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THEORETICAL ESSAYS ON OPTIMAL SOURCING
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SANTOSH KUMAR MAHAPATRA

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Ram Narasimhan

Major Professor's Signature

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THEORETICAL ESSAYS ON OPTIMAL SOURCING STRATEGY UNDER PRICE UNCERTAINTY

By

SANTOSH KUMAR MAHAPATRA

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ABSTRACT

THEORETICAL ESSAYS ON OPTIMAL SOURCING STRATEGY UNDER PRICE UNCERTAINTY

By

SANTOSH KUMAR MAHAPATRA

This study examines the issue of sourcing strategy in the context of price uncertainty. The extant literature has not addressed this issue adequately while accounting for a risk averse buyer's concern for magnitude and uncertainty of cost. This dissertation investigates the issue analytically when the buying firm adopts "contractual" sourcing arrangement with deterministic prices and "open market" sourcing arrangement with stochastic prices to procure a product. The investigation utilizes a disutility minimization perspective to analyze the problem. The study utilizes the optimal control theory and proposes three alternative models.

The first two models solve the problem in continuous time to find the optimal fraction of expenses and the optimal fraction of units to procure from the alternative arrangements when the buying firm does not incur switching cost for shifting the units between the two alternative arrangements. The third essay solves the problem in discrete time when the buying firm incurs switching cost for shifting the units across the two alternative arrangements.

The study provides theoretical and managerial insights that have implications for designing the sourcing mechanisms across different market states that can be characterized by the varied price dynamics of the procured product.

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Dedicated to my beloved daughter *Meghna*, my joy, inspiration and support.

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It gives me immense pleasure to express my humble appreciation to my advisor, Professor Ram Narasimhan. You were a constant source of inspiration, support and guidance in completing this dissertation. I also express my earnest gratitude for your care in preparing me for the future challenges. My sincere thanks to my co-advisor Professor Shlomo Levental who helped me in learning the stochastic calculus and checked my mathematical calculations meticulously. I am deeply indebted to my other committee members, Professor Shawnee K. Vickery, Professor Srinivas Talluri and Professor Kenneth K. Boyer for providing insightful feedback and constructive comments to improve the quality of my dissertation. My thanks to Mr. Santosh Kar who supported me by developing the computer code for the discrete time numerical analyses.

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Finally, I express my deepest gratitude to my wife and daughter for their constant support and patient endurance during my graduate studies.

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Chapter 1

Introduction

Purchased goods and services are one of the largest elements of cost for many firms. Accordingly, purchasing has emerged as a strategic activity. Bowersox et al. (2002, pp. 135) noted:

> 'In the average manufacturing firm in North America, purchased goods and services account for approximately 55 cents of every sales dollar.... it is clear that the potential savings from strategic management of purchasing and sourcing can be considerable.'

A strategic activity involves establishment of operational objectives and means to achieve competitiveness. Consequently, competitive priorities should guide a firm's procurement strategy (Narasimhan and Carter, 1998) and the procurement strategy should be changed according to the changes in competitive priorities with the evolution of product-market dynamics.

The manufacturing strategy literature suggests five competitive priorities: cost, quality, delivery, flexibility, and innovation that a firm may focus on to achieve competitive advantage (Wheelwright and Hayes, 1985). Among these, cost is the common denominator against which benefits of other non-price value elements (quality, delivery performance, flexibility etc.) are evaluated. Consequently, cost is considered as the most important parameter in sourcing decisions and supplier firms manipulate the price element to win against competition and attempt to improve the non-price elements over a long-term since these are difficult to adjust in the short-term.

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Cohen and Agrawal (1999) noted:

> '...managers have been slow to invest in long-term and cooperative relationships, even though they claim to seek such relationships. Furthermore, despite the extensive amounts of data collected from suppliers on various technical and financial factors, most supplier selection procedures are fairly subjective, with purchase price being the most significant factor.'

The price of a product is influenced by its stage in the Product Life Cycle (PLC), which influences the customer preferences, technological changes, and the competitive dynamics at the market place (Reed, 2002, pp. 91; Thorelli and Burnnet, 1981). These factors lead to varied patterns of price dynamics that make formulation of long-term procurement strategy difficult. Consequently, procurement arrangements that are developed with a long-term perspective are likely to be sub-optimal in many circumstances (Cohen and Agrawal, 1999).

The impact of input price dynamics on a firm can be evaluated in terms of the magnitude and uncertainty of cost of procurement and their relative significance vis-à-vis other priorities. Firms adopt various procurement arrangements to address the issues of procurement costs, changing priorities and uncertainty in procurement due to the market dynamics of the procured product. These arrangements may be broadly categorized into two types: long-term contracts or partnership, and short-term contracts or open market arrangements (Cohen and Agrawal, 1999). It is observed that the 'open market' arrangements typically offer a lower average price than 'contract' arrangements but expose the firm to a higher degree of price variability (Smith et al., 1999).

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Despite the lower average prices in the open market, the long-term contractual arrangements have traditionally been preferred to the open market arrangements. This is because, the open market arrangements involve multiple arrangements over time leading to high transaction cost as a result of higher search cost, monitoring cost and coordination cost than that of long-term contractual arrangements. Moreover, the long-term contractual arrangements offer better control mechanisms to address the supply side uncertainties relating to input price, quality, quantity etc. However, it may be noted that the long-term arrangements may lead to over-dependence on external supplier(s) that may result in loss of control and exploitation by the supplier(s) in the long-term. Furthermore, the long-term contracts prevent the buying firm to gainfully readjust sourcing arrangements to realize the benefits of competition, which may result in lower input-prices and superior input attributes. Therefore, buying firms should have the flexibility to avoid the disadvantages of over dependence on a few suppliers. Towards this end, firms may adopt a mix of supply arrangements, which can lower purchasing costs and provide options to revise decisions as conditions warrant (Narasimhan and Das, 1999).

In recent years, the emergence of electronic procurement mechanisms has enabled buying firms to have access to a wider market and to benefit from market efficiency (i.e., reduced cost of transaction and increased transparency). Economic theory suggests that efficient markets maximize consumer surplus. Thus it may be in the interest of the buying firm to procure from open market, which is likely to be more efficient due to competitive influences. There is

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empirical evidence that provides support to the above theoretical premise. Several firms in automotive, chemical, electronics, retail, energy, semiconductor, and beverage sector have benefited from reduction in procurement cost, ordering cost, and ordering time by successfully adopting electronic procurement mechanisms that allow for sourcing efficiently both from contractual and open market arrangements (Wu and Kleindorfer, 2005; Neef, 2001, pp. 84). In typical electronic procurement settings a buying firm engages with a restricted supplier base (e.g., one to five suppliers) for the long-term contract purchases, while using the open market as a secondary source of supply and as a means to evaluate the supplies received through long-term contractual arrangements. This provides an opportunity to suitably integrate the long-term and short-term/open market arrangements to safeguard against various supply side uncertainties while achieving procurement efficiency.

The foregoing discussion on sourcing context and the need for integrated sourcing arrangement may be summarized in the conceptual framework presented in Figure 1.1. According to this framework, the PLC of the procured product influences a buying firm's purchasing priorities and uncertainty in the business environment, which affect the suitability of 'contract' and 'open market' sourcing arrangements for the firm. The recent emergence of electronic procurement mechanisms presents opportunities for gainful integration of the 'relatively certain' long-term contract arrangement with the 'relatively uncertain' open market arrangements. In this context it is imperative for a buying firm to identify the appropriate combination of sourcing arrangements (e.g., contractual and open market

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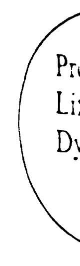

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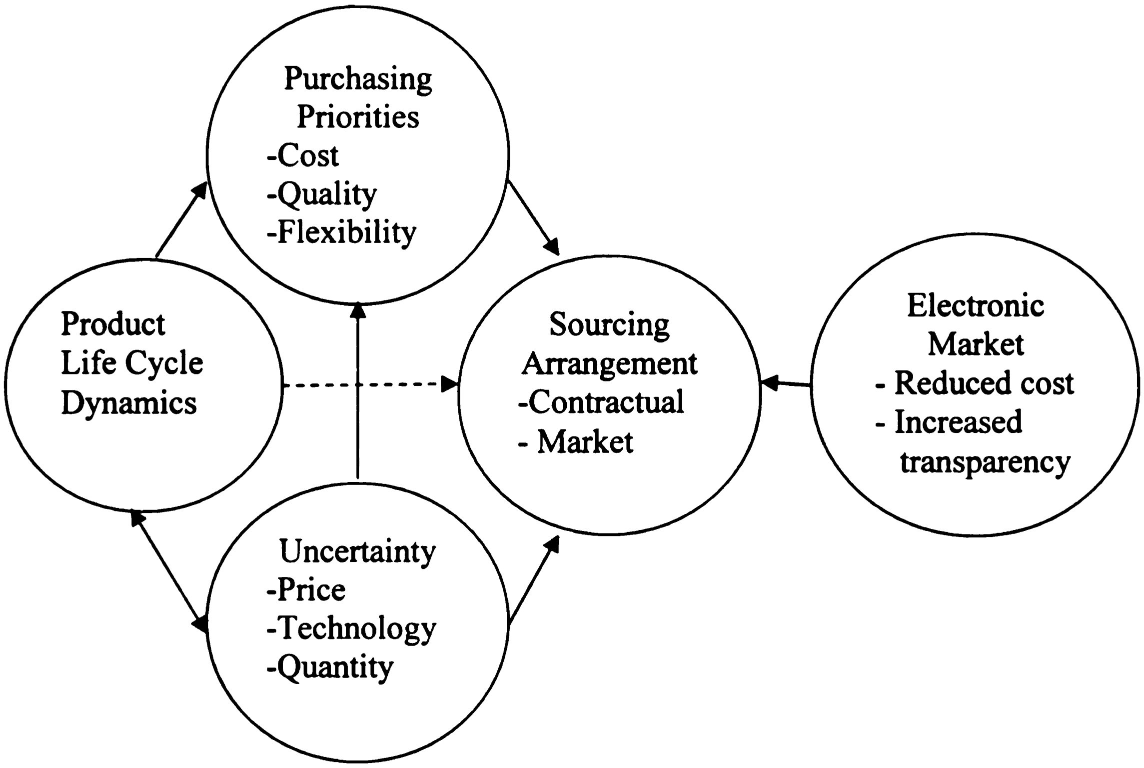
market sourcing) to balance the cost and uncertainties inherent in both the arrangements.

Figure 1.1: A conceptual framework to describe the influence of sourcing context on sourcing arrangement alternatives.



In the extant literature, while there are theories (i.e., transaction cost theory, resource dependency theory etc.) that explain various sourcing arrangements, these theories do not suggest suitability of alternative sourcing arrangements in the context of varying importance of cost vis-à-vis other priorities such as quality, delivery speed etc. across different stages of the PLC of the procured product. This emphasizes the importance of examining two important issues in sourcing that are the focus of the present research: a) deciding the op

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the optimal pattern of procurement across the two most commonly adopted sourcing arrangements i.e., 'long-term contractual purchase' and 'open/spot market purchase' in different stages of the procured product's life cycle that may be characterized in terms of the product's price dynamics and the relative cost priority of the buying firm over time, and b) identifying optimal time to switch among sourcing arrangements. I examine these two issues in this dissertation. Since, the issues pertain to optimal strategy, the research would use analytical methodology.

The research is presented in six chapters. The following chapter discusses the key literature on sourcing strategy under uncertainty that have implications on the research issues addressed in this dissertation. The subsequent three chapters present three analytical models and illustrate their applications in finding optimal sourcing strategies in varied procurement contexts. The last chapter highlights the key theoretical and managerial contributions of the research and suggests directions for future research.

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Chapter 2

Literature Review

In this chapter the key literature on sourcing strategy under uncertainty is reviewed with a special emphasis on the literature on strategy under price uncertainty. There are several studies that deal with the issue of sourcing strategy in general. However, there is hardly any study that has examined the issue of sourcing strategy under uncertainty in the context of dynamically evolving priorities of a risk averse buying firm.

The relevant literature may be studied along five broad themes: a) sourcing strategies under uncertainty, b) price dynamics of the procured product and sourcing strategy, c) optimal sourcing strategy without price uncertainty considerations, d) optimal sourcing strategy with price uncertainty considerations, and e) impact of the emergence of electronic market mechanisms on sourcing strategy.

2.1 Strategic sourcing arrangements under uncertainty

The literature on sourcing strategy under uncertainty may be examined in terms of the appropriateness of various sourcing arrangements under varied types of uncertainty that are experienced by the buying firm. The life cycle of a product describes the composite effects of its product-market dynamics that influence the nature of uncertainty, and intensity of competition. For example, in the early stage.

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stage, uncertainty due to product characteristics is higher than that in the later stages. Accordingly, the product life cycle (PLC) offers a useful framework to examine the sourcing strategy of a firm. Purchasing being a boundary spanning activity, the usefulness of such a framework is particularly high when the life cycles of components and end products are strongly interlinked. The product life cycles of electronic components, memory chips, hard drives, ZIP drives and of their end products e.g., computers, electronic gadgets etc. are examples of such strong linkages (Tibben-Lembke, 2002).

Based on empirical research, Rink and Fox (1999) developed a PLC oriented procurement framework which suggests distinct patterns of sourcing strategy for inputs that are closely linked to sales of end product across the five stages (i.e., pre-commercialization, introduction, growth, maturity, and decline) of the end product's PLC. According to the framework, in the pre-commercialization stage, the component product standards are not yet developed; consequently costs should be subordinated to the objective of prompt service on trial (small) orders by vendors such that evaluation of new vendors and supplier development efforts can be initiated. During the introduction stage, buying firms should employ (short-term) contracts until a supplier's capability with respect to the component product's varied attributes is demonstrated. This would encourage vendors to develop appropriate capabilities and set the stage for long-term supply arrangements by facilitating an orderly changeover from short-term contracts to long-term supply arrangements. In the growth stage, buying firms should focus on optimizing fixed costs by sourcing from in-house facilities or through long-term

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supply arrangements with one or a few key and large volume suppliers. In the maturity stage, elimination of single sources of supply should be considered in order to stimulate competition among vendors for achieving the lowest possible cost. Their study implicitly suggests that across these stages, a buying firm's relative cost priority with respect to the components (importance of cost vis-à-vis other non-price attributes) increases from introduction to maturity phase as the end product becomes increasingly standard and that a firm's sourcing strategy is governed by efficiency maximization and uncertainty minimization objectives.

The findings of Rink and Fox (1999) converge with the normative suggestions about the sourcing strategies made by the theoretical frameworks in the literature. This convergence may be attributed to the common perspective of efficiency maximization and uncertainty minimization in the theoretical frameworks. Transaction cost theory, agency theory and resource dependency theory are the three principal theories that explain the sourcing strategies of firms. These theories have their origin in the field of institutional economics and classify the sourcing strategies into two broad types: proactive strategy (long-term arrangements) and reactive strategy (short-term/open market arrangements) (Noordewier et al. 1990; Mabert and Venkatraman, 1998; Smeltzer and Sifred, 1998; Krause, 1999; McCutcheon and Stuart, 2000).

According to transaction cost theory, all transaction arrangements are associated with costs and uncertainties due to the transaction specific investments, the searching cost for the best supplier, the costs of establishing, monitoring and enforcing the contracts, and the risk of opportunism (Williamson, 1985). The theory

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theory assumes the transacting parties to be *risk neutral, limitedly rational* and *opportunistic* and the various transaction related costs to be *subjective* at the time of setting up the arrangement (Chiles and McMackin, 1996). Agency theory differs from transaction cost theory in terms of the assumptions held about the transacting parties. It assumes the transacting parties to be *risk averse* and to operate in a context of *information asymmetry* (Eisenhardt, 1989). Resource dependency theory states that organizations should *acquire and control resources* to *avoid risky dependency* on external organizations to minimize the chance of exploitation (Ulrich and Barney, 1984). Application of these theories in a sourcing context suggests efficiency and efficacy of transactions are realized by matching the transaction governance structure (i.e., hierarchical, market oriented or contractual governance structure with characteristics of both hierarchy and market governance) with the contextual aspects of transaction (McCutcheon and Stuart, 2000; Krause et al., 2000).

Proactive sourcing involves long-term arrangements that emphasize potential for future benefits by sharing risks and current knowledge (e.g., collaborative technological innovations) with a few key suppliers. In contrast, reactive sourcing involves short-term or open market arrangements that aim at reduction of administrative (acquisition) and material expenditures by allowing for opportunistic switch among suppliers in the rapidly changing business contexts (e.g., discontinuous technological changes that may make technology oriented alliances obsolete and ineffective). In a typical business context, a buying firm may choose to adopt any one of the above two types of sourcing arrange

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arrangements or some combination of these two types arrangements depending on the fit among the sourcing arrangement, context and procurement priorities.

Proactive sourcing necessitates considerable investment of time, effort and money to build the requisite trust and commitment for developing close partnership type of arrangements. Thus these arrangements are associated with higher risks of resource dependency, forward integration by supplier, and failure of exchange mechanisms than that of reactive (open market sourcing) arrangements (McCutcheon and Stuart, 2000). The risk of failure is particularly high when the elements of uncertainty are out side the range of *pre-specified* tasks of exchange. In such circumstances, the adjustment mechanisms of partnership type of governance may not mitigate the risks, and the goals of exchange become difficult to achieve (Noordewier et al., 1990). Consequently, true supplier partnerships are somewhat uncommon and firms often engage in long-term contractual relationships as a substitute to partnership type of arrangements.

It may thus be argued that proactive sourcing arrangements involve higher development cost and entail a higher risk of failure if not designed and managed properly. On the other hand, reactive (open market) sourcing is likely to be beneficial when proactive sourcing is likely to be less cost-effective (i.e., in later stages of the PLC with standard products and intense price-based competition) or when managing the proactive sourcing arrangement is difficult (i.e., in early stages of the PLC with unstable product characteristics and technology standards). However, open market arrangements may be fraught with issues such as high search and transaction costs and inconsistent quality and delivery standards due to the mu

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the multiplicity of supply arrangements. This calls for appropriate configuration of sourcing arrangements while accounting for the varied types of uncertainties and costs.

Sutcliffe and Zaheer (1998) categorized sourcing related uncertainties into three broad types: primary uncertainty (e.g., uncertainty due to changes in technology, customer preferences and other exogenous factors), competitive uncertainty (e.g., price uncertainty and other product-attribute related competitive actions), and supplier-related uncertainty (e.g., volume of supply, inconsistent operations of suppliers etc.). They argued that the higher the primary and competitive uncertainties, the lesser should be the emphasis on vertical integration that may result in inappropriate investments, and the greater the supplier-related uncertainty, the higher should be the propensity for vertical integration that may encourage asset specific investments.

It may be conjectured that primary uncertainty is likely to be higher in the early stages of the PLC, supplier uncertainty is likely to be higher in the intermediate phases i.e., growth phases of PLC, and competitive uncertainty is likely to be higher during the later stages of the PLC due to intense price-based competition around a standard product. Moreover, long-term contracts that are analogous to vertical integration would be involve high development costs (e.g., long-term supply arrangement cost and penalty costs for contract modification) and would be difficult to design under high degrees of primary and competitive uncertainties. Consequently, long-term contracts would be more useful during the intermediate stages than in the earlier and later stages of the PLC.

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Treleven and Schweikhart (1988) identified single and multiple sourcing as two basic sourcing strategies that are useful in addressing various types of supply related uncertainties and costs. Single sourcing is somewhat similar to proactive sourcing and addresses supply uncertainty by establishing partnership/long-term contract arrangements with one or two suppliers; this facilitates a stable and high quality supply of key inputs at a fair price. It involves supplier development efforts, paring of the supplier base, and works well with a contingency plan to source from an alternative source. In contrast to single sourcing, multiple sourcing may make use of spot market (reactive sourcing) and/or multiple supply contracts simultaneously to achieve competitive price, quality and delivery performance. Their paper did not discuss the relative efficacy of the two strategies with reference to any specific pattern of uncertainty. Nevertheless, it may be argued that single sourcing would be useful when the uncertainty and cost concerns are limited and could be addressed by the supplier in question, and multiple sourcing would be useful when the uncertainty and cost concerns are wider and may not be within the capabilities of the supplier in question (e.g., discontinuous technological changes that may nullify the advantages of the existing supplier competent in an existing technology).

The foregoing discussion on sourcing arrangements suggests uncertainty and cost considerations influence the efficacy of various sourcing arrangements. Long-term/partnership (proactive) type sourcing (e.g., a single or very few supply arrangements) is likely to be more effective during growth and early maturity (intermediate) stages of the PLC that are characterized by moderate levels of risk across

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across primary, competitive and supplier based uncertainties. Short-term/transactional (reactive) type sourcing (e.g. spot market, multiple supply arrangements etc.) is likely to be more effective in the introduction and maturity stages of the PLC that are characterized by high levels of risk due to primary and competitive uncertainties respectively.

However, such conceptual generalization is inadequate to suggest appropriate sourcing arrangements (e.g., how much to buy from proactive/long-term and/or reactive/short-term open market arrangements over time) to address the cost and uncertainty concerns of the buying firm across different phases of the life cycle of a product. This calls for studies to suggest optimal sourcing strategies across the varied procurement contexts of the different phases of PLC. Since, the product-market structure evolves dynamically, it is natural that the sourcing arrangements are appropriately restructured over time according to the evolving purchasing priorities of the buying firm.

2.2 Price dynamics and sourcing strategy

The price of a product is the key factor against which other product attributes are evaluated. Consequently, the sourcing strategy of firms should take into account the characteristic price pattern of the procured product over time. Price dynamics of a product are often explained in terms of the economic theory of price, suppliers' pricing strategies, and PLC dynamics (Curry and Riesz, 1988). Among these, the PLC represents the composite effects of several factors that impact the product prices over time. Elasticity of demand, rate of demand diffusion, rate of demand saturation, rate of innovation, experience curve

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dynamics, inflation, supplier's planning horizon, technological changes, and competition are the common factors that influence the price pattern over a PLC (Hofer, 1975; Dolan and Jeuland, 1981; Clarke and Dolan, 1984; Rao and Bass, 1985; Krishnan et al., 1999, Smith et al., 1999). For example, the impact of experience curve effect (i.e., reduction of price due to lower cost over time) is evident from the empirical study by Lambkin and Day (1989). This study found that specialist pioneers with small scale operations are likely to be the best performers in the early stage of market evolution; however, large-scale generalist followers with established interest in related markets are likely to be best performers in the developing and maturing stages of market.

Literature suggests that contractual prices are higher than competitive market prices (Klein and Leffler, 1981; Horowitz, 1986; Rao and Bergen, 1992; Rao and Monroe, 1996). Buying firms are likely to pay higher prices in contracts for two principal reasons: one, they lack complete information about the true cost of production of the supplier, and two, they give incentives to motivate suppliers to reduce the supply uncertainty and meet the contractual terms with respect to quality. Smith et al. (1999) examined the issue of price dynamics and the buying firm's willingness to pay price premiums in contractual arrangements empirically. They observed that in high-tech, short PLC product markets, prices decline over time as producers lower the prices of newer versions to reduce the switching costs of the buyer. In contrast, in low-tech long PLC product markets, prices may increase, remain constant or decline depending on the intensity of competition. However, the standard deviation and rate of change of prices in both the product-

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market contexts generally decline over time. They also indicated that buying firms are likely to pay a price premium in the presence of disadvantages due to supplier monopoly, information asymmetry with respect to quality and cost of production, and transaction specific investments. Nevertheless, the buying firm's propensity to pay a contractual premium is likely to decline with the decline of these uncertainties.

It is thus apparent that prices offered by suppliers are dynamic, and standard deviation of prices and contract premiums paid by the buyer decline as the procured product becomes increasingly standard over time (e.g., competition and customer's awareness level about quality and cost of production increase). The buying firm needs to consider these aspects while designing sourcing arrangements and negotiating contracts with suppliers (Smith et al., 1999).

2.3 Sourcing strategy without price uncertainty considerations

The typical sourcing arrangements are contractual arrangements with specific terms and conditions that define the supply arrangement. These terms and conditions specify the pattern of prices over time, duration of contract, and degree of flexibility with respect to conditions of supply. Thus contractual arrangements eliminate or reduce supply related uncertainties significantly in exchange for a more or less deterministic price pattern that the buying firm commits to abide by. The existing analytical literature on supply contracts mainly focuses on deriving the optimal price and payment structure (e.g., linear pricing, two part-non linear pricing, options, discounts, risk sharing through quantity flexibility, purchase commitments, returns etc.) as key coordination mechanisms between the buying firm ar

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firm and supplier under stochastic demand (Pasternack, 1985; Eppen and Iyer, 1997; Anupindi and Bassok, 1999; Lariviere, 1999; Corbett and Tang, 1999, Tsay et al.1999). For example, Schuster et al. (2002) analyzed the benefits of options in improving channel performance in the face of demand uncertainty in a two period buyer-supplier system. The approaches for optimal contracts in this stream of literature do not explicitly consider the issue of market price uncertainty, and examine contractual arrangements with a single/limited number of suppliers that preclude transactions in the open market. However, in a contractual purchasing context, buyers consider the market price pattern as a benchmark for effective negotiation or for deciding limits of contract price premiums (Smith et al. 1999; Kleindorfer and Wu, 2003). Consequently, analytical models on the design of contractual sourcing arrangements should appropriately examine the relative cost and benefits of contractual arrangements vis-à-vis open market arrangements.

2.4 Sourcing strategy with price uncertainty considerations

The studies on sourcing arrangements that account for price uncertainty are predominantly influenced by the 'real options' literature (Dixit and Pindyck, 1994; McDonald and Siegel, 1986; Brennan and Schwartz, 1985; Kogut and Kulatilaka, 1994). According to 'real options' perspective, a firm's ability to adapt sourcing strategies in response to altered economic or operating contexts can increase its value by improving its upside potential while limiting its downside losses. However, any change in strategy may involve additional transaction costs, which is known as switching costs in the real options literature. Thus real option valuation procedures are useful in deciding the timing, source,

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and quantity of sourcing (Dixit, 1989; Huchzermeier and Cohen, 1996; Cohen and Huchzermeier, 1999; Murthy et al., 2002; Kouvelis, 1999).

Murthy et al. (2002) used the real option approach to analyze the dynamic procurement and switching behavior of a buying firm that experiences different relationship specific fixed costs with suppliers located in a foreign country. The investigation assumed that the buying firm's output price/revenue to be deterministic and the firm can procure only from one of the suppliers in a particular time-period. Kouvelis (1999) also used the real options approach to propose a general modeling framework to examine multi-supplier sourcing strategy with switching costs. The study derived the threshold price levels for switching the preference from one supplier to another in a two-supplier sourcing context, in a two-supplier sourcing context, and indicated that the buyer should source simultaneously from both the suppliers only if the least cost supplier's capacity is inadequate. Li and Kouvelis (1999) developed models for minimizing the procurement cost when the buyer uses risk-sharing (flexible) supply contracts in a single or two suppliers sourcing context. They used a numerical (binomial lattice) approach to identify optimal sourcing strategies for flexible contracts while accounting for inventory costs and capacity constraints.

Two of the key assumptions in applying the real option approach to the procurement context are the existence of security portfolios, which replicate the rates of change of sourcing costs, and the existence of risk-less assets with a specified risk free rate of return. Often, these assumptions are difficult to implement, thereby limiting the applicability of these models. Furthermore, most of the

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of the existing analytical studies on sourcing in the context of price uncertainty derive the optimal strategy for a risk-neutral buying firm. In contrast, as discussed earlier, the theories on sourcing strategy justify the contractual arrangements on the basis of the buying firm's concern for uncertainty. Accordingly, optimal strategies need to be derived from the perspective of a risk averse buying firm.

Cohen and Agrawal (1999) examined the issue of optimal sourcing from the perspective of a risk averse buying firm perhaps for the first time. They adopted a utility maximization perspective to compare the advantages of short-term contracts that have price uncertainty, but provide the flexibility to switch to alternative suppliers, with that of long-term contracts that involve fixed cost of developing the arrangement, but provide certainty of prices. Their results defied the conventional notion that long-term contracts are superior to short-term contracts and highlighted the fact that the advantage of a specific type of arrangement is governed by transaction specific investments, length of planning horizon, uncertainty of market prices, opportunity for cost reduction over time, and decision makers' risk preference. They employed the 'mean-variance' technique for modeling the buying firm's utility function; this technique associates only with quadratic utility function. In reality, a buying firm's utility function may be more varied than the 'quadratic utility function', which does not model the theoretical relationships between risk aversion and magnitude of expense appropriately (Eeckhoudt and Gollier, 1995, pp. 48).

The models in the extant studies solve for the case of adopting a single sourcing arrangement at a time. In reality, a buying firm may have existing contrac

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contract arrangements that may be difficult to cancel at a given time; consequently it may be beneficial to suitably mix a number of sourcing arrangements simultaneously (Billington, 2002). This is particularly of significance due to the recent emergence of electronic procurement mechanisms that provide for quick configuration of supply arrangements. Accordingly, models need to be developed for optimally integrating multiple arrangements from the perspective of a risk averse buying firm.

2.5 The impact of the emergence of electronic market on sourcing strategy

The emergence of electronic market has affected the sourcing strategy of firms significantly (Martinez-de-Albeniz and Simchi-Levi, 2005). To make a proper assessment of its implications for sourcing strategy, its characteristics must be assessed with reference to the sourcing strategy frameworks suggested in literature. The various sourcing arrangements adopted by firms may be viewed to lie on a continuum of long-term relational type to short-term transactional type arrangements (Williamson, 1991; Hobbs, 1996; Melnyk and Swink, 2003). Characteristically, the relational types are more suitable for non-standard/ strategic products with controllable but uncertain supply contexts, and the transactional types are more suitable for standard/non-strategic products or for non-standard/strategic products with less uncertain supply contexts (Kraljic 1983; Martinez-de-Albeniz and Simchi-Levi, 2005). The standard products are usually the products that have easy to specify attributes (Kleindorfer and Wu, 2003).

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Relational arrangements ensure certainty of price and supply, and are likely to result in low transaction cost in situations of high asset specificity, high transaction frequency and uncertain supply contexts. However, true relational arrangements are difficult to develop; consequently, long-term contracts are used as a substitute to relational arrangements to achieve certainty of supply (Dyer, Cho and Chu, 1998). From a buying firm's perspective, the certainty through contractual arrangements may involve two main costs: a) sacrificing the flexibility to switch to an alternative source to benefit from favorable developments in competitive market (McCutcheon and Stuart, 2000; Sanchez, 2003; and Martinez-de-Albeniz and Simchi-Levi, 2005), b) payment of a price premium (Klein and Leffler 1981; Horowitz, 1986; Smith et al. 1999). Thus long-term contractual arrangements may not always be optimal from a buying firm's perspective (Peleg et al., 2002; Cohen and Agrawal 1999). Rather, (open) market arrangements with higher flexibility to switch to alternative sources are likely to be particularly useful in situations of high relationship development cost, or when there are risks of higher price premium and lock-ins in long-term contract commitments (Sanchez, 2003). Since the upper and lower bounds of risks and relationship development cost are context specific and difficult to quantify in clear terms, complete dependence on short-term or open market arrangements to benefit from the competitive market may expose a firm to an inordinate amount of price and supply side uncertainty. In this context, integration of long-term contracts and short-term contracts or open market arrangements can be effective in reducing the magnitude and uncertainty of cost of procurement without affecting the nonmonetar

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monetary performance (Kleindorfer and Wu, 2003; Sanchez 2003; Martinez-de-Albeniz and Simchi-Levi, 2005).

The emergence of variety electronic procurement mechanisms (e.g., private e-exchange, industry consortia, public e-market places, and ubiquitous third party tools such as e-tendering, e-ordering, e-catalogues, e-reverse auctions etc.) has facilitated such integration in several industries by increasing transparency and transaction efficiency through reduced search cost and coordination cost, and lower switching cost for new supply arrangements (Boer et al., 2001; Billington, 2002, CAPS Research and McKinsey & Company, 2002, Jap and Mohr, 2002; Kaplan and Sawhney, 2000; Johnson and Whang, 2002; CAPS Research, 2003; Rabinovich et al., 2003). These mechanisms enable to develop *product specific* sourcing arrangements for a wide range of product-market contexts (Skjott-Larsen et al., 2003; Lancioni et al., 2003). For example, industry consortia type electronic (B2B) exchanges such as Covisint have successfully evolved to provide infrastructure to facilitate transaction of complex product and service bundles (Wise and Morrison, 2000).

An analytical explanation of the benefits of integrated sourcing arrangement may be found in the study of Peleg et al. (2002), who analyzed the relative efficiency of the spot market purchase, the long-term contracts and the combined strategy of spot market purchase and long-term contracts in a two period stochastic demand context. They observed that while no particular strategy has a clear dominance over the others, in situations of negligible supplier search cost, a combined strategy is superior to long-term contracts.

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To summarize the literature: open market purchase or short-term contracts, and long-term contracts are two principal alternatives for sourcing; these may be combined to achieve optimality against the uncertainties over time. The composite effects of market dynamics of a product are reflected in its price dynamics. Most of the extant analytical investigations have derived optimal solutions with a cost minimization perspective while considering the buying firm to be risk neutral. In reality, as evident from the empirical and sourcing strategy literature, firms are concerned about uncertainty and seek to reduce the risk. Furthermore, buyers evaluate monetary cost of procurement against other value elements or strategic considerations; consequently, their relative concern for procurement cost vis-à-vis other (non-price) value elements changes over time across the different phases of the PLC. For example, a buying firm may want to transact with multiple suppliers, or to buy in the spot market to avoid over-dependence on a particular supplier. Buying firms in above situations may view the importance of procurement cost differently and may have different cost related utility functions; thus the optimization perspective needs to accommodate such considerations (Cohen and Agrawal, 1999).

In this context, it is imperative to investigate the issue of optimal sourcing strategy taking into account the buying firm's risk aversion and price uncertainty across different phases of the PLC when the firm uses (open) market or short-term contracts and long-term contracts for procurement. This dissertation addresses this gap in the existing literature. It examines the following issues: a) what is the optimal pattern of allocating supplies across the two most commonly adopted sourcin

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sourcing arrangements i.e., ‘contractual purchase’ and ‘open/spot market purchase’ while accounting for typical price patterns, transaction cost, and cost disutility function of the buying firm in various stages of PLC, and b) what are the implications of adopting optimal pattern of sourcing on developing the sourcing arrangements.

The results of the investigation can provide insights on optimal sourcing arrangements across different stages of the PLC that may be characterized by varied patterns of input prices, buying firm’s cost disutility functions and transaction cost or switching cost. These insights can be used for designing sourcing strategies and generating hypotheses about sourcing arrangements that may subsequently be verified through empirical investigations.

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Chapter 3

Sourcing Strategy For Minimizing Expense Disutility

3.1 The problem context

The research examines the optimal sourcing strategy for a risk averse buying firm that uses long-term contractual arrangement and short-term market arrangements in the following procurement context. The firm buys a *standardized* product continuously for satisfying the specified levels of *demand* across the different stages of the product's life cycle. The attributes of the product can be specified objectively in each procurement cycle, and there would be several potential suppliers in the market who can supply according to the specifications. The proposed models are based on the premise that the price of the product is the common denominator against which other non-price value elements can be evaluated over time (Melnyk and Swink, 2003). This premise enables to link the price with the degree of product *standardization* which influences the various elements of transaction specific costs and uncertainties e.g., cost of transaction, investment in transaction specific assets, supply-demand instability, and premium for guaranteed supply (Klein and Leffler, 1981; Horowitz, 1986; Smith et al., 1999). This argument is somewhat similar to the conceptual framework offered by Kleindorfer and Wu (2003), which suggests a direct relationship between cost of codifiability (i.e., possibility to objectively specify the product/service attributes) and relative cost of contract establishment and production.

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In the extant literature, there are studies that examine the issue of integration of short-term and long-term arrangements, which provide opportunities to trade-off certainty and higher cost against flexibility and lower cost of transaction (Cohen and Agrawal, 1999; Kleindorfer and Wu, 2003; Martinez de-Albeniz & Simchi-Levi, 2005; Wu and Kleindorfer, 2005). However, these studies either consider an option theoretic framework in a discrete time context in which only one type of arrangement is active at a time (Cohen and Agrawal, 1999) or, consider the firm to be risk neutral in the context of stationary open market price distribution during a specific operating horizon (Peleg et al., 2002; Kleindorfer and Wu, 2003; Martinez de-Albeniz & Simchi-Levi, 2005). Both of these considerations do not reflect the realities in the actual problem context.

The recent emergence of B2B exchanges has motivated firms to adopt a combined sourcing strategy, involving a dynamic mix of short-term contracts or, open market purchase (with stochastic prices) and long-term contracts to buy the product (CAPS Research and Mc Kinsey Consulting 2002). Such sourcing arrangements help firms to manage risk and reduce cost in several industries such as aluminium, auto-components, electricity, electronic components, chemical and semiconductors (Kleindorfer and Wu 2003; Martinez de-Albeniz and Simchi-Levi, 2005; Wu and Kleindorfer, 2005). For example, Hewlett Packard optimizes the procurement cost of electricity or, memory products using such an integrated approach (Billington, 2002). Past studies have recommend continuous time modeling of such problems for better analytical insights (Cohen and Agrawal

1999; Kleindorfer and Wu 2003). The proposed model in this research addresses these considerations.

The proposed modeling framework is distinct from existing studies in that it explicitly considers the buying firm's concern for uncertainty in terms of a utility maximization perspective (Keeney and Raiffa, 1976, pp. 148). This approach has been extensively used in the portfolio optimization problems in finance (Korn and Korn, 2001, pp. 203). The efficacy of this approach lies in estimating the subjective utility function appropriately. There are several alternative approaches such as maximizing the value of 'mean-variance' or the 'max-min' function that can also address the issue of 'concern for uncertainty'. However, the utility maximization criterion scores over others due to the following reasons. While, the 'max-min' criterion does not consider all possible outcomes and thereby deforms the actual probabilities (Eeckhoudt and Gollier, 1995, chapter 4, pp. 39), the 'mean-variance' criterion is more justified when the outcomes are normally distributed (Meyer, 1987). Consequently, optimization based on maximization of suitable utility functions would be a more appropriate approach to address the concerns of the magnitude and uncertainty of outcomes.

Utility functions being subjective are often difficult to assess across multiple objectives; utility literature suggests substitution of easy-to-use simple utility functions for such contexts (Hammond, 1974). Using the above rationale, single attribute cost disutility functions that are polynomial in expense can be derived to effectively describe the buying firm's over all preference structure for a product with specifiable attributes (Keeney and Raiffa, 1976, Chapter 5 and 6).

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These disutility functions can be used to identify optimal sourcing strategies in procurement contexts with specified price dynamics.

For the problem context in this study, it is assumed that a process of vendor certification has already taken place, and thus the firm has available to it a pool of contracts from which it must choose its sources of supply. The set of certified vendors may be different in different stages of PLC. The supply management decision made by the firm has two components i.e., strategic and tactical. The strategic decision involves selection of contractual suppliers and types of contract (e.g., fixed, flexible, or quantity buy-back etc.) that are reviewed over a longer period of time may be once a year. Tactical decisions are made after the contractual supplier is selected for actual ordering of the quantity of supply over shorter time frames (e.g., may be weekly or even daily) to match firm's operational requirements as per the existing demand and price conditions in the market. Thus it allows for contract price renegotiation and/or procuring variable quantity of supplies from alternative arrangements towards satisfying the exact demand requirement over the tactical review period. It is further assumed that the firm uses a single source of supply for each type of arrangement at a time. Consequently, a long-term contractual supplier remains fixed for the entire strategic review period, and open market supplier, contract prices and amount of procurements from each of the sources may be adjusted across the tactical review periods. These assumptions are consistent with the standard industry practices, used in contractual sourcing arrangements (Cohen and Agrawal, 1999; Martinez de-Albeniz and Simchi-Levi, 2005). Accordingly, for the above problem context

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the optimal mix of the two extreme sourcing arrangements (e.g., open market arrangements and long-term contracts or partnerships) is being determined.

It is assumed that at the beginning of any review period, contract price patterns are determined after accounting for the 'contract premium' that is justified on the basis of supplier's likely demand for such a premium for ensuring stable supply according to contract terms, and buyer's propensity to pay a premium due to the lack of awareness about supplier's true cost (Klein and Leffler, 1981; Horowitz, 1986; Smith et al. 1999). One of the ways to achieve this could be, for the same contract and (expected) open market price at the beginning of planning horizon, the contract price would increase (decrease) at a rate higher (lower) than the expected growth (decline) rate of the open market price. From a practical standpoint, this is reasonable as the expected market price growth (decline) rate is likely to be lower than long-term contract due to the moderating effect of competition. This approach to contract price determination is comparable to the approach suggested by Cohen and Agrawal (1999). It is assumed that the market is efficient with a competitive price, and the buying firm does not incur any inventory holding cost i.e., the demand for the procured product may be viewed as flow units per time or the product may be considered as non-storable (i.e., electricity, services etc.). It is also assumed that there is no suppliers' capacity and buyer's monetary constraint that limit sourcing over time. Such assumptions are typical of stochastic optimal control problems in extant literature (e.g., Aytekin and Birge, 2004; Kouvelis, 1999). The buying firm can have flexibility to procure from the contract arrangement a quantity that may vary to some e

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some extent around a specified average volume of procurement at the negotiated price. While, the contract arrangement provides certainty of supply, the open market sourcing arrangement enables the buying firm to benefit from lower competitive market price.

3.2 The model

Mathematically, the above problem can be described as follows. Let P_1 and P_2 be the open market and contract prices with α_1 and α_2 the respective mean growth rates, and β_1 be the standard deviation parameter of the open market price process that follows a geometric Brownian motion (GBM) in a specified time horizon. GBM is a stochastic process, commonly used in modeling uncertain price and exchange rate dynamics (Dixit and Pindyck, 1994), and is considered appropriate for analytical examination of integrated supply arrangement problems such as the present one (Cohen and Agrawal, 1999; Kouvelis, 1999; Li and Kouvelis, 1999 etc.).

The firm searches for admissible and optimal sourcing arrangements across the various procurement contexts in different stages of the PLC. The procurement contexts may be characterized by varied price patterns and by the firm's subjective concern for uncertainty and magnitude of cost. The firm inexpensively adjusts the percentage of expenses incurred on supplies from each sourcing arrangement continuously over time in order to maximize (minimize) the value (cost disutility) of the procurement strategy.

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The geometric Brownian motion with α_1 and β_1 as drift and standard deviation parameter and W_t characterizing the Brownian process at time 't' is represented as:

$$\frac{dP_1}{P_1} = \alpha_1 \, dt + \beta_1 dW_t; \qquad \text{and } P_1(t = 0) = P_{10} \tag{3.1}$$

In the above price process, for a known initial price 'P_{10}' and price parameters α_1 and β_1, prices remain uncertain during the operating horizon and the buying firm pays the uncertain prices over time.

The deterministic long-term contract price process with α_2 as the expected price growth rate is represented as:

$$\frac{dP_2}{P_2} = \alpha_2 dt \text{ ; and } P_2(t = 0) = P_{20} \tag{3.2}$$

The above price processes are continuous in time. This implies if at any instant one of the prices is greater than the other, then the prices need to be equal before the originally greater price becomes smaller than the other price. During this period of unequal prices, it is obviously optimal to buy all requirements from the less expensive alternative until the prices become equal. It is to be noted that the total procurement expense corresponding to any possible optimal arrangement (including the singular arrangements that involve only one of the arrangements) involving two continuous price processes would be continuous (i.e., the minimum of two continuous functions is always continuous). While the optimal procurement strategy is obvious when prices are likely to be unequal over the tactical review period, in a competitive stochastic market price context it would be diff

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be difficult to decide which type of arrangement would remain optimal right after the instant when the prices are equal. Besides, an instantaneous complete switch ('*bang-bang*' solution) to the cheaper alternative may be impractical due to the significant amount of transaction or penalty costs

A feasible strategy at this instant may therefore be to wait until the prices become unequal and to completely switch ('*bang-bang*' solution) in favor of the lower prices once the uncertainty of prices are resolved. However, waiting longer than the instant when the prices become unequal would not be optimal since the buying firm is losing an opportunity to spend less due to such waiting. In case of such sub-optimal waiting, the amount of money that could have been saved by a completely optimal solution would be governed by the duration of time for which the purchases are made at an unfavorable price. Moreover, for such waiting and subsequent switching, the procurement expense pattern will be discontinuous. Thus the strategy to wait to switch completely in favor of lower price is not optimal, and would result in discontinuous expense pattern. The extent of deviation from optimality is governed by the threshold values when switching decisions are made. These decisions are tricky to make in a stochastic price context. Besides, as discussed earlier an instantaneous complete switch to the cheaper alternative may be impractical due to the significant amount of transaction or, penalty costs. To summarize, implementing the intuitively appealing '*bang-bang*' policy is difficult to implement and sub-optimal.

Thus in situations of competitive stochastic market price context, it is difficult to identify and instantaneously switch to the expense-minimizing altern

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alternative over the planning horizon. The above phenomenon of 'non-determinism' due to irregular behavior of market price at an instant is recognized as the 'local time' problem in stochastic literature (Oksendal, 1996, pp. 138). The issue is structurally similar to the infeasibility of 'Stop-Loss-Start-Gain' trading strategy in financial literature (Carr and Jarrow, 1990). A mathematical description of the issue is presented in the Appendix A.

Since the price realizations that make the '*bang-bang*' policy optimal is difficult to identify *a-priori* at each instant of transaction in such continuous time stochastic problems, it is justified to derive the optimal sourcing strategy for continuous expense over time. Accordingly in the following model, the expense pattern is considered to be continuous over the planning horizon $(0,T)$. As discussed earlier, for the same starting open market and contract prices, the condition $\alpha_2 > \alpha_1$ is required to be satisfied for the contract price premium requirement. Though this is not a modeling requirement, this needs to be satisfied for drawing meaningful insights about the sourcing arrangements.

Let, $X_t > 0$ be the total procurement expenses of the firm across the two sourcing arrangements at time $t \geq 0$; let '*u*' be the fraction of expense to be spent on purchase at open market price (P_1) and '*1-u*' be the fraction of expense to be spent on purchase at long-term contract price (P_2) at time *t* to minimize expense disutility. It is assumed that at time '*t*' the buyer can select and inexpensively adjust the fractions allocated to market sourcing and long-term contract sourcing based on expenses at time X_t. This renders the procurement expense X_t to follow the Ito's diffusion process with Markovian control '*u*'. Accordingly, as the

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buying firm allocates expenses across alternative arrangements optimally over time, the expenses and units procured from alterative arrangements are random. This random behavior is governed by the underlying price dynamics and disutility function of the buying firm that influence the optimal fractions of expenses to allocate across alternative arrangements.

The differential equation for expense X_t can be written as follows. If 'u' is the fraction allocated to market, then:

$$X_t = uX_t + (1-u)X_t \tag{3.3}$$

Furthermore, if N_{1t} and N_{2t} are units procured from market and contracts then, change in expense at an instant is: $dX_t = N_{1t}dP_{1t} + N_{1t}dP_{2t}$

$$= uX_t \frac{dP_{1t}}{P_{1t}} + (1-u)X_t \frac{dP_{2t}}{P_{2t}} \tag{3.4}$$

Using equations (3.1) and (3.2), Equation (3.4) can be written as:

$$dX_t = uX_t\alpha_1 dt + uX_t\beta_1 dW_t + X_t(1-u)\alpha_2 dt \tag{3.5}$$

It may be noted that at each instant of time in the planning horizon under consideration, the change in expense over '*dt*' is a function of change in prices. It is assumed that resources required to accommodate the change in expense dX_t over '*dt*' are immediately and continuously available as the firm adjusts the fraction allocated to alternative sourcing arrangements to minimize the cost disutility. After each '*dt*' the firm rebalances the fraction spent on alternative sources in order to minimize its cost disutility across the sourcing arrangements.

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Thus with continuous expense, the equilibrium condition at the time of rebalance is the following: *expense before reallocation of fractions between arrangements is same as expense after reallocation of fractions at each instant.* This implies marginal rate of substitution (MRS) at the time of rebalance is equal to ratio of prices from alternative sourcing arrangements. This would result in changing the number of units sourced from a particular source. Thus the exchange process should be such that the overall number of units after exchange is at least equal to the overall number of units procured at the initial state (if overall number of units procured is of significance). The modeling of explicit consideration of the '*units procured to be equal to demand*' constraint is hard and complex.

Consequently the problem is formulated first without such constraint (referred as the base model in subsequent discussion) and subsequently the conditions for verification of sufficiency of units are derived. The conditions for the sufficiency of units to be satisfied at any instant in the operating period will depend on the cost disutility minimization objective that influence the fraction of expense spent on alternative sourcing arrangements, the initial prices, and the price dynamics of alternative sourcing arrangements. The mathematical expressions for the above conditions are presented in Appendices B and C. The mathematical formulation of the base model (i.e., without the explicit consideration of the units sufficiency condition) to derive the optimal expense pattern is as follows. The buying firm wants to maximize (minimize) the value (disutility) of procurement expenses over a time horizon [*t*, *T*]. Let $F(t, x_t^u)$ be the co

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the continuous disutility rate function at time t, and $S(x_T)$ be the terminal cost disutility function at time T.

Then the overall (expected) cost disutility functional can be written as:

$$J^u(t,x) = E^{t,x}\left[\int_t^T F(\tau, x_t^u)d\tau + S(x_T)\right] \tag{3.6}$$

where, $E(.)$ is the expectation operator and x_t^u is the expense at 't' when fraction of expense allocated to market is 'u'.

The buying firm wants to maximize (minimize) the value (cost disutility) of procurement expenses over the entire time horizon under consideration. Let, $C(t,x)$ be the minimum of cost disutility functional at time 't' due to procurement expenses. It is assumed that the Markovian control parameter i.e., fraction purchased, $u \in U$ would lie between 0 and 1. The Hamilton-Jacobi-Bellman (HJB) equation is used to solve the above stochastic control problem.

The HJB equation for the relevant (expected) cost disutility functional given in equation (3.6) is:

$$\inf_u \left\{L^u C + F(t, x^u)\right\} = 0 \text{ for } t < T \text{ and } x > 0; C(T,x) = S(x) \tag{3.7}$$

In the above equation the differential operator L^u uses the Ito's Lemma and has the following form:

$$\left(L^u C\right)(t,x) = \frac{\partial C}{\partial t} + x(\alpha_1 u + \alpha_2(1-u))\frac{\partial C}{\partial x} + \frac{1}{2}\beta_1^2 u^2 x^2 \frac{\partial^2 C}{\partial x^2} \tag{3.8}$$

For obtaining the optimal solution, Equation (3.7) needs to be solved for Markovian control u.

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Let, $F(t,x^u)=x^n$, $n>1$ in (3.7), where 'n' represents the cost disutility parameter i.e., the higher the n, the higher the cost disutility of the buyer. Equation (3.7) is solved to obtain $u(t,x)$:

$$u(t,x) = -\frac{(\alpha_1 - \alpha_2)C_x}{\beta_1^2 x C_{xx}} \tag{3.9}$$

It is to be noted that the disutility functional $C(t,x)$ should be convex and increasing in x which makes both C_x and C_{xx} (first and second order derivatives with respect to x) positive, thus rendering $u(t,x)$ nonnegative and valid.

Substitution of the expression for optimizing $u(t,x)$ from (3.9) in (3.7) yields:

$$x^n + C_t + \alpha_2 x C_x - \frac{(\alpha_1 - \alpha_2)^2 C_x^2}{2\beta_1^2 C_{xx}} = 0\text{; for } t < T \text{ and } x > 0; C(T,x) = S(x) \tag{3.10}$$

Equation (3.10) is non-linear in $C(t,x)$ and is difficult to solve for general $S(.)$. Consequently, it is solved for a special type of increasing convex function given by:

$$S(x) = x^n, n > 1 \tag{3.11}$$

For this, let the trial solution be:

$$C(t,x) = f(t)x^n; \tag{3.12}$$

While, the above power functional form is a mathematical convenience and is structurally similar to the utility function used in wealth maximization problems in financial literature (Korn and Korn, 2001), it has also the desired properties from a practical standpoint. The convex disutility function (x^n) is monot

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monotonically increasing in 'x', represents the buying firms' risk aversion characteristics very well and ensures that higher the procurement expense the greater is the disutility and risk aversion (Eeckhoudt and Gollier, 1995). Furthermore, it facilitates incorporation of alternative values for n over distinct time phases to allow for varied disutility patterns of the buyer over the PLC of the procured product. Equation (3.10) is solved for the special case of the increasing convex function given in (3.11), and $C(t,x)$ given in (3.12) and the solution for $C(t,x)$ is obtained as:

$$C(t,x) = [e^{\theta(T-t)} + e^{-\theta t}\int_t^T e^{\theta\tau}\, d\tau](x^n) \tag{3.13}$$

where, $\theta = \alpha_2 n - \dfrac{(\alpha_1 - \alpha_2)^2 n}{2\beta_1^2 (n-1)}$

Substituting $C(t,x)$ in (3.13) into (3.10) the optimal control expression is obtained as:

$$u^*(t,x) = \frac{\alpha_2 - \alpha_1}{\beta_1^2 (n-1)} \text{ where, } n > 1 \text{ and valid range for } u^* \text{ is } 0 \le u^* \le 1 \tag{3.14}$$

If for specified parameter values, u^* lies outside the limits of 0 or 1, then it would imply, the total expenses (X_t) would be completely spent on contract purchase or market purchase respectively.

It may be observed that the optimal control function for expense disutility minimization in the above model is constant and the fraction allocated to market varies directly with difference in mean growth rates of prices, and varies inversely with the variance of random market price and disutility parameter. The control function influences the characteristics of expenses and units procured from alternative arrangements. Consequently, for specified patterns of price dynamics and buying firm's cost disutility parameter n, the optimal control function, pattern of expenses, and the units procured from alternative arrangements can be obtained.

The above base model does not consider the '*units procured to be equal to demand*' constraint to maintain mathematical tractability. However, for specified parameters the conditions for sufficiency units over the planning horizon is derived in Appendices B and C without and with the '*unit sufficiency*' constraint respectively. As per the results in Appendix B, for specified parameters, there is a minimum operating horizon of length, 't^*', beyond which the units procured would always be sufficient if the expenses are allocated as per optimal fractions. It is observed that 't^*' equals zero (i.e., units procured as per optimal fractions will always be sufficient) when the starting prices of both the sourcing arrangements are equal. However, for unequal but comparable range of starting prices at any instant before 't^*' there is a probability that total units procured as per optimal fractions with the expense X_t may be less than units procured at time '0'. It is observed for reasonable parameter values 't^*' and likely shortage are very small to be of practical significance. If the possibility of having such shortfall before 't^*' is unacceptable, then the buying firm may make provision for additional expense to meet the shortfall, which may be allocated among alternative arrangements as per the optimal fractions (e.g., u^*, $1-u^*$). Alternatively, as demonstrated in the Appendix C, the constraint for sufficiency of units can be incorp

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incorporated into the model. However, as the results indicate, incorporation of sufficiency of units constraint leads 'u^*' to depend on stochastic expense (X_t) and market price (P_{1t}); this would require continuous monitoring and control of the expense pattern across alternative arrangements.

It is to be noted that even if the model minimizes the expense disutility, it may sometimes lead the buyer to accrue surplus units beyond the specified demand. This is counterintuitive, and may be explained as follows. As per the model, units sourced from alternative arrangements depend upon both the price processes and the (optimal) expense fractions while the expense remains continuous in time. The continuity of overall expense implies, the '*expense incurred before reallocation between arrangements is same as the expense after the reallocation*' at each instant of adjustment. As a result, savings that may occur due to rebalance at an instant is not withdrawn and is reutilized in purchasing leading to surplus purchase.

For reasonable range of parameter values, and length of (tactical) review period, the surplus units are observed to be of small order. However, to avoid such a situation in practice, the buyer may start a fresh operating cycle at suitable intervals (i.e., tactical review periods) with a revised initial expense and (surplus) inventory-adjusted demand to procure the required units while maintaining the optimal fractions (u^*, $1-u^*$) at the instant. This is feasible as per optimal results presented in Appendix C that suggest the expressions for optimal fractions are independent of planning horizon effect. Alternatively the surplus units may be sold back in the open markets (Martinez de-Albeniz & Simchi-Levi, 2005). The emerg

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emergence of electronic markets provides opportunities for such sell back opportunities in many industrial situations (Mc Kinsey & Company, and CAPS Research, 2000; Martinez de-Albeniz & Simchi-Levi, 2005).

It also needs to be recognized that procurement as per the optimal fractions would make expenses and units procured from alternative arrangements to behave randomly and such behavior is a function of the price dynamics, and the degree of risk aversion that influence the disutility function of the buying firm. The stochastic behavior of units to be procured from alternative arrangements may thus necessitate the contracts to be of flexible types (Lariviere, 1999; Martinez de-Albeniz & Simchi-Levi, 2005). In a given circumstance for specified parameter values, the expected magnitude of procured units and their variance across alternative arrangements can be computed as per the formulae (28), (29), (31), and (32) in Appendix B. These results can be used for designing appropriate flexible contractual arrangements.

3.3 Illustration of the model application

The model can be used for finding the optimal sourcing strategy in varied procurement contexts that can be described in terms of specific price parameters of market and contract price processes and the buying firm's risk attitude. Thus for appropriately chosen parameter values, the model may be employed for finding the optimal strategy both in a specific stage or over multiple stages of the PLC of the procured product. The application of the model is illustrated in three stages. In the first stage, the impact of changes in price patterns and cost disutility parameters of the buying firm on patterns of change in optimal fractions allocated to mar

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to market (u^*) is illustrated separately. In the second stage, the implications of characteristic price patterns and cost disutility factors across different stages of the PLC on (optimal) sourcing pattern are demonstrated. Finally, for a specific degree of cost disutility, the implications of price processes, and the duration tactical review period on units sourced from alternative arrangements are evaluated across early, intermediate and late stages of the procured product's life cycle.

The hypothetical data in Table 3.1 and Table 3.2 are used for the above analyses. While, the data are arbitrary, these are stylized to represent the typical contract and market price dynamics (Klein and Leffler 1981; Horowitz, 1986; Smith et al. 1999) and relative cost preference of firms (Reed, 2002, pp. 91; Thorelli and Burnnet, 1981).

Table 3.1: Illustrative data representing the price dynamics across the PLC stages

State of the market across the PLC stages	Expected market price annual growth rates 'α_1'	Contract price annual growth rates 'α_2'	Difference in contract and market price growth rates 'α'	Standard deviation parameter of market price process 'β_1'	Expected market price: initial price: $100	Contract price: initial price: $100	Standard deviation of market price
1	-0.25	-0.2	0.05	0.75	77.88	81.87	52.70
2	-0.232	-0.185	0.047	0.67	61.75	64.73	36.86
3	-0.214	-0.17	0.044	0.6	49.86	52.10	26.50
4	-0.196	-0.155	0.041	0.54	40.98	42.70	19.60
5	-0.178	-0.14	0.038	0.49	34.30	35.63	14.96
6	-0.16	-0.125	0.035	0.45	29.23	30.27	11.80
7	-0.142	-0.11	0.032	0.42	25.36	26.18	9.66
8	-0.124	-0.095	0.029	0.39	22.40	23.06	8.02
9	-0.106	-0.08	0.026	0.35	20.15	20.68	6.54
10	-0.088	-0.065	0.023	0.3	18.45	18.88	5.19
11	-0.07	-0.05	0.02	0.25	17.20	17.55	4.07
12	-0.052	-0.035	0.017	0.2	16.33	16.61	3.13
13	-0.034	-0.02	0.014	0.15	15.79	16.01	2.30

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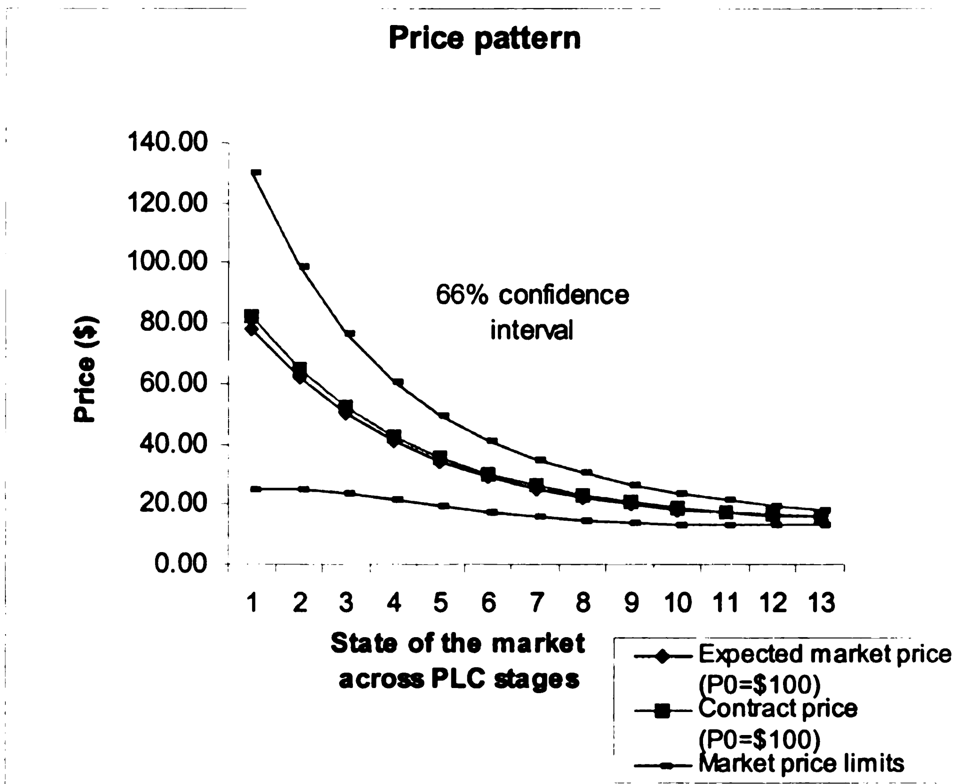
Thus in the hypothetical data in Table 3.1, a positive contract and market price change rate differential is maintained to account for 'contract premiums', and the 'contract premiums' and standard deviation parameter of market prices are set to decline. The prices are constructed in line with approaches suggested in Cohen and Agrawal (1999) and are graphically represented in Figure 3.1.

Figure 3.1: Characteristic (hypothetical) price pattern across different stages of the PLC



To illustrate, using the formulae (24) and (26) for prices in Appendix B, the expected market and contract prices are first constructed with an unit initial price of \$100, specified growth rates of -0.25 (α_1) and -0.2 (α_2), and standard deviat

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deviation parameter of 0.75 (β_1) for a planning horizon of one year. At the end of one year, the contract price is equated to the expected market price (e.g., indicating the result of renegotiation); these negotiated prices are subsequently used to compute the prices for the second year using the parameters (α_1, β_1 and α_2) for the second year. The process was repeated for each of the subsequent years. The total duration of the PLC until the maturity stage in the hypothetical dataset is considered to be of 13 years. Table 3.2 presents the cost disutility factors (n) for two scenarios (cases) of buying firm's preference over the 13 years of PLC, and the corresponding optimal solutions.

Table 3.2: The impact of price pattern and cost disutility factors on optimal fraction (u^*) of expense spent on market sourcing

State of the market across the PLC stages	Difference in contract and market price growth rates 'α'	Standard deviation parameter of market price process 'β_1'	Cost disutility factor 'n' (case 1: varying disutility factor)	Cost disutility factor 'n' (case 2: constant disutility factor)	Fraction spent on market sourcing (u^*) (varying disutility factor)	Fraction spent on market sourcing (u^*) (constant disutility factor $n = 2$)
1	0.05	0.75	1.10	2.00	0.89	0.09
2	0.047	0.67	1.20	2.00	0.52	0.10
3	0.044	0.6	1.30	2.00	0.41	0.12
4	0.041	0.54	1.40	2.00	0.35	0.14
5	0.038	0.49	1.50	2.00	0.32	0.16
6	0.035	0.45	1.60	2.00	0.29	0.17
7	0.032	0.42	1.70	2.00	0.26	0.18
8	0.029	0.39	1.75	2.00	0.25	0.19
9	0.026	0.35	1.80	2.00	0.27	0.21
10	0.023	0.3	1.85	2.00	0.30	0.26
11	0.02	0.25	1.90	2.00	0.36	0.32
12	0.017	0.2	1.95	2.00	0.45	0.43
13	0.014	0.15	2.00	2.00	0.62	0.62

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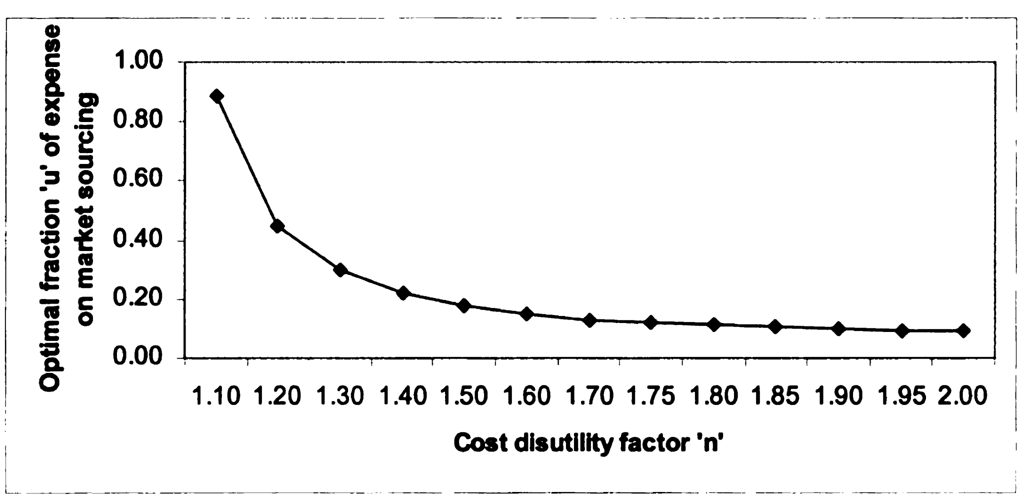
It may be observed that the cost disutility parameters are greater than 1 to account for buying firm's risk aversion; and, cost disutility parameters in case 1 are set to increase to represent the buying firm's increasing concern for cost as the product advances in the PLC.

For first stage of analysis, the base case considers a cost disutility factor (n) of 2.00, spot (market) price standard deviation parameter (β_1) of 0.75, and expected price change rate difference $(\alpha_2 - \alpha_1)$ of 0.05. Subsequently, the patterns of optimal fractions to source from market are obtained by separately changing cost disutility factor, standard deviation parameter of market price, and expected price growth rates differences. These results are plotted in Figure 3.2, Figure 3.3 and Figure 3.4 respectively. The results are data dependent; however, similar computation can be performed using relevant contextual data to gain industry or product specific managerial insights.

Figure 3.2: Optimal fraction of expense 'u^*' on market sourcing for varying disutility factor 'n' and constant price parameters ($\alpha_2 - \alpha_1 = 0.05, \beta_1 = 0.75$)



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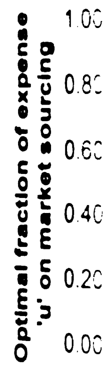

Figure 3.4: Op
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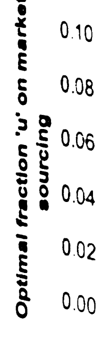


Figure 3.3: Optimal fraction of expense 'u^*'on market sourcing for varying market price standard deviation 'β_1', and constant price growth rate differential ($\alpha_2 - \alpha_1 = 0.05$) and cost disutility factor (n=2)

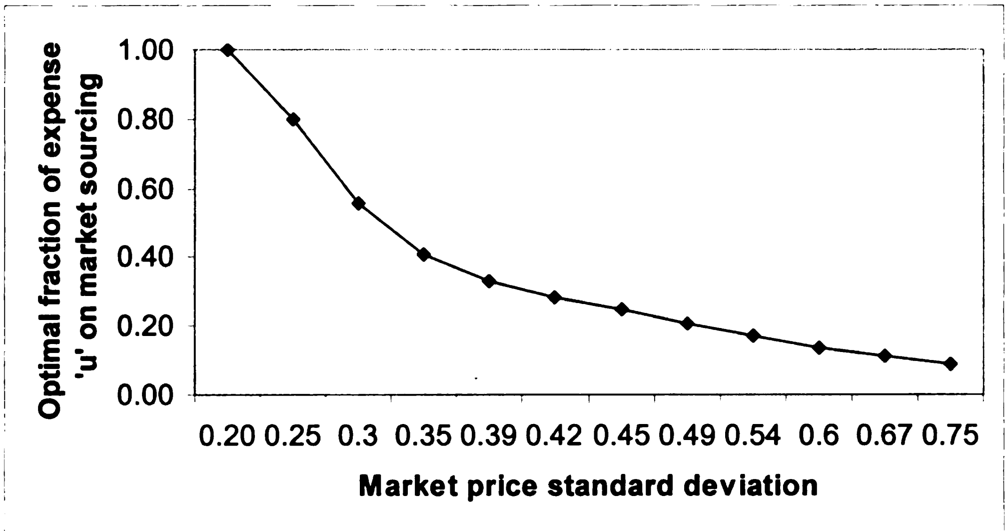
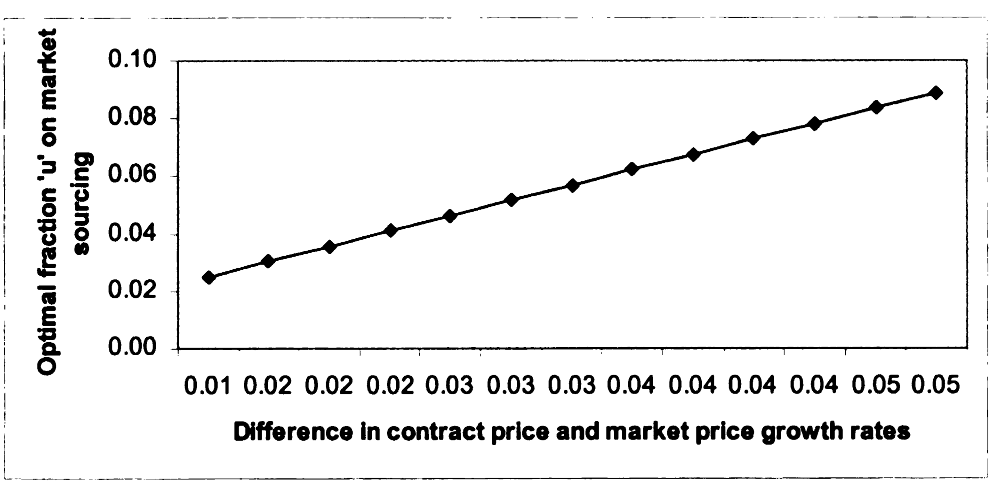


Figure 3.4: Optimal fraction 'u^*' of expense on market sourcing with varying price growth rate differential 'α', and constant standard deviation parameter ($\beta_1 = 0.75$) and cost disutility factor ($n = 2$)



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The plots for the illustrative dataset indicate the following general patterns:

a) the higher the cost disutility parameter, the lesser the fraction of expense spent on market for a given set of price parameters; this implies buyers, concerned about uncertainty would procure less from market,

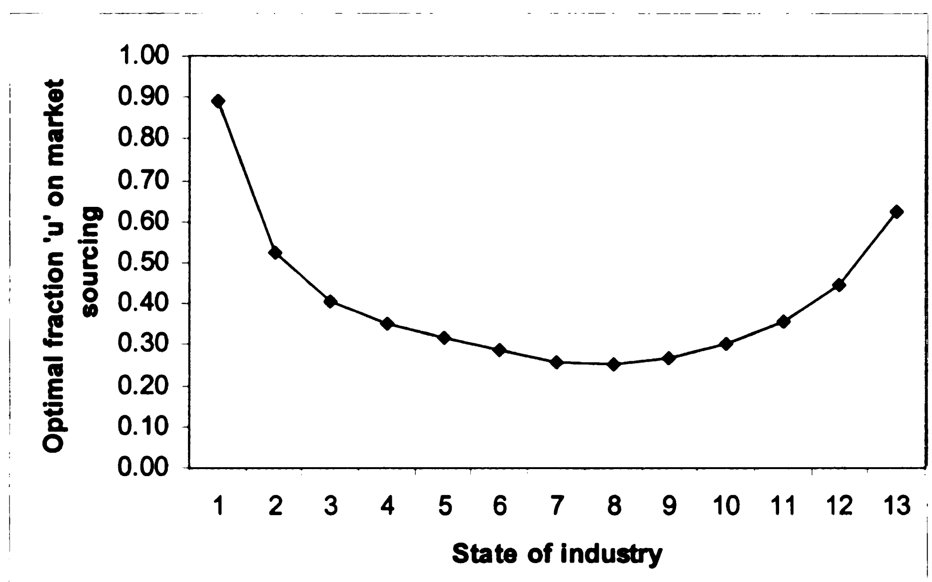
b) the greater the spot (market) price uncertainty (β_1), the lesser the fraction spent on market when the price growth rate differential and cost disutility parameter remain unchanged; and

c) the higher the expected price growth rate differential, the larger the fraction sourced from open market when the market price uncertainty (β_1) and cost disutility parameter remain unchanged.

These patterns suggest the need for appropriate sourcing strategies for different procurement contexts as characterized by cost disutility parameters and price regimes. For example, it may be noted from Figure 3.2 that buyers with lower cost disutility parameter have higher risk of adopting sub-optimal strategy; this is because, the optimal fraction curve has a higher slope in the region of lower disutility factor representing higher sensitivity to inaccurate specification of disutility parameter.

The implications of characteristic price patterns and buyer's cost disutility on optimal fractions to source from alternative arrangements across various stages of the PLC is examined with reference to the illustrative data set in Table 3.1 and Table 3.2. As noted earlier, the data are stylized to reflect the typical pattern of price of a product and the likely increase in cost disutility as a product shifts from early stages of PLC to the mature phases of PLC. The optimal fractions of expense to allocate to market are plotted in Figure 3.5 and Figure 3.6 for cases of increasing cost disutility and constant cost disutility respectively.

Figure 3.5: Optimal fraction of expense 'u^*' on market sourcing with characteristic price dynamics and cost disutility factors (i.e., n varies from 1.1 to 2.0) across different stages of the PLC



It is observed that for the case of increasing disutility factor, the optimal fractions for market expense have a spoon shape indicating phases of initial decline (i.e., from $u^* = 0.89$ at the early stage), a low level of (i.e., $u^* = 0.25$ at the intermediate stage) followed by an increase towards the later stages (i.e., $u^* = 0.62$ at the late stage). However, as evident from Figure 3.6, the optimal fractions for a firm having constant disutility factor across various stages of the PLC have a 'J' pa

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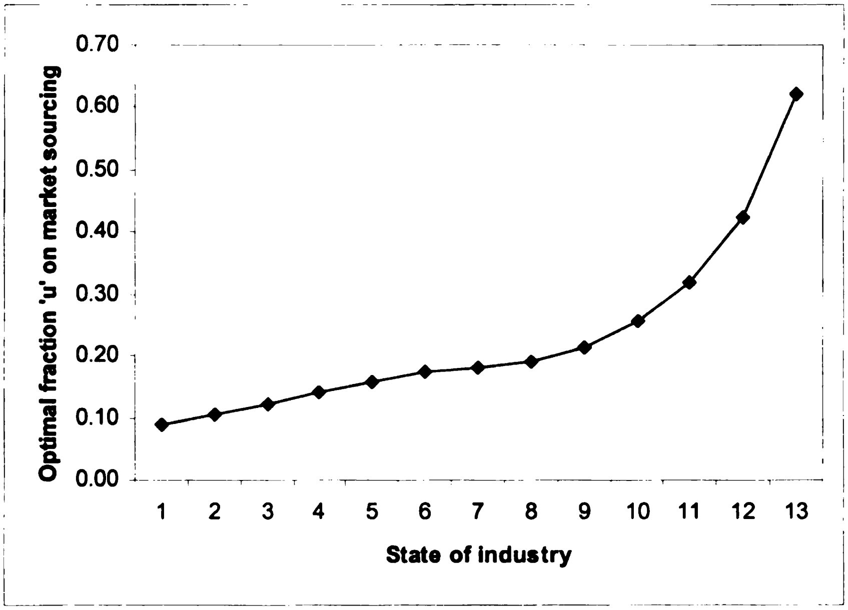
pattern indicating a gradual increase over time (i.e., from $u^{*} = 0.09$ at the early stage to 0.62 at the late stage for disutility factor $n = 2$).

Figure 3.6: Optimal fraction of expense 'u^{*}' on market sourcing for characteristic price dynamics across different stages of the PLC and for constant disutility factor ($n = 2$)



Comparison of Figure 3.5 and Figure 3.6 indicates for specified price dynamics, optimal fractions to spend on alternative arrangements are dependent on buying firm's degree of risk aversion that is represented by the cost disutility factor. For given price dynamics in a planning period, a firm with higher cost disutility would spend proportionately less on market arrangement.

The above patterns have a variety of implications. These indicate the advantage of sourcing from open market in the early stages of the PLC (introduction to early growth stages) that are characterized by high contract and market price growth rate differential, high standard deviation of market prices and

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lower cost disutility. The plots highlight the advantage of contract-price based sourcing during the intermediate (moderate price growth rate differential) stage when long-term suppliers are likely to charge a reasonable price premium over the moderately uncertain spot (market) price due to competition. It also suggests the increased importance of spot (market) price based sourcing towards the later stage of PLC when the product becomes standardized and functional as competition intensifies, and companies become more cost conscious.

The impact of contract and market price dynamics and duration of tactical review periods on units sourced may be evaluated by computing the mean and standard deviation of number of units sourced from alternative arrangements at different times using the formulae 28, 29, 31, 32 specified in Appendix B. Illustrative results of such computation for the price growth rates (α_1 and α_2) and standard deviation parameters (β_1) across early (state 1), intermediate (state 7) and late (state 13) stages of market state specified in Table 3.1 are given in Table 3.3 and are plotted in Figure 3.7. These results are obtained for demand of 100 units, and starting prices of $100.00 at the beginning of planning horizon across each of the stages for a risk averse firm with $n = 2$.

The results indicate that the average total units sourced from both the arrangements in each of the stages are sufficient to meet the demand, but increase with the duration of tactical review period (i.e., for the early stage, the average units sourced increased from 100.23 for $t = 0.05$ to 106.24 for $t = 1.0$; for the late stage, the average units sourced increased from 100.03 for $t = 0.05$ to 100.54 for $t = 1.0$). This suggests that the longer the review period the higher the expected surplus, and the magnitude of expected surplus is more in the earlier stages of the PLC as compared to the later stages of the PLC due to higher volatility of market prices in the earlier stages that presents opportunity to buy at lower prices.

Table 3.3: Impact of the price pattern over time on units sourced from alternative arrangements over time (Demand = 100 units, n=2, P_{10}= 100.00, P_{20}= \$100.00, X_0=\$10000.00)

Units sourced from alternative arrangements at various times										
State of Industry	time in year									
	t=0		t=0.05		t=0.25		t=0.5		t=1	
	Market	Cont--ract	Market	Cont--ract	Market	Cont--ract	Market	Cont--ract	Market	Cont--ract
Early Stage (1)										
Mean	8.89	91.11	9.14	91.09	10.22	91.01	11.75	90.91	15.53	90.71
Standard deviation	0.00	0.00	1.44	1.36	4.14	3.03	7.97	4.28	20.94	6.03
Intermediate Stage (7)										
Mean	18.14	81.86	18.30	81.84	18.93	81.74	19.76	81.62	21.51	81.39
Standard deviation	0.00	0.00	1.42	1.39	3.42	3.11	5.31	4.39	9.04	6.17
Late Stage (13)										
Mean	62.22	37.78	62.26	37.76	62.44	37.70	62.65	37.62	63.08	37.45
Standard deviation	0.00	0.00	0.79	0.79	1.78	1.76	2.53	2.47	3.63	3.47

Accordingly, even when the buying firm minimizes the expense disutility there is a probability of surplus due to higher sourcing from market arrangements. These surpluses may be traded back in the open market for additional indirect savings or, may be carried over to the subsequent period and the buying firm may procure units after adjusting for any such surplus from the previous period. The results show that the variability of units sourced from alternative arrangements declines towards later stages in the PLC and increases with the duration of tactical review period, and the variability is more pronounced for open market purchases.

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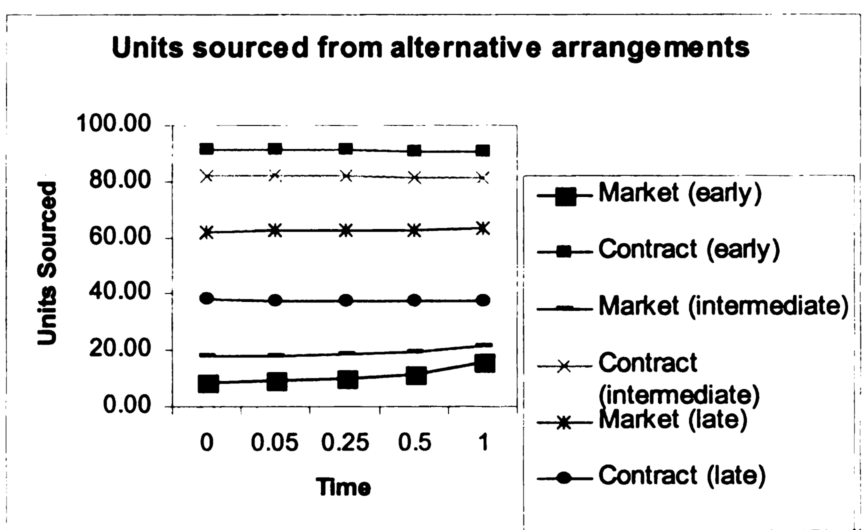
Furthermore, the average units sourced from contractual arrangement marginally declines with longer review periods (e.g., from 91.09 for $t = 0.0$ to 90.71 for $t = 1.0$ in the early stage); this is due to the higher sourcing from open market arrangement, which is expected to provide a price advantage. These results highlight the significance of quantity flexible contracts and periodic renegotiation of contracts at suitable intervals such that the expectations of buying firm and the supplier firm are appropriately matched over time. For example, the buying firm may renegotiate a long-term contract as a combination of short-term contracts for reducing a variance in purchases made from alternate arrangements. The formulae presented in Appendix B for expected magnitude and variance of units sourced from alternative arrangements would enable the buying firm to design such contracts for varied tactical review periods.

Figure 3.7: Impact of the price pattern on units sourced from alternative arrangements over time across different states of the industry for cost disutility factor $n = 2$



The increase in emphasis on market sourcing as evident from the optimal results, seem to conform with the PLC based framework of Rink and Fox (1999) that recommends a shift in emphasis of purchasing approaches for key components that have a close correspondence with the PLC of the end product: gradual paring of supply base (in favor of reduced number of suppliers) as the products progresses towards growth stages, emphasis on contractual arrangements in intermediate/growth stages for assured supply, and increased sourcing from spot market to benefit from competitive market in later stages of PLC. Accordingly, the changing pattern of optimal fractions for varied disutility patterns as suggested in Figure 3.5 may be explained in terms of contextual characteristics and buying firm's likely risk attitude in the three major phases of PLC i.e., introduction-early growth, late growth, and maturity phases.

In the early stages, technological and customer preference uncertainty would be high; there may not be adequate number of capable suppliers for contractual arrangements to meet buyer's (unsteady) requirements. While, this may lead to higher price differential between contract price and spot market price, the buying firm may be relatively less risk averse and more willing to try with multiple suppliers due to lesser degree of standardization of the product. Therefore, some purchase from open market at this stage would facilitate supplier assessment, and would foster competition among potential suppliers. This would drive down the price differential and standard deviation of market price. Moreover, it may encourage suppliers to develop appropriate capabilities to match buyer's expectations thus setting the stage for long-term contract arrangements towards the intermediate (growth) phases that are typically characterized by product availability uncertainty. The decline in fractions on contract purchase in favor of open market purchase as the product advances from intermediate phase to mature phase may be justified on the basis of higher cost sensitivity and price-based competition towards later phases that reduces price growth rate differential and spot (market) price uncertainty.

In parctice such reallocation is plausible only if the market price becomes truly competitive with highly capable suppliers, reduced supplier search cost and switching cost, and stagnation in innovations such that the product becomes more or less a standard product. Consequently, the pattern of sourcing will be a function of the duration of various stages of the PLC that may be governed by technological changes, market competition, and emergence of frictionless efficient markets.

The degree of smoothness of the optimal fraction curve would indicate the severity of the impact of transitions across different phases of the PLC that has implications on designing appropriate arrangements and organizational capabilities for optimal sourcing. Thus to increase allocation of share to market arrangement with minimal severity may necessitate the buying firm to proactively indicate the contract suppliers about the emphasis on open market purchase towards later phases, to stimulate timely competition among suppliers such that the market becomes efficient with reduced cost without any compromise on non-price attributes, and to train procurement managers with capabilities for tracking and forecasting of open (spot) market prices, designing of flexible contracts, and valuation of a

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valuation of a portfolio of sourcing arrangements. It is to be recognized that the increasing shift towards open market arrangement towards the later stages may however be avoided if the buying firm can induce the existing contractual supplier to match the prices in the competitive open market periodically. To illustrate, Anheuser Busch adopts such an approach in sourcing aluminium for producing cans in the beverages industry (Wu and Kleindorfer, 2005). In this scenario, the optimal solution in the model, which reflects the integrated valuation of contract and market offers, can be used as a benchmark to negotiate contractual arrangements.

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Chapter 4

Sourcing Strategy For Minimizing Expense Disutility While Procuring The Exact Number of Units

4.1 The problem context

The model described in the previous chapter identifies the optimal fraction of expenses to be allocated between contract and open market sourcing arrangements to minimize the expense disutility while ensuring that the units procured are at least equal to the requirement. In the model, it was found that for specified initial market and contract prices, the planning horizon should be at least equal to a specified duration to ensure that the units procured are sufficient to meet the requirement. Furthermore, it was observed that for planning horizons greater than this minimum, the total units procured might exceed the original requirement due to the stochastic nature of market price process. While such accrual presents an opportunity for additional indirect savings by way of trading back in the spot market, a buying firm may want to avoid such accrual of surplus if there is a net cost in handling such materials. Consequently, there is a need for modeling the optimal sourcing problem such that the total units procured do not result in any accrual of surplus at any time. The corresponding models in a context of continuous and cost-less readjustment process are developed in this chapter.

As discussed in the previous chapter, the absolutely optimal procurement strategy would have continuous expense and would be of '*bang-bang*' type (i.e., to buy all re

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to buy all requirements from the less expensive alternative until the prices become equal) if the arrangement with favorable prices can be known in advance at all times of transaction. However, in a stochastic market price context it is difficult to decide which type of arrangement would remain favorable when the prices are equal. Since the price realizations that make the '*bang-bang*' policy optimal is difficult to identify '*a-priori*' at each instant of transaction in such continuous time stochastic problems, it is justified to derive the optimal sourcing strategy for continuous expense over time. Accordingly, the current model also considers the expense pattern to be continuous. The model is developed for the same problem context as that in the previous chapter and is stated below for brevity.

A risk averse buying firm considers adopting a dual (combined) sourcing strategy involving a mix of long-term contract with 'deterministic prices' and open market arrangement with 'stochastic prices' to procure a *standardized* product over time to satisfy specified levels of *demand* over time while minimizing the expense disutility. As noted in the previous chapter the proposed modeling framework is distinct from existing analytical research in that it explicitly considers the buying firm's concern for uncertainty in terms of a utility maximization perspective (Keeney and Raiffa 1976, pp. 148; Eeckhoudt and Gollier, 1995, chapter 4, pp. 39). The firm has available to it a pool of suppliers from a set of certified vendors from which it must choose its sources of supply. The sourcing arrangements are implemented through a series of 'strategic' and 'tactical' decisions in lines similar to standard industry practices (Cohen and Agrawal, 1999; Martinez de-Albeniz & Simchi-Levi, 2005). It is assumed that at the beginning

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the beginning of any 'tactical' review period, contract price patterns are chosen while accounting for the issue of 'contract premium', which is justified on the basis of supplier's likely demand for such a premium for stability of supply as per contract terms while absorbing various risks, and the buyer's propensity to pay the premium due to the lack of awareness about supplier's true cost, and for being able to avoid market price uncertainty (Klein and Leffler, 1981; Horowitz, 1986; Smith et al. 1999). One of the ways to achieve this would be: for the same starting contract price and (expected) open market price, the contract price is negotiated to increase (decrease) at a rate higher (lower) than the expected growth (decline) rate of the open market price. From a practical standpoint, this is reasonable as the expected market price growth (decline) rate is likely to be lower than long-term contract due to the moderating effect of competition. Accordingly, in this problem context the optimal mix of two extreme sourcing arrangements (e.g., open market arrangements and long-term contracts or partnerships) in the continuum of buyer supplier arrangements is being determined for minimizing the expense disutility.

4.2 The model

Mathematically, the model can be described as follows. The market price process P_1 is described as a geometric Brownian motion with α_1 and β_1 as the drift and standard deviation parameters and W_t characterizing the Brownian process. Thus the governing equation is:

$$dP_1 / P_1 = \alpha_1 \, dt + \beta_1 dW_t \text{; and } P_1(0) = P_{10} \qquad (4.1)$$

The deterministic long-term contract price process P_2 is described with α_2 as the expected price growth rate. Hence, the corresponding equation is:

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$$dP_2 / P_2 = \alpha_2 dt \text{; and } P_2(0) = P_{20} \tag{4.2}$$

As discussed earlier, for the same starting open market and contract prices, the condition $\alpha_2 > \alpha_1$ is required to be satisfied for the contract price premium requirement. Though this is not a modeling requirement, this needs to be satisfied for drawing meaningful insights about the sourcing arrangements in typical procurement contexts.

Let $X_t > 0$ be the total procurement expenses of the firm across two sourcing strategies at time $t \geq 0$; and '$u_1(t)$' be the fraction of units to purchase at open market price (P_1) and '$u_2(t)$' be the fraction of units to purchase at long-term contract price (P_2) at time t to minimize the expense disutility. Furthermore, in the planning horizon under consideration, for a given '$u_1(t)$' and '$u_2(t)$', the change in expense 'dX_t'over infinitesimal time 'dt' is a function of change in prices. It is assumed that resources required to accommodate the change in expense over time are immediately and continuously available as the firm adjusts the fractions allocated to alternate strategies to minimize the cost disutility. After each 'dt' the firm can adjust the fractions sourced from alternative arrangements (without withdrawing money from the system) to minimize the expense disutility over the planning horizon. Thus the equilibrium condition at the time of rebalance is: expense before reallocation of fractions among sources is same as expense after reallocation of fractions' at each discrete time point.

Correspondingly, the following equations hold good at any instant 't'.

$$X_t = u_1(t)P_1(t) + u_2(t)P_2(t) \tag{4.3}$$

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$$dX(t) = u_1(t)dP_1(t) + u_2(t)dP_2(t) \tag{4.4}$$

$$1 = u_1(t) + u_2(t) \tag{4.5}$$

The above equations define the expenses, the units procured from alternative sources at any instant 't', and how the expenses change over the planning horizon. It may be noted that at any instant of time over the planning horizon, the buying firm can readjust the fractions procured continuously or in jumps across the market and long-term contract arrangements without incurring any additional expense. In the following discussion the analysis is limited to continuous adjustment of the fractions procured from the arrangements. The implications of adjustment in discrete jumps at an instant are presented in Appendix D. It may be observed from the following analysis that for such continuous adjustment, units procured from alternative arrangements remain constant when equations (4.3), (4.4) and (4.5) hold good. The analysis involves standard Ito Calculus (Oksendal, 1996) and is presented as a mathematical proof to the following claim.

Claim: For fixed total purchase, units purchased from alternative arrangements do not change over time in a planning horizon ($0 \le s \le t$).

Thus if conditions (4.3), (4.4) and (4.5) hold good then $u_1(t) = u_1(s) = u_1(0)$ and correspondingly $u_2(t) = u_2(s) = u_2(0)$.

Proof:

The market and contract price processes are defined as in (4.1) and (4.2).

It is to be proved that $u_1(t) = u_1(0) \Leftrightarrow du_1 = 0$.

Let the argument and the subscript 't' of the functions be dropped in the following discussion for notational convenience.

Equations (4.3), (4.4) and (4.5) can be rewritten as:

$X = u_1 P_1 + u_2 P_2$, $dX = u_1 dP_1 + u_2 dP_2$ and $1 = u_1 + u_2$ respectively.

$$dX = u_1 dP_1 + u_2 dP_2 = u_1 dP_1 + u_2 P_2 \alpha_2 dt \tag{4.6}$$

Let $\overline{X} = X / P_2$, $\overline{P_1} = P_1 / P_2$; Hence, $d\overline{P_1} = \overline{P_1}[(\alpha_1 - \alpha_2)dt + \beta_1 dW)]$

Then, $d\overline{X} = d(X \cdot P_2^{-1}) = Xd(P_2^{-1}) + P_2^{-1} dX$

$$= Xd(P_2^{-1}) + P_2^{-1}[u_1 dP_1 + u_2 dP_2]$$

$$= -\alpha_2 \overline{X} dt + P_2^{-1} u_1 dP_1 + u_2 \alpha_2 dt$$

$$= u_1(-\alpha_2 \overline{P_1} dt + P_2^{-1} dP_1), \text{ (since, } \overline{X} = u_1 \overline{P_1} + u_2 \text{)}$$

$$= u_1 d\overline{P_1} \tag{4.7}$$

Since, the market price is stochastic, for optimal sourcing, the (required) fractions of units procured from arrangements at an instant should also behave stochastically. Let the continuous change of units sourced from market arrangement be represented as:

$$du_1 = du_1^c = A(t)dt + B(t)dW_t \tag{4.8}$$

Here, $A(t)$ and $B(t)$ are the drift and standard deviation parameters, and W_t represents the standard Weiner process.

$$\overline{X} = u_1 \overline{P_1} + u_2$$

$$\Rightarrow d\overline{X} = u_1 d\overline{P_1} + (\overline{P_1} - 1) du_1^c + du_1^c \cdot d\overline{P_1} \tag{4.9}$$

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Equating (4.7) and (4.9) one obtains:

$$0 = (\overline{P}_1 - 1)du_1^c + du_1^c \cdot d\overline{P}_1$$

$$= (\overline{P}_1 - 1)du_1^c + du_1^c \cdot \overline{P}_1[(\alpha_1 - \alpha_2)dt + \beta_1 dW_t)] \qquad (4.10)$$

Using (4.8) in (4.10), and ignoring the higher order terms of dt it can be written:

$$0 = (\overline{P}_1 - 1)[A(t)dt + B(t)dW_t] + B(t)\overline{P}_1\beta_1 dt \qquad (4.11)$$

It may be observed that, since W_t is random RHS of (4.11) can be zero only when both $A(t)$ and $B(t)$ are zero. Thus du_1^c is zero, and correspondingly, for the restriction $u_1(t) + u_2(t) = 1$ to hold good, one must have, $du_2 = 0$.

Hence, it is observed that units procured from an arrangement under conditions (4.3), (4.4), and (4.5) remain unchanged over time, i.e.,

$u_1(t) = u_1(0)$ and $u_2(t) = u_2(0)$.

This implies for price processes defined by (4.1) and (4.2), a buying firm intending to minimize disutility of expenses while satisfying (4.3), (4.4) and (4.5) would source a fixed number of units from a particular arrangement.

The optimal fraction of units to be procured from alternative arrangements when the firm minimizes the cost disutility is derived next. The derivation is cumbersome for general disutility function; consequently, results for a specific disutility function are derived to illustrate the implications of degree of risk aversion on optimal sourcing.

Let $F(t, x_t^{u_1})$ be the instantaneous procurement disutility of a buying firm at time t, when the buying firm incurs a total expense of X_t and chooses to procure a fraction of u

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fraction of u_1 from the open market arrangement over the planning horizon $[t, T]$. The buying firm intends to minimize the overall procurement cost disutility of over the planning horizon. Accordingly, the minimum of overall (expected) cost disutility functional can be written as

$$C^{u_1}(t,x) = \min_{0<u_1<1} E^{t,x}\left[\int_t^T F(s, x_s^{u_1})ds\right] \tag{4.12}$$

where, $E(.)$is the expectation operator.

Let x^n be the disutility rate function $F(.)$ in (4.12), where $n(>1)$ and represents the cost disutility parameter i.e., the higher the n, the higher the cost disutility of the buyer. As discussed in the previous chapter the above power functional form is a mathematical convenience and is structurally similar to the utility function used in the wealth maximization problems in financial literature (Korn and Korn, 2001). Though this functional form is a mathematical convenience, it has also the desired properties from a practical and theoretical standpoint. The monotonically increasing convex function characterizes the buying firms' risk aversion and ensures that the higher the procurement expense the greater is the disutility. Furthermore it facilitates incorporation of alternative values for n over distinct time phases to address the issue of varied disutility function of buyers over the PLC. Equation (4.12) requires to be solved to obtain to the optimal $u_1(t,x)$. Since obtaining the optimal $u_1(t,x)$ for general x^n is cumbersome, it is derived for the disutility function with $n = 2$ below to illustrate the approach and only the results for the disutility function with $n = 3$ are presented. From a practical standpoint, the power di

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the power disutility functions with n greater than 3 are unlikely to be specified by the buying firm.

Earlier in (4.7) it was seen that $d\overline{X} = u_1 d\overline{P_1}$, where

$d\overline{P_1} = \overline{P_1}[(\alpha_1 - \alpha_2)dt + \beta_1 dW)]$.

Let $(\alpha_1 - \alpha_2) = \alpha$, in the following discussion. Also, let the subscript 1 in $\overline{P_1}$ be dropped and the limits of planning horizon be changed from [t, T] to [0, t] for notational convenience.

Integrating both sides of $d\overline{X} = u_1 d\overline{P}$, one obtains:

$$\int_0^t d\overline{X} = u_1 \int_0^t d\overline{P}$$

$$\Rightarrow \overline{X_t} = \overline{X_0} + u_1(\overline{P_t} - \overline{P_0}) = \overline{X_t} = \overline{X_0} - u_1\overline{P_0} + u_1\overline{P_t}$$

$$= u_1(\overline{P_0} - 1) + 1 - u_1\overline{P_0} + u_1\overline{P_t}$$

$$= (1 - u_1) + u_1\overline{P_t}$$

For a disutility function with $n = 2$, the objective functional will be:

$$\min_{u_1} E\left(\int_0^t \overline{X_s^2}\, ds\right) = \min_{u_1} E\left[\int_0^t (1-u_1)^2 ds + \int_0^t u_1^2 \overline{P_s^2}\, ds + \int_0^t 2u_1(1-u_1)\overline{P_s}\, ds\right]$$

$$= \min_{u_1}\left[\int_0^t (1-u_1)^2 ds + u_1^2 \int_0^t E(\overline{P_s^2})ds + 2u_1\,(1-u_1)\int_0^t E(\overline{P_s})ds\right]$$

$$= \min_{u_1}\left[\int_0^t (1-u_1)^2 ds + u_1^2 \int_0^t \overline{P_0^2} e^{(2\alpha+\beta_1^2)s} ds + 2u_1\,(1-u_1)\int_0^t \overline{P_0} e^{\alpha s} ds\right]$$

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Taking the first order condition for optimization:

$$-2(1-u_1)t+\frac{2(1-2u_1)\overline{P_0}}{\alpha}(e^{\alpha t}-1)+\frac{2u_1\overline{P_0^2}}{(2\alpha+\beta_1^2)}(e^{(2\alpha+\beta_1^2)t}-1)=0$$

$$\Rightarrow 2u_1 t-\frac{4u_1\overline{P_0}}{\alpha}(e^{\alpha t}-1)+\frac{2u_1\overline{P_0^2}}{(2\alpha+\beta_1^2)}(e^{(2\alpha+\beta_1^2)t}-1)=2t-\frac{2\overline{P_0}}{\alpha}(e^{\alpha t}-1)$$

$$\Rightarrow u_1^* = \frac{t-\frac{\overline{P_0}}{\alpha}(e^{\alpha t}-1)}{t-\frac{2\overline{P_0}(e^{\alpha t}-1)}{\alpha}+\frac{\overline{P_0^2}}{(2\alpha+\beta_1^2)}(e^{(2\alpha+\beta_1^2)t}-1)} \qquad (4.14)$$

The expression for optimal fraction of units sourced from market at each instant can be computed by construing the problem as zero planning horizon; correspondingly u_1^* can be determined by using L'Hospital's rule and can be written as:

$$u_1^* = \frac{\overline{P_0}(\alpha_2-\alpha_1)}{\overline{P_0^2}[\beta_1^2-2(\alpha_2-\alpha_1)]+2\overline{P_0}(\alpha_2-\alpha_1)} \qquad (4.15)$$

The above expression indicates that instantaneously optimal allocation to market varies directly with difference in mean price growth rates, and varies inversely with the variance of the stochastic price.

To verify that first order condition indeed gives the minimum, it is to be ensured that the second derivative is positive.

The appropriate condition for second derivative of (4.13) may be written as:

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$$2t - \frac{4\overline{P_0}}{\alpha}(e^{\alpha t} - 1) + \frac{2\overline{P_0^2}}{(2\alpha + \beta_1^2)}(e^{(2\alpha+\beta_1^2)t} - 1) > 0$$

$$\Rightarrow t + \frac{2\overline{P_0}}{\alpha} + \frac{\overline{P_0^2}}{(2\alpha + \beta_1^2)}e^{(2\alpha+\beta_1^2)t} > \frac{2\overline{P_0}}{\alpha}e^{\alpha t} + \frac{\overline{P_0^2}}{(2\alpha + \beta_1^2)}$$

$$\Rightarrow \frac{\overline{P_0^2}}{(2\alpha + \beta_1^2)t}(e^{(2\alpha+\beta_1^2)t} - 1) + 1 > \frac{2\overline{P_0}}{\alpha t}(e^{\alpha t} - 1) \tag{4.16}$$

The above expression indicates that existence of minimum will depend upon the specific parameter values. Since, as per market condition α (i.e., difference between market price and contract price) is always negative, RHS and the part with exponential term in LHS are always positive, convex and increasing in *t*. Thus (4.16) is likely to be satisfied when $\beta_1^2 > (\alpha_2 - \alpha_1)$. Correspondingly, the likely range of values for u_1 is:

$$u_1^* = \begin{cases} < 0 & \text{if } \alpha_2 < \alpha_1 \\ \in (0,1) & \text{if } \beta_1^2 > \alpha_2 - \alpha_1 \\ > 1 & \text{if } \beta_1^2 < \alpha_2 - \alpha_1 \end{cases}$$

In lines similar to above, the expression for optimal fraction for disutility function with $n = 3$ (i.e., x^3) can be derived as:

$$u_1^*(t) = \frac{-Y + \sqrt{Y^2 - 4XZ}}{2X}; \tag{4.17}$$

where,

$$\begin{cases} X = -3t + \cdots \\ Y = 6t + \cdots \\ Z = 3c\overline{P_0} \cdots \end{cases}$$

and, $\begin{cases} A = \cdots \\ B = \cdots \\ C = \cdots \end{cases}$

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$$\begin{cases} X = -3t + 3A\overline{P_0^3} - 9B\overline{P_0^2} + 9c\overline{P_0} \\ Y = 6t + 6B\overline{P_0^2} - 12c\overline{P_0} \\ Z = 3c\overline{P_0} - 3t \end{cases}$$

and, $$\begin{cases} A = \dfrac{e^{3(\alpha+\beta_1^2)t} - 1}{3(\alpha+\beta_1^2)} \\ B = \dfrac{e^{(2\alpha+\beta_1^2)t} - 1}{(2\alpha+\beta_1^2)} \\ C = \dfrac{e^{\alpha t} - 1}{\alpha} \end{cases}$$

It is to be noted that, if u_1^* lies out side the limits of 0 or 1 for specified parameter values, then it would imply, all the procurements would be from either from contract purchase or market purchase respectively. It is evident from equation (4.14) and (4.17) that the optimal fraction of units to procure from a source is governed by the price parameters, duration of planning horizon and the prices at the initial instant.

It is important to note that as per the model, for continuous adjustment of units sourced from alternative arrangements, the optimal fraction of units to procure from a source depends on the duration of planning horizon; however, for a specified planning horizon it remains constant during the planning horizon.

4.3 Illustration of model application

As discussed in the previous chapter, the model can be used for finding the optimal sourcing strategy in varied procurement contexts that can be described in terms of specific price parameters of market and contract price processes and the buying firm

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buying firm's risk attitude. Thus for appropriately chosen parameter values, the model may be employed for finding the optimal strategy both in a specific stage of the procured product's life cycle and over multiple stages of the PLC. The application of the model is illustrated in two stages. In the first stage, the patterns of optimal fractions (u_1) sourced from open market arrangement are examined over time across different stages of the PLC, when the risk averse buying firm has disutility functions x^n with $n = 2$ and $n = 3$. In the second stage, the implications of the changing concern for cost (i.e., varied disutility) on (optimal) sourcing across various stages of the PLC of the procured product is investigated.

Towards this end the pattern of change in optimal fractions to source from market arrangement when the buying firm's cost disutility function changes from x^n with $n = 2$ in earlier stages to x^n with $n = 3$ towards the later stages of PLC is studied. It is to be noted that since a disutility function with $n = 3$ has a higher index of power, it would represent a higher degree of risk aversion than a disutility function with $n = 2$ (Keeney and Raiffa, 1976). The hypothetical price data set used in the previous chapter is also used for the above analyses and is presented in Table 4.1 for reference.

Table 4.1: Illustrative data representing the price dynamics across the PLC stages

State of the market across the PLC stages	Expected market price annual growth rates 'α_1'	Contract price annual growth rates 'α_2'	Difference in contract and market price growth rates 'α'	Standard deviation parameter of market price process 'β_1'	Expected market price: initial price: $100	Contract price: initial price: $100	Standard deviation of market price
1	-0.25	-0.2	0.05	0.75	77.88	81.87	52.70
2	-0.232	-0.185	0.047	0.67	61.75	64.73	36.86
3	-0.214	-0.17	0.044	0.6	49.86	52.10	26.50
4	-0.196	-0.155	0.041	0.54	40.98	42.70	19.60
5	-0.178	-0.14	0.038	0.49	34.30	35.63	14.96
6	-0.16	-0.125	0.035	0.45	29.23	30.27	11.80
7	-0.142	-0.11	0.032	0.42	25.36	26.18	9.66
8	-0.124	-0.095	0.029	0.39	22.40	23.06	8.02
9	-0.106	-0.08	0.026	0.35	20.15	20.68	6.54
10	-0.088	-0.065	0.023	0.3	18.45	18.88	5.19
11	-0.07	-0.05	0.02	0.25	17.20	17.55	4.07
12	-0.052	-0.035	0.017	0.2	16.33	16.61	3.13
13	-0.034	-0.02	0.014	0.15	15.79	16.01	2.30

The dataset has been stylized to reflect the effect of PLC on the price pattern. The price pattern is graphically presented in Figure 4.1. As evident from the Figure 4.1, the average market prices is lower than contract prices, and both the market and contract prices, and the variance of the market prices decline over time in line with the theoretical and empirical literature (Klein and Leffler, 1981; Horowitz, 1986; Smith et al, 1999).

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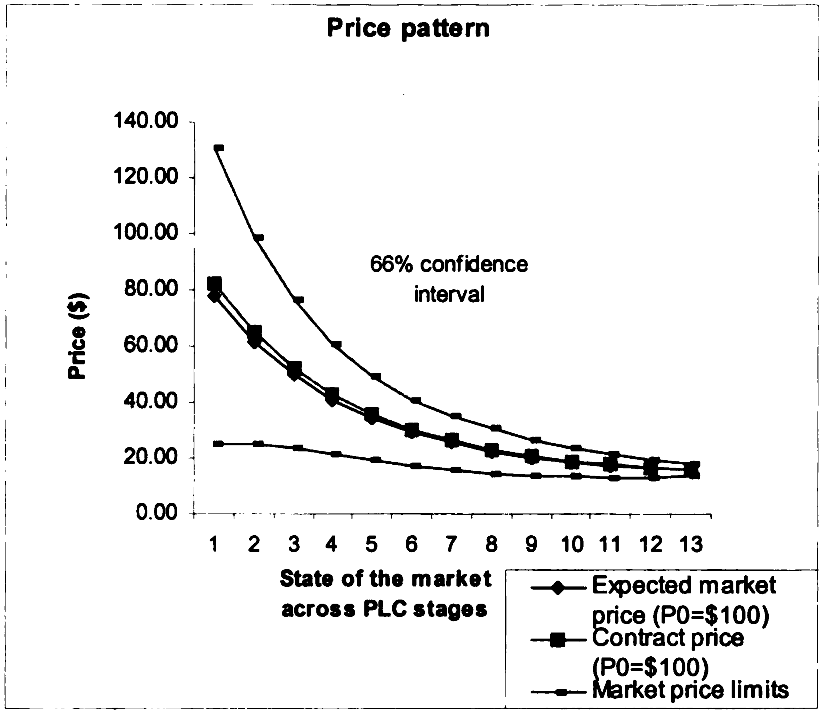
Figure 4.1: Characteristic (hypothetical) price pattern across different stages of the PLC



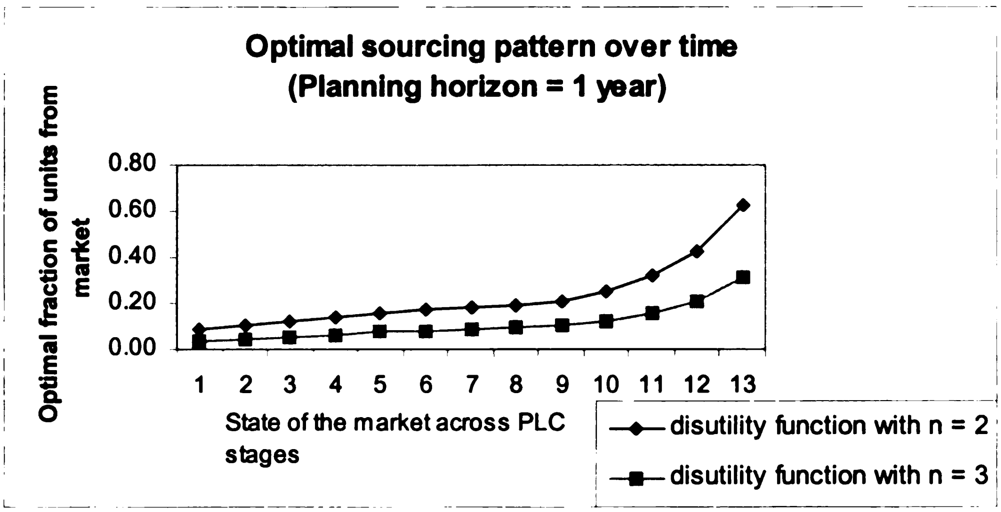
For the above price pattern, a risk neutral buying firm should consistently buy from the market as the expected market price is always lower than the contract price. However, a risk averse buying firm would act differently. Table 4.2 presents the cost disutility factors (n) for two scenarios (cases) of buying firm's risk preference over the 13 years of PLC, and the corresponding optimal fractions of units to source from market arrangement for a one year planning horizon. The optimal fractions are computed using formulae (4.14) and (4.17).

Table-4.2: The impact of price pattern and cost disutility factor (n) on optimal fraction (u_1^*) sourced from market

State of the market across the PLC stages	Difference in contract and market price growth rates 'α'	Standard deviation parameter of market price process 'β_1'	Cost disutility factor 'n'= 2	Cost disutility factor 'n'= 3	Fraction spent on market sourcing (u_1^*) for 'n'= 2	Fraction spent on market sourcing (u_1^*) for 'n'= 3
1	0.05	0.75	2.00	3.00	0.08	0.04
2	0.047	0.67	2.00	3.00	0.09	0.05
3	0.044	0.6	2.00	3.00	0.11	0.06
4	0.041	0.54	2.00	3.00	0.13	0.06
5	0.038	0.49	2.00	3.00	0.15	0.07
6	0.035	0.45	2.00	3.00	0.17	0.08
7	0.032	0.42	2.00	3.00	0.18	0.09
8	0.029	0.39	2.00	3.00	0.19	0.09
9	0.026	0.35	2.00	3.00	0.21	0.10
10	0.023	0.3	2.00	3.00	0.25	0.13
11	0.02	0.25	2.00	3.00	0.32	0.16
12	0.017	0.2	2.00	3.00	0.42	0.21
13	0.014	0.15	2.00	3.00	0.62	0.31

The pattern of optimal fractions to source from the market for disutility functions (x^n) with $n = 2$ and $n = 3$ are obtained using formulae (4.14) and (4.17). The results are presented in Table 4.2 and are plotted in Figure 4.2. The results are data dependent; however, similar computations can be carried out using relevant contextual data to gain firm, industry or product specific (managerial) insights.

Figure 4.2: Patterns of optimal fraction to source for disutility function with $n = 2$ and disutility function with $n = 3$ for a one year planning horizon in different stages of the PLC



The plots for the illustrative dataset indicate the following general patterns:

a) *ceteris paribus*, the higher the risk aversion (i.e., higher the power of disutility function), the lesser the fraction procured from open market; this implies buying firms with higher cost concern would procure less from the open market,

b) *ceteris paribus*, the greater the ratio of market price uncertainty (β_1) to the difference in contract and market prices (growth rates), the lesser the fraction procured from open market; consequently, for a specific degree of risk aversion, contract arrangement is more important in the earlier stages of the PLC and market arrangement gradually becomes more important in the later stages of PLC,

c) the general characteristics of optimal trajectories are similar. Hence for disutility functions with index '*n*' between 2 and 3, the optimal fractions may be obtained by appropriate interpolation. The results of such interpolation are presented in Table 4.3 and Figure 4.3.

The above results suggest that the optimal sourcing strategies should be dynamic, and need to be adjusted in accordance with the degree of risk aversion and price pattern across different procurement contexts.

The implications of varied planning horizons for the characteristic price patterns and buyer's cost disutility function on optimal sourcing strategy in different stages of the PLC can be examined in terms of the optimal results obtained using formulae (4.14) and (4.17). These results are presented in Table 4.4 and Figure 4.4.

Table 4.3: Optimal fractions of units to source from market for gradually varying disutility function over the PLC stages (the fractions during transition phase are computed by interpolation and hence approximate)

State of the market across the PLC stages	Optimal fraction (u_1^*) for disutility function $n=2$	Optimal fraction (u_1^*) for disutility function with $n=3$	Disutility function	Optimal fractions (u_1^*) from market
1	0.08	0.04	Disutility function with $n=2$	0.08
2	0.09	0.05	Disutility function with $n=2$	0.09
3	0.11	0.06	Disutility function with $n=2$	0.11
4	0.13	0.06	Disutility function with $n=2$	0.13
5	0.15	0.07	Intermediate between disutility function with $n=2$ and disutility function with $n=3$	$\frac{0.15+0.07}{2}=0.11$
6	0.17	0.08	Intermediate between disutility function with $n=2$ and disutility function with $n=3$	$\frac{0.17+0.08}{2}=0.12$
7	0.18	0.09	Intermediate between disutility function with $n=2$ and disutility function with $n=3$	$\frac{0.18+0.09}{2}=0.13$
8	0.19	0.09	Intermediate between disutility function with $n=2$ and disutility function with $n=3$	$\frac{0.19+0.09}{2}=0.14$
9	0.21	0.10	Intermediate between disutility function with $n=2$ and disutility function with $n=3$	$\frac{0.21+0.10}{2}=0.16$
10	0.25	0.13	Disutility function with $n=3$	0.13
11	0.32	0.16	Disutility function with $n=3$	0.16
12	0.42	0.21	Disutility function with $n=3$	0.21
13	0.62	0.31	Disutility function with $n=3$	0.31

Figure 4.3: Pattern of optimal fractions of units to source from market for gradually varying disutility function across the PLC stages

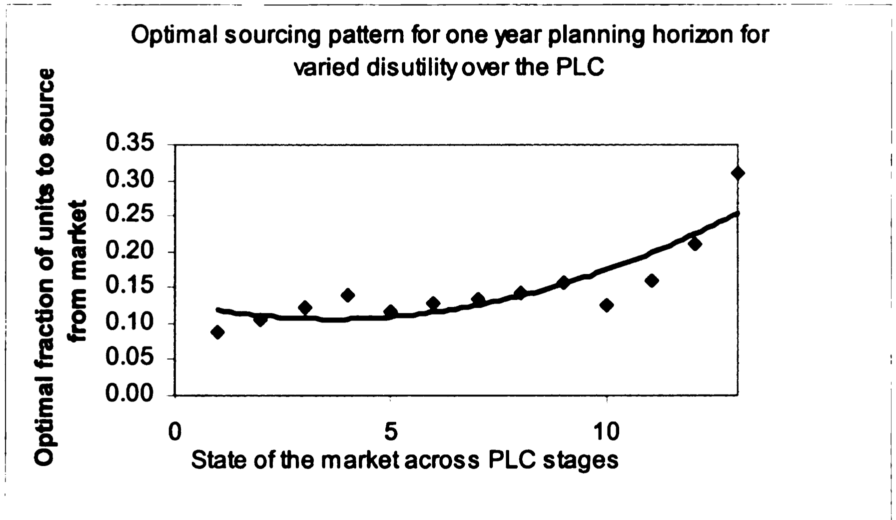
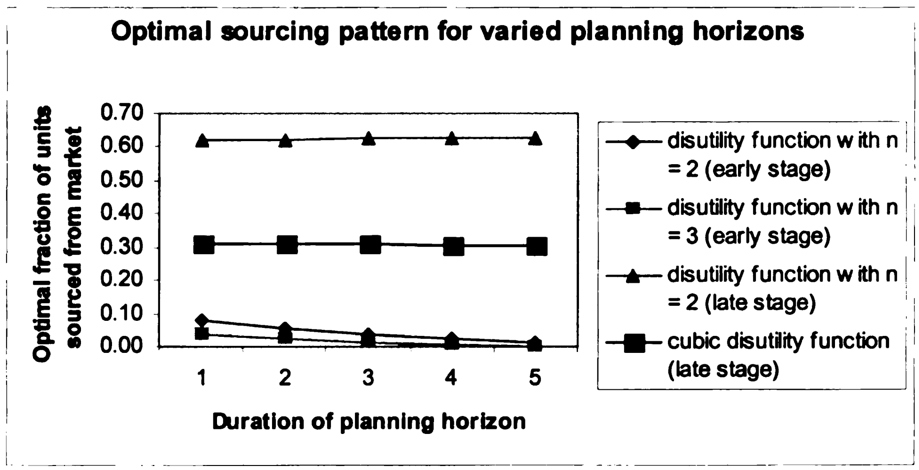


Table 4.4: Optimal fraction of units to source from market for varied planning horizons in different stages of the PLC

Planning horizon	Early state of industry (1)		Late state of industry (13)	
	Optimal fraction (u_1^*) for disutility function with $n=2$	Optimal fraction (u_1^*) for disutility function with $n=3$	Optimal fraction (u_1^*) for disutility function with $n=2$	Optimal fraction (u_1^*) for disutility function with $n=3$
1	0.08	0.04	0.62	0.31
3	0.05	0.02	0.62	0.31
5	0.04	0.01	0.62	0.31
7	0.02	0.00	0.63	0.31
9	0.01	0.00	0.63	0.31

Figure 4.4: Patterns of optimal fraction (u_1^*) of units to source from market for varied planning horizons (the values represent results in Table 4.3)



It is evident from the results that the optimal fractions to source from market arrangement diminishes fast with the duration of planning horizon in the earlier stage of PLC as compared to the later stage of PLC. This suggests that in the earlier stages, the risk averse buying firm can minimize the expected cost disutility either by sourcing higher fractions form market with shorter planning horizons or by sourcing lesser fractions from market with a longer planning horizon. Any of these approaches would enable to maintain a balance between the exposure to cost uncertainty and overpayment to the contractual supplier. It is also observed that for similar durations of planning horizons the rate of decline is faster for lesser degree of risk aversion (e.g., disutility function with $n = 2$) as compared to that for higher degree of risk aversion (e.g., disutility function with $n = 3$). This implies the duration of planning horizon moderates the impact of the degree of risk aversion. Consequently, the propensity to source from market and to readjust the optimal fractions by firms with higher and lesser degree of risk aversion would reduce with increase in planning horizon.

The above patterns have a variety of strategic implications. First, these highlight the significance of mixed sourcing strategy to balance uncertainty of cost with the opportunity to reduce cost according to the risk aversion characteristics of the buying firm. The results illustrate the advantages of some sourcing from open market even in the earlier stages of the PLC that are characterized by high price growth rate differential, and high market price variance. Such dual strategy enables the buying firm to present a competitive tension to the contractual suppliers; this would discourage them to unreasonably overcharge the buying firm. Second, the patterns highlight the relative importance of contract based sourcing during the early/intermediate (with moderate price growth rate differential) stage to ensure certainty of supply even when long-term contractual suppliers are likely to charge a price premium over the expected (competitive) market prices. Third, the patterns suggest the increased importance of market price based sourcing towards the later stages of PLC (when the product becomes standardized and functional) as competition intensifies, and firms become more cost conscious. The decline in fraction of contract purchase in favor of open market purchase (as illustrated in Figure 4.2) as the product advances from intermediate phase to mature phase may be justified on the basis of higher cost sensitivity and price-based competition towards later phases that reduces price growth rate differential and market price uncertainty. However, in reality such reallocation is plausible only if the market becomes truly competitive with highly capable suppliers, and supplier search cost and switching cost are insignificant. Thus the pattern of optimal sourcing is a function of price patterns across various stages of the PLC that may be governed by technological changes, market competition, and emergence of efficient transaction mechanisms. The above results seem to conform to the framework of Rink and Fox (1999) that suggests varied emphasis on alternative sourcing approaches along the PLC of end product that often has a close correspondence with the PLC of the components. The presence of close correspondence between the life cycles influences the relative cost priorities of components over time and results in paring of supply base as the products progress from early stage to growth stage, higher preference for long-term contractual arrangements in intermediate stages to reduce fixed cost by enabling the suppliers to derive economies of scale, and higher purchase from market to stimulate competition among suppliers in later stages of the life cycles.

While the proposed model helps to identify optimal fraction of units to source from alternative arrangements across different stages of the PLC, the extent of smoothness of the trajectory would indicate the nature of transition between the stages and the need for planned changes in sourcing arrangements. Thus reallocation of share to market sourcing arrangement over time may necessitate the buying firm to proactively indicate or signal the suppliers about the emphasis on price based open market purchase towards later phases. This would stimulate timely competition among them such that the market becomes truly efficient (i.e., reduced price uncertainty and growth rate differentials) with reduced transaction cost and without the buying firm having to compromise with other non-price attributes. As noted in the previous chapter, the increasing shift towards open market arrangement towards the later stages may however be avoided if the buying firm can induce the existing contractual supplier to match the prices in the competitive open market periodically. In this scenario, the optimal solution in the model would reflect the integrated valuation of contract and market price processes and may be used as a mechanism for benchmarking the expense patterns for minimizing the expense disutility over time.

Chapter 5

Optimal Sourcing Strategy With Proportional Switching Cost

5.1 The problem context

The models discussed in the previous two chapters for a typical (single product) procurement context of a firm that adopts a dual (combined) sourcing strategy involving a mix of long-term contract and open market arrangements over continuous time consider the procurement expense function to be continuous and ignore the presence of switching cost to readjust the units sourced between arrangements. These models are appropriate for a procurement context in which the costs for switching allocations between alternative arrangements are negligible and the firm procures continuously over time.

However, in practice there are procurement contexts in which a firm may incur a noticeable cost of switching units procured between arrangements due to contract penalty clauses or structural and infrastructural change requirements. Moreover firms may procure materials at a significant interval of time such that the patterns of expenses are discontinuous in time. In extant literature, these issues have been addressed to an extent primarily from the perspective of a risk neutral buying firm (Kouvelis, 1999; Li and Kouvelis (1999). There seems to be only one study (Cohen and Agrawal, 1999) that examines the relative advantages of long and short-term contracts from a risk averse buying firm's perspective when the buying firm chooses one of the alternative arrangement at a time. Thus this model does not consider the possibility of simultaneous consideration of both alternatives. Moreover their model uses the 'mean-variance approach' to evaluate the cost; this approach is less general in a sense that the formulation implies the variance is the only measure of risk and the utility function of that characterizes the risk aversion is of quadratic nature only that does not satisfy the theoretical relationships between risk aversion and magnitude of expense appropriately (Eeckhoudt and Gollier, 1995).

In this chapter, the procurement problem with switching costs and transactions in discrete time is modeled for minimizing procurement expense disutility over a specified time horizon. In contrast to the 'mean-variance approach', the model formulated here considers a power disutility function that can address more general cases of risk aversion. In this model, it is assumed that the costs for switching allocations between arrangements are proportional to the magnitude of switch. Such a situation is plausible when the supplier charges a premium for changing the volume sourced in the previous period or the buying firm can translate the impact of switching cost as a percentage of the unit price of the product.

5.2 The model

As discussed in earlier models, the market price process (P_1) is continuous and may be described as a geometric Brownian motion with α_1 and β_1 as drift and standard deviation parameters and W_t characterizing the Brownian process. Thus the governing equation is:

$$\frac{dP_1}{P_1} = \alpha_1 \, dt + \beta_1 dW_t; \qquad \text{and} \qquad P_1(0) = P_{10} \tag{5.1}$$

The long-term contract price process (P_2) is a continuous process with α_2 as the expected price growth rate. Hence, the corresponding equation is:

$$\frac{dP_2}{P_2} = \alpha_2 dt \,. \qquad \text{and} \qquad P_2(0) = P_{20} \tag{5.2}$$

As discussed in earlier chapters, the contract prices would be higher than the average competitive market prices over time (Klein and Leffler, 1981; Smith et al., 1999). Correspondingly, it may be assumed that $\alpha_2 > \alpha_1$ will hold good for the same beginning prices. It is to be noted that the condition '$\alpha_2 > \alpha_1$' is not a requirement for the development of the model here. However, it is a plausible outcome of competitive market economics and may be used for drawing meaningful theoretical managerial insights.

For the above context, the relative price process $P = P_1 / P_2$ may be described as:

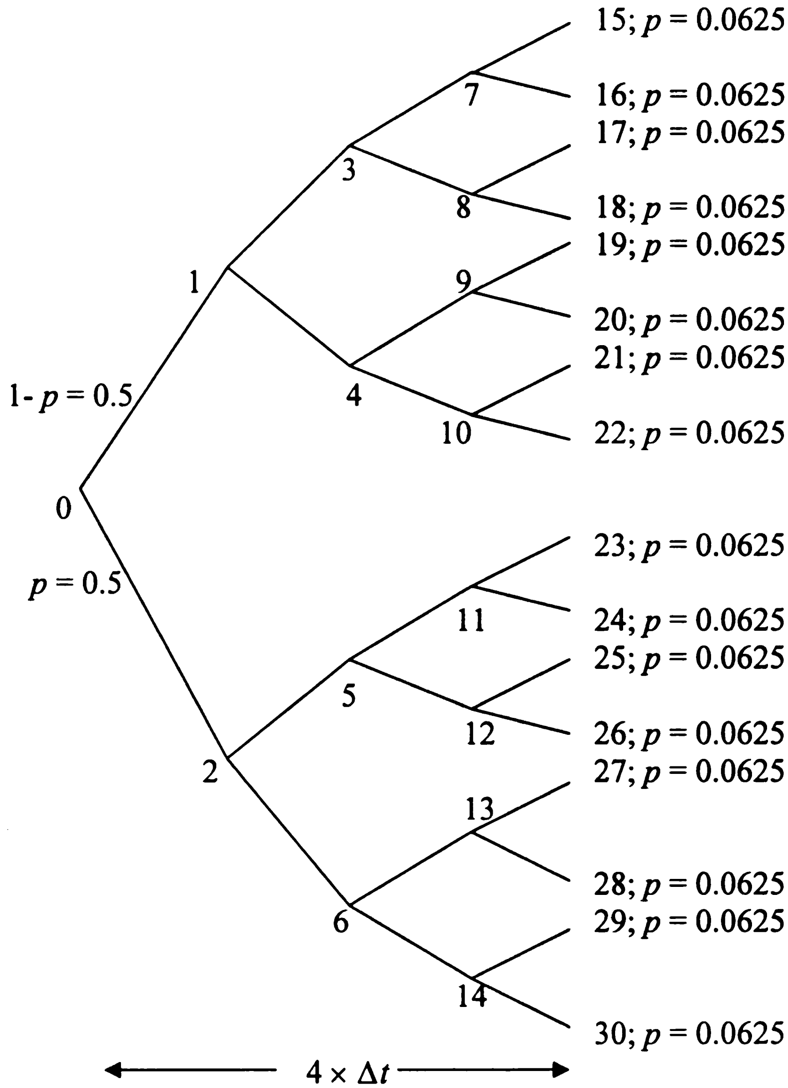
$$dP = P \, [(\alpha_1 - \alpha_2)dt + \beta_1 dW)] \tag{5.3}$$

The above relative price process may be considered to evolve as an equivalent binomial process with probability of moving up and down ($p, 1-p$) (Hull, 2003, p. 407). Thus the relative price process for a time interval of Δt between two successive nodes may be described as:

$$P = P_0 e^{(\alpha_1 - \alpha_2)\Delta t \pm \beta_1 \sqrt{\Delta t}}; \tag{5.4}$$

where, p=1/2 and 1-p=1/2 and P_0 is the ratio of prices at the beginning of operating/planning horizon i.e., at t = 0. Thus a binomial tree with five time stages will represent transactions over four time intervals in a planning horizon of one year (Figure 5.1). For example, for a planning horizon of one year, these four intervals may be considered as four quarters in a year.

Figure 5.1: Binomial Tree: The binomial representation of the evolution of relative price process $P = P_1 / P_2$ over four time intervals. The integers represent the nodes corresponding to a specific price state with a specific probability of occurrence.



Let 'u' and '1-u' refer to the optimal fraction of units to be sourced from the market and contract arrangements respectively at an instant. At any transaction time stage 'j', the risk averse buying firm may change the pattern of sourcing from $(u_{j-1}, 1-u_{j-1})$ to $(u_{j}, 1-u_{j})$. Correspondingly, the minimum of expense disutility C (.) at any instant (j) of transaction may be described as:

$$C(u_{j-1}, j) = \min_{0 \le u_j \le 1} \left\{ [u_j \cdot P_j + 1 - u_j + (\lambda_1 P_j + \lambda_2) |u_j - u_{j-1}|]^n + E_j[C(u_j, j+1)] \right\} \quad (5.5)$$

Here, $E_j[C(u, j+1)]$ refers to the expected minimum disutility at instant 'j' due to future sourcing pattern and is zero for terminal nodes. Let λ_1 and λ_2 be the coefficients of switching cost (expressed in percentage of unit cost) for a unit change in allocation pattern across market and contractual arrangements at an instant. 'n' is the cost disutility index, which can be of any value greater than 1 for the risk averse buying firm. In the current model, the formulations are developed for n = 2. Formulations for higher values of n can be carried out in similar ways. It is to be noted that at the beginning instant (t=0), the buying firm does not incur any switching cost because it starts the procurement arrangements fresh. Let 'N' be the number of time stages in the operational planning horizon when the transaction occurs. So the optimal cost disutility equations for the entire procurement process over the operational planning horizon becomes:

$$C \equiv \min_{0 \le u_0,\ldots u_N \le 1} \left\{ [u_0 \cdot P_0 + 1 - u_0]^2 + E\{\sum_{j=1}^{N} [u_j \cdot P_j + 1 - u_j + (\lambda_1 P_j + \lambda_2) | u_j - u_{j-1} |]^2 \} \right\} \quad (5.6)$$

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$$C \equiv \min_{0<u_0<1} \left\{ [u_0 \cdot P_0 + 1 - u_0]^2 + E_0\{C(u_0, 1)\} \right\} \tag{5.7}$$

For $1 \leq j \leq N-1$ we have:

$$C(u_{j-1}, j) = \min_{0 \leq u \leq 1} \left\{ [u_j \cdot P_j + 1 - u + (\lambda_1 P_j + \lambda_2) | u_j - u_{j-1} |]^2 + E_j[C(u, j+1)] \right\} \tag{5.8}$$

For $j = N$ at terminal time instant,

$$C(u_{N-1}, N) = \min_{0 \leq u_j \leq 1} \left\{ [u_j \cdot P_N + 1 - u_j + (\lambda_1 P_N + \lambda_2) | u_j - u_{N-1} |]^2 \right\} \tag{5.9}$$

It is to be noted that the above equations use the Bellman's principle of dynamic optimization in a recursive manner (Dixit and Pindyck, 1994, p.100). For the above cases optimal 'u_j' values at every instant of transaction are to be computed for obtaining C (.). However, it may be observed that deriving closed form expressions for optimal 'u_j' values becomes difficult beyond the last two-time stages of the planning horizon because the optimal 'u_j' values are path dependent. As a result, 'u_j' values at subsequent stages are influenced by the previous values; consequently, the expressions become unwieldy as one proceeds beyond two stages from the terminal stage. Hence, as an alternative to applying the dynamic optimization principle recursively for the entire operating horizon, a combination of computational and analytical procedures is adopted for obtaining optimal 'u_j' values over the operating horizon.

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First, in line with the principles of dynamic optimization, the optimal actions (i.e., transaction pattern 'u_N' and '1-u_N') at the nodes in the terminal stage are obtained for arbitrarily specified patterns of transaction 'u_{N-1}' in the penultimate stage. Subsequently, using the optimal expressions for 'u_N' at the terminal state, optimal expressions 'u_{N-1}' at the nodes in the penultimate stages are computed with respect to the arbitrarily specified pattern of transaction in the previous stage 'u_{N-2}'. This process could have been continued until the beginning instant 't = 0'. However, as discussed in the previous paragraph the expressions become unwieldy after this stage. Consequently a numerical procedure is adopted after this stage. As per this procedure, the disutility corresponding to alternative actions (i.e., discrete values of '$u_j \in [0,1]$') at each of the previous stages (i.e., u_0, u_1, u_2, $u_j, \ldots u_{N-2}$) are evaluated to obtain C at the initial instant $t = 0$. These results represent an optimal pattern of transactions as time progresses.

It is to be noted that the computational approach used to obtain 'u_j' values in stages previous to terminal two stages is robust but naïve. As a result, although in principle a purely numerical approach as described above can be used for obtaining optimal values at all nodes from start to end, the exponential increase in numerical operations with higher number of discrete values of '$u_j \in [0,1]$' prohibits the use of the computational approach beyond three time stages (i.e., 0-6 nodes). Thus in the absence of a more efficient alternative numerical approach, the naïve approach is used here for a limited range of discrete

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discrete values '$u_j \in [0,1]$' for the first three time stages and an analytical approach is used for the terminal two stages. The computational process is implemented using a computer software code written in C-language. The code is presented in Appendix E. The algebraic expressions for optimal transaction patterns for terminal two stages are derived using the dynamic optimization principle and are presented below.

Conditions for optimality at terminal nodes (j=N)

The condition for minimizing the expense disutility function at the terminal stage described in expression (5.9) gives the optimal $0 \le u_N \le 1$ as:

$u_N = u_{N-1} + s_N$;

Where,

$s_N = \dfrac{1 + (P_N - 1)u_{N-1}}{(1+\lambda_1)\{\dfrac{1-\lambda_2}{1+\lambda_1} - P_N\}}$ represents a positive action i.e., increase in market allocation with respect to u_{N-1}, and is valid when $0 \le s \le 1 - u_{N-1}$;

or,

$s_N = \dfrac{1 + (P_N - 1)u_{N-1}}{(1-\lambda_1)\{\dfrac{1+\lambda_2}{1-\lambda_1} - P_N\}}$ represents a negative action i.e., decrease in market allocation, and is valid when $-u_{N-1} \le s < 0$.

The above two conditions can be simplified as:

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$$u_N = \begin{cases} 0 & \text{if } P_N > \dfrac{1+\lambda_2}{1-\lambda_1} \\ u_{N-1} & \text{if } \dfrac{1-\lambda_2}{1+\lambda_1} < P_N \le \dfrac{1+\lambda_2}{1-\lambda_1} \\ 1 & \text{if } P_N \le \dfrac{1-\lambda_2}{1+\lambda_1} \end{cases} \tag{5.10}$$

Thus for any u_{N-1}, u_N can be computed. Correspondingly,

$$C(u_{N-1}, N) = \begin{cases} [1+(\lambda_1 P_N + \lambda_2)u_{N-1}]^2 & \text{if } P_N > \dfrac{1+\lambda_2}{1-\lambda_1} \\ (u_{N-1}P_N + 1 - u_{N-1})^2 & \text{if } \dfrac{1-\lambda_2}{1+\lambda_1} < P_N \le \dfrac{1+\lambda_2}{1-\lambda_1} \\ [P_N + (\lambda_1 P_N + \lambda_2)(1-u_{N-1})]^2 & \text{if } P_N \le \dfrac{1-\lambda_2}{1+\lambda_1} \end{cases} \tag{5.11}$$

Conditions for optimality at penultimate nodes (j=N-1)

The condition for minimizing the expense disutility at the penultimate stage as given in expression (8) can be used to obtain the optimal $0 \le u_{N-1} \le 1$ such that:

$u_{N-1} = u_{N-2} + s_{N-1}$; where, s_{N-1} is the change in allocation with respect to previous pattern of transaction 'u_{N-2}'. The expressions for s_{N-1} can be written as follows.

Let $A_j = (\lambda_1 P_j + \lambda_2)$, $B_j = P_j - 1$, and $C_j = P_j + A_j$

The relative price states at terminal nodes may be represented as the following:

$$P_{N-1} \begin{cases} P_N^i \text{ (up state)} \\ P_N^{i+1} \text{ (down state)} \end{cases}$$

For the above scenario, there can be six plausible scenarios.

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Case 1

$$P_N^i, P_N^{i+1} > \frac{1+\lambda_2}{1-\lambda_1}$$

if action (s_{N-1}) is positive, then

$$s_{N-1} = -\frac{[2(B_{N-1}u_{N-2}+1)(A_{N-1}+B_{N-1})+(A_N^i+A_N^{i+1})+u_{N-2}\{(A_N^i)^2+(A_N^{i+1})^2\}]}{2(A_{N-1}+B_{N-1})^2+\{(A_N^i)^2+(A_N^{i+1})^2\}}$$

if action (s_{N-1}) is negative, then

$$s_{N-1} = -\frac{[2(B_{N-1}u_{N-2}+1)(B_{N-1}-A_{N-1})+(A_N^i+A_N^{i+1})+u_{N-2}\{(A_N^i)^2+(A_N^{i+1})^2\}]}{2(B_{N-1}-A_{N-1})^2+\{(A_N^i)^2+(A_N^{i+1})^2\}}$$

If the results are inconsistent (i.e., s_{N-1} des not have the expected sign) then no change in allocation pattern is required. Thus $u_{N-1} = u_{N-2}$.

Case 2

$$P_N^i > \frac{1+\lambda_2}{1-\lambda_1} \text{ and } \frac{1-\lambda_2}{1+\lambda_1} < P_N^{i+1} < \frac{1+\lambda_2}{1-\lambda_1}$$

if action (s_{N-1})is positive, then

$$s_{N-1} = -\frac{[2(B_{N-1}u_{N-2}+1)(A_{N-1}+B_{N-1})+(A_N^i+B_N^{i+1})+u_{N-2}\{(A_N^i)^2+(B_N^{i+1})^2\}]}{2(A_{N-1}+B_{N-1})^2+\{(A_N^i)^2+(B_N^{i+1})^2\}}$$

if action (s_{N-1}) is negative, then

$$s_{N-1} = -\frac{[2(B_{N-1}u_{N-2}+1)(B_{N-1}-A_{N-1})+(A_N^i+B_N^{i+1})+u_{N-2}\{(A_N^i)^2+(B_N^{i+1})^2\}]}{2(B_{N-1}-A_{N-1})^2+\{(A_N^i)^2+(B_N^{i+1})^2\}}$$

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Case 3

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If the results are inconsistent (i.e., s_{N-1} des not have the expected sign) then no change in allocation pattern is required. Thus $u_{N-1} = u_{N-2}$.

Case 3

$$\frac{1-\lambda_2}{1+\lambda_1} < P_N^i,\ P_N^{i+1} < \frac{1+\lambda_2}{1-\lambda_1}$$

if action (s_{N-1}) is positive, then

$$s_{N-1} = -\frac{[2(B_{N-1}u_{N-2}+1)(A_{N-1}+B_{N-1}) + (B_N^i + B_N^{i+1}) + u_{N-2}\{(B_N^i)^2 + (B_N^{i+1})^2\}]}{2(A_{N-1}+B_{N-1})^2 + \{(B_N^i)^2 + (B_N^{i+1})^2\}}$$

if action (s_{N-1}) is negative, then

$$s_{N-1} = -\frac{[2(B_{N-1}u_{N-2}+1)(B_{N-1}-A_{N-1}) + (B_N^i + B_N^{i+1}) + u_{N-2}\{(B_N^i)^2 + (B_N^{i+1})^2\}]}{2(B_{N-1}-A_{N-1})^2 + \{(B_N^i)^2 + (B_N^{i+1})^2\}}$$

If the results are inconsistent (i.e., s_{N-1} des not have the expected sign) then no change in allocation pattern is required. Thus $u_{N-1} = u_{N-2}$.

Case 4

$$\frac{1-\lambda_2}{1+\lambda_1} < P_N^i < \frac{1+\lambda_2}{1-\lambda_1}, \text{ and } P_N^{i+1} < \frac{1-\lambda_2}{1+\lambda_1}$$

if action (s_{N-1}) is positive, then

$$s_{N-1} = -\frac{[2(B_{N-1}u_{N-2}+1)(A_{N-1}+B_{N-1}) + (B_N^i - A_N^{i+1}C_N^{i+1}) + u_{N-2}\{(B_N^i)^2 + (A_N^{i+1})^2\}]}{2(A_{N-1}+B_{N-1})^2 + \{(B_N^i)^2 + (A_N^{i+1})^2\}}$$

if action (s_{N-1}) is negative, then

$$s_{N-1} = -\frac{[2(B_{N-1}u_{N-2}+1)(B_{N-1}-A_{N-1})+(B_N^i - A_N^{i+1}C_N^{i+1})+u_{N-2}\{(B_N^i)^2+(A_N^{i+1})^2\}]}{2(B_{N-1}-A_{N-1})^2+\{(B_N^i)^2+(A_N^{i+1})^2\}}$$

If the results are inconsistent (i.e., s_{N-1} des not have the expected sign) then no change in allocation pattern is required. Thus $u_{N-1} = u_{N-2}$.

<u>Case 5</u>

$$P_N^i, P_N^{i+1} < \frac{1-\lambda_2}{1+\lambda_1}$$

if action (s_{N-1}) is positive, then

$$s_{N-1} = -\frac{[2(B_{N-1}u_{N-2}+1)(A_{N-1}+B_{N-1})-(C_N^i A_N^i + A_N^{i+1}C_N^{i+1})+u_{N-2}\{(A_N^i)^2+(A_N^{i+1})^2\}]}{2(A_{N-1}+B_{N-1})^2+\{(A_N^i)^2+(A_N^{i+1})^2\}}$$

if action (s_{N-1}) is negative, then

$$s_{N-1} = -\frac{[2(B_{N-1}u_{N-2}+1)(B_{N-1}-A_{N-1})-(C_N^i A_N^i + A_N^{i+1}C_N^{i+1})+u_{N-2}\{(A_N^i)^2+(A_N^{i+1})^2\}]}{2(B_{N-1}-A_{N-1})^2+\{(A_N^i)^2+(A_N^{i+1})^2\}}$$

If the results are inconsistent (i.e., s_{N-1} des not have the expected sign) then no change in allocation pattern is required. Thus $u_{N-1} = u_{N-2}$.

<u>Case 6</u>

$$P_N^i > \frac{1+\lambda_2}{1-\lambda_1}, \text{ and } P_N^{i+1} < \frac{1-\lambda_2}{1+\lambda_1}$$

if action (s_{N-1}) is positive, then

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$$= -\frac{[2(B_{N-1}u_{N-2}+1)(A_{N-1}+B_{N-1})+(A_N^i - A_N^{i+1}C_N^{i+1})+u_{N-2}\{(A_N^i)^2+(A_N^{i+1})^2\}]}{2(A_{N-1}+B_{N-1})^2+\{(A_N^i)^2+(A_N^{i+1})^2\}}$$

if action (s_{N-1}) is negative, then

s_{N-1}

$$= -\frac{[2(B_{N-1}u_{N-2}+1)(B_{N-1}-A_{N-1})+(A_N^i - A_N^{i+1}C_N^{i+1})+u_{N-2}\{(A_N^i)^2+(A_N^{i+1})^2\}]}{2(B_{N-1}-A_{N-1})^2+\{(A_N^i)^2+(A_N^{i+1})^2\}}$$

If the results are inconsistent (i.e., s_{N-1} des not have the expected sign) then no change in allocation pattern is required. Thus $u_{N-1} = u_{N-2}$.

Using the above expressions for any u_{N-2}, u_{N-1} can be computed. Thus the solution procedure evaluates optimal u_{N-1} and u_N for alternative actions at each of the previous stages (i.e., u_0, u_1, u_2, u_j, $\ldots u_{N-2}$), and for the combination of actions that obtains C are the optimal set of actions.

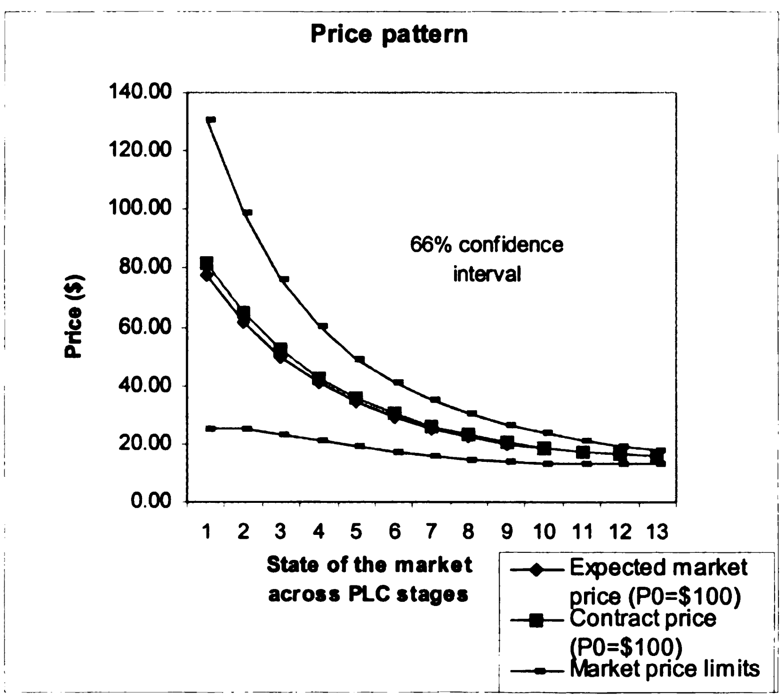
5.3 Illustration of the model application

The application of the above numerical model for 4 time stages as described in Figure 5.1 is illustrated with respect to the hypothetical data set presented in Table 5.1. This is the same dataset that is used in previous chapters. This data represent the typical price pattern of a product over time across different stages of the PLC. The price pattern is graphically presented again in Figure 5.2 for brevity. It may be noted that the competitive market price is on an average lower than the contract prices.

Table 5.1: Illustrative data representing the price dynamics across the PLC stages

State of the market across the PLC stages	Expected market price annual growth rates 'α_1'	Contract price annual growth rates 'α_2'	Difference in contract and market price growth rates 'α'	Standard deviation parameter of market price process 'β_1'	Expected market price: initial price: $100	Contract price: initial price: $100	Standard deviation of market price
1	-0.25	-0.2	0.05	0.75	77.88	81.87	52.70
2	-0.232	-0.185	0.047	0.67	61.75	64.73	36.86
3	-0.214	-0.17	0.044	0.6	49.86	52.10	26.50
4	-0.196	-0.155	0.041	0.54	40.98	42.70	19.60
5	-0.178	-0.14	0.038	0.49	34.30	35.63	14.96
6	-0.16	-0.125	0.035	0.45	29.23	30.27	11.80
7	-0.142	-0.11	0.032	0.42	25.36	26.18	9.66
8	-0.124	-0.095	0.029	0.39	22.40	23.06	8.02
9	-0.106	-0.08	0.026	0.35	20.15	20.68	6.54
10	-0.088	-0.065	0.023	0.3	18.45	18.88	5.19
11	-0.07	-0.05	0.02	0.25	17.20	17.55	4.07
12	-0.052	-0.035	0.017	0.2	16.33	16.61	3.13
13	-0.034	-0.02	0.014	0.15	15.79	16.01	2.30

Figure 5.2: Characteristic (hypothetical) price pattern across different stages of the PLC



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The computational illustration is carried out in two stages. First, the model is applied to a base case of switching cost ratio (SCR) (e.g., $\frac{\lambda_2}{\lambda_1}$) and initial relative price (P_0). Subsequently, switching cost ratios and initial relative price are varied to conduct sensitivity analysis to examine the impact of changes in switching cost and initial prices. In all cases of analysis, the model considers $'u_j'$ at an interval of 0.1 (i.e.,$'u_j \in [0,\ 0.1,....1.0]'$) for the first six nodes. Thus the base case considers beginning relative price of $P_0 = 1$ (i.e., $P_1(0) = P_2(0)$), switching costs of $\lambda_1 = 0.01$, and $\lambda_2 = 0.1$ and $\Delta t = 0.25$. Subsequently, sensitivity analysis is carried out for a range of relative price and switching cost ratios (e.g., $P_0 = [0.95, 0.96, 0.97, 0.98, 0.99]$ and $\frac{\lambda_2}{\lambda_1} = [1, 5, ...50]$ when $\lambda_1 = 0.01$) to examine how the optimal sourcing patterns change in different stages of PLC.

Since, the supply market is likely to be more competitive in later stages, contract readjustment costs are likely to be smaller towards the later stages of PLC. Hence, the upper limit of switching cost ratios in later stages of the PLC is kept smaller than in earlier stages. Thus the application of the model is illustrated in terms of finding the optimal sourcing pattern for a risk averse buying firm with expense disutility function (x^n) with $n = 2$, when the firm procures four times in a planning horizon while incurring switching costs to reallocate units sourced from alternative arrangements.

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In the previous two chapters, it was seen that continuous time models with continuous expense that do not consider switching costs yield optimal fractions of procurement across arrangements to be somewhat constant for a specified planning horizon. In contrast, the current discrete time model with switching costs allows the optimal pattern of procurement to vary over the planning horizon. This is because the procurement pattern depends upon past history i.e., the state of prices and previous decision patterns influence the optimal pattern in the current time period. The characteristic pattern of optimal fractions of units sourced from the market over time across early, intermediate and late stages of PLC for a varied range of beginning relative price and switching cost ratios are presented in Tables 5.2-5.7. Graphical representations of the price and sourcing patterns are presented in Figures 5.3-5.12.

Table

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1

Table 5.2: Optimal fraction of units to source from market for beginning relative price $P(0)=1$ and varied switching cost ratio (SCR) $\frac{\lambda_2}{\lambda_1}$ with $\lambda_1 = 0.01$ at the early market state '1'.

Nodes	Relative price $P = P_1 / P_2$ over time and switching cost ratio 'SCR'						
	Relative Price	**SCR: 1**	**SCR: 20**	**SCR: 30**	**SCR: 35**	**SCR: 40**	**SCR: 50**
0	1	0	0	0	0	0	0
1	1.44	0	0	0	0	0	0
2	0.68	1	1	1	0.9	0.6	0.3
3	2.06	0	0	0	0	0	0
4	0.98	0	0	0	0	0	0
5	0.98	1	1	1	0.9	0.6	0.3
6	0.46	1	1	1	1	1	1
7	2.97	0	0	0	0	0	0
8	1.40	0	0	0	0	0	0
9	1.40	0	0	0	0	0	0
10	0.66	1	1	1	1	1	0.6
11	1.40	0	0	0	0	0	0.0
12	0.66	1	1	1	1	1	0.8
13	0.66	1	1	1	1	1	1.0
14	0.31	1	1	1	1	1	1.0
15	4.26	0	0	0	0	0	0.0
16	2.01	0	0	0	0	0	0.0
17	2.01	0	0	0	0	0	0.0
18	0.95	1	0	0	0	0	0.0
19	2.01	0	0	0	0	0	0.0
20	0.95	1	0	0	0	0	0.0
21	0.95	1	1	1	1	1	0.6
22	0.45	1	1	1	1	1	1.0
23	2.01	0	0	0	0	0	0.0
24	0.95	1	0	0	0	0	0.0
25	0.95	1	1	1	1	1	0.8
26	0.45	1	1	1	1	1	1.0
27	0.95	1	1	1	1	1	1.0
28	0.45	1	1	1	1	1	1.0
29	0.45	1	1	1	1	1	1.0
30	0.21	1	1	1	1	1	1.0

Table 5.3: Optimal fraction of units to source from market for switching cost ratio $\frac{\lambda_2}{\lambda_1} = 10$ with $\lambda_1 = 0.01$ and varied beginning relative price $P(0)$ at the early market state '1'.

	Relative Price $P = P_1 / P_2$ at time t=0					
Nodes	**0.95**	**0.96**	**0.97**	**0.98**	**0.99**	**1**
0	1	1	1	0.3	0	0
1	0	0	0	0	0	0
2	1	1	1	1	1	1
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	1	1	1	1	1	1
6	1	1	1	1	1	1
7	0	0	0	0	0	0
8	0	0	0	0	0	0
9	0	0	0	0	0	0
10	1	1	1	1	1	1
11	0	0	0	0	0	0
12	1	1	1	1	1	1
13	1	1	1	1	1	1
14	1	1	1	1	1	1
15	0	0	0	0	0	0
16	0	0	0	0	0	0
17	0	0	0	0	0	0
18	0	0	0	0	0	0
19	0	0	0	0	0	0
20	0	0	0	0	0	0
21	1	1	1	1	1	1
22	1	1	1	1	1	1
23	0	0	0	0	0	0
24	0	0	0	0	0	0
25	1	1	1	1	1	1
26	1	1	1	1	1	1
27	1	1	1	1	1	1
28	1	1	1	1	1	1
29	1	1	1	1	1	1
30	1	1	1	1	1	1

Table

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Table 5.4: Optimal fraction of units to source from market for beginning relative price $P(0)=1$ and varied switching cost ratio $\frac{\lambda_2}{\lambda_1}$ with $\lambda_1 = 0.01$ at the intermediate market state '7'.

Nodes	Relative price $P = P_1 / P_2$ over time and switching cost ratio 'SCR'						
	Relative Price	SCR: 1	SCR: 20	SCR: 25	SCR: 30	SCR: 35	SCR: 40
0	1	0	0	0.1	0.1	0	0
1	1.22	0	0	0	0	0	0
2	0.80	1	1	0.9	0.5	0.2	0.1
3	1.50	0	0	0	0	0	0
4	0.98	0	0	0	0	0	0
5	0.98	1	1	0.9	0.5	0.2	0.1
6	0.65	1	1	1	1	1	1
7	1.83	0	0	0	0	0	0
8	1.20	0	0	0	0	0	0
9	1.20	0	0	0	0	0	0
10	0.79	1	1	1	1	0.50	0
11	1.20	0	0	0	0	0.03	0.1
12	0.79	1	1	1	1	0.63	0.1
13	0.79	1	1	1	1	1	1
14	0.52	1	1	1	1	1	1
15	2.24	0	0	0	0	0	0
16	1.47	0	0	0	0	0	0
17	1.47	0	0	0	0	0	0
18	0.97	1	0	0	0	0	0
19	1.47	0	0	0	0	0	0
20	0.97	1	0	0	0	0	0
21	0.97	1	1	1	1	0.50	0
22	0.64	1	1	1	1	1	0
23	1.47	0	0	0	0	0	0
24	0.97	1	0	0	0	0.03	0.1
25	0.97	1	1	1	1	0.63	0.1
26	0.64	1	1	1	1	1	0.1
27	0.97	1	1	1	1	1	1
28	0.64	1	1	1	1	1	1
29	0.64	1	1	1	1	1	1
30	0.42	1	1	1	1	1	1

Table 5.5: Optimal fraction of units to source from market for switching cost ratio $\frac{\lambda_2}{\lambda_1} = 10$ with $\lambda_1 = 0.01$ and varied beginning relative price $P(0)$ at the intermediate market state '7'.

Nodes	Relative Price $P = P_1 / P_2$ at time t=0					
	0.95	0.96	0.97	0.98	0.99	1
0	1	1	1	1	0.2	0
1	0	0	0	0	0	0
2	1	1	1	1	1	1
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	1	1	1	1	1	1
6	1	1	1	1	1	1
7	0	0	0	0	0	0
8	0	0	0	0	0	0
9	0	0	0	0	0	0
10	1	1	1	1	1	1
11	0	0	0	0	0	0
12	1	1	1	1	1	1
13	1	1	1	1	1	1
14	1	1	1	1	1	1
15	0	0	0	0	0	0
16	0	0	0	0	0	0
17	0	0	0	0	0	0
18	0	0	0	0	0	0
19	0	0	0	0	0	0
20	0	0	0	0	0	0
21	1	1	1	1	1	1
22	1	1	1	1	1	1
23	0	0	0	0	0	0
24	0	0	0	0	0	0
25	1	1	1	1	1	1
26	1	1	1	1	1	1
27	1	1	1	1	1	1
28	1	1	1	1	1	1
29	1	1	1	1	1	1
30	1	1	1	1	1	1

Table 5.6: Optimal fraction of units to source from market for beginning relative price $P(0) = 1$ and varied switching cost ratio $\frac{\lambda_2}{\lambda_1}$ with $\lambda_1 = 0.01$ at the late market state '13'.

	Relative price $P = P_1 / P_2$ over time and switching cost ratio 'SCR'					
Nodes	**Relative Price**	**SCR: 1**	**SCR: 5**	**SCR: 10**	**SCR: 15**	**SCR: 20**
0	1	0	0	0.100	0.4	0.4
1	1.07	0	0	0.000	0.4	0.4
2	0.92	1	1	1.000	0.5	0.4
3	1.15	0	0	0.000	0	0
4	0.99	0	0	0.000	0.4	0.4
5	0.99	1	1	1.000	0.5	0.4
6	0.85	1	1	1.000	1	1
7	1.24	0	0	0.000	0	0
8	1.07	0	0	0.000	0	0
9	1.07	0	0	0.000	0.4	0.4
10	0.92	1	1	1.000	0.59	0.4
11	1.07	0	0	0.017	0.5	0.4
12	0.92	1	1	1.000	0.66	0.4
13	0.92	1	1	1.000	1	1
14	0.79	1	1	1.000	1	1
15	1.33	0	0	0.000	0	0
16	1.15	0	0	0.000	0	0
17	1.15	0	0	0.000	0	0
18	0.99	0	0	0.000	0	0
19	1.15	0	0	0.000	0.4	0.4
20	0.99	0	0	0.000	0.4	0.4
21	0.99	1	1	1.000	0.59	0.4
22	0.85	1	1	1.000	0.59	0.4
23	1.15	0	0	0.000	0.5	0.4
24	0.99	0	0	0.017	0.5	0.4
25	0.99	1	1	1.000	0.66	0.4
26	0.85	1	1	1.000	0.66	0.4
27	0.99	1	1	1.000	1	1
28	0.85	1	1	1.000	1	1
29	0.85	1	1	1.000	1	1
30	0.73	1	1	1.000	1	1

Table 5.7: Optimal fraction of units to source from market for switching cost ratio $\frac{\lambda_2}{\lambda_1} = 10$ with $\lambda_1 = 0.01$ and varied beginning relative price $P(0)$ at the late market state '13'.

	Relative Price $P = P_1 / P_2$ at time t=0					
Nodes	0.95	0.96	0.97	0.98	0.99	1
0	1	1	1	1	1	0
1	1	1	1	1	0.6	0
2	1	1	1	1	1	1
3	0	0	0	0	0	0
4	1	1	1	1	0.6	0
5	1	1	1	1	1	1
6	1	1	1	1	1	1
7	0	0	0	0	0	0
8	0	0	0	0	0	0
9	1	1	1	1	0.6	0
10	1	1	1	1	1	1
11	1	1	1	1	1	0
12	1	1	1	1	1	1
13	1	1	1	1	1	1
14	1	1	1	1	1	1
15	0	0	0	0	0	0
16	0	0	0	0	0	0
17	0	0	0	0	0	0
18	0	0	0	0	0	0
19	1	1	0	0	0	0
20	1	1	1	1	0.6	0
21	1	1	1	1	1	1
22	1	1	1	1	1	1
23	1	1	0	0	0	0
24	1	1	1	1	1	0
25	1	1	1	1	1	1
26	1	1	1	1	1	1
27	1	1	1	1	1	1
28	1	1	1	1	1	1
29	1	1	1	1	1	1
30	1	1	1	1	1	1

Figure 5.3: The binomial representation of the evolution of relative price process over one year for $P(0) = P_1 / P_2 = 1.0$ in the early market state '1'of PLC.

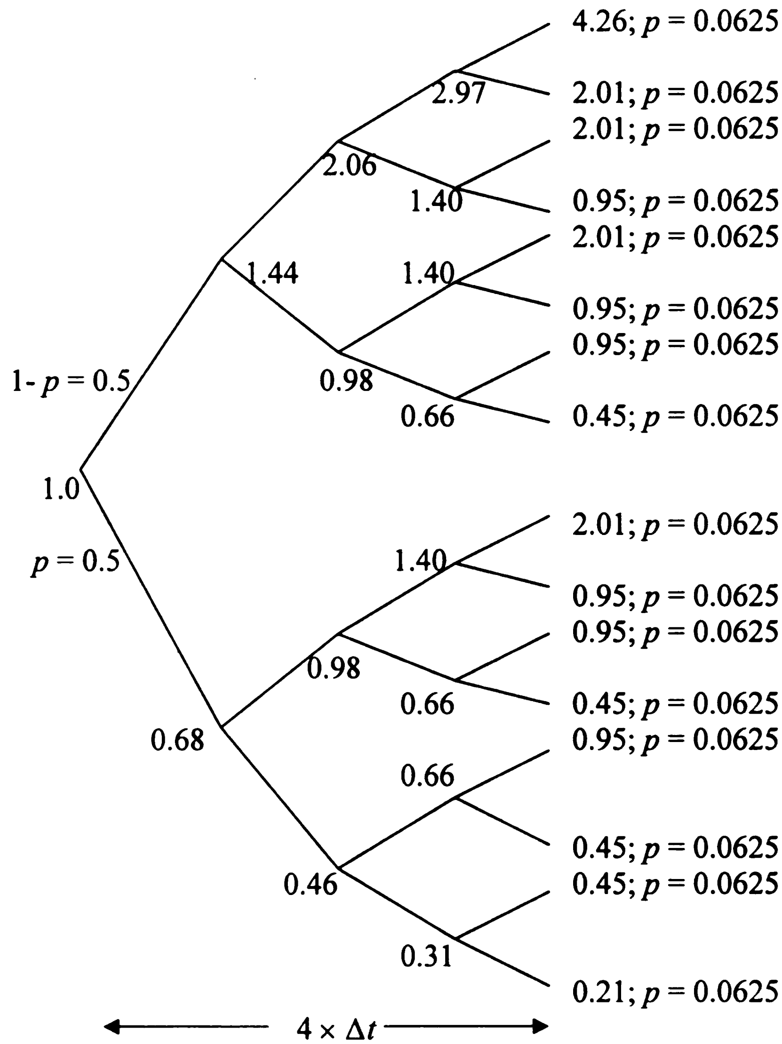

Figure 5.4: Optimal fraction of units 'u' to source from market over one year for $P(0) = P_1 / P_2 = 1.0$ and switching cost ratio $\frac{\lambda_2}{\lambda_1} = 1$ with $\lambda_1 = 0.01$ in the early market state '1' of PLC.

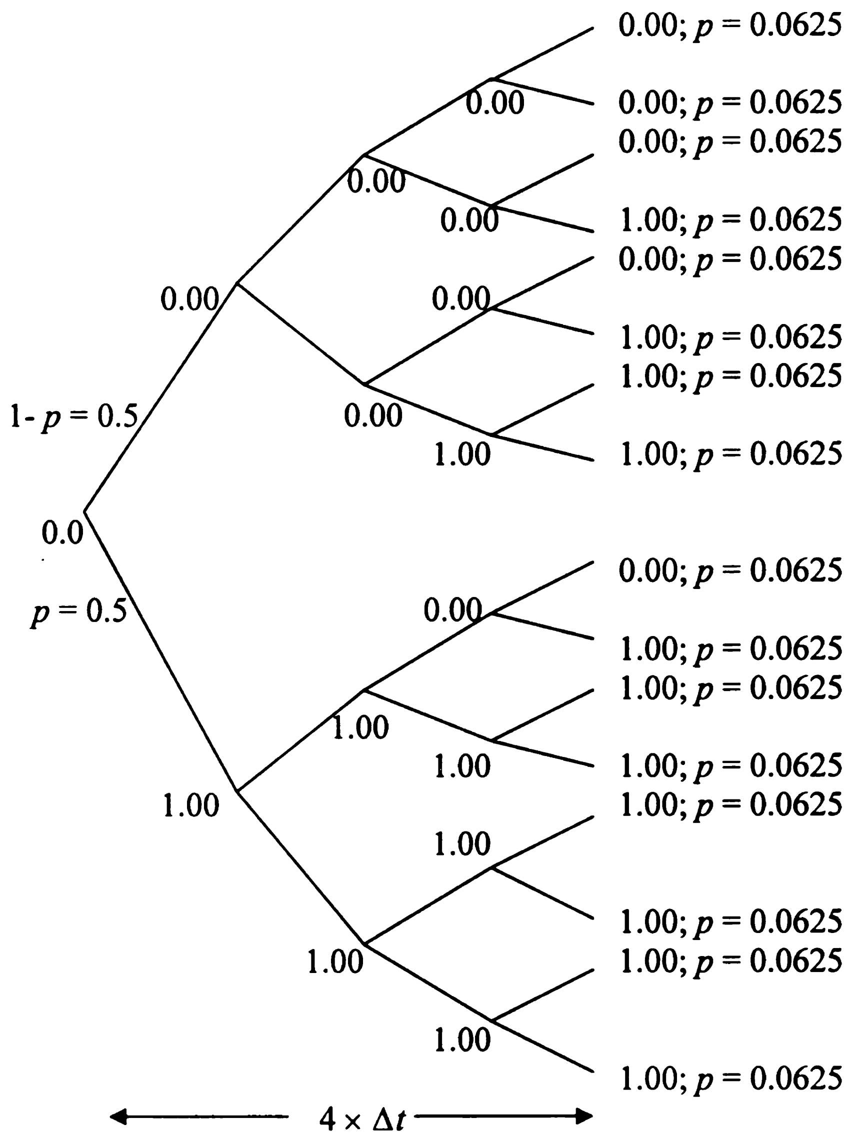

Figure 5.5: Optimal fraction of units 'u' to source from market over one year for $P(0) = P_1 / P_2 = 1.0$ and switching cost ratio $\frac{\lambda_2}{\lambda_1}$=50 with $\lambda_1 = 0.01$ in the early market state '1' of PLC.

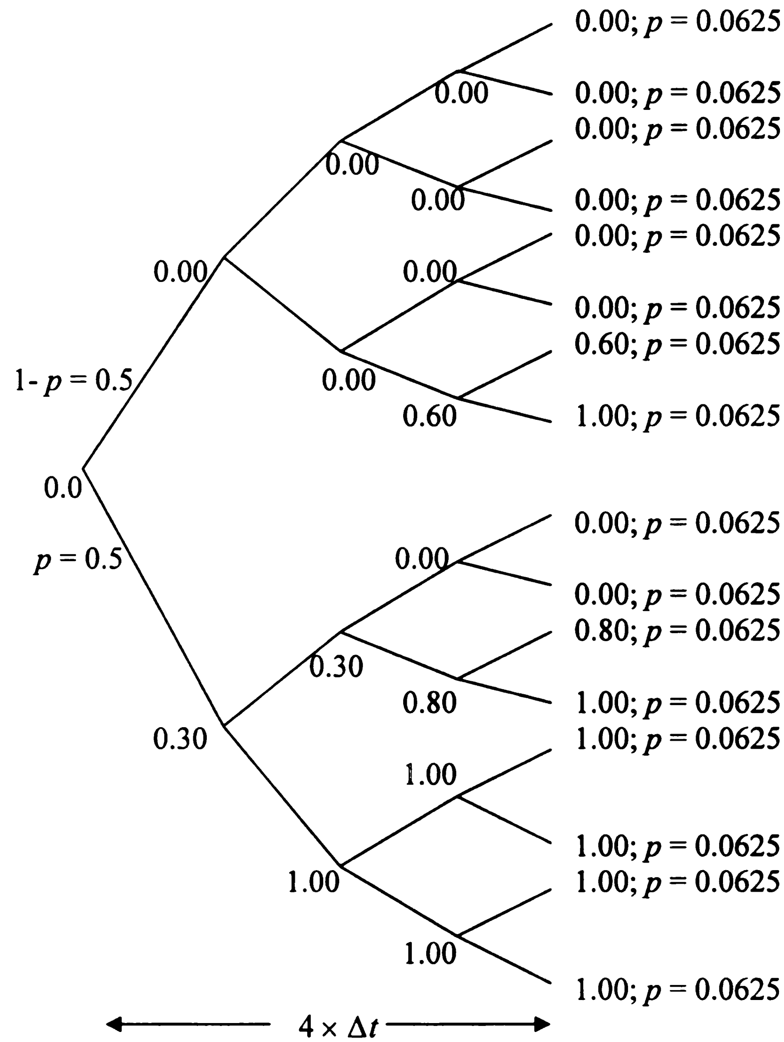

Figure 5.6: Optimal fraction of units 'u' to source from market over one year for $P(0) = P_1 \; / \, P_2 = 1.0$ and switching cost ratio $\dfrac{\lambda_2}{\lambda_1}$=10 with $\lambda_1 = 0.01$ in the early market state '1' of PLC.

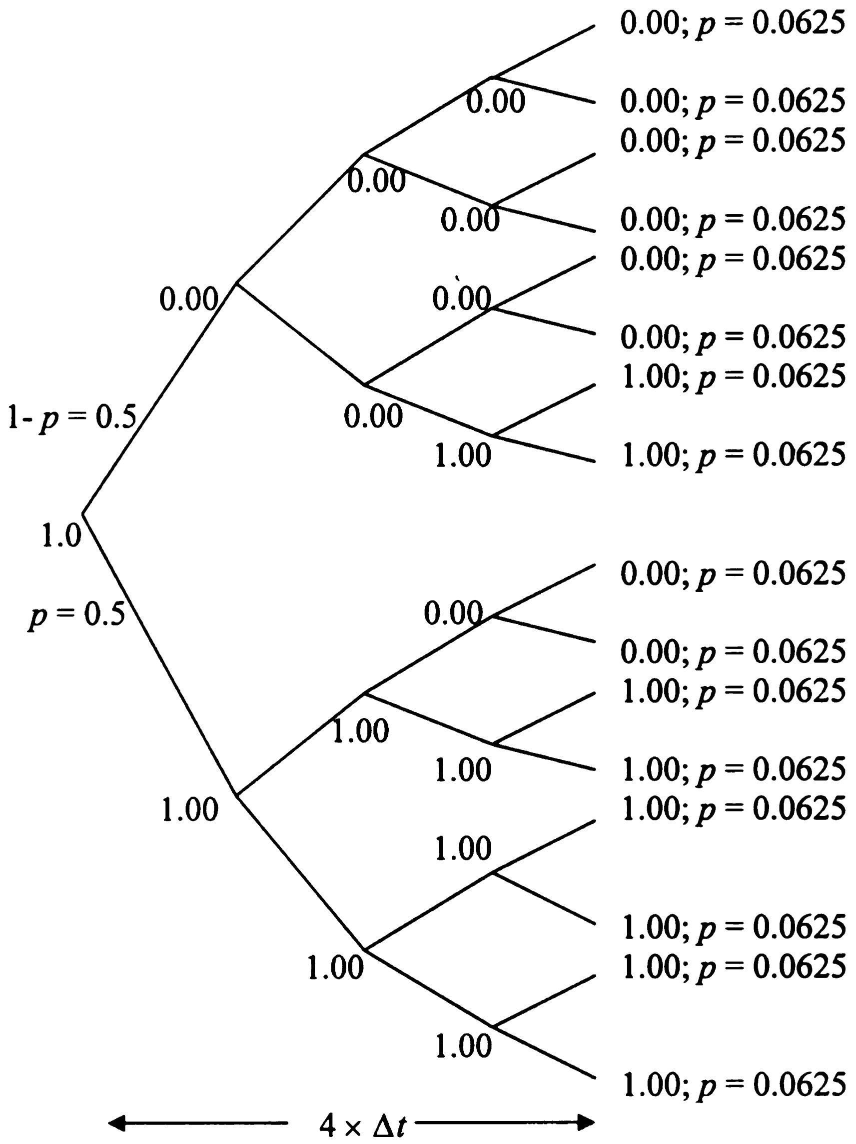

Figure 5.7: Optimal fraction of units 'u' to source from market over one year for $P(0) = P_1 / P_2 = 1.0$ and switching cost ratio $\frac{\lambda_2}{\lambda_1}$=25 with $\lambda_1 = 0.01$ in the intermediate market state '7' of PLC.

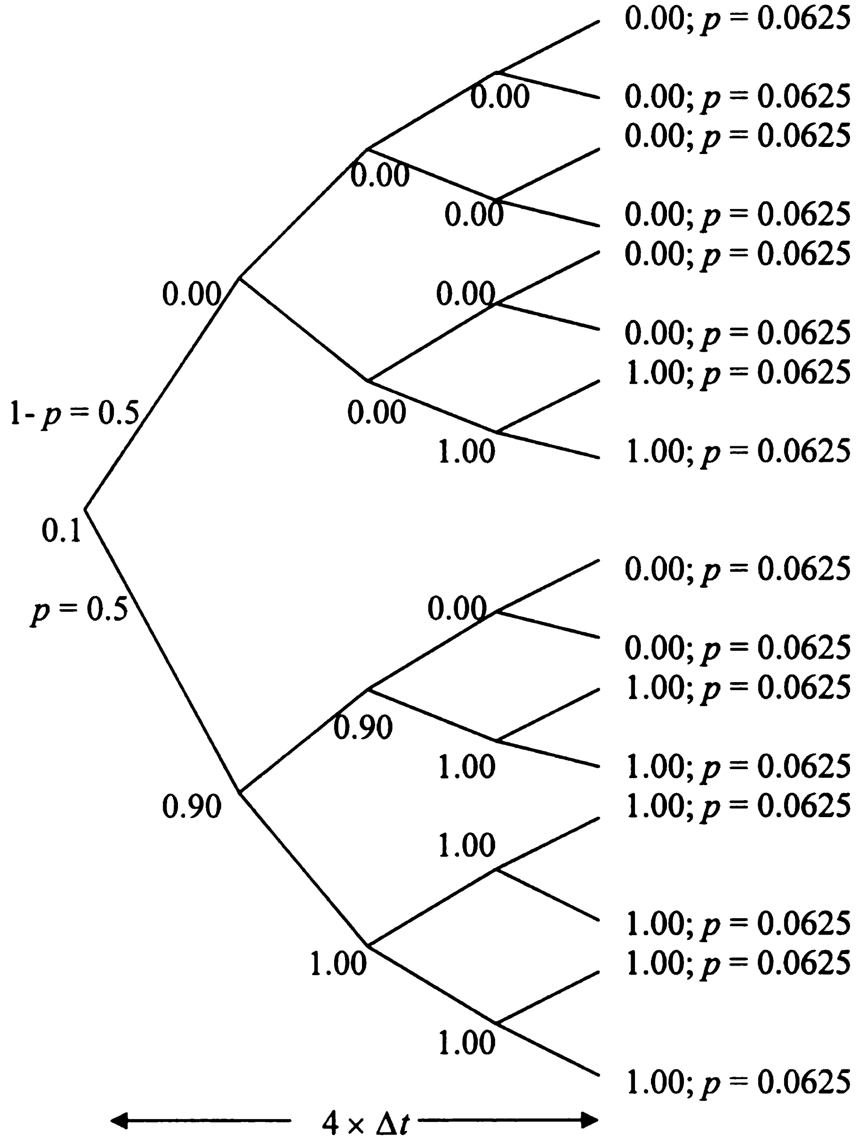

Figure 5.8: Optimal fraction of units 'u' to source from market over one year for $P(0) = P_1 / P_2 = 1.0$ and switching cost ratio $\frac{\lambda_2}{\lambda_1} = 40$ with $\lambda_1 = 0.01$ in the intermediate market state '7' of PLC.

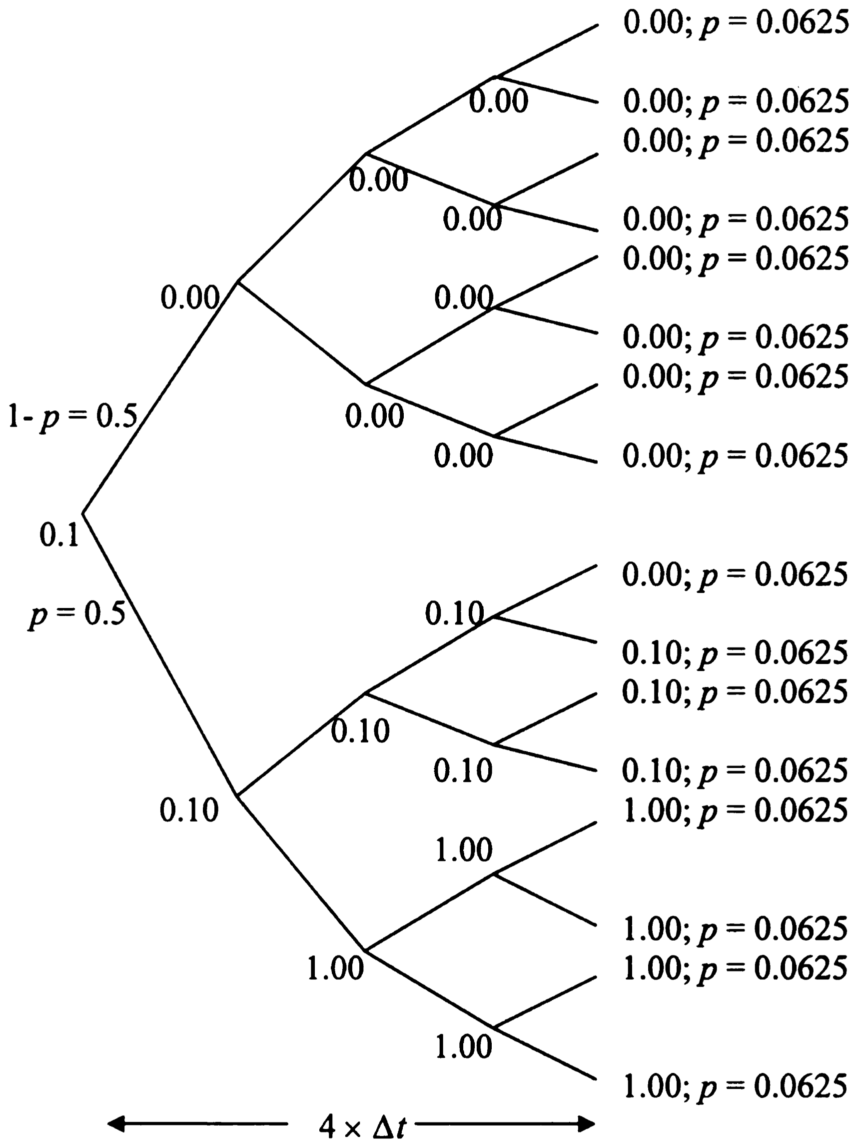

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Figure 5.9: Optimal fraction of units 'u' to source from market over one year for $P(0) = P_1 \ / P_2 = 1.0$ and switching cost ratio $\frac{\lambda_2}{\lambda_1} = 1$ and 5 with $\lambda_1 = 0.01$ in the late market state '13'of PLC.

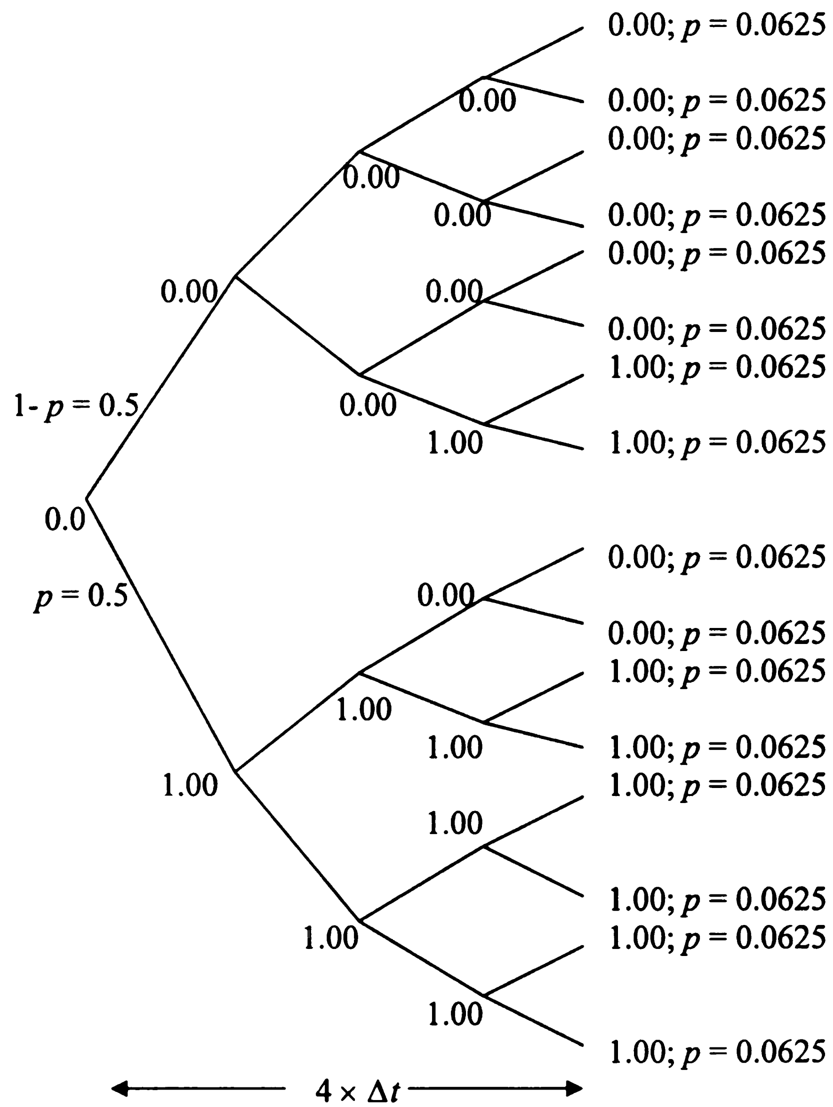

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Figure 5.10: Optimal fraction of units 'u' to source from market over one year for $P(0) = P_1 / P_2 = 1.0$ and switching cost ratio $\frac{\lambda_2}{\lambda_1} = 20$ with $\lambda_1 = 0.01$ in the late market state '13'of PLC.

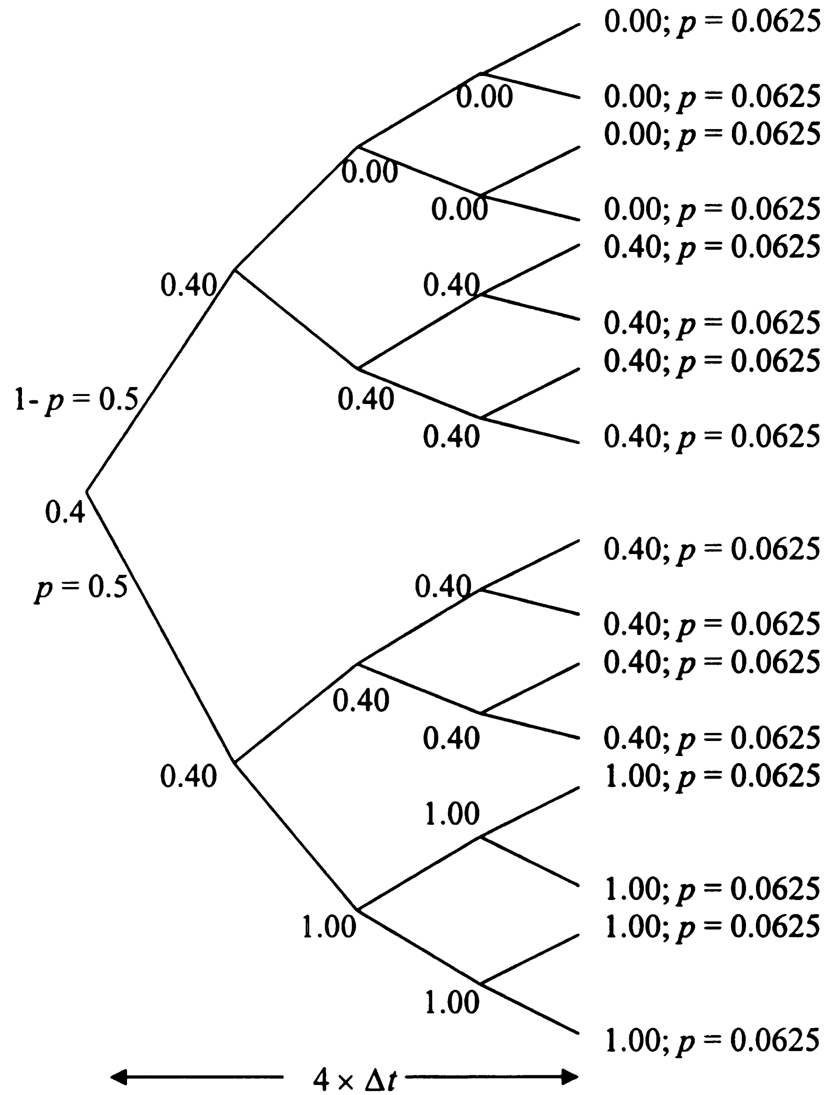


Figure 5.11: Optimal fraction of units 'u' to source from market over one year for $P(0) = P_1 / P_2 = 1.0$ and switching cost ratio $\frac{\lambda_2}{\lambda_1} = 20$ with $\lambda_1 = 0.01$ in the alternative market states '1', '7', and '13' of PLC. The numbers in the parentheses represent the optimal fractions in states 1, 7, and 13 respectively.

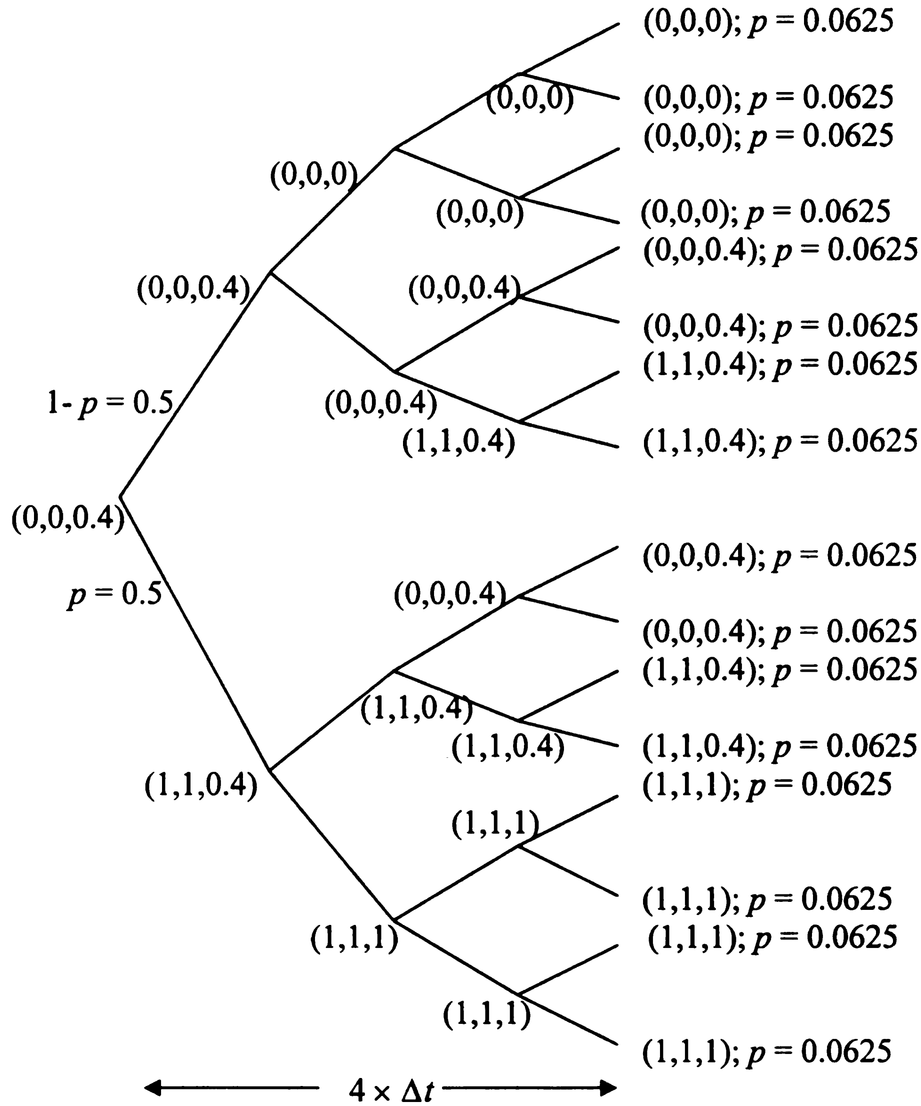
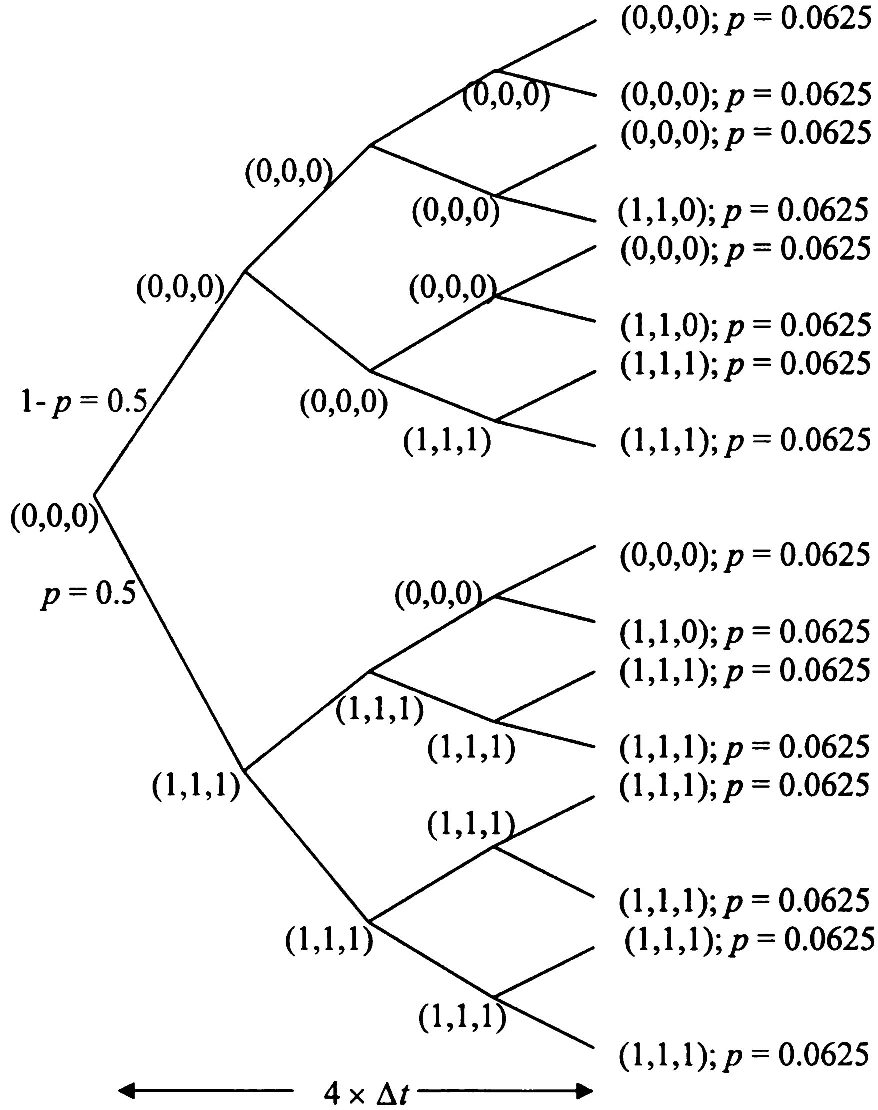


Figure 5.12: Optimal fraction of units 'u' to source from market over one year for $P(0) = P_1 / P_2 = 1.0$ and switching cost ratio $\frac{\lambda_2}{\lambda_1} = 1$ with $\lambda_1 = 0.01$ in the alternative market states '1', '7', and '13' of PLC. The numbers in the parentheses represent the optimal fractions in states 1, 7, and 13 respectively.



The results are data dependent; however, similar computations can be carried out using relevant contextual data to gain firm, industry or product specific (managerial) insights. The results indicate the following general patterns.

a) The optimal sourcing pattern is not always of the '*bang-bang*' variety i.e., it is optimal to procure all units from the cheaper alternative at a given time. This result is in contrast to the bang-bang type of outcome that occurs when the buying firm is assumed to be risk neutral (Kouvelis, 1999; Li and Kouvelis 1999). It is observed that for specified price parameters, switching costs, planning horizon and initial prices, there are threshold price levels or price bands at which the optimal fractions to procure from an arrangement need to be readjusted over time. For example, for a typical price pattern and a set of switching cost ratio, it is observed from the sourcing pattern in the early stage of the PLC as presented in Table 5.2 and described in Figures 5.4-5.6, that though the optimal sourcing arrangement is purely contract type at the beginning, it is optimal to switch a specified fraction (i.e., 1 or 0.3) of procurement to market arrangement when the relative price falls to 0.68, and to continue with the specified level of market arrangement until the relative price goes up to 1.40 or drops further. Thus when the relative price lies between 0.68 and 1.40, it is optimal to maintain the status quo until the last transaction at the terminal time. These patterns are somewhat similar to the concept of target optimal portfolio boundaries described in financial literature

(Shreve and Soner, 1994). As per the aforesaid financial literature, the portfolio (e.g., ratio of risky shares to risk less bonds) space can be divided into three disjoint regions, which can be specified as the buy region, the sell region, and the no trading or status quo region. The implications are: the initial portfolio structure need not be changed until prices of equity shares hit specified limits; when prices hit these limits, portfolio can be rebalanced for certain target structure by buy or sell actions. It is also observed that the optimal pattern of sourcing over time depends upon the duration of (remaining) operating horizon, and the switching cost ratios. The longer the (remaining) time horizon, the wider is the price band in which the status quo may be maintained; and the greater is the switching cost ratios, the lesser the magnitude of readjustment for a specified price band. These characteristics are similar to the concept of hysteresis that describes the inertia against (quick) readjustment in the presence of transaction cost and price uncertainty (Dixit, 1989). It may be noted that the optimal pattern of sourcing also depends upon the initial prices. Typically, the lesser the market to contract price ratio the greater is the initial allocation to market (Table 5.3). As evident from Tables 5.4-5.7 and Figures 5.7-5.10, the above general patterns of sourcing are observed across intermediate (market state '7') and late stages (market state '13') of PLC as well.

b) While, the general patterns of optimal fractions to source from alternative arrangements as described above across different market states are similar, the sensitivity of fractions to switching costs and initial prices vary across different market states. For example, comparing the results in Tables 5.3, 5.5 and 5.7 for varied relative prices at a specified switching cost ratio of 10 across early, intermediate and late stages of the PLC, it may be concluded that the propensity to procure from market arrangement is higher towards the later stages of the PLC for the same relative price at the beginning of planning horizon. This may be attributed to relatively greater (cost) benefit compared to the uncertainty by sourcing from the market towards the later stages. Similarly, a comparison of fractions sourced from alternative arrangements for switching cost ratios of 20 and 1 from Tables 5.2, 5.4, and 5.6 indicates that the buying firm should source more from the market arrangement in the later stages of PLC than in the earlier stages in a situation of relatively higher switching cost of contract arrangement. This implies relatively higher switching costs or penalties for readjusting units sourced from contractual arrangements are likely to be counterproductive for the contractual supplier in later stages of PLC. Consequently, it is in the interest of the contractual supplier to provide more contract flexibility, and provide matching market price in later stages.

c) The greater the ratio of market price uncertainty (β_1) to the difference in contract and market prices (growth rates), the lesser the probability of procurement from open market; accordingly, contract sourcing is more important in the early stage of the PLC; however, towards the later stages of the PLC, market sourcing gradually becomes more important. In the illustrative dataset presented in Table 5.1, the ratio of market price uncertainty (β_1) to the difference in contract and market prices (growth rates) declines from 15 in market state '1' through 13 in market state '7' to 10.7 in market state '13'. The optimal patterns of sourcing across early, intermediate and late stages of the PLC for specific switching cost ratios (i.e., 20 and 1) are graphically presented in Figures 5.11 and 5.12. It is apparent that, when the contract switching costs are relatively higher, as per the optimal strategy, the buying firm will have a higher propensity to buy from market throughout the operational planning horizon, and the sourcing arrangements would be more stable (e.g., number of switches are less and magnitudes of switch are smaller) in later stages of PLC than in earlier stages of PLC. However, it is to be noted that this stability is achieved with a price both for the buying and the contractual supplier firm. The supplying firm gets lesser allocation and the buying firm has to procure more from market thus being subjected to higher uncertainty.

The above characteristics of sourcing patterns suggest that the optimal strategy for a risk averse buying firm is not of '*bang-bang*' type. Furthermore, the sourcing strategies need to be redesigned in accordance with the attitudes towards risk, relative switching cost and state of prices over time across different procurement contexts. The results suggest that in the earlier stages it is generally optimal to buy from contractual arrangements and it is only advantageous to switch to market arrangements under very favorable market price conditions when the relative switching cost ratios are smaller. However, in later stages of PLC, it is optimal to buy more from market when the contract readjusting costs are relatively higher and market prices are more favorable. This implies the contractual supplier may be able to charge a higher penalty for contract readjustment only during the earlier stages and not towards the later stages. Correspondingly, the contractual supplier needs to be more responsive to market price patterns in the later stages than in the earlier stages. Cohen and Agrawal (1999) observed a somewhat similar pattern for cases of high fixed cost and stable market conditions in that it is optimal to buy from market arrangement (e.g., short term contract) at the beginning of planning horizon and not to switch to long-term arrangement in the planning horizon. The increased importance of market price based sourcing towards the later stage of PLC (when the product becomes standardized and functional) as competition intensifies and firms become more cost conscious conforms to the industry practice (Rink and Fox, 1999; Cohen and Agrawal 1999).

The model discussed here may be improved along the following lines. First, the proposed model helps to identify optimal fraction of units to source from alternative arrangements in the presence of proportional switching cost. The numerical results indicate there are many situations when it is optimal to completely procure from either contract or market states during the intermediate period of the operating horizon. This implies complete abandonment of a sourcing arrangement in those situations. It is to be noted that in reality a complete abandonment of a sourcing arrangement under very favorable prices of alternative arrangement though rational may be difficult to implement considering the implications on buyer-supplier relationships or the enormity of cost to restart the arrangement again in the future. Thus a simultaneous consideration of fixed and proportional switching cost would be more appropriate and may be considered as an area for future research. Second, the model illustrated here considers a power disutility function of cost. In reality, a buying firm's utility function may be more complex. Thus studies may be carried out for obtaining solutions for more general utility functions. The current numerical solution approach is limited by the exponential increase in numerical operations due to increase in solution space with finer discretization in time and fractions allocated. Research should be undertaken to develop efficient solution techniques that can substantially reduce the solution space thereby providing more precise estimation of optimal outcomes.

Chapter 6

Conclusion

This research makes a contribution to the sourcing literature by proposing a set of analytical models for formulating integrated contract and open market sourcing strategy. The modeling framework resembles the portfolio optimization in the finance literature. It is different from the existing approaches in the sourcing literature in that it considers the buying firm to be risk averse and allows for the simultaneous use of both the sourcing mechanisms. In contrast, the related sourcing literature predominantly considers the buying firm to be risk neutral and assumes the use of one sourcing alternative at a time unless there is a capacity constraint with the least cost supplier that justifies multiple arrangements simultaneously (Li and Kouvelis, 1999; Peleg and Lee, 2002; Kleindorfer and Wu, 2003; Martinez-de-Albeniz and Simchi-Levi, 2005).

The study by Cohen and Agrawal (1999) is probably the only one that considers the buying firm to be risk averse. Their model examines the trade-off between a high fixed cost long-term contract with allowance for cost reduction due to continuous learning (an adjustment mechanism), and price-uncertain short-term contract arrangement without any cost reduction opportunity, when these are adopted singularly. In practice, a buying firm is more likely to use a portfolio of long-term and the short-term contract arrangements simultaneously (Billington, 2002; Kleindorfer and Wu, 2003; Kleindorfer and Wu, 2005). This is because the market conditions are dynamic and uncertain, and single type of sourcing arrangement would entail either the risk of excessive supplier opportunism or unstable supply conditions. From this perspective, the proposed models with dual sourcing arrangement have more practical relevance.

In the present study three analytical models were developed. Two of these were continuous time models that do not consider the (indirect) transaction/switching cost and the other is a discrete time model that explicitly considers the (indirect) transaction/switching cost. These models solve for optimal fractions to be sourced from open market arrangements with 'uncertain prices' and contractual arrangements with 'deterministic prices'. A comparative discussion of the key characteristics of models, their relative advantages and limitations is presented in Figure 6.1. The models provide insights on two key areas: a) how do the tradeoffs between procurement cost and uncertainty influence the sourcing strategy of a risk averse buying firm, and b) what are the likely challenges in implementing the sourcing strategy, and how to address these challenges over time. The models are applicable to procurement contexts where product and service attributes can be specified objectively in a verifiable manner.

The proposed models make several theoretical contributions and provide significant managerial insights regarding the optimal sourcing strategy. These insights and directions for future research are discussed next.

Figure 6.1: A descriptive comparison of the three analytical models.

Model description	**Key findings**	**Advantages**	**Limitations**
Model-1 The continuous time model developed in Chapter 3 that derives optimal fractions of expense to allocate across contractual and market sourcing arrangements to minimize the procurement expense disutility, when the expense is continuous over time.	1. For equal contract and market prices at the beginning of planning horizon, the optimal fractions are constant and depend on price parameters, and buying firm's risk attitude. 2. For unequal contract and market prices at the beginning of planning horizon, the optimal fractions are stochastic and depend on price parameters, buying firm's risk attitude, instantaneous expense, and units of demand. 3. The optimal fractions do not depend on the duration of planning horizon. 4. The average units procured from contractual arrangement decline over time; the average units procured from market arrangement increase over time. However, the variance of units procured from both arrangements increase over time. The rates of these decline and increase are smaller for the contractual arrangement than for the market arrangement.	1. The model can obtain optimal fractions of expense for power disutility function x^n with n>1. Consequently, it can provide solutions for a wide range of power disutility functions of the type x^n. 2. The solution results in accrual of surplus units over time even when the expense disutility is minimized. This provides opportunity for indirect savings by selling back the surplus in secondary markets.	1. Accrual of surplus may be problematic when inventory cost is high. 2. The units procured from alternative arrangements are stochastic; this calls for quantity flexible contractual arrangements. 3. The approach involves solving of the HJB equation, which calls for trial functions that are difficult to guess for general disutility functions that are different from power disutility function.

Figure6.1 (cont'd)

Model description	**Key findings**	**Advantages**	**Limitations**
Model-2 The continuous time model in Chapter 4 that derives optimal fractions of units to be sourced across contractual and market sourcing arrangements to minimize the procurement expense disutility, when the expense is continuous over time.	1. The optimal fractions depend on price parameters, buying firm's risk attitude, and duration of planning horizon. 2. The fractions procured from market arrangements decline for longer planning horizons. 3. The fractions may be adjusted in jumps when the prices are equal during a planning horizon.	1. The approach can be extended to obtain optimal fractions for disutility function that are more general in functional form than the power disutility function x^n. 2. The solution exactly meets the demand and the units procured from the arrangements are deterministic.	1. The model is not amenable to power disutility function x^n with non-integer values for $n > 1$.

Figure 6.1 (cont'd)

Model description	**Key findings**	**Advantages**	**Limitations**
Model-3 The discrete time model time model in Chapter 5 that derives optimal fractions of units sourced across contractual and market sourcing arrangements to minimize the procurement expense disutility, when there is proportional switching cost due to the shift between arrangements	1. The optimal fractions depend on price parameters, buying firm's risk attitude, switching cost, and remaining duration of planning horizon. 2. The optimal fractions are path dependent and can be adjusted in jumps when the prices are favorable during the planning horizon. 3. The higher the switching cost, and the longer the remaining planning horizon, the greater the inertia against readjustment of fraction of units sourced across arrangements.	1. Accounts for the effect of proportional switching cost in shifting across the sourcing arrangements. 2. Provides for savings due to readjustment of units across the arrangements in discrete time.	1. The solution approach is numerical and requires finer discretization for precise results. However, exponential increase in computational effort with increase in discretization limits the precision. 2. Does not address the issue of fixed cost of switching across arrangements.

6.1 Theoretical contributions

The proposed models build on the theoretical frameworks on sourcing strategy and provide insights on the combined impact of price dynamics, risk aversion, and switching cost on a firm's sourcing strategy. The results suggest that a dual sourcing arrangement that provides strategic flexibility is preferable to single sourcing in uncertain procurement contexts.

The models indicate the optimal sourcing strategy described in terms of fractions of units to source from alternative arrangements depends upon the degree of risk aversion, price parameters, the switching cost, and the duration of the planning horizon. However, the optimal strategy described in terms of fractions to spend on alternative arrangements additionally depends upon units of demand and instantaneous expense. As a result, the units sourced from alternative arrangements are stochastic when the firm controls the fraction of expense. It is found from the models in Chapters 3 and 4 that *ceteris paribus,* the propensity to source from market is directly proportional to rate of contract price premium and inversely proportional to market price uncertainty and the buying firm's degree of risk aversion. Moreover, the propensity to source from market is convex in market price uncertainty and degree of risk aversion i.e., the rate of decline in market sourcing increases with increase in price uncertainty and degree of risk aversion. It is to be noted that the rate of contract price premium is governed by the relative bargaining strength and negotiation between buying firm and contractual supplier; this can act as a managerial lever to influence the sourcing strategy.

Consequently, the optimal sourcing strategy should be an outcome of uncertain product-market environment, risk attitude and managerial capability.

The application of the models on the illustrative data indicates that the sourcing strategy is dynamic and a buying firm needs to adjust the amount sourced from contractual and open market arrangement over time as the procured product advances through its product life cycle. In general, for a firm with constant risk attitude the amount sourced from open market should increase over time. This may be explained as the increasing attractiveness of open market procurement when the product becomes increasingly standard and the market becomes intensely competitive and liquid towards the later stages of the product life cycle (Mendeleson and Tunca, 2003). The results also conform to Kleindorfer and Wu (2003)'s conceptual framework that justifies greater suitability of market arrangement when the product attributes are easy to codify and cost of developing contractual arrangement is relatively high.

The dual arrangement with some sourcing from market arrangement in introduction stage, dominance of contractual arrangement in growth stage followed by the dominance of open market arrangement in the maturity stage of the procured product's life cycle indicates the importance of managing suppliers through a varying mix of relational and competitive arrangements across different stages of the PLC. Examination of the above results in light of transaction cost theory, agency theory, and resource based theory justifies the significance and scope for implementing such integrated sourcing strategy in terms of contextual relevance and functional appropriateness across different stages of the PLC.

To illustrate, the introduction stage of the PLC may be characterized by the simultaneous existence of technological, supplier capability, product design, and price uncertainties. In the face of these varied types of uncertainties, risk averse buying firms may significantly differ in terms of their risk attitudes towards the magnitudes and uncertainty of cost. While a relatively more aggressive buying firm with less concern for cost uncertainty may experiment with several suppliers to assess the future potential of building a long-term supply arrangement with a specific set of suppliers, a relatively less aggressive buying firm with more concern for cost uncertainty may limit the interactions to a few well-known suppliers. Correspondingly, the former type may emphasize trial purchases using short-term arrangements, which are comparable to market oriented sourcing arrangements. In contrast, the latter type may emphasize relatively more long-term contractual arrangements. Interestingly, the application of the models on the illustrative dataset of PLC oriented price patterns suggests such alternative sourcing strategies in the early stage the PLC.

These strategies can also be justified on the following theoretical grounds. Agency theory assumes that the actions of contracting parties are guided by opportunism, bounded rationality and risk aversion. It explains the principal-agent relationship in a contractual arrangement in terms of organization of information sharing for verification of actual behavior and for sharing of risk. Accordingly, there are two types of contracts: outcome based contracts e.g., market governance, and behavior based contracts e.g., hierarchical governance (Eisenhardt, 1989). In a sourcing context, it may be reasoned that short-term open market arrangement and long-term contract arrangement would correspond with Eisenhardt's outcome-based contract and behavior-based contract respectively.

Eisenhardt posits that there will be a preference for outcome-based contracts in the presence of goal incongruence between transacting firms; however, in the presence of difficulty in specifying tasks, there will be a preference for behavior-based contracts in the presence lower outcome (performance) measurability and higher outcome (performance) certainty. In the early stage of the procured product's life cycle, a buying firm may not settle on the product requirements and may therefore need to redefine the task definition frequently; furthermore, the supplying firm's goals may not clearly converge on that of the buying firm, when the product characteristics are incompletely described. Consequently, agency theory would suggest that a buying firm that recognizes the merit of changing specifications and measurement system for task performance in the early stage, and the capability of implementing such change in specifications and measurement system will emphasize a market arrangement. In contrast, a buying firm not capable of implementing such change in specifications and measurement system will emphasize a contract arrangement. Thus, it may be contended that the optimal strategy identified by the model in the introduction stage is in alignment with agency theory.

Transaction cost theory and resource-dependency theory make a case for alternative sourcing strategies (e.g., relatively higher and lower emphasis on contract arrangement) on the basis of a buying firm's relative attitude for reducing transaction costs and uncertainties (Noordewier et al., 1990). While more emphasis on trial purchases in the early stage of the PLC from multiple suppliers enables the buying firm to avoid the cost of risky dependence and idiosyncratic investments in (uncertain) product/technology required of contractual arrangements, more emphasis on contractual arrangements reduces uncertainty of transaction cost due to multiplicity of arrangements and facilitates development of relational arrangement, which if successful would be of higher value in future. Consequently, a buying firm may choose either of the arrangements, depending on the perceived value of the trade-off between transaction cost and high uncertainty. In the early stage, contractual arrangement may increase the transaction cost due to lesser transaction frequency, but this increased cost may be worthwhile considering the higher degree of uncertainty from open market sourcing arrangement.

The resource-based theory posits that firms need to acquire rare, tacit, difficult to imitate, causally ambiguous, and heterogeneously distributed resources to achieve sustainable competitive advantage (Barney, 1991). Since, in the early stage, it is not apparent which supplier would provide access to such critical resources, the resource-based perspective would suggest the buying firm should engage in exploring strengths and weakness of alternative suppliers through direct experience in early stages of the PLC. Accordingly, it may be conjectured that firms focusing on building resource based strategic capabilities are more likely to have lower cost related risk aversion and may gain from trial supply arrangements with multiple suppliers in the early stage of the PLC.

As the product advances to the growth stage, the sourcing context changes to one of greater stability regarding technological changes, increasingly standardized product design, higher competitive intensity, and higher uncertainty regarding availability of adequate amount of supply. Consequently, buying firms tend to be increasingly concerned about magnitude and uncertainty of input cost and volume of supply, and are likely to look for opportunities for cost reduction and increased certainty of supply and input cost (Rink and Fox, 1999). The applications of model on the data corresponding to this stage for a risk averse buying firm suggests a sourcing strategy with relatively higher emphasis on contractual arrangement. These results may also be justified from transaction cost, resource based and agency theory perspectives.

While, transaction theory suggests that transaction arrangements should reduce the direct and indirect costs of transaction and uncertainty, the resource-based theory recommends that a firm should have access to difficult to imitate critical resources. The buying firm's transactions costs increase with supplier specific (idiosyncratic) investments and uncontrollable transaction uncertainty, and decrease with frequency of transaction with a specific supplier. In the intermediate (growth) stage, the uncertainty regarding supplier capabilities, product features and functionalities decrease over time. There are opportunities for greater goal congruence between the buying and the supplier firms, lesser difficulty in task specification and performance measure and lesser information asymmetry. Also, there exists opportunities for higher volume of transaction over a sustained period of time with specific suppliers who have proven capabilities to satisfy the requirements of the product in the introduction stage. According to agency theory under such circumstance, the buying firm should choose long-term contractual arrangements. Such arrangement can: reduce the impact of asset specific investments over time; present opportunities for mutual learning by information sharing and sustained cost reduction through collaborative planning and control initiatives; offer learning curve benefits, and reduce uncertainties regarding supplies. These sequential outcomes can reduce transaction costs, and provide for the critical resources to the advantage of the buying firm. However, long-term arrangements can entail the risk of supplier opportunism, and lock-ins. Agency theory suggests that in case of such dysfunctional relationship with a supplier it may be in the interest of the buying firm to rely on outcome (market) based contracts to address the negative outcomes in a contract (Eisenhardt, 1989). This would enable the buying firm to maintain a competitive mechanism for the contractual supplier by making purchases through short-term or open market arrangements. For a buying firm this may however mean practices such as alternative sourcing arrangement, inducing short-term suppliers to make idiosyncratic investments, flexible/reversible supplier specific investments to reduce excessive dependence on a supplier, and redesign of incentives through periodic renegotiation of contracts etc. The optimal sourcing strategy in the intermediate stages suggested by the models is consistent with these theoretical suggestions. The models' results suggest integration of competitive open market and contractual sourcing arrangements that can lead to long-term competence building and process innovation while preventing dysfunctional contractual outcomes.

In the maturity phase of the PLC, the product becomes highly standardized with increase in competition among more or less homogeneous suppliers. The uncertainties regarding various non-price attributes, asset specificity, and availability of the product are negligible during this stage. Furthermore, the buying firm becomes aware of the true cost structure of suppliers and does not experience uncertainty regarding the availability of the product, leading to emphasis on cost-based procurement. Consequently, suppliers tend to compete on prices. Rink and Fox (1999) noted that buying firms should increase the market orientation during this stage. The application of models on the illustrative data corresponding to this stage also suggests a sourcing strategy with relatively higher emphasis on market-oriented arrangement, which may also be explained from transaction cost, resource based and agency theoretic perspectives.

The sourcing context in the maturity phase of the PLC is characterized by highest order of information symmetry, performance verifiability and certainty of outcomes. According to agency theory, the outcome (market) oriented contracts would be appropriate in such circumstances (Eisenhardt, 1989). It can also be argued that, the information symmetry, performance verifiability and certainty of outcomes in this stage can facilitate structured specification of tasks and goal congruence between the buying firm and the supplier, which can justify verifiable, long-term (behavior based) contractual arrangement (Eisenhardt, 1989). Thus agency theory does not clearly explain the suitability of a sourcing arrangement; however, the transaction costs theory perspective provides a better answer for the appropriate sourcing strategy in this context.

According to transaction cost theory, for a limitedly rational buying firm, competitive mechanism is likely to reduce transaction cost and uncertainty in a situation of lower risk of asset specificity and higher risk of supplier opportunism (Williamson, 1985). This implies in the maturity phase, since a buying firm is mainly exposed to the risk of supplier opportunism, market competition led contractual arrangements are likely to be most efficacious in ensuring certainty and minimal cost of transaction. Consequently the appropriate mechanism should be a combination of open market and market driven contractual arrangements. A buying firm may achieve this integration either by simultaneously procuring through both open market and contractual arrangements or by frequently modifying the terms of contractual arrangements according to competitive market conditions. The requirement for relatively higher emphasis on market arrangement would necessitate a buying firm to have unique capabilities to quickly understand the developments in the product market and readjust sourcing arrangements with supplier(s). Accordingly, from the resource-based perspective, the critical resource to acquire in this stage would be the capability to quickly reconfigure the supply arrangements and/or to modify the contractual terms with the existing suppliers. A firm having sourcing arrangements with highly capable supply base is likely to have the greatest opportunity to integrate the contractual and market arrangements in the maturity phase of the PLC. For example, firms such as Anheuser Busch and Hewlett Packard use such approach while procuring aluminium, electricity, electronic components, and semiconductors (Billington, 2002; Wu and Kleindorfer, 2005).

The foregoing theoretical interpretation of the results supports the argument that applicability of the dominantly held view that 'supplier alliance or partnerships is crucial for effective supply management' is not universal and is rather context specific (McCutcheon and Stuart, 2000). The findings are in line with the perspective that characteristics of inter-organizational alliances evolve dynamically in accordance with the nature of competition and criticality of external resources over time (Gulati, 1998). In the early stages of the procured product's life cycle there are likely to be a small number of capable suppliers who can meet the buying firm's requirements. Consequently, the key factors that would affect the success of a sourcing arrangement during this stage are: commitment of suppliers for a sustained relationship, existence of inter-organizational communication systems, and existence of a mechanism to balance the costs and risks in the arrangement. Adoption of long-term contractual arrangements with scope for periodic renegotiation can provide mechanisms for continuity, trust, and commitment in relationship while facilitating risk and reward sharing (Krause, 1999; McCutcheon and Stuart, 2000; Carr and Smeltzer, 2002). Consequently, the relational arrangements in managing suppliers would be of high utility during this stage (Rink and Fox, 1999; Dyer, 1997, Saeed et al., 2005).

While the long-term contractual arrangements can enable the development of relational mechanisms for supplier management, the use of short-term open market arrangements can enable the buying firm to maintain a competitive and constructive tension with the contractual supplier that may be useful for market development, improvement of the contractual supplier, and achieving strategic flexibility. The empirical sourcing literature also highlights the efficacy of such an approach in buying firm's efforts to improve supplier performance (Krause, et al., 2000). This line of research suggests that supplier development activities can be divided into two groups: externalized activities and internalized activities. While externalized activities entail use of a combination of competitive pressure, supplier assessment, and supplier incentives in the form of purchase volumes to motivate suppliers to improve, internalized activities involve direct involvement of the buying firm in supplier's development through various types of investments. Their study observes that systematic use of externalized mechanisms can be effective in supplier development. Thus by maintaining an alternative market sourcing arrangement, the buying firm can employ such *externalized mechanisms* to induce the contract suppliers to continuously improve in line with the development in competitive market place.

In view of above discussion and on the basis of results of the numerical analysis of the illustrative dataset with the PLC perspective, it can be hypothesized that while relational arrangements are likely to be more relevant in earlier stages of the PLC, competitive arrangements would be more relevant towards the later stages of the PLC. The execution of such a souring strategy with a dynamic perspective to match with the internal and external contingencies would however require appropriate implementation of systems that facilitate boundary-spanning activities (e.g., information exchange, supplier selection, assessment and monitoring, purchasing etc.) that manage the buyer-supplier relationships. The related issue is how should the buying firm go about adopting and implementing the requisite (electronic) procurement mechanisms that are to support a mix of relational and competitive buyer-supplier relationships across different phases of the procured product's life cycle.

Studies on the impact of information technology and inter-organizational systems on process efficiency and sourcing leverage (e.g., close buyer-supplier relationship and benefits from competition among suppliers) indicate, while extensive information sharing mechanisms facilitate integration of buyer-supplier systems for higher efficiency, systems for higher number of interconnections with suppliers lead to higher sourcing leverage (Stump and Sriram, 1997; Saeed et al., 2005). Furthermore, buying firms who drive the information integration processes with many suppliers can develop increased awareness of suppliers' capabilities. This knowledge can subsequently be used for achieving sourcing leverage/competitive efficiency among suppliers. Consequently, firms with long-term contractual arrangements will have the opportunity to revise the transaction terms according to the market conditions. This may however require the buying firm to dynamically structure and sequence the various functionalities of the electronic systems for advantageous execution of relational and competitive mechanisms over time. This is crucial because many firms have tended to focus on collaboration oriented electronic procurement mechanisms that support the pre-existing long-term relationships which has been the traditional mode of exchange process; often these firms are not inclined to adopt the alternative of efficient open market sourcing mechanisms even when the supporting electronic mechanisms are available for adoption (Wang and Archer, 2004). Accordingly, there is a need for examining the correspondence between the alternative types of sourcing strategy and electronic procurement functionalities with a contingency perspective.

In addition to above strategic implications, the models suggest several insights that have tactical and operational implications. The continuous time model in Chapter 3 indicates that units procured from alternative arrangements are variable and the variability increases with the duration of planning horizon. Furthermore, the average units sourced from contractual arrangement declines marginally with longer review periods due to the higher sourcing from the open market arrangement, which is expected to provide a price advantage over contractual arrangement. These results highlight the significance of quantity flexible contracts with higher flexibility for longer-term contracts. In case a supplier firm is not willing for such contracts, the buying firm may consider renegotiating the long-term contract as a sequence of short-term contracts for reducing the variance in purchases made from alternative arrangements.

The continuous time model in Chapter 4 exactly matches the units constraint, suggests that while the market arrangement is generally attractive towards later stages of PLC, a risk averse firm with longer planning horizon would buy lesser fraction of units from market arrangement than that with shorter planning horizon. This is because the arrangements with longer planning horizon incorporates more market uncertainty than the one with shorter planning horizon. Since, the earlier stages of the PLC are likely to have higher price uncertainty, this implies a buying firm may choose to employ short-term tactical planning horizons in the earlier stages of the PLC as compared to the later stages of PLC to benefit from competitive market price. This would allow the buying firm to minimize the expected cost disutility by limiting the exposure to cost uncertainty. The results thus suggest that long-term contracts may have to be renegotiated more frequently during the earlier stages than in the later stages.

The optimal sourcing strategy with switching cost as per the discrete time model in Chapter 5 is somewhat different. This is because the firm readjusts fractions between arrangements infrequently and the decision to readjust depends upon both the price dynamics, and the cost and benefits of switching over the remaining time period. It is observed that a firm's switching frequency (i.e., the number of time the fraction of units sourced from alternative arrangements are adjusted) falls (rises) when the switching cost from/to the contract arrangement is high (small). However, in case of a switch, the magnitude of switch is high (small) if the price differences between arrangements are high (small). Thus, higher switching cost and lower price differential present higher inertia against switching and provide a case for greater stability of the sourcing arrangement. This implies relatively high switching costs/penalties for readjusting units sourced from contractual arrangements are likely to be counterproductive for the contractual supplier in later stages of PLC when the product is very standardized to command a price premium.

The research extends the contingency perspective in sourcing strategy by explicitly including the buyer's risk attitude, transaction/switching cost and competitive price dynamics in the analysis. The results have normative implications on how contracts should be designed and renegotiated, how buyer-supplier relationships are to be managed, how electronic procurement mechanisms are to be implemented, and how much importance is to be given to market arrangements over time. This conceptualization aids in understanding the importance and operationalization of dynamic readjustment of sourcing strategy over time and contributes to future theory-building activities in the broad area of managing buyer-supplier relationships and adopting electronic procurement mechanisms. Some of the insights developed from the study are presented below as hypotheses regarding the sourcing strategy for a standardized product.

1. The buying firm should have a higher emphasis on long-term contractual sourcing mechanisms with scope for renegotiation in the earlier stages of the procured product's life cycle.
2. The buying firm should have a higher emphasis on open market sourcing mechanisms in later stages of the procured product's life cycle.
3. The long-term contractual arrangements in later stages of the procured product's life cycle do not necessitate frequent renegotiation.
4. The contractual arrangements are likely to have lesser penalty/switching cost in later stages of the PLC.

5. The buying firm is likely to emphasize open market oriented functionalities offered in public electronic procurement mechanisms in later stages of the procured product's life cycle.
6. The buying firm is likely to emphasize relation building functionalities offered in private or consortia based electronic procurement mechanisms in earlier/intermediate stages of the procured product's life cycle.

6.2 Managerial contributions

The models formulated in this dissertation can be applied to many product-market contexts (e.g., steel, chemicals, electricity, semi-conductors etc.) in which the product attributes are easy to standardize and codify. The applications to context specific data can enable managers to formulate context specific sourcing strategies. For example, in light of the results of models applied to the illustrative data set, it may be observed that for a buying firm with negligible switching cost and transaction cost, it is optimal to have dual sourcing, and the fraction of sourcing through the open market arrangement should increase over time due to the changes in procurement contexts across different stages in the PLC.

The procurement contexts that affect the optimal fractions can be described in terms of contractual and open market price dynamics and risk aversion. The optimal sourcing patterns have practical relevance in the context of the recent emergence of electronic procurement arrangements. These arrangements have enabled firms to achieve information transparency and to reduce the costs and time of identifying new suppliers, switching and conducting transactions (Boer et al., 2001; CAPS Research, 2003; Flynn, 2004). A joint study by McKinsey and CAPS Research mentions 'B2B e-market places can provide real value to buyers and the buyer should move quickly in finding ways to capture that value. The best approach is to view B2B e-market places from a portfolio perspective, blending traditional purchasing best practices with the specific online benefits that each type of B2B market places can provide' (McKinsey and CAPS Research, 2000). A subsequent study further indicates that e-procurement can accommodate a total cost of ownership perspective (TCO) perspective when the buying firm's requirements across price and non-price factors are clearly specified and could be implemented for direct, indirect, capital goods and services that represent the commodities, leverage (e.g., high value and low supply complexity) and bottleneck (e.g., low value and high supply complexity) products in the purchasing strategy matrix suggested in Kraljic's 2003 study (CAPS Research, 2003).

Accordingly, the key issue is how does a firm implement the suggested optimal pattern of sourcing over time without negative outcomes. This highlights the significance of developing appropriate strategic (e.g., conducting spend analysis, identifying new opportunities through analysis of information, managing suppliers etc.), tactical (e.g., setting specifications, selecting suppliers, negotiating and contracting etc.), and operational (e.g., ordering, tendering, expediting etc.) approaches to actualize the identified 'optimal sourcing strategy' effectively. It implies that the buying firm needs to develop appropriate organizational arrangements (e.g., structure, process, people, and technology) for contractual and market sourcing over time. Successful implementation of the optimal strategy may thus call for appropriate adoption and execution of e-procurement alternatives over time such that there exists alignment among the strategy and organizational structure, culture, operational processes, and portfolio of e-supply tools over time (CAPS Research study, 2002; Flynn, 2004). These studies highlight the role of sourcing strategy in achieving a rationalized supply base, appropriate buyer-supplier relationships, and economies of scale and scope, and recommend that sourcing strategy should drive the e-sourcing strategy. Such a perspective is critical from the buying firm's perspective as several studies raise concerns about the negative implications of (purely cost oriented) e-procurement mechanisms on buyer-supplier relationships and long-term relation specific investments by suppliers (Jap, 2003; Williams et al., 2002; Emiliani, 2000).

Research indicates there are several possible alternative e-sourcing arrangements (e.g., private e-sourcing exchanges, industry consortia, public e-market places, and ubiquitous third party *non-collaborative* tools such as e-tendering, e-ordering, e-catalogues, e-reverse auctions etc.), which can facilitate *product specific* sourcing arrangements spanning long-term contractual sourcing and spot market sourcing in a varied range of product-market contexts (Skjott-Larsen et al., 2003; Lancioni et al., 2003; CAPS Research and Mc Kinsey & Company, 2002). These arrangements help identify a possible supply base, establish terms of purchase while accounting for price transparency over time, aggregate demand across firms, facilitate transactions and execute purchases with the key objective of optimizing the direct and indirect costs and benefits from both contract and open market sourcing (Mc Kinsey & Company and CAPS Research, 2000). There are studies that suggest alternative frameworks to assess the impact of various e-procurement tools/techniques (Boer et al. 2001; CAPS Research and Mc Kinsey & Company, 2002; Skjott-Larsen et al., 2003, Harnik, 2005). These frameworks may be used to identify most befitting procurement arrangements for adopting both contractual and market arrangements over time.

To illustrate, it is apparent that the buying firm needs to focus on developing internal capabilities (e.g., ability and skills for effective negotiation of flexible contracts, IT infrastructure for effective B2B transactions etc.) to implement both contractual and market arrangements over time. Towards this end, buying firms may emphasize customized e-procurement mechanisms (e.g. ubiquitous e-procurement tools for a specific function, or private exchanges with reverse auctions, e-catalogs, and data-mining tools) for handling varied non-standard information requirements in earlier stages of the procured product's PLC, industry consortia for industry-wide collaboration to manage (increasingly) standardized information and large volume of procurement in the intermediate stages, and public market exchanges for commodity products in the later stages (CAPS Research and Mc Kinsey & Company, 2002). Such a strategy would enable buying firms to cost-effectively deal with products in different stages of the PLC. For example, Hewlett Packard used the private auction mechanism 'TradingHubs' and private exchange called 'GetSupply' when dealing with non-standardized or proprietary products and the public exchange Converge when dealing with relatively more standardized products.

The pattern of optimal sourcing indicating decline in fraction of contract purchase in favor of open market purchase as the product advances in the PLC highlights the importance of building proactive buyer-supplier relationship over time. This is because it is plausible to increasingly source from market only if the market price becomes truly competitive with highly capable suppliers that make the product more or less a standard product over time. This may necessitate the buying firm to proactively indicate to the contract suppliers about the likely emphasis on open market purchase over time unless they outperform the alternatives in the competitive market on a sustained basis. It may thus require the buying firm to stimulate appropriate type of competition among suppliers in a manner such that they gain insights about their strengths and weaknesses in the efficient market, which may be characterized by reduced price, insignificant transaction cost, and superior non-price attributes (CAPS Research, 2003).

A firm needs to take several measures towards achieving the above objectives. While undertaking such measures the firm may explore opportunities for achieving economies of scale and scope across multiple products and across firms (e.g., auto-parts exchange by the auto manufacturers). Some of the required measures are:

a) Integrating finance and operations to implement the dual sourcing arrangement. This implies the focus of sourcing functions should expand beyond cost, quality, dependability issues to include financial and operational risk management activities such as tracking and forecasting of open (spot) market prices, design of flexible contracts, and valuation of a portfolio of sourcing arrangements.

b) Designing contracts with appropriate clauses for revising and/or terminating contracts, and timely re-bidding and re-negotiation of contracts to adjust with changes in the competitive market place.

c) Cost-effective adoption and modification of e-procurement mechanisms over time, which is challenging given the fact that firms find it hard to change the existing practices to leverage the technology (CAPS Research and Mc Kinsey & Company, 2002). This means development of standardized, robust procurement processes both for market and contract sourcing, and integration of requisite e-procurement tools to organizational structure, culture, people and information systems of the buyer and participating suppliers (Flynn, 2004; Harnik, 2005).

d) Rationalizing the pre-qualified supplier base such that constructive competition is stimulated among the suppliers in the earlier stages (CAPS Research study, 2003; Flynn, 2004). This would involve inducing adequate number of capable suppliers to participate in e-procurement process in a responsible and economically sustainable manner.

e) Developing clear specifications for products and practicing clear communication. The greater the codifiability of a product's attributes and the clearer the communication the easier its adoption to e-procurement mechanisms (Kleindorfer and Wu 2003; CAPS Research, 2003; Huber et al., 2005).

f) Handling internal and inter-organizational challenges. These challenges have to be met while implementing e-procurement strategies (CAPS Research and Mc Kinsey & Company, 2002; Booz-Allen and Hamilton, 2000; and Huber et al., 2005). The challenges may be due to significant capital investment, resistance to change, increased influence of IT departments, inter-organizational incompatibility, and mismatch between existing practices and roles of managers and new practices and roles. There is a possibility that purchasing managers would feel of some loss of control due to paring of the traditional work and may feel overwhelmed by the requirement of strategic negotiating, decision-making, and order allocation skills. Moreover, given the fact there is simultaneous existence of two types of purchasing roles: one for contractual purchase and the other for market purchase, appropriate training of purchasing professionals are to be undertaken to suit the skills with job requirements in different procurement contexts.

6.3 Directions for future research

The thesis addresses several important issues in formulating long-term sourcing strategy by proposing analytical models to account for buying firm's purchasing priorities, risk attitude and relative bargaining strength, and competitive dynamics of the product-market as reflected in market price dynamics and switching cost between arrangements. The models aid in generation of normative insights and optimal sourcing plans for varied procurement contexts. Several extensions can be considered for this research theme and some of them are discussed below.

The models consider a disutility minimization perspective to express the buying firm's purchasing priorities and risk attitude. It is assumed that a buying firm's utility function can be described as a single attribute utility function when utilities of each transaction attributes can be specified objectively in terms of the common denominator 'cost'. The approach presumes that the utility functions of non-cost attributes to be preferentially independent. However, in reality the buying firm's utility functions with respect to different attributes may not be preferentially independent and additive. In such circumstances the appropriate multi-criteria utility functions should be considered in formulating the models. Accordingly, efforts may be undertaken to develop more realistic models while incorporating more accurate preference structure of the buying firm.

The utility functions in the proposed models are described as a simple power function of the expense for mathematical tractability. In reality, the elicitation of the preference of the buying firm may result in a utility function, which may be more complex. Thus as an extension of the base models, more general single attribute utility functions may be considered for deriving meaningful and more context specific insights.

The market price process is modeled as a continuous geometric Brownian motion process. This process has been found to replicate the empirical price data for commodity products reasonably well. However, there may be alternative price processes, which may fit empirical data more closely particularly the ones with sudden jumps. Investigations may be carried out for exploring optimal strategies for such price processes.

The model in Chapter 5 assumes the switching costs to be proportional to the magnitude of quantity readjustments between arrangements. However, switching costs may be of varied types. In particular, these may have a fixed cost component, which may be independent of the magnitude of adjustment. This modification would provide more realistic insights on the impact of switching/transaction cost on sourcing strategy.

The model with switching cost in Chapter 5 is implemented numerically. This is because an analytical implementation of the model is difficult. However, the accuracy of results is influenced by the granularity of discretization. The illustrative numerical experiment on the hypothetical data is limited to 4 time intervals and 11 discrete values for fractions sourced from market. Further refinement of state space was difficult due to the exponential growth of computational efforts with the increase in time steps and alternative values for fractions sourced from market. Consequently, the results are unlikely to converge with the continuous time results. Adoption of efficient computational algorithm will enable to reduce search space and facilitate better convergence to more precise continuous time results. Hence, efforts may be directed to implement a more efficient computational algorithm.

The models in Chapters 3, 4 and 5 are based on a conceptual framework that the sourcing arrangement of a firm are guided by the firm's purchasing priorities, product-market uncertainty, life cycle of the procured product, and increase in market efficiency with the emergence of electronic market. Survey and case study based research may be carried out for empirical validation of this framework and for verifying the propositions generated from the application of the analytical models on the PLC oriented data set. Results of such investigation would enable the modification of the conceptual model and to revise the analytical models for more realistic examination of the research theme.

Finally, the proposed models consider the operating context to be unaffected by issues such as portfolio of products, demand uncertainty, capacity limits, inventory costs, relationship development cost, possibility of quantity discounts, size of supply base, contract price limits etc. From a very long-term strategic perspective these issues may be ignored assuming that the unit price can reflect the impact of these factors. However, in a short-medium term contexts these issues may not be too small to ignore. Consequently, from a tactical and operational strategy perspective, research may be undertaken to develop models that appropriately consider these issues for a closer representation of the procurement contexts.

Appendix A

In the specified problem context, the objective can be described as: to find the optimal fractions of units to source at open market price of $P_1(t)$ and at contract price of $P_2(t)$ to minimize the disutility of the procurement expenses $X(t)$. Let $n_1(t)$ and $n_2(t)$ be the respective fractions. The governing equations that describe the situation are:

$$X(t) = n_1(t)P_1(t) + n_2(t)P_2(t) \tag{1}$$

$$dX(t) = n_1(t)dP_1(t) + n_2(t)dP_2(t) + \Delta n_1(t)P_1(t) + \Delta n_2(t)P_2(t) \tag{2}$$

$$1 \equiv n_1(t) + n_2(t) \tag{3}$$

Let at any time 't', $\overline{P_1} = P_1 / P_2$. One can use a strategy of switching completely from one source to another depending if $\overline{P_1} = 1 + a$ or $1 - a$ for $a > 0$, the threshold level fixed in advance (i.e., n_1 changes from 1 to 0 if $\overline{P_1} = 1 + a$, etc.). With this strategy at a time t of switch the savings would be:

$$\Delta n_1(t)P_1(t) + \Delta n_2(t)P_2(t) = a.P_2(t) \tag{4}$$

To maximize the saving one has to choose 'a' as small as possible which naturally leads to the limit of $\sum_k a \cdot P_2(t_k)$ as $a \to 0$, where $\{t_k\}$ are the switching times between arrangements. It turns out that at the limit the following solution is obtained:

$$n_1(t) = \begin{cases} 0 & \text{if } \overline{P_1}(t) > 1 \\ 1 & \text{if } \overline{P_1}(t) \le 1 \end{cases} \tag{5}$$

$$n_2(t) = 1 - n_1(t) \tag{6}$$

$$X(t) = P_1(t) \wedge P_2(t) \tag{7}$$

$$dX(t) = n_1(t)dP_1(t) + n_2(t)dP_2(t) - P_2(t)dL(t) \tag{8}$$

$$\sum_k a \cdot P_2(t_k) \xrightarrow[a \to 0]{} \int_0^T P_2(t)dL(t) \tag{9}$$

where, $L(t)$ is known in stochastic analysis as the "local time" (Oksendal, 1996, pp. 138) of $\overline{P_1}(t)$ at 1. It reflects the amount of time that $P_1 = P_2$ in the planning horizon. It is interesting that $L(t)$ is continuous, increasing function that changes only when $P_1 = P_2$. In particular at the limit, $X(t)$ is continuous even though it has jumps when $a > 0$. It has to be noted that the above solution is theoretical in nature, since, $L(t)$ is random and cannot be expressed explicitly. That is why the solution above is difficult to implement in practice.

Appendix B

Derivation Of Conditions On Minimum Duration Of Tactical Planning Horizon For Sufficiency Of The Procured Units

The purchases from alternative arrangements and rebalance (i.e., control) process to ensure sufficiency of units over time may be explained in terms of the following diagram in discrete time. Let units procured (*N*), prices (*P*), and expenses (*X*) at an instant (t=0,1,2), from open market arrangement (1), and contract arrangement (2) be represented with subscripts 't', 1 and 2 respectively.

Time	0	1	2
Price (market)	P_{10}	P_{11}	P_{12}
(contract)	P_{20}	P_{21}	P_{22}
Units (market)	N_{10}	N_{11}	N_{12}
(contract)	N_{20}	N_{21}	N_{22}

Therefore, expense at $t = 0$:

$$X_0 = P_{10}\,N_{10} + P_{20}\,N_{20} \tag{10}$$

And, expense at $t = 1$:

$$X_0 + dX_0 = (P_{10} + dP_{10})\,N_{10} + (P_{20} + dP_{20})\,N_{20} = X_1 \tag{11}$$

$$\Rightarrow X_1 = P_{11}\,N_{10} + P_{21}\,N_{20} \tag{12}$$

After rebalance, $X_1 = P_{11}\,N_{11} + P_{21}\,N_{21}$ (13)

It is to be noted that the equilibrium condition at the time of rebalance is the following: *expense before reallocation of fractions between arrangements is same as expense after reallocation of fractions at each instant.* Thus, equating, (12) and (13), the marginal rate of substitution (MRS) between two sources is:

$$\frac{dN_{11}}{dN_{21}} = -\frac{P_{21}}{P_{11}} \tag{14}$$

The rebalance should be such that after rebalance, the total units are at least equal to total number of units at time $t = 0$.

In general terms at any time 't' equation (10) and (11) can be written as:

$$X_t = N_{1t}P_{1t} + N_{2t}P_{2t} \tag{15}$$

$$dX_t = N_{1t}dP_{1t} + N_{2t}dP_{2t} \tag{16}$$

The differential equation for expense 'dX_t' in (16) can be written as follows. If 'u' is the fraction allocated to market, then one can write (15) and (16) as:

$$X_t = uX_t + (1-u)X_t \tag{17}$$

And, $dX_t = N_{1t}dP_{1t} + N_{2t}dP_{2t}$

$$= uX_t \frac{dP_{1t}}{P_{1t}} + (1-u)X_t \frac{dP_{2t}}{P_{2t}} \tag{18}$$

Using equations (3.1) and (3.2), equation (18) can be written as:

$$\Leftrightarrow \quad dX_t = X_t(u\alpha_1 + (1-u)\alpha_2)dt + u\beta_1 X_t dW_t \tag{19}$$

The condition for sufficiency of units procured over the time horizon under consideration may be derived as follows.

According to equation (19) the total procurement expense at 't' while sourcing as per the optimal fractions (e.g., u in open market, and $1-u$ in contract arrangements) across the two sourcing arrangements starting with an initial expense of X_0 will follow geometric Brownian motion and can be written as:

$$X_t = X_0 e^{\{\alpha_1 u+\alpha_2(1-u)-\frac{1}{2}\beta_1^2 u^2\}t+\int_0^t \beta_1 u dW_t} . \tag{20}$$

Accordingly, the mean and variance of X_t can be written as:

$$E(X_t) = X_0 e^{\{\alpha_1 u+\alpha_2(1-u)\}t} \tag{21}$$

$$Var(X_t) = X_0^2 e^{2\{\alpha_1 u+\alpha_2(1-u)\}t}[e^{\beta_1^2 u^2 t} - 1] \tag{22}$$

Similarly, the open market unit price at 't' will follow geometric Brownian motion with initial price P_{10} and can be written as:

$$P_{1t} = P_{10} e^{(\alpha_1 - \frac{1}{2}\beta_1^2)t+\int_0^t \beta_1 dW_t} . \tag{23}$$

Correspondingly, the mean and variance of P_{1t} can be written as:

$$E(P_{1t}) = P_{10} e^{\alpha_1 t} \tag{24}$$

$$Var(P_{1t}) = (P_{10})^2 e^{2\alpha_1 t}[e^{\beta_1^2 t} - 1] \tag{25}$$

The contract purchase unit price at 't' will have a steady rate of change with initial price P_{20} and can be written as:

$$P_{2t} = P_{20} e^{\alpha_2 t} \tag{26}$$

If the buying firm allocates fraction 'u' of total expense to open market sourcing, then, units procured from open market at 't' can be written as:

$$N_{1t} = \frac{X_0}{P_{10}} u e^{\{\alpha_1 u + (\alpha_2 - \alpha_1) - \alpha_2 u - \frac{1}{2}\beta_1^2 u^2 + \frac{1}{2}\beta_1^2\}t + \beta_1 (u-1)\int_0^t dW_t}$$

$\Leftrightarrow$

$$N_{1t} = \frac{X_0}{P_{10}} u e^{\{(\alpha_1 - \alpha_2)u + (\alpha_2 - \alpha_1) + \frac{1}{2}\beta_1^2 (1-u^2)\}t - \beta_1 (1-u) W_t} \tag{27}$$

The mean and variance of N_{1t} can be written as:

$$E(N_{1t}) = N_{10} e^{\{(\alpha_2 - \alpha_1)(1-u) + \beta_1^2 (1-u)\}t} \tag{28}$$

$$Var(N_{1t}) = (N_{10})^2 e^{2\{(\alpha_2 - \alpha_1)(1-u) + \beta_1^2 (1-u)\}t} [e^{\beta_1^2 (u-1)^2 t} - 1] \tag{29}$$

Consequently, units procured from contractual arrangement at 't' can be written as:

$$N_{2t} = \frac{(1-u)}{P_{20}} X_0 \, e^{\{(\alpha_1 - \alpha_2)u - \frac{1}{2}\beta_1^2 u^2\}t + \beta_1 u W_t} \tag{30}$$

The mean and variance of N_{2t} can be written as:

$$E(N_{2t}) = N_{20} e^{(\alpha_1 - \alpha_2)ut} \tag{31}$$

$$Var(N_{2t}) = (N_{20})^2 e^{2(\alpha_1 - \alpha_2)ut} [e^{\beta_1^2 u^2 t} - 1] \tag{32}$$

It may be noted that since $\alpha_2 > \alpha_1$ (for positive contract premium requirement) and variability of market prices increase with longer planning horizon or tactical review period, the average units sourced from market 'N_{1t}' increases continuously with 't', and the average units sourced from contract 'N_{2t}' decreases continuously with 't'; however the variance of units sourced from both the arrangements increase continuously with 't'. These patterns of mean and variance of units sourced from alternative arrangements will have implications for contract design.

The total units procured at instant 't', $N_t = N_{1t} + N_{2t}$ (33)

Setting,

$\frac{dN_t}{dW_t} = 0$, for the W_t^* that gives minimum N_t;

$$W_t^* = \frac{1}{\beta_1} \ln[\frac{P_{20}}{P_{10}} e^{\{(\alpha_2 - \alpha_1) + \frac{1}{2}\beta_1^2\}t}] \tag{34}$$

The condition for number of units at any time 't' to be more than initial units at $t = 0$ with probability 1 is given by:

$$min(N_{1t} + N_{2t}) - N_{10} - N_{20} \geq 0 \tag{35}$$

Solving the above equation, after plugging the above value of W_t^* to (27) and (30), it is found:

$$t^* \equiv \frac{2}{\beta_1^2 u(1-u)} \left[(1-u) \ln \frac{P_{20}}{P_{10}} - \ln P - \ln(\frac{1}{u}) \right] \vee 0; \tag{36}$$

In (36), P is the proportion of units sourced from open market at $t = 0$, and t^* is the critical time instant at which W_t renders the total units procured to be at least equal to the initial (at, $t = 0$) units procured when the buyer allocates expenses across market and contractual sourcing arrangements as per fractions [u, $1-u$]. Consequently, for $t \geq t^*$ equation (35) is satisfied with probability 1.

The corresponding value of critical W_t^* is given by:

$$W_{t^*}^* = \frac{1}{\beta_1} \ln[\frac{P_{20}}{P_{10}} e^{\{(\alpha_2-\alpha_1)+\frac{1}{2}\beta_1^2\}t^*}] \tag{37}$$

It is to be noted from (36) that if $P_{10} = P_{20}$ then $t^* = 0$. Thus for such a situation, the units procured at any time $(t > 0)$ would always be as much as the units procured at $t = 0$.

However, for $P_{10} \neq P_{20}$ and $0 < t < t^*$ there would be a positive probability that, $N(t)$ may be less than $N(0)$ for $t < t^*$ due to the dominance of stochastic influence in short-term in a Brownian process. Accordingly, procuring as per the optimal fractions might render the total units procured to be less than original units procured (at $t = 0$), for very short time horizons where $t < t^*$ (i.e., very close to initial time instant). During this period, at any instant 't', the actual probability of units procured to be more than original units procured at the initial point can be computed by solving the condition:

$$N_{1t} + N_{2t} \geq N_0 \tag{38}$$

For this, the necessary condition becomes:

$$R.e^{-bW_t} + S.e^{dW_t} \geq 1 \tag{39}$$

where, $R = \dfrac{uX_0}{P_{10}.N_0}.e^{\left[(\alpha_2-\alpha_1)(1-u)+\frac{1}{2}.\beta_1^2(1-u^2)\right]t}$,

$S = \dfrac{(1-u)X_0}{P_{20}.N_0}.e^{\left[(\alpha_1-\alpha_2)(1-u)-\frac{1}{2}.\beta_1^2u^2\right]t}$,

$b = \beta_1(1-u)$, and $d = \beta_1 u$.

Solving equation (38) for W_t, it is found:

there exists $W_1 < W_2$ so that (39) holds if $W_t \le W_1$ or $W_t \ge W_2$.

Subsequently the probability for sufficiency of units can be computed as:

$$prob(N_{1t} + N_{2t} \ge N_0) = prob(W_t \le W_1) + prob(W_t \ge W_2) \qquad (40)$$

It has been found through simulation study that the duration of likely shortage ($t < t^*$), and corresponding shortage in units are very small to be of practical significance for reasonable price and disutility parameter values.

Appendix C

Derivation Of Conditions For Sufficiency Of Units Procured At Any Instant 't' Over The Operating Horizon

The conditions on tactical planning horizon 't' in Appendix B ensure that total units procured are sufficient when the tactical planning horizon exceeds 't'. The conditions for sufficiency of units at any instant without such a restriction of minimum tactical planning horizon may be derived using Kuhn-Tucker conditions as demonstrated below (Chang, 2003, pp. 241).

The optimization problem with condition on sufficiency of units can be stated by modifying the equation (3.7) and (3.8) as the following:

$$\underset{u}{Min}\{\ x^n + C_t + \alpha_2 x C_x + u(\alpha_1 - \alpha_2) x C_x + \frac{1}{2}\beta_1^2 u^2 x^2 C_{xx}\ \} = 0; \quad \text{for} \quad t < T \text{ and}$$

$$x > 0 \quad C(T, x) = S(x)$$

Subject to:

$\dfrac{uX_t}{P_{1t}} + \dfrac{(1-u)X_t}{P_{2t}} \geq D$; where D is the units of demand at time $t = 0$.

Using Lagrange-Multiplier technique the Lagrange can be written as:

$$L = x^n + C_t + \alpha_2 x C_x + u(\alpha_1 - \alpha_2) x C_x + \frac{1}{2}\beta_1^2 u^2 x^2 C_{xx}$$

$$-\lambda\left[\frac{uX_t}{P_{1t}} + \frac{(1-u)X_t}{P_{2t}} - D\right] \qquad (41)$$

Using the Kuhn-Tucker conditions:

$$\frac{\partial L}{\partial u^*} = 0$$

$$\Rightarrow x(\alpha_1 - \alpha_2)C_x + \beta_1^2 u x^2 C_{xx} - \lambda[\frac{X_t}{P_{1t}} - \frac{X_t}{P_{2t}}] = 0 \quad (42)$$

$$\lambda \geq 0 \quad (43)$$

$$\lambda[\frac{uX_t}{P_{1t}} + \frac{(1-u)X_t}{P_{2t}} - D] = 0 \quad (44)$$

and, $\frac{uX_t}{P_{1t}} + \frac{(1-u)X_t}{P_{2t}} - D \geq 0$ (45)

It may be observed that as per the sufficiency constraint, if $\frac{uX_t}{P_{1t}} + \frac{(1-u)X_t}{P_{2t}} - D > 0$, then from (44), $\lambda = 0$ and from (42):

$u = -\frac{(\alpha_1 - \alpha_2)C_x}{\beta_1^2 x C_{xx}}$; as in the unconstrained case in (3.9).

The condition $\frac{uX_t}{P_{1t}} + \frac{(1-u)X_t}{P_{2t}} - D > 0$ after simplification can be written as:

$u > \dfrac{P_{2t}D - X_t}{P_{2t}X_t[\frac{1}{P_{1t}} - \frac{1}{P_{2t}}]}$, if $[\frac{1}{P_{1t}} - \frac{1}{P_{2t}}]$ is positive; (46)

or,

$u < \dfrac{P_{2t}D - X_t}{P_{2t}X_t[\frac{1}{P_{1t}} - \frac{1}{P_{2t}}]}$, if $[\frac{1}{P_{1t}} - \frac{1}{P_{2t}}]$ is negative. (47)

Alternately, if $\frac{uX_t}{P_{1t}}+\frac{(1-u)X_t}{P_{2t}}-D=0$, then from (42) and (43) it may be written

that $\left\{ \frac{(\alpha_1-\alpha_2)xC_x+\beta_1^2ux^2C_{xx}}{[\frac{X_t}{P_{1t}}-\frac{X_t}{P_{2t}}]}=\lambda \right\} \geq 0$ (48)

For (48) to hold good if, $[\frac{1}{P_{1t}}-\frac{1}{P_{2t}}]$ is positive, then the numerator of LHS should be positive.

Correspondingly, $(\alpha_1-\alpha_2)xC_x+\beta_1^2ux^2C_{xx}\geq 0$

$\Rightarrow u \geq -\frac{(\alpha_1-\alpha_2)C_x}{\beta_1^2xC_{xx}}$; since C_{xx} is positive. (49)

Alternately, for (48) to hold good if, $[\frac{1}{P_{1t}}-\frac{1}{P_{2t}}]$ is negative then the numerator of LHS should be negative.

Correspondingly, $(\alpha_1-\alpha_2)xC_x+\beta_1^2ux^2C_{xx}\leq 0$

$\Rightarrow u \leq -\frac{(\alpha_1-\alpha_2)C_x}{\beta_1^2xC_{xx}}$; since, C_{xx} is positive. (50)

The condition $\frac{uX_t}{P_{1t}}+\frac{(1-u)X_t}{P_{2t}}-D=0$ after simplification can be written as:

$$u=\frac{P_{2t}D-X_t}{P_{2t}X_t[\frac{1}{P_{1t}}-\frac{1}{P_{2t}}]}$$

Using (3.9) and (3.14), $-\frac{(\alpha_1-\alpha_2)C_x}{\beta_1^2xC_{xx}}=\frac{\alpha_2-\alpha_1}{\beta_1^2(n-1)}$ (51)

Thus, for $\frac{uX_t}{P_{1t}} + \frac{(1-u)X_t}{P_{2t}} \geq D$ to hold good, the necessary conditions are:

$$u(x,t) = \min\left\{ \frac{P_{2t}D - X_t}{P_{2t}X_t[\frac{1}{P_{1t}} - \frac{1}{P_{2t}}]}, \frac{\alpha_2 - \alpha_1}{\beta_1^2(n-1)} \right\} \text{ when, } P_{1t} > P_{2t}; \tag{52}$$

or,

$$u(x,t) = \max\left\{ \frac{P_{2t}D - X_t}{P_{2t}X_t[\frac{1}{P_{1t}} - \frac{1}{P_{2t}}]}, \frac{\alpha_2 - \alpha_1}{\beta_1^2(n-1)} \right\} \text{ when, } P_{2t} > P_{1t} \tag{53}$$

Appendix D

Derivation For Continuous And Jump Adjustment Of Units

It is observed that for the specified price processes the units purchased from alternative arrangements can change only in jumps when the prices across alternative channels are equal. This finding is presented as a mathematical proof to the following claim.

Claim:

Let, $X_t > 0$ be the total procurement expenses of the firm across two sourcing strategies at time $t \geq 0$; and, let '$u_1(t)$' be the fraction of units to purchase at open market price (P_1) and '$u_2(t)$' be the fraction of units to purchase at long-term contract price (P_2) at time t to minimize expense disutility. Thus for continuous expense:

$$X_t = u_1(t)P_1(t) + u_2(t)P_2(t) \tag{54}$$

$$dX(t) = u_1(t)dP_1(t) + u_2(t)dP_2(t) \tag{55}$$

$$1 \equiv u_1(t) + u_2(t) \tag{56}$$

and, u_1 and u_2 can be changed only at those times when $P_1 = P_2$.

Proof:

The market and contract price processes are defined as in (4.1) and (4.2).

In the following discussion the argument or subscript 't' is dropped sometimes for notational convenience.

Then (54)-(56) can be written as:

$X = u_1 P_1 + u_2 P_2$, $dX = u_1 dP_1 + u_2 dP_2$ and $1 \equiv u_1 + u_2$

It is to be noted that since, $u_1(t) + u_2(t) = 1$, at any instant $\Delta u_1(t) = -\Delta u_2(t)$

Let $\overline{X} = X / P_2$, $\overline{P_1} = P_1 / P_2$, hence

$d\overline{P_1} = \overline{P_1}[(\alpha_1 - \alpha_2)dt + \beta_1 dW_t)]$

(54) can be written as $\overline{X} = u_1 \overline{P_1} + u_2 = u_1(\overline{P_1} - 1) + 1$

(55) can be written as $dX = u_1 dP_1 + (1 - u_1)dP_2$

Accordingly, $d\overline{X} = d(X \cdot P_2^{-1}) = Xd(P_2^{-1}) + P_2^{-1}dX$

$$= -\frac{X}{P_2}\frac{dP_2}{P_2} + \frac{1}{P_2}[u_1 dP_1 + (1 - u_1)dP_2]$$

$$=-[u_1(\overline{P_1} - 1) + 1]\alpha_2 dt + \frac{1}{P_2}[u_1 dP_1 + (1 - u_1)dP_2]$$

After simplification:

$$= u_1 d\overline{P_1} \tag{57}$$

Let the dynamics of units sourced from market arrangement be represented as:

$du_1(t) = K(t)dt + L(t)dW_t + \Delta u_1(t)$,

where $\Delta u_1(t)$ denote the instantaneous change at *t*.

Hence, the continuous change in u_1 is:

$$du_1^c = K(t)dt + L(t)dW_t; \tag{58}$$

where, $K(t)$ and $L(t)$ are the drift and standard deviation parameters, and W_t represents the Weiner process.

Since, $\overline{X} = u_1(\overline{P_1} - 1) + 1$

$$\Rightarrow d\overline{X} = u_1 d\overline{P}_1 + (\overline{P}_1 - 1)du_1^c + du_1^c \cdot d\overline{P}_1 + (\overline{P}_1 - 1)\Delta u_1 + \Delta u_1 d\overline{P_1} \tag{59}$$

Equating (57) and (59):

$$0 = (\overline{P}_1 - 1)du_1^c + du_1^c \cdot d\overline{P}_1 + \Delta u_1 d\overline{P_1} + (\overline{P}_1 - 1)\Delta u_1 \tag{60}$$

$$= (\overline{P}_1 - 1)du_1^c + du_1^c \cdot \overline{P}_1[(\alpha_1 - \alpha_2)dt + \beta_1 dW_t)] + \Delta u_1 d\overline{P_1} + (\overline{P}_1 - 1)\Delta u_1$$

Since, $\overline{X}$ is continuous one must have $(\overline{P}_1 - 1)\Delta u_1$=0, which makes,

$\Delta u_1 \neq 0$ when, $\overline{P}_1 = 1$; this implies contract price equals market price.

Thus (60) becomes: $(\overline{P}_1 - 1)du_1^c + du_1^c \cdot \overline{P}_1[(\alpha_1 - \alpha_2)dt + \beta_1 dW)] + \Delta u_1 d\overline{P_1}$ (61)

Further, since, $\overline{P}_1$ is continuous $\Delta u_1 d\overline{P_1} = 0$.

Hence, (61) becomes:

$$0 = (\overline{P}_1 - 1)du_1^c + du_1^c \cdot \overline{P}_1[(\alpha_1 - \alpha_2)dt + \beta_1 dW_t)] \tag{62}$$

Using (58) in (62), and ignoring the higher order terms of dt and terms with $dW_t dt$:

$$0 = (\overline{P}_1 - 1)[K(t)dt + L(t)dW_t] + L(t)\overline{P}_1\beta_1 dt \tag{63}$$

It may be observed that, since W_t is random RHS of (63) can be zero only when both $K(t)$ and $L(t)$ are zero.

Thus, du_1^c is zero, and correspondingly, for restriction $u_1(t) + u_2(t) = 1$, and continuous expense X_t, the fraction of re-adjustments across alternative arrangements can be made in jumps (lumps) at discrete time points only when prices across channel are equal. However, for such adjustment, the expense remains unchanged at the instant. In light of above proof, it is apparent that the fraction of units derived in (4.14) and (4.17) for a planning horizon $(0,t)$ remains optimal only if prices do not become equal at any instant during the planning horizon. And, if prices become equal at an instant during the planning horizon, then it would be prudent to revise the optimal fraction at that instant using (4.14) and (4.17) for the remaining part of planning horizon. Thus in a given sourcing context, the sourcing strategy (in terms of optimal fraction $u_1(t)$ and $u_2(t)$) would benefit from revision (when prices are equal) over time for the remaining part of planning horizon. This implies the buying firm may have to renegotiate the contract as per optimal fractions from time to time by matching the contract price with competitive market price.

For given initial prices $(P_1(0), P_2(0))$, the expected time at which the prices become equal can be computed as below.

Let t be the earliest time at which the prices are equal.

Then, $P_1(0)e^{(\alpha_1 - \frac{\beta_1^2}{2})t + \beta_1 W_t} = P_2(0)e^{\alpha_2 t}$.

$$\text{Then, } E(\tau) = \frac{\ln(\frac{P_2(0)}{P_1(0)})}{(\alpha_2 - \alpha_1 + \frac{\beta_1^2}{2})}\text{; where } \tau = \inf\{t : P_1(t) = P_2(t)\} \qquad (64)$$

Appendix E

Computer Code To Obtain The Optimal Sourcing Strategy With Proportional Switching Cost

```
/*
 * FILE: optim.c
*/

#include <stdlib.h>
#include <stdio.h>
#include <math.h>

FILE *inFile, *outFile;


void computePenultimateS(int node, double lam1, double lam2, double price[],
double uParent, double *s);
double a(double price, double lam1, double lam2);
double b(double price);
double c(double price, double lam1, double lam2);
void findUC(int noOfLevel, int origLevel, double uOptParentN, double lam1,
double lam2,
                    double price[], double u[], double expectUp, double
expectDown, double *C0);
void findOptUC_7Node(int noUNode, double delU, int noOfLevel, int origLevel,
double uOptParentN, double lam1, double lam2,
                                    double price[], double expectUp, double
expectDown, double *COpt0, double *UOpt0);
void findOptUC_3Node(int noUNode, double delU, int noOfLevel, int origLevel,
double uOptParentN, double lam1, double lam2,
                                    double price[], double expectUp, double
expectDown, double *COpt0, double *UOpt0);
void findOptUC_1Node(int noUNode, double delU, int noOfLevel, int origLevel,
double uOptParentN, double lam1, double lam2,
                                    double price[], double expectUp, double
expectDown, double *COpt0, double *UOpt0);
void findReducedPrice(int noOfLevel, int j, double price[], double
reducedPrice[]);
void findPenultimateUltimateU(int noOfLevel, double lam1, double lam2, double
price[], double u[]);

void main(){
```

```
	int noOfLevel = 0;
	double alpha1 = 0.0;
	double alpha2 = 0.0;
	double beta = 0.0;
	double delt = 0.0;
	double p0 = 0.0;
	double price[32] = {0.0};
	int start = 0, end = 0, i = 0,
		j = 0, totalNodes = 0, parentN = 0;
	double up = 0.0;
	double down = 0.0;
	double exp1 = 0.0, exp2 = 0.0;
	// double u[32] = {0.0};
	double uParent = 0.0;
	double lam1 = 0.0;
	double lam2 = 0.0;
	double sNode = 0.0;
	double high = 0.0, low = 0.0;
	double marketC = 0.0, contractC = 0.0, penaltyC = 0.0, upChildC = 0.0,
downChildC = 0.0;
	int upChild = 0, downChild = 0;
	double expectUp = 0.0, expectDown = 0.0;
	double C[32] = {0.0};
	int i0 = 0, i1 = 0, i2 = 0, i3 = 0, i4 = 0, i5 = 0, i6 = 0;
	int noUNode = 0;
	double delU = 0.0;
	double minC0 = 100000000.0;
	double uMin[32] = {0.0};

	double COpt0 = 0.0, UOpt0 = 0.0;
	double COpt[32] = {0.0};
	double UOpt[32] = {0.0};
	double uOptParentN = 0.0;
	double reducedPrice[32] = {0.0};



	// FILE *inFile, *outFile;

	inFile = fopen("input.txt", "r");
	outFile = fopen("output.txt", "w");

	/* User Inputs
	printf("No of levels: ");
	scanf("%d", &noOfLevel);
	printf("Alpha1: ");
```

```
        scanf("%lf", &alpha1);
        printf("Alpha2: ");
        scanf("%lf", &alpha2);
        printf("Beta: ");
        scanf("%lf", &beta);
        printf("Lambda1: ");
        scanf("%lf", &lam1);
        printf("Lambda2: ");
        scanf("%lf", &lam2);
        printf("delt: ");
        scanf("%lf", &delt);
        printf("p0: ");
        scanf("%lf", &p0);
        printf("expectUp: ");
        scanf("%lf", &expectUp);
        printf("expectDown: ");
        scanf("%lf", &expectDown);
        printf("noUNode: ");
        scanf("%d", &noUNode);

    */



        fscanf(inFile, "%d %lf %lf %lf %lf %lf %lf %lf %lf %lf %d",
                        &noOfLevel, &alpha1, &alpha2, &beta, &lam1, &lam2,
                              &delt, &p0, &expectUp, &expectDown,
&noUNode);
        /* Verify user input */

        fprintf(outFile, "No of levels: %d Alpha1: %f Alpha2: %f Beta: %f
Lambda1: %f Lambda2: %f delt: %f p0: %f expectUp: %f expectDown: %f
noUNode: %d\n",
                      noOfLevel, alpha1, alpha2, beta, lam1, lam2, delt, p0, expectUp,
expectDown, noUNode);

        /* Compute price array */

        exp1 = (alpha1-alpha2) * delt;
        exp2 = beta * sqrt(delt);

        // printf("exp1: %lf exp2: %lf\n", exp1, exp2);

        up = exp(exp1 + exp2);
        down = exp(exp1 - exp2);
```

```
fprintf(outFile,"up: %lf down: %lf\n", up, down);

price[0] = p0;


for (i= 0; i < noOfLevel; i++){
        start = pow(2, i) - 1;
        end = 2 * start;

        for (j = start; j < end+1 ; j++){
                price[2*j+1] = price[j] * up;
                price[2*j+2] = price[j] * down;

        }
}

totalNodes = pow(2, noOfLevel+1) - 1;


//fprintf(outFile,"\n**** PRICE VALUES ****\n");

//for (i= 0; i < totalNodes; i++)
//      fprintf(outFile,"price[%d]: %lf \n", i, price[i]);

delU = 1.0/(noUNode-1);

for (i = 0; i < noOfLevel-1; i++){
        start = pow(2, i) - 1;
        end = 2 * start;

        for (j = start; j < end+1; j++){

                fprintf(outFile,"******************************\n");
                fprintf(outFile,"**** OUTPUT for NODE:  %d ****\n",
j);
                fprintf(outFile,"******************************\n");

                if ( j > 0){
                        parentN = (j -1 )/2;
                        uOptParentN = UOpt[parentN];
                        findReducedPrice(noOfLevel-i, j, price,
reducedPrice);
                }

                if (i == 0)
```

```
                                        findOptUC_7Node(noUNode, delU, noOfLevel-i, i,
uOptParentN, lam1, lam2, price, expectUp, expectDown, &COpt0, &UOpt0);
                                else if ( i == 1)
                                        findOptUC_3Node(noUNode, delU, noOfLevel-i, i,
uOptParentN, lam1, lam2, reducedPrice, expectUp, expectDown, &COpt0,
&UOpt0);
                                else if (i == 2)
                                        findOptUC_1Node(noUNode, delU, noOfLevel-i, i,
uOptParentN, lam1, lam2, reducedPrice, expectUp, expectDown, &COpt0,
&UOpt0);

                                COpt[j] = COpt0;
                                UOpt[j] = UOpt0;
                        }
                }

        findPenultimateUltimateU(noOfLevel, lam1, lam2, price, UOpt);

        fprintf(outFile,"********************************\n");
        fprintf(outFile,"**** OPTIMAL U values   ******\n");
        fprintf(outFile,"********************************\n");

        for (i= 0; i < totalNodes; i++)
                fprintf(outFile,"UOpt[%d]: %lf \n", i, UOpt[i]);

}

void computePenultimateS(int node, double lam1, double lam2, double price[],
double uParent, double *s){
        double lim1 = 0.0, lim2 = 0.0;
        double high = 0.0, low = 0.0, upPrice = 0.0, downPrice = 0.0, nodePrice =
0.0;
        int upChild = 0, downChild = 0;
        double aUp = 0.0, aDown = 0.0, aNode = 0.0, bNode = 0.0, cUp = 0.0,
cDown = 0.0, bDown = 0.0, bUp = 0.0;
        double den2 = 0.0, num2 = 0.0, num3 = 0.0, num11 = 0.0, num = 0.0, den
= 0.0, sNode = 0.0;

        high = (1. + lam2)/(1. - lam1);
        low = (1. - lam2)/(1. +lam1);

        upChild = 2 * node + 1;
        downChild = 2 * node + 2;

        upPrice = price[upChild];
        downPrice = price[downChild];
```

```
nodePrice = price[node];

/* Case 1 */

if (upPrice > high && downPrice > high){

        /* Compute common expressions */
        aUp = a(upPrice, lam1, lam2);
        aDown = a(downPrice, lam1, lam2);

        den2 = aUp * aUp + aDown * aDown;
        num2 = aUp + aDown;
        num3 = uParent * den2;

        aNode = a(nodePrice, lam1, lam2);
        bNode = b(nodePrice);
        num11 = 2. * (bNode * uParent + 1);


        /* Check for +ve S */

        num = num11 * (aNode + bNode) + num2 + num3;

        den = (2. * (aNode + bNode) * (aNode + bNode) + den2);

        sNode = -num/den;

        if (sNode > 0.0){
                /* +ve S valid */
                *s = sNode;
        }
        else{
                /* +ve S invalid; check for -ve S */
                /* Check for +ve S */

                num = num11 * (bNode - aNode) + num2 + num3;

                den = (2. * (bNode - aNode) * (bNode - aNode) + den2);

                sNode = -num/den;

                if (sNode < 0.0){
                        /* -ve S valid */
                        *s = sNode;
                }
                else{
```

```
                    /* both +ve S and -ve invalid; s = 0 */
                    *s = 0;
                }
        }
}

/* Case 2 */

if (upPrice > high && low < downPrice  && downPrice < high){

        /* Compute common expressions */
        aUp = a(upPrice, lam1, lam2);
        bDown = b(downPrice);

        den2 = aUp * aUp + bDown * bDown;
        num2 = aUp + bDown;
        num3 = uParent * den2;

        aNode = a(nodePrice, lam1, lam2);
        bNode = b(nodePrice);
        num11 = 2. * (bNode * uParent + 1);


        /* Check for +ve S */

        num = num11 * (aNode + bNode) + num2 + num3;

        den = (2. * (aNode + bNode) * (aNode + bNode) + den2);

        sNode = -num/den;

        if (sNode > 0.0){
                /* +ve S valid */
                *s = sNode;
        }
        else{
                /* +ve S invalid; check for -ve S */
                /* Check for +ve S */

                num = num11 * (bNode - aNode) + num2 + num3;

                den = (2. * (bNode - aNode) * (bNode - aNode) + den2);

                sNode = -num/den;

                if (sNode < 0.0){
```

```
                        /* -ve S valid */
                        *s = sNode;
                }
                else{
                        /* both +ve S and -ve invalid; s = 0 */
                        *s = 0;
                }
        }
}

/* Case 3 */

if (low < upPrice  && upPrice < high && low < downPrice  &&
downPrice < high){

        /* Compute common expressions */
        bUp = b(upPrice);
        bDown = b(downPrice);

        den2 = bUp * bUp + bDown * bDown;
        num2 = bUp + bDown;
        num3 = uParent * den2;

        aNode = a(nodePrice, lam1, lam2);
        bNode = b(nodePrice);
        num11 = 2. * (bNode * uParent + 1);


        /* Check for +ve S */

        num = num11 * (aNode + bNode) + num2 + num3;

        den = (2. * (aNode + bNode) * (aNode + bNode) + den2);

        sNode = -num/den;

        if (sNode > 0.0){
                /* +ve S valid */
                *s = sNode;
        }
        else{
                /* +ve S invalid; check for -ve S */
                /* Check for +ve S */

                num = num11 * (bNode - aNode) + num2 + num3;
```

```
            den = (2. * (bNode - aNode) * (bNode - aNode) + den2);

            sNode = -num/den;

            if (sNode < 0.0){
                /* -ve S valid */
                *s = sNode;
            }
            else{
                /* both +ve S and -ve invalid; s = 0 */
                *s = 0;
            }
        }
}

/* Case 4 */

if (low < upPrice  && upPrice < high && downPrice < low){

        /* Compute common expressions */
        bUp = b(upPrice);
        aDown = a(downPrice, lam1, lam2);
        cDown = c(downPrice, lam1, lam2);

        den2 = bUp * bUp + aDown * aDown;
        num2 = bUp - aDown * cDown;
        num3 = uParent * den2;

        aNode = a(nodePrice, lam1, lam2);
        bNode = b(nodePrice);
        num11 = 2. * (bNode * uParent + 1);


        /* Check for +ve S */

        num = num11 * (aNode + bNode) + num2 + num3;

        den = (2. * (aNode + bNode) * (aNode + bNode) + den2);

        sNode = -num/den;

        if (sNode > 0.0){
            /* +ve S valid */
            *s = sNode;
        }
        else{
```

```
                /* +ve S invalid; check for -ve S */
                /* Check for +ve S */

                num = num11 * (bNode - aNode) + num2 + num3;

                den = (2. * (bNode - aNode) * (bNode - aNode) + den2);

                sNode = -num/den;

                if (sNode < 0.0){
                        /* -ve S valid */
                        *s = sNode;
                }
                else{
                        /* both +ve S and -ve invalid; s = 0 */
                        *s = 0;
                }
        }
}

/* Case 5 */

if (upPrice < low && downPrice < low){

        /* Compute common expressions */
        aUp = a(upPrice, lam1, lam2);
        aDown = a(downPrice, lam1, lam2);
        cDown = c(downPrice, lam1, lam2);
        cUp = c(upPrice, lam1, lam2);

        den2 = aUp * aUp + aDown * aDown;
        num2 = cUp * aUp + cDown * aDown;
        num3 = uParent * den2;

        aNode = a(nodePrice, lam1, lam2);
        bNode = b(nodePrice);
        num11 = 2. * (bNode * uParent + 1);


        /* Check for +ve S */

        num = num11 * (aNode + bNode) - num2 + num3;

        den = (2. * (aNode + bNode) * (aNode + bNode) + den2);

        sNode = -num/den;
```

```
        if (sNode > 0.0){
                /* +ve S valid */
                *s = sNode;
        }
        else{
                /* +ve S invalid; check for -ve S */
                /* Check for +ve S */

                num = num11 * (bNode - aNode) - num2 + num3;

                den = (2. * (bNode - aNode) * (bNode - aNode) + den2);

                sNode = -num/den;

                if (sNode < 0.0){
                        /* -ve S valid */
                        *s = sNode;
                }
                else{
                        /* both +ve S and -ve invalid; s = 0 */
                        *s = 0;
                }
        }
}

/* Case 6 */

if (upPrice > high && downPrice < low){

        /* Compute common expressions */
        aUp = a(upPrice, lam1, lam2);
        aDown = a(downPrice, lam1, lam2);
        cDown = c(downPrice, lam1, lam2);
        cUp = c(upPrice, lam1, lam2);

        den2 = aUp * aUp + aDown * aDown;
        num2 = aUp - cDown * aDown;
        num3 = uParent * den2;

        aNode = a(nodePrice, lam1, lam2);
        bNode = b(nodePrice);
        num11 = 2. * (bNode * uParent + 1);


        /* Check for +ve S */
```

```
            num = num11 * (aNode + bNode) + num2 + num3;

            den = (2. * (aNode + bNode) * (aNode + bNode) + den2);

            sNode = -num/den;

            if (sNode > 0.0){
                /* +ve S valid */
                *s = sNode;
            }
            else{
                /* +ve S invalid; check for -ve S */
                /* Check for +ve S */

                num = num11 * (bNode - aNode) + num2 + num3;

                den = (2. * (bNode - aNode) * (bNode - aNode) + den2);

                sNode = -num/den;

                if (sNode < 0.0){
                    /* -ve S valid */
                    *s = sNode;
                }
                else{
                    /* both +ve S and -ve invalid; s = 0 */
                    *s = 0;
                }
            }
        }

}

double a(double price, double lam1, double lam2){
        return (lam1 * price + lam2);
}

double b(double price){
        return (price - 1.);
}

double c(double price, double lam1, double lam2){
        return (price + a(price, lam1, lam2));
}
```

```
void findUC(int noOfLevel, int origLevel, double uOptParentN, double lam1,
double lam2,
                    double price[], double u[], double expectUp, double
expectDown, double *C0){

        int start = 0, end = 0, i = 0,
                j = 0, parentN = 0;
        double exp1 = 0.0, exp2 = 0.0;
        // double u[32] = {0.0};
        double uParent = 0.0;
        double sNode = 0.0;
        double high = 0.0, low = 0.0;
        double marketC = 0.0, contractC = 0.0, penaltyC = 0.0, upChildC = 0.0,
downChildC = 0.0;
        int upChild = 0, downChild = 0;
        // double expectUp = 0.0, expectDown = 0.0;
        double C[32] = {0.0};
        int totalNodes = 0;


        /* Compute u values at penultimate and ultimate level */

        findPenultimateUltimateU(noOfLevel, lam1, lam2, price, u);

        totalNodes = pow(2, noOfLevel+1) - 1;

        //fprintf(outFile,"\n**** U VALUES (0-30) ****\n");

        //for (i= 0; i < totalNodes; i++)
        //        fprintf(outFile,"%lf \t", u[i]);
        //fprintf(outFile,"\n");

        /* C values */

        for (i = noOfLevel; i >= 0; i--){
                start = pow(2, i) - 1;
                end = 2 * start;

                for (j = start; j < end+1 ; j++){
                        marketC = u[j] * price[j];
                        contractC = 1. - u[j];

                        /* Except for the first layer, all other layers have parent */

                        if ( i != 0){
```

```
                        parentN = (j -1)/2;
                        penaltyC = (lam1 * price[j] + lam2) * fabs (u[j] -
u[parentN]);
                }
                else if ( i == 0){
                        if (origLevel == 0)
                                penaltyC = 0.0;
                        else {
                                // parentN = origParent;
                                penaltyC = (lam1 * price[j] + lam2) * fabs
(u[j] - uOptParentN);
                        }
                }

                /* Except for the last layer, all other layers have children */
                if ( i != noOfLevel){
                        upChild = 2 * j + 1;
                        downChild = 2 * j + 2;
                        upChildC = expectUp * C[upChild];
                        downChildC = expectDown * C[downChild];
                }
                else{
                        upChildC = 0.0;
                        downChildC = 0.0;
                }
                C[j] = (marketC + contractC + penaltyC) * (marketC +
contractC + penaltyC)
                         + upChildC + downChildC;
                // fprintf(outFile,"marketC: %lf contractC: %lf penaltyC:
%lf upChildC: %lf downChildc: %lf C[%d]: %lf\n",
                //              marketC, contractC, penaltyC, upChildC,
downChildC, j, C[j]);

            }
    }

    //fprintf(outFile,"\n**** C VALUES (30 - 0) ****\n");

    //for (i = totalNodes - 1; i >= 0; i--)
    //      fprintf(outFile,"%lf \t", C[i]);
    //fprintf(outFile,"\n");

    *C0 = C[0];
}
```

```
void findOptUC_7Node(int noUNode, double delU, int noOfLevel, int origLevel,
double uOptParentN, double lam1, double lam2,
                                        double price[], double expectUp, double
expectDown, double *COpt0, double *UOpt0){

        int i0 = 0, i1 = 0, i2 = 0, i3 = 0, i4 = 0, i5 = 0, i6 = 0;
        // int noUNode = 0;
        // double delU = 0.0;
        // double minC0 = 100000000.0;
        double u[32] = {0.0};
        double C0 = 0.0;

        int totalNodes = 0, i = 0;
        double uMin[32] = {0.0};
        totalNodes = pow(2, noOfLevel+1) - 1;


        *COpt0 = 100000000.0;

        /* Loops start here for U-discretizations */

        for (i0 = 0; i0 < noUNode; i0++){
                u[0] = i0 * delU;
                for (i1 = 0; i1 < noUNode; i1++){
                        u[1] = i1 * delU;
                        for (i2 = 0; i2 < noUNode; i2++){
                                u[2] = i2 * delU;
                                for (i3 = 0; i3 < noUNode; i3++){
                                        u[3] = i3 * delU;
                                        for (i4 = 0; i4 < noUNode; i4++){
                                                u[4] = i4 * delU;
                                                for (i5 = 0; i5 < noUNode; i5++){
                                                        u[5] = i5 * delU;
                                                        for (i6 = 0; i6 < noUNode;
i6++){
                                                                u[6] = i6 * delU;



                                                                findUC(noOfLevel,
origLevel, uOptParentN, lam1, lam2, price, u, expectUp, expectDown, &C0);

                                                                // fprintf(outFile,"%lf
\n", C0);
                                                                // fprintf(outFile,"\n");
                                                                /* Find the min C0 */
```

```
                                                            if (C0 < *COpt0){
                                                                  *COpt0 = C0;
                                                                  *UOpt0 =
u[0];
                                                                  for (i= 0; i <
totalNodes; i++)

      uMin[i] = u[i];

                                                            }
                                                      }
                                                }
                                          }
                                    }
                              }
                        }
      }

      /* MinC0 and u-set for the parameter combo */
      //fprintf(outFile, "\n **** Optimum Values ****\n");
      //fprintf(outFile, "minC0: %lf\n", *COpt0);
      //fprintf(outFile, "minU0: %lf\n", *UOpt0);
      //fprintf(outFile,"\n**** OPTIMUM U VALUES (0-30) ****\n");

      for (i= 0; i < totalNodes; i++)
            fprintf(outFile,"%lf \t", uMin[i]);
      fprintf(outFile,"\n");
}


void findOptUC_3Node(int noUNode, double delU, int noOfLevel, int origLevel,
double uOptParentN, double lam1, double lam2,
                              double price[], double expectUp, double
expectDown, double *COpt0, double *UOpt0){

      int i0 = 0, i1 = 0, i2 = 0, i3 = 0, i4 = 0, i5 = 0, i6 = 0;
      // int noUNode = 0;
      // double delU = 0.0;
      // double minC0 = 100000000.0;
      double u[16] = {0.0};
      double C0 = 0.0;

      int totalNodes = 0, i = 0;
      double uMin[32] = {0.0};
```

```
totalNodes = pow(2, noOfLevel+1) - 1;

*COpt0 = 100000000.0;

// fprintf(outFile,"\n**** U VALUES (0-30) & C0 ****\n\n");
/* Loops start here for U-discretizations */

for (i0 = 0; i0 < noUNode; i0++){
        u[0] = i0 * delU;
        for (i1 = 0; i1 < noUNode; i1++){
                u[1] = i1 * delU;
                for (i2 = 0; i2 < noUNode; i2++){
                        u[2] = i2 * delU;



                        findUC(noOfLevel, origLevel, uOptParentN, lam1,
lam2, price, u, expectUp, expectDown, &C0);

                        //fprintf(outFile,"\n**** C VALUES (30 - 0)
****\n");

                        //for (i = totalNodes - 1; i >= 0; i--)
                        //        fprintf(outFile,"%lf \t", C[i]);
                        //fprintf(outFile,"\n");

                        //fprintf(outFile,"%lf \n", C[0]);
                        //fprintf(outFile,"\n");
                        /* Find the min C0 */

                        if (C0 < *COpt0){
                                *COpt0 = C0;
                                *UOpt0 = u[0];
                                for (i= 0; i < totalNodes; i++)
                                        uMin[i] = u[i];
                        }
                }
        }
}
/* MinC0 and u-set for the parameter combo */
//fprintf(outFile, "\n **** Optimum Values ****\n");
//fprintf(outFile, "minC0: %lf\n", *COpt0);
//fprintf(outFile, "minU0: %lf\n", *UOpt0);
//fprintf(outFile,"\n**** OPTIMUM U VALUES (0-30) ****\n");

for (i= 0; i < totalNodes; i++)
```

```
                fprintf(outFile,"%lf \t", uMin[i]);
        fprintf(outFile,"\n");
}

void findOptUC_1Node(int noUNode, double delU, int noOfLevel, int origLevel,
double uOptParentN, double lam1, double lam2,
                                        double price[], double expectUp, double
expectDown, double *COpt0, double *UOpt0){

        int i0 = 0, i1 = 0, i2 = 0, i3 = 0, i4 = 0, i5 = 0, i6 = 0;
        // int noUNode = 0;
        // double delU = 0.0;
        // double minC0 = 100000000.0;
        double u[8] = {0.0};
        double C0 = 0.0;

        int totalNodes = 0, i = 0;
        double uMin[32] = {0.0};
        totalNodes = pow(2, noOfLevel+1) - 1;

        *COpt0 = 100000000.0;

        // fprintf(outFile,"\n**** U VALUES (0-30) & C0 ****\n\n");
        /* Loops start here for U-discretizations */

        for (i0 = 0; i0 < noUNode; i0++){
                u[0] = i0 * delU;


                findUC(noOfLevel, origLevel, uOptParentN, lam1, lam2, price, u,
expectUp, expectDown, &C0);

                //fprintf(outFile,"\n**** C VALUES (30 - 0) ****\n");

                //for (i = totalNodes - 1; i >= 0; i--)
                //      fprintf(outFile,"%lf \t", C[i]);
                //fprintf(outFile,"\n");

                //fprintf(outFile,"%lf \n", C[0]);
                //fprintf(outFile,"\n");
                /* Find the min C0 */

                if (C0 < *COpt0){
                        *COpt0 = C0;
                        *UOpt0 = u[0];
                        for (i= 0; i < totalNodes; i++)
```

```
                        uMin[i] = u[i];
                }
        }
        /* MinC0 and u-set for the parameter combo */
        //fprintf(outFile, "\n **** Optimum Values ****\n");
        //fprintf(outFile, "minC0: %lf\n", *COpt0);
        //fprintf(outFile, "minU0: %lf\n", *UOpt0);
        //fprintf(outFile,"\n**** OPTIMUM U VALUES (0-30) ****\n");

        for (i= 0; i < totalNodes; i++)
                fprintf(outFile,"%lf \t", uMin[i]);
        fprintf(outFile,"\n");
}

void findReducedPrice(int noOfLevel, int j, double price[], double
reducedPrice[]){

        int totalNodes = 0, i = 0, k = 0;
        int arr[32] = {0};

        totalNodes = pow(2, noOfLevel) - 1;

        reducedPrice[0] = price[j];
        arr[0] = j;

        for (i = 0; i < totalNodes; i++){
                k = arr[i];
                arr[2*i+1] = 2*k+1;
                arr[2*i+2] = 2*k+2;
                reducedPrice[2*i+1] = price[2*k+1];
                reducedPrice[2*i+2] = price[2*k+2];
        }
        //fprintf(outFile,"\n**** REDUCED PRICE VALUES ****\n");

        //for (i= 0; i < 2*totalNodes+1; i++)
        //      fprintf(outFile,"price[%d]: %lf \n", i, reducedPrice[i]);
}

void findPenultimateUltimateU(int noOfLevel, double lam1, double lam2, double
price[], double u[]){

        int start = 0, end = 0,
                j = 0, parentN = 0;
        double uParent = 0.0;
        double sNode = 0.0;
        double high = 0.0, low = 0.0;
```

```
	start = pow(2, noOfLevel-1) - 1;
	end = 2 * start;

	// fprintf(outFile,"\n**** penultimate U VALUES (unnormalized)
****\n");
	for (j = start; j < end+1 ; j++){
		parentN = (j -1)/2;
		uParent = u[parentN];
		computePenultimateS(j, lam1, lam2, price, uParent, &sNode);
		/* Find u at node */
		u[j] = uParent + sNode;

		// fprintf(outFile,"uParent: %lf sNode: %lf u[%d]: %lf \n", uParent,
sNode, j, u[j]);

		/* check and limit u value */

		if (u[j] < 0.0)
			u[j] = 0.0;
		if (u[j] > 1.0)
			u[j] = 1.0;
	}

	/* Compute u and C values at ultimate level */

	start = pow(2, noOfLevel) - 1;
	end = 2 * start;

	high = (1. + lam2)/(1. - lam1);
	low = (1. - lam2)/(1. +lam1);

	// fprintf(outFile,"high: %lf low: %lf\n", high, low);

	for (j = start; j < end+1 ; j++){
		if (price[j] < low)
			u[j] = 1.0;
		else if (price[j] > high)
			u[j] = 0.0;
		else{
			parentN = (j -1)/2;
			uParent = u[parentN];
			u[j] = uParent;
		}
	}
}
```

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