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THREE ESSAYS ON POLICIES TOWARDS RISK

By

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ABSTRACT

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By

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Payment systems facilitate all types of trade in monetary economies by providing a range of mechanisms including instruments and operating procedure through which transactions are settled. A break down or malfunction of the system can be seriously costly. In this regard, governments may implement policies to keep the payment system stable. As the former Federal Reserve Board Chairman Greenspan (2004) suggests, IT IS IMPORTANT TO UNDERSTAND THE MANY SOURCES OF *RISK* AND *UNCERTAINTY THAT POLICYMAKERS* FACE.

Chapter 1: "Intraday credit risks in a real time gross settlement system" identifies credit risks in the market and analyzes intraday credit policy in a large-value real time gross settlement payment system. A large-value interbank payment (LVIP) system such as Fedwire is the backbone of the payment systems, since it provides finality of settlements. Usually the LVIP system depends on the central bank for intraday credit (or daylight overdrafts). Credit risks in the market induce the central bank to implement several policies to manage risks on intraday credit. The apprehension of an economy about potential loss from credit risks is a determinant of the policy choice. Impediments to conducting monetary policy toward shocks from payment system credit risks can be a source of this apprehension. Quantitative limits (or caps) can be used when the economy fails to take into account potential loss from credit risks in the payment system. Price can be used when the economy puts greater weight on this potential loss. Since there are two types of credit risk in the market, it is enough to employ two tools. The model illustrates

that a price and collateral policy achieves the highest social welfare among other sets of polices.

Chapter 2: "Payment instruments and the central bank" explores the sensitivity of payment system stability to macroeconomic shocks through the assets (from lenders' viewpoint) used as media of exchange. Specifically, this chapter studies a "payment instrument policy," such as the one implemented by the Bank of Korea, to promote usage of a certain type of payment instrument to stabilize the payment system. This chapter analyzes two payment instruments, promissory notes (non-intermediated credit) and bills of exchange (intermediated credit) using a simple partial equilibrium model of the credit market with moral hazard. The model shows that bills of exchange have a lower market risk than promissory notes under negative macroeconomic shocks. If, therefore, both promissory notes and bills of exchange are used in addition to fiat money, systems with a greater proportion of promissory notes in circulation will be riskier and more prone to collapse than systems with fewer promissory notes. The model also shows that a subsidy for bills of exchange is better than a tax on promissory notes.

Chapter 3: "Production uncertainty and private information in a search theoretic model" is a methodological exercise looking at production uncertainty and private information in a search theoretic model based on Williamson and Wright (1994). Different types of uncertainty generate different effects: the model shows that there exist more equilibria in the production uncertainty case than under the no production uncertainty case. Comparative statics show that an increase of production uncertainty raises possible returns from choosing the bad technology. These findings support agents' choice under a macroeconomic shock and generate important implications for designing policies to alleviate risk as in Chapter 2.

To my parents

And to my wife

Jeongjin

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Intraday credit risks in a real time gross settlement system

1. Introduction

Payment systems are the instruments, organizations, operating procedures, and information and communications systems used to initiate and transmit payment messages from payer to payee and to settle payments (Balino et al. 1994). The payment system often refers only to a large-value interbank payment system such as Fedwire of the Federal Reserve System or TARGET of the European Central Bank. Payment systems facilitate all types of trade in monetary economies, and accordingly a breakdown[1] of a system can be seriously costly. For example, if there is a severe shock such as settlement failure from one or more banks which makes a payment system fail, then the ability to make payments would be impaired. This would cause serious disruptions in goods markets and instability in financial markets. In order to remove or alleviate this possibility or "risks", the central bank implements payment polices. In this paper, I analyze the intraday credit policy tools implemented by the central bank under a large-value real time gross settlement (hereafter RTGS) system with several types of exogenous risks.

[1] Bank failures occur when banks are unable to meet the demands of their creditors (in earlier times these were note holders; later on, they were more often depositors) [Grossman 2003]. During the U.S. National Banking Era (1863-1913), one method of dealing with banking panics in light of the payment system was for the bankers of a city to pool their resources, through the local bankers' clearinghouse and to jointly guarantee the payment of every member banks' liabilities (Gorton 1985a, b).

As Greenspan (1996) suggests, the central bank needs to improve risk management within payment systems themselves. This rationale shows the importance of a careful study of how to implement intraday credit policy more effectively. Though many combinations of policy exist in payment systems, I consider the three basic elements employed by the central banks: price, quantitative limits (also known as 'caps') and collateral. Among 15 RTGS systems to which the central banks provide intraday credit, 12 systems employ a price[2] and collateral policy and the other three[3] employ a price, caps and collateral policy (Bank for International Settlements [BIS] 2005). In this regard, the questions that I am going to study are why every central bank employs multiple tools for managing the risks within a seemingly simple interbank payment system and which set of tools is efficient to the economy.

Existing studies do not pay much attention to these questions[4] but mainly focus on choosing efficient policy tools under limited types of credit risk. For price, Mills (2004b) and Lacker (1997) are typical. Mills (2004b) considers endogenized credit risk committed by debtors and justifies charging interest on intraday credit by arguing monitoring is costly. Lacker (1997) supports the interest charge policy not only for overnight but also for intraday credit. But in his model, the interest charge policy is only an outcome of the non-interest bearing overnight reserve requirement. Some other economists such as Faulhaber et al. (1990) and Angell (1991) also argue for charging interest on intraday credit. There are also arguments for quantitative constraints in Rochet and Tirole (1996),

[2] Only the Federal Reserve charges it in the form of explicit interest, other central banks charge it in the form of a haircut on collateral.

[3] These three systems employ all three tools: the Fedwire system of U.S. (partial collateralization), CHAPS Euro system of U.K. and E-RIX system of Sweden.

[4] Furfine and Stehm (1998) analyze G-10 countries' intraday credit policy tools as of 1996. They identify private and social costs regarding RTGS system. They demonstrate the cost structure of a central bank determines the choice of intraday credit policy tools.

Kahn and Roberds (2001), and Martin (2004). Rochet and Tirole (1996) show the same rationale for the price of intraday credit as Mills (2004b), but they implicitly consider credit risk committed by borrowers of intraday credit and argue that a cap (or collateral) is a better means to control the overuse of intraday credit than price because of incomplete information. Kahn and Roberds (2001) implicitly consider credit risk committed by borrowers of intraday credit from the central bank (CB) since they do not consider private IOUs. They argue that collateral with free intraday credit can eliminate the liquidity shortage and restore the first-best consumption allocation. Martin (2004) considers an endogenous choice of risk committed by borrowers of intraday credit. He supports a zero intraday interest rate. He argues collateral is the effective means of controlling risk.

In the real world the credit provided by the central bank (or daylight overdraft) is used to measure the central bank's potential exposure to an end-of-day settlement failure (Milano 1991). Recent huge increases in the volume and the value of payments put central banks at greater potential loss as they provide intraday credit to the system. For instance, the average *daily* volume of funds transfer through Fedwire[5] was 296.4 thousand transactions, and average daily value of transfers was $857.5 billion as of 4th quarter of 1994. The figure became 513.2 thousand transactions and $2,007.2 billion as of 4th quarter of 2004. During the same period, the amount of intraday credit (daylight overdraft) provided by the Federal Reserve has grown at a slower pace[6]. The amount of peak daily intraday credit increased[7] from $66.4 billion as of 4th quarter of 1994 to $111.5

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[5] http://www.federalreserve.gov/paymentsystems/fedwire/fedwirefundstrfqtr.htm

[6] The fee for the use of intraday credit has been changed: 24 basis points in 1994, 36 basis points since 1995.

[7] http://www.federalreserve.gov/PaymentSystems/psr/dlodpeakqtr.htm.

billion as of 4th quarter of 2004. These dramatic increases also highlight the importance of the RTGS system and accordingly intraday credit policy to the financial systems.

I modify Freeman's isolated island overlapping generation (OLG) model (1996b, 1999) to analyze intraday credit policy. Freeman's model is useful because 1) as agents are spatially separated, money works as a medium of exchange; 2) private debt is incurred between two parties as a payment instrument; 3) private money (IOUs) is repaid by fiat money; and 4) there can arise liquidity problems which lead to a role for a CB[8]. Because his model was originally designed to study the use of open-market operations and the discount window in conducting monetary policy in environments with a liquidity shortage, I adjust several of its components. I adjust the timing of events so that an explicit private intraday credit market emerges. This is important because the price in the private intraday credit market works as a benchmark for the CB's pricing rule. The risk premium for intraday credit is a major ingredient for this benchmark. I substitute 'one settlement stage and one consumption stage' with two groups of stages each consisting of one consumption stage followed by one settlement stage. During each settlement stage the intraday credit market is open: one for financing intraday credit and the other for repaying it. Then, following Zhou (2000), I reinterpret the debt settlement market as a payment clearing market, resale of debt as private intraday borrowing/lending of reserves, and the CB's injection of liquidity as intraday credit lending rather than an open market operation or discount window lending. Finally, I introduce an additional type of credit risk. Creditors provide overnight consumption credit (in other words, consumption debt)

[8] As Mills (2004b) noted, one can interpret a central bank as a private clearinghouse that is separate from the other agents. As noted in Green (1997), the liquidity-providing institution in the model can be either a public or private one. This is a payment system version of the inside and outside money debate which remains an open question. Refer to Williamson (1999) for a further review on historical debates about inside and outside money.

to debtors. Creditors face the risk that debtors will not repay this loan (I will call this type I risk). Users of intraday credit (in other words, payment debt, which arises only in the process of payment and settlement) may not repay the intraday credit (I will call this type II credit risk). As a payment model without explicit production processes, creditors have no opportunity to invest. The type II credit risk, however, can be a proxy for creditors' moral hazard in the model. I analyze the intraday credit policy tools under these two types of credit risks.

One contribution of the essay is to identify credit risks in the market which explain the combined usage of intraday credit policy tools in the real world. The model considers two types of credit risk. This consideration has more meaning than a simple addition of two credit risks. It serves as a qualitatively different approach to modeling the RTGS system. Removing or alleviating type I credit risk and type II credit risk are the goals of intraday credit policy. In order to manage a couple of credit risks, a combination of tools is implemented. Price deals with both type I and II credit risks. A cap works for type I credit risk. Collateral is an effective tool to avoid type II credit risk.

A practical contribution is a direct calculation and comparison of welfare under possible sets of three tools. Under credit risk free[9] circumstances, the Friedman rule works in the intraday credit market. Different from that implication, however, under credit risks the model shows that a positive interest rate on intraday credit set by the CB could achieve higher social welfare than a zero interest rate. Free intraday credit with caps policy can be implemented by the CB, when the CB does not take serious consideration of potential loss from credit risks. Even though there may be some

[9] Strictly speaking, we need not analyze the RTGS system, since the DNS system is better under credit risk free circumstances.

justification for caps, the model illustrates that collateral is better than caps in terms of social welfare. A price policy is implemented when the CB places significant weight on potential loss. Finally, one of the policy implications from the illustration of the model is that it is desirable to employ a price and collateral policy to achieve higher social welfare.

1.1. Overview of payment system

There are two alternative systems[10]: RTGS and delayed net settlement (DNS) system. An RTGS system is one in which payment messages are processed in a timely manner and settlement occurs immediately on bilateral and gross bases. The RTGS system has advantages in finality of payment, security and timeliness. A DNS system is one in which payment messages are processed continuously but are settled at a given time (usually end of the day) at which incoming and outgoing messages are netted. Under the DNS system, synchronization of payment messages does not matter. The DNS system is efficient but vulnerable to systemic risk. For example, when a participant of the DNS system commits settlement failure, then all messages from him should be unwound and all net amounts should be recalculated. Some other participants may be forced to commit settlement failure which otherwise would not happen. This leads the system to be shut down.

There are five payment system risks: credit risk, liquidity risk, systemic risk, operational risk and legal risk. According to the BIS (2001), credit risk is the risk that a party within the system will be unable fully to meet its financial obligations within the system either when due or at any time in the future. Liquidity risk is the risk that a party within the system will have insufficient funds to meet financial obligations within the system as and when expected, although it may be able to do so at some time in the future.

[10] There is a hybrid of the two systems. This is, however, beyond our scope of discussion.

Systemic risk is the risk that the inability of one of the participants to meet its obligations, or a disruption in the system itself, could result in the inability of other system participants or of financial institutions in other parts of the financial system to meet their obligations as they become due. Such a failure could cause widespread liquidity or credit problems and, as a result, could threaten the stability of the system or of financial markets. Operational risk is related to operational factors such as technical malfunctions or operational mistakes, and legal risk is related to a poor legal framework or legal uncertainties.

In order 1) to remove or alleviate the systemic risk; or 2) to enhance the efficiency of the payment system; or, more fundamentally, 3) to maintain the payment system (or monetary economy), payment (system) policy is implemented. If there are no credit risks, the DNS system is better, and we need no further payment policy. Once there are credit risks, however, the DNS system is inferior. A settlement failure may need a huge amount of money from the CB as a lender of last resort to restore the system, since the DNS system uses multilateral netting at the end of the day. Furthermore, because of the netting procedure, the DNS system may have much higher probability of failure. In the RTGS system, however, settlement failure can be resolved at 'relatively' low cost. As a result[11] of the payment policy, most central banks choose the RTGS system. The RTGS system has one drawback: liquidity shortages owing to imperfect synchronization of ordering of payment messages. If the intraday balance available for payments is too small relative to the value of payments to be made in a given time, it could bring about gridlock which

[11] Finality of the payment is major component to the alleviation of the systemic risk. Even a privately operated, a used-to-be typical DNS system, CHIPS (Clearing House Interbank Payment System) has employed a hybrid system by borrowing the finality of payment from the RTGS system since January 2001 (Coleman 2002).

prevents payments from being executed, and therefore the system becomes less efficient. Central banks, therefore, implement another policy: provision of intraday liquidity in order to enhance the efficiency of the payment system. If a bank cannot repay the intraday credit until the end of the day, the CB suffers a loss. Accordingly, central banks are exposed to credit risks under the RTGS system.

The paper is organized as follows. Section 2 provides the environment of the model. Section 3 solves agents' decisions and presents optimal allocation without intervention. Intraday payment policies will be analyzed in section 4. Section 5 summarizes findings and concludes. Lengthy proofs are in the appendix.

2. The environment

A large number of outer islands are arranged in I pairs around central islands. Each pair contains both of two types of islands, which are called 'bank' and 'debtor' islands. Time is discrete and infinite. There are two types of consumption goods C and D in each period $t \geq 1$. On each island, N two-period-lived agents are born in each period $t \geq 1$. In the first period each island also has N agents (the initial old) who live only in the first period. For simplicity, N is normalized to 1. The central islands consist of three islands: a redemption island where normal repayments of IOUs occur; and an intraday island where an intraday credit market opens, and a payment policy is implemented; and a fraud island where agents who are deviated from their given route arrive before they travel to debtor islands to trade with young debtors. There exists on the intraday island a monetary authority able to issue fiat money, which is noncounterfeitable, unbacked, intrinsically useless, and costlessly exchanged. This authority issues an initial stock of M dollars at period t = 0 to initial old banks.

Each agent born on a bank island ("bank") is endowed at birth with 1 unit of a non-storable good C specific to this island (and with nothing when old). He wishes to consume the good C when young and good D when old. No other consumption is desired. I assume that no agents want to leave a bequest in any form. Because of imperfect synchronization in receipt and consumption, some banks can borrow intraday credit and become 'debtors in the intraday credit market (d.i. in short),' and others can lend intraday credit and become 'creditors in the intraday credit market (c.i. in short).' The utility of a bank is given by the function $U = u(C_{ct}) + u(D_{c,t+1})$ where C_{ct} and $D_{c,t+1}$ represent his consumption of good C when young and good D when old respectively.

Each agent born on a debtor island (each "debtor") is endowed at birth with 1 unit of a non-storable good D specific to his island (and with nothing when old). Agents wish to consume the goods D and C when young without uncertainty, good D when old with some probability.

There is a credit risk in the form of a fraud shock. A fraud is defined as a situation in which the ships of old debtor or of old d.i. banks deviate from the given route on the way to repay their debts (in case of old debtors) or their intraday credit (in case of old d.i. banks); they arrive at the fraud island instead; after that they proceed to trade with young debtors. The agents who arrive at the fraud island are called to be dishonest. Every old debtor has a probability $\Delta_d \in (0,1)$, with which the ships deviate from the given route and therefore old debtors cannot arrive at the redemption island (I call this type I credit risk). Each debtor has an equal chance to have a fraud shock and it is not known until a debtor departs his island. Old d.i. banks have a probability $\Delta_c \in (0,1)$, with which the d.i. banks do not repay intraday credit at the intraday island due to the deviation of the ships from the given route on the way to the intraday island (I call this type II credit risk). Each d.i. bank has an equal chance to have a fraud shock, and it is not known until a d.i. bank departs from the redemption island. Each probability is independent of the other. Therefore, the intraday credit risk is union of two types of risks: the credit risk (type I) committed by debtors for overnight credit as in Freeman (1999), Mills (2004a) and Martin (2004), and the credit risk (type II) committed by intraday credit users. The agents in the fraud island have additional chance to trade with young debtors. Notice that the number of borrowers who experience fraud shocks are equal to that of lenders who are not repaid; therefore, young debtors do not suffer from loss of trading partners from the fraud shock.

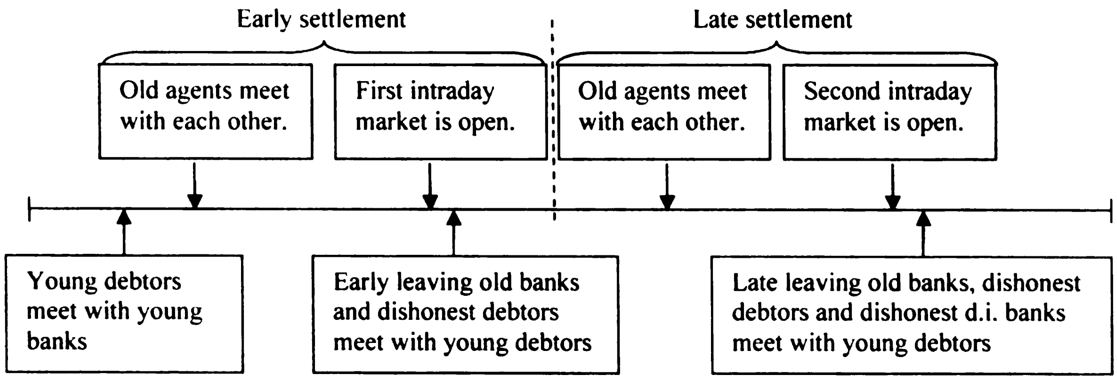
The expected utility of a debtor is given by the function $V = v(D_{dt}) + v(C_{dt}) + \Delta_d v(D_{d,t+1})$, where D_{dt}, C_{dt} and $D_{d,t+1}$ represent his consumption of good D and of good C at period t, and of good D at period t+1, respectively. I use logarithmic utility functions for both debtor and bank.

As in the timing of meeting, the arrivals and departures are assumed not to be fully synchronized. At the settlement stage, all banks arrive in the central island, but only a fraction of debtors arrive. Let $\lambda \in [0,1)$ be the intrinsic fraction of early arrival debtors. Since this probability is independent of the probability of a fraud shock, the actual fraction of early arrival debtors under credit risk is $\lambda^* \equiv \lambda (1 - \Delta_d) \in [0, 1 - \Delta_d)$. Now I describe the timing of banks' departure. Before the remaining $1 - \lambda^*$ debtors arrive, $1 - \alpha$ old banks leave the central island, where $\alpha \in [0,1]$. For an individual agent, the timing of his or her arrival or departure (early or late) is completely random and is learned only before the early settlement. Table 1.1 summarizes the timing of arrival and departure. Old banks may fall into one of the three groups: Y, banks who benefit from leaving early; Y^*, banks who benefit from leaving early and not repaying the intraday credit; and Z, banks who benefit from leaving late.

Table 1.1: Timing of arrival and departure with fraud risks (banks' action)

bank \ debtor		λ^* (early-arriving) $\equiv \lambda(1-\Delta_d)$	$1-\lambda^*$ (late-arriving)	
			$(1-\lambda)(1-\Delta_d) = 1-\lambda^*-\Delta_d$	Δ_d (fraud)
$(1-\alpha)$ (early-leaving: group Y)	$(1-\Delta_c)$ (not fraud)	Consume.	Borrows from late leaving Does repay intraday credit	bank and consumes **Cannot repay**
	Δ_c (fraud)		**Does not repay his intraday credit and proceeds to consume (Y^*)**	**Cannot and does not repay**
α (late-leaving: group Z)		Lend to an early leaving bank. Sometimes is repaid and consume; sometimes is not repaid.	Consume	Can not consume.

Figure 1.1: Modeling timing



3. The model

This section studies the intraday market under credit risks. Specifically, I start with intraday credit traded in the market without intervention into the intraday credit market by the CB.

3.1. Debtors' problem

Debtors want to consume both good C and good D when they are young and have an additional chance to consume good D with a probability of Δ_d when old. Let p_{Dt} denote the price of good D and p_{Ct} denote the price of good C at period t. Because of a fraud shock, when a debtor buys good C, he borrows overnight credit at a discount factor $\delta^*_t < 1$ from the trading partner bank. More specifically, at period t, a young debtor visits his companion bank's island and buys C_{dt} of good C at the price of p_{Ct} and pays $\delta^*_t \cdot h^*_t$ dollars with his newly-issued IOU h^*_t discounted by δ^*_t. After consumption, he returns to his home island. The possible trading partner of a young debtor is either an old bank or an old debtor. The young debtor sells $(1 - D_{dt})$ units of his endowment at the price of p_{Dt}. and consumes D_{dt}. Therefore the young debtor receives m^*_t units of fiat money from the transaction. Here m^*_t is the debtor's nominal acquisition of fiat money or is the debtor's nominal demand for fiat money. The next morning, the debtor visits the redemption island, where he pays back the debt, h^*_t with his fiat money m^*_t with probability $1 - \Delta_d$. If he has a fraud shock, he arrives at the fraud island instead. He can use the fiat money to buy good D. The representative debtor maximizes his utility by choosing the consumption bundle $(D_{dt}, C_{dt}, D_{d,t+1})$ to maximize his utility subject to his budget constraints. The following are optimization problem and budget constraints.

$$\max_{C_{dt}, D_{dt}, D_{d,t+1}} \log D_{dt} + \log C_{dt} + \Delta_d \log D_{d,t+1} \quad (1.1)$$

subject to:

$$p_{Dt} D_{dt} + p_{Ct} C_{dt} + (1-\delta_t^*) h_t^* + \Delta_d p_{D,t+1} D_{d,t+1} = p_{Dt} + \Delta_d h_t^* . \quad (1.2)$$

A debtor's income (right hand side of budget constraint (1.2)) consists of one endowment ($p_{Dt} \cdot 1$) and one conditional income. When the debtor has a fraud shock, he gets h_t^*, therefore his expected income from the fraud shock is $\Delta_d h_t^*$. From the trade itinerary, one can find the following feasibility relations for the debtors' budget constraint:

$$p_{Ct} C_{dt} = \delta_t^* h_t^* \quad (1.3)$$

$$p_{Dt}(1 - D_{dt}) = m_t^* \quad (1.4)$$

$$p_{D,t+1} D_{d,t+1} = m_t^* . \quad (1.5)$$

From the budget constraint (1.2) and the relations of (1.3) through (1.5), the following is derived

$$m_t^* = h_t^* . \quad (1.6)$$

Using (1.3) through (1.6), the optimization problem becomes the choice of money demand m^*_t from the choice of consumption bundle:

$$\max \log\left(1 - \frac{m_t^*}{p_{Dt}}\right) + \log \frac{\delta_t^* m_t^*}{p_{Ct}} + \Delta_d \log \frac{m_t^*}{p_{D,t+1}} . \quad (1.7)$$

The resulting first order condition for the debtor's problem is

$$-\frac{1}{p_{Dt}-m_t^*}+\frac{1}{m_t^*}+\frac{\Delta_d}{m_t^*}=0. \qquad (1.8)$$

3.2. Banks' problem

At period t, a young bank sells $(1 - C_{ct})$ units of his endowment to a young debtor in exchange for l^*_t dollars of IOU in the morning. He consumes remaining C_{ct} units of C good. At period t + 1, he goes to the redemption island to get his l^*_t paid back. When the debtors repay their overnight loans, the bank can consume. The bank proceeds to trade with young debtors. Let $D^G_{c,t+1}$ denote the amount of good D consumed by the old banks in group G = Y, Y^* and Z. An old bank in group Y can borrow liquidity (payment debt) at discount factor $\rho^*_{t+1} < 1$ at the intraday market from banks in group Z. Let q^*_{t+1} denote the amount of intraday credit transacted in the intraday market and valued at the end of the market. The total amount of intraday credit in the market, $\rho^*_{t+1}q^*_{t+1}$, cannot exceed cash balances available to a bank at the settlement stage, $\lambda^* l^*_t$. Every early-leaving old bank returns to the redemption island to be repaid by the late-arriving old debtors who do not have a fraud shock.

As I assumed, there is a probability Δ_c with which the d.i. bank does not repay its intraday credit in the second settlement stage. When a d.i. bank has a fraud shock, he can have one more chance to visit a debtor island to consume good D again (group Y^*). Let $\Delta_I \equiv \Delta_c + \frac{(1-\Delta_c)\Delta_d}{1-\lambda^*}$ denote the overall fraud rate in the intraday market.

When banks become old, they can participate in the intraday credit market and consume good D. If they are revealed to be early-leaving, then they have utility

$\log(D^{Y}_{c,t+1})$. In addition to that, if they have fraud shock after they are repaid by late arrival debtors, they can enjoy utility $\log(D^{Y*}_{c,t+1})$ with a probability $\Delta_c - \frac{\Delta_c \Delta_d}{1-\lambda^*}$. If they are revealed to be late-leaving, then they can enjoy utility $\log(D^{Z}_{c,t+1})$; but they can only consume when their intraday credit is repaid [$1 - \Delta_d - \left(\Delta_c - \frac{\Delta_c \Delta_d}{1-\lambda^*}\right)$]. Because of the risk premium, all consumption after second settlement stage is discounted with discount factor $\rho^*_{t+1} \in (0, 1]$. Now the bank's problem is to choose a consumption bundle (C_{ct}, $D^{Y}_{c,t+1}$, $D^{Y*}_{c,t+1}$, $D^{Z}_{c,t+1}$) to maximize his utility subject to budget and liquidity constraints.

$$\max_{\substack{C_{ct}, D^{Y}_{c,t+1}, \\ D^{Y*}_{c,t+1}, D^{Z}_{c,t+1}}} \log C_{ct} + \left\{ \begin{array}{l} (1-\alpha)\left[\log D^{Y}_{c,t+1} + \left(\Delta_c - \frac{\Delta_c \Delta_d}{1-\lambda^*}\right)\rho^*_{t+1} \log D^{Y*}_{c,t+1}\right] \\ +\alpha\left[1-\Delta_d - \left(\Delta_c - \frac{\Delta_c \Delta_d}{1-\lambda^*}\right)\right]\rho^*_{t+1}\log D^{Z}_{c,t+1} \end{array} \right\} \tag{1.9}$$

subject to:

$$p_{Ct} = p_{Ct} C_{ct} + \delta^*_t l^*_t \text{ (when young)} \tag{1.10}$$

$$(1-\lambda^*)\rho^*_{t+1} l^*_t + \lambda^* l^*_t = p_{D,t+1} D^{Y}_{c,t+1} \text{ (early-leaving old)} \tag{1.11}$$

- When his IOU is repaid late, he needs to borrow l^*_t units of money with discount factor ρ^*_{t+1}. When his IOU is repaid early, he just proceeds to consume.

$$(1-\Delta_d) l^*_t = p_{D,t+1} D^{Y*}_{c,t+1} \text{ (early-leaving old: additional dishonest)} \tag{1.12}$$

- When he uses intraday credit as he leaves the intraday island early before he is repaid by the debtor in the redemption island, he needs to repay intraday credit with his own non-fraud overnight loan l^*_t. But with a probability Δ_c, he has fraud shock on the intraday credit market after he is repaid by the corresponding late arrival debtor. If he has a fraud shock, he has another chance to consume good D.

$$(1-\Delta_d)l_t^* + [1-\Delta_l - \rho_{t+1}^*]q_{t+1}^* = p_{D,t+1}D_{c,t+1}^Z \text{ (late-leaving old)} \tag{1.13}$$

- When a corresponding debtor does not have a fraud shock, a late-leaving old bank can be repaid l^*_t, some of the late-leaving banks lend this money as intraday credit to early-leaving banks. This intraday credit can earn $(1 - \rho^*_{t+1})$ q^*_{t+1} when the d.i. bank repays safely, but loses q^*_{t+1} if the corresponding d.i. bank cannot repay.

$$\lambda^* l_t^* - \rho_{t+1}^* q_{t+1}^* \geq 0 \text{ (intraday liquidity constraint)} \tag{1.14}$$

- The total amount of the intraday credit in the market, $\rho^*_{t+1}\, q^*_{t+1}$, cannot exceed cash balances available to a bank at the first settlement stage, $\lambda^*\, l^*_t$.

Using the binding budget constraints the optimization problem can be adjusted from the choice of consumption bundle to the choice of financial asset bundle, l^*_t and q^*_{t+1}. Now Lagrangian function of the optimization problem is:

$$\max_{l_t^*, q_{t+1}^*} L_c = \log\left(1 - \frac{\delta_t^* l_t^*}{p_{Ct}}\right) + \left\{ \begin{array}{l} (1-\alpha)\left[\log\left(\dfrac{(1-\lambda^*)\rho_{t+1}^* l_t^* + \lambda^* l_t^*}{p_{D,t+1}}\right) + \left(\Delta_c - \dfrac{\Delta_c \Delta_d}{1-\lambda^*}\right)\rho_{t+1}^* \log\left(\dfrac{(1-\Delta_d)l_t^*}{p_{D,t+1}}\right)\right] \\ +\alpha\left[1 - \Delta_d - \left(\Delta_c - \dfrac{\Delta_c \Delta_d}{1-\lambda^*}\right)\right]\rho_{t+1}^* \log\left(\dfrac{(1-\Delta_d)l_t^* + \left[1-\Delta_l - \rho_{t+1}^*\right]q_{t+1}^*}{p_{D,t+1}}\right) \end{array} \right\}$$
$$+\mu[\lambda^* l_t^* - \rho_{t+1}^* q_{t+1}^*] \tag{1.15}$$

where μ is a Lagrangian multiplier.

First order conditions for the bank's problem are

$$-\frac{\delta_t^*}{p_{Ct}-\delta_t^* l_t^*}+\left\{\begin{array}{l}(1-\alpha)\left[\frac{1}{l_t^*}+\left(\Delta_c-\frac{\Delta_c\Delta_d}{1-\lambda^*}\right)\rho_{t+1}^*\frac{1}{l_t^*}\right]\\ +\alpha\left[1-\Delta_d-\left(\Delta_c-\frac{\Delta_c\Delta_d}{1-\lambda^*}\right)\right]\rho_{t+1}^*\frac{(1-\Delta_d)}{(1-\Delta_d)l_t^*+\left[1-\Delta_l-\rho_{t+1}^*\right]q_{t+1}^*}\end{array}\right\}+\mu\lambda^*=0 \tag{1.16}$$

and

$$\alpha\left[1-\Delta_d-\left(\Delta_c-\frac{\Delta_c\Delta_d}{1-\lambda^*}\right)\right]\frac{\left[1-\Delta_l-\rho_{t+1}^*\right]}{(1-\Delta_d)l_t^*+\left[1-\Delta_l-\rho_{t+1}^*\right]q_{t+1}^*}-\mu=0. \tag{1.17}$$

3.3. Equilibrium

In this section, I consider a symmetric competitive equilibrium.

Definition *Given* α, λ^*, *and M, a symmetric competitive equilibrium is a set of prices* (p_{Dt}, p_{Dt+1}, p_{Ct}, δ^*_t, ρ^*_{t+1}) *and a set of allocations* (D_{dt}, $D_{d,t+1}$, $D^Y_{c,t+1}$, $D^{Y*}_{c,t+1}$, $D^Z_{c,t+1}$, C_{dt}, C_{ct}) *such that*

(i) Given prices, a set of allocations (D_{dt}, $D_{d,t+1}$, $D^Y_{c,t+1}$, $D^{Y*}_{c,t+1}$, $D^Z_{c,t+1}$, C_{dt}, C_{ct}) *solves maximization problems of (1.1) and (1.9).*

(ii) Markets clear:

For overnight credit market,

$$h_t^* = l_t^*. \tag{1.18}$$

For good D and good C markets

$$D_{dt}+\Delta_d D_{d,t+1}+(1-\alpha)\left[D^Y_{c,t+1}+\left(\Delta_c-\frac{\Delta_c\Delta_d}{1-\lambda^*}\right)D^{Y*}_{c,t+1}\right]+\alpha\left[1-\Delta_d-\left(\Delta_c-\frac{\Delta_c\Delta_d}{1-\lambda^*}\right)\right]D^Z_{c,t+1}=1 \tag{1.19}$$

$$C_{ct} + C_{dt} = 1. \quad (1.20)$$

For money supply and demand,

$$m_t^* = M. \quad (1.21)$$

If liquidity constraint (1.14) is not binding (or $\mu = 0$), intraday credit will be provided at a price ρ^*_i which reflects only the intraday credit's risk of fraud. Banks who leave early will inelastically demand intraday credit at any price so the price of intraday credit will be determined by the supply of that by late-leaving banks (1.17), which yields

$$\rho_i^* = 1 - \Delta_l = (1 - \Delta_c)(1 - \frac{\Delta_d}{1 - \lambda^*}). \quad (1.22)$$

In order to find ρ^*_{t+1} when liquidity constraint (1.14) is binding (or $\mu > 0$), notice that the clearing condition[12] of the intraday credit market requires

$$\alpha q_{t+1}^* = (1 - \alpha)(1 - \lambda^*) l_t^*. \quad (1.23)$$

If liquidity constraint (1.14) is binding (or $\mu > 0$), from (1.14) and (1.23) we have

$$\rho_{t+1}^* = \frac{\alpha \lambda^*}{(1 - \alpha)(1 - \lambda^*)}. \quad (1.24)$$

This means the proportion of intraday suppliers $[\alpha \lambda^*]$ is smaller than the proportion of intraday credit demanders among agents adjusted by the ρ^*_i: $[(1 - \alpha)(1 - \lambda^*)] \rho^*_i$. This result can be extended to the following equilibrium results of intraday credit market price ρ^*_{t+1}:

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[12] The intraday credit is traded at a certain price. In the second opening of the intraday market, the intraday credit should be repaid by the d.i. banks to the c.i. banks. The potential amount of intraday credit repaid is $(1 - \alpha)(1 - \lambda^*) l^*_t$; therefore at the end of the second opening of the intraday credit market, demand side of the intraday market implies $\alpha q^*_{t+1} = (1 - \alpha)(1 - \lambda^*) l^*_t$.

$$\rho^*_{t+1} = \frac{\alpha\lambda^*}{(1-\alpha)(1-\lambda^*)} < \rho^*_t \text{ if and only if } \alpha < \frac{(1-\Delta_c)(1-\Delta_d-\lambda^*)}{1-\Delta_d-\Delta_c(1-\Delta_d-\lambda^*)} \qquad (1.25)$$

$$\rho^*_{t+1} = \rho^*_t \text{ if and only if } \alpha \geq \frac{(1-\Delta_c)(1-\Delta_d-\lambda^*)}{1-\Delta_d-\Delta_c(1-\Delta_d-\lambda^*)}. \qquad (1.26)$$

4. Intraday credit policy

This section analyzes intervention into the intraday credit market by the CB and intraday credit policy including pricing rules under credit risks. I have analyzed the payment system in which liquidity needs arise due to the imperfect synchronization at the settlement stage. When the liquidity constraint is binding (liquidity shortages), there is a role for the CB to improve social welfare by providing liquidity to the system. I restrict attention to two different objectives of the CB. According to the Fed charter, for example, the Fed is supposed to supply an elastic currency and insure financial stability. In this regard, firstly I consider the case when the CB simply provides sufficient liquidity, or liquidity-only objective. If there is no credit risk, then the intraday credit is repaid on the same day. A liquidity shortage can easily be corrected by the simple intraday credit policy by the CB: free lending at zero interest on intraday credit. This implies the liquidity-only objective is sufficient for the CB to resolve the problem. Under credit risks, however, this may not be an appropriate policy, since this objective may cause several problems including possibility of losses from credit risk which may make the financial system unstable. Therefore, the CB needs to provide the necessary amount of liquidity under some constraint considering risks. This is the second objective of the CB. The objectives of the CB are characterized by a pricing rule of intraday credit in the next subsection.

The liquidity provision by the CB is characterized by endogenization of α in the model. When there are liquidity shortages at the settlement stage, receipts and payments are not synchronized. As in (1.24), liquidity shortages (or binding liquidity constraint) is expressed by the synchronization parameters (λ^*, α) in the model. Since the CB provides liquidity to the banks and relaxes the binding liquidity constraint on them, the provision by the CB implies synchronization of receipts and payments. In other words, the CB modifies the payment timing of banks [α (λ^*)] under given receipt (debtors' repayment) timing. Once the CB relaxes the binding liquidity constraint by providing intraday liquidity to the system this liquidity can enable the economy to obtain an equilibrium which would otherwise not be achieved. One may consider this as one rationale of intervention by the CB.

4.1. The central bank and pricing rule

A safe and efficient payment system is critical for the effective functioning of financial systems [BIS, 2001]. Central banks put safety[13] as the first criterion for designing payment systems; therefore most central banks choose RTGS systems. However, the RTGS system is inefficient. Shortages of liquidity owing to imperfect synchronization of the ordering of payment messages are typical. Central banks provide intraday liquidity to enhance the efficiency of the payment system as well as social welfare.

[13] Finality of the payment is major component to the alleviation of the systemic risk.

Since intraday credit policy implies the CB supplying fiat money during the day and absorbing it at the end of the day, if there are credit risks[14], an intervention into the intraday credit market by the CB affects the money stock. If there is a fraud shock on the intraday credit market, then the amount extracted by fraud is not repaid to the CB at the end of the day and therefore the money stock as of the end of day increases by that amount; therefore there is a positive change in money stock over all. If the CB provides intraday credit at a discount factor ρ^*_{CB}, then the CB earns $(1 - \rho^*_{CB})$ and therefore money stock decreases by that proportion; therefore there is a negative change in the money stock over all. During the day, besides these changes there can be additional changes in the money stock by the CB as a result of monetary policy. Now we have following law of motion in money supply

$$M^*_{t+1} = M^*_t + \left[\Delta_t - (1 - \rho^*_{CB})\right] N^*_t + \text{change in } M^*_t \text{ from monetary policy}$$

where N^*_t denotes the amount of intraday credit provided by the CB at period t.

Following Greenspan's (1996) suggestion that the central bank needs to improve risk management within payment systems themselves, I consider only a change in money stock during the payment procedure. Therefore, I assume that there is no smoothing operation by the CB, or the change in M^*_t from monetary policy equals zero. Now we have

$$M^*_{t+1} = M^*_t + \left[\Delta_t - (1 - \rho^*_{CB})\right] N^*_t . \tag{1.27}$$

[14] I consider end of day money stock. In the credit risk free case, the provision of intraday credit by the CB can affect the money supply. Since the price of intraday credit is $\rho = 1$ (or zero interest), however, and the intraday credit is repaid at the end of the day for certain, the money supply does not change.

I consider two cases of CBs whose objectives are presumably different from each other: a CB with a liquidity-only objective (CB_L) and a CB providing necessary liquidity under risk (more precisely, loss from risk) management[15] objective (CB_R). Necessary liquidity means that the amount of liquidity the CB provides does not decrease the overall money stock in (1.27). Since the only concern of the CB_L is liquidity, I restrict the case in which the CB_L ignores the risk, therefore the CB_L provides liquidity free (or $\rho_{CB}^{*} = 1$). Since CB_R provides at least necessary liquidity and minimizes its loss from the provision of intraday credit at the same time, CB_R sets the price at $1 - \Delta_l$ or $\rho_{CB}^{*} = 1 - \Delta_l$. Notice that the price of intraday credit set by the CB_R is equivalent to the price of intraday credit in the private market when the liquidity constraint is not binding. In other words, the CB_R comes to set its price by taking the risk premium into consideration. Since the CB's loss[16] is $\left[\Delta_l - (1 - \rho_{CB}^{*})\right] N_l^{*}$ as in (1.27), when the CB_R sets the price as $\rho_{CB}^{*} = 1 - \Delta_l \left(= \rho_i^{*}\right)$, the CB_R does not experience expected losses from the intraday credit provision.

The CB_L makes a private intraday credit market collapse; the price of the intraday credit the CB sets is lower than that in the competitive market, therefore the CB crowds out private lenders (no late-leaving banks lend their money to the market). As a result, the demand for the intraday liquidity is solely supplied by the CB.

[15] The loss from the risk in the RTGS system becomes the burden of the general public either in the form of increased tax or higher prices (Clair 1991)

[16] I implicitly take this as a policy loss function.

4.2. Optimization and equilibrium

I keep all the assumptions in the previous section. The budget constraint of the debtor's problem is the same as in the previous section,

$$p_{Dt}D_{dt} + p_{Ct}C_{dt} + (1-\delta_t^*)h_t^* + \Delta_d p_{D,t+1}D_{d,t+1} = p_{Dt} + \Delta_d h_t^*.$$

We have an identical problem to the previous section for a debtor.

In the banks' case, we need to adjust the objective of late-leaving banks. Under the CB_L, since late-leaving banks do not provide intraday credit, they can enjoy utility $\log(D_{c,t+1}^Z)$ once the relevant IOU is repaid; but under the CB_R, late-leaving banks can only consume when their intraday credit is safely repaid[17] [or $1 - \Delta_d - \left(\Delta_c - \frac{\Delta_c\Delta_d}{1-\lambda^*}\right)$].

The bank's problem is

$$\max_{\substack{C_{ct}, D_{c,t+1}^Y, \\ D_{c,t+1}^{Y*}, D_{c,t+1}^Z}} \log C_{ct} + \left\{ \begin{array}{l} (1-\alpha)\left[\log D_{c,t+1}^Y + \left(\Delta_c - \frac{\Delta_c\Delta_d}{1-\lambda^*}\right)\rho_{CB}^* \log D_{c,t+1}^{Y*}\right] \\ +\alpha(1-\Delta_d - A)\rho_{CB}^* \log D_{c,t+1}^Z \end{array} \right\} \quad (1.28)$$

where $A = \begin{cases} 0, \text{ under the CB\_L} \\ \left(\Delta_c - \frac{\Delta_c\Delta_d}{1-\lambda^*}\right), \text{ under the CB\_R} \end{cases}$

subject to:

$$p_{Ct} = p_{Ct}C_{ct} + \delta_t^* l_t^* \text{ (when young)} \quad (1.29)$$

$$(1-\lambda^*)\rho_{CB}^* l_t^* + \lambda^* l_t^* = p_{D,t+1}D_{c,t+1}^Y \text{ (early-leaving old)} \quad (1.30)$$

$$(1-\Delta_d)l_t^* = p_{D,t+1}D_{c,t+1}^{Y*} \text{ (early-leaving old: additional dishonest)} \quad (1.31)$$

[17] I implicitly consider that the CB_R employs a cap. If the CB_R does not employ a cap, this probability becomes $1 - \Delta_d$.

$$(1-\Delta_d)l_t^* = p_{D,t+1}D_{c,t+1}^Z \text{ (late-leaving old).} \tag{1.32}$$

Notice that the provision of the intraday credit by the CB relaxes the liquidity constraint. In other words, the CB can adjust a synchronization parameter α given λ^* to relax the constraint. The optimization problem can be adjusted from the choice of consumption bundle to the choice of financial asset, l^*_t. Now the maximum problem becomes:

$$\max L_c = \log\left(1-\frac{\delta_t^* l_t^*}{p_{Ct}}\right)+\left\{\begin{array}{l}(1-\alpha)\left[\begin{array}{l}\log\left(\dfrac{(1-\lambda^*)\rho_{CB}^* l_t^* + \lambda^* l_t^*}{p_{D,t+1}}\right)\\ +\left(\Delta_c - \dfrac{\Delta_c\Delta_d}{1-\lambda^*}\right)\rho_{CB}^* \log\left(\dfrac{(1-\Delta_d)l_t^*}{p_{D,t+1}}\right)\end{array}\right]\\ +\alpha(1-\Delta_d - A)\rho_{CB}^* \log\left(\dfrac{(1-\Delta_d)l_t^*}{p_{D,t+1}}\right)\end{array}\right\}. \tag{1.33}$$

The first order condition for the bank's problem is

$$\frac{\delta_t^*}{p_{Ct}-\delta_t^* l_t^*} = \frac{1}{l_t^*}\left\{(1-\alpha)\left[1+\Delta_c\left(1-\frac{\Delta_d}{1-\lambda^*}\right)\rho_{CB}^*\right]+\alpha(1-\Delta_d - A)\rho_{CB}^*\right\} \tag{1.34}$$

Since now the money stock is not constant and the timing of the bank's (α) is determined under the equilibrium condition, I modify the definition of equilibrium.

Definition *Given λ^*, M^*_t and ρ^*_{CB}, a symmetric intervention equilibrium is a set of prices (p_{Dt}, $p_{D,t+1}$, p_{Ct}, δ^*_t) and a set of allocations (D_{dt}, $D_{d,t+1}$, $D_{c,t+1}^Y$, $D_{c,t+1}^{Y*}$, $D_{c,t+1}^Z$, C_{dt}, C_{ct}) such that*

(i) Given prices, a set of allocations (D_{dt}, $D_{d,t+1}$, $D_{c,t+1}^Y$, $D_{c,t+1}^{Y*}$, $D_{c,t+1}^Z$, C_{dt}, C_{ct}) solves maximization problems of a representative debtor and a representative bank.

(ii) Markets clear:

For overnight credit market,

$$h_t^* = l_t^* . \tag{1.35}$$

For good D and good C markets

$$D_{dt} + \Delta_d D_{d,t+1} + (1-\alpha)\left[D_{c,t+1}^Y + \left(\Delta_c - \frac{\Delta_c \Delta_d}{1-\lambda^*} \right) D_{c,t+1}^{Y*} \right] + \alpha\left[1 - \Delta_d - A\right] D_{c,t+1}^Z = 1 \tag{1.36}$$

$$C_{ct} + C_{dt} = 1. \tag{1.37}$$

For money supply and demand,

$$m_t^* = M_t^* . \tag{1.38}$$

From the clearing condition for the good D market, we have the following after some algebra

$$T_{CB} \cdot l_t^* = m_{t+1}^* - \Delta_d m_t^* \tag{1.39}$$

where $T_{CB} = (1-\alpha)\left[\left[(1-\lambda^*)\rho_{CB}^* + \lambda^* \right] + \Delta_c \left(1 - \frac{\Delta_d}{1-\lambda^*} \right)(1-\Delta_d) \right] + \alpha(1-\Delta_d - A)(1-\Delta_d)$.

Lemma 1. *When the CB sets price of intraday credit at one, or* $\rho_{CB}^* = 1$ *(or free intraday credit), there exists an equilibrium if* $\alpha = \dfrac{\Delta_c\left[\lambda^*(1-\lambda^*) - \Delta_d(1-\Delta_d) \right]}{\Delta_c(1-\lambda^*)\lambda^* + \Delta_d(1-\Delta_d)(1-\Delta_c-\lambda^*)} (\equiv \alpha_L)$.

Proof: See the Appendix (A.1). □

Lemma 2. *When the CB sets price of intraday credit at* ρ_i^*, *or* $\rho_{CB}^* = 1-\Delta_I$, *there exists an equilibrium if*

$$\alpha = \frac{\Delta_c\left[\lambda^*(1-\lambda^*)-\Delta_d(1-\Delta_d)\right]}{(1-\Delta_d)\Delta_d(1-\lambda^*)+\Delta_c\left[(1-2\Delta_d)(1-\Delta_d)-\lambda^*(\lambda^*-\Delta_d)\right]}\left(\equiv \alpha_{R|\text{with caps}}\right) \quad (1.40)$$

or

$$\alpha = \frac{\Delta_c\left[\lambda^*(1-\lambda^*)-\Delta_d(1-\Delta_d)\right]}{\Delta_c(1-\lambda^*)\lambda^*+\Delta_d(1-\Delta_d)(1-\Delta_c-\lambda^*)}\left(\equiv \alpha_{R|\text{without caps}}\right). \quad (1.41)$$

Proof: See the Appendix (A.2). □

Lemmas 1 and 2 describe equilibria which are the result of the CB's intraday credit policy.

Lemma 3. *There is an equilibrium under the intervention by the CB when* $\lambda^* \in [\Delta_d, 1-\Delta_d)$ *or* $\lambda \in [\frac{\Delta_d}{1-\Delta_d}, 1)$.

Proof: See the Appendix (A.3). □

Lemma 3 provides a limitation of the intervention by the CB. When the proportion of early-arriving debtors is too low, the CB cannot achieve an equilibrium.

4.3. Quantitative restrictions

This subsection studies two quantitative policy tools: caps and collateral. In addition to the pricing rules above, these are two possible policy tools for risk management. Different central banks use different policy tools. For example, the Federal Reserve mainly depends on caps and (partial) collateral, whereas the European Central Bank (ECB) uses collateral.

Until now, the two policy tools, price and quantity, have been considered to be substitutes; therefore, the arguments on price vs. quantity are dichotomous. For example, Faulhaber et al. (1990) argue against a cap because a cap is distortionary. Rochet and Tirole (1996), on the contrary, argue for the quantitative restrictions. Quantitative restrictions such as caps or collateral may be better means to control the overuse of intraday credit than price. They argue that price may induce moral hazard or adverse selection in situations where information is incomplete. Furthermore, Kahn and Roberds (2001) argue optimally set quantitative restrictions at zero interest rate can eliminate the liquidity shortages and restore the first-best consumption allocation.

4.3.1. Caps

The CB relaxes the liquidity constraint by providing intraday liquidity. Banks can shift their potential credit risks onto the CB by using as much intraday credit from the CB as possible; therefore, imposing quantitative restrictions helps to control potential loss. In the model, the caps are set as $(1 - \lambda^*)\ (1 - \alpha)\ l^*_t$, which is sufficient to relax the liquidity shortage. If the size of caps is larger than this, all other banks may try to borrow intraday credit from the CB in order to shift their potential type I credit risk to the CB. If there are more early arriving debtors in an economy, then caps are low. Notice that the CB needs to know the information about λ^* which consists of λ and $(1 - \Delta_d)$. This implies the CB has information about the assets of commercial banks in real world. If the CB is unable to access the information on the assets of commercial banks, then the CB can not manage risks effectively through caps.

4.3.2. Collateral

I consider full collateralization in this subsection. Institutionally, banks put up collateral before their types are revealed in order to use the facilities provided by the central bank; specifically, for access to the discount window. According to Furfine and Stehm (1998), the nature of these collateral arrangements typically involve either pledging collateral to the central bank or entering into an intraday repurchase agreement (repo) with the central bank. In the model, since the commodities are perishable and there are no other assets except IOUs, only the IOUs can be used as collateral[18]. Therefore, I focus on repo type collateral in this section. Specifically, the banks sell their IOUs to the CB in the form of a repo; therefore they do not put up collateral before their types are revealed which is different from usual institutional set up of collateral. In this repo set up, collateral has the value of repayment from the debtor just as does an IOU. I assume that the "haircut" on collateral is calculated by the pricing rule in the previous section. In the model, therefore, a collateral policy always goes with a price policy. Notice that I restrict attention to collateral for the CB; that is I do not consider a case where collateral is allowed in the private intraday market.

Collateral changes the market timing so the event of fraud shock by the d.i. bank no longer occurs. As I consider a repo type collateral, a d.i. bank hands over the IOU to the CB. Since the CB holds the IOU now, according to the sequence of events, borrowers

[18] Since there were concerns about deficiency of collateral, the ECB entitled a wide range of assets including commercial loans for collateral.

of intraday credit have no possibility to have another chance to consume. The CB can be repaid directly from the issuer of the IOU in this case. Notice that there still remains a risk that the consumption debt will not be repaid by the debtor. Therefore, this policy can not remove all fraud shocks in the economy. This means collateral cannot control shifts of potential risks from banks to the CB. This is an effective policy, however, to improve social welfare as well as to limit losses from credit risk.

By imposing collateral, the equilibrium has the following properties. First, since now Δ_c is zero in the model, collateral makes Δ_I decrease which results in ρ^*_i increasing. As a result, the CB_R can set higher ρ^*_{CB} (or lower intraday interest) according to the pricing rule. The economy can sustain higher social welfare than before. In the case of CB_L, the decreased Δ_I makes the price of good C higher than before. The bank is better off from the price change, but the debtor is worse off. This has unclear effects on social welfare. Notice that collateral perfectly removes type II credit risk.

Second, the equilibrium under the intervention of the CB with charging interest on intraday credit exists[19] only when $\alpha(\lambda^*) = 0$.

4.4. Calibration of social welfare and policy analysis

This subsection calibrates social welfare of the combinations of the policy tools. I compare social welfares under equilibrium. I use $\Delta_d = 0.03$, $\Delta_c = 0.01$, $M^*_t = 1$. Using Lemma 1 and 2, I calculate $\alpha(\lambda^*)$ given the synchronization parameter of debtors ($\lambda^* = 0.814$). Consumption bundles under each policy set are calculated using above parameter values.

[19] See the Appendix (A.4).

The result of calibration is summarized in Table 1.2. Under competitive equilibrium social welfare is 10.57 with a debtor's being 5.42 and a bank's being 5.15. When the CB provides intraday credit at zero interest rate without caps, social welfare becomes 10.60 with a debtor's being 5.43 and a bank's being 5.17. Once the economy (or the CB as a representative of the economy) takes into account, however, its loss from this policy, social welfare is reduced according to the weight the CB places on the loss. For example, if the CB takes the loss into full consideration, social welfare becomes 9.10 (= 10.60 – 1.50).

Property 1. *When the CB places a low weight on the loss from free intraday credit, the CB can make the economy better off just by providing intraday liquidity at zero interest.*

As the CB considers the loss more, the CB may use quantitative restrictions, caps or collateral. According to Table 1.2, when the CB puts less than 2% weight on its loss from the risks, social welfare under zero interest is higher than competitive equilibrium (10.57).

Property 2. *There is a critical weight w_1 to the CB's loss such that if a weight is less than w_1, caps with free intraday credit can improve social welfare.*

According to Table 1.2, when the society puts less than 2.1% of weight to its loss, social welfare under caps with free intraday credit is higher than competitive equilibrium.

Property 3. *Price tool should be applied with quantitative restriction, specifically collateral, in order to improve social welfare.*

The economy becomes worse off under a price only policy whose social welfare is 10.54. This is not a good policy, because this does not improve social welfare compared to that of no intervention. Under priced intraday credit with caps, however, social welfare becomes 10.58 with a debtor's being 5.42 and a bank's being 5.16. This is one justification of caps along with positive interest rate.

Property 4. *There is a critical weight w_2 on the CB's loss such that if a weight is greater than w_2, a price and caps policy dominates a caps with free intraday credit policy in terms of social welfare.*

According to Table 1.2, when the CB puts more than 1.6% of weight to its loss from the risks, social welfare under a price and caps policy (10.58) is higher than a caps policy at zero interest.

Property 5. *When intraday credit is costly, collateral is a better tool than caps in terms of social welfare.*

Property 6. *Social welfare from a price and collateral policy is the same as that from a price, caps and collateral policy.*

Price considers both types I and II credit risks, and caps deal with type I credit risk, and collateral controls type II credit risk. As long as the CB places sufficient weight on potential loss from the risks, Property 6 implies that using a price and collateral policy is more efficient than any other policies in terms of social welfare.

Table 1.2: Consumption bundles and utility in equilibrium

	without intervention		intervention					
	(Risk free)	(Risky)	Free	Free+Caps	Priced	(Priced+ Caps)	(Priced+ Collateral)	(Priced+caps Collateral)
Debtors' Utility	5.4071	5.4158	5.4269	5.4269	5.4202	5.4216	5.4269	5.4269
Banks' Utility	5.2407	5.1545	5.1734	5.1734	5.1178	5.1567	5.3911	5.3911
The CB Loss			0.0316	0.0257				
Disutility			1.4991	1.4106				
Social Welfare[20]	10.6479	10.5703	10.6003 (9.1012)	10.6003 (9.1897)	10.5380	10.5783	10.8180	10.8180

Figures in ( ) are the utility after full consideration of the loss.

5. Concluding remarks

This paper has analyzed the intraday credit policy tools implemented by the central bank under a large-value RTGS system with several types of exogenous risks.

This paper identifies credit risks in the market which explain joint usage of elements of intraday credit policy under RTGS system in the real world. Price deals with both type I and II credit risks; a cap works for managing type I credit risk; even though this chapter does not consider moral hazard problem, collateral removes type II credit risk just as in Martin[21] (2004).

From the analysis of intraday credit policy, the model suggests the following. When the economy (or the CB as a representative of the economy) does not place significant weight on the potential loss from credit risks, the CB can make the economy

[20] Because of logarithmic functional form, I multiply consumption amount by 1000 in order to make positive utility.

[21] More specifically, Martin argues that collateral lending with zero interest can prevent moral hazard committed by banks.

better off simply by providing intraday liquidity at zero interest. Caps may be used only when the economy takes little consideration of the potential loss from the credit risks in the RTGS system. The CB starts to use the price tool when there is an increase in the economy's consideration of the potential loss from credit risks. The price tool, however, should be used with other quantitative tools specifically with collateral. When intraday credit is costly, collateral is a better tool than caps in terms of social welfare. Since there are two types of credit risk in the market, it is enough to employ two tools. The model illustrates that a price and collateral policy achieves the highest social welfare among other sets of polices.

The model demonstrates that intraday credit policy implemented by the CB may fully displace a private market for intraday credit. Specifically, collateral makes the private market collapse. This is why an active private market for intraday credit is not common. According to BIS (2005), there are only 5 RTGS systems (Germany, Japan, Singapore, and two systems in Sweden) out of 16 systems (including TARGET of the ECB) to which the CB provides intraday credit that have additional private market.

Now I discuss sources of apprehension of the society to the potential loss from the credit risks in the payment systems. The first source can be limitations in the conduct of monetary policy. Potential loss from the payment policy is revealed in the form of change in fiat money stock. If the CB faces any impediment to implementing monetary policy for this purpose, then the society would pay much attention to the potential loss. This can be an explanation of the payment system policy tools of the ECB. According to Pollard (2003), the Maastricht Treaty, under which the ECB is established, does not mention a lender of last resort function for the ECB and the ECB has been criticized for lacking this function. Another possible limitation of the ability originates from the procedure[22] of open market operations. The ECB conducts its main open market operations only once per week. Additionally, open market operations are decentralized in the euro area. From these environments, the ECB should pay a lot more attention to the potential loss from the payment system rather than Federal Reserve.

The second source can be the size of potential loss (daylight overdraft). As the size of the potential loss increased, for example, the Federal Reserve started to employ caps and to price intraday credit.

As the model separates senders (debtors) and receivers (banks), a price policy does not affect the synchronization parameter in this paper. Some studies[23], however, show that a price policy may cause a network externality which may induce banks to postpone payment activity.

Since there are too many parameters, one disadvantage of the paper is that it does not analyze welfare in a clear analytical form, but in calibrated ones. Another limitation is the model simplifies settlements procedure too much. One more disadvantage is that this paper considers exogenous risks. One possible extension of research, therefore, is to endogenize the risks. In this case there may be a cost for committing fraud. If the CB has full information of the payment timing and ability to supervise the banks, the cost for committing fraud by the bank can be sufficiently large enough to prevent commitment of fraud by the intraday users. This gives the same result as collateral.

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[22] The Federal Reserve deals exclusively in U.S. government securities, whereas the ECB has a broader range of assets including commercial loans. Since the policy makers of the Fed concern that collateralized intraday loan may impair the ability to conduct monetary policy through open market operations, the Fed does not heavily depend on collateral among payment policy tools.

[23] See Angelini (1998), Kobayakawa (1997).

Appendix

A.1. Proof of Lemma 1

In case of the CB_L, $T_{CB}=(1-\alpha)\left[1+\Delta_c\left(1-\frac{\Delta_d}{1-\lambda^*}\right)(1-\Delta_d)\right]+\alpha(1-\Delta_d)^2$. From (1.27), (1.39) and money supply and demand equilibrium condition, we have

$$T_{CB}\cdot l_t^* = (1-\Delta_d)M_t^* + \Delta_l N_t^* \tag{1.A.1}$$

Under the CB_L, the only provider of the intraday credit is the CB. This implies $N_t^* = (1-\alpha)(1-\lambda^*)l_t^*$. Applying this into (1.A.1) and rearranging gives

$$\left\{T_{CB}-\left[\Delta_c+\frac{(1-\Delta_c)\Delta_d}{1-\lambda^*}\right](1-\alpha)(1-\lambda^*)\right\}l_t^* = (1-\Delta_d)M_t^*. \tag{1.A.2}$$

From (1.A.2), if $T_{CB}-\left[\Delta_c+\frac{(1-\Delta_c)\Delta_d}{1-\lambda^*}\right](1-\alpha)(1-\lambda^*)=1-\Delta_d$, we have $l^*_t = h^*_t = m^*_t = M^*_t$. Solving $T_{CB}-\left[\Delta_c+\frac{(1-\Delta_c)\Delta_d}{1-\lambda^*}\right](1-\alpha)(1-\lambda^*)=1-\Delta_d$ for α, we get

$$\alpha=\frac{\Delta_c\left[\lambda^*(1-\lambda^*)-\Delta_d(1-\Delta_d)\right]}{\Delta_c(1-\lambda^*)\lambda^*+\Delta_d(1-\Delta_d)(1-\Delta_c-\lambda^*)}(\equiv\alpha_L). \square \tag{1.A.3}$$

A.2. Proof of Lemma 2

We have two different forms of T_{CB} in the CB_R:

$T_{CB}=\alpha(1-\Delta_d)^2+(1-\alpha)\left[(1-\Delta_c)(1-\lambda^*-\Delta_d)+\lambda^*\right]+(1-2\alpha)\Delta_c\left(1-\frac{\Delta_d}{1-\lambda^*}\right)(1-\Delta_d)$ (in case of the CB_R with caps) or

$T_{CB} = (1-\alpha)\left[(1-\Delta_c)(1-\lambda^* - \Delta_d) + \lambda^* + \Delta_c\left(1-\frac{\Delta_d}{1-\lambda^*}\right)(1-\Delta_d)\right] + \alpha(1-\Delta_d)^2$ (in case of the CB_R without caps)

In either case of T_{CB}, we have $T_{CB}\cdot l_t^* = (1-\Delta_d)M_t^*$ from (1.39) and money supply and demand equilibrium condition. When $T_{CB} = (1-\Delta_d)$, we have $l^*_t = h^*_t = m^*_t = M^*_t$.

Solving $T_{CB} = (1-\Delta_d)$ for α, we get

$$\alpha = \frac{\Delta_c\left[\lambda^*(1-\lambda^*) - \Delta_d(1-\Delta_d)\right]}{(1-\Delta_d)\Delta_d(1-\lambda^*) + \Delta_c\left[(1-2\Delta_d)(1-\Delta_d) - \lambda^*(\lambda^* - \Delta_d)\right]}\left(\equiv \alpha_{R|\text{with caps}}\right) \quad (1.A.4)$$

or

$$\alpha = \frac{\Delta_c\left[\lambda^*(1-\lambda^*) - \Delta_d(1-\Delta_d)\right]}{\Delta_c(1-\lambda^*)\lambda^* + \Delta_d(1-\Delta_d)(1-\Delta_c-\lambda^*)}\left(\equiv \alpha_{R|\text{without caps}}\right). \quad (1.A.5)$$

A.3. Proof of Lemma 3

Since (1.A.5) is identical to (1.A.3), I look at (1.A.3) and (1.A.4). From the property of α, we need to have $0 \le \alpha \le 1$. Applying (1.A.3) and (1.A.4) to this property, we have

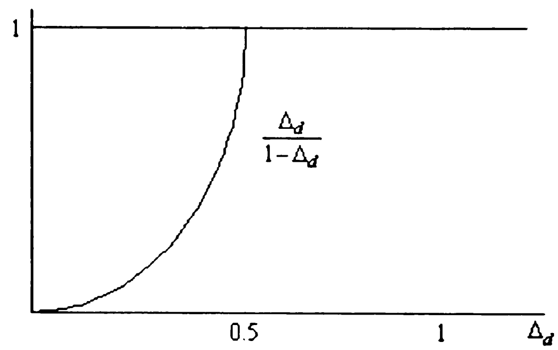
$$0 \le \frac{\Delta_c\left[\lambda^*(1-\lambda^*) - \Delta_d(1-\Delta_d)\right]}{\Delta_c(1-\lambda^*)\lambda^* + \Delta_d(1-\Delta_d)^2(1-\Delta_c-\lambda^*)} \le 1$$

and

$$0 \le \frac{\Delta_c\left[\lambda^*(1-\lambda^*) - \Delta_d(1-\Delta_d)\right]}{(1-\Delta_d)\Delta_d(1-\lambda^*) + \Delta_c\left[(1-2\Delta_d)(1-\Delta_d) - \lambda^*(\lambda^* - \Delta_d)\right]} \le 1.$$

The numerators of both inequalities are identical and are smaller than denominators. Therefore, these inequalities hold once the numerator is non-negative. The numerator is a quadratic equation in λ^*. We have $\alpha = 0$ when $\lambda^* = 1 - \Delta_d$ or $\lambda^* = \Delta_d$. Therefore, the equilibrium under intervention by the CB exists in the region of $\lambda^* \in [\Delta_d, 1 - \Delta_d)$ or $\lambda \in [\frac{\Delta_d}{1-\Delta_d}, 1)$. The lower bound $\frac{\Delta_d}{1-\Delta_d}$ should be less than 1. This implies $\Delta_d < \frac{1}{2}$. See Figure 1.A.1.

Figure 1.A.1: Equilibrium range $\lambda^* \in [\Delta_d, 1 - \Delta_d)$ or $\lambda \in [\frac{\Delta_d}{1-\Delta_d}, 1)$ under intervention by the CB



A.4. Equilibrium under collateral

In the equilibrium where M is changing (The CB_L: the CB sets ρ^*_{CB}=1), we have $l^*_t = h^*_t = m^*_t = M^*_t$. This implies that

$$T_L \cdot l_t^* = (1-\Delta_d)M_t^* + \frac{\Delta_d}{1-\lambda^*} N_t^* \tag{1.A.6}$$

where $T_L = (1-\alpha) + \alpha(1-\Delta_d)^2$.

When the CB sets ρ^*_{CB}=1, then the only provider of the intraday credit is the CB. This implies[24] $N^*_t = (1-\alpha)(1-\lambda^*)l^*_t$. Applying this and rearranging gives us

$$[T_L - \Delta_d(1-\alpha)]l^*_t = (1-\Delta_d)M^*_t \qquad (1.A.7)$$

Only when $T_L - \Delta_d(1-\alpha) = 1-\Delta_d$, does there exist an equilibrium. Solving this for α, we get $\alpha = 0$.

In the equilibrium where M is fixed (the CB_R), we have $l^*_t = h^*_t = m^*_t = M^*_t$. This implies that $T_R \cdot l^*_t = (1-\Delta_d)M^*_t$ where $T_R = (1-\alpha\Delta_d)(1-\Delta_d)$. Only when $T_R = 1 - \Delta_d$, does there exist an equilibrium. Solving this for α, we get $\alpha = 0$.

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[24] I assume that the CB imposes a cap. If there is no cap, we have $N^*_t = (1-\lambda^*)l^*_t$. The result, however, does not change.

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Chapter 2

Payment instruments and the central bank

1. Introduction

This chapter explores the sensitivity of payment system stability to macroeconomic shocks through the assets (from lenders' viewpoint) or liabilities (from borrowers' viewpoint) used as media of exchange. A safe and efficient payment system is a gateway to the effective functioning of the financial system in monetary economies. In this regard, governments may implement policies to keep the payment system stable. Specifically, this chapter studies a "payment instrument policy," such as the one implemented by the Bank of Korea, to promote usage of a certain type of payment instrument to stabilize the payment system.

In addition to fiat money, there are several types of credit which circulate as payment instruments (or "private money [1] "). Examples of these instruments are promissory notes, which are simple credits in the sense that sellers provide credit [2] directly to buyers; or bills of exchange, which are intermediated credits in the sense that a commercial bank is necessary to provide credit and repay the bill between buyers and sellers. These two payment instruments coexist and are used by different agents. In practice direct credits are used by relatively wealthy agents about whom much is publicly

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[1] In the real world, these payment instruments are circulated when they are endorsed.

[2] There are a group of papers regarding "trade credit" which means goods lent by suppliers to their customers. I consider a transaction under "quid pro quo" principle in this chapter; therefore no transaction occurs without any (private) money. [Refer Petersen and Rajan (1997) for comprehensive review of theories on trade credit].

known, whereas others demand more information-intensive intermediated credits (Holmstrom and Tirole, 1997), relying on the financial intermediary's reputation to secure funds.

In contrast to fiat money, these private monies are risky. Specifically unlike fiat money the market value of these payment instruments is affected by moral hazard and macroeconomic shocks such as recessions or oil shocks. The moral hazard may be accurately priced, but macroeconomic shocks may not. In case of macroeconomic shocks, expectations of them are a major determining factor of the price. If there are fewer agents who anticipate macroeconomic shocks, then the shocks would not be accurately priced and, consequently, the distribution of product return would be wide. Thus, the shocks alter production choices of agents. As a result, the economy has a higher risk when there are unanticipated macroeconomic shocks. For example, the Korean currency crisis[3] of November 1997 is considered an unanticipated shock (Radelet and Sachs 1998). As a result, the bankruptcy rate of firms skyrocketed as represented by the historically high dishonored bill ratio[4] at the end of 1997 (Hahm and Mishkin 2000). I call this a market risk. A market risk can be considered as systemic risk[5], since all agents are affected by the risk and cannot fully control it at the individual level.

In order to alleviate overall market risk from private monies, the government may implement a "payment instrument policy." In this chapter, the government may implement a payment instrument policy by subsidizing or taxing a certain type of

[3] Refer to Hahm and Mishkin (2000) for details.

[4] The dishonored bill ratio is the percentage of corporate promissory notes, checks and bills in default out of total promissory notes, checks and bills cleared at the clearing house during the day.

[5] Systemic risk is the risk that the inability of one of the participants to meet its obligations, or a disruption in the system itself, could result in the inability of other system participants or of financial institutions in other parts of the financial system to meet their obligations as they become due. Such a failure could cause widespread liquidity or credit problems and, as a result, could threaten the stability of the system or of financial markets (Bank for International Settlements [BIS] 2001).

payment instrument in order to achieve a safe and effectively functioning payments system. For example, in 2000 after the currency crisis, the Bank of Korea (BOK) introduced a new facility, the "Corporation Procurement Loan," for subsidizing bills of exchange in order to promote the usage of bills of exchange and reduce systemic risk caused by the market risk from promissory notes.

The main question of this chapter is why the BOK chose to promote the usage of bills of exchange instead of promissory notes as a part of its "payment instrument policy." Usually negative macroeconomic shocks hit small size debtors the hardest (Holmstrom and Tirole, 1997). As stated above, promissory notes (or direct credit) are used by relatively wealthy agents. One may think from this fact that the government intervention can be justified by supporting the economically relatively weak. This, however, does not answer the question sufficiently from the viewpoint of a "payment instrument policy." I look at market risk of each payment instrument in order to answer this question. The next question is an extension of the first one: why these two payment instruments incur different market risk; and which one is better in terms of risk management.

This chapter studies the role of the government regarding private money in the presence of macroeconomic shocks. Two types of private money are studied: promissory notes (in short PN) and bills of exchange (in short BE). I modify a simple partial equilibrium moral hazard model of wealth constrained agents. The model features (i) two types of consumption credit (IOUs); (ii) two different types of lenders: creditors who provide a direct consumption credit and financial intermediaries which provide intermediated credit and monitoring services; (iii) the central bank [6] which may implement a "payment instrument policy" with a tax or subsidy; (iv) a macroeconomic shock which affects incentives of debtors and lenders. I first consider an economy where PN and BE coexist and work as consumption credit and study the critical level of debtors' wealth. I introduce a macroeconomic shock and assume that agents do not anticipate the shock; I then examine a pure PN economy and a pure BE economy to study market risk of each payment instrument. Then, I look at two different payment instrument policy tools: tax and subsidy. Finally, I relax the non-anticipation assumption of the macroeconomic shock.

This chapter provides a different perspective on intervention for macroeconomic risk management by the government. The model shows that bills of exchange have a lower market risk than promissory notes under negative macroeconomic shocks. If, therefore, both promissory notes and bills of exchange are used in addition to fiat money, systems with a greater proportion of promissory notes in circulation will be riskier and more prone to collapse than systems with fewer promissory notes. Payment instruments play an important role in payment system stability in the presence of moral hazard. This is a justification for a policy which supports usage of bills of exchange. The model also shows that a subsidy for bills of exchange is better than a tax on promissory notes. Another contribution is the identification of types of risk: credit risk from the production technology (moral hazard) and market risk caused by a macroeconomic shock. The

[6] As Mills (2004b) noted, one can interpret a central bank as a private clearinghouse that is separate from the other agents. As noted in Green (1997), the liquidity-providing institution in the model can be either a public or private one. This is a payment system version of inside and outside money debate which remains an open question. Refer to Williamson (1999) for a further review on historical debates about inside and outside money.

model demonstrates that unanticipated macroeconomic shocks bring higher market risk than anticipated shocks.

In contrast to this chapter, existing studies on payment policy mainly pay attention to intraday credit policy. See, for example, Mills (2004b), Lacker (1997), Faulhaber et al. (1990), Angell (1991), Rochet and Tirole (1996), Kahn and Roberds (2001), and Martin (2004). However, none lend insight into why governments implement a policy which encourages certain types of payment instrument in order to achieve payment system stability.

This chapter is organized as follows. Section 2 describes the environment of the model; section 3 analyzes an economy with two types of private money. Section 4 analyzes an economy with a macroeconomic shock and discusses the government's role, and section 5 summarizes our findings and concludes. Lengthy proofs are in the appendix.

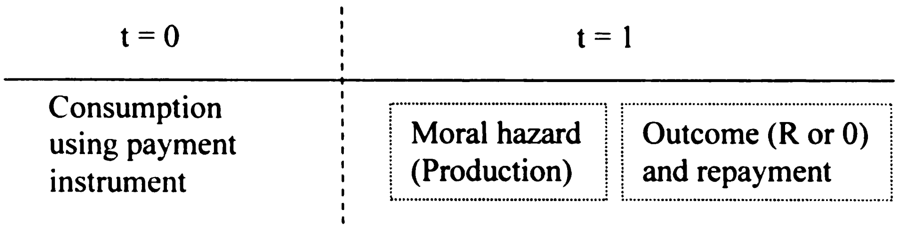
2. The environment

The model shares environments of Holmstrom and Tirole (1997, 1998). Consider a two-period economy with three types of risk neutral agents: debtors, creditors and banks. Debtors have an initial endowment of W, which comes as a random draw from a uniform distribution with support [0, sup (W)]. The amount of wealth is assumed to be public information. The utility of a debtor is given by $U = u(E) + u(R_d)$ where E is an amount of consumption at date 0, exogenously set such that sup (W) < E; where $R_d \geq 0$ is an amount of consumption at date 1 when production succeed.

Hence, debtors need to borrow resources from other agents. These resources are repaid at date 1, but repayment is subject to a moral hazard problem. Precisely, at date 1 each debtor has the option between a good and a bad technology of production and this choice is unverifiable. The good (bad) technology leads to a probability p (p – Δp, where Δp > 0) of a successful production but the bad technology provides a private benefit to the debtor. Returns from production are verifiable and success (failure) means that an output of R (0) is produced where R > 0. Creditors and banks only consume at date 1. Moreover, they start with an amount of endowment sufficient to meet the debtors' demands. Banks have a more costly but more efficient monitoring technology. In particular, banks (creditors) face a monitoring cost of C (0) and induce a private benefit of b (B) to the debtor with a bad technology, with B > b. If a creditor lends to a debtor, the payment instrument that underlies this transaction is denoted as a PN. Alternatively, when the debtor borrows from the bank, the corresponding payment instrument is a BE. These are the only payment instruments available in the economy. Attention is focused on debt contracts where (i) neither party is paid anything if the production fails; (ii) if the production succeeds and there is no systemic risk, $R_d \geq 0$, and the creditor (bank) is repaid $R_c > 0$ or $R_b > 0$, where

$$R_d + R_c = R \ \ (R_d + R_b = R).$$

Figure 2.1: Timing of events (debtor's viewpoint)



Creditors and banks are assumed to have sufficient initial wealth to be able to provide consumption credit to debtors at date 0 which is repaid at date 1. The utility of a creditor (bank) is $V = u(R_c)$ ($V = u(R_b - C)$). There is an authority which enforces a debt contract. The risk free (gross) interest rate is normalized to 1. Let γ be the (endogenous) expected rate of return on promissory notes; and β be the (endogenous) expected rate of return on bills of exchange. I also assume that consumption credit is small relative to the overall size of a bank's asset portfolio. This implies that a default by an individual debtor will not compel the bank into a default.

3. Simple model for payment instruments

There are two different types of payment instruments, promissory notes and bills of exchange. PN is a non-intermediated credit provided by individual creditors, whereas BE is an intermediated credit provided by banks. Payment instruments are issued at date 0 and settled at date 1 after production of output R.

3.1. Promissory notes

In this subsection, debtors use PN for consumption. Incentive compatibility constraint for debtors to choose a good production technology is

$$p \cdot R_d \geq (p - \Delta p) \cdot R_d + B. \qquad (IC_d)$$

Individual rationality constraint for a creditor is

$$p \cdot R_c \geq \gamma \cdot (PN) \qquad (IR_c)$$

Lemma 1. There is a critical level of debtor's wealth $\bar{W}$ such that creditors provide consumption credit only to debtors whose wealth is above the critical level $\bar{W}$.

Proof: From (IR_c) we have an upper bound on PN

$$\left(PN\right) \le \frac{p \cdot R_c}{\gamma},$$

and from (IC_d) this becomes

$$\left(PN\right) \le \frac{p \cdot R_c}{\gamma} \le \frac{p}{\gamma}\left[R - \frac{B}{\Delta p}\right].$$

In order to consume debtors have

$$W + \left(PN\right) \ge E.$$

Rearranging this for W and using the upper bound on PN above, we have

$$W \ge \bar{W}$$

where $\bar{W} \equiv E - \frac{p}{\gamma}\left[R - \frac{B}{\Delta p}\right]$; therefore, debtors who are sufficiently wealthy can use promissory notes.

3.2. Bills of exchange

In this subsection, I study debtors who use BE for consumption. Banks can help a wealth constrained debtor to consume. Now the banks monitor debtors. Since monitoring reduces private benefit of a debtor from B to b, the incentive constraint of a debtor is

$$p \cdot R_d \ge (p - \Delta p) \cdot R_d + b. \qquad (IC'_d)$$

Participation constraint for the bank is:

$$p \cdot R_b \ge \beta \cdot \left(BE\right) \qquad (IR_b)$$

Lemma 2. There is a critical level of debtor's wealth $\underline{W}$ such that banks provide consumption credit only to debtors whose wealth is above the critical level $\underline{W}$.

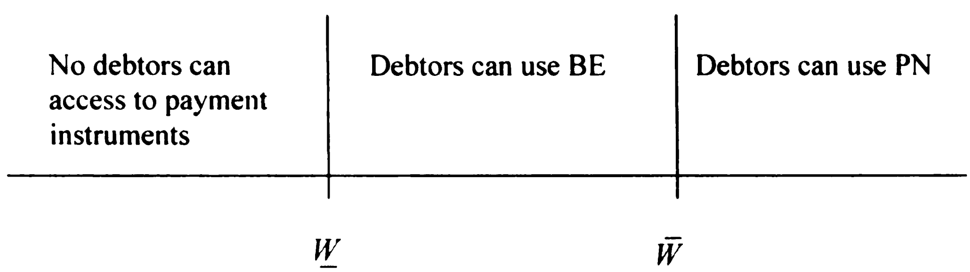
Proof: Similarly as in Lemma 1, we have $W \geq \underline{W}$

where $\underline{W} \equiv E - \frac{p}{\beta}\left[R - \frac{b}{\Delta p}\right]$; therefore, debtors who are wealthy enough can use bills of exchange. □

Lemma 3. When γ < β, there is a critical output level $\bar{R}$ such that if $R \in (0, \bar{R})$, we have $\underline{W} < \bar{W}$.

Proof: See the Appendix. □

Figure 2.2: Critical levels of wealth for private monies



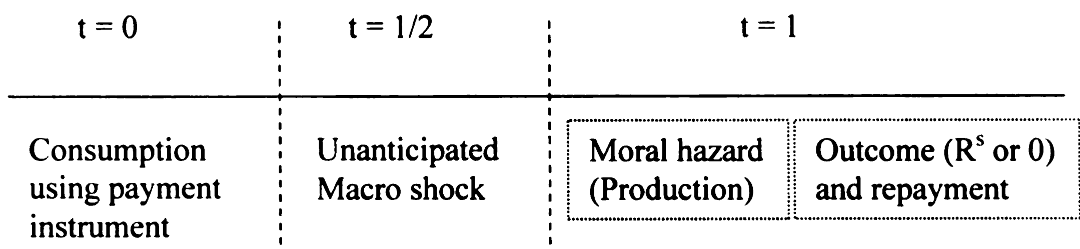
Lemma 3 implies that the level of output R is in a reasonable range. If the output is too abundant such as air or water, then size of wealth does not matter. If $\underline{W} > \bar{W}$, there is no demand for BE. This implies that the monitoring technology is too costly to be socially useful. Since I consider an economy with coexisting PN and BE, I assume that $R \in (0, \bar{R})$. Since PN is cheaper than BE[7], debtors whose wealth is greater than $\bar{W}$ have no incentive to use BE; debtors whose wealth is between $\underline{W}$ and $\bar{W}$ can only use BE and be monitored by the banks.

[7] See Lemma 8.

4. Macroeconomic shock and payment instrument policy

This section studies market risk from macroeconomic shocks and payment instrument policy of the central bank (CB). In order 1) to remove or alleviate the systemic risk; or 2) to enhance the efficiency of the payment system; or, more fundamentally, 3) to maintain the payment system (or monetary economy), payment (system) policy is implemented.

Figure 2.3: Timing of events (debtor's viewpoint)



Following Holmstrom and Tirole (1998), a macroeconomic shock is introduced by assuming that at an interim date (t = 1/2), each debtor suffers from an output shock. For simplicity, the economy is assumed to consist of homogenous agents who do not anticipate macro shocks. The output R decreases to R^s. This shock is non-diversifiable event like oil shock or a sharp economic contraction after a currency crisis. In order to verify market risks of each payment instrument, I study a PN only economy and a BE only economy under macroeconomic shock.

4.1. Pure promissory notes economy

There are two types of market risk from the macro shock in the payment and settlement system: let η_{PN} and η_{BE} be the endogenous market risk of PN and BE from the macro shock respectively (these are calculated in next subsection). If the probability of success from the bad technology ($p - \Delta p$) is lower than the market risk from the macro shock, then the expected amount of repayment of consumption debt is less than zero. This implies that the monetary economy cannot exist, therefore I rule out this case and restrict analysis to the case[8] where the market risk from the macro shock is lower than the probability of success from the bad technology (p – Δp) or

$$p - \Delta p > \eta_{PN}$$

and

$$p - \Delta p > \eta_{BE} .$$

Lemma 4. There is an upper limit $\bar{B}$ for private benefit B such that if $B \leq \bar{B}$, the market risk η_{PN} is negatively related to the macro shock.

Proof: See the Appendix. □

Since the same logic is applied, there is an upper limit $\bar{b}$ for private benefit b such that $b \leq \bar{b}$, the market risk η_{BE} is negatively related to the macro shock. From now on, I assume that the private benefit B and b are not too big.

Because of the macro shock we have the following adjusted (IC^S_d) and (IR^S_c) constraints:

$$p \cdot R^s{}_d \geq (p - \Delta p) \cdot R^s{}_d + B \qquad (IC^S_d)$$

$$(p - \eta_{PN}) \cdot R^s{}_c \geq \gamma \cdot (PN) . \qquad (IR^S_c)$$

From Lemma 1, a new critical wealth for PN is

[8] A sufficient condition for this restriction is

$$p > 2 \cdot \Delta p .$$

$$\bar{W}^s \equiv E - \frac{(p-\eta_{PN})}{\gamma}\left[R^s - \frac{B}{\Delta p}\right]. \qquad (2.1)$$

There are two criteria of wealth for decisions of agents in this model: a lending criterion for lenders at date 0 and a technology choice criterion for debtors at date 1. A lending criterion determines the type of payment instrument, PN or BE as in Lemma 1 and 2. Lenders (creditors or banks) provide either direct credit or intermediated credit according to the critical level of wealth at date 0. As a result, for example, debtors for whom $W > \bar{W}$ can use PN for their consumption credit. A technology choice criterion determines the type of production technology, p or (p – Δp). For example, debtors with $W > \bar{W}$ who uses PN choose a high technology (p) consistent with the (IC_d) constraint at date 1. When there is no macro shock, these two criteria are identical. The macro shock, however, makes these two criteria differ from each other. As the economy consists of homogeneous agents, the lending criterion at date 0 is the same as before: debtors with $W > \bar{W}$ can use PN. A technology choice criterion, however, differs from the case of no macro shock. Since the agents do not anticipate the macro shock, the critical level of wealth after the macro shock increases to $\bar{W}^s$ as in (2.1); therefore now debtors with $W > \bar{W}^s$ choose a high technology (p) consistent with the new (IC^s_d) constraint at date 1. To sum up, the macro shock separates the two criteria of wealth for decisions of agents.

Lemma 5. When a negative macro shock occurs, the critical wealth $\bar{W}^s$ for promissory notes increases.

Proof: Derivative of $\bar{W}^s$ with respect to R^s has negative sign:

$$\frac{\partial \bar{W}^s}{\partial R^s} = -\frac{(p-\eta_{PN})}{\gamma} + \frac{R^s}{\gamma} \underbrace{\frac{\partial \eta_{PN}}{\partial R^s}}_{-} + \frac{(p-\eta_{PN})}{\gamma^2}\left[R^s - \frac{B}{\Delta p}\right] \underbrace{\frac{\partial \gamma}{\partial R^s}}_{-} < 0. \square$$

4.2. Pure bills of exchange economy

As in PN case, because of the macro shock we have following adjusted (IR^s_b) constraint:

$$p \cdot R^s{}_d \geq (p-\Delta p) \cdot R^s{}_d + b, \qquad (IC'^s{}_d)$$

$$(p-\eta_{BE}) \cdot R^s{}_c \geq \beta \cdot (BE). \qquad (IR^s{}_b)$$

From Lemma 2, a new critical wealth for BE is

$$\underline{W}^s \equiv E - \frac{(p-\eta_{BE})}{\beta}\left[R^s - \frac{b}{\Delta p}\right]. \qquad (2.2)$$

Lemma 6. When a negative macro shock occurs, the critical wealth $\underline{W}^s$ for bills of exchange increases.

Proof: Derivative of $\underline{W}^s$ with respect to R^s has negative sign:

$$\frac{\partial \underline{W}^s}{\partial R^s} = -\frac{(p-\eta_{BE})}{\beta} + \frac{R^s}{\beta} \underbrace{\frac{\partial \eta_{BE}}{\partial R^s}}_{-} + \frac{(p-\eta_{BE})}{\beta^2}\left[R^s - \frac{b}{\Delta p}\right] \underbrace{\frac{\partial \beta}{\partial R^s}}_{-} < 0. \square$$

4.3. Preference of central bank

First I consider credit risk from the production technology p, and then consider market risk from the macro shock. From the lender's point of view we have the following default rate δ for each payment instrument without a macro shock:

$$\delta = 1 - p \begin{cases} W \geq \bar{W} \text{ in case of PN} \\ W \geq \underline{W} \text{ in case of BE} \end{cases} \tag{2.3}$$

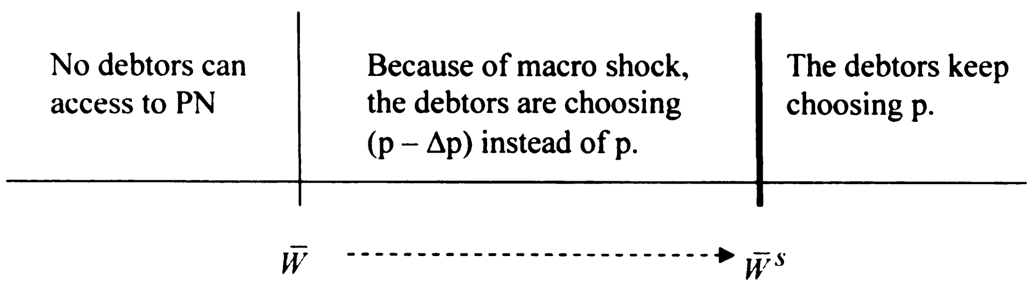
Notice that without a macro shock, both payment instruments incur identical default rates, even though the range of wealth is different from each other.

One corollary from the default rate from the production technology (2.3) is that if the wealth level is lower than the critical levels, the default rate becomes higher such as

$$1 - (p - \Delta p).$$

This corollary helps us to consider market risk after a macro shock, since debtors have a new critical level of wealth after the macro shock which determines debtors' production technology choice.

Figure 2.4: Critical level of wealth and macro shock (PN case)



From Lemma 5 and Lemma 6, we have two "grey areas" of wealth level for debtors, $\bar{W} \sim \bar{W}^S$ for PN and $\underline{W} \sim \underline{W}^S$ for BE. Since there is a macro shock in the economy, debtors whose wealth is in the "grey area" change their action from choosing a good technology (p) to choosing a bad technology (p – Δp) as in the above corollary (See Figure 2.4).

Lemma 7. When $\gamma < \beta$, there is a production technology $\underline{p}$ such that if $p > \underline{p}$, the critical wealth $\bar{W}^S$ for promissory notes increases more than $\underline{W}^S$ for bills of exchange when a negative macro shock occurs.

Proof: See the Appendix. □

Proposition. When $R < \bar{R}$ and $p > \underline{p}$, market risk from bills of exchange is lower than promissory notes in the case of a macro shock.

Proof: From Lemma 7, the critical wealth $\bar{W}^S$ for promissory notes increases more than $\underline{W}^S$ for bills of exchange when a negative macro shock occurs. This means that the "grey area" for promissory notes ($\bar{W} \sim \bar{W}^S$) is wider than that of bills of exchange ($\underline{W} \sim \underline{W}^S$). □

Proposition provides a rationale for the "payment instrument policy" of the BOK to support bills of exchange.

4.4. Payment instrument policy

I start by studying the equilibrium prices of promissory notes (γ) and bills of exchange (β). In the absence of monitoring and settlement cost, the gross interest rate on PN at equilibrium is obtained such that

$$(1-\delta)\cdot\gamma=1,$$

where the risk free gross interest rate is 1. I introduce a settlement cost which occurs in the process of payment and settlement. This cost contains all costs related to payment and settlement such as costs from reserve requirements, from non-synchronization of receipts and payments, and admission or membership fees for payment systems. Let σ be a settlement cost. Since the expected return on the promissory notes fully covers the cost of the credit as above, the expected rate of return on PN at equilibrium becomes

$$(1-\delta_{PN})\cdot\gamma=1+\sigma_{PN}. \tag{2.4}$$

Assuming perfect competition between banks, similarly, the rate of return on bills of exchange at equilibrium is determined by the break-even condition

$$(1-\delta_{BE})\cdot\beta=1+C+\sigma_{BE} \tag{2.5}$$

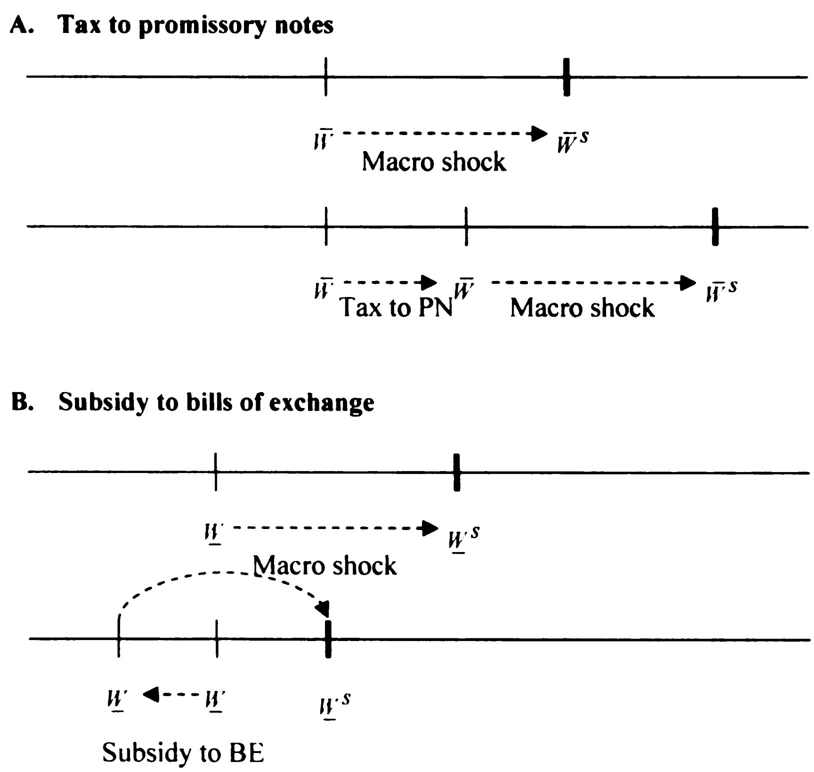
For the time being, I assume that the settlement costs of PN and BE are identical. This assumption will be relaxed when the payment instrument policy is implemented.

Lemma 8. When agents do not anticipate a macro shock, γ < β.

Proof: When agents do not anticipate macro shock, both payment instruments are priced by the default rates which are determined by debtors' moral hazard. According to (2.3), both payment instruments have identical default rates. The remaining is straightforward.

Now let me discuss the payment instrument policy tool of the BOK. I argue that subsidy for BE is better than tax to PN in terms of credit risk control. The critical level of wealth changes as the payment instrument policy is implemented. Taxing PN makes the critical wealth $\bar{W}$ increase, since $\frac{\partial \bar{W}}{\partial \gamma} > 0$. This makes the critical wealth $\bar{W}^s$ increases from the level which would otherwise reach after the macro shock. This makes the grey area ($\bar{W} \sim \bar{W}^s$) widen and therefore credit risk increase.

Figure 2.5: Payment instrument policies



Subsidizing BE makes the critical wealth $\underline{W}$ decrease, since $\frac{\partial \underline{W}}{\partial \beta} > 0$. This makes the critical wealth $\underline{W}^S$ decrease from the level which would otherwise reach after the macro shock. This makes the grey area ($\underline{W} \sim \underline{W}^S$) narrow and therefore credit risk does not increase.

Until now the economy is assumed to consist of homogenous agents who do not anticipate macroeconomic shocks. If this assumption is relaxed so that a certain proportion of the agents anticipate this shock at date 0, then the model has a different implication on market risks: the greater the proportion of agents that anticipate the shock, the lower market risk. Let q ∈ [0,1] be the portion of the agents that anticipate the shock. The economy has weighted critical wealth for lending standard for each types of consumption credit:

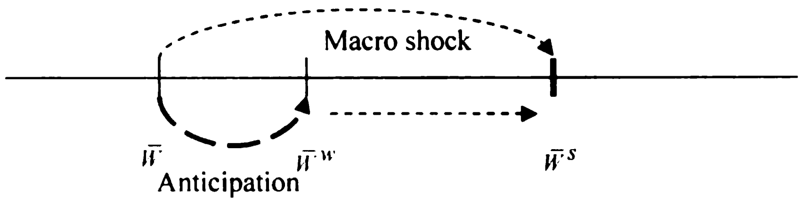
$$\bar{W}^W = (1-q) \cdot \bar{W} + q \cdot \bar{W}^S$$

and

$$\underline{W}^W = (1-q) \cdot \underline{W} + q \cdot \underline{W}^S$$

where $\bar{W}^W$ and $\underline{W}^W$ are weighted critical wealth of debtors for PN and weighted critical wealth for BE, respectively. As agents anticipate the macro shock, the economy has new critical levels of wealth for payment instruments. Now debtors of $W > \bar{W}^W$ can access PN, and debtors of $W > \underline{W}^W$ can use BE. This change makes the grey area ($\bar{W}^W \sim \bar{W}^S$ for PN or $\underline{W}^W \sim \underline{W}^S$ for BE) narrow and therefore lowers the market risk.

Figure 2.6: Macro shock and anticipation



5. Concluding remarks

This chapter has studied the role of the government on private money without legal restriction concerning usage of certain types of private money under macroeconomic shocks.

Payment systems are defined as organizations, operating procedures, and information and communications systems used to initiate and transmit payment messages from payer to payee and to settle payments (Balino et al. 1994). Since there occurs some cost in the process of payment and settlement of private monies, governments can manage market risk of private monies by adjusting their cost in payment and settlement. Payment instruments connect payer and payee from the beginning of a transaction. Different from other central bank polices[9], the effect of the "payment instrument policy" falls directly on the agents in a sense that it alters the initial asset choice problem of the individual.

By assumption, the private benefit of choosing a bad technology is higher under promissory notes as compared to bills of exchange; therefore, given a set of parameters, the range of values of wealth such that lending and good technology choice occurs is higher under bills of exchange than promissory notes. If we introduce a negative macro

[9] For example, monetary policy (open market operations) changes distribution of assets in the economy, and therefore it affects the economy as a whole not the individual agent.

shock such that the incentive to choose the good technology goes down, this shock is going to have a bigger impact in an economy with promissory notes.

Since promissory notes have a direct relationship between debtors and creditors, one may think that the use of promissory notes has a contagion effect[10]. However, the bottom line of the problem with promissory notes is shown to be moral hazard. Intrinsic market risk of promissory notes is higher than bills of exchange in the presence of macroeconomic shocks, and this property of promissory notes may cause collapse of payment system. This is a justification for a policy which supports usage of bills of exchange. The model also shows that a subsidy for bills of exchange is better than a tax on promissory notes. The "payment instrument policy" of the BOK can be justified by these results.

Another contribution is that this chapter provides a simple model for policy analysis. This partial equilibrium model is generally said to be too primitive to be used for policy analysis (Holmstrom and Tirole 1997); but if we care about risks alone, the model provides a framework which is sufficient for the analysis of risk management.

There are too many exogenous variables in the model and too few choices available to the agents. If we relax some of these relationships, one can find an interesting implication: what would happen if C and b, instead of being independent from each other, were negatively correlated? This seems like a natural relation if one considers a scenario where monitoring is an endogenous variable, i.e., we would expect that an increase in the monitoring effort would lead to both an increase in the cost of monitoring and a reduction

[10] Since trade credit also has a direct relationship between debtors and creditors, the use of trade credit causes shocks to propagate in the economy [Refer to Kiyotaki and Moore (1997) or Boissay (2006)]. In addition to that, the BOK mentioned this contagion effect (or in their term 'chain of defaults') of promissory notes as one reason to introduce the "Corporate Procurement Loan" (BOK, 2001).

in the private benefit of the bad technology. As a result, the economy has a lower critical level of wealth $\underline{W}$, and therefore more agents who are relatively poorer can use payment instruments.

One can extend this model in several ways: one can consider the moral hazard problem committed by the banks. In relation to this, one may extend the model by considering the size of capital of the banks which may limit the size of lending.

Appendix

A.1. Proof of Lemma 3

We have $\overline{W} \equiv E - \frac{p}{\gamma}\left[R - \frac{B}{\Delta p}\right]$ and $\underline{W} \equiv E - \frac{p}{\beta}\left[R - \frac{b}{\Delta p}\right]$. Subtracting $\underline{W}$ from $\overline{W}$ gives:

$$\overline{W} - \underline{W} = E - \frac{p}{\gamma}\left[R - \frac{B}{\Delta p}\right] - E + \frac{p}{\beta}\left[R - \frac{b}{\Delta p}\right],$$

which is simplified

$$= \frac{p}{\beta \cdot \gamma}\left\{\underbrace{(\gamma - \beta)}_{-} R + \underbrace{(B \cdot \beta - b \cdot \gamma)}_{+}\frac{1}{\Delta p}\right\}.$$

If $(\gamma - \beta) R + (B \cdot \beta - b \cdot \gamma)\frac{1}{\Delta p} > 0$ or

$$R < \frac{(B \cdot \beta - b \cdot \gamma)}{(\beta - \gamma)} \frac{1}{\Delta p} \equiv \overline{R},$$

we have $\overline{W} - \underline{W} > 0$. □

A.2. Proof of Lemma 4

I calculate the default rate after a macro shock, then prove Lemma 4. Let w be the weight of the "grey area" with respect to the "accessible area" in which the agent can use a payment instrument. We have

$$w_{PN} = \frac{G(\overline{W}^{S}) - G(\overline{W})}{1 - G(\overline{W})}.$$

Total default rate is

$$\left[1 - (p - \Delta p)\right] \cdot w_{PN} + (1 - p) \cdot (1 - w_{PN}) \quad (2.A.6)$$

The first term is default rate from "grey area", and the second term is default rate from remaining area (= "accessible area" – "grey area"). Rearranging (2.A.6), we have

$$(1-p)+\Delta p\cdot w_{PN} \tag{2.A.7}$$

$(1-p)$ is default rate from production technology (or moral hazard), and $\Delta p\cdot w_{PN}$ is default rate from the macro shock. Therefore, market risk from the macro shock is

$$\eta_{PN}=\Delta p\cdot w_{PN} \tag{2.A.8}$$

Derivative of market risk with respect to the macro shock R^s is

$$\begin{aligned}\frac{\partial\eta_{PN}}{\partial R^s}&=\Delta p\cdot\frac{\partial w_{PN}}{\partial\overline{W}^s}\cdot\frac{\partial\overline{W}^s}{\partial R^s}\\&=\Delta p\cdot\frac{\partial w_{PN}}{\partial\overline{W}^s}\cdot\left(-\frac{(p-\eta_{PN})}{\gamma}+\frac{R^s}{\gamma}\frac{\partial\eta_{PN}}{\partial R^s}+\frac{(p-\eta_{PN})}{\gamma^2}\left[R^s-\frac{B}{\Delta p}\right]\frac{\partial\gamma}{\partial R^s}\right)\end{aligned}$$

since $\dfrac{\partial\overline{W}^s}{\partial R^s}=-\dfrac{(p-\eta_{PN})}{\gamma}+\dfrac{R^s}{\gamma}\dfrac{\partial\eta_{PN}}{\partial R^s}+\dfrac{(p-\eta_{PN})}{\gamma^2}\left[R^s-\dfrac{B}{\Delta p}\right]\dfrac{\partial\gamma}{\partial R^s}$.

Solving the above equation for $\dfrac{\partial\eta_{PN}}{\partial R^s}$, we have

$$\frac{\partial\eta_{PN}}{\partial R^s}=\frac{\Delta p\cdot\frac{\partial w_{PN}}{\partial\overline{W}^s}}{1-\Delta p\cdot\frac{\partial w_{PN}}{\partial\overline{W}^s}\cdot\frac{R^s}{\gamma}}\cdot\left(-\underbrace{\frac{(p-\eta_{PN})}{\gamma}}_{+}+\underbrace{\frac{(p-\eta_{PN})}{\gamma^2}\left[R^s-\frac{B}{\Delta p}\right]}_{+}\underbrace{\frac{\partial\gamma}{\partial R^s}}_{-}\right).$$

We know that $\dfrac{\partial w_{PN}}{\partial\overline{W}^s}$ has positive sign from the definition of weight w_{PN}. The only thing that decides the sign of $\dfrac{\partial\eta_{PN}}{\partial R^s}$ is the sign of the denominator, $1-\Delta p\cdot\frac{\partial w_{PN}}{\partial\bar{W}^s}\cdot\frac{R^s}{\gamma}$. If $1-\Delta p\cdot\frac{\partial w_{PN}}{\partial\bar{W}^s}\cdot\frac{R^s}{\gamma}>0$, then it will suffice. Let me specify the distribution of wealth. Wealth is uniformly distributed with lower limit of zero and upper limit of N. The cumulative distribution function of wealth, G(·) is defined as

$$G(W)=\frac{W}{N}$$

for $0 \le W \le N$. The weight, therefore, w_{PN} is

$$w_{PN}=\frac{G(\bar{W}^s)-G(\bar{W})}{1-G(\bar{W})}=\frac{\bar{W}^s-\bar{W}}{N-\bar{W}},$$

and

$$\frac{\partial w_{PN}}{\partial\bar{W}^s}=\frac{1}{N-\bar{W}}.$$

The denominator is

$$1-\Delta p\cdot\frac{\partial w_{PN}}{\partial\bar{W}^s}\cdot\frac{R^s}{\gamma}=1-\Delta p\cdot\frac{1}{N-\bar{W}}\cdot\frac{R^s}{\gamma}=1-\frac{\Delta p\cdot R^s}{\gamma(N-E)+p\left[R-\frac{B}{\Delta p}\right]}. \quad (2.A.9)$$

If the private benefit B is not too big, or

$$B<\frac{\Delta p\cdot\left[(1+\sigma)(N-E)+p^2\cdot R-p\cdot\Delta p\cdot R^s\right]}{p^2}\left(\equiv\bar{B}\right),$$

we have positive sign. This is straightforward from (2.A.9). We have

$$\frac{\gamma(N-E)+p\left[R-\frac{B}{\Delta p}\right]-\Delta p\cdot R^s}{\gamma(N-E)+p\left[R-\frac{B}{\Delta p}\right]}>0$$

or

$$B < \frac{\gamma(N-E) + p \cdot R - \Delta p \cdot R^s}{p} \cdot \Delta p .$$

Since the minimum of γ is $\frac{1+\sigma}{p}$, we have positive sign of the denominator when

$$B < \frac{\frac{1+\sigma}{p} \cdot (N-E) + p \cdot R - \Delta p \cdot R^s}{p} \cdot \Delta p .$$

A.3. Proof of Lemma 7

Since we consider a negative macro shock, both of the derivatives of critical values of wealth are negative. I compare absolute value of derivatives of wealth with respect to macro shock.

$$\left|\frac{\partial \bar{W}^s}{\partial R^s}\right| - \left|\frac{\partial \underline{W}^s}{\partial R^s}\right| = \frac{(p-\eta_{PN})}{\gamma} - \frac{R^s}{\gamma}\frac{\partial \eta_{PN}}{\partial R^s} - \frac{(p-\eta_{PN})}{\gamma^2}\left[R^s - \frac{B}{\Delta p}\right]\frac{\partial \gamma}{\partial R^s}$$
$$-\frac{(p-\eta_{BE})}{\beta} + \frac{R^s}{\beta}\frac{\partial \eta_{BE}}{\partial R^s} + \frac{(p-\eta_{BE})}{\beta^2}\left[R^s - \frac{b}{\Delta p}\right]\frac{\partial \beta}{\partial R^s}$$

Rearranging this and solving for p, we have

$$\frac{(p-\eta_{PN})}{\gamma^2}\left[\gamma - A_1\frac{\partial \gamma}{\partial R^s}\right] + \frac{(p-\eta_{BE})}{\beta^2}\left[-\beta + A_2\frac{\partial \beta}{\partial R^s}\right] + R^s\left[-\frac{1}{\gamma}\frac{\partial \eta_{PN}}{\partial R^s} + \frac{1}{\beta}\frac{\partial \eta_{BE}}{\partial R^s}\right]$$

where $A_1 = R^s - \frac{B}{\Delta p}$ and $A_2 = R^s - \frac{b}{\Delta p}$.

Since $\frac{\partial \eta_{PN}}{\partial R^s} < 0$ and $\frac{\partial \eta_{BE}}{\partial R^s} < 0$ are very small, and we have $\frac{1}{\gamma} > \frac{1}{\beta}$ from Lemma 8; the last term has positive sign. We have $\left|\frac{\partial \bar{W}^s}{\partial R^s}\right| - \left|\frac{\partial \underline{W}^s}{\partial R^s}\right| > 0$, when

$$\frac{1}{\gamma^2\beta^2}\left\{\beta^2(p-\eta_{PN})\left[\gamma - A_1\frac{\partial\gamma}{\partial R^s}\right]+\gamma^2(p-\eta_{BE})\left[-\beta + A_2\frac{\partial\beta}{\partial R^s}\right]\right\}>0$$

or

$$p > \frac{\beta^2\left[\gamma - A_1\frac{\partial\gamma}{\partial R^s}\right]\eta_{PN} - \gamma^2\left[\beta - A_2\frac{\partial\beta}{\partial R^s}\right]\eta_{BE}}{\beta^2\left[\gamma - A_1\frac{\partial\gamma}{\partial R^s}\right] - \gamma^2\left[\beta - A_2\frac{\partial\beta}{\partial R^s}\right]}\left(\equiv \underline{p}\right). \qquad (2.A.10)$$

Since the maximum weights of PN and BE are one from (2.A.8) and its corollary for η_{BE}, a sufficient probability $\underline{p}$ is Δp, or

$$p > \frac{\left\{\beta^2\left[\gamma - A_1\frac{\partial\gamma}{\partial R^s}\right] - \gamma^2\left[\beta - A_2\frac{\partial\beta}{\partial R^s}\right]\right\}\Delta p}{\beta^2\left[\gamma - A_1\frac{\partial\gamma}{\partial R^s}\right] - \gamma^2\left[\beta - A_2\frac{\partial\beta}{\partial R^s}\right]} \geq \underline{p}. \square \qquad (2.A.11)$$

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Chapter 3

Production technology and private information in a search theoretic model

1. Introduction

Risk management is a core element of payment policy. In order to implement an effectual policy, it is necessary to understand properties of the risks. As former Federal Reserve Chairman Greenspan mentioned in his 2004 AEA meeting address[1], it is important to understand the many sources of risk and uncertainty. In Chapter 2, market risk from moral hazard of debtors' choice of production technology plays an important role in the sensitivity of the payment system when there is a macroeconomic shock.

Since Akerlof (1970), qualitative uncertainty in the trading sector has been studied in repeated game setups. With a search theoretic approach, Williamson and Wright (1994, hereafter WW) develop a model on the same topic. This has inspired other research using the search theoretic approach to an economy with private information[2]. They all focus on informational frictions which arise in the trading sector. There are also various types of uncertainty in production processes. For example, farmers sow good

[1] "As a consequence, the conduct of monetary policy ... has come to involve, at its core, crucial elements of risk management. This conceptual framework emphasizes understanding as much as possible the many sources of risk and uncertainty that policymakers face, quantifying those risks when possible, and assessing the costs associated with each of the risks."
[2] Kim (1996) studies the endogenous information structure with fixed acquisition cost; Trejos (1997) studies the effects of monetary matters on the incentives to produce high-quality output; and Trejos (1999) studies informational friction economy with divisible goods. Some other related research includes Cuadras-Morato (1994) and Li (1995, 1998).

quality seeds, and grow them with care, but unanticipated events, such as drought, flood, or disease affects farm production. Or, because there are so many constituent parts, there is a multitude of ways in which a car can be defective. The first production uncertainty can be characterized as bad luck, and the latter as a limitation of technology. Both have the common feature that agents cannot completely control the outcome. This gives rise to quality control to reduce the number of defective products produced, and warranties to protect consumers from suffering should they purchase, unwittingly, a defective product.

I study the role of production uncertainty in a search theoretic model with private information. A search theoretic model impedes any possibility to compensate or punish a willful cheat since one will never meet the same trading partner twice. As a result of this property, the equilibrium trading pattern differs from that of a Walrasian market. Search models amplify the effect of frictions and uncertainty in the sense that they return a narrower and smaller range of parameter values in which an equilibrium exists than in a repeated game setup[3].

There are several search-theoretic papers dealing with the production sector, although not with private information. Diamond (1982) designs a search model with a production sector in which production costs are randomly drawn from a given distribution. Since his model generates multiple steady state equilibria, even without lags in the ability of the government to affect private decisions, an economic policy for stabilization may not have the power to achieve instantaneously its goal. Wallace and Zhou (1997) and Boyarchenko (2000) deal with heterogeneous agents who have different production costs or productivity. Both papers show that higher productivity agents dominate trade. The findings in Diamond (1982) depend on the assumption of an

[3] See the Appendix (A.1)

increasing returns to scale meeting technology. Johri (1999) relaxes this assumption and generates multiple equilibria as Diamond did. Johri uses the production sector to show the different equilibrium price levels are associated with different levels of the reservation production cost.

I extend the WW model by introducing exogenous uncertainty in the production process. The production uncertainty implies that a product or process will not meet the quality requirements, therefore it can be referred as probability of defect. Agents in the model choose between good and bad production technologies. Because of production uncertainty, agents may get a bad product even under a good production technology. Different types of uncertainty generate different effects: the model shows that there exist more equilibria in the production uncertainty case than in the no production uncertainty case. Comparative statics show that an increase of production uncertainty increases possible returns from choosing the bad technology. These findings generate important implications for designing policies to alleviate risk. When the economy experiences a deterioration of technology such as a macroeconomic shock, it amplifies the tendency of the economy to choose the bad technology. This may lead the economy to autarchy.

This chapter is organized as follows. Section 2 describes the basic model, section 3 analyzes equilibria, section 4 analyzes welfare and section 5 discusses comparative statics. Section 6 summarizes our findings and concludes. Lengthy proofs are in the appendix.

2. Basic Model

This model is essentially the same as in WW. The economy is populated by a continuum of homogeneous, infinitely-lived agents distributed on the unit interval. There are two objects: a good quality commodity and a bad quality commodity. The good commodity and the bad commodity can be produced by all agents. All commodities are indivisible, freely disposable and storable at zero cost, but only one unit at a time. There are two types of production processes: a good process which generates a good quality commodity, and a bad process which generates a bad quality commodity. Because of an imperfection in the production technology there is a defect in the good process so agents get the quality that they intended to produce with probability $\theta_p \in [0, 1]$. This probability is independent across agents. The disutility of producing one unit of a good under the good process equals $\gamma > 0$. Given the production technology, this implies that the average unit cost of production[4] is γ / θ_p. The cost of using the bad process is equal to zero. I assume that an agent cannot consume his own produced commodity and has full information on the quality of his commodity after production. Consumption of a good commodity yields utility $U > 0$ and consumption of a bad commodity yields zero utility. All agents are risk neutral.

Each period, agents are pairwise matched under a uniform random matching technology. Trade takes place if and only if mutually beneficial. Moreover, trade always involves a one-for-one swap of inventories since all types of objects are indivisible. So I do not consider the prices of the objects in this paper. An agent recognizes the quality of

[4] Because of the change of production cost, one may say the distribution of production cost can return the same result as the present paper. This is true when we consider type A, B and D equilibria. Under the distribution of the production cost γ, we cannot get type E equilibrium as in the present paper.

a good produced by another agent with probability $\theta \in [0, 1]$. This probability is independent across commodity traders when they meet. Agents do not know whether agents recognize their commodities and do not know anything about other agents' histories. Once a trade decision is made, it is irreversible. There are neither private credit arrangements nor promises, since agents who meet in one period will meet again in another period with probability zero. Let p denote the proportion of traders holding good output. The following summarizes the sequence of events within a period:

1. Agents meet pairwise at random. They check the quality of the other trader's commodity. Traders swap their inventories only if mutually agreeable.
2. After trade, agents separate and the types of commodities are revealed. Agents decide to consume, dispose or store the commodity. If they consume or dispose of the commodity, they choose the production process for the next period.
3. If the good process is chosen, a good output obtains with probability θ_p. Under a bad process, only bad commodities are produced.

I look for stationary Nash equilibria in which the meeting probabilities are summarized by p, the fraction of goods holders with a good output. Moreover, when making his decision, an agent takes as given the probabilities with which he believes other traders with good commodities will accept commodities of unrecognized quality.

3. Equilibrium

In this chapter I restrict attention to non-monetary (or barter) equilibria where at least some good commodities are produced and consumed. I denote these equilibria as active equilibria. I assume that $\theta < 1$, and $\theta_p < 1$, so that agents are not able to recognize the quality of the other potential trader's commodity in some meetings and get a quality of commodity different from their original intention in good production processes. An agent with a bad commodity is willing to trade with any agent at every possibility because at worst he gets another bad commodity, however an agent with a good commodity is not willing to do so. Agents decide whether to choose the good or bad production process, and whether to accept or reject a commodity of unrecognized quality when they are currently holding a good commodity themselves.

Let Σ denote the probability with which an agent believes that other traders with good commodities will accept commodities of unrecognized quality, and let σ be an individual's best response.

Lemma 1. *$0 < p \leq \theta_p < 1$ in any active equilibrium.*

Proof. Given the production technology θ_p, we cannot have $p > \theta_p$. When every agent tries to produce good commodities then we have $p = \theta_p$. Moreover, $p = 0$ is not an active equilibrium. □

Let V_j denote the value function at the end of period for an agent holding object j, where j = g, b denotes a good commodity and a bad commodity respectively. Let $r > 0$ be the discount rate. Let $W = \max\{-\gamma + \theta_p V_g + (1-\theta_p)V_b, V_b\}$ represent the value function for a producer with nothing in inventory, who is deciding which production process to use. The first element of W is when agents choose a good production process. Usually this effort leads agents to a good outcome, but there is some noise in the process. So, agents would get low quality output with probability $1 - \theta_p$, even though they made an effort in terms of γ units of disutility.

An individual's best response problem is described as follows:

$$rV_g = \theta p \Delta_N (U + W - V_g) + (1-\theta)\max_{\sigma} \sigma[p\Delta_N (U + W - V_g) + (1-p)(W - V_g)] \quad (3.1)$$

$$rV_b = p(1-\theta)\Sigma(U + W - V_b) \quad (3.2)$$

where $\Delta_N \equiv \theta + (1-\theta)\Sigma$ is the probability that an agent with a good commodity is willing to trade. The first term on the right hand side of (3.1) is the probability that the individual meets someone with good output he can identify ($\theta \cdot p$) multiplied by the probability that his partner is willing to trade (Δ_N) times the gain from trading, $U + W - V_g$. The second term is the probability that the individual meets someone he can not identify, $1-\theta$, multiplied by the net expected gain from choosing the acceptance probability σ. Expression (3.2) can be explained in a similar way. By Lemma 1, $p > 0$ and $W = -\gamma + \theta_p V_g + (1-\theta_p)V_b \geq V_b$ in any active equilibrium. Hence, the difference between the value functions from the good process and the bad process is greater than or equal to the expected cost of a unit of good quality good. We have:

$$V_g - V_b \geq \frac{\gamma}{\theta_p}. \quad (3.3)$$

Axiom *$V_g - V_b > 0$.*

Definition. *An active symmetric Nash equilibrium is a set (V_g, V_b, p, Σ) such that:*

(i) $-\gamma + \theta_p V_g + (1-\theta_p)V_b > V_b$ *implies* $p = \theta_p$

and $p < \theta_p$ *implies* $-\gamma + \theta_p V_g + (1-\theta_p)V_b = V_b$

(ii) *given p, σ = Σ must solve the maximization problem in (3.1).*

According to the range of p and Σ, there are potentially five types of equilibria that may occur. Table 3.1 shows types of equilibria by p and Σ. Plugging $W = -\gamma + \theta_p V_g + (1-\theta_p)V_b$ into (3.1) and (3.2), we have

$$\begin{aligned} rV_g &= \theta p \Delta_N [U - \gamma - (1-\theta_p)(V_g - V_b)] \\ &\quad + (1-\theta)\max_{\sigma} \sigma\{p\Delta_N[U-\gamma-(1-\theta_p)(V_g - V_b)] + (1-p)[-\gamma-(1-\theta_p)(V_g-V_b)]\} \end{aligned} \tag{3.1′}$$

and

$$rV_b = p(1-\theta)\Sigma[U - \gamma + \theta_p(V_g - V_b)]. \tag{3.2′}$$

Now I will check the existence of equilibria. The procedure for checking the existence of each equilibrium is as follows: first I plug the parameters of Table 3.1. into the value functions (3.1′) and (3.2′), and then solve the value functions for parameters, and then check restrictions for parameters in a way that conditions for each equilibrium are satisfied. Consider type A equilibrium. Type A is the case of every agent choosing the good production process, and every agent accepts unrecognized commodity: $p = \theta_p$ and $\Sigma = 1$. After plugging these two parameters into the value functions, I check the condition under which choosing a good production process is a best response, or i.e. choosing a good production process is unimprovable by a one-shot deviation. One can show that condition (3.3) holds if and only if $\theta \geq \theta_1$ where $\theta_1 = \dfrac{(r+1)\gamma}{\theta_p(\theta_p U - \gamma) + \gamma}$. Type B, D and E equilibria can be checked with the same procedure as described above for type A equilibrium. Proposition 1 summarizes the existence condition for each equilibrium[5].

Table 3.1: Types of equilibria

	$p = \theta_p$	$p \in (0, \theta_p)$
$\Sigma = 1$	A	B
$\Sigma \in (0,1)$	C	D
$\Sigma = 0$	E	-

Proposition 1. *From the existence condition for each type of equilibrium, each type of equilibrium has following critical values of* θ*:*

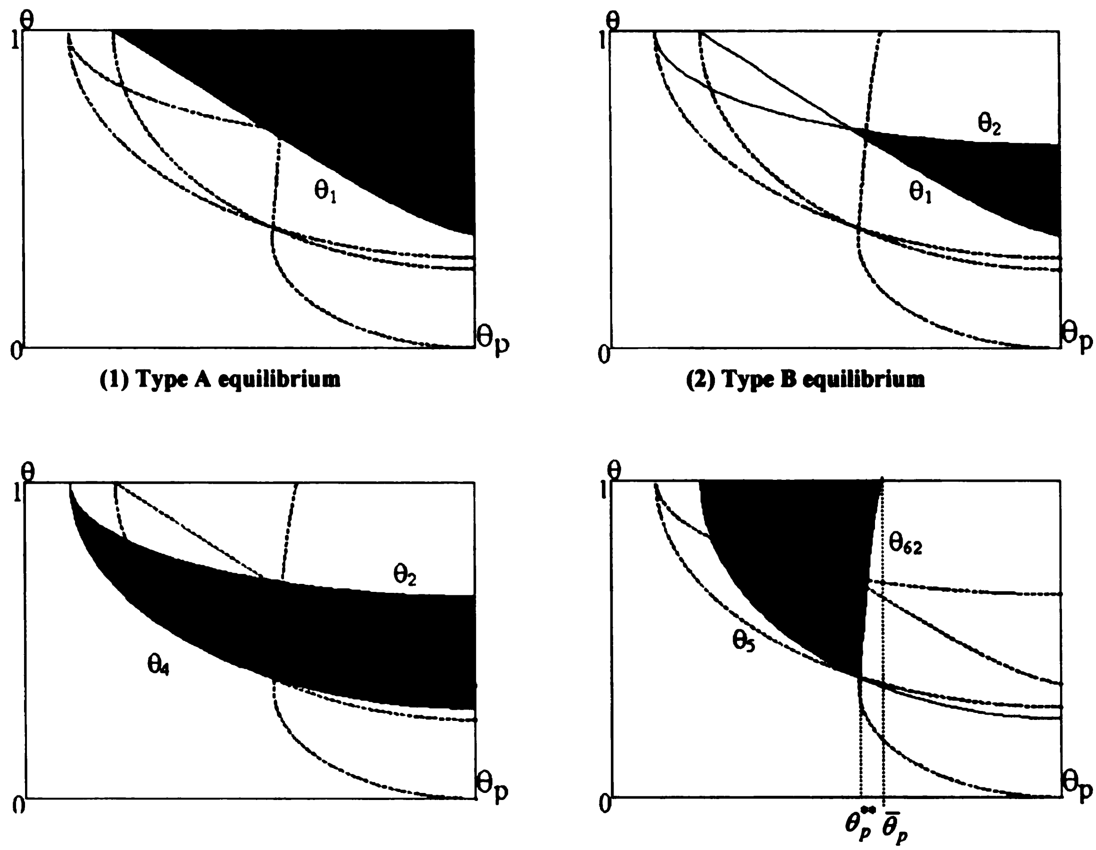
1. *Type A equilibrium for* $\theta_1(r, \theta_p) \leq \theta$
2. *Type B equilibrium for* $\theta_1(r, \theta_p) < \theta \leq \theta_2(r, \theta_p)$
3. *Type D equilibrium for* $\theta_4(r, \theta_p) < \theta < \theta_2(r, \theta_p)$
4. *Type E equilibrium for* $\theta_5(r, \theta_p) \leq \theta < 1$ *when* θ_p *is low,*

   and for $\theta_{62}(r, \theta_p) \leq \theta < 1$ *or* $\theta_5(r, \theta_p) \leq \theta < \theta_{61}(r, \theta_p)$ *when* θ_p *is high.*

Proof. See the Appendix. □

[5] One may consider type C equilibrium: every agent chooses a good production process, but some agents would not accept to trade with unrecognized commodity. Unfortunately, however, it is impossible to check the existence of the equilibrium.

Figure 3.1: Production technology and critical values of θ



(1) Type A equilibrium (2) Type B equilibrium

(3) Type D equilibrium (4) Type E equilibrium

Proposition 1 shows that types of equilibria depend on critical values of θ, and those critical values are functions of production technology θ_p and discount rate r. Figure 3.1 shows that the relationship between θ and θ_p with arbitrary values[6] of U, γ and r. Type E equilibrium exists $\theta_5(r,\theta_p) \le \theta$ when production technology is lower than θ_p^{**}, the solution to

$$D_{E1} \equiv r^2\theta_p{}^2(U-\gamma)^2 - 4(1-\theta p)^3 r\theta_p U\gamma,$$

and exists $\theta_{62}(r,\theta_p) \le \theta$ or $\theta_5(r,\theta_p) \le \theta < \theta_{61}(r,\theta_p)$ when production technology is higher than θ_p^{**}. Notice that if the production technology is higher than $\bar{\theta}_p$, then the type

[6] I use U = 2.5, γ = 0.75 and r = 0.05.

E equilibrium does not exist any more. WW is a special case of this. Lower boundaries of each equilibrium have negative slope (except θ_{62} for type E), so one can find a substitute relation between inspection technology θ and production technology θ_p.

***Lemma** 2. When the equilibria exist, traders in type A equilibrium hold more good products than traders in type B. Also, traders in type B equilibrium hold more good products than traders in type D equilibrium; i.e.* $p_A(=\theta_p) > p_B > p_D$

Proof. Notice that $p_A = \theta_p$ is the highest proportion which the economy can achieve, therefore $p_A(=\theta_p) > p_B$ is trivial. Next, I will show that $p_B > p_D$.

We have

$$p_B = \frac{(r+1-\theta)\gamma}{\theta(\theta_p U - \gamma)}$$

and

$$p_D = \frac{[r+(1-\theta)\Sigma_D]\gamma}{\{[\theta+(1-\theta)\Sigma_D]^2 - (1-\theta)\Sigma_D\}(\theta_p U - \gamma)}$$

where $\Sigma_D = \dfrac{(\theta-r)\theta(\theta_p U-\gamma)-r\gamma}{(1-\theta)[(r-\theta+1)(\theta_p U-\gamma)+\gamma]}$ [See the Appendix (A.2)]. The ratio p_B to p_D is

$$\frac{p_B}{p_D} = \frac{(r+1-\theta)[\theta(\theta_p U-\gamma)^2 + \gamma\theta_p U]}{\theta(\theta_p U-\gamma)[\gamma(\theta-r)+\theta_p U(1+r-\theta)]}.$$

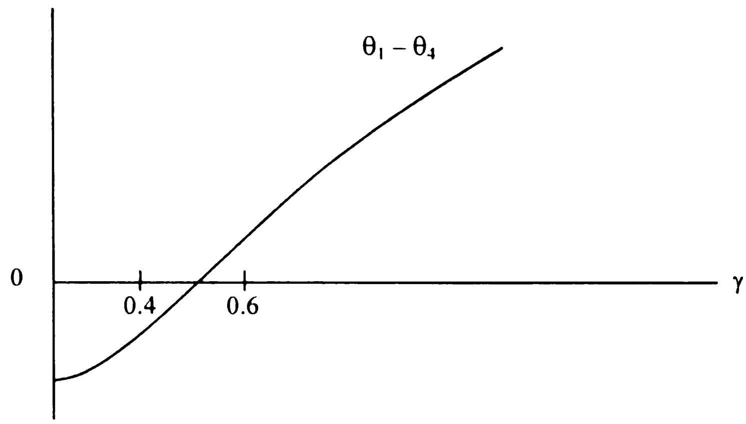
The numerator of $\frac{p_B}{p_D}$ minus the denominator of $\frac{p_B}{p_D}$ gives

$$\gamma(\gamma\theta + \theta_p U + r\theta_p U - 2\theta\theta_p U).$$

From 2. and 3. of Proposition 1 and $\gamma > 0$, one can show $\gamma\theta + \theta_p U + r\theta_p U - 2\theta\theta_p U > 0$.

When the private information problem is severe under perfect production technology ($\theta_p = 1$; WW), WW argue that type D has the greatest chance of surviving; because $\Sigma < 1$ imposes the greatest discipline on producers of the bad commodity. WW explain that the existence of the type D equilibrium depends on the willingness of agents with good output sometimes to give up a trade for something that they cannot recognize and wait to trade later for something that they can recognize. They emphasize trade process and the role of discount rate r for this willingness to wait. I argue that production cost γ also be considered as one factor for this willingness to wait. When comparing the critical regions for each type equilibrium, one can find $\theta_1|_{\theta_p=1} < \theta_4|_{\theta_p=1}$, which means type A or B equilibria exist under more severe private information problem than type D. Figure 3.2 shows this possibility. For example, there are two types of water in the economy; filtered water (good output) and tap water (bad output). The production cost of filtered water is very small compared with its utility. In this economy, if there are no other frictions, agents would accept the trade even though they cannot recognize the quality. In this example type A or type B equilibrium survives under more severe private information conditions. For another example, there are two types of cars in the economy; plums (good output) and lemons (bad output). I define the production cost of plums is sum of maintenance cost for a car, such as timely change of lube, brake and transmission, etc., and careful driving; In this case production cost is high compared with its utility. In this economy more agents would not accept the trade when they cannot recognize the quality. In this example type D has more possibility to survive.

Figure 3.2: A graph of $\theta_1 - \theta_4$ when $\theta_p = 1$ (U = 2.5, r = 0.05)



4. Welfare Analysis

Let Z_i denote social welfare in type i equilibrium, defined as the expected utility of the representative agent in the initial period. Then social welfare Z_i is measured by

$$Z_i = (1-p_i)V_b + p_i\left(-\frac{\gamma}{\theta_p} + V_g\right). \tag{3.4}$$

As Li (1998) argues welfare is determined by the frequency of trade and the probability of getting high-quality commodities from those transactions. The frequency of trade depends on, among other things, Σ. When we plug in the values in Table 3.1 to equation (3.4), we have following social welfares

$$rZ_A = \theta_p(1-\theta)(U-\gamma) - r\gamma + \theta_p[(1-\theta)\theta_p + r]\cdot K_A$$

$$\text{where } K_A\left(\equiv V_g^A - V_b^A\right) = \frac{\theta_p\theta(U-\gamma)-(1-\theta)(1-\theta_p)\gamma}{r+(1-\theta)(1-\theta_p)+\theta_p\theta(1-\theta_p)+\theta_p{}^2(1-\theta)}\left(\equiv\frac{NK_A}{DK_A}\right), \quad (3.5)$$

$$rZ_B = \frac{\gamma(1+r-\theta)(1-\theta)U}{\theta(\theta_p U-\gamma)},$$

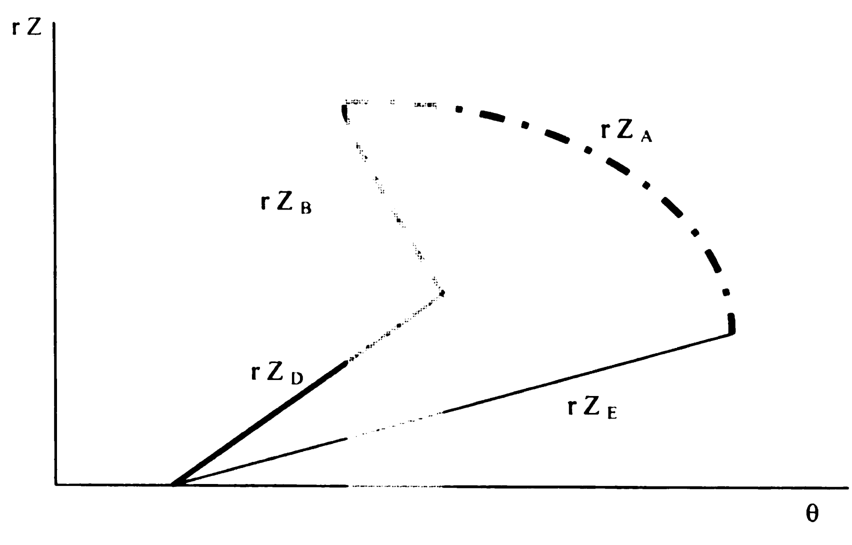
$$rZ_D = \frac{\gamma U[-r\gamma + r\gamma\theta - \gamma\theta^2 - r\theta\theta_p U + \theta^2\theta_p U]}{\gamma^2\theta + \gamma\theta_p U - 2\gamma\theta\theta_p U + \theta\theta_p{}^2 U^2},$$

$$rZ_E = -r\gamma + \frac{r\theta^2\theta_p^2(U-\gamma)}{r+\theta^2\theta_p(1-\theta_p)}.$$

***Proposition 2**. When utility U is sufficiently large relative to the average production cost (γ / θ_p), and the discount rate r is not too high, then welfare Z_A and welfare Z_B are decreasing in θ, welfare Z_D and welfare Z_E, however, are increasing in θ.*

Proof. See the Appendix (A.3). □

Figure 3.3: Social welfare by types of equilibria (U = 2.5, γ = 0.75, θ_p = 0.75 and r = 0.05)



Social welfare at type A equilibrium is decreasing in θ. This comes from the fact that payoffs of agents who have bad output decreases because it takes more time to conduct a successful trade. Social welfare at type B equilibrium is decreasing in θ. As WW point out, this results from the fact that the fraction of high quality goods, p_B, is decreasing in θ, as can be seen in rZ_B. Figure 3.3 represents the properties of social welfare Z_i using given but arbitrary values of U, γ, θ_p, and r. One can verify $Z_A > Z_B > Z_D > Z_E$ in the coexistence region (shaded area in Figure 3.3). I will show this in the Appendix (A.4).

Type D and type E equilibria have a couple of advantages. First, as inspection technology improves social welfare at these equilibria increases, whereas welfare at other types of equilibria decreases. Second, type D equilibrium is the most feasible choice when agents choose their trading strategy. $\Sigma < 1$ imposes a discipline on those who choose a bad production process in a way that the agent takes more time to have a successful trade. In type A and type B equilibria, by accepting every unrecognized commodity can give society higher welfare, but it makes the lemons problem more severe.

5. Comparative statics

In this section I analyze comparative statics with a production technology shock and a change in agents' patience.

I start with the change of production technology. A type D equilibrium has some interesting properties, so I discuss type D separately.

5.1. *Production technology and type A, B and E equilibria.* When the utility U is sufficiently large relative to the average production cost (γ / θ_p), an improvement in production technology extends the critical regions for equilibrium A, B and E. This is straightforward from the result of derivatives on critical values of θ in Proposition 1 with respect to θ_p. See the Appendix (A.5) for detail.

Proposition 3. *When the utility U is sufficiently large relative to the average production cost (γ / θ_p), an improvement in the production technology increases the social welfare at type A, B, E equilibria.*

Proof. See the Appendix (A.6). □

5.2. *Production technology and type D equilibrium.* I consider the critical region first. I keep the condition that the utility U is sufficiently large relative to the average production cost (γ / θ_p). We have following relation between the derivatives of the critical values of θ for type D equilibrium with respect to θ_p,

$$\frac{\partial\theta_2(r,\theta_p)}{\partial\theta_p}-\frac{\partial\theta_4(r,\theta_p)}{\partial\theta_p}=\frac{\gamma U\left[r(2\theta_pU-\gamma)^2-(1+r)(\theta_pU-\gamma)\sqrt{r(\theta_pU-\gamma)[4\gamma+r(\theta_pU-\gamma)]}\right]}{(2\theta_pU-\gamma)^2(\theta_pU-\gamma)\sqrt{r(\theta_pU-\gamma)[4\gamma+r(\theta_pU-\gamma)]}}.$$

Table 3.2: Production technology interval for type D equilibrium when production technology improves

	r = 0.01		r = 0.05		r = 0.1	
	$\underline{\theta}_p$	$\bar{\theta}_p$	$\underline{\theta}_p$	$\bar{\theta}_p$	$\underline{\theta}_p$	$\bar{\theta}_p$
$\gamma = 0.1$	0.0488	1(1.0415)	0.0721	0.1687	n.a.	n.a.
$\gamma = 0.5$	0.2437	1(5.2076)	0.3606	0.8436	n.a.	n.a.
$\gamma = 0.75$	0.3656	1(7.8114)	0.5408	1(1.265)	n.a.	n.a.
$\gamma = 1.0$	0.4875	1(10.415)	0.7211	1(1.687)	n.a.	n.a.
$\gamma = 1.5$	0.7312	1(15.623)	n.a.	n.a.	n.a.	n.a.

n.a. means there is no production technology between 0 and 1 to reduce the existence region of type D.

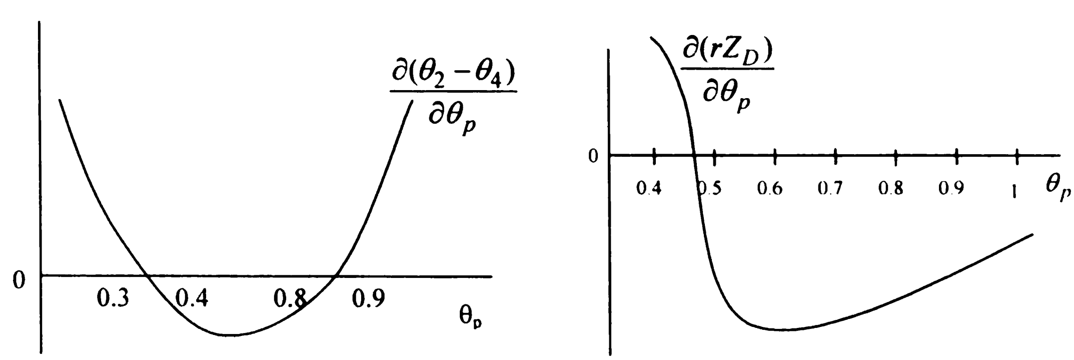
We can verify from Table 3.2 the value depends on the initial production technology level [Figure 3.4 (a) uses U = 2.5, γ = 0.5 and r = 0.05, and show that when the initial production technology is in (0.36, 0.84), the derivative of critical values of θ for type D has a negative sign; therefore the existence region for type D shrinks as the production technology improves].

Now I consider social welfare at type D. The derivative of welfare at type D equilibrium is

$$\frac{\partial(rZ_D)}{\partial\theta_p} = \gamma U^2[r\gamma^2 - 3r\gamma^2\theta + \gamma^2\theta^2 + r\gamma^2\theta^2 - \gamma^2\theta^3 + 2r\gamma\theta\theta_p U - 2r\gamma\theta^2\theta_p U + 2\gamma\theta^3\theta_p U + r\theta^2\theta_p{}^2U^2 - \theta^3\theta_p{}^2U^2] \div [\gamma^2\theta + \gamma\theta_p U - 2\gamma\theta\theta_p U + \theta\theta_p{}^2U^2]^2 .$$

Look at Figure 3.4 (b) (U = 2.5, γ = 0.5, r = 0.05 and θ = 0.75). It shows that if the initial production technology is high enough, the derivative of welfare at type D has negative sign; therefore the welfare of type D equilibrium decreases as the production technology improves.

Figure 3.4: Derivatives of type D equilibrium with respect to θ_p.



(a) Derivative of critical values of existence region (b) Derivative of social welfare

5.3. *Discount rate and equilibria.* Proposition 4 summarizes the relationship between the discount rate and the existence of each equilibrium and Proposition 5 deals with the relationship between the discount rate and social welfare at each equilibrium.

Proposition 4. *When the utility U is sufficiently large relative to the average production cost (γ / θ_p), if the discount rate r increases then the existing regions for equilibria shrink except for the case of type B.*

Proof. See the Appendix (A.7). □

Proposition 5. *When the utility U is sufficiently large relative to the average production cost (γ / θ_p), the social welfares of type B, D and E equilibria decrease in the discount rate r. The welfare of type A, however, increases in r.*

Proof: We will check the sign of derivatives of social welfare with respect to r. In case of type A equilibrium,

$$\frac{\partial Z_A}{\partial r} = \frac{\theta_p(1-\theta)(U-\gamma)}{r^2} - \frac{\theta_p{}^2(1-\theta)}{r^2} \cdot \frac{\partial K_A}{\partial r}$$

where $\frac{\partial K_A}{\partial r} = \frac{-(-\gamma + \gamma\theta_p + \gamma\theta - 2\gamma\theta\theta_p + \theta\theta_p U)}{[r + (1-\theta)(1-\theta_p) + \theta_p\theta(1-\theta_p) + \theta_p{}^2(1-\theta)]^2}$. Since $\frac{\partial K_A}{\partial r}$ has negative sign from (3.5, $\frac{\partial Z_A}{\partial r}$ has positive sign. Other derivatives are straight forward;

$$\frac{\partial Z_B}{\partial r} = \frac{-\gamma(1-\theta)^2 U}{r^2\theta(\theta_p U - \gamma)} < 0,$$

$$\frac{\partial Z_D}{\partial r} = \frac{-\gamma\theta^2 U(\theta_p U - \gamma)}{r^2(\gamma^2\theta + \gamma\theta_p U - 2\gamma\theta\theta_p U + \theta\theta_p{}^2 U^2)} < 0,$$

$$\frac{\partial Z_E}{\partial r} = \frac{-\theta^2\theta_p{}^2(U - \gamma)}{(r + \theta^2\theta_p - \theta^2\theta_p{}^2)^2} < 0. \square$$

Welfare Z_A increases when agents become less patient. This is because type A equilibrium has no lemons problem, so less patient agents trade more frequently which helps welfare increase. Intuition for Proposition 5 is that when there is a private information problem, an economy consisting of less patient agents has lower welfare than that consists of patient agents. Table 3.3 describes Proposition 4 and 5 when discount rate r decreases. When agents become patient without a private information problem, critical regions for equilibria become larger. From the social welfare point of view, not accepting unrecognized commodity works in this case, therefore the social welfare at the type E equilibrium increases under this circumstance. When agents become patient with a private information problem, social welfare increases. The critical region for the type D equilibrium, however, expands, because there are agents who deny the trade when they cannot recognize the quality.

Table 3.3: Discount rate and equilibria

r decreases		Welfare	
		Increase	Decrease
Critical Region	Expand	D, E	A
	Shrink	B	-

6. Concluding Remarks

This chapter has studied the role of production uncertainty in a search model with private information. In active equilibria, there exist multiple equilibria by the degree of production technology. Furthermore, the introduction of another type of noise, production technology, brings two additional types of equilibria which do not exist in the original model; type C – an equilibrium where all agents choose good production processes, but some of them will not accept unrecognized commodity – and type E – an equilibrium where all agents choose good production processes, but all of them do not accept unrecognized commodities. These equilibria can be Pareto-ranked. An increase of production uncertainty makes social welfare decrease and existing regions for type A, B and E equilibria shrink. Type D – some agents choose bad production processes, and some others will not accept unrecognized commodity – has a different property. The model has shown that both welfare and existing region of type D equilibrium depends on initial production technology. Considering that type D equilibrium is most feasible equilibrium under private information problem, a macroeconomic shock which would increase production uncertainty makes agents choose a bad process. When agents become patient, not accepting an unrecognized commodity is a helpful means for improving social welfare.

APPENDIX

A.1. An infinitely repeated game

In this subsection, I study an infinitely repeated game with production uncertainty. In this game I show that punishment affects current behavior. There are two agents in the economy; agent A and B. They meet each other to trade their product. I hold the same cost and utility structure as in the search model, and I assume that they cannot consume their own product. They can choose either a good production process (GP) in which they can get a good product with probability of θ_p or a bad production (BP) process in which they only get a bad output. I suppose that for each time t, the outcomes of the t − 1 preceding choice are observed before the choice at time t. Let β be discount factor which defined as 1 / (1 + r) where r is discount rate. Table 3.A.1 shows the payoffs of each agent.

Let agent A's strategy be as follows: (a) He starts with GP, (b) From t =2 and on if agent B chose GP on the previous time then choose GP, otherwise choose BP. In the same manner, agent B's strategy is as follows: (a) He starts with GP, (b) From t =2 and on if agent A chose GP on the previous time then choose GP, otherwise choose BP.

Table 3.A.1: A table of payoffs

A \ B	GP	BP
GP	$(-\gamma + \theta_p U, -\gamma + \theta_p U)$	$(-\gamma, \theta_p U)$
BP	$(\theta_p U, -\gamma)$	(0, 0)

Now I consider the present value of the payoff. When an agent chooses to stay GP, his present value of the payoff is

$$(-\gamma + \theta_p U)(1 + \beta + \beta^2 + \cdots) = \frac{-\gamma + \theta_p U}{1 - \beta}. \quad (3.A.1)$$

Present value of payoff when agent exercises one time deviation from GP is

$$\theta_p U . \tag{3.A.2}$$

The given strategies are Nash equilibrium if and only if (3.A.1) $\geq$ (3.A.2) (unimprovability criteria). As a result one can get

$$\frac{\theta_p U - \gamma}{\gamma} \geq r . \tag{3.A.3}$$

I look at the critical value of type A equilibrium in the search model. Rearranging the critical value, we have

$$\frac{\theta[\theta_p(\theta_p U - \gamma) + \gamma] - \gamma}{\gamma} \geq r .$$

When there is no private information problem (θ = 1) in the search model, the equilibrium condition in an infinitely repeated game (3.A.3) is weaker than that in the search model.

A.2. Proof of Proposition 1

The value functions of type B ($\Sigma = 1$, $0 < p < \theta_p$) equilibrium are;

$$rV_g = pU - (\theta \cdot p + 1 - \theta)[\gamma + (1 - \theta_p)(V_g - V_b)],$$

$$rV_b = p(1 - \theta)U .$$

Solving for p with the above value functions, we have p_B. From the condition of $0 < p_B < \theta_p$, and the condition of $\sigma = \Sigma = 1$ being a best response or i.e.

$p_B[U - \gamma - (1 - \theta_p)(V_g - V_b)] + (1 - p_B)[-\gamma - (1 - \theta_p)(V_g - V_b)] \geq 0$, we have

$$\frac{(1+r)\gamma}{\theta_p U(\theta_p U-\gamma)+\gamma}<\theta \le \frac{(1+r)\theta_p U}{2\theta_p U-\gamma}(\equiv \theta_2).$$

The value functions of type D ($0 < \Sigma < 1$, $0 < p < \theta_p$) equilibrium are

$$rV_g = [\theta+(1-\theta)\Sigma]^2 p(U-K)-(1-\theta)\Sigma(1-p)(V_g - V_b),$$

$$rV_b = p(1-\theta)\Sigma U.$$

Solving for p using these value functions, we have p_D. We can get the Σ_D from the condition of $\sigma = \Sigma_D \in (0,1)$ being a best response or i.e.

$p_D[\theta+(1-\theta)\Sigma_D](U-V_g+V_b)=(1-p_D)(V_g-V_b)$, and then solving $0 < \Sigma_D < 1$, we have

$$\frac{(\theta_p U-\gamma)r+\sqrt{(\theta_p U-\gamma)^2 r^2+4r\gamma(\theta_p U-\gamma)}}{2(\theta_p U-\gamma)}(\equiv \theta_4)<\theta<\frac{(r+1)\theta_p U}{2\theta_p U-\gamma}.$$

The value functions of Type E ($\Sigma = 0$, $p = \theta_p$) equilibrium becomes

$$V_g = \frac{\theta^2\theta_p(U-\gamma)}{r+\theta^2\theta_p(1-\theta_p)},$$

$$V_b = 0.$$

Choosing the good production process ($p = \theta_p$) is a best response if and only if condition (3.3) holds. When solving this, we have

$$\sqrt{\frac{r\gamma}{\theta_p(\theta_p U-\gamma)}}(\equiv \theta_5)\le \theta. \tag{3.A.4}$$

From the condition for $\sigma = \Sigma = 0$ being a best response, or i.e.

$\theta\theta_p[U-\gamma-(1-\theta_p)(V_g-V_b)]+(1-\theta_p)[-\gamma-(1-\theta_p)(V_g-V_b)]<0$, we have

$$(1-\theta_p)^2\theta_p U\theta^2 - r\theta_p(U-\gamma)\theta + r\gamma(1-\theta_p) > 0. \qquad (3.A.5)$$

Before solving the inequality for θ, we need to check its determinant:

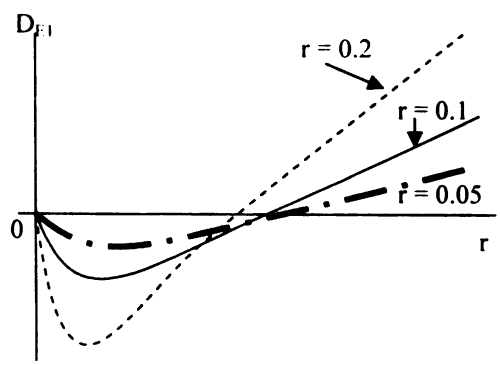
$$D_{E1} = r^2\theta_p^{\,2}(U-\gamma)^2 - 4(1-\theta_p)^3 r\theta_p U\gamma = r\theta_p[r\theta_p(U-\gamma)^2 - 4(1-\theta_p)^3 U\gamma].$$

If $D_{E1} < 0$ or $r\theta_p(U-\gamma)^2 < 4(1-\theta_p)^3 U\gamma$ (in case the production technology θ_p is low, look Figure 3.A.1), then (3.A.5) always holds, and therefore we need to consider condition (3.A.4) only. If

$$D_{E1} > 0 \text{ or } r\theta_p(U-\gamma)^2 > 4(1-\theta_p)^3 U\gamma \qquad (3.A.6)$$

(in case the production technology θ_p is high), then we have to consider second condition at the same time. Figure 3.A.1 (U = 2.5, γ = 1) shows that production technology affects the sign of determinant.

Figure 3.A.1: Negative determinant



Now, let us solve (3.A.5) for θ when $D_{E1} > 1$; we have

$$\theta < \frac{r\theta_p(U-\gamma) - \sqrt{r^2\theta_p^{\,2}(U-\gamma)^2 - 4(1-\theta_p)^3 r\theta_p U\gamma}}{2(1-\theta_p)^2\theta_p U} (\equiv \theta_{61}) \qquad (3.A.7)$$

and

$$\theta > \frac{r\theta_p(U-\gamma)+\sqrt{r^2\theta_p^{\,2}(U-\gamma)^2-4(1-\theta_p)^3 r\theta_p U\gamma}}{2(1-\theta_p)^2\theta_p U} (\equiv \theta_{62}) \qquad (3.A.8)$$

When (3.A.6) holds, we have a condition (3.A.4) ∩ [(3.A.7) U (3.A.8)]; therefore, we need to compare θ_5, θ_{61} and θ_{62}. It is hard to show analytically, but we can verify $\theta_5 < \theta_{62}$. We consider U = 2.5, γ = 0.5, 1, 1.5, r = 0.05. 0.10, 0.15, 0.20 and θ_p = 0.5, 0.6, 0.7, 0.8, 0.9. We have $\theta_{61} < \theta_5$ in all cases but when U = 2.5, γ = 0.5, r = 0.05, θ_p = 0.7. Therefore, we have $\theta_5(r,\theta_p) < \theta$ when θ_p is low (or D_{E1} < 0), and for $\theta_{62}(r,\theta_p) \le \theta$ or $\theta_5(r,\theta_p) \le \theta < \theta_{61}(r,\theta_p)$ when θ_p is high (or D_{E1} > 0). □

A.3. Proof of Proposition 2

We need to check sign of $\frac{\partial(rZ_i)}{\partial\theta}$. First, differentiating rZ_A with respect to θ, we have

$$\frac{\partial(rZ_A)}{\partial\theta} = -\theta_p(U-\gamma)-\theta_p^{\,2}\cdot K_A + \frac{\theta_p[(1-\theta)\theta_p+r][NK_A'\cdot DK_A - NK_A\cdot DK_A']}{DK_A^{\,2}} \qquad (3.A.9)$$

where $NK_A' = \theta_p(U-\gamma)+(1-\theta_p)\gamma > 0$, $DK_A' = -1+2\theta_p-2\theta_p^{\,2} < 0$.

Dividing (3.A.9) by θ_p and rearranging it, we have

$$\frac{-(1-\theta_p)[g_U(r,\theta,\theta_p)U - g_\gamma(r,\theta,\theta_p)\gamma]}{DK_A^{\,2}} \qquad (3.A.10)$$

where $g_U(r,\theta,\theta_p) = 1-\theta_p+\theta_p^{\,2}+2r-r\theta_p+r\theta_p^{\,2}+r^2-2\theta+4\theta\theta_p-4\theta\theta_p^{\,2}$
$+2\theta\theta_p^{\,3}-2r\theta+2r\theta\theta_p+\theta^2-3\theta^2\theta_p+4\theta^2\theta_p^{\,2}-2\theta^2\theta_p^{\,3}$,

$$g_\gamma(r,\theta,\theta_p)=1+\theta_p{}^2+2r+2r\theta_p+2r^2-2\theta+2\theta\theta_p-2\theta\theta_p{}^2-2r\theta+\theta^2-2\theta^2\theta_p+2\theta^2\theta_p{}^2.$$

It is hard to show (3.A.10) has negative sign analytically, but one can show that if $g_U(r,\theta,\theta_p)-g_\gamma(r,\theta,\theta_p)>0$, then (3.A.10) has negative sign. Differentiating rZ_B with respect to θ is straightforward;

$$\frac{\partial(rZ_B)}{\partial\theta}=\frac{-\gamma U(1+r-\theta^2)}{\theta^2(\theta_p U-\gamma)}<0$$

Differentiating rZ_D with respect to θ, we have

$$\frac{\partial(rZ_D)}{\partial\theta}=\frac{-\gamma U(\theta_p U-\gamma)(\gamma^2 r-\gamma^2\theta^2-2\gamma\theta_p\theta U+2\gamma\theta_p\theta^2 U-\theta_p{}^2\theta^2 U^2)}{(\gamma^2\theta+\gamma\theta_p U-2\gamma\theta_p\theta U+\theta_p{}^2\theta U^2)^2}$$

It is obvious that the denominator of $\frac{\partial(rZ_D)}{\partial\theta}$ has positive sign. Rearranging the numerator, we have

$$-\gamma U(\theta_p U-\gamma)[-\gamma\underbrace{(2\theta\theta_p U-\gamma r)}_{+}-(\theta_p U-\gamma)^2\theta^2]>0.$$

Differentiating rZ_E with respect to θ, we have

$$\frac{\partial(rZ_E)}{\partial\theta}=\frac{2\theta_p{}^2 r^2\theta(U-\gamma)}{(r+\theta_p\theta^2-\theta_p{}^2\theta^2)^2}>0.\ \square \qquad (3.A.11)$$

A.4. Pareto rank of social welfares Z_A, Z_B, Z_D and Z_E

This part shows Pareto-ranked social welfares in analytic and graphical way. First, I compare rZ_A with rZ_B;

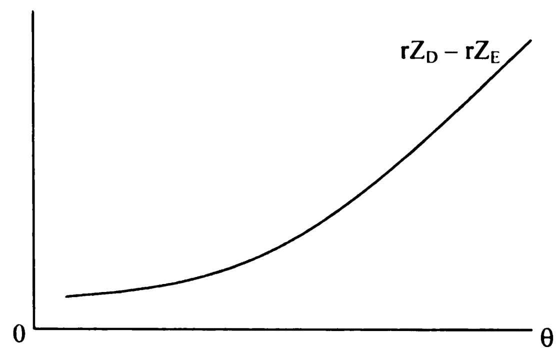
$$rZ_A - rZ_B = \frac{(-\gamma - \gamma r + \gamma\theta - \gamma\theta_p\theta + \theta_p{}^2\theta U)\times[\theta_p(1-\theta)+r]\theta(\theta_p U-\gamma)+U(1-\theta)\cdot DK_A}{[DK_A \cdot \theta(\theta_p U - \gamma)]}$$

From $0 < p_B < \theta_p$, I can show the first part of the first term of the numerator in $rZ_A - rZ_B$ has positive sign, and the second part of the first term and denominator have positive sign. I can show $rZ_B - rZ_D > 0$ by Lemma 2. Finally, I compare rZ_D with rZ_E;

$$rZ_D - rZ_E = \frac{\gamma U[-r\gamma + r\gamma\theta - r\theta^2 - r\theta\theta_p U + \theta^2\theta_p U]}{\gamma^2\theta + \gamma\theta_p U - 2\gamma\theta\theta_p U + \theta\theta_p{}^2 U^2} + r\gamma - \frac{r\theta^2\theta_p{}^2(U-\gamma)}{r+\theta^2\theta_p(1-\theta_p)}.$$

It is hard to show analytically, but I can show this with graph with $U = 2.5$, $r = 0.1$, $\theta_p = 0.6$ (Figure 3.A.2).

Figure 3.A.2: Private information and equilibria type D and E ($U = 2.5, \gamma = 0.75, \theta_p = 0.75, r = 0.05$)



A.5. Production technology and existence region for equilibrium type A, B and E

In order to check the relationship between the production technology and the existence region for each equilibrium, we get derivatives of critical values of θ for each equilibrium with respect to θ_p. We have following derivatives of critical values of θ;

$$\frac{\partial\theta_1(r,\theta_p)}{\partial\theta_p}=\frac{-(r+1)\gamma(2\theta_pU-\gamma)}{[\theta_p(\theta_pU-\gamma)+\gamma]^2}<0,$$

$$\frac{\partial\theta_2(r,\theta_p)}{\partial\theta_p}=\frac{-\gamma(r+1)U}{(2\theta_pU-\gamma)^2}<0,$$

$$\frac{\partial\theta_4(r,\theta_p)}{\partial\theta_p}=\frac{-r\gamma U}{(\theta_pU-\gamma)\sqrt{r(\theta_pU-\gamma)[4\gamma+r(\theta_pU-\gamma)]}}<0,$$

$$\frac{\partial\theta_5(r,\theta_p)}{\partial\theta_p}=\frac{-r\gamma(2\theta_pU-\gamma)}{2\theta_p^{\,2}(\theta_pU-\gamma)^2\sqrt{\dfrac{r\gamma}{\theta_p(\theta_pU-\gamma)}}}<0,$$

$$\frac{\partial\theta_{62}(r,\theta_p)}{\partial\theta_p}=\frac{\begin{bmatrix}2r(1-\theta_p)^2\theta_pU(U-\gamma)+[r\theta_p(U-\gamma)+\sqrt{D_{E1}}]2(1-\theta_p)(-1+3\theta_p)U\\+2r(1-\theta_p)^2\theta_pU[r\theta_p(U-\gamma)^2+2(1-\theta_p)^2U\gamma(-1+4\theta_p)]/\sqrt{D_{E1}}\end{bmatrix}}{[2(1-\theta_p)^2\theta_pU]^2}>0$$

Type A is trivial. Let us look at type B equilibrium. We need to check the sign of $\frac{\partial\theta_2(r,\theta_p)}{\partial\theta_p}-\frac{\partial\theta_1(r,\theta_p)}{\partial\theta_p}$ for type B. We have,

$$\frac{\partial\theta_2(r,\theta_p)}{\partial\theta_p}-\frac{\partial\theta_1(r,\theta_p)}{\partial\theta_p}=\frac{\gamma(r+1)[(2\theta_pU-\gamma)^3-U(\theta_p^{\,2}U-\theta_p\gamma+\gamma)^2]}{(2\theta_pU-\gamma)^2(\theta_p^{\,2}U-\theta_p\gamma+\gamma)^2}.$$

The denominator of $\frac{\partial\theta_2(r,\theta_p)}{\partial\theta_p}-\frac{\partial\theta_1(r,\theta_p)}{\partial\theta_p}$ has positive sign; therefore it is enough to show the numerator is positive, i.e. $(2\theta_pU-\gamma)^3-U(\theta_p^{\,2}U-\theta_p\gamma+\gamma)^2>0$. Letting $J = \theta_pU-\gamma>0$ and rearranging the numerator, we have

$$J^3+U[J^2\theta_p(3-\theta_p)+\theta_pJ(3\theta_pU-2\gamma)+(\theta_p^{\frac{3}{2}}U-\gamma)(\theta_p^{\frac{3}{2}}U+\gamma)]>0.$$

When the utility U is sufficiently large relative to the average production cost (γ / θ_p), we have a positive sign for the numerator; therefore $\frac{\partial\theta_2(r,\theta_p)}{\partial\theta_p} - \frac{\partial\theta_1(r,\theta_p)}{\partial\theta_p} > 0$. In case of type E, the critical region is mainly related to θ_5 and θ_{62}. θ_5 has a negative derivative, whereas θ_{62} has positive derivative. When the initial production technology is low, only θ_5 is related to the existence of type E. When the initial production technology is high, θ_{62} is related; however the equilibrium disappears quickly as the production technology improves. Therefore, when the initial production technology is low, the type E equilibrium also expands as production technology improves. □

A.6. Proof of Proposition 3

Derivatives of welfare of type A, B and E with respect to θ_p are straightforward;

$$\frac{\partial(rZ_A)}{\partial\theta_p} = (1-\theta)(U-\gamma) + [2(1-\theta)\theta_p + r]K_A + [(1-\theta)\theta_p{}^2 + r\theta_p]\frac{\partial K_A}{\partial\theta_p} > 0$$

where
$$\frac{\partial K_A}{\partial\theta_p} = \frac{[\theta(U-\gamma)+(1-\theta)\gamma][r+(1-\theta)(1-\theta_p)+\theta\theta_p(1-\theta_p)+\theta_p{}^2(1-\theta)]}{[r+(1-\theta)(1-\theta_p)+\theta\theta_p(1-\theta_p)+\theta_p{}^2(1-\theta)]^2} + \frac{[\theta\theta_p(U-\gamma)-(1-\theta)(1-\theta_p)\gamma](1-2\theta)(1-2\theta_p)}{[r+(1-\theta)(1-\theta_p)+\theta\theta_p(1-\theta_p)+\theta_p{}^2(1-\theta)]^2} > 0 \quad . \quad (3.A.12)$$

The first term of $\frac{\partial K_A}{\partial\theta_p}$ is positive, and the second term is positive when θ and θ_p are both greater than 1/2.

$$\frac{\partial(rZ_B)}{\partial\theta_p} = \frac{\gamma(1+r-\theta)(1-\theta)\theta U^2}{\theta^2(\theta_p U-\gamma)^2} > 0,$$

$$\frac{\partial(rZ_E)}{\partial\theta_p} = \frac{r\theta^2\theta_p(2r+\theta^2\theta_p)(U-\gamma)}{(-r-\theta^2\theta_p+\theta^2\theta_p{}^2)^2} > 0. \square$$

A.7. Proof of Proposition 4

The derivatives of critical values for each equilibrium from Proposition 1 with respect to discount rate r have positive values, therefore it is trivial that existing regions for type A equilibrium shrinks by the increase of discount rate r. Critical region for type E equilibrium is mainly related to θ_5 and θ_{62}. They have positive derivatives; therefore the existing region for type E equilibrium also shrinks as r increases.

$$\frac{\partial\theta_1(r,\theta_p)}{\partial r} = \frac{\gamma}{\theta_p(\theta_p U-\gamma)+\gamma} > 0,$$

$$\frac{\partial\theta_2(r,\theta_p)}{\partial r} = \frac{\theta_p U}{2\theta_p U-\gamma} > 0,$$

$$\frac{\partial\theta_4(r,\theta_p)}{\partial r} = \frac{2\gamma+r(\theta_p U-\gamma)+\sqrt{r(\theta_p U-\gamma)[4\gamma+r(\theta_p U-\gamma)]}}{2\sqrt{r(\theta_p U-\gamma)[4\gamma+r(\theta_p U-\gamma)]}} > 0,$$

$$\frac{\partial\theta_5(r,\theta_p)}{\partial r} = \frac{\gamma}{2\theta_p(\theta_p U-\gamma)\sqrt{\dfrac{\gamma r}{\theta_p(\theta_p U-\gamma)}}} > 0 \quad ,$$

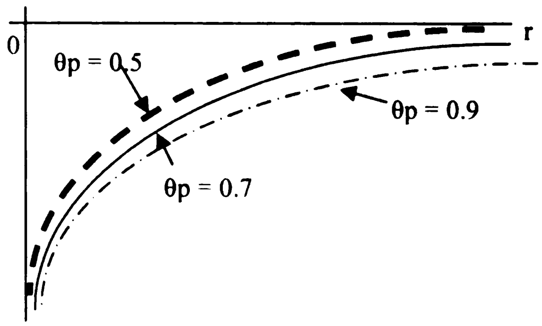
$$\frac{\partial\theta_{62}(r,\theta_p)}{\partial r} = \frac{(U-\gamma)+\dfrac{r\theta_p(U-\gamma)^2-2(1-\theta_p)^3 U\gamma}{\sqrt{r^2\theta_p{}^2(U-\gamma)^2-4(1-\theta_p)^3 r\theta_p U\gamma}}}{2(1-\theta_p)^2 U} > 0 \quad .$$

Type B equilibrium case is:

$$\frac{\partial\theta_2(r,\theta_p)}{\partial r} - \frac{\partial\theta_1(r,\theta_p)}{\partial r} = \frac{\theta_p U[\theta_p(\theta_p U-\gamma)+\gamma]-\gamma(2\theta_p U-\gamma)}{(2\theta_p U-\gamma)[\theta_p(\theta_p U-\gamma)+\gamma]}. \qquad (3.A.13)$$

We have positive denominator in (3.A.13), and we can show numerator > 0. Rearranging the numerator of (3.14 gives $(\theta_p{}^2 U - \gamma)(\theta_p U - \gamma)$, and this is true when U is sufficiently large.

Figure 3.A.3: Derivative of critical values of θ for type D equilibrium with respect to r



In case of type D, I am going to use a graph to show its shrinkage. Type D equilibrium case is;

$$\frac{\partial\theta_2(r,\theta_p)}{\partial r} - \frac{\partial\theta_4(r,\theta_p)}{\partial r}$$
$$= \frac{r\gamma(\theta_p U - \gamma)[4\gamma + r(\theta_p U - \gamma)] - (2\theta_p U - \gamma)[2\gamma + r(\theta_p U - \gamma)]\sqrt{r(\theta_p U - \gamma)[4\gamma + r(\theta_p U - \gamma)}}{2(2\theta_p U - \gamma)r(\theta_p U - \gamma)[4\gamma + r(\theta_p U - \gamma)]}$$

When discount rate r is not too big (r = 0.1), One can verify that $\frac{\partial\theta_2(r,\theta_p)}{\partial r} - \frac{\partial\theta_4(r,\theta_p)}{\partial r}$ has negative sign as in Figure 3.A.3 (U = 2.5). □

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