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METHODS OF META-ANALYZING REGRESSION STUDIES: APPLICATIONS OF GENERALIZED LEAST SQUARES AND FACTORED LIKELIHOODS

By

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ABSTRACT

METHODS OF META-ANALYZING REGRESSION STUDIES: APPLICATIONS OF GENERALIZED LEAST SQUARES AND FACTORED LIKELIHOODS

By

Meng-Jia Wu

Regression is one of the most commonly used quantitative methods for exploring the relationship between predictor(s) and the outcome of interest. One of the challenges meta-analysts may face when intending to combine results from regression studies is that the predictors are usually different from study to study, even though the primary researchers may have been studying the same outcome. In the current study, two methods, generalized least squares (GLS) and factored likelihoods through the sweep operator (SWP), for combining results were examined for their ability to reduce the problems arising from regression models that contain different predictors in the meta-analysis.

Both of the methods utilize the zero-order correlations among the variables in the regression studies. The GLS method treats the correlations from each study as a subset of multivariate outcomes, and combines the results with the consideration of the dependencies of the correlations across studies (Raudenbush, Becker, & Kalaian, 1988; Becker, 1992). The SWP method in this study applies the concept of missing data to the regression models that contained different predictors included in the synthesis. An empirical study was conducted by creating a set of regression studies using a subset of NELS:88 dataset. The correlations among the created studies were combined. A final regression model with standardized slopes was calculated for each of the predictors using

each of the two methods. The results from this empirical study showed that SWP produced less biased estimates of slopes in most situations.

The precision of the results from those two methods could be impacted by the features of studies included in the meta-analysis. Therefore, a simulation was conducted to investigate the impacts of missing-data patterns, intercorrelations among the predictors and the outcome, and the sample size. The results indicated that the difference between the two methods was not large. SWP consistently performed slightly better at estimating the slope of the predictor that was fully observed in all studies in the synthesis. Generally, SWP performed well when the sample sizes were equal and small across all studies, and GLS performed better when the sample sizes were equal and large.

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CHAPTER 1

INTRODUCTION

Meta-analysis is a quantitative procedure that allows researchers to summarize a myriad of studies focusing on one topic. This technique helps to address the challenges introduced by the existence of multiple answers to a given research question. The essential feature of meta-analysis is adopting the same type of effect size across studies, so the results from different studies are comparable. As Lipsey and Wilson (2001) summarized:

> The various effect size statistics used to code different forms of quantitative study findings in meta-analysis are based on the concept of standardization. The effect statistic produces a statistical standardization of the study findings such that the resulting numerical values are interpretable in a consistent fashion across all the variables and measures involved. (p.4)

The commonly used effect sizes in meta-analysis fall into one of two families: The *d* family and the *r* family (Rosenthal, 1994). Generally speaking, the *d* family includes proportions or mean differences between groups; the *r* family includes the Pearson product moment correlation (*r*), as well as the Fisher's transformation of *r*.

1.1 Absence of Methods for Meta-analyzing Regression Results

For more than two decades, methods for synthesizing mean differences and correlations have been broadly studied and clearly documented in several major publications (see Cooper, 1998; Cooper & Hedges, 1994; Hunter & Schmidt, 2001;

Lipsey & Wilson, 2001; Sutton, Abrams, Jones, Sheldon, & Song, 2000). Methods for synthesizing cumulative evidence from studies using regression, however, have not yet been well studied.

Regression has been widely used by researchers in different fields for predicting and explaining the variation in outcomes of interest. Regression can also be considered as a more sophisticated method, compared to correlation, because it involves more statistical controls when studying relationships among variables. Without appropriate methods to combine results using regression, a great deal of evidence can not be used. Excluding regression studies sabotages the thorough understanding of research questions when conducting a meta-analysis.

1.2 Potential Effect Sizes from Regression Studies

Statistics that can be found in regression studies are the raw regression coefficient or slope and sometimes its standard error, the t statistic for testing the significance of the slope, the standardized slope, and the R^2, which is the proportion of variance explained by the model.

A raw regression coefficient (slope) represents the expected increment in the dependent variable when the focal independent variable increases one unit, while controlling for other independent variables in the model. The magnitude of the raw coefficient changes when the scales of the dependent and independent variables change. This characteristic means the slope cannot be compared directly across models, unless all the models use the same scales to measure both the dependent and independent variables. The t statistic associated with the slope is more like a standardized estimate because each

t is the raw regression coefficient scaled by its own standard error. The t statistic itself has less power to explain the magnitude of the slope, and it depends on the sample size. The standardized slope is the raw regression coefficient standardized by the standard deviations of the predictor and the outcome. It is scaled in a standardized unit. Therefore, it can be compared directly across models. The explained variance (R^2) represents the proportion of the variance in the outcome accounted for by all the predictors combined in a regression model. If we wish to focus on variance explained by a certain predictor, then the partial R^2 value for that predictor will need to be computed by withdrawing the effects of other independent variables. Hunter and Schmidt (1990) argued that using R^2 to represent the magnitude of effect loses the direction of the effect. They also stated "variance-based indices of effect size make [variables that account for small percentages of the variance which might be] important effects appear much less important than they actually are, misleading both researchers and consumers of research" (p. 190).

1.3 Potential Problems of Synthesizing Regression Studies

One of the major problems arising when synthesizing regression studies is that the potential effect sizes discussed above are not comparable if the models included in the synthesis do not all use the same predictors. That is, the effects of different variables are partialed out, or held constant, when computing the effect of a focal predictor. Therefore, the focal slope, no matter whether it is a standardized or a raw slope, has different meanings across studies. This problem becomes complicated quickly when models contain many predictors. Unless the extra predictors in some models are absolutely independent of the focal predictor, which is never true, comparing the slopes or other effect sizes from unparallel models is comparing apples to oranges. One solution to this issue might be including only models that contain the same variables. However, it is unrealistic to expect to find parallel models created for the same research question, especially in education, where large numbers of variables are typically used to investigate one phenomenon.

Another problem that arises when meta-analyzing raw slopes from a set of regression studies is that the magnitude of the raw slope can change when the scales of the outcome and the predictors change. This implies that only when all variables are measured using the same scales, and all models contain the same predictors in the studies included in the synthesis, can slopes be compared directly.

1.4 Purpose of This Research

Since the solution of including only models that contain the same variables measured in the same scales (so the raw slopes can be comparable) is impractical, the current study focused on investigating methods for reducing the impact of unparallel regression models by synthesizing scale-free correlations among the variables in the model, which the standardized slopes are based on. Then the synthesized correlations can be used to create a final regression model with standardized slopes as the synthesis result.

Two methods were examined in this study. One method uses a non-model based multivariate generalized least squares (GLS) approach; the other method uses model based factored likelihood estimation. These two methods were first examined empirically by creating and analyzing four pseudo studies based on samples that were drawn from a selected sub sample from a large national dataset. Then a Monte Carlo stimulation was conducted to test the precision and stability of the two methods under different scenarios.

CHAPTER 2

LITERATURE REVIEW

Researchers in several fields have been trying to include regression studies in their meta-analyses. Most of these syntheses have either oversimplified the situation, or the methods proposed were limited to other fields and may not be applicable to education. Among those methods, a more universal technique that was proposed in the early 1970s to investigating regression coefficients at one time was to create a hierarchical-linear-model-like model for modeling the variance among the coefficients (Hanushek, 1974). However, the method focused on quantifying the variance among the slopes and required raw data along with some infrequently reported summarized statistics, which may not be applicable in the meta-analysis context.

2.1 In Psychology

Raju, Fralicx, and Steinhaus (1986) proposed a "regression slope model" to adjust for the variability of the slopes found among studies that originates from the use of unreliable measures. The model presented by the authors is

$$b_{yx} = B_{yx} r_{xx} + \mathrm{e}$$

where b_{yx} is the observed regression coefficient for predicting y from x, B_{yx} is the unattenuated and unrestricted population regression coefficient, r_{xx} is the unrestricted population reliability of predictor x, and e is the sample error associated with b_{yx} (p. 197).

The ultimate goal for assessing validity generalization (VG) is to estimate the mean and the variance of the regression slope parameter (B_{yx}) using the mean of B_{yx} (M_B) and its variance (V_B). As Raju et al. pointed out, regression slope models "should theoretically be affected by scale differences in either or both of the predictor or criterion instruments used across the separate validity studies.....The use of the new models for studying validity generalization, therefore, required that the scales for the predictor and criterion variables be comparable across studies" (p. 199). As Raju, Pappas, and Williams (1989) also pointed out, "[w]ithout the common metrics for the criterion and predictor variables, it is almost impossible to interpret credibility intervals of the type used with the correlation model. The use of the new models for studying VG, therefore, requires that scales for the predictor and criterion variables be comparable across studies"(p. 903). In addition to limiting scale comparability, the other requirement that is implied by Raju and colleagues' model is that only one predictor is involved in the model. This condition might easily be achieved when studying validity generalization, yet it is usually not the case in education studies.

2.2 In Epidemiology

Several reviews have been done that synthesize the slopes from dose-response models, which are widely used for evaluating the relationship between dose (e.g., of a drug or other treatment) and response. To create a dose-response model, researchers assign values for different dose levels and use those values as a predictor to predict a targeted response which is in the form of an odds ratio. Greenland and Longnecker (1992) combined the slopes from dose-response models based on 10 published datasets. They used techniques analogous to the standard inverse-variance weighting techniques that are used in contingency data to analyze the differences among the slopes. The same approach was adopted to study the relationship between individual consumption of chlorinated drinking water and bladder cancer (Villanueva, Fernandez, Malats, Grimalt, & Kogevinas, 2003). In both meta-analyses, the dose levels in different studies were relabeled with new values according to the same standards, and the outcomes were all odds ratios. Therefore, the slopes are comparable.

2.3 In Economics

Meta-analyses of regression studies can be found in syntheses of demand studies. The characteristic of demand studies that facilitates conducting a meta-analysis is that the demand elasticities from different studies are typically all on the same scale, because a demand elasticity, which is a regression slope, expresses the relationship between demand and its determinant as the percentage change in demand caused by a 1% change in the determinant. Crouch (1995) conducted a meta-analysis to synthesize 80 studies of international tourism demand. Those studies produced 1,964 observations (i. e., regression equations) and 10,078 regression coefficients. The majority of included demand elasticities concerned income, price, exchange rates, transportation cost, and marketing expenditures. The author adopted the synthesis method proposed by Raju et al. (1986), mentioned in the previous section. However, the regression coefficients were obtained from international tourism demand models that were not parallel and contained more than one independent variable, which violated the requirement of Raju and his colleagues' methods. The author was actually aware of the violation and stated that "the value of b_j may be affected by the inclusion of other explanatory variables" (p. 109), but he did not justify his decision to include unparallel models. A series of articles pertinent to meta-analyzing regression studies focusing on elasticity in economics can be found in the special issue of *Journal of Economic Surveys* published in 2005 (Vol. 19, Issue 3).

2.4 In Ecology

A recent review combining regression results is focused on summarizing the relationship between population density and body size for mammals and birds (Bini, Coelho, & Diniz-Filho, 2001). The authors used a conventional weighting scheme to weight the slope by its standard error. It is not clear whether "body size" and "population density" were measured on the same scales though they could have been. It might be safe to assume that population density was measured on one scale across studies. However, body size could be measured in terms of length, weight, body mass, or some other measure. The authors did not mention how they dealt with the different units for the predictor. Moreover, it is not clear if all 74 regression models included in this meta-analysis used "body size" as the only predictor.

2.5 In Political Science

Lau, Sigelman, Heldman and Babbitt (1999) tried to combine the results from both group comparison studies and regression studies that focused on the effect of negative political advertisements on political campaigns. They found that about one-quarter of their data points "come from ordinary least squares (OLS) or logistic regression equations, and there is no universally accepted method for handling such data in a meta-analysis" (p. 855). To avoid losing data, they decided to use *t* statistics associated with the regression coefficients from regression studies to represent the treatment (exposure to negative advertisements) versus control (exposure to no advertisements or positive advertisements) mean difference effect, and then converted each t value into d, by using $d = 2t/(df)^{1/2}$. They argue that therefore the converted ds can be combined with other ds from group comparisons. The authors cited Stanley and Jarrell (1989), who claim the t statistic has no dimensionality therefore can be combined directly when the units of independent and dependent variables are not the same across studies, to justify the usage of synthesizing t statistic for the slopes from regression models in their synthesis. However, the impact from different independent variables being used in different models still exists.

2.6 In Education

Hanushek (1989) summarized 187 studies studying the impact of differential expenditures on school performance in 38 separately published articles or books, using the "vote-count" method, which simply ignores the magnitudes of effects, and counts the numbers of studies with significant positive estimates, significant negative estimates, nonsignificant positive estimates, or nonsignificant negative estimates. To avoid the poor statistical properties of the vote-count method (Hedges & Olkin, 1980), Greenwald, Hedges and Laine (1996) tried to summarize half-standardized slopes in a review of educational production functions examining the same topic as Hanushek. However, fundamental problems for synthesizing the results from the production function still exist: The models usually do not involve the same predictors; different outcomes might be used in different studies; and the scales of all the variables may not be identical across studies.

2.7 The Most Recent Study

In a recently article, Peterson and Brown (2005) conducted an empirical study and derived a formula for converting standardized slopes (often denoted as βs even though they are sample estimates) reported in regression studies into Pearson's correlations (rs) in order to include slopes and analyze them with other correlations using conventional methods designed for synthesizing correlations. The authors searched 35 journals from disciplines including psychology, consumer behavior, management, marketing, and sociology from the period of 1975-2001. They included only studies with both βs and rs reported at the individual level. A total of 1,504 corresponding βs and rs were identified from 143 articles containing 160 data sets and 270 regression models. Given the relationships shown in the βs and rs they collected, the authors derived an equation $r = .98\beta + .05\lambda$, where λ is an indicator variable that equals 1 when β is nonnegative and 0 when β is negative.

Peterson and Brown's research is the first published study that mainly focused on incorporating the estimates from regression studies with those from correlational studies in the meta-analysis context. The authors did notice the relationship between βs and rs can be impacted by features such as sample size and numbers of predictors in the regression model. However, they oversimplified the situation and did not really utilize those features to create their formula for converting βs to rs.

CHAPTER 3

METHODOLOGIES

As mentioned in the introduction, two major problems for incorporating regression studies in meta-analysis are that 1) different predictors may be used in different primary studies studying the same topic, and 2) predictors and the outcome are often measured in different scales across studies. As presented in the literature review section, most of the meta-analyses that have been done in different fields are either solely focused on simple regression studies where the same scales for the predictor and the outcome are comparable across studies, or the meta-analyst simply ignored the fact that the slopes may have different meanings because different predictors are used across studies. In order to combine the results from regression models in a general way and to be more precise in estimating the effect of the predictors by considering the impact from unparallel models, the currently research focuses on utilizing the zero-order correlation matrix from each study included in the meta-analysis to calculate summarized standardized slopes for a final regression model, which is the result of synthesizing regression studies.

3.1 Focusing on the Zero-order Correlations

Instead of synthesizing slopes directly, the two methods examined in this study both start by summarizing the zero-order correlations among variables in regression models. As Hunter and Schmidt (2001) pointed out:

> A multiple regression analysis of a primary study is based on the full zero-order

correlation (or covariance) matrix for the set of predictor variables and the criterion variable. Similarly, a cumulation of multiple regression analyses must be based on a cumulative zero-order correlation matrix. (p. 475)

Three major reasons make combining correlations beneficial. First, focusing on the correlations among the predictors and the outcome across studies, rather than trying to combine the slopes directly, disposes of the problem that slopes have different meaning when the models contain different predictors. This is because correlations among the variables used in a regression model are "zero-order measures" for the relationships, which means that the correlations between two variables will not change when other predictors are added into the model. Other than the advantage of stability, focusing on correlations also allows us to get by the problem of different scales used in measuring the same predictors in different models, because correlations are metric free and can be combined directly (under certain assumptions, which will be discussed later). Moreover, with the focus on correlations among the variables in the regression model, the results from correlational studies can be easily combined with the results from regression studies. This expands the set of studies that can be synthesized, because studies reporting the relationship between any pairs of variables of interest can be included.

3.2 Constructing the Standardized Regression Model

Once the selected correlations that the regression models are based on are combined appropriately, the summarized correlations are used to create a final regression model with standardized slopes, because standardized slopes are functions of the associated correlations. The relative importance of the predictors can then be appraised.

Also, the variance explained by each predictor (e.g., the partial R^2) based on the final model can also be calculated if it is of interest.

3.3 Proposed Methods

Two methods for summarizing regression results based on the zero-order correlations that allow unparallel models to be combined were investigated in this research. One method uses a non-model based multivariate generalized least squares (GLS) approach (see Becker, 1992; Becker & Schram, 1994; Gleser & Olkin, 1994; Hedges & Olkin,1985; Raudenbush, Becker, & Kalaian, 1988); the other method uses model based factored likelihood estimation through the sweep operator (SWP). As indicated in Becker (2000), the GLS method has been typically used in multivariate meta-analysis. This method was used in the current research to compare with the SWP method, a new application to meta-analysis. Details for each method will discussed separately.

Before the methods are presented, it should be noted that, as with all parametric statistical methodologies, the methods proposed here require certain assumptions. A general assumption that is required for each model included in the meta-analysis is that all the predictors and the outcome are measured appropriately, and they are related to each other approximately linearly, except for the presence of dummy variables. Those are the major assumptions for any regression study. In addition, we have to assume that multicollinearity is not a problem for each of the regression models. That is, the predictors are not highly correlated with each other in one model. In the primary studies, the authors may or may not report checking these assumptions. Yet we have to assume the condition of linearity is not violated to work on the correlations in the meta-analysis, and we have to assume the absence of multicollinearity to build a meaningful final model and estimate the synthesized standardized slopes based on individual ones. Other specific assumptions for each method will be discussed in the presentation of each method.

3.3.1 Multivariate Generalized Least Squares

If we think about the zero-order correlations between variables (predictors and outcomes) from regression models as the effect sizes in a synthesis, the problem of meta-analyzing those correlations is similar to meta-analyzing multivariate effect sizes from studies. Each study may contain some similar predictors and some different ones, that makes the correlations produced in each study a subset of the correlations from the final model, which need to be determined.

Several methods for synthesizing correlations, in terms of the correlation matrices, have been investigated and discussed (e.g., Becker & Schram, 1994; Furlow & Beretvas, 2005). To combine subsets of multivariate outcomes in order to calculate the standardized slopes for the predictors in the final model, the method first proposed by Raudenbush, Becker, and Kalaian (1988) based on generalized least squares (GLS) is adopted in the current research. To illustrate the application of GLS to synthesize regression results, an auxiliary example is used. The same example will be used to illustrate the next method as well.

Suppose four regression studies are to be included in a synthesis. All of them studied the same outcomes Y_{kl}, where k is study number ($k = 1$ to 4) and l represents subject l in study k. Study 1 contains only predictor X_1; Study 2 contains both X_1 and X_2;

Study 3 contains X_1, X_2, and X_3; Study 4 contains X_1, X_2, X_3,and X_4. The estimated regression models with standardized slopes ($\hat{B}$ s) are as shown below for the four studies.

Study 1: $Y_{1l} = B_{11}X_{11l}$ for $l = 1$ to n_1

Study 2: $Y_{2l} = B_{21}X_{21l} + B_{22}X_{22l}$ for $l = 1$ to n_2

Study 3: $Y_{3l} = B_{31}X_{31l} + B_{32}X_{32l} + B_{33}X_{33l}$ for $l = 1$ to n_3

Study 4: $Y_{4l} = B_{41}X_{41l} + B_{42}X_{42l} + B_{43}X_{43l} + B_{44}X_{44l}$ for $l = 1$ to n_4

where,

X_{k1l} is the value of variable X_1 for subject l in study k,

X_{k2l} is the value of variable X_2 for subject l in study k,

X_{k3l} is the value of variable X_3 for subject l in study k,

X_{k4l} is the value of variable X_4 for subject l in study k, and

n_k is the sample size of study k.

Following the example above, the vectors of zero-order correlations of the four studies ($\mathbf{r}_k$, k=1, 2, 3, or 4) with elements $r_{k\,(\text{variable 1 variable 2})}$ in the vectors are as follows. In the following expressions, for simplicity, only the numerical part of the variable label is used inside of the parentheses (i.e., 4 indicates of X_4).

$$\mathbf{r_1} = \begin{bmatrix} r_{1(Y1)} \end{bmatrix}, \mathbf{r_2} = \begin{bmatrix} r_{2(Y1)} \\ r_{2(Y2)} \\ r_{2(12)} \end{bmatrix}, \mathbf{r_3} = \begin{bmatrix} r_{3(Y1)} \\ r_{3(Y2)} \\ r_{3(Y3)} \\ r_{3(12)} \\ r_{3(13)} \\ r_{3(23)} \end{bmatrix}, \text{ and } \mathbf{r_4} = \begin{bmatrix} r_{4(Y1)} \\ r_{4(Y2)} \\ r_{4(Y3)} \\ r_{4(Y4)} \\ r_{4(12)} \\ r_{4(13)} \\ r_{4(14)} \\ r_{4(23)} \\ r_{4(24)} \\ r_{4(34)} \end{bmatrix}.$$

To use the GLS method to summarize multivariate outcomes, we need an identity matrix, **W**, to identify which correlation is estimated in each study. The relationships among the correlation vector, indicator matrix, and the population correlation vector (**ρ**) is shown below.

$$\mathbf{r} \quad = \quad \mathbf{W} \quad \text{x} \quad \boldsymbol{\rho} \quad + \quad \mathbf{e} \tag{1}$$

$$\begin{bmatrix} r_{1(Y1)} \\ r_{2(Y1)} \\ r_{2(Y2)} \\ r_{2(12)} \\ r_{3(Y1)} \\ r_{3(Y2)} \\ r_{3(Y3)} \\ r_{3(12)} \\ r_{3(13)} \\ r_{3(23)} \\ r_{4(Y1)} \\ r_{4(Y2)} \\ r_{4(Y3)} \\ r_{4(Y4)} \\ r_{4(12)} \\ r_{4(13)} \\ r_{4(14)} \\ r_{4(23)} \\ r_{4(24)} \\ r_{4(34)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \rho_{(Y1)} \\ \rho_{(Y2)} \\ \rho_{(Y3)} \\ \rho_{(Y4)} \\ \rho_{(12)} \\ \rho_{(13)} \\ \rho_{(14)} \\ \rho_{(23)} \\ \rho_{(24)} \\ \rho_{(34)} \end{bmatrix} + \begin{bmatrix} e_{1(Y1)} \\ e_{2(Y1)} \\ e_{2(Y2)} \\ e_{2(12)} \\ e_{3(Y1)} \\ e_{3(Y2)} \\ e_{3(Y3)} \\ e_{3(12)} \\ e_{3(13)} \\ e_{3(23)} \\ e_{4(Y1)} \\ e_{4(Y2)} \\ e_{4(Y3)} \\ e_{4(Y4)} \\ e_{4(12)} \\ e_{4(13)} \\ e_{4(14)} \\ e_{4(23)} \\ e_{4(24)} \\ e_{4(34)} \end{bmatrix}.$$

The estimated population correlation vector ($\hat{\boldsymbol{\rho}}$) contains the synthesized correlations for the full model, if we assume the final model contains the outcome Y and all the four predictors. It can be computed as:

$$\hat{\boldsymbol{\rho}} = (\mathbf{W}'\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{W})^{-1}\mathbf{W}'\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{r}\,, \tag{2}$$

where $\hat{\boldsymbol{\Sigma}}$ is the large variance-covariance matrix containing the variance-covariance matrices ($\hat{\mathbf{\Sigma}}_k$ s) of all the studies included in the meta-analysis on the diagonal, and zeros in the upper and lower triangles. That is,

$$\hat{\mathbf{\Sigma}} = \begin{bmatrix} \hat{\mathbf{\Sigma}}_1 & 0 & 0 & 0 \\ 0 & \hat{\mathbf{\Sigma}}_2 & 0 & 0 \\ 0 & 0 & \hat{\mathbf{\Sigma}}_3 & 0 \\ 0 & 0 & 0 & \hat{\mathbf{\Sigma}}_4 \end{bmatrix}, \tag{3}$$

The components in $\hat{\mathbf{\Sigma}}_k$ depend on the intercorrelations of the predictors and the outcome, as well as the sample size in study *k*. The variance of each correlation in each study can be obtained based on second-order and fourth-order moments of the samples based on large sample theory (Pearson & Filon, 1898; Olkin & Siotani, 1976), which can be simplified to

$$\hat{\sigma}_k^2(r_{ij}) = \frac{(1 - r_{k(ij)}{}^2)^2}{n_k}$$

for $k = 1, 2, 3$ and 4; $i = Y, X_1, X_2, X_3$, and X_4; $j = Y, X_1, X_2, X_3$, and X_4; $i \neq j$. (4)

The covariance between any two correlations in which there is a common variable is

$$\hat{\sigma}_k(r_{ij}, r_{ij'}) =$$

$$[\frac{1}{2}(2r_{k(jj')} - r_{k(ij)}r_{k(ij')})(1 - r_{k(ij)}^2 - r_{k(ij')}^2 - r_{k(jj')}^2) + r_{k(jj')}^3]/n_k. \tag{5}$$

The covariance between any two correlations that do not involve any variables in common is

$$\hat{\sigma}_k(r_{ij}, r_{i'j'}) =$$

$$\begin{aligned}
&[\frac{1}{2} r_{k(ij)} r_{k(i'j')} (r^2_{k(ii')} + r^2_{k(ij')} + r^2_{k(j'j)} + r^2_{k(i'j)}) + r_{(ii')} r_{k(jj')} + r_{k(ij')} r_{k(ji')} - \\
&(r_{k(ij)} r_{k(ii')} r_{k(ij')} + r_{k(ji)} r_{k(ji')} r_{k(jj')} + r_{k(i'i)} r_{k(i'j)} r_{k(i'j')} + r_{k(j'i)} r_{k(j'j)} r_{k(j'i')})] / n_k.
\end{aligned} \tag{6}$$

Therefore, to fit on a page, the full variance-covariance matrices for the first two studies stacked to form the large matrix for GLS look like

$$\hat{\boldsymbol{\Sigma}} = \begin{bmatrix} \hat{\boldsymbol{\Sigma}}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \hat{\boldsymbol{\Sigma}}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \hat{\boldsymbol{\Sigma}}_3 \end{bmatrix}$$

$$= \begin{bmatrix} \hat{\sigma}_1^2(r_{Y_1X_1}) & 0 & 0 & 0 & 0 & 0 \\ 0 & \hat{\sigma}_2^2(r_{Y_2X_1}) & \hat{\sigma}_2(r_{Y_2X_1}, r_{Y_2X_2}) & \hat{\sigma}_2(r_{Y_2X_1}, r_{X_1X_2}) & 0 & 0 \\ 0 & \hat{\sigma}_2(r_{Y_2X_1}, r_{Y_2X_2}) & \hat{\sigma}_2^2(r_{Y_2X_2}) & \hat{\sigma}_2(r_{Y_2X_2}, r_{X_1X_2}) & 0 & 0 \\ 0 & \hat{\sigma}_2(r_{Y_2X_1}, r_{X_1X_2}) & \hat{\sigma}_2(r_{Y_2X_2}, r_{X_1X_2}) & \hat{\sigma}_2^2(r_{X_1X_2}) & 0 & 0 \\ 0 & 0 & 0 & 0 & \underset{6*6}{\hat{\boldsymbol{\Sigma}}_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \underset{10*10}{\hat{\boldsymbol{\Sigma}}_4} \end{bmatrix} \quad (7)$$

Once the population correlations are estimated based on the correlations from all the studies, the standardized slopes can be calculated for the final model with all four predictors in it based on the estimated correlations. The value of the standardized slope is a function of the correlation between that predictor and the outcome, as well as the correlations that exist among the predictors themselves. According to Cooley and Lohnes (1971) as well as the discussion that is more focused on the synthesis context in Becker and Schram (1994), we can set up the full correlation matrix (**R**) containing all the pairs of synthesized correlations in the final model, where there are (p-1) predictors and the pth variable is the outcome variable. To simplely use algebra to calculate the standardized slopes, the **R** is partitioned as

$$\mathbf{R} = \left(\begin{array}{ccccc|c} 1.0 & r_{12} & r_{13} & \cdots & r_{1,p-1} & r_{1p} \\ r_{21} & 1.0 & r_{23} & \cdots & r_{2,p-1} & r_{2p} \\ r_{31} & r_{32} & 1.0 & \cdots & r_{3,p-1} & r_{3p} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ r_{p-1,1} & r_{p-1,2} & r_{p-1,3} & \cdots & 1.0 & r_{p-1,p} \\ \hline r_{p1} & r_{p2} & r_{p3} & \cdots & r_{p,p-1} & 1.0 \end{array}\right)$$

$$= \left(\begin{array}{c|c} \mathbf{R}_{11} & \mathbf{R}_{12} \\ \hline \mathbf{R}_{21} & 1.0 \end{array}\right) \tag{8}$$

The standardized slopes vector (**B**) can be calculated by

$$\mathbf{B} = \mathbf{R}_{11}^{-1}\mathbf{R}_{12}. \tag{9}$$

3.3.2 Factored Likelihoods through Sweep Operators

The second approach examined for synthesizing regression results, based on a multivariate normal distribution model, uses likelihood-based estimation. The maximum likelihood estimation investigated in the current study is based on the factored likelihood, which was first proposed by Anderson (1957) to deal with missing data.

The usage of factor likelihood allows us to obtain the maximum likelihood estimates noniteratively, and no imputation will really be needed to fill in the data on the predictors that are missing from the model. This original idea of factored likelihood can be understood more easily by illustration using the bivariate case. Suppose two variables X and Y have a bivariate normal distribution. There are n observations on both X and Y, and an additional m observations on X. The data may look like:

$X_1, \ldots, X_n, X_{n+1}, \ldots X_{n+m}$, and

$Y_1, \ldots, Y_n$.

The bivariate normal density for the two variables with mean μ_X and μ_Y, variances σ_X^2 and σ_Y^2, and covariance σ_{XY} can be denoted as

$$\mathrm{N}(X, Y \mid \mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \sigma_{XY}). \tag{10}$$

The density function of the data can then be revised as the product of the marginal density of X and the conditional density of Y given X, which is

$$\mathrm{N}(X, Y \mid \mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \sigma_{XY}) = \mathrm{N}(X \mid \mu_X, \sigma_X^2)\mathrm{N}(Y \mid \alpha + \beta_{YX} X, \sigma_{Y \cdot X}^2), \tag{11}$$

where α is the regression constant, and β_{YX} is the regression coefficient of Y on X, and $\sigma^2_{Y \cdot X}$ is the residual variance of Y given X. The relationships among the parameters on the left-hand side and those on the right-hand side in the density function (11) are

$$\begin{aligned}
&\alpha = \mu_Y - \beta_{YX}\mu_X = \mu_Y - \mu_X\sigma_{XY} / \sigma^2_X, \\
&\beta_{YX} = \sigma_{XY} / \sigma^2_X, \text{ and} \\
&\sigma^2_{Y \cdot X} = \sigma^2_Y[1 - (\beta_{YX}\sigma_X / \sigma_Y)^2] = \sigma^2_Y - \beta^2_{YX}\sigma^2_X = \sigma^2_Y - \sigma^2_{XY} / \sigma^2_X.
\end{aligned} \tag{12}$$

The five parameters on the right-hand side in (11) are one-to-one functions of the five parameters on the left-hand side.

Based on the density function in (11), the likelihood function for the dataset with n cases of Y and $(n+m)$ cases of X can be factored into a product of likelihoods

$$\begin{aligned}
&\prod_{i=1}^{n} N(X_i, Y_i \mid \mu_X, \mu_Y, \sigma^2_X, \sigma^2_Y, \sigma_{XY}) \prod_{i=n+1}^{n+m} N(X_i \mid \mu_X, \sigma^2_Y) \\
&= \left[\prod_{i=1}^{n+m} N(X_i \mid \mu_X, \sigma^2_X)\right]\left[\prod_{i=1}^{n} N(Y_i \mid \alpha + \beta_{YX}X_i, \sigma^2_{Y \cdot X})\right].
\end{aligned} \tag{13}$$

The factoring permits the independent maximization of the two bracketed expressions in (13), since the parameters in the expressions are distinct. That is, μ_X and σ^2_X do not occur in both products as they do in the first line of (13). By maximizing the likelihood in the second line of (13), the maximum likelihood estimates of those parameters can be obtained and the original parameters can be derived as shown in (12). In other words, the parameters (i.e., the $\mu_X, \mu_Y, \sigma^2_X, \sigma^2_Y$, and σ_{XY} in (13)) are transformed in the way that the likelihood function is factorized into distinct factors (i.e., the μ_X, σ_X^2, α, β_{YX}, and $\sigma_{Y \cdot X}^2$ in (13)). Therefore, the original estimation problem is decomposed into a series of smaller estimation problems that can be solved by computing and manipulating the variance-covariance matrices for selected subsets of observations.

One important assumption of using this method is that the mechanism of missing predictor(s), which produces blocked missing data, is assumed ignorable. As the first depicted by Rubin (1976) and later elaborated by Little and Rubin (2002) and many other researchers studying missing data methods, the missing-data mechanism is ignorable if the parameter of missing data and the parameter of observed data are not functionally related. In the current application to synthesize regression models, this assumption means that a predictor that is not included in a model is left out randomly and the missingness is not related to any of the existing predictors. This can easily happen when we include studies using government released large-scale datasets, in which many variables are measured and can be used, along with some other studies, in which the data are collected by individual researchers and fewer variables are measured because of the constraints of time and money.

To illustrate how exactly the factored maximum likelihood method for missing data can be applied to the issue of combining regression results, the example used to illustrate the GLS method is used again to facilitate the explanation.

Figure 3.1 portrays how the issue of combining regression results is similar to the issue of handling missing data. The left column in Figure 3.1 contains four regression models for synthesis, as described earlier. The columns on the right side show the data structure for each model.

		Y	*1*	*2*	*3*	*4*
Study 1: $\hat{Y}_{1l} = \hat{B}_{11} X_{11l}$ for $l = 1$ to 58	**Study 1**	Y_{11} Y_{12} $\vdots$ Y_{1n_1}	X_{111} X_{112} $\vdots$ X_{11n_1}	Missing	Missing	Missing
Study 2: $\hat{Y}_{2l} = \hat{B}_{21} X_{21l} + \hat{B}_{22} X_{22l}$ for $l = 1$ to 66	**Study 2**	Y_{21} Y_{22} $\vdots$ Y_{2n_2}	X_{211} X_{212} $\vdots$ X_{21n_2}	X_{221} X_{222} $\vdots$ X_{22n_2}		
Study 3: $\hat{Y}_{3l} = \hat{B}_{31} X_{31l} + \hat{B}_{32} X_{32l} + \hat{B}_{33} X_{33l}$ for $l = 1$ to 74	**Study 3**	Y_{31} Y_{32} $\vdots$ Y_{3n_3}	X_{311} X_{312} $\vdots$ X_{31n_3}	X_{321} X_{322} $\vdots$ X_{32n_3}	X_{331} X_{332} $\vdots$ X_{33n_3}	
Study 4: $\hat{Y}_{4l} = \hat{B}_{41} X_{41l} + \hat{B}_{42} X_{42l} + \hat{B}_{43} X_{43l}$ $+ \hat{B}_{44} X_{44l}$ for $l = 1$ to 82	**Study 4**	Y_{41} Y_{42} $\vdots$ Y_{4n_4}	X_{411} X_{412} $\vdots$ X_{41n_4}	X_{421} X_{422} $\vdots$ X_{42n_4}	X_{431} X_{432} $\vdots$ X_{43n_4}	X_{441} X_{442} $\vdots$ X_{44n_4}

Note. $\hat{Y}_{kl}$: estimated score for person l in study k; X_{kil} : score on variable i for person l in study k ; $\hat{B}_{ki}$: the estimated standardized slope for variable i in study k; n_k : sample size for study k.

Figure 3.1. The Models for Four Created Studies and the Structure of the Data

Since the ultimate goal of using this method is to produced a final regression model with standardized slopes, it is helpful to think of all the hypothetical *X*s and *Y*s presented in Figure 3.1 as standardized scores (*z* scores) with a bivariate distribution (Study 1) or multivariate distributions (Studies 2, 3, and 4). When the original data from three studies are concatenated in the way shown in the columns on the right side in Figure 3.1, a predictor that is not included in a study-specific model (e.g., X_3 in study 2) can be seen as a missing predictor from the final full model, as described earlier. The final model in this example is a model with all four predictors in it. The data for the missing predictors in studies constitute missing blocks in the multivariate dataset after combining studies 1, 2, 3, and 4. Therefore, the factored likelihood method described for the bivariate cases earlier can be applied to estimate the parameters of interest (which will be the correlations among Y, X_1, X_2, and X_3) in this multivariate example.

As previous discussion suggested, when a multivariate data set contains blocks of missing observations, the original estimation problem can be decomposed into smaller estimation problems by factoring the likelihood of the observed data into a product of likelihoods whose parameters are distinct. Therefore, the likelihood of the estimates of the parameters in the current example with four studies can be written as

$$
\begin{aligned}
&\prod_{l=1}^{n_4} \mathrm{N}(Y_{4l}, X_{41l}, X_{42l}, X_{431}, X_{44l} \mid \mu_Y, \mu_1, \mu_2, \mu_3, \mu_4, \sigma_Y^2, \sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_{Y1}, \sigma_{Y2}, \sigma_{Y3}, \sigma_{Y4}, \sigma_{12}, \sigma_{13}, \sigma_{14}, \sigma_{23}, \sigma_{24}, \sigma_{34}) \\
&\prod_{l=1}^{n_3} \mathrm{N}(Y_{3l}, X_{31l}, X_{32l}, X_{33l} \mid \mu_Y, \mu_1, \mu_2, \mu_3, \sigma_Y^2, \sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_{Y1}, \sigma_{Y2}, \sigma_{Y3}, \sigma_{12}, \sigma_{13}, \sigma_{23}) \\
&\prod_{l=1}^{n_2} \mathrm{N}(Y_{2l}, X_{21l}, X_{22l} \mid \mu_Y, \mu_1, \mu_2, \sigma_Y^2, \sigma_1^2, \sigma_2^2, \sigma_{Y1}, \sigma_{Y2}, \sigma_{12}) \\
&\prod_{l=1}^{n_1} \mathrm{N}(Y_{1l}, X_{11l} \mid \mu_Y, \mu_1, \sigma_Y^2, \sigma_1^2, \sigma_{Y1}) \\
= &\prod_{l=1}^{n_1+n_2+n_3+n_4} \mathrm{N}(Y_l, X_{1l} \mid \mu_Y, \mu_1, \sigma_Y^2, \sigma_1^2, \sigma_{Y1}) \\
&\prod_{l=1}^{n_2+n_3+n_4} \mathrm{N}(X_{2l} \mid \hat{\beta}_{20\cdot Y1} + \beta_{2Y\cdot 1} Y_l + \beta_{21\cdot Y} X_{1l}, \sigma_{2\cdot Y1}^2) \\
&\prod_{l=1}^{n_3+n_4} \mathrm{N}(X_{3l} \mid \hat{\beta}_{30\cdot Y12} + \beta_{3Y\cdot 12} Y_l + \beta_{31\cdot Y2} X_{1l} + \beta_{32\cdot Y1} X_{2l}, \sigma_{3\cdot Y12}^2)\ . \\
&\prod_{l=1}^{n_4} \mathrm{N}(X_{4l} \mid \hat{\beta}_{40\cdot Y123} + \beta_{4Y\cdot 123} Y_l + \beta_{41\cdot Y23} X_{1l} + \beta_{42\cdot Y13} X_{2l} + \beta_{43\cdot Y12} X_{3l}, \sigma_{4\cdot Y123}^2) \qquad (14)
\end{aligned}
$$

This method eventually yields ML estimates of the synthesized mean vector ($\hat{\boldsymbol{\mu}}$) and the variance-covariance matrix ($\hat{\boldsymbol{\Xi}}$) based on the four studies:

$$\hat{\boldsymbol{\mu}} = \begin{bmatrix} \hat{\mu}_Y \\ \hat{\mu}_{X_1} \\ \hat{\mu}_{X_2} \\ \hat{\mu}_{X_3} \\ \hat{\mu}_{X_4} \end{bmatrix}, \quad \hat{\boldsymbol{\Xi}} = \begin{bmatrix} \hat{\sigma}_Y^2 & \hat{\sigma}_{Y1} & \hat{\sigma}_{Y2} & \hat{\sigma}_{Y3} & \hat{\sigma}_{Y4} \\ & \hat{\sigma}_1^2 & \hat{\sigma}_{12} & \hat{\sigma}_{13} & \hat{\sigma}_{14} \\ & & \hat{\sigma}_2^2 & \hat{\sigma}_{23} & \hat{\sigma}_{24} \\ & & & \hat{\sigma}_3^2 & \hat{\sigma}_{34} \\ & & & & \hat{\sigma}_4^2 \end{bmatrix}.$$

Since the values of the predictors and the outcomes were treated as *z* scores with means equal to zero and standard deviations equal to one, the mean vector ($\hat{\boldsymbol{\mu}}$) would be a vector of zeros; the variances of variables Y, X_1, X_2, X_3, and X_4 on the diagonal of matrix $\hat{\boldsymbol{\Xi}}$ are expected to be 1s; the covariance of any two variables on the upper triangle in matrix $\hat{\boldsymbol{\Xi}}$ would be the synthesized correlation between two variables based on four studies. Once the correlations are combined across studies, they can be used to calculate the standardized slopes for the final model with all four predictors.

To determine the factored likelihood estimates of $\hat{\boldsymbol{\Xi}}$, sweep operators were adopted to figure out the regression coefficients (the αs and βs at the right side of equal sign in expression (14)) as well as the residual variances ($\hat{\sigma}^2$ s in the same expression). Then the reverse sweep was used to obtain the estimates of the parameters of interest as shown in (16).

The sweep operator and reverse sweep operator were originally defined by Beaton (1964) and later redefined by Dempster (1969) in the missing data context. In this research, the sweep operator from Dempster (1969) was adopted, which is defined as follows:

A $p*p$ matrix **M** is said to have been swept on row and column c if **M** is replaced by another $p*p$ matrix **N** whose element n_{ij} is related to the ijth element m_{ij} of **M** as follows:

$$
\begin{aligned}
&n_{cc} = -1/m_{cc} \\
&n_{ic} = m_{ic}/m_{cc} \\
&n_{cj} = m_{cj}/m_{cc} \\
&n_{ij} = m_{ij} - m_{ic} * m_{cj}/m_{cc}
\end{aligned}
\tag{15}
$$

for $i \neq c$ and $j \neq c$. That is, if **M** is a 3*3 matrix,

$$
\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix}.
$$

When we apply the rules above and sweep row and column 1 out of **M** to get another matrix **N**, the matrix **N** will look like

$$
\mathbf{N} = \begin{bmatrix} -1/m_{11} & m_{12}/m_{11} & m_{13}/m_{11} \\ m_{12}/m_{11} & m_{22} - m_{12}^2/m_{11} & m_{23} - m_{13}m_{12}/m_{11} \\ m_{13}/m_{11} & m_{23} - m_{13}m_{12}/m_{11} & m_{33} - m_{13}^2/m_{11} \end{bmatrix}.
$$

For brevity, using the terminology defined in Beaton (1964), the matrix **N** can be denoted as **N**=SWP[c]**M**. So, the example above can be expressed as

$$\mathbf{N} = \mathrm{SWP}[1]\mathbf{M} = \begin{bmatrix} -1/m_{11} & m_{12}/m_{11} & m_{13}/m_{11} \\ m_{12}/m_{11} & m_{22} - m_{12}^2/m_{11} & m_{23} - m_{13}m_{12}/m_{11} \\ m_{13}/m_{11} & m_{23} - m_{13}m_{12}/m_{11} & m_{33} - m_{13}^2/m_{11} \end{bmatrix}.$$

The result of successively applying the operations SWP[c_1], SWP[c_2],... SWP[c_t] to matrix **M** can be denoted by SWP[$c_1, c_2,... c_t$]. The operations are carried out successively, and each stage uses the output of only the previous stage.

One of the important properties of the sweep operator is that it is very easy to undo or reverse. Based on algebra, we can reverse sweep by replacing the *ij*th element n_{ij} in **N** matrix with the *ij*th element m_{ij} and obtain the original **M** matrix as

$$\begin{aligned} m_{cc} &= -1/n_{cc} \\ m_{ic} &= -n_{ic}/n_{cc} \\ m_{cj} &= -n_{cj}/n_{cc} \\ m_{ij} &= n_{ij} - n_{ic} * n_{cj}/n_{cc}. \end{aligned} \tag{16}$$

The reverse sweep is denoted as **M**=RSW[c]**N**.

$$\mathrm{RSW}[1] \begin{bmatrix} -1/n_{11} & -n_{12}/n_{11} & -n_{13}/n_{11} \\ -n_{12}/n_{11} & n_{22} - n_{12}^2/n_{11} & n_{23} - n_{13}n_{12}/n_{11} \\ -n_{13}/n_{11} & n_{23} - n_{13}n_{12}/n_{11} & n_{33} - n_{13}^2/n_{11} \end{bmatrix}$$

$$= \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix} = \mathbf{M}.$$

One important application of the sweep operator is to obtain maximum likelihood estimates for regression. That is, if we have a square matrix of intercorrelations of certain variables and choose one variable to sweep out, we are going to adjust all the remaining variables by removing the regression on that variable, which is identical to regressing the remaining variables on the swept-out variable.

For example, let matrix **R** be the correlation matrix of predictor X and outcome Y

$$\begin{array}{cc} & \begin{array}{cc} X & Y \end{array} \\ \mathbf{R} = & \begin{bmatrix} 1 & r_{XY} \\ r_{XY} & 1 \end{bmatrix}. \end{array}$$

After the operation of sweeping on X (row and column 1), the element h_{12} (r_{12}) in the matrix **H** below becomes the standardized coefficient of X, and the element h_{22} $(1-r_{12}^2)$ is the residual variance.

$$\mathbf{H} = \mathrm{SWP}[1]\mathbf{R} = \begin{bmatrix} -1 & r_{XY} \\ r_{XY} & 1-r_{XY}^2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{12} & h_{22} \end{bmatrix}.$$

The sweep operator can also be used to find the maximum likelihood estimates for multivariate regression. For example, matrix **Q** represents the correlation matrix for predictor variables X_1, X_2, and the outcome Y.

$$\begin{array}{cc} & \begin{array}{ccc} X_1 & X_2 & Y \end{array} \\ \mathbf{Q} = & \begin{bmatrix} 1 & r_{12} & r_{1Y} \\ r_{12} & 1 & r_{2Y} \\ r_{1Y} & r_{2Y} & 1 \end{bmatrix}. \end{array}$$

To calculate the standardized slopes for the multivariate linear regression of X_2 and Y on X_1, we sweep on the row and column 1.

$$\mathrm{SWP}[1]\mathbf{Q} = \mathrm{SWP}[1]\begin{bmatrix} 1 & r_{12} & r_{1Y} \\ r_{12} & 1 & r_{2Y} \\ r_{1Y} & r_{2Y} & 1 \end{bmatrix} = \begin{bmatrix} -1 & r_{12} & r_{1Y} \\ r_{12} & 1-r_{12}^2 & r_{2Y}-r_{1Y}r_{12} \\ r_{1Y} & r_{2Y}-r_{1Y}r_{12} & 1-r_{1Y}^2 \end{bmatrix}$$

$$= \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{12} & g_{22} & g_{23} \\ g_{13} & g_{23} & g_{33} \end{bmatrix} = \mathrm{G}.$$

The elements g_{12} (r_{12}) and g_{13} (r_{1Y}) in the matrix **G** are the multivariate standardized regression coefficients for regressing X_2 and Y each on X_1; $\begin{bmatrix} g_{22} & g_{23} \\ g_{23} & g_{33} \end{bmatrix}$ is the residual covariance matrix of X_2 and Y.

If we go further and sweep matrix **G** on row and column 2, and obtain a new matrix **F**,

$$\mathrm{SWP}[2]\mathbf{G} = \mathrm{SWP}[2]\mathrm{SWP}[1]\mathbf{Q} = \mathrm{SWP}[1,2]\mathbf{Q} = \mathrm{SWP}[2]\begin{bmatrix} -1 & r_{12} & r_{1Y} \\ r_{12} & 1-r_{12}^2 & r_{2Y}-r_{1Y}r_{12} \\ r_{1Y} & r_{2Y}-r_{1Y}r_{12} & 1-r_{1Y}^2 \end{bmatrix}$$

$$= \begin{bmatrix} -1-r_{12}^2/(1-r_{12}^2) & r_{12}/(1-r_{12}^2) & r_{1Y}-r_{12}(r_{2Y}-r_{1Y}r_{12})/(1-r_{12}^2) \\ r_{12}/(1-r_{12}^2) & -1/(1-r_{12}^2) & (r_{2Y}-r_{1Y}r_{12})/(1-r_{12}^2) \\ r_{1Y}-r_{12}(r_{2Y}-r_{1Y}r_{12})/(1-r_{12}^2) & (r_{2Y}-r_{1Y}r_{12})/(1-r_{12}^2) & (1-r_{1Y}^2)-(r_{2Y}-r_{1Y}r_{12})^2/(1-r_{12}^2) \end{bmatrix}$$

$$= \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{12} & f_{22} & f_{23} \\ f_{13} & f_{23} & f_{33} \end{bmatrix} = \mathbf{F},$$

the elements $f_{13} = (r_{1Y} - r_{12}(r_{2Y} - r_{1Y}r_{12})/(1-r_{12}^2))$ and $f_{23} = ((r_{2Y} - r_{1Y}r_{12})/(1-r_{12}^2))$ in matrix **F** above are the standardized regression coefficients for Y regressed on X_1 and X_2; $f_{33} = ((1-r_{1Y}^2) - (r_{2Y} - r_{1Y}r_{12})^2/(1-r_{12}^2))$ is the residual.

Below are the explicit steps showing how the sweep operators and reverse sweep operator can be applied to find the synthesized correlation matrix of the variables included in the four studies, and to create the final model as a summary, using the auxiliary example created in the previous section.

1. Find the maximum likelihood estimates of the correlations between the variables that are used in all studies included in the synthesis. In the example described earlier, Y and X_1 are present in all four studies. The correlation between the two variables can be estimated by calculating the weighted mean correlation

   $$\bar{r}_{\cdot(Y1)} = (n_1 \times r_{1(Y1)} + n_2 \times r_{2(Y1)} + n_3 \times r_{3(Y1)} + n_4 \times r_{4(Y1)})/(n_1 + n_2 + n_3 + n_4),$$

   and the mean correlation is stored in matrix O as $\begin{bmatrix} 1 & \bar{r}_{\cdot(Y1)} \\ \bar{r}_{\cdot(Y1)} & 1 \end{bmatrix}$.

2. Find the maximum likelihood estimates of the standardized slopes ($\hat{B}_{2Y.Y1}$ and $\hat{B}_{21.Y1}$) and error variance ($\hat{\sigma}_{2.Y1}^2$) for regressing X_2, which is the second-most-used variable, on Y and X_1 based on the samples containing all those variables. To use the sweep operator to obtain these estimates, we first create a correlation matrix, S_{234}, with weighted mean correlations among variables Y, X_1, and X_2, based on studies 2, 3, and 4. That is,

   $$\bar{r}_{\cdot(Y1)} = (n_2 \times r_{2(Y1)} + n_3 \times r_{3(Y1)} + n_4 \times r_{4(Y1)})/(n_2 + n_3 + n_4),$$

   $$\bar{r}_{\cdot Y2} = (n_2 \times r_{2Y2} + n_3 \times r_{3Y2} + n_4 \times r_{4Y2})/(n_2 + n_3 + n_4),$$

$\bar{r}_{\bullet(12)} = (n_2 \times r_{2(12)} + n_3 \times r_{3(12)} + n_4 \times r_{4(12)})/(n_2 + n_3 + n_4)$, and

$$\mathbf{S}_{234} = \begin{bmatrix} 1 & \bar{r}_{\bullet(Y1)} & \bar{r}_{\bullet(Y2)} \\ \bar{r}_{\bullet(Y1)} & 1 & \bar{r}_{\bullet(12)} \\ \bar{r}_{\bullet(Y2)} & \bar{r}_{\bullet(12)} & 1 \end{bmatrix}.$$

To obtain the slopes, we sweep Y and X_1 out of $\mathbf{S}_{234}$ as defined in (15).

$$\mathrm{SWP}[\mathrm{Y},1]\begin{bmatrix} 1 & \bar{r}_{\bullet(Y1)} & \bar{r}_{\bullet(Y2)} \\ \bar{r}_{\bullet(Y1)} & 1 & \bar{r}_{\bullet(12)} \\ \bar{r}_{\bullet(Y2)} & \bar{r}_{\bullet(12)} & 1 \end{bmatrix} = \begin{bmatrix} sweeped & sweeped & \hat{B}_{2Y.1} \\ sweeped & sweeped & \hat{B}_{21.Y} \\ \hat{B}_{2Y.1} & \hat{B}_{21.Y} & \hat{\sigma}^2_{2.Y1} \end{bmatrix}.$$

The "sweeped" in the matrix above indicated the elements, which are were of interest, were "swept out" the matrix using (15). The last column/row in the matrix show the estimates of interest, $\hat{B}_{2Y.1}$, $\hat{B}_{21.Y}$, and $\hat{\sigma}^2_{2.Y1}$.

3. Sweep Y and X_1 out of matrix $\mathbf{O}$ using (15) to obtain a new matrix $\mathbf{A}$

$$\mathbf{A} = \mathrm{SWP}[\mathrm{Y},1]\mathbf{O} = \begin{bmatrix} -1-(\bar{r}_{\bullet(Y1)}{}^2/1-\bar{r}_{\bullet(Y1)}{}^2) & \bar{r}_{\bullet(Y1)}/1-\bar{r}_{\bullet(Y1)}{}^2 \\ \bar{r}_{\bullet(Y1)}/1-\bar{r}_{\bullet(Y1)}{}^2 & -1/(1-\bar{r}_{\bullet(Y1)}{}^2) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} = \mathbf{A},$$

and augment matrix $\mathbf{A}$ to form a new matrix $\mathbf{P}$, with the estimated standardized slopes ($\hat{B}_{2Y.1}$ and $\hat{B}_{21.Y}$) and error variance ($\hat{\sigma}^2_{2.Y1}$) from the previous step.

$$\mathbf{P} = \begin{bmatrix} a_{11} & a_{12} & \hat{B}_{2Y.1} \\ a_{12} & a_{22} & \hat{B}_{21.Y} \\ \hat{B}_{2Y.1} & \hat{B}_{21.Y} & \hat{\sigma}_{22.Y1} \end{bmatrix}.$$

The matrix above looks like the matrix we would obtain when sweeping out the rows and columns 1 and 2 when we have a 3*3 matrix—even though the estimates in matrix **A** and the estimates of slopes and the variance were not based on exactly the same samples. Establishing desired statistics based on different samples allows us to "borrow" the information from other studies when we reverse the operator at the end of all calculations.

4. Find the maximum likelihood estimates of the standardized slopes ($\hat{B}_{3Y.12}$, $\hat{B}_{31.Y2}$, and $\hat{B}_{32.Y1}$) and error variance ($\hat{\sigma}^2_{3.Y12}$) for regressing X_3, which is the third-most-used variable, on Y, X_1, and X_2. Again, to use the sweep operator to obtain these estimates, we create a correlation matrix, $\mathbf{S}_{34}$, with weighted mean correlations among variables Y, X_1, X_2, and X_3, based on study 3 and study 4. That is,

$\bar{r}_{\bullet(Y1)} = (n_3 \times r_{3(Y1)} + n_4 \times r_{4(Y1)})/(n_3 + n_4)$,

$\bar{r}_{\bullet(Y2)} = (n_3 \times r_{3(Y2)} + n_4 \times r_{4(Y2)})/(n_3 + n_4)$,

$\bar{r}_{\bullet(Y3)} = (n_3 \times r_{3(Y3)} + n_4 \times r_{4(Y3)})/(n_3 + n_4)$,

$\bar{r}_{\bullet(12)} = (n_3 \times r_{3(12)} + n_4 \times r_{4(12)})/(n_3 + n_4)$,

$\bar{r}_{\bullet(13)} = (n_3 \times r_{3(13)} + n_4 \times r_{4(13)})/(n_3 + n_4)$,

$\bar{r}_{\bullet(23)} = (n_3 \times r_{3(23)} + n_4 \times r_{4(23)})/(n_3 + n_4)$, and

$$\mathbf{S}_{34} = \begin{bmatrix} 1 & \bar{r}_{\bullet(Y1)} & \bar{r}_{\bullet(Y2)} & \bar{r}_{\bullet(Y3)} \\ \bar{r}_{\bullet(Y1)} & 1 & \bar{r}_{\bullet(12)} & \bar{r}_{\bullet(13)} \\ \bar{r}_{\bullet(Y2)} & \bar{r}_{\bullet(12)} & 1 & \bar{r}_{\bullet(23)} \\ \bar{r}_{\bullet(Y3)} & \bar{r}_{\bullet(13)} & \bar{r}_{\bullet(23)} & 1 \end{bmatrix}.$$

To obtain the slopes, we sweep Y, X_1, and X_2 out of $\mathbf{S}_{34}$.

$$\text{SWP[Y,1,2]}\ \mathbf{S}_{34} = \begin{bmatrix} sweeped & sweeped & sweeped & \hat{B}_{3Y.12} \\ sweeped & sweeped & sweeped & \hat{B}_{31.Y2} \\ sweeped & sweeped & sweeped & \hat{B}_{32.Y1} \\ \hat{B}_{3Y.12} & \hat{B}_{31.Y2} & \hat{B}_{32.Y1} & \hat{\sigma}^2_{3.Y12} \end{bmatrix}.$$

The last column/row in the matrix show the estimates of interest, which are $\hat{B}_{3Y.12}, \hat{B}_{31.Y2}, \hat{B}_{32.Y1}$, and $\hat{\sigma}^2_{3.Y12}$.

5. Sweep X_2 out of matrix **P** to obtain a new matrix **B**,

$$\text{SWP[2]}\mathbf{P} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{bmatrix} = \mathbf{B},$$

and augment matrix **B** with the estimated standardized slopes ($\hat{B}_{3Y.12}, \hat{B}_{31.Y2}$ and $\hat{B}_{32.Y1}$) and error variance ($\hat{\sigma}^2_{3.Y12}$) obtained from previous step to for a new matrix **T**,

$$\mathbf{T} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & \hat{B}_{3Y.12} \\ b_{12} & b_{22} & b_{23} & \hat{B}_{31.Y2} \\ b_{13} & b_{23} & b_{33} & \hat{B}_{32.Y1} \\ \hat{B}_{3Y.12} & \hat{B}_{31.Y2} & \hat{B}_{32.Y1} & \hat{\sigma}_{33.Y12} \end{bmatrix}.$$

6. Find the maximum likelihood estimates of the standardized slopes ($\hat{B}_{4Y.123}$, $\hat{B}_{41.Y23}$, $\hat{B}_{42.Y13}$, and $\hat{B}_{43.Y12}$) and error variance ($\hat{\sigma}^2_{4.Y123}$) for regressing X_4 on Y, X_1, X_2, and X_3 based on the studies with all those variables in the model. Since only the last study uses all five variables, the sweep operator will be applied to the correlation matrix based on study 4 only. We sweep Y, X_1, X_2, and X_3 out of the correlation matrix of study 4.

$$
\text{SWP[Y,1,2,3]}\begin{bmatrix} 1 & r_{4(Y1)} & r_{4(Y2)} & r_{4(Y3)} & r_{4(Y4)} \\ r_{4(Y1)} & 1 & r_{4(12)} & r_{4(13)} & r_{4(14)} \\ r_{4(Y2)} & r_{4(12)} & 1 & r_{4(23)} & r_{4(24)} \\ r_{4(Y3)} & r_{4(13)} & r_{4(23)} & 1 & r_{4(34)} \\ r_{4(Y4)} & r_{4(14)} & r_{4(24)} & r_{4(34)} & 1 \end{bmatrix}.
$$

$$
= \begin{bmatrix} Sweeped & Sweeped & Sweeped & Sweeped & \hat{B}_{4Y.123} \\ Sweeped & Sweeped & Sweeped & Sweeped & \hat{B}_{41.Y23} \\ \hat{B}\,Sweeped & Sweeped & Sweeped & Sweeped & \hat{B}_{42.Y13} \\ Sweeped & Sweeped & Sweeped & Sweeped & \hat{B}_{43.Y12} \\ \hat{B}_{4Y.123} & \hat{B}_{41.Y23} & \hat{B}_{42.Y13} & \hat{B}_{43.Y12} & \hat{\sigma}^2_{4.Y123} \end{bmatrix}.
$$

The last column/row in the matrix show the estimates $\hat{B}_{4Y.123}$, $\hat{B}_{41.Y23}$, $\hat{B}_{42.Y13}$, $\hat{B}_{43.Y12}$, and $\hat{\sigma}^2_{4.Y123}$.

7. Sweep X_3 out of matrix **T** to obtain a new matrix **C**,

$$\mathrm{SWP}[3]\mathbf{T} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{12} & c_{22} & c_{23} & c_{24} \\ c_{13} & c_{23} & c_{33} & c_{34} \\ c_{14} & c_{24} & c_{34} & c_{44} \end{bmatrix} = \mathbf{C},$$

and store the new matrix with the results of step 6 as follows and denote it as **U**.

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & \hat{B}_{4Y.123} \\ c_{12} & c_{22} & c_{23} & c_{24} & \hat{B}_{41.Y23} \\ c_{13} & c_{23} & c_{33} & c_{34} & \hat{B}_{42.Y13} \\ c_{14} & c_{24} & c_{34} & c_{44} & \hat{B}_{43.Y12} \\ \hat{B}_{4Y.123} & \hat{B}_{41.Y23} & \hat{B}_{42.Y13} & \hat{B}_{43.Y12} & \hat{\sigma}^2_{4.Y123} \end{bmatrix} = \mathbf{U}.$$

8. To obtain the maximum likelihood estimates of the correlation matrix of Y, X_1, X_2, X_3, and X_4, we conduct the reverse sweep operation on the matrix **U** as defined in (16). The reversed matrix is the summarized matrix with the off diagonal elements equal to the combined correlations.

The procedure described above can be represented concisely by the expression

$$\mathrm{RSW}[3,2,1,Y]\left\{\mathrm{SWP}[4]\left\{\mathrm{SWP}[3]\left\{\mathrm{SWP}[Y,1]\begin{array}{ccccc} Y & X_1 & X_2 & X_3 & X_4 \\ 1 & \bar{r}_{\cdot(Y1)} & \hat{B}_{2Y.1} & \hat{B}_{3Y.12} & \hat{B}_{4Y.123} \\ \bar{r}_{\cdot(Y1)} & 1 & \hat{B}_{21.Y} & \hat{B}_{31.Y2} & \hat{B}_{41.Y23} \\ \hat{B}_{2Y.1} & \hat{B}_{21.Y} & \hat{\sigma}^2_{2.Y1} & \hat{B}_{32.Y1} & \hat{B}_{42.Y13} \\ \hat{B}_{3Y.12} & \hat{B}_{31.Y2} & \hat{B}_{32.Y1} & \hat{\sigma}^2_{3.Y12} & \hat{B}_{43.Y12} \\ \hat{B}_{4Y.123} & \hat{B}_{41.Y23} & \hat{B}_{42.Y13} & \hat{B}_{43.Y12} & \hat{\sigma}^2_{4.Y123} \end{array}\right\}\right\}\right\}$$

Before the resulting matrix can then be used to calculate the standardized coefficients of the final model with Y as the outcome and X_1, X_2, X_3, X_4 as the predictors, an adjustment is needed to calculate the coefficients using the summarized matrix. That is, the diagonal elements in the summarized matrix need to be adjusted to 1 if they were not one after reversing the matrix using the reverse sweep operation. In this example, Y and X_1 are the two variables used in all four samples and in the correlations with themselves should exactly equal to 1. Once the values on the diagonal in the synthesized correlation matrix have been adjusted, the expression (8) can again be used to obtain the standardized regression slopes.

3.4 Data Generation

SAS/IML (SAS Institute, 2002) version 9.1 was used to generate the desired data to test and compare the results from GLS and SWP. The precision of the results from those two methods could be impacted by the features of the models included in the meta-analysis, such as the number of predictors, intercorrelations among the predictors and the outcome, and the sample size in each model. SAS/IML was programmed according to the designed parameters, as described below, to generate subject-level data based on the assumption of normality within each study included in the synthesis. The Cholesky decomposition was used to obtain data with the desired relationships defined in the intercorrelation matrix assigned to each study in the synthesis. Once the data for four studies with the desired sample sizes were obtained, the two methods were used to calculate the summarized correlation matrices. The standardized slopes for the four predictors and their standard errors were then computed based on the summarized correlation matrices to compare the precision and stability of the two methods. The example SAS codes programmed for GLS and SWP can be found in Appendix A and Appendix B.

3.4.1 Choice of Parameters

The parameters that were varied were the number of predictors in each model (p), the intercorrelations among the predictors and the outcome (ρs), and the sample sizes of the studies included in a synthesis (Ns). The parameters that did not change were the number of the predictors in the final model (four), and the number of studies included in the synthesis (four).

Number of predictors. The numbers of predictors in the models in this simulation ranged from one (p=1 is a simple regression or Pearson's correlation) to four (p=4 is multiple regression with four predictors). More than four predictors are often used in many regression studies. For the purpose of the current research, four predictors are sufficient to capture the different patterns of missing predictors from the final model. As shown in Figure 3.2, four sets of regression models (Patterns I, II, III, and IV), each with different missing predictor patterns for the four studies included in one meta-analysis, were synthesized using the proposed methods. The studies in each of the four sets of regression models were arranged in certain patterns, from the study containing fewer predictor(s) to the study containing more predictors, to show different numbers of predictors missing in each of the four included studies. The shaded blocks for each of the four studies in each pattern indicate the predictors that were included in that study. For example, in Pattern I, the first study used only predictor X_1 to predict the outcome Y, while the fourth study used predictors X_1, X_2, X_3, and X_4 to predict the outcome Y.

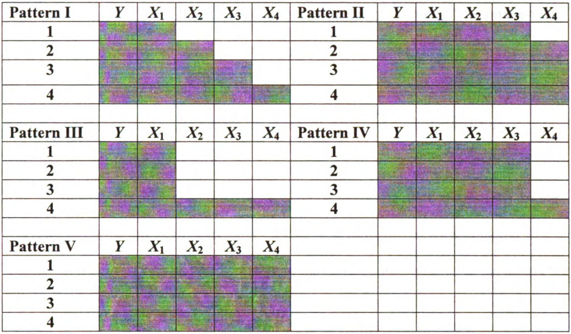


Figure 3.2. Five Sets of Regression Models with Different Numbers of Predictors Missing from Studies.

Intercorrelation matrix. The intercorrelation matrix, containing the correlations among the outcome and predictor(s) in each study, was set to be the same (fixed effects) or varied (random effects) across the studies in a synthesis. In each matrix, the correlations were designed based on one principal: If there was more than one predictor in the model, the correlation between any pair of predictors was designed to be equal to or smaller than any correlation between any predictor and the outcome. This is consistent with the idea that multicollinearity was not a problem in the original studies.

According to Cohen (1988), correlations of .1, .3, and .5 are small, medium, and large, respectively. In social science, the results from the correlation studies for testing validity of a measure can easily go beyond 0.5. Therefore, in the first matrix, the largest correlation between a predictor and the outcome in the current simulation research was designed to be .6 to represent a large predictive effect from a variable. The smallest correlation between a predictor and the outcome was designed to be .25 to capture a trivial relation but worth keeping in the regression model. The correlation between any pair of predictors was designed to be .25 or less to represent small but existing collinearity among the predictors.

Following the rules described above, the first intercorrelation matrix ($\mathbf{R}_1$) for the outcome Y and the four predictors X_1, X_2, X_3, X_4, was formed. The corresponding standardized slopes (β_1s) and the R^2 based on the correlations specified in the matrix are

$$\begin{array}{c} \\ \mathbf{R}_1 = \end{array} \begin{array}{c} \begin{array}{ccccc} Y & X_1 & X_2 & X_3 & X_4 \end{array} \\ \begin{bmatrix} 1 & .6 & .4 & .3 & .25 \\ .6 & 1 & .25 & .1 & .05 \\ .4 & .25 & 1 & .15 & .1 \\ .3 & .1 & .15 & 1 & .15 \\ .25 & .05 & .1 & .15 & 1 \end{bmatrix} \end{array}$$

β_{11}=0.5161; β_{12}=0.2253; β_{13}=0.1886; β_{14}=0.1734.
R^2=.500.

To test the condition where there was no multicollinearity present, the second intercorrelation matrix ($\mathbf{R}_2$) was designed with correlations identical to those in the first matrix between the outcome and the predictors, but removing all the correlations among predictors. The intercorrelation matrix and the corresponding standardized slopes (β_2s) as well as the R^2 are

$$\mathbf{R}_2 = \begin{bmatrix} 1 & .6 & .4 & .3 & .25 \\ .6 & 1 & 0 & 0 & 0 \\ .4 & 0 & 1 & 0 & 0 \\ .3 & 0 & 0 & 1 & 0 \\ .25 & 0 & 0 & 0 & 1 \end{bmatrix}$$

β_{21}=0.6; β_{22}= 0.4; β_{23}=0.3; β_{24}= 0.25.
R^2=.673.

To test the scenario that important predictors were being left out in some studies, the third matrix was designed by reversing the order of correlations between the outcome and the predictors in $\mathbf{R}_1$. The intercorrelations of the *X*s were rearranged accordingly. The intercorrelation matrix and the corresponding standardized slopes (β_3s) as well as the R^2 are

$$\mathbf{R}_3 = \begin{bmatrix} 1 & .25 & .3 & .4 & .6 \\ .25 & 1 & .15 & .1 & .05 \\ .3 & .15 & 1 & .15 & .1 \\ .4 & .1 & .15 & 1 & .25 \\ .6 & .05 & .1 & .25 & 1 \end{bmatrix}$$

β_{31}=0.1734; β_{32}=0.1886; β_{33}=0.2253; β_{34}=0.5161.
R^2=.500.

Then the correlations among predictors in $\mathbf{R}_3$ were removed to form $\mathbf{R}_4$ to test the condition where no multicollinearity was present when the predictor with a stronger relationship with the outcome tended to be left out. The intercorrelation matrix and the corresponding standardized slopes (β_4s) as well as the R^2 are

$$\mathbf{R}_4 = \begin{bmatrix} 1 & .25 & .3 & .4 & .6 \\ .25 & 1 & 0 & 0 & 0 \\ .3 & 0 & 1 & 0 & 0 \\ .4 & 0 & 0 & 1 & 0 \\ .6 & 0 & 0 & 0 & 1 \end{bmatrix}$$

β_{41}=0.25; β_{42}=0.3; β_{43}=0.4; β_{44}=0.6.
R^2=.673

Combinations of matrices conditions. The same correlation matrix will be applied to all four studies in a meta-analysis (under fixed effects), in Condition 1 to Condition 4 (in Figure 3.3). The two methods were also examined when the matrices were not all the same across studies. A mixed-effects model for the synthesis that contains the studies based on the matrices with and without correlations among predictors was investigated in Condition 5 to Condition 8.

Sample size sets. According to Cohen and Cohen (1975), at least 124 participants are needed to maintain 80% power with a single predictor that in the population correlates with the dependent variable at .30. Therefore, in the current research the minimal sample size for a study was chosen as 150. Since many studies adopt regression techniques to analyze data from big datasets, the maximum sample size is designed to be 2000 in this study. Four sets of sample sizes for four studies were investigated:

$N1 = \{150, 150, 150, 150\}$,

$N2 = \{2000, 2000, 2000, 2000\}$,

$N3 = \{150, 500, 1000, 2000\}$, and

$N4 = \{2000, 1000, 500, 150\}$.

The first two sample size sets represented equally small ($N1$) and large ($N2$) samples in a synthesis. The other two sets represented unequal sample sizes across studies. With different patterns of the sizes of studies in the synthesis, the impact of different missing rates for variables was examined.

	Fixed-effects				Mixed-effects			
Study	1	2	3	4	5	6	7	8
1	$\mathbf{R}_1$	$\mathbf{R}_2$	$\mathbf{R}_3$	$\mathbf{R}_4$	$\mathbf{R}_1$	$\mathbf{R}_2$	$\mathbf{R}_3$	$\mathbf{R}_4$
2	$\mathbf{R}_1$	$\mathbf{R}_2$	$\mathbf{R}_3$	$\mathbf{R}_4$	$\mathbf{R}_1$	$\mathbf{R}_2$	$\mathbf{R}_3$	$\mathbf{R}_4$
3	$\mathbf{R}_1$	$\mathbf{R}_2$	$\mathbf{R}_3$	$\mathbf{R}_4$	$\mathbf{R}_2$	$\mathbf{R}_1$	$\mathbf{R}_4$	$\mathbf{R}_3$
4	$\mathbf{R}_1$	$\mathbf{R}_2$	$\mathbf{R}_3$	$\mathbf{R}_4$	$\mathbf{R}_2$	$\mathbf{R}_1$	$\mathbf{R}_4$	$\mathbf{R}_3$

Figure 3.3. Eight Combinations of the Four Intercorrelation Matrices for Four Studies.

3.4.2 Missing Rate

With the patterns and sample sizes designed above, the percentage of missingness for each variable in each scenario was calculated and presented in Table 3.1. The missing rates for *N*1 and *N*2 are always the same within a pattern because those two sample size sets assumed equal sample sizes across all the four studies included in a synthesis.

3.4.3 Replications in the Simulation

Each of the two methods (2) were tested for five patterns of regression model (5) with eight different intercorrelation matrices (8) based on four sets of sample sizes (4). This yielded 320 (2*5*8*4) scenarios for the simulation in this research. In each scenario, the synthesized correlation matrix for the summarized model was calculated, and it was used to compute the standardized slopes for X_1, X_2, X_3 and X_4. The procedure was replicated 1000 times and produced 1000 syntheses for each of the 320 conditions. The means of the summarized slopes, and their standard errors in each condition from the 1000 replications, were used to evaluate the two methods.

Table 3.1

Percentages of Missingness for Each Pattern with Each Sample Size

Pattern	Sample Size	Missingness X_1	X_2	X_3	X_4
Pattern I	*N*1	0%	25%	50%	75%
	*N*2	0%	25%	50%	75%
	*N*3	0%	4%	18%	45%
	*N*4	0%	55%	82%	96%
Pattern II	*N*1	0%	0%	0%	25%
	*N*2	0%	0%	0%	25%
	*N*3	0%	0%	0%	4%
	*N*4	0%	0%	0%	55%
Pattern III	*N*1	0%	75%	75%	75%
	*N*2	0%	75%	75%	75%
	*N*3	0%	45%	45%	45%
	*N*4	0%	96%	96%	96%
Pattern IV	*N*1	0%	0%	0%	75%
	*N*2	0%	0%	0%	75%
	*N*3	0%	0%	0%	45%
	*N*4	0%	0%	0%	96%
Pattern V	*N*1	0%	0%	0%	0%
	*N*2	0%	0%	0%	0%
	*N*3	0%	0%	0%	0%
	*N*4	0%	0%	0%	0%

3.5 Data Analysis

The estimated mean slopes and their standard errors from 1,000 replications were calculated for each predictor under different patterns of missingness for each sample size set using GLS and SWP procedures. The mean slopes calculated using the two methods were compared to the population values that generated the data. Under the fixed-effects model (Condition 1 through Condition 4), the population slopes were calculated based on the correlation matrices $\mathbf{R}_1$ though $\mathbf{R}_4$, and they are presented in the note in the end of each result table. When studies in a synthesis were based on different matrices (Condition 5 through Condition 8), the weighted mean correlation matrix was first calculated by weighting the elements that were not missing in the matrix by their sample sizes. Then the population slopes were calculated based on this weighted mean correlation matrix.

The standard errors from the two methods can be compared to each other to see which method produced more stable estimates. This was a reasonable comparison under each specific scenario because the data generated for the two methods were controlled to be identical using "seed" in SAS in order for the data to be comparable.

The relative percentage bias of each slope was also computed in each scenario to quantify the difference between the calculated value and the population value. It is defined as

$$B(\hat{\theta}) = \frac{\bar{\hat{\theta}} - \theta}{\theta} \times 100\%$$

where θ is the population value of the parameter and $\bar{\hat{\theta}}$ is the mean of the estimates of the parameters across the replications. In this research, the θ is the population slope and the $\bar{\hat{\theta}}$ is the mean slope obtained from averaging the sample slopes from 1000 replications. Good estimation methods should have relative bias less than 5% (Hoogland & Boomsma, 1998).

To investigate the impacts of patterns, sample sizes, and the correlations among variables on the estimates of each of the predictors based the two methods, a factorial Analysis of Variance (ANOVA) was conducted on results from the fixed-effects model (Condition 1 though Condition 4) for which the two methods were invented. The outcome was the difference between the slopes from GLS and SWP (GLS slope minus SWP slope). The difference was calculated for each of the 1000 generated data under each scenario. The factors included in the ANOVA were the five missing-data patterns, four correlations, and four sample size sets. These factors resulted in 80 exclusive scenarios. The η^2 statistic was computed for each factor and their interactions as the effect size for representing the proportion of variance explained in the slope differences from two methods.

CHAPTER 4

EMPIRICAL EXAMINATION

Before conducting the simulation study, the two methods were examined empirically. The detailed steps for synthesizing results from regression studies, as described in Chapter 3, were demonstrated in this chapter using four pseudo studies created from a large dataset. The studies created for the pseudo meta-analysis were designed to follow Pattern I, as discussed in the simulation plan, which reflects the situation that is most likely to happen when synthesizing regression studies, in which different numbers of variables were used to predict the outcome across studies.

4.1 Sample Creation

The two methods were demonstrated by focusing on the primary regression studies investigating factors that impact *student achievement* using one of the major nationwide datasets, the National Education Longitudinal Study:1988 (NELS:88. Ingels, Scott, Taylor, Owings & Quinn, 1998). Nationwide datasets usually contain more information and larger samples, which attract researchers who plan to use multiple regression. According to Wu and Becker (2004), 103 different predictors were used in eleven studies based on NELS:88 data to model *student achievement* that also included a variety of teacher qualifications as predictors. Those studies were published before 2002 and used at least one indicator of teacher qualification as a predictor in the models.

In the current investigation, four studies based on real data were created for synthesis using a subset of the NELS:88 data. The sub dataset contains 2508 students in

10^{th} grade in 1990, and has complete data on first follow-up standardized math scores (*F1math*), base year standardized math scores (*BYmath*), social economic status (*SES*), whether the student's teacher has a bachelor degree in math or not (*BSdegree*), and 10^{th} grade drop out rate (*Drop*) of the school the student attends. *F1math* is one of the outcomes that has been studied the most in previous studies using NELS:88 data. Therefore, it is adopted as the outcome for the regressions in the current study. The other four variables are adopted as the predictors to create the regression models. These predictors represent the effects of characteristics of the student (*BYmath*), and their family (*SES*), teacher (*BSdegree*), and school (*Drop*). These represent four dimensions that researchers have studied extensively in models of student achievement in previous regression studies using the NELS:88 dataset (Wu & Becker, 2004).

Four samples were randomly drawn from the subset of the large dataset to create four pseudo regression studies. According to Green (1991), the suggested sample size (n) used to create a regression model, with .8 power, should be $n = 50 + 8*p$, where p is the number of predictors in the model. The first study contains *BYmath* as the only predictor; a second study contains both *BYmath* and *SES* as predictors; the third study contains *BYmath*, *BSdegree* and *Drop* as predictors; the fourth study uses all the predictors to explain the variation in the outcomes. Therefore, the sample sizes for four studies in the current example are 58 (50+8*1 for study 1), 66 (50+8*2 for study 2), 74 (50+8*3 for study 3), and 82 (50+8*4 for study 4) respectively.

The estimated standardized slopes, with *F1math* as the outcome, for the four randomly selected samples are shown in Table 4.1. The correlations among the variables for the total sample and for each of the sub samples are shown in Table 4.2.

Table 4.1

Sample Sizes and Standardized Regression Coefficients for Four Studies

	n	*BYmath*	*SES*	*BSdegree*	*Drop*
Study 1	58	0.861			
Study 2	66	0.868	0.012		
Study 3	74	0.857	0.025	0.055	
Study 4	82	0.785	0.187	0.111	0.027

Table 4.2

Correlations among Five Variables

<table>
<tr><th></th><th>F1math</th><th>BYmath</th><th>SES</th><th>BSdegree</th><th>Drop</th></tr>
<tr><td rowspan="4">F1math</td><td rowspan="4">1</td><td>.861 (n_1=58)</td><td></td><td></td><td></td></tr>
<tr><td>.874 (n_2=66)</td><td>.456</td><td></td><td></td></tr>
<tr><td>.884 (n_3=74)</td><td>.407</td><td>.323</td><td></td></tr>
<tr><td>.856 (n_4=82)</td><td>.466</td><td>.135</td><td>-.074</td></tr>
<tr><td rowspan="4">BYmath</td><td rowspan="4">.867
(N=2508)</td><td rowspan="4">1</td><td></td><td></td><td></td></tr>
<tr><td>.511</td><td></td><td></td></tr>
<tr><td>.433</td><td>.308</td><td></td></tr>
<tr><td>.370</td><td>.046</td><td>-.094</td></tr>
<tr><td rowspan="4">SES</td><td rowspan="4">.441</td><td rowspan="4">.418</td><td rowspan="4">1</td><td></td><td></td></tr>
<tr><td></td><td></td></tr>
<tr><td>.196</td><td></td></tr>
<tr><td>-.064</td><td>-.118</td></tr>
<tr><td rowspan="4">BSdegree</td><td rowspan="4">.180</td><td rowspan="4">-.136</td><td rowspan="4">.077</td><td rowspan="4">1</td><td></td></tr>
<tr><td></td></tr>
<tr><td></td></tr>
<tr><td>-.042</td></tr>
<tr><td>Drop</td><td>-.082</td><td>.133</td><td>-.176</td><td>.018</td><td>1</td></tr>
</table>

Note. Elements in the upper triangle are correlations based on each of the four random selected samples; elements in the lower triangle are correlations based on the total sample of 2508 students.

4.2 Application of Multivariate Generalized Least Squares

As mentioned earlier, synthesizing the zero-order correlations between variables (predictors and outcomes) from regression models is like synthesizing multivariate data points from studies. Each study may contain some similar predictors and some different ones, which makes the correlations produced in each study a subset of the correlations from the final model, that is determined by the meta-analyst based on the studies included in the synthesis. In this examination, the final model used to summary the four regression studies was the model containing all four predictors. That is, the final estimated model for person l is:

$$\hat{Y}_{F1math\_l} = \hat{B}_1 X_{BYmath\_l} + \hat{B}_2 X_{SES\_l} + \hat{B}_3 X_{BSdegree\_l} + \hat{B}_4 X_{Drop\_l},$$

where $\hat{\beta}$ s are the estimated standardized slopes for the predictors.

The vectors of zero-order correlations for each of the four studies ($\mathbf{r}_k$, l=1, 2, 3, or 4) with elements $r_{k\,(i,j)}$, where i = *F1math (Y)*, *BYmath* (*1*), *SES* (*2*), *BSdegree* (*3*), and *Drop* (*4*); j = *F1math (Y)*, *BYmath* (*1*), *SES* (*2*), *BSdegree* (*3*), and *Drop* (*4*); $i \neq j$, are:

$$\mathbf{r}_1 = \begin{bmatrix} r_{1(Y1)} \end{bmatrix} = \begin{bmatrix} .861 \end{bmatrix}, \mathbf{r}_2 = \begin{bmatrix} r_{2(Y1)} \\ r_{2(Y2)} \\ r_{2(12)} \end{bmatrix} = \begin{bmatrix} .874 \\ .456 \\ .511 \end{bmatrix}, \mathbf{r}_3 = \begin{bmatrix} r_{3(Y1)} \\ r_{3(Y2)} \\ r_{3(Y3)} \\ r_{3(12)} \\ r_{3(13)} \\ r_{3(23)} \end{bmatrix} = \begin{bmatrix} .884 \\ .407 \\ .323 \\ .433 \\ .308 \\ .196 \end{bmatrix}, \text{ and}$$

$$\mathbf{r_4} = \begin{bmatrix} r_{4(Y1)} \\ r_{4(Y2)} \\ r_{4(Y3)} \\ r_{4(Y4)} \\ r_{4(12)} \\ r_{4(13)} \\ r_{4(14)} \\ r_{4(23)} \\ r_{4(24)} \\ r_{4(34)} \end{bmatrix} = \begin{bmatrix} .856 \\ .466 \\ .135 \\ -.074 \\ .370 \\ .046 \\ -.094 \\ -.064 \\ -.118 \\ -.042 \end{bmatrix}.$$

To use the GLS method to summarize multivariate outcomes, the identity matrix **W** for this example is

$$
\mathbf{W} = \begin{array}{c}
\begin{array}{cccccccccc} r_{Y1} & r_{Y2} & r_{Y3} & r_{Y4} & r_{12} & r_{13} & r_{14} & r_{23} & r_{24} & r_{34} \end{array} \\
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\end{array}
\text{ for }
\begin{bmatrix}
r_{1(Y1)} \\ r_{2(Y1)} \\ r_{2(Y2)} \\ r_{2(12)} \\ r_{3(Y1)} \\ r_{3(Y2)} \\ r_{3(Y3)} \\ r_{3(12)} \\ r_{3(13)} \\ r_{3(23)} \\ r_{4(Y1)} \\ r_{4(Y2)} \\ r_{4(Y3)} \\ r_{4(Y4)} \\ r_{4(12)} \\ r_{4(13)} \\ r_{4(14)} \\ r_{4(23)} \\ r_{4(24)} \\ r_{4(34)}
\end{bmatrix}.
$$

Based on the correlations from the four studies reported in Table 4.2 (the elements above the diagonal), the variance-covariance matrix estimated in each study ($\hat{\boldsymbol{\Sigma}}_k$) was calculated using the formulas (4), (5), and (6). Then the full variance-covariance matrix $\hat{\boldsymbol{\Sigma}}$ for all the studies in the meta-analysis was constructed as follows.

$$\hat{\boldsymbol{\Sigma}} = \begin{bmatrix} \hat{\boldsymbol{\Sigma}}_1 & 0 & 0 & 0 \\ 0 & \hat{\boldsymbol{\Sigma}}_2 & 0 & 0 \\ 0 & 0 & \hat{\boldsymbol{\Sigma}}_3 & 0 \\ 0 & 0 & 0 & \hat{\boldsymbol{\Sigma}}_4 \end{bmatrix} =$$

$$\left(\begin{array}{cccccccccccccccccccc}
0.001 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.001 & 0.001 & 0.001 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.001 & 0.010 & 0.007 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.001 & 0.007 & 0.008 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.001 & 0.001 & 0.000 & 0.001 & 0.001 & 0.000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.001 & 0.009 & 0.001 & 0.008 & 0.001 & 0.003 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.000 & 0.001 & 0.011 & 0.001 & 0.010 & 0.004 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.001 & 0.008 & 0.001 & 0.009 & 0.001 & 0.003 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.001 & 0.001 & 0.010 & 0.001 & 0.011 & 0.005 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.000 & 0.003 & 0.004 & 0.003 & 0.005 & 0.012 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.001 & 0.000 & 0.000 & 0.000 & 0.001 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.000 & 0.007 & -0.001 & -0.001 & 0.007 & -0.001 & -0.001 & 0.001 & 0.000 & 0.000 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.000 & -0.001 & 0.012 & 0.000 & -0.001 & 0.010 & 0.000 & 0.006 & 0.000 & -0.001 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.000 & -0.001 & 0.000 & 0.012 & -0.001 & 0.000 & 0.010 & 0.000 & 0.006 & 0.002 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.001 & 0.007 & -0.001 & -0.001 & 0.009 & -0.001 & -0.001 & 0.001 & -0.001 & 0.000 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.000 & -0.001 & 0.010 & 0.000 & -0.001 & 0.012 & 0.000 & 0.004 & 0.000 & -0.001 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.000 & -0.001 & 0.000 & 0.010 & -0.001 & 0.000 & 0.012 & 0.000 & 0.004 & 0.001 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.000 & 0.001 & 0.006 & 0.000 & 0.001 & 0.004 & 0.000 & 0.012 & -0.001 & -0.001 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.000 & 0.000 & 0.000 & 0.006 & -0.001 & 0.000 & 0.004 & -0.001 & 0.012 & -0.001
\end{array}\right)$$

The estimated population correlations, which were the synthesized correlation based on the four regression studies, were computed using $\hat{\boldsymbol{\rho}} = (\mathbf{W}'\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{W})^{-1}\mathbf{W}'\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{r}$. The calculated correlations were shown in the matrix form

$$\begin{array}{cccccc} & BYmath & SES & BSdegree & Drop & F1math \\ \hat{\boldsymbol{\rho}} = & \left[\begin{array}{ccccc} 1 & 0.432 & 0.195 & -0.109 & 0.869 \\ 0.432 & 1 & 0.061 & -0.137 & 0.443 \\ 0.195 & 0.061 & 1 & -0.068 & 0.225 \\ -0.109 & -0.137 & -0.068 & 1 & -0.078 \\ 0.869 & 0.443 & 0.225 & -0.078 & 1 \end{array}\right] & & & & \end{array}.$$

The standardized slopes were be calculated based on the correlation matrix above. The final estimated regression model by GLS method is

$$\hat{Y}_{F1math\_l} = 0.828X_{BYmath\_l} + 0.082X_{SES\_l} + 0.090X_{BS\deg ree\_l} + 0.031X_{Drop\_l}.$$

4.3 Application of Factored Likelihood Method through the Sweep Operators

The information needed for obtaining the synthesized correlations through the sweep operators were the sample size and correlations among the predictors and the outcome from each study created for this investigation. Those information were displayed in Table 4.2.

The first step to adopt the concept of factored likelihood was to find the maximum likelihood estimate of the correlations between the variables that are used in *all* four studies included in the synthesis. In this example, *F1math* (*Y*) and *BYmath* (1) are in all four studies. The weighted mean correlation is the estimated maximum likelihood of the correlation in this example, which was

$$\bar{r}_{.Y1} = .861*58+.874*66+.884*74+.856*82)/(58+66+74+82) = .869.$$

The estimated value was stored in the matrix form and denoted as **O**.

$$\mathbf{O} = \begin{array}{c} \begin{array}{cc} \scriptstyle F1math & \scriptstyle BYmath \end{array} \\ \begin{bmatrix} 1 & .869 \\ .869 & 1 \end{bmatrix} \end{array}.$$

The second step is to find the maximum likelihood estimates of the standardized slopes ($\hat{B}_{2Y.Y1}$ and $\hat{B}_{21.Y1}$) and error variance ($\hat{\sigma}^2_{2.Y1}$) for regressing *SES* (*2*), which is the second-most-used variable, on *F1math* (*Y*) and *B1math* (1) based on the studies containing all those variables. Before finding those estimates, a correlation matrix, $\mathbf{S}_{234,}$ was created to store the weighted mean correlations among variables *F1math* (*Y*), *BYmath* (1), and *SES* (2), based on studies 2, 3, and 4.

$$\bar{r}_{.Y1} = (.874*66+.884*74+.856*82)/(66+74+82)= .871$$

$$\bar{r}_{.Y2} = (.456*66+.407*74+.466*82)/(66+74+82)= .443$$

$$\bar{r}_{.12} = (.511*66+.433*74+.37*82)/(66+74+82)= .433$$

$$
\begin{array}{cc}
 & \begin{array}{ccc} \mathit{F1math} & \mathit{BYmath} & \mathit{SES} \end{array} \\
\mathbf{S}_{234} = & \begin{bmatrix} 1 & .871 & .443 \\ .871 & 1 & .433 \\ .443 & .433 & 1 \end{bmatrix}.
\end{array}
$$

To obtain the standardized slopes and the error variance, *F1math* (*Y*) and *BYmath* (1) were swept out of $\mathbf{S}_{234}$, as described in Chapter 3. This sweep-out process was recorded as follow

$$
\mathrm{SWP}[\mathrm{Y},1]\begin{bmatrix} 1 & .871 & .443 \\ .871 & 1 & .433 \\ .443 & .433 & 1 \end{bmatrix} = \begin{bmatrix} -4.134 & 3.599 & 0.275 \\ 3.599 & -4.134 & 0.194 \\ 0.275 & 0.194 & 0.794 \end{bmatrix}.
$$

The last column/row in the swept-out matrix showed the estimates of interest, which are

$$
\begin{aligned}
&\hat{B}_{2Y.1} = 0.275 \\
&\hat{B}_{21.Y} = 0.194 \\
&\hat{\sigma}^2_{2.Y1} = 0.794.
\end{aligned}
$$

Next, *F1math* (*Y*) and *BYmath* (1) were swept out of matrix **O** to obtain a new matrix, denoted as **A.**

$$
\mathbf{A} = \mathrm{SWP}[\mathrm{Y},1]\begin{bmatrix} 1 & .869 \\ .869 & 1 \end{bmatrix} = \begin{bmatrix} -4.075 & 3.540 \\ 3.540 & -4.075 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}.
$$

The matrix **A** then was augmented with the estimated standardized slopes ($\hat{B}_{2Y.1}$

and $\hat{B}_{21.Y}$) and error variance ($\hat{\sigma}^2_{2.Y1}$) obtained earlier to form a new matrix **P**:

$$\mathbf{P} = \begin{bmatrix} -4.075 & 3.540 & 0.275 \\ 3.540 & -4.075 & 0.194 \\ 0.275 & 0.194 & 0.794 \end{bmatrix}.$$

The matrix **P** is the matrix that could be obtained when sweeping the rows and columns 1 and 2 when we have a 3*3 matrix.

The next step was to find the maximum likelihood estimates of the standardized slopes ($\hat{B}_{3Y.12}$, $\hat{B}_{31.Y2}$, and $\hat{B}_{32.Y1}$) and error variance ($\hat{\sigma}^2_{3.Y12}$) for regression of *BSdegree* (3), which was the third-most-used variable, on *F1math* (*Y*), *B1math* (1), and *SES* (2). Again, to use the sweep operator to obtain these estimates, another correlation matrix, $\mathbf{S}_{34}$, was created with weighted mean correlations among variables *F1math* (*Y*), *BYmath* (1), SES (2), and *BSdegree* (4), based on studies 3 and 4. That is,

$\bar{r}_{\cdot Y1}$ = (.884*74+.856*82)/(74+82) =.869,

$\bar{r}_{\cdot Y2}$ = (.407*74+.466*82)/(74+82) =.438,

$\bar{r}_{\cdot Y3}$ = (.323*74+.135*82)/(74+82) =.224,

$\bar{r}_{\cdot 12}$ = (.433*74+.37*82)/(74+82) =.400,

$\bar{r}_{\cdot 13}$ = (.308*74+.046*82)/(74+82) =.170,

$\bar{r}_{\cdot 23}$ =(.196*74+(-.064)*82)/(74+82) =.059, so

$$\mathbf{S}_{34} = \begin{array}{c} \begin{array}{cccc} \textit{F1math} & \textit{BYmath} & \textit{SES} & \textit{BSdegree} \end{array} \\ \begin{bmatrix} 1 & .869 & .438 & .224 \\ .869 & 1 & .400 & .170 \\ .438 & .400 & 1 & .059 \\ .224 & .170 & .059 & 1 \end{bmatrix} \end{array}.$$

To obtain the slopes of interest in this step, *F1math* (*Y*), *BYmath* (*1*), and *SES* (*2*) were swept out of $\mathbf{S}_{34}$.

$$\mathrm{SWP}[Y,1,2] \begin{array}{c} \begin{array}{cccc} \text{F1math} & \text{BYmath} & \text{SES} & \text{BSdegree} \end{array} \\ \begin{bmatrix} 1 & .869 & .438 & .224 \\ .869 & 1 & .400 & .170 \\ .438 & .400 & 1 & .059 \\ .224 & .170 & .059 & 1 \end{bmatrix} \end{array} = \begin{bmatrix} -4.262 & 3.522 & 0.459 & 0.329 \\ 3.522 & -4.100 & 0.097 & -0.097 \\ 0.459 & 0.097 & -1.240 & -0.046 \\ 0.329 & -0.097 & -0.046 & 0.946 \end{bmatrix}.$$

The last column in the matrix shows the estimates, which are

$$\begin{aligned} \hat{B}_{3Y.12} &= 0.329 \\ \hat{B}_{31.Y2} &= \text{-}0.097 \\ \hat{B}_{32.Y1} &= \text{-}0.046 \\ \hat{\sigma}^2_{3.Y12} &= 0.946. \end{aligned}$$

Then *SES* (2) was swept out of matrix **P** to obtain a new matrix **B**,

$$\mathrm{SWP}[2] \begin{bmatrix} -4.075 & 3.540 & 0.275 \\ 3.540 & -4.075 & 0.194 \\ 0.275 & 0.194 & 0.794 \end{bmatrix} = \begin{bmatrix} -4.170 & 3.473 & 0.346 \\ 3.473 & -4.122 & 0.224 \\ 0.346 & 0.224 & -1.259 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{bmatrix} = \mathbf{B},$$

and matrix **B** was augmented with the estimated standardized slopes ($\hat{B}_{3Y.12}, \hat{B}_{31.Y2}$ and

$\hat{B}_{32.Y1}$) and error variance ($\hat{\sigma}^2_{3.Y12}$) and for a new matrix denoted as **T**.

$$\mathbf{T}=\begin{bmatrix} -4.170 & 3.473 & 0.346 & 0.329 \\ 3.473 & -4.122 & 0.244 & -0.097 \\ 0.346 & 0.244 & -1.259 & -0.046 \\ 0.329 & -0.097 & -0.046 & 0.946 \end{bmatrix}.$$

Next we find the maximum likelihood estimates of the standardized slopes ($\hat{B}_{4Y.123}$, $\hat{B}_{41.Y23}$, $\hat{B}_{42.Y13}$, and $\hat{B}_{43.Y12}$) and error variance ($\hat{\sigma}^2_{4.Y123}$) for regressing *Drop* (4) on *F1math* (*Y*), *B1math* (*1*), *SES* (*2*), and *BSdegree* (*3*) based on the studies with all those variables in the models. Since only the last sample uses all five variables, the sweep operation was applied to the correlation matrix based on study 4 only. The outcome *F1math* (*Y*), *BYmath* (1), *SES* (2), and *BSdegree* (3) were swept out of the correlation matrix of study 4.

$$\begin{array}{cc} & \begin{array}{ccccc} \mathit{F1math} & \mathit{BYmath} & \mathit{SES} & \mathit{BSdegree} & \mathit{Drop} \end{array} \\ \mathrm{SWP}[Y,1,2,3] & \begin{bmatrix} 1 & .856 & .466 & .135 & -.074 \\ .856 & 1 & .370 & .046 & -.094 \\ .466 & .370 & 1 & -.064 & -.118 \\ .135 & .046 & -.064 & 1 & -.042 \\ -.074 & -.094 & -.118 & -.042 & 1 \end{bmatrix} \end{array}$$

$$=\begin{bmatrix} 4.361 & -3.415 & -0.800 & -0.483 & 0.113 \\ -3.415 & 3.839 & 0.190 & 0.297 & -0.143 \\ -0.800 & 0.190 & 1.314 & 0.183 & -0.121 \\ -0.483 & 0.297 & 0.183 & 1.063 & -0.058 \\ 0.113 & -0.143 & -0.121 & -0.058 & 0.978 \end{bmatrix}.$$

The last column in the matrix above shows the estimates of interest in this step, which were

$$\begin{aligned}
\hat{B}_{4Y.123} &= 0.113 \\
\hat{B}_{41.Y23} &= -0.143 \\
\hat{B}_{42.Y13} &= -0.121 \\
\hat{B}_{43.Y12} &= -0.058 \\
\hat{\sigma}^2_{4.Y123} &= 0.978.
\end{aligned}$$

Then, *BSdegree* (3) was swept out of matrix **T** to obtain a new matrix **C**,

$$\text{SWP[3]}\begin{bmatrix} -4.170 & 3.473 & 0.346 & 0.329 \\ 3.473 & -4.122 & 0.244 & -0.097 \\ 0.346 & 0.244 & -1.259 & -0.046 \\ 0.329 & -0.097 & -0.046 & 0.946 \end{bmatrix} = \begin{bmatrix} -4.284 & 3.507 & 0.362 & 0.348 \\ 3.507 & -4.132 & 0.239 & -0.103 \\ 0.362 & 0.239 & -1.261 & -0.048 \\ 0.348 & -0.103 & -0.048 & -1.058 \end{bmatrix}$$

$$= \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{12} & c_{22} & c_{23} & c_{24} \\ c_{13} & c_{23} & c_{33} & c_{34} \\ c_{14} & c_{24} & c_{34} & c_{44} \end{bmatrix} = \mathbf{C},$$

and the matrix **C** was augmented and the new matrix was denoted as **U**.

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & \hat{\beta}_{4Y.123} \\ c_{12} & c_{22} & c_{23} & c_{24} & \hat{\beta}_{41.Y23} \\ c_{13} & c_{23} & c_{33} & c_{34} & \hat{\beta}_{42.Y13} \\ c_{14} & c_{24} & c_{34} & c_{44} & \hat{\beta}_{43.Y12} \\ \hat{\beta}_{4Y.123} & \hat{\beta}_{41.Y23} & \hat{\beta}_{42.Y13} & \hat{\beta}_{43.Y12} & \hat{\sigma}^2_{4.Y123} \end{bmatrix}$$

$$= \begin{bmatrix} -4.284 & 3.507 & 0.362 & 0.348 & 0.113 \\ 3.507 & -4.132 & 0.239 & -0.103 & -0.143 \\ 0.362 & 0.239 & -1.261 & -0.048 & -0.121 \\ 0.348 & -0.103 & -0.048 & -1.058 & -0.058 \\ 0.113 & -0.143 & -0.121 & -0.058 & 0.978 \end{bmatrix} = \mathbf{U}.$$

In order to obtain the maximum likelihood estimate of the correlation matrix of *F1math* (*Y*), *BYmath* (1), *SES* (2), *BSdegree* (3), and *Drop* (4), the reverse sweep operation was applied to the matrix **U**.

$$\begin{array}{c} \\ \mathrm{RSW}[3,2,1,Y] = \end{array} \begin{array}{c} \begin{array}{ccccc} \textit{F1math} & \textit{BYmath} & \textit{SES} & \textit{BSdegree} & \textit{Drop} \end{array} \\ \begin{bmatrix} -4.284 & 3.507 & 0.362 & 0.348 & 0.113 \\ 3.507 & -4.132 & 0.239 & -0.103 & -0.143 \\ 0.362 & 0.239 & -1.261 & -0.048 & -0.121 \\ 0.348 & -0.103 & -0.048 & -1.058 & -0.058 \\ 0.113 & -0.143 & -0.121 & -0.058 & 0.978 \end{bmatrix} \end{array}$$

$$= \begin{bmatrix} 1 & 0.869 & 0.443 & 0.224 & -0.078 \\ 0.869 & 1 & 0.432 & 0.169 & -0.107 \\ 0.443 & 0.432 & 1 & 0.058 & -0.137 \\ 0.224 & 0.169 & 0.058 & 1 & -0.064 \\ -0.078 & -0.107 & -0.137 & -0.064 & 1 \end{bmatrix}.$$

The reversed matrix is the synthesized matrix based on the four studies with different numbers of predictors involved. The steps described above can be represented concisely by the expression

$$\mathrm{RSW}[3,2,1,Y]\left(\mathrm{SWP}[4]\left(\mathrm{SWP}[3]\left(\mathrm{SWP}[Y,1]\begin{array}{ccccc} Y & 1 & 2 & 3 & 4 \\ \hline 1.000 & 0.869 & 0.275 & 0.329 & 0.113 \\ 0.869 & 1.000 & 0.194 & -0.097 & -0.143 \\ 0.275 & 0.194 & 0.794 & -0.046 & -0.121 \\ 0.329 & -0.097 & -0.046 & 0.946 & -0.058 \\ 0.113 & 0.143 & -0.121 & -0.058 & 0.978 \end{array}\right)\right)\right).$$

As mentioned earlier, the diagonal elements in the summarized matrix need to be adjusted to 1, before the resulting matrix can then be used to calculate the standardized regression coefficients of the final model. In this investigation, *F1math* and *BYmath* were the two variables used in all four samples, and the correlations of each with itself were exactly equal to 1. The correlations for *SES* and *BSdegree* to themselves in the summarized matrix (0.9997 and 1.0002 respectively) were also 1 after rounding.

Using the synthesized correlation matrix, the final model with the standardized coefficients is

$$\hat{Y}_{F1math\_l} = 0.820X_{BYmath\_l} + 0.087X_{SES\_l} + 0.082X_{BSdegree\_l} + 0.027X_{Drop\_l}.$$

4.4 Results from the Empirical Examination

Table 4.3 shows the coefficients for the final synthesized models based on GLS and SWP methods, and from the regression model based on the total set of 2508 participants in the sub sample of NELS:88.

Table 4.3 Estimated Standardized Regression Coefficients from both Methods

	n	*BYmath*	*SES*	*BSdegree*	*Drop*
GLS	280	0.828	0.082	0.090	0.031
SWP	280	0.820	0.087	0.082	0.027
Complete	*2508*	*0.822*	*0.101*	*0.062*	*0.046*

Compared to the estimates from the GLS method, the estimated standardized slopes based on the SWP methods are closer to the estimates from the complete sample. The SWP method is especially accurate at estimating the slope for *BYmath*, which was the most observed (used frequently) predictor. The GLS method produced an overestimate of the slope on this fully observed predictor.

The estimated slope of *Drop* using SWP did not get the chance to be adjusted because there is only one regression model (sample 4) containing that predictor. That is, the synthesized slope of *Drop* ($\hat{B}_{Drop}$ =0.027) did not change from the estimated slope based on study 4 in Table 1. On the other hand, GLS adjusted the slope through the variance-covariance matrix during the calculation, and the estimate for that predictor is closer to the estimated slope based on the complete set of cases.

CHAPTER 5

SIMULATION RESULTS

The simulation results based on the fixed-effects model (Condition 1 through Condition 4) are presented separately in this chapter. A brief discussion is given of four other conditions (Condition 5 through Condition 8) that do not represent fixed-effect cases. More attention was paid to the first four conditions, for which GLS and SWP methods were originally invented. The mean slopes for each predictor and their standard errors from the two procedures based on the fixed-effects model were first compared for different patterns and different correlation matrices. The percentage relative bias values for slopes for each scenario assuming fixed effects can be found in Appendix C. Similar estimates were calculated based on the more complex mixed-effects models and a brief review of those results is then presented. The bulk of these results as well as the percentage relative bias for slopes for each scenario can be found in Appendix D.

5.1 Fixed-effects Model (Condition 1 through Condition 4)

The mean estimated slopes ($\bar{\hat{B}}$ s) and their standard errors (*SE*s) based on matrices $\mathbf{R}_1$ though $\mathbf{R}_4$ under a fixed-effects model for each study pattern are shown in Table 5.1.1 through Table 5.1.20. For each pattern based on each correlation matrix, the estimated mean slope for each predictor and its standard error based on GLS and SWP methods are listed for each sample size set (*N*1 through *N*4).

5.1.1 Correlation Matrix $\mathbf{R}_1$

Pattern I. When Pattern I is combined with correlation matrix $\mathbf{R}_1$, more data were missing as the relationships between variables and the outcome became weaker. As shown in Table 5.1, the mean slopes from SWP for X_1, which was fully observed, were consistently closer to the population slope (β_{11}=0.5161) under all different sample sizes. Most of the time the SWP underestimated the slope for X_1 while GLS tended to overestimate it. When the total sample size was large (3650 from four studies) and the maximum amount of data were missing for a predictor (96% for X_4 in $N4$), GLS produced a better estimate for that predictor ($\bar{\hat{B}}_4$ =0.1744, Bias $\bar{\hat{B}}_4$ =0.565%) than SWP ($\bar{\hat{B}}_4$ =0.1767, Bias $\bar{\hat{B}}_4$ =1.909%). When sample sizes were equal across all studies in a meta-analysis in this pattern, SWP seemed to perform better with small equal sample sizes ($N1$) than with large equal sample sizes (N2). The percentage relative biases were all less than 5% for slopes from both methods in all scenarios, but the magnitude of the bias was much larger for GLS slopes than for SWP slopes in most cases. Generally speaking, SWP produced more stable estimates (smaller *SE*s) than GLS in Pattern I. However, the *SE*s for $\bar{\hat{B}}_4$, was present in only one study and had the smallest correlations with the outcome in this pattern, showed slightly more stability when estimated via GLS.

Pattern II. The combination of Pattern II with $\mathbf{R}_1$ has missing data only on the last variable X_4, which had the weakest relation to the outcome, and was missing in only one study included in the meta-analysis. As shown in Table 5.2, SWP still estimated the slope of X_1 better than GLS as in Pattern I. In contrast to the results in Pattern I, when the sample sizes were small and equal across studies in the synthesis ($N1$), GLS produced slightly better estimates for X_3 ($\bar{\hat{B}}_3$ =0.1894, Bias $\bar{\hat{B}}_3$ =0.445%) and X_4 ($\bar{\hat{B}}_4$ =0.1737, Bias $\bar{\hat{B}}_4$ =0.173%) in this pattern. When 55% value were missing on X_4 with other predictors fully observed, SWP produced a mean slope on X_4 (0.1735) that was very close to the population value (0.1734). While the percentage relative bias was larger than 1% for $\bar{\hat{B}}_1$ in *N*1 using GLS, all other biases were less than 1%. The differences in *SE*s between the two methods were very minor for most predictors. The estimates from SWP were consistently more stable than GLS when sample sizes varied.

Pattern III. For the combination of Pattern III with $\mathbf{R}_1$, the predictors that were weakly related to the outcome (X_2, X_3, and X_4) were present in only the last study in the synthesis. As shown in Table 5.3, the mean slope for X_1, which was fully observed in all studies, was consistently better estimated through SWP. Both GLS and SWP tended to result in overestimation of the X_1 slope when sample sizes varied. SWP also performed better for estimating the slope of X_2, which was the variable that had the second strongest relationship with the outcome. SWP tended to perform better overall when sample sizes were equal and small across studies (*N*1); with larger equal samples (*N*2), GLS performed slightly better at estimating the slope of X_3 ($\bar{\hat{B}}_3$ =0.1891, Bias $\bar{\hat{B}}_3$ =0.249%) and X_4 ($\bar{\hat{B}}_4$ =0.1742, Bias $\bar{\hat{B}}_4$ =0.496%), which have weaker relationships with the outcome. SWP resulted in more precisely estimated slopes when much missingness occurred (*N*4) in this pattern. However, in an absolute sense, the differences in the *SE*s produced by the two methods were trivial. Most of the time, GLS produced slightly more stable estimates in this pattern.

Pattern IV. The combination of Pattern IV with correlation matrix $\mathbf{R}_1$ had predictors X_1 to X_3 presented in all four studies. Predictor X_4, which had the weakest relation to the outcome, was present only in the last study included in the synthesis. As shown in Table 5.4, SWP still performed better at estimating the slope of X_1 with the percentage relative bias below 0.2% across all sample sizes, and GLS tended to consistently overestimate it with the percentage relative bias larger than 1% in this pattern. With small and equal sample sizes (*N*1), SWP produced better estimates of the slopes of all four variables; with large and equal sample sizes (*N*2), GLS performed better in estimating the slopes for X_2, X_3, and X_4. When 96% of values were missing on X_4 in *N*4, GLS estimated its slope more accurately; when X_4 was missing less (45%), SWP performed better. SWP tended to result in more stable estimates on the fully observed variables X_1, X_2, and X_3; GLS resulted in more stable estimates of the slope of X_4.

Pattern V. In this pattern, all the studies in the synthesis included all four predictors and there is no missing data for any predictor. Under this scenario, as shown in Table 5.5, both SWP and GLS consistently overestimated the slope of X_1. SWP produced mean slopes of X_1 that were closer to the population value when sample sizes varied. With large and equal sample sizes (*N*2), GLS produced mean slopes for X_2, X_3, and X_4 that were close or identical to the population values. Similar to the results in Pattern II, the percentage relative bias was larger than 1% for X_1 in *N*1 using GLS, and all other biases were less than 1%. Generally speaking, SWP produced more stable results for all predictors under different sample sizes.

Table 5.1

Missing Percentage, Estimated Mean Slopes and Standard Errors for Each Predictor for Pattern I and Correlation Matrix $\mathbf{R}_1$

Method	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
	0.5161	**0.2253**	**0.1886**	**0.1734**				
*N*1	0%	25%	50%	75%				
GLS	0.5218	0.2271	0.1902	0.1688	0.001061	0.001278	0.001367	0.001800
SWP	0.5161[a]	0.2248	0.1891	0.1731	0.001048	0.001224	0.001357	0.001848
*N*2	0%	25%	50%	75%				
GLS	0.5169	0.2250	0.1884	0.1731	0.000275	0.000324	0.000361	0.000490
SWP	0.5164	0.2248	0.1884	0.1734[a]	0.000274	0.000323	0.000362	0.000493
*N*3	0%	4%	18%	45%				
GLS	0.5170	0.2255	0.1888	0.1728	0.000364	0.000425	0.000431	0.000485
SWP	0.5160	0.2252	0.1888	0.1734[a]	0.000362	0.000423	0.000430	0.000487
*N*4	0%	55%	82%	96%				
GLS	0.5167	0.2260	0.1899	0.1744	0.000649	0.000841	0.001043	0.001883
SWP	0.5159	0.2249	0.1881	0.1767	0.000653	0.000843	0.001038	0.001928

Note. The bolded values are the population slopes for predictors.

a. Mean estimated slope is equal to the population value.

Table 5.2

Missing Percentage, Estimated Mean Slopes and Standard Errors for Each Predictor for Pattern II and Correlation Matrix $\mathbf{R}_1$

Method	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
	0.5161	**0.2253**	**0.1886**	**0.1734**				
*N*1	0%	0%	0%	25%				
GLS	0.5230	0.2269	0.1894	0.1737	0.000945	0.001050	0.001008	0.001109
SWP	0.5156	0.2247	0.1876	0.1729	0.000887	0.000998	0.000950	0.001064
*N*2	0%	0%	0%	25%				
GLS	0.5171	0.2250	0.1887	0.1733	0.000240	0.000267	0.000266	0.000282
SWP	0.5165	0.2248	0.1885	0.1733	0.000239	0.000266	0.000265	0.000281
*N*3	0%	0%	0%	4%				
GLS	0.5180	0.2252	0.1889	0.1732	0.000359	0.000401	0.000386	0.000369
SWP	0.5167	0.2249	0.1885	0.1733	0.000358	0.000396	0.000382	0.000366
*N*4	0%	0%	0%	55%				
GLS	0.5171	0.2250	0.1889	0.1743	0.000373	0.000427	0.000394	0.000550
SWP	0.5158	0.2246	0.1889	0.1735	0.000369	0.000424	0.000389	0.000542

Note. The bolded values are the population slopes for predictors.

Table 5.3

Missing Percentage, Estimated Mean Slopes and Standard Errors for Each Predictor for Pattern III and Correlation Matrix $\mathbf{R}_1$

Method	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
	0.5161	**0.2253**	**0.1886**	**0.1734**				
*N*1	0%	75%	75%	75%				
GLS	0.5243	0.2237	0.1875	0.1704	0.001248	0.001925	0.001810	0.001795
SWP	0.5170	0.2263	0.1896	0.1723	0.001242	0.001947	0.001826	0.001818
*N*2	0%	75%	75%	75%				
GLS	0.5171	0.2249	0.1891	0.1742	0.000340	0.000519	0.000511	0.000491
SWP	0.5165	0.2251	0.1892	0.1744	0.000340	0.000520	0.000513	0.000492
*N*3	0%	45%	45%	45%				
GLS	0.5169	0.2251	0.1890	0.1729	0.000398	0.000521	0.000506	0.000476
SWP	0.5158	0.2254	0.1893	0.1732	0.000398	0.000523	0.000508	0.000477
*N*4	0%	96%	96%	96%				
GLS	0.5191	0.2243	0.1860	0.1724	0.001099	0.001930	0.001902	0.001786
SWP	0.5173	0.2250	0.1864	0.1728	0.001104	0.001940	0.001907	0.001797

Note. The bolded values are the population slopes for predictors.

Table 5.4

Missing Percentage, Estimated Mean Slopes and Standard Errors for Each Predictor for Pattern IV and Correlation Matrix $\mathbf{R}_1$

Method	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
	0.5161	**0.2253**	**0.1886**	**0.1734**				
*N*1	0%	0%	0%	75%				
GLS	0.5218	0.2283	0.1915	0.1688	0.001009	0.001096	0.001098	0.001799
SWP	0.5151	0.2254	0.1882	0.1742	0.000986	0.001067	0.001063	0.001862
*N*2	0%	0%	0%	75%				
GLS	0.5169	0.2247	0.1886[a]	0.1737	0.000251	0.000277	0.000280	0.000478
SWP	0.5164	0.2245	0.1884	0.1741	0.000251	0.000275	0.000278	0.000479
*N*3	0%	0%	0%	45%				
GLS	0.5173	0.2255	0.1895	0.1724	0.000356	0.000413	0.000401	0.000478
SWP	0.5161[a]	0.2251	0.1890	0.1731	0.000356	0.000411	0.000399	0.000480
*N*4	0%	0%	0%	96%				
GLS	0.5171	0.2257	0.1895	0.1742	0.000584	0.000622	0.000633	0.001833
SWP	0.5158	0.2251	0.1886[a]	0.1763	0.000593	0.000627	0.000638	0.001855

Note. The bolded values are the population slopes for predictors.

a. Mean estimated slope is equal to the population value.

Table 5.5

Missing Percentage, Estimated Mean Slopes and Standard Errors for Each Predictor for Pattern V and Correlation matrix $\mathbf{R}_1$

Method	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
	0.5161	**0.2253**	**0.1886**	**0.1734**				
*N*1	0%	0%	0%	0%				
GLS	0.5240	0.2272	0.1892	0.1737	0.000927	0.001030	0.000988	0.000982
SWP	0.5166	0.2247	0.1878	0.1719	0.000879	0.000979	0.000935	0.000931
*N*2	0%	0%	0%	0%				
GLS	0.5170	0.2253[a]	0.1887	0.1734[a]	0.000249	0.000269	0.000260	0.000248
SWP	0.5165	0.2251	0.1885	0.1733	0.000248	0.000267	0.000259	0.000248
*N*3	0%	0%	0%	0%				
GLS	0.5179	0.2252	0.1889	0.1736	0.000367	0.000404	0.000374	0.000367
SWP	0.5165	0.2248	0.1885	0.1734[a]	0.000362	0.000397	0.000369	0.000362
*N*4	0%	0%	0%	0%				
GLS	0.5177	0.2252	0.1890	0.1736	0.000366	0.000401	0.000372	0.000366
SWP	0.5166	0.2248	0.1886[a]	0.1734[a]	0.000361	0.000397	0.000369	0.000362

Note. The bolded values are the population slopes for predictors.

a. Mean estimated slope is equal to the population value.

5.1.2 Correlation Matrix $\mathbf{R}_2$

Pattern I. The combination of Pattern I with $\mathbf{R}_2$ showed more missing data as the relationship between the predictor and the outcome became weaker, and there was no correlation among predictors with $\mathbf{R}_2$. As shown in Table 5.6, SWP estimated the slope for X_1 better than GLS under different sample sizes. GLS always overestimated the slope for X_1. Different from the results for this pattern with $\mathbf{R}_1$, GLS estimated the slope of X_3 precisely when the sample sizes were small and equal across studies (*N*1) in the synthesis. As was true for correlation matrix $\mathbf{R}_1$, GLS was superior when much data was missing (96% in *N*4 on X_4). Compared to the scenario where the correlation was $\mathbf{R}_1$, the results from GLS were more stable with smaller *SE*s than those from SWP, yet the differences in *SE*s between the two methods were small.

Pattern II. The combination of Pattern II with $\mathbf{R}_2$ had missingness only on the last variable X_4, the weakest predictor, in one study included in the meta-analysis, and there was no correlation among the predictors. As shown in Table 5.7, SWP gave better estimates of the slope for X_1 and GLS still overestimated the slope for this variable. GLS usually did better in estimating slopes for X_2 and X_3, while SWP did well in estimating the slopes for X_4. SWP produced more stable estimates for the slopes for all variables, except for X_4 with sample size defined by *N*2, where GLS produced a slightly smaller *SE* than SWP.

Pattern III. The combination of Pattern III with $\mathbf{R}_2$ had the predictors that were weakly related to the outcome (X_2, X_3, and X_4) were present in only the last study in the synthesis and there was no correlation among predictors. As shown in Table 5.8, SWP still worked better than GLS in estimating the slope for fully observed variable X_1, and

GLS still overestimated the slope for this variable. SWP also performed better in estimating the slopes for the four variables when the sample sizes were small and equal across the four studies ($N1$) in the synthesis. The variables that were less strong related with the outcome (X_3 and X_4) were more often missing. GLS started to show better estimates. Similar to the condition when the correlation was $\mathbf{R}_1$, GLS produced slightly more stable estimates than SWP.

Pattern IV. The combination of Pattern IV with correlation matrix $\mathbf{R}_2$ had predictors X_1 to X_3 present in all four studies. Predictor X_4, which had the weakest relation to the outcome, was present only in the last study included in the synthesis. As shown in Table 5.9, SWP performed better in estimating the slope for X_1, while GLS kept overestimating the slope for this variable. SWP still worked better than GLS with small equal sample sizes ($N1$) in this pattern. When sample sizes were large with much missing data (e.g., X_4 with 75% missing in $N2$ and 96% in $N4$), GLS tended to work better than SWP. GLS also produced more stable estimates for $\bar{\hat{B}}_4$ when sample sizes varied, as well as for all four variables in $N4$.

Pattern V. In Pattern V with correlation matrix $\mathbf{R}_2$, all the studies in the synthesis included all four predictors. There was no missing data for any of the predictors and there was no correlation among those predictors. In this scenario, as shown in Table 5.10, SWP produced estimates closer to the population values on X_1, while GLS continued overestimating the slope for this variable. However, comparing to the results from the correlation matrix $\mathbf{R}_1$, the mean slopes from GLS were closer to the population values for most of the predictors than those from GLS, when sample sizes varied. As in the conditions where the correlation matrix was $\mathbf{R}_1$, SWP produced more stable estimates than GLS.

Table 5.6

Missing Percentage, Estimated Mean Slopes and Standard Errors for Each Predictor for Pattern I and Correlation Matrix $\mathbf{R}_2$

Method	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
	0.6	**0.4**	**0.3**	**0.25**				
*N*1	0%	25%	50%	75%				
GLS	0.6041	0.4013	0.3000[a]	0.2442	0.001040	0.001082	0.001123	0.001419
SWP	0.5998	0.3992	0.3003	0.2514	0.001041	0.001079	0.001123	0.001467
*N*2	0%	25%	50%	75%				
GLS	0.6005	0.3997	0.2999	0.2497	0.000277	0.000288	0.000299	0.000378
SWP	0.6002	0.3995	0.2999	0.2502	0.000279	0.000288	0.000300	0.000384
*N*3	0%	4%	18%	45%				
GLS	0.6006	0.4000[a]	0.2999	0.2492	0.000333	0.000359	0.000354	0.000393
SWP	0.5999	0.3998	0.3001	0.2501	0.000331	0.000358	0.000354	0.000397
*N*4	0%	55%	82%	96%				
GLS	0.6006	0.4010	0.3016	0.2513	0.000753	0.000842	0.000902	0.001411
SWP	0.5996	0.3997	0.3001	0.2546	0.000775	0.000850	0.000915	0.001487

Note. The bolded values are the population slopes for predictors.

a. Mean estimated slope is equal to the population value.

Table 5.7

Missing Percentage, Estimated Mean Slopes and Standard Errors for Each Predictor for Pattern II and Correlation Matrix $\mathbf{R_2}$

Method	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
	0.6	**0.4**	**0.3**	**0.25**				
*N*1	0%	0%	0%	25%				
GLS	0.6043	0.4015	0.3005	0.2497	0.000823	0.000863	0.000847	0.000890
SWP	0.5991	0.3990	0.2987	0.2499	0.000772	0.000824	0.000799	0.000854
*N*2	0%	0%	0%	25%				
GLS	0.6006	0.3997	0.3000[a]	0.2500[a]	0.000211	0.000223	0.000221	0.000230
SWP	0.6002	0.3995	0.2999	0.2500[a]	0.000209	0.000221	0.000220	0.000233
*N*3	0%	0%	0%	4%				
GLS	0.6012	0.3998	0.3001	0.2496	0.000312	0.000329	0.000306	0.000306
SWP	0.6003	0.3994	0.2998	0.2499	0.000310	0.000326	0.000305	0.000305
*N*4	0%	0%	0%	55%				
GLS	0.6005	0.3999	0.3005	0.2514	0.000335	0.000370	0.000341	0.000436
SWP	0.5996	0.3994	0.3003	0.2504	0.000332	0.000366	0.000337	0.000435

Note. The bolded values are the population slopes for predictors.

a. Mean estimated slope is equal to the population value.

Table 5.8

Missing Percentage, Estimated Mean Slopes and Standard Errors for Each Predictor for Pattern III and Correlation Matrix $\mathbf{R}_2$

Method	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
	0.6	**0.4**	**0.3**	**0.25**				
*N*1	0%	75%	75%	75%				
GLS	0.6067	0.3977	0.2981	0.2472	0.001321	0.001464	0.001446	0.001450
SWP	0.6021	0.4014	0.3005	0.2491	0.001328	0.001499	0.001452	0.001464
*N*2	0%	75%	75%	75%				
GLS	0.6006	0.3996	0.3005	0.2506	0.000351	0.000397	0.000401	0.000398
SWP	0.6002	0.3999	0.3006	0.2508	0.000351	0.000402	0.000404	0.000399
*N*3	0%	45%	45%	45%				
GLS	0.6007	0.3997	0.3002	0.2497	0.000375	0.000408	0.000406	0.000394
SWP	0.6000 [a]	0.4003	0.3006	0.2500 [a]	0.000375	0.000412	0.000408	0.000394
*N*4	0%	96%	96%	96%				
GLS	0.6023	0.4006	0.2990	0.2507	0.001283	0.001434	0.001498	0.001452
SWP	0.6007	0.4008	0.2992	0.2508	0.001281	0.001464	0.001502	0.001459

Note. The bolded values are the population slopes for predictors.

a. Mean estimated slope is equal to the population value.

Table 5.9

Missing Percentage, Estimated Mean Slopes and Standard Errors for Each Predictor for Pattern IV and Correlation Matrix $\mathbf{R}_2$

Method	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
	0.6	**0.4**	**0.3**	**0.25**				
*N*1	0%	0%	0%	75%				
GLS	0.6034	0.4020	0.3014	0.2440	0.000968	0.000968	0.000977	0.001367
SWP	0.5987	0.3994	0.2993	0.2532	0.000967	0.000965	0.000954	0.001435
*N*2	0%	0%	0%	75%				
GLS	0.6006	0.3995	0.3002	0.2501	0.000244	0.000253	0.000253	0.000371
SWP	0.6002	0.3993	0.3000[a]	0.2508	0.000246	0.000245	0.000253	0.000376
*N*3	0%	0%	0%	45%				
GLS	0.6008	0.4003	0.3006	0.2488	0.000318	0.000352	0.000337	0.000389
SWP	0.6000[a]	0.3999	0.3003	0.2501	0.000320	0.000352	0.000335	0.000393
*N*4	0%	0%	0%	96%				
GLS	0.6013	0.4010	0.3008	0.2526	0.000680	0.000698	0.000669	0.001365
SWP	0.6001	0.4003	0.3002	0.2550	0.000705	0.000708	0.000675	0.001407

Note. The bolded values are the population slopes for predictors.

a. Mean estimated slope is equal to the population value.

Table 5.10

Missing Percentage, Estimated Mean Slopes and Standard Errors for Each Predictor for Pattern V and Correlation Matrix $\mathbf{R}_2$

Method	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
	0.6	**0.4**	**0.3**	**0.25**				
*N*1	0%	0%	0%	0%				
GLS	0.6053	0.4017	0.3002	0.2500[a]	0.000798	0.000825	0.000803	0.000806
SWP	0.6000[a]	0.3989	0.2987	0.2487	0.000758	0.000790	0.000760	0.000771
*N*2	0%	0%	0%	0%				
GLS	0.6006	0.3999	0.3001	0.2500[a]	0.000210	0.000221	0.000210	0.000207
SWP	0.6002	0.3997	0.2999	0.2499	0.000209	0.000220	0.000209	0.000206
*N*3	0%	0%	0%	0%				
GLS	0.6011	0.3998	0.3002	0.2502	0.000313	0.000331	0.000302	0.000304
SWP	0.6002	0.3994	0.2998	0.2499	0.000309	0.000326	0.000297	0.000300
*N*4	0%	0%	0%	0%				
GLS	0.6010	0.3999	0.3003	0.2501	0.000311	0.000330	0.000299	0.000303
SWP	0.6002	0.3994	0.2999	0.2499	0.000308	0.000327	0.000298	0.000300

Note. The bolded values are the population slopes for predictors.

a. Mean estimated slope is equal to the population value.

5.1.3 Correlation Matrix $\mathbf{R}_3$

Pattern I. The combination of Pattern I with $\mathbf{R}_3$ had more missing data as the relationships between predictors and the outcome became stronger. As shown in Table 5.11, GLS outperformed SWP in estimating the slope of X_1 in $N4$ where a large portion of data were missing in X_2, X_3, and X_4. Consistent with previous results, GLS tended to result in slight overestimation of the slope for X_1 when sample sizes varied in this pattern, and so did SWP. When the sample sizes were small and equal across studies in the synthesis ($N1$), SWP performed better. When the sample sizes were large and equal ($N2$), GLS tended to do better. When a large portion of data were missing on $X4$ (e.g., in $N4$), which had a high correlation with the outcome, GLS generally performed better. SWP tended to be more stable when the sample size was small and equal across studies ($N1$) and when missingness occurred less ($N3$); GLS seemed to be more stable when sample size was large ($N2$) or when more data were missing ($N4$).

Pattern II. The combination of Pattern II with correlation matrix $\mathbf{R}_3$ had missing data only on the last variable X_4, which had the strongest relation to the outcome, in only one study included in the meta-analysis. As shown in Table 5.12, SWP produced mean slopes for X_1 that were closer to the population value (β_{31}=0.1734) than the GLS means, except in $N4$ where there were more missing values on the last predictor. That was the same finding as in Pattern I with correlation matrix $\mathbf{R}_3$. For all the different sample sizes, SWP produced better slopes for X_4 than did GLS. When the overall sample size was large and data were more complete (e.g. $N2$ and $N3$), SWP precisely reproduced the population value for the slope for X_4. Also, SWP resulted in more stable estimates.

Pattern III. In Pattern III with the correlation matrix $\mathbf{R}_3$, the predictors that were more strongly related to the outcome (X_2, X_3, and X_4) were present in only the last study in the synthesis. As shown in Table 5.13, SWP tended to perform better than GLS in estimating the slope for X_1 when sample sizes varied, except in *N*3, where the sole information on X_2, X_3, and X_4 was based on a study with a large sample size. Generally speaking, GLS and SWP produced very similar mean slopes for several variables under different sample size sets (e.g., X_2 in *N*2, *N*3; X_4 in *N*2) in this pattern. GLS and SWP produced similar *SE*s. Yet GLS was slightly more stable than SWP in most of the conditions.

Pattern IV. The combination of Pattern IV with correlation matrix $\mathbf{R}_3$ had predictors X_1 to X_3 present in all four studies, and X_4, which had the strongest relation to the outcome, was present only in the last study included in the synthesis. As shown in Table 5.14, SWP consistently resulted in better estimates of the slope of X_1 than GLS, which tended to result in overestimation of the slope of X_1 as well as slopes of other variables. SWP also performed better than GLS in *N*1, *N*3, and *N*4 at estimating the slopes of X_2 and X_3. For X_4, GLS tended to perform better than SWP. GLS consistently came up with more stable estimates for the slopes for X_4.

Pattern V. In Pattern V, all the studies in the synthesis included all four predictors and there was no missing data for any of the predictors. Under this scenario, as shown in Table 5.15, SWP produced better estimate of the X_1 slope most of the time. In contrast to previous findings in this pattern, GLS produced a mean slope for X_1 that was the same as the population value when the sample size was large and equal across studies (*N*2). When the sample size was small and equal across studies (*N*1), SWP tended to perform better than GLS at estimating the slopes of all variables. SWP also better estimated the slopes for X_3 and X_4, that were more strongly related to the outcome, in this pattern. Moreover, SWP produced more stable estimates than GLS for slopes of all variables when sample sizes varied.

Table 5.11

Missing Percentage, Estimated Mean Slopes and Standard Errors for Each Predictor for Pattern I and Correlation Matrix $\mathbf{R}_3$

Method	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
	0.1734	**0.1886**	**0.2253**	**0.5161**				
*N*1	0%	25%	50%	75%				
GLS	0.1758	0.1914	0.2285	0.5121	0.001387	0.001460	0.001599	0.001640
SWP	0.1736	0.1882	0.2240	0.5184	0.001386	0.001434	0.001584	0.001656
*N*2	0%	25%	50%	75%				
GLS	0.1737	0.1882	0.2253[a]	0.5161[a]	0.000383	0.000391	0.000425	0.000438
SWP	0.1736	0.1880	0.2250	0.5165	0.000383	0.000391	0.000426	0.000444
*N*3	0%	4%	18%	45%				
GLS	0.1736	0.1887	0.2258	0.5157	0.000438	0.000467	0.000472	0.000456
SWP	0.1732	0.1883	0.2254	0.5163	0.000437	0.000465	0.000470	0.000457
*N*4	0%	55%	82%	96%				
GLS	0.1735	0.1892	0.2277	0.5177	0.001146	0.001316	0.001389	0.001637
SWP	0.1731	0.1879	0.2242	0.5226	0.001160	0.001327	0.001396	0.001695

Note. The bolded values are the population slopes for predictors.

a. Mean estimated slope is equal to the population value.

Table 5.12

Missing Percentage, Estimated Mean Slopes and Standard Errors for Each Predictor for Pattern II and Correlation Matrix $\mathbf{R}_3$

Method	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
	0.1734	**0.1886**	**0.2253**	**0.5161**				
*N*1	0%	0%	0%	25%				
GLS	0.1750	0.1901	0.2262	0.5215	0.001052	0.001073	0.001089	0.001031
SWP	0.1729	0.1878	0.2238	0.5164	0.000998	0.001020	0.001024	0.000968
*N*2	0%	0%	0%	25%				
GLS	0.1738	0.1883	0.2254	0.5166	0.000276	0.000282	0.000276	0.000260
SWP	0.1737	0.1881	0.2252	0.5161[a]	0.000274	0.000280	0.000275	0.000260
*N*3	0%	0%	0%	4%				
GLS	0.1743	0.1885	0.2259	0.5166	0.000394	0.000398	0.000397	0.000348
SWP	0.1739	0.1881	0.2252	0.5161[a]	0.000393	0.000395	0.000394	0.000344
*N*4	0%	0%	0%	55%				
GLS	0.1733	0.0188	0.2254	0.5188	0.000455	0.000479	0.000472	0.000495
SWP	0.1730	0.1879	0.2256	0.5165	0.000448	0.000473	0.000467	0.000493

Note. The bolded values are the population slopes for predictors.

a. Mean estimated slope is equal to the population value.

Table 5.13

Missing Percentage, Estimated Mean Slopes and Standard Errors for Each Predictor for Pattern III and Correlation Matrix $\mathbf{R}_3$

Method	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
	0.1734	**0.1886**	**0.2253**	**0.5161**				
*N*1	0%	75%	75%	75%				
GLS	0.1789	0.1887	0.2269	0.5123	0.001531	0.001930	0.001866	0.001695
SWP	0.1757	0.1891	0.2272	0.5138	0.001521	0.001933	0.001870	0.001706
*N*2	0%	75%	75%	75%				
GLS	0.1741	0.1886[a]	0.2256	0.5170	0.000408	0.000533	0.000540	0.000457
SWP	0.1739	0.1886[a]	0.2257	0.5170	0.000408	0.000534	0.000540	0.000459
*N*3	0%	45%	45%	45%				
GLS	0.1735	0.1883	0.2259	0.5159	0.000440	0.000505	0.000521	0.000452
SWP	0.1731	0.1883	0.2260	0.5161[a]	0.000440	0.000505	0.000521	0.000451
*N*4	0%	96%	96%	96%				
GLS	0.1746	0.1893	0.2221	0.5170	0.001423	0.001923	0.001987	0.001705
SWP	0.1736	0.1894	0.2223	0.5176	0.001424	0.001925	0.001994	0.001728

Note. The bolded values are the population slopes for predictors.

a. Mean estimated slope is equal to the population value.

Table 5.14

Missing Percentage, Estimated Mean Slopes and Standard Errors for Each Predictor for Pattern IV and Correlation Matrix $\mathbf{R}_3$

Method	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
	0.1734	**0.1886**	**0.2253**	**0.5161**				
*N*1	0%	0%	0%	75%				
GLS	0.1747	0.1920	0.2311	0.5124	0.001381	0.001366	0.001433	0.001623
SWP	0.1723	0.1883	0.2236	0.5205	0.001372	0.001361	0.001423	0.001655
*N*2	0%	0%	0%	75%				
GLS	0.1740	0.1880	0.2257	0.5163	0.000353	0.000361	0.000374	0.000425
SWP	0.1738	0.1878	0.2252	0.5169	0.000345	0.000362	0.000375	0.000429
*N*3	0%	0%	0%	45%				
GLS	0.1738	0.1889	0.2267	0.5150	0.000430	0.000450	0.000448	0.000440
SWP	0.1735	0.1885	0.2257	0.5159	0.000430	0.000449	0.000446	0.000442
*N*4	0%	0%	0%	96%				
GLS	0.1741	0.1899	0.2268	0.5180	0.001110	0.001158	0.001202	0.001601
SWP	0.1733	0.1888	0.2240	0.5226	0.001127	0.001169	0.001215	0.001631

Note. The bolded values are the population slopes for predictors.

Table 5.15

Missing Percentage, Estimated Mean Slopes and Standard Errors for Each Predictor for Pattern V and Correlation Matrix $\mathbf{R}_3$

Method	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
	0.1734	**0.1886**	**0.2253**	**0.5161**				
*N*1	0%	0%	0%	0%				
GLS	0.1760	0.1902	0.2264	0.5219	0.000987	0.001013	0.001014	0.000936
SWP	0.1740	0.1879	0.2246	0.5147	0.000952	0.000961	0.000949	0.000887
*N*2	0%	0%	0%	0%				
GLS	0.1734[a]	0.1886[a]	0.2255	0.5166	0.000264	0.000262	0.000268	0.000231
SWP	0.1737	0.1885	0.2253[a]	0.5160	0.000263	0.000261	0.000268	0.000229
*N*3	0%	0%	0%	0%				
GLS	0.1740	0.1883	0.2258	0.5174	0.000401	0.000400	0.000388	0.000347
SWP	0.1737	0.1880	0.2253[a]	0.5162	0.000395	0.000393	0.000382	0.000340
*N*4	0%	0%	0%	0%				
GLS	0.1739	0.1884	0.2259	0.5174	0.000399	0.000398	0.000384	0.000343
SWP	0.1737	0.1880	0.2253[a]	0.5162	0.000394	0.000394	0.000382	0.000340

Note. The bolded values are the population slopes for predictors.

a. Mean estimated slope is equal to the population value.

5.1.4 Correlation Matrix $\mathbf{R}_4$

Pattern I. The combination of Pattern I with correlation matrix $\mathbf{R}_4$ led to more missing data occurred as the relationships between the predictor variables and the outcome became stronger. Also, there was no correlation among the predictors in $\mathbf{R}_4$. As shown in Table 5.16, both GLS and SWP performed well in estimating the slope of X_1. SWP estimated the slope of X_1 precisely when the sample sizes were equal across studies ($N1$ and $N2$). As was true for other patterns and correlations, GLS tended to overestimate the slope of X_1 all the time. SWP did not estimate the slope well when large amounts of data were missing on the variable that related strongly to the outcome (e.g., the slope for X_4 in $N4$). Similar to earlier findings, GLS always produced more stable estimates of the slope for X_4 and resulted in a smaller *SE*. The differences in *SE*s between the two methods were similar to those found in Pattern I with correlation matrix $\mathbf{R}_2$.

Pattern II. The combination of Pattern II with correlation matrix $\mathbf{R}_4$ had missing data only on the last variable X_4, which had the strongest relation to the outcome and appeared in only one study included in the meta-analysis. There was no correlation among predictors. As shown in Table 5.17, SWP produced better estimates of the slope for X_1 most of the time and GLS still tended to overestimate the slope of X_1. GLS performed better than SWP at estimating the slope of X_2. GLS also produced very precise estimates of the slope of X_4 when there was little missing data (4%) in $N3$. SWP produced more stable estimates than GLS most of the time, except the SE_4 values calculated via GLS in $N2$ and $N4$ were smaller than those produced by SWP.

Pattern III. When Pattern III is combined with correlation matrix $\mathbf{R}_4$, the predictors that were more strongly related to the outcome (X_2, X_3, and X_4) were present in only the last study in the synthesis and there was no correlation among predictors. As shown in Table 5.18, SWP gave better estimates of the slope for X_1 most of the time and GLS tended to overestimate the slope of X_1. Compared with the results from other scenarios, neither method performed particularly well at estimating the slope of X_1 in $N1$. When the sample size was larger and equal across studies ($N2$), GLS and SWP overestimated the slopes for all four variables. Both methods produced good estimates for slopes of the variables when less missing data occurred ($N3$); when there was more missing data occurred ($N4$), SWP produced better estimates for X_1, which was the only variable that was fully observed. Both methods produced equally stable estimates in most situations.

Pattern IV. In the combination of Pattern IV with correlation matrix $\mathbf{R}_4$, predictors X_1 to X_3 were present in all four studies, whereas X_4, which was related to the outcome the most strongly, was present only in the last study included in the synthesis. As shown in Table 5.19, SWP estimated the slope of X_1 better across all different sample size patterns, except in $N1$ when GLS estimates were less bias. Both methods tended to overestimate the slope of X_1. SWP gave better estimates of the slope of X_3, which was a variable that was fully observed and related to the outcome most strongly. When sample sizes were equal and large across studies ($N2$), GLS produced precisely estimate of the slope of X_4. When many values were missing on X_4 (96% in $N4$), GLS also better estimated the slope of X_4. When the proportion of missingness was smaller (45% in $N3$), SWP tended to do better. GLS generally produced more stable estimates than SWP.

Pattern V. In Pattern V with the correlation matrix $\mathbf{R}_4$, all the studies in the synthesis included all four predictors. No missing data occurred for any of the predictors and there was no correlation among those predictors. As shown in Table 5.10, SWP consistently produced estimates of the slope for X_1 that were closer to the population value, while GLS always resulted in overestimation of the slope of this variable when sample sizes varied. SWP worked well when the sample sizes were small and equal (N1). SWP produced less stable estimates of the slopes for X_4. SWP also produced less stable estimates when variables highly related to the outcome were based on smaller sample sizes (*N*4).

Table 5.16

Missing Percentage, Estimated Mean Slopes and Standard Errors for Each Predictor for Pattern I and Correlation Matrix $\mathbf{R}_4$

Method	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
	0.25	**0.3**	**0.4**	**0.6**				
*N*1	0%	25%	50%	75%				
GLS	0.2522	0.3023	0.4022	0.5954	0.001271	0.001312	0.001370	0.001240
SWP	0.2500[a]	0.2996	0.3998	0.6034	0.001283	0.001308	0.001371	0.001312
*N*2	0%	25%	50%	75%				
GLS	0.2502	0.2997	0.4000	0.5999	0.000356	0.000361	0.000369	0.000330
SWP	0.2500[a]	0.2995	0.3998	0.6004	0.000356	0.000362	0.000370	0.000354
*N*3	0%	4%	18%	45%				
GLS	0.2501	0.3001	0.4000[a]	0.5994	0.000388	0.000406	0.000399	0.000371
SWP	0.2498	0.2998	0.4000[a]	0.6002	0.000387	0.000405	0.000398	0.000387
*N*4	0%	55%	82%	96%				
GLS	0.2506	0.3015	0.4029	0.6015	0.001145	0.001283	0.001287	0.001130
SWP	0.2497	0.3001	0.4009	0.6066	0.001161	0.001293	0.001298	0.001321

Note. The bolded values are the population slopes for predictors.

a. Mean estimated slope is equal to the population value.

Table 5.17

Missing Percentage, Estimated Mean Slopes and Standard Errors for Each Predictor for Pattern II and CorrelationMatrix $\mathbf{R_4}$

Method	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
	0.25	**0.3**	**0.4**	**0.6**				
*N*1	0%	0%	0%	25%				
GLS	0.2509	0.3015	0.4009	0.6027	0.000883	0.000894	0.000897	0.000844
SWP	0.2491	0.2993	0.3980	0.6002	0.000843	0.000852	0.000849	0.000807
*N*2	0%	0%	0%	25%				
GLS	0.2502	0.2998	0.4001	0.6003	0.000234	0.000239	0.000234	0.000216
SWP	0.2502	0.2996	0.3999	0.6001	0.000232	0.000238	0.000233	0.000222
*N*3	0%	0%	0%	4%				
GLS	0.2505	0.2999	0.4003	0.6000[a]	0.000326	0.000329	0.000320	0.000303
SWP	0.2502	0.2996	0.3998	0.5999	0.000325	0.000327	0.000318	0.000302
*N*4	0%	0%	0%	55%				
GLS	0.2499	0.2999	0.4007	0.6021	0.000399	0.000428	0.000425	0.000374
SWP	0.2495	0.2995	0.4002	0.6004	0.000395	0.000422	0.000421	0.000392

Note. The bolded values are the population slopes for predictors.

a. Mean estimated slope is equal to the population value.

Table 5.18

Missing Percentage, Estimated Mean Slopes and Standard Errors for Each Predictor for Pattern III and Correlation Matrix $\mathbf{R}_4$

Method	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
	0.25	**0.3**	**0.4**	**0.6**				
*N*1	0%	75%	75%	75%				
GLS	0.2545	0.3010	0.4013	0.5976	0.001407	0.001568	0.001519	0.001438
SWP	0.2523	0.3012	0.4017	0.5986	0.001402	0.001572	0.001522	0.001449
*N*2	0%	75%	75%	75%				
GLS	0.2505	0.3001	0.4004	0.6005	0.000369	0.000439	0.000434	0.000390
SWP	0.2503	0.3001	0.4004	0.6006	0.000369	0.000439	0.000434	0.000393
*N*3	0%	45%	45%	45%				
GLS	0.2501	0.2998	0.4006	0.5999	0.000383	0.000413	0.000422	0.000394
SWP	0.2498	0.2999	0.4007	0.6000[a]	0.000383	0.000413	0.000422	0.000393
*N*4	0%	96%	96%	96%				
GLS	0.2511	0.3021	0.3987	0.6026	0.001375	0.001555	0.001589	0.001463
SWP	0.2502	0.3018	0.3985	0.6026	0.001375	0.001557	0.001591	0.001486

Note. The bolded values are the population slopes for predictors.

a. Mean estimated slope is equal to the population value.

Table 5.19

Missing Percentage, Estimated Mean Slopes and Standard Errors for Each Predictor for Pattern IV and Correlation Matrix $\mathbf{R}_4$

Method	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
	0.25	**0.3**	**0.4**	**0.6**				
*N*1	0%	0%	0%	75%				
GLS	0.2511	0.3029	0.4037	0.5948	0.001295	0.001275	0.001314	0.001146
SWP	0.2487	0.2999	0.3993	0.6052	0.001305	0.001282	0.001316	0.001272
*N*2	0%	0%	0%	75%				
GLS	0.2505	0.2996	0.4004	0.6000[a]	0.000333	0.000343	0.000349	0.000304
SWP	0.2503	0.2994	0.4001	0.6007	0.000335	0.000345	0.000351	0.000337
*N*3	0%	0%	0%	45%				
GLS	0.2503	0.3004	0.4008	0.5987	0.000378	0.000397	0.000383	0.000356
SWP	0.2500[a]	0.3000[a]	0.4002	0.6001	0.000380	0.000397	0.000382	0.000373
*N*4	0%	0%	0%	96%				
GLS	0.2516	0.3023	0.4020	0.6029	0.001129	0.001183	0.001181	0.001033
SWP	0.2505	0.3010	0.4002	0.6067	0.001145	0.001194	0.001186	0.001215

Note. The bolded values are the population slopes for predictors.

a. Mean estimated slope is equal to the population value.

Table 5.20

Missing Percentage, Estimated Mean Slopes and Standard Errors for Each Predictor for Pattern V and Correlation Matrix $\mathbf{R}_4$

Method	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
	0.25	**0.3**	**0.4**	**0.6**				
*N*1	0%	0%	0%	0%				
GLS	0.2516	0.3015	0.4008	0.6036	0.000808	0.000825	0.000797	0.000803
SWP	0.2499	0.2993	0.3986	0.5987	0.000784	0.000787	0.000750	0.000763
*N*2	0%	0%	0%	0%				
GLS	0.2502	0.3001	0.4001	0.6003	0.000215	0.000217	0.000218	0.000203
SWP	0.2502	0.3000[a]	0.3999	0.5999	0.000214	0.000216	0.000217	0.000202
*N*3	0%	0%	0%	0%				
GLS	0.2503	0.2998	0.4003	0.6008	0.000329	0.000323	0.000316	0.000305
SWP	0.2500[a]	0.2995	0.3998	0.5999	0.000324	0.000318	0.000311	0.000299
*N*4	0%	0%	0%	0%				
GLS	0.2502	0.2998	0.4004	0.6007	0.000327	0.000321	0.000313	0.000302
SWP	0.2500[a]	0.2995	0.3999	0.5999	0.000323	0.000318	0.000311	0.000299

Note. The bolded values are the population slopes for predictors.

a. Mean estimated slope is equal to the population value.

Table 5.21 presents the bias ranges for each slope for each method across scenarios. When important variables tended to be missing from the model (**R**$_3$ and **R**$_4$), the estimates for X_1, X_2, and X_3 from both departed from the population values relatively large. The ranges of the bias for the slope of X_4 were largest among the four predictor slopes for both methods. Since it was missing the most across studies, it was more difficulty to estimate it precisely.

Table 5.21

Ranges of Percentage Relative Bias Produced by GLS and SWP

	SWP		GLS	
	Largest negative	Largest positive	Largest negative	Largest positive
X_1				
Value	-0.6229	1.3324	-0.06	3.1782
Scenario	Pat4N_1R_3	Pat3N_1R_3	Pat2N_4R_4	Pat3N_1R_3
X_2				
Value	-0.4375	0.6067	-0.71	1.7976
Scenario	Pat4N_2R_3	Pat3N_4R_4	Pat3N_1R_1	Pat4N_1R_3
X_3				
Value	-1.3313	0.8432	-1.4334	1.5561
Scenario	Pat3N_4R_3	Pat3N_1R_3	Pat3N_4R_3	Pat4N_1R_3
X_4				
Value	-0.8479	2.00	-2.6648	1.1121
Scenario	Pat5N_1R_1	Pat4N_4R_2	Pat4N_1R_1	Pat5N_1R_3

Note. Eight characters denote a scenario. The first four characters indicate the pattern (Pat1 through Pat5); the following two characters indicate the sample size set (N1 through N4); the last two characters indicate the correlation matrix (**R**$_1$ through **R**$_4$).

5.2 ANOVA Results

The ANOVA results for each predictor were summarized in Table 5.22 to Table 5.25. Noted that the scale for the marginal means for pattern IV was different from other patterns to present the large negative differences between two methods in estimating X_4 slope. The small adjusted R squares for modeling the difference between two methods for each predictor (ranging from .029 for X_2 to .088 for X_1) indicated that only a small portion of the differences between two methods was attributable to the missing data patterns, correlation matrices, sample size sets, and their interactions. Because of the large amount of data that was generated for this research, the significance values were all less than .0001, which indicates the significance of all factors. The largest η^2 estimate (E^2) among the four ANOVAs for four predictors was .06 for sample sizes (N) for the slopes for X_1. Different missing patterns explained less than 0.01% in the variance of the different estimates between the GLS and the SWP methods for X_1. Correlation matrices (**R**s) also explained less than 0.01% variances of the different estimates between two methods for X_2. For X_4, missing data patterns explained most amount of variance (E^2=.034) of the outcome, while the pattern and sample size interaction also contributed 2.8% of the variance.

The interactions were also significant at the .0001 level. To show the nature of the interaction, the correlation matrix (**R**)*sample sizes (N) interactions were plotted for the five missing data patterns for each of the four predictors in Figure 5.1 to Figure 5.4. In most of the plots for the X_1 slope, large discrepancies arose for the four predictors in the sample size set $N1$. Across all five patterns, the largest differences between the methods of estimating the X_1 slope were present with the matrix $\mathbf{R_1}$. The differences between the two methods for estimating the slope of X_2 were smaller. For the predictors X_3 and X_4 when more data were missing, the two methods were similar at estimating the slope under *N2* and *N3*. The two methods were different at estimating the slopes for these two predictors when important correlations tended to be missing more frequently ($\mathbf{R}_3$ and $\mathbf{R}_4$).

Table 5.22

Analysis of Variance for the Differences in Estimates of the Slope of X_1

Dependent Variable: $\bar{\hat{B}}_1(\text{GLS}) - \bar{\hat{B}}_1(\text{SWP})$

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	.287[a]	79	.004	99.23	.000	.089
Intercept	.177	1	.177	4848.50	.000	.057
Pattern	.001	4	.000	6.18	.000	.000
R	.041	3	.014	370.09	.000	.014
N	.186	3	.062	1696.83	.000	.060
Pattern * R	.003	12	.000	6.34	.000	.001
Pattern * N	.002	12	.000	4.97	.000	.001
R * N	.051	9	.006	153.87	.000	.017
Pattern * R * N	.003	36	9.44E-005	2.58	.000	.001
Error	2.922	79920	3.66E-005			
Total	3.386	80000				
Corrected Total	3.208	79999				

a. R Squared = .089 (Adjusted R Squared = .088)

Table 5.23

Analysis of Variance for the Differences in Estimates of the Slope of X_2

Dependent Variable: $\bar{\hat{B}}_2(\text{GLS}) - \bar{\hat{B}}_2(\text{SWP})$

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	.110[a]	79	.001	30.88	.000	.030
Intercept	.034	1	.034	750.78	.000	.009
Pattern	.032	4	.008	175.87	.000	.009
R	.002	3	.001	12.19	.000	.000
N	.031	3	.010	230.63	.000	.009
Pattern * R	.004	12	.000	6.68	.000	.001
Pattern * N	.035	12	.003	64.42	.000	.010
R * N	.002	9	.000	5.30	.000	.001
Pattern * R * N	.005	36	.000	2.96	.000	.001
Error	3.602	79920	4.51E-005			
Total	3.745	80000				
Corrected Total	3.712	79999				

a. R Squared = .030 (Adjusted R Squared = .029)

Table 5.24

Analysis of Variance for the Differences in Estimates of the Slope of X_3

Dependent Variable: $\bar{\hat{B}}_3(\text{GLS}) - \bar{\hat{B}}_3(\text{SWP})$

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	.156[a]	79	.002	43.75	.000	.041
Intercept	.046	1	.046	1014.26	.000	.013
Pattern	.039	4	.010	213.81	.000	.011
R	.013	3	.004	93.08	.000	.003
N	.033	3	.011	243.15	.000	.009
Pattern * R	.009	12	.001	16.07	.000	.002
Pattern * N	.044	12	.004	81.41	.000	.012
R * N	.012	9	.001	28.90	.000	.003
Pattern * R * N	.007	36	.000	4.52	.000	.002
Error	3.604	79920	4.51E-005			
Total	3.806	80000				
Corrected Total	3.760	79999				

a. R Squared = .041 (Adjusted R Squared = .041)

Table 5.25

Analysis of Variance for the Differences in Estimates of the Slope of X_4

Dependent Variable: $\bar{\hat{B}}_4(\text{GLS}) - \bar{\hat{B}}_4(\text{SWP})$

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	.652[a]	79	.008	82.690	.000	.076
Intercept	.067	1	.067	669.80	.000	.008
Pattern	.278	4	.070	696.66	.000	.034
R	.007	3	.002	22.01	.000	.001
N	.055	3	.018	182.15	.000	.007
Pattern * R	.033	12	.003	27.78	.000	.004
Pattern * N	.231	12	.019	193.06	.000	.028
R * N	.015	9	.002	16.91	.000	.002
Pattern * R * N	.033	36	.001	9.20	.000	.004
Error	7.982	79920	9.99E-005			
Total	8.702	80000				
Corrected Total	8.635	79999				

a. R Squared = .076 (Adjusted R Squared = .075)

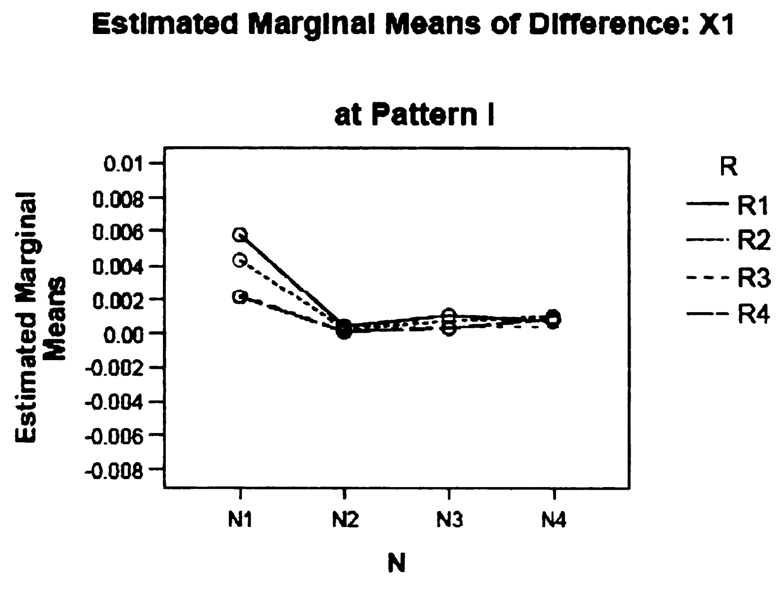
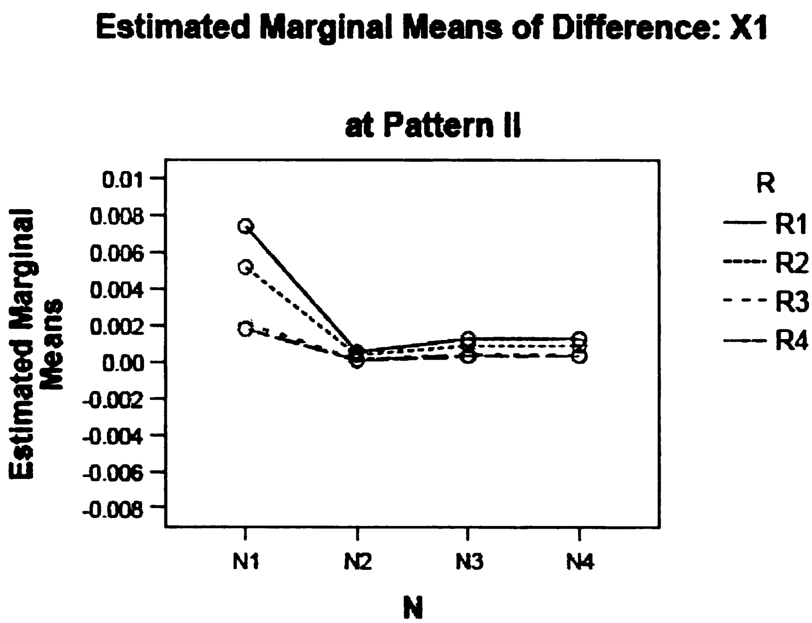


Figure 5.1. Interactions of Sample Size Sets and Correlation Matrices for Five Patterns for Differences in Slopes of X_1



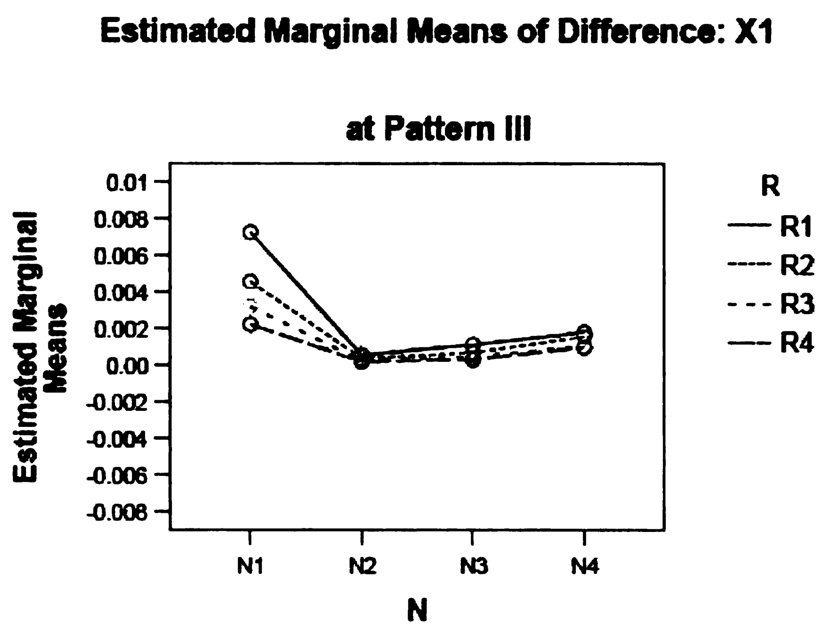
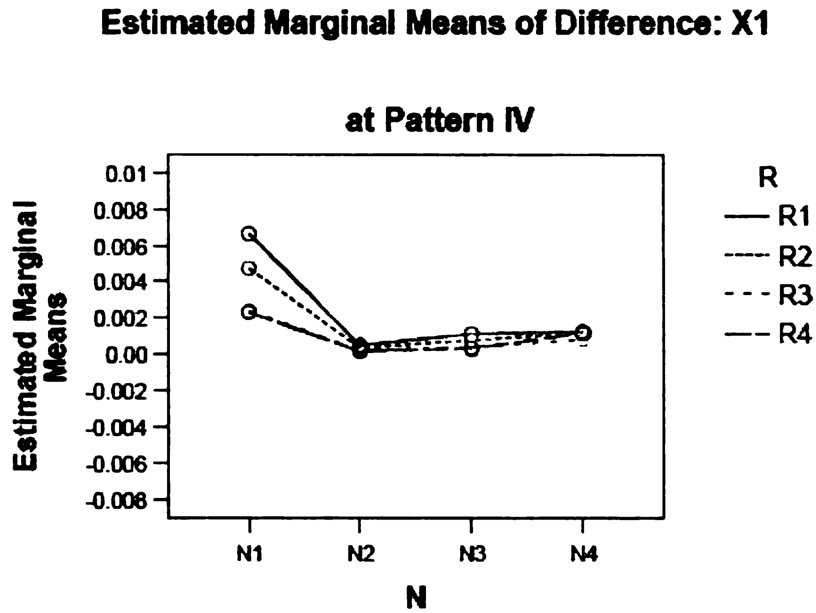


Figure 5.1. (cont'd) Interactions of Sample Size Sets and Correlation Matrices for Five Patterns for Differences in Slopes of X_1



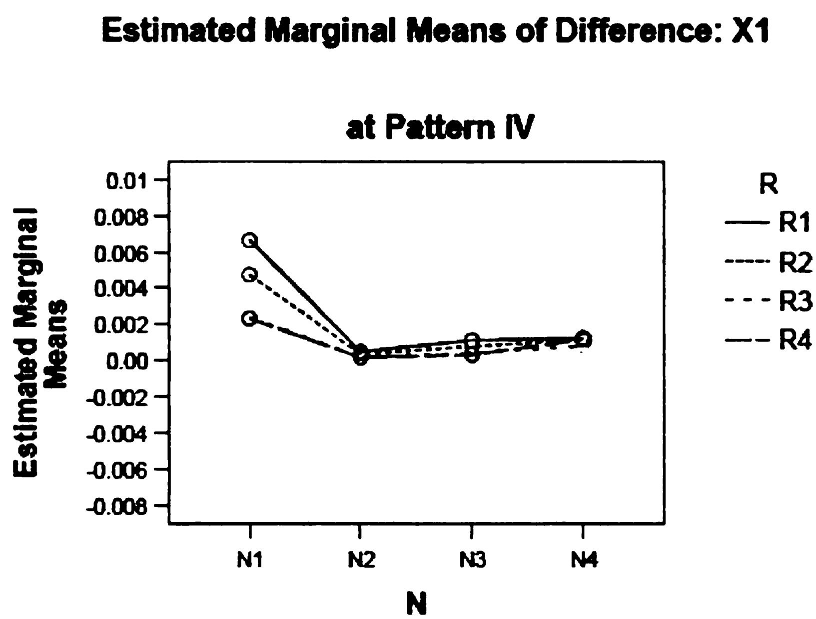


Figure 5.1. (cont'd) Interactions of Sample Size Sets and Correlation Matrices for Five Patterns for Differences in Slopes of X_1

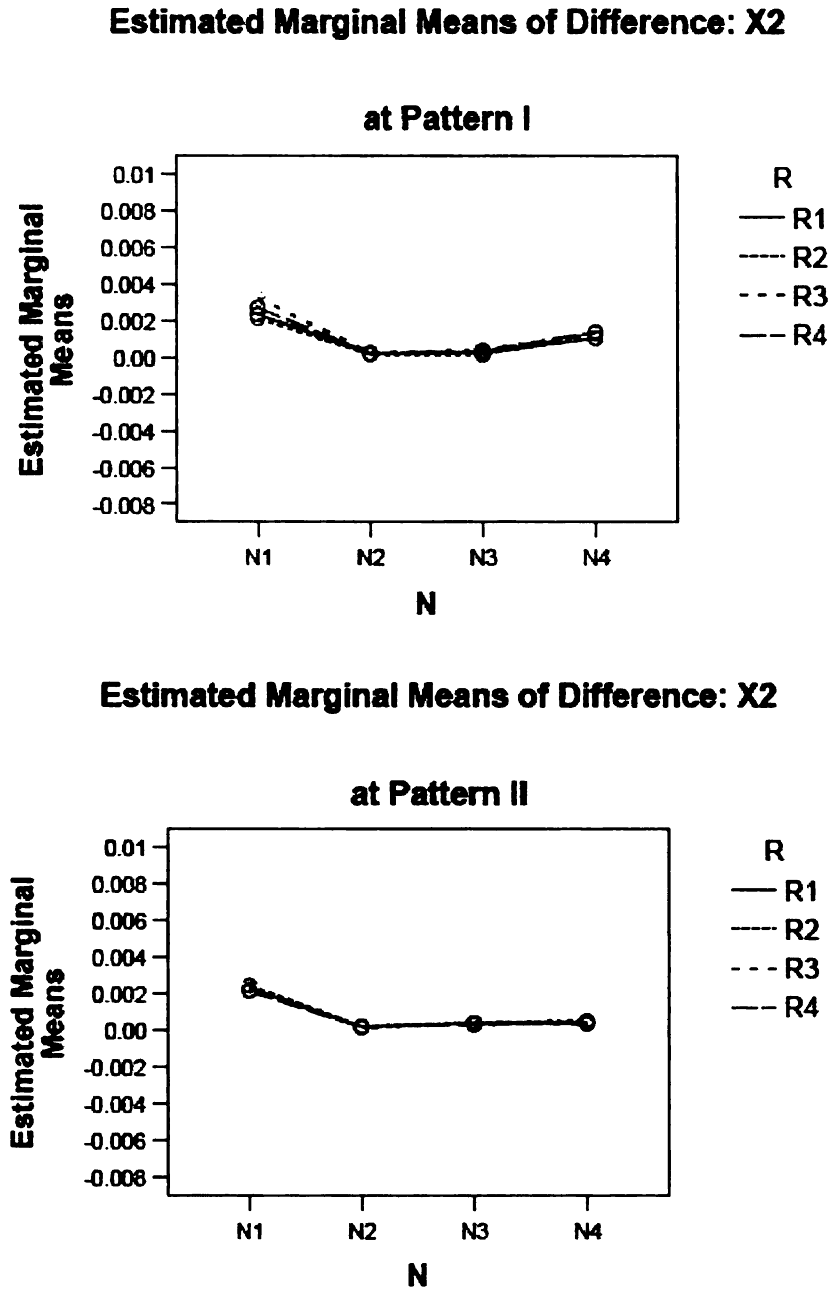
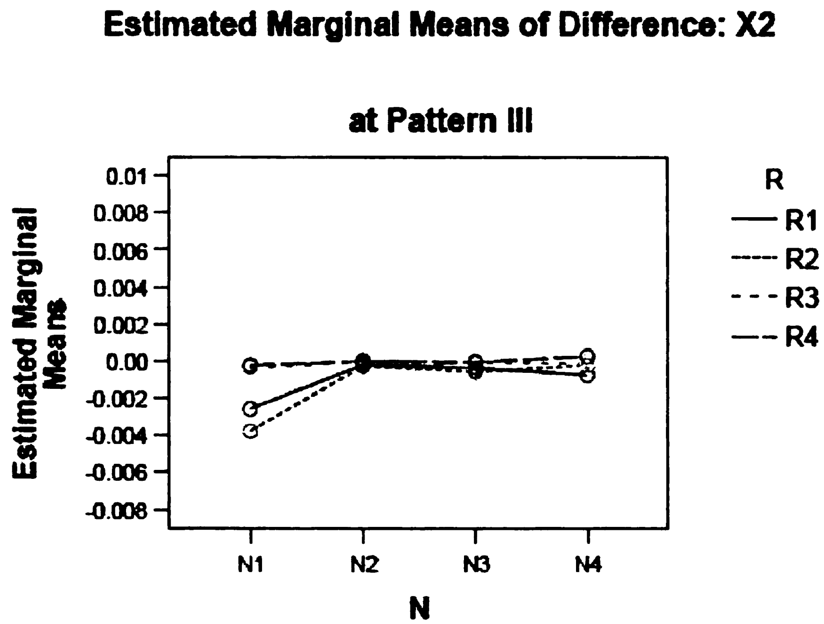


Figure 5.2. Interactions of Sample Size Sets and Correlation Matrices for Five Patterns for Differences in Slopes of X_2



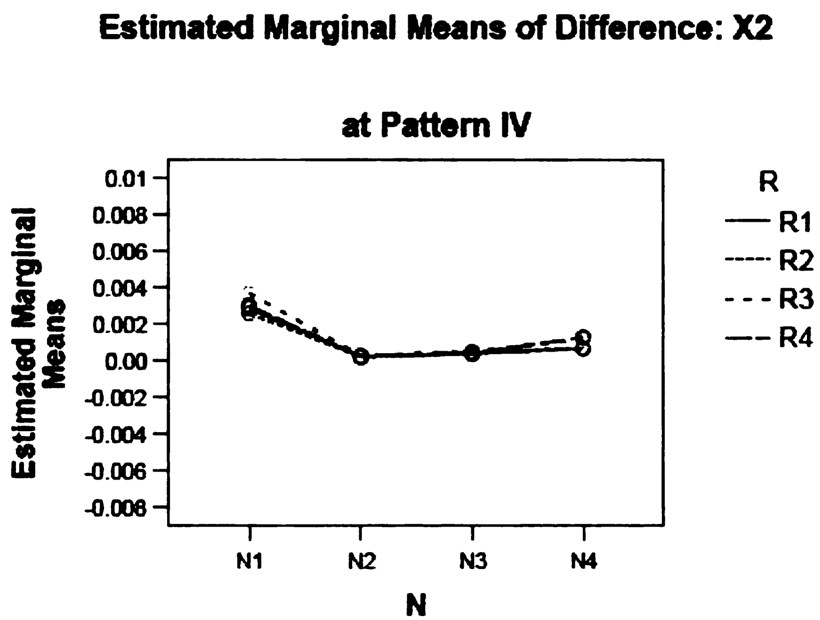


Figure 5.2. (cont'd) Interactions of Sample Size Sets and Correlation Matrices for Five Patterns for Differences in Slopes of X_2

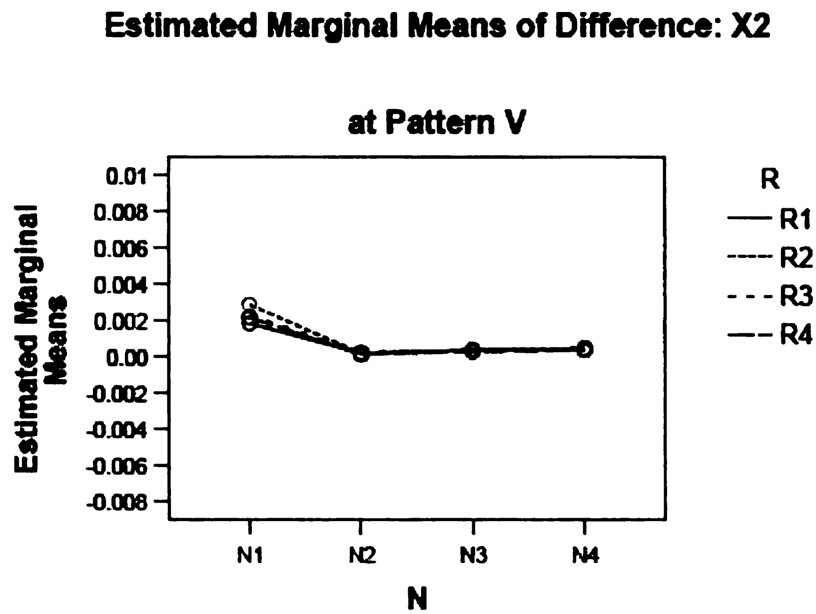


Figure 5.2. (cont'd) Interactions of Sample Size Sets and Correlation Matrices for Five Patterns for Differences in Slopes of X_2

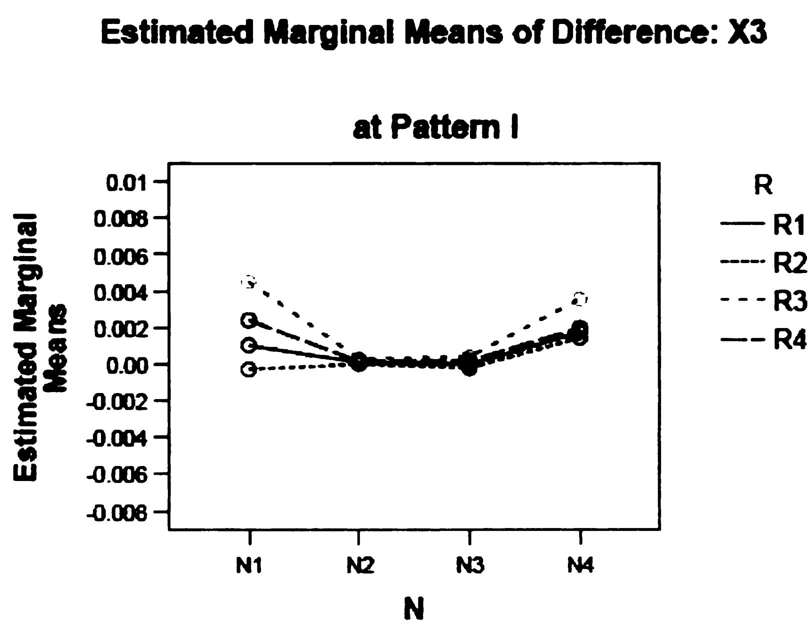
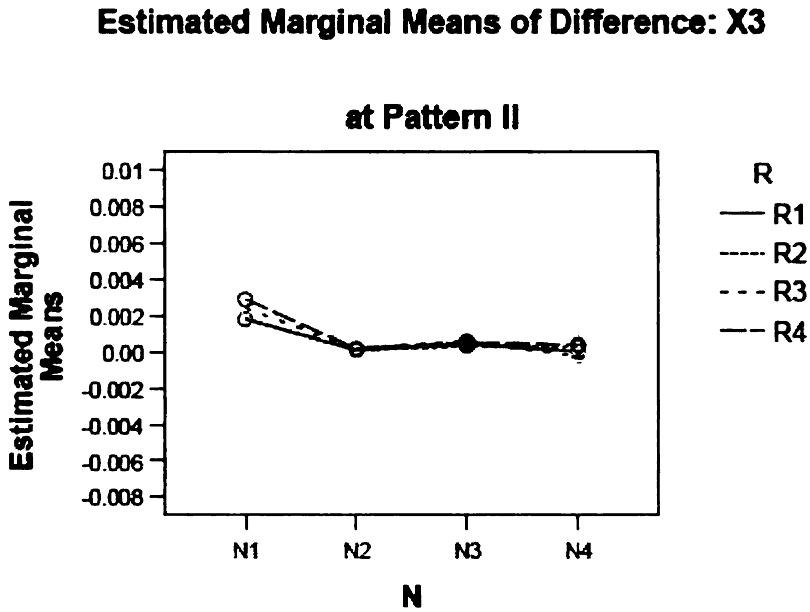


Figure 5.3. Interactions of Sample size Sets and Correlation Matrices in Five Patterns for Differences in Slopes of X_3



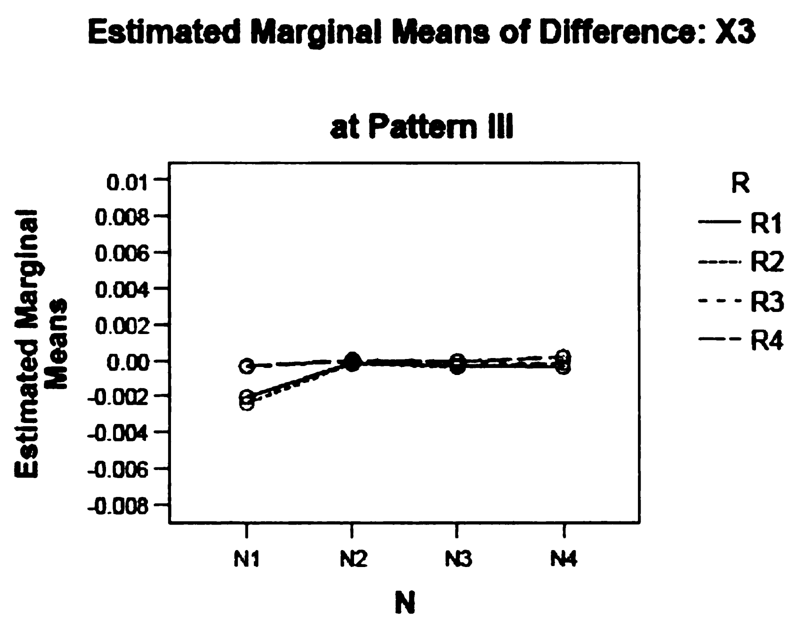
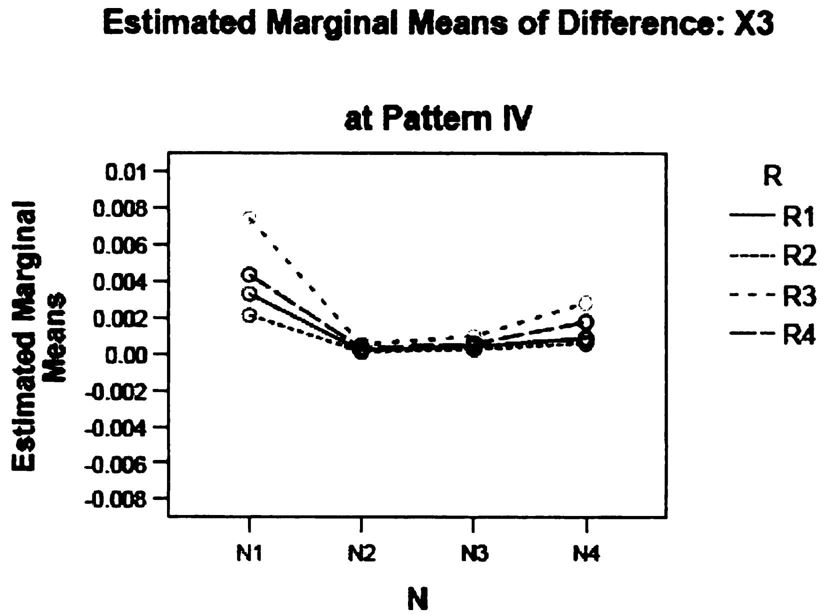


Figure 5.3. (cont'd) Interactions of Sample size Sets and Correlation Matrices in Five Patterns for Differences in Slopes of X_3



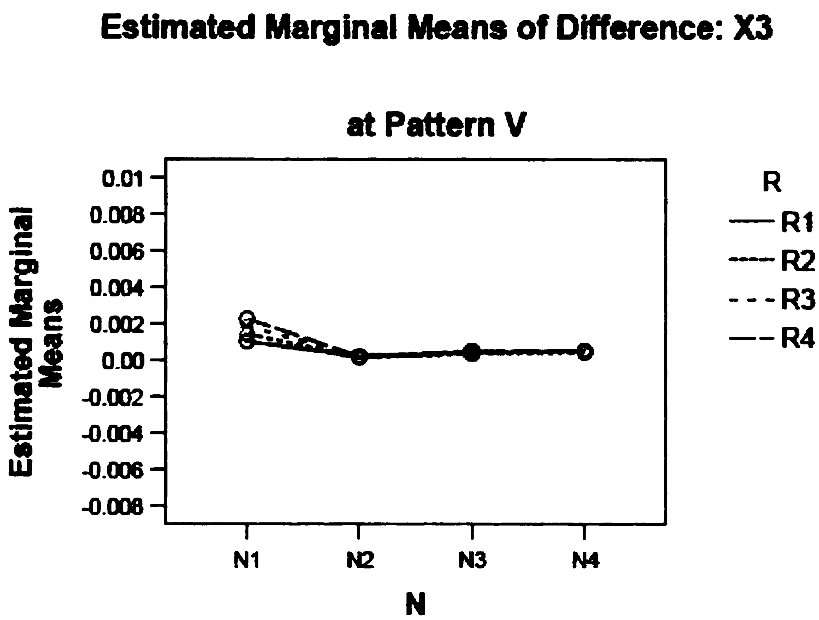
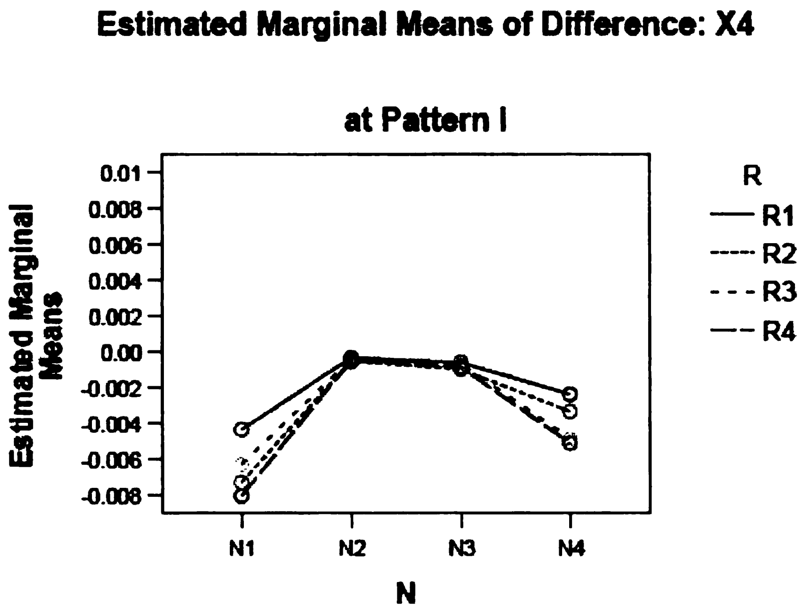


Figure 5.3. (cont'd) Interactions of Sample size Sets and Correlation Matrices in Five Patterns for Differences in Slopes of X_3



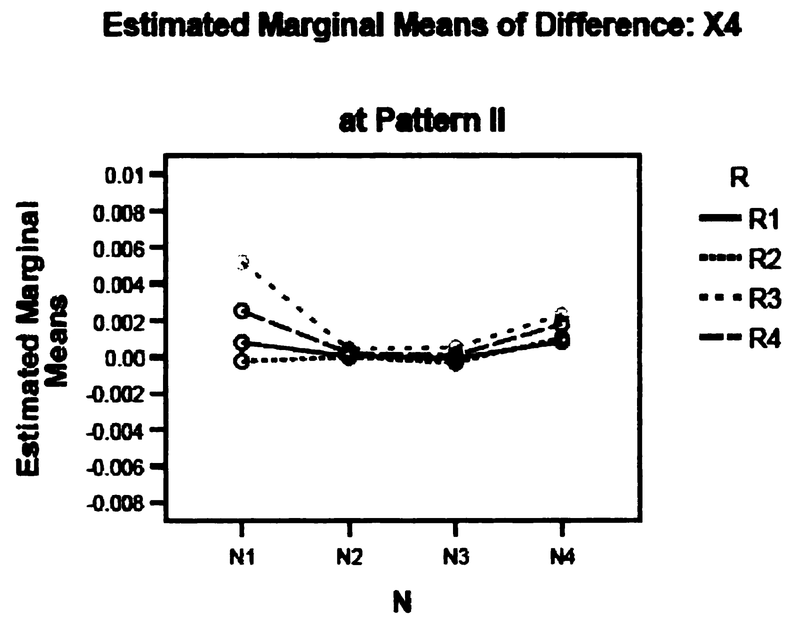
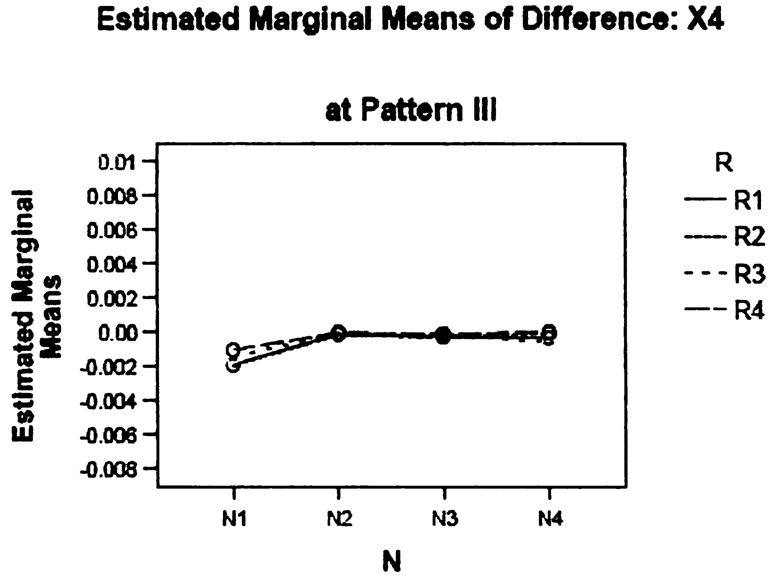


Figure 5.4. Interactions of Sample size Sets and Correlation Matrices in Five Patterns for Differences in Slopes of X_4



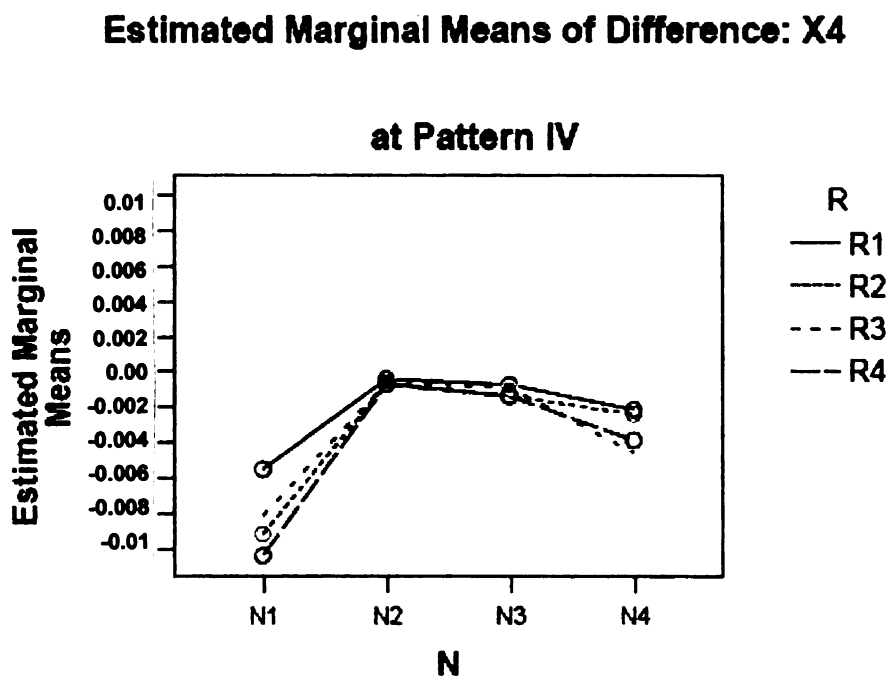


Figure 5.4. (cont'd) Interactions of Sample size Sets and Correlation Matrices in Five Patterns for Differences in Slopes of X_4

Estimated Marginal Means of Difference: X4

at Pattern V

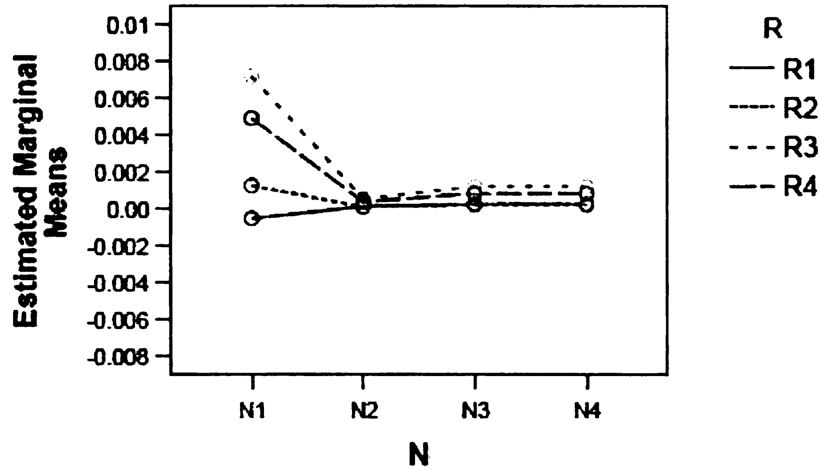


Figure 5.4. (cont'd) Interactions of Sample size Sets and Correlation Matrices in Five Patterns for Differences in Slopes of X_4

5.3 Mixed-effects Model (Condition 5 through Condition 8)

In this part of simulation, I made the models more complex by choosing different correlation matrices for the first two and the last two studies. This represents a more complex fixed effects model, with two groups of studies. The results based on different matrices ($\mathbf{R}_5$ though $\mathbf{R}_8$) under mixed-effects model in the syntheses for each of the five missing patterns are shown in Table 5.26 through Table 5.30. Within each pattern, the mean slope for each predictor and the standard errors based on GLS and SWP methods were reported for each sample size set (*N*1 thought *N*4) for each of the four conditions. Note that the population values for the slopes for the variables were always the same for *N*1 and *N*2. This is because *N*1 and *N*2 both had equal sample sizes across the four studies included in the synthesis, and the summarized correlation matrix used for calculating the slopes weighted by sample size as shown in the methods section.

5.3.1 Pattern I

In this pattern, the relative bias between of the estimated slopes was less than 5% most of the time for both methods. However, the relative bias was much greater in some conditions. SWP generally performed better than GLS, and produced slopes closer to the population values. The worst estimation from SWP in this pattern was in condition 5 when the sample size set was *N*4. Here correlations among predictors (X_1 and X_2) existed only in one study (study 2) based on a somewhat large sample (the sample size for the second study was 1000 in *N*4). The relative bias of the slopes for X_2, X_3, and X_4 were 9.55%, 11.48%, and 12.64% respectively. Both GLS and SWP produced smaller relative bias when the sample sizes were from *N*2 and *N*3. They performed well especially in condition 8 when the sample size was equal to *N*3. The relative biases of estimates of all the slopes produced by both methods were all less than 1% in that condition. The stability of the estimates of both methods was similar to those based on fixed-effects model.

5.3.2 Pattern II

In this pattern, SWP produced much closer estimates than did the GLS procedure. Most of the time, SWP resulted in less than 1% relative bias in estimating the slopes. For GLS, with sample size sets *N*1 and *N*2, the relative biases were greater than 5% all the time for all variables, while SWP produced bias values under 1% most of the time. With sample size sets *N*3 and *N*4, GLS performed only slightly better for a few slopes with the relative bias less than 5%. The bias for those with relative bias less than 5% by GLS method ranged from 2.35% (slope for X_4 with sample size *N*3 in condition 6) to 4.98%

(X_3 with sample size $N4$ in condition 8). SWP did not perform as well as in other scenarios when the sample size set was $N4$ in condition 8. The relative biases of slopes for all four predictors were all above 1%. However, they were still smaller than the values for estimates from the GLS method. The stability of the estimates of both methods was similar to those based on the simple fixed-effects model.

5.3.3 Pattern III

The results for the each condition presented in Table 5.28 were identical to those presented in Table 5.8 (same as Condition 5), Table 5.18 (same as Condition 6), Table 5.3 (same as Condition 7), and Table 5.13 (same as Condition 8). The identity arose because, in this pattern, the intercorrelations among X_2, X_3, and X_3 were provided by only the last study in the synthesis, which made it the same as Pattern III under the simple fixed-effects model. The comparisons between GLS and SWP under each condition for different sample size sets can be found in the previous sections.

5.3.4 Pattern IV

In this pattern, GLS produced large relative percentage bias values in most conditions. The largest bias produced by GLS was in estimating the slope of X_1 (bias=18.33%) with the sample size set $N4$ in Condition 6. SWP also produced the largest bias in the same scenario (bias=17.86%). Actually, when the sample size equaled $N4$ in this pattern in Condition 5 and Condition 6, where X_4 had zero correlation with other variables, the estimated slopes for all the four variables from both methods had rather larger relative biases. SWP consistently resulted in large bias, ranging from 8.35% (in $N3$)

to 35.95% (in $N4$), for $\bar{\hat{\beta}}_4$ in Condition 5. Contrary to the large biases found in those situations, SWP consistently produced small biases for all four variables across the sample size sets in Condition 7.

5.3.5 Pattern V

No missing data occurred in this pattern, and the results in this pattern were similar to those found for Pattern II, where the missingness occurred only in one variable in the first study. Most of the time, the SWP slopes showed less than 1% relative percentage bias. In Condition 7 and Condition 8, when sample sizes were equal across studies ($N1$ and $N2$), GLS estimated had more than 10% relative bias for the slopes of X_1, X_2 and X_3. Large biases in estimating the slopes of X_2, X_3 and X_4 from GLS could be found in Condition 5 and Condition 6 when the sample sizes were equal, as well as in Condition 6 with sample size set $N3$ and in Condition 5 with sample size set $N4$.

Table 5.26

Parameters, Estimated Mean Slopes and Standard Errors for Each Predictor under Mixed-effect Models with Different Sample Sizes in Pattern I

Method		$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Condition 5: $R_1R_1R_2R_2$									
		0.5706	**0.3524**	**0.3000**	**0.2500**				
N1	GLS	0.5880	0.3739	0.3084	0.2508	0.001059	0.001117	0.001155	0.001465
	SWP	0.5808	0.3682	0.3203	0.2669	0.001080	0.001114	0.001207	0.001566
N2	GLS	0.5842	0.3724	0.3085	0.2565	0.000279	0.000297	0.000308	0.000388
	SWP	0.5808	0.3681	0.3200	0.2656	0.000287	0.000303	0.000323	0.000410
		0.5865	**0.3791**	**0.3000**	**0.2500**				
N3	GLS	0.5942	0.3875	0.3027	0.2514	0.000337	0.000365	0.000359	0.000398
	SWP	0.5908	0.3858	0.3091	0.2571	0.000335	0.000361	0.000368	0.000408
		0.5521	**0.3164**	**0.3000**	**0.2500**				
N4	GLS	0.5687	0.3537	0.3223	0.2678	0.000775	0.000880	0.000958	0.001495
	SWP	0.5690	0.3466	0.3344	0.2816	0.000846	0.000933	0.001045	0.001658
Condition 6: $R_3R_3R_4R_4$									
		0.2356	**0.2882**	**0.4000**	**0.6000**				
N1	GLS	0.2466	0.2981	0.4033	0.5976	0.001272	0.001312	0.001374	0.001245
	SWP	0.2447	0.2952	0.4017	0.6076	0.001290	0.001314	0.001381	0.001326
N2	GLS	0.2447	0.2956	0.4011	0.6022	0.000355	0.000361	0.000370	0.000331
	SWP	0.2445	0.2950	0.4018	0.6046	0.000357	0.000364	0.000373	0.000357
		0.2437	**0.2948**	**0.4000**	**0.6000**				
N3	GLS	0.2478	0.2983	0.4007	0.6003	0.000389	0.000407	0.000399	0.000372
	SWP	0.2474	0.2978	0.4009	0.6021	0.000388	0.000404	0.000399	0.000388
		0.2246	**0.2796**	**0.4000**	**0.6000**				
N4	GLS	0.2406	0.2940	0.4053	0.6061	0.001145	0.001282	0.001296	0.001134
	SWP	0.2403	0.2926	0.4043	0.6142	0.001174	0.001303	0.001319	0.001338

Table 5.26 (cont'd)

Parameters, Estimated Mean Slopes and Standard Errors for Each Predictor under Mixed-effect Models with Different Sample Sizes in Pattern I

Method		$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Condition 7: $R_2R_2R_1R_1$									
		0.5288	**0.2677**	**0.1815**	**0.1696**				
N1	GLS	0.5438	0.2765	0.1773	0.1571	0.001029	0.001272	0.001297	0.001682
	SWP	0.5298	0.2627	0.1819	0.1664	0.001007	0.001214	0.001312	0.001781
N2	GLS	0.5398	0.2752	0.1756	0.1612	0.000271	0.000327	0.000341	0.000460
	SWP	0.5304	0.2632	0.1811	0.1666	0.000265	0.000311	0.000349	0.000474
		0.5207	**0.2434**	**0.1857**	**0.1718**				
N3	GLS	0.5294	0.2487	0.1821	0.1665	0.000360	0.000427	0.000420	0.000470
	SWP	0.5212	0.2414	0.1859	0.1706	0.000357	0.000419	0.000424	0.000480
		0.5443	**0.3035**	**0.1751**	**0.1662**				
N4	GLS	0.5518	0.3091	0.1708	0.1569	0.000612	0.000822	0.000953	0.001710
	SWP	0.5460	0.2957	0.1739	0.1632	0.000613	0.000811	0.000963	0.001791
Condition 8: $R_4R_4R_3R_3$									
		0.1822	**0.1967**	**0.2234**	**0.5154**				
N1	GLS	0.1839	0.1983	0.2273	0.5092	0.001379	0.001458	0.001592	0.001636
	SWP	0.1809	0.1938	0.2232	0.5160	0.001378	0.001428	0.001578	0.001651
N2	GLS	0.1817	0.1951	0.2242	0.5132	0.000383	0.000392	0.000423	0.000437
	SWP	0.1809	0.1937	0.2243	0.5140	0.000382	0.000389	0.000424	0.000442
		0.1771	**0.1920**	**0.2245**	**0.5158**				
N3	GLS	0.1773	0.1918	0.2252	0.5144	0.000436	0.000466	0.000471	0.000455
	SWP	0.1763	0.1906	0.2250	0.5153	0.000436	0.000465	0.000470	0.000457
		0.1900	**0.2040**	**0.2217**	**0.5147**				
N4	GLS	0.1874	0.2011	0.2258	0.5129	0.001135	0.001311	0.001378	0.001632
	SWP	0.1868	0.1986	0.2227	0.5180	0.001115	0.001323	0.001383	0.001687

Note. The bold values were the parameter slopes for the condition.

Table 5.27

Parameters, Estimated Mean Slopes and Standard Errors for Each Predictor under Mixed-effect Models with Different Sample Sizes in Pattern II

Method		$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Condition 5: $R_1R_1R_2R_2$									
		0.5461	**0.3065**	**0.2388**	**0.2187**				
N1	GLS	0.5798	0.3430	0.2690	0.2453	0.000894	0.000944	0.000910	0.000986
	SWP	0.5463	0.3080	0.2386	0.2276	0.000821	0.000897	0.000871	0.000989
N2	GLS	0.5764	0.3417	0.2691	0.2461	0.000231	0.000243	0.000239	0.000254
	SWP	0.5475	0.3084	0.2397	0.2280	0.000223	0.000241	0.000242	0.000270
		0.5772	**0.3636**	**0.2750**	**0.2348**				
N3	GLS	0.5931	0.3797	0.2901	0.2458	0.000317	0.000342	0.000318	0.000315
	SWP	0.5781	0.3638	0.2752	0.2379	0.000316	0.000334	0.000317	0.000322
		0.5246	**0.2545**	**0.2073**	**0.1998**				
N4	GLS	0.5511	0.2885	0.2334	0.2449	0.000391	0.000443	0.000397	0.000572
	SWP	0.5244	0.2555	0.2081	0.2060	0.000360	0.000408	0.000374	0.000530
Condition 6: $R_3R_3R_4R_4$									
		0.2063	**0.2415**	**0.3248**	**0.5614**				
N1	GLS	0.2324	0.2738	0.3552	0.5954	0.000945	0.000938	0.000941	0.000916
	SWP	0.2093	0.2454	0.3261	0.5672	0.000913	0.000918	0.000901	0.000862
N2	GLS	0.2322	0.2726	0.3545	0.5931	0.000250	0.000251	0.000252	0.000235
	SWP	0.2105	0.2456	0.3277	0.5674	0.000251	0.000256	0.000245	0.000236
		0.2319	**0.2757**	**0.3677**	**0.5813**				
N3	GLS	0.2443	0.2904	0.3832	0.5950	0.000333	0.000338	0.000334	0.000311
	SWP	0.2335	0.2767	0.3685	0.5830	0.000338	0.000338	0.000326	0.000310
		0.1849	**0.2103**	**0.2771**	**0.5397**				
N4	GLS	0.2126	0.2440	0.3073	0.5890	0.000460	0.000483	0.000470	0.000474
	SWP	0.1873	0.2130	0.2799	0.5443	0.000432	0.000456	0.000440	0.000456

Table 5.27 (cont'd)

Parameters, Estimated Mean Slopes and Standard Errors for Each Predictor under Mixed-effect Models with Different Sample Sizes in Pattern II

Method		$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Condition 7: $R_2R_2R_1R_1$									
		0.5444	**0.3020**	**0.2313**	**0.1886**				
N1	GLS	0.5780	0.3348	0.2603	0.2094	0.000887	0.000978	0.009340	0.001014
	SWP	0.5437	0.3002	0.2299	0.1839	0.000814	0.000904	0.000883	0.000968
N2	GLS	0.5741	0.3341	0.2603	0.2099	0.000226	0.000254	0.000248	0.000272
	SWP	0.5446	0.3005	0.2310	0.1841	0.000218	0.000240	0.000244	0.000254
		0.5239	**0.2519**	**0.2028**	**0.1799**				
N3	GLS	0.5481	0.2810	0.2279	0.1952	0.000367	0.000413	0.000379	0.000378
	SWP	0.5244	0.2512	0.2026	0.1794	0.000347	0.000384	0.000373	0.000356
		0.5751	**0.3590**	**0.2678**	**0.2087**				
N4	GLS	0.5919	0.3744	0.2827	0.2294	0.000343	0.000384	0.000353	0.000472
	SWP	0.5737	0.3561	0.2672	0.1970	0.000329	0.000370	0.000344	0.000454
Condition 8: $R_4R_4R_3R_3$									
		0.2010	**0.2282**	**0.2844**	**0.5307**				
N1	GLS	0.2241	0.2571	0.3140	0.5645	0.001001	0.001028	0.001047	0.000963
	SWP	0.1985	0.2250	0.2806	0.5282	0.000940	0.000957	0.000949	0.000904
N2	GLS	0.2235	0.2561	0.3147	0.5619	0.000267	0.000274	0.000268	0.000252
	SWP	0.1992	0.2254	0.2824	0.5277	0.000258	0.000262	0.000258	0.000245
		0.1823	**0.2022**	**0.2483**	**0.5216**				
N3	GLS	0.2010	0.2251	0.2758	0.5420	0.000396	0.000400	0.000401	0.000365
	SWP	0.1825	0.2015	0.2479	0.5213	0.000383	0.000386	0.000383	0.000335
		0.2261	**0.2633**	**0.3346**	**0.5522**				
N4	GLS	0.2398	0.2806	0.3513	0.5837	0.000427	0.000453	0.000463	0.000421
	SWP	0.2209	0.2572	0.3310	0.5453	0.000415	0.000436	0.000427	0.000428

Note. The bold values were the parameter slopes for the condition.

Table 5.28

Parameters, Estimated Mean Slopes and Standard Errors for Each Predictor under Mixed-effect Models with Different Sample Sizes in Pattern III

Method		$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Condition 5: $R_1R_1R_2R_2$									
	0.5706	**0.6**	**0.4**	**0.3**	**0.25**				
N1	GLS	0.6067	0.3977	0.2981	0.2472	0.001321	0.001464	0.001446	0.001450
	SWP	0.6021	0.4014	0.3005	0.2491	0.001328	0.001499	0.001452	0.001464
N2	GLS	0.6006	0.3996	0.3005	0.2506	0.000351	0.000397	0.000401	0.000398
	SWP	0.6002	0.3999	0.3006	0.2508	0.000351	0.000402	0.000404	0.000399
N3	GLS	0.6007	0.3997	0.3002	0.2497	0.000375	0.000408	0.000406	0.000394
	SWP	0.6000	0.4003	0.3006	0.2500	0.000375	0.000412	0.000408	0.000394
N4	GLS	0.6023	0.4006	0.2990	0.2507	0.001283	0.001434	0.001498	0.001452
	SWP	0.6007	0.4008	0.2992	0.2508	0.001281	0.001464	0.001502	0.001459
Condition 6: $R_3R_3R_4R_4$									
		0.25	**0.3**	**0.4**	**0.6**				
N1	GLS	0.2545	0.3010	0.4013	0.5976	0.001407	0.001568	0.001519	0.001438
	SWP	0.2523	0.3012	0.4017	0.5986	0.001402	0.001572	0.001522	0.001449
N2	GLS	0.2505	0.3001	0.4004	0.6005	0.000369	0.000439	0.000434	0.000390
	SWP	0.2503	0.3001	0.4004	0.6006	0.000369	0.000439	0.000434	0.000393
N3	GLS	0.2501	0.2998	0.4006	0.5999	0.000383	0.000413	0.000422	0.000394
	SWP	0.2498	0.2999	0.4007	0.6000	0.000383	0.000413	0.000422	0.000393
N4	GLS	0.2511	0.3021	0.3987	0.6026	0.001375	0.001555	0.001589	0.001463
	SWP	0.2502	0.3018	0.3985	0.6026	0.001375	0.001557	0.001591	0.001486

Table 5.28 (cont'd)

Parameters, Estimated Mean Slopes and Standard Errors for Each Predictor under Mixed-effect Models with Different Sample Sizes in Pattern III

Method		$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Condition 7: $R_2R_2R_1R_1$									
		0.5161	**0.2253**	**0.1886**	**0.1734**				
N1	GLS	0.5243	0.2237	0.1875	0.1704	0.001248	0.001925	0.001810	0.001795
	SWP	0.5170	0.2263	0.1896	0.1723	0.001242	0.001947	0.001826	0.001818
N2	GLS	0.5171	0.2249	0.1891	0.1742	0.000340	0.000519	0.000511	0.000491
	SWP	0.5165	0.2251	0.1892	0.1744	0.000340	0.000520	0.000513	0.000492
N3	GLS	0.5169	0.2251	0.1890	0.1729	0.000398	0.000521	0.000506	0.000476
	SWP	0.5158	0.2254	0.1893	0.1732	0.000398	0.000523	0.000508	0.000477
N4	GLS	0.5191	0.2243	0.1860	0.1724	0.001099	0.001930	0.001902	0.001786
	SWP	0.5173	0.2250	0.1864	0.1728	0.001104	0.001940	0.001907	0.001797
Condition 8: $R_4R_4R_3R_3$									
		0.1734	**0.1886**	**0.2253**	**0.5161**				
N1	GLS	0.1789	0.1887	0.2269	0.5123	0.001531	0.001930	0.001866	0.001695
	SWP	0.1757	0.1891	0.2272	0.5138	0.001521	0.001933	0.001870	0.001706
N2	GLS	0.1741	0.1886	0.2256	0.5170	0.000408	0.000533	0.000540	0.000457
	SWP	0.1739	0.1886	0.2257	0.5170	0.000408	0.000253	0.000540	0.000459
N3	GLS	0.1735	0.1883	0.2259	0.5159	0.000440	0.000505	0.000521	0.000452
	SWP	0.1731	0.1883	0.2260	0.5161	0.000440	0.000505	0.000521	0.000451
N4	GLS	0.1746	0.1893	0.2221	0.5170	0.001423	0.001923	0.001987	0.001705
	SWP	0.1736	0.1894	0.2223	0.5176	0.001424	0.001925	0.001994	0.001728

Note. The bold values were the parameter slopes for the condition.

Table 5.29

Parameters, Estimated Mean Slopes and Standard Errors for Each Predictor under Mixed-effect Models with Different Sample Sizes in Pattern IV

Method		$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Condition 5: $R_1R_1R_2R_2$									
	0.5706	**0.5484**	**0.3128**	**0.2491**	**0.2500**				
N1	GLS	0.5779	0.3475	0.2760	0.2635	0.001017	0.001040	0.001026	0.001474
	SWP	0.5577	0.3304	0.2592	0.3075	0.001090	0.001078	0.001071	0.001766
N2	GLS	0.5756	0.3457	0.2752	0.2697	0.000260	0.000271	0.000269	0.000404
	SWP	0.5588	0.3291	0.2591	0.3047	0.000274	0.000289	0.000286	0.000463
		0.5787	**0.3668**	**0.2799**	**0.2500**				
N3	GLS	0.5923	0.3811	0.2922	0.2538	0.000322	0.000360	0.000341	0.000398
	SWP	0.5824	0.3725	0.2837	0.2709	0.000327	0.000362	0.000346	0.000428
		0.5273	**0.2640**	**0.2241**	**0.2500**				
N4	GLS	0.5532	0.3050	0.2526	0.3023	0.000754	0.000796	0.000758	0.001600
	SWP	0.5441	0.2956	0.2417	0.3399	0.000925	0.000969	0.000884	0.000193
Condition 6: $R_3R_3R_4R_4$									
		0.2123	**0.2563**	**0.3702**	**0.6000**				
N1	GLS	0.2363	0.2848	0.3918	0.6098	0.001303	0.001277	0.001323	0.001161
	SWP	0.2322	0.2810	0.3864	0.6377	0.001355	0.001328	0.001367	0.001352
N2	GLS	0.2364	0.2821	0.3889	0.6150	0.000336	0.000345	0.000352	0.000311
	SWP	0.2335	0.2799	0.3868	0.6332	0.000347	0.000358	0.000366	0.000357
		0.2355	**0.2833**	**0.3882**	**0.6000**				
N3	GLS	0.2453	0.2941	0.3968	0.6033	0.000379	0.000396	0.000384	0.000358
	SWP	0.2432	0.2923	0.3947	0.6120	0.000383	0.000400	0.000386	0.000383
		0.1921	**0.2325**	**0.3555**	**0.6000**				
N4	GLS	0.2273	0.2727	0.3821	0.6337	0.001149	0.001196	0.001194	0.001045
	SWP	0.2264	0.2731	0.3818	0.6596	0.001244	0.001297	0.001280	0.001328

Table 5.29 (cont'd)
Parameters, Estimated Mean Slopes and Standard Errors for Each Predictor under Mixed-effect Models with Different Sample Sizes in Pattern IV

Method		$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Condition 7: $R_2R_2R_1R_1$									
		0.5433	**0.2992**	**0.2266**	**0.1589**				
N1	GLS	0.5717	0.3246	0.2508	0.1368	0.000932	0.001009	0.001026	0.001495
	SWP	0.5433	0.2967	0.2251	0.1565	0.009080	0.001002	0.001000	0.001688
N2	GLS	0.5681	0.3218	0.2486	0.1412	0.000230	0.000262	0.000262	0.000397
	SWP	0.5445	0.2966	0.2259	0.1561	0.000231	0.000255	0.000260	0.000435
		0.5235	**0.2508**	**0.2008**	**0.1686**				
N3	GLS	0.5419	0.2704	0.2184	0.1559	0.000355	0.000424	0.000405	0.000444
	SWP	0.5239	0.2498	0.2009	0.1674	0.000346	0.000399	0.000391	0.000466
		0.5723	**0.3529**	**0.2583**	**0.1474**				
N4	GLS	0.5861	0.3621	0.2692	0.1339	0.000490	0.000551	0.000556	0.001441
	SWP	0.5730	0.3493	0.2571	0.1430	0.000507	0.000575	0.000576	0.001536
Condition 8: $R_4R_4R_3R_3$									
		0.1961	**0.2161**	**0.2473**	**0.5068**				
N1	GLS	0.2019	0.2238	0.2559	0.4935	0.001355	0.001343	0.001422	0.001628
	SWP	0.1929	0.2130	0.2409	0.5077	0.001346	0.001344	0.001409	0.001646
N2	GLS	0.2006	0.2193	0.2501	0.4979	0.000345	0.000355	0.000370	0.000424
	SWP	0.1944	0.2128	0.2428	0.5039	0.000347	0.000355	0.000370	0.000427
		0.1809	**0.1977**	**0.2325**	**0.5131**				
N3	GLS	0.1846	0.2015	0.2367	0.5071	0.000428	0.000454	0.000450	0.000444
	SWP	0.1802	0.1968	0.2313	0.5116	0.000428	0.000446	0.000443	0.000442
		0.2140	**0.2373**	**0.2650**	**0.4993**				
N4	GLS	0.2140	0.2373	0.2629	0.4919	0.001064	0.001123	0.001182	0.001588
	SWP	0.2097	0.2326	0.2560	0.4994	0.001089	0.001146	0.001206	0.001614

Note. The bold values were the parameter slopes for the condition.

Table 5.30
Parameters, Estimated Mean Slopes and Standard Errors for Each Predictor under Mixed-effect Models with Different Sample Sizes in Pattern V

	Method	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Condition 5: $R_1R_1R_2R_2$									
	0.5706	**0.5452**	**0.3041**	**0.2347**	**0.2036**				
N1	GLS	0.5803	0.3415	0.2670	0.2292	0.000862	0.000915	0.000881	0.000887
	SWP	0.5455	0.3034	0.2337	0.2022	0.000814	0.000874	0.000852	0.000867
N2	GLS	0.5762	0.3404	0.2676	0.2290	0.000225	0.000244	0.000237	0.000227
	SWP	0.5455	0.3038	0.2346	0.2035	0.000226	0.000239	0.000236	0.000229
		0.5769	**0.3629**	**0.2739**	**0.2311**				
N3	GLS	0.5929	0.3793	0.2897	0.2435	0.000322	0.000346	0.000308	0.000310
	SWP	0.5771	0.3623	0.2738	0.2311	0.000316	0.000339	0.000309	0.000317
		0.5239	**0.2523**	**0.2033**	**0.1827**				
N4	GLS	0.5487	0.2838	0.2305	0.2037	0.000372	0.000416	0.000372	0.000375
	SWP	0.5243	0.2518	0.2034	0.1827	0.000350	0.000384	0.000357	0.000350
Condition 6: $R_3R_3R_4R_4$									
		0.2036	**0.2347**	**0.3041**	**0.5452**				
N1	GLS	0.2303	0.2687	0.3409	0.5792	0.000892	0.000911	0.000911	0.000872
	SWP	0.2040	0.2342	0.3031	0.5436	0.000886	0.000877	0.000848	0.000817
N2	GLS	0.2293	0.2675	0.3406	0.5757	0.000233	0.000234	0.000242	0.000218
	SWP	0.2038	0.2345	0.3039	0.5451	0.000243	0.000239	0.000240	0.000213
		0.2311	**0.2739**	**0.3629**	**0.5769**				
N3	GLS	0.2435	0.2892	0.3798	0.5927	0.000337	0.000334	0.000331	0.000311
	SWP	0.2312	0.2734	0.3629	0.5769	0.000341	0.000334	0.000319	0.000306
		0.1827	**0.2033**	**0.2523**	**0.5239**				
N4	GLS	0.2034	0.2300	0.2846	0.5490	0.000400	0.000401	0.000387	0.000358
	SWP	0.1830	0.2029	0.2523	0.5239	0.000384	0.000382	0.000366	0.000329

Table 5.30 (cont'd)
Parameters, Estimated Mean Slopes and Standard Errors for Each Predictor under Mixed-effect Models with Different Sample Sizes in Pattern V

Method		$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Condition 7: $R_2R_2R_1R_1$									
		0.5452	**0.3041**	**0.2347**	**0.2036**				
N1	GLS	0.5808	0.3411	0.2672	0.2281	0.000872	0.000934	0.000896	0.000887
	SWP	0.5454	0.3029	0.2335	0.2022	0.000802	0.000876	0.000849	0.000853
N2	GLS	0.5760	0.3403	0.2674	0.2291	0.000234	0.000250	0.000229	0.000229
	SWP	0.5454	0.3038	0.2346	0.2035	0.000226	0.000240	0.000235	0.000229
		0.5239	**0.2523**	**0.2033**	**0.1827**				
N3	GLS	0.5495	0.2841	0.2308	0.2037	0.000373	0.000412	0.000377	0.000376
	SWP	0.5244	0.2517	0.2032	0.1826	0.000350	0.000379	0.000358	0.000351
		0.5769	**0.3629**	**0.2739**	**0.2311**				
N4	GLS	0.5937	0.3795	0.2899	0.2434	0.000318	0.000343	0.000310	0.000310
	SWP	0.5772	0.3622	0.2737	0.2309	0.000315	0.000334	0.000312	0.000316
Condition 8: $R_4R_4R_3R_3$									
		0.2036	**0.2347**	**0.3041**	**0.5452**				
N1	GLS	0.2304	0.2680	0.3401	0.5784	0.000897	0.000899	0.000895	0.000875
	SWP	0.2038	0.2338	0.3028	0.5440	0.000867	0.000878	0.000834	0.000809
N2	GLS	0.2293	0.2676	0.3406	0.5758	0.000243	0.000242	0.000244	0.000221
	SWP	0.2038	0.2346	0.3040	0.5451	0.000239	0.000238	0.000241	0.000212
		0.1827	**0.2033**	**0.2523**	**0.5239**				
N3	GLS	0.2041	0.2304	0.2848	0.5490	0.000402	0.000394	0.000394	0.000362
	SWP	0.1830	0.2027	0.2521	0.5240	0.000383	0.000380	0.000368	0.000329
		0.2311	**0.2739**	**0.3629**	**0.5769**				
N4	GLS	0.2438	0.2894	0.3800	0.5926	0.000334	0.000332	0.000329	0.000309
	SWP	0.2312	0.2732	0.3627	0.5769	0.000338	0.000332	0.000321	0.000305

Note. The bold values were the parameter slopes for the condition.

CHAPTER 6

DISCUSSION

This chapter summarizes the major findings based on the two methods investigated in this research, and compares the two methods in a more general way. Suggestions for choosing from the two methods are provided, as well as the limitations and further investigations of the current study.

This research extends the factored likelihood method through the sweep operator (SWP), which was originally designed for handling missing data, to the meta-analysis context. The results from the SWP method were compared to the results from the GLS method, which is a typical procedure for synthesizing multivariate data in meta-analysis. The major difference between the two methods is that the SWP utilizes the concept of maximum likelihood while GLS is not a likelihood-based approach and focuses on weighting the correlations by their variability. Exploring the SWP method provides another point of view and possible way to deal with the missing information that often occurs in meta-analyses.

In the current study, the correlation matrices from regression studies were combined in order to obtain the synthesized standardized slopes as a summary of the included regression models in the synthesis. The two methods investigated in this study allow the information from regression studies to be combined with correlational studies, which can be considered as simple regression studies. Being able to incorporating regression studies with correlational studies helps to improve the understanding of the relationship found in correlational studies, because more variables are held constant in regression studies than in correlational studies while exploring the relationship between the outcome and the predictor.

As the results presented in Chapter 5 show, each of the GLS and SWP methods has its own strength in synthesizing regression studies with different patterns of missing data, missing rates, and differences in what was missed (in terms of the strength of the correlations remaining in the matrix). The methods were first examined assuming fixed effects (Condition 1 through Condition 4), when all the four studies included in a meta-analysis were based on the same population correlation matrix. The major finding assuming fixed effects across studies was that SWP consistently performed slightly better than GLS when estimating the slope of the variable that was present in all regression models (X_1), while GLS consistently overestimated it in all the five missing patterns with different sample sizes. The empirical examination using the pseudo studies in Chapter 4 also confirmed this finding. This result makes SWP a more desirable method especially when a researcher's focus is on the relationship between the outcome and one specific predictor. In that case, the bivariate relationship of interest can be adjusted appropriately by other variables that were controlled in the regression studies when using SWP.

The estimated slopes obtained from two methods were very close to the population values, indicating that both methods produced good estimates of the slopes for the final model. There were a few tendencies found for the two methods in terms of the impact of the study patterns, sample sizes, and the strength of correlations that were missing. For example, SWP tended to perform better in estimating β_2 when the sample size was small and equal across studies (*N*1) in all patterns; GLS tended to perform better on estimating β_2 when the sample size was large and equal across studies (*N*2). When more missing data occurred ($N4$), SWP tended to produce more precise estimates of the slope for X_3 in all patterns, no matter what correlation matrix the studies were based on; GLS tended to perform better on the same variable when the sample size was large and equal across studies ($N2$). When there were less missing data within studies ($N3$), SWP tended to estimate the slope for X_4 better, while GLS tended to do better with more data missing ($N4$) on this variable.

The percentage relative biases were calculated to quantify the differences between estimated slopes and the population slopes. In all the scenarios of simulation, both methods produced the bias under 5%, which made the two methods good estimation methods (Hoogland & Boomsma, 1998). Yet the ranges of bias from GLS were consistently larger than the ranges from SWP which made GLS less desirable.
The largest positive bias values for estimating the slope for X_1 produced by both methods were both under Pattern III with the sample size $N1$ and correlation matrix $\mathbf{R}_3$. This indicates that when the sample size was small and equal across studies and when important variables were missing more, comparing to missingness for less important variables, both methods did not do as well as in other scenarios for estimating the X_1 slope. On the other hand, SWP did very well (zero bias) in estimating B_1 in this pattern with the sample size $N3$ and the correlation matrix $\mathbf{R}_2$. That implies when there were mostly correlational studies but only a few regression studies with big sample sizes included in the meta-analysis, SWP can estimate the slope for the most observed variable very well, especially when the predictors that were missing from the correlational studies were related to each other and the outcome less strongly and when there are no intercorrelations among the predictors. Another summary and comparison of the results

from two methods can be found in Appendix E.

In the factorial ANOVAs, relatively little of the variance of the differences between slopes estimated by the two methods were explained by the patterns, the correlation matrices, and the sample size sets. In all cases, less than 10 percent of the variability is explained by all the factors. For X_1, which is of the most of interest, the sample sizes seemed to be the most important factor for explaining the variation of the differences and patterns seemed to be the least important factor. The implication for this finding is that when the researcher is making the decision of the method to be used, the most important thing to keep in mind is the sample sizes of the primary studies included in the synthesis. S Ordinal interactions existed in the current analyses for X_1. The interaction plots showed that, when separating by patterns, GLS and SWP produced more different estimates of the X_1 slope in sample size set $N1$ than in other sample size sets. Combining this result with the previous finding that SWP consistently produced closer slopes on X_1, the SWP method is especially preferred when the sample sizes are small and equal across studies, no matter what the correlation matrix is.

The two methods were also examined under mixed-effects models (Condition 5 through Condition 8). By assuming mixed effects, the relationships among variables in the four studies in the synthesis were not all based on the same population correlation matrix. Since methods for meta-analyzing multivariate data under this model have not been well developed (due to the difficulty of estimating between-study variances with multivariate data appropriately), the population slopes calculated for each scenario were based on the weighted mean correlations using the existing correlations in the current research. As a consequence, the estimates from SWP showed lower relative percentage bias most of the time, because SWP depends more on the weighted mean correlations at the beginning of the calculations than does the GLS procedure.

When comparing the results for the fully observed predictor X_1 under the mixed-effects model to the results of same scenarios but under a fixed-effects model, both methods showed the largest negative differences (mixed-effect results minus fixed-effects results) in Condition 8 with sample size equaled $N1$ and the largest positive differences in Condition 7 with the sample size defined by $N3$ in Pattern III. This finding indicates that when the important variables (e.g., the X_4 the correlation matrices $\mathbf{R}_3$ and $\mathbf{R}_4$ in this research) were more likely to be missing (e.g., X_4 is missing in study 1 to study 3 in pattern III), the estimate of the slope of the fully observed variable can be very different (when using either methods) under fixed- and mixed- effects conditions. More investigations on producing appropriate estimates under non-fixed effect models will be needed.

As all research has limitations, this study is constrained in several ways. First, using the factored likelihood estimation through the sweep operator requires the predictors included in the models in the synthesis to be arranged in a monotone pattern somewhat like those shown in Figure 3.2. Those patterns help to obtain the maximum likelihood of the correlations without an iterative process, which made SWP an easy method to use. The desired pattern sometimes can be achieved by rearranging the order of the predictors in the models. Or, some variables may have to be excluded from models in order to obtain the desired pattern. The GLS method, on the contrary, is more flexible in this matter, and can be used with any correlations that are available in the studies. Other methods for handling missing data that might be useful for synthesizing regression studies in the meta-analysis context, such as multiple imputations, might be worth while to investigate, since combining regression studies is somewhat similar to dealing with missing information from primary studies.

Second, the correlations used in the synthesis from the primary studies were assumed to be perfectly measured in the current studies. That is, the errors from the instruments used to measure the variables in the regression models were not taken into account. I made this simplification because I wanted to focus on eliminating the impact of the unparallel situations that occurs when regression models do not contain all the same predictors. Further research should investigate possible solutions, such as utilizing the concept of structural equation modeling, to incorporate measurement errors in meta-analysis.

Third, the correlations among the variables in the regression studies are required in order to use the methods investigated in this research. Unfortunately, it is very likely the information about the zero-order correlations may not be reported or may be only partially reported. In this matter, Bayesian perspectives might provide a possible direction for obtaining the correlations needed for synthesizing regression studies based on other information, such as slopes reported in the regression study. A possible method is to use the Gibbs Sampler (Casella & George, 1992; Gelfand & Smith, 1990), that is based on elementary properties of Markov chains, to generate possible correlations based on the observed distribution of the slopes of regression models.

APPENDIX A:

SAS Macro for GLS

Below is the example of SAS macro for generating the data under Pattern I for four sample sizes and four correlations and calculating the standardized slopes and standard errors using GLS.

****

N= Sample size set for four studies ($N1$ through $N4$);

R= Correlation matrix $\mathbf{R}_1$ through $\mathbf{R}_5$;

n_k=sample size for study k in a synthesis, k=1 to 4;

$r_{ii'}$=correlation between variables i, i=y,1,2,3,or 4; $i \neq i'$.

****

```
Libname GLSp1 'C:\';
%Macro GLSp1(N,R,n1,n2,n3,n4,ry1,ry2,ry3,ry4,r12,r13,r14,r23,r24,r34);
Title GLS PATTERN1 &N &R ;
Proc IML;
nseed=125;nrep=1000;
Pat1=j(nrep,8,0);
do sim=1 to nrep;
    s1=j(&n1,2,0);
        do i=1 to &n1;
        s1[i,1]=rannor(nseed);
        s1[i,2]=rannor(nseed);
        end;
    s1r={1 &ry1,&ry1 1};
    co1=root(s1r);
    z1=s1*co1;
    r1=corr(z1);

    s2=j(&n2,3,0);
        do i=1 to &n2;
        s2[i,1]=rannor(nseed);
        s2[i,2]=rannor(nseed);
        s2[i,3]=rannor(nseed);
        end;
    s2r={1 &ry1 &ry2,&ry1 1 &r12,&ry2 &r12 1};
    co2=root(s2r);
    z2=s2*co2;
    r2=corr(z2);

    s3=j(&n3,4,0);
        do i=1 to &n3;
        s3[i,1]=rannor(nseed);
        s3[i,2]=rannor(nseed);
```

```
        s3[i,3]=rannor(nseed);
        s3[i,4]=rannor(nseed);
        end;
    s3r={1 &ry1 &ry2 &ry3,
         &ry1 1 &r12 &r13,
         &ry2 &r12 1 &r23,
         &ry3 &r13 &r23 1};
    co3=root(s3r);
    z3=s3*co3;
    r3=corr(z3);

    s4=j(&n4,5,0);
        do i=1 to &n4;
        s4[i,1]=rannor(nseed);
        s4[i,2]=rannor(nseed);
        s4[i,3]=rannor(nseed);
        s4[i,4]=rannor(nseed);
        s4[i,5]=rannor(nseed);
        end;
    s4r={1 &ry1 &ry2 &ry3 &ry4,
            &ry1 1 &r12 &r13 &r14,
            &ry2 &r12 1 &r23 &r24,
            &ry3 &r13 &r23 1 &r34,
            &ry4 &r14 &r24 &r34 1};
    co4=root(s4r);
    z4=s4*co4;
    r4=corr(z4);

r1_y1=r1[1,2];
r2_y1=r2[1,2];r2_y2=r2[1,3];r2_12=r2[2,3];
r3_y1=r3[1,2];r3_y2=r3[1,3];r3_y3=r3[1,4];r3_12=r3[2,3];r3_13=r3[2,4];r
3_23=r3[3,4];
r4_y1=r4[1,2];r4_y2=r4[1,3];r4_y3=r4[1,4];r4_y4=r4[1,5];r4_12=r4[2,3];r
4_13=r4[2,4];r4_14=r4[2,5];r4_23=r4[3,4];r4_24=r4[3,5];r4_34=r4[4,5];
***********************************************************************
*****************************************;
p1=ncol(r1);
dim1=p1*(p1-1)/2;
cov1=j(dim1,dim1,0);
mat=j(dim1,2,0);
    k=1;
    do i=2 to p1;
        do j=1 to (i-1);
            cov1[k,k]=(1-r1[i,j]##2)##2/&n1;
            mat[k,1]=i;
            mat[k,2]=j;
            k=k+1;
        end;
    end;
    do i=2 to dim1;
        do j=1 to (i-1);
            s=mat[i,1];
            t=mat[i,2];
            u=mat[j,1];
            v=mat[j,2];

    cov1[i,j]=(0.5*r1[s,t]*r1[u,v]*(r1[s,u]##2+r1[s,v]##2+r1[t,u]##2+r1
```

```
[t,v]##2)+r1[s,u]*r1[t,v]+r1[s,v]*r1[t,u]-(r1[s,t]*r1[s,u]*r1[s,v]+r1[t
,s]*r1[t,u]*r1[t,v]+r1[u,s]*r1[u,t]*r1[u,v]+r1[v,s]*r1[v,t]*r1[v,u]))/&
n1;
          end;
     end;
     do i=2 to dim1;
          do j=1 to (i-1);
          cov1[j,i]=cov1[i,j];
          end;
     end;

p2=ncol(r2);
dim2=p2*(p2-1)/2;
cov2=j(dim2,dim2,0);
mat=j(dim2,2,0);
     k=1;
     do i=2 to p2;
          do j=1 to (i-1);
               cov2[k,k]=(1-r2[i,j]##2)##2/&n2;
               mat[k,1]=i;
               mat[k,2]=j;
               k=k+1;
          end;
     end;
     do i=2 to dim2;
          do j=1 to (i-1);
               s=mat[i,1];
               t=mat[i,2];
               u=mat[j,1];
               v=mat[j,2];

     cov2[i,j]=(0.5*r2[s,t]*r2[u,v]*(r2[s,u]##2+r2[s,v]##2+r2[t,u]##2+r2
[t,v]##2)+r2[s,u]*r2[t,v]+r2[s,v]*r2[t,u]-(r2[s,t]*r2[s,u]*r2[s,v]+r2[t
,s]*r2[t,u]*r2[t,v]+r2[u,s]*r2[u,t]*r2[u,v]+r2[v,s]*r2[v,t]*r2[v,u]))/&
n2;
          end;
     end;
     do i=2 to dim2;
          do j=1 to (i-1);
          cov2[j,i]=cov2[i,j];
          end;
     end;

p3=ncol(r3);
dim3=p3*(p3-1)/2;
cov3=j(dim3,dim3,0);
mat=j(dim3,2,0);
     k=1;
     do i=2 to p3;
          do j=1 to (i-1);
               cov3[k,k]=(1-r3[i,j]##2)##2/&n3;
               mat[k,1]=i;
               mat[k,2]=j;
               k=k+1;
          end;
     end;
     do i=2 to dim3;
```

```
        do j=1 to (i-1);
            s=mat[i,1];
            t=mat[i,2];
            u=mat[j,1];
            v=mat[j,2];

    cov3[i,j]=(0.5*r3[s,t]*r3[u,v]*(r3[s,u]##2+r3[s,v]##2+r3[t,u]##2+r3
[t,v]##2)+r3[s,u]*r3[t,v]+r3[s,v]*r3[t,u]-(r3[s,t]*r3[s,u]*r3[s,v]+r3[t
,s]*r3[t,u]*r3[t,v]+r3[u,s]*r3[u,t]*r3[u,v]+r3[v,s]*r3[v,t]*r3[v,u]))/&
n3;
        end;
    end;
    do i=2 to dim3;
        do j=1 to (i-1);
        cov3[j,i]=cov3[i,j];
        end;
    end;

p4=ncol(r4);
dim4=p4*(p4-1)/2;
cov4=j(dim4,dim4,0);
mat=j(dim4,2,0);
    k=1;
    do i=2 to p4;
        do j=1 to (i-1);
            cov4[k,k]=(1-r4[i,j]##2)##2/&n4;
            mat[k,1]=i;
            mat[k,2]=j;
            k=k+1;
        end;
    end;
    do i=2 to dim4;
        do j=1 to (i-1);
            s=mat[i,1];
            t=mat[i,2];
            u=mat[j,1];
            v=mat[j,2];

    cov4[i,j]=(0.5*r4[s,t]*r4[u,v]*(r4[s,u]##2+r4[s,v]##2+r4[t,u]##2+r4
[t,v]##2)+r4[s,u]*r4[t,v]+r4[s,v]*r4[t,u]-(r4[s,t]*r4[s,u]*r4[s,v]+r4[t
,s]*r4[t,u]*r4[t,v]+r4[u,s]*r4[u,t]*r4[u,v]+r4[v,s]*r4[v,t]*r4[v,u]))/&
n4;
        end;
    end;
    do i=2 to dim4;
        do j=1 to (i-1);
        cov4[j,i]=cov4[i,j];
        end;
    end;

p=dim1+dim2+dim3+dim4;
bigmtx=j(p,p,0);
bigmtx[1:dim1,1:dim1]=cov1;
bigmtx[dim1+1:dim1+dim2,dim1+1:dim1+dim2]=cov2;
bigmtx[dim1+dim2+1:dim1+dim2+dim3,dim1+dim2+1:dim1+dim2+dim3]=cov3;
bigmtx[dim1+dim2+dim3+1:dim1+dim2+dim3+dim4,dim1+dim2+dim3+1:dim1+dim2+
dim3+dim4]=cov4;
```

```
rvec1=r1_y1;

Start rvec2;
    k=1;
    rvec2=j(dim2,1,0);
    do i=2 to p2;
        do j=1 to (i-1);
        rvec2[k]=r2[i,j];
        k=k+1;
        end;
    end;
 finish;
 run rvec2;

Start rvec3;
    k=1;
    rvec3=j(dim3,1,0);
    do i=2 to p3;
        do j=1 to (i-1);
        rvec3[k]=r3[i,j];
        k=k+1;
        end;
    end;
 finish;
 run rvec3;

Start rvec4;
    k=1;
    rvec4=j(dim4,1,0);
    do i=2 to p4;
        do j=1 to (i-1);
        rvec4[k]=r4[i,j];
        k=k+1;
        end;
    end;
 finish;
 run rvec4;

outcome=rvec1//rvec2//rvec3//rvec4;

w=j(p,10,0);
w[1,1]=1;
w[2,1]=1;
w[3,2]=1;
w[4,3]=1;
w[5,1]=1;
w[6,2]=1;
w[7,3]=1;
w[8,4]=1;
w[9,5]=1;
w[10,6]=1;
w[11,1]=1;
w[12,2]=1;
w[13,3]=1;
w[14,4]=1;
w[15,5]=1;
```

```
w[16,6]=1;
w[17,7]=1;
w[18,8]=1;
w[19,9]=1;
w[20,10]=1;
rho=inv(t(w)*inv(bigmtx)*w)*t(w)*inv(bigmtx)*outcome;

Start backmtx;
syncor=j(5,5,1);
k=1;
do i=2 to 5;
    do j=1 to (i-1);
    syncor[i,j]=rho[k];
    syncor[j,i]=rho[k];
    k=k+1;
    end;
end;
finish;
run backmtx;

R11=syncor[2:5,2:5];
R12=syncor[2:5,1];
SLOPE=inv(R11)*R12;

Pat1[sim,1]=&n1;
Pat1[sim,2]=&n2;
Pat1[sim,3]=&n3;
Pat1[sim,4]=&n4;
Pat1[sim,5]=SLOPE[1,1];
Pat1[sim,6]=SLOPE[2,1];
Pat1[sim,7]=SLOPE[3,1];
Pat1[sim,8]=SLOPE[4,1];
end;
Create GLSp1.GLSPAT1&N&R from Pat1 [colname={n1 n2 n3 n4 x1 x2 x3 x4}]; Append
from Pat1;
run;
quit;

%Mend GLSp1;

/*R1*/
%GLSp1(N1,R1,150,150,150,150,0.6,0.4,0.3,0.25,0.25,0.1,0.05,0.15,0.1,0.
15);
%GLSp1(N2,R1,2000,2000,2000,2000,0.6,0.4,0.3,0.25,0.25,0.1,0.05,0.15,0.
1,0.15);
%GLSp1(N3,R1,150,500,1000,2000,0.6,0.4,0.3,0.25,0.25,0.1,0.05,0.15,0.1,
0.15);
%GLSp1(N4,R1,2000,1000,500,150,0.6,0.4,0.3,0.25,0.25,0.1,0.05,0.15,0.1,
0.15);

/*R2*/
%GLSp1(N1,R2,150,150,150,150,0.6,0.4,0.3,0.25,0,0,0,0,0,0);
%GLSp1(N2,R2,2000,2000,2000,2000,0.6,0.4,0.3,0.25,0,0,0,0,0,0);
%GLSp1(N3,R2,150,500,1000,2000,0.6,0.4,0.3,0.25,0,0,0,0,0,0);
%GLSp1(N4,R2,2000,1000,500,150,0.6,0.4,0.3,0.25,0,0,0,0,0,0);

/*R3*/
```

```
%GLSp1(N1,R3,150,150,150,150,0.25,0.3,0.4,0.6,0.15,0.1,0.05,0.15,0.1,0.
25);
%GLSp1(N2,R3,2000,2000,2000,2000,0.25,0.3,0.4,0.6,0.15,0.1,0.05,0.15,0.
1,0.25);
%GLSp1(N3,R3,150,500,1000,2000,0.25,0.3,0.4,0.6,0.15,0.1,0.05,0.15,0.1,
0.25);
%GLSp1(N4,R3,2000,1000,500,150,0.25,0.3,0.4,0.6,0.15,0.1,0.05,0.15,0.1,
0.25);

/*R4*/
%GLSp1(N1,R4,150,150,150,150,0.25,0.3,0.4,0.6,0,0,0,0,0,0);
%GLSp1(N2,R4,2000,2000,2000,2000,0.25,0.3,0.4,0.6,0,0,0,0,0,0);
%GLSp1(N3,R4,150,500,1000,2000,0.25,0.3,0.4,0.6,0,0,0,0,0,0);
%GLSp1(N4,R4,2000,1000,500,150,0.25,0.3,0.4,0.6,0,0,0,0,0,0);

quit;
```

APPENDIX B:

SAS Macro for SWP

Below is the example of SAS macro for generating the data under Pattern I for four sample sizes and four correlations and calculating the standardized slopes and standard errors using SWP. Among the five patterns studied in this research, Pattern I has the most complicated codes because of the numbers of steps the calculation needed to be carried out.

****

N= Sample size set for four studies (*N*1 through *N*4);

R= Correlation matrix $\mathbf{R}_1$ through $\mathbf{R}_5$;

n_k=sample size for study k in a synthesis, k=1 to 4;

$r_{ii'}$=correlation between variables *i*, *i*=*y*,1,2,3,or 4; *i*≠*i*'.

****

```
Libname SWPp1 'C:\';
%Macro
pattern1(N,R,n1,n2,n3,n4,ry1,ry2,ry3,ry4,r12,r13,r14,r23,r24,r34);
Title PATTERN1 &N &R ;
Proc IML;
nseed=125;nrep=1000;
Pat1=j(nrep,8,0);
do sim=1 to nrep;
    s1=j(&n1,2,0);
        do i=1 to &n1;
        s1[i,1]=rannor(nseed);
        s1[i,2]=rannor(nseed);
        end;
    s1r={1 &ry1,&ry1 1};
    co1=root(s1r);
    z1=s1*co1;
    r1=corr(z1);
    /*print r1;*/
    s2=j(&n2,3,0);
        do i=1 to &n2;
        s2[i,1]=rannor(nseed);
        s2[i,2]=rannor(nseed);
        s2[i,3]=rannor(nseed);
        end;
    s2r={1 &ry1 &ry2,&ry1 1 &r12,&ry2 &r12 1};
```

```
    co2=root(s2r);
    z2=s2*co2;
    r2=corr(z2);
    s3=j(&n3,4,0);
        do i=1 to &n3;
        s3[i,1]=rannor(nseed);
        s3[i,2]=rannor(nseed);
        s3[i,3]=rannor(nseed);
        s3[i,4]=rannor(nseed);
        end;
    s3r={1 &ry1 &ry2 &ry3,
         &ry1 1 &r12 &r13,
         &ry2 &r12 1 &r23,
         &ry3 &r13 &r23 1};
    co3=root(s3r);
    z3=s3*co3;
    r3=corr(z3);
    s4=j(&n4,5,0);
        do i=1 to &n4;
        s4[i,1]=rannor(nseed);
        s4[i,2]=rannor(nseed);
        s4[i,3]=rannor(nseed);
        s4[i,4]=rannor(nseed);
        s4[i,5]=rannor(nseed);
        end;
    s4r={1 &ry1 &ry2 &ry3 &ry4,
             &ry1 1 &r12 &r13 &r14,
             &ry2 &r12 1 &r23 &r24,
             &ry3 &r13 &r23 1 &r34,
             &ry4 &r14 &r24 &r34 1};
    co4=root(s4r);
    z4=s4*co4;
    r4=corr(z4);

r1_y1=r1[1,2];
r2_y1=r2[1,2];r2_y2=r2[1,3];r2_12=r2[2,3];
r3_y1=r3[1,2];r3_y2=r3[1,3];r3_y3=r3[1,4];r3_12=r3[2,3];r3_13=r3[2,4];r
3_23=r3[3,4];
r4_y1=r4[1,2];r4_y2=r4[1,3];r4_y3=r4[1,4];r4_y4=r4[1,5];r4_12=r4[2,3];r
4_13=r4[2,4];r4_14=r4[2,5];r4_23=r4[3,4];r4_24=r4[3,5];r4_34=r4[4,5];
*********************************************************************
;
AVEy1=(r1_y1*&n1+r2_y1*&n2+r3_y1*&n3+r4_y1*&n4)/(&n1+&n2+&n3+&n4);
O=j(2, 2, 1);
O[1,2]=AVEy1;
O[2,1]=AVEy1;
AVEy1=(r2_y1*&n2+r3_y1*&n3+r4_y1*&n4)/(&n2+&n3+&n4);
AVEy2=(r2_y2*&n2+r3_y2*&n3+r4_y2*&n4)/(&n2+&n3+&n4);
AVE12=(r2_12*&n2+r3_12*&n3+r4_12*&n4)/(&n2+&n3+&n4);
S234=j(3,3,1);
S234[1,2]=AVEy1;
S234[2,1]=S234[1,2];
S234[1,3]=AVEy2;
S234[3,1]=S234[1,3];
S234[2,3]=AVE12;
S234[3,2]=S234[2,3];
R11=S234[1:2,1:2];
```

```
R12=S234[1:2,3];
slope234=inv(R11)*R12;
var234=1-t(slope234)*R12;

A_y=j(2,2,1);
A_y[1,1]=-1/O[1,1];
A_y[1,2]=O[1,2]/O[1,1];
A_y[2,2]=O[2,2]-O[1,2]*O[1,2]/O[1,1];
A_y[2,1]=A_y[1,2];
A=j(2,2,1);
A[2,2]=-1/A_y[2,2];
A[1,2]=A_y[1,2]/A_y[2,2];
A[1,1]=A_y[1,1]-A_y[1,2]*A_y[1,2]/A_y[2,2];
A[2,1]=A_y[1,2];
P=j(3,3,1);
P[1:2,1:2]=A;
P[1:2,3]=slope234[1:2,1];
P[3,1:2]=T(slope234[1:2,1]);
P[3,3]=var234[1];

AVEy1=(r3_y1*&n3+r4_y1*&n4)/(&n3+&n4);
AVEy2=(r3_y2*&n3+r4_y2*&n4)/(&n3+&n4);
AVEy3=(r3_y3*&n3+r4_y3*&n4)/(&n3+&n4);
AVE12=(r3_12*&n3+r4_12*&n4)/(&n3+&n4);
AVE13=(r3_13*&n3+r4_13*&n4)/(&n3+&n4);
AVE23=(r3_23*&n3+r4_23*&n4)/(&n3+&n4);
Vec=j(6,1,1);
Vec[1,1]=AVEy1;
Vec[2,1]=AVEy2;
Vec[3,1]=AVE12;
Vec[4,1]=AVEy3;
Vec[5,1]=AVE13;
Vec[6,1]=AVE23;
    Start matrix;
    S34=j(4,4,1);
    k=1;
    do i = 2 to 4;
        do j = 1 to (i-1);
        S34[i,j]=Vec[k];
        S34[j,i]=Vec[k];
        k=k+1;
        end;
    end;
    Finish matrix;
    run matrix ;
    /*print S34;*/
R11=S34[1:3,1:3];
R12=S34[1:3,4];
slope34=inv(R11)*R12;
var34=1-t(slope34)*R12;

B=j(3,3,1);
B[3,3]=-1/P[3,3];
B[1,3]=P[1,3]/P[3,3];
B[3,1]=B[1,3];
B[2,3]=P[2,3]/P[3,3];
B[3,2]=B[2,3];
```

```
B[1,1]=P[1,1]-P[1,3]*P[3,1]/P[3,3];
B[1,2]=P[1,2]-P[1,3]*P[3,2]/P[3,3];
B[2,1]=B[1,2];
B[2,2]=P[2,2]-P[2,3]*P[3,2]/P[3,3];

T=j(4,4,1);
T[1:3,1:3]=B;
T[1:3,4]=slope34[1:3,1];
T[4,1:3]=T(slope34[1:3,1]);
T[4,4]=var34;
S4=j(5,5,1);
S4[1,2]=r4_y1;
S4[2,1]=S4[1,2];
S4[1,3]=r4_y2;
S4[3,1]=S4[1,3];
S4[1,4]=r4_y3;
S4[4,1]=S4[1,4];
S4[1,5]=r4_y4;
S4[5,1]=S4[1,5];
S4[2,3]=r4_12;
S4[3,2]=S4[2,3];
S4[2,4]=r4_13;
S4[4,2]=S4[2,4];
S4[2,5]=r4_14;
S4[5,2]=S4[2,5];
S4[3,4]=r4_23;
S4[4,3]=S4[3,4];
S4[3,5]=r4_24;
S4[5,3]=S4[3,5];
S4[4,5]=r4_34;
S4[5,4]=S4[4,5];
R11=S4[1:4,1:4];
R12=S4[1:4,5];
slope4=inv(R11)*R12;
var4=1-t(slope4)*R12;

C=j(4,4,1);
C[4,4]=-1/T[4,4];
C[1,4]=T[1,4]/T[4,4];
C[2,4]=T[2,4]/T[4,4];
C[3,4]=T[3,4]/T[4,4];
C[4,1]=C[1,4];
C[4,2]=C[2,4];
C[4,3]=C[3,4];
C[1,1]=T[1,1]-T[1,4]*T[4,1]/T[4,4];
C[2,2]=T[2,2]-T[2,4]*T[4,2]/T[4,4];
C[3,3]=T[3,3]-T[3,4]*T[4,3]/T[4,4];
C[1,2]=T[1,2]-T[1,4]*T[4,2]/T[4,4];
C[1,3]=T[1,3]-T[1,4]*T[3,4]/T[4,4];
C[2,1]=C[1,2];
C[3,1]=C[1,3];
C[2,3]=T[2,3]-T[2,4]*T[3,4]/T[4,4];
C[3,2]=C[2,3];

U=j(5,5,1);
U[1:4,1:4]=C;
U[1:4,5]=slope4;
```

```
U[5,1:4]=T(slope4);
U[5,5]=var4;

***********************************
REVERSE Operators matrix U on y 1 2 3
***********************************;

Uy=j(5,5,1);
Uy[1,1]=-1/U[1,1];
Uy[1,2]=-U[1,2]/U[1,1];
Uy[1,3]=-U[1,3]/U[1,1];
Uy[1,4]=-U[1,4]/U[1,1];
Uy[1,5]=-U[1,5]/U[1,1];
Uy[2,1]=Uy[1,2];
Uy[3,1]=Uy[1,3];
Uy[4,1]=Uy[1,4];
Uy[5,1]=Uy[1,5];
Uy[2,2]=U[2,2]-U[1,2]*U[2,1]/U[1,1];
Uy[2,3]=U[2,3]-U[1,2]*U[3,1]/U[1,1];
Uy[2,4]=U[2,4]-U[1,2]*U[4,1]/U[1,1];
Uy[2,5]=U[2,5]-U[1,2]*U[5,1]/U[1,1];
Uy[3,2]=Uy[2,3];
Uy[4,2]=Uy[2,4];
Uy[5,2]=Uy[2,5];
Uy[3,3]=U[3,3]-U[1,3]*U[3,1]/U[1,1];
Uy[3,4]=U[3,4]-U[1,3]*U[4,1]/U[1,1];
Uy[3,5]=U[3,5]-U[1,3]*U[5,1]/U[1,1];
Uy[4,3]=Uy[3,4];
Uy[5,3]=Uy[3,5];
Uy[4,4]=U[4,4]-U[1,4]*U[4,1]/U[1,1];
Uy[4,5]=U[4,5]-U[1,4]*U[5,1]/U[1,1];
Uy[5,4]=Uy[4,5];
Uy[5,5]=U[5,5]-U[1,5]*U[5,1]/U[1,1];

Uy1=j(5,5,1);
Uy1[1,1]=Uy[1,1]-Uy[1,2]*Uy[2,1]/Uy[2,2];;
Uy1[1,2]=-Uy[1,2]/Uy[2,2];
Uy1[1,3]=Uy[1,3]-Uy[1,2]*Uy[2,3]/Uy[2,2];
Uy1[1,4]=Uy[1,4]-Uy[1,2]*Uy[2,4]/Uy[2,2];
Uy1[1,5]=Uy[1,5]-Uy[1,2]*Uy[2,5]/Uy[2,2];
Uy1[2,1]=Uy1[1,2];
Uy1[3,1]=Uy1[1,3];
Uy1[4,1]=Uy1[1,4];
Uy1[5,1]=Uy1[1,5];
Uy1[2,2]=-1/Uy[2,2];
Uy1[2,3]=-Uy[2,3]/Uy[2,2];
Uy1[2,4]=-Uy[2,4]/Uy[2,2];
Uy1[2,5]=-Uy[2,5]/Uy[2,2];
Uy1[3,2]=Uy1[2,3];
Uy1[4,2]=Uy1[2,4];
Uy1[5,2]=Uy1[2,5];
Uy1[3,3]=Uy[3,3]-Uy[2,3]*Uy[3,2]/Uy[2,2];
Uy1[3,4]=Uy[3,4]-Uy[2,3]*Uy[4,2]/Uy[2,2];
Uy1[3,5]=Uy[3,5]-Uy[2,3]*Uy[5,2]/Uy[2,2];
Uy1[4,3]=Uy1[3,4];
Uy1[5,3]=Uy1[3,5];
Uy1[4,4]=Uy[4,4]-Uy[2,4]*Uy[4,2]/Uy[2,2];
```

```
Uy1[4,5]=Uy[4,5]-Uy[2,4]*Uy[5,2]/Uy[2,2];
Uy1[5,4]=Uy1[4,5];
Uy1[5,5]=Uy[5,5]-Uy[2,5]*Uy[5,2]/Uy[2,2];

Uy12=j(5,5,1);
Uy12[1,1]=Uy1[1,1]-Uy1[1,3]*Uy1[3,1]/Uy1[3,3];
Uy12[1,2]=Uy1[1,2]-Uy1[1,3]*Uy1[3,2]/Uy1[3,3];
Uy12[1,3]=-Uy1[1,3]/Uy1[3,3];
Uy12[1,4]=Uy1[1,4]-Uy1[1,3]*Uy1[3,4]/Uy1[3,3];
Uy12[1,5]=Uy1[1,5]-Uy1[1,3]*Uy1[3,5]/Uy1[3,3];
Uy12[2,1]=Uy12[1,2];
Uy12[3,1]=Uy12[1,3];
Uy12[4,1]=Uy12[1,4];
Uy12[5,1]=Uy12[1,5];
Uy12[2,2]=Uy1[2,2]-Uy1[2,3]*Uy1[3,2]/Uy1[3,3];
Uy12[2,3]=-Uy1[2,3]/Uy1[3,3];
Uy12[2,4]=Uy1[2,4]-Uy1[2,3]*Uy1[3,4]/Uy1[3,3];
Uy12[2,5]=Uy1[2,5]-Uy1[2,3]*Uy1[3,5]/Uy1[3,3];
Uy12[3,2]=Uy12[2,3];
Uy12[4,2]=Uy12[2,4];
Uy12[5,2]=Uy12[2,5];
Uy12[3,3]=-1/Uy1[3,3];
Uy12[3,4]=-Uy1[3,4]/Uy1[3,3];
Uy12[3,5]=-Uy1[3,5]/Uy1[3,3];
Uy12[4,3]=Uy12[3,4];
Uy12[5,3]=Uy12[3,5];
Uy12[4,4]=Uy1[4,4]-Uy1[4,3]*Uy1[3,4]/Uy1[3,3];
Uy12[4,5]=Uy1[4,5]-Uy1[4,3]*Uy1[3,5]/Uy1[3,3];
Uy12[5,4]=Uy12[4,5];
Uy12[5,5]=Uy1[5,5]-Uy1[5,3]*Uy1[3,5]/Uy1[3,3];

Uy123=j(5,5,1);
Uy123[1,1]=Uy12[1,1]-Uy12[1,4]*Uy12[4,1]/Uy12[4,4];
Uy123[1,2]=Uy12[1,2]-Uy12[1,4]*Uy12[4,2]/Uy12[4,4];
Uy123[1,3]=Uy12[1,3]-Uy12[1,4]*Uy12[4,3]/Uy12[4,4];
Uy123[1,4]=-Uy12[1,4]/Uy12[4,4];
Uy123[1,5]=Uy12[1,5]-Uy12[1,4]*Uy12[4,5]/Uy12[4,4];
Uy123[2,1]=Uy123[1,2];
Uy123[3,1]=Uy123[1,3];
Uy123[4,1]=Uy123[1,4];
Uy123[5,1]=Uy123[1,5];
Uy123[2,2]=Uy12[2,2]-Uy12[2,4]*Uy12[4,2]/Uy12[4,4];
Uy123[2,3]=Uy12[2,3]-Uy12[2,4]*Uy12[4,3]/Uy12[4,4];
Uy123[2,4]=-Uy12[2,4]/Uy12[4,4];
Uy123[2,5]=Uy12[2,5]-Uy12[2,4]*Uy12[4,5]/Uy12[4,4];
Uy123[3,2]=Uy123[2,3];
Uy123[4,2]=Uy123[2,4];
Uy123[5,2]=Uy123[2,5];
Uy123[3,3]=Uy12[3,3]-Uy12[3,4]*Uy12[4,3]/Uy12[4,4];
Uy123[3,4]=-Uy12[3,4]/Uy12[4,4];
Uy123[3,5]=Uy12[3,5]-Uy12[3,4]*Uy12[4,5]/Uy12[4,4];
Uy123[4,3]=Uy123[3,4];
Uy123[5,3]=Uy123[3,5];
Uy123[4,4]=-1/Uy12[4,4];
Uy123[4,5]=-Uy12[4,5]/Uy12[4,4];
Uy123[5,4]=Uy123[4,5];
Uy123[5,5]=Uy12[5,5]-Uy12[5,4]*Uy12[4,5]/Uy12[4,4];
```

```
do i=1 to 5;
    Uy123[i,i]=1;
end;
R11=Uy123[2:5,2:5];
R12=Uy123[2:5,1];
SLOPE=inv(R11)*R12;

Pat1[sim,1]=&n1;
Pat1[sim,2]=&n2;
Pat1[sim,3]=&n3;
Pat1[sim,4]=&n4;
Pat1[sim,5]=SLOPE[1,1];
Pat1[sim,6]=SLOPE[2,1];
Pat1[sim,7]=SLOPE[3,1];
Pat1[sim,8]=SLOPE[4,1];
end;
Create SWPp1.SWPPAT1&N&R from Pat1 [colname={n1 n2 n3 n4 x1 x2 x3 x4}]; Append
from Pat1;
run;
quit;
%Mend pattern1;


/*R1*/
%pattern1(N1,R1,150,150,150,150,0.6,0.4,0.3,0.25,0.25,0.1,0.05,0.15,0.1
,0.15);
%pattern1(N2,R1,2000,2000,2000,2000,0.6,0.4,0.3,0.25,0.25,0.1,0.05,0.15
,0.1,0.15);
%pattern1(N3,R1,150,500,1000,2000,0.6,0.4,0.3,0.25,0.25,0.1,0.05,0.15,0
.1,0.15);
%pattern1(N4,R1,2000,1000,500,150,0.6,0.4,0.3,0.25,0.25,0.1,0.05,0.15,0
.1,0.15);

/*R2*/
%pattern1(N1,R2,150,150,150,150,0.6,0.4,0.3,0.25,0,0,0,0,0,0);
%pattern1(N2,R2,2000,2000,2000,2000,0.6,0.4,0.3,0.25,0,0,0,0,0,0);
%pattern1(N3,R2,150,500,1000,2000,0.6,0.4,0.3,0.25,0,0,0,0,0,0);
%pattern1(N4,R2,2000,1000,500,150,0.6,0.4,0.3,0.25,0,0,0,0,0,0);

/*R3*/
%pattern1(N1,R3,150,150,150,150,0.25,0.3,0.4,0.6,0.15,0.1,0.05,0.15,0.1
,0.25);
%pattern1(N2,R3,2000,2000,2000,2000,0.25,0.3,0.4,0.6,0.15,0.1,0.05,0.15
,0.1,0.25);
%pattern1(N3,R3,150,500,1000,2000,0.25,0.3,0.4,0.6,0.15,0.1,0.05,0.15,0
.1,0.25);
%pattern1(N4,R3,2000,1000,500,150,0.25,0.3,0.4,0.6,0.15,0.1,0.05,0.15,0
.1,0.25);

/*R4*/
%pattern1(N1,R4,150,150,150,150,0.25,0.3,0.4,0.6,0,0,0,0,0,0);
%pattern1(N2,R4,2000,2000,2000,2000,0.25,0.3,0.4,0.6,0,0,0,0,0,0);
%pattern1(N3,R4,150,500,1000,2000,0.25,0.3,0.4,0.6,0,0,0,0,0,0);
%pattern1(N4,R4,2000,1000,500,150,0.25,0.3,0.4,0.6,0,0,0,0,0,0);
quit;
```

APPENDIX C

Table C.1 Relative Percentage Bias and Standard Errors for Each Predictor When the Correlation = $\mathbf{R}_1$

	Percentage Relative Bias							
	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Pattern I								
*N*1								
GLS	1.104	0.794	0.843	-2.659	0.001061	0.001278	0.001367	0.001800
SWP	-0.017	-0.249	0.292	-0.179	0.001048	0.001224	0.001357	0.001848
*N*2								
GLS	0.143	-0.169	-0.074	-0.133	0.000275	0.000324	0.000361	0.000490
SWP	0.058	-0.253	-0.122	0.040	0.000274	0.000323	0.000362	0.000493
*N*3								
GLS	0.169	0.058	0.133	-0.329	0.000364	0.000425	0.000431	0.000485
SWP	-0.037	-0.058	0.133	-0.006	0.000362	0.000423	0.000430	0.000487
*N*4								
GLS	0.103	0.280	0.673	0.565	0.000649	0.000841	0.001043	0.001883
SWP	-0.041	-0.186	-0.260	1.909	0.000653	0.000843	0.001038	0.001928
Pattern II								
*N*1								
GLS	1.331	0.706	0.445	0.173	0.000945	0.001050	0.001008	0.001109
SWP	-0.105	-0.306	-0.530	-0.277	0.000887	0.000998	0.000950	0.001064
*N*2								
GLS	0.178	-0.146	0.037	-0.017	0.000240	0.000267	0.000266	0.000282
SWP	0.074	-0.226	-0.032	-0.040	0.000239	0.000266	0.000265	0.000281
*N*3								
GLS	0.353	-0.049	0.180	-0.081	0.000359	0.000401	0.000386	0.000369
SWP	0.108	-0.217	-0.069	-0.035	0.000358	0.000396	0.000382	0.000366
*N*4								
GLS	0.184	-0.146	0.180	0.531	0.000373	0.000427	0.000394	0.000550
SWP	-0.062	-0.315	0.154	0.058	0.000369	0.000424	0.000389	0.000542
Pattern III								
*N*1								
GLS	1.581	-0.710	-0.562	-1.736	0.001248	0.001925	0.001810	0.001795
SWP	0.172	0.430	0.530	-0.646	0.001242	0.001947	0.001826	0.001818
*N*2								
GLS	0.178	-0.195	0.249	0.496	0.000340	0.000519	0.000511	0.000491
SWP	0.072	-0.111	0.334	0.577	0.000340	0.000520	0.000513	0.000492
*N*3								
GLS	0.149	-0.129	0.212	-0.271	0.000398	0.000521	0.000506	0.000476
SWP	-0.064	0.031	0.382	-0.110	0.000398	0.000523	0.000508	0.000477
*N*4								
GLS	0.575	-0.466	-1.363	-0.554	0.001099	0.001930	0.001902	0.001786
SWP	0.227	-0.138	-1.156	-0.352	0.001104	0.001940	0.001907	0.001797

Table C.1 (cont'd) Relative Percentage Bias and Standard Errors for Each Predictor When the Correlation = $\mathbf{R}_1$

	Percentage Relative Bias							
	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Pattern IV								
*N*1								
GLS	1.089	1.291	1.570	-2.665	0.001009	0.001096	0.001098	0.001799
SWP	-0.198	0.022	-0.196	0.502	0.000986	0.001067	0.001063	0.001862
*N*2								
GLS	0.155	-0.266	0.021	0.202	0.000251	0.000277	0.000280	0.000478
SWP	0.056	-0.364	-0.095	0.427	0.000251	0.000275	0.000278	0.000479
*N*3								
GLS	0.217	0.075	0.467	-0.559	0.000356	0.000413	0.000401	0.000478
SWP	-0.006	-0.102	0.228	-0.156	0.000356	0.000411	0.000399	0.000480
*N*4								
GLS	0.176	0.169	0.477	0.479	0.000584	0.000622	0.000633	0.001833
SWP	-0.070	-0.111	0.000	1.702	0.000593	0.000627	0.000638	0.001855
Pattern V								
*N*1								
GLS	1.521	0.839	0.345	0.185	0.000927	0.001030	0.000988	0.000982
SWP	0.079	-0.280	-0.430	-0.848	0.000879	0.000979	0.000935	0.000931
*N*2								
GLS	0.174	-0.040	0.053	0.017	0.000249	0.000269	0.000260	0.000248
SWP	0.072	-0.124	-0.037	-0.052	0.000248	0.000267	0.000259	0.000248
*N*3								
GLS	0.333	-0.084	0.180	0.144	0.000367	0.000404	0.000374	0.000367
SWP	0.077	-0.240	-0.037	0.000	0.000362	0.000397	0.000369	0.000362
*N*4								
GLS	0.308	-0.049	0.239	0.138	0.000366	0.000401	0.000372	0.000366
SWP	0.079	-0.240	-0.016	-0.006	0.000361	0.000397	0.000369	0.000362

Table C.2 Relative Percentage Bias and Standard Errors for Each Predictor When the Correlation = $\mathbf{R}_2$

	Percentage Relative Bias							
	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Pattern I								
*N*1								
GLS	0.688	0.317	0.000	-2.340	0.001040	0.001082	0.001123	0.001419
SWP	-0.037	-0.205	0.097	0.556	0.001041	0.001079	0.001123	0.001467
*N*2								
GLS	0.075	-0.080	-0.037	-0.136	0.000277	0.000288	0.000299	0.000378
SWP	0.027	-0.118	-0.037	0.060	0.000279	0.000288	0.000300	0.000384
*N*3								
GLS	0.103	-0.008	-0.027	-0.316	0.000333	0.000359	0.000354	0.000393
SWP	-0.018	-0.043	0.043	0.056	0.000331	0.000358	0.000354	0.000397
*N*4								
GLS	0.097	0.245	0.517	0.512	0.000753	0.000842	0.000902	0.001411
SWP	-0.075	-0.080	0.040	1.828	0.000775	0.000850	0.000915	0.001487
Pattern II								
*N*1								
GLS	0.717	0.370	0.167	-0.136	0.000823	0.000863	0.000847	0.000890
SWP	-0.147	-0.245	-0.440	-0.040	0.000772	0.000824	0.000799	0.000854
*N*2								
GLS	0.097	-0.078	0.013	0.004	0.000211	0.000223	0.000221	0.000230
SWP	0.035	-0.128	-0.030	0.008	0.000209	0.000221	0.000220	0.000233
*N*3								
GLS	0.202	-0.060	0.037	-0.156	0.000312	0.000329	0.000306	0.000306
SWP	0.055	-0.155	-0.080	-0.024	0.000310	0.000326	0.000305	0.000305
*N*4								
GLS	0.083	-0.015	0.173	0.564	0.000335	0.000370	0.000341	0.000436
SWP	-0.067	-0.143	0.083	0.148	0.000332	0.000366	0.000337	0.000435
Pattern III								
*N*1								
GLS	1.115	-0.583	-0.647	-1.132	0.001321	0.001464	0.001446	0.001450
SWP	0.353	0.357	0.153	-0.348	0.001328	0.001499	0.001452	0.001464
*N*2								
GLS	0.092	-0.095	0.150	0.248	0.000351	0.000397	0.000401	0.000398
SWP	0.035	-0.030	0.213	0.304	0.000351	0.000402	0.000404	0.000399
*N*3								
GLS	0.112	-0.080	0.070	-0.140	0.000375	0.000408	0.000406	0.000394
SWP	0.000	0.062	0.200	-0.016	0.000375	0.000412	0.000408	0.000394
*N*4								
GLS	0.377	0.157	-0.327	0.276	0.001283	0.001434	0.001498	0.001452
SWP	0.118	0.210	-0.277	0.320	0.001281	0.001464	0.001502	0.001459

Table C.2 (cont'd) Relative Percentage Bias and Standard Errors for Each Predictor When the Correlation = $\mathbf{R}_2$

	Percentage Relative Bias							
	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Pattern IV								
*N*1								
GLS	0.567	0.502	0.480	-2.392	0.000968	0.000968	0.000977	0.001367
SWP	-0.223	-0.140	-0.237	1.264	0.000967	0.000965	0.000954	0.001435
*N*2								
GLS	0.092	-0.120	0.060	0.052	0.000244	0.000253	0.000253	0.000371
SWP	0.032	-0.168	0.010	0.308	0.000246	0.000245	0.000253	0.000376
*N*3								
GLS	0.135	0.065	0.193	-0.492	0.000318	0.000352	0.000337	0.000389
SWP	0.003	-0.033	0.097	0.044	0.000320	0.000352	0.000335	0.000393
*N*4								
GLS	0.220	0.240	0.260	1.044	0.000680	0.000698	0.000669	0.001365
SWP	0.022	0.070	0.053	2.000	0.000705	0.000708	0.000675	0.001407
Pattern V								
*N*1								
GLS	0.875	0.432	0.050	-0.016	0.000798	0.000825	0.000803	0.000806
SWP	0.005	-0.285	-0.427	-0.512	0.000758	0.000790	0.000760	0.000771
*N*2								
GLS	0.095	-0.027	0.020	0.016	0.000210	0.000221	0.000210	0.000207
SWP	0.033	-0.078	-0.030	-0.024	0.000209	0.000220	0.000209	0.000206
*N*3								
GLS	0.177	-0.063	0.063	0.060	0.000313	0.000331	0.000302	0.000304
SWP	0.032	-0.155	-0.053	-0.028	0.000309	0.000326	0.000297	0.000300
*N*4								
GLS	0.162	-0.038	0.087	0.044	0.000311	0.000330	0.000299	0.000303
SWP	0.033	-0.158	-0.047	-0.032	0.000308	0.000327	0.000298	0.000300

Table C.3 Relative Percentage Bias and Standard Errors for Each Predictor When the Correlation = $\mathbf{R_3}$

	Percentage Relative Bias							
	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Pattern I								
*N*1								
GLS	1.425	1.501	1.402	-0.775	0.001387	0.001460	0.001599	0.001640
SWP	0.133	-0.217	-0.599	0.434	0.001386	0.001434	0.001584	0.001656
*N*2								
GLS	0.202	-0.196	-0.009	-0.014	0.000383	0.000391	0.000425	0.000438
SWP	0.110	-0.323	-0.142	0.062	0.000383	0.000391	0.000426	0.000444
*N*3								
GLS	0.144	0.064	0.186	-0.079	0.000438	0.000467	0.000472	0.000456
SWP	-0.092	-0.143	0.004	0.037	0.000437	0.000465	0.000470	0.000457
*N*4								
GLS	0.075	0.313	1.056	0.310	0.001146	0.001316	0.001389	0.001637
SWP	-0.162	-0.376	-0.524	1.242	0.001160	0.001327	0.001396	0.001695
Pattern II								
*N*1								
GLS	0.952	0.795	0.391	1.044	0.001052	0.001073	0.001089	0.001031
SWP	-0.277	-0.408	-0.697	0.041	0.000998	0.001020	0.001024	0.000968
*N*2								
GLS	0.271	-0.175	0.022	0.083	0.000276	0.000282	0.000276	0.000260
SWP	0.202	-0.260	-0.058	0.000	0.000274	0.000280	0.000275	0.000260
*N*3								
GLS	0.508	-0.053	0.249	0.097	0.000394	0.000398	0.000397	0.000348
SWP	0.283	-0.233	-0.044	-0.002	0.000393	0.000395	0.000394	0.000344
*N*4								
GLS	-0.023	-0.207	0.022	0.513	0.000455	0.000479	0.000472	0.000495
SWP	-0.231	-0.371	0.120	0.066	0.000448	0.000473	0.000467	0.000493
Pattern III								
*N*1								
GLS	3.178	0.085	0.670	-0.742	0.001531	0.001930	0.001866	0.001695
SWP	1.332	0.270	0.843	-0.463	0.001521	0.001933	0.001870	0.001706
*N*2								
GLS	0.415	-0.005	0.129	0.159	0.000408	0.000533	0.000540	0.000457
SWP	0.283	0.011	0.146	0.172	0.000408	0.000534	0.000540	0.000459
*N*3								
GLS	0.092	-0.175	0.249	-0.045	0.000440	0.000505	0.000521	0.000452
SWP	-0.162	-0.154	0.271	-0.017	0.000440	0.000505	0.000521	0.000451
*N*4								
GLS	0.721	0.387	-1.433	0.161	0.001423	0.001923	0.001987	0.001705
SWP	0.110	0.451	-1.331	0.281	0.001424	0.001925	0.001994	0.001728

Table C.3 (cont'd) Relative Percentage Bias and Standard Errors for Each Predictor When the Correlation = $\mathbf{R}_3$

	Percentage Relative Bias							
	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Pattern IV								
*N*1								
GLS	0.756	1.798	2.556	-0.734	0.001381	0.001366	0.001433	0.001623
SWP	-0.623	-0.159	-0.759	0.835	0.001372	0.001361	0.001423	0.001655
*N*2								
GLS	0.363	-0.297	0.169	0.027	0.000353	0.000361	0.000374	0.000425
SWP	0.242	-0.437	-0.067	0.141	0.000345	0.000362	0.000375	0.000429
*N*3								
GLS	0.219	0.191	0.608	-0.223	0.000430	0.000450	0.000448	0.000440
SWP	0.075	-0.069	0.160	-0.052	0.000430	0.000449	0.000446	0.000442
*N*4								
GLS	0.404	0.705	0.648	0.358	0.001110	0.001158	0.001202	0.001601
SWP	-0.069	0.106	-0.617	1.242	0.001127	0.001169	0.001215	0.001631
Pattern V								
*N*1								
GLS	1.488	0.864	0.475	1.112	0.000987	0.001013	0.001014	0.000936
SWP	0.335	-0.350	-0.342	-0.281	0.000952	0.000961	0.000949	0.000887
*N*2								
GLS	0.006	0.016	0.053	0.083	0.000264	0.000262	0.000268	0.000231
SWP	0.196	-0.064	-0.036	-0.021	0.000263	0.000261	0.000268	0.000229
*N*3								
GLS	0.340	-0.133	0.186	0.244	0.000401	0.000400	0.000388	0.000347
SWP	0.185	-0.308	-0.018	0.010	0.000395	0.000393	0.000382	0.000340
*N*4								
GLS	0.311	-0.111	0.226	0.242	0.000399	0.000398	0.000384	0.000343
SWP	0.179	-0.297	-0.004	0.006	0.000394	0.000394	0.000382	0.000340

Table C.4 Relative percentage bias and standard errors for each predictor when the correlation = $\mathbf{R}_4$

	Percentage Relative Bias							
	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Pattern I								
*N*1								
GLS	0.864	0.773	0.545	-0.772	0.001271	0.001312	0.001370	0.001240
SWP	0.008	-0.130	-0.055	0.560	0.001283	0.001308	0.001371	0.001312
*N*2								
GLS	0.068	-0.087	-0.008	-0.018	0.000356	0.000361	0.000369	0.000330
SWP	0.012	-0.153	-0.048	0.058	0.000356	0.000362	0.000370	0.000354
*N*3								
GLS	0.052	0.020	0.005	-0.095	0.000388	0.000406	0.000399	0.000371
SWP	-0.076	-0.080	0.000	0.040	0.000387	0.000405	0.000398	0.000387
*N*4								
GLS	0.236	0.490	0.712	0.253	0.001145	0.001283	0.001287	0.001130
SWP	-0.124	0.030	0.220	1.098	0.001161	0.001293	0.001298	0.001321
Pattern II								
*N*1								
GLS	0.356	0.487	0.230	0.452	0.000883	0.000894	0.000897	0.000844
SWP	-0.360	-0.220	-0.503	0.028	0.000843	0.000852	0.000849	0.000807
*N*2								
GLS	0.092	-0.073	0.022	0.057	0.000234	0.000239	0.000234	0.000216
SWP	0.060	-0.127	-0.030	0.022	0.000232	0.000238	0.000233	0.000222
*N*3								
GLS	0.192	-0.043	0.080	0.003	0.000326	0.000329	0.000320	0.000303
SWP	0.068	-0.147	-0.058	-0.012	0.000325	0.000327	0.000318	0.000302
*N*4								
GLS	-0.060	-0.033	0.168	0.357	0.000399	0.000428	0.000425	0.000374
SWP	-0.192	-0.163	0.060	0.058	0.000395	0.000422	0.000421	0.000392
Pattern III								
*N*1								
GLS	1.796	0.320	0.332	-0.402	0.001407	0.001568	0.001519	0.001438
SWP	0.908	0.397	0.415	-0.228	0.001402	0.001572	0.001522	0.001449
*N*2								
GLS	0.180	0.037	0.090	0.087	0.000369	0.000439	0.000434	0.000390
SWP	0.112	0.040	0.095	0.092	0.000369	0.000439	0.000434	0.000393
*N*3								
GLS	0.036	-0.063	0.150	-0.017	0.000383	0.000413	0.000422	0.000394
SWP	-0.080	-0.047	0.168	0.005	0.000383	0.000413	0.000422	0.000393
*N*4								
GLS	0.452	0.690	-0.330	0.430	0.001375	0.001555	0.001589	0.001463
SWP	0.072	0.607	-0.373	0.432	0.001375	0.001557	0.001591	0.001486

Table C.4 (cont'd) Relative percentage bias and standard errors for each predictor when the correlation = $\mathbf{R}_4$

	Percentage Relative Bias							
	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Pattern IV								
*N*1								
GLS	0.420	0.973	0.922	-0.865	0.001295	0.001275	0.001314	0.001146
SWP	-0.512	-0.027	-0.170	0.863	0.001305	0.001282	0.001316	0.001272
*N*2								
GLS	0.204	-0.123	0.105	0.007	0.000333	0.000343	0.000349	0.000304
SWP	0.132	-0.187	0.020	0.122	0.000335	0.000345	0.000351	0.000337
*N*3								
GLS	0.116	0.140	0.195	-0.217	0.000378	0.000397	0.000383	0.000356
SWP	-0.016	0.013	0.047	0.010	0.000380	0.000397	0.000382	0.000373
*N*4								
GLS	0.656	0.757	0.492	0.482	0.001129	0.001183	0.001181	0.001033
SWP	0.184	0.333	0.043	1.123	0.001145	0.001194	0.001186	0.001215
Pattern V								
*N*1								
GLS	0.620	0.497	0.205	0.592	0.000808	0.000825	0.000797	0.000803
SWP	-0.024	-0.227	-0.363	-0.225	0.000784	0.000787	0.000750	0.000763
*N*2								
GLS	0.096	0.027	0.025	0.052	0.000215	0.000217	0.000218	0.000203
SWP	0.068	-0.017	-0.025	-0.010	0.000214	0.000216	0.000217	0.000202
*N*3								
GLS	0.100	-0.080	0.080	0.127	0.000329	0.000323	0.000316	0.000305
SWP	0.012	-0.173	-0.040	-0.010	0.000324	0.000318	0.000311	0.000299
*N*4								
GLS	0.080	-0.063	0.095	0.120	0.000327	0.000321	0.000313	0.000302
SWP	0.012	-0.173	-0.033	-0.015	0.000323	0.000318	0.000311	0.000299

APPENDIX D

Table D.1 Relative Percentage Bias and Standard Errors for Each Predictor in Pattern I

	Percentage Relative Bias							
	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Condition 5:								
$R_1R_1R_2R_2$								
GLS (*N*1)	3.049	6.101	2.803	0.312	0.001059	0.001117	0.001155	0.001465
SWP(*N*1)	1.786	4.478	6.760	6.760	0.001080	0.001114	0.001207	0.001566
GLS (*N*2)	2.380	5.687	2.823	2.580	0.000279	0.000297	0.000308	0.000388
SWP(*N*2)	1.781	4.441	6.673	6.256	0.000287	0.000303	0.000323	0.00041
GLS (*N*3)	1.309	2.205	0.907	0.568	0.000337	0.000365	0.000359	0.000398
SWP(*N*3)	0.733	1.770	3.033	2.832	0.000335	0.000361	0.000368	0.000408
GLS (*N*4)	3.010	11.792	7.433	7.108	0.000775	0.00088	0.000958	0.001495
SWP(*N*4)	3.068	9.548	11.480	12.644	0.000846	0.000933	0.001045	0.001658
Condition 6:								
$R_3R_3R_4R_4$								
GLS (*N*1)	4.660	3.446	0.835	-0.403	0.001272	0.001312	0.001374	0.001245
SWP(*N*1)	3.850	2.429	0.432	1.262	0.00129	0.001314	0.001381	0.001326
GLS (*N*2)	3.862	2.582	0.285	0.360	0.000355	0.000361	0.00037	0.000331
SWP(*N*2)	3.795	2.370	0.442	0.758	0.000357	0.000364	0.000373	0.000357
GLS (*N*3)	1.682	1.174	0.165	0.055	0.000389	0.000407	0.000399	0.000372
SWP(*N*3)	1.514	1.007	0.215	0.342	0.000388	0.000404	0.000399	0.000388
GLS (*N*4)	7.106	5.154	1.318	1.023	0.001145	0.001282	0.001296	0.001134
SWP(*N*4)	6.999	4.653	1.078	2.362	0.001174	0.001303	0.001319	0.001338
Condition 7:								
$R_2R_2R_1R_1$								
GLS (*N*1)	2.835	3.291	-2.314	-7.347	0.001029	0.001272	0.001297	0.001682
SWP(*N*1)	0.195	-1.853	0.226	-1.916	0.001007	0.001214	0.001312	0.001781
GLS (*N*2)	2.078	2.790	-3.251	-4.935	0.000271	0.000327	0.000341	0.00046
SWP(*N*2)	0.310	-1.681	-0.248	-1.787	0.000265	0.000311	0.000349	0.000474
GLS (*N*3)	1.677	2.161	-1.966	-3.073	0.00036	0.000427	0.00042	0.00047
SWP(*N*3)	0.096	-0.842	0.097	-0.710	0.000357	0.000419	0.000424	0.00048
GLS (*N*4)	1.372	1.852	-2.439	-5.614	0.000612	0.000822	0.000953	0.00171
SWP(*N*4)	0.307	-2.570	-0.714	-1.829	0.000613	0.000811	0.000963	0.001791
Condition 8:								
$R_4R_4R_3R_3$								
GLS (*N*1)	0.906	0.813	1.750	-1.201	0.001379	0.001458	0.001592	0.001636
SWP(*N*1)	-0.724	-1.464	-0.076	0.109	0.001378	0.001428	0.001578	0.001651
GLS (*N*2)	-0.285	-0.819	0.345	-0.435	0.000383	0.000392	0.000423	0.000437
SWP(*N*2)	-0.703	-1.535	0.380	-0.274	0.000382	0.000389	0.000424	0.000442
GLS (*N*3)	0.102	-0.120	0.321	-0.279	0.000436	0.000466	0.000471	0.000455
SWP(*N*3)	-0.446	-0.708	0.236	-0.097	0.000436	0.000465	0.00047	0.000457
GLS (*N*4)	-1.353	-1.402	1.840	-0.346	0.001135	0.001311	0.001378	0.001632
SWP(*N*4)	-1.705	-2.662	0.456	0.635	0.001115	0.001323	0.001383	0.001687

Table D.2 Relative Percentage Bias and Standard Errors for Each Predictor in Pattern II

	Percentage Relative Bias							
	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Condition 5:								
$R_1R_1R_2R_2$								
GLS (*N*1)	6.169	11.896	12.659	12.149	0.000894	0.000944	0.00091	0.000986
SWP(*N*1)	0.042	0.480	-0.101	4.088	0.000821	0.000897	0.000871	0.000989
GLS (*N*2)	5.547	11.498	12.697	12.506	0.000231	0.000243	0.000239	0.000254
SWP(*N*2)	0.247	0.604	0.360	4.262	0.000223	0.000241	0.000242	0.00027
GLS (*N*3)	2.762	4.425	5.473	4.689	0.000317	0.000342	0.000318	0.000315
SWP(*N*3)	0.161	0.063	0.065	1.299	0.000316	0.000334	0.000317	0.000322
GLS (*N*4)	5.046	13.340	12.590	22.553	0.000391	0.000443	0.000397	0.000572
SWP(*N*4)	-0.038	0.409	0.396	3.103	0.00036	0.000408	0.000374	0.00053
Condition 6:								
$R_3R_3R_4R_4$								
GLS (*N*1)	12.651	13.387	9.360	6.056	0.000945	0.000938	0.000941	0.000916
SWP(*N*1)	1.469	1.602	0.400	1.031	0.000913	0.000918	0.000901	0.000862
GLS (*N*2)	12.564	12.865	9.141	5.652	0.00025	0.000251	0.000252	0.000235
SWP(*N*2)	2.036	1.694	0.884	1.062	0.000251	0.000256	0.000245	0.000236
GLS (*N*3)	5.334	5.332	4.213	2.348	0.000333	0.000338	0.000334	0.000311
SWP(*N*3)	0.668	0.345	0.220	0.296	0.000338	0.000338	0.000326	0.00031
GLS (*N*4)	15.003	16.001	10.913	9.137	0.00046	0.000483	0.00047	0.000474
SWP(*N*4)	1.303	1.303	0.996	0.845	0.000432	0.000456	0.00044	0.000456
Condition 7:								
$R_2R_2R_1R_1$								
GLS (*N*1)	6.170	10.874	12.551	11.002	0.000887	0.000978	0.00934	0.001014
SWP(*N*1)	-0.121	-0.609	-0.610	-2.471	0.000814	0.000904	0.000883	0.000968
GLS (*N*2)	5.461	10.636	12.555	11.288	0.000226	0.000254	0.000248	0.000272
SWP(*N*2)	0.035	-0.493	-0.147	-2.381	0.000218	0.00024	0.000244	0.000254
GLS (*N*3)	4.619	11.552	12.387	8.499	0.000367	0.000413	0.000379	0.000378
SWP(*N*3)	0.097	-0.278	-0.113	-0.278	0.000347	0.000384	0.000373	0.000356
GLS (*N*4)	2.916	4.298	5.568	9.919	0.000343	0.000384	0.000353	0.000472
SWP(*N*4)	-0.247	-0.811	-0.239	-5.630	0.000329	0.00037	0.000344	0.000454
Condition 8:								
$R_4R_4R_3R_3$								
GLS (*N*1)	11.473	12.673	10.415	6.369	0.001001	0.001028	0.001047	0.000963
SWP(*N*1)	-1.259	-1.394	-1.329	-0.477	0.00094	0.000957	0.000949	0.000904
GLS (*N*2)	11.194	12.244	10.658	5.885	0.000267	0.000274	0.000268	0.000252
SWP(*N*2)	-0.900	-1.218	-0.714	-0.560	0.000258	0.000262	0.000258	0.000245
GLS (*N*3)	10.280	11.311	11.063	3.903	0.000396	0.0004	0.000401	0.000365
SWP(*N*3)	0.082	-0.361	-0.181	-0.059	0.000383	0.000386	0.000383	0.000335
GLS (*N*4)	6.037	6.578	4.982	5.708	0.000427	0.000453	0.000463	0.000421
SWP(*N*4)	-2.291	-2.332	-1.076	-1.242	0.000415	0.000436	0.000427	0.000428

Table D.3 Relative Percentage Bias and Standard Errors for Each Predictor in Pattern III

	Percentage Relative Bias							
	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Condition 5:								
$R_1R_1R_2R_2$								
GLS (*N*1)	1.115	-0.583	-0.647	-1.132	0.001321	0.001464	0.001446	0.00145
SWP(*N*1)	0.353	0.357	0.153	-0.348	0.001328	0.001499	0.001452	0.001464
GLS (*N*2)	0.092	-0.095	0.150	0.248	0.000351	0.000397	0.000401	0.000398
SWP(*N*2)	0.035	-0.030	0.213	0.304	0.000351	0.000402	0.000404	0.000399
GLS (*N*3)	0.112	-0.080	0.070	-0.140	0.000375	0.000408	0.000406	0.000394
SWP(*N*3)	0.000	0.062	0.200	-0.016	0.000375	0.000412	0.000408	0.000394
GLS (*N*4)	0.377	0.157	-0.327	0.276	0.001283	0.001434	0.001498	0.001452
SWP(*N*4)	0.118	0.210	-0.277	0.320	0.001281	0.001464	0.001502	0.001459
Condition 6:								
$R_3R_3R_4R_4$								
GLS (*N*1)	1.796	0.320	0.332	-0.402	0.001407	0.001568	0.001519	0.001438
SWP(*N*1)	0.908	0.397	0.415	-0.228	0.001402	0.001572	0.001522	0.001449
GLS (*N*2)	0.180	0.037	0.090	0.087	0.000369	0.000439	0.000434	0.00039
SWP(*N*2)	0.112	0.040	0.095	0.092	0.000369	0.000439	0.000434	0.000393
GLS (*N*3)	0.036	-0.063	0.150	-0.017	0.000383	0.000413	0.000422	0.000394
SWP(*N*3)	-0.080	-0.047	0.168	0.005	0.000383	0.000413	0.000422	0.000393
GLS (*N*4)	0.452	0.690	-0.330	0.430	0.001375	0.001555	0.001589	0.001463
SWP(*N*4)	0.072	0.607	-0.373	0.432	0.001375	0.001557	0.001591	0.001486
Condition 7:								
$R_2R_2R_1R_1$								
GLS (*N*1)	1.589	-0.692	-0.573	-1.753	0.001248	0.001925	0.00181	0.001795
SWP(*N*1)	0.180	0.448	0.520	-0.663	0.001242	0.001947	0.001826	0.001818
GLS (*N*2)	0.186	-0.178	0.239	0.479	0.00034	0.000519	0.000511	0.000491
SWP(*N*2)	0.079	-0.093	0.323	0.559	0.00034	0.00052	0.000513	0.000492
GLS (*N*3)	0.157	-0.111	0.201	-0.288	0.000398	0.000521	0.000506	0.000476
SWP(*N*3)	-0.056	0.049	0.371	-0.127	0.000398	0.000523	0.000508	0.000477
GLS (*N*4)	0.583	-0.448	-1.373	-0.571	0.001099	0.00193	0.001902	0.001786
SWP(*N*4)	0.234	-0.120	-1.166	-0.369	0.001104	0.00194	0.001907	0.001797
Condition 8:								
$R_4R_4R_3R_3$								
GLS (*N*1)	3.160	0.074	0.688	-0.734	0.001531	0.00193	0.001866	0.001695
SWP(*N*1)	1.315	0.260	0.861	-0.455	0.001521	0.001933	0.00187	0.001706
GLS (*N*2)	0.398	-0.016	0.146	0.167	0.000408	0.000533	0.00054	0.000457
SWP(*N*2)	0.265	0.000	0.164	0.180	0.000408	0.000253	0.00054	0.000459
GLS (*N*3)	0.075	-0.186	0.266	-0.037	0.00044	0.000505	0.000521	0.000452
SWP(*N*3)	-0.179	-0.164	0.289	-0.010	0.00044	0.000505	0.000521	0.000451
GLS (*N*4)	0.704	0.376	-1.416	0.169	0.001423	0.001923	0.001987	0.001705
SWP(*N*4)	0.092	0.440	-1.314	0.289	0.001424	0.001925	0.001994	0.001728

Table D.4 Relative Percentage Bias and Standard Errors for Each Predictor in Pattern IV

	Percentage Relative Bias							
	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Condition 5: $R_1R_1R_2R_2$								
GLS (*N*1)	5.379	11.103	10.815	5.412	0.001017	0.00104	0.001026	0.001474
SWP(*N*1)	1.692	5.627	4.047	22.992	0.00109	0.001078	0.001071	0.001766
GLS (*N*2)	4.951	10.531	10.466	7.868	0.00026	0.000271	0.000269	0.000404
SWP(*N*2)	1.896	5.217	4.006	21.880	0.000274	0.000289	0.000286	0.000463
GLS (*N*3)	2.350	3.904	4.391	1.520	0.000322	0.00036	0.000341	0.000398
SWP(*N*3)	0.639	1.540	1.372	8.352	0.000327	0.000362	0.000346	0.000428
GLS (*N*4)	4.906	15.519	12.726	20.932	0.000754	0.000796	0.000758	0.0016
SWP(*N*4)	3.192	11.962	7.867	35.948	0.000925	0.000969	0.000884	0.000193
Condition 6: $R_3R_3R_4R_4$								
GLS (*N*1)	11.291	11.108	5.832	1.630	0.001303	0.001277	0.001323	0.001161
SWP(*N*1)	9.378	9.622	4.384	6.290	0.001355	0.001328	0.001367	0.001352
GLS (*N*2)	11.347	10.066	5.059	2.507	0.000336	0.000345	0.000352	0.000311
SWP(*N*2)	9.991	9.220	4.471	5.528	0.000347	0.000358	0.000366	0.000357
GLS (*N*3)	4.166	3.826	2.205	0.543	0.000379	0.000396	0.000384	0.000358
SWP(*N*3)	3.278	3.177	1.685	1.993	0.000383	0.0004	0.000386	0.000383
GLS (*N*4)	18.329	17.303	7.468	5.612	0.001149	0.001196	0.001194	0.001045
SWP(*N*4)	17.860	17.445	7.390	9.930	0.001244	0.001297	0.00128	0.001328
Condition 7: $R_2R_2R_1R_1$								
GLS (*N*1)	5.222	8.473	10.666	-13.896	0.000932	0.001009	0.001026	0.001495
SWP(*N*1)	-0.007	-0.829	-0.684	-1.498	0.00908	0.001002	0.001	0.001688
GLS (*N*2)	4.557	7.543	9.704	-11.158	0.00023	0.000262	0.000262	0.000397
SWP(*N*2)	0.217	-0.859	-0.318	-1.775	0.000231	0.000255	0.00026	0.000435
GLS (*N*3)	3.513	7.799	8.745	-7.550	0.000355	0.000424	0.000405	0.000444
SWP(*N*3)	0.084	-0.399	0.040	-0.712	0.000346	0.000399	0.000391	0.000466
GLS (*N*4)	2.411	2.596	4.216	-9.186	0.00049	0.000551	0.000556	0.001441
SWP(*N*4)	0.124	-1.012	-0.461	-2.958	0.000507	0.000575	0.000576	0.001536
Condition 8: $R_4R_4R_3R_3$								
GLS (*N*1)	2.978	3.545	3.465	-2.628	0.001355	0.001343	0.001422	0.001628
SWP(*N*1)	-1.657	-1.416	-2.588	0.170	0.001346	0.001344	0.001409	0.001646
GLS (*N*2)	2.269	1.472	1.148	-1.754	0.000345	0.000355	0.00037	0.000424
SWP(*N*2)	-0.887	-1.513	-1.820	-0.578	0.000347	0.000355	0.00037	0.000427
GLS (*N*3)	2.023	1.917	1.824	-1.164	0.000428	0.000454	0.00045	0.000444
SWP(*N*3)	-0.409	-0.445	-0.533	-0.286	0.000428	0.000446	0.000443	0.000442
GLS (*N*4)	0.019	0.004	-0.789	-1.492	0.001064	0.001123	0.001182	0.001588
SWP(*N*4)	-2.000	-1.972	-3.389	0.028	0.001089	0.001146	0.001206	0.001614

Table D.5 Relative Percentage Bias and Standard Errors for Each Predictor in Pattern V

	Percentage Relative Bias							
	$\bar{\hat{B}}_1$	$\bar{\hat{B}}_2$	$\bar{\hat{B}}_3$	$\bar{\hat{B}}_4$	SE_1	SE_2	SE_3	SE_4
Condition 5:								
$R_1R_1R_2R_2$								
GLS (*N*1)	6.440	12.292	13.767	12.593	0.000862	0.000915	0.000881	0.000887
SWP(*N*1)	0.048	-0.240	-0.409	-0.702	0.000814	0.000874	0.000852	0.000867
GLS (*N*2)	5.679	11.930	14.031	12.471	0.000225	0.000244	0.000237	0.000227
SWP(*N*2)	0.053	-0.112	-0.064	-0.054	0.000226	0.000239	0.000236	0.000229
GLS (*N*3)	2.765	4.514	5.765	5.370	0.000322	0.000346	0.000308	0.00031
SWP(*N*3)	0.038	-0.154	-0.044	-0.004	0.000316	0.000339	0.000309	0.000317
GLS (*N*4)	4.738	12.481	13.384	11.472	0.000372	0.000416	0.000372	0.000375
SWP(*N*4)	0.073	-0.210	0.025	0.000	0.00035	0.000384	0.000357	0.00035
Condition 6:								
$R_3R_3R_4R_4$								
GLS (*N*1)	13.109	14.474	12.114	6.236	0.000892	0.000911	0.000911	0.000872
SWP(*N*1)	0.182	-0.222	-0.319	-0.297	0.000886	0.000877	0.000848	0.000817
GLS (*N*2)	12.618	13.975	11.996	5.598	0.000233	0.000234	0.000242	0.000218
SWP(*N*2)	0.113	-0.068	-0.062	-0.018	0.000243	0.000239	0.00024	0.000213
GLS (*N*3)	5.383	5.601	4.649	2.741	0.000337	0.000334	0.000331	0.000311
SWP(*N*3)	0.030	-0.183	-0.014	0.000	0.000341	0.000334	0.000319	0.000306
GLS (*N*4)	11.336	13.153	12.806	4.787	0.0004	0.000401	0.000387	0.000358
SWP(*N*4)	0.142	-0.212	0.008	0.004	0.000384	0.000382	0.000366	0.000329
Condition 7:								
$R_2R_2R_1R_1$								
GLS (*N*1)	6.532	12.164	13.860	12.014	0.000872	0.000934	0.000896	0.000887
SWP(*N*1)	0.042	-0.391	-0.507	-0.693	0.000802	0.000876	0.000849	0.000853
GLS (*N*2)	5.655	11.914	13.945	12.529	0.000234	0.00025	0.000229	0.000229
SWP(*N*2)	0.044	-0.099	-0.038	-0.059	0.000226	0.00024	0.000235	0.000229
GLS (*N*3)	4.892	12.620	13.532	10.958	0.000373	0.000412	0.000377	0.000376
SWP(*N*3)	0.088	-0.258	-0.064	-0.586	0.00035	0.000379	0.000358	0.000351
GLS (*N*4)	2.914	4.563	5.853	5.335	0.000318	0.000343	0.00031	0.00031
SWP(*N*4)	0.052	-0.196	-0.091	-0.078	0.000315	0.000334	0.000312	0.000316
Condition 8:								
$R_4R_4R_3R_3$								
GLS (*N*1)	13.163	14.197	11.841	6.097	0.000897	0.000899	0.000895	0.000875
SWP(*N*1)	0.088	-0.392	-0.427	-0.218	0.000867	0.000878	0.000834	0.000809
GLS (*N*2)	12.633	13.997	12.003	5.614	0.000243	0.000242	0.000244	0.000221
SWP(*N*2)	0.108	-0.034	-0.020	-0.026	0.000239	0.000238	0.000241	0.000212
GLS (*N*3)	11.719	13.310	12.866	4.787	0.000402	0.000394	0.000394	0.000362
SWP(*N*3)	0.137	-0.295	-0.079	0.019	0.000383	0.00038	0.000368	0.000329
GLS (*N*4)	5.504	5.666	4.701	2.716	0.000334	0.000332	0.000329	0.000309
SWP(*N*4)	0.022	-0.248	-0.055	-0.009	0.000338	0.000332	0.000321	0.000305

APPENDIX E

This appendix provides another way to summarize and compare the results from the GLS and SWP methods. In the column X_{ps} (where p represents predictors 1 to 4), the notation indicates which method (G=GLS; S=SWP) is superior in estimating the slope of X_p, along with the sign that shows whether the method underestimated (-) the population slope or overestimated it (+). In the column X_{pi}, the inferior method is recorded. If "=" is in X_{ps}, it indicates the two methods produced equal estimate for that predictor in that scenario and the next cell and the X_{pi} next to it showed the overestimation or underestimation of each method. If "0" follows "G" or "S", it indicates the method produced a mean slope that is identical to the population slope for that variable in that scenario. In the SE columns, the method that produced more stable estimates was recorded.

R_1		X_{1s}	X_{1i}	X_{2s}	X_{2i}	X_{3s}	X_{3i}	X_{4s}	X_{4i}	SE_1	SE_2	SE_3	SE_4
Pattern I	*N*1	S-	G+	S-	G+	S+	G+	S-	G-	S	S	S	G
	*N*2	S+	G+	G-	S-	G-	S-	S+	G-	S	S	G	G
	*N*3	S-	G+	=	S-G+	=	S+G+	S0	G-	S	S	S	G
	*N*4	S-	G+	S-	G+	S-	G+	G+	S+	G	G	S	G
Pattern II	*N*1	S-	G+	S-	G+	G+	S-	G+	S-	S	S	S	S
	*N*2	S+	G+	G-	S-	=	S-G+	G0	S-	S	S	S	S
	*N*3	S+	G+	G-	S-	S-	G+	=	S-G-	S	S	S	S
	*N*4	S-	G+	G-	S-	=	S+G+	S+	G+	S	S	S	S

R$_1$		X_{1s}	X_{1i}	X_{2s}	X_{2i}	X_{3s}	X_{3i}	X_{4s}	X_{4i}	SE$_1$	SE$_2$	SE$_3$	SE$_4$
Pattern III	*N*1	S+	G+	S+	G-	S+	G-	S-	G-	S	G	G	G
	*N*2	S+	G+	S-	G-	G+	S+	G+	S+	=	G	G	G
	*N*3	S-	G+	S+	G-	G+	S+	S-	G-	=	G	G	G
	*N*4	S	G+	S-	G-	S-	G-	S-	G-	G	G	G	G
Pattern IV	*N*1	S-	G+	S0	G+	S-	G+	S+	G-	S	S	S	G
	*N*2	S+	G+	G-	S-	G0	S-	G+	S+	=	S	S	G
	*N*3	S=	G+	=	S-G+	S+	G+	S-	G+	=	S	S	G
	*N*4	S+	G+	S-	G+	S=	G+	G+	S+	G	G	G	G
Pattern V	*N*1	S+	G+	S-	G+	G+	S-	G+	S-	S	S	S	S
	*N*2	S+	G+	G-	S-	=	S-G+	G0	S-	S	S	S	=
	*N*3	S+	G+	G-	S-	S-	G+	S0	G+	S	S	S	S
	*N*4	S+	G+	G-	S-	S0	G+	S0	G+	S	S	S	S
R$_2$		X_{1s}	X_{1i}	X_{2s}	X_{2i}	X_{3s}	X_{3i}	X_{4s}	X_{4i}	SE$_1$	SE$_2$	SE$_3$	SE$_4$
Pattern I	*N*1	S-	G+	S+	G+	G0	S+	S+	G-	G	S	=	G
	*N*2	S+	G+	G+	S-	=	S-G-	S+	G-	G	=	G	G
	*N*3	S-	G+	G0	S-	=	S+G-	S+	G-	S	S	=	G
	*N*4	S-	G+	S-	G+	S+	G+	G+	S+	G	G	G	G
Pattern II	*N*1	S-	G+	S-	G+	G+	S-	S-	G-	S	S	S	S
	*N*2	S+	G+	G-	S-	G0	S-	=	0	S	S	S	G
	*N*3	S+	G+	G-	S-	G+	S-	S-	G-	S	S	S	S
	*N*4	S-	G+	G-	S-	S+	G+	S+	G+	S	S	S	S
Pattern III	*N*1	S+	G+	S+	G-	G+	S-	S-	G-	G	G	G	G
	*N*2	S+	G+	S-	G-	S+	G+	G+	S+	=	G	G	G
	*N*3	S0	G+	S+	G-	S+	G+	S0	G-	=	G	G	=
	*N*4	S+	G+	G+	S+	G-	S-	G+	S+	S	G	G	G

R_2		X_{1s}	X_{1i}	X_{2s}	X_{2i}	X_{3s}	X_{3i}	X_{4s}	X_{4i}	SE_1	SE_2	SE_3	SE_4
Pattern IV	*N*1	S-	G+	S-	G+	G-	S+	S+	G-	S	S	S	G
	*N*2	S+	G+	G-	S-	S0	G+	G+	S+	G	S	=	G
	*N*3	S0	G+	S-	G+	S+	G+	S+	G-	G	=	S	G
	*N*4	S+	G+	S+	G+	S+	G+	G+	S+	G	G	G	G
Pattern V	*N*1	S0	G+	S-	G+	G+	S-	G0	S-	S	S	S	S
	*N*2	S+	G+	G-	S-	=	S-G+	G0	S-	S	S	S	S
	*N*3	S+	G+	G-	S-	=	S-G+	=	S-G+	S	S	S	S
	*N*4	S+	G+	G-	S-	S-	G+	=	S-G+	S	S	S	S
R_3		X_{1s}	X_{1i}	X_{2s}	X_{2i}	X_{3s}	X_{3i}	X_{4s}	X_{4i}	SE_1	SE_2	SE_3	SE_4
Pattern I	*N*1	S+	G+	S-	G+	S-	G+	S+	G-	S	S	S	G
	*N*2	S+	G+	G-	S+	G0	S-	G-	S+	=	=	G	G
	*N*3	S-	G+	G+	S-	S0	G+	S+	G-	S	S	S	G
	*N*4	G+	S-	G+	S-	S-	G+	G+	S+	G	G	G	G
Pattern II	*N*1	S-	G+	S-	G+	G+	S-	S+	G+	S	S	S	S
	*N*2	S+	G+	G-	S-	G0	S-	S0	G+	S	S	S	=
	*N*3	S+	G+	G-	S-	S-	G+	S0	G+	S	S	S	S
	*N*4	G0	S-	G-	S-	G0	S+	S+	G+	S	S	S	S
Pattern III	*N*1	S+	G+	G+	S+	G+	S+	S-	G-	S	G	G	G
	*N*2	S+	G+	=	0	=	S+G+	G+	S+	=	G	=	G
	*N*3	G+	S-	=	S-G-	=	S+G+	S-	G-	=	=	=	S
	*N*4	S+	G+	G+	S+	S-	G-	G+	S+	G	G	G	G
Pattern IV	*N*1	S-	G+	S-	G+	S-	G+	G-	S+	S	S	=	G
	*N*2	S+	G+	G-	S-	S-	G+	G+	S+	S	G	G	G
	*N*3	S+	G+	S-	G+	S+	G+	S-	G-	=	S	S	G
	*N*4	S-	G+	S+	G+	S-	G+	G+	S+	G	G	G	G

R_3		X_{1s}	X_{1i}	X_{2s}	X_{2i}	X_{3s}	X_{3i}	X_{4s}	X_{4i}	SE_1	SE_2	SE_3	SE_4
Pattern V	*N1*	S+	G+	S-	G+	S-	G+	S-	G+	S	S	S	S
	N2	G0	S+	G0	S-	=	S-G+	S-	G+	S	S	=	S
	N3	S+	G+	G-	S-	S0	G+	S0	G+	S	S	S	S
	N4	S+	G+	G-	S+	S0	G+	S0	G+	S	S	S	S
R_4		X_{1s}	X_{1i}	X_{2s}	X_{2i}	X_{3s}	X_{3i}	X_{4s}	X_{4i}	SE_1	SE_2	SE_3	SE_4
Pattern I	*N1*	S0	G+	S-	G+	S	G+	S+	G-	G	S	G	G
	N2	S0	G+	G-	S-	G0	S-	G-	S+	=	G	G	G
	N3	G+	S-	G+	S-	=	0	S+	G-	S	S	S	G
	N4	S-	G+	S+	G+	S+	G+	G+	S+	G	G	G	G
Pattern II	*N1*	=	S-G+	S-	G+	G+	S-	S+	G+	S	S	S	S
	N2	S+	G+	G-	S-	=	S-G+	S+	G+	S	S	S	G
	N3	S+	G+	G-	S-	S-	G+	G0	S-	S	S	S	S
	N4	G-	S-	G-	S-	S+	G+	S+	G+	S	S	S	G
Pattern III	*N1*	S+	G+	G+	S+	G+	S+	S-	G-	S	G	G	G
	N2	S+	G+	=	S+G+	=	+	S+	G+	=	=	=	G
	N3	G+	S-	S+	G-	G+	S+	S0	G-	=	=	=	S
	N4	S+	G+	S+	G+	G-	S-	=	+	=	G	G	G
Pattern IV	*N1*	G+	S-	S-	G+	S-	G+	=	S+G-	G	G	G	G
	N2	S+	G+	G-	S-	S+	G+	G0	S+	G	G	G	G
	N3	S0	G+	S0	G+	S+	G+	S+	G-	G	=	S	G
	N4	S+	G+	S+	G+	S+	G+	G+	S+	G	G	G	G
Pattern V	*N1*	S0	G+	S-	G+	S-	G+	S+	G-	G	S	G	G
	N2	S0	G+	G-	S-	G0	S-	G-	S+	=	G	G	G
	N3	G+	S-	G+	S-	=	0	S+	G-	S	S	S	G
	N4	S-	G+	S+	G+	S+	G+	G+	S+	G	G	G	G

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