

**This is to certify that the
thesis entitled**

DEVELOPMENT OF A JOINT ORDER
INVENTORY MODEL

presented by

HARISH L. VERMA

**has been accepted towards fulfillment
of the requirements for**

Ph.D. degree in MANAGEMENT

Date September

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ABSTRACT

DEVELOPMENT OF A JOINT ORDER INVENTORY MODEL

by Harish L. Verma

The purpose of this study is to develop and analyze a Joint Order Inventory Model. The term joint ordering, as used here, implies ordering a number of items in a single purchase order. The potential savings resulting from the inclusion of many items on one purchase order is quite large. Some of the advantages of joint orders are:

1. Ordering cost can be reduced by including several items in one purchase order.
2. Shipping costs can be decreased if the total order is of a convenient size, e.g. a truck load. Since a number of items are ordered jointly, it is more likely that the total order will be of a convenient size.
3. When a number of items are ordered jointly the dollar value of the order is larger. Hence, there is an increased opportunity to take advantage of quantity discounts offered by a vendor.

The Joint Ordering Model that is proposed is characterized by three parameters, $\underline{S}$, $\underline{s}$, and $\underline{s}'$, where $\underline{S}$ is the maximum inventory level, $\underline{s}$ is the reorder point, and $\underline{s}'$ determines the items

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to be included in the order. Note that $\underline{S}$ is greater than $\underline{s}'$, which is greater than $\underline{s}$. Thus, any item whose inventory level is equal to or less than $\underline{s}'$ should be included in the joint order; $\underline{s}'$ to $\underline{s}$ is defined as the reorder range. The joint ordering rule is defined as follows:

When the inventory level (inventory on hand and on order) of any item in the group has dropped to the reorder level $\underline{s}$, all items which have inventory levels within the reorder range $\underline{s}'$ to $\underline{s}$ are ordered jointly. The order quantity for each item ordered is given by $(S - I)$ where $\underline{I}$ is the inventory on hand and on order.

Two hypotheses were proposed for testing.

Major Hypothesis: There exist a number of situations in which the application of the joint ordering rule, i.e. ordering a group of items in a single order, results in lower costs when compared with the use of the fixed order quantity rule.

Minor Hypothesis: If the joint ordering rule is defined by the three parameters $\underline{S}$, $\underline{s}$, and $\underline{s}'$, as defined earlier, then there exists some optimum value for each of the parameters, such that the total cost of the inventory control system is minimized. The costs that are included are ordering costs, inventory carrying costs, and stock out costs.

A considerable portion of this study was concerned with

the development of a joint ordering rule and the study of the properties of the rule, especially the performance of the rule under extreme conditions. Hence, it was necessary to generate some hypothetical data which represented these extreme conditions. In generating this data it was assumed that the product group consists of twelve items, and that demand for each item is normally distributed with a certain mean and standard deviation. Although hypothetical data was used to study the properties of the model, it was felt that the model should be tested with real data. Real data were obtained from two sources, The Steel Service Center Institute, and a million dollar farmers' cooperative. However, the data from the farmers' cooperative was not complete, and sales records for certain periods of time were missing. For this and other reasons, the data from the farmers' cooperative could not be used to test the proposed joint ordering model.

There were basically two approaches to the development of the joint ordering model; the analytical approach, and the simulation approach. A simulation approach was used in this research, mainly because it was believed that the mathematical complexity of the problem would make it extremely difficult to formulate the problem and obtain an analytical solution. This view was supported by a number of researchers.

Computer programs for the analysis of the data were developed since no standard routines applicable to the specific

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nature of the problem were available. These programs were used to simulate the inventory control system when the fixed order rule and the joint ordering rule were used to control inventory. In order to compare the performance of the joint ordering rule with the fixed order rule, the same data set was used to compute the total cost per year.

The results obtained from the hypothetical data and the real data indicated that there was sufficient evidence to conclude that the major and minor hypotheses were true. Moreover, inspection of the minimum cost values of the three parameters $\underline{S}$, $\underline{s}$, and $\underline{s}'$ indicated that three distinct cases of the joint ordering rule could be identified. These were:

1. $S = s' > s$: The proposed joint ordering rule is now characterized by two parameters, $\underline{S}$ (or s') and $\underline{s}$.
2. $S > s' > s$: The joint ordering rule is characterized by three parameters, $\underline{S}$, $\underline{s}'$, and $\underline{s}$.
3. $S > s' = s$: As in case 1, the joint ordering rule is characterized by two parameters. However, the two parameters are $\underline{S}$ and $\underline{s}'$ (or $\underline{s}$).

The most important conclusion drawn from these three cases was that the proposed Joint Ordering Rule is more general, and incorporates within itself a number of other reordering rules, both individual item ordering rules such as the two bin inventory control system, and some joint ordering rules proposed by other researchers.

Besides the three cases mentioned above, it was found that

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there were some situations where it was more economical to use the fixed order quantity rule rather than the proposed joint ordering rule. This occurred when the mean demands of the items in the group were widely dispersed. Hence, it was concluded that when the mean demands of the items in the group are widely dispersed, it is not only uneconomical to use the joint ordering rule to control inventory, but even including the items in a group is questionable. Lastly, some criteria for grouping items is suggested.

DEVELOPMENT OF A JOINT ORDER INVENTORY MODEL

By

Harish L. Verma

A THESIS

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in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

College of Business
Department of Management

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CHAPTER I

INTRODUCTION

Inventories are one of the most important assets in the average company, and their control is one of the most difficult and challenging tasks of management. The typical manufacturing corporation has about twenty-four percent of its assets invested in inventories, compared with thirty-nine percent in net property, plant, and equipment.¹ Further, of all the business assets, inventories are the least stable, and the most difficult to control. Even the most successful companies are rarely able to manage their inventories as well as they would like to. Either they have too much inventory of certain items, too little of others, or a combination of both. It is generally believed that inventory control can make or break a company. The importance of inventories is explained by Maynard.²

The control of inventories is one of the most complex and far-reaching of all business activities. It is the focal point of many seemingly conflicting interests and considerations--both short-and long-range. Its planning and execution involve participation by most of the functional segments of a business: sales, production, purchasing, finance and accounting. The end result achieved has a major bearing on the company's financial strength and competitive position, since it directly affects quality of service to customers, production costs, earnings, and soundness of working-capital position.

¹Ammer, C. Materials Management (Homewood, Ill. : Irwin, 1962,) p. 7.

²Maynard, H.B. Industrial Engineering Handbook (New York: McGraw-Hill Book Company, 1956) pp. 6-55

In recent years, the number of items held in inventory has been constantly growing. It is not surprising to find a company with over five hundred individual items in inventory. This is because of the increasing technical nature of the items, a demand for greater variety by customers, and lastly a demand for better service. Further, it is because of these and other factors that the number of dollars invested in inventory is increasing at an even faster rate than the number of items. Hence, in most companies, management has begun to place greater emphasis on the control of inventories. The word control, as used in this thesis, implies minimizing the sum of the ordering costs, inventory carrying costs, and stock out costs.

It is not surprising therefore, that in the last twenty years there has been a rapid growth of interest in what is referred to as scientific inventory control—the use of mathematical models to obtain rules for operating inventory systems.¹ More has been written about this subject than any other in the field of production management.

A survey of the literature on this subject of inventory control reveals that:

1. Both practitioners and theorists agree on the importance of sound inventory control.

¹ Hanssman, F., Operations Research in Production and Inventory Control (New York: John Wiley and Sons, Inc. 1962) p. 6

2. A growing number of researchers are working with inventory models because they present challenging theoretical problems in mathematics.¹ The technical journals are filled with theoretical and rigorous discussions of inventory control problems. Unfortunately, such articles are not meaningful to most businessmen who are faced with pragmatic problems of running the business. This is because the models are either abstract and too mathematical or certain parameters used in the model are not operationally defined. Often, assumptions made in the model are unrealistic. Balintfy's multi-item inventory model serves as a good example. In his article Balintfy admits that the assumptions were made so as to get an analytical solution to the problem. At the same time, he believes that these assumptions restrict the application of his model to practical problems.

3. With the development of high speed computers and the increased emphasis on computer based inventory control systems, it has become possible to apply complex decision rules to maintain tight control over several thousand different items in inventory. With the decreasing cost of computer time, the trend will continue in this direction.

4. The inventory control systems that are generally discussed in the literature are the fixed order quantity

¹Hanssman, F., op. cit., p. (vii)

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system, the fixed interval system, and lastly, the S and s system.

The Fixed Order System:

The oldest and most commonly used inventory control system is the fixed order system, or the two-bin system. The system uses a fixed-order quantity, which may be an economic order quantity (EOQ), and a variable order interval. With this system, the same order quantity (EOQ) of an item is ordered each time. But the time that an order is placed varies with fluctuations in usage. The reordering rule can be stated as follows:

The economic order quantity is ordered each time the inventory on hand plus the inventory on order equals the expected demand during lead time, plus the safety stock. The latter is the inventory needed to protect against possible demand in excess of that expected during the lead time.

The following numerical illustration¹ gives a detailed description of this system.

Assume that:

demand D = 1000 units per year

cost per unit C = \$2.00

ordering cost A = \$15.00 per order

inventory carrying cost I = 15 per cent of cost per year

weekly demand: Normal distribution with mean of 15 units and

demand is normally distributed with mean of 15 units and

¹Stockton, R.S. Basic Inventory Systems (Boston: Allyn and Bacon, Inc., 1965), p. 65

standard deviation of 5 units.

lead time = 1 week

Management believes that this item should not be out of stock more than once a year.

$$\begin{aligned}\text{Economic order quantity} &= \sqrt{\frac{2AD}{CI}} \\ &= \sqrt{\frac{2(1000)15}{2(0.15)}} = \underline{316 \text{ units}}\end{aligned}$$

Therefore the number of orders per year

$$= \frac{1000}{316} = 3.2 \text{ approximately.}$$

Therefore the number of stockouts per order

$$= \frac{1}{3.2} = 0.312$$

From the table for a normal distribution,

0.312 corresponds to 0.49 standard deviations beyond the mean.

$$\begin{aligned}\text{Hence, the reorder point} &= (15 \times 1) + (0.49 \times 5) \\ &= 15 + 2.45 = 17.45 = \underline{15 \text{ units.}}\end{aligned}$$

The fixed order system works well when a continuous review of inventory records is maintained and when the demand for the items in stock is stable. Lee and Dobler¹ list the essential characteristics of items controllable with a fixed order quantity system.

¹Lee, L. Jr. and Dobler, D.W. Purchasing and Materials Management (New York: McGraw-Hill Book Company, 1965) p. 220

1. The item must experience a reasonably stable usage.
2. The item should have a lead time which does not exhibit radical variation.
3. The item must be acquired from a supplier who is able to accept irregularly and unscheduled orders.

The Fixed Interval System:

In the fixed interval system, the reorder cycle is fixed while the order quantity varies depending on demand. Thus the inventory level is checked at fixed intervals, e.g. once a month, and a replenishment order is placed based on the amount used since the last review. The replenishment order is equal to the expected demand during lead time plus review period less the stock on hand and on order.

The following numerical illustration demonstrates the operation of the periodic ordering system.

Assume that the review period for an item has already been established and is equal to 5 days. Assume that the lead time is equal to 2 days and is constant. Further, assume that a study of user demand during the last year indicated that demand per day is normally distributed with a mean of 100 units and standard deviation of 27 units. Then, if at a particular review the stock on hand and on order is 200 units, the size of the order to be placed (Q) should be equal to

$$Q = (\text{expected demand during lead time plus review period})$$

- (inventory on hand and on order) + buffer stock

$$\text{Buffer stock} = \sqrt{2} \times 27 \times 2$$

if a 98% service level is desired

$$\begin{aligned} Q &= 100 \times 7 - 200 + 1.41 \times 27 \times 2 \\ &= \underline{576 \text{ units}} \end{aligned}$$

Periodic reordering system is used where a book inventory control is maintained, and where it is possible to examine inventory stocks on a fixed time cycle so as to reveal when inventory level reaches the reorder point and an order is placed, for example in a warehouse.

Lee and Dobler¹ list criteria for selecting items which should be controlled with a fixed-interval system.

1. Those exhibiting highly irregular usage and/or lead time.
2. Items whose purchases must be scheduled in advance because of various conditions within the suppliers' operations.
3. Perhaps items with volatile prices.
4. Perhaps a group of items which are all purchased from the same supplier and can be ordered on one purchase order and shipped together.

The (s, S) System:

The third inventory control system is a compromise between the fixed-interval and the fixed-quantity system. The ordering rule can be simply stated as follows:

"When the inventory on hand plus the inventory on order is equal to or less than s units, order a quantity sufficient

¹Lee, L. Jr. and Dobler, D.W. op. cit., p. 220

to bring stock up to a level $\underline{S}$, otherwise, do not order."

¹
Magee and Boodman describe the (s, S) rule when applied to a fixed-interval system.

1. Choose two inventory levels $\underline{S}$ and $\underline{s}$, $\underline{S}$ larger than $\underline{s}$.
2. At each review period, compare the available inventory $\underline{I}$ with $\underline{S}$ and $\underline{s}$.
3. If $\underline{I}$ lies between $\underline{S}$ and $\underline{s}$, place no order.
4. If $\underline{I}$ is at or below the level $\underline{s}$, place an order for an amount equal to $\underline{S} - \underline{I}$.

For example, assume that $\underline{S}$ is equal to 100 units and $\underline{s}$ is equal to 30 units (see Figure 1.1). Assume that at a certain review period the inventory on hand is equal to 25 units. Then, since the inventory on hand is less than $\underline{s}$, order an amount equal to 100 ($\underline{S}$) minus 25, that is 75 units will be ordered.

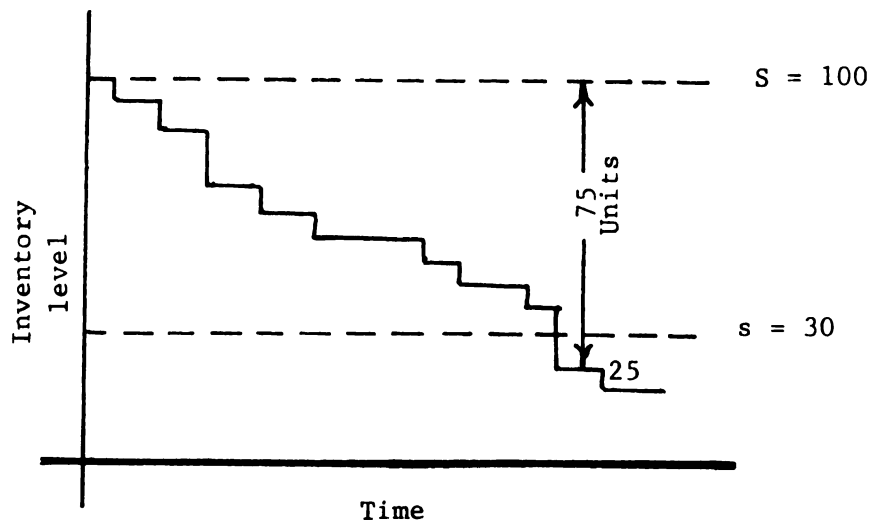
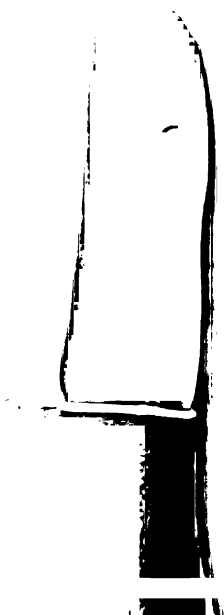


Figure 1.1 : Order Quantity under a (s, S) inventory control system

¹ Magee, J.F. and Boodman, D.M. Production Planning and Inventory Control (New York: McGraw-Hill Book Company, 1967)
p. 137

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In this case, the reorder point $\underline{s}$ should be large enough so that whenever the inventory on hand and on order is greater than $\underline{s}$, the system is protected from run-out to the desired degree over a period equal to the lead time plus the review period. The system of control is particularly useful where the cost of making a review and the cost of placing an order are separate and significant.

The ordering rules described above are most appropriate when there are few items in inventory. Thus, a reorder point and an economic reorder quantity (the fixed order system), or reorder interval and reorder quantity (the fixed interval system), could be easily calculated for each item, and the inventory controlled accordingly. However, inventories are seldom composed of a single or even a few items. Typically, several thousand different items are carried in stock. Even for a single product, it is not unusual to have an assortment of shapes, sizes, colors, etc. For example, the product category "screws" in a typical manufacturers inventory will include screws of various lengths, diameters, number of threads to the inch, wood screws, machine screws, brass screws, and so on. In the same way, a department store will carry many different sizes, colors, materials, and styles of home appliances, men's clothing, women's clothing, etc. A supermarket carries stocks of a variety of virtually all items stocked. In these cases it becomes virtually impossible to establish an economic order quantity and reorder point for

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each stock-item and more so to use these parameters to control these inventories. What is desired is a method by which similar items could be grouped. A rule might then be established for the group with common parameters (reorder point, order quantity) used to control the inventory of each item in the group.

The task of calculating the economic order quantity and reorder point for each item is physically impossible. Moreover, the conventional reordering rules (fixed order quantity, fixed interval and the $\underline{S}$ and $\underline{s}$ reordering rules) are based on the assumption that the inventory level of each item is controlled separately, and that each item in inventory is ordered independently from others. This is frequently not the case. Fetter and Dalleck¹ recognize this limitation of most inventory models. They state:

If we now add items to our inventory management problem, the item-by-item computations for R and Q remain the same, but we are often faced with new difficulties. These stem primarily from the fact that the model assumes that each set of item decisions is independent, when in general, it is not. (R = reorder point, Q = EOQ)

Often, one supplier is the source of a variety of related items. Savings can be, and often are, achieved by including a number of such items on one order. Magee and Boodman² discuss this principle advantage of joint orders.

¹ Fetter, R.B. and Dalleck, W. Decision Models for Inventory Management (Homewood, Illinois: Irwin, 1961), p. 63

² Magee, J.F. and Boodman, D.M., op. cit., pp. 152-153

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For example, a field warehouse may obtain a large number of items from a single source. It may be desirable to have any shipment from the source to the warehouse equal an economical size, such as a carload, in total, but the mix of items in the order may not affect the cost of making the shipment measurable. Frequently, a distributor may wish to order a group of items supplied by a single vendor not all of which he needs at the present. He may order some items early from the vendor in order to take advantage of a variety of forms of discount offered, to reduce total order costs in those cases where the cost of an additional line on an order is less than the cost of a one-line order, or to meet a vendor constraint such as minimum order size.

Sometimes in manufacturing operations the cost of setting up a process may indicate the size of a total run or batch of an item, but the run can be split among a number of individual package sizes, etc. For example, in textile manufacture, it may be desirable to dye or print a large quantity of cloth which can be put up in a number of different-width bolts.

Another advantage of joint ordering is the increased opportunity of taking advantage of quantity discounts offered by a vendor. Thus, according to Prichard and Eagle,¹

...(another) benefit of joint ordering is the discount the vendor may offer on a large dollar purchase. Some suppliers offer price reductions on individual items bought in large quantities. Other suppliers allow significant discount on the total value of single large orders, even if many items, each of low value, are included in the order. There are situations in which a discount of this type becomes economically attractive only if several items are ordered jointly.

Hence, there are many instances in which it is worthwhile to treat items jointly rather than independently, and to order them as a group. The potential savings resulting from the

¹ Prichard, J.W. and Eagle, R.H., Modern Inventory Management (New York: John Wiley and Sons, Inc., 1965) p.360

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the inclusion of many items on one purchase order are quite large.

Silver¹ summarizes the advantages of joint orders.

1. A reduction in ordering costs may be achieved because several items are processed under a single order.
2. The supplier may offer a discount if an order exceeds a certain quantity. One way of achieving this discount is to lump several items under one order.
3. Shipping costs may be significantly reduced if an order is of a convenient size, e.g. a box car. It might be necessary to order several items simultaneously to achieve such a quantity.

The three basic reordering rules discussed earlier are therefore inappropriate. What is now required is some sort of a joint ordering rule. The term joint ordering as used here implies ordering a number of items in a single purchase order.

A review of the literature indicates that much analytical work has been done in the cases of independent ordering strategies. The rules most frequently discussed are those mentioned earlier. But the literature is almost void of discussion about a joint-ordering strategy. It is surprising to note that most general references about inventory theory do not mention this strategy. Some writers recognize the advantages of the joint-ordering strategy but fail to develop any joint-ordering rules.

Hence, though much has been written about inventory control generally and independent ordering rules in particular, there is a real need to research the joint-ordering rule. The

¹Silver, E.A. "Some characteristics of a special joint-order inventory model", (O.R. Vol. 13, No. 2, March-April, 1965) p. 319

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development of a joint-ordering rule seems an appropriate topic for a doctoral dissertation.

STATEMENT OF PROBLEM

The purpose of this study was to develop a joint-ordering rule which could be used to order a group of items at one time. The two important decisions in any single item inventory control system are the selection of the time to order, and the quantity to order. In the case of joint ordering, it is necessary to add a third decision, namely which items should be ordered jointly. It was believed that these basic decisions could be made if the joint ordering rule was defined in terms of three parameters, $\underline{S}$, $\underline{s}$, and $\underline{s}'$, where $\underline{S}$ is the maximum inventory level, $\underline{s}$ is the reorder point or trigger point, and $\underline{s}'$ determines the items to be reordered. Any item whose inventory level is equal to or less than $\underline{s}'$ should be ordered; $\underline{s}'$ to $\underline{s}$ is defined as the reorder range. Note that $\underline{S}$ is greater than $\underline{s}'$, which is greater than $\underline{s}$ (see Figure 1.2). It illustrates the variations in the inventory levels of three items. At time t_0 the inventory level of item 1 is less than $\underline{s}$, the reorder point. This triggers the reordering process. The inventory level of the rest of the items in the group is checked, and as is evident from the figure, the inventory level of item 2 lies between $\underline{s}'$ and $\underline{s}$, the reorder range. Hence, items 1 and 2 are ordered jointly. The inventory level of item 3 lies outside the reorder range. Hence, item 3 is not included in the joint order. It is assumed that the inventory on order for

each of the items is equal to zero.

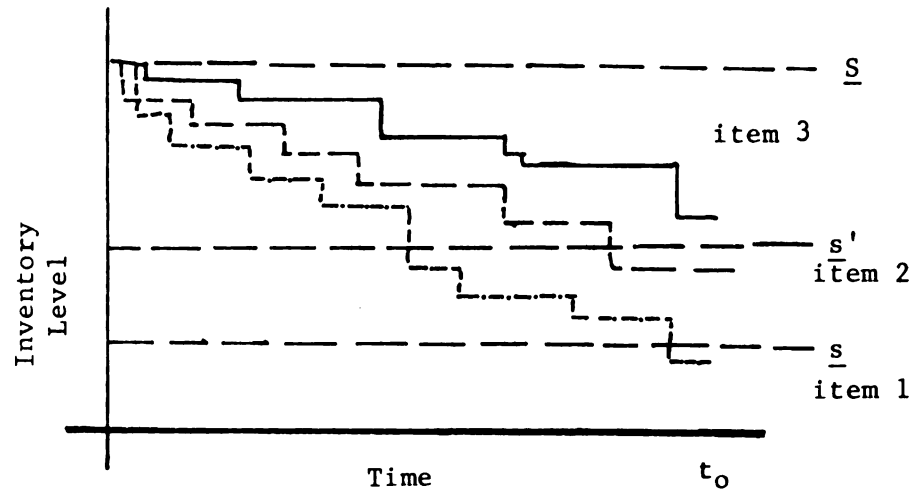


Figure 1.2: Variation in inventory level of three items

The joint-ordering rule proposed is:

Whenever the stock of any item in a group has dropped to the reorder level, the inventory level of all the items in the group is checked; all items with inventory levels in the reorder range ($\underline{s'}$ to $\underline{s}$) are ordered jointly.¹

In terms of the three parameters, $\underline{s}$, $\underline{s}$ and $\underline{s'}$ the rule is defined as follows:

When the inventory level of any item in the group has dropped to the reorder level $\underline{s}$, all items which have inventory levels (inventory on hand plus on order) within the reorder range $\underline{s'}$ to $\underline{s}$ are ordered jointly. The order quantity for

¹The writer wishes to state that the proposed joint-ordering rule is not "original". The basic idea of the rule was obtained from the article by Balintfy.

each item ordered is given by $(S-I)$ where I is the inventory on hand and on order.

The proposed joint ordering rule is an extension of the (s,S) rule discussed earlier. Instead of two parameters, S and s , the proposed rule has three parameters, S , s , and s' . It was necessary to introduce s' to determine which items to order jointly. Recall that in the case of the (s,S) system only two decisions, when to order, and how much to order had to be made. Hence, two parameters were sufficient. In the case of joint orders there are three decisions to be made, when to order, how much to order, and which items to order. Hence, the need for the third parameter s' . A detailed discussion of this rule is included in Chapter three.

HYPOTHESES

The following hypotheses were proposed for testing:

Major Hypothesis:

There exist a number of situations in which the application of the joint ordering rule, i.e. ordering a group of items in a single order, results in lower costs when compared with the use of the fixed order quantity rule.

Minor Hypothesis:

If the joint ordering rule is defined by the three parameters S , s , and s' , as defined above, then there exists some optimum value for each of the parameters such that the total



cost of the inventory control system is minimized. The costs that are included are ordering costs, inventory carrying costs, and stock out costs.

The major part of this research is concerned with testing these hypotheses and hence establishing a joint ordering rule.

There is one question that yet remains unanswered. The proposed joint ordering rule can be applied only after the items have been grouped. Hence, the immediate question is what criteria should be used to group the items. The following criteria are suggested.

1. Natural grouping: Some items can be grouped on the basis of their physical characteristics. For example, all screws of various lengths, diameters, and the number of threads per inch could be included in one group. In a farmer's cooperative visited by the writer, such a natural grouping did exist. All fertilizers were included in one group, all farm fences were included in another group, all kinds of animal feed were included in a third group, and so on. Thus, physical characteristics provide a very simple and natural way of grouping items.
2. On the basis of demand: The criteria suggested here is very similar to the conventional A, B, C

classification of items in inventory. Magee and Boodman¹ describe this classification as follows,

Close examination of a large number of multi- item inventories has revealed a useful statistical regularity in the distribution of the demand rates of the items in an inventory. It has been observed that item demand rates follow lognormal distribution; the logarithms of the item demands are normally distributed, that is, they fall into the well-known bell-shaped normal distribution pattern. Consequently, most items have relatively low demand and a few high demand; proportionately few items account for the major part of total demand.

The dispersion in demand rates suggests that high-volume items should be handled differently from low-volume items. One approach is to segment stock into what is called an ABC classification:

- Class A : The top 5 to 10 per cent of items, which account for the highest dollar inventory investment.
- Class B : The middle 20 to 30 per cent of items, which account for a moderate share of investment.
- Class C : The large remaining group of stock-keeping items, which accounts for a small fraction of total investments.

Thus, it is possible to group items on the basis of demand. In addition to the broad A, B, C classification, each class could be further sub-divided into groups. The joint ordering could then be applied to each group.

3. Nature of the source of supply: Frequently a number of items are supplied by a single vendor. Hence, all such items could be put into one group and

¹ Magee , J.B. and Boodman, D.M., op. cit., p. 156

ordered jointly so as to take advantage of a variety of forms of discount offered by the vendor. In the case of the farmers cooperative mentioned earlier, it was found that all farm fences came from a single supplier. This provides an additional reason for including all farm fences in one group.

4. Nature of production process. There are a number of production processes where the law of fixed proportions applies to a considerable extent. This is especially true in the chemical industry where for example one pound of item A would require two pounds of item B so as to get one and a half pounds of a product C. In this case item A and B could be included in one group. Similarly, in a manufacturing operation every nut used requires a bolt. Hence, nuts and bolts could be included in a group.

These are some of the methods that could be used to group items. The best one seems to be the grouping of items on the basis of demand. However, this or any other method could be used in conjunction with another. For example, in the farmers cooperative visited by the writer, the items were already classified on the basis of their physical characteristics (e.g. farm fencing, fertilizers, etc.). Each group of items was then subdivided into a number of smaller

groups on the basis of the expected demand for each item.

The joint ordering rule could then be applied to each smaller group.

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CHAPTER II

SURVEY OF LITERATURE

More work has been done in the area of inventory control than in any other area in production management. Starr and Miller believe that, "bookshelves are creaking in despair at the weight of volumes on this subject". There are literally hundreds of articles and volumes on the subject of inventory management. Some of these publications are devoted to the application of inventory theory to practical problems. But, an increasing number of researchers are working with inventory models because they present interesting theoretical problems in mathematics.¹

It would be almost impossible to list all the volumes and articles on the subject of scientific inventory control. Moreover most of these publications are not relevant to the research topic in particular. Hence, a general bibliography about inventory control is included. These publications have been selected on the basis of two criteria. In the first place they are well known and a serious student of inventory control should have knowledge about them. And secondly, some of the publications touch upon the chosen research area, joint ordering rules. However, a survey of

¹Hanssman, F. Operations Research in Production and Inventory Control (New York: John Wiley and Sons, Inc., 1962) p. vii

the literature indicated that on joint order inventory models in particular, there are very few publications. The discussion in this chapter will be restricted to publications relevant to this thesis.

Following is a list of joint order inventory models of specific interest.

Balintfy's Random Joint Order Model¹

Balintfy analyzed multi-item inventory problems, where joint order of several items might lead to savings in setup cost. He also developed a new ordering policy called "random joint order policy" which is characterized by a reorder range within which several items can be ordered. Balintfy described this policy as follows:¹

As is known, the individual orders in most random output systems are triggered by the inventory level, in particular by the reorder level of the items. This level or the corresponding point on the time scale, the reorder point, constitutes a dead line when the order for new items must be issued.

Now, it is easy to define another (higher) inventory level which may be called "can-order" level, or "can-order" point - - such that the range determined by difference between "can-order" and reorder points will replace the triggering role of the reorder point. The rule then would say that whenever an order for a particular item must be issued, i.e., the stock of any item has dropped to the reorder level, the inventory level of the rest of the items will be checked, and all items which are in the reorder range shall be ordered jointly.

¹Balintfy, J. L., "On a basic class of Multi-item Inventory Problems", (Management Science, Vol. 10, No. 2, January 1964) pp. 287-297

To arrive at an analytical solution to the multi-item inventory problem, Balintfy made certain assumptions. One assumption was that $\underline{T}$, the reorder period for an item, is defined as a negative exponentially distributed random variable with an expected value $\underline{T}_0$. Another assumption was, $\underline{T}_0$ for all items should be the same. This implies that in general all items are reordered everytime an order is placed. The author admits the limitation of his analysis because of these and other assumptions. According to him,¹

These restrictions were needed to clear the way for the application of [equation] (28). [The equation gives the probability distribution of the number of joint orders] Their (assumptions) removal means that we have to attempt to solve a machine interference problem with general service time distributions, non-Poisson arrivals, and different arrival rates in different channels. It is a known fact that an analytical solution has not been found for this problem thus far. Yet, it cannot be overlooked that most of the practical problems will fall in this category.

He then goes on to suggest that one way to determine reorder ranges in the general case, is to resort to simulation techniques.

²

Magee and Boodman's Multi-item Model

Magee and Boodman analyzed the multi-item problem as a production problem. That is, they were interested in determining how a single production run could be split among a number of

¹ Balintfy, J.F. op. cit., p. 296

² Magee, J.F. and Boodman, D.M. Production Planning and Inventory Control (New York: McGraw-Hill Book Company, 1967) pp. 152-157

individual items or package size. This problem is faced by almost all manufacturing operations. For example, in chemical manufacture, process economies may determine the total size of a batch of an item to be produced, but this batch could be split among a number of different container sizes or package sizes. Magee and Boodman suggest procedures to determine when to produce a batch and how to balance the batch or shipment among individual items. The procedure is as follows:¹

1. A batch, run, or shipment must be started to be completed before any individual item runs out.
2. The sum or the amounts of the individual items made or shipped must equal the total desired economical batch or order.
3. The quantity made or shipped should be balanced among individual items to delay need for the next run or shipment as long as possible.

An approach that can be used follows these lines:

1. A reorder point is set on each individual end item, e.g. each package size. This is set in the usual way, to cover maximum demand or to give the desired service protection on the individual item over procurement lead time.
2. A new run or shipment is made whenever the inventory on hand or in process of an individual item reaches a reorder point.
3. The new run or shipment is distributed among the individual items as follows:

Let I_i = inventory on hand or in process of one item,
the i^{th} item.

¹Magee, J.F. and Boodman, D.M., op. cit., p. 153

p_i = reorder point, the i^{th} item

s_i = expected usage rate, the i^{th} item

Q_i = amount of the run or shipment given over
to the i^{th} item

Q = total run or shipment size

I = total inventory, all items

P = total of the individual reorder points

S = total of the individual usage rates

Then the amount of any individual product shipped
would be

$$Q_i = \frac{s_i(Q + I - P)}{S} - I_i + P_i$$

Magee and Boodman have analyzed the multi-item reorder problem as a production problem rather than an inventory control problem, although the model could be used for multi-item inventory control. However, the model has certain limitations. In the first place, they do not indicate how the formula for Q_i was derived. It could be based on experience. Further, the model lacks details, and this severely limits the application of the model. For example, the authors do not indicate how Q , the total shipment size would be determined. Lastly, though they do illustrate how this procedure works, they fail to demonstrate any economic advantage of their procedure over the conventional individual item ordering rules.

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Maynard's Group Ordering Rules¹

Maynard refers to the multi-item reordering rule as Group Ordering Rules. His version of the group-ordering rule is as follows:¹

All the items in a group are coded to designate the particular ordering group, and each item is assigned an individual order point and order quantity. In addition, a total order quantity is assigned for the group of items. When the stock on hand and previously on order of any one of the items in the group falls to its order point, a new order is placed. The order is made up by ordering the established order quantity for each item in the group, starting with the item that reached its order point and adding items that are near their order points until the size of the over-all order reaches the assigned total order quantity.

This joint ordering rule seems to be heuristic. The author does not go into details about how the total order quantity for the group, or the individual order quantities for each item within the group, can be determined. Opportunities for obtaining quantity discounts were not considered, although this may be the principal advantage of the joint ordering rule. Lastly, Maynard did not indicate what cost savings will result when this rule is compared with the conventional individual item ordering rules.

Prichard and Eagle - The Economics of Joint Orders²

¹Maynard, H. B. Industrial Engineering Handbook (New York: McGraw-Hill Book Company, 1956) pp. 7-64

²Prichard, J. W., Eagle, R. H. Modern Inventory Management (New York: John Wiley and Sons, Inc. 1965) pp. 360

Prichard and Eagle have by far the best analysis of joint orders. They include a discussion about the advantages and disadvantages of joint orders, and then by a numerical illustration have demonstrated the cost savings resulting from joint orders. They considered a group of four items ordered from one supplier and three ways of controlling inventories of the four items. The three inventory control methods were:

- a) Periodic review and separate replenishment method.
- b) Separate replenishment with continual review method.
- c) Joint replenishment with periodic review method.

The authors then compared the total cost of the three alternative methods of controlling the inventory of the four items.

The analysis is excellent because it takes into account a number of details which surprisingly lead to significant changes in total costs. Some of the details considered were,

- a) A portion of the total ordering cost was assumed to depend on the number of items on the order.
- b) The safety-stock required under each replenishment policy was calculated so as to minimize the value of back-ordered demands. The safety stock required turned out to be different for the different methods of controlling inventory.
- c) Quantity discounts on large orders were included.

The calculation of the safety-stock required under each replenishment policy led the authors to conclude that¹

The increased chance of shortage due to the higher frequency of replenishment coupled with the need to guard against demand variation over the review cycle and during lead time leads to a much higher safety stock for joint replenishment than for separate replenishment.

On the basis of a numerical illustration, the authors finally concluded that in general joint replenishment is cheaper than separate replenishment.

The analysis has one serious limitation. It is assumed implicitly that all items are ordered every time a joint order is placed. That is, every replenishment order is a joint order and includes all four items. This does not seem necessary. It should be possible to achieve additional savings through decrease in total inventory by ordering some items less frequently than others. The joint ordering rule proposed in this thesis is not limited by Prichard and Eagle's assumption. This is because, according to the proposed joint ordering rule, when an order is placed, only those items which have inventory levels within the reorder range (s' to s) are included in the order.

Silvers's Two Item Rule²

Silver analyzed a joint-order rule involving two items.

¹Prichard, J. W., Eagle, R. H., op. cit., pp. 363

²Silver, E. A. "Some characteristics of a Special Joint-Order Inventory Model" (O.R. Vol. 13, No. 2, March-April, 1965) pp. 319-322

The two items were assumed to have identical characteristics, including unit Poisson demand. The joint ordering in this case was to order both items up to a level Q each time the level of either item dropped to zero. The author compared his joint ordering rule with the alternative, where the two items are ordered separately. He finally concluded that,¹

For any given value of $\frac{A\lambda}{cr}$ [where A is the cost of placing an order for one item, c is the unit cost of an item, r is the inventory carrying cost, and λ is the mean demand for each item] there is a critical value of m [mA is the cost of placing an order for two items, where $1 \leq m \leq 2$] below which it is preferable to use joint ordering, above which we prefer independent ordering.

.... for example, for $\frac{A\lambda}{cr}$ equal to 5, the critical m is 1.23. Therefore, in this case, if the cost of placing a joint order is less than 1.23 times the cost of placing an order for a single item, we use a joint ordering strategy.

Silver's analysis is useful and well conceived, but it too suffers from certain limitations. They are,

1. The analysis considers only two items.
2. The two items are assumed to have identical cost characteristics and unit Poisson demand. Each time an item is demanded, it is assumed that the demand is for one unit. In addition to this, the mean demand for the two items is assumed to be identical.
3. The reorder point is zero as the delivery lead time is assumed to be zero, i.e., an order is assumed to arrive instantaneously.

4. It follows from (3) that there are no stockouts and hence the cost of lost sales is not considered.

These assumptions indicate, that the analysis is of limited theoretical significance. It may be that the joint ordering problem was analyzed by Silver because it presents an interesting mathematical problem. It is doubtful that his joint ordering rule could be applied given his constraints.

Starr's Constrained Control of Multiple Items¹

Starr views the inventory control of multiple items as a constrained problem, the constraints being limited company resources or limited capacity of the ordering department. According to him²,

The company's resources are limited. It is frequently unreasonable to carry the total average dollar inventory that the individual item's optimal policies would require. The capacity of the ordering department may be overtaxed, storage facilities may be filled to capacity; the amount of capital invested in inventory may exceed the amount that the company has available. These limitations, if they exist, require a modification of inventory policy. That is, the theoretical system's optimal is not feasible because it violates other practical system's constraints.

The author then developed a procedure for handling such problems. The problem that is analyzed by Starr is different from that considered in this thesis. The principal difference being that there are no constraints to the problem dealt with

¹Starr, Martin, K. Production Management (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1963) pp. 321

²Starr, M. K., op. cit., pp. 323

in this thesis. In this thesis it was assumed that there was both sufficient capital available to invest in inventory and sufficient storage facilities to store the goods. Lacking such constraints, the problem was simply to develop a joint ordering rule that minimized total costs. Starr, on the other hand, assumed that there were certain limitations, for example, limited company resources available for inventory which require a modification of the inventory policy. That is, it was not feasible to order the economical order quantity for each item because it led to a total average inventory investment greater than that available. Starr, then developed a procedure for calculating what the order quantity for each time should be under the constrained conditions. As is evident, the problem is clearly different from that analyzed in this thesis.

Fetter and Dalleck: Managing Multi-item Inventory¹

Fetter and Dalleck's analysis of the management of a multi-item inventory is similar to Starr's. Like Starr, these authors developed a least cost solution in which the total number of orders, setup, and total order cost or setup cost were restricted in some way. The problem was to find the order quantity for each item under these constrained conditions. Fetter and Dalleck's solution to this problem was very similar to Starr's. As mentioned earlier, the problem analyzed

¹Fetter, Robert B., and Dalleck, W. Decision Models for Inventory Management (Homewood, Illinois: Irwin, 1961) p. 63

in this thesis was not a constrained problem. The problem was simply to develop a joint ordering rule that minimized total costs.

Starr and Miller: Multiple Items, One Supplier¹

Starr and Miller considered the problem of joint orders under the heading, Multiple Items from one supplier. They took an analytical approach to the problem. However, their joint ordering rule was very similar to Maynard's Group Ordering Rules mentioned earlier.

They considered all the items in the group as representing one unit, and calculated the ordering policy for this one unit. That is, they developed an EOQ formula when the group of items was considered as one unit (the EOQ formula was developed in terms of t , the months between orders). They then compared the total cost obtained by this approach with the total costs when each item was "optimized separately", i.e. when each item was considered separately and the order quantity was determined so as to minimize the total costs for each item. Lastly, Starr and Miller analyzed the particular case when one of the items in the group accounted for a large proportion of the total dollar demand for the group of items, or, when the items were divided into two sub-groups and each sub-group was optimized separately. They concluded that a criterion could be set up which would

¹Starr, M.K. and Miller, D.W. Inventory Control: Theory and Practice (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1962) pp. 104-110



determine when the $\underline{t}$ (months between orders) should be the same for all items. For example, the criteria as to when the items should be divided into two groups was as follows:

Let,

the total number of items be = $\underline{n}$

the total dollar demand for these items be = $\underline{D}$

Let the cost of preparing an order be equal to a fixed cost $\underline{c}$ and a variable cost which depends on the number of items in the order.¹ The variable cost accounts for the cost of physical inspection, cost of pulling out a stock card, etc., and also depends on the number of items in the order. Let the review cost for the individual item be expressed as a multiple of the fixed cost, $\underline{h} \times \underline{c}$, where $\underline{h}$ is a constant.

Let the items be divided into two groups, $\underline{A}$ and $\underline{B}$.

The number of items in group $\underline{A}$ = $\underline{g}$.

Therefore, the number of items in group $\underline{B}$ = $(n - g)$.

Assume that group $\underline{A}$ accounts for a fraction $\underline{a}$ of the total dollar demand.

Therefore,

¹These assumptions were made by Starr and Miller in their analysis.

Group A has a total dollar demand = $a \times D$

Group B has a total dollar demand = $(1 - a)D$

Starr and Miller's criteria for determining when the items should be divided into two groups was as follows:¹

$$a > \frac{gh + 1}{nh + 1}$$

Hence, whenever this inequality was true for some g , then t , the months between orders should be determined separately for each of the two groups, A and B. If, the inequality was not true, then it would be more economical to use the same t for all items.

One other interesting conclusion reached by the authors was:²

....that the potential savings resulting from the incorporation of many items on one order are generally quite large. Of course, most companies which are able to amalgamate items on single orders automatically do so. However, they do not necessarily do so in the optimal way, so further savings can be accomplished by utilizing the methods we have been developing (in this chapter). The point for future reference is that, since this is so, any theoretical inventory system which does not permit such multiple item orders is incurring a cost of the kind we called, (in the first chapter) systemic costs.

The analysis is sound and useful, but it too has its limitations. They are:

¹ Starr, M.K. and Miller, D.W. op. cit., p. 108

² Starr, M.K. and Miller, D.W. op. cit., p.109

1. The analysis assumes that demand for a particular item is constant over time, i.e. certainty is assumed. Hence, inventory carrying costs and ordering costs were the only costs that were included in the total cost. The cost of lost sales is not considered.
2. Although an EOQ for the group as a unit can be calculated, the problem as to how much of each item within the group should be ordered so that the total order is equal to the EOQ for the entire unit remains unsolved.
3. The authors calculate the value of t , the number of months between orders, which is assumed to be the same for all items within the group. Hence every item is ordered every t months. This is not necessary. There may be instances where it may be more economical to order some items more often than others.

Thus, while Starr and Miller's analysis is impressive it does have theoretical limitations and hence, restricted application value.

Thus, the survey of the literature indicated that although there are definite advantages of the joint ordering over the conventional independent ordering strategies, no such rule has been developed. The few joint ordering models that were proposed have theoretical limitations and hence, restricted application value. In some instances the joint ordering problem was analyzed because it represents an interesting mathematical problem.

CHAPTER III

ANALYSIS OF THE MODEL

Before getting into the experimentation of the model, it was felt that it would be worthwhile at this stage to analyze the model in some detail. The purpose of such an analysis was to get a better understanding of the proposed joint ordering rule.

SPECIFICATION OF THE RULE

The joint ordering rule was defined in terms of three parameters, $\underline{S}$, $\underline{s}$, and $\underline{s}'$, where $\underline{S}$ is the maximum inventory level, $\underline{s}$ the reorder point or trigger point, and $\underline{s}'$ determines the items to be included in the joint order. $\underline{s}'$ to $\underline{s}$ was defined as the reorder range. Note that $\underline{S}$ is greater than $\underline{s}'$, which is greater than $\underline{s}$ as shown in the figure below.

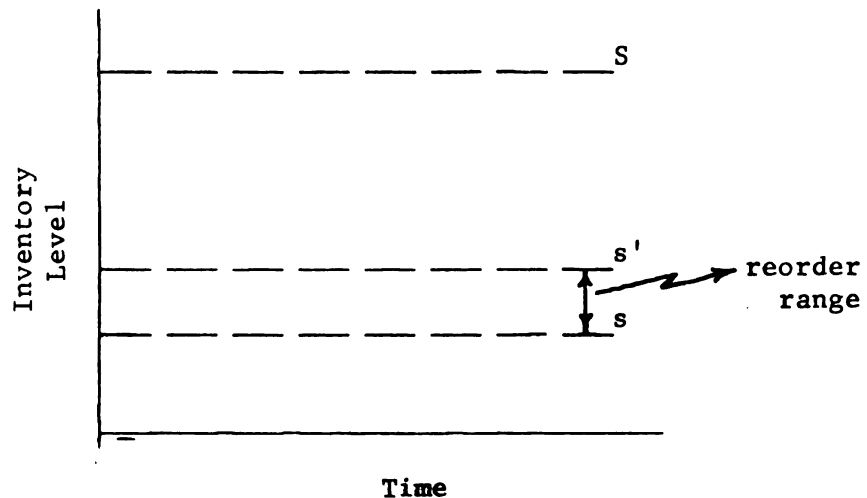


Figure 3.1: The Joint Ordering Rule characterized by three parameters, $\underline{S}$, $\underline{s}$ and $\underline{s}'$.

The proposed joint ordering rule was stated as follows:

Whenever the stock of any item (inventory on hand and on order) in a group has dropped to the reorder level $\underline{s}$, the inventory level of the rest of the items in the group is checked and all items with inventory levels in the reorder range $\underline{s}'$ to $\underline{s}$ are ordered jointly. The quantity of each item ordered is given by $(S - I)$ where $\underline{I}$ is the inventory on hand and on order.

In general, joint ordering will lead to an increase in inventory carrying costs. This is because some items in the group, which have inventory levels between $\underline{s}'$ and $\underline{s}$, the reorder range, are ordered although they should be ordered when the inventory level reached the reorder point $\underline{s}$. This more frequent ordering of some items before the reorder point is reached, leads to an increase in inventory carrying costs.

On the other hand, joint ordering will lead to a savings in ordering costs because a number of items are included in a single order. Further, since a number of items are included in a single order, the total dollar value of the order is larger. This provides an opportunity to take advantage of quantity discounts.

The inventory carrying cost and the ordering cost will depend on the size of the reorder range. An increase in the width of the reorder range will increase the probability of joint orders. This will lead to a savings in ordering costs.

But the same effect (increasing the width of the reorder range) will increase the average inventory level due to the "lifted trigger levels of some items."¹ This suggests that an optimum value of the three parameters $\underline{S}$, $\underline{s}'$, and $\underline{s}$ exist and can be found. The optimum will occur when the difference between the savings due to joint orders and increased costs due to increased inventory is maximum.

PROPERTIES OF THE MODEL

The proposed joint ordering rule was characterized by three parameters, $\underline{S}$, $\underline{s}$, and $\underline{s}'$, where $\underline{S}$ is greater than $\underline{s}'$, and $\underline{s}'$ is greater than $\underline{s}$. Hence, by definition $\underline{s}'$ lies between $\underline{S}$ and $\underline{s}$, and can have a maximum value of $\underline{S}$ and a minimum value of $\underline{s}$. Hence, four distinct cases can be identified depending on the value of $\underline{s}'$.

Case 1 : When $\underline{s}' = \underline{S}$

By definition, the reorder range $\underline{s}'$ to $\underline{s}$ determined the number of items that would be included in a single order, i.e. ordered jointly. The larger the size of the reorder range, the greater the number of items that will be ordered jointly. But for a fixed value of $\underline{s}$, the size of the reorder range is determined by $\underline{s}'$. The closer $\underline{s}'$ is to $\underline{S}$ rather than $\underline{s}$, the larger the size of the reorder range.

¹Balintfy, J.L., "On a basic class of multi-item inventory problems" (Management Science, Vol. 10, No. 2, January 1964) p. 292

Since, by definition $\underline{s}'$ can have a maximum value of $\underline{S}$, the reorder range would be maximum when $\underline{s}'$ is equal to $\underline{S}$. Further, since the maximum inventory for any item in the group is equal to $\underline{S}$ (this includes inventory on hand and on order), all items will have inventory levels (inventory on hand and on order) between $\underline{s}'$ (equal to $\underline{S}$) and $\underline{s}$. That is, all items will have inventory levels within the reorder range $\underline{s}'$ to $\underline{s}$ and hence, all items will be ordered every time a joint order is placed. The joint ordering rule now becomes,

Whenever the stock of any item in a group has dropped to the reorder level, all items will be included in the order.

The three parameter joint ordering has now been reduced to a two parameter case, the two parameters being $\underline{S}$ (or $\underline{s}'$) and $\underline{s}$. This situation is illustrated in Figure 3.2.

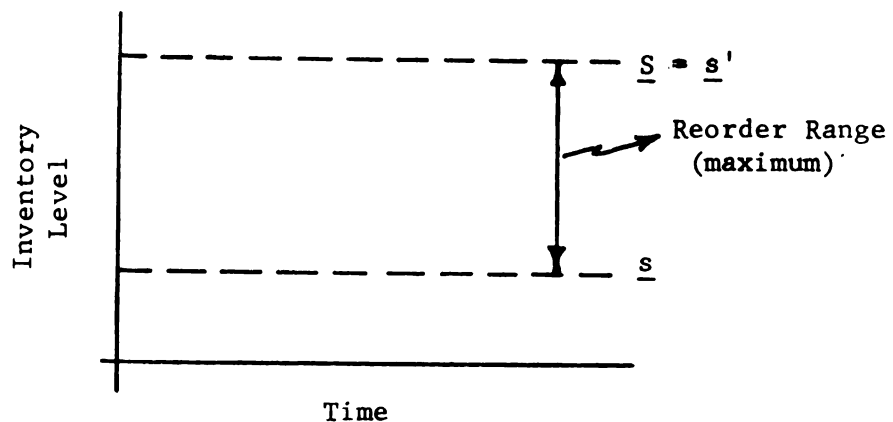


Figure 3.2: A special case of the Joint Ordering Rule. The Rule is now characterized by two parameters $\underline{S}$ (or $\underline{s}'$) and $\underline{s}$.

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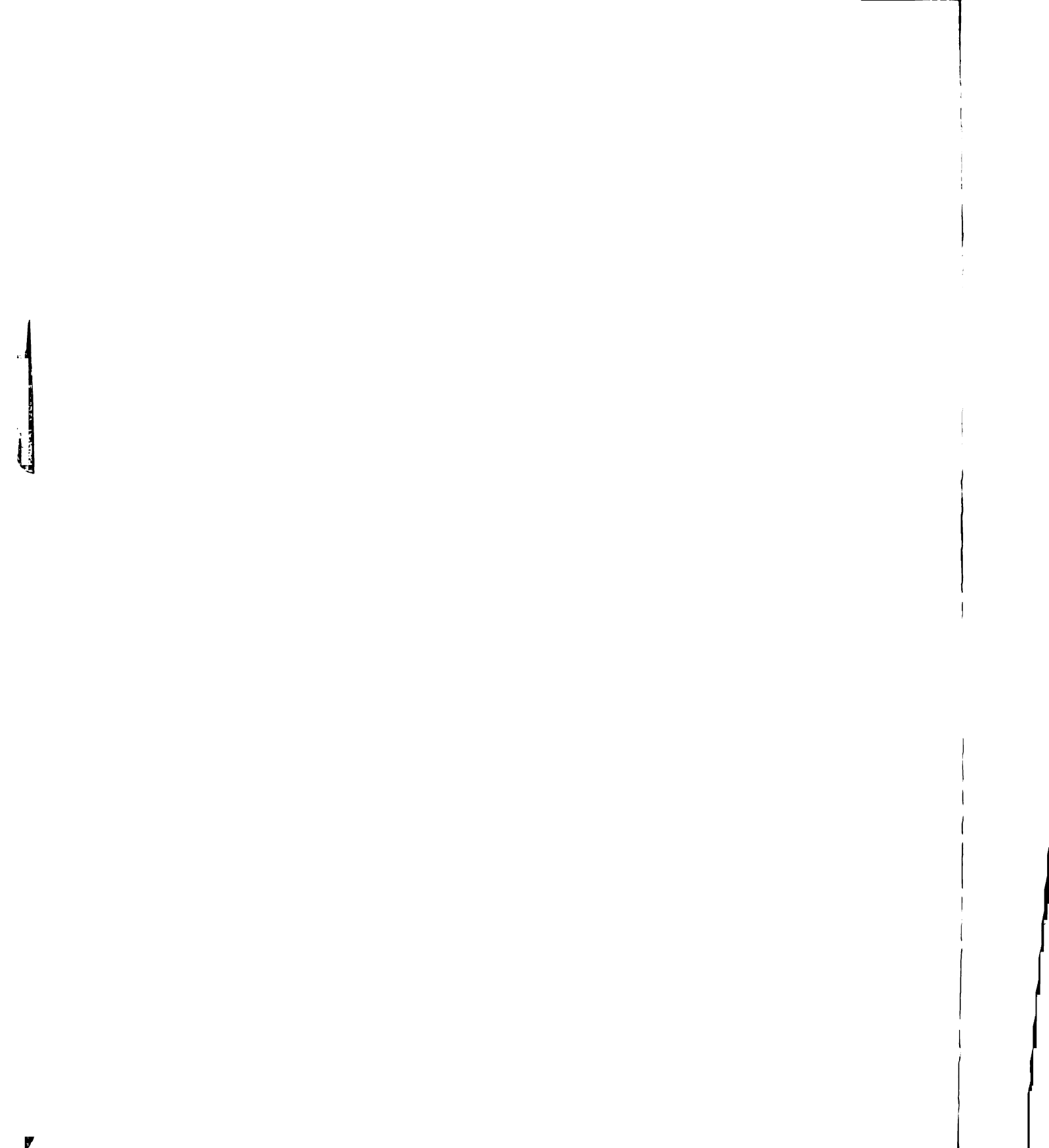
Next, under what conditions will $\underline{s}'$ be equal to $\underline{S}$? This situation, namely $\underline{s}'$ equal to $\underline{S}$, such that all items are included in every order, will arise when the mean demands for the items making up the group are almost equal. That is, the dispersion of the mean demand is relatively small.¹ If the standard deviation of mean demands is used as a measure of this dispersion, and if the standard deviation is relatively small, it would be more economical to order all items whenever an order is placed. This is because, if the mean demands are almost equal, and if the inventory carrying cost is assumed to be almost the same for all items, then the reorder points and economic order quantities for all the items in the group will be almost equal.

Case 2: When $s' = s$

The other extreme case will occur when $\underline{s}'$ is equal to $\underline{s}$. Hence, the reorder range $\underline{s}'$ to $\underline{s}$ will be minimum and equal to zero. The situation is illustrated in Figure 3.3.

When the reorder range is equal to zero, it means that only those items which have inventory levels less than or equal to $\underline{s}$, the reorder point, will be included in the order. The joint ordering rule now becomes,

¹The term "relatively" small or "relatively" large is used because no single figure can be specified. The next section in this chapter includes a discussion of what factors determine the range of the standard deviation of the mean demands within which each of the four cases will occur.



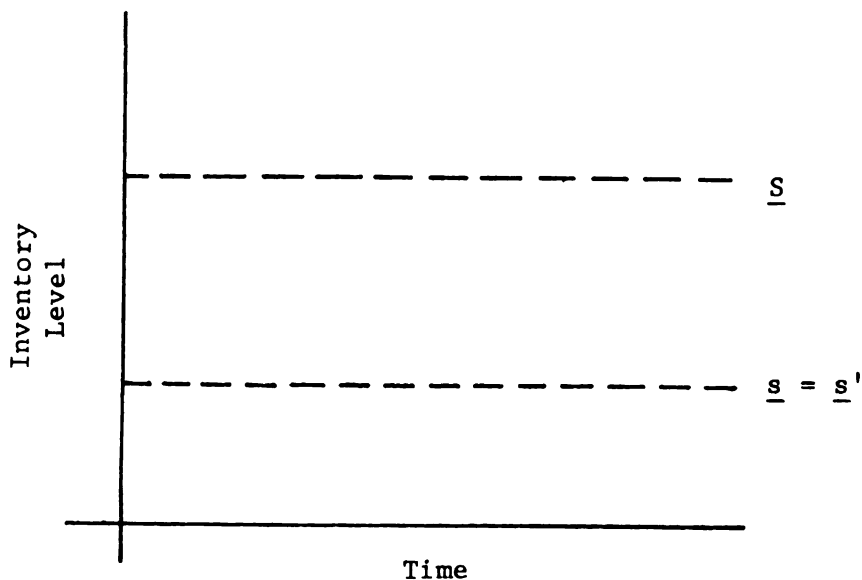


Figure 3.3: Another special case of the Joint Ordering Rule. The Rule is characterized by two parameters $\underline{S}$ and $\underline{s}$ (or $\underline{s}'$).

Whenever the inventory level of any item in a group has dropped to the reorder level, that item alone is reordered.

As in case (1) the three parameter rule is reduced to a two parameter case, the two parameters being $\underline{S}$ and $\underline{s}$ (or $\underline{s}'$). Moreover, the joint ordering rule is now very similar to the simple individual item rule, namely the $(\underline{s}, \underline{S})$ rule or the familiar two bin inventory control system. There is however, one important difference between the simple $(\underline{s}, \underline{S})$ rule and the proposed joint ordering rule in the modified form $(\underline{s}'=\underline{s}, \underline{S})$. In the $(\underline{s}, \underline{S})$ system, the parameters $\underline{S}$ and $\underline{s}$ are computed for each individual item, and these parameters are then used to

control the inventory of the respective items. For example, if there were twelve items in the group, it would be necessary to compute twenty-four parameters, two for each item. But in the proposed joint ordering rule, in the modified form ($\underline{s}' = \underline{s}, \underline{S}$), the values of the two parameters $\underline{s}$ and $\underline{S}$ are common to all items in the group. Hence, unlike the ($\underline{s}, \underline{S}$) system, only two parameters are sufficient to control the inventory of the entire group.

Next, under what conditions will $\underline{s}'$ equal $\underline{s}$? $\underline{s}'$ will equal $\underline{s}$ when it is more economical to control the inventory of each item separately. This in turn will occur when mean demand for the items is widely dispersed. That is, when the standard deviation of the mean demands for the items is relatively large. This means that there are some slow moving items, some medium selling items, and some fast moving items in the group. Hence, if the economical order quantity and the reorder point were calculated for each item, they would be so widely dispersed that it would be more economical to treat the items individually.

Case 3: When $\underline{S} > \underline{s}' > \underline{s}$

The two extreme cases were those mentioned under case (1) and case (2). In case (1), $\underline{s}'$ was equal to $\underline{S}$, and the reorder range $\underline{s}'$ to $\underline{s}$ was maximum. In case (2), $\underline{s}'$ was equal to $\underline{s}$ and the reorder range $\underline{s}'$ to $\underline{s}$ was minimum. In case (3), $\underline{s}'$ is neither equal to $\underline{S}$ nor $\underline{s}$ but lies between $\underline{S}$ and $\underline{s}$, and the statement of the joint ordering rule remains the same as that proposed in Chapter 1 of this thesis. The rule is:

Whenever the stock of any item in a group has dropped to the reorder level $\underline{s}$, all items which have inventory levels in the reorder range $\underline{s}'$ to $\underline{s}$ are ordered jointly.

As is probably evident, this situation will arise when the mean demands for the items are neither widely dispersed as in case (2), nor are the mean demands almost equal as in case (1). In short, the dispersion of the mean demands would lie somewhere between the two extremes. If the standard deviation of mean demands is used as a measure of dispersion, then case (3) would occur when the standard deviation would neither be too large nor too small, but somewhere in between.

Case 4:

There is another situation that is conceivable. In some situations it may not be economical to use the joint ordering rule in any form what so ever. That is, it would be more economical to control the inventory level of each item independently of the others, using one of the three independent ordering rules discussed in Chapter 1. This will occur when the mean demands of the items are so widely dispersed that including the items in a single group is also questionable. Hence, the standard deviation of demands for the items will be extremely large.

The figure below summarizes the four cases discussed so far. Using the standard deviation of mean demands for the items as a measure of dispersion, the figure illustrates the four



cases that are possible.

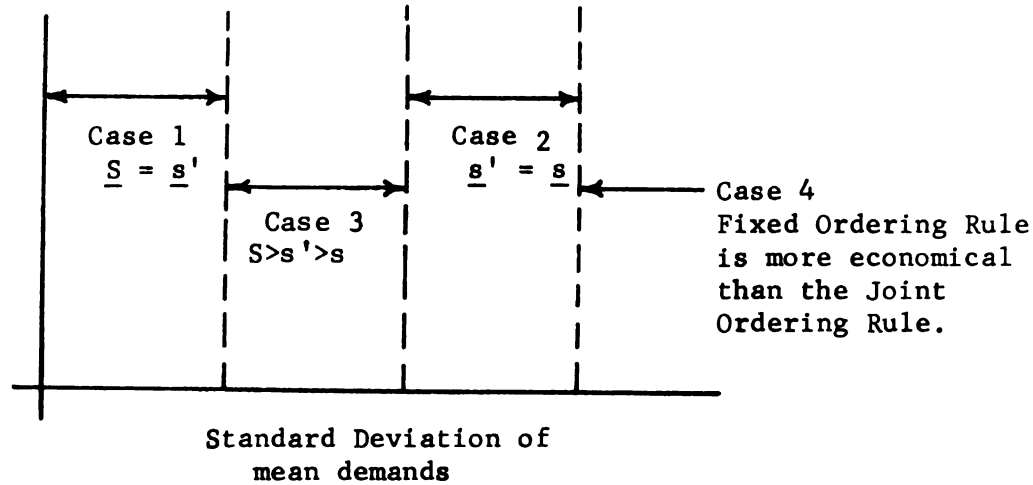


Figure 3.4: A summary of the four distinct cases of the Joint Ordering Rule.

Range of the Standard Deviation

In the last section four cases were identified, depending on the value of the standard deviation of mean demands of the items. For example, in case (1) the minimum cost values of the three parameters $\underline{S}$, $\underline{s}'$ and $\underline{s}$ would be such that $\underline{s}'$ equalled $\underline{S}$ and the reorder range was maximum. This would occur when the standard deviation of the demands of the items was "relatively" small. In fact the term relative was used to define the order of magnitude of the standard deviation in case (2), case (3), and case (4). No attempt was made to specify a specific range for the standard deviation within which case (1) would occur.

It is almost impossible to specify a specific range of the standard deviation for each of the four cases. But a number of

factors which do affect the magnitude of the range can be identified.

1. Nature of the demand distribution: The most important factor that will influence the magnitude of the range of the standard deviation for each of the four cases, is the probability distribution of demand for each item. For example, the magnitude of the range of the standard deviation will be different when the demand for each item is normally distributed than when the demand has a poisson distribution.
2. Inventory carrying costs: The joint ordering rule leads to an increase in average inventory. Hence, the lower the inventory carrying cost per unit per year, the more economical it is to carry more inventory and hence order items more frequently by including a larger number of items in a single order. This could be done by increasing the size of the reorder range $\underline{s}'$ to $\underline{s}$, as the size of the reorder range determines the items that should be included in an order. Hence, keeping all other conditions the same, the size of the reorder range (at minimum total cost) will be greater when the inventory carrying cost per unit is decreased. This means that the range of standard deviation of mean demands under case (1) will be greater, the greater the inventory carrying cost. Thus, the inventory carrying cost will

have a considerable influence on the magnitude of the range of the standard deviation of demands in question.

3. Ordering costs: The greater the ordering cost, the greater the savings by including a number of items in a single order. This can be achieved by increasing the size of the reorder range $\underline{s}'$ to $\underline{s}$. Hence, assuming that all other conditions remain the same, the size of the reorder range (corresponding to minimum total cost) for Case 1 will be greater, the greater the cost of re-ordering. In short, the range of the standard deviation of demands under each of the four cases will be influenced by the magnitude of the ordering cost.

4. Cost of lost sales: The greater the penalty for unfilled demands, the greater the savings that can be achieved by maintaining a higher level of inventory, or more correctly by raising the reorder point $\underline{s}$. This is exactly what happens when a large number of items are included in a single order. This is because some items in the group which have inventory levels within the reorder range, are ordered although they should be ordered when the inventory level reached the reorder point $\underline{s}$. In short, the effect of joint orders results in "lifted trigger levels for some items."¹ Hence, the greater the size of the reorder

¹ Balintfy, J.L., op. cit. p. 292

range the less the losses through unfilled demands.

The cost of lost sales therefore effects the size of the reorder range, and hence the magnitude of the range of the standard deviation within which each of the four cases would occur.

These are the important factors that determine exactly what the term "relative" used throughout the four cases means. No specific values that are universally valid can be determined. In this thesis, demand was assumed to be normally distributed, and certain values for the ordering cost, inventory carrying cost, etc. were assumed. Under these specific assumptions, the range of the standard deviation for each of the four cases was determined.

CHAPTER IV

EXPERIMENTATION

This chapter is divided into three sections. The first section considers the development of the model, and experimentation with it. Since a simulation approach was adopted, the section includes an explanation as to why this particular approach was preferred. The second section considers the nature of the data used to test the model, and the sources of the data. The last section discusses the methodology and the detailed steps taken in the research.

DEVELOPMENT AND EXPERIMENTATION OF THE MODEL

On the basis of the literature search, there were basically two approaches to the development of the joint ordering model.

The analytical approach: This approach begins with the identification of the important, exogeneous variables, endogenous variables, parameters, etc. Mathematical relations between these variables and parameters are then established. The system is then said to be described in terms of a set of mathematical equations. A solution is then obtained by solving the equations analytically, using such techniques as calculus, algebra, etc.

In general, analytical techniques are most suitable for

solving deterministic models. Naylor¹ defines the term deterministic models and explains why deterministic models can be solved analytically.

In deterministic models, neither the exogenous variables nor the endogenous variables are permitted to be random variables, and the operating characteristics are assumed to be exact relationships rather than probability density functions. Deterministic models are less demanding computationally than stochastic models, and can frequently be solved analytically by such techniques as the calculus of maxima and minima. Most of the traditional models in microeconomic theory are deterministic models in which complete certainty is an implicit assumption.

Although simulation, and in particular Monte Carlo analysis can be used to obtain solutions to strictly deterministic models..., in most cases analytical techniques are more efficient, computationally speaking, than simulation models...

The Simulation approach:

Simulation is essentially a technique that involves setting up a model of a real system, and then performing experiments on the model so as to obtain data about the probable behavior of the real system.

In general, simulation techniques are most suitable for solving stochastic models which are more complex than deterministic models. According to Naylor²,

Those models in which at least one of the operating

¹Naylor, T.H., Balintfy, J.L., Burdick, D.S., and Chu, K. Computer Simulation Techniques (New York: John Wiley and Son, Inc., 1966) p.16

²Naylor, T.H. et. al. op. cit. p.17

characteristics is given by a probability function are said to be stochastic models. Because stochastic models are considerably more complex than deterministic models, the adequacy of analytical techniques for obtaining solutions to these models is quite limited. For this reason, simulation is much more attractive as a method for analyzing and solving stochastic models than deterministic models.

Each of these approaches has its advantages and shortcomings. It would be worthwhile at this stage to discuss these very briefly.

Advantages and Disadvantages of the analytical approach:

The analytical approach is very general, precise, and more accurate than the simulation approach. It is general because it applies to a whole class of problems. Its generality is its principle advantage. It is precise because all definitions, functions, and relationships are stated unambiguously. Lastly, the solutions to problems obtained analytically are also unambiguous and accurate.

However, mathematical analysis is often not powerful enough to yield general analytical solutions to situations as complex as are encountered in business. Often, the observed system is so complex that it becomes almost impossible to describe it in terms of mathematical equations. For example, it is virtually impossible to describe the operations of a firm or industry (at the micro level) in terms of a set of algebraic equations. And even though such a mathematical model can be formulated, it may be almost impossible to obtain a solution

to the model by straight forward analytical means. Complex queuing problems provide examples of these kinds of difficulties. Queuing problems are both difficult to set up in terms of algebraic equations, and even more difficult to solve the set of equations to obtain an analytical solution. Buffa¹ believes that most real queuing problems are beyond an analytical solution.

The main limitation of the practical application of waiting line theory, is the fact that existing formulations for standard mathematical distributions often do not fit the actual distributions of arrival and service rates in specific real problems. The mathematical complexity increases with nonstandard distributions and as the basic problem departs from the simple single-channel situation to multichannel problems in tandem. Fortunately, these more complex waiting line problems can be handled by the general techniques of simulation, regardless of how complex they might be mathematically.

Advantages and Disadvantages of the Simulation approach:

The principal reason for choosing computer simulation, is it's ability to overcome the above mentioned difficulties. It is possible to avoid the complex mathematics, and yet get fairly accurate solutions to problems using a simulation approach. Schmidt and Taylor² list four advantages of simulation.

1. The model of a system once constructed, may be

¹Buffa, E. Modern Production Management (New York: John Wiley and Sons, Inc., 1962) p. 67

²Schmidt, J.W. and Taylor, R.E. Simulation and Analysis of Industrial Systems (Homewood, Ill.: Irwin, Inc., 1970) p.5

- employed as often as desired to analyze different situations.
2. Simulation methods are handy for analyzing proposed systems in which information is sketchy at best.
 3. Usually data for further analysis can be obtained from a simulation model much more cheaply than it can from the real world system.
 4. ---simulation methods are often easier to apply than pure analytic methods, and hence, can be employed by many more individuals.

Buffa¹ refers to another very important advantage of simulation. He believes that for the decision maker, the ideal situation is to be able to try out an idea without first risking or committing company funds. With a simulation model and a computer, the manager can try out dozens of alternatives. Thus, simulation, with the aid of high speed computers, makes available an "experimental laboratory" to management personnel. Another advantage of computer simulation is that through simulation, one can study the effects of certain changes (informational, environmental, etc.) on the system, by making alterations on the system's behavior. At the same time, no changes are made in the actual physical system itself. Further, the effect of all such changes can be studied within a matter of minutes.

Although simulation seems such a useful technique, it is not without limitations. In the first place, it is not as general as the analytic approach. This seems to be the principle limitation. Secondly, a simulation model does not produce an

¹Buffa, E. op. cit. p. 68

optimum answer, as some mathematical models. Mize and Cox¹ summarize this limitation of simulation methods as follows:

Simulation methods are used more broadly than in deriving a solution from a mathematical model of a process. The expressed purpose of certain simulation studies is to provide a means of observing the behavior of the components of a system under varying conditions. No solution in the mathematical sense is sought; rather, the objective is to gain an understanding of the relationships among components of the system.

Thirdly, simulation generally leads to the solution of a given problem by an iterative process. Compared to solving equations, the iterative process is not so neat, and definitely requires more computation. If the model is very complicated, it may be necessary to expend a great deal of computer time so as to obtain trustworthy answers. This is another drawback of the simulation approach. Schmidt and Taylor² believe this to be a very important limitation. Thus, according to them:

1. Simulation models for computers are very costly to construct and to validate. In general, a different program must be constructed for each separate system. Special purpose simulation languages--- have helped to reduce this factor. However, this is still a formidable disadvantage.
2. The running of the simulation program, once constructed, can involve a great deal of computer time, which is also very costly.

¹Mize, J.H. and Cox, J.G. Essentials of Simulation (Englewood Cliff, New Jersey: Prentice-Hall) p.2

²Schmidt, J.W. and Taylor, R.E. Simulation and Analysis of Industrial Systems (Homewood, Ill. : Irwin, 1970) p. 6

Lastly, with the introduction of stochastic variables into a simulation, the variables that are used to measure the system performance become stochastic variables. Hence, the problem of gauging the significance of the results must be considered, as the values measured are no more than a sample. Wagner¹ believes that this is one of the reasons why most operations research analysts look upon digital computer simulation as a method of last resort.

When the model includes uncertain events, the answers stemming from a particular simulation must be viewed only as estimates subject to statistical error. For example, a simulated queuing model yields only an estimate of a waiting line's average length or the associated probability of a delay. Therefore, when you draw conclusions about the relative merit of different specific trial policies as tested by a simulation model, you must be careful to assess the accompanying random variations.

Although no attempt will be made to get into this problem at this stage, it must be mentioned that a large number of computer runs have to be made so as to get reliable results. This means more computer time and hence, more costs.

Simulation approach preferred:

There were basically three reasons for using a simulation approach in this research.

1. It was believed that the mathematical complexity

¹Wagner, H.M. Principles of Management Science (Englewood Cliffs, New Jersey: Prentice-Hall, 1970) p. 500

of the problem would make it extremely difficult to formulate the problem and obtain an analytical solution. This view was supported by a number of researchers. Balintfy¹ defines certain parameters and makes certain assumptions so as to arrive at an analytical solution to a problem very similar to the one analyzed in this dissertation. However, he concluded that an analytical solution would be difficult to obtain for this problem when the assumptions are dropped. Yet, most of the real problems are not constrained problems. That is, the assumptions are invalid. Balintfy then suggests that a simulation approach would be appropriate. Brown² also believes that joint order problems reach a degree of complexity that is unmanageable and beyond analytical solution. In view of these opinions, it was felt that a simulation approach was the only practical alternative.

2. It was also believed that some of the demand data that would be used to test the model would not fit the mathematically defined probability distributions, e.g., the normal distribution, the poisson distribution, etc. This is especially true when one works with real data. Here again, simulation is the only

¹Balintfy, J.L. "On a basic class of multi-item inventory problems" (Management Science Vol. 10, NO. 2, January, 1964) p. 296

²Brown, R.B. Decision Rules for Inventory Management (New York: Holt, Rinehar and Winston, 1967) p. 203

approach available.

3. Lastly, since it was likely that some experimentation would be involved, i.e., changing the values of the parameters, and recording the consequences over time, simulation would be necessary. A good example of such an experimentation would be the search for the optimum values of the three parameters, $\underline{S}$, $\underline{s}$, $\underline{s'}$, so as to minimize total cost.

DATA SOURCE

Hypothetical Data

A considerable portion of this research was concerned with the development of a joint ordering rule, and the study of the properties of the rule, especially the performance of the rule under extreme conditions. Hence, it was necessary to generate some hypothetical data which represented these extreme conditions. In generating this data, the following assumptions were made.

1. The product group consists of twelve items.¹
2. The demand for each item is normally distributed with a certain mean and standard deviation.

Nine data sets were generated. These are listed in Table 4.1 through Table 4.9 As is evident from the tables,

¹Costs of computer runs and the time factor in analyzing the results were major constraints in extending the analysis to a larger number of items.

Table 4.1:

Data Set 1

The means and standard deviations of the demands for the twelve items in the group¹.

ITEM	MEAN	STANDARD DEVIATION	PERCENTAGE DEMAND
1	8	1	7.619
2	10	1	9.5238
3	10	2	9.5238
4	6	1	5.7143
5	10	1	9.5238
6	11	1	10.4762
7	12	1	11.4286
8	9	1	8.5714
9	9	2	8.5714
10	5	1	4.7619
11	8	1	7.619
12	7	1	6.6667
			<u>100.00</u>

The Standard Deviation of the mean demands = 1.96

Range of mean demands = 12 - 5 = 7

¹It was assumed that the demand for each item was normally distributed with some mean and standard deviation.

Table 4.2 :

Data Set 2

The means and standard deviations of the demands for the twelve items in the group¹.

ITEM	MEAN	STANDARD DEVIATION	PERCENTAGE DEMAND
1	5	1	4.8544
2	7	1	6.7961
3	9	2	8.7379
4	8	1	7.767
5	6	1	5.8252
6	11	2	10.6796
7	12	1	11.6505
8	9	1	8.7379
9	10	1	9.7087
10	11	2	10.6796
11	8	1	7.767
12	7	1	6.7961
			<u>100.00</u>

The Standard Deviation of the mean demands = 2.15

Range of mean demands = 12 - 5 = 7

¹It was assumed that the demand for each item was normally distributed with some mean and standard deviation.

1

2

Table 4.3 :

Data Set 3

The means and standard deviations of the demands for the twelve items in the group¹.

ITEM	MEAN	STANDARD DEVIATION	PERCENTAGE DEMAND
1	13	2	6.7708
2	15	1	7.8125
3	16	2	8.3333
4	19	1	9.8958
5	21	1	10.9375
6	11	1	5.7292
7	17	2	8.8542
8	21	1	10.9275
9	23	2	11.9792
10	14	1	7.2917
11	10	1	5.2083
12	12	1	6.25
			<u>100.00</u>

The Standard Deviation of the mean demands = 4.26

Range of mean demands = 23 - 10 = 13

¹It was assumed that the demand for each item was normally distributed with some mean and standard deviation.

1

1

Table 4.4 :

Data Set 4

The means and standard deviations of the demands for the twelve items in the group¹.

ITEM	MEAN	STANDARD DEVIATION	PERCENTAGE DEMAND
1	20	1	5.3333
2	23	2	6.1333
3	26	3	6.9333
4	28	2	7.4667
5	30	4	8.00
6	29	1	7.7333
7	33	2	8.80
8	35	4	9.3333
9	37	3	9.8667
10	40	2	10.6667
11	40	3	10.6667
12	34	3	<u>9.0667</u>
			100.00

The Standard Deviation of the mean demands = 6.39

Range of mean demands = 40 - 20 = 20

¹It was assumed that the demand for each item was normally distributed with some mean and standard deviation.

Table 4.5 :

Data Set 5

The means and standard deviations of the demands for the twelve items in the group¹.

ITEM	MEAN	STANDARD DEVIATION	PERCENTAGE DEMAND
1	4	1	2.2989
2	6	1	3.4483
3	9	2	5.1724
4	12	1	6.8966
5	15	2	8.6207
6	17	2	9.7701
7	19	3	10.9195
8	21	2	12.069
9	23	2	13.2184
10	25	3	14.3678
11	10	1	5.7471
12	13	2	<u>7.4713</u>
			100.00

The Standard Deviation of the mean demands = 6.69

Range of mean demands = 25 - 4 = 21

It was assumed that the demand for each item was normally distributed with some mean and standard deviation.

Table 4.6 :

Data Set 6

The means and standard deviations of the demands for the twelve items in the group¹.

ITEM	MEAN	STANDARD DEVIATION	PERCENTAGE DEMAND
1	5	1	2.2222
2	7	1	3.1111
3	11	2	4.8889
4	14	2	6.2222
5	17	2	7.5556
6	21	3	9.3333
7	24	2	10.6667
8	27	3	12.00
9	30	3	13.3333
10	32	3	14.2222
11	12	1	5.3333
12	25	2	<u>11.1111</u>
			100.00

The Standard Deviation of the mean demands = 9.04

Range of mean demands = 32 - 5 = 27

¹It was assumed that the demand for each item was normally distributed with some mean and standard deviation.

Table 4.7 :

Data Set 7

The means and standard deviations of the demands for the twelve items in the group¹.

ITEM	MEAN	STANDARD DEVIATION	PERCENTAGE DEMAND
1	10	1	3.2787
2	15	2	4.918
3	17	2	5.5738
4	19	1	6.2295
5	20	3	6.5574
6	25	2	8.1967
7	27	2	8.8525
8	29	3	9.5082
9	30	3	9.8361
10	35	3	11.4754
11	38	2	12.459
12	40	3	<u>13.1148</u>
			100.00

The Standard Deviation of the mean demands = 9.47

Range of mean demands = 40 - 10 = 30

¹It was assumed that the demand for each item was normally distributed with some mean and standard deviation.

Table 4.8 :

Data Set 8

The means and standard deviations of the demands for the twelve items in the group¹.

ITEM	MEAN	STANDARD DEVIATION	PERCENTAGE DEMAND
1	5	1	1.2887
2	21	1	5.4124
3	27	2	6.9588
4	31	2	7.9897
5	34	3	8.7629
6	40	2	10.3093
7	48	3	12.3711
8	54	2	13.9175
9	62	2	15.9794
10	12	1	3.0928
11	24	1	6.1856
12	30	1	<u>7.732</u>
			100.00

The Standard Deviation of the mean demands = 16.65

Range of mean demands = 62 - 5 = 57

¹It was assumed that the demand for each item was normally distributed with some mean and standard deviation.

Table 4.9

Data Set 9

The means and standard deviations of the demands for the twelve items in the group.¹

ITEM	MEAN	STANDARD DEVIATION	PERCENTAGE DEMAND
1	60	3	14.7059
2	10	3	2.451
3	21	3	5.1471
4	30	2	7.3529
5	39	3	9.5588
6	53	3	12.9902
7	64	3	15.6863
8	3	1	.7353
9	25	1	6.1275
10	26	1	6.3725
11	33	2	8.0882
12	44	1	<u>10.7843</u>
			100.00

The Standard Deviation of the mean demands = 18.90

Range of mean demands = 64 - 3 = 61

¹It was assumed that the demand for each item was normally distributed with some mean mean and standard deviation.

the standard deviation of the mean demands of the twelve items is a minimum for data set 1. The standard deviation then gradually increases and is a maximum for data set 9. Thus, it was possible to study the behavior of the proposed reordering rule under various demand levels. The standard deviation could not be increased beyond that of data set 9, because then the economical order quantity for items with a high mean demand becomes less than the reorder point. However, it was strongly believed that though hypothetical data was used for the experimentation, the results obtained are general. The hypothetical data is in no way different from real data.¹ If actual data closely resembling the hypothetical data were available, the results obtained would remain unchanged.

Real Data

Although hypothetical data was used to study the properties of the joint ordering model, it was felt that the model should be tested with real data. Real data was obtained from two sources.

Steel Service Center Institute: One set of data was obtained from the Steel Service Center Institute. Sales data from this source was not obtained directly, but from the work of Basic.² Basic selected twelve items of a typical Steel Service Center which had relatively high volume and which contributed significantly to the firm's profit. The demand for each item, in units per month,

¹The inventory carrying cost and ordering cost assumed, were obtained from a production handbook. (see p. 66)

²Basic, M.K., "Development and Application of a Gamma-based Inventory Management Theory" (Unpublished Ph.D. dissertation, Michigan State University, East Lansing, Michigan, 1965)

was recorded for 59 months, from January , 1960 through November, 1965. Basic then fitted a gamma distribution function to the sales data for each item using standard acceptable statistical methods. The selected items were:

<u>Description</u>	<u>Price per bar</u> ¹
1" round steel bar, C1018, 12' long	\$9.43
1½" round steel bar, C1018, 12' long	\$14.68
1½" round steel bar, C1018, 12' long	\$21.14
1 3/4 round steel bar, C1018, 12' long	\$28.72
½" square steel rod, C1018, 12' long	\$1.00
1" square steel rod, C1018, 12' long	\$12.63
½" x 3/4" rectangular steel flat, C1018, 12' long	\$3.00
½" x 1" rectangular steel flat, C1018, 12' long	\$3.56
½" x 4" rectangular steel flat, C1018, 12' long	\$14.07
½" x 1" rectangular steel flat, C1018, 12' long	\$6.57
½" x 3" rectangular steel flat, C1018, 12' long	\$19.65
½" x 6" rectangular steel flat, C1018, 12" long	\$40.15

Basic concluded that the demand for each item was gamma distributed. The values of the parameters² r and λ for the gamma distribution which best fit the data for each item were

¹The prices of the twelve items were not available from Basic's thesis. Hence, these were obtained from a steel supplier in Lansing, Michigan. The prices are as of March 13, 1970.

²Appendix A has a general description of a gamma distribution and the meaning of the parameters r and λ.

therefore readily available in the dissertation. Since the primary purpose of this research was to test the joint ordering rule developed earlier, and not fitting a probability distribution to demand data (unless necessary), it was felt that data from Basic's dissertation would be most appropriate. Knowing the values of the gamma parameters and the cumulative gamma probabilities for each item, demand could be simulated fairly easily.¹ Further, the twelve items selected by Basic were essentially similar in nature; steel bars, round, square, or rectangular in shape. And lastly, the items could be ordered from the same supplier as a joint order.

The profit margin, inventory carrying costs, and the ordering costs were not available from Basic's thesis. After consultation with individuals experienced in the buying and selling of steel, the following values were assumed:

Inventory carrying cost	= 30% of the selling price of the item per unit per year
Profit margin	= 20% of the selling price
Ordering cost	= \$15.00 per order
Delivery lead time	= 3 weeks

Farmers' Cooperative: One year sales figures were obtained from a million dollar farmers' cooperative. The firm is a large

¹Appendix D has a detailed description of the procedure for the generation of gamma variates.

organization geared mainly to the needs of Michigan farmers. It carries a very broad line of products, estimated to include about five thousand individual items. These items are grouped into fifteen categories on the basis of physical characteristics. Some typical product groups are farm hardware, fertilizers, and animal feed. Each product group includes fifteen to two hundred individual items. Sales data were obtained for the farm fence group consisting of fourteen items. All the fourteen items were essentially different kinds of farm fence. They were all ordered from the same supplier. This product group was selected for several reasons:

1. The items in the group were similar in nature except for minor physical differences. Hence, they were appropriate for this research.
2. The items were ordered from the same supplier and hence, could be ordered as a joint order.
3. Sales records for these fourteen items were available. This was not true for the other product groups.
4. The product group included fourteen items, which was neither too small nor too large a number.

The list of fourteen items, along with the cost of each item, is provided in Table 4.10. All the items are different kinds of farm fence, and hence, no product description is included.

After consultation with individuals in charge of the

warehouses, the inventory carrying costs, and delivery lead time were assumed as follows:

Inventory carrying costs = 28% of the cost of the item per
unit per year.

Ordering costs = \$10.00 per order

Delivery lead time = 1 week

Profit margin = 10% of the cost of the item.

METHODOLOGY

The step by step procedure adopted when hypothetical data was used, was different as compared to the procedure when real data was used to carry out the simulation. Hence, these are listed separately.

Hypothetical Data

Step 1: Computer programs for the analysis of the data were first developed, since no standard routines applicable to the specific nature of the problem were available. The programs were written in FORTRAN, suitable for the CDC-6500 and CDC-3600 computers at Michigan State University. These are listed in Appendix F of this thesis.

The following assumptions were made in preparing the programs:

Starting inventory = 100 units for each of the twelve items

Ordering Cost = \$15.00 per order¹

¹See Appendix B for an explanation of the basis of this assumption.

Value of each item = \$10.00

Inventory Carrying

Cost = \$2.08 per unit per year, which is
approximately 20% of the value of
an item¹

Profit margin = 10% of the value of an item
= \$1.00 per unit

Value of lost

sales = \$1.00 x (number of units demanded
but not supplied)²

Delivery lead

time = 3 weeks

It was assumed that demand for each item is expressed only once a week. Moreover, the demand was assumed to occur at the beginning of each week. Hence, the inventory carried over from the preceeding week, less the demand at the beginning of the week. Therefore, the inventory carrying cost for the week will be equal to balance inventory times 0.04, where 0.04 is the inventory carrying cost per week. For example,

Initial inventory = 100 units

¹See Appendix B for an explanation of the basis for this assumption.

²If an item is demanded but not supplied because of insufficient inventory, the sale is assumed to be lost. That is, it is assumed that the customer is not willing to wait for fresh supplies to arrive, and goes to a competitor to make his purchase.

Assume that the demand at the beginning of week one = 12 units.

Therefore, the inventory for the remainder of week one = $100 - 12 = 88$ units.

Therefore, the inventory carrying cost for week one = $88 \times 0.04 = \underline{\$3.52}$.

If the demand at the beginning of week two = 10 units, then, the inventory for the rest of week two = $88 - 10 = 78$.

Therefore, the inventory carrying cost for week two = $78 \times 0.04 = \underline{\$3.12}$.

Hence, the total inventory carrying cost at the end of two weeks = $\$3.52 + \$3.12 = \underline{\$6.64}$.

This process is repeated for the third week, and so on.

Lastly, all the calculations in the simulation were made on a weekly basis. Again, the cost of computer time was the major constraint preventing the use of a time step of one day.

As mentioned earlier, demand for each item was assumed to be normally distributed with a certain mean and standard deviation. Hence, it was necessary to generate normally distributed random variates with a specified mean and standard deviation. Naylor¹ has developed a procedure and a FORTRAN

¹Naylor, T.H. et.al. op. cit. p. 95

subroutine for the generation of normal variates. The subroutine was quite general, and was used directly. The procedure involves taking the sum of twelve uniformly distributed random variates between zero and one. Then, if $\underline{x}$ is a normally distributed random variable with standard deviation σ and mean μ

$$\chi = \sigma \times (\text{sum} - 6.0) + \mu$$

where SUM = sum of the twelve uniformly distributed random variates between zero and one.

A listing of the subroutine is provided in Appendix C.

In all, four programs were written. Program EOQ simulates the operation of the system assuming that the inventory level of the items is controlled using the fixed order quantity system. This system is characterized by two parameters. These are:

$$\text{Economical order quantity EOQ} = \sqrt{\frac{2 \times A \times D}{i}}$$

where $\underline{A}$ = ordering cost (in dollars)

$\underline{D}$ = demand per unit time (in units)

$\underline{i}$ = inventory carrying costs (in dollars per unit per unit time)

Reorder point ROP = ($\underline{a} \times \sqrt{3}$ x standard deviation of weekly demand) + (3 x average weekly demand)

where $\underline{a}$ = a parameter to be determined by management.

It determines the allowable risk of service failure.

= 2 (assumed)

Since the demand occurs once a week and the delivery lead time is three weeks, the expected demand during the lead time will be three times the average weekly demand. The square root of three results, because the standard deviation of demand during lead time (three weeks) is equal to square root of three times the weekly standard deviation of demand. Recall that the ordering cost in this case was assumed to be fifteen dollars per item ordered.

Program EOQ Modified was the same as the program EOQ except for one difference. The ordering cost was now assumed to be fifteen dollars per order regardless of the number of items included in the order.

Program REPEAT was a search program which searched for the optimum values of the three parameters $\underline{S}$, $\underline{s}$, and $\underline{s}'$.

Lastly, program CONVERG was used to simulate the system for 500 years,¹ so as to get a reasonable degree of convergence with respect to the total cost per year.

¹A detailed discussion of why 500 years was selected is included on p. 80.

Step 2: In step one, program REPEAT was written, which searched for the optimum values of the three parameters $\underline{S}$, $\underline{s}$, and $\underline{s}'$. But before the search could be started, it was necessary to calculate the initial (or starting) values of the three parameters. Hence, in step two these initial values were calculated.

Computation of $\underline{s}$: In the proposed joint ordering rule, $\underline{s}$ was defined as a trigger point for the entire group. When the inventory level of any item in the group is equal to or less than $\underline{s}$, a joint order is placed. In the simple $(\underline{s}, \underline{S})$ inventory control system, $\underline{s}$ is the trigger point for an individual item. Hence, the only difference between $\underline{s}$ in the $(\underline{s}, \underline{S})$ rule and $\underline{s}$ in the $(\underline{s}, \underline{s}', \underline{S})$ rule is that in the former, $\underline{s}$ is the trigger point for an item while in the latter, $\underline{s}$ is the trigger point for the entire group. Hence, one way to compute $\underline{s}$ for the group would be to compute $\underline{s}$ for each individual item in the group (using the traditional method) and then use these values to compute $\underline{s}$ for the entire group. Hence, as a first approximation, the reorder point for the group, $\underline{s}$ could be computed by taking the weighted average of the individual reorder points. The procedure adopted for computing the weights assigned to the individual reorder points and the initial value of $\underline{s}$ was as follows:

1. The reorder point for each item in the group was computed using the conventional method such as

$$\begin{aligned} (ROP)_j &= \text{reorder point for item } j \\ &= (\text{expected demand during lead time}) \\ &\quad + (\underline{a} \times \text{standard deviation of demand} \\ &\quad \text{during lead time}) \end{aligned}$$

where $\underline{a}$ = a constant. It determines the allowable risk of stock out
 $= 2$ (assumed)

2. Once the individual reorder points had been calculated, the value of $\underline{s}$ for the group was computed as follows:

$$\underline{s} = \sum_j w_j \times (ROP)_j$$

where w_j = the weight assigned to item j
 $= \frac{\text{demand for item } j}{\text{total demand for the entire group.}}$

Computation of S:

In the simple ($\underline{s}$, $\underline{S}$) inventory control system, $\underline{S}$ determines the quantity that should be ordered for each item, since $(S-I)$ is the order quantity where $\underline{I}$ is the inventory on hand. As an approximation, the average order quantity will equal the difference $(\underline{S}-\underline{s})$. In the ($\underline{s}$, $\underline{s}'$, $\underline{S}$) system, $\underline{S}$ determines the order quantity for all items in the group, as $\underline{S}$ is common for the entire group. It would be desirable that the order quantity for each item be equal to an economic order

quantity. That is, the difference ($\underline{S} - \underline{s}$) for each item in the group be equal to an economic order quantity. But since the parameters $\underline{S}$ and $\underline{s}$ are common for the entire group, the difference ($\underline{S} - \underline{s}$) will also be common for the entire group. Hence, it is necessary to compute a single value of ($\underline{S} - \underline{s}$), i.e., a single value of the economic order quantity for each item in the group (using the traditional method), and then use these values to compute an economic order quantity that is common for the entire group. As a first approximation, the economic order quantity for the group could be computed by taking the weighted average of the individual economic order quantities. The procedure adopted for computing the economic order quantity common to the group and hence computing $\underline{S}$ was as follows:

1. The economic order quantity for each item in the group was first computed using the conventional lot size formula.

$$\begin{aligned}(\text{EOQ})_j &= \text{economic order quantity for item } j \\ &= \sqrt{\frac{2AD}{I}}\end{aligned}$$

where $\underline{A}$ = ordering cost

$\underline{D}$ = demand per unit time

$\underline{I}$ = inventory carrying cost per unit per unit time.

2. Next, the individual item economic order quantities

were used to compute an economic order quantity common to the entire group.

$$(EOQ) = \sum_j w_j \times (EOQ)_j$$

where w_j = weight assigned to item j
= $\frac{\text{demand for item } j}{\text{total demand for the entire group}}$

3. Recall that as an approximation, the difference ($\underline{S} - \underline{s}$) was equal to an economic order quantity. Therefore,

$$S - s = EOQ$$

i.e. $S = s + EOQ$

where s = the parameter calculated previously.

Calculation of s' :

By definition of the joint ordering rule, $\underline{s}'$ lies between $\underline{S}$ and $\underline{s}$ and has a maximum value of $\underline{S}$ and a minimum value of $\underline{s}$. Hence, the most appropriate initial value of $\underline{s}'$ would be it's maximum value $\underline{S}$. The initial value of $\underline{s}'$ was therefore set equal to $\underline{S}$. Starting with this maximum value, $\underline{s}'$ was decreased in steps up to a minimum value of $\underline{s}$. Details of this procedure are described in step 3.

The initial (or starting) values of the three parameters have now been computed. In step three, these initial values were used to search for the optimum values of the three parameters,

S, s, and s'.

Step 3: The next step was simulating the operation of the system using three inventory control systems and the hypothetical data. The three inventory control system were:

1. The Fixed Order System:¹ This inventory control system is characterized by two parameters, the economic order quantity and the reorder point. With this system the same order quantity (EOQ) of an item is ordered each time; but the time that an order is placed varies with fluctuations in usage. The ordering cost was assumed to be \$15 per item ordered.
2. The Modified Fixed Order System: The inventory control system in this case is the same as in (1) except for one difference. A number of items can be included in one order. But a separate economic order quantity and reorder point is computed for each item. The ordering cost was assumed to be \$15 per order regardless of the number of items included in the order.
3. The Joint Ordering System: In this case, the proposed joint ordering rule characterized by three parameters, S, s, and s', is used to control inventory. The ordering cost was assumed to be \$15 per order as in (2).

¹See p. 4 for details.

In order to compare the performance of the three inventory control systems, the same data set was used to compute the total cost per year when each inventory control system was used. Thus, a data set was first used to determine the total cost under the conventional EOQ system. The program EOQ was used to simulate the system for $\underline{n}$ years. Next, the same data set was used in the EOQ (Modified) program to determine the total cost when a number of items could be included in one order, although a separate economic order quantity and reorder point was used for each item. The system was simulated for $\underline{n}$ years. Lastly, the same data were used to simulate the inventory control system when the proposed joint ordering rule was used to control inventory. In this case, two steps were involved. First, program REPEAT was used to search for the minimum cost values of the three parameters, $\underline{S}$, $\underline{s}$, and $\underline{s}'$. Next, once the optimum values of the parameters were found, the program CONVERG was used to simulate the system for $\underline{n}$ years. In all three cases, conventional EOQ, modified EOQ, and the joint ordering rule, a single estimate for the total cost per year was assumed to be the mean of the last twenty of the $\underline{n}$ yearly costs.

There are two things that need further explanation. First, the term $\underline{n}$ years was used throughout the earlier discussion. Hence, it is necessary to explain why the system should be simulated for $\underline{n}$ years and what the appropriate value of $\underline{n}$ should be. Second, it is necessary to explain the method adopted to search

for the minimum cost values of the three parameters, $\underline{S}$, $\underline{s}$, and $\underline{s}'$ using the program REPEAT.

The Value of n:

In any simulation experiment certain initial or starting conditions have to be assumed. This gives rise to what is known as an initial bias in the results. That is, the results obtained immediately after time zero are influenced by the starting conditions and hence, cannot be used as a measure of system performance. Moreover, for the first few time periods wide variability in results are generally obtained. Both these problems make it difficult to get an estimate of the time results.

Gordon¹ suggests a procedure for overcoming these problems:

The more common approach to removing initial bias is to eliminate an initial section of the run. The run is started from an idle state and stopped after a certain period of time. The entities existing in the system at that time are left as they are. The run is then restarted with statistics being gathered from the point of restart. As a practical matter, it is usual to program the simulation so that statistics are gathered from the beginning, and simply wipe out the statistics gathered up to the point of restart. No simple rules can be given to decide how long an interval should be eliminated.

Another approach to the problem of estimating the precision of simulation results does not rely upon repetition, but uses a single long run, preferably with the initial bias removed. The run is divided into a number of segments to separate them into batches of equal size. The mean of each batch is taken and the individual batch means are regarded as independent observations. The estimated value of the variable being measured is the

¹Gordon, G. System Simulation (Englewood Cliffs, New Jersey: Prentice-Hall, 1969) p. 285

mean of the batch means.

The method adopted in this thesis was similar to the second approach recommended by Gordon. In order to eliminate the initial bias and the initial variability the system was simulated for five hundred years (each year consisting of 52 weeks and hence, 52 iterations). Hence, $\underline{n}$ was equal to 500. The cost of computer time was a major constraint in extending the length of the run. Moreover, it was found that convergence with respect to total cost was obtained to a reasonable degree within five hundred years. Then, a single estimate for the total cost per year was assumed to be the mean of the last twenty yearly total costs.

The Search Process:

An exhaustive search for the minimum cost values of the three parameters $\underline{S}$, $\underline{s}$, and $\underline{s}'$ is almost impossible because of the large number of combinations of the three parameters that is possible. For example, assume that the search is started with the following initial values of two parameters.

$$\underline{S} = 180$$

$$\underline{s} = 60$$

Then $\underline{s}'$ can take on a maximum value of 180 and a minimum value of 60. Assuming that the values of the parameters are changed in steps of 10, the number of possible combinations of the parameters is equal to

19 x 19! x 19!

Moreover, for each set of parameters it is necessary to simulate the system for a sufficiently long period of time so as to get a reasonable degree of convergence, and hence, a good estimate of the total cost per year. It is clear that not only is the task physically impossible, but the cost of computer time would run to several thousand dollars. Hence, only a limited search was conducted in most cases, using the following procedure.

The parameters $\underline{S}$ and $\underline{s}$ were held constant and $\underline{s}'$ varied between¹ $\underline{S}$ and $\underline{s}$ until the minimum total cost was obtained. The value of $\underline{s}'$ was changed in steps of 10 and for each set of parameters the system was simulated for 50 years², so as to get a reasonable degree of convergence. A single estimate for the total cost per year was assumed to be the mean of the last twenty yearly costs. Next $\underline{s}'$ was held constant at it's minimum cost value along with $\underline{s}$ and $\underline{S}$ was varied between a maximum (the first estimate of $\underline{S}$) and a minimum of $\underline{s}'$. Once a minimum cost value of $\underline{S}$ was obtained, $\underline{S}$ and $\underline{s}'$ were held constant and $\underline{s}$ was varied between a maximum of $\underline{s}'$ and a minimum of zero. In most cases the process was stopped at this stage, and it was assumed that the minimum cost values of the three parameters had

¹Recall that by definition $\underline{s}'$ can have a maximum value of $\underline{S}$ and a minimum value of $\underline{s}$.

²The cost of computer time was a major constraint in limiting the simulation to 50 years.

been determined. The search was therefore limited. But, in some cases, the whole process mentioned earlier was repeated a number of times until the minimum cost values of the three parameters were obtained. It must be mentioned that the limited search procedure did give fairly accurate results. That is, the minimum yearly total costs did not differ significantly from the minimum costs obtained through limited search.

Step 4: The next step was the analysis of the results. The total yearly costs under the three reordering rules¹ using the same data set were compared, and a percentage difference in costs with respect to the proposed joint ordering rule was calculated. Finally, from the results obtained , the necessary conclusions were drawn.

REAL DATA

Data from the Farmers Cooperative:

Step 1: The first step was the collection of sales data from the farmers cooperative. The data were obtained from sales receipts or order forms, then tabulated and finally punched into computer cards.

Step 2: Next, the items were divided into three groups, the high-volume items, the low-volume items, and the medium selling items, using the ABC classification approach.² The

¹See p. 73 of this thesis for a detailed statement of the three reordering rules.

²For an explanation of this approach, see p. 17 of this thesis.

medium selling items consisting of ten items were selected for further study since it was felt that inventory for the high volume and low volume items could be controlled using the traditional methods. Moreover, management at the farmer's cooperative was more concerned with these medium selling items.

Step 3: The sales data for these items were then tabulated and the monthly sales for each item were computed. Since exact dates of the sales were not available, the weekly sales figures could not be obtained. The monthly sales for each item were then plotted on graph paper so as to study the sales pattern for each item over the year.

Step 4: A study of these graphs indicated that the sales of each item was highly seasonal, being maximum during the months of May through November, and almost zero during the period of January through March.¹ In order to simulate this data it was necessary to fit a probability distribution to the demand data. However, it was found that this was impossible for several reasons.

1. In the first place, only one year's sales data were available. And since the sales per month was computed, only twelve data points per item were available. It was impossible to fit a probability distribution to these

¹See graphs on p. 151 and 152.

Table: 4.10

Product Code	Cost			Weight of 1 roll lbs./roll
	30,000 lbs. and over \$/roll	20,000 lbs. and over \$/roll	Less than 20,000 lbs. \$/roll	
1003	24.22	24.64	25.70	212
1004	21.53	21.91	22.85	188
1005	19.01	19.35	20.18	166
1006	32.44	33.00	34.40	280
1007	28.57	29.07	30.30	246
1008	24.87	25.30	26.37	214
1009	16.98	17.28	18.01	146
1010	15.31	15.58	16.24	132
1011	13.54	13.78	14.36	116
1012	22.46	22.84	23.79	190
1013	19.90	20.24	21.08	168
1014	17.49	17.79	18.53	148
1017	13.72	13.94	14.49	109
1018	11.94	12.13	12.61	95

data points because of the small number of points available.

2. Secondly, if at all a probability distribution function was fitted to the sales data, it would be difficult to test the accuracy of this probability function since no other sales data were available. This would mean using the same data to do both obtain a probability distribution function and next, to test the probability distribution function. This procedure is questionable, especially in this case where the sales patterns are not consistent from year to year.
3. Thirdly, during the process of computing the monthly sales it was found that sales records for certain periods of time were missing. Hence, the accuracy of the data in general was questionable.

It was for these reasons that the data obtained from the farmers cooperative was not used to test the proposed joint ordering model. However, since considerable time was spent on the collection and analysis of this data (a worthwhile experience indeed), it was felt that whatever results were obtained should be included in the thesis.

Data from The Steel Service Center Institute:

Step 1: As mentioned earlier, Basic¹ fitted a gamma distribu-

¹Basic, M.E. op. cit.

tion function to the demand data for twelve items. The twelve items were relatively high volume basic items which a metal service center is expected to have on hand to satisfy customer needs. Hence, the data from the Steel Service Center were already tabulated. Moreover, since a gamma distribution had been fitted to the data, tables showing the gamma cumulative probability distribution for each item were readily available.¹ These tables are reproduced in Tables 4.11 through 4.22 of this thesis. The gamma cumulative probability distribution was used to generate the demand for each item.

Step 2: Computer programs for the analysis of the data were developed, since no standard routines applicable to the specific nature of the problem were available. The programs were written in FORTRAN, suitable for the CDC-6500 and CDC-3600 computers at Michigan State University. The programs are listed in Appendix F.

The following assumptions were made in preparing the programs.

Starting inventory = 100 units for each of the twelve items.

If an item was demanded but not supplied because of insufficient inventory, the sale was assumed to be lost. The

¹Basic, M.E. op. cit. pp. 47-56

Table 4.11 :

Cumulative gamma probability distribution for 1" round steel bar,
C1018, 12' long

Units	Cum. Prob.
10	.00024
20	.00479
30	.02329
40	.06385
50	.12839
60	.21288
70	.30995
80	.41152
90	.51058
100	.60206
110	.68292
120	.75188
130	.80897
140	.85504
150	.89144
160	.91966
170	.94117
180	.95735
190	.96935
200	.97815
210	.98454

Table 4.12:

Cumulative gamma probability distribution for $1\frac{1}{4}$ " round steel
bar, C1018, 12' long

Units	Cum. Prob.
10	.21298
20	.38938
30	.52823
40	.63635
50	.72011
60	.78482
70	.83470
80	.87309
90	.90262
100	.92531
110	.94274
120	.95611
130	.96637
140	.97423
150	.98026
160	.98489
170	.98843
180	.99114
190	.99322
200	.99481
210	.99603
220	.99696
230	.99767
240	.99822

Table 4.13:

Cumulative gamma probability distribution for $1\frac{1}{2}$ " round steel
bar, C1018, 12' long

Units	Cum. Prob.
2	.0072
4	.00481
6	.01399
8	.02892
10	.04960
12	.07561
14	.10629
16	.14088
18	.17854
20	.21849
22	.25995
24	.30226
26	.34479
28	.38704
30	.42856
32	.46900
34	.50806
36	.54555
38	.58128
40	.61515
42	.64711
44	.67712
46	.70518
48	.73132
50	.75559
52	.77805
54	.79878
56	.81784
58	.83535
60	.85138
62	.86602
64	.87937
66	.89152
68	.90256
70	.91257
72	.92164
74	.92983
76	.93723
78	.94390
80	.94990
82	.95530
84	.96015
86	.96450
88	.96840
90	.97189
92	.97501
94	.97780

Table 4.14:

Cumulative gamma probability distribution for 1 3/4" round steel
bar, C1018, 12' long

Units	Cum. Prob.
2	.11593
4	.25434
6	.38247
8	.49410
10	.58860
12	.66726
14	.73199
16	.78484
18	.82773
20	.86238
22	.89027
24	.91265
26	.93056
28	.94487
30	.95628
32	.96536
34	.97258
36	.97831
38	.98286
40	.98646
42	.98931
44	.99157
46	.99335
48	.99476
50	.99587

Table 4.15:

Cumulative gamma probability distribution for $\frac{1}{4}$ " square steel
rod, C1018, 12' long

Units	Cum. Prob.
10	.42120
20	.65681
30	.79535
40	.87760
50	.92666
60	.95600
70	.97358
80	.98412
90	.99045
100	.99526

Table 4.16:

Cumulative gamma probability distribution for 1" square steel
rod, C1018, 12' long

Units	Cum. Prob.
2	.00820
4	.03651
6	.08230
8	.14066
10	.20681
12	.27673
14	.34731
16	.41622
18	.48187
20	.54323
22	.59970
24	.65102
26	.69718
28	.73833
30	.77474
32	.80675
34	.83472
36	.85904
38	.88010
40	.89825
42	.91385
44	.92720
46	.93860
48	.94831
50	.95655
52	.96354
54	.96944
56	.97443
58	.97863
60	.98216
62	.98512
64	.98760

Table 4.17:

Cumulative gamma probability distribution for $\frac{1}{4}$ " x $\frac{3}{4}$ "
rectangular steel flat, C1018, 12' long

Units	Cum. Prob.
10	.35363
20	.49802
30	.59749
40	.67193
50	.72984
60	.77593
70	.81316
80	.84355
90	.86856
100	.88926
110	.90649
120	.92088
130	.93294
140	.94308
150	.95162
160	.95883
170	.96493
180	.97010
190	.97448
200	.97820
210	.98137
220	.98407
230	.98637
240	.98832
250	.99000
260	.99143
270	.99265
280	.99369
290	.99459
300	.99535
310	.99601
320	.99657
330	.99705

Table 4.18 :

Cumulative gamma probability distribution for $\frac{1}{4}$ " x 1"
rectangular steel flat, C1018, 12' long

Units	Cum. Prob.
10	.00028
20	.00335
30	.01303
40	.03216
50	.06192
60	.10202
70	.15107
80	.20710
90	.26785
100	.33115
110	.39501
120	.45776
130	.51808
140	.57500
150	.62787
160	.67630
170	.72014
180	.75940
190	.79422
200	.82485
210	.85159
220	.87477
230	.89474
240	.91186
250	.92643
260	.93880
270	.94924
280	.95802
290	.96537
300	.97151
310	.97661
320	.98085
330	.98435
340	.98724
350	.98961
360	.99156
370	.99315
380	.99446

Table 4.19 :

Cumulative gamma probability distribution for $\frac{1}{4}$ " x 4"
rectangular steel flat, C1018, 12' long

Units	Cum. Prob.
10	.05307
20	.20366
30	.38551
40	.55313
50	.68853
60	.78966
70	.86138
80	.91039
90	.94297
100	.96418
110	.97774
120	.98630
130	.99163
140	.99492
150	.99694
160	.99817

Table 4.20:

Cumulative gamma probability distribution for $\frac{1}{2}$ " x 1"
rectangular steel flat, C1018, 12' long

Units	Cum. Prob.
5	.05855
10	.16992
15	.29440
20	.41453
25	.52255
30	.61568
35	.69379
40	.75802
45	.81007
50	.85177
55	.88487
60	.91094
65	.93136
70	.94727
75	.95960
80	.95960
85	.96913
90	.97646
95	.98639
100	.98968

Table 4.21 :

Cumulative gamma probability distribution for $\frac{1}{2}$ " x 3"
rectangular steel flat, C1018, 12' long

Units	Cum. Prob.
2	.00087
4	.00643
6	.01946
8	.04098
10	.07079
12	.10794
14	.15110
16	.19880
18	.24956
20	.30204
22	.35502
24	.40752
26	.45870
28	.50794
30	.55476
32	.59883
34	.63998
36	.67807
38	.71311
40	.74515
42	.77427
44	.80061
46	.82433
48	.84560
50	.86460
52	.88151
54	.89652
56	.90980
58	.92152
60	.93182
62	.94087
64	.94880
66	.95573
68	.96177
70	.96703
72	.97160
74	.97557
76	.97901
78	.98198
80	.98455

Table 4.22 :

Cumulative gamma probability distribution for $\frac{1}{2}$ " x 6"
rectangular steel flat, C1018, 12' long

Units	Cum. Prob.
2	.03403
4	.10151
6	.18286
8	.26840
10	.35260
12	.43231
14	.50583
16	.57242
18	.63190
20	.68446
22	.73051
24	.77059
26	.80526
28	.83510
30	.86069
32	.88254
34	.90114
36	.91694
38	.93031
40	.94162
42	.95115
44	.95917
46	.96592
48	.97157
50	.97631
52	.98028
54	.98360
56	.98637

loss was assumed to be equal to the profit margin on each item not supplied.

It was assumed that demand for each item is expressed only once a week. Moreover, the demand was assumed to occur at the beginning of each week. The procedure adopted to compute the inventory carrying cost was the same as when hypothetical data was used.¹

However, there was one problem in this case. Basic expressed the demand for each item in units per month, not units per week. Hence, if the gamma cumulative probability distribution tables were used to generate demand, the demand would be in units per month not units per week. But, as mentioned earlier, it was assumed that the demand for each item occurred once a week.² Hence, it was necessary to convert the demand per month into demand per week. The following procedure was adopted. It was assumed that there are $4 \frac{1}{3}$ weeks in a month. Then, every week demand was generated using the gamma cumulative distribution tables. The demand obtained was therefore demand in units per month. This figure was then divided by $4 \frac{1}{3}$ to obtain the demand in units per week. Appendix E includes a short proof of the validity of this procedure.

The final assumption made was that all the calculations in the simulation were made on a weekly basis. The cost of computer

¹For details see p. 70.

²Further, all calculations in the simulation were made on a weekly basis (see p. 71).

time was the major constraint preventing the use of a time step of one day.

In all, four programs were written.¹ They were:

1. Program EOQ simulates the operation of the system, assuming that the inventory level of the items was controlled using the fixed order quantity inventory control system. The order quantity was assumed to be \$15 per item ordered.
2. Program EOQ Modified was the same as the above program, except that in this case the ordering cost was assumed to be \$15 per order regardless of the number of items included in the order.
3. Program REPEAT searches for the optimum values of the three parameters S, s, and s'.
4. Program CONVERG was used to simulate the system for n years, using the optimum values of the parameters found by program REPEAT.

These four programs were identical to those developed earlier for the analysis of the hypothetical data. But there were two differences.

First, in the case of the hypothetical data, the demand for each item was assumed to be normally distributed with a certain

¹A listing of the programs is provided in Appendix F.

mean and standard deviation. The mean and standard deviation of the normal distribution were used to generate normal variates, using the FORTRAN subroutine developed by Naylor.¹ But, in the case of the data from the Steel Service Center, demand for each item was gamma distributed, and the tables of the gamma cumulative probabilities for each item were available. Hence, instead of using the mean and standard deviation of the gamma distribution to generate gamma variates, the cumulative probability tables were used. The procedure was therefore slightly different. Naylor² recommends a procedure for generating random variates from a particular statistical population whose cumulative distribution function is known. The procedure is called the Inverse Transformation Method.³ This procedure was used to generate gamma variates.

The second difference was in the method used to compute the reorder point for each item. Instead of using the formula,

$$\begin{aligned}(\text{ROP}) &= \text{Reorder point for item } j \\ &= (\underline{a} \times \text{standard deviation of demand during lead time}) \\ &\quad + (\text{expected demand during lead time})\end{aligned}$$

where a = a parameter to be determined by management.

It determines the allowable risk of service failure

¹See p. 71.

²Naylor, T.H. et. al. op. cit. p. 70

³See Appendix D for a detailed description of this procedure.

as done previously, (pp. 70-71), Basic's method¹ was used to compute the reorder point for each item. This method was as follows:

1. The economic order quantities for each item were first computed using the formula

$$\begin{aligned}(\text{EOQ})_j &= \text{economic order quantity for item } j \\ &= \sqrt{\frac{2 A D}{I}}\end{aligned}$$

where A = ordering cost

D = expected demand per unit time

I = inventory carrying cost per unit per
unit time

2. Next, the optimum probability of stock out for each item was computed using the formula²

$$= \frac{(\text{UHC}) \times (\text{CPU}) \times (\text{EOQ})}{(\text{STKCOS}) \times (\text{DPY}) + (\text{UHC}) \times (\text{CPU}) \times (\text{EOQ})}$$

where UHC = unit holding cost expressed as a percentage
of the cost per unit

CPU = cost per unit

EOQ = economic order quantity

STKCOS = stock out cost per unit short

DPY = expected demand per year

3. Given the mean and standard deviation of the demand

¹Basic, E.M. op. cit. p. 98

²Basic, E.M. op. cit. p. 94

for each item, the two gamma parameters $\underline{r}$ and $\underline{\lambda}$ were computed.

$$\underline{r} = \frac{(\text{Mean})^2}{(\text{Standard Deviation})^2}$$

$$\underline{\lambda} = \frac{(\text{Mean})}{(\text{Standard Deviation})^2}$$

4. Having computed the optimum probability of stock out and the gamma parameter $\underline{r}$, the table of cumulative gamma probabilities¹ was used to compute a parameter $\underline{m}$, where $m = (\lambda) \times (X)$ where $\underline{X}$ is the quantity whose probability is under study.

5. Using this value of $\underline{m}$ and the value of the gamma parameter $\underline{\lambda}$, the reorder point was computed as follows,

$$X = \frac{\underline{m}}{\underline{\lambda}}$$

where $\underline{X}$ = reorder point.

Step 3: In this step the starting values of the three parameters $\underline{S}$, $\underline{s}$ and $\underline{s}'$ were computed using the economic order quantity, and the reorder point of each item computed in step (2). The procedure adopted was the same as that described on pages 70 to 73. As a first approximation, the value of $\underline{s}$ was set equal to the weighted average of the reorder points of the individual items in the group, while the value of $\underline{S}$ was set equal to the starting value of $\underline{s}$ computed earlier, plus the weighted average of the

¹ Basic, E.M. op. cit. p. 38

economic order quantities of the individual items. The initial value of $\underline{s}'$ was set equal to $\underline{S}$, since by definition of the joint ordering rule $\underline{s}'$ has a maximum value of $\underline{S}$.

Step 4 : In this step, the operation of the system was simulated using the three reordering rules and the real data. The reordering rules were the same as those described on pages 73 and 74. Briefly, these were:

1. The Fixed Order Rule : In this case the ordering cost was assumed to be \$15 per item ordered.
2. The Modified Fixed Order Rule : Ordering cost was assumed to be \$15 per order regardless of the number of items included in the order.
3. The Joint Ordering Rule : This was the proposed joint ordering rule characterized by three parameters, $\underline{S}$, $\underline{s}$, and $\underline{s}'$. Ordering cost was assumed to be \$15 per order.

In order to make a comparison, the same data set was used to compute the total cost per year under each reordering rule. Hence, a data set was first used to determine the total cost when the fixed order system was used to control inventory, and the program EOQ was used to simulate the system for $\underline{n}$ years. Next, the same data set was used in the modified EOQ program to determine the total cost when the modified fixed order system was used to control inventory. Once again the system was simulated for $\underline{n}$ years. Lastly, the same data set was used to deter-

mine the total cost when the joint ordering rule was used. First, program REPEAT was used to search for the minimum values of the three parameters $\underline{S}$, $\underline{s}$, and $\underline{s}'$. Next, the optimum values of the parameters were used to simulate the system for $\underline{n}$ years using program CONVERG.

As mentioned earlier,¹ it was necessary to simulate the system for $\underline{n}$ years so as to eliminate the initial bias and initial variability. The value of $\underline{n}$ selected was 500 years (as before). As regards the search process, the procedure was once again the same as that described on page 81. But, in this case the search was not partial but complete. That is, the parameters $\underline{S}$ and $\underline{s}$ were held constant and $\underline{s}'$ varied until the minimum total cost was obtained. Next, $\underline{s}'$ (corresponding to the minimum cost) and $\underline{s}$ were held constant, and $\underline{S}$ was varied until the minimum total cost was obtained. Then $\underline{s}'$ and $\underline{S}$ (corresponding to minimum total cost) were held constant and $\underline{s}$ varied until minimum total cost was obtained. The whole procedure was repeated until the minimum cost values of the three parameters were obtained.

The data from the Steel Service Center was used in two ways. First, it was assumed that the product group consisted of only 8 items. The items selected were:

1½" round steel bar, C1018, 12' long

1½" round steel bar, C1018, 12' long

¹See p.80.

$\frac{1}{2}$ " square steel rod, C1018, 12' long

1" square steel rod, C1018, 12' long

$\frac{1}{2}$ " x $\frac{3}{4}$ " rectangular steel flat, C1018, 12' long

$\frac{1}{2}$ " x 4" rectangular steel flat, C1018, 12' long

$\frac{1}{2}$ " x 1" rectangular steel flat, C1018, 12' long

$\frac{1}{2}$ " x 3" rectangular steel flat, C1018, 12' long

These items were selected so as to decrease the dispersion of the mean demands of the items. The standard deviation of the mean demands for the 8 items was equal to 8.26 (See Table 5.10).

In the second case, it was assumed that the product group consisted of all twelve items. The standard deviation of the mean demands for the 12 items was equal to 35.4 (See Table 5.11).

Thus, with one set of real data that was available, it was possible to test the proposed joint ordering rule when the standard deviation of mean demands was both relatively large and relatively small.

Step 5 : The last step was the analysis of the results. The total yearly costs corresponding to the three reordering rules, using the same data set, were compared. The savings, if any, resulting from the use of the proposed joint ordering rule were computed. Finally, the necessary conclusions were drawn.

CHAPTER 5

RESULTS

This chapter is divided into two sections. The results obtained from the hypothetical data are reported in the first section. The results obtained from real data are reported in the second section.

HYPOTHETICAL DATA

Table 5.2 through 5.10 have a detailed account of the results. However, they are summarized in Table 5.1. The results indicated that:

1. The average total cost per year when the proposed Joint Ordering Rule was used to control inventory was less than the average total cost per year when the Fixed Order System was used. This was true of all data sets. The savings resulting from the use of the Joint Ordering Rule were substantial and ranged from a minimum of 11.64% in the case of Data Set 7 to a maximum of 48.16% for Data Set 9.
2. The average total cost per year when the proposed Joint Ordering Rule was used to control inventory was in general, less than the average total cost per year when the Modified Fixed Order System was used. There were only two situations (Data Sets 8 and 9) out of nine where this was not true, and the proposed Joint Ordering Rule resulted in higher costs

Table 5.1

DATA SET	STANDARD DEVIATION OF MEAN DEMANDS	AVERAGE TOTAL COST PER YEAR FIXED ORDER SYSTEM	AVERAGE TOTAL COST PER YEAR MODIFIED FIXED ORDER SYSTEM	PROPOSED JOINT ORDERING RULE			SAVINGS OVER	MODIFIED FIXED ORDER SYSTEM			
				OPTIMUM OF PARAMETERS	VALUES	AVERAGE TOTAL COST PER YEAR					
REAL DATA											
1	1.96	\$2387.66	\$1871.18	80	28	80	\$1412.79	\$975	40.82	\$459	24.49
2	2.15	\$2361.64	\$1892.89	50	30	50	\$1224.04	\$1137	48.16	\$668	35.33
3	4.26	\$3338.64	\$2603.40	80	50	80	\$1829.86	\$1508	45.19	\$773	29.71
4	6.39	\$5009.27	\$3815.28	170	105	160	\$3367.37	\$1641	32.77	\$447	11.73
5	6.69	\$3156.39	\$2509.29	90	50	80	\$2131.01	\$1025	32.47	\$378	15.06
6	9.04	\$3616.00	\$2833.75	107	63	87	\$2639.27	\$976	27.01	\$194	6.86
7	9.47	\$4292.20	\$3280.45	128	83	108	\$3103.51	\$1188	27.69	\$176	5.39
8	16.65	\$5153.85	\$3991.50	220	130	160	\$4553.85	\$600	11.64	-\$562	-14.08
9	18.90	\$5549.74	\$4384.99	246	139	156	\$4693.79	\$855	15.42	-\$308	-7.04
1	8.26	\$2881.97	\$2620.97	65	15	65	\$1616.19	\$1265	43.92	\$1004	38.33
2	35.40	\$4902.56	\$4335.96	60	40	60	\$3991.23	\$911	18.58	\$344	7.95

than the Modified Fixed Order System. The reasons for this are analyzed later.

The savings resulting from the use of the Joint Ordering Rule were not as large as in the previous case (see (1) above). The savings ranged from a minimum of 6.86% for Data Set 10, to a maximum of 35.33% for Data Set 9.

The above two conclusions indicated that there was sufficient evidence to believe that in general, the proposed Joint Ordering Rule characterized by three parameters, $\underline{S}$, $\underline{s}$, and $\underline{s}'$ results in lower total costs, as compared to both the Fixed Order System and the Modified Fixed Order System. Hence, whenever possible, it would be more economical to group items and order them jointly using the proposed Joint Ordering Rule.

The major hypothesis was therefore true.

3. Inspection of the minimum cost values of the three parameters $\underline{S}$, $\underline{s}$, and $\underline{s}'$ obtained for the various data sets indicated that for Data Sets 1, 2, and 3, $\underline{S}$ was equal to $\underline{s}'$, and both $\underline{S}$ and $\underline{s}'$ were greater than $\underline{s}$.

That is,

$$\underline{S} = \underline{s}' > \underline{s}$$

But for Data Sets 4, 5, 6, and 7, $\underline{S}$ was greater than $\underline{s}'$, which in turn was greater than $\underline{s}$. That is,

$$\underline{S} > \underline{s}' > \underline{s}$$

Hence, two distinct cases could be identified.

Case 1: $s' = S$

When $\underline{S}$ is equal to $\underline{s'}$ the Joint Ordering Rule is modified and becomes:¹

When the inventory level of any item in the group is equal to or less than the reorder point $\underline{s}$, all items in the group should be reordered jointly.

Analysis of the data sets which gave rise to these particular cases (Data Sets 1, 2, and 3) indicated that the standard deviations of mean demands for the items in the group were relatively small. For example, the standard deviation of mean demands for Data Set 1 was less than one fourth that of Data Set 6 (standard deviation = 9.04). The standard deviation of mean demands for Data Set 3 (standard deviation = 4.26) was less than half that of Data Set 8, (standard deviation = 16.65). These relatively small values of the standard deviation of mean demands for the items in the group, indicates that the mean demands are not widely dispersed. Hence, it was concluded that:

If the standard deviation of mean demands for the item is relatively small, it is not only economical to group the

¹ See p. 37.

items and order them jointly, but when an order is placed, it is more economical to include all items in every joint order. That is, when the inventory level of any item is less than or equal to the reorder point, all items in the group should be ordered jointly.

Case 2: $S > s' > s$:

In this case the Joint Ordering Rule is as follows:¹

When the inventory level of any one item is equal to or less than the reorder point s, the inventory level of the rest of the items in the group is checked, and all items that have inventory levels between s' and s are reordered jointly.

Inspection of the data which gave rise to this situation (Data Sets 4, 5, 6, and 7) indicated that the standard deviations of mean demands were neither too small as in Case (1), nor relatively large. For example, the standard deviation of mean demands for Data Set 6 (standard deviation = 9.04) was greater than that of Data Set 1, (standard deviation = 1.96). In fact, the standard deviations of demands for Data Sets 4, 5, 6, and 7 were greater than that of data sets included in Case 1, discussed earlier.² But when compared to Data Sets 8 and 9, the standard deviations of

¹See p. 36.

²See p. 111.

mean demands were relatively smaller. All this means is, that in the case of Data Sets 4, 5, 6, and 7, the mean demands for the items in the group are neither as widely dispersed as in Data Sets 8 and 9, nor are they as narrowly dispersed as in Data Sets 1, 2, and 3. Hence, it was concluded that:

When the standard deviation of mean demands of the items in the group is neither too large nor too small, it is not economical to include all items in every order. Instead, it is more economical to include some items in the joint order. The items that should be included is determined by the reorder range $\underline{s}'$ to $\underline{s}$. In short, the joint ordering rule that would be most economical would be the one stated on p. 112.

4. According to the definition of the Joint Ordering Rule, $\underline{s}'$ could have any value between $\underline{S}$ and $\underline{s}$. That is, $\underline{s}'$ could have a maximum value of $\underline{S}$ and a minimum value of $\underline{s}$. Hence, three distinct cases were identified.

Case 1: $\underline{s}' = \underline{S}$

Data Sets 1, 2, and 3 illustrated this particular case.

The standard deviation of the mean demands for the items ranged from 1.96 to 6.0, approximately.

Case 2: $\underline{S} > \underline{s}' > \underline{s}$:

Data Sets 4, 5, 6, and 7 illustrated this case. The standard

deviation of mean demands for the items ranged from 6.0 to 14, approximately.

Case 3: $s' = s$:

No data sets gave rise to this case.

In addition to these three, there was another distinct case.

Case 4:

Here, the total cost per year when the proposed Joint Ordering Rule was used was greater than the total cost per year when the Modified Fixed Order System was used to control inventory. Data Sets 8 and 9 were examples of this case. The standard deviations of mean demands were greater than 16.50.

Hence, using the standard deviation as a measure of the dispersion of the mean demands (of the items in a group), the approximate ranges of standard deviation within which each of the four cases would occur are as shown:

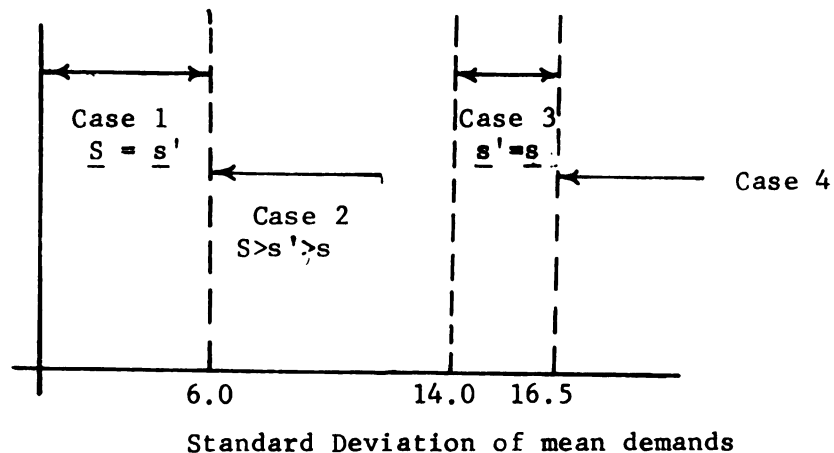


Figure 5.1: Approximate ranges for the four cases of the Joint Ordering Rule. The ranges are expressed in terms of the mean demands.

It must be recognized that these approximate ranges are valid only under the assumptions made in the analysis. If these assumptions are changed, new estimates of these ranges will have to be determined. Some of the assumptions made in arriving at the approximate ranges are:¹

- i) The demand for each item is normally distributed with a certain mean and standard deviation.
 - ii) Ordering cost is equal to \$15 per order regardless of the number of items included in the order.
 - iii) Inventory carrying cost per unit per year is approximately equal to 20 per cent of the value of an item. The value of the item was assumed to be \$10. Hence, the inventory carrying cost was \$2.08 per unit per year.
 - iv) The profit margin is equal to 10 percent of the value of an item, which is equal to \$1 per unit.
 - v) The value of lost sales is equal to \$1 times the number of units demanded but not supplied.
5. As mentioned in (4), no data sets gave rise to the particular case where $s' = s$. It would be worthwhile to investigate the reason for this.

When $\underline{s}'$ is equal to $\underline{s}$, the reorder range $\underline{s}'$ to $\underline{s}$ is minimum and is equal to zero. Hence, no items can have inventory levels in the reorder range. This means that the

¹See pp. 55, 69.

Joint Ordering Rule is modified as follows:

Whenever the inventory level of an item is equal to, or less than the reorder point $\underline{s}$, that item alone should be reordered.

It is evident that the Joint Ordering Rule has been reduced to a simple individual item reorder rule. Moreover, instead of three parameters, the rule is characterized by two parameters, namely $\underline{S}$ and $\underline{s}$ (or $\underline{s}'$). The rule is now very similar to the simple (S, s) rule, or the familiar two bin inventory control system.

There is however, one important difference between the simple (S, s) rule and the Joint Ordering Rule in the modified form, $(s' = s, S)$. In the simple (S, s) rule, the parameters $\underline{S}$ and $\underline{s}$ are computed for each individual item, and these are used to control the inventory of the respective items. Thus, if there were twelve items in the group, it would be necessary to compute twenty-four values, two for each item. But, in the modified Joint Ordering Rule $(s' = s, S)$, the parameters $\underline{S}$ and $\underline{s}$ (or $\underline{s}'$) are common to all items in the group. Hence, unlike the simple (S, s) system, two parameters are sufficient to control the inventory of the entire group.

It is this difference between the simple (S, s) system and the modified form of the Joint Ordering Rule that is

responsible for a less frequent occurrence of this case, namely $\underline{s}'$ equal to $\underline{s}$. This is because, though the demands for the items are widely dispersed,¹ a single reorder point $\underline{s}$ and a single maximum inventory level $\underline{S}$ are used to control the inventory of all items in the group. In most cases the use of these single values for the entire group would be less economical, and lead to higher total costs than when the simple (S, s) system or two bin system is used. For example, since the mean demands for the items are widely dispersed, the reorder points computed for each of these items independently will be widely dispersed. Hence, the use of a single reorder point for the entire group, will lead to increased back orders for some items and increased average inventory for others. The net effect would be an increase in total costs, or more correctly, the total costs would be greater than when the two bin system is used. This corresponds to Case (4) mentioned earlier.

On the other hand, if at all, it was more economical to use the Joint Ordering Rule. Then, it would (generally) be more economical to order more than one item at a time, at least sometimes, if not always. This in turn implies a non zero reorder range, i.e., $\underline{s}'$ not equal to $\underline{s}$, and corresponds to

¹See p. 111. This particular case where $\underline{s}'$ is equal to $\underline{s}$ should occur when the standard deviations of mean demands is between 14.0 and 16.5, approximately.

²See p. 113.

1

case (2) mentioned earlier.¹ In short, when the standard deviation of mean demands is relatively large, either case (2), ($S > s' > s$), or case (4), (the Fixed Order system is more economical than the Joint Ordering Rule) would occur. That is, situations where $\underline{s}'$ is equal to $\underline{s}$ would be less frequent.

6. In order to determine the optimum² values of the parameters, $\underline{S}$, $\underline{s}'$ and $\underline{s}$, which characterize the Joint Ordering Rule, a search was conducted. For reasons mentioned on p. 81, a limited search procedure was conducted in most cases. Because of this limited search, the optimum values of the parameters were only local optimums. But, for all data sets where such a limited search was conducted, a local optimum was found.

For Data Sets 4 and 5 an extensive search for the optimum values of the parameters was conducted. For both data sets the optimum values of the three parameters were found. The minor hypothesis was therefore true.

But, during the search process (extensive or limited) an interesting thing was noticed; the optimum values of the parameters were not unique. For example, when Data Set 7 was used, two sets of optimum values of the three parameters

¹See page 112.

²The word "optimum" as used here means the values of the parameters corresponding to minimum total cost.

were obtained. These were:

1. $\underline{S} = 128$

$$\underline{s}' = 128$$

$$\underline{s} = 83$$

Average total cost per year = \$3100.

2. $\underline{S} = 128$

$$\underline{s}' = 83$$

$$\underline{s} = 108$$

Average total cost per year = \$3103.

Hence, it was concluded that for a certain data set there may not necessarily be a unique optimum set of parameters.

REAL DATA

Table 5.11 and 5. 12 have a detailed account of the results obtained when real data was used to simulate the inventory control system. The results indicated that:

1. The average total cost per year when the proposed Joint Ordering Rule was used to control inventory was less than the average total cost per year when the Fixed Order System was used. This was true for both Real Data Set 1, when there were 8 items in the group, and Real Data Set 2, when the number of items in the group was equal to 12. The savings resulting from the use of the Joint Ordering Rule were 43.92% for Real Data Set 1, and 18.58% for Real Data Set 2.

2. The average total cost per year when the proposed Joint Ordering Rule was used to control inventory was less than the average total cost per year when the Modified Fixed Order System was used. This was true for both Real Data Sets 1 and 2. The savings resulting from the use of the Joint Ordering Rule were 38.33% for Real Data Set 1, and 7.95% for Real Data Set 2. As is evident, the savings are less than when the Joint Ordering rule is compared with the Fixed Order System.

The above two conclusions once again indicated that the proposed Joint Ordering resulted in lower total cost per year, as compared to both the Fixed Order System and the Modified Fixed Order System. Hence, if possible, items should be grouped and ordered jointly so as to minimize total costs. The major hypothesis was therefore true.

3. Inspection of the minimum cost values of the parameters $\underline{S}$, $\underline{s}'$, and $\underline{s}$, obtained for the two data sets, indicated that for both data sets $\underline{S}$ was equal to $\underline{s}'$, and both $\underline{S}$ and $\underline{s}'$ were greater than $\underline{s}$. That is, $(S = s' > s)$. Hence, for both data sets it was not only more economical to order the items jointly, but, when an order was placed, additional savings were achieved by including all items in every joint order. This corresponds to case (1) discussed on p. 111.
4. Inspection of the two data sets indicated that the standard deviation of mean demands for Real Data Set 1 was 8.26, while

that for Real Data Set 2 was 35.4. It is evident that the standard deviation for Real Data Set 2 is very large as compared to the standard deviation for Real Data Set 1. Yet, in both cases it was more economical to include all items in every order. This is contrary to the conclusions reached when the hypothetical data was used to simulate the inventory control system.¹ According to those conclusions, Real Data Set 2 would probably come under Case 4. That is, since the standard deviations of mean demands for Real Data Set 2 is so large, the mean demands are widely dispersed, and hence, it would probably be more economical to treat each item individually and use an individual item rule to manage inventory. But, as mentioned earlier (see p.115), the approximate ranges of the standard deviation determined for each of the four cases were only valid under certain conditions. These conditions are not valid for the real data. For example, in the case of the hypothetical data, demand for each item was assumed to be normally distributed. For the real data, demand for each item has a gamma distribution. In addition to this, for the hypothetical data, inventory carrying cost was assumed to be 20 percent of the value of an item. In the case of the real data, inventory carrying cost was 30 percent of the value of the item. These are but a few differences between

¹See p. 114.

the assumptions made in the hypothetical data and the real data. Hence, the ranges established for the four cases using hypothetical data will not be valid for the real data. Lastly, the ranges of the standard deviation for the four different cases could not be established for the real data, because of the limited data available.

5. In order to determine the optimum values of the parameters $\underline{S}$, $\underline{s}'$ and $\underline{s}$, which characterize the Joint Ordering Rule, a search procedure was adopted. The search in this case was not limited , but extensive. The search procedure is outlined on p.81. For both data sets the optimum values of the three parameters were found. The minor hypothesis was therefore true. But, as in the case of the hypothetical data, the optimum values of the parameters were not unique, especially for Real Data Set 1. For this data set, the two sets of parameters which lead to the same minimum cost are:

1. $\underline{S} = 65$ $\underline{s}' = 65$ $\underline{s} = 15$
2. $\underline{S} = 65$ $\underline{s}' = 60$ $\underline{s} = 15$

Table 5.1 :

Data Set 1

The means and standard deviations of the demands for the twelve items in the group.

ITEM	MEAN	STANDARD DEVIATION	E.O.Q.	REORDER POINT	PERCENTAGE DEMAND
1	8	1	77	27	7.619
2	10	1	86	33	9.5238
3	10	2	86	36	9.5238
4	6	1	67	21	5.7143
5	10	1	86	33	9.5238
6	11	1	90	36	10.4762
7	12	1	94	39	11.4386
8	9	1	82	20	8.5714
9	9	2	82	33	8.5714
10	5	1	61	18	4.7619
11	8	1	77	27	7.619
12	7	1	72	24	6.6667
					<u>100.00</u>

The Standard Deviation of the mean demands = 1.96
Range of ~~mean~~ demands = 12 - 5 = 7

The Weighted Average E.O.Q. = 82.0667
The Weighted Average Reorder Point = 31.1143

1. The Fixed Order System:

Cost per item ordered = \$15
Average total cost per year = \$2387.66

2. The Modified Fixed Order System:

Cost per order = \$15
Average total cost per year = \$1871.18

3. The Proposed Joint Ordering Rule:

Optimum Values of the Parameters:
S = 80 s = 28 s' = 80
Average total cost per year = \$1412.79

Savings in Total Cost with respect to The Fixed Order System = \$974.87
= 40.82%

Savings in Total Cost with respect to The Modified Fixed
Order System = \$458.39
= 24.49%

Table 5.2:

Data Set 2

The means and standard deviations of the demands for the twelve items in the group.

ITEM	MEAN	STANDARD DEVIATION	E.O.Q.	REORDER POINT	PERCENTAGE DEMAND
1	5	1	61	18	4.8544
2	7	1	72	24	6.7961
3	9	2	82	33	8.7379
4	8	1	77	27	7.767
5	6	1	67	21	5.8252
6	11	2	90	39	10.6796
7	12	1	94	39	11.6505
8	9	1	82	30	8.7379
9	10	1	86	33	9.7087
10	11	2	90	39	10.6796
11	8	1	77	27	7.767
12	7	1	72	24	6.7961
					<u>100.00</u>

The Standard Deviation of the mean demands = 2.15
 Range of mean demands 12 - 5 = 7

The Weighted Average E.O.Q. = 81.466
 The Weighted Average Reorder Point = 31.1359

1. The Fixed Order System :

Cost per item ordered = \$15
 Average total cost per year = \$2361.64

2. The Modified Fixed Order System:

Cost per order = \$15
 Average total cost per year = \$1892.89

3. The Proposed Joint Ordering Rule:

Optimum Values of the Parmeters:
 $\underline{s} = 50$ $\underline{s} = 30$ $\underline{s}' = 50$
 Average total cost per year = \$1224.04

Savings in Total Cost with respect to The Fixed Order System = \$1137.60
= 48.16%

Savings in Total Cost with respect to The Modified Fixed
 Order System = \$668.85
= 35.33%

Table 5.3:

Data Set 3

The means and standard deviations of the demands for the twelve items in the group.

ITEM	MEAN	STANDARD DEVIATION	E.O.Q.	REORDER POINT	PERCENTAGE DEMAND
1	13	2	98	45	6.7708
2	15	1	106	48	7.8125
3	16	2	109	54	8.3333
4	19	1	119	60	9.8958
5	21	1	125	66	10.9375
6	11	1	90	36	5.7292
7	17	2	112	57	8.8542
8	21	1	125	66	10.9375
9	23	2	131	75	11.9792
10	14	1	102	45	7.2917
11	10	1	86	33	5.2083
12	12	1	94	39	6.25
					<u>100.00</u>

The Standard Deviation of the mean demands = 4.26
 Range of mean demands = 23 - 10 = 13

The Weighted Average E.O.Q. = 111.6771
 The Weighted Average Reorder Point = 55.2031

1. The Fixed Order System:

Cost per item ordered = \$15
 Average total cost per year = \$3338.64

2. The Modified Fixed Order System:

Cost per order = \$15
 Average total cost per year = \$2603.40

3. The Proposed Joint Ordering Rule:

Optimum Values of the Parameters:
 S = 80 s = 50 s' = 80
 Average total cost per year = \$1829.86

Savings in Total Cost with respect to The Fixed Order System = \$1508.78
 = 45.19%

Savings in Total Cost with respect to The Modified Fixed
 Order System = \$773.54
 = 29.71%

Table 5.4:

Data Set 4

The means and standard deviations of the demands for the twelve items in the group.

ITEM	MEAN	STANDARD DEVIATION	E.O.Q.	REORDER POINT	PERCENTAGE DEMAND
1	20	1	122	63	5.3333
2	23	2	131	75	6.1333
3	26	3	139	88	6.9333
4	28	2	144	90	7.4667
5	30	4	149	103	8.00
6	29	1	147	90	7.7333
7	33	2	157	105	8.80
8	35	4	162	118	9.3333
9	37	3	166	121	9.8667
10	40	2	173	126	10.6667
11	40	3	173	130	10.6667
12	34	3	159	112	9.0667
					<u>100.00</u>

The Standard Deviation of the mean demands = 6.39

Range of mean demands = 40 - 20 = 20

The Weighted Average E.O.Q. = 154.856

The Weighted Average Reorder Point = 105.6347

1. The Fixed Order System:

Cost per item ordered = \$15

Average total cost per year = \$5009.27

2. The Modified Fixed Order System:

Cost per order = \$15

Average total cost per year = \$3815.28

3. The Proposed Joint Ordering Rule:

Optimum Values of the Parameters:

S = 170 s = 105 s' = 160

Average total cost per year = \$3367.37

Savings in Total Cost with respect to The Fixed Order System = \$1641.90

= 32.77%

Savings in Total Cost with respect to The Modified Fixed
Order System

= \$447.91

= 11.73%

Table 5.5 :

Data Set 5

The means and standard deviations of the demands for the twelve items in the group.

ITEM	MEAN	STANDARD DEVIATION	E.O.Q.	REORDER POINT	PERCENTAGE DEMAND
1	4	1	54	15	2.2989
2	6	1	67	21	3.4483
3	9	2	82	33	5.1724
4	12	1	94	39	6.8966
5	15	2	106	51	8.6207
6	17	2	112	47	9.7701
7	19	3	119	67	10.9195
8	21	2	125	69	12.069
9	23	2	131	75	13.2184
10	25	3	136	85	14.3678
11	10	1	86	33	5.7471
12	12	2	98	45	7.4713
					<u>100.00</u>

The Standard Deviation of the mean demands = 6.69

Range of mean demands = 25 - 4 = 21

The Weighted Average E.O.Q. = 111.5575

The Weighted Average Reorder Point = 58.4958

1. The Fixed Order System:

Cost per item ordered = \$15

Average total cost per year = \$3156.39

2. The Modified Fixed Order System:

Cost per order = \$15

Average total cost per year = \$2509.29

3. The Proposed Joint Ordering Rule:

Optimum Values of the Parameters:

S = 90 s = 60 s' = 90

Average total cost per year = \$2131.01

Savings in Total Cost with respect to The Fixed Order System = \$1025.38

= 32.47%

Savings in Total Cost with respect to The Modified Fixed
Order System

= \$378.28

= 15.06%

Table 5.6 :

Data Set 6

The means and standard deviations of the demands for the twelve items in the group.

ITEM	MEAN	STANDARD DEVIATION	E.O.Q.	REORDER POINT	PERCENTAGE DEMAND
1	5	1	61	18	2.2222
2	7	1	72	24	3.1111
3	11	2	90	39	4.8889
4	14	2	102	48	6.2222
5	17	2	112	57	7.5556
6	21	3	125	73	9.3333
7	24	2	134	78	10.6667
8	27	3	142	91	12.00
9	30	3	149	100	13.3333
10	32	3	154	106	14.2222
11	12	1	94	39	5.3333
12	25	2	136	81	11.1111
					<u>100.00</u>

The Standard Deviation of the mean demands = 9.04

Range of mean demands = 32 - 5 = 27

The Weighted Average E.O.Q. = 127.6978

The Weighted Average Reorder Point = 75.8889

1. The Fixed Order System:

Cost per item ordered = \$15

Average total cost per year = \$3616.00

2. The Modified Fixed Order System:

Cost per order = \$15

Average total cost per year = \$2833.75

3. The Proposed Joint Ordering Rule:

Optimum Values of the Parameters:

$\underline{S} = 107$ $\underline{s} = 63$ $\underline{s}' = 87$

Average total cost per year = \$2639.27

Savings in Total Cost with respect to The Fixed Order System = \$976.73

= 27.01%

Savings in Total Cost with respect to The Modified Fixed
Order System

= \$194.48

= 6.86%

Table 5.7:

Data Set 7

The means and standard deviations of the demands for the twelve items in the group.

ITEM	MEAN	STANDARD DEVIATION	E.O.Q.	REORDER POINT	PERCENTAGE DEMAND
1	10	1	86	33	3.2787
2	15	2	106	51	4.918
3	17	2	112	57	5.5738
4	19	1	119	60	6.2295
5	20	3	122	70	6.5574
6	25	2	136	81	8.1967
7	27	2	142	87	8.8525
8	29	3	147	97	9.5082
9	30	3	149	100	9.8361
10	35	3	162	115	11.4754
11	38	2	168	120	12.459
12	40	3	173	130	13.1148
					<u>100.00</u>

The Standard Deviation of the mean demands = 9.47

Range of mean demands = 40 - 10 = 30

The Weighted Average E.O.Q. = 144.2492

The Weighted Average Reorder Point = 93.6918

1. The Fixed Order System:

Cost per item ordered = \$15.

Average total cost per year = \$4292.20

2. The Modified Fixed Order System:

Cost per order = \$15

Average total cost per year = \$3280.45

3. The Proposed Joint Ordering Rule:

Optimum Values of the Parameters:

S = 128 s = 83 s' = 108

Average total cost per year = \$3103.51

Savings in Total Cost with respect to The Fixed Order System = \$1188. 69

= 27.69%

Savings in Total Cost with respect to The Modified Fixed
Order System

= \$176.94

= 5.39%

Table 5.8 :

Data Set 8

The means and standard deviations of the demands for the twelve items in the group.

ITEM	MEAN	STANDARD DEVIATION	E.O.Q.	REORDER POINT	PERCENTAGE DEMAND
1	5	1	61	18	1.2887
2	21	1	125	66	5.4124
3	27	2	142	87	6.9588
4	31	2	152	99	7.9897
5	34	3	159	112	8.7629
6	40	2	173	126	10.3093
7	48	3	189	154	12.3711
8	54	2	201	168	13.9175
9	62	2	215	192	15.9794
10	12	1	94	39	3.0928
11	24	1	134	75	6.1856
12	30	1	149	93	7.732
					<u>100.00</u>

The Standard Deviation of the mean demands = 16.65

Range of mean demands = 62 - 5 = 57

The Weighted Average E.O.Q. = 169.7732

The Weighted Average Reorder Point = 126.7216

1. The Fixed Order System:

Cost per item ordered = \$15

Average total cost per year = \$5153.85

2. The Modified Fixed Order System:

Cost per order = \$15

Average total cost per year = \$3991.50

3. The Proposed Joint Ordering Rule:

Optimum Values of the Parameters

S = 220 s = 130 s' = 160

Average total cost per year = \$4553.85

Savings in Total Cost with respect to The Fixed Order System = \$600.00

= 11.64%

Savings in Total Cost with respect to The Modified Fixed
Order System

= -\$562.35

= -14.08%

Table 5.9 :

Data Set 9

The means and standard deviations of the demands for the twelve items in the group.

ITEM	MEAN	STANDARD DEVIATION	E.O.Q.	REORDER POINT	PERCENTAGE DEMAND
1	60	3	212	190	14.7059
2	10	3	86	40	2.451
3	21	3	125	73	5.1471
4	30	2	149	96	7.3529
5	39	3	171	127	9.5588
6	53	3	199	169	12.9902
7	64	3	219	202	15.6863
8	3	1	47	12	.7353
9	25	1	136	78	6.1275
10	26	1	139	81	6.3725
11	33	2	157	105	8.0882
12	44	1	181	135	10.7843
					<u>100.00</u>

The Standard Deviation of the mean demands = 18.90
Range of mean demands = 64 - 3 = 61

The Weighted Average E.O.Q. = 176.9779
The Weighted Average Reorder Point = 138.598

1. The Fixed Order System:

Cost per item ordered = \$15
Average total cost per year = \$5549.74

2. The Modified Fixed Order System:

Cost per order = \$15
Average total cost per year = \$4384.99

3. The Proposed Joint Ordering Rule:

Optimum Values of the Parameters:
S = 246 s = 139 s = 156
Average total cost per year = \$4693.79

Savings in Total Cost with respect to The Fixed Order System = \$855.95
= 15.42%

Savings in Total Cost with respect to The Modified Fixed
Order System = -\$308.80
= -7.04%

Table 5.10 :

REAL Data Set 1:

The means and standard deviations of the demands for the eight items in the group.

ITEM	MEAN	STANDARD DEVIATION	E.O.Q.	REORDER POINT	PERCENTAGE DEMAND
1	39.27	1452	56.6	249.8	15.1
2	37.81	491	46.3	190.1	14.6
3	18.84	374	150.3	123.5	7.3
4	21.51	195	45.2	112.9	8.3
5	40.19	2869	126.7	306.1	15.5
6	42.05	701	59.8	217.8	16.2
7	29.02	464	72.8	161.6	11.2
8	30.93	309	43.4	153.6	11.9
					<u>100.0</u>

The Standard Deviation of the mean demands = 8.264

Range of mean demands = 23.21

The Weighted Average E.O.Q. = 72.6

The Weighted Average Reorder Point = 202.8

1. The Fixed Order System:

Cost per item ordered = \$15

Average total cost per year = \$2881.97

2. The Modified Fixed Order System:

Cost per order = \$15

Average total cost per year = \$2620.97

3. The Proposed Joint Ordering Rule:

Optimum Values of the Parameters:

S = 65 s = 15 s' = 65

Average total cost per year = \$1616.19

Savings in Total Cost with respect to The Fixed Order System = \$1265.78

= 43.92%

Savings in Total Cost with respect to The Modified Fixed
Order System

= \$1004.78

= 38.33%

Table 5.11 :

REAL Data Set 2:

The means and standard deviations of the demands for the twelve items in the group.

ITEM	MEAN	STANDARD DEVIATION	E.O.Q.	REORDER POINT	PERCENTAGE DEMAND
1	95.27	1844	110.1	432.5	18.3
2	39.27	1452	56.6	249.8	7.5
3	37.81	491	46.3	190.1	7.3
4	10.63	85	21.0	63.8	2.0
5	18.84	374	150.3	123.5	3.6
6	21.51	195	45.2	112.9	4.1
7	40.19	2869	126.1	306.1	7.7
8	138.64	4945	216.1	659.5	26.6
9	42.05	701	59.8	217.8	8.1
10	29.02	464	72.8	161.6	5.6
11	30.93	309	43.4	153.6	5.9
12	16.9	162	22.4	94.7	<u>3.2</u>
					100.00

The Standard Deviation of the mean demands = 35.4

Range of mean demands = 128.01

The Weighted Average E.O.Q. = 114.9

The Weighted Average Reorder Point = 360.3

1. The Fixed Order System:

Cost per item ordered = \$15

Average total cost per year = \$4902.56

2. The Modified Fixed Order System:

Cost per order = \$15

Average total cost per year = \$4335.96

3. The Proposed Joint Ordering Rule:

Optimum Values of the Parameters:

$\underline{S} = 60$ $\underline{s} = 40$ $\underline{s}' = 60$

Average total cost per year = \$3991.23

Savings in Total cost with respect to The Fixed Order System = \$911.33

= 18.58%

Savings in Total Cost with respect to The Modified Fixed
Order System

= \$344.73

= 7.95%

CHAPTER 6

SUMMARY OF CONCLUSIONS

The purpose of this research was to develop a joint ordering rule which could be used to order a group of items at one time. The potential savings resulting from the inclusion of many items on one purchase order are quite large. Some of the advantages of joint orders are:

1. Ordering costs can be reduced by including several items in a single order. This is especially true when one supplier is the source of a variety of related items.
2. Shipping costs can be decreased if the total order is of a convenient size, e.g. a truck load. Since a number of items are ordered jointly, it is more likely that the total order will be of a convenient size.
3. By including a number of items in a single order, there is an increased opportunity of taking advantage of quantity discounts offered by a vendor.

A review of the literature indicates that much analytical work has been done in the cases of independent ordering strategies such as the fixed order rule, the periodic ordering rule, and the (s, S) inventory control rule. But, the literature is almost void of discussion about a joint ordering rule. Hence, it was felt

that there was a real need to research the joint ordering rule.

The proposed joint ordering rule¹ was defined in terms of three parameters, $\underline{S}$, $\underline{s}$, and $\underline{s}'$, where $\underline{S}$ is the maximum inventory level, $\underline{s}$ is the reorder point, and $\underline{s}'$ determines the items that should be included in the order. $\underline{s}'$ to $\underline{s}$ was defined as the reorder range. Note that $\underline{S}$ is greater than $\underline{s}'$, which is greater than $\underline{s}$ as shown in the figure.

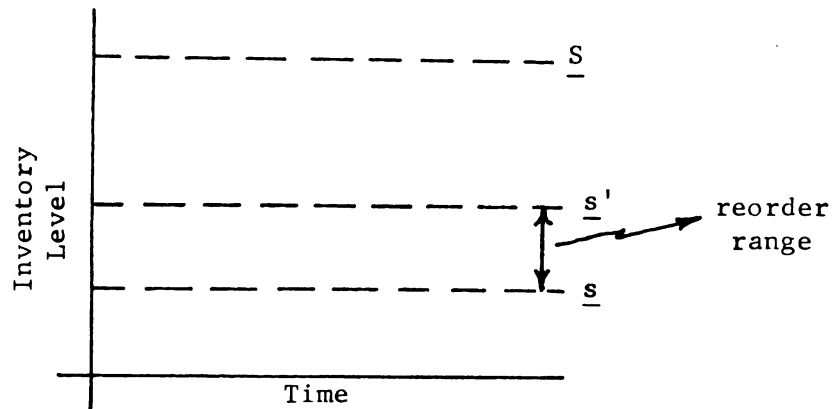


Figure 6.1: The Joint Ordering Rule characterized by three parameters, $\underline{S}$, $\underline{s}$ and $\underline{s}'$.

The joint ordering rule was defined as follows:

When the inventory level of any item in the group has dropped to the reorder level $\underline{s}$, the inventory level of all items in the group is checked; all items which have inventory levels (inventory

¹The writer wishes to state that the proposed joint ordering rule is not "original", but was first proposed by Balintfy.

on hand and on order) within the reorder range $\underline{s}'$ to $\underline{s}$ are ordered jointly. The order quantity for each item ordered is given by $(S-I)$, where $\underline{I}$ is the inventory on hand and on order.

HYPOTHESES

Besides a joint ordering rule, two hypotheses were proposed for testing. These were:

Major Hypothesis:

There exist a number of situations in which the application of the joint ordering rule to control inventory, i.e. ordering a group of items in a single order, results in lower costs when compared with the use of the fixed order quantity rule.

Minor Hypothesis:

If the joint ordering rule is defined by the three parameters $\underline{S}$, $\underline{s}$, and $\underline{s}'$ as mentioned above, then there exists some optimum value for each of the parameters, such that the total cost of the inventory control system is minimized. The costs that are included are ordering costs, inventory carrying costs, and stock out costs.

METHODOLOGY

On the basis of the literature search, there were basically two approaches to the development of the joint ordering model: the analytical approach and the simulation approach. A simulation approach was used in this research, mainly because it was believed

that the mathematical complexity of the problem would make it extremely difficult to formulate the problem and obtain an analytical solution. This view was supported by a number of researchers.

DATA SOURCE

Two kinds of data were used to test the proposed joint ordering rule.

Hypothetical data: A considerable portion of this research was concerned with the study of the properties of the joint ordering rule, especially under extreme conditions. Hence, nine sets of hypothetical data which represented these extreme conditions were generated. In generating this data it was assumed that a product group consists of twelve items, and the demand for each item is normally distributed with a specified mean and standard deviation. The standard deviation of the mean demands of the twelve items was a minimum for Data Set 1 and a maximum for Data Set 9. Data Sets 1 and 9 correspond to the extreme conditions mentioned earlier.

Real Data: Real data were obtained from the Steel Service Center Institute, and a farmers cooperative. Unfortunately, only limited data were available from the farmers cooperative, and hence it was almost impossible to fit a probability distribution to the data. Thus, this data was not used.

Sales data from the Steel Service Center were not obtained

directly, but from the work of Basic.¹ Basic selected twelve items of a typical Steel Service Center and found that the demand for each item was gamma distributed. Hence, the cumulative gamma probabilities for each item were available from Basics' dissertation.

EXPERIMENTATION

The proposed joint ordering policy was compared with two other ordering policies. These were essentially independent ordering policies; that is, each item is assumed to be independent of the others, and hence, the inventory level of each item is controlled independently from the others. In all, three ordering policies were tested.

1. The Fixed Order System: In this case, the same quantity (economic order quantity) of an item is ordered each time the inventory level of the item is equal to or less than the reorder point. The time interval between orders varies with fluctuations in usage. The ordering cost was assumed to be \$15 per item ordered.
2. The Modified Fixed Order System: The inventory control system in this case is the same as in (1). But the ordering cost was assumed to be \$15 per order regardless

¹Basic, M.K., "Development and Application of a Gamma-based Inventory Management Theory" (Unpublished Ph.D. dissertation, Michigan State University, East Lansing, Michigan, 1965)

of the number of items included in the order.

1. Program EOQ simulates the operation of the inventory control system when the Fixed Order System is used to manage inventory.
2. Program EOQ (Modified) was the same as program EOQ. But the ordering cost was now assumed to be \$15 per order regardless of the number of items included in the order.
3. Program REPEAT searches for the optimum values of the three parameters $\underline{S}$, $\underline{s}$, and $\underline{s}'$, that define the Joint Ordering Rule.
4. Once the optimum values of the parameters were found, program CONVERG was used to simulate the system for $\underline{n}$ years, so as to get a reasonable degree of convergence with respect to the total cost per year.

In order to compare the performance of the three inventory control systems, the same data set was used to compute the total cost per year when each inventory control system was used. Thus, a data set was first used to determine the total cost per year when the Fixed Order System was used to control inventory. Program EOQ was used here. Next, the same data set was used in the EOQ (Modified) program to determine the total cost per year when

the Modified Fixed Order System was used to control inventory. Lastly, the same data set was used to determine the total cost per year when the proposed joint ordering rule was used to control inventory. Program REPEAT was first used to search for the minimum cost values of the three parameters $\underline{S}$, $\underline{s}'$, and $\underline{s}$. Then program CONVERG was used to simulate the system for $\underline{n}$ years, using the optimum values of the parameters.

CONCLUSIONS

The results obtained are presented in Table 5.1 through 5.11. An inspection of the results led to the following conclusions:

1. The most important conclusion was, that for certain values of the parameters $\underline{S}$, $\underline{s}'$ and $\underline{s}$, the Joint Ordering Rule resulted in lower total costs per year when compared to the two individual item rules,¹ the Fixed Order Rule and the Modified Fixed Order Rule. Moreover, for certain optimum² values of the three parameters, the savings resulting from the use of the Joint Ordering Rule were substantial. For example, for Data Set 2 the savings were 55.33%.

There was sufficient evidence to support this conclusion. The use of the Joint Ordering Rule resulted in savings for all

¹These are referred to as individual item rules because the inventory level of each item is controlled independently from the others.

²Optimum , means minimum cost values of the parameters.

hypothetical data sets except Data sets 8 and 9. When the data from the Steel Service Center was used, the Joint Ordering Rule once again resulted in lower total cost per year than either the Fixed Order System or the Modified Fixed Order System. Hence, it was concluded that the major hypothesis was true.

2. By definition of the Joint Ordering Rule, $\underline{s}'$ can have a maximum value of $\underline{S}$ and a minimum of $\underline{s}$. Hence, three distinct cases can be identified.

Case 1: $\underline{s}' = \underline{S}$:

When $\underline{s}'$ is equal to $\underline{S}$, the three parameter Joint Ordering Rule is then defined by two parameters, $\underline{S}$ (or $\underline{s}'$) and $\underline{s}$. And, since $\underline{S}$ is the maximum inventory level and $\underline{s}'$ to $\underline{s}$ is defined as the reorder range, the reorder range now becomes $\underline{S}$ to $\underline{s}$ and is maximum. Moreover, the inventory level of an item (inventory on hand and on order) will always lie between the maximum $\underline{S}$ and the reorder point $\underline{s}$. Therefore, when $\underline{S}$ is equal to $\underline{s}'$, the inventory level will always lie in the reorder range $\underline{S}$ to $\underline{s}$. All this means is, that the Joint Ordering Rule is modified and now becomes:

When the inventory level (inventory on hand and on order) of any item in the group is equal to or less than the trigger point $\underline{s}$, all items in the group should be reordered. The quantity of each item ordered is equal to $(S - I)$, where $\underline{I}$ is equal to the inventory on hand and on order.

This modified form of the Joint Ordering Rule is very similar to the one proposed by Starr and Miller.¹ Starr and Miller compute a value of $\underline{t}$, the number of months between orders which is assumed to be the same for all items within the group. Hence, according to the authors, every item should be ordered every $\underline{t}$ months. Starr and Miller's joint ordering rule is simply a particular case of the Joint Ordering Rule proposed in this thesis.

During the experimentation process it was found that for Data Sets 1, 2, and 3, and Real Data Sets 1 and 2, the least cost values of the parameters were such that $\underline{s}'$ was equal to $\underline{S}$. Hence, these data sets illustrate this particular case. Analysis of the four hypothetical data sets indicated that the standard deviations of the mean demands for the items were relatively small (between 1.96 and 4.26). This means that the mean weekly demands for the items in the group were not widely dispersed. Hence, it was concluded that when the mean demands for the items are not widely dispersed, it is not only economical to group the items and order them jointly, but additional savings may be achieved by including all items in every joint order.

Case 2: $S > s' > s$:

In this case, the Joint Ordering Rule is characterized by three parameters, $\underline{S}$, $\underline{s}'$ and $\underline{s}$, and is as stated in the major hypothesis:

¹Starr, M.K. and Miller, D.W., Inventory Control: Theory and Practice (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1966) pp. 104-110

When the inventory level (inventory on hand and on order) of any item in the group is equal to or less than the reorder point $\underline{s}$, the inventory level (inventory on hand and on order) of the rest of the items in the group is checked, and all items that have inventory levels (inventory on hand and on order) between $\underline{s}'$ and $\underline{s}$, the reorder range, are ordered jointly.

The results indicate that Data Sets 4, 5, 6, and 7 are illustrations of this particular case. Analysis of these three data sets indicated that the standard deviations of the mean demands for the items (between 6.39 and 9.47) were neither as small as those of Data Sets 1,2,3, or 4 (i.e. Case 1) nor were they as large as those of Data Sets 8 and 9. This means that the mean demands of the items were neither too widely dispersed nor too narrowly dispersed. Hence, it was concluded that if the mean demands of the items are neither too widely nor too narrowly dispersed, it is more economical to include only some items in the joint order. The items that should be included are those which have inventory levels (inventory on hand and on order) within the reorder range $\underline{s}'$ to $\underline{s}$, when an order is placed.

Case 3: $\underline{s}' = \underline{s}$:

When $\underline{s}'$ is equal to $\underline{s}$ the Joint Ordering Rule is once again defined by two parameters, $\underline{s}$ and $\underline{s}'$ (or $\underline{s}$) as in Case (1). Since $\underline{s}'$ to $\underline{s}$ is the reorder range, when $\underline{s}'$ is equal to $\underline{s}$ the reorder range is equal to zero and is minimum. The Joint Ordering Rule

now becomes:

When the inventory level (inventory on hand and on order) of an item is equal to or less than the reorder point $\underline{s}$, only that single item should be reordered.

The joint ordering rule is therefore reduced to an individual item rule such as the simple (s, S) rule. However, there is one difference between this modified form of the Joint Ordering Rule and the (s, S) , or the two bin inventory control system. In the simple (s, S) inventory control system, the parameters $\underline{s}$ and $\underline{S}$ are computed for each individual item, and these parameters are then used to control the inventory of the respective items. In the proposed Joint Ordering Rule in the modified form, the values of the two parameters $\underline{s}$ and $\underline{S}$ are common to all items in the group. Other than this, the two reordering rules are identical and both are essentially individual item rules.

Hence, it was concluded that, if the value of the two parameters $\underline{s}$ and $\underline{S}$ of (s, S) inventory control system is the same for all items, then the (s, S) individual ordering rule is a particular case of the joint rule characterized by three parameters, $\underline{S}$, $\underline{s}'$ and $\underline{s}$.

Out of the nine data sets used to test the proposed joint ordering rule, in no case were the minimum cost parameters such that $\underline{s}'$ was equal to $\underline{s}$. This was indeed surprising. An explan-

ation as to why examples of this particular case would be rare is given on p. 115.

The most important conclusion that was drawn from these three cases was that the Joint Ordering Rule proposed in this thesis is more general and incorporates within itself a number of other reordering rules. Both individual item ordering rules such as the two bin inventory control system, and some joint ordering rules such as the one proposed by Starr and Miller are incorporated.

3. Besides the 3 cases identified above, there is another case which is conceivable. As mentioned in the major hypothesis, there will exist a number of situations where the joint ordering rule will lead to lower total costs than the individual item ordering rule. However, there will also be certain situations where the joint ordering rule will lead to total costs greater than the individual item ordering rules. During the experimentation process it was found that for Data Sets 8 and 9 the average total cost per year when the Joint Ordering Rule was used to manage inventory was greater than the average total cost when the Modified Joint Ordering Rule was used.

Analysis of Data Sets 8 and 9 indicated that the standard deviation of the mean demands for the items was relatively large. For example, for Data Set 8 the standard deviation was equal to

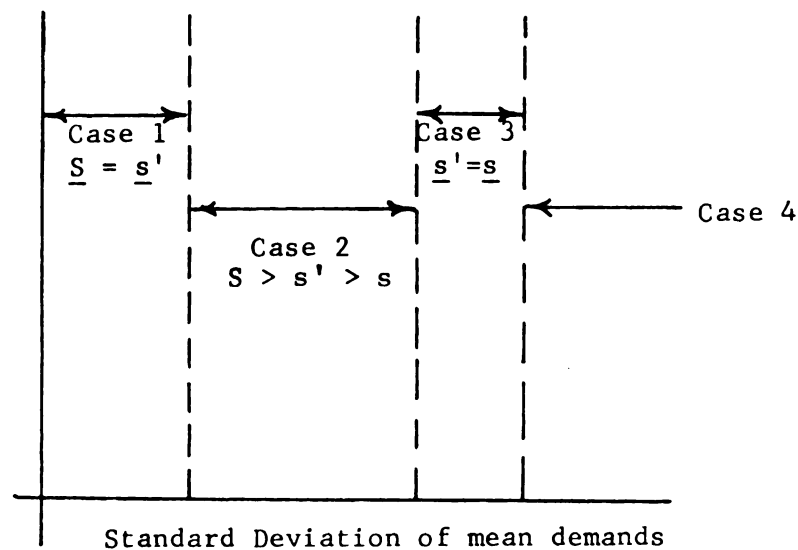


Figure 6.2 : Approximate ranges for the four cases of the Joint Ordering Rule. The ranges are expressed in terms of the standard deviation of the mean demands.

5. In Chapter 4, a method for calculating the starting values of the parameters $\underline{S}$, $\underline{s}'$, and $\underline{s}$ was suggested (see p. 74). It was necessary to calculate these values so as to have some initial values to begin the search for the optimum values of the three parameters $\underline{S}$, $\underline{s}$, and $\underline{s}'$. The starting value of $\underline{s}$ was assumed to be the weighted average of the individual reorder points. The starting value of $\underline{S}$ was assumed to be the weighted average of the individual economic order quantities, plus the starting value of $\underline{s}$ computed earlier. Lastly, the starting value of $\underline{s}'$ was assumed to equal $\underline{S}$, the maximum value of $\underline{s}'$.

However, the results indicated that minimum cost values of the three parameters were not close to these starting values. This was true for all data sets. For example, for Real Data Set 1 the starting values of the three parameters were:

$$\underline{S} = 113$$

$$\underline{s}' = 113$$

$$\underline{s} = 31$$

But the minimum cost values of the three parameters were:

$$\underline{S} = 65$$

$$\underline{s}' = 65$$

$$\underline{s} = 15$$

It was therefore concluded that the procedure for calculating the starting values of the three parameters suggested in Chapter 4 was not the best. The writer believes that this conclusion will be most useful if further investigation of the proposed Joint Ordering Rule is planned.

6. Joint ordering will lead to a savings in ordering costs because a number of items are included in a single order. Additional savings can be achieved through quantity discounts, since the dollar value of a joint order is larger. On the other hand, joint ordering will lead to an increase in inventory carrying costs because some items which have inventory levels within

the reorder range are ordered, though they should be ordered when the inventory level reaches the reorder point $\underline{s}$. This more frequent ordering of some items leads to an increase in average inventory, and hence, an increase in inventory carrying costs.

These opposite costs will depend on the value of the three parameters $\underline{S}$, $\underline{s}'$, and $\underline{s}$. This suggests that there should be some values of the three parameters such that the total cost is minimum.¹ These optimum values were found for all data sets, including Real Data Sets 1 and 2. When the search for the optimum values was essentially a limited search, "local" optimums were found. In the case of Real Data Sets 1 and 2, and hypothetical Data Sets 4 and 5, an extensive search was conducted.² Hence, the optimum values of the parameters were found. All this indicated that the minor hypothesis was true. However, during the search it was found that the optimum values of the three parameters are not necessarily unique.

7. Attempts to find an analytical method for computing the optimum values of $\underline{S}$, $\underline{s}$, and $\underline{s}'$ so as to minimize total cost of the system meet with little success. The only method that can be suggested to compute these optimum values, is the search method used in this research.³ It must be pointed out that this procedure is not as time consuming as it appears. The average

¹This was the minor hypothesis.

²See pp. 36-37 for details.

³See p. 81 for details.

time for computing the minimum cost values of the three parameters on the CDC 6500 at Michigan State University was about 20 to 30 minutes for each data set. The cost of this computer time is a very small fraction of the potential savings resulting from the use of the Joint Ordering Rule.

Lastly, as mentioned earlier, one important advantage of the Joint Ordering Rule is that there is an increased opportunity for taking advantage of quantity discounts offered by a vendor. However, quantity discounts have not been included in the analysis. Yet, the savings resulting from the use of the Joint Ordering Rule to control inventory were found to be substantial. Hence, if quantity discounts were included in the analysis, the savings would probably be greater still. This would merely provide additional evidence that the major hypothesis is true.

There is considerable scope for further research. Firstly, some method for computing the minimum cost values of the three parameters must be developed. Secondly, a certain parameter (or ratio) should be defined which would indicate which of the four cases of the Joint Ordering Rule is likely to occur. This parameter would be a function of the ordering cost, the inventory carrying cost, most of unfilled demand, and some measure of the dispersion of the mean demands of the items in the group. This parameter could also be used to indicate whether the Joint Ordering Rule should be used at all. Lastly, the ultimate goal would be to develop an analytical solution to the joint ordering problem.

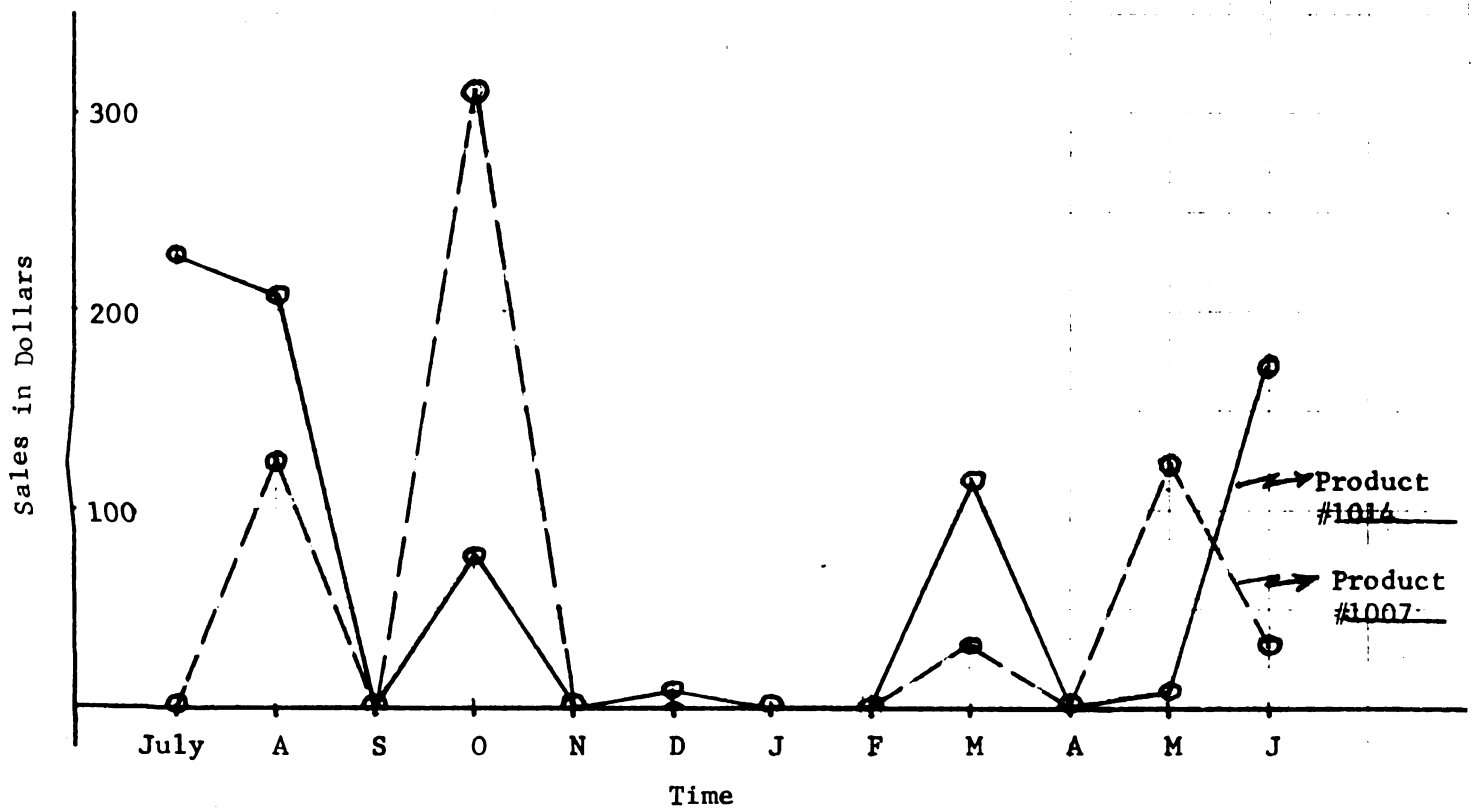
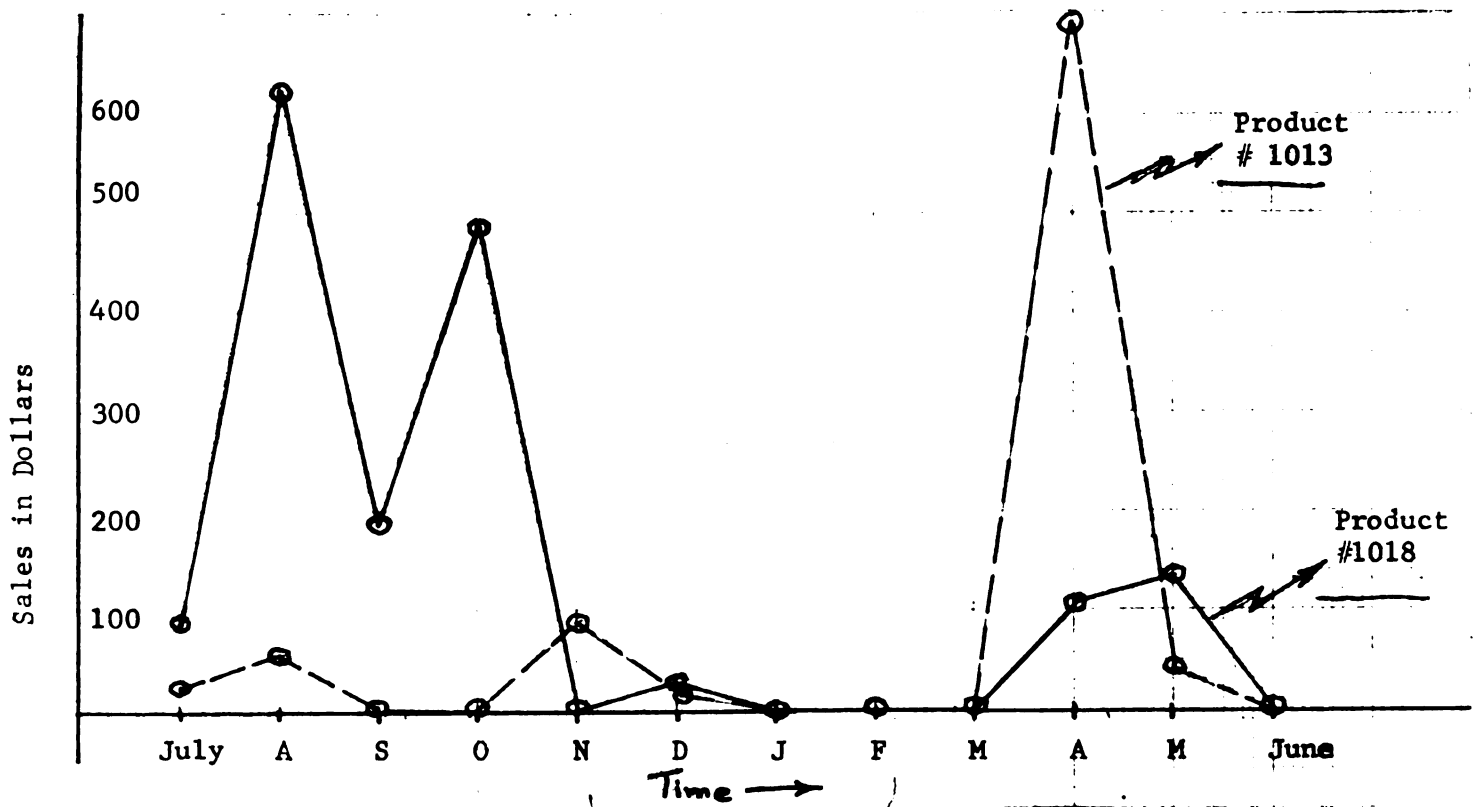
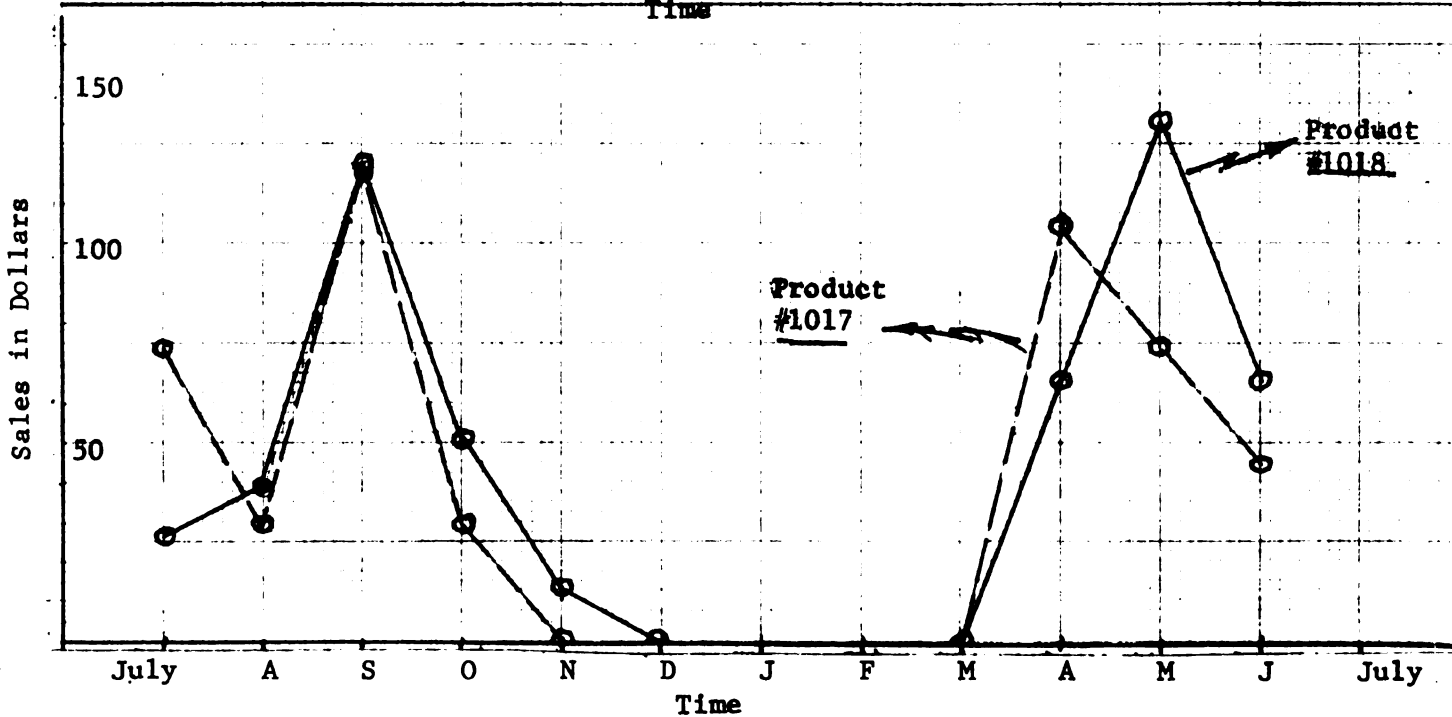
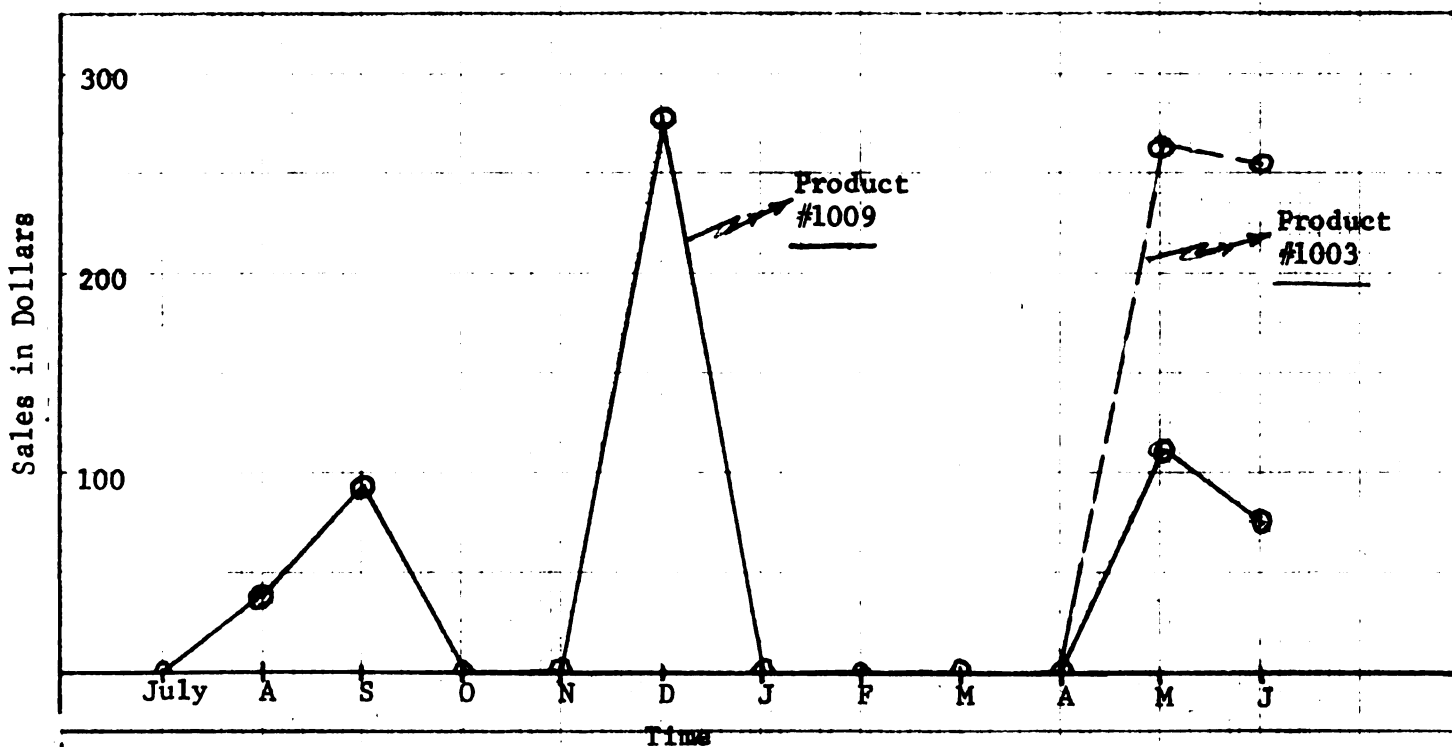
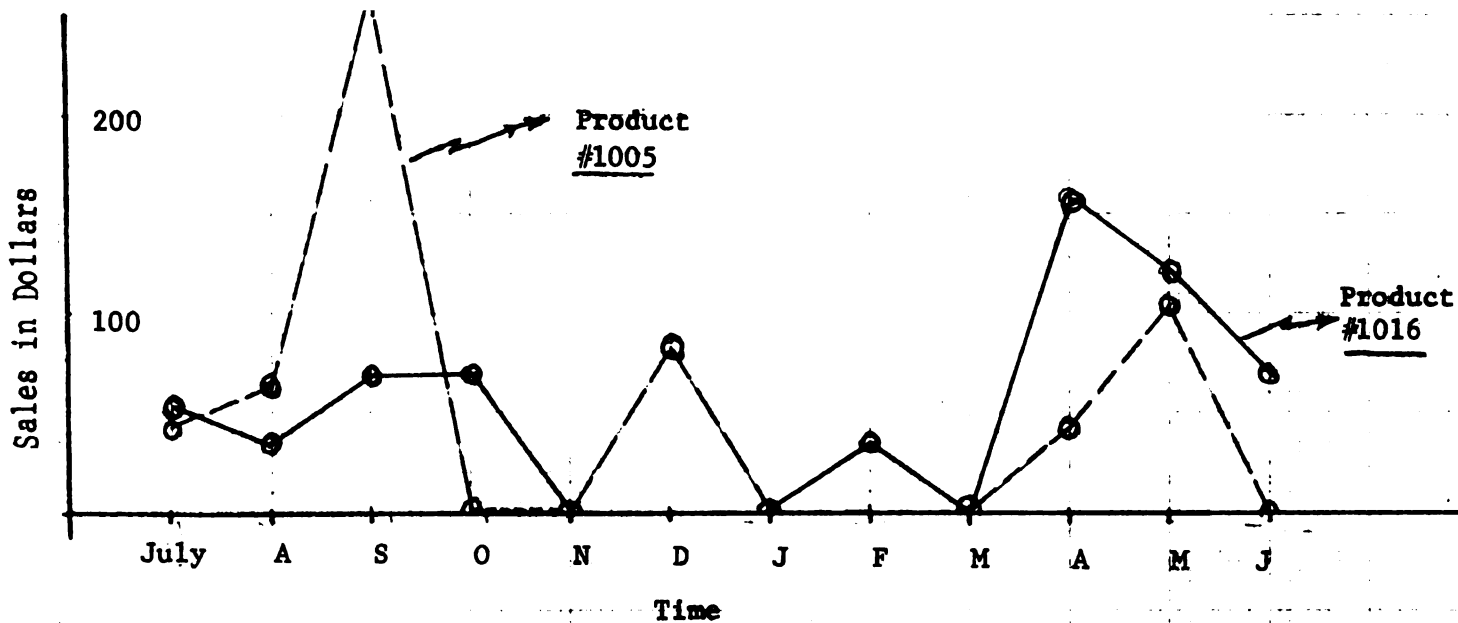


Figure 6.3 : A plot of the sales data for ten items.



(Figure 6.3 continued)

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BIBLIOGRAPHY

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APPENDICES

APPENDIX A
THE GAMMA DISTRIBUTION

Most real processes in business can be approximated by either the normal distribution or the poisson distribution. But both these distributions suffer from certain limitations. The normal distribution becomes inappropriate when the ratio of the standard deviation to the mean of the process is larger than about 1/3.¹ This is because the normal variates generated will often be negative when this ratio of 1/3 is exceeded. The poisson process does not generate negative variables. But the mean and the variance in the poisson distribution are always equal. This restricts the usefulness of the poisson distribution.

The gamma distribution does not suffer from these limitations and hence is very useful. The probability density function of the gamma distribution is:

$$f(x) = \frac{\lambda^r x^{r-1} e^{-\lambda x}}{(r-1)!}$$

where $\underline{r}$ and $\underline{\lambda}$ are the gamma parameters. The gamma distribution results from an effort to determine the probability of $\underline{x}$ units of length between one success and the $\underline{r}$ th succeeding success. The mean and the standard deviation of a gamma distribution are:

¹ McMillan, C. and Gonzalez, R. F. Systems Analysis (Homewood, Illinois: Richard D. Irwin, Inc., 1968), p. 261.

$$\text{Mean} = r/\lambda$$

$$\text{Standard Deviation} = \sqrt{r}/\lambda$$

From this it follows that:

$$r = \frac{(\text{mean})^2}{(\text{standard deviation})^2}$$

$$\lambda = \frac{(\text{mean})}{(\text{standard deviation})^2}$$

Hence a gamma distribution, like a normal distribution, can be uniquely specified by its mean and its standard deviation.

When $r=1$, the gamma distribution is identical to the exponential distribution. If $\underline{r}$ is a positive integer, the gamma distribution is identical to the Erlang distribution. As $\underline{r}$ increases, the gamma distribution approaches a normal distribution asymptotically.

APPENDIX B

ESTIMATION OF INVENTORY CARRYING COSTS AND ORDERING COSTS

In order to carry out the simulation with the hypothetical data, it was necessary to assume certain values for the inventory carrying costs and the ordering costs. The inventory carrying cost per unit per year was assumed to be approximately 20 percent of the value of the item, while the ordering cost was assumed to be \$15 per order. These values were based on the estimates provided by Carson.¹

According to Carson, the inventory carrying costs usually runs from 10 percent to 25 percent of the value of the inventory per year. One commonly used estimate of the percentage cost per year of carrying inventory is

Interest on Investment	3.0%
Shrinkage (waste, scrap, losses, theft, etc.)	5.0%
Storage (rent, heat, light, etc.)	2.0%
Taxes	1.5%
Insurance	0.5%
Depreciation on Capital Assets	2.0%
Material Handling and Record Keeping	<u>4.0%</u>
TOTAL	18.0%

¹ Carson, G. B. Production Handbook (New York: The Ronald Press, 1960) sections 4-55 to 4-58.

Since the inventory carrying costs are estimated to run from 10 percent to 25 percent, a value of 20 percent was assumed in this research.

Carson estimated the ordering costs to range from \$8 to \$20. Hence, an ordering cost of \$15 per order was assumed in this research.

APPENDIX C

FORTTRAN SUBROUTINE FOR THE GENERATION OF NORMAL VARIATES

The following subroutine¹ was used for the generation of normal variates with some mean (EX) and standard deviation (STDY)

```
SUBROUTINE NORMAL (EX,STDY,X)
SUM = 0.0
DO 5 I = 1,12
R = RANF (-1)
SUM = SUM + R
X = STDY * (SUM - 6.0) + EX
RETURN
END
```

¹ Naylor, T. H., Balintfy, J. L., Burdick, D. S. and Chu, K. Computer Simulation Techniques (New York: John Wiley and Sons, Inc., 1968), p. 95.

APPENDIX D

GENERATION OF RANDOM VARIATES FROM SOME STATISTICAL POPULATION WHOSE CUMULATIVE PROBABILITY DISTRIBUTION IS GIVEN

Naylor¹ has developed a procedure called "The Inverse Transformation Method" for the generation of random variates from some particular statistical population whose cumulative distribution function is known. The procedure is as follows:

Let $F(x)$ be the cumulative distribution function of a random variate $\underline{x}$. Then, since $F(x)$ is defined over the range zero to one, we can generate uniformly distributed random numbers $\underline{r}$ between zero and one and set

$$F(x) = r$$

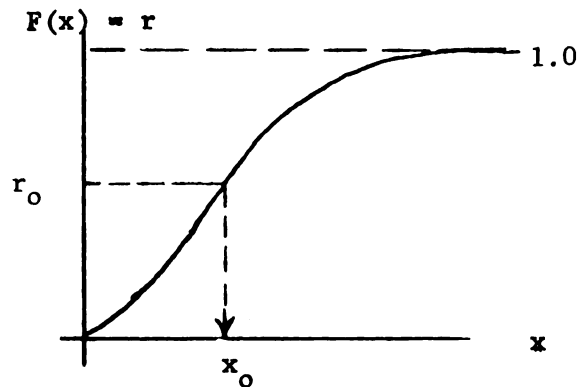
Hence $\underline{x}$ is uniquely determined by $r = F(x)$. Also, for any particular value of $\underline{r}$ say $\underline{r}_0$ generated, it is possible to find the corresponding value of $\underline{x}$, say $\underline{x}_0$ by the inverse function of $\underline{F}$. That is,

$$\underline{x}_0 = F^{-1}(\underline{r}_0)$$

where $F^{-1}(r)$ is the inverse transformation of $\underline{r}$.

¹ Naylor, T. H., Balintfy, J. L., Burdick, D. S., and Chu, K. Computer Simulation Techniques (New York: John Wiley and Sons, Inc., 1968), pp. 70-71.

Graphically the procedure is as follows:



Hence the procedure involves the generation of a uniformly distributed random variable $\underline{r_0}$ between zero and one, and then reading off the corresponding value of x , i.e., $\underline{x_0}$, from the cumulative distribution function $F(x)$.

The values of the cumulative probabilities (ARG) for various values of $\underline{x}$ (VAL) were first read in. Then the following FORTRAN FUNCTION was used to determine the appropriate value of $\underline{x}$, given $\underline{r}$.

```

FUNCTION TABLI(VAL,ARG,DUMMY,K)
  DIMENSION VAL(1),ARG(1)
  DUM=AMXL(AMIN1(DUMMY,ARG(K)),ARG(1))
  DO 1 I=2,K
    IF (DUM-ARG(I))2,2,1
  2  TABLI=(DUM-ARG(I-1))*(VAL(I)-VAL(I-1))/(ARG(I)-ARG(I-1))+VAL(I-1)
  TABLI=TABLI*3/13
  RETURN
1  CONTINUE
  RETURN
END

```

APPENDIX B

CONVERSION OF DEMAND PER MONTH TO DEMAND PER WEEK

Let D_i = demand during month i

Then,

$$\text{the mean demand per month} = \frac{\sum_{i=1}^m D_i}{m}$$

$$\text{and the variance of the demand per month is} = \frac{\sum_{i=1}^m D_i^2}{m} - \left(\frac{\sum_{i=1}^m D_i}{m} \right)^2$$

Assume that there are 4 1/3 weeks in a month. It will be shown that the procedure adopted in this thesis, namely generating a monthly demand (given the mean and standard deviation) and then computing the weekly demand as 4 1/3 the monthly demand, is valid.

Let d_i = demand during week i

Then,

$$\text{the mean of the weekly demand} = \frac{\sum d_i}{m}$$

$$\text{and the variance of the weekly demand} = \frac{\sum (d_i)^2}{m} - \left(\frac{\sum d_i}{m} \right)^2$$

$$\text{But } d_i = (3/13) D_i$$

$$\text{Therefore mean of the weekly demand} = \frac{\sum 3/13 D_i}{m}$$

$$= 3/13 \frac{\sum D_i}{m}$$

$$= 3/13 (\text{mean of monthly demand})$$

Similarly,

The variance of the weekly demand

$$\begin{aligned} &= \frac{\sum (3/13 D_i)^2}{m} - \left[\frac{\sum 3/13 D_i}{m} \right]^2 \\ &= (3/13)^2 \frac{\sum (D_i)^2}{m} - (3/13)^2 \left[\frac{\sum D_i}{m} \right]^2 \\ &= (3/13)^2 \left[\frac{\sum D_i^2}{m} - \left(\frac{\sum D_i}{m} \right)^2 \right] \\ &= (3/13)^2 (\text{variance of monthly demand}) \end{aligned}$$

Therefore,

Mean of the weekly demand = 3/13(mean of monthly demand)

Variance of the weekly demand = 3/13 (variance of monthly demand)

APPENDIX F

LISTING OF THE FORTRAN PROGRAMS

```

PROGRAM CONVERG (INPUT,OUTPUT)
  DIMENSION INV (12,75),IEOQ(12,75),IDEM(12),MARGIN(12,IEX(12,ISTD(
112,IHOO(12,IGIT(12,4),IDUM(12,100)
  COMMON IDEM,IEX,ISTD
  DO 21 I=1,12
21 READ 22,IEX(I),ISTD(I)
22 FORMAT (2I10)
  DO 35 J=1,100
  DO 36 J1=1,12
36 IDUM(J1,J)=0
35 CONTINUE
  DO1 I=1,12
  MARGIN(I)=1
  IGIT(I,3)=0
  IGIT(I,2)=0
  IGIT(I,1)=0
  INV(I,1)=100
  IEOQ(I)=0
1 CONTINUE
  ITC3=0
  ITC2=0
  TC1=0.
  TC=0.
  IORDER=0
  ICAPS=50
  IDASHS=50
  ISMALLS=30
  DO600 J6=1,500
  DO 100 I=1,52
  IF (I-1)9,8,9
9 DO 10 J=1,12
10 INV(J,I)=INV(J,I-1)+IGIT(J,3)
8 CALL DEMAND
  DO 11 J=1,12
  INV(J,I)=INV(J,I)-IDEM(J)
  IF (INV(J,I)-0)12,11,11
12 ITC3=ITC3-(INV(J,I)*MARGIN(J))
  INV(J,I)=0
11 TC1=TC1+INV(J,I)*0.04
  DO 14 J=1,12
  IHOO(J)=INV(J,I)+IGIT(J,3)+IGIT(J,2)+IGIT(J,1)

```

```
14 CONTINUE
   DO 3 J=1,12
     IF (IHOO(J)-ISMALLS(15,15,3
3 CONTINUE
   GO TO 101
15 IORDER=IORDER+1
   ITC2=ITC2+15
   DO 16 J1=1,12
     IF (ITHOO(J1)-IDASHS)17,17,16
17 IEOQ(J1,I)=ICAPS-INV(J1,I)
   IDUM (J1,I)=IEOQ(J1,I)
16 CONTINUE
101 DO 30 I2=1,12
   IGIT(12,3)=IGIT(12,2)
   IGIT(12,2)=IGIT(12,1)
   IGIT(12,1)=LDUM(12,I0
30 IDUM(12,I)=0
100 CONTINUE
   TC=ITC3+ITC2+TC1
   PRINT 200,ICAPS,ISMALLS,IDASHS,ITC3,ITC2,TC1,TC
200 FORMAT (5(10X,I7),2(10X,F8.2))
   ITC3=ITC2=0
   TC1=TC=0.
600 CONTINUE
   PRINT 800,(IEX(I),I=1,12)
   PRINT 800,(ISTD(I),I=1,12)
800 FORMAT (///,12(5X,16))
   END
   SUBROUTINE DEMAND
   DIMENSION ID(12),IS(12),IE(12)
   COMMON ID,IE,IS
   DO 10 I1=1,12
     SUM=0.
     DO 20 I=1,12
       R=RANF(-1)
20 SUM=SUM+R
10 ID(I1)=ABS(IS(I1)*(SUM-6.0)+IE(I1))
   RETURN
   END
```

```
PROGRAM REPEAT (INPUT,OUTPUT)
   DIMENSION INV (12,75),IEOQ(12,75),IDEM(12),MARGIN(12),IEX(12),ISTD(
112),IHOO(12),IGIT(12,4),IDUM(12,100)
   COMMON IDEM,IEX,ISTD
   DO 21 I=1,12
21 READ 22,IEX(I),ISTD(I)
22 FORMAT (2I10)
   ICAPS=110
   ISMALLS=30
   IDASHS=110
```

```
DO 53 K1=1,9
DO 35 J=1,100
DO 36 J1=1,12
36 IDUM(J1,J)=0
35 CONTINUE
DO 1 I=1,12
MARGIN(I)=1
IGIT(I,3)=0
IGIT(I,2)=0
INV(I,1)=100
IGIT(I,1)=0
IEOQ(I)=0
1 CONTINUE
ITC3=0
ITC2=0
TC1=0.
TC=0.
IORDER=0
SUM=0.
DO 222 K6=1,50
DO 100 I=1,52
IF (I-1)9,8,9
9 DO 10 J=1,12
10 INV(J,I)=INV(J,*-1)+IGIT(J,3)
8 CALL DEMAND
DO 11 J=1,12
INV(J,I)=INV(J,I)-IDEM(J)
IF (INV(J,I)-0)12,11,11
12 ITC3=ITC3-(INV(J,I)*MARGIN(J))
INV(J,I)=0
11 TC1=TC1+INV(J,I)*0.04
DO 14 J=1,12
IHOO(J)=INV(J,I)+IGIT(J,3)+IGIT(J,3)+IGIT(J,1)
14 CONTINUE
DO 3 J=1,12
IF (IHOO(J)-ISMALLS)15,15,3
3 CONTINUE
GO TO 101
15 IORDER=IORDER+1
ITC2=ITC2+15
DO 16 J1=1,12
IF (IHOO(J1)-IDASHS(17,17,16
17 IEOQ(J1,I)=ICAPS-INV(J1,I)
IDUM J1,I)=IEOQ(J1,I)
16 CONTINUE
101 DO 30 I2=1,12
IGIT(12,3)=IGIT(12,2)
IGIT(12,2)=IGIT(12,1)
IGIT(12,1)=IDUM(12,I)
30 IDUM(12,I)=0
100 CONTINUE
```

1

```
TC=ITC3+ITC2+TC1
PRINT 200,ICAPS,ISMALLS,IDASHS,ITC3,ITC2,TC1,TC
200 FORMAT (5(10X,I7),2(10X,F8.2))
IF (K6-31)19,18,18
18 SUM=SUM+TC
19 ITC3+ITC2=0
TC1=TC=0.
222 CONTINUE
XMEAN=SUM/20.
PRINT 202,XMEAN
202 FORMAT (*///*,* THE MEAN OF LAST TWENTY TC IS *,F10.3)
53 IDASHS=IDASHS-10
END
SUBROUTINE DEMAND
DIMENSION ID(12),IS(12),IE(12)
COMMON ID,IE,IS
DO 10 I1=1,12
SUM=0.
DO 20 I=1,12
R=RANF(-1)
20 SUM=SUM+R
10 ID(I1)+ABS(IS(I1)*(SUM-6.0)+IE(I1))
RETURN
END
```

```
PROGRAM EOQ (INPUT,OUTPUT)
DIMENSION INV(12,75),IEOQ(12),IDEM(12),MARGIN(12),IEX(12),ISTD(12)
1,IHOO(12),IGIT(12,4),IDUM(12,100),IR(12)
COMMON IDEM,IEX,ISTD
C THIS IS THE MODIFIED EOQ PROGRAM
IFLAG=0
DO 21 I=1,12
21 READ 22,IEX(I),ISTD(I)
22 FORMAT (2110)
DO 35 J=1,100
DO 36 J1=1,12
36 IDUM(J1,J)=0
35 CONTINUE
DO 1 I=1,12
MARGIN(I)=1
IGIT(I,3)=0
IGIT(I,2)=0
IGIT(I,1)=0
IEOQ(I)=((15*2*IEX(I))/0.04)**0.5
IR(I)=3*IEX(I)+2*(3**0.5)*ISTD(I)
INV(I,1)=100
1 CONTINUE
ITC3=0
ITC2=0
TC1=TC=0
```

```

IORDER=0
DO 600 J6=1,500
DO 100 I=1,52
IF (I-1)9,8,9
9 DO 10 J=1,12
10 INV(J,I)=INV(J,I-1)+IGIT(J,3)
8 CALL DEMAND
DO 11 J=1,12
INV(J,I)=INV(J,I)-IDEM(J)
IF (INV(J,I)-0)12,11,11
12 ITC3=ITC3-(INV(J,I)*MARGIN(J))
INV(J,I)=0
11 TC1=TC1+INV(J,I)*0.04
DO 14 J=1,12
IHOO(J)=INV(J,I)+IGIT(J,3)+IGIT(J,2)+IGIT(J,1)
14 CONTINUE
DO 3 J=1,12
IF (IHOO(J)-IR(J))15,15,3
15 IDUM(J,I)=IEOQ(J)
IORDER=IORDER+1
IFLAG=1
3 CONTINUE
IF (IFLAG-0)23,101,23
23 ITC2=ITC2+15
IFLAG=0
101 DO 30 I2=1,12
IGIT(12,3)=IGIT(12,2)
IGIT(12,2)=IGIT(12,1)
IGIT(12,1)=IDUM(12,I)
30 IDUM(12,I)=0
100 CONTINUE
TC=ITC3+ITC2+TC1
PRINT 200,ITC3,ITC2,TC1,TC
200 FORMAT (2(10X,I7),2(10X,F8.2))
ITC3=ITC2=0
TC1=TC=0.
600 CONTINUE
PRINT 71,(IEOQ(I),I=1,12)
PRINT 71,(IR(I),I=1,12)
71 FORMAT (12(15,5X((
PRINT 800,(IEX(I),I=1,12)
PRINT 800,(ISTD(I),I=1,12)
800 FORMAT (///,12(5X,I6))
END
SUBROUTINE DEMAND
DIMENSION ID(12),IS(12),IE(12)
COMMON ID,IE,IS
DO 10 I1=1,12
SUM=0.
DO 20 I=1,12
R=RANF(-1)

```

```
20 SUM=SUM+R
10 ID(I1)=ABS(IS(I1)*(SUM-6.0)*IE(I1))
   RETURN
   END
```

```
PROGRAM EOQ (INPUT,OUTPUT)
  DIMENSION INV(12,75),IEOQ(12),IDEM(12),MARGIN(12,IEX(12),ISTD(12)
1, IHOO(12),ISIT(12,4),IDUM(12,100),IR(12)
  COMMON IDEM,IEX,ISTD
  DO 21 I=1,12
21 READ 22,IEX(I),ISTD(I)
22 FORMAT (2I10)
  DO 35 J=1,100
  DO 36 J1=1,12
36 IDUM(J1,J)=0
35 CONTINUE
  DO 1 I=1,12
  MARGIN(I)=1
  IGIT(I,3)=0
  IGIT(I,2)=0
  IGIT(I,1)=0
  IEOQ(I)=((15*2*IEX(I))/0.04)**0.5
  IR(I)=3**IEX(I)+2*(3**0.5)*ISTD(I)
  INV(I,1)=100
1 CONTINUE
  ITC3=0
  ITC2=0
  TC1=TC=0.
  IORDER=0
  DO 600 J6=1,500
  DO 100 I=1,52
  IF (I-1)9,8,9
9 DO 10 J=1,12
10 INV(J,I)=INV(J,I-1)=ITIT(J,3)
8 CALL DEMAND
  DO 11 J=1,12
  INV(J,I)=INV(J,I)-IDEM(J)
  IF (INV(J,I)-0)12,11,11
12 ITC3=ITC3-(INV(J,*)*MARGIN(J))
  INV(J,I)=0
11 TC1=TC1+INV(J,I)*0.04
  DO 14 J=1,12
  IHOO(J)=INV(J,I)+IGIT(J,3)+IGIT(J,2)+IGIT(J,1)
14 CONTINUE
  DO 3 J=1,12
  IF (IHOO(J)-IR(J))15,15,3
15 IDUM(J,I)=IEOQ(J)
  IORDER=IORDER+1
  ITC2=ITC2+15
3 CONTINUE
101 DO 30 I2=1,12
```

```
      IGIT(12,3)=IGIT(12,2)
      IGIT(12,2)=IGIT(12,1)
      IGIT(12,1)=IDUM(12,1)
30  IDUM(12,1)=0
100 CONTINUE
      TC=ITC3+ITC2+TC1
      PRINT 200,ITC3,ITC2,TC1,TC
200 FORMAT (2(10X,I7),2(10X,F8.2))
      ITC3=ITC2=0
      TC1=TC=0.
600 CONTINUE
      PRINT 71,(IEOQ(I),I=1,12)
      PRINT 71,(IR(I),I=1,12)
71  FORMAT (12,(15,5X))
      PRINT 800,(IEX(I),I=1,12)
      PRINT 800,(ISTD(I),I=1,12)
800 FORMAT (///,12(5X,I6))
      END
      SUBROUTINE DEMAND
      DIMENSION ID(12),IS(12,IE(12))
      COMMON ID,IE,IS
      DO 10 I1=1,12
      SUM=0.
      DO 20 I=1,12
      R=RANF(-1)
20  SUM=SUM+R
10  ID(I1)=ABS(IS(I1)*(SUM-6.0)+IE(I1))
      RETURN
      END
```

```
      PROGRAM CONVERG (INPUT,OUTPUT)
      DIMENSION INV(12,75),IEOQ(12,75),IDEM(12),XMARGIN(12),IHOO(12),IGI
1T(12,4),IDUM(12,100),CPU(12)
      DIMENSION VAL1(22),ARG1(22),VAL2(25),ARG2(25),VAL3(48),ARG3(48),VA
1L4(26),ARG4(26),VAL5(11),ARG5(11),VAL6(33),ARG6(33),VAL7(34),ARG7(
234),VAL8(39),ARG8(39),VAL9(17),ARG9(17),VAL10(21),ARG10(21),VAL11(
341),ARG11(41),VAL12(29),ARG12(29)
      IFLAG=0
      K2=25
      K3=48
      K5=11
      K6=33
      K7=34
      K9=17
      K10=21
      K11=41
20  FORMAT (F10.7)
      VALX=0.
      DO 811 I=1,25
      VAL2(I)=VALX
```

```
      VALX=VALX+10
811 READ 20,ARG2(I)
      VALX=0.
      DO 812 I=1,48
        VAL3(I)=VALX
        VALX=VALX+2.
812 READ 20,ARG3(I)
      VALX=0.
      DO 814 I=1,11
        VAL5(I)=VALX
        VALX=VALX+10.
814 READ 20,ARG5(I)
      VALX=0.
      DO 815 I=1,33
        VAL6(I)=VALX
        VALX=VALX+2.
815 READ 20,ARG6(I)
      VALX=0.
      DO 816 I=1,34
        VAL7(I)=VALX
        VALX=VALX+10.
816 READ 20,ARG7(I)
      VALX=0.
      DO 818 I=1,17
        VAL9(I)=VALX
        VALX=VALX+10.
818 READ 20,ARG9(I)
      VALX=0.
      DO 819 I=1,21
        VAL10(I)=VALX
        VALX=VALX+5.
819 READ 20,ARG10(I)
      VALX=0.
      DO 821 I=1,41
        VAL11(I)=VALX
        VALX=VALX+2.
821 READ 20,ARG11(I)
      DO 21 I=1,8
21 READ 22,CPU(I)
22 FORMAT (F10.4)
      ICAPS=65
      IDASHS=65
      ISMALLS=15
      DO 35 J=1,100
      DO 36 J1=1,12
36 IDUM(J1,J)=0
35 CONTINUE
      DO 1 I=1,12
      XMARGIN(I)=0.2
      IGIT(I,3)=0
      IGIT(I,2)=0
      INV(I,1)=100
```

```
IGIT(I,1)=0
IEOQ(I)=0
1 CONTINUE
TC1=TC=XTC3=0.
ITC2=0
IORDER=0
SUM=0.
DO 600 J6=1,500
DO 100 I=1,52
IF (I-1)9,8,9
9 DO 10 J=1,8
10 INV(J,I)=INV(J,I-1)+IGIT(J,3)
8 X=RANF(-1)
IDEM(1)=TABLI(VAL2,ARG2,X,K2)
IDEM(2)=TABLI(VAL3,ARG3,X,K3)
IDEM(3)=TABLI(VAL5,ARG5,X,K5)
IDEM(4)=TABLI(VAL6,ARG6,X,K6)
IDEM(5)=TABLI(VAL7,ARG7,X,K7)
IDEM(6)=TABLI(VAL9,ARG9,X,K9)
IDEM(7)=TABLI(VAL10,ARG10,X,K10)
IDEM(8)=TABLI(VAL11,ARG11,X,K11)
DO 11 J=1,8
INV(J,I)=INV(J,I)-IDEM(J)
IF (INV(I)-0)12,11,11
12 XTC3=XTC3-(INV(J,I)*XMARGIN(J)*CPU(J))
INV(J,I)=0
11 TC1=TC1+(INV(J,I)*0.3*CPU(J)/52.)
DO 14 J=1,8
THOO(J)=INV(J,I)+IGIT(J,3)+IGIT(J,2)+IGIT(J,1)
14 CONTINUE
DO 3 J=1,8
IF (IHOO(J)-ISMALLS)15,15,3
3 CONTINUE
GO TO 101
15 IORDER=IORDER+1
ITC2=ITC2+15
DO 16 J1=1,8
IF (IHOO(J1)-IDASHS)17,17,16
17 IEOQ(J1,I)=ICAPS-INV(J1,I)
IDUM(J1,I)=IEOQ(J1,I)
16 CONTINUE
101 DO 30 I2=1,8
IGIT(I2,3)=IGIT(I2,2)
IGIT(I2,2)=IGIT(I2,1)
IGIT(I2,1)=IDUM(I2,I)
30 IDUM(I2,I)=0
100 CONTINUE
TC=XTC3+ITC2+TC1
PRINT 200,ICAPS,ISMALLS,IDASHS,XTC3,TTC2,TC1,TC
200 FORMAT (3(10X,I7),10X,F8.2,10X,I7,2(1X,F14.2))
IF(J6-481)19,18,18
18 SUM=SUM+TC
19 TC1=TC=XTC3=0.
```

```

        ITC2=0
600 CONTINUE
        XMEAN=SUM/20.
        PRINT 202,XMEAN
200 FORMAT (*//*,* THE MEAN OF LAST TWENTY TC IS *,F10.3)
        END
        FUNCTION TABLI(VAL,ARG,DUMMY,K)
        DIMENSION VAL(1),ARG(1)
        DUM=AVAX1(AMIN1(DUMMY,ARG(K)),ARG(1))
        DO 1 I=2,K
            IF (DUM-ARG(I))2,2,1
2        TABLI=(DUM-ARG(I-1))*(VAL(I)-VAL(I-1))/(ARG(I)-ARG(I-1))+VAL(I-1)
        TABLI=TABLI*3/13
        RETURN
1 CONTINUE
        RETURN
        END

```

```

        PROGRAM REPEAT (INPUT,OUTPUT)
        DIMENSION INV(12,75),IEOQ(12,74),IDEM(12),XMARGIN(12),IHOO(12),IGI
IT(12,4),IDUM(12,100),CPU(12)
        DIMENSION VAL1(22),ARG1(22),VAL2(25),ARG2(25),VAL3(48),ARG3(48),VA
1L4(26),ARG4(26),VAL5(11),ARG5(11),VAL6(33),ARG6(33),VAL7(34),ARG7(
234,VAL8(39),ARG8(39),VAL9(17),ARG9(17,VA
341),ARG11(41),VAL12(29),ARG12(29)
        IFLAG=0
        K2=25
        K3=48
        K5=11
        K6=33
        K7=34
        K9=17
        K10=21
        K11=41
20 FORMAT (F10.7)
        VALX=0.
        DO 811 I=1,25
            VAL2(I)=VALX
            VALX=VALX+10.
811 READ 20,ARG2(I)
            VALX=0.
            DO 812 I=1,48
                VAL3(I)=VALX
                VALX=VALX+2.
812 READ 20,ARG3(I)
            VALX=0.
            DO 814 I=1,11
                VAL5(I)=VALX
                VALX=VALX+10.
814 READ 20,ARG5(I)

```

```
VALX=0.
DO 815 I=1,33
VAL6(I)=VALX
VALX=VALX+2.
815 READ 20,ARG6(I)
VALX=0.
DO 816 I=1,34
VAL7(I)=VALX
VALX=VALX+10.
816 READ 20,ARG7(I)
VALX=0.
DO 818 I=1,17
VAL9(I)=VALX
VALX=VALX+10.
818 READ 20,ARG9(I)
VALX=0.
DO 819 I=1,21
VAL10(I)=VALX
VALX=VALX+5.
819 READ 20,ARG10(I)
VALX=0.
DO 821 I=1,41
VAL11(I)=VALX
VALX=VALX+2.
821 READ 20,ARG11(I)
DO 21 I=1,8
21 READ 22,CPU(I)
22 FORMAT (F10.4)
IDASHS=105
ISMAILS=15
ICAPS=105
DO 53 KK1=1,10
DO 35 J=1,100
DO 36 J1=1,12
36 IDUM(J1,J)=0
35 CONTINUE
DO 1 I=1,12
XMARGIN(I)=0.2
IGIT(I,3)=0
IGIT(I,2)=0
INV(I,1)=100
IGIT(I,1)=0
IEOQ(I)=0
1 CONTINUE
TC1=TC=XTC3=0.
ITC2=0
IORDER=0
SUM=0.
DO 222 KK6=1,50
DO 100 I=1,52
IF (I-1)9,8,9
9 DO 10 J=1,8
10 INV(J,I)=INV(J,I-1)+IGIT(J,3)
```

```

8 X=RANF(-1)
  IDEM(1)=TABLI (VAL2,ARG2,X,K2)
  IDEM(2)=TABLI (VAL3,ARG3,X,K3)
  IDEM(3)=TABLI (VAL5,ARG5,X,K5)
  IDEM(4)=TABLI (VAL6,ARG6,X,K6)
  IDEM(5)=TABLI (VAL7,ARG7,X,K7)
  IDEM(6)=TABLI (VAL9,ARG9,X,K9)
  IDEM(7)=TABLI (VAL10,ARG10,X,K10)
  IDEM(8)=TABLI (VAL11,ARG11,X,K11)
  DO 11 J=1,8
    INV(J,I)=INV(J,I)-IDEM(J)
    IF (INV(J,I)-0)12,11,11
12  XTC3=XTC3-(INV(J,I)*XMARGIN(J)*CPU(J))
    INV(J,I)=0
11  TC1=TC1+(INV(J,I)*0.3*CPU(J)/52.)
    DO 14 J=1,8
      IHOO(J)=INV(J,I)+IGIT(J,3)+IGIT(J,2)+IGIT(J,1)
14  CONTINUE
    DO 3 J=1,8
      IF (IHOO(J)-ISMALLS)15,15,3
3    CONTINUE
    GO TO 101
15  IORDER=IORDER+1
    ITC2=ITC2+15
    DO 16 J1=1,8
      IF (IHOO(J1)-IDASHS)17,17,16
17  IEQ(J1,I)=ICAPS-INV(J1,I)
    IDUM(J1,I)=IEQ(J1,I)
16  CONTINUE
101 DO 30 I2=1,8
    IGIT(12,3)-IGIT(I2,2)
    IGIT(I2,2)=IGIT(I2,1)
    IGIT(I2,1)=IDUM(I2,I)
30  IDUM(I2,I)=0
100 CONTINUE
    TC=XTC3+ITC2+TC1
    PRINT 200,ICAPS,ISMALLS,IDASHS,XTC3,ITC2,TC1,TC
200 FORMAT (3,(10X,I7),10X,F8.2,10X,I7,29IX,F14.2))
    IF (KK6-31)19,18,18
18  SUM=SUM+TC
19  TC1=TC=XTC3=0.
    ITC2=0
222 CONTINUE
    XMEAN=SUM/20.
    PRINT 202,XMEAN
202 FORMAT (*///*,* THE MEAN OF LAST TWENTY TC IS *,F10.3)
53  IDASHS=IDASHS-10
    END
    FUNCTION TABLI (VAL,ARG,DUMMY,K)
    DIMENSION VAL(1),ARG(1)
    DUM=AMAX1 (AMIN1 (DUMMY,ARG(K)),ARG(1))
    DO 1 I=2,K
      IF (DUM-ARG(I))2,2,1

```

```
2 TABLI=(DUM-ARG(I-1))*(VAL(I)-VAL(I-1))/(ARG(I)-ARG(I-1))+VAL(I-1)
  TABLI=TABLI*3/13
  RETURN
1 CONTINUE
  RETURN
  END
```

```
PROGRAM EQQ (INPUT,OUTPUT)
  DIMENSION INV(12,75),IEOQ(12,IDEQ(12),XMARGIN(12),IHOO(12),IGIT(1
12,4),IDUM(12,100),CPU(12),IR(12)
  DIMENSION VAL1(22),ARG1(22),VAL2(25,ARG2(25),VAL3(48),ARG3(48,VA
114(26),ARG4(26),VAL5(11),ARG5(11),VAL6(33),ARG6(33),VAL7(34),ARG7(
234(,VAL8(39),ARG8(39),VAL9(17),ARG9(17),VAL10(21),ARG10(21),VAL11(
341),ARG11(41),VAL12(29),ARG12(29)
C  THIS IS THE MODIFIED EQQ PROGRAM
  IFLAG=0
  K2=25
  K3=48
  K5=11
  K6=33
  K7=34
  K9=17
  K10=21
  K11=41
20  FORMAT (F10.7)
  VALX=0.
  DO 811 I=1,25
    VAL2(I)=VALX
    VALX=VALX+10.
811 READ 20,ARG2(I)
  VALX=0.
  DO 812 I=1,48
    VAL3(I)=VALX
    VALX=VALX+2.
812 READ 20,ARG3(I)
  VALX=0.
  DO 814 I=1,11
    VAL5(I)=VALX
    VALX=VALX+10.
814 READ 20,ARG5(I)
  VALX=0.
  DO 815 I=1,33
    VAL6(I)=VALX
    VALX=VALX+2.
815 READ 20,ARG6(I)
  VALX=0.
  DO 816 I=1,34
    VAL7(I)=VALX
    VALX=VALX+10.
816 READ 20,ARG7(I)
```

```
VALX=0.
DO 818 I=1,17.
VAL9(I)=VALX
VALX=VALX+10.
818 READ 20,ARG9(I)
VALX=0.
DO 819 I=1,21
VAL10(I)=VALX
VALX=VALX+5.
819 READ 20,ARG10(I)
VALX=0.
DO 821 I=1,41
VAL11(I)=VALX
VALX=VALX+2.
821 READ 20,ARG11(I)
DO 101 I=1,8
101 READ 102,CPU(I),IEOQ(I),IR(I)
102 FORMAT (F10.3,I10,I10)
DO 35 J=1,100
DO 36 J1=1,8
36 IDUM(J1,J)=0
35 CONTINUE
DO 1 I=1,8
XMARGIN(I)=0.2
IGIT(I,3)=0
IGIT(I,2)=0
IGIT(I,1)=0
INV(I,1)=100
1 CONTINUE
TC1=TC=XTC3=0.
ITC2=0
IORDER=0
DO 600 J6=1,500
DO 100 I=1,52
IF (I-1)9,8,9
9 DO 10 J=1,8
10 INV(J,I)=INV(J,I-1)+IGIT(J,3)
8 X=RANF(-1)
IDEM(1)=TABLI(VAL2,ARG2,X,K2)
IDEM(2)=TABLI(VAL3,ARG3,X,K3)
IDEM(3)=TABLI(VAL5,ARG5,X,K5)
IDEM(4)=TABLI(VAL6,ARG6,X,K6)
IDEM(5)=TABLI(VAL7,ARG7,X,K7)
IDEM(6)=TABLI(VAL9,ARG9,X,K9)
IDEM(7)=TABLI(VAL10,ARG10,X,K10)
IDEM(8)=TABLI(VAL11,ARG11,X,K11)
DO 11 J=1,8
INV(J,I)=INV(J,I)-IDEM(J)
IF(INVOJ,I)-0)12,11,11
12 XTC3=XTC3-(INV(J,I)*XMARGIN(J)*CPU(J))
INV(J,I)=0
11 TC1=TC1+(INV(J,I)*0.3*CPU(J)/52.)
```

```

      IHOO(J)=INV(J,I)+IGIT(J,3)+IGIT(J,2)+IGIT(J,1)
14  CONTINUE
      DO 3 J=1,8
      IF (IHOO(J)-ISMALLS)15,15,3
15  IDUM(J,I)=IEOQ(J)
      IORDER=IORDER+1
      IFLAG=1
      3  CONTINUE
      IF (IFLAG-0) 23,10)),23
23  ITC2=ITC2+15
      IFLAG=0
1011 DO 30 I2=1,8
      IGIT(I2,3)=IGIT(I2,2)
      IGIT(I2,2)=IGIT(I2,1)
      IGIT(I2,1)=IDUM(I2,I)
      30 IDUM(I2,I)=0
100  CONTINUE
      TC=XTC3+ITC2+TC1
      PRINT 200,XTC3,ITC2,TC1,TC
200  FORMAT(F18.2,10X,I7,2(iX,F14.2))
      ITC2=0
      TC1=TC=XTC3=0.
600  CONTINUE
      PRINT 71,(IEOQ(I),I=1,8)
      PRINT 71,(IR(I),I=1,8)
71  FORMAT (8(I5,5X))
      END
      FUNCTION TABLI(VAL,ARG,DUMMY,K)
      DIMENSION VAL(1),ARG(1)
      DUM=AMAX1(AMIN1(DUMMY,ARG(K)),ARG(1))
      DO 1 I=2,K
      IF (DUM-ARG(I))2,2,1
2  TABLI=(DUM-ARG(I-1))*(VAL(I)-VAL(I-1))/(ARG(I)-ARG(I-1))+VAL(I-1)
      TABLI=TABLI*3/13
      RETURN
1  CONTINUE
      RETURN
      END

      PROGRAM EOQ (INPUT,OUTPUT)
      DIMENSION INV(12,75),IEOQ(12),IDEM(12),XMARGIN(12),IHOO(12),IGIT(1
12,4),IDUM(12,100),CPU(12),IR(12)
      DIMENSION VAL1(22),ARG1(22),VAL2(25),ARG2(25),VAL3(48),ARG3(48,VA
114(26),ARG4(26),VAL5(11),ARG5(11),VAL6(33),ARG6(33),VAL7(34),ARG7(
234),VAL8(39),ARG8(39),VAL9(17),ARG9(17),VAL10(21),ARG10(21),VAL11(
341),ARG11(41),VAL12(29),ARG12(29)
      K2=25
      K3=48
      K5=11
      K6=33
      K7=34

```

```
K9=17
K10=21
K11=41
20 FORMAT (F10.7)
VALX=0.
DO 811 I=1,25
VAL2(I)=VALX
VALX=VALX+10.
811 READ 20,ARG2(I)
VALX=0.
DO 812 I=1,48
VAL3(I)=VALX
VALX=VALX+2.
812 READ 20,ARG3(I)
VALX=0.
DO 814 I=1,11
VAL5(I)=VALX
VALX=VALX+10.
814 READ 20,ARG5(I)
VALX=0.
DO 815 I=1,33
VAL6(I)=VALX
VALX=VALX+2.
815 READ 20,ARG6(I)
VALX=0.
DO 816 I=1,34
VAL7(I)=VALX
VALX=VALX+10.
816 READ 20,ARG7(I)
VALX=0.
DO 818 I=1,17
VAL9(I)=VALX
VALX=VALX+10.
818 READ 20,ARG9(I)
VALX=0.
DO 819 I=1,21
VAL10(I)=VALX
VALX=VALX+5.
819 READ 20,ARG10(I)
VALX=0.
DO 821 I=1,41
VAL11(I)=VALX
VALX=VALX+2.
821 READ 20,ARG11(I)
DO 101 I=1,8
101 READ 102,CPU(I),IEOQ(I),IR(I)
102 FORMAT (F10.2,I10,I10)
DO 35 J=1,100
DO 36 J1=1,8
36 IDUM(J1,J)=0
35 CONTINUE
DO 1 I=1,8
```

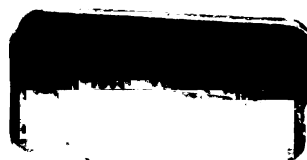


```

XMARGIN(I)=0.2
IGIT(I,3)=0
IGIT(I,2)=0
IGIT(I,1)=0
INV(I,1)=100
1 CONTINUE
TC1=TC=XTC3=0.
ITC2=0
IORDER=0
DO 600 J6=1,500
DO 100 I=1,52
IF (I-1)9,8,9
9 DO 10 J=1,8
10 INV(J,I)=INV(J,I-1)+IGIT(J,3)
8 X=RANF(-1)
IDEM(1)=TABLI(VAL2,ARG2,X,K2)
IDEM(2)=TABLI(VAL3,ARG3,X,K3)
IDEM(3)=TABLI(VAL5,ARG5,X,K5)
IDEM(4)=TABLI(VAL6,ARG6,X,K6)
IDEM(5)=TABLI(VAL7,ARG7,X,K7)
IDEM(6)=TABLI(VAL9,ARG9,X,K9)
IDEM(7)=TABLI(VAL10,ARG10,X,K10)
IDEM(8)=TABLI(VAL11,ARG11,X,K11)
DO 11 J=1,8
INV(J,I)=INV(J,I)-IDEM(J)
IF ~(INV(J,I)-0)12,11,11
12 XTC3=XTC3-(INV(J,*)*XMARGIN(J)*CPU(J))
INV(J,I)=0
11 TC1=TC1+(INV(J,I)*0.3*CPU(J)/52.)
DO 14 J=1,8
IH00(J)=INV(J,I)+IGIT(J,3)+IGIT(J,2)+IGIT(J,1)
14 CONTINUE
DO 3 J=1,8
IF (IH00(J)-IR(J))15,15,3
15 IDUM(J,I)=IEOQ(J)
IORDER=IORDER+1
ITC2=ITC2+I5
3 CONTINUE
1011 DO 30 I2= 1,8
IGIT(I2,3)=IGIT(I2,2)
IGIT(I2,2)=IGIT(I2,1)
IGIT(I2,1)=IDUM(I2,I)
30 IDUM(I2,I)=0
100 CONTINUE
TC=XTC3+ITC2+TC1
PRINT 200,XTC3,ITC2,TC1,TC
FUNCTION TABLI(VAL,ARG,DUMMY,K)
DIMENSION VAL(1),ARG(1)
DUM=AMAX1(AMIN1(DUMMY,ARG(K)),ARG(1))
DO 1 I=2,K
IF (DUM-ARG(I))2,2,1

```

```
2 TABLI=(DUM-ARG(I-1))*(VAL(I)-VAL(I-1))/(ARG(I)-ARG(I-1))+VAL(I-1)
  TABLI=TABLI*3/13
  RETURN
1 CONTINUE
  RETURN
  END
```



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