ESSAYS ON PLATFORMS AND TWO-SIDED MARKETS

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ABSTRACT

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Platform providers connect two groups that interact through, or on, a platform. For example, Apple develops smartphones that connect consumers with app, music, and video producers. Once consumers own a smartphone they can purchase this content and use it on their phone. Other examples of platform industries are video game consoles that connect gamers with video game developers, eReaders that connect readers with book publishers, ride-sharing platforms like Uber that connect drivers with passengers, and online marketplaces that connect buyers with sellers.

A critical feature of these two-sided markets is that there are indirect network externalities between the two sides of the market. In the case of smartphones, consumers benefit from greater content availability on their smartphone while content providers benefit from greater sales when there are more consumers that own smartphones. In this dissertation I develop three chapters that investigate two-sided platforms. Study of these markets is important because of their rapid rise in the economy and their unique structure. In fact, many of my results differ from those found in traditional industrial organization models that do not incorporate two-sidedness.

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Introduction

Many platforms are updated over time with new generations. For example, the iPhone is now in its sixth generation while the video game console industry began its eighth generation in November 2013. Similarly, new and updated personal computers and eReaders are updated regularly by their developers. By updating a platform, software, app, and game developers on the content provider side of the market can develop new content of higher quality while the platform makes additional sales to its existing customer base. For consumers that own the previous generation, the new generation often provides more than new content and updated technologies. Platforms develop their new generation so that content purchases made for the previous generation can still be used, that is, they are backward compatible. This carryover can be understood as the cloud storage by a platform for its customers. For example, content purchased on iTunes is stored for consumers to use on any Apple device over many generations of a device; the same is true for software on personal computers and ebooks for eReaders. Similarly, using previous generation video games on new consoles serves as the same type of cloud storage over time.

I show how backward compatibility and cloud storage affect dynamic platform competition when an incumbent platform provider is faced with entry. When platforms update to a new generation, a consumer that purchased the previous generation can use all of their previous content on that platform's new generation. Thus, a consumer that stays with a single platform across generations of the platform receives carryover utility from their previously purchased content. This carryover utility affects future consumer demand for the platform. Furthermore, the incumbent's first period prices to the consumer side of the market and the content provider side of the market affect consumer participation and the amount of content available to consumers, which determines the extent of the carryover utility that consumers receive in future periods. Thus, the incumbent is able to use its first period prices to consumers and content providers to endogenously determine the carryover utility its consumers face when considering an entrant's platform in future periods. In determining the equilibria for a dynamic game of platform entry, I show that the equilibria that exist conform to evidence found across many two-sided industries including smartphones, video game consoles, eReaders, personal computers, internet video subscriptions, and online marketplaces.[1] Furthermore, these equilibria are not fully explained by the existing literature on platform competition.

There is a large contemporary literature on platform competition: Rochet and Tirole (2003), Caillaud and Jullien (2003), Armstrong (2006), Hagiu (2006), Jullien (2011), Lee (2013), Jeitschko and Tremblay (2015) and White and Weyl (2015), all of which model static competition. A common equilibrium concern in this literature is the tipping equilibrium where an incumbency advantage, sometimes favorable beliefs, results in all participation occurring on a single platform. In the model presented here, this tipping equilibrium is akin to the entrant failing to enter the market and all participation occurring on the incumbent's platform. I find that entry often occurs and platform providers compete in equilibrium, but the incumbent locks in the more profitable consumers and earns greater profits than the entrant. This suggests that an asymmetric equilibrium will occur without tipping. Namely, an equilibrium in which participation occurs on each platform but platform prices, profits, and market share differ.

[1]For example, in the market for smartphones, the Blackberry was the first platform to market. The Blackberry was extremely successful in maintaining market share while being profitable over several generations until Apple entered the market with the iPhone. With Apple's higher quality smartphone, Blackberry quickly lost share and profits. Since then the iPhone has sustained relatively high prices even though their market share has decreased over time. Furthermore, Apple has faced entrants that charge lower prices (e.g., Google Android and Microsoft Windows Phone) some of which have succeeded and gained market share (Google) and others who have struggled to gain share in the market for smartphones (Microsoft). Nevertheless, even successful entrants have not been as profitable as the Apple iPhone. Similarly, the market for video game consoles has seen entrants succeed over multiple generations (Microsoft's Xbox) and others that have failed to reach a second generation (Saga's Dreamcast). The model presented here is the first model of platform competition that effectively predicts which of the multiple entry equilibria seen within these industries will occur and shows how platform competition takes places when entry occurs.

Little research has been done on dynamic competition in two-sided platform markets. The main contributions to this literature have been empirical. Iansiti and Zhu (2012) and Lee (2013) investigate entry in the video game console industry, and Kim et al. (2015) investigate entry in the daily deal promotions market. One theoretical contribution to this literature is Hałaburda et al. (2015), who develop a dynamic model with network effects where the platform that "won" in the previous period is *focal* in the current period. That is, consumers see which platform was available in the previous period and form favorable beliefs toward that platform in the current period. This essentially acts as an exogenously given switching cost that makes it more difficult for the non-focal platform to convince consumers to join their platform even if it is of higher quality.[2] In these dynamic platform papers, backward compatibility and the resulting effects on an incumbent endogenizing the switching costs that consumers face is not considered.

This paper also makes a contribution to the existing literature on markets with consumer switching costs and lock-in [see Farrell and Shapiro (1988) and Padilla (1995), or Farrell and Klemperer (2007) for an overview]. The endogenously determined carryover utility for consumers that exists through backward compatibility is akin to a traditional switching cost: a loss of utility that a consumer faces when switching to the entrant's platform. Thus, the carryover utility helps the incumbent platform provider lock in its consumers in future periods. I show how two-sidedness allows the incumbent to affect consumer lock-in in two dimensions. First, the incumbent affects consumer lock-in on the traditional extensive margin where a lower price to consumers results in more consumers purchasing the platform in the first period. Second, the incumbent also affects the intensity of the lock-in through the content side of the market. By providing more content the platform is more valuable to consumers, which increases the strength of the lock-in. Thus, a lower fee to content providers strengthens consumer lock-in. These effects result in the incumbent having a greater advantage over a potential entrant than it would in a traditional model of switching

[2] Caillaud and Jullien (2003), Hagiu (2006), Jullien (2011) and Jeitschko and Tremblay (2015) investigate focal platforms that have favorable beliefs in static models where switching costs are not considered.

costs.

There is some recent research on dynamic competition with switching costs that leads to an incumbency advantage [Biglaiser and Crémer (2011) and Biglaiser et al. (2013)]. However, in these models it remains the case that, as in the traditional switching cost literature, the incumbent only affects the extensive margin of switching costs. Thus, this paper contributes to the literature on switching costs and consumer lock-in by identifying a new marketplace where switching costs exist (two-sided markets), and by showing how two-sidedness with backward compatibility result in the incumbent endogenously determining the switching costs that exist. In this case with two-sidedness, the incumbent has a greater advantage over a potential entrant than in a traditional one-sided model of switching costs.

A Model of Dynamic Platform Competition

In this section, dynamic platform competition where an incumbent platform provider uses content carryover across platform generations to compete against a potential entrant is analyzed. In the first period the incumbent is the only platform provider that exists and in the second period there are potentially two platform providers, the incumbent and the entrant. Depending on the strength of the backward compatibility and the relative quality of the incumbent's platform compared to the entrant's platform, different equilibria arise.

The timing of the game is as follows. In the first period, the incumbent first sets prices of its first generation to consumers, P_1^I, and to content sellers, f_1^I. The consumers and content providers observe these prices and then simultaneously make participation decisions, where N_{C1}^I (N_{S1}^I) denotes the number of consumers (sellers) that join the incumbent's platform in the first period. Once agents join the platform, the consumers purchase content from sellers in the exchange market (purchase apps from app providers through the app store). Each seller has one item of content, that it develops and each seller sets the price of their content.[3]

[3]The sellers', as opposed to the platform's, ability to set prices for their content varies across platform marketplaces; this is examined by Hagiu and Lee (2011). It is also possible that the platform provider is the

At the beginning of the second period, the incumbent introduces a new generation of its platform to consumers and sellers or exits the market. The new platform is backward compatible so that previous content purchases by consumers can still be used on the new platform.[4] At the same time there exists a potential entrant that decides whether or not to enter the market with its own platform; however, previous content is not compatible with the entrant's platform. Given the entry decisions by the platform providers, the platform providers simultaneously set prices to consumers and sellers (P_2^I and f_2^I for the incumbent and P_2^E and f_2^E for the entrant). Consumers and sellers observe platform entry and prices and then make participation decisions simultaneously. Once agents join the platforms in the second period, consumers purchase new content from sellers in the exchange market for their platform.

The first generation platform is obsolete in the second period. If consumers want to use the incumbent's platform in the second period, then they must purchase the new generation.[5] Agents are forward looking, and the incumbent cannot commit to future prices or price discriminate in the second period. For simplicity, assume that there is no discounting.

Consumers, Sellers, and the Platforms

In developing consumers' and sellers' gains from joining a platform, first consider the gains that occur through the exchange market for content once consumers and sellers are on a platform. A consumer that is interested in an item of content draws a value υ for that item of content, where $\upsilon \sim U[0, \sigma]$. Sellers set the price for their content, p, and a consumer purchases the content if their value is greater than the price, $\upsilon > p$.

seller of some content which is examined by Hagiu and Wright (2014) and Johnson (2014). In this paper the sellers have all of the pricing power for their content and all of the content is developed by these third party sellers.

[4]Alternatively, the new generation could be the renewal of a platform membership subscription where consumer preferences within the platform will carryover. As in the case with Netflix where movie and TV show preferences are maintained over time for Netflix customers.

[5]In the second period there is a used first generation platform market in practice. This analysis abstracts away from the used market but this aspect can be incorporated to this model as discussed in the next subsection. Alternatively, this dynamic platform can be thought of as charging reoccurring prices for a subscription over time as in the case of Netflix.

Not all consumers want all content; some consumers are interested in many items of content while other consumers are only interested in a few.[6] Let τ denote a consumer's type, and suppose that the total mass of all consumers is distributed uniformly on $[0,1]$. A consumer of type τ is interested in an item of content with probability $(1-\tau)$. Thus, with probability τ consumer τ's value for a given content is zero. In other words, for each item of content a consumer randomly draws whether or not they find the content interesting and then randomly draws their value for the content if they are interested.[7] This valuation process for content implies that if N_{Ct} consumers join a platform, the demand for an item of content is linear:

$$q_t = (a - b \cdot p) \cdot g(N_{Ct}), \tag{1}$$

where a and b are parameters, the expected quantity sold of a given item in period t is q_t, and $g(N_{Ct})$ is a polynomial function of degree two with $g^{'}(N_{Ct}) > 0$ (more consumers result in more sales) and $g^{''}(N_{Ct}) < 0$ (the additional consumers that join a platform are the least likely to purchase the item). The exact specification of consumer demand for an item of content is shown in the appendix.

In maximizing profits, sellers take the number of consumers on the platform as given when setting their price. Consistent with many digital markets, and for ease of exposition, assume that the seller's marginal cost of production is zero. Since sellers face the same marginal cost and demand, the equilibrium price of an item of content, p^*, is the same for all content. The resulting expected consumer surplus per item is denoted by $CS(\tau, p^*)$ and the resulting expected revenues from content sales for a seller is denoted by $\pi(N_{Ct}, p^*)$. The entire exchange equilibrium is solved in the appendix, and there it is shown that $\frac{\partial CS(\tau,p)}{\partial \tau} < 0$

[6] Consumers differing in their interest for the content side of the market is confirmed in the market for apps on smartphones, Bresnahan et al. (2014), and in the market for video games for consoles, Lee (2013).

[7] This valuation process results in the equilibrium seller price being independent of platform prices. This provides tractability. Furthermore, consumers differ in the number of items of content that they purchase, something found empirically in the market for apps and video games. A more natural valuation process may be to assume that τ directly corresponds to consumer values. For example, consumer τ's value of an item of content is simply $(1-\tau)$. In this case, the equilibrium seller price depends on platform prices and solving even the static monopoly platform model is no longer tractable.

and $\frac{\partial \pi(N_{Ct},p)}{\partial N_{Ct}} > 0$. That is, consumers that are less likely to be interested in an item of content have a lower expected marginal benefit from more content and more consumers on a platform generates greater expected revenues for a content seller.

In the first period, consumers and sellers consider joining the incumbent's platform in light of the expected gains from the exchange market and the observed prices set by the incumbent. First consider consumers. In addition to gaining surplus from the exchange market, consumers obtain stand-alone utility from a platform. This utility is denoted by V^I for the incumbent's platform. This stand-alone utility can be large, as in the case of smartphones where there are many uses for the phone outside of the apps (making calls, checking email, surfing the web), or close to zero, as in the case of online marketplaces where there is little gain from a platform outside of interaction with sellers. Thus, a consumer of type τ that joins the incumbent's platform in the first period has utility

$$u_{C1}(\tau) = V^I + CS(\tau, p^*) \cdot N_{S1}^I - P_1^I. \tag{2}$$

The consumers who purchased the incumbent's platform in the first period also purchase content. Through cloud storage and backward compatibility, the content can be used on the incumbent's new platform in the second period. This creates a carryover utility from the first period into the second period for consumers that stay on the incumbent's platform. The degree of carryover is captured by a factor of $\phi \in [0, 1]$. Here ϕ can be thought of as the strength of the carryover effect.[8] If $\phi = 0$ then there is no carryover, the incumbent's platform is not backward compatible, and the previously purchased content provides no added value. For $\phi > 0$, the carryover exists and consumers receive utility from continued use of their previously purchased content.

[8] If the durable first period market is considered, i.e., the first period platform can be used in the second period, then the use of previous content on an old generation provides less utility than if they were used on the new platform. For example, with a new smartphone the existing apps are updated and the new smartphone has updated graphics and software that is not available to the previous smartphone. In this case, ϕ represents the added utility from using the previously purchased content on the new platform instead of the old platform. Thus, the durable platform can be modelled with an alternative interpretation of the value of ϕ and the results presented here will follow.

If consumer τ purchased the incumbent's platform in the first period, then the expected carryover utility that consumer τ receives from their first period content purchases is the expected number of purchases times the expected utility from each purchase. The expected carryover utility for consumer τ is denoted by $\overline{u}(\tau, e_1^\tau, N_{S1})$ where e_1^τ is consumer τ's first period participation decision. The carryover utility is explicitly determined in the appendix. Thus, utility for a consumer of type τ from joining the incumbent's platform in the second period is given by:

$$u_{C2}^I(\tau) = V^I + CS(\tau, p^*) \cdot N_{S2}^I - P_2^I + \phi \cdot \overline{u}(\tau, e_1^\tau, N_{S1}^I). \tag{3}$$

Alternatively, the utility for a consumer of type τ from joining the entrant's platform in the second period is given by:

$$u_{C2}^E(\tau) = V^E + CS(\tau, p^*) \cdot N_{S2}^E - P_2^E. \tag{4}$$

Let the difference in quality between the entrant's platform and the incumbent's platform be denoted by $\triangle = V^E - V^I$. Thus, if $\triangle > 0$ then the entrant's platform is of higher quality, and if $\triangle < 0$ then the incumbent's platform is of higher quality.[9] For simplicity, assume consumers single-home; that is, consumers join only one platform.[10] However, sellers can either single-home or multi-home.

Now consider the sellers' decision to join the platforms. The sellers are indexed by $\theta \in [0, \infty)$ which corresponds to their development costs, θ. That is, low θ-type sellers have lower development costs than high θ-sellers. Sellers are short lived, and a new seller of type θ

[9]The membership benefit for a platform may also incorporate all of the preloaded content and content that is exclusive to a platform. For example, if the entrant has exclusive content that is only available on its platform, then $V^E > V^I$. This occurred with Microsoft's Xbox video game platform, where Microsoft developed a popular game in-house that was exclusive to their platform; however, exclusive content can also be provided by a third party through an exclusive contract. Exclusive contracts are beyond the scope of this paper; however, this is one way to incorporate such deals into the model.

[10]Allowing consumers to multi-home is possible here but Jeitschko and Tremblay (2015) show that the equilibrium where consumer single-home always exists. This allocation also coincides with the industries of interest in this paper.

develops new content in the second period, with a fixed cost θ to develop the content. Thus, the utility functions for sellers in the first period and in the second period are given by:

$$u^I_{St}(\theta) = (1 - f^I_t)\pi(N^I_{Ct}, p) - \theta, \tag{5}$$

$$u^E_{S2}(\theta) = (1 - f^E_2)\pi(N^E_{C2}, p) - \theta, \tag{6}$$

$$u^M_{S2}(\theta) = u^I_{S2}(\theta) + u^E_{S2}(\theta), \tag{7}$$

Sellers generate profits from selling their content to a platform's consumers and platforms charge sellers a percentage of these profits to join the platforms in each period. This is the fee structure used in many platform industries. For example, app stores take a percent of app sales revenue, as do video game platforms and many online marketplaces. Note that in multi-homing a seller incurs the cost θ twice. This can be thought of as the cost for content to be synchronized to an additional platform's operating system.

Lastly, platform profits for the incumbent and the entrant are given by:

$$\Pi^I = N^I_{C1} \cdot (P^I_1 - C) + N^I_{S1} \cdot f^I_1 \cdot \pi(N^I_{C1}, p) + N^I_{C2} \cdot (P^I_2 - C) + N^I_{S2} \cdot f^I_2 \cdot \pi(N^I_{C2}, p), \tag{8}$$

$$\Pi^E = N^E_{C2} \cdot (P^E_2 - C) + N^E_{S2} \cdot f^E_2 \cdot \pi(N^E_{C2}, p), \tag{9}$$

where C is the marginal cost of an additional consumer for each of the platforms.

Equilibrium

In each period, consumers and sellers simultaneously make participation decisions after platform prices are set and observed by all agents (I will refer to each of these as a Participation Game while the whole two period game is the Platform Game). Let $e^\tau_1, e^\theta_1 \in \{0, I\}$ be the first period participation decisions and let $e^\tau_2, e^\theta_2 \in \{0, I, E, M\}$ be the second period participation decisions. In terms of notation, $e_t = I$ denotes that an agent decides to join the incumbent's platform, $e_t = E$ denotes that an agent decides to join the entrant's platform,

$e_t = M$ denotes that a seller decides to multi-home and join both platforms (recall that only sellers can multi-home), and $e_t = 0$ denotes that an agent decides to not participate.

In the first period, a consumer of type τ has strategies $s_1^\tau : (P_1^I, f_1^I) \to \{0, I\}$. That is, a consumer's strategy in the first period maps the observed prices set by the incumbent into the consumer's first period participation decision. In the second period, a consumer of type τ has strategies $s_2^\tau : (e_1^\tau, P_2^I, f_2^I, P_2^E, f_2^E) \to \{0, I, E\}$.[11] That is, a consumer's strategy in the second period maps all (available) platform prices and the consumer's previous participation decision into the consumer's second period participation decision. Let $s^\tau = (s_1^\tau, s_2^\tau)$ denote consumer τ's strategy profile.

In the first period, a seller of type θ has strategies $s_1^\theta : (P_1^I, f_1^I) \to \{0, I\}$. That is, a seller's strategy in the first period maps the observed prices set by the incumbent into the seller's first period participation decision. In the second period, a seller of type θ has strategies $s_2^\theta : (N_{C1}^I, P_2^I, f_2^I, P_2^E, f_2^E) \to \{0, I, E, M\}$. That is, a seller's strategy in the second period maps the number of consumers that purchased the incumbent's platform in the first period and the second period platforms' prices into the seller's second period participation decision.

The consumers and sellers are sequentially rational. For consumers, this implies that for all consumer types τ, the s^τ maximize $u_{C1}(\tau) + u_{C2}(\tau)$ for the given price constellations by the platforms and given the seller strategies. For sellers, sequential rationality implies that for all seller types θ, the s_t^θ maximize $u_{St}(\theta)$, $t = 1, 2$, for the given price constellations by the platforms and given consumer strategies. Note that in either period the no-trade outcome where no consumers and no sellers join a platform is an equilibrium in the participation game.[12] If these no-trade equilibria exist in the participation games on and off the equilibrium

[11] A second period strategy depends on the full history of play, which the agents observe. However, the optimal behavioral strategies in period 2 are invariant over P_1, f_1, and what other consumers did in period 1. For ease of exposition, I consider strategies in which period 2 actions only depend on consumer's first period participation choice, P_2, and f_2. Among these strategies I find an equilibrium that is robust to the other strategies with full histories. Similar reduced second period strategies will be used for sellers and the platforms.

[12] That is, if prices are low enough so that there are gains from trade, then the outcome where no agent joins a platform (a coordination failure), is still a Nash Equilibrium in the participation game.

path, then there exists a continuum of trivial equilibria in the platform game where an arbitrary price constellation is an equilibrium in the platform game because no-trade occurs off path (for all other price constellations). Hence, I rule out the no-trade outcome in each participation game on and off the equilibrium path when there exist gains from trade for a given price constellation. Thus, consumers and sellers optimize on and off the equilibrium path and the no-trade allocation does not occur on and off the equilibrium path if there exists gains from trade.

The incumbent is forward looking when setting its prices. This implies that the incumbent takes into account the effect that its first period prices will have on the consumer's carryover utility, the entry decisions, and the resulting second period equilibrium when making its pricing decisions in the first period. In the first period, the incumbent has strategies $s_1^I = (P_1^I, f_1^I)$. That is, an incumbent's strategy is the first period prices. In the second period, the incumbent has strategies $s_2^I = \left(N_{C1}^I\right) \rightarrow (P_2^I, f_2^I)$. That is, the incumbent's second period strategy maps the number of first period consumers that joined the incumbent's platform into the incumbent's second period prices. Let $s^I = (s_1^I, s_2^I)$ denote the platform's strategy profile. The incumbent chooses s^I to maximize profits (Π^I) given consumers', sellers', and entrant's strategies. Exit occurs at the beginning of the second period if incumbent profits are less than zero.

After the first period, the entrant makes its entry decision given the carryover utility consumers have for the incumbent's platform. If entry occurs, the entrant and incumbent simultaneously choose prices. Given entry occurs, the entrant has strategies $s^E = \left(N_{C1}^I\right) \rightarrow (P_2^E, f_2^E)$. That is, the entrant's strategy maps the number of first period consumers that joined the incumbent's platform into the entrant's second period prices. The entrant chooses s^E to maximize profits (Π^E) given consumers', sellers', and incumbent's strategies. Entry occurs when the maximized profit is greater than zero.

All agents are sequentially rational and the no trade outcome within the participation game is ruled out on and off path when there exist gains from trade. If a strategy profile

($s^I, s^E, s^\tau, s_1^\theta, s_2^\theta$ for all τ and θ) satisfies these properties, then it is an equilibrium of the platform game with resulting prices and participation levels ($P_1^I, f_1^I, N_{C1}^I, N_{S1}^I, P_2^I, f_2^I, N_{C2}^I, N_{S2}^I$, $P_2^E, f_2^E, N_{C2}^E, N_{S2}^E$).

Theorem 1. *There are at most two equilibria (s^I, s^E, and a collection of s^τ, s_1^θ, and s_2^θ for all τ and θ), for all $\triangle$. When entry occurs, the resulting equilibrium are:*

$$\overline{P}_2^I = V^I + CS(N_{C2}^I)N_{S2}^I + \phi\overline{u}(\tau, e_1^\tau, N_{S1}^I), \ \textit{with}$$

$$N_{St}^I = (1 - f_t^I) \cdot \pi(N_{Ct}^I, p^*), \ \textit{for } t = 1, 2,$$

$$N_{S2}^E = (1 - f_2^E) \cdot \pi(N_{C2}^E, p^*),$$

$$P_1^I = V^I + CS(N_{C1}^I)N_{S1}^I, f_1^I = \frac{N_{C1}^I - 3\phi(1 - N_{C2}^I)N_{C2}^I}{2(2 - N_{C1}^I)N_{C1}^I}, \ f_2^I = \frac{1}{2(2 - N_{C2}^I)},$$

and $f_2^E, N_{C2}^I, N_{C2}^E, N_{C1}^I, P_2^E$ that depend on whether or not a corner solution occurs.

In addition, there exists a second equilibrium where the higher priced platform to consumers has a reduced higher consumer price than the above equilibria.

All proofs are in the appendix.

The second, reduced consumer price equilibria, exists as there are two potential allocations of consumers and sellers for the prices in the first equilibrium: the allocation given in the equilibrium and the allocation where all consumers, and thus all sellers, join the lower priced platform. However, if the higher priced consumer platform lowers its price to consumers then this tipping allocation will no longer be a potential equilibrium allocation. This restricted second period consumer price for the higher priced platform gives the second equilibrium. For example, take the case where the incumbent is the higher consumer price platform; in this case the incumbent must reduce its price to consumers in the second period to ensure the collective switching allocation does not exist. In markets where there is uncertainty regarding the allocation equilibrium, the reduced consumer price equilibrium is unique as the potential loss from no participation outweighs the gain from higher prices. In what follows, the first equilibrium will be the focus of the analysis as many of the results are consistent across the two equilibria.

To better understand the dynamic competitive equilibria, I first investigate competition when platforms are symmetric in quality, $V^I = V^E$ ($\triangle = 0$). Then, a full analysis of entry across quality differences is considered.

Symmetric Quality Competition

As a base case, suppose $\triangle = 0$ and the only difference between the platforms is that the incumbent has previous customers with carryover utility. This case provides much of the intuition regarding dynamic platform competition.[13] The equilibrium for the symmetric quality case is summarized by the following theorem:

Theorem 2. *When platforms are of equal quality, entry occurs and the entrant does not charge consumers a markup, $P_2^E = V^E < P_2^I$.*

Furthermore, there exists $\tilde{\phi} \in (0,1)$ such that when the carryover is weak, $\phi \leq \tilde{\phi}$, the incumbent aggressively competes with the entrant by setting a relatively low consumer price that results in all previous customers returning to the incumbent's platform in the second period.

However, when the carryover is strong, $\phi > \tilde{\phi}$, the incumbent sets a high consumer price in the second period and some consumers switch platforms.

If there are no quality differences between the two platforms ($\triangle = 0$), then entry occurs. The entrant sets a low consumer price equal to consumer's membership value of the platform which makes it attractive to consumers, especially those consumers who value the platform more as a product unto itself rather than for the content the seller side provides. Both platforms make positive profits; however, the incumbent has higher profits as it charges a higher price to consumers while retaining more consumer participation than the entrant.

[13] Note that when there is no backward compatibility, $\phi = 0$, then the incumbent has no advantage over the entrant in the second period. In this case, dynamics do not play a role. The focus of this paper is on dynamic platform competition, so let $\phi > 0$.

That is, the incumbent not only locks in consumers but the incumbent locks in the most profitable consumers (the consumers that purchase a lot of content and value the platform the most), and the entrant is left with the residual consumers that buy very few items of content and generate little profits.

When the carryover is weak, $\phi \leq \tilde{\phi}$, there is little gain for consumers from staying with the incumbent's platform and competition between platform providers is strong. The incumbent aggressively competes with the entrant, its consumer price is relatively low but still larger than the entrant's, and the incumbent captures all of its previous consumers. Thus, the incumbent uses backward compatibility to ensure that no consumers switch platforms in the second period. However, as the carryover increases, it becomes more costly for the incumbent to keep a low consumer price to ensure that no switching occurs.

As the carryover becomes strong, $\phi > \tilde{\phi}$, it becomes more attractive for the incumbent to set a higher second period price to consumers and extract some of the larger carryover surplus from its previous consumers. This increase in the incumbent's consumer price in the second period results in some of the marginal consumers switching to the entrant's platform. Furthermore, as the carryover increases, the incumbent's consumer price increases and more consumers switch to the entrant's platform. However, while the incumbent has fewer second period consumers, the incumbent also obtains greater profit through greater rent extraction from the most profitable consumers. This implies that it is optimal for the incumbent to let some consumers switch and lose market share on the consumer side. This loss of market share by the incumbent while maintaining high prices has occurred in the market for smartphones with Apple's iPhone.[14]

Theorem 3. *When the carryover is strong, $\phi > \tilde{\phi}$, an increase in the carryover results in an increase in consumer switching.*

[14] Theorem 2 also provides insight regarding why an incumbent would like to sell multiple, slightly differentiated platforms at different price points. In the market for smartphones this is often seen where multiple products of a new generation are unveiled and only differ in hard drive space but are priced differently. Although a proper analysis is outside the scope of this paper, it is clear that such a strategy allows platform providers to target different types of consumers.

This result is somewhat surprising as switching become less desirable to consumers when ϕ is larger. However, with a stronger carryover more consumers switch platforms. With greater carryover utility, the incumbent has an incentive to increase its price to consumers to extract some of this additional surplus. For the marginal consumer, this increased price outweighs the gains from a better carryover and so more consumers switch.

In this base case where platform quality is equal, the threat of entry is not substantial as the incumbent uses backward compatibility to ensure that it captures the most profitable consumers in the second period. In this sense, the incumbent platform can be considered focal or favored. The effects of a focal platform competing with a non-focal platform originated with Caillaud and Jullien (2003) and Hagiu (2006). This concept continues to have an important influence on models of platform competition; see Jullien (2011), Hałaburda et al. (2015) and Jeitschko and Tremblay (2015). However, in these papers the magnitude of the parameter determining which platform is focal is exogenous. In this model, the focality or strength of incumbency is endogenously determined, and it affects the extent to which the incumbent is focal. The extent of the incumbent's advantage depends on the strength of the backward compatibility and the differences in quality between the platforms. In the case where platforms are of the same quality, the results found here show that the non-focal platform (i.e., the entrant) successfully enters the market but only captures the remaining least profitable consumers, makes minimal profits, and has less participation than the focal or incumbent platform. The case when platforms differ in quality is considered in the following subsection.

Asymmetric Quality Competition

Now consider platform entry and exit when platforms differ in quality. When the two platforms are of different quality it follows that:

Theorem 4 (Platform Entry and Exit)**.** *There exist* $\triangle_E, \triangle_S, \triangle_{EM}$ *with* $\triangle_E < 0 < \triangle_S < \triangle_{EM}$ *such that*

1. *For $\triangle \leq \triangle_E$, entry is foreclosed.*

2. *For $\triangle_E < \triangle \leq \triangle_S$, entry occurs, the entrant does not charge a markup to consumers ($P_2^E = V^E$), and the entrant makes little profit relative to the incumbent.*

3. *For $\triangle_S < \triangle \leq \triangle_{EM}$, entry occurs and the entrant charges a markup to consumers, $P_2^E > V^E$. The incumbent retains the most profitable consumers; however, as the quality difference increases the incumbent captures fewer consumers and the entrant earns greater profits.*

4. *For $\triangle_{EM} < \triangle$, entry occurs and the incumbent exits.*

When the platforms differ in quality there exist three critical quality difference levels that determine the entry equilibrium. When the entrant's quality is low, $\triangle \leq \triangle_E$, entry is foreclosed and only the incumbent's platform is available. Note that when $\triangle < 0$ the entrant is subsidizing the consumer side of the market at a cost of $P_2^E - C = V^E - V^I = \triangle < 0$ per consumer. At the boundary case when $\triangle = \triangle_E$, the cost exactly equals the maximum revenues that the entrant generates from the seller side of the market. If the entrant's quality is greater, so that $\triangle > \triangle_E$, entry is profitable.

When the entrant's quality is moderate, $\triangle_E < \triangle \leq \triangle_S$, entry occurs but the entrant charges consumers a relatively low price which is equal to the consumer's membership benefit for the entrant's platform, $P_2^E = V^E$. This equilibrium is characterized in the previous subsection by Theorem 2.

If the entrant's platform has a significant quality advantage over the incumbent's platform, $\triangle_S < \triangle \leq \triangle_{EM}$, then the entrant charges consumers a markup above the membership benefit, $P_2^E > V^E$. As the quality difference increases more switching occurs.

Lastly, in the extreme case when the entrant's platform dominates the incumbent's platform in terms of quality, $\triangle > \triangle_{EM}$, the incumbent acts as a single period monopolist in the first period and exits the market in the second period. That is, the high quality entrant replaces the low quality incumbent in the second period.

Thus, Theorem 4 demonstrates that the entrant is able to overcome the incumbent's advantage only when the quality difference favors the entrant's platform. Otherwise the focality that the incumbent gains from being the first to market is enough to skew competition in favor of the incumbent in the second period. Moreover, the theorem shows the importance of entering the market with a better platform or with a platform that has exclusive content that consumers find desirable enough to prefer the entrant's platform over the incumbent's existing platform.

Furthermore, Theorem 4 has strong predictive power regarding the different equilibrium configurations found within and across platform industries. When the iPhone entered the smartphone market to compete with the incumbent, Blackberry, the iPhone had tremendous success and overtook the Blackberry. This occurred because the iPhone was of significantly superior quality which is shown to be a necessary condition in the fourth case of Theorem 4.

More recently in the smartphone market, Google's Android entered to compete with Apple's iPhone and the Android software was touted as being of better quality. Android successfully entered the market; however, Apple was able to charge a higher price and maintain many of its consumers even though Android's price was much lower. This equilibrium follows the spirit of the second case of Theorem 4 where the Android phone was of higher quality, but the difference in quality was not enough for it to dominate the iPhone.

In the market for video game consoles, Microsoft's Xbox entered the market with a popular video game (Halo) that was exclusive to Xbox. This exclusive deal improved the quality of Microsoft's Xbox for consumers and Microsoft was able to successfully enter the market and charge a consumer price that was similar to the prices set by Playstation and Nintendo. The three giants split the market in shares and profits. This equilibrium follows the spirit of the third case of Theorem 4.[15]

A generation before Microsoft's successful entry, Saga entered the video game market with its Dreamcast console. This console was considered to be of lower quality and entry

[15] For more on the successful entry of Xbox see Lee (2013).

failed. This scenario is best described in the first case of the theorem.

Conclusion

This paper examines dynamic platform competition and entry when consumers experience carryover utility across platform generations through backward compatibility in platform content. Platform providers enable their consumers, through cloud storage or backward compatibility, to use content purchased on previous generations (apps, software, games or ebooks) on the current platform generation. This creates an opportunity cost in the form of lost content that helps an incumbent platform provider lock-in consumers when facing a potential entrant.

The incumbent's ability to lock in consumers depends on the carryover that its consumers receive from previously purchased content. The incumbent affects the carryover utility using two pricing mechanisms. First, the incumbent uses the first period price to consumers to influence first period consumer participation levels. This is the same mechanism seen in the traditional consumer lock-in literature where a lower price in the first period affects the extent of consumer lock-in. Second, the incumbent uses its first period price to the content side of the market to make more content available to consumers in the first period. Expanded content selection for consumers increases the intensity of the carryover utility. Thus, two-sidedness generates an additional mechanism for the incumbent to affect the intensity of the consumer lock-in.

I find that the use of backward compatibility is critical in dynamic platform competition, entry, and exit. In a dynamic game where the incumbent develops a platform in the first period and then competes with a potential entrant in future periods, the incumbent uses its first period prices to increase both the extent and the intensity of the consumer lock-in resulting in a competitive advantage in future periods. If the second period platforms are of the same quality, then entry occurs with the entrant charging lower prices. However, the incumbent locks in the most profitable consumers (the consumers that value and purchase a lot of content who also have a higher willingness to pay for the platform itself), resulting in the incumbent making the majority of the profits.

If the platforms differ in quality, then the extent to which entry is successful varies. The incumbent's ability to endogenously lock in consumers generates a strong incumbency advantage in competition with an entrant. The entrant is able to overcome the incumbent's advantage only with a significantly better platform in terms of standalone quality. Depending on the platform industry (e.g., smartphones, video game consoles, personal computers, and eReaders), there has been a wide range of entry success in terms of gaining share and profitability. The equilibria found in this model conform to many of the equilibria seen in these industries over the past decade.

APPENDIX

Exchange Market

Consider the exchange market. The consumer valuation assumptions imply that the demand for an item of content is given by:

$$q_t \equiv \int_0^{N_{Ct}} Pr(\tau \text{ buys}|p)d\tau = \int_0^{N_{Ct}} \left(1 - \frac{p}{\sigma}\right)(1-\tau)d\tau = \left(1 - \frac{p}{\sigma}\right)\left(1 - \frac{N_{Ct}}{2}\right) \cdot N_{Ct}. \quad (10)$$

To get consumer demand for content as specified in Equation 1, note that $a = 1$, $b = \frac{1}{\sigma}$, and $g(N_{Ct}) = \left(1 - \frac{N_{Ct}}{2}\right) \cdot N_{Ct}$. Thus, assuming consumer's valuations follow the uniform distribution results in consumers having linear demand for an item of content. Sellers take the number of consumers that join the platform, N_{Ct}, as given when maximizing profits. This leads to the following equilibrium for the exchange market between consumers and sellers:

$$p^* = \frac{\sigma}{2}, \quad (11)$$

$$\pi^*(N_{Ct}, p^*) = \frac{\sigma}{8} \cdot (2 - N_{Ct}) \cdot N_{Ct}, \quad (12)$$

$$CS(\tau, p^*) \equiv E[v - p^*|v \geq p^*] \cdot Pr(\tau \text{ buys}|p^*) = \frac{\sigma}{8}(1-\tau). \quad (13)$$

For the carryover utility that a consumer τ receives, note that the consumer already owns an item of content. Thus, the carryover utility for consumer τ, $\overline{u}(\tau, e_1^\tau, N_{S1})$, is given by:

$$\overline{u}(\tau, e_1^\tau, N_{S1}) \equiv E(v|v > p^*) \cdot \Pr(v > p^*) \cdot N_{S1} = \frac{3\sigma}{8}(1-\tau)N_{S1}. \quad (14)$$

Appendix of Proofs

Proof of Theorem 1: In the first period consumers and sellers make participation decisions for given prices where $u^I_{C1}(N^I_{C1}) = 0$ and $u^I_{S1}(N^I_{S1}) = 0$ give the best reply functions that provide the anticipated participation levels so long as prices are not too high. In the second period, for given incumbent and entrant prices and consumer previous participation decisions, there is a consumer and seller participation equilibrium. The best reply function for consumer participation on the incumbent's platform is given by $u^I_{C2}(\tau) > u^E_{C2}(\tau)$ and $u^I_{C2}(\tau) > 0$; and the best reply function for consumer participation on the entrant's platform is given by $u^I_{C2}(\tau) < u^E_{C2}(\tau)$ and $u^E_{C2}(\tau) > 0$. A consumer does not participate in the second period if $u^I_{C2}(\tau), u^E_{C2}(\tau) < 0$. Seller participation best replies are given by $u^I_{C2}(\tau) > 0$ for the incumbent's platform and $u^E_{C2}(\tau) > 0$ for the entrant's platform. Given the participation decisions by the agents, the platforms play the price competition game and entry occurs when the anticipated equilibrium results in the entrant having positive profits.

In solving the platforms' problem, note that the last consumer τ to join the incumbent's platform, $\tau = N^I_{C2}$, is given by $u^I_{C2}(N^I_{C2}) - u^E_{C2}(N^I_{C2}) = 0$. Let N_{C2} be the total number of consumers that join a platform; that is, $N_{C2} = N^I_{C2} + N^E_{C2}$. For the entrant, the last consumer τ that joins the entrant's platform, $\tau = N_{C2}$, is given by $u^E_{C2}(N_{C2}) = 0$. That is, given that both platforms have consumer participation then the consumers that join the incumbent's platform in the second period are the consumers that have the most to lose by switching platforms and the consumers that join the entrant's platform are either new consumers or the incumbent's previous consumers that have less to lose by switching platforms. Thus, platform k maximize individual profits with respect to N^k_{C2} and f^k_2 taking the other platform's decisions as given. This generates platform best reply functions that give the unique dynamic entry equilibrium. To simplify computations, let V^I be constant over the two periods and let V^I equal C.[16]

If entry occurs, then rational expectation, the marginal agent participation constraints,

[16] This assumption is not critical but makes computations simpler.

and Equations (3), (4), (5), and 6 imply:

$$N^I_{St} = (1 - f^I_t) \cdot \pi(N^I_{Ct}, p^*), \text{ for } t = 1, 2,$$
$$N^E_{S2} = (1 - f^E_2) \cdot \pi(N^E_{C2}, p^*),$$
$$P^I_1 = V + CS(N^I_{C1})N^I_{S1}, \; P^I_2 = V + CS(N^I_{C2})N^I_{S2} + \phi\overline{u}(\tau, e^\tau_1, N^I_{S1}).$$

Using these to solve the platforms' problem implies that

$$f^I_1 = \frac{(3 - N^I_{C1})N^I_{C1} - 3\phi(1 - N^I_{C2})N^I_{C2}}{4(2 - N^I_{C1})N^I_{C1}}, \; f^I_2 = \frac{3 - N^I_{C2}}{4(2 - N^I_{C2})},$$

and $f^E_2, N^I_{C2}, N^E_{C2}, N^I_{C1}, P^E_2$ that depend on whether or not a corner solution occurs. The next two proofs detail these solutions.

For the second equilibrium, first consider the case when the incumbent is the higher consumer priced platform. In this case, the maximum price to ensure consumer participation is given by comparing Equations (3) and (4) when the incumbent does not have any seller participation. This requires: $V^I + 0 - P^I_2 + \phi\overline{u}(\tau, e^\tau_1, N^I_{S1}) > V^E + CS(\tau)N^E_{S2} - P^E_2$. Solving for the incumbent's price implies that this equilibrium requires $P^I_2 = \max\{P^E_2, P^E_2 - \triangle + \overline{u}(\tau, e^\tau_1, N^I_{S1}) - CS(\tau)N^E_{S2}\}$ when solving the platforms' problems.

Similarly, in considering the case when the entrant is the higher consumer priced platform we compare Equations (3) and (4) when the entrant does not have any seller participation. This requires: $V^I + CS(\tau)N^I_{S2} - P^I_2 + \phi\overline{u}(\tau, e^\tau_1, N^I_{S1}) > V^E + 0 - P^E_2$. Solving for the entrant's price implies that this equilibrium requires $P^E_2 = \max\{P^I_2, \triangle + P^E_2 - \overline{u}(\tau, e^\tau_1, N^I_{S1}) - CS(\tau)N^I_{S2}\}$ when solving the platforms' problems. □

Proof of Theorem 2: In solving the incumbent's problem there are two conditions that arise in the maximization problem. First, the function $P^I_2(N^I_{C1}, N^I_{C2}, f^I_1, f^I_2)$ given by $u^I_{C2}(N^I_{C2}) - u^E_{C2}(N^I_{C2}) = 0$ is discontinuous at $N^I_{C2} = N^I_{C1}$ and unlike in the monopoly platform's problem this condition does bind in the case for $\phi \leq \tilde{\phi}$. Second, it must be that the consumer side is at most covered by the two platforms, $N_{C2}^I + N_{C2}^E \leq 1$.

The entrant's problem is simpler; the entrant maximizes profits given the incumbents choice variables. The last consumer τ that joins the entrant's platform, $\tau = N_{C2}$, is given by $u_{C2}^E(N_{C2}) = 0$ which implies $P_2^E = V^E + \frac{\sigma}{8}(1 - N_{C2}^I - N_{C2}^E) \cdot N_{S2}^E$. Similarly, the last seller θ that joins the entrant's platform, $\theta = N_{S2}^E$, is given by $u_{S2}^E(N_{S2}^E)$ which implies $N_{S2}^E = (1 - f_2^E) \cdot \frac{\sigma}{8}(2 - N_{C2}^E)N_{C2}^E$. Maximizing the entrant's profits with respect to f_2^E and N_{C2}^E given N_{C2}^I implies:

$$f_2^E = \frac{1 + N_{C2}^I}{2(2 - N_{C2}^E)},$$

$$N_{C2}^E \cdot (3 - 2N_{C2}^E - N_{C2}^I) \cdot (3 - 4N_{C2}^E - N_{C2}^I) = -\triangle \cdot \frac{128}{\sigma^2} = 0, \tag{15}$$

where the last equality is since $\triangle = 0$. The left two terms in Equation (15) are each positive which implies it must be that $0 = [3 - 4N_{C2}^E - N_{C2}^I)]$. This implies that

$$N_{C2}^E = \frac{3 - N_{C2}^I}{4},$$

which implies that $N_{C2}^I + N_{C2}^E \leq 1$ becomes $N_{C2}^I \leq \frac{1}{3}$.

In solving the incumbent's problem it takes the entrant's choices as given. Using the utility functions to measure the last agent, $u_{it}^I(N_{it}) = 0$, gives P_1^I, N_{S1}^I and N_{S2}^I as functions of N_{C1}^I, N_{C2}^I, f_1^I, and f_2^I. Lastly, $u_{C2}^I(N_{C2}^I) - u_{C2}^E(N_{C2}^I) = 0$ implies that $P_2^I = \frac{\sigma}{8}(1 - N_{C2}^I)(N_{S2}^I - N_{S2}^E) + P_2^E + \mathbf{1}_{\{\tau \leq N_{C1}^I\}} \cdot \phi \cdot \frac{3\sigma}{8}(1 - N_{C2})N_{S1}^I$. In maximizing the incumbent's profits with respect to f_1^I, f_2^I, N_{C1}^I and N_{C2}^I, given the entrant's choice variables, the first order conditions for f_1^I and f_1^E imply:

$$f_1^I = \frac{N_{C1}^I - 3\phi(1 - N_{C2}^I)N_{C2}^I}{2(2 - N_{C1}^I)N_{C1}^I},$$

$$f_2^I = \frac{1}{2(2 - N_{C2}^I)}.$$

Using the above equation for f_1^I and the first order condition for N_{C1}^I we have $N_{C1}^I = 3/4$.

Lastly, the first order condition for N^I_{C2} when $N^I_{C2} < N^I_{C1}$ implies the following:

$$\begin{aligned}(1-f^E_2)(2-N^E_{C2})N^E_{C2}(1-2N^I_{C2}) - \tfrac{64}{\sigma^2}(P^E_2 - V^E) \\ = \tfrac{1}{2}\cdot N^I_{C2}(3-4N^I_{C2})(3-2N^I_{C2}) + \tfrac{3\phi}{2}(1-2N^I_{C2})[3/4 + 3\phi(1-N^I_{C2})N^I_{C2}].\end{aligned} \tag{16}$$

This equation will also be important in proving the next theorem when $\triangle$ is nonzero. If an interior solution, $N^I_{C2} + N^E_{C2} < 1$, exists then Equations (15) and (16) define the equilibrium. However, in solving these equations no solution exists for $\phi \in [0,1]$; that is, the solution to the system of equations violates the constraint that $N^I_{C2} + N^E_{C2} \leq 1$. Thus, we have a corner solution for all ϕ. This implies $P^E_2 = V^E$ and the consumers that do not join the incumbents platform will join the entrants platform.

Given the corner solution and the entrant's consumer price, $P^E_2 = V^E$, in maximizing entrant's profits we have $f^E_2 = \frac{1}{2}$. Given this, the incumbent's first order condition for N^I_{C2}, Equation (16), becomes:

$$[1 - 14N^I_{C2} + 19(N^I_{C2})^2 - 4(N^I_{C2})^3](3-2N^I_{C2}) = \tfrac{15\phi}{2}\phi(1-2N^I_{C2})[9/8 + 3\phi(1-N^I_{C2})N^I_{C2}].$$

This equation implicitly defines N^I_{C2} when $N^I_{C2} < N^I_{C1} = 3/4$. However, for $\phi \leq 0.16428 := \tilde{\phi}$ it is the case that $N^I_{C2} > 3/4 = N^I_{C1}$. Thus, when $\phi \leq \tilde{\phi}$ we must solve the incumbent's problem when $N^I_{C2} \leq N^I_{C1}$ is binding. In solving the new incumbent's problem an implicit function for $N^I_{C1} = N^I_{C2}$ of N^E_{C2} is found and combined with Equation (15) there does not exist a solution for $N^I_{C2} + N^E_{C2} \leq 1$. Thus, it is again the case there is a corner solution and $P^E_2 = V^E$.

Thus, for $\phi \leq \tilde{\phi}$ the incumbent platform captures all its first period consumers and the entrant prices to consumers so that all consumers participate in the second period. In this case the incumbent's first order condition for N^I_{C1} implies:

$$N^I_{C1}(3-2N^I_{C1})(3-4N^I_{C1})+N^I_{C1}(3-4N^I_{C1})\cdot[3-2N^I_{C1}+3\phi(1-N^I_{C1})]$$
$$+3\phi N^I_{C1}(1-2N^I_{C1})[3-2N^I_{C1}+3\phi(1-N^I_{C1})]= \ 1-2N^I_{C1}-3(N^I_{C1})^2+4(N^I_{C1})^3.$$

This equation gives the optimal number of first period and second period consumers the incumbent platform attracts when $\phi \leq \tilde{\phi}$. This completes the equilibrium outcomes over $\phi \in (0,1]$. The entrant always sets its consumer price equal to the consumer membership benefit so that it captures all the remaining consumers that do not join the incumbent's platform, $P^E_2 = V^E$. □

Proof of Theorem 3: This follows directly from the fact that N^I_{C1} is decreasing in ϕ for $\phi > \tilde{\phi}$. □

Proof of Theorem 4: To determine the existence of the cutoff points found in this theorem it is easiest to begin from $\triangle = 0$ and the results found in Theorem 2. As $\triangle$ becomes negative, the equations used in Theorem 2 that implicitly define the interior solution, $N^I_{C2} + N^E_{C2} < 1$, also define the interior solution for $\triangle < 0$ when it exists, Equations (15) and (16). Initially, for $\triangle$ just below zero, these equations continue to not give a solution and the corner solution, $N^I_{C2} + N^E_{C2} = 1$, exists with $P^E_2 = V^E < C$. This pricing scheme implies that for $\triangle < 0$ the entrant is supporting consumers at a cost. Eventually, as $\triangle$ decreases, an interior solution exists where the entrant sets $P^E_2 > V^E$ as keeping the low consumer price is too costly. In this case, the entrant's platform loses participation on each side of the market and eventually, for $\triangle = \triangle_E$, entrant profits go to zero and for $\triangle < \triangle_E$ the entrant fails to enter.

As $\triangle$ becomes positive, Equations (15) and (16) continue to fail to provide a solution and the equilibrium continues to be the corner solution with $P^E_2 = V^E > C$. However, with $\triangle > 0$ the entrant is making profits on the consumer side of the market. Eventually, as $\triangle$ increases, Equations (15) and (16) provide an interior solution for $\phi \in (0,1]$. Let $\triangle_S$ be the $\triangle > 0$ when this interior solution exists. This implies for all $\triangle \in [\triangle_E, \triangle_S)$ the entrant's platform enters the market with $P^E_2 = V^E$.

In the case where $\triangle > \triangle_S$ we have $P_2^E > V^E$, that is the incumbent is optimally restricting consumer participation to increase profits. To this end, the entrant has some market power and the platforms are splitting the market where the incumbent earns profits from the more profitable consumers while the entrant earns profits from the less valuable consumers. Platforms compete over the location of the marginal consumer, $\tau = N_{C2}^I$. Equation (16) implies the number of consumers that join the entrant's platform, N_{C2}^E, is increasing in $\triangle$. This implies N_{C2}^I is decreasing in $\triangle$ since $N_{C2} = N_{C2}^I + N_{C2}^E$ is decreasing in $\triangle$. Eventually, as $\triangle$ increases, there exists $\triangle_{EM} > \triangle_S$ so that the incumbent fails to enter the market in the second period for all $\phi \in (0,1]$. Thus, the entrant is a monopoly in the second period for all $\triangle > \triangle_{EM}$. $\square$

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Platform Competition with Endogenous Homing

Introduction

Video game consoles and smartphones are two platforms that have grown in importance in many people's lives: Two thirds of U.S. households own a video game console, and the average time spent by gamers on their consoles is eight hours a week—the equivalent of a full work day. While this is a considerable amount of time, it pales in comparison to college students who spend as much as eight to ten hours on their phones *every day*—amounting to half their waking hours. Nearly 85 percent of 18–29 year olds in the U.S. have smartphones, and Nielson data show a 65 percent increase in time spent using apps by Android and iPhone users over the last two years, with 18–44 year olds using close to thirty different apps each month. These trends are not confined to the U.S., they are present in Europe and Asia as well.[1]

The market structures in which platforms operate vary considerably in terms of the participation decisions that people make. When several platforms offer competing services, participants may join only one platform (called single-homing)—e.g., most consumers own only one smartphone—or they may patronize several platforms (called multi-homing)—e.g., many app developers make their apps available across competing smartphones.

The literature on platform competition often abstracts away from the issue of endogenous homing decisions by exogenously fixing agents homing decisions on each side of the market; Caillaud and Jullien (2003), Armstrong (2006), Hagiu (2006), Rochet and Tirole (2006) and subsequent work. However, this leads to allocation specific pricing decisions: exogenously fixed multi-homers face high prices as the cannot exit one or both platforms. Furthermore,

[1] The data on gaming come from ESRP (2010), those on college students' phone usage from Roberts et al. (2014), and the data on prevalence of smartphones at Smith (2012). The Nielson data is at Nielsen (2014a,b).

these homing assumptions prevent the allocation where each side of the market has a mix of homing decisions in equilibrium, such an allocation is seen in many platform industries (e.g., video game consoles). To truly understand the elements of platform competition (how prices on each side relate to homing decisions and which homing decisions should be expected in equilibrium), one must allow for endogenous homing decisions by agents.

We consider platform competition in which consumers and firms endogenously choose which platforms to join. We show that in equilibrium, different allocations of consumers and firms emerge, mirroring the configurations found on many platforms, including those for smartphones, game consoles, and ridesharing services. In comparing the multiple equilibrium allocations with allocations seen in platform industries we identify the parameters that are significant in determining which allocations are more likely to occur for a particular platform industry. We further identify under which circumstances a monopoly platform generates higher welfare than competing platforms across the potential equilibrium allocations.

The literature on platforms can be traced back to work on markets where network externalities are prominent. In the case of platforms, network effects carry over across the platform from one side of the market to the other; see, e.g., Evans (2003), Ellison and Fudenberg (2003), Rochet and Tirole (2003, 2006), Armstrong (2006); and sometimes also within one side of the platform, e.g., Deltas and Jeitschko (2007). While much of the literature considers monopoly platforms, competition between platforms is recognized to be an important characteristic that shapes these markets; and this is a necessary focus in order to understand endogenous homing decisions.

As agents choose which platforms to join, the availability of agents on the other side of the platform is critical in determining the equilibrium and potential coordination issues across the two sides of the platform can arise. Caillaud and Jullien (2003) assume that with platform competition coordination favors the incumbent platform; otherwise platforms may fail to gain a critical mass, i.e. "fail to launch." They argue this solves the "chicken and egg" problem of each side's action depending on the other side's action. Hagiu (2006) shows the chicken and egg problem does not occur when sides join platforms sequentially; and Jullien (2011) investigates this further over a broader class of multi-sided markets. Ambrus and Argenziano (2009) show how prices can endogenize heterogeneity and steer agents to asymmetric allocation configurations; and Lee (2013) investigates the video game market, and shows that Xbox was able to enter the video game market because exclusive contracts with game developers allowed Microsoft to overcome the coordination issue. Moreover, the role of beliefs and information play an important role in determining the equilibria as examined by Hagiu and Hałaburda (2014), who consider 'passive' price expectations on one side in contrast to complete information about prices on the second side, and Gabszewicz and Wauthy (2014) who also consider active and passive beliefs in determining platform allocations, or Hałaburda and Yehezkel (2013), who show how multi-homing alleviates coordination issues tied to asymmetric information.[2]

These models of platform competition generally exogenously fix whether agents single-home or multi-home. A common model that is used is "competitive bottlenecks," originally developed by Armstrong (2006), where consumers single-home and sellers multi-home. This allocation is the most commonly seen in platform industries and Armstrong and Wright (2007) show how it can exist endogenously in Armstrong's framework.[3] However, a common feature of many of these markets is that there are a mix of single-homers and multi-homers. In order to understand how this affects competition, agents' allocation decisions must be derived endogenously as part of the equilibrium.

One of the early papers that allows for endogenous homing is Rochet and Tirole (2003), where buyers and sellers engage in a matched transaction that takes place on a platform.

[2]In normative analyses Weyl (2010) and White and Weyl (2016) fully mitigate coordination issues through insulating tariffs in which prices are contingent upon participation, thus resolving failure-to-launch and multiple equilibrium concerns.

[3]We show that this allocation is always possible in equilibrium and under what conditions it is the equilibrium allocation that is likely to occur.

The model is best illustrated by the credit card market: it is assumed that card-issuers only charge per transaction and do not charge card-users or merchants any membership fees, so all agents can costlessly multi-home. However, because card-users choose which card to use when they make a purchase, merchants may wish to single-home in order to limit the customers' options of which card to use. In contrast to per-transaction fees, many platform markets—including smartphones and gaming systems—are characterized by access or membership fees. Although we allow for both usage and stand-alone membership benefits, we consider platforms that compete by setting membership/access fees, as this relates more closely to the markets that we are concerned with.

There is also a nascent literature on multi-homing in media markets. There the focus is on determining the pricing structure on the advertising-side of the platform. Thus, in Anderson et al. (2015) consumers are not charged to join a platform, so the only cost is a nuisance cost of advertising. However, as consumers do not observe the prices platforms charge to advertisers, platforms cannot affect consumer participation. In Athey et al. (2014) the focus is on endogenous homing on the ad-side, while assuming that consumer allocations are exogenously fixed; and in a related model Ambrus et al. (2014) allow platform pricing to affect consumer participation; however, participation on any given platform does not affect demand on other platforms, so there is no competition between platforms for consumers. In contrast to the studies of media markets, we explicitly model the competition that takes place between platforms to attract agents on both sides of the market and we allow agents from both sides to endogenously make their homing decisions.

In allowing for endogenous homing decisions, we find a unique pricing equilibrium that emerges in platform competition. However, even with unique prices, the allocation equilibria of firms and consumers need not be unique. Interestingly, though, the case where agents on both sides of the market choose to single-home and the case where all agents multi-home do not actually arise in equilibrium, even though the former is a constellation that is frequently postulated in the literature. Instead, there are as many as three possible types of equilibrium allocation configurations, each reflecting a commonly found market structure.

The first allocation equilibrium—which can always arise in equilibrium—has all consumers single-homing, whereas all firms multi-home. This is the allocation generally observed in the market for smartphones. Indeed, Bresnahan et al. (2014) find that the practice of multi-homing by app producers—that is their simultaneous presence on competing platforms—insures against a tipping in the market that would concentrate all economic activity on a single platform.

The second equilibrium allocation is one in which there is a mix of single-homing and multi-homing on both sides. This is the division found in the market for game consoles (Lee (2013)). Many consumers have only one system, but others will buy competing platforms; and while some games are available only on one system, others can be purchased for multiple machines.

In the third equilibrium allocation all firms single-home, whereas some or all consumers multi-home. An example of this is found in ride-sharing services such as Uber and Lyft or the nascent restaurant reservations market with opentable.com and the anticipated launch of Yelp's seatme. Here firms (drivers or restaurants) are members of one system, whereas consumers either have a preferred platform and single-home or they make use of both platforms and multi-home.

When comparing welfare between competing platforms and a monopoly platform, we find that lower prices and stronger platform differentiation favors competition, whereas more concentrated network effects and potential cost savings from not having to make apps compatible across multiple platforms tend to favor a monopoly platform. In addition to these factors, we find that when firms' presence on the platform greatly benefit consumers, then a monopoly platform may increase welfare by subsidizing firm entry. Due to non-appropriability if firms multi-home in competition, competing platforms forgo such welfare enhancing investments, which tends to also favor the monopoly platform market in terms of total welfare.

Given the interplay of these factors, we show that a monopoly platform can generate more welfare than that obtained in any of the three competitive equilibrium allocations. Conversely, however, any of the three competitive allocations may also result in competition leading to greater welfare.

Lastly, we sketch how entry into the platform market or declining marginal network effects impact welfare; how exclusive deals can lead to asymmetric equilibrium configurations or even tipping of the market to monopoly; and how endogenous multi-homing can prevent a market from tipping even if there is a focal platform.

The Model

Two groups of agents can benefit from interaction, but require an intermediary in order to do so. The benefits from the interaction to an agent in one group depends on the number of agents of the other group that are made available through the intermediary. This intermediary—the platform—charges agents in each group a price to participate on the platform and in exchange brings these groups together. We consider two platforms, indexed by $X \in \{A, B\}$.

Platforms

Agents on each side of the platform are described by continuous variables. Agents on Side 1 are consumers or buyers, and agents on Side 2 are firms or sellers. The number of consumers that join Platform X is $n_1^X \in [0, \overline{N}_1]$, and the number of firms on Platform X is $n_2^X \in [0, \overline{N}_2]$.

The cost to the platform of accommodating an agent on side $i \in \{1, 2\}$ who joins the platform is $f_i \geq 0$, and there are no fixed costs. Platform X has profits of

$$\Pi^X = n_1^X(p_1^X - f_1) + n_2^X(p_2^X - f_2), \tag{1}$$

where p_i^X is the price that platform X charges to the agents on side i.

If a platform announces that it will enter the market but fails to launch (i.e. the platform has no participation), then that platform incurs a cost. That is, suppose that there are reputational consequences or costs to a breach in production contracts or an inability to recoup capital investments that occurs when a platform is tipped out of the market by a competitor after announcing that they will enter the market. This reputational cost is denoted by F and is only incurred by a platform that announce entry but fails to obtain users.

Side 1: Consumers

Consumers on Side 1 are indexed by $\tau \in [0, \overline{N}_1]$. The utility for a consumer of type τ from joining Platform X is

$$u_1^X(\tau) = v + \alpha_1(\tau) \cdot n_2^X - p_1^X. \tag{2}$$

Here $v \geq 0$ is the membership value every consumer receives from joining the platform. This is the stand alone utility of being a member of the platform that one gets even if no firms join the platform. Note that it is possible for $v = 0$, but for smartphones and video game consoles $v > 0$. For smartphones v is the utility from using a smartphone as a phone, including the preloaded features, and for video game consoles v is the utility from using the console to watch Blu-ray discs. Consumers are homogeneous in their membership benefit to the platform; so v does not depend on consumer type τ; and the stand-alone value of joining a platform is the same regardless of which platform is joined.

Consumers are heterogeneous in their marginal benefit from firms. The network effect or the marginal benefit to a consumer of type τ for an additional firm on the platform is constant and given by $\alpha_1(\tau)$; and the number of firms that join the platform is n_2^X. We focus on the case when network effects are positive so $\alpha_1(\tau) \geq 0$ for all τ, where $\alpha_1(\cdot)$ is a decreasing, twice continuously differentiable function. Since $\alpha_1(\tau)$ is decreasing, it orders consumers by their marginal benefits. Consumers whose type τ is close to zero have marginal benefits that are high relative to those consumers whose type is located far from zero.

The platform knows v and $\alpha_1(\cdot)$ but cannot distinguish the individual values for each consumer τ. Thus, it cannot price discriminate between consumers, so the price or membership fee that consumers pay the platform is a uniform price given by p_1^X.

With there being two platforms in the market, consumers and firms can either join a single platform (single-home) or join multiple platforms (multi-home). A consumer who multi-homes has utility

$$u_1^{AB}(\tau) = (1+\delta)v + \alpha_1(\tau) \cdot N_2 - p_1^A - p_1^B. \tag{3}$$

Notice that if a consumer participates on two platforms the intrinsic benefit from membership to the second platform diminishes by $\delta \in [0,1]$ so that the total stand-alone membership benefit from the two platforms is $(1+\delta)v$. If $\delta = 0$, then there is no additional membership benefit from joining the second platform, and when $\delta = 1$ the membership benefit is unaffected by being a member of another platform.[4]

Apart from the positive membership value of being on a second platform, the main gain to joining a second platform is access to additional firms. Letting n_2^m denote the number of multi-homing firms, a consumer that multi-homes has access to $N_2 := n_2^A + n_2^B - n_2^m$ distinct

[4]One may also consider the possibility that owning a second platforms is tedious so that $\delta < 0$, but this doesn't impact the main results; and depreciation in network benefits, α_1, is also a possibility, see, e.g., Ambrus et al. (2014).

firms: these are all the firms that join at least one platform. The above utility function implies a multi-homing firm provides only a one-time gain to a consumer that multi-homes. Having a firm available on both platforms to which the consumer has access provides no added benefit.

Side 2: Firms

On the other side of the platform, Side 2, are firms that are indexed by $\theta \in [0, \overline{N}_2]$. A firm's payoff from joining Platform X is

$$u_2^X(\theta) = \alpha_2 \cdot n_1^X - c\theta - p_2^X. \tag{4}$$

The marginal benefit firms receive from an additional consumer on the platform is α_2—which is the same for all firms, so firms' marginal benefits for an additional consumer are homogeneous across firm type. The logic here is that an additional consumer will (in expectation) shift the demand curve for a firm's app upward in the same way for all firms. The assumption we are making here is that each consumer sees firm products—their app, or game—as homogeneous, but consumers differ in their preferences, resulting in different willingness to purchase apps and games.[5]

Firms incur a cost of $c > 0$ to join the platform. This cost reflects development and synchronization costs associated with programming and formatting their product to fit the platform. Firms are heterogeneous with respect to their development and synchronization costs. Firms with type θ close to zero have lower costs compared to those with higher θ.[6]

[5] Note that we abstract from the transactions costs that may occur between consumers and firms. Thus, the benefits that accrue to consumers from interacting with firms—apps and games—can be viewed as being net of prices paid to firms. Deltas and Jeitschko (2007) consider auctioneers setting optimal reserves on an auction hosting platform, and Reisinger (2014) generalizes Armstrong (2006) and considers tariff-pricing with heterogeneous trading. See Tremblay (2015) for a more detailed analysis of pricing across the platform in a framework that is more similar to our current setting.

[6] The model is isomorphic to assuming there is a limited number of app developers with increasing costs

The platform knows the firm's profit structure but cannot identify firms individually; hence, it cannot price discriminate between firms and the price or membership fee the firms pay to the platform is given by p_2^X for all firms.

A firm that multi-homes has payoff

$$u_2^{AB}(\theta) = \alpha_2 \cdot N_1 - (1+\sigma)c\theta - p_2^A - p_2^B, \tag{5}$$

where $N_1 := n_1^A + n_1^B - n_1^m$ is the number of distinct consumers to which the firm gains access; these are all the consumers that join at least one platform. As noted above, when a firm's product is available to the multi-homing consumer on both platforms, a consumer will only purchase the product at most once. Therefore a firm only cares about the number of distinct consumers that are available to it through the platforms.

When a firm participates on two platforms its development and synchronization cost for joining the second platform diminishes to $\sigma \in [0, 1]$. Thus, σ represents the amount of 'duplication economies' that exist when synchronizing an app or game to a second platform. If $\sigma = 1$ then there are no economies of duplication and as σ decreases, there exists economies of duplication.[7]

Equilibrium

The sequence of play is as follows: first the platforms simultaneously (and non-cooperatively) announce whether or not they will enter the market. The platforms that are present in the market then choose consumer and firm prices, p_i^X for $X = A, B$ and $i = 1, 2$. Lastly,

of developing apps so the equilibrium number of apps brought to market is endogenous.

[7]The relative lack of duplication economies played a role in providing an app for Facebook in the tablet market. For some time the app 'Friendly for Facebook' was used by Facebook users because Facebook itself had not developed an app for the tablet. It was rumored that Apple later 'assisted' Facebook in developing the official Facebook app.

consumers and firms simultaneously choose whether to join and which platforms to join, yielding n_i^X and $N_i = n_i^A + n_i^B - n_i^m$, $i = 1, 2$. The model is solved using backward induction and we focus exclusively on pure strategy solutions. We first investigate the allocation subgame of consumers and firms in joining platforms for given (arbitrary) prices charged by the platforms; and then we determine the equilibria for the entire game by considering price competition between the two platforms, in light of the profits obtained in the allocation subgame.[8]

The Allocation Equilibrium for Arbitrary Prices

By allowing consumers and firms to make homing decisions there are potentially many allocations that can occur for a given set of prices. To ease the analysis we make a standard assumption that is consistent with markets of price competition. Suppose that Platform X has prices that are no worse than Platform Ys ($p_i^X \leq p_i^Y, \forall i$), then we assume that Platform X must have some firms join its platform ($n_2^X \neq 0$). In other words we preclude dis-coordinated allocation configurations in which despite having worse (i.e., higher) prices a platform corners the market on the firm side. Note that this assumption says nothing about the equilibrium allocation of consumers. And, importantly, it says nothing about consumers or firms for the case that one platform has a lower price on one side, but the rival platform has the lower price on the other side.

The rationale for why the assumption pertains only to a minimum participation of the firms—rather than guaranteeing minimum participation by consumers—is twofold. First, a platform can always attract some consumers when it prices sufficiently low, because the platform offers a stand-alone value to consumers; and second, in the contexts we have in

[8] For simplicity, we focus exclusively on the cases when prices are sufficiently low for at least some participation to exist. Constellations in which a platform prices itself out of the market are easily derived, but are merely a distraction as they do not arise in any of the pricing equilibrium configurations of the entire game. An implication of this is that the total participation of agents on each side across both platforms is positive, $N_i > 0$ for $i = 1, 2$.

mind it is reasonable to assume that firms are aware of pricing on both sides of the platform, whereas consumers are likely to only observe prices on Side 1. Hence, firms are able to observe any price-advantages regardless of the side on which they are offered.[9]

First consider the case where platforms have chosen symmetric pricing strategies.

Proposition 1 (Allocations under Symmetric Pricing). *If $p_i^X = p_i^Y = p_i$ and $p_1 > \delta v$ and $p_2 > 0$ then $n_i^X = n_i^Y = n_i$.*

The set of multi-homing consumers is given by $\tau \in [0, n_1^m]$, and the set of single-homing consumers is given by $\tau \in [n_1^m, N_1]$, with

$$n_1^m = \alpha_1^{-1}\left(\frac{p_1 - \delta v}{n_2 - n_2^m}\right) \quad \textit{and} \quad N_1 = \alpha_1^{-1}\left(\frac{p_1 - v}{n_2}\right). \tag{6}$$

The set of multi-homing firms is given by $\theta \in [0, n_2^m]$, and the set of single-homing firms is given by $\theta \in (n_2^m, N_2]$, with

$$n_2^m = \min\left\{\frac{\alpha_2 \cdot N_1 - 2p_2}{(1+\sigma)c}, \frac{\alpha_2 \cdot (n_1 - n_1^m) - p_2}{\sigma c}\right\} \quad \textit{and} \quad N_2 = \min\left\{\frac{\alpha_2 n_1 - p_2}{c}, n_2^m\right\}. \tag{7}$$

For each side of the market at least one set is non-empty, and it is possible for both sets to be non-empty; as a result there exist multiple equilibrium allocations.

Proposition 1 says that when platforms set equal prices and prices are above the subsidy level on each side ($p_1 > \delta v$ and $p_2 > 0$), the platforms split both sides of the market equally. However, this equal division does not determine the extent to which consumers and firms multi-home in equilibrium. In fact, the allocation of one side of the market depends on the allocation on the other side; and this results in the possibility of multiple equilibrium allocations—depending on parameter values as well as the platform prices chosen.

[9] Hagiu and Hałaburda (2014) use this fact to differentiate between information that firms have in contrast to beliefs that consumers have about prices.

Consider first consumers. Consumers always obtain an added benefit from joining a second platform, namely δv. Hence, if prices to consumers are low enough, $p_1 < \delta v$, then all consumers join both platforms: $n_1^m = N_1 = \overline{N}_1$.[10] For consumer prices above this threshold, but still below the stand-alone utility from a single platform membership, $\delta v < p_1 < v$, all consumers will join one platform, $N_1 = \overline{N}_1$; but whether any consumers join a second platform (multi-home) depends on whether firms multi-home. In particular, if the number of multi-homing firms is large ($n_2 - n_2^m$ is small), then consumers have access to many firms when joining the first platform and so the number of multi-homing consumers is small, or even zero. For even higher consumer prices, consumers with large values of τ even refrain from joining a single platform, $N_1 < \overline{N}_1$.

Unlike consumers, firms do not obtain a stand-alone benefit from joining a platform. However, they experience duplication economies in production when joining a second platform. This implies that a firm will multi-home only when the marginal gain from joining a second platform and the total payoff from being on two platforms are both positive. And hence, the set of multi-homing firms depends on the number of consumers that multi-home. If all consumers multi-home then $n_1^m = n_1$ and no firm will multi-home, unless they are paid to do so (which requires $p_2 < 0$). Second, the firms that choose to multi-home instead of single-home are the firms with sufficiently low synchronization costs, θ close to zero. As the synchronization cost gets larger the marginal cost for joining another platform becomes larger than the marginal gain from having access to additional consumers. Hence, for firms with higher synchronization costs, θ farther from zero, it becomes too costly to join more than one platform. Thus, a firm is more likely to multi-home if it faces a lower synchronization cost to join a platform.

Note finally that if few consumers multi-home (n_2^m is small) and there are strong du-

[10]Note that since $\alpha(\cdot)$ is positive and decreasing, so is $\alpha^{-1}(\cdot)$ and therefore when $p_1 - \delta v < 0$ the corner solution obtains in which $n_1^m = N_1 = \overline{N}_1$.

plication economies (small σ), then it is possible that no firms single-home and all firms multi-home.

We now determine the allocations that occur with unequal price constellations.

Proposition 2 (Allocations with Price-Undercutting)**.** *If $p_i^Y \leq p_i^X$ with at least one strict inequality then there exists a unique allocation equilibrium. In this equilibrium $n_i^Y = N_i$, $i = 1, 2$, with $n_1^X = n_1^m > 0$ only when $p_1^X \leq \delta v$ and $n_2^X = n_2^m > 0$ only when $p_2^X < 0$.*

Proposition 2 shows that when one platform has better prices (at least one better price, and the other price no worse), then all agents—consumers and firms alike—will join the platform with the price advantage. Whether agents also join the second platform (and, thus, multi-home) depends on the prices on the second platform. Consumers will join the second platform only if the price is below their marginal stand-alone benefit from joining a second platform, $p_1^X < \delta v$, because they already have access to all active firms through the first platform so that any firm presence on the second platform is of no value to consumers. Similarly, because firms already have complete market access to all consumers on one platform, they will only join the second platform if they are paid to do so, $p_2^X < 0$.

Lastly, consider the case when prices are unequal and neither platform has a clear price advantage.

Proposition 3 (Allocations under Orthogonal Pricing)**.** *If $p_1^X > p_1^Y$ and $p_2^X < p_2^Y$ for $X \neq Y \in \{A, B\}$ then the following are possible equilibrium allocations. If $p_1^Y \leq \delta v$ then $n_1^Y = n_1^m \leq n_1^X$ and $n_2^X > n_2^Y$. Otherwise the following allocations are possible:*

- *All participation occurs on Platform X for $X = A$ or B (these two allocations always exist).*

- *Consumers single-home so that there exists a τ^c where the consumers of type $\tau < \tau^c$ join Platform X (the higher consumer priced platform), and the other consumers join Platform Y; additionally, $n_2^m = n_2^Y < n_2^X$. This allocation may not exist.*

When prices are unequal and neither platform has the lower price on both sides of the market there are three possible ways consumers and firms can divide themselves onto the two platforms. Thus, it is possible to have multiple equilibrium allocations.

In the first case listed in the proposition, we have the tipping outcome which can occur for either platform. That is, all participation occurs on one of the platforms exclusively. Both of these equilibrium allocations exist as equilibria in the participation subgame when there are orthogonal prices set by the platforms.

In the second case listed, consumers single-home so that the consumers that have a higher marginal benefit from the firm side of the market $\big($the lower τ type consumers who have larger $\alpha_1(\tau)\big)$ join the platform with the higher consumer price, Platform X. Platform X also has more firms then Platform Y who only captures the multi-homing firms and the consumers with lower marginal benefits from firm participation. This is an asymmetric allocation that does not exist for all orthogonal price constellations.

Equilibria of the Pricing Game

Before solving the platform game, notice that Proposition 3 gives rise to multiple distinct allocation equilibria when platforms set orthogonal prices. For the platforms pricing game to be well defined, the beliefs for the platforms must be specified for the price constellations where there exist multiple equilibrium allocations. Specifically, for the cases in Proposition 3 where platform profits differ across the two and potentially three different equilibrium allocations.

In light of these multiple equilibrium allocations for given prices, we consider pure strategy pricing strategies by platforms with belief systems for price constellations with multiple equilibrium allocations. Thus, when there exist multiple equilibrium allocations for a given price constellation, the expected profits for each price constellation is well defined and the platforms face risk across which allocation will occur.

A recurring theme in the allocation configurations was whether a platform sets prices low enough to attract consumers merely for the marginal stand-alone value. This pricing decision often plays a special role in determining whether consumers multi-home. In particular, if a platform sets $p_1^X < \delta v$, then it is sure to capture all consumers—regardless of all other prices and homing decisions. In light of this, when determining the platforms' pricing decisions it is important to consider the relationship between the cost of providing service to a consumer and the consumer's marginal stand-alone value for the second platform, i.e., $f_1 \gtreqless \delta v$.

We first suppose that $f_1 < \delta v$. In this case a platform can charge a consumer price of $p_1^X = \delta v > f_1$ and guarantee itself positive profits since consumers will either single-home on platform X or if a consumer is already on platform $Y \neq X$ then they will be willing to multi-home even absent any firms on platform X. Hence, both platforms are guaranteed profits and, in equilibrium, all consumers $\tau \in [0, \overline{N}_1]$ join at least one platform.

Theorem 1 (Weak Competition; $f_1 < \delta v$)**.** *There exists a unique symmetric equilibrium with $p_1^A = p_1^B = \delta v$ and $p_2^A = p_2^B = f_2$. All consumers multi-home, $n_1^m = \overline{N}_1$, and firms that join a platform single-home, $n_2^m = 0$. Platform profits are $\Pi^A = \Pi^B = \overline{N}_1(\delta v - f_1) > 0$.*

The case of 'weak competition' implies that failure to launch issues are not encountered, since both platforms are able to establish themselves on the consumer side of the market. This occurs because platforms are differentiated from the consumers' perspectives—there is a positive marginal value from joining a second platform—and so consumers are willing to join a platform even if there are no apps available on that platform.

However, for many products the membership benefit depreciates almost to zero when a consumer multi-homes, $\delta \approx 0$. This implies that even for small f_1 the marginal cost of accommodating an additional consumer on the platform is greater than the additional membership benefit from joining another platform. In the smartphone case, for example, the membership benefit is the ability to make calls and use the phone's preloaded features. Since most phones have similar pre-loaded features, δ is close to zero and any additional benefit from a second phone would not overcome the production cost of an additional phone.

We now consider the case $f_1 \geq \delta v$, so the cost of attracting a consumer who has already joined the rival platform exceeds the platform's stand-alone value to the consumer. As a result, platforms compete head-to-head for single-homers, rather than trying to attract multi-homers. We refer to this as 'strong competition,' which leads to fierce price-competition resulting in a form of Bertrand Paradox. While this implies symmetric prices in equilibrium, there are potentially three different allocations of consumers and firms in equilibrium.

Theorem 2 (Strong Competition; $f_1 \geq \delta v$)**.** *There exist unique duopoly equilibrium prices:* $p_1^A = p_1^B = f_1$ *and* $p_2^A = p_2^B = f_2$ *so that* $\Pi^A = \Pi^B = 0$.

There exists at least one and possibly as many as three types of allocations in equilibrium:

I. *All active consumers single-home and all active firms multi-home:* $n_1^m = 0$, $n_2^m = N_2$. *This is always an equilibrium.*

II. *A mix of multi-homing and single-homing consumers with multi-homing and single-homing firms:* $n_i^m \in (0, N_i)$.

III. *All active firms single-home and many, potentially all, active consumers multi-home:* $n_2^m = 0$, $n_1^m \in (0, N_1]$.

The formal proof is in the appendix; however, it is important to note that the duopoly equilibrium requires that under orthogonal pricing, each platform faces the potential of being shut out of the market. With symmetric pricing at the marginal costs, neither platform would deviate by charging a higher price because they would lose all participation to the competing platform. Similarly, neither platform would deviate by charging a lower price because they would be pricing below marginal cost on one side of the market without a mark up on the other side, resulting in negative profits.

The intriguing deviation is that of cross-subsidizing, which results in an orthogonal price constellation. With an orthogonal price constellation, the allocation that occurs is indeterminant across the three potential allocations in Proposition 3. Thus, if there is a sufficient cost of being shut out of the market, then the duopoly equilibrium is supported when platforms face some probability of being shut out of the market under orthogonal prices. Alternatively, if there is no probability that the shut out allocation occurs for one of the platforms when prices are orthogonal, then the unique pure strategy equilibrium is a monopoly platform equilibrium where one of the platforms does not enter. This will be discussed in greater detail in Theorem 3.

The three allocations that occur resemble several two-sided market industries. Allocation I mirrors the two-sided market for smartphones. Almost all consumers single-home, they own only one phone; and almost all firms multi-home, the vast majority of apps are available across all types of smartphones. This is also the 'competitive bottleneck' allocation in Armstrong (2006). There, however, the homing decisions are exogenously assumed, rather than endogenously derived. As a result, firms face high prices, whereas in our model—in which platforms compete to attract firms—firms face marginal cost pricing.

Allocation II resembles current allocations seen in many two-sided markets, including those for game consoles: For video game platforms, there exist consumers who multi-home—buying several game consoles—and others that single-home; and there exists game designers whose games are available across platforms, i.e., they multi-home, while others are available on only one system, i.e., they single-home.

Allocation III is best characterized when considering the sufficient condition of $v = 0$. An example of this type of configuration is seen with the ride sharing companies Uber and Lyft. These are platforms that connect drivers (i.e., firms) with passengers seeking transportation (consumers). Drivers offer their services through one ride sharing company (i.e., they single-home); whereas many customers seeking rides use both companies and compare availability and prices (i.e., they multi-home). Since there is no benefit from linking to a ride sharing company that has no drivers $v = 0$, consumers can join for free $p_1 = 0$, and an additional consumer joining Uber is costless for the platform we have an clear example of an industry where Allocation III occurs.[11]

With the possibility of multiple equilibrium allocations, it is important to understand which allocation to expect in specific platform industries. To further show the importance of the implications of Theorem 2 we discuss some upcoming results now, as they pertain to the equilibrium allocations that should be expected.

Note that Allocation I, the competitive bottleneck allocation that is used in much of the literature, always exists. In the case for smartphones, where we see this allocation with consumers single-homing and app providers multi-homing, it is unlikely that Allocation II exists as a possible equilibrium allocation in this case. For smartphones, it is reasonable to assume that v is relatively large, consumers receive a high level of utility from using their smartphone as a phone with texting and preloaded features, and δ is close to zero, carrying around an additional phone just to have the same calling ability and preloaded features has little or no added benefit. Under these conditions it is unlikely that Allocation II exists since, as we show in Theorem 5 and its proof, Allocation II only exists when $f(\alpha_1(\cdot), \alpha_2) > (1-\delta)\cdot v$. Notice that this set of parameters $\{\alpha_1(\cdot), \alpha_2\}$ becomes more restricted as v increases or δ decreases. Thus, in the market for smartphones where δ is small and v is large relative to the network benefits, Allocation II is unlikely to exist as a possible equilibrium allocation

[11]Another example are antique malls with many individual stalls each rented out to individual antiques dealers (i.e., firms), and consumers who visit the mall to browse the individual stalls. Vendors sell their antiques in only one mall (single-home), yet consumers browse at different malls (multi-home). There is no benefit from going to a vacant antique mall so $v = 0$. Also, Yelp's seatme restaurant reservation system may enter into competition with opentable.com. In this case, restaurants would work with one or the other system, but patrons could search either.

and it follows that Allocation I should be the expected equilibrium allocation.

The fact that we see the mixed allocation instead of Allocation I in the market for video game consoles with consumers and game developers is twofold. First, it is more likely the case that Allocation II exists in the console industry as v, the utility from using the console without games, is relatively small compared to the network benefits. This implies there exist multiple allocation equilibrium, Allocations I and II. Second, there exist exclusive contracts between consoles and game developers and this pushes the allocation equilibrium away from Allocation I and toward the mixed homing allocation equilibrium, Allocation II. Thus, it is not surprising that the mixed allocation occurs in the video game industry.

The conditions for which Allocation III exists are quite strong. For the case when $v = 0$ and $p_1 = f_1 = 0$ it is reasonable to expect that Allocation III will occur over Allocations I and II as a zero price to consumers will induce consumers to multi-home. This, in turn, induces drivers to single-home and so it is not surprising that Allocation III is the equilibrium allocation that arises in the ride sharing marketplace. Given this analysis on which allocations should be expected to occur in a certain industry, it is clear how the industry specific parameters drive the equilibrium allocation that occurs when multiple equilibrium allocations exist.

It is worth noting that the equilibrium allocation configurations in Theorem 2 are exhaustive; that is, there are no other equilibrium allocations. In particular, while many papers on platform competition assume single-homing, there does not exist an equilibrium in which all active consumers and all active firms single-home. When all consumers single-home, then firms optimally multi-home in order to reach all consumers. Also, there is no equilibrium allocation in which all active firms and consumers multi-home. If all consumers are multi-homing, firms optimally respond by single-homing.

Theorem 3 (A Monopoly Platform)**.** *If there is no probability that the shut out allocation* *occurs for either of the platforms when prices are orthogonal, then the unique pure strategy equilibrium is a monopoly platform equilibrium where one of the platforms does not enter.*

With no probability of being shut out, the duopoly equilibrium will not exist and only one platform will enter the market, and the monopoly platform sets profit maximizing prices. In the following section we compare welfare across these platform equilibria.

Monopoly versus Strong Competition

Does strong competition between two platforms result in higher welfare when compared to a monopoly platform? The answer is not readily apparent. On the one hand competition results in lower prices and additional stand-alone membership benefits to consumers who multi-home. On the other hand, however, competition can increase synchronization costs, as well as destroy network surplus by fragmenting the market. Moreover—as we show in this section—competition may also undermine welfare-increasing cross-subsidization that takes place in the monopoly setting.

Since weak competition leads to all consumers multi-homing due to the positive incremental value of joining another platform, we consider the case of strong competition, and show that even in this case the monopoly equilibrium may welfare-dominate.

To obtain closed form solutions and welfare we assume that τ is distributed uniformly on $[0, {}^{a}\!/\!_{b}]$, which implies that $\alpha_1(\cdot)$ is linear: $\alpha_1(\tau) = a - b\tau$. The number of potential consumers is then $\overline{N}_1 = \frac{a}{b}$. We assume that θ is distributed uniformly, with $\overline{N}_2$ sufficiently large so that the platform can always attract more app producers. That is, there exists many potential app producers, many of which end up not developing an app because their synchronization costs are too high. To simplify calculations, we further assume $v = f_1$ and $f_2 = 0$ (which implies the case of strong competition since $f_1 = v > \delta v$).[12]

[12]These assumptions are not that critical and they make computations straightforward: In the market

Monopoly

For given prices, the agents' participation decisions are implied by the marginal agents on both sides being indifferent between participation and opting out, in light of their expectations about the participation decisions on the opposite side of the platform. Thus, on the consumer side $u_1(\tau = N_1) \equiv 0$ implies $p_1 = v + (a - bN_1) \cdot N_2$ (see 2); and on the firm side $u_2(\theta = N_2) \equiv 0$ yields $p_2 = \alpha_2 \cdot N_1 - cN_2$ (see 4). We maintain the natural assumptions of consistent beliefs and no coordination failure.

Using these relations between participation and prices in conjunction with the platform's profit function (1), the monopolist's objective is to chose the implied participation levels, N_1 and N_2 to maximize

$$\Pi^M = N_1(v + (a - bN_1) \cdot N_2 - f_1) + N_2(\alpha_2 \cdot N_1 - cN_2 - f_2). \tag{8}$$

With $\alpha_1(\tau) = a - b\tau$, the highest marginal benefit any consumer has (namely a consumer of type $\tau = 0$) for firm participation is a. If the firms' constant marginal valuation of consumer participation exceeds that of consumers, $\alpha_2 > a \geq \alpha_1(\tau)$, then firms' gross willingness to pay (gross of the synchronization costs $c\theta$) exceeds that of all consumers. Hence the optimal platform strategy is to attract as many consumers as possible in order to make the platform as valuable as possible to firms. In turn, this allows the platform to extract a larger surplus from firms than was the cost of attracting consumers. Hence, whenever $\alpha_2 \geq a$ a corner solution is obtained in which the platform prices consumer participation such that all consumers join. The highest price to consumers that still attracts all consumers is $p_1 = v$. Given this price and the implied consumer participation of $N_1 = a/b = \overline{N}_1$, the platform

for smartphones and video game consoles it would be the case that both the marginal cost to produce the platform and the membership gains consumers receive are positive and approximately equal; and the cost to platforms of adding an additional app or game is nearly costless.

maximizes profits with respect to N_2 with $p_2 = p_2(N_1 = a/b, N_2) = \alpha_2 \frac{a}{b} - cN_2$. This yields

$$p_1^{M_C} = v, \quad p_2^{M_C} = \frac{a\alpha_2}{2b}, \text{ and} \tag{9}$$

$$N_1^{M_C} = \frac{a}{b} = \overline{N}_1, \quad N_2^{M_C} = \frac{a\alpha_2}{2bc}; \tag{10}$$

where M_C is a mnemonic that denotes the monopoly corner solution (with respect to consumers).

In contrast, when $\alpha_2 \leq a$, the consumers with the highest marginal benefit from firm participation have a higher willingness to pay than any firm has for consumers. As a result, consumers are charged higher prices and an interior equilibrium emerges, in which some consumers do not join the platform.

The second order conditions hold for this problem and it is straightforward to show that for the interior equilibrium the prices and allocations are

$$p_1^{M_I} = v + \frac{1}{16bc}(a+\alpha_2)^2(a-\alpha_2), \quad p_2^{M_I} = \frac{1}{8b}(a+\alpha_2)(3\alpha_2 - a), \text{ and} \tag{11}$$

$$N_1^{M_I} = \frac{1}{2b}(a+\alpha_2), \quad N_2^{M_I} = \frac{1}{8bc}(a+\alpha_2)^2; \tag{12}$$

where M_I denotes the interior monopoly solution.

There are two things worth noting in this equilibrium. First, recall the usual monopoly problem with linear inverse demand $P = a - bQ$ and marginal cost equal to zero, yielding the monopoly output of $Q^M = \frac{a}{2b}$. Now notice that $N_1^{M_I} > Q^M$, so in equilibrium, a monopoly platform will price to have more consumers than a traditional (one-sided) monopolist—even when there is no corner solution on the consumer side. This is because the added consumers generate additional surplus on the platform which makes the platform more attractive to firms.

More important is the second observation, namely that prices charged to firms can be negative, $p_2^{M_I} < 0$. That is, firms may be subsidized to join the platform. This occurs when consumers with a high willingness to pay for apps (small τ) value firm presence on the platform especially high compared to the value of a consumer to the firm, i.e., $a > 3\alpha_2$. The monopolist then invests in the attractiveness of the platform to consumers by paying firms to join. This investment is then recouped through higher prices to consumers.

Welfare Comparison

Consider now competing platforms. Because we are dealing with the case of strong competition, Theorem 2 holds and so $p_1^A = p_1^B = f_1 = v \geq 0$ and $p_2^A = p_2^B = f_2 = 0$. Moreover, from Theorem 2 we know that there can be up to three distinct allocations of consumers and firms in equilibrium.

Allocation I is an equilibrium allocation that exists for all parameter values. All consumers single-home and all firms multi-home: $n_1^m = 0$ and $n_2^A = n_2^B = n_2^m = N_2$. Given the prices from Theorem 2 in conjunction with our functional form assumptions, $n_1 := n_1^A = n_1^B = \frac{1}{2}\overline{N}_1 = \frac{1}{2}\frac{a}{b}$ and $n_2 := n_2^A = n_2^B = n_2^m = \frac{\alpha_2}{(1+\sigma)c}\frac{a}{b}$. From this follows:

Theorem 4 (Allocation I v. Monopoly)**.** *Whenever $\alpha_2 \geq a$ all consumers join a platform regardless of the market structure (corner monopoly solution); and there exists $\sigma^C := \frac{a+2\alpha_2}{3a+2\alpha_2} \in$ $(3/5, 1)$ such that monopoly generates more welfare than competition* iff $\sigma \geq \sigma^C$.

When $\alpha_2 < a$ competition serves all consumer types, whereas the monopoly limits consumer participation (interior monopoly solution); and yet there exists $\sigma^I := \frac{64\alpha_2 a^2}{5(a+\alpha_2)^3} - 1 \in$ $(-1, 1)$ such that monopoly generates more welfare than competition iff $\sigma \geq \sigma^I$.

Notice that competition always leads to all consumers joining a platform, $N_1^{AI} = \overline{N}_1$, whereas a monopoly excludes some when the willingness to pay is sufficiently large for the consumers with the highest marginal benefits from firm participation (the interior solution), $N_1^{M_I} < \overline{N}_1$.

Despite the (weakly) greater market coverage on the consumer side when platforms compete, whenever $\sigma > \max\left\{\sigma^I, \sigma^C\right\}$ the monopoly generates higher surplus than competition. The intuition for a superior outcome under monopoly when duplication costs are high is quite straightforward: in the competitive equilibrium of Allocation I all firms that join a platform end up multi-homing, and therefore incur the duplication cost of $\sigma c\theta$. Hence, the larger are the duplication costs, the more costly is the competitive solution—so much so that for sufficiently high duplication costs the monopoly platform welfare-dominates; even when it reduces participation on the consumer side.

Notice, however, that it is possible that a monopoly generates greater welfare even when there are no duplication costs ($\sigma = 0$) so there are no additional costs associated with firms joining a second platform. This can happen when the monopoly interior solution obtains ($\alpha_2 < a$) and $\sigma^I \leq 0$. The reason for this is that whenever $\sigma^I \leq 0$ then it must be that $3\alpha_2 < a$;[13] and for this case the monopolist subsidizes firm entry (i.e., $p_2^{M_I} < 0$, see 11). The reason for the firm subsidy is to increase the total welfare that the platform generates so that the monopolist can appropriate this through higher prices to consumers.[14]

In contrast, in the competitive equilibrium the welfare-enhancing investment in firm-entry does not take place: a platform that subsidizes firms must make up for the cost of doing so by charging higher consumer prices. However, when $\sigma = 0$, all firms that obtain the investment subsidy will join both platforms; and so the platform that doesn't make the investment in firm participation reaps the same reward as does the rival platform that does subsidize firm entry—thus placing the non-investing platform at a competitive advantage. Therefore welfare-increasing firm subsidies do not occur when platforms compete.

Turning to the comparison between monopoly and Allocation II under competition, note

[13]For $\alpha_2 < a$ (the interior solution) σ^I is strictly increasing in α_2. However, when $\alpha_2 = \frac{1}{3}a$, $\sigma^I > 0$, so whenever $\sigma^I \leq 0$, it follows that $\alpha_2 < \frac{1}{3}a$.

[14]This internalization of welfare considerations by the monopoly platform and not by competing platforms also occurs in comparing "closed" platforms with open platforms; see Hagiu (2007) for more information.

from Theorem 2 that a type-II Allocation may not exist. Indeed, it only occurs when network effects are sufficiently strong, viz., $(1-\delta)v < \frac{a\alpha_2\overline{N}_1}{8c}$.[15]

Theorem 5 (Allocation II v. Monopoly)**.** *When duplication costs are zero ($\sigma = 0$) competition always generates more total surplus; but regardless of whether the monopoly has an interior or a corner solution, there exists mixed allocations such that the welfare from the competitive mixed allocation equilibrium is greater than the welfare with the monopoly platform; even when there are no savings in duplication ($\sigma = 1$).*

Thus, when under competition the mixed allocation emerges in equilibrium then sufficiently strong duplication economies (small enough σ) will assure greater welfare from competition than in monopoly, because firms are able to cheaply multi-home. However, Theorem 5 also makes clear that the converse need not hold. That is, even when it is costly for firms to multi-home in terms production and synchronization costs ($\sigma = 1$), competition can generate greater welfare in the mixed allocation.

Taken together, Theorems 4 and 5 show that when duplication economies are small (large σ) monopoly is preferred to competition, unless under competition a mixed allocation emerges in which not all firms multi-home. Notice also that for the case of an interior monopoly solution and parameters such that $\sigma^I < 0$, then for sufficiently large duplication effects (σ small) competition generates less welfare than monopoly if with competition all consumers single-home, but competition generates more welfare if a mixed-homing equilibrium emerges, because multi-homing is not expensive and the lower prices increase network effects.

Consider now Allocation III. To assure existence of this equilibrium, let $v = f_1 = 0$. Prices are $p_1 = 0$ and $p_2 = 0$, all firms single-home, and all consumers multi-home.

[15]This follows from Equations 13 and 14 and since x in the proof of Theorem 2 must be in $[0, 1]$. Also see the proof of Theorem 5.

Theorem 6 (Allocation III v. Monopoly)**.** *Whenever $\alpha_2 \geq \frac{1}{4}a$ the competitive equilibrium in which all firms single-home and all consumers multi-home generates greater welfare than the monopoly outcome.*

Notice that the welfare comparison between monopoly and competition does not involve duplication economies σ, since in either case firms only single-home. Also, when $\alpha_2 \in \left(\frac{1}{4}a, \frac{1}{3}a\right)$, then the monopoly platform subsidizes firms to increase the value of the platform and increase overall welfare, but the lower prices on the consumer side that occur under competition more than offset this so that competition leads to greater welfare.

Lastly, because Allocation III does not involve any multi-homing by firms it is possible that it generates greater welfare than Allocation I.[16] This then implies that welfare can be non-monotone in duplication economies across the degree of competitiveness in the market: In particular, there are parameters under which for small enough σ competition with Allocation I generates the highest welfare, whereas increases in σ lead to the monopoly generating the greatest welfare, only to be dominated by the competitive Allocation III upon further increases in σ.

Conclusion

In many markets in which platforms compete against each other agents choose to join either one platform (single-home) or they join several platforms (multi-home). While most of the previous literature on platform competition has assumed this decision to be given exogenously, we allow participants on both sides of the platform to endogenously make an optimal homing decision.

When platforms are sufficiently differentiated in the sense that membership at a second platform bestows an additional stand-lone benefit apart from access to more agents on the

[16]This occurs when both equilibrium configurations exist and $\sigma > \frac{\alpha_2}{3\alpha_2+4a}$.

other side, then platforms set prices to attract multi-homers, blunting head-to-head price competition between platforms (weak competition). In contrast, when added stand-along values are small relative to the cost of providing membership benefits, platforms engage in fierce competition for single-homers (strong competition), which leads to a zero-profit equilibrium with marginal-cost pricing, akin to the Bertrand Paradox. While we find that there is a unique pricing equilibrium, there are potentially multiple equilibrium allocations concerning how consumers and firms dived themselves onto the competing platforms.

One equilibrium allocation that always exists entails all consumers single-homing and all firms multi-homing. This mirrors the allocation in the market for smartphones, where virtually all consumers own only a single phone, but virtually all apps are available across competing smartphones.

When network effects are strong enough, another type of equilibrium allocation emerges in which there is a mix of multi-homing and single-homing on both sides of the platforms. This constellation is found in the market for video game consoles: while many consumers have only one console, serious gamers often have more than one system; and while some games are available across providing platforms, others are exclusive to one system.

The third possible equilibrium constellation has all firms single-homing, whereas some or all consumers multi-home. This occurs when there is no stand-alone benefit to consumers of accessing the platform, that is, consumers are exclusively interested in the service provided by firms. This market structure is found with the rideshare services Uber and Lyft, where drivers sign up with one or the other platform (single-home), but some consumers multi-home to compare prices and availability across the services.

Compared to the lower prices and possibly greater access to services provided by competition between platforms, a monopoly platform may (but need not) generate higher welfare compared to any of the three possible allocations under competition. This is because the monopoly may better concentrate network effects and prevents cost-redundancies when firms multi-home. Moreover, the monopolist may invest in the value of the platform by subsidizing firm entry and thereby increase total welfare—whereas such investments are not undertaken in competition due to non-appropriability of the investments.

We find that decreasing marginal network effects adversely affect welfare under monopoly more than with competition, but that increased entry beyond two platforms is unlikely to increase welfare. Finally we show that due to endogenous homing exclusive contracts and focal platforms may skew market shares, but a complete tipping of the market is often prevented as disadvantaged platforms can set prices so as to attract some multi-homers, which proves to be a sufficiently strong foothold to preserve competition.

APPENDIX

Appendix of Proofs

Proof of Proposition 1 Suppose that $p_i^X = p_i^Y$, but there are non-equal participation levels on either side of the market result. In this case, agents on the smaller platform will deviate to the larger platform. That is, if $n_2^X > n_2^Y$ then $u_1^X(\tau) > u_1^Y(\tau)$ for all τ which results in all consumers joining Platform X and none joining Platform Y; all consumers joining Platform X implies that $u_2^X(\theta) > u_2^Y(\theta)$ so that all firms will also join Platform X, but this violates are assumption regarding no dis-coordination. Similarly, if $n_1^X > n_1^Y$ then all firms will join Platform X and none will join Platform Y and this violates our assumption. Thus, it must be that $n_1^A = n_1^B$ and $n_2^A = n_2^B$ in equilibrium. Condition (6) is generated from the marginal agents found using Equations (2) and (3), i.e. solving for τ when $u_1^X(\tau) = 0$ and $u_1^{AB}(\tau) = 0$. Similarly, Condition (7) is generated from the marginal agents found with Equations (4) and (5). □

Proof of Proposition 2 Given Platform Y has better prices, our no dis-coordination assumption implies that $n_2^Y > 0$. If Platform X does not have prices so low that it induces multi-homing on either side of the market ($p_1^X > \delta v$ and $p_2^X > 0$), then it must be that all participation occurs on Platform Y. If not, suppose Platform X has participation, then either $u_2^X(\theta) > u_2^Y(\theta)$ and each agent on Platform Y deviates to Platform X, this contradicts $n_2^Y > 0$, or $u_2^X(\theta) < u_2^Y(\theta)$ and each agent on Platform X deviates and join Platform Y. Thus, the only equilibrium allocation is when all (non-subsidized) participation occurs on Platform Y. This implies that on the consumer side we have $n_1^Y = N_1$ and $n_1^X = 0$ only when $p_1^X > \delta v$ and if $p_1^X \leq \delta v$ then $n_1^X = n_1^m > 0$. On the firm side of the market, this means that $n_2^Y = N_2$ and $n_2^X = 0$ only when $p_2^X \geq 0$ and if $p_1^X < 0$ then $n_1^X = n_1^m > 0$. □

Proof of Proposition 3 In this case, there are several outcomes that can occur. First, either tipping equilibria can occur. That is, all consumers and all firms joining Platform X for $X = A$ or B while no one joins the other platform, unless they are subsidized to multi-home, are equilibrium allocations.

Furthermore, the case where consumers segregate (low type τ join the high consumer priced platform, Platform X, and high type τ consumers join Platform Y) is characterized by the following equations and also requires that $n_2^X > n_2^Y$.

$$u_1^X(\tau = n_1^X) = v + \alpha_1(n_1^X)n_2^X - p_1^X = v + \alpha_1(n_1^X)n_2^Y - p_1^Y = u_1^Y(\tau = n_1^X),$$

$$u_1^Y(\tau = N_1) = v + \alpha_1(N_1)n_2^Y - p_1^Y = 0,$$

$$n_2^X = \frac{\alpha_2 n_1^X - p_2^X}{c},$$

$$n_2^m = n_2^Y = \frac{\alpha_2 n_1^Y - p_2^Y}{(1+\phi)c}.$$

The first equation gives the marginal agent that is indifferent between facing the higher priced Platform X that has more firms available and the lower priced Platform Y that has fewer firms available. Note that if $n_2^X < n_2^Y$ then no such agent exists and this equilibrium does not exist. The second equation gives the last consumer to join Platform Y where the consumers that join Platform Y are the $\tau \in (n_1^X, N_1]$ so that $n_1^Y = N_1 - n_1^X$. Consumers single-homing results in firms multi-homing. The firms that join Platform X are given by Equation (4) set equal to zero which generates the third equation above. Lastly, given the firms multi-home and that $n_2^X > n_2^Y$ is required, it must be that the last firm to join Platform Y is the last multi-homing firm given in the fourth equation which is generated from Equation (5). This constellation does not exist when the consumer price difference is so large that either $n_2^X > n_2^Y$ fails or the first equation above does not give a $\tau \in [0, \overline{N}_1]$. $\square$

Proof of Theorem 1 Given these prices it is clear that both platform's profits are $\Pi^X = \Pi^Y = \overline{N}_1(\delta v - f_1) > 0$. Furthermore, the allocation follows since, $p_1 = \delta v$ all consumers will multi-home which implies all the firms that participate will single-home. To prove unique prices, we first show that this is the unique symmetric price constellation that can occur in equilibrium and then we show that there cannot exist an asymmetric price constellation in equilibrium.

When the platforms set equal prices on both sides of the market, then $p_1^X = p_1^Y = \delta v$ and $p_2^X = p_2^Y = f_2$ is the only equal price constellation where neither platform has an incentive to deviate. At any $p_1 < \delta v$ both platforms will increase their consumer price. If $p_1 > \delta v$ then both platforms have an incentive to undercut the other platform as $N_1(p_1 - \epsilon - f_1) > {}^{N_1}\!/_2(p_1 - f_1)$. Similarly, for any $p_2 \neq f_2$.

When $p_i^X \geq p_i^Y$, $i = 1, 2$, with at least one inequality being strict, if $\Pi^Y > 0$ then Platform X will undercut Platform Y's prices and make positive profits, and if $\Pi^Y = 0$ then Platform Y will increase its prices but still undercut Platform X's prices. Thus, a price deviation occurs.

When $p_1^X > p_1^Y$, $p_2^X < p_2^Y$, and $p_1^X, p_1^Y > \delta v$ then at least one of the platforms faces a positive probability of facing the failure to launch reputation cost, in which case profits will be negative and they will deviate. Lastly, when $p_1^X > p_1^Y$, $p_2^X < p_2^Y$, and $p_1^X, p_1^Y \leq \delta v$ then Platform Y will lower their price to firms and increase profits.

Thus, the unique set of prices that occurs in equilibrium is $p_1^X = p_1^Y = \delta v$ and $p_2^X = p_2^Y = f_2$. □

Proof of Theorem 2 Given these prices it is clear that both platforms make zero profits. Furthermore, each platform having the same number of consumers and the same number of firms must occur in equilibrium; otherwise, if one platform has more participation but prices are symmetric then every agent on the smaller sized platform has an incentive to deviate to the larger platform. Furthermore, the no-discoordination assumption implies that the tipping outcomes cannot occur in equilibrium as symmetric prices imply it must be that $n_2^X, n_2^Y > 0$. Thus, participation must be symmetric across platforms in equilibrium with symmetric prices. To prove the remainder of this theorem, we first show that the equilibrium prices must be the specified symmetric price constellations with price equal to marginal cost on each side of the market, and then we show the equilibrium allocations that can occur under symmetric prices.

Consider the pricing game platforms. When $p_i^X \geq p_i^Y$, $i = 1, 2$, with at least one inequality being strict, if $\Pi^Y > 0$ then Platform X will undercut Platform Y's prices and make positive profits, and if $\Pi^Y = 0$ then Platform Y will increase its prices but still undercut Platform X's prices. Thus, a price deviation occurs.

Now consider the orthogonal pricing ($p_1^X > p_1^Y$, $p_2^X < p_2^Y$ when $p_1^X, p_1^Y > \delta v$). In this case, if each of the platforms has a positive probability of facing the failure to launch reputation cost then there is some reputation cost that is large enough so that expected profits will be negative. As a result, a price or entry announcement deviation will occur. Thus, an orthogonal price constellation cannot occur in equilibrium.

To prove that the unique symmetric price constellation is where the price on each side of the market is equal to the corresponding marginal cost, consider the symmetric price constellation where one side's price is less than marginal cost while the other side is greater than marginal cost ($p_i < f_i$ and $p_j > f_j$). In this case, each platform has an incentive to undercut the price on the profitable side of the market while capping the participation on the subsidized side of the market. This results in greater profits while keeping costs the same; a net increase in profits. Thus, $p_i = f_i$ with $i = 1, 2$ is the unique price constellation in equilibrium.

We now show the equilibrium allocations for general symmetric prices p_1 and p_2.

Allocation I: Allocations (7) imply all firms multi-home when all consumers single-home, since $n_2^m > N_2$ i.e. $[n_2^m, N_2]$ is empty when $n_1^m = 0$. Furthermore, when $n_2^m = n_2^A = n_2^B$, allocation (6) implies no consumer multi-homes. Hence, all consumers single-home if and only if all firms multi-home. Thus, the allocation where all firms multi-home and all consumers single-home is a Nash Equilibrium.

Allocation II**:** Since $p_1 > \delta v$, allocation (6) implies the set of multi-homing consumers is non-empty when the number of multi-homing firms is not to large. Let $x \in [0,1]$ be the percent of consumers who multi-home of those n_1^X who join platform X so that in expectation $n_1^m = xn_1^X = xn_1^Y$. This implies $N_1 = (2-x)n_1^X$ since $N_1 = n_1^X + n_1^Y - n_1^m$ and in expectation $n_1^X = n_1^Y$. From the Allocation I $x > 0$ occurs when not all of the firms are multi-homing. From Equation 7, this occurs when $\min\left\{\frac{\alpha_2\cdot(2-x)n_1^X-2p_2}{(1+\sigma)c}, \frac{\alpha_2\cdot(1-x)n_1^Y-p_2}{\sigma c}\right\} < \frac{\alpha_2 n_1^X - p_2}{c}$.

In the remainder of this proof we assume $\sigma = 1$, no economies of duplication. Using allocation (7) there exists x^m such that for $x > x^m$ no firm will multi-home. Allocation (7) implies $0 = \alpha_2(1-x^m)n_1^Y - p_2$. Thus, $x^m = 1 - \frac{p_2}{\alpha_2 n_1^Y}$. And for all $x > x^m$ no firm multi-homes. Note, $p_2 < \alpha_2 n_1^Y$ since otherwise the market collapses, hence $x^m \in (0,1)$.

If $0 < x < x^m$ then some firms will single-home and some firms will multi-home. Allocation (7) implies $n_2^m = \frac{\alpha_2(1-x)n_1^Y - p_2}{c}$ and allocation (7) implies $n_2^Y = (1/2)(N_2 + n_2^m) = (1/2c)[\alpha_2(2-x)n_1^X - 2p_2]$. Similarly, allocation (6) defines the number of multi-homing consumers: $0 = \delta v + \alpha_1(n_1^m)(n_2^Y - n_2^m) - p_1$; using this equation and the equations for n_2^m, n_2^Y, and $n_1^m = xn_1^X = xn_1^Y$ we can characterize x by:

$$\begin{aligned} 0 &= \delta v + \alpha_1(xn_1^X)(1/2c)[\alpha_2(2-x)n_1^X - 2p_2 - 2\alpha_2(1-x)n_1^Y + 2p_2)] - p_1 \\ &= \delta v + \alpha_1(xn_1^X)(1/2c)[\alpha_2 \cdot xn_1^X] - p_1, \end{aligned} \tag{13}$$

Furthermore, allocation (6) defines N_1, the number of consumers on Platform X: $0 = v + \alpha_1(N_1)n_2^X - p_1$. Thus we have:

$$0 = v + \alpha_1(N_1)n_2^X - p_1 = v + \alpha_1((2-x)n_1^X)(1/2c)(\alpha_2 \cdot (2-x)n_1^X - 2p_2) - p_1. \tag{14}$$

Thus, we have two equations (13) and (14) and two unknowns, x and n_1^X. If the solution is $x \in (0, x^m)$ then we have a Nash Equilibrium. Note, this equilibrium does not exist when $x \notin (0, x^m)$.

Allocation III**:** Allocation (7) implies all firms single-home when the number of multi-homing consumers is $n_1^Y \leq n_1^m + p_2/\alpha_2$. If $p_2 = 0$, then this holds when all consumers multi-home. By allocation (6), this will only be an equilibrium when $\delta v = p_1$. If $p_2 > 0$, then allocation (6) implies there exists an equilibrium where all firms single-home and a large portion of consumers multi-home given prices such that $N_1 - n_1^m = \alpha_1^{-1}(\frac{p_1 - v}{n_2^X}) - \alpha_1^{-1}(\frac{p_1 - \delta v}{n_2^X}) \leq \frac{2p_2}{\alpha_2}$, i.e., $\delta v + \epsilon = p_1$ for small $\epsilon > 0$.

Thus, there exists at least one and potentially three allocations that occur in equilibrium with unique equilibrium prices $p_1^X = p_1^Y = f_1$ and $p_2^X = p_2^Y = f_2$. □

Proof of Theorem 3 Consider the case when one of the platforms, say Platform X, does not have a positive probability of facing the shut out allocation when prices are orthogonal. In this case, we must rule out that the symmetric equilibrium found in Theorem 2 will not occur. If prices are symmetric then Platform X would deviate so that $p_1^X > p_1^Y = f_1$ and $p_2^X < p_2^Y = f_2$ and either participation tips so that Platform X gains all participation and makes positive profits, or the second allocation in Proposition 3 occurs where Platform X makes positive profits as it captures the high demand consumers (lower τ types), who are willing to pay a higher consumer price while only needing to reduce the firm price, p_2^X, by ϵ below marginal cost. Thus, the only pure strategy equilibrium is for Platform X to be the only platform that announces entry and then acts as a monopolist. □

Proof of Theorem 4 Given the monopoly corner solution, we calculate the standard welfare measures.

$$\Pi^{M_C} = N_1(p_1^{M_C} - f_1) + N_2(p_2^{M_C} - f_2) = N_1 \times 0 + N_2 p_2^{M_C} = \frac{a^2\alpha_2^2}{4b^2c}, \tag{15}$$

$$CS^{M_C} = \int_0^{N_1^{M_C}} \left(v + \alpha_1(\tau)N_2^{M_C} - p_1^{M_C}\right) d\tau = \frac{a^3\alpha_2}{4b^2c}, \tag{16}$$

$$FS^{M_C} = \int_0^{N_2^{M_C}} \left(\alpha_2 N_1^{M_C} - c\theta - p_2^{M_C}\right) d\theta = \frac{a^2\alpha_2^2}{8b^2c}, \tag{17}$$

$$W^{M_C} = \frac{a^2\alpha_2}{8b^2c}(3\alpha_2 + 2a). \tag{18}$$

For the monopoly interior case: given equilibrium prices, the number of consumers, and the number of firms, we calculate platform profits, consumer, firm, and welfare.

$$\Pi^{M_I} = N_1(p_1^{M_I} - f_1) + N_2(p_2^{M_I} - f_2) = \frac{\left(a + \alpha_2\right)^4}{64b^2c}, \tag{19}$$

$$CS^{M_I} = \int_0^{N_1^{M_I}} \left(v + \alpha_1(\tau)N_2^{M_I} - p_1^{M_I}\right) d\tau = \frac{(a + \alpha_2)^4}{64b^2c}, \tag{20}$$

$$FS^{M_I} = \int_0^{N_2^{M_I}} \left(\alpha_2 N_1^{M_I} - c\theta - p_2^{M_I}\right) d\theta = \frac{(a + \alpha_2)^4}{128b^2c}, \tag{21}$$

$$W^{M_I} = \frac{5(a + \alpha_2)^4}{128b^2c}. \tag{22}$$

Lastly, the welfare results for Allocation I with competing platforms.

$$\Pi^A = \Pi^B = 0, \tag{23}$$

$$CS^{AI} = \int_0^{a/b} (a - b\tau)\frac{2\alpha_2 \cdot n_1}{(1+\sigma)c} d\tau = \frac{a^3\alpha_2}{2(1+\sigma)cb^2}, \tag{24}$$

$$FS^{AI} = \int_0^{n_2} (\alpha_2 \overline{N}_1 - 2c\theta - 2p_2) d\theta = \frac{a^2\alpha_2^2}{2(1+\sigma)cb^2}, \tag{25}$$

$$W^{AI} = \frac{a^2\alpha_2}{2(1+\sigma)cb^2}(\alpha_2 + a); \tag{26}$$

where the superscript AI denotes Allocation I.

A monopoly corner solution occurs when $\alpha_2 \geq a$. Using the welfare equations (18) and (26), $W^{AI} < W^{M_C}$ occurs when $\sigma > \frac{a+2\alpha_2}{3a+2\alpha_2}$. A monopoly interior solution occurs when $\alpha_2 < a$. Using welfare equations (22) and (26), $W^{AI} < W^{M_I}$ occurs when $\sigma > \frac{64\alpha_2 a^2}{5(a+\alpha_2)^3} - 1$. $\square$

Proof of Theorem 5 We first show that Allocation II in Theorem 2 exists when $\frac{b(1-\delta)vc}{a^2\alpha_2} \in (0, \frac{1}{8})$: Equations (13) and (14) imply we have two equations and two unknowns, x and n_1^A. Solving these equations implies x is implicitly defined by: $t \equiv \frac{b(1-\delta)vc}{a^2\alpha_2} = \frac{(1-x)x}{(2-x)^2}$. This implies $0 = (1+t)x^2 - (1+4t)x + 4t$. Solving for x as a function of t and using the quadratic formula such that $x \in (0,1)$ implies we must have $t \in (0, \frac{1}{8})$. Thus, Allocation II exists if and only if $(1-\delta)v < \frac{a\alpha_2\overline{N}_1}{8c}$.

Consider now the Theorem. When $x = 1/2$, equations (13) and (14) imply half of firms and a third of consumers will multi-home. The welfare from this allocation is greater than the welfare from the monopoly interior solution if and only if $0 > 135a^4 - 484a^3\alpha_2 - 150a^2\alpha_2^2 + 540a\alpha_2^3 + 135\alpha_2^4$. This occurs when $\alpha_2 \in [r_1 \cdot a, r_2 \cdot a]$ where $r_1 \cdot a = \alpha_2$ and $r_2 \cdot a = \alpha_2$ are the roots of the preceding polynomial. However, the welfare for $x = 1/2$ is never greater than the monopoly corner solution.

When $x = .9$, equations (13) and (14) imply a tenth of firms and (8/11)s of consumers will multi-home. The welfare from this allocation is greater than the welfare from the monopoly corner solution for all $\alpha_2 \geq a$ since $3.3388\alpha_2 + 3.5823a > 3\alpha_2 + 2a$. $\square$

Proof of Theorem 6 In Allocation III all firms single-home and all consumers multi-home; we have $n_2^m = 0$ and $n_1^A = n_1^B = n_1^m = \overline{N}_1 = \frac{a}{b}$. With $p_1 = 0$ and $p_2 = 0$ we have

$n_2^A = n_2^B = (1/2)N_2 = \frac{\alpha_2 a}{bc}$; resulting in

$$\Pi^A = \Pi^B = 0, \tag{27}$$

$$CS^{AIII} = \int_0^{a/b} (a - b\tau)\frac{\alpha_2 a}{bc} d\tau = \frac{a^3\alpha_2}{2(cb^2}, \tag{28}$$

$$FS^{AIII} = \int_0^{\frac{\alpha_2 a}{bc}} \alpha_2 \overline{N}_1 - c\theta d\theta = \frac{3a^2\alpha_2^2}{8cb^2}, \tag{29}$$

$$W^{AIII} = \frac{a^2\alpha_2}{8cb^2}(3\alpha_2 + 4a); \tag{30}$$

where the superscript $AIII$ denotes Allocation III.

By comparing equations (18) and (30) we see that $W^{AIII} > W^{M_C}$ always holds. The corner solution for the monopoly platform is implemented when $\alpha_2 \geq a$; thus, $W^{AIII} > W^M$ when $\alpha_2 \geq a$. When an interior solution occurs, equations (22) and (30) imply $W^{AIII} > W^{M_I}$ if and only if $0 > 5a^4 - 44a^3\alpha_2 - 18a^2\alpha_2^2 + 20a\alpha_2^3 + 5\alpha_2^4$. This occurs when $\alpha_2 > (1/4)a$. Thus, $W^{AIII} > W^M$ when $\alpha_2 > (1/4)a$. $\square$

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Vertical Relationships within Platform Marketplaces

Introduction

Over the last ten years smartphones have become ubiquitous. In 2013, the smartphone market reached 1 billion units sold to consumers worldwide.[1] In addition to being an important consumer good in its own right, the smartphone also provides consumers with the opportunity to purchase applications (apps) that can be used on the smartphone. For example, consumers can purchase gaming apps, weather apps, map apps, ridesharing apps, etc. and these apps make the platform more valuable to consumers. Apps are often provided by third party developers that do not produce the smartphone. Thus, the smartphone acts as a platform that creates a marketplace where consumers and app providers interact within sub-markets, the individual markets for different app types.

This structure exists in many economic markets: video game consoles provide consumers with sub-markets for many different genres of games that are developed by competing game developers; eReaders connect readers with book publishers creating sub-markets for different genres of books; and online marketplaces like Amazon connect consumers with product sub-markets ranging across many retail items. In these platform marketplaces, consumers care about the variety of content (the availability of many sub-markets) and the prices for content within these sub-markets. If there are many sub-markets on the platform and there is considerable competition within each sub-market so that the price of content is low, then the platform is very desirable for consumers. However, more competition within sub-markets also implies that joining the platform for a seller will be less profitable. Lower seller profits implies less entry by sellers onto the platform which results in fewer sub-markets on the platform marketplace. Thus, the competition structure affects network effects for both consumers and sellers that join the platform.

[1] For more statistics on smartphones, see Bouchard et al. (2014).

In this paper I show how competition among sellers affects the platform's pricing strategies, the resulting levels of content and consumer participation on the platform, and the welfare that is generated on the platform. Participation by consumers and the availability of products on the platform are endogenously determined through the membership prices that the platform charges consumers and sellers. For a given set of platform prices, more competition reduces deadweight loss within each sub-market which increases total surplus. At the same time, more competition lowers profits which implies fewer sub-markets on the platform marketplace which decreases total surplus. Thus, there exists a tradeoff from changes in competition within a sub-market and the platform's pricing strategies, the amount of participation by consumers and sellers, and the welfare generated by the platform depend on the mode of competition that exists within each sub-market.

The tradeoff between deadweight loss and the number of sub-markets available to consumers leads to interesting pricing strategies by the platform. I find that when sellers have more market power within a sub-market, the platform lowers its prices to consumers and sellers so that there is more participation on each side of the platform. As participation is what generates surplus on the platform, this pricing strategy by the platform leads to several interesting results regarding welfare.

I find that less competition within each sub-market causes an increase in the number of sub-markets on the platform marketplace. The added surplus from additional sub-markets is enough to overcome the additional deadweight loss from increased seller market power. In other words, the total deadweight loss across all of the sub-markets from less competition at the seller level is overcome by the additional surplus generated from more sub-markets on the platform marketplace resulting in greater welfare. However, if the gains to consumer surplus from an increase in competition within a sub-market are sufficiently large then this argument fails and welfare improves. These findings show the importance of the sub-market structure when investigating pricing and welfare on a two-sided platform.

Many of the original papers on platforms and two-sided markets, Rochet and Tirole

(2003, 2006), Hagiu (2006), and Armstrong (2006), as well as subsequent work, assume that agents have homogeneous network effects. Furthermore, the magnitude of the network effect on each side of the market is independent of the network effect on the other side of the market. That is, the platform literature has abstracted from the sub-market structure that generates the network effects between consumers and sellers.

Solving for equilibria when agents have homogeneous network effects on each side of the market requires assumptions on participation decisions of agents. Another concern is that homogeneous network effects do not coincide with the empirical evidence of Lee (2013) and Bresnahan et al. (2015), who find that the network benefits that consumers receive from video games and apps vary across consumers. Thus, allowing for heterogeneity is important in modelling platforms. Jeitschko and Tremblay (2015) develop a model with heterogeneous consumers and find equilibria that correspond to many platform markets, including smartphones and video game consoles. However, they do not consider the pricing relationship that exists between consumers and sellers.[2] This paper allows for heterogeneity and aims to illustrate the importance of the sub-market structure that exists within a platform in analyzing network effects, platform pricing strategies, and platform participation.

The relationship that exists between a platform and its sellers relates to the traditional models on vertical relationships which dates back to Spengler (1950). He shows that when a wholesaler and a retailer each have market power, double marginalization occurs, where each firm along the supply chain adds a market-power markup and final prices exceed the simple monopoly price. He finds that a vertical merger between wholesalers and retailers improve efficiency by lowering the final price while increasing profits and consumer surplus.

The research on platforms relating to vertical relationships is very limited. Lee (2013) investigates empirically exclusive deals, which can be interpreted as vertical integration. He shows how exclusive deals between video game platforms and video game developers enable entry into the market for video game consoles. Lee finds that vertical integration helps a

[2]Deltas and Jeitschko (2007) also show how heterogeneity on the consumer side plays a critical role in the platform profit maximization problem.

platform enter the market and compete with an incumbent platform. However, his model does not lend itself to investigating the effect of vertical relationships on prices.

The vertical relationship in a two-sided market for credit cards is analyzed in a different model by Shy and Wang (2011). The focus of their work is on the fee structures for consumers and merchants used by credit card companies. They find that greater merchant competition leads to lower prices for consumers and increased welfare when there is a monopoly platform. This is consistent with the usual double marginalization result. However, Shy and Wang assume homogeneous agents on each side of the market and a network benefit structure that does not coincide with the empirical findings of Lee (2013) and Bresnahan et al. (2015). Furthermore, participation on the consumer side of the market is exogenously given: consumers do not choose whether or not to join the platform. This paper is not limited by these assumptions.

The Model

There are three types of players: a platform, consumers who join the platform on one side of the market, and sellers who make up the other side. Consumers benefit by purchasing products that are available on the platform. Sellers must join the platform to make their products available to consumers. The platform earns profits by charging consumers and sellers to join the platform.[3] The sub-market structure that exists between the consumer and seller sides of the platform is developed first in the following subsection.

Consumers and Sellers

On Side 1 there exists a mass of consumers, normalized to 1, with individual consumers indexed by $\tau \in [0, 1]$. The number of consumers that decide to join the platform is denoted

[3] I will use the terminology "join the platform" in two ways. Depending on the type of platform market, consumers either join the platform (e.g., Netflix, Amazon, Pandora, etc.) or consumers purchase the platform (e.g., video game consoles, smartphones, digital devices, etc.).

by $N_1 \in [0,1]$. Once a consumer joins the platform they can engage in transactions on the platform marketplace. If sub-market θ is available on the platform marketplace then the consumer surplus for consumer τ from sub-market θ, when there are n sellers in the sub-market whose competition structure is defined by $\mathcal{C}$, is given by $cs(\tau, \theta, n, \mathcal{C})$.[4]

Consumers are heterogeneous in their likelihood to value a sub-market (type of product or category of content). That is, consumers differ in the number of sub-markets that they demand. For example, teens have many types of apps on their smartphones relative to their parents. Suppose that a consumer of type τ is only interested in a particular sub-market θ with probability $(1-\tau)$. Thus, τ captures the probability that a consumer has any interest in participating in a given sub-market.[5] More formally, suppose that for sub-market θ a consumer of type τ has $cs(\tau, \theta, n, \mathcal{C}) > 0$ with probability $(1-\tau)$ and has $cs(\tau, \theta, n, \mathcal{C}) = 0$ with probability τ. Thus, the expected consumer surplus for consumer τ from sub-market θ is:

$$E[cs(\tau, \theta, n, \mathcal{C})] = cs(\theta, n, \mathcal{C}) \cdot (1-\tau), \tag{1}$$

where $cs(\theta, n, \mathcal{C})$ is the expected surplus a consumer receives from sub-market θ, given that the consumer is interested in the product. That is, a consumer who is interested in sub-market θ may not purchase the product if the price is too high. Hence, $cs(\theta, n, \mathcal{C})$ is an expected surplus.

Side 2 contains products and their sellers. Products are indexed by $\theta \in [0, \infty)$ and the number of products (sub-markets) that are available on the platform is denoted by N_2. I use θ to represent both a product and its sub-market.[6] Each seller of sub-market θ receives profits from a consumer of type τ given by $\pi(\tau, \theta, n, \mathcal{C})$. That is, profitability differs across sub-markets and across consumers, and profitability depends on the competitive structure

[4]Note, consumer surplus as a function of n and $\mathcal{C}$ is more general than as a function of the set of prices set by the n sellers as the type of competitive structure can affect consumer surplus. One case occurs when products sold by the n sellers are differentiated. Hence, consumer surplus is simply defined by n and $\mathcal{C}$ and subsumes price changes.

[5]This is common in platform marketplaces where consumers vary in the number of products that interest them.

[6]Thus, more products on the platform coincide with more sub-markets on the platform.

and the number of sellers within the sub-market.[7] I assume profits are twice differentiable in n for a given competitive structure, $\mathcal{C}$.

For sellers, this consumer structure implies that some consumers simply will not make a purchase as they are not interested in the product. Thus, the expected profits for a seller of sub-market θ from a consumer τ is given by

$$E[\pi(\tau, \theta, n, \mathcal{C})] = \pi(\theta, n, \mathcal{C}) \cdot (1 - \tau), \tag{2}$$

where $\pi(\theta, n, \mathcal{C})$ is the expected profit that a seller would make from a consumer that is interested in the product.[8] Equation (2) implies that when there are N_1 available consumers on the platform, a seller of sub-market θ will have profits given by:

$$\int_0^{N_1} \pi(\theta, n, \mathcal{C}) \cdot (1 - \tau) d\tau = \pi(\theta, n, \mathcal{C}) \cdot \left(1 - \frac{N_1}{2}\right) N_1. \tag{3}$$

Notice how seller profits change with the number of consumers who join the platform. Sellers always prefer more consumers on the platform, $\frac{\partial \pi}{\partial N_1} > 0$, as this raises demand for their products. However, consumer heterogeneity implies that the marginal consumer who joins the platform is the least likely to purchase their products. Hence, sellers have decreasing marginal benefits from consumer participation on the platform, $\frac{\partial^2 \pi}{\partial N_1^2} < 0$. This differs from the previous literature, including Rochet and Tirole (2003, 2006), Armstrong (2006) and subsequent work, including more recent papers by Shy and Wang (2011) and Jeitschko and Tremblay (2015).

When considering whether or not to join the platform, consumers and sellers take expectations over the popularity of a product, consistent with the "experience good" nature of many relevant products.[9] Thus, the expected consumer surplus and the expected profit for

[7]Notice that demand for products differ across sub-markets as both consumer surplus and seller profits are functions of θ.

[8]Similar to consumer surplus, $cs(\theta, n, \mathcal{C})$, the term $\pi(\theta, n, \mathcal{C})$ is an expected profit as some consumers that are interested in the product may not purchase it.

[9]This is the case for many apps and games where the quality is unclear upon release to the market and

a seller from a given product are given by:

$$\int_0^{N_2} cs(\theta, n, \mathcal{C}) \cdot (1-\tau) d\theta = cs(n, \mathcal{C}) \cdot (1-\tau), \tag{4}$$

$$\int_0^{N_2} \pi(\theta, n, \mathcal{C}) \left(1 - \frac{N_1}{2}\right) N_1 d\theta = \pi(n, \mathcal{C}) \left(1 - \frac{N_1}{2}\right) N_1, \tag{5}$$

where $cs(n, \mathcal{C})$ is the expected consumer surplus across products for a consumer, given the consumer is interested in a product, and $\pi(n, \mathcal{C})$ is the expected profit from a consumer across product popularity, given that consumer is interested in the product.

When consumers join the platform they have access to all of the products that are available on the platform, N_2. Thus, by joining the platform consumer τ's expected utility is given by:

$$u_1(\tau, n, \mathcal{C}) = cs(n, \mathcal{C}) \cdot (1-\tau) \cdot N_2 - P_1, \tag{6}$$

where N_2 is the number of sub-markets that are available on the platform and P_1 is the price a consumer pays to join the platform. Every consumer has a reservation utility that is normalized to zero. Thus, a consumer τ joins the platform when $u_1(\tau) \geq 0$.

To allow for endogenous entry of sub-markets, sellers have different sunk costs. Let $c \cdot \theta$ be the sunk cost of developing a product in sub-market θ. That is, low θ-type sellers have lower sunk costs than high θ-sellers. Sellers earn profits by joining the platform and selling their products. The marginal cost of production is set to zero.

Thus, a seller of sub-market θ has expected utility from joining the platform which is given by:

$$u_2(\theta, n, \mathcal{C}) = \pi(n, \mathcal{C}) \left(1 - \frac{N_1}{2}\right) N_1 - c \cdot \theta - P_2, \tag{7}$$

where P_2 is the price that a seller pays to join the platform. Every seller has a reservation utility that is normalized to zero. Thus, a seller of type θ joins the platform when $u_2(\theta) \geq 0$.

it takes time for consumers to experience and review a new product.

The Platform and Timing of Play

The platform connects consumers with sellers. Every consumer pays a membership fee, P_1, to join the platform. For example, P_1 could be the monthly fee consumers pay to join Netflix or Hulu, or the retail price that consumers pay to purchase a smartphone or video game console. Similarly, the platform charges a membership fee, P_2, to sellers which gives sellers access to the consumers that join the platform.

The platform then maximizes profits with respect to prices P_1 and P_2. Platform profits are given by:

$$\Pi = N_1 \cdot P_1 + n \cdot N_2 \cdot P_2, \tag{8}$$

where N_1 is the number of consumers that join the platform, n is the number of sellers in each sub-market, and N_2 is the number of sub-markets so that $n \cdot N_2$ is the total number of sellers that join the platform. For simplicity, I will assume that the platform's marginal and fixed costs are zero.

The timing of play is as follows. First, the number of sellers in each sub-market, n, and the type of competition between sellers in a sub-market, $\mathcal{C}$, are given. The popularity of products is realized after sunk participation decisions are made. Thus, once consumers and sellers observe the sub-market structure they take expectations over the network gains from joining the platform. Given the expected gains to consumers and sellers from joining the platform, the platform sets prices, P_1 and P_2, which can be less than zero. Lastly, participation decisions are made and payoffs are realized.

Equilibrium

The aim of this paper is to determine how the amount of competition between sellers within sub-markets, characterized by the number of sellers and the competitive structure, affects the platform's pricing strategies and the welfare generated on the platform. In principle, consumer surplus and seller profits move in opposite directions as they are splitting the total surplus generated within a sub-market. However, the redistribution of surplus with changes in competition need not be one to one. For example, when sellers of homogeneous products compete *a la* Cournot then an increase in the number of sellers increases total surplus by eliminating deadweight loss. Similarly, if sellers have differentiated products within a sub-market then an increase in the number of sellers can increase consumer surplus more than it decreases sellers' profit. Thus, standard demand assumptions are used: $\frac{\partial cs(n,\mathcal{C})}{\partial n} \geq 0$ and $\frac{\partial \pi(n,\mathcal{C})}{\partial n} \leq 0$.[10] This gives a general characterization of sub-markets that allows for many types of demand and competitive structures.

Given the number of sellers and the competitive structure within each sub-market, $\langle n, \mathcal{C} \rangle$, the platform sets prices P_1 and P_2 to maximize profit given by Equation (8) and consumers and sellers anticipate the size of the network, N_1 and N_2, in making participation decisions. Notice that the marginal agents for the platform decision on each side of the market, τ^c for consumers and θ^c for sub-markets, identify the total number of consumers and sub-markets on the platform. That is, Equations (6) and (7) imply $u_1(N_1, n, \mathcal{C}) \equiv 0$ and $u_2(N_2, n, \mathcal{C}) \equiv 0$. Thus, the platform solves for each price as a function of N_1 and N_2 so that its profit is a function of N_1 and N_2. For ease of exposition, the $\mathcal{C}$ in $\pi(n, \mathcal{C})$ and $cs(n, \mathcal{C})$ will be suppressed so that $\pi(n)$ and $cs(n)$.

Solving the platform's problem gives the equilibrium of the entire game:[11]

$$N_1^* = \frac{n\pi(n) + cs(n)}{n\pi(n) + 2cs(n)}, \tag{9}$$

$$N_2^* = \frac{[n\pi(n) + cs(n)]^2}{4cn[n\pi(n) + 2cs(n)]}, \tag{10}$$

$$P_1^* = \frac{cs(n)^2 \cdot [n\pi(n) + cs(n)]^2}{8c[n\pi(n) + 2cs(n)]^2}, \tag{11}$$

$$P_2^* = \frac{n\pi(n) + cs(n)}{8[n\pi(n) + 2cs(n)]^2} \cdot [3n^2\pi(n)^2 + 9n\pi(n)cs(n) - 2cs(n)]. \tag{12}$$

[10] This implies that for any competition structure within sub-markets, when the number of sellers increases within a sub-market then consumer surplus for each consumer increases and each seller's profit decreases.

[11] The second-order conditions hold for profit maximization.

An examination of the equilibrium produces the following theorem.

Theorem 1 (Consumer Participation)**.** *The equilibrium number of consumers that the platform serves is decreasing in the number of sellers within sub-markets,* $\frac{\partial N_1^*}{\partial n} < 0$*.*

Theorem 1 states that an increase in the number of sellers within a sub-market always induces the platform to serve fewer consumers. An increase in the number of sellers creates two important incentives on the platform that drive this result. First, greater competition implies that the platform's existing consumers each receive additional surplus which raises the platform's marginal profit for an increase in the consumer price. Second, greater competition implies that an additional consumer generates less profit on the seller side which reduces in the marginal cost due to a decrease in the number of consumers on the platform.[12] Thus, more competition implies that the marginal benefit from an increase in the consumer price increases while the marginal cost, in terms of fewer consumers for the seller side of the market, decreases. As a result, the platform raises its price so that fewer consumers join the platform.

Note that all else equal (i.e., the platform does not change its price with a change in the number of sellers), an increase in the number of sellers results in greater consumer surplus and less profit for sellers. More consumers join the platform but there are fewer sub-markets. Theorem 1 implies that the platform raises its consumer price so that the number of consumers that join the platform actually decreases. This occurs since the gains to the platform from keeping its consumer price low and capturing additional marginal consumers are insufficient to overcome the two effects, greater marginal profits and lesser marginal costs, from a higher consumer price with fewer consumers.

An alternative perspective is to consider the price elasticity of demand for the platform.

[12]Note, if there exists platform competition then this marginal cost is increasing and the extent depends on the platform's market power. In this case, the result is contingent on the platform having sufficient market power.

Consumer elasticity of demand for the platform is given by:

$$\epsilon = \frac{\partial N_1}{\partial P_1} \cdot \frac{P_1}{N_1} = -\frac{1}{cs(n) \cdot N_2} \cdot \frac{P_1}{N_1}, \tag{13}$$

where the last equality holds since $P_1 = cs(n)(1 - N_1)N_2$. As consumer surplus increases, holding $\frac{P_1}{N_1 \cdot N_2}$ fixed, consumer elasticity of demand for platform membership becomes inelastic, or relatively unresponsive to changes in price. Thus, the platform's optimal response is to increase its consumer price resulting in fewer consumers or to decrease the number of sub-markets. The network effects that exist in this two-sided market imply that there also exist indirect effects, but the consumer elasticity of demand for the platform suggests that the platform has an incentive to decrease the number of sub-markets. The following theorem shows that this is often the case.

Theorem 2 (Sub-Markets)**.** *The equilibrium number of sub-markets on the platform is decreasing in the number of sellers within sub-markets unless the increase in consumer surplus that results from additional sellers within sub-markets is relatively large. That is* $\frac{\partial N_2^*}{\partial n} < 0$, *unless* $\frac{\partial cs(n)}{\partial n} > \frac{cs(n)}{2} + \left(-\frac{\partial \pi(n)}{\partial n}\right) \cdot \left(\frac{n^2 \pi(n)}{2cs(n)} + \frac{3n}{2}\right)$.

On the seller side of the market, more competition has two first order effects. First, having lower seller profit implies that the platform must reduce its price to sellers to attract or maintain the number of sub-markets which reduces the platforms incentive to provide sub-markets. Second, an increase in consumer surplus within each sub-market implies that the platform has an incentive to provide more sub-markets. Thus, the platform only provides more sub-markets when the increase to consumer surplus is the larger of the two effects.

Given this equilibrium, welfare and the effects on welfare from changes in the sub-markets are investigated. Total welfare generated by the platform is given by:

$$W^* = \frac{[n\pi(n) + cs(n)]^4}{32cn[n\pi(n) + 2cs(n)]^3} \cdot [3n\pi(n) + 10cs(n)]. \tag{14}$$

In platform markets, surplus is generated by the interaction between the two sides of the market. Thus, holding network benefits fixed, a reduction in the number of consumers or in the number of sub-markets on the platform results in lower surplus (welfare). With this in mind, consider how an increase in the number of sellers in a sub-market affects total welfare generated by the platform. From Theorem 1 the number of consumers decreases, which lowers welfare. Furthermore, the number of sub-markets decreases unless the increase in consumer surplus is relatively large. Thus, when the number of sellers in sub-markets, n, increases then total welfare generated by the platform, W^*, will decrease unless the increase in consumer surplus, $cs'(n)$, is relatively large.

Theorem 3 (Welfare)**.** *The total welfare generated by the platform is decreasing in the number of sellers in a sub-market,* $\frac{\partial W^*}{\partial n} < 0$*, unless the increase in consumer surplus from a greater number of sellers in a sub-market,* $\frac{\partial cs(n)}{\partial n}$*, is sufficiently large.*

Theorem 3 implies that one can expect the platform to generate less welfare unless there is a significant increase to consumer surplus that results from additional sellers (e.g., if sellers have differentiated products and the sub-markets are not saturated).

Content Provided by the Platform

Now suppose that the platform provides the content on the platform. That is, given the competitive structure that exists within each sub-market, $\mathcal{C}$, the platform chooses the number of products that are available to its consumers in that sub-market, $n(\theta)$, and the price that consumers pay to join the platform, P_1. In this case, the platform chooses the number of sellers in each sub-market θ to maximize total surplus generated in that sub-market minus the total cost. This implies that the platform sets each product's price equal to marginal cost so that there is no deadweight loss within a sub-market and sets $n(\theta)$ to maximize

$cs(n, \mathcal{C}) - c\theta n$. Thus, the platform solves the following problem:

$$\max_{N_1, N_2} \Pi = N_1 \cdot P_1 - \int_0^{N_2} c \cdot \theta \cdot n(\theta) \; d\theta \tag{15}$$

$$\text{s.t.} \quad P_1 = CS(N_1) \cdot N_2. \tag{16}$$

When the platform provides content it is able to reduce two forms of deadweight loss. First, there no longer exists any deadweight loss within a sub-market as there are no longer product price markups by sellers that have market power. The platform sets product prices equal to marginal cost which eliminates deadweight loss. Second, the platform provides the optimal number of products within each sub-market by accounting for the cost of the sub-market, $c\theta n$. Thus, the platform reduces the total costs of providing content on the platform. Consequently, that when the platform provides content, welfare is improved.[13] Thus, the following corollary follows:

Theorem 4 (Welfare with Integration)**.** *When the platform and the product side of the market are fully integrated, welfare increases.*

The term, fully integrated means that the platform provides all the content. In digital marketplaces, fully integrated platforms do not exist. However, Costco and Sam's Club resemble this case. These marketplaces sell products to their members with prices equal to marginal costs and then charge members a membership fee to shop in the marketplace. Thus, the platform implements the same two-tier pricing scheme that is used by Costco and Sam's Club when it acts as the marketplace product provider.

In the two previous sections, a general formalisation of how the sub-market structure affects the platform is analyzed. In order to explore this relationship further, several specific sub-market structures are investigated in the following section and the effects of the sub-market structure on network effects and the platform are determined.

[13]For more on the platform's decision to act as a marketplace or as a retailer see Hagiu and Wright (2014) and Johnson (2014).

Conclusion

In this paper, the relationship between a platform and its sub-markets is considered. The mode of competition that exists within sub-markets affects the network effects between consumers and sellers, which in turn affect agents' participation decisions and platform pricing strategies. The network benefits that consumers and sellers receive from joining the platform are determined by consumer demand and the competitive structure that exists among sellers for these products. If there is less competition within a sub-market, then the price of the product will be higher, resulting in greater network gains for sellers but lower network gains for consumers. However, the size of the network also matters. More consumers on the platform increases demand for a product, and more products available on the platform makes participation on the platform more desirable for consumers.

When the number of sellers within a sub-market increases, competition within a sub-market increases, each seller receives less expected profit from a given consumer; thus, the platform has less of an incentive to provide sellers with additional consumers resulting in the platform serving fewer consumers. I find that the platform reduces consumer participation when the number of sellers increases, and this result is robust to many types of competitive structures within sub-markets.

I also find that if each product is provided by a monopoly seller, then welfare is greater than if products are provided by any number Cournot or Bertrand competing sellers. More generally, when the number of sellers increases, welfare generated by the platform decreases unless the gains to consumer surplus are significant. For example, if sellers within sub-markets sell differentiated products and these sub-markets have only a few sellers then the increase in consumer surplus from additional sellers may be sufficiently large so that welfare increases with the number of sellers. However, it is often the case that more competition within sub-markets results in lower total welfare generated on the platform.

Furthermore, by comparing Cournot competition with Bertrand competition, I find that welfare is always greater with Cournot. This implies that the platform's creation of deadweight loss from restricting the number of consumers and sub-markets on the platform is of a higher magnitude than the sellers' creation of deadweight loss within each sub-market through price markups. That is, having more consumers that make purchases within each sub-market and having more products available to consumers generates more surplus than the total deadweight loss across all sub-markets that is created by less competition within each sub-market. This provides the intuition for the welfare results found in this paper.

In many platform markets, mergers between the platform and the product or seller side of the market are common. For example, in its online marketplace, Amazon connects consumers with sellers but it is often a seller itself. Similarly, many video games are developed by the console developers. I find that efficiency increases when the platform integrates with the seller side of the market. With integration two forms of surplus destruction are mitigated. First, the platform maximizes the surplus generated within each sub-market with respect to the number of product sellers within that sub-market; hence, surplus destruction through redundant sunk costs is minimized. Second, the platform sets the price of each product equal to marginal cost so that there is no distortion in product provision. The distortion that remains is the platform's market power on the consumer side of the market. I find that integration leads to greater platform profits, total consumer surplus, and total welfare generated on the platform. Even though full integration is unlikely to occur in many platform markets, this serves as a base case for policymakers where such integration is a concern.

APPENDIX

Appendix of Proofs

Proof of Theorem 1: Taking the derivative of Equation (9) with respect to n implies that $\frac{\partial N_1^*}{\partial n} < 0$ if and only if $\frac{\pi'(n)}{\pi(n)} < \frac{cs'(n)}{cs(n)}$ which holds since $cs'(n) \geq 0 \geq \pi'(n)$ and $cs(n), \pi(n), \geq 0$. $\square$

Proof of Theorem 2: Taking the derivative of Equation (10) with respect to n implies that $\frac{\partial N_2^*}{\partial n} > 0$ if and only if $\frac{\partial cs(n)}{\partial n} > \frac{cs(n)}{2} + \left(-\frac{\partial \pi(n)}{\partial n}\right) \cdot \left(\frac{n^2\pi(n)}{2cs(n)} + \frac{3n}{2}\right)$. $\square$

Proof of Theorem 3: Taking the derivative of Equation (14) with respect to n implies that $\frac{\partial W^*}{\partial n} > 0$ if and only if

$$\begin{aligned}
&4n \cdot cs'(n)[n^2\pi(n)^2 + 4n\pi(n)cs(n) + 10cs(n)^2] + n\pi(n)[3n^2\pi(n)^2 + 34n\pi(n)cs(n) + 10cs(n)^2] \\
&\quad > \; n^2\pi(n)^2[19cs(n) - \pi'(n)6n^2] - \pi'(n)34n^3\pi(n)cs(n) \\
&\qquad +cs(n)^2[36n\pi(n) - \pi'(n)56n + 10cs(n)].
\end{aligned}$$

Essentially this requires that $cs'(n)$ be large relative to $n\pi(n)$ and $cs(n)$ and that $n\pi'(n)$ is non-negative. $\square$

Proof of Theorem 4: When the platform provides content it is able to reduce two forms of deadweight loss. First, there no longer exists any deadweight loss within a sub-market as there are no longer product price markups by sellers that have market power. The platform sets product prices equal to marginal cost which eliminates deadweight loss. Second, the platform provides the optimal number of products within each sub-market by accounting for the cost of the sub-market, $c\theta n$. Thus, the platform reduces the total costs of providing content on the platform. Consequently, that when the platform provides content, welfare is improved. $\square$

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