SOME IMPROVEMENTS OF THE IMAGE-PARAMETER METHOD FOR THE DESIGN OF L.C FILTERS

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AN ABSTRACT

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ABSTRACT

The image parameter method for filter design is not yet excelled for its simplicity. Darlington's technique, although quite straightforward, has the following restrictions: (1) A special type of insertion-loss requirement is considered in the block-band. (2) The determination of element values involves long calculations.

In recent years the image parameter method is being increasingly exploited to overcome the above difficulties. Tuttle has shown by means of image parameter techniques that in the special case of two cascaded Zobel sections, one can get Darlington's type of insertion-loss characteristics. Belevitch has pointed out that there is promise of the two apparently differing techniques coming closer to each other.

In this thesis, the image parameter method is studied in detail with the specific purpose of eliminating the drawbacks which are withholding its wider application in filter design. The results of this study have yielded a definite improvement over the existing method. The salient features of the method defined in this thesis are:

(i) The difficulties of Zobel's decomposition formula are eliminated by considering a new formulation.

(ii) A method due to Feldtkeller, greatly extended by Belevitch, applies the image parameter method to the design in the pass-band. This formulation is now extended to the block-band and a detail study of this is presented.

(iii) Exact requirements on the transfer-loss function in the block-band are given. Consequently, the number of intermediate sections in the filter could be minimized. This important conse-

quence of the new formulation is demonstrated by means of an example.

(iv) Properties of the general terminating sections are considered. General formulas are derived for the calculation of the element values of the terminating sections.

(v) The formulas for the insertion function valid for the cut-off frequency are given. These general formulas cover Belevitch's formulas as a special case.

(vi) The effect of dissipation on the insertion function is determined by means of the electronic digital computer. A general program is written for this purpose.

The new results presented in this thesis are a consequency of a detailed study of the image parameter method, viewed from more than one angle. It can be safely concluded that the study has certainly raised hopes of making use of image parameter method - with the suggested improvements - for a wider application in filter design.

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TABLE OF CONTENTS

I. INTRODUCTION

An image parameter filter is obtained by cascading sections (four-terminal network) with matched image impedances. The image-transfer function of this filter is the sum of the image-transfer functions of the individual sections. If the filter contains L's and C's only and operates between its image impedances, it will transmit without any loss over a certain band (or bands) of frequency (pass-band) and attenuate for all other frequencies (block-band).

In actual practice, we are interested in knowing either the operation loss* or insertion loss of the filter. In general, neither of these loss functions is the same as the image transfer loss of the filter. Zobel has given an expression for insertion loss of the filter in terms of image parameters. When the insertion loss is specified and the filter is to be found, the different factors in Zobel's expression must be considered. The investigation of Zobel's formula shows that it is possible to make some approximations on the insertion loss of the filter, and consequently the design procedures can be simplified.

Bode has made further investigations on the image parameter filter theory and has shown how the filter can be constructed by considering only one image impedance and the poles of image transfer loss. His matched cascaded sections generally differ from those of Zobel's, but for the practical case, he arrives at Zobel's composite filter but with more complicated terminating sections.

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* "Operation loss" is defined as the logarithm of the absolute value of the ratio of two voltages or currents. One of these measurements is at the output of the filter when it operates between the terminal resistance; the other is measured when the source is connected directly to a resistance equal in value to its internal resistance. The term "insertion loss" is defined in Chapter II.

Darlington and others have considered the cut-and-try method involved in the image parameter filter design as cumbersome. Darlington's method of filter design based on insertion loss is well-known. In this method, a special type of insertion loss function is considered and after finding a characteristic function, ϕ, the problem is reduced to finding the element values of the filter.

Darlington's special insertion loss function has been extended by Fromageot and others. But, in the determination of the ϕ - function, some approximations must be made, and after designing the filter, it must be checked as to how good this approximation is. In spite of this generalization, the inherent disadvantage of the method, viz., of determining the element values of the filter, still exists.

The above disadvantage of insertion loss filter design (Darlington) compels the designer to look for more practical methods. Classical image parameter theory does not have this disadvantage of Darlington's method, i.e., after the design parameters are chosen, the element values of the filter can be determined very easily. Tuttle has shown that two cascaded, matched Zobel sections can produce Darlington's type of insertion loss function by proper choice of the design parameters. Tuttle's method is based on image parameter theory. Consequently, the calculation of the element values, which is the difficult part of Darlington's method, is made extremely simple for this particular case.

Belevitch considers a new formulation for the insertion loss function. Based on this formulation, he discusses the insertion loss function in only the pass-band.

In this thesis:

(1) Belevitch's formulation is here generalized to include a discussion of the insertion function of the block-band. On the basis of new properties obtained from this generalized formulation, procedures for filter design are described.

(2) At the cut-off frequency, the general expression for the insertion function is given.

(3) Low pass filter terminating half sections are considered in general: general expressions are found for determining the element values of the terminating half sections.

(4) After designing the filter, the effect of dissipation on insertion loss and phase functions are investigated by using a digital computer.

II. SURVEY OF INSERTION-LOSS FILTER DESIGN TECHNIQUES

2.1 INSERTION FUNCTION

The insertion function, P_s, of a four terminal network is defined in terms of two currents, I_R and I_R' (or two voltages V_R and V_R') of fig. 2.1.1 and by the ratio

$$P_s = \ell n \frac{I_R'}{I_R} = \ell n \frac{V_R'}{V_R} \qquad (2.1.1)$$

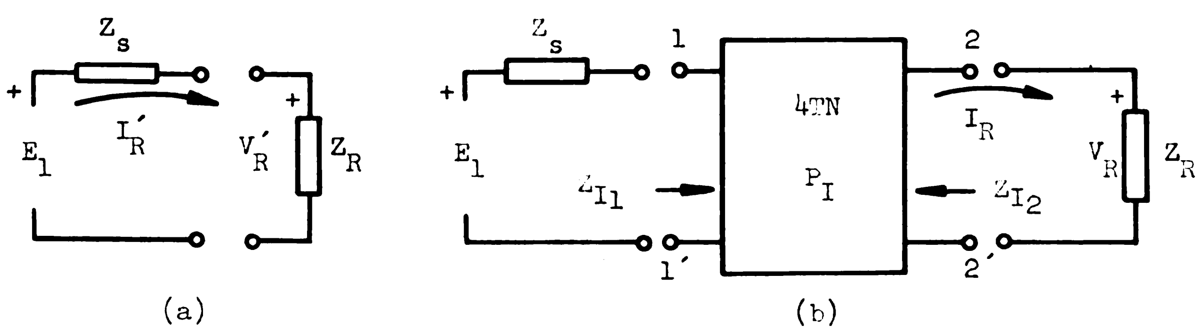


Fig. 2.1.1

Since in general the voltages or currents in eq. 2.1.1 are complex, then P_s is also complex and can be written as

$$P_s = A_s + j\,B_s \tag{2.1.2}$$

where the insertion loss function A_s is in nepers and the insertion phase function B_s is in radians. Therefore, from eq. 2.1.1 we also have,

$$e^{2A_s} = \left|\frac{I_R'}{I_R}\right|^2 = \left|\frac{V_R'}{V_R}\right|^2, \quad B_s = \arg\left(\frac{I_R'}{I_R}\right) = \arg\left(\frac{V_R'}{V_R}\right) \tag{2.1.3}$$

The expression for insertion function in terms of the image parameters was first given by Zobel [1]. In this thesis, we shall use the same notations as used in reference [2]. In terms of these notations,

$$e^{P_s} = \left(\frac{2\sqrt{Z_s Z_R}}{Z_s + Z_R}\right)\left(\frac{Z_s + Z_{I2}}{2\sqrt{Z_s Z_{I1}}}\right)\left(\frac{Z_R + Z_{I2}}{2\sqrt{Z_R Z_{I1}}}\right)\left(1 - K_s K_R e^{-2P_I}\right) e^{P_I} \tag{2.1.4}$$

The terms within the parenthesis on the right hand side of eq. 2.1.4 are designated, from left to right, as the: 1) Transformer, 2) Input reflection, 3) Output reflection, 4) Interaction, and 5) Image transfer. The parameters K_s and K_R appearing in eq. 2.1.4 are, respectively, the input and output reflection coefficients. The logarithms of the factors 1) through 5) correspond to certain functions whose properties are well known [2].

As may be seen from eq. 2.1.4, if Z_s and Z_R are equal respectively to the image impedances, Z_{I_1} and Z_{I_2}, of the four terminal network, then

$$e^{P_s} = \frac{2\sqrt{Z_{I_1} Z_{I_2}}}{Z_{I_1} + Z_{I_2}}\, e^{P_I} \tag{2.1.5}$$

On the other hand, if only, say, Z_s, is equal to the image impedance Z_{I_1}, then

$$e^{P_s} = \frac{Z_{I_1}}{Z_{I_2}} \, \frac{Z_{I_2} + Z_R}{Z_{I_1} + Z_R} \, e^{P_I} \qquad (2.1.6)$$

For a symmetrical four terminal network, where $Z_{I_1} = Z_{I_2} = Z_I$, eqs. 2.1.5 and 2.1.6 yield

$$e^{P_s} = e^{P_I}$$

or

$$P_s = P_I \qquad (2.1.7)$$

Therefore, for a symmetrical four terminal network, if there is a matching at least at one pair of terminals, the insertion function and the image transfer function are identical. Also, P_s is independent of the load impedance Z_R.

The result in eq. 2.1.7 is also obvious from the definition of P_s. For consider a symmetrical four terminal network with $Z_{I_1} = Z_{I_2} = Z_s$ as in fig. 2.1.1b. The replacement of the sub-network to the left of the terminals 2 - 2′ by its Thévenin equivalent will yield the network in fig. 2.1.1a. The only difference is that such a network will have a different voltage source.

In practice, the terminating impedances, Z_s and Z_R, of the filter are mostly pure resistances. On the other hand, the image impedances of a filter are real in the pass-band but not constant. Therefore, to provide a matching in the pass-band, the image impedance must be as constant as possible in this region. This can be done to a certain degree of approximation by increasing the order of image impedance (e.g., by use of Zobel's multiple-derived sections). If perfect matching is possible, then eq. 2.1.7 is valid and the design of the filter is reduced to the design of an uncorrected image parameter filter.

2.2 GENERAL DISCUSSIONS ON THE INSERTION LOSS OF SYMMETRICAL REACTIVE (LC) FILTERS

In this section we shall have a general look at the filter synthesis technique on an insertion loss basis. The purpose of this thesis will then be clarified.

The filter synthesis technique on an insertion loss basis has two main parts:

(A) To find an approximating function, ϕ, which is related to the insertion loss function as [3].

$$e^{2A_s} = \Lambda\ [1 + |\phi|^2] \tag{2.2.1}$$

such that ϕ must satisfy all the imposed conditions on the A_s-function.

(B) Find the element values of a lattice or ladder filter which is to be obtained from the above approximating function, ϕ.

These two parts are considered in the following.

(A) Determination of the ϕ-Function:

From the general theory of insertion loss filters, the insertion power ratio for a symmetrical filter with equal terminations is of the form [3]

$$e^{2A_s} = 1 + |\phi|^2 \tag{2.2.2}$$

where ϕ is an odd rational function of p (= jω). The poles of ϕ are the poles of the A_s-function and the zeros of ϕ are the zeros of the A_s-function.

For example, let

$$|\phi| = H \left| \frac{\omega(\omega_1^2 - \omega^2) \ldots (\omega_n^2 - \omega^2)}{(1 - \alpha_1^2\omega^2) \ldots (1 - \alpha_n^2\omega^2)} \right| \tag{2.2.3}$$

where H, ω_i's and α_i's are arbitrary positive constants. In order to obtain low-pass filter characteristics with a desired effective cut-off at

ω_o, it is necessary to take all ω_i's less than ω_o and all α_i's less than $1/\omega_o$. The insertion loss characteristics of this low-pass filter are of the general form of fig. 2.3.1 --- poles of A_s are all greater than ω_o and zeros all less than ω_o.

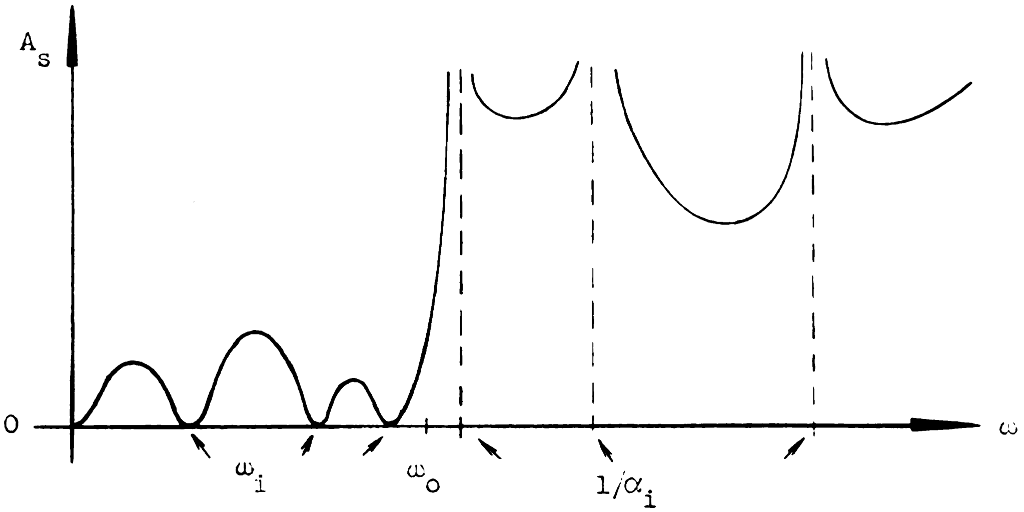


Fig. 2.2.1

If ϕ is chosen as an odd degree polynomial of frequency [15], then the well-known characteristics [4] (Butterworth and Tschebyscheff) in the effective pass-band can be obtained by imposing the proper requirements on the location of the zeros of the ϕ-function.

In addition, equal ripple characteristics in the pass-band can be obtained if ϕ is defined by

$$|\phi| = \sqrt{e^{2\alpha_p} - 1} \quad |\text{Sinh } P_I| \tag{2.2.4}$$

where P_I is the image transfer function of any image parameter antimetrical (sec. 3.3) filter (the so-called "reference filter"), and α_p is an arbitrary positive constant. The factor $(e^{2\alpha_p} - 1)$ is thereby always made positive. Since the reference filter is antimetrical, P_I is an even function of $p = j\omega$ and Sinh P_I is an odd function of p.

In the pass-band of the reference filter, $P_I = j\,B_I$ ($A_I = 0$), hence eq. 2.2.4 becomes

$$|\phi| = \sqrt{e^{2\alpha_p} - 1}\;\left|\mathrm{Sin}\,B_I\right| \qquad (2.2.5)$$

Since B_I is real, $|\phi|$ varies between zero and $\sqrt{e^{2\alpha_p} - 1}$ in the pass-band. Therefore, from eq. 2.2.2 we can see that A_s varies between zero and α_p.

In the block-band of the reference filter, $P_I = A_I + j\,k\,\pi$ ($k = 0, \pm 1, \pm 2, \ldots$) where $A_I \geq 0$, hence

$$\phi = \sqrt{e^{2\alpha_p} - 1}\;\;\mathrm{Sinh}\,A_I \qquad (2.2.6)$$

which is always positive and has poles at the poles of the A_I - function. Therefore, A_s is positive in the block-band and has poles at the same frequencies at which A_I has poles.

If the ϕ - function is chosen as

$$|\phi| = H\left|\frac{\Omega(\Omega^2 - \Omega_1^{\,2})\;\ldots\;(\Omega^2 - \Omega_n^{\,2})}{(1-\Omega_1^{\,2}\Omega^2)\;\;\ldots\;(1-\Omega_n^{\,2}\Omega^2)}\right| \qquad (2.2.7)$$

where

$$\Omega = \frac{\omega}{\sqrt{\omega_1\omega_2}}$$

ω_1 and ω_2 correspond, respectively, to the cut-off and limit of the effective block-band. If A_s has equal ripple characteristics in the effective pass-band ($0 - \omega_1$), then A_s will have equal valley heights in the effective block-band ($\omega_2 - \infty$) as shown in fig. 2.2.2.

These characteristics can be obtained if the parameters Ω_i's are chosen as follows:

$$\Omega_i = \sqrt{k}\quad \mathrm{sn}\left[\frac{2iK}{2n + 1}\,,\,k\right]\;\; (i = 1,2,\ldots,n) \qquad (2.2.8)$$

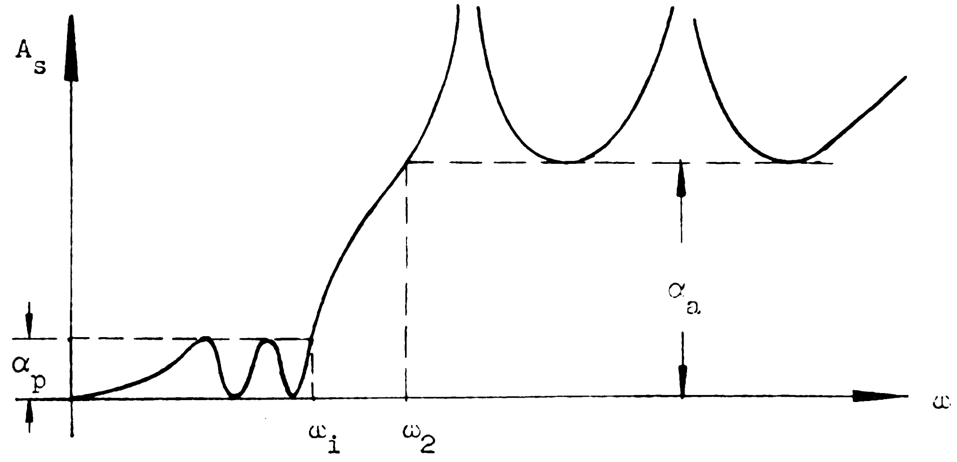


Figure 2.2.2

where $k = \omega_1/\omega_2$ the modulus of the elliptic sine function and K is the corresponding complete elliptic integral.

In a general case, in actual practice, the values of the insertion loss A_s, is to be less than a given value α_p, in the effective pass-band. On the other hand, in the effective block-band, A_s is not always necessarily greater than a given value α_a.

If the contour of requirement for A_s in the effective block-band is not a horizontal line (flat), it is difficult to find a corresponding ϕ - function satisfying this given non-flat requirement. In such a case, as is usually done, an equation similar to eq. 2.2.4 can be used.

In the effective block-band, for frequencies of high A_s, there is a linear relation between A_s and A_I [3]. Therefore, A_I can approximately be determined. The problem now is reduced to finding a reference filter which will have an image attenuation in the block-band, approximated to A_I just determined.

Since the poles of A_I are known, poles of A_s are identical. With this information the ϕ - function can be determined [3], [7].

In a recent article [5], the determination of ϕ - function is considered corresponding to the requirements on A_s in the effective block-band as: (1) A_s will not be less than a given set of different values of αM_i's in the corresponding frequency bands as shown for a low-pass filter in fig. 2.2.3, and (2) in the effective pass band $A_s \leq \alpha_m$.

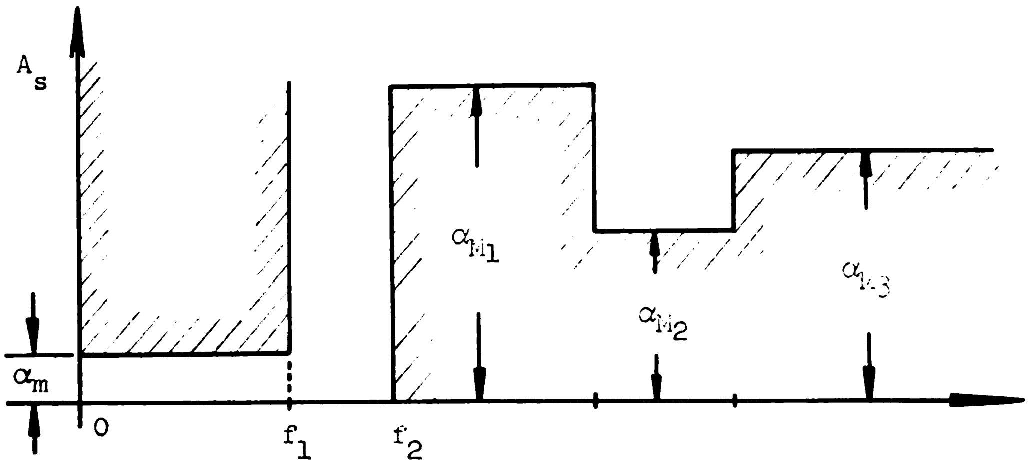


Fig. 2.2.3

Also, ϕ - functions for different types of filters (i.e., symmetrical, antimetrical, and dissymetrical) which satisfy the imposed restrictions are investigated. This investigation follows exactly the same procedures stated earlier, i.e., of determining the approximate locations of the poles of A_s. Since the form of the ϕ - function is similar to that as in eq. 2.2.4, A_s has equal-ripple characteristics in the effective pass-band.

Now, in conclusion, it can be summarized that in filter design on the basis of insertion loss, the important part is that a ϕ - function must be found which satisfies the imposed restrictions on the A_s function.

(B) Determination of Element Values:

After finding a ϕ - function, the next problem is to find the element values of the filter, either with lattice or ladder configuration, corresponding to this ϕ - function. Explicit formulas for these element values exist for some special cases such as: 1) Butterworth or Tschbyscheff behavior in the effective pass band, 2) Tschebyscheff behavior in both effective bands (Darlington's filter) [6], [8], [9], [10], [11], [12], [13], [36].

Although Darlington's method of determining element values includes the general case, [3], [7], [18], the technique entails long calculations and hence is laborious for any practical application. However, the digital computer overcomes this difficulty somewhat, [6], [37].

2.3 ADVANTAGE OF IMAGE PARAMETER METHOD

The possible use of digital computers with the image parameter method is mentioned only in passing. Although it is believed that such a digital investigation holds a promise, it is not the main idea of this thesis.

The key notion of this thesis, is the utilization of the image parameter method to overcome the difficulty encountered in the calculation of the element values for the general case, as given by Darlington. Tuttle [14] and others have emphasized the need for a fuller investigation of the classical image parameter method.

Specifically, one of the main advantages of the image parameter method is that once design parameters are found, the element values can easily be determined since there exist simple relationships between the element values and the design parameters. However, this method also suffers from a serious draw-back, i.e., the cut-and-try method, necessitated because of the fact that the design parameters are closely interrelated.

The above draw-back is a direct consequence of Zobel's decomposition formula on which the existing image parameter method is based.

2.4 SOME DISADVANTAGES OF ZOBEL'S DECOMPOSITION FORMULA

Zobel's decomposition formula is considered in section (2.1). This formula is rewritten here for convenience.

$$e^{P_s} = \left(\frac{2\sqrt{Z_s Z_R}}{Z_s + Z_R}\right)\left(\frac{Z_s + Z_{I_2}}{2\sqrt{Z_s Z_{I_1}}}\right)\left(\frac{Z_R + Z_{I_2}}{2\sqrt{Z_R Z_{I_2}}}\right)\left(1-K_s K_R e^{-2P_I}\right) e^{P_I} \quad (2.4.1)$$

The main disadvantages are:

(1) In the block band, the interaction term is commonly neglected.

(2) It is not at all clear how each term affects the insertion function. Specifically, consider the example of a symmetrical filter with equal terminating resistances. For this case, the insertion loss function in the pass band is zero at frequencies where the image impedance is equal to the terminating resistance, i.e., when the second and third factors within parantheses in eq. (2.4.1) are equal to unity. But this fact, will not enable us to conclude that these are the only frequencies at which A_s is zero. In fact, other frequencies do exist at which A_s is zero in the pass band. However, these other frequencies cannot be determined by considering conditions at which the logarithm of the individual factors in eq. 2.4.1 vanish. Any determination of these frequencies, for this particular example, can only be accomplished by

considering the combined effect of the interaction and reflection terms [2].

(3) Zobel's decomposition formula does not include the possibility of considering the case of degenerated four terminal networks [15].

(4) At cut-off frequency, the reflection function (factors 2, 3, 4 in eq. 2.4.1) tend to infinity while A_I tends to zero. Consequently, the A_s - function has an indeterminate form at cut-off frequency. However, it is well known that the A_s - function at the cut-off frequency has a finite value, a fact which does not manifest in Zobel's formulation.

On the basis of Feldtkeller's study [16], Belevitch has expressed the insertion loss function in a different form than Zobel's decomposition [15], [17]. This new formulation of insertion loss, A_s, is used in investigating the A_s in only the pass-band of the filter. In Chapter III of this thesis, Belevitch's formulation is derived for different types of filters, i.e., symmetrical, antimetrical, and dissymmetrical including lossy cases. For the symmetrical filter case, the factors which appear in this new formulation are discussed in general, and new properties of these factors obtained in both pass- and stop-band. Also, a design procedure on the basis of this new investigation is described in Chapter VI.

III. A NEW FORMULATION FOR INSERTION FUNCTION OF IMAGE PARAMETER FILTERS

The insertion function is here first expressed in a different form than Zobel's decomposition formula. This new form is derivable either by starting from eq. 2.1.4, or from the definition of the insertion function.

3.1 THE NEW FORMULATION OF INSERTION FUNCTION

Consider a passive general four terminal network, N, as in Fig. 3.1.1. The input-output voltage and current relationships in terms of the image parameters are [2].

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \sqrt{\dfrac{Z_{I_1}}{Z_{I_2}}}\ \text{Cosh}\ P_I & \sqrt{Z_{I_1}\ Z_{I_2}}\ \text{Sinh}\ P_I \\ \dfrac{\text{Sinh}\ P_I}{\sqrt{Z_{I_1}\ Z_{I_2}}} & \sqrt{\dfrac{Z_{I_2}}{Z_{I_1}}}\ \text{Cosh}\ P_I \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad (3.1.1)$$

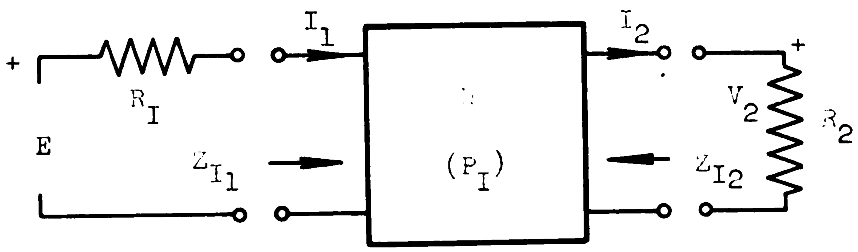


Fig. 3.1.1

Eq. 3.1.1 in symbolic form can be written as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad (3.1.2)$$

In addition, we have from the fig. 3.1.1,

$$V_2 = R_2\ I_2 \qquad (3.1.3)$$

$$V_1 = E - R_1\ I_1 \qquad (3.1.4)$$

From Eqs. 3.1.2, 3.1.3 and 3.1.4, after eliminating V_1, I_1 and V_2, a relation for I_2 in terms of E is given by

$$I_2 = \frac{E}{C\ R_1 R_2 + AR_2 + DR_1 + B} \qquad (3.1.5)$$

On the other hand, when the source E is directly connected to the load, (N is taken away in Fig. 3.1.1), we have

$$I_2' = \frac{E}{R_1 + R_2} \qquad (3.1.6)$$

Therefore, the ratio which determines the insertion function is

$$\frac{I_2'}{I_2} = \frac{C\,R_1R_2 + AR_2 + DR_1 + B}{R_1 + R_2} \qquad (3.1.7)$$

Let the parameters,

$$z_1 = \frac{Z_{I_1}}{R_1}$$
$$z_2 = \frac{Z_{I_2}}{R_2} \qquad (3.1.8)$$

be the normalized image impedances with respect to the terminating resistances. Substituting the values of A, B, C and D from eq. 3.1.1 into eq. 3.1.7 and from eq. 3.1.8, we have

$$\frac{I_2'}{I_2} = e^{P_s} = \frac{\sqrt{R_1 R_2}}{R_1 + R_2}\left[\frac{1 + Z_1Z_2}{\sqrt{Z_1 Z_2}}\,\text{Sinh}\,P_I + \frac{Z_1 + Z_2}{\sqrt{Z_1 Z_2}}\,\text{Cosh}\,P_I\right] \qquad (3.1.9)$$

From eq. 2.1.3 and 2.1.16, we have

$$e^{2A_s} = \frac{R_1R_2}{(R_1 + R_2)^2}\left|\frac{1 + Z_1Z_2}{\sqrt{Z_1Z_2}}\,\text{Sinh}\,P_I + \frac{Z_1 + Z_2}{\sqrt{Z_1Z_2}}\,\text{Cosh}\,P_I\right|^2 \qquad (3.1.10)$$

$$B_s = \arg\left[\frac{1 + Z_1Z_2}{\sqrt{Z_1Z_2}}\,\text{Sinh}\,P_I + \frac{Z_1 + Z_2}{\sqrt{Z_1Z_2}}\,\text{Cosh}\,P_I\right] \qquad (3.1.11)$$

The formulas (3.1.10) and (3.1.11) for insertion loss and insertion phase are used throughout this thesis. In the following sections, discussions of these formulas either in general or in some special form are presented.

3.2 SYMMETRICAL FILTERS

For symmetrical filters, $Z_{I_1} = Z_{I_2}$, let us assume that equal terminating resistances are always used*, i.e.,

$$R_1 = R_2 = R \tag{3.2.1}$$

then

$$Z_1 = Z_2 = Z \tag{3.2.2}$$

Substituting the relations (3.2.1) and (3.2.2) into the eqs. 3.1.10 and 3.1.11, yields the following

$$e^{2A_s} = \left| \frac{1 + Z^2}{2Z} \operatorname{Sinh} P_I + \operatorname{Cosh} P_I \right|^2 \tag{3.2.3}$$

$$B_s = \arg \left[\frac{1 + Z^2}{2Z} \operatorname{Sinh} P_I + \operatorname{Cosh} P_I \right] \tag{3.2.4}$$

(A) If N is a purely reactive four terminal network, then, in the pass-band,

$$P_I = j\, B_I \quad (A_I = 0)$$

Therefore, eqs. 3.2.3 and 3.2.4 take the forms

$$e^{2A_s} = 1 + \left(\frac{1 + Z^2}{2Z} \right) \operatorname{Sin}^2 B_I \tag{3.2.5}$$

$$B_s = \arctan \left[\frac{1 + Z^2}{2Z} \tan B_I \right] \tag{3.2.6}$$

respectively. In the block-band, since

z is purely imaginary

$$P_I = A_I + j\, k\, \pi \qquad (k = 0, \pm 1, \pm 2, \ldots)$$

---

* Otherwise the term $\frac{1}{2}\, \ln \dfrac{\sqrt{R_1 R_2}}{R_1 + R_2}$ must be added to the insertion loss expression.

then

$$\text{Sinh } P_I = \pm \text{ Sinh } A_I$$
$$\text{Cosh } P_I = \pm \text{ Cosh } A_I$$

where if the plus (minus) sign is used in one expression it must also be used in the other; therefore, eqs. 3.2.3 and 3.2.4 in this case take the following forms:

$$e^{2A_s} = 1 - \left(\frac{1 - Z^2}{2Z}\right)^2 \text{Sinh}^2 A_I \tag{3.2.7}$$

$$B_s = \arctan \left[j \frac{1 - Z^2}{2Z} \tan A_I \right] + k\,\pi \ (k = 0, \pm 1, \pm 2, \ldots) \tag{3.2.8}$$

Therefore, in both cases, i.e., in the pass-band and block-band, for insertion loss function we have from eqs. 3.2.5 and 3.2.7,

$$e^{2A_s} = 1 - \left(\frac{1 - Z^2}{2Z}\right)^2 \text{Sinh}^2 P_I \tag{3.2.9}$$

(B) If N is a lossy four terminal network, then we cannot distinguish between pass- and stop-bands since z and P_I are complex quantities for all ω. Let,

$$z = r + j\,x \tag{3.2.10}$$

$$P_I = A_I + j\,B_I \tag{3.2.11}$$

then

$$\frac{1 + Z^2}{2Z} = \frac{1 + (r + jx)^2}{2\,(r + jx)} = \frac{r}{2}\left(1 + \frac{1}{r^2 + x^2}\right) + j\,\frac{x}{2}\left(1 - \frac{1}{r^2 + x^2}\right)$$

$$= W_1 + jW_2 \tag{3.2.12}$$

and

$$\left.\begin{aligned} \text{Sinh } P_I &= \text{Sinh } A_I \text{ Cos } B_I + j \text{ Cosh } A_I \text{ Sin } B_I \\ \text{Cosh } P_I &= \text{Cosh } A_I \text{ Cos } B_I + j \text{ Sinh } A_I \text{ Sin } B_I \end{aligned}\right\} \tag{3.2.13}$$

Substituting eqs. 3.2.12 and 3.2.13 into eqs. 3.2.3 and 3.2.4, it can be shown that

$$e^{2A_s} = (W_1^2 + W_2^2 + 1)\ \mathrm{Cosh}^2\ A_I - (W_1^2 + W_2^2 - 1)\ \mathrm{Cos}^2\ B_I + W_1\ \mathrm{Sinh}\ (2A_I) - W_2\ \mathrm{Sin}\ (2B_I) - 1 \tag{3.2.14}$$

$$B_s = \arctan \frac{W_2\ \mathrm{Sinh}\ A_I\ \mathrm{Cos}\ B_I + W_1\ \mathrm{Cosh}\ A_I\ \mathrm{Sin}\ B_I + \mathrm{Sinh}\ A_I\ \mathrm{Sin}\ B_I}{W_1\ \mathrm{Sinh}\ A_I\ \mathrm{Cos}\ B_I - W_2\ \mathrm{Cosh}\ A_I\ \mathrm{Sin}\ B_I + \mathrm{Sinh}\ A_I\ \mathrm{Cos}\ B_I} \tag{3.2.15}$$

3.3 . ANTIMETRICAL FILTERS (INVERSE IMPEDANCE FILTERS)

For an antimetrical filter, image impedances are the inverse of each other with respect to the product R_1R_2, i.e.,

$$Z_{I1}\ Z_{I2} = R_1R_2 \tag{3.3.1}$$

then

$$Z_1Z_2 = 1 \tag{3.3.2}$$

and let

$$Z = Z_1 = \frac{1}{Z_2} \tag{3.3.3}$$

Substituting the eqs. 3.3.1 and 3.3.3 into eqs. 3.1.10 and 3.1.11, we have

$$e^{2A_s} = \left| \mathrm{Sinh}\ P_I + \frac{1 + Z^2}{2Z}\ \mathrm{Cosh}\ P_I \right|^2 \tag{3.3.4}$$

$$B_s = \arctan \left[\mathrm{Sinh}\ P_I + \frac{1 + Z^2}{2Z}\ \mathrm{Cosh}\ P_I \right] \tag{3.3.5}$$

(A) If N is a purely reactive four terminal network, then in the pass-band,

z is real

$P_I = j\ B_I\ (A_I = 0)$

In this case, eqs. 3.3.4 and 3.3.5 can be written as follows

$$e^{2A_s} = 1 + \left(\frac{1 + Z^2}{2Z}\right)^2 \text{Cos}^2 B_I \qquad (3.3.6)$$

$$B_s = \arctan \left[\left(\frac{2Z}{1 + Z^2}\right) \tan B_I\right] \qquad (3.3.7)$$

In the block band

z is purely imaginary

$P_I = A_I + j\, k\, \pi \;(k = 0, \pm 1, \pm 2, \ldots)$

therefore, eqs. 3.3.4 and 3.3.5 can be written as

$$e^{2A_s} = 1 + \left(\frac{1 - Z^2}{2Z}\right)^2 \text{Cosh}^2 A_I \qquad (3.3.8)$$

$$B_s = \arctan \left[j \frac{1 + Z}{2Z} \text{Coth}\, Z_I\right] + k\,\pi \;(k = 0, \pm 1, \pm 2, \ldots) \qquad (3.3.9)$$

Eqs. 3.3.6 and 3.3.8 can be given as one equation. Therefore, for antimetrical reactive filters, the expression for insertion loss is

$$e^{2A_s} = 1 + \left(\frac{1 - Z^2}{2Z}\right)^2 \text{Cosh}^2 P_I \qquad (3.3.10)$$

(B) If N is a lossy antimetrical four terminal network, then the image transfer loss function does not vanish in some interval, i.e., there is no pass-band. For all values of ω, z and P_I are complex quantities. Letting

$z = r + j\, x$

$P_I = A_I + j\, B_I$

and substituting eqs. 3.2.12 and 3.2.13 into eqs. 3.3.4 and 3.3.5, we have, for a lossy antimetrical four terminal network,

$$\begin{aligned} e^{2A_s} = {} & (W_1^{\,2} + W_2^{\,2} + 1)\, \text{Sinh}^2 A_I - (W_1^{\,2} + W_2^{\,2} - 1)\, \text{Sin}^2 B_I \\ & + W_1 \,\text{Sinh}\,(2A_I) + W_2 \,\text{Sin}\,(2B_I) + (W_1^{\,2} + W_2^{\,2}) \end{aligned} \qquad (3.3.11)$$

$$B_s = \arctan \frac{\text{Cosh } A_I \text{ Sin } B_I + W_2 \text{ Cosh } A_I \text{ Cos } B_I + W_1 \text{ Sinh } A_I \text{ Sin } B_I}{\text{Sinh } A_I \text{ Cos } B_I + W_1 \text{ Cosh } A_I \text{ Cos } B_I - W_2 \text{ Sinh } A_I \text{ Sin } B_I} \tag{3.3.12}$$

3.4 DISSYMMETRICAL LOSSLESS FILTERS

In this section, a general expression for insertion loss and phase for a general lossless filter is given.

In the pass-band:

z_1, z_2 are real

$P_I = j\, B_I \ (A_I = 0)$

Therefore, eqs. 3.1.10 and 3.1.11 give

$$e^{2A_s} = \frac{R_1 R_2}{(R_1 + R_2)^2} \left[\frac{(1 + Z_1 Z_2)^2}{Z_1 Z_2} \text{Sin}^2 B_I + \frac{(Z_1 + Z_2)^2}{Z_1 Z_2} \text{Cos}^2 B_I \right] \tag{3.4.1}$$

$$B_s = \arctan \left[\frac{1 + Z_1 Z_2}{Z_1 + Z_2} \tan B_I \right] \tag{3.4.2}$$

In the block-band:

z_1, z_2 are purely reactive

$P_I = A_I + j\, k\, \pi \quad (k = 0, \pm 1, \pm 2, \ldots)$

then in this case

$$e^{2A_s} = \frac{R_1 R_2}{(R_1 + R_2)^2} \left[\frac{(1 + Z_1 Z_2)^2}{Z_1 Z_2} \text{Sinh}^2 A_I - \frac{(Z_1 + Z_2)^2}{Z_1 Z_2} \text{Cosh}^2 A_I \right] \tag{3.4.3}$$

$$B_s = \arctan \left[- j \frac{Z_1 + Z_2}{1 + Z_1 Z_2} \text{Coth } A_I \right] \tag{3.4.4}$$

For all ω's, insertion loss can be given in one equation as follows:

$$e^{2A_s} = \frac{R_1 R_2}{(R_1 + R_2)^2} \left[\frac{(1 + Z_1 Z_2)^2}{Z_1 Z_2} \text{Sinh}^2 P_I - \frac{(Z_1 + Z_2)^2}{Z_1 Z_2} \text{Cosh}^2 P_I \right] \tag{3.4.5}$$

or

$$e^{2A_s} = \frac{4 R_1 R_2}{(R_1 + R_2)^2} \left[1 + \frac{(1 - Z_1 Z_2)^2}{4 Z_1 Z_2} \operatorname{Sinh}^2 P_I - \frac{(Z_1 - Z_2)^2}{4 Z_1 Z_2} \operatorname{Cosh}^2 P_I \right] \tag{3.4.6}$$

IV. DISCUSSION OF THE ϕ - FUNCTION OF SYMMETRICAL FILTERS IN TERMS OF IMAGE PARAMETERS

In the preceding chapter, insertion function is reformulated differently from Zobel's decomposition. The insertion loss function, A_s, as so reformulated, permits an ease of investigation of A_s not possible with Zobel's formulation. Belevitch [17] has used this formulation in studying the A_s function in only pass-band. In this chapter the complete discussion of this formulation for symmetrical filters is considered. The general properties obtained from this wider investigation also includes Belevitch's results. These general results are subsequently applied to filter design.

4.1 THE ϕ - FUNCTION IN TERMS OF IMAGE PARAMETERS

For symmetrical, lossless filters with equal terminating resistances at both terminal pairs, it is shown in section 3.2 that the insertion loss in either bands (pass or block) can be given by the single formula

$$e^{2A_s} = 1 - \left[\frac{1 - z^2}{2z}\right]^2 \operatorname{Sinh}^2 P_I \tag{4.1.1}$$

In insertion loss theory, it has been found convenient to let

$$\phi^2 = - \left[\frac{1 - z^2}{2z}\right]^2 \operatorname{Sinh}^2 P_I \tag{4.1.2}$$

The hyperbolic term in eq. 4.1.2 can be modified and expressed in terms of a ratio function, H, of the filter. Letting

$$H = \operatorname{Tanh} \frac{P_I}{2} \tag{4.1.3}$$

then,

$$\sinh^2 P_I = \left(\frac{2h}{1 - h^2}\right)^2 \qquad (4.1.4)$$

Substituting eq. 4.1.4 into eq. 4.1.2,

$$\phi = -\left(\frac{1 - z^2}{2z}\right)^2 \left(\frac{2H}{1 - H^2}\right)^2 \qquad (4.1.5)$$

In eq. 4.1.5, ϕ^2 has two factors which are reciprocals, each of which is expressed in a different variable. In the following discussion it is shown that the first factor is effective in the pass-band only, and the second is effective in the block-band only.

4.2 A USEFUL CONFORMAL MAPPING

As already noted, in eq. 4.1.5 the two factors $\left(\frac{1 - z^2}{2z}\right)^2$ and $\left(\frac{2H}{1 - h^2}\right)^2$ are reciprocal in mathematical form but are expressed in different variables. Therefore, the study of one of them yields the properties of the other. In this section we consider only the first factor in eq. 4.1.5 and investigate its properties.

Consider the following function of a complex variable z

$$W = \left(\frac{1 - z^2}{2z}\right)^2 \qquad (4.2.1)$$

If we let

$$\lambda = \tfrac{1}{2}\left(z - \frac{1}{z}\right) \qquad (4.2.2)$$

then, we have

$$w = \lambda^2 \qquad (4.2.3)$$

Therefore, we first consider the well-known function of eq. 4.2.2.

In eq. 4.2.2, let

$$z = \rho \cdot e^{i\theta} \qquad (4.2.4)$$

and

$$\lambda = u + iv \qquad (4.2.5$$

If the variable z in eq. 4.2.2 represents the normalized image impedance, the real part of z must be positive. Therefore, in the following discussion the open left half of z - plane is not considered.

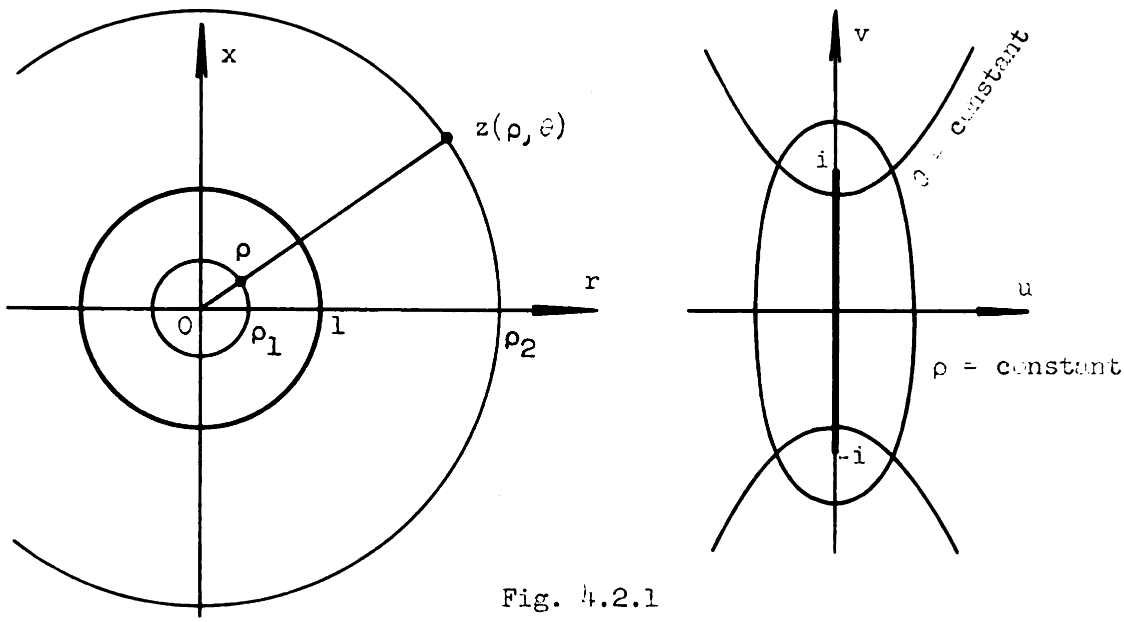


Fig. 4.2.1

The positive part of real axis in the z - plane is mapped into the whole real axis of λ - plane. But imaginary axis of the z - plane is mapped into the same part of the imaginary axis of λ - plane as shown in fig. 4.2.2.

Once the λ - function is determined, W is given by the simple relationship in eq. 4.2.3. The mapping of the z - plane into the W - plane is also shown in fig. 4.2.2.

From eq. 4.2.1 and the fig. 4.2.2 we can see that the positive real axis of the z - plane is mapped into the positive real axis of the W - plane (but not in one-to-one correspondence) and the imaginary axis of the z - plane is mapped into a portion of the negative real axis of the W - plane.

Substituting eqs. 4.2.4 and 4.2.5 into eq. 4.2.2, we can obtain the following relations between the real and imaginary parts of z and λ.

$$u = \frac{1}{2} \left(\rho - \frac{1}{\rho}\right) \mathrm{Cos}\ \theta$$
$$v = \frac{1}{2} \left(\rho + \frac{1}{\rho}\right) \mathrm{Sin}\ \theta \qquad (4.2.6)$$

From eq. 4.2.6, the following properties of the λ - function can be found:

(1) The circles (ρ = constant) in the z - plane are mapped into the homofocal ellipses in the λ - plane. Their equations are given by

$$\frac{u^2}{\left(\rho - \frac{1}{\rho}\right)^2} + \frac{v^2}{\left(\rho + \frac{1}{\rho}\right)^2} = \frac{1}{4} \qquad (4.2.7)$$

The common focii of these ellipses are the (i) and (-i) points of the λ - plane.

(2) Two circles with the radii ρ_1 and ρ_2 respectively are mapped into the same ellipse if $\rho_1\ \rho_2 = 1$. Therefore, the inverse function, i.e., z = f (λ) is not single valued. The λ - plane, actually is a two-sheeted Riemann surface. The unit circle in the z - plane is mapped into the section of straight line between the points (i) and (-i) in the λ - plane. Therefore, along this section of straight line a cut can be made and the two sheets of Riemann surface can be considered separately, each of which corresponds to either the inside or the outside region of the unit circle in the z - plane.

(3) The straight lines passing through the origin in the z - plane (θ = constant lines) are mapped into homofocal hyperbolas in the λ - plane. The focii of these hyperbolas are the same as those of ellipses.

This mapping of eq. 4.2.2 is illustrated in fig. 4.2.1.

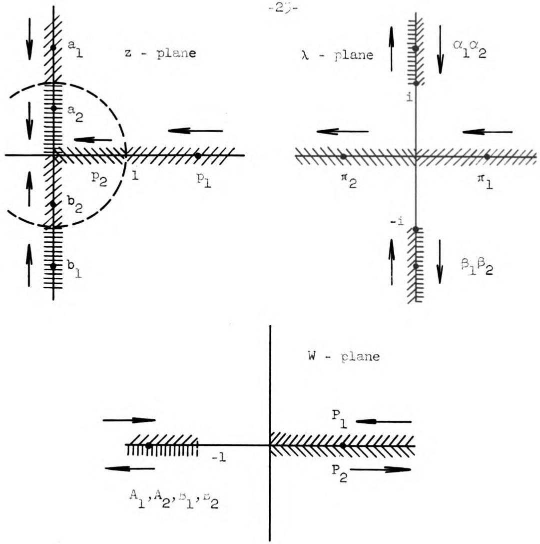


Fig. 4.2.2

Since z represents the normalized image impedance of a filter, z is real and positive in the pass-band and pure imaginary in the block-band. Therefore, the main properties of the factor $(\frac{1 - z^2}{2z^2})^2$ can be stated in a theorem. Thus:

<u>Theorem I.</u> The factor (for an LC network)

$$W = (\frac{1 - z^2}{2z})^2$$

is real and is always positive in the pass-band and always negative and not greater than (-1) in the block-band. At the cut-off frequency, this factor increases without limit.

Now, we can consider the second factor in eq. 4.1.5, i.e.,

$$\eta = \left(\frac{2H}{1 - H^2}\right)^2 \qquad (4.2.8)$$

Since this factor is the reciprocal of the first factor in eq. 4.1.5 except for the difference in variables, the mapping in fig. 4.2.1 can be extended once more by taking the inverse of the W - function. In this case the mapping from the H - plane into η - plane can be given as in fig. 4.2.3.

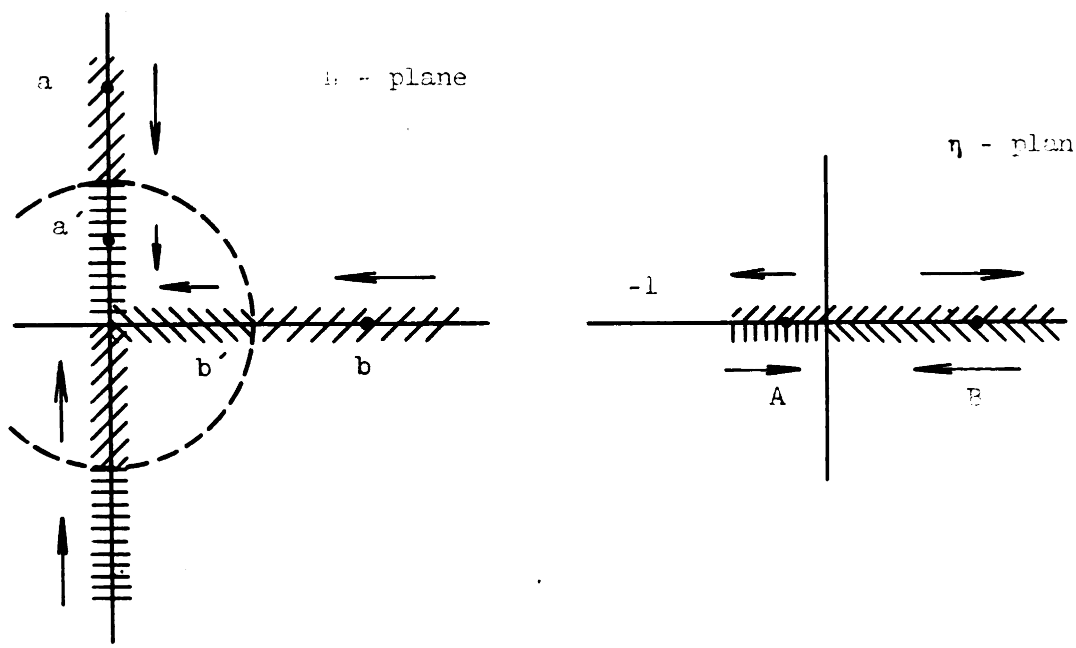


fig. 4.2.3

Fig. 4.2.3 indicates that the positive real axis in the H - plane is mapped into the positive real axis of the η - plane. The imaginary axis of the H - plane is mapped into the line segment (-1, 0) on the negative real axis of the η - plane.

The properties of the H - function are well-known [2]: H is purely imaginary in the pass-band of the filter and it is real and positive in the block-band. H has a zero or pole at the cut-off frequency. Therefore, considering the mapping in fig. 4.2.3 and the properties of the H - function, the important properties of the factor η, in eq. 4.2.8 can be summarized and stated in the form of a theorem:

Theorem II. The factor (for an LC-network)

$$\eta = \left(\frac{2H}{1 - H^2}\right)^2$$

is real and is always positive in the block-band, and its value varies between (0) and (-1) in the pass-band. At the cut-off frequency, η is zero.

In order to illustrate the properties of the two factors in eq. 4.1.5, two figures, fig. 4.2.4 and fig. 4.2.5, are presented. Further properties of these two factors can easily be observed from these figures.

Further properties on the factor W:

(1) At the critical frequencies of the z - function, W has poles.

(2) At the cut-off and infinite frequencies, W has poles.

(3) In the pass-band, the abscissa of the intersection points (matching points) of z - curve and the horizontal line, z = 1 correspond to the zeros of W.

(4) In the block-band, the abscissa of the intersection points of z - curve (imaginary) and the horizontal lines, $z = \mp i$, correspond to the maximum points of W - curve. As its known from Theorem I, these maxima are the same and equal to (-1).

(5) For two values, z_1 and z_2, of z such that $z_1 z_2 = \mp 1$, the corresponding values of W are the same.

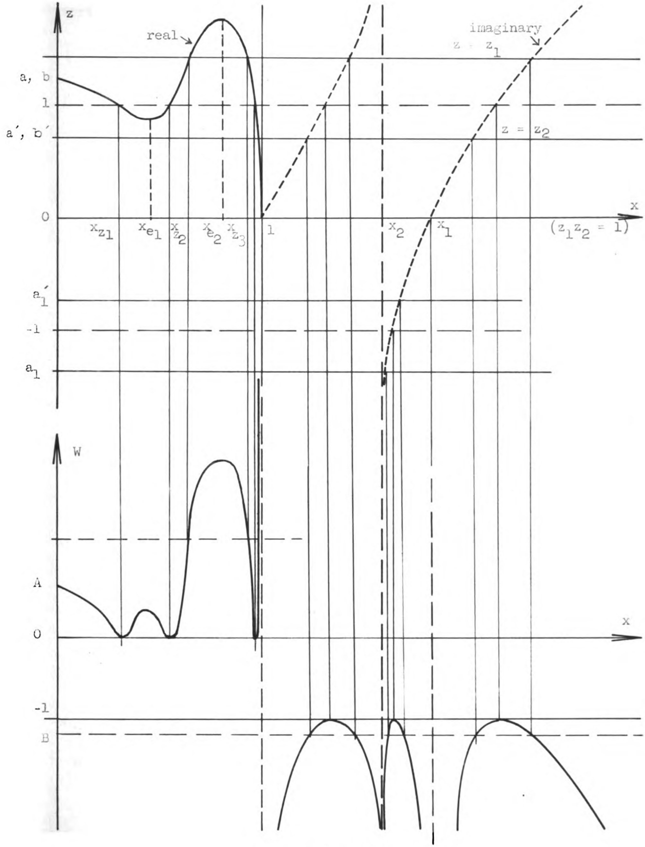

z
real
imaginary
$z = z_1$
a, b
1
a′, b′
$z = z_2$
0
x
x_{z_1}
x_{e_1}
x_{z_2}
x_{e_2}
x_{z_3}
1
x_2
x_1
$(z_1 z_2 = 1)$
a_1'
-1
a_1
W
A
0
x
-1
B

Fig. 4.2.4

Further properties of the η - factor:

(1) At the critical frequencies (poles and zeros) of H, η has zeros.

(2) At the cut-off frequency, η is zero.

(3) At the infinite frequency, η has a limiting value* of:

$$\eta_{\infty} = \frac{1}{4}\left[\frac{\prod_{i=1}^{m}(1+m_i)^2 - \prod_{i=1}^{n}(1-m_i)^2}{\prod_{i=1}^{m}(1-m_i^2)}\right]^2$$

where n is the number of sections in the filter and m_i represents the m- parameter of each individual section. As can be seen from the expression for η_{∞}, if the filter contains at least one constant-k section, then η_{∞} tends to infinity with ω (in this case H (∞) = 1).

(4) In the block-band, the intersection points of H - curve and the horizontal line H_1 = 1 correspond to the poles of η - function.

(5) In the pass-band, the abscissa of the intersection points of the H - curve (H is imaginary) and the horizontal lines $H_1 = \pm i$ correspond to the minimum points of the η - curve. From Theorem II, it is known that these minimums are the same and equal to (-1).

(6) For the values of H_1 and H_2 of H, such that $H_1 H_2 = \pm 1$, the corresponding values of η are the same.

---

*This formula can be derived if the relation between A_I and H is considered [2]. For an image parameter filter A_I is the sum of all image attenuation functions of the individual sections and at infinite frequency these individual functions have the value [2] of

$$A_{I_i}(\infty) = \ln\left(\frac{1+m_i}{1-m_i}\right)$$

4.3 FORMULAS FOR THE CHARACTERISTIC POINTS OF W AND η CURVES

In the preceding chapter, general W- and η- functions are considered and the general properties of these two functions are obtained. For the characteristic points of W- and η- curves such as, x_1, x_2, ..., x_{a1}, x_{a2}, ..., x_{z1}, x_{z2}, ..., x_{h1}, x_{h2}, ..., x_{e1}, x_{e2}, ..., and x_{p1}, x_{p2}, ... in fig. 4.2.4 and fig. 4.2.5, it is difficult to obtain a mathematical expression. But for particular z- and H- functions these formulas exist and are given in the following [7], [2] (LC network).

A) z - function is in the form of a geometric-mean variation (Tschebyscheff - approximation).

Let the number of critical frequencies of z be n. Then,

Critical frequencies (poles and zeros) of z occur at (block region)

$$x_i = \frac{1}{\mathrm{sn}\left[\frac{iK}{n+1}\right]} \quad (i = 1, 2, \ldots, n) \qquad (4.3.1)$$

Unit values of z occur at (pass band)

$$x_{z_i} = x_u \,\mathrm{sn}\left[\frac{2(n-i)+3}{2(n+1)} K\right] \quad (i = 1, 2, \ldots, n+1) \qquad (4.3.2)$$

Extremal points of z occur at (pass band)

$$x_{e_i} = x_u \,\mathrm{sn}\left[\frac{iK}{n+1}\right] \quad (i = 0, 1, \ldots, n+1) \qquad (4.3.3)$$

$$(x_{e_0} = 0, \quad x_{e\,n+1} = x_u)$$

B) H - function is in the form of a geometric-mean variation (Tschebyscheff - approximation).

Let the number of critical frequencies of H be n. Then,

Critical frequencies of H occur at (pass band)

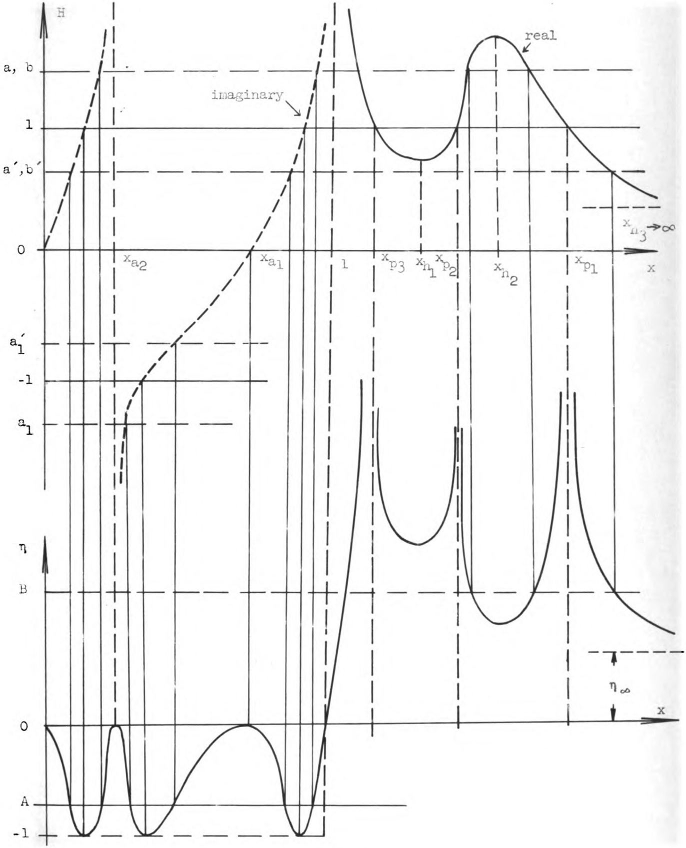

H
a, b
1
a´,b´
0
imaginary
real
x_{a_2}
x_{a_1}
1
x_{p_3}
x_{n_1}
x_{p_2}
x_{n_2}
x_{p_1}
x
$x_{n_3} \to \infty$
a_1'
-1
a_1
η
B
0
A
-1
η_∞
x

Fig. 4.2.5

$$x_{a_i} = \text{sn}\left[\frac{n+1-i}{n+1} K\right] \quad (i = 1, 2, \ldots, n) \qquad (4.3.4)$$

Poles of A_I (K = 1) occur at (block band)

$$x_{p_i} = x_{u_A} \frac{1}{\text{sn}\left[\frac{2i-1}{2(n+1)} K\right]} \quad (i = 1, 2, \ldots, n+1) \qquad (4.3.5)$$

Extremal points of H occur at (block band)

$$x_{h_i} = \frac{x_{u_A}}{\text{sn}\left[\frac{n+1-i}{n+1} K\right]} \quad (i = 0, 1, \ldots, n+1) \qquad (4.3.6)$$

$$(x_{h_0} = x_{u_A} \quad , \quad x_{h_{n+1}} = \infty)$$

4.4 FORMULAS FOR THE INSERTION LOSS AND PHASE OF A SYMMETRICAL IMAGE PARAMETER FILTER AT CUT-OFF FREQUENCY

The two factors, W and η, appearing in the expression for ϕ^2 in eq. 4.1.2, gives an indeterminate form for ϕ^2 at cut-off frequency because the factor W tends to infinity while the factor η tends to zero when the frequency approaches the cut-off frequency. It is shown in this section that ϕ^2 has a finite value at the cut-off frequency. This problem is discussed by Belevitch [19] and insertion loss and phase expressions at cut-off are given but for only the special case where the image impedance of the filter is an m - derived type. In this section, we generalize Belevitch's results to a general image impedance case.

In anticipation of a detailed discussion of the next chapter, assume now that terminating sections are used in filter design. Therefore, the image impedance of the terminating section is the image impedance of the filter. Using eqs. 5.4.20 and 5.4.21, and in these equations letting

$$p = j\omega \quad , \quad x = \frac{\omega}{\omega_0}$$

we have,

$$Z_I = \frac{k\sqrt{1-x^2}\left(1 - \frac{x^2}{x_2^2}\right) \cdots \left(1 - \frac{x^2}{x_{\gamma-1}^2}\right)}{\left(1 - \frac{x^2}{x_1^2}\right)\left(1 - \frac{x^2}{x_3^2}\right) \cdots \left(1 - \frac{x^2}{x_\gamma^2}\right)} \quad (\text{for } \gamma \text{ odd}) \qquad (4.4.1)$$

$$Z_I = \frac{k\left(1 - \frac{x^2}{x_2^2}\right) \cdots \left(1 - \frac{x^2}{x_\gamma^2}\right)}{\sqrt{1 - x^2}\left(1 - \frac{x^2}{x_1^2}\right) \cdots \left(1 - \frac{x^2}{x_{\gamma-1}^2}\right)} \quad (\text{for } \gamma \text{ even}) \qquad (4.4.2)$$

On the other hand, the H function of a symmetrical image parameter filter can be found as follows:

Each half section in the composite filter, including terminating sections, has the following H_i function

$$H_i = \operatorname{Tanh} \frac{P_{I_i}}{2} = m_i \operatorname{Tanh} \frac{P_o}{2} = m_i H_o \qquad (4.4.3)$$

where P_o and H_o represent, respectively, the image transfer and ratio functions of a constant-k filter section. From eq. 4.4.3, we have

$$e^{P_{I_i}} = \frac{1 + H_i}{1 - H_i} = \frac{1 + m_i H_o}{1 - m_i H_o} \qquad (4.4.4)$$

For the filter consists of n sections (includes terminating sections), since

$$P_I = \sum_{i=1}^{n} P_{I_i} \quad \text{and} \quad H = \operatorname{Tanh} \frac{P_I}{2}$$

From eqs. 4.4.3 and 4.4.4, we find

$$H = \frac{\prod_{i=1}^{n} (1 + m_i H_o) - \prod_{i=1}^{n} (1 - m_i H_o)}{\prod_{i=1}^{n} (1 + m_i H_o) + \prod_{i=1}^{n} (1 - m_i H_o)} \qquad (4.4.5)$$

But since, in general,

$$\prod_{i=1}^{n} (1 \pm a_i) = A_o \pm A_1 \pm A_2 \pm \cdots + (-1)^n A_n$$

where

$$A_o = 1$$

$$A_1 = a_1 + a_2 + \dots + a_n$$

$$A_2 = a_1 a_2 + a_1 a_3 + \dots + a_{n-1}\, a_n \qquad (4.4.6)$$

$$\dots\dots\dots\dots\dots$$

$$A_n = a_1 a_2 \dots\dots a_n$$

and if we let

$$a_i = m_i H_o \qquad (4.4.7)$$

then eq. 4.4.5 can be written as follows:

(1) n is even;

$$H = \frac{A_1 + A_3 + \dots\dots + A_{n-1}}{A_o + A_2 + \dots\dots + A_n} \qquad (4.4.8)$$

(2) n is odd;

$$H = \frac{A_1 + A_3 + \dots\dots + A_n}{A_o + A_2 + \dots\dots + A_{n-1}} \qquad (4.4.9)$$

On the other hand

$$H_o = \frac{x}{\sqrt{x^2 - 1}}$$

Therefore, as x approaches the value of unity, eqs. 4.4.8 and 4.4.9 show that H approaches the following values:

(1) n is even

$$H = \frac{A_{n-1}}{A_n} = \frac{1}{H_o}\left(\frac{1}{m_1} + \frac{1}{m_2} + \dots + \frac{1}{m_n}\right)$$

or

$$H = \frac{\sqrt{x^2 - 1}}{x} \sum_{i=1}^{n} \frac{1}{m_i} \qquad (4.4.10)$$

(2) n is odd

$$H = \frac{A_n}{A_{n-1}} = H_o \frac{1}{\frac{1}{m_1} + \frac{1}{m_2} + \ldots + \frac{1}{m_n}}$$

or

$$H = \frac{x}{\sqrt{x^2 - 1}} \quad \frac{1}{\sum_{i=1}^{n} \frac{1}{m_i}} \qquad (4.4.11)$$

Now, consider the value of φ^2 - function at cut-off. From eqs. 4.4.1 and 4.4.2 for normalized image impedance,

$$z = \frac{k}{R_T} \sqrt{1 - x^2} \;\; P_1(x) \qquad \text{(for } \gamma \text{ odd)} \qquad (4.4.12)$$

and

$$z = \frac{k}{R_T} \; \frac{1}{\sqrt{1 - x^2}} \;\; P_2(x) \qquad \text{(for } \gamma \text{ even)} \qquad (4.4.13)$$

Eqs. 4.4.10 and 4.4.11 can be written as,

$$H = \frac{\sqrt{x^2 - 1}}{x} \;\; Q_1(x) \qquad \text{(for n even)} \qquad (4.4.14)$$

and

$$H = \frac{x}{\sqrt{x^2 - 1}} \;\; Q_2(\lambda) \qquad \text{(for n odd)} \qquad (4.4.15)$$

with

$$Q_1(x) \;\; Q_2(x) = 1 \qquad (4.4.16)$$

where n is the number of full-sections in the filter (including the terminating sections) while γ is the number of half sections in one of the terminating half sections. Therefore, $n \geq \gamma$.

For different number of n and γ, different types of z- and H-functions are obtained. However, there are only four such different cases as follows:

Case 1 -

$$z = \frac{k}{R_T} \sqrt{1 - x^2} \quad P_1(x) \qquad (\gamma \text{ odd})$$

$$H = \frac{\sqrt{x^2 - 1}}{x} \quad Q_1(x) \qquad \text{(n even)}$$

Case 2 -

$$z = \frac{k}{R_T} \sqrt{1 - x^2} \quad P_1(x) \qquad (\nu \text{ odd})$$

$$H = \frac{x}{\sqrt{x^2 - 1}} \quad Q_2(x) \qquad \text{(n odd)}$$

Case 3 -

$$z = \frac{k}{R_T} \frac{1}{\sqrt{1 - x^2}} \quad P_2(x) \qquad (\nu \text{ even})$$

$$H = \frac{\sqrt{x^2 - 1}}{x} \quad Q_2(x) \qquad \text{(n even)}$$

Case 4 -

$$z = \frac{k}{R_T} \frac{1}{\sqrt{1 - x^2}} \quad P_2(x) \qquad (\nu \text{ even})$$

$$H = \frac{x}{\sqrt{x^2 - 1}} \quad Q_2(x) \qquad \text{(n odd)}$$

If z and H are substituted into the eq. 4.1.5, i.e.,

$$\phi^2 = -\left(\frac{1 - z^2}{2z}\right)^2 \left(\frac{2H}{1 - H^2}\right)^2$$

and when x tends to unity, ϕ^2 will have the following values corresponding to the cases mentioned above:

$$\phi^2(\omega_o) = \left(\frac{Q_1}{\frac{k}{R_T} P_1}\right)^2 \qquad \text{(for case-1)}$$

$$\phi^2(\omega_o) = \left(\frac{1}{\frac{k}{R_T} P_1 Q_2}\right)^2 \qquad \text{(for case-2)}$$

$$\phi^2(\omega_o) = \left(\frac{k}{R_T} P_2 Q_1\right)^2 \qquad \text{(for case-3)}$$

$$\phi^2(\omega_o) = (\frac{K}{K_T} P_2 \frac{1}{Q_2})^2 \qquad \text{(for case-4)}$$

but, from eq. 4.4.16, all four of these ϕ^2 are not different. In fact, only the following two cases exist:

(1) For γ odd

$$\phi^2(\omega_o) = \left(\frac{Q}{\frac{K}{K_T} P_I}\right)^2 \qquad (4.4.17)$$

(2) For γ even

$$\phi^2(\omega_o) = (\frac{K}{K_T} P_2 Q_1)^2 \qquad (4.4.18)$$

The value of Q_1 is known from eq. 4.4.10 as

$$Q_1 = \sum_{i=1}^{m} \frac{1}{m_i} \qquad (4.4.19)$$

To calculate the value of P_1 and P_2 it is sufficient to consider eqs. 4.4.1 and 4.4.2. Indeed, we have for P_1 and P_2 the following:

Since each factor in eqs. 4.4.1 and 4.4.2 can be written, when x tends to unity, as

$$\lim_{x \to 1} (1 - \frac{x}{x_i^2}) = \lim_{x \to 1} (1 - (1-m_i^2)\, x^2) = m_i^2$$

then

$$P_1 = \left[\frac{m_2\ m_4\ \dots\ m_{\gamma-1}}{m_1\ m_3\ \dots\ m_\gamma}\right]^2 \qquad (4.4.20)$$

and

$$P_2 = \left[\frac{m_2\ m_4\ \dots\dots\ m_\gamma}{m_1\ m_3\ \dots\ m_{\gamma-1}}\right]^2 \qquad (4.4.21)$$

Therefore, substituting eqs. 4.4.19 and 4.4.20 with 4.4.21 into eqs. 4.4.17 and 4.4.18 yield

$$Q^2(\omega_c) = \left\{ \frac{R_T}{k} \left[\frac{m_1 m_3 \cdots m_\gamma}{m_2 m_4 \cdots m_{\gamma-1}}\right]^2 \sum_{i=1}^{n} \frac{1}{m_i} \right\}^2 \quad \text{(for } \gamma \text{ odd)} \quad (4.4.22)$$

and

$$Q^2(\omega_c) = \left\{ \frac{k}{R_T} \left[\frac{m_2 m_4 \cdots m_\gamma}{m_1 m_3 \cdots m_{\gamma-1}}\right]^2 \sum_{i=1}^{n} \frac{1}{m_i} \right\}^2 \quad \text{(for } \gamma \text{ even)} \quad (4.4.23)$$

Consider eqs. 4.1.1 and 4.1.5. Substituting eqs. 4.4.22 and 4.4.23 into these equations, we have, for insertion loss at cut-off:

$$A_s = \frac{1}{2}\ln\left\{1 + \left[\frac{R_T}{k}\left(\frac{m_1 m_3 \cdots m_\gamma}{m_2 m_4 \cdots m_{\gamma-1}}\right)^2 \sum_{i=1}^{n} \frac{1}{m_i}\right]^2\right\} \quad \text{(for } \gamma \text{ odd)} \quad (4.4.24)$$

and

$$A_s = \frac{1}{2}\ln\left\{1 + \left[\frac{k}{R_T}\left(\frac{m_2 \cdots m_\gamma}{m_1 \cdots m_{\gamma-1}}\right)^2 \sum_{i=1}^{n} \frac{1}{m_i}\right]^2\right\} \quad \text{(for } \gamma \text{ even)} \quad (4.4.25)$$

We now calculate the insertion phase of an image parameter filter at the cut-off frequency.

For a symmetrical filter, we have for the insertion phase, eq. 3.2.4. i.e.,

$$B_s = \arg\left[\frac{1+z^2}{2z}\sinh P_I + \cosh P_I\right] \quad (4.4.26)$$

At the cut-off frequency, $P_I = jB_I = js\pi$ ($A_I = 0$), where s is an integer. Therefore, only the first term in eq. 4.4.26 is indeterminate.

Repeating the above discussion for the first term in eq. 4.4.26, i.e.,

$$\Psi = \frac{1+z^2}{2z}\sinh P_I \quad (4.4.27)$$

we can calculate the value of Ψ at cut-off. Since

$$\sinh P_I = \frac{2H}{1-H^2}$$

we have for Ψ,

$$\Psi = \left[\frac{1 + z^2}{2z} \cdot \frac{2z}{1 - z^2}\right] \qquad (4.4.27a)$$

From the above discussion, the values of z and P are known. Therefore, we can immediately obtain the following.

$$\Psi(\omega_o) = \pm j \frac{Q_1}{\frac{k}{R_T} P_I} \qquad (\text{for } \gamma \text{ odd})$$

$$\Psi(\omega_o) = \pm j \frac{k}{R_T} P_2 Q_1 \qquad (\text{for } \gamma \text{ even})$$

or

$$\Psi(\omega_o) = \pm j \frac{R_T}{k} \left(\frac{m_1 \, m_3 \, \cdots \, m_\gamma}{m_2 \, m_4 \, \cdots \, m_{\gamma-1}}\right)^2 \sum_{i=1}^{n} \frac{1}{m_i} \quad (\text{for } \gamma \text{ odd}) \quad (4.4.28)$$

$$\Psi(\omega_o) = \pm j \frac{k}{R_T} \left(\frac{m_2 \, m_4 \, \cdots \, m_\gamma}{m_1 \, m_3 \, \cdots \, m_{\gamma-1}}\right)^2 \sum_{i=1}^{n} \frac{1}{m_i} \quad (\text{for } \gamma \text{ even}) (4.4.29)$$

Therefore, considering eq. 4.4.26 where Cosh $(js\pi) = \mp 1$, at cut-off:

$$B_s = \pm \arctan \left[\frac{R_T}{k} \left(\frac{m_1 \, m_3 \, \cdots \, m_\gamma}{m_2 \, m_4 \, \cdots \, m_{\gamma-1}}\right)^2 \sum_{i=1}^{n} \frac{1}{m_i}\right] + s\pi \quad (\text{for } \gamma \text{ odd}) \quad (4.4.30)$$

$$B_s = \pm \arctan \left[\frac{k}{R_T} \left(\frac{m_2 \, m_4 \, \cdots \, m_\gamma}{m_1 \, m_3 \, \cdots \, m_{\gamma-1}}\right)^2 \sum_{i=1}^{n} \frac{1}{m_i}\right] + s\pi \quad (\text{for } \gamma \text{ even}) \quad (4.4.31)$$

4.5 APPROXIMATE FORMULA FOR A_s IN THE BLOCK-BAND

Consider the ϕ - function in eq. 4.1.2 in the block-band, the expression for ϕ^2 is,

$$\phi^2 = -\left(\frac{1 - z^2}{2z}\right)^2 \text{Sinh}^2 A_I \qquad (4.5.1)$$

where z is purely imaginary. From Theorem I in section 4.2, the first factor in eq. 4.5.1 with negative sign in front of it, is always real and positive and not less than unity. Therefore, the value of the ϕ^2 - function is always greater than, or at least equal to, the value of $\text{Sinh}^2 A_I$.

To find an approximate formula for A_s in the block-band, one can then write.

$$e^{2A_s} = 1 + \phi^2 \geq 1 + \mathrm{Sinh}^2 A_I = \mathrm{Cosh}^2 A_I$$

hence.

$$e^{2A_s} \geq \tfrac{1}{2} (e^{A_I} + e^{-A_I})^2 \qquad (4.5.2)$$

If A_I is very large, e.g., at the vicinity of transfer poles, eq. 4.5.2 yields

$$A_s \geq A_I - \ln 2 \qquad (4.5.3)$$

Therefore, the insertion loss can be less than the image transfer loss in the block-band. In the symmetrical filter design, to retain the imposed restrictions on A_s, the last term in eq. 4.5.3 is always considered, i.e., ln 2 = 0.693 nepers or 6.02 decibels loss is added to the given contour of requirement for A_s in the block-band.

For symmetrical filters, the formula 4.5.3 is well-known and can also be obtained from the Zobel's decomposition formula, eq. 2.1.4, i.e., neglecting the interaction term and considering the minimum of one of the reflection loss which is very close to -3 decibels, the formula 4.5.3 follows.

V. FILTER TERMINATING SECTIONS

5.1 General

A low-pass filter-terminating section is here represented symbolically as TS. It is merely a low pass four terminal - LC diagram. At one pair of terminals, the image impedance is of the simplest order, i.e., with constant-k, at the other pair of terminals, the image impedance is of higher order. The order of an image impedance is defined as a number of unit values of z in the pass band.

A simple example of a TS is the Zobel's M-derived half section shown in fig. 5.1.1. At the pair of terminals 1-1′, the order of the image impedance is equal to two. The image impedance at the terminals 2-2′ is constant-K and of order one.

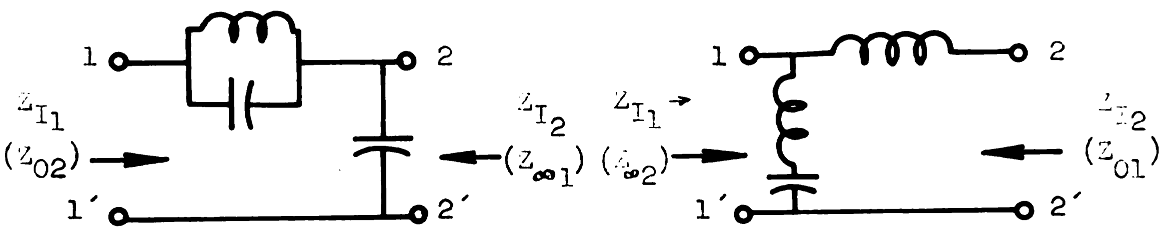


Fig. 5.1.1

To be able to increase the order of one of the image impedances of a TS, Zobel used a transformation and obtained a set of new higher order derived sections called MM′, MM′M″, ... types [20]. Each section is obtained from the previous one and has the following properties.

(a) An n-th order derived section has only one attenuation pole which corresponds to the parameter $m = m_1 m_2 \dots m_n$.

(b) The difference between the orders of the two image impedances is unity.

(c) There are two different sequences of derived sections. The image impedances of the sections in one sequence are reciprocal to the corresponding image impedances of the sections of the second sequence with respect to a constant, K^2: $K^2 = L_o/C_o$, where L_o and C_o represent, respectively, series inductance and parallel capacitance of the constant-K half section from which derived sections are obtained.

The property in (b) indicates that any n^{th} order derived Zobel section cannot be used for a terminating section, TS, except for the case where $n = 2$. Because, for $n > 2$, if Z_{I_1} is the higher order image impedance, then the order of Z_{I_2} is not a constant-k type image impedance.

It is possible, however, to obtain for a TS with $n > 2$, a constant-k image impedance by considering a set of cascaded Zobel sections on a matched basis. The matching is such that the last section in the set is an M-derived half section fig. 5.1.2.

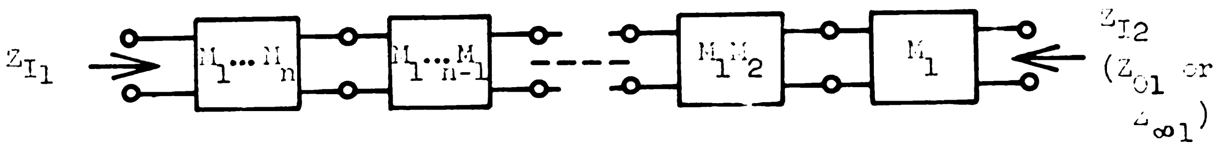


Fig. 5.1.2

This new section is, then, a TS. In order to make the discussion clear, some of the higher order derived sections are given in fig. 5.1.3. As is seen from fig. 5.1.3, each derived section has two main branches, namely, series and shunt.

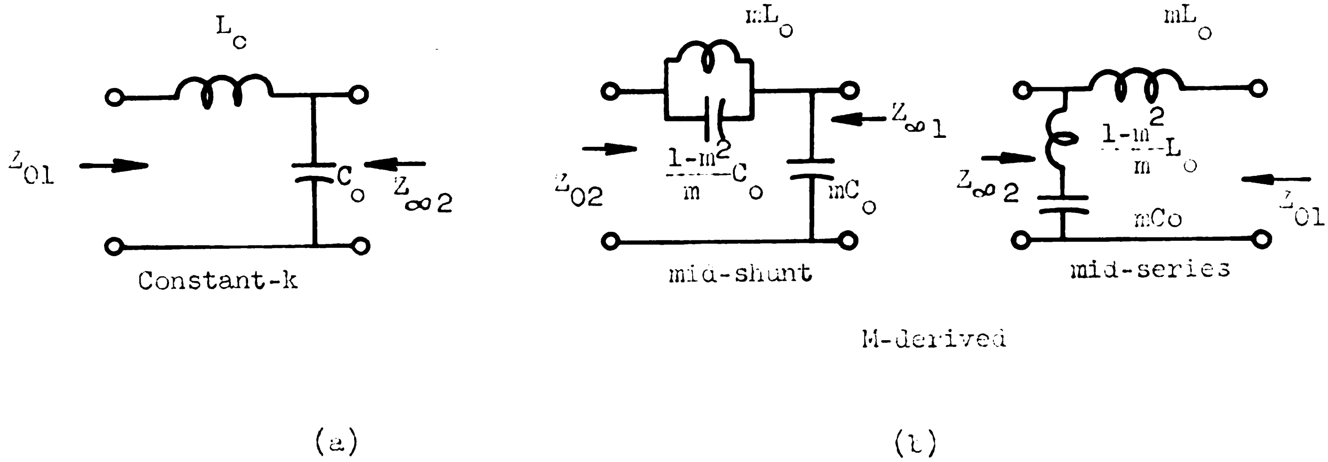
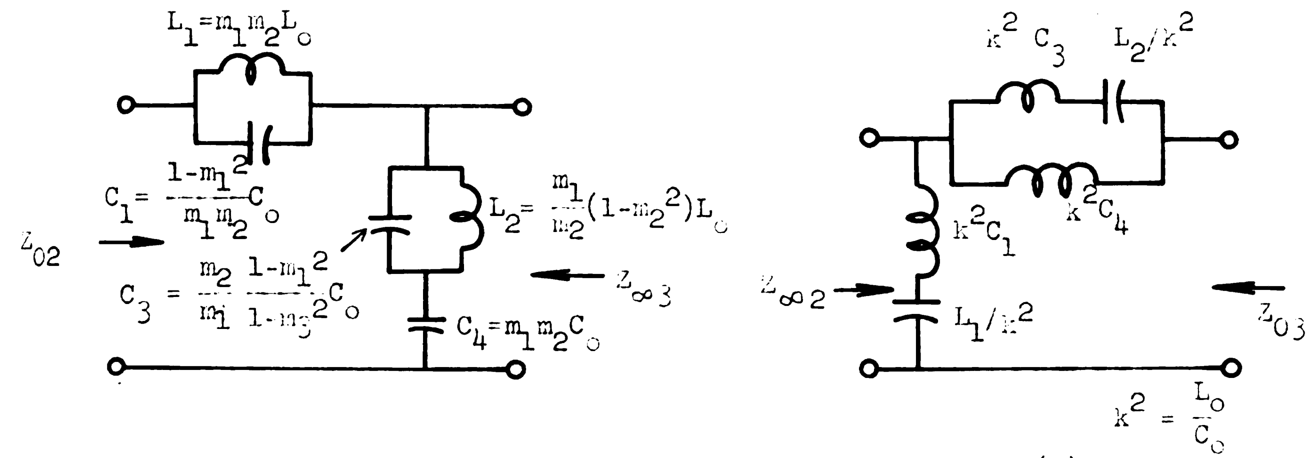
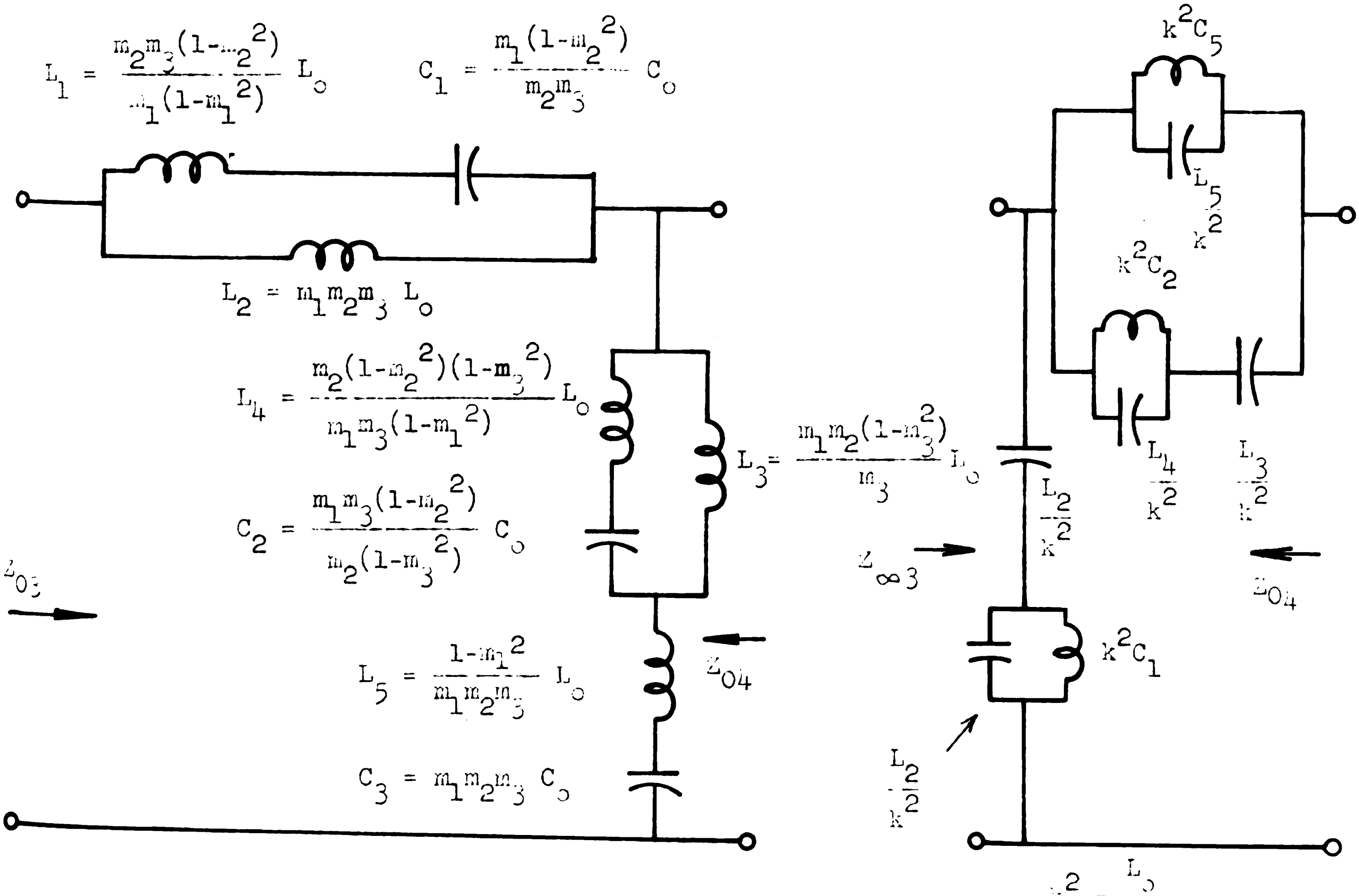


Fig. 5.1.3



(c)

$M_1 M_2$-derived Sections



$k^2 = \frac{L_o}{C_o}$

$M_1 M_2 M_3$-derived Sections

Fig. 5.1.3 (Cont'd.)

But both of them have complicated combinations of L and C elements. Of course, by use of Foster's [21] or Cauer's expansion method, each branch can be replaced by its canonical configuration. The TS obtained by cascading these sections will be in ladder form but every branch of this ladder TS will not be a simple L or C element. In practical design, a ladder network with branches consisting of simple L, C, parallel resonator, and series resonator is desired. Since Zobel's higher order derived sections do not correspond to the above design requirements, they are not henceforth considered here.

5.2 A USEFUL TRANSFORMATION

One of the higher order TS can be obtained from ladder-lattice transformation as follows: Consider a half section with arbitrary impedances Z_1 and Z_2 as in fig. 5.1.1a. From the following procedure, this section can be converted into another section(s), as in fig. 5.2.1b and c, with a higher order impedance but having the same transfer function.

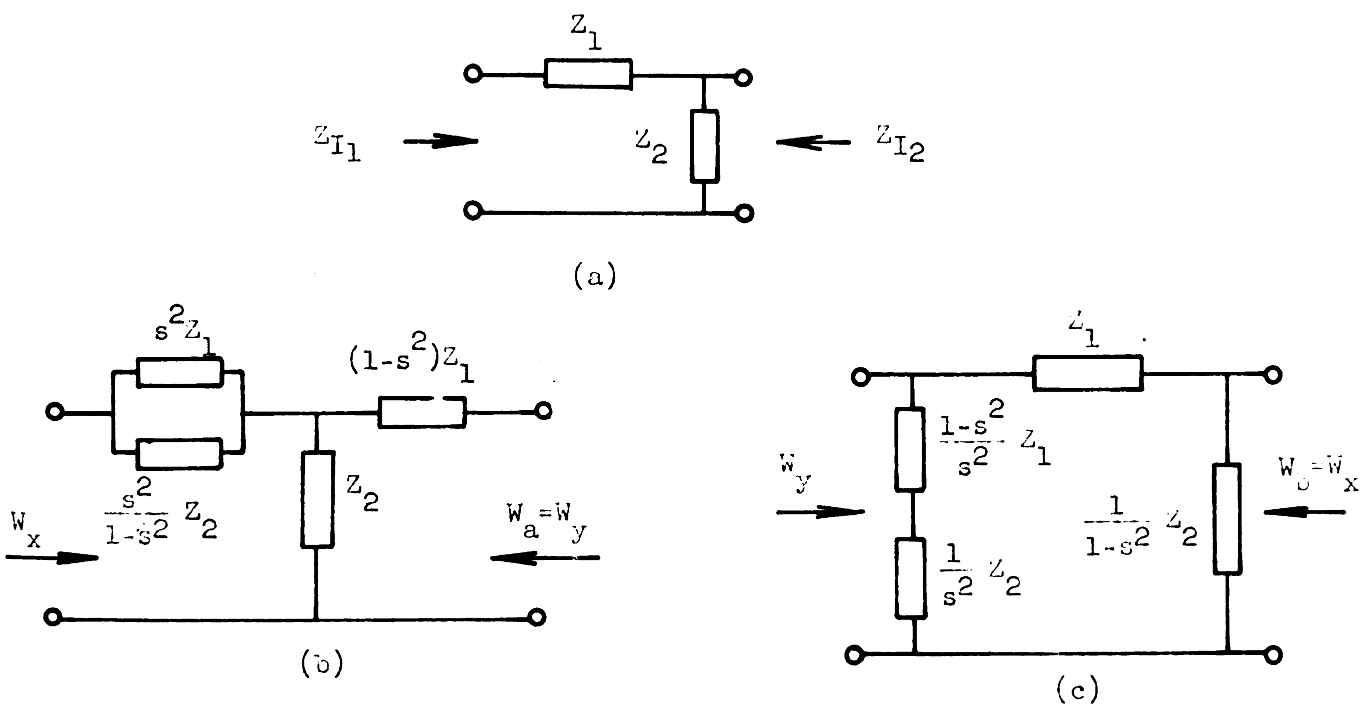


Fig. 5.2.1

Procedure:

1- Find the corresponding symmetrical lattice which has the same image impedance Z_{I1} (Z_{I2}) with the ratio function of 2H.

2- Multiply the series arms of this lattice by a constant, s, and divide the cross arms by the same factor. In this process, the ratio function of the lattice becomes 2sH but the image impedance is still the same, Z_{I1} (Z_{I2}).

3- Find the ladder equivalent to this lattice. Ladder networks exist if, and only if, $0 < s \leq 1$. Consequently, consider the half section which has one image impedance equal to Z_{I1} (Z_{I2}), the other having higher order, say, W_x (W_y) with the ratio function sH.

4- Now consider the half section in 3 and find another symmetrical lattice which has W_x (W_y) as its image impedance with the ratio function of 2sH.

5- Multiply cross arms by the same factor as in 2, i.e., s ($0 < s \leq 1$) and divide the series arms by s. The final lattice has the same image impedance, W_x (W_y), as in 4, but the ratio function is now 2H.

6- Find the ladder equivalent to this lattice and consider the half section. This final half section(s) will have the image impedances W_x (or W_y), W_a (or W_b) and the ratio function of H.

It can be shown that, $W_x = W_b$ and $W_y = W_a$. The expressions of Z_{I1}, Z_{I2}, W_x and W_y are given as follows:

$$Z_{I_1} = Z_1 \sqrt{1 + \frac{Z_2}{Z_1}} \tag{5.2.1}$$

$$Z_{I2} = \frac{Z_2}{\sqrt{1 + \frac{Z_2}{Z_1}}} \tag{5.2.2}$$

$$W_x = \frac{Z_2\sqrt{1 + \frac{Z_2}{Z_1}}}{(1 - s^2) + \frac{Z_2}{Z_1}} \tag{5.2.3}$$

$$W_y = \frac{Z_1\left[(1 - s^2) + \frac{Z_2}{Z_1}\right]}{\sqrt{1 + \frac{Z_2}{Z_1}}} \tag{5.2.4}$$

It can be seen from the above formulas that.

$$Z_{I_1} Z_{I_2} = W_x W_y = Z_1 Z_2 \tag{5.2.5}$$

The sections in fig. 5.2.1b and c are derived by Bode [22] in a somewhat different way and called h-derivations. Higher order h-derivations do not result in a ladder network consisting of only simple L, C, parallel resonator and series resonator as its arms. Hence, they are not the one of the TS that we want to consider.

Since the transformed sections in fig. 5.2.1b and c have higher order image impedances at both of the terminal pairs, it is necessary to cascade these sections with one or more sections on a matched basis, in order to obtain a simple image impedance at one of the terminal pairs of the final section.

Let us consider that fig. 5.2.1a is a half mid-shunt M-derived section, with the parameter m. Therefore, Z_1 corresponds to a parallel resonator and Z_2 to a capacitor. The transformed section, therefore, can be obtained from fig. 5.2.1 and given in fig. 5.2.2.

Since

$$Z_1 = j\,\frac{mkx}{1 - (1 - m^2)\,x^2}$$

$$Z_2 = -\,j\,\frac{k}{m\,x}$$

where

$$k = \sqrt{L_o / C_o}\ ; \qquad x = \omega / \omega_o\ , \qquad \omega_o = 1 \ / \ \sqrt{L_o C_o}$$

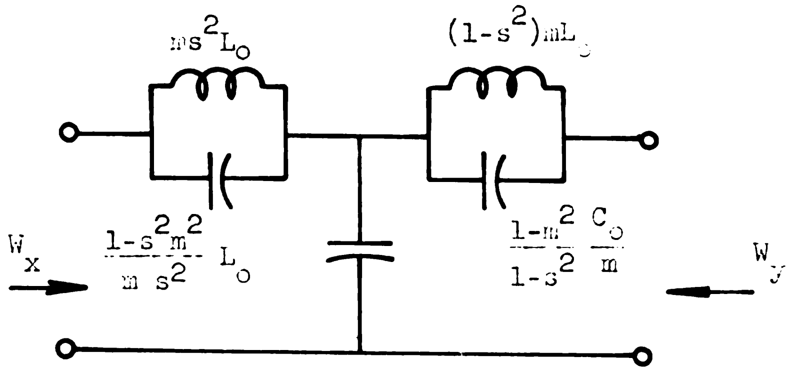


Fig. 5.2.2

Then from the formulas (5.2.3) and (5.2.4), we have

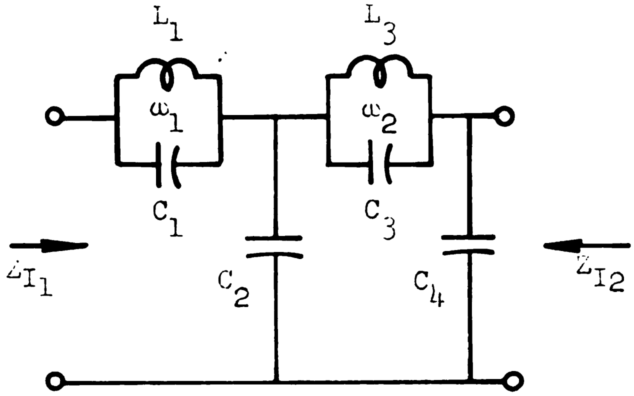
$$W_x = \frac{k\sqrt{1 - x^2}}{1 - (1 - s^2\ m^2)\ x^2} \qquad (Z_{02} \text{ - type})$$

$$W_y = \frac{k\left[1 - (1 - s^2\ m^2)\ x^2\right]}{\sqrt{1 - x^2}\left[1 - (1 - m^2)\ x^2\right]} \qquad (Z_{\infty 2} \text{ - type})$$

As we know from the above discussion, this section has the same transfer function as the original section. In order to reduce the order of W_x, we connect another section in cascade on a matched basis to this section. This final section is simply an M-derived half section with the parameter of sm. If we let

$$m = m_1$$
$$sm = m_2 \quad (m_2 < m_1)$$

then the final TS will be obtained as in fig. 5.2.3. Element values of this complete section are given on the figure in terms of the parameters m_1 and m_2. Image impedances are also indicated on the same figure.



$$L_1 = \frac{m_1^2 - m_2^2}{m_1} L_o$$

$$L_3 = \frac{m_2}{m_1} (m_1 + m_2) L_o$$

$$C_1 = \frac{m_1 (1 - m_1^2)}{m_1^2 - m_2^2} C_o$$

$$C_2 = m_1 C_o$$

$$C_3 = \frac{m_1 (1 - m_2^2)}{m_2 (m_1 + m_2)} C_o$$

$$C_4 = m_2 C_o$$

$$Z_{I_2} = \frac{k}{\sqrt{1 - x^2}}$$

$$Z_{I_1} = \frac{k \left[1 - (1 - m_2^2) x^2\right]}{\sqrt{1-x^2} \left[1 - (1-m_1^2) x^2\right]}$$

Fig. 5.2.3

Since this TS is obtained simply by cascading two matched sections, its transfer function is the sum of the transfer functions of the individual sections, each of which has an M-derived half section transfer function. This section may be obtained with a different procedure as given elsewhere [2].

5.3 LADDER TYPE TERMINATING SECTIONS, TS.

In the following discussion, we shall restrict ourselves to a special ladder TS with the configurations given in Fig. 5.3.1a and b. These TS are assumed to be obtained by cascading the simple mid-series or mid-shunt half-sections on a mismatched basis since increasing the order of one of the image impedances is impossible if matched sections are used [23].

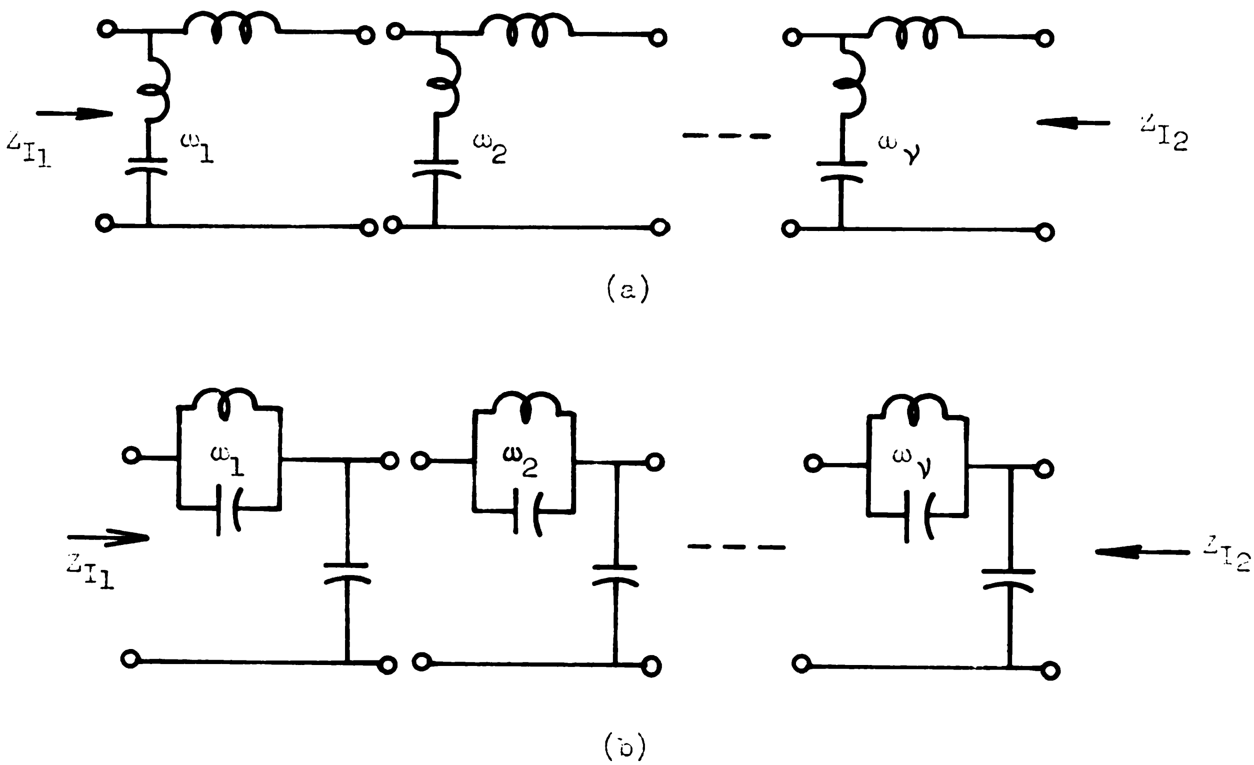


Fig. 5.3.1

For practical purposes (e.g., minimum number of coils in the filter) we deal here with only the mid-shunt type half-section. Since this section is the dual of the other mid-series TS, once we know the element values of one of them, those of the other can be calculated easily [3].

A cascaded connection of simple mid-shunt half sections with the same cut-off frequency, as in fig. 5.3.1b, generally does not lead to a low pass filter. This problem is already discussed in reference [23].

In the following discussions, the TS in fig. 5.3.1b as a whole is analyzed and some more remarks are made on the properties of this TS. Finally, we shall calculate the element values of this TS.

5.4 PROPERTIES OF MID-SHUNT TYPE TERMINATING SECTIONS

A mid-shunt type TS is repeated in Fig. 5.4.1, and the elements are labelled. If a voltage driver, E_1, is applied to the terminal pairs 1-1′ and a load resistance, R_L, is connected to the terminal pairs 2-2′, then the following properties are known [24], [2].

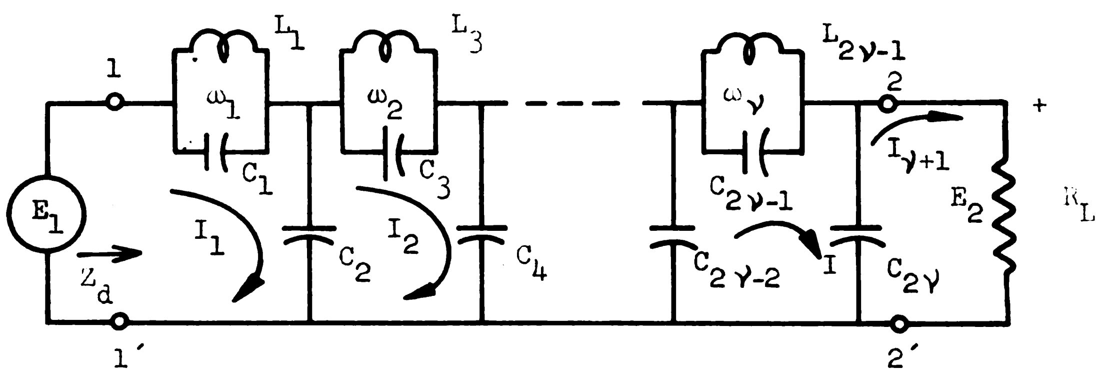


Fig. 5.4.1

Properties:

1- The output voltage, E_2, is zero at, and only at, the resonant frequencies, ω_i, of the parallel resonators (i.e., at the poles of transfer loss).*

2- At the resonance frequencies of the resonators, ω_i, the driving point impedance, Z_d, seen at the terminal pairs 1-1′, is independent of R_L, therefore

$$Z_{I1} = Z_{oc1} = Z_{sc1}$$

and at $\omega = \omega_1$, Z_{I_1}, Z_{oc_1} and Z_{sc_1} have a pole.

---

*This property is valid only under certain conditions. These conditions are discussed in detail later in this chapter.

3- Z_{oc1} and Z_{sc1} cannot be equal at other than the resonant frequencies, ω_i, of the resonators.

4- All critical frequencies of Z_{I_1} occur at some or all of the resonant frequencies, ω_i, of the resonators. Therefore, the number of the critical frequencies of Z_{I_1} is less than, or at most equal to, the number of resonators.

Although the property as stated in (4) is of value, further clarification is necessary. Such a clarification is given in the following section.

5.5 GENERAL DISCUSSION OF THE LOCATIONS OF THE CRITICAL FREQUENCIES OF THE IMAGE IMPEDANCES OF A TS

Consider the TS in Fig. 5.4.1. The mesh system of equations is written to include the voltage E_2 across the element R_L rather than R_L itself. The system of equations is given by,

$$\begin{bmatrix} Z_1+Z_2 & -Z_2 & 0 & \cdots & 0 & 0 \\ -Z_2 & Z_2+Z_3+Z_4 & -Z_4 & & 0 & 0 \\ 0 & -Z_4 & Z_4+Z_5+Z_6 & \ddots & 0 & 0 \\ \vdots & & & \ddots & \vdots & \vdots \\ \vdots & & & & -Z_{2\nu-2} & 0 \\ & & & & Z_{2\nu-2}+Z_{2\nu-1}+Z_{2\nu} & -Z_{2\nu} \\ 0 & 0 & 0 & \cdots & -Z_2 & Z_{2\nu} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_{\nu-1} \\ I_\nu \\ I_{\nu+1} \end{bmatrix} = \begin{bmatrix} E_1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ -E_2 \end{bmatrix} \tag{5.5.1}$$

where

$$\left.\begin{aligned} Z_{2i} &= \frac{1}{p\,C_{2i}} \\ Z_{2i-1} &= \frac{p\,L_{2i-1}}{1+\frac{p^2}{\omega_i^2}} \\ \omega_i^2 &= 1/\,L_{2i-1}\,C_{2i-1} \\ p &= j\omega \quad , \quad i = 1,2,\ldots,\nu\,. \end{aligned}\right\} \tag{5.5.2}$$

The image parameters for this TS can be written in terms of the determinants of the coefficient matrix or its sub-matrices in eq. 5.5.1 as follows [2].

$$Z_{I_1} = \sqrt{\frac{\Delta_{\nu+1,\nu+1}\ \Delta}{\Delta_{11}\ \Delta_{11,\nu+1,\nu+1}}} \qquad (5.5.3)$$

$$Z_{I_2} = \sqrt{\frac{\Delta_{11}\ \Delta}{\Delta_{\nu+1,\nu+1}\ \Delta_{11,\nu+1,\nu+1}}} \qquad (5.5.4)$$

$$H = \tanh P_I = \sqrt{\frac{\Delta\ \Delta_{11,\nu+1,\nu+1}}{\Delta_{11}\ \Delta_{\nu+1,\nu+1}}} \qquad (5.5.5)$$

From eqs. 5.5.3, 5.5.4 and 5.5.5, we can obtain a relation among the image parameters:

$$\frac{Z_{I_2}}{Z_{I_1}} \cdot (1 - H^2) = \left(\frac{\Delta_{1,\nu+1}}{\Delta_{\nu+1,\nu+1}}\right)^2 \qquad (5.5.6)$$

In the following discussion, eq. 5.5.6 plays an important role. For a symmetrical determinant, since

$$\Delta_{11}\ \Delta_{\nu+1,\nu+1} - \Delta_{1,\nu+1}^2 = \Delta\Delta_{11,\nu+1,\nu+1}$$

then, from eq. 5.5.5, we have

$$1 - H^2 = \frac{\Delta_{1,\nu+1}^2}{\Delta_{11}\Delta_{\nu+1,\nu+1}} \qquad . \qquad (5.5.7)$$

Let us calculate the determinants in eq. 5.5.7. From eq. 5.5.1, we have

$$\Delta_{1,\nu+1} = (-1)^{\nu}\ (-Z_2)(-Z_4)\ \ldots\ (Z_{2\nu}) = (-1)^{2\nu} \prod_{i=1}^{\nu} Z_{2i}$$

$$= \left(\frac{1}{p}\right)^{\nu} \prod_{i=1}^{\nu} \frac{1}{C_{2i}} \qquad (5.5.8)$$

On the other hand, Δ_{11} and $\Delta_{\nu+1,\nu+1}$ are the special types of determinants called "continuant." They can be reduced, by using some useful transformations into a simpler form called "simple continuants" [25]. By use of the properties of continuants, Δ_{11} and $\Delta_{\nu+1,\nu+1}$ can be investigated*, but for our purpose these properties are not needed.

The diagonal entries of Δ_{11} and $\Delta_{\nu+1,\nu+1}$ contain all of the other entries in these respective determinants. By definition, a determinant, Δ, of order n is the sum of all possible products of its entries taken n at a time with a proper sign. Therefore, in the expansion of Δ, the term which has the highest ordered denominator-polynomial in p^2 is the product of all the diagonal entries of this determinant, e.g., for $\Delta_{\nu+1,\nu+1}$, we have

$$\left(\frac{p\,L_1}{1+\frac{p^2}{\omega_1{}^2}} + \frac{1}{p\,C_2}\right)\left(\frac{1}{pC_2} + \frac{p\,L_3}{1+\frac{p^2}{\omega_2{}^2}} + \frac{1}{pC_4}\right)(\ldots)\ \ldots$$

$$\left(\frac{1}{p\,C_{2\nu-2}} + \frac{p\,L_{2\nu-1}}{1+\frac{p^2}{\omega_\nu{}^2}} + \frac{1}{pC_2}\right) = \frac{p_\nu(p^2)}{p^\nu\ C_{2\nu}\prod_{i=1}^{\nu-1} C_{2i}^2\ \prod_{i=1}^{\nu}\left(1+\frac{p^2}{\omega_i{}^2}\right)}$$

where, $p\,(p^2)$ indicates a polynomial function of p^2 of order ν. Therefore, Δ_{11} and $\Delta_{\nu+1,\nu+1}$ can be expressed in the following forms:

$$\Delta_{\nu+1,\nu+1} = \frac{Q_\nu(p^2)}{p^\nu\ C_{2\nu}\prod_{i=1}^{\nu-1} C_{2i}^2\ \prod_{i=1}^{\nu}\left(1+\frac{p^2}{\omega_i{}^2}\right)} \qquad (5.5.9)$$

$$\Delta_{11} = \frac{R_{\nu-1}(p^2)}{p^\nu\ C_2\prod_{i=2}^{\nu} C_{2i}^2\ \prod_{i=2}^{\nu}\left(1+\frac{p^2}{\omega_i{}^2}\right)} \qquad (5.5.10)$$

---

* "Principles of Circuit Synthesis," by E. S. Kuh and D. O. Pederson, McGraw-Hill Book Co., 1959. This book contains an appendix on the applications simple continuants to ladder networks.

where again $Q_\nu(p^2)$ and $R_{\nu-1}(p^2)$ are polynomial functions in p^2 of orders ν and $\nu - 1$ respectively. Cancellations may occur between the numerator and denominator polynomials in eqs. 3.4.8 and 3.4.9. This is considered later.

Substituting eqs. 5.5.8, 5.5.9 and 5.5.10 into eq. 5.5.7, we obtain the following relation:

$$1 - H^2 = \frac{(1 + \frac{p^2}{\omega_1{}^2}) \prod_{i=2}^{\nu} (1 + \frac{p^2}{\omega_i{}^2})^2}{C_2 \, C_{2\nu} \, Q_\nu(p^2) \, R_{\nu-1}(p^2)} \tag{5.5.11}$$

In 5.5.11, since the denominator is a simple polynomial in p^2, then $(1 - H^2)$ can have only zeros at $\omega = \omega_i$, $i=1,2, \ldots, \nu$. On the other hand, ω_i's are the poles of transfer loss function, A_I, and since [2],

$$A_I = \ln \left| \frac{1 + H}{1 - H} \right|$$

then, $(1 - H^2)$ must have a zero at, and only at, these frequencies, ω_i. From this short discussion we have the following conclusions:

(I) - Both $Q_\nu(p^2)$ and $R_{\nu-1}(p^2)$ cannot have a factor of the form $(1 + \frac{p^2}{\omega_1{}^2})$.

(II) - If $i=2,3, \ldots, \nu$, then either $Q_\nu(p^2)$ or $R_{\nu-1}(p^2)$ but not both have a simple factor of the form $(1 + \frac{p^2}{\omega_i{}^2})$. Of course, $Q_\nu(p^2)$ and $R_{\nu-1}(p^2)$ may have common factors other than $(1 + \frac{p^2}{\omega_i{}^2})$, $i=1,2, \ldots, \nu$.

On the other hand, H is purely imaginary in the pass band and H has its poles and zeros only in this region (pass band critical frequencies). Since the poles of H also appear in the denominator of $(1 - H^2)$ but doubled, from eq. 5.5.11 we have another conclusion:

(III) - The poles of K are some of the zeros of $Q_\nu(p^2)$ or $R_{\nu-1}(p^2)$ or both.

For a TS section, one of the requirements here being forced is that Z_{I_2} must have a simple form, i.e., Z_{I_2} is a constant-k image impedance. We assume that

$$Z_{I_2} = \frac{k}{\sqrt{1 + \frac{p^2}{\omega_o^2}}} \qquad (5.5.12)$$

On the other hand, since Z_{I_1} also has the factor $\sqrt{1 + \frac{p^2}{\omega_o^2}}$ in its numerator or denominator, the image impedance ratio, Z_{I_2}/Z_{I_1}, may or may not have the factor $(1 + \frac{p^2}{\omega_o^2})$ in its denominator.

Another possibility is to choose

$$Z_{I_2} = k\sqrt{1 + \frac{p^2}{\omega_o^2}}$$

This case is considered later.

Let us substitute eqs. 5.5.11, 5.5.8 and 5.5.9 into eq. 5.5.6. After making necessary simplifications, we obtain

$$\frac{Z_{I_2}}{Z_{I_1}} \cdot \frac{(1 + \frac{p^2}{\omega_1^2}) \prod_{i=2}^{\nu} (1 + \frac{p^2}{\omega_i^2})^2}{R_{\nu-1}(p^2)} = \frac{\prod_{i=1}^{\nu} (1 + \frac{p^2}{\omega_i^2})^2}{K \quad Q_\nu(p^2)} \qquad (5.5.13)$$

where K is a positive constant:

$$K = \frac{1}{C_2} \prod_{i=1}^{\nu-1} \frac{1}{C_{2i}}$$

Since eq. 5.5.13 must be satisfied for the considered TS, then we can find the restrictions on the Z_{I_1} impedance when Z_{I_2} is given by eq. 5.5.12.

From the conclusion (I), since $R_{\nu-1}(p^2)$ and $Q_\nu(p^2)$ do not contain the factor $(1 + \frac{p^2}{\omega_1^2})$, then eq. 5.4.13 shows that (Z_{I_2}/Z_{I_1}) must have this factor with power equal to unity. Since Z_{I_2} is chosen as in eq. 5.5.12, then this factor must appear in the denominator of Z_{I_1}, i.e.,

(a) Z_{I_1} has a pole at $\omega = \omega_1$.

Since for a FS, Z_{I_1} is required to be a higher ordered image impedance, we next investigate the maximum possible order for Z_{I_1}. From conclusion (III), if $R_{\gamma-1}(p^2)$ has a zero (that should be simple) at any ω_i, then $Q_\gamma(p^2)$ can not have a zero at this frequency, or, if $Q_\gamma(p^2)$ has a zero at any ω_i, $R_{\gamma-1}(p^2)$ can not have a zero at this frequency. Therefore, assume that none of the polynomials, $R_{\gamma-1}(p^2)$ and $Q_\gamma(p^2)$ have a zero at $\omega = \omega_i$'s. Then, looking at eq. 5.5.13, we can conclude that (Z_{I_2}/Z_{I_1}) can, in general, be equal to a rational function with the same degree, say n, of numerator and denominator polynomials, i.e., either

$$\frac{Z_{I_2}}{Z_{I_1}} = \frac{(1 + \frac{p^2}{\omega_1^2})(1 + \frac{p^2}{\lambda_3^2}) \ \ldots \ (1 + \frac{p^2}{\lambda_{2n-1}^2})}{(1 + \frac{p^2}{\lambda_2^2})(1 + \frac{p^2}{\lambda_4^2}) \ \ldots \ (1 + \frac{p^2}{\lambda_{2n}^2})} \tag{5.5.14}$$

or,

$$\frac{Z_{I_2}}{Z_{I_1}} = \frac{(1 + \frac{p^2}{\omega_1^2})(1 + \frac{p^2}{\lambda_1^2}) \ \ldots \ (1 + \frac{p^2}{\lambda_{2n-3}^2})}{(1 + \frac{p^2}{\omega_0^2})(1 + \frac{p^2}{\lambda_2^2}) \ \ldots \ (1 + \frac{p^2}{\lambda_{2n-2}^2})} \tag{5.5.15}$$

where, since Z_{I_2} is a simple image impedance, the λ_i^2's, are the zeros and poles of Z_{I_1} which are all real and positive numbers. The factors $(1 + \frac{p^2}{\lambda_i^2})$ do not appear either on the left hand side of eq. 5.5.13 or on its right hand side; therefore, necessarily, these factors in eqs. 5.5.14 and 5.5.15 cancel such that the degree of numerator and denominator remain equal to unity, since (Z_{I_2}/Z_{I_1}) must contain the factor $(1 + \frac{p^2}{\omega_1^2})$. Hence, we have either,

$$\frac{Z_{I_2}}{Z_{I_1}} = \frac{1 + \frac{p^2}{\omega_1^2}}{1 + \frac{p^2}{\lambda_2^2}} \tag{5.5.16}$$

or,

$$\frac{Z_{I_2}}{Z_{I_1}} = \frac{1 + \frac{p^2}{\omega_1^2}}{1 + \frac{p^2}{\omega_o^2}} \tag{5.5.17}$$

For these cases, $(1 + \frac{p^2}{\lambda_2^2})$ and $(1 + \frac{p^2}{\omega_o^2})$ must be contained in $Q_\nu(p^2)$. Therefore, considering eqs. 5.5.12, we have for Z_{I_1} either

$$Z_{I_1} = \frac{k\,(1 + \frac{p^2}{\lambda_2^2})}{\sqrt{1 + \frac{p^2}{\omega_o^2}}\,(1 + \frac{p^2}{\omega_1^2})} \tag{5.5.18}$$

or

$$Z_{I_1} = \frac{k\sqrt{1 + \frac{p^2}{\omega_o^2}}}{1 + \frac{p^2}{\omega_1^2}} \tag{5.5.19}$$

Therefore, if neither $Q_\nu(p^2)$ nor $R_{\nu-1}(p^2)$ has a zero at $\omega = \omega_i$ ($i = 2, 3, \ldots, \nu$) then because Z_{I_2} is as in eq. 5.5.12, the order of Z_{I_1} cannot be increased beyond that indicated by eqs. 5.5.18 and 5.5.19.

Since the roots of $1 - H^2 = 0$ (ω_i's) and ω_o are known, from a theorem which is stated by Bode [22], the H function can be determined uniquely. Therefore, the two forms of Z_{I_1} in eqs. 5.5.18 and 5.5.19 cannot exist simultaneously. If H contains the factor $\sqrt{1 + \frac{p^2}{\omega_o^2}}$ in its numerator, then Z_{I_1} as in eq. 5.5.18 is possible. In the other case where the factor $\sqrt{1 + \frac{p^2}{\omega_o^2}}$ does appear in the denominator of H function, then Z_{I_1} as in eq. 5.5.19 is possible.

From the foregoing discussion, finally we can see that in order to increase the order of Z_{I_1}, keeping Z_{I_2} as in eq. 5.5.12, some of the simple zeros of either $R_{\nu-1}(p^2)$ or $Q_\gamma(p^2)$ (but not both - see conclusion II) must coincide with some of the poles of the transfer loss function, i.e., with the ω_i's. Then we have two possibilities:

A) Suppose: $Q_\gamma(p^2)$ has simple zeros at some ω_i's.

In this case, one of the factors, $(1 + \frac{p^2}{\omega_i^2})$ on the numerator of the right hand side in eq. 5.5.13, does not appear; but this factor still appears in the numerator of $1 - h^2$ as double zeros. Since $R_{\gamma-1}(p^2)$ can not contain the factor $(1 + \frac{p^2}{\omega_i^2})$, this factor must be in the denominator of the ratio (Z_{I_2}/Z_{I_1}). In other words, it must present a zero of Z_{I_1}. Therefore:

(b) Conclusion: If $Q_\gamma(p^2)$ has simple zeros at some of the ω_i's, then these zeros are also the zeros of the Z_{I_1} image impedance.

B) Suppose: $R_{\gamma-1}(p^2)$ has a simple zero at any $\omega = \omega_i$.

In this case, the factor $(1 + \frac{p^2}{\omega_i^2})$ does not appear in the numerator of $1 - h^2$. But this factor still appears on the right hand side of eq. 5.5.13 as a double factor. Hence, (Z_{I_2}/Z_{I_1}) must contain this factor, i.e., $(1 + \frac{p^2}{\omega_i^2})$ must be a zero of this image impedance ratio or we can state the following:

(c) Conclusion: If $R_{\gamma-1}(p^2)$ has simple zeros at some of the ω_i's, then these zeros are the poles of Z_{I_1} image impedance.

From the above results, (b) and (c), we can see that to be able to increase the order of Z_{I_1}, i.e., to be able to increase the number of zeros and poles for Z_{I_1}, the polynomials $Q_\gamma(p^2)$ and $R_{\gamma-1}(p^2)$ must have as many zeros as possible at the ω_i's. From a property of the image impedance [2], the degrees of numerator and denominator polynomials in p^2, excluding the factor $\sqrt{1+\frac{p^2}{\omega_o^2}}$, can differ only by unity. Therefore, only the $Q_\nu(p^2)$ or $R_{\nu-1}(p^2)$ cannot have all the ω_i's for its zeros. Therefore, maximum obtainable order for Z_{I_1} occurs if all zeros and poles of Z_{I_1} (block band critical frequencies) occur at the poles of transfer loss. To prove this, the above argument can be used, i.e., assuming that Z_{I_1} has more zeros and poles. In other words, the number of critical frequencies of Z_{I_1} is larger than ν . Then, in (Z_{I_2}/Z_{I_1}), we will have some factors as $(1+\frac{p^2}{\lambda_i^2})$. Since these factors do not appear either on the right hand side of eq. 5.5.13 or in the numerator or denominator of $1-H^2$, they must cancel. Hence, the maximum obtainable order for the Z_{I_1} function is that one which will have all ω_i's as its critical frequencies. The conclusion of this discussion follows:

> Let the Z_{I_2} image impedance have a simple form (constant-k type) as in eq. 5.5.12. In order to increase the order of Z_{I_1}, it is necessary that the critical frequencies of Z_{I_1} be located at the poles of transfer loss. Otherwise, the order of Z_{I_1} cannot be increased.

From the above discussion, because of Z_{I_2} as in eq. 5.5.12, the simplest form of Z_{I_1} is either as in eq. 5.5.18 or eq. 5.5.19, depending upon whether the factor $\sqrt{1+\frac{p^2}{\omega_o^2}}$ is in the numerator or denominator of the H - function respectively. On the other hand, the form of the highest ordered Z_{I_1}, not only depends on the location of the factor $\sqrt{1+\frac{p^2}{\omega_o^2}}$ in H, but also on the oddness or evenness of the number ν. So far, no restrictions on the ω_i's are imposed. However, there is a restriction on these ω_i's as the following property shows: From a property of the image impedance [2], we know that its poles and zeros must alternate. Hence, since some of the

ω_i's are the zeros of Z_{I_1} and some other ω_i's are the poles of Z_{I_1} then there should be some orders between these ω_i's. If we now impose that (but it is not necessary)

$$\omega_1 > \omega_2 > \ldots > \omega_\nu$$

then two possibilities for Z_{I_1} exist*:

If ν is odd:

$$Z_{I_1} = \frac{k\sqrt{1 + \frac{p^2}{\omega_o^2}}\ (1 + \frac{p^2}{\omega_2^2}) \ldots (1 + \frac{p^2}{\omega_{\nu-1}^2})}{(1 + \frac{p^2}{\omega_1^2}) \ \ldots \ (1 + \frac{p^2}{\omega_\nu^2})} \tag{5.5.20}$$

If ν is even:

$$Z_{I_1} = \frac{k\ (1 + \frac{p^2}{\omega_2^2}) \ \ldots \ (1 + \frac{p^2}{\omega_\nu^2})}{\sqrt{1 + \frac{p^2}{\omega_o^2}}\ (1 + \frac{p^2}{\omega_1^2}) \ldots (1 + \frac{p^2}{\omega_{\nu-1}^2})} \tag{5.5.21}$$

These two forms of image impedances are used in the following discussions.

Now, let us consider other simplest form for Z_{I_2}, i.e.,

$$Z_{I_2} = k\sqrt{1 + \frac{p^2}{\omega_o^2}} \tag{5.5.22}$$

If the foregoing discussion is repeated for this case, it can be seen from eq. 5.5.13 that Z_{I_1} cannot have the factor $\sqrt{1 + \frac{p^2}{\omega_o^2}}$ in its denominator. Therefore, Z_{I_1} has the form of eq. 5.5.20. In addition to this, the following conditions must hold:

---

* Özker [24] has studied different types of low pass sections including the TS which we are considering here, with the assumption $\omega_1 > \omega_2 > \ldots > \omega_\nu$ and indicated that maximum ordered image impedance can be obtained if all the critical frequencies of this image impedance occur at ω_i's. Darlington [3] has also mentioned these properties for mid-series types of ladder filters.

(1) The H function of the considered TS has the factor $\sqrt{1 - \frac{\omega^2}{\omega_c^2}}$ in its denominator.

(2) For the highest ordered Z_{I_1}, ν is odd.

(3) The degrees of the numerator and denominator polynomials of the image impedance ratio, Z_{I_2}/Z_{I_1}, must be the same.

It is seen from eqs. 5.5.20 and 5.5.22 that the ratio of Z_{I_2}/Z_{I_1} does not satisfy the condition (3). Therefore, for the TS as in fig. 5.4.1, Z_{I_2} cannot be taken as in eq. 5.5.22.

The image impedances in eqs. 5.5.20 and 5.5.21 give the following sequence:

$$Z_{02}\ ;\ Z_{04}\ ,\ \cdots\ ;\ Z_{0,\nu+1} \qquad (\nu\ \text{odd})$$
$$Z_{\infty 3}\ ,\ Z_{\infty 5}\ ,\ \cdots\ ,\ Z_{\infty,\nu+1} \qquad (\nu\ \text{even})$$

Another sequence, i.e.,

$$Z_{\infty 2}\ ,\ Z_{\infty 4}\ ,\ \cdots\ ,\ Z_{\infty,\nu+1} \qquad (\nu\ \text{odd})$$
$$Z_{03}\ ;\ Z_{05}\ ;\ \cdots\ ,\ Z_{0,\nu+1} \qquad (\nu\ \text{even})$$

can be obtained from the dual ladder (mid-series type) to the considered TS.

If Z_{I_2} is not restricted (constant-k) to a simple form, then Z_{I_1} and Z_{I_2} will have critical frequencies that occur other than the poles of transfer loss. But from eq. 5.5.13, it is clear that these critical frequencies Z_{I_1} and Z_{I_2} must be the same. In a recent article [26], this problem is considered such that a section as in fig. 5.4.1 (not a TS any more) must have Z_{01}, Z_{02}, Z_{03}, ... (or $Z_{\infty 1}$, $Z_{\infty 2}$, $Z_{\infty 3}$, ...) type image impedances at both its terminal pairs, but the poles of transfer loss function must not coincide with the critical frequencies (reflection poles) of the image impedances. Belevitch gives some low pass filter sections for only the Z_{02} or $Z_{\infty 2}$ types of mid-shunt ladder networks and their element values, but also indicates that it is not known whether such ladder sections exist if the order of image impedances is higher than Z_{02} or $Z_{\infty 2}$. This problem is not discussed in this thesis, since we are concerned with the TS. But perhaps a discussion similar to the one above could be used to clarify the many points in this problem. With a given set of image impedance and attenuation poles, it is possible to find symmetrical LC lattice networks but their ladder equivalents may not exist. As Belevitch indicated in his article [26], the existence of a ladder equivalent to this lattice can be checked by the Fujisawa [27] criterion. This criterion is extended by Meinguet and Belevitch [28]. It is worthwhile to note that for the existence of ladder mid-series type low pass filters (i.e., for positive element values) Darlington gave two sufficient conditions [3]. These two conditions are actually the same conditions that we wanted to impose for our TS. Therefore, the considered TS will exist (i.e., all elements have positive value) if Z_{I_2} is a constant-k type image impedance or Z_{I_1} has the highest obtainable order. In this connection, a general existence theorem for ladder mid-series low pass networks is given by Fujisawa [27] and these conditions are extended for band-pass ladder filters by Watanabe [29].

Our problem is now almost clarified. All we have to do now is to determine the element values of the TS as in Fig. 3.3.1. This problem is considered next.

3.6 ELEMENT VALUES OF TS

In this section we consider the TS as in Fig. 3.3.1 with the following restrictions.

(I)

$$Z_{I_2} = \frac{k}{\sqrt{1 + \frac{p^2}{\omega_0^2}}} \qquad (5.6.1)$$

(II)

$$\omega_1 > \omega_2 > \ldots > \omega_\nu > \omega_0 \qquad (5.6.2)$$

(III)

$$Z_{I_1} = \frac{k\sqrt{1 + \frac{p^2}{\omega_0^2}}\ (1 + \frac{p^2}{\omega_2^2}) \ldots (1 + \frac{p^2}{\omega_{\nu-1}^2})}{(1 + \frac{p^2}{\omega_1^2}) \ \ldots \ (1 + \frac{p^2}{\omega_\nu^2})} \qquad (5.6.3)$$

If ν is even

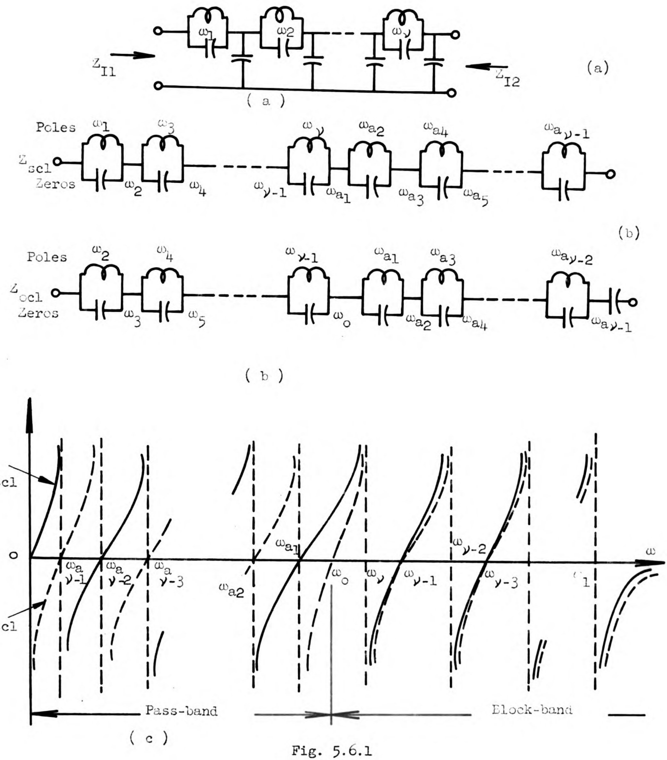
$$Z_{I_1} = \frac{k \ (1 + \frac{p^2}{\omega_2^2}) \ldots (1 + \frac{p^2}{\omega_\nu^2})}{\sqrt{1 + \frac{p^2}{\omega_0^2}}\ (1 + \frac{p^2}{\omega_1^2}) \ldots (1 + \frac{p^2}{\omega_{\nu-1}^2})} \qquad (5.6.4)$$

With the above restrictions, the equivalent Foster forms of the short circuit and open circuit impedances of this TS are given as in fig. 5.6.1. Where ν is taken as an odd number. If ν is even, similar two terminal LC Foster forms for Z_{oc_1} and Z_{sc_1} and the reactance pattern can be given.

The impedances Z_{oc_1} and Z_{sc_1} can be expressed in terms of Z_{I_1} and H as [2]:

$$\left.\begin{aligned} Z_{oc_1} &= \frac{1}{H} \ Z_{I_1} \\ Z_{sc_1} &= H \ Z_{I_1} \end{aligned}\right\} \qquad (5.6.5)$$

Since the ω_i's and ω_0 are known, H can be uniquely determined, and since the form Z_{I_1} is also given, Z_{oc_1} or Z_{sc_1} can be found from eq. 5.6.5. In fig. 5.6.1b, the only unknowns are the ω_{a_i}'s (ν -1 in number). These frequencies may be found as follows:



Z_{I1}
Z_{I2}
ω_1
ω_2
ω_ν
(a)
(a)
Poles
Zeros
Z_{sc1}
ω_1
ω_3
ω_2
ω_4
ω_ν
$\omega_{\nu-1}$
ω_{a2}
ω_{a4}
ω_{a1}
ω_{a3}
ω_{a5}
$\omega_{a\nu-1}$
(b)
Poles
Zeros
Z_{oc1}
ω_2
ω_4
ω_3
ω_5
$\omega_{\nu-1}$
ω_o
ω_{a1}
ω_{a3}
ω_{a2}
ω_{a4}
$\omega_{a\nu-2}$
$\omega_{a\nu-1}$
(b)
Z_{sc1}
Z_{oc1}
o
ω
$\omega_{a\nu-1}$
$\omega_{a\nu-2}$
$\omega_{a\nu-3}$
ω_{a2}
ω_{a1}
ω_o
ω_ν
$\omega_{\nu-1}$
$\omega_{\nu-2}$
$\omega_{\nu-3}$
ω_1
Pass-band
Block-band
(c)

Fig. 5.6.1

Since $H^2 = Z_{sc_1} / Z_{oc_1}$, then from Fig. 5.6.1b, we have

$$H^2 = h \frac{p^2 \left(1 + \frac{p^2}{\omega_{a_1}^2}\right)^2 \left(1 + \frac{p^2}{\omega_{a_3}^2}\right)^2 \cdots \left(1 + \frac{p^2}{\omega_{a_{\nu-1}}^2}\right)^2}{\left(1 + \frac{p^2}{\omega_o^2}\right) \left(1 + \frac{p^2}{\omega_{a_2}^2}\right)^2 \cdots \left(1 + \frac{p^2}{\omega_{a_{\nu-2}}^2}\right)^2} \tag{5.6.6}$$

Since H^2 must have a unit value for $\omega = \omega_i$, then from eq. 5.6.6 we have the following ν relation

$$\left(1 + \frac{p^2}{\omega_o^2}\right)\left(1 + \frac{p_i^2}{\omega_{a_2}^2}\right)^2 \cdots \left(1 + \frac{p_i^2}{\omega_{a_{\nu-2}}^2}\right)^2 - hp^2 \left(1 + \frac{p_i^2}{\omega_{a_1}^2}\right)^2 \cdots \left(1 + \frac{p_i^2}{\omega_{a_{\nu-1}}^2}\right) \quad (i = 1, 2, \ldots, \nu) \tag{5.6.7}$$

where, as mentioned before, $p_i = j\,\omega_i$.

Actually, to be able to determine the H - function, the set of relations 5.6.7 must be solved and ω_{a_i}'s and h must be found. If ν is a small number, e.g., $\nu = 2$ or 3, an attempt can be made to solve this system, but in general this is difficult since the system is non-linear. Bode [22], has proved that H is unique. Therefore, there should be one solution to this system of relations 5.6.7.

Now, suppose that we have found the H function, therefore from eq. 5.6.5 we can determine Z_{oc_1} or Z_{sc_1} since Z_{I_1} is also known. One can consider Darlington's method [3] of determining the element values of the mid-shunt ladder TS by use of either Z_{oc_1} or Z_{sc_1}. By this method, we can really find all the element values if we know the H function. But we only know that it has a unit value at $\omega = \omega_1$. Therefore, L_1 can be calculated without any difficulty. But to find the values of the other elements, we need the derivatives of H function since Darlington's method requires a knowledge of the derivatives of Z_{oc_1} or Z_{sc_1}. Unfortunately, as mentioned above, although the H function is unique, it is difficult to determine it from the zeros of $1 - H^2$ and ω_o. Obviously, if H cannot be determined, neither can its derivatives.

Before starting to calculate the element values, we introduce a new parameter, ζ, as follows [3]: let

$$p^2 = -\omega^2 = -\frac{1}{\zeta} \tag{5.6.8}$$

and then

$$\omega_i^2 = \frac{1}{\zeta_i} \tag{5.6.9}$$

$$1 + \frac{p^2}{\omega_i^2} = -\frac{1}{\zeta}(\zeta_i - \zeta) \tag{5.6.10}$$

$$1 + \frac{p^2}{\omega_o^2} = -\frac{1 - \omega_o^2 \zeta}{\omega_o^2 \zeta} \tag{5.6.11}$$

Equations 5.6.3 and 5.6.4, then, can be rewritten in terms of ζ as

$$Z_{I_1}(\zeta) = \frac{k \zeta \sqrt{1 - \omega_o^2 \zeta}\ (\zeta_2 - \zeta) \ldots (\zeta_{\nu-1} - \zeta)}{\omega_o \sqrt{-\zeta}\ (\zeta_1 - \zeta) \ldots\ldots (\zeta_\nu - \zeta)} \tag{5.6.12}$$

and

$$Z_{I_1}(\zeta) = \frac{\omega_o k \sqrt{-\zeta}\ (\zeta_2 - \zeta) \ldots (\zeta_\nu - \zeta)}{\sqrt{1 - \omega_o^2 \zeta}\ (\zeta_1 - \zeta) \ldots (\zeta_{\nu-1} - \zeta)} \tag{5.6.13}$$

A new set of parameters, m_i, are introduced in the following way: At $\omega = \omega_i$, the factor $\sqrt{1 - \omega_o^2}$ appears in the expression of $Z_{I_1}(\zeta)$ is designated by

$$\sqrt{1 - \omega_o^2 \zeta_i} = \sqrt{1 - \frac{\omega_o^2}{\omega_i^2}} = \sqrt{m_i^2} = m_i \tag{5.6.14}$$

Therefore, the Z_{oc_1} and Z_{sc_1} functions of the TS in fig. 5.6.1a can be written in terms of the ζ - variable as follows:

$$Z_{oc_1}(\zeta) = \sqrt{-\zeta}\left[\frac{L_1}{\zeta_1 - \zeta} + \cfrac{1}{C_2 + \cfrac{1}{\cfrac{L_3}{\zeta_2 - \zeta} + \cfrac{1}{\cdots\cdots\cdots \cfrac{L_{2\nu-1}}{\zeta_\nu - \zeta} + \cfrac{1}{C_{2\nu}}}}}\right] \qquad (5.6.15)$$

Z_{sc_1} has the same form in which only the last term $(1 / C_{2\nu})$ does not appear. For Z_{I_2}, we have

$$Z_{I_2} = \frac{\omega_o k \sqrt{-\zeta}}{\sqrt{1 - \omega_o^2}} \qquad (5.6.16)$$

Since at $\omega = \omega_i$ (or $\zeta = \zeta_i$), $Z_{I_2} = Z_{oc_2} = Z_{sc_2}$, then eq. 5.6.16 gives

$$Z_{I_2}(\zeta_i) = \frac{\omega_o k \sqrt{-\zeta_i}}{\sqrt{1 - \omega_o^2 \zeta_i}} = \frac{\omega_o k \sqrt{-\zeta_i}}{m_i} \qquad (5.6.17)$$

On the other hand, at $\zeta = \zeta_i$, from fig. 5.6.1a, we have

$$Z_{oc_2} = Z_{sc_2} = \frac{1}{p\, C_{2\nu}} = \frac{\sqrt{-\zeta_\nu}}{C_{2\nu}} \qquad (5.6.18)$$

Therefore, equating equations 5.6.17 and 5.6.18 yields

$$C_{2\nu} = \frac{m_\nu}{\omega_o k}$$

or, finally, since $\omega_o^2 = 1 / L_o C_o$ and $k^2 = L_o / C_o$, we have the general expression for $C_{2\nu}$, whether ν is odd or even, that

$$C_{2\nu} = m_\nu C_o \qquad (5.6.19)$$

In order to calculate L_1, consider the relation

$$Z_{I_1}^2 = Z_{oc_1} \cdot Z_{sc_1}$$

Let ν be an odd number, then from eqs. 3.5.12 and 3.5.15, we have

$$\frac{k^2 \zeta^2 (1 - \omega_o^2 \zeta)(\zeta_2 - \zeta)^2 \cdots (\zeta_{\nu-1} - \zeta)^2}{- \omega_o^2 \zeta (\zeta_1 - \zeta)^2 \cdots (\zeta_\nu - \zeta)^2} =$$

$$-\zeta \left[\frac{L_1}{\zeta_1 - \zeta} + \cfrac{}{\ddots \; \cfrac{L_{2\nu-1}}{\zeta_\nu - \zeta} + \cfrac{1}{C_{2\nu}}} \right] \left[\frac{L_1}{\zeta_1 - \zeta} + \cfrac{}{\ddots \; -\cfrac{1}{C_{2\nu}}} \right]$$

Multiplying both sides of this equation by the factor $(\zeta_1 - \zeta)^2$ and then taking the limit as $\zeta \longrightarrow \zeta_1$, we have

$$\frac{k^2 (1 - \omega_o^2 \zeta_1)(\zeta_2 - \zeta_1)^2 \cdots (\zeta_{\nu-1} - \zeta)^2}{\omega_o^2 (\zeta_3 - \zeta_1)^2 \cdots\cdots (\zeta_\nu - \zeta_1)^2} = L_1^2 \qquad (5.6.20)$$

But, from eq. 5.6.14, since

$$\zeta_i - \zeta_j = \frac{1}{\omega_o^2} \left[\frac{\omega_o^2}{\omega_i^2} - \frac{\omega_o^2}{\omega_j^2} \right] = \frac{1}{\omega_o^2} (m_j^2 - m_i^2) \qquad (5.6.21)$$

and

$$1 - \omega_o^2 \zeta_i = m_i^2$$

eq. 5.6.20 can finally be put into the form

$$L_1 = \frac{m_1 (m_1^2 - m_2^2)(m_1^2 - m_4^2) \cdots (m_1^2 - m_{\nu-1}^2)}{(m_1^2 - m_3^2)(m_1^2 - m_5^2) \cdots (m_1^2 - m_\nu^2)} \qquad (5.6.22)$$

(For ν odd)

In a similar way, if ν is even, we can find

$$L_1 = \frac{(m_1^2 - m_2^2) \cdots (m_1^2 - m_\nu^2)}{m_1 (m_1^2 - m_3^2) \cdots\cdots (m_1^2 - m_{\nu-1}^2)} L_o \qquad (5.6.23)$$

Note that from the requirement (II), i.e.,

$$\omega_1 > \omega_2 > \cdots\cdots > \omega_\nu$$

and the definition of m_i, we have

$$m_1 > m_2 > \ldots\ldots\ldots > m_\nu \qquad (5.6.24)$$

For the following special cases, from eqs. 5.6.22 and 5.6.23, we can find that

If $\nu = 1$ (taking $m_2 = m_3 = \ldots = m_\nu = 0$)

$$L_1 = m_1 L_o \qquad (5.6.25)$$

If $\nu = 2$ (taking $m_3 = m_4 = \ldots = m_\nu = 0$)

$$L_1 = \frac{m_1^2 - m_2^2}{m_1} L_o \qquad (5.6.26)$$

If $\nu = 3$ (taking $m_4 = m_5 = \ldots = m_\nu = 0$)

$$L_1 = \frac{m_1 (m_1^2 - m_2^2)}{(m_1^2 - m_3^2)} \qquad (5.6.27)$$

At $\zeta = \zeta_2$, in either case, i.e., ν odd or even, $Z_{I_1} = Z_{oc_1} - Z_{sc_1} = 0$. Therefore, from eq. 5.6.15, we have

$$\frac{L_1}{\zeta_1 - \zeta_2} + \frac{1}{C_2} = 0 \qquad (5.6.28)$$

Substituting the values of L_1 from eqs. 5.6.22 and 5.6.23 into eq. 5.6.28, and considering eq. 5.6.21, we have

$$C_2 = \frac{(m_1^2 - m_3^2)\,(m_1^2 - m_5^2)\,\ldots\ldots\,(m_1^2 - m_\nu^2)}{m_1\,(m_1^2 - m_4^2)\,\ldots\ldots\,(m_1^2 - m_{\nu-1}^2)} \quad (\text{for } \nu \text{ odd}) \qquad (5.6.29)$$

and

$$C_2 = \frac{m_1\,(m_1^2 - m_3^2)\,\ldots\ldots\ldots\,(m_1^2 - m_{\nu-1}^2)}{(m_1^2 - m_4^2)\,\ldots\ldots\ldots\,(m_1^2 - m_\nu^2)} \quad (\text{for } \nu \text{ even}) \qquad (5.6.30)$$

For the following special cases, from eqs. 5.6.29 and 5.6.30, we find

If $\nu = 1$ (taking $m_2 = m_3 = m_\nu = 0$)

$$C_2 = m_1 C_o \qquad (5.6.31)$$

If $\nu = 2$ (taking $m_3 = m_4 = \ldots\ldots = m_\nu = 0$)

$$C_2 = m_1 C_o \tag{5.6.32}$$

If $\nu = 3$ (taking $m_4 = m_5 = \ldots = m_\nu = 0$)

$$C_2 = \frac{m_1^2 - m_3^2}{m_1} C_o \tag{5.6.33}$$

etc...

If $\nu = 1$, the element values are determined by the foregoing, since L_1 and C_2 are given by eqs. 5.6.26 and 5.6.31 which corresponds to an M-derived simple half section.

If $\nu = 2$, we have only to calculate L_3. This can be done by considering the expressions of Z_{I_1}, Z_{oc_1}, Z_{sc_1} and the relation, $Z_{I_1} = Z_{oc_1} \cdot Z_{sc_1}$. But the following alternate argument makes this calculation simple. From the eqs. 5.6.3 and 5.6.4, when p approaches zero, we find that

$$Z_{I_1}^2 \;(0) = k^2 = \frac{L_o}{C_o} \tag{5.6.34}$$

On the other hand, when p approaches zero, from fig. 3.5.1a we can see that Z_{sc_1} is inductive and approaches the value of $(L_1 + L_3 + \ldots + L_{2\nu-1})$ p, and Z_{oc_1} is capacitive and approaches to the value of $1 / (C_2 + C_4 + \ldots + C_{2\nu})$ p. Therefore, the product $Z_{oc_1} \cdot Z_{sc_1}$ approaches to a constant value and by use of eq. 5.6.34 we finally have

$$\frac{L_1 + L_3 + \ldots\ldots + L_{2\nu-1}}{C_2 + C_4 + \ldots\ldots + C_{2\nu}} = \frac{L_o}{C_o} \tag{5.6.35}$$

Therefore, if $\nu = 2$, L_3 can be found from eq. 5.6.35 as

$$L_3 = \frac{m_2}{m_1} (m_1 + m_2) L_o \tag{5.6.36}$$

This section, i.e., $\nu = 2$, is the same as in fig. 5.2.3.

In order to calculate C_1 for a TS, in both cases, i.e., ν is odd or even, we can use the relation

$$\omega_1^2 = 1 / L_1 C_1$$

Therefore, considering the value of L_1, we have

$$C_1 = \frac{(1 - m_1^2)(m_1^2 - m_3^2)(m_1^2 - m_5^2) \ldots (m_1^2 - m_\nu^2)}{m_1 (m_1^2 - m_2^2)(m_1^2 - m_4^2) \ldots (m_1^2 - m_{\nu-1}^2)} C_o \qquad (5.6.37)$$

(for ν odd)

$$C_1 = \frac{m_1 (1 - m_1^2)(m_1^2 - m_3^2) \ldots\ldots (m_1^2 - m_{\nu-1}^2)}{(m_1^2 - m_2^2)(m_1^2 - m_4^2) \ldots\ldots (m_1^2 - m_\nu^2)} C_o \qquad (5.6.38)$$

(for ν even)

So far we were able to calculate only four elements of the general TS. In order to calculate the rest of the element values of the TS, unfortunately, we can not continue this process, i.e., considering only the relation

$$Z_{I_1}^2 = Z_{oc1} \; Z_{sc1}$$

This method actually permits calculation of Z_{oc1} or Z_{sc1} from the Z_{I_1} and H function, and all the element values can be calculate from Z_{oc1} [3]. Therefore, at this point, we have to change our techniques. A formal calculation technique of the element values for a TS is described by applying to a TS with $\nu = 2$, the method considered by Reed [2]. This technique gives a set of non-linear equations relating the element values of the TS, the latter of which are to be calculated. Even for $\nu = 2$, too much algebra is involved; therefore, for $\nu = 3$ it is more difficult since systems of non-linear equations containing 6 unknowns are formed. But this technique can still be used little higher ν's since in the previous discussion we have already calculated some of the element values. This will reduce the number of unknowns in the system of non-linear equations. Element values of TS corresponding $\nu = 1$, 2 and 3 are given on Table I. For practical purposes, this

Table can be employed. From Table I we notice that the following relations

$$\sum_{i=1}^{\nu} L_{2i-1} = L_o \sum_{i=1}^{\nu} m_i$$
$$\sum_{i=1}^{\nu} C_{2i} = C_o \sum_{i=1}^{\nu} m_i \qquad (5.6.39)$$

hold for $\nu = 1, 2,$ and 3. If we have shown the validity of this relation, the element values of TS for $\nu = 3$ could be calculated easily since the system of non-linear equations would only contain two unknowns. No easy method has been found that eqs. 5.6.39 hold for $\nu > 3$.

In practice, the use of a terminating section with $\nu = 1$, mostly gives satisfactory results. Hence, we end our discussion on filter terminating half sections here since more complicated sections have no general use in practice.

Note that the TS given in Table I have image impedances at the terminal pair 1 - 1′ of the form

$$Z_{02} , Z_{\infty 3} , Z_{04} , Z_{\infty 5} , \ldots \text{ etc.} \qquad (5.6.40)$$

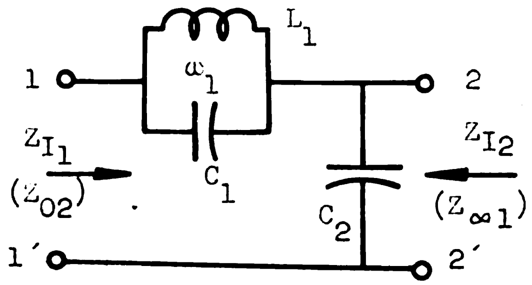
The other forms, i.e.,

$$Z_{\infty 2} , Z_{03} , Z_{\infty 4} , Z_{05} , \ldots \text{ etc.} \qquad (5.6.41)$$

as mentioned before, can be obtained if we consider the dual ladder to the TS considered here [3]. But dual the TS contain more inductances and on the other hand, as it is seen in Chapter II, the insertion function will be the same whether the image impedance of the filter is in the form of $Z_{0,2\nu}$; $Z_{\infty, 2\nu-1}$ or its reciprocal $Z_{\infty, 2\nu}$, $Z_{0,2\nu-1}$. Hence, the TS which has the image impedance of the form as in eq. 5.6.41 is of reduced practical importance.

TABLE -- I

(a)

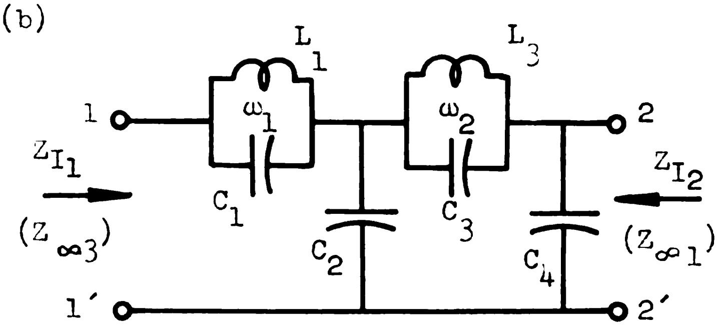


$$Z_{I_1} = \frac{k\sqrt{1 + \frac{p^2}{\omega_o^2}}}{1 + \frac{p^2}{\omega_1^2}}$$

$$\frac{k}{\sqrt{1 + \frac{p^2}{\omega_o^2}}}$$

$$L_1 = m_1 L_o, \quad C_1 = \frac{1 - m_1^2}{m_1} C_o, \quad C_2 = m_1 C_o.$$

(b)



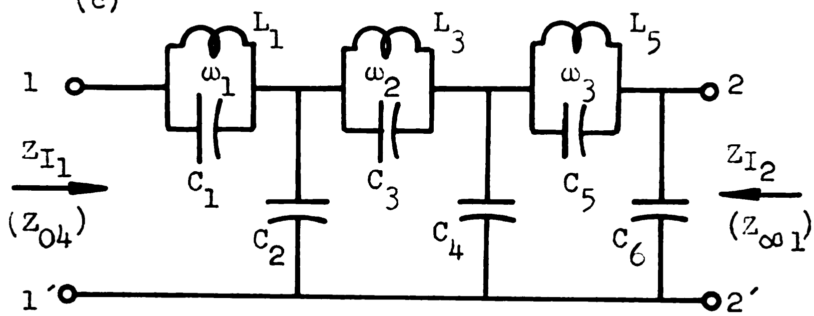
$$Z_{I_1} = \frac{k\left(1 + \frac{p^2}{\omega_2^2}\right)}{\sqrt{1 + \frac{p^2}{\omega_o^2}}\left(1 + \frac{p^2}{\omega_1^2}\right)}$$

$$Z_{I_2} = \frac{k}{\sqrt{1 + \frac{p^2}{\omega_o^2}}}$$

$$L_1 = \frac{m_1^2 - m_2^2}{m_1} L_o, \quad L_3 = \frac{m_2}{m_1}(m_1 + m_2) L_o, \quad C_1 = \frac{m_1(1 - m_1^2)}{m_1^2 - m_2^2} C_o,$$

$$C_2 = m_1 C_o, \quad C_3 = \frac{m_1(1 - m_2^2)}{m_2(m_1 + m_2)} C_o, \quad C_4 = m_2 C_o$$

(c)



$$Z_{I_1} = \frac{k\sqrt{1 + \frac{p^2}{\omega_o^2}}\left(1 + \frac{p^2}{\omega_2^2}\right)}{\left(1 + \frac{p^2}{\omega_1^2}\right)\left(1 + \frac{p^2}{\omega_3^2}\right)}$$

$$Z_{I_2} = \frac{k}{\sqrt{1 + \frac{p^2}{\omega_o^2}}}$$

$$L_1 = \frac{m_1(m_1^2 - m_2^2)}{m_1^2 - m_3^2} L_o, \quad L_3 = \frac{m_1^2(m_1 + m_2)(m_2^2 - m_3^2)}{(m_1^2 - m_3^2)(m_1 m_2 + m_3^2)} L_o,$$

$$L_5 = \frac{m_3(m_1 + m_3)(m_2 + m_3)}{m_1 m_2 + m_3^2} L_o, \quad C_3 = \frac{1 - m_1^2)(m_1^2 - m_3^2)}{m_1(m_1^2 - m_2^2)} C_o,$$

$$C_2 = \frac{m_1^2 - m_3^2}{m_1} C_o, \qquad C_3 = \frac{(1-m_2^2)(m_1^2 - m_3^2)(m_1 m_2 + m_3^2)}{m_1^2 (m_1 + m_2)(m_2^2 - m_3^2)} C_o,$$

$$C_4 = \frac{m_1 m_2 + m_3^2}{m_1} C_o, \quad C_5 = \frac{(1 - m_3^2)(m_1 m_2 + m_3^2)}{m_3 (m_1 + m_3)(m_2 + m_3)} C_o, \quad C_6 = m_3 C_o$$

VI. A DESIGN PROCEDURE FOR SYMMETRICAL LOW-PASS IMAGE PARAMETER FILTER

6.1 GENERAL REVIEW OF IMAGE PARAMETER FILTERS

If cascaded four terminal LC networks on a matched basis constitute a filter, then it is well-known that this filter has an image transfer function equal to the sum of the transfer functions of the individual sections constituting the filter. Also, the image impedances of this filter are the same as the image impedances of the end sections.

In filter design, generally, the insertion loss function, A_s, is given. In the effective pass-band, usually it is desired that A_s must be less than a given constant value A_{sp}. In the effective block-band, A_s must be greater than a given contour of requirement which is generally not a horizontal straight line.

If the filter works between its image impedances, insertion loss and image transfer loss are identical. Hence, in the pass-band, the insertion loss is identically zero. Under such an operating condition, the design of image parameter filter is simple. In this case, when the transfer loss is desired to be flat in the block-band, Cauer [7] has given explicit formulas for the location of the poles of transfer loss function. On the other hand, for the case of arbitrary transfer loss functions, the locations of these poles has to be determined by cut-and-try methods. The cut-and-try method involved is considerably simplified by use of some transformations, e.g., Rumpelt's* templet method [17] or Saraga and Fosgate's transformation [30].

In normal practice, when a filter works between two pure resistances, and not between two image impedances, the insertion loss is not identical to the image transfer loss. Zobel's decomposition formula is an expression of the insertion function for the general operating condition. From this formula, it is clear that in the pass-band if the image impedances of the filter can be made to approximate the terminating resistances, the insertion loss will not differ much from the image transfer loss function in this region, i.e., A_s can be made as small as possible, since A_I is identically zero in the pass-band. Because of this idea, Zobel introduced the composite filters [31] which are actually image parameter filters but having two extra end sections called terminating half sections, each of which has higher order image impedances.

By introducing the terminating half sections, TS, which are discussed in detail in Chapter V, the A_s function can be made very close to A_I in the pass-band ($A_I \equiv 0$). But, in the block-band, it may not be possible to make A_s and A_I very close to each other, since, generally, at the critical frequencies of the image impedance of the terminating half sections, A_s has poles but may not A_I. As is shown in section 4.5, regardless of the image impedance of the symmetrical filter, subtracting 0.02 db. loss from the image transfer loss, A_I, will yield a lower bound to A_s in the block-band. The image transfer losses of terminating half sections are also included in A_I. This approximation is always done since it considerably

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* Rumpelt, E., "Über den Entwurf electrischer Wellenfilter mit vorgeschriebenem Betriebsverhalten," Doctoral dissertation, Technische Hochschule, Munich, Germany, 1947.

simplifies the image parameter filter design.

In general, the commonly used approximate design procedure just described, although simple, necessitates the use of more sections than is necessary. This fact is demonstrated by a "precise design procedure" resulting in an economy of the sections to be used. This precise design procedure is demonstrated in the following discussions.

6.2 THE DESIGN PROCEDURE

A low-pass filter operates between two resistances of value R_T. The insertion loss is not to be greater than A_{ps} between the frequencies $0 - f_1$ (effective pass-band). It is not to be less than a given contour C_s for frequencies larger than f_2 (effective block-band). These requirements on the A_s - function are indicated in fig. 6.2.1.

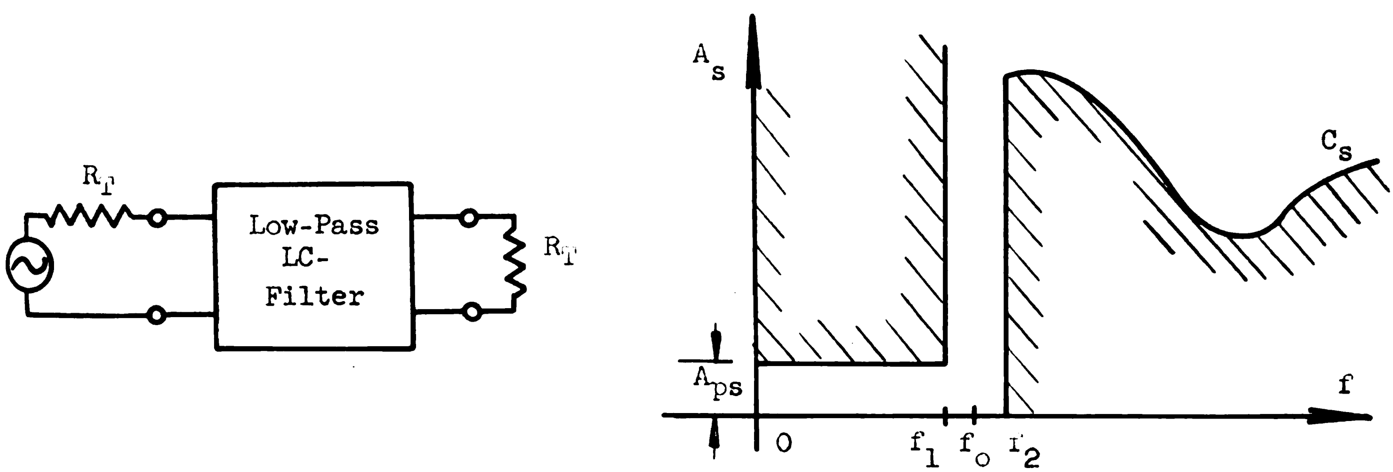


Fig. 6.2.1

(A) CHOICE AND CALCULATION OF TERMINATING HALF SECTIONS

A cut-off frequency, f_o, must be chosen. The difficulties in choosing f_o are well-known [2]. If f_o is close to f_1, complicated terminating half sections must be used. In this case, critical frequencies of the image impedance of the terminating half section become numerous and are crowded very close to f_o, in the block-band. These critical frequencies produce some poles of A_s.

On the other hand, if f_o is close to f_2, then simpler terminating sections can be used, with fewer critical frequencies of the image impedance. Correspondingly, only a few locations of poles of A_s are fixed. This gives more flexibility on the location of poles of A_s. But, if f_o is really very close to f_2, then to provide a sharp cut-off, we may have to use more intermediate sections than are saved by using simpler terminating sections.

Since A_{rs} is given, we may check first from the Graph-I (at the end of this chapter) to determine whether one, two, ... critical frequencies are required to assure that f_1 will correspond to x_{uz}, and from which the location of f_o is determined. After fixing the location of f_o and the type of TS, the critical frequencies of the image impedance of this terminating half section can be calculated from the formula (4.3.1).

With the above information, the element values of TS can be calculated completely. Because from the Graph-I, U is known, i.e.,

$$U^{\pm 1} = k/R_T \quad (k^2 = L_o/C_o) \tag{6.2.1}$$

then k is determined. Also, since f_o is chosen

$$L_o C_o = 1/\ 4\pi\ f_o^{\ 2} \tag{6.2.2}$$

Therefore, a knowledge of the values of k, f_o and U yields

$$L_o = \frac{k}{2\pi f_o} = \frac{U^{\pm 1} R_T}{2\pi\ f_o} \tag{6.2.3}$$

$$C_o = \frac{1}{2\pi\ k f_o} = \frac{1}{2\pi U^{\pm 1} R_T f_o} \tag{6.2.4}$$

From the Table-I in Chapter V, element values of TS can easily be found.

(B) CALCULATION OF INTERMEDIATE SECTIONS

Since the expression of insertion loss, in the block-band, is given by

$$e^{2A_s} = -\left(\frac{1 - z_1^2}{2z}\right)^2 \mathrm{Sinh}^2 A_I$$

the image attenuation, A_I, can be found as

$$A_I = \mathrm{argsinh}\left[-\left(\frac{2z}{1 - z^2}\right)^2 e^{2A_s}\right]^{\frac{1}{2}} \tag{6.2.5}$$

The TS, already designed, determines z. Therefore, the factor

$$E = -\left(\frac{2z}{1 - z^2}\right)^2 \tag{6.2.6}$$

is known. This factor, in the block-band, is positive and varies between zero and unity. On the other hand, the requirement on A_s in the block-band is also given by C_s of Fig. 6.2.1. Hence, e^{2A_s} is known. Therefore, eq. 6.2.5 gives the requirement on the A_I - function in the block-band.

Calculation to meet the requirement on A_I can be simplified if a digital computer is used. Indeed, first by choosing a set of points on the given contour of requirement, C_s, and using the Least-square routine, we can find an approximate polynomial for the equation of this contour. Since z is known, a program can also be written for the factor in eq. 6.2.6. Hence, in the block-band, the whole expression, i.e., eq. 6.2.5, can be found by including additional routines for argsinh and square root.

An example is shown in Fig. 6.2.2 to illustrate the requirement on A_I. The requirement on the A_I - function is indicated by the solid line on this figure. This A_I - curve is obtained by calculation from eq. 6.2.1.

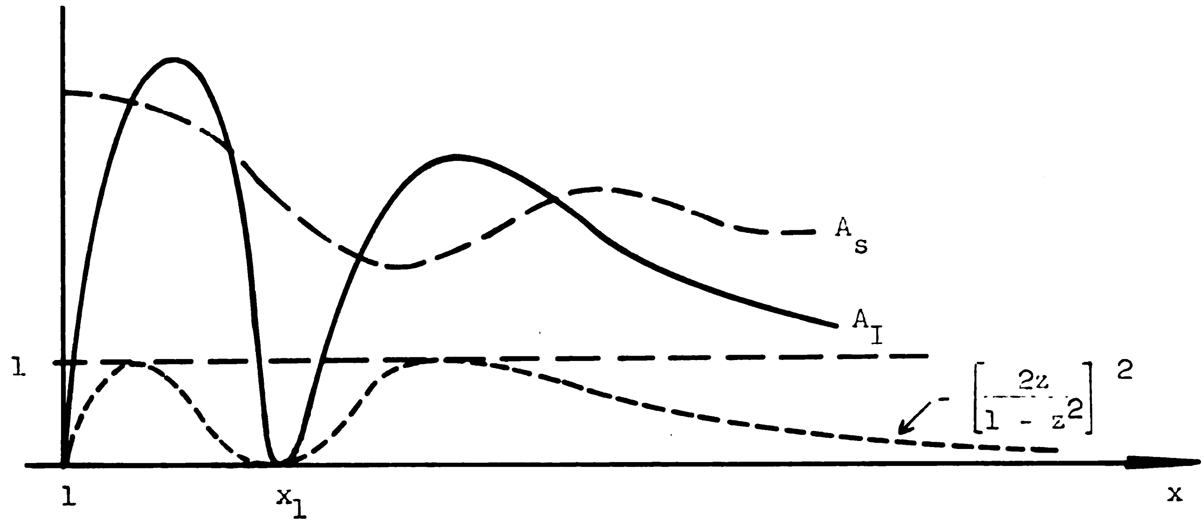


Fig. 6.2.2

Now the problem is reduced to find the necessary number of intermediate sections and the location of their transfer loss poles. This part of the design is considerably simplified if one uses Rummelt's templet method. The description of this method can be found elsewhere [17], [32], [33]. By using Belevitch's notations [17], on the transformed γ-axis, the requirement on A_I is shown in fig. 6.2.3. Now, since the TS is known, the corresponding poles of A_I are given. These attenuation curves are drawn by using a templet. The symmetry axis of the templet curves, T_i, coincide with the critical points of the image impedance of the TS on the new γ-axis (γ_i). In fig. 6.2.2, since we assume that the image impedance of the TS has only one critical frequency, there is only one such curve, T_i, in fig. 6.2.3. Subtracting the curve T_i from A_I, the remaining curve A_{I_1} is next to be obtained from the intermediate m - derived sections.

It may happen that, when a complicated TS is used, the A_{I_1} curve would be under the γ-axis. In such a case, we do not need to use intermediate sections and the filter will consist of only the terminating half-sections.

In the general case, the A_I curve will have some positive portion in some intervals of γ as seen in fig. 6.2.3. Therefore, by using the templet,

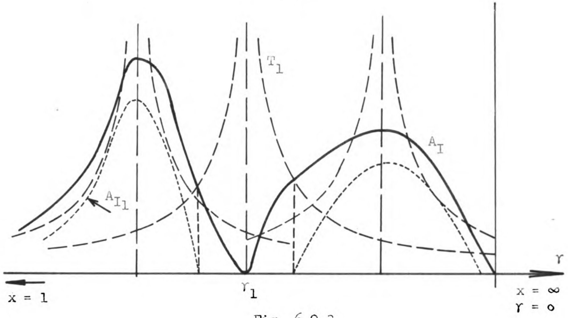


Fig. 6.2.3

it is easy to determine graphically how many curves, T_i, must be used and where the critical frequencies of the T_i's are located (γ_i). If we consider the example in fig. 6.2.3, the probable locations of critical points, γ_i, of curves, T_i, must be very close to the abscissa of the maximum points of the A_{I_1} curve. After locating the necessary number of T_i curves on the γ-axis by this templet of Graph-II, we can directly find from γ_i the x_i or m_i. Hence, on finding the m_i's, the design procedure is completed; because the element values of intermediate sections can be found in terms of L_o, C_o and m_i in Table-I.

6.3 EXAMPLE

A symmetrical, reactive (LC) low-pass filter is to satisfy the following insertion loss requirements

$$A_s \leq A_{p_s} = 0.045 \text{ db} \quad (0 \leq x \leq x_{u_z})$$

$$A_s \geq A_{b_s} = 32 \text{ db} \quad (x_{u_A} \leq x)$$

with

$$x_{u_z} = 0.96593$$

$$x_{u_A} = 1.01920$$

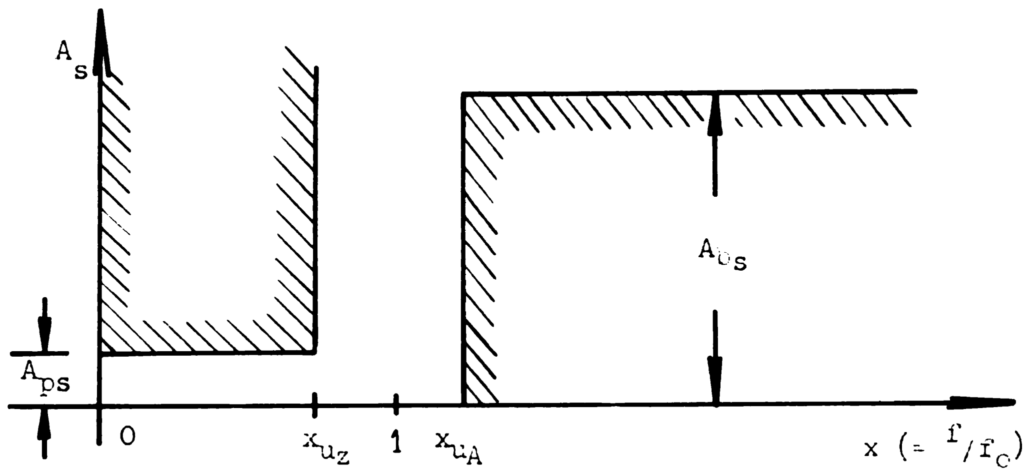


Fig. 6.3.1

The above requirements are also indicated in Fig. 6.3.1.

From Graph-I (at the end of this chapter), it is evident that a terminating half-section with only one critical frequency could be selected. This terminating half-section is the first one in Table-I. The approximate value of U can be found from Graph-I. On the other hand, a more precise value may be obtained by the following calculations.

From the tabulation 4.14.1 and 4.14.2 of the reference [2], we have

$$x_1 = \frac{1}{\mathrm{sn}\ \frac{K}{2}} = \frac{1}{0.8913}$$

and since

$$(1 - U^4)^{\frac{1}{2}} = \frac{x_{u_z}^{\ 2}}{x_1^{\ 4}}$$

then

$$U = \left(1 - \frac{x_{u_z}}{x_1^{\ 2}}\right)^{\frac{1}{4}} = 1.100369339 \ldots$$

The normalized image impedance of the filter is

$$z = U \frac{x_1^2\sqrt{1 - x^2}}{x_1^2 - x^2}$$

Therefore, the factor

$$E = -\left(\frac{2z}{1 - z^2}\right)^2$$

can be calculated easily. The corresponding E curve is given in fig. 6.3.2. From eq. 6.2.1. A_I can be found. This has been done and the A_I - curve is given in fig. 6.3.3. Now, by using the templet method, it is found that two intermediate sections having the parameters

$$m_2 = 0.213$$
$$m_3 = 0.83$$

must be used. Therefore, the filter contains two intermediate sections.

If the approximate in the classical method discussed in section 4.5 is used for this example, A_{bs} = 32 db, hence the image transfer loss in the block-band is taken as

$$A_I = A_{bs} + 6 = 38 \text{ db.}$$

From fig. 4.15.2 of reference [2], more than 3 sections total are required (terminating half sections are included). Since the method presented in this thesis requires exactly three sections total to meet the requirements, the approximation stated above certainly leads to at least one superfluous section.

In this particular example, the image transfer loss, A_I, is considered as "flat-loss" in the block-band. Fig. 4.15.2 in reference [2] is then used to determine the necessary number of sections in this low-pass LC-filter. To satisfy the flat-loss property, the locations of the poles of

A_I are determined. On the other hand, we have already chosen terminating half sections to meet the given requirements for the pass-band. Since these terminating half sections fix certain poles of A_I, we have to check the following:

1) It has to be verified whether one of the poles of flat-loss A_I, determined in the foregoing, is also the required transfer pole of the terminating half sections. In general, it will not be.

If the A_I-pole for the terminating section is not also a flat-loss pole, we may proceed as follows:

2) Shift the image transfer pole closest to the A_I-pole into coincidence. Since now the flat-loss property will not be valid, we have to check whether the block-band requirement is still satisfied or not.

In general, the block-band requirement on image transfer loss is given by an arbitrary contour. Therefore, fig. 4.15.2 in [2] cannot be used directly in this case. It can be employed if one replaces the given contour of requirement on A_I by a horizontal line which is drawn at the maximum point of this requirement. Although this can be done, the result is not so satisfactory since the number of necessary sections is unnecessarily increased.

It is, of course, possible to avoid the unnecessary sections used. The following remedies are suggested.

1) By using a templet method a good approximation can be made and the number of necessary intermediate sections can be found from the block-band requirement. On the other hand, the terminating half sections will also produce some additional transfer loss in the block-band over that produced by the intermediate sections. Therefore, the extra transfer loss

when considered separately results in more sections.

2) A more precise method is as follows: By considering the terminating half sections, and their image transfer loss, this loss can be subtracted from the given requirement on A_I to determine a new requirement, A_I'. This new A_I' is then provided by intermediate sections, with a saving in the number of sections.

In conclusion, it can be inferred that in all cases, discussed in the classical method, since A_I is found from A_{is} by adding 6 db., it may not always be possible to use the minimum number of sections. The above example is chosen with the specific purpose of demonstrating this conclusion.

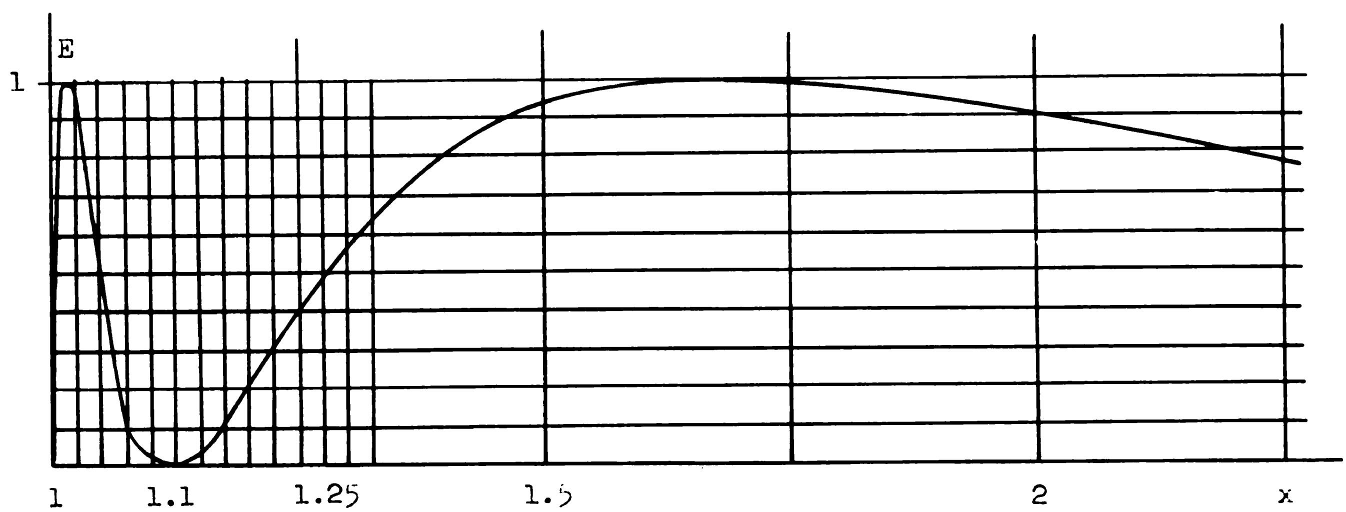


Fig. 6.3.2

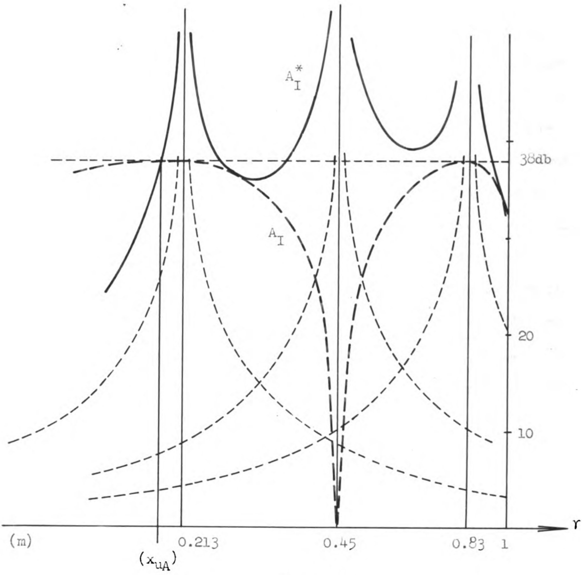


Fig. 6.3.3

6.4 SOME REMARKS ON THE IMAGE PARAMETER METHOD

On a close investigation of the Fig. 6.2.2, we can see that at the critical frequency, x_1, of the image impedance of TS, $A_I = 0$. But a pole of image attenuation of TS occurs at this frequency. This is a disadvantage, since we are unnecessarily locating one of the largest values of A_I at which the requirement on A_I is zero. This always happens since we are using a particular type of TS: In Chapter V a class of TS is considered and it is found there that if one of the image impedances of this TS is a constant-k type, to increase the order of the other image impedance of this TS, the critical frequencies of the latter image impedance must occur at the poles of transfer loss function.

The above property of the location of the critical frequencies of the image impedance of terminating half section indicates that no other terminating half sections with the mid-series or mid-shunt type of configuration could be found, such that its critical frequencies do not occur at the locations of the attenuation poles of these half sections.

Let us keep the configuration of filter terminating half sections as mid-series or mid-shunt form and impose the condition that the critical frequencies of one of the image impedances do not occur at the transfer poles of this section. This implies that the other image impedance cannot be taken as constant-k type. Hence, this violates the definition of terminating section, i.e., the section will not be a terminating section as it is defined in Chapter V.

The above problem was recently considered for only m - derived type of image impedance case [26], where Rowland's equivalent network transformations [34] are applied mostly to the mid-shunt type configurations. These configurations constitute filter, whereas our discussion pertains to terminating sections. Since the procedure in [26] is not the exact answer to our problem, it will not be considered here.

Returning to the earlier discussion on terminating half sections, the disadvantage results in the increase of the number of elements in the filter. This disadvantage could be overcome by adopting a different design procedure. However, the simplicity of the image parameter method is thereby lost. In practice, in most of the cases, only one critical frequency for the image impedance of terminating half section gives satisfactory results. Consequently, the above disadvantage of terminating half sections is not so serious.

If the disadvantage is considered formidable, some alternate methods are available:

(1) Use of Darlington's method. This method imposes a special contour of requirements on A_s in the block-band. In this case, the following can be done:

Find the maximum point of the given contour, C_s, and draw a horizontal line at this point. Choose this horizontal straight line as the contour of requirement. Then apply the classical method.

But in this case, necessarily more elements in the filter will be used, since the degree of the filter is increased.

(2) Use of Fromageot's method [5]. As indicated in the foregoing, this method involves searching for a ϕ - function which contains approximate requirements on the A_s - function. It is more general than Darlington's method, but does not differ from it when the element values are to be calculated.

6.5 TUTTLE'S PROBLEM

It is considered of interest here to mention in this section Tuttle's two mid-shunt type Zobel's sections which gives a Tschebyscheff type of insertion loss characteristic in both effective pass- and block-band [35]. An explanation is given as to why the extension of this problem to more than two sections does not give the same type of insertion loss characteristics.

Consider ν m - derived mid-shunt sections in cascade with different parameters m_i. The corresponding ϕ - function will be found from

$$\phi^2 = -\left(\frac{1 - z^2}{2z}\right)^2 \operatorname{Sinh}^2 P_I \tag{6.5.1}$$

where

$$z = \frac{\alpha}{\sqrt{1 - x^2}} \quad \text{with } \alpha = k/R_T \tag{6.5.2}$$

and

$$P_I = \sum_{i=1}^{\nu} P_{I_i} \tag{6.5.3}$$

The ratio function, H_i, of each section can be written as follows,

$$H_i = \operatorname{Tanh} \frac{P_{I_i}}{2} = \frac{e^{P_{I_i}} - 1}{e^{P_{I_i}} + 1} = m_i H_o$$

hence,

$$e^{P_{I_i}} = \frac{1 + m_i H_o}{1 - m_i H_o} \tag{6.5.4}$$

where H_o is the ratio function of the half prototype section, i.e.,

$$H_o = \operatorname{Tanh} \frac{P_o}{2} = \frac{x}{\sqrt{x^2 - 1}} \tag{6.5.5}$$

Consider

$$\operatorname{Sinh} P_I = \tfrac{1}{2}\left(e^{P_I} - e^{-P_I}\right) \tag{6.5.6}$$

and substitute eqs. 6.5.3 and 6.5.4 in eq. 6.5.6, we have

$$\operatorname{Sinh} P_I = \tfrac{1}{2}\left[\prod_{i=1}^{\nu} \frac{1 + m_i H_o}{1 - m_i H_o} - \prod_{i=1}^{\nu} \frac{1 - m_i H_o}{1 + m_i H_o}\right]$$

or

$$\operatorname{Sinh} P_I = \frac{2\left[1 + \sigma_{2\nu} H_o^2 + \sigma_{4\nu} H_o^4 + \cdots + \sigma_{2n,\nu} H_o^{2n}\right]\left[\sigma_{1\nu} H_o + \sigma_{3\nu} H_o^3 + \cdots + \sigma_{s,\nu} H_o^s\right]}{\prod_{i=1}^{\nu} \left(1 - m_i^2 H_o^2\right)} \tag{6.5.7}$$

where, if ν is even, then $2n = \nu$, $s = \nu - 1$,

if ν is odd, then $2n = \nu - 1$, $s = \nu$, and $\sigma_{i,\nu}$ is the symmetrical function of ν variables, i.e.,

$$\sigma_{i,\nu} = \sum_{j_1 j_2 \cdots j_\nu} m_{j1}\, m_{j2} \cdots m_{j\nu} \qquad (6.5.8)$$

On the other hand, from eqs. 6.5.2 and 6.5.5, we have

$$z^2 = \alpha^2 (1 - H_o^{\,2})$$

Hence,

$$\left(\frac{1 - z^2}{2z}\right)^2 = \frac{\left[(1 - \alpha^2) + \alpha^2 H_o^{\,2}\right]^2}{4\,\alpha^2 (1 - H_o^{\,2})} \qquad (6.5.9)$$

Substituting eqs. 6.5.7 and 6.5.9 into eq. 6.5.1 yields

$$\phi^2 = - \frac{\left[(1-\alpha^2) + \alpha^2 H_o^2\right]^2 \left[1+ \sigma_{2\nu} H_o^2 + \cdots + \sigma_{2n,\nu} H_o^{2n}\right]^2 \left[\sigma_{1\nu} H_o + \sigma_{3\nu} H_o^3 + \cdots + \sigma_{s\nu} H_o^s\right]^2}{\alpha^2 (1 - H_o^{\,2}) \prod_{i=1}^{\nu} (1 - m_i^{\,2} H_o^{\,2})^2} \qquad (6.5.10)$$

If $\nu = 2$, then substituting eq. 6.5.5 into eq. 6.5.10, we obtain

$$\phi^2 = \frac{(m_1 + m_2)^2\, x^2 \left[(1 - \alpha^2) - x^2\right]^2 \left[1 - (1 + m_1 m_2)\, x^2\right]^2}{\alpha^2 \left[1 - (1 - m_1^{\,2})\, x^2\right]^2 \left[1 - (1 - m_2^{\,2})\, x^2\right]^2} \qquad (6.5.11)$$

Tuttle compared eq. 6.5.11 with the ϕ_D - function of a two section Darlington filter and showed that the parameters m_1, m_2, α, f_{oc} can be determined uniquely in terms of the parameters of ϕ_D - function, (where f_{oc} is a factor with which the frequency scale to be multiplied). Indeed, the comparison of the ϕ - and ϕ_D - functions gives four non-linear equations of four variables (parameters) from which the parameters can be determined uniquely.

If $\nu > 2$, the number of parameters is thereby increased, but the number of non-linear equations increases faster, i.e., if $\nu = k > 2$, the number of unknowns is $k + 2$, but number of non-linear equations is $2k$ which is greater than $k + 2$ when $k > 2$.

It can be shown that if $\kappa > 2$, the nonlinear system of equations is inconsistent. This implies that with the cascaded m - derived sections, if $\gamma > 2$, it is not possible to obtain Darlington's type of insertion loss filter characteristics - "flat" in both pass and block regions.

VII. EFFECT OF DISSIPATION ON INSERTION LOSS AND PHASE OF A SYMMETRICAL IMAGE PARAMETER FILTER

7.1 GENERAL

In actual practice, where the L and C elements in the filter are lossy, the insertion loss and phase of the filter are slightly different from the lossless form.

In this chapter, the effect of dissipation is considered and discussed through the use of an example. This discussion is simplified by use of a digital computer. The program which is written and used for the example is general; that is, it can be used for any symmetrical low-pass image parameter filter.

In the lossy case, the formulas for A_s and B_s functions are given by eqs. 3.2.14 and 3.2.15 in Chapter III. A "flow diagram" for the computer program is shown in fig. 7.1.1 at the end of this chapter.

7.2 EXAMPLE

The example considered here is taken from Reed's book*.

Data: A low-pass filter operates between two pure resistances, each of which is 75 ohms. The following are its design details:

---

* Reference [2], pp. 197 - 207.

f_o = 555 kc (ω_o = 3.487 x 10^6 rad/sec)

$$\left.\begin{array}{lcl} x_u & = & 0.994054 \\ x_1 & = & 1.1957288 \\ x_2 & = & 1.0137135 \\ m_1 & = & 0.5482568 \\ m_2 & = & 0.1639298 \end{array}\right\} \text{ for terminating half sections}$$

$$\left.\begin{array}{lcl} m_3 & = & 0.1597463637 \\ m_4 & = & 0.1970563839 \\ m_5 & = & 0.2752527275 \\ m_6 & = & 0.3780835545 \end{array}\right\} \text{ for intermediate sections}$$

From the relation [2], [7]

$$\sqrt{1 - U^{-4}} = x_u^{\,2} \;/\; x_1^{\,4}$$

we have

$$U = 1.06733063$$

Since the image impedance is of the Z_{∞_3} form, in eqs. 6.2.13 and 6.2.14, the negative power for U must be used. Therefore,

$$L_o = 0.20141232265 \text{ X } 10^{-4} \text{ henry}$$

$$C_o = 0.40829545 \text{ X } 10^{-8} \text{ farad}$$

On the data tape the following parameters appear:

$$R_T = 75 \text{ ohms}$$

$$L_o = 0.20141232265 \text{ X } 10^{-4}$$

$$C_o = 0.40829545 \text{ X } 10^{-8}$$

$$\omega_o = 3.48716764 \text{ X } 10^{6}$$

$$d_L = (1/\ Q_L)$$

$$d_C = (1/\ Q_C)$$

n = (Number of divisions)

x_i = (Frequency interval) = 1.3

m_1 = 0.548256659

. . . .

m_c = 0.3780835545

where Q_L and Q_C are the Q-factors of the L and C elements of the filter. In this example, Q_C is taken as infinity (d_C = 0) and for different values of Q_L (= 50, 100, 200, 500, 20,000, 100,000,000) a set of insertion loss curves is obtained as shown in figs. 7.2.1-3.

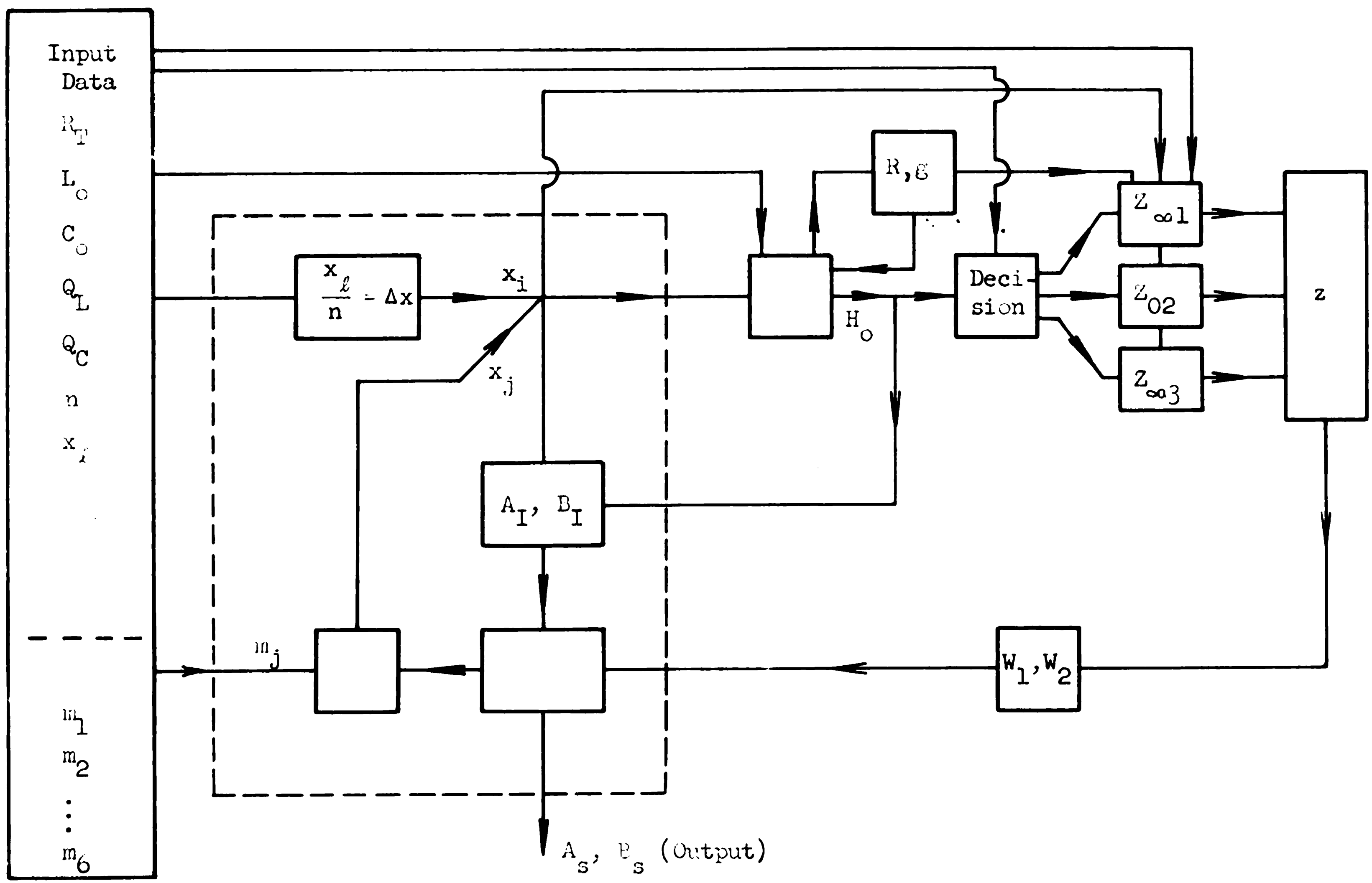

Input Data
R_T
L_o
C_o
Q_L
Q_C
n
x_ℓ
m_1
m_2
...
m_6
$\frac{x_\ell}{n} - \Delta x$
x_i
x_j
m_j
A_I, B_I
H_o
R, ε
Decision
$Z_{\infty 1}$
Z_{02}
$Z_{\infty 3}$
z
W_1, W_2
A_s, B_s (Output)

Fig. 7.2.1

ADDITIONAL FORMULAS REQUIRED FOR CALCULATIONS:

$$A_I = \sum_i A_i \quad , \quad B_I = \sum_i B_i$$

$$e^{2A_i} = \frac{(1 + m_i\, a_o)^2 + m_i^2\, b_o^2}{(1 - m_i\, a_o)^2 + m_i^2\, b_o^2}$$

$$B_i = \arctan \frac{m_i\, b_o}{1 + m_i b_o} + \arctan \frac{m_i\, b_o}{1 - m_i b_o}$$

$$H_o = a_o + j\, b_o$$

$$Z_1 = R + j\, L\, \omega$$

$$Z_2 = \frac{1}{g + j\, C\, \omega}$$

$$R_T\, Z_{\infty 1} = Z_2\, H_o$$

$$R_T\, Z_{02} = \frac{Z_2}{H_o \left[(1 - m^2) + \frac{Z_2}{Z_1} \right]}$$

$$R_T\, Z_{\infty 3} = \frac{Z_2\, H_o \left[(1 - m_2^2) + \frac{Z_2}{Z_1} \right]}{(1 - m_1^2) + \frac{Z_2}{Z_1}}$$

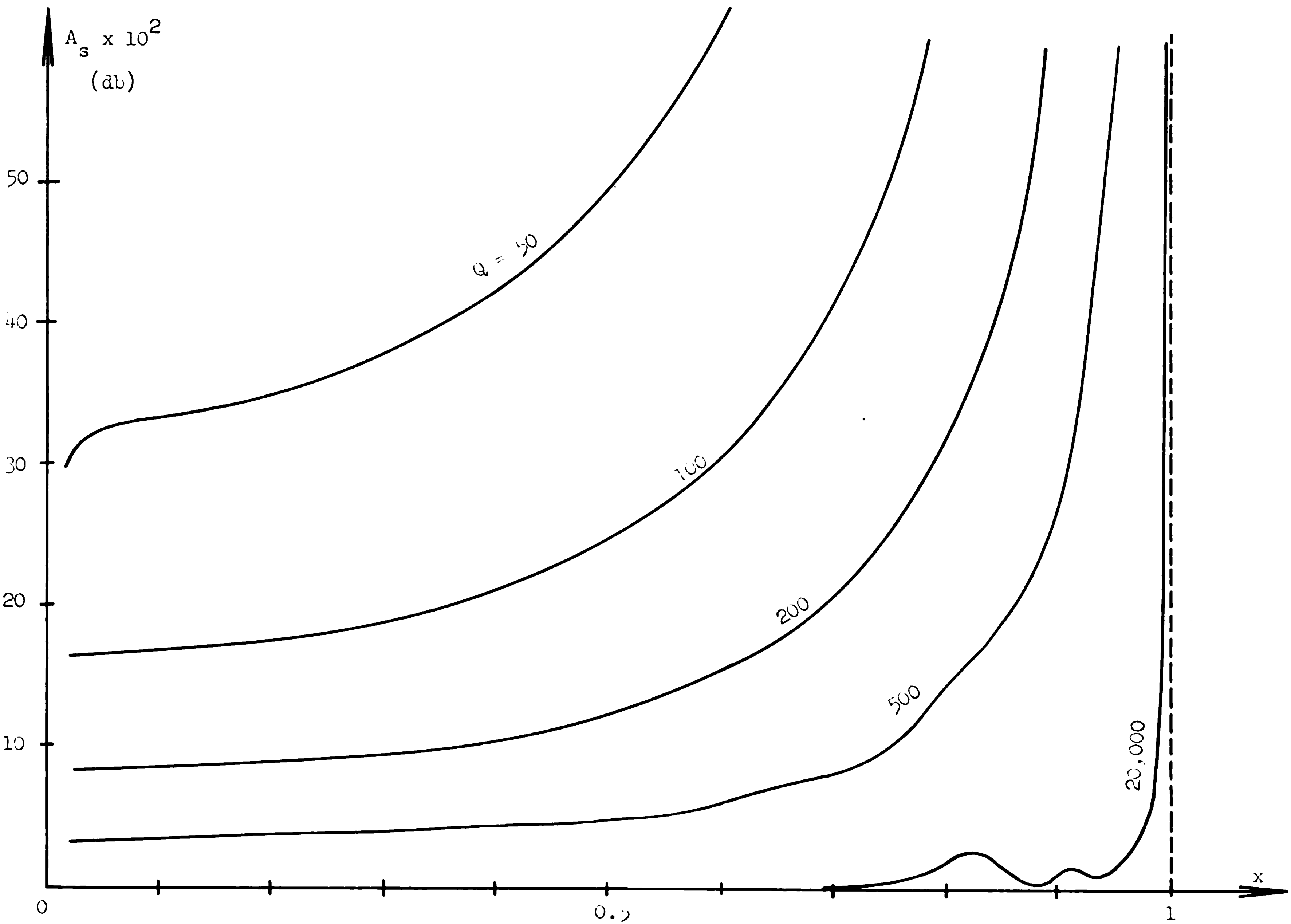


Fig. 7.2.1

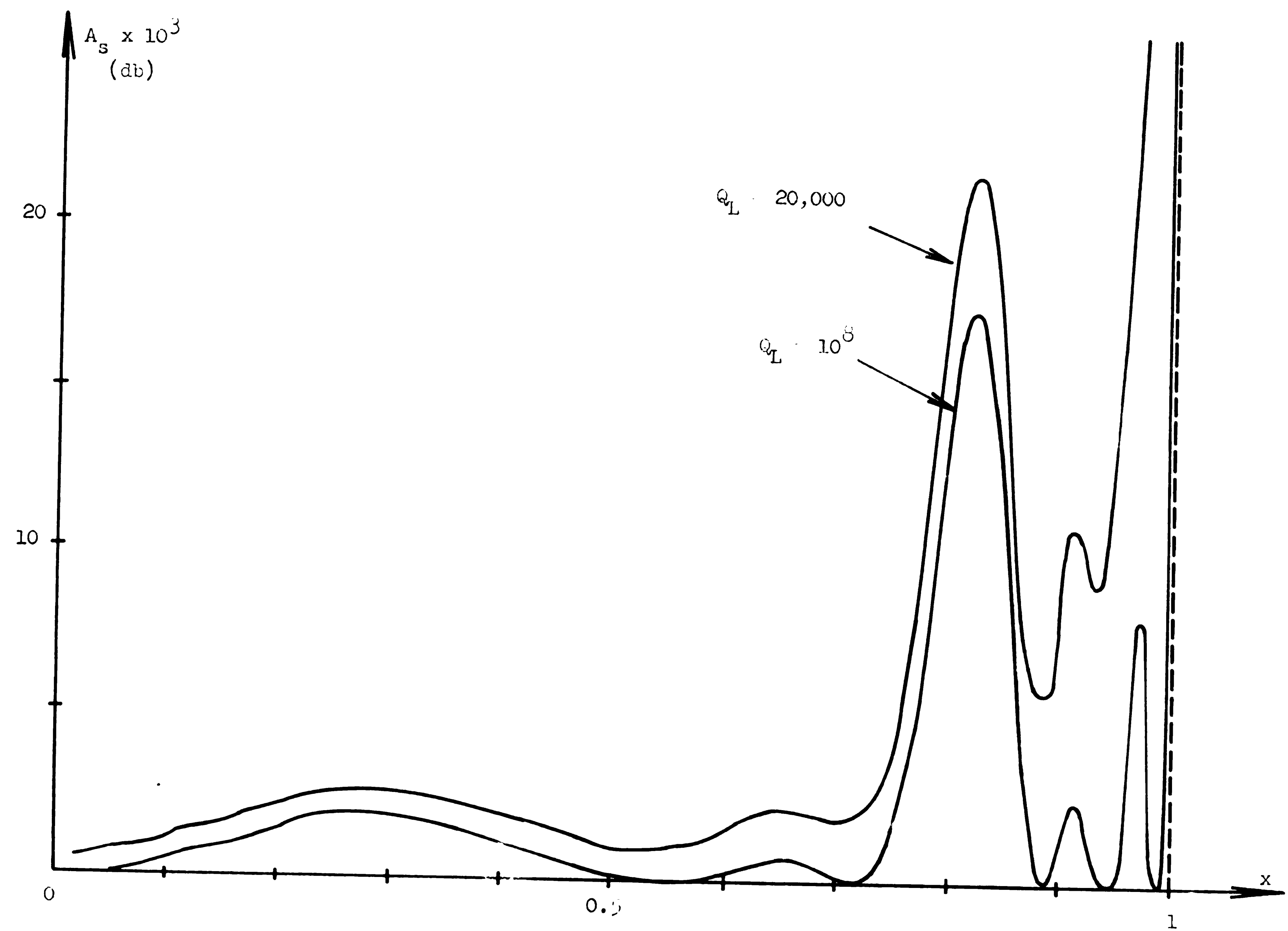


Fig. 7.2.2

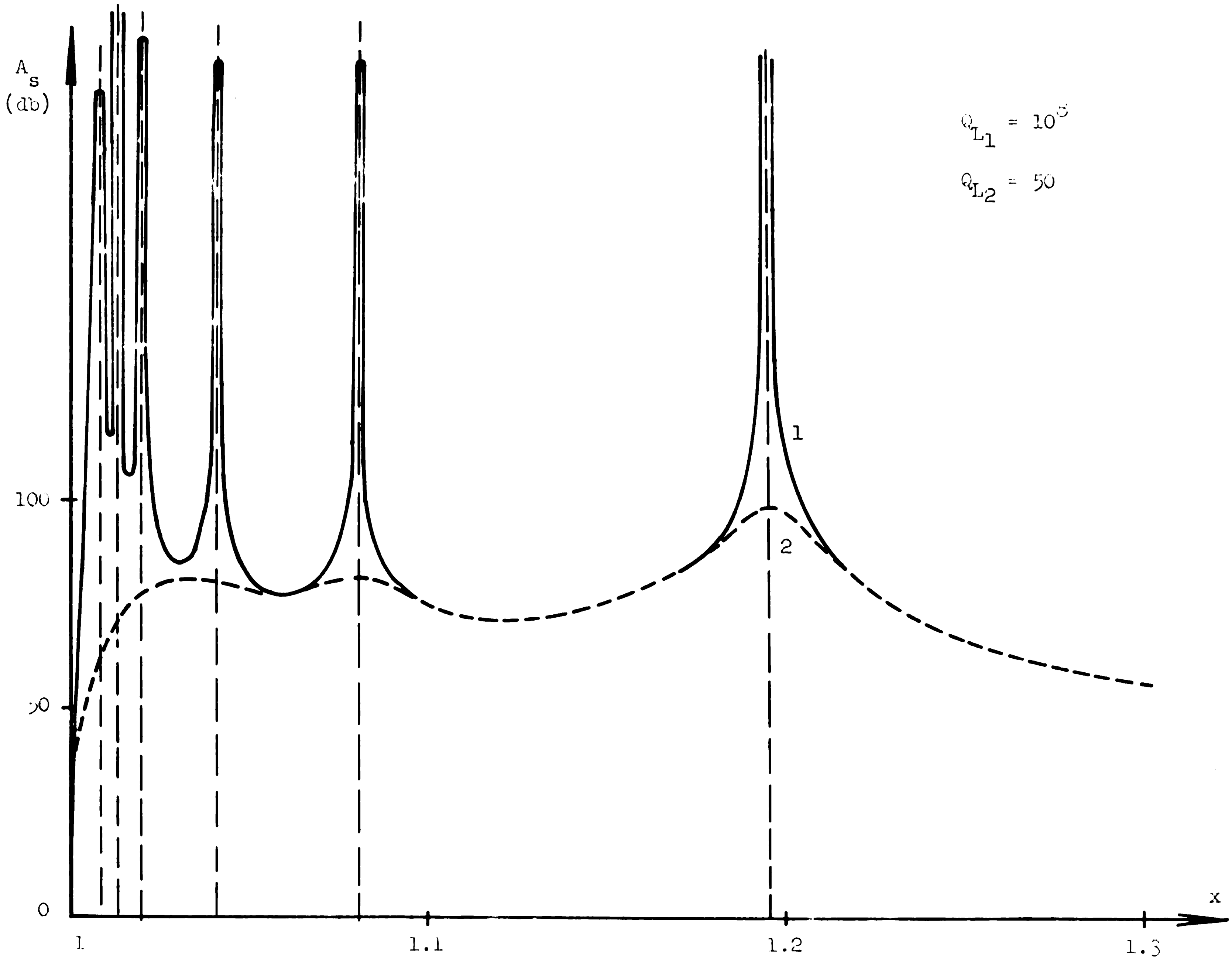
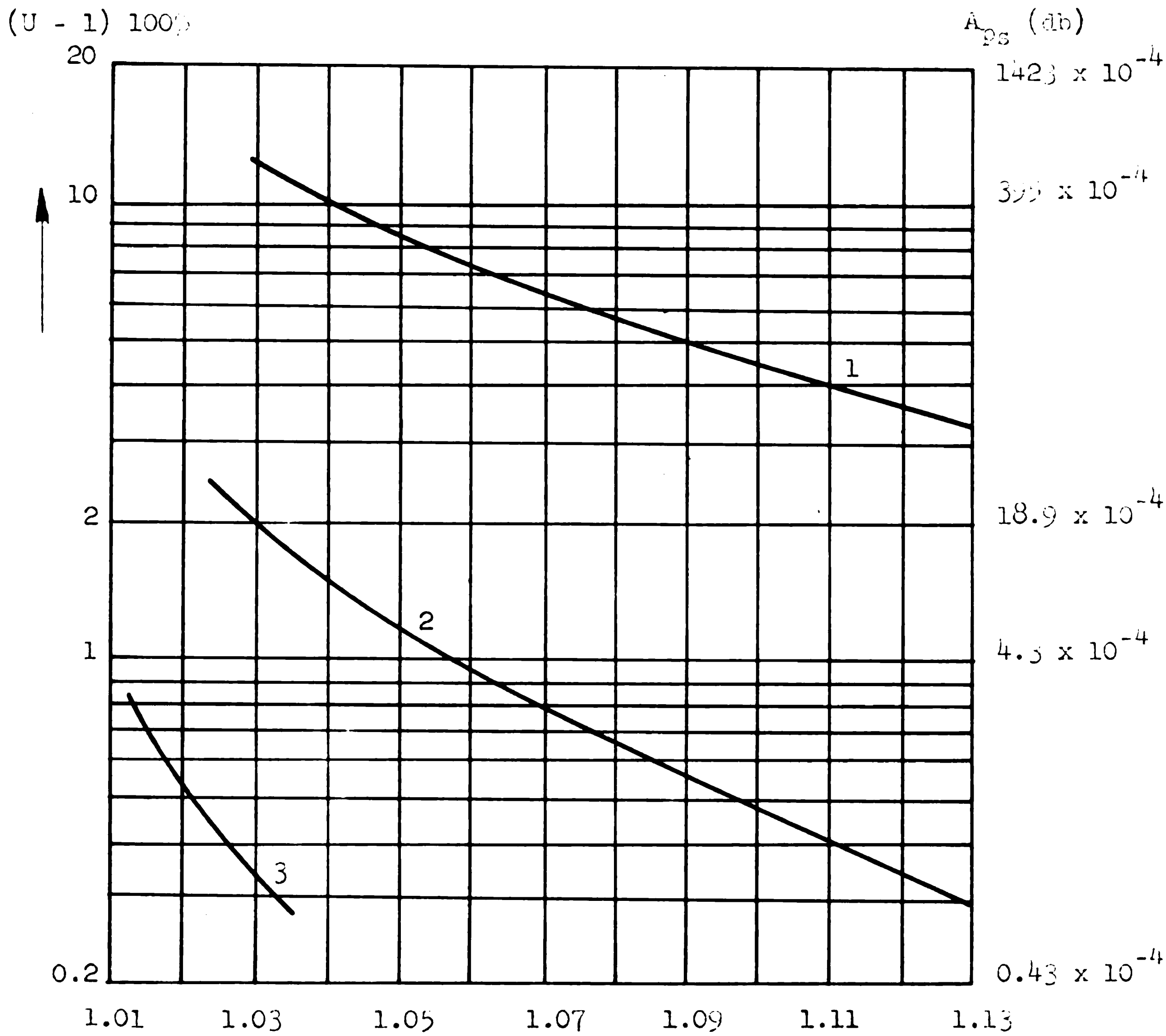


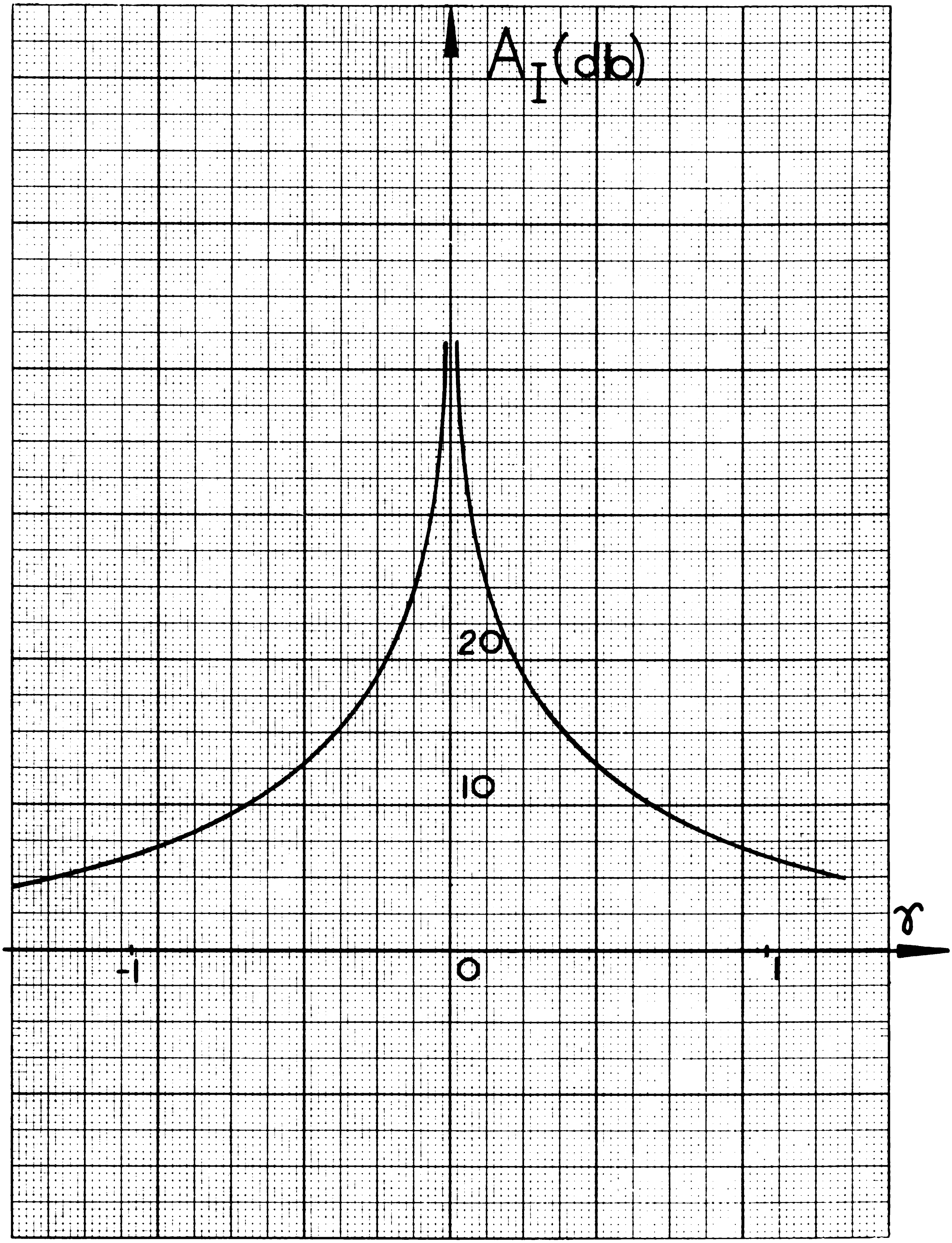
Fig. 7.2.

GRAPH I



$$A_{ps} = 8.686 \ \ln \frac{1}{2} \left(U + \frac{1}{U}\right)$$

GRAPH - II-A



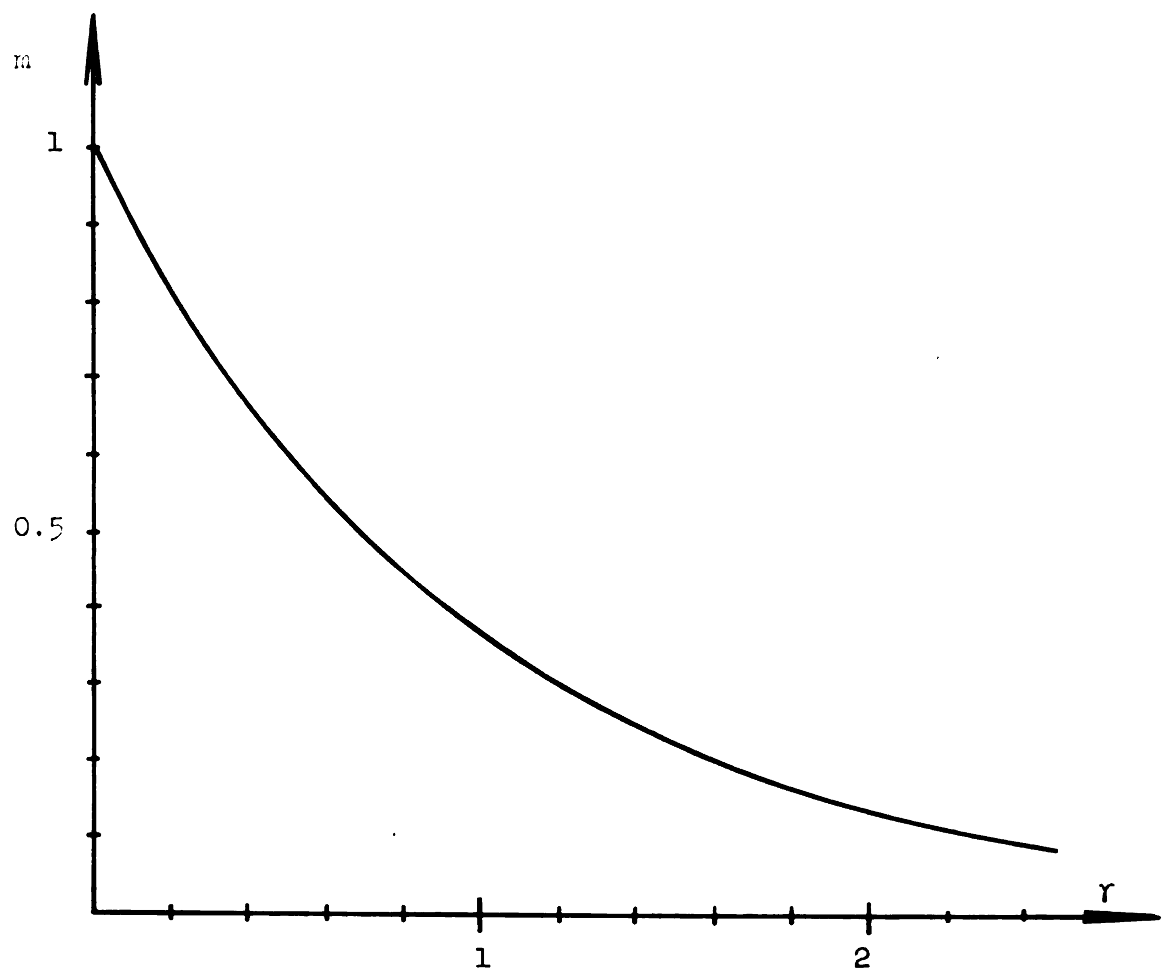
$$A_I = 8.682 \ \ln \left| \frac{1 + e^{\gamma}}{1 - e^{\gamma}} \right|$$

GRAPH - II-3

$$\gamma_i = -\ln m_i$$

$$x_i = \frac{1}{\sqrt{1 - m_i^2}}$$

x_i	m_i	γ_i
1.00	0.00	infinity
1.005	0.1	2.30259
1.02	0.2	1.60944
1.05	0.3	1.20397
1.09	0.4	0.91629
1.15	0.5	0.69315
1.25	0.6	0.51083
1.40	0.7	0.35667
1.69	0.8	0.22314
2.29	0.9	0.10536
infinity	1.0	0.00000



CONCLUSIONS

A new formulation for insertion loss and phase of a filter is considered. The design procedure of lossless image parameter filters is described by using this new formulation. The discussion is focused on the symmetrical low-pass filter. For high-pass and band-pass filter design, the well-known frequency transformations can be used and the problem reduced to the low-pass filter design.

Formulas for insertion function of a symmetrical image parameter filter at cut-off frequency with a general terminating half section are given.

The terminating half sections are also considered in detail and some formulas for the element values of terminating half sections are derived.

In the classical procedures, the block-band requirements on insertion loss function are reduced to the transfer-loss function by means of approximation formulas. This procedure results in allowing tolerances which, although sufficient, are by no means necessary. In this thesis, precise formulas are developed for reducing the insertion loss function to the transfer-loss function. Consequently, it is possible to minimize the number of sections to be used in filter design. This new procedure is demonstrated through the use of an example.

After designing the filter in which losses are not considered, the effect of dissipation on the filter elements is considered. A digital computer program is written for determining the insertion loss and phase characteristics for various values of Q_L and Q_C. This computer program simplifies the investigation on insertion loss and phase characteristics.

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