This is to certify that the
dissertation entitled

LEARNING AND EXPERIMENTATION IN DIETING AND HEALTH

presented by

SAHARAT PONGSREE

has been accepted towards fulfillment
of the requirements for the

Ph.D. degree in Economics

Thomas D. Jeitschko

Major Professor's Signature

11/10/06

Date

LEARNING AND EXPERIMENTATION IN DIETING AND HEALTH

By

Saharat Pongsree

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Economics

2006

ABSTRACT

LEARNING AND EXPERIMENTATION IN DIETING AND HEALTH

By

Saharat Pongsree

This dissertation is composed of three essays that examine the behavior of dieters regarding diet choice, intensity, and maintenance.

In the first chapter, a rationale for the weight cycling phenomenon—sharp weight loss followed by weight regain—is put forward. Using a three-period utility maximization model, the chapter examines a weight loss program in which the dieter is required to follow rules regarding the amount of food consumed. The health benefits of the diet plan are uncertain and may be revealed after it is implemented. I find that the agent never follows the plan exactly. But since following the plan more strictly yields stronger signals concerning the health benefits of the diet, the dieter does follow the diet plan more strictly than he would in the absence of learning, i.e. the dieter experiments with the diet program. Under certain conditions, for instance, having found out that the health benefits are inadequate or having not learned and found that the cost of dieting is too high to continue, the dieter relapses in the second period by not following the prescription as strictly as in the first period. This helps explain the high dropout rates in some diet programs and the weight cycling phenomenon as an outcome of experimenting and learning since the dieter changes the amount of food consumed from one period to the next.

The second chapter investigates how a dieter chooses among competing diet programs which are characterized by different entry costs and their corresponding

effectiveness. It examines how this choice affects the effort put forth in losing weight and the amount of weight loss. It also provides an explanation of the growing revenue within the weight-loss industry despite poor weight-loss outcomes. Before purchasing a diet plan, the agent faces uncertain diets' effectiveness. The diets' distribution of effectiveness can be stochastically ranked with regard to the entry costs. Once the entry cost is paid for the chosen diet, the true effectiveness of the plan is revealed. The agent then puts forth effort to lose weight. I find that the greater the initial overweightness, the more expensive the program chosen. Moreover, the initially less overweight agent is more likely to quit the diet after the purchase than the initially more overweight agent due to the smaller marginal benefit of weight loss. As a result, weight losses among the less overweight agents tend to be minimal. This helps explain the aforementioned growing revenue in the presence of poor weight-loss results.

The third chapter allows a time-discounting, infinitely-lived agent to switch back and forth between two diet plans. It examines how well the agent adheres to a diet plan which promises certain weight loss but becomes boring with repetition. Two types of agents are examined: those with long memory and those with short memory. I find that the short-memory agent abandons the superior plan more frequently if the plan produces greater boredom. The switch to the inferior plan occurs even if that plan has a zero success rate. Under the assumption that the agent chooses to be on the diet plan or off it, I also establish that the long-memory agent is on the diet plan only every other period if he finds the plan sufficiently boring. In comparison to the short-memory agent, the long-memory agent fares worse in terms of adhering to the diet plan. This helps explain the low adherence rates among fad diets and the weight cycling phenomenon.

ACKNOWLEDGMENTS

First of all, I thank Dr. Rowena A. Pecchenino for her critical role in this dissertation from beginning to end. Without her initial encouragement and the extra mile she was willing to go, the dissertation would not even have begun. I have a profound appreciation for her support, patience, inspiration, and kindness. Her knowledge and insightful comments and feedback week after week are invaluable. I feel very fortunate to have undertaken this study under her guidance.

I thank Dr. Thomas D. Jeitschko to whom I owe my mathematical and technical knowledge. His countless comments and tremendous expertise have greatly contributed to the study from the start, and especially during the final stages. The challenge he gave has strengthened me and this dissertation. His dedication is truly appreciated.

I am grateful for advice given by Dr. Carl Davidson. His suggestions and instructions are very valuable and have allowed me to more carefully think about the issues. I thank Dr. Sharon Hoerr for her involvement in this study and valuable discussions that we had. They have opened my eyes to many related issues.

I thank Dr. John Goddeeris for his involvement in the initial stages. I also thank other faculty members at the Department of Economics, MSU for their general encouragement every time they see me. I thank Dr. Gerhard Glomm and MEA participants for their helpful comments for the first chapter.

Lastly, I am truly and forever indebted to my mom and dad. All my achievements would be impossible without their unconditional love, support, and inspiration. I owe my sister for always believing in me. I thank Chayawan Sonchaeng for her love. Her faith, support and encouragement have given me mental strength to finish the study.

TABLE OF CONTENTS

LIST OF FIGURES

REVIEW OF RELEVANT LITERATURE

I. Introduction

"Call toll-free 1-800-GET-SLIM, and it will be the last diet plan you are on!" Such an advertisement is not uncommon as obesity has become a public health crisis. Diet plans and weight loss programs are plenty. Some plans sound deceptively simple: low-carb diet, low-fat diet. Some have names: Atkins, South Beach, Bob Greene, Jenny Craig. There are even the popcorn diet, the peanut butter diet, and the chocolate diet! Think of your favorite food and they probably have a diet plan designed just for you. Yet more than 60 percent of Americans are either overweight or obese. Less than ten percent of American dieters are successful in sustaining the weight loss.

A study of dieters' behavior under an economic framework is necessary in order to better understand why diet programs do not seem to work well. Necessary public policies can be designed accordingly. Such an investigation necessarily draws upon previous work in the economic literature and other related fields such as the medical literature with regard to being overweight. The following sections summarize the body of literature relevant to weight cycling, weight maintenance, and choice of diets.

II. Relevant Medical Literature

Much of the medical literature with regard to overweightness focuses on its detrimental effects on physiological health. Excess body weight is directly related to several diseases. Type II diabetes, hypertension, and osteoarthritis are but a few examples of diseases that are more likely to develop in overweight individuals.[1] The National Institutes of Health (NIH), in a Consensus Conference, as far back as 1985, include cholelithiasis, obstructive sleep apnea, hypoventilation, degenerative arthritis and psychosocial impairments in the list.[2] For these reasons, weight-reduction procedures have been taken seriously among health-care professionals.

A. Weight Maintenance and Weight Cycling

While losing weight is quite an ordeal, sustaining the weight loss is even more difficult. Jeffery, et al. (2000) find that long-term weight losses resulting from being on diet plans are only small to modest. Furthermore, they contend that most dieters who lose weight will regain the lost weight.

Several medical researchers empirically study the causes of weight cycling. Elfhag and Rossner (2004) author a summary of factors that affect weight maintenance and weight regain. They find that realistic weight goals facilitate weight maintenance afterward. Thus, initiating a diet plan promising significant but unrealistic weight loss within a short period of time may not be wise if one wants to sustain weight loss. They also find that a history of weight cycling is a cause of future weight regain. Small portion sizes and low frequency of snacking are also found to be important in weight management. Other factors mentioned are psychological in nature. Eckel (2005) suggests a few explanations for the weight cycling phenomenon which include the findings that appetite for energy food among dieters increases and that metabolic rates

---

[1] *Overweight, Obesity, and Health Risk.* National Task Force on the Prevention and Treatment of Obesity. Arch Intern Med 2000;160: 898-904.
[2] *Health Implications of Obesity.* NIH Consensus Development Conference Statement. Ann Int Med, 1985. 103: 1073-77.

drop. However, he concludes that knowledge on why individuals keep regaining the lost weight needs further research. Steen, et al. (1988) also find evidence supporting the hypothesis that weight cycling leads to better food efficiency and lower resting metabolic rates. Spriet and Peters (1998) find that extreme dieting methods such as very low-carb or very low-fat diets lead to changes in metabolism rates and sources of energy burned while exercising. These metabolic responses may potentially lead to weight cycling as one can gain weight while eating less because of the increased efficiency with which the body uses energy. Muls, et al. (1995) establish a potential positive impact of weight cycling on dieters' preference for fat and on the likelihood of binge eating. The implication is that weight cycling fuels on itself. Nonetheless, they conclude that further research on weight cycling is needed and that losing weight, even with the risk of regaining the lost weight, still is better than remaining overweight.

On the theoretical side, Goldbeter (2006) puts forth a dynamic model of weight cycling. In this model, dieters suffer from weight oscillations due to a threshold weight above which dieters decide to decrease caloric intake. The premise relies solely on psychological aspects. Furthermore, weight cycling occurs only in agents who are sufficiently overweight initially.

Other researchers empirically establish the prevalence of dieters' high attrition rates among commercial weight-reduction programs which could potentially lead to weight cycling. In 1993, a medical conference panel concluded that almost all non-surgical weight-reduction treatments result in weight loss for the first four to six months followed by weight regain due to low adherence.[3] Dansinger, et al. (2005) study dieters

[3] *Methods for Voluntary Weight Loss and Control.* NIH Technology Assessment Conference Panel. Ann Intern Med 1993;119: 764-770.

in four commercial diet programs—Zone, Weight Watchers, Ornish, and Atkins. They conclude that dropout rates were high for all four programs but especially high for the Atkins program (a low-carb plan) and the Ornish program (a very-low-fat plan). Among the non-dropouts, results are not impressive either. Dieters started out doing what the plans asked them to do. However, after a month, the amount of carbohydrates consumed by Atkins dieters was more than three times the amount that Dr. Atkins prescribes. Dieters on the Ornish plan, after a month, consumed almost twice as much fat as prescribed. Overall, the authors find that the adherence rates are the highest for the Weight Watcher programs after one year. Foster, et al. (2003) also find similar results in a randomized trial of low-carb dieters in that the attrition rates are very high. Samaha, et al. (2003) contend that attrition rates are high for dieters on both low-carb and low-fat diets. Dr. Wadden, in an interview by Stephenson (2003), asserts that both low-carb and low-fat diets lead to only short-term weight loss while failing to facilitate long-term weight maintenance.

There is little consensus on the harm actually done by weight cycling. Muls, et al. (1995) find a link between weight cycling and excess mortality. Some studies find that weight cycling leads to higher risk in developing high blood pressure, high cholesterol, and gallbladder disease.[4] Other studies indicate that there is no sufficient evidence of harm caused by weight cycling.[5] However, negative psychological effects of weight cycling may emerge as dieters "feel like a failure."[6] Overall, medical researchers

---

[4] NIH Publication No. 01-3901

[5] For example, *Weight Cycling.* National Task Force on the Prevention and Treatment of Obesity. JAMA 272: 1196-1202; *Weight Loss and Nutrition Myths.* National Institute of Diabetes and Digestive and Kidney Diseases (NIDDK). NIH Publication No. 00-4561.

[6] ibid.

agree that a commitment to long-term weight maintenance is critical to a person's physiological health.

B. Variety and Choice of Diet Plans and Other Weight-Loss Methods

Choices of weight-reduction methods range from financially costless self-help to expensive surgical procedures. Between the two extremes lie commercial diet books and programs. These can be differentiated along the cost dimension. Diet books entail minimal financial costs. Diet programs which may include support meetings and professional consulting are more expensive. Some have additional supporting pre-packaged meals which necessarily raise the cost. Heshka, et al. (2003) study and compare the efficacy of a weight-loss program (Weight Watchers) and that of self-help methods. The self-help dieters were also directed to publicly available information on how to effectively lose weight in order to replicate well-informed dieters in reality. They find that, over the period of two years, the program is more effective than self-help methods in terms of reductions in BMI, waist circumferences, and weight. Although one may feel that weight can be lost effectively by merely eating less and exercising more without professional help, the study shows that weight-loss programs are costly for good reasons.

For the morbidly obese, the last resort is the surgical procedures. These are extremely costly. The outcomes are outstanding. Buchwald, et al. (2004) establish that 61 percent of bariatric surgery patients experience significant and effective weight losses. Moreover, a substantial majority of patients with obesity-related diseases such as diabetes, hyperlipidemia, hypertension, and sleep apnea experience complete resolution or substantial improvement. Blackburn (2005) finds that more than 90 percent of gastric bypass surgery patients achieve permanent weight loss.[7] Furthermore, these weight losses are significant.[8] It seems that weight-loss successes and the financial burden of being on the program are correlated in some ways.

These weight-reduction methods are also different in terms of the philosophy adopted. Many diet books, for example, focus on the 'good-food, bad-food' philosophy. These are known as 'fad' diets. Riley (2006) summarizes several low-carb diets such as the Atkins and the Protein Power diets on one end of the spectrum and provides a brief discussion of low-fat diets such as the Dean Ornish and the Pritikin diets on the other end. The author finds that anti-fat and anti-carb programs have significant negative impacts on health, metabolism, and sustainability of lost weight. She also recommends a few criteria by which diet plans can be evaluated. Baker (2006) investigates popular fad diets by categories. She describes adverse effects of low-carb diets in particular which include vitamin deficiencies, kidney stones, renal insufficiency, and altered cognitive functions.

However, most studies find that low-carb and low-fat diets lead to significant weight losses within the first 6 months.[9] Meckling, et al. (2004) find that most of these short-term weight losses are merely results of extreme daily energy deficit regardless of types of food consumed and restricted. They also find that only 5 percent of fad dieters precisely follow the particular diet in terms of what and how much to consume. Riley

---

[7] Permanent weight loss is assessed at longer than 14-year follow-up.

[8] 'Significant weight loss' in this context is defined as weight loss of greater than 50% of initial body weight.

[9] For example, Samaha, et al. (2003), Foster, et al. (2003), and Avenell, et al. (2004).

(2006) and Eckel (2005) suggest that even in the short-term, weight loss results also depend on whether the diet is the 'right' match for the dieter.

There are several studies that directly and specifically investigate low-carb diets. Goff, et al. (2006) study the South Beach Diet book (a low-carb diet). The authors summarize nutritional facts claimed in the book and assess whether they are scientifically supported in peer-reviewed journals. Only one-third of the facts studied were supported. Among those claimed facts unsupported are that the diet has been "scientifically studied and proven effective" and that "eight to thirteen pounds will be lost in two weeks." This suggests that popular diet books may be misleading and claiming unfounded scientific 'facts'. Tapper-Gardzina, et al. (2002) examine the safety issues of following a low-carb diet. They assess that even health care professionals have difficulty giving advice to dieters about true effects of low-carb diets. They provide a list of potential negative side effects of low-carb diets on long-term health considerations such as levels of Cholesterol, insulin resistance, etc. Cheuvront (2003) examines the Zone diet, a low-to-moderate carb diet. He reports that the Zone theory is based on selective scientific observations and that, similar to Meckling, short-term weight losses among Zone dieters rely on severe energy restriction. However, these authors conclude that at the current state of medical research, a definite answer on the safety of low-carb diets has not been obtained due to lack of long-run studies. Nonetheless, they suggest that recommendation of low-carb diets be avoided.

Kennedy, et al. (2001) study the effects of categorized popular diets on health as measured by USDA-established Healthy Eating Index (HEI) which has ten components based primarily on the Food Pyramid and the U.S. Dietary Guidelines. The analysis shows that HEI is the lowest for the low-carb diet group. Similar to Meckling and Cheuvront, they find that short-term weight losses are independent of diet composition and rely mainly on energy restriction.

Realizing that recidivism among fad diets is very high, most medical professionals attempt to promote approaches that encourage dieters to have a long-term commitment to weight management. These methods usually include gradual adjustment in eating behavior and result in only small short-term weight losses. Hill, et al. (2003) find that even small energy deficits can result in beneficial weight losses for the obese over a period of one year. Thus, the objective should be small changes over the long haul in order for the dieter to realize appreciable and sustainable long-term weight reduction. These gradual changes are unlikely to lead to large and abrupt weight losses observed in fad diets that severely restrict energy intake. Volpe (2006) emphasizes the critical role of regular physical activities in weight-maintenance success. She contends that there is no quick fix when it comes to weight reduction. Foody (2005) suggests a 'no-fad' diet where the dieter picks a diet and sticks to it. This implies that dieters should avoid diets that have proven to result in attrition. Baker (2006) contends that portion sizes are most important and that, in the presence of 'good-food, bad-food' philosophy, dieters have the mindset that size does not matter.

III. Relevant Economic Literature

Studies of individuals' behavior regarding diet plans are lacking in the economic literature. The existing studies focus on what causes overweightness and obesity. Although it is true that a problem is best tackled at its roots, it is unwise to disregard individuals who already have become overweight and are trying to get in shape. Treating overweightness is as important as preventing it. Nevertheless, the economic studies of the causes of overweightness provide a good framework from which a study of diet plans and dieters' behavior can be developed.

A. Rational Models

In rational choice models, over-consumption and/or under-exercise are results of rational and optimal choice. These cause individuals to become overweight. Several of these models explain the upward trend in weight with technological change. Philipson and Posner (1999) and Lakdawalla and Philipson (2002) establish that technological improvement lowers the price of food consumption by lowering the price of food itself and by reducing time spent on cooking and preparing food. In response to this lower price, consumers eat more. Lakdawalla and Philipson also argue that technology generally reduces the level of physical activities by making work more sedentary. Better technology serves as an economic incentive for agents to under-exercise and overeat. Lakdawalla and Philipson also provide empirical analyses to support their hypotheses. Jeitschko and Pecchenino (2006) investigate consumers' choices of portion sizes and food expenditures. In particular, they examine the effects of volume discounts on the consumers' choices to 'super size' their food portions at restaurants. Consumers in their model rationally choose to super size and consequently increase their waistlines. Food portions may be a factor contributing to the rise in obesity; however, choices of portion sizes lie in consumers' hands.

Bednarek, Jeitschko, and Pecchenino (2006) develop a one-period model where agents derive utility from food consumption, goods consumption, leisure time, and health. The agent faces time and budget constraints. The agent comes into the period with accustomed levels of leisure and food consumption. Should they be excessive relative to the optima, adjusting consumption and leisure time below these levels is costly. They find that wealthy individuals rationally choose levels of food consumption and leisure that are higher than physiological optima due to the trade-off between physiological health and consumption/leisure. These optima are called "bliss" optima which lead to overweightness, suggesting that, even though these individuals may be considered overweight, they are happy. Second, agents with lower income consume less and spend less time as leisure than wealthier agents. Hence they may be physiologically healthier, albeit less happy, than the wealthier agents. This is consistent with the trends of becoming increasingly overweight as the society becomes wealthier. However, with regard to overall health, overweightness can be a better state than the physiological optimum in that it improves the 'mental' health by more than physiological health sacrificed.

Lastly, they establish that, any agent that comes into the period with high accustomed levels of leisure and food consumption will not attain the bliss optima due to the adjustment cost. Obesity, or at least overweightness, is a persistent problem. This actually is equivalent to saying dieting is a difficult task because dieters are in some sense 'addicted' to overeating and under-exercising.

B. Addiction Models

Because individuals' behavior in consuming addictive substances seemingly suggests irrationality due to harm inflicted upon themselves, it appears difficult to model calorie addiction as a rational choice. However, there have been papers that investigate the issue. Becker and Murphy (1988) characterize an addictive good as one whose current consumption is significantly complemented by past consumption of the good. They suggest two requirements for a good to be addictive: Reinforcement—the more you consume, the more you want to consume that good, and Tolerance—the more you consume today, the more you need tomorrow to attain the same level of utility. In the context of overeating as an addiction, this means that if you overate yesterday, you are bound to overeat today as a result of being 'addicted' to food consumption. The authors raise food consumption and addiction as an example and the model does well in explaining binge-eating and severe-dieting behaviors. However, that food is addictive by the authors' definitions is debatable. Nonetheless, the main point of the paper is that whether food is addictive is irrelevant because the rational framework will be useful regardless.

The model also investigates overeating in the current period as a result of agents discounting the future heavily. In particular, agents consider the current benefits from food consumption to exceed the future penalties. In a primarily empirical study, Offer (1998) also asserts the adverse effects of hyperbolic discounting and 'self-control' on individuals' weights.

Also under the rational framework, Orphanides and Zervos (1995) establish that individuals may become 'hooked' on an addictive good as a result of experimenting with the good and failing to realize its additive power early enough. In this model, there is a subjective belief on the harmful effect of the addictive good. In the context of food consumption, obesity may be a result of individuals' experimentation. In contrast to the model studied by Becker and Murphy, rational agents in this model regret their initial beliefs about how potentially harmful the food is whereas Becker and Murphy's addicts are happy. In this sense, it is not surprising to see the rise in the number of dieters each year. Moreover, if food is truly addictive, a study of individuals' behavior while on a diet plan is crucial to understand the process of reversing these harmful effects.

C. Models of Health Investment

Health investment models involve multi-period optimization. These models treat health as a capital stock which may rise or fall depending on prior choices of some other variables such as food consumption, medical care, exercise, etc. These other variables are viewed as 'health investments' (or disinvestments in some cases). These papers are highly relevant because diet plans are essentially a form of investment in which immediate outlays and effort are transformed into some future weight-loss payoffs.

The pioneer paper is the seminal work of Grossman (1972a, b). Grossman clarifies the difference between health capital and other forms of human capital in that better health, in addition to increasing one's productivity, expands the amount of time one can spend producing commodities other than health which enter one's utility function. The original work involves a finite-horizon optimization. In his working paper (1999), Grossman modifies the process to an infinite-horizon optimization problem. While Grossman was mostly concerned with medical care as an input for health stock, diet programs resemble his concepts of health investment. Nonetheless, the model involves no uncertainty frequently encountered in most dieting procedures.

Although not explicitly considering diet plans, a model of rational eating put forward by Levy (2002) explains the cycles in food consumption which may explain weight cycling. Levy addresses the problems of overweightness and underweightness with a finite time-horizon model while dealing directly with the agent's weight. In steady-state, consumers are rationally overweight as compared to the physiological optimum.

D. Uncertainty and Experimentation Models

There are economic models that involve uncertainty in the consumption decision. Some of these models explain the consumers' learning and experimentation process. Although these models do not directly and explicitly deal with diet plans, they describe how making choice under uncertainty could potentially lead to new information at some future date. The dieting process may be regarded in this way in that choosing to follow a diet plan today may lead the dieter to obtain information about the plan or the food in the future.

Mirman (1971) studies optimal consumption and investment decisions under uncertainty in which the agent maximizes expected utility over two periods. In the model, uncertainty exists in how current choice affects future production function. He finds that consumption under uncertainty may be more or less than consumption under deterministic production function depending on the measure of relative risk aversion. In this model, the learning aspect of the choice variable is not discussed. Learning may occur; however, information obtained is valueless because the model does not allow the agent to make use of it.

Grossman, Kihlstrom, and Mirman (1977) examine a dynamic, finite-horizon model of experimentation in a consumption good (in their case, a drug) whose quality is unknown. The unknown parameters are discrete and the observable variable is a simple linear regressor of the choice variable. The level of current consumption has an impact on the amount of information obtained about the quality of the good. In this model, the information obtained has some use although it is imperfect and noisy. They find that, with the learning opportunity, the amount of drug consumed is greater than what it would be without learning. This finding shows the benefits of experimenting and learning new information although incomplete learning is optimal. Kiefer and Nyarko (1989) consider a slightly more complicated model with infinite time horizon in which the space of uncertain parameters is continuous. The model also employs a simple linear regression model of the observable variable. The findings are similar in that it is optimal for the agent to deviate from what would be optimal if the learning opportunity did not exist and that incomplete learning is optimal.

The process of learning and experimentation is highly complex. This often invites criticism of how rationality may be bounded. In an empirical work, Miravete and Palacios-Huerta (2003) study households in Kentucky who are offered a choice which requires them to estimate some future demand. In this setting, consumers can either attempt to experiment and learn if the new scheme results in saving or act as if learning opportunities did not exist. They find that, for a saving of merely $5 a month, consumers undertake the experimentation, learn quickly to save by using the information obtained, and make no systematic errors. This has a critical implication for the investigation of experimentation in general and for the study of learning process while on a diet plan in particular.

There are several other studies that focus on causes of obesity. Cutler, Glaeser, and Shapiro (2003) empirically confirm the positive impact of technological innovations on weight which is presumably caused by an increase in food intake and a reduction in energy expenditure. Skinner, Miller, and Bryant (2005) summarize causes of obesity and investigate how food labeling and advertising in the UK contribute to the rise in obesity because information and knowledge about food are far from perfect. They also seem to seriously consider a Pigovian 'fat' tax to correct the negative externality of eating fatty food (as hospital services are free in the UK). This primarily would discourage individuals from becoming fat in the first place. Cawley (2004) suggests a very general, yet highly relevant economic framework for the study of dieters' behavior in which individuals make decisions about what and how much to consume subject to simple time and budget constraints. Arguably, most, if not all, rational economic models with food consumption are some modification of Cawley's. Since preferences are different across individuals, some people are more predisposed to gaining weight than others. Needless to say, there is no economic justification to 'fix' these preferences even though they are direct causes of obesity. Government interventions are appropriate only when markets fail. Information problems, untruthful advertising and externalities are a few examples that Cawley raises.

These economic studies have greatly contributed to the understanding of the prevalent overweightness problem. Public policies can be designed to prevent people to become overweight. Nonetheless, for the most part, they do not deal with individuals' choice of diet plans and how they behave while dieting. It is the purpose of my study to fill this gap and examine individuals' dieting behavior where they attempt to eliminate the harm already done.

IV. Concluding Remarks

Medical studies have long and firmly established negative effects of excess body weight. Being overweight or obese has long been viewed as a public health crisis. The most recent report indicates that 1.5 billion people worldwide are overweight, approximately 21 percent of whom are obese.[10] Diseases related to overweightness are many. Clearly the problems are not to be taken lightly.

Among phenomena associated with being overweight is weight cycling. Although debates are on-going among medical researchers as to whether weight cycling does any direct harm, it is an indicator of attrition in weight management. As indicated in the medical literature review, weight cycling is more common and severe among fad dieters. Fad dieters are also known to quit the diet more often than non-fad dieters.

In this regard, a few questions naturally arise. What causes fad dieters to choose patterns of food consumptions that lead to weight cycling? Are there rational economic explanations for these patterns? A theoretical, psychological explanation has been

---

[10] *IASO Media Release*. International Association for the Study of Obesity. 10th International Congress on Obesity, September 2006, Sydney, Australia.

offered. Within the economic literature, however, the consumers' choices under a diet plan remain largely uninvestigated.

Lastly, diet plans and procedures require one to pay certain fees upfront. Since there are plenty of diets, consumers are required to assess certain characteristics of these diets and make a choice of what fee to pay and which diet to initiate. These fees have generated an incredible level of revenue within the diet industry even when most dieters fail to manage weight. Economic studies are lacking and lagging in investigating the choice of one diet plan among several with respect to cost differentials and how the dynamics of these choices over time may lead to inferior weight management.

In this dissertation I present a study that fills these voids. Two chapters explain the prevalence of weight cycling phenomenon under rational behavior. The other chapter describes the mechanism by which dieters choose diet plans with regard to fees and the consequences of such actions.

The first chapter employs a model involving uncertainty which is closely related to the model by Grossman, Kihlstrom, and Mirman (1977). However, I do not utilize the simple linear regression model in the observable variable. This is appropriate since in the dieting world, observable health is not linear in food consumption. Instead, a moderation in food consumption may be best for one's health. The model also is similar to many economic models aforementioned in that it employs the idea of costly but attainable physiological optimal weight.[11] This optimum represents the main purpose of dieting—to move toward it. The dieting process also represents a form of health investment in Grossman's sense. However, the dimension of uncertainty in dieting is a critical addition. This may lead to a sub-optimal investment in health and consequently a sub-

[11] For example, Bednarek, Jeitschko, and Pecchenino (2006) and Levy (2002).

optimal level of dieting. Consequently, the implications regarding possible regulations may arise. Economic justifications for public policies as suggested by Cawley (2004) are also explored.

The third chapter also explains weight cycling under rationality. As suggested by medical researchers, fad dieters are more likely to engage themselves in yo-yo dieting, it is interesting to see how dieters can initiate a diet plan and then easily but rationally abandon it shortly after even if the plan has proven successful.

The second chapter describes the choice set faced by the dieters when costs, financial and otherwise, are different across diet programs/procedures. This is critical since losing weight always entails some sacrifice, most notably in terms of the money paid to the program administrators and the necessary effort while losing weight. The chapter describes the underlying systematic differences among diet plans and provides some explanation to the observed failures in weight loss attempts in the presence of these costs and the ever-growing revenue within the diet industry.

REFERENCES

Avenell, Allison, Brown, Tamara J., McGee Margaret.A., et al. "What Are the Long-Term Benefits of Weight Reducing Diets in Adults? A Systematic Review of Randomized Controlled Trials." Journal of Human Nutrition and Dietetics. 2004; 17: 317-335.

Baker, Barbara. "Weight Loss and Diet Plans: Several Types of Diet Plans Produce At Least Short-Term Weight Loss; Portion Size May Matter More Than What We Eat." American Journal of Nursing. 2006; 106 (6): 52-59.

Becker, Gary S. and Murphy, Kevin M. "The Theory of Rational Addiction." Journal of Political Economy. 1988; 96: 675-700.

Bednarek, Heather L., Jeitschko, Thomas D., and Pecchenino, Rowena A. "Gluttony and Sloth: Symptoms of Trouble or Signs of Bliss?—A Theory of Choice in the Presence of Behavioral Adjustment Cost." Contributions to Economic Analysis & Policy. 2006; 5(1): Article 6. http://www.bepress.com/bejeap/contributions/vol5/iss1/art6.

Blackburn, George. "Solution in Weight Control: Lessons from Gastric Surgery." American Journal of Clinical Nutrition. 2005; 82 (1): 248S-252S.

Buchwald, Henry, et al. "Bariatric Surgery: A Systematic Review and Meta Analysis." Journal of the American Medical Association. 2004; 292: 1724-1737.

Cawley, John. "An Economic Framework for Understanding Physical Activity and Eating Behaviors." American Journal of Preventive Medicine. 2004; 27: 117-25.

Cheuvront, Samuel N. "The Zone Diet Phenomenon: A Closer Look at the Science behind the Claims." Journal of American College of Nutrition. 2003; 22 (1): 9-17.

Cutler, David M., Glaeser, Edward L., Shapiro, Jesse M. "Why Have Americans Become More Obese." NBER Working Paper 9446. 2003.

Dansinger, Michael L., et al. "Comparisons of the Atkins, Ornish, Weight Watchers, and Zone Diets for Weight Loss and Heart Disease Risk Reduction: A Randomized Trial." Journal of the American Medical Association. 2005; 291: 43-53.

Eckel, Robert H. "The Dietary Approach to Obesity." Journal of the American Medical Association. 2005; 293: 96-97.

Elfhag, Kristina. and Rössner, Stephan. "Who Succeeds in Maintaining Weight Loss? A Conceptual Review of Factors Associated with Weight Loss Maintenance and Weight Regain." Obesity Review. 2005; 6: 67-85.

Foody, JoAnne M. "Pick a Diet, Then Stick with It." Cardiology Journal Watch. February 11, 2005.

Foster, Gary D., et al. "A Randomized Trial of a Low-Carbohydrate Diet for Obesity." New England Journal of Medicine. 2003; 348: 2082-2090.

Goff, Sarah L., et al. "Brief Report: Nutrition and Weight Loss Information in a Popular Diet Book: Is It Fact, Fiction, or Something in Between?" Journal of General Internal Medicine. 2006; 21 (7): 769-774.

Goldbeter, Albert "A Model for the Dynamics of Human Weight Cycling." Journal of Biosciences. 2006; 31: 129-136.

Grossman, Michael. "The Demand for Health: A Theoretical and Empirical Investigation." NBER Occasional Paper 119. 1972(a)

Grossman, Michael. "On the Concept of Health Capital and Demand for Health." Journal of Political Economy. 1972(b); 80: 223-255.

Grossman, Michael. "The Human Capital Model of the Demand for Health." NBER Working Paper 7078. 1999.

Grossman, Sanford J., Kihlstrom, Richard E., and Mirman Leonard J. "A Bayesian Approach to the Production of Information and Learning by Doing." Review of Economic Studies. 1977; 44 (3): 533-547.

Heshka, Stanley, et al. "Weight Loss with Self-Help Compared with a Structured Commercial Program: A Randomized Trial." Journal of the American Medical Association. 2003; 289: 1792-1798.

Hill, James O., Wyatt, Holly R., Reed, George W., et al. "Obesity and the Environment: Where Do We Go from Here?" Science. 2003; 299: 853-855.

International Association for the Study of Obesity. *IASO Media Release.* 10th International Congress on Obesity, Sydney, Australia. September 2006.

Jeffery, Robert W., et al. "Long-Term Maintenance of Weight Loss: Current Status." Health Psychology. 2000; 19: 5-16.

Jeitschko, Thomas D. and Pecchenino, Rowena A. "Do You Want Fries with That? An Exploration of Serving Size, Social Welfare, and Our Waistlines." Economic Inquiry. 2006; 44(3): 442-450.

Kennedy, Eileen T., et al. "Popular Diets: Correlation to Health, Nutrition, and Obesity." Journal of American Dietetic Association. 2001; 101: 411-420.

Kiefer, Nicholas M., Nyarko, Yaw. "Optimal Control of an Unknown Linear Process with Learning." International Economic Review. 1989; 30 (3): 571-586.

Lakdawalla, Darius and Philipson, Tomas J. "The Growth of Obesity and Technological Change: A Theoretical and Empirical Examination." NBER Working Paper 8946. 2002.

Levy, Amnon. "Rational Eating: Can It Lead to Overweightness or Underweightness?" Journal of Health Economics. 2002; 21: 887-899.

Meckling, Kelly A., O'Sullivan, Caitriona, and Saari, Dayna. "Comparison of a Low-Fat Diet to a Low-Carbohydrate Diet on Weight Loss, Body Composition, and Risk Factors for Diabetes and Cardiovascular Disease in Free-Living, Overweight Men and Women." Journal of Clinical Endocrinology & Metabolism. 2004; 89 (6): 2717-2723.

Miravete, Eugenio and Palacios-Huerta, Ignacio. "Rational Attention and Deliberation Costs in a Repeated Decision Problem." mimeo. Brown University. 2003.

Mirman, Leonard J. "Uncertainty and Optimal Consumption Decisions." Econometrica. 1971; 39: 179-185.

Muls, Erik, et al. "Is Weight Cycling Detrimental to Health? A Review of the Literature in Humans." International Journal of Obesity and Related Metabolic Disorders. 1995; 19 (Suppl. 3): S46-50.

National Institute of Diabetes and Digestive and Kidney Diseases (NIDDK). "Weight Loss and Nutrition Myths." NIH Publication No. 00-4561. 2000.

NIH Consensus Development Conference Statement. "Health Implications of Obesity." Annals of Internal Medicine. 1985; 103: 1073-77.

NIH Technology Assessment Conference Panel. "Methods for Voluntary Weight Loss and Control." Annals of Internal Medicine. 1993; 119: 764-770.

National Institutes of Health. "Weight Cycling." NIH Publication No. 01-3901. 2004.

National Task Force on the Prevention and Treatment of Obesity. "Overweight, Obesity, and Health Risk." Archives of Internal Medicine. 2000; 160: 898-904.

National Task Force on the Prevention and Treatment of Obesity. "Weight Cycling." Journal of the American Medical Association. 1994; 272: 1196-1202.

Offer, Avner. "Epidemic of Abundance: Overeating and Slimming in the USA and Britain Since the 1950s." University of Oxford Discussion Papers in Economic and Social History. 1998; 25.

Orphanides, Athanasios and Zervos, David. "Rational Addiction with Learning and Regret." Journal of Political Economy. 1995; 103: 739-58.

Philipson, Tomas J. and Posner, Richard A. "The Long-Run Growth in Obesity as a function of Technological Change." NBER Working Paper 7423. 1999.

Riley, Rosemary E. "Popular Weight Loss Diets: Health and Exercise Implications." Clinic in Sports Medicine. 1999; 18 (3): 691-701.

Samaha, Frederick F., et al. "A Low-Carbohydrate as Compared with a Low-Fat Diet in Severe Obesity." New England Journal of Medicine. 2003; 348: 2074-2081.

Skinner, Tom, Miller, Howard, and Bryant, Chris. "The Literature on the Economic Causes of and Policy Responses to Obesity." Acta Agriculturae Scand Section C. 2005; 2: 128-37.

Spriet, Lawrence L. and Peters, Sandra J. "Influence of Diet on the Metabolic Responses to Exercise." Proceedings of the Nutrition Society. 1998; 57: 25-33.

Steen, Suzanne N., Oppliger, Robert A., and Brownell, Kelly D. "Metabolic Effects of Repeated Weight Loss and Regain in Adolescent Wrestlers." Journal of the American Medical Association. 1988; 260: 47-50.

Stephenson, Joan. "Low-Carb, Low-Fat Diet Gurus Face Off." Journal of the American Medical Association. 2003; 289: 1767-1773.

Tapper-Gardzina, Yvonne B., Cotugna, Nancy, and Vickery Connie E. "Should You Recommend a Low-Carb, High-Protein Diet?" Nurse Practitioner. 2002; 27 (4): 52-59.

Volpe, Stella L. "Popular Weight Reduction Diets." Journal of Cardiovascular Nursing. 2006; 21 (1): 34-39.

CHAPTER 1

THE DIETER'S DILEMMA: EXPERIMENTATION, WEIGHT LOSS, AND WEIGHT GAIN

I. Introduction and Motivation

A person's weight is certainly one of the most important aspects of his health. As a whole, however, Americans seem to be moving in the wrong direction when it comes to weight. According to the U.S. Department of Health and Human Services, more than 60 percent of Americans are either overweight or obese. According to a recent report by the International Association for the Study of Obesity, 1.5 billion people worldwide are overweight, over 20 percent of whom are obese.[12] These numbers are growing every year.

Approximately fifty million Americans initiate a diet program every year. Of these, only five to ten percent will lose the extra pounds and keep them off. One-third of those who do initially lose weight regain more pounds than they had lost.

A great number of overweight individuals on diet programs experience a phenomenon known as 'weight cycling'. Within the medical community, weight cycling is defined as repeated weight losses and weight regains over time, especially a large weight loss when starting a program followed by a weight regain. Although medical researchers are still debating the true effects of weight cycling, they agree that a commitment to long-term weight maintenance is critical to one's physiological health. There is some evidence that weight cycling leads to a higher risk in developing obesity-

[12] *IASO Media Release*. International Association for the Study of Obesity. 10th International Congress on Obesity, September 2006, Sydney, Australia.

related diseases and causes excess mortality.[13] More importantly, it may signify a lack of commitment to long-term weight maintenance.

Dieting is a choice. But, given well-documented cases of weight cycling, what are the reasons that dieters make these choices that lead to health-deteriorating outcomes? The purpose of this paper is to examine dieting behavior.

Diet plans in this paper are modeled as a health investment with an uncertain outcome, weight loss, which occurs after the investment is undertaken. The concept of health investment is related to the work by Grossman (1972a, b). Since the diet plan yields a random outcome, the dieter has an opportunity to learn about the quality of the diet plan while on it. Because the dieter also cares about losing weight in the future, the benefit of learning plays an important role in the dieting decision. The learning aspect of being on a diet plan has some similarity to work done by Grossman, Kihlstrom, and Mirman (1977) and that done by Kiefer and Nyarko (1989).

The agent maximizes utility over a three-period horizon. Utility in each period is defined over three arguments: food consumption, consumption of other goods, and the level of health. The level of health depends on the amount of food consumed in the previous period. The agent does not know for certain the degree to which the food consumption affects future health. The utility represents a fundamental trade-off between gratification from eating now and the subsequent physiological health. This notion has been studied by Bednarek, Jeitschko, and Pecchenino (2006) and Levy (2002).

I argue that, within the confines of this model, weight cycling is a rational response by a dieter to some diet plans. The reason behind this is that the opportunity to learn leads to experimentation. To learn, the agent first conforms to the diet plan then

[13] Muls, et al. (1995) and NIH Publication No. 01-3901 (2004).

relapses after having learned. This helps explain weight cycling as the agent, instead of dieting consistently over time, may lose a significant amount of weight during the learning phase only to regain it once the quality of the plan has been learned.

Section II provides a detailed description of the model. Section III solves the model. Sections IV and V discuss some important results. Section VI concludes and discusses some policy implications.

II. Model setup

A. Utility and Preferences

The agent's preferences in any period t are defined over three arguments. First, food consumption F_t benefits the agent instantaneously. With respect to this instantaneous benefit only, the more food, the better. The instantaneous benefit derived from food also demonstrates diminishing marginal utility. Second, the agent also benefits from consuming a composite good G_t. The agent buys food and the composite good with his income y. Lastly, the level of health stock at the beginning of the period h_t benefits the agent in that period. The health stock in any period is influenced by the past choice of food intake, a process which is described in the next section.

Consider the following period-t utility function

$$U_t = \lambda \ln F_t + \mu \ln G_t + \eta \ln h_t; \qquad \lambda, \mu, \eta \in (0,1).$$

The agent faces a three-period horizon. In each period, the agent chooses the food consumption F_t and the consumption of the composite good G_t to maximize expected utility for all the remaining periods subject to the following budget constraint:

$$y = p\,F_t + G_t,$$

where y denotes the agent's stationary income, p denotes the relative price of food, and G_t serves as the numeraire good.

The agent starts the first period with an exogenously given (overweight) level of h_1. At the beginning of each period thereafter, the agent observes the level of health stock and benefits from it for the period.

Since, in the third period, the agent has no concern of future health, food consumption yields only immediate utility. The observed level of health in this period, h_3, benefits the agent; however, it has no bearing on the choices of food consumption and the consumption of the composite good. Hence, in the second period, expected utility obtained in the third period is contingent upon h_3 observed. However, h_3 is influenced by food consumption in the second period. In principle, the agent then chooses food consumption and the consumption of the composite good in the second period, realizing that they have an impact on the expected utility to be obtained in the third period while taking the observed h_2 as given. Similarly, h_2 is influenced by the choice of food consumption in the first period and the agent chooses the food consumption in the first period while being aware that it affects all future utility. As such, the model is solved by backward induction.

In reality, people realize that current food consumption does impact future health. The multi-period model is meant to capture that aspect. However, the extent to which they value health depends on personal preferences. This aspect is also captured in the utility function by allowing different values of η.

The influences of current food consumption on future health are associated with the concept of health investment. The outcomes of the investment are uncertain. The details of this process are described next.

B. The Health Investment Function—The Diet Program

The variable h_t, the health stock in period t, is defined as a stock variable which is not chosen in any period. Rather, h_t is observed at the beginning of the period t and the agent gets to enjoy utility derived from that level of observed health for the current period.

This health stock variable can be a measure which is a combination of (an inverse of) weight measurement or BMI (Body Mass Index) measurement and other aspects that one considers 'health' benefits such as the effect on the odds of getting a heart attack, the impact on blood pressure, etc. There are medical studies that establish links between overweightness/obesity and harmful medical conditions.[14] For example, an overweight person is more likely to develop type II diabetes, heart diseases, strokes, high blood pressure, high cholesterol level, gallstones, osteoarthritis, sleep apnea, and even cancer.

[14] http://www.medterms.com/script/main/art.asp?articlekey=4608; see Review of Relevant Literature.

Even a modest weight loss of ten to twenty pounds can significantly improve one's health.[15]

The level of health capital stock observed at the beginning of any period depends, among other things, on the level of food intake in the previous period as compared to the prescribed amount. In general, the closer the agent is to the prescribed amount, the more improvement in health.

The health investment function is given by:

$$h_t = h_{t-1} + \alpha\, e^{-(F_{t-1}-\hat{F})^2} + \varepsilon_{t-1};$$

where α is a (positive) parameter that is unknown to the agent. However, the probability distribution of the α parameter is known. α is a coefficient of matching which shows how well the plan benefits the agent health-wise. For a particular diet program, α may be different for different agents due to idiosyncratic factors such as genetics. According to Eckel (2005) and Riley (2006), weight loss successes depend on whether the diet program is the 'right' match for the particular dieter.

The health investment function encompasses the idea that one must limit the amount of food consumed in order to improve one's health. However, less food does not always mean better health. The dieter's resting metabolic rate may also fall when food intake is dramatically low.[16] This means that if food consumption is too low, health can deteriorate and weight can actually increase. The health investment function reflects the fact that a moderate amount of food is best as denoted by $\hat{F}$, which is the amount of food intake prescribed by the diet program.

---

[15] http://www.webmd.com/content/article/46/2731_1658

[16] http://www.nahanniriverherbs.com/94,203, see review of literature.

The learning process of the parameter α is related to the work by Grossman, Kihlstrom, and Mirman (1977). However, the amount of learning is not linear in the choice variable. This is due to the premise that a moderate amount of food is best for one's physiological health.

The health benefit with regard to the amount of food intake is unknown due to the uncertainty in the amount of weight being lost as a result of following the prescription. In this sense, the model assumes that (1) the dieter has never undertaken the diet before, or (2) the dieter has undertaken the diet before but has not learned how well-matched the diet is to him.

For any agent, α could take on either a high value (well matched and highly beneficial) or a low value (poorly matched and only somewhat beneficial). Let α_H denote the high α and α_L denote the low α with $\alpha_H > \alpha_L > 0$. The prior belief and probability distribution of α is that α will take on the value α_H with probablity π_1, and with complementary probability, $1-\pi_1$, it takes on the low value of α_L.

The noise term ε_{t-1} is an unobservable random shock which takes place in period t−1. With a (bad) draw, a plan that is significantly beneficial to the agent on average may lead to a health reduction for the period. Generally, ε_{t-1} has a mean of zero, is independently and identically distributed across time, and its density function $f(\varepsilon)$ satisfies the Monotone Likelihood Ratio Property (MLRP), i.e. $\dfrac{f'(\varepsilon)}{f(\varepsilon)}$ is weakly monotone in ε (either non-decreasing or non-increasing). The property implies that the updated beliefs that the plan is well-matched are monotone in the signal which is observable as health. The expression being non-decreasing amounts to saying that a higher health signal leads to greater beliefs that α is high.

In particular, let ε_{t-1} be uniformly distributed over the interval $[-a, a]$ for all t, in which case MLRP is satisfied. (*See Appendix*).

The level of health capital stock h_{t-1} is observed at the beginning of the period t−1. The initial level of health stock at the beginning of the first period, h_1, is exogenously given.

At the end of each period, the ending health level is observed. This generally helps the agent infer the level of α. However, due to the unobservable random shock, the observed health level provides only noisy information regarding the quality of matching. The agent updates the belief about α using Bayes' rule.

III. The Solution

The solution is found by backward induction. The maximizing problem in the third period is solved first.

$$\max U_3 = \max \ \lambda \ln F_3 + \mu \ln G_3 + \eta \ln h_3 \ ; \ h_3 \text{ is observed}$$

subject to the budget constraint: $y = p F_3 + G_3$.

The budget constraint implies that

$$G_3 = y - p F_3 .$$

Therefore, the agent maximizes

$$\lambda \ln F_3 + \mu \ln [y - p F_3] + \eta \ln h_3.$$

The first-order condition is

$$\frac{\lambda}{F_3} - \frac{p\mu}{y - pF_3} = 0.$$

The solution is $F_3 = \dfrac{\lambda y}{(\lambda + \mu)p}$.

Substituting F_3 back into the budget constraint yields

$$G_3 = \frac{\mu y}{(\lambda + \mu)}.$$

Period 2

In the second period, the agent chooses F_2 and G_2 to maximize the instantaneous utility and the expected utility to be obtained in period 3. The solution depends on whether or not the agent has learned anything from the first period. The learning process is discussed first. Note that, under the uniform distribution of the random shock ε, the agent either learns completely, in which case the true α is realized or the agent learns nothing, in which case, the prior and posterior beliefs are the same.

The intuition is that, a health signal greater than some cutoff point could not have happened with α being α_L; and a health signal smaller than another cutoff point could not have happened with α being α_H. In these two cases, the agent learns completely that α is either α_H or α_L respectively. If the health signal falls between the two thresholds, the probabilities of the signal being drawn from α_L and α_H are identical. Hence, the agent does not learn anything in this case and the posterior belief is exactly the prior belief. (*See Appendix for a detailed proof*).

Recall the health investment function:

$$h_t = h_{t-1} + \alpha\, e^{-(F_{t-1}-\hat{F})^2} + \varepsilon_{t-1};$$

the prior beliefs:

$$\text{Prob}\ (\alpha = \alpha_H) = \pi_1\ ,$$

and the distribution of ε_{t-1} which is uniform over the interval $[-a, a]$, i.i.d. across time.

After the first period, F_1 and G_1 have been chosen and h_2 has been observed;

$$h_2 = h_1 + \alpha\, e^{-(F_1-\hat{F})^2} + \varepsilon_1.$$

For given values of α_H, α_L, and a; and a chosen value of F_1, the probability of learning is

$$\text{Prob (learn completely)} = \frac{(\alpha_H - \alpha_L)}{2a}\, e^{-(F_1-\hat{F})^2};$$

and the probability that the agent learns nothing is

$$\text{Prob (learn nothing)} = 1 - \frac{(\alpha_H - \alpha_L)}{2a}\, e^{-(F_1-\hat{F})^2}.$$

The agent learns the true value of α:

In the second period, the agent chooses F_2 and G_2 to maximize the expected utility over the remaining two periods. In the case in which the agent realizes the true value of α, the agent maximizes $U_2 + U_3$, using the realized α.

The first-order condition is:

$$\frac{\lambda}{F_2} - \frac{p\mu}{y - pF_2} - \frac{2\eta\alpha.(F_2 - \hat{F})e^{-(F_2 - \hat{F})^2}}{h_2 + \alpha.e^{-(F_2 - \hat{F})^2}} = 0 \qquad (1)$$

Let F_{2H}^* denote F_2 which solves Equation (1) with α_H and F_{2L}^* denote F_2 which solves the first-order condition with α_L. Let U_{2H}^* denote the total expected utility (U_2+U_3) which is obtained by consuming F_{2H}^* when $\alpha = \alpha_H$. Finally, let U_{2L}^* denote the total expected utility (U_2+U_3) which is obtained by consuming F_{2L}^* when $\alpha = \alpha_L$.

The agent does not learn the true value of α:

In this case, $\pi_2 = \pi_1$ and the expected health level in the third period depends on π_1. The first-order condition of the second-period maximizing problem is:

$$\frac{\lambda}{F_2} - \frac{p\mu}{y - pF_2} - \frac{2\pi_1\eta\alpha_H(F_2 - \hat{F})e^{-(F_2 - \hat{F})^2}}{h_2 + \alpha_H e^{-(F_2 - \hat{F})^2}}$$

$$- \frac{2(1-\pi_1)\eta\alpha_L(F_2 - \hat{F})e^{-(F_2 - \hat{F})^2}}{h_2 + \alpha_L e^{-(F_2 - \hat{F})^2}} = 0 \qquad (2)$$

Let $F_{2\alpha}^{*}$ denote F_2 which solves the Equation (2) and let $U_{2\alpha}^{*}$ denote the total expected utility (U_2+U_3) which is obtained by consuming $F_{2\alpha}^{*}$ when $\pi_2 = \pi_1$.

Equations (1) and (2) are examined in Sections IV.B. and IV. D. in order to make a comparison among food consumptions under different scenarios and to investigate if the agent rationally chooses to vary food consumptions across periods.

Period 1

In the first period, the agent chooses the level of F_1 and G_1 to maximize the expected total utility over three periods without discounting. The expected total utility obtained in periods 2 and 3 depends on the probability of learning. The expected total utility from three periods is:

$$\lambda \ln F_1 + \mu \ln(y - pF_1) + \eta \ln h_1 + \frac{(\alpha_H - \alpha_L)}{2a} e^{-(F_1-\hat{F})^2} [\pi_1 U_{2H}^{*} + (1-\pi_1) U_{2L}^{*}]$$

$$+ [1 - \frac{(\alpha_H - \alpha_L)}{2a} e^{-(F_1-\hat{F})^2}] U_{2\alpha}^{*} \qquad \text{(U1)}$$

The first three terms express instantaneous utility obtained in period 1 with h_1 exogenously given. The fourth term expresses the expected utility for periods 2 and 3 when the agent learns multiplied by the probability that the agent learns. The last term represents the expected utility for periods 2 and 3 when no learning occurs multiplied by the probability that the agent does not learn.

The first-order condition is:

$$\frac{\lambda}{F_1} - \frac{p\mu}{y - pF_1} - 2[\pi_1 U_{2H}^* + (1-\pi_1)U_{2L}^* - U_{2\alpha}^*]\frac{(\alpha_H - \alpha_L)}{2a}(F_1 - \hat{F})\, e^{-(F_1-\hat{F})^2}$$

$$+\frac{\partial U_{2\alpha}^*}{\partial F_1} = 0 \tag{3}$$

Let F_1^* denote the solution to period-one optimizing problem.

Equation (3) is useful in the derivation of the condition under which the agent overeats and an investigation of whether or not the agent's food consumption fluctuates from one period to the next. These analyses are in Sections IV.A. and IV.D.

IV. Some Interesting Results

A. Food Consumption in the First Period

<u>Lemma 1</u>: (Initial Adherence to the Diet). The agent does not consume the amount of food prescribed in the first period. If $\lambda > \dfrac{p\mu\hat{F}}{y - p\hat{F}}$ (i.e. the instantaneous benefit of food is sufficiently large), then the agent overeats in the first period, $F_1^* > \hat{F}$.

Proof:

Recall Equation (3). The task is to figure out the signs for the third and last terms. First consider the last term: $\dfrac{\partial U_{2\alpha}^*}{\partial F_1}$.

(A proof for Equation (3a) is in the appendix).

$$\frac{\partial U_{2\alpha}^{*}}{\partial F_1} = -\frac{2\bar{\alpha}\eta(F_1 - \hat{F})e^{-(F_1 - \hat{F})^2}}{h_2} - \frac{2\pi_1\alpha_H\eta(F_1 - \hat{F})e^{-(F_1 - \hat{F})^2}}{h_{3H}}$$

$$-\frac{2(1-\pi_1)\alpha_L\eta(F_1 - \hat{F})e^{-(F_1 - \hat{F})^2}}{h_{3L}} \tag{3a}$$

which is positive if $F_1 < \hat{F}$, and is negative if $F_1 > \hat{F}$.

The third term is positive if $F_1 < \hat{F}$, and is negative if $F_1 > \hat{F}$.

(a) If the first-period food consumption exceeds the prescribed amount, $\hat{F}$:

In this case the sign of the first term is positive and the sign of each of the last three terms is negative. Evaluated at $\hat{F}$, the first-order condition is positive since the last two terms become zero. This means that, at $\hat{F}$, the first term is necessarily greater than the second: $\frac{\lambda}{\hat{F}} > \frac{p\mu}{y - p\hat{F}}$.

(b) If the first-period food consumption is below the prescribed amount, $\hat{F}$:

In this case the sign of the second term is negative and the sign of each of the other three terms is positive. Evaluated at $\hat{F}$, the first-order condition is negative since the last two terms become zero. This means that, at $\hat{F}$, the first term is necessarily smaller than the second: $\frac{\lambda}{\hat{F}} < \frac{p\mu}{y - p\hat{F}}$.

Hence, the condition under which $F_1 > \hat{F}$ is derived as $\frac{\lambda}{\hat{F}} > \frac{p\mu}{y - p\hat{F}}$, or

$$\lambda > \frac{p\mu\hat{F}}{y - p\hat{F}} \tag{3b}$$

QED.

This means that the agent chooses to overeat (as compared to the prescribed amount) if λ is large relative to μ. That is, the agent has a strong preference for food relative to his preference for other goods. As this paper is an attempt to explain and describe dieters, it is sensible to assume that the agent in some sense has revealed a strong preference for food as that probably was the reason that the agent became overweight to begin with. Note that, according to Eckel (2005), if the agent has had past episodes of weight cycling, appetite for food increases, or the preference for food is higher.

Assumption A1: $\lambda > \dfrac{p\mu\hat{F}}{y - p\hat{F}}$ and the agent overeats compared to the prescribed amount $\hat{F}$.

B. Experimentation

Definition 1: A **non-learning solution (NL)** in period 1 is given by the amount of food F_1 and the amount of other goods G_1 that maximize current utility in period 1 and expected utility in the two following periods with a belief that no learning will occur, that is, the posterior beliefs are the same as the prior beliefs.

Definition 2: (Experimentation). The agent is said to **experiment** if the amount of food chosen differs from the myopic solution.

Lemma 2: The agent experiments in the first period.

Proof:

For period 1, the non-learning solution is the level of F_1 that maximizes

$$\lambda \ln F_1 + \mu \ln(y - pF_1) + \eta \ln h_1 + U^*_{2\alpha} \qquad \text{(U1NL)}$$

The first-order condition to the problem is

$$\frac{\lambda}{F_1} - \frac{p\mu}{y - pF_1} + \frac{\partial U^*_{2\alpha}}{\partial F_1} = 0 \qquad (4)$$

Let F_1^{NL} denote the non-learning solution in the first period.

Recall Equation (3), the first-order condition for F_1^*

$$\frac{\lambda}{F_1} - \frac{p\mu}{y - pF_1} - 2[\pi_1 U^*_{2H} + (1-\pi_1)U^*_{2L} - U^*_{2\alpha}]\frac{(\alpha_H - \alpha_L)}{2a}(F_1 - \hat{F})\, e^{-(F_1 - \hat{F})^2}$$

$$+ \frac{\partial U^*_{2\alpha}}{\partial F_1} = 0$$

Evaluated at F_1^{NL}, Equation (3) only has the third term since F_1^{NL} makes the other three terms zero. The third term is negative if $F_1 > \hat{F}$. This implies that $F_1^{NL} > F_1^*$ and the agent experiments by manipulating the choice of the amount of food in period one. In this case experimentation is done by consuming *less* food in period one or, in other words, the agent sticks to the plan more closely and strictly. QED.

This result is similar to that in the study by Grossman, Kihlstrom, and Mirman (1977) in that the agent manipulates the choice variable in order to gain some information. In their model, however, experimentation is executed by consuming more of the choice variable (a drug), whereas in this model, experimentation occurs when the agent consumes less of the choice variable (food). This is due to the specification of the health investment function that food in moderation is best for one's health.

When there is uncertainty in the quality of matching, following the diet plan has two distinct sources of expected benefit. First, it leads to an expected weight loss and improvement in health which benefits the agent immediately. Second, it allows the agent to obtain some information about the quality of matching. Although, specific to the model studied, the agent only probabilistically expects to obtain such information, the expectation of learning represents an added benefit to the immediate health benefit. Consequently, the agent follows the diet plan more closely than he would without this learning opportunity. Similar to Grossman, Kihlstrom, and Mirman (1977) and Kiefer and Nyarko (1989), imperfect learning is optimal. The imperfect learning in this model, however, is signified by the probability that the agent may not learn at all.

C. Comparisons of Food Consumptions in the Second Period

Lemma 3: The agent overeats in the second period.

Proof:

First, the first-order conditions for all food consumptions in the second period under different scenarios are recalled (Equations (1) and (2)):

$$F_{2H}^{*}: \frac{\lambda}{F_2} - \frac{p\mu}{y - pF_2} - \frac{2\eta\alpha_H (F_2 - \hat{F}) e^{-(F_2 - \hat{F})^2}}{h_2 + \alpha_H e^{-(F_2 - \hat{F})^2}} = 0 \tag{5}$$

$$F_{2L}^{*}: \frac{\lambda}{F_2} - \frac{p\mu}{y - pF_2} - \frac{2\eta\alpha_L (F_2 - \hat{F}) e^{-(F_2 - \hat{F})^2}}{h_2 + \alpha_L e^{-(F_2 - \hat{F})^2}} = 0 \tag{6}$$

$$F_{2\alpha}^{*}: \frac{\lambda}{F_2} - \frac{p\mu}{y - pF_2} - \frac{2\pi_1\eta\alpha_H (F_2 - \hat{F}) e^{-(F_2 - \hat{F})^2}}{h_2 + \alpha_H e^{-(F_2 - \hat{F})^2}}$$

$$- \frac{2(1-\pi_1)\eta\alpha_L (F_2 - \hat{F}) e^{-(F_2 - \hat{F})^2}}{h_2 + \alpha_L e^{-(F_2 - \hat{F})^2}} = 0 \tag{2}$$

Under Assumption A1, all three first-order conditions are positive when evaluated at $\hat{F}$. This implies that F_{2H}^{*}, F_{2L}^{*}, and $F_{2\alpha}^{*}$ are greater than $\hat{F}$. QED.

Lemma 3 shows that the agent will overeat in the second period, provided that he overeats in the first period. In this sense, the agent consistently and persistently overeats because of the relatively strong preference for food.

<u>Lemma 3A</u>: In the second period, food consumption for agents who learn that the plan is a bad match is greater than that for agents who learn that the plan is a good match (i.e. $F_{2L}^{*} > F_{2H}^{*}$).

Proof:

Evaluated at F_{2L}^{*}, the first-order condition for F_{2H}^{*}, Equation (5), reduces to:

$$\frac{2\eta\alpha_L (F_2 - \hat{F}) e^{-(F_2 - \hat{F})^2}}{h_2 + \alpha_L e^{-(F_2 - \hat{F})^2}} - \frac{2\eta\alpha_H (F_2 - \hat{F}) e^{-(F_2 - \hat{F})^2}}{h_2 + \alpha_H e^{-(F_2 - \hat{F})^2}}$$

which is negative. Hence, $F_{2L}^{*} > F_{2H}^{*}$. The agent sticks to the plan less closely if he finds that the plan is a bad match than he would if the plan were a good match. QED.

Lemma 3A shows that the agent consumes more food if the diet plan is less well-matched. This is intuitive as the primary benefit of eating less food is the health improvement which depends on the quality of matching.

Lemma 3B: In the second period, food consumption for agents who learn nothing about the plan is greater than that for agents who learn that the plan is a good match (i.e. $F_{2\alpha}^{*} > F_{2H}^{*}$).

Proof:

Evaluated at F_{2H}^{*}, the first-order condition for $F_{2\alpha}^{*}$, Equation (2), reduces to:

$$\frac{2(1-\pi_1)\eta\alpha_H(F_2-\hat{F})e^{-(F_2-\hat{F})^2}}{h_2+\alpha_H e^{-(F_2-\hat{F})^2}} - \frac{2(1-\pi_1)\eta\alpha_L(F_2-\hat{F})e^{-(F_2-\hat{F})^2}}{h_2+\alpha_L e^{-(F_2-\hat{F})^2}}$$

which is positive. Hence, $F_{2\alpha}^{*} > F_{2H}^{*}$. The agent sticks to the plan more closely if he learns that the plan is a good match than he would if no learning occurred after the first period. QED.

Lemma 3B shows that food consumption when the agent does not learn the quality of matching is greater than food consumption when the plan is well-matched. Again, the intuition is that the benefit of consuming less food in the present is the health improvement. The expected health improvement when the plan is of uncertain quality is lower than that when the plan is well-matched, thereby encouraging the agent to consume more food.

Lemma 3C: In the second period, food consumption for agents who learn that the plan is a bad match is greater than that for agents who learn nothing about the plan (i.e. $F_{2L}^{*} > F_{2\alpha}^{*}$).

Proof:

Evaluated at F_{2L}^{*}, the first-order condition for $F_{2\alpha}^{*}$, Equation (2), reduces to:

$$\frac{2\pi_1 \eta \alpha_L (F_2 - \hat{F}) e^{-(F_2 - \hat{F})^2}}{h_2 + \alpha_L e^{-(F_2 - \hat{F})^2}} - \frac{2\pi_1 \eta \alpha_H (F_2 - \hat{F}) e^{-(F_2 - \hat{F})^2}}{h_2 + \alpha_H e^{-(F_2 - \hat{F})^2}}$$

which is negative. Hence, $F_{2L}^{*} > F_{2\alpha}^{*}$. The agent sticks to the plan less closely if it has been learned that the plan is a bad match than he does if no learning occurred. QED.

Similar to Lemma 3B, Lemma 3C shows that food consumption when the plan is poorly-matched is greater than that when the quality of the plan is uncertain due to lower expected health improvement.

In conclusion, $\hat{F} < F_{2H}^{*} < F_{2\alpha}^{*} < F_{2L}^{*}$. The interpretation of Lemmas 3A, 3B, and 3C is that, the agent, in the second period, adheres to the food prescription most strictly if it is learned that the plan is a good match. Food consumption is the furthest away from the prescribed amount $\hat{F}$ when the agent learns that the plan is a bad match. Certainly, this makes perfect sense as the benefit from being on the diet plan and doing what is being prescribed depends crucially on the quality of the match. At the start of the second period, the information obtained thereafter is valueless. Therefore, the benefit of learning and experimentation no longer exists. This is true even if the agent did not learn at the end of the first period because the agent lacks the opportunity to act upon any information obtained at the end of the second period.

In all cases, the agent still never does exactly what the plan prescribes. Surely, every dieter wants to lose weight. But following the diet plan more strictly requires some sacrifice of immediate utility from food consumption. Lemma 3 suggests that the marginal health benefit of following the plan exactly does not justify the lost benefit as a result of sacrificing immediate food consumption. Furthermore, the agent intuitively follows the plan more closely if the plan has proven to be a good match as suggested by these three lemmas.

D. Subsequent Relapse in Dieting

Definition 3: (Relapse). The agent relapses (or 'slacks off') in period 2 if the agent chooses a higher level of food consumption in period 2 than in period 1 (i.e. $F_2 > F_1$).

Lemma 4A: Agents who learn that the plan is a bad match and agents who learn nothing about the plan unconditionally relapse in period 2.

Proof:

First, the case in which the agent does not learn the value of α is considered. Recall the first-order condition for $F_{2\alpha}^{*}$, Equation (2):

$$\frac{\lambda}{F_2} - \frac{p\mu}{y - pF_2} - \frac{2\pi_1\eta\alpha_H(F_2 - \hat{F})e^{-(F_2 - \hat{F})^2}}{h_2 + \alpha_H e^{-(F_2 - \hat{F})^2}}$$

$$- \frac{2(1-\pi_1)\eta\alpha_L(F_2 - \hat{F})e^{-(F_2 - \hat{F})^2}}{h_2 + \alpha_L e^{-(F_2 - \hat{F})^2}} = 0$$

And the first-order condition for F_1^*, Equation (3):

$$\frac{\lambda}{F_1} - \frac{p\mu}{y - pF_1} - 2[\pi_1 U_{2H}^* + (1-\pi_1)U_{2L}^* - U_{2\alpha}^*]\frac{(\alpha_H - \alpha_L)}{2a}(F_1 - \hat{F})\, e^{-(F_1 - \hat{F})^2}$$

$$+ \frac{\partial U_{2\alpha}^*}{\partial F_1} = 0$$

where $$\frac{\partial U_{2\alpha}^*}{\partial F_1} = -\frac{2\overline{\alpha}\eta(F_1 - \hat{F})e^{-(F_1 - \hat{F})^2}}{h_2} - \frac{2\pi_1\alpha_H\eta(F_1 - \hat{F})e^{-(F_1 - \hat{F})^2}}{h_{3H}}$$

$$- \frac{2(1-\pi_1)\alpha_L\eta(F_1 - \hat{F})e^{-(F_1 - \hat{F})^2}}{h_{3L}} \qquad \text{(3a)}$$

Rewrite Equation (3) as: $$\frac{\lambda}{F_1} - \frac{p\mu}{y - pF_1} - \frac{2\pi_1\alpha_H\eta(F_1 - \hat{F})e^{-(F_1 - \hat{F})^2}}{h_{3H}}$$

$$- \frac{2(1-\pi_1)\alpha_L\eta(F_1 - \hat{F})e^{-(F_1 - \hat{F})^2}}{h_{3L}}$$

$$- 2[\pi_1 U_{2H}^* + (1-\pi_1)U_{2L}^* - U_{2\alpha}^*]\frac{(\alpha_H - \alpha_L)}{2a}(F_1 - \hat{F})\, e^{-(F_1 - \hat{F})^2}$$

$$- \frac{2\overline{\alpha}\eta(F_1 - \hat{F})e^{-(F_1 - \hat{F})^2}}{h_2} = 0$$

The first four terms together are the first-order condition for $F_{2\alpha}^{*}$, Equation (2). Hence the first-order condition for F_1^{*}, evaluated at $F_{2\alpha}^{*}$, has a sign of the last two terms of the last expression:

$$-2[\pi_1 U_{2H}^{*} + (1-\pi_1)U_{2L}^{*} - U_{2\alpha}^{*}]\frac{(\alpha_H - \alpha_L)}{2a}(F_1 - \hat{F})\, e^{-(F_1-\hat{F})^2}$$

$$-\frac{2\bar{\alpha}\eta(F_1 - \hat{F})e^{-(F_1-\hat{F})^2}}{h_2}$$

Both terms are negative and hence the first-order condition for F_1^{*}, evaluated at $F_{2\alpha}^{*}$, has a negative sign and it follows that $F_{2\alpha}^{*} > F_1^{*}$. The agent relapses in period 2 if he does not learn the true value of α.

By Lemma 3C, $F_{2L}^{*} > F_{2\alpha}^{*}$. Therefore, it follows immediately that the agent also relapses if he learns that the program is a bad match. QED.

Lemma 4A provides the insights that the agent always relapses in the second period if (1) he does not learn the value of α, or (2) if he learns that it was a poor match. The latter case clearly is intuitive. A poorly-matched diet plan produces low health benefit. In reality, if the dieter learns that a diet program is inefficient for him, it would be pointless to keep sacrificing current consumption when the improvement in health is known to be insignificant.

The agent also relapses even when the true value of α is not learned. In this case, the difference in food consumption in the first period and that in the second period is due solely to the benefit of obtaining information which exists only in the first period. As stated earlier, the agent does not have an opportunity to act upon any information learned in the second period or after. As a result, the benefit of following the plan falls in the second period when learning does not occur. The agent adjusts and follows the plan accordingly with the lower benefit.

Lemma 4B: Agents who learn that the plan is a good match relapse in the second period whenever $\pi_1 > 0.5$. Moreover, if α_H is sufficiently large, relapse occurs over larger values of π_1.

Proof:

Recall the first-order condition for F_{2H}^*, Equation (5):

$$\frac{\lambda}{F_2} - \frac{p\mu}{y - pF_2} - \frac{2\eta\alpha_H (F_2 - \hat{F})e^{-(F_2 - \hat{F})^2}}{h_2 + \alpha_H e^{-(F_2 - \hat{F})^2}} = 0.$$

Rewrite Equation (3) as:

$$\frac{\lambda}{F_1} - \frac{p\mu}{y - pF_1} - \frac{2\alpha_H \eta(F_1 - \hat{F})e^{-(F_1 - \hat{F})^2}}{h_{3H}} + \frac{2(1-\pi_1)\alpha_H \eta(F_1 - \hat{F})e^{-(F_1 - \hat{F})^2}}{h_{3H}}$$

$$- \frac{2(1-\pi_1)\alpha_L \eta(F_1 - \hat{F})e^{-(F_1 - \hat{F})^2}}{h_{3L}}$$

$$- 2[\pi_1 U_{2H}^* + (1-\pi_1)U_{2L}^* - U_{2\alpha}^*]\frac{(\alpha_H - \alpha_L)}{2a}(F_1 - \hat{F})\, e^{-(F_1 - \hat{F})^2}$$

$$- \frac{2\overline{\alpha}\eta(F_1 - \hat{F})e^{-(F_1 - \hat{F})^2}}{h_2} = 0$$

The first three terms together are the first-order condition for F_{2H}^{*}. The sign of the above expression, evaluated at F_{2H}^{*}, has the sign of the last four terms, which can be reduced to:

$$(F_1-\hat{F})\, e^{-(F_1-\hat{F})^2}\left[\frac{(1-\pi_1)\eta\alpha_H}{h_{3H}}-\frac{(1-\pi_1)\eta\alpha_L}{h_{3L}}-(\Delta U)\frac{(\alpha_H-\alpha_L)}{2a}-\frac{\eta\bar{\alpha}}{h_2}\right]$$

$$\Delta U=\pi_1 U_{2H}^{*}+(1-\pi_1)U_{2L}^{*}-U_{2\alpha}^{*} \text{ and } \bar{\alpha}=\pi_1\alpha_H+(1-\pi_1)\alpha_L$$

The term outside the brackets is positive. The terms inside the brackets will be negative if

$$h_2 < 2\pi_1 h_2+\pi_1\alpha_H\, e^{-(F_1-\hat{F})^2} \tag{7}$$

(*A sufficient but not necessary condition*)

(*A detailed proof is in the appendix*).

Note that if $\pi_1 > 0.5$, Expression (7) is immediately true. However, for smaller π_1, the expression is still true if the last term of the right hand side is large enough. A sufficiently large α_H ($\alpha_H > \dfrac{(1-2\pi_1)h_2}{\pi_1 e^{-(F_1-\hat{F})^2}}$) would make this true. QED.

Lemma 4B suggests that, in addition, the agent relapses sometimes even when the learned information indicates that the program is a good match and highly beneficial. For the agent not to relapse, π_1 has to be below 0.5 or α_H has to be low enough. This may be surprising. The result, however, exemplifies the benefit and the value of information. A low α_H generates low expected value of information in the first period, other things equal, because learning leads to the knowledge that the plan is only *slightly better* than expected. As a result, the dieter does not follow the plan as strictly in the first period. In the second period, with a sufficiently low α_H, a well-matched plan is followed more strictly as the deemed benefit of good matching dominates the disappearing benefit of learning. Similarly, the value of information is increasing in π_1. Therefore a high π_1 leads to more experimentation in the first period. In the second period, when the benefit of learning disappears, the dieter ceases to follow the plan so strictly.

Almost every diet plan in the real world suffers from some form of relapse on the dieters' part. According to Dansinger et al. (2005) in one study of 160 participants and four diet programs published by the Journal of the American Medical Association, 34 subjects completely quit after only two months and 61 quit after six months. According to the researchers, plans which suffer a low adherence rate are those with high initial weight loss among participants who did complete the study. This may suggest that dieters follow the prescription very strictly initially. Overall, the dropout rate is 42 percent. The average weight loss is computed by looking at only participants who completed the program.

The relapse effect can also be observed from weight loss, weight re-gain and weight fluctuation in general. According to several studies, only about one to three percent of weight loss is ever maintained in the long run. [17] In some studies, 95 percent

[17] For example: F.M. Kramer's long-term follow-up study published in a 1989 *International Journal of Obesity*.

of dieters regain some weight that was initially lost.[18] By four to five years after the diet program is initiated, almost all dieters have regained most or all the weight that has been lost. And most of these dieters weigh *more* than they did when starting the program.[19] According to Wadden and Stunkard et al.(1988), the larger and faster the initial weight loss, the faster the weight regain. In fact, they assert that the faster one loses weight, the higher the chance that one will weigh more after dieting. All these studies basically suggest that most dieters do relapse after the initial period. This is exactly what the model establishes—the agent follows the prescription very closely initially only to significantly relapse later on.

The model also gives freedom on how a dieter assesses what π_1 is. In reality, this is executed by wandering onto the internet, reading books, articles, etc. Given the way most diet programs advertise their products, claim scientific proof misleadingly, and selectively publish successful testimonials, a lot of dieters may mistakenly assess π_1 as being high.[20] As mentioned earlier, higher π_1 leads to higher strictness with which the prescription is followed initially. This adds to the problem and more dieters follow the prescription closely for a while just to relapse and regain weight later on.

---

[18] http://studentweb.tulane.edu/~cleblanc/dietindustry.html
[19] http://www.techcentralstation.com/071803B.html
[20] See, for example, Tapper-Gardzina, et al. (2002), Cheuvront (2003), and Goff, et al. (2006).

V. Some Comparative Statics

Proposition 1: The more disperse the random shock, the greater the amount of food intake in period one, i.e. $\frac{\partial F_1}{\partial a} > 0$.

Proof:

Let f denote

$$\frac{\lambda}{F_1} - \frac{p\mu}{y - pF_1} - 2[\pi_1 U_{2H}^* + (1-\pi_1)U_{2L}^* - U_{2\alpha}^*]\frac{(\alpha_H - \alpha_L)}{2a}(F_1 - \hat{F})\, e^{-(F_1 - \hat{F})^2} + \frac{\partial U_{2\alpha}^*}{\partial F_1}$$

which is the expression on the left-hand side of the first-order condition in the first period, Equation (3).

$$\frac{\partial F_1}{\partial a} = -\frac{f_a}{f_{F_1}}, \text{ where}$$

$$f_a = 2[\pi_1 U_{2H}^* + (1-\pi_1)U_{2L}^* - U_{2\alpha}^*]\frac{(\alpha_H - \alpha_L)}{2a^2}(F_1 - \hat{F})\, e^{-(F_1 - \hat{F})^2}$$

which is positive when $F_1 > \hat{F}$, and f_{F_1} is negative by concavity. Hence $\frac{\partial F_1}{\partial a}$ is positive when the agent's first-period food intake exceeds the prescription. QED.

The proposition is with regard to the random noise which interferes with the learning process. When there is more random noise, the agent consumes more food in the first period, taking himself away from the prescription.

In reality, there are factors beyond the dieters' control while losing weight on a diet program. The greater the influence of these factors, the less strictness with which the dieter follows the plan. This is because the value of information depends on how clear the signal is. Noises interfere with the dissemination of information and reduce its clarity. As a result, the value of information and the benefits of learning fall. The agent adheres to the prescription less strictly by consuming more food accordingly.

Proposition 2: The greater the difference between α_H and α_L, which amounts to higher likelihood of learning, the smaller the amount of food intake in period 1; the agent adheres more strictly to the diet prescription when the difference between benefit derived from the good match and that from the bad match is larger, i.e. $\frac{\partial F_1}{\partial(\alpha_H - \alpha_L)} < 0$.

Proof:

$$\frac{\partial F_1}{\partial(\alpha_H - \alpha_L)} = -\frac{f_{(\alpha_H - \alpha_L)}}{f_{F_1}}$$

$$f_{(\alpha_H - \alpha_L)} = -\frac{1}{a}[\pi_1 U_{2H}^* + (1-\pi_1)U_{2L}^* - U_{2\alpha}^*](F_1 - \hat{F})\, e^{-(F_1 - \hat{F})^2}$$

which is negative when $F_1 > \hat{F}$, and f_{F_1} is negative by concavity. Hence $\frac{\partial F_1}{\partial(\alpha_H - \alpha_L)}$ is negative when the agent's first-period food intake exceeds the prescription. QED.

True α_H and true α_L exist for every plan. The model assumes that these values are known. What is unknown is the probability that the dieter is of one type and not the other. On the Jenny Craig Diet website, for example, one can find numerous dramatic success stories (always with a fine-print disclaimer: "results not typical").[21] These suggest that the true α_H is high for the plan, if you happen to be among the lucky few.

[21] http://www.jennycraig.com

Considering the fine print, one can easily infer that there are people whom the plan has failed because they are of α_L type. No wonder dieters closely follow the prescription for a while with weight being gained back afterwards. Other fad diet programs offer similar allures. For example, the South Beach and the Atkins programs persistently report eight to fifteen pounds in the first week—very high α_H. Proposition 2 suggests that dieters would follow these diets very strictly to learn. Fad dieters do exactly what the proposition suggests.

Virtually all diet programs have success pages on their websites—even the Weight Watchers website.[22] However, the claim that comes with Weight Watchers is always humble, in the range of one to two pounds per month. This suggests that, if there is any difference between α_H and α_L for Weight Watchers, it is probably small. If so, Proposition 2 suggests that dieters on the program would not follow the diet plan so closely to begin with. Conversely, they experience less relapse.

Proposition 3: When the value of information from learning increases, the amount of food intake in the first period drops and the agent sticks to the plan more strictly.

Proof:

The value of information can be represented by the gain in expected utility that results from learning such information. This can be written as

[22] http://www.weightwatchers.com

$\pi_1 U_{2H}^* + (1-\pi_1)U_{2L}^* - U_{2\alpha}^*$. Note that this gain is positive by concavity (in α) of the utility function. The first two terms are E(U(α)) and the last term is U(E(α)). Let ΔU denote this gain. The proposition is that

$$\frac{\partial F_1}{\partial \Delta U} < 0, \qquad \text{where } \frac{\partial F_1}{\partial \Delta U} = -\frac{f_{\Delta U}}{f_{F_1}}$$

$$f_{\Delta U} = -\frac{(\alpha_H - \alpha_L)}{a}(F_1 - \hat{F})\, e^{-(F_1 - \hat{F})^2}$$

which is negative when $F_1 > \hat{F}$, and f_{F_1} is negative by concavity. Hence $\frac{\partial F_1}{\partial \Delta U}$ is negative when the agent's first-period food intake exceeds the prescription. QED.

Again, this proposition exhibits how the value of information affects the agent's choice. The strictness with which the agent follows the plan is intuitively increasing in the value of information.

In comparison to fad diets, there is not much to gain in terms of information when one is on a gradual plan such as the Weight Watchers Program. In essence, there is no secret with Weight Watchers. What you see is what you get. You will lose one to two pounds a month if you are on the plan. The only things required are patience and perseverance. The Weight Watchers Program is essentially the antithesis of the 'get thin quick' scheme. The dieters do not experiment as much as they do with fad diet plans.

Proposition 4: When the prior belief that the diet plan is a good match is higher, the amount of food chosen in period 1 falls and the agent initially adheres to the diet prescription more strictly, i.e. $\frac{\partial F_1}{\partial \pi_1} < 0$.

Proof:

$$\frac{\partial F_1}{\partial \pi_1} = -\frac{f_{\pi_1}}{f_{F_1}}$$

$$f_{\pi_1} = -[U_{2H}^* - U_{2L}^* - \eta \ln h_{3H} + \eta \ln h_{3L}] \frac{(\alpha_H - \alpha_L)}{a} (F_1 - \hat{F})\, e^{-(F_1 - \hat{F})^2}$$

$$- 2\eta\, (F_1 - \hat{F}) [\frac{\alpha_H e^{-(F_1 - \hat{F})^2}}{h_{3H}} + \frac{\alpha_L e^{-(F_1 - \hat{F})^2}}{h_{3L}}]$$

Since $U_{2H}^* - U_{2L}^* - \eta \ln h_{3H} + \eta \ln h_{3L} = 0$, the first term is zero. The second term is negative when $F_1 > \hat{F}$. Hence, f_{π_1} is negative when the agent eats more than the amount prescribed in period 1. QED.

This is very intuitive since the agent sticks more closely to the prescription primarily because there is a higher chance that the plan is of a well-matched/highly beneficial type. The higher the chance (or the belief that it is), the more closely the agent is to the prescription.

In the real world, prior beliefs are assessed subjectively. Diet plan providers have an incentive to make dieters believe that π_1 is high. As mentioned earlier, self-posted and selective success stories on websites, unfounded 'scientific' claims, and false advertisement facilitate would-be dieters to perceive that π_1 is high. In this sense, competition among commercial diet plans actually leads to an outcome where dieters choose to closely follow the diet prescription in the beginning and experience weight cycling as a result.

In conclusion, the agent chooses to adhere more closely to the diet prescription in the first period when (1) the likelihood of learning increases, (2) the value of information generated by learning increases, or (3) the prior belief that the plan is well-matched increases. As discussed, fad diets provide greater likelihood of learning, yield greater value of information, and foster prior belief that highly favors the well-matched type than do long-term weight management programs (non-fad). Consequently, fad dieters initially adhere to the diet plan more closely than non-fad dieters. This may lead one to conclude that fad diets yield better weight-loss outcomes than non-fad diets. However, the following proposition shows that, over multiple periods, the conclusion may be otherwise.

Proposition 5: Among the agents whose food consumption in period 2 is greater than that in period 1 (i.e. 'relapsing' or 'slacking off'), the greater the α_L parameter, the lower the food consumption in period 2 (i.e. following the prescription more closely in period 2). That is, $\frac{\partial F_{2L}^*}{\partial \alpha_L} < 0$ and $\frac{\partial F_{2\alpha}^*}{\partial \alpha_L} < 0$.

Proof:

First, note that $\frac{\partial F_{2H}^*}{\partial \alpha_L} = 0$, as α_L has no bearing on the choice of food consumption in period 2 if the agent has learned that the plan is highly beneficial (i.e. of α_H type).

$$\frac{\partial F_{2L}^*}{\partial \alpha_L} = -\frac{f_{\alpha_L}}{f_{F2L}}$$

Recall Equation (6): $f = \frac{\lambda}{F_2} - \frac{p\mu}{y - pF_2} - \frac{2\eta\alpha_L(F_2 - \hat{F})e^{-(F_2 - \hat{F})^2}}{h_2 + \alpha_L e^{-(F_2 - \hat{F})^2}}$. Note that $f_{F_{2L}}$ is negative by concavity.

$$f_{\alpha_L} = -\frac{h_2[2\eta\alpha_L(F_2 - \hat{F})e^{-(F_2 - \hat{F})^2}]}{h_3^2} \text{ which is negative.}$$

Therefore, $\frac{\partial F_{2L}^*}{\partial \alpha_L} = -\frac{f_{\alpha_L}}{f_{F_{2L}}}$ is negative. $\frac{\partial F_{2\alpha}^*}{\partial \alpha_L} < 0$ can be proven in a similar fashion.

QED.

As discussed earlier, fad diet plans most likely have a lower α_L than that of non-fad diet plans. Proposition 5 suggests that, among dieters who relapse in the second period, fad dieters adhere to the diet plan less closely than non-fad dieters.

Among dieters who relapse (i.e. F_2 is greater than F_1), Propositions 2, 3, 4, along with Proposition 5 show that, from the first period to the second period, amounts of food consumed swing more greatly among fad dieters than they do among non-fad dieters. With greater swings in food consumption come greater weight fluctuations and weight cycling. This is intuitive as non-fad diet plans most likely offer an average weight loss without much deviation—one will not experience a dramatic weight-loss but the weight-loss will be sustained while fad diet plans offer dramatic weight loss as long as the plan fits you well. If not, one wasted time and energy for nothing. Fad diet plans are experimented and 'crashed' initially while the opportunity of learning exists. Once the initial period is over, the opportunity to experiment and learn ceases to exist and the dieter naturally and rationally moves away from the prescription or stops following the diet plan so closely. This suggests higher attrition rates among fad diet plans than those among non-fad diet plans. It is well-known that the drop-out rate among Weight Watchers (a non-fad program) followers is very low which is consistent with these propositions.[23]

There are studies regarding dropouts among some popular plans. According to studies done in 2004, 40 percent of Atkins dieters dropped out of the plan before completing.[24] Comparable figures have been reported for the South Beach diet where 45 percent dropped out after one year.[25]

With this in mind, it is not surprising to see weight fluctuations that numerous dieters have experienced. Fad diets are named 'fad' because they are short-lived. Dieters start them and abandon them shortly after. Pounds leave their bodies and then they come back. These propositions explain why dieters experience weight cycling under some diet programs.

Weight fluctuations or weight cycling have several impacts on how bodies function over the long run. Steen, Oppliger and Brownell (1988) find that weight cycling results in significantly lower resting metabolism rates—about 20 percent less energy per kilogram of lean body mass per hour is expended during resting periods. This has a significant implication on the process of losing weight for any overweight agent. As dieters go through many different fad diets, experiencing weight cycles along the way, it becomes increasingly hard to lose weight even when a well-matched plan comes along

---

[23] For example, Dansinger, et al. (2005).
[24] http://www.thedietchannel.com/atkins.htm
[25] For example, Foster, et al. (2003).

because the body becomes increasingly efficient at expending energy and the metabolism rates become dramatically low. With other negative impacts of being obese itself (type II diabetes, blood pressure, etc.), weight cycling certainly is not something any dieter ever wants to experience.

Weight cycling also has medical side effects on other aspects of life. Lee and Paffenbarger (1992) find significant increases in risk in mortality from all causes and risk in mortality from coronary diseases among weight cycling subjects.

There are also medical articles that express concerns about possible adverse psychological impacts resulting from weight cycling. However, there have been no official studies done to support the argument. These articles simply warn against 'feeling like a failure' in the face of weight cycling.[26]

VI. Conclusion

In this chapter, I develop a three-period, utility-maximizing dieting model in which the dieting process is viewed as a health investment with uncertain outcomes and the agent observes only a noisy health signal.

This chapter illustrates an impact of uncertainty of a dieting process where food consumption is restricted. The agent chooses the amount of food to consume. First, I establish that, under a reasonable condition, the agent never exactly follows the prescription. With uncertainty about the health benefit, I show that the agent experiments with the diet program by adhering to the plan more strictly than he would if there were no

[26] For example, *Weight Cycling.* National Task Force on the Prevention and Treatment of Obesity. JAMA 272: 1196-1202; *Weight Loss and Nutrition Myths.* National Institute of Diabetes and Digestive and Kidney Diseases (NIDDK). NIH Publication No. 00-4561.

possibility of learning. The model also suggests that, in most cases, the agent relapses after learning has occurred.

In the dieting world, reports and studies find that fad diet plans are likely to lead to sudden and drastic weight losses immediately after the dieter has started the diet program. This is usually followed by a period of weight regain. The outcomes established by the model are consistent with these stylized facts.

Some comparative statics exercises have also been conducted. These help explain the success that is seen with some long-term plans such as the Weight Watchers program. They also help explain the observed failure of many fad diets.

More importantly, Proposition 4 shows that the severity of the weight cycling phenomenon increases with agents' optimistic prior beliefs. Greater optimism only leads to more experimentation which eventually leads to more weight cycling and worse long-run physiological health.

In reality, optimism is created by the diet industry's marketing and advertising effort. Therefore, it is perhaps advisable that these advertising strategies and the dieting results that the diet programs claim be regulated and monitored in some way. As suggested by several medical studies, fad diets often falsely claim scientific results. False advertisement of diet plans only distorts the true pictures for dieters. Specifically, truthful information about each diet plan such as the probability of success and the magnitude of success needs to be conspicuously disclosed in order for dieters to make good, well-informed decisions. This includes all the negative, short-term and long-term side effects of the diet plan. If a diet plan, in helping dieters lose ten pounds in the short run, has a possibility of leading to a long-run weight cycling, then dieters should be informed of this true likelihood. With this kind of regulating effort, individual welfare and personal long-run health can be improved.

In comparison to the food and pharmaceutical industries, the diet industry is lagging in terms of truth in labeling. With regard to food and drugs, the Food and Drug Administration (FDA—a division of the U.S. Department of Health and Human Services) regulates virtually everything from food safety and nutrition to drug and toxicology research and evaluation.[27] The Truth in Labeling Act, enforceable on all foods, drugs, and cosmetics, was initiated in 1938 and was amended in 1964, 1990 and 2004.[28] As a result, we have everything labeled on our food and drug packages ranging from amounts and nutritional values of different substances in our food to nearly all the conceivable side effects of each drug available to the public.

The concerns about overweightness and obesity are also within the responsibility of the U.S. Department of Health and Human Services. However, there is no division within it that directly oversees the diet programs. Consequently, there are currently no regulations to speak of when it comes to any kind of labeling information about diet programs. Perhaps there should be an independent division solely in charge of regulating and overseeing the diet industry. Part of its mission could be to conduct and monitor unbiased scientific research that supports (or nullifies) a particular claim and to facilitate the truthful disclosure and labeling of information about diet programs. Dieters would have a better idea of what to expect, negative and positive, when starting a diet.

Information is naturally valuable. Dieters need to 'pay' to obtain this valuable asset. One way is to experiment in order to learn about the health benefit. This chapter

---

[27] http://www.fda.gov/default.htm
[28] http://encarta.msn.com/encyclopedia_761560675_4/Food_Processing_and_Preservation.html

suggests one way in which the mechanism may work. It also explains weight cycling under rational behavior as a result of experimentation. It is hoped that the study helps in identifying the problems and some regulatory issues in diet plans and the diet industry as a whole.

Appendix

On MLRP of the uniform distribution

For ε uniformly distributed over the interval $[-a, a]$, i.e.

$$f(\varepsilon) = \frac{1}{2a}; -a \le \varepsilon \le a$$

$$= 0 \text{ ; elsewhere}$$

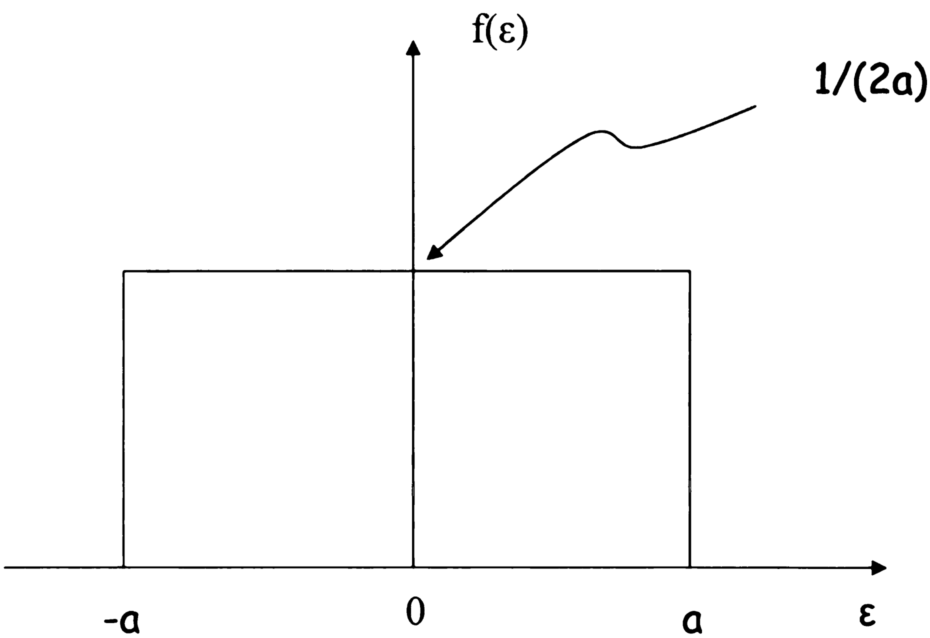

Figure 1.1: Uniform Distribution of the Noise Term ε

Recall the health investment function in the first period:

$$h_2 = h_1 + \alpha\, e^{-(F_1 - \hat{F})^2} + \varepsilon_1.$$

At the end of the first period, F_1 has been chosen and h_2 has been observed. Let the posterior belief (that $\alpha = \alpha_H$) be denoted by π_2.

Rewrite the health function as

$$\varepsilon_1 = h_2 - h_1 - \alpha\, e^{-(F_1 - \hat{F})^2}$$

If $\alpha = \alpha_H$, we have

$$\varepsilon_{1H} = h_2 - h_1 - \alpha_H\, e^{-(F_1 - \hat{F})^2} \qquad \text{(A1)}$$

And if $\alpha = \alpha_L$, we have

$$\varepsilon_{1L} = h_2 - h_1 - \alpha_L\, e^{-(F_1 - \hat{F})^2} \qquad \text{(A2)}$$

<u>Lemma A1</u>: If (A1) $< -a$, then $\pi_2 = 0$.

That is, if ε_{1H}, the implied noise when $\alpha = \alpha_H$, is less than the lower limit of its distribution, then the agent's updated belief is that the plan is poorly matched for certain. This is true because such a low level of health (and hence, noise) could not have been observed if the plan was well matched.

<u>Lemma A2</u>: If (A2) $> a$, then $\pi_2 = 1$.

That is, if ε_{1L}, the implied noise when $\alpha = \alpha_L$, is greater than the upper limit of its distribution, then the agent's updated belief is that the plan is well matched for certain. Similar to the argument for Lemma A1, such a high level of health and noise could not have been observed if the plan was poorly matched.

Lemma A3: If (A1) $> -a$, and (A2) $< a$, then $\pi_2 = \pi_1$.

If the level of implied noise and the level of health observed could have come from either distribution, then the agent updates the belief according to Bayes' Rule as follows:

$$\pi_2 = \frac{\pi_1 * p(h_2 \mid \alpha_H)}{\pi_1 * p(h_2 \mid \alpha_H) + (1-\pi_1) * p(h_2 \mid \alpha_L)}$$

In this overlapping area, $p(h_2 \mid \alpha_H) = p(h_2 \mid \alpha_L) = \frac{1}{2a}$. Substituting $\frac{1}{2a}$ into the above expression yields $\pi_2 = \pi_1$, that is, the agent's posterior beliefs are exactly the priors.

For the Monotone Likelihood Ratio Property (MLRP) to (weakly) hold, the following has to be satisfied:

$$\frac{d}{dh_2}\left(\frac{f_L}{f_H}\right) \leq (\text{or} \geq)\ 0;\ \text{where } f_L = p(h_2 \mid \alpha_L) \text{ and } f_H = p(h_2 \mid \alpha_H).$$

Since, (1) $f_L = 1$ and $f_H = 0$ for low observed health by Lemma A1,

(2) $f_L = f_H = \frac{1}{2a}$ for moderate observed health by Lemma A3, and

(3) $f_L = 0$ and $f_H = 1$ for high observed health by Lemma A2,

$\left(\frac{f_L}{f_H}\right)$ is non-increasing with respect to h_2, i.e. $\frac{d}{dh_2}\left(\frac{f_L}{f_H}\right) \leq 0$. MLRP is satisfied with uniform distribution. This amounts to saying that higher h_2 brings about higher expectation that the plan is well matched, i.e. α was high (α_H).

Furthermore, Lemma A1 shows that, for a sufficiently high level of observed health, the agent completely learns that the plan is well-matched ($\alpha = \alpha_H$) and Lemma A2 shows that the agent completely learns that the plan is poorly matched ($\alpha = \alpha_L$)

when the level of observed health is sufficiently low. Lastly, Lemma A3 shows that, for a moderate level of observed health, the agent learns nothing as the posterior beliefs are exactly the prior.

<u>Proof of Equation (3a)</u>:

$$U_{2\alpha}^{*} = U_{2\alpha}(F_{2\alpha}^{*})$$

$$= \lambda \ln F_{2\alpha}^{*} + \mu \ln(y - pF_{2\alpha}^{*}) + \eta \ln h_2$$

$$+ \lambda \ln F_3 + \mu \ln(y - pF_3) + \eta \pi_1 \ln h_{3H} + \eta(1-\pi_1)\ln h_{3L}$$

where h_2 is observed at the beginning of the second period,

$$h_{3H} = h_1 + \alpha_H e^{-(F_1 - \hat{F})^2} + \alpha_H e^{-(F_{2\alpha}^{*} - \hat{F})^2} + \varepsilon_1 + \varepsilon_2$$

$$h_{3L} = h_1 + \alpha_L e^{-(F_1 - \hat{F})^2} + \alpha_L e^{-(F_{2\alpha}^{*} - \hat{F})^2} + \varepsilon_1 + \varepsilon_2$$

$$\frac{\partial U_{2\alpha}^{*}}{\partial F_1} = \frac{\eta}{h_2}\,\overline{\alpha}\,(-2)\; e^{-(F_1 - \hat{F})^2}\;(F_1 - \hat{F}) + \frac{\eta \pi_1}{h_{3H}}\,\alpha_H(-2)\; e^{-(F_1 - \hat{F})^2}\;(F_1 - \hat{F})$$

$$+ \frac{\eta(1-\pi_1)}{h_{3L}}\,\alpha_L(-2)\; e^{-(F_1 - \hat{F})^2}\;(F_1 - \hat{F})$$

$$= -\frac{2\overline{\alpha}\eta(F_1 - \hat{F})e^{-(F_1 - \hat{F})^2}}{h_2} - \frac{2\pi_1\alpha_H\eta(F_1 - \hat{F})e^{-(F_1 - \hat{F})^2}}{h_{3H}}$$

$$- \frac{2(1-\pi_1)\alpha_L\eta(F_1 - \hat{F})e^{-(F_1 - \hat{F})^2}}{h_{3L}},$$

where $\overline{\alpha} = \pi_1\alpha_H + (1-\pi_1)\alpha_L$ QED.

Proof of Condition (7):

$$(F_1-\hat{F})\, e^{-(F_1-\hat{F})^2}\, [\frac{(1-\pi_1)\eta\alpha_H}{h_{3H}} - \frac{(1-\pi_1)\eta\alpha_L}{h_{3L}} - (\Delta U)\frac{(\alpha_H-\alpha_L)}{2a} - \frac{\eta\bar{\alpha}}{h_2}] \text{ is negative}$$

$$\text{if } \frac{(1-\pi_1)\eta\alpha_H}{h_{3H}} - \frac{(1-\pi_1)\eta\alpha_L}{h_{3L}} - (\Delta U)\frac{(\alpha_H-\alpha_L)}{2a} - \frac{\eta\bar{\alpha}}{h_2} \text{ is negative}$$

since $(F_1-\hat{F})\, e^{-(F_1-\hat{F})^2}$ is positive.

$$\frac{(1-\pi_1)\eta\alpha_H}{h_{3H}} - \frac{(1-\pi_1)\eta\alpha_L}{h_{3L}} - (\Delta U)\frac{(\alpha_H-\alpha_L)}{2a} - \frac{\eta\bar{\alpha}}{h_2}$$

$$= \frac{h_2 h_{3L}(1-\pi_1)\eta\alpha_H - (h_{3H}h_{3L}\eta\pi_1\alpha_H + h_{3H}h_{3L}\eta(1-\pi_1)\alpha_L) - h_2 h_{3H}(1-\pi_1)\eta\alpha_L}{h_{3H}h_{3L}h_2}$$

$$- (\Delta U)\frac{(\alpha_H-\alpha_L)}{2a},$$

which is negative if the first term is negative, or if

$$h_2 h_{3L}(1-\pi_1)\eta\alpha_H - (h_{3H}h_{3L}\eta\pi_1\alpha_H + h_{3H}h_{3L}\eta(1-\pi_1)\alpha_L) < 0$$

$$\rightarrow h_{3L}\eta\pi_1\,(h_2(1-\pi_1) - h_{3H}\pi_1) < 0$$

$$\rightarrow (h_2(1-\pi_1) - (h_2 + \alpha_H\, e^{-(F_1-\hat{F})^2} + \varepsilon_2)\,\pi_1 < 0$$

Rearranging,

$$h_2 < 2\pi_1 h_2 + \pi_1\alpha_H\, e^{-(F_1-\hat{F})^2} + \varepsilon_2,\ \mathrm{E}(\varepsilon_2) = 0$$

which is Condition (7), a *sufficient but not necessary condition* as the two negative terms are ignored. QED.

REFERENCES

Bednarek, Heather L., Jeitschko, Thomas D., and Pecchenino, Rowena A. "Gluttony and Sloth: Symptoms of Trouble or Signs of Bliss?—A Theory of Choice in the Presence of Behavioral Adjustment Cost." Contributions to Economic Analysis & Policy. 2006; 5(1): Article 6. http://www.bepress.com/bejeap/contributions/vol5/iss1/art6.

Cheuvront, Samuel N. "The Zone Diet Phenomenon: A Closer Look at the Science behind the Claims." Journal of American College of Nutrition. 2003; 22 (1): 9-17.

Dansinger, Michael L.; Gleason, Joi A.; Griffith John L.; Selker, Harry P.; Schaefer, Ernst J. "Comparison of the Atkins, Ornish, Weight Watchers, and Zone Diets for Weight Loss and Heart Disease Risk Reduction." Journal of the American Medical Association. 2005; 293: 43-53.

Eckel, Robert H. "The Dietary Approach to Obesity." Journal of the American Medical Association. 2005; 293: 96-97.

Food and Drug Administration Home Page (2006). U.S. Department of Health and Human Services. 18 Feb. 2006. < http://www.fda.gov/default.htm>

Foster, Gary D., et al. "A Randomized Trial of a Low-Carbohydrate Diet for Obesity." New England Journal of Medicine. 2003; 348: 2082-2090.

Goff, Sarah L., et al. "Brief Report: Nutrition and Weight Loss Information in a Popular Diet Book: Is It Fact, Fiction, or Something in Between?" Journal of General Internal Medicine. 2006; 21 (7): 769-774.

Goodwin, Kathy, RD. "The Atkins Diet—A Comprehensive Analysis." The Diet Channel. 21 Nov. 2005. <http://www.thedietchannel.com/atkins.htm>

Grossman, Michael. "The Demand for Health: A Theoretical and Empirical Investigation." NBER Occasional Paper 119. 1972(a)

Grossman, Michael. "On the Concept of Health Capital and Demand for Health." Journal of Political Economy. 1972(b); 80: 223-255.

Grossman, Sanford J., Kihlstrom, Richard E., and Mirman Leonard J. "A Bayesian Approach to the Production of Information and Learning by Doing." Review of Economic Studies. 1977; 44 (3): 533-547.

Haines, Cynthia. "Health Risk Associated with Obesity." WebMD. October 2005. The Cleveland Clinic. 12 Dec. 2005. <http://www.webmd.com/content/article/46/2731_1658>.

International Association for the Study of Obesity. *IASO Media Release.* 10th International Congress on Obesity, Sydney, Australia. September 2006.

Kiefer, Nicholas M., Nyarko, Yaw. "Optimal Control of an Unknown Linear Process with Learning." International Economic Review. 1989; 30 (3): 571-586.

Lee, I-Min., Paffenbarger Jr., Ralph S. "Change in Body Weight and Longevity." Journal of the American Medical Association. 1992; 268: 2045-2049.

Levy, Amnon. "Rational Eating: Can It Lead to Overweightness or Underweightness?" Journal of Health Economics. 2002; 21: 887-899.

Metabolic Rate, MMR and RMR-Balance Your Metabolic Rate (2006). Nahanni River Herbs. 18 Feb. 2006. <http://www.nahanniriverherbs.com/94,203>.

Muls, Erik, et al. "Is Weight Cycling Detrimental to Health? A Review of the Literature in Humans." International Journal of Obesity and Related Metabolic Disorders. 1995; 19 (Suppl. 3): S46-50.

National Institute of Diabetes and Digestive and Kidney Diseases (NIDDK). "Weight Loss and Nutrition Myths." NIH Publication No. 00-4561. 2000.

National Institutes of Health. "Weight Cycling." NIH Publication No. 01-3901. 2004.

Obesity-Related Disease Definition (2005). MedicineNet, Inc. 12 Dec. 2005. <http://www.medterms.com/script/main/art.asp?articlekey=4608>.

Riley, Rosemary E. "Popular Weight Loss Diets: Health and Exercise Implications." Clinic in Sports Medicine. 1999; 18 (3): 691-701.

Steen, Suzanne N., Oppliger, Robert A., Brownell, Kelly D. "Metabolic Effects of Repeated Weight Loss and Regain in Adolescent Wrestlers." Journal of the American Medical Association. 1988; 260: 47-50.

Szwarc, Sandy. "The Diet Problem." TCS Daily. July 2003. 18 Feb. 2006. < http://www.tcsdaily.com/article.aspx?id=071803B>.

Tapper-Gardzina, Yvonne B., Cotugna, Nancy, and Vickery Connie E. "Should You Recommend a Low-Carb, High-Protein Diet?" Nurse Practitioner. 2002; 27 (4): 52-59.

Taylor-Davis, Stephanie, Stone, Martha B. “Food Processing and Preservation.” MSN Encarta 2006. Microsoft Corporation. 18 Feb. 2006. <http://encarta.msn.com/encyclopedia_761560675_4/Food_Processing_and_Preservation.html>.

Weight-Loss Programs – Diet Plans – Lose Weight with Jenny Craig (2006). Jenny Craig, Inc. 18 Feb. 2006. <http://www.jennycraig.com>.

Wadden, Thomas A., Stunkard, Albert J., et al. “Body fat distribution in adult obese women II: Changes in fat distribution accompanying weight loss.” American Journal of Clinical Nutrition. 1988; 47: 229-234.

CHAPTER 2

ON THE MECHANISM OF CHOOSING TO PAY FOR A DIET PLAN

I. Introduction and Motivation

Overweightness and obesity have become significant health issues in recent years. As a result, individuals devote more time to examining and implementing methods to achieve a perceived healthy weight: they go on diets and the weight loss industry offers a diet plan for every dieter.

Yet, empirically, these programs do not lead to substantial weight loss. According to the U.S. Department of Health and Human Services, fifty million Americans initiate a diet program yearly. However, only five to ten percent of dieters are able to lose the extra pounds and keep them off, and more Americans are becoming overweight each year. This results in an ever-growing weight loss industry which currently generates approximately $44 billion annually.[29] The weight loss outcomes for dieters do not look promising. Why do dieters continue to purchase diet programs given their poor performance? This chapter provides an answer.

Consider an agent who is attempting to lose weight. First the dieter chooses a diet program which involves incurring an initial cost. Under the plan's guidance, the dieter subsequently puts forth effort to lose weight. The dieter benefits in terms of weight loss while the costs include the initial cost and the effort. Clearly, the amount of weight loss depends critically on effort and the effectiveness of the diet plan. In most cases, the plan-specific effectiveness is random before the initiation of the plan.

---

[29] The revenue for 2006 is projected to be $48 billion.

The random effectiveness of a diet plan is modeled as positively dependent upon the initial cost paid. This is true in reality as the dieter will purchase a more expensive diet plan only if the plan generates a greater effectiveness. It is assumed that this relationship has a specific and known probability distribution. These probability distributions, and thus the potential effectiveness of the diet plans, are stochastically ranked, either in the first or in the second order sense. A higher initial cost leads to a stochastically dominant distribution of potential effectiveness. The initial cost provides the dieter with the distribution that corresponds to the diet program purchased. After the purchase, the effectiveness of the program is randomly drawn from the distribution and revealed to the dieter. The dieter then puts forth effort to lose weight.

I argue that there are casual dieters who incur an initial cost but then choose to exert minimal or no effort in losing weight once on the plan. The outcome is primarily due to the initial uncertainty of the effectiveness of the diet program. Once on the plan and the initial uncertainty is resolved, the initial costs are sunk and the decision to exert effort to lose weight depends only on the revealed effectiveness of the diet plan. For some dieters, the actual effectiveness obtained is not adequate to induce positive levels of effort. In this model, the initially marginally overweight are most likely to be casual dieters. Since the obese are not the majority of the overweight, this helps explain the lack of success among dieters who contribute to the growing revenue of the weight-loss industry.[30]

Section II describes the setup of the model. Section III provides the solutions and discusses some results. Section IV discusses an alternative model. Section V concludes.

[30] A recent report by the International Association for the Study of Obesity (*IASO Media Release*, September 2006) indicates that approximately 21 percent of overweight individuals are considered obese.

II. The Model

A. Utility and Weight Preferences

Consider the choice problem of an individual agent with initial weight W_0. Assume that the agent is happiest when he is at his ideal weight, $\hat{W}$, where $\hat{W} < W_0$. If the agent loses weight, then for any ending weight W_1, the agent suffers a disutility of

$$\text{Disutility}(W_1) = (W_1 - \hat{W})^2 \quad \text{[31]} \qquad \text{(A)}$$

For simplicity, assume that the agent's weight remains unchanged at W_0 if he chooses not to lose weight.

B. Weight Loss and Effort

Let e denote effort. The relationship between weight loss and effort is given by $\Delta W = W_0 - W_1 = \alpha\, e$, where $\alpha \geq 0$ is a random parameter drawn from a diet-specific probability distribution. More effort, therefore, leads to greater weight loss. However, the magnitude of weight loss depends on how large α is. Effort is costly to the agent.

C. Initial Cost and the Distribution of α

Let C denote the initial cost. In order to initiate a diet program, the agent has to pay this initial cost. The distribution of α depends on the initial cost incurred. Once the investment is made, α is drawn from the distribution and the true effectiveness of the program with which the agent loses weight is revealed. Since, among different diet

[31] See Section IV Alternative Model

programs with identical distributions of α, the agent will always choose the one with the lowest initial cost, it is assumed that a higher initial cost leads to a better distribution of α in some stochastic sense. Two specifications, first-order and second-order stochastic dominance, of the distribution will be investigated in this paper. In all cases, the agent knows the distribution of any diet program available. Hence, for a given initial cost C, the agent knows the distribution that will be obtained. The agent incurs a disutility of C as a result of paying the initial cost.

In reality, initial costs are most obvious for bariatric surgery where the patient would have his stomach stapled before losing an ounce. The monetary costs of the surgery are high, and the surgery is painful. This pain and suffering are also included in the concept of initial costs as they precede the process of losing weight.

Nonetheless, initial costs are seen in other diet programs as well. Membership fees to join programs and those to join gyms, costs to obtain and read a book, costs of scheduling the exercise prescribed, and costs of buying an exercise equipment can be interpreted as initial costs. For example, it costs about $45 initially to join the Weight Watchers program. The fees include costs of dieting guidelines and twelve weekly meetings. Additional meetings would cost the dieter extra $10-14 per week.[32] "A-Personal Dietitian" program costs about $45 initially also. The Diet Divas program costs anywhere from $50 to $119 to join depending on the type of membership desired. Global Health and Fitness (GHF) program costs about $60 to join.[33] The dieters are given a customized plan, fitness tracking software, unlimited consulting, etc. "Your Last Diet" program costs about $97 in initial fees. The Jenny Craig plan initially costs the dieter

[32] http://www.freep.com/news/health/wwatch22_20020422.htm
[33] http://www.global-fitness.com/nonmem_survey.php

well over $100 to join. With a 'platinum' lifetime membership, the dieter pays $290 to join the plan. Some diet plans cost less. Dietsmart.com, for instance, costs only an initial fee of $25 to join. It is clear that the dieters pay these hefty fees essentially to purchase a better probability to benefit in terms of weight loss afterward.

Some diet plans sell the dieters a "secret book." The South Beach Diet book (Agatston, 2003), for example, costs $25. It probably costs the dieter about three more hours or so (depending on how fast he reads) to read the book and get familiar with what the plan is all about—these non-monetary costs are included in the model as well. Different levels of initial costs within the same diet program also exist. For example, it would cost the dieter about $15 more to purchase the South Beach Diet Cookbook which certainly would improve the effectiveness with which the dieter adopts the South Beach program. The Atkins for Life book (Atkins, 2003) also costs about $25.[34] There is also a choice of paying an additional $30 fee for the first two weeks to obtain personalized, one-on-one help. An additional book to help the dieter in the weight loss process costs an additional $10.[35] It seems what the dieters buy in these cases is the mysterious secret that is revealed by reading the books.

---

[34] There are other variations of books written by Dr. Atkins, all of which contain essentially the same dieting philosophy.

[35] The Atkins Essentials: A Two-Week Program to Jump Start Your Low Carb Lifestyle, Atkins Nutritional, Inc., 2004

D. Timing of Events

The agent pays the initial cost C to initiate the chosen diet program with a known α distribution. The variable α then is drawn from the distribution and revealed to the agent. The agent then puts forth effort e to lose weight according to the relationship stated in part B. Therefore, the weight loss is deterministic after a diet program is chosen and the initial cost is incurred.

The agent minimizes disutility in two stages. Disutility is defined over three arguments: initial cost, effort, and weight:

$$\text{Disutility} \equiv (W_1 - \hat{W})^2 + C + e$$

In the first stage, the agent chooses a level of C to minimize the total disutility, at which point α is a random variable with known distribution. In the second stage, α is revealed[36] and the agent chooses a level of e to lose weight, given α.

[36] In this sense, it is assumed that the amount of weight loss is certain after the realization of α. In reality, this may be untrue; however, we choose to treat the weight loss as being certain. To incorporate uncertain weight loss, random noise can be added. Nevertheless, the following analysis holds if the certain weight loss is replaced by the expected weight loss, given that noise has a mean of zero.

III. Solution and Some Results

The Second Stage:

The solution is found by backward induction. First, we solve for e. At this point, the initial cost has been incurred and has no bearing on the choice of e. The random parameter α has also been revealed and is a known constant. Hence, the solution is a level of effort e contingent on α.

Specifically, the agent chooses e to minimize

$$(W_1 - \hat{W})^2 + e.$$

Since weight loss

$$\Delta W = W_0 - W_1 = \alpha\, e,$$

$$W_1 = W_0 - \alpha\, e.$$

That is, the ending weight W_1 naturally depends on the starting weight W_0, effort e, and α which indicates the effectiveness of the plan.

Substituting, the agent minimizes

$$(W_0 - \alpha\, e - \hat{W})^2 + e$$

The first-order condition is

$$-2\alpha\,(W_0 - \alpha\, e - \hat{W}) + 1 = 0.$$

The solution is

$$e*(\alpha) = \frac{2(W_0 - \hat{W})\alpha - 1}{2\alpha^2} \qquad (1)$$

$$\text{if } \alpha > \frac{1}{2(W_0 - \hat{W})} \qquad (1A)$$

$$= 0, \text{ otherwise.}$$

With positive effort $e*(\alpha)$, i.e. if (1A) holds, the amount of weight lost is

$$\alpha\, e*(\alpha) = (W_0 - \hat{W}) - \frac{1}{2\alpha}.$$

And the minimized disutility is

$$(W_0 - \alpha\, e*(\alpha) - \hat{W})^2 + e*(\alpha) = \frac{(W_0 - \hat{W})}{\alpha} - \frac{1}{4\alpha^2} \tag{2}$$

Equation (2) is critical in the analysis in the first stage as the agent obtains this minimized disutility provided that he puts in positive effort to lose weight.

Some results from the second stage

Proposition 1: For α sufficiently high, higher α leads to lower effort. However, higher α unconditionally leads to greater weight loss.

Proof:

$$\frac{\partial}{\partial \alpha} e*(\alpha) = \frac{1-(W_0 - \hat{W})\alpha}{\alpha^3}$$

which is positive if

$$\frac{1}{2(W_0 - \hat{W})} < \alpha < \frac{1}{(W_0 - \hat{W})}$$

and negative if

$$\alpha > \frac{1}{(W_0 - \hat{W})}.$$

The amount of weight loss is $\alpha\ e*(\alpha\ )$.

$$\frac{\partial}{\partial\alpha}\alpha\cdot e*(\alpha)= e*(\alpha\ )+\frac{\partial}{\partial\alpha}e*(\alpha)$$

$$=\frac{1}{2\alpha^2}>0. \qquad \text{QED.}$$

The first part of Proposition 1 states that, if the weight loss process is adequately efficient (α sufficiently high), then effort is decreasing in α. This may seem counterintuitive. However, with a more effective weight-loss program, less effort is needed to generate any desired weight loss. That is, program effectiveness and effort are substitutes. Furthermore, the second part of Proposition 1 shows that, even with less effort, a more effective program leads to greater weight loss. The interpretation is that program effectiveness is more important than effort when it comes to weight-loss results.

Proposition 2: The more overweight the agent initially is, the greater effort put forth and the greater weight loss experienced.

Proof:

$$\frac{\partial}{\partial(W_0-\hat{W})}e*(\alpha)=\frac{1}{\alpha}\ >\ 0.$$

The amount of weight loss is

$$\alpha\ e*(\alpha\ )=(W_0-\hat{W})-\frac{1}{2\alpha}$$

$$\frac{\partial}{\partial(W_0-\hat{W})}\alpha\cdot e*(\alpha)=1>0. \qquad \text{QED.}$$

A more overweight agent has more to gain from weight loss on the margin since disutility is quadratic in weight deviation. As a result, he chooses to expend more effort in losing weight than a less overweight agent.

Real-world dieters assess the benefit of losing weight by determining how overweight they initially are. The result suggests that the less overweight the dieter is, the lower the level of effort put forth while trying to lose weight. That is, less overweight agents tend to diet only casually. In reality, one may observe individuals who put in half the effort because the excess weight is small.

On the other hand, dieters who are obese will try harder to lose weight. The marginal benefit of losing a pound is much greater for the obese than it is for individuals who are less overweight.[37]

Proposition 2 also suggests that the agent who is more overweight will lose more weight than a less overweight agent if they are on programs with equal effectiveness. This naturally follows from the first part since more overweight agents exert more effort than less overweight agents.

<u>Proposition 3</u>: (<u>Quitters</u>) Given α , agents with lower initial overweightness are more likely to quit the diet plan than agents with higher initial overweightness.

---

[37] Note that this result and others depend critically on the assumption that disutility is quadratic in weight deviation. However, this assumption is not unrealistic. A person who is slightly overweight is likely to disregard the excess weight as insignificant and be content with it. On the other hand, an obese person is more likely to literally always have weight on his mind and would benefit more from a pound lost. See also Section IV: Alternative Model, which discusses a case in which disutility is expressed as weight deviation as a percentage of the ideal weight.

Effort is positive only if $\alpha > \frac{1}{2(W_0 - \hat{W})}$. As $(W_0 - \hat{W})$ falls, $\frac{1}{2(W_0 - \hat{W})}$ increases. Therefore, as $(W_0 - \hat{W})$ falls, the α realized is more likely to be less than $\frac{1}{2(W_0 - \hat{W})}$ which would lead to no effort in the second stage. QED.

Proposition 3 shows that, given a level of effectiveness in the weight-loss process, individuals who are less overweight are more likely to quit the plan than their more overweight counterparts. In other words, there exist slightly overweight dieters who incur initial costs only to experience no weight loss. This is because the marginal benefit of losing weight is lower, the less overweight one is. Consequently, given a diet plan (and thus a distribution of α), it is more likely for the less overweight agents to find that the benefit from losing weight is just not worth the effort.

The First Stage:

In the first stage, the agent chooses the initial cost C to minimize the total expected disutility which is composed of the disutility from incurring the initial cost C and the expected disutility that results in the second stage. At this point, α is a random variable with a probability distribution. The solution depends on the distribution of α that is obtained once the initial cost is incurred. Two different classes of uniform distributions will be investigated in this paper.

A. First-Order Stochastic Dominance

In the first case, α has a uniform distribution $[0, 2C^{\beta}]$, where $\beta \in (0, 1]$ is a known parameter. This implies that the upper limit of the uniform distribution is strictly increasing and weakly concave in β. This assumption is reasonable since the agent will be willing to pay a higher initial cost only if the distribution improves. In this case, the distribution improves in a first-order stochastic sense. Consequently, the higher the initial cost, the greater the mean of the distribution.

Mathematically speaking, a higher initial cost buys a more effective program in expectation. The term $2C^{\beta}$ represents the degree to which the expected diet effectiveness increases with the initial cost. As the initial cost C rises, the expected effectiveness rises at a decreasing rate due to the fact that $\beta \in (0, 1]$. Note, however, that the assumption is not that a higher initial cost automatically buys a more effective program. Rather, the assumption is that, given a level of expected effectiveness, the agent chooses the least expensive diet. This leads the agent to incur a higher initial fee only if there is an improvement in the diet effectiveness.

There are two possibilities for the disutility that results in the second stage. The disutility in the second stage is as expressed in Equation (2) if Condition (1A) holds. However, if Condition (1A) does not hold, the agent chooses $e = 0$, therefore experiencing no weight loss and obtaining a disutility of $(W_0 - \hat{W})^2$. For a given level of C chosen, the following diagram illustrates the distribution obtained from the diet program:

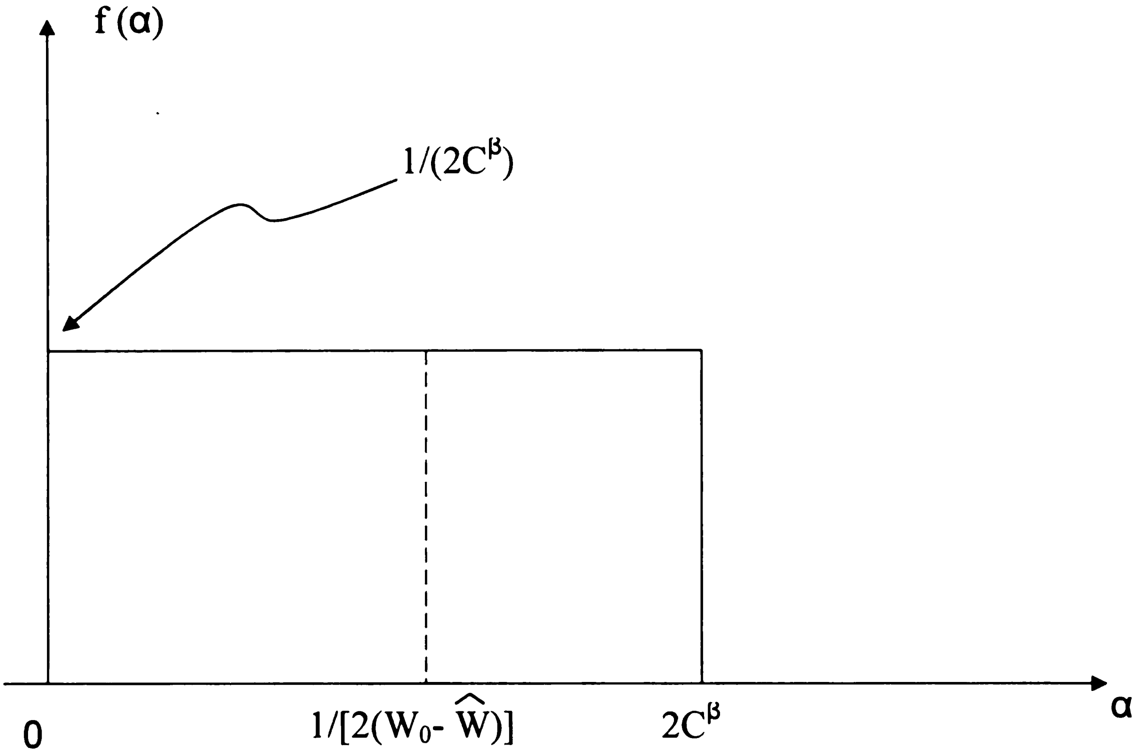

Figure 2.1: Uniform Distribution of α : First-Order Stochastic Dominance

For a given C and, hence, a given distribution,

$$P\,(\alpha \le \frac{1}{2(W_0 - \hat{W})}) = \frac{1}{4(W_0 - \hat{W})C^{\beta}},$$

in which case the disutility in the second stage is $(W_0 - \hat{W})^2$. And

$$P\,(\alpha > \frac{1}{2(W_0 - \hat{W})}) = 1 - \frac{1}{4(W_0 - \hat{W})C^{\beta}},$$

in which case the disutility in the second stage is

$$\frac{(W_0 - \hat{W})}{\alpha} - \frac{1}{4\alpha^2}.$$

Therefore, the agent minimizes:

$$\frac{(W_0-\hat{W})^2}{4(W_0-\hat{W})C^{\beta}} + E[1-\frac{1}{4(W_0-\hat{W})C^{\beta}}][\frac{(W_0-\hat{W})}{\alpha}-\frac{1}{4\alpha^2}]+C$$

$$= \frac{(W_0-\hat{W})}{4C^{\beta}} + \frac{4(W_0-\hat{W})C^{\beta}-1}{4C^{\beta}} E(\frac{1}{\alpha} \mid \alpha > \frac{1}{2(W_0-\hat{W})})$$

$$- \frac{1}{4}[1-\frac{1}{4(W_0-\hat{W})C^{\beta}}] E(\frac{1}{\alpha^2} \mid \alpha > \frac{1}{2(W_0-\hat{W})}) + C \qquad (3)$$

$$\text{where } E(\frac{1}{\alpha} \mid \alpha > \frac{1}{2(W_0-\hat{W})}) = \frac{1}{2C^{\beta}} \ln 4(W_0-\hat{W})C^{\beta} \qquad (4),$$

$$E(\frac{1}{\alpha^2} \mid \alpha > \frac{1}{2(W_0-\hat{W})}) = -\frac{1}{2C^{\beta}}[\frac{1}{2C^{\beta}}-2(W_0-\hat{W})] \qquad (5)$$

Substituting (4) and (5) in (3), rearranging and collecting terms, the agent minimizes

$$\frac{(W_0-\hat{W})[\ln 4(W_0-\hat{W})+\beta\ln C]}{2C^{\beta}} - \frac{2[\ln 4(W_0-\hat{W})+\beta\ln C-1]}{(4C^{\beta})^2} - \frac{1}{(4C^{\beta})^3(W_0-\hat{W})} + C$$

The first-order condition is:

$$\frac{\beta(W_0-\hat{W})}{2C^{\beta+1}}[1-\ln 4(W_0-\hat{W})+\beta\ln C] + \frac{\beta}{8C^{2\beta+1}}[2\ln 4(W_0-\hat{W})+2\beta\ln C-3]$$

$$+ \frac{3\beta}{64C^{3\beta+1}(W_0-\hat{W})} + 1 = 0 \qquad (6)$$

Note that the agent chooses $C = 0$ if the expected disutility resulting from incurring the initial cost is greater than the disutility resulting from staying at the initial weight W_0. In other words, the initial cost is positive if and only if the minimized disutility is less than the status quo disutility:

$$C > 0 \leftrightarrow \frac{(W_0 - \hat{W})}{4C^\beta} + \frac{4(W_0 - \hat{W})C^\beta - 1}{4C^\beta} E(\frac{1}{\alpha}) - \frac{1}{4} [1 - \frac{1}{4(W_0 - \hat{W})C^\beta}] E(\frac{1}{\alpha^2}) + C$$

$$< (W_0 - \hat{W})^2 \text{ , or}$$

$$(W_0 - \hat{W}) [1 - \frac{1}{4(W_0 - \hat{W})C^\beta}][E(\frac{1}{\alpha}) - (W_0 - \hat{W})]$$

$$-\frac{1}{4} [1 - \frac{1}{4(W_0 - \hat{W})C^\beta}] E(\frac{1}{\alpha^2}) + C < 0 \qquad \text{(6A)}$$

Therefore, $C(W_0, \hat{W}, \beta)$ is implicitly defined by Equation (6) if Condition (6A) holds, and is zero otherwise.

Equation (6) is explored further in the following proposition to investigate how the initial cost chosen changes with respect to how overweight the agent initially is. This also leads to further examination of how the agent may choose to quit the program.

<u>Proposition 4A</u>: For α ~ uniform $[0, 2C^\beta]$, $\beta \in (0, 1]$, greater initial overweightness leads to a larger initial cost chosen if the agent is initially sufficiently overweight.

Let f denote the left-hand side of Equation (6).

$$\frac{\partial}{\partial(W_0 - \hat{W})} C(W_0, \hat{W}, \beta) = -\frac{f_{(W_0 - \hat{W})}}{f_C},$$

f_C is positive by second-order condition.

$$f_{(W_0 - \hat{W})} < 0 \text{ if}$$

$$(W_0 - \hat{W}) > \frac{1}{2C^{\beta} \ln[4(W_0 - \hat{W})C^{\beta}]} \tag{7}$$

(*A detailed proof is in the appendix*).

Therefore, $\frac{\partial}{\partial(W_0 - \hat{W})} C(W_0, \hat{W}, \beta)$ is positive if Condition (7) holds, that is, if the agent is sufficiently overweight.

Proposition 4A states that the initial cost incurred is larger for more overweight agents than it is for less overweight agents. This is intuitive as more overweight agents have more to gain from a better distribution of α than do less overweight agents. Combined with Proposition 2, the model suggests that there are agents who consider themselves to be only slightly overweight. Consequently, these agents opt for inexpensive programs and put forth relatively little effort. In other words, Propositions 4A and 2 together suggest that these marginally overweight will be casual dieters who experience little weight loss. This is further supported by Proposition 5A below.

Proposition 5A: For $\alpha \sim$ uniform $[0, 2C^{\beta}]$, $\beta \in (0, 1]$, the expected weight loss is increasing in the level of initial overweightness.

In the first stage, the expected weight loss is

$$(W_0 - \hat{W}) - \frac{1}{2} E\,(\frac{1}{\alpha} \mid \alpha > \frac{1}{2(W_0 - \hat{W})})$$

$$\frac{\partial}{\partial (W_0 - \hat{W})} E\,[(W_0 - \hat{W}) - \frac{1}{2\alpha}] > 0.$$

(*A detailed proof is in the appendix*).

Proposition 5A shows that the expected weight loss is increasing in the level of initial overweightness. This result follows intuitively from Propositions 2 and 4A. The expected weight loss depends on the effort expended as well as the initial cost incurred. Since, in comparison to less overweight agents, more overweight agents expend greater effort and incur a higher initial cost to purchase a stochastically dominant program, expected weight loss is greater. Furthermore, a less overweight agent has a greater chance of drawing an α that leads to no effort and no weight loss for a given diet program. This is summarized in the following proposition.

Proposition 6A: For $\alpha \sim$ uniform $[0, 2C^{\beta}]$, $\beta \in (0, 1]$, the probability of quitting after incurring an initial cost is decreasing in the level of initial overweightness.

The probability of abandoning the program after the purchase is the probability that effort expended is zero in the second stage. This probability is

$$\mathrm{P}\left(\alpha \leq \frac{1}{2(W_0 - \hat{W})}\right) = \frac{1}{4(W_0 - \hat{W})C^{\beta}}.$$

$$\frac{\partial}{\partial(W_0 - \hat{W})}\mathrm{P}\left(\alpha \leq \frac{1}{2(W_0 - \hat{W})}\right) = \frac{-1}{4(W_0 - \hat{W})^2 C^{\beta}} < 0.$$

Therefore, less overweight agents are more likely to abandon a diet program after paying for it than more overweight agents. QED.

Dieters differ with regard to their initial overweightness. One extreme is the morbidly obese who undergo surgical procedures such as bariatric surgery. These are extremely expensive. The effectiveness of the surgery is, however, proven.[38] On the other hand, dieters who are only slightly overweight often find it hard to 'lose the last 5 pounds'. These dieters are likely to purchase or initiate a diet program only to make no effort in losing weight, for instance, paying for the gym membership only to rarely visit. Essentially, if the program obtained is not effective enough, the marginal benefit from weight loss will be so low that it is not worth the effort. For these less overweight dieters, the probability of drawing an insufficiently effective program is higher. Consequently, the probability of quitting the program is higher. Similar to Proposition 3, these less overweight dieters incur an initial cost with a greater likelihood of abandoning the program when the time comes to put forth effort.

[38] See Buchwald et al. (2004) and Blackburn (2005).

B. Second-Order Stochastic Dominance

In the second case, the variable α is uniformly distributed over the interval $[\frac{1}{2C^{\beta}}, 2C^{\beta}]$, where $\beta \in (0, 1]$ and $C^{\beta} \geq ½$. This implies that a higher initial cost can lead to a worse realization of α, as the higher initial cost leads to a distribution with both a higher mean and a higher variance. The agent can always choose a plan that yields α = 1 with certainty. This can be done by paying C^{β} = ½.

The solution to the problem in the second stage is identical to that of case A. Hence, we will be utilizing Equations (1) and (2), and Condition (1A).

For an initial cost C chosen, the following diagram illustrates the distribution of α obtained:

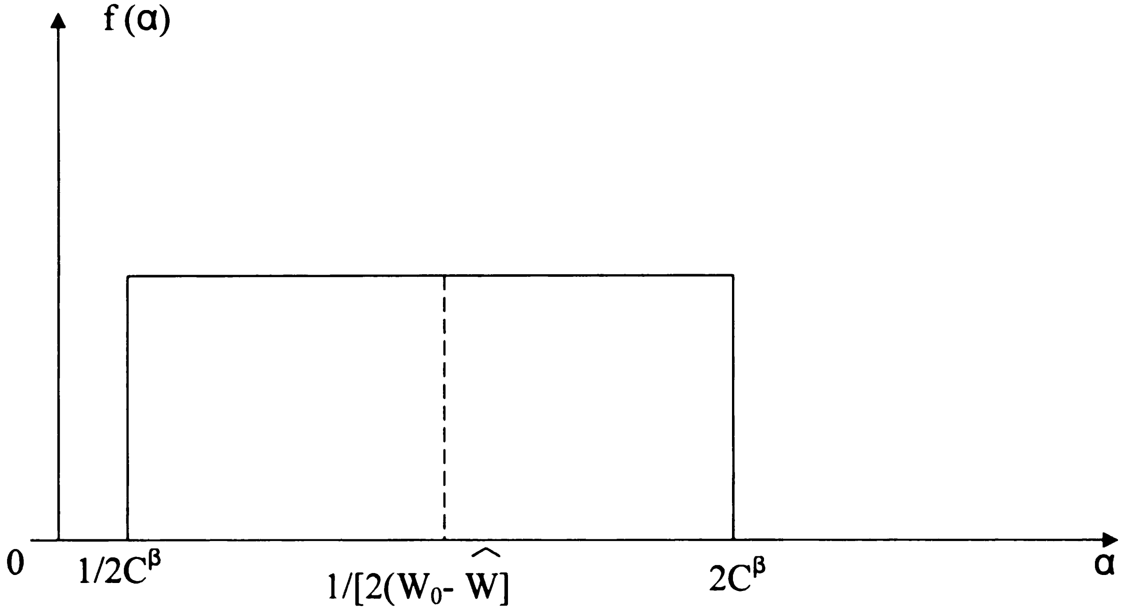

Figure 2.2: Uniform Distribution of α : Second-Order Stochastic Dominance

$$\text{where } f(\alpha) = \frac{2C^{\beta}}{(2C^{\beta})^2 - 1}, \quad \frac{1}{2C^{\beta}} \le \alpha \le 2C^{\beta},$$

$$= 0, \text{ otherwise.}$$

The agent chooses to put forth no effort in the second stage if

$$\alpha < \frac{1}{2(W_0 - \hat{W})}.$$

In this case, the agent obtains a disutility of $(W_0 - \hat{W})^2$ because no weight is lost. The probability that this occurs is:

$$P\,(\alpha \le \frac{1}{2(W_0 - \hat{W})}) = (\frac{1}{2(W_0 - \hat{W})} - \frac{1}{2C^{\beta}})(\frac{2C^{\beta}}{(2C^{\beta})^2 - 1})$$

$$= \frac{C^{\beta} - (W_0 - \hat{W})}{(W_0 - \hat{W})(4C^{2\beta} - 1)}$$

If $\alpha > \frac{1}{2(W_0 - \hat{W})}$,

the agent obtains the disutility $\frac{(W_0 - \hat{W})}{\alpha} - \frac{1}{4\alpha^2}$ (Equation (2)). This happens with

probability

$$P(\alpha > \frac{1}{2(W_0 - \hat{W})}) = 1 - P(\alpha \leq \frac{1}{2(W_0 - \hat{W})})$$

$$= 1 - \frac{C^\beta - (W_0 - \hat{W})}{(W_0 - \hat{W})(4C^{2\beta} - 1)}.$$

It is worth noting that if the agent chooses C such that $C^\beta \leq (W_0 - \hat{W})$, then

$\alpha > \frac{1}{2(W_0 - \hat{W})}$ with probability one, that is, the agent will, with certainty, exert

positive effort in the second stage.

Therefore, provided that $C^\beta > (W_0 - \hat{W})$, the agent chooses the initial cost C to

minimize

$$(W_0 - \hat{W})^2 \left(\frac{C^\beta - (W_0 - \hat{W})}{(W_0 - \hat{W})(4C^{2\beta} - 1)}\right)$$

$$+ E\left[1 - \frac{C^\beta - (W_0 - \hat{W})}{(W_0 - \hat{W})(4C^{2\beta} - 1)}\right]\left[\frac{(W_0 - \hat{W})}{\alpha} - \frac{1}{4\alpha^2}\right] + C,$$

where $E(\frac{1}{\alpha} \mid \alpha > \frac{1}{2(W_0 - \hat{W})}) = \frac{2C^\beta}{(2C^\beta)^2 - 1} \ln 4(W_0 - \hat{W})C^\beta$

$$E(\frac{1}{\alpha^2} \mid \alpha > \frac{1}{2(W_0 - \hat{W})}) = \frac{2C^\beta}{(2C^\beta)^2 - 1}\left[\frac{1}{2C^\beta} - 2(W_0 - \hat{W})\right]$$

Substituting, rearranging, and collecting terms, the agent minimizes

$$(W_0 - \hat{W}) \left(\frac{C^{\beta} - (W_0 - \hat{W})}{4C^{2\beta} - 1}\right) + \frac{2C^{2\beta}(4C^{\beta}(W_0 - \hat{W}) - 1)}{(4C^{2\beta} - 1)^2} (\ln 4(W_0 - \hat{W}) + \beta \ln C - \frac{1}{2})$$

$$+ \frac{C^{\beta}[4C^{\beta}(W_0 - \hat{W}) - 1]}{4(W_0 - \hat{W})(4C^{2\beta} - 1)^2} + \mathrm{C}$$

The first-order condition is

$$\frac{(W_0 - \hat{W})(\beta C^{\beta-1})[(4C^{2\beta} - 1) - 8C^{\beta}(C^{\beta} - (W_0 - \hat{W}))]}{(4C^{2\beta} - 1)^2}$$

$$+ \frac{4\beta C^{\beta-1}[(W_0 - \hat{W})(4C^{2\beta} - 1) - 4C^{\beta}(4C^{\beta}(W_0 - \hat{W}) - 1)]}{(4C^{2\beta} - 1)^3}$$

$$[2C^{2\beta} (\ln 4(W_0 - \hat{W}) + \beta \ln C - \frac{1}{2}) + \frac{C^{\beta}}{4(W_0 - \hat{W})}]$$

$$+ \frac{[4C^{\beta}(W_0 - \hat{W}) - 1]\beta C^{\beta-1}}{(4C^{2\beta} - 1)^2} [4C^{\beta} \ \ln 4(W_0 - \hat{W})C^{\beta} + \frac{1}{4(W_0 - \hat{W})}] + 1 = 0 \qquad (8)$$

Similar to Case A, the agent chooses not to incur any initial cost if the expected disutility resulting from incurring the initial cost is greater than the disutility resulting from staying at the initial weight W_0. Thus, the initial cost is positive if and only if Condition (6A) holds,

$$C > 0 \;\leftrightarrow\; (W_0 - \hat{W})\,[1 - \frac{1}{4(W_0 - \hat{W})C^{\beta}}][E\,(\frac{1}{\alpha}) - (W_0 - \hat{W})]$$

$$-\frac{1}{4}\,[1 - \frac{1}{4(W_0 - \hat{W})C^{\beta}}]\;E\,(\frac{1}{\alpha^2}) + C \;<\; 0$$

Note that, even though the condition is the same as that in part A, the parameter restriction is different due to the differences in $E\,(\frac{1}{\alpha})$ and $E\,(\frac{1}{\alpha^2})$.

Therefore, provided that $C^{\beta} > (W_0 - \hat{W})$, $C(W_0, \hat{W}, \beta)$ is implicitly defined by Equation (8) if Condition (6A) holds, and is zero otherwise.

Lemma 1: At the optimum, $C^{\beta} > (W_0 - \hat{W})$, provided that the agent is sufficiently overweight.

Substituting $C^{\beta} = (W_0 - \hat{W})$ into the first-order condition, Equation (8), we obtain:

$$\frac{(W_0 - \hat{W})^2(\beta C^{-1})[4(W_0 - \hat{W})^2 - 1]}{(4C^{2\beta} - 1)^2}$$

$$\frac{-12\beta C^{-1}(W_0 - \hat{W})^2}{(4C^{2\beta} - 1)^2}\,[2(W_0 - \hat{W})^2\,(\ln 4(W_0 - \hat{W})^2 - \frac{1}{2}) + \frac{1}{4}]$$

$$+\frac{[4(W_0 - \hat{W})^2 - 1]\beta(W_0 - \hat{W})C^{-1}}{(4C^{2\beta} - 1)^2}$$

$$[4(W_0 - \hat{W})\,\ln 4(W_0 - \hat{W})^2 + \frac{1}{4(W_0 - \hat{W})}] + 1 \;<\; 0, \quad \text{if}$$

$$(W_0 - \hat{W}) > (\frac{\beta}{4})^{\beta} \qquad \text{(8A),}$$

that is, if the agent is sufficiently overweight. QED.

If Condition (8A) holds, then Equation (8), evaluated at $(W_0 - \hat{W})$ is negative. Hence, the optimal initial cost C is such that $C^{\beta} > (W_0 - \hat{W})$. The optimal initial cost in this case is implicitly defined by Equation (8).

All the results for Case A are robust to the change in the distribution of α as Propositions 4A, 5A, and 6A hold for case B as well.

First, the initial cost chosen is increasing in the initial overweightness. This is intuitive as the more overweight agents have much more to gain on the margin than the less overweight agents even when on a relatively ineffective program due to the quadratic disutility in weight deviation. This is true even when a possibility of a less ineffective program being realized is introduced in part B. In reality, the more overweight dieters are more likely to undertake more expensive regimens such as professional consultation.

Second, the more overweight agents can expect to lose more weight than the less overweight agents due to three factors. First, they incur a higher initial cost than the less overweight dieters in order to purchase a stochastically dominant program. This yields a higher mean for α. Second, for a given α, they also exert more effort than the less overweight agents.

The third reason has to do with the probability of quitting the diet plan altogether. The less overweight dieters, again, have less to gain from a given program than the more overweight dieters. With lower initial cost incurred and an inferior distribution, the probability that the less overweight dieters end up with a plan for which optimal effort is zero is higher.

All proofs for case B are included in the appendix.

In reality, the less overweight dieters are more likely to give up on a plan after paying for it. This may help explain the enormous revenue within the weight loss industry with minimal weight-loss results; it is not that the programs themselves fail the dieters, it may be that these casual dieters continue to financially contribute to the programs but put forth little or no effort to lose weight. The intuition behind this lies in the fact that purchasing a diet plan and exerting effort are two distinct activities. At the point of purchase, the expected benefits exceed the expected costs. However, once the plan is obtained, the actual benefits may be lower than the actual cost of effort, resulting in no effort.

IV. Alternative Model

In this section, we briefly discuss an alternative expression of disutility. Arguably the disutility expression with regard to weight in Section II A, Equation (A), may not truly represent most personal preferences although a justification of the expression is provided in Footnote 37. The expression describes disutility as absolute weight deviation squared. This means that two persons who are, for instance, 10 pounds overweight suffer the same disutility regardless of how much they weigh and regardless of the level of ideal weight. One may argue that a person who weighs 210 pounds when the ideal weight is 200 pounds does not suffer as much disutility as another person who weighs 110 pounds when the ideal weight is 100 pounds. The former person in some sense suffers a smaller disutility since 10 pounds is smaller compared to the ideal weight.

One way to represent such a notion is to express the disutility as relative weight deviation squared. For example, disutility can be expressed as weight deviation as a percentage of the ideal weight as follows:

$$\text{Disutility} = \left(\frac{W_1 - \hat{W}}{\hat{W}}\right)^2 \tag{B}$$

In the example given above, the person with an ideal weight of 200 pounds has to be 20 pounds overweight to suffer the same disutility as the person who is 10 pounds overweight with an ideal weight of 100 pounds.

In the second period, the agent minimizes

$$\left(\frac{W_1 - \hat{W}}{\hat{W}}\right)^2 + e\,.$$

Substituting, the agent minimizes

$$\left(\frac{W_0 - \alpha \cdot e}{\hat{W}} - 1\right)^2 + e$$

The first-order condition is

$$-2\frac{\alpha}{\hat{W}}\left(\frac{W_0 - \alpha \cdot e}{\hat{W}} - 1\right) + 1 = 0.$$

The solution is

$$e*(\alpha) = \frac{2(W_0 - \hat{W})\alpha - \hat{W}}{2\alpha^2} \tag{9}$$

$$\text{if } \alpha > \frac{\hat{W}}{2(W_0 - \hat{W})} \tag{9A}$$

$$= \; 0, \text{ otherwise.}$$

Note the similarity of this solution to the solution in Section III, Equations (1) and (1A). Furthermore, Propositions 1, 2, and 3 hold qualitatively with minor changes in parameter restrictions. Proofs are provided in the appendix.

V. Conclusion

This paper provides an explanation for the apparent lack of weight-loss success among dieters on diet plans while the weight-loss industry's revenue continues to grow. I develop a model which describes dieters' choice of a diet program and its implementation.

The paper illustrates some important points. First, I show that the initially less overweight dieters will put forth relatively little effort, given realized diet effectiveness. Consequently, they experience relatively little weight loss. Furthermore, given a diet plan, less overweight agents are more likely to quit the plan than more overweight agents.

I also find that lower program effectiveness leads to higher effort but that weight loss is increasing in the level of effectiveness. Consequently, there will probabilistically always be some unlucky dieters who end up with relatively ineffective programs and keep on trying hard without seeing a satisfying weight-loss result. Even though program effectiveness and effort are substitutes, realized weight loss depends more on effectiveness than on individual effort.

For both classes of distributions investigated, I also establish that the more overweight dieters incur a higher initial cost than the less overweight dieters and expect to lose more weight. They also are less likely to quit the program after the purchase takes place.

The explanation of the large and growing revenue in the weight loss industry is among the less overweight dieters. The chapter suggests that the less overweight dieters will incur some initial cost in order to be on a diet plan with the intention of losing weight. The less overweight they are, the less effort they exert and the more likely they are to quit the program altogether. These casual dieters, in other words, spend their money to obtain the diet plan, learn about the costs and benefits of dieting, and then choose not to lose weight. Due to the fact that most overweight are not categorized as obese (21 percent of the overweight are obese), this naturally leads to lack of success coinciding with the growth of the industry's revenue.[39]

Furthermore, this also explains the popularity of diet books, most of which are inexpensive. Most likely, casual dieters pick up one of these books but never find it beneficial to put forth the effort needed to lose weight. Some casual dieters pay for the gym membership only to visit rarely, again, because the marginal benefit of actual weight loss does not justify the additional cost of effort. On the other hand, the more overweight counterparts most likely opt for expensive plans which may include personal consultation and clinical visits.

Even though the weight loss industry has been under assault lately due to unsatisfying success rates in light of its rising profits, this paper shows that this outcome can be a natural consequence of dieters choosing among diet programs and putting forth effort according to their personal perception of their own well-being. In other words, there may be nothing wrong with the diet industry after all.

---

[39] *IASO Media Release*. International Association for the Study of Obesity. 10th International Congress on Obesity, September 2006, Sydney, Australia.

Appendix

Proof of Propositions for Case A

Proposition 4A: For α ~ uniform $[0, 2C^{\beta}]$, $\beta \in (0, 1]$, greater initial overweightness leads to a larger initial cost chosen if the agent is initially sufficiently overweight.

Let f denote the left-hand side of Equation (6).

$$\frac{\partial}{\partial(W_0 - \hat{W})} C(W_0, \hat{W}, \beta) = -\frac{f_{(W_0 - \hat{W})}}{f_C},$$

f_C is positive by second-order condition.

$$f_{(W_0 - \hat{W})} = -\frac{\beta}{2C^{\beta+1}} \ln[4(W_0 - \hat{W})C^{\beta}]$$

$$+\frac{\beta}{4C^{2\beta+1}(W_0 - \hat{W})} - \frac{3\beta}{64C^{3\beta+1}(W_0 - \hat{W})^2}$$

$$= \frac{\beta}{2C^{\beta+1}} \{-\ln[4(W_0 - \hat{W})C^{\beta}] + \frac{1}{2C^{\beta}(W_0 - \hat{W})} - \frac{3}{32C^{2\beta}(W_0 - \hat{W})^2}\},$$

which is negative if the terms in the brackets are negative, or if

$$32C^{2\beta}(W_0 - \hat{W})^2 \ln[4(W_0 - \hat{W})C^{\beta}] > 16C^{\beta}(W_0 - \hat{W}),$$

which holds if

$$(W_0 - \hat{W}) > \frac{1}{2C^{\beta} \ln[4(W_0 - \hat{W})C^{\beta}]}$$

which is Condition (7). QED.

Proposition 5A: For $\alpha \sim$ uniform $[0, 2C^{\beta}]$, $\beta \in (0, 1]$, the expected weight loss is increasing in the level of initial overweightness.

If Condition (1A) holds, the amount of weight loss is

$$\alpha\, e*(\alpha) = (W_0 - \hat{W}) - \frac{1}{2\alpha}.$$

Therefore, the expected weight loss in the first stage is

$$E[(W_0 - \hat{W}) - \frac{1}{2\alpha}] = (W_0 - \hat{W}) - \frac{1}{2} E(\frac{1}{\alpha} \mid \alpha > \frac{1}{2(W_0 - \hat{W})})$$

$$= \{(W_0 - \hat{W}) - \frac{1}{4C^{\beta}}[\ln 4(W_0 - \hat{W}) + \beta \ln C]\}\{1 - \frac{1}{4(W_0 - \hat{W})C^{\beta}}\}$$

$$\frac{\partial}{\partial(W_0 - \hat{W})} E[(W_0 - \hat{W}) - \frac{1}{2\alpha}] =$$

$$[1 - \frac{1}{4(W_0 - \hat{W})C^{\beta}}]^2 + [(W_0 - \hat{W}) - \frac{1}{4C^{\beta}} \ln 4(W_0 - \hat{W})C^{\beta}][\frac{1}{4C^{\beta}(W_0 - \hat{W})^2}]$$
$$+ \frac{\beta}{4C^{\beta+1}} \frac{\partial C}{\partial(W_0 - \hat{W})}\{\ln 4(W_0 - \hat{W})C^{\beta} - \frac{[2\ln 4(W_0 - \hat{W})C^{\beta} - 1]}{4C^{\beta}(W_0 - \hat{W})}\} > 0. \quad \text{QED.}$$

Proof of Propositions for Case B

Proposition 4B: For $\alpha \sim$ uniform $[\frac{1}{2C^{\beta}}, 2C^{\beta}]$, where $\beta \in (0, 1]$ and $C^{\beta} \geq ½$, greater initial overweightness leads to a higher initial cost chosen.

Let f denote the left-hand-side of Equation (8).

$$\frac{\partial}{\partial(W_0 - \hat{W})} C(W_0, \hat{W}, \beta) = -\frac{f_{(W_0 - \hat{W})}}{f_C}$$

$$f_{(W_0-\hat{W})} = -\frac{\beta C^{\beta-1}(4C^{2\beta}+1)}{(4C^{2\beta}-1)^2} + \frac{16\beta C^{2\beta-1}(W_0 - \hat{W})}{(4C^{2\beta}-1)^2}$$

$$-\frac{8\beta C^{3\beta-1}(3+4C^{2\beta})}{(4C^{2\beta}-1)^3}[\ln 4(W_0 - \hat{W})C^{\beta} + 1] + \frac{4\beta C^{2\beta-1}(4C^{2\beta}+1)}{(W_0 - \hat{W})(4C^{2\beta}-1)^3}$$

$$-\frac{4\beta C^{3\beta-1}(12C^{2\beta}+1)}{(4C^{2\beta}-1)^3} - \frac{\beta C^{\beta-1}(12C^{2\beta}+1)}{4(W_0 - \hat{W})^2(4C^{2\beta}-1)^3},$$

which is negative if $\ln 4(W_0 - \hat{W})^2 > 1$.

Therefore, $\frac{\partial}{\partial(W_0 - \hat{W})} C(W_0, \hat{W}, \beta)$ is positive if the agent is sufficiently overweight. More overweight agents choose a higher initial cost than do less overweight agents. QED.

<u>Proposition 5B</u>: For $\alpha \sim \text{uniform}\left[\frac{1}{2C^{\beta}}, 2C^{\beta}\right]$, where $\beta \in (0, 1]$ and $C^{\beta} \geq ½$, expected weight loss is increasing in the level of initial overweightness.

If Condition (1A) holds, the weight loss is

$$\alpha\, e*(\alpha) = (W_0 - \hat{W}) - \frac{1}{2\alpha}.$$

In the first stage, α is a random variable. Therefore, the expected weight loss in the first stage is

$$E[(W_0-\hat{W})-\frac{1}{2\alpha}]=(W_0-\hat{W})-\frac{1}{2}E(\frac{1}{\alpha}\mid\alpha>\frac{1}{2(W_0-\hat{W})})$$

$$=\{(W_0-\hat{W})-(\frac{C^{\beta}}{4C^{2\beta}-1})\ln 4(W_0-\hat{W})C^{\beta}\}\{1-\frac{C^{\beta}-(W_0-\hat{W})}{(W_0-\hat{W})(4C^{2\beta}-1)}\}$$

$$\frac{\partial}{\partial(W_0-\hat{W})}E[(W_0-\hat{W})-\frac{1}{2\alpha}]=$$

$$\{(W_0-\hat{W})-(\frac{C^{\beta}}{4C^{2\beta}-1})\ln 4(W_0-\hat{W})C^{\beta}\}\{\frac{C^{\beta}}{(W_0-\hat{W})^2(4C^{2\beta}-1)}\}$$

$$+\{1-\frac{C^{\beta}-(W_0-\hat{W})}{(W_0-\hat{W})(4C^{2\beta}-1)}\}\{1-\frac{C^{\beta}}{(W_0-\hat{W})\ (4C^{2\beta}-1)}\}>0.\qquad \text{QED.}$$

Proposition 6B: For $\alpha \sim$ uniform $[\frac{1}{2C^{\beta}}, 2C^{\beta}]$, where $\beta \in (0, 1]$ and $C^{\beta} \geq ½$, the probability that the agent will abandon the diet program (i.e. exert no effort) after incurring the initial cost is decreasing in the level of initial overweightness.

The probability of abandoning the program is the probability that effort expended is zero is the second stage. This probability is

$$\mathrm{P}(\alpha \leq \frac{1}{2(W_0-\hat{W})})=\frac{C^{\beta}-(W_0-\hat{W})}{(W_0-\hat{W})(4C^{2\beta}-1)}.$$

$$\frac{\partial}{\partial(W_0-\hat{W})}\mathrm{P}(\alpha \leq \frac{1}{2(W_0-\hat{W})})=\frac{-C^{\beta}}{(W_0-\hat{W})^2(4C^{2\beta}-1)}<0.$$

Therefore, less overweight agents are more likely to quit the diet program and exert no effort in the second stage after the purchase than more overweight agents. QED.

Proofs for Section IV: Alternative Model

Proposition IV 1: For α sufficiently high, higher α leads to lower effort. However, higher α unconditionally leads to greater weight loss.

Proof:

$$\frac{\partial}{\partial\alpha} e*(\alpha) = \frac{\hat{W} - (W_0 - \hat{W})\alpha}{\alpha^3},$$

which is positive if

$$\frac{\hat{W}}{2(W_0 - \hat{W})} < \alpha < \frac{\hat{W}}{(W_0 - \hat{W})},$$

and negative if

$$\alpha > \frac{\hat{W}}{(W_0 - \hat{W})}.$$

The amount of weight loss is $\alpha\ e*(\alpha)$.

$$\frac{\partial}{\partial\alpha}\alpha \cdot e*(\alpha) = e*(\alpha) + \frac{\partial}{\partial\alpha} e*(\alpha)$$

$$= \frac{\hat{W}}{2\alpha^2} > 0.$$

QED.

Proposition IV 2: The more overweight the agent initially is, the greater effort put forth and the greater weight loss experienced.

Proof:

$$\frac{\partial}{\partial(W_0 - \hat{W})} e*(\alpha) = \frac{1}{\alpha} > 0.$$

The amount of weight loss is

$$\alpha\ e*(\alpha\ ) = (W_0 - \hat{W}) - \frac{\hat{W}}{2\alpha}$$

$$\frac{\partial}{\partial(W_0 - \hat{W})} \alpha \cdot e*(\alpha) = 1 > 0.$$

QED.

Proposition IV 3: (Quitters) Given α, agents with lower initial overweightness are more likely to quit the diet plan than agents with higher initial overweightness.

Effort is positive only if $\alpha > \frac{\hat{W}}{2(W_0 - \hat{W})}$. As $(W_0 - \hat{W})$ falls, $\frac{\hat{W}}{2(W_0 - \hat{W})}$ increases. Therefore, as $(W_0 - \hat{W})$ falls, the α realized is more likely to be less than $\frac{\hat{W}}{2(W_0 - \hat{W})}$ which leads to no effort in the second stage. QED.

REFERENCES

Agatston, Arthur. The South Beach Diet: the Delicious, Doctor-Designed, Foolproof Plan for Fast and Healthy Weight Loss. New York: Random House, 2003.

Atkins Nutritional Inc. The Atkins Essentials: A Two-Week Program to Jump Start Your Low Carb Lifestyle. New York: HarperCollins, 2004.

Atkins, Robert C. Atkins for Life: The Complete Controlled Carb Program for Permanent Weight Loss and Good Health. New York: St. Martin's, 2003.

Blackburn, George. "Solution in Weight Control: Lessons from Gastric Surgery." American Journal of Clinical Nutrition. 2005; 82 (1): 248S-252S.

Buchwald, Henry, et al. "Bariatric Surgery: A Systematic Review and Meta Analysis." Journal of the American Medical Association. 2004; 292: 1724-1737.

Dupuis, Armand. "Secrets the Weight Loss Industry Are Hiding from You." 2006. Weight Loss Center. 26 Feb. 2006. <http://colonhealth.net/weight_loss/Secrets_the_Weight_Loss_Industry_are_Hiding_from_You.html>.

International Association for the Study of Obesity. *IASO Media Release.* 10th International Congress on Obesity, Sydney, Australia. September 2006.

Obesity Rate in USA. Intelihealth.com - Harvard Medical School. February 2003. 26 Feb. 2006. < http://www.annecollins.com/weight_health/obesity-rate.htm>.

Small Step (2006). U.S. Department of Health and Human Services. 26 Feb. 2006. < http://www.smallstep.gov/index.htm>.

CHAPTER 3

CHOICE OF DIETS AND BOREDOM PREFERENCE

I. Introduction and Motivation

The scene is familiar. A dieter starts a new fad diet. A few weeks later, the dieter finds himself abandoning the diet and starting a new one. And there are always plenty of fad diets to switch to. Yet, medical researchers find that the most important determinant in losing weight is adherence and commitment to one single diet plan.[40]

A fad diet is a restrictive weight-loss plan that promises dramatic weight-loss results typically in a short period of time.[41] Most fad diets rely on restrictions of certain food groups—low fat, low carb, liquid diet, etc. Some dieters are successful while on these plans. However, studies of these fad diets conclude that adherence rates are low. Multiple studies find that attrition rates among fad diets are as high as 40 percent after six months.[42] Besides being unsuccessful in losing weight in the long run, the dieters also could experience weight cycling where weight losses and weight regains occur repeatedly over time. Although it is still debatable, the phenomenon is believed by some medical researchers to be worse than being overweight itself in some cases.[43]

This chapter is an examination of the mechanism by which dieters switch between two fad diet plans, asking, for example, whether it is possible for dieters to stick with a fad diet that is sure to bring a success. Since, in reality, fad diets restrict which food groups are consumed, they tend to create craving for the taboo foods. Thus, diet plans in

---

[40] For example, Dansinger, et al. (2005) and Eckel (2005).
[41] http://familydoctor.org/784.xml
[42] For example, Dansinger, et al. (2005), Foster, et al. (2003) and Samaha, et al. (2003) http://www.centre4activeliving.ca/publications/research_update/2003/December.htm
[43] For example, Muls, et al. (1995) and NIH Publication No. 01-3901 (2004).

this paper incorporate the boredom caused by sticking to a single diet plan over time. I assume that diet plans are different in terms of their success rates but are equally boring. The chapter has significant implications on adherence to diet plans, successful weight maintenance, and weight cycling.

Consider the problem faced by an infinitely-lived agent who attempts to lose weight by implementing a diet plan. In each period, there are two diet programs from which to choose. Each program yields a success or a failure at the end of the period in which it is implemented. The success rates are different and known. The agent discounts the future and experiences boredom if a single plan is repeatedly implemented. Specifically, the payoff in each period is additionally discounted by a boredom factor which depends on the pattern of prior choices of diet programs up to that period. The agent maximizes the infinite sum of time-discounted and boredom-adjusted payoffs.

Under a given pattern of boredom discounting, I find that the agent never repeats the inferior plan, using it only to "reset" the boredom factor of the superior plan. Also, the pattern of choices repeats itself for a given set of parameters.

I argue that, for agents with short memory, there exists a period in which the agent abandons the superior diet plan and implements the inferior plan for one period. That is, as long as both plans create some boredom, the agent does not repeat the superior plan indefinitely. I also establish that there exists a boredom level above which the agent never repeats the superior plan regardless of the success rate. I find that the agent stays on the superior plan longer when the boredom level falls and when the time-discounting factor rises.

Under the assumption that one plan of the two is a sure failure—interpreted as not being on any diet plan—agents with long memory have a very simple optimal path. The path is one on which the agent either switches in every period if the diet plan is sufficiently boring or is always on the diet plan.

Section II describes the model. Section III discusses the agents with short memory and the solution. Section IV discusses the agents with long memory and the solution. Section V provides some extensions. Section VI concludes.

II. Model and Setup

There are two diet programs, A and B. The agent chooses which diet program to implement in each period. The number of trials/periods is infinite. Let δ denote the time-discounting factor. Each program, when implemented, generates one of two outcomes: success or failure, which is observed at the end of the period. The value of success is normalized to one; while the value of failure is zero.

Each program has a corresponding known success rate. Let α denote the success rate of Plan A and let β denote the success rate of Plan B, where α and $\beta \in [0, 1]$. Without loss of generality, let $\beta > \alpha$.

The agent also experiences boredom when a diet plan is repeated. In general, when the agent is on one plan for $(t+1)$ consecutive periods, the value of success in the current period is discounted at λ^t, where $\lambda \in (0, 1]$ is a boredom-discounting factor which is applied uniformly to both plans. The higher the λ, the lower the boredom created by repeating a plan. Overall boredom is increasing in t; the more times the agent repeats a plan consecutively, the higher the boredom in the period.

The agent maximizes the time-discounted and boredom-discounted expected value of successes.

III. Case I: Agents with Short Memory

Consider the first case in which t is reset to zero and the boredom discount λ^t is reset to 1 for the plan if the agent is off the plan for at least one period. This is to say that the boredom factor matters only for consecutive periods on the same plan. The agent is said to have a short memory as only one period off the plan completely eliminates the boredom accumulated. The diagram below illustrates an example of the framework.

Plan chosen	B	B	B	A	A	B	B	A
Time discount	1	δ	δ^2	δ^3	δ^4	δ^5	δ^6	δ^7
Payoff with Boredom	β	$\lambda\beta$	$\lambda^2\beta$	α	$\lambda\alpha$	β	$\lambda\beta$	α

Figure 3.1: Short-Memory Agents: An Illustration

A. Solution

Lemma 1: On the optimal path, the agent repeats the pattern of A and B, that is, if it is initially optimal for the first $(n + m)$ periods to choose B in the first n periods and then choose A in the next m periods, then the pattern of n B's followed by m A's continues indefinitely.

Proof:

Let Ω denote the set of parameters $(\alpha, \beta, \lambda, \delta)$. Let Φ denote the path of choices of B and A. Let $\Pi(\Phi;\Omega)$ denote the total payoff resulting from the set of parameters Ω and the path Φ.

Suppose on the optimal path Φ^*, plan B is chosen in the first period. Suppose the pattern of n B's and m A's is initially optimal, i.e. the optimal path starts with this pattern. Let the total payoff for these $n + m$ periods be denoted by $\pi(n,m;\Omega)$. Let ν and μ be members of the set of integers. Since the pattern of n B's and m A's is initially optimal,

$$\pi(n,m;\Omega) + \delta^{n+m}\,\pi^* \quad \geq \quad \pi(n+\nu, m+\mu;\Omega) + \delta^{n+m+\nu+\mu}\,\pi^*; \qquad \forall \nu, \forall \mu \in I \quad (1)$$

where π^* denotes the payoff on the optimal path thereafter. Note that, in either case, the optimal path 'thereafter' starts with Plan B.

Since the pattern of n B's and m A's is assumed to be optimal initially, it is the pattern initially chosen. Now consider two strategies after the first $(n + m)$ periods. First, consider 'repeating the pattern' strategy, where the pattern of n B's and m A's is repeated in the second $(n + m)$ periods and the optimal path is followed thereafter. In this case, the payoff is

$$\Pi_{repeat}(\Phi;\Omega) \ = \pi(n,m;\Omega) + \ \delta^{n+m} \ \pi(n,m;\Omega) + \ \delta^{2n+2m} \ \pi^* \qquad (1R)$$

Second, consider 'not repeating the pattern' strategy. In this case, the agent does not follow the *n-m* pattern, obtaining the payoff of

$$\Pi_{non-repeat}(\Phi;\Omega) \ = \pi(n,m;\Omega) + \ \delta^{n+m} \ \pi(n+\nu, m+\mu;\Omega) + \ \delta^{2n+2m+\nu+\mu} \ \pi^* \quad (1NR)$$

for integers v and μ.

Since Condition (1) is assumed to hold, $\Pi_{repeat} \ \geq \ \Pi_{non-repeat}$. Hence, repeating the pattern which is found to be initially optimal is the optimal path.

The proof is similar if it is initially optimal to start with Plan A. QED.

Lemma 2: On the optimal path, Plan A is never repeated.

(Sketch of proof)

Suppose that implementing Plan B in n consecutive periods is optimal. Let $\pi(n,1;\Omega)$ denote the payoff in the first $(n+1)$ periods that results from implementing Plan B in the first n consecutive periods and then Plan A in one period. If $(n,1)$ is initially optimal, then, by Lemma 1, $(n,1)$ is repeated indefinitely. Let $\pi(n,2;\Omega)$ denote the payoff in the first $(n+2)$ periods that results from implementing Plan B in the first n consecutive periods and then Plan A in two consecutive periods, i.e. repeating Plan A. Similarly, by Lemma 1, if $(n,2)$ is initially optimal, then it is repeated indefinitely. Therefore,

$$\Pi(n,1;\Omega) = \pi(n,1;\Omega) + \delta^{n+1}\,\pi(n,1;\Omega) + \delta^{2(n+1)}\,\pi(n,1;\Omega) + \ldots \tag{2A}$$

$$\Pi(n,2;\Omega) = \pi(n,2;\Omega) + \delta^{n+2}\,\pi(n,2;\Omega) + \delta^{2(n+2)}\,\pi(n,2;\Omega) + \ldots \tag{2B}$$

Since $\Pi(n,2;\Omega) - \Pi(n,1;\Omega) < 0$, it is optimal to never repeat Plan A.

If it is initially optimal to start the first period with Plan A, the proof is similarly structured and the agent, on the optimal path, will also never repeat Plan A. QED.

(*A detailed proof is in the appendix*).

Lemma 3: On the optimal path, the agent always starts with Plan B in the first period.

Proof:

Suppose that implementing Plan B in n consecutive periods is optimal. By Lemmas 1 and 2, we can limit our attention to the case in which the agent implements Plan B in n consecutive periods and Plan A in only one period. On the optimal path, the agent either starts with 1A or n B's. Recall the assumption that $\beta > \alpha$.

Suppose the optimal path starts with Plan A in the first period. This implies that

$$\Pi^*(\Phi^*;\Omega) = \Pi(\Phi_1;\Omega) > \Pi(\Phi_2;\Omega),$$

where $\Phi_1 = \Phi^*$ = Starting with Plan A in the first period and then Plan B in n periods, and Φ_2 = any other path.

The total payoff on the optimal path is

$$\Pi^*(\Phi^*;\Omega) = \alpha + \delta\beta + \delta^2\lambda\beta + \ldots + \delta^{n-1}\lambda^{n-2}\beta + \delta^n\alpha + \ldots$$

Now consider the following path Φ_2. Implement Plan B in the first period, then Plan A in the second period. Implement Plan B for the following $(n-1)$ period. Then resume Φ^*. The total payoff on path Φ_2 is

$$\Pi(\Phi_2;\Omega) = \beta + \delta\alpha + \delta^2\beta + \delta^3\lambda\beta + \ldots + \delta^{n-1}\lambda^{n-3}\beta + \delta^n\alpha + \ldots$$

Since $\beta > \lambda\beta$, for the 3rd period through the n^{th} period, the payoffs on the optimal path Φ^* are smaller than those on path Φ_2. Hence,

$$\Pi^*(\Phi^*;\Omega) > \Pi(\Phi_2;\Omega) \text{ implies that } \alpha + \delta\beta > \beta + \delta\alpha \text{, or } \alpha > \beta,$$

which is a contradiction. Therefore, the optimal path does not start with Plan A; it has to start with Plan B. QED.

The Optimal n

Since, on the optimal path, Plan A is never repeated (i.e. $m = 1$), we consider only a class of paths which are fully characterized by n, the number of consecutive periods in which B is implemented. Let $\Pi(n;\Omega)$ denote the total payoff on such a path.

With Lemmas 1, 2, and 3, the series of payoffs where n B's and 1 A pattern is adopted is as follows:

Time discount	1	δ	δ^2	$\cdots$	δ^{n-1}	δ^n	δ^{n+1}	$\cdot\cdot$	δ^{2n}	δ^{2n+1}	$\cdots$
Payoffs	β	$\lambda\beta$	$\lambda^2\beta$	$\cdots$	$\lambda^{n-1}\beta$	α	β	$\cdot\cdot$	$\lambda^{n-1}\beta$	α	$\cdots$

[----------n B's and 1 A--------------][..................................][...

Figure 3.2: A General Solution for Short-Memory Agents

The total payoffs for all A's are:

$$\delta^n \alpha + \delta^{2n+1} \alpha + \delta^{3n+2} \alpha + \delta^{4n+3} \alpha + \ldots$$

$$= \delta^n \alpha (1 + \delta^{n+1} + \delta^{2n+2} + \delta^{3n+3} + \ldots)$$

$$= \delta^n \alpha \sum_{t=0}^{\infty} \delta^{t(n+1)} = \frac{\delta^n \alpha}{1-\delta^{n+1}} \tag{4}$$

The total payoffs for the first n B's are:

$$\sum_{t=0}^{n-1} \delta^t \lambda^t \beta \qquad = \beta \sum_{t=0}^{n-1} \delta^t \lambda^t \tag{5}$$

Let b denote the sum $\beta \sum_{t=0}^{n-1} \delta^t \lambda^t$. This sum occurs repeatedly. The sum of these sums is $b + \delta^{(n+1)} b + \delta^{2(n+1)} b + \delta^{3(n+1)} b + \ldots$

$$= b\sum_{t=0}^{\infty} \delta^{t(n+1)} = \frac{b}{1-\delta^{n+1}} = \frac{\beta}{1-\delta^{n+1}}\sum_{t=0}^{n-1} \delta^t\lambda^t \tag{6}$$

Therefore, the infinite sum of all payoffs is the sum of (4) and (6):

$$\text{Total payoff} \equiv \Pi(n;\Omega) = \frac{\delta^n\alpha}{1-\delta^{n+1}} + \frac{\beta}{1-\delta^{n+1}}\sum_{t=0}^{n-1} \delta^t\lambda^t \tag{7}$$

The agent chooses n (i.e. the number of consecutive periods in which B is implemented) that maximizes $\Pi(n;\Omega)$.

By substituting $(n+1)$ in Equation (7) and subtracting $\Pi(n;\Omega)$ from it, we obtain:

$$\Delta\Pi_n \equiv \Pi(n+1;\Omega) - \Pi(n;\Omega) = \alpha\left(\frac{\delta^{n+1}}{1-\delta^{n+2}} - \frac{\delta^n}{1-\delta^{n+1}}\right) + \beta\left(\frac{\sum_{t=0}^{n}(\delta\lambda)^t}{1-\delta^{n+2}} - \frac{\sum_{t=0}^{n-1}(\delta\lambda)^t}{1-\delta^{n+1}}\right) \tag{8}$$

If $\Delta\Pi_n \equiv \Pi(n+1;\Omega) - \Pi(n;\Omega) > 0$, then increase n by one, otherwise the current n is optimal. Therefore, the solution for n is that: n is the smallest integer for which

$\Delta\Pi_n \equiv \Pi(n+1;\Omega) - \Pi(n;\Omega)$ is less than zero.

Let n^* denote the optimal n. Then,

$\Delta\Pi_{n^*} \equiv \Pi(n^*+1;\Omega) - \Pi(n^*;\Omega) < 0$, and

$\Delta\Pi_{n^*-1} \equiv \Pi(n^*;\Omega) - \Pi(n^*-1;\Omega) > 0$,

or n^* is n that satisfies

$$\alpha\left(\frac{\delta^{n+1}}{1-\delta^{n+2}}-\frac{\delta^{n}}{1-\delta^{n+1}}\right)+\ \beta\left(\frac{\sum_{t=0}^{n}(\delta\lambda)^{t}}{1-\delta^{n+2}}-\frac{\sum_{t=0}^{n-1}(\delta\lambda)^{t}}{1-\delta^{n+1}}\right)<0 \qquad \text{(9A)}$$

$$\text{and}\quad \alpha\left(\frac{\delta^{n}}{1-\delta^{n+1}}-\frac{\delta^{n-1}}{1-\delta^{n}}\right)+\ \beta\left(\frac{\sum_{t=0}^{n-1}(\delta\lambda)^{t}}{1-\delta^{n+1}}-\frac{\sum_{t=0}^{n-2}(\delta\lambda)^{t}}{1-\delta^{n}}\right)>0 \qquad \text{(9B)}$$

After collecting terms and rearranging, the optimal n is such that

$$\frac{1}{\lambda^{n-1}(1+\delta+\ldots+\delta^{n-1})-\delta(1+\delta\lambda+\ldots+\delta^{n-2}\lambda^{n-2})} < \frac{\beta}{\alpha}$$

$$< \frac{1}{\lambda^{n}(1+\delta+\ldots+\delta^{n})-\delta(1+\delta\lambda+\ldots+\delta^{n-1}\lambda^{n-1})} \qquad \text{(9)}$$

Note that, if $\alpha=\beta$, that is, if the diet plans are equally effective, then the agent switches in every period to avoid boredom.[44] In this case, however, the agent does not suffer from weight cycling because weight loss success is identical in every period.

Furthermore, in the presence of a third plan, the agent will only ever choose the two best plans. This is true because the agent switches to the inferior plan only to reset the boredom factor for the superior plan. Consequently, the agent will always be better off resetting the boredom while on the second best plan than on the worst plan.[45]

---

[44] Replacing α with β in Condition 9A, we obtain $\beta(1-\lambda)(\delta^{3}-\delta)$ which is less than 0, i.e. the optimal n is 1.
[45] In reality, even though most dieters do switch plans, the two plans chosen are different across dieters. This is most likely due to the fact that diets' success rates are dieter-specific.

B. Some Interesting Results

As mentioned earlier, medical professionals agree that sticking to one single diet plan is the most important factor in losing weight. The following two propositions provide an insight to the extreme cases in which the agent never sticks with a diet plan in any two consecutive periods. These occur for the plan that comes with great boredom—low λ. As suggested in the medical literature, boring fad diets are likely to cause attrition and result in poor weight loss outcomes.

<u>Proposition 1A</u>: If $\beta[\lambda(1+\delta)-\delta] < \alpha$, then the agent switches in every period, i.e. $n^* = 1$.

<u>Proposition 1B</u>: If $\lambda < \frac{\delta}{1+\delta}$, the agent switches in every period regardless of the sizes of β and α.

Proof:

Substituting optimal $n = 1$ in (9), we obtain

$$\frac{\beta}{\alpha} < \frac{1}{\lambda(1+\delta)-\delta}, \text{ or } \beta[\lambda(1+\delta)-\delta] < \alpha \tag{10A}$$

For any α and β, Condition (10A) holds if $[\lambda(1+\delta)-\delta] < 0$, or

$$\lambda < \frac{\delta}{1+\delta} \tag{10B}$$

QED.

Proposition 1A provides a condition under which the agent never sticks with one single plan. First, note that $[\lambda(1+\delta)-\delta]$ is increasing in λ. Hence, Condition (10A) is more likely to hold, and the agent is more likely to always switch, when λ is smaller, or when the diet plan is more boring. Second, $[\lambda(1+\delta)-\delta]$ is decreasing in δ. This implies that Condition (10A) is more likely to hold when δ is bigger, or when the future means more to the agent. This is intuitive as switching more frequently yields the highest payoff of β earlier and more often.

Proposition 1B provides a condition under which the agent never sticks with a plan in any consecutive periods regardless of how big β is and how small α is. This means that if the plan is sufficiently boring (Condition (10B) holds), even if Plan A is a certain failure and Plan B is a sure success, the agent still never sticks with Plan B.

In order to examine the cases in which the agent repeats the superior plan, the following assumption is made for the analyses that follow.

Assumption A1: $\lambda > \dfrac{\delta}{1+\delta}$.

Lemma 4: $\Lambda_n = \dfrac{1}{\lambda^n(1+\delta+\ldots+\delta^n)-\delta(1+\delta\lambda+\ldots+\delta^{n-1}\lambda^{n-1})}$ is increasing in ***n***,

provided that $\lambda^n(1+\delta+\ldots+\delta^n)-\delta(1+\delta\lambda+\ldots+\delta^{n-1}\lambda^{n-1})$ is positive.

(*A detailed proof is in the appendix*).

Under Assumption A1, i.e.

$$\lambda^n(1+\delta+...+\delta^n)-\delta(1+\delta\lambda+...+\delta^{n-1}\lambda^{n-1})>0 \quad \text{when } n=1,$$

Lemma 4 shows that the solution to Condition (9) exists as

$$\frac{1}{\lambda^{n-1}(1+\delta+...+\delta^{n-1})-\delta(1+\delta\lambda+...+\delta^{n-2}\lambda^{n-2})}$$

$$<\frac{1}{\lambda^n(1+\delta+...+\delta^n)-\delta(1+\delta\lambda+...+\delta^{n-1}\lambda^{n-1})}.$$

<u>Proposition 2</u>: If $\alpha = 0$, the pattern of switching is independent of the size of β.

Moreover, the agent switches more often if the plan is sufficiently boring.

Proof:

If $\alpha = 0$, then Equation (8) for any n is:

$$\Delta\Pi_n = \beta\left(\frac{\sum_{t=0}^{n}(\delta\lambda)^t}{1-\delta^{n+2}}-\frac{\sum_{t=0}^{n-1}(\delta\lambda)^t}{1-\delta^{n+1}}\right)$$

$$=\beta\left(\frac{1+\delta\lambda+(\delta\lambda)^2+...+(\delta\lambda)^{n-1}+(\delta\lambda)^n}{1-\delta^{n+2}}-\frac{1+\delta\lambda+(\delta\lambda)^2+...+(\delta\lambda)^{n-1}}{1-\delta^{n+1}}\right)$$

$$=\beta\left(\frac{1}{1-\delta\lambda}\right)\left(\frac{1-\delta^{n+1}\lambda^{n+1}}{1-\delta^{n+2}}-\frac{1-\delta^n\lambda^n}{1-\delta^{n+1}}\right)$$

Let Γ denote $\left(\frac{1-\delta^{n+1}\lambda^{n+1}}{1-\delta^{n+2}}-\frac{1-\delta^n\lambda^n}{1-\delta^{n+1}}\right)$.

The optimal n is the smallest integer that makes Γ negative. But Γ is independent of β.

Furthermore,

$$\frac{\partial}{\partial \lambda}\Gamma = \delta\left(\frac{n\delta^n \lambda^{n-1}}{1-\delta^{n+1}} - \frac{(n+1)\delta^{n+1}\lambda^n}{1-\delta^{n+2}}\right), \text{ which is} > 0 \text{ if}$$

$$n(1-\delta^{n+2}) > (n+1)\delta\lambda(1-\delta^{n+1}),$$

which holds for small λ. This means that Γ will be negative at a smaller n and the agent will switch more often if the diet plan is sufficiently boring. QED.

The first part of this proposition implies that the agent will switch to Plan A at some point even if the plan is a failure with certainty and even if Plan B is a sure success. $\alpha = 0$ can also be interpreted as 'off the diet plan'. In this sense, the agent will be off the diet plan in some period even if the diet plan brings weight-loss success every time.

The second part of the proposition is intuitive. All else equal, the frequency with which the agent is off the diet plan is increasing in the boredom that the plan causes.

As stated in the medical literature review, medical studies suggest that fad diets do result in significantly sizable weight losses in the short run.[46] They also establish that the attrition rates are extremely high for these plans. In the presence of great short-term successes, it may seem irrational for dieters to quit the plan. However, considering the boredom, dieters may shy away from even the most successful plan.

[46] For example, Samaha, et al. (2003), Foster, et al. (2003), and Avenell, et al. (2004). See Relevant Medical Literature section.

Let Λ_n denote $\dfrac{1}{\lambda^n(1+\delta+...+\delta^n)-\delta(1+\delta\lambda+...+\delta^{n-1}\lambda^{n-1})}$.

Proposition 3: Less boring plans (higher λ) cause the agent to switch less often, i.e.

$\dfrac{\partial}{\partial\lambda}\Lambda_n < 0$. (*A detailed proof is in the appendix*).

This means that both upper- and lower- limits in (9) decrease as λ increases, or as diet plans become less boring. The fall in the limits implies a larger optimal n and that the agent switches less often.

This is intuitive as the primary reason the agent switches is the boredom caused by being on one plan repeatedly. One real-world factor that might correspond to the boredom factor is the restrictiveness of diet plans. Fad diets whose restrictions focus on food groups from which to refrain tend to create greater boredom (lower λ). Consequently, according to the model, dieters on fad diets switch more often than those on less restrictive plans such as the Weight Watchers program. Again, this is consistent with the findings in the medical literature.

Proposition 4: A greater time discount (δ) causes the agent to switch more often, i.e.

$\frac{\partial}{\partial \delta} \Lambda_n > 0$. (*A detailed proof is in the appendix*).

This means that both upper- and lower- limits in (9) increase as δ increases, or as future gets greater weight. The rise in the limits implies a smaller optimal n and that the agent switches more often.

Future payoffs depend a great deal on how frequently the agent switches. Switching back and forth is the means to eliminate boredom and return to the highest payoff of β. The intuition behind Proposition 4 is that sticking to Plan B costs the dieters in terms of delaying the payoff without boredom. With a greater time-discounting factor, the cost of delaying increases, hence the agent switches more frequently.

It is not unusual to learn of a fad diet's claim in terms of pounds lost per week while non-fad diets focus on longer time durations. For example, the Weight Watchers program claims pounds lost per month. In this regard, the paper suggests that, all else equal, the dieter will switch more often on a fad diet plan than he would on a non-fad diet. This is, again, consistent with the findings that the adherence rates among fad diets are much lower than those among non-fad diets such as the Weight Watcher program.

IV. Case II: Agents with Long Memory

Recall the boredom discount of λ^t. Recall that t is the indicator of boredom created by the plan, where a higher t indicates greater boredom. Consider the second case in which t does not reset to zero and therefore the boredom discount λ^t does not reset to one after being off the plan for one period. Instead, t is reduced in some predetermined pattern. Let this pattern be such that t is reduced by one for each period off the plan. Compared to Case I, the agent in Case II has a longer memory in a sense that being off the plan for one period does not completely eliminate the boredom that has been accumulated. Instead, if the agent has been on the plan for three consecutive periods, for instance, he would have to be off the plan for three consecutive periods to reset t to zero and completely eliminate boredom. The payoff after one period off the plan improves only by a factor of λ instead of λ^t as is the case for short-memory agents.

Assume that $\alpha = 0$. This could be interpreted as the agent having only one plan (Plan B) to implement. But he can be on it or off it in any period. During any period off the plan, the agent experiences no weight loss. One would expect that being off the plan generates no boredom, i.e. $\lambda = 1$. However, since payoffs are zero when the agent is off the plan, the boredom discounting factor makes no difference in all these periods.

The following diagram illustrates an example of how the boredom discount is affected by being off the plan. Here, Plan A indicates that the dieter is off the diet plan, receiving a payoff of zero.

Period #	1	2	3	4	5	6	7	8	9
Plan chosen	B	B	B	A	B	A	A	B	B
Time discount	1	δ	δ^2	δ^3	δ^4	δ^5	δ^6	δ^7	δ^8
Payoff with Boredom	β	$\lambda\beta$	$\lambda^2\beta$	0 ($\lambda^3\beta$)	$\lambda^2\beta$	0 ($\lambda^3\beta$)	0 ($\lambda^2\beta$)	$\lambda\beta$	$\lambda^2\beta$

Figure 3.3: Long-Memory Agents: An Illustration

Note: Bracketed payoffs in the last row are those that would have been received, had the agent chosen to implement Plan B.

From the first period through the third, boredom increases according to the rule specified in Case I where, in each period, the payoff is discounted at λ^t, where t is the number of consecutive periods on the plan prior to the period. In the fourth period, if the dieter chose Plan B, the payoff would be $\lambda^3\beta$. But since the dieter chooses to be off the diet plan, boredom falls and the boredom discount rises to λ^2 for the following, fifth, period, yielding a payoff of $\lambda^2\beta$. In other words, being off the plan in one period serves to reduce t by one. This situation is repeated in the sixth period. The payoff on the diet plan in the seventh period would be $\lambda^2\beta$. However, since the agent is off the plan in the seventh period, the boredom discount rises to λ for the following, eighth, period. In summary, therefore, t is reduced by m, where m is the number of consecutive periods off the plan.

A. Solution

First, consider the path on which the agent always switches. Let Φ_{AS} denote this path. In this case, the agent receives a payoff of β in every other period. In all the rest of the periods, the agent receives zero.

The total payoff in this case is

$$\Pi(\Phi_{AS};\Omega) = \beta \sum_{t=0}^{\infty} \delta^{2t} = \frac{\beta}{1-\delta^2} \tag{11}$$

Next, consider another class of paths where repeating the diet plan occurs. If the agent is on the diet plan in n consecutive periods, then the agent will be off the plan at most n consecutive periods. As an illustration, suppose n = 3. The possible streams of payoffs are as follows (time discounting factor omitted):

a) Off the plan in one period: $\beta \quad \lambda\beta \quad \lambda^2\beta \quad 0 \quad \lambda^2\beta \quad 0 \quad \lambda^2\beta \quad 0.....$

b) Off the plan in two periods: $\beta \quad \lambda\beta \quad \lambda^2\beta \quad 0 \quad 0 \quad \lambda\beta \quad \lambda^2\beta \quad 0 \quad 0 \quad \lambda\beta \quad .....$

c) Off the plan in three periods: $\beta \quad \lambda\beta \quad \lambda^2\beta \quad 0 \quad 0 \quad 0 \quad \beta \quad \lambda\beta \quad \lambda^2\beta \quad 0 \quad 0 \quad 0....$

The total payoff depends on which scenario is chosen. For any n, there are n scenarios. Let m denote consecutive periods off the diet plan. Let Φnm denote the path on which n and m are chosen. Let the payoff for each of these scenarios be denoted by $\Pi(\Phi nm;\Omega)$. The payoff for any n and m is

$$\Pi(\Phi nm;\Omega) = \beta \sum_{t=0}^{n-m-1} (\delta\lambda)^t + \frac{\delta^{n-m}\lambda^{n-m}\beta + ... + \delta^{n-1}\lambda^{n-1}\beta}{1-\delta^{2m}} \tag{12}$$

Note that the first term disappears if $m = n$.

Let Θ_Φ denote the class of such paths for any positive integer $n \geq 2$ and for any positive integer $m \leq n$.

Lastly, consider the path on which the agent never switches, i.e. the agent is on the diet plan in every period. Let Φ_{NS} denote this path.

The total payoff in this case is

$$\Pi(\Phi_{NS};\Omega) = \beta \sum_{t=0}^{\infty} (\delta\lambda)^t = \frac{\beta}{1-\delta\lambda} \tag{13}$$

<u>Lemma 5</u>: If $\lambda < \delta$, then $\Pi(\Phi_{AS};\Omega) > \Pi(\Phi_{NS};\Omega)$ and $\Pi(\Phi_{AS};\Omega) > \Pi(\Phi nm;\Omega)$, for any $\Phi nm \in \Theta_\Phi$.

(*A detailed proof is in the appendix*).

Lemma 5 states that if the boredom discounting factor is smaller than the time discounting factor, that is, if the diet plan is sufficiently dreadful, then the agent will be off the plan in every other period.

<u>Lemma 6</u>: If $\lambda > \delta$, then $\Pi(\Phi_{NS};\Omega) > \Pi(\Phi_{AS};\Omega)$ and $\Pi(\Phi_{NS};\Omega) > \Pi(\Phi nm;\Omega)$, for any $\Phi nm \in \Theta_\Phi$.

(*A detailed proof is in the appendix*).

Lemma 6 states that if the boredom discounting factor is greater than the time discounting factor, that is, if the diet plan does not significantly bore the agent, then the agent will always be on the diet plan.

Lemmas 5 and 6 are intuitive. First, due to the specification of how boredom is reduced while off the diet plan, the class of paths Θ_{Φ} is always inferior to either Φ_{NS} or Φ_{AS}. While off the diet plan, the agent accepts the payoff of zero (i.e. no weight loss) in order to receive the improved payoff—with boredom reduced by a factor of λ—in the following period which is discounted at δ. This means that if it is beneficial to go off the diet in any period after n periods on it, it was also beneficial to be off it after $n-1$ periods on it. In other words, if it is worthwhile to go off it at any point, it is better sooner than later—Φ_{AS} is superior to $\Phi nm \in \Theta_{\Phi}$. And if it is not worthwhile to go off it after only one period on it, then it will never pay off to be off it—Φ_{NS} is superior to $\Phi nm \in \Theta_{\Phi}$.

The benefit of being off the plan is to avoid boredom today and to receive the payoff without boredom tomorrow. But since the boredom-less payoff comes tomorrow, being off the plan in every other period is beneficial only when the boredom discounting factor dominates the time discounting factor ($\lambda < \delta$—Lemma 5). Otherwise, it is beneficial never to be off the plan ($\lambda > \delta$—Lemma 6).

Lemmas 5 and 6 together provide the solution to the problem as follows:

The agent always switches (i.e. be on the plan only every other period) if $\lambda < \delta$, obtaining a total payoff of

$$\Pi(\Phi_{AS};\Omega) = \beta\sum_{t=0}^{\infty}\delta^{2t} = \frac{\beta}{1-\delta^2},$$

and the agent never switches (i.e. be on the plan in every period) if $\lambda > \delta$, obtaining a total payoff of

$$\Pi(\Phi_{NS};\Omega) = \beta\sum_{t=0}^{\infty}(\delta\lambda)^t = \frac{\beta}{1-\delta\lambda}.$$

B. Comparisons with Case I

Consider the case in which $\alpha = 0$, that is, the agent has only one diet plan to implement—Plan B. Choosing Plan A in any period is interpreted as being off the plan for the period.

For Case I—agents with short memory, Proposition 1B provides a lower limit of the boredom factor below which the agent never sticks to the plan in any two consecutive periods. This cutoff boredom factor is $\frac{\delta}{1+\delta}$.

In comparison to Case I, agents with long memory (Case II) do more poorly in terms of adherence to the diet plan. The lower limit of the boredom factor is δ which is greater than $\frac{\delta}{1+\delta}$. For example, if $\delta = 0.75$, then the long-memory agent will be off the plan in every other period for any $\lambda < 0.75$. On the other hand, the short-memory agent will be off the plan in every other period for any $\lambda < \frac{0.75}{1+0.75} \approx 0.43$.

The boredom discounting factor can be diet-specific. Therefore, given a diet program, the long-memory agent is more likely to experience weight cycling due to the greater likelihood of constantly being on and off the diet plan.

However, if the plan is not sufficiently boring (i.e. $\lambda > \delta$), the long-memory agent will never be off the plan while the short-memory agent will eventually be off the plan at some point. This is to say that the long-memory agent fares better in such cases. Nevertheless, if the plan is not so boring, the short-memory agent is off the plan so infrequently that the weight-cycling effect may not be significant.

V. Extension

Further comparisons between short-memory agents and long-memory agents are possible for the case in which $\alpha > 0$. For long-memory agents, this yields an added benefit of the payoff in the period off the superior plan (Plan B). Consequently, switching to Plan A should occur with greater frequency—that is the cutoff point for the boredom discounting factor λ should be greater than δ. This has two opposing effects on the weight-cycling phenomenon. First, the greater frequency of switching leads to greater weight-cycling effect as being on the inferior Plan A does not bring as great a weight-loss success. However, the fact that α is now positive and Plan A has some success leads to smaller swings in weight.

A more general specification of how boredom is reduced may also be introduced. For an agent whose current boredom discount is λ^t, being off the plan in the following period leads to a boredom discount of λ^τ in the following period if the plan is implemented, where $\tau \in \{0, 1, \ldots, t)$. For short-memory agents, $\tau = 0$. For long-memory agents, $\tau = t$. An intermediate-memory agent may be defined as having $0 < \tau < t$. Due to its success rate of zero, Plan A still should never be repeated. However, the switching pattern should be less extreme than in the long-memory case. The intermediate-memory agents would most likely adopt a pattern similar to those implemented by short-memory agents. However, the number of consecutive periods on the diet program (Plan B) should be smaller than that in the short-memory case.

In the short-memory case, a brief discussion is provided for a case in which there are more than two diets from which to choose. In that case, the third-best plan is never chosen because once the second-best plan is chosen, the best plan immediately becomes completely non-boring. As a result, the agent switches back to the best plan in the very next period and as soon as the boredom has worn off. Since any plan beside the best plan is implemented solely to eliminate boredom of the best plan, being on the second-best plan is always better than being on any other plan.

A similar extension can also be applied to long-memory agents. In the presence of a third-best plan, it may be used to reduce the boredom that has been accumulated for the best and the second-best plans at once. The agent starts out with the best plan. The boredom for the best plan accumulates. At some point, the second-best plan, which is non-boring at the moment, will become attractive. The agent switches to the second-best plan. The boredom for the second-best plan accumulates while boredom for the best plan reduces. Two scenarios can occur at this point. If the third-best plan is sufficiently ineffective, i.e. its success rate is sufficiently low, then the best plan will become more attractive than the now-boring second-best plan before the third-best plan is ever attractive. In such cases, the third-best plan is never implemented. However, if the third-best plan is sufficiently effective, the agent will switch to the third-best plan as soon as the second-best plan is sufficiently boring, when compared to the non-boring third-best plan. In the process, the agent is reducing the boredom for the best plan and that for the second-best plan simultaneously. After some time, the third-best plan becomes sufficiently boring as well. The agent then switches back to the best plan accordingly. In the presence of a fourth-best plan, the same scenarios apply. In principle, with many plans, the agent can rank these plans with regard to their success rates. The agent then proceeds down the list. At some point, the next-best plan is sufficiently ineffective and the agent switches back to the best plan.

Note also that switching should occur more frequently when diet plans are more equally successful. This is because the next-best plan will become attractive sooner in such cases as its attractiveness depends primarily on its success rate when compared to that of the current plan, that is, how inferior it is relative to the current plan.

To the extent that real world dieters have somewhat long memory, dieters can potentially be on some inferior and ineffective diets for a while. This should not come as a surprise as long as diets are boring. Considering only the health and weight-loss outcomes, one may argue that the presence of these inferior diets harms the agent in that the weight-loss results are not as great as they could be. However, the presence of these inferior plans clearly allows the dieters to be more flexible in their choices and, more importantly, to avoid boredom. Since preferences are also defined over boredom, these plans, even with poor weight-loss outcomes, make them happier. Moreover, as discussed, the agent switches to these inferior plans only if they are sufficiently effective.

In this sense, the agent has carefully weighed the benefits and costs of losing weight and being bored.

VI. Conclusion

In this chapter, we develop a model that explains the pattern of choices between two diet plans when repeating one diet plan causes the agent to become bored. Diet plans are plenty but weight loss outcomes are poor. In light of several medical studies indicating that the most significant determinant in any attempt to lose weight is the adherence to one diet plan over time, this paper provides some interesting results.

The chapter examines two types of agents: those with short memory and those with long memory. The difference between the two types lies in how easily the accumulated boredom is eliminated. For the short-memory agents, boredom is completely eliminated with one period off the plan whereas for the long-memory agents it takes much longer.

We find that, among the short-memory agents, the greater the boredom, the lower the adherence to the superior plan over time. Furthermore, if the agent is choosing between being on or off a successful diet plan, the agent will switch (i.e. be off the diet plan) at some point no matter how successful the plan is. We also find that, among the long-memory agents, if the boredom-discounting factor is lower than the time-discounting factor, then the agent never repeats the successful diet plan.

In terms of real-world diet plans, boredom can easily be caused by the restrictions of food groups that are deemed taboo. Most fad diets do restrict certain food groups. The Atkins and the South Beach plans, for instance, are low-carb diets where intake of carbohydrates is severely restricted. Eating meat without potatoes is bound to get old after some time. Some plans are even more restrictive. The Grapefruit diet, for example, tells the dieters to eat a lot of grapefruit. The Healthy Soy diet suggests that the dieters replace most meat with soy bean. The boredom created by such plans causes the dieters to be on and off the chosen plan even though the success rates in the short run are indeed very high. This is consistent with what the model suggests.

Switching diet plans or being on and off a diet plan also contributes to the weight cycling phenomenon as well. A restrictive, fad diet plan that leads to a significant weight loss in the short run will only lead to the agent being on and off the plan in the long run. This eventually leads to weight cycling where the agent loses and regains weight repeatedly.

A non-fad diet plan most likely creates less boredom as it is less restrictive than a fad diet. The Weight Watchers regimen, for instance, allows the dieters to consume any type of food as long as they stay within the 'points' allowed each day. Hence, the dieters form their own food constraints depending on their personal taste. This alleviates the problem of boredom encountered in most fad diets. As is well known within the diet industry, the Weight Watcher program is among the best in terms of long-term weight loss and weight maintenance.

Moreover, fad diets rely on a very short time span per period while non-fad diets rely on a longer time span. Consequently, the time-discounting factor is relatively large when compared to that of non-fad diets. According to the model, for agents with short memory, this leads to more switching when on a fad diet. For agents with long memory, this leads to a greater likelihood that the boredom-discounting factor is smaller than the time-discounting factor which leads to a higher probability that the agent always switches.

This chapter suggests a framework under which dieters systematically get on and off fad diets constantly, leading to a disappointing overall weight-loss and weight-maintenance outcome. Clearly the overall picture can be improved and the American Heart Association was onto something when it published "No Fad Diet" in 2005. It is also hoped that this chapter contributes to the investigation of dieting endeavors as well.

Appendix

Proof of Lemma 2: On the optimal path, Plan A is never repeated.

Recall Equations (2A) and (2B),

$$\Pi(n,1;\Omega) = \pi(n,1;\Omega) + \delta^{n+1}\pi(n,1;\Omega) + \delta^{2(n+1)}\pi(n,1;\Omega) + \ldots \quad \text{(2A)}$$

$$\Pi(n,2;\Omega) = \pi(n,2;\Omega) + \delta^{n+2}\pi(n,2;\Omega) + \delta^{2(n+2)}\pi(n,2;\Omega) + \ldots \quad \text{(2B)}$$

$$\pi(n,1;\Omega) = \sum_{t=0}^{n-1} \delta^t\lambda^t\beta + \delta^n\alpha$$

$$\pi(n,2;\Omega) = \sum_{t=0}^{n-1} \delta^t\lambda^t\beta + \delta^n\alpha + \delta^{(n+1)}\lambda\alpha.$$

Therefore,

$$\text{(2B)} - \text{(2A)} = \lambda\alpha(\delta^{(n+1)} + \delta^{(2n+3)} + \delta^{(3n+5)} + \ldots)$$

$$- \delta\pi(n,1;\Omega) - \delta^2\pi(n,1;\Omega) - \delta^3\pi(n,1;\Omega) - \delta^4\pi(n,1;\Omega) - \ldots$$

$$= \lambda\delta^{(n+1)}\alpha \sum_{t=0}^{\infty} \delta^{t(n+2)} - \delta\pi(n,1;\Omega)\sum_{t=0}^{\infty}\delta^t$$

$$= \delta\left(\frac{\lambda\delta^n\alpha}{1-\delta^{n+2}} - \frac{\pi(n,1;\Omega)}{1-\delta}\right) < 0 \text{ since}$$

$$\lambda\delta^n\alpha < \pi(n,1;\Omega) \text{ and } 1-\delta^{(n+2)} > 1-\delta.$$

Hence, Plan A is never repeated on the optimal path. QED.

Proof of Lemma 4: $\Lambda_n = \dfrac{1}{\lambda^n(1+\delta+...+\delta^n)-\delta(1+\delta\lambda+...+\delta^{n-1}\lambda^{n-1})}$ is increasing in n, provided that $\lambda^n(1+\delta+...+\delta^n)-\delta(1+\delta\lambda+...+\delta^{n-1}\lambda^{n-1})$ is positive.

Consider only the denominator of the upper limit of Condition (9).

$$\lambda^n(1+\delta+...+\delta^n)-\delta(1+\delta\lambda+...+\delta^{n-1}\lambda^{n-1}) \tag{14A}$$

When n rises by one, the denominator becomes

$$\lambda^{n+1}(1+\delta+...+\delta^{n+1})-\delta(1+\delta\lambda+...+\delta^{n}\lambda^{n}) \tag{14B}$$

The denominator rises by (14B) – (14A) =

$$\lambda^{n+1}(1+\delta+...+\delta^{n+1})-\delta(1+\delta\lambda+...+\delta^{n}\lambda^{n})$$

$$-\lambda^n(1+\delta+...+\delta^n)-\delta(1+\delta\lambda+...+\delta^{n-1}\lambda^{n-1})$$

$$=\lambda^{n+1}(1+\delta+...+\delta^{n+1})-\lambda^{n}(1+\delta+...+\delta^{n+1})<0.$$

That is, the denominator falls when n rises. Therefore,

$\dfrac{1}{\lambda^n(1+\delta+...+\delta^n)-\delta(1+\delta\lambda+...+\delta^{n-1}\lambda^{n-1})}$ is increasing in n. QED.

Proof of Proposition 3: Less boring plans (higher λ) cause the agent to switch less often, i.e. $\frac{\partial}{\partial\lambda}\Lambda_n<0$.

$$\Lambda_n = \frac{1}{\lambda^n(1+\delta+...+\delta^n)-\delta(1+\delta\lambda+...+\delta^{n-1}\lambda^{n-1})}.$$

For $n \geq 2$,

$$\frac{\partial}{\partial\lambda}\Lambda_n = -n\lambda^{n-1}(1+\delta+...+\delta^n)+\delta^2(1+2\delta\lambda+3\delta^2\lambda^2+...+(n-1)\delta^{n-2}\lambda^{n-2})<0.$$

For $n = 1$,

$$\frac{\partial}{\partial \lambda} \Lambda_n = -(1+\delta) < 0.$$

That is, as λ rises, the limits in Condition (9) fall and the agent switches less frequently.

QED.

Proof of Proposition 4: A greater time discount (δ) causes the agent to switch more often,

i.e. $\frac{\partial}{\partial \delta} \Lambda_n > 0$.

$$\frac{\partial}{\partial \delta} \Lambda_n = -(\lambda^n + 2\lambda^n\delta + \ldots + n\lambda^n\delta^{n-1}) + (1 + 2\lambda\delta + \ldots + n\lambda^{n-1}\delta^{n-1})$$

$$= (1-\lambda^n) + 2(\lambda - \lambda^n)\delta + \ldots + n(\lambda^{n-1} - \lambda^n)\delta^{n-1} \quad > 0.$$

That is, as δ rises, the limits in Condition (9) rise and the agent switches more frequently.

QED.

Proof of Lemma 5: If $\lambda < \delta$, then $\Pi(\Phi_{AS};\Omega) > \Pi(\Phi_{NS};\Omega)$ and $\Pi(\Phi_{AS};\Omega) >$ $\Pi(\Phi nm;\Omega)$, for any $\Phi nm \in \Theta_{\Phi}$.

Recall

$$\Pi(\Phi_{AS};\Omega) = \beta\sum_{t=0}^{\infty}\delta^{2t} = \frac{\beta}{1-\delta^2} \tag{11}$$

$$\Pi(\Phi nm;\Omega) = \beta\sum_{t=0}^{n-m-1}(\delta\lambda)^t + \frac{\delta^{n-m}\lambda^{n-m}\beta + \ldots + \delta^{n-1}\lambda^{n-1}\beta}{1-\delta^{2m}} \tag{12}$$

$$\Pi(\Phi_{NS};\Omega) = \beta\sum_{t=0}^{\infty}(\delta\lambda)^t = \frac{\beta}{1-\delta\lambda} \tag{13}$$

Suppose $\lambda < \delta$. Then

(i) $\Pi(\Phi_{AS};\Omega) - \Pi(\Phi_{NS};\Omega) = \dfrac{\beta}{1-\delta^2} - \dfrac{\beta}{1-\delta\lambda} > 0$, and

(ii) $\Pi(\Phi_{AS};\Omega) - \Pi(\Phi nm;\Omega)$

$$= \frac{\beta}{1-\delta^2} - \beta \sum_{t=0}^{n-m-1} (\delta\lambda)^t + \frac{\delta^{n-m}\lambda^{n-m}\beta + \ldots + \delta^{n-1}\lambda^{n-1}\beta}{1-\delta^{2m}}$$

$$= \beta\left(\frac{1}{1-\delta^2} - \frac{1-(\delta\lambda)^{n-m}}{1-\delta\lambda} - \frac{(\delta\lambda)^{n-m} - (\delta\lambda)^n}{(1-\delta\lambda)(1-\delta^{2m})}\right)$$

$$= \beta\left(\frac{1-(\delta\lambda)^{n-m}}{1-\delta^2} - \frac{1-(\delta\lambda)^{n-m}}{1-\delta\lambda}\right)$$

$$+ \beta\left(\frac{(\delta\lambda)^{n-m}}{1-\delta^2} - \frac{(\delta\lambda)^{n-m}}{(1-\delta\lambda)(1-\delta^{2m})} + \frac{(\delta\lambda)^n}{(1-\delta\lambda)(1-\delta^{2m})}\right)$$

$$= \beta\left(\frac{1-(\delta\lambda)^{n-m}}{1-\delta^2} - \frac{1-(\delta\lambda)^{n-m}}{1-\delta\lambda}\right)$$

$$+ \frac{\beta(\delta\lambda)^{n-m}}{(1-\delta^2)(1-\delta^{2m})}\left[(1-\delta^{2m}) - \frac{1-\delta^2}{1-\delta\lambda} + \frac{(1-\delta^2)\delta^m\lambda^m}{1-\delta\lambda}\right]$$

$$= \beta\left(\frac{1-(\delta\lambda)^{n-m}}{1-\delta^2} - \frac{1-(\delta\lambda)^{n-m}}{1-\delta\lambda}\right) + \frac{\beta(\delta\lambda)^{n-m}}{(1-\delta^2)(1-\delta^{2m})}\left[(1-\delta^{2m}) - \frac{(1-\delta^2)(1-\delta^m\lambda^m)}{1-\delta\lambda}\right]$$

$$= \beta\left(\frac{1-(\delta\lambda)^{n-m}}{1-\delta^2} - \frac{1-(\delta\lambda)^{n-m}}{1-\delta\lambda}\right)$$

$$+ \frac{\beta(\delta\lambda)^{n-m}}{(1-\delta^2)(1-\delta^{2m})}\{(1-\delta^2)\,[(1+\delta^2+\ldots+(\delta^2)^{m-1}) - (1+\delta\lambda+\ldots+(\delta\lambda)^{m-1}]\}$$

$> 0.$

Therefore, 'always switching' is the optimal path if $\lambda < \delta$. QED.

Proof of Lemma 6: If $\lambda > \delta$, then $\Pi(\Phi_{NS};\Omega) > \Pi(\Phi_{AS};\Omega)$ and $\Pi(\Phi_{NS};\Omega) >$ $\Pi(\Phi nm;\Omega)$, for any $\Phi nm \in \Theta_{\Phi}$.

Recall

$$\Pi(\Phi_{AS};\Omega) = \beta \sum_{t=0}^{\infty} \delta^{2t} = \frac{\beta}{1-\delta^2} \tag{11}$$

$$\Pi(\Phi nm;\Omega) = \beta \sum_{t=0}^{n-m-1} (\delta\lambda)^t + \frac{\delta^{n-m}\lambda^{n-m}\beta + \ldots + \delta^{n-1}\lambda^{n-1}\beta}{1-\delta^{2m}} \tag{12}$$

$$\Pi(\Phi_{NS};\Omega) = \beta \sum_{t=0}^{\infty} (\delta\lambda)^t = \frac{\beta}{1-\delta\lambda} \tag{13}$$

Suppose $\lambda > \delta$. Then

(i) $\Pi(\Phi_{NS};\Omega) - \Pi(\Phi_{AS};\Omega) = \dfrac{\beta}{1-\delta\lambda} - \dfrac{\beta}{1-\delta^2} > 0$, and

(ii) $\Pi(\Phi_{NS};\Omega) - \Pi(\Phi nm;\Omega)$

$$= \beta\left(\frac{1}{1-\delta\lambda} - \frac{1-(\delta\lambda)^{n-m}}{1-\delta\lambda} - \frac{(\delta\lambda)^{n-m} - (\delta\lambda)^n}{(1-\delta\lambda)(1-\delta^{2m})}\right)$$

$$= \frac{\beta}{1-\delta\lambda}\left[1-1 + \delta^{n-m}\lambda^{n-m} - \frac{\delta^{n-m}\lambda^{n-m} - \delta^n\lambda^n}{1-\delta^{2m}}\right]$$

$$= \frac{\beta\delta^{n-m}\lambda^{n-m}}{1-\delta\lambda}\left(1 - \frac{1}{1-\delta^{2m}} + \frac{\delta^m\lambda^m}{1-\delta^{2m}}\right)$$

$$= \frac{\beta\delta^{n-m}\lambda^{n-m}}{1-\delta\lambda}\left(\frac{\delta^m\lambda^m - \delta^{2m}}{1-\delta^{2m}}\right) > 0.$$

Therefore, 'never switch' is the optimal path if $\lambda > \delta$. QED.

REFERENCES

American Heart Association. No-Fad Diet: A Personal Plan for Healthy Weight Loss. New York: Clarkson Potter, 2005.

Dansinger, Michael L.; Gleason, Joi A.; Griffith John L.; Selker, Harry P.; Schaefer, Ernst J. "Comparison of the Atkins, Ornish, Weight Watchers, and Zone Diets for Weight Loss and Heart Disease Risk Reduction." Journal of the American Medical Association. 2005; 293: 43-53.

Eckel, Robert H. "The Dietary Approach to Obesity: Is It the Diet or the Disorder?" Journal of the American Medical Association. 2005; 293: 96-97.

Fad Diets: What You Need to Know (April 2005). Familydoctor.org. American Academy of Family Physicians. <http://familydoctor.org/784.xml>.

Foster, Gary D. et al. "A Randomized Trial of a Low-Carbohydrate Diet for Obesity." New England Journal of Medicine. 2003; 348: 2082-2090.

Hutchinson, Fred. "Yo-Yo Dieting May Weaken Immune System." 2004. About, Inc., New York Times. 5 April 2006. <http://sportsmedicine.about.com/od/sportsnutrition/a/060304.htm>.

Muls, Erik, et al. "Is Weight Cycling Detrimental to Health? A Review of the Literature in Humans." International Journal of Obesity and Related Metabolic Disorders. 1995; 19 (Suppl. 3): S46-50.

National Institutes of Health. "Weight Cycling." NIH Publication No. 01-3901. 2004.

Samaha, Frederick F. et al. "A Low-Carbohydrate as Compared with a Low-Fat Diet in Severe Obesity." New England Journal of Medicine 2003; 348: 2074-2081.

Thirsk, Jayne. "Sorting through Recent Developments in Dietary Advice for Weight Loss." Research Update, December 2003. Alberta Centre for Active Living. 5 April 2006. <http://www.centre4activeliving.ca/publications/research_update/2003/December.htm>.

Ulrich, Cornelia M. et al. "Frequent Intentional Weight Loss Is Associated with Lower Natural Killer Cell Cytotoxicity in Postmenopausal Women: Possible Long-Term Immune Effects." Journal of American Dietetic Association. 2004; 104: 903-912.