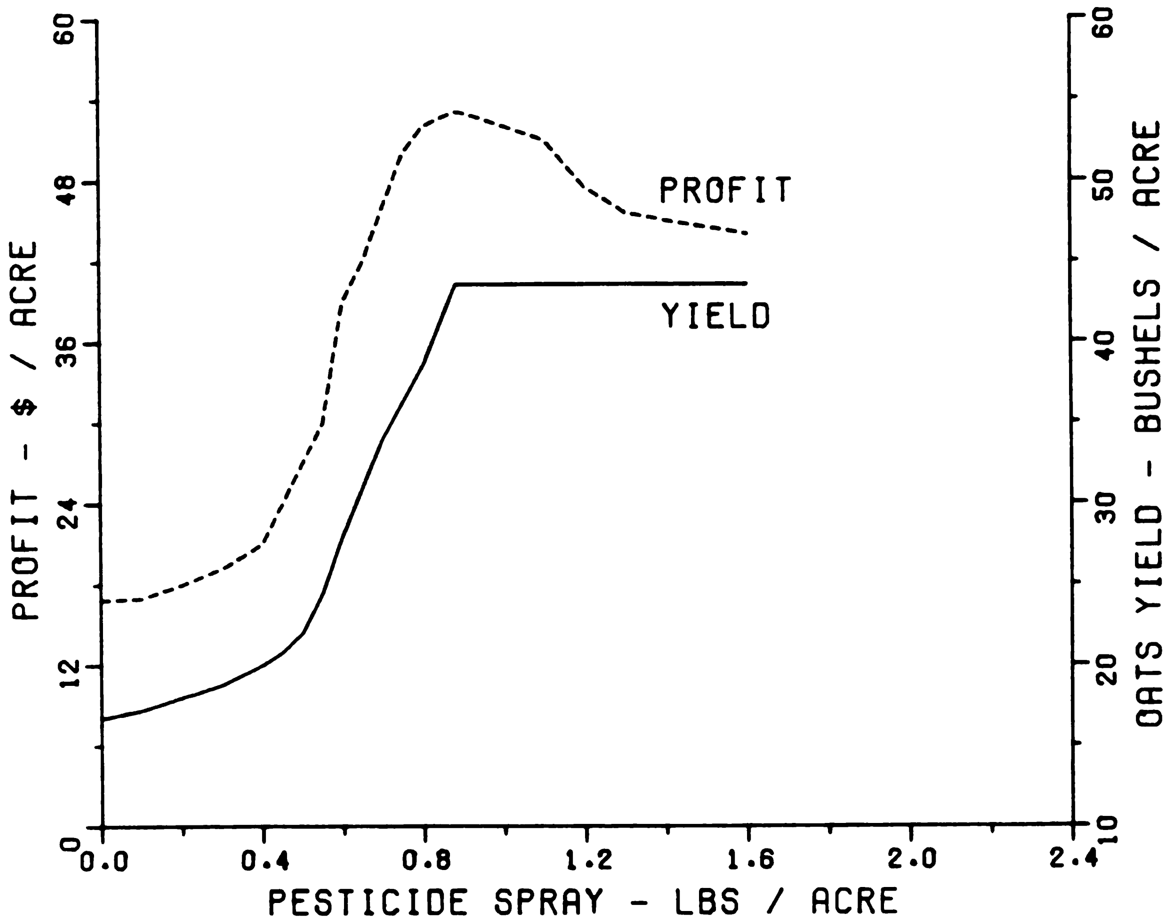


FIGURE 33. Oats yield and profit under different regulatory managements of the quantity of pesticide sprayed.

The taxation policy is more complicated, and we will consider only a very simple case in which there is a constant tax per lb of pesticide used. Equivalently, this can be viewed as an increase in the cost of pesticide and as a variable cost of production. In this case, the profit maximization problem (considered before) is transformed to:

Max [revenue - control cost - tax].

The optimization problem, with initial conditions set as above, is solved for different levels of tax per lb of pesticide used. The results are illustrated in Figures 34 through 36 in which the non-optimal case represents a user assumed to spray 1 lb/acre regardless of the tax levied. With the optimal scheme, 0.88 lb/acre continues to be the optimal amount of spray until the tax levied is as high as $40/lb of pesticide used, and the pesticide use does not decline gradually with the increase in taxes. This is because any considerable reduction (down from 0.88 lb/acre) in pesticide use and the associated saving in taxes are more than offset by a reduction in revenue. This situation is critically dependent on the nature of the dosage response characteristics and the relationship between yield and pesticide use (see discussions on sensitivity analysis with reference to timing and amount of spray). A tax over and above $41/lb leads to the optimal rule being to not spray at all (Figure 34). This may also imply that the spraying strategies decided by the farmer are essentially binary strategies--to spray some fixed amount of pesticide or not spray at all.

The optimal user always receives larger yield (Figure 35) and higher profits (Figure 36) as compared to the non-optimal user. More importantly, in comparison with the optimal user, the profits of the non-optimal

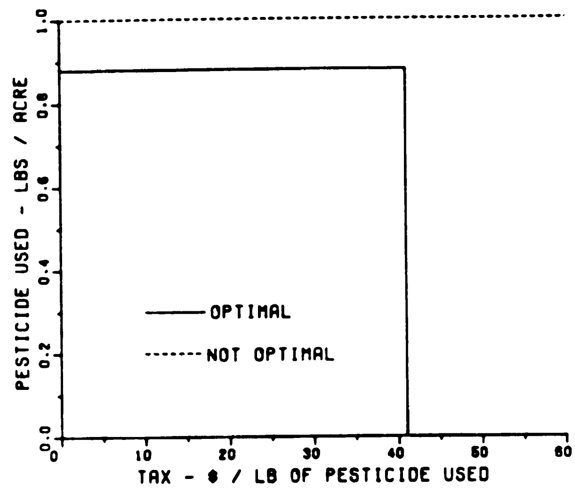


FIGURE 34. Quantity of pesticide sprayed by optimal and non-optimal users as a function of tax imposed.

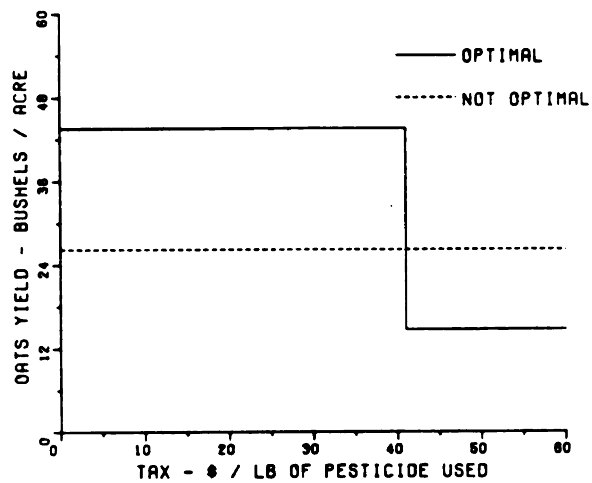


FIGURE 35. Oats yield obtained by optimal and non-optimal users as a function of tax imposed.

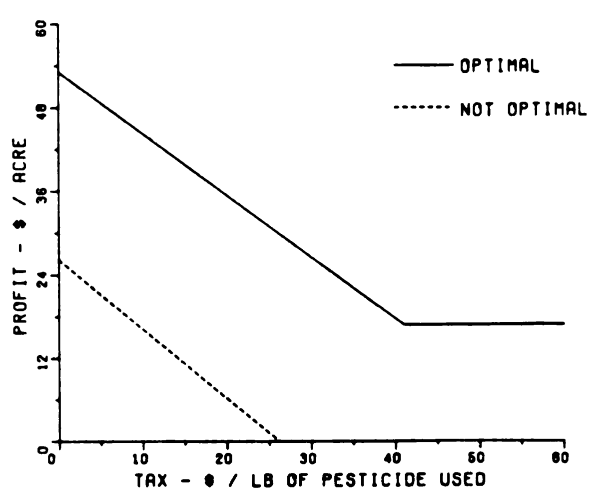


FIGURE 36. Profit obtained by optimal and non-optimal users as a function of tax imposed.

user decline at a faster rate as the taxes are increased. This is because in the non-optimal case the farmer uses more pesticide in comparison to the optimal user. It is obvious that a large tax is required to reduce pesticide use, but in the problem under consideration, we have only two levels of pesticide use for the optimal case--0.88 lb/acre and no spray. This can be changed by using taxation schemes where the rate of taxation increases drastically with the increased use of pesticides. However, it will be extremely difficult to justify any one particular basis used in structuring the taxes.

EFFECTS OF CHANGE IN CROP PRICE ON PESTICIDE USE

As a matter of general interest, the effect of varying the price of the crop on the use of pesticide is also viewed within the framework of the profit maximization problem (results are illustrated in Figure 37). As expected, the increased crop price provides an incentive to the farmer to increase pesticide usage even when it is not warranted. Although the effective profit increases due to the increased crop price (and the increased pesticide use) it can be seen that the oats yield eventually stabilizes.

We would like to emphasize once again that there is no easy solution to the externality problem--all of the approaches, direct regulation, taxes and subsidies, pricing, etc., are complex and are difficult to implement. More research is definitely needed in this area.

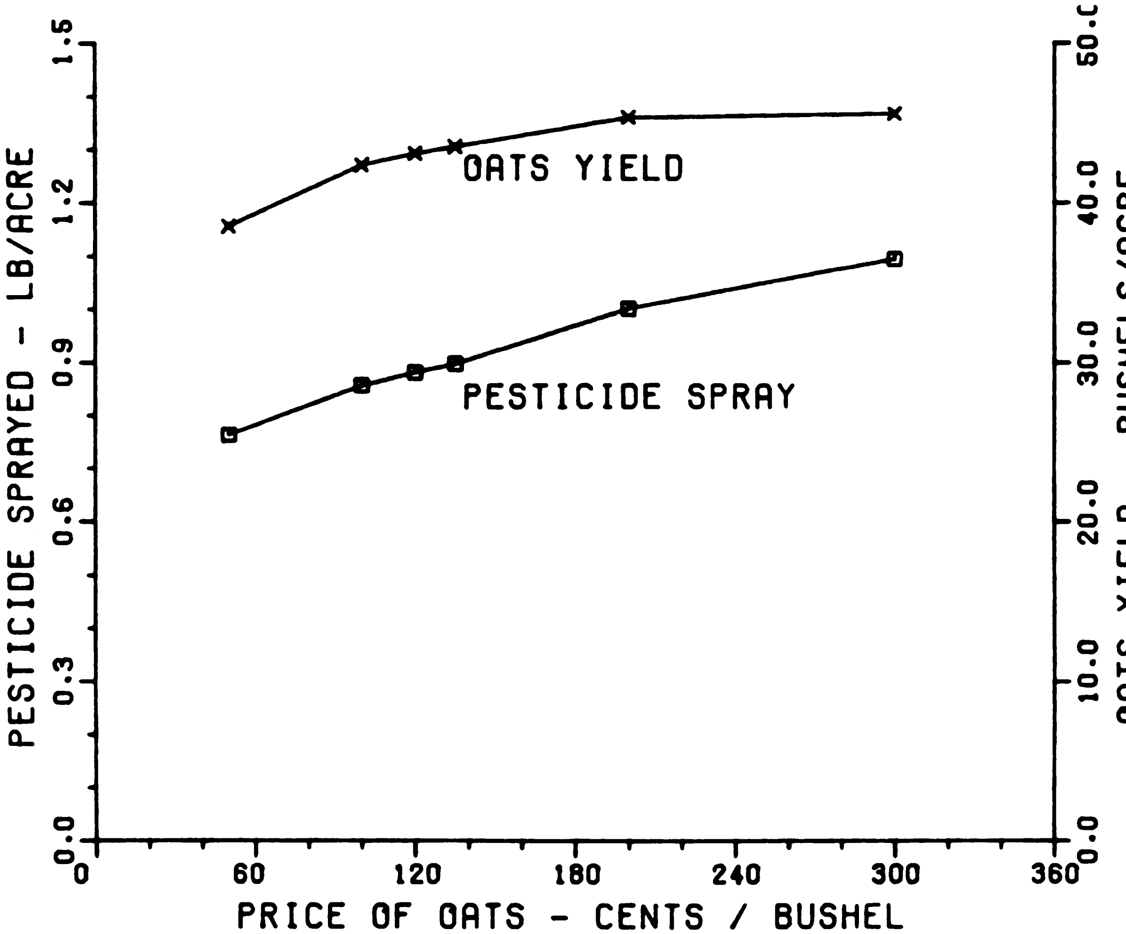


FIGURE 37. Sensitivity of oats yield and amount of pesticide sprayed with reference to changes in the price of oats.

SUMMARY AND CONCLUSIONS

In this research work, we have developed a comprehensive model of the CLB ecosystem with all its major components--the CLB, its larval parasite *T. julis*, and the oats plant. Both chemical control and biological control aspects are incorporated into the model so that it can be tested within the framework of integrated pest management.

A first order, successive approximation algorithm is utilized to develop optimal control strategies for the integrated control of the CLB ecosystem. The optimal control strategies are characterized by emphasis on biological control and reduction in the use of chemical control. The optimal strategies are compared with the conventional spraying schemes currently utilized. Such analysis are carried out for both single season and multiple season pest management problems. The optimal policy leads to higher yields and greater profits, reinforces the increase of *T. julis* populations, and is more effective in suppressing the CLB damage; more significantly, all of the aforementioned gains are obtained with a reduced pesticide use as compared to the conventional spraying practice.

Optimal strategies may not always lead to spectacular gains over conventional policies; however, optimal strategies will *always* be at least as good as, and often times better than, the conventional schemes. This is because the optimization algorithm routinely searches numerous policy options with reference to the timing and amount of pesticide spray, and chooses the one that maximizes yield and profit with minimal pesticide use. In the event the conventional spraying schemes happen to be optimal, the optimization scheme will automatically choose it.

A sensitivity analysis is carried out with reference to timing and amount of pesticide spray, and it is found that the optimal timing is at odds with the timing of spray under the conventional schemes: the conventional strategy is timed earlier in the growing season and is aimed at the CLB spring adults and the CLB eggs. On the other hand, the optimal spray is timed later in the season and is targeted for the early larval instars of the CLB. Regarding the amount of spray, the optimal scheme results in an average reduction of about 10% in the use of pesticides as compared to the conventional strategy. This may not seem impressive at first glance, but considering the fact that the acreage of oats harvested in the United States is close to 13.5 million acres (Michigan Agricultural Statistics, June 1978) and assuming 40% of these are infested by CLB, a 10% reduction will result in a reduction of 500,000 lbs of pesticide use annually--for oats alone. This will be an enormous reduction in terms of pesticide use and associated environmental pollution.

Unlike the CLB infestation which usually requires just a single spray, optimal schemes will lead to greater savings in pesticide use when used with pest management problems in which frequent sprayings are common--like the onion maggot problem currently under investigation at Michigan State University. Furthermore, the optimal scheme achieves such a reduction in pesticide usage with minimal reduction in crop yield and profits.

A simple analysis is carried out, based on direct regulation and taxation approaches to regulating the environmental pollution caused by the pesticide sprays. It is found that, for the problem under consideration, the pesticide use is not very sensitive to taxes--heavy taxation is required to reduce pesticide use. The direct regulatory approach, in which absolute limits are prescribed as the level of pesticide use, is also discussed. In general, it is extremely difficult to formulate an equitable policy based on either direct regulation or taxation.

The analyses carried out in this research point to several areas where improvements can be made by conducting more field (or laboratory) experiments. Among them, the dosage response characteristics are the most important. The amount of spray used is directly dependent on the dosage response characteristics. Currently available data on dosage response are incomplete, especially with reference to low level dosages. Further, no experimental data is available on the effect of pesticides on _T. julis_. Field and laboratory experiments are needed to acquire the necessary data in these areas. Further, field experiments have to be carried out to evaluate conventional and optimal spraying strategies with reference to their effect on crop yield, profit, CLB feeding, parasite densities, etc.

In the present research work we have designed an optimal controller for pest management problems. However, it is confined to a deterministic framework, but the real world pest management problem is actually stochastic--due to variations in climatic factors, sampling errors, etc.

Thus, the deterministic optimal controller will not be optimal in real world stochastic situations. Further, we would like to have on-line features incorporated into the optimal controller so that the pest management strategies can be implemented on-line. We propose the Linear-Quadratic-Gaussian (L-Q-G) methodology (Athans 1971, 1974) for the design of such an on-line controller. A brief description of the L-Q-G methodology is presented in Appendix B. Essentially the L-Q-G design consists of two components, (1) a deterministic optimal controller, and (2) a stochastic estimator. The complete design of the deterministic optimal controller has been accomplished in this research. The design of the stochastic estimator must await another work.

The essential features of the on-line controller for pest management are illustrated in the block diagram of Figure 38. The deterministic optimal controller utilizes the ecosystem model to compute (off-line) the determinisitc optimal control strategy. On the other hand, the stochastic estimator (filter) combines model estimates and actual field data (containing errors) of biological and climatological variables of the ecosystem, to give a new improved estimate of the states of the ecosystem. The deviations between the actual states of the ecosystem from its ideal, deterministic response generated by the model are used to generate the on-line correction strategy. One of the most significant features of the L-Q-G design is that it fits the general guidelines established for the on-line pest management systems (Haynes and Tummala 1977). Furthermore, the methodology developed in this research can be easily extended to the L-Q-G design.

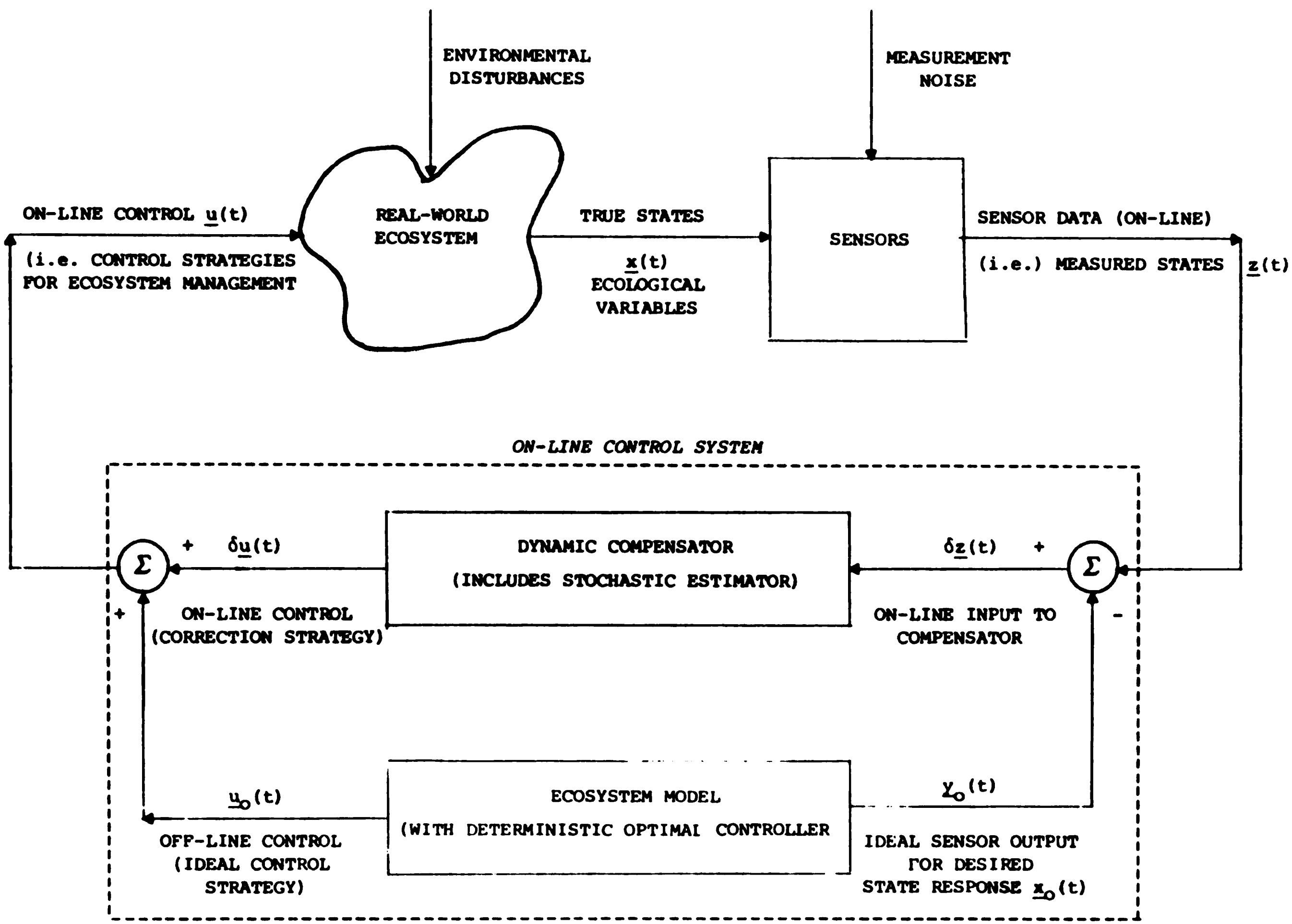


FIGURE 38. Block diagram illustrating L-Q-G design for on-line control of a pest ecosystem.

Finally, we wish to emphasize that optimization is <u>not</u> a substitute for the decision-making process, but rather a powerful tool in aiding it. Optimization has been proven successful in a wide range of engineering and physical problems. The same ideas can be carried over to the management of biological problems--the major difference being that biological systems are much more complicated, and mathematical descriptions for their behavior do not exist but for a few cases. Nevertheless, even some of the simplified models along with optimization schemes can provide valuable insights to the management of biological systems. It will be a while before optimal control strategies are "directly" put into use in pest management. With the advent of more quantitative approaches to biological problems, and the interdisciplinary systems approaches becoming increasingly popular, such a situation is likely in a conceivable future.

APPENDIX A

COMPUTER PROGRAM FOR THE OPTIMIZATION ALGORITHM

COMPUTER PROGRAM FOR THE OPTIMIZATION ALGORITHM

PROGRAM STRUCTURE

The structure of the computer program used in implementing the optimization algorithm (refer to earlier discussions) is illustrated in Figure 39. It consists of a main program and subroutines.

MAIN program coordinates all the subroutines. Subroutine INPUT reads in the initial conditions for the state variables and all the other parameters required for the optimization algorithm.

Subroutine MODEL generates the state variable trajectories using the state-space model of the system. It also computes the performance index.

Subroutine FX computes the matrix of partial derivatives, F_x (i.e., partial derivatives, $\frac{\partial F}{\partial X}$) with reference to state variables X.

Subroutine FU computes the matrix of partial derivatives F_u (i.e., partial derivatives, $\frac{\partial F}{\partial U}$) with reference to control variables U.

Subroutine OUTPUT prints optimal controls and optimal state trajectories for every time step. It also prints the performance index and other variables (if any) for every iteration. The subroutine OUTPUT also writes all these outputs in catalogued tapes that can be accessed for graphing at a later point in time.

CONVERGENCE PROPERTIES

Convergence of the optimization algorithm is illustrated in Figure 40 in which the two cases correspond to two different nominal controls.

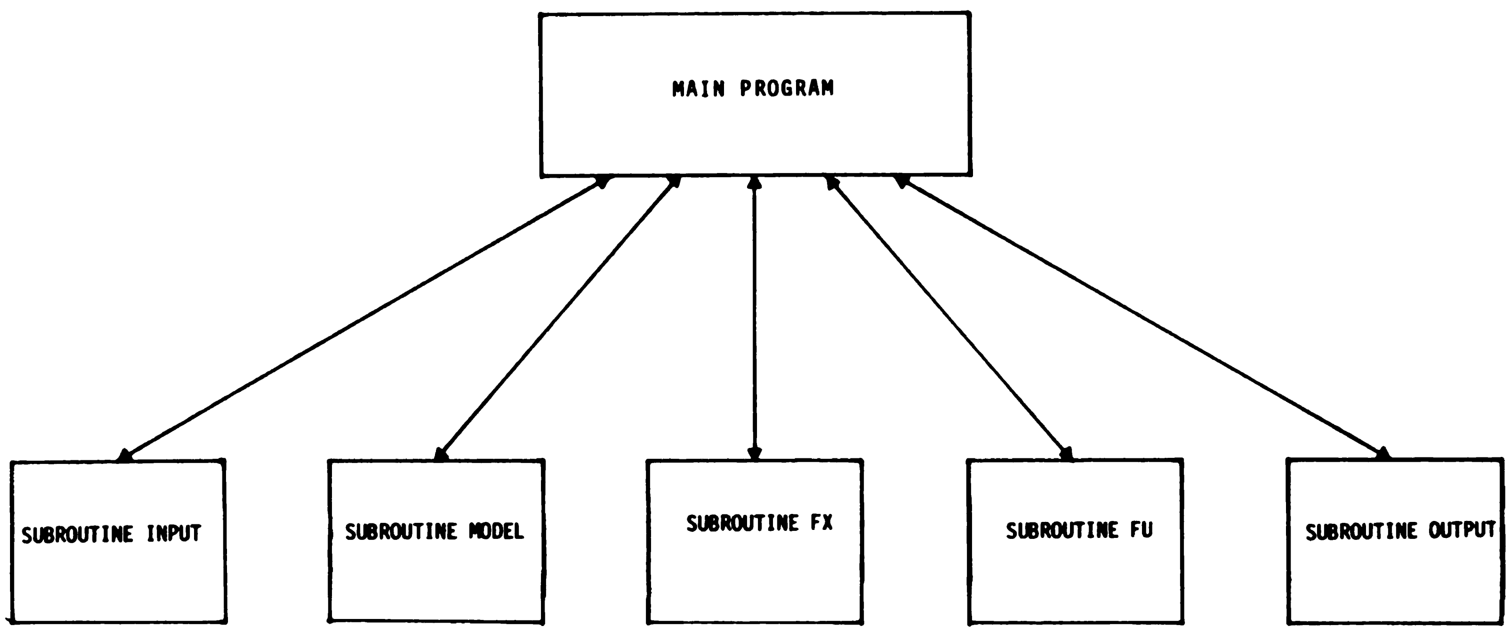


FIGURE 39. Block diagram illustrating the structure of the computer program.

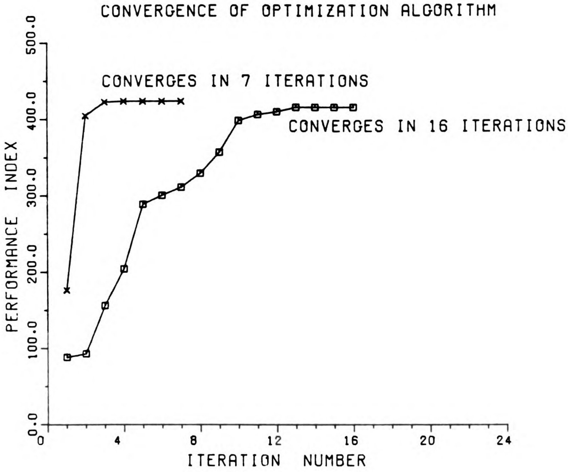


FIGURE 40. Convergence of the optimization algorithm.

It can be observed that in both cases the optimum value of the performance index is almost identical (within 0.05% error), thereby implying good convergence. On the average, the algorithm takes about 15 iterations to converge. Of course, convergence of any optimization algorithm depends on a variety of factors that include functional forms of the model, performance index, constraints on the problem, characteristic features of the algorithm (i.e., first order or second order algorithm) programming efficiency, etc. As such it will be rather difficult to draw any general conclusions about the convergence of the algorithm.

Using the CDC 6500 computer system at the Michigan State University the optimization algorithm takes about 0.8 cp sec (central processor seconds) per iteration. Total memory requirements are about 20,000 octal units, which is quite modest, especially so in view of the dimensions of the problems handled--over 30 state variables.

A flow chart for the computational algorithm is illustrated in Figure 41, and a complete listing of the FORTRAN program is also attached.

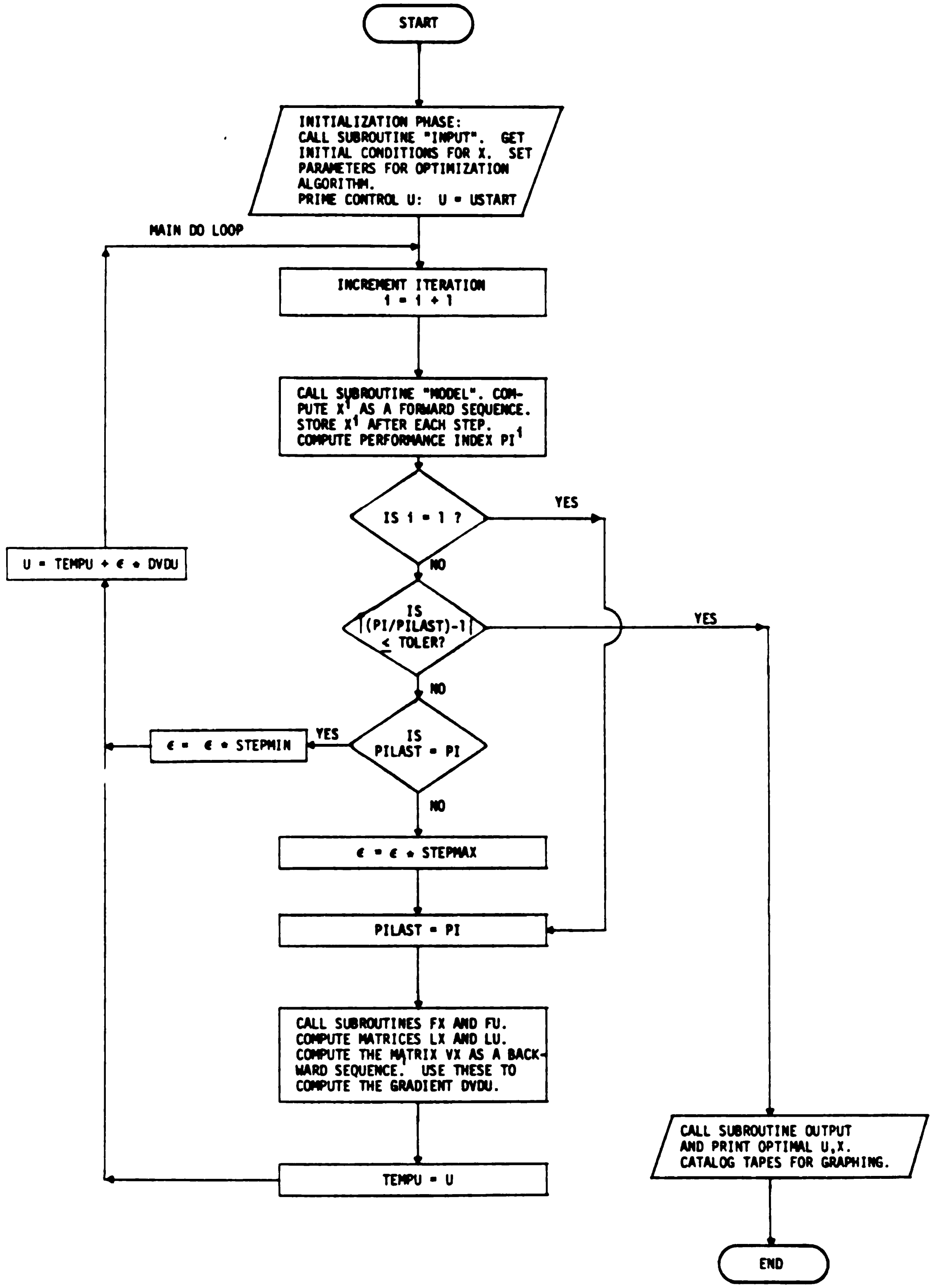


FIGURE 41. Computer flow chart for the optimization algorithm.

FORTRAN LISTING

```
      PROGRAM MATOPT (INPUT=65, OUTPUT=129, TAPE3=65, TAPE4=65,
     +        TAPE5=65, TAPE6=65, TAPE7=65, TAPE77=65, TAPE78=65,
     +        ZZZZOT=65, TAPE9=ZZZZOT, TAPE61=OUTPUT)
      INTEGER TLAST,YR,YEAR,TD,TSPRAY
      REAL LX,LU,JULISIN,INTERM,MX,MU,INCOME
      REAL K1,K3,K4,K6,K7,K8,MORTFN,MORTFN2
      DIMENSION VX(35,35),LX(35,35),PRVXFX(35,35),DVDU(35,2)
     +,PRVXFU(35,2),TERM1(35,2),TERM2(35,2),LU(35,2),TEMPU(35,2)
      DIMENSION MX(35,35),WX(35,35),MU(40,2),PRWXFU(40,2),
     + PRWXFX(35,35),DWDU(35,2)
      COMMON X(35,35),U(35,2)
      COMMON /PLANT/ P(4,4)
      COMMON /ENOUGH/ C1,C2,C3,C4,D1,D2,D3,D4,D5
      COMMON /ENOUGH/ DF1,DF2,DF3,DF4,DF5,DF6,DF7,DF8,DF9
      COMMON /ENOUGH/ A,B,C,D,F,K1,K3,K4,K6,K7,K8,FEEDQ
      COMMON /ENOUGH/RP,CP,E1,E2,E3,ECP,ONECP,TD,IDIA,IFEED,NTOATS
      COMMON /BLOCK/ MORTFN(30),MORTFN2(30),SURVFN(30),SURVFN2(30)
      COMMON /BLOCK/ SL1(30),SL4(30),F1(30),DIAFFN(30),DIFMFN(30)
      COMMON /BLOCK/ DIFMFN2(30),F1DR(30),F2(30),DSL1(30),DSL4(30)
      COMMON /FX/ DRSQ,DRSQ2,F1DRCP,F1DRCP1,X18CP,X18CP1
      COMMON /INPUT/ IFLAG, NITER, EPSILON, STEPMIN, STEPMAX, Q, R
      COMMON /INPUT/ USTART, TVALUE, TOLER, ET, TLAST, CLBIN
      COMMON /INPUT/ JULISIN, FINALTJ, NYEAR, TSPRAY, SPRAY
      COMMON /INPUT/ PRICE, TAX
      COMMON /MODEL/ XREF(35),DIFU,SUMU,SUMUSQ,DIFUSQ,FI
      COMMON /MODEL/ TEGG, TJ2, TLAR3, TUNPAR4, TPAR4
      COMMON /TIME/ YR, NN, NSTEP, NX, NU
      DATA ( P(1,I), I=1,4 ) / 8.035127 E-01 , 5.170637 E-01,
     +                         -5.640611 E-02 , 0.0             /
      DATA ( P(2,I), I=1,4 ) /-1.127719 E-01 , 1.240871 E+00,
     +                         -1.876073 E-02 , 6.548521 E-02   /
      DATA ( P(3,I), I=1,4 ) / 0.0            , 3.638312 E-02,
     +                         7.656064 E-01 , 6.145583 E-01    /
      DATA ( P(4,I), I=1,4 ) / 0.0            , 4.779900 E-02,
     +                         -1.726377 E-01 , 1.303389 E+00 /
      DATA K1,K3,K4,K6,K7,K8 / 0.1, 0.3, 0.45, 0.4, 0.5, 0.5 /
      DATA A,B,C,D,F / 0.7, 22.0, 1.0, 0.6, 1.0 /
      DATA FEEDQ,RP,CP / 0.002029, 0.0, 0.75 /
      DATA TD,IDIA,IFEED,NTOATS / 13, 1, 1, 13 /
      DATA C1,C2,C3,C4 /9.684848E-01, 1.829393E-03, -3.108558E-05,
     +     6.604507E-08 /
      DATA D1,D2,D3,D4,D5 / 9.402374E-01, 1.683252E-02,
     +     2.763500E-04, -2.314922E-06, 7.326632E-09 /
      DATA DF1,DF2,DF3,DF4,DF5,DF6,DF7,DF8,DF9 /
     +     4.97031E-01, -3.09390E+00, 3.95259E+00,
     +     -1.15387E+00, 1.66972E-01, -1.23781E-02,
     +     4.86900E-04, -9.71720E-06, 7.76777E-08 /
      DATA E1,E2,E3 / 20.0, 5.0, 100.0 /
      DATA DELTAU / 1.0 /
```

```
C *******************************************
C *                                         *
C *        DICTIONARY OF STATE VARIABLES    *
C *                                         *
C *******************************************
C
C X1       SPRING ADULT CLB DENSITY
C X2
C X3       CLB EGG DENSITY
C X4
C X5       FIRST INSTAR CLB DENSITY
C X6       SECOND INSTAR CLB DENSITY
C X7       THIRD INSTAR CLB DENSITY
C X8       UNPARASITIZED FOURTH INSTAR CLB DENSITY
C X9
C X10
C X11
C X12      UNPARASITIZED CLB PUPA DENSITY
C X13
C X14
C X15
C X16      SUMMER ADULT CLB DENSITY
C X17      DIAPAUSING TJ DENSITY
C X18      ADULT TJ DENSITY
C X19      PARASITIZED FOURTH INSTAR CLB DENSITY
C X21
C X22      PARASITIZED CLB PUPA DENSITY
C X23
C X24
C X20
C X25
C X26      PARASITE PER PEST INDIVIDUAL IN DIFFERENT STAGES
C X27
C X28
C X29      CLB LARVAL FEEDING
C X30      WEIGHT OF OATS PLANT
C X31      LEAF SURFACE AREA OF OATS PLANT
C X32      WEIGHT OF GRAIN HEAD
C X33      SURFACE AREA OF GRAIN HEAD
C
```

```
C ***************************************************
C *                                                 *
C *               SYSTEM PARAMETERS                 *
C *                                                 *
C ***************************************************
C
C NAME          DESCRIPTION                           VALUE
C -----------------------------------------------------------
C
C A       SPRING ADULT SURVIVAL                        .70
C B       CLB EGGS / CLB FEMALE / 60 DD               22.00
C C       SUMMER ADULT SURVIVAL                       1.00
C D       T.JULIS ADULT SURVIVAL                      0.60
C E1      MAX EGGS / TJ ADULT / 60 DD                 2[illegible].00
C E2      MAX TJ EGGS / CLB LARVA / 60 DD             5.00
C E3      TJ SEARCHING CONSTANT                       1[illegible]0.00
C RF      TJ MORTALITY INSIDE CLB                     [illegible].00
C K1      MORTALITY OF CLB EGGS                       0.10
C K2      MORTALITY OF CLB L1                         VARIABLE
C K3      MORTALITY OF CLB L2                         0.30
C K4      MORTALITY OF CLB L3                         0.45
C K5      MORTALITY OF CLB L4                         VARIABLE
C K6      MORTALITY OF CLB PUPAE                      0.40
C K7      MORTALITY OF OVERWINTERING CLB              0.77
C K8      MORTALITY OF OVERWINTERING TJ               0.50
C CP      EXPONENT IN PARASITISM EQUATION             0.75
C FEEDQ   FEEDING FUNCTION COEFFICIENT                .002[illegible]29
C P       MATRIX DIMENSIONED (4,4) CONTAINING
C         PARAMETERS FOR THE OATS PLANT MODEL
C         OBTAINED THROUGH TIME-SERIES ANALYSIS
C
```

```
C                          F MATRIX VALUES:
C -------------------------------------------------------------------
C  8.035127E-01     5.170637E-01    -5.640611E-02     0.0
C -1.127719E-01     1.240871E+00    -1.806073E-02     6.548521E-02
C  0.0              3.638312E-02     7.656064E-01     6.145583E-01
C  0.0              4.779900E-02    -1.726377E-01     1.303389E+00
C -------------------------------------------------------------------
C
C
C REWIND ALL OUTPUT FILES
C
      REWIND 3  $  REWIND 4  $  REWIND 5  $  REWIND 6  $  REWIND 7
      REWIND 77 $ REWIND 78

C DEFINE ALL VARIABLES IN /INPUT/ COMMON BLOCK IN ORDER OF
C DECLARATION.  PROMPT USER FOR EACH, THEN READ VARIABLE FREE
C FORMAT, ONE NUMBER PER LINE.  SEE SUBROUTINE INPUT.
      CALL INPUT

      ECF = E2**CF
      ONECP = 1.0 - CP
C NUMBER OF TIME STEPS FROM START TO HARVEST.
      NSTEP=27
      NN=NSTEP+1
      N=NSTEP
C NUMBER OF STATE VARIABLES.
      NX=33
      NU=1
C **** DO LOOP FOR MULTIPLE YEAR RUNS ****
      DO 1 YEAR = 1, NYEAR
      YR = YEAR
C CLEAR ARRAYS
      DO 5 I=1,NN
      DO 5 J=1,NX
      LX(I,J)=0.0
      VX(I,J)=0.0
      MX(I,J)=0.0
      WX(I,J)=0.0
      X (I,J)=0.0
5     CONTINUE
      DO 6 I=1,NN
      XREF(I) = 1000.0
      IF (I .LE. TLAST) XREF(I) = ET
6     CONTINUE
      DO 7 I=1,NSTEP
      DO 7 J=1,NU
      MU(I,J)=0.0
      LU(I,J)=0.0
      DWDU(I,J)=0.0
      DVDU(I,J)=0.0
      U(I,J)=USTART
7     CONTINUE
C **** USTART IS ADJUSTED TO BE SAME AS CONVENTIONAL SPRAY ****
```

```
C **** U(3,1) = 1.0
      U(TSPRAY,1) = SPRAY
C DEFINE NON-ZERO INITIAL CONDITIONS (IF ANY ) FOR THE
C STATE VARIABLES.
      X(6,18)=JULISIN
      X(1,1 )=CLBIN
      X(6,30) = 3.192963 E-01
      X(6,31) = 4.103903 E+00
C INITIALIZATION FOR SUMMATION VARIAPLES
      TEGG = 1.0E-7
      TLAR3  = 0.0
      TFAR4  = 0.0
      TUNFAR4= 0.0
      TJ2    = 0.0
      SUMU  = 0.0
      SUMUSQ   = 0.0
C DEFINE TERMINAL VALUES FOR X (IF ANY), VX, WX.
      X(NN,15) = FINALTJ
C *****************************************************************
C ********** MAXIMIZATION PROBLEM ****************************
      VX(NN,32) = 1.0
C **** DO LOOP FOR OPTIMIZATION STARTS HERE ****
      DO 10 ITER=1,NITER
C WRITE THE TERMINAL CONSTRAINT EQUATION

C **** CALL SUBROUTINE MODEL TO COMPUTE STATE VARIABLES ****
      CALL MODEL

C ADD ANY TERMINAL (NON-INTEGRAL) TERM TO PI.
C CHECK FOR VERY FIRST ITERATION
      IF (ITER .EQ. 1) GO TO 62
      IF ( ABS(PI/PILAST - 1.0) .LT. TOLER ) GO TO 5
      IF (PI .LE. PILAST) GO TO 57
C PI IS LESS THAN PILAST
      EPSILON = EPSILON*STEPMAX
      GO TO 62
C PI IS GREATER THAN PILAST
57    EPSILON = EPSILON*STEPMIN
      GO TO 177
62    PILAST = PI
C COMPUTE  PARTIAL DERIVATIVES FOR THE RETURN FUNCTION BY
C BACKWARD INTEGRATION.
C FIRST EVALUATE FX,LX,MX MATRICES.
C NOTE: FX AND FU ARE EVALUATED AS FUNCTION SUPPROGRAMS.
      DO 16 K=1,NSTEP
      LX (K,32) = 2.0 * X(K,32) * ( Q * PRICE / 1.35 )
      IF ( IFLAG .NE. 1 ) GO TO 16
      LU(K,1) = - 2.0 *  (R+TAX) * U(K,1)
16    CONTINUE
      DO 17 K=1,NSTEP
      MU(K,1) =   2.0 *  (R+TAX) * U(K,1)
17    CONTINUE
C DEFINE TERMINAL VALUES  FOR X (IF ANY), VX,WX
```

```
C NEXT EVALUATE WX,VX MATRICES.
C CLEAR PRWXFX,PRVXFX ARRAYS.
      DO 18 I=1,NSTEP
      DO 18 J=1,NX
      PRVXFX(I,J)=0.0
      PRWXFX(I,J)=0.0
18    CONTINUE
      DO 20 K=1,NSTEP
      N1K= N+1-K
      N2K= N+2-K
      DRSQ = F1DR(N1K)**(2*CP)
      DRSQ2 = F1DR(N1K)**(2.0*ONECP)
      F1DRCP = F1DR(N1K)** CP
      F1DRCP1 = 1.0/(F1DR(N1K) ** (1.0-CP) )
      X18CP = X(N1K,18) ** CP
      X18CP1 = 1.0 / (X(N1K,18) ** (1.0-CP) )
      IF ( X(N1K,18) .EQ. 0.0 ) X18CP = 1E-5
      IF ( X(N1K,18) .EQ. 0.0 ) X18CP1= 1E+4
      DO 21 I=1,NX
      DO 22 J=1,NX
      FXX=FX(J,I,N1K)
      PRVXFX(N1K,I)=PRVXFX(N1K,I)+VX(N2K,J)*FXX
22    PRWXFX(N1K,I)=PRWXFX(N1K,I)+WX(N2K,J)*FXX
      VX(N1K,I)=PRVXFX(N1K,I)+LX(N1K,I)
21    WX(N1K,I)=PRWXFX(N1K,I)+MX(N1K,I)
20    CONTINUE
C TO FIND THE MULTIPLIER  V  FOR CONSTRAINED PROBLEMS.
C FIRST COMPUTE THE SUM TERMS.
      DO 28 I=1,NSTEP
      DO 28 J=1,NU
      PRVXFU(I,J)=0.0
      PRWXFU(I,J) =0.0
      TERM1(I,J) = 0.0
      TERM2(I,J)=0.0
28    CONTINUE
      SUM1 = 0.0
      SUM2 = 0.0
      DO 30 K=1,NSTEP
      KP1=K+1
      DO 30 I=1,NU
      DO 32 J=1,NX
      FUU= FU(J,I,K)
      PRVXFU(K,I)= PRVXFU(K,I)+ VX(KP1,J) *FUU
32    PRWXFU(K,I)= PRWXFU(K,I)+ WX(KP1,J) *FUU
      TERM1(K,I)=PRWXFU(K,I)+MU(K,I)
      TERM2(K,I)=PRVXFU(K,I)+LU(K,I)
      SUM1 = SUM1 + (TERM1(K,I) ** 2 )
      SUM2 = SUM2 + (TERM1(K,I) + TERM2(K,I) )
30    CONTINUE
      DO 41 I=1,NSTEP
      DO 41 J=1,NU
      TEMPU(I,J)= U(I,J)
```

```
41    CONTINUE
      IF (IFLAG .EQ. 1) GO TO 177
      INTERM = 1.0 / (EPSILON * SUM1)
      V = INTERM * (DIFUSQ - EPSILON * SUM2 )
C COMPUTE GRADIENT WRT CONTROL
C NOTE: DVDU(I,J)=TERM2(I,J); DWDU(I,J)=TERM1(I,J)
C UPDATE CONTROL
177   DELTAU = 0.0
      DO 40 I=1,NSTEP
      DO 40 J=1,NU
      U(I,J) = TEMPU(I,J) + EPSILON*TERM2(I,J)
      IF (IFLAG .EQ. 0) U(I,J) = U(I,J) + EPSILON*V*TERM1(I,J)
      U(I,J) = ABS(U(I,J))
      DELTAU = DELTAU + ( U(I,J) - TEMPU(I,J) )
40    CONTINUE
      PRINT 47,ITER,PI,PILAST,EPSILON,SUMU,SUMUSQ,X(NN,18)
47    FORMAT ("0ITER=",I2,5X,"PI=",1PE10.3,5X,"PILAST=",1PE10.3,
     +5X,"EPSILON",1PE10.3,5X,"SUMU",1PE10.3,5X,"SUMUSQ",1PE10.3,
     +2X,"X(NN,18)=",2X,1PE10.3)
10    CONTINUE

C COMPUTE INCOME FROM OATS YIELD
C REF EMMETT *** SEED WT = HEAD DRY WT * 0.9119 - 0.003035
50    SEEDWT = X(NN,32) * 0.9119 - 0.003035
C YIELD IN BUSHELS/ACRE = SEED WT IN GMS/SQFT * 3.000992
      YIELD = SEEDWT * 3.000992
C INCOME $/ACRE : YIELD IN BU/ACRE * PRICE OF OATS IN $
C CURRENT BUYING PRICE OF OATS IN MICHIGAN AS BOUGHT FROM FARMERS
C $1.35 / BUSHEL  REF: MASON ELEVATOR COMPANY
      INCOME = YIELD * PRICE
C COMPUTE PESTICIDE COSTS, INCLUDING MATERIAL AND APPLICATION COST
C COST OF MALATHION : $ 3.00 / LB
C COST OF APPLICATION : $ 3.00 / ACRE
C TAXES CAN ALSO BE IMPOSED ON PESTICIDE USE
      COST = SUMU * ( PRICE + TAX ) + 3.00
C NET PROFIT/ACRE : INCOME - COST
      PROFIT = INCOME - COST
C COMPUTE PERCENT PARASITISM
      PARA =  ( TPAR4 / ( TPAR4 + TUNPAR4 ) ) * 100.0
C INITIALIZE CLB AND T JULIS DENSITIES FOR NEXT YEAR
C NOTE : OVERWINTERING MORTALITY FOR CLB IS SET AT 77 BASED ON
C         REFERENCES ( YUN, 1964; WELLSO ET AL., 1970 )
C         OVERWINTERING MORTALITY OF T.JULIS IS SET AT 50
      CLBIN  =  0.23  * X(NN,16)
      JULISIN=  0.50  * X(NN,17)
C WRITE YEARLY OUTPUTS IN A SEPERATE TAPE # 77
      WRITE ( 77,444) YEAR, X(1,1), TEGG, X(6,18), TJ2,TLAR3, PARA
444   FORMAT ("-",T11,I2,T19,1PE10.3,T34,1PE10.3,T49,1PE10.3,
     +                 T64,1PE10.3,T79,1PE10.3,T94,1PE10.3 )
C WRITE INCOME-RELATED OUTPUTS ON A SEPARATE TAPE # 78
      WRITE ( 78,499) YEAR, YIELD, INCOME, SUMU, COST, PROFIT
499   FORMAT ("-",T11,I2,T24,1PE10.3,T44,1PE10.3,T64,1PE10.3,
     +               T87,1PE10.3,T109,1PE10.3  )
C
```

```
C ******* FRINT TABLES *******
C
C SUBROUTINE OUTFUT WRITES UNHEADERED TABLES ON OUTFUT TAPES AND
C ALSO FRINTS HEADERED TAPLES ON OUTPUT.
C
C TABLE 1 IS THE CONTROL VARIABLES "U", PRINTED YEARLY.
C THE UNHEADERED CONTROL VAFIABLES ARE WRITTEN TO TAFE3.
C TABLE 2 IS FOUR TABLES OF STATE VARIABLES, FRINTED YEARLY:
C         1: X1-X10     2: X11-X20     3: X21-X30     4: X31-X34
C ON TAPE    TAPE4         TAFE5          TAPE6          TAPE7
      CALL OUTFUT (1)
      CALL OUTFUT (2)
1     CONTINUE
C
C TABLE 3 IS A DIRECT COFY OF TAFE77, WHICH IS WRITTEN ABOVE.
C TABLE 4 IS A DIRECT COFY OF TAFE78, WHICH IS WRITTEN ABOVE.
C BOTH OF THESE ARE WRITTEN AT THE END OF RUN ONLY.
C NOTE THAT TAPE77 AND TAFE78 ARE THE UNHEADERED VERSIONS OF
C THE FRINTED TABLES 3 AND 4 RESFECTIVELY.
      CALL OUTPUT (3)
      CALL OUTPUT (4)
      END
```

```
      FUNCTION FX(J,I,K)
C
C ***********************************************
C *                                             *
C *               FUNCTION FX                   *
C *                                             *
C ***********************************************
C
C FUNCTION FX COMPUTES THE MATRIX OF PARTIAL DERIVATIVES
C (DF/DX) WITH REFERENCE TO STATE VARIABLES X.
C
      REAL K1,K3,K4,K6,K7,K8,MORTFN,MORTFN2
      COMMON X(35,35),U(35,2)
      COMMON /PLANT/ P(4,4)
      COMMON /ENOUGH/ C1,C2,C3,C4,D1,D2,D3,D4,D5
      COMMON /ENOUGH/ DF1,DF2,DF3,DF4,DF5,DF6,DF7,DF8,DF9
      COMMON /ENOUGH/ A,B,C,D,F,K1,K3,K4,K6,K7,K8,FEEDG
      COMMON /ENOUGH/RF,CF,E1,E2,E3,ECF,ONECF,TD,IDIA,IFEED,NTOATS
      COMMON /BLOCK/ MORTFN(30),MORTFN2(30),SURVFN(30),SURVFN2(30)
      COMMON /BLOCK/ SL1(30),SL4(30),F1(30),DIAFFN(30),DIFMFN(30)
      COMMON /BLOCK/ DIFMFN2(30),F1DR(30),F2(30),DSL1(30),DSL4(30)
      COMMON /FX/ DRSQ,DRSQ2,F1DRCP,F1DRCP1,X18CP,X18CP1
      FX=0.0
C FUNCTIONS ARE DEFINED BELOW FOR NONZERO FUNCTIONS ONLY.
      GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,
     +       21,22,23,24,25,26,27,28,29,30,31,32,33,34) J
1     IF (I.EQ.1) FX= A*SURVFN2(K)                           $RETURN
2     IF (I.EQ.1) FX= B                                      $RETURN
3     IF (I.EQ.2) FX=1.0                                     $RETURN
4     IF (I.EQ.3) FX=1.0                                     $RETURN
5     IF (I.EQ.4) FX=(1.0-K1)*SURVFN(K)                      $RETURN
6     IF (I.EQ.5) FX= DSL1(K)*SURVFN(K)                      $RETURN
7     IF (I.EQ.6) FX= (1.0-K3)*SURVFN(K)                     $RETURN
8     IF (I.EQ.7) FX= (1.0-K4)*SURVFN(K)*(1.0 - (((1.0/ECP)
     +  *F1DRCP*X18CP - X18CP*X(K,7)*CP*F1DRCP*(1.0/E1)) / DRSQ))
      IF ( I.EQ.18 ) FX = - (1.0-K4) * SURVFN(K) * (1.0/ECP) *
     +   (( F1DRCP * CP * X18CP1 * X(K,7) - X18CP * X(K,7) * CP *
     +  F1DRCP1 * (1.0/E2) ) / DRSQ )                        $RETURN
9     IF (I.EQ.8) FX=DSL4(K)                                 $RETURN
10    IF (I.EQ.9) FX=1.0                                     $RETURN
11    IF (I.EQ.10) FX=1.0                                    $RETURN
12    IF (I.EQ.11) FX=1.0                                    $RETURN
13    IF (I.EQ.12) FX=1.0                                    $RETURN
14    IF (I.EQ.13) FX=1.                                     $RETURN
15    IF (I.EQ.14) FX=1.0
16    IF (I.EQ.15) FX=(1.0-K6)*SURVFN2(K)
      IF (I.EQ.16) FX=C*SURVFN2(K)                           $RETURN
17    IF (I.EQ.17) FX= 1.
      IF (I.EQ.24) FX=DIAFFN(K)*X(K,28)
      IF (I.EQ.28) FX=DIAFFN(K)*X(K,24)                      $RETURN
```

```
18     IF (K.LT.6) RETURN
       IF (I.EQ.12) FX=D*SURVFN(K)
       IF (I.EQ.24) FX=(1.0-DIAPFN(K))*X(K,28)*SURVFN(K)
       IF (I.EQ.28) FX=(1.0-DIAPFN(K))*X(K,24)*SURVFN(K)     $RETURN
19     IF ( I.EQ.7 ) FX = (1.0-K4)*SURVFN(K)*( ( (1.0/ECP)*
      +     F1DRCF*X18CF - X18CP*X(K,7)*CP*F1DRCF1*(1.0/E1))
      +     /  DRSQ )
       IF (I.EQ.18 ) FX = (1.0-K4) * SURVFN(K) * (1.0/ECP) *
      +     ( ( F1DRCP*CF*X18CF1*X(K,7) - X18CP*X(K,7)*CF*
      +     F1DRCF1*(1.0/E2) ) / DRSQ )                     $RETURN
20     IF ( X(K,7).EQ.0.0 .OR. X(K,18).EQ.0.0 ) RETURN
       IF ( I.EQ.7 ) FX = - ECP * ( ( (1.0/X18CF1) * ONECF *
      +      (1.0/F1DRCF) * (1.0/E1) ) / DRSQ2 )
       IF (I.EQ.18) FX = ECP*(((1.0/F1DRCP1)*ONECP*(1.0/X18CF)
      +   - (1.0/X18CF1)*ONECP*(1.0/F1DRCP)*(1.0/E2))
      +   /DRSQ2)                                             $RETURN
21     IF (I.EQ.19) FX=DSL4(K)                                $RETURN
22     IF (I.EQ.21) FX=1.0                                    $RETURN
23     IF (I.EQ.22) FX=1.0                                    $RETURN
24     IF (I.EQ.23) FX=1.0                                    $RETURN
25     IF ( X(K,20) .EQ. 0.0 ) RETURN
       IF ( I.EQ.2  ) FX = 1.0                                $RETURN
26     IF (I.EQ.25) FX=1.0                                    $RETURN
27     IF (I.EQ.26) FX=1.0                                    $RETURN
28     IF (I.EQ.27) FX=1.0                                    $RETURN
29     IF (I.EQ.5) FX=FEEDQ
       IF (I.EQ.6) FX=FEEDQ*2.87
       IF (I.EQ.7) FX=FEEDQ*5.97
       IF (I.EQ.8) FX=FEEDQ*24.23
       IF (I.EQ.19) FX=FEEDQ*24.23                            $RETURN
30     IF (K.LT.6) RETURN
       IF (I.EQ.30) FX = P(1,1)
       IF (I.EQ.31) FX = F(1,2)
       IF (I.EQ.32) FX = F(1,3)
       IF (I.EQ.33) FX = F(1,4)
       IF (I.EQ.29) FX = -0.25                                $RETURN
31     IF(K.LT.6) RETURN
       IF (I.EQ.30) FX = F(2,1)
       IF (I.EQ.31) FX = F(2,2)
       IF (I.EQ.32) FX = F(2,2)
       IF (I.EQ.33) FX = F(2,3)
       IF (I.EQ.29) FX = -1.0                                 $RETURN
32     IF(K.LT.14) RETURN
       IF (I.EQ.30) FX = F(3,1)
       IF (I.EQ.31) FX = F(3,2)
       IF (I.EQ.32) FX = F(3,3)
       IF (I.EQ.33) FX = F(3,4)                               $RETURN
33     IF(K.LT.14) RETURN
       IF (I.EQ.30) FX = F(4,1)
       IF (I.EQ.31) FX = F(4,2)
       IF (I.EQ.32) FX = F(4,3)
       IF (I.EQ.33) FX = F(4,4)                               $RETURN
```

```
34    IF ( I.EQ.2 ) FX = 1.
      IF ( I.EQ.3 ) FX = 1.
      IF ( I.EQ.4)  FX = 1.
      IF ( I.EQ.5 ) FX = 1.
      IF ( I.EQ.6 ) FX = 1.
      IF ( I.EQ.7 ) FX = 1.
      IF ( I.EQ.8 ) FX = 1.
      IF ( I.EQ.19) FX = 1.                                        $RETURN
      END
```

```
      FUNCTION FU (J,I,K)
C
C **************************************************
C *                                                *
C *                  FUNCTION FU                   *
C *                                                *
C **************************************************
C
C FUNCTION FU COMPUTES THE MATRIX OF PARTIAL DERIVATIVES
C (DF/DU) WITH REFERENCE TO CONTROL VARIABLES U.
C
      REAL K1,K3,K4,K6,K7,K8,MORTFN,MORTFN2
      COMMON X(35,35),U(35,2)
      COMMON /PLANT/ P(4,4)
      COMMON /ENOUGH/ C1,C2,C3,C4,D1,D2,D3,D4,D5
      COMMON /ENOUGH/ DF1,DF2,DF3,DF4,DF5,DF6,DF7,DF8,DF9
      COMMON /ENOUGH/ A,B,C,D,F,K1,K3,K4,K6,K7,K8,FEEDQ
      COMMON /ENOUGH/RF,CF,E1,E2,E3,ECF,ONECP,TD,IDIA,IFEED,NTOATS
      COMMON /BLOCK/ MORTFN(30),MORTFN2(30),SURVFN(30),SURVFN2(30)
      COMMON /BLOCK/ SL1(30),SL4(30),F1(30),DIAFFN(30),DIFMFN(30)
      COMMON /BLOCK/ DIFMFN2(30),F1DR(30),F2(30),DSL1(30),DSL4(30)
      FU=0.0
      IF (I.NE.1) RETURN
      GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,
     +       21,22,23,24,25,26,27,28,29,30,31,32,33,34) J
C FUNCTIONS ARE DEFINED BELOW FOR NONZERO FUNCTIONS ONLY.
1     FU=-X(K,1)*A*DIFMFN2(K)                                  $RETURN
2     RETURN
3     RETURN
4     RETURN
5     FU=-X(K,4)*(1.0-K1)*DIFMFN(K)                            $RETURN
6     FU=-X(K,5)*SL1(K)*DIFMFN(K)                              $RETURN
7     FU=-X(K,6)*(1.0-K3)*DIFMFN(K)                            $RETURN
8     FU=-X(K,7)*(1.0-K4)*(1.0-F2(K))*DIFMFN(K)                $RETURN
9     RETURN
10    RETURN
11    RETURN
12    RETURN
13    RETURN
14    RETURN
15    RETURN
16    FU=-(X(K,16)*C+X(K,15)*(1.0-K6))*DIFMFN(K)               $RETURN
17    RETURN
18    FU=-(X(K,18)*D+(1.0-DIAPFN(K))*X(K,24)*X(K,28))*DIFMFN(K)
      RETURN
19    FU=-X(K,7)*(1.0-K4)*F2(K)*DIFMFN(K)                      $RETURN
20    RETURN
21    RETURN
22    RETURN
23    RETURN
24    RETURN
25    RETURN
```

```
26      RETURN
27      RETURN
28      RETURN
29      RETURN
30      RETURN
31      RETURN
32      RETURN
33      RETURN
34      RETURN

        END
```

```
      SUBROUTINE OUTPUT (IT)
C
C ************************************************************
C *                                                          *
C *                  SUBROUTINE OUTPUT                       *
C *                                                          *
C ************************************************************
C
C SUBROUTINE OUTPUT PRINTS OPTIMAL CONTROLS AND OPTIMAL STATE
C TRAJECTORIES FOR EVERY TIME STEP.  IT ALSO PRINTS THE
C PERFORMANCE INDEX AND OTHER VARIABLES FOR EVERY ITERATION.
C SUBROUTINE OUTPUT ALSO WRITES ALL THESE OUTPUTS ON CATALOGUED
C TAPES THAT CAN BE ACCESSED FOR GRAPHING AT A LATER TIME.
C
      COMMON X(35,35),U(35,2)
      COMMON /TIME/ YR, NN, NSTEP, NX, NU
      INTEGER YR,LINE(14)

C
C THE PARAMETER -IT- IS THE TABLE NUMBER TO PRINT, AND IS ALSO
C A FLAG TO PRINT THE TABLE ON A NEW PAGE (FINAL TABLE PRINTOUTS),
C OR TO PRINT SEVERAL TABLES PER PAGE (DEBUGGING PURPOSES).
C IF (IT .LT. 0) PRINT TABLE NUMBER IABS(IT) ON THE SAME PAGE,
C ELSE PRINT TABLE NUMBER (IT) ON A NEW PAGE.
C
C     TABLE 1, PRINT -U- ARRAY FOR WHOLE YEAR.
C     TABLE 2, PRINT -X- ARRAY FOR WHOLE YEAR.
C     TABLE 3, PRINT TAPE77 TABLE.
C     TABLE 4, PRINT TAPE78 TABLE.

      IABSIT = IABS(IT)
      GO TO (100, 200, 300, 400) IABSIT
100   IF (IT .LT. 0) PRINT 64, "-", YR
      IF (IT .GT. 0) PRINT 64, "1", YR
64    FORMAT (A1," YEAR",I3,/,"-  TIME",8X,"CONTROL",//)
      PRINT 65, (II,U(II,1),II=1,NSTEP)
      WRITE (3,65) (II,U(II,1),II=1,NSTEP)
65    FORMAT (3X,I4,5X,1PE15.6)
      RETURN
200   IF (IT .GT. 0) PRINT 74, "1"
      IF (IT .LT. 0) PRINT 74, "-"
74    FORMAT (A1,2X,"TIME",5X,"X1",9X,"X2",9X,"X3",9X,"X4",
     +9X,"X5",9X,"X6",9X,"X7",9X,"X8",9X,"X9",8X,"X10"//)
      PRINT 95, (II,(X(II,LL),LL=1,10),II=1,NN)
      WRITE (4,95)    (II,(X(II,LL),LL=1,10),II=1,NN)
      IF (IT .GT. 0) PRINT 84, "1"
      IF (IT .LT. 0) PRINT 84, "-"
84    FORMAT (A1,2X,"TIME",5X,"X11",8X,"X12",8X,"X13",8X,"X14",
     +8X,"X15",8X,"X16",8X,"X17",8X,"X18",8X,"X19",8X,"X20"//)
      PRINT 95, (II,(X(II,LL),LL=11,20),II=1,NN)
      WRITE (5,95)    (II,(X(II,LL),LL=11,20),II=1,NN)
      IF (IT .GT. 0) PRINT 94, "1"
      IF (IT .LT. 0) PRINT 94, "-"
```

```
94    FORMAT (A1,2X,"TIME",5X,"X21",8X,"X22",8X,"X23",8X,"X24",
     +8X,"X25",8X,"X26",8X,"X27",8X,"X28",8X,"X29",8X,"X30"//)
      PRINT 95,  (II,(X(II,LL),LL=21,30),II=1,NN)
      WRITE (6,95)    (II,(X(II,LL),LL=21,30),II=1,NN)
      IF (IT .GT. 0) PRINT 97, "1"
      IF (IT .LT. 0) PRINT 97, "-"
97    FORMAT (A1,3X,"TIME",5X,"X31",8X,"X32",8X,"X33",8X,"X34")
      PRINT 96,  (II,(X(II,LL),LL=31,34),II=1,NN)
      WRITE (7,96)    (II,(X(II,LL),LL=31,34),II=1,NN)
95    FORMAT (I5,1PE12.3,9E11.3)
96    FORMAT (I5,1PE12.3,3E11.3)
      RETURN
300   REWIND 77
      PRINT 667
667   FORMAT ("1", T10,"YEAR",T21,"OW CLB",T35,"CLB EGGS",T51,
     + "TJ(1ST)",T66,"TJ(2ND)",T80,"CLB TLAR3",T97,"PARA" )
310   READ (77,666) LINE
      IF (EOF(77) .NE. 0) RETURN
      PRINT 666, LINE
      GO TO 310
400   REWIND 78
      PRINT 777
777   FORMAT ("1",T10,"YEAR",T22,"YIELD BU/ACRE",T43,
     +         "YIELD $/ACRE",T60," PESTICIDE LB/ACRE",T85,
     +         "TOTAL COST $/ACRE",T1.8,"PROFIT $/ACRE" )
410   READ (78,666) LINE
      IF (EOF(78) .NE. 0) RETURN
      PRINT 666, LINE
      GO TO 410
666   FORMAT (13A10,A7)
      END
```

```
      SUBROUTINE INPUT
C
C ***********************************************************
C *                                                         *
C *                  SUBROUTINE INPUT                       *
C *                                                         *
C ***********************************************************
C
C SUBROUTINE INPUT READS IN THE INITIAL CONDITIONS FOR
C THE STATE VARIABLES, AND ALL OTHER PARAMETERS REQUIRED
C FOR THE OPTEMIZATION ALGORITHM.
C
      COMMON /INPUT/ IFLAG, NITER, EPSILON, STEPMIN, STEPMAX, Q, R
      COMMON /INPUT/ USTART, TVALUE, TOLER, PT, TLAST, CLPIN
      COMMON /INPUT/ JULISIN, FINALTJ, NYEAR, TSPRAY, SPRAY
      COMMON /INPUT/ PRICE, TAX
      COMMON /TIME/ YR, NN, NSTEP, NX, NU
      REAL JULISIN
      INTEGER TLAST,TSPRAY

C     THIS IS THE COMMON INPUT PROCESSOR.  THIS INITIALIZES ALL
C     OF THE VARIABLES IN THE /INPUT/ COMMON BLOCK BY PROMPTING
C     AND READING THEM IN.  THEN THEY ARE PRINTED OUT ON THE
C     OUTPUT FILE ON A SEPARATE PAGE.  THE VARIABLES ARE SET
C     IN ORDER OF THEIR SPECIFICATION IN THE /INPUT/ BLOCK.
C     -TAPE9- IS THE TELETYPE PROMPTING FILE.  THIS IS DECLARED
C     TO BE THE FILE -ZZZZOT- ON THE PROGRAM CARD.  IF UNIT 9
C     (ZZZZOT) IS CONNECTED, IT USUALLY IS FOR INTERACTIVE RUNS,
C     ALL THE PROMPTING LINES WILL BE DISPLAYED ON TERMINAL,
C     AND THE PROGRAM WILL READ USER TYPEINS.  IF RUN FROM
C     BATCH, ALL PROMPTING LINES WILL BE WRITTEN TO LOCAL
C     FILE -ZZZZOT- AND NOT PRINTED, AND INPUT VARIABLES
C     WILL BE READ FROM DATA CARDS.

      CALL CONNEC (9)

C IFLAG DETERMINES THE TYPE OF OPTIMIZATION PROBLEM :
C IFLAG = 0 IMPLIES EXPLICIT CONSTRAINT ON CONTROL
C IFLAG = 1 IMPLIES NO EXPLICIT CONSTRAINT ON CONTROL
      PRINT (9,*)"PROBLEM TYPE (IFLAG):"
      READ *,IFLAG
      PRINT (9,*)"NUMBER OF INTERATIONS (NITER):"
      READ *,NITER
      PRINT (9,*)"INITIAL STEP-SIZE (EPSILON):"
      READ *,EPSILON
      PRINT (9,*)"EPSILON STEP DECREMENT (STEPMIN):"
      READ *,STEPMIN
      PRINT (9,*)"EPSILON STEP INCREMENT (STEPMAX):"
      READ *,STEPMAX
      PRINT (9,*)"WEIGHTING FACTOR (Q):"
      READ *,Q
      PRINT (9,*)"WEIGHTING FACTOR (R):"
      READ *,R
      PRINT (9,*)"NOMINAL CONTROL (USTART):"
```

```
      READ *,USTART
      PRINT (9,*)"CONSTRAINT LEVEL FOR CONTROL U (TVALUE):"
      READ *,TVALUE
      PRINT (9,*)"PERFORMANCE INDEX TOLERANCE (TOLER):"
      READ *, TOLER
      PRINT (9,*)"ECONOMIC INJURY LEVEL (ET):"
      READ *,ET
      PRINT (9,*)"TIME UNIT (TLAST):"
      READ *,TLAST
      PRINT (9,*)"STARTING DENSITY FOR SPRING-ADULT CLB (CLBIN):"
      READ *,CLBIN
      PRINT (9,*)"STARTING DENSITY FOR ADULT T.JULIS (JULISIN):"
      READ *,JULISIN
      PRINT (9,*)"VALUE FOR OVERWINTERING T JULIS (FINALTJ):"
      READ *,FINALTJ
      PRINT (9,*) "ENTER # YEARS FOR MODEL RUN (NYEAR):"
      READ *, NYEAR
      PRINT (9,*) "ENTER TIME FOR CONVENTIONAL SPRAY (TSPRAY):"
      READ *, TSPRAY
      PRINT (9,*) "ENTER SPRAY AMOUNT LBS/ACRE (SPRAY):"
      READ *, SPRAY
      PRINT (9,*) "PRICE OF PESTICIDE $/LB (PRICE):"
      READ *, PRICE
      PRINT (9,*) "TAX IMPOSED $/LB OF PESTICIDE USE (TAX):"
      READ *, TAX
      PRINT 342, IFLAG, NITER, EPSILON, STEPMIN, STEPMAX, Q, R,
     +          USTART, TVALUE, TOLER, ET, TLAST
342   FORMAT ("-",/,"-",/,"-",/,
     + "-",T25,"DEFINE TYPE OF PROBLEM (IFLAG)",T75,I2,/,
     + "-",T25,"NUMBER OF ITERATIONS (NITER)",T75,I3,/,
     + "-",T25,"INITIAL STEP SIZE (EPSILON)",T75,1PE10.3,/,
     + "-",T25,"EPSILON STEP DECREMENT (STEPMIN)",T75,E10.3,/,
     + "-",T25,"EPSILON STEP INCREMENT (STEPMAX)",T75,E10.3,/,
     + "-",T25,"WEIGHTING FACTOR (Q)",T75,E10.3,/,
     + "-",T25,"WEIGHTING FACTOR (R)",T75,E10.3,/,
     + "-",T25,"NOMINAL CONTROL (USTART)",T75,E10.3,/,
     + "-",T25,"CONSTRAINT LEVEL FOR CONTROL - U (TVALUE)",T75,
     +     E10.3,/,
     + "-",T25,"PERFORMANCE INDEX TOLERANCE (TOLER)",T75,E10.3,/,
     + "-",T25,"ECONOMIC INJURY LEVEL (ET)",T75,E10.3,/,
     + "-",T25,"TIME UNIT (TLAST)",T75,I3)
      PRINT 343,PRICE, TAX, JULISIN, FINALTJ, NYEAR, TSPRAY, SPRAY
343   FORMAT
     +("-",T25,"STARTING DENSITY SPRING-ADULT CLB (CLBIN)",T75,
     +     E10.3,/,
     + "-",T25,"STARTING DENSITY ADULT T.JULIS (JULISIN)",T75,
     +     E10.3,/,
     + "-",T25,"VALUE FOR OVERWINTERING T.JULIS (FINALTJ)",T75,
     +     E10.3,/,
     + "-",T25,"# YEARS FOR MODEL RUN (NYEAR)",T75,I3,/,
     + "-",T25,"TIME STEP FOR CONVENTIONAL SPRAY (TSPRAY)",T75,I2/
     + "-",T25,"SPRAY AMOUNT LBS/ACRE (SPRAY) :",T75,E10.3,/,
     + "-",T25,"PRICE OF OATS - $/BUSHEL (PRICE) :",T75,E10.3,/,
     + "-",T25,"TAX - $/LB OF PESTICIDE SPRAY (TAX)",T75,E10.3,/)
```

```
      PRINT 344
344   FORMAT (1H1)
      RETURN
      END
```

```
      SUBROUTINE MODEL
C
C ***************************************************
C *                                                 *
C *                SUBROUTINE MODEL                 *
C *                                                 *
C ***************************************************
C
C SUBROUTINE MODEL GENERATES THE STATE VARIABLE TRAJECTORIES
C USING THE STATE-SPACE MODEL OF THE SYSTEM.  IT ALSO COMPUTES
C THE PERFORMANCE INDEX.
C
      REAL JULISIN,K1,K3,K4,K6,K7,MORTFN,MORTFN2
      INTEGER TLAST
      COMMON X(35,35),U(35,2)
      COMMON /PLANT/ P(4,4)
      COMMON /ENOUGH/ C1,C2,C3,C4,D1,D2,D3,D4,D5
      COMMON /ENOUGH/ DF1,DF2,DF3,DF4,DF5,DF6,DF7,DF8,DF9
      COMMON /ENOUGH/ A,B,C,D,F,K1,K3,K4,K6,K7,K8,FEEDQ
      COMMON /ENOUGH/RF,CP,E1,E2,E3,ECP,ONECP,TD,IDIA,IFEED,NTOATS
      COMMON /BLOCK/ MORTFN(30),MORTFN2(30),SURVFN(30),SURVFN2(30)
      COMMON /BLOCK/ SL1(30),SL4(30),F1(30),DIAPFN(30),DIFMFN(30)
      COMMON /BLOCK/ DIFMFN2(30),F1DR(30),F2(30),DSL1(30),DSL4(30)
      COMMON /FX/ DRSQ,DRSQ2,F1DRCP,F1DRCP1,X18CP,X18CP1
      COMMON /INPUT/ IFLAG, NITER, EPSILON, STEPMIN, STEPMAX, Q, R
      COMMON /INPUT/ USTART, TVALUE, TOLER, ET, TLAST, CLBIN
      COMMON /INPUT/ JULISIN, FINALTJ, NYEAR, TSPRAY, SPRAY
      COMMON /INPUT/ PRICE, TAX
      COMMON /TIME/ YR, NN, NSTEP, NX, NU
      COMMON /MODEL/ XREF(35),DIFU,SUMU,SUMUSQ,DIFUSQ,PI
      COMMON /MODEL/ TEGG, TJ2, TLAR3, TUNPAR4, TPAR4

C     RUN THE MODEL FOR A SINGLE STEP.  COPIED FROM OLD MAIN
C     PROGRAM, ALL ADDITIONAL VARIABLES DEFINED IN /MODEL/ BLOCK.

C ********* SPECIFY VALUE (1/0) FOR THE VARIABLE -- IMORT ********
      IMORT = 1
C INITIALIZE PERFORMANCE INDEX.
      PI=0.0
      SUMU   = 0.0
      SUMUSQ =0.0
C GET STATE TRAJECTORY.
      DO 15 I=1,NSTEP
      IF ( I.EQ.NTOATS ) X(I,1) = 0.0
      I1= I+1
C DIFFERENTIAL KILL FOR ADULT AND LARVAE.
       CTRLDR=403.4
       CTRL2=4915.0
      IF (U(I,1).GT.0.0) CTRLDR=EXP(-(10.0*U(I,1)-6.0))
      IF (U(I,1).GT.0.0) CTRL2=EXP(-(10.0 *U(I,1)-8.5))
      MORTFN(I)=1.0/(1.0+CTRLDR)
      MORTFN2(I)=1.0/(1.0+CTRL2)
      SURVFN(I)=1.0-MORTFN(I)
      SURVFN2(I)=1.0-MORTFN2(I)
```

```
      DIFMFN(I)=10.0*((1.0+CTRLDR)**(-2))*CTRLDR
      DIFMFN2(I)=10.0*((1.0+CTRL2)**(-2))*CTRL2
      F1DR(I) = X(I,7)/E1 + X(I,18)/E2 + 1.0/E3
      F1(I) = (X(I,7)*X(I,18))/F1DR(I)
      F2(I) = ( X(I,18) / (E2*F1DR(I)) ) ** CP
C NOTE: IDIA = 1 IMPLIES ROGER'S FUNCTION
C NOTE: IDIA = 0 IMPLIES LAL'S FUNCTION
      IF ( IDIA .EQ. 0 ) GO TO 12
      DIAPFN(I) = (DF9*I**8 + DF8*I**7 + DF7*I**6 + DF6*I**5 +
     + DF5*I**4 + DF4*I**3 + DF3*I**2 + DF2*I + DF1)*0.01
      GO TO 14
12    IF ( I.LE.TD ) DIAPFN(I) = 0.0
      DTERM = TD -I
      IF ( I.GT.TD ) DIAPFN(I) = 1.0 - EXP ( DTERM )
14    X(I1,1)= X(I,1)*A *SURVFN2(I)
      X(I1,2)= X(I,1)*B
C TOTAL EGG INPUT
      TEGG  = TEGG + X(I,2)
C DENSITY DEPENDENT MORTALITY MORTALITY FOR L1 AND L4.
C NOTE: IMORT = 1 IMPLIES ROGER'S FUNCTION
C NOTE: IMORT = 0 IMPLIES LAL'S FUNCTION
      IF ( IMORT .EQ. 0 ) GO TO 22
      SL1(I)= C1+C2*(X(I,5))+C3*(X(I,5)**2)+C4*(X(I,5)**3)
      SL4(I) = DF5*X(I,8)**4 + DF4*X(I,8)**3 + DF3*X(I,8)**2 +
     + DF2*X(I,8) + DF1
      DSL1(I) = C1 + 2.0 * C2 * X(I,5) + 3.0 * C3 * X(I,5)**2 +
     + 4.0 * C4 * X(I,5)**3
      DSL4(I) = D1 + 2.0 * D2 * X(I,8) + 3.0 *D3 * X(I,8)**2 +
     + 4.0 * D4 * X(I,8)**3 + 5.0 * D5 * X(I,8)**4
      GO TO 11
22    SL1(I)=1.0 - AMAX1( 0.0, AMIN1(0.99,0.46*ALOG10(TEGG)-0.85))
      SL4(I)=1.0 - AMAX1( 0.0, AMIN1(0.99,0.28*ALOG10(TEGG)-0.18))
11    X(I1,3)= X(I,2)
      X(I1,4)=X(I,3)
      X(I1,5)=X(I,4)*(1.0-K1)*SURVFN(I)
      X(I1,6)=  SL1(I)*X(I,5)*SURVFN(I)
      X(I1,7)= X(I,6)*(1.0-K3)*SURVFN(I)
      X(I1,8)= X(I,7)* (1.0-K4)* (1.0-F2(I)) *SURVFN(I)
      X(I1,9)= SL4(I)*X(I,8)
      X(I1,10)= X(I,9)
      X(I1,11)= X(I,10)
      X(I1,12)= X(I,11)
      X(I1,13)= X(I,12)
      X(I1,14)= X(I,13)
      X(I1,15)= X(I,14)
      X(I1,16)= (X(I,16)*C+X(I,15)*(1.0-K6))*SURVFN2(I)
      X(I1,17)= X(I,17)  * F + DIAPFN(I) * X(I,24)*X(I,28)
      IF ( I.LT.6 ) GO TO 29
      SECTJ  = ( 1.0 - DIAPFN(I) ) * X(I,24) * X(I,25)
      X(I1,18) = ( X(I,18) * D + SECTJ ) * SURVFN(I)
      GO TO 39
29    SECTJ = 0.0
      X(I,18) =0.0
```

```
39    X(I1,19) = (1.0-K4)*F2(I)*X(I,7)*SURVFN(I)
      IF ( X(I,7).EQ.0.0 .OR. X(I,18).EQ.0.0 ) GO TO 1
      X(I1,20) = ECF*((X(I,18)/F1DR(I))**ONECF)
      GO TO 2
1     X(I1,20) = 0.0
2     X(I1,21)= SL4(I) *X(I,19)
      X(I1,22)= X(I ,21)
      X(I1,23)= X(I,22)
      X(I1,24)= X(I,23)
      X(I1,25)= X(I,20)*(1.0-RF)
      X(I1,26)= X(I,25)
      X(I1,27)= X(I,26)
      X(I1,28)= X(I,27)
      X(I1,29)= FEEDQ* (X(I,5)+2.87*X(I,6)+5.97*X(I,7)+24.23
     +*(X(I,8)+X(I,19)))
      IF ( I.LT.6 ) GO TO 59
      X(I1,30) = P(1,1) * X(I,30) + P(1,2) * X(I,31) + P(1,3) *
     + X(I,32) + P(1,4) * X(I,33) - 0.25 * X(I,29) * IFEED
      X(I1,31) = P(2,1) * X(I,30) + P(2,2) * X(I,31) + P(2,3) *
     + X(I,32) + P(2,4) * X(I,33) - X(I,29) * IFEED
      IF (I.LT.14) GO TO 59
      X(I1,32) = P(3,1) * X(I,30) + P(3,2) * X(I,31) + P(3,3) *
     + X(I,32) + P(3,4) * X(I,33)
      X(I1,33) = P(4,1) * X(I,30) + P(4,2) * X(I,31) + P(4,3) *
     + X(I,32) + P(4,4) * X(I,33)
      X(I1,34) = X(I,2) + X(I,3) + X(I,4) + X(I,5) + X(I,6) +
     + X(I,7) + X(I,19)
C SUM PERFORMANCE INDEX OVER TIME.
C ******************* MAXIMIZATION PROBLEM *********************
59    PI = PI + ( Q * PRICE / 1.35 )  * X(I,32) * X(I,32)
      IF (IFLAG .NE. 1 ) GO TO 17
      PI = PI - (R+TAX) * U(I,1) * U(I,1)
17    SUMU = SUMU + U(I,1)
      SUMUSQ = SUMUSQ + U(I,1) * U(I,1)
C SUMS UP 2ND GENERATION T JULIS
      TJ2  = TJ2 + SECTJ
C TOTAL 3RD INSTAR POPULATION
      TLAR3 = TLAR3 + X(I,7)
C TOTAL UNPARASITIZED 4TH INSTAR POPULATION
      TUNPAR4 = TUNPAR4 + X(I,8)
C TOTAL PARASITIZED 4TH INSTAR POPULATION
      TPAR4 = TPAR4 + X(I,19)
C TOTAL PESTICIDE USE FOR A SINGLE GROWING SEASON
15    CONTINUE
      PI = PI + X(NN,32)
      DIFU= SUMU-TVALUE
      DIFUSQ = SUMUSQ - TVALUE
      RETURN
      END
```

APPENDIX B

L-Q-G DESIGN FOR ON-LINE CONTROL

L-Q-G DESIGN FOR ON-LINE CONTROL

The basic problem in real-world control system design almost invariably involves the on-line (i.e., real-time) feedback control of an uncertain, usually non-linear, physical process. This is especially true with pest management problems--pest ecosystems are usually characterized by non-linear, dynamic processes; stochastic variations in abiotic factors affecting the ecosystem adds uncertainty to the process. Further, modeling errors are invariably present since models are just simple abstractions representing only the dominant features of the pest ecosystem under consideration.

To recapitulate our discussions from an earlier section, one of the major drawbacks associated with the currently available optimization models in pest management is that, for the most part, they have ignored the stochastic aspects of the control and concentrated instead on the deterministic problem. In the deterministic optimal controller design, one assumes that there is no uncertainty, and that <u>exact</u> measurement of <u>all</u> state variables are available. This is seldom the case is almost all practical applications and especially so in pest management problems. For example, while it is generally easier to take measurement of larval stages of an insect, it is difficult to measure densities of pupae and adult. This problem is further compounded by the fact that certain age-classes (within life-stages of an insect) introduced for modeling purposes, cannot be distinguished in the field, and hence, cannot be measured. Even if all the state variables could be measured, there will be measurement errors introduced by physical sensors, human errors in carrying out the measurements, and sampling errors. This uncertainty in measurement should be taken into account in the design of the optimal controller. Also, in real-world situations there are likely to be disturbance inputs acting on the physical process described by the system model (e.g. climatological changes affecting an ecosystem). It is obvious that a deterministic optimal controller will not be optimal in a real-world stochastic situation. In order that we may take into account the stochastic aspects of the problem, the design of the optimal controller should include a stochastic estimator/filter and a scheme for stochastic feedback control.

Figure 42 illustrates a generalized control problem encountered in the real-world situation. It is modeled as a feedback control system subject to persistent disturbances. The entire system consists of three basic components:

(i) The physical process--In the pest management problem it will be represented by the system model for the pest ecosystem. The ecosystem is subject to environmental disturbance.

(ii) Measurement unit--This represents the sensors or human measurements carried out in the field. All measurements are subject to measurement noise.

(iii) Decision-making unit--This consists of an optimal controller, a stochastic estimator, and provisions for stochastic feedback control.

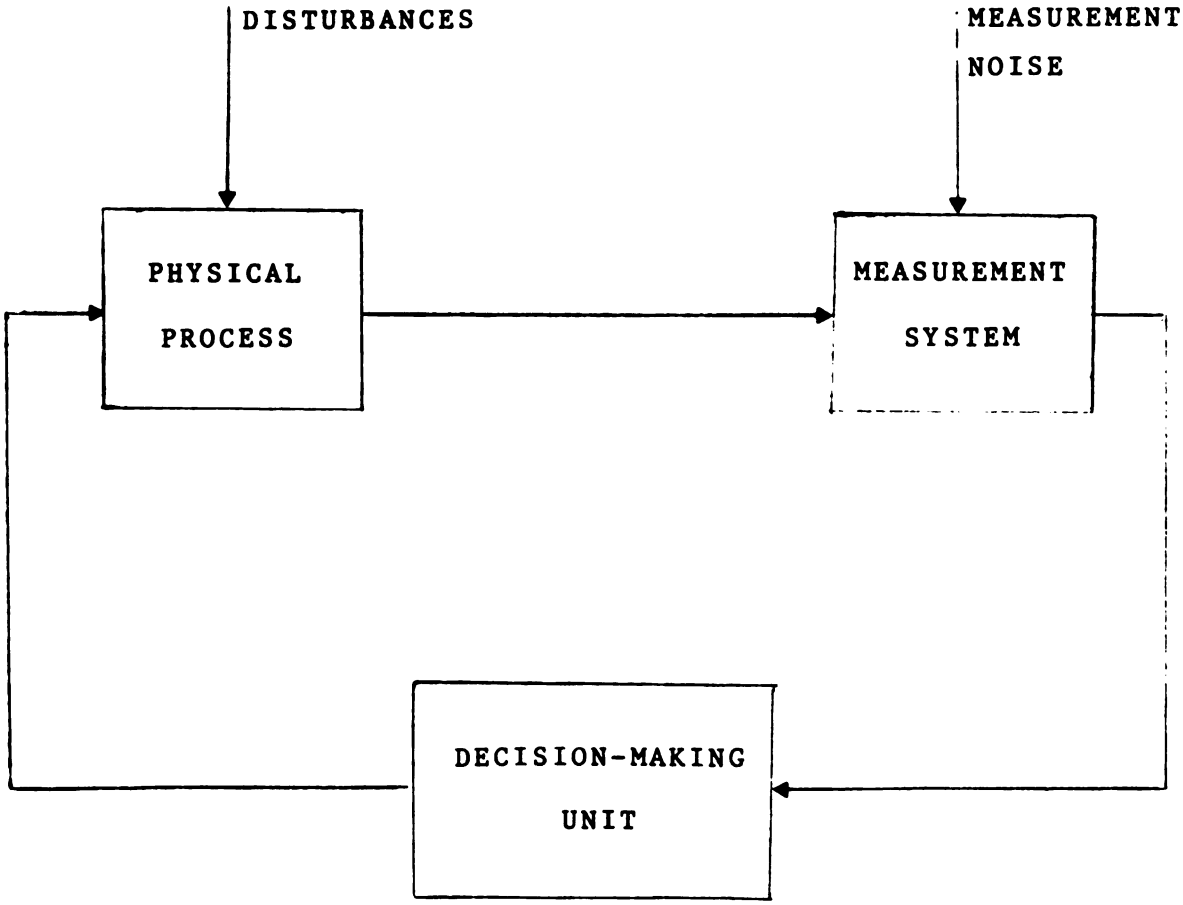


FIGURE 42. Schematic illustrating a generalized control problem.

The decision-making unit, which is called the "compensator" in control literature, has the task of translating the actual sensor measurements into the actual commanded inputs (i.e. control strategies for managing the pest ecosystem) in such a manner that the performance criterion specified for the problem is optimized. It is obvious that the design of the compensator will be dependent on:

(i) Natural dynamics of the system under consideration, both in the absence of uncertainty (deterministic), and in the presence of uncertainty (stochastic).

(ii) The level of uncertainty associated with the system.

(iii) The performance criterion specified for the problem.

Clearly the design issue is clouded because it involves the interplay between the natural dynamics of the physical process, the stochastic nature of the uncertainties, and the effects of the deterministic commanded inputs. Nonetheless, one can adopt a design philosophy, known as the Linear-Quadratic-Gaussian methodology, that involves the following three basic steps:

Step 1. Deterministic Ideal Response Analysis and Design.

Step 2. Stochastic Estimation Analysis and Design.

Step 3. Stochastic Feedback Control System Design.

We outline here the L-Q-G approach to the stochastic optimal controller design. Detailed discussion on all aspects of this problem can be found in the excellent papers by Athans (1971, 1972) (see also Meier et al 1971, Kramer and Athans 1972, and Saridis 1977.)

TECHNIQUES FOR IMPLEMENTING THE L-Q-G ALGORITHM

Step 1 involves the design of a discrete-time, deterministic optimal controller. As mentioned earlier, a variety of approaches are possible (i.e., dynamic programming, discrete maximum principle, non-linear programming, differential dynamic programming/successive sweep method, and their variations). For the reasons discussed earlier, we can adopt the first order successive approximation technique of McReynolds and Dyer (1970).

Step 2 involves the design of a state estimator. It should be emphasized here that a linearized perturbation model is obtained prior to step 2. The linearization is done with reference to the optimal state and trajectories obtained as a solution to the deterministic optimal control problem in Step 1. Based on the linearized perturbation model, the deterministic linear quadratic problem can be formulated. This problem can be readily solved using the matrix Riccati equation (see references on Optimal Control) and leads to a linear time-varying/time-invariant feedback relationship between the state perturbation vector $\delta x(t)$ and the control perturbation vector $\delta u(t)$. Detailed discussions on the perturbation problem, the choice of quadratic criterion, and the solution of the deterministic, linear-quadratic problem can be found in Athans (1971, 1972).

In the stochastic estimation problem of Step 2, the uncertainties arising out of disturbances, measurement errors, as well as the input uncertainty, will be modeled by the use of <u>white noise</u>. The resulting linear-gaussian estimation problem can be solved through the use of the Kalman-Bucy filter that generates the best estimate $\delta\hat{x}(t/t)$ of the deviation of the true state vector x(t) from its ideal deterministic response $x_o(t)$. Since excellent treatment of the Kalman-Bucy filter is available in the literature (see references cited under Optimal Control and Estimation Techniques) it will not be repeated here. However, the key equations involved in its implementation will be summarized toward the end of this section.

In short, the Kalman filter combines two independent estimates of a state vector to provide "best" (minimum variance) estimate of the system state. The two independent estimates of the states of the system are given by:

(i) a process model based on apriori understanding of the prototype system, and

(ii) measurements of some or all state variables.

As discussed earlier, estimates from the process model contain uncertainty due to model errors and other limitations. Also measurements contain sampling and analytical errors. The filter combines the model and sensor estimates by weighting them according to the uncertainties associated with each of them in such a manner that the uncertainty associated with the filter estimate is less than the uncertainty associated with either independent estimate individually. Output from the filter consists of a new improved estimate of the states of the system and the variance associated with that estimate. The Kalman filter utilizes a recursive algorithm for its implementation. Because of its recursive nature only the most recent measurements are stored in the computer. Thus the memory and computational requirements are minimal.

Step 3 involves the design of the stochastic controller. The linearized Kalman-Bucy filter can be designed so as to generate on-line the estimated deviation $\delta\hat{x}(t/t)$ of the actual plant state $x(t)$ from its ideal deterministic response $x_o(t)$. It is to be noted that $\delta x(t)$ also depends on the control correction vector $\delta u(t)$. Hence, one can now think of the final step of the desing process as the techniques necessary for generating on-line the control correction vector $\delta u(t)$ as a function of the measurements so as to keep $\delta x(t)$ small.

The remarkable property of the Linear-Quadratic-Gaussian control problem is that the optimal control correction $\delta u(t)$ is generated from the estimated state deviation $\delta\hat{x}(t/t)$ generated by the Kalman filter by means of the relationship:

$$\delta u(t) = -G_o(t)\,\delta\hat{x}(t/t)$$

where the gain matrix $G_o(t)$ is precisely the one determined in the solution of the deterministic linear-quadratic problem. (Note that the deterministic linear-quadratic problem has the solution:

$$\delta u(t) = -G_o(t)\,\delta x(t)$$

with the assumption that the entire state perturbation vector $\delta x(t)$ is measured exactly.)

This valuable feature of the Linear-Quadratic-Gaussian problem arises due to the so-called "separation theorem" that can be stated as follows:

> In linear systems with quadratic performance criteria and subjected to Gaussian inputs, the stochastic optimal feedback controller is synthesized by cascading an optimal estimator (Kalman-Bucy filter) with the determinisitic optimal controller. (Sar 1977)

The separation theorem is always valid for "neutral systems"* which includes the class of Linear-Quadratic-Gaussian problems. For detailed discussions on the separation principle and related topics, see Bryson and Ho (1975), Meier et al (1971), Kramer and Athans (1972), Patchell and Jacobs (1971), and Saridis (1977).

SUMMARY OF THE L-Q-G APPROACH

We summarize here the key steps involved in the Linear-Quadratic-Gaussian approach to the design of stochastic optimal feedback controller (see also Figures 43 and 44):

Part A. Deterministic Modeling:

Step 1. Determine a deterministic model of the plant; this yields:

$$x(t + 1) = f(x(t), u(t), t).$$

Step 2. Determine a deterministic model of the sensors; this yields:

$$y(t) = g(x(t), t).$$

Step 3. Determine ideal input-state-output sequences using deterministic optimal control techniques:

$\{u_o(t)\}$: ideal input sequence

$\{x_o(t)\}$: ideal state sequence

$\{y_o(t)\}$: ideal output sequence

for all $t = 0, 1, 2, \ldots, T$.

*The class of neutral control problems (Fel'dbaum 1965) includes all problems where the rate of reduction of uncertainty is unaffected by the control signals.

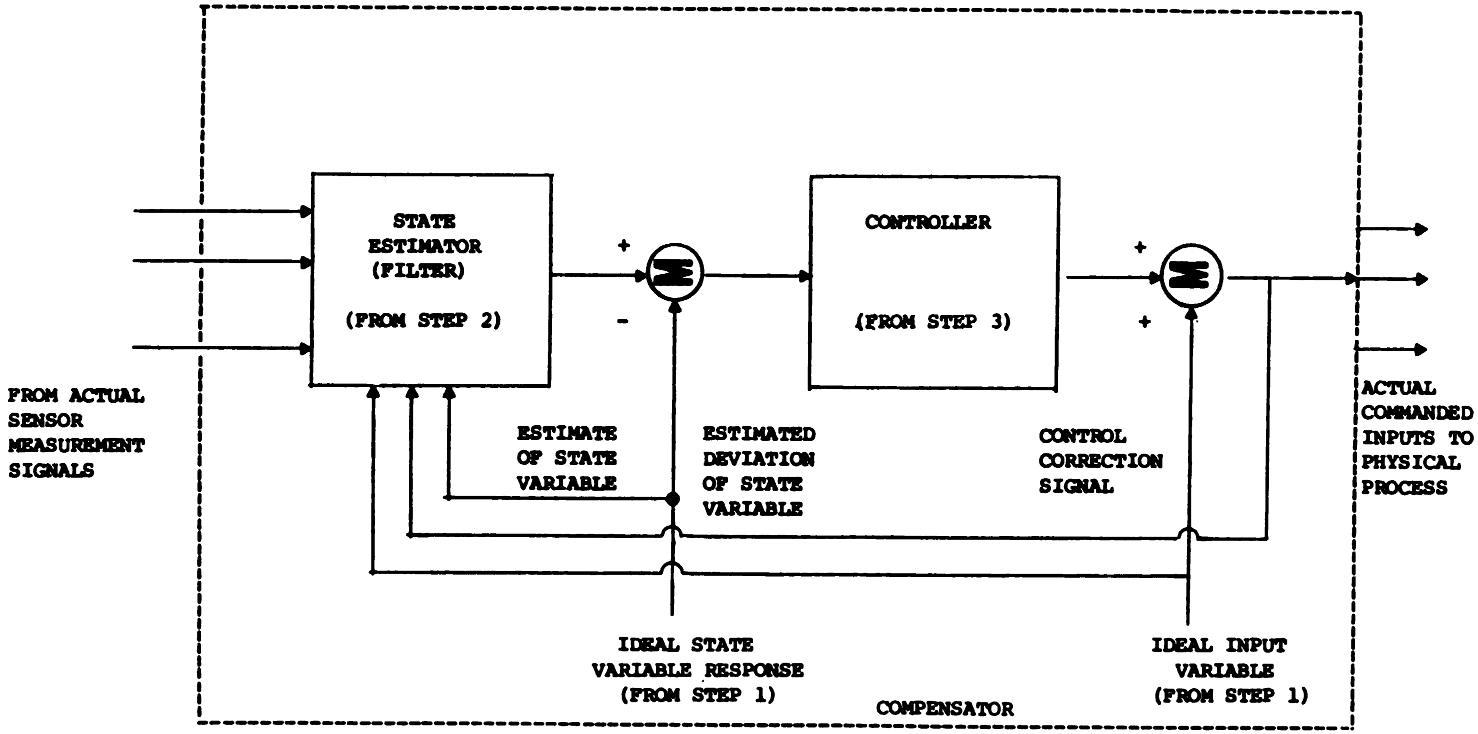


FIGURE 43. Internal structure of a compensator.

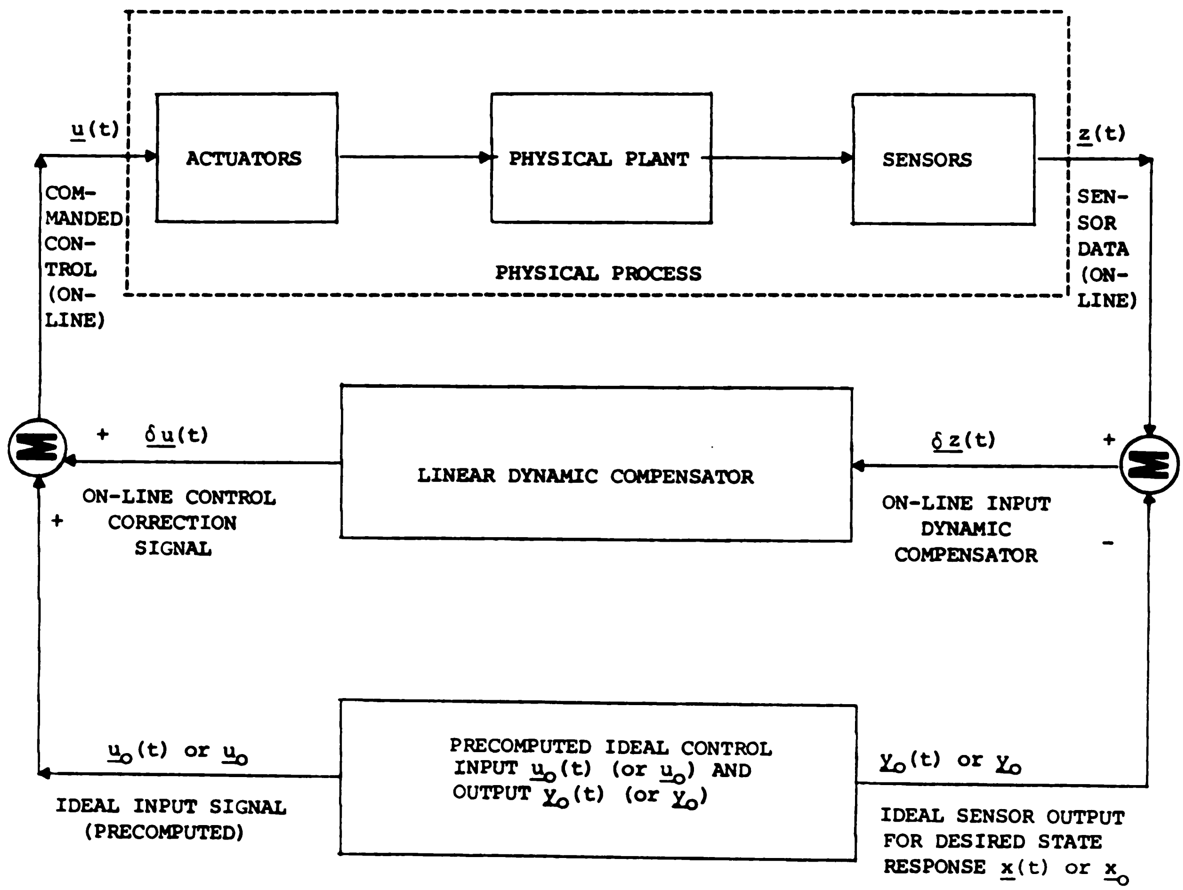


FIGURE 44. Schematic for L-Q-G design.

Part B. Stochastic Modeling:

Step 4. Model uncertainty in initial plant state:

Select mean: $x_o = E(x(0)$

Select covariance: $\sum_o = \text{cov}[x(0);x(0)]$

Step 5. Model input uncertainty:

Select covariance: $\Xi(t)\delta_{t\tau} = \text{cov}[\xi(t);\xi(\tau)]$

Step 6. Model sensor uncertainty:

Select covariance: $\Theta(t)\delta_{t\tau} = \text{cov}[\theta(t);\theta(\tau)]$

Part C. Linearization:

Step 7. Establish matrices $A_0(t)$, $B_o(t)$, and $C_o(t)$ from information in Steps 1, 2, and 3:

$$A_o(t) = \left.\frac{\partial f}{\partial x(t)}\right|_o, \quad B_o(t) = \left.\frac{\partial f}{\partial u(t)}\right|_o, \quad C_o(t) = \left.\frac{\partial g}{\partial x(t)}\right|_o.$$

Step 8. Depending on "degree of nonlinearity" select weighting matrices $Q_o(t)$, $R_o(t)$ with due consideration of the values of $\sum_o, \Xi(t)$, and $\Theta(t)$.

Part D. Control Problem Calculations (Off-Line):

Step 9. Using the weighting matrices $Q_o(t)$, $R_o(t)$ of Step 8 and the matrices $A_o(t)$, $B_o(t)$ of Step 7 solve backward in time the matrix difference equation:

$$K_o(t) = Q_o(t) + A_o'(t)K_o(t + 1)A_o(t) - A_o'(t)K_o(t + 1)B_o(t)$$
$$* [B_o'(t)K_o(t + 1)B_o(t) + R_o(t)]^{-1}B_o'(t)K_o(t + 1)A_o(t)$$

with $K_o(T) = Q_o(T)$.

Step 10. Compute the control gain matrix $G_o(t)$:

$$G_o(t) = [B_o'(t)K_o(t + 1)B_o(t) + R_o(t)]^{-1}B_o'(t)K_o(t + 1)A_o(t).$$

Part E. Filter Problem Calculations (Off-Line)

Step 11. Using the covariance matrices $\Sigma_o, \Xi(t)$ and $\Theta(t)$, established in Steps 4-6, and the matrices $A_o(t)$, $C_o(t)$ of Step 7, solve forward in time the matrix difference equations:

$$\Sigma_o(t + 1|t) = A_o(t)\Sigma_o(t|t)A_o'(t) + \Xi(t)$$

$$\Sigma_o(t + 1|t + 1) = \Sigma_o(t + 1|t) + \Sigma_o(t + 1|t + 1)C_o'(t + 1)$$

$$* [C_o(t + 1)\Sigma_o(t + 1|t)C_o'(t + 1) + \Theta(t + 1)]^{-1}C_o(t + 1)\Sigma_o(t + 1|t)$$

with $\Sigma_o(0|0) = \Sigma_o$.

Step 12. Compute the filter gain matrix:

$$H_o(t + 1) = \Sigma_o(t + 1|t + 1)C_o'(t + 1)\Theta^{-1}(t + 1).$$

Part F. On-Line Calculations

From actual measurements z(1), z(2), ...

(a) Compute $\delta z(1), \delta z(2), \ldots,$by:

$$\delta z(t) = z(t) - g(x_o(t),t).$$

(b) Compute estimated deviations $\delta x(t/t)$ and control correction $\delta u(t)$ by:

$$\delta\hat{x}(t + 1|t) = A_o(t)\delta\hat{x}(t|t) + B_o(t)\delta u(t)$$

$$\delta r(t + 1) = \delta z(t + 1) - C_o(t + 1)\delta\hat{x}(t + 1|t)$$

$$\delta\hat{x}(t + 1|t + 1) = \delta\hat{x}(t + 1|t) + H_o(t + 1)\delta r(t + 1)$$

$$\delta u(t) = -G_o(t)\delta\hat{x}(t|t)$$

$$\delta\hat{x}(0|0) = E\{x(0)\} - x_o(0).$$

(c) Compute actual control u(t) by:

$$u(t) = u_o(t) + \delta u(t).$$

It is worth noting here that the on-line computational requirements are minimal, thus leading to a simpler and economical controller.

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