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TWO ESSAYS ON PRODUCT BUNDLING AND ONE ESSAY ON VERTICAL INTEGRATION

By

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ABSTRACT

TWO ESSAYS ON PRODUCT BUNDLING AND ONE ESSAY ON VERTICAL INTEGRATION

By

Kyonghwa Jeong

Discounts are often offered to consumers attached to competitors in various industries where consumers make repeated purchases. Chapter 1 studies such a practice in a vertically related industry, i.e. U.S. telecom industry, where firms offer a package of handsets and phone service plan with long-term contracts of one or two years. I show that the equilibrium incentives for bundling can be explained with the practice of consumer poaching. The reason is that bundling acts as a commitment device through which firms can lock in consumers by creating artificial switching costs and firms can commit to future prices in combination with long-term contracts.

Chapter 2 studies the strategic use of mixed bundling of two independent goods in duopoly. I show that mixed bundling may arise in equilibrium in markets with heterogeneous consumers' valuations. Mixed bundling dominates pure bundling in duopoly when a number of customers with low valuations switch to rivals when offered a bundle only. This result is consistent with that in a monopoly where mixed bundling is preferred to pure bundling when consumers' valuations of two goods are not perfectly correlated.

In high tech industries, a competitive concern about a vertical merger is that merged entity transfers proprietary information of a competitor's innovation to its downstream or upstream division. In several recent vertical merger cases, FTC/DOJ requires Firewalls to prevent the passage of competitively-sensitive information within an merged entity. Chapter 3 studies the welfare consequences of such a merger and related information transfers in markets with vertical R&D spillovers. The main result is that a Firewalls provision does not necessarily improve social welfare. The reason is that Firewalls may affect a nonintegrated firm' decision for an input supplier.

To My Parents

ACKNOWLEDGEMENTS

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Chapter 1

Brand Switching and Product Bundling

1.1 Introduction

Firms often offer discounts to consumers attached to competitors in subscription markets where consumers can be easily distinguished into groups, first time and repeated buyers. Early termination fees are also fairly standard in a subscription-based business. The whole point of early termination fees is that firms offer new customers a discounted price if they agree to stay for a minimum period. Hence, early termination fee acts like switching costs for customers who may switch between brands in the middle of a contract. In the U.S. wireless industry, the persistence of the pattern of discounts of handsets suggests that discounts on handsets are used to lock-in consumers. Such strategic behavior of loyalty inducing discounts can arise in order for consumers to incur costs when they switch between suppliers. Customers must be connected to one operator for at least one- or two-year term arrangements. If they switch between operators and breach a contract, they are obligated to pay a release fee.[1] The phone itself does not work if used on a different network. SIM cards are

[1]According to FCC(http://ftp.fcc.gov/cgb/NumberPortability/#earlyfee), customers are obligated to pay any early termination fees they may have under an existing contract. In the U.S. wireless industry, an early termination fee ranges from $150 to $240, depending on the company. As of Sep. 22, 2005, Verizon charges $175 per line as an early termination fee. Other examples of early termination fees are $200 per line of T-mobile, $240 or $150 of Cingular, $200 of Alltel, and $150 of

typically locked so that handsets only work on one network (hardware lock). So, if consumers switch between providers, they also need to purchase another handsets locked by a new provider.[2]

With consumers' recognition, however, intertemporal price discrimination generates a more competitive price and even more in the presence of switching costs (Klemperer (1987), Caminal and Claici (2005)). Hence bargaining and rip-off strategy is not optimal for firms to pursue in a market with switching costs. This result critically depends on an important assumption: whether firms can commit themselves to future behavior or not.

In the literature on intertemporal price discrimination based on consumers' past behavior, it is common to assume that firms cannot commit themselves to future behavior, not to discriminate between loyal customers and new customers. There might be many reasons why firms may not be able to commit themselves to future behavior. For instance, there may be uncertainty about demand, costs, or possible entry into the market where customers make repeated purchases. Under such circumstances, firms are in prisoners' dilemma when firms engage in price discrimination purely based on consumers' preferences even in the absence of switching costs (Villas-Boas (1999); Fudenberg and Tirole (2000)). Fudenberg and Tirole (2000) use a two-period model where the first period choices leaves firms with two different types of consumers in the second period: its own customers and those of its competitor. Since the competitor's customers can be targeted with discounts in the second period, consumer poaching, consumers are less sensitive to prices in the first period. By offering a long-term contract in addition to a short-term contract, firms are able to have smaller portion of customers who switch between brands. Nevertheless, firms make less profit than when they offer a sequence of short-term contracts because each firm competes with itself over its turf by offering both long-and short-term contracts. Consumer poaching with an infinite horizon model is also studied by Villas-Boas (1999). The general result

Sprint & Nextel.

[2]Some further switching costs relevant to mobile users can be the subscriber number (stationary costs), and special links with other services. Since those switching costs do not affect our main result, I ignore them in the present paper.

that each firm will offer discounts to the other firm's customers remains unchanged. On the other hand, Chen (1997) studies a two-period model of a duopoly where firms offer discounts to customers attached to the other firm in the presence of switching costs. He finds that both firms will offer discounts to customers with a history of purchasing from the other firm. Shaffer and Zhang (2000) extend the work of Chen (1997) by generalizing the demand side. They find that if the loyalties of customer groups differ, i.e. if its own customers are not very loyal, it may be optimal for a firm to offer discounts to its own customers rather than to poach customers from competitors. Taylor (2000) also extends Chen's (1997) work in several ways by allowing for an arbitrary number of periods and randomly varying switching costs. The modeling of many periods enables Taylor to be explicit about strategic switching on the consumer side. This occurs when a customer switches solely to establish a history of frequent switching, thereby securing better future offers from his suppliers. In those papers, consumer poaching results in prisoners' dilemma. The reason is that intertemporal price discrimination based on purchase histories intensifies price competition either when consumers are already locked-in or when they are not yet locked-in. The critical assumption is that firms are unable to commit not to poach customers attached to competitors

This paper extends the existing studies on consumer poaching based on consumers' past behavior and connects two important streams of brand switching and product bundling in a complementary market where a good/service is perishable and the other good is durable. Since consumers switch between brands as a result of consumer poaching, firms may have strategic incentives to lock-in consumers. Bundling can act as such a locking-in device because bundling creates artificial switching costs in a complementary market.

Our main results are as follows. I show that bundling can be an effective commitment device through which firms can lock-in consumers and firms will offer long-term contracts in the presence of consumer poaching. That is, the equilibrium incentives for bundling can be explained with the presence of consumer poaching. The reason is that bundling creates an artificial switching cost that locks in customers. Such artificial switching costs are different from learning (switching) costs to use new brands. Learning costs can arise when customers get to know a new word processing system or when customers receive an added benefit if they stays with the previous word-processing-system providers in the second period. Tying can be a profitable strategy in the presence of such learning-by-doing switching costs (Carlton and Waldman, 2005).[3] On the other hand, artificial switching costs in the present paper arise as results of firms' actions, bundling. When customers are locked-in with such switching costs after the first purchase, firms may have an incentive to take advantage of locked-in customers. However, this intensifies competition for market share in the first period and results in lower profits than in the absence of switching costs. Hence, firms have a collective incentive to commit themselves to future prices by offering long-term contracts in the presence of switching costs. On the other hand, when customers are not locked-in with switching costs after the first purchase, pure bundling with short-term contracts may arise in equilibrium as a result of bundling game. When customers are not locked-in with switching costs, firms have incentives to offer a sequence of short-term contracts because long-term contracts intensify competition for market base in the first period. Nevertheless, firms make lower profits in pure bundling than in no bundling. The reason is that firms face more competitive market in both periods in the presence of both consumer poaching and switching costs, while firms face less elastic demand in the first period, but more elastic market in the second period in the absence of switching costs.

In the following section, I set up the benchmark model where firms are unable to price discriminate between customers and across periods. In section 4, I analyze a model where firms may have an incentive to offer discounts to customers attached to competitors under alternative scenarios, no bundling, pure bundling. In appendix A, I show the case where one firm bundles while the other firm does not. Conclusions are at the end.

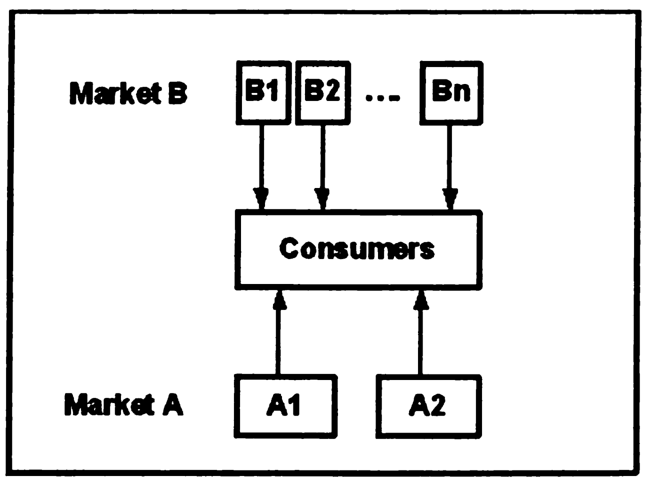
[3]An example of Learning-by-doing costs in Carlton and Waldman (2005) paper: If a consumer purchased Word 2002 rather than Word Perfect 11 in the first period, then he will prefer Word 2003 over Word Perfect 12 in the second period because of the similar ways in which Word 2002 and Word 2003 operate.

1.2 Model

There are two firms, A1 and A2, in market A with a horizon of two periods. They are located at extremes of an interval of $[0, 1]$ in a traditional Hotelling model. Firm A1 is at 0 and A2 is at 1. Product A is perishable and it must be repurchased in each period. The firms are symmetric in the sense that they have the same marginal cost, $c_A \geq 0$. Consumers are uniformly located on the interval of $[0, 1]$. A consumer at θ, where $\theta \in [0, 1]$, incurs the transportation cost of $t\theta$ in each period if good A is purchased. To consume a product A, each customer needs to buy a product B that lasts for two periods. There are $n \geq 2$ firms in market B. Each firm produces a homogeneous good B_i, where $i = 1, 2, ..., n$, at the same marginal cost of $c_B > 0$. Each consumer enjoys a gross utility of U from consuming goods/services A and B together per period. I assume U to be large enough that in equilibrium all consumers in fact purchase one unit of both A_i and B_j, where $i, j = 1, 2$, in each period.

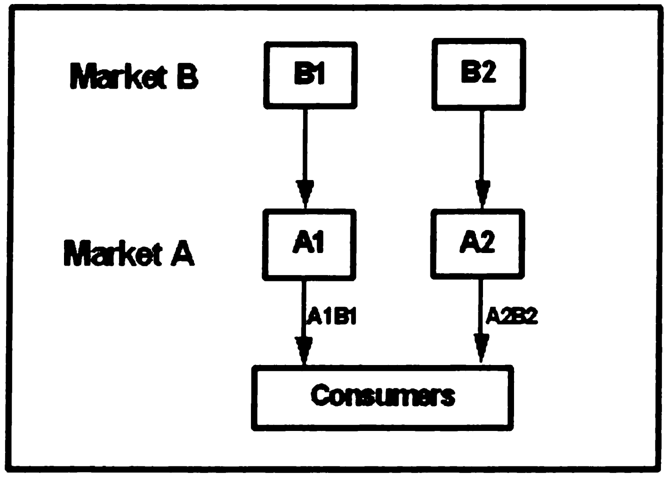
I assume that firms in market A make the decision of whether to bundle or

Figure 1.1: No Bundling



not and the bundling decisions are irreversible. In no bundling, firms in market A sell their products A_i, $i = 1, 2$, separately from products B_j, $j = 1, 2, ..., n$, in both periods. Consumers independently purchase products B_j, $j = 1, 2, ..., n$, in market

Figure 1.2: Pure Bundling



B. Product B is durable and technically compatible with any product A, and hence can be used with any A in the second period. On the other hand, in pure bundling, I assume that firm A1 purchases B_1 and firm A2 buys B_2, at the price of c_B and pack it together with A_1 and A_2, respectively, making the bundle of A_iB_i, $i = 1, 2$, at the cost of $c_A + c_B$. Firms in market A sell only a package of A_iB_i, $i = 1, 2$, to all first time consumers. Hence consumers who may switch between providers need to buy a package of A_iB_i , $i = 1, 2$, rather than A_i, $i = 1, 2$. Product B_i purchased in the first period cannot be used with A_j, $i, j = 1, 2$, $i \neq j$, in the second period. For second time users, firms sell A_i, $i = 1, 2$, because product B is durable.

Consumers are rational in the sense that they can correctly anticipate the consequences of current actions on future decisions. Both firms and consumers discount their second-period payoffs with the discount factor of $\delta \in (0, 1]$.

I consider three different types of contract arrangements in the present paper, short-term contract, long-and short-term contracts, and long-term contract. In a short-term contract, firms offer the first period and second period spot prices sequentially. When firms offer both long-and short-term contracts, firms offer a schedule of prices for customers who purchase long-term contracts in the first period. Poaching prices to new customers and the second period spot prices for loyal customers who purchase a sequence of short-term contracts are determined in the second period. In a long-term contract, firms offer discounts of d_i, where $i = 1, 2$, in exchange for long term contracts with a schedule of prices. Since discounts are given to customers in return for a long-term contract, customers are obligated to pay a termination fee of s_i, where $i = 1, 2$, if they breach the contract earlier than the contract term. The early termination fee is a charge to compensate firms for customers' failure to satisfy the commitment on which the price of a service/good is based, $s_i = \gamma d_i$, where $\gamma \in (0, \infty]$. In other words, an early termination fee allows firms to recoup the costs of providing customers with goods/services and discounts. In this sense, I assume that a customer is subject to an early termination fee equal to the amounts of damages caused by terminating the contract early. Klemperer (1995) points out that "Loyalty Contracts" are implemented by offering customers discounts in exchange for long term contracts committing the customers to pay damages if they terminate the contract. Signing such a contract that specifies damages of s is equivalent to paying s for a discount coupon of present-value s that is offered in exchange for the next purchase. In the present paper, it is equivalent to the sum of the fixed amounts of discounts and an interest of the discounts, $s_i = (1 + r)d_i = \frac{1}{\delta}d_i$, where $\delta \in (0, 1]$, $i = 1, 2$, r is an interest rate.

There are two assumptions that hold throughout this paper: (i) Each consumer's preferences are constant over time. (ii) Firms in market A cannot commit not to poach consumers from competitors.

The timing of the game is the following. In the first stage, firms make bundling decisions. In the second stage, firms choose whether to offer a sequence of short-term contracts, both long- and short-term contracts, or a long-term contract. The proper solution concept for this two-stage game is Subgame Perfect Nash equilibrium.

1.3 Benchmark: No price discrimination

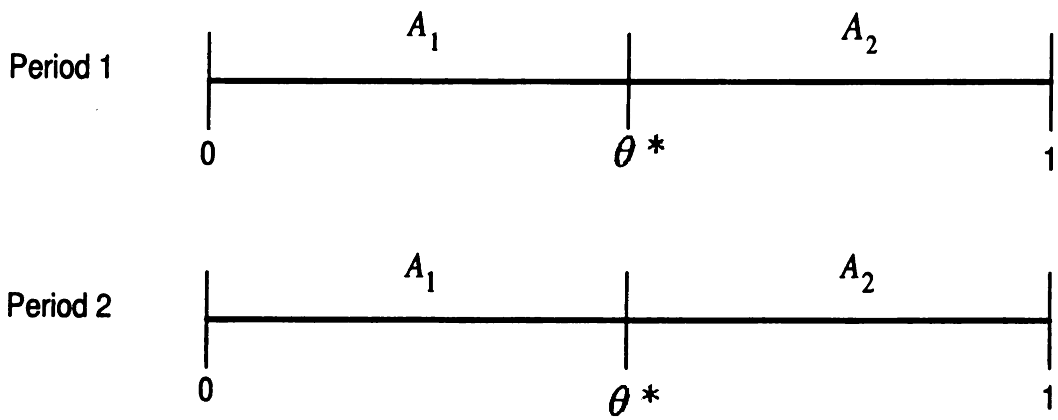
1.3.1 No bundling

This is the static model where firms do not price discriminate between customers and over time. Since consumers' preferences are constant over time, in each period, a consumer at θ^* is indifferent between A_1 and A_2 if

$$\begin{aligned} U - c_B - p^i_{A1} - t\theta^* &= U - c_B - p^i_{A2} - t(1-\theta^*) \\ \Rightarrow \theta^* &= \frac{1}{2t}(t + p^i_{A2} - p^i_{A1}), \quad i = 1, 2. \end{aligned} \tag{1.1}$$

where p^i_{A1} and p^i_{A2} are the prices of A_1 and A_2, respectively, in period $i = 1, 2$. Customers in $[0, \theta^*]$ and $[\theta^*, 1]$ purchase A_1 and A_2, respectively, in each period.

Figure 1.3: No bundling in Benchmark



By maximizing the following profit functions with respect to prices, p^i_{A1}, p^i_{A2}, $i = 1, 2$,

$$\pi_{A1} = (p^1_{A1} - c_A)\theta^* + \delta(p^2_{A1} - c_A)\theta^*, \tag{1.2}$$

$$\pi_{A2} = (p^1_{A2} - c_A)(1-\theta^*) + \delta(p^2_{A2} - c_A)(1-\theta^*), \tag{1.3}$$

The equilibrium prices and profits in market A are as follows.

$$p_{A1}^1 = p_{A2}^1 = p_{A1}^2 = p_{A2}^2 = t + c_A, \tag{1.4}$$

$$\pi_{A1}^* = \pi_{A2}^* = (1+\delta)\frac{t}{2}. \tag{1.5}$$

In market B, since products B are durable, there is no sale in the second period. The market price and profits are the general Bertrand competition outcomes as follows.

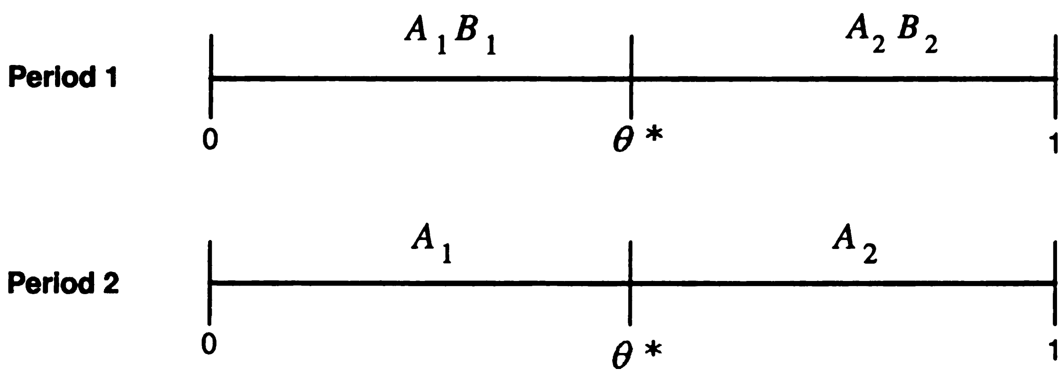
$$p_{Bi}^* = c_B, \tag{1.6}$$

$$\pi_{Bi}^* = 0, \quad i = 1, 2, ..., n. \tag{1.7}$$

1.3.2 Pure bundling

Firms in market A offer a package of A_iB_i, $i = 1, 2$, but not A_i separately, $i = 1, 2$, in the first period. In the second period, firms offer A_i, $i = 1, 2$, to their own customers because products B are durable. Customers in $[0, \theta^*]$ and $[\theta^*, 1]$ purchase the bundle A_1B_1 and A_2B_2, respectively, in the first period, and A_1 and A_2, respectively, in the second period in the absence of price discrimiation.

Figure 1.4: Pure bundling in Benchmark



A customer at θ^* is indifferent A_1B_1 and A_2B_2 in the first period and A_1 and

A_2 in the second period if

$$p^B_{A1} + t\theta^* = p^B_{A2} + t(1-\theta^*) \Rightarrow \theta^* = \frac{1}{2} + \frac{p^B_{A2} - p^B_{A1}}{2t}, \quad (1.8)$$

$$p^2_{A1} + t\theta^* = p^2_{A2} + t(1-\theta^*) \Rightarrow \theta^* = \frac{1}{2} + \frac{p^2_{A2} - p^2_{A1}}{2t}, \quad (1.9)$$

where p^B_{A1} and p^B_{A2} are the bundle prices of A_1B_1 and A_2B_2 of firm A1 and A2, respectively, in the first period and p^2_{A1} and p^2_{A2} are the prices of A_1 and A_2 of firm A1 and A2, respectively, in the second period. By maximizing the following profit functions with respect to prices, $\{p^B_{A1}, p^2_{A1}\}$ and $\{p^B_{A2}, p^2_{A2}\}$, respectively,

$$\pi_{A1} = (p^B_{A1} - c_A - c_B)\theta^* + \delta(p^2_{A1} - c_A)\theta^*, \quad (1.10)$$

$$\pi_{A2} = (p^B_{A2} - c_A - c_B)(1-\theta^*) + \delta(p^2_{A2} - c_A)(1-\theta^*), \quad (1.11)$$

The equilibrium prices and profits in market A are as follows.

$$p^B_{A1} = p^B_{A2} = t + c_A + c_B, \quad (1.12)$$

$$p^2_{A1} = p^2_{A2} = t + c_A, \quad (1.13)$$

$$\pi^*_{A1} = \pi^*_{A2} = (1+\delta)\frac{t}{2}. \quad (1.14)$$

In market B, firm A1 purchases B_1 and firm A2 buys B_2 at c_B due to Bertrand competition in market B. The market price and equilibrium profits are as follows.

$$p^*_{B1} = p^*_{B2} = c_B, \quad (1.15)$$

$$\pi^*_{B1} = \pi^*_{B2} = 0. \quad (1.16)$$

1.3.3 No bundling vs. Pure bundling

The equilibrium outcomes of pure bundling are the same as in no bundling when firms engage in no price discrimination between consumers and over periods. The reason is that no positive effects of bundling such as a production differentiation role of bundling can take place in the present paper because products B are complementary to goods/services A in a Bertrand competition model.

1.4 Model with consumer poaching

Price discrimination between loyal and new consumers is well observed in various industries where consumers make repeated purchases. I analyze such a practice in a vertically related industry. Consumers correctly anticipate the second period prices and maximize their total utilities when sellers are able to price discriminate based on consumers' past behavior. Firms can offer three different types of contract arrangements: short-term, long-and short-term, and long-term contracts.

1.4.1 No bundling

Firms in market A and B sell independently to consumers. Consumers purchase $A_i + B_j$, $i, j = 1, 2$, in the first period and purchase A_i, $i = 1, 2$, in the second period.

In market B, firms have no sale in the second period under any contract arrangements because customers can use product B purchased in the first period, even when they switch between brands. Firms sell product B at the marginal cost of c_B and make zero profits.

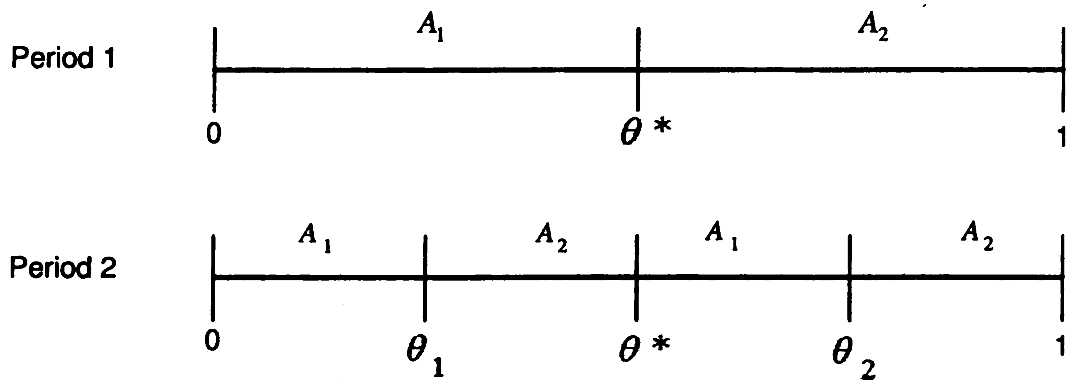
$$p^*_{Bi} = c_B, \tag{1.17}$$

$$\pi^*_{Bi} = 0\ ,\ \ i = 1, 2, ..., n. \tag{1.18}$$

(1) Short-term contract

Firms offer a short-term contract each period. Customers in $[0,\ \theta^*]$ purchase A_1 and customers in $[\theta^*,\ 1]$ purchase A_2 in the first period. After observing consumer preferences in the first period, firms in market A offer discounts to consumers attached to competitors in the second period. That is, consumers in $[\theta_1,\ \theta^*]$ and $[\theta^*,\ \theta_2]$ are targeted with discounts by competitors with A_2 and A_1, respectively, in the second period.

Figure 1.5: No bundling and Short term contract



I solve the problem by backward induction. In the second period, given θ^*, a consumer at θ_1 and θ_2 is indifferent between A_1 and A_2 if

$$p_{A1}^2 + t\theta_1 = \hat{p}_{A2}^2 + t(1-\theta_1) \Rightarrow \theta_1 = \frac{1}{2t}(t + \hat{p}_{A2}^2 - p_{A1}^2)\,, \tag{1.19}$$

$$\hat{p}_{A1}^2 + t\theta_2 = p_{A2}^2 + t(1-\theta_2) \Rightarrow \theta_2 = \frac{1}{2t}(t + p_{A2}^2 - \hat{p}_{A1}^2)\,, \tag{1.20}$$

where p_{A1}^2 and p_{A2}^2 are the second period prices of A_1 and A_2 for loyal consumers of firms A1 and A2, respectively. $\hat{p}_{A1}^2$ and $\hat{p}_{A2}^2$ are the second period poaching prices of A_1 and A_2, respectively. Consumers in the intervals $[0,\ \theta_1]$ and $[\theta_2,\ 1]$ are loyal customers to firms, A1 and A2, respectively. They do not switch between suppliers even in the presence of consumer poaching. On the other hand, consumers in the intervals $[\theta_1,\ \theta^*]$ and $[\theta^*,\ \theta_2]$ switch between brands in the second period, from A1 to A2 and from A2 to A1, respectively. The profit functions of the firms in the second period are

$$\pi_{12}^{NB(S)} = (p_{A1}^2 - c_A)\theta_1 + (\hat{p}_{A1}^2 - c_A)(\theta_2 - \theta^*), \tag{1.21}$$

$$\pi_{22}^{NB(S)} = (p_{A2}^2 - c_A)(1-\theta_2) + (\hat{p}_{A2}^2 - c_A)(\theta^* - \theta_1). \tag{1.22}$$

By maximizing the profit functions with respect to the second period prices, The second period prices as a function of θ^* are as follows.

$$\begin{aligned} p_{A1}^2 &= c_A + \left(\frac{2\theta^*+1}{3}\right)t,, \quad p_{A2}^2 = c_A + \left(\frac{3-2\theta^*}{3}\right)t\,, \\ \hat{p}_{A1}^2 &= c_A + \left(\frac{3-4\theta^*}{3}\right)t\,, \quad \hat{p}_{A2}^2 = c_A + \left(\frac{4\theta^*-1}{3}\right)t\,, \end{aligned} \tag{1.23}$$

In the first period, a consumer at θ^* is indifferent between A_1 and A_2 if

$$\begin{aligned} p_{A1}^1 &+ t\theta^* + \delta(\hat{p}_{A2}^2 + t(1-\theta^*)) = p_{A2}^1 + t(1-\theta^*) + \delta(\hat{p}_{A1}^2 + t\theta^*) \\ \Rightarrow \theta^* &= \frac{1}{2t} + \frac{(p_{A2}^1 - p_{A1}^1) - \delta(\hat{p}_{A2}^2 - \hat{p}_{A1}^2)}{2(1-\delta)t} = \frac{1}{2} + \frac{3(p_{A2}^1 - p_{A1}^1)}{2(3+\delta)t}. \end{aligned} \tag{1.24}$$

Firms maximize the following profit functions with respect to p_{A1}^1 and p_{A2}^1, respectively.

$$\pi_1^{NB(S)} = (p_{A1}^1 - c_A)\left(\frac{1}{2} + \frac{3(p_{A2}^1 - p_{A1}^1)}{2(3+\delta)t}\right) + \delta\pi_{12}^{NB(S)}\,, \tag{1.25}$$

$$\pi_2^{NB(S)} = (p_{A2}^1 - c_A)\left(\frac{1}{2} - \frac{3(p_{A2}^1 - p_{A1}^1)}{2(3+\delta)t}\right) + \delta\pi_{22}^{NB(S)}\,. \tag{1.26}$$

Note that at $\theta^* = 1/2$, $\frac{\partial \pi_{i2}^{NB(S)}}{\partial \theta^*}\frac{\partial \theta^*}{\partial p_{Ai}^1} = -\frac{5(2\theta^*-1)}{3(3+\delta)} = 0$, where $i = 1, 2$. When firms have the same market share of 1/2, an increase in the first period prices do not affect the second period profits. The reason is that in the presence of consumer poaching, the gains from a market with loyal customers are exactly offset by the costs from the other market with new buyers. In equilibrium, the market prices are as follows.

$$\begin{aligned} p_{A1}^1 &= p_{A2}^1 = t + c_A + \frac{\delta}{3}t\,, \\ p_{A1}^2 &= p_{A2}^2 = \frac{2}{3}t + c_A\,, \\ \hat{p}_{A1}^2 &= \hat{p}_{A2}^2 = \frac{1}{3}t + c_A\,. \end{aligned} \tag{1.27}$$

Note that the first period prices of p_{A1}^1 and p_{A2}^2 are higher than those in the benchmark model. This reflects the fact that consumers are willing to pay more in the first period (less elastic demand) because they can take advantage of discounts offered by competing firms in the second period (more elastic demand). However, the second period prices are lower than those in the benchmark model.[4]

In equilibrium, each firm shares the market evenly in both periods. By substituting the optimal prices into eq.(1.19) and eq. (1.20), the followings are derived.

$$\theta_1 = \frac{1}{3}\,, \quad \theta^* = \frac{1}{2}\,, \quad \theta_2 = \frac{2}{3}\,. \tag{1.28}$$

1/3 of consumers switch between brands as a result of consumer poaching. The portion of loyal customers is 2/3. By offering discounts to consumers attached to competitors, each firm is worse off than in the benchmark model where firms do not price discriminate based on customers' past behavior. The equilibrium profits with short-term contracts are as follows.

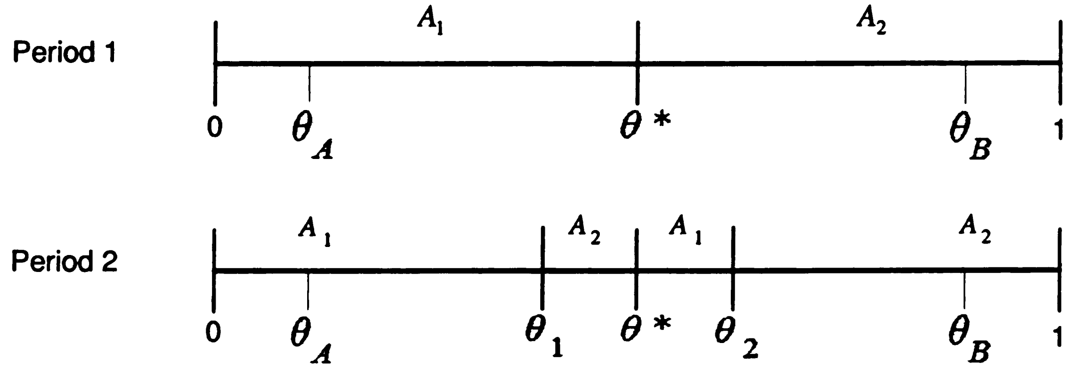
$$\pi_{A1}^{NB(S)} = \pi_{A2}^{NB(S)} = \left(1 + \frac{8\delta}{9}\right)\frac{t}{2}\,. \tag{1.29}$$

(2) Long- and Short-term contracts

Firms offer both long-and short-term contracts. With long-term contracts, firms offer a schedule of prices for the first and second period. Firms do not commit to second period prices for customers who purchase a short-term contract in the first period. Since customers who have high preferences for A_1 will be more willing to commit themselves to consuming A_1, I assume that long-term contracts are purchased by the consumers who most prefer that firm's product. This tie-breaking assumption is needed because our model is deterministic and hence a consumer who plans to purchase from firm A1 chooses between a long-term contract and a sequence of short-term contracts based on his costs. Consumers in $[0,\ \theta^*]$ and $[\theta^*,\ 1]$ purchase A_1 and A_2, respectively, in the first period. Consumers in the interval of $[0,\ \theta_A]$ and $[\theta_B,\ 1]$ purchase long-term contracts at the price of $\tilde{p}_{A1}$ and $\tilde{p}_{A2}$ from firm A1 and A2, respectively. Hence, competition in the second period is over the interval

[4] These are the main results of Fudenberg and Tirole (2000)

Figure 1.6: No bundling and Long and Short term contracts



$[\theta_A,\ \theta_B]$. Consumers in the intervals $[\theta_A,\ \theta_1]$ and $[\theta_2,\ \theta_B]$ buy a sequence of short-term contracts from A1 and A2, respectively. Consumers in the intervals $[\theta_1,\ \theta^*]$ and $[\theta^*,\ \theta_2]$ are targeted with discounts by rivals and switch between brands in the second period, from A1 to A2, from A2 to A1, respectively.

In the second period, given θ^*, a consumer at θ_1 and θ_2 is indifferent between A_1 and A_2 if

$$p_{A1}^2 + t\theta_1 = \hat{p}_{A2}^2 + t(1-\theta_1) \ \Rightarrow\ \theta_1 = \frac{1}{2t}(t + \hat{p}_{A2}^2 - p_{A1}^2), \tag{1.30}$$

$$\hat{p}_{A1}^2 + t\theta_2 = p_{A2}^2 + t(1-\theta_2) \ \Rightarrow\ \theta_2 = \frac{1}{2t}(t + p_{A2}^2 - \hat{p}_{A1}^2), \tag{1.31}$$

where p_{A1}^2 and p_{A2}^2 are the second period prices of A_1 and A_2 for loyal customers of firm A1 and A2, respectively. $\hat{p}_{A1}^2$ and $\hat{p}_{A2}^2$ are the second period poaching prices of A_1 and A_2, respectively. Firms maximize the following profit functions with respect to second period prices, $\{p_{A1}^2, \hat{p}_{A1}^2\}$ and $\{p_{A2}^2, \hat{p}_{A2}^2\}$,

$$\pi_{12}^{NB(L\&S)} = (p_{A1}^2 - c_A)(\theta_1 - \theta_A) + (\hat{p}_{A1}^1 - c_A)(\theta_2 - \theta^*), \tag{1.32}$$

$$\pi_{22}^{NB(L\&S)} = (p_{A2}^2 - c_A)(\theta_B - \theta_2) + (\hat{p}_{A2}^2 - c_A)(\theta^* - \theta_1). \tag{1.33}$$

Then, we have

$$p^2_{A1} = c_A + \left(\frac{2\theta^* + 1}{3}\right)t - \frac{4}{3}\theta_A t\,, \;\; p^2_{A2} = c_A - \left(\frac{2\theta^* + 1}{3}\right)t + \frac{4}{3}\theta_B t\,, \qquad (1.34)$$

$$\hat{p}^2_{A1} = c_A - \left(\frac{4\theta^* - 1}{3}\right)t + \frac{2}{3}\theta_B t\,, \;\; \hat{p}^2_{A2} = c_A + \left(\frac{4\theta^* - 1}{3}\right)t - \frac{2}{3}\theta_A t\,. \qquad (1.35)$$

In the first period, a consumer at θ^* is indifferent between A_1 and A_2 if

$$\begin{aligned} p^1_{A1} + t\theta^* + \delta(\hat{p}^2_{A2} + t(1-\theta^*)) &= p^1_{A2} + t(1-\theta^*) + \delta(\hat{p}^2_{A1} + t\theta^*) \\ \Rightarrow \theta^* &= \frac{1}{2} + \frac{p^1_{A2} - p^1_{A1}}{2(1-\delta)t} - \frac{\delta(\hat{p}^2_{A2} - \hat{p}^2_{A1})}{2(1-\delta)t}, \\ \Rightarrow \theta^* &= \frac{1}{2} + \frac{p^1_{A2} - p^1_{A1}}{2t} + \frac{\delta(p^2_{A2} - p^2_{A1})}{4t}\,. \qquad (1.36) \end{aligned}$$

A consumer at θ_A and θ_B is indifferent between a long-term contract and a sequence of short-term contracts if

$$\begin{aligned} &\tilde{p}_{A1} + (1+\delta)t\theta_A = p^1_{A1} + t\theta_A + \delta(p^2_{A1} + t\theta_A) \\ &\Rightarrow \tilde{p}_{A1} = p^1_{A1} + \delta p^2_{A1}, \qquad (1.37) \\ &\tilde{p}_{A2} + (1+\delta)t(1-\theta_B) = p^1_{A2} + t(1-\theta_B) + \delta(p^2_{A2} + t(1-\theta_B)) \\ &\Rightarrow \tilde{p}_{A2} = p^1_{A2} + \delta p^2_{A2}, \qquad (1.38) \end{aligned}$$

where $\tilde{p}_{A1}$ and $\tilde{p}_{A2}$ are the prices of A_1 and B_1 for a long-term contract, respectively. Firms are maximizing the following profit functions with respect to the first period prices, p^1_{A1} and p^1_{A2}.

$$\begin{aligned} \pi_1^{NB(L\&S)} &= (p^1_{A1} - c_A)\theta^* + \delta(p^2_{A1} - c_A)\theta_1 + \delta(\hat{p}^2_{A1} - c_A)(\theta_2 - \theta^*)\,, \qquad (1.39) \\ \pi_2^{NB(L\&S)} &= (p^1_{A2} - c_A)(1-\theta^*) + \delta(p^2_{A2} - c_A)(1-\theta_2) \\ &\quad + \delta(\hat{p}^2_{A2} - c_A)(\theta^* - \theta_1). \qquad (1.40) \end{aligned}$$

In equilibrium, the first and second period prices are lower than in a sequence of short-term contracts. The reason is that each firm competes with itself over its turf by offering both long-and short-term contracts simultaneously.

$$\begin{aligned}
\tilde{p}_{A1} = \tilde{p}_{A2} &= \left(1 + \frac{1}{2}\delta\right) t + (1+\delta)c_A, \\
p^1_{A1} = p^1_{A2} &= t + c_A, \\
p^2_{A1} = p^2_{A2} &= \frac{1}{2}t + c_A, \\
\hat{p}^2_{A1} = \hat{p}^2_{A2} &= \frac{1}{4}t + c_A.
\end{aligned} \tag{1.41}$$

The fraction of consumer poaching is 1/4, which is less than in a sequence of short-term contracts, where consumer poaching is 1/3. By substituting the optimal prices into eq.(1.30) and eq. (1.31), I can find $\{\theta_1, \ \theta_2\}$. I can derive $\{\theta_A, \ \theta_B\}$ from the first order conditions of eq. (1.39) and eq. (1.40) at $\frac{\partial \theta^*}{\partial p^1_{Ai}} = 0$, where first-period sales are unaffected if firms increase p^1_{Ai}, but decrease $\tilde{p}_{Ai}$ by the same amount, $i = 1, 2$.

$$\theta_A = \frac{1}{8}, \quad \theta_1 = \frac{3}{8}, \quad \theta^* = \frac{1}{2}, \quad \theta_2 = \frac{5}{8}, \quad \theta_B = \frac{7}{8}. \tag{1.42}$$

The equilibrium profits of firms are

$$\pi_1^{NB(L\&S)} = \pi_2^{NB(L\&S)} = \left(1 + \frac{7}{16}\delta\right)\frac{t}{2}. \tag{1.43}$$

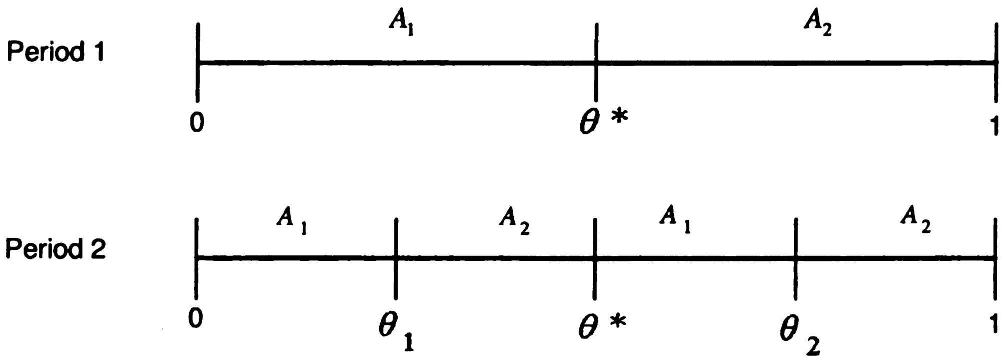
Proposition 1.1 When offering both long-and short-term contracts under no bundling, firms have positive portions of consumers who buy long-term contracts, 1/4. There are also consumers who switch between brands in the second period, 1/4, which is smaller than in a sequence of short-term contracts. The equilibrium prices and profits are lower than in a sequence of short-term contracts.

Proof. See appendix B.

(3) Long-term contract

To prevent any consumer poaching, rather than simple long-term contracts where firms offer a schedule of the first and second period prices for repeated buyers, firms may have an incentive to offer long-term contracts in which firms offer a schedule of prices of $\{p_{A1}^1,\ p_{A1}^2\}$ and $\{p_{A2}^1,\ p_{A2}^2\}$ with discounts of d_i, $i = 1, 2$, in exchange for long-term contracts. In this long-term contract, I allow customers who committed to a long-term contract to switch between brands. But they are subject to early termination fee of s_i, $i = 1, 2$, under the long-term contract that they signed up for.

Figure 1.7: No bundling and Long term contract



Customers in $[0,\ \theta^*]$ and $[\theta^*,\ 1]$ purchase long term contracts of A_1 and A_2, respectively, in the first period. After observing consumer preferences in the first period, firms in market A may offer discounts to consumers committed to competitors. Consumers in $[\theta_1,\ \theta^*]$ and $[\theta^*,\ \theta_2]$ are targeted with discounts by competitors with A_2 and A_1, respectively, in the second period.

In the second period, given θ^*, customers are indifferent between A_1 and A_2 at θ_1 and θ_2 if

$$p_{A1}^2 + t\theta_1 = \hat{p}_{A2}^2 + s_1 + t(1-\theta_1) \ \Rightarrow\ \theta_1 = \frac{1}{2t}(t + \hat{p}_{A2}^2 + s_1 - p_{A1}^2), \quad (1.44)$$

$$\hat{p}_{A1}^2 + s_2 + t\theta_2 = p_{A2}^2 + t(1-\theta_2) \ \Rightarrow\ \theta_2 = \frac{1}{2t}(t + p_{A2}^2 - \hat{p}_{A1}^2 - s_2), \quad (1.45)$$

where p_{A1}^2 and p_{A2}^2 are the second period prices of A_1 and A_2 for consumers of firm A1 and A2, respectively, who committed to long-term contracts. $\hat{p}_{A1}^2$ and $\hat{p}_{A2}^2$ are the second period prices of A_1 and A_2 for consumers who switch between providers in the second period. s_i, where $i = 1, 2$, is an early termination fee that customers are obligated to pay when they breach a long-term contract. By maximizing the following profit functions with respect to the second period prices for new customers,

$$\max \quad (\hat{p}^2_{A1} - c_A)(\theta_2 - \theta^*), \tag{1.46}$$

$$\max \quad (\hat{p}^2_{A2} - c_A)(\theta^* - \theta_1), \tag{1.47}$$

I have the second period prices to new buyers as follows.

$$\hat{p}^2_{A1} = \frac{1}{2}\left(t + c_A - s_2 - 2t\theta^* + p^2_{A2}\right), \quad \hat{p}^2_{A2} = \frac{1}{2}\left(2t\theta^* - t + c_A - s_1 + p^2_{A1}\right). \tag{1.48}$$

In the first period, a consumer at θ^* is indifferent between A_1 and A_2 if

$$\begin{aligned} p^1_{A1} \; + \; & t\theta^* - \delta s_1 + \delta(\hat{p}^2_{A2} + s_1 + t(1-\theta^*)) \\ & = p^1_{A2} + t(1-\theta^*) - \delta s_2 + \delta(\hat{p}^2_{A1} + s_2 + t\theta^*) \\ \Rightarrow \quad \theta^* & = \frac{1}{2} + \frac{p^1_{A2} - p^1_{A1}}{2(1-\delta)t} - \frac{\delta(\hat{p}^2_{A2} - \hat{p}^2_{A1})}{2(1-\delta)t} \\ \Rightarrow \quad \theta^* & = \frac{1}{2} + \frac{p^1_{A2} - p^1_{A1}}{2t} + \frac{\delta(p^2_{A2} - p^2_{A1})}{4t} - \frac{\delta(s_2 - s_1)}{4t}, \end{aligned} \tag{1.49}$$

where p^1_{A1} and p^1_{A2} are the first period prices of A_1 and A_2 in long-term contracts, respectively. Note that a customer receives a fixed amount of discounts of $d_i = \delta s_i$ in return for a long-term contract. Hence, if terminating early, the customer is obligated to pay back s_i, $i = 1, 2$, as an early termination fee.

Firm A1 and A2 maximize the following profit functions with respect to $\{p^1_{A1}, p^2_{A1}\}$ and $\{p^1_{A2}, p^2_{A2}\}$, respectively, for customers committed to long-term contracts.

$$\begin{aligned} \max \quad & (p^1_{A1} - c_A - \delta s_1)\theta^* + \delta(p^2_{A1} - c_A)\theta_1 + \delta(\hat{p}^2_{A1} - c_A)(\theta_2 - \theta^*) + \delta s_1(\theta^* - \theta_1), \\ & s.\,t. \quad \theta_1 = \theta^*. \end{aligned} \tag{1.50}$$

$$\begin{aligned} \max \quad & (p^1_{A2} - c_A - \delta s_2)(1-\theta^*) + \delta(p^2_{A2} - c_A)(1-\theta_2) \\ & + \delta(\hat{p}^2_{A2} - c_A)(\theta^* - \theta_1) + \delta s_2(\theta_2 - \theta^*), \\ & s.\,t. \quad \theta_2 = \theta^*. \end{aligned} \tag{1.51}$$

By substituting the optimal prices and the condition, $\theta_1 = \theta_2 = \theta^*$, into eq. (1.44) and eq. (1.45), I have the optimal amount of discounts of d^* and early termination fee of s^* as follows.

$$d_1^* = d_2^* = \delta t, \tag{1.52}$$

$$s_1^* = s_2^* = t. \tag{1.53}$$

And the equilibrium prices are as follows.

$$\begin{aligned} p_{A1}^1 &= p_{A2}^1 = t + c_A + \frac{\delta}{2} t, \\ p_{A1}^2 &= p_{A2}^2 = t + c_A, \\ \hat{p}_{A1}^2 &= \hat{p}_{A2}^2 = c_A. \end{aligned} \tag{1.54}$$

The first period price is higher than in the benchmark model. However, with discounts, profit margin, price-cost-discounts, is lower than in the benchmark model. At the price of c_A, no consumer switches between brands due to early termination fee $s^* = t$. With loyalty contracts, firms allow no consumer poaching. However, they face intensified competition in the first period and are worse off than in short-term contracts. The equilibrium profits of firms are

$$\pi_1^{NB(L)} = \pi_2^{NB(L)} = \left(1 + \frac{1}{2}\delta\right) \frac{t}{2}. \tag{1.55}$$

Proposition 1.2 In no bundling with long-term contracts, (i) The optimal discounts are $d_1^* = d_2^* = \delta t$ and early termination fees are $s_1^* = s_2^* = t$. (ii) The first period profit margin, price-cost-discounts, is lower than in a sequence of short-term contracts while the second period profit margin is higher than in short-term contracts. (iii) By offering discounts in exchange for long-term contracts, firms are worse off than in a sequence of short-term contracts.

Proof: See appendix B.

Firms in market A may not allow any consumer poaching by offering loyalty inducing discounts in return for long-term contracts. That is, by creating an artificial switching cost to block consumer poaching, no customer switches between brands. However, this intensifies competition for market share in the first period and hence firms are worse off than in a sequence of short-term contracts while payoffs from offering long-term contracts are higher than from offering both long-and short-term contracts.

(4) Short-term contract vs. Long-term contract

In the presence of consumer poaching, firms may have an incentive to offer discounts to customers in exchange for long-term contracts in order not to allow any consumer poaching by competitors. However, this intensifies competition for market base in the first period, which results in fewer profits than in short-term contracts where positive customers switch between brands.

In Table 1.1, I compare payoffs from three different types of contract arrange-

Table 1.1: Payoffs in No Bundling ($\delta = 1$)

Firm A2 / Firm A1	Short-term	Long-term	Long&Short
Short-term	$0.944t$, $0.944t$	$0.963t$, $0.790t$	$0.880t$, $0.816t$
Long-term	$0.790t$, $0.963t$	$0.750t$, $0.750t$	$0.667t$, $0.800t$
Long&Short	$0.816t$, $0.880t$	$0.800t$, $0.667t$	$0.719t$, $0.719t$

ments at $\delta = 1$. Short-term contract arises in equilibrium in the presence of consumer poaching. The reason is that by offering both long-and short-term contracts, each firm faces intensified competition due to its own competition between long-and short-term contracts. Long-term contract may not allow any consumer poaching, but intensifies competition for a market base in the first period. In a short-term contract, while facing intensified competition in the second period, firms are able to benefit from softened competition in the first period because customers become less elastic in the first period in the presence of consumer poaching. Hence, firms are better off when they offer a sequence of short-term contracts and allow consumer poaching by competitors than either long-term contracts or long-and short-term contracts.

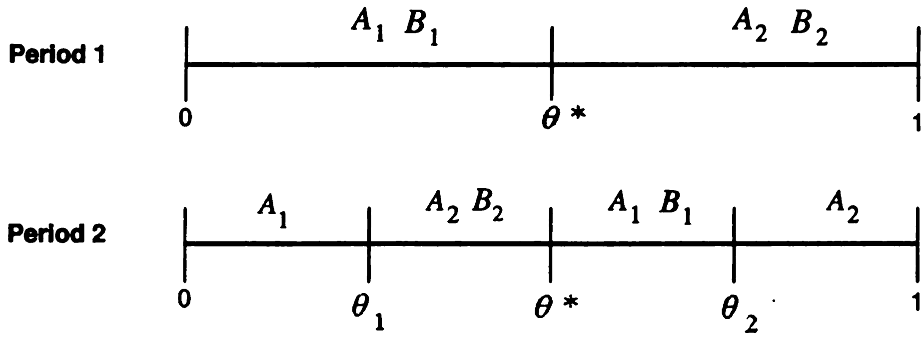
1.4.2 Pure bundling

I consider a model where firms in market A offer a package of A_iB_i, where $i = 1, 2$, to all first time consumers. Consumers who switch between brands incur switching costs because they need to buy a whole package of A_iB_i, rather than A_i, where $i = 1, 2$. Note that I assumed that firm A1 purchases B_1 and firm A2 buys B_2 at the price of c_B and pack it together with A_1 and A_2, respectively, making the bundle of A_iB_i, $i = 1, 2$, at the cost of $c_A + c_B$.

(1) Short-term contract

I consider the case where firms offer a sequence of short-term contracts in the presence of consumer poaching in market A. Consumers in $[0,\ \theta^*]$ and $[\theta^*,\ 1]$ buy the bundle A_1B_1 and A_2B_2, respectively, in the first period. In the second period, consumers in the intervals $[\theta_1,\ \theta^*]$ and $[\theta^*,\ \theta_2]$ are targeted with discounts by competitors and switch between brands, purchasing the whole package of A_iB_i, rather than A_i separately, where $i = 1, 2$. On the other hand, consumers in the intervals $[0,\ \theta_1]$ and $[\theta_2,\ 1]$ do not switch suppliers. In the second period, given θ^*, a customer

Figure 1.8: Pure bundling and Short term contract



is indifferent between A_1 and A_2B_2 at θ_1 and between A_1B_1 and A_2 at θ_2 if

$$p^2_{A1} + t\theta_1 = \hat{p}^B_{A2} + t(1-\theta_1) \Rightarrow \theta_1 = \frac{1}{2t}(t + \hat{p}^B_{A2} - p^2_{A1}), \qquad (1.56)$$

$$\hat{p}^B_{A1} + t\theta_2 = p^2_{A2} + t(1-\theta_2) \Rightarrow \theta_2 = \frac{1}{2t}(t + p^2_{A2} - \hat{p}^B_{A1}), \qquad (1.57)$$

where p^2_{A1} and p^2_{A2} are the second period prices of A_1 and A_2 for loyal consumers of firm A1 and A2, respectively. $\hat{p}^B_{A1}$ and $\hat{p}^B_{A2}$ are the second period prices of A_1B_1 and A_2B_2 for consumers who switch between providers. The profit functions of the firms in the second period are as follows.

$$\pi^{PB(S)}_{12} = (p^2_{A1} - c_A)\theta_1 + (\hat{p}^B_{A1} - c_A - c_B)(\theta_2 - \theta^*), \tag{1.58}$$
$$\pi^{PB(S)}_{22} = (p^2_{A2} - c_A)(1 - \theta_2) + (\hat{p}^B_{A2} - c_A - c_B)(\theta^* - \theta_1). \tag{1.59}$$

By maximizing the profit functions with respect to the second period prices, I have the second period prices as a function of θ^* as follows.

$$p^2_{A1} = c_A + \frac{1}{3}c_B + \left(\frac{2\theta^* + 1}{3}\right)t, \ p^2_{A2} = c_A + \frac{1}{3}c_B + \left(\frac{3 - 2\theta^*}{3}\right)t, \tag{1.60}$$
$$\hat{p}^B_{A1} = c_A + \frac{2}{3}c_B + \left(\frac{3 - 4\theta^*}{3}\right)t, \ \hat{p}^B_{A2} = c_A + \frac{2}{3}c_B + \left(\frac{4\theta^* - 1}{3}\right)t.$$

In the first period, a consumer at θ^* is indifferent between A_1B_1 and A_2B_2 if

$$p^B_{A1} + t\theta^* + \delta(\hat{p}^B_{A2} + t(1 - \theta^*)) = p^B_{A2} + t(1 - \theta^*) + \delta(\hat{p}^B_{A1} + t\theta^*)$$
$$\Rightarrow \theta^* = \frac{1}{2t} + \frac{(p^B_{A2} - p^B_{A1}) + \delta(\hat{p}^B_{A1} - \hat{p}^B_{A2})}{2(1-\delta)t} = \frac{1}{2} + \frac{3(p^B_{A2} - p^B_{A1})}{2(3+\delta)t}. \tag{1.61}$$

Firms set the first period prices, p^1_{A1} and p^1_{A2}, by maximizing the following profit functions,

$$\pi^{PB(S)}_{1} = (p^B_{A1} - c_A - c_B)\left(\frac{1}{2} + \frac{3(p^B_{A2} - p^B_{A1})}{2(3+\delta)t}\right) + \delta\pi^{PB(S)}_{12}, \tag{1.62}$$
$$\pi^{PB(S)}_{2} = (p^B_{A2} - c_A - c_B)\left(\frac{1}{2} - \frac{3(p^B_{A2} - p^B_{A1})}{2(3+\delta)t}\right) + \delta\pi^{PB(S)}_{22}. \tag{1.63}$$

Note that at $\theta^* = 1/2$, $\frac{\partial\pi^{PB(S)}_{i2}}{\partial\theta^*}\frac{\partial\theta^*}{\partial p^B_{Ai}} = -\left[\frac{5(2\theta^*-1)}{3(3+\delta)} + \frac{c_B}{(3+\delta)t}\right] = -\frac{c_B}{(3+\delta)t}$, where $i = 1, 2$, which implies that when there are switching costs of c_B created by bundling, firms compete more aggressively for market share in the first period than in the absence of switching costs. The equilibrium prices are as follows.

$$\begin{aligned} p_{A1}^{B} &= p_{A2}^{B} = t + c_A + c_B + \frac{\delta}{3}(t - 2c_B), \\ p_{A1}^{2} &= p_{A2}^{2} = \frac{2}{3}t + c_A + \frac{1}{3}c_B, \\ \hat{p}_{A1}^{B} &= \hat{p}_{A2}^{B} = \frac{1}{3}t + c_A + \frac{2}{3}c_B. \end{aligned} \tag{1.64}$$

By substituting the optimal prices into eq.(1.56) and eq. (1.57), I derive

$$\theta_1 = \frac{1}{3} + \frac{c_B}{6t}, \ \theta^* = \frac{1}{2}, \ \theta_2 = \frac{4}{6} - \frac{c_B}{6t}. \tag{1.65}$$

(1.1) $t > c_B$

Customers are not locked-in after the first purchase if $t > c_B$. Consumers who switch between brands are located in the intervals $[\theta_1, \theta^*]$ and $[\theta^*, \theta_2]$. The portion of consumer poaching is $\frac{1}{3}(1 - \frac{c_B}{t})$, which is less than in the absence of switching costs. In no bundling, 1/3 of consumers switch between providers in the second period. By creating switching costs, bundling locks-in some of consumers who may otherwise switch between providers. However, the equilibrium profits are lower than in no bundling because firms face intensified competition in the first period in the presence of switching costs. The equilibrium profits are

$$\pi_1^{PB(S)} = \pi_2^{PB(S)} = \left(1 + \frac{8\delta}{9}\right)\frac{t}{2} - \frac{\delta c_B}{9t}(2t - c_B). \tag{1.66}$$

In appendix A, I show the case where firm A1 offers the bundle A_1B_1 to new customers while firm A2 does not bundle. Firm A1 makes higher profits in pure bundling at the cost of firm A2. Hence, in equilibrium, both firms engage in bundling as a result of bundling game and are worse off than in no bundling (See Table 1.2 & 1.3).

Table 1.2: Payoffs in short-term contract under $\left(\frac{27+5\delta}{27-11\delta}\right) c_B < t$ $(\delta = 1)$

	NB(Firm A2)	PB(Firm A2)
NB (Firm A1)	$0.9444t$, $0.9444t$	$0.9444t - 0.28c_B + 0.15\frac{c_B^2}{t}$, $0.9444t + 0.06c_B + 0.15\frac{c_B^2}{t}$
PB (Firm A1)	$0.9444t + 0.06c_B + 0.15\frac{c_B^2}{t}$, $0.9444t - 0.28c_B + 0.15\frac{c_B^2}{t}$	$0.9444t - 0.22c_B + 0.11\frac{c_B^2}{t}$, $0.9444t - 0.22c_B + 0.11\frac{c_B^2}{t}$

Table 1.3: Payoffs in short-term contract under $c_B < t \leq \left(\frac{27+5\delta}{27-11\delta}\right) c_B$ $(\delta = 1)$

	NB(A2)	PB (A2)
NB(A1)	$0.9444t$, $0.9444t$	$0.7897t$, $0.9633t$
PB(A2)	$0.9633t$, $0.7897t$	$0.944t - 0.22c_B + 0.11\frac{c_B^2}{t}$, $0.944t - 0.22c_B + 0.11\frac{c_B^2}{t}$

Proposition 1.3 In pure bundling with a sequence of short-term contracts, if $c_B < t$, (i) $\frac{1}{3}(1 - \frac{c_B}{t})$ of consumers switch between brands (ii) The first and second period prices are given in eq.(1.64). (iii) If $c_B < t \leq 2c_B$, the first and second period profit margins, *price − marginal costs*, are lower than in the benchmark model. If $2c_B \leq t$, the first period profit margin is higher than in the benchmark model while the second period profit margin is lower than in the benchmark model. (iv) Pure bundling with a sequence of short-term contracts arises in equilibrium as a result of bundling game in the presence of consumer poaching, resulting in prisoners' dilemma.

Proof: See appendix B.

In market B, firms B1 and B2 have a positive sale, $\frac{1}{3}(1 - \frac{c_B}{t})$, where $c_B < t$, in the second period because some customers switch between brands and need to purchase a package of A_iB_i, $i = 1, 2$. The market prices of B_1 and B_2 for both periods are the marginal cost of c_B. Each firm makes zero profit.

(1.2) $t \leq c_B$

No consumer switches between brands in the second period if $t \leq c_B$. Knowing that customers are locked-in after the first purchase, firms may have an incentive to charge locked-in customers as much as they can, $U - t\theta^*$ for A_1 and $U - t(1-\theta^*)$ for A_2, where U is the consumers' reservation value for A_iB_i, $i = 1, 2$. This leads to intensified competition in the first period and hence results in less profit in pure bundling than in no bundling.

In the first period, given $t \leq c_B$, a consumer at θ^* is indifferent between A_1B_1 and A_2B_2 if

$$\begin{aligned} p^B_{A1} + t\theta^* + \delta(U - t\theta^* + t\theta^*) &= p^B_{A2} + t(1-\theta^*) + \delta(U - t(1-\theta^*) + t(1-\theta^*)) \\ \Rightarrow \theta^* &= \frac{1}{2t} + \frac{(p^B_{A2} - p^B_{A1})}{2t} \,. \end{aligned} \tag{1.67}$$

Firms maximize the following profit functions with respect to p^B_{A1} and p^B_{A2}.

$$\pi_1^{PB(S)} = (p^B_{A1} - c_A - c_B)\theta^* + \delta(U - t\theta^* - c_A)\theta^* \,, \tag{1.68}$$

$$\pi_2^{PB(S)} = (p^B_{A2} - c_A - c_B)(1-\theta^*) + \delta(U - t(1-\theta^*) - c_A)(1-\theta^*) \,. \tag{1.69}$$

The equilibrium market prices and profits are as follows.

$$\begin{aligned} p^B_{A1} &= p^B_{A1} = t + c_A + c_B - \delta(U - t - c_A), \\ p^2_{A1} &= p^2_{A2} = U - \frac{t}{2} \,, \\ \pi_1^{PB(S)} &= \pi_2^{PB(S)} = \left(1 + \frac{\delta}{2}\right)\frac{t}{2}. \end{aligned} \tag{1.70}$$

Proposition 1.4 In pure bundling with a sequence of short-term contracts, if $t \leq c_B$, (i) No customer switches between brands due to high switching costs. (ii) The first period price of the bundle is lower than in the benchmark model while the second period price is higher than in the benchmark model. (iii) Firms make lower profits than in no bundling.

Proof: See appendix B.

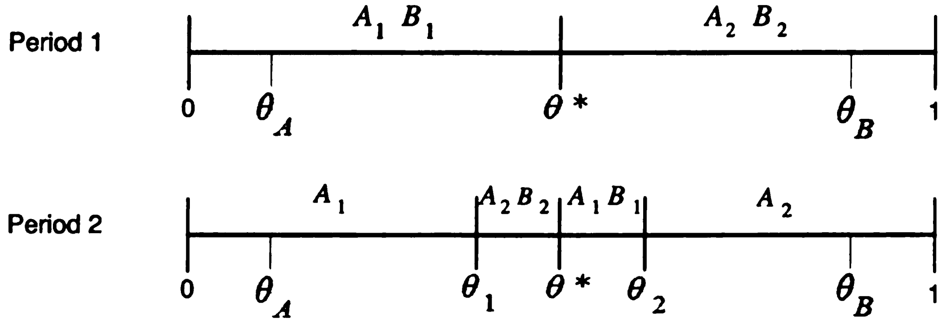
The intuition behind proposition 1.4 is that firms face lessened competition in the second period due to locking-in effects of switching costs while switching costs intensify competition for market base in the first period. The loss from intensified competition in the first period cannot be covered by the gains from softened competition in the second period.

Firms in market B have no sale in the second period while in the first period firms B1 and B2 have half of market share at the price of c_B, making zero profits.

(2) Long- and Short-term contracts

Firms offer long-term contracts as well as short-term contracts. Consumers in $[0,\ \theta^*]$ and $[\theta^*,\ 1]$ buy the bundle A_1B_1 and A_2B_2, respectively, in the first period. Consumers in $[0,\ \theta_A]$ and $[\theta_B,\ 1]$ commit to a long-term contract at the price of $\tilde{p}_{A1}$ and $\tilde{p}_{A2}$ from firm A1 and A2, respectively. Consumers in $[\theta_A,\ \theta^*]$ and $[\theta^*,\ \theta_B]$ buy a short-term contract at p^B_{A1} and p^B_{A2}, from firm A1 and A2, respectively. In the second period, some consumers who bought a short-term contract in the first period, $[\theta_1,\ \theta^*]$ and $[\theta^*,\ \theta_2]$, are targeted with discounts and switch between providers, and pay the second period poaching price $\hat{p}^B_{A2}$ and $\hat{p}^B_{A1}$ for A_2B_2 and A_1B_1, respectively. Consumers in $[\theta_A,\ \theta_1]$ and $[\theta_2,\ \theta_B]$ stay with the previous providers and pay p^2_{A1} and p^2_{A2} for A_1 and A_2, respectively. Since customers in $[0,\ \theta_A]$ and $[\theta_B,\ 1]$ committed to

Figure 1.9: Pure bundling and Long and Short term contracts



a long term contract at $\tilde{p}_{A1}$ and $\tilde{p}_{A2}$, respectively, competition in the second period is over the interval $[\theta_A,\ \theta_B]$. Given θ^*, a customer is indifferent between A_1 and A_2B_2

at θ_1 and between A_1B_1 and A_2 at θ_2 if

$$p^2_{A1} + t\theta_1 = \hat{p}^B_{A2} + t(1-\theta_1) \;\Rightarrow\; \theta_1 = \frac{1}{2t}(t + \hat{p}^B_{A2} - p^2_{A1}) \,, \tag{1.71}$$

$$\hat{p}^B_{A1} + t\theta_2 = p^2_{A2} + t(1-\theta_2) \;\Rightarrow\; \theta_2 = \frac{1}{2t}(t + p^2_{A2} - \hat{p}^B_{A1}) \,. \tag{1.72}$$

Firms are maximizing the following profit functions with respect to the second period prices, $\{p^2_{A1}, \hat{p}^B_{A1}\}$ and $\{p^2_{A2}, \hat{p}^B_{A2}\}$.

$$\pi^{PB(L\&S)}_{12} = (p^2_{A1} - c_A)(\theta_1 - \theta_A) + (\hat{p}^B_{A1} - c_A - c_B)(\theta_2 - \theta^*) \,, \tag{1.73}$$

$$\pi^{PB(L\&S)}_{22} = (p^2_{A2} - c_A)(\theta_B - \theta_2) + (\hat{p}^B_{A2} - c_A - c_B)(\theta^* - \theta_1). \tag{1.74}$$

Then, I have

$$\begin{aligned}
p^2_{A1} &= c_A + \frac{1}{3}c_B + \left(\frac{2\theta^*+1}{3}\right)t - \frac{4}{3}\theta_A t,\\
p^2_{A2} &= c_A + \frac{1}{3}c_B - \left(\frac{2\theta^*+1}{3}\right)t + \frac{4}{3}\theta_B t,\\
\hat{p}^B_{A1} &= c_A + \frac{2}{3}c_B - \left(\frac{4\theta^*-1}{3}\right)t + \frac{2}{3}\theta_B t,\\
\hat{p}^B_{A2} &= c_A + \frac{2}{3}c_B + \left(\frac{4\theta^*-1}{3}\right)t - \frac{2}{3}\theta_A t.
\end{aligned} \tag{1.75}$$

In the first period, a consumer at θ^* is indifferent between A_1B_1 and A_2B_2 if

$$\begin{aligned}
&p^B_{A1} + t\theta^* + \delta(\hat{p}^B_{A2} + t(1-\theta^*)) = p^B_{A2} + t(1-\theta^*) + \delta(\hat{p}^B_{A1} + t\theta^*)\\
&\Rightarrow \theta^* = \frac{1}{2} + \frac{p^B_{A2} - p^B_{A1}}{2(1-\delta)t} - \frac{\delta(\hat{p}^B_{A2} - \hat{p}^B_{A1})}{2(1-\delta)t}\\
&\Rightarrow \theta^* = \frac{1}{2} + \frac{p^B_{A2} - p^B_{A1}}{2t} + \frac{\delta(p^2_{A2} - p^2_{A1})}{4t}.
\end{aligned} \tag{1.76}$$

A consumer at θ_A and at θ_B is indifferent between a long-term contract and a sequence of short-term contracts if

$$\tilde{p}_{A1} + (1+\delta)t\theta_A = p^B_{A1} + t\theta_A + \delta(p^2_{A1} + t\theta_A) \Rightarrow \tilde{p}_{A1} = p^B_{A1} + \delta p^2_{A1}, \quad (1.77)$$

$$\tilde{p}_{A2} + (1+\delta)t(1-\theta_B) = p^B_{A2} + t(1-\theta_B) + \delta(p^2_{A2} + t(1-\theta_B))$$
$$\Rightarrow \tilde{p}_{A2} = p^B_{A2} + \delta p^2_{A2}. \quad (1.78)$$

Firm A1 and A2 maximize the following profit functions with respect to p^B_{A1} and p^B_{A2}, respectively.

$$\pi_1^{PB(L\&S)} = (p^B_{A1} - c_A - c_B)\theta^* + \delta(p^2_{A1} - c_A)\theta_1$$
$$+\delta(\hat{p}^B_{A1} - c_A - c_B)(\theta_2 - \theta^*), \quad (1.79)$$
$$\pi_2^{PB(L\&S)} = (p^B_{A2} - c_A - c_B)(1-\theta^*) + \delta(p^2_{A2} - c_A)(1-\theta_2)$$
$$+\delta(\hat{p}^B_{A2} - c_A - c_B)(\theta^* - \theta_1). \quad (1.80)$$

The equilibrium prices are

$$\tilde{p}_{A1} = \tilde{p}_{A2} = \left(1 + \frac{\delta}{2}\right)t + (1+\delta)c_A + c_B,$$
$$p^B_{A1} = p^B_{A2} = t + c_A + c_B\left(1 - \frac{\delta}{2}\right), \quad (1.81)$$
$$p^2_{A1} = p^2_{A2} = \frac{1}{2}t + c_A + \frac{1}{2}c_B,$$
$$\hat{p}^B_{A1} = \hat{p}^B_{A2} = \frac{1}{4}t + c_A + \frac{3}{4}c_B.$$

By substituting the optimal prices into eq. (1.71) and eq. (1.72), I have $\{\theta_1, \theta_2\}$. I can also derive $\{\theta_A, \theta_B\}$ from the first order conditions of eq. (1.79) and eq. (1.80) at $\frac{\partial\theta^*}{\partial p^B_{Ai}} = 0$, where first-period sales are unaffected if firms increase p^B_{Ai}, but decrease $\tilde{p}_{Ai}$ by the same amount, $i = 1, 2$.

$$\theta_A = \frac{1}{8}\left(1 - \frac{c_B}{t}\right), \ \theta_1 = \frac{3}{8}\left(1 + \frac{c_B}{3t}\right),$$
$$\theta^* = \frac{1}{2}, \ \theta_2 = \frac{5}{8}\left(1 - \frac{c_B}{5t}\right), \ \theta_B = \frac{7}{8}\left(1 + \frac{c_B}{7t}\right). \quad (1.82)$$

(2.1) $t > c_B$

By offering both long-and short-term contracts, firms have a smaller portion of customers who switch between brands in pure bundling than in no bundling. The reason is that it costs more for firms to poach in the presence of switching costs of c_B. It is also smaller than in a sequence of short-term contracts where $\frac{1}{3}\left(1-\frac{c_B}{t}\right)$ of consumers switch. I can find that a positive portion of consumers commits to long-term contracts, $\frac{1}{4}\left(1-\frac{c_B}{t}\right)$, where $t > c_B$. The portion of consumers who purchases a sequence of short-term contracts is $\frac{1}{2}\left(1-\frac{c_B}{t}\right)$, where $t > c_B$. Nevertheless, firms make less profit than in short-term contracts.

The equilibrium profits are

$$\pi_1^{PB(L\&S)} = \pi_2^{PB(L\&S)} = \left(1+\frac{7}{16}\delta\right)\frac{t}{2} - \frac{\delta c_B}{32t}\left(2t-3c_B\right). \tag{1.83}$$

Proposition 1.5 In pure bundling with both long-and short-term contracts, (i) A positive number of consumers purchases long-term contracts, $\frac{1}{4}\left(1-\frac{c_B}{t}\right)$, where $t > c_B$. The fraction of consumer poaching is $\frac{1}{4}\left(1-\frac{c_B}{t}\right)$, where $t > c_B$, which is less than in a short-term contact. (ii) Profits are lower than in no bundling if $\frac{3}{2}c_B \leq t$, but higher than in no bundling if $c_B < t \leq \frac{3}{2}c_B$, while no firm necessarily makes more profits than in a short-term contract.

Proof. See appendix B.

Firms face more intensified competition when offering both long-and short-term contracts than a sequence of short-term contracts because each firm competes with itself on its own turf. Hence, payoffs from short-term contracts are higher than in long-and short-term contracts

In market B, firms have a positive sale, $\frac{1}{4}(1-\frac{c_B}{t})$, where $c_B < t$, in the second period because some customers switch between brands and need to purchase a package of A_iB_i, $i = 1, 2$. The market prices of B_1 and B_2 for both periods are the marginal cost of c_B. Each firm makes zero profit.

(2.2) $t \leq c_B$

Long-and short-term contracts may not arise in equilibrium if $t \leq c_B$. No consumers switch between brands and purchase long-term contracts in equilibrium. All consumers purchase a sequence of short-term contracts. Since customers are completely locked-in in the second period, firms severely compete for market share in the first period, resulting in the same equilibrium outcomes as in the previous case of offering a sequence of short-term contracts.

(3) Long-term contract

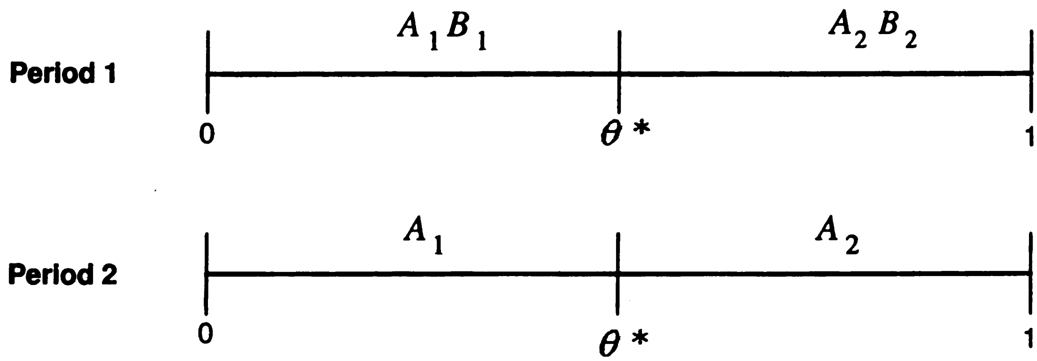
Since firms offer a package of A_iB_i, where $i = 1, 2$, firms in market A offer discounts on the purchase of A_iB_i, where $i = 1, 2$. I consider long term contracts in which firms in market A offer a schedule of prices and a fixed amount of discounts of d^* in return for a long term contract of good/service A, and establish a fee of $s^* = \frac{1}{\delta}d^*$, where $0 < \delta \leq 1$, when consumers terminate the contract earlier than the contract term. Such strategic discounts is well observed in markets with repeated purchase. In the U.S. telecom industry, firms offer discounts on the purchase of handsets in exchange for one or two year long-term contract on a service plan.

In pure bundling, customers face switching costs of c_B when they switch providers. Hence, there are two cases that we need to consider when firms in market A have an incentive to offer long-term contracts. From the previous case of short-term contracts, we know that if $t \leq c_B$, no customer switches between brands due to high switching costs of c_B, and if $t > c_B$, customers are not yet locked-in and hence some customers switch between brands in the presence of consumer poaching.

(3.1) $t \leq c_B$

Customers are locked-in in the second period. Unlike a sequence of short-term contracts, firms commit themselves to future prices in long-term contracts. Consumers in [0, θ^*] and [θ^*, 1] purchase a long-term contract of $\{A_1B_1,\ A_1\}$ and $\{A_2B_2,\ A_2\}$, respectively. Let us assume that firms in market A offer a price for both periods, $\bar{p}_{A1}$ for firm A1 and $\bar{p}_{A2}$ for firm A2. In the first period, given $t \leq c_B$, a customer at θ^*

Figure 1.10: Pure bundling and Long term contract



is indifferent between $\{A_1B_1,\ A_1\}$ and $\{A_2B_2,\ A_2\}$ if

$$\bar{p}_{A1} + t\theta^* + \delta(\bar{p}_{A1} + t\theta^*) = \bar{p}_{A2} + t(1-\theta^*) + \delta(\bar{p}_{A2} + t(1-\theta^*))$$
$$\Rightarrow \theta^* = \frac{1}{2} + \frac{\bar{p}_{A2} - \bar{p}_{A1}}{2t}\,. \tag{1.84}$$

Each firm maximizes the following profit function with respect to $\bar{p}_{A1}$ and $\bar{p}_{A2}$, respectively.

$$\pi_1^{PB(L)} = (\bar{p}_{A1} - c_A - c_B)\theta^* + \delta(\bar{p}_{A1} - c_A)\theta^*, \tag{1.85}$$
$$\pi_2^{PB(L)} = (\bar{p}_{A2} - c_A - c_B)(1-\theta^*) + \delta(\bar{p}_{A2} - c_A)(1-\theta^*). \tag{1.86}$$

I have the equilibrium market price and profits as follows.

$$\bar{p}_{A1} = \bar{p}_{A2} = t + c_A + \frac{c_B}{1+\delta}, \tag{1.87}$$
$$\pi_1^{PB(L)} = \pi_2^{PB(L)} = (1+\delta)\frac{t}{2}, \tag{1.88}$$

where $t \leq c_B$. Discounts on the purchase of product B emerge endogenously, $d^* = \frac{\delta}{1+\delta}c_B$. However, these discounts are recouped in the second period. Remember that I assumed that when a customer is offered discounts in return for a long-term contract, then the customer needs to pay back the discounts plus interest as an early termination fee if he terminates early. Since customers get discounts of $\frac{\delta}{1+\delta}c_B$ on the purchase of good B, they need to pay back $\frac{c_B}{1+\delta}$ as an early termination fee if switching between brands in the middle of contract term.

Let us check whether customers have any incentives to breach a long-term contract and switch between brands in the presence of consumer poaching. In the second period, consumers who may switch between brands need to pay the sum of a discounted price of the bundle and the early termination fee, $c_A + c_B + \frac{c_B}{1+\delta}$, which is higher than the price of $t + c_A + \frac{c_B}{1+\delta}$, where $t \leq c_B$, and hence no customer is willing to switch between brands.[5] The idea is that in the presence of consumer poaching, bundling can act as a commitment device through which firms are able to lock-in customers by creating switching costs and firms offer long-term contracts in the presence of such switching costs. That is, bundling permits neither customer switching nor consumer poaching.

An example can be found in the U.S. telecom industry where firms offer handsets at a price lower than the marginal cost in exchange for a long-term contract of phone-service plan for 1 or 2 years. When customers terminate a long-term contract, they are obligated to pay an early termination fee, ranging from \$150 to \$240. The early termination fee discourages customers from switching between brands. This practice is observed through bundling of handsets and service plan.

Now let us check whether firms have collective incentives to bundle with long-term contracts when $t \leq c_B$.[6] In Table 1.4, I show that long-term contracts may arise in equilibrium.

Table 1.4: Payoffs in long-term contract under $t \leq c_B$

	No Bundling (Firm A2)	Pure Bundling (Firm A2)
No Bundling (Firm A1)	$(1+\frac{\delta}{2})\frac{t}{2},\ (1+\frac{\delta}{2})\frac{t}{2}$	$(1+\frac{\delta}{2})\frac{t}{2},\ (1+\frac{\delta}{2})\frac{t}{2}$
Pure Bundling (Firm A1)	$(1+\frac{\delta}{2})\frac{t}{2},\ (1+\frac{\delta}{2})\frac{t}{2}$	$(1+\delta)\frac{t}{2},\ (1+\delta)\frac{t}{2}$

Proposition 1.6 In pure bundling with long-term contacts, if $t \leq c_B$, the equilibrium price is given in eq. (1.87). Pure bundling with long-term contracts arises in

[5]The poaching price of bundle to new customers is $\hat{p}^B_{A1} = \hat{p}^B_{A2} = c_A + c_B$.

[6]In appendix A, I show the case where firm A1 offers the bundle with loyalty contracts while firm A2 does not.

equilibrium. Firms are able to earn the same equilibrium profits as in the benchmark model.

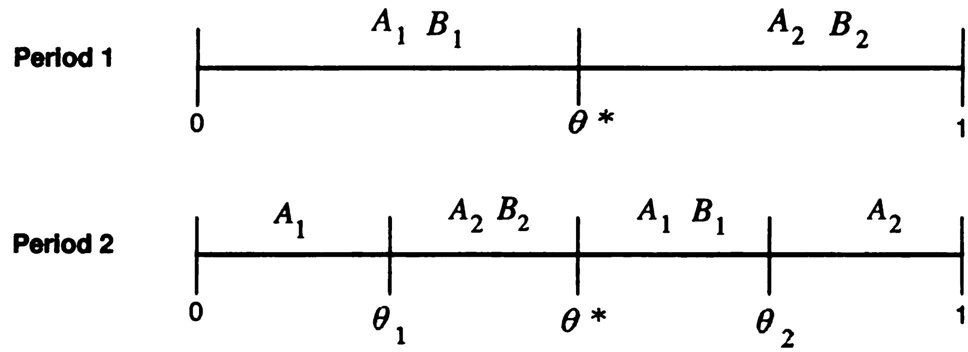
Proof: See appendix B.

The intuition behind proposition 1.6 is that firms have incentives to bundle with loyalty contracts in order to create artificial switching costs in the presence of consumer poaching and to commit to future prices in the presence of such switching costs.

(3.2) $c_B < t$

This is the case where firms offer a fixed amount of discounts of d^* in exchange for a long-term contract of good/service A with a schedule of prices, $\{p^B_{A1},\ p^2_{A1}\}$ and $\{p^B_{A2},\ p^2_{A2}\}$ when customers are not yet locked-in after the first purchase. Consumers in $[0,\ \theta^*]$ and $[\theta^*,\ 1]$ buy the bundle A_1B_1 and A_2B_2, respectively, in the first period. In the second period, consumers in the intervals $[\theta_1,\ \theta^*]$ and $[\theta^*,\ \theta_2]$ are targeted with discounts by competitors and may switch between brands, paying an early termination fee of s_i and purchasing the whole package of A_iB_i, rather than A_i separately, where $i = 1, 2$. On the other hand, consumers in the intervals $[0,\ \theta_1]$ and $[\theta_2,\ 1]$ stay with their previous suppliers and purchase A_1 and A_2, respectively. In the second

Figure 1.11: Pure bundling and Long term contract with $c_B < t$



period, given θ^*, a customer at θ_1 and at θ_2 is indifferent between A_1 and A_2B_2 and between A_1B_1 and A_2, respectively, if

$$p_{A1}^2 + t\theta_1 = \hat{p}_{A2}^B + s_1 + t(1-\theta_1) \;\Rightarrow\; \theta_1 = \frac{1}{2t}(t + \hat{p}_{A2}^B + s_1 - p_{A1}^2), \quad (1.89)$$

$$\hat{p}_{A1}^B + s_2 + t\theta_2 = p_{A2}^2 + t(1-\theta_2) \;\Rightarrow\; \theta_2 = \frac{1}{2t}(t + p_{A2}^2 - \hat{p}_{A1}^B - s_2), \quad (1.90)$$

where p_{A1}^2 and p_{A2}^2 are the second period prices of A_1 and A_2 for consumers of firm A1 and A2, respectively, who committed to a long-term contract. $\hat{p}_{A1}^B$ and $\hat{p}_{A2}^B$ are the second period prices of A_1B_1 and A_2B_2 for consumers who switch between providers, paying an early termination fee of s_i, $i = 1, 2$. Firms maximize the following profit functions with respect to the their own poaching prices.

$$max \quad (\hat{p}_{A1}^B - c_A - c_B)(\theta_2 - \theta^*), \quad (1.91)$$

$$max \quad (\hat{p}_{A2}^B - c_A - c_B)(\theta^* - \theta_1). \quad (1.92)$$

Then, I have the second period prices for new customers as follows.

$$\hat{p}_{A1}^B = \frac{1}{2}(t + p_{A2}^2 - s_2 - 2t\theta^* + c_A + c_B), \quad (1.93)$$
$$\hat{p}_{A2}^B = \frac{1}{2}(2t\theta^* - t - s_1 + p_{A1}^2 + c_A + c_B).$$

In the first period, a consumer at θ^* is indifferent between A_1B_1 and A_2B_2 if

$$\begin{aligned} p_{A1}^B + t\theta^* - \delta s_1 + \delta(\hat{p}_{A2}^B + t(1-\theta^*) + s_1) \\ = p_{A2}^B + t(1-\theta^*) - \delta s_2 + \delta(\hat{p}_{A1}^B + t\theta^* + s_2) \\ \Rightarrow \theta^* = \frac{1}{2} + \frac{p_{A2}^B - p_{A1}^B}{2(1-\delta)t} - \frac{\delta(\hat{p}_{A2}^B - \hat{p}_{A1}^B)}{2(1-\delta)t} \\ \Rightarrow \theta^* = \frac{1}{2} + \frac{p_{A2}^B - p_{A1}^B}{2t} + \frac{\delta(p_{A2}^2 - p_{A1}^2)}{4t} - \frac{\delta(s_2 - s_1)}{4t}, \quad (1.94) \end{aligned}$$

where p_{A1}^B and p_{A2}^B are the first period prices of A_1B_1 and A_2B_2 in long-term contracts, respectively. δs_i is a fixed amount of discounts of d_i offered to customers in return for a long-term contract, $i = 1, 2$. s_i is an early termination fee imposed on customers who terminate the contract early, $i = 1, 2$.

Firm A1 maximizes the following profit function with respect to $\{p^B_{A1},\ p^2_{A1}\}$.

$$\begin{aligned}\max \quad & (p^B_{A1} - c_A - c_B - \delta s_1)\theta^* + \delta(p^2_{A1} - c_A)\theta_1 \\ & + \delta(\hat{p}^B_{A1} - c_A - c_B)(\theta_2 - \theta^*) + \delta s_1(\theta^* - \theta_1)\ , \\ & s.\,t. \quad \theta_1 = \theta^*. \end{aligned} \tag{1.95}$$

Firm A2 maximizes the following profit function with respect to $\{p^B_{A2},\ p^2_{A2}\}$.

$$\begin{aligned}\max \quad & (p^B_{A2} - c_A - c_B - \delta s_2)(1 - \theta^*) + \delta(p^2_{A2} - c_A)(1 - \theta_2) \\ & + \delta(\hat{p}^B_{A2} - c_A - c_B)(\theta^* - \theta_1) + \delta s_2(\theta_2 - \theta^*)\ , \\ & s.\,t. \quad \theta_2 = \theta^*\ . \end{aligned} \tag{1.96}$$

By substituting the optimal prices and the condition, $\theta_1 = \theta_2 = \theta^*$, into eq. (1.89) and eq. (1.90), I have the optimal discounts of d^* and early termination fee of s^* as follows.

$$d^*_1 = d^*_2 = \delta(t - c_B), \tag{1.97}$$

$$s^*_1 = s^*_2 = t - c_B. \tag{1.98}$$

I have a positive early termination fee, $s^* > 0$, where $c_B < t$, high enough that firms are unable to gain additional customers attached to competitors. In other words, firms need to offer a positive amount of discounts of $d^* = \delta(t - c_B)$ in order to lock-in customers in the second period.

The equilibrium prices are

$$\begin{aligned} p^B_{A1} &= p^B_{A1} = t + c_A + c_B + \frac{\delta}{2}t - \delta c_B\ , \\ p^2_{A1} &= p^2_{A2} = t + c_A\ , \\ \hat{p}^B_{A1} &= \hat{p}^B_{A2} = c_A + c_B\ , \end{aligned} \tag{1.99}$$

where $\{p^B_{A1},\ p^2_{A1}\}$ and $\{p^B_{A2},\ p^2_{A2}\}$ are a schedule of prices of the first and second period in long-term contracts. Firms also include early termination fee s^* in long-term contracts.

The equilibrium profits are as follows.

$$\pi_1^{PB(L)} = \pi_2^{PB(L)} = (1+\frac{\delta}{2})\frac{t}{2}\,. \tag{1.100}$$

Firms offer discounts in the first period in order to lock-in customers in the second period. This intensifies competition for market base in the first period and hence payoffs are lower than in short-term contracts. Even though the discounts are smaller than in no bundling, the equilibrium profits are the same as in no bundling. The reason is that since the first period price reflects the amount of discounts offered to customers and the difference between prices for loyal customers and new customers, profit margin in pure bundling, price-marginal costs-discounts, is the same in no bundling.

Proposition 1.7 In pure bundling with long-term contacts, if $t > c_B$, the optimal discount is $d^* = \delta(t - c_B)$ and early termination fee is $s^* = t - c_B$. It then follows that no consumer has an incentive to switch between brands. Each firm gains the same equilibrium profits as in no bundling, but lower than in a sequence of short-term contracts.

Proof: See appendix B.

In market B, since no consumer switches between brands, there is no sale in the second period. The market price to firms in market A is c_B. Firms in market B make zero profits.

(4) Short-term contract vs. Long-term contract

With high switching costs, i.e. $t \leq c_B$, customers are locked-in after the first purchase. Hence, firms have an incentive to commit themselves to future prices through long-term contracts in the presence of such switching costs because otherwise firms face more competitive market in the first period and are worse off than in no bundling.

In Table 1.5, I compare payoffs under different types of contract arrangements

Table 1.5: Payoffs in Pure Bundling under $t \leq c_B$ ($\delta = 1$)

	Short-term (Firm A2)	Long-term (Firm A2)
Short-term (Firm A1)	$0.750t$, $0.750t$	$0.826t$, $0.893t$
Long-term (Firm A1)	$0.893t$, $0.826t$	t, t

Table 1.6: Payoffs in Pure Bundling under $c_B < t$ ($\delta = 1$)

	Short-term (Firm A2)	Long-term (Firm A2)	Long&Short (Firm A2)
S (A1)	$0.94t-0.22c_B+0.11\frac{c_B^2}{t}$, $0.94t-0.22c_B+0.11\frac{c_B^2}{t}$	$0.96t-0.25c_B+0.09\frac{c_B^2}{t}$, $0.79t-0.11c_B+0.10\frac{c_B^2}{t}$	$0.88t-0.15c_B+0.09\frac{c_B^2}{t}$, $0.82t-0.16c_B+0.12\frac{c_B^2}{t}$
L (A1)	$0.79t-0.11c_B+0.10\frac{c_B^2}{t}$, $0.96t-0.25c_B+0.09\frac{c_B^2}{t}$	$0.75t$, $0.75t$	$0.67t+0.02c_B+0.06\frac{c_B^2}{t}$, $0.80t-0.14c_B+0.09\frac{c_B^2}{t}$
L&S (A1)	$0.82t-0.16c_B+0.12\frac{c_B^2}{t}$, $0.88t-0.15c_B+0.09\frac{c_B^2}{t}$	$0.80t-0.14c_B+0.09\frac{c_B^2}{t}$, $0.67t+0.02c_B+0.06\frac{c_B^2}{t}$	$0.72t-0.06_B+0.09\frac{c_B^2}{t}$, $0.72t-0.06c_B+0.09\frac{c_B^2}{t}$

at $\delta = 1$. When customers are locked-in by switching costs, firms have incentives to offer long-term contracts. Nevertheless, when customers are not locked-in after the first purchase, $c_B < t$, a sequence of short-term contracts is an optimal strategy in the presence of consumer poaching (See Table 1.6). The reason is that firms face more competitive market in the first period (or in both periods) than in short-term contracts by offering long-term contracts (or long-and short-term contracts).

1.4.3 No bundling vs. Pure bundling

Pure bundling with long-term contracts may arise in equilibrium. The equilibrium outcomes are the same as in the benchmark model with no price discrimination. The reason is that bundling creates an artificial switching cost. When customers are locked-in with such switching costs, i.e. $t \leq c_B$, long-term contacts permit firms to commit to future prices in the presence of such switching costs. Hence, firms in a vertically related industry may have an incentive to bundle in order to lock-in consumers in the presence of consumer poaching and to offer long-term contracts in order to commit to future prices.

On the other hand, pure bundling with short-term contracts may arise in equilibrium as a result of bundling game when bundling creates switching costs that cannot lock-in customers in the presence of consumer poaching, i.e. $c_B < t$. Firms are worse off in pure bundling than in no bundling. The reason is that firms face intensified competition for market base in the first period in the presence of switching costs created by bundling. The intuition is that firms face more competitive market in the second period, but softened market in the first period in the absence of switching costs in markets with consumer poaching, while firms face more competitive market in the first period as well as in the second period in the presence of switching costs in those markets.

1.5 Conclusions

This paper extends the existing studies on consumer poaching based on consumers' past behavior and connects two important streams of brand switching and product bundling in a complementary market where a good/service is perishable and the other good is durable. This study shows that firms in a market where consumers make repeated purchases may have a collective incentive to bundle two complementary goods/services in combination with long-term contracts in the presence of consumer poaching. The reason is bundling creates artificial switching costs and hence firms are able to lock-in consumers. Long-term contracts permit firms to commit to future prices in the presence of such switching costs. That is, bundling acts as a commitment device through which firms are able to lock-in customers and firms offer introductory discounts on a durable good in exchange for a long-term contract on the other good that needs to be repurchased in each period. In the U.S. telecom industry where firms offer a package of handsets and mobile service plan, discounts are given on the purchase of handsets in return for a long-term contract of mobile service plan. I also show that pure bundling with short-term contracts may arise in equilibrium as a result of bundling game when customers face switching costs, but are able to switch between brands when targeted with discounts by competitors. Firms are in prisoners' dilemma in the sense that firms make less profit than in no bundling because they face more competitive markets in the presence of switching costs than in the absence of switching costs.

Our paper has limitations. First, I assume that consumers have constant preferences over periods. I expect that the results would not be different from our paper even when I consider the case where consumers change their preferences over periods. Second, I do not consider a model with overlapping generation consumers. New customers may enter the market in the second period. In this case, firms may find it difficult to distinguish between customers that have just entered the market and customers attached to competitors. Third, I consider only pure bundling, but not mixed bundling, in the present paper. Models with the last two ideas would be extensions of our paper. Another extension would be in the possibility of entry.

By the construction in the present paper, bundling may seem to result in positive effect on welfare, eliminating inefficient brand switching and yielding the same outcomes as in the benchmark model. However, if I consider possibility of entry in the second period, bundling may not necessarily improve social welfare in the presence of consumer poaching. The reason is that it is hard for an entrant to gain a positive market share, less competitive than it would be with new entry (preemption). This suggests that the practice of bundling of phone service plan and handsets should be regarded with suspicion.

Chapter 2

Strategic Mixed Bundling in Duopoly

2.1 Introduction

Bundling is the selling of two or more goods as a package, and there are two types of this practice, pure and mixed bundling. In pure bundling, the individual goods are sold only as a package. Mixed bundling refers to the practice of offering consumers the option of either buying two goods separately or buying a package of two different goods, generally at a discounted price. Mixed bundling includes the case where a firm offers good A or B as well as the bundle of AB. For example, local exchange carriers in the U.S. tend to offer local service as well as a package of local service and Internet service, but do not offer Internet service alone.[1]

Most of the previous analysis of bundling has focused either on the ability of bundling to achieve price discrimination or its ability to foreclose competition. A classic analysis in price discrimination by a monopolist is that bundling leads monopolists to do better than selling independently when consumers' valuations of two goods are not perfectly correlated (Stigler, 1968; Adams and Yellen, 1976; Schmalensee, 1982; McAfee, McMillan, and Whinston, 1989). The idea is that bundling reduces the vari-

[1]SBC Internet service (DSL) requires SBC local service. Qwest Choice DSL is available only to Qwest local service customers for residential use. AT&T DSL Service is available only to residential AT&T local service customers.

ations of customers' valuations. The argument of the foreclosure is that a monopolist of one product increases its profits by earning monopoly profits in the monopolized tied market. For a long time, this was quite controversial because the monopolist needs not monopolize the tied market to earn all the potential monopoly profits (Posner, 1976; Bork, 1978). However, Whinston (1990) shows that bundling can be an effective foreclosure device under a monopoly in the first market and duopoly in the second market.[2] Under the same market structure, Carbajo et al (1990) show that a monopolist has a strategic incentive to bundle when valuations are perfectly correlated. Their idea is that bundling can be used as a product differentiation device. Chen (1997) also has the same idea in markets where there is duopoly in a primary market and more than two firms in the other. Other explanations of bundling regard cost savings and R&D. Choi (1996) studies the effects of bundling on research and development and shows that bundling serves as a channel to monopolize the second market when taking R&D investments into account. Salinger (1995) examines the role of costs in explaining why products are bundled.

In monopoly models, mixed bundling is preferred to pure bundling because the bundle is priced to extract all the surplus of consumers who value both goods high and the separate markets are used to extract the surplus of consumers who value one high, but the other low. Adams and Yellen (1976) were the first to show that if the firm has a monopoly over both goods, profits may be strictly higher under mixed bundling than pure bundling. Schmalensee (1984) has shown, using the Adams-Yellen (1976) framework, that when reservation prices for the two goods are independently normally distributed, mixed bundling is preferred, by a multi-product monopolist, to either pure bundling or no bundling. However, there are papers that show mixed bundling is not optimal in a more competitive environment, unlike a monopoly. In the Carbajo et al (1990) and Chen (1997) papers where bundling acts as a product differentiation device, mixed bundling is dominated by pure bundling. They recognize the role of bundling as a product differentiation device. Mixed bundling undermines the role of product differentiation, which intensifies price competition and leaves firms

[2] Choi (2003) and Nalebuff (1999) study on entry deterrence in different environments.

worse off. Symmetric duopolies in complement markets are considered in the Matutes and Regibeau (1992) and Anderson and Leruth (1993) papers. They show that mixed bundling is more likely to be associated with monopoly due to the benefits obtained through price discrimination. In a duopoly, however, mixed bundling is dominated by pure bundling, unlike the monopoly situation since firms face more competitive markets under mixed bundling. Economides (1993) shows that when goods are not very close substitutes, using mixed bundling leads to lower profits for both firms in comparison to the profits achieved when both firms avoid using mixed bundling. In these papers, mixed bundling leads to a typical prisoners' dilemma situation.

In contrast to the previous works on mixed bundling in more competitive markets, I show that mixed bundling is a dominant strategy in duopoly like in the monopoly situation where mixed bundling allows the monopolist to extract more consumers' surplus than pure bundling due to the benefits reaped via price discrimination. In this paper, I show that an integrated firm (firm 1) strategically uses mixed bundling in markets with consumers' heterogenous valuations. I consider a model where there are two independent markets, i.e. Internet access and local/long-distance phone providers (or cable TV providers). In each market, there are a horizontally integrated firm and an independent firm: firm 1 and A2 competing in a Hotelling model in market A and firm 1 and B2 competing in a Bertrand model in market B.[3] In Matutes and Regibeau (1992), Anderson and Leruth (1993), and Economides (1993), two multi-product firms produce complementary products. Their result is that mixed bundling can decrease the equilibrium profits for a duopolist. However, Choi (2003) shows that mixed bundling is better than pure bundling for a vertically integrated firm in a system market because the vertically integrated firm has a strategic incentive to sell inputs to its rival in the downstream market. I consider a partial mixed bundling where firm 1 offers product A_1 as well as A_1B_1 . It is important to consider this type

[3]Kovac (2004) explores tying in a market where a multi-product firm in independent markets competes against several single-product firms. He shows that a multi-product firm has an incentive to use pure bundling. However, he only analyzes a specific partial mixed bundling where a multi-product firm offers a bundle of A_1B_1 and a homogeneous good of B_1 in a market where consumer's valuations of good B are heterogeneous. In my model, I analyze a partial mixed bundling where an integrated firm offers a bundle of A_1B_1 and a differentiated good of A_1.

of mixed bundling because the integrated firm might have an incentive to foreclose a market by practicing the mixed bundling when pure bundling is not permitted.[4]

My main result is as follows. With heterogenous valuations of good B, bundling has two effects: substitution effect and competition softening effect. Substitution effect of bundling arises from a situation where consumers of firm 1 who value good B less than the bundle price minus the price of good A_1 switch to its rival, A_2, when firm 1 offers A_1 only in the bundle of A_1B_1. On the other hand, competition softening effects occur in both markets A and B, resulting from interdependence of two goods in price and product differentiation role of bundling. The bundle A_1B_1 is differentiated from good B_2. Bundling also interrelates two independent goods in price in market A. Pure bundling is profitable when products A are highly differentiated so that competition softening effect of bundling in market B dominates substitution effect of bundling occurred in market A. Firm 1 is better off in pure bundling than in no bundling at the costs of firm A2 because firm A2 needs to decrease its price to compete against the bundle as bundling lessens competition in market B. On the other hand, mixed bundling is profitable even when products A are close substitutes. The reason is that mixed bundling dampens the substitution effect of bundling. In addition, when products A are highly differentiated, mixed bundling is preferred, by firm 1, to either pure bundling or no bundling because with high magnitude of product differentiation degree, t, competition softening effect of bundling is significant compared the substitution effect of bundling. As in pure bundling, firm A2 is also worse off in mixed bundling than in no bundling. Suppose $\pi_{A2}^{PB} < F < \pi_{A2}^{NB}$ and $\pi_{A2}^{MB} < F < \pi_{A2}^{NB}$ where F is an entry cost. Then, mixed bundling can act as an entry barrier where pure bundling is not allowed by an anti-trust authority. In the previous example, local exchange carriers might be interested in offering local phone service separately as well as a package of Internet access service and local phone service, but not Internet service alone, to deter an entry in local service. Therefore, mixed bundling in duopoly should be regarded with suspicion, like pure bundling.

[4]In addition, firm 1 has no incentive to offer a mixed bundling of $\{B_1, A_1B_1\}$ and $\{A_1, B_1, A_1B_1\}$ because products B are homogeneous in my model.

In the following sections, I set up a model with discrete consumer's valuations of good B in section 2. As a special case, I analyze a model with homogeneous valuations of good B in appendix A. For the robustness of our results, a model with continuous valuations of good B is analyzed in section 3. Conclusions are at the end.

2.2 Model with discrete valuations of good B

There are two independent markets for indivisible products A and B. In market A, there are two firms: firm 1 and A2. Product A is horizontally differentiated in a simple Hotelling model. It is assumed that firm 1 is located at point 0 and firm A2 at 1. Each firm produces goods at the same constant marginal costs of c_A. A continuum of consumers is normalized its total mass to 1. Each consumer is described by her location on an interval, $x \in [0, 1]$. The consumers are uniformly distributed on the interval. Each consumer enjoys a gross utility of u from good/service A. I assume the value, u, of the good to be large enough that in equilibrium, every consumer has a unit demand for product A. A consumer at x, where $x \in [0,\ 1]$, incurs the transportation cost of tx, or disutility associated with the difference between the purchased product and the consumer's most preferred product. If a consumer located at x purchases from firm 1, then that consumer gains net utility of $u - p_{A1} - tx$. If that same consumer purchases from firm A2 then that consumer gains net utility $u - p_{A2} - t(1 - x)$. The marginal utility from any additional unit of good A is zero. This ensures that a consumer will buy one unit of product A.

In market B, firm 1 and B2 produce homogeneous goods at the constant marginal cost of c_B and compete in price. There are two types of consumers in market B, high($\bar{v}$) and low($\underline{v}$) valuations. That is, the utility from the first unit of good B is heterogeneous. I assume that $\underline{v} < c_B < \bar{v}$ and there are λ consumers with valuation $\bar{v}$ and $1 - \lambda$ with $\underline{v}$. For tractability and simplicity, I set $\underline{v} = 0$ and $\bar{v} = 1$, with $\lambda = 1/2$. As a special case, I show in appendix A the case where consumers have homogeneous valuations of good B, $\bar{v} = 1$ with $\lambda = 1$. For robustness of our results, I will consider continuous consumers' valuations of good B in section 3.

There are assumptions that hold throughout the paper. If firm 1 is indifferent between bundling and no bundling, firm 1 chooses to bundle. If it is indifferent between mixed bundling and pure bundling, firm 1 chooses mixed bundling.

The timing of the game is the following. In the first stage, firms make bundling decisions: No-bundling, Pure-bundling, and Mixed-bundling and these decisions are irreversible. In the second stage, firms compete in price. The proper solution concept for this two-stage game is Subgame Perfect Nash equilibrium.

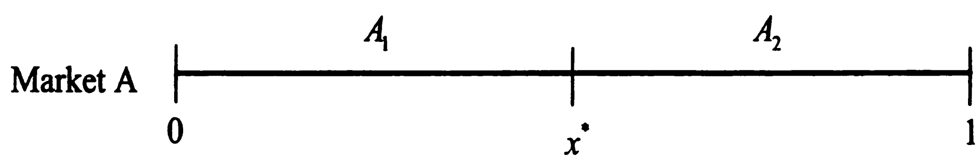
2.2.1 No Bundling

Under No Bundling, firm 1 sells A_1 separately from B_1. Hence, there are products A_1, A_2, B_1, B_2 available in the markets. In market A, firms compete in a Hotelling model. A consumer at x^* is indifferent between A_1 and A_2 if

$$\begin{aligned} & u - p_{A1} - tx^* = u - p_{A2} - t(1 - x^*) \\ & \Rightarrow \theta^* = \frac{1}{2t}(t + p_{A2} - p_{A1}). \end{aligned} \tag{2.1}$$

where p_{A1} and p_{A2} are the prices of A_1 and A_2, respectively.

Figure 2.1: No bundling in discrete valuations of good B



Firms 1 and A2 maximize the following profits function with respect to p_{A1} and

p_{A2} respectively.

$$\pi_1 = \frac{1}{2t}(p_{A1} - c_A)(t + p_{A2} - p_{A1}), \quad (2.2)$$
$$\pi_{A2} = \frac{1}{2t}(p_{A2} - c_A)(t - p_{A2} + p_{A1}). \quad (2.3)$$

In equilibrium, each firm has $\frac{1}{2}$ of market share. The equilibrium price and profits for firm 1 and A2 are as follows.

$$p^*_{A1} = p^*_{A2} = t + c_A, \quad (2.4)$$
$$\pi^*_1 = \pi^*_{A2} = \frac{t}{2}. \quad (2.5)$$

Since firms in market B compete in price with homogeneous products, the market price of good B is the marginal cost and each firm makes zero profit.

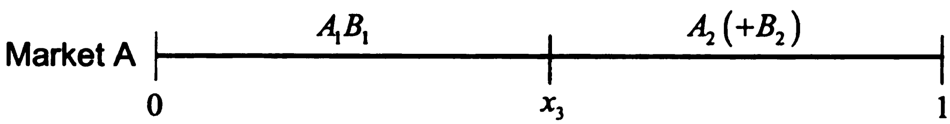
$$p_B{}^* = c_B, \quad (2.6)$$
$$\pi^*_1 = \pi^*_{B2} = 0. \quad (2.7)$$

2.2.2 Pure-Bundling

Firm 1 offers only the bundle of A_1B_1, but not A_1 and B_1 separately. In the markets, the bundle A_1B_1 and separate goods A_2 and B_2 are available. Since firm 1 offers only the bundle of A_1B_1, firm B2 is the only one producing good B. However, it cannot enjoy a monopoly profit since there exists the bundle of A_1B_1. In market A, firm A2 competes against the bundle A_1B_1. Since a positive number of consumers who have low valuations of good B switches to A_2 when A_1 is offered to them only as a package of A_1B_1, firm A2 can get benefits from pure bundling.

A consumer at x_3 is indifferent between A_1B_1 and $A_2 + B_2$ when $\tilde{p} + tx_3 = p_{A2} + p_{B2} + t(1 - x_3)$. $\tilde{p}$ is the price of the bundle of A_1B_1. p_{A2} and p_{B2} are the prices of A_2 and B_2 respectively.

Figure 2.2: Pure bundling in discrete valuation of good B



Considering heterogeneous consumer's valuations of good B: $\underline{v} = 0$, $\bar{v} = 1$, with $\lambda = \frac{1}{2}$, the demand for A_1B_1 is $\tilde{q} = \frac{1}{2t}(t + p_{A2} + \frac{1}{2}p_{B2} - \tilde{p})$, the demand for A_2 is $q_{A2} = \frac{1}{2t}(t - p_{A2} - \frac{1}{2}p_{B2} + \tilde{p})$, and the demand for B_2 is $q_{B2} = \frac{1}{4t}(t - p_{A2} - p_{B2} + \tilde{p})$. The profit functions are

$$\pi_1 = \frac{1}{2t}(\tilde{p} - c_A - c_B)(t + p_{A2} + \frac{1}{2}p_{B2} - \tilde{p}), \tag{2.8}$$

$$\pi_{A2} = \frac{1}{2t}(p_{A2} - c_A)(t - p_{A2} - \frac{1}{2}p_{B2} + \tilde{p}), \tag{2.9}$$

$$\pi_{B2} = \frac{1}{4t}(p_{B2} - c_B)(t - p_{A2} - p_{B2} + \tilde{p}). \tag{2.10}$$

There are three possible pricing policies for firm B2 to consider.

$$p_{B2} = \begin{cases} c_B \\ c_B < p_{B2} < 1 \\ 1 \end{cases}$$

I assume that if a firm cannot make a positive profit, its best pricing policy is to set the price equal to the marginal cost. In Table 2.1, I show the equilibrium outcomes of pure bundling.

Table 2.1: Equilibrium outcomes of Pure Bundling

	$p_{B2} = c_B$	$c_B < p_{B2} < 1$	$p_{B2} = 1$
p_{B2}		$\frac{1}{5}(3t + 4c_B)$	
$\tilde{p}$	$t + c_A + \frac{5}{6}c_B$	$\frac{11}{10}t + c_A + \frac{4}{5}c_B$	$t + c_A + \frac{1}{6} + \frac{2}{3}c_B$
p_{A2}	$t + c_A + \frac{1}{6}c_B$	$\frac{9}{10}t + c_A + \frac{1}{5}c_B$	$t + c_A - \frac{1}{6} + \frac{1}{3}c_B$
π_1^{PB}	$\frac{1}{2t}(t - \frac{1}{6}c_B)^2$	$\frac{1}{200t}(11t - 2c_B)^2$	$\frac{1}{2t}(t + \frac{1}{6} - \frac{1}{3}c_B)^2$
π_{A2}^{PB}	$\frac{1}{2t}(t + \frac{1}{6}c_B)^2$	$\frac{1}{200t}(9t + 2c_B)^2$	$\frac{1}{2t}(t - \frac{1}{6} + \frac{1}{3}c_B)^2$
π_{B2}^{PB}	0	$\frac{1}{100t}(3t - c_B)^2$	$\frac{1}{4t}(1 - c_B)(t - \frac{2}{3} + \frac{1}{3}c_B)$

(1) $p^*_{B2} = c_B$

This would be the case if $t < \frac{1}{3}c_B$.[5] When goods/services A are close substitutes such as $t < \frac{1}{3}c_B$, the competition between the bundle of A_1B_1 and $A_2 + B_2$ is so severe that the best strategy for firm B2 is to set $p^*_{B2} = c_B$. The integrated firm 1 has no incentive to offer bundling because when firm 1 only offers the bundle of A_1B_1, consumers who value product B less than $\tilde{p} - p_{A1}$ switch from firm 1 to firm A2. That is, the substitution effect of bundling arises in market A, but no competition softening effect of bundling occurs in market B.

(2) $c_B < p^*_{B2} < 1$

If $\frac{1}{3}c_B \leq t < \frac{5-4c_B}{3}$, firm B2 has an incentive to set $c_B < p^*_{B2} < 1$, which results in a positive competition softening effect of bundling in market B.[6] When $t \geq 2c_B$, firm 1 has an incentive to bundle its products. The condition ensures for firm 1 that competition softening effect of bundling dominates substitution effect of bundling that occurs in market A. Combining the two conditions, I can derive $2c_B \leq t < \frac{5-4c_B}{3}$, where $c_B < \frac{1}{2}$, in which firm 1 offers the bundle of A_1B_1 and firm B2 sets its price at a price higher than the marginal cost. On the other hand, firm A2 is worse off because it competes against the bundle of A_1B_1, rather than a separate good A_1. As p^*_{B2} is priced more than the marginal cost, firm A2 needs to lower p_{A2} in order

[5] $min\{\frac{1}{3}c_B, \frac{2-c_B}{3}\} = \frac{c_B}{3}$, where $0 < c_B < 1$

[6] $p^*_{B2} < 1$, if $t < \frac{5-4c_B}{3}$

to compete against A_1B_1. That is, with high magnitude of product differentiation degree, t, positive substitution effect of bundling, from a view point of firm A2, is dominated by negative competition softening effect of bundling. Note that the bundle price is lower than the sum of prices of two separate products: $\tilde{p} < p_{A2}^{PB} + p_{B2}^{PB}$, and that if $t \geq 2c_B$, the bundle price is higher than the sum of prices of two separate products in no bundling: $p_{A1}^{NB} + p_{B1}^{NB} \leq \tilde{p}$, but the price of firm A2 is less than that in no bundling: $p_{A2}^{PB} \leq p_{A2}^{NB}$. The following proposition summarizes the equilibrium outcomes of pure bundling.

Proposition 2.1 If $2c_B \leq t < \frac{5-4c_B}{3}$, where $c_B < \frac{1}{2}$, firm B2 has an incentive to set its price at $\frac{1}{5}(3t + 4c_B)$. Profits of firm 1 and B2 are higher in pure bundling than no bundling. However, profit of firm A2 is lower in pure bundling than in no bundling.

Proof: See appendix D.

The intuition behind proposition 2.1 is that bundling is profitable because the condition of $2c_B \leq t < \frac{5-4c_B}{3}$, where $c_B < \frac{1}{2}$, ensures for firm 1 that bundling yields a significant competition softening effect in comparison to a substitution effect. On the other hand, if $\frac{1}{3}c_B \leq t \leq 2c_B$, the opposite is true: bundling is not profitable because the substitution effect dominates the competition softening effect even when firm B2 sets its price higher than the marginal cost.

(3) $p_{B2}^* = 1$

Firm B2 has an incentive to set its price at 1 if $\frac{2-c_B}{3} \leq t$. The necessary condition for firm 1 to bundle is $c_B \leq \frac{1}{2}$. The condition of $c_B \leq \frac{1}{2}$ ensures for firm 1 that in market B, the positive competition softening effect of bundling dominates the negative substitution effect of bundling. Both firm 1 and B2 are better off in pure bundling than in no bundling whereas firm A2 is worse off because when $\frac{2-c_B}{3} \leq t$, from a view point of firm A2, there is little positive effect of bundling (substitution effect), but significant negative effect of bundling(competition softening effect).

Proposition 2.2 If $\frac{2-c_B}{3} \leq t$, where $c_B \leq \frac{1}{2}$, firm B2 has an incentive to set its price at 1. Profits of firm 1 and B2 in pure bundling are higher than in no bundling while that of firm A2 is lower than in no bundling.

Proof: See appendix D.

The idea behind proposition 2.2 is the same as in the previous case of $c_B < p^*_{B2} < 1$. Pure bundling is optimal only when products A are somewhat differentiated so that bundling lessens competition in market B significantly enough to cover the costs that result from heterogeneous consumers' valuations of good B.

2.2.3 Mixed-Bundling

There are three types of mixed bundling strategies for firm 1 to choose: $\{A_1, A_1B_1\}$, $\{B_1, A_1B_1\}$, and $\{A_1, B_1, A_1B_1\}$. If firm 1 offers $\{B_1, A_1B_1\}$, or $\{A_1, B_1, A_1B_1\}$, there is no competition softening effect of bundling in market B. The results of these mixed bundling are the same as the case of no bundling. Therefore, our interests are in the partial mixed bundling of $\{A_1, A_1B_1\}$. Firm 1 offers A_1 and A_1B_1 in this subgame. By offering A_1 as well as A_1B_1, firm 1 holds consumers with valuations of good B less than $\tilde{p} - p_{A1}$ who otherwise may switch to A_2 in pure bundling. However, firm 1 competes against itself: A_1B_1 competes against $A_1 + B_2$. Firm 1 has to satisfy the condition of $\tilde{p} \leq p_{A1} + p_{B2}$ in order to have positive sales of A_1B_1. A consumer at x_3 is indifferent between A_1B_1 and $A_2 + B_2$ when

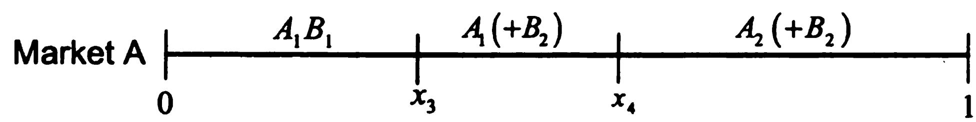
$$\tilde{p} + tx_3 = p_{A2} + p_{B2} + t(1 - x_3) \tag{2.11}$$

And a consumer at x_4 is indifferent between $A_1 + B_2$ and $A_2 + B_2$ when

$$p_{A1} + p_{B2} + tx_4 = p_{A2} + p_{B2} + t(1 - x_4) \tag{2.12}$$

From (2) and (3) with discrete consumer's valuations of good B, I can derive $\hat{x}_3 = \frac{1}{2t}\{t + p_{A2} + \frac{1}{2}p_{B2} - \tilde{p}\}$ and $x_4 = \frac{1}{2t}\{t + p_{A2} - p_{A1}\}$. In addition to market share in pure bundling, firm 1 can gain additional market share of $x_4 - \hat{x}_3$ by setting prices such as $p_{A1} + \frac{1}{2}p_{B2} \leq \tilde{p} \leq p_{A1} + p_{B2}$. The idea is that mixed bundling permits firm 1 to keep those who may otherwise switch to A_2 when A_1 is offered to them only as a package of A_1B_1.

Figure 2.3: Mixed bundling in discrete valuations of good B



Firm 1 has positive demands for both A_1B_1 and A_1, $\tilde{q} = \frac{1}{2t}(t + p_{A2} + \frac{1}{2}p_{B2} - \tilde{p})$ and $q_{A1} = \frac{1}{2t}(\tilde{p} - p_{A1} - \frac{1}{2}p_{B2})$, respectively. Demands for firm A2 and B2 are $q_{A2} = \frac{1}{2t}(t - p_{A2} + p_{A1})$ and $q_{B2} = \frac{1}{4t}(t - p_{A2} - p_{B2} + \tilde{p})$, respectively. The profit functions are

$$\begin{aligned}
\pi_1 &= \frac{1}{2t}(\tilde{p} - c_A - c_B)(t + p_{A2} + \frac{1}{2}p_{B2} - \tilde{p}) \\
&\quad + \frac{1}{2t}(p_{A1} - c_A)(\tilde{p} - p_{A1} - \frac{1}{2}p_{B2}), \quad (2.13) \\
\pi_{A2} &= \frac{1}{2t}(p_{A2} - c_A)(t - p_{A2} + p_{A1}), \quad (2.14) \\
\pi_{B2} &= \frac{1}{4t}(p_{B2} - c_B)(t - p_{A2} - p_{B2} + \tilde{p}). \quad (2.15)
\end{aligned}$$

As in pure bundling, there are three possible pricing policies for firm B2 to consider. Table 2.2 shows equilibrium outcomes.

Table 2.2: Equilibrium outcomes of Mixed Bundling

	$p_{B2} = c_B$	$c_B < p_{B2} < 1$	$p_{B2} = 1$
p_{B2}		$\frac{7}{9}t + \frac{8}{9}c_B$	
$\tilde{p}$	$\frac{6}{5}t + c_A + \frac{9}{10}c_B$	$\frac{23}{18}t + c_A + \frac{8}{9}c_B$	$\frac{6}{5}t + c_A + \frac{1}{10} + \frac{4}{5}c_B$
p_{A1}	$\frac{3}{5}t + c_A + \frac{1}{5}c_B$	$\frac{4}{9}t + c_A + \frac{2}{9}c_B$	$\frac{3}{5}t + c_A - \frac{1}{5} + \frac{2}{5}c_B$
p_{A2}	$\frac{4}{5}t + c_A + \frac{1}{10}c_B$	$\frac{13}{18}t + c_A + \frac{1}{9}c_B$	$\frac{4}{5}t + c_A - \frac{1}{10} + \frac{1}{5}c_B$
π_1^{MB}	$\frac{3}{50t}(6t - \frac{1}{2}c_B)(t - \frac{1}{2}c_B)$ $+\frac{1}{2t}(\frac{3}{5}t + \frac{1}{5}c_B)^2$	$\frac{1}{54t}(\frac{23}{2}t - c_B)(\frac{5}{2}t - c_B)$ $+\frac{1}{2t}(\frac{4}{9}t + \frac{2}{9}c_B)^2$	$\frac{3}{50t}(6t - c_B + \frac{1}{2})(t - c_B + \frac{1}{2})$ $+\frac{1}{2t}(\frac{3}{5}t + \frac{2}{5}c_B - \frac{1}{5})^2$
π_{A2}^{MB}	$\frac{1}{2t}(\frac{4}{5}t + \frac{1}{10}c_B)^2$	$\frac{1}{2t}(\frac{13}{18}t + \frac{1}{9}c_B)^2$	$\frac{1}{2t}(\frac{4}{5}t - \frac{1}{10} + \frac{1}{5}c_B)^2$
π_{B2}^{MB}	0	$\frac{1}{4t}(\frac{7}{9}t - \frac{1}{9}c_B)^2$	$\frac{1}{4t}(1 - c_B)(\frac{7}{5}t - \frac{4}{5} + \frac{3}{5}c_B)$

(1) $p^*_{B2} = c_B$

If $t < \frac{1}{7}c_B$, firm B2 would set $p^*_{B2} = c_B$.[7] When $t \leq \frac{1}{2}c_B$ or $\frac{7}{4}c_B \leq t$, firm 1 gains more in mixed bundling than in no bundling even in the absence of competition softening effect, $p^*_{B2} = c_B$, in market B. The reason is that mixed bundling minimizes the substitution effect and the bundle option softens competition in market A. When $t \leq \frac{1}{7}c_B$, firm A2 also makes a higher profit than in no bundling due to interdependence in prices of two independent goods A and B while there is no negative competition softening effect of bundling in market B.[8] I can check that in equilibrium, the bundle price is less than the sum of prices of separate goods: $\tilde{p} \leq p_{A1}^{MB} + p_{B2}^{MB}$, where $t < \frac{1}{7}c_B$. It also holds that $p_{A1} + \frac{1}{2}p_{B2} < \tilde{p}$.

Proposition 2.3 If $t < \frac{1}{7}c_B$, (i) Firm B2 sets its price at the marginal cost c_B. Both firms 1 and A2 gain more in mixed bundling than in no bundling. (ii) Firm 1 is better off in mixed bundling than in pure bundling. Firm A2 makes less in mixed bundling than in pure bundling.

Proof: See appendix D.

When products A are close substitutes, i.e. $t < \frac{1}{7}c_B$, mixed bundling permits

[7] $min\{\frac{c_B}{7}, \frac{4-3c_B}{7}\} = \frac{1}{7}c_B$, $0 < c_B < 1$

[8] If $t \leq \frac{1}{2}c_B$, $p_{A1}^{NB} \leq p_{A1}^{MB} \Longleftrightarrow t + c_A \leq \frac{3}{5}t + c_A + \frac{1}{5}c_B$; $p_{A2}^{NB} \leq p_{A2}^{MB} \Longleftrightarrow t + c_A \leq \frac{4}{5}t + c_A + \frac{1}{10}c_B$.

little substitution effect and hence is preferred to either no bundling or pure bundling by firm 1. Firm A2 is also better off in mixed bundling than no bundling due to the competition softening effect of bundling in market A.

(2) $c_B < p^*_{B2} < 1$

If $\frac{1}{7}c_B \leq t < \frac{9-8c_B}{7}$, firm B2 has an incentive to set its price at $p^*_{B2} = \frac{1}{9}(7t+8c_B)$ and gains a positive profit. Firm 1 can gain higher profits in mixed bundling than in no bundling if $t \leq \frac{2}{5}c_B$ or $\frac{14}{17}c_B \leq t$. Firm A2 also gains a higher profit in mixed bundling than in no bundling if $t \leq \frac{2}{5}c_B$ due to interdependence of two goods in price (competition softening effect), $p^{NB}_{A1} \leq p^{MB}_{A1}$ and $p^{NB}_{A2} \leq p^{MB}_{A2}$ if $t \leq \frac{2}{5}c_B$. However, if $t \geq \frac{14}{17}c_B$, firm A2 is worse off in mixed bundling than in no bundling due to the competition softening effect of bundling in market B. In order to have positive sales of the bundle, it must hold that $\tilde{p} \leq p_{A1} + p_{B2}$, where $t \leq 4c_B$. Therefore, We need $\frac{1}{7}c_B \leq t < \frac{2}{5}c_B$ or $\frac{14}{17}c_B \leq t < \min\{4c_B, \frac{9-8c_B}{7}\} = t_1$, where $c_B < \frac{17}{26}$.

Proposition 2.4 If $\frac{1}{7}c_B \leq t \leq \frac{2}{5}c_B$, or If $\frac{14}{17}c_B \leq t < \min\{4c_B, \frac{9-8c_B}{7}\}$, where $c_B < \frac{17}{26}$, firm B2 sets its price at $p^*_{B2} = \frac{1}{9}(7t + 8c_B)$, making positive profits and firm 1 makes higher profits in mixed bundling than in no bundling. Firm A2 gains less profits in mixed bundling than in no bundling if $\frac{14}{17}c_B \leq t < \min\{4c_B, \frac{9-8c_B}{7}\}$, where $c_B < \frac{17}{26}$, but higher profits if $\frac{1}{7}c_B \leq t \leq \frac{2}{5}c_B$.

Proof: See appendix D.

The idea behind Proposition 2.4 is that both firms gain more in mixed bundling than no bundling when products A are close substitutes. The reason is that both firms can enjoy positive competition softening effect of bundling in market A. On the other hand, when products A are sufficiently differentiated, i.e. $\frac{14}{17}c_B \leq t < t_1$, firm A2 is worse off in mixed bundling than in no bundling because it needs to decrease its price far more than what it can benefit from the competition softening effect of bundling in market A.

There are intervals where mixed bundling is more profitable than no bundling, while pure bundling is not. For example, firm 1 has no incentive to offer pure bundling in the interval of $\frac{1}{7}c_B \leq t < \frac{2}{5}c_B$ or $\frac{14}{17}c_B \leq t < 2c_B$ where mixed bundling is preferred, by firm 1, to no bundling.

Proposition 2.5 If $\frac{1}{7}c_B \leq t \leq \frac{2}{5}c_B$ or $\frac{14}{17}c_B \leq t < t_1$, where $t_1 = \min\{4c_B, \frac{9-8c_B}{7}\}$, $c_B < \frac{17}{26}$, firm 1 is better off in mixed bundling than in pure bundling while firm A2 gains less profit in mixed bundling than in pure bundling. Firm B2 makes higher profits in mixed bundling than in pure bundling if $\frac{7}{31}c_B \leq t \leq \frac{2}{5}c_B$ or if $\frac{14}{17}c_B \leq t < \min\{4c_B, \frac{9-8c_B}{7}\}$, where $c_B < \frac{17}{26}$, while it makes more profits in pure bundling than mixed bundling if $\frac{1}{7}c_B \leq t \leq \frac{7}{31}c_B$.

Proof: See appendix D.

(3) $p^*_{B2} = 1$

When $\frac{4-3c_B}{7} \leq t$, firm B2 sets $p^*_{B2} = 1$. Firm 1 has an incentive to offer mixed bundling if $c_B \leq \frac{1}{2}$. Unlike in pure bundling, firm 1 also prefers mixed bundling to no bundling if $t \leq c_B - \frac{1}{2}$ or $\frac{7}{2}(c_B - \frac{1}{2}) \leq t$, where $c_B > \frac{1}{2}$, due to significant competition softening effect of bundling. Firm 1 has a positive demand for the bundle, $\tilde{p} \leq p_{A1} + p_{B2}$, if $t \leq \frac{7-4c_B}{6}$. The necessary condition for $p_{A1} + \frac{1}{2}p_{B2} \leq \tilde{p}$ is $t \geq \max\{\frac{1-2c_B}{3}, 0\}$. This holds under $\frac{4-3c_B}{7} \leq t \leq \frac{7-4c_B}{6}$.

Proposition 2.6 (i) (a) If $\frac{4-3c_B}{7} \leq t \leq \frac{7-4c_B}{6}$, where $c_B \leq \frac{1}{2}$, (b) If $t_2 \leq t \leq \frac{7-4c_B}{6}$, where $t_2 = \max\left\{\frac{4-3c_B}{7}, \frac{7(2c_B-1)}{4}\right\}$, $\frac{1}{2} < c_B \leq \frac{7}{10}$, or (c) If $\frac{4-3c_B}{7} \leq t \leq c_B - \frac{1}{2}$, where $\frac{3}{4} \leq c_B$, firm B2 has an incentive to set its price at 1, making positive profits in mixed bundling. Firm 1 makes higher profits in mixed bundling than in no bundling while firm A2 gains less in mixed bundling than in no bundling except the case of (c). (ii) If $t_3 \leq t \leq \frac{7-4c_B}{6}$, where $t_3 = \max\left\{\frac{4-3c_B}{7}, \frac{19(1-2c_B)}{12}\right\}$, $\frac{1}{6} \leq c_B < \frac{1}{2}$, or If $\frac{4-3c_B}{7} \leq t \leq \frac{7-4c_B}{6}$, where $\frac{1}{2} \leq c_B$, firm 1 is better off in mixed bundling than in pure bundling. Firm A2 gains less in mixed bundling than in pure bundling, while firm B2 makes higher profits in mixed bundling than in pure bundling.

Proof: See appendix D.

Proposition 2.6 implies that mixed bundling has business stealing effect in the sense that firm 1 gains higher profits in mixed bundling than in either no bundling or pure bundling at the expenses of firm A2. Mixed bundling is preferred, by firm 1, to pure bundling even when the marginal cost of good B is significant, i.e. $\frac{1}{2} \leq c_B$, unlike in pure bundling. The reason is that mixed bundling allows little substitution effect. High competition softening effect in comparison to substitution effect of bundling, from a view point of firm 1, occurs when products A are highly differentiated. This can also occur when products A are close substitutes if the marginal cost of good B is high.

2.2.4 Sub-game perfect equilibrium

Mixed bundling may arise as a sub-game perfect equilibrium. When products A are close substitutes, i.e. $t < \frac{1}{7}c_B$ in which firm B2 sets its price at the marginal cost, firm 1 is better off in mixed bundling than in pure bundling. The reason is that mixed bundling dampens substitution effect of bundling. When $\frac{1}{7}c_B < t < \frac{2}{5}c_B$ or $\frac{14}{17}c_B \leq t < \min\left\{4c_B, \frac{9-8c_B}{7}\right\} = t_1$, where $c_B \leq \frac{17}{26}$, in which firm B2 has an incentive to set its price at $c_B < p^*_{B2} < 1$, firm 1 also makes higher profits in mixed bundling than in pure bundling due to competition softening effects in both markets and reduced substitution effect in market A. When $\frac{4-3c_B}{7} \leq t \leq \frac{7-4c_B}{6}$, where $\frac{1}{2} \leq c_B$ or when $t_3 \leq t \leq \frac{7-4c_B}{6}$, where $t_3 = \max\left\{\frac{4-3c_B}{7}, \frac{19(1-2c_B)}{12}\right\}$, $\frac{1}{6} \leq c_B < \frac{1}{2}$, in which firm B2 has an incentive to set its price at $p_{B2} = 1$, mixed bundling is preferred, by firm 1, to pure bundling.

With mixed bundling, firm B2 gains a higher profit than in pure bundling except in the interval of $\frac{1}{7}c_B \leq t \leq \frac{7}{31}c_B$. Note also that firm A2 is better in pure bundling than mixed bundling. In addition, it prefers no bundling to mixed bundling unless $t \leq \frac{2}{5}c_B$ or $\frac{4-3c_B}{7} \leq t \leq c_B - \frac{1}{2}$, where $\frac{3}{4} \leq c_B$. This implies that firm 1 may have an incentive to offer mixed bundling in order to deter entry. In the previous example, local exchange carriers might be interested in offering local phone service separately as well as a package of Internet access service and local phone service, but not Internet service alone, to deter an entry into local service.

2.3 Model with continuous valuations of good B

In this section, I consider a case in which the consumer's valuation of good B is uniformly distributed, $v \sim Unif[0,1]$. I follow the assumptions made in the previous model where consumers have discrete valuations of good B in exception of continuous valuations of good B.

2.3.1 No-Bundling

This is the case where firm 1 sells goods A_1 and B_1 separately. Hence, equilibrium outcomes of firms in market A and B are the same as in the model with discrete valuations of good B.

2.3.2 Pure bundling

Firm 1 offers only the bundle of A_1B_1. A consumer at x_5 is indifferent between A_1B_1 and A_2+B_2 when $\tilde{p}+tx_5 = p_{A2}+p_{B2}+t(1-x_5)$. $\tilde{p}$ is the price of A_1B_1. p_{A2} and p_{B2} are the prices of A_2 and B_2, respectively. Considering consumers' valuations of good B: $v \sim Unif[0,1]$, the demand functions for the bundle, A_2 and B_2, are

$$\tilde{q} = \int_0^1 x_5^*(v)dv = \frac{1}{2t}\left(t + p_{A2} - \tilde{p} + p_{B2} - \frac{1}{2}p_{B2}^2\right), \tag{2.16}$$

$$q_{A2} = 1 - \tilde{q} = \frac{1}{2t}(t - p_{A2} + \tilde{p} - p_{B1} + \frac{1}{2}p_{B2}^2), \tag{2.17}$$

$$q_{B2} = (1 - x_5^*(p_{B2}))(1 - p_{B2}) = \frac{1}{2t}(t - p_{A2} + \tilde{p} - p_{B2})(1 - p_{B2}). \tag{2.18}$$

where $x_5^*(v) = \frac{1}{2t}(t + p_{A2} + \min\{p_{B2}, v\} - \tilde{p})$. Clearly, firm B2 has no incentive to set $p_{B2}^* = 1$. The profit functions are as follows.

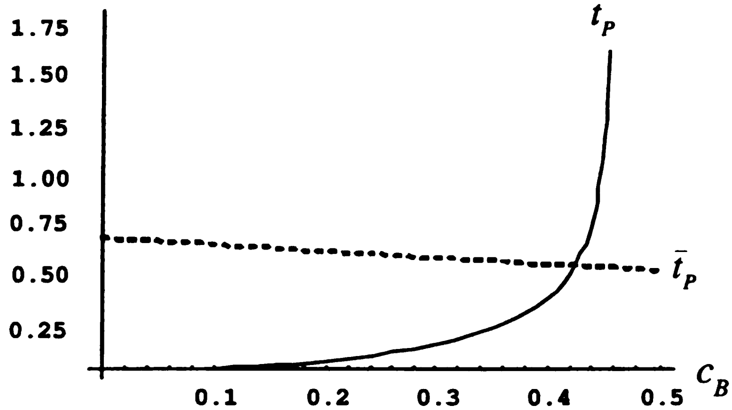
$$\pi_1 = \frac{1}{2t}(\tilde{p} - c_A - c_B)\left(t + p_{A2} - \tilde{p} + p_{B2} - \frac{1}{2}p_{B2}^2\right), \tag{2.19}$$

$$\pi_{A2} = \frac{1}{2t}(p_{A2} - c_A)\left(t - p_{A2} + \tilde{p} - p_{B2} + \frac{1}{2}p_{B2}^2\right), \tag{2.20}$$

$$\pi_{B2} = \frac{1}{2t}(p_{B2} - c_B)(t - p_{A2} + \tilde{p} - p_{B2})(1 - p_{B2}). \tag{2.21}$$

I do simulation analysis for these outcomes since they are rather messy in expression. In appendix C, I show profits of firms when firm 1 offers only the bundle of A_1B_1. Even though I do not have closed forms of the equilibrium outcomes, I can show conditions where firm B2 sets its price $c_B < p^*_{B2} < 1$ and firm 1 offers pure bundling (see Figure 2.4).

Figure 2.4: Pure bundling in continuous valuations of good B



Proposition 2.7 If $t_p < t < \bar{t}_p$, optimal price of firm B2 is at $c_B < p^*_{B2} < 1$. Firm 1 and B2 make higher profits in pure bundling than in no bundling whereas firm A2 gains less profits in pure bundling than in no bundling.

$$t_p = \frac{8c_B + 4(1-c_B)\sqrt{1-2c_B} - 4 - c_B^2}{2\sqrt{1-2c_B} - (1-c_B)},$$
$$\bar{t}_p = \frac{2-c_B}{3}, \quad where \ c_B < \frac{1}{2}.$$

Proof: From the profit functions of firm 1, $\pi_1^{PB} \geq \pi_1^{NB}$, if $\alpha \geq c_B$, where $\alpha = p^*_{B2} - \frac{1}{2}(p^*_{B2})^2$, p^*_{B2} is the optimal price of good B: $p^*_{B2} - \frac{1}{2}(p^*_{B2})^2 \geq c_B$. The equality of $p^*_{B2} - \frac{1}{2}(p^*_{B2})^2 \geq c_B$ holds at $1 - \sqrt{1-2c_B} = \bar{p}$, where $c_B < \frac{1}{2}$.[9] So, the condition of $\bar{p} < p^*_{B2}$ is needed to hold in order for firm 1 to bundle. By substituting $\bar{p}$ into the F.O.C. of π_{B2}, I can find a condition for $p^*_{B2} > \bar{p}$: (i) $t > \frac{8c_B+4(1-c_B)\sqrt{1-2c_B}-4-c_B^2}{2\sqrt{1-2c_B}-(1-c_B)} = t_p$. Now let's find conditions for firm B2 to set its price at $c_B < p^*_{B2} < 1$ to make positive profits. Firm B2 has a positive profit

[9]Note that $c_B < \bar{p}$.

margin, $p^*_{B2} - c_B > 0$, if $\bar{p} < p^*_{B2}$ because $c_B < \bar{p}$. (ii) In order for firm B2 to sell a positive amount of B_2, $q_{B2} = (1 - x^*(p^*_{B2})(1 - p^*_{B2}) > 0$, it needs to hold that $p^*_{B2} < \hat{p}$, where $\hat{p} = min\{\frac{1}{2}(\sqrt{1 + 4(3t + c_B)} - 1),\ 1\}$. Hence, combining the two conditions for pure bundling equilibrium, it needs to hold that $\bar{p} < p^*_{B2} < \hat{p}$. This holds if $t_p < t < \frac{2-c_B}{3} = \bar{t}_p$, where $c_B < \frac{1}{2}$.

Pure bundling vs No bundling The intuition behind proposition 2.7 is the same as in the previous case of discrete valuations of good B. Bundling has a substitution effect in market A and competition softening effects in both markets A and B. When products A are close substitutes: i.e. $t < t_p = \frac{8c_B + 4(1-c_B)\sqrt{1-2c_B} - 4 - c_B{}^2}{2\sqrt{1-2c_B} - (1-c_B)}$, competition between A_1B_1 and $A_2 + B_2$ is so severe that firm B2 is less willing to increase its price while pure bundling causes substitution effect due to heterogenous customers' valuations of good B. In this case, firm 1 has no incentive to offer pure bundling. On the other hand, if products A are sufficiently differentiated, i.e. $t > \bar{t}_p$, in which substitution effect is dominated by the competition softening effects of bundling, firm 1 has an incentive to offer pure bundling. I compare profits of firms in pure bundling with those in no bundling in Appendix E.

2.3.3 Mixed bundling

Firm 1 offers A_1 as well as the bundle of A_1B_1. There are A_1, A_2, B_2, and A_1B_1 in the market. A consumer at x_5 is indifferent between A_1B_1 and $A_2 + B_2$ when

$$\tilde{p} + tx_5 = p_{A2} + p_{B2} + t(1 - x_5) \Rightarrow x_5 = \frac{1}{2t}(t + p_{A2} + p_{B2} - \tilde{p}),$$

And a consumer at x_6 is indifferent between $A_1 + B_2$ and $A_2 + B_2$ when

$$p_{A1} + p_{B2} + tx_6 = p_{A2} + p_{B2} + t(1 - x_6) \Rightarrow x_6 = \frac{1}{2t}(t + p_{A2} - p_{A1}),$$

And, to sell a positive amount of A_1B_1, the bundle price is less than the sum of the two separate products: $\tilde{p} \leq p_{A1} + p_{B2}$. With continuous valuations of good

B, $\hat{x}_5 = \int_0^1 x_5(v)dv = \frac{1}{2t}(t + p_{A2} - \tilde{p} + p_{B2} - \frac{1}{2}p_{B2}^2)$, where $x_5(v) = \frac{1}{2t}(t + p_{A2} + \min\{p_{B2}, v\} - \tilde{p})$. By offering A_1 as well as A_1B_1, firm 1 can gain an additional market share of $x_6 - \hat{x}_5$ with $p_{A1} + p_{B2} - \frac{1}{2}p_{B2}^2 \leq \tilde{p} \leq p_{A1} + p_{B2}$. The demand for the bundle is $\tilde{q} = \int_0^1 x_5(v)dv = \frac{1}{2t}\left(t + p_{A2} - \tilde{p} + p_{B2} - \frac{1}{2}p_{B2}^2\right)$, demand for A_1 is $q_{A1} = x_6 - \int_0^1 x_5(v)dv = \frac{1}{2t}\left(\tilde{p} - p_{A1} - p_{B2} + \frac{1}{2}p_{B2}^2\right)$, demand for firm A2 is $q_{A2} = 1 - \tilde{q} - q_{A1} = \frac{1}{2t}(t - p_{A2} + p_{A1})$, firm B2 has a market share of $q_{B2} = (1 - x_5(p_{B2}))(1 - p_{B2}) = \frac{1}{2t}(t - p_{A2} + \tilde{p} - p_{B2})(1 - p_{B2})$. Clearly, firm B2 has no incentive to set $p_{B2} = 1$ in order to sell positive amounts of B_2. The profit functions are as follows.

$$\begin{aligned}
\pi_1 &= \frac{1}{2t}\left[(\tilde{p} - c_A - c_B)\left(t + p_{A2} - \tilde{p} + p_{B2} - \frac{1}{2}p_{B2}^2\right)\right] \\
&\quad + \frac{1}{2t}\left[(p_{A1} - c_A)\left(\tilde{p} - p_{A1} - p_{B2} + \frac{1}{2}p_{B2}^2\right)\right], && (2.22) \\
\pi_{A2} &= \frac{1}{2t}(p_{A2} - c_A)(t - p_{A2} + p_{A1}), && (2.23) \\
\pi_{B2} &= \frac{1}{2t}(p_{B2} - c_B)(t - p_{A2} + \tilde{p} - p_{B2})(1 - p_{B2}). && (2.24)
\end{aligned}$$

In Appendix E, I do simulation analysis when firm 1 offers A_1 and A_1B_1.

Example 1. (Mixed bundling vs No bundling) Simulation analysis shows us that firm 1 has an incentive to offer mixed bundling, i.e. when products A are sufficiently differentiated with small marginal cost of good B. It is also preferred to no bundling even when products A are close substitutes if the marginal cost of good B is high. Let's suppose that firm B2 sets $p_{B2}^* = c_B$. If $t \leq \frac{1}{2}c_B^2$ or $\frac{7}{4}c_B^2 \leq t$, mixed bundling is preferred, by firm 1, to no bundling even when $p_{B2}^* = c_B$. From pure bundling case, I know that bundling is profitable if $t_p < t$. Hence, if $t \leq t_p$, pure bundling cannot arise in equilibrium while this example suggests that mixed bundling may arise in equilibrium, i.e. $t \leq \frac{1}{2}c_B^2$ (see Figure 2.5).

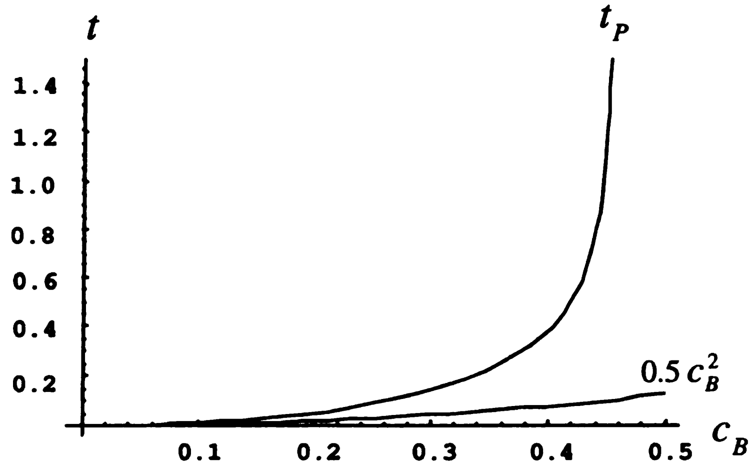
Proof: We know that $\pi_1^{MB} - \pi_1^{NB} \geq 0$ if $t + \frac{9}{2}(\beta - c_B) + \frac{7}{2t}(\beta - c_B)^2 \geq 0$, where $\beta = p_{B2} - \frac{1}{2}p_{B2}^2$. Suppose that $p_{B2}^* = c_B$. Then it becomes $t - \frac{9}{4}(c_B)^2 t + \frac{7}{8t}(c_B)^4 \geq 0$.

So, if $t \le \frac{1}{2}c_B^2$, or $\frac{7}{4}c_B^2 \le t$, $\pi_1^{MB} \ge \pi_1^{NB}$ even when $p^*_{B2} = c_B$.

$$\tilde{p} = \frac{6}{5}t + c_A + \frac{4}{5}c_B + \frac{1}{5}\beta,\ p_{A1} = \frac{3}{5}t + c_A + \frac{2}{5}c_B - \frac{2}{5}\beta,\ p_{A2} = \frac{4}{5}t + c_A + \frac{1}{5}c_B - \frac{1}{5}\beta.$$
$$\pi_1 = \frac{1}{50t}((6t + (\beta - c_B))(3t + 3(\beta - c_B)) + (3t - 2(\beta - c_B))^2),$$
$$\pi_{A2}^{MB} = \frac{1}{50t}(4t - (\beta - c_B))^2.$$

where $\beta = p^*_{B2} - \frac{1}{2}(p^*_{B2})^2$, p^*_{B2} is the optimal price of good B_2.

Figure 2.5: Mixed bundling vs No bundling



Example 2. (Mixed bundling vs Pure bundling) Now, let's illustrate an example where mixed bundling may arise in equilibrium while pure bundling may not. If $p^*_{B2} \ge 1 - \sqrt{1 - 2c_B} = \bar{p}$, where $c_B < \frac{1}{2}$, firm 1 has an incentive to offer mixed bundling.[10] When $p^*_{B2} = \bar{p}$, then $\beta - c_B = 0$, where $\beta = p^*_{B2} - \frac{1}{2}(p^*_{B2})^2$, p^*_{B2} is the optimal price of good B_2. Note also that the F.O.C. of π_{B2}^{MB} increases at $\bar{p}$, $p^*_{B2} > \bar{p}$, if $t \ge t_M$, where $\frac{5}{7}\left(\frac{8c_B + 4(1-c_B)\sqrt{1-2c_B} - 4 - c_B^2}{2\sqrt{1-2c_B} - (1-c_B)}\right) = \frac{5}{7}t_p = t_M$, where $c_B < \frac{1}{2}$. Now let's find some intervals where firm B2 sets its price at $c_B < p^*_{B2} < 1$ to make positive profits. Firm B2 has a positive profit margin, $p^*_{B2} - c_B > 0$, if $\bar{p} < p^*_{B2}$ because

[10] From example 1, $\pi_1^{MB} - \pi_1^{NB} \ge 0$ if $t + \frac{9}{2}(\beta - c_B) + \frac{7}{2t}(\beta - c_B)^2 \ge 0$, where $\beta = p_{B2} - \frac{1}{2}p_{B2}^2$. Hence, if $p^*_{B2} \le 1 - \sqrt{1 - 2c_B + 2t}$ or $1 - \frac{\sqrt{7}}{7}\sqrt{7 - 14c_B + 4t} \le p^*_{B2}$, firm 1 makes higher profits in mixed bundling than in no bundling. The conditions for mixed bundling equilibrium are rather messy in expression. Hence, I show examples where mixed bundling is preferred to either pure bundling or no bundling. Note that if $p^*_{B2} \ge 1 - \sqrt{1 - 2c_B} = \bar{p}$, firm 1 has an incentive to offer mixed bundling because $\left(1 - \frac{\sqrt{7}}{7}\sqrt{7 - 14c_B + 4t}\right) < \bar{p}$.

$c_B < \bar{p}$. In order for firm B2 to sell a positive amount of B_2, it needs to hold that $p^*_{B2} < \ddot{p}$, where $\ddot{p} = min\{\frac{1}{2}(\sqrt{9+28t+12c_B} - 3),\ 1\}$. Hence, combining the two conditions for mixed bundling equilibrium, it needs to hold that $\bar{p} < p^*_{B2} < \ddot{p}$. This holds if $t_M < t < \frac{4-3c_B}{7} = \bar{t}_M$, where $c_B < \frac{1}{2}$. Therefore, if $t_M \leq t < \bar{t}_M$, where $c_B < \frac{1}{2}$, firm B2 has an incentive to set $\bar{p} < p^*_{B2} < \ddot{p}$ and firm 1 gains more in mixed bundling than in no bundling. Remember that the necessary condition for firm 1 to offer pure bundling is $t_p < t$, where $c_B < \frac{1}{2}$. This example suggests that there are some intervals where mixed bundling is profitable, but not pure bundling, due to a large substitution effect of bundling compared to competition softening effects of bundling: i.e. $t_M \leq t < t_p$. I compare profits of firms in mixed bundling with those in pure bundling in appendix C. Mixed bundling is preferred to pure bundling when the marginal cost c_B is high, resulting in both significant substitution effect and competition softening effect of bundling. Firm B2 is better off in mixed bundling when marginal cost is small while it prefers pure bundling to mixed bundling when marginal cost is high. Firm A2 is worse off in mixed bundling than in pure bundling.

2.3.4 Sub-game perfect equilibrium

If $t_p < t < \bar{t}_p$, where $c_B < \frac{1}{2}$, pure bundling may arise in equilibrium. I illustrated that mixed bundling is a profitable strategy while pure bundling is not in some intervals such as $t_M < t < t_p$, where $c_B < \frac{1}{2}$. I also find that firm 1 has an incentive to offer mixed bundling even when firm B2 sets its price at $p^*_{B2} = c_B$. Simulation analysis shows that mixed bundling is preferred to pure bundling when there exists a competition softening effect, i.e. high c_B, and a significant substitution effect of bundling, i.e. high marginal cost of good B and small degree of product differentiation.

2.4 Conclusions

This paper studies strategic use of mixed bundling in duopoly where there is a multi-product firm and an independent firm in each market. I show that in duopoly, mixed bundling may arise in equilibrium like in a monopoly model where mixed bundling dominates pure bundling when consumers' valuations of two goods are not perfectly correlated. In monopoly, the bundle is priced to extract all the surplus of consumers who value both goods highly, while the separate markets are used to extract the surplus of consumers who value one high, but the other low. On the other hand, in the previous duopoly models, pure bundling dominates mixed bundling because firms fear the extra degree of competition inherent in mixed bundling.

In this paper, I show that an integrated firm strategically uses the mixed bundling strategy in a market where bundling has two effects: substitution effect and competition softening effect. Substitution effect of bundling arises from a situation where consumers in market B have heterogeneous valuations of good B. If firm 1 only sells the bundle of A_1B_1, consumers of firm 1 who value good B less than the bundle price minus the price of A_1 switch to firm A2. This effect is significant when products A are close substitutes and/or the marginal cost of good B is high. On the other hand, competition softening effects of bundling occur in both markets A and B, resulting from interdependence in price of two independent goods and production differentiation role of bundling. I show that mixed bundling is preferred, by the integrated firm 1, to pure bundling not only when products A are close substitutes, but also when products A are sufficiently differentiated due to competition softening effects of bundling. Firm 1 gains more profits in either mixed bundling or pure bundling at the expense of firm A2. Firm A2 is worse off in either mixed bundling or pure bundling than in no bundling due to the interdependence of two goods in price. Our result has an important implication on anti-trust policies. When $\pi_{A2}^{PB} < F < \pi_{A2}^{NB}$ and $\pi_{A2}^{MB} < F < \pi_{A2}^{NB}$, where F is an entry cost, mixed bundling can be used as an entry barrier in a market where pure bundling is not allowed by an anti-trust authority. For instance, local exchange carriers might be interested in offering a package of Internet access service and local service, but not Internet service alone, to deter an entry into local service. Therefore, mixed bundling in duopoly should be regarded with suspicion, like pure bundling.

Chapter 3

Vertical Integration and Firewalls with R&D Spillovers

3.1 Introduction

Vertical integrations are mergers of noncompeting companies where one's good is a complementary good to the other's. Such mergers can achieve procompetitive efficiency benefits in the sense that vertical integration can reduce costs or enhance innovation efforts through several ways. When both the input and output markets are imperfectly competitive, a vertical merger of a firm with an input supplier can get efficiency by eliminating one of the two markups and hence reducing output prices. Vertical mergers also facilitate better coordination between input suppliers and output producers with respect to product design and make it possible to share technological information common to separate stages within an industry. Design coordination can lead to lower costs and higher product quality. There may be positive R&D externalities, vertical R&D spillovers, through the sharing proprietary technological information between two different production stages within a vertically integrated firm engaging in innovation. Even though a vertical merger can increase the costs to nonintegrated competitors (rasing rival's cost), it does not necessarily represent inefficiency of vertical merger. Vertical merger efficiency can arise from the fact that inefficient output firm serves smaller output market share or is out of the market after the merger.

Vertical acquisitions can be also anticompetitive under certain circumstances. By engaging in exclusionary conducts in its pricing and purchasing decisions that create or raise entry barriers at both upstream and downstream levels, vertical mergers can lead to higher prices or lower innovation effort levels. In upstream market, this may involve the upstream division raising the input price it charges to competitors of the downstream division or refusing to supply them. In downstream market, this may involve refusing to purchase from rival input suppliers. And also, a vertical merger raises competitive concerns regarding competitively sensitive information transfers within an entity. Since vertical acquisitions involve companies making products one of which is a necessary complement to the other's, vertical integrations can give the vertically integrated entity the ability to obtain rival' proprietary information in either market. The information can involve nonpublic pricing or demand information, in which case the competitive concern is that collusion in one of the markets is facilitated as a result of the merger. For this matter, a settlement involves firewalls, a provision of Department of Justice (DOJ) or Federal Trade Commission (FTC) to prevent the sharing of competitively sensitive information of competitor's between different divisions of a vertically combined entity. For example, in Lilly's acquisition of PCS, a pharmacy benefit management company, the Commission's settlement requires that the merged firm constructs a firewalls provision to prevent information transfer to Lilly such as other drug producers' prices, bids, or other conditions of sale. In high tech industries where upstream and downstream companies work closely, a vertically integrated firm's division in one market can pass on competitors' innovation information in the other market to its division in that other market. A competitive concern here is that the merged firm can free ride off its competitors' innovation effort, potentially reducing the rivals' R&D effort levels. For this matter, in the the Silicon Graphics, Inc. (SGI) acquisition of Alias & Wavefront, one of conditions in the settlement with Silicon Graphics from the Commission in order to maintain fair competition at both upstream and downstream levels while at the same time allowing the achievement of potential vertical merger efficiencies is a firewalls provision preventing information from passing nonpublic information from SGI (upstream supplier) to its Alias (downstream producer).[1] For the concern of vertical foreclosure, the Commission requires as one of the conditions on approval of the merger that SGI is to refrain from discriminating between its downstream division (Alias and Wavefront) and rivals, and also to maintain an open architecture and provide its application programming interfaces for its workstations.

There is, however, little economic research regarding the competitive effects of firewalls in imperfect competition markets following a vertical integration. Riordan and Salop (1995) study that vertical integration may allow downstream units to act as conduits for the exchange of pricing, output demand, and other competitively sensitive information by upstream suppliers. They offer a perspective that a vertical merger can raise competitive concerns regarding competitively sensitive information exchange under three conditions: projectable, nonpublic, and appropriate upstream market structure for successful pricing coordination. Hughes and Kao (2001) consider the consequences of information transfer within an entity on competition and organizational structure, and assess the impact of a firewalls provision on social welfare. In their analysis, they consider a market structure with competition in both input and output markets and refer to the downstream competitors private output demand information. Their idea is that there are costs of obtaining private demand information for a vertically integrated firm, i.e. input price concession in exchange for a private output demand information to its competitor in downstream market. The price concession results in lower marginal costs of downstream competitors that leads to consumers' surplus and social welfare gains. This is the reason that Firewalls decrease both consumer surplus and total social welfare while they increase the producer surplus. In a more recent paper, Milliou (2004) has investigated the impact of Firewalls on innovation incentives and on social welfare in a partially vertically integrated industry, in which a vertically integrated firm provides a nonintegrated downstream firm with an input in upstream market and at the same time competes

[1] SGI is the dominant graphics workstation provider with a 90% market base and Alias and Wavefront are two of the three dominant Unix-based software developers that operate on those workstations.

against the nonintegrated firm in the final output market. The upstream division of the integrated firm pass on private information such as the nonintegrated firm' R&D efforts obtained in the process of trade to its downstream division. That is, the integrated upstream division acts as a conduit for horizontal R&D spillovers. Due to the elimination of double marginalization, a vertically integrated firm is more efficient in the sense of smaller marginal cost of output than its rival. When allowed information flow within a vertically integrated firm, horizontal R&D spillovers, the more efficient vertically integrated firm is willing to invest more in R&D and can serve more market share and vice versa for less efficient nonintegrated firm, which leads to consumers' surplus and social welfare gain. This is the idea behind his paper to conclude that Firewalls do not necessarily improve R&D investments and total social welfare.

This paper is different from the previous works in several ways. First, there are symmetric firms in the upstream market competing in a Bertrand model rather than a monopoly as in Milliou (2004). Second, unlike Hughes and Kao (2001) where information transfer refers to demand side, I consider proprietary information of competitor' R&D efforts, horizontal R&D spillovers. Third, firms in the downstream market engage in R&D investments and there are vertical R&D spillovers once firms are vertically integrated with one of upstream firms. There is lots of evidence of significant inter-industry spillovers such as vertical R&D spillovers. Technological links among different stages within a vertically related industy are important features of many industries. Information about technology, design, or specific characteristics of products can be shared between upstream and downstream divisions within a firm. This seems to be most of the R&D intensive industries, where the coordination of the upstream and downstream divisions is necessary in order for products to be compatible and to avoid extra adjustment costs. Examples are electric utility, defense, telecommunications, and energy industries. However, this perspective of vertical R&D spillovers with a Firewalls provision on social welfare has not been considered in the previous works.

The main result is that such firewalls do not necessarily improve social welfare while it may increases R&D incentives. The result here is partially consistant with the general idea of FCC/DOJ that Firewalls may improve R&D incentives (Pitofsky, 2001). However, the result regarding social welfare is different from the general idea that Firewalls improve overall social welfare. Under Firewalls, the nonintegrated firm has a strategic incentive to purchase from the upstream division of the integrated firm in order to soften competition in the final output market because the integrated firm faces the nonintegrated in both upstream and downstream markets and hence competes less aggressively in downstream market.[2] However, industry profit (producers surplus) is smaller than under Information Transfer because firms in downstream market compete in R&D as well as in price. They severely compete each other in R&D game and as a result, engage in a highly inefficient R&D investment. This occurs when outputs are close substitutes. Consumers surplus is larger than under Information Flow due to severe R&D competition. Hence, total social welfare is smaller than under Information Transfer under certain curcumstances, i.e. high γ and β. When final products are close substitutes, the nonintegrated firm is willing to take high risk R&D investment because competition softening effect obtained by purchasing from the upstream division of the integrated firm is significantly high. Without the provision, firm D2 is not willing to trade with the integrated firm because information flow within the different stages of the integrated firm do harm itself.

The remainder of the paper is organized as follows. Section 2 describes the basic model of the present paper. In the following sections, equilibrium outcomes under Information Transfer (No Firewalls) and under Firewalls are analyzed. I conclude with welfare analysis at the end.

3.2 Model

Consumers purchase an output in a market where one of two downstream firms, D1, is vertically integrated with an upstream firm, U1, while firm D2 is independent from upstream firms. There is a fixed-coefficient technology such that each unit of

[2] Chen (2001) studies that a nonintegrated firm may strategically choose the integrated firm as an input supplier due to competition softening effect.

output in downstream market requires one unit of input from upstream market. Demand functions are as follows: $q_i = 1 - p_i + \gamma(p_j - p_i)$, where $\gamma > 0$, $i, j = 1, 2$, and $i \neq j$. The parameter γ represents the measure of product differentiation. The products are highly differentiated if γ is close to 0. The products are almost homogeneous as γ goes to ∞. There are upstream producers $(U1, U2, ..., Un)$, where $n \geq 2$, producing a homogeneous input for the downstream industry at a constant marginal cost of production, $c_u > 0$ and competing in a Bertrand model. I assume that upstream firms set linear prices for their products. Two-part tariffs are typically set in a model where a single upstream firm supplies each downstream firm because downstream firms are assumed to have no bargaining power. But in my model, I want to model cases where there are upstream firms competing in the Bertrand framework $(w^* = w_1 = w_2 = ... = w_n = c_u)$ to diminish the role of foreclosure and hence to focus on the consequences of information transfer on R&D incentives and social welfare.

Each downstream firm makes an investment in process innovating R&D that is cost reducing. There is no uncertainty. Two downstream firms produce differentiated products at the cost of $c_{di} = c_d - x_i$, where $x_i < c_d$, $i = 1, 2$. The cost of R&D is given by $I_i = \lambda \frac{x_i^2}{2}$, $i = 1, 2$, reflecting the existence of diminishing returns to R&D expenditures. The parameter λ is related to the marginal cost of R&D. x_i is the amount of R&D undertaken by a downstream firm, Di, $i = 1, 2$. There is vertical R&D spillovers of β, where $0 < \beta < 1$, from the integrated downstream division to its upstream division. Hence, the cost of upstream division of the integrated firm is $c_{u1} = c_u - \beta x_1$, where $\beta x_1 < c_u$.

The timing of the game is the following. In the first stage, D1 and D2 simultaneously choose their R&D effort levels, x_1 and x_2. In the second stage, the integrated firm sets the input price for firm D2 and firm D2 chooses its input supplier. I assume that if firm D2 decides to purchase from one of other alternative suppliers, $U2, ..., Un$, firm D2 chooses firm U2. Finally, D1 and D2 compete in prices in the final output market. To make sure that the second order conditions are satisfied and that R&D effort levels are positive, I make the following assumption.

Assumption 1: given γ and β, the degree of λ in R&D satisfies the following condition.[3]

$$\lambda > \underline{\lambda} = \max\{E_1, E_2\} \tag{3.1}$$

where,

$$E_1 = 2(1+\gamma)(1+\beta)^2 \left[\frac{2+4\gamma+\gamma^2}{4(1+\gamma)^2-\gamma^2}\right]\left[\frac{2+4\gamma+\gamma^2+\gamma(1+\gamma)}{4(1+\gamma)^2-\gamma^2}\right],$$

$$E_2 = \frac{\beta\gamma(1+\gamma+\beta)}{4(1+\gamma)^2-\gamma^2} + \left\{\frac{2[2\beta\gamma(1+\gamma)+(1+\beta)(2+4\gamma+\gamma^2)+\gamma(1+\gamma+\beta)]}{[4(1+\gamma)^2-\gamma^2]^2}\right\} \times \left\{\frac{(1+\gamma-\beta\gamma)(2\beta\gamma(1+\gamma)+(1+\beta)(2+4\gamma+\gamma^2))}{[4(1+\gamma)^2-\gamma^2]^2}\right\}.$$

Note that E_1 and E_2 increase in γ and β. That is, this assumption assures that firm D2 is willing to take R&D investments with high marginal costs when outputs are close substitutes in the presence of high vertical R&D spillovers.

Assumption 2: The upstream division of the integrated firm is unable to foreclose the nonintegrated downstream firm.

This assumption is reasonable because FCC/DOJ generally concerns about vertical foreclosure and requires as a condition on approval of a vertical merger that the integrated firm must maintain fair competition in both markets as in the SGI acquisition of Allias & Wavefront example.

3.3 Equilibrium outcomes

The nonintegrated firm D2 chooses its input supplier under two different regimes, Firewalls and Information Transfer. Under Information Transfer, firm D2 would face cost disadvantage due to information flow (horizontal R&D spillovers) between two different production stages within the integrated firm if it purchases an input from the upstream division of the integrated firm (U1) and hence has an incentive to buy

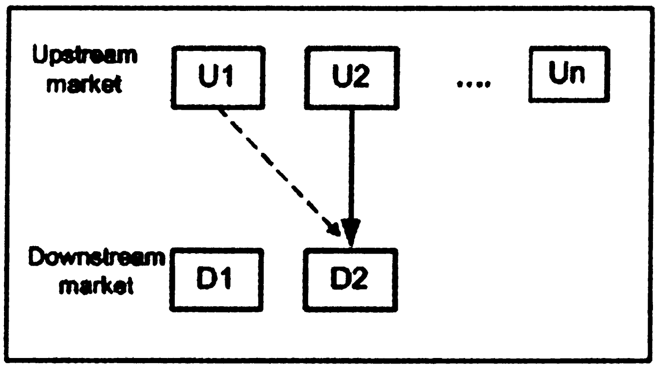
[3] E_1 is for $x_2^{nf} > 0$ under Information Transfer and E_2 is for $x_2^f > 0$ under Firewalls.

an input from other than U1. On the other hand, under Firewalls, the integrated firm U1 is no longer able to transfer firm D2' proprietary information to its downstream division. Firm D2 may have an incentive to purchase an input from the integrated firm in order to soften competition in the final output market. The integrated firm U1 cannot foreclose firm D2 by assumption 2 and cannot charge more than the marginal cost c_u due to alternative input suppliers.

3.3.1 Equilibrium Outcomes with Information Transfer

Firm D2 can purchase an input from the integrated firm U1 or from other than firm U1. Under no Firewalls provision, firm D2 would purchase an input from firm U2, rather than the integrated firm U1 as in Figure 3.1 because the upstream division of the integrated firm may pass on firm D2' nonpublic information to its downstream division once firm D2 purchases from firm U1.

Figure 3.1: Under Information Transfer



I solve the game by Backward Induction. In the third stage, the best price response functions are chosen to maximize the following profit functions.

$$\pi_{D1} = (p_1 - \tilde{c}_1)q_1 - \lambda\frac{x_1^2}{2}, \quad \pi_{D2} = (p_2 - \tilde{c}_2)q_2 - \lambda\frac{x_2^2}{2}, \tag{3.2}$$

where $\tilde{c}_1 = c_u + c_d - (1+\beta)x_1$, $\tilde{c}_2 = c_u + c_d - x_2$. The best price response functions are

$$R_i(p_j) = \frac{1 + \gamma p_j + (1+\gamma)\tilde{c}_i}{2(1+\gamma)}. \tag{3.3}$$

where $i = 1, 2$, $i \neq j$. As we can see in eq. (3.3), price is a strategic complement in the sense that p_i increases in p_j, $i, j = 1, 2$, $i \neq j$. From the first order condition, I can derive demand for each firm, $q_i = (1+\gamma)(p_i - \tilde{c}_i)$, $i = 1, 2$.

In the second stage, R&D levels are chosen to maximize the following profit function.

$$\pi_{Di} = (1+\gamma)(p_i - \tilde{c}_i)^2 - \lambda\frac{x_i^2}{2}. \tag{3.4}$$

where $i = 1, 2$. By differentiating the profit functions, the best R&D response functions are as follows.

$$R_1(x_2) = \frac{(1+\beta)v(A - ux_2)}{\lambda' - (1+\beta)v^2}, \quad R_2(x_1) = \frac{v(A - (1+\beta)ux_1)}{\lambda' - v^2}. \tag{3.5}$$

where $\lambda' = \frac{\lambda}{2(1+\gamma)}$, $A = \frac{(2+3\gamma)(1-c)}{4(1+\gamma)^2-\gamma^2}$, $v = \frac{2+4\gamma+\gamma^2}{4(1+\gamma)^2-\gamma^2}$, $u = \frac{\gamma(1+\gamma)}{4(1+\gamma)^2-\gamma^2}$. Unlike price, R&D is a strategic substitute in the sense that x_i decreases in x_j, $i, j = 1, 2$, $i \neq j$. The simultaneous Nash R&D levels are as follows.

$$x_1^{nf} = \frac{(1+\beta)vT_1}{\lambda' - (1+\beta)^2 vS_1}, \quad x_2^{nf} = \frac{vT_2'}{\lambda' - vS_2'}. \tag{3.6}$$

where $T_1 = A\left(1 - \frac{uv}{\lambda' - v^2}\right)$, $S_1 = v\left(1 + \frac{u^2}{\lambda' - v^2}\right)$, $T_2' = A\left(1 - \frac{(1+\beta)^2 uv}{\lambda' - (1+\beta)^2 v^2}\right)$, $S_2' = v\left(1 + \frac{(1+\beta)^2 u^2}{\lambda' - (1+\beta)^2 v^2}\right)$.

The R&D effort levels are positive under assumption 1. The integrated firm makes higher R&D effort than the nonintegrate firm. The reason is that firm D1 has positive vertical R&D spillovers β and hence has higher incentives to invest in R&D than firm

D2. The profit for each firm is as follows.

$$\pi_{D1}^{nf} = (1+\gamma)\lambda' \left(\frac{\lambda'}{(1+\beta)^2 v^2} - 1\right)(x_1^{nf})^2, \tag{3.7}$$

$$\pi_{D2}^{nf} = (1+\gamma)\lambda' \left(\frac{\lambda'}{v^2} - 1\right)(x_2^{nf})^2. \tag{3.8}$$

The profits are quadratic pay-offs of R&D effort levels. A firm with higher R&D investments makes higher profits. In the presence of a vertical spillover β, firm D1 is more willing to invest in R&D and hence makes higher profit than firm D2.

Proposition 3.1 Under Information Transfer, the amount of R&D effort level of the integrated firm D1 are larger than that of nonintegrated firm D2. If $\lambda > \tilde{\lambda}$, the profit of the integrated firm D1 is higher than that of nonintegrated firm D2. where,

$$\tilde{\lambda} = \frac{2(1+\gamma)(2+4\gamma+\gamma^2)(2+5\gamma+2\gamma^2)[(1+\beta)^2+1]}{(4(1+\gamma)^2-\gamma^2)^2(2+6\gamma+3\gamma^2)} \tag{3.9}$$

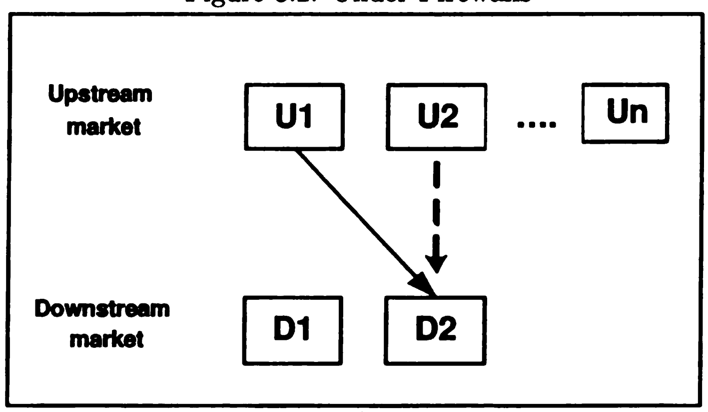
Proof: $x_1^{nf} - x_2^{nf} > 0$, because $(1+\beta)vT_1 > vT_2'$ and $\lambda' - (1+\beta)^2 vS_1 < \lambda' - vS_2'$. $sign[\pi_{D1}^{nf} - \pi_{D2}^{nf}] = sign[(\frac{\lambda}{2(1+\gamma)} - (1+\beta)^2v^2)\left(\frac{\lambda}{2(1+\gamma)} - v(v+u)\right)^2 - (\frac{\lambda}{2(1+\gamma)} - v^2)\left(\frac{\lambda}{2(1+\gamma)} - (1+\beta)^2 v(v+u)\right)^2] = sign[\lambda'(\lambda'(v+2u) - v(v+u)^2[(1+\beta)^2+1]) + (1+\beta)^2v^3(v+u)^2]$ (i) $\pi_{D1}^{nf} - \pi_{D2}^{nf} > 0$ if $\lambda > \frac{2(1+\gamma)(2+4\gamma+\gamma^2)(2+5\gamma+2\gamma^2)[(1+\beta)^2+1]}{(4(1+\gamma)^2-\gamma^2)^2(2+6\gamma+3\gamma^2)} = \tilde{\lambda}$. Note that $\tilde{\lambda}$ increases in γ and β and that $\lambda > E_1$ is the condition for firm D2 to make positive amounts of R&D effort under Information Transfer, $T_2' > 0$, in which the integrated firm D1 also makes positive R&D effort levels. By comparison, we know $\tilde{\lambda} > E_1$. (ii) Hence, it may be possible that the nonintegrated firm D2 makes higher profits than the integrated firm if $\tilde{\lambda} > \lambda > E_1$.

3.3.2 Equilibrium Outcomes with Firewalls

This is the case where the upstream division of the integrated firm U1 cannot pass on the proprietary information of firm D2 to its downstream division D1 and is not allowed to foreclose firm D2. Hence, firm D2 may have an incentive to purchase an input from the integrated firm U1 since it softens competition in the downstream market because price is a strategic complement.

Under Firewalls, let's check whether firm D2 has an incentive to purchase an input from the integrated firm. When firm D2 purchases an input from U2, the problem under Firewalls is the same as that in the previous case of Information Flow. Hence, I solve by backward induction the case where firm D2 purchases an input from the integrated firm as in Figure 3.2.

Figure 3.2: Under Firewalls



As in the previous scenario, the cost function for the integrated firm is $\tilde{c}_1 = c_u + c_d - (1+\beta)x_1$, where $0 < \beta < 1$, while that of firm D2 is $\tilde{c}_2 = w^* + c_d - x_2$. The profit functions of the integrated and of the nonintegrated firms are respectively given by

$$\pi_{D1} = (p_1 - \tilde{c}_1)q_1 + (w_1 - c_{u1})q_2 - \lambda\frac{x_1^2}{2}, \tag{3.10}$$

$$\pi_{D2} = (p_2 - \tilde{c}_2)q_2 - \lambda\frac{x_2^2}{2}, \tag{3.11}$$

where $c_{u1} = c_u - \beta x_1$. The integrated firm sets the input price at c_u due to alternative suppliers and cannot foreclose firm D2 by assumption 2. Notice that the profits of the vertically integrated firm comes from both upstream and downstream markets, in which the firm operates. Solving the production stage, demand for each firm is $q_1 = (1+\gamma)(p_1 - \tilde{c}_1) - \gamma(w_1 - c_{u1})$ and $q_2 = (1+\gamma)(p_2 - \tilde{c}_2)$. The integrated firm loses a portion of market share in the downstream market. This is because it serves the upstream market and makes positive profits, and hence is less aggressive in the downstream market. The best price reaction functions are as follows.

$$R_1(P_2) = \frac{1 + \gamma p_2 + (1+\gamma)\tilde{c}_1 + \gamma(w_1 - c_{u1})}{2(1+\gamma)}, \tag{3.12}$$

$$R_2(P_1) = \frac{1 + \gamma p_1 + (1+\gamma)\tilde{c}_2}{2(1+\gamma)}. \tag{3.13}$$

Note that the price reaction function of the integrated firm has an extra term that does not appear in eq. (3.3). The extra term will not be zero unless the market price of the integrated firm is more than the competitive market price, $w_1 = c_u$. This implies that firm D2 may have an incentive to purchase from the integrated firm because the integrated firm is less aggressive in the final product market in the sense that $R_1(p_2)$ in eq. (3.12) $>$ $R_1(p_2)$ in eq. (3.3). Hence, when firm D2 buys from the upstream division of the integrated firm, the integrated firm maximize the following profit function.

$$\begin{aligned} \pi_{D1} &= (1+\gamma)(p_1 - \tilde{c}_1)^2 + (w_1 - c_{u1})[(1+\gamma)(p_2 - \tilde{c}_2) - \gamma(p_1 - \tilde{c}_1)] - \lambda\frac{x_1^2}{2}, \\ &\quad \textit{subject to } w_1 = c_u. \end{aligned} \tag{3.14}$$

and firm D2 maximizes the following profit function.

$$\pi_{D2} = (1+\gamma)(p_2 - \tilde{c}_2)^2 - \lambda\frac{x_2^2}{2}. \tag{3.15}$$

By differentiating the profit functions with respect to x_i, $i = 1, 2$, the best R&D

response functions are as follows.

$$R_1(x_2) = \frac{[2(1+\gamma)v_1^* + \beta]A - [2(1+\gamma)v_1^* u - \beta\gamma u - \beta(1+\gamma)v]x_2}{\lambda + 2\beta\gamma v_1^* + 2\beta(1+\gamma)u_2^* - 2(1+\gamma)(v_1^*)^2}, \quad (3.16)$$

$$R_2(x_1) = \frac{v(A - u_2^* x_1)}{\lambda' - v^2}, \quad (3.17)$$

where $v_1^* = (1+\beta)v + 2\beta u$, $u_2^* = \left(1 + \frac{\beta}{1+\gamma}\right)u$. By substituting, I can derive simultaneous Nash R&D levels as follows.

$$x_1^f = \frac{[2(1+\gamma)v_1^* + \beta]A - [2(1+\gamma)v_1^* u - \beta\gamma u - \beta(1+\gamma)v](\frac{vT_2^*}{\lambda' - vS_2^*})}{\lambda + 2\beta\gamma v_1^* + 2\beta(1+\gamma)u_2^* - 2(1+\gamma)(v_1^*)^2}, \quad (3.18)$$

$$x_2^f = \frac{vT_2^*}{\lambda' - vS_2^*}. \quad (3.19)$$

where,

$$T_2^* = A\left(1 - \frac{u_2^*[2(1+\gamma)v_1^* + \beta]}{\lambda + 2\beta\gamma v_1^* + 2\beta(1+\gamma)u_2^* - 2(1+\gamma)(v_1^*)^2}\right),$$

$$S_2^* = v + \frac{u_2^*[2(1+\gamma)v_1^* u - \beta\gamma u - \beta(1+\gamma)v]}{\lambda + 2\beta\gamma v_1^* + 2\beta(1+\gamma)u_2^* - 2(1+\gamma)(v_1^*)^2}.$$

Note that each firm has positive R&D effort level, $x_i^f > 0$, $i = 1, 2$, under the assumption 1. For all β and γ, the integrated firm D1 has higher R&D effort level than the nonintegrated firm D2. The total profit functions of the integrated firm and of the non-integrated firm are as follows.

$$\pi_{D1}^f = (1+\gamma)(\lambda' - \beta^2)\left(\frac{\lambda' - \beta^2}{(v_1^*)^2} - 1\right)(x_1^f)^2, \quad (3.20)$$

$$\pi_{D2}^f = (1+\gamma)\lambda'\left(\frac{\lambda'}{v^2} - 1\right)(x_2^f)^2. \quad (3.21)$$

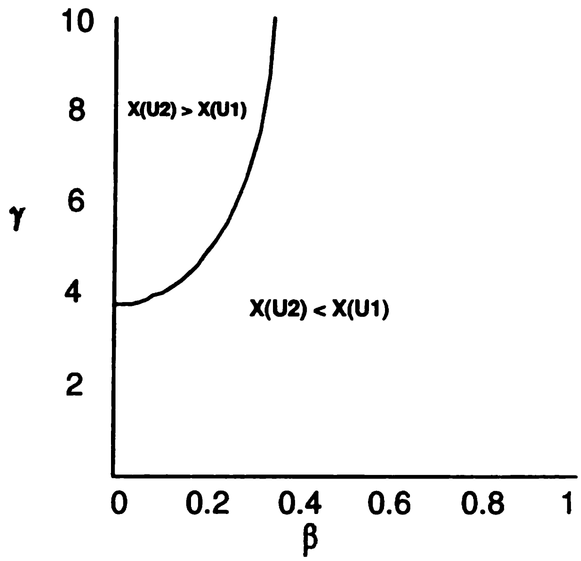
When firm D2 purchases an input from U1 under Firewalls, for moderate β and γ, the integrated firm makes high profit than the nonintegrated firm and vice versa for sufficiently high β and γ even when firm D1 has higher R&D effort level. [4] The

[4]For profit comparison between the integrated firm and nonintegrated firm, see appendix F. The

reason is that for sufficiently high β and γ, in which outputs are close substitues in the presence of high vertical R&D spillovers, firms in downstream market engage in inefficient R&D game in the sense of high value of λ.

Now, let's check when firm D2 would like to purchase an input from firm U1 rather than firm U2. When firm D2 purchases from firm U2, the analysis is the same as that in the previous case of Information Transfer. Since the expressions here are rather messy, I do simulation analysis. Figure 3.3 shows firm D2' R&D level comparison between purchasing from U1 and from U2. Since profit is a quadratic function of R&D effort level, firm D2 makes higher profit by purchasing from firm U1 in the are of $X(U2) < x(U1)$ in Figure 3.3.

Figure 3.3: Firm D2' R&D level Comparison with $\lambda = 25$, $c = 0.5$



Proposition 3.2 Under Firewalls, there exists a $\overline{\lambda}$ such that for $\overline{\lambda} \leq \lambda$ where $\lambda(T_2^* - T_2^{'}) - 2(1+r)v(S_2^* T_2^{'} - S_2^{'} T_2^*) \geq 0$, the nonintegrated firm D2 has an incentive to buy an input from the integrated firm U1. The R&D level and the profit of firm D2 are higher when it buys from the integrated firm than from other alternative suppliers.

simulations are made with $\lambda = 25$, $c = 0.5$.

Sketch of Proof: if $\lambda(T_2^* - T_2^{'}) - 2(1+r)v(S_2^* T_2^{'} - S_2^{'} T_2^*) \geq 0$, in which firm D2 purchases an input from firm U1 and makes higher R&D effort levels, firm D2 makes higher profits. As we can see from Figure 3.3, for sufficiently high β and γ given $\lambda = 25$ and $c = 0.5$, firm D2 makes higher R&D effort levels when it purchases an input from the integrated firm U1 than from firm U2. The intuition here is that the extra term in the price response function eq. (3.12) is a increasing function of γ and β. That is, when outputs are close substitutes (high γ) and vertical R&D spillovers are high, competition softening effect is significant and hence firm D2 can get benefits from purchasing an input from firm U1 under no pressure of information flow within the integrated firm.

3.3.3 Information Transfer vs. Firewalls

In a vertically related market where a vertically integrated firm competes with other input suppliers in upstream market for nonintegrated downstream firm D2 while competing with nonintegrated firm D2 in downstream market, the nonintegrated downstream firm D2 make a decision whether to buy an input from the integrated firm U1 or firm U2 under two different policy regimes, Firewalls vs. Information Transfer. As we can see from Table 3.1, payoffs of firm D2 depend on its R&D effort levels. The higher R&D investments, the higher payoffs it can get. Under Information Transfer, firm D2 buys an input from firm D2 rather than firm U1 due to cost disadvantage resulted from horizontal R&D spillovers. Under Firewalls, firm D2 may have an incentive to purchase an input from the integrated firm due to competition softening effect.

Table 3.1: Payoffs of nonintegrated firm D2

	Buy from U1	Buy from U2
No Firewalls	$(1+\gamma)\lambda^{'}\left(\frac{\lambda^{'}}{v^2} - 1\right)(x_2^{nf1})^2$	$(1+\gamma)\lambda^{'}\left(\frac{\lambda^{'}}{v^2} - 1\right)(x_2^{nf2})^2$
Firewalls	$(1+\gamma)\lambda^{'}\left(\frac{\lambda^{'}}{v^2} - 1\right)(x_2^{f1})^2$	$(1+\gamma)\lambda^{'}\left(\frac{\lambda^{'}}{v^2} - 1\right)(x_2^{f2})^2$

note: $x_2^{nf2} = x_2^{f2}$.

When $\overline{\lambda} \leq \lambda$, firm D2 makes higher profit as well as higher R&D effort levels when purchasing an input from the integrated firm than from firm U2, $x_2^{f1} > x_2^{f2} = x_2^{nf2}$. Hence, in equilibrium, the nonintegrated firm D2 purchases an input from the integrated firm U1 under Firewalls.

On the other hand, as we can see in Appendix F, when $\overline{\lambda} \leq \lambda$, in which firm D2 makes higher R&D effort levels and profits under Firewalls than under Information Transfer, for low γ and β, firm D1 makes higher profits under Firewalls than under Information Transfer and vice versa for high γ and β. And also, firm D1 makes higher profits than firm D2 for low γ and β and vice versa for high γ and β.

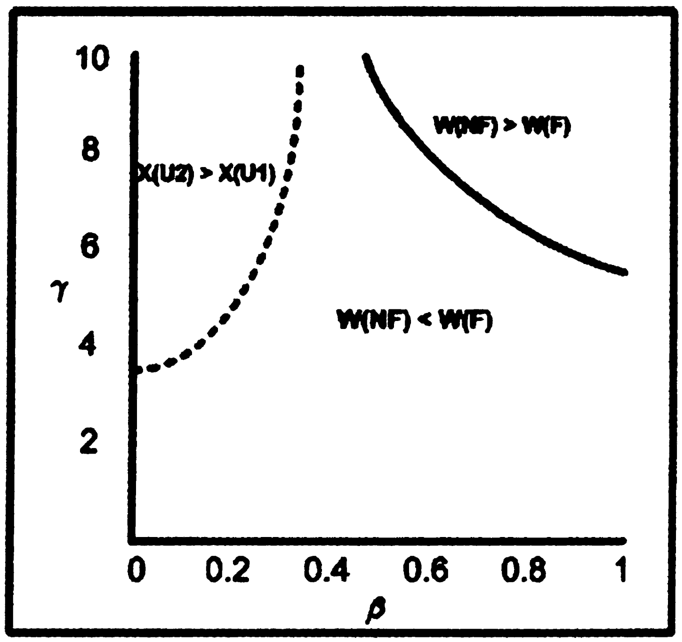
3.4 Welfare

Total social welfare is defined as the sum of the consumers' surplus and the producers' surplus, $W = CS + PS$.

$$\begin{aligned} W(NF) &= \frac{1+\gamma}{2}((1-p_1)(p_1-\tilde{c})+(1-p_2)(p_2-\tilde{c})) \\ &\quad +(1+\gamma)\left((p_1-\tilde{c}_1)^2-\lambda\frac{x_1^2}{2}+(p_2-\tilde{c}_2)^2-\lambda\frac{x_2^2}{2}\right), \qquad (3.22) \\ W(F) &= \frac{1+\gamma}{2}((1-p_1)(p_1-\tilde{c})+(1-p_2)(p_2-\tilde{c})) \\ &\quad +(1+\gamma)\left((p_1-\tilde{c}_1)^2+(\beta x_1)^2-\lambda\frac{x_1^2}{2}+(p_2-\tilde{c}_2)^2-\lambda\frac{x_2^2}{2}\right) \qquad (3.23) \end{aligned}$$

In order to compare total social welfare under two different policy regimes, Information Transfer (W(NF)) vs. Firewalls W(F), I do simulation analysis. Figure 3.4 shows that for sufficiently high β and γ, social welfare under Information Transfer is higher than under Firewalls and vice versa for moderate β and γ. When final products are close substitues (high γ), firms engages in inefficient R&D game, which leads to welfare losses. If $\overline{\lambda} \leq \lambda$, in equilibrium, firm D2 purchases an input from the integrated firm, in which consumers surplus and R&D levels are higher under Firewalls than under Information Transfer. This result is consistant with that a Firewalls provision can retain the integrated firm' incentives to free ride on competitors' hard work and hence improve final output competition as well as R&D incentives. However, my result regarding producers' surplus and overall social welfare tells a different story. Final output producers suffer from severe R&D competition with sufficiently high value of λ. A Firewalls provision may lead to inefficient R&D game by affecting firm D2' decision for an input supplier and hence producers' surplus losses and overall social welfare losses.[5]

Figure 3.4: Welfare Comparison with $\lambda = 25$, $c = 0.5$



Proposition 3.3 For all β and γ, consumers surplus is larger under Firewalls than under Information Transfer. However, for sufficiently high β and γ, producers surplus is smaller under Firewalls than under Information Transfer. As a result, for sufficiently high β and γ, total social welfare is smaller under Firealls than under Information Transfer.

Sketch of Proof: As downstream firms engage in severe R&D game, consumers get benefits while firms suffer from high costs of R&D. From a social point of view,

[5]see the appendix for consumers surplus, producers surplus, and R&D comparisons.

Firewalls may lead to inefficient R&D game and as a result not necessarily improve total social welfare.

3.5 Conclusion

I analyze the consequence of a Firewalls provision on social welfare and on R&D incentives in a market where a vertically integrated firm and a nonintegrated firm compete in price and engage in R&D in downstream market. Firms in upstream markets are symmetric, except the integrated firm has a cost advantage in the sense that there exists vertical R&D spillovers. In such circumstances, the nonintegrated firm needs to make a decision whether to buy an input from the upstream division of the integrated firm or other alternative input suppliers under two different policy regimes, Information Transfer vs. Firewalls. When authority allows Information Transfer, i.e. horizontal R&D spillovers from firm D2 to firm D1 through the upstream division of the integrated firm, firm D2 has no incentive to purchase an input from the integrated firm due to cost disadvantage of horizontal R&D spillovers. On the other hand, when authority does not allow Information Transfer by establishing a Firewalls provision, in equilibrium, firm D2 may have an incentive to purchase an input from the upstream division of the integrated firm. The reason is that the integrated firm competes less aggressively in downstream market because in both upstream and downstream market, it operates and faces firm D2 as an input provider as well as a final output competitor. The integrated firm can make positive profits in both upstream and downstream markets.

FCC and DOJ generally allows a vertical merger under conditions of Firewalls and Nonforeclosure in a high tech industry where upstream and downstream firms work closely. My study shows that for high β and γ, a Firewalls provision with nonforeclosure condition alters the nonintegrated firm' decision for an input supplier and leads to total social welfare smaller under Firewalls than under Information Transfer due to inefficient R&D investment (sufficiently large value of λ). Nevertheless consumers surplus is larger than under Information Transfer. The result regarding final output market competition in the present paper is consistant with the competition concerns suggested by FOC/DOJ. The commission suggests that Firewalls along with non-foreclosure provision may improve both upstream and downstream competition and hence increase consumers' surplus and social welfare. However, my study regarding overall social welfare indicates that under certain curcumstances Firewalls may not necessarily improve social welfare due to produecers' surplus losses.

My result is based on two critical assumptions. First, with significantly high value of λ, I assumed that each downstream firm makes positive R&D investments in order to make the anyalsis simple. Second, the integrated firm is unable to foreclose the nontegrated downstream firm. Given high γ and β, in which case the integrated firm makes less profit by supplying an input to the nonintegrated firm, the integrated firm simply stop supplying an input to the nonintegrated firm if it may. However, under the assumption, the integrated firm must supply an input to the nonintegrated firm if the nonintegrated firm make an order to the integrated firm. Nevertheless, my result suggests that when the Commission evaluates vertical merger efficiency in line with competition concerns in both markets, overall social welfare should be carefully considered in high tech industries with significantly high R&D investments.

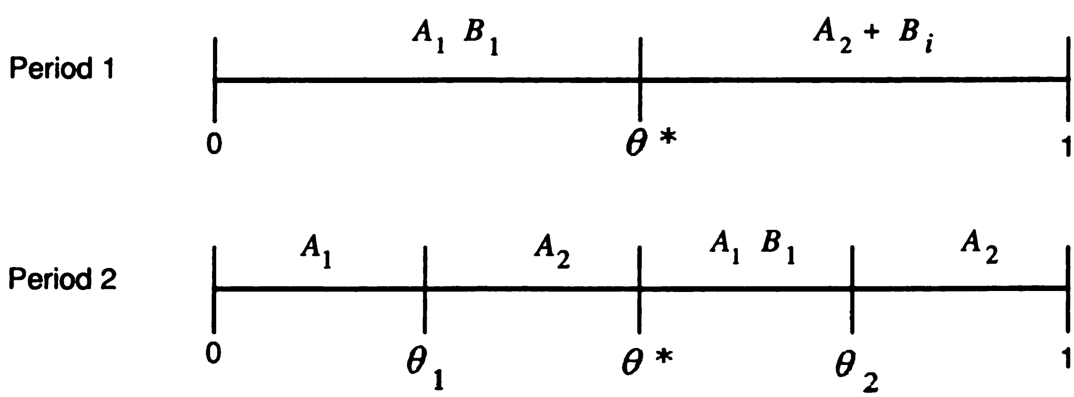
Appendix A

Appendix: Unilateral Bundling

In this section, I consider the case where one firm offers the bundle A_iB_i, $i = 1, 2$ while the other firm does not bundle. I assume that firm A1 offers the bundle of A_1B_1 to new customers, while firm A2 does not practice bundling.

A.1 Short-term contract

Consumers in [0, θ^*] buy the bundle A_1B_1 and customers in [θ^*, 1] purchase A_2 and B_i, where $i = 2, ..., n$, in the first period. In the second period, consumers in the intervals [θ_1, θ^*] and [θ^*, θ_2] are targeted with discounts by competitors and switch between brands, purchasing A_2 and A_1B_1, respectively. On the other hand, consumers in the intervals [0, θ_1] and [θ_2, 1] do not switch their suppliers.



In the second period, given θ^*, a customer is indifferent between A_1 and A_2 at θ_1

and between A_1B_1 and A_2 at θ_2 if

$$p_{A1}^2 + t\theta_1 = \hat{p}_{A2}^2 + t(1-\theta_1) \Rightarrow \theta_1 = \frac{1}{2t}(t + \hat{p}_{A2}^2 - p_{A1}^2), \tag{A.1}$$
$$\hat{p}_{A1}^B + t\theta_2 = p_{A2}^2 + t(1-\theta_2) \Rightarrow \theta_2 = \frac{1}{2t}(t + p_{A2}^2 - \hat{p}_{A1}^B), \tag{A.2}$$

where p_{A1}^2 and p_{A2}^2 are the second period prices of A_1 and A_2 for loyal customers of firm A1 and A2, respectively. $\hat{p}_{A1}^B$ and $\hat{p}_{A2}^2$ are the second period prices of A_1B_1 and A_2 for customers who switch between providers. The profit functions of firms in the second period are as follows.

$$\pi_{12}^{PB(S)} = (p_{A1}^2 - c_A)\theta_1 + (\hat{p}_{A1}^B - c_A - c_B)(\theta_2 - \theta^*), \tag{A.3}$$
$$\pi_{22}^{NB(S)} = (p_{A2}^2 - c_A)(1-\theta_2) + (\hat{p}_{A2}^2 - c_A)(\theta^* - \theta_1). \tag{A.4}$$

By maximizing the profit functions with respect to the second period prices, I have the second period prices as follows.

$$p_{A1}^2 = c_A + \left(\frac{2\theta^* + 1}{3}\right)t, \quad p_{A2}^2 = c_A + \frac{1}{3}c_B + \left(\frac{3 - 2\theta^*}{3}\right)t, \tag{A.5}$$
$$\hat{p}_{A1}^B = c_A + \frac{2}{3}c_B + \left(\frac{3 - 4\theta^*}{3}\right)t, \quad \hat{p}_{A2}^2 = c_A + \left(\frac{4\theta^* - 1}{3}\right)t. \tag{A.6}$$

In the first period, a consumer at θ^* is indifferent between A_1B_1 and $A_2 + B_i$, $i = 2, ..., n$, if

$$\begin{aligned} p_{A1}^B + t\theta^* &+ \delta(\hat{p}_{A2}^2 + t(1-\theta^*)) = p_{A2}^1 + c_B + t(1-\theta^*) + \delta(\hat{p}_{A1}^B + t\theta^*) \\ \Rightarrow \theta^* &= \frac{1}{2} + \frac{(p_{A2}^1 + c_B - p_{A1}^B) - \delta(\hat{p}_{A2}^2 - \hat{p}_{A1}^B)}{2(1-\delta)t} \\ &= \frac{1}{2} + \frac{3(p_{A2}^1 - p_{A2}^B)}{2(3+\delta)t} + \frac{3+2\delta}{2(3+\delta)t}c_B. \end{aligned} \tag{A.7}$$

Firms set the first period prices, p_{A1}^B and p_{A2}^1, by maximizing the following profit functions,

$$\pi_1^{PB(S)} = (p_{A1}^B - c_A - c_B)\theta^* + \delta\pi_{12}^{PB(S)} , \quad (A.8)$$

$$\pi_2^{NB(S)} = (p_{A2}^1 - c_A)(1-\theta^*) + \delta\pi_{22}^{PB(S)} . \quad (A.9)$$

By substituting the optimal prices into eq. (A.1) and eq. (A.2), we derive

$$\theta_1 = \frac{1}{3} + \frac{4\delta c_B}{3(27-11\delta)t}, \quad \theta^* = \frac{1}{2} + \frac{4\delta c_B}{(27-11\delta)t}, \quad \theta_2 = \frac{2}{3} - \frac{(27-19\delta)c_B}{6(27-11\delta)t} . \quad (A.10)$$

We need the condition of $t > \left(\frac{27+5\delta}{27-11\delta}\right) c_B$ for $\theta_2 > \theta^*$. Otherwise, firm A1 is unable to gain customers attached to firm A2 in the second period while firm A2 is able to gain a positive portion of customers attached to firm A1. That is, customers who purchased from firm A2 in the first period are locked-in if $t \leq \left(\frac{27+5\delta}{27-11\delta}\right) c_B$. Firm A2 becomes a monopoly over the interval $[\theta^*, 1]$ and charge $p_{A2}^2 = U - t(1-\theta^*)$ to its locked-in customers in the second period. Hence, we need to consider two cases, $t > \left(\frac{27+5\delta}{27-11\delta}\right) c_B$ and $t \leq \left(\frac{27+5\delta}{27-11\delta}\right) c_B$.

(1) $t > \left(\frac{27+5\delta}{27-11\delta}\right) c_B$

Both firms are able to gain a positive portion of customers attached to competitors. The equilibrium profits are as follows.

$$\pi_1^{PB(S)} = \left(1+\frac{8\delta}{9}\right)\frac{t}{2} + \left(2\xi - \frac{4}{9}\delta\xi - \frac{3}{9}\delta\right)c_B + \left(2\xi^2 - \frac{4}{9}\delta\xi^2 + \frac{1}{18}\delta\right)\frac{c_B^2}{t} , \quad (A.11)$$

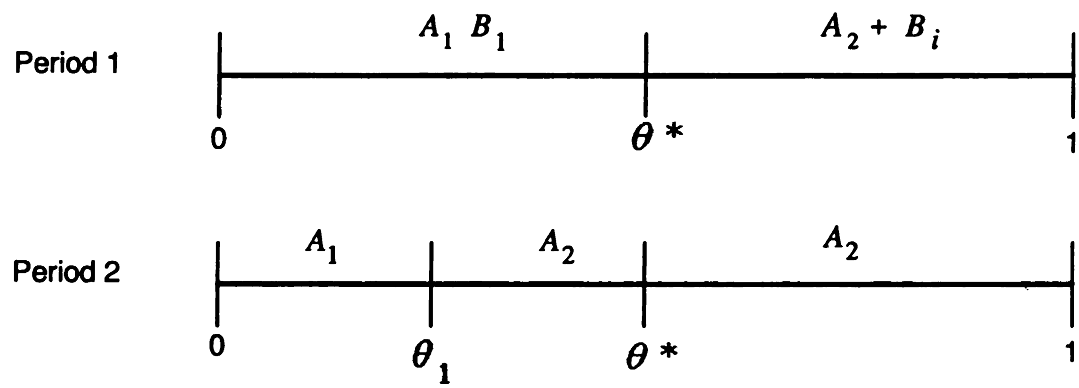
$$\pi_2^{NB(S)} = \left(1+\frac{8\delta}{9}\right)\frac{t}{2} - \left(2\xi - \frac{4}{9}\delta\xi - \frac{1}{9}\delta\right)c_B + \left(2\xi^2 - \frac{4}{9}\delta\xi^2 + \frac{1}{18}\delta\right)\frac{c_B^2}{t} , \quad (A.12)$$

$$where \quad \xi = \frac{4\delta}{27-11\delta} .$$

(2) $t \leq \left(\frac{27+5\delta}{27-11\delta}\right) c_B$

Customers of firm A2 are locked-in after the first purchase, $\theta_2 = \theta^*$. Consumers in $[0, \theta^*]$ buy the bundle A_1B_1 and customers in $[\theta^*, 1]$ purchase A_2 and B_i, where

$i = 2, ..., n$, in the first period. After observing customers' behavior, consumers in the intervals $[\theta_1,\ \theta^*]$ are targeted with discounts by firm A2 and switch to A_2. On the other hand, consumers in the intervals $[0,\ \theta_1]$ and $[\theta^*,\ 1]$ do not switch their suppliers.



In the second period, given θ^*, a customer is indifferent between A_1 and A_2 at θ_1 if

$$p^2_{A1} + t\theta_1 = \hat{p}^2_{A2} + t(1 - \theta_1) \Rightarrow \theta_1 = \frac{1}{2t}(t + \hat{p}^2_{A2} - p^2_{A1}), \tag{A.13}$$

where p^2_{A1} is the second period price of A_1 for loyal customers of firm A1 and $\hat{p}^2_{A2}$ is the second period price of A_2 for firm A1' consumers who switch to firm A2. The profit functions of firms in the second period are as follows.

$$\pi_{12}^{PB(S)} = (p^2_{A1} - c_A)\theta_1, \tag{A.14}$$

$$\pi_{22}^{NB(S)} = (p^2_{A2} - c_A)(1 - \theta^*) + (\hat{p}^2_{A2} - c_A)(\theta^* - \theta_1), \tag{A.15}$$

where $p^2_{A2} = U - t(1 - \theta^*)$. By maximizing the profit functions with respect to the second period prices, p^2_{A1}, $\hat{p}^2_{A2}$, I have the second period prices as follows.

$$p^2_{A1} = c_A + \left(\frac{2\theta^* + 1}{3}\right)t\,,\ \hat{p}^2_{A2} = c_A + \left(\frac{4\theta^* - 1}{3}\right)t\,. \tag{A.16}$$

In the first period, a consumer at θ^* is indifferent between A_1B_1 and $A_2 + B_i$,

$i = 2, ..., n$, if

$$p_{A1}^{B} + t\theta^* + \delta(\hat{p}_{A2}^{2} + t(1-\theta^*)) = p_{A2}^{1} + c_B + t(1-\theta^*) + \delta(U - t(1-\theta^*) + t(1-\theta^*))$$
$$\Rightarrow \theta^* = \frac{3-2\delta}{6+\delta} + \frac{3(p_{A2}^{1} + c_B - p_{A1}^{B})}{(6+\delta)t} + \frac{3\delta(U - c_A)}{(6+\delta)t}. \quad \text{(A.17)}$$

Firms set the first period prices, p_{A1}^{B} and p_{A2}^{1}, by maximizing the following profit functions,

$$\pi_1^{PB(S)} = (p_{A1}^{B} - c_A - c_B)\theta^* + \delta(p_{A1}^{2} - c_A)\theta_1, \quad \text{(A.18)}$$
$$\pi_2^{NB(S)} = (p_{A2}^{1} - c_A)(1-\theta^*) + \delta(U - t(1-\theta^*) - c_A)(1-\theta^*)$$
$$+\delta(\hat{p}_{A1}^{2} - c_A)(\theta^* - \theta_1). \quad \text{(A.19)}$$

By substituting the optimal prices into eq. (A.13), I derive

$$\theta_1 = \frac{1}{6} + \frac{27+13\delta}{3(54+7\delta)}, \quad \theta^* = \frac{27+13\delta}{54+7\delta}. \quad \text{(A.20)}$$

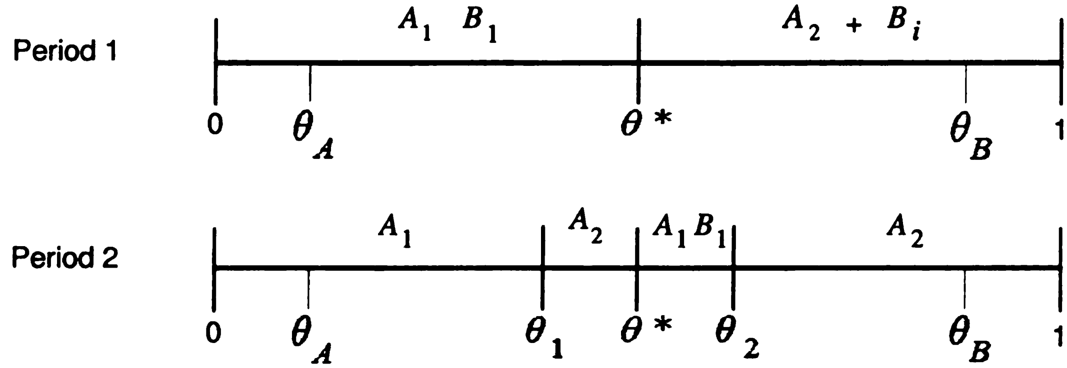
The equilibrium prices are as follows.

$$p_{A1}^{B} = c_A + c_B - \frac{2}{9}\delta t + \left(\frac{18-\delta}{9}\right)\left(\frac{27+13\delta}{54+7\delta}\right)t,$$
$$p_{A2}^{1} = 2t + c_A + \frac{17}{9}\delta t - \delta U + \delta c_A - \left(\frac{18+5\delta}{9}\right)\left(\frac{27+13\delta}{54+7\delta}\right)t, \quad \text{(A.21)}$$
$$p_{A1}^{2} = c_A + \frac{1}{3}t + \frac{2}{3}\left(\frac{27+13\delta}{54+7\delta}\right)t, \quad p_{A2}^{2} = U - \left(\frac{27-6\delta}{54+7\delta}\right)t,$$
$$\hat{p}_{A2}^{2} = c_A - \frac{1}{3}t + \frac{4}{3}\left(\frac{27+13\delta}{54+7\delta}\right)t.$$

The equilibrium profits are as follows.

$$\pi_1^{PB(S)} = \frac{\delta}{18}t + \left(\frac{18+\delta}{9}\right)\left(\frac{27+13\delta}{54+7\delta}\right)^2 t, \quad \text{(A.22)}$$
$$\pi_2^{NB(S)} = t\left(\frac{36+17\delta}{18}\right) - t\left(\frac{36+8\delta}{9}\right)\left(\frac{27+13\delta}{54+7\delta}\right)$$
$$+t\left(\frac{18+4\delta}{9}\right)\left(\frac{27+13\delta}{54+7\delta}\right)^2. \quad \text{(A.23)}$$

A.2 Long- and Short-term contracts



Consumers in $[0,\ \theta^*]$ and $[\theta^*,\ 1]$ buy the bundle A_1B_1 and $A_2 + B_i$, respectively, where $i = 2, ..., n$, in the first period. Consumers in $[0,\ \theta_A]$ and $[\theta_B,\ 1]$ commit to a long-term contract of $\{A_1B_1,\ A_1\}$ and $\{A_2,\ A_2\}$, at the price of $\tilde{p}_{A1}$ and $\tilde{p}_{A2}$, respectively. Consumers in $[\theta_A,\ \theta^*]$ and $[\theta^*,\ \theta_B]$ buy a short-term contract at p^B_{A1} and p^1_{A2}, from firm A1 and A2, respectively. In the second period, some consumers who bought a short-term contract in the first period, $[\theta_1,\ \theta^*]$ and $[\theta^*,\ \theta_2]$, are targeted with discounts and switch between providers and pay the second period poaching prices $\hat{p}^2_{A2}$ and $\hat{p}^B_{A1}$ for A_2 and A_1B_1, respectively. Consumers in $[\theta_A,\ \theta_1]$ and $[\theta_2,\ \theta_B]$ stay with the previous providers and pay p^2_{A1} and p^2_{A2} for A_1 and A_2, respectively. Since customers in $[0,\ \theta_A]$ and $[\theta_B,\ 1]$ committed to a long term contract at $\tilde{p}_{A1}$ and $\tilde{p}_{A2}$, respectively, competition in the second period is over the interval $[\theta_A,\ \theta_B]$.

In the second period, given θ^*, a customer is indifferent between A_1 and A_2 at θ_1 and between A_1B_1 and A_2 at θ_2 if

$$p^2_{A1} + t\theta_1 = \hat{p}^2_{A2} + t(1 - \theta_1) \ \Rightarrow\ \theta_1 = \frac{1}{2t}(t + \hat{p}^2_{A2} - p^2_{A1})\,, \qquad \text{(A.24)}$$

$$\hat{p}^B_{A1} + t\theta_2 = p^2_{A2} + t(1 - \theta_2) \ \Rightarrow\ \theta_2 = \frac{1}{2t}(t + p^2_{A2} - \hat{p}^B_{A1})\,. \qquad \text{(A.25)}$$

Firms are maximizing the following profit functions with respect to the second period prices, $\{p^2_{A1}, \hat{p}^B_{A1}\}$ and $\{p^2_{A2}, \hat{p}^2_{A2}\}$.

$$\pi_{12}^{PB(L\&S)} = (p^2_{A1} - c_A)(\theta_1 - \theta_A) + (\hat{p}^B_{A1} - c_A - c_B)(\theta_2 - \theta^*) , \tag{A.26}$$

$$\pi_{22}^{NB(L\&S)} = (p^2_{A2} - c_A)(\theta_B - \theta_2) + (\hat{p}^2_{A2} - c_A)(\theta^* - \theta_1) . \tag{A.27}$$

Then, I have

$$\begin{aligned} p^2_{A1} &= c_A + \frac{1}{3}t - \frac{4}{3}t\theta_A + \frac{2}{3}t\theta^* , \quad p^2_{A2} = c_A - \frac{1}{3}t + \frac{1}{3}c_B + \frac{4}{3}t\theta_B - \frac{2}{3}t\theta^* , \\ \hat{p}^B_{A1} &= c_A + \frac{2}{3}c_B + \frac{1}{3}t + \frac{2}{3}t\theta_B - \frac{4}{3}t\theta^* , \quad \hat{p}^2_{A2} = c_A - \frac{1}{3}t - \frac{2}{3}t\theta_A + \frac{4}{3}t\theta^* . \end{aligned} \tag{A.28}$$

In the first period, a consumer at $\theta^*$is indifferent between A_1B_1 and $A_2 + B_i$, $i = 2, ..., n$, if

$$\begin{aligned} p^B_{A1} + t\theta^* + \delta(\hat{p}^2_{A2} + t(1-\theta^*)) &= p^1_{A2} + c_B + t(1-\theta^*) + \delta(\hat{p}^B_{A1} + t\theta^*) \\ \Rightarrow \theta^* &= \frac{1}{2} + \frac{p^1_{A2} + c_B - p^B_{A1}}{2(1-\delta)t} - \frac{\delta(\hat{p}^2_{A2} - \hat{p}^B_{A1})}{2(1-\delta)t} \\ \Rightarrow \theta^* &= \frac{1}{2} + \frac{p^1_{A2} + c_B - p^B_{A1}}{2t} + \frac{\delta(p^2_{A2} - p^2_{A1})}{4t} + \frac{\delta c_B}{4t} . \end{aligned} \tag{A.29}$$

A consumer at θ_A and at θ_B is indifferent between a long-term contract and a sequence of short-term contracts if

$$\tilde{p}_{A1} + (1+\delta)t\theta_A = p^B_{A1} + t\theta_A + \delta(p^2_{A1} + t\theta_A) \Rightarrow \tilde{p}_{A1} = p^B_{A1} + \delta p^2_{A1}, \tag{A.30}$$

$$\begin{aligned} \tilde{p}_{A2} + (1+\delta)t(1-\theta_B) &= p^1_{A2} + t(1-\theta_B) + \delta(p^2_{A2} + t(1-\theta_B)) \\ \Rightarrow \tilde{p}_{A2} &= p^1_{A2} + \delta p^2_{A2}. \end{aligned} \tag{A.31}$$

Firms maximize the following profit functions with respect to the first period prices.

$$\pi_1^{PB(L\&S)} = (p_{A1}^B - c_A - c_B)\theta^* + \delta(p_{A1}^2 - c_A)\theta_1 + \delta(\hat{p}_{A1}^B - c_A - c_B)(\theta_2 - \theta^*), \tag{A.32}$$

$$\pi_2^{NB(L\&S)} = (p_{A2}^1 - c_A)(1-\theta^*) + \delta(p_{A2}^2 - c_A)(1-\theta_2) + \delta(\hat{p}_{A2}^2 - c_A)(\theta^* - \theta_1). \tag{A.33}$$

The equilibrium prices are

$$\begin{aligned}
\tilde{p}_{A1} &= c_A + c_B - \frac{1}{4}\delta c_B + t + 2\psi c_B - \delta\psi c_B + \delta\left(\frac{1}{2}t + c_A\right), \\
\tilde{p}_{A2} &= c_A - \frac{1}{4}\delta c_B + t - 2\psi c_B + \delta\psi c_B + \delta\left(\frac{1}{2}t + c_A + \frac{1}{2}c_B\right), \\
p_{A1}^B &= c_A + c_B - \frac{1}{4}\delta c_B + t + 2\psi c_B - \delta\psi c_B, \\
p_{A2}^1 &= c_A - \frac{1}{4}\delta c_B + t - 2\psi c_B + \delta\psi c_B, \\
p_{A1}^2 &= \frac{1}{2}t + c_A, \quad p_{A2}^2 = \frac{1}{2}t + c_A + \frac{1}{2}c_B, \\
\hat{p}_{A1}^B &= \frac{1}{4}t + c_A + \frac{3}{4}c_B - \psi c_B, \quad \hat{p}_{A2}^2 = \frac{1}{4}t + c_A + \psi c_B, \quad where\ \psi = \frac{3\delta}{24 - 8\delta}.
\end{aligned} \tag{A.34}$$

By substituting the optimal prices into eq. (A.24) and eq. (A. 25), I can derive $\{\theta_1, \theta_2\}$. I can also get $\{\theta_A, \theta_B\}$ from the first order conditions of eq. (A.32) and eq. (A.33) at $\frac{\partial\theta^*}{\partial p_{Ai}^1} = 0$, $i = 1, 2$, where first-period sales are unaffected if firm A1 and A2 increase p_{A1}^B and p_{A2}^1, but decrease $\tilde{p}_{A1}$ and $\tilde{p}_{A2}$ by the same amount, respectively.

$$\begin{aligned}
\theta_A &= \frac{1}{8} + \frac{\psi c_B}{2t}, \quad \theta_1 = \frac{3}{8} + \frac{\psi c_B}{2t}, \quad \theta^* = \frac{1}{2} + \frac{\psi c_B}{t}, \\
\theta_2 &= \frac{5}{8} - \frac{c_B}{8t} + \frac{\psi c_B}{2t}, \quad \theta_B = \frac{7}{8} + \frac{c_B}{8t} + \frac{\psi c_B}{2t}, \quad where\ \psi = \frac{3\delta}{24 - 8\delta}.
\end{aligned} \tag{A.35}$$

The equilibrium profits are

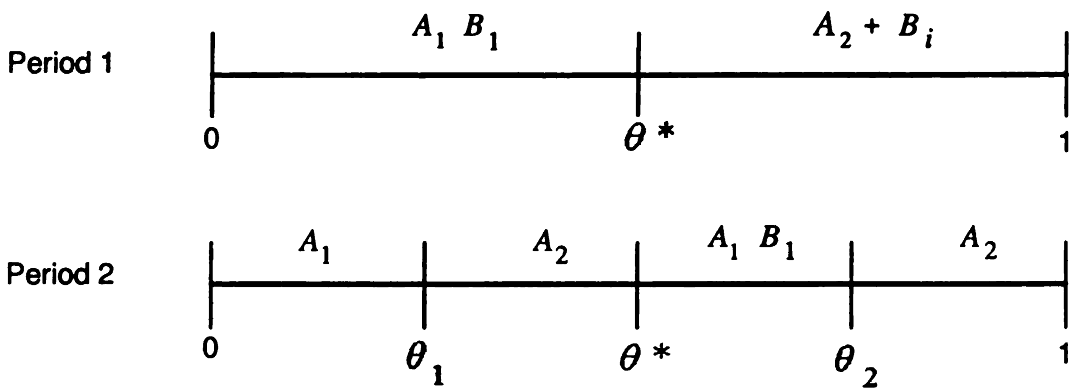
$$\pi_1^{PB(L\&S)} = \left(1+\frac{7}{16}\delta\right)\frac{t}{2}+\left(2\psi-\frac{1}{2}\delta\psi-\frac{3}{16}\delta\right)c_B + \left(\frac{1}{32}\delta+2\psi^2-\frac{1}{2}\delta\psi^2\right)\frac{c_B^2}{t}, \quad (A.36)$$

$$\pi_2^{NB(L\&S)} = \left(1+\frac{7}{16}\delta\right)\frac{t}{2}-\left(2\psi-\frac{1}{2}\delta\psi-\frac{1}{8}\delta\right)c_B + \left(\frac{1}{16}\delta+2\psi^2-\frac{1}{2}\delta\psi^2\right)\frac{c_B^2}{t}, \quad (A.37)$$

$$where\ \psi=\frac{3\delta}{24-8\delta}.$$

A.3 Long-term contract

Consumers in $[0,\ \theta^*]$ buy the bundle A_1B_1 and customers in $[\theta^*,\ 1]$ purchase A_2 and B_i, where $i=2,....,n$, in the first period. In the second period, consumers in the intervals $[\theta_1,\ \theta^*]$ and $[\theta^*,\ \theta_2]$ are targeted with discounts by competitors and switch between brands, purchasing A_2 and A_1B_1, respectively. On the other hand, consumers in the intervals $[0,\ \theta_1]$ and $[\theta_2,\ 1]$ do not switch their suppliers.



In the second period, given θ^*, a customer is indifferent between A_1 and A_2 at θ_1 and between A_1B_1 and A_2 at θ_2 if

$$p_{A1}^2+t\theta_1=\hat{p}_{A2}^2+s_1+t(1-\theta_1)\Rightarrow\theta_1=\frac{1}{2t}(t+\hat{p}_{A2}^2-p_{A1}^2+s_1)\ , \quad (A.38)$$

$$\hat{p}_{A1}^B+s_2+t\theta_2=p_{A2}^2+t(1-\theta_2)\Rightarrow\theta_2=\frac{1}{2t}(t+p_{A2}^2-\hat{p}_{A1}^B-s_2)\ , \quad (A.39)$$

where p^2_{A1} and p^2_{A2} are the second period prices of A_1 and A_2 for loyal customers of firm A1 and A2, respectively. $\hat{p}^B_{A1}$ and $\hat{p}^2_{A2}$ are the second period prices of A_1B_1 and A_2 for customers who switch between providers. s_i , $i = 1, 2$, is an early termination fee that customers who breach a long-term contract are obligated to pay. By maximizing the following profit functions with respect to poaching prices,

$$\max\ (\hat{p}^B_{A1} - c_A - c_B)(\theta_2 - \theta^*) , \tag{A.40}$$
$$\max\ (\hat{p}^2_{A2} - c_A)(\theta^* - \theta_1) ,$$

I have $\hat{p}^B_{A1}$ and $\hat{p}^2_{A2}$ as follows.

$$\hat{p}^B_{A1} = \frac{1}{2}(t + \beta - s_2 - 2t\theta^* + c_A + c_B) , \ \hat{p}^2_{A2} = \frac{1}{2}(2t\theta^* - t + \alpha + c_A - s_1) . \tag{A.41}$$

In the first period, a consumer at θ^* is indifferent between A_1B_1 and $A_2 + B_i$, $i = 2, ..., n$, if

$$\begin{aligned} & p^B_{A1} - \delta s_1 + t\theta^* + \delta(\hat{p}^2_{A2} + s_1 + t(1 - \theta^*)) \\ & \quad = p^1_{A2} - \delta s_2 + c_B + t(1 - \theta^*) + \delta(\hat{p}^B_{A1} + s_2 + t\theta^*) , \\ \Rightarrow \quad & \theta^* = \frac{1}{2} + \frac{(p^1_{A2} + c_B - p^B_{A1}) - \delta(\hat{p}^2_{A2} - \hat{p}^B_{A1})}{2(1 - \delta)t} , \\ \Rightarrow \quad & \theta^* = \frac{1}{2} + \frac{(p^1_{A2} + c_B - p^B_{A2})}{2t} + \frac{\delta(p^2_{A2} - p^2_{A1})}{4t} - \frac{\delta(s_2 - s_1)}{4t} + \frac{\delta c_B}{4t} . \end{aligned} \tag{A.42}$$

By maximizing the following profits functions with respect to $\{p^B_{A1}, p^2_{A1}\}$ and $\{p^1_{A2}, p^2_{A2}\}$,

$$\begin{aligned} \max\ & (p^B_{A1} - c_A - c_B - \delta s_1)\theta^* \\ & + \delta(p^2_{A1} - c_A)\theta_1 + \delta(\hat{p}^B_{A1} - c_A - c_B)(\theta_2 - \theta^*) + \delta s_1(\theta^* - \theta_1) \\ s.t.\ \ & \theta_1 = \theta^* , \end{aligned} \tag{A.43}$$

$$\begin{aligned} \max\ & (p^1_{A2} - c_A - \delta s_2)(1 - \theta^*) + \delta(p^2_{A2} - c_A)(1 - \theta_2) \\ & + \delta(\hat{p}^2_{A2} - c_A)(\theta^* - \theta_1) + \delta s_2(\theta_2 - \theta^*) \\ s.t.\ \ & \theta_2 = \theta^* . \end{aligned} \tag{A.44}$$

the equilibrium prices are as follows.

$$\begin{aligned}
p^B_{A1} &= t + c_A + c_B + \frac{1}{2}\delta t\,, & p^1_{A2} &= t + c_A + \frac{1}{2}\delta t - \delta c_B\,, \\
p^2_{A1} &= t + c_A\,, & p^2_{A2} &= t + c_A\,, \\
\hat{p}^B_{A1} &= c_A + c_B\,, & \hat{p}^2_{A2} &= c_A\,.
\end{aligned} \tag{A.45}$$

By substituting the optimal prices and the condition $\theta_1 = \theta^* = \theta_2$ into eq.(A.38) and eq. (A.39), the optimal amount of discounts and early termination fees are obtained as follows.

$$d^*_1 = \delta t\,, \quad d^*_2 = \delta t(t - c_B)\,, \tag{A.46}$$

$$s^*_1 = \ t\,, \quad s^*_2 = t - c_B\,. \tag{A.47}$$

The equilibrium profits are as follows.

$$\pi_1^{PB(S)} = \pi_2^{NB(S)} = \left(1+\frac{\delta}{2}\right)\frac{t}{2}\,. \tag{A.48}$$

In market B, firms have no sale in the second period. The market prices of B_1 and B_2 for both periods are the marginal cost of c_B . Each firm makes zero profit.

Appendix B

Appendix: Proof of chapter 1

Proof of Proposition 1.1: $\theta^* = \frac{1}{2} + \frac{p^1_{A2} - p^1_{A1}}{2t} + \frac{\delta(p^2_{A2} - p^2_{A1})}{4t} \Rightarrow \frac{\partial\theta^*}{\partial p^1_{A1}} = -\frac{1}{2t} - \frac{\delta}{4t}\frac{\partial p^2_{A1}}{\partial p^1_{A1}}$
$\Rightarrow$ (i) If $\frac{\partial p^2_{A1}}{\partial p^1_{A1}} = -\frac{2}{\delta}$, then, $\frac{\partial\theta^*}{\partial p^1_{A1}} = 0$.

$$\max(p^1_{A1} - c_A)\theta^* + \delta(p^2_{A1} - c_A)\theta_1 + \delta(\hat{p}^2_{A1} - c_A)(\theta_2 - \theta^*) \tag{B.1}$$

F.O.C. of eq.(B.1) : $\theta^* + (p^1_{A1} - c_A)\frac{\partial\theta^*}{\partial p^1_{A1}} + \delta(p^2_{A1} - c_A)\frac{\partial\theta_1}{\partial p^1_{A1}} + \delta\theta_1\frac{\partial p^2_{A1}}{\partial p^1_{A1}} - \delta(\hat{p}^2_{A1} - c_A)\frac{\partial\theta^*}{\partial p^1_{A1}} = 0$.[1] From (i), we know that $\frac{\partial p^2_{A1}}{\partial p^1_{A1}} = -\frac{2}{\delta} \Leftrightarrow \frac{\partial\theta^*}{\partial p^1_{A1}} = 0$, and an increase in p^1_{A1} has no effect on θ_2. Hence, F.O.C. : $\theta^* + \delta(p^2_{A1} - c_A)\frac{\partial\theta_1}{\partial p^1_{A1}} + \delta\theta_1(-\frac{2}{\delta}) = 0$. By substituting the second period prices of eq.(1.34) and eq.(1.35)into eq.(1.30), I have

$$\theta_1 = \frac{1}{6} + \frac{1}{3}(\theta^* + \theta_A) \Rightarrow \frac{\partial\theta_1}{\partial p^1_{A1}} = \frac{1}{3}\left(\frac{\partial\theta^*}{\partial p^1_{A1}} + \frac{\partial\theta_A}{\partial p^1_{A1}}\right) = \frac{1}{3}\frac{\partial\theta_A}{\partial p^1_{A1}}.$$

$\frac{3(p^2_{A1} - c_A)}{t} = (2\theta^* + 1 - 4\theta_A) \Rightarrow \theta_A = \frac{2\theta^* + 1}{4} - \frac{3(p^2_{A1} - c_A)}{4t}$, $\theta_1 = \frac{1}{4} + \frac{\theta^*}{2} - \frac{p^2_{A1} - c_A}{4t}$. $\frac{\partial\theta_1}{\partial p^1_{A1}} = -\frac{1}{4t}\frac{\partial p^2_{A1}}{\partial p^1_{A1}} = \frac{1}{2\delta t}$. Hence F.O.C. : $\theta^* + \delta(p^2_{A1} - c_A)\frac{1}{2\delta t} + \delta\theta_1(-\frac{2}{\delta}) = 0$ $\Rightarrow p^2_{A1} - c_A = (2\theta_1 - \theta^*)2t$. By substituting $p^2_{A1} - c_A$ into θ_A and θ_1, I have $\theta_A = \frac{1}{2} + \frac{3}{2}\theta^* - 3\theta_1$ and $\theta_1 = \frac{1}{2}(\frac{1}{4} + \theta^*) = \frac{1}{8} + \frac{\theta^*}{2}$. Hence, at $\theta^* = 1/2$, $\theta_1 = 3/8$, $\theta_A = 1/8$. Since firms in market A are symmetry, I can derive $\theta_2 = 5/8$, $\theta_A = 7/8$.

In order to calculate p^1_{A1} and p^1_{A2}, suppose that firm A1 considers increasing both p^1_{A1} and p^{12}_{A1} by the same amount, which will leave $\frac{\partial p^2_{A1}}{\partial p^1_{A1}} = 0$ and hence

[1] $\frac{\partial\hat{\alpha}}{\partial p^1_{A1}} = 0$ by envelope theorem.

$\frac{\partial\theta^*}{\partial p^1_{A1}} = -\frac{1}{2t}$. F.O.C. of eq.(B.1) : $\theta^* + (p^1_{A1} - c_A)\frac{\partial\theta^*}{\partial p^1_{A1}} + \delta(p^2_{A1} - c_A)\frac{\partial\theta_1}{\partial p^1_{A1}} - \delta(\hat{p}^2_{A1} - c_A)\frac{\partial\theta^*}{\partial p^1_{A1}} = 0$. Since $\frac{\partial\theta_1}{\partial p^1_{A1}} = \frac{1}{2}\frac{\partial\theta^*}{\partial p^1_{A1}}$, F.O.C. becomes $\theta^* + (a - c_A)\frac{\partial\theta^*}{\partial p^1_{A1}} = \delta\left((\hat{p}^2_{A1} - c_A) - \frac{1}{2}(p^2_{A1} - c_A)\right)\frac{\partial\theta^*}{\partial p^1_{A1}}$. The right side is zero in a symmetric equilibrium and hence $p^1_{A1} = p^1_{A2} = t + c_A$.

By substituting cutoff points into eq.(1.34) and eq.(1.35), I can find the second period prices.

Proof of Proposition 1.2: (i) (1) F.O.C. $\{p^1_{A1}\}$: $\theta^* - \frac{1}{2t}(p^1_{A1} - c_A - \delta s_1) - \frac{\delta}{4t}(p^2_{A1} - c_A) + \frac{\delta}{4t}(\hat{p}^2_{A1} - c_A) = 0$.

(2) F.O.C. $\{p^2_{A1}\}$: $\delta[\theta_1 - \frac{1}{4t}(p^2_{A1} - c_A) - \frac{\delta}{8t}(p^2_{A1} - c_A) - \frac{1}{4t}(p^1_{A1} - c_A - \delta s_1) + \frac{\delta}{8t}(\hat{p}^2_{A1} - c_A)] = 0$.

(3) F.O.C. $\{s_1\}$: $\delta[-\theta^* + \frac{1}{4t}(p^1_{A1} - c_A - \delta s_1) + \frac{1}{4t}(p^2_{A1} - c_A) + \frac{\delta}{8t}(p^2_{A1} - c_A) - \frac{\delta}{8t}(\hat{p}^2_{A1} - c_A)] = 0$.

By (1) and (2), in a symmetric equilibrium, $p^2_{A1} = t + c_A$ and $p^2_{A2} = t + c_A$. By substituting the equilibrium price of p^2_{A1} and p^2_{A2} into eq.(1.44) and eq. (1.45) with the condition $\theta_1 = \theta_2 = \theta^*$, I have $s^*_1 = s^*_2 = t$.

Proof of Proposition 1.3: (i) Customers in the interval $[\theta_1, \theta^*]$ and $[\theta^*, \theta_2]$ switch between brands. $\theta^* - \theta_1 = \frac{1}{6} - \frac{c_B}{6t} = \theta_2 - \theta^*$. Hence, consumer poaching= $\frac{1}{3}(1 - \frac{c_B}{t})$, where $t > c_B$.

Proof of Proposition 1.4: (i) $\theta_1 = \frac{1}{3} + \frac{c_B}{6t} \le \theta^* = 1/2$, if $t \le c_B$. In other words, customers are locked-in after the first purchase if $t \le c_B$.

Proof of Proposition 1.5: (i) $\theta^* = \frac{1}{2} + \frac{p^B_{A2} - p^B_{A1}}{2t} + \frac{\delta(p^2_{A2} - p^2_{A1})}{4t} \Rightarrow \frac{\partial\theta^*}{\partial p^B_{A1}} = -\frac{1}{2t} - \frac{\delta}{4t}\frac{\partial p^2_{A1}}{\partial p^B_{A1}} \Rightarrow$ (i) If $\frac{\partial p^2_{A1}}{\partial p^B_{A1}} = -\frac{2}{\delta}$, then $\frac{\partial\theta^*}{\partial p^B_{A1}} = 0$.

$$\max(p^B_{A1} - c_A - c_B)\theta^* + \delta(p^2_{A1} - c_A)\theta_1 + \delta(\hat{p}^B_{A1} - c_A - c_B)(\theta_2 - \theta^*) \qquad \text{(B.2)}$$

F.O.C. of eq.(B.2) : $\theta^* + (p^B_{A1} - c_A - c_B)\frac{\partial\theta^*}{\partial p^B_{A1}} + \delta(p^2_{A1} - c_A)\frac{\partial\theta_1}{\partial p^B_{A1}} + \delta\theta_1\frac{\partial p^2_{A1}}{\partial p^B_{A1}} - \delta(\hat{p}^B_{A1} - c_A - c_B)\frac{\partial\theta^*}{\partial p^B_{A1}} = 0$. From (i), we know that $\frac{\partial p^2_{A1}}{\partial p^B_{A1}} = -\frac{2}{\delta} \Leftrightarrow \frac{\partial\theta^*}{\partial p^B_{A1}} = 0$. Hence, F.O.C. : $\theta^* + \delta(p^2_{A1} - c_A)\frac{\partial\theta_1}{\partial p^B_{A1}} + \delta\theta_1(-\frac{2}{\delta}) = 0$. By substituting the second period prices of eq.(1.75) into eq.(1.71), I have

$$\theta_1 = \frac{1}{6} + \frac{1}{6t}c_B + \frac{1}{3}(\theta^* + \theta_A) \Rightarrow \frac{\partial \theta_1}{\partial p_{A1}^B} = \frac{1}{3}\left(\frac{\partial \theta^*}{\partial p_{A1}^B} + \frac{\partial \theta_A}{\partial p_{A1}^B}\right) = \frac{1}{3}\frac{\partial \theta_A}{\partial p_{A1}^B}. \quad \text{(B.3)}$$

$\frac{3(p_{A1}^2 - c_A)}{t} = (2\theta^* + 1 - 4\theta_A) + \frac{c_B}{t} \Rightarrow \theta_A = \frac{2\theta^*+1}{4} - \frac{3(p_{A1}^2 - c_A)}{4t} + \frac{c_B}{4t} \Rightarrow \frac{\partial \theta_A}{\partial p_{A1}^B} = \frac{-3}{4t}\frac{\partial p_{A1}^2}{\partial p_{A1}^B}$, $\theta_1 = \frac{1}{4} + \frac{\theta^*}{2} - \frac{p_{A1}^2 - c_A}{4t} + \frac{c_B}{4t}$. Hence, I have $\frac{\partial \theta_1}{\partial p_{A1}^B} = \frac{1}{2\delta t}$. F.O.C. : $\theta^* + \delta(p_{A1}^2 - c_A)\frac{1}{2\delta t} + \delta\theta_1(-\frac{2}{\delta}) = 0 \Rightarrow p_{A1}^2 - c_A = (2\theta_1 - \theta^*)2t$. By substituting $p_{A1}^2 - c_A$ into θ_A and θ_1, I have, at $\theta^* = \frac{1}{2}$, $\theta_1 = \frac{3}{8}(1 + \frac{c_B}{3t})$, $\theta_A = \frac{1}{8}(1 - \frac{c_B}{t})$. Since firms in market A are symmetry, I can derive $\theta_2 = \frac{5}{8}(1 + \frac{c_B}{5t})$, $\theta_B = \frac{7}{8}(1 + \frac{c_B}{7t})$.

In order to calculate p_{A1}^B and p_{A2}^B, suppose that firm A1 considers increasing both p_{A1}^B and $\bar{p}_{A1}$ by the same amount, which will leave $\frac{\partial p_{A1}^2}{\partial p_{A1}^B} = 0$ and hence $\frac{\partial \theta^*}{\partial p_{A1}^B} = -\frac{1}{2t}$. F.O.C. of eq.(B.2) : $\theta^* + (p_{A1}^B - c_A - c_B)\frac{\partial \theta^*}{\partial p_{A1}^B} + \delta(p_{A1}^2 - c_A)\frac{\partial \theta_1}{\partial p_{A1}^B} - \delta(\hat{p}_{A1}^B - c_A - c_B)\frac{\partial \theta^*}{\partial p_{A1}^B} = 0$.[2] Since $\frac{\partial \theta_1}{\partial p_{A1}^B} = \frac{1}{2}\frac{\partial \theta^*}{\partial p_{A1}^B}$, F.O.C. becomes $\theta^* + (p_{A1}^B - c_A - c_B)\frac{\partial \theta^*}{\partial p_{A1}^B} = \delta\left((\hat{p}_{A1}^B - c_A - c_B) - \frac{1}{2}(p_{A1}^2 - c_A)\right)\frac{\partial \theta^*}{\partial p_{A1}^B} = -\frac{\delta c_B}{2}\frac{\partial \theta^*}{\partial p_{A1}^B}$. In a symmetric equilibrium, I have $p_{A1}^B = p_{A2}^B = t + c_A + c_B - \frac{\delta}{2}c_B$.

Proof of Proposition 1.6: Each firm has an incentive to bundle and ends up being in {bundling, bundling} situation (see Table 1.4).

Proof of Proposition 1.7: (ii) (1) F.O.C. $\{p_{A1}^B\}$: $\theta^* - \frac{1}{2t}(p_{A1}^B - c_A - c_B - \delta s_1) - \frac{\delta}{4t}(p_{A1}^2 - c_A) + \frac{\delta}{4t}(\hat{p}_{A1}^B - c_A - c_B) = 0$. (2) F.O.C. $\{p_{A1}^2\}$: $\delta[\theta_1 - \frac{1}{4t}(p_{A1}^2 - c_A) - \frac{\delta}{8t}(p_{A1}^2 - c_A) - \frac{1}{4t}(p_{A1}^B - c_A - c_B - \delta s_1) + \frac{\delta}{8t}(\hat{p}_{A1}^B - c_A - c_B)] = 0$. (3) F.O.C. $\{s_1\}$: $\delta[-\theta^* + \frac{1}{4t}(p_{A1}^B - c_A - c_B - \delta s_1) + \frac{1}{4t}(p_{A1}^2 - c_A) + \frac{\delta}{8t}(p_{A1}^2 - c_A) - \frac{\delta}{8t}(\hat{p}_{A1}^B - c_A - c_B)] = 0$. By (1) and (2), $p_{A1}^2 = t + c_A$ and in a symmetric equilibrium, $p_{A2}^2 = t + c_A$. By substituting the optimal prices into eq.(1.89) and eq. (1.90) with $\theta_1 = \theta_2 = \theta^*$, I have $s_1^* = s_2^* = t - c_B$. □

[2] $\frac{\partial \hat{\alpha}}{\partial p_{A1}^1} = 0$ by envelope theorem.

Appendix C

Appendix: Model with homogenous valuations of good B

I consider that consumers have homogenous valuations of good B, $v > c_B$. For simplicity, I assume that each consumer in market A purchases at most one unit of good B. That is, consumers buy one unit of good/service A.

C.1 Pure Bundling

The integrated firm 1 bundles its products A_1 and B_1 in this sub-game. There are A_1B_1, A_2, and B_2 available in the markets. Since firm 1 offers only the bundle of A_1B_1, firm B2 is only one producing good B. However, it cannot enjoy a monopoly profit since there exists the bundle of A_1B_1. On the other hand, firm A2 competes against A_1B_1, rather than A_1 . Hence, pricing decision of firm A2 depends on firm B1 as well as firm A1 in pure bundling.

A consumer is indifferent between A_1B_1 and $A_2 + B_2$ at x_1 when $\tilde{p} + tx_1 = p_{A2} + p_{B2} + t(1 - x_1)$. $\tilde{p}$ is the price of A_1B_1 . p_{A2} and p_{B2} are the prices of A_2 and B_2 , respectively. The profit functions are

$$\pi_1 = \frac{1}{2t}(\tilde{p} - c_A - c_B)(t + p_{A2} + p_{B2} - \tilde{p}) , \tag{C.1}$$
$$\pi_{A2} = \frac{1}{2t}(p_{A2} - c_A)(t - p_{A2} - p_{B2} + \tilde{p}) , \tag{C.2}$$
$$\pi_{B2} = \frac{1}{2t}(p_{B2} - c_B)(t - p_{A2} - p_{B2} + \tilde{p}) , \tag{C.3}$$

The equilibrium prices and profits in pure bundling are

$$\tilde{p} = \frac{5}{4}t + c_A + c_B\,, \quad p_{A2} = \frac{3}{4}t + c_A\,, \quad p_{B2} = \frac{3}{4}t + c_B\,, \tag{C.4}$$

$$\pi_1 = \frac{25}{32}t\,, \quad \pi_{A2} = \frac{9}{32}t\,, \quad \pi_{B2} = \frac{9}{32}t\,. \tag{C.5}$$

Note that in equilibrium, $p_{A2}^{NB} + p_{B2}^{NB} > \tilde{p}$. Firm B2 charges a price higher than the marginal cost. The profit margin for firm 1 is higher in pure bundling than in no bundling: $p_{A1}^{NB} + p_{B1}^{NB} < \tilde{p}$. These results can be explained by competition softening effect in market B resulting from a product differentiation role played by bundling. However, the profit margin for firm A2 is lower in pure bundling than in no bundling. Since firm A2 competes against $A_1 B_1$ rather than A_1, firm A2 has to reduce the price of A_2 as firm B2 sets the price of B_2 at a price higher than the marginal cost of good B. No substitution effect of bundling occurs because consumer's valuation of good B is homogeneous and is high enough that all consumers in market A buy one unit of good B.

Proposition C.1 When firm 1 offers only a package of $A_1 B_1$, the profits of firms 1 and B2 are higher than in no bundling, while that of firm A2 is lower than in no bundling.

Proof: It is clear from direct comparisons of profits and prices.

The intuition behind proposition A.1 is that bundling has a business stealing effect as a result of a product differentiation role of bundling in market B.

C.2 Mixed-Bundling

Firm 1 offers A_1 as well as $A_1 B_1$. A consumer at x_1 is indifferent between the bundle $A_1 B_1$ and $A_2 + B_2$ when

$$\tilde{p} + t x_1 = p_{A2} + p_{B2} + t(1 - x_1) \Rightarrow x_1 = \frac{1}{2t}(t + p_{A2} + p_{B2} - \tilde{p}). \tag{C.6}$$

A consumer at x_2 is indifferent between $A_1 + B_2$ and $A_2 + B_2$ when

$$p_{A1} + p_{B2} + tx_2 = p_{A2} + p_{B2} + t(1 - x_2) \Rightarrow x_2 = \frac{1}{2t}(t + p_{A2} - p_{A1}). \quad \text{(C.7)}$$

And also, firm 1 can sell positive amounts of A_1B_1 if $p_{A1} + p_{B2} \geq \tilde{p}$. Otherwise, consumers would buy $A_1 + B_2$ rather than the bundle of A_1B_1. From eq.(C.6) and eq.(C.7), I have $x_1 \geq x_2$ under $p_{A1} + p_{B2} \geq \tilde{p}$. That is, by offering A_1 separately to consumers who value product A highly and the bundle of A_1B_1 at a discounted price to the consumers who have less preference over A_1, firm 1 can gain an additional market share of $x_1 - x_2$. The demand of the bundle is $\tilde{q} = \frac{1}{2t}(p_{A1} + p_{B2} - \tilde{p})$, the demand of A_1 is $q_{A1} = \frac{1}{2t}(t + p_{A2} - p_{A1})$, the market share of firm A2 is $q_{A2} = \frac{1}{2t}(t - p_{A2} - p_{B2} + \tilde{p})$, and that of firm B2 is $q_{B2} = 1 - \tilde{q} = \frac{1}{2t}(2t - p_{A1} - p_{B2} + \tilde{p})$. Profit functions of firms are as follows:

$$\begin{aligned}
\pi_1^{MB} &= \frac{1}{2t}(\tilde{p} - c_A - c_B)(p_{A1} + p_{B2} - \tilde{p}) + \frac{1}{2t}(p_{A1} - c_A)(t + p_{A2} - p_{A1}) && \text{(C.8)} \\
\pi_{A2}^{MB} &= \frac{1}{2t}(p_{A2} - c_A)(t - p_{A2} - p_{B2} + \tilde{p}), && \text{(C.9)} \\
\pi_{B2}^{MB} &= \frac{1}{2t}(p_{B2} - c_B)(2t - p_{A1} - p_{B2} + \tilde{p}), && \text{(C.10)}
\end{aligned}$$

The equilibrium prices and profits are

$$\begin{aligned}
\tilde{p} &= \frac{9}{8}t + c_A + c_B\,, \quad p_{A1} = \frac{11}{8}t + c_A\,, \quad p_{A2} = \frac{5}{8}t + c_A\,, \quad p_{B2} = \frac{7}{8}t + c_B\,, \\
\pi_1^{MB} &= \frac{103}{128}t\,, \quad \pi_{A2}^{MB} = \frac{25}{128}t\,, \quad \pi_{B2}^{MB} = \frac{49}{128}t. && \text{(C.11)}
\end{aligned}$$

Note that the bundle price is less than $p_{A1} + p_{B2}$ and $p_{A2} + p_{B2}$. The price of A_2 is less than in pure bundling, while the price of B_2 is higher than in pure bundling. The reason is that the integrated firm 1 becomes aggressive in price by offering discounts for the bundle option to consumers who have less preference over A_1, while the bundle option softens competition in market B. Note also that with homogeneous valuations of good B, no substitution effect of bundling arises in market A.

Proposition C.2 By offering mixed bundling, firm 1 makes higher profits than in either no bundling or pure bundling. Firm A2 is worse off. Firm B2 is better off with mixed bundling. (i) $\pi_1^{MB} > \pi_1^{PB} > \pi_1^{NB}$, (ii) $\pi_{A2}^{MB} < \pi_{A2}^{PB} < \pi_{A2}^{NB}$, (iii) $\pi_{B2}^{MB} > \pi_{B2}^{PB} > \pi_{B2}^{NB}$

Proof: It is clear from direct comparisons of equilibrium profits under alternative scenarios.

The idea behind proposition A.2 is that mixed bundling acts as a price discriminatory device based on consumers' preferences.

C.3 Sub-game perfect equilibrium

With homogeneous valuations of both goods, the outcomes of (partial) mixed bundling arise in equilibrium.

Appendix D

Appendix: Proof of Chapter 2

Proof of proposition 2.1: $\pi_1 = \frac{1}{200t}(11t - 2c_B)^2 = \frac{101}{200}t - \frac{11}{50}c_B + \frac{1}{50t}c_B^2 \geq \frac{t}{2}$, iff $t \geq 2c_B$. $\pi_{A2}^{MB} = \frac{1}{200t}(9t + 2c_B)^2 < \frac{t}{2} \Leftrightarrow (9t + 2c_B)^2 \leq 100t^2 = (10t)^2 \Leftrightarrow 9t + 2c_B \leq 10t$, iff $t \geq 2c_B$. $\pi_{B2}^{MB} = \frac{1}{100t}(3t - c_B)^2 > 0$, iff $t \geq \frac{1}{3}c_B$. $t \geq 2c_B > \frac{1}{3}c_B \rightarrow \pi_{B2} > 0$. if $t < \frac{5-4c_B}{3}$, I have $p_{B2}^* < 1 \Rightarrow$ If $2c_B \leq t < \frac{5-4c_B}{3}$, where $c_B < \frac{1}{2}$, firm B2 would set its price such as $c_B < p_{B2} < 1$ and firm 1 makes higher profits in pure bundling than no bundling.

Proof of proposition 2.2: $\pi_1 = \frac{1}{2t}(t + \frac{1}{6} - \frac{1}{3}c_B)^2 \geq \frac{t}{2}$ and $\pi_{A2} = \frac{1}{2t}(t - \frac{1}{6} + \frac{1}{3}c_B)^2 \leq \frac{t}{2}$, iff $c_B \leq \frac{1}{2}$. $\pi_{B2} = \frac{1}{4t}(1 - c_B)(t - \frac{2}{3} + \frac{1}{3}c_B) \geq 0$, iff $t \geq \frac{1}{3}(2 - c_B)$.

Proof of proposition 2.3: (i) $\pi_{B2}^{MB} - \pi_{B2}^{NB} = \frac{1}{4t}(\frac{7}{9}t - \frac{1}{9}c_B)^2 - 0 \geq 0$, if $t \geq \frac{1}{7}c_B$. $\pi_1^{MB} - \pi_1^{NB} = \frac{1}{25t}(t^2 - \frac{9}{4}c_B t + \frac{7}{8}c_B{}^2) \geq 0$, iff $t \leq \frac{1}{2}c_B$ or $t \geq \frac{7}{4}c_B$. $\pi_{A2}^{MB} - \pi_{A2}^{NB} = \frac{1}{2t}(\frac{4}{5}t + \frac{1}{10}c_B)^2 - \frac{t}{2} = \frac{1}{2t}(t + \frac{1}{10}(c_B - 2t))^2 - \frac{t}{2} \geq 0$, if $t \leq \frac{1}{2}c_B$.

(ii) $\pi_1^{PB} - \pi_1^{NB} < 0$, if $t < \frac{1}{3}c_B$. $\pi_1^{MB} - \pi_1^{NB} \geq 0$, if $t \leq \frac{1}{2}c_B$ or $t \geq \frac{7}{4}c_B$. Hence, if $t \leq \frac{1}{7}c_B$, $\pi_1^{PB} < \pi_1^{NB} \leq \pi_1^{MB}$. $\pi_{A2}^{PB} - \pi_{A2}^{MB} = \frac{1}{2t}(t + \frac{1}{6}c_B)^2 - \frac{1}{2t}(\frac{4}{5}t + \frac{1}{10}c_B)^2 > 0$, for all c_B and t .

Proof of proposition 2.4: $\pi_{B2}^{MB} - \pi_{B2}^{NB} = \frac{1}{4t}(\frac{7}{9}t - \frac{1}{9}c_B)^2 - 0 \geq 0$,if $t \geq \frac{1}{7}c_B$. $\pi_1^{MB} - \pi_1^{NB} = \frac{85}{648t}(t^2 - \frac{104}{85}c_B t + \frac{28}{85}c_B{}^2) \geq 0$, if $t \leq \frac{2}{5}c_B$ or $t \geq \frac{14}{17}c_B$. And, $\pi_{A2}^{MB} - \pi_{A2}^{NB} = \frac{1}{2t}(t - \frac{5}{18}t + \frac{2}{18}c_B)^2 - \frac{t}{2} \geq 0$ if $t \leq \frac{2}{5}c_B$. Now, let's check the condition of $\tilde{p} \leq p_{A1} + p_{B2} \Leftrightarrow \frac{23}{18}t + c_A + \frac{8}{9}c_B \leq \frac{4}{9}t + c_A + \frac{2}{9}c_B + \frac{7}{9}t + \frac{8}{9}c_B$ if $t \leq 4c_B$. Hence, when $\frac{1}{7}c_B \leq t \leq \frac{2}{5}c_B$, or $\frac{14}{17}c_B \leq t \leq \min\{4c_B, \frac{9-8c_B}{7}\}$, where $c_B \leq \frac{17}{26}$, mixed bundling arises in equilibrium.

Proof of proposition 2.5: $\pi_1^{MB} - \pi_1^{PB} = (\frac{409}{648t}t^2 - \frac{26}{162t}c_B t + \frac{7}{162}\frac{c_B^2}{t}) - (\frac{1}{200t}(11t -$

$2c_B)^2) \geq 0$, for all t and c_B. $\pi_{A2}^{MB} - \pi_{A2}^{PB} = \frac{1}{2t}(\frac{13}{18}t + \frac{1}{9}c_B)^2 - \frac{1}{2t}(\frac{9}{10}t + \frac{2}{10}c_B)^2 < 0$, for all t and c_B. $\pi_{B2}^{MB} - \pi_{B2}^{PB} = \frac{1}{4t}(\frac{7}{9}t - \frac{1}{9}c_B)^2 - \frac{1}{100t}(3t - c_B)^2 \geq 0$ if $t \geq \frac{7}{31}c_B$. Hence, if $\frac{1}{7}c_B \leq t \leq \frac{7}{31}c_B$, $\pi_{B2}^{MB} - \pi_{B2}^{PB} \leq 0$. If $\frac{7}{31}c_B \leq t \leq \frac{2}{5}c_B$ or If $\frac{14}{17}c_B \leq t < \min\{4c_B, \frac{9-8c_B}{7}\}$, where $c_B < \frac{17}{26}$, $\pi_{B2}^{MB} - \pi_{B2}^{PB} \geq 0$

Proof of proposition 2.6: (i) For $\tilde{p} \leq p_{A1}+p_{B2}$, I need $t \leq \frac{7-4c_B}{6}$. $\pi_{B2}^{MB} - \pi_{B2}^{NB} = \frac{1}{4t}(1 - c_B)(\frac{7}{5}t - \frac{3}{5}c_B - \frac{4}{5}) \geq 0$, if $\frac{4-3c_B}{7} \leq t$. $\pi_1^{MB} - \pi_1^{NB} = \frac{1}{25t}(t^2 + (\frac{9-18c_B}{4})t + (\frac{7}{2}c_B^2 - \frac{7}{2}c_B + \frac{7}{8})) > 0$ if $c_B \leq \frac{1}{2}$. Firm 1 also makes higher profits in mixed bundling than in no bundling if $t \geq \frac{7(2c_B-1)}{4}$ or $t \leq c_B - \frac{1}{2}$, where $c_B > \frac{1}{2}$. Hence if $\max\{\frac{4-3c_B}{7}, \frac{7(2c_B-1)}{4}\} \leq t \leq \frac{7-4c_B}{6}$, where $\frac{1}{2} < c_B \leq \frac{7}{10}$, or if $\frac{4-3c_B}{7} \leq t \leq \min\{\frac{7-4c_B}{6},\ c_B - \frac{1}{2}\} = c_B - \frac{1}{2}$, where $\frac{3}{4} \leq c_B$, then $\pi_1^{MB} - \pi_1^{NB} \geq 0$.

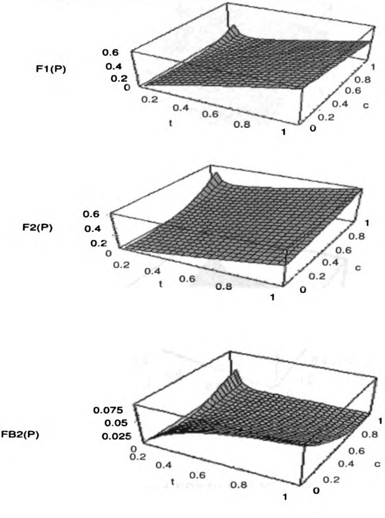
Firm A2 makes more profits in mixed bundling than in no bundling, $\pi_{A2}^{MB} - \pi_{A2}^{NB} = \frac{1}{2t}(\frac{4}{5}t + \frac{1}{5}c_B - \frac{1}{10})^2 - \frac{t}{2} = \frac{1}{2t}(t + \frac{1}{5}(c_B - t - \frac{1}{2}))^2 - \frac{t}{2} \geq 0$, iff $c_B - \frac{1}{2} \geq t \geq \frac{4-3c_B}{7}$ where $c_B \geq \frac{3}{4}$. Otherwise, firm A2 is strictly better off in pure bundling than in mixed bundling. (ii) It is clear that $\pi_1^{MB} - \pi_1^{PB} = \frac{1}{25t}(t^2 + (\frac{9-18c_B}{4})t + (\frac{7}{2}c_B^2 - \frac{7}{2}c_B + \frac{7}{8})) - \frac{1}{2t}(t + \frac{1}{6} - \frac{1}{3}c_B)^2 = \frac{1}{25t}(t^2 - (\frac{23(1-2c_B)}{12})t + \frac{19}{9}(c_B - \frac{1}{2})^2) \geq 0$, if $c_B \geq \frac{1}{2}$. Hence, combining the conditions for firm 1 and B2, (*) if $\frac{4-3c_B}{7} \leq t \leq \frac{7-4c_B}{6}$, where $c_B \geq \frac{1}{2}$, then, $\pi_1^{MB} - \pi_1^{PB} \geq 0$. And also, $\pi_1^{MB} - \pi_1^{PB} \geq 0$, if $t \leq \frac{1}{3}(1 - 2c_B)$ or $t \geq \frac{19(1-2c_B)}{12}$, where $c_B < \frac{1}{2}$. Combining these conditions with the condition for firm B2 to set $p_{B2}^* = 1$, I derive conditions with which firm 1 is better off in mixed bundling than in pure bundling, (**) $t_3 \leq t \leq \frac{7-4c_B}{6}$, where $t_3 = max\{\frac{4-3c_B}{7}, \frac{19(1-2c_B)}{12}\}$, $c_B < \frac{1}{2}$. (On the other hand, $\frac{4-3c_B}{7} \leq t \leq \min\{\frac{7-4c_B}{6},\ \frac{1}{3}(1 - 2c_B)\}$, where $c_B \geq \frac{1}{2}$, cannot hold because $\frac{4-3c_B}{7} > \frac{1}{3}(1 - 2c_B)$).

Firm A2 makes less in mixed bundling than in pure bundling. $\pi_{A2}^{PB} - \pi_{A2}^{MB} = \frac{1}{2t}((t - \frac{1}{6} + \frac{1}{3}c_B)^2 - (\frac{4}{5}t + \frac{1}{5}c_B - \frac{1}{10})^2)) > 0$. $sign[\pi_{A2}^{PB} - \pi_{A2}^{MB}] = sign[3t + 2c_B - 1] > 0$, under (*) and (**). Firm B2 makes higher profits in mixed bundling than in pure bundling, $\pi_{B2}^{MB} - \pi_{B2}^{PB} = \frac{1}{30t}(1 - c_B)(3t + 2c_B - 1) > 0$, under (*) and (**).

Appendix E

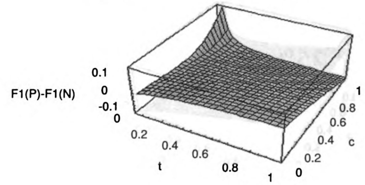
Appendix: Figures of Chapter 2

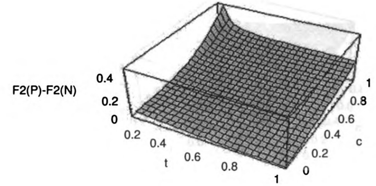
Figure E.1: Profits of firms in Pure Bundling

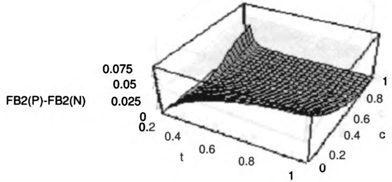


Note: F1(P), F2(P), and FB2(P) are profits of firms 1, 2, and B2 respectively.

Figure E.2: Pure vs. No Bundling

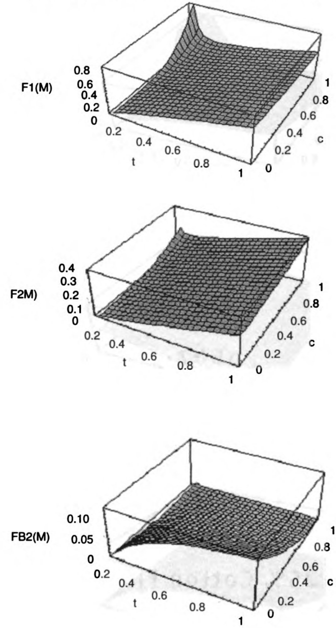






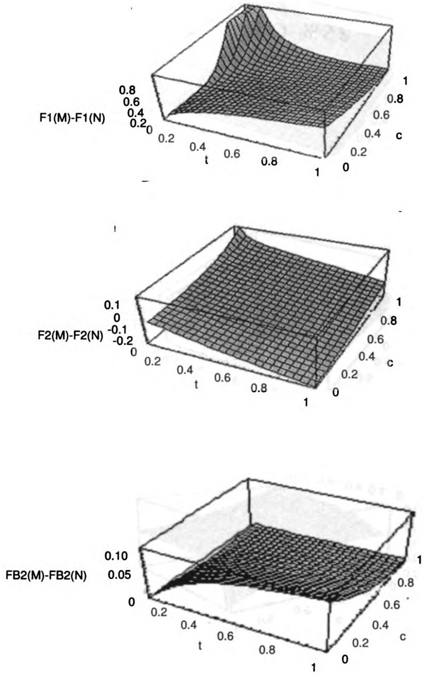
Note: F1(P)-F1(N), F2(P)-F2(N), and FB2(P)-FB2(N) are firm 1's, firm 2's, and firm B2's profit difference between pure bundling and no bundling respectively.

Figure E.3: Profits of firms in Mixed Bundling



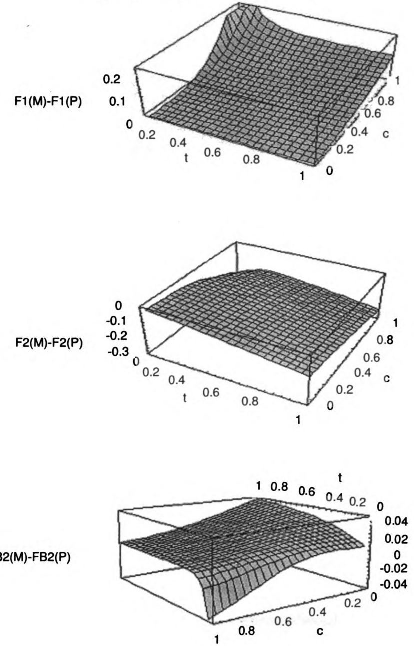
Note: F1(M), F2(M), and FB2(M) are profits of firm 1, firm 2, and firm B2 respectively.

Figure E.4: Mixed vs. No Bundling



Note: F1(M)-F1(N), F2(M)-F2(N), and FB2(M)-FB2(N) are firm 1's, firm 2's, and firm B2's profit difference between mixed bundling and no bundling respectively.

Figure E.5: Mixed vs.Pure Bundling

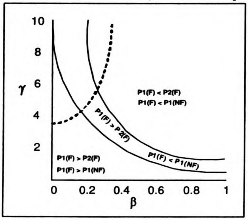


Note: F1(M)-F1(P), F2(M)-F2(P), and FB2(M)-FB2(P) are firm 1's, firm 2's, and firm B2's profit difference between mixed bundling and pure bundling respectively.

Appendix F

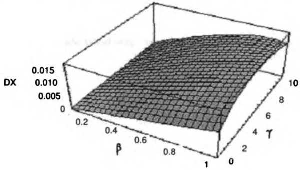
Appendix: Figures of Chapter 3

Figure F.1: Under Firewalls, Profits Comparison



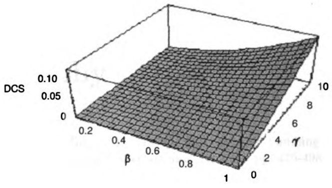
Note: P1(F) and P2(F) are profits of firm 1 and D2 under Firewalls, respectively. P1(NF) and P2(NF) are profits of firm 1 and D2 under Information Flow, respectively.

Figure F.2: Total R&D Level Comparison (Firewalls vs. Information Transfer)



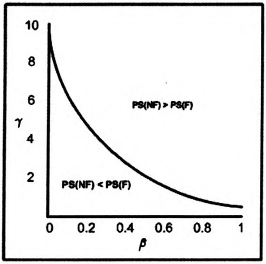
Note: DX is R&D(Firewalls)-R&D(Information Flow)

Figure F.3: Consumers Surplus Comparison (Firewalls vs. Information Transfer)



Note: DCS is CS(Firewalls)-CS(Information Flow)

Figure F.4: Producers Comparison (Firewalls vs. Information Transfer)



Note: PS(F) and PS(NF) are producer' surplus under Firewalls and under Information Flow, respectively.

Bibliography

[1] Adams, W.J. and Yellen, J.L. (1976), "Commodity bundling and the burden of monopoly", Quarterly journal of economics, 90, Aug., pp. 475-498.

[2] Anderson, S.P. and L. Leruth (1993), "Why firms may prefer not price discriminate via mixed bundling", International Journal of Industrial Organization 11, pp.49-61.

[3] Atallah, Gamal (2000), "Vertical Spillovers, Cooperation, Market Structure, and Innovation", Working paper, CIRANO.

[4] Brander, J. and B.J. Spender (1983), "Strategic commitment with R&D: the symmetric case", The Bell Journal of Economics, 225-235.

[5] Buehier, S. and A. Schmutzler (2003), "Who Integrates?", CEPR Discussion Paper No. 4066.

[6] Carbajo, J., D. deMeza, and D.J. Seidmann (1990), "A strategic motivation for commodity bundling", Journal of industrial economics, 38, pp.282-298.

[7] Carlton, D. and M. Waldman (2005), "Tying, upgrades, and switching costs in durable-goods markets", NBER Working paper 11407.

[8] Chen, Y. (1997), "Equilibrium product bundling", Journal of business, Vol. 70, No 1, pp.85-103.

[9] Chen, Y (1997), "Paying customers to switch", Journal of economics and management strategy, Vol. 6, pp. 877-897.

[10] Chen, Y. (2001), "On vertical mergers and their competitive effects", RAND Journal of Economics, Vol. 32, No. 4, 667-685.

[11] Choi, J.P. (1996), "Preemptive R&D, rent dissipation, and the leverage theory", Quarterly journal of economics, 111, pp.1153-1183.

[12] Choi, J.P. (2003), "Antitrust analysis of mergers with bundling in complementary markets: implications for pricing, innovation, and compatibility choice", working paper, Michigan State University.

[13] Caminal, R. and A. Claici (2005), "Are loyalty-rewarding pricing schemes anti-competitive?", Working paper.

[14] Economides, N. (2004), "Telecommunications regulation: An introduction", New York university.

[15] Fudenberg, D. and J. Tirole (2000), "Customer poaching and brand switching", Rand journal of economics, Vol. 31, NO. 4, pp. 634-657.

[16] Harhoff, D. (1996), "Strategic Spillovers and Incentives for Research and Development", management Science, Vol. 42, No. 6, 907-925.

[17] Klemperer, P. (1987), "Markets with consumer switching costs", The Qurterly Journal of Economics, May, pp. 375-394.

[18] Klemperer, P. (1995), "Competition when consumers have switching cost: An overview with applications to Industrial Organization, Macroeconomics, and Iternational Trade", Review of Economic Studies, 62, pp. 515-539.

[19] Kovac, E. (2004), "Tying by a non-monopolist", working paper

[20] McAfee, R.P., J. McMillan, and M.D. Whinston (1989), "Multiproduct monopoly, commodity bundling, and correlation of values", The quarterly journal of economics, 104, pp.371-383.

[21] MaLaren, J. (2000), " 'Globalization' and Vertical Structure", American Economic Review, 90, 1239-1254.

[22] Matutes, C. and P. Regibeau (1992), "compatibility and bundling of complementary goods in a duopoly", The journal of industrial economics, Vol. XL, pp.37-54.

[23] Milliou, C. (2004), "Vertical Integration and R&D Spillover: Is There a Need for 'Firewalls'?", International Journal of Industrial Organization, Vol. 22, Issue 1, 25-43.

[24] Nalebuff, B. (1999), "Bundling", Working paper, Yale University.

[25] Ordover, J.A., Saloner, G., and Salop, S.C. (1990), "Equilibrium Vertical Foreclosure", American Economic Review, Vol 80, 127-142.

[26] Pepall, L., and G. Norman (2001), "Product Differentiation and Upstream-Downstream Relations", Journal of Economics & Management Strategy, Vol. 10, No. 2, 201-233.

[27] Pitofsky R.(2001), "Antitrust and Intellectual Property: Unresolved issues at the heart of the new economy", Antitrust, Technology and intellectual Proerty Conference at University of California, Berkeley.

[28] Riordan, M. (1998), "Anticompetitive Vertical Integration by a Dominant Firm", The American Economic Review, Vol. 88, No. 5, 1232-1248.

[29] Riordan and Salop (1995), "Evaluationg Vertical Mergers: A Post-Chicago Approach", Antitrust Law Journal, 63, 513-568.

[30] Salinger, M. (1995), "A graphical analysis of bundling", Journal of business, Vol. 68,no. 1, pp.85-98.

[31] Shaffer, G. and Z.J. Zhang (2000), "Pay to switch or pay to stay: Preference-based price discrimination in markets with switching costs", Journal of economics and management strategy, Vol. 9, No. 3, pp.397-424.

[32] Taylor, C. (2000), "Supplier surfing: Competition and consumer behavior in subscription markets", Rand journal of economics, Vol. 34, No. 2, pp.223-246.

[33] Tirole, J. (2000), "The theory of industrial organization", The MIT press.

[34] Villas-Boas, J.M. (1999), "Dynamics competition with customer recognition", Rand journal of economics, Vol. 30, pp. 604-631.

[35] Ward, M.R., and Dranove, D. (1995), "The vertical chain of research and development in the pharmaceutical industry", Economic Inquiry, 33(1), 70-87.