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THEORETICAL AND EMPIRICAL STUDIES IN STRATEGIC PRICING

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ABSTRACT

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This dissertation provides two theoretical studies of a firm maximizing profits under demand uncertainty, these studies are preceded by a literature review on similar research; and lastly, one empirical essay where oil prices are used to measure the efficiency of OPEC as a cartel.

Chapter 1 presents a literature review comparing the models that will be developed in Chapters 2 and 3. The first half of the chapter reviews models that involve a firm's choices from amongst its price level, production output, or both variables simultaneously. The review is then extended in the second half, to incorporate the models developed in Chapter 3, to include research on storable goods and network effects.

Chapter 2 studies a two-period model in which a firm faces uncertainty in the slope of the demand function and is able to choose either the price or the production level of its good. The model is solved in two different scenarios: one in which the firm sells a storable good, hence, second period demand decreases when first period demand is high; and the other in which consumers experience a network effect. The main result is that welfare increases when the firm sets prices instead of quantity in either scenario. Also, the myopic solution generates greater first period welfare than with the dynamic outcome.

Chapter 3 examines a single firm that maximizes expected profits. Two models are analyzed, one in the spirit of the "newsboy problem" where the firm must commit to the production level before realizing the state of demand, and the other where a price or quantity combination, or a commitment to both variables must be chosen before realizing the true demand.

Conditions for the optimal price and quantity combination are found to depend on the probability of the states of demand and the marginal cost. The main result is that pre-committing to only the production level gives higher expected profits than when there is pre-commitment in the two variables, however, production levels are lower. In the model with pre-commitment in the two variables, the optimal price is constrained by the monopoly outcome of the two possible states of demand under certainty while on the other hand, the optimal quantity is constrained by the monopoly outcome of the low demand. However, under certain conditions the optimal quantity can be greater than the monopoly quantity with the high demand.

Chapter 4 provides a test for the cooperative behavior hypothesis for OPEC during the period 1974 to 2004. A modification of the Green and Porter model allows non-OPEC producers to be treated as a competitive fringe. The proposition of whether oil price fluctuations are a consequence of noncooperative behavior within the cartel or only shocks in the demand for oil is examined. A Three Stage Least Square estimation and a simplified version of the E-M algorithm are implemented after constructing a cooperative behavior variable. The main finding is that, overall, OPEC has not been effective in keeping price and quantity above competitive levels. Moreover, oil prices are significantly higher in periods of collusion among OPEC members.

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1 Firm Strategies: Setting Prices and Setting Output – a Short Literature Review

1.1 Introduction

The idea that a firm can adjust its pricing and production strategy due to changes in the market, number and quality of consumers and rivals, or new information gives more flexibility to models. When the firm is restricted to make some of these decisions prior to entering the market, either in terms of production or pricing, the problem becomes more complicated due to the possibility of overproduction or underproduction if the firm chooses output, or over and underpricing with a price-setting firm. In the quantity-setting case overproduction leads to unnecessary extra costs, while the underproduction case represents an inefficient allocation. Whereas when the firm chooses prices, overpricing leads to zero profits since it is charging a price that exceeds the reservation price of any consumer, surpassing the willingness to pay for any type of consumer; and underpricing leads to an inefficient rationing because it is not maximizing revenues.

The following two chapters study these limitations on firms, particularly focusing on the case in which the firm faces uncertainty in the demand for its product. Firms having to make production and/or pricing decisions under conditions of market uncertainty are found in almost all industries. Uncertainty in this study is modelled in three frameworks where the difference lies in the assumptions placed on the uncertainty of the demand function. All models assume linear demand curves. The first model considers a possible uniformly higher demand, the second model addresses uncertainty in the saturation quantity, and the third considers uncertainty in the reservation price.

Chapters 2 and 3 solve each of the three frameworks when the firm chooses either prices or quantities for its product, and each addresses a different question. In Chapter

2, two cases are differentiated. First, we consider goods that are storable, and second, we suppose that the good exhibits network effects. Furthermore, a static benchmark is compared to a two-period model, in order to give insight into how the introduction of dynamics in the model affects the firm's decisions. Chapter 3, following a similar procedure, compares models when the firm can choose either prices or quantities, or is restricted to commit to a price-quantity combination before realizing demand. The benchmark for this case follows the classic "newsvendor problem".

The following section briefly describes the main works in the existing literature related to Chapters 2 and 3.

1.2 Description of Literature

A firm setting prices or production levels presents different issues to all parties involved. From the firm's perspective, it imposes a different approach on how to maximize profits, since dealing with overpricing has different consequences than overproduction. As it is shown in Chapter 2, overpricing can lead to zero profits if the price charged exceeds the reservation price of every consumer, whereas with overproduction the firm can still earn profits but has an associated larger cost of production due to the unnecessary leftover produced. From the consumers' perspective, consumer surplus changes if the market does not clear and it depends on who gets the sold units (e.g. with efficient rationing the good is only sold to the consumers with the highest valuation). Finally, the government's perspective also changes since the regulation policies have different meanings; they can be imposed in terms of quotas for a quantity-setting firm or in terms of price floors or ceilings to a price-setting firm; and depending on the industry's characteristics, this could result in distinctly different outcomes. The standard consensus is that if a firm chooses prices, then the efficient output will always be chosen as its profit maximization outcome (Bertrand competition). However, this remark holds depending on the type and number of consumers to whom the product is being sold or how profits are defined, as is explained in the following Chapters.

Weitzman (1974) finds that there is no difference between a price- or quantity-setting firm when the same information is needed for choosing one of the two variables. In Miller and Pazgal (2001), with a two-stage, differentiated-product, oligopoly model, assuming linear demand and constant marginal cost, the final outcome, when prices are chosen versus quantities, is the same. Addressing the issue of prices versus quantities in specific industries, in Menanteau, Finon and Lamy (2003), a quantity-based regulation policy for promoting the development of renewable energy is found to be more efficient since it is based on a quota system allowing better control of social costs. If the renewable resources are complements, as in Gaudet and Moreaux (1990), then a price-decision strategy dominates instead. This result is also substantiated in Singh and Vives (1984) and Cheng (1985). Hence, the study of price- versus quantity-setting models is extremely sensitive to the type of assumptions applied to the industry and consumers, and this produces ambiguous results.

In Chapter 2, demand is modelled for a storable good or a network effect good. With a storable good, each consumer buys the product only once; that is, if a consumer purchases the good in period 1, she will leave the market at the beginning of period 2. Classic examples for durable goods include the purchase of houses, boats, appliances or furniture which have a life span long enough to be considered lifetime purchases. However, storable goods are a much broader group, also including goods that can be bought and stored for a period of time, making unnecessary its purchase in more than one period; among durable goods examples are cleaning and hygienic products (e.g. razor blades, dishwashing detergent, paper towels). The amount of sales in period 1 decreases demand for period 2, therefore, the firm must evaluate the trade-off of having high profits in one period versus how that reduces demand in the next. The study of storable goods has been of great interest over the last several decades, not only for academic purposes but from the industry side as well. As stated in Waldman (2003), durable goods accounted for 60% of aggregate production for the manufacturing sector in the United States, in the year 2000.

From the theoretical perspective, there have been different approaches to solving models with storable goods. One direction is with the durability of the good, as in Coase (1972), where the issue is the competition and market trade-off between new and used goods in a multi-period framework. Another direction is with asymmetric information, as described in Akerlof (1970) for the 'lemons' market. A different approach for firms dealing with storable goods was first addressed in Bulow (1982), where the firm's decision to sell or lease its good and the trade-off between these options is developed. Furthermore, Bhaskaran and Gilbert (2003) find conditions when a firm chooses to combine a leasing and selling strategy versus choosing one or the other, following the model first implemented in Bulow (1982). Another area of study deals with the firm having the option of learning about demand over time through experimentation, as developed in Mirman, Samuelson and Urbano (1993), where different incentives are presented to a price- or quantity-setter monopolist. Experimentation can also be useful for learning how consumers vary their tastes over time depending on their experiences, as analyzed in Conlisk, Gerstner and Sobel (1984) and Paredes (2006). More recently, Gowrisankaran and Rysman (2005) have estimated dynamic consumer preferences in the context of storable goods. In Bagnoli, Salant and Swierzbinski (1989), assuming a finite number of consumers instead of a continuum, gives an opposite result from the Coase conjecture, where the monopolist does not lose its monopoly power in a dynamic setup. Before turning to network effects, it is worth mentioning that durable goods can be produced in advance and stored in order to be sold in a later period, hence, they are different than perishable goods which have a shorter life duration.

With a network effect, the more sold in the initial period, the more is demanded in the next. This indicates that overproducing is the best strategy for the firm. However, overproduction implies lower prices, hence, it might compromise profits. Therefore, the firm must once again compare the trade-off between producing at the myopic solution and deviating from it.

Goods that present network effects or network externalities, have the special characteristic of attracting more consumers if everyone else in their environment are using them. A classic example for goods with network externalities can be found in the telecommunication industry, where the greater the number of people that are part of a network, the cheaper the calls are since they are originating from the same carrier. This is taken to the limit with most carriers within the wireless industry now-a-days; for example, if a consumer originates and terminates a call with another customer of the same carrier, it is completely free for both users. The strategy involved is that each user encourages everyone around him to belong to the same carrier in order to have a greater common satisfaction for the good. Other examples include the Bandwagon effect, which is interpreted as the desire to be in style or fashion; the use of the same operating system in personal computer, or the use of e-mails at the workplace.

The pioneer in studying network effects on consumers' expectations and production decisions for the firm is the seminal model in Katz and Shapiro (1985). In Mason (1999), network effects are related to learning-by-doing, confirming the perception of network externalities as economies of scale on the demand side, due to its incentive to increase future demand. Lemley and McGowand (1998) describe the legal perspective of network effects, and address the question of in which cases and contexts should laws involving network effects be modified. Notice that because network externalities can be interpreted as economies of scales, then a one-product firm might have an advantage that would not be fair in other markets since the network effect is not incorporated in the antitrust laws. In Spiegel, Ben-Zion and Tavor (2005), conditions are found where a network effect monopolist achieves the social welfare maximum, making any type of regulation unnecessary. One of the most recent studies is Economides, Mitchell and Skrzypacz (2005), where it is estimated that in a duopoly a small network effect will help the higher quality firm set higher prices, whereas, a strong network effect gives the advantage to the firm with the largest market share. In Cabral, Salant and Woroch (1999), the intuition of the Coase conjecture is overturned with network externalities since prices rise over time. This happens when the network effect is strong enough to make the firm underprice in order to earn higher profits in the future. The empirical study of network effects can be best described in the telephone and internet industries, as addressed in Werden (2001), Madden, Coble-Neal and Dalzell (2004), and Birke and Swann (2005).

For Chapter 3, the classic "newsvendor problem" is used as the static benchmark in order to compare it to a less flexible model when the firm has to commit to prices and production levels for its good, before knowing the true demand. The "newsvendor problem" or "newsboy problem" considers a newspaper vendor that has to choose the amount of newspaper copies to have in stock to be sold the next morning. The size of the stock has to be chosen without knowing how many people are going to stop and purchase a newspaper. Consequently, a firm must find the optimal quantity to produce that maximizes expected profits without knowing the exact quantity demanded. Therefore, in the general setting of the "newsvendor problem", prices are not a control variable, rather they are determined by the market depending on the chosen production quantity.

The first to solve a version of the "newsvendor problem" was Whitin (1955). He assumes that the firm simultaneously chooses the price of the good and the stocking quantity. The solution to his model consists in finding the optimal stocking output and then finding the corresponding optimal price, however, in this model demand is known. A similar approach is used in Dada and Petruzzi (1999), where their single period additive uncertainty resembles our model except for two main points. First, they assume the distribution of the stochastic term depends on the price strategy of the firm, as in Whitin (1955). Second, they do not allow for price-quantity combinations that result in zero profits if the chosen price is too high and the realized demand is low. Khouja (1999) provides an comprehensive review of the "newsvendor problem" with possible extensions in terms of a multi-period framework, managing inventories and firms choosing price-quantity combinations.

In Lau (1980), a variation of the "newsvendor problem" is solved with a firm facing uncertainty and maximizing for the optimal probability of achieving a predetermined level of expected profits. In this case, the control variable is the probability of the states of demand instead of prices or quantities. In a dynamic setting, as in Lazear (1986), a firm facing uncertainty in the consumer's valuation and unit-demand, adjusts the price of the good from the myopic solution in the first period in order to use the information obtained for the second-period pricing. If the good was not sold in the first period, it is assumed that the price was too high, and the optimal approach is to reduce it in the second period. In this framework, information is useful as the firm is able to update its beliefs about the consumers' valuation and get higher expected profits than with the myopic solution. Wolinsky (1991) found conditions in a two-period model where a monopolist should hold inventories for strategic considerations under a continuum amount of consumers with unit demand and unknown reservation values. However, in a static model learning and information have no value. This raises the question of under what conditions the firm should overproduce.

The use of models where a firm has to commit to a price-quantity combination is becoming more popular in the literature. Li and Atkins (2005) solve a model where the headquarters of a firm is in charge of the pricing and replenishment strategy. Their finding is that both price and service levels decrease with demand variability. In contrast to the model solved in this study, they assume that the pricing decision affects the level of uncertainty, implying a correlation with the realized demand.

Chen and Levy (2004) solve a finite horizon model for a price- and quantity-setting firm and find the optimal inventory policy based on the inventory and variable cost of production. Khouja (2000) finds an optimal price strategy to sell the leftover sequentially from the first period, in a typical "newsvendor problem" setting.

In contrast with the bulk of the literature, the dynamic model in this study is solved assuming that high demand is realized with certainty in the second period. In order to maximize expected profits, the firm must make a price and production level decision simultaneously, before realizing the true demand and not solve for only one of the variables assuming the other as given or to be determined by the market. Hence, there is less strategy and flexibility involved from the firm's standpoint where none of the previous mainstream approaches can be used. For example, in this model the firm does not have the option between leasing and selling its product, as in Bulow (1986) and Tirole (2000), since the good is sold for one period. Price dispersion, as described in Wilson (1988), is not optimal either because the product is offered only once to the consumers available at that particular moment. Experimentation is also of no value to the firm, as addressed in Mirman, Samuelson and Urbano (1993), or to consumers, as in Riordan (1986) and Paredes (2006), since neither firm nor consumers gain profits or utility from static outcome deviations in order to learn about the future. Also, clearance sales are not an option for the firm as detailed in Nocke and Peitz (2005), since there is certainty in the second period regardless of production or pricing decisions. If a firm can sell its product with information on the valuation of its consumers, a price discrimination approach as described in Segal (2003) can be used. Finally, Mills (1959) find that for a one-period model, a price setter firm facing uncertainty in the demand for its product with a constant marginal cost, charges a lower price compare to the case with certainty.

Subsequently, Chapter 2 presents conditions for each of the three models assuming uncertainty in different intercepts of the demand function: the first scenario examines a uniformly higher demand, while the other two scenarios analyze uncertainty in one of the two intercepts. The latter scenarios are interpreted as uncertainty in the saturation production (vertical intercept) or consumers' reservation price (horizontal intercept). Each model is then solved in a static and dynamic setting for either a storable good or network effect. Chapter 3 solves the same three models, but assumes a price-quantity setting firm rather than a firm that can only choose one of the two variables without the inclusion of storable goods or network effects.

2 Demand Uncertainty with Storable Goods and Network Effects

2.1 Introduction

A firm faces crucial decisions regarding the pricing and output level of its product before introducing it to the market. The more information about consumers available, the more accurate these decisions will be. However, perfect information about demand is not always feasible or available. If the firm's decisions are going to affect the number of their total potential consumers, then a different approach might be necessary where the main objective is not only to maximize present profits, but to maximize future profits as well.

This study examines a firm's choices when it faces uncertainty in the demand for the good it sells. The firm must choose either a price or quantity to be produced before realizing the true demand. After making the production or pricing decisions for the first period, the firm realizes the actual demand. At the beginning of the second period, the realized demand might fluctuate but by now the firm has full information about its consumers.

The models developed address the following questions for storable goods and network effect goods, independently. By how much should the firm deviate from the myopic solution in a static setting? Is it different if the firm commits to prices or quantities in a static versus a dynamic setting? In a dynamic setting, is the firm better off by choosing prices or quantities? Finally, what are the welfare implications? The purpose of this chapter is not only to compare a myopic equilibrium to its dynamic counterpart, but also, to estimate welfare changes when the firm can choose either prices or the production level, in the presence of uncertainty and varying demand across time. As discussed in the previous section, in the case of a storable good, the number of second period consumers is reduced if sales are high in the first period, while with a network effect, the number of consumers increases in the second period if sales are high in the first.

The contribution of this study is to compare firm decisions and welfare implications when consumer preferences are modeled for storable goods and network effects, in a static and dynamic setting, using three different settings for the demand function. This provides an intuition for in which settings the firm and/or consumers are better off when the firm chooses either prices or quantities, and how assuming a storable good demand or a network effect changes the results.

The chapter is divided into three sections. Each section covers a different model with every model addressing both a static and dynamic setting. For the dynamic models there are two possible assumptions for the demand function; it is modelled either as a network effect or a storable good. The purpose of modelling demand with either of the two previous assumptions is to provide insights on how the firm and/or consumers are better off when the firm chooses either prices or quantities in different industries. For example, consider that when a firm sells a storable good, such as a car, or a network effect good, such as a cellular phone, the resulting profits the firm earns and satisfaction the consumers receive might not be the same if the firm sets prices or production level since it depends on the type of product demand. The type of demands to be considered are addressed in this chapter as follows: Section 2.2 assumes a model where there is uncertainty with uniform demands. Section 2.3 assumes uncertainty in the saturation quantity. Section 2.4 considers uncertainty in the vertical intercept with different slopes and finally, Section 2.5 summarizes the main results.

2.2 Uncertainty with Uniform Demands

The model solved in this section is used as a benchmark for the static model in the remainder of this section and the first section of Chapter 3.

Assume a profit maximizing firm chooses either the price or production level of the good it sells. Following this decision, in a traditional framework with full information the market determines the remaining variable and clears. But if the firm faces uncertainty in the demand for its good in a static setting, its optimal strategy might deviate from the full information model. It can be expected that the firm decides not take any risks due to the uncertainty and produce at the full information level of the low demand if it believes that the low demand is more likely. Nonetheless, with a possible higher uniform demand, there is the chance that the firm can sell more at a higher price.

If the firm is instead assumed to sell for two periods, then by the beginning of the second period it will have full information about the demand curve. Hence, in contrast with models where experimentation or learning by doing is involved, it will be assumed that there is known demand at the beginning of the second period, regardless of the firm's output in the first period. However, consumers change their tastes depending on the previous demand for the good. That is, the demand of the second period might adjust depending on how many consumers bought the good in the first period. But, by that point, the firm is aware and has full knowledge of the adjustment. Consider a firm maximizing expected profits where the demand is assumed to be linear in the form $q = a - p$. For simplicity, it is assumed that the uncertainty lies in the intercept of the demand function, and the slope of the two possible demand curves is normalized. The demand function can be $q = 2 - p$ (a good, or high state of demand) with probability γ, or $q = 1 - p$ (a bad, or low state of demand) with probability $1 - \gamma$. Assume a marginal cost $c \in (0, 1)$ and no fixed cost.[1] Figure (2.1) presents the two possible demands and the marginal cost. Due to the symmetry of the change in both intercepts, it can be expected that there is no difference between maximizing with respect to quantities or prices.[2]

It is assumed that the firm chooses the quantity to be produced before realizing the true state of demand and then the market determines the price. The uncertainty has a Bernoulli distribution, and the loss in profits associated with underproduction is used as a penalty instead of an explicit shortage penalty parameter.[3]

The firm maximizes expected profits as follow:

$$E(\pi)_q^s = \begin{cases} \gamma\,(2-q-c)\,q + (1-\gamma)(1-q-c)q & if\ q \in [0,1] \\ \gamma\,(2-q)\,q - cq & if\ q \in [1,2]. \end{cases} \tag{2.1}$$

The first branch of Equation (2.1) restricts the production decision to less than or equal to 1. With probability γ, the firm gets the profits of the high state of demand and with probability $1-\gamma$, profits from the low demand. The second branch is never going to be an optimal decision since it loses all the profits from the low state of demand ($\gamma = 0$); generating lower revenues and generating a cost of c per unit produced regardless of if it is sold. In order to have profits in the event of the low state of demand, the quantity to be produced must lie in the interval $[0,1]$. Maximizing the first branch of Equation (2.1) with respect to quantity yields:

$$q^* = \frac{1-c+\gamma}{2}. \tag{2.2}$$

The optimal quantity, q^*, lies between $[0,1]$ since the probability and marginal cost are also restricted by the same interval. It is also greater than or equal to the optimal

[1]The assumption of no fixed cost is only for simplicity as it would not affect any of the results.

[2]In fact, if the axis names are switched, the same model is obtained.

[3]With a shortage penalty parameter, if there are D consumers and prodution is $P < D$ then there will be a penalty imposed to the firm because $D-P$ consumers were not able to purchase the good. The profit function would then have an extra term of $\bar{c}(D-P)$ where $\bar{c}$ is the penalty for each consumer not satisfied.

output of the low demand under certainty. If the firm knows that it is facing the low demand ($\gamma = 0$) then the optimal output is $q^* = \frac{1-c}{2}$. If the firm knows that the high demand is realized ($\gamma = 1$) then the firm produces at the optimal output of the high state of demand under certainty $\left(q^* = \frac{2-c}{2}\right)$. In Equation (2.3), the optimal expected price for the static model, $E(p^*)^s$, is calculated as the weighted optimal price associated with each state of demand when q^* is the production level. The high demand has an associated price of $p_h^* = 2 - q^*$ and the low demand of $p_l^* = 1 - q^*$. Expected price is calculated as: $\gamma p_h^* + (1-\gamma)\, p_l^*$. It must be noticed that $E(p^*)^s$ also equals the optimal price of each state of demand when γ equals 0 or 1.[4]

$$E(p^*)^s = \frac{1+\gamma+c}{2}. \tag{2.3}$$

Equation (2.4) presents the static expected profits associated with the optimal output q^*. This is calculated by plugging in q^* in the first branch of Equation (2.1).

$$E(\pi^*)_q^s = \frac{(1-c)^2}{4} + \frac{\gamma}{2} + \frac{\gamma^2}{4} - \frac{\gamma c}{2} = \frac{(1+\gamma-c)^2}{4} \tag{2.4}$$

The first term of $E(\pi^*)^s$ is the monopoly profits when the low state of demand is realized with certainty. The other three terms of the profit function can be interpreted as by how much profits adjust with respect to the probabilities. If γ equals 0, we obtain profits associated with the low state of demand with certainty; if γ equals 1, then π^* equals $\frac{1}{4}(2-c)^2$ which are the profits associated with the high state of demand under certainty. Equality to the output with certainty only holds when γ equals zero. Therefore, profits with production pre-commitment are defined in the interval between the optimal profits under certainty for each state of demand.

[4] If the model is solved assuming the firm is instead committing to a price and quantity is determined by the market, the same results are obtained.

2.2.1 Storable Goods

This section solves the previous model in a dynamic setting rather than a static framework. Second period demand is modelled as a storable good. The amount of consumers in the second period are reduced depending on first period sales. Hence, the second period intercept, a_2, is defined as $a_2 = \lambda(q)a_1$, where a_1 is the first period intercept ($a_1 \subseteq \{1,2\}$). The parameter $\lambda(q)$ measures the change in the number of consumers. For a storable good $\lambda(q) \in (0,1]$, in order to represent a reduction in second period demand. The derivative of $\lambda(q)$ with respect to first period sales, measures the effect of the first period sales on second period demand. This derivative is defined in the interval $[-1,0)$ to represent a demand decrease of no more than one unit in the second period per unit sold in the first period. By the beginning of the second period, the firm has full information about demand (i.e. knows if it is high or low), but the firm has full information about $\lambda(q)$ at all times. Second period profits are defined as $\pi_2(a) = \frac{1}{4}\lambda(q)^2(a-c)^2$,where differentiating with respect to q equals $\frac{1}{2}\frac{\partial\lambda}{\partial q}\lambda(a-c)^2$.

The problem solved by the firm is:

$$\max_q E(\pi)_q^d = \gamma\left[(2-q-c)q + \delta\pi_2(2)\right] + (1-\gamma)\left[(1-q-c)q + \delta\pi_2(1)\right] \tag{2.5}$$

where δ, the discount factor for second period demand, is assumed to be in the interval $(0,1)$. If δ equals 0, the dynamic problem is identical to the static model. Also, δ is assumed to be less than 1 to have a discount on second period profits. The first branch represents the expected profits of the high state of demand plus the discounted profits that the firm will earn in the second period. Similarly, the second branch represents the expected profits with the low state of demand.

Lemma 2.1 *With a storable good and a quantity-setting firm, expected profits in a static setting are higher than the first period profits of a two-period model*

Proof. *Maximizing Equation (2.5) with respect to quantity the following results are obtained:*[5]

$$q_d = \frac{1+\gamma-c}{2} + \frac{\delta\lambda}{4}\frac{\partial\lambda}{\partial q}\left(\gamma(2-c)^2 + (1-\gamma)(1-c)^2\right) \tag{2.6}$$

$$E(p)_d = \frac{1+\gamma+c}{2} - \frac{\delta\lambda}{4}\frac{\partial\lambda}{\partial q}\left(\gamma(2-c)^2 + (1-\gamma)(1-c)^2\right), \tag{2.7}$$

which evaluated in Equation (2.5) results in associated profits of

$$E(\pi_1^*)_q^d = \frac{(1+\gamma-c)^2}{4} - \frac{1}{4}\left(\delta\lambda\frac{\partial\lambda}{\partial q}\left(\gamma(2-c)^2 + (1-\gamma)(1-c)^2\right)\right)^2 \tag{2.8}$$

since the first term of $E(\pi_1^*)_q^d$ equals π^*, and the second term is always negative, then $\pi^* \geq E(\pi_1^*)_q^d$. ■

Lemma 2.1 shows that for a two-period model and demand for a storable good, the firm will underproduce in the first period. In order to have higher second period profits, the firm sacrifices production and first period profits. Hence, profits from the static model are always higher than the first period profits from the dynamic model. While this result is trivially true, because the myopic firm can always decide to mimic the first-period production plan of the forward-looking dynamic firm, it is not immediately clear how total welfare in the first period compares across models.

In order to measure the surplus consumers obtain, consumer surplus for each state of demand is defined in the usual form: the maximum of the consumers' willingness to pay, the area below the demand curve, minus the price he actually pays, $E(p^*)^s$. For the high and low demand, these areas are defined as $q^* \int_0^{q^*} (2-q- \; E(p^*))\,dq$ and $q^* \int_0^{q^*} (1-q- \; E(p^*))\,dq$, respectively. Expected consumer surplus is the consumer surplus from the high state of demand weighted by γ, plus the consumer surplus

[5] See Appendix

corresponding to the low state of demand weighted by $1-\gamma$. For the benchmark model expected consumer surplus is defined as:

$$E(CS^*)^s_q \;=\frac{(1+\gamma-c)^2}{8}. \tag{2.9}$$

When γ equals 0 or 1, $E(CS^*)^s_q$ is the consumer surplus from the low or high state of demand, respectively. As in a classic microeconomics model, expected consumer surplus equals half of the associated expected profits, therefore all the analysis from Equation (2.4) follows through for $E(CS^*)^s_q$.

Expected welfare for the static model is defined as the sum of the static profit function from Equation (2.4), plus the expected consumer surplus from Equation (2.10):

$$E(W)^s_q = \frac{3}{8}(1+\gamma-c)^2. \tag{2.10}$$

As with the associated profits and expected consumer surplus from Equations (2.9) and (2.4), expected welfare is bound by the welfare under certainty with the low and high state of demand and they are equal when γ equals 0 or 1, respectively. To shed some light on this, Lemma 2.2 compares the associate welfare from Equation (2.10) and the first period welfare of the two-period model.

Lemma 2.2 *With a storable good and a quantity-setting firm, welfare in the static model is greater than first-period welfare from the dynamic model.*

Proof. *Expected consumer surplus from the dynamic model is:*

$$E(CS^*)^d_q \;=\frac{(1+\gamma-c)^2}{8}+A\left(\frac{1+\gamma-c}{2}+\frac{A}{2}\right) \tag{2.11}$$

where $A=\frac{1}{4}\delta\lambda\frac{\partial\lambda}{\partial q}\left(\gamma(2-c)^2+(1-\gamma)(1-c)^2\right).$

The first term of $E(CS^*)^d_q$ *equals* $E(CS^*)^s_q$. *Since* $\frac{\partial\lambda}{\partial q}$ *is contained in the interval* $[-1,0)$, *the first factor of the second term is always negative. Moreover, the second* *factor is always positive, resulting in a negative second term for* $E(CS^*)^d_q$. *Hence,* $E(CS^*)^s_q \geq E(CS^*)^d_q$. *The equality only holds when* λ, $\frac{\partial \lambda}{\partial q}$, *or* δ *equal 0.*

First period welfare in the dynamic model is defined as $W^d_q = E(\pi^*_1)^d_q + E(CS^*)^d_q$. *Then,* $W^d_q - W^s_q = A\left(q^* - \frac{A}{2}\right)$. *Since* $\frac{\partial \lambda}{\partial q} \leq 0$ *implies* $A \leq 0$, *then* $A\left(q^* - \frac{A}{2}\right) \leq 0$. *Therefore, welfare from the static model,* W^s_q, *is greater than first period welfare of the two-period model,* W^d_q. ■

Comparing a static versus a two-period model, it is obtained that profits and consumer surplus are greater in the static setting; therefore, welfare is greater with the myopic solution for the first period.

Lemma 2.1 and Lemma 2.2 can be summarized as follows: since the firm is under-producing in the two-period model, it will earn less profits than in the static setting. The change in consumer surplus is explained in the same manner; a lower production implies a higher price resulting in less welfare for consumers. Since profits and consumer surplus are lower in the first period of the dynamic model, welfare is also lower.

2.2.2 Network Effect

When the consumers demand is modelled with a network effect instead of a storable good, the more sold, the more the good will be demanded. In particular, when first period sales increase, it will result in an increase in second period demand. In contrast with the previous model, second period demand faces an augmentation instead of a reduction. The intercept of the second period is again defined as $a_2 = \lambda(q)a_1$, but since second period demand increases, $\lambda(q)$ is defined in the interval $[1, 2]$. The parameter $\lambda(q)$ cannot be less than 1 since this would represent a reduction instead of an augmentation. Restricting $\lambda(q)$ to be less than or equal to 2 lets the second period demand be modelled assuming that the increase in demand will not be more than double the first period demand. The effect of first period sales in second period demand, $\frac{\partial \lambda}{\partial q}$, has to be positive and less than 1, then each unit sold in the first period will attract at most one more unit to be demanded in the second period.

The following Lemma compares the static versus the two-period welfare when a network effect is assumed instead of a storable good demand as in the previous Lemmas.

Lemma 2.3 *With a network effect and a quantity-setting firm, first period welfare of the dynamic model is greater than welfare in the static model.*

Proof. *From Equation (2.8) it is obtained that Lemma 2.1 also holds for a network effect, even though the firm is producing more than in the static model. That is, the firm earns higher profits in the static model versus in the first-period of the dynamic model.*

However, consumer surplus from the dynamic setting is much larger. With a network effect $\frac{\partial \lambda}{\partial q} \geq 0$, *from Equation (2.11) it is obtained that the second term is always positive resulting in an opposite inequality for the consumer surpluses (in this case* $E(CS^*)_q^s \leq E(CS^*)_q^d$*).*

Comparing the two welfares in question, the difference $W_q^d - W_q^s = A\left(q^* - \frac{A}{2}\right)$ *is positive since* $A \geq 0$ *and* $q^* \geq \frac{A}{2}$. ■

Lemma 2.3 provides a result opposite of Lemma 2.2. When a network effect is assumed instead of a storable good, welfare in the first period of the two-period model is greater than welfare from the static model. Even though first period profits are always smaller than profits from the static model, consumer surplus in the dynamic model is greater. Moreover, the difference in consumer surplus is much larger than the difference in profits, resulting in greater welfare in the dynamic model. The result is explained by the tendency of the firm to overproduce with a network effect in order to have higher sales in the second period. The overproduction also results in higher consumer surplus for each period since it has an associated lower expected price which benefits consumers.

The possibility of having an unknown uniformly higher demand before the firm makes a marketing decision, makes the firm indifferent between choosing prices or production levels. The following Lemma shows this comparison.

Lemma 2.4 *With parallel demands and uncertainty in the intercept, a firm is indifferent between choosing prices or quantity.*

Proof. *When the firm chooses prices in a static setting, it solves the following problem:*

$$\max_p E(\pi)_p^s = \gamma(2-p)(p-c) + (1-\gamma)(1-p)(p-c) \tag{2.12}$$

after maximizing Equation (2.12) with respect to prices, p_s^* is obtained

$$p_s^* = \frac{1+\gamma+c}{2} \tag{2.13}$$

$$E(q^*)^s = \frac{1+\gamma-c}{2} \tag{2.14}$$

As with the quantity-setting firm model, for this case after the optimal price has been estimated, the expected production level is calculated as the weighted average of the two state of demand optimal quantities which is $\gamma(2-p_s^*)+(1-\gamma)(1-p_s^*)$. Expected profits for the price-setting firm are calculated as $\gamma(2-p_s^*)(p_s^*-c)+(1-\gamma)(1-p_s^*)(p_s^*-c)$ which simplifies to:

$$E(\pi^*)_p^s = \frac{(1+\gamma-c)^2}{4} \tag{2.15}$$

Finnaly expected consumer surplus is calculated in the same manner as before, that is as the weighted average of the consumer surplus from the high and low states of demand

$$E(CS^*)_p^s = \frac{(1+\gamma-c)^2}{8}. \tag{2.16}$$

which is the same outcome associated with the static model and a quantity-setting firm solved above.

For the dynamic model we obtain the same results since differentiating $\pi_2(a)$ with *respect to p results in* $\frac{1}{2}\frac{\partial\lambda}{\partial q}\frac{\partial q}{\partial p}\lambda(a-c)^2 = -\frac{1}{2}\lambda\frac{\partial\lambda}{\partial q}(a-c)^2$. ■

Consumers are also indifferent if they are purchasing the good from a price- or quantity-setting firm. Due to the symmetry of the model, the firm's price or production level does not change depending on which of the two variables it chooses, therefore, the benefit consumers obtain from buying the good also remains the same. The following Lemma states this result.

Lemma 2.5 *With parallel demands and uncertainty in the intercept, consumers are indifferent between purchasing from a price- or quantity-setting firm.*

Proof. *The proof follows directly from comparing Equation (2.9) and Equation (2.16).* ■

The simplicity of the proof arises from the fact that the symmetry in the model allows the variables to switch on the axis without affecting the consumer surplus. The next section shows how changing the assumption of the demand functions changes the preferences of consumers and firms, when the firm chooses price or production levels.

2.3 Uncertainty in the Saturation Quantity

This section assumes a common intercept but the uncertainty lies in the slopes, giving two possible saturation quantities. The demand function for the good produced by the firm is assumed to be linear, and for simplicity, with a normalized vertical intercept. It can be either a high (good) state of demand, $p = 1 - \underline{b}q$, with probability γ, or a low (bad) state, $p = 1 - \overline{b}q$, with probability $1-\gamma$, where $\overline{b} > \underline{b}$. Hence, the uncertainty lies in the slope of the demand function. The two states of demand have an intercept in the vertical axis at 1; prices greater than 1 will always result in no profits for the firm. The saturation quantities for each demand are $\frac{1}{\underline{b}}$ and $\frac{1}{\overline{b}}$ for the high and low state, respectively. Figure (2.2) presents the two possible states of demand with the constant marginal cost. If the output level is set above $\frac{1}{\bar{b}}$ and the realized demand is low, then the firm will make profits up to where the price equals the low demand function. If the output is set between $\frac{1}{\underline{b}}$ and $\frac{1}{\bar{b}}$ and the realized demand is high, then the firm underproduced and will have lower profits than if it would have chosen a price-quantity combination lying on the high demand. In addition, it will be assumed that the firm has a constant marginal cost $c \in (0, 1)$. Without loss of generality $\bar{b}$ is normalized to 1, and $\underline{b} \in [.4, 1]$. The assumption of having $\underline{b}$ on the defined interval is necessary for the Lemmas stated in this section.

The firm maximizes expected profit in a two-period framework. In the first period, the firm has to choose its quantity or price before realizing actual demand but knowing the possible states and their probabilities. At the beginning of the second period, the firm learns the true demand function and is able to allocate the monopoly outcome. However, the second period slope is the first period slope parametrized by a factor, λ, that depends on the first period sales;[6] hence, the second period slope is defined as $b_2 = \frac{b_1}{\lambda(q)}$. Profits in the second period are then defined as $\pi_2(b) = \frac{\lambda}{4b}(1 - c)^2$, where b could be either the high or low state slope. The parameter λ represents one of two possible case scenarios:

A storable good: if a consumer buys the good in the first period he will not consume it again in the second period; hence, demand in the second period is reduced by first period sales.

A network effect: demand in the second period will be augmented from the first period; for stationary purposes second period demand will be assumed to, at most, double the first period demand. Also, with a network effect it is assumed that an increase in first period consumption has a positive impact on second period demand.

[6]When the market clears, the quantity produced equals sales so there is no leftover.

The derivation of the myopic optimal outcome when the firm chooses prices or quantity in a static setting will be used as a benchmark. The expected profit maximization problem for the firm choosing quantity is:

$$\max_q E(\pi)_q^s = \gamma(1 - \underline{b}q) + (1 - \gamma)(1 - \bar{b}q) - cq \tag{2.17}$$

which yields the following results:

$$q_s = \frac{1 - c}{2} \frac{1}{\gamma \underline{b} + (1 - \gamma)\bar{b}} \tag{2.18}$$

$$E(p)_s = \frac{1 + c}{2} \tag{2.19}$$

The static quantity, q_s, equals the monopoly outcome of maximizing the average of the two possible states. The expected average price $E(p)_s$ is calculated as the weighted average of the optimal price for each state of demand. However, as is explained below, the optimal price for each state of demand equals $\frac{1+c}{2}$. Expected profits are derived as the sum of the profits obtained from each state of demand weighted by its respective probability. Expected consumer surplus is calculated in a similar manner as Equation (2.16):

$$E(\pi)_q^s = \frac{(1 - c)^2}{4} \frac{1}{(\gamma \underline{b} + (1 - \gamma)\bar{b})} \tag{2.20}$$

$$E(CS)_q^s = \frac{(1 - c)^2}{8} \frac{1}{(\gamma \underline{b} + (1 - \gamma)\bar{b})}. \tag{2.21}$$

Maximizing expected profits with a price-setting firm we obtain:

$$\max_p E(\pi)_p^s = \gamma \left(\frac{1 - p}{\underline{b}} \right) (p - c) + (1 - \gamma) \left(\frac{1 - p}{\bar{b}} \right) (p - c) \tag{2.22}$$

resulting in:

$$p_s = \frac{1+c}{2} \quad (2.23)$$

$$E(q)_s = \frac{1-c}{2}\left(\frac{\gamma\bar{b}+(1-\gamma)\underline{b}}{\bar{b}\underline{b}}\right) \quad (2.24)$$

From Equation (2.23) it is obtained that prices are the same, independent of which variable the firm chooses, but the expected production level is lower than the output level from the previous model as the following Lemma states.

$$E(\pi)_p^s = \frac{(1-c)^2}{4}\left(\frac{\gamma}{\bar{b}}+\frac{1-\gamma}{\underline{b}}\right) \quad (2.25)$$

$$E(CS)_p^s = \frac{(1-c)^2}{8}\left(\frac{\gamma}{\bar{b}}+\frac{1-\gamma}{\underline{b}}\right). \quad (2.26)$$

Expected profits and consumer surplus are calculated in the same manner as in Equations (2.20) and (2.21). The following Lemma compares the outcome between a price setter and quantity setter firm in a static model.

Lemma 2.6 *In a static setting, total welfare is greater when the firm sets prices instead of quantity.*

Proof. *Welfare is calculated as the sum of profits and consumer surplus. Comparing Equations (2.20) and (2.4) it can be concluded that profits and consumer surplus are greater with a price-setting firm since* $\frac{1}{\gamma\underline{b}+(1-\gamma)\bar{b}} < \frac{\gamma}{\bar{b}} + \frac{1-\gamma}{\underline{b}}$ *which can be interpreted as by how much profits are reduced in a price- or quantity-setting model. Therefore, welfare is greater with a price setter firm.* ■

The intuition of having greater welfare from a price setter firm, as stated in Lemma 2.1, is that the optimal price for the high and low state of demand are the same ($\frac{1+c}{2}$). Hence, the average price $E(p)_s$, equals the optimal price p_s. But expected quantity $E(q)_s$ is larger than the optimal solution for the quantity-setting problem, q_s, since the former is the quantity obtained after choosing the optimal price for either state of demand. The static output, q_s, is the optimal outcome for maximizing expected profits but is not the optimal output level for either of the two states of demand. This result resembles Mirman, Samuelson and Urbano (1993) where experimentation is of no value in a similar framework, and the firm would prefer to set prices instead of quantity for the same reason. The next section presents the two-period framework assuming that the firm sells a storable good. This result matches Fudenberg and Tirole (1983), where learning by doing increases first period output compared to the myopic quantity in order to have higher first period profits and shift downward constant marginal cost in the second period.

2.3.1 Storable Good

In this section, comparisons between dynamic and static models for price and quantity setters are provided when a storable good assumption is used in the second period demand. Since second period demand is reduced with a storable good, the slope parameter $\lambda(q)$, will be defined in the interval $(0, 1]$. Therefore, second period demand is bound between the first period demand and no demand at all. Moreover, an increase in first period sales has a negative effect on the second period demand which can be represented as defining the marginal effect of first period sales on second period demand, $\frac{\partial \lambda}{\partial q}$, to the interval $[-1, 0)$.

Lemma 2.7 *With a storable good and a quantity-setting firm, welfare in the static model is greater than first-period welfare in the dynamic model.*[7]

Proof. *The maximization problem in a dynamic setting is:*

$$\max_q E(\pi)_q^d = \gamma \left[[1 - \underline{b}q - c]\, q + \delta \pi_2(\underline{b}) \right] + (1 - \gamma) \left[(1 - \bar{b}q - c)q + \delta \pi_2(\bar{b}) \right] \quad (2.27)$$

[7] The proofs assume $\bar{b}$ normalized to 1, and $\underline{b} \in [.4, 1]$

resulting in

$$
\begin{aligned}
q_1 &= \frac{1}{2(\gamma\underline{b} + (1-\gamma)\bar{b})}\left(1 - c + \delta\left[\gamma\frac{\partial\pi_2(\underline{b})}{\delta q} + (1-\gamma)\frac{\partial\pi_2(\bar{b})}{\delta q}\right]\right), \quad (2.28)\\
E(p)_1 &= \frac{1-c}{2} - \frac{\delta}{2}\left[\gamma\frac{\partial\pi_2(\underline{b})}{\delta q} + (1-\gamma)\frac{\partial\pi_2(\bar{b})}{\delta q}\right]. \quad (2.29)
\end{aligned}
$$

Second period profits are defined as $\pi_2(b) = \frac{1}{4b}\lambda(q)(1-c)^2$, *and assumed to be discounted by the factor* δ. *Using the chain rule with respect to first period quantity we have*

$$
\frac{\partial\pi_2(b)}{\delta q} = \frac{\partial\lambda}{\delta q}\frac{1}{4b}(1-c)^2. \quad (2.30)
$$

The optimal dynamic first period quantity, q_1, *consists of two terms: the first is the optimal quantity for the static model solved above, the second term represents the discounted adjustment of the second period profits to changes in* q_1. *Notice that* $q_1 < q_s$ *since with a storable good the second term is negative because* $\frac{\partial\lambda}{\delta q}$ *is negative. Also,* $q_1 = q_s$ *if second period profits are not taken into account for the maximization problem (i.e.* $\delta = 0$*); or if there is no relationship between the slope parameter,* λ, *and the first period quantity* $\left(\frac{\partial\lambda}{\delta q} = 0\right)$.

Similarly, the first period expected dynamic price, $E(p)_1$, *consists of the optimal static price plus a dynamic term, that for the case of a storable good is positive, resulting in a higher dynamic price compared to the myopic solution.*

$$
E(\pi_1)^d_q = \frac{(1-c)^2}{4(\gamma\underline{b} + (1-\gamma)\bar{b})} - \frac{\delta^2}{4}\left[\gamma\frac{\partial\pi_2(\underline{b})}{\delta q} + (1-\gamma)\frac{\partial\pi_2(\bar{b})}{\delta q}\right]^2 \frac{1}{\gamma\underline{b} + (1-\gamma)\bar{b}}, \quad (2.31)
$$

where $E(\pi_1)^d_q$ *is the expected profits in the first period.* $E(\pi_1)^s_q > E(\pi)^d_q$ *since the second term of Equation (2.31) is always negative.*

Expected consumer surplus in the dynamic model is:

$$E(CS_1)^d_q = \frac{(1-c)^2}{8(\gamma\underline{b}+(1-\gamma)\bar{b})} + \frac{\delta(1-c)}{4}\left[\gamma\frac{\partial\pi_2(\underline{b})}{\delta q} + (1-\gamma)\frac{\partial\pi_2(\bar{b})}{\delta q}\right] + \qquad (2.32)$$
$$+\frac{\delta^2}{8}\left[\gamma\frac{\partial\pi_2(\underline{b})}{\delta q} + (1-\gamma)\frac{\partial\pi_2(\bar{b})}{\delta q}\right]^2 \frac{1}{\gamma\underline{b}+(1-\gamma)\bar{b}}.$$

Comparing the expected consumer surplus of the one-period versus the two-period model we obtain:

$E(CS_1)^d_q - E(CS)^s_q =$

$$\begin{aligned}
&= \frac{\delta(1-c)}{4}\left[\gamma\frac{\partial\pi_2(\underline{b})}{\delta q} + (1-\gamma)\frac{\partial\pi_2(\bar{b})}{\delta q}\right] + \\
&\quad +\frac{\delta^2}{8}\left[\gamma\frac{\partial\pi_2(\underline{b})}{\delta q} + (1-\gamma)\frac{\partial\pi_2(\bar{b})}{\delta q}\right]^2 \frac{1}{\gamma\underline{b}+(1-\gamma)\bar{b}} \\
&= \frac{\delta(1-c)}{4}\left[\gamma\frac{\partial\pi_2(\underline{b})}{\delta q} + (1-\gamma)\frac{\partial\pi_2(\bar{b})}{\delta q}\right] + \\
&\quad +\frac{\delta^2}{8}\left[\gamma\frac{\partial\pi_2(\underline{b})}{\delta q} + (1-\gamma)\frac{\partial\pi_2(\bar{b})}{\delta q}\right]^2 \frac{1}{\gamma\underline{b}+(1-\gamma)\bar{b}} \\
&= \frac{\delta(1-c)^3}{16}\frac{\partial\lambda}{\partial q}\left[\frac{\gamma}{\underline{b}} + \frac{(1-\gamma)}{\bar{b}}\right] + \\
&\quad +\frac{\delta^2}{8}\frac{\partial\lambda^2}{\partial q}\frac{(1-c)^4}{16}\left[\frac{\gamma}{\underline{b}} + \frac{(1-\gamma)}{\bar{b}}\right]^2 \frac{1}{\gamma\underline{b}+(1-\gamma)\bar{b}} \\
&= \frac{\delta(1-c)^3}{16}\frac{\partial\lambda}{\partial q}\left[\frac{\gamma}{\underline{b}} + \frac{(1-\gamma)}{\bar{b}}\right]\left[1 + \frac{\delta}{8}\frac{\partial\lambda}{\partial q}\frac{(1-c)^4}{16}\left[\frac{\gamma}{\underline{b}} + \frac{(1-\gamma)}{\bar{b}}\right]^2 \frac{1}{\gamma\underline{b}+(1-\gamma)\bar{b}}\right]
\end{aligned}$$

where $1+\frac{\delta}{128}\frac{\partial\lambda}{\partial q}(1-c)^4\left[\frac{\gamma}{\underline{b}}+\frac{(1-\gamma)}{\bar{b}}\right]^2\frac{1}{\gamma\underline{b}+(1-\gamma)\bar{b}} > 0$ *since the second term is less than 1 in absolute value.*

Hence, the same result is obtained for first period expected consumer surplus as in profits: $E(CS_1)^d_q < E(CS)^s_q$. *Therefore, first period welfare for the static setting is larger than welfare in the dynamic model.* ■

In Lemma 2.6 it is demonstrated that the firm in the dynamic setting underproduces and, as a consequence, charges a higher price compared to the myopic solution.

In order to get higher profits in the second period, the firm's underproduction leads to lower profits in the first period. In addition, with a reduction in consumer surplus in the dynamic model, we obtain lower first period welfare when the firm chooses the optimal output of the two-period maximization problem over the myopic output level. This is a similar result to Lemma 2.2 in the previous Section. The following Lemma proves that this result also holds with a price-setting firm.

Lemma 2.8 *With a storable good and a price-setting firm, welfare in the static model is greater than first-period welfare in the dynamic model.*

Proof. *In this case the firm solves the following maximization problem:*

$$\max_p E(\pi)_p^d = \gamma\left[\left(\frac{1-p}{\underline{b}}\right)(p-c)+\delta\pi_2(\underline{b})\right]+(1-\gamma)\left[\left(\frac{1-p}{\bar{b}}\right)(p-c)+\delta\pi_2(\bar{b})\right]$$

differentiating with respect to prices and making the derivative equal to zero, the optimal price is defined as:

$$\begin{aligned} p_1 &= \frac{1-c}{2}+\frac{\delta}{2}\left[\gamma\frac{\partial\pi_2(\underline{b})}{\delta p}+(1-\gamma)\frac{\partial\pi_2(\bar{b})}{\delta p}\right], && (2.33)\\ E(q)_1 &= \frac{1-c}{2}\left(\frac{\gamma\bar{b}+(1-\gamma)\underline{b}}{\bar{b}\underline{b}}\right)-\frac{\delta}{2}\left[\gamma\frac{\partial\pi_2(\underline{b})}{\delta p}+(1-\gamma)\frac{\partial\pi_2(\bar{b})}{\delta p}\right]. && (2.34)\end{aligned}$$

The expected production level was calculated as in the previous models, as the weighted average of the optimal quantity for each state of demand. Using the chain rule on second period profits this time with respect to price gives

$$\frac{\partial\pi_2(b)}{\delta p}=\frac{\partial\pi_2(b)}{\delta\lambda}\frac{\partial\lambda}{\delta q}\frac{\delta q}{\delta p}=\frac{\partial\lambda}{\delta q}\frac{(1-c)^2}{4b}\left(-\frac{1}{b}\right).$$

Then first period expected profit and consumer surplus are calculated as in Equation (2.30) and Equation (2.31):[9]

$$E(\pi_1)_p^d = \frac{(1-c)^2}{4}\left(\frac{\gamma}{\bar{b}} + \frac{1-\gamma}{\underline{b}}\right) - \frac{\delta^2}{4}\left[\gamma\frac{\partial\pi_2(\underline{b})}{\delta p} + (1-\gamma)\frac{\partial\pi_2(\bar{b})}{\delta p}\right]^2 \frac{\bar{b}\underline{b}}{\gamma\bar{b} + (1-\gamma)\underline{b}}, \tag{2.35}$$

$$\begin{aligned} E(CS_1)_p^d &= \frac{(1-c)^2}{8}\left(\frac{\gamma}{\bar{b}} + \frac{1-\gamma}{\underline{b}}\right) + \frac{\delta^2}{8}\left[\gamma\frac{\partial\pi_2(\underline{b})}{\delta p} + (1-\gamma)\frac{\partial\pi_2(\bar{b})}{\delta p}\right]^2 \frac{\bar{b}\underline{b}}{\gamma\bar{b} + (1-\gamma)\underline{b}} - \\ &\quad -\frac{\delta(1-c)}{4}\left[\gamma\frac{\partial\pi_2(\underline{b})}{\delta p} + (1-\gamma)\frac{\partial\pi_2(\bar{b})}{\delta p}\right]. \end{aligned} \tag{2.36}$$

First period expected profits with a price setter firm present a similar structure as a quantity setter firm. The first term is the static price setter profits, whereas, the second term represents the discounted profits obtained from the second period. In the case of a storable good, the second term is negative, resulting in lower first period dynamic profits.

Comparing the expected consumer surpluses of the static and dynamic model:
$E(CS_1)_p^d - E(CS)_p^s =$

$$\begin{aligned} &= \frac{\delta^2}{8}\left[\gamma\frac{\partial\pi_2(\underline{b})}{\delta p} + (1-\gamma)\frac{\partial\pi_2(\bar{b})}{\delta p}\right]^2 \frac{\bar{b}\underline{b}}{\gamma\bar{b} + (1-\gamma)\underline{b}} - \frac{\delta(1-c)}{4}\left[\gamma\frac{\partial\pi_2(\underline{b})}{\delta p} + (1-\gamma)\frac{\partial\pi_2(\bar{b})}{\delta p}\right] \\ &= \frac{\delta^2}{16}\frac{\partial\lambda}{\partial q}\frac{(1-c)^4}{16}\left(\frac{\gamma}{\underline{b}^2} + \frac{(1-\gamma)}{\bar{b}^2}\right)\left[1 + \frac{\delta(1-c)}{8}\frac{\partial\lambda}{\partial q}\left(\frac{\gamma}{\underline{b}^2} + \frac{(1-\gamma)}{\bar{b}^2}\right)\frac{\bar{b}\underline{b}}{\gamma\bar{b} + (1-\gamma)\underline{b}}\right] \end{aligned}$$

$$\textit{and}\left|\frac{\delta(1-c)}{8}\frac{\partial\lambda}{\partial q}\left(\frac{\gamma}{\underline{b}^2} + \frac{(1-\gamma)}{\bar{b}^2}\right)\frac{\bar{b}\underline{b}}{\gamma\bar{b} + (1-\gamma)\underline{b}}\right| < 1,$$

therefore $E(CS_1)_p^d < E(CS)_p^s$.

Expected consumer surplus in the dynamic setting is also smaller, hence, welfare in the static model is greater than first period welfare in the dynamic setting. ■

[9] See Appendix for proof of Equations (2.30) and (2.31)

Lemma 2.7 presents a result similar to Lemma 2.2; first period expected profit and consumer surplus in the dynamic setting, are smaller than in the static model. Hence, welfare in the myopic case is greater than first period welfare in the dynamic model. Therefore, with a storable good, regardless of whether the firm sets quantity or prices, welfare is greater in the static case versus welfare in the first period of the dynamic model.

2.3.2 Network Effect

The following two Lemmas make the same comparisons as the previous section but instead use a network effect. With a network effect, second period demand increases compared to the first period. In the model this assumption is included by assuming the slope parameter, $\lambda(q)$, is defined in the interval $[1, 2]$ so that the augmentation in second period demand is no more than double the period one demand. Consequently, an increase in first period consumption has a positive effect on second period demand; it is assumed that the demand will not increase by more than a one-to-one ratio, meaning that each unit purchased in the first period will attract, at most, one more consumer which is represented as the marginal effect of the first period sales on second period demand, $\frac{\partial \lambda}{\partial q}$, being bound in the interval $[0, 1]$.

Lemma 2.9 *With a network effect and a quantity-setting firm, first period welfare in a dynamic model is greater than the static model welfare.*

Proof. *From Equation (2.28) the dynamic quantity, q_1, is larger than the static optimal output q_s, because with a network effect, since $\frac{\partial \lambda}{\partial q}$ is positive, the second term of q_1, is also positive.*

First period profits in the dynamic setting ($E(\pi_1)_q^d$) are smaller than in the static model, but in contrast with Lemma 2.2, with a network effect first period consumer surplus from the dynamic setting is greater than consumer surplus from the static model. Moreover, $E(CS_1)_q^d - E(CS_1)_q^s > E(\pi_1)_q^s - E(\pi_1)_q^d$. Thus, first period welfare *increases in the dynamic model.* ■

Since a network effect implies that the more consumed in the first period the greater second period demand will be, the firm overproduces compared to the myopic case in order to increase second period demand. This increase in output implies a lower first period price resulting in greater expected consumer surplus. The gain in consumer surplus is greater than the losses in profits implying greater first period welfare in the dynamic scenario.

Lemma 2.10 *With a network effect and a price-setting firm, first period welfare in a dynamic model is greater than in a static model.*

Proof. *Using Equation (2.33), the dynamic price is lower than the myopic price since $\frac{\partial \lambda}{\delta q}$ is positive. Also, first period profits of the dynamic model are lower than the static expected profits. However, consumer surplus is greater and $E(CS_1)_p^d - E(CS_1)_p^s > E(\pi_1)_p^s - E(\pi_1)_p^d$, so welfare increases with a network effect.* ■

As with an output setter firm, a network effect also implies greater welfare in the dynamic case with a price setter firm because of the increase in expected consumer surplus.

A network effect gives a result opposite of a storable good: first period welfare for the dynamic setting is greater than the myopic profits regardless if the firm sets prices or quantity. However, all the previous comparisons have been between dynamic and static models. The next Lemma examines the best decision for the firm between setting price or quantity in a dynamic framework.

Lemma 2.11 *Welfare is greater when the firm sets prices instead of quantities in a two-period model.*

Proof. *Comparing Equations (2.28) and (2.33), the expected quantity obtained from the price-setting problem, $E(q)_1$, is always larger than the optimal output for the* *quantity setter problem regardless of whether it is a storable good or network effect. However, prices do not have the same behavior. With a storable good, the expected price obtained from the quantity setter problem is larger than the optimal price from the price-setting model. With a network effect the result is reversed. In terms of expected profits, the firm is better off choosing prices instead of quantity:*

$$E(\pi)_q^d - E(\pi)_p^d =$$

$$\frac{(1-c)^2}{4}\left[\frac{1}{\gamma\underline{b}+(1-\gamma)\bar{b}} - \left(\frac{\gamma}{\underline{b}}+\frac{(1-\gamma)}{\bar{b}}\right)\right] +$$
$$+\frac{\delta^2}{4}\frac{\partial\lambda^2}{\partial q}\frac{(1-c)^4}{16}\left\{\left(\frac{\gamma}{\underline{b}^2}+\frac{(1-\gamma)}{\bar{b}^2}\right)^2\frac{\bar{b}\underline{b}}{\gamma\bar{b}+(1-\gamma)\underline{b}} - \left[\frac{\gamma}{\underline{b}}+\frac{(1-\gamma)}{\bar{b}}\right]^2\frac{1}{\gamma\underline{b}+(1-\gamma)\bar{b}}\right\}$$

which with the assumed slope values is always positive.

Comparing the expected consumer surplus of the price versus quantity setter firm we get:

$$E(CS)_q^d - E(CS)_p^d =$$

$$\frac{(1-c)^2}{8}\left[\frac{1}{\gamma\underline{b}+(1-\gamma)\bar{b}} - \left(\frac{\gamma}{\underline{b}}+\frac{(1-\gamma)}{\bar{b}}\right)\right] +$$
$$+\frac{\delta(1-c)^3}{16}\frac{\partial\lambda}{\partial q}\left[\frac{\gamma}{\underline{b}}+\frac{(1-\gamma)}{\bar{b}}+\frac{\gamma}{\underline{b}^2}+\frac{(1-\gamma)}{\bar{b}^2}\right] +$$
$$+\frac{\delta}{8}\frac{\partial\lambda}{\partial q}(1-c)\left[\left[\frac{\gamma}{\underline{b}}+\frac{(1-\gamma)}{\bar{b}}\right]^2\frac{1}{\gamma\underline{b}+(1-\gamma)\bar{b}} - \left(\frac{\gamma}{\underline{b}^2}+\frac{(1-\gamma)}{\bar{b}^2}\right)^2\frac{\bar{b}\underline{b}}{\gamma\bar{b}+(1-\gamma)\underline{b}}\right].$$

The first and third terms are negative and the second is positive, however, the difference is always positive for a storable good, furthermore, if $\frac{\delta}{2}(1-c)\frac{\partial\lambda}{\delta q} > .14$*, the second term is big enough for the difference to still be positive.*

The comparison of expected consumer surplus gives a similar result except for a new constraint imposed in some of the parameters: $E(CS)_p^d > E(CS)_q^d$ *with a storable good; when a network effect is assumed, the inequality is also true when*

$\delta(1-c)\frac{\partial\lambda}{\delta q} > .28$. *The constraint on the network effect case can be interpreted as the combination of a small marginal cost and discount factor, and a relative large marginal effect of the first period production on the second period slope parameter. Therefore, welfare increases with a price setter monopolist regardless of with a storable good or network effect.* ■

Lemma 2.10 corroborates the finding in Lemma 2.1, regardless of whether it is a storable good or network effect, the firm and consumers are better off when the firm sets prices instead of quantities. This result holds in either a static or dynamic framework. As previously discussed, if the firm faces uncertainty only in the saturation quantity then when it maximizes expected profits with respect to prices, the obtained price is also the optimal price for each state of demand when the firm knows the realized demand. On the other hand, when the firm chooses quantity, the optimal output that maximizes expected profits is neither of the optimal outputs with the states of demand, resulting in lower expected profits. This analysis explains why first-period profits are always higher with a price-setting firm. Consumer surplus presents a similar behavior, with a storable good consumers are always better off with a price-setting firm since it is associated with a lower price and a larger expected output than in a quantity-setting firm. With a network effect the same inequality holds but it is restricted to having a large effect on second period sales and/or discount factor compared to the marginal cost. This restriction is explained by the consumer surplus increase associated with a network effect that compensates for the asymmetric relation of prices and quantities in the possible states of demand.

2.4 Uncertainty in the Reservation Price

The model in the previous section solved for expected profits when the slope of the possible demand curve was unknown with the same vertical intercept, thereby affecting only the saturation quantity. This section solves a similar model but assumes uncertainty in the vertical intercept. This uncertainty is interpreted as an unknown reservation price, or the price at which consumers will no longer be interested in purchasing the good since that price would be greater than the 'happiness' generated by the good. The high demand is defined as $p = 2 - 2q$, with probability γ, and the low demand as $p = 1 - q$, with probability $1 - \gamma$. The marginal cost remains constant between 0 and 1. Figure (2.3) presents the defined states of demand with the marginal cost. The vertical axis is similar to the model solved in Section 2.2 but for this scenario the firm will never produce more than 1. It also resembles the model from Section 2.3, if prices and quantities are shifted on the axis.

2.4.1 Model with Production Decision

Equation (2.37) presents the problem that the firm faces when there is uncertainty in the vertical intercept and the firm chooses its production level in a static setting.

$$\max_{q} E(\pi)_q^s = \gamma\left[2 - 2q - c\right]q + (1 - \gamma)\left[1 - q - c\right]q \tag{2.37}$$

which results in:

$$q_s = \frac{1 - c + \gamma}{2(\gamma + 1)} \tag{2.38}$$

$$E(p)_s = \frac{1 + c + \gamma}{2}. \tag{2.39}$$

For this model, the optimal quantity depends on the probability of the states of demand. Notice that q_s and $E(p)_s$ equal the optimal output level and price when the firm faces the low demand with certainty ($\gamma = 0$):

$$E(\pi)_q^s = \frac{(1-c+\gamma)^2}{4(\gamma+1)} \tag{2.40}$$

$$E(CS)_q^s = \frac{(1-c+\gamma)^2}{8(\gamma+1)}. \tag{2.41}$$

In a dynamic setting, as in the previous sections, once the firm has made its optimization decision and the market clears, the firm has full information about the demand function its product faces. For the second period the demand function is $p = a_2 - bq$, where the second period intercept is the first period intercept parametrized by $\lambda(q)$, $a_2 = \lambda(q)a_1$ with the same slope (b). Hence, second period profits are defined as $\pi_2(a) = \frac{1}{4a}\lambda(q)^2(a-c)^2$.

Lemma 2.12 *With a storable good, welfare in the static model is greater than first period welfare of the two-period model.*

Proof. *The dynamic optimization solved by the firm is:*

$$\max_q E(\pi)_q^d = \gamma\left[[2-2q-c]\,q + \delta\pi_2(2)\right] + (1-\gamma)\left[(1-1q-c)q + \delta\pi_2(1)\right] \tag{2.42}$$

resulting in the following outcome:

$$q^d = \frac{1-c+\gamma}{2(\gamma+1)} + \frac{\delta}{2(\gamma+1)}\left[\gamma\frac{\partial\pi_2(2)}{\delta q} + (1-\gamma)\frac{\partial\pi_2(1)}{\delta q}\right] \tag{2.43}$$

$$E(p)_d = \frac{1+c+\gamma}{2(\gamma+1)} - \frac{\delta}{2(\gamma+1)}\left[\gamma\frac{\partial\pi_2(2)}{\delta q} + (1-\gamma)\frac{\partial\pi_2(1)}{\delta q}\right] \tag{2.44}$$

with the following associated expected profits and consumer surplus:

$$E(\pi_1)_q^d = \frac{(1-c+\gamma)^2}{4(\gamma+1)} - [A]^2 \tag{2.45}$$

$$E(CS)_q^d = \frac{(1-c+\gamma)^2}{8(\gamma+1)} + A\left[\frac{A}{8(\gamma+1)} + 1 - c + \gamma\right] \tag{2.46}$$

$$where\ A = \delta\left[\gamma\frac{\partial\pi_2(2)}{\delta q} + (1-\gamma)\frac{\partial\pi_2(1)}{\delta q}\right].$$

As in the previous sections, profits from the static model are greater than first period profits from the two-period model. Since A is negative with a storable good and the term in parenthesis is positive then consumer surplus from the static model is greater than from the two-period model. Since profits and consumer surplus are greater in the static model, it follows that welfare is also greater. ■

Once more, a storable good implies lower first period welfare for the dynamic model. With a storable good the marginal effect of second period demand on first period sales is negative, hence, the static quantity is equal or greater than the dynamic output. This results in a greater expected dynamic price than its static counterpart. Therefore, the decrease in second period demand makes the firm produce at a lower output in the first period in order to sell more in the second period. As a consequence, consumers are worse off since they are paying a higher price which results in overall lower welfare.

In the case of a network effect, as before, it is obtained that first period welfare of the two-period model is greater than welfare in the static model. The following Lemma proves this finding for the model with uncertainty in the reservation price.

Lemma 2.13 *With a network effect first period welfare of the two-period model is greater than welfare in the static model.*

Proof. *It follows from the results of the previous Lemma that profits from the dynamic model are also going to be less than the associated profits of the static model. The change in consumer surplus, however, is not the same. Since* $\frac{\partial\lambda}{\delta q} \geq 0$, *A is positive resulting in a larger consumer surplus for the dynamic model. Moreover, the difference in welfare* $W_d - W_s = A\left[-\frac{A}{8(\gamma+1)} + 1 - c + \gamma\right]$ *is positive which results in greater welfare for the dynamic model.* ■

When a network effect is assumed, similar to previous sections, the first period welfare associated with the dynamic model is greater than the welfare from the static model. Even though profits are always going to be lower in the first period of the dynamic model, as Equation (2.31) shows, with a network effect the optimal dynamic output is greater than the static output, having an associated lower expected price which increases the consumer surplus in the first-period dynamic model.

2.4.2 Static Model with Pricing Decision

When the firm chooses prices instead of quantity, it runs the risk of pricing itself out; specifically, if the firm chooses a price greater than 1 and the realized demand is low there are no consumer willing to pay that much. The problem to be solve by the firm for this setting is:

$$\max_p E(\pi)^s_p = \gamma\,[p-c]\left(\frac{2-p}{2}\right) + (1-\gamma)\,[p-c]\max\{0, 1-p\}. \tag{2.47}$$

There are two possible solutions for this model, depending on if the firm charges a price greater or less than 1. If $p < 1$, then Equation (2.47) is:

$$\max_p E(\pi)^{s1}_p = \gamma\,[p-c]\left(\frac{2-p}{2}\right) + (1-\gamma)(p-c)(1-p). \tag{2.48}$$

When Equation (2.48) is maximized with respect to prices, the following pricing level is obtained

$$p^1_s = \frac{(2+2c-\gamma c)}{2(2-\gamma)}. \tag{2.49}$$

With a price of p_s^1, the associated expected production level and expected profits are calculated as the weighted average used in the previous models, this results in

$$E(q)_s^1 = \frac{2-2c+2\gamma c}{4} \tag{2.50}$$

$$E(\pi)_p^{s1} = \frac{(2-2c+\gamma c)^2}{8(2-\gamma)}. \tag{2.51}$$

If $p > 1$, then Equation (2.47) reduces to maximizing only in the high demand since the price exceeds the reservation price:

$$\max_p E(\pi)_p^s = \gamma(p-c)\left(\frac{2-p}{2}\right). \tag{2.52}$$

Therefore, the firm only earns profits in the case of realizing the high state of demand. The optimal outcome for this scenario is:

$$p_s^2 = \frac{2+c}{2} \tag{2.53}$$

$$E(q)_s^2 = \frac{\gamma(2-c)}{4} \tag{2.54}$$

the associated expected profits and expected consumer surplus are defined as:

$$E(\pi)_p^{s2} = \frac{\gamma}{2}\frac{(2-c)^2}{4} \tag{2.55}$$

$$E(CS)_p^{s2} = \frac{\gamma}{4}\frac{(2-c)^2}{4}. \tag{2.56}$$

As was previously stated, the firm only earns profits if the high demand is realized. Moreover, this result also extends to consumers, since a price greater than one is too expensive for them, their optimal decision is to not purchase the good, producing no benefit or surplus. The following Lemma compares the firms earnings when it sets either prices or production levels.

Lemma 2.14 *With an unknown vertical intercept the firm earns more profits when* *choosing production levels instead of prices if* $c < \frac{1}{2+\gamma}\left(2+2\gamma-\sqrt{2\gamma+2\gamma^2}\right)$.

Proof. *Comparing the associated profit functions for each case we get that* $E(\pi)_q^s$ *is always equal or greater than* $E(\pi)_p^{s1}$ *for any values of* c *and* γ. *Comparing expected profits associated with the two pricing options, it is obtained that* $E(\pi)_p^{s1} > E(\pi)_p^{s2} \Longleftrightarrow c < \frac{1}{2-\gamma}\left(2-\gamma-\sqrt{2\gamma-\gamma^2}\right) = c_1$. *Finally, comparing the profits from the quantity-setting model with the profits obtained with a price greater than 1, yields* $E(\pi)_q^s > E(\pi)_p^{s2} \Longleftrightarrow c < \frac{1}{2+\gamma}\left(2+2\gamma-\sqrt{2\gamma+2\gamma^2}\right) = c_2$. *But* c_1 *is less than* c_2, *as shown in Figure (2.4), so profits for the quantity-setting firm are always higher when the marginal cost is less than* c_2. ■

The preceding Lemma compares profits between a price and quantity-setting firm. In contrast with Lemmas 2.4 and 2.5 from Section 2.3, switching the uncertainty from the reservation price to the saturation quantity does not reflect the same change in terms of choosing prices versus quantities. That is, when the uncertainty lies in the vertical axis, the firm is better off choosing prices instead of the production level. But when the firm faces uncertainty in the horizontal intercept of the demand for its good, it is not always better off choosing quantity instead of prices. With high marginal costs (approximately .8) and almost any probability ($\gamma > .2$), the firm is better off charging a price greater than 1 rather than choosing quantities.

2.5 Conclusion

This study provides comparisons amongst three different models where the firm can choose either price or production level in a two-period model. It is assumed that demand in the second period is related to sales in the first period. This connection comes about in one of two ways: either there are network externalities in consumption across periods, or the good is storable so that consumers can purchase in either period, regardless of when exactly they wish to benefit from the consumption of the good.

In addition to the previous assumption, the firm has uncertainty about the demand for its good. This uncertainty is modelled in three different cases. When the firm faces two possible parallel demand functions, the firm and consumers are indifferent if the firm chooses prices or quantities. With a storable good, welfare from the static model is greater than first period welfare of the two-period model. If a network effect is assumed instead of a storable good, first period welfare from the dynamic model is greater than welfare from the static model since the firm will overproduce in order to increase second period sales.

A different setting presents the model when there is uncertainty in the saturation quantity (i.e. unknown horizontal intercept). The same result is obtained as in the first model except that the firm and consumers are better off when the firm chooses prices instead of quantities. This is because the optimal price for a price setter firm, under uncertainty, is the optimal price for each of the possible states of demand. Whereas, the optimal quantity that maximizes expected profits is not the optimal output for any of the possible demand functions. Consumers are also better off when the firm sets prices, and the gain in consumer surplus is even greater with the presence of a network effect.

Finally, a third model is solved where the uncertainty lies in the reservation price allowing for the possibility that the firm be priced out of the market. This uncertainty is modelled with an unknown vertical intercept. In this case, the firm is better off choosing quantity instead of price except with high marginal costs, since the repercussions for overproducing are not as severe.

Figure 2.1: Uncertainty in the Intercept with Parallel Demands

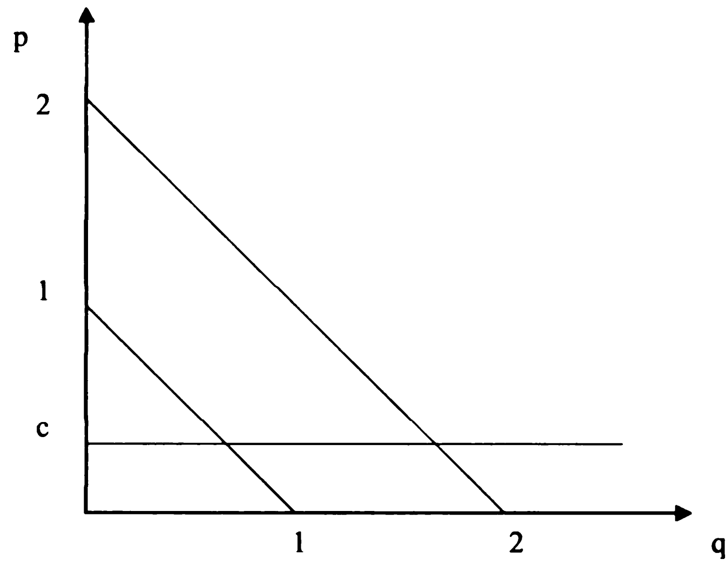


Figure 2.2: States of Demand with Uncertainty in the Saturation Quantity

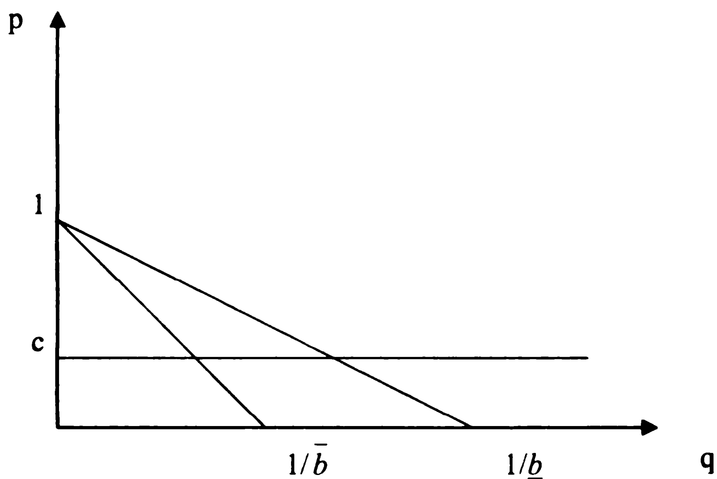


Figure 2.3: States of Demand with Uncertainty in the Reservation Price

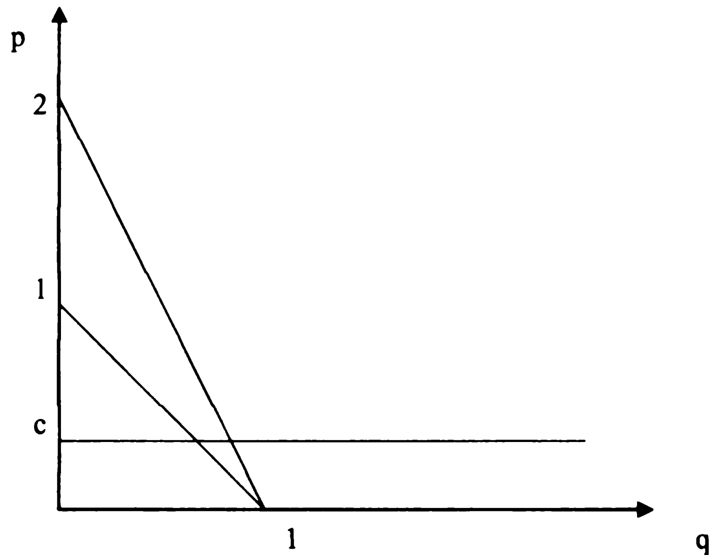
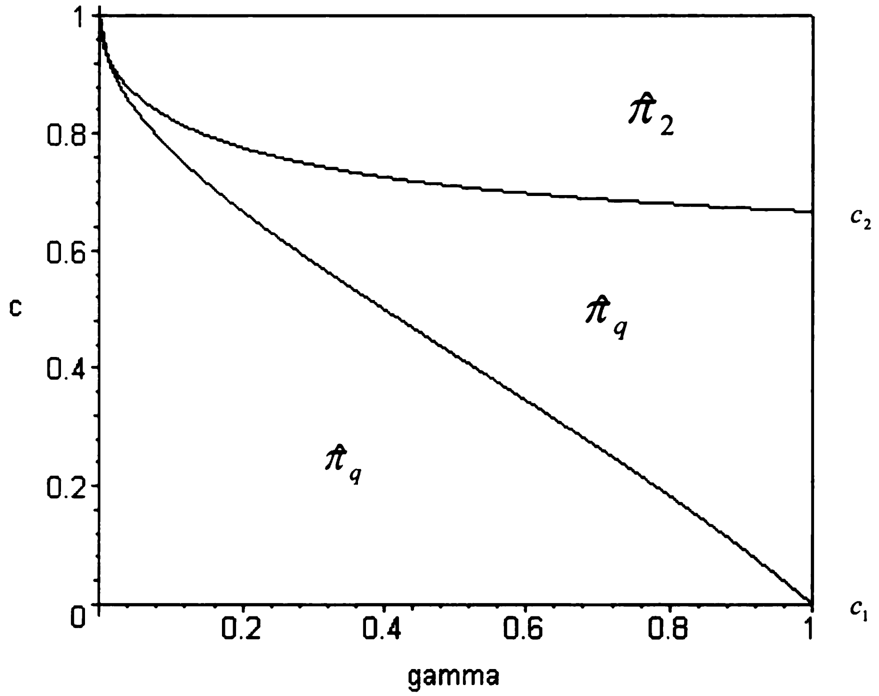


Figure 2.4: Regions for Profits with Unknown Horizontal Intercept



2.6 Appendix

Derivation of Equation 2.6-2.8. Differentiating Equation (2.5) with respect to output and setting it equal to 0, it is obtained that:

$$\frac{\partial}{\partial q}\gamma\left[(2-q-c)q+\delta\pi_2(2)\right]+(1-\gamma)\left[(1-q-c)q+\delta\pi_2(1)\right]=$$

$$\frac{\partial}{\partial q}\gamma\left[(2-q-c)q+\delta\frac{\lambda(q)^2(2-c)^2}{4}\right]+(1-\gamma)\left[(1-q-c)q+\delta\frac{\lambda(q)^2(1-c)^2}{4}\right]=$$

$$\gamma\left[2-2q-c+\frac{\partial\lambda}{\partial q}\frac{(2-c)^2}{2}+\right](1-\gamma)\left[1-2q-c+\delta\frac{\partial\lambda}{\partial q}\frac{(1-c)^2}{2}\right]=0.$$

After some algebra, q_d is obtained. Equation (2.7) is defined as $\gamma(2-q_d)+(1-\gamma)(1-q_d)$. Finally, expected profits from Equation (2.8) are calculated as:

$$E(\pi)_q^d=\gamma\left[(2-q_d-c)q_d+\delta\frac{\lambda(q_d)^2(2-c)^2}{4}\right]+(1-\gamma)\left[(1-q_d-c)q_d+\delta\frac{\lambda(q_d)^2(1-c)^2}{4}\right].$$

Derivation of Equations 2.28-2.31. Expected profits in Equation (2.27) are defined as:

$$\max_q E(\pi)_q^d=\gamma\left[\left[1-\underline{b}q-c\right]q+\delta\pi_2(\underline{b})\right]+(1-\gamma)\left[(1-\bar{b}q-c)q+\delta\pi_2(\bar{b})\right].$$

The first order condition after differentiating with respect to quantity becomes:

$$\gamma\left[1-2\underline{b}q-c+\delta\frac{\partial\pi_2(\underline{b})}{\partial q}\right]+(1-\gamma)\left[1-2\bar{b}q-c+\delta\frac{\partial\pi_2(\bar{b})}{\partial q}\right]=0.$$

After rearranging terms, q_1 is obtained. Following the same approach as before, the expected price is calculated as a weighted average of the form: $E(p)_1=\gamma(1-\underline{b}q_1)+(1-\gamma)(1-\bar{b}q_1)$.

The associated expected profits from Equation (2.30) are calculated in the same manner as for the static model of Equation (2.20). That is, in this case evaluating q_1 in Equation (2.27).

Expected consumer surplus for the dynamic model is defined as the sum of the consumer surplus from each state of demand weighted by its associated probability:: $\frac{\gamma}{2}\int_0^{q_1}[1-(1-\underline{b}q_1)q]dq + \frac{(1-\gamma)}{2}\int_0^{q_1}[1-(1-\bar{b}q_1)q]dq = \frac{\left(\gamma\underline{b}+(1-\gamma)\bar{b}\right)}{2}q_1^2$.

3 Price-Quantity Commitment Under Demand Uncertainty

3.1 Introduction

A firm facing uncertainty in the demand for its good might prefer to choose either quantities or prices in order to earn higher profits. But in some cases they have to commit to both variables in advance which brings about different issues for the firm to consider. These issues, or limitations, arise from the flexibility that is lost by having to choose a price-quantity combination; making it less likely to achieve a market clearance for its goods, as is the case when only one of the two variables is chosen.

Consider a firm launching a new advertising campaign in order to attract more consumers; if the campaign is successful there will be higher demand for the product, if the campaign fails current demand for the product will be retained. However, in many instances, the firm must choose the price and production level of the product before realizing the effect of its advertising campaign. The limitation of having to commit to production and price before entering the market can be explained by various factors: the firm might have to include the price and production data in the advertising campaign or market studies; the dealers of the product may have to make an inventory decision that depends on the price (due to budget constraints) before they realize the demand for the product; or, the firm might have to submit a forecast of assets or future sales before getting the results of the advertising campaign. As such, it faces the risk of overproducing if the campaign fails, or underproducing and then losing profits if the campaign is successful but the selected price-quantity combination was not adequate.

Another example is a car dealer that offers a new model and is not sure if it is going to attract consumers with high or low valuation for the car. The dealer has information about the tastes of consumers and their willingness to pay for the car, including how likely each state of demand is, but it must choose the price and quantity of cars to be kept at the dealership before knowing what kind of consumers are actually going to purchase it. After pricing and ordering the number of cars, consumers will go to see it, and the dealer will learn which of the two types of consumers are purchasing the vehicle.

Following the structure presented in Chapter 2, the same three models are going to be used. The first model presents a uniform case where the possibility of higher demand exists. In this case, consumers could purchase more of the good even at higher prices. The second model presents uncertainty in the reservation price that consumers are willing to pay. The third model's uncertainty lies in the saturation quantity, so that the firm does not know the amount of consumers willing to buy its product at a specific price. Instead of assuming a storable good or network effect in the second period, as the previous chapter, the models are solved when the firm has to choose a price and production level combination before knowing the realized demand. Section 3.2 solves the model where the two possible demands are parallel. Section 3.3 assumes uncertainty in the reservation price and solves the two-variable model providing comparative statics with its one-variable model counterpart. Section 3.4 assumes instead uncertainty in the saturation quantity and follows the same procedure. Section 3.5 solves the model in a two-period framework and compares it with Section 3.3. The main finding is that in the two-period framework, overproduction is more likely since there is the possibility of selling the leftover in the second period. In Section 3.6 a different variation is presented in which the static model is solved assuming the uncertainty is an additive term with an uniform distribution. Section 3.7 summarizes the main results.

3.2 Uncertainty with Uniform Demands

3.2.1 Price and Quantity Commitment

As was defined in Chapter 2, when the uncertainty lies in the intercept of the demand function, a firm has to choose either its production output or price for its product under a high or low demand as presented in Figure (2.1).

For the "newsvendor problem" from Equations (2.2) and (2.3) from Chapter 2, the optimal outcome is:

$$q^* = \frac{1-c+\gamma}{2} \tag{3.1}$$

$$p^* = \frac{1+c+\gamma}{2} \tag{3.2}$$

which yields profits of

$$\pi^* = \frac{(1-c)^2}{4} + \frac{\gamma}{2} + \frac{\gamma^2}{4} - \frac{\gamma c}{2}. \tag{3.3}$$

As was explained in Section 2.2, the result from Equation (3.1) is indifferent if the firm chooses prices or quantities, since there is no change if the variables are switched to the other axis. Using Equation (3.1) as a benchmark, the next section solves the same model when the firm commits, in advance, to a price-quantity combination.

3.2.2 Price-Quantity Commitment

The problem of committing to a price-quantity combination, as presented above, is solved in two steps. First, maximizing expected profits with respect to quantity q, assuming a given price p. Subsequently, in the second step, solving to find the optimal price.

Figure (3.1) shows a graphic representation of the high and low state of demand. If the firm chooses the price-quantity combination (p_A, q_A), the firm gains no profits if the low state is the realized demand; that is, because the associated price p_A, is greater than 1, it is too high for any consumer to purchase the product. If instead the high state is the realized demand, then the firm earns revenues of $p_A * q_A$ and profits of $(p_A - c)q_A$.

Expected profits are defined as:

$$E(\pi) = \begin{cases} (p-c)q & q \leq 1-p \\ \gamma pq + (1-\gamma)p\max\{0, 1-p\} - cq & if \quad 1-p < q < 2-p \\ \gamma p(2-p) + (1-\gamma)p\max\{0, 1-p\} - cq & q \geq 2-p \end{cases} \tag{3.4}$$

Initially, the firm could choose any price-quantity combination in the space, either in-between or outside of the two demand functions since there are no restrictions for the quantity or price. Each branch of Equation (3.4) represents the expected profits for the firm if it chooses a quantity within the defined intervals. The first branch is the profit that the firm obtains if it chooses a quantity less than or equal to the low demand. Since expected profits are linear and increasing in q, as shown in Equation (3.5), not producing at its greatest possible value results in a shortage of production. As is shown in Lemma 3.1, this branch reduces to the problem solved by a firm under certainty with the low state of demand. Similarly, the third branch is also linear but decreasing in q, in which case, the optimal choice of quantity is $q = 2 - p$. The firm cannot sell more than $q = 2 - p$, since producing more than that increases costs but not revenues.

The first term of the second branch of Equation (3.4) is the expected revenue of choosing a quantity between the two demands, if the realized state is the high demand which happens with probability γ. If the realized state is the low demand with probability $1-\gamma$, implying that the firm overproduced, then there are two cases: first, the price associated with the quantity chosen is less than 1, resulting in revenues of $p(1-p)$; or second, that the price associated with the quantity is greater than 1, resulting in zero revenues since no quantity is sold at that price. In either case the total cost is the quantity produced times the marginal cost. Notice that in the first case the firm ends up with a leftover of $q - (1 - p)$, and in the second case since nothing was sold the leftover is q (all that was produced). Because this is a static model, the leftover is disregarded for now (see Section 2.5). The following first order conditions with respect to the quantity to be produced are:

$$\frac{\partial \pi}{\partial q} \begin{cases} p - c & q \leq 1 - p \\ \gamma p - c \qquad if & 1 - p < q < 2 - p \\ -c & q \geq 2 - p. \end{cases} \tag{3.5}$$

The second branch of Equation (3.5) could be positive or negative depending on if $\gamma p - c > 0$. Figure (3.2) gives an interpretation of each case. If $\gamma p - c > 0$ (Case A), the maximum is attained at $q = 2 - p$. If $\gamma p - c < 0$ (Case B), the maximum is attained at $q = 1 - p$. Because of the linearity in quantity, the maximum is attained at one of the two states of demand. Hence, the firm is not going to choose any price-quantity combination off one of the two demand functions.

The model has three possible solutions: one under Case A, and two under Case B depending on if the given price is greater or less than 1, as implied in Equation (3.4). As previously mentioned, Case A is the regular monopoly solution under certainty with low demand. The following three Lemmas give the optimal solution for each of the three cases.

Lemma 3.1 states the condition necessary for the monopoly outcome of the low state of demand with certainty ($\widehat{p}_m$, $\widehat{q}_m$) to be the optimal price-quantity combination. Notice that this is identical to the problem solved by a firm with the low state of demand and certainty.

Lemma 3.1 *The outcome of the low state of demand with certainty is the optimal price-quantity allocation that maximizes expected profits if* $c > \frac{\gamma}{2-\gamma}$.

Proof. *As shown in Figure (3.2), if* $\gamma p - c < 0$, *then* $q = 1 - p$. *Maximizing the* *first branch of Equation (3.4), the point* $(\widehat{p}_m, \ \widehat{q}_m)$ *is obtained which gives profits of* $\widehat{\pi}_m = \frac{1}{4}(1-c)^2$. *For* $\widehat{p}_m$ *to be a maximum, the condition* $\gamma p - c < 0$ *must hold which is true with the assumption* $c > \frac{\gamma}{2-\gamma}$. ■

For Case B, and a price less than 1, we get the following result from the high state of demand.

Lemma 3.2 *The price-quantity combination* $\widehat{p}_1 = \frac{1+c+\gamma}{2}$, $\widehat{q}_1 = \frac{3-c-\gamma}{2}$ *maximizes expected profits if* $\frac{\gamma(\gamma+1)}{2-\gamma} > c$ *when a price less than 1 is chosen*

Proof. *Rewriting the second branch of Equation (3.4) we get:* $\gamma p(2-p) + (1-\gamma)p\max\{0, 1-p\} - c(2-p)$. *Assuming* $1-p>0$ *it reduces to:* $\gamma p(2-p) + (1-\gamma)p(1-p) - c(2-p)$. *Maximizing with respect to prices and evaluating at the high demand the combination* $\widehat{p}_1, \widehat{q}_1$ *is obtained. For* $\widehat{p}_1, \widehat{q}_1$ *to be a maximum, the following two conditions must be satisfied:* $\gamma \ \widehat{p}_1 - c > 0 \Longleftrightarrow \frac{\gamma(\gamma+1)}{2-\gamma} > c$. *The condition* $\frac{\gamma(\gamma+1)}{2-\gamma}$ *is increasing and convex in* γ. *Notice that* $\widehat{p}_1 \leq 1 \Longrightarrow c + \gamma \leq 1$.

The expected profits associated with this price-quantity combination are $\widehat{\pi}_1 = \frac{1}{4}\left[(1-c)^2 + \gamma^2\right] + \frac{\gamma}{2}(1+c) - c$. ■

The price-quantity combination $(\widehat{p}_1, \widehat{q}_1)$, obtained in Lemma 3.2, is only feasible when the marginal cost is relatively small compared to the probability of the high demand. Since $\frac{\gamma(\gamma+1)}{2-\gamma}$ is increasing and convex then with a high probability of the high demand, $(\widehat{p}_1, \widehat{q}_1)$ is the optimal outcome with any marginal cost.

Notice that $\widehat{p}_1$ is bound by the price of each of the states of demand under certainty. On the contrary, $\widehat{q}_1$ is greater (or equal) than the output when the high demand is realized with certainty. This is because a high price implies zero profits in case of realizing the low demand, whereas a large output would only imply a surplus without penalizing the expected profits of the firm.

The first term of the expected profits, $\widehat{\pi}_1$, is the monopoly profits with certainty from the low demand. Hence, as is shown below, when the probability of the good demand is high enough, $\widehat{\pi}_1$ is always greater than $\widehat{\pi}_m$.

The remaining possibility in Case B, assumes a price greater than 1. Now the firm is taking the risk of selling nothing if the realized state is the low demand.

Lemma 3.3 *The price-quantity combination* $\widehat{p}_2 = \frac{2\gamma+c}{2\gamma}$, $\widehat{q}_2 = \frac{2\gamma-c}{2\gamma}$, *is the optimal outcome that maximizes expected profits if* $2\gamma > c$ *and a price greater than 1 is chosen.*

Proof. *From* $\gamma p(2-p) + (1-\gamma)p\max\{0, 1-p\} - c(2-p)$ *using* $p > 1$ *and* $q = 2-p$ *the second branch can be rewritten as* $\pi = (\gamma p - c)(2-p)$. *Hence,* $\widehat{p}_2 = \arg\max_p \{(\gamma p - c)(2-p)\}$. *For* $\widehat{p}_2$ *to be a maximum, the condition* $\gamma\, \widehat{p}_2 - c > 0$ *must be satisfied, which holds with the assumption* $2\gamma > c$. *Assuming* $2\gamma > c$ *with* $\gamma > .5$, *for example, does not impose any condition on c but for other values it requires a small c for a small* γ.

For this outcome the associated profits are $\widehat{\pi}_2 = \gamma + \frac{1}{4\gamma}c^2 - c$. ■

The price $\widehat{p}_2$ obtained in Lemma 3.3 requires a strictly positive γ (there is at least a small chance of lying in the high demand) and is upper bound by price of the good state of demand with certainty.

The price-quantity combination with a price less than 1 equals $(\widehat{p}_2, \widehat{q}_2)$ when $\gamma = 1$, but since $\widehat{p}_1 \leq 1$ and $\widehat{p}_2 \geqslant 1$, this case is only feasible when $c = 0$.

Hence, depending on which of the two assumptions holds, a different solution can be found. Nevertheless, the regions that these assumptions represent overlap each other so the profit functions must be compared in order to determine the global maximum.

Lemma 3.4 *The relevant intervals for each profit function are:*

$$\begin{aligned}
\widehat{\pi}_1 &\implies c \in \left(0, \frac{\gamma(\gamma+2)}{2-\gamma}\right) \; if \;\; \gamma \leq \gamma^* and \; c \in (0, -\gamma + \sqrt{\gamma}) \; if \; \gamma \geq \gamma^* \quad (3.6)\\
\widehat{\pi}_2 &\implies c \in (-\gamma + \sqrt{\gamma}, \min\{c1, 1\}) \; and \; \gamma \geq \gamma^*\\
\widehat{\pi}_m &\implies c \in \left(\frac{\gamma(\gamma+2)}{2-\gamma}, 1\right) \; if \;\; \gamma \leq \gamma^* \; and \; c \in (c_1, 1) \; if \;\; \gamma \geq \gamma^*
\end{aligned}$$

where $\gamma^* = \arg\left\{-\gamma + \sqrt{\gamma} = \frac{\gamma(\gamma+2)}{2-\gamma}\right\} = .6 - 4\sqrt{2} \approx .34$ *and* $c_1 = \frac{1}{1-\gamma}\left(\gamma - \sqrt{\gamma - 4\gamma^2 + 4\gamma^3}\right)$.

Proof. *Comparing each of the profit function with each other we get the following conditions:*

$$\begin{aligned}
\widehat{\pi}_1 &> \widehat{\pi}_2 \implies c < -\gamma + \sqrt{\gamma}\\
\widehat{\pi}_1 &> \widehat{\pi}_m \implies c < \frac{\gamma(\gamma+2)}{2-\gamma}\\
\widehat{\pi}_1 &> \widehat{\pi}_m \implies c < c_1 = \frac{\gamma - \sqrt{\gamma - 4\gamma^2 + 4\gamma^3}}{1-\gamma} \; and \; c > c_2 = \frac{\gamma + \sqrt{\gamma - 4\gamma^2 + 4\gamma^3}}{1-\gamma}.
\end{aligned}$$

The result is obtained comparing the conditions $\left(\frac{\gamma(\gamma+1)}{2-\gamma} > c \text{ and } 1 - \gamma > c\right)$ *for each price-quantity combination to be feasible, with the three conditions obtained above.* ■

From Lemma 3.4 the global maximum can be calculated depending on the values of the marginal cost and the probability γ, as illustrated in Figure (3.3). For costs greater than approximately .22 and any γ, the choice is between the profits from the low state of demand ($\widehat{\pi}_m$), realized with certainty, and the profits obtained with a price greater than 1 ($\widehat{\pi}_2$). The constant γ^* is the threshold between the different profit expressions. Notice that at $\gamma = \gamma^*$ and $c = .24$ the three profit functions are equal.

When $\gamma < \gamma^*$, $\widehat{\pi}_2$ is not an option, this is because the good state is not likely enough for the firm to take the risk of charging a price greater than 1, not even with costs close to 1. For γ between γ^* and .5 all three profit functions are feasible and the choice depends on the marginal cost: low costs (less than approximately .22) imply $\widehat{\pi}_1$, middle costs imply $\widehat{\pi}_2$ (between .22 and c_1) and high costs result in the profits of the low demand with certainty. For values of γ above .5 (the high state of demand is more likely), the choice is only between $\widehat{\pi}_2$ and $\widehat{\pi}_1$: with low costs it is optimal to use a price less than 1 ($\widehat{p}_1$), for large costs it is optimal to charge a price greater than 1 ($\widehat{p}_2$). $\widehat{\pi}_2$ is always greater than $\widehat{\pi}_m$, making the difference nearly irrelevant when the marginal cost is between .45 and .55 and probabilities near γ^*.

Finally, a price equal to 1 is never an optimal solution. If the realized demand is low the firm has zero profits and if the realized demand is high the firm makes profits but less than the profits of the high demand with certainty. Hence, the result from maximizing profits when the high demand is realized with certainty, gives higher expected profits than a price equal to one, regardless of the probabilities of the states of demand.

In order to compare in which scenario consumers are better off, consumer surplus is calculated in two different ways. Since the market does not clear when the firm chooses a price-quantity combination there are two possible cases. If the firm overproduces then consumer surplus has the usual triangle area formula; but if there is underproduction, then only some consumers will get the good. Under efficient rationing, consumers with the highest willingness to pay will be able to purchase the good. Under least efficient rationing, consumers with the lowest valuation will have access to purchase the product. Hence, efficient rationing consumer surplus $(\overline{CS})$, is always greater than least efficient rationing consumer surplus $(\underline{CS})$, since there is more satisfaction for the consumers with the highest valuation.

The following Lemma compares the expected consumer surplus associated with each of the price-quantity combinations found above.

Lemma 3.5 *Expected consumer surplus is the greatest when the firm chooses the* *price-quantity combination* $(\widehat{p}_1, \ \widehat{q}_1)$.

Proof. *Expected consumer surplus is calculated as the consumer surplus of the associated price-quantity combination in the high state multiplied by* γ*, plus the associated consumer surplus of the low state multiplied by* $1-\gamma$*.*

The consumer surplus associated with the price-quantity combination $(\widehat{p}_m, \ \widehat{q}_m)$ *is* $E(\overline{CS})_m = \frac{\gamma}{2}\int_0^{\widehat{q}_m}(2-\widehat{p}_m-q)dq + \frac{(1-\gamma)}{2}(1-\widehat{p}_m)\ \widehat{q}_m = \frac{1}{8}(1-c)^2 + \frac{\gamma}{8}\left(\frac{3}{2}+5c-\frac{1}{2}c^2\right)$. *For the combination* $(\widehat{p}_1, \ \widehat{q}_1)$*, the associated expected consumer surplus is* $E(\overline{CS})_1 = \frac{\gamma}{2}(2-\widehat{p}_1)\ \widehat{q}_1 + \frac{(1-\gamma)}{2}(1-\widehat{p}_1)\ \widehat{q}_1 = \frac{1}{8}(1-c)^2 + \frac{(1-c)}{4} + \frac{\gamma}{4}\left(1-\frac{\gamma}{2}\right)$.

The first term of $E(CS)_1$ *and* $E(CS)_m$ *are the same; however, for* $E(CS)_1$ *to be feasible, c must be less than* $\frac{\gamma(\gamma+2)}{2-\gamma}$*. For this interval* $E(\overline{CS})_1 \geqslant E(\overline{CS})_m$*.*

For the price-quantity combination with a price greater than 1 $(\widehat{p}_2, \ \widehat{q}_2)$*, the associated expected consumer surplus is* $E(CS)_2 = \frac{\gamma}{2}(2-\widehat{p}_2)\widehat{q}_2 = \frac{1}{8}\frac{(2\gamma-c)^2}{\gamma}$ *which is always less than* $E(CS)_1$*.* ■

When the firm chooses $(\widehat{p}_1, \ \widehat{q}_1)$, it implies a price less than 1 and a production level above any of the other possible outcomes, making it the best one for consumers. If least efficient rationing is assumed instead, it would only make the difference between $E(\underline{CS})_1$ and $E(\underline{CS})_m$ even larger, since $E(\overline{CS})_m \geq E(\underline{CS})_m$. Moreover, $E(\overline{CS})_1 = E(\underline{CS})_1$ since all consumers are able to purchase the good in both scenarios, making the consumer surplus under efficient and least efficient rationing identical. Expected consumer surplus behaves similarly when a price greater than 1 is assumed instead, since all consumers are able to purchase the good only in the case of realizing the low demand.

Lemma 3.6 compares the results when the firm chooses a price-quantity combination versus when the firm commits in advance to only one of the two variables as solved in the previous two sections.

Lemma 3.6 *When a firm commits only to production levels under demand uncertainty and prices are determined by the market, profits are always higher than when* *the firm commits to a price-quantity combination before realizing the state of demand except when $\gamma \in \{0, 1\}$ or $c = 0$.*

Proof. *Since the first term of π^* is $\widehat{\pi}_m = \frac{1}{4}(1-c)^2$ and the sum of the other three terms is always positive, π^* is always larger than the profits of the low state of demand with certainty except when $\gamma = 0$ (then they are equal).*

Comparing π^* with $\widehat{\pi}_1$ we obtain:

$$\pi^* - \widehat{\pi}_1 = \frac{(1-c)^2}{2} + \frac{\gamma}{2} + \frac{\gamma^2}{4} - \frac{\gamma c}{2} - \left(\frac{(1-c)^2}{4} + \frac{\gamma}{2} + \frac{\gamma^2}{4} + \frac{\gamma c}{2} - c\right) = c(1-\gamma)$$

which is always positive except when $\gamma = 1$ and $c = 0$.

Finally, π^* is always greater than $\widehat{\pi}_2$, except when $c = \gamma + \sqrt{\gamma}$ but by the definition of $\widehat{\pi}_2$ that is not feasible, hence they are equal only when $\gamma = 1$. ■

This Lemma follows the intuition of an optimization problem of choosing one variable versus choosing two, that is, since the firm has to commit only to one variable, it has more flexibility to adjust to the uncertainty and the risk is not penalized in profits as in the two-variable model. Therefore, expected profits in the "newsvendor problem" setting gives higher profits than when the firm commits to a price-quantity combination.

3.2.3 Comparative Statics

For the static model solved in the previous sections, we obtain the following limits:

$$\begin{aligned} \lim_{\gamma \longrightarrow 0} \widehat{p}_1 &= \frac{1+c}{2} = \widehat{p}_m \\ \lim_{\gamma \longrightarrow 0} \widehat{q}_1 &= \frac{3-c}{2} = 1 + \widehat{q}_m \end{aligned}$$

When γ approaches 0 (the low state of demand is extremely likely), $\widehat{p}_1$ goes to $\widehat{p}_m$. However, $\widehat{q}_1$ is the output when the low demand is realized with certainty plus 1 because it comes from the monopoly price of the low demand with certainty evaluated on the high demand. Hence, when the firm knows that the low demand is going to be realized, its best strategy is to produce at the certainty price as it would with perfect information. The overproduction is feasible since there is no leftover cost, therefore, if the high demand is going to be realized, the firm has an output level high enough to satisfy consumers, but if demand is low the firm wastes resources at no extra cost.

When the high state of demand is most likely ($\gamma \longrightarrow 1$), $\widehat{p}_1$ and $\widehat{p}_2$ converge to the price under certainty from the high demand ($\widehat{p}_m^2$). Also, $\widehat{q}_1$ and $\widehat{q}_2$ converge to the high demand quantity with certainty ($\widehat{q}_m^2$).

$$\begin{aligned}
\lim_{\gamma \longrightarrow 1} \widehat{p}_1 &= \frac{2+c}{2} = \lim_{\gamma \longrightarrow 1} \widehat{p}_2 = \widehat{p}_m^2 \\
\lim_{\gamma \longrightarrow 1} \widehat{q}_1 &= \frac{2-c}{2} = \lim_{\gamma \longrightarrow 1} \widehat{q}_2 = \widehat{q}_m^2
\end{aligned}$$

where $\widehat{p}_m^2$, $\widehat{q}_m^2$ is the optimization result from the high demand under certainty. For this scenario, if the firm knows that the high demand is going to be realized with certainty, then the best outcome is to produce at the maximization outcome of the high state of demand with certainty. This generates a level of profits as if there was no uncertainty.

Figure (3.4) presents the results obtained above. Prices are bound by the price of each demand function with certainty. Quantity is bound between the output of the high state of demand with certainty and the output level evaluating the monopoly price of the low demand in the high demand. Figure (3.4) shows this behavior where M_1, M_2 are the outcomes with certainty for the low and high demand, respectively.

With low costs and a high probability of the high demand, the firm produces even above the optimal output of the high demand with certainty. Producing at $1 + \widehat{q}_m$

gives the firm the chance of making profits even in the case of realizing that the 'true' demand is low.

3.3 Uncertainty in the Saturation Quantity

Assume a profit maximizing firm chooses either the price or production level of the good it sells. In a traditional framework with full information, following this decision, the market determines the remaining variable and clears.

The demand function for the good produced by the firm is assumed to be linear, and for simplicity in this model with a normalized slope. It can be either a high (good) state of demand, $p = 1 - \frac{1}{2}q$, with probability γ, or a low (bad) state, $p = 1 - q$, with probability $1 - \gamma$. Hence, uncertainty lies in the slope of the demand function. This is the same model that was solved in Section 2.3, but here it is evaluated at $\bar{b} = 1$, $\underline{b} = \frac{1}{2}$. As in the previous model, it is assumed that the firm has a constant marginal cost $c \in (0, 1)$. Comparing it to the previous model, notice that unlike before the price is never going to be greater than 1; that is because the firm knows the reservation price (no consumer would buy with a price greater than 1). The quantity axis resembles the q axis from the model of the previous section, that was presented in Figure (2.2).

3.3.1 Price versus Quantity Commitment

If the firm sets quantity, then its maximization problem is the following:

$$\max_q E(\pi)_q^b = \gamma(1 - \frac{q}{2}) + (1 - \gamma)(1 - q) - cq. \tag{3.7}$$

As before, the market will clear since the firm is only choosing quantities and the price will be determined depending which of the two states of demand is realized.

The optimal quantity for the problem defined in Equation (3.7) is:

$$q_b = \frac{1-c}{2-\gamma}. \tag{3.8}$$

Notice that Equation (3.8) has the same solution as Equation (2.13) evaluated at $\bar{b} = 1$, $\underline{b} = \frac{1}{2}$. For this output the associated expected price is $E(p_b) = \frac{1+c}{2}$. It is worth recalling that the optimal expected price does not depend on the probability of the states of demand. That is because the optimal price associated with q_s for each of the states of demand is the same, hence, its expected value is itself. This result is further corroborated when the problem is solved for a price-setting firm, simplified as follows:

$$\max_p E(\pi)_p^b = \gamma 2(1-p)(p-c) + (1-\gamma)(1-p)(p-c) = (1+\gamma)(1-p)(p-c) \tag{3.9}$$

where the optimal price is $p_b = \frac{1+c}{2}$ and the associated expected output is $E(q_b) = \frac{1-c}{2}(1+\gamma)$. Again, the expected quantity, $E(q_b)$, is greater than the optimal output, q_b, from the quantity-setting problem. This is because, as was previously explained, it is associated with the optimal price from either state of demand, while the optimal output is not the optimal quantity level for either of the states of demand but for maximizing expected profits. Lemma 2.5 from Section 2.3 still holds since the only difference is that values have been assigned to the slope parameters. Hence, $E(\pi)_p^b = \frac{1}{4}(1-c)^2(2-\gamma) \geq E(\pi)_q^b$. The results obtained are used as a benchmark in order to compare a more flexible model where the firm can choose one of the variables versus a model where the firm must choose both variables before realizing the true demand.

3.3.2 Price-Quantity Commitment

In this section the firm has to set a price-quantity combination before knowing the true demand. Equation (3.10) presents the associated expected profits with this optimization problem.

$$E(\pi) = \begin{cases} (p-c)q & q \leq 1-p \\ \gamma pq + (1-\gamma)p\,(1-p) - cq & if \quad 1-p < q < 2(1-p) \\ \gamma p2(1-p) + (1-\gamma)p(1-p) - cq & q \geq 2(1-p) \end{cases} \tag{3.10}$$

In contrast to Equation (3.4), the firm will never charge a price greater than 1 and does not have the risk of ending with zero profits. The first branch of Equation (3.10) is still the same as maximizing the low state of demand under certainty. The second branch, represents the profits that the firm would earn if it chose an output level between the two possible demand functions; that is, with probability γ, the firm earns pq, and with probability $1-\gamma$, the realized state is low so that the firm earns revenues up to the point where the market clears in the low demand. With either state of demand the firm has cost, c, for each unit produced.

The first order conditions of Equation (3.10) are identical to Equation (3.5), hence, the firm has two possible solutions, one when $\gamma p - c < 0$, and the other when $\gamma p - c > 0$.

Lemma 3.7 *If $c > \frac{\gamma}{2}(\gamma+1)$, the firm achieves maximum expected profits at the optimal outcome with certainty of the low state of demand.*

Proof. *When $\gamma p - c < 0$, the firm achieves a maximum at $q = 1-p$, hence, the first and second branch of Equation (3.10) are identical providing the outcome of the low demand under certainty (i.e. $q_1^* = \frac{1-c}{2}$, $p_1^* = \frac{1+c}{2}$ and $\pi_1^* = \frac{1}{4}(1-c)^2$). When $\gamma p - c > 0$, the optimal production level must lie in the high state of demand ($q = 2(1-p)$), resulting in $q_2^* = \frac{\gamma+1-2c}{2(\gamma+1)}$, $p_2^* = \frac{\gamma+1+2c}{2(\gamma+1)}$ with associated expected profits of $\pi_2^* = \frac{1}{4(\gamma+1)}(\gamma+1-2c)^2$.*

After comparing the two expected profit expressions it is found that $\pi_1^* \geq \pi_2^* \iff c \geq \frac{\gamma}{2}(\gamma+1)$. ■

When the marginal cost is less than $\frac{\gamma}{2}(\gamma+1)$, the firm earns higher expected profits when choosing the optimal outcome of the low state of demand under certainty. Therefore, when marginal costs are high enough, the firm will choose the outcome associated with the high state of demand (p_2^*, q_2^*), regardless of the probability of realizing it. This is because with a marginal cost close to 1 and producing in the low state of demand, with probability γ, it is going to earn significantly lower profits since the price is too close to the cost, and with probability $1-\gamma$, it is underproducing, thereby wasting possible revenues. Whereas, if the firm produces in the high state of demand, and the realized state is low it will earn low profits but with probability $1-\gamma$, it will earn greater profits.

For the allocation (p_1^*, q_1^*) the associate consumer surpluses under efficient and least efficient rationing are:

$$E\left(\overline{CS}_1\right) = \frac{(1-\gamma)(1-p_1^*)q_1^*}{2} + \gamma \int_0^{q_1^*} (1 - q_1^* - p_1^*)\, dq = \frac{(2+\gamma)(1-c)^2}{16} \quad (3.11)$$

$$E\left(\underline{CS}_1\right) = \frac{(1-\gamma)(1-p_1^*)q_1^*}{2} + \frac{\gamma}{2}(1 - \frac{q_1^*}{2} - p_1^*)(2 - 2p_1^* - q_1^*) = \frac{(1-c)^2}{8} \quad (3.12)$$

Since the firm produces in the low state of demand, with probability $1-\gamma$, the market clears. With probability γ, the first q_1^* are sold under efficient rationing, and with least efficient rationing, the the last q_1^* units are sold, which are defined as $(2 - 2p_1^* - q_1^*)$. $E\left(\overline{CS}_1\right)$ implies higher expected surplus, as explained above.

When the firm chooses the combination on the high state of demand (p_2^*, q_2^*) then $E\left(\overline{CS}_2\right) = E\left(\underline{CS}_2\right) = E(CS_2)$, because the firm is producing on the high state of demand, therefore, there is no underproduction in either case (all consumers can purchase the good), but there is the possibility of the firm ending with a leftover stock.

$$E(CS_2) = \frac{\gamma}{2}(1-p_2^*)(2-p_2^*) + \frac{(1-\gamma)}{2}(1-p_2^*)^2 = \frac{(\gamma+1-2c)^2}{8(\gamma+1)} \quad (3.13)$$

Notice that $E(CS_2)$ equals half of its associated profits as is the usual case under certainty.

Comparing consumer surplus we obtain the same relationships as comparing expected profits. If $c \leq \frac{\gamma}{2}(\gamma + 1)$, consumers earn greater consumer surplus when the firm chooses the price-quantity combination in the high state of demand.

3.3.3 Comparative Statics

In this section comparative statics are provided with the myopic solution and the two possible outcomes when the firm has to commit, in advance, to a price-quantity combination. Comparing outputs, we show that the myopic solution equals q_1^* and is less than q_2^* for any marginal cost (and equal with $c = 0$). This corroborates the understanding that when the low state of demand is most likely, the firm will produce in the low state of demand, and q_1^* approaches the myopic output as the low demand becomes more likely.

$$\lim_{\gamma \longrightarrow 0} q_b = \frac{1-c}{2} = q_1^* \geq \lim_{\gamma \longrightarrow 0} q_2^* = \frac{1-2c}{2}$$

When the high state of demand is going to be realized almost with certainty, the myopic output is greater than the associated output when the firm has to commit to both variables. Therefore, when the firm has to commit to both variables, it will underproduce when the high state is most likely because of the loss of flexibility in the model.

$$\lim_{\gamma \longrightarrow 1} q_b = 1 - c \geq \lim_{\gamma \longrightarrow 1} q_2^* = q_1^*$$

Subsequently, prices follow a similar but much simpler behavior when the firm sets a price on the high demand.

$$\lim_{\gamma \longrightarrow 0} p_2^* = \frac{1+2c}{2} \geq p_b = p_1^* = \lim_{\gamma \longrightarrow 1} p_2^*$$

When $\gamma \longrightarrow 0$, the price associated with the high demand (p_2^*) is greater, however, this price has lower associated profits, as was demonstrated in the previous Lemma. Notice that when the high state of demand is most likely all prices are the same, in which case, the highest output is associated with the highest profits and, as should be expected, it is the benchmark output.

3.4 Uncertainty in the Reservation Price

This section uses the same model introduced in Section 2.4 where there is a high state of demand, $p = 2-2q$, with probability γ, and a low state of demand, $p = 1-q$, with probability $1-\gamma$. As before, if the firm overprices it risks earning zero profits. In particular, with probability $1-\gamma$, the firm earns no profits if the price is greater than 1 since the low state of demand is realized. A representation of this model is shown in Figure (2.3). In contrast to Section 2.4, the model is solved when the firm commits to prices and production levels before knowing the realized demand instead of assuming a dynamic model with storable good or network effects.

3.4.1 Price versus Production Commitment

As in the previous sections, the model where the firm chooses only prices or production levels is used as a benchmark for comparative statics. If the firm chooses production, then the optimal outcome is $q_b = \frac{1-c+\gamma}{2(\gamma+1)}$, $p_b = \frac{1+c+\gamma}{2}$. When the firm sets prices instead, there are two possible solutions, depending on if the price is greater or less than 1. The solutions are $p_b^1 = \frac{2+2c-\gamma c}{2(2-\gamma)}$ if the price is less than 1 and $p_b^2 = \frac{2+c}{2}$ if the price is greater than 1. Since a price-setting firm generates greater welfare, the prices p_b^1 and p_b^2 are going to be the benchmark.

3.4.2 Price-Quantity Commitment

This section solves the model that the firm faces when committing in advance to both variables. It is first solved finding the optimal output in terms of prices and then finding the optimal combination. For this model expected profits are defined as follows:

$$E(\pi) = \begin{cases} (p-c)q & if \;\; q \leq 1-p \\ \gamma pq + (1-\gamma)p \max\{0, 1-p\} - cq & 1-p < q < \frac{(2-p)}{2}. \end{cases} \tag{3.14}$$

Since the first order conditions are the same as Equation (3.5), there are three possible solutions: one when $\gamma p - c < 0$, with associated profits, π^*, and two with $\gamma p - c > 0$ since the price can be less (π_1^*) or greater (π_2^*) than 1. The following Lemma provides the conditions for each case.

Lemma 3.8 *The relevant intervals for each profit function are:*

$$\begin{aligned} \pi^* &\iff c \leq 2\gamma \\ \pi_1^* &\iff c \in \left(2\gamma, \frac{\gamma}{2-\gamma}\right) \\ \pi_2^* &\iff c \geq \frac{\gamma}{2-\gamma}. \end{aligned}$$

Proof. *From the first order conditions in Equation (3.5), if $\gamma p - c < 0$ then maximum profits are achieved at $q = 1-p$, in which case the first and second branch of Equation (3.14) are identical. Maximizing $(p-c)q$, with respect to prices subject to $q = 1-p$, yields $p^* = \frac{1+c}{2}$, $q^* = \frac{1-c}{2}$, and associated profits of $\pi^* = \frac{1}{4}(1-c)^2$.*

If instead $\gamma p - c > 0$, then the maximum is achieved at $q = \frac{2-p}{2}$. If the price is greater than 1, the second branch of Equation (3.14) becomes $(\gamma p - c)\left(\frac{2-p}{2}\right)$; an expression that is maximized at $p_1^* = \frac{2+c}{2(2-\gamma)}$ with the following associated output, $q_1^* = \frac{6-4\gamma-c}{2(2-\gamma)}$, and profits of $\pi_1^* = \frac{1}{8(2-\gamma)}(2-c)^2 - \frac{1}{2-\gamma}c(1-\gamma)$. If $\gamma p - c > 0$, and the price is *less than 1, the problem to be solved becomes* $\gamma p\left(\frac{2-p}{2}\right)+(1-\gamma)p(1-p)-c\left(\frac{2-p}{2}\right)$, *which results in* $p_2^* = \frac{2\gamma+c}{2\gamma}$, $q_2^* = \frac{2\gamma-c}{4\gamma}$, $\pi_2^* = \frac{1}{8\gamma}(2\gamma-c)^2$. *For the condition* $\gamma p_1^* - c > 0$ *to hold, the restriction* $c < \frac{\gamma}{2-\gamma}$ *is needed. Also, for* $\gamma p_2^* - c > 0$ *to hold, it must be true that* $c < 2\gamma$. *But* $\pi_1^* \geq \pi_2^* \geq \pi^*$; *hence the conditions are determined depending on in which interval each profit function is relevant. The proposed results are obtained since* $2\gamma \geq \frac{\gamma}{2-\gamma}$. ■

This Lemma defines the interval for each of the possible profit functions, presented in Figure (3.6). Notice that with low probabilities and high marginal costs, the firm is better off choosing the outcome of the low state of demand with certainty. As the low state of demand is more likely and there are high costs, the firm is better off not taking the risk of overproducing. When the probability of the high state of demand is high, the firm tends to produce in the high state of demand with a price less than 1. Nonetheless, for certain values in-between the firm is better off risking production in the high state of demand with a price greater than 1, since the expected profits with the high demand overcome the risk of no profits if the low demand is realized.

In order to measure the changes in consumer surplus, the same procedure as in Section 2.5 will be used. The consumer surpluses associated with (p^*, q^*) are:

$$E(\overline{CS}^*) = \frac{(1-\gamma)(1-p^*)^2}{2} + \gamma\int_0^{q^*}(2-2q-c)dq = \frac{(1-\gamma)(1-c)^2}{8} + \frac{\gamma(1-c^2)}{4}. \quad (3.15)$$

With a probability of $1-\gamma$, consumers receive the usual consumer surplus from the low state of demand. With probability of γ, they earn less than the consumer surplus of the high demand since the good is not sold to all consumers.

$$E(\underline{CS}^*) = \frac{(1-\gamma)(1-p^*)^2}{2} + \gamma\int_{q^*}^{2-2q^*}(2-2q-c)dq = \frac{(1-\gamma)(1-c)^2}{8} - \frac{\gamma(1-c)^2}{16} + \frac{\gamma c}{4} \quad (3.16)$$

As expected, consumer surplus with least efficient rationing is less than the consumer surplus associated with efficient rationing. This is due to an associated smaller area representing surplus as the good is sold to consumers with the lowest valuation in the case of high demand.

For the two solutions in the high state of demand, there is no need of rationing as all consumers are able to purchase the product. Expected consumer surplus is defined as:

$$E(CS_1^*) = \frac{(1-\gamma)(1-p_1^*)^2}{2} + \frac{\gamma(2-p_1^*)^2}{4} = \frac{(2-c)^2}{16(2-\gamma)} + \frac{\gamma(1-\gamma)}{2(2-\gamma)} \quad (3.17)$$

$$E(CS_2^*) = \frac{\gamma(2-p_2^*)^2}{4} = \frac{(2\gamma-c)^2}{16\gamma}. \quad (3.18)$$

The first expression represents consumer surplus when the firm sets a price less than 1 in the high demand, hence, there is a positive surplus for each state of demand. On the contrary, when the firm sets a price above 1, it will only sell its good if the realized demand is high, which happens only when the marginal cost is large enough $\left(c \geq \frac{\gamma}{2-\gamma}\right)$.

3.4.3 Comparative Statics

This section presents comparisons for the prices and quantities obtained when the probability of the states of demand approach the limits. When the high state is going to be realized with certainty, the price in the high state of demand is greater than the low demand price which equals its benchmark equivalent.

$$\lim_{\gamma \longrightarrow 0} p_1^* = \frac{2+c}{4} \leq \frac{1+c}{2} = p^* = \lim_{\gamma \longrightarrow 0} p_b^1$$

As shown in Figure (3.6), when $\gamma \longrightarrow 0$ the firm is better off choosing the allocation with p^*, except when $c = 0$, due to all profits being equal. If the probability approaches 1, making the high state of demand more likely, then the firm is indifferent as the optimal price of each state of demand is the same, even when the firm only chooses a price.

$$\lim_{\gamma \longrightarrow 1} p_1^* = \frac{2+c}{2} = \lim_{\gamma \longrightarrow 1} p_2^* = p_b^2 = \lim_{\gamma \longrightarrow 1} p_b^1$$

However, the firm earns higher profits with a price less than 1 in the high state of demand, as was proven in Lemma 2.8. For this case, expected profits are closest to the associated profits when the firm only chooses a price less than 1 and quantity is determined by the market.

3.5 A Two-Period Model

This section extends the model presented in Section 3.3 to a two-period model where the firm is able to sell the leftover production from the first period in the second. For simplicity, demand in the second period is assumed to be the high state with certainty. The leftover is realized at the beginning of the second period, hence, the firm solves the following problem in period 2: $\max_{p,q} pq - c(q-e)$ subject to $q = 2 - p$, where e is the leftover from the first period and p, q are the price and quantity demanded in the second period. Since e units have already been produced in the first period, the firm only has to produce $q - e$ units. The optimal outcome for this problem is:

$$\widehat{p}^2 = \frac{2+c}{2} \tag{3.19}$$

$$\widehat{q}^2 = \frac{2-c}{2} \tag{3.20}$$

which results in associated profits of

$$\widehat{\pi}^2(e) = \frac{(2-c)^2}{4} + ce. \tag{3.21}$$

The present discounted value of the inter-temporal profit function is:

$$\pi = \begin{cases} (p-c)q + \delta\, \widehat{\pi}^2(0) & if\ \ q \leq 1-p \\ \\ \gamma\left[pq + \delta\, \widehat{\pi}^2(0)\right] + & if\ \ 1-p < q < 2-p \\ +(1-\gamma)\left[p\max\{0, 1-p\} + \delta\, \widehat{\pi}^2(q - \max\{0, 1-p\})\right] - cq & \\ \\ \gamma\left[p(2-p) + \delta\, \widehat{\pi}^2(0)\right] + & if\ \ q \geq 2-p \\ +(1-\gamma)\left[p\max\{0, 1-p\} + \delta\, \widehat{\pi}^2(q - \max\{0, 1-p\})\right] - cq & \end{cases} \tag{3.22}$$

where δ is a discount factor for the profits of the second period. Following the same approach as before, assuming a given price, the profit function is linear in quantity.

The first order condition with respect to q is:

$$\frac{\partial \pi}{\partial q} = \begin{cases} p-c & q \leq 1-p \\ \gamma(p-c\ \ ) + \delta c - \gamma\delta c \quad if & 1-p < q < 2-p \\ \gamma\delta c + \delta c - \gamma\delta c - c & q \geq 2-p. \end{cases} \tag{3.23}$$

As in the previous section, the first branch of Equation (3.23) is always positive and the third is always negative. For the second branch to be positive, the following condition is needed:

$$\gamma p - c + \delta c - \gamma\delta c > 0 \Leftrightarrow \frac{\gamma p}{\gamma\delta + 1 - \delta} > c. \tag{3.24}$$

Case A (producing in the low demand) from Section 3.2 now is defined when Equation (3.5) does not hold. The following Lemma provides the optimal price and production decision when the output decision lies in the low demand.

Lemma 3.9 *The price-quantity combination* $\widehat{p}_m = \frac{1+c}{2}$, $\widehat{q}_m = \frac{1-c}{2}$, *is the optimal allocation that maximizes first-period expected profits if* $\frac{\gamma}{2\delta\gamma+2-2\delta-\gamma} > c$.

Proof. *Under Case A, the optimal choice of quantity is* $q = 1 - p$. *The result is obtained maximizing the first branch of Equation (3.22) with respect to the price, similar to Lemma 2.1 from the previous chapter.*

To assure that it is a maximum, the condition of Equation (3.24) must not hold and that is true with the assumption of $\frac{\gamma}{2\delta\gamma+2-2\delta-\gamma} > c$.

Resulting profits are the first period profits with the low demand with certainty plus the discounted high demand profits with certainty as well: $\widehat{\pi}_m = \frac{1}{4}(1-c)^2 + \frac{\delta}{4}(2-c)^2$. ■

Case B represents when Equation (3.24) holds, resulting in a quantity of $q = 2-p$. Again there are two possible solutions for Case B, depending if the price is greater or less than 1. Lemma 3.10 shows the optimal combination decision restricting the optimal price to less than 1.

Lemma 3.10 *If the optimal price is less than 1, then the price-quantity combination* $\widehat{p}_1 = \frac{1+\gamma+c}{2}$, $\widehat{q}_1 = \frac{3-\gamma-c}{2}$ *is the local maximum expected profits if* $\frac{\gamma(\gamma+1)}{2\delta\gamma+2-2\delta-\gamma} > c$.

Proof. *Assuming the condition in Equation (3.24), the optimal quantity is attained at* $q = 2 - p$. *The second branch of Equation (3.22) can be written as:*

$$\gamma p(2-p)+(1-\gamma)\left[p\max\{0,1-p\} + \delta\ c((2-p) - \max\{0,1-p\})\right]-c(2-p)+\frac{\delta}{4}(2-c)^2. \tag{3.25}$$

Assuming, $1 - p > 0$, *Equation (3.25) reduces to:*

$$\gamma p(2-p) + (1-\gamma)\left[p(1-p) + \delta\ c[(2-p) - (1-p)] - c(2-p) + \frac{\delta}{4}(2-c)^2. \tag{3.26}\right.$$

Maximizing with respect to the price, $\widehat{p}_1$ *is obtained, however, for* $\widehat{p}_1$ *to be a maximum Equation (3.24) is required which holds with the condition* $\frac{\gamma(\gamma+1)}{2\delta\gamma+2-2\delta-\gamma} > c$. *The associated profits are* $\widehat{\pi}_1 = \frac{1}{4}\left[(1-c)^2 + \gamma^2\right] + \frac{\gamma}{2}(1+c) - \delta\ \gamma c$. ■

In order to compare the outcome when a price greater or less than 1 is chosen, Lemma 3.11 solves for the second possibility of Case B. That is, a price greater than 1.

Lemma 3.11 *If the optimal price is greater than 1, then the price-quantity combination* $\widehat{p}_2 = \frac{2\gamma-\delta c+\delta\gamma c+c}{2}$, $\widehat{q}_2 = \frac{2\gamma+\delta c-\delta\gamma c-c}{2}$ *is the local maximum expected profits if* $\frac{2\gamma}{\delta\gamma+1-\delta} < c$.

Proof. *Using the condition of a price greater than 1, Equation (3.22) reduces to:*

$$\gamma p(2-p) + (1-\gamma)\delta\ c(2-p) - c(2-p) + \frac{\delta}{4}\ (2-c)^2.$$

Maximizing with respect to prices, $\widehat{p}_2$ *is obtained. For* $\widehat{p}_2, \widehat{q}_2$ *to be a maximum, Equation (3.24) must hold which is true with the assumption of* $\frac{2\gamma}{\delta\gamma+1-\delta} < c$. *The associated profits are:*

$$\widehat{\pi}_2 = \gamma + \frac{\delta^2 c^2}{4\gamma} - \frac{\delta^2 c^2}{2} - \frac{\delta c^2}{2\gamma} + \frac{\gamma\delta^2 c^2}{4} + \frac{3\gamma\delta c^2}{4} + \frac{c^2}{4\gamma} - \delta\ \gamma c - c + \delta.$$

■

Before giving an explicit solution for this model, three examples are considered with different values of δ.

- $\delta = 0$ is the one-period model case solved in the previous section. In this case, the firm gains nothing from trying to sell the leftover in the second period.
- $\delta = 1$ is the case in which there is no discount factor. The solution is:[10]

$\widehat{\pi}_1 \Longrightarrow c \in (0, -\gamma + \sqrt{\gamma})$ and $\widehat{\pi}_2 \Longrightarrow c \in (-\gamma + \sqrt{\gamma}, 1)$. Notice that $\widehat{\pi}_m$ is never chosen, even though γ could be quite small making the good state most unlikely. However, the leftover can always be sold in the second period and will give as much profit as in the first period since there is no discount factor.

[10]The intervals are found comparing the two profit functions.

- $\delta = 1/2$. The solution for this case is:

$$\widehat{\pi}_1 \implies c \in \left(0, \min\left\{\gamma + \frac{\gamma^2}{2}, \frac{4\gamma - 2\sqrt{2\gamma^2 + \gamma^3 + \gamma}}{\gamma - 1}\right\}\right) \tag{3.27}$$

$$\widehat{\pi}_2 \implies c \in \left(\frac{4\gamma - 2\sqrt{2\gamma^2 + \gamma^3 + \gamma}}{\gamma - 1}, \min\{c_3, 1\}\right) \text{ and } \gamma \geqslant \gamma^* \tag{3.28}$$

$$\widehat{\pi}_m \implies c \in \left(\gamma + \frac{\gamma^2}{2}, 1\right) \text{ if } \gamma \leq \gamma^* \text{ and } c \in (c_3, 1) \text{ if } \geqslant \gamma^* \tag{3.29}$$

where γ^* is the solution to $\gamma + \frac{1}{2}\gamma^2 = \frac{1}{\gamma - 1}\left(4\gamma - 2\sqrt{2\gamma^2 + \gamma^3 + \gamma}\right)$, and equals approximately .284.

Figure (3.7) shows these results where the dotted line is the solution for the model just solved, and the solid line is the graph from Figure 3.3 (static model). As before, the firm never chooses $\widehat{\pi}_2$ (profits with the price greater than 1) if the probability of the high state is less than γ^*. $\widehat{\pi}_1$ is the optimal choice when the marginal cost is small enough. Furthermore, this latter result is almost independent of the choice of γ. Notice that γ^* is .284 and in the static mode it was .34. Now the firm is more likely to choose the high state of demand with a price greater than 1.

Another observation emerging from Figure 5 is that for the static model when the probability of the good demand was greater than 1/2, the decision was only between $\widehat{\pi}_1$ and $\widehat{\pi}_2$. Here, with a discount factor of 1/2, $\gamma = 1/3$ is the threshold for the profits of the low demand ($\widehat{\pi}_m$). In other words, with the static model until the probability of the high state was 1/2, the outcome of the low demand with certainty was attainable. In the dynamic model, the low state of demand is feasible up to $\gamma = 1/3$.

It has been showed that the possibility of selling the leftover in the second period increases the "willingness" of the firm to choose the high state of demand.

The solution for the general model is:

$$
\begin{aligned}
\widehat{\pi}_1 &\implies c \in \left(0, \min\left\{\frac{\gamma(\gamma+2)}{2(2\delta\gamma+2-2\delta-\gamma)}, A\right\}\right) \\
\widehat{\pi}_2 &\implies c \in (A, \min\{c_4, 1\}) \text{ and } \gamma \geqslant \gamma^* \\
\widehat{\pi}_m &\implies c \in \left(\frac{\gamma(\gamma+2)}{2(2\delta\gamma+2-2\delta-\gamma)}, 1\right) \text{ if } \gamma \leqslant \gamma^* \text{and } c \in (c_4, 1) \text{ if } \gamma \geqslant \gamma^* \\
\text{where } A &= \frac{\gamma - \sqrt{\delta\gamma^3 - 2\delta^2\gamma^2 + \gamma + 2\delta\gamma^2 - 2\delta\gamma + \delta\gamma^3}}{\delta\gamma - 1 + 2\delta - \delta^2}
\end{aligned}
$$

$$
c_4 = \frac{2\delta\gamma^2 + 2\gamma - 2\gamma\delta - \sqrt{-4\gamma^3\delta + \delta\gamma^3 - 4\gamma^2 + 6\gamma^2\delta - 2\gamma^2\delta + \delta^2\gamma + \gamma + 4\gamma^3 - 2\delta\gamma}}{\delta - 2\delta^2\gamma + 1 + \gamma^2\delta^2 + 2\delta\gamma - \gamma - 2\delta}
$$

$$
\text{and } \gamma^* = \arg\max_{\gamma} \left\{\frac{\gamma(\gamma+2)}{2(2\delta\gamma+2-2\delta-\gamma)}, \frac{\gamma - \sqrt{\delta\gamma^3 - 2\delta^2\gamma^2 + \gamma + 2\delta\gamma^2 - 2\delta\gamma + \delta\gamma^3}}{\delta\gamma - 1 + 2\delta - \delta^2}\right\}.
$$

3.6 The Model with a Continuous Distribution

As a last extension, the model from Section 3.3 is solved using an uniform distribution instead of a Bernoulli distribution. Consider the following demand function: $q^D = \frac{3}{2} - p + \epsilon$ with $\epsilon \backsim U\left[-\frac{1}{2}, \frac{1}{2}\right]$. Profits are maximized in the region between the same two demand functions from Section 3.2. Since for this section a continuous distribution is used instead, there is not only a high or low demand, but rather the realized demand could be any curve with a normalized slope in the interval between the high and low states of demand.

Profits are defined as:

$$
\pi = \max_{p,q} \left\{p \min\left\{q^D, q^S\right\} - cq^S\right\} = \max_{p,q} \left\{p \min\left\{\frac{3}{2} - p + \epsilon, q^S\right\} - cq^S\right\} \quad (3.30)
$$

where q^D is the quantity demanded and q^S is the quantity supplied or produced.

Assume that q^S must be chosen before realizing ϵ. Hence, if $q^D > q^S$, the firm is underproducing, which means that it could have had higher profits by increasing the quantity supplied. If $q^D < q^S$, the firm is overproducing and ends up with a leftover of $q^S - q^D$. As before, the model is solved for the production level assuming a given price, and then solving for the optimal price.

Since the stochastic term is distributed with an uniform distribution in the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$, the smallest possible value for the quantity demanded q^D is in the low state of demand $1 - p$; therefore, if $q^S < 1 - p$, then $\pi = pq^S - cq^S = (p - c)q^S$.

If we take the largest possible value of ϵ, then q^D is $2 - p$. Now, if $q^S > 2 - p$, then

$$\pi = pq^D - cq^S = p\left(\frac{3}{2} - p + \epsilon\right) - cq^S. \tag{3.31}$$

Hence, expected profits are defined as:

$$E_\epsilon(\pi) = \begin{cases} (p-c)q^S & q^S \leq 1-p \\ pq^S \Pr(q^S < q^D) + p\left(\frac{3}{2} - p\right)\Pr(q^S > q^D) - cq^S & if \quad 1-p < q^S < 2-p \\ p\left(\frac{3}{2} - p\right) - cq^S & q^S \geq 2-p \end{cases} \tag{3.32}$$

where $\Pr(q^S < q^D) = 2 - q^S - p$, $\Pr(q^S > q^D) = 1 - \Pr(q^S < q^D) = q^S + p - 1$.

After some algebra we can rewrite Equation (3.32) as:

$$E_\epsilon(\pi) = \begin{cases} (p-c)q^S & q^S \leq 1-p \\ \frac{3}{2}p + \frac{7}{2}pq^S - p(q^S)^2 - 2p^2q^S + \frac{p^2}{2} - p^3 - cq^S & if \quad 1-p < q^S < 2-p \\ p\left(\frac{3}{2} - p\right) - cq^S & q^S \geq 2-p \end{cases} \tag{3.33}$$

As in the discrete case, the first branch of Equation (3.33) is an increasing line with slope of $p - c$. The third branch is a decreasing line with slope of $-c$. The second branch could go either way so it must be maximized with respect to q to find the optimal output. The first and second derivatives with respect to the quantity produced are:

$$\frac{\partial E_\epsilon(\pi)}{\partial q^S} = \frac{7}{2}p - 2pq^S - 2p^2 - c, \quad \frac{\partial^2 E_\epsilon(\pi)}{\partial^2 q^S} = -2p < 0 \tag{3.34}$$

$$\frac{\partial E_\epsilon(\pi)}{\partial q^S} = 0 \Longrightarrow \widehat{q} = \frac{\frac{7}{2}p - 2p^2 - c}{2p}. \tag{3.35}$$

From the second derivative we know that this turning point is a maximum. However, it must be verified that this maximum is achieved in the interval $(1-p, 2-p)$. The upper bound $\widehat{q} \leq 2-p$ always holds. The lower bound has the following restriction: $1 - p \leq \widehat{q} \Longleftrightarrow c \leq \frac{3}{2}p$.

To get the optimal price, $\widehat{q}$ must be replaced on the second branch of the profit function, hence, profits depend on price and cost from above:

$$\pi(p) = \frac{25p^2 - 16p^3 - 28pc + 16p^2c + 4c^2}{16p}. \tag{3.36}$$

Taking the first order derivative of this new profit function we obtain:

$$\frac{\partial \pi(p)}{\partial p} = \frac{25p^2 - 32p^3 + 16p^2c - 4c^2}{16p^2} = 0 \Longleftrightarrow 25p^2 - 32p^3 + 16p^2c - 4c^2 = 0 \tag{3.37}$$

Two of the three roots of the polynomial are ruled out for been imaginary numbers. The remaining third root is:

$$\widehat{p} = \frac{A^{2/3} + 625 + 800c + 256c^2 + 25A^{1/3} + 16cA^{1/3}}{A^{1/3}} \tag{3.38}$$

where $A = -36096c^2 + 15625 + 30000c + 4096c^3 +$

$$+192\sqrt{-3c^2(15625 + 30000c - 8448c^2 + 4096c^3)}.$$

The optimal price with certainty is defined as $p^M = \frac{3+2c}{2}$, $q^M = \frac{3-2c}{2}$ and $\pi^M = \frac{1}{2}(3-2c)^2$. Figure (3.8) shows the price obtained above compared to the monopoly price under certainty. Even though the obtained optimal price $\widehat{p}$, has a complicated expression, it resembles a linear function that intersects the price with certainty exactly at $c = .5$. Another important observation is that $\widehat{p}$ is greater than p^M for small values of c. In other words, the obtained pricing path charges a price greater than p^M when the cost is less than .5.

Evaluating $\widehat{p}$ using Equation (3.38), the resulting quantity is:

$\widehat{q} = (-2059776c^2 + 484375 + 680000c + 409600c^3 + 7552\sqrt{3B} + 19375C^{1/3} + 14800cC^{1/3} + 13568C^{1/3}c^2 + 775C^{2/3} - 1440cC^{2/3} - 65536c^4 - 256C^{2/3}c^2 - 2048c\sqrt{3B} - 4096c^3C^{1/3} - 64C^{1/3}\sqrt{3B})/D$

$B = -c^2(15625 + 30000c - 8448c^2 + 4096c^3)$

$C = -36096c^2 + 15625 + 30000c + 4096c^3 + 193\sqrt{3B}$

$D = 32C^{1/3}(C^{2/3} + 625 + 800c + 256c^2 + 25C^{1/3} + 16cC^{1/3})$.

Figure (3.9) shows the obtained quantity plotted when there is certainty. The same behavior is observed for quantity as for prices, as was shown in Figure (3.8). Choosing to produce above the output with certainty when the cost is sufficiently small and determining a threshold intersection at $c = .5$.

Evaluating the price from Equation (3.36), we attain the profit function in terms of the marginal cost. Figure (3.10) shows this profit function, $\widehat{\pi}$, and the profit function under certainty. $\widehat{\pi}$ is always higher than the profits under certainty making the difference diminutive when c is between .45 and .55.

As previously mentioned, with low marginal costs the firm should produce and charge a price greater than the price under certainty, taking the risk to achieve higher expected profits. For example at $c = 0$, the firm should have a price of .78 this is greater than p^M (.75) and a quantity of .96 (q^M is .75). This will lead to expected profits of .61 which is also greater than the associated profits with certainty (.56). However, if an arbitrarily large c such as .8 is chosen instead, from the graphs we can tell that the price and quantity are lower than the monopoly outcomes, but expected profits are still going to be higher than π^M.

3.7 Conclusion

When a firm faces uncertainty in the demand curve and must commit to a price-quantity combination before realizing the true state of demand, expected profits are higher when the pre-commitment involves only the production level, as in the "newsvendor problem", assuming a possible uniformly higher demand. The exception to this is when one of the possible states is going to be realized with certainty or there is no marginal cost. In the pre-commitment case, expected profits are always greater than what a firm would earn with the low state of demand and certainty. Moreover, the optimal price lies between the monopoly prices of each state of demand under certainty. The optimal production level lies above the production level with certainty of the low state of demand, but can be much greater than the output of the high state of demand.

When the uncertainty lies instead in the reservation price or saturation output, conditions are found where it is best to choose the optimal outcome of the low state of demand with certainty, or to choose a combination in the high state of demand with the risk of overproducing and, in some cases, zero profits due to overpricing.

Dynamic and continuous distribution models are considered as extensions for the case of a possible uniformly higher demand. Compared to the discrete case, the choice of choosing a price-quantity combination in the high state of demand increases when a dynamic model is considered instead of a static one. To be more precise, in the dynamic game, the low demand optimal outcome with certainty is less likely to be chosen since the firm can better bear the risk of overproduction by selling the leftover in the second period, making it more likely to charge a price above 1. The static model shows an expected result: when the probability of the high demand is greater than .5, the solution lies on the high state of demand which converges to the high demand outcome with certainty as the probability of high demand increases. However, in the dynamic model a probability greater than 1/3 is enough for the same shift when a discount factor of 1/2 is used. The continuous static model shows also that the firm is always worse off choosing the outcome under certainty of the low state of demand rather than the optimal combination of the static model.

Figure 3.1: High and Low States of Demand

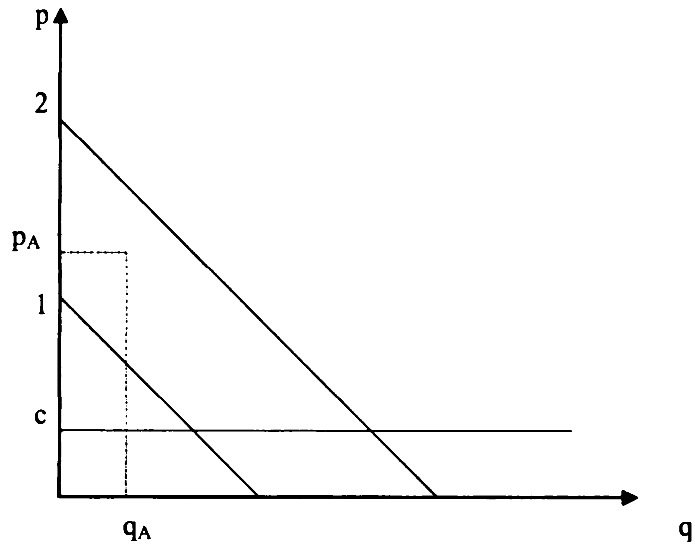
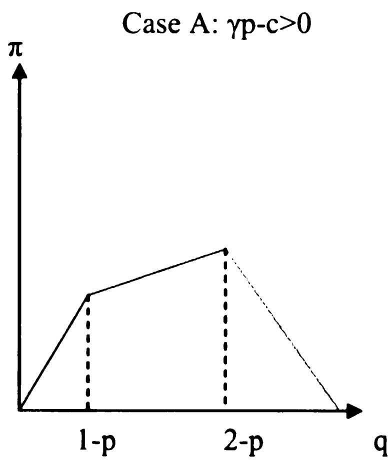


Figure 3.2: Profits as a Function of Produced Quantity



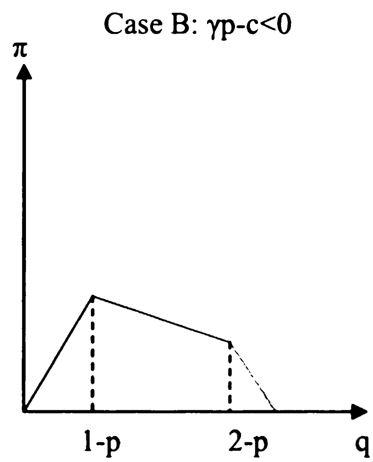


Figure 3.3: Regions for the Profit Functions

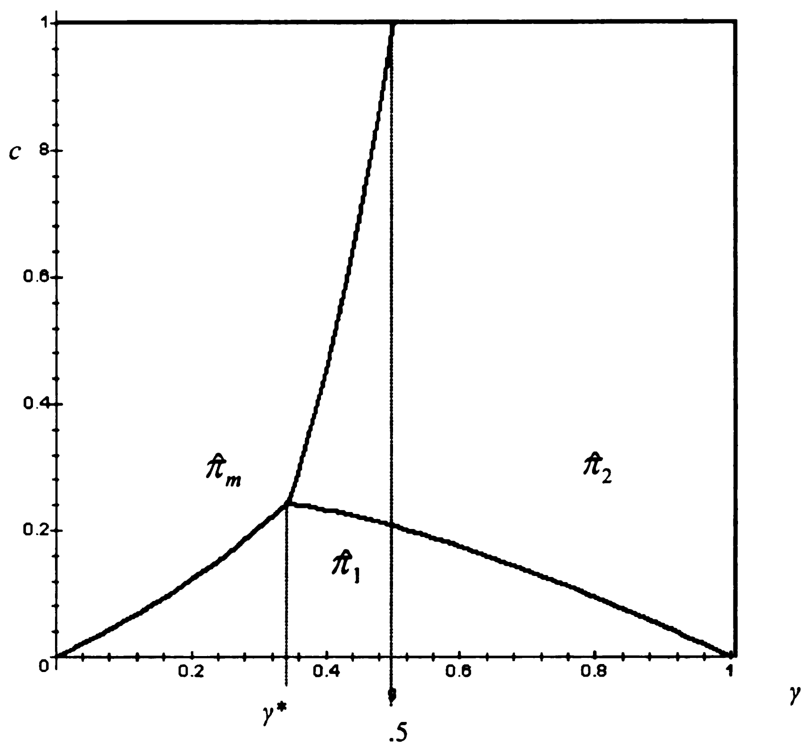


Figure 3.4: Optimal Pricing Path

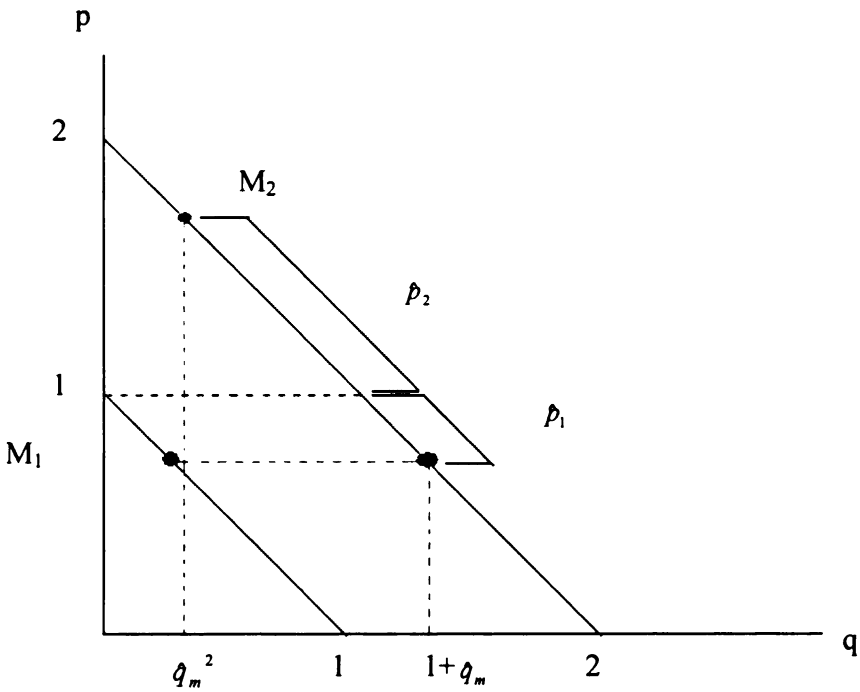


Figure 3.5: Myopic versus High State Profits

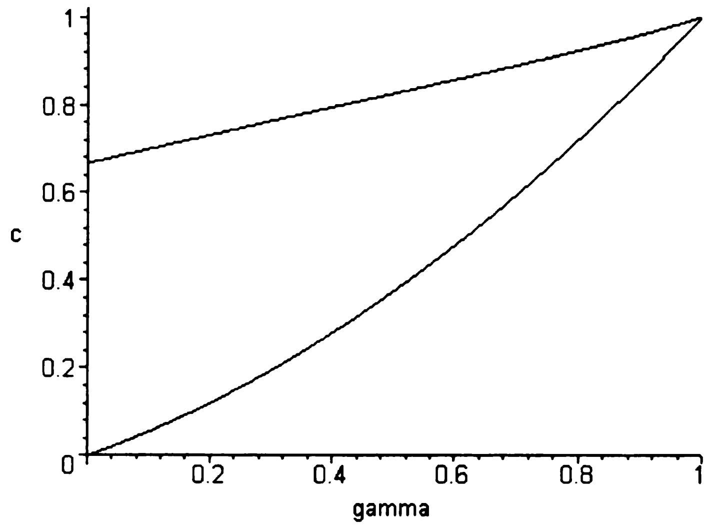


Figure 3.6: Intervals for the Profit Functions with Uncertainty in the Reservation Price

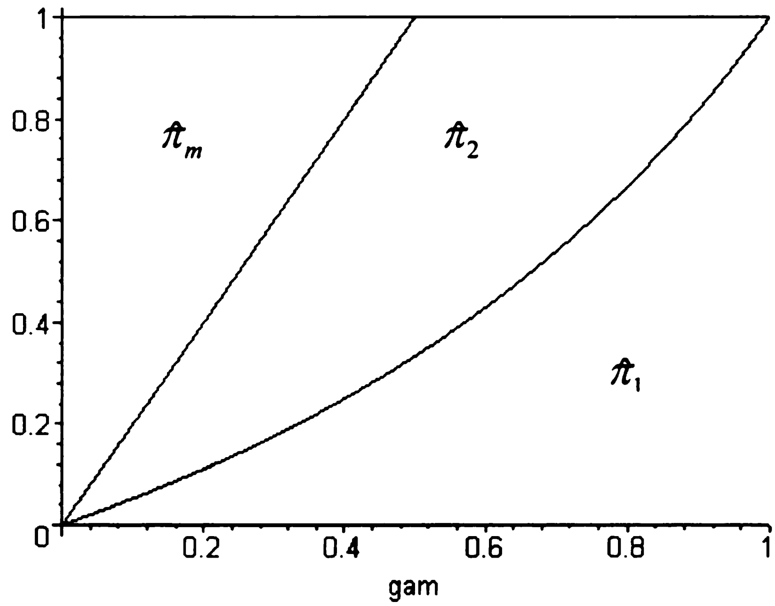


Figure 3.7: Profit Functions for the Static and Dynamic Model

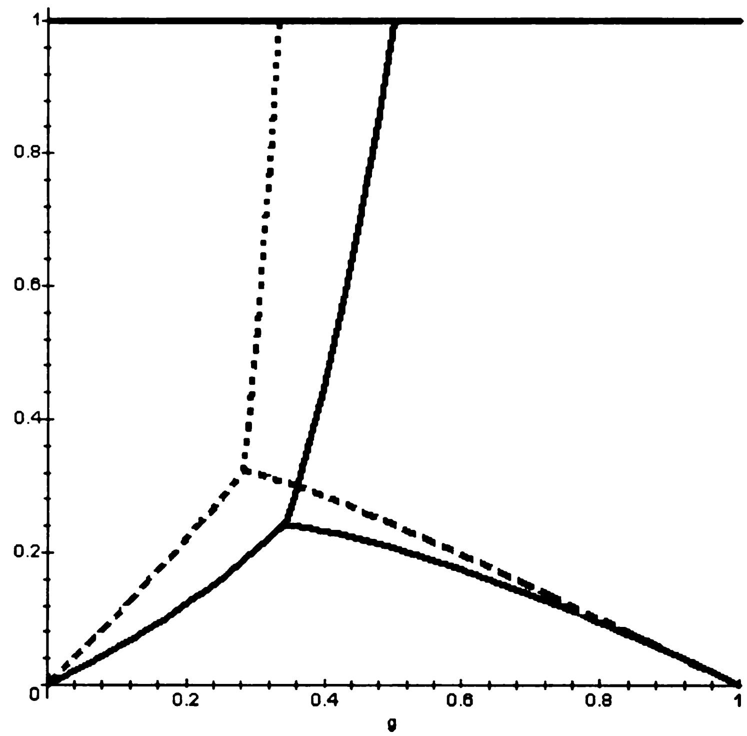


Figure 3.8: Optimal versus Monopoly Price

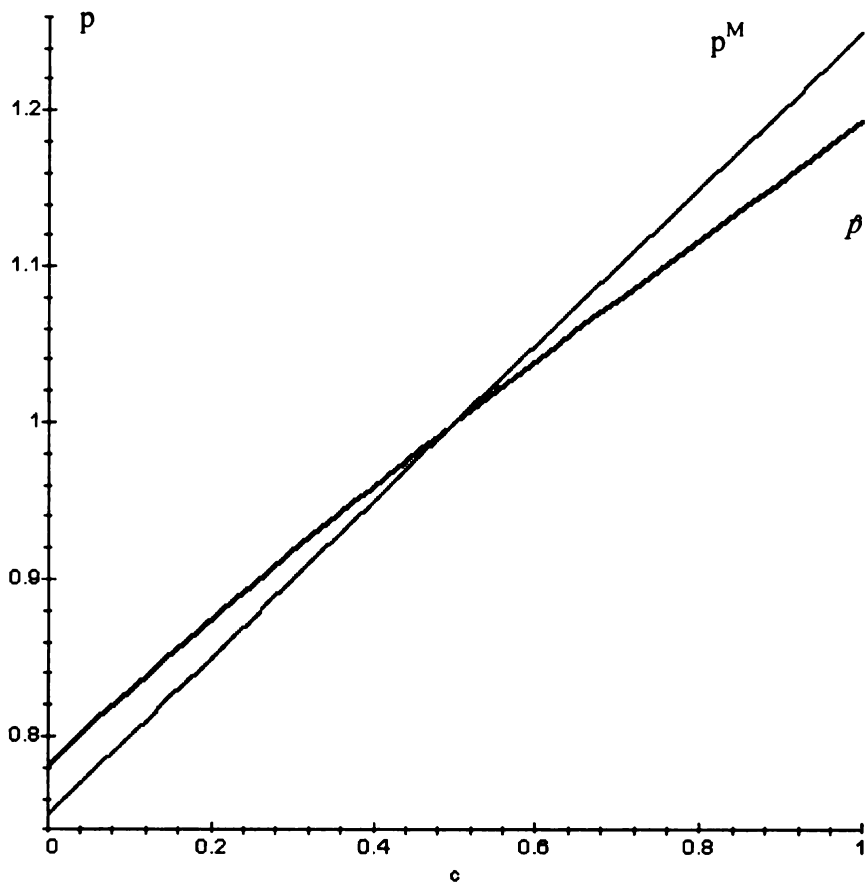


Figure 3.9: Optimal versus Monopoly Quantity

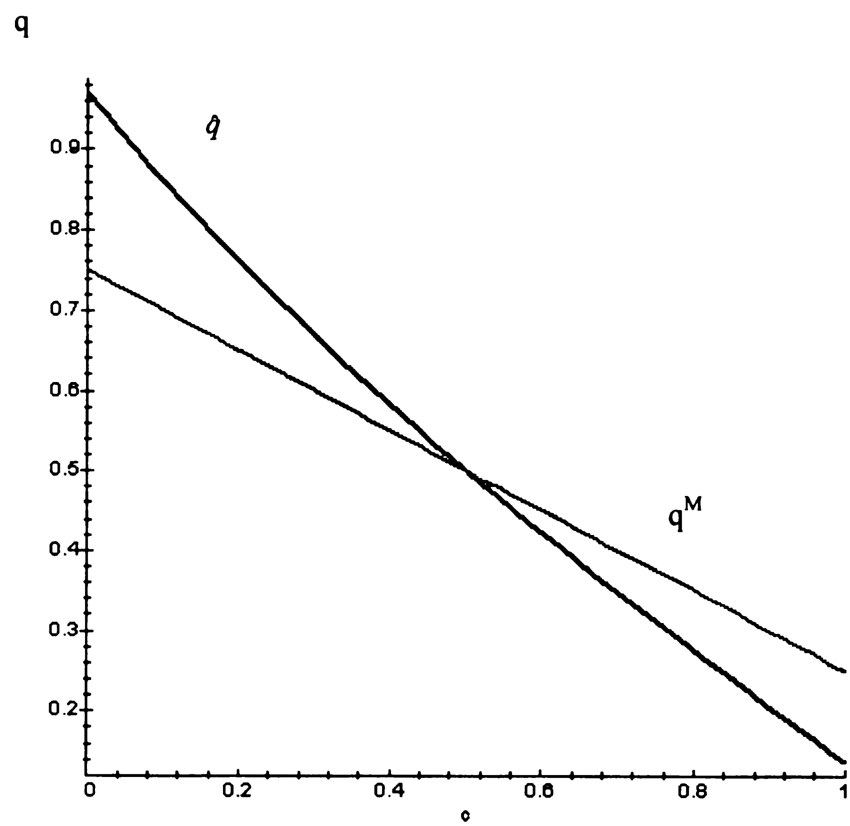
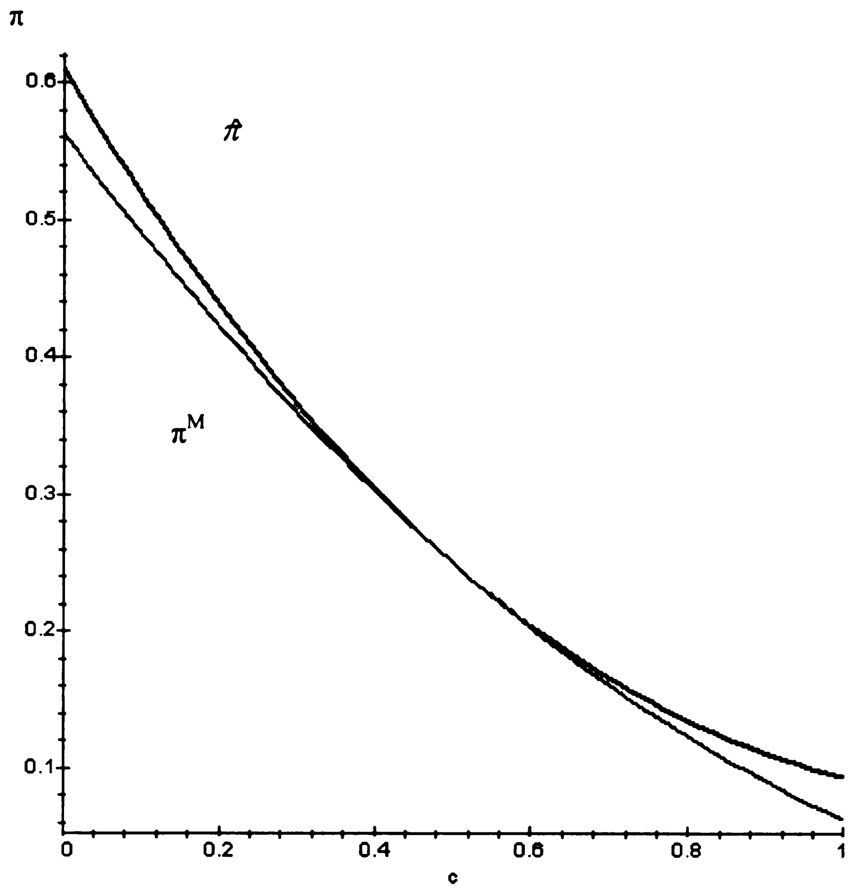


Figure 3.10: Optimal versus Monopoly Profits



4 A Study of OPEC Cartel Stability

4.1 Introduction

The Organization of Petroleum Exporting Countries (OPEC) was formed in 1960 at the Baghdad Conference.[11] One of the stated objectives of OPEC is to "coordinate and unify petroleum policies among Member Countries, in order to secure fair and stable prices for petroleum producers,"[12] which has been taken to mean maximize OPEC countries' profits. In spite of this goal, significant price fluctuations over the history of the organization suggest to some a competitive market perhaps abetted by occasional attempts to restrict output that invariably leads individual countries defections. On the other hand, it is well-known that in an environments with significant unanticipated demand fluctuations, the optimal strategy for a cartel results in periods of price wars that help enforce the cartel in the long run (Green and Porter 1984). In particular, using data for the Joint Executive Committee railroad cartel, Porter (1983) has tested and rejected the null hypothesis that "no switch took place, so that price and quantity movements were solely attributable to exogenous shifts in the demand and cost functions" (pp.301). The question then is which better explains OPEC: whether such price fluctuations reflect switches between collusive and non-cooperative periods in the cartel or whether the price behavior is indicative of OPEC's overall inability to affect the world's oil prices. The goal of this paper is to test for significant switches from collusive to non-cooperative behavior by OPEC.

Not surprisingly, there has been a good deal of work testing for collusion behavior in OPEC. What is surprising is that with notable exceptions, previous work has used a reduced form approach and have focused on individual country behavior rather than looking at OPEC as a whole. That is, previous work does not use a measure

[11]The five founding members are Saudi Arabia, Iran, Iraq, Kuwait and Venezuela. The current members also include Algeria, Indonesia, Nigeria, Libya, Qatar and the United Arab Emirates.

[12]See http://www.opec.org/aboutus/history/history.htm

of collusion for OPEC members, nor does it focus on the overall behavior of OPEC. In contrast, this paper follows Porter (1983), that is, we test for a structural model based on an optimal mechanism to enforce a cartel that faces unobserved demand fluctuations. In addition, by modifying the model to reflect how OPEC differs for the cartel examined by Porter (1983), this paper makes a second contribution. Specifically, we allow for non-OPEC producers to be treated as a competitive fringe. Because OPEC's production accounts for only 40% of the world's supply of crude, this modification is key in testing for switches in cartel behavior. Specifically, while a cartel might keep to a quota until enough periods have passed to switch to a regime of punishment, the fringe is not so constrained. Thus, an OPEC member in considering whether to deviate from the collusive outcome must take into account that the fringe will respond sooner to the resulting price decrease.

A novel result of this paper is our finding of significant switches between collusive and noncooperative behavior between 1974 and 2004. We estimate that the probability of being in a cooperative period was 35%, with periods of collusion resulting in 28% higher oil prices relative to periods of quantity competition. It is worth noting that the magnitude of this price increase is about half of that estimated to result from wars involving any OPEC member (60%). Furthermore, we find statistical evidence that OPEC does not behave as an effective cartel but as a non-cooperative oligopoly. Our estimates are consistent with OPEC behaving as in Cournot competition in the presence of a competitive fringe.

Regarding the overall market structure, our results are in line with the extant literature's findings that OPEC members produce at levels between quantity competition and the perfect collusive outcome. Although a number of studies in the 1980s and early 1990s suggest that OPEC behaves as a collusive cartel, recent work seems to point towards a different market structure. The seminal paper of Griffin (1985) tests individual countries' behavior among both OPEC and non-OPEC members for the period 1971-1983. He finds that most OPEC members behave as if they were part of a collusive cartel, while non-OPEC countries behave as if in Bertrand competition. Jones (1990) uses the same model for the 1983-1988 period with analogous findings. Using an ARIMA specification, Loderer (1984) finds that OPEC did not influence oil prices during the 1974-1980 period, but did so between 1981 and 1983. His results provide some evidence supporting the hypothesis that OPEC was a dominant producer during the second subsample.

In contrast, Spilimbergo (2001) uses a dynamic framework and rejects the hypothesis that OPEC was a market sharing cartel during the 1983-1991. Similar results have been found by other authors using different techniques. For instance, Gülen (1996) examines whether OPEC is a cartel whose members agree on their role assigned by the organization in an output rationing framework, and whether OPEC has the power to affect the market price of oil by adjusting its production. Using cointegration tests for the period 1965-1993, he finds that overall OPEC is not a cohesive organization. Griffin and Xiong (1997) calculate joint-profit-maximizing price paths and find evidence that –for some countries– it is more profitable to cheat on the assigned quota, despite the possibility of punishment if caught. Alhajji (2001) tests the dominant producer hypothesis for: (a) OPEC overall, (b) the OPEC core and (c) Saudi Arabia when non-OPEC oil producers are treated as a competitive fringe. All but the final model are rejected. More recently Smith (2002) concluded that OPEC's market structure lies between a non-cooperative oligopoly and a cartel. In the work closest to ours, Yang (2004) also uses Green and Porter (1984) as a starting point. However, in Yang (2004) individual supply functions for each member country are calculated so that instead of having a collusive variable indicator for OPEC as a whole, the switches between regimes are based on specific characteristics of each member and the total production of the rest of the organization, as in Griffin (1985). While not considering perfectly competitive market outcomes, his main finding is that each OPEC member produces between the collusive and Cournot outcomes.

In contrast, we remain closer to Green and Porter, by considering OPEC as a single entity, rather than a collection of individual members. There are two compelling reasons for doing so. First, there is ample evidence that individual members frequently respond directly to other members' outputs in such a way that overall OPEC behavior vis-à-vis the world market remains unchanged. And second, to the extent that oil prices are calculated as the average of the members' crude oil streams, a constant elasticity of variable cost appears to be a reasonable assumption.[13] Hence, there is an incentive to share technology among members to reduce costs since they are the oil producers with the lowest cost as explained in Adelman (1993).

A second difference with respect to Yang (2004) is that our specification allows us to test across many potential market structures — including perfect competition and alternative specifications of imperfectly competitive market structures with and without competitive fringes.

The remainder of this paper is organized as follows. Section 4.2 presents the model and the hypotheses that are tested. In Section 4.3 the data used is described. In Section 4.4, the results and interpretations obtained are discussed and Section 4.5 concludes.

4.2 The Model

Using a model first proposed by Green and Porter (1984), Porter (1983) tests the cooperative behavior hypothesis on the Joint Executive Committee of railroads — a cartel in operation from 1880 to 1886. His methodology estimates the level of competition in a specific industry or market where each of the cartel members assumes that a drop in demand could be explained either as an exogenous shock to the demand

[13] As of June 2005, OPEC's reference basket now consists of eleven crude streams representing the main export crudes of all member countries.

function or as a member's deviation from the collusive output, which may trigger a punishment stage.[14]

While the Joint Executive Committee controlled virtually all of the freight shipments during the 1880's in the U.S., OPEC has a significantly lower market share (around 42%), that has been decreasing since the late 1980's due to increased production from non-OPEC producers. To account for the role of non-OPEC countries in the oil market, we modify Porter's model. Specifically, Porter's supply function is modified so as to allow non-OPEC members to be treated as a competitive fringe. This modification also allows us to test the fringe's statistical significance. Using a constant elasticity of demand model, we assume that demand for OPEC oil is given by a loglinear function of price in addition to a set of demand shifters:

$$\ln opec_t = \alpha_0 + \alpha_1 \ln p_t + \alpha_2 OECD_t + \alpha_3 \ln nopec_t + \alpha_4 dummies_t + U_{1t}, \qquad (4.1)$$

where $\ln opec_t$ is the logarithm of OPEC oil supplied, $\ln p_t$ is the logarithm of world oil prices, $OECD_t$ is the first difference of the logarithm of GDP for OECD countries, $\ln nopec_t$ is the logarithm of non-OPEC major producers and $dummies_t$ is a vector of seasonal dummies. U_{1t} is an error term assumed to be i.i.d. normal with mean zero and variance σ_1^2.

The coefficient α_1 is the price elasticity of demand for OPEC oil, which is expected to be negative and greater than 1 in absolute value if OPEC indeed maximizes profits as a dominant producer operating in the elastic segment of the demand curve.[15] $OECD_t$ is intended to proxy for world income.[16] The sign of α_2 could be positive or negative since $OECD_t$ is measured as a growth rate. Thus, a negative sign merely indicates that the (positive) rate at which demand for OPEC oil grows decreases

[14]See Tirole (1988) pp. 262 for a brief description of the model.

[15]See Alhajji (2001).

[16]Although world GDP would have been a better measure for world income, data is not available at a quarterly frequency.

when the income for OECD countries increases; nonetheless oil is assumed to be a normal good. The coefficient on $\ln nopec_t$, α_3, is expected to be negative, since it is the substitute good for OPEC oil.

For the supply equation, it is assumed that OPEC chooses the quantity to be produced (which is consistent with the stated policy at a majority of their meetings). Then the world price is set and the competitive fringe produces where the world price equals their marginal cost. World demand, Q^w, is given as the sum of non-OPEC output, Q^{no}, and OPEC output, Q^o, where OPEC output is the sum of each OPEC country's production:

$$Q^w = Q^o + Q^{no}, \text{ where } Q^o = \sum q_i. \tag{4.2}$$

Each of OPEC's member countries, indexed by i, maximize the usual profit function,

$$\pi_i = p_t q_{it} - C_i(q_{it}). \tag{4.3}$$

Let $C_i(q_{it}) = a_i q_{it}^{\delta} + F_i$ be the cost function for OPEC member i, δ is the constant elasticity of variable costs with respect to output, a_i is a firm-specific shift parameter, and F_i is the fixed cost of firm i.

Hence Equation (4.5) is obtained following Porter's derivation and including the presence of the competitive fringe as in Church and Ware (1999). The market share for OPEC is defined as $s^o = Q^o/Q^w$ and the market share of country i within the organization as $s_i = q_i/Q^o$. The price elasticity of world oil demand is defined as:

$$\epsilon^w = \frac{\partial Q^w}{\partial p}\frac{p}{Q^w}. \tag{4.4}$$

In line with Porter's derivation, the parameter θ_{it} is included in Equation (4.5) and defines the possible five market structure cases as follows: [17]

$$p_t \left[1 + \frac{\theta_{it} \times s^o}{\epsilon^w}\right] = MC_i, \tag{4.5}$$

- if OPEC members exhibit noncooperative behavior and they price at marginal cost, then we have evidence of Bertrand competition or, what is the same, a perfectly competitive industry. In this case, $\theta_{it} = 0, \forall i, t$;
- if OPEC members maximize joint profits or, what is the same, follow efficient cartel behavior using the collusive outcome in presence of a competitive fringe, then $\theta_{it} = 1$;[18]
- for an efficient cartel without a fringe, $\theta_{it} = 1/s^0$;
- for Cournot competition with a competitive fringe, $\theta_{it} = s_i$; and
- for Cournot behavior without a fringe, $\theta_{it} = s_i/s^o, \forall i, t$.

We assume that the oil produced by each country is of similar (i.e., equivalent) quality; therefore, in equilibrium, each OPEC country obtains the same price.

After multiplying Equation (4.5) by each country's market share and summing across countries, we obtain:

$$\sum_i s_i p_t \left[1 + \frac{\theta_{it} \times s^0}{\epsilon^w}\right] = p_t \left[1 + \frac{s^0 \times \theta_t}{\epsilon^w}\right] = \sum_i s_i MC_i, \tag{4.6}$$

with $\theta_t = \sum_i s_i \theta_{it}$.

On the other hand, the marginal cost for firm i can be rewritten as

$$\sum s_i MC_i(q_{it}) = DQ^{\delta - 1}. \tag{4.7}$$

[17] A detailed explanation of Equation (4.5) is on the Appendix.

[18] See Church and Ware (1999) for a detailed explanation of a dominant firm with a competitive fringe model.

Notice that the left hand side of (4.7) equals the right hand side of Equation (4.6), thus,

$$p_t\left[1+\frac{\theta_t \times s^0}{\epsilon^w}\right] = DQ^{\delta-1}, \tag{4.8}$$

where D is a function of the country-specific shift parameter and the constant elasticity of demand.

Taking the logarithm of Equation (4.8), the aggregate supply relationship is

$$\ln p_t = \beta_0 + \beta_1 Q_t + \beta_2 Z_t + \beta_3 I_t + U_{2t}, \tag{4.9}$$

where I_t is a dummy variable that equals 1 when the industry is in a cooperative period and equals 0 when it is in a reversionary period. The error term U_{2t} is independent of time and normally distributed with zero mean and variance σ_2^2. The variable Z_t is included to control for other factors of the supply function that would otherwise be part of the error term, therefore it minimizes the omitted variable bias. The parameters β_0, β_1, β_3 are given by

$$\begin{aligned} \beta_0 &= \ln D, \\ \beta_1 &= \delta - 1, \\ \beta_3 &= -\ln\left(1+\frac{s^0 \times \theta_t}{\epsilon^w}\right). \end{aligned} \tag{4.10}$$

Although, the market structure for an individual OPEC member could be estimated from Equation (4.5), we opt instead to estimate and test a model of overall OPEC collusive behavior. The motivation for this choice is twofold. First, there is evidence that some countries have adjusted their quotas for a short period of time in order to compensate for a shortage of production from another member country, thus keeping the production level of the organization unchanged. This behavior would give a wrong signal of overproduction for some countries in cases where the increased production was clearly aimed at stabilizing OPEC's total output. Second, the objective of this paper is to test OPEC's collusive behavior and the market structure as a whole, focusing on possible switches between collusive and non-cooperative behavior over time.

Hypotesis 4.1 *Given the assumption that* $\theta_t = \theta$ *for all* t, β_3 *reduces to* $-\ln(1 + s^0\theta/\epsilon^w)$. *Letting* $H = \sum s_i^2$ *denote the Herfindahl index we can test the market structure as follows:*

1. $\theta = 0$ *for perfectly competitive behavior,*

2. $\theta = H \approx .15$ *for Cournot behavior with a fringe,*

3. $\theta = H/s^0 \approx .3642$ *for Cournot behavior without a fringe,*

4. $\theta = 1$ *for a perfectly collusive cartel with a fringe, and*

5. $\theta = 1/s^0 \approx 2.428$ *for an efficient monopolistic cartel.*

In Porter (1985), where there is no competitive fringe, the parameter θ only controls for Bertrand, Cournot or perfectly collusive firms. In the present model the market structure parameter allows us to test not only for these three market structures, but also for the significance of the competitive fringe represented by non-OPEC producers within the previous market structures. Thus, we have five possible cases.

In the OPEC framework, we *a priori* expect the competitive fringe to be significant due to the size of non-OPEC production. Unless, however, the market is perfectly competitive with OPEC and non-OPEC countries behaving as in Bertrand competition. Moreover, if there is no fringe and OPEC is an effective cartel, the price and quantity used by the organization should be the same as those of a profit-maximizing monopolist.

Equilibrium implies that quantity demanded equals quantity supplied. Therefore, from Equation (4.1), $\epsilon^w = \alpha_1 \times s^0$, since α_1 is the price elasticity of demand for OPEC oil, and

$$\theta = \alpha_1 \left[\exp(-\beta_3) - 1\right]. \tag{4.11}$$

After obtaining the price elasticity of demand from (4.1) and the collusive behavior coefficient (β_3) from (4.9), the value of θ can be found from Equation (4.11).

In the OPEC setting, the supply function is given by

$$\ln p_t = \beta_0 + \beta_1 \ln opec_t + \beta_2 history_t + \beta_3 I_t + \beta_4 break_t + \beta_5 dummies_t + U_{2t}, \tag{4.12}$$

where $history_t$ is a dummy variable that controls for wars involving an OPEC country, and U_{2t} is a normally distributed error term.

As mentioned above, the coefficient β_1 is expected to be positive. The coefficient on the collusive behavior dummy variable, β_3, is expected to be positive and significant if collusive periods have higher prices. The dummy variables, $dummies_t$, capture seasons.

Equations (4.1) and (4.12) constitute a simultaneous equation model (SEM), in which $\ln p_t$ and $\ln opec_t$ are the two endogenous variables. If the I_t variable is the true indicator of collusion for the cartel, the system can be estimated by three stage least squares. If, instead, the indicator of collusion is assumed to be unknown but independent of time and following a Bernoulli distribution, then the estimation is done using the E-M algorithm first proposed by Kiefer (1980).[19]

Specifically, following Porter's notation, assume that I_t equals 1 with probability λ and 0 with probability $1 - \lambda$. Rewriting Equations (4.1) and (4.12) in a matrix

[19] While periods of cheating by OPEC members have never been sustained for a prolonged period of time, we nevertheless hope to further substantiate our findings in future work by adapting the model for a Markov process on switches instead of using the memoryless Bernoulli distribution of the current paper.

form:

$$By_t = \Gamma X_t + \Psi I_t + U_t, \tag{4.13}$$

where $y_t = (\ln opec_t, \ln p_t)'$, $X_t = (1, OECD_t, \ln nopec_t, history_t, break_t, dummies_t)'$, $U_t = (U_{1t}, U_{2t})'$ and $dummies_t$ is a 3×1 vector consisting of the seasonal dummies. The error vector U_t is identically and independently distributed $N(0, \Sigma)$.

$$B = \begin{pmatrix} 1 & -\alpha_1 \\ -\beta_1 & 1 \end{pmatrix}, \Psi = \begin{pmatrix} 0 \\ \beta_3 \end{pmatrix}, \Gamma = \begin{pmatrix} \alpha_0 & \alpha_2 & \alpha_3 & 0 & 0 & \boldsymbol{\alpha}_4 \\ \beta_0 & 0 & 0 & \beta_2 & \beta_4 & \boldsymbol{\beta}_5 \end{pmatrix}, \Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}, \tag{4.14}$$

where $\boldsymbol{\alpha}_4$ and $\boldsymbol{\beta}_5$ are 1×3 vectors.

Because the variable I_t is unknown, the probability density function is defined as:

$$\begin{aligned} f(y_t|X_t, I_t) &= \left[\lambda \exp\left\{-\frac{1}{2}(By_t - \Gamma X_t - \Psi)\Sigma^{-1}(By_t - \Gamma X_t - \Psi)\right\} + \right. \\ &\left. (1-\lambda)\exp\left\{-\frac{1}{2}(By_t - \Gamma X_t)\Sigma^{-1}(By_t - \Gamma X_t)\right\}\right](2\pi)^{-1/2}|\Sigma|^{-1/2}|B|. \end{aligned} \tag{4.15}$$

With an initial estimate of the regime classification sequence $(w_1^0, .., w_T^0)$ where $w_t^0 = \Pr(I_t = 1)$, an initial estimate of λ is constructed as the mean of the classification sequence.[20] Hence, maximizing the product of the density functions, initial estimates $B^0, \Gamma^0, \Psi^0, \Sigma^0$ are obtained. Using Bayes' formula the new classification series is updated. Thus,

$$w_t^1 = \Pr(I_t = 1|y_t, X_t, \Psi^0, \Gamma^0, \Sigma^0, B^0, \lambda^0) = \frac{\lambda^0 h(y_t|I_t = 1)}{\lambda^0 h(y_t|I_t = 1) + (1-\lambda^0)h(y_t|I_t = 0)}, \tag{4.16}$$

where $h(y_t|I_t)$ is the probability density function of y_t given I_t.

Then the switching probability λ is updated: $\lambda^1 = \frac{1}{T}\sum w_t^1$. This procedure is repeated until the correlation between two consecutive estimates of w_t exceeds .999.

[20]The previously constructed I_t is used to construct the initial classification sequence.

4.3 The Data

The data used in this paper spans the period between 1974:1 and 2004:4, where each quarterly observation is calculated as the middle month of that quarter. Oil quantities and prices are obtained from the U.S. Department of Energy.[21] The variable $\ln opec_t$ is the logarithm of OPEC oil supplied in period t measured in thousands of barrels per day. The variable $\ln p_t$ is the logarithm of world oil prices measured as the Refiner Acquisition Cost of imported crude oil in U.S. dollars per barrel.

The set of demand shifters comprises the logarithm of the GDP for the OECD, $OECD_t$; the logarithm of the quantity of oil sold by the major non-OPEC producers, $\ln nopec_t$; a dummy that controls for the U.S. price decontrols since the beginning of 1981, $break_t$; and a vector of seasonal dummy variables, $dummies_t$.

As mentioned in the previous section, $OECD_t$ is intended as a proxy for world income. In fact, the real GDP of the OECD amounts to 78.4% of world GDP and its oil demand accounts for 66% of world's oil demand. This variable is computed as the first difference of the logarithm of GDP for OECD countries, measured in millions of U.S. dollars. The data source for $OECD_t$ is the OECD Economic Outlook.

The variable $\ln nopec_t$ is measured in thousands of barrels per day and comprises the production of Canada, China, Egypt, Mexico, Norway, the U.S.S.R. and the nations that formerly comprised it, the United Kingdom, and the United States. The variable $break_t$ takes the value of 1 for the quarter 1981:1 onwards, and 0 otherwise.

The collusive behavior variable I_t, and the dummy variable $history_t$ serve as controls in the supply equation. The variable I_t takes on the value of 1 when there is evidence that OPEC was in a cooperative period. This variables is computed using the Oil Price Chronology of the U.S. Department of Energy. Specifically, the production quotas assigned by OPEC are compared to the actual production levels: If actual production in period t is at least 5% over the quota established for that period, and

[21] nominal prices were converted to real using CPI from the BLS webpage.

there is no evidence that overproduction was a consequence of an increase in world demand, I_t takes the value of zero. Finally, the dummy variable $history_t$ equals 1 when there is a war involving an OPEC country at time t. This variable includes the Iran-Iraq war, and the invasion of Kuwait in 1990.

Before proceeding to the econometric analysis, consider the historical evolution and time series properties of the variables of interest. Figure 4.1 plots world, OPEC, and non-OPEC oil production. The world production has been roughly constant between 50 and 70 millions barrels per day with a slightly increasing trend. OPEC decreased its production from 1980 to 1983, but has been increasing it ever since. Non-OPEC members have had more constant behavior also with an increasing trend overall, surpassing OPEC's production in 1979.

In Figure 4.2, real world prices and the collusive behavior indicator I_t are plotted. The figure also reports major world events that lead to large fluctuations in oil prices. The Iranian revolution resulted in a drop of 3.9 million barrels per day of crude oil production between 1978 and 1981. Even though other OPEC members raised their production seeking to maintain the same total output, the revolution still caused an increase in oil prices. This trend was reversed when the U.S. removed the price and allocation controls on the oil industry. As a consequence oil prices dropped. This, together with a decrease in demand, an increase in non-OPEC production, and Saudi overproduction, resulted in OPEC loosing control of world oil prices by 1982. By December 1985, and despite several cutbacks in OPEC production, an abundance of oil in the market was evident. This situation triggered a price decline that ended in the so-called crash of 1986. Since 1987 oil prices have been more stable, except for the five-month peak caused by the Kuwait invasion at the end of 1990.

As Figure 4.2 illustrates, the indicator variable I_t seems to be a fair indicator of collusive behavior. Even though oil prices have been more stable after 1987, the variable indicates that there has been significant overproduction compared to levels of effective cartel output.

Table 4.1 reports statistics about oil production by OPEC members.[22] The OPEC " core," which is formed by Saudi Arabia, the United Arab Emirates (U.A.E.), Kuwait, Qatar, and Libya, account for over 50% of total OPEC production, with about 30% of OPEC's oil being produced by Saudi Arabia alone. Note that minimum observed values of zero are reported for Iraq and Kuwait. These values correspond to the following periods of no production: February and March of 1991 for Iraq, and February to May of 1991 for Kuwait (due to the Persian Gulf War).

Table 4.2 reports summary statistics for the variables used in the empirical analysis. The quantity supplied by OPEC accounts for 41.2% of the world production for this period which is around 59 millions of barrels per day. According to the variable I_t, there are 39 identified cooperative periods out of 124 observations, that is 33.8% of the sample. The average price per barrel is 35.2 December 2002 U.S. Dollars.

As a measure of industry concentration, consider the Herfindahl index. This index is a more accurate measure of concentration than the concentration-ratio since it gives more weight to large firms. Schmalensee and Willing (1989) state that the Herfindahl index gives the answer to: "By how much would welfare rise if we could perturb the industry a small amount in the optimal welfare-improving direction?" Using the U.S. Department of Justice classification of concentrated industries, Herfindahl indices between 1000 and 1800 are moderately concentrated, those above 1800 indicate concentrated market structures. Figure 4.3 plots the Herfindahl index for OPEC across time. Note that although the period 1981–1983 is indicated as concentrated, the large magnitude of the index reflects the decline in Iran and Iraq's supply, which was compensated by Saudi Arabia. In this manner, total OPEC output remained essentially at the same level, with the resulting increase in share of Saudi Arabia (see

[22] Gabon and Ecuador are excluded from the analysis, even though they were part of OPEC for most of the period to be examined (Ecuador 1973:11-1992:11, Gabon 1973:12-1995:01), their combined production was less than 1% of total OPEC oil production.

Figure 4.4). Similar behavior can be observed during the Gulf War where Saudi Arabia made up for Kuwait's and Iraq's shortages. Nevertheless, when the production of Iraq, Iran, Kuwait and Saudi Arabia is taken as a whole, the market share remains fairly constant over time (see Figure 4.5). Indeed, the pattern of Saudi Arabia's share is almost identical to that of the index, given that this country has by far the largest share.

Table 4.3 shows OPEC quotas, production allocations and the Herfindahl index for four select periods: (i) the last quarter of 1978, before the Iranian Revolution and the U.S. decontrol; (ii) 1983 when OPEC started using quotas instead of royalty rates; (iii) 1990, before the Gulf War and (iv) November 1997.[23] As shown in Figure 4.3, in 1983 the Herfindahl index was at its highest, this is reflected in Saudi Arabia's share of over 40% of total OPEC production. After 1984, Saudi Arabia's share declined to a level between 25% and 30% of total production, even though the output level had been constantly increasing. At the same time, the share of some of the smaller producers, that were already producing close to their maximum capacity, decreased. Over time, other countries, like Venezuela, Nigeria and the U.A.E. slightly increased their production share, even though OPEC's output has also increased over time. From the late 1990's, the index goes back to its average level during the 1970's.

Lastly, Table 4.4 reports the results for Elliott and Jansson (2003) unit root test for the two main variables of interest, the logarithm of OPEC production, $\ln opec_t$, and the logarithm of oil prices, $\ln p_t$, as well as for non-OPEC production, $\ln nopec_t$. The motivation for using this test over the commonly used Dickey-Fuller test is twofold. First, this test has been shown to have excellent properties in small samples such as the one in this paper. Second, Elliott and Jansson (2003) demonstrate that very large power gains over the Dickey-Fuller test are obtained when relevant stationary covariates are modeled with the potentially integrated variable. Indeed, given the

[23]For 1978 production as of the fourth quarter of the year, all other years production is quota allocated. The Herfindahl index is presented in percentage terms.

structure of our model the choice of covariates appears to be straightforward. According with equation (4.1), we use the rate of growth of GDP in the OECD, $OECD_t$, as the stationary covariate in testing for a unit root in the quantities. We include a constants, but no time trend in testing for a unit root in $\ln opec_t$, and constants and time trends when testing $\ln nopec_t$. For the estimated R^2 of 0.43 for $\ln opec_t$ and 0.20 for $\ln nopec_t$, the test statistics are 3.2535 and 3.7062 which are less than the respective critical values of 4.1057 and 5.9976.[24] Thus, in both cases we reject the null of a unit root for a 5% test. Equation (4.12) and the rejection of a unit root in $\ln opec_t$ would suggest using this variable in testing for a unit root in prices. Including constants and time trends in both prices and quantities, for the estimated R^2 0.68, the critical value is 12.0812, so with a test value of 9.8399 we reject the null of a unit root. We reach the same conclusion if instead -not having any prior information on the stationarity of $\ln opec_t$- we are cautious and use the first difference of $\ln opec_t$ as a covariate. In that case, even though the estimated R^2 is considerably smaller (R^2 =0.01), the test leads us to reject the null of a unit root. Thus, in the following section we proceed to estimate the model under the assumption that all the variables in the system (4.13) are stationary.

4.4 Empirical Results

The estimation results are presented in Table 4.5. The first two columns report the parameter estimates for Equations (4.1) and (4.12) obtained by Three Stage Least Squares ($3SLS$), under the assumption that the constructed cheating variable is an accurate classification of the regimes. The third and fourth columns correspond to the estimates obtained using the maximum likelihood algorithm described in the previous section, when the structural break is not taken into account. The last two columns present the maximum likelihood estimates when controlling for the structural

[24] The R^2 is the correlation between the covariate and the potentially integrated variable.

break.

Focusing first on the $3SLS$ estimates, note that in the demand equation we obtain a negative price elasticity that is significantly less than one in absolute value, thus suggesting an inelastic demand for OPEC's oil. According to these estimates, OPEC does not maximize profits as a dominant firm given that it does not produce on the elastic part of its demand curve. When non-OPEC production increases by 1%, OPEC production is estimated to decrease by 1.4%. The coefficient on the OECD's GDP is not significantly different from zero. The estimates of the supply equation predict 28% higher oil prices in cooperative periods, and higher prices when an OPEC country is involved in a war. None of the seasonal dummies are statistically significant.

Regarding OPEC's behavior, using Equation (4.10), we obtain an estimate of $\theta = 0.5419$, which is consistent with Cournot behavior without the fringe (see Hypothesis 4.1.3). Notice that the closest market structure would be an efficient cartel. This result also matches the estimated elasticity of demand.

Consider now the estimates obtained when we allow for the regime classification to be estimated by maximizing the likelihood function, Equation (4.15). If we do not take into account the structural break in 1981:1 (Columns 3 and 4 in Table 4.5), the estimated price elasticity in the demand equation is negative and less than one in absolute value. Hence, the profit maximizing condition does not hold. As it is the case for $3SLS$, OPEC's production is estimated to decrease with increases in non-OPEC production. However, raises in the world's income, as proxied by $OECD_t$, are estimated to have a negative and statistically significant effect on the demand for OPEC's oil ($\alpha_2 = -0.03$). On the other hand, war periods involving an OPEC country increase prices by 41%. The main difference with respect to the $3SLS$ estimate is the effect of cooperative periods on oil prices: the maximum likelihood estimates give a 14% increase, about half the magnitude obtained from $3SLS$.

Regarding the probability of being in a cooperative period, the estimate obtained from the maximum likelihood procedure equals 0.353 (see Table 4.5). This probability is slightly higher than the initial value at time t ($\lambda = 0.336$), which is equivalent to the mean of the constructed collusive variable I_t used in the $3SLS$ estimation. The corresponding θ equals .1059, a result also consistent with Cournot behavior with the presence of a competitive fringe (Hypothesis 4.1.2).[25] Note that given this estimated value for θ, the closest alternative possible market structure would be competitive behavior (see Hypothesis 4.1.1).These result corroborates the prior beliefs that non-OPEC producers having a significantly larger market share, take some of OPEC's market power.

Finally, the last two columns of Table 4.5 report the estimates obtained when we take into account the structural break in 1981:1 in the supply equation. The inclusion of the $break_t$ dummy increases the log likelihood by 108.9; thus, providing statistical evidence of a structural break in the mid-1980s. The demand equation presents a similar intercept for the pre-1981 period, however, the price elasticity of demand is smaller in magnitude. Now a 1% increase in non-OPEC production decreases OPEC production by almost 1% which is expected assuming that OPEC and non-OPEC countries are the only two oil producers. Also the coefficient of GDP for OECD countries is still negative but not significant, hence OPEC demand does not depend on the growth rate of OECD countries. Note that the finding of price inelastic oil demand is consistent with Alhajji (2000), who finds that OPEC is not a dominant producer since it operates in the elastic part of its demand function.

Regarding the supply equation, the intercept appears to be overestimated by the ML with no break. More importantly, the inclusion of the break results in a significantly larger value for the coefficient on the collusive dummy which is almost the same as in OLS. In fact, these results are consistent with a more realistic scenario

[25]A t-statistic was constructed using the delta method for the standard error of θ. The only hypohesis that we failed to reject was Cournot competition with a competitive fringe.

where periods of collusion are estimated to result in a 28% increase in oil prices, and the coefficient on the *history* dummy reflects a 64% increase in OPEC prices during periods of wars. Furthermore, when the break is taken into account, the value of θ equals .116. This value is smaller than the $3SLS$ estimate but larger than the ML estimate with no break, however still consistent with Cournot behavior with the competitive fringe (see Hypothesis 4.1.2). In other words, estimates for the ML model with the break dummy, suggest that OPEC has not been effective in raising prices over quantity competition levels.

Note that in this model the probability of collusion ($\lambda = 0.3524$) is closer to the average number of collusive periods in the data, and lower than the estimated probability when the structural break is not taken into account. All in all, accounting for the structural break renders the fringe insignificant since OPEC and non-OPEC producer appear to have the same market structure. Yet, the statistical significance of β_3 provides evidence of switches between collusive and non-cooperative behavior at the interior of OPEC. This result is confirmed by the fact that a likelihood ratio test comparing this model and one where β_3 is restricted to zero equals 217.98.

Summarizing, accounting for the structural break originated by the U.S. price decontrol leads one to conclude that OPEC behavior is better described by quantity competition between OPEC and non-OPEC producers (Hypothesis 4.1.2). Regardless of the model, the price elasticity of demand indicates that OPEC is not a dominant producer.

4.5 Conclusion

The estimations in this paper examine world crude oil prices for the period 1974:2-2002:4 in order to gain insight into OPEC behavior. It is demonstrated that changes in oil prices are a result not only of shocks to demand, but also of switches from collusive to non-collusive behavior among members of the cartel. That is, we present statistical evidence that switches in the behavior of the organization lead to price changes with prices being significantly higher in periods of cooperative behavior. Hence, the hypothesis that only cooperative or noncooperative periods are observed is rejected in favor of switches between regimes.

Indeed, throughout OPEC's history, there has been evidence of deliberate cheating among members and our findings suggest that these deviations are significant enough to drive the overall behavior of the organization away from the collusive outcome, in spite of efforts from some of its members like Saudi Arabia. In other words, despite spells of collusive behavior, OPEC cannot be viewed as an effective cartel during the time period examined. Therefore, the world crude oil market is best described by the Cournot model of quantity competition where OPEC acts as an inefficient dominant producer and non-OPEC producers as a competitive fringe.

4.6 Appendix

Derivation of Equation (4.5):

From the profit function of member i, $\pi_i = p_t q_{it} - C_i(q_{it})$, the first order condition with respect to the production quantity is

$$\frac{\partial \pi_i}{\partial q_{it}} = p_t + \frac{\partial p_t}{q_{it}} q_{it} - MC_i = 0 \Longleftrightarrow p_t + \frac{\partial p_t}{q_{it}} q_{it} = MC_i.$$

$$\text{Using the chain rule we get } \frac{\partial p_t}{q_{it}} = \frac{\partial p_t}{\partial Q^W} \frac{\partial Q^W}{\partial Q^O} \frac{\partial Q^O}{q_{it}},$$

$$\text{where } \frac{\partial Q^W}{\partial Q^O} = \frac{\partial Q^O}{q_{it}} = 1.$$

Then $\partial p_t / \partial q_{it} = \partial p_t / \partial Q^W$, hence, we can rewrite the first order condition as

$$p_t + \frac{\partial p_t}{\partial Q^W} q_{it} = MC_i \text{ or } p_t + \frac{\partial p_t}{\partial Q^W} \frac{Q^W}{p_t} q_{it} \frac{p_t}{Q^W} = MC_i,$$

using the definition of the world elasticity of demand,

$$\epsilon^w = \frac{\partial Q^w}{\partial p} \frac{p}{Q^w},$$

and $s_i = q_i / Q^o$, the first order condition becomes

$$p \left[1 + \frac{s_i s^o}{\epsilon^w} \right] = MC_i. \tag{4.17}$$

Writing the first order conditions of each of the possible market structures we obtain:

1. Bertrand competition implies $p = MC_i$. Note that Bertrand competition maximizes with respect to prices instead of quantity, however in the derivation prices are not replaced by the demand function (e.g. $p = 1 - q$), hence we can still compare the first order condition, since it is solved as an implicit derivative.

2. Cooperative cartel points to: $p\left[1 + 1/\epsilon^w\right] = MC_i$. This is the first order condition for a regular monopoly and can be derived as follows:

$$\pi = (p-c)q \tag{4.18}$$

$$\frac{\partial \pi}{\partial q} = \frac{\partial p}{\partial q}q + p - c = 0 \Longleftrightarrow p\left[1 + \frac{\partial p}{\partial q}\frac{q}{p}\right] = c. \tag{4.19}$$

The inverse price elasticity of demand is $\frac{1}{\epsilon} = \frac{\partial p}{\partial q}\frac{q}{p}$, hence we obtain the specified condition. From Equation (4.17) it implies that s_i and s^o equal 1, where s^o represents the market share of the monopolist, that in this case equals 1 as there is no fringe. And s_i is the market share of each producer, that in the case of a monopoly also equals 1 because there is only one producer.

3. Cooperative cartel in presence of a competitive fringe indicates:

   $p\left[1 + s^o/\epsilon^w\right] = MC_i$. This is the same as a dominant producer in the presence of a competitive fringe, as in Church and Ware (1999). The result is obtained defining the price elasticity of demand in terms of the elasticity of the dominant firm and of the competitive fringe as $\epsilon^w = \epsilon^o s^o + \epsilon^f(1 - s^o)$, where ϵ^f is the price elasticity of demand for the good of the competitive fringe. This market structure can be interpreted from Equation (4.17), s_i also equals 1 since there is only one dominant producer, but s^o represents the market share of that producer in world production because the competitive fringe has a market share of $1 - s^o$.

4. Cournot competition in presence of a competitive fringe implies:

   $p\left[1 + s_i s^o/\epsilon^w\right] = MC_i$. This is the case of the first order condition derived above in Equation (4.17).

5. Cournot competition without the fringe indicates: $p\left[1 + s_i/\epsilon^w\right] = MC_i$. Following the same derivation as the previous market structure, but with no competitive fringe, the oligopoly has all the market share ($s^o = 1$), hence the price

elasticity of demand depends only on the producers of the oligopoly.

Therefore including the parameter θ_{it} in Equation (4.17), the first order condition becomes:

$$p\left[1+\frac{\theta_{it}s^{o}}{\epsilon^{w}}\right]=MC_{i}, \tag{4.20}$$

where θ_{it} can be tested for each of the five market structures for each member. As the purpose of this paper is to test for an overall behavior of the organization rather than for each member country, instead of directly testing Equation (4.20) we find an aggregate relationship to test OPEC's behavior.

Table 4.1: OPEC Members Production (thousands of barrels per day).

Variable	Mean	Std. Dev.	Min	Max
Algeria	1168.77	174.18	860.66	1715
Indonesia	1440.76	145.24	1088	1698.66
Iran	3442.39	1236.12	887.33	6604.66
Iraq	1731.6	924.48	85.34	3661
Kuwait	1723.66	590.65	16.89	2889.33
Libya	1412.86	327.94	654.33	2154.66
Nigeria	1891.38	361.67	820	2546.66
Qatar	485.63	153.26	208.33	835
Saudi Arabia	7488.79	1814.88	2686.66	10298
U.A.E.	1909.25	427.78	1053.66	2602
Venezuela	2376.27	479.34	1490	3423.57

Table 4.2: Descriptive Statistics

	Mean	Std. Dev.	Min	Max
OPEC production	25070.93	4557.95	15159.33	33556.67
World Prices	35.19	16.59	11.95	80.45
history	.2825	.4519	0	1
I	.3387	.4751	0	1
Non-OPEC production	35646.33	4238.63	25110	42443.73
GDP for OECD countries	53.93	34.98	8.4	132.84

Table 4.3: Herfindahl Index for OPEC Members for Related Years

	1978		1983		1990		1997	
	Share	Production	Share	Production	Share	Production	Share	Production
Total Production	1	31	1	17.15	1	21.61	1	27.5
Saudi Arabia	.32	9.97	.416	7.15	.249	5.38	.318	8.76
Iran	.12	3.81	.07	1.2	.145	3.14	.143	3.94
Iraq	.09	2.98	.07	1.2	.145	3.14	.047	1.31
Venezuela	.07	2.32	.087	1.5	.089	1.94	.093	2.58
Nigeria	.07	2.17	.075	1.3	.074	1.61	.074	2.04
Indonesia	.05	1.58	.075	1.3	.063	1.37	.052	1.45
Kuwait	.07	2.31	.046	.8	.069	1.5	.079	2.19
Libya	.06	2.13	.043	.75	.056	1.23	.055	1.52
U.A.E.	.05	1.83	.058	1	.05	1.09	.085	2.36
Algeria	.04	1.37	.037	.65	.037	.82	.032	.9
Qatar	.01	.52	.017	.3	.017	.37	.014	.41
Herfindahl Index	.1476		.212		.1342		.1594	

Table 4.4: Elliott and Jansson Unit Root Test

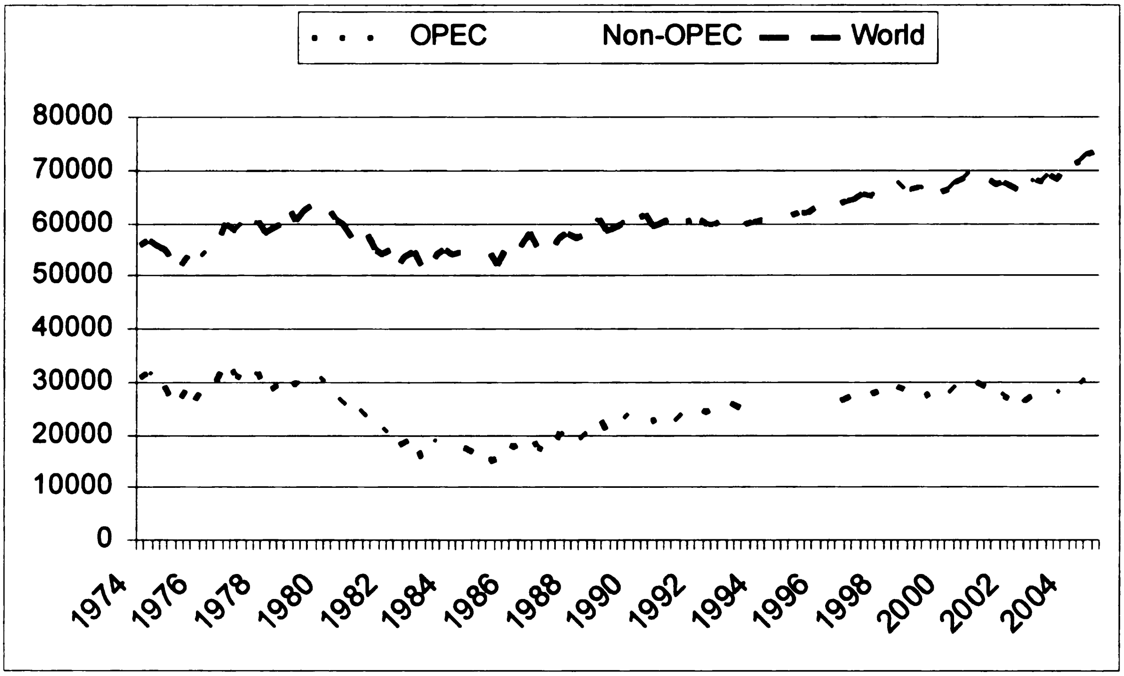
Variable	Test Value	5% Critical Value	R^2	Covariate
$ln\,(opec)$	3.2535	4.1057	0.4324	$OECD$
$ln(nopec)$	3.7062	5.9976	0.2044	$OECD$
$ln(p)$	9.8399	12.0812	0.6825	$ln\,(opec)$
$ln(p)$	5.5940	5.7094	0.0104	$\Delta ln\,(opec)$

Table 4.5: SEM Estimates

	3SLS* (1)		MLE (2)		MLE with breaks (3)	
Variables	Demand	Supply	Demand	Supply	Demand	Supply
constant	26.5591†	-34.55 †	16.8413†	2.322†	21.5946†	1.6773†
	(2.3555)	(8.37)	(1.1712)	(.2658)	(2.3028)	(.371)
ln (opec)		3.7024 †		.096†		.176†
		(.8212)		(.0238)		(.0411)
ln(p)	-.5419†		-.8021†		-.4696†	
	(.1)		(.1883)		(.0746)	
OECD	2.7172		-.0320†		-.0218	
	(3.8897)		(.0309)		(.0378)	
ln(nopec)	-1.395 †		-.3893†		-.9486†	
	(.2107)		(.1029)		(.2305)	
history		1.6899 †		.4144†		.6456†
		(2941)		(.0799)		(.0732)
I		.2896 †		.1416†		.2838†
		(0841)		(.0513)		(.0494)
break						-.3713†
						(.1296)
january	-.0529	.0773	-.0407	-.0139	-.0439	.0011
	(.055)	(.0996)	(.1125)	(.1263)	(.0534)	(.085)
april	-.0565	.139	-.0366	.0076	-.0436	.0257
	(..0552)	(1020)	(.0764)	(.0817)	(.0596)	(.0982)
july	-.0253	.0225	-.0101	.007	-.0197	.0013
	(.055)	(.0982)	(.1716)	(.1953)	(.0736)	(.1282)
theta	.5419		.1059		.116	
	(0.0016)		(.0286)		(.0204)	
Loglikelihood			625.9		734.89	

* Standard errors are in parenthesis. †Significant at 1%.

Figure 4.1: Oil Production (thousands of barrels per day)



*Prices are measured in dollars of December 2002. The I_t dummy series is scaled to match prices in the same graph, when $I_t$equals 10, OPEC is in a cooperative period

Figure 4.2: Real Prices and Cooperative Period Indicator

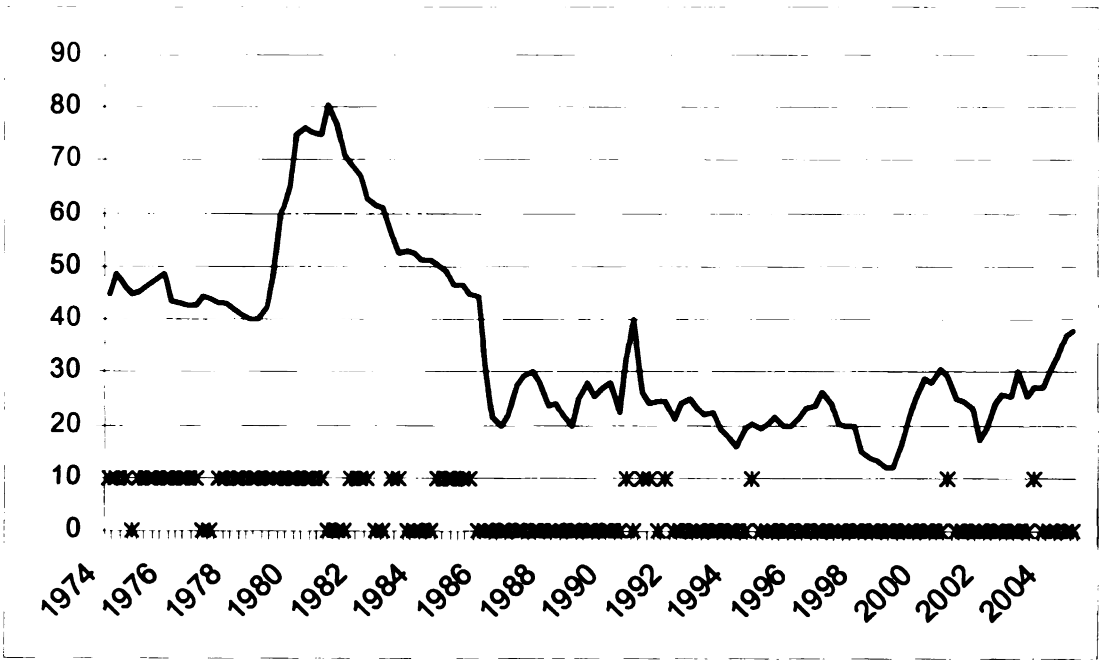


Figure 4.3: OPEC's Herfindahl Index

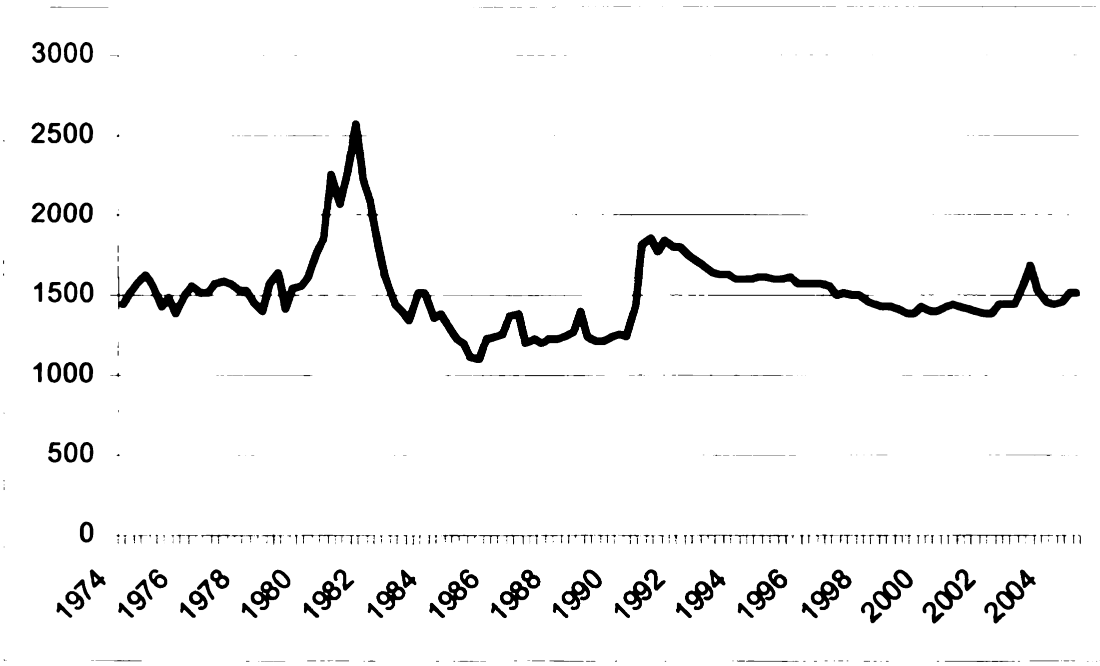


Figure 4.4: Market Shares

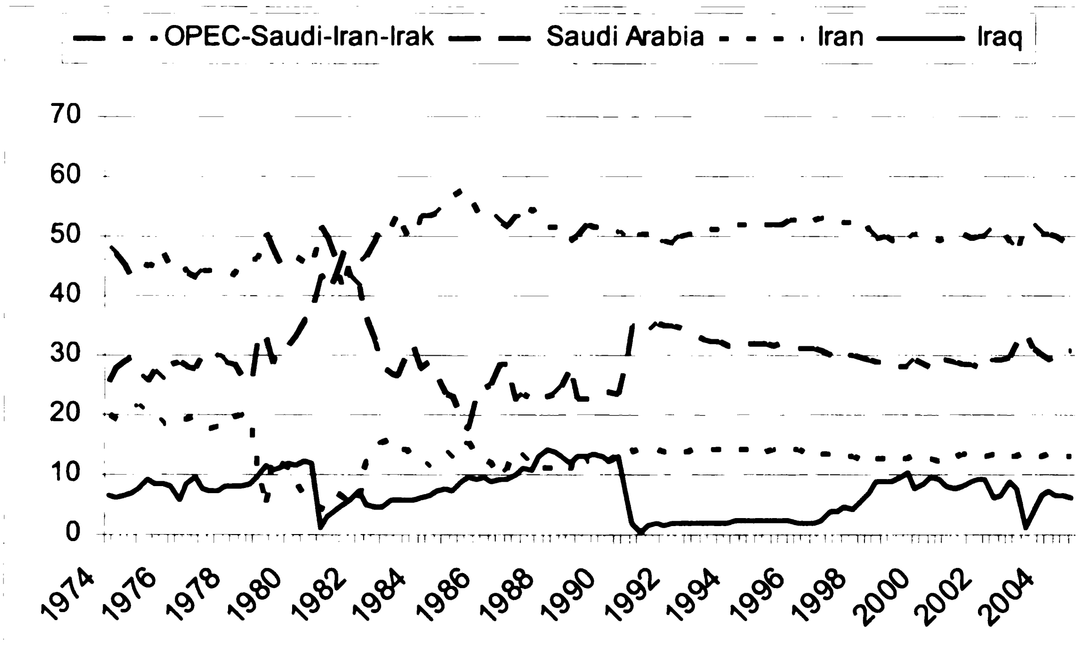
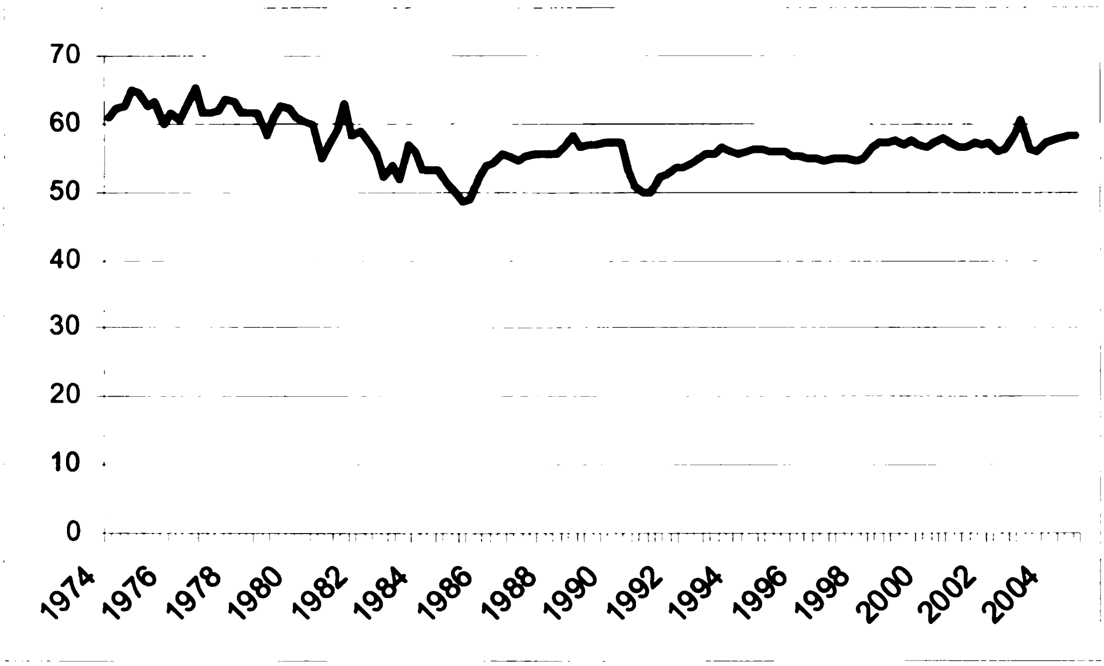


Figure 4.5: Combined Share for Iran, Iraq, Kuwait and Saudi Arabia



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