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By

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ABSTRACT

DISCURSIVE POSSIBILITIES: RE-IMAGINING REFORM AND EQUITY IN ELEMENTARY MATHEMATICS

By

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Much of the research on preservice education has focused on finding and remediating the problematic beliefs and inadequate knowledge of individual beginning teachers. Research on children engaged in elementary mathematics has generally been seen as a separate body of work. This dissertation seeks to explore the work of preservice education by drawing the lines around it differently. First, it uses discourse as a theoretical frame, which locates practices like teaching, problem-solving and understanding mathematics, outside of individual heads and disperses them into multiple, but always interacting, fields. Second, it uses children's interactions with elementary mathematics as a catalyst for thinking in new ways about how we might educate those who teach them.

To explore these issues, I spent a year in a third-grade urban classroom, primarily during math class, where I observed the students, the teacher, and the student teacher. I also observed the elementary mathematics methods course that the student teacher attended during the first semester of the year. Throughout this dissertation, I draw on my experiences in these two classrooms, as well as on relevant written documents, such as curricula, state standards, and academic writing in mathematics education.

In particular, this dissertation is intended to respond to two prominent strands within the conversation about preservice education in elementary mathematics: those about equity and reform-oriented teaching. In addition, the dissertation is also designed to respond to current calls for "evidence-based" or "scientific" education research by drawing on ethnographic, genealogical, and rhetorical research traditions. By highlighting different research traditions in different chapters, the dissertation makes it possible to see the analytical affordances of each of these research strands. In particular, the ethnographic chapter argues that students who do not share the linguistic, cultural or ethnic background of their teacher may have more difficulty answering open-ended questions in competent ways. The chapters that draw on genealogical and rhetorical traditions examine the role that metaphors of hierarchy have played in defining the ways that we think about student learning in mathematics, the persuasive powers of different kinds of problems commonly used in mathematics classrooms, and the purposes that multiple genres of teaching may play in the mathematics classroom. The overarching argument of the dissertation is that the current narrowly-focused consensus on the definitions of equity and reform-teaching limits, rather than expands, possibilities for students.

DEDICATION

For Perry,

Who kept asking me to explain Foucault,

And for Sophie,

Who didn't.

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This dissertation would not have been possible without the intellectual and emotional support of my advisors, Lynn Fendler and Sandra Crespo. Lynn introduced me to new ways of seeing not just my work, but my life, and seemed to have an instinctive knowledge of when to push and when to back off. Sandra made room in the world of mathematics education for an elementary school teacher and encouraged me to pursue work that mattered to me, even if it put me at odds with the majority mathematics educators. Always, they had my back.

I would also like to thank the two intellectual communities who significantly contributed to my thinking about this work. I am deeply grateful to my friends at Critical Studies, particularly Cleo Cherryholmes, Steve Tuckey, Brett Merritt, and Sharon Strickland, who took me seriously -- even though they were so many turtles farther along than I -- and who challenged me and agreed with me, depending on what I needed. Finally, I want to acknowledge the role that Anne Haas Dyson and the students in her Advanced Qualitative Research seminars played in the work of conceiving of this study. You held my hand through the early days.

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CHAPTER ONE

INTRODUCTION

In E.B. White's novel *Charlotte's Web*, a gray barnyard spider manages to save the life of a pig intended for the dinner table. She accomplishes this by writing a series of words in her web extolling the pig's virtues. She calls Wilbur "radiant," "terrific," and "humble," rejecting the rat's suggestion of "crunchy" for obvious reasons. The farmer and the fair-going pubic agree that Wilbur is "some pig" – not to be squandered as bacon – and he lives out his days taking care of Charlotte's descendants. Of all the characters in the story, only the farmer's wife suggests that it is not the subject of the web messages who is remarkable, but the author.

In this dissertation, I am concerned with the spider. I am not interested in whether Wilbur *is* an unusual pig, but in how he came to be seen as an unusual pig in a particular time and place; or, laying the analogy aside, I am not so much interested in elementary students themselves, as with texts produced about them. To muddle things up even more, I am interested in these texts not for what they can tell us about students, but for what they can help us understand about preservice education. This dissertation is designed to explore how students are constructed -- through problems, through teaching, and through conversations about them -- and to consider what these constructions tell us about the way we currently do preservice education in elementary mathematics.

This is a different way of thinking about the problem of preservice education because the literature in this area has tended to focus on the deficiencies of individual teachers. That is, it has mostly identified the work of preservice education as finding and remediating the problematic beliefs and inadequate knowledge of individual beginning teachers, while research on children engaged in elementary mathematics has generally been seen as a separate body of work (often produced by psychologists, rather than teacher educators). This dissertation seeks to explore the work of preservice education by drawing the lines around it differently. First, it uses discourse as a theoretical frame, which locates practices like teaching, problem-solving and understanding mathematics, outside of individual heads and disperses them into multiple, but always interacting, fields. Second, it uses children's interactions with elementary mathematics as a catalyst for thinking in new ways about how we might educate those who teach them.

To explore these issues, I spent a year in a third-grade classroom, primarily during math class. In addition to spending time with the students, teacher and student teacher in this urban elementary school, I also observed the elementary mathematics methods course that the student teacher attended on Thursdays during the fall semester of the school year. The course was taught by an experienced mathematics educator with a background in mathematics. Throughout this dissertation, I draw on my experiences in these two classrooms, as well as on relevant written documents, such as curricula, state standards, and academic writing in mathematics education.

In particular, this dissertation is intended to respond to two prominent strands within the conversation about preservice education in elementary mathematics: those about equity and those about reform-oriented teaching. Currently, prospective teachers are expected to develop knowledge, skills and dispositions that will allow them to teach mathematics in ways that create learning communities, encourage discussion, and promote reasoning and problem-solving (Ball & Cohen, 1999; National Council of Teachers of Mathematics, 1991, 2000; Putnam & Borko, 1997). In addition, beginning teachers are expected to develop understandings of students that will allow them to teach in ways that work against current disparities in mathematics achievement, which have been described primarily in relation to race, ethnicity, class and gender (Cochran-Smith, 1999; Gutstein et al, 2005; NCTM, 2005). Despite wide agreement about the goals of reform-teaching and equity, we know little about how working toward either of these goals impacts the other during the teaching of preservice teachers. The NCTM Standards assume that work toward equity is “interwoven” (p. 12) with the goals of coherent curricula, learning with understanding and collaborative environments (NCTM, 2000). Teacher educators may make similar assumptions, but we don’t know if these assumptions are warranted. Popkewitz (2004) argued that the language of equity in the Standards documents actually works against the stated goal of helping all children to learn mathematics because the word “all” does *not* signify all children, but instead calls to mind a certain type of child --one who is “disadvantaged,” and therefore unlike the child who has the “capacities to learn, problem solve, and achieve” (p.23). His argument raises questions about the role reform teaching plays in defining what equity is in the teacher education classroom. Similarly, conversations about race, gender or class, which may be held in methods classrooms to address the goal of equity, may impact the meaning of reform teaching in mathematics.

For example, in the *Principles and Standards for School Mathematics* (2000) the phrase “all students” occurs repeatedly in the section on “The Equity Principle.” On just one page (p. 13), the report reiterates the phrase several times: “mathematics can and must be learned by *all students*,” “*all students* should have access to an excellent and equitable mathematics program,” “high expectations for mathematics learning must be communicated to *all students*," and "schools have an obligation to ensure that *all students* participate in a strong instructional program" (italics added). However, other language around these phrases seems to suggest that the authors are not referring to all students, but to students with certain characteristics. The report's synonym for *all students* is not "students," but "students who live in poverty, students who are not native speakers of English, students with disabilities, females, and many non-white students" (p. 13). The linking of these categories of students with the concern about helping *all* students to understand mathematics works to reinforce the idea that certain kinds of students can be expected to have difficulty learning mathematics and therefore require extra attention.

The relative absence of the phrase "all students" from the sections of the report that address various mathematical strands reinforces this interpretation of all students. The qualifier "all" is dropped in almost all descriptions of students in the sections on mathematics content. Instead, it says "students must become fluent in arithmetic computations," (p. 35), "students should learn to formulate convincing explanations," (p. 42), and "By the end of the second grade, students should be able to organize and display their data through both graphical displays and numerical summaries" (p. 109). In these and similar cases, the single word "students" seems to signify all students in the report's discussion of reform teaching. The student constructed through the description of reform teaching is one who is fluent in computations, can form convincing explanations and can organize and display data. Students who do not meet these criteria become "all students" and need some sort of remediation or special attention. Thus, despite the frequent exhortations to teach all students, the report works against a notion of equity that includes a recognition that students come to school with varied ways of understanding and acting on mathematics. If "students should be able to organize and display their data" or "formulate convincing explanations," those who cannot are inadequate. By emphasizing what students *should* be able to do, the report works to create two kinds of students -- those who meet reform expectations and those who do not. The way educators talk about certain students' failures to meet these expectations and the expectations set out for students through problems and teaching situations are topics that will be explored in the dissertation.

Looking closely at how the subject of the elementary student gets constructed in both the elementary and the methods contexts will allow me to explore ways that notions about reform teaching --- which can be similar to or different than those in Standards documents --- have consequences for how one thinks about providing equitable educations for students in mathematics. My decision to take a critical stance toward commonly held notions of equity and reform mathematics is based in part on my recognition that the audience for this dissertation will be composed primarily of university teacher educators, many of whom seek to change the attitudes and practices of novice and expert teachers in relation to ideas of equity and reform-mathematics, while seeing their own understanding of these ideas as unproblematic. Like Charlotte, who considered the impact of "crunchy" on her audience, I am choosing to look at equity and reform critically because I believe for this audience, it is the most educative stance. Were I talking to teachers, politicians or parents, I might discuss these ideas in a different way. In particular, I see the critical stance I am adopting as a way of creating openings in the dense discourses around reform mathematics and equity in teacher education. I see this move as productive because widespread agreement about goals, methods, and projects necessarily limits innovation and possibility, and I believe that unpredictable innovations are important to creating opportunities for change. When I am working in K-12 settings, I often aggressively promote reform mathematics as a way of creating openings in the dense discourse around traditional, algorithmic conceptions of mathematics. For the same reason, when talking to teacher educators, I want to be critical of reform mathematics to create openings for unpredictable innovations in the dense positive discourse around Standards-based teaching.

In addition to contributing to the conversation about reform mathematics and equity, I would also like this dissertation to respond to current calls for "evidence-based" or "scientific" education research (National Center for Education Evaluation and Regional Assistance, 2003, p. 1). Exactly what these calls mean is still under negotiation. For instance, while the policy statements of some agencies have included only large studies that use randomized control groups as appropriate (e.g., National Center for Education Evaluation and Regional Assistance, 2003), the National Research Council (2002) classified ethnographic as well as experimental designs as evidence-based. Despite these differences in interpretation, most commentators on these policies have generally accepted the principle of evidence-based research, although they have argued simultaneously for a definition that is both clear and inclusive. For instance, in their essay which leads the *Educational Researcher* focus issue on the NRC report, Feuer, Towne and Shavelson (2002, p. 4) said "Almost everyone can appreciate intuitively the advantages of evidence-based policy; it is another matter entirely to make this concept clear, operational and valid." Others have argued that evidenced-based policies must include room for small ethnographic studies (Erickson & Gutierrez, 2002), for the messy ways of researchers who work in chaotic schools rather than orderly labs (Berliner, 2002, p. 18), and for closer links between researchers and teachers (Burkhardt & Schoenfeld, 2003). While critical of the NRC report in various ways, these authors do not question whether it is truly valuable to make a research in education "scientific."

In the same focus issue, St. Pierre (2002, p. 27) offered a somewhat strident -- "The NRC report should scare us all to death." -- response to the calls for evidence-based research. Drawing on postmodern philosophies, she wrote "This latest attempt to marginalize certain epistemologies and methodologies in order to control science, to reduce it, to *center* it, cannot go unanswered" (italics in original). Her argument was that science ought to include educational research that is not evidence-based, generalizable or methods-dependent. I want to make a slightly different move in this dissertation, although one that also draws on postmodern thinkers, by ceding the right to claim my work as social science. Unlike the researchers cited above, I am not trying to broaden the government definition of science, but to see what it might be possible to learn about children and teaching by playing another sort of game.

Fendler (2006) offered one possible alternative to social science as the means of understanding the work of teaching when she suggested that the field of rhetoric could provide useful tools for analyzing interactions in classrooms. Following this tradition, teaching can be seen as "a kind of persuasion, or perhaps a way of moving people" (p. 2) and analysis can examine the techniques used in this act of persuasion, including beautiful language, shared assumptions, and available genres (discussion, lecture, groupwork, etc.). In an essay on rhetoric as a research perspective, Leff (1987, p. 24) wrote:

> (Rhetoric) implies a pluralism in which methods of inquiry and argument are adapted to the particular subject under investigation. It seeks to solve situated problems rather than to formulate abstract theoretical principles. Explanation replaces prediction as the standard for verifying arguments. And such verification depends on agreement within the community of those concerned about the subject, not upon a process of matching evidence against a disinterested criterion of proof. All abstract categories are treated with suspicion.

On first reading, this stance may not seem so different from those taken by many qualitative researchers. For instance, Erickson (1986) described interpretive research as skeptical of predictions, focused on the local, and methodologically diverse. However rhetorical analyses differ from qualitative ones in important ways. Most significant may be differences in claims made about the kind of work being undertaken. Ethnographers collect data as evidence for arguments they will make about how people operate in social situations. Although most ethnographers now acknowledge that the way they are positioned in the world as human beings and as researchers impacts the way that data is collected and interpreted, many also argue that careful attention to methods can produce "findings" that reveal (at the very least, situational,) truths about the world (Bogdan & Biklen, 2003; Emerson, Fretz & Shaw, 1995; Erickson, 1986). By framing teaching as persuasion, or as an argument, rhetoric makes different claims about the work being done by researchers. Rather than collecting evidence about the real world, a rhetorical analysis identifies arguments and seeks to make explicit in a variety of ways the strategies used to make these arguments.

Aristotle, one of the original rhetoricians, argued that persuasive power depends on the character of the speaker and the perspectives of the audience, as well as on the quality of the argument. Throughout the Rhetoric, Aristotle used examples to show that decision-making (and therefore persuasion) is intensely personal. He wrote that blinding a one-eyed man is worse than blinding a man with two eyes because the first man "has been robbed of what he dearly prized" (1365b, ln 17-18). This is not an argument based on logic; it is an emotional one. Because the eye was valued more, desired more, prized more, destroying it was worse. In this dissertation, I will draw on a handful of rhetorical tools – persuasion, genre, and metaphor – to make arguments about the elementary and methods classrooms I observed. To be persuasive to readers, Aristotle argued that I will have to be able to "(1) to reason logically, (2) to understand human character and goodness in their various forms, and (3) to understand the emotions-that is, to name them and describe them, to know their causes and the way in which they are excited" (1356a, ln 16-19). As you read this dissertation, I hope you will find the logical leaps I make sensible; the ways in which I describe the human characters to be considerate and my mobilization of texts honest; and the text, occasionally worthy of inspiring emotion, including both anger and laughter. These are the means by which I seek to persuade you. I am not – except for the second chapter, which I will discuss in a moment – asking that you believe me because I followed standard ethnographic methods. I do not expect that you or anyone else would be able to replicate this study, follow my ethnographic path, or reach identical conclusions. In other words, I am not mobilizing the conventions of social science in order to persuade you.

Poster-child for the postmodern Michel Foucault said he was not interested in efforts to make Marxism or psychoanalysis into science. Instead, he asked "What types of knowledge do you want to disqualify in the very instant of your demand: 'Is it a science?'" (Foucault, 1980, p. 85). It is not just supporters of the *No Child Left Behind* policy who are seeking to disqualify certain kinds of knowledge, but also those who claim that ethnography or critical race theory are scientific. The insistence that one form of knowledge production is valid always carries with it the shadow claim that another form is not. Foucault's work intentionally focused on these forms of disqualified knowledge. He called this work *genealogy*, which he described as entertaining "the claims to attention of local, discontinuous, disqualified, illegitimate knowledges against the claims of a unitary body of theory which would filter, hierarchies and order them in the name of some true knowledge and some arbitrary idea of what constitutes a science and its objects" (Foucault, 1980, p. 83). In other words, the emphasis in genealogy is not on what *is* true, but on how what we now know as true has come to be produced. Examining truth as production requires attention to local, discontinuous and disqualified knowledges because taking seriously these shifting, plausible, overlooked alternatives provides a way of seeing the rules of our own time and place as mutable, rather than fixed.

I began my writing of this dissertation from within commonly accepted ethnographic traditions. I collected data, coded and analyzed it, and made assertions. The second chapter, which examines questioning practices in an urban third-grade classroom, is the result of this work. However, I felt deeply uncomfortable about the ways in which this chapter positioned the children in the classroom and myself as a researcher, particularly in regard to issues of race and equity. The conclusions of the chapter – that some children had more trouble than others answering open-ended questions (and I bet you can guess which children) – seemed preordained, and my own work, despite its talk of equity, seemed to reinforce long-standing assumptions. I turned toward rhetorical and genealogical analyses in the following chapters as a way of unsettling the predictable outcomes that the methodological traditions of social science (fieldnotes, data, evidence,) and the accompanying theoretical constructs (culture, beliefs, learning) seemed to produce. Despite this turn toward genealogy, I included this ethnographic chapter in the dissertation for a couple of reasons. First, it provides a record of where my thinking began and offers a contrast with the genealogical/rhetorical chapters that follow. This serves a pedagogical purpose because the ways in which equity and methods of inquiry are approached in this chapter differ from those that follow. For instance, in the ethnographic chapter equity is coupled with the notion of culturally relevant pedagogy, and therefore, the expectation that attending to the race, gender and economic status of children is central to teaching them. Also, the ethnographic chapter relies on data, coding, and assertions, while the following chapters make arguments without mobilizing these terms. This contrast allows the reader to think about what he or she finds persuasive and why and opens the possibility for thinking about what non-scientific modes of inquiry might offer. A second reason for including this chapter, despite my reservations, is that I see it as essential to my entry into the research community. Ethnographic methods (unlike rhetorical and genealogical modes of inquiry) are widely understood and accepted by educational researchers. As a novice, I wanted to demonstrate my mastery of these methods before adopting a critical stance toward ethnography. In this way, the chapter serves an important rhetorical purpose.

The majority of this dissertation is located within rhetorical and genealogical traditions, rather than ethnographic ones, because these traditions create possibilities for unpredictable outcomes. E.B. White's Wilbur was the runt of the litter. Charlotte's response to his imminent death could have been to study the situation as it was and then to work on remediating a pig, whose test scores for weight and height were well below average, so that he might eventually have been seen as worthy of an award at the fair. Instead, she thought about how truths are produced in the human world and deployed this knowledge to come up with a solution that could not have been imagined based on all the data collected up to the moment of her writing. In similar ways, my hope is that this dissertation will create unpredictable openings for myself and my readers to act in our own lives by both analyzing the problems of teacher education in elementary mathematics in new ways and by demonstrating the possibilities for understanding the work of education offered by non-scientific ways of knowing. To do this, I organized the dissertation in the following way:

The second chapter is an ethnographic look at teacher questioning in the elementary classroom I observed. I include this chapter both to discuss the ways that different kinds of questions include and exclude students, but also to make it possible for readers to contrast ethnographic and rhetorical/genealogical approaches to studying teaching and learning.

The third chapter is a short follow-up to the ethnographic chapter, which examines the ways in which I have come to see standard ethnographic arguments as problematic and discusses my own postmodern turn.

The fourth chapter discusses in detail Foucault's notion of discourse and its relationship to research in mathematics education. This chapter lays the groundwork for the following three chapters, which use rhetorical tools to pursue genealogical questions.

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Each of these chapters seeks to examine a commonly accepted notion – such as, the belief that process problems are better than algorithmic problems – in a way that makes the concept seem less inevitable, natural and right. The common goal of each of these chapters is to create more possibilities for teachers, children, and researchers. Although each chapter is quite different, certain rhetorical maneuvers can be seen across the chapters, such as the tracing of ideas through discourse broadly in embodied and written texts and the avoidance of charts and categories. These strategies can be read as coming out of rhetorical and genealogical traditions.

In particular, the fifth chapter addresses the issue of equity by examining through the lens of metaphor, rather than belief, comments made about elementary students.

The sixth chapter addresses reform mathematics through the examination of the mathematics problems used in the elementary classroom, the teacher education classroom, and two elementary curricula, and focuses on the ways in which different kinds of students are constructed by different kinds of problems.

The seventh chapter simultaneously addresses reform mathematics and equity through a description and analysis of the genres of teaching present in the elementary classroom, focusing on the opportunities for student participation in each genre.

The final chapter returns to the problems of reform teaching, equity and evidence-based education to re-examine these issues in light of the arguments presented in the dissertation.

CHAPTER TWO

HOW MIGHT EXPLICIT QUESTIONING SUPPORT THE LEARNING OF ALL CHILDREN IN MATHEMATICS?

Over the last two decades, most mathematics educators have moved toward a shared belief that teacher talk which is open-ended, probing and neutral toward right and wrong answers offers more opportunities for students to learn significant mathematics than talk which is narrow, directive, and authoritative (Forman & Ansell, 2001; Lampert, 2001; Moyer & Milewicz, 2002; National Council of Teachers of Mathematics, 1991; 2000; Simon & Shifter, 1991). More recently, educators concerned with equity issues have begun to question whether this more open teaching style, which I will call reform teaching, is effective for all children, particularly for children who belong to minority groups, who speak English as a second language, and who are poor (Ball, Goffney & Bass, 2005; Lubienski, 2000). The purpose of this chapter is to explore one intersection of reform teaching and equity issues by examining teacher questioning practices in a reform-oriented elementary classroom located in an urban school with a diverse student body.

The NCTM Standards documents (1991; 2000), which have played a major role in shaping the shared conception of reform teaching, promote indirect teaching practices, such as encouraging student discussion, asking for justification, and providing time for exploration of challenging problems, rather than direct teaching practices, such as lecturing, modeling, and drilling of correct procedures. These indirect teaching practices are seen as strategies that promote process skills, such as reasoning, communicating and problem solving, in addition to content knowledge of mathematical strands such as number, geometry and data. Throughout this chapter I will use the word *reform* to describe teaching that includes process skills as well as content and the word *traditional* to describe teaching that is more narrowly focused on content. Many have argued that reform practices lead to deeper learning of mathematics by all children (e.g., Blanton, 2002; Cobb et. al., 1997; Heaton, 2000; Lampert, 1990; Van de Walle, 2004; Yackel & Hanna, 2003), and others have claimed that these practices are particularly important for children who belong to groups who traditionally have not been successful in mathematics classrooms (Davidson & Hammerman, 1993; Ladson-Billings, 1997; Moses & Cobb, 2001; Trafton & Claus, 1994; White, 2000). For instance, Haberman (1991, p. 290) contrasted practices like the offering of challenging problems and the asking of open-ended questions with the "pedagogy of poverty," which "emphasizes repetition; drill; convergent right-answer thinking; and predictability."

In contrast, others have suggested that these reform practices, which may be productive for majority, middle-class children, may not best serve children from non-dominant cultures (Ball, Goffney & Bass, 2005; Lubienski, 2000). Both inside and outside of mathematics education, many have argued that children excluded from the dominant culture need explicit instruction in content and practices that may be automatic for others. In her book on literacy, Delpit (1995) argued that children who are not taught "the codes of power" (p. xvi) at home need direct instruction about these codes, which, among other social practices, include ways of speaking and listening, ways of behaving, and attitudes toward adults. Although Delpit has sometimes been cited as an advocate of skills-based teaching over reform teaching, her argument is more subtle. Calling the opposition between reform and traditional teaching a "false dichotomy," (p. 46) she advocated a middle ground where children engage in work that promotes creativity and critical thinking as well as in work that makes the standards of the larger community explicit. Researchers in mathematics education have just begun to explore what this middle ground might look like. In particular, Boaler (2002) has suggested that the fact that some reform practices may disadvantage some students does not necessarily mean that traditional practices would provide more learning opportunities for these same students. Instead, she suggested that "the differences between equitable and inequitable teaching lie *within* the different methods commonly discussed by researchers" (p. 240). In her work, she identified several practices -- such as making real world contexts accessible and teaching children to explain and justify -- that made mathematical discussions productive for children in an urban school. She recommended that researchers continue to explore ways that teachers can enact practices such as mathematical discussions and problem solving with an eye toward variations that promote equity. This chapter seeks to build on this work by exploring the ways that various kinds of questions can support students in learning significant mathematics.

Research on teachers' questioning practices has tended to emphasize moves away from direct, single-answer questions toward open, probing questions (Lampert, 2001; McCrone, 2005; NCTM, 1991; Vacc, 1993). For example, Lampert (2001) talked about beginning lessons with questions like "Okay, who has something to say about A?" as a way of teaching students "that mathematical talk can have a broad range, and not just be about right and wrong answers to teachers' questions" (p. 145). The 1991 NCTM Standards document presents a list of similar questions to Lampert's as examples of good questioning, including questions like: "How did you think about the problem?" and "Does that make sense?" (NCTM, 1991, pp. 3-4). In addition to promoting reasoning and justification, these questions can also be considered implicit, rather than explicit. That is, not only are a wide range of answers possible, but a wide range of *kinds* of answers are possible. As Lampert said, students could respond to her question of "Who has something to say about A?" by offering an answer, providing a description of their thought processes, asking a question, or evaluating the problem in some way. The question itself does not provide clues about the kind of answer desired by the teacher. Thus, it can be considered an implicit question, as opposed to an explicit one which does provide clues about the kind of answer the teacher expects.

Research on questioning has not paid a great deal of attention to the ways that context or differences in students might impact what is considered competent questioning (Carpenter et al, 1999; Mewborn & Huberty, 1999; Moyer & Milewicz, 2002). In their study of preservice teachers' question-asking during diagnostic interviews, Moyer and Milewicz (2002) describe three hierarchical categories of questioning -- checklisting, instructing rather than assessing, and probing and follow-up – which they use to evaluate the questioning skills of the preservice teachers they studied. Their focus remained tightly on the preservice teachers during the interviews, rather than on the ways that different students responded to questions. Similarly, in their discussions of questioning, Carpenter and his colleagues (1999) emphasized the skills teachers must have in choosing problems and selecting open-ended questions, rather than the skills teachers must have in adapting their instructional practices to meet diverse needs in their classrooms. The current study seeks to re-examine teacher questioning in reform classrooms in light of equity concerns about the need for explicit language and direct instruction by keeping both the teacher and the students in focus. To do this, I asked the following research questions:

- What kinds of questions do teachers ask in a reform-oriented elementary classroom located in a minority-majority urban school?
- What learning opportunities do various kinds of questions make possible for students?

Theoretical framework: Talking about race and language

This chapter is framed by the lens of culturally relevant mathematics pedagogy (Gutstein, Lipman, Hernandez & de los Reyes, 1997; Ladson-Billings, 1995; 1997), which asks researchers to attend to ways that students' racial and cultural identities interact with their learning of mathematics. Ladson-Billings (1995) described culturally relevant pedagogy as teaching that was committed to developing students' academic excellence, cultural competence, and critical consciousness. Most relevant to this chapter is the notion of cultural competence. Students of different cultural backgrounds and language practices are likely to respond in different ways to classroom language and instructional practices (Cazden, 1988; Dyson, 1997; 2003; Heath, 1983), and, therefore, are likely to feel different degrees of cultural competence in the classroom. Ladson-Billings (1997) argued that European American, middle-class students are more likely to feel culturally competent in mathematics because current teaching practices tend to draw on familiar cultural values, such as "efficiency, consensus, abstraction and rationality" (p. 699) while many African American students might feel more competent working in classroom cultures that emphasize other values, such as "orality," "communalism," and "movement" (p. 700).

In this chapter, I paid attention to race in the analysis of student-teacher interactions in a reform-oriented classroom because I wanted to make it possible to think about ways that questioning practices might advantage some students and disadvantage others. I am aware that pursuing this inquiry is dangerous work. Other researchers with equity-oriented intentions have ended up in problematic places as a result of their efforts to examine the ways that students of various races respond to language in the classroom. In her seminal study, *Ways with Words*, Heath (1983) described differences in the ways that children positioned differently in terms of race and class responded to the kinds of questions commonly asked at school. By studying how children interacted with their parents at home, Heath was able to argue that cultural differences produced these varied responses and to suggest ways in which teachers could interact more productively with all of their children. This work was important in challenging cultural deficit models, where poor and minority students were seen as inadequate in reference to middle-class, majority students. However, it also inadvertently suggested that race was an essential quality, which could produce certain kinds of behaviors. More troubling, Orr (1987), who studied the failure of African American students in mathematics and science at a private, progressive school, concluded that Black Vernacular English prevented students from thinking in mathematical ways. She suggested that this dialect, used by many African-American students in the school, did not have adequate vocabulary or grammar structures to support high-level quantitative thinking. Although this work was harshly critiqued by linguists, in particular Baugh (1988, 1994), for both the quality of the scholarship and the deficit-oriented stance, researchers have cited it (and continue to cite it) as part of the useful literature on language in the mathematics classroom (e.g. Ball, 1990; Ma &

Kishor, 1997). My challenge in this chapter was to write about the ways that the African American and Asian American students in my study responded to various questions without (as much as is possible) either essentializing race and ethnicity or suggesting that the ways that students responded were in some way inadequate.

I do not want to treat race, even implicitly, as an independent variable. That is, I do not assume that there is a causal relationship between a child's racial identity and his or her performance in the mathematics classroom. However, I do want to acknowledge the relevance of race in classroom interactions. In socio-cultural ethnographic research, human interactions are seen as causal, rather than traits such as race, gender or mathematical ability. Omi and Winant (2004) wrote about this as "the performative aspect of race" (p. 10), where race is seen as continually acted out in various ways by various people. Thus, race is understood as performed rather than as a fixed biological category; however, "the enormous number of effects race thinking (and race acting) have produced" (p. 9) are possible objects of study.

This is the perspective I adopt as I work to describe classroom interactions between European American teachers and the (mostly) minority children in their classroom. In doing so, I acknowledge that race is tied up in other features of classroom life that may be important to understanding these classroom interactions. Race is highly visible (to American eyes); however, children's differential knowledge of the rules for participation in classrooms may be more important to understanding instructional interactions, but harder to see. For instance, Cazden (1988) argued that children's familiarity with the ways of speaking expected by teachers impacted how successful children were in participating in classroom conversations. These ways of speaking, which can be thought of as registers, include word choice, timing, and syntax. Children who speak in appropriate registers are more likely to be heard and responded positively to by teachers. This is not to say that children's ways of speaking are unrelated to their race and culture, but that these ways of speaking may be one way of performing race and that it may be as important to pay attention to patterns of language as to other performances of race and culture (such as experiences at home, notions of appropriate adult-child relationships, notions of self and other, etc.). My major concern in this chapter is to explore the ways that various kinds of questions provided and constrained learning opportunities for the particular children I studied, rather than to detail a pedagogy for African American or other minority students. For this reason, I attend to the ways that language was used in the classroom; however, because of the effects of "race thinking" and "race acting" in our schools, I do not want to make race invisible.

Methods

Interpretive, ethnographic traditions emphasize close attention to the particularities of human interactions within specific, and complicated, contexts. Erickson (1986) said that one of the benefits of ethnographic work is that it acts against the "invisibility of everyday life" by "making the familiar strange and interesting" (p. 121). These methods allowed me to closely examine the ways teachers spoke to children during reform-oriented practices and to consider unfamiliar interpretations of their words – for instance, that an explicit question promoted more rather than less mathematical thinking. As a researcher, I attended to the speech of participants, their interactions with each other, and their use (and non-use) of mathematical texts and resources.

Setting and Participants

Blythe Elementary is a small K-5 school serving about 300 children. Like many urban schools, Blythe's student population is minority-majority. During the year of the study, about 60 percent of its students were African-American and about 20 percent were European-American. Asian American and Hispanic students made up nearly all of the remaining 20 percent. About two-thirds of Blythe's students received free- or reduced-lunch during the year data was collected. The demographics of the class I studied mirrored the population of the school as a whole. Diana Emerson, the teacher in the third-grade class where my study was situated, had a great deal of experience, both as a classroom teacher and as a part of a university teacher education program. She had been teaching at Blythe Elementary for nearly 30 years; and throughout that time, she participated in university study groups about mathematics, and welcomed student teachers, university undergraduates and faculty into her classroom. Sara, the intern assigned to Diana during the year of my study, took a year off in between her graduation and her internship, which she spent working for an after-school program in another city in the same state. Sara was described by former professors as a strong student who was committed to teaching. Both Sara and Diana were European American, as were the five education undergraduates who visited Diana's classroom one morning a week to teach problem-solving lessons to small groups as part of their coursework.

Diana's classroom was large, bright and well-stocked, although the carpet, ceiling and visible walls revealed the age of the building, which had just celebrated its 40th anniversary. The seating arrangement changed throughout the year, but student desks were always arranged in groups, angled so students could see the board and overhead screen at the front of the room. Shelves stuffed with books, math manipulatives and science supplies lined the walls. Large desks for Diana and Sara occupied the back of the room. One bulletin board ran along a side wall and typically displayed art work by each student in the class, including self portraits, drawings of the future, and attempts to represent the styles of various artists.

The year of the study was the first in the district's adoption of a new elementary mathematics curriculum, *Math Advantage*, by Harcourt School (Andrews et al, 2004). Diana received 300-plus-page math books for each student, which sat on a shelf in the back of the room for the first six weeks of the year until Diana decided she needed to pass them out to avoid attracting the attention of her principal or other school district visitors. These books were used for the first time in January during Sara's lead teaching. In addition to student books and teachers' guides, Diana received a student manipulatives kit for each child, which contained materials such as place value blocks, dice and rulers. She did not ever pass these out, preferring to use the class sets of similar supplies she already owned. In addition to the *Math Advantage* curriculum, Diana owned a complete third-grade set of the Investigations curriculum by TERC, which both she and Sara referred to occasionally.

To guide my observations of the mathematics classroom for the dissertation, I chose five focal students in Diana's classroom as an "anchor points" (Dyson & Genishi, 2005, p. 49) for my attention. I alternated observations among my focal students, which allowed me to see the classroom from multiple perspectives. When observing students, I typically sat next to them or behind them. I took notes in a notepad on my lap and placed my tape recorder in a visible but discrete location, such as on an empty table nearby. Because I wanted to observe diverse experiences in the classroom, I purposefully chose focal students who were situated differently in terms of ethnicity, gender, perceived mathematical ability, frequency of participation in whole-class conversations, and physical location in the classroom. The chart below describes my five focal students.

Table 1: Focal Students

Name	Race	Gender	Additional Information
Aliah	Bi-racial: African American/European American	Female	Aliah left the classroom twice a week to attend the district's gifted and talented program. She spoke rarely in whole-group settings.
Caitlin	African American	Female	Caitlin spoke infrequently in whole-group conversations, but was often an enthusiastic participant in small-group settings. Diana described her as "trying hard."
Ben	European American	Male	Ben spoke occasionally in whole-group discussions and tended to work by himself in small-group settings. He was described as an excellent math student by Diana, Sara, and many of the children.
Jerome	African American	Male	Jerome participated occasionally in whole-group discussions and preferred to work

			alone in small-group settings. He was described as "struggling" by Diana.
Marcus	African American	Male	Marcus participated frequently in whole-class discussions as well as in small-group settings. Marcus was seen as a good, although not exemplary, student.

The class as a whole had 19 students for most of the year, including nine boys and ten girls. Of these children, ten were identified as African American on enrollment forms; four were identified as European American; three were identified as bi-racial; and two as Asian American. Both Asian students were Hmong. One of these students, Mia, was the only student in the class currently learning English as a second language, although Charlie, the other Hmong student, spoke fluent English as his second language.

I decided to use the above chart as a rhetorical move, recognizing that readers may need a quick way of keeping track of the students I spent an academic year getting to know. However, it is important to note that the clean lines and categories of this chart are problematic. Although it is not my intention, labeling Jerome as African-American, male and struggling works to make each of these categories stable (and worse yet, works to conflate them). One of the challenges of this chapter was the struggle to write about ethnicity, gender and ability in ways that recognized the role these categories played in the life of the classroom while simultaneously working toward a theoretical stance that recognizes these categories as continually performed rather than fixed.

Data Collection

At the end of September, I started weekly observations of Diana's classroom during math time. I visited weekly until Sara left the classroom at the end of April, alternating observations among focal children in the classroom. I audio taped observations and wrote fieldnotes immediately afterward, following ethnographic traditions (Emerson, Fretz & Shaw, 1995; Erickson, 1986). I made copies of assigned student work, assigned pages of the textbook, and pages of teacher's guides, standards documents and assessments that were referenced. I also took notes on what was written on the board or the overhead. In addition, I took notes about the physical setting of the classroom each week. The chart below summarizes data I collected.

Table 2: Summary of Data

Type	Quantity
Observations of Elementary Classroom	22 observations of math classes Breakdown of observations of focal students: Aliah 3; Ben, 4; Caitlin, 6; Jerome, 5; Marcus, 4. 1 observation of science 2 observations of literacy 1 observation of recess
Lesson Plans	Copies of 4 months of plans from Diana Copies of 6 months of plans from Sara
Interviews	5 informal interviews of Sara 3 informal interviews of Diana 1 formal interview of Sara and Diana together
Mathematics Curricula	Mathematics Advantage by Hartcourt School, Third Grade student book and teachers' guide Investigations by TERC, teachers' guide
State Standards for Elementary Mathematics	Grade Level Content Expectations by the Michigan State Department of Education
Student work	15 pages of journal entries by each elementary focal child 2 classroom assignments by every elementary student in 2 classroom assignments by each elementary focal child 1 methods assignment by each student 3 lesson plans for methods class by Sara

Data Analysis

When I completed data collection, I began analysis by using open coding (Emerson, Fretz & Shaw, 1995; Erickson, 1986) on my fieldnotes. I developed several major categories based on this coding. The most relevant for this chapter was "genres of teaching," which included whole-class mathematical discussions, small group work, practice sessions, whole-class question-answer sessions and teacher-child individual work. For the purposes of this chapter, I focused on episodes of teacher-child individual work and whole-class discussions because these two genres allowed me to look at practices with opportunities for reform-oriented learning, such as mathematical discussions and problem solving, and at teacher questions. I coded episodes within these two genres from across the year for kinds of questions asked, types of student responses, and evidence of learning, and used this analysis to develop the assertions in this chapter. The chart below lists the most frequent codes I used for the analysis; the meanings of many of these codes will be explored in the following section of the chapter.

Table 3: Most Frequent Codes

What kinds of questions do teachers use in their work with individual children in a reform-oriented elementary classroom located in a minority-majority urban school?	Explicit question, Single-answer question, Multi-purpose question, Implicit question, Reasoning question, Analytic question, Teaching question
What learning opportunities do various kinds of questions make possible for students?	Student confused, student solves problem, student makes meaningful mathematical statement, student continues to work, student frustrated, student gives up

When coding the data, I sometimes assigned multiple codes to a single segment of data. For instance, a question might have been both "implicit" and "analytic." After coding the data, I sorted chunks of data by code. For example, I took all of episodes where explicit questions were asked and compared them across students, kinds of tasks, and types of responses. I made notes on key similarities and differences among several chunks of similarly coded data and used these notes to write this chapter.

Researcher Role

At one time or another, I have occupied the roles of each of my participants. I have been a third-grade student, a third-grade teacher, and a preservice teacher. The result of this is both that I have some insight into (and some blind spots about) the challenges and perspectives of each of these roles and that my own experiences shaped what I saw during my observations. As a child, I attended a large integrated school in Milwaukee. The school's population was almost evenly split between neighborhood children, who were mostly working-class and African American, and children bussed in, who were mostly middle-class and European American (myself included). In junior high, I moved to a wealthy, nearly all-white suburb, an experience which made both social class and race visible for me. As an elementary teacher, I worked only in minority-majority schools in both suburban and urban areas. In each of these roles, I've struggled with mathematics, finding it distasteful as a child, boring as an adolescent, and sometimes mystifying as an educator. I chose to work in mathematics not because I have found it easy, but because I have found it difficult. My own struggles bring me insight into the struggles of students and remind me of what it feels like to be an outsider.

As I approached my participants, I found myself drawing on each of these roles, sharing my experiences as a beginning teacher with Diana and Sara and laughing with students about the difficulty of a certain problem. My history also shaped my observations. I found myself drawn to students who struggle in mathematics and looked for undercurrents related to both race and class. Additionally, I found that I had to hold in check my reactions to teaching. I have strong tendencies to like teaching that resembles my own and to find teaching that strays too far from my own practice distasteful. My goal was come in with a real curiosity that would help me to learn about the people in the room, rather than to hold them up against my own experiences.

Thinking about Kinds of Questions

Initially, I had expected to find two broad categories of questions in my data: implicit, reform-oriented questions and explicit, traditional questions. That is, I expected that questions aimed at encouraging students to reason, communicate and problem solve (reform questions) would be open-ended and unspecific, such as "why?" or "what do you notice?" I also expected to find explicit, traditional questions -- questions where only one correct answer was expected, where the teacher clearly signaled what kind of answer she was expecting, and which focused on mathematical content, rather than process skills. However during analysis, I found these two categories to be unproductive because they contained too many disparate ideas. For instance, Diana frequently asked individual children: "Do you agree with this solution?" when another child presented an answer to a problem. This was an explicit question. It identified one student who was expected to respond, narrowed the range of appropriate responses, and directed the student's attention to evaluating another student's answer. However, it was also a question that required children to analyze the work of their classmates and, following the norms of this classroom, to offer a sentence or two about their own reasoning. It was a question that seemed to be both explicit and reform-oriented. As a result of trying to classify questions like this one, I decided that I needed to work along two axes: from *reform* to *traditional* and from *implicit* to *explicit*. Questions that I considered explicit were firmly embedded in a context from which children could draw possible answers; whereas, implicit questions required students to decide on a context in which to locate their answers. The chart below classifies a few exemplar questions that Diana and Sara used in whole-class lessons.

Table 4: Categories of Questions

	Implicit	Explicit
Reform	Why? What do you notice about this? Why does this make sense? What's a prediction you could make? What can you tell me about this? What do you think?	Caitlin, can you say why you disagree with Sienna's answer? Tell me why you're adding 32 and 32. Why would 26 not make any sense as an answer?
Traditional	What do you do to add two-digit numbers with regrouping? If you haven't memorized your facts, what can you do to get the answer?	What is four groups of two? What digit is in the one's place, everybody? What do we call the name of this coin? Okay, in Celsius, what temperature does water freeze at?

The questions in the top row require students to communicate their reasoning about a problem or to offer justifications for their own thinking, while the questions in the bottom row require students to compute or recite memorized information or to discuss strategies for doing so. In some ways, it is difficult to evaluate these questions outside of the classroom context in which they occurred. For instance, the question “If you haven’t memorized your facts, what can you do to get the answer?” might be considered a reform question because it offers the possibility for students to reason or justify their thinking. However, this question was asked as part of the introduction to a timed test with the stated goal of offering students ways to find answers if they had not memorized their facts. The purpose was to promote fast computation, not to explore student thinking. I classified it as implicit because the question did not signal the kinds of responses that were expected, so some students did offer commentaries on ways they broke apart and put together numbers before being redirected by the teacher toward test-taking strategies such as counting on fingers, looking for multiple problems with the same answer, and starting with easy problems.

Questions in the first column forced students to figure out a way to answer the question from multiple possibilities. For instance, when asked “What do you notice?” students might focus on patterns, the way numbers are written, correctness or incorrectness, or other features of the problem. Similarly, the answer to “What do you do to add two-digit numbers with regrouping?” might begin with lining up the problem or with adding the ones column or with an explanation of the regrouping process. In contrast, questions in the second column, direct students attention much more narrowly. Rather than saying what she noticed, Caitlin must say whether she agrees or disagrees with a given answer.

The questions in the reform-implicit box resemble those most often recommended to teachers as effective questions (e.g., Vacc, 1993; NCTM, 1991) because they are seen as eliciting higher-level thinking and because teachers of many grade levels and content areas can incorporate them easily into their repertoire. That is, their acontextuality is seen as a strength because these questions can be "good questions" in many kinds of lessons. Many of the explicit-reform questions were more specific versions of their implicit counterparts. For instance, instead of simply asking "why," Diana asked a student to tell her why he was adding 32 and 32. However, this difference in phrasing was important -- not only because the second question asked children to attend to particular features of the mathematics being discussed, but also because students in the classroom responded differently depending on the type of question asked. The following two sections explore these different responses to both implicit and explicit reform questions. Understanding how these questions operated in the classroom requires looking at their use in the context of classroom interactions, which takes space. To provide this space, the role that traditional questions played in the classroom is not discussed here.

Difficulties in responding to implicit reform questions

By far, implicit reform questions were the most likely to be met with silence when asked during whole-class lessons. Early in the year, Diana pointed to the numbers 5, 10, 15, 20, 25, 30, 35, 40, 45, 50 on the board asked students to tell her what they noticed. No raised his or her hand except Ben, a European American child frequently identified as "good at math" by the teachers. After a few moments, Diana remarked "I know more than one person knows this." When she made the question more specific by asking students to focus on the ones place, several students raised their hands. Diana called on Marcus, an African American child, who said that the ones place would "always be a zero or a five in that pattern."

In another lesson, Diana wrote the problem:

$$\begin{array}{r} 25 \\ \underline{+27} \end{array}$$

on the board and asked: "What if you had to do this problem without pencil and paper?" Many students, who had been eagerly raising their hands to answer earlier questions, put their hands down and stared blankly at Diana. Ben and Charlie, who was Asian American, raised their hands to answer and Diana remarked: "Honestly, sometimes I think there's only two people in here." With her question, Diana had wanted to direct students toward practice with mental addition (and in fact when she asked students for "mental math" ways of solving the problem, several children raised their hands). However, most students did not seem to immediately connect the idea of not having pencil and paper to mental arithmetic. The question itself left a wide range of responses possible responses (use a pen, ask to borrow some, get a calculator, etc.) and most students seemed unsure of Diana's goal.

These frequent interactions worked to place a spotlight on Ben who was often in the position of rescuing the class when no one else would raise his or her hand to speak. (Incidentally, this was not a position that Ben occupied unwillingly. Typically, Ben would not raise his hand to answer most questions asked, but would wait until he saw that no one was willing or able to answer the question.) After one of these interactions in January, Diana asked the class to listen closely to Ben's explanation. Then she asked the class, "Why should you listen to Ben?" Charlie called out: "Because you think he's smart." Jerome added: "You think he's right." Diana quickly disagreed, saying that the others should listen so they could learn to answer these questions as well.

At the end of the year, most students were no more likely to answer these kinds of implicit-reform questions than they were at the beginning. However, many of them had deeply entrenched beliefs in Ben's mathematical ability, which were revealed in interactions such as the one described above and in small group work, where students routinely deferred to Ben, even when they had correct answers and Ben did not. Just as these exchanges worked to frame Ben as a good math student, they also worked to frame other students as struggling, at least in the public discourse. Marcus, an African American student, routinely solved all problems asked of him, participated frequently in small groups and quickly mastered his multiplication facts. However, in whole-class discussions, he often appeared to be struggling when responding to implicit-reform questions.

At the beginning of a lesson on rounding, Diana wrote the following on the board:

730	700
73	100
703	700

She asked students to write in the journals about what they observed and then raise their hands. After a few moments, Marcus raised his hand and Diana called on him.

Marcus: It's even.

Diana: Why?

Marcus: Odd?

Diana: Why?

Marcus: Even?

Diana: Now they're back to even. When I say 'why' do I mean you're wrong?

Marcus: No?

Diana: You need to tell me why. If you don't know, you need to say 'I just guessed. I don't know.'

Marcus: I don't know.

Diana drew a number line on the board from 0 to 800, with slashes at each hundred. She asked the students to look at the numbers and think about rounding. Marcus put 73, 730 and 703 in correct places on the number line. He drew arrows from these numbers to the hundreds numbers they rounded to; however, he did not raise his hand again when Diana asked students what they had done.

In this episode, Marcus did not get to demonstrate his mathematical competence publicly, although he did privately in his journal and Diana made a point of looking at his work and praising him. Marcus interpreted Diana's "why" as a cue that his answer was wrong. Diana expressly denied that this was her intent; however, like many teachers she was slightly more likely to ask "why" when students answered incorrectly. In this case, it is not clear if Diana was asking why Marcus said the numbers were even as opposed to odd; why he chose the category of even/odd as important; or why he chose to focus on the numbers in the second column (which were all even) as opposed to the numbers in the first column. Because her question did not narrow possible responses for Marcus, he had a wide range of interpretations to choose from, including the one he chose: that "why" was code for wrong. Given his prompt response to the number line, it seems likely that Marcus could have made a statement about the relationship between the numbers on the board and the nearest hundred; however, the open question "why" left him struggling over Diana's intent, rather than the mathematics.

A conversation like this in the public space has consequences beyond Marcus. For other students trying to decide whether they too might raise their hands and participate, Marcus's eventual admission of "I don't know" after the confusing even-odd exchange could seem discouraging. Throughout the year, only five students responded to implicit-reform questions in whole-class discussions, despite the fact that all but two students participated in whole-class discussions nearly every day when other types of questions were asked.

It is tempting in looking at this episode to second-guess Diana's teaching decisions. However, she was working hard to enact the values of reform mathematics as she understood them. Marcus opened the episode by saying the numbers were "even." This is true of at least one column of the numbers presented and Diana could have agreed and then led him toward where she was heading with rounding numbers. Instead, she said "why" because it was her practice to ask this question frequently and because she wanted to offer Marcus the opportunity to make the connection between rounding and his answer himself. In this episode, Diana also attempted to teach the children about her use of the question "why?" by saying that it did not mean she thought the answer was wrong. Diana frequently made these sorts of commentaries on her teaching strategies; however, many children did not seem to internalize them. In April, after nearly a year of similar statements by Diana, Jerome responded to a why question by immediately erasing his answer.

In analyzing this and other episodes where students had difficulty answering implicit-reform questions, it seemed that some students – most often, those whose race, ethnicity, or ways of talking were different from Diana's -- had to divide their attention between the mathematics at hand and the interpretation of the language being used. Sometimes, this interpretive work resulted in amusing miscommunications. For example, after presenting the following problem:

You're going to the store to buy carpet for a room that is 6 feet by 9 feet. The perimeter is 30 feet. How much carpet do you need?

Diana asked: "What is most important here?" Charlie, an Asian-American student who had learned English as his second language, shouted out: "To be polite!" However, more often, these episodes seemed to leave children confused and, occasionally, the teachers frustrated.

Supporting mathematical thinking through explicit questions

In contrast to implicit reform questions, explicit reform questions seemed to offer students opportunities to reason, but also to provide the support necessary for more students to enter conversations productively. In whole-group discussions, many more children chose to respond to these kinds of questions and their participation tended to be seen as far more competent by the teachers. For example, Caitlin, an African American student who rarely participated in whole-class discussions and never raised her hand to answer an implicit reform question, spoke with confidence when Diana asked her to say whether she thought another student's expansion of 730 as 700 + 30 + 1 was correct.

Caitlin: No. No, it's wrong.
Diana: Can you say why you *disagree* with Sienna?
Caitlin: There's an extra one. 700 plus 30 *is* 730. There should be a zero, not a one.

Here Caitlin publicly evaluated another student's answer and provided a reason for her correction, which demonstrated knowledge of place value as well as an ability to put her mathematical thinking into words. Diana's question focused Caitlin's attention on the answer to a particular problem and gave her specific directions about the kind of answer expected. This confident exchange stood in contrast to a small-group interaction that occurred after Caitlin had solved a problem that asked how many outfits could be made with four shirts and three pairs of pants. Caitlin drew four shirts and wrote “3 outfits” on each one. Then she wrote “3 x 4 = 12.” The final prompt on the worksheet said: “Explain your work.” Caitlin, confused, questioned the student teacher, who replied: “What do you mean you don’t get it? You already did the whole thing.” Caitlin said: “I know how to get the answer, but I don’t get how to *solve* this problem.” For Caitlin, the requirements of “solve” and “explain your work” were mysterious, and she seemed to classify these questions as being a kind of mathematical work that she could not do, unlike finding the answer to particular problems.

Unlike implicit questions such as “Why?” “What do you notice?” and “Explain your work,” explicit reform questions removed ambiguity about the teacher’s purpose in asking questions, which is what seemed to provide children like Caitlin with the confidence to join the conversation. A number of specific strategies seemed to be at play in the asking of these kinds of questions. Often, Diana attached a student’s name to the question, which worked to invite particular children to speak. Also, these explicit questions tended to identify what the teacher saw as important in the problem under discussion (mental math strategies, rounding, Sienna’s answer). These questions also seemed to follow other questions. That is, rather than opening a discussion, they were almost always used in the midst of a conversation, which at least some of the time helped students to think about their answers in terms of what others had said previously, although sometimes these questions were used to redirect conversations that were not going where the teacher intended, as Diana did when she directed students to focus on the one’s place in the pattern episode.

Finally, many of these questions allowed students to respond silently as a group before they committed themselves to speech. Diana frequently asked students to give her a thumbs up or a thumbs down in response to questions. It is impossible to answer a question like "why" in this way; however, Diana found ways to encourage students to think when asking yes and no questions. Examples include: "Is there a way to solve this problem without regrouping?" and "Did he find all of the arrays?" All children responded to questions like these (admittedly, because Diana demanded that each student take a stand). Some children looked carefully around the room before making a decision; however, Diana frequently called on these students to explain why they had answered yes or no. These students, buoyed by the knowledge that many of their classmates had interpreted and answered the question in similar ways, often were able to articulate their thoughts more clearly than when asked a more open question or when asked a question that was unexpected.

Jerome, an African American boy whom both Diana and Sara described as "struggling," rarely answered questions as expected in whole-group discussions. However, he seemed to be better able to participate in conversations that began with a public yes/no question that allowed him to focus in on a specific issue and to feel relatively confident that he was right, as evidenced by the answers provided by the rest of the class. Early on in a discussion about a problem that asked students to figure out how many vegetable pieces had been used in a pot of soup, Jerome shrugged and refused to answer when Diana asked him to tell the class something he had done to solve the problem. However, a few minutes later she asked the class to put their thumbs up or down to show whether they should add 50 carrots to the total number of vegetable pieces in the soup. After looking around the room and seeing that most students had their thumbs pointed down, Jerome pointed his down as well.

> Diana called on him and asked why they couldn't just write down 50 in their list.
> Jerome: You have to double it! Double it!
> Diana: Why do you have to double the carrots?
> Jerome: Because it said to cut them in half.

Jerome had solved this problem in his journal before the discussion began; however, when asked to say "something" about his solution, he had nothing to say. However, when asked a specific question about what he had done and when given the additional support of seeing that his classmates agreed, he participated with both competence and confidence.

Concluding Thoughts

This study contributes to current conversations about whether reform teaching is best for all children (Ball, Goffney & Bass, 2005; Boaler, 2002; Lubienski, 2000) by suggesting that explicit questions are not in opposition to reform practices, but a possible support for them. Open, ambiguous questions such as "what do you notice?" may be appropriate for students who do not have to work to interpret the teachers' language or cultural practices but may provide an obstacle for students who must think about language in addition to mathematics. In addition, students who are not fluent in the register of school speech, as described by Cazden (1988), may need more explicit clues about the timing, syntax and content of their responses. Future studies are needed to identify what it is that allows some students to respond to implicit questions in ways that are perceived as competent by the teacher. It may be that shared cultural practices between teachers and students provide these supports or it may be that students' familiarity with the register of school speech, regardless of the race or culture of the teacher, provides these supports.

This study demonstrated that implicit questions can construct some math students as successful and others as needing support. In a diverse classroom, overlaps between culture, dialect and race, may cause teachers and children to make problematic assumptions about mathematical ability based on students' responses to implicit questions. This study does not suggest that students from non-dominant cultures or who use non-dominant dialects and languages cannot engage in abstract thinking and problem solving. Rather, it fosters Boaler's (2002) claim that they way teachers enact instructional practices is central to whether these practices are equitable or inequitable. Questions that require students to interpret teachers' intentions and instructional goals, as implicit questions do, seem to advantage some students.

In addition, this study also suggests that it would be fruitful to explore the possibilities of implicit traditional questions (such as "What do you do to add two-digit numbers with regrouping?). Although it was not the focus of this chapter, questions that dealt with computation and memorization in ways that asked students to talk about processes for performing these procedures seemed to operate in productive ways by causing students to verbalize their thinking. Studying these sorts of questions would contribute to the problem raised by Star (2005), who argued that we know very little about what high-quality procedural knowledge looks like, because procedural knowledge often has been conflated with weak understanding of mathematics.

Teaching mathematics in ways that are culturally responsive requires that educators think both about the ways that race and culture might be impacting mathe

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mathematics teaching and the ways that mathematics teaching might be impacting race and culture. In this study, students' familiarity with the language and practices of the teacher seemed to have an impact on their willingness and ability to answer questions. Thus, question answering became a way to "perform" race, which makes it important that mathematics educators find ways to engage students in mathematical discussions so that it is possible for all students to have competent public performances.

CHAPTER THREE

Down the rabbit hole: My own postmodern turn

"'Then you should say what you mean,' the Mad Hatter went on.
'I do,' Alice hastily replied; 'at least – at least I mean what I say – that's the same thing, you know.'
'Not the same thing a bit!' said the Hatter. 'Why, you might just as well say that 'I see what I eat' is the same thing as 'I eat what I see'! "
-- Lewis Carroll in *Alice's Adventures in Wonderland*

"Q: You were ahead of others.
MF: Ahead of others? Not at all. I was below them."
-- Michel Foucault in *Foucault Live*

My early encounters with postmodern thinkers left me feeling quite a bit like Alice at the tea party: surrounded by people talking nonsense, picking on my language, and playing games with serious work. Still, I found myself drawn in by the elegant prose, the humor, and the unfamiliar ideas. The more I read, the deeper into the rabbit hole I found myself, until eventually, my sensibilities changed and writers up on the surface began to seem just as mad. The purpose of this chapter is to explore a few ideas in the warren of postmodern philosophy that I have found meaningful with the goal of transitioning between the ethnographic stance taken in the previous chapter and the genealogical approach that will be adopted in the following ones.

Although I write now as a researcher, it was through my teaching that I came to understand the possibilities of Foucault's work. I entered graduate school after six years as an elementary teacher, and in the beginning, I interpreted all writers through this lens. At first, I was not sure what this French philosopher had to say about my time in the classroom. His discussions of power, knowledge and subjectivity seemed both distant and obtuse. In one of the first Foucault texts I encountered, I read: "power is neither given, nor exchanged, nor recovered, but rather exercised, and that it only exists in action" (1989/1996, p. 89). My reaction to this was something along the lines of "Maybe. But so what?"

It was Popkewitz (2002) who answered this question for me. In an essay on the teaching of school subjects, Popkewitz discussed the attention teacher educators have given to the "urban child," saying: "Teaching is to rescue the child with low self-esteem, a division from the unspoken characteristics of high self-esteem. All the rescuings are paradoxical. The system of reason makes it so that the child can never be of the average" (p. 266). For me, reading this essay was like walking into a brick wall. The story I had constructed about myself as a teacher began to unravel, and as I began to tell a new one, I learned to understand and to value the conception of power Foucault had articulated.

Throughout my teaching career, I taught classrooms of predominantly African American students, first in a rundown, city school and later in a suburban showpiece. In both places, I brought passion and certainty to what I did and saw myself as an advocate for my students, whom I believed were oppressed both by society at large and by the school in particular. In the first setting, my advocacy manifested itself in a fierce commitment to the belief that my students could pass the state's standardized tests. In pursuit of this goal, I attended workshops, went on home visits, devoted half the day to literacy, and learned ways to engage in mathematics that went beyond the four most common algorithms. And many of my children did pass those tests. In my suburban school, I lost some of my urgency, but still fought the gate-keeping of the gifted program, refused to track my children and chose to work with the remedial classes during intersessions. I was angry a lot of the time, but the truth is, I enjoyed the fights. I got into teaching because I was on a mission.

When I read Popkewitz's description of rescuings as "paradoxical," it occurred to me for the first time that my efforts to save my children might not have been wholly positive. I saw that identifying children as poor, lacking in self-esteem, or without academic resources emphasized what was "other" about them. Children who need rescuing begin their school careers seen as deficient, powerless and needy. Popkewitz's argument helped me to understand Foucault's (1989/1996) idea of power as a force that is not limited to people who have been privileged by race, class or gender but that can be exercised by nearly all people in nearly all situations. When I looked back on the children I had taught, I began to redefine them as people who were at all times exercising power in various ways and were not the helpless victims of their oppressors. Rather than thinking about how to save them, I had to ask myself why it had been so important to me to teach from the belief that the children in my classroom needed me to rescue them.

Working with this question caused me to redefine myself as a teacher and a researcher. Rather than a savior, I now see myself as a story teller, whose words invite others to recognize the ways in which they and those they interact with exercise power, particularly in the classroom. I no longer see power as a scarce resource that I must redistribute with my teaching and research, and as a result, I no longer need to rescue school children or (to say the same thing in other words) to see them as inadequate in order to justify my own work. Foucault said "to reveal relations of power is, in my opinion at any rate, to put them back in the hands of those who exercise them"

(1989/1996, p. 144). This idea has become central to my redefinition of my work as something other than a rescue mission. Right now teacher educators act in certain ways – for instance, they "do" problem solving, promote discussions, and advocate "culturally relevant" practices. These ways of acting, which can be thought of as *technologies*, are means by which teacher educators exercise power. Exploring how and in what circumstances these technologies are employed is a way of calling attention to power relations and making it possible for teacher educators, including myself, to think consciously about how we do our work. The justification for my work is not saving, but creating openings for change by decreasing the sense that our current ways of acting are inevitable.

You can't spend much time reading about Foucault's conception of power before you have to come to terms with "power/knowledge." (Thinking "tens of thousands of words in the language and he has to make up new ones" is emotionally satisfying but ultimately unhelpful.) The slash between power and knowledge reminds us not to go around thinking that these are two separate things. Karl Von Clausewitz famously said that war is the continuation of politics by other means; Foucault turned this around to say that politics is the continuation of war by other means.

> [I]t implies that the relations of power that function in a society such as ours essentially rest upon a definite relation of forces that is established at a determinate, historically specifiable moment, in war and by war. ... The role of political power, on this hypothesis, is perpetually to reinscribe this relation through a form of unspoken warfare; to re-inscribe it in social institutions, in economic inequalities, in language, in the bodies themselves of each and every one of us (Foucault, 1980, p. 90).

What does this have to do with knowledge? What we *know* is a way of exercising power – that is, research-based best practices are ways of reinscribing certain relations of power within the social institution of the schools. What we know is also a product of the ways that power has been exercised in the past – that is, we know about the American Revolution rather than the American Rebellion because of power relations established at a specific moment "in war and by war." We *know* George Washington is a hero and not a terrorist because of these power relations and each time we tell the story, use a dollar or view Mount Rushmore, we inscribe the story a bit deeper. All of this means that the ways in which we speak/think/act create words, categories, and ideas that function as truth. (I borrowed Foucault's slashes to make the point here, but he used the word *discourse* to cover speech/action/thought, and I will adopt this efficiency from now on.) Recognizing the conflation of knowledge and power has implications for how we think as researchers about the creation of knowledge. Following Foucault, truth is not an absolute we uncover, but a way of exercising power. If theories, facts and interpretations become true through constant reinscription in discourse, then researchers have ethical obligations both to analyze the ideas being reinscribed in their own communities and to consider the stories that their own work will make true. This is a different ethical obligation than that required by positivist theories, which ask that researchers *tell* the truth, no matter how unpalatable; and it is different from the obligations required by modernist critical theories, which demand that researchers *reframe* current truths in light of social and political inequities. Postmodern theories see research as a *will-to-truth*. This is not so much the production of knowledge that socio-cultural theorists talk about, but the recognition that all knowledge is the exercise of power.

Unlike the relationship between rescue and power, this is an idea that I understood intellectually before I got it emotionally. It wasn't until I picked up a Forbes magazine left on my living room table by my mother-in-law that I began to take seriously the idea that much of my own writing and teaching was working to reinscribe the truths I believed I was arguing against. The Forbes piece was a typical right-wing rant about the education establishment, which ran under the subhead: "The latest preposterous idea in educationland is 'closing the achievement gap.' Educators everywhere are enlisting in the campaign and somehow not noticing that it can't possibly succeed" (Seligman, 2005, p. 120). The gist of the argument was that the achievement gap can never be closed because some kids -- those from the middle and upper classes -- are simply smarter than other kids -- those who are "disadvantaged." The story ran (charmingly) with a drawing of two children, one with a head full of brains and another with, literally, nothing upstairs. I went through several stages of vocal rage during my initial reading of this article, which included a typical left-wing rant about people who conflate biology, culture, environment, and upbringing. Then, in the calm after the storm, I saw it. It is only because of liberals like me, who have worked – tirelessly – to produce the achievement gap that Seligman and his colleagues at Forbes have anything to write about.

Just a few months before the Forbes piece, NCTM ran their position on "Closing the Achievement Gap" (NCTM, 2005, p. 4). In it, they defined the achievement gap as:

> disparities among groups of students usually identified (accurately or not) by racial, ethnic, linguistic, or socio-economic status with respect to a variety of measures, including attrition and enrollment rates, alienation from school and society, attitudes toward mathematics, and test scores.

The constant repetition of the phrase "the achievement gap" works to create a real phenomenon. An ERIC search for documents with the phrase in the abstract turned up more than 500 texts; a Google search produced over a million hits. This is reinscription on its most basic level. Measuring the achievement gap etches this particular relation of power into our social institutions, our language, and our bodies more deeply. What we measure (alienation, attitude and math test scores), how we measure (multiple choice, surveys, short answer) and what we report (scores by race and gender) are technologies that allow educators to exercise power relations that were established in war and by war. They remind some individuals of their inferiority (identifiable by "racial, ethnic, linguistic, or socio-economic status," [NCTM, 2005, p. 4]) and others of their accomplishments. In fact, it is because of these relations of power that NCTM can refer vaguely to racial, ethnic, linguistic and socio-economic categories and we all know who they're talking about. (Hint: it's not upper-class white males.)

These technologies – the methods of measurement, the content being measured, and the way scores are (or are not) disaggregated – are not innocent, neutral or natural; they do not simply measure what is true; they produce it. Typically, scores are not reported by income-level, educational attainment of parents, hair color or height. We choose which categories to make important. In one study on stereotype threat, researchers told European American males that Asian Americans typically outperform whites in mathematics. Researchers then gave these men a challenging math test. These participants performed significantly worse on the test than men who had taken the test without having this stereotype invoked beforehand (Aronson et al, 1998). For minority students, the act of taking a standardized test may work to invoke stereotypes about performance even without an explicit reminder because phenomena like the achievement gap are so widely accepted as real. Thus, the achievement gap works in two ways in these situations. First, its acceptance as a real phenomenon impacts student performance on tests, and second, the tests then go on to produce evidence, in the form of test scores, that the phenomenon is, in fact, real. That's power/knowledge.

This relationship is also an example of what Canadian philosopher Ian Hacking called "the looping effects of human kinds," (Hacking, 1999a; 1999b). As social scientists, the categories we use to describe those we study create kinds of people, and unlike species of plants or animals, human kinds (minorities, females, urban children) can learn about their kind and can change their behavior as a result. Hacking wrote:

> They can make tacit or even explicit choices, adapt or adopt ways of living so as to fit or get away from the very classification that may be applied to them. These very choices, adaptations or adoptions have consequences for the very group, for the kind of people that is involved. What was known about people of a kind may become false because people of that kind have changed in virtue of what they believe about themselves (Hacking, 1999a, p. 34).

In the mathematics education literature today, many human kinds exist. Researchers discuss African-American students (Strutchens, Johnson & Tate (eds.), 2000; Ladson-Billings, 1997; Lee, 1998), Latino/Latina students (Ortiz-Franco, Hernandez & De La Cruz, 1999; Gutstein, 2003), children with disabilities, (Thornton & Bley (eds.), 1994), girls (Jacobs, Becker & Gilmer eds., 2001; McGraw, Lubienski & Strutchens, 2006), English language learners (Lee & Jung, 2004), and urban students (Fuson, Smith & Cicero, 1997; Secada, 1996). As intuitive as these categories may appear, they were not always available.

Early research in mathematics education tended to differentiate students in far fewer ways than we do today. Edward Thorndike (1922) referred only to "the pupil." For instance, we should try to find problems "which not only stimulate the pupil to reason, but also direct his reasoning to useful channels" (p. 20) or "it is undesirable that the pupil should regard 'the crutch' response as essential" (p. 112). Very rarely, Thorndike discussed students in terms of their mathematical ability, calling students "gifted" (p. 54) and referring to "the dullest twentieth of pupils" (p. 57). Writers today would probably not use this phrasing (preferring perhaps *students with disabilities*); but, however it was phrased, mathematical ability was one of very few criteria for Thorndike to use in differentiating among students. Similarly, other writers who discussed problem solving in the early part of the 20th century tended to see students in more unitary ways than we do today. William Brownell (1938; 1948; Brownell & Chazal, 1935) did not discuss differences among students, except in regard to ways that they had been taught – through drill or through meaningful instruction. George Katona (1940), who wrote about adult problem solving, took a similar stance toward problem solvers. He discussed the differences in his subjects' abilities to remember card tricks in terms of the methods by which they had been taught (rote versus with an organized structure), but not in terms of demographic characteristics of the subjects themselves. Theorist Lynn Fendler (1999) pointed out that this increased interest in demographics was part of a broad change across disciplines in education, which was linked to increased use of statistical measures designed to regulate increasing numbers of immigrant students in the school system.

No doubt, the proliferation of kinds of students has helped many of those working in mathematics and mathematics education today to see themselves in the students being written about in ways that were not possible in the research of the early 20th century. However, as individuals recognize and read about their kinds, they must also find ways of living in response to the category. Stereotype threat isn't possible without the stereotype, and research explicitly focused on kinds of students who have trouble in mathematics can, without intending to, force individuals to "fit or get away from the very classification that may be applied to them" (Hacking, 1999a, p. 34). It is because of the proliferation of student kinds that the phrase "all students" has become so common in equity literature. Thorndike and Brownell could write "pupils" or "students" and assume that their readers would understand them to be talking about all students. However, now we have many different kinds of students in our heads – boys and girls; Blacks, Whites and Asians; urban and suburban – that the word "students" is likely to cause many readers to ask "which kind?" Thus, the word "all" is added in order to work against the lines that have been drawn. However, the word "all" now also acts to call to mind all of these categories.

My time in the postmodern rabbit hole has led me to a new stance toward equity. In my writing, I no longer want to engage in arguments about the size of the achievement gap, the reasons for it, or the way to close it. I no longer wish to be a part of reinscribing the relations of power embedded in the notion of the achievement gap. I do not want to repeat the story of race determining achievement – as I did, to some extent, in the previous chapter. When I conceived of this dissertation, I imagined an ethnographic work informed by postmodern thinkers, with a collection of chapters very similar to the previous one on questioning. However, from where I stand now, I am acutely aware of both the stance of rescue I adopted in that chapter and of the ways in which I reiterated the beliefs I was seeking to write in opposition to.

This tension is evident in my treatment of the dilemma of rejecting race as a causal factor and arguing for a culturally relevant pedagogy (which names race as a key factor in thinking about teaching). I tried to get around this problem in the chapter by doing some serious back pedaling. For instance, I used a chart to summarize children's demographic characteristics and then acknowledged it to be problematic; I talked about African American culture, but then said that I was not treating race as an independent variable; I warned readers of the trouble that others have gotten into, and then proceeded to get in that same trouble myself. This sort of back pedaling is common in this kind of work. Ladson-Billings, whom I cited in the previous chapter for her description of African American culture as valuing "rhythm, orality, communalism, and spirituality" (1997, p. 700), included footnotes in her own work where she stepped away from her description of culture. She wrote "I do not mean to imply there is one monolithic African American culture" and "It is important that this notation not be read as the stereotypical 'all Black people got rhythm'" (ibid).

I do not know what motivated Ladson-Billings to include these disclaimers; however, I do know why I followed up the chart describing the demographics of my focal students by apologizing for it. Whether I wished it or not, a chart that highlighted just four qualities about students (their race, gender, mathematical ability, and level of participation) reinforced the belief that these are the qualities that matter about human beings and are important in making educational decisions. What I recognize in *this* chapter is that my writing: "Labeling Jerome as African-American, male and struggling works to make each of these categories stable (and worse yet, works to conflate them)" did not undo the work of the chart. The (not very) hidden argument of the previous chapter is that African American children are different from European American children in important ways that have to do with the learning of mathematics. This is not a story I want to tell because telling it reinscribes power relations that draw on racial inequity. That is, the story makes race an important factor in thinking about who children are in school, encourages people to make judgments about others based on race, and suggests that being White is linked to successful performance in math class. Despite these problems, I include the chapter in the dissertation for pedagogical purposes – aimed at myself and my readers – because it will allow me to talk about the ways that genealogical approaches to thinking about education might offer other perspectives on equity than those made possible by ethnographic texts.

This strategy of simultaneously (or at least in sequence) writing in argument for and against an idea is what French philosopher Jacques Derrida called writing "under erasure" (Derrida, 1976, p. 60) and is undertaken when the concept under consideration is both necessary and rejected. For instance, when talking about the relationship between the sign and the signified, Derrida wrote: "But we cannot do without the concept of the sign, for we cannot give up this metaphysical complicity without also giving up the critique we are directing against this complicity," (Derrida, 1978, p. 281). Here he argued that we cannot talk about the pointlessness of differentiating between the sign (e.g., the written word "child") and the signified (e.g., the small humans we see in schoolyards) without necessarily calling into play the concept of signs (as separate from signified). Writing under erasure is a way of coping with this difficulty by explicitly pointing at the tension. Similarly, the previous chapter was necessary to my argument for a number of reasons. First, in order to establish credibility as a beginning researcher, I needed to show that I could work within accepted traditions for qualitative work. Second, to illustrate the problems that I see caused by calling race into play, I needed to call race into play.

Having accomplished these goals, I have now spent this chapter unwriting my previous work. Derrida sometimes signaled erasure by leaving a word that had been crossed out within the text – showing it to be both necessary and challenged. This is how I see the previous chapter in the context of the larger dissertation.

Although I am crossing out my use of race discourse, I am not arguing that one should never mobilize the achievement gap, gender, or race to fight for equal funding, better teachers or fairer tests. I recognize that I am giving some things up when I adopt a postmodern critical stance. First, I lose the ability to speak directly about racism, sexism and class difference, which in turn might make it easier for some to deny that these are common ways of exercising power in our country today. In addition, I need to give up some comfortable and relatively efficient tools. In the previous chapter, I drew on ethnographic methods. I asked research questions, collected data in the form of fieldnotes and student work, coded my data, and wrote assertions, which I backed up with evidence. Following these forms, which are common in academic texts in education, provided me with certain advantages. Readers could use these forms to identify the genre of my work and to determine whether they wanted to trust my conclusions. The organization of the chapter was easy because I could adopt a commonly accepted skeleton; and, I could distance myself from my conclusions because they were what I "found" through analysis of the data, recognizing, of course, the commonly accepted ethnographic caveat of reflexivity, where my role as a researcher in collecting, writing up and making sense of the data is openly acknowledged.

In writing about ethnography, anthropologist Clifford Geertz (1973) asked readers to think about the difference between a wink and a twitch as a way of understanding culture, the "think description" of which Geertz argued is the central work of ethnographic anthropology. He wrote:

> The thing to ask about a burlesqued wink or a mock sheep raid is not what their ontological status is. It is the same as that of rocks on the one hand and dreams on the other – they are things of this world. The thing to ask is what their import is: what is this, ridicule or challenge, irony or anger, snobbery or pride, that, in their occurrence and through their agency, is getting said (Geertz, 1973, p. 10).

With tools like fieldnotes, coding and analysis, ethnography is good at closely describing human interactions and making connections between these interactions. These descriptions make convincing arguments about, for example, whether an observed muscle spasm is a wink that indicates insider status or an involuntary reflex. As a result, ethnographies do a good job of exploring questions about the meaning of people's interactions. For instance, studies have examined how children co-opted characters from popular culture for their own purposes (Dyson, 1997); how young men in a low-income neighborhood experienced race and schooling (MacLeod, 1995); and how the funds of knowledge present in minority communities can be brought into the public schools (Gonzalez, Moll & Armanti, 2005). These and similar studies have done equity-oriented work by challenging cultural deficit theories, by articulating ways that race has been made meaningful in our schools and society, and by describing culture as continually shifting and negotiated, rather than as stagnant and monolithic.

However, there are other kinds of equity work to be done. In discussing the differences between ethnography and genealogy, theorist Erica McWilliam (2003) wrote that one of the greatest challenges of adopting a genealogical perspective is letting go of the role of advocate. As an example, she noted that deciding to ask the genealogical questions "Why bullying now?" and "How bullying now?" rather than the ethnographic questions "What is bullying?" and "How do we stop it?" can be unsettling (McWilliam, 2003, p. 60). In similar ways, the genealogical questions of "How has it become possible to think about home culture as different from school culture?" or "Why has it become common to explain students' schooling experiences in terms of race?" might seem like less compelling ways of working toward an equity agenda than the ethnographic questions that framed the studies described above.

As with ethnographic studies, genealogical studies do their equity work through thick description. However, the object of description is not only human interactions in a particular time and place, but also discourse in many times and places. The object of this work (unlike ethnographic studies) is not to document the present through the mobilization of evidence, method and theory, but to bracket the present through argument so that the object of study appears contingent, permeable and historically dependent. Like Geertz, the genealogist does not seek to determine the ontological status of the wink; however, she also does not want to describe how it functions in its cultural context. Rather, her goal might be to trace the use of winking as communication to show how it became possible in a certain moment for one person to express "ridicule or challenge, anger or irony" through the twitch of the eye. By revealing the historical contingency of winking, it becomes possible to imagine a world without winking. This is how genealogy does its equity work. Rather than reinscribing the importance of current social categories, it makes it possible, however briefly, to imagine the world otherwise. The laying aside of social science for rhetorical or genealogical tools is central to this work. Scientific constructs such as data, analysis, and coding all function to make the objects under study real, rather than contingent. For this reason, in the following chapters, I will be using rhetorical tools such as argument, persuasion, metaphor and genre to get out from under the weight of social science traditions – or perhaps, to go below them.

CHAPTER FOUR

Discourse in Mathematics Education

"Words, words, words." – Hamlet (Shakespeare, II, ii)

Mathematics educators have paid a lot of attention to discourse. Researchers have looked at the discourse of reform classrooms in elementary schools (Cobb, 2000; Cobb, Boufi, McClain & Whitenack, 1997; Forman & Ansell, 2001; Sherin, 2002; Stevens, 2000; Zack & Graves, 2001), the discourse of preservice classrooms (Blanton, 2002; Crespo & Nicol, 2006; Danielewicz, 1998) and the discourse of practicing teachers engaged in professional development (Fernandez, Cannon, Chokshi, 2003; Remillard & Geist, 2002). In almost every case, the focus has been on the *words* of participants – both those uttered in conversation, as in most of the classroom studies, and those uttered in interviews as presumed proxies for thought (e.g., Ben Yehuda, Lavy, Linchevski & Sfard, 2005; Sfard, 2001).

Although some of the studies cited above do not use the word *discourse*, the mathematics education community has come to define any study that focuses on communication, language or conversation as a discourse study. In their literature review for the *Research Companion to the Principles and Standards*, Lampert and Cobb (2003) used "communication," "language," and "discourse" as near synonyms. In another piece, Cobb (2000) defined classroom discourse as "the ways in which the teacher and students talked" (p. 55). His definition of discourse as speech is shared by most of the authors cited above. In addition, discourse studies are often justified by reference to the National Council of Teachers of Mathematics recommendations that students engage in discussions where mathematical arguments are made and critiqued, as Lampert and Cobb (2003, p. 237) did in the introduction to their literature review:

> If school lessons are to involve learners doing mathematical work, classrooms will not be silent places where each learner is privately engaged with ideas. If students are to engage in mathematical argumentation and produce mathematical evidence, they will need to talk or write in ways that expose their reasoning to one another and to their teacher. These activities are about communication and use of language.

Here, Lampert and Cobb made the argument that discourse studies are important because of the role language plays in the learning of mathematics. Reform mathematics practices, such as classroom discussions, small group work and student presentations, have both driven much of the research on discourse in the classroom and have helped to define how discourse is thought of in the community. Discourse and language have become synonymous partly because studies that look at discourse tend to be located in conversation-rich classrooms.

Most of these studies draw on one of two theoretical traditions, both primarily developed in studies of language and literacy. *Socio-linguistic analyses* (Hymes, 1972; Lampert, 1990; Mehan, 1982) tend to examine patterns of questions and answers, exchanges of ideas or vocabulary, or conversational roles of teachers and students. *Socio-cultural analyses* (e.g., Ben-Yehuda, Lavy, Linchevski & Sfard, 2005; Cobb, 2000; Forman & Ansell, 2001; Heath, 1982; Lave, 1988) tend to examine the communicative purposes of teachers and children, meaning-making by individuals in social contexts and tensions between home and school discourses. Often relationships are drawn between these phenomena and the participation structures and content of conversations. Although socio-cultural analyses in mathematics pay more attention to cultural tools (i.e., the use of calculators) and practices (i.e., sharing journals), discourse is usually seen primarily as language.

Cobb and his colleagues have done many socio-cultural analyses of interactions in primary classrooms. I describe one of these studies here to highlight some of the intellectual consequences of adopting a discourse-as-language framework. In this study, Cobb, Stephan, McClain and Gravemeijer (2001) described first graders' participation in a measuring unit where children were asked to measure various objects using unifix cubes. The context for the unit involved stories about tiny characters – Smurfs – who used food cans the size of unifix cubes to measure. Together with the teacher, the children also developed a ten-unit stick as a measure, which they called a "Smurf stick." Cobb and his colleagues reported the following exchange to argue that one child, Nancy, saw measuring as an accumulation of distance and that her explanation, along with other interventions by the researchers, helped two other children, Mitch and Megan, to adopt Nancy's (correct) way of reasoning. The episode opens with Nancy and Megan measuring the white board with a Smurf stick and the teacher making marks at 10 and 20, as the girls say those numbers. (The "R" in the dialogue stands for researcher.)

> R: Where's the 20? What does 20 mean?
> Megan: 20 means 20 food cans.
> R: That means 20 food cans? How much space would that be? Can somebody show me how much space 20 cans would take up there? Mitch?
> Mitch: About that long. [Indicates the space between 10 and 20].
> Nancy: No. [Indicates the space from the edge of the white board to 20.] This is because he [Mitch] did 10, not 20. [Mitch indicates he has changed his mind.] (Ibid., p. 141)

Using this episode, the authors argued that Nancy's way of reasoning became commonly accepted in the classroom through the social participation of the students, the teacher, and the researchers. They wrote:

> In the case at hand, Nancy's explanation was constituted as legitimate, whereas Mitch's proposal was treated as illegitimate. In responding to Mitch, Nancy therefore contributed to the third mathematical practice, *measuring by iterating the Smurf bar*. Similarly, Mitch contributed by making his proposal and by indicating that he changed his mind. (Ibid, p. 142).

By looking at discourse in this way, the authors were able to make arguments about what kinds of conversational and pedagogical practices might be important for helping children to understand measurement and make mathematical arguments. In addition, they used the concept of social space to get around arguments about transfer. Rather than suggesting that ideas leapt out of Nancy's head and into Mitch's, they argued that ideas about measurement in the classroom were constructed by Nancy and Mitch together in collaboration with the other human beings present. These ideas then become available for the teachers, researchers and students in this community to draw on during future learning.

This analysis (and similar ones by other authors) bring mathematical learning out of individual heads into the social space of the classroom, allow researchers to consider ways that social norms contribute to (or inhibit) mathematical learning, and make it possible for researchers to think about how ideas are constructed in the classroom. However, analyses like this one, especially when they read together as a body of work, have other consequences as well. First, the narrow focus on language ignores many of the complexities of interactions in the classroom. Students respond not only to each other's words, but also to social and academic status, which can be marked by past interactions with the teacher, clothing, playground activities, accent, etc. As readers, we have none of this information to interpret Mitch's abrupt change of mind, and this lack of information serves a purpose: it implies that the mathematical idea carried the day; however, *measuring by iterating the Smurf stick* may not have been the only idea with mathematical consequences constructed in Nancy and Mitch's interaction. In one study, choosing to focus on the explicitly mathematical idea, rather than on other ideas, may be a small limitation. After all, researchers cannot study everything all at once. (A map the size of the area is of very little analytic use.) However, when the majority of studies in mathematics education define discourse in this way, they work to remove status (as well as race, class, and other "silent" constructs) from the literature.

Second, the emphasis on the construction of ideas in the space of the classroom appears to emphasize the social; however, it also works to reinforce the idea of the individual existing independently from the community in which he or she is situated. In the Cobb and colleagues piece, the authors describe Megan as a student who "reorganized her reasoning" (p. 142) in response to Nancy and Mitch's interaction. In describing Megan in this way, the authors locate her as an individual within a social space. This theoretical movement – which Latour (1999, p. 7) described as the change from "mind-in-a-vat" thinking to "many minds in many vats" – doesn't get us very far. It implies, although the authors wrote that this was not their intent, that Mitch, Nancy and Megan are variously able to appropriate ideas from the discussion not only because their individual understandings about mathematics and measurement, but also because of how each of them is situated in broader conversations. That is, their gender, race, family background, kindergarten teachers, breakfast, and tolerance for Smurfs all play a role in shaping their verbal participation in math class. A steady diet of studies that emphasize the ways individuals participate in conversation works to create a theoretical world where individual knowledge, skills and beliefs are seen as the appropriate sites for intervention and other possible sites (the openness of the classroom to students' home cultures, definitions of mathematical thinking, ways of relating on the playground) become invisible.

In his historical analysis of the mathematical practices of abacus masters, Radford (2003) critiqued the current emphasis on language in mathematics education research for similar reasons, arguing that researchers have ignored the role that social practices play in defining what mathematics is and who is good at it. He did this by looking at the kinds of problems abacists solved during the Renaissance. For instance, he argued that the notion of fair trade (where some amount of wax could be understood as mathematically equal to some amount of wool) needed to be in place in order for certain problems to be posed. Thus, the mathematical practice of students learning to use the abacus was shaped not just by the conversations that went on between masters and their apprentices, but by the social world of the time. In his conclusion, Radford argued that discourse was not a robust enough tool to theorize mathematical understanding. He said researchers must "take other elements into consideration – elements found *beyond* discourse itself" (p. 140, italics in the original).

It is true that discourse-as-language is not an adequate tool to make sense of the production of mathematical (or pedagogical) knowledge. As Radford (p. 141) said: "[K]nowledge is also objectified by other means, such as sculptural forms, graphics and plastics, as well as by the habitual and historically constituted manner in which one acts toward things and interacts with individuals." However, a theoretical tool does exist that takes all of the forms Radford mentioned (and more) into account – a tool that not only makes it unnecessary for us to move "beyond" discourse, but, in fact, makes it impossible to do so. Foucault (1978/1990; 1983) defined discourse quite differently than most mathematics education researchers writing today. In his work, which looked at the ways that big ideas such as sexuality and madness got constructed at certain moments in history, Foucault read a wide range of activities and artifacts as discourse, including published books, spoken language, symbolic language, the organization of physical objects and people's actions. In the first volume of *The History of Sexuality*, Foucault (1978/1990) argued that the commonly-told story about western sexuality – that we've moved from a repressive era where sex could not be spoken about to a more open one -- is invalid. Along the way, he discussed secondary schools in the eighteenth century:

> On the whole, one can have the impression that sex was hardly spoken of at all in these institutions. But one only has to glance over the architectural layout, the rules of discipline, and the whole internal organization: the question of sex was a constant preoccupation. … The space for classes, the shape of the tables, the planning of the recreation lessons, the distributions of the dormitories (with or without partitions, with or without curtains), the rules for monitoring bedtime and sleep periods – all this referred, in the most prolix manner, to the sexuality of children (Ibid, p. 27-28).

In context of this dissertation, there are two important ideas to notice here. First, the physicality of Foucault's notion of discourse demonstrates the limits of notions of discourse that consider only language. Whether or not the word "sex," is spoken, hanging curtains between beds brings sexuality into the conversation. Second, this wide-ranging take on discourse makes it impossible to think about standing *outside* of discourse for purposes of analysis (or for any other purpose). Everything is discourse: architectural plans and recreation curriculum. It is impossible for any of us to step outside of our cultural and geographic time and place to make sense of an object. This should not be read as an argument that innate objects – like rocks – do not exist, but as an argument that we can only understand rocks -- and children, and, yes, mathematics too -- through discourse. The purpose of bringing Foucaultian discourse to bear on social institutions, such as schools, is to call attention to ideas that have become a taken-for-granted part of our intellectual landscape, and thus, to make it possible to see those ideas as mutable constructions.

Valerie Walkerdine (1988) engaged in this sort of analysis when she described the discourses of rationality, desire and control at play in the teaching of mathematics to young children. She used this analysis to argue that current notions about child development and mathematics are neither inevitable nor natural. In talking about her work, she said:

> My argument has been precisely that 'language' and 'cognitive development' are not descriptions of a real which takes place outside practices: all language, all signs, concepts, and so forth are produced as and by relations in specific practices. These practices therefore produce and read children as 'the child.' I will use the concept of positioning to examine further what happens when such readings are produced and how children become *normal* and *pathological*, fast and slow, rote-learning and displaying real understanding, and so forth. In other words the practices provide systems of signs which are at once systems of classification, regulation and normalization (Walkerdine, 1988, p. 204).

Here Walkerdine described "the child" as a subject produced through discourse. Her goal in doing so was to show that our notions about who children are and what they can do are constructed through practices like teaching, writing and parenting. For example, Walkerdine described how popular theories of children and mathematics constructed the subject of the child as someone who must be "developmentally ready" to engage in particular mathematical practices, such as learning place value. She showed how one teacher's practice both contributed to and was shaped by this construction. In doing this analysis, Walkerdine made it possible for readers to ask themselves: "What if there is no such thing as developmental readiness?" and to consider the possibilities that this question raises for the teaching of children. For instance, if developmental readiness does not exist, then waiting for children to become ready is no longer a viable pedagogical practice. Teachers must attend to children whom they may previously have considered unteachable because the children were not ready. By using subjectivity (looking at the child as a subject rather than at the identity of individual children) to emphasize how current "truths" were produced, Walkerdine made it possible to think about teaching young children in ways that went against the norms of the historical moment.

Following Foucault and Walkerdine, I talk about subjects and subjectivity in this dissertation, rather than identity. This is a deliberate choice meant to signify a postmodern stance. Structural analyses portray individuals as shaped by social institutions and forces, most particularly by class position. Many more recent critical analyses (e.g., Levinson & Holland, 1996) see the relationship between individuals and society as more fluid, where individuals are both shaping and being shaped by the societal institutions with which they interact. Researchers operating from this paradigm often use the term *identity* to describe the individuals they study. From my perspective, the trouble with identity (and the theoretical traditions from which it is drawn) is that it reinforces an opposition between individuals and society, implying that either one could exist without the other. Subjectivity is meant to suggest a being defined discursively. There is not an independent entity influencing and influenced by; rather, there is a subject continually under construction.

A non-mathematical example might help to illustrate the analysis of the construction of subjects through discourse. As I write this, my kitchen is a mess, and because I am sitting here typing rather than cleaning it, I feel guilty. As an academic, I could examine the source of this guilt in multiple ways. Perhaps, the most obvious choice would be the psycho-analytic route, where I would ask about the experiences and traumas in my past that have contributed to my current feelings about the kitchen. Alternatively, I could do a content analysis of the commercials I watch, looking at the messages conveyed to me about cleaning the kitchen and also observing my own reactions to these messages. I could look at my participation (or lack there of) in the rituals of cleaning the kitchen and at the social meaning of cleaning in my household or at the relationship of house cleaning to my identity as a white, middle-class woman.

Or, I could engage in the sort of discourse analysis I am proposing here, examining the idea of clean kitchens by constructing the subject of the housekeeper. If I pursued this project, I might find that at this time and place, there is a relatively dense discourse around the idea of clean kitchens as important for housekeepers. That is, the idea is reiterated in multiple ways. For instance, I do see many commercials advertising products for washing dishes and mopping floors. Also, when I moved into my house, there was a gadget in the basement designed to hold mops and brooms, and there was a latch on the cabinet under the sink, designed to protect children from the cleaning products to be stored there. Routinely, I receive fliers in the mail from people offering to clean my kitchen for me. And even when friends admit to messy kitchens, this admitting reinforces the idea that we need to have clean kitchens. (My friends do not "admit" that they do not sew their own clothing or fold their dinner napkins into swans).

If I wanted to pursue this problem, I might delve into historical records looking for when it became possible to talk about keeping the kitchen floor clean --- sometime after pigs and chickens were banished to the barnyard, I presume. I might also look for other texts to examine: other people's cleaning practices, the placement of products at the grocery store, the conversations of mothers on the playground. Through this work, I would be able to show how the subject of the housekeeper has been constructed and to challenge the notion that clean kitchen floors are right, good and natural. In looking at myself as a subject, I would not privilege my own thoughts, beliefs and actions about housekeeping (identity-thinking); instead, I would see these as part of a larger discourse that constructs a housekeeper.

Choosing to use a Foucaultian notion of discourse, rather than a socio-cultural or socio-linguistic one for this dissertation allowed me to address different sorts of questions than those that have previously been explored in relation to teacher education in mathematics. Socio-cultural and socio-linguistic studies both have the goal of understanding practices as they are currently enacted. This work is valuable because it can capture the complexity of something like reform teaching in ways that studies that seek to show relationships between variables cannot. For instance, the work of Paul Cobb and his colleagues has helped to show that reform teaching does not simply arise from using a problem-oriented curricula or from allowing periods for student discussion, but requires certain moves on the part of the teacher, such as the use of mathematizing language to support students' moves from concrete to abstract representations (e.g., Cobb, Boufi, Mc Clain & Whitenack, 1997; Cobb, Gravemeijer, Yackel, McClain & Whitenack, 1997). Socio-cultural or socio-linguistic uses of discourse can also show how practices that have been assumed to be homogeneous can vary in important ways across contexts, as Gutstein, Lipman, Hernandez and de los Reyes (1997) did when they described instances of reform teaching in mathematics that were shaped by the social activism concerns of a group of urban teachers.

However, I am proposing a different sort of project. Rather than trying to understand practices around reform teaching and equity as they currently are or trying to show how certain instances of practice might vary from norms in the literature, I am trying to highlight the unintended consequences of our current understandings of reform and equity in the context of teacher education, to make visible norms that have become taken-for-granted, and to create a space where we can think about teacher education in ways that are not currently possible. To do this, I need a notion of discourse that supports the tracing of big ideas across texts and contexts. In the three following chapters of this dissertation, I use a Foucaultian notion of discourse to:

- Examine metaphors used to talk about children and to discuss the consequences of these metaphors for how beginning teachers see the students in their classrooms.

- Analyze the mathematical problems posed in both the methods classroom and the elementary classroom to see how the student is constructed in both sites by various kinds of problems and to explore the consequences of these constructions.
- Identify the variety of teaching genres present in an elementary classroom and examine the possibilities each of the genres opens up for competent participation by students.

As I worked on these projects, the Foucaultian notion of discourse provided new opportunities for me to consider the ethical obligations of being a researcher in the classroom. Initially, I conceived of my role in both classrooms as one near the observer end of the participant-observer continuum. I expected that my presence would certainly be noted by the people in the room, but thought that because I did not plan to speak, my impact would be insignificant. Working with Foucault's broad notion of discourse caused me to understand my role in a new way: as part of the discourse whether or not I chose to speak. In fact, it was not only Foucault who taught me this, but the children themselves. About two months into the school year, I followed Caitlin to a neighboring table to watch her work with a partner on one of the problems assigned by the university seniors. After about ten minutes of work, her partner, Krystal, looked up at me and asked: "Why are you watching us? Are we bad?" I assured her that they were not bad, but "interesting." Krystal seemed to accept this and went back to work; however, as I continued to reflect on this exchange, I realized that while none of my focal children had ever asked why I was watching them, this did not mean that they did not have their own explanations for why they were chosen. (Similarly, other children in the classroom almost certainly had their own explanations for my choices about whom to watch.) Some of these explanations were likely to have been related to the very phenomena I was there to study: who is good at math, who is struggling, what mathematical practices are worthy of attention.

The realization that I had no choice but to be part of the discourse caused me to abandon my stance as a silent observer at some times. In early February, I observed Jerome using tiles to discover how many arrays could be made from 17 tiles. After several minutes of experimentation, he made one long row and wrote "1 x 17 = 17" in his journal. As other children continued to work, Jerome stacked his tiles into two vertical towers. Sara, the student teacher, walked by at this point, looked at Jerome's notebook and said: "Jerome, you can't just copy that. You need to figure it out with the tiles." Jerome looked up, scowling. "I did do it," he said and then he looked at me.

I had a choice then. As a researcher, it might have been interesting to see how Sara responded to Jerome's claim without an adult to back it up, since this would be more typical of student-teacher interactions in the classroom. But Jerome knew I had seen his work. He had watched me sketch it in my own notebook. To evade Jerome's silent plea at this point would have taught him something about the trustworthiness of adults in the classroom and about their perceptions of him; I would have added density to a discourse to which I did not care to contribute. So I said: "He *did* do it. He just didn't draw the picture." Sara briefly apologized to Jerome and moved on. If we imagine that discourse is something that one can stand outside of and analyze, this interaction becomes problematic. The classroom is a different place after my interaction than it was before it; in my study, I have to acknowledge the ways in which I as a researcher interacted with the people in the room, detailing those interactions and the possible consequences.

However, if we acknowledge that we cannot get outside of discourse, then it becomes apparent that keeping silent would have been no less of a contribution to the discourse of the classroom than speaking out. Only the content of my contribution would have changed. In the following chapters, I analyze discourse not as an independent observer, but as someone always in the midst of it.

In doing these analyses, I examined language used in both the elementary and methods classrooms in addition to physical setting and social practices. In order to dig deeply into the words of my participants, I borrowed some theoretical tools from Russian literary critic, Mikhail Bakhtin. His work appealed to me because his view of language recognizes the social world in which words are situated. He wrote:

> All words have the 'taste' of a profession, a genre, a tendency, a party, a particular work, a particular person, a generation, an age group, the day and hour. Each word tastes of the context and contexts in which it has lived its socially charged life; all words and forms are populated by intentions. (Bakhtin, 1981, p. 293).

Considering the "taste" of words provided a useful way of thinking about which words got said where and to whom. Some words taste of reform mathematics, others of the public school, and some of both. In looking at multiple sites – two classrooms, standards documents and curricula -- I wanted to examine the ways that words used in particular sites carried the intentions of those not present. For instance, Diana often asked students "Why?" when students offered an answer to a problem. For Diana, "Why?" carried the taste of reform, of the many university-sponsored study groups she participated in; it was a question asked by good teachers who want their students to think. For students on the receiving end of the question, the word carried different intentions. When asked "Why?" most students assumed they were wrong and changed their answers.

In these exchanges, both Diana and the children responded to the taste that "Why?" had picked up in earlier interactions.

Many of my own words in this dissertation carry the taste of other writers. Most particularly, I am informed by postmodern thinkers, including Fendler (2004), Foucault (1977) and Popkewitz (2004). Given this influence, the goal of my project is not to critique the current state of affairs to propose alternatives, nor is it to save either students or preservice teachers from poor teaching, society or each other. Rather, through my description and analysis, I hope to disrupt "old habits of thought that have been limiting what it is possible to think" (Fendler, 2004, p. 451). This can mean bringing a critical stance to practices commonly seen as beneficial in teacher education, such as reflection, practice-based learning, and collaboration, or it can mean bringing a similar stance toward attitudes and practices in teacher education commonly seen as promoting equity. These critiques can then serve as points of departure for others to re-imagine teaching and learning. However, solutions will not be prescribed --- both because the offering of solutions forecloses possibilities for unpredictable innovations and because the ability of intellectuals to create emancipatory futures for others is called into question. This skepticism toward progressive leaders is based on a history of failed attempts to liberate the less fortunate. As Blacker (1998) said: "They promise us 'Good,' but things turn out bad. This happens again and again and again. Maybe we shouldn't believe their promises anymore" (p. 351). Researchers working out of this tradition are not confident that they possess the solutions to other people's problems.

In taking this stance for the dissertation, I am not abdicating my responsibility as a teacher, a parent, a researcher or a citizen to act against practices that I see as inequitable. In fact, I expect that my work in this dissertation will help me to figure out appropriate ways to act on the problems I am closest to in the contexts I best understand. Nor am I suggesting that previous work in mathematics education that draws on linguistic or socio-cultural traditions of discourse is misguided; it is merely different. What taking this stance does mean is that I will not argue that the solutions that make sense to me -- in the classes I teach or in the schools where I send my daughter -- should be universally employed. In addition, it means that I see diversity of research perspectives as productive for the field of mathematics education. Drawing on postmodern notions of discourse can broaden, rather than supplant, understandings developed in other research programs. The goal of the dissertation is to provide academics (including myself) a way of thinking about the problems of teacher education in ways that will allow us to approach the problems we understand best in new ways. It is an activist stance, although not a prescriptive one.

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CHAPTER FIVE

BAD BELIEFS OR DENSE METAPHORS?

Sara, Diana's student teacher, left Blythe each Thursday to attend methods classes in mathematics and literacy at the university with a cohort of 20 other beginning teachers, called "interns" during their year of student teaching. Four of these beginning teachers shared Sara's placement at Blythe, while the other 16 interns had been placed in suburban schools near the university. Early in the year, David, the instructor of the math methods course, told his students that he would like to take them on a field trip to one of the placement schools so they could watch him teach young children with the kinds of problem-solving lessons that he had been demonstrating in class. Because he had relationships with some of the teachers at one of the suburban schools, David arranged for the class to meet there one afternoon to observe him teach geometry lessons to fourth-grade and first-grade classes.

The fourth-grade room, which was large enough to easily accommodate the visiting interns as well as the 22 students, had been arranged with the desks in groups of four to five angled toward the white board at the front of the room. David began the lesson by telling the children that definitions were important in mathematics and asking them to help him come up with a definition of a chair. Students called out characteristics and David challenged them through the use of examples and counter-examples until the class agreed on a list of minimally defining characteristics. David then led them through similar exercises with squares, rectangles, parallelograms and rhombuses, and finally asked them to state relationships between these figures (i.e., a rectangle is a special kind of parallelogram). After the lesson, the interns gathered in the hallway to wait until the end of recess, when they would go to see David teach a lesson in a first-grade classroom.

> While waiting, Sara approached Jack, an intern assigned to another third-grade classroom at Blythe, and remarked, "Can you picture one of our kids saying the square is a polygon? They don't even know that that means." Jack said "no," and Sara turned to Katie, another Blythe intern, and continued: "This just shows me how far behind our kids are. I mean, they're third grade, but they won't be here next year. Diana is always saying our kids are so good, but they're not like *this*. Even when things are going well, when they're excited about something, they just don't have the attention span."
>
> Katie: "They don't raise their hands. Or rather they blurt out first and then they raise their hands."
>
> Sara agreed.

This episode struck me because I was surprised by Sara's comments -- first, because she had always seemed to show a great deal of pleasure and pride in the accomplishments of the students in her classroom, and second, because her and Katie's perceptions of the lesson that David had taught were so different from mine. I had observed several students who called out without being called on; in fact, David had seemed to encourage this by asking questions of the whole class and waiting for a choral answer. In addition, while the class had produced a great deal of technical vocabulary, including "polygon," "quadrilateral" and "symmetrical," most of these words had come from just few students. In the lesson I observed Sara teach at Blythe the following week, her students used the words "square number," "repeated addition" and "strategy" while describing their work. These words may not have been as unusual as the geometric ones, but they showed appropriate uses of mathematical vocabulary for a third-grade unit on multiplication.[1] Yet, Sara was clearly struck by differences between her class at Blythe and the suburban class she observed David teach. And she did not see these differences as the results of participating in a different kind of lesson with an experienced mathematics educator, of being a year older, or of studying a different content strand within mathematics. Instead, she described the differences as innate in the students, calling her own class "behind," "not like *this*," and lacking in "attention span."

Bad Beliefs

The literature in mathematics education and in teacher education more broadly has often explained episodes such as the one I described above as instances of beginning (or experienced) teachers revealing their problematic beliefs. For instance, the literature has described teachers as holding beliefs that embody dysconscious racism (King, 1991), that promote color-blindness (Bell, 2002), and that link low achievement to inadequacies in the students' culture (Cooper & Jordan, 2003). In an review of research on prospective teachers' beliefs about teaching children of different races, ethnicities, and socio-economic backgrounds, Gomez (1993) described a host of problematic teacher beliefs that have been documented in dozens of studies, including beliefs that economic rewards are fairly distributed in the United States, that low-income families do not support their children's learning, and that some students cannot learn.

Most studies of teachers' beliefs in mathematics education have focused on teachers' beliefs about how mathematics should be taught or about what mathematics is, rather than on beliefs teachers hold about students (e.g., Barlow & Cates, 2006; Mingus

[1] My goal here is not to say that Sara did not see the lesson for what it "really" was, but to point out that she constructed differences that were meaningful to her. I also saw differences between this lesson and those in Diana's classroom: the teacher talked far more, students did no independent work, and children almost never spoke to each other. My production of these differences could equally serve as a point of departure for study. That, however, remains a challenge for another day.

& Grassl, 1999; Stipek, Givven, Salmon & MacGyvers, 2001; Timmerman, 2004; Warfield, Wood & Lehman, 2005; Wilkins & Brand, 2004). However, a few studies (Fennema, Peterson, Carpenter & Lubinski, 1990; Sztajn, 2003; Tiedemann, 2002) have looked at interactions between these belief systems. For example, Sztajn (2003, pp. 53-54) wrote that teachers "have to put together, according to their own beliefs and interpretations of existing rhetoric what they consider to be the best mathematics education for their children." To do this, he argued, teachers draw on both their sense of what mathematics is and on their beliefs about who the children in their classrooms are. Based on a case study analysis of two experienced teachers, he argued that the teachers' beliefs about mathematics and children caused them to use drill-oriented work with classrooms of low-income students and to assign activities that promoted higher-order thinking to classrooms with children from more affluent families.

Framing teaching decisions and comments like Sara's as instances of problematic beliefs coming to light has meant that the interventions proposed by teacher educators have often focused on finding ways to successfully change individuals' beliefs (e.g., Artiles & McClafferty, 1998; Bondy, Schmitz & Johnson, 1993; Brown, 2004; Sleeter, 2001). For example, in their study of mentor-novice pairs, Achinstein and Barret (2004) discussed beginning teachers' tendencies to see their diverse classrooms as collections of management problems. The authors of the study described ways that mentors were able to help beginners reframe their thinking about students to include political and social considerations, which caused the novice teachers to change their beliefs about their students' academic capabilities. Alternatively, studies have focused on identifying qualities in beginning teachers that lead to either problematic or productive beliefs, such as previous experiences with cultural diversity, empathy for others, or valuing of diversity (Garmon, 2004; Pohan, 1996; Smith, Moallem & Sherrill, 1997).

My concern with framing comments like Sara's and questionable teaching practices as the products of problematic beliefs is that this theoretical stance places too much emphasis on the individual. Although most studies acknowledge that individuals' beliefs are situated in larger social contexts, methods, such as surveys and interviews, work to highlight the responsibility of single individuals. Similarly, interventions that seek to change the beliefs of individuals through classes on cultural diversity, field experiences, or reflections on experience make what goes on inside individual heads the primary locus of change. I see two problems with this. Practically, it is slow work. Following this model, teacher educators must diagnose and treat each prospective teacher they encounter from now until the end of time. Theoretically, this model is also troublesome. Whether one follows Vygotsky (1978, p. 88) who wrote that learning "presupposes a specific social nature," Bakhtin (1981, p. 293) who said that the word "exits in other people's mouths, in other people's contexts, serving other people's intentions," or the host of other thinkers who have argued that researchers cannot consider the individual without the social (e.g., Lave, 1988; Rogoff, 1995; Wenger, 1998), studies of beliefs seem to give far too much weight to the individual, both in terms of responsibility for problematic ideas and in terms of proposed interventions. Educational philosophers Blacke, Smeyers, Smith and Standish, (1998, p. 88) in one of the most eloquent articulations of this idea, wrote that "In part what is given in teaching, in the initiation into a culture, is a gift that cannot be refused. What we come to know in this way precedes the possibility of our autonomy." In this chapter, I argue that Sara's words comparing the children at her urban school with those she saw in the suburban classroom did not reveal her "*own* beliefs" because the way she came to see her children as "behind" preceded the possibility of her autonomy.

Dense Metaphors

Rather than framing this chapter as an analysis of Sara's beliefs, I want to examine Sara's comments as a re-articulation of a common metaphor in the current discourse about children. My hope is that by approaching the analysis in this way, I will be able to simultaneously consider the actions of individuals as well as the social soup[2] in which those actions are located. To do this, I draw on the notion of metaphor articulated by George Lakoff and Mark Johnson (1980), who argued that metaphors are more than ways of using language, but also ways of understanding concepts. They used the metaphor of *argument as war* to demonstrate both the pervasiveness of metaphorical language in our everyday communications and also the ways in which metaphors can influence how we see the world. Lakoff and Johnson pointed out that in our culture people talk about attacking, demolishing, shooting down and winning arguments. They plan counterattacks and defend their points. In contrast, the authors asked readers to imagine a culture where *argument as dance* was the dominant metaphor for speaking and thinking about disagreements.

> [T]he participants are seen as performers, and the goal is to perform in a balanced and aesthetically pleasing way. In such a culture, people would view arguments differently, experience them differently, carry them out differently, and talk about

[2] Speaking of metaphors, our lives as "social soup" is one of my favorites. Rather than seeing individuals and discourse/society as separate, the soup metaphor merges these two constructs. While it is possible to call individuals or discourse to the foreground (as a chef might consider the amount of salt or potatoes) the soup cannot be separated from its component parts. Nor can we, as ingredients, get a vantage point outside of the soup to make observations. Like the force, soup is always in motion.

> them differently. But we would probably not view them as arguing at all: they would simply be doing something different (Lakoff & Johnson, 1980, p. 5).

Using this example, the authors suggested that metaphors are not simply instances of poetic language that writers use to make their work more aesthetically pleasing, but are ways of seeing the world that become more and more dense as they are re-articulated by multiple speakers. These dense metaphors then become part of everyday language and are often not registered as metaphors by listeners. That is, we hear the metaphor literally so that if we hear a colleague say "I fought with the chairman over the budget," we typically would not consider the word "fought" to be metaphorical language. We interpret fighting as the use of words in an angry way, not as physical struggle. However, Lakoff and Johnson argued that the notion of "fight" works in the same way as more unusual metaphors by highlighting some aspects of the concept of argument (the disagreement, the expectation of winners and losers) while diminishing other qualities (the shared purpose, the turn-taking). The frequency with which metaphors of war are used to talk about arguments makes it easier for people to see only the aspects of argument that are consistent with that metaphor.

Building on this work, Santa Ana (1999), who examined the prevalence of the metaphor *immigrants as animals* in the media, argued that the more commonplace metaphors become and the more prosaic they seem, the more powerful they become in shaping the way people interact with the world. He wrote:

> When an original, truly novel metaphor is used, the reader of the turn of phrase is prompted by its novelty to evaluate the metaphor for its appropriateness, creativity and utility. The mindful reader can choose to reject the linkage. If, however, the metaphor does not draw attention to itself, then the reader is most

often unaware that a conceptual linkage has been reproduced and is being reinforced (Santa Ana, 1999, p. 237).

Lakoff and Johnson argued that many of our most common metaphors – those that do not "draw attention" to themselves -- are both based on our physical experience of the world and can influence the meaning of other metaphors by drawing on the same physical experiences. Many concepts we talk about metaphorically are structured in terms of up and down, and forward and behind. That is, these metaphors draw on our notion of a physical path that can be traveled in two directions, either vertically or horizontally, in order to explain some other concept. For instance, conscious is "up" (wake up, early-riser, up and at 'em) and unconscious is "down" (fell asleep, dropped off, sank into a coma).

Coherence among metaphors can be created when multiple metaphors draw on similar physical concepts. For example, many concepts (e.g., good, more, awake, happy) map onto "up" while other concepts (e.g., bad, less, asleep and sad) map onto "down" in many of our commonly used expressions (Lakoff and Johnson, 1980). Sfard and Lavie (2005) offered one empirical example of this kind of coherence when they told the story of two four-year-olds who insisted that the mystery box they chose was "more" than an identical rejected box. When questioned, the children agreed that the two boxes were the same size; however, they stood by their claim that the chosen box was "more huge." The adults present could not understand the children's statements, but Sfard and Lavie argued that the children associated the word more with good and used it because they believed that the chosen item must be by definition "more" than the less desirable item. This is an example of coherence between the concepts of good and more, both of which are often discussed in ways that draw on the physical concepts of "up" and "ahead." In the following section, I will describe the ways in which Sara's comments that her children are "*behind*" and are "not going *to be here* next year" draw on the physical world to shape thinking about students and mathematics and examine the prevalence of other metaphors that draw on the physical world in similar ways. In addition, I will discuss the aspects of children and mathematics that these metaphors emphasize as well as the aspects they tend to hide.

Talking about children: Metaphors of hierarchy

In order for Sara to make her relatively straightforward comment that watching the children in the model lesson made her realize "how far behind our children are," she needed to make several decisions: first, that an appropriate action when observing two sets of children is to compare them; second, that an appropriate next step is to rank them according to some criteria; and third, that an effective way to express that ranking is to use a metaphor that draws on front-to-back directionality. Neither Jack nor Katie, the interns Sara spoke to, seemed perplexed by her use of the word "behind." They did not seem to stop and evaluate the metaphor, as Santa Ana suggested that people may do when confronted with novel metaphorical language. Both seemed to understand that Sara was ranking the children in some way and Katie, at least, seemed to agree with the ranking, offering her own evidence to support it: that the children in her Blythe classroom called out without raising their hands. In this section, I would like to examine the prevalence of metaphors of hierarchy in discussions about children in mathematics education literature, elementary mathematics textbooks, standards documents, conversations in David's methods classroom, and Sara's elementary classroom at Blythe.

In the methods classroom, both David and the other interns often talked about students in ways that created hierarchies by drawing on metaphors of the physical world. For example, at various times in the semester, the following comments were made:

- "What do you do with a second grader who's really awesome in math – who's *really far ahead* of everyone else?"
- "She's in special ed. She's really *low*."
- "Obviously you need to set up groups of kids in problem solving that can help each other and *pull people along* who are having trouble."
- "Some of your kids – they're going to be *way up here* (holding hand above head). You have to *rise up* to meet them."
- "I'm afraid he'll have to be retained. He can't *keep up*."

Each of these comments draw on a notion of children ordered in physical space, whether horizontally ("really far ahead") or vertically ("low"). Some of the comments put students in motion. The line of students is marching forward and children must either keep up or be pulled along. This is drawing on a similar sense of space as Sara's comment that her children "won't be here" next year. In both cases, the mathematics is portrayed as a path that must be traveled. Sara is saying that her third-graders will not have made enough progress along the path in order to be in the same place as the fourth-graders in a year's time. The remark about "pulling children along," which was made by David, implies that some children will not be able to travel the path under their own power and must be dragged forward by their more able classmates. Initially, it may seem strange to think of these ordinary bits of language as metaphors; however, like more poetic figures of speech, each of these phrases asks the listener to think of children in non-literal ways. Children are not really ordered in physical space. There is no path. No one (contrary to political rhetoric) is actually being left behind. As Santa Ana argued, one of the reasons this sort of language seems unremarkable is because it is so common. Sara encountered these kinds of physical metaphors for children in her elementary classroom, in her textbook and in documents from the state Department of Education.

For example, in one lesson Diana said: "Ben, you did it in a really sophisticated way, but that might *bring it back* to other people," "Come on, Jerome! You need to *keep up* with the rest of the class" and (to the whole class) "You're letting Tyler and Ben *carry you*." This last variation echoes David's comment about pulling kids along. The children in Diana and Sara's class also used these kinds of metaphors. During independent work, Aliah remarked that "Mia is *ahead of* me. She knows her division and her multiplication." Once, Sara even physically acted out the metaphor of mathematical achievement as physical ordering on a path by calling children to line up at the end of the lesson in an order that ranked their participation in a discussion. Students who spoke frequently were at the front of the line, while Mia, who at this point in the year spoke very little English, brought up the rear.

Although the language in state documents and mathematics textbooks tends to be more formal than the spoken language, examples of phrases that drew on metaphors of crossing physical space to talk about children's learning of mathematics did occur. In a handout for parents about mathematics standards in third grade, the authors wrote, "The expectations were designed to ensure that students receive *seamless* instruction from one grade level to the next, leaving *no gaps* in any child's education" (Michigan State Department of Education, 2006). As in the phrase *achievement gap*, the language in the parent handout implicitly draws on the crossing of physical space as a metaphor for children's learning in mathematics, where gaps in the path can be seen as dangerous. The metaphor of physical space was also implicitly drawn on in the methods textbook that Sara often used when planning lessons. Van de Walle (2004, p. 95) wrote "It remains true that students *will rise or fall* to the level of our expectations." Here students are compared to an independent criterion (expectations) rather than each other, but still moving up is good and going back, or falling, is bad.

All of the metaphors discussed above hang together coherently because each uses the idea of a line in physical space, where being closer to the front or the top (or moving in that direction) is good. This metaphor structures thinking about children and their learning of mathematics in particular ways. Although saying a student is "really far ahead" of everyone else hardly seems poetic, like other metaphors, it connects one concept (knowing math) to another dissimilar concept (being physically in front). And, like all metaphors, this connection makes some features of knowing math more salient than others. Repeatedly using language that describes children's learning of mathematics as a journey or a ranking in physical space emphasizes the idea that students' thinking, learning and progress can be compared to each other. Just as location or progress along a path can be identified through the use of landmarks, children's mathematical progress can be identified by where they stand relative to each other or by where they stand relative to the content. This is how children can be thought of as "behind" or "ahead."

These metaphors also work to portray mathematics as narrow path, which can be traveled in only one direction (up and ahead). Sara observed David teach a geometry lesson in a fourth-grade classroom and said that her children were "behind" the ones she had just observed. Geometry had not yet been taught in her classroom at Blythe; yet Sara assumed that the suburban children's performance in this geometry lesson revealed their understanding of other mathematical strands as well. She did not say "Boy, they know a lot of geometric vocabulary." The ease with which she could perform the ordering of the children in these two classes and the ease with which her ordering was accepted by the other interns in part relies on the metaphor of mathematical achievement as a journey along a narrow path. One cannot simultaneously be "ahead" in geometry and "behind" in number. Each person has one location on the path.

The metaphor of travel along a path makes it difficult to see a student who struggles with number as mathematically competent because of an ability to visualize three-dimensional objects. This metaphor obscures multiple-entry points into mathematics and makes it difficult for teachers to see student work as varied without also seeing it as ordered. On a path, one is always nearer or farther from the front. Similarly, this metaphor can cause teachers to interpret chunks of mathematical content as necessarily ordered. In the methods class, one intern said that she could not move on to multiplication because many of her students could not subtract with regrouping. David asked what one had to do with the other and the intern replied "Subtraction with regrouping comes first." Subtraction with regrouping did come first in the mathematics books used in Sara's classroom (as in most third-grade math books); however, there is no mathematical reason to assume that competence in the former is a prerequisite for competence in the later. Children's performance in different mathematical strands – such as, geometry, measurement, data, and operations – can be quite varied, but using a metaphor that portrays mathematics as a "seamless" path with no "gaps" can make if difficult to recognize this.

The metaphor of the path also contributes to understanding children in ways that promote the idea that some children are good at mathematics and some children are not. When mathematics is seen as a linear list of skills that must be mastered one after the other so as to progress toward a single goal (the end of the path), then some children must always be closer to that goal and others must be further away. When Ben offered a new solution to an addition problem, Diana did not merely identify the solution as different, but labeled it as "sophisticated," and told Ben his thinking would "bring it back" to the rest of the class. Similarly, students who are in different places than their classmates in terms of understanding mathematics are seen as problems for the teacher and other students. They must be "pulled along" or work to "keep up." The metaphor of traveling along a path obscures ways of understanding student differences as interesting, valuable, or natural.

These ways of thinking are reinforced in mathematics education by many concepts that may not use words that explicitly refer to positioning in the physical world, but that nonetheless draw on the root idea that learning mathematics has one path that individuals are more or less ably suited to traveling. In mathematics education, the notion of development, which underlies theories and practices that expect all children to progress through identical stages in identical orders (like travel along a path) is present in much of the writing about children. Piaget's theory (1928; 1950; 1952) that children develop the abilities to realize that objects do not disappear, to use symbols, and to conserve mass (among other abilities) at particular ages and in a predictable order has been particularly influential. David mentioned Piagetian concepts several times during his lectures, suggesting that the prospective teachers use simple activities, such as asking children to identify the larger of two sets of identical but differently spaced counters, in order to make decisions about what sort of instruction might be appropriate.

Similarly, the van Hiele levels of geometric thought, which David lectured about in class and which the author of Sara's methods textbook devoted nearly forty pages to, are grounded in a notion of development where all children are seen as progressing through the same levels in the same order. Calling the van Hiele theory "the most influential factor in the American geometry curriculum," Van de Walle (2004, p. 347-348) described the theory as saying that "the levels are sequential. To arrive at any level above level 0, students must move through all prior levels." In elementary school, children are expected to move from Level 0, where they can identify shapes only because they look just like other shapes they have seen ("It's a square because it looks like one.") to Level 2, where they can reason about the properties of shapes ("Because it's a square, all the angles must be 90 degrees."). Both the van Hiele levels and Piagetian stages support ways of thinking that allow some children to be seen as "behind" and other to be seen as "ahead," either relative to their classmates or to benchmarks delineated in the theories. In fact, this happened in the methods class on the day of David's van Hiele lecture when one of the interns volunteered that her daughter's kindergarten homework had been to name the features that made a circle a circle. David replied, "I don't think kindergarteners or even most first graders could do that." The intern responded that her daughter had said that circles don't have corners, that they go all the way around, and that they don't have straight lines. David shrugged, adding "She must be very advanced."

Here, a 5-year-old's ideas about circles were labeled as advanced because the uptake of the van Hiele levels has been that kindergarteners are focused on visualization. The theory created the opportunity for this child to be seen as ahead and at the same time created opportunities for other children to be seen as behind. My goal here is not to argue that the van Hiele levels are inaccurate, or that David's use of them was incorrect, only that the prevalence of developmental thinking in the mathematics education discourse reinscribes the metaphors of hierarchy that Sara drew on to talk about her children.

In similar ways, our system of organizing students in classrooms by age reinforces thinking about mathematics as a narrow path. Michigan's Grade Level Content Expectations "provide a set of clear and rigorous expectations for all students and provide teachers with clearly defined statements of what students should know and be able to do as they progress through school" (Michigan State Department of Education, 2006). In other words, these standards define the path; they lay out what a third-grader must do and make it possible for teachers to identify children for remediation (those who are behind) and enrichment (those who are ahead). As Fendler (1999) has pointed out, this is a different enterprise than looking at actual third-graders to figure out what it is they know and are doing.

The metaphor of schooling as a path with grade-levels as landmarks dominates not only practices in the classroom – Diana and Sara used textbooks written for third-graders, prepared for the third-grade standardized test and followed the third-grade standards – but also frames how educators talk about children. Diana frequently asked her students to "act like," "write like," and "sit like" third graders. On one particularly noisy afternoon, she looked slowly around the classroom until students quieted. Then she remarked. "Now it's quieting down. That's good because I was a little worried. I was thinking somebody else might have snuck in here." Concerned, Marcus asked "Who?" Diana said: "Someone who wasn't quite ready to be in third grade." Jerome, who was frequently chided, asked: "Like me?" Jerome had gone through three and half years of schooling in the same building with many of the same classmates. He was eight years old and knew himself to be in third-grade. Yet, the frequency with which his behavior was corrected and answers revealed to be incorrect caused him to identify himself as someone the teacher thought was not ready for third grade. In this instance, Diana reassured him that he was, but Jerome seemed to not only recognize the metaphor of schooling as a path, but also to be able to identify his place along it.

The third grade math books, which Sara used occasionally, also drew on this metaphor. Each page in the math book represented one lesson. Students were expected to proceed along a path from the first page to the last. In addition, each page in the math book came with three additional worksheets, labeled "re-teaching" "practice" and "enrichment." Teachers were expected to assign each student one of these pages based on the students' position relative to the intended lesson. In a professional development session about the curriculum, teachers were told that a child should never be assigned more than one of these worksheet pages. Each student could occupy only one of these positions. In addition, each page of the math book also had a section called "quick review," which students in the classroom completed at varying rates. This was a constant source of frustration to Sara, who once exclaimed: "It's *Quick* Review! What's it supposed to be? Quick!" Third graders, who were all supposed to be at roughly the same point along the path, were not expected to take different amounts of time to complete the same task.

In the methods classroom, the prospective teachers frequently identified children in their stories by grade level and if they did not when asking a question, such as "How do you talk about least common multiples?" David would ask for the grade level. This was not one of many bits of information solicited in order to make an informed teaching decision, but most often, the only one. A students' grade level was assumed to reveal what a student would know and not know (his or her place along the path), which allowed comments like "This is way too hard for first-graders" and "Fourth graders can see relationships among shapes." Although these statements do not explicitly use language that compares mathematical learning to travel on a path, they do suggest that all first-graders or fourth-graders should be in the same place in relation to learning mathematics and thus are coherent with physical space metaphors.

David sometimes drew on notions of ranking and progress even when discussing children beyond mathematics. As an assignment, David asked his interns to find out something that their students did well outside of the classroom. When asked to report back the following week, Jack said that he had learned that one of his students had a beautiful singing voice. David then asked if she sang in a choir or a competition. Jack said no; he had heard her on the playground. Then Sara offered that one her students (Jerome) spent his weekends on a farm milking goats. She said this was unusual because she taught at an urban school and she thought that most of her kids had never seen a farm. David replied, "That's a cool experience. So you got that category of cool experiences and that other category of someone does something really well. You can't milk a goat really well. I don't know how you would do that." With both Jack and Sara, David seemed to diminish the importance of their stories because they did not support ordering. David responded with much greater enthusiasm to stories about child gymnasts, bicycle racers, and competitors in Irish dance, which all make it easy to talk about children in hierarchical ways.

Coherence among metaphors: Or what else can we think of as ordered?

The prevalence of language that explicitly draws on the image of traveling along a path to describe children's mathematical learning as well as the saturation of mathematics education with practices that reinforce this metaphor create certain discursive possibilities and close down others. In other words, Sara's choice to compare and order the children in her placement classroom and the children in the model lesson classroom emerged from a discourse community in which hierarchical thinking about children was common. Sara's adoption of that language can be seen as evidence of her participation in that community. However, the prevalence of hierarchical thinking does not explain the order in which Sara ranked the classrooms. That is, why did she see the students in her own classroom as behind, rather than ahead?

The grade level of the students may have played a role. As discussed in the previous section, most educators would probably see fourth graders as "ahead of" third graders. However, Sara explicitly rejected this as a reason saying "I mean, they're third grade, but they won't *be here* next year." In addition, because the notion of grade levels as markers along the path is so common, third graders who did not know as much as fourth graders would not be seen as behind by most educators. They would be seen as making adequate progress, relative to when they began their journey. Sara offered the use of geometric vocabulary and long attention spans as reasons for her judgment, and Katie suggested that the frequency of hand-raising before answers as a criterion for ranking. These factors may certainly have played a role. However, I would like to entertain another possibility based on Lakoff and Johnson's notion of metaphorical coherence.

Sara's class at Blythe Elementary, a small urban school, was comprised of 19 students. Four of these students had been identified as White/Caucasian on school enrollment forms. The other 15 students had been identified as African American, Asian American, or Bi-racial. Many of these children had brown skin and brown eyes. In addition, many of the children of all ethnicities spoke non-dominant forms of English. In some cases, this was the result of English being a second language. In other cases, this was the result of speaking a dialect. For some students, the variations in their speech were quite subtle – such as slight southern accents and using words like "y'all." For others, variations included grammar often deemed unacceptable in school, such as "He don't know." In the class of 22 students at Northside (which hosted David's model lesson), most students (about three-quarters of the class) had the pale skin and facial features that most Americans would probably identify as European-American. All of the children who spoke used variations of English that would probably be considered standard. That is, they used contractions frequently, but used grammatical constructions that would be considered correct in formal, written English. Most children's accents and inflections mirrored those of David (and Sara and me).

In the preceding paragraph, I sketched out a few of the differences between the classes at Blythe and at Northside. Of course, there were many other differences. The architecture of the buildings and arrangement of furniture in the rooms was different. As group, the children at Northside seemed slightly taller. The children at Blythe tended to talk to each other while the children at Northside tended to address only the teacher. However, I did not randomly choose the differences I highlighted in the previous paragraph. I described them because I believe they were differences that Sara noticed and drew on in making her assessment. I would now like to examine some examples of discourse about children from the mathematics education community with the goal of demonstrating how the ethnic, racial and class differences I identified above may have become salient for Sara and may have contributed to her ranking the two classes in the order she did.

More than a decade ago, NCTM issued what it calls its "Every Child" Statement, which is posted on the NCTM website and quoted in Van de Walle's methods textbook (2004, p. 95), in order to underscore the council's commitment to the ability of all children to learn mathematics. The second half of the statement goes like this:

> We emphasize that "every child" includes--
>
> - learners of English as a second language and speakers of English as a first language;
> - members of underrepresented ethnic groups and members of well-represented groups;
> - students who are physically challenged and those who are not;
> - females and males;
> - students who live in poverty and those who do not;
> - students who have not been successful and those who have been successful in school and in mathematics.

The way the subjects of each of the above bullets are ordered is not coincidental. In the first position are the kinds of students who have trouble learning mathematics: speakers of English as a second language, minorities, those with disabilities, girls, the poor, and the unsuccessful. These statements would not have what Lakoff and Johnson called "coherence" if they were all jumbled about. The category of "males" does not go with the category of "poor" and "minority." Although the language of ordering was not used explicitly, the pattern of the bullets implies a continuum of students who range from "those who have not been successful" to "those who have." In addition, ordering the subjects of each bullet in this way works to attach the categories of minority, female, poor and physically challenged to unsuccessful.

Adding It Up, a report on elementary mathematics in the United States which was authored by some of the most prominent mathematics educators in the field, also invoked metaphors of hierarchy and linked notions of race and class with the ordering. First of all, the section of the report that addresses issues of equity is called "Equity and Remediation." Immediately, this phrasing invokes the metaphor of the narrow path. Remediation is only possible when some students are behind. The challenge of equity becomes catching these students up to where they are "supposed" to be, rather than on, say, learning to value where these students are. This section of the report goes on to say:

> A number of children, however, particularly those from **low socioeconomic** groups enter school with specific *gaps* in their mathematical proficiency. … Overall, the research shows that **poor and minority** children entering school do possess some informal mathematical abilities, but that many of these abilities have developed at *slower rates* than **middle-class** children. The *immaturity* of their mathematical development may account for the problems **poor and minority** children have understanding the basis for simple arithmetic and solving

simple word problems (pp. 172-173, Adding It Up, 2005, italics and bolding mine).

The authors of this paragraph explicitly invoked metaphors of travel along a path with the use of the words "gaps" and "slower rates." In addition, they drew on notions of development by referring to the "immaturity" of some students. Not only does the phrasing of this paragraph support ways of thinking that place some children ahead of other children, but the words chosen also attach demographic characteristics to positions on the path. Poor and minority children are slow and immature. Another rhetorical move in this paragraph is to conflate poor with minority, not only by using them in connection with each other, but also by occasionally using one to stand for the other. The authors wrote that "Overall, the research shows that **poor and minority** children entering school do possess some informal mathematical abilities, but that many of these abilities have developed at *slower rates* than **middle-class** children." In the second half of this sentence, the authors qualify the noun "children" with only the adjective "middle-class," although they began the sentence by referring to "poor and minority children." This is a little sleight of hand (or text). Consider the alternative: Poor and minority children develop at slower rates than white children.

Replacing "white" with "middle-class" makes the sentence less jarring, but by using "poor and minority" at the beginning of the sentence and by repeating the same two adjectives in the following sentence, the authors order race along with mathematical ability.[3] In addition words like "poor," "minority," "immature," and "slow" become

[3] Astute readers might detect slightly less compassion on my part for the authors of this report than I show for Sara and might (justly) point out that the authors of *Adding It Up* are no more reporting their "own"

attached to each other in ways that one can stand as a proxy for the others. This substitution of various qualities for race occurred in conversations in the methods classroom as well as in written texts. For instance, when telling her story about Jerome milking goats, she said: "This is an urban student – that's what our school mostly is. You would never guess by looking at him that he does this." Jerome, who typically wore glasses, button-down shirts and jeans belted at the waist, did not look like a stereotypical farmer; however, he did not look particularly "urban" either, with the exception of his dark skin. Sara's comment demonstrates the ways in which race can get noticed and remarked upon without using racially explicit language.

When multiple texts attach (however circuitously) concepts that typically would be considered categorical rather than ordered – such as race and gender -- onto metaphors of hierarchy, they work to create discursive expectations. The result is that when the authors of the NCTM Principles and Standards for School Mathematics (2000, p. 12) wrote that "all students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study – and support to learn – mathematics," readers can understand this to mean that some students – those who "live in poverty, students who are not native speakers of English, students with disabilities, females, and many nonwhite students" (p. 13) can be thought of as struggling, or perhaps behind. Through similar reiterations, the metaphor of students distributed along a path

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beliefs than Sara is. All of us are situated in the current discourse about children in mathematics. However, I do hold the authors of *Adding It Up* slightly more accountable for their reiterations of this hierarchical discourse than I do Sara. First, they are speaking formally in a considered and public report, as opposed to having a casual hallway conversation. Second, because of their stature, their contributions to the discourse carry more weight.

begins to describe not just individual children, but kinds of children. I would argue that as a result of these layerings of discourse, Sara, as well as most other educators, would have been inclined to position the students from Blythe as "behind" the students from Northside because this ordering had more coherence with other metaphors of hierarchy within the discourse of mathematics education than the reverse order. That is, it would have been conceptually surprising to think of a class of mostly minority students from poorer homes as "ahead of" a class of mostly majority students from middle-class homes.

Possibilities for Teacher Educators

This is not an argument intended to absolve Sara, or any of us, of responsibility for our words. In fact, it is just the opposite. Seeing Sara's positioning of her student as the result of dense metaphors in mathematics education rather than as the problematic beliefs of individuals changes the responsibility of teacher educators. Instead of (or perhaps, in addition to) searching for ways to change beginning teachers, we can think about the ways that our own teaching, writing, and conversation work for or against hierarchical ways of thinking about children. In addition, we can be conscious of the ways that we use demographic labels. This is not easy. As Lakoff and Johnson wrote, metaphors are not "just talk" (1980, p. 4); they influence how we understand the world, and with dense metaphors, like learning mathematics as travel along a narrow path, revisioning the world requires new words, new metaphors and new practices that may seem troubling, precisely because they are at odds with the ways that we have always understood mathematics and children. In closing, I would ask readers to consider the following three questions as a way of beginning a conversation about re-imagining our language and practices as teacher educators.

1. *How could we talk about learning in mathematics without using metaphors of hierarchy?*

School learning, and not just in mathematics, may seem to necessarily contain within it the notion of progress and of more or less, which is what leads to the ordering of mathematical concepts and the ranking of children. One way of creating new metaphors to talk about learning may be to talk about difference rather than progress. For instance, Gardner's (1983; 1999) work on multiple intelligences offered one possible language for talking about intelligence without ordering. By identifying specific ways of thinking, such as visual, logical and linguistic, Gardner argued that intelligence is not one continuum, but a diverse grab-bag of abilities. Drawing on this metaphor, learning might be looked at as a practice that adds diversity to one's grab-bag. In mathematics, where visualizing, making sense of the world quantitatively, arguing, asking questions, creating representations, following procedures, and much more, all can be seen as essential, drawing on metaphors that value diverse ways of understanding number, data and space ought to be possible. Another possibility may be to hold each other accountable for the meanings of the words we use to describe children. We can ask each other to explain what it is we mean when we call children "low" or "immature." What are we saying he doesn't understand? What is it that he does know?

2. *How can we talk about learning in mathematics over time without drawing on notions of development?*

Stage theories seem particularly dangerous in terms of positioning children. When theories, like the van Hiele levels, are repeated in classrooms and textbooks, they take on the power of physical law. Students who do not achieve stages on time or in the correct order are seen as requiring remediation. Fendler (1999) asked why we choose to see these children as behind, rather than seeing them as empirical evidence that theory is wrong. Rather than labeling poor and minority children as "immature" because they, as a group, do not solve problems in similar ways to middle-class, majority children, we might instead ask explore these differences as a path toward understanding the variety of ways that children come to understand the world quantitatively. For some time, literacy researchers (e.g., Cazden, 1988; Dyson, 1995; 1997; 2003; Heath, 1983) have moved away from talking about differences in the ways that children come to the written and spoken language as evidence of maturity (or not). Instead, they have documented differences with the goal of showing the many ways that children move toward understanding. Dyson (2003, p. 5) wrote

> I hope to turn this [developmental] view inside out, as I look from inside a particular child culture out toward school demands. In this way, I aim to provide conceptual substance for a different theoretical view of written language development, one that normalizes variations in (as well as broadens conceptions of) children's literacy resources and learning pathways.

Similar work in mathematics education offers rich possibilities for new research.

3. *How can we talk about the interactions of race, class and gender with the learning of mathematics without reincribing metaphors of hierarchy?*

Schooling resources, including money, challenging curricula and skilled teachers, have historically not been equally distributed across all demographic groups of children. Acts of racism, sexism, and discrimination against the poor have occurred and will occur again. School cultures can be varyingly welcoming to children depending on their lives at home, and as researchers we need a language to talk about these phenomena. However, by repeatedly singling out certain demographic groups when talking about difficulties in learning mathematics, we create a shared understanding that some groups, but not others, have trouble learning. One rhetorical strategy may be to consciously create "incoherence" in our writing, such as in the following phrase: "boys as well as girls, minority students as well as majority students." Small moves like this may at least work to challenge some of the unconscious connections reinforced in a lot of writing. Another strategy may be to become more localized in the ways that we talk about students so that we write not about low-income, minority students, but about particular students in particular places -- who have certain language practices, schooling histories, family lives, friends, and personal preferences in addition to races and socio-economic statuses. The more complicated students become in our discourse the more difficult they will become to rank based on any one characteristic.

My responses to the questions I posed are not intended to be answers so much as openings to conversations. Like everyone else, I am immersed in the discourse I am trying to change. The primary intervention of this chapter is to make visible metaphors of hierarchy that previously may have gone unnoticed. As Santa Ana pointed out, the more common our metaphors are, the less likely we are to question their usefulness and accuracy. As long as our language of ranking remains invisible, it works to reinscribe a particular set of power relations – one where some students (identifiable by physical characteristics) are seen as mathematically capable and others are not. Foucault (1983/1996, p. 144) said that to identify power relations was "to put them back in the hands of those who exercise them" and that is my hope for this chapter – that mathematics educators recognize the ways in which our words and practices exercise power in the shaping of how we -- and our students -- think about young children.

CHAPTER SIX

PROBLEM SOLVING AS PERSUASION

"We problem solve because we're problem solvers."
-- George Bush, in the aftermath of Hurricane Katrina

Many references to problem solving in education literature contain little more information or complexity than this tautology offered by the president. In the *Principles and Standards* (2000) written by the National Council of Teachers of Mathematics, the authors told us that "one cannot solve problems without understanding and using mathematical content," (p. 7) and that "problem solving in grades 6-8 should promote opportunities for mathematical learning" (p. 256). The authors of *Adding It Up*, a report on the state of elementary mathematics, contribute the statement that "problem solving ability is enhanced when students have opportunities to solve problems themselves and to see problems being solved" (2005, p. 420). It is difficult to take exception to or to be surprised by the arguments, frequently made by those advocating reform mathematics, that in order to solve math problems one has to use math; that students should learn math when doing problems; or that to learn to solve problems students should solve problems and see people solve problems. Few have argued, recently or in the past, that students in mathematics should not do problems or that they should not learn from them. This begs the question of why there has been such a great need to make arguments in favor of problem solving and problem-based teaching when there is so little evidence of anyone seeking to do away with the use of problems in school mathematics.

In fact, the argument in the literature is not actually about the need for problem solving in mathematics, but about the *kind* of problem solving that students should be doing. This can be seen in the modifying adjectives many writers attach to problems and problem solving in an attempt to differentiate the kind of problems they are talking about from those described by some unnamed other. For instance, there are "real problems" (Ball, 1988, p. 41), "genuine problems" (Hiebert et al, 1997), "non-routine problems" (Adding it up, 2005, p. 126), "contextualized problems" (Ibid, p. 327), "challenging, but accessible problems" (Ibid, p. 412), "interesting and well-selected problems" (NCTM, 2000, p. 182), and "rich and appropriate problems" (Ibid, p. 185). These adjectives might be taken together to describe desired problem solving -- with routine, unchallenging, boring, and fake problems as the opposite kind, although arguments in favor of these sorts of problems are hard to find. In addition, most of the reform literature argues that the good kind of problem solving is that which has content embedded in it. That is, problem solving should not be taught as separate from content, such as number, geometry or data, but used in the teaching of these mathematical strands (NCTM, 1991, 2000; NCR, 2001; Romberg, 1999). The purpose of this chapter is to examine from a rhetorical perspective the kinds of problems used in the elementary class that Diana and Sara taught and the methods class Sara attended. My aim is to provide new ways for thinking about the kinds of problems educators use. By looking at problems as means of persuasion, I hope to break out of binaries, where regardless of the particular adjectives chosen (rich/boring, challenging/routine, genuine/decontextualized) some problems are seen as good and others as bad. Instead, I want to offer new language for talking about the work that problems are doing in mathematics classrooms.

If we see teaching as an act of persuasion designed – to list a few possibilities -- to cause students to adopt a belief, accept an idea as true, practice a skill, or feel pleasure while working (Fendler, 2006), then we can see problems as one of the means of persuasion available to teachers. They are not, by any stretch of the imagination, the only means available. Teachers could lecture, demonstrate, model, ask students to read expository text, or employ a host of other pedagogical practices. Aristotle called rhetoric as "the faculty of observing in any given case the available means of persuasion" (Book I, Chpt. 2, ln. 1). This is one way of seeing the work of teaching – surveying and choosing available means to achieve desired ends. In the case of the third grade classroom, the desired ends were spelled out pretty clearly for Diana and Sara by the Grade Level Content Expectations offered by the state, the Pacing Guide provided by the district, and, should there be need for further guidance, the standards offered by NCTM. These documents asked that third-graders be persuaded to do a number of things, including recognize benchmark temperatures, read thermometers, use multiplication to describe grouping situations, know multiplication facts from memory, talk about mathematical ideas, explain solution methods, and make and test conjectures (Lansing School District, 2005). The extent to which these documents were persuasive to Diana and Sara is another question, and one that is laid aside in this chapter to focus on other issues. Certainly, both Diana and Sara did refer to these documents in their lesson plans and in conversations with me, and many, if not all, of the lessons taught in their classroom could be reasonably described as working toward one of the goals stated in the standards documents. David, the university methods instructor, laid out his desired outcomes in his syllabus. He said he wanted his intern-year students to "make problem solving an important part of their mathematics curriculum," to help children realize "that mathematics is all around them," and to understand "important mathematical concepts." In setting out these goals, he also referred to the state and NCTM standards, and, like Diana and Sara, routinely used problems as a means of persuasion in his classroom.

Problems as Persuasion

This chapter examines three kinds of problems used in both the elementary and methods classrooms, which differed in the arguments they made and in their modes of persuasion: *practice problems*, *process problems* and *concept problems*. I want to point out here that I am not examining *how* teachers persuaded their students to complete these problems. That is, I am not interested in how the teachers got their students to do their work in the classroom. Rather, I am naming the argument that each of these problems makes in relation to mathematics and examining the means by which the problems work to persuade students. To do this, I adopted some analytic tools from Aristotle, who described three modes of persuasion open to speakers: logos, or logical argument; pathos, or appeals to emotions; and ethos, or authority of the speaker and the context. Examining these modes of persuasion allows differences in problems other than those typically described (genuine or false; challenging or routine) to be discussed. The following section of this chapter looks closely at each of these kinds of problems, identifying key features, the arguments being made, and the modes of persuasion.

Practice Problems

The problems used in the elementary and methods classrooms that I identified as practice problems are those many adults today would recognize as having made up the primary mathematical work of their childhoods and are probably what many mathematics educators have in mind when they issue calls for different kinds of problem solving. These problems tended to be short, mostly (although not always) presented as "naked numbers," assigned in sets, evaluated for correctness as well as speed, and little discussed. Diana and Sara both frequently assigned practice problems in the elementary classroom. Sometimes they were a feature of the textbook, which began each lesson with a "quick review," where students were asked to solve four or five similar problems, such as these:

1. 7 x __ = 42
2. ___ x 5 = 45
3. ___ x 8 = 32
4. 9 x ___ = 63 (Andrews et al., Hartcourt School, p. 368).

Sara, who taught the lesson where these problems were assigned, responded to students both in terms of the correctness of their answers and the speed with which they offered them. In the following episode, Sara asked Mia to solve the first problem.

Mia: (almost instantly): Six.
Sara: Six. Good for you. See how quick Mia was with that. Did she have to skip count in her head or use her fingers?"
A few children said "no." And Sara added: "No::."
Sara: She knows that fact and you should all be *knowing* your facts. Good job, Mia.
Sara called on Ashanti next, after pulling her name out of a jar.
Ashanti: (after about ten seconds) "Nine."
Sara: "You're right. Good for you. Did you skip count in your head there?"
Ashanti nodded slowly.
Sara: That's an *okay* way to do it, but you should still know the facts because it's way faster.

Unlike her responses to students' solutions of other kinds of problems, here Sara did not explicitly value varied solution methods. In fact, rather than asking Mia and Ashanti to explain how they solved the problems, Sara assumed what their process had been and evaluated their effectiveness in terms of speed as well as accuracy. The role of speed in defining practice problems can be seen both in teachers' attention to how quickly students completed the tasks as well as in the number of tasks given. In the lesson above, Sara expected students to solve four problems in about four minutes. Timed tests of addition, subtraction and multiplication facts were another common site for practice problems in the elementary classroom. During these activities, students were expected to complete a hundred problems in five minutes. Students tried each week to complete a higher number of correct problems than in their previous attempts.

Although most practice problems involved one of the four operations emphasized in elementary mathematics, some did not. For instance, over the course of the year students completed sets of fifteen to twenty problems that involved reading thermometers, measuring lines, and identifying shapes. The lack of discussion surrounding these problems, the length of time given, the number of problems assigned, and their repetitive nature marked these as practice problems despite non-operational content. David assigned practice problems less often in the methods classroom than Sara and Diana did in the elementary school. However, he did on a few occasions engage his students in mental arithmetic, where he flashed a problem on the overhead for ten to twenty seconds and expected students to rapidly compute an answer. As in the elementary classroom, these problems, such as "28 x 31 =" and "46 + 87 =" were relatively simple for students to interpret and each problem in the set was quite similar to others. Like Diana and Sara, David explicitly valued speed when assigning these kinds of problems. David called on Sara after flashing a two-digit addition problem on the overhead. Sara reported her answer (which was correct), saying that she had just carried the numbers in her head. David replied: "But that's not why we're doing it. It will take *longer*. You just did it by the algorithm, mentally. I'd like you to get good at doing it the other way." It would be possible to assign problems for mental arithmetic and to not have them be practice problems. For instance, conversation afterward might focus on ways to add that reinforced important place value concepts. However, here, David rejected Sara's method, not because it lacked explanatory power, but because it took longer.

If practice problems are a *means* of persuasion, then the argument they are making is that a certain kind of mathematical knowledge needs to be automatic, inexplicit, and unthinkable. The reading of thermometers, single-digit addition and subtraction for third graders and double-digit addition for beginning teachers are not problems that should be figured out but things that, in Sara's words, you need to "be *knowing*," a statement with which the Lansing School District Pacing Guide concurs. Van de Walle, author of the most widely-used elementary mathematics methods textbook, wrote about fact mastery in terms of students "recalling the fact without being conscious of using a strategy" (Van de Walle, 2006, p. 183). Mathematician Wu, who probably would not agree with much in Van de Walle's reform-oriented methods textbook, took a similar stand on basic facts, writing that "the automaticity in putting a skill to use frees up mental energy to focus on the more rigorous demands of a complicated problem" (1999, p. 2). Practice problems are a means to an end; in order to pursue higher mathematics some mathematical acts must become automatic and unconscious so that attention can be turned to other things.

If this is so, we can then ask about how these problems go about persuading students to make certain knowledge bits (multiplication facts or mental arithmetic processes) unconscious and automatic. The logical argument behind these problems is that if students think a thought over and over again, it will soon become automatic. This is mathematics as athletics. Hawhee (2002), a professor of English, talked about the learning of rhetoric as "a bodily art: an art learned, practiced, and performed by the body as well as the mind" (p. 144). Similarly, the knowledge intended to be created through practice problems is a bodily art, where the response to a presented problem flows automatically from the pen, rather than the mind. Like learning to shoot a basket or hit a baseball, one small skill is practiced over and over, leading to mastery. The argument is not this these problems are real mathematics, any more than taking swings in a batting cage is real baseball, but that repetition will lead to habits that will be useful in other pursuits. A student who completes a sheet of ten multiplication facts each day may be persuaded to automatically recite that two times five is ten.

The emotional aspects of persuasion related to practice problems do not need to be vocalized by either teachers or students in order to play a role in convincing students. When doing long sets of practice problems, students routinely checked their own work against that of neighboring students. Generally, this checking focused on the quickness with which problems were completed rather than the answers being produced. Once while working on a set of division problems, Aliah looked over at Mia's paper and said: "Dang. Mia's on 21. I'm on 17. I do *not* get this." During another assignment, Jerome finished a worksheet filled with problems on reading thermometers and looked around the room. He noticed that nearly everyone else was still working and commented. "I'm smart at math today!" In both cases, Aliah and Jerome's emotional responses to the assignment centered around speed rather than correctness, even though neither Sara nor Diana made explicit reference to finishing quickly on those particular assignments. Because speed was an important criterion on practice problems, students could feel pride (or shame) by monitoring their own progress in relation to their peers without public acknowledgement (or criticism) from the teachers. The emotional payoff of pride for finishing quickly was available to anyone who worked steadily through an assignment and worked to persuade students to come to know certain information by repeatedly doing similar problems.

Practice problems drew on a number of accepted norms to persuade students to come to know certain mathematical facts. The ethos of the elementary school in particular is central to the persuasive power of practice problems. For example, the student math book was filled with practice problems. In addition to the quick review on each page, nearly every page offered a set of ten to twenty similarly-constructed problems. Although these problems could possibly have been assigned for purposes other than practice, the large number of problems and the relatively straightforward directions on each page clearly marked them for this purpose. In addition, students came in to the third grade expecting to learn their multiplication facts. Many reported using flash cards at home and told stories about their older brothers and sisters learning the multiplication facts. That these sorts of problems were an entirely expected part of third grade mathematics played an important role in the problems persuasiveness. Students did not question having to do these problems over and over again. The repetition confirmed their expectations. This was also true in the methods class, where interns routinely challenged the instructor when doing problems related to proofs about prime numbers or algebraic explanations of number games, but never asked David why they needed to be able to add two-digit numbers in their heads. The expectedness of the problems was in and of itself persuasive.

Process Problems

Unlike practice problems, process problems were not concerned with getting students to know a particular bit of mathematical information, but with causing them to execute certain ways of thinking that are currently seen as mathematical, such as organizing information, finding patterns, explaining one's reasoning, and using multiple solution methods. Although Diana and Sara did not name the other kinds of problems used in their classroom, they did call these "problem-solving problems." Similarly, David would mark these problems as separate from the other work that went on in the methods classroom by announcing that they were about to do "problem solving." These problems could be recognized as a kind because they were planned, were not linked to any particular mathematical strand, and were always followed by discussions that focused on solution methods rather than answers. In addition, when introducing these problems, teachers called on students to solve them in multiple ways, unlike when introducing other kinds of problems. A couple of examples of process problems from the elementary classroom include:

- You have 50 carrots broken into 2 pieces, 15 celery stalks broken into 4 pieces, 9 potatoes cut into 10 pieces and 2 onions cut into 50 pieces. How many whole vegetables did we have? How many vegetable pieces did we have?

- If A = 1 cent, B = 2 cents, C = 3 cents and so on, what is the value of your first name? Are there any words worth $1.00?

And from the methods classroom:

- In a room with 20 people how many handshakes would it take for all the people to meet with each other? What if there were n people?

- During a census, a man told the census taker that he had three children. When asked their ages he replied, "The product of their ages is seventy-two. The sum of their ages is my house number." The census taker turned ran outside to look at the house number and said, "Using the information you have given me, I cannot tell their ages." The man said, "I should have told you the oldest likes angel food cake." Hearing this, the census taker promptly replied, "Now I know their ages!" What was his solution?[4]

For each of these problems, students were encouraged to solve them in multiple ways, and, in fact, much of the discussion afterward focused on identifying the variety of strategies students used. In both the methods and the elementary classroom, students were called to the front of the room to present their work, which never happened with practice problems and rarely occurred with concept problems. During these kinds of discussions, the most common teacher question was: "Who has another way to solve this problem?" Unlike practice problems, the revelation of the answer was not considered the final word.

That these problems were planned was also an important identifying characteristic. Unlike practice problems or concept problems, these problems were

[4] For those of you scratching your heads over this one, David's intended solution involved students generating sets of three numbers where the product was 72. The sets "2, 6, 6" and "3, 3, 8" both result in a sum of 14 (presumably the house number); thus, the census taker's confusion. The angel food cake clue is intended to signal that there *is* an oldest so the answer must be "3, 3, 8."

always written ahead of time -- in plan books and on overheads for Sara and Diana and on handouts for David; these problems were never produced on the fly in either classroom. For the most part, these problems were also not written by the teachers ahead of time, but culled from other sources. In fact, many of these problems have a handed-down feel to them, which David referred to when he introduced these sorts of problems, saying they were for "special lessons. And you've got to have file of these problems for these special lessons, like when someone's coming in to observe you and you want to show real problem solving." The "handshake problem," which David used in the methods class, is a particularly clear example of a "special problem." The authors of *Adding It Up* (2005, p. 107) used the problem as an example, noting that it is one of the more popular problems found in the mathematics education literature. It can also be found in the Van de Walle text, in a lesson shown on the instructional *video Powerful Practices in Mathematics and Science* put out by Carpenter & Romberg (2004), and in many other books, articles, and curricula about problem solving (e.g., Krantz, 1997; Slavit, 1999; Yarema, Adams & Cagle, 2000). With the guidance of a teacher, young students may be able to find an algebraic expression or a numeric pattern that predicts answers. However, the problem is not generally recommended for use with middle or high school students, who may be working on formalizing patterns algebraically, but with much younger children. (The lesson in the video involves third graders; David suggested it was appropriate for children in the primary and intermediate grades.) This problem helps to illustrate the focus on processes rather than mathematical concepts in these kinds of problems. (Of course, the problem could be used for content-oriented purposes in high school or undergraduate mathematics classes.) If the purpose of the handshake problem was to teach a concept, we might reasonably ask what content this problem is supposed to teach. Certainly to solve the problem children would have to understand addition and potentially numerical patterns. It could be used, with guidance from the teacher, to build algebraic reasoning. However, this problem's prevalence in the elementary mathematics education literature seems out of proportion with its usefulness in getting at any of these concepts. In fact, the authors of *Adding It Up* speculate that the problem is seen so often "because it can be solved in so many ways" (2005, p. 107). This virtue has little to do with the problem's effectiveness at revealing important mathematical ideas within a particular strand, such as algebra, number or geometry, and much to do with the learning of problem solving as a domain in and of itself.

Similarly, the vegetable soup and adding up names problems used in the elementary classroom were not designed to teach students particular mathematical concepts. These problems were assigned in the first weeks of the year, when Diana alternated between doing these kinds of problems and reviewing for the state's standardized test, which was scheduled to be given in October. Although each of these problems *could* have been used to teach particular mathematical knowledge, this was not the focus of the discussions that occurred after problems had been solved. In the vegetable soup problem, Diana called four students to the board whom she had determined had each solved the problems in different ways. The first three students had grouped numbers in various ways to add. Ben had used multiplication. In commenting on Ben's solution, Diana said:

> Ben, you did this in a really sophisticated way. But that might bring it back to the other people. This would be a wonderful thing to take notes on because Ben did something that most of the rest of you didn't do. Because you're in third grade

> and you haven't done times a lot ... I'm thinking a lot of third graders in early October, you don't know any times yet and that's okay. But I still thought this was a problem you could do, and some of you did do it in different ways. That's what matters.

Here, Diana did not use Ben's solution to build on the connection between multiplication and addition, as she and Sara would do later when they started the unit on multiplication. Instead, Diana reinforced that there were many ways to solve the problem. She valued Ben's solution for being "something that most of the rest of you didn't do," and suggested that students take notes on it for this reason, even while noting that most of the children probably would not have enough experience to understand the multiplication. She then went on to discuss other children's solutions, in each case highlighting the strategies students used, such as skip counting, drawing pictures or grouping numbers, which made them different from the previous ones.

The argument these problems make is that the way one goes about solving a problem in mathematics is as important as the answer produced. Process problems seek to persuade students both to *do* certain things, such as to record their thinking, to explain their reasoning in words, and to interpret complicated problems; and to *believe* certain things, such as to value multiple solutions, originality, and showing one's work. The logical argument here is that if a problem is complicated enough students will not solve it in identical ways. Thus, the complexity allows students to see multiple solutions, to pursue originality, and to perfect particular mathematical strategies or behaviors. Unlike practice problems, which draw on athletic metaphors, process problems present mathematics as a simulation. Students cannot actually do the work of mathematicians, because they are not working on the edge of what is known; however, process problems are designed to create an environment where students feel and act as if they are doing real mathematics. These simulations of mathematical work allow the behaviors and beliefs revealed to become objects of discussion, which contributes to the problem's ability to persuade students to think and act in certain ways.

As in practice problems, feelings of pride and shame are drawn on as ways of persuading students to adopt desired beliefs and to engage in expected processes, although the way these emotions are incited is different. In process problems, students are encouraged to feel pride when their work is revealed in public, especially if it shows an original (or "sophisticated") solution. When Diana called for volunteers to demonstrate their answers to the vegetable soup problem, many children who raised their hands were turned down because their solution was "the same as" someone else's. Denying them the opportunity to show off their work for this reason served to motivate students to attempt something original in the future. An additional emotional aspect of persuasion in these problems involved the ability of the problems to invoke feelings of "smartness" or "dumbness" in students. Much more so than the practice problems, process problems seemed to appeal to feelings of competence. This may be because all students were able to have some success with practice problems, so that the speed with which they were completed marked only degrees of competence. Whereas with process problems, students who could not figure out what to do often had extreme emotional reactions. This was true in the both the elementary classroom and the methods classroom, where, after reading the angel food cake problem, one intern remarked: "I haven't the faintest idea what to do. And so now I feel dumb … in math … again." In contrast, this same student's inability to compute the mental arithmetic problems as fast as David expected produced genuine laughter on her part. The teachers also had different expectations for competence with these kinds of problems. When doing practice problems as a class, Diana and Sara drew names randomly out of a jar to call on students, which demonstrated that they expected all students to be able to solve these problems. However, with process problems the sticks were never used and often Diana and Sara checked a students' paper before asking him or her to contribute to the discussion, communicating that all students were not expected to be able to solve these problems, or at the very least that they were not expected to do so in a way that would be interesting to the rest of the class.

The ethos of process problems was probably not as persuasive to children as that of the practice problems. Using Bahktin's language, process problems have the "taste" of the university about them. These problems often trickle in from universities, either from reform curricula written by mathematics educators or else from more informal interactions. In fact, of the four lessons I observed that were centered on process problems, two were introduced by university seniors who had been encouraged to use the problems by their methods instructor. Process problems do not look like the mathematics expected in the elementary classroom. They are not straightforward algorithms; nor are they the typical three-sentence word problems seen most commonly in traditional textbooks. Neither the third graders nor the interns saw these problems as expected parts of the elementary mathematics curricula, and as a result, teachers in both contexts had to do some extra work to persuade students to engage in these problems. Diana and Sara did this by linking these problems with appealing activities, such as making vegetable soup or eating watermelon (after doing a problem that required estimating the weight and length of the fruit). Diana also made explicit statements to children about the value of these problems saying things like "with these problems you have to really use your brain!" David made similar appeals to the interns, describing the angel food cake problem as "not a real-world problem. This is a really-get-you-thinking problem." Because process problems were not an expected part of mathematics class, teachers had to draw more explicitly on their own authority as experts when assigning these kinds of problems.

Concept Problems

Concept problems were the most difficult to identify because they had much greater diversity in their textual features than the other two types and because they revealed themselves as concept problems only within the contexts of particular lessons. They are also a kind of problems that seems to disappear from binary discussions of problems that compare only practice problems and process problems. Concept problems are not designed to make certain knowledge automatic like practice problems; nor are they aimed at promoting particular mathematical strategies or beliefs. Concept problems were designed to helped students build understandings of particular bits of mathematical content, often by building representations, vocabulary, or connections among ideas. Concept problems were typically assigned during a particular unit of study in the elementary classroom or as part of a lecture on a particular topic in the methods classroom. Sometimes they looked like process problems, as in these examples:

- A million seconds is about 11 days. How many days do you think a trillion seconds would be? (methods class)

- You have 3 different shirts and 4 different pants. How many outfits can you make? (elementary class)

- Mrs. Emerson bought 36 cookies with green frosting. She wants everyone in our class to get an equal amount. How can she do that? Show your work. (elementary class)

Sometimes they looked like practice problems:

- Which is greater: $\frac{34}{55}$ or $\frac{55}{89}$ (methods)
- $21 \div 7 =$ (elementary)

And sometimes they didn't look like problems at all:

- How can you show children that you can't divide by zero? (methods)
- Why do people usually start adding from the right? (elementary)

What all these problems have in common is that they were aimed at helping students to build particular concepts. Rather than valuing diverse ways of solving the problems, the teachers in each of these cases were looking for students to call on particular ways of representing or talking about the mathematics and used the discussions afterward to work toward this purpose.

Concept problems seemed to embody the calls in Standards documents to teach mathematics through problem solving (NCTM, 1991, 2000); however, these problems did not often provide opportunities for students to solve problems in original ways or to practice particular strategies, like organizing data or finding patterns. Because teachers working on concept problems used the discussion time to build ideas related to the content under study, mathematical processes often took a backseat. Students were encouraged to solve concept problems in similar ways so as to build understanding of the content under study. In the following episode, which occurred during the first week of a unit on multiplication, Sara gave students the problem: There are three flowers, each with six petals. How many petals were there?" She asked students to represent their answer in three ways.

After being called on, Ben reported that he did six times three and got 24. Sara expressed surprise and then Ben corrected himself, saying "No, I mean 18." Sara wrote the expression "6 x 3 = 18" on the board and asked students how else they had represented this problem. Charlie suggested counting by sixes.

> Sara: Skip count by sixes, if you're good. It's kind of hard. That's kind of a high number to skip count with. But you could go 6 … 12 … 18.
>
> Sara: How else can you *write* it?
>
> Chacoria, Charlie, Khassan, and Evan raised their hands, and Sara called on Chacoria, who said "Three groups of six."
>
> Sara: Right. Cause we have three groups of six (Sara wrote "3 groups of 6" on the board.) You can easily say three groups of six. I can think of two other ways to write what you have up here.
>
> Caitlin and Khassan raised their hands. Jennifer called on Khassan.
>
> Khassan: Count by twelves.
>
> Jerome (calling out): Count by *twelves*?
>
> Sara: Are there an even number of twelves in there?
>
> Khassan: We already know the six plus six equals twelve, so-
>
> Sara: Whoa, whoa, whoa. We can only skip count if there are an even number of those things. Do you want to think about that a little bit, Khassan? (turning) Caitlin?
>
> Caitlin: You know six and six is twelve. So you can do twelve plus six is eighteen.
>
> Sara: So Caitlin does know that six plus six is twelve (gesturing at picture) so she's going to make these two twelve. And she's going to add the last six *to* twelve. You *could* do that.

Aliah, whispering to herself: Six plus six plus six.

Sara: What Aliah?

Aliah: Six plus six plus six.

Sara (writing it on the board): What do we call that?

Many children, including Aliah: Repeated addition.

Sara: Good. How about you guys do one on your own?

In the two lessons previous to this one, students had been asked to draw pictures of multiplicative situations and to write an addition sentence, a multiplication sentence and a word sentence to describe the picture. According to Sara's lesson plans, the goal of these lessons was to help students develop an understanding of multiplication as a process of adding equal-sized groups. In the episode above, Sara responded in different ways to answers that used the vocabulary of the previous days' lessons and those that did not. Charlie suggested skip counting, and Khassan and Caitlin pursued this idea. These answers could be considered original in reference to this problem because most children in their journals used one of the representations that the class had used the previous day. However, unlike with process problems, originality was not publicly valued here. Sara cut Khassan off as he attempted to articulate his thinking about the skip counting. She acknowledged Caitlin's explanation of the solution as correct, but did not record it on the board. Nor did she record Charlie's skip counting by sixes. Instead, she responded to Aliah's whispers in order to get repeated addition on the board because the connection between multiplication and repeated addition was an important part of this lesson. After solving the problem in this way, Sara assigned a similar problem to the students. As they worked, she encouraged them to produce as many of the representations that had already been produced as possible, asking questions like: "Can you show it with a picture? What would the addition sentence look like?" The focus of the instruction was on building links between various representations of multiplication rather than on problem-solving strategies. This was not because Sara was unable to focus on mathematical processes like reasoning and justification. She had learned from Diana to ask questions like: "How did you get that answer?" and "Who thought about this in a different way?" however, in this lesson, she did not ask these kinds of questions.

Over time, as children became more familiar with multiplication and the kinds of story problems that are typically used to represent these ideas, problems similar to the flower petal problem came to be treated as practice problems. During early lessons in the unit, Sara did not encourage students to solve problems quickly. She rarely assigned more than two or three problems in a 45-minute lesson. As students worked, she asked them to go back and count, to draw lines between numbers and pictures, and to think about how to describe the problem in words. However, a few weeks later, problems like these did become practice problems for most students. Toward the end of the multiplication unit, Sara passed out a worksheet of ten story problems all dealing with multiplication. Although students were asked to "show their work," most completed this worksheet quickly and none of the problems were discussed as a whole class.

The logical argument behind content problems is that important concepts in mathematics are best learned through solving problems that related to those concepts. In our time, this seems like an obvious argument; however, in the early days of mathematics education, educators believed that students learned mathematics deductively, that is, by following a step-by-step argument. Edward Thorndike, an early mathematics educator whose name has become synonymous with skill and drill, provided examples of both deductive and inductive ways of teaching mathematics in his book, *The Psychology of Arithmetic*. A deductive explanation of multiplication (for the problem 623 x 3) would be:

> For convenience we write the multiplier under the multiplicand and begin with units to multiply. 3 times are units are 9 units. We write the nine in the tens' place in the product. 3 times 6 hundreds are 18 hundreds, or 1 thousand and 8 hundreds. The 1 thousand we write in the thousands' place and the 8 hundreds in the hundreds' place in the product. Therefore, the product is 1 thousand 8 hundreds, 6 tens and 9 units, or 1869 (Thorndike, 1922, p. 61).

In contrast, Thorndike argued that inductive ways of learning mathematics would be more effective. For example, he described an inductive method of teaching with multiplication as beginning with a problem like "The children of the third grade are to have a picnic. How many sandwiches will they need if each of the 32 children has four sandwiches?" (Thorndike, 1922, p. 61). He suggested following problems like this up with multiplication problems written in algorithmic form, in words, and as repeated addition. The process he described is not so very different from what went on in Diana and Sara's classroom; nor is it so very different from the argument that many advocates of problem solving make today. In their book on Cognitively Guided Instruction, Carpenter and his colleagues offer problems like: "Our class has 5 boxes of doughnuts. There are 10 doughnuts in each box. We also have 3 extra doughnuts? How many do we have all together?" (Carpenter, et al, 1999, p. 60) as a strategy for teaching multiplication to children. The extra three doughnuts make this problem slightly more complicated than the ones offered by Sara and Thorndike; however, the heart of the argument, that children learn mathematical concepts through working problems that connect those concepts to concrete representations (flowers, sandwiches, doughnuts) remains the same. To be sure, Carpenter and his colleagues advised that students have plenty of time to invent algorithms before being introduced to the formal procedures, but the differences in beliefs behind how children learn do not seem to be as significant as are sometimes described by current advocates of problem solving (NCTM, 2000) or algorithmic practice (Wu, 1999).

Concept problems based their emotional appeal by linking feelings of pride to inclusion and shame to exclusion. Because the goal of these problems was often to develop shared representations of particular ideas, students worked much more collectively on these problems than on others. In the counting petals problem, Sara wrote some solutions on the board, but not others, despite the fact that they were equally correct. The desire to be included in the classroom community for these problems was a mode of persuading students to offer expected and shared representations of the mathematics. This was as true in David's classroom as in the elementary one. For example, when he asked his students to develop a child-friendly argument for why division by zero was impossible, he did not break students into groups or partners, as he typically did when doing process problems. Instead, he asked students to work independently for a few minutes and then to share ideas with the class. He guided students toward two arguments, one based on repeated subtraction and one based on the impossibility of imagining zero groups of six. He outlined these two arguments on the board as people added to them, but did not make notes about an argument based on arrays put forward by one of the students. Inclusion and exclusion were also promoted through the use of "we-oriented" language. For instance, when Chacoria and Aliah responded to

Sara with expected representations, Sara replied to them with "we" statements. However, when Caitlin made her argument about skip counting, Sara located it as something that *Caitlin* was doing, although it would work. Similarly, if you go back and look at the conversation presented in the section on practice problems, you will see that solutions are attached to individual students, rather than a "we."

As a mode of persuasion, the authority of the teacher was drawn upon most in concept problems. Much more than with the other two kinds, Sara, David, and Diana were likely to write these kinds of problems in the midst of a lesson, rather than relying on plan books or texts. Often these problems were written to address a misunderstanding or perceived need on the part of students. For instance, once in the methods class during the discussion of a process problem, a student suggested comparing two fractions by seeing how far apart the numerator and denominator were. David stopped the discussion about the process problem and wrote on the overhead: "Prove or disprove Hannah's claim." This problem was designed in the moment, in response to a particular need; it was not a "special" problem that needed to be taken from a file. The responsiveness of the teacher in assigning these problems became a way of persuading students to learn the concepts because the problems could be produced just in time. Similarly, in debriefing discussions, the teacher's authority on what was mathematically useful was much more in play during these kinds of problems. Practice problems tended to have single answers, so there was little role for the teacher to differentiate between subtle interpretations. In process problems, virtually any student action might have been picked up as mathematically interesting as long as it was different. But in concept problems, Diana, Sara and David generally made decisions ahead of time about which representations or ideas would be most pedagogically useful and they used their authority as teachers to focus student attention on these predetermined ideas.

The Alchemy of Problem Solving

In discussing school subjects, Popkewitz (2002, p. 262) argued that disciplines, such as mathematics, science and music, undergo a transformation in schools so that the mathematics taught in classrooms is no longer the discipline of mathematics as practiced by mathematicians, but an entirely new discipline, that of school mathematics. He wrote:

> [A]n alchemy occurs as the knowledge of an academic field moves into the school. School subjects are organized in relation to expectations related to the school timetable, conceptions of childhood, and organizational theories of teaching. The question of academic or disciplinary fields is transmogrified into school psychologies of instruction and theories for changing the dispositions and characteristics of the teacher and child. The magic of the transformation is to reconfigure the academic fields in schools so that only the namesake appears, as a ubiquitous doorplate to mark a house.

Just as entire disciplines underwent alchemy when they were brought into the schoolhouse, I want to argue that the practice of problem solving underwent a similar alchemy. The problem-solving that mathematics educators write about today is not a practice that has always been present in schools or in mathematics, but one that emerged at a particular time in history through the work of researchers in mathematics education.

The presence of this new kind of problem solving can be seen in the three kinds of problems that showed up in the classes I observed. My identification of practice, process and concept problems is not an odd artifact of the particular contexts I studied, rather it is a reflection of the broader discourse in mathematics education. The existence of this new kind of problem solving – where the processes that students use to solve problems are considered as much a part of the content to be learned as the mathematical ideas the problems address – is deeply entrenched. Process problems are assigned in schools because practice problems and concept problems are not seen as adequately addressing the content entailed in this new conception of problem solving. Initially, this statement may not seem credible because of quantity of statements made that argue that problem solving cannot be separated from mathematical content. For instance, Van de Walle (2004, p. 38) wrote: "It is important to understand that mathematics is to be taught *through* problem solving. That is, problem-based tasks or activities are the vehicle by which the desired curriculum is developed." Similarly, in the section on problem-solving, the writers of the *Principles and Standards* said that problem solving "should not be an isolated part of the mathematics program" (p. 52). However, if one looks more closely at how these (and other) texts are set up, at the type of problems discussed, and at what is expected to be learned from problem-solving, the claim of the inseparability of problem solving from content becomes less credible. And, it is in this separation of problem solving from content that the emergence of this new kind of problem solving can be seen.

For instance, although the *Principles and Standards* describes problem solving as intertwined with the learning of mathematical content, the document separates discussions of problem solving from discussions of content. This means that in the discussion of mathematics teaching in the primary grades, number and operations is discussed from pages 79 to 88, geometry from pages 96 to 101 and problem solving from pages 116 to 121. If problem solving was truly intertwined with the learning of content, there would be no need for a separate section because there would be very little to say without a content area to frame the discussion. In the section on problem solving, the writers recommend that children learn "to develop a broad range of problem-solving strategies, to pose (formulate) challenging problems, and to learn to monitor and reflect on their own ideas in problem solving" (NCTM, 2000, p. 116). Similarly, both the Michigan Grade Level Content Expectations and the Lansing School District's Pacing Guide list "problem solving" as a separate content area from number, data, and geometry. All of these documents expect students to learn a particular content *about* problem solving. In the discussion of problem solving in the middle school, NCTM describes this content as building "important problem solving dispositions – an orientation toward problem finding and problem posing; an interest in, and capacity for, explaining and generalizing; and a propensity for reflecting on their work and monitoring their solutions" (Ibid, p. 258). To achieve these goals teachers need ways of monitoring students' dispositions as well as knowledge, and become likely to select problems for use in their classroom that make this monitoring possible.

It would probably be impossible to pinpoint exactly when this new kind of problem solving emerged, but there can be little doubt that Polya's (1957; 1968) work on problem-solving heuristics was an important turning point. In a review of research on problem solving, Kilpatrick (1985) claimed that "everyone in mathematics education who works on problem solving must come to terms with Polya's view of problem solving" (p. 7). Polya (1968, p. x) called heuristics "the study of means and methods of problem solving" and argued that in addition to information (or mathematical content knowledge) students should be taught "know-how" (or problem-solving strategies). In his work, Polya outlined a general series of steps that problem-solvers might go through to solve a problem, no matter what the content, including understanding the problem, devising a plan, carrying out the plan and looking back. The articulation of these steps led to research that examined the impact of explicit teaching of heuristics and other general problem-solving strategies to children. Cyert (1980, p. 5), in setting a goal for this line of research, wrote "it would be highly desirable to have a way of teaching the problem solving process." One of the results of this work in classrooms was the emergence of problem-solving posters, which outlined the steps of successful problem solvers. Students could be expected to memorize these steps and to produce evidence of them in their solutions to mathematical problems. In addition, they could be assessed on their ability to problem solve, separately from their abilities to articulate understandings about number, geometry or algebra. Although Polya studied the ways that mathematicians solved problems in order to articulate his heuristics, few mathematicians have posters of problem solving steps on their walls which they move through in linear ways. In his journalistic account of mathematician Andrew Wiles' struggle to prove Fermat's Last Theorem, Singh (1997) described Wiles' solution path as including long pauses in the work, walks through the park, and pursuits of seemingly unrelated mathematical ideas. Many mathematicians might relate to these strategies, but it unlikely any of them will become codified in schools as part of the curriculum of problem solving. This anecdote points to one of the ways that we can understand school problem solving as a different element than problem solving in the discipline of mathematics, for all that researchers might draw connections between the two.

Lester (1980, p. 302), who reviewed the body of work that explored the teaching of the problem solving process, listed the key problem solving behaviors identified by mathematics education researchers: "divergent thinking, blind guessing, identifying a pattern, ability to employ abstract analytical reasoning, making use of drawings and looking carefully at details." Researchers working on heuristics tried to figure out how these processes might be taught to children and to what extent teaching them might support children's efforts to solve mathematical problems. This work, along with Polya's and his followers, helped to create a content area for the teaching of problem solving that was not part of the available discourse when Edward Thorndike and his contemporaries were writing about mathematics education in the early part of the 20^{th} century. For them, mathematical knowledge could only be discussed in terms of the problems students could solve. The way they went about solving them was not an object of study. For Brownell (1938, 1945), a student's inability to articulate how she found the answer to 8 + 3 was not indicative of a lack of problem-solving abilities, but of mathematical content knowledge, which teachers would need to address by building students' understandings of arithmetic. Only after Polya's work did it became possible for teachers to instruct children on a series of strategies, such as making a chart or drawing a picture, in isolation from content in particular problems. The separation between content and process, which was reinforced through studies of problem solving and problem solving standards, created the necessity for claims today that problem solving and content must be interwoven. They would have been impossible to make before this new kind of problem solving, which focused on process rather than content, came into being.

What does it mean that we have this new kind of problem solving? One consequence is that it is now common in conversations about reform mathematics to talk about real problems, non-routine problems, and challenging problems. I believe these words are intended to signify the difference between practice problems on the one hand

and process problems on the other. Concept problems are overlooked because they do not represent extremes in the binary description. Problems that require children to draw on algorithmic or teacher-supported knowledge are seen as bits of potentially essential content, but not as examples of problem solving. Thus, students who can successfully engage with mathematical problems in wide range of content areas can also be seen as lacking *true understanding* if they do not also demonstrate problem-solving skills. Few educators, politicians or parents would expect 8-year-olds to be fluent in the sort of algebraic knowledge required to generalize a solution to the handshake problem; yet, because the problem is seen as requiring problem-solving knowledge rather than content knowledge, it is seen as appropriate for third-graders.

In addition, multiple kinds of problems provide educators with multiple ways of comparing students. While practice problems may not differentiate a great deal among students – only revealing some to be faster than others – process problems offer a wider scale for ranking, with some students providing original, "sophisticated" solutions and others offering those that are the same as everyone else's. Success at process problems shows who is really good at math. Because this new kind of problem solving, which is valorized in discussions of reform mathematics, requires that students reveal *their* thinking (and preferably thinking that is different from their classmates), students may have more difficulty learning how to be successful at this kind of problem solving. It is far easier to teach someone how to represent a multiplication problem as a picture than to teach her to be original.

Another consequence of the creation of this kind of problem solving – where process is seen as separate from content – is that what it means to know mathematics can be mapped in binary terms, with the mathematically correct on one side (practice) and the mathematically sane (process) on the other. Depending on one's perspective, algorithmic practice problems are "simple but sound" (Wu, 1999, p. 2) or "following rules without thinking," (Van de Walle, 2007, p. 166) and process problems that encourage invented algorithms are "potentially harmful" (Wu, 1999, p. 7) or pathways to "rich understanding" (Van de Walle, 2007, p. 219). Because these kinds of problems have been set up in opposition to each other, there is little discussion of the role both kinds of problems might play in the classroom; nor has there been much effort to develop a vocabulary to talk about problems, such as those I call concept problems, that do not fit neatly into a dichotomy between procedural knowledge and problem-solving strategies.

In writing about language, Bakhtin (1981) described authoritative and internally persuasive discourses. He called authoritative discourses "acknowledged truths," and "the official line," while he defined internally persuasive discourses as those we believe to be true even though they are "backed up by no authority at all" (Bakhtin, 1981, pp. 342-344). Bakhtin hypothesized that loud and frequent repetitions of authoritative discourses indicate that the speaker does not find these discourses to be internally persuasive. This notion raises questions about the reasons for the repeated calls to teach mathematical content through interesting, appropriate and genuine problems. This is authoritative discourse in mathematics education; it is the party line, repeated loudly and often. The discourse that is internally persuasive to mathematics educators is less apparent. The practice, process, and content problems identified in this study can be found in many mathematical texts and assessments. In this chapter, I argue that each kind of problem persuades students to adopt a different stance toward mathematics – whether it is making knowledge automatic as in practice problems, adopting mathematical behaviors as in process problems, or developing understanding of new mathematical content, as in concept problems. The alchemy that created the new kind of problem solving emphasizes process problems rather than the other kinds. As a result, good teaching is that which emphasizes process problems and good students are those who successfully solve them. However, the fact that mathematics educators repeatedly separate out content, such as number, geometry and patterning, from problem solving suggests that the community is internally persuaded that other kinds of problems might be equally important. Acknowledging this in the community's party line on problem solving might provide a broader field for defining school mathematics, which might in turn make spaces for more students to see themselves as successful.

Mathematician Wu has been a continual critic of the movement in reform mathematics, particularly those aspects that emphasize this new kind of problem solving, where student thinking and explanations are seen as equally or more valuable than the learning of time-tested algorithms. Wu (1999) has argued that students do not need to make meaning through original invented algorithms, because standard algorithms are standard precisely because they embed mathematical meaning within them. For some time, I have been puzzled by the vitriol that Wu has directed toward the reform mathematics movement. However, I have come to see that while Wu's argument (which echoes Thorndike's) that meaning emerges from repeatedly solving practice problems is counter-intuitive to the way that I came to understand mathematics, it may very well represent how *he* came to make sense of the discipline. If this is the case, then his hostility to reform mathematics and its valorization of "problem solving" (as opposed to the solving of problems) may lie in the fact that his ways of knowing have been denied a place in the party line – however internally persuasive these ways of knowing may be.

I began this chapter by saying that I wanted to draw on rhetoric to find new ways of talking about problem solving. My goal was to show the practice of problem solving as one that emerged at a particular time, rather than as one that has been a part of mathematics since Plato first pulled the discipline down from the ether. By bracketing problem solving in this way and by demonstrating the ways in which other kinds of mathematical work persuaded students to enter into the discipline, I hoped to broaden conceptions of what effective mathematics teaching might be. The greater the consensus is around the kinds of problems deemed appropriate for schools; the louder and more narrow the authoritative discourse; and the fewer the possibilities open to students.

CHAPTER SEVEN

GENRES OF TEACHING

For some time now, teacher educators have been calling on teachers to make substantial changes in the way they work with students in mathematics. Teachers have been asked to create mathematical communities, to promote reasoning and problem-solving, and to allow students to explore their own conjectures (National Council of Teachers of Mathematics, 1991, 2000; National Research Council, 2005). To support teachers in making these changes, researchers have paid a great deal of attention to ways that classroom teachers interact with students. Many of these classroom interaction studies, as well as the discussions about them, have tended to talk about observed classroom episodes as representative of teachers' practices as a whole. In other words, studies have used particular classroom episodes to locate teachers on a continuum between old teaching styles, which are seen as focused on procedures and right answers, and new teaching styles, which are seen as focused on conceptual understandings and mathematical processes. Similarly, procedural classrooms have been described as producing students with algorithmic understandings; whereas, conceptual classrooms have been described as producing students who can problem solve, reason, and justify (e.g., Cobb et al, 1991; Heaton, 2000; Hiebert, Morris & Glass, 2003; NCR, 2004; Stigler & Hiebert, 1999).

Most of these interaction studies have been shaped by a classic anthropologic notion of culture, where features of classroom cultures (such as questioning practices, treatment of right and wrong answers, and use of curriculum) are analyzed to see if they support or work against students' development of meaningful understandings of mathematics (e.g., Boaler, 1998; 2000; 2002; Cobb, Gravemeijer, Yackel, McClain & Whitenack, 1997; Forman & Ansell, 2001; Lampert, 1990; 2001; Lubienski, 2000; McCrone, 2005; Spillane, 1999). In many of these studies, classrooms are presented as having one culture at all times, which has certain attributes. For instance, Lampert (2001) described her classroom as a place where she was working toward the norm that "multiple ideas, not right answers, were the 'coin of the realm' for buying attention" (p. 62). In a large-scale study, Spillane (1999) categorized the cultures of 25 different mathematics classrooms as falling into one of three groups: classrooms that dealt with significant mathematics in ways that supported students' conceptual understandings, classrooms that dealt with significant mathematics, but in ways that often undermined conceptual understandings, and classrooms that focused on procedural skills. Similarly, Boaler (2002, p. 255) contrasted one school culture, which she described as unproductive for children because it "offered a structured, procedural approach," with another culture, which she described in positive ways because it emphasized "open-ended work." These sorts of studies, grounded on the classic notion of culture, make it easy for researchers to see relationships between certain kinds of practices and beliefs and desired outcomes for children. However, this static view of culture has been challenged by many anthropologists, including Rosaldo, who wrote:

> Although the classic vision of unique cultural patterns has proven merit, it also has serious limitations. It emphasizes shared patterns at the expense of processes of change and internal inconsistencies, conflicts and contradictions. By defining culture as a set of shared meanings, classic norms of analysis make it difficult to study zones of difference within and between cultures (Rosaldo, 1989, pp. 27-28)

Rosaldo's critique pointed to some of the assumptions underlying the classic notion of culture. With its emphasis on "shared patterns," the anthropological notion of culture implies that classrooms or societies have a set of unchanging characteristics. Because it supports the description of classrooms in fixed ways, using culture to talk about teaching practices can also encourage hierarchical ways of thinking about cultures and classrooms. For instance, anthropologists might have referred to cultures as being arrayed on a continuum from primitive to sophisticated, while mathematics educators have used the words traditional and reform as well as procedural and conceptual to convey similar ideas. Rosaldo's solution to the dilemma of cultural descriptions that appear stagnant and unitary is to put culture "into motion" (p. 91) by studying the ways in which individuals within a culture cross borders, act in opposition to norms, and vary from one and other.

In this chapter, I am calling on the rhetorical concept of genre to do similar work. I want to examine the diversity that existed within one classroom, rather than talking about the teaching practice there in holistic ways. Instead of characterizing the culture of the classroom I studied, I want to look at different genres of teaching that existed within this single classroom. In particular, I use Bakhtin's (1940, p. 945) notion of *speech genres,* which he defined as the "relatively stable types of utterances" that develop in particular spheres where language is used. (An utterance is one speaker's turn in a conversation.) For example, lecturing, ordering in a restaurant, chatting with a friend, and testifying in court are all speech genres where certain types of behaviors and utterances are expected, but not others. In each of these situations, we do not choose what to say from the infinitely-possible ways of arranging our words, but transform our ideas into what we perceive as the appropriate genre, considering content, word choice, and tone (in addition to other qualities). In Bakhtin's words:

> We learn to cast our speech in generic forms and, when hearing others' speech, we guess its genre from the very first words; we predict a certain length (that is the approximate length of the speech whole) and a certain compositional structure. … If speech genres did not exist and we had not mastered them, if we had to originate them during the speech process and construct each utterance at will for the first time, speech communication would be almost impossible (Ibid., p. 956).

Like all other social spaces, the classroom has its own expected speech genres, with students and teachers continually in the process of predicting the genre of others' speech and determining appropriate responses. In this chapter, I analyze classroom interactions at the level of genre (rather than at the level of utterance or of classroom culture), which makes it possible to think about the kinds of genres present in elementary mathematics classrooms, the opportunities for participation within various genres, and the expectations that different genres have for the student. By taking the spotlight off the words of individuals and placing it on the *kinds* of conversations that people have, Bakhtin's concept of speech genres makes it possible to closely consider language in the classroom in ways that support, rather than worked against, Foucault's notion of discourse, where both words and context are considered meaningful and where individuals and communities are understood only in relation to each other.

Unlike the classic notion of culture, which emphasizes similarities within a group or context, looking at genres emphasizes differences. Using this framework, one classroom can be seen as having many kinds of interactions, which sometimes work at cross purposes from each other, rather than having a single culture that could be described as either *reform-oriented* or *traditional*. It is probably worth noting here that I am using speech genre differently than Forman (1996) did in her analysis of classroom interactions. She described the *mathematics register* as a particular genre of speech that students had more or fewer opportunities to participate in depending on the speech practices in their classrooms. The analysis in this chapter is slightly closer up, looking at the multiple genres that exist in an elementary classroom, all of which might be considered part of the mathematics register.

In the following section, I describe a few of the genres present in the elementary mathematics classroom and examine the subject of the student constructed by these genres, drawing on Foucault's genealogical framework, which identifies particular points of focus, such as invitations to participate and expected practices, as useful in describing a subject (Foucault, 1983). The goal here is to reveal some of the multiple subject positions available for children in an elementary mathematics classroom. The conversations I analyzed for this chapter came from more than twenty classroom visits between September and April; these conversations were recorded during my observations and transcribed shortly after each visit. I began my textual analysis of the transcripts by drawing on Tannen's (1984) work in conversational analysis. After identifying speech episodes, which I defined by topic of conversation and continuity of participants, I examined these episodes for key features. These included topics of conversation, kinds of questions asked by the teacher, kinds of responses offered by students, ways of physically attracting (or deflecting) attention, kinds of mathematical work getting done, and pacing. I grouped episodes that were similar across many or all of these characteristics as genres. This resulted in my identification of eleven different genres in the elementary classroom, although I will look closely only at three in this chapter. I chose these three because they were the most frequently employed in Diana's classroom. I did analyze the genres of the methods classroom as well, and found them to be quite similar; however, I give examples only from the elementary classroom here to help illustrate the difference between genres is not that the teachers are different, but that the kinds of interactions are.

Genres of Teaching

This section of the chapter explores three genres of teaching observed in a third-grade, urban classroom, which had been described as reform-oriented by university faculty who had visited over several years. The teacher, Diana, had been teaching for more than thirty years and had been involved in numerous university-led professional development activities in elementary mathematics. The three genres described in this chapter were not the only ones observed over the course of the year, and the goal here is not to provide a comprehensive summary of all of the interactions that went on in this classroom, but to explore the intellectual possibilities offered by thinking about classrooms in ways that highlight the differences among teaching practices in one site rather than discussing teachers' practices in holistic ways.

Discussions

One common genre when Diana was teaching was the mathematical discussion. During these conversations, the object of discussion was generally a mathematical idea or process, rather than an answer; Diana frequently challenged students' answers by asking them to explain or by presenting counter examples; and many students participated, sometimes talking to each other. The following episode occurred as Diana was reviewing addition with regrouping before the state standardized test. Children had discussed their solution strategies for the problem:

$$\begin{array}{r} 25 \\ \underline{+\,27} \end{array}$$

Diana then wrote the following on the board:

$$\begin{array}{l} 25 \\ \;1 \\ \underline{+27\quad} \end{array}$$

and asked if this would be an okay way to solve the problem. Many students called out "No!" Many also put their hands up in the air.

Diana:	I'm going to call on someone and it might not be someone with their hand up. It might be someone I'm curious about how they're thinking. Okay, Brian, I'm going to call on you. You have your hand up, but you have a puzzled look on your face. What do you think about this problem?
Brian:	Um … you should switch them?
Diana:	Why would you switch them?
Charlie:	(calling out to Brian) It doesn't matter. It's just the same.
Diana:	Brian, can you explain to Charlie what you're thinking?
Brian:	You would add up that side first.
Diana	Don't say side; use the place value name.
Brian:	You would add up the ones first.
Diana:	Do you want to show how you would do it?

Brian nodded and came to the front of the room. He rewrote the problem in the conventional way, writing a "1" at the top of the tens place and writing "52" as the answer.

Diana:	Why did you do that?
Brian:	Seven and five is twelve. The "1" is the ten of the twelve (pointing to the "1" and "2" in the problem). You have to put it in the tens place.

Diana: Charlie, you said it didn't matter, where the one is. What about what Brian is saying?

Charlie: The one is still a ten -- even if it's in the middle. It's the same. It doesn't matter.

Many children shook their heads.

Diana: It looks like some of you disagree. What do you think about Brian and Charlie's ideas?

In mathematical discussions, the invitations to speak came in the form of unusual problems or ideas, challenges from the teacher, and requests for students to reveal their thinking. For example, in the episode above, Diana told Brian she was calling on him because he had a "confused look" on his face. Instead of looking for certainty on the part of students, Diana wanted to encourage students who had thoughts about the problem to voice them to the class. In addition, Diana invited students to speak to each other, both through direct requests and by attaching ideas to particular children. In this episode, when Charlie called out without raising his hand, Diana did not silence him as she sometimes did during other teaching genres, but instead asked Brian to explain his thinking to Charlie. Later, she opened up the conversation to the rest of the class by asking them to comment on Brian's and Charlie's ideas, rather than on the problem on the board. This move communicated to students that participation required engagement with each other's ideas, not just the teacher and the problem.

During the genre of discussions, students were expected to engage in the practices of reasoning, justifying and communicating, much more than in practices closely linked to particular mathematical domains, such as adding with regrouping. The conversation about how students actually solved the problem 25 + 27 fell into a different genre, one I call presenting student work. In this genre, students took turns describing the steps they executed to find the answer and the teacher rarely encouraged students to speak to each other and instead focused their attention on useful strategies that they might want to record in their journals. However, in the discussion episode above, Diana brought in Charlie's idea about the placement of the 1 not mattering after Brian correctly used the standard algorithm to solve the problem. At her prompting, Charlie said: "It's still a ten, even if it's in the middle. It's still the same. It doesn't matter." Diana's goal in eliciting this comment was not to teach children a new solution method for addition problems. She later ended the discussion by saying that it was just easier to write the "1" on top, noting that this is why most people solved the problem in the standard way. She introduced the idea of writing the "1" in the middle of the problem to give students a chance to justify their ideas about place value and to reason about whether their classmates' ideas made sense. This is a different practice from solving problems. The discussion was designed to provide an opportunity for students to make mathematical arguments and to critique the arguments of others using mathematical reasoning.

Related to this emphasis on process skills, Diana's concern during the genre of discussions was with how students made their arguments or responded to the statements of others, rather than with whether students obtained correct answers. Diana corrected Brian's language when he said "that side," asking him to use the place value name instead. Diana did not ask him what that column was called; she assumed he knew the answer. However, by correcting Brian, Diana demonstrated the need to use precise and mathematical language when making an argument. Similarly, in mathematical discussions, Diana and the children often used the words "agree" and "disagree" to take positions. In other genres, Diana might occasionally ask the class if an individual's answer was "right." However, she rarely used this language in discussions, instead using words that emphasized students' commitments to particular positions, which she would then call on them to defend.

The ideal student constructed during discussions is a mathematical apprentice, someone who is learning the norms and conventions of the discipline. The genre of discussions constructs a student who engages in the process skills, such as offering proof, reasoning and justifying, described in the standards documents (NCTM, 1991; 2000) and other reform-oriented texts (NCR, 2005; Van de Walle, 2004). The student is expected not just to learn content, but to learn how to speak about the content in ways that echo those of the community of mathematicians. Ball and Bass (2003, p. 29) wrote that reasoning "is more than individual sense making. … Reasoning, as we use it, comprises a set of practices and norms that are collective, not merely individual or idiosyncratic, and rooted in the discipline." In the discussion above, Diana's students were expected to engage in a collective process, where individual students did not produce answers to be evaluated by the teacher, but where the class as a whole contributed to a shared understanding. Students were expected to listen to and to comment on each other's points and the focus of the discussion as well as the vocabulary used was expected to be rooted in the discipline. The third graders in Diana's class will probably never be confronted with a similar problem to the one she raised on a standardized test or in adding situations in their outside-of-school lives. The problem was designed so that students could act like mathematicians, or at least act like mathematicians as described by mathematics educators, by evaluating a novel problem and making public sense of it with the goal of increasing their own and others' understandings of mathematics.

Game Shows

The expectations during other genres within this same class were often quite different from those enacted during discussions, particularly in the genre I am calling game shows. Like discussions, this genre involved the whole class; however, the content involved and the norms for participation differed. During game shows, students were expected to answer quickly and correctly; students rarely spoke to each other, and the purpose of the discussion was often to allow students to demonstrate what they already knew rather than to explore new ideas together. The following episode occurred during a lesson on rounding.

Diana: What is three hundred-sixty-two rounded?

Many children raised their hands.

Diana: Everybody?

Children: Four hundred!

Diana: Who can tell us why it's four hundred and not three hundred?

Eight children raised their hands, some leaning forward in their seats. Others, including Caitlin and Jerome, looked down at the ground.

Diana: Marcus?

Marcus: It's past three hundred fifty and three hundred fifty is in the middle of 300 and 400.

Diana: Very good. Everybody, what does nine hundred and seventy-eight round off to? (About five seconds go by.) This is hard. Nine. Hundred. Seventy. Eight. (said slowly).

Two children raised their hands. Diana called on LaTonya.

LaTonya: Ten thousand – wait (she looked at the ceiling).

Diana: I know you have the right answer. You just got confused.

LaTonya: One thousand.

Diana: Good. You got it!

Ben (quietly): LaTonya's smart.

Aliah (to Ben): Yeah. LaTonya *is* smart.

Diana: What does five hundred ninety six round off to?

This conversation continued for several more minutes, with Diana asking similar questions, praising students when they answered correctly, and asking students to try again when they answered incorrectly. After the conversation, students went on to solve several similar problems on their own in their notebooks.

In the game show genre, students who believed they knew the correct answers were invited to speak, while students who believed they did not know the answers were invited to remain silent. In the episode above, many students raised their hands to signal to Diana that they wanted to speak because they knew the right answers. Unlike in the discussion, when Diana expressly said that she might call on someone who did not raise their hand or who had a "confused look," during game shows, Diana only called on students who indicated that they had a correct answer to share – just as Alex Trebek only asks for answers from contestants who have buzzed in. At the same time, students who believed that they did not know the correct answer not only refrained from raising their hands, but also often attempted to remain inconspicuous, as Jerome and Caitlin did by looking at the ground.

The practices students were expected to engage in during the game show genre involved mentally solving mathematical problems, sharing answers, and sometimes being able to explain the steps taken to achieve these answers. For instance, after the class collectively answered that three hundred sixty-two rounded to four hundred, Diana asked someone to explain how he or she knew. After being called on, Marcus said that three hundred sixty-two was above three hundred fifty so the correct answer would be four hundred. This explanation, as well as Diana's response, was quite different than those put out by Brian and Charlie in the discussion example. Marcus was not expected to justify his reasoning about why it *made sense* that three hundred sixty-two rounded to four hundred, instead he was supposed to explain the procedure he used to find the answer. Other children were not supposed to evaluate this procedure or to suggest that they were thinking about it differently. Diana closed off conversation by saying "very good" after Marcus's explanation; whereas, in the discussion example, she asked Brian to think about Charlie's idea when he argued that the "1" could legitimately be placed in between the addends. The genre of game shows was not concerned with the practices of reasoning and justification, but with solving mathematically problems quickly and correctly. Marcus's explanation was a tool that might help some students to engage in the valued practice, rather than an example of a child thinking like a mathematician.

During game shows, a great deal of concern was devoted to evaluating children's mathematical knowledge. This can be seen in comments from the teacher – like "very good" and "I know you have the right answer." – as well as in comments from students – like Ben's remark: "LaTonya is smart." When LaTonya started off by incorrectly answering "ten thousand," Diana could have asked her if this answer made sense, a question she was fond of asking in other contexts. However, in this case she made a different choice. The comment "I know you have the right answer" indicated that Diana was assessing LaTonya's ability to round numbers, rather than her ability to explain it or even to say the correct answer aloud, although LaTonya did go on to do this. Ben's evaluative comment about LayTonya was based not just on her correct answer, but on her ability to answer a question that had hung in the air for several seconds because other students had not been able to offer a response. These kinds of comments from students, both positive and negative, were common during the game show genre, but were absent from many other kinds of conversations, which suggests that the students as well as the teacher saw this format as particularly relevant for evaluating students' competence.

The student constructed during game shows is one who takes standardized tests. In Michigan, as in many other states, state standards demand that students be able to perform a long list of mathematical procedures. In third grade, this includes identifying the place value of a digit, comparing and ordering numbers to ten thousand, and estimating the sum and difference of two numbers (Michigan State Department of Education, 2006) – all skills touched on in the rounding conversation. Each year, from third grade through high school, Michigan students take a standardized test, where they are expected to perform these skills on a series of multiple-choice questions. During these tests, students answer one question after another; they do not have opportunities to discuss their thinking with others; and the ability to generate a right answer is prized. Although the test in Michigan does ask a few open-ended questions, for the most part the ability to reason, justify or prove is not tested. The genre of game show in the classroom is a natural partner to the format of the standardized test, mirroring the evaluative purposes as well as the tone.

Groupwork

In Diana's classroom, about half of each lesson was spent in whole-class conversations and about half the time was spent with students working at their desks. Sometimes students worked by themselves, although they were always allowed to speak with their neighbors, and sometimes students were explicitly asked to work in groups. Most often these groups were comprised of the four to five students sitting at a cluster of desks; however occasionally Diana would assign students to work with particular partners or would mix up children in the classroom. During groupwork, students usually attempted a single or a small number of problems and they were expected to speak only to members of their group. The teacher intervened very rarely, sometimes answering questions when called over and sometimes managing logistical problems; however, most conversation during groupwork occurred among students. In the following episode, Caitlin and Evan had been assigned to work together on the following problem: Use tiles to make three figures, each with an area of 16 but with different perimeters. Record your answers on graph paper.

> Before sitting down, Caitlin complained quietly about having to work with Evan primarily because he was a boy and her best friend was getting to work with a girl. She threw her books down on the table before sitting down and sweeping the tiles on the middle of the table toward her. She and Evan both took sheets of graph paper. Caitlin arranged the tiles into a three by five array with one extra tile in the last column. She turned toward Evan.
>
> Caitlin: Can it be like that?
>
> Evan: It can be like that, can't it? (leaning over the table to look at the tiles).
>
> They both started drawing the shape on their paper. When she finished, Caitlin started to rearrange the blocks even though Evan was not done. Evan looked over at Caitlin's drawing, but she had drawn hers vertically and Evan had drawn his horizontally. He continued to draw on his own as Caitlin worked on rearranging the tiles. Caitlin then colored her shape in, while Evan wrote the date on his paper. Then she looked at his drawing.
>
> Caitlin: Don't do that! We're supposed to do it the same way!
>
> Evan ignored her and continued to draw.
>
> Caitlin: You did it wrong! (She pointed to his picture, where he had drawn two rows of five instead of three rows of five.) You missed some.

Evan: I don't get it. My head hurts (putting his head in his hands).

Caitlin: Fine. We were supposed to work together, but I guess that's not going to happen. Fine!

Caitlin rearranged the blocks into a four-by-four square. She started to draw the shape and then looked back at Evan's paper.

Caitlin: You did that wrong! I hope you know that. There's supposed to be another one down there. (She pointed, but Evan ignored her.) Okay, you just do what you do.

Caitlin began drawing on her own paper. Evan looked at her work for a moment and then wrote "A = 16" by his first drawing, even though the drawing showed an area of eleven. He looked at his drawing for a moment and then counted the tiles. When he got eleven, he added another row to his drawing and then counted the perimeter. He wrote "P=18." Caitlin looked at his paper and then counted the perimeter and area on her first drawing. She wrote "A =16" and "P = 18." She started to count the second one, writing "A=16" and "P=17" for the four-by-four square. She looked at Evan's paper.

Caitlin: What did you get for the second one?

Evan: I got sixteen. The perimeter is sixteen.

Caitlin: Okay. I'll count again. (She did.) You do get sixteen (surprised).

Evan: So I was right.

Caitlin: Yeah (smiling). Let's see if we can do three and then the ones for extra credit.

Invitations to participate in groupwork came in more informal ways than in genres that involved the whole class. This was true both of invitations issued by the teacher and of those issued by other students. In many of the groupwork problems, Diana and Sara provided students with physical materials to use, which in themselves were a kind of invitation because many students wanted to touch the materials. In addition, the more relaxed atmosphere was a sort of invitation to participate in the mathematics. Because students could chat with their classmates, although always about the task, many students engaged more actively in groupwork than in whole-class interactions. Also, the small number of students in a group exerted a certain amount of social pressure to participate because students could not count on classmates to fill the conversational space. In the small group, students who opted not to participate were likely to be taken to task by their classmates, as Caitlin did with Evan in the episode above. During the whole-class discussion, Jerome and Caitlin could both avoid participating and avoid attracting the negative attention of their peers, but Caitlin got vocally angry with Evan when his body language suggested that he was going to disengage. Once he began to work again, she conversationally asked him to report his answer. Reluctant students were often pulled into groups in a variety of ways, including cajoling, threatening and requests for help. The emotional tone of the invitations varied depending on the compositions of the group, the problem at hand, and children's moods. In this episode, Caitlin took a challenging tone with Evan when he made a mistake and when he declined to respond to her criticisms. However, when working with her best friend, Caitlin often took a playful tone and with Ben, whom she viewed as being good at math, she was quite deferential. Much more than in the teacher-led genres, students did not know what to expect in terms of invitations when they entered into groupwork with their classmates. A wider range of speech, gesture and tone was accepted in this genre than in those in which the teacher was a central participant.

During groupwork, students were expected to have conversations with each other, to solve mathematical problems, to represent what their group had done in writing, and to resolve any difficulties that arose. Unlike the gameshow and discussion genres, groupwork almost always involved the production of a product, such as the graph paper drawing Caitlin and Evan worked on in the above episode, a poster to share with the class, or paragraphs written in individual journals. The practices important in groupwork generally revolved around the production of this product. For instance, when Caitlin observed that Evan had made a mistake in drawing his first figure, she insisted that he change it. This would be a very unusual move in the genre of individual seatwork, where students produced their own products and showed little concern for the work of others. However, because of the genre, Caitlin perceived Evan's work as a reflection of her own understandings and became emotionally involved in his competence.

In the genre of groupwork, both students' social and mathematical skills became objects of concern. Diana frequently worked with group members around the mathematics of the task at hand, sometimes suggesting a strategy to try, sometimes clarifying directions; however, she did not engage with students on regulating how much individuals should participate or how they should speak. Thus, when Caitlin grew angry with Evan for making a mistake and when Evan grew frustrated with Caitlin for yelling at him, neither called the teacher for help. Both knew that they were expected to solve social problems on their own. Caitlin's strategy was to keep working, while Evan's was to complain briefly about a headache and then to continue working. Diana said that one of her goals during groupwork was that students would learn to work together in productive ways; thus, these sorts of interactions were not seen as being less important than the work around the mathematical task, but also part of the intended instruction.

The genre of groupwork constructs a student who is a collaborative worker. The conversations that occurred during groupwork often resembled those that go on in many offices. As with most adult workers, students did not choose their colleagues nor did they have a great deal of control about the task they were supposed to accomplish, the tools for doing it, or the time frame. The desire to get the job done shaped many groupwork interactions. Although Caitlin had been initially irritated about working with Evan, she began by immediately responding to the task posed by Diana and by seeking Evan's validation that her response was correct. It was only when Caitlin saw Evan as getting in the way of completing the job that she raised her voice to him. Toward the end of the episode, Caitlin smiled at Evan and softened her tone when she recognized the value he was bringing to the task in the form of correct answers. Her final comment proposing that they attempt the "extra credit" also reflected a worker-like attitude. Rather, than engaging in conversations about the mathematical relationships they may have been uncovering or about a topic unrelated to school, Caitlin directed Evan's attention toward getting more work done. The mathematics in groupwork often functioned as a job students needed to complete rather than as a source of interesting puzzles and questions. It was during discussions that Diana took the tasks completed by groups and transformed them into objects of curiosity.

Multiple genres, Multiple subjects

The three genres described in detail here do not by any means comprise the total number of genres observed. In addition to individual seat work and presentations of student work, which were mentioned briefly above, genres included private work between teachers and individual students as well as public work between teachers and individual students, where the teacher worked to remediate the misconception of a particular child before turning attention back to the class as a whole. The genres used in the methods classroom were remarkably similar to those in the elementary class. Like the third graders, the student teachers engaged in discussions, game shows, groupwork and individual seat work. Lecturing, which was rarely used in the third grade classroom, was a frequently used genre at the university. An analysis of these genres that drew on the classic version of culture would probably seek to locate Diana's practice on a continuum that ranged from traditional to reform and would suggest possible ways of moving her along that continuum. The goal would be to reduce the amount of time spent on genres like game shows and increase the amount of time spent on genres like group work and problem solving. However, I want to do something different.

Rather than seeing the variety of genres used as a negative feature of Diana's teaching, I would like to suggest that the multiple genres she drew on allowed many students to find ways to engage in mathematics in her classroom. For instance, Caitlin, who rarely spoke during whole-class discussions, demonstrated in the groupwork episode described here that she could be an enthusiastic and dominant mathematical learner. An analysis that focused solely on whole-class interactions might describe Caitlin as passive; however, her leadership role with Evan works against that classification. Other students participated reluctantly in groupwork, but enjoyed the chance to show off their mathematical knowledge during game shows. As discussed in the beginning of the chapter, calls in the mathematics education literature for teaching based on reasoning, problem-solving, and other process skills have contributed to the idea that there is one kind of good mathematical practice and have suggested that the goal of mathematics educators ought to be to move all teachers toward one kind of practice. I would like to suggest that it might be productive to work toward the goal of teachers having diverse practices. Just as the genres of discussions, game shows and group work construct particular kinds of students, so do all other genres used in the classroom. The more subject positions available for students, the greater the likelihood of all students finding places for themselves in the mathematics classroom.

The concept of genre supports further inquiry along these lines, by allowing researchers to look for features of particular episodes of teaching practice and to consider what possibilities these genres make open up for students. In addition to exploring genres commonly used in mathematics classrooms, researchers might also look for ways to incorporate genres rarely seen. For instance, in neither the elementary classroom nor the methods classroom did I observe a genre of teaching that primarily constructed the students as appreciators of the beauty of mathematics. This value is commonly recognized by mathematicians; however, it is rarely communicated to students either directly or through our ways of teaching. At one point in the methods classroom, David led the students through some shortcuts for finding the sum of all the factors of a number. In frustration, a student shouted out "What is the point! … What is the *real* point of finding the sum of the factors of a number? What is that going to *do* in your life for you?" David turned on the student, revealing more emotion than he had all semester and nearly shouted: "What is the real point of going to look at a Van Gogh painting? Of trying to analyze the color? What's that going to *do* for you? There is a part of the mathematics that should be sort of aesthetic. We've got this amazing relationship between numbers." However, none of the genres used in the methods class had constructed students as appreciators of the beauty of mathematics. In fact, this episode was the first time in either classroom that the word "aesthetic" was used. "Beauty" and "beautiful" were never used. Although David did occasionally mention a proof that he found "elegant" in off hand ways, students in neither the methods nor the elementary classroom spent any time talking about parts of mathematics they found beautiful, mystical or pleasing. Nor did they engage in assignments which required them to judge mathematical problems or solutions on aesthetic grounds. In her book on mathematics and beauty, Sinclair (2006) offered some suggestions of what genres that constructed students as appreciators of mathematics might look like. For example, she suggested that students might engage in critical comparisons of solutions in ways that made aesthetic mathematical values explicit. This practice would be different than the presentations of student work that I saw in the two classrooms I observed (as well as in many other classrooms) where the goal of instruction seems to be to show that there are many equally correct ways of solving the problem. Instead, students would evaluate two or more correct solutions for aesthetic properties. Teachers might make this aesthetic visible to children by asking specific questions, such as "Does it explain better? Is it more general or predictive? Is it more transparent?" (Sinclair, 2006, p. 174). Teaching in ways that construct students as appreciators of mathematics could work to broaden the possibilities of what it means to be a student in mathematics. Students who enjoy interpretive work with literature, music and art might find genres that focus on developing an aesthetic sense of mathematics to be ways to find room for themselves in the discipline.

Another set of genres that might work to make mathematics classrooms more inclusive are those that would construct students as users of mathematics in the everyday world. This is different from using real-world contexts in mathematics problems. As Gerofsky (1996) pointed out in her analysis of word problems as a genre, despite their use of real world objects and situations, these problems often have very little to do with how people draw on mathematical ideas in their everyday lives. For instance, I never do put six oranges in my fruit basket, discover than I now have fourteen and wonder how many I started with. Recently, when I described my work to my hairdresser, she responded that she had never been very good at math in school and had not believed her teachers when they told her she would need it in her adult life. She said that as it turned out, she did use quite a bit of math, but it wasn't anything like what she had done in school. As Lave's work (1988) demonstrated, people's use of mathematics in their daily lives is often more fluid and less resolved than much of school mathematics. People grocery shopping stop halfway through calculations because they realize they are close enough, and they care not about the lowest unit price in the world, but about which of their two favorite brands is the better buy. Teaching genres that construct students as users of math in the everyday world would have to allow students to work in mathematical situations – such as cooking and carpentry – without requiring that the situations be represented as school mathematics.

The Standards movement in mathematics education has worked to build the definition of what school mathematics is on the work of mathematicians. These documents (NCTM 1991; 2000), and those who draw on them, argue that mathematics must include proof, justification and reasoning, and much of the literature in the field has been aimed at getting teachers to adopt these values and the practices that go with them. These are not bad values. However, the consensus about what teachers should be doing is dangerous because it narrows the possibilities of what it means to be a student who does mathematics. This is not an argument that mathematics should be more like game shows, more like art, or more like grocery shopping. It is an argument that mathematics is all of these things and that teachers who draw on multiple genres create more spaces for their students to find productive relationships with mathematics than those who foreclose possibilities by teaching in genres that represent a narrow view of mathematics, even a Standards-based one.

CHAPTER EIGHT

Discursive Possibilities Or:

How I Learned to Stop Worrying and Love the Achievement Gap

"The whole political thrust of postmodern criticism is to decrease the weight of managerial discourses. Policy and solutions usually add to that weight. The tricky part, for people relatively new to postmodern thinking, is to regard critique as a positive, productive, generative, and helpful contribution. Conversely, to regard proposed 'solutions' as authoritarian, managerial, instrumental, and exclusionary." – Lynn Fendler, personal communication.

"We were worried about a Doomsday gap." – Russian Ambassador, Dr. Strangelove

In Stanley Kubrick's (1964) movie, *Dr. Strangelove*, the entire world is plunged into nuclear winter when a wacky American general sends a squadron of bombers to attack the Soviet Union, triggering a Doomsday device that cannot be shut down. The annihilation of all life on Earth occurs because of, rather than despite, the multiple policies aimed at preventing just such an occurrence. Driven by the fear of a weapons gap, both countries produced more and more weapons and implemented policies designed to deter their opponent from ever putting these weapons to use. However, each of the proposed solutions to the weapons gap -- the Doomsday machine, with its automated total destruction, the emergency back-up plan to launch planes that cannot be recalled, and the locking down of communication to military bases in order to prevent sabotage -- all contributed to the horrible (if comic) climax. In the end, it was not merely the atomic bombs themselves that were dangerous, but the lack of options available for response.

This dissertation -- while not a love song to the achievement gap -- is an attempt to reduce the weight of the managerial solutions that have been proposed in the wake of concerns about both equity and reform mathematics. Following Kubrick's movie, I want to argue that the closing down of possibilities for teachers and students – even as part of an effort to reduce the achievement gap or to improve mathematics instruction – could ultimately be dangerous in unpredictable ways. In the second chapter of the dissertation, I argued that students who did not share linguistic or cultural background of the teacher were likely to be disadvantaged by the open-ended reform questions that are described as part of quality mathematics teaching by much of the research community. I proposed that instead teachers ask more explicit questions by naming students who should respond, by identifying the part of the problem on which to focus, and by offering a narrow range of possible answers. These proposed solutions might indeed allow more students to participate in mathematics, but they could also work to close down possibilities for students in the classroom. Students might never have the opportunity to tackle open-ended questions or to find their own ways toward sensible conclusions. In addition, teachers might assume that they can look at the skin color of a child or listen to his or her speech and know which kinds of questions are most appropriate. Students would have another – albeit new – set of expectations to which they would be expected to conform. This is an example of proposed solutions adding to managerial weight – what teachers are supposed to know and do in order to demonstrate best practices – and of closing down positions that students might occupy in the classroom.

In contrast, the other three chapters which describe classroom practice, while dealing with quite different topics, all seek to open up possibilities for teachers and students. In the chapter on metaphors, discourses of hierarchy are critiqued with the goal of creating a space where educators might begin to imagine talking about children's learning of mathematics in ways that do not rank and order students. This act of imagination allows not only new ways of thinking about children as having multiple abilities in terms of the ways that they interact with the world quantitatively, but also allows for new ways of conceiving of mathematics, where the learning of the discipline can be imagined not as proceeding linearly, but as wandering over a terrain where paths and speed of travel might depend on individual interests or goals. In the chapter on problem solving, I argued that the three kinds of problems seen in the methods and elementary classrooms serve different persuasive purposes in reference to mathematics and that the privileging of one kind of problem – those that emphasize mathematical processes – is a relatively recent historical move in mathematics and one that is likely to exclude some students. In the final chapter on genre, I argued that the diversity of genres present in Diana's classroom opened up opportunities to participate competently in mathematics that would not be present if her classroom had included only those genres approved by the Standards community.

My goal in each of these chapters was to challenge common assumptions held by the teacher education community: 1.) That beginning teachers are the source of problematic beliefs about students and that they need the interventions of mathematics educators to change. 2.) That process problems resemble the work of real mathematicians and are the kinds of problems all teachers should be focusing on in their practices. 3.) That good mathematics teaching always has certain key features and that the work of teacher educators is to get teachers practices' as close as possible to this ideal. These assumptions have certain discursive consequences that go beyond efforts to close the achievement gap or to reform mathematics instruction.

The current conversation around equity has focused on identifying groups of children who have trouble in mathematics and describing ways that their difficulties might be remediated (NRC, 2001; NCTM, 2005; Struchens, 2000). The discursive consequences of this line of reasoning include the expectation that looking at children will reveal something meaningful about their ability to do mathematics, the belief that children can be categorized and linked with appropriate (or culturally relevant) pedagogies, and the presentation of mathematics as a discipline that proceeds in linear ways with some content and some ways of thinking as being more valuable than others. This current discourse around equity contributes to Sara saying that you would never guess by looking at him that one of her students could milk a goat and to the authors of *Adding Up* saying that poor and minority children are developmentally behind. In different ways, all three of the genealogical chapters seek to unsettle this discourse, whether it is by arguing that certain problems (and therefore the children who successfully solve them) serve valuable mathematical purposes or by arguing that the commonly used metaphor of the achievement gap contributes to, rather than works against, the ordering of children, even if the discussion is about how to close the gap.

In similar ways, this dissertation also worked to unsettle discourse around reform mathematics. In the mathematics education community there has been broad agreement that teachers should teach mathematics in ways that create learning communities, encourage discussion, and promote reasoning and problem-solving (Ball & Cohen, 1999; NCTM, 1991, 2000; Putnam & Borko, 1997). By analyzing the genres used in Diana's classroom and the persuasive purposes of different kinds of problems, I wanted to challenge this shared agreement and to create a space where it was possible to consider the dictum that *good mathematical practices are those that promote discussion, reasoning and problem-solving* as not being universally true. Because of the widespread agreement about what mathematics classrooms *should* look like (NCTM, 1991, 2000), Diana felt it necessary to apologize when she taught in ways that deviated from the practice described in Standards documents, and opportunities for Jerome to feel "smart at math," as he did when completing a temperature handout, came fewer and farther between (possibly even as a result of my observations, as I was situated as a university mathematics educator). This is another example of proposed solutions (have discussions, use problems that focus on reasoning, hold children publicly accountable) adding to the managerial weight. Diana felt that she had fewer degrees of freedom in the classroom because of the widespread agreement of the mathematics educators who want to tell her and other teachers what to do.

I struggled to wrap up each of my genealogical chapters without offering solutions in ways that seemed meaningful and that rhetorically resembled endings. I was not entirely successful in this. For instance, I recommend that we find ways of talking about mathematical learning that do not rely on hierarchies; that we, as a mathematics education community, articulate the purposes that we see problems serving in our classrooms; and that we look for diversity of teaching genre in teachers' practices. In part, these recommendations are a failure of conviction and imagination on my part. As I think about communicating my work to the larger education community, I know they will expect to read both a paragraph that resembles an ending on the last page of an article and also that they will expect to get some insight from me, as the author, about what my analysis means to them. However, even with my postmodern influences, I am comfortable with these conclusions because I see them as working to open up possibilities in already dense discourses rather than shutting them down. That is, my call for diversity in teachers' practices does not ask that teachers engage in or cut down on any particular kinds of behavior. In fact, in pursuit of diversity, some teachers may adopt practices that I would rather not see in mathematics classrooms. However, I believe the recommendation to mathematics educators to support teachers' development of diverse practices could serve to give teachers room to move, experiment, and choose in their own classrooms, rather than work to direct teachers and teacher educators to act in particular ways in order to solve identified problems.

In the introduction to the dissertation, I stated that I wanted to explore the possibilities that drawing on genealogical and rhetorical research traditions offered in contrast to those offered by traditions that draw on social science. I believe that one important role that rhetorical and genealogical traditions played in my study was that of providing ways to think about educational research that worked against, rather than for, the creation of categories. Social science research historically has drawn on scientific paradigms, such as collecting data, forming theories, and gathering evidence. More recently the "science" part of social science research in education has been emphasized by governmental agencies (National Center for Education Evaluation and Regional Assistance, 2003; NRC, 2002). The expectation that research in this tradition should produce clear findings can lead to studies that categorize children and teaching practices. For instance, teachers have been labeled as reform-oriented or traditional (Boaler, 2002; Spillane, 1999) and students have been attached to particular pedagogies based on their learning styles (Wilson, 2006), races (Ladson-Billings, 1995), and genders (Ambrose, 2002). These categories reduce discursive possibilities for students. In other words, they create expectations that students will learn (or not learn) in particular ways and it becomes more difficult for students to act in ways not provided for by these categories.

Genealogical and rhetorical traditions, because they do not call on scientific traditions of methods and findings in order to add persuasive power to their arguments, are less likely to produce firm categories backed up by data. For instance, in the genres chapter, I do not make an effort to create a comprehensive list of classrooms genres and their features. I recognize that each teacher will enact genres in particular ways. Nor, do I make any attempt to link particular kinds of children to particular genres. I suggest genre as a rhetorical tool because I believe that it can support mathematics education researchers in looking at teaching practice in ways that value diversity. I do not argue that the genres that I "found" in Diana's classroom are objective categories that "really exist" or that other researchers would find the same ones. In fact, I would resist these sorts of interpretations of the argument I present in that chapter or future studies that would try to draw these kinds of conclusions. I believe it would be particularly dangerous to attempt to proscribe genres for children based on race, gender, learning style, or temperament because these descriptions would reduce rather open up possibilities for children and teachers in mathematics classrooms. By drawing on literary rather than scientific traditions, rhetoric offers researchers the opportunity to talk about the world in ways that cause others to think differently about social interactions without etching boundaries between people as the result of "evidence."

I want to clearly state that the dissertation is *not* an argument that genealogical and rhetorical perspectives are better than ethnographic or quantitative ones. Nor is it an argument that traditional ways of doing mathematics are better than reform-oriented ones.

It is an argument that dense, dominant ways of thinking close down possibilities for human beings living in these ways of thinking and that efforts to make currently accepted truths seem permeable and historically contingent promote equity precisely because they allow for more ways of knowing and being. This is why it is productive for mathematics educators to challenge traditional ways of learning mathematics in many elementary schools. And it is why it is productive for me to challenge reform-oriented teaching and ethnographic methods in teacher education. And, why, someday, it may be productive for someone else to challenge the dominance of rhetorical and genealogical ways of thinking.

To close, I want to turn my attention to the enterprise of teacher education. In the opening of the dissertation, I said that I wanted to theorize preservice education by drawing on discourse and by turning my attention toward elementary children in the mathematics classroom. Descriptions of the methods classroom showed up very rarely throughout the dissertation, so it may seem a stretch to argue that I have met this goal. However, I would like to spend this last section discussing the ways in which mathematics teacher education could be informed by this work.

Whether or not the phrase is used, much of the research about teacher education in general and mathematics teacher education in particular has been informed by the idea of best practices. Researchers have set out to describe particular competencies that teachers must develop (Ball & Bass, 2000; Hiebert, Morris & Glass, 2003; Lampert, 1990; Ma, 1999), to identify practices that will help them develop these competencies (Blanton, 2002; Empson & Junk, 2004; Feiman-Nemser, 1990; Fernandez, 2002), and to identify practices that will promote the beliefs seen as necessary for these competencies (Ambrose, 2004; Warfield, Wood & Lehman, 2005). For the most part, researchers have drawn the particular competencies, explicitly or implicitly, from the NCTM standards documents, which say that teachers should to create learning communities, encourage discussion, and promote reasoning and problem-solving (Ball & Cohen, 1999; NCTM, 1991, 2000; Putnam & Borko, 1997; Stein, Grover & Henningsen,1996). Researchers have recommended a handful of practices to help teachers develop these competencies, including using multi-media tools (Van Es & Sherin, 2002), interviewing (Moyer & Milewicz, 2002), participating in lesson study (Fernandez, 2003), and letter-writing (Crespo, 2003). In general, these recommended practices involve teachers looking at artifacts, such as video tapes, transcripts and student work, and having conversations about these artifacts over time.

This dense, unitary discourse around the practices which researchers want beginning teachers to adopt as well as the relative agreement about the best pedagogical practices for attaining these goals have certain consequences. First, this Standards-based agreement offers a relatively narrow vision of good teaching to prospective teachers. So much so, that the image of the "good teacher" is often personified in the form a single person (Deborah Ball) -- through transcripts, video tapes and research articles. Second, beginning teachers often experience the same sorts of learning activities over and over again in their preparation classes[5], which privilege some students (those who like participating in public conversations and private reflections) and disadvantage others (those who have a distaste for self-analysis, regardless of their strengths in mathematics or in working with children). Finally, teacher educators and their students often begin to view their field placements in elementary schools and their methods courses at

[5] When I taught my first methods course, I asked my students to tell me what they wanted to learn about and what they were tired of. Overwhelmingly, they told me that they didn't want to talk in groups and write things on chart paper.

universities in oppositional ways. For instance, in their argument for reforming teacher education, Ball and Cohen (1999, p. 6) wrote that teacher education needed "to become sufficiently powerful to immunize teachers against the conservative lessons that most learn from practice." Here, teaching in the schools is framed as a disease that university folk need to eradicate. This kind of metaphor is an expected outcome of a discourse about teaching that valorizes only a narrow range of practices.

The discussions of children's interactions in the elementary classroom in this dissertation call into question the usefulness of viewing teacher education through the lens of best practices. Just as Diana's use of the game show genre and of practice problems allowed some students to feel competent in ways not offered by other genres and problems, a more diverse vision of good teaching might not only welcome in more prospective teachers, but might also support them in welcoming more students. Also, a broader vision of quality teaching might make it easier not to draw such clear boundaries between the teaching practices studied at the university and the teaching practices observed in elementary schools. This more open discourse might free prospective teachers from the expectation that they have to choose between their cooperating teachers and methods instructors as mentors. Methods courses that themselves reflect a diversity of practice in terms of assignments might also contribute to the opening up of visions of good teaching. This opening up of assignments might require recognizing that a prospective teacher could work with children in mathematically meaningful ways while at the same time might be unable to write a compelling reflection about the lesson (a heretical notion, I know). Opening up the discourse around teacher education would require operating on multiple levels, including giving assignments that recognized mathematical knowledge and teaching practice as well as self-analysis, doing research about teacher education in ways that framed practices not as *best*, but as possible, and explicitly working to expand – in teaching and in writing – visions of good teaching and acceptable beliefs.

When asked about thinking in terms of good and evil, Foucault (1989/1996, p. 137) said:

> I think it is important to shift the boundaries, to make them indefinite, shake them up, make them fragile, allow for crossovers and osmosis. It isn't possible not to think in terms of good and evil, true and false. But you have to say every time: and if it were the opposite, what if the lines were elsewhere …

Here, he acknowledged the human desire to categorize events, people, feelings and teaching as good and bad, while at the same time arguing for the intellectual payoff in considering the permeability of these boundaries. This is a maneuver I think of as "But what if that's not the question?" What if -- "How do we get beginning teachers to adopt Standards-based practices?" -- isn't the question. What if -- "How do we close the achievement gap?" – isn't it either.

Many years ago, when one of my closest friends was engaged to be married, her mother decided to sew the wedding dress herself. Two months before the wedding, after a particularly exasperating fitting, her mother exclaimed: "This is it. If it doesn't fit next time, then it's *you* who's going to change – not the dress!" It seems to me that in teacher education we have a beautiful wedding dress and that much of the research we've done has been focused on getting the bride to fit. Many of our research questions have centered on how we can make others change – whether we want them to adopt new beliefs, score better on tests, succeed at different kinds of mathematics problems, or promote discussions, reasoning, and higher-level thinking. All of these questions are based on "gap" thinking, with our current situation and our desired outcome arrayed along a continuum with more or less distance between. What this dissertation did for me – and I hope for others – is to bring some of the consequences of this kind of thinking into focus and to suggest that there may be more possibilities for all of us if we abandon the notion of gaps, whether they be about weapons or achievement.

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