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ESSAYS IN SUPPLY CHAIN CONTRACTS

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WENMING CHUNG

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ESSAYS IN SUPPLY CHAIN CONTRACTS

By

Wenming Chung

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ABSTRACT

ESSAYS IN SUPLPY CHAIN CONTRACTS

By

Wenming Chung

This dissertation investigates two supply chain contracts that are widely utilized in various industries: the quantity flexibility contract and the price discount scheme. Motivated by the need of a more through understanding of the implications of supply chain contracts, we develop three supply chain contract models in three different types of supply chain structures: (1) the quantity flexibility contract with incentives in a one- supplier, one-buyer supply chain (2) the quantity flexibility contract and price markdown scheme in a two-supplier, one-buyer supply chain (3) the price discount scheme in a three-echelon supply chain consisting of a supplier, an original equipment manufacturer (OEM), and a buyer. The first essay focuses on designing a new quantity flexibility that seeks to balance the inventory risk between the traditional QF contract and the price-only contract for both the buyer and the supplier. The second essay analyzes the competition between the quantity flexibility contract and the price discount scheme. The third essay investigates the impact of price discount on retail price, capacity planning, and stocking policy decisions.

We found in essay 1 that the new quantity flexibility contract with incentives is able to achieve supply chain coordination. It also allows firms to identify areas where the new contract is able to Pareto-improve from the traditional quantity flexibility contract under certain circumstances. In essay 2, we present the conditions under which flexibility is more desirable and areas where the price discount has an

advantage. We found that in most cases the buyer will be better off simply using only one supplier. In essay 3, we show that firms will need to increase the capacity and stocking level to cope with the impact of price discount. We found that when price difference is not significantly large and the component's arrival time is uncertain, firms will be better off keeping higher inventory beyond the effective time of the new price. We address discussions, managerial implications, and provide directions for future research opportunities in each of the three essays.

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CHAPTER 1 INTRODUCTION

This dissertation seeks to contribute to better understanding of how firms can improve their profits in their best interests or in line with the system objective via proper design of contractual agreements. It is well documented that when firms' actions are aligned with the global objective in the presence of a contract, a decentralized supply chain may possibly achieve the same level of profits as that of a centralized supply chain. Thus in this case, the supply chain is considered to be coordinated. When the agents in a supply chain act in their best interests, a well-know phenomenon referred to as "double marginalization" occurs, and supply chain efficiency decreases. Thus, supply chain coordination is not possible. Nonetheless, research in both areas is equally important, because the former provides guidance for firms to achieve higher supply chain performance and the latter describes phenomena that are most commonly observed in real-world business practices.

The benefits of information sharing and the lengthened physical spans of supply chains have allowed utilization of forecasting mechanisms between firms to mitigate the effect of supply chain uncertainties. On the other hand, buffer stock remains an effective way for firms to cope with unexpected demand. It is common that firms negotiate agreements on both forecasting mechanisms and buffer stock level to facilitate their buy-and-sell activities. This can be best described by the well-known "quantity flexibility" contract. Although it is widely utilized by firms in various industries, the quantity flexibility contract has received little attention in the supply chain contract literature.

On the other hand, rapid technology innovations have brought about benefits such as frequent launch of new products and constant reduction of production costs to name a few. Among the consequences is the frequent price discount scheme that is widely observed in the high tech industry, e.g., the personal computer and cell phone industries. The margins in the manufacturing sector have been known to be low. With short product lifecycles and frequent price discounts, inventory management remains a major challenge for firms in supply chains in today's dynamic business environment. Although work in price discount or quantity discount is rich, extant research does not thoroughly address the various issues revealed by the presence of price discounts.

This dissertation intends to study the effects ofthe quantity flexibility contract and the price discount scheme on the operational decisions in supply chains. The bulk of research in supply chain contracts appeared in the past decade. However, the majority has focused on a simplified supply chain that consists of one buyer and one supplier. More complex systems that involve multiple suppliers or three echelons have received relatively little attention. Specifically, supply chain contract research involved with competing suppliers is rare.

This dissertation contains three separate, analytic essays on three different supply chain structures. Each essay studies either the quantity flexibility contract or the price discount scheme, or a mixture of both. Additionally, each essay takes a position from the buyer's view, the supplier's perspective, or from both standpoints. The goal is to study supply chain decisions in the presence of a contract from different perspectives under a specific supply chain structure. It provides foundations to future research for expanding research problems into more complicated supply chains. It adds a new design of the quantity flexibility contract to the literature, in the presence of competition in the contractual setting, and in a contract that involves multi-echelon supply chains.

Chapter 2, entitled "The Quantity Flexibility Contract with Incentives", studies a quantity flexibility problem in a one buyer, one supplier supply chain in which the supplier offers discount price for any units sold above the contracted quantity. The purpose is to shift portion of the inventory burden back to the buyer, thus creating higher profits than what the original quantity flexibility contract would allow. Chapter 2 intends to identify areas where it is most appropriate for the supplier to introduce the discount incentives. On the other hand, the buyer will choose not to execute the discount scheme if not expecting higher profits than the quantity flexibility contract would offer. As such, when the new contract is executed, it needs to allow Pareto improvement from the traditional quantity flexibility contract.

Another goal of this research is to examine whether the system can achieve supply chain coordination under the new quantity flexibility contract. It is known that the quantity flexibility contract allows a decentralized supply chain to reach the centralized efficiency. This research intends to design a new quantity flexibility contract with incentives that performs equally well as the traditional version.

The analysis makes very general assumptions; it is applicable to any demand distribution. The new contract offers "flexibility" in terms of the ability to identify the coordinating conditions, yet this can be achieved with easy access to information needed. Much emphasis is placed on comparing differences in inventory decisions and supply chain performance between the original and the proposed new quantity flexibility contracts. It aims to establish a framework that can assist managers to easily identify areas where either one has an advantage over the other, thus guiding them to make the right decisions.

Chapter 3, entitled " Quantity Flexibility Contract in a Two-Supplier-One-Buyer Supply Chain", considers supplier competition in a two-echelon supply chain. In this supply chain, one supplier offers the quantity flexibility contract while the other offers better price for the same, but not identical, component. This chapter analyzes the effects of the competition on the buyer's and supplier's decisions. It intends to examine the tradeoffbetween the benefits from the quantity flexibility and from cost saving. It takes the buyer's standpoint and investigates his decision making behavior in the presence of the two options. It also provides insights to the suppliers as to whether the incentives that they offer are attractive enough to the buyer.

Similarly, chapter 3 investigates the performance of the focused supply chain with competing suppliers. One can expect that the buyer will take advantage of the competition and make the most benefit out of it. However, from the system's point of view, such a multi-competing-supplier supply chain may likely result in supply chain deficiency. This research intends to specify the optimal conditions and the corresponding profits for each agents and the entire system, thus analyzing what contributes to the supply chain deficiency should it occur. A further step is to propose managerial insights as to actions or strategies that allow system performance to improve.

The analysis needs no specific demand distribution. However, certain types of distributions may be favored due to constraints in computational power of computers. Commonly, buyers have multiple sources of supply for the same components. Contract research in this regard is rare primarily because of technical restrictions. As such, this paper not only adds a significant contribution to literature but also provides managerial insights to practitioners. It enables managers to estimate how much flexibility is worth and the ideal allocation between the two competing suppliers.

Chapter 4, entitled "Price Markdown Scheme in a Three-Echelon Supply Chain", considers a price discount scheme involved in a supply chain consisting of a supplier, an original equipment manufacturer (OEM), and a buyer (retailer). The supplier is able to offer price discount on a regular basis due to innovation in manufacturing technologies. This research extends the discount literature by considering three agents in a supply chain as well as the emerging issues of delivery and demand uncertainties that play a critical role in agents' decisions.

When a supplier takes the initiative and offers a new, lower, all-unit price markdown, it resembles price-only contract rather than the quantity discount mechanism. Thus, such a supply chain discount problem is known not being able to achieve supply chain coordination. However, this does not degrade the importance of the problem, because the price markdown scheme is executed frequently by many firms across various industries, especially in the high tech industry. As such, this research is concerned with supply chain decisions at the individual level rather than from the system's view. It analyzes how the new pricing should be specified and how the corresponding capacity planning and inventory stocking strategies along the chain can be decided.

The development of this research is based on stochastic and price sensitive demand functions. A variety of price-sensitive demand functions are available in the economic literature, a common property of which is that demand decreases as price increases. The only additional requirement for the model in chapter 4 is that the demand function needs to be concave. Specifically, this research examines the effect of the price discount on the inventory stocking level beyond the price break by explicitly considering the role of lead-time uncertainty in such decisions. The analysis can facilitate manager's decisions when the material's arrival time is not somewhat unpredictable.

The concluding remarks are provided in chapter 5, where we review the contributions of this dissertation. The study of supply chain contracts enhances the understanding of the impact of contractual agreements on firms' financial performance, thus providing the fundamental know-how in designing efficiency-enhancing and Pareto-improving contracts. This dissertation is most appropriate for a potential audience such as supply chain managers, logistics managers, sales representatives, as well as academicians in the field of operations management or management science.

CHAPTER 2 QUANTITY FLEXIBILITY CONTRACT WITH INVENTICES

2.1 Introduction

Wistron Corporation, a Taiwan-based company that employs 20,000 workers worldwide, is one of world's largest PC original equipment manufacturer (OEM) based on revenue, with a large customer base of global, branded IT companies. Wistron develops high-technology products such as notebook and desktop computer systems, servers and storage systems, and networking and communication systems. To better the relationship with its customers and secure a high market share of the PC business, Wistron agrees to fully commit to the rolling 13-week forecast provided by its customers if material supply is not an issue. Furthermore, Wistron is also willing to prepare additional buffer inventory based on the forecast in case of unforeseen demand. On the other hand, Wistron's customers are not free of material liability. If obsolete inventory occurs from the rolling forecast, Wistron's customer is obligated to absorb the material burden to a certain extent. Such a contractual mechanism is called the Quantity Flexibility (QF) contract in the supply chain management literature. The QF contract is widely observed in various industries. For example, Sun Microsystems, IBM (High tech/PC), Toyota (automobile), Nippon Otis (Elevator) are all reported to employ the QF contract with their suppliers or customers (Tsay, 1999).

In general terms, the QF contract can be described as following: the buyer first provides forecast q to the supplier, the supplier then decides her production quantity Q. When the demand is realized, the buyer will place a purchase order to the supplier at quantity r. Based on the QF contract, the supplier will need to produce a certain percentage α of buffer stock above the forecasted quantity, and the buyer needs to buy a certain percentage $(1-\omega)$ of the forecasted number, no matter what the demand is (Tsay, 1999).

It is clear that the QF contract works in favor of the buyer compared with the traditional price-only business behavior. Under the price-only contract, the buyer places orders to the supplier to fulfill demand observed from the market; the supplier produces exactly the quantity that the buyer orders. As such, the buyer bears all the inventory riskwhile the supplier's profit is linear to the order size. With the QF contract in place, the buyer does not need to assume 100% liability for the forecasted quantity, yet the supplier will have to ensure that extra buffer stock is available for the buyer above the forecasted quantity. As a result, the supplier shares certain level of risk under the QF contract. Whenthe level of competition is high and the buyer has many sources of supply to choose from, the supplier is likely to offer incentives such as the QF contract to secure business from the buyer.

In this research, we intend to study a revision of the QF contract that can potentially partially reduce the burden of inventory risk for the supplier that she would have to bear when offering the QF contract. We investigate a supply chain in which the buyer and the supplier agree upon a QF contract for the buy-and-sell activities. To encourage the buyer to purchase more, the supplier also offers a discount for any units sold above the QF-contracted quantity. However, these units at the discount price are not protected by the QF contract; they can not be returned even if they are unused eventually. As such, the supplier offers a menu of contracts with different component (transaction) prices, and the buyer needs to decide how many units to buy under the QF contract and how many to purchase at the discount price. We name this contract as the quantity flexibility with incentives, hereafter, the QF*i* contract. The lower price will encourage the buyer to consider to buy more than the contracted quantity. However, having to bear all the risk of inventory of the discounted units may prevent the buyer to purchase all that is needed at the discounted price. Consequently, the tradeoff for the buyer is to value the cost saving for the additional, cheaper units and the potential loss from carrying too much inventory should the demand be lower than the estimated level.

The motivation of this study is to take the supplier's perspective and identify the conditions under which the supplier will be benefit more by offering the QFi contract contract than by offering the traditional QF contract. When supplier offers the QFi contract, she expects to stimulate the buyer to increase the order size, and thus raising her profit level. However, as will be detailed in the later sections, the QFi contract can actually hurt the supplier if not introduced at the right conditions. In such cases, the supplier will be better-off to stick to the traditional QF contract and not to offer any discount incentive. We intend to identify the conditions under which the supplier should and should not offer the QFi contract.

One common theme in the research stream of supply chain contracts is to seek the conditions under which the individually motivated buyer and/or supplier can act in line with the optimal system-wide profit through proper designs of mutually-agreed contracts. It is well-known that the whole chain's profits level can reach the highest level when decisions are made centrally in a supply chain. This condition is termed as supply chain coordination in the supply chain contract literature (Pasterneck, 1985; Weng, 1995; Tsay, 1999; Taylor. 2002; Cachon and Lariviere, 2005; Berstein and Federgruen, 2005). The system-wide optimal profit level normally is used as a benchmark to evaluate the performance of a supply chain. Supply chain efficiency, defined as decentralized chain profit divided by the centralized chain profit, is the most cited metric to estimate the performance of a supply chain when there is a contract in place. It is easy to show that the total profit of a decentralized supply chain is always smaller than or equal to the centralized profit. When a supply chain efficiency reaches 100%, the supply chain is considered coordinated.

The QF contract is one of the several contracts in literature that have been proven to be able to achieve supply chain coordination. Tsay (1999) showed that if the transaction price between the buyer and the supplier is set under certain conditions, the supplier will be induced to produce the quantity that will result in the highest expected profit level for the entire chain. We will show that the revised QF*i* contract will be able to achieve supply chain coordination as well. The remainder of the chapter is organized as follows. Section 2.2 discusses the related literature in the area of supply chain contracts. The problem setting is laid out in section 2.3. The new QF*i* contract is forwarded in section 2.5. In section 2.5, the topic of supply chain efficiency under the QF*i* contract is addressed. In section 2.6, the QF*i* contract is compared with the traditional QF contract. In section 2.7, a series of numerical examples are presented and managerial implications are discussed. The chapter concludes in section 2.8 with the contributions of this research and suggests future research opportunities.

2.2 Literature Review

It is well-documented that when supply chain's decision making is aligned with the global objectives of the entire chain, it is considered a coordinated supply chain (Pasterneck, 1985; Taylor. 2002; Sahin and Robinson, 2002; Cachon and Lariviere, 2005). This can be achieved relatively easily when there is one single owner of the entire chain. However, it is generally the case that a supply chain consists of independent agents who act only in their own best interests, i.e., a decentralized supply chain. As such, their goal pursuing often leads to sub-optimal results for the entire system, thus supply chain deficiency occurs. For instance, a buyer and a supply in a supply chain may utilize inventory policies differently from a central owner, resulting in the well-known phenomenon of "double marginalization" (Spengler, 1950; Tirole, 1988).

A common cure for supply chain deficiency is to utilize supply chain contracts that could possibly allow Pareto improvement in the supply chain or even achieve supply chain coordination (Iyer and Bergen, 1997; Cachon, 2004). The general idea is to install rules that guide business conduct between agents in a supply chain to be aligned with the best interest of the system (Tsay, 1999). Supply chain contracts may consist of financial or non-financial mechanisms. The former are the most studied. Common supply chain contracts include mechanisms such as quantity discount (Chiang et al., 1994; Weng, 1995; Cachon, 2004; Tomlin, 2003), price discount and rebate (Bernstein and Federgruen, 2005), sales rebate (Taylor, 2002; Krishnan et al., 2004), buybacks (Pasternact, 1985; Lariviere, 1999), revenue sharing (Cachon and Lariviere, 2005), quantity flexibility (Tsay, 1999), forecast-commitment (Durango-Cohen and Yano, 2006), quick response (Iyer and Gergen, 1997), to name a few.

We focus on the QF contract in this research. As mentioned, the QF contract is widely observed in various industries. However, it has received relatively little attention in literature. The QF contract allows the buyer's ultimate purchase quantity to deviate from the original estimation (Lariviere, 1999). As such, the supplier agrees to bear the risk of inventory to a certain extent that she would not be liable for if the QF contact is not implemented. Such a mechanism functions similarly to the buyback contract in the sense that in both cases, the supplier absorbs unsold units. But the extent to which the supplier resumes inventory responsibility may differ between the QF contact and the buyback contract, based on the contract agreements.

The seminal work in the QF contract is attributed to Tsay (1999). He considers a supply chain consisting of one buyer and one supplier. The buyer provides a forecast to the supplier and the supplier decides the production quantity based on the forecast information as well as the contract restriction. Tsay (1999) formulates a general QF problem in which not only the buyer is granted additional availability above the forecasted quantity, but also he is liable for only portion of the estimated demand. He found that under the QF contract, the supplier will produce exactly the contracted quantity. As a result, the forecast directly affects the final availability of the component (product). Furthermore, Tsay showed that the supply chain can be coordinated utilizing the QF mechanism. He identified the component pricing scheme that will induce the buyer to respond in a manner aligned with the system's desirable outcome. However, identifying the component price that coordinates the supply chain via the QF contract requires the knowledge of the buyer's demand distribution.

Sethi et al. (2003) studied the QF contract in a setting with multi-periods and two sources of component supplies. The buyer is allowed to purchase at two distinct time periods, with the size of the second order being constrained by the QF contractual parameter. However, these units for the second order are sold at a higher price than that in the first order. On the other hand, the buyer will also have an option to buy from the spot market for any desired quantity at the market price, which can be higher or lower than the supplier's price. Under these conditions, they identified the optimal order quantities from the supplier in both periods, as well as the order size from the spot market in the second period While their findings are not unexpected, they demonstrated how one can incorporate information updates into the QF contract decisions. Wu (2005) conducted a similar research on the QF contract with Bayesian updating. He followed Tsay's (1999) framework and allowed multiple updates of demand information before the ultimate purchase order. He concluded that QF contract works in favor of the buyer. It allows both agents to share the benefits from information updating.

Tsay and Lovejoy (1999) studied the QF contract under rolling-horizon planning. In this research, they focus on understanding how the greatest benefits can be achieved under the QF contract along the planning horizon and how much to pay for it. They explored issues such as the impact of the system flexibility on inventory characteristics and the patterns by which forecast and order variability propagate along the supply chain. They found that, all else equal, increasing flexibility will reduce the buyer's cost but at the expense of the supplier's cost. They also found that the QF contract may dampen the transmission of order variability throughout the chain, thus potentially retarding the well-known "bullwhip effect".

Motivated by these findings, we study a variation of the QF contract by adding the discount scheme on top of the QF mechanism from the supplier's perspective, in the hope of reducing the increased inventory risk that the supplier needs to absorb in the QF contract agreements. The purpose of this research is to design a new QFi contract that can potentially reduce the inventory risk level and increase profits for the supplier compared with that from a traditional QF contract setting. With this new QFi contract, the supplier will have an option as to whether to offer the new QFi contract or to stick to the original QF agreements. We will show in our analyses that this new QFi contract is able to achieve the supply chain coordination, thus performing equally well when compared to the traditional QF contract. But the two QF contracts achieve 100% supply chain efficiency in different conditions. The ultimate goal is to identify the conditions under which the supplier should propose to offer the new contract and the conditions under which the supplier should not.

2.3 The Traditional QF Contract and The Performance Benchmark

We consider a supply chain that consists of one supplier and one buyer. The supplier produces and sells components to the buyer, and the buyer makes the final product and sells it to the end customers. The QF contract, (α, ω), is in place in this supply chain. As such, the buyer observes the market demand D, which is assumed stochastic, and releases forecast q to the supplier; the supplier will then decide the production quantity Q. Based on the QF contract, the supplier is obligated to produce at least $(1+\alpha)q$ units and the buyer is liable for buying at least $(1-\omega)q$ units. To simplify the analysis, we consider only one component and one product in single period.

Tsay (1999) showed that under the QF contract, the supplier's optimal production quantity is $(1+\alpha)q$. In other words, the supplier will produce only the minimum contracted quantity and no more than that. As such, the buyer is able to decide the total availability of the component via the forecasted quantity. So under the traditional QF contract, the task for the buyer is to decide the optimal forecast q so as to lead the supplier to produce $(1+\alpha)q$ so that the buyer's expected profit can be maximized. Consequently, the buyer is clearly the leader of this Stackelberg game.

2.3.1 The centralized system: a benchmark

The centralized system is normally used as a benchmark to evaluate the efficiency of a decentralized supply chain. When both the supplier and the buyer are owned bya single firm, the entire supply chain has a single decision-maker. The total expected profitof the centralized chain can be expressed as following:

$$\Pi_c = (p-c)Q - (p-v)E(Q-D)^+$$

where Q is the quantity produced, p is the retail price, c is the production cost, v is the salvage value, D is the random demand, and F is the cumulative density function (c.d.f.) of D. It is well documented that when the firm produces $Q_c^* = F^{-1}(p-c/p-v)$, the centralized supply chain achieves the highest expected profit: $\Pi_c^* = (p-v)\int_0^{Q_c^*} x dF(x)$ (Taylor, 2002). Any production/ordering quantity other than Q_c^* will disadvantage either buyer or supplier, or both, thus decreasing the entirechain's profit. Notice that the transaction cost w plays no role when there is a single owner for the supply chain, because a markup of the component price between the two parties no longer exists. Therefore, unlike the decentralized supply chain, the centralized chain is free of the issue of "double marginalization" (Spengler, 1950).

As mentioned earlier, the most cited metric for evaluating a supply chain's performance is the supply chain efficiency, defined as: $(\Pi_b + \Pi_s)/\Pi_c^*$. When the supply chain efficiency is equal to 1, the decentralized chain achieves supply chain coordination (Pasterneck, 1985; Weng, 1995; Tsay, 1999; Donohue, 2000; Taylor, 2002; Cachon and Lariviere, 2005; Krishanan et al., 2004; Berstein and Federgruen, 2005). In the next section, we will describe and develop the new QF*i* contract model. Throughout chapter 2, we will utilize the following notation:

Notation

Π_c : Total expected profit of a centralized supply chain

Π_s : Supplier's total expected profit in a decentralized supply chain

Π_b : Buyer's total expected profit in a decentralized supply chain

D : Demand, a random variable

F : Cumulative density function (c.d.f.) of the random demand

f : Probability density function (p.d.f.) of the random demand

q : Forecast released by the buyer to the supplier under the traditional QF contract

q_1 : Forecast released by the buyer to the supplier under the QF contract with incentives

q_2 : Order size at the discount price under the QF*i* contract

p : Retail price of the final product

w_1 : Original component price under the new QF contract, $w_1 = w$.

w_2 : Discounted component price

v : Salvage revenue of the component per unit for both the buyer and the supplier

c : Supplier's production cost

Note: $p > w_1 > w_2 > c > v > 0$

2.4 The Quantity Flexibility Contract with Incentives

Consider the QF contract with incentives. In order to earn more than she could under the traditional QF contract, the supplier proposes that any unit sold above the contracted quantity will be sold at a discounted price w_2. However, these additional units can not be returned no matter what the final demand. In other words, the supplier provides a menu of contracts that combines the traditional QF contract and the discount scheme, presented as $(\alpha, \omega, w_1, w_2)$. The motivation is to encourage the buyer to buy more so that the supplier can earn higher profits. Under this QF*i* contract, the buyer gets to decide the **forecast** quantity q_1 under the QF contract and the additional **firm** order q_2 that he wants to purchase at the discount price. Based on the contract, the supplier will produce $(1+\alpha)q_1 + q_2$ and the buyer is obligated to buy $(1-\omega)q_1 + q_2$. Any unit within the QF contracted quantity $(1+\alpha)q_1$ is sold at w_1 and any unit of q_2 is sold at w_2; $w_1 > w_2$, $\alpha > 0, 1 \geq \omega > 0$. Notice that the buyer can choose to buy all that is needed via q_2 as well. However, the full responsibility of inventory risk may prevent the buyer from doing so. If the buyer places only a firm order q_2 and chooses not to execute the QF contract, i.e., $q_1 = 0$, the problem becomes a price-only contract and it is well-known that underproduction will occur. Besides, the entire chain will suffer from supply chain deficiency and supply chain coordination can not be achieved. It is also straightforward that when q_2 =0, the problem becomes exactly the traditional QF contract problem. As a result, the QFi contract seeks a balance of inventory risk somewhere in the middle between the traditional QF contract and the price-only contract for both the buyer and the supplier.

Notice that the buyer remains the leader of the Stackelberg game under the new QFi contract and has the control over the forecasting and purchasing decision making. As such, we formulate the problem by maximizing the buyer's total expected profit as follows:

$$
\begin{aligned}
\Pi_b &= (p-w_1)(1+\alpha)q_1 + (p-w_2)q_2 \\
&\quad -(p-w_1)[E((1+\alpha)q_1+q_2-D)^+ - E((1-\omega)q_1+q_2-D)^+] \\
&\quad -(p-v)E((1-\omega)q_1+q_2-D)^+ \qquad (2.1)
\end{aligned}
$$

The first two terms describe the buyer's profits if everything is sold. Under the QF contract, the buyer can return as many as $(\alpha+\omega)q_1$ units. The second term describes the potential unsold units falling within this limit that is eligible for return and full refund from the supplier. For any unsold units that exceed the returnable allowance, the buyer will have to absorb the loss himself. These items are to be salvaged and the loss is described by the third term. All unsold item's revenue are deducted as indicated in last two terms. The decision variables are q_1 and q_2. Notice that q_1 represents the forecast under the QF contract agreement while q_2 refers to a firm order at the discount price. Next, we explore the property of this unconstrained nonlinear objective function.

Theorem 2.1

(1) The buyer's expected profit function is jointly concave in q_1 and q_2.

(2) Let $\chi = \dfrac{1+\alpha}{1-\omega}$. If $\chi > \underline{\chi} = \dfrac{(w_1 - v)(p - w_2)}{(w_2 - v)(p - w_1)}$, buyer will release a non-zero forecast

q_1 to the supplier.

(3) Following (2), if $\chi > \underline{\chi}$, the optimal forecast $q_1^* = \dfrac{F^{-1}(M) - F^{-1}(N)}{(\chi - 1)(1-\omega)}$, where

$$M = \frac{(p - w_1)\chi - (p - w_2)}{(\chi - 1)(p - w_1)} \text{ and } N = \frac{\chi(w_1 - w_2)}{(\chi - 1)(w_1 - v)}.$$

(i) if $q_1^* \geq q_{QF}$, then $q_1^* = q_{QF}$, the optimal discount order $q_2^* = 0$

(ii) otherwise, the optimal discount order $q_2^* = \dfrac{\chi F^{-1}(N) - F^{-1}(M)}{\chi - 1}$

(4) Following (2)&(3), if $\chi \leq \underline{\chi}$, then $q_1^* = 0$ and $q_2^* = F^{-1}(\dfrac{p - w_2}{p - v})$.

Proof. See Appendix of chapter 2.

Theorem 2.1(1) guarantees that if optimal solution q_1^* and q_2^* exist, they are unique to maximize the buyer's profit. However, technically, there is no guarantee that both q_1^* and q_2^* will be positive at the optimum. Theorem 2.1(2) employs the measure of flexibility introduced by Tsay (1999). Both α and ω are contractual parameters and are exogenous to the model. It is straightforward to see that χ must not be smaller than 1. Theorem 2.1(2) suggests that the flexibility needs to be large enough to attract the buyer to remain his interest in using the QF contract, i.e., $q_1^* > 0$. Otherwise, the buyer will choose to purchase all that is needed at the discount price. In addition, there are two extreme sets of parameters. First, when $(\alpha, \omega) = 0$, the QF contract does not exist and the problem becomes a price-only contract problem. It is easy to show that under the price-only contract, the order quantity is always less than Q_c^*, thus underproduction will occur and it will not achieve the maximum profit for the entire chain (Tsay, 1999). In this case the buyer assumes the full risk of inventory. Second, when $\omega = 1$, $\chi \to \infty$, over-forecasting will occur, thus leading to overproduction. In this case, the buyer does not have any liability for the forecast quantity so he will be tempted to exaggerate the forecast that will force the supplier to produce more than what is possibly needed. In either case supply chain deficiency occurs. Detail discussion can also be found in Tsay (1999).

Finally, Theorems 2.1(3) and (4) present the solution forms of q_1^* and q_2^*. As discussed, q_1^* is the optimal forecast under the QF contract and q_2^* is the optimal firm order at the discount price. Note that q_1^* and q_2^* are both non-negative. So should negative values occur for either q_1^* and q_2^*, they should be set to zero. When $q_1^* = 0$, the problem is equivalent to the price only contract, thus $q_2^* = F^{-1}(\frac{p - w_2}{p - v})$. When $q_2^* = 0$, the problem degenerates into the traditional QF contract, thus q_1^* will be the same as the optimal q_{QF} identified in Tsay (1999). Notice that the total availability is $(1+\alpha)q_1^* + q_2^*$ and the final purchase quantity can be expressed as $r^* = D \perp [(1-\omega)q_1^* + q_2^*, (1+\alpha)q_1^* + q_2^*]$. The expression denotes the point in the interval that is closest to D. So when $D < (1-\omega)q_1^*$, $r^* = (1-\omega)q_1^* + q_2^*$; when $D > (1+\alpha)q_1^* + q_2^*$, $r^* = (1+\alpha)q_1^* + q_2^*$; otherwise, $r^* = D$.

The result of Theorem 2.1(2) provides a lower bound of the discount price w_2 as shown in the following Corollary.

Corollary 2.1 If $w_2 > \underline{w}_2 = \dfrac{\chi(p-w_1)v + p(w_1 - v)}{\chi(p-w_1) + (w_1 - v)}$, the buyer will remain interested in the executing the QF contract.

When the discounted price is lower than the lower bound $\underline{w}_2$, the price discount becomes so attractive to the buyer so that he will choose to execute his orders all at the discount price. Additionally, w_2 must also be greater than the production cost c. Otherwise, the discounted units will cause a loss per unit sold and the purpose of making higher profits via the combination of the QF contract and discount pricing will not be met. Note that w_1 serves as the upper bound for w_2.

Corollary 2.2

(1) M is increasing in χ

(2) N is decreasing in χ.

Proof. M can be rewritten as $1-\frac{w_1-w_2}{(\chi-1)(p-w_1)}$ so it is clearly decreasing as χ increases. On the other hand, it is straightforward that $\frac{\chi}{\chi-1}$ is decreasing in χ. Thus, N is decreasing in χ. □

In theorem 2.1 we solve for the equilibrium solution for the optimal forecast and optimal discount order for the buyer and identify conditions under which optimal forecast is feasible and non-zero. Since the supplier is the follower, she simply produces the quantity that fulfills the contracted quantity and the discount order. In the next section we discuss the coordination condition under the QF*i* contract.

2.5 The QF*i* Contract and Supply Chain Coordination

As indicated in section 2.3, Q_c^* indicates the quantity that will lead to the highest possible profit level for the entire supply chain. In this section, we investigate a pricing scheme set by the supplier that incentivizes the buyer to choose a combination of forecast and firm order quantities in line with the system-wide optimal solution.

Theorem 2.2 Under the QF*i* contract (α,ω,w_1,w_2), when the discount price w_2 is set by the following equation, the buyer will be induced to release (q_1^c,q_2^c) so that the total availability Q_{QFi}^* will be equal to Q_c^*, thus the individually motivated supply chain will be coordinated:

$$w_2 = w_1 - \frac{(\chi-1)(p-w_1)(c-v)}{p-v}$$

Proof. The total availability given (q_1^c, q_2^c) is $Q_{QFi}^* = (1+\alpha)q_1^c + q_2^c = Q_c^*$ at supply chain coordination. Thus, from (A2.3) in the Appendix we have

$$F(Q_{QFi}^*) = \frac{(p-w_1)\chi-(p-w_2)}{(p-w_1)\chi-(p-w_1)} = \frac{p-c}{p-v} = F(Q_c^*)$$

$$\Rightarrow 1 - \frac{w_1 - w_2}{(p-w_1)(\chi-1)} = 1 - \frac{c-v}{p-v}$$

$$\Rightarrow w_1 - w_2 = \frac{(p-w_1)(\chi-1)(c-v)}{p-v}$$

$$\Rightarrow w_2 = w_1 - \frac{(p-w_1)(\chi-1)(c-v)}{p-v} \qquad \square \qquad (2.2)$$

Condition (2.2) defines the discount pricing scheme for the QF*i* contract. Given the flexibility negotiated as well as the retail price, salvage value and the supplier's production cost, w_2 in (2.2) will be able to induce the buyer to respond with optimal inventory policies that are in line with the centralized decision. As such, the decentralized supply chain will be able to achieve the same profit level as the centralized chain does, thus achieving the state of supply chain coordination. Consequently, the supply chain efficiency is at 100% under the QF*i* contract at the pricing scheme in condition (2.2). In addition, it can be observed that w_2 is decreasing in flexibility measure χ. When χ is large, the buyer is in a more powerful position. So the supplier is willing to sacrifice by assuming higher inventory risk in order to gain orders from the buyer. As such, the supplier will be inclined to offer a larger discount (lower w_2) to encourage the buyer to enlarge the firm order size q_2. By doing so the inventory burden shifts partially back to the buyer's side.

We can also observe that there is one-to-one mapping between the discount price w_2 and flexibility χ. As such, when χ is agreed upon by both parties, there will be a unique w_2 that will achieve the supply chain coordination. Conversely, when w_2 is given, there will be a unique χ that achieves supply chain coordination.

Proposition 2.1

(1) Given w_2, there exists a unique $\chi_c = 1 + \dfrac{(w_1 - w_2)(p - v)}{(c - v)(p - w_1)}$ such that $Q^*_{QFi} = Q^*_c$

(2) When the supply chain is coordinated under the QFi contract, the QF mechanism must be active between the buyer and supplier.

Proof. Prop. 2.1(1) is straightforward. Rearranging the terms in (2.2) we have

$\chi_c = 1 + \dfrac{(w_1 - w_2)(p - v)}{(c - v)(p - w_1)}$. For Prop.2.1(2) it suffices to show $\chi_c \geq \underline{\chi}$.

Rearrange R.H.S:

$$\frac{(w_1 - v)(p - w_2)}{(w_2 - v)(p - w_1)} = \frac{(w_2 - v)(p - w_1) + (w_1 - w_2)(p - v)}{(w_2 - v)(p - w_1)} = 1 + \frac{(w_1 - w_2)(p - v)}{(w_2 - v)(p - w_1)}$$

$$w_2 > c \Rightarrow \frac{(w_1 - w_2)}{(c - v)} - \frac{(w_1 - w_2)}{(w_2 - v)} > 0 .$$

$$\chi_c > \frac{(w_1 - v)(p - w_2)}{(w_2 - v)(p - w_1)} = \underline{\chi} .$$

Thus by Theorem 2.1, q_1^c must be non-zero. □

Proposition 2.1 indicates that the coordination status fulfills the conditions that is necessary for the unique optimal solutions described in theorem 2.1 and corollary 2.1. Although there is a unique χ_c when achieving the optimal system profits, the associated (α,ω) do not need to be unique. There will be a menu of (α,ω) that can result in the same χ_c. Consequently, the pricing decision and the flexibility negotiation can work two ways: flexibility parameters can be decided first, followed by the pricing scheme, or price can be decided first, followed by the decisions of the flexibility parameters. Either way there is a chance to achieve 100% supply chain efficiency.

Finally, both the traditional QF contract (α,ω,w) and the revised QF*i* contract (α,ω,w_1,w_2) can achieve supply chain coordination via the pricing schemes identified in Tsay (1999) and in (2.2), respectively. Notice that w in Tsay's price scheme equals w_1 in our model. A major difference between these two contracts is that we have two transactional prices (w_1,w_2) while there is only one under the traditional QF contract (Tsay, 1999). More importantly, our pricing scheme does NOT require knowledge of the demand distribution for achieving the coordination status, while such knowledge is a must under the traditional QF contract. This implies that the QF*i* contract's coordinating price scheme will achieve system-wide profit no matter what the demand distribution is, while the QF contract needs to decide the demand distribution before one can identify the coordinating pricing. Furthermore, the QF*i* contract can identify the coordinating discount price given any QF contract price. But the QF contract may result in a coordinating price that deviates radically from original QF component price. As such, our QF*i* contract has the advantage of requiring relatively less information than the traditional QF contract, and a wide range of coordinating pricing schemes available which are not possible under the traditional QF contract. These are critical factors for successful implementation of any supply chain contract.

2.6 Comparative Analysis Between $(\alpha, \omega, w_1, w_2)$ and (α, ω, w): The Supplier's Perspective

We have thus far identified the optimal conditions that allow the decentralized supply chain to achieve the highest profit level that a centralized chain can achieve. However, the QF*i* contract $(\alpha, \omega, w_1, w_2)$ does not necessarily always benefit the supplier, even though the intention of the supplier's offering the QF*i* contract is to increase her profit level. In this section we analyze the conditions under which the supplier will benefit from offering the QF*i* contract and the conditions under which the supplier should stick to the traditional QF contract (α, ω, w). We analyze these conditions both in the coordinated QF*i*/QF supply chains and the general non-coordinated QF*i*/QF supply chains.

2.6.1 The coordinated QF*i*/QF supply chains

Define Π_s^{QF} and Π_s^{QFi} the supplier's total expected profit when coordinating under the QF contract and under the QF*i* contract, respectively. Let (w_c, Q_c, q_c) be the component price, the production quantity, and the forecast quantity under the traditional QF contract that coordinates the supply chain. Readers can refer to Tsay (1999) and (Lariviere, 1999) for the details of the coordination pricing scheme and the optimal quantities. Notice that $Q_c = (1+\alpha)q_c$, $Q_c^* = (1+\alpha)q_1^c + q_2^c$. To facilitate the comparison, we let $w_1 = w_c$. w_2 can be determined by the pricing scheme in Theorem 2.2. By definition, Π_s^{QF} and Π_s^{QFi} can be calculated as following:

$$\Pi_s^{QF} = (w_1 - c)Q_c - (w_1 - v)[E(Q_c - D)^+ - E((1-\omega)q_c - D)]$$

$$= w_1 Q_c^* - cQ_c^* - (w_1 - v)\int_{(1-\omega)q_c}^{Q_c^*} F(x)dx \qquad (2.3)$$

$$\Pi_s^{QFi} = (w_1 - c)(1+\alpha)q_1^c + (w_2 - c)q_2^c$$

$$- (w_1 - v)[E((1+\alpha)q_1^c + q_2^c - D)^+ - E((1-\omega)q_1^c + q_2^c - D)]$$

$$= (w_1(1+\alpha)q_1^c + w_2 q_2^c) - cQ_c^{\bullet} - (w_1 - v)\int_{(1-\omega)q_1^C + q_2^C}^{Q_C^{\bullet}} F(x)dx \qquad (2.4)$$

The following properties in Corollary 2.3 are useful to facilitate the comparisons.

Corollary 2.3

(1) $w_1 Q_c^{\bullet} \ge w_1(1+\alpha)q_1^c + w_2 q_2^c$

(2) $\int_{(1-\omega)q_C}^{Q_C^{\bullet}} F(x)dx \ge \int_{(1-\omega)q_1^C + q_2^C}^{Q_C^{\bullet}} F(x)dx$

Proof. (1) $w_1 Q_c^{\bullet} = w_1(1+\alpha)q_1^c + w_1 q_2^c \ge w_1(1+\alpha)q_1^c + w_2 q_2^c$

(2) $Q_c^{\bullet} = (1+\alpha)q_1^c + q_2^c = (1+\alpha)q_c$, so $q_1^{\bullet} + \frac{1}{1+\alpha}q_2^{\bullet} = q_c$

$\Rightarrow (1-\omega)q_1^c + \frac{1-\omega}{1+\alpha}q_2^c = (1-\omega)q_c$

From proposition 2.1, we know $\frac{1-\omega}{1+\alpha} = \chi_c$ at coordination

$\Rightarrow (1-\omega)q_1^c + \frac{1-\omega}{1+\alpha}q_2^c = (1-\omega)q_1^c + \frac{1}{\chi_c}q_2^c \le (1-\omega)q_1^c + q_2^c$ □

Corollary 2.3 indicates that there is a mixed relationship between Π_s^{QF} and Π_s^{QFi}; Π_s^{QFi} can be larger than, equal to, or smaller than Π_s^{QF}. This suggests that providing the QF*i* contract $(\alpha, \omega, w_1, w_2)$ to the buyer does not necessarily always benefit the supplier. In some circumstances, the supplier will be better off to simply stay with the traditional QF contract (α, ω, w). To better understand this phenomenon, define $\Delta\Pi_s = \Pi_s^{QFi} - \Pi_s^{QF}$:

$$\Delta\Pi_s = (w_1(1+\alpha)q_1^c + w_2q_2^c) - w_1Q_c^* - (w_1 - v)[\int_{(1-\omega)q_1^C+q_2^C}^{Q_C^*} F(x)dx - \int_{(1-\omega)q_C}^{Q_C^*} F(x)dx]$$

$$= -(w_1 - w_2)q_2^c + (w_1 - v)[\int_{(1-\omega)q_1^C+\frac{1}{\chi_C}q_2^C}^{Q_C^*} F(x)dx - \int_{(1-\omega)q_1^C+q_2^C}^{Q_C^*} F(x)dx]$$

$$= -(w_1 - w_2)q_2^c + (w_1 - v)\int_{(1-\omega)q_1^C+\frac{1}{\chi_C}q_2^C}^{(1-\omega)q_1^C+q_2^C} F(x)dx \tag{2.5}$$

Recall that under the QF contract, the supplier needs to bearthe risk of unsold units as high as $(\alpha+\omega)q_c$. With the new QF*i* contract, the highest possible unsold inventory that the supplier will claim for a loss is $(\alpha+\omega)q_1^c$. Because $q_c \geq q_1^c$, $(\alpha+\omega)q_c \geq (\alpha+\omega)q_1^c$. Therefore, the loss from expected unsold units that belongs to supplier's responsibility is less in the QF*i* contract than in the traditional QF contract. From (2.5), we can clearly observe the tradeoff between the loss of profits from discounted units and the benefit from lesser expected unsold units.

The first term refers to the total loss of profit (revenue) for units sold at the discount price. The second term indicates the savings from lesser responsibility if the volume of units produced is greater than the final demand. As such, when the expected saving from lesser responsibility on the unsold units outweighs the loss caused by the discounted units, i.e.,$\Delta\Pi_s > 0$, the supplier will favor the new QF*i* contract. In contrast, when the expected saving from lesser responsibility on the unsold items is not significant enough to cover the loss of profits by offering discount, , i.e.,$\Delta\Pi_s < 0$, the supplier should stick to the traditional QF contract and offer no discount options to the buyer. When $\Delta\Pi_s = 0$, the supplier is indifferent between the two QF contracts.

Proposition 2.2 As the flexibility increases under the traditional QF contract, the supplier will be inclined toward offering discount units above the contracted quantity.

Proof. It suffices to show that $\frac{d\Delta\Pi_s}{d\chi_c} > 0$. Alternatively, $(1-\omega)q_1^c + \frac{1}{\chi_c}q_2^c$ decreases in χ_c. Thus, $\int_{(1-\omega)q_1^C + \frac{1}{\chi_C}q_2^C}^{(1-\omega)q_1^C + q_2^C} F(x)dx$ increases in χ_c, so is $\Delta\Pi_s$. □

Theorem 2.3. If $(w_1 - v)\int_{(1-\omega)q_1^C}^{(1-\omega)q_1^C + q_2^C} F(x)dx > (w - w_2)q_2^c$, there exists a unique minimum flexibility $\chi_{\min}$, such that when $\chi_c > \chi_{\min}$, the supplier should offer the QF*i* contract $(\alpha, \omega, w_1, w_2)$. Otherwise, the supplier should remain offering the traditional QF contract (α, ω, w).

Proof. $\frac{d\Delta\Pi_s}{d\chi_c} = (w_1 - v)[-F((1-w)q_1^c + \frac{1}{\chi_c}q_2^c)(\frac{-1}{\chi_c^2})q_2^c] > 0$

$$\frac{d^2\Delta\Pi_s}{d\chi_c^2} = (w_1 - v)(\frac{-2q_2^c}{\chi_c^3})F((1-w)q_1^c + \frac{1}{\chi_c}q_2^c) - (w_1 - v)(\frac{q_2^c}{\chi_c^2})^2 f(x) < 0$$

So $\Delta\Pi_s$ is nondecreasing in χ_c

When χ_c=1, $\Delta\Pi_s = -(w_1 - w_2)q_2^c < 0$. In this case q_1^c will be 0.

When $\chi_c \to 1$=1, $\Delta\Pi_s = -(w_1 - w_2)q_2^c + (w_1 - v)\int_{(1-\omega)q_1^C}^{(1-\omega)q_1^C + q_2^C} F(x)dx$ >0 under the assumption. As a result, $\Delta\Pi_s$ will intersect with $\Delta\Pi_s$=0 only once.

Let $\chi_{\min}$ be the value of χ at the intersection. Then $\Delta\Pi_s \geq 0$, if $\chi_c \geq \chi_{\min}$; $\Delta\Pi_s < 0$, otherwise. □

Once the buyer and supplier agree upon the contract parameters(α, ω), thus the flexibility χ under the QF contract, the supplier can calculate the discount pricing w_2 via (2.2), followed by finding χ_c and $\chi_{\min}$ indicated in Theorem 2.3. The final decision as to whether to offer the discount prices above the contracted quantity or not can then be decided through the rule stated in Theorem 2.3. This determines when and under what conditions the supplier should propose discounted units on the top of the QF contract. When $(w_1 - v)\int_{(1-\omega)q_1^C}^{(1-\omega)q_1^C+q_2^C} F(x)dx > (w_1 - w_2)q_2^c$, $\chi_{\min}$ can be solved by setting $\Delta\Pi_s$ in (2.5) equal to zero. However, when $(w_1 - v)\int_{(1-\omega)q_1^C}^{(1-\omega)q_1^C+q_2^C} F(x)dx \le (w_1 - w_2)q_2^c$, it is the buyer who is going to benefit more from the QFi contract than from the traditional QF contract. In this case, there is no reason for the supplier to offer thediscount incentives and she should stick to offering only the traditional QF contract.

2.6.2 The general QF*i*/QF supply chains

A more general decision rule can be developed in a similar manner for the QFi/QF supply chain when the supply chain efficiency is not specified at 100%. Let $\Delta\pi_s = \pi_s^{QFi} - \pi_s^{QF}$, where π_s^{QFi} and π_s^{QF} are the supplier's expected profit under the QFi contract and the traditional QF contract, respectively. So, π_s^{QFi} and π_s^{QF} can be expressed as following:

$$\pi_s^{QFi} = (w_1(1+\alpha)q_1^* + w_2q_2^*) - c((1+\alpha)q_1^* + q_2^*) - (w_1 - v)\int_{(1-\omega)q_1^*+q_2^*}^{(1+\alpha)q_1^*+q_2^*} F(x)dx \qquad (2.6)$$

$$\pi_s^{QF} = (w_1 - c)(1+\alpha)q^* - (w_1 - v)\int_{(1-\omega)q^*}^{(1+\alpha)q^*} F(x)dx \qquad (2.7)$$

Thus, $\Delta\pi_s = \pi_s^{QFi} - \pi_s^{QF}$

$$= (w_1(1+\alpha)q_1^* + w_2 q_2^*) - c((1+\alpha)q_1^* + q_2^*) - (w_1 - c)(1+\alpha)q^*$$

$$+ (w_1 - v)[\int_{(1-\omega)q^*}^{(1+\alpha)q^*} F(x)dx - \int_{(1-\omega)q_1^*+q_2^*}^{(1+\alpha)q_1^*+q_2^*} F(x)dx] \tag{2.8}$$

So, when $\Delta\pi_s > 0$, supplier will benefit from providing the new QF*i* contract; when $\Delta\pi_s \le 0$, the supplier should stick to the traditional QF contract. Notice that q^* is the optimal forecast quantity under the traditional QF contract; refer to Tsay (1999) for the solution method. When the supply chain is not coordinated, $(1+\alpha)q_1^* + q_2^*$ and $(1+\alpha)q^*$ do not necessarily have a relationship as they do in the coordination condition. As such, (2.8) can not be rearranged into a format similar to (2.5) in which the difference of profits can be presented as a function of the flexibility measure χ in stead of (α, ω). Consequently, there is no χ analogous to $\chi_{\min}$ in (2.5) that can be identified in the general case.

Note q_1^* and q_2^* must be non-negative. Since the buyer decides the optimal forecasting and purchasing quantity under the QF*i* contract, it is easy to see that the solutions presented in Theorems 2.1(3) and (4) will bring the buyer at least the profit level that he can earn under the QF contract. On the other hand, $\Delta\pi_s$ must be positive to provide the supplier incentive to propose the QF*i* contract. As such, when the optimal solution q_1^* and q_2^* are both positive and will result in positive $\Delta\pi_s$, the QF*i* contract achieves Pareto improvement from the QF contract. In this case, the QFi contract creates a win-win situation for both the buyer and the supplier. We formally state this in the next proposition.

Proposition 2.3 When $q_1^* > 0$, $q_2^* > 0$, and $\Delta\pi_s > 0$ are true, the QF*i* contract achieves Pareto improvement from the QF contract.

In the next section, we introduce numerical examples to add to the development of the theoretical grounding of the new QF*i* contract and discuss managerial implications.

2.7 Numerical Experiments

We consider a base dataset for the numerical experiments as follows: $p = 50$, $c = 30$, $v = 20$, $w_1 = 42$, $w_2 = 40$. For convenience of the analysis and to enable closed forms for decisions variables in the QF*i* contract, we consider the market demand to follow a uniform distribution over the interval [400, 800]. Our analysis mainly focuses on comparison between the the QF*i* and the traditional QF contract. We examine the inventory decisions and the profit levels for both the buyer and the supplier across a range of discount pricing and flexibility agreements. We identify the conditions under which the supplier will benefit from offering additional discount units and conditions under which the supplier will be better off sticking to the QF contract. Then we analyze these areas of interest at the coordination condition.

2.7.1 Comparing the QF*i* and the QF contracts: varying discount pricing

We set the flexibility parameters at $\alpha = 0.2$, $\omega = 0.25$ in this experiment. The component price is \$42 under the QF agreement. Intended to earn more, the supplier proposes that the buyer can purchase any additional units above the contracted quantity $Q_1 = (1+\alpha)q_1$ at a cheaper price w_2. Figure 2.1 presents the buyer's decisions when the supplier decides to offer discount incentives in addition to the QF contract. As one can observe, it will not take much discount for the buyer to decide to simply go for the discount scheme for all that is needed. As the discount price w_2 decreases from \$42, it takes about \$1 of discount for the buyer to be interested in taking the offer. Between \$39 and \$41, the order size of the discount order q_2 increases and the forecast quantity decreases tremendously, and the buyer loses interest in the flexibility very quickly. When the discount price is below \$39, the buyer will discard the flexibility and only purchase discount units.

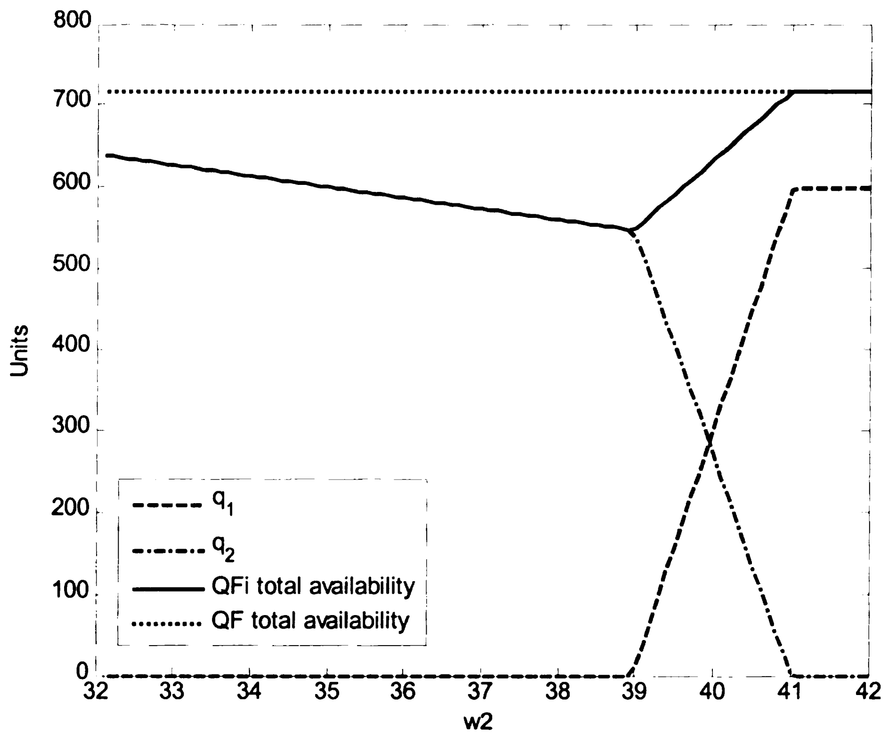

Figure 2.1 q_1 v.s. q_2 under QF*i*: w_1=42

From Figure 2.2, we can clearly see that the supplier can actually benefit from the QF*i* contract when she offers the discount price in the range between \$40.3 and \$41. In this range, the supplier is expected to earn more than what she could from the QF contract. On the other hand, we found that when $40.3 \leq w_2 \leq 41$, the buyer can increase his profit by ordering nonzero q_1 and q_2 at the same time. In other words, with w_1 =\$41 and \$$40.3 \leq w_2 \leq$ \$41, the QF*i* contract is able to create the Pareto improvement from the QF contract. The supplier's profit decreases as the discount price decreases. If she offers the discount price below \$39, the buyer will only be interested in the discount scheme and the supplier will be worse off compared with her expected profit when there is no discount, i.e., the QF contract. Note that between \$41 and \$42, the buyer will only be interested in executing the QF mechanism. So the result suggests that the supplier can propose the QF*i* contract, but the discount price should be somewhat close to the QF price. Otherwise, the QF*i* could only benefit the buyer more at the expense of the supplier.

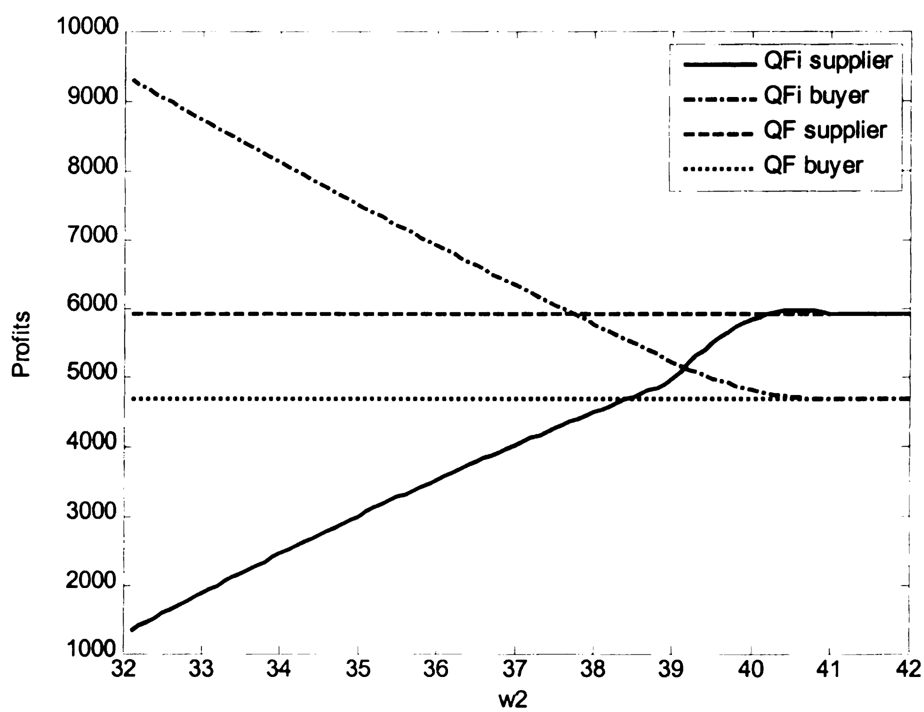

Figure 2.2 Supplier/buyer profits QF*i* v.s. QF: w_1 =42

Figure 2.3 presents the minimum level of flexibility against the discount pricing so that the buyer remains interested in executing the QF mechanism. As one can expect, the higher the discount, the higher the flexibility needed. Note that at each discount price, the supplier will be able to estimate the expected profit if all units are sold under the discount price. As such, the supplier will be able to understand the "value" of the flexibility and how much it will cost her by agreeing upon a set of flexibility parameter values (α , ω). This helps the supplier to estimate the magnitude of the flexibility offered and thus, leads to decisions as to whether to offer the QFi contract or not.

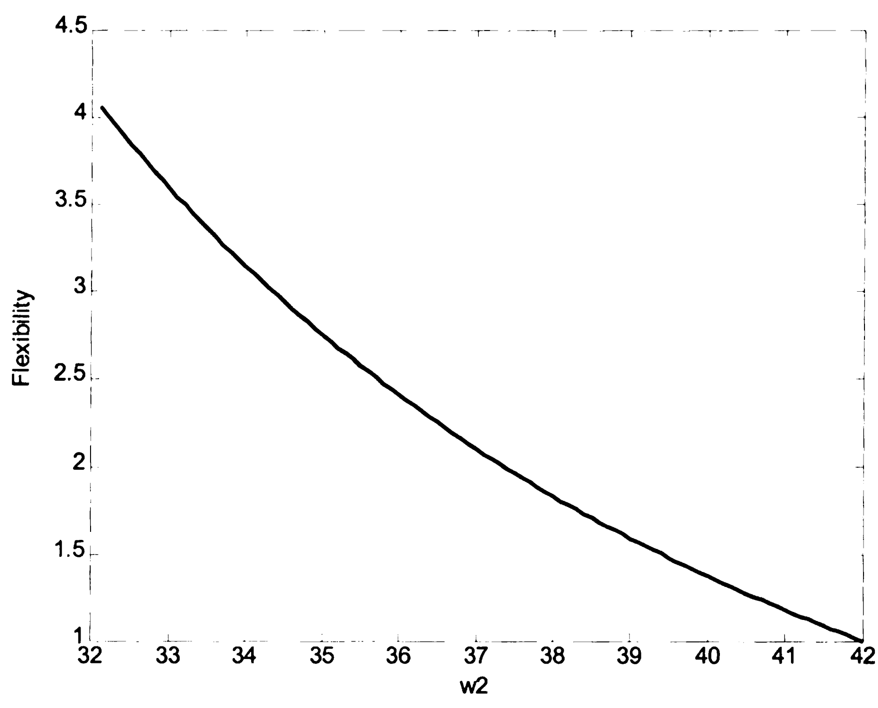

Figure 2.3 Minimum flexibility for QF to remain executed

Figure 2.4 presents the supply chain efficiency considering the buyer and the supplier's profits combined. The result indicates that there is a mixed performance compared with the

QF contract for the entire chain, as indicated in section 2.6. When the discount price is between \$39.8 and \$41, the QF*i* contract outperforms the QF contract. However, as the price continues to go down to \$38.9, the entire chain's profit starts declining down to an efficiency of 95.05% before it starts to climb up again. Combining this with the result in Figure 2.2, we can see that although the supply chain gets higher efficiency when the discount price continues to decrease from \$39.8, it is the buyer who gains all the benefits from the lower price. As the discount price decreases towards the component cost of \$30, the supply chain efficiency moves towards 100%, which is literately equivalent to a centralized chain in which the supplier makes no markup of the component cost when selling it to the buyer. The readers can also observe that the QF*i* contract achieves supply chain coordination when the discount price is at \$40.4, which is within the interval of the Pareto improvement. It is in this range that the supplier should target at setting the discount price.

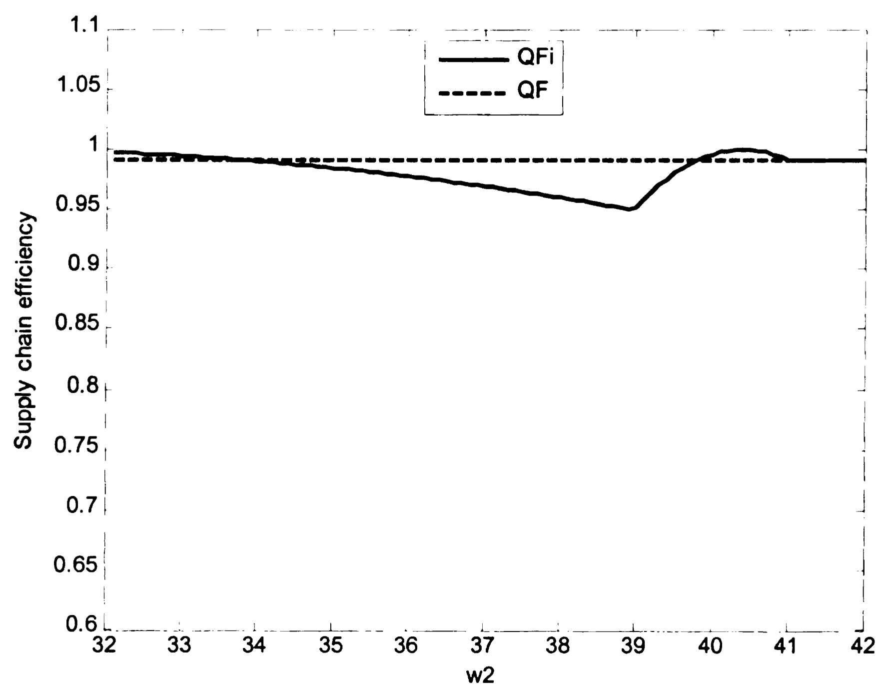

Figure 2.4 Supply chain performance: QF*i* vs. QF: w_1=42

2.7.2 Comparing the QF*i* and the QF contracts: varying the flexibility parameter

The magnitude of flexibility in the QF contract is another key factor that affects the supplier's decisions in offering the discount on additional units. Flexibility defined in the QF contract contains two parameters α and ω. We analyzed both parameters' impact on the QF*i* contract decisions. We found that increasing α and increasing ω, both enlarging flexibility, imposes similar impact on the inventory decisions in this experiment. Thus, we present only results from varying the ω value in this section. In this experiment, we set $w_1 = 42$, $w_2 = 40$, and $\alpha = 0.1$.

Figure 2.5 presents the QF*i* decisions varied with the magnitude of QF flexibility indicated by ω. When ω increases, flexibility increases as well. As one can observe, when ω is small, the \$2 discount is more attractive. So the buyer will only purchase firm order q_2 at the discount price $w_2 = \$40$. Recall that these units are non-returnable. As ω increases, i.e., flexibility increases, the buyer gains interest in executing the QF contract as the protection of inventory risk provided will outweigh the cost savings from the discount scheme. However, this result simply presents the buyer's decisions across the various levels of flexibility. Results from the previous section clearly indicate that when only the discount scheme is executed, the supplier is actually worse-off. Our next experiment presents the ideal range of flexibility to offer the discount.

From Figure 2.6, we can observe that the supplier can benefit from offering discount at $w_2 = \$40$ when $\omega \geq 0.33$. As we know, when flexibility is high, the supplier's inventory responsibility is high under the traditional QF contract. So, with the units sold at the discount price, the inventory risk for the supplier is mitigated and its expected financial impact (gain) is greater than the sales loss of \$2 per unit. Therefore, the supplier will benefit from offering the QF*i* contract in the high flexibility range as indicated in the figure.

On the other hand, one can also observe that the buyer can always benefit if the discount scheme is also offered. As such, when $\omega \geq 0.33$, both supplier and buyer can benefit from the QFi contract, yielding the Pareto improvement from the QF contract. Although the numerical results may vary with different parameter values, this observation supports that in some circumstances, the QFi contract can mostly outperform the traditional QF contract when the flexibility is high. Figure 2.7 shows explicitly the profit difference between the two contracts for the supplier.

Finally, Figure 2.8, provides a clear view to support the benefits of the QFi contract. Both the QFi contract and the QF contract achieve 100% efficiency at a point within the presented range of ω. From the system profit point of view, the QFi contract outperforms the QF contract when $\omega \geq 0.3$ in this specific experiment. Note that from Figure 2.6, when $\omega \geq 0.33$, the QFi contract achieves Pareto improvement from the QF contract. On the other hand, when $\omega < 0.3$, the QF contract achieves higher system profit for the entire chain. Specifically, when $\omega \leq 0.2$, under the QFi contract, the buyer will buy only q_2 and is not interested in executing the QF mechanism. In this range, the buyer gains profit, but the supplier will suffer from financial loss, the magnitude of which is so large that it causes the entire chain's efficiency to decline. In this case, the system will favor the QF contract over the QFi contract.

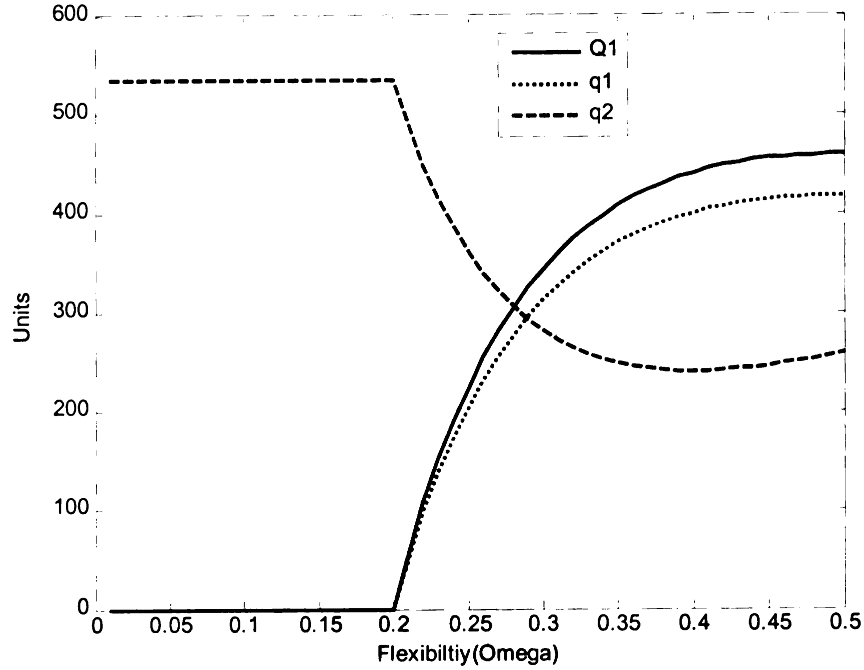

Figure 2.5 q_1 v.s. q_2 under QF*i*: w_1=42, w_2=40

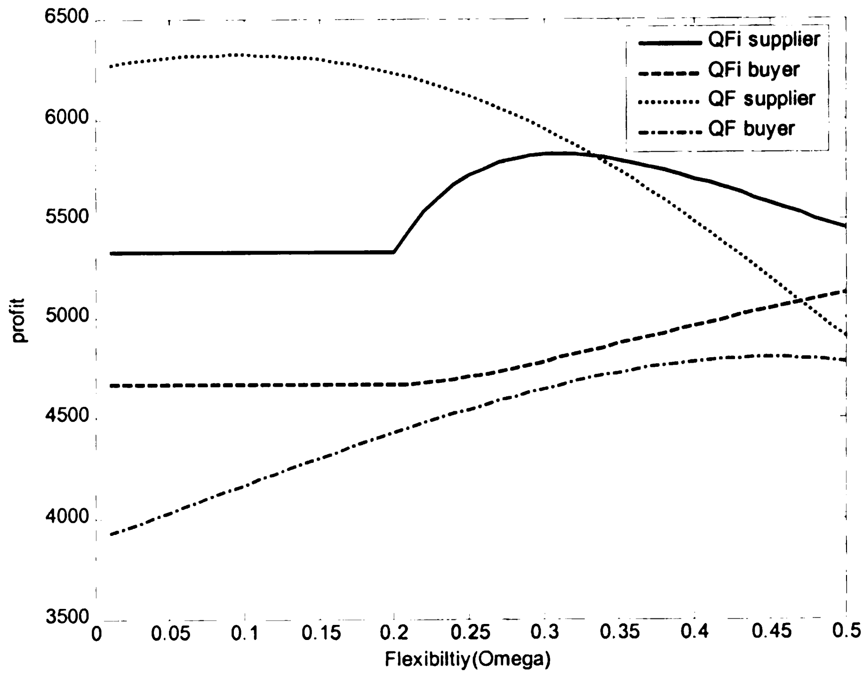

Figure 2.6 Supplier/buyer profits QF*i* v.s. QF: w_1=42, w_2=40

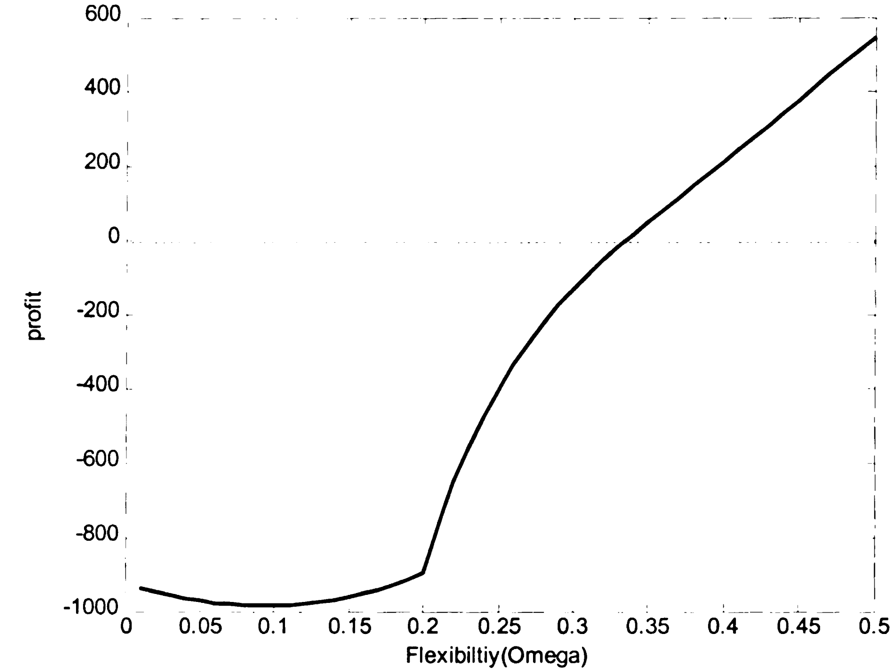

Figure 2.7 Supplier profit difference (QFi-QF): w_1=42, w_2=40

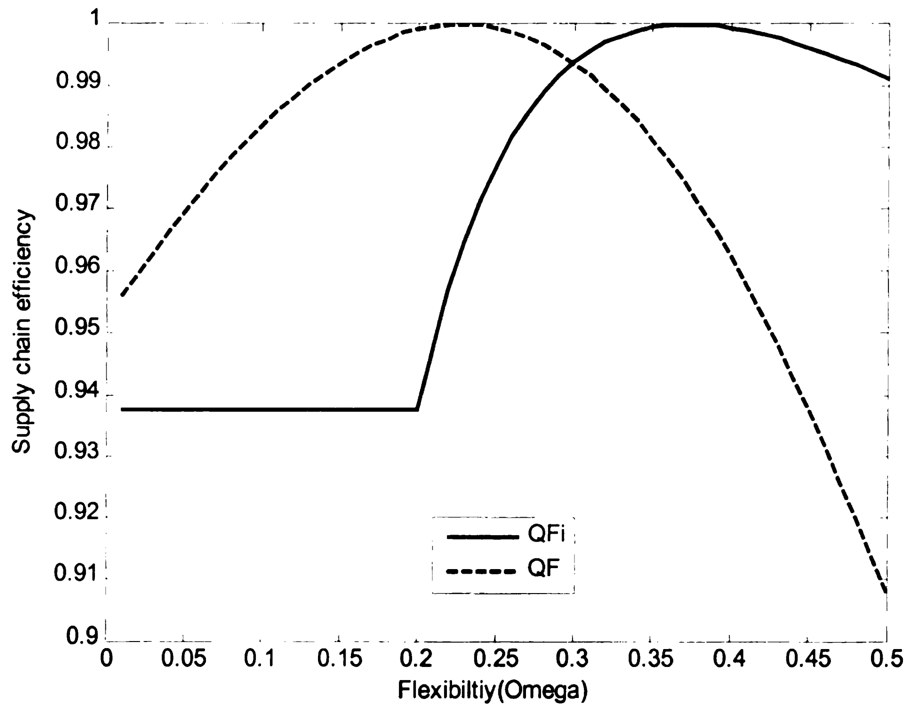

Figure 2.8 Total profits QFi v.s. QF: w_1=42, w_2=40

2.7.3 Comparing the QF*i* and the QF contracts when the supply chain is coordinated

Recall that the QF*i* can achieve supply chain coordination by identifying w_2 via (2.2), given w_1. So, we vary w_1 in this experiment and calculate the corresponding coordinating w_2 via (2.2). The benefit of the QF*i* contract is that it allows coordination literately for any given w_1, without loss of generality, although the coordination might come at the discount price that is undesirable for the supplier. In these experiments, we set $\alpha = 0.1$ and $\omega = 0.1$.

Figure 2.9 presents the buyer and the supplier's profits under both the QF*i* and QF contracts when both coordinate the supply chain. With the given set of parameter values, the coordinating price scheme under the QF contract is \$35.85. The coordinated QF contract brings the buyer a profit close to \$8000 and the supplier around \$3000. As indicated in the figure, as w_1 increases, supplier's profit can also increase. Intuitively, the supplier's profit will increase when both w_1 and w_2 increase. However, there is an important managerial implication here as discussed below.

In our initial problem setting, the component price w_1 is set at \$42. To achieve the supply chain efficiency under the QF contract, supplier has to agree to sell them at \$35.85 so that the entire supply chain's expected profit can reach its maximum. However, the improved supply chain performance comes at the expense of the supplier, because she has to provide the same level of flexibility, yet lowering the price to induce the buyer to act in line with the global objective. As such, the supply chain coordination becomes an ideal condition and is unlikely to happen when the buyer is taking all the benefits. The QF*i* contract provides a solution for this. The supplier can offer any price w_1 in between the original price \$42 and the QF coordinating price \$35.85, and offer a discount price w_2 via (2.2) to increase her profit, yet without causing a deficiency for the whole chain. The profit allocation between the buyer and the supplier is clearly presented in Figure 2.9. Notice that each w_1 is associated with a coordinating discount scheme w_2; these discount prices w_2 are not presented in Figure 2.9.

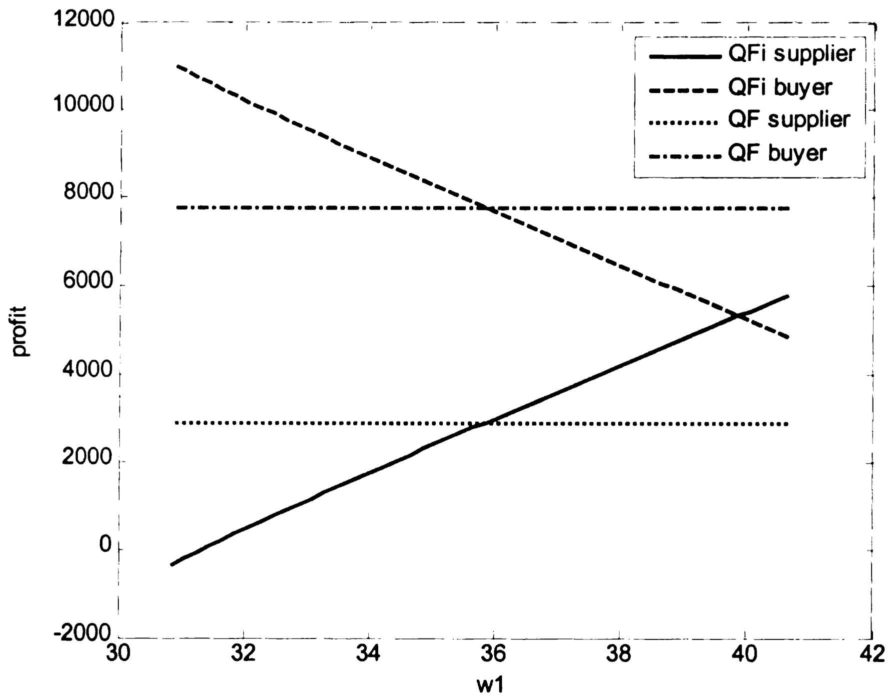

Figure 2.9 Supplier/buyer profits QFi v.s. QF at coordination: w_{QF} =38.85

Finally, we examine the performance of the QFi contract at coordination when the magnitude of flexibility (ω) changes. Both the coordinating component price w_{QF} under the QF contract and the discount price w_2 under the QFi contract vary with flexibility. As such, w_{QF} and w_2 are not constant in this experiment. But w_1 is fixed at \$42. As shown in Figure 2.10, when $\omega \leq 0.22$, the QFi contract will not result in non-zero q_2^* at coordination. As such, the buyer and the suppliers' profit curves are overlap in Figure 2.11. However, once ω exceeds 0.22, the buyer will execute both the QF and price discount scheme to achieve the 100% efficiency. However, Figure 2.11 exhibits that the supplier is actually worse-off from the QF contract when coordinating in the high ω region. This is opposite to the results from previous experiments in Figure 2.6 when supply chain is not coordinated. So, does the QF*i* contract result in undesirable discount scheme for achieving **system-wide** profit for the supplier when flexibility is high? Not really. A deeper investigation on the coordinating pricings (Figure 2.12) reveals that under the current experiment setting, the coordinating w_{QF} under the QF contract increases toward the retail price ($50) as ω increases. In other words, when the supplier offers large flexibility under the QF contract, the coordinating component price needs to be very close to the retail price, and the supplier expects the majority of profit as indicated in Figure 2.11. However, due to its capability of coordinating the supply chain at any (reasonable) w_1, the QF*i* contract can identify a discount scheme w_2 so that, together with w_1, it can reach the 100% supply chain efficiency. Although w_2 varies with ω, it is relatively stable across the various level of flexibility, as indicated in Figure 2.12 As a result, the QFi contract does not result in radical change in pricing scheme as the QF contract does, when the system-wide profit is the goal.

Furthermore, we can observe in Figure 2.10 that as ω exceeds 0.22, the decisions under the QF*i* contract start deviating from those under the QF contract. However, the resulting buyer and suppliers' profits in Figure 2.11 do not vary much. In contrast, under the QF contract, it is obvious that one party (in this case, the supplier) tends to get all the benefit to allow an efficient supply chain to exist. In other words, the profit allocation between the two parties changes dramatically as flexibility grows. Consequently, when flexibility is relatively high, we found that the QF*i* contract is a more desirable contract from the system's perspective. It can easily achieve global optimal profit at the original price combined with a relative stable and reasonable discount scheme. It also avoids dramatic changes in profit split between the two agents in the supply chain.

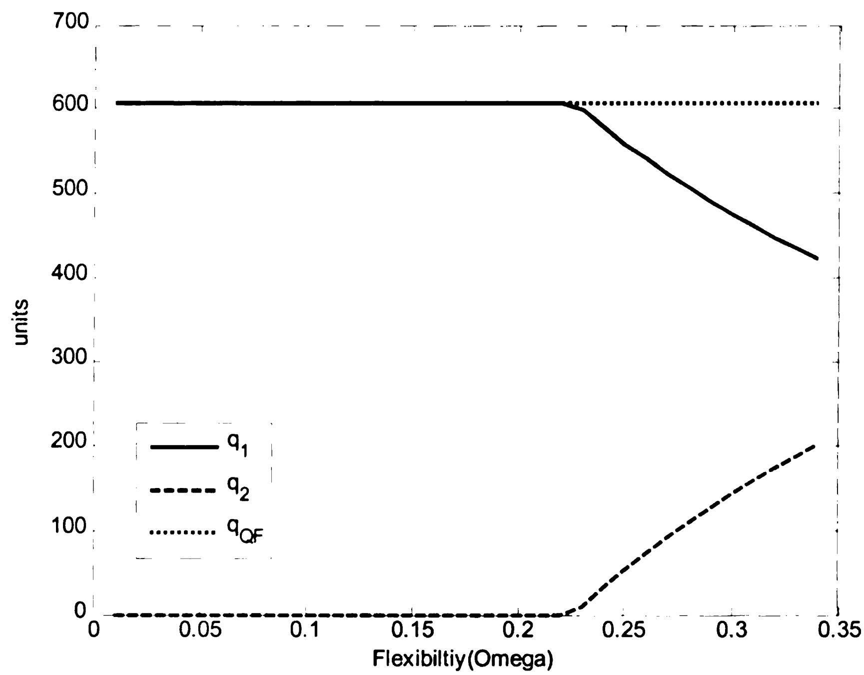

Figure 2.10 q_1 v.s. q_2 under QF*i* at coordination: $w_1 = w_{QF}$

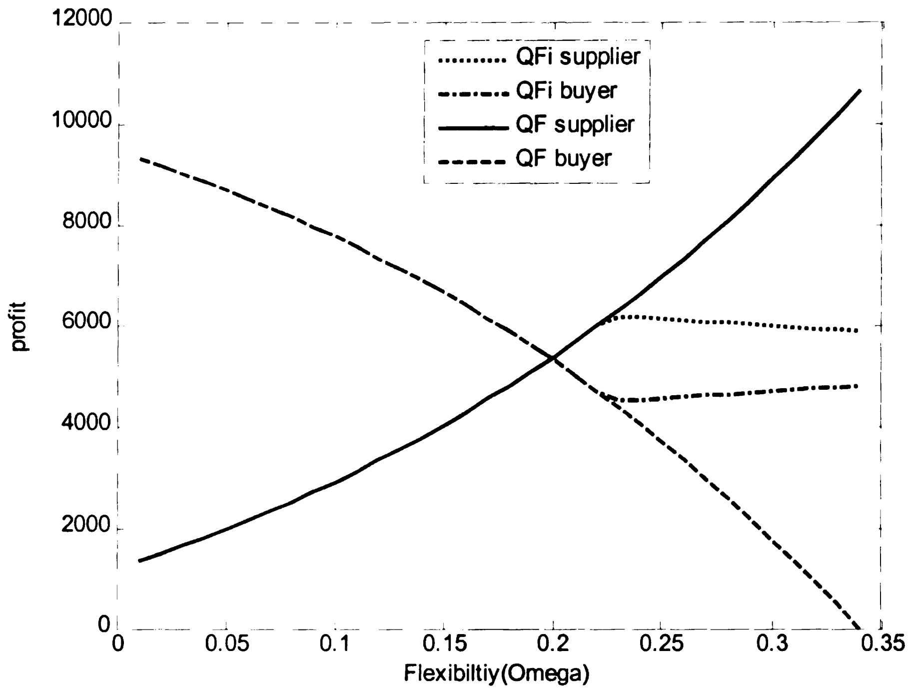

Figure 2.11 Supplier/buyer profits QF*i* v.s. QF: w_1=42

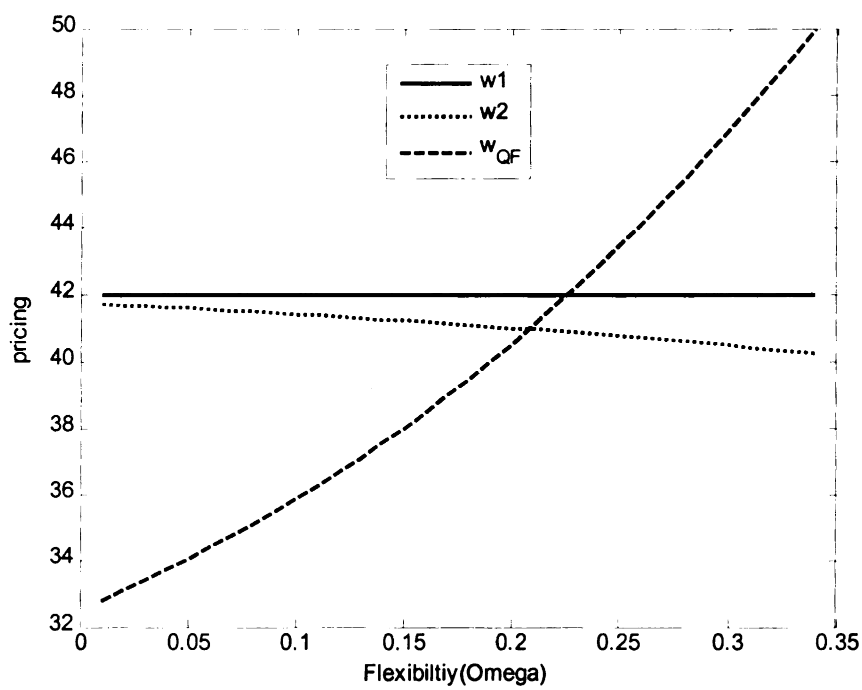

Figure 2.12 Coordinating pricing in QF*i* and QF contracts

2.8 Conclusions and Discussions

We considered a decentralized supply chain in which the traditional QF contract is in employed to govern the buy and sell activities between the buyer and the supplier in this supply chain. Under the QF contract, the supplier agrees to share the burden of inventory risk with the buyer to a certain extent. To mitigate the impact of the returnable stock on her total expected profit, the supplier proposes to offer a discount price for any unit sold above the QF-contracted quantity. The purpose is to reduce the supplier's inventory burden (loss) imposed by the QF contract so that her total expected profit may increase from the QF contract settings. We termed this contract as the QF contract with incentives, the QF*i* contract.

We analyzed how the buyer reacts to the QF*i* contract so that the conditions under which the QF*i* contract can work in favor of the supplier are identified. We have shown that when the QF*i* contract is executed, it can actually benefit both the supplier and the buyer at the same time. Specifically, we found that the QF*i* contract works the best when the supplier offers a slightly lower price than the QF contract price for the same component. In this case, the QF*i* contract can create Pareto improvement compared with the QF contract, thus resulting in a win-win situation for both the buyer and the supplier.

We have shown that the QF*i* contract is able to achieve supply chain coordination. More importantly, it can achieve the system-wide profit literally at any component price agreed upon by both parties under the QF agreements. In contract to the QF contract, the QF*i* contract does not require the knowledge of demand distribution for obtaining the coordinating price scheme (Tsay, 1999; Cachon and Lariviere, 2005). .

We conducted a series of numerical experiments to demonstrate various strategies that can be utilized to improve the supplier's profit via the QF*i* contract. We identified the areas where the QF*i* contract achieves Pareto improvement compared with the QF contract. We showed how the supplier can estimate the "worth" of flexibility that she offers to the buyer. More importantly, we showed that the QF*i* achieves supply chain coordination in a much more "flexible" manner than the QF contract.

In summary, the main contributions of this chapter are threefold. First, we designed a new QF*i* contract that can work as well as the QF contract, yet it provides an option for the buyer and supplier to compromise the inventory risk level between the QF contract and the price-only contract. We developed the theoretical foundation of the QF*i* contract, and demonstrated a series of numerical experiments to demonstrate how the QF*i* contract can be utilized to improve the supplier's profit from the QF contract. Although we developed this contract from the supplier's perspective, we have shown that at the correctly chosen discount pricing, both parties can benefit when QF*i* is executed. The supplier can stick to the QF contract, or introduce the QF*i* contract when appropriate.

Second, we are able to identify areas where it is more beneficial to implement the QF*i* contract than the QF contract from the supplier's point of view. We conducted thorough comparisons and analyses between the QF*i* contract and the QF contract. We presented conditions where the QF*i* contract outperforms the traditional QF contract, at both the individual and the system level. We provided a framework for the supplier to evaluate the value of flexibility. More importantly, we showed that the QFi contract can possibly result in Pareto improvement.

Third, we have shown that QF*i* contract has the "flexibility" to achieve supply chain coordination. As mentioned earlier, the QFi contract has the advantage of achieving system-wide profit at multiple pricing schemes, yet it is not restricted by the need for the knowledge of demand distribution. We consider this a major improvement from the QF contract. Specifically, under the QF contract the coordination might heavily favor one specific agent in the supply chain. Thus, the disadvantaged party will be reluctant to act in line with the system's goal under the QF contract. With the QF*i* contract, such a problem will easily be fixed. As shown in our experiments, implementation of the QFi contract towards the 100% supply chain efficiency does not result in radical change of system profit allocation between the buyer and the supplier. The coordination can be achieved at the very component price that is agreed upon by the two parties under the QF contract. As such, the coordinating profits for both parties will not deviate significantly from where they stand when there is no discount incentive. As such, the buyer and supplier should not be against strongly aligning their actions with the system's goal. Consequently, from the system's perspective, we believe that the QFi contract is more desirable than the QF contract due to these advantages.

From the numerical experiments, we found that the supplier should offer small incentives off the QF contract price rather than offering a large discount, or buyer will be tempted to the cost saving and lose interest in the benefit of flexibility very quickly. As the discount enlarges, the buyer will not hesitate to simply buy everything at the discount price, if no other restriction apply. In this case, the QFi contract becomes a price-only contract and the discount price will likely hurt the supplier more than expected. As such, the supplier should not "overdo" the discount; a more conservative discount scheme is shown most favorable for the supplier to achieve the desirable results.

The QF*i* contract is designed to allow the supplier to pose no restriction to the buyer in terms of forecasting under the QF agreement and the firm order under the price discount scheme. As mentioned, it might not take much discount before the buyer becomes completely uninterested in executing the QF mechanisms. Posing a minimum required forecast under the QF contract may dampen the effects that the discount pricing may have on the buyer's loss of interest in the QF contract and on the sudden decrease of supply chain efficiency , as observed in our results. This provides an opportunity for future research to further investigate the performance of the QFi contract. Additionally, future research can apply the QFi contract on a multiple period problem setting or a more complicated supply chain structure. When multi- period scenario is considered, the need for offering discount might not be at the same level as it is in the single period. Furthermore, when there are multiple buyers in the supply chain, the supplier's behavior is likely to change as well. Finally, the inventory decisions and the coordinating discount scheme in the QFi contract will change if the demand is price-sensitive. More research on the areas of the QFi/QF contracts is warranted.

Appendix of Chapter 2

Proof of Theorem 2.1 (1)

$$\frac{d\Pi_b}{dq_1} = (p-w_1)(1+\alpha) - (p-w_1)[F((1+\alpha)q_1+q_2)(1+\alpha) - F((1-\omega)q_1+q_2)(1-\omega)]$$

$$-(p-v)F((1-\omega)q_1+q_2)(1-\omega)$$

$$= (p-w_1)(1+\alpha)\overline{F}((1+\alpha)q_1) + (p-w_1)(1-\omega)F((1-\omega)q_1+q_2)$$

$$-(p-v)(1-\omega)F((1-\omega)q_1+q_2)$$

$$= (p-w_1)(1+\alpha)\overline{F}((1+\alpha)q_1+q_2) - (w_1-v)(1-\omega)F((1-\omega)q_1+q_2)$$

$$\frac{d^2\Pi_b}{dq_1^2} = -(p-w_1)(1+\alpha)^2 f((1+\alpha)q_1+q_2)$$

$$-(w_1-v)(1-\omega)^2 f((1-\omega)q_1+q_2) < 0$$

$$\frac{d\Pi_b}{dq_2} = (p-w_2) - (p-w_1)[F((1+\alpha)q_1+q_2) - F((1-\omega)q_1+q_2)]$$

$$-(p-v)F((1-\omega)q_1+q_2)$$

$$= (w_1 - w_2) + (p - w_1)\overline{F}((1+\alpha)q_1 + q_2) - (w_1 - v)F((1-\omega)q_1 + q_2))$$

$$\frac{d^2\Pi_b}{dq_2^2} = -(p - w_1)f((1+\alpha)q_1 + q_2) - (w_1 - v)f((1-\omega)q_1 + q_2) < 0$$

$$\frac{d^2\Pi_b}{dq_1 dq_2} = \frac{d^2\Pi_b}{dq_2 dq_1} = -(p - w_1)(1+\alpha)f((1+\alpha)q_1 + q_2)$$

$$-(w_1 - v)(1-\omega)f((1-\omega)q_1 + q_2) < 0$$

So by examining the Hessian matrix $|H|$ and its leading principle minors $|H_1|$ and $|H_2|$

we have: $|H_1| = \frac{d^2\Pi_b}{dq_1^2} < 0$ and

$$|H_2| = |H| = \begin{vmatrix} \frac{d^2\Pi_b}{dq_1^2} & \frac{d^2\Pi_b}{dq_1 dq_2} \\ \frac{d^2\Pi_b}{dq_2 dq_1} & \frac{d^2\Pi_b}{dq_2^2} \end{vmatrix}$$

$$= (p - w_1)^2(1+\alpha)^2 f^2((1+\alpha)q_1 + q_2) + (p - w_1)(w_1 - v)(1-\omega)^2 f((1-\omega)q_1 + q_2)$$

$$+ (p - w_1)(1+\alpha)^2(w_1 - v)f((1+\alpha)q_1 + q_2)f((1-\omega)q_1 + q_2)$$

$$+ (1-\omega)^2(w_1 - v)^2 f^2((1-\omega)q_1 + q_2) - (p - w_1)^2(1+\alpha)^2 f^2((1+\alpha)q_1 + q_2)$$

$$- 2(p - w_1)(1+\alpha)(w_1 - v)(1-\omega)f((1+\alpha)q_1 + q_2)f((1-\omega)q_1 + q_2)$$

$$- (w_1 - v)^2(1-\omega)^2 f^2((1-\omega)q_1 + q_2)$$

$$= (p - w_1)(w_1 - v)f((1+\alpha)q_1 + q_2)f((1-\omega)q_1 + q_2)[(1+\alpha)^2$$

$$+ (1-\omega)^2 - 2(1+\alpha)(1-\omega)]$$

$$= (p - w_1)(w_1 - v)f((1+\alpha)q_1 + q_2)f((1-\omega)q_1 + q_2)(\alpha^2 + \omega^2 + 2\alpha\omega)$$

$$= (p-w_1)(w_1-v)f((1+\alpha)q_1+q_2)f((1-\omega)q_1+q_2)(\alpha+\omega)^2 > 0$$

So $|H|$ is negative-definite, thus Π_b is concave in q_1 and q_2 □

Proof of Theorem 2.1 (2)

Here we consider the non-extreme cases only, i.e. $(\alpha,\omega)\neq 0$ and $\omega \neq 1$. At optimum, we have the first order conditions: $\dfrac{d\Pi_b}{dq_1}=0$ and $\dfrac{d\Pi_b}{dq_2}=0$. Thus,

$$(p-w_1)(1+\alpha)\overline{F}((1+\alpha)q_1)-(w_1-v)(1-\omega)F((1-\omega)q_1+q_2)=0 \tag{A2.1}$$

$$(w_1-w_2)+(p-w_1)\overline{F}((1+\alpha)q_1+q_2)-(w_1-v)F((1-\omega)q_1+q_2))=0 \tag{A2.2}$$

Operate (A2.2)* $(1-\omega)$ − (A2.1), then divided by $(1-\omega)$ we get

$$(w_1-w_2)+(p-w_1)\overline{F}((1+\alpha)q_1+q_2)-(p-w_1)\chi\overline{F}((1-\omega)q_1+q_2))=0$$

$$\Rightarrow (w_1-w_2)=(\chi-1)(p-w_1)\overline{F}((1+\alpha)q_1+q_2)$$

$$\Rightarrow \overline{F}((1+\alpha)q_1+q_2)=\frac{(w_1-w_2)}{(\chi-1)(p-w_1)}=1-F((1+\alpha)q_1+q_2)$$

$$\Rightarrow F((1+\alpha)q_1+q_2)=1-\frac{(w_1-w_2)}{(\chi-1)(p-w_1)}=\frac{(p-w_1)\chi-(p-w_2)}{(\chi-1)(p-w_1)}$$

$$=\frac{(p-w_1)\chi-(p-w_2)}{(p-w_1)\chi-(p-w_1)} \tag{A2.3}$$

Operate (A2.2)* $(1+\alpha)$ − (A2.1), then divided by $(1-\omega)$ we get

$$\chi(w_1-w_2)-\chi(w_1-v)F((1-\omega)q_1+q_2)+(w_1-v)F((1-\omega)q_1+q_2))=0$$

$$\Rightarrow (\chi-1)(w_1-v)F((1-\omega)q_1+q_2)=\chi(w_1-w_2)$$

$$\Rightarrow F((1-\omega)q_1+q_2)=\frac{\chi(w_1-w_2)}{(\chi-1)(w_1-v)} \qquad \text{(A2.4)}$$

Since F is the p.d.f. of the random variable D, the following must be true:

$$0\le F((1-\omega)q_1+q_2)\le F((1+\alpha)q_1+q_2)\le 1$$

From (A2.3) and (A2.4), it is easy to verify that $F((1+\alpha)q_1+q_2)\le 1$ and

$F((1-\omega)q_1+q_2)\ge 0$. From (A2.3) and (A2.4) we also have

$$\frac{\chi(w_1-w_2)}{(\chi-1)(w_1-v)}\le\frac{(p-w_1)\chi-(p-w_2)}{(p-w_1)\chi-(p-w_1)}=\frac{(p-w_1)\chi-(p-w_2)}{(\chi-1)(p-w_1)}$$

$$\Rightarrow \frac{\chi(w_1-w_2)}{(w_1-v)}=\chi-\frac{\chi(w_2-v)}{(w_1-v)}\le\frac{(p-w_1)\chi-(p-w_2)}{(p-w_1)}=\chi-\frac{(p-w_2)}{(p-w_1)}$$

$$\Rightarrow \chi\ge\underline{\chi}=\frac{(p-w_2)(w_1-v)}{(p-w_1)(w_2-v)} \quad \square \qquad \text{(A2.5)}$$

Proof of Theorem 2.1 (3) & (4)

Let $M=\dfrac{(p-w_1)\chi-(p-w_2)}{(\chi-1)(p-w_1)}$ and $N=\dfrac{\chi(w_1-w_2)}{(\chi-1)(w_1-v)}$. From (A2.3) and (A2.4) we have

$$(1+\alpha)q_1+q_2=F^{-1}(M) \qquad \text{(A2.6)}$$

$$(1-\omega)q_1+q_2=F^{-1}(N) \qquad \text{(A2.7)}$$

Thus, $q_1^*=\dfrac{1}{\alpha+\omega}(F^{-1}(M)-F^{-1}(N))=\dfrac{(1-\omega)(F^{-1}(M)-F^{-1}(N))}{(\chi-1)}$

$$q_2^* = \frac{1}{\alpha+\omega}((1+\alpha)F^{-1}(N) - (1-\omega)F^{-1}(M)) = \frac{\chi F^{-1}(N) - F^{-1}(M)}{\chi - 1}$$

To show that q_1^* is constrained by q_{QF}, let $q_2^* > 0$ optimal firm order under the QFi contract. Thus, we can express the buyer's expected profit as

$$\begin{aligned}\Pi_{QFi}(q_1, q_2^*) &= (p-w_1)(1+\alpha)q_1 + (p-w_2)q_2^* \\ &\quad - (p-w_1)[E((1+\alpha)q_1 + q_2^* - D)^+ - E((1-\omega)q_1 + q_2^* - D)^+] \\ &\quad - (p-v)E((1-\omega)q_1 + q_2^* - D)^+ \end{aligned} \tag{A2.8}$$

Let $D' = D - q_2^*$, (A2.8) can then be rewritten as

$$\begin{aligned}\Pi_{QFi}(q_1, q_2^*) &= (p-w_1)(1+\alpha)q_1 + (p-w_2)q_2^* \\ &\quad - (p-w_1)[E((1+\alpha)q_1 - D')^+ - E((1-\omega)q_1 - D')^+] \\ &\quad - (p-v)E((1-\omega)q_1 - D')^+ \end{aligned} \tag{A2.9}$$

It is clear that the q_1^* of (A2.9) follows the QF contract solution, only the demand function is D' rather than D. Since $D' < D$, $q_1^* < q_{QF}$ must be true when $q_2^* > 0$.

Next, we show that q_2^* is constrained by $q_{w2} = F^{-1}(\frac{p-w_2}{p-v})$. Let $q_1^* > 0$ the optimal forecast under the QFi contract. Then we can express the buyer's expected profit as

$$\begin{aligned}\Pi_{QFi}(q_1^*, q_2) &= (p-w_1)(1+\alpha)q_1^* + (p-w_2)q_2 \\ &\quad - (p-w_1)[E((1+\alpha)q_1^* + q_2 - D)^+ - E((1-\omega)q_1^* + q_2 - D)^+] \\ &\quad - (p-v)E((1-\omega)q_1^* + q_2 - D)^+ \end{aligned}$$

$$=(p-w_2)q_2-(p-v)E(q_2-D_2)^+$$

$$-(p-w_1)[E(q_2-D_1)^+-E(q_2-D_2)^+]$$

$$+(p-w_1)(1+\alpha)q_1^* \tag{A2.10}$$

where $D_1=D-(1+\alpha)q_1^*$ and $D_2=D-(1-\omega)q_1^*$. Also, the last term is a constant.

Define $\Pi_{QFi}^1(q_2)=(p-w_2)q_2-(p-v)E(q_2-D_2)^+$

$$-(p-w_1)[E(q_2-D_1)^+-E(q_2-D_2)^+]$$

$$\Pi_{QFi}^2(q_2)=(p-w_2)q_2-(p-v)E(q_2-D_2)^+$$

It is clear that the optimal solution for $\Pi_{QFi}^2(q_2)$ is $F_{D2}^{-1}(\frac{p-w_2}{p-v})$, where F_{D2} is the c.d.f. of D_2. Thus, $F_{D2}^{-1}(\frac{p-w_2}{p-v})<F^{-1}(\frac{p-w_2}{p-v})$ must be true because $D_2<D$.

$\Pi_{QFi}^1(q_2)$ shares the same optimal solution with $\Pi_{QFi}(q_1^*,q_2)$. Note that $[E(q_2-D_1)^+-E(q_2-D_2)^+]$ in $\Pi_{QFi}^1(q_2)$ is always greater than or equal to 0. It is also increasing in q_2. So the optimal solution of $\Pi_{QFi}^1(q_2)$ is always less than or equal to $F_{D2}^{-1}(\frac{p-w_2}{p-v})$, because a larger q_2 will result in reduced value of both $(p-w_2)q_2-(p-v)E(q_2-D_2)^+$ and $-(p-w_1)[E(q_2-D_1)^+-E(q_2-D_2)^+]$.

Thus, we conclude that the optimal q_2^* in $\Pi_{QFi}(q_1^*,q_2)$ of (A2.10) is always less than $q_{w2}=F^{-1}(\frac{p-w_2}{p-v})$ when $q_1^*>0$. □

CHAPTER 3 QUANTITY FLXIBILITY CONTRACT IN A TWO-SUPPLIER-ONE-BUYER SUPPLY CHAIN

3.1 Introduction and Literature Review

Recent trends in business partnerships and reduction in the size of supply base in various industries has increased the need for developing solutions and directions for improving a supply chain's performance from a system's view. Among them is the stream of supply chain contract research. Utilizing the newsvendor framework as backbone, many have reported that through proper design of supply chain contract(s), the decentralized decision making can be aligned with the global goal of the entire supply chain. Through mutually-agreed contractual mechanisms, buyers and suppliers can possibly coordinate their business and create a win-win situation compared with the transactional-based relationship.

Although numerous studies on supply chain contracts can be found in literature, the majority have a focus on either one-buyer-one-supplier (1-1) supply chains or one supplier-many-buyer(retailer) (1-N) supply chains. However, a firm can very likely have more than one supplier for the same set of components. For example, a PC manufacturer can normally buy Hard Disk Drive (HDD) from more than two HDD suppliers; an automobile manufacturer can have multiple sources for car lamps; and finally, a grocery store has different brands of cereal on shelves. These suppliers are competing suppliers and their components are "substitutable". Additionally, the buyer may have different types of relationships with these suppliers that produce the same (substitutable) components. For example, Acer, a PC maker from Taiwan, announced its partnership with an original equipment manufacturer, Wistron Corporation, yet Acer employees two other OEMs as alternative sources at the same time. The supply chain contract literature has given relatively little attention to the larger segments of manufacturing and most consumer goods in which multiple competing suppliers are available to the buying firm (Choi, 1999).

Tomlin (2003) investigated the price-only contracts considering capacity investments between a component supplier and a manufacturer. He identified a quantity-premium price-only contract that coordinates one-supplier-one-manufacturer supply chains, and extended the results to an N-supplier assembly system. Each of the N suppliers produces a different component and the manufacturer assembles the N components into the end product. In other words, they are complimentary suppliers, rather than competing suppliers. Gerchak and Wang (2004) studied the revenue sharing contract with incentive to suppliers and the wholesale price contracts with buyback, each under a supply chain consisting of multiple complementary suppliers and one assembler. They identified conditions under which the channel is coordinated for each contract. They found that the channel performance of the latter can be degraded with the number of suppliers, while it is not the case for the former contract. Zou et al. (2004) argued that, due to variations of order processing time and cost structures, supply chains consisting of multiple suppliers and one assembler need to be synchronized to improve system efficiency. The N suppliers in their research are also complementary suppliers. They found that risk sharing and proper safety stock placement lead to better system coordination and improve system performance.

Several researchers dealt with competing suppliers in difference problem settings. Choi (1991) analyzed three channel competition problems in a supply chain that consist of two competing suppliers and single powerful buyer. Two of the three problems resemble the Stackelberg game and one resembles the Nash game, resulting from difference in the power structure of the buyer and the two suppliers. He investigated the effect of cost differences on equilibrium prices and profits. He found that the forms of the demand function play an important role in such decisions: the buyer has an incentive to use multiple suppliers when demand is linear while it is more desirable for the buyer to use only one supplier if the demand function is non-linear.

A more "contract-oriented" mutli-supplier and single buyer problem can be seen in Serel et al. (2003) and Sethi et al (2003). Serel et al. (2003) studied sourcing decisions of a buyer that employs the "capacity reservation contract" with a long-term supplier and an option to buy the same component from the spot market. Under the contract, the buyer essentially buys the rights to order up to a certain number of units from the supplier in each period. They concluded that inclusion of the spot market alternative significantly reduces the capacity commitments from the contracted supplier. Sethi et al. (2003) studied the QF contract in a multiperiod, two sources of supply setting. The buyer is allowed to purchase at two distinct time periods, with the size of the second order being constrained by the QF contractual parameter. However, the units for the second order are sold at a higher price than that in the first order. On the other hand, the buyer will also have an option to buy from the spot market in the next period for any desired quantity at the market price, which can be higher or lower than the supplier's price. In other words, the spot market is not considered a source of supply in the first period. They solved for the optimal orders from the supplier for both periods, as well as the order from the spot market. They also examined the impact of information quality and the flexibility on optimal decisions.

The problem of interest in this research has a similar structure to Sethi et al's (2003) work. The key difference is that in our model, we intend to examine the buyer's inventory policies in a supply chain consisting of two competing suppliers that can supply the components in the same time period. Specifically, one supplier offers the QF contract as an incentive to share the inventory risk while the other supplier offers financial incentives with a cheaper component price. The components from both suppliers have the same functionality and features; they are not identical but are substitutable. As a result, the buyer can use either one to assemble the final product, which will be sold at the same retail price to the end customers no matter what the source of the component is. The research question then is, what is the buyer's optimal allocation of forecasting and firm orders for the two suppliers? Will using the alternative supplier always benefit the buyer? How much flexibility should the supplier offer, facing the price competition? Will such a supply chain achieve the optimal system-wide profit?

The main contributions of this research are threefold. First, our study examines the tradeoff between the level of flexibility and the cost saving benefits for the buyer, which sheds some light on the value of flexibility. As will be indicated in later sections, when flexibility is high, the buyer will be inclined towards increasing the forecast to the QF supplier and decreasing the size of firm order to the price-only supplier. Our models can assist buyers to make such decisions when having multiple sources. Second, our analysis indicates that from the system's standpoint, the 1-1 QF supply chain is favored over the 2-1 QF supply chain. Our results indicate that a 2-1 QF supply chain will fail to achieve the global profit level. We found that in general, the system profit in a 2-1 QF supply chain is always lower than or equal to that in a 1-1 QF supply chain. This finding provides support to the recent trend of firms' shrinking their supply base and maintaining closer relationship with selected suppliers. Third, our results also provide insights to the suppliers: when facing competition, how should they adapt their pricing policies or flexibility allowance so that they can win more business from the buyer over their competitor. It is important for the suppliers to understand how they could better initiate business strategies and production plans to cope with competition.

The remainder of the paper is organized as follows. We first discuss the problem in section 3.2. Then we formulate the 2-1 QF contract problem with two criteria in section 3.3. We then develop the solution and identify the optimal conditions for the two cases in sections 3.4 and 3.5, respectively. We provide numerical examples and discuss managerial implications in section 3.6. Finally, in section 3.7, we conclude this research and suggest future research opportunities.

3.2 Problem Setting

We consider a supply chain that consists of two suppliers and one buyer. The two suppliers produce substitutable components that provide same functionality and same features. As such, there is no difference in the final product using either one. The components are substitutable, however, they are not identical. They may differ in price, production cost, salvage value, quality performance, etc. To simplify the analysis, we consider only the difference in financial terms and assume everything else as equal.

The buyer has a stronger relationship with one of the supplier, say, supplier 1, and she has agreed upon a QF contract to govern joint business activities. As a result, the buyer provides forecast to supplier 1 and the supplier 1 makes the component available complying with the QF contract agreement. On the other hand, the buyer has a transaction-based relationship with supplier 2, and uses it as an alternative source. Owing to the production lead-time, the buyer needs to provide forecast to supplier 1 or places a purchase order to supplier 2, or both, in advance. Both suppliers will not be able to fulfill sudden orders should final demand exceed the previous estimation.

The component prices between these two suppliers are different When supplier 2's price is higher than or equal to supplier 1's, it is not difficult to see that the buyer will disregard supplier 2 and release his order (forecast) to supplier 1 only, as supplier 1 provides flexibility and shares portion of the inventory risk, yet her component price is cheaper. In this case, the problem degenerates into the traditional 1-1 QF supply chain problem. However, when supplier 2 can offer a cheaper price to the buyer, the problem becomes more interesting because there is clearly a tradeoff between the cost saving and risk sharing for the buyer. As such, the buyer needs to decide the forecast quantity tosupplier 1 as well as the ordering quantity to supplier 2 so that the total availability of the components will best benefit the buyer. Consequently, we will only analyze the 2-1 QF supply chain problem when supplier 2's price is lower than supplier 1's price. This is most reasonable because the flexibility may not come free of charge to the buyer. Throughout this chapter we use the following notation:

Π_b : Buyer's total expected profit in a decentralized supply chain

D : Demand, a random variable

q_1 : Buyer's forecast to supplier 1 (the QF supplier)

q_2 : Buyer's order to supplier 2

p : Retail price of the final product

w_1 : Component price from supplier 1.

w_2 : Component price from supplier 2.

v_1 : Salvage revenue of the component from supplier 1

v_2 : Salvage revenue of the component from supplier 2

c_1 : Supplier 1's production cost

c_2 : Supplier 2's production cost

F : c.d.f. of the random demand

f : p.d.f. of the random demand

Note : $p > w_1 > w_2 > c_1 > v_1 > 0$, $p > w_1 > w_2 > c_2 > v_2 > 0$

3.3 The 2-1 QF Models

Under this QF contract, the buyer releases forecast q_1 to supplier 1. Supplier 1 will then produce $(1+\alpha)q_1$ and the buyer needs to purchase at least $(1-\omega)q_1$; $\alpha \geq 0, 1 \geq \omega > 0$. Supplier 1 sells the component to the buyer at w_1 per unit. On the other hand, the buyer may place a firm order q_2 to supplier 2 at w_2. This order is not returnable, so the buyer bears all the inventory risk for any unit in the order q_2. As a result, the total availability of the component is $(1+\alpha)q_1 + q_2$ and the supplier is obligated to own $(1-\omega)q_1 + q_2$. Notice that $w_1 > w_2$, but the component price difference does not affect the final product's retail price p. On the other hand, should overstocking occur, the buyer can return at most $(\alpha+\omega)q_1$ to supplier 1 and salvage the rest of excessive inventory if there is any. Consequently, the buyer's priority is to use up either $(1-\omega)q_1$ units from supplier 1 or q_2 units from supplier 2 before further consuming the remaining units of $(\alpha+\omega)q_1$ from supplier 1. We assume that the cost of such a prioritizing control is marginal and thus can be ignored. As for whether to feed $(1-\omega)q_1$ or q_2 first to the production depends on the salvage values of the two components.

Although the two components are substitutable, they are not identical. As such, they may have difference salvage values v_1 and v_2, respectively. For example, a PC make can use either Seagate's or Maxtor's 60G hard drive disk (HDD) in a specific model. A PC equipped with either one will sell at the same retail price, but the two HDDs may differ in the component price as well as in the resale value if obsolete inventory should occur.

Furthermore, there is no guarantee that the resale value of the more expensive component is absolutely going to be higher than the cheaper one. As a result, v_1 can be greater than, equal to, or smaller than v_2 in our analysis. When realized demand is lower than expected and there are more unsold units than the returnable allowance $(\alpha+\omega)q_1$, the higher the salvage value is, the higher the priority is for salvage, thus the lower the priority for usage. So when $v_1 > v_2$, q_2 units should be used for production; when $v_1 \le v_2$, $(1-\omega)q_1$ units from supplier 1 should be consumed first, followed by q_2. In both cases, the buyer should save $(\alpha+\omega)q_1$ units from supplier 1 before using any q_2 from supplier 2.

Consequently, we formulate the 2-1 QF problems for the two cases:

Case I: $v_1 > v_2$

$$\begin{aligned}\Pi_b &= (p-w_1)(1+\alpha)q_1 + (p-w_2)q_2 \\ &\quad -(p-w_1)[E((1+\alpha)q_1+q_2-D)^+ - E((1-\omega)q_1+q_2-D)^+] \\ &\quad -(p-v_1)[E((1-\omega)q_1+q_2-D)^+ - E(q_2-D)^+] \\ &\quad -(p-v_2)E(q_2-D)^+ \qquad (3.1)\end{aligned}$$

The first term in (3.1) describes the total profit if inventory is sold out. The second term deducts the profits from the potentially unsold units that are returnable under the QF contract. The third term subtracts revenue of the potentially unsold, non-returnable units from supplier 1 and adds the salvage revenue of those units. Finally, the last term further deductsthe revenue of potential unsold, non-returnable units purchased from supplier 2 and adds the salvage revenue. As such, the priority for component return and salvage follows the order of the terms subtracted in (3.1), because of the fact that $w_1 > v_1 > v_2$. Conversely, the priority of component usage follows the reverse order; they all create the same revenue p per unit, if consumed, but the one that creates the least salvage revenue should be used first. So in this case, the buyer should use up all q_2 before using any $(1+\alpha)q_1$:

Case II: $v_1 \leq v_2$

$$\begin{aligned}\Pi_b &= (p-w_1)(1+\alpha)q_1 + (p-w_2)q_2 \\ &\quad -(p-w_1)[E((1+\alpha)q_1+q_2-D)^+ - E((1-\omega)q_1+q_2-D)^+] \\ &\quad -(p-v_2)[E((1-\omega)q_1+q_2-D)^+ - E((1-\omega)q_1-D)^+] \\ &\quad -(p-v_1)E((1-\omega)q_1-D)^+ \qquad (3.2)\end{aligned}$$

The difference between (3.2) and (3.1) is that any units in q_2 have higher priority for salvage than any non-returnable $(1-\omega)q_1$, because of the higher salvage value. As such, the component usage priority follows $(1-\omega)q_1 \rightarrow q_2 \rightarrow (\alpha+\omega)q_1$. In other words, after consuming $(1-\omega)q_1$, the buyer should start using q_2 before he sends any of remaining $(\alpha+\omega)q_1$ units into production.

Our models are formulated with the newsvendor problem (NVP) framework. The salvage values are attributed to part of the overstocking cost. To simplify the analysis, we omit inclusion of the understocking cost in our models. As one can observe, there are two decision variables (q_1, q_2) in the models. Our first step is to analyze the property of the objective functions for profit maximization.

Theorem 3.1 The buyer's total expected profit function in either case is jointly concave in q_1 and q_2.

Proof. See Appendix of chapter 3

Theorem 3.1 assures that the optimal solution q_1^* and q_2^* are unique if they do exist. The readers may observe the similarity between the models in (3.1), (3.2) and (2.1) However they differ in at least four different aspects. First, in this research, the components are from two different suppliers and they are substitutable but not identical while in the QF*i* model they are identical and from the same supplier. Second, the suppliers have control over their own price only whereas in the QF*i* contract the supplier decides both prices. The suppliers do not know each other's price, either. Third, the components from the two suppliers have different salvage revenue. Fourth, this research is analyzed from the buyer's perspective while the previous chapter is taking the supplier's position. Nonetheless, the buyer maintains leader's position in this problem as well.

On the other hand, the traditional QF contract and the price-only contract in a 1-1 supply chain provide boundaries for the optimal solution of the 2-1 QF contract problem. When the buyer decides not to purchase any units from supplier 2, i.e., $q_2^* = 0$, then the optimal forecast quantity q_1^* should follow the optimal forecast q_{QF} identified in Tsay (1999). On the other hand, if the buyer decides to not buy from the QF supplier, i.e., $q_1^* = 0$, then following the classic newsvendor problem solution, $q_2^* = F^{-1}(\frac{p-c_2}{p-v_2}) = q_{w2}$. Thus, $q_1^* + q_2^*$ is bounded by $q_{QF} + q_{w2}$.

3.3.1. The system view: the benchmark

In a 1-1 supply chain, the centralized expected profit can be expressed as

$$\Pi_c = (p-c)Q - (p-v)(Q-D)^+$$

It follows the same notation as that in section 2.3.1. The optimal system profit Π_c^* occurs when $Q_c = F^{-1}(p-c/p-v)$. In a 2-1 supply chain, the system-wide expected profit can be calculated in a similar manner. The key difference is that when there are two suppliers, we have two pairs of (c_1, v_1) and (c_2, v_2). It is easy to see that Π_c increases in v_1 and decreases in c_1. So ideally, in a 2-1 supply chain, Π_c will reach optimal with $\min(c_1, c_2)$ and $\max(v_1, v_2)$. However, it is more reasonable to use the pair of (c, v) from the same supplier. As such, the system-wide profit will be either $\Pi_c(c_1, v_1)$ or $\Pi_c(c_2, v_2)$. To simplify the analysis, we assume both suppliers have the same production cost, i.e., $c_1 = c_2 = c$. As such, when $v_1 > v_2$, $\Pi_c = \Pi_c(v_1)$ and $Q_c =$ $F^{-1}(p-c/p-v_1)$. When $v_1 \le v_2$, $\Pi_c = \Pi_c(v_2)$ and $Q_c = F^{-1}(p-c/p-v_2)$.

Finally, our research has a focus on the QF contract and intends to analyze inventory policies and system performance with the existence of the QF contract in a 2-1 supply chain. We are specifically interested in comparing the supply chain decisions and performance between the 1-1 QF and 2-1 QF supply chains. Consequently, we will discuss the topic of supply chain performance only for case I, because in this case the benchmark profit $\Pi_c = \Pi_c(v_1)$ shares the same form as in the 1-1 QF chain. We develop the models of the two cases separately.

3.4 Solutions and Optimality Conditions for Case I

As mentioned earlier, buyer decides the forecasting and ordering quantities. Buyer's total expected was expressed in (3.1). We also have shown that the objective function is jointly concave in (q_1, q_2). Next, we discuss the optimality conditions.

Lemma 3.1 Let $\chi = \dfrac{1+\alpha}{1-\omega}$.

(1) If optimal solution exists, $(w_1 - w_2) - (v_1 - v_2)F(q_2^*) \geq 0$ must be true.

(2) At optimum, $\dfrac{1}{1+\alpha}(F^{-1}(A) - q_2^*) = \dfrac{1}{1-\omega}(F^{-1}(B) - q_2^*)$ must be true, where

$$A = \frac{-(w_1 - w_2) + (\chi - 1)(p - w_1) + (v_1 - v_2)F(q_2^*)}{(\chi - 1)(p - w_1)}$$

$$B = \frac{\chi[(w_1 - w_2) - (v_1 - v_2)F(q_2^*)]}{(\chi - 1)(w_1 - v_1)}$$

(3) If $\chi > \underline{\chi}' = \dfrac{(w_1 - v_1)(p - w_2)}{(w_2 - v_1)(p - w_1)}$, buyer will release a non-zero forecast q_1^* to the supplier.

Proof. See Appendix of chapter 3.

Lemma 3.1 presents the necessary conditions for optimality if it exists. Both q_1^* and q_2^* must be greater than or equal to zero. If negative optimal solution occurs, the optimal solution should be either $(q_{QF}, 0)$ if $q_2^* \leq 0$ or $(0, q_{w2})$ if $q_1^* \leq 0$, as discussed previously. Note that both A and B are a function of q_2. Rearranging the terms in the equality in Lemma 3.1 (2) we obtain the following equality: $(\chi - 1)q_2 = \chi F^{-1}(B) - F^{-1}(A)$.

Theorem 3.2 Let $m(q_2) = \chi F^{-1}(B) - F^{-1}(A)$. Given χ

(1) If $m(0) > 0$, there exists optimal positive q_2^* that solves

$(\chi - 1)q_2 = \chi F^{-1}(B) - F^{-1}(A)$. But q_2^* must fulfill the boundary constraints:

(i) if $q_{w2} \le q_2^*$, $q_1^* = 0$, $q_2^* = q_{w2}$

(ii) if $0 < q_2^* \le q_{w2}$, $q_1^* = \dfrac{1}{1+\alpha}(F^{-1}(A(q_2^*)) - q_2^*)$

$$= \frac{1}{1-\omega}(F^{-1}(B(q_2^*) - q_2^*)$$

(2) If $m(0) \le 0$, the optimal solution is (q_{QF}, 0).

Proof. $m(q_2)$ is obvious continuous in q_2. Given χ, $m(q_2)$ is monotonically decreasing in q_2, because B decreases in q_2 and A increases in q_2. Thus, as long as $m(0) > 0$, the two functions $m(q_2) = \chi F^{-1}(B) - F^{-1}(A)$ and $m(q_2) = (\chi - 1)q_2$ must intersect at a point where q_2 is positive, since $m(q_2) = (\chi - 1)q_2$ is linearly increasing in q_2. The fact that q_2^* is bounded by q_{w2} leads to (i) and (ii). Theorem 3.1 assures uniqueness of the optimal solutions. If $m(0) \le 0$, the two functions intersect at negative q_2, thus $q_2^* = 0$, the problem degenerates into the 1-1 QF problem. Thus, $q_1^* = q_{QF}$ must be true. □

Theorem 3.2 presents the unique optimal solution for the model I. Because of the inverse function of the demand distribution, close form solution q_2^* is not possible. Nonetheless, when optimality conditions in Theorem 3.2 are met, one can solve for q_1^* and

q_2^* accordingly. As a result, the model is able to assist the buyer to identity an optimal solution as to how much forecast to release to supplier 1 and how many units to buy from supplier 2 at the cheaper price. In addition, Lemma 3.1(3) serves as a useful tool for the suppliers to analyze the relationship between the flexibility offered and the discount price. It provides the minimum level of flexibility that supplier 1 should offer to be able to compete against the price discount offered by supplier 2. It allows the supplier 1 to examine whether the QF contract is attractive enough, and supplier 2 to understand if the price difference is good enough to the buyer. The total availability is $(1+\alpha)q_1^* + q_2^*$ and the final purchase quantity can be expressed as $r^* = D \perp \ [(1-\omega)q_1^* + q_2^*, (1+\alpha)q_1^* + q_2^*]$. The expression on the high hand side denotes the point in the interval that is closest to D.

Next, we discuss the 2-1 QF supply chain from the system's perspective. We intend to investigate the supply chain efficiency that a 2-1 QF supply chain can achieve.

Theorem 3.3 In a 2-1 QF supply chain, if the buyer purchases components from both suppliers, this supply chain will not be able to achieve the system-wide profit.

Proof.

$$\Pi_{S1} = (w_1 - c)(1+\alpha)q_1$$

$$- (w_1 - v_1)[E((1+\alpha)q_1 + q_2 - D)^+ - E((1-\omega)q_1 + q_2 - D)^+] \tag{3.3}$$

$$\Pi_{S1} = (w_2 - c)q_2 \tag{3.4}$$

Add (3.1),(3.3), (3.4) we get the total profit of the 2-1 QF supply chain as following:

$$\Pi_{2-1QF} = (p-c)((1+\alpha)q_1 + q_2)$$

$$-(p - v_1)[E((1+\alpha)q_1 + q_2 - D)^+ - E(q_2 - D)^+]$$

$$-(p-v_2)E(q_2-D)^+ \tag{3.5}$$

It can be shown that $\Pi_{2\text{-}1QF}$ is jointly concave in (q_1, q_2). We omit the derivation of this property. Then exploring the First Order Conditions (FOCs), we get

$$\frac{d\Pi_{2\text{-}1QF}}{q_1} = (p-w_1)(1+\alpha)$$

$$+(p-v_1)[F((1+\alpha)q_1+q_2)(1+\alpha)-F((1-\omega)q_1+q_2)(1-\omega)=0$$

$$\Rightarrow F((1+\alpha)q_1+q_2)=\frac{p-c}{p-v_1} \tag{3.6}$$

$$\frac{d\Pi_{2\text{-}1QF}}{q_2} = (p-c)-(p-v_1)[F((1+\alpha)q_1+q_2)-F(q_2)]-(p-v_2)Fq_2)=0$$

$$\Rightarrow (p-c)-(p-v_1)F((1+\alpha)q_1+q_2)-(v_1-v_2)F(q_2)=0 \tag{3.7}$$

Substituting (3.6) in (3.7) results in $F(q_2)=0$, so $q_2^*=0$. Thus,

$F((1+\alpha)q_1)=\dfrac{p-c}{p-v_1}=F(Q_c)$, which leads $q_1^*=q_{QF}$. So if $q_2^*>0$, the 2-1 QF chain fails to achieve the system-wide profit □

From Theorem 3.3, we see that only if the buyer does not buy from supplier 2 it will allow the system to reach the centralized profit level. In this case, the 2-1 QF supply chain becomes the traditional 1-1 QF supply chain. As a result, from the system's perspective, the 2-1QF supply chain is less efficient than a 1-1 QF supply chain. We will analyze this phenomenon in greater detail via numerical examples in section 3.7. In brief, we found that the competition between the two suppliers can only benefit the buyer. The system's efficiency maybe at a high level, however, majority of the system's profit goes to the buyer. The suppliers suffer from competition badly.

In the proof of Theorem 3.3, we show that when at optimum, the following is true:

$F((1+\alpha)q_1) = \dfrac{p-w_1}{p-v_1} = F(Q_c)$, i.e., $(q_1^*, q_2^*) = (q_{QF}, 0)$. However, we found that

$(q_{QF}, 0)$ is not the only optimal solution for the 2-1 QF chain that results in the total availability at Q_c.

Theorem 3.4

When the following conditions holds, the total availability in the 2-1 QF supply chain reaches Q_c:

(1) $\chi F^{-1}(K) - F^{-1}(\dfrac{p-c_1}{p-v_1}) = (\chi - 1)F^{-1}(H)$, where

$$K = \frac{\chi(p-w_1)(c_1-v_1)}{(p-v_1)(w_1-v_1)}, \quad H = \frac{w_1-w_2}{v_1-v_2} - \frac{(\chi-1)(p-w_1)(c_1-v_1)}{(v_1-v_2)(p-v_1)}$$

(2) The flexibility χ must be in the interval $[\underline{\chi}_{\min}, \overline{\chi}_{\max}]$, where

$$\underline{\chi}_{\min} = \frac{(w_1-v_1)[(p-w_1)(c_1-v_1)+(p-v_1)(w_1-w_2)]}{(p-w_1)(c_1-v_1)(w_1-v_2)}, \quad \overline{\chi}_{\max} = \frac{(p-c)(w_1-v_1)}{(p-w_1)(c-v_1)}$$

(3) $w_2 \le w_1 - \dfrac{(\chi-1)(p-w_1)(c_1-v_1)}{p-v_1}$

Proof. See Appendix of chapter 3.

Corollary 3.1

$$q_1^* = \frac{1}{1+\alpha}[F^{-1}(\frac{p-c_1}{p-v_1}) - F^{-1}(H)] \quad \text{and} \quad q_2^* = F^{-1}(H).$$

Proof. This is a direct result from Theorem 3.4(1).

Theorem 3.4 presents the conditions that need to hold when the total availability $(1+\alpha)q_1^* + q_2^*$ equals Q_c in the 2-1 QF supply chain. Note that w_2 needs to be decided by the equality shown in Theorem 3.4 (1). This pricing scheme also needs to fulfill the constraint presented in Theorem 3.4 (3). w_2 that does not fulfill this constraint will lead the buyer to decide $q_1^* = q_{QF}$ and $q_2^* = 0$.

Two key areas differentiate the meaning that Q_c carries between the 2-1 QF chain and the 1-1 QF*i* contract models. First, Q_c allows a 1-1 supply chain to achieve supply chain coordination. However, it will not achieve the same profit level in a 2-1QF supply chain. It can be easily shown that when the total availability $(1+\alpha)q_1^* + q_2^*$ equals Q_c, the 2-1 QF system's profit is $(v_1 - v_2)\int_0^{q_2^*} F(x)dx$ off from the centralized profit. As one can observe, the higher q_2^* is, the greater the 2-1 system profit will deviate from the centralized profit. Second, in the 1-1 QF*i* model, the supplier has total control over both w_1 and w_2. However, in the 2-1 QF problem, suppliers can decide only their own component price. As such, they both will determine the pricing policy in their own interest, the result of which will not necessarily fulfill the conditions in Theorem 3.4.

3.5 Solutions and Optimality Conditions for Case II

In this case supplier 2's component is cheaper, yet it has a higher salvage revenue than the components from supplier 1. Therefore, supplier 2's component is more attractive to the buyer in this case than in the previous case. As a result, supplier 1's flexibility needs to be raised to an even higher level than that in case 2 so that the buyer will be willing to continue to purchase from supplier 1.

The buyer's total expected profit function was presented in (3.2). We also have shown that it is jointly concave in (q_1, q_2). We follow similar procedures for case 1 to explore the solution for case II.

Lemma 3.2

(1) If optimal solution exists, the following must be true:

$$F((1-\omega)q_1^*) \leq \frac{\chi(p-w_1)(w_2-v_2)-(p-w_2)(w_1-v_2)}{(p-v_2)(v_2-v_1)}$$

(2) At optimum, $(\alpha+\omega)q_1^* = F^{-1}(A') - F^{-1}(B')$, where

$$A' = 1 - \frac{(w_1-w_2)+(v_2-v_1)F((1-\omega)q_1^*)}{(\chi-1)(p-w_1)},$$

$$B' = \frac{\chi(w_1-w_2)+(v_2-v_1)F((1-\omega)q_1)}{(\chi-1)(w_1-v_2)}$$

Proof. See Appendix of chapter 3.

Lemma 3.2 presents the necessary conditions for optimality if it exists. Notice that both A' and B' are a function of q_1. Similar to model I, let $m(q_1) = F^{-1}(A') - F^{-1}(B')$, we have the following properties:

Corollary 3.2

(1) $A^{'}$ is decreasing in q_1; $B^{'}$ is increasing in q_1

(2) $A^{'}$ is increasing in χ; $B^{'}$ is decreasing in χ

(3) $m(q_1)$ is increasing in χ and decreasing in q_1

We can observe that $(\alpha+\omega)q_1$ is a linearly increasing function of q_1 while $F^{-1}(A^{'})-F^{-1}(B^{'})$ is non-linear and decreasing function of q_1. Similarly, if the two functions intersect in the area where q_1 is positive, non-zero optimal q_1^* exists. Otherwise the buyer will not provide any forecast q_1 to the supplier and will buy all that is needed from supplier 2. We discuss the optimality conditions and solution in Theorem 3.5.

Theorem 3.5

(1) When $\chi > \chi^{'}_{\min} = \dfrac{(p-w_2)(w_1-v_2)}{(p-w_1)(w_2-v_2)}$, there exists optimal $q_1^* > 0$ that solves

$(\alpha+\omega)q_1 = F^{-1}(A^{'})-F^{-1}(B^{'})$. q_1^* must fulfill the boundary constraints:

(i) if $0 \le q_1^* < q_{QF}$, $q_2^* = F^{-1}(A^{'})-(1+\alpha)q_1^* = F^{-1}(B^{'})-(1-\omega)q_1^*$

(ii) if $q_1^* \ge q_{QF}$, $q_1^* = q_{QF}$, $q_2^* = 0$

(2) When $\chi \le \chi^{'}_{\min}$, $q_1^* = 0$ and $q_2^* = q_{w2}$.

Proof. With properties in Corollary 3.1, when $m(0) > 0$, the two functions $m(q_1) = F^{-1}(A') - F^{-1}(B')$ and $m(q_1) = (\alpha + \omega)q_1$ will intersect and intersect only once. When $m(0) > 0$, $A' > B'$ must be true ,so

$$A'(0) = 1 - \frac{(w_1 - w_2)}{(\chi - 1)(p - w_1)} > B'(0) = \frac{\chi(w_1 - w_2)}{(\chi - 1)(w_1 - v_2)}$$

Rearrange terms will lead to $\chi > \chi'_{\min} = \dfrac{(p - w_2)(w_1 - v_2)}{(p - w_1)(w_2 - v_2)}$. If $\chi \leq \chi'_{\min}$, the two function will not intersect at a positive q_1. Thus $q_1^* = 0$, $q_2^* = q_{w2}$. Other conditions are direct result of the boundaries for q_1^* and q_2^* □

Theorem 3.5 suggests that when there is a competitor who can offer a cheaper price for the component, and the component has a better resale value, supplier 1 has to offer flexibility at least at $\chi'_{\min}$ for the buyer to be interested in placing an forecast (order) to her. Otherwise, supplier 2's cheaper price and the higher resale value will lead the buyer to take the full inventory risk and buy everything from supplier 2. As the flexibility level increases, $m(q_1) = F^{-1}(A') - F^{-1}(B')$ increases. This implies that the intersection of $m(q_1) = F^{-1}(A') - F^{-1}(B')$ and $m(q_1) = (\alpha + \omega)q_1$ will move up towards the right, thus resulting in a larger q_1^*. Note that q_1^* is bounded by the q_{QF}, given QF parameters (α, ω). Again, the total availability is $(1 + \alpha)q_1^* + q_2^*$ and the final purchase quantity is $r^* = D \perp [(1 - \omega)q_1^* + q_2^*, (1 + \alpha)q_1^* + q_2^*]$.

In summary, supplier 1 can affect the buyer's decision by changing the flexibility level under the QF contract and supplier 2 can affect the buyer's decision by reducing its price. Nonetheless, the buyer can utilize our models to decide the best combination of q_1^* and q_2^* to make the most profit when there are multiple sources of component supply. In the next section, we conduct a series of numerical experiments to supplement our theoretical models.

3.6 Numerical Examples

We consider a base dataset for the numerical experiments as follows: $p = 50$, $c_1 = c_2 = 30$, $\alpha = 0.1, \omega = 0.1$, $w_1 = 42$, $w_2 = 40$, $v_1 = 20$, $v_2 = 18$. For convenience of analysis and to enable closed forms for decisions variables, we consider the market demand to follow a uniform distribution over the interval [400, 800]. We also present only the results of numerical experiments for the first model in which supplier 1's component is more expensive but has a higher salvage value. Our analysis mainly focuses on how the price difference between the two suppliers affects the buyer's decision and how the magnitude of the flexibility offered by supplier 1 can affect such decisions. The key areas we will examine include the profit for each party, the optimal ordering and forecasting policies for the buyer, and most importantly, the advantages and disadvantages of having an alternative supplier from the buyer's perspective and from the system's perspective.

3.6.1 Competitive pricing and its impact on the buyer's decisions

In this experiment, we examine the situation when the component price from supplier 1 is set at w_1=$42 in both the 1-1 QF and the 2-1 QF supply chains. Supplier 2's price w_2 is lower than w_1. We vary w_2 and investigate how the price difference would affect the buyer's decision. From Figure 3.1 we observe that as supplier 2's price changes to below \$41.2, the cost saving of 80 cents per unit will start to outweigh the benefit of the flexibility provided by supplier 1. As w_2 continues to decrease, supplier 2 starts to gain orders rapidly. Once w_2 reaches \$40.6, the buyer will favor buying only from supplier 2 even if the buyer has to take inventory burden. As w_2 continues to decrease, the buyer will order more units from supplier 2. As one can observe, it takes a small amount of price difference for the buyer to be interested only in buying from supplier 2. Notice that the buyer's decision also depends on the salvage value of the component from supplier 2. The intersection of q_1 and q_2 curves in Figure 3.1 will move towards the left if v_2 is higher than the current value of \$18. Comparing q_1 with q_{QF}, we can see that supplier 1's business fades away rapidly when she faces a competitor that takes the discount strategy against her flexibility strategy.

Figure 3.2 presents the profits of each party in this supply chain. As we can see, the buyer will truly enjoy the competition between the two suppliers. If the price discount is not large enough, he stays with supplier 1's QF contract. But when the price difference is high, buyer can only benefit from buying more or buying solely from supplier 2. For supplier 2, our result suggests that its best discount price is at around \$40.6, which will secure all the buyer's business. As one can see, any further discount below \$40.6 will only worsen supplier 2's profit; it will only benefit the buyer even if supplier 2 gets the whole pie. On the other hand, supplier 1 will need to offer a larger flexibility to induce the buyer to remain interested in the flexibility that she offers. Notice that the magnitude of flexibility in this experiment is set at 1.222.

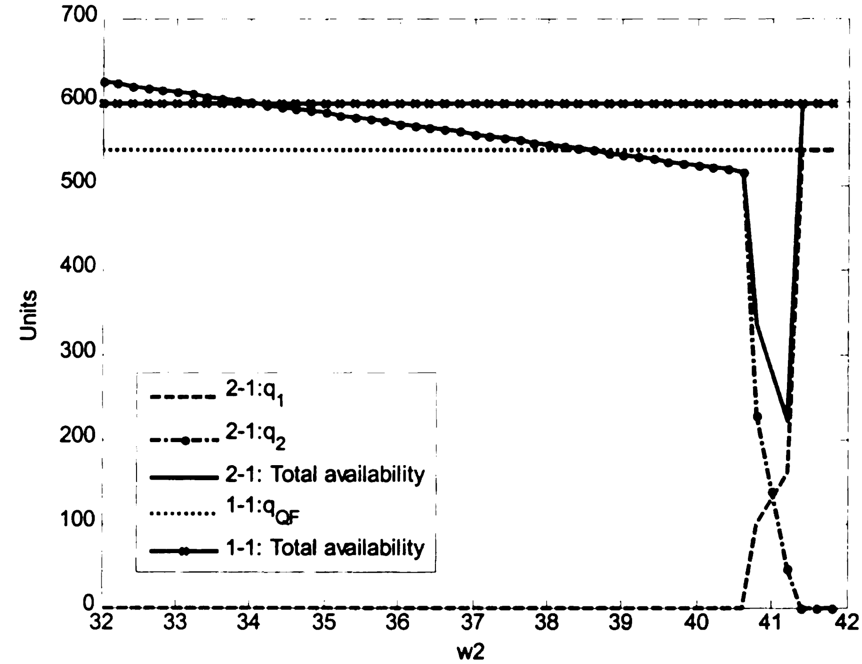

Figure 3.1 Forecast/ordering quantities 2-1 QF v.s. 1-1 QF: w_1=42

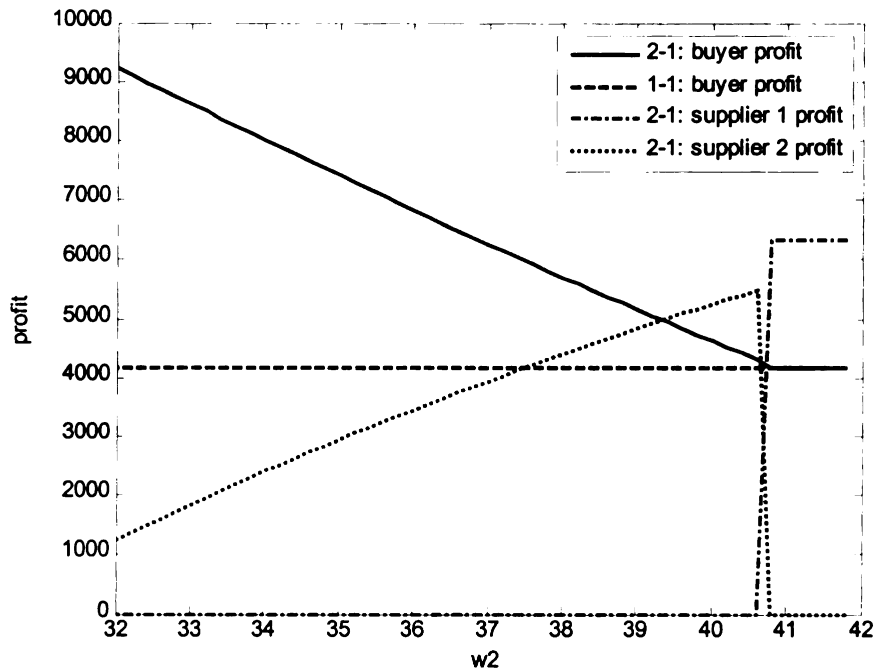

Figure 3.2 Buyer/supplier profits 2-1 QF v.s. 1-1 QF: w_1=42

Figure 3.3 presents the minimum required flexibility for supplier 1 to at least gain some business from the buyer against the various levels of the discount price offered by supplier 2. When w_2 is fairly close to w_1, the minimum-required flexibility is relatively low. As the price difference increases, the required flexibility goes up. Our numerical results indicates that when w_2 is at \$39.4, i.e., \$2.6 or 6.2% cheaper than w_1, the minimum required flexibility for supplier 1 is 1.5 to stay in business with the buyer. When w_2 is at \$37.4, i.e., \$4.6 or 11% cheaper than w_1, the minimum required flexibility for supplier 1 is 2. These results assist supplier 1 to understand better the value of flexibility she offers.

From the system's standpoint, such a competition between the two suppliers actually leads to supply chain deficiency, indicated in Figure 3.4. If using only one supplier under the QF contract, the entire chain's profit can reach \$10,492 at the current setting, which achieves 98.36% supply chain efficiency. In this case the buyer's estimated profit is \$4172.8 and the supplier's expected profit is \$6,319.1. As one can see from previous results, when supplier 2 is joining the game and offers a lower price, the buyer's profit starts roaring as supplier 2's price decreases. As indicated in Figure 3.2, supplier 2's highest profit is lower than what supplier 1 can earn under the 1-1 QF contract without competition. Meanwhile, the buyer's profit actually increases slower than the loss of suppliers' total profits at that point. This contributes to the sharp decrease in efficiency that we observe in Figure 3.4. As w_2 continues to decrease, the entire chain's efficiency starts to increase. But as mentioned, in the low discount price range, it is the buyer who gains all the benefits. If w_2 drops down to \$30, which is equivalent to the production cost c_2, it reaches the condition that is equivalent to the centralized chain, thus will achieve centralized profit.

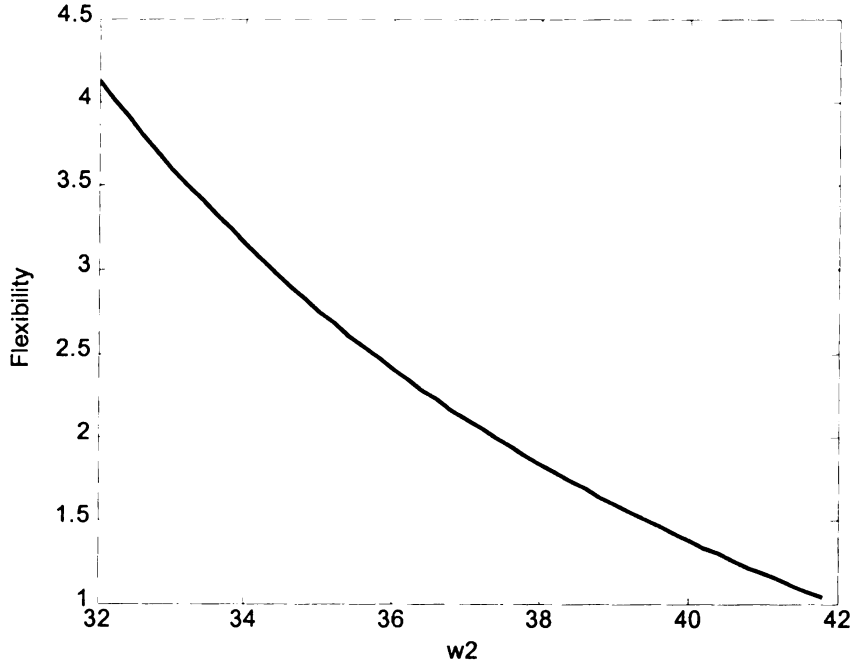

Figure 3.3 Minimum flexibility for the QF mechanism: w_1=42

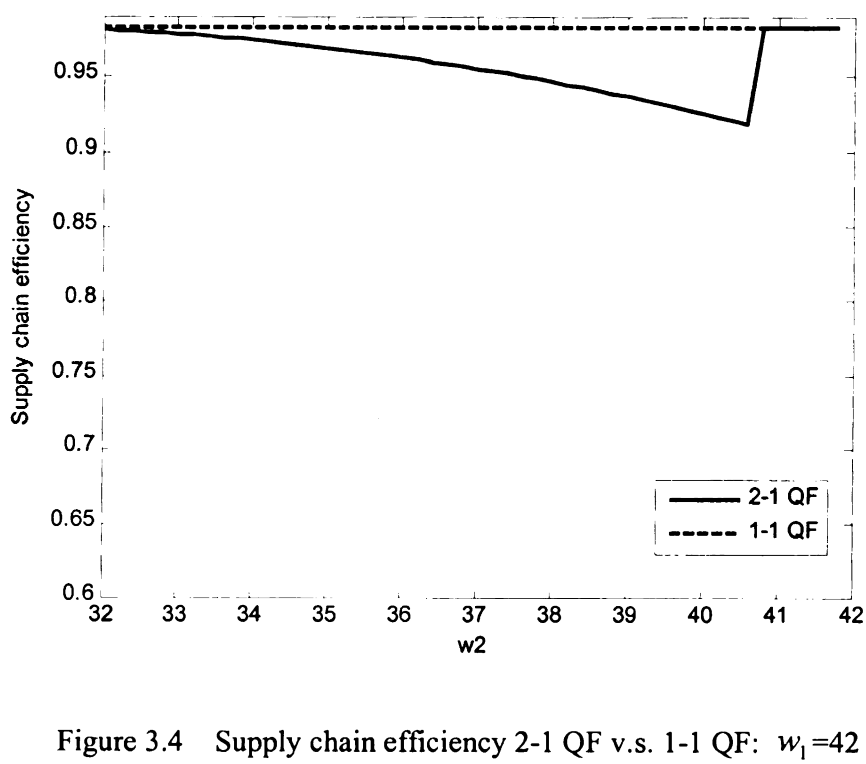

Figure 3.4 Supply chain efficiency 2-1 QF v.s. 1-1 QF: w_1=42

These results clearly describe the buyer's behavior when he can choose from two suppliers for the substitutable components and the benefit of having a second source of material supply. In contrast to the previous model in chapter 2, where the single supplier has full control of the two pricing schemes, the two suppliers in this chapter can only control their own prices. However, the buyer will take the advantage of getting both price quotations from the two suppliers. As mentioned, under the 1-1 QF contract, the buyer's profit may not be at his desired level. Unless supplier 2's price is actually higher than supplier 1's, having an alternative choice will always benefit the buyer. In the next section we investigate the magnitude of flexibility and its impact on the buyer's decisions.

3.6.2 Flexibility and its impact on the buyer's decisions

In this experiment, we intend to investigate the impact of flexibility on the 2-1 QF supply chain decisions. We examine the situation where the component price from supplier 1 is set w_1=\$42 in both 2-1 QF and 1-1 QF supply chain and supplier 2's price is w_2= \$40. We vary the magnitude of flexibility by fixing α=0.1 and changing the ω value. From Figure 3.5, we observe that as flexibility increases, the buyer will start to appreciate the flexibility and is willingness to share the inventory burden offered by supplier 1. With the \$2 difference in the component cost, if supplier 1 agrees that she prepares 10% of buffer stock above forecast and the buyer is only liable for around 79% of his forecasted, i.e., a flexibility of 1.39, supplier 1 will be able to acquire the buyer's business for at least 79% of the forecasted quantity. On the other hand, if the buyer needs to be responsible for 80% of the forecast, the buyer will not be interested in what the QF contract can offer and will switch all orders to supplier 2. Notice that in the previous experiment we found that when $w_2 \leq \$40.6$, the buyer will not place any forecast to supplier 1. In this experiment, we demonstrate that supplier 1 can cope with the supplier 2's low pricing strategy (w_2 = \$40) by increasing the level of flexibility.

The result in Figure 3.2 indicates that supplier 1 can either increase the buffer stock α or lower the buyer's liability ω to be able to compete against supplier 2, if keeping the same price. However, as ω continues to increase beyond 0.48, supplier 1 actually will suffer from offering too much flexibility and the buyer will begin placing some orders to supplier 2. This is attributed to the structure of the buyer's expected profit function. Technically, when ω is large, a combination of large ω and positive q_1 and q_2 will achieve higher expected profit than a combination of large ω , positive q_1 and zero q_2. This is somewhat counterintuitive because one would think that as flexibility increases, the buyer will definitely favor the flexibility over the price discount. A further investigation reveals that when the ω value is high, the buyer's responsibility from his forecasted quantity is so low that it is safe for him to buy some non-returnable units from the supplier 2 to a certain extent. The final demand is likely to be close to the mean of previous estimation, so these non-returnable units will likely be consumed. This explains why the supplier 1 will, surprisingly, suffer from offering too much flexibility, as indicated in Figure 3.5.

Additionally, Figure 3.5 indicates that when compared with the 1-1 QF supply chain, having two suppliers with differenct prices is likely to result in less total available units to fulfill the end-customer demand. This might cause problems to the buyer if the stockout cost is considered by the buyer for decision making. A high stockout cost will lower the total expected profit. One can further investigate the impact of the stockout cost on the 2-1 QF problem by incorporating stockouts into our framework.

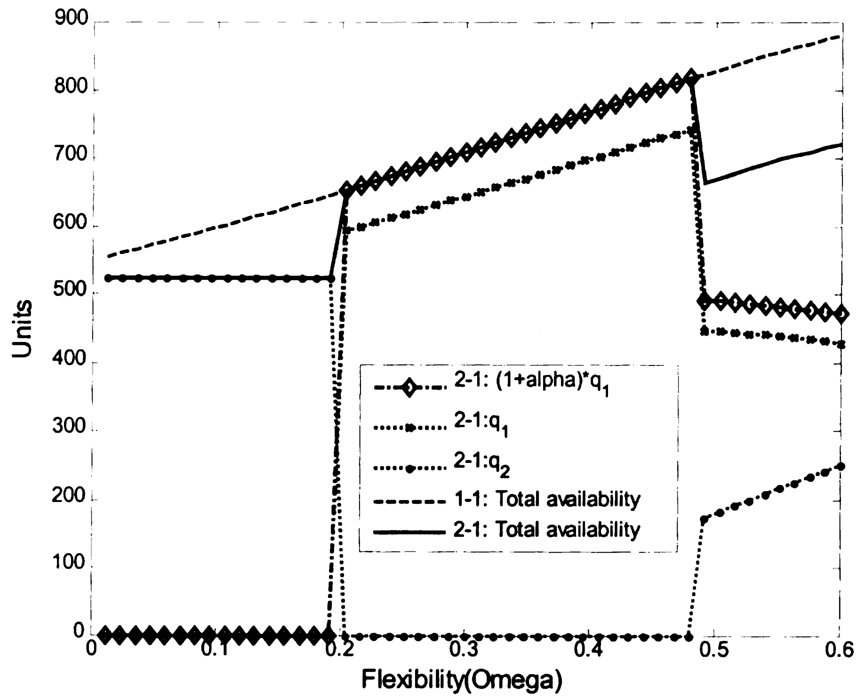

Figure 3.5 Forecast/ordering quantities 2-1 QF v.s. 1-1 QF: w_1=42, w_2=40, varying ω

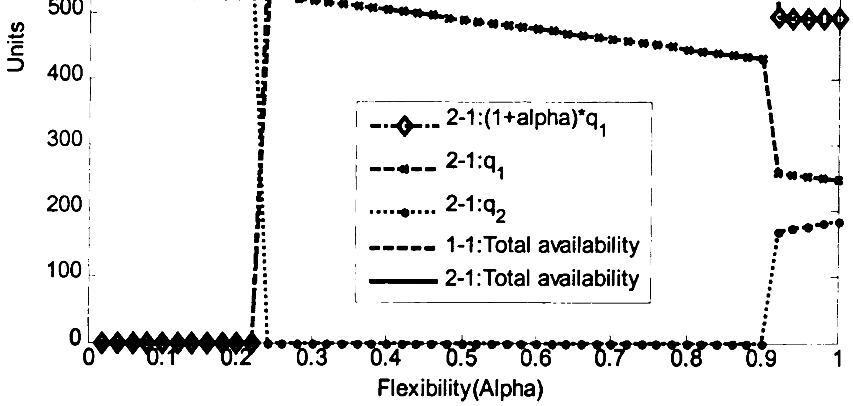

Figure 3.6 Forecast/ordering quantities 2-1 QF v.s. 1-1 QF: w_1=42, w_2=40, varying α

Figure 3.6 shows results from the same experiment except here, we fixω value at 0.1 and vary the α value. It reveals similar patterns to those in Figure 3.5. However, while $(1+\alpha)q_1$ reveals the same patterns in both figures, q_1 does not. In Figure 3.5, q_1 is moving upwards in the middle segment but it is moving downwards in Figure 3.6. This is due to the fact that in both experiments we have the same frontier $(1+\alpha)q_1$, but enlarging α allows smaller q_1 to reach the same $(1+\alpha)q_1$ value.

Figure 3.7 presents the financial consequences to the buyer due to the magnitude of the flexibility, with everything else being equal. As we can see, with the optional QF supplier 1 and price-competitive supplier 2, the buyer will always enjoy higher profit than he could in the 1-1 QF supply chain. Interestingly, under the 1-1 QF contract, the buyer's total expected profit goes down as ω increases to 1. This is due to the characteristics of the buyer's expected function under the QF contract. When ω increases, supplier's (supplier 1's) inventory responsibility increases, which is being deducted from both the buyer's and supplier 1's total expected profits. As a result, the buyer's expected profits decreases as ω increases under the QF contract, so does the QF supplier's expected profit.

On the other hand, the suppliers' profits are presented in Figure 3.8. As one can observe, when ω is below 0.2, supplier 2's lower pricing captures the business. When ω is greater than 0.2, supplier 1's flexibility has an advantage. As ω exceeds 0.48, combing with Figure 3.8, we observe that both the buyer's and suppliers' total profits in the 2-1 QF chain increase. The higher total suppliers' profit comes from supplier 2's increasing profit.

Finally, Figure 3.9 summarizes the experiment results observed in this section. When flexibility is low (ω <0.2), the buyer obtains his components only from supplier 2, due to the cheaper price. The sum of the buyer and supplier 2's profit will be lower than the total profits in the 1-1 QF chain. Thus, in this area, the 1-1 QF supply chain outperforms the 2-1 QF supply chain, from the system's perspective. When ω value falls in between 0.2 and 0.48, the two supply chains perform equally well. However, when ω exceeds 0.48, the 2-1QF supply chain starts outperforming the 1-1 QF supply chain.

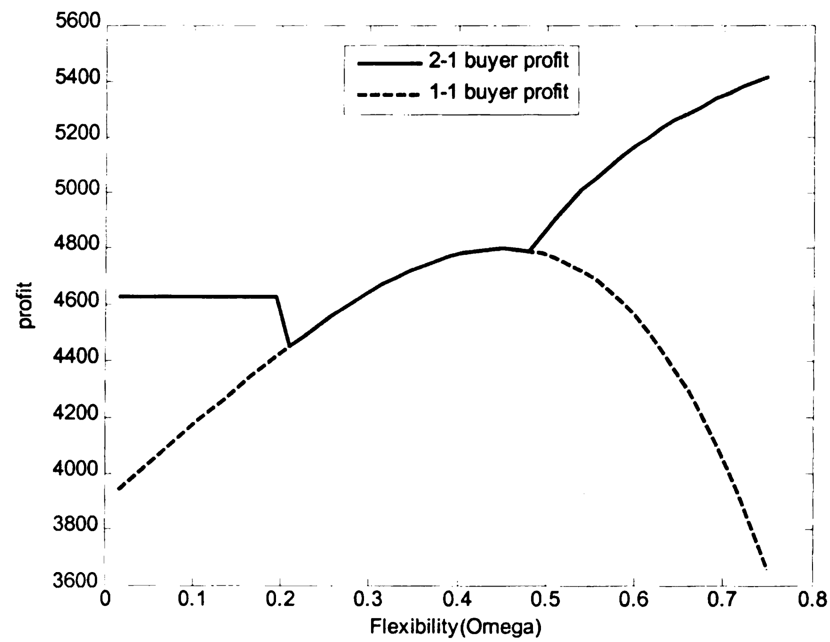

Figure 3.7 Buyer profits 2-1 QF v.s. 1-1 QF: w_1=42, w_2=40

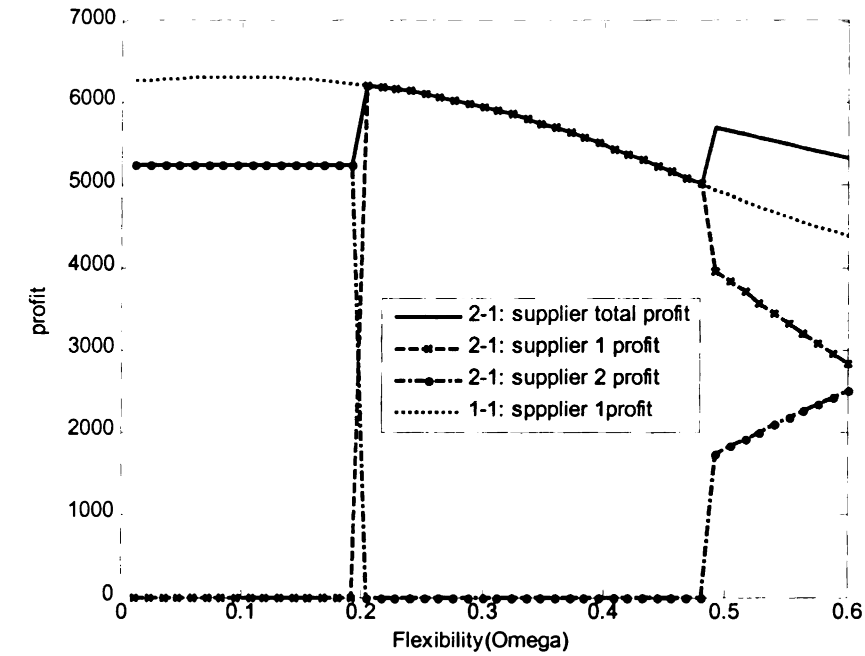

Figure 3.8 Supplier profits 2-1 QF v.s. 1-1 QF: w_1=42, w_2=40

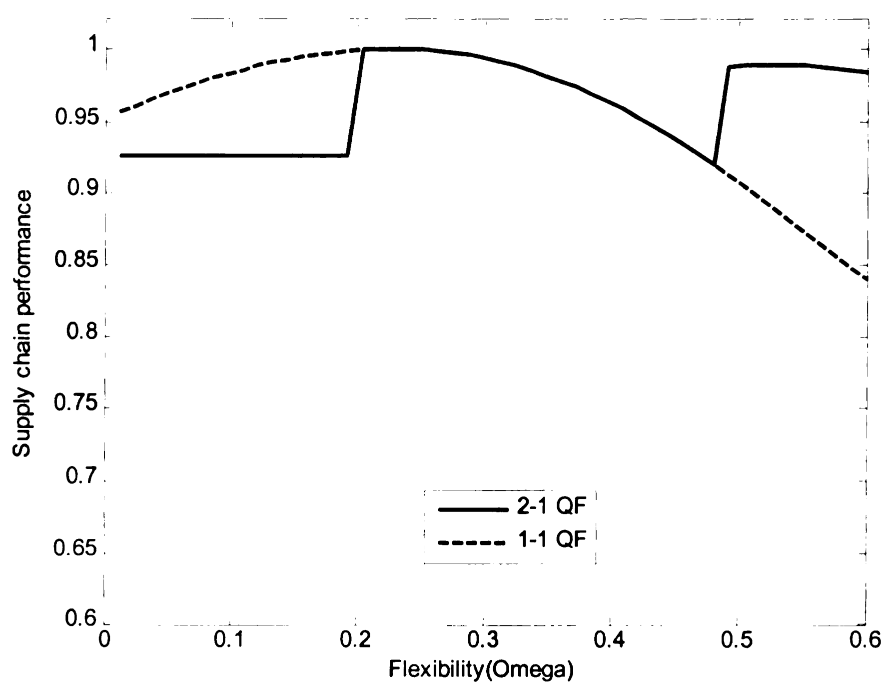

Figure 3.9 2-1 QF chain performance: w_1=42, w_2=40

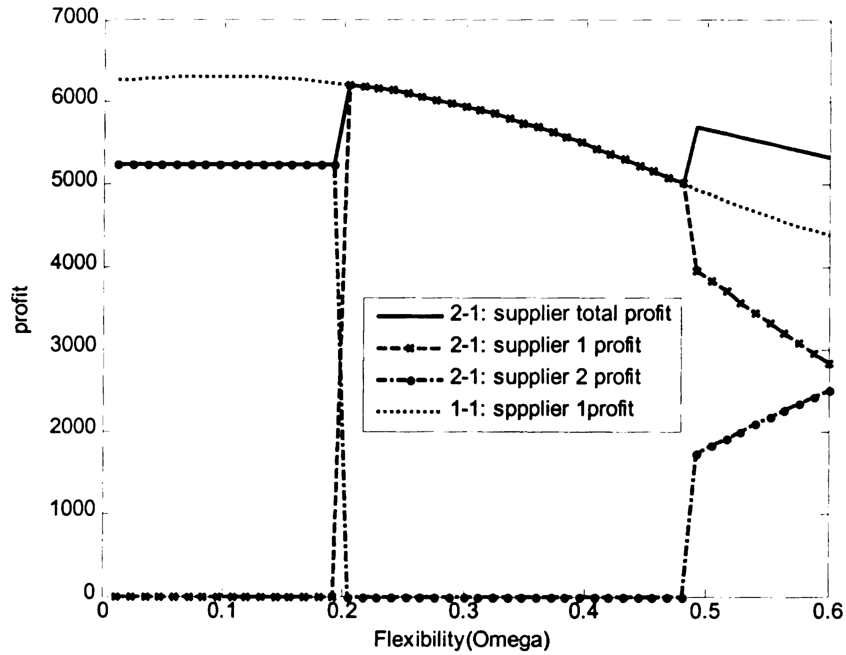

Figure 3.8 Supplier profits 2-1 QF v.s. 1-1 QF: w_1=42, w_2=40

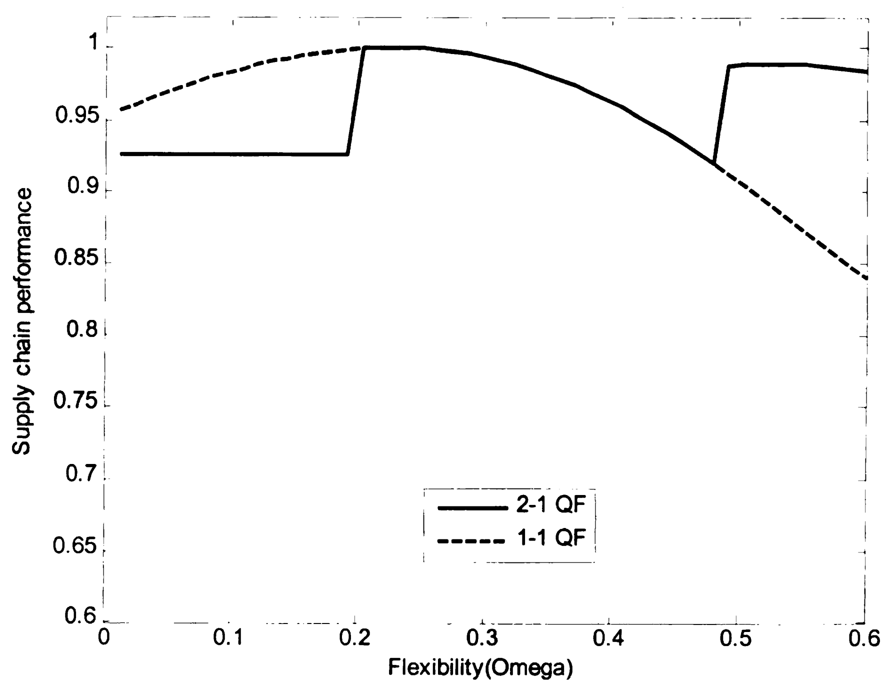

Figure 3.9 2-1 QF chain performance: w_1=42, w_2=40

3.6.3 Discussion

One important observation from these two sets of experiments is that, surprisingly, the results seem to suggest that the buyer will most likely be better off simply keeping one supplier. From Figure 3.1 and 3.5, we can see that in most cases our model suggests that the buyer purchases from either supplier 1 or supplier 2. The area that suggests buying from both suppliers simultaneously is relatively small. In the past decade, there is a trend observed in business practices that firms are downsizing the supply base and maintaining a closer relationship with selected suppliers. Our study provides theoretical explanation of this phenomenon and offers support to this strategy. Notice that in our model, we examined tradeoffs between two forces: price and (volume) flexibility. Other key factors to the supply chain sourcing decisions such as quality and delivery capabilities will play a role if incorporated into the model. This provides an avenue for future research on the topic of supply chain contracts.

Additionally, we found that from the systems' view, a multi-supplier system is likely to be outperformed by the single-supplier supply chain, unless the flexibility is extremely high. In a reasonable range of price difference and small magnitude of flexibility, existence of multiple competing suppliers is not favored by the system, as shown in Figures 3.4 and 3.9. This finding suggests that a more complex supply chain structure may not perform as well as a simple, streamlined supply chain. This, again, supports the benefit of a small supply base and close partnerships from the system's perspective. However, as mentioned previously, the buyer is likely to enjoy the benefits brought about from the competition between suppliers. As such, the supplier's task is to convince the buyer to be willing to sacrifice a little to allow the supplier to earn a reasonable profit, thus enhancing the system's performance and creating a win-win situation.

When flexibility increases to a certain level, $\omega > 0.48$ in Figure 3.8, the 2-1 QF supply chain efficiency is observed to increase. However, supplier 1 does not benefit from offering higher flexibility that improves the system's performance. As a matter of fact, Figure 3.8 shows that supplier 1's expected profit is decreasing; it is supplier 2 who gets the benefit from receiving orders from the buyer in the range of higher flexibility. Although we assumed that the suppliers only know their own prices in our models, commonly in real-world setting, firms may actually gather information of their competitor's prices in the same industry. For example, AMD has information about Intel's CPU prices, and Micron is aware that its memory module is a little more expensive than Hynix's. So, if supplier 1 somehow is able to find out (estimate) that supplier 2's price is at \$40, there is no incentive for supplier 1 to further increase flexibility (ω) above 0.21, where she starts to win business over supplier 2. Therefore, if the QF supplier knows her competitor's price, the better system performance to the right in Figure 3.9 is unlikely to occur. Thus, the system performance of the 2-1 QF chain will likely be no better than that of the 1-1 QF chain, if not worse.

Finally, our numerical experiments were conducted based on a uniformly distributed demand function. A different distribution may result in different decision patterns and characteristics for the buyer and suppliers. A random demand variable with a complex probability density function may cause computational issues such as failing to identify close form solutions. Nonetheless, our models are developed on a strong theoretical ground and conceptually, should work for all distributions.

3.7 Conclusions and Future Research Directions

We study a decentralized supply chain in which there are two competing suppliers and one single buyer. One supplier offers the QF contract while the other provides a lower price. We study this problem from the buyer's perspective and solve for the buyer's optimal buying and forecasting decisions between the two suppliers. We investigate how well the QF contract can compete against the price discount scheme, everything else being equal. We identify areas where flexibility will be favored and areas where the discount price will receive more attention by the buyer. We found that the buyer will be better-off simply using either supplier in most cases. Compared with the situation where the buyer has the QF supplier as the single source, the buyer can only improve his profit having an alternative supplier who can offer a cheaper price.

However, from the system point of view, we found that the 2-1 QF supply chain is always outperformed by the 1-1 QF supply chain. The competition for the buyer's business between the two suppliers will result in a lower total profit that suppliers can earn than that in the 1-1 QF supply chain. Furthermore, the loss of the supplier's total profit is larger than the buyer's gain in the 2-1 QF supply chain. As a result, the system's profit declines and the supply chain performance worsens from the 1-1 QF supply chain. Although from the buyer's perspective, having an alternative source of supply is always beneficial. Our finding suggests that supply chain deficiency occurs with the existence of multiple suppliers.

We successfully developed models that are able to examine a two-supplier-one-buyer problem in a supply chain contract setting. Past research in the supply chain contract literature primarily focused on single-buyer-single-supplier, single-supplier-multiple-buyer, or multi-(complementary)supplier-single-buyer supply chain problems. Few looked at the multi-competing-supplier scenario that is common in a variety of business. Our analysis yields insights not only in multiple sourcing decisions in the presence of the QF contract, but also has implications in the technique to modeling supply chain contract problems involving competing suppliers.

In this research, we focused on supply chain sourcing decisions regarding the tradeoff between discount pricing and the quantity flexibility from two competing suppliers. However, such decisions may involve other important factors such as quality performance, logistics capability, cost sharing, promotional efforts, or service agreements to name a few. Our study indicates that a simple 1-1 supply chain is more desirable from the systems' point of view. The extra benefits that a buyer expects to receive from having competing suppliers may lead the supply chain to become less efficient. As such, a fruitful avenue for future research is to put more efforts to studying the supply chain contract problems that involve multiple competing suppliers. In addition, in our research, we assumed that the component supply is not an issue. However, this may not be true in a real-world setting. Two key factors that will affect the component's availability is lead-time and capacity. When a buyer has multiple customers, the component availability issue becomes even more critical. Furthermore, we constrained our focus to a single period problem. When buyers and suppliers continue their business relationship on the same product over a longer period, the models need to be modified and the problems need to be revisited. Future research efforts can target these areas.

Appendix of Chapter 3

Proof of Theorem 3.1

It suffices to show that the Hessian matrix is negative definite for both cases.

(1) Case I: $v_1 > v_2$

$$\frac{d\Pi_b}{dq_1} = (p - w_1)(1+\alpha)$$

$$- (p - w_1)[F((1+\alpha)q_1 + q_2)(1+\alpha) - F((1-\omega)q_1 + q_2)(1-\omega)]$$

$$-(p-v_1)F((1-\omega)q_1+q_2)(1-\omega)$$

$$=(p-w_1)(1+\alpha)\overline{F}((1+\alpha)q_1+q_2)-(w_1-v_1)(1-\omega)F((1-\omega)q_1+q_2) \quad \text{(A3.1)}$$

$$\frac{d^2\Pi_b}{dq_1^2}=-(p-w_1)(1+\alpha)^2 f((1+\alpha)q_1+q_2)-(w_1-v_1)(1-\omega)^2 f((1-\omega)q_1+q_2)<0$$

$$\frac{d\Pi_b}{dq_2}=(p-w_2)-(p-w_1)[F((1+\alpha)q_1+q_2)-F((1-\omega)q_1+q_2)]$$

$$-(p-v_1)[F((1-\omega)q_1+q_2)-F(q_2)]-(p-v_2)F(q_2)$$

$$=(w_1-w_2)+(p-w_1)\overline{F}((1+\alpha)q_1+q_2)$$

$$-(w_1-v)F((1-\omega)q_1+q_2))-(v_1-v_2)F(q_2) \quad \text{(A3.2)}$$

$$\frac{d^2\Pi_b}{dq_2^2}=-(p-w_1)f((1+\alpha)q_1+q_2)-(w_1-v_1)f((1-\omega)q_1+q_2)$$

$$-(v_1-v_2)f(q_2)<0$$

$$\frac{d^2\Pi_b}{dq_1dq_2}=\frac{d^2\Pi_b}{dq_2dq_1}=-(p-w_1)(1+\alpha)f((1+\alpha)q_1+q_2)$$

$$-(w_1-v_1)(1-\omega)f((1-\omega)q_1+q_2)$$

So by examining the Hessian matrix $|H|$ and its leading principle minors $|H_1|$ and $|H_2|$

we have: $|H_1|=\dfrac{d^2\Pi_b}{dq_1^2}<0$ and

$$|H_2|=|H|=\begin{vmatrix} \dfrac{d^2\Pi_b}{dq_1^2} & \dfrac{d^2\Pi_b}{dq_1dq_2} \\ \dfrac{d^2\Pi_b}{dq_2dq_1} & \dfrac{d^2\Pi_b}{dq_2^2} \end{vmatrix}$$

$$= (p-w_1)^2(1+\alpha)^2 f^2((1+\alpha)q_1+q_2)$$

$$+(p-w_1)(w_1-v_1)(1-\omega)^2 f((1+\alpha)q_1+q_2)f((1-\omega)q_1+q_2)$$

$$+(p-w_1)(1+\alpha)^2(w_1-v_1)f((1+\alpha)q_1+q_2)f((1-\omega)q_1+q_2)$$

$$+(1-\omega)^2(w_1-v_1)^2 f^2((1-\omega)q_1+q_2)$$

$$+(p-w_1)(v_1-v_2)(1+\alpha)^2 f((1+\alpha)q_1+q_2)f(q_2)$$

$$+(w_1-v_1)(v_1-v_2)(1-\omega)^2 f((1-\omega)q_1+q_2)f(q_2)$$

$$-(p-w_1)^2(1+\alpha)^2 f^2((1+\alpha)q_1+q_2)$$

$$-2(p-w_1)(1+\alpha)(w_1-v_1)(1-\omega)f((1+\alpha)q_1+q_2)f((1-\omega)q_1+q_2)$$

$$-(w_1-v_1)^2(1-\omega)^2 f^2((1-\omega)q_1+q_2)$$

$$= (p-w_1)(w_1-v_1)(\alpha+\omega)^2 f((1+\alpha)q_1+q_2)f((1-\omega)q_1+q_2)$$

$$+(p-w_1)(v_1-v_2)(1+\alpha)^2 f((1+\alpha)q_1+q_2)f(q_2)$$

$$+(w_1-v_1)(v_1-v_2)(1-\omega)^2 f((1-\omega)q_1+q_2)f(q_2) > 0$$

So $|H|$ is negative-definite.

(2) Case II: $v_1 \le v_2$

$$\frac{d\Pi_b}{dq_1} = (p-w_1)(1+\alpha)$$

$$-(p-w_1)[F((1+\alpha)q_1+q_2)(1+\alpha) - F((1-\omega)q_1+q_2)(1-\omega)]$$

$$-(p-v_2)[F((1-\omega)q_1+q_2)(1-\omega) - F((1-\omega)q_1)(1-\omega)]$$

$$-(p-v_1)F((1-\omega)q_1)(1-\omega)$$

$$= (p - w_1)(1+\alpha)\overline{F}((1+\alpha)q_1 + q_2) - (w_1 - v_2)(1-\omega)F((1-\omega)q_1 + q_2)$$

$$- (v_2 - v_1)(1-\omega)F((1-\omega)q_1) \tag{A3.3}$$

$$\frac{d^2\Pi_b}{dq_1^2} = -(p - w_1)(1+\alpha)^2 f((1+\alpha)q_1 + q_2) - (w_1 - v_2)(1-\omega)^2 f((1-\omega)q_1 + q_2)$$

$$-(v_2 - v_1)(1-\omega)^2 f((1-\omega)q_1) < 0$$

$$\frac{d\Pi_b}{dq_2} = (p - w_2) - (p - w_1)[F((1+\alpha)q_1 + q_2) - F((1-\omega)q_1 + q_2)]$$

$$- (p - v_2)F((1-\omega)q_1 + q_2)$$

$$= (w_1 - w_2) + (p - w_1)\overline{F}((1+\alpha)q_1 + q_2) - (w_1 - v_2)F((1-\omega)q_1 + q_2)) \tag{A3.4}$$

$$\frac{d^2\Pi_b}{dq_2^2} = -(p - w_1)f((1+\alpha)q_1 + q_2) - (w_1 - v_2)f((1-\omega)q_1 + q_2) < 0$$

$$\frac{d^2\Pi_b}{dq_1 dq_2} = \frac{d^2\Pi_b}{dq_2 dq_1} = -(p - w_1)(1+\alpha)f((1+\alpha)q_1 + q_2)$$

$$- (w_1 - v_2)(1-\omega)f((1-\omega)q_1 + q_2)$$

So we have: $|H_1| = \frac{d^2\Pi_b}{dq_1^2} < 0$ and

$$|H_2| = |H| = \begin{vmatrix} \frac{d^2\Pi_b}{dq_1^2} & \frac{d^2\Pi_b}{dq_1 dq_2} \\ \frac{d^2\Pi_b}{dq_2 dq_1} & \frac{d^2\Pi_b}{dq_2^2} \end{vmatrix}$$

$$= (p - w_1)^2(1+\alpha)^2 f^2((1+\alpha)q_1 + q_2)$$

$$+ (p - w_1)(w_1 - v_2)(1+\alpha)^2 f((1+\alpha)q_1 + q_2) f((1-\omega)q_1 + q_2)$$

$$+(p-w_1)(w_1-v_2)(1-\omega)^2 f((1+\alpha)q_1+q_2)f((1-\omega)q_1+q_2)$$

$$+(w_1-v_2)^2(1-\omega)^2 f^2((1-\omega)q_1+q_2)$$

$$+(p-w_1)(v_2-v_1)(1-\omega)^2 f((1+\alpha)q_1+q_2)f((1-\omega)q_1)$$

$$+(w_1-v_2)(v_2-v_1)(1-\omega)^2 f((1-\omega)q_1+q_2)f((1-\omega)q_1)$$

$$-(p-w_1)^2(1+\alpha)^2 f^2((1+\alpha)q_1+q_2)$$

$$-2(p-w_1)(w_1-v_2)(1+\alpha)(1-\omega)f((1+\alpha)q_1+q_2)f((1-\omega)q_1+q_2)$$

$$-(w_1-v_2)^2(1-\omega)^2 f^2((1-\omega)q_1+q_2)$$

$$=(p-w_1)(w_1-v_2)(\alpha+\omega)^2 f((1+\alpha)q_1+q_2)f((1-\omega)q_1+q_2)$$

$$+(p-w_1)(v_2-v_1)(1-\omega)^2 f((1+\alpha)q_1+q_2)f((1-\omega)q_1)$$

$$+(w_1-v_2)(v_2-v_1)(1-\omega)^2 f((1-\omega)q_1+q_2)f((1-\omega)q_1)>0$$

So $|H|$ is negative-definite, thus Π_b is concave in q_1 and q_2 □

Proof of Lemma 3.1

Exploring the FOCs, we have:

$$(p-w_1)(1+\alpha)\overline{F}((1+\alpha)q_1+q_2)-(w_1-v_1)(1-\omega)F((1-\omega)q_1+q_2)=0 \quad \text{(A3.5)}$$

$$(w_1-w_2)+(p-w_1)\overline{F}((1+\alpha)q_1+q_2)-(w_1-v_1)F((1-\omega)q_1+q_2))$$
$$-(v_1-v_2)F(q_2)=0 \quad \text{(A3.6)}$$

Operate (A3.6)* $(1-\omega)-$(A3.5), then divided by $(1-\omega)$ we get

$$(w_1 - w_2) - (\chi - 1)(p - w_1) + (\chi - 1)(p - w_1)F((1+\alpha)q_1 + q_2) - (v_1 - v_2)F(q_2) = 0$$

$$\Rightarrow F((1+\alpha)q_1 + q_2) = \frac{-(w_1 - w_2) + (\chi - 1)(p - w_1) + (v_1 - v_2)F(q_2)}{(\chi - 1)(p - w_1)} = A$$

$$\Rightarrow (1+\alpha)q_1 + q_2 = F^{-1}(A)$$

$$\Rightarrow q_1 = \frac{1}{1+\alpha}(F^{-1}(A) - q_2) \tag{A3.7}$$

Operate (A3.6)* $(1+\alpha)$ − (A3.5), then divided by $(1-\omega)$ we get

$$\chi(w_1 - w_2) - (\chi - 1)(w_1 - v_1)F((1-\omega)q_1 + q_2) - \chi(v_1 - v_2)F(q_2) = 0$$

$$\Rightarrow F((1-\omega)q_1 + q_2) = \frac{\chi(w_1 - w_2) - \chi(v_1 - v_2)F(q_2)}{(\chi - 1)(w_1 - v_1)} = B$$

$$\Rightarrow (1-\omega)q_1 + q_2 = F^{-1}(B)$$

$$\Rightarrow q_1 = \frac{1}{1-\omega}(F^{-1}(B) - q_2) \tag{A3.8}$$

Notice that $0 \le B \le A \le 1$, so we obtain

(a) $0 \le B$

$$\Rightarrow \quad \frac{\chi(w_1 - w_2) - \chi(v_1 - v_2)F(q_2)}{(\chi - 1)(w_1 - v_1)} \ge 0$$

$$\Rightarrow \chi(w_1 - w_2) - \chi(v_1 - v_2)F(q_2) \ge 0$$

$$\Rightarrow w_1 - w_2 \ge (v_1 - v_2)F(q_2)$$

$$\Rightarrow F(q_2) \le \frac{w_1 - w_2}{v_1 - v_2}$$ must be true at optimum

(b) $A \leq 1$

$$\Rightarrow \frac{-(w_1 - w_2) + (\chi - 1)(p - w_1) + (v_1 - v_2)F(q_2)}{(\chi - 1)(p - w_1)} \leq 1$$

$$\Rightarrow -(w_1 - w_2) + (\chi - 1)(p - w_1) + (v_1 - v_2)F(q_2) \leq (\chi - 1)(p - w_1)$$

$$\Rightarrow w_1 - w_2 \geq (v_1 - v_2)F(q_2)$$

(c) $B \leq A$

$$\Rightarrow \frac{\chi(w_1 - w_2) - \chi(v_1 - v_2)F(q_2)}{(\chi - 1)(w_1 - v_1)}$$

$$\leq \frac{-(w_1 - w_2) + (\chi - 1)(p - w_1) + (v_1 - v_2)F(q_2)}{(\chi - 1)(p - w_1)}$$

$$\Rightarrow \chi \geq \frac{(w_1 - v_1)[(p - w_2) - (v_1 - v_2)F(q_2)]}{(p - w_1)[(w_2 - v_1) + (v_1 - v_2)F(q_2)]} \tag{A3.9}$$

(detailed derivation skipped)

$$F(q_2) \leq \frac{w_1 - w_2}{v_1 - v_2}$$

$$\Rightarrow \frac{(w_1 - v_1)[(p - w_2) - (v_1 - v_2)F(q_2)]}{(p - w_1)[(w_2 - v_1) + (v_1 - v_2)F(q_2)]} \geq$$

$$\frac{(w_1 - v_1)[(p - w_2) - (v_1 - v_2)\frac{(w_1 - w_2)}{(v_1 - v_2)}]}{(p - w_1)[(w_2 - v_1) + (v_1 - v_2)\frac{(w_1 - w_2)}{(v_1 - v_2)}]} = 1$$

So we conclude that at the optimal condition, $F(q_2^*) \leq \dfrac{w_1 - w_2}{v_1 - v_2}$ must be true for all χ.

The second part of the Lemma is a direct result (2) is a by letting (A3.7)=(A3.8). When

$q_2^* = 0$, from (A3.9) we get $\chi \geq \dfrac{(p-w_2)(w_1-v_1)}{(p-w_1)(w_2-v_1)}$. Thus if

$\chi > \underline{\chi}' = \dfrac{(p-w_2)(w_1-v_1)}{(p-w_1)(w_2-v_1)}$, B<A, thus $q_1^* > 0$ must bet true. □

Proof of Theorem 3.4

When $(1+\alpha)q_1 + q_2 = Q_c$, $F((1+\alpha)q_1 + q_2) = F(Q_c) = \dfrac{p-c}{p-v_1}$. Thus from (A3.5)

and (A3.6) we have

$$F((1-\omega)q_1 + q_2) = \frac{\chi(p-w_1)(c-v_1)}{(p-v_1)(w_1-v_1)} = K$$

$$F(q_2) = \frac{w_1-w_2}{v_1-v_2} - \frac{(\chi-1)(p-w_1)(c-v_1)}{(v_1-v_2)(p-v_1)} = H$$

So $q_2 = F^{-1}(H)$, $q_1 = \dfrac{1}{1+\alpha}[F^{-1}(K) - F^{-1}(H)] = \dfrac{1}{1-\omega}[F^{-1}(\dfrac{p-c}{p-v_1}) - F^{-1}(H)]$

Rearrange the terms we get $\chi F^{-1}(K) - F^{-1}(\dfrac{p-c}{p-v_1}) = (\chi-1)F^{-1}(H)$.

Notice that $0 \leq H \leq K \leq \dfrac{p-c}{p-v_1}$:

(a) $0 \leq H$

$$\Rightarrow \frac{w_1-w_2}{v_1-v_2} - \frac{(\chi-1)(p-w_1)(c-v_1)}{(v_1-v_2)(p-v_1)} \geq 0$$

$$\Rightarrow w_2 \leq w_1 - \frac{(\chi-1)(p-w_1)(c-v_1)}{p-v_1}$$

(b) $H \leq K$

$$\Rightarrow \frac{w_1 - w_2}{v_1 - v_2} - \frac{(\chi - 1)(p - w_1)(c - v_1)}{(v_1 - v_2)(p - v_1)} \leq \frac{\chi(p - w_1)(c - v_1)}{(p - v_1)(w_1 - v_1)}$$

$$\Rightarrow \frac{w_1 - w_2}{v_1 - v_2} + \frac{(p - w_1)(c - v_1)}{(v_1 - v_2)(p - v_1)} \leq \frac{\chi(p - w_1)(c - v_1)}{(p - v_1)(w_1 - v_1)} + \frac{\chi(p - w_1)(c - v_1)}{(v_1 - v_2)(p - v_1)}$$

$$\Rightarrow \chi = \frac{(w_1 - v_1)[(p - w_1)(c - v_1) + (p - v_1)(w_1 - w_2)]}{(p - w_1)(c - v_1)(w_1 - v_2)} = \underline{\chi}_{\min}$$

(c) $K \leq \dfrac{p - c}{p - v_1}$

$$\Rightarrow \frac{\chi(p - w_1)(c - v_1)}{(p - v_1)(w_1 - v_1)} \leq \frac{p - c}{p - v_1} \quad \Rightarrow \chi \leq \frac{(p - c)(w_1 - v_1)}{(p - w_1)(c - v_1)} = \overline{\chi}_{\max} \qquad \square$$

Proof of Lemma 3.2

Exploring the FOCs, we have:

$$(p - w_1)(1 + \alpha)\overline{F}((1 + \alpha)q_1 + q_2) - (w_1 - v_2)(1 - \omega)F((1 - \omega)q_1 + q_2)$$

$$-(v_2 - v_1)(1 - \omega)F((1 - \omega)q_1) = 0 \qquad \text{(A3.10)}$$

$$(w_1 - w_2) + (p - w_1)\overline{F}((1 + \alpha)q_1 + q_2) - (w_1 - v_2)F((1 - \omega)q_1 + q_2)) = 0 \qquad \text{(A3.11)}$$

Operate (3.8)* $(1 - \omega) -$ (3.7), then divided by $(1 - \omega)$ we get

$$(w_1 - w_2) - (\chi - 1)(p - w_1)\overline{F}((1 + \alpha)q_1 + q_2) + (v_2 - v_1)F((1 - \omega)q_1) = 0$$

$$\Rightarrow F((1 + \alpha)q_1 + q_2) = 1 - \frac{(w_1 - w_2) + (v_2 - v_1)F((1 - \omega)q_2)}{(\chi - 1)(p - w_1)} = A' \qquad \text{(A3.12)}$$

$$\Rightarrow (1 + \alpha)q_1 + q_2 = F^{-1}(A')$$

$$\Rightarrow q_2 = F^{-1}(A') - (1 + \alpha)q_1$$

Now operate (3.8)* $(1 + \alpha) -$ (3.7), then divided by $(1 - \omega)$ we get

$$\chi(w_1 - w_2) - (\chi - 1)(w_1 - v_2)F((1-\omega)q_1 + q_2) + (v_2 - v_1)F((1-\omega)q_1) = 0$$

$$\Rightarrow F((1-\omega)q_1 + q_2) = \frac{\chi(w_1 - w_2) + (v_2 - v_1)F((1-\omega)q_1)}{(\chi - 1)(w_1 - v_2)} = B' \qquad \text{(A3.13)}$$

$$\Rightarrow (1-\omega)q_1 + q_2 = F^{-1}(B')$$

$$\Rightarrow q_2 = F^{-1}(B') - (1-\omega)q_1$$

Thus, $F^{-1}(A') - (1+\alpha)q_1 = F^{-1}(B') - (1-\omega)q_1$ leads to

$(\alpha + \omega)q_1^* = F^{-1}(A') - F^{-1}(B')$

Also, $0 \le B' \le A' \le 1$:

(a) Its straightforward that $0 \le B'$ and $A' \le 1$ are always true..

(b) $B' \le A'$

$$\Rightarrow \frac{\chi(w_1 - w_2) + (v_2 - v_1)F((1-\omega)q_1)}{(\chi - 1)(w_1 - v_2)} \le 1 - \frac{(w_1 - w_2) + (v_2 - v_1)F((1-\omega)q_2)}{(\chi - 1)(p - w_1)}$$

$$\Rightarrow \quad F((1-\omega)q_1^*) \le \frac{\chi(p - w_1)(w_2 - v_2) - (p - w_2)(w_1 - v_2)}{(p - v_2)(v_2 - v_1)} \qquad \square$$

CHAPTER 4 PRICE MARKDOWN SCHEME IN A THREE-ECHELON SUPPLY CHAIN

4.1 Introduction

Quantity discount pricing, or price-break schemes are fundamental strategies common to the high-technology sector (PC assembly, semiconductor manufacturing), the service sector (cell phone services, transportation services), and the consumer packaged goods sector (breakfast cereals, dairy products) among others. The topic of quantity discount hasbeen the subject of both managerial debate and academic research. These include: 1) suppliers' motivation to offer price discount incentives in the hope of stimulating sales volume, 2) excess inventories that lead to high carrying costs and obsolescence, 3) buying organizations' imperative to reduce procurement spending while contributing to corporate profitability, and 4) buying organizations' tendency to place large-sized special orders when offered cost-saving price-breaks by suppliers (Ramasesh and Rachamadugu, 2001). In general, the main task has been to find the total cost minimizing order quantity when the seller offers a price discount schedule.

The three most common price-break structures in the literature are the all-units price-break, the incremental quantity price-break, and price markdown (Hu and Munson, 2002; Weng, 1995; Gupta, 1988; Madan et al., 1993; Christoph and LaForge, 1989; Diaby and Martel, 1993; Arcelus and Srinivasan, 1995; Wee and Yu, 1997; Fazel et al., 1998; Abad, 2003; Lin & Kroll, 1997; Khouja, 1995). In an all-units scheme, the buyer pays the same unit price for every unit ordered with this unit price being determined by the quantity range into which the order fits (Madan et al., 1993). In the incremental discounting scheme, discounts are offered only on the additional units ordered beyond a specified quantity (Gupta, 1988). In the price markdown scheme, the buyer pays the same unit prices regardless of the ordering quantity. All three discount schemes are in nature a quantity discount scheme such that the greater the purchasing volume, the higher the saving.

In this chapter we study a price markdown contract in a supply chain that consists of a supplier, an original equipment manufacturer (OEM), and a buyer (retailer). The supplier produces the component that is needed for the product that the buyer sells to end-customers and the OEM is hired by the buyer to produce or assemble the product. A supply chain characterized with an OEM can be found in various industries such as the PC, cell phone, high-tech, and grocery/retailing. The component price is negotiated between the supplier and buyer. The supplier sells the component directly to the OEM and the buyer pays to the OEM the component costs and a fixed rate per unit of final product made. In this supply chain, the supplier is able to develop new technologies so that it can cut down the production cost. Demand is price sensitive; it increases as price decreases. Thus, the supplier takes the initiative to offer a price discount and the buyer will adjust the retail price downwards accordingly. Both parties expect to benefit from higher demand induced by a lower price. Analogous to the Stackelberg game, the supplier is a leader of this game and the buyer plays the role of a follower.

Price markdown problems in a three-echelon supply chain can be found in many industries. For example, in the personal computer assembly sector of the computer industry, PC makers routinely negotiate price reductions with their suppliers on a quarterly or even monthly basis, because of the relatively short life-cycles of the components. The price reduction is made possible because component suppliers are able to develop new technologies so that they can produce the same components at a lower cost. An excellent example of the price markdown scheme that we consider is Intel's CPU/Chipset price-breaks. Intel offers price markdown roughly once every two months. The price difference between the current price and the new price can approach 35% of the original price of the same CPUs.

Intel's price breaks are typically announced well in advance and the new prices become effective on a pre-determined date. As a result, the PC makers also reduce the prices of PCs to reflect the CPU price breaks. Many other electronic component manufacturers, such as Seagate, Western Digital (Hard drive disks), Hitachi, Sony (optical drives), Micron, Infenion, Hynix, Samsung (Semiconductor memory chips/ modules), and LG/Philips (LCD) also offer the same type of price markdown schemes to the PC makers.

When a supplier decides to offer a price discount, her task is to specify the new pricing so that she can benefit the most from the increased demand. In addition, the supplier needs to reserve enough capacity to cope with the higher demand. On the other hand, the buyer will adjust the retail price to reflect the lowered component cost in order to induce higher demand, hoping to create higher profits. The increasing demand will also induce the need to adjust the stocking level of the final product. These decisions are effective in the next period with the new, lower pricing for both the supplier and buyer.

In contrast, the OEM's problem deals with when the new price becomes effective. The OEM's profits per unit of product made is not affected by the price changes, however, the price difference will affect its holding cost and stockout cost for any units of components carried beyond the price break point. In addition, new components' arrival time is uncertain due to weather condition, airport congestion, or custom inspection just to name a few. Moreover, the delivery uncertainty is further complicated by a mismatch between the buyer's operating schedules and the carrier's delivery schedules. In this context, the OEM is mainly concerned with the safety inventory level to be carried beyond the price break because the price difference may cause substantial finance loss if a wrong stocking decision is made.

In this chapter, we develop a price-break model that involves a complete supply chain. This decision problem has not been considered thoroughly in previous discount contract research. The model discussed in the paper extends the price-break literature. The results offer significant managerial insights to practitioners in industries where the "cost-price-performance squeeze" creates slim profit margins and in lean procurement environments.

The remainder of the paper is organized as follows. We begin with reviewing the related literature. We then describe the research problem of interest. Next, we formulate the supplier's and buyer's problems, followed by the OEM's problem. Weprovide a numerical example to demonstrate the usefulness of themodel. The data utilized in the numerical illustration were obtained from a major OEM firm in the PC industry. We also present a series of sensitivity analyses, which yield several interesting managerial insights. The final section summarizes our results and presents several extensions to consider in future research.

4.2 Literature Review

Research on the price-break problem is rich and has been addressed in such diverse fields as economics, marketing, and procurement. We focus primarily on those studies in the inventory and procurement literature. In this research stream, several researchers have studied the problem from the buyer's perspective which seeks to minimize the total cost by determining the optimal ordering quantity (Gupta, 1988; Ardalan, 1988; Christoph and LaForge, 1989; Aull-Hyde, 1992; Madan et al., 1993; Diaby and Martel, 1993; Arcelus and Srinivasan, 1995; Wee and Yu, 1997; Fazel et el., 2003; Rubin and Benton, 2003). In contrast, several researchers have examined the problem of determining anoptimal schedule for price discounts from the supplier perspective. This perspective seeks to maximize the profit for the supplier (Kim and Hwang, 1988; Wang and Wu, 2000; Klastorin et al., 2002; Rubin and Benton, 2003; Burnetas et al., 2007). Yet another stream of research in quantity discount has paid much attention to how buyer and supplier can jointly determine the optimal discount and ordering policies that achieves the system profit; see Weng (1995), Corbett and Groote (2000), Wang (2005).

While various research aim at designing a quantity discount mechanism that can achieve the system-wide profit for the individually motivated buyer and supplier, such a goal may not be viable in a decentralized, wholesale-price-based supply chain, which is prevalent in numerous industries (Wang, 2005). A price-markdown scheme is virtually a price-only contract. A more thorough understanding of the decision making processes in the presence of the price markdown scheme in a decentralized supply chain is warranted. The majority of work in the discount literature considered a two-echelon supply chain that consists of a supplier and a buyer (retailer). However, it is common that a supply chain contains more than two agents. In this case, the price discount's impact is beyond the two echelons. In this chapter, we study a price markdown problem on a three-echelon supply chain in which an OEM provides production service to the buyer. Such a price markdown scheme is induced by manufacturing technology innovation and is widely observed in the hi-tech industry (Lee et al., 2000)

Various forms of the demand function have been utilized in the quantity discount research stream. In those focusing on ordering cost and lot-sizing in the presence of quantity discount, market demand was assumed to be a constant that is independent of the price discount (Chiang, et al., 1994). On the other hand, those studying how the joint profits can be affected by quantity discounts have considered demand as a deterministic decreasing function of price (Weng, 1995; Viswanathan and Wang, 2003; Wang, 2005; Yue et al., 2006). In contrast, others have studied quantity discount problems by considering stochastic demand; for a detailed discussion of stochastic demand functions and their assumptions, refer to Petruzzi and Dada (1999), Ray et al. (2006) and Zhou (2007), Burnetas et al (2007), and Lau et al. (2007). Finally, stochastic demand that incorporate delivery uncertainty has been well documented, i.e., the lead-time demand (Zipkin, 2000). However, work in the discount literature that considers both demand and lead-time uncertainties is rare.

Similar to most work in the supply chain contract and mechanism literature, the price markdown scheme that we consider in this paper has a model structure equivalent to the newsvendor problem. Interest in the newsvendor problem continues unabated, as many extensions to it have been proposed (Lau and Lau, 1988a,b; Nahmias and Smith, 1994). These extensions have considered alternative pricing schedules and lot-sizing rules, multiple locations, and progressive multiple discounts. These characterizations of the problem have been observed not only in the consumer packaged goods and airline industries, but also in retail apparel and automotive industries (Khouja, 1995; Cherikh, 2000). A comprehensive taxonomy of the newsvendor problem is available in Khouja (1999).

In this chapter, we study a price markdown problem with price-sensitive, stochastic demand from a supplier, the buyer, and the OEM perspectives. We formulate our models following the Stackelberg game framework, with the supplier being the leader of the game. Our model has a similar structure to Ray et al's (2006) research. Ray et al. (2005) considered a pricing and stocking problem in a supply chain that consists of a manufacture, a distributor, and a retailer. But they focused on the logistics side of this supply chain; they modeled and solved for the optimal pricing and stocking policies for the distributor and the retailer, omitting the manufacturer's problem. Our research, on the other hand, considers the entire supply chain in which we study each party's problem in this chain. We study the supplier's optimal component pricing and capacity planning decisions, the buyer's optimal retail pricing and stocking policies, and the OEM's optimal stocking policy in the presence of the price markdown scheme. Additionally, we incorporate the interplay of the OEM's operational hours and local carrier's delivery schedules in our models and examine its impact on the OEM's optimal ending inventory decisions when offered a price markdown. The problem we study is common in the hi-tech industry and an emerging practice in other industries, however, little attention has been paid to this discount problem in the literature. Extant literature does not consider the emerging issue of delivery uncertainty in the optimum ordering policy models with price-breaks. We seek to fill this gap. In the next section, we describe the general problem setting and include our assumptions specific to the price-break models.

4.3 Problem Setting

We consider a supply chain that consists of a supplier, a retailer (buyer), and an OEM for the buyer. The OEM buys component(s) from the supplier and manufactures the product for the buyer. The buyer pays the OEM a fixed rate per unit made as well as the cost of the components. In this supply chain, the supplier decides the component price and the buyer decides the retail price, given the component price. The OEM charges the buyer at a fixed rate per unit built; this service rate is not affected by the prices of the component and the product. The supplier ships material(s) to the OEM. The OEM then produces and ships the product(s) to the buyer. To simplify our analysis, we assume that there is one component and one product in this supply chain.

Demand is assumed price-sensitive in this supply chain. Specifically, demand increases as price decreases. As discussed previously, the supplier is able cut down her production cost by innovation in manufacturing technologies (Lee et al., 2000). Therefore, the supplier offers a price markdown scheme to the buyer on a regular basis. As the component price decreases, the buyer will adjust the retail price accordingly. The goal of cutting down the retail price is to induce a higher level of demand in the hope of achieving a higher level of profits. On the other hand, the OEM's service charge is assumed not affected by the price changes of the component and the product; OEM earns the same fixed rate per unit built for the buyer.

We assume that the buyer shares full information of market demand observation with the supplier and the OEM, i.e., they are aware of the demand patterns and distribution. So the supplier's challenge is to decide the optimal component price to maximize her profit level and the planned capacity to cope with increased demand. On the other hand, the buyer's challenge is to decide the optimal retail pricing policy and the optimal stocking policies to maximize his own profit. Notice that both supplier's and buyer's problems are associated with the whole next period with the new pricing. Although not explicitly specified in our model, the length of the next period with the new pricing can be two to four months which is, for example, the case in the PC industry.

However, the OEM's challenge is not only to identify the optimal stocking level for the next period, but also the optimal "ending" inventory level right before the price break. The price difference of the components will tremendously increase the carrying cost and decrease the stockout cost for the OEM, which will lead to a different stocking level from that in the regular time. After the new (lower) price becomes effective, the OEM's carrying cost will be back to the original level. Therefore, the OEM's decision making focuses on the very short period of time when the price markdown becomes effective. OEM is known to have a low margin. Proper ending inventory control can possibly create substantial savings for the OEM. Both the supplier and the buyer can decide the timing of price discount. The supplier can announce a price discount after she produces the component at the lower cost; the buyer can announce a price markdown after he uses up the existing inventory. So the ending inventory control is not as critical to the supplier and buyer as it is to the OEM.

4.4 The Decentralized Supply Chain Model in The Presence of a Price Discount

We model our problem via a wholesale-price-only discount contract, following the Stackelberg framework (Ray et al., 2005). We first focus on the supplier's and the buyer's problem and solve for the optimal (reduced) pricing for both parties as well as the optimal capacity investment for the supplier and optimal stocking policy for the buyer. We then turn our attention to the OEM's problem, given the new (lower) component and retail prices. The following notation will be utilized throughout this chapter.

Notation:

Π_S : Supplier's total expected profit

Π_B : Buyer's total expected profit

D : Demand

p_i : retail price in period i, i=0,1. 0: current period; 1: next period

w_i : Component price in period i

w_m : OEM's service charge per unit of product made

c_i : Supplier's production cost in period i

c_k : cost per unit of planned capacity

c_m : OEM's production cost per unit

K_i : Planned capacity in period i

ϕ : p.d.f. of standard normal distribution

Φ : c.d.f. of standard normal distribution

ϕ_D : p.d.f. of random variable D

Φ_D : c.d.f. of random variable D

h : buyer's holding cost per unit of product

h_m : OEM's holding cost per unit of component

s : supplier's understocking cost per unit

s_m : OEM's understocking cost per unit

s_B : Buyer's understocking cost per unit

I_i : Buyer's stocking level per unit time in period i

x_i : OEM's stocking level per unit time in period i

4.4.1 Demand structure

We consider a price-sensitive, end-customer demand $D(p)$ arriving at the buyer per unit time, where p is the retail price of the product. $D(p)$ consists of a deterministic term and a stochastic error term (Zipkin, 2000; Ray et al., 2005). Two types of price-sensitive demand functions have been widely utilized in the economic and management science literature: the additive and the multiplicative demand functions (Petruzzi and Dada, 1999). To simplify the development of the models, we consider only the addictive demand function; the optimal policies in our models can easily be revised to adapt the multiplicative demand functions.

The typical additive demand function has the following format: $D(p) = y(p) + \varepsilon$, where $y(p)$ is deterministic, and decreasing and concave in p (Ray et al., 2005), and ε is a continuous random variable that follows a normal distribution with mean u_ε and variance σ_ε^2, i.e., $N(\mu_\varepsilon, \sigma_\varepsilon^2)$. As a result, $D(p)$ also follows the normal distribution with mean $\lambda(p) = y(p) + u_\varepsilon$ and variance σ_ε^2, i.e., $N(\lambda(p), \sigma_\varepsilon^2)$. Examples of the forms of $y(p)$ include (1) $(A - Bp^k)^\gamma, A > 0, B > 0, k \geq 1, \gamma \leq 1$ (2) $A - Bp^k, A > 0, B > 1$ (3) $\ln[(A - Bp)^\gamma], \gamma \geq 0, B > 0$ (Ray et al., 2005). Detailed discussions of additive and multiplicative $D(p)$ can be found in Mills (1959), Karlin and Carr (1962), Petruzzi and Dada (1999), Ray et al. (2005), Arcelus et al. (2005), and Bernstein and Federgruen (2005).

4.4.2 The supplier's model

Let (w_0, c_0) be the supplier's component price and production cost in the current period, say, period 0; p_0 is the retail price set by the buyer, given w_0. The supplier is able to produce the same component at a lower cost c_1 and is planning to reduce the price to w_1 in the next period, say, period 1. As a result, the buyer will reduce the retail price from p_0 to p_1 to induce the demand to increase from $D(p_0)$ to $D(p_1)$. To cope with the increased demand, the supplier will need to decide a capacity level K_1, with current capacity being K_0. The supplier's task is to decide the optimal pricing w_1^* and optimal capacity K_1^* so that the expected profit at the new price will increase from the expected profit when the component price is kept at w_0. In other words, the supplier's objective is to maximize the following:

$$\text{Max}\, \Delta\Pi_S = \Pi_S(w_1, c_1, K_1) - \Pi_S(w_0, c_1, K_0), \text{ where, } w_0 > w_1,\ K_0 < K_1, \&$$

$$\Pi_S(w_i,c_1,K_i)=(w_i-c_1)E(D(p_i))-c_kK_i-sE(D(p_i)-K_i)^+$$

Lemma 4.1 Maximizing $\Delta\Pi_S$ is equivalent to maximizing $\Delta\Pi_S(w_1,c_1)$.

Proof. The newer, lower-cost manufacturing technology has been developed, therefore, c_1 is treated as a parameter in the profit function. One can clearly see that $\Pi_S(w_0,c_1,K_0)$ is independent of the decision variables (w_1,K_1). As such, $\Delta\Pi_S=\Pi_S(w_1,c_1,K_1)-\text{constant}$. This completes the proof □

As a result of lemma 4.1, the all-unit price discount model is virtually a wholesale-price-only model, because in our model we have only one reduced component price, regardless of order quantity. Market demand at the buyer's location will be passed onto the OEM and the supplier under the assumption of full information sharing. Therefore, the supplier's model can then be expressed as following:

$$\text{Max}\,\Pi_S(w_1,c_1,K_1)=(w_1-c_1)\lambda(p_1)-c_kK_1-sE(D(p_1)-K_1)^+ \tag{4.1}$$

The first terms indicates the expected profit per unit time with the new component price and the new retail price. The second term is the cost of capacity per unit time planned for the next period (period 1). Notice that there is difference between c_1 and c_k. The former refers to the cost associated with production activities; c_1 will not occur if there is no production, e.g., material cost, machine time, etc. On the other hand, c_k is the cost associated with reserving the capacity; the planned capacity will cost the supplier even if there is no production for the component. Examples of c_k can be labor hired, machine purchased, and facility depreciation. Finally, the model is structured in a manner that supplier will fulfill all demand. When the demand rate exceeds the planned capacity rate, it costs the supplier extra cost s per unit to satisfy buyer's orders. Examples of s include working overtime, rescheduling production, and expediting a shipment.

Corollary 4.1 $\Pi_S(w_1, c_1, K_1)$ is jointly concave in w_1 and K_1.

Proof. It can be easily shown that $\frac{d\Pi_S}{dw_1} = \lambda(p_1), \frac{d^2\Pi_S}{dw_1^2} = \lambda'(p_1)\frac{dp_1}{dw_1} < 0$,

$$\frac{d\Pi_S}{dK_1} = s - c_k - s\Phi_D(K_1),\ \frac{d^2\Pi_S}{dK_1^2} = -s\phi_D(K_1) < 0,\ \frac{d^2\Pi_S}{dw_1 dK_1} = 0.$$

Therefore, $|H_1| = H_{11} = \frac{d^2\Pi_S}{dw_1^2}$<0 and $|H_2| = |H| = \begin{vmatrix} \frac{d^2\Pi_S}{dw_1^2} & 0 \\ 0 & \frac{d^2\Pi_S}{dK_1^2} \end{vmatrix} > 0$.

So the Hessian matrix H is negative definite. This completes the proof. □

4.4.3 The buyer's model

The buyer (retailer) directly faces the end-customer and market demand. He sells the product that is made by the OEM he hires. He bears the material costs and pays the OEM at a fixed rate per unit of product made. The buyer decides the retail price that maximizes his expected profit, given the material cost and the OEM service charges. The buyer also needs to decide a stocking policy so that he will be expecting the highest benefit considering the understocking and overstocking costs. As such, the buyer's problem can be modeled as following:

$$\text{Max}\,\Pi_B(p_1,w_1,I_1) = (p_1 - w_1 - w_m)\lambda(p_1) - [hE((I_1 - D(p_1))^+ + s_B E(D(p_1) - I_1)^+] \tag{4.2}$$

The first term refers to the expected profits from selling the product to end-customers. The second terms includes the carrying cost and undertocking penalty from the discrepancy between the safety stocking level and demand rate per unit time. Notice that the decision variables in the buyer's model are (p_1, I_1). w_1 is exogenous and is decided by the supplier. Since the buyer is the follower of the Stackelberg game, his goal is to maximize the expect profit in the next period by finding the optimal (p_1, I_1), given w_1. Therefore, the buyer's model also has the same structure as a wholesale-price-only contract model. Parameter values (p_0, w_0, I_0) in current period serve as a boundary of the optimal solution and will not affect the decision of (p_1, I_1), unless (p_1, I_1) are biding to the boundary value.

Corollary 4.2

(1) $\Pi_B(p_1, w_1, I_1)$ is jointly concave in p_1 and I_1

(2) Given component price w_1, there is a unique retail price $p_1(w_1)$ and I_1 that will maximize the buyer's expected profit.

Proof. (1) The proof is skipped as it is similar to the proof of corollary 4.1.

(2) This is the direct result of corollary 4.2(1).

4.4.4. Finding the optimal pricing and stocking policies

The solution method follows the backward induction that is well documented in the contract literature (Tsay, 1999; Ray et al., 2005). We will solve the buyer's model, followed by solving the supplier's model.

4.4.4.1 Optimal pricing and stocking policies for the buyer

Lemma 4.2 $I_1^* = y(p_1) + \mu_\varepsilon + z_1^* \sigma_\varepsilon$, where $z_1^* = \Phi^{-1}(\frac{s_B}{h + s_B})$

Proof.

To solve I_1^*, we first rearrange the terms in the bracket in (42) as follows:

$$hE(I_1 - D(p_1))^+ + s_B E(D(p_1) - I_1)^+$$

$$= hE(I_1 - y(p_1) - \varepsilon)^+ + s_B[E(I_1 - y(p_1) - \varepsilon)^+ + \mu_\varepsilon - (I_1 - y(p_1))]$$

$$= (h + s_B)E(I_1 - y(p_1) - \varepsilon)^+ + s_B \mu_\varepsilon - s_B(I_1 - y(p_1))$$

Applying the standard normalization procedure, let $I_1 - y(p_1) = \mu_\varepsilon + z_1 \sigma_\varepsilon$

$$\Rightarrow E(I_1 - y(p_1) - \varepsilon)^+ = E(z_1 \sigma_\varepsilon - (\varepsilon - u_\varepsilon))^+ = \sigma_\varepsilon E(z_1 - \frac{\varepsilon - u_\varepsilon}{\sigma_\varepsilon})^+$$

Notice that $\frac{\varepsilon - u_\varepsilon}{\sigma_\varepsilon} \sim N(0,1)$. So the objective function can be expressed as follows:

$$\Pi_B$$

$$= (p_1 - w_1 - w_m)\lambda(p_1) - (h + s_B)\sigma_\varepsilon E(z_1 - \frac{\varepsilon - \mu_\varepsilon}{\sigma_\varepsilon})^+ - s_B \mu_\varepsilon + s_B(\mu_\varepsilon + z_1 \sigma_\varepsilon)$$

$$\Rightarrow \frac{d\Pi_B}{dz_1} = -(h+s_B)\sigma_\varepsilon \Phi(z_1) + s_B \sigma_\varepsilon = 0$$

$$\Rightarrow z_1^* = \Phi^{-1}(\frac{s_B}{h+s_B})$$

$$\Rightarrow I_1^* = y(p_1) + \mu_\varepsilon + z_1^* \sigma_\varepsilon \qquad \square \qquad (4.3)$$

The advantage of utilizing the standard normal z_1 is that when ε is not normal, the solution process can be used to approximate the optimal stocking policy, as long as the mean and variance of ε are known. In addition, one can observe from (4.3) that buyer's optimal stocking policy per unit time is affected by the retail price as well as bythe distribution parameters of the error term in the demand function. Finally, once I_1^* is identified, the buyer will know how much change in the stocking level the new, lower price has caused.

Corollary 4.3 $\Delta I = I_1^* - I_0^* > 0$.

Proof. It's straightforward to show that $I_0^* = y(p_0) + \mu_\varepsilon + z_0^* \sigma_\varepsilon$, where

$z_0^* = \Phi^{-1}(\frac{s_B}{h+s_B}) = z_1^*$. Therefore, $\Delta I = I_1^* - I_0^* = y(p_1) - y(p_0) > 0$ $\square$

Next we solve for the optimal pricing policy.

Theorem 4.1 Given component price w_1, the buyer's optimal retail price is the unique solution to the following: $p_1 + \frac{\lambda(p_1)}{\lambda'(p_1)} = w_1 + w_m$

Proof. First we rearrange the terms in the bracket in (4.2) as following:

$$hE(I_1 - D(p_1))^+ + s_B E(D(p_1) - I_1)^+$$

$$= hE[(\varepsilon - (I_1 - y(p_1)))^+ + (I_1 - y(p_1)) - \mu_\varepsilon] + s_B E(\varepsilon - (I_1 - y(p_1)))^+$$

$$= (h + s_B)E(\varepsilon - (I_1 - y(p_1)))^+ + h(I_1 - y(p_1)) - h\mu_\varepsilon$$

Substituting in $I_1^* = y(p_1) + \mu_\varepsilon + z_1^*\sigma_\varepsilon$ we get

$$hE(I_1 - D(p_1))^+ + s_B E(D(p_1) - I_1)^+$$

$$= (h + s_B)\sigma_\varepsilon E(\frac{\varepsilon - \mu_\varepsilon}{\sigma_\varepsilon} - z_z^*)^+ + h(I_1 - y(p_1)) - h\mu_\varepsilon$$

$$= (h + s_B)\sigma_\varepsilon I_N(z_1^*) + h(\mu_\varepsilon + z_1^*\sigma_\varepsilon) - h\mu_\varepsilon,$$

where $I_N(z_1^*) = \phi(z_1^*) - z_1^*(1 - \Phi(z_1^*))$ and $z_1^* = \Phi^{-1}(\frac{s_B}{h + s_B})$

(Porteus, 2002)

So we can rewrite the buyer's expected profit function as following:

$$\Pi_B = (p_1 - w_1 - w_m)\lambda(p_1) - (h + s_B)\sigma_\varepsilon I_N(z_1^*) - h(\mu_\varepsilon + z_1^*\sigma_\varepsilon) + h\mu_\varepsilon$$

$$= (p_1 - w_1 - w_m)\lambda(p_1) - (h + s_B)\sigma_\varepsilon[\phi(z_1^*) - z_1^*(1 - \Phi(z_1^*))] - hz_1^*\sigma_\varepsilon$$

$$= (p_1 - w_1 - w_m)\lambda(p_1) - (h + s_B)\sigma_\varepsilon\phi(z_1^*)$$

FOC: $\frac{d\Pi_B}{dp_1} = (p_1 - w_1 - w_m)\lambda'(p_1) + \lambda(p_1) = 0$

$\Rightarrow$ The optimal retail price p_1^* solves $p_1 + \frac{\lambda(p_1)}{\lambda'(p_1)} = w_1 + w_m$ □ (4.4)

Notice that $p_1^* \geq w_1 + w_m$ due to the fact that $\lambda'(p_1^*) < 0$. It is also clear that the retail price p_1 is increasing in the component price w_1. Once the supplier decides the component price w_1, the buyer will be able to obtain the optimal retail price via (4.4). Interestingly, during the standardization procedure of the normal random variable, the terms that are dependent on price p_1 the terms of overstocking and understanding costs of (4.2) get eliminated. As such, the optimal stocking policy I_1^* is affected by the retail price (4.3), but the expected total overstocking and understocking costs are not.

4.4.4.2 Optimal pricing and stocking policies for the supplier

Now we turn our attention to (4.1). Similarly to the solution procedure of the buyer's model, we first solve for the optimal capacity level for the supplier, then we analyze the optimal pricing policy and the optimal conditions.

Lemmal 4.3 If $s > c_k$, $K_1^* = y(p_1) + \mu_\varepsilon + z_2^* \sigma_\varepsilon$, where $z_2^* = \Phi^{-1}(\frac{s - c_k}{s})$.

Otherwise, $K_1^* = y(p_1) + \mu_\varepsilon$.

Proof. Supplier's expected profit function can be rewritten as

$$\Pi_S(w_1, c_1, K_1) = (w_1 - c_1)\lambda(p_1) + (s - c_k)K_1 - sE(K_1 - y(p_1) - \varepsilon)^+ - s\mu_\varepsilon$$

Let $K_1 - y(p_1) = \mu_\varepsilon + z_2 \sigma_\varepsilon$, we have

$$\Pi_S = (w_1 - c_1)\lambda(p_1) + (s - c_k)(y(p_1) + \mu_\varepsilon + z_2\sigma_\varepsilon) - s\sigma_\varepsilon E(z_2 - \frac{\varepsilon - \mu_\varepsilon}{\sigma_\varepsilon})^+ - s\mu_\varepsilon$$

FOC: $\dfrac{d\Pi_B}{dz_2} = (s - c_k) - s\Phi(z_2) = 0$

$$\Rightarrow \text{ If } s-c_k>0,\ z_2^*=\Phi^{-1}(\frac{s-c_k}{s})=0. \text{ Otherwise, } z_2^*=0$$

$$\Rightarrow K_1^*=y(p_1)+\mu_\varepsilon+z_2^*\sigma_\varepsilon \qquad \square \tag{4.5}$$

As one can see, the optimal capacity is also dependent on the retail price, as the retail price affects the demand level, which in turn affects the supplier's planned capacity. Additionally, it is clear that the optimal capacity is increasing in the retail price. Furthermore, the capacity needs to be greater than or equal to zero. When $s<c_k$, the z_2^* will approach negative infinity, thus resulting in K_1^* to be negative infinite. In this case, we simply reset the K_1^* to be zero. Finally, we expect that K_1^* to increase from K_2^* because when a price discount is offered, the demand is expected to increase, thus resulting in a larger capacity reservation to accommodate the potentially larger demand.

Corollary 4.4 $\Delta K=K_1^*-K_0^*>0$.

The next step is to solve the optimal component pricing for the next period. Solving w_1^* directly from the first order condition of Π_S can be problematic because we will not be able to convert p_1 to w_1 in a clean format without knowing the exact form of $\lambda(p_1)$. So instead of solving for optimal w_1^*, we convert w_1 to p_1 via (4.4) and rewrite the supplier's expected profit as a function of p_1. By doing so, we can solve for the optimal p_1^* that benefits the supplier the most. Through (4.4), we can then identify the optimal

w_1^* that will induce the buyer to decide the retail price as p_1^*. This technique has been also employed by Ray et al. (2005).

Lemma 4.4 $\underset{p_1}{Max}\Pi_S$ is equivalent to $\underset{p_1}{Max}(p_1 + \frac{\lambda(p_1)}{\lambda'(p_1)} - w_m - c_1 - c_k)\lambda(p_1)$

Proof. Utilize (4.5), (4.6) and rewrite (4.1) as

$$\Pi_S = (p_1 + \frac{\lambda(p_1)}{\lambda'(p_1)} - w_m - c_1)\lambda(p_1) - c_k(y(p_1) + \mu_\varepsilon + z_2^*\sigma_\varepsilon) - s\sigma_\varepsilon I_N(z_2^*)$$

$$= (p_1 + \frac{\lambda(p_1)}{\lambda'(p_1)} - w_m - c_1 - c_k)\lambda(p_1) - c_k z_2^*\sigma_\varepsilon - s\sigma_\varepsilon I_N(z_2^*) \tag{4.6}$$

where $I_N(z_2^*) = \phi(z_2^*) - z_2^*(1 - \Phi(z_2^*))$ and $z_2^* = \Phi^{-1}(\frac{s - c_k}{s})$. It is clear that the last two terms are independent of p_1, thus the proof is completed. □

Theorem 4.2

(1) If $\lambda''' > \frac{4\lambda'^3}{\lambda^2} + \frac{3\lambda''^2}{\lambda'} - \frac{5\lambda'\lambda''}{\lambda}$, then there exists a unique p_1 such that $\Pi_S(p_1, z_2)$ is minimized

(2) Buyer's desired p_1^* solves the following:

$$p_1 - \frac{\lambda^2(p_1)\lambda''(p_1)}{\lambda'^3(p_1)} + \frac{3\lambda(p_1)}{\lambda'(p_1)} = c_1 + c_k + w_m$$

Proof. To prove the uniqueness of optimal p_1, it suffices to show that Π_S is unimodal,

i.e., $\frac{d^2\Pi_S}{dp_1^{\,2}} < 0$.

$$\frac{d\Pi_S}{dp_1} = \frac{d}{dp_1}[(p_1 + \frac{\lambda}{\lambda'} - w_m - c_1 - c_k)\lambda]$$

$$= (p_1 + \frac{\lambda}{\lambda'} - w_m - c_1 - c_k)'\lambda + (p_1 + \frac{\lambda}{\lambda'} - w_m - c_1 - c_k)\lambda'$$

$$= (2 - \frac{\lambda\lambda''}{\lambda'^2})\lambda + (p_1 + \frac{\lambda}{\lambda'} - w_m - c_1 - c_k)\lambda'$$

$$= 3\lambda - \frac{\lambda^2\lambda''}{\lambda'^2} + (p_1 - w_m - c_1 - c_k)\lambda' = 0$$

$$\therefore \quad (p_1 - w_m - c_1 - c_k) = -\frac{3\lambda}{\lambda'} + \frac{\lambda^2\lambda''}{\lambda'^3}$$

$$\frac{d^2\Pi_S}{dp_1^{\,2}} = 4\lambda' - \frac{2\lambda\lambda'^2\lambda'' + \lambda^2\lambda'\lambda''' - 2\lambda^2\lambda''}{\lambda'^2} + (p_1 - w_m - c_1 - c_k)\lambda''$$

$$= 4\lambda' - \frac{2\lambda\lambda'^2\lambda'' + \lambda^2\lambda'\lambda''' - 2\lambda^2\lambda''^2}{\lambda'^3} + (-\frac{3\lambda}{\lambda'} + \frac{\lambda^2\lambda''}{\lambda'^3})\lambda''$$

$$= 4\lambda' + \frac{\lambda^2\lambda''^2}{\lambda'^3} - \frac{3\lambda\lambda''}{\lambda'} - \frac{2\lambda\lambda'^2\lambda'' + \lambda^2\lambda'\lambda''' - 2\lambda^2\lambda''^2}{\lambda'^3}$$

Since λ' and λ'' are both negative, it is straightforward that $\frac{d^2\Pi_S}{dp_1^{\,2}} < 0$ requires

$\lambda''' > \frac{4\lambda'^3}{\lambda^2} + \frac{3\lambda''^2}{\lambda'} - \frac{5\lambda'\lambda''}{\lambda}$ to be true. □

When λ''' satisfies the inequality, there is an unique p_1^* such that Π_S is maximized. This optimal p_1^* is desired by the supplier, and anticipated by (4.4). As such, by specifying the desirable p_1^*, the supplier will set the optimal component price w_1^* through (4.4), which will consequently induce the buyer to set the retail price at p_1^*. Notice that each term on the right hand side of the inequality above is negative, under the assumption that λ is decreasing and concave. So as long as $\lambda''' \geq 0$, Π_S is guaranteed unimodal, thus having a unique optimal solution.

We have thus far solved for supplier's optimal pricing and capacity planning policies, as well as the buyer's optimal retail pricing and stocking policies. Both buyer and supplier's decisions are effective for the next whole period. In contrast, the OEM's problem has a slightly different focus. As mentioned previously, the OEM's problem falls on deciding the optimal stocking level to be carried beyond the price break. The price difference does change tremendously both the holding and stockout costs specifically to the inventory being carried beyond the price break point, which is termed "ending inventory" hereafter. Once the new (lower) component price becomes effective, both carrying cost and stockout cost will be back to the normal level. Furthermore, observations from the PC industry suggest that lead-time variation needs to be incorporated into the ending inventory decision, because the supplier tends not to make component available before the price break. We present the OEM's model in the next section.

4.5 The OEM's Model

The OEM's problem is twofold. We first present the model that aims to solve the optimal "regular" stocking policy of the component for the OEM for the next period with the new pricing. We then present the second model which facilitates the OEM's decision in deciding the "ending" inventory level to cope with the impact of price difference and the delivery uncertainty. Furthermore, we analyze how the match and mismatch of the carrier's schedule and the OEM's operation schedule, which further complicates the delivery uncertainty, will affect such ending inventory decisions.

4.5.1. Model I: deciding optimal stocking policy for the new period

The OEM's expected profit function has the same structure as the buyer's expected profit function:

$$\Pi_m(w_m, c_m, x_1) = (w_m - c_m)\lambda(p_1) - [h_m E((x_1 - D(p_1))^+ + s_m E(D(p_1) - x_1)^+]$$

As mentioned, the OEM is not involved in deciding p_1, thus $\lambda(p_1)$ is exogenous to the OEM's model. Additionally, w_m comes from negotiation between the OEM and the buyer, and the production cost c_m is assumed not changing with the component price. As a result, the first term in the OEM's expected profit function is considered exogenous. Consequently, OEM's goal to decide the optimal stocking policy can be done by minimizing the holding and stockout costs. Therefore, the objective function is as follows:

$$Min\ C_m = h_m E[(x_1 - D(p_1)]^+ + s_m E[D(p_1) - x_1]^+ \tag{4.7}$$

The expected total cost function follows the typical newsvendor problem framework, thus it is convex and has a unique optimal solution that minimizes the objective function. Following the same solution approach presented in section 4.4.4, it can be easily shown that $x_1^* = y(p_1) + \mu_\varepsilon + z_3^* \sigma_\varepsilon$, where $z_3^* = \Phi^{-1}(\frac{s_m}{h_m + s_m})$. One can observe that x_1^* is also dependent on p_1, which is similar to buyer's I_1^*. When there is a price discount on the component price, retail price is expected to be lowered. Thus, the OEM's stocking policy should be adjusted accordingly in order to accommodate the increased demand. Notice that x_1^* is the optimal stocking policy at regular time after the price break. Next, we turn our attention to the optimal "ending inventory" to be carried beyond the price break. The ending inventory is purchased in the current period at the current component price in order to fulfill any demand incurred before the new component's arrival after the price break.

4.5.2. Model II: deciding optimal "ending" inventory policy

Inventory to be carried beyond the price break are purchased at the old price w_0. As mentioned the new component price will be w_1. So each unit of component carried beyond the price break will result in a loss of $\Delta = w_0 - w_1$. As a result, the carrying cost for the ending inventory is increased from h_m to $h_m + \Delta$. On the other hand, the stockout cost is actually decreased from s_m to $s_m - \Delta$, because each unit of product not fulfilled avoids a loss of price difference Δ. As seen in the previous section, both carrying cost and stockout cost are key elements to the optimal stocking policy, given the retail price. When price difference Δ is large, its impact on the stocking policy can be substantial.

Additionally, it is common in the PC industry that the component suppliers will not make the new components at the new (lower) price available before the price break point. They tend to make aggressive shipments of the new component to target at arriving on the price break effective day. Component suppliers such as Intel, AMD, and Seagate all have practiced this way for years. However, shipments of new components normally come from oversea countries and the arrival time can be easily delayed due to weather, airport congestion, or custom inspection. As a result, the arrival time is uncertain and this uncertainty is critical to the safety stock decision right before the price break. Therefore, in this model we consider both demand and delivery uncertainties. As such, the objective function can be expressed as follows:

$$Min\ C_m = (h_m + \Delta)E[(x - D(p_1, L)]^+ + (s_m - \Delta)E[D(p_1, L) - x]^+ \tag{4.8}$$

Stochastic demand that incorporates lead-time uncertainty is well documented in literature (Zipkin, 2000; Ray et al., 2005); such demand has been termed 'lead-time demand" (LTD). We employ the treatment for the lead-time demand, however, our L is slightly different from that in literature. While L refers to a random variable "lead-time" in literature, it is referred as the random variable "arrival time" (after price break point) in our model. In other words, what we consider in this model is the randomness of the component's arrival time after price break, rather than the variation of the length of time in transit. Nonetheless, thisdoes not cause any change of the treatment for the lead-time demand technically.

Assume that L has mean $E(L)$ and variance $Var(L)$, but its distribution is unknown. Hence the lead-time demand $D(p_1, L)$ can be expressed as $\mu_L = \lambda(p_1)E(L)$ and $\sigma_L^2 = \lambda(p_1)^2 Var(L) + \sigma_\varepsilon^2 E(L)$ (Ross, 1989; Zipkin, 2000; Ray et al., 2005).

Corollary 4.5 $x^* = y(p_1) + \mu_L + z_4^* \sigma_L$, where $z_4^* = \Phi^{-1}(\dfrac{s_m - \Delta}{h_m + s_m})$

Here we follow the same procedure as that presented in section 4.4.4 and 4.5.1 to obtain the optimal ending inventory x^*. However, in section 4.4.1 and 4.5.1 we dealt with normal random variable ε. Here we deal with a random variable withan unknown distribution. As a result, the standardization process presents an "approximated" solution to the true optimal stocking policy.

It is clear that x^* is decreasing in Δ; the price break induces the OEM to lower the inventory level due to the lower stockout cost. Interestingly, the increased carrying cost does not have any effect in the optimal ending inventory; it gets balanced out by the effect of the decreased stockout cost, thus resulting in the same denominator in z_4^* as that in z_3^* . Although the price difference Δ ensures $z_3^* > z_4^*$, x^* can be greater than, equal to, or smaller then x_1^*, depending on the value of μ_L and σ_L. As the mean and variance increase, x^* will increase to cope with the delivery uncertainty. Finally, it is easy to see that if one considers the lead-time uncertainty for the entire next period, i.e., model I, x_1^* will be expressed as $y(p_1) + \mu_L + z_3^* \sigma_L$. In this case $x^* < x_1^*$ will always be true.

So far we have developed the OEM's price break model by assuming that the shipment of the new component at the new (lower) price can arrive any time after the price break. However, in realty this tends not to be the case. It is common in practice for the freight companies to co-locate with the OEM (for example, in an industrial park). The components shipped from the supplier first arrive at the freight company's local depot before they are delivered to the OEM's facility. The carrier and the OEM may have different operating hours on each working day. Deliveries are not possible during the carrier's non-delivery (ND) hours, nor are they possible during the OEM's non-working (NW) hours, assuming receiving is not possible during the NW hours. As a result, the mismatch of the OEM's and carrier's schedules is worth studying as it affects the size of total demand over time, thus affecting the optimal ending inventory level.

Without loss of generality, and to simplify the presentation, we assume that the OEM is near the freight company's <u>local</u> depot such that the transportation lead-time between these two facilities is short enough to be ignored. Under this assumption, the component's arrival time at the OEM is the same as the freight company's time of delivery from its local depot. This assumption is not restrictive and can be easily relaxed by adding a constant local transportation lead-time into the model.

In the next sections we analyze two mismatch cases: NW>ND and ND>NW. The model we have developed in section 4.5.2 is considered a "full-time" model in which both carrier and OEM operate 24 hours a day.

4.5.2.1 ND>NW

In the non-24-hour cases, the random variable L is defined only on the carrier's operating hours; no delivery is available during the ND hours. On the other hand, demand $D(p_1)$ exists only during the working hours. In other words, we assume that there will be no demand when the manufacturer is not operating. In addition, we assume that ND and NW are fixed on each working day and that both the carrier and OEM have the same starting time on each working day.

Let L_0=ND-NW. Demand associated with L and L_0 are labeled D_L and D_{L0}, and each has mean and variance of (μ_L, σ_L^2) and $(\mu_{L0}, \sigma_{L0}^2)$, respectively. Notice that D_L and D_{L0} are no longer unit-time demand; they are total demand for L and L_0. Additionally, D_L and D_{L0} are clearly independent.

Corollary 4.6 $\mu_L = \lambda(p_1)E(L), \sigma_L^2 = \lambda(p_1)^2 Var(L) + \sigma_\varepsilon^2 E(L)$, $\mu_{L0} = \lambda(p_1)L_0$, and $\sigma_{L0}^2 = L_0\sigma_\varepsilon^2$. (Proof skipped)

The total demand, D, prior to the arrival of the new shipment of new components will be the sum of D_L and D_{L0}. Letting the time of arrival fall on the *i-th* day after the price-break's effective date, the demand by the arrival time can be expressed as $D = D_L + (i-1)D_{L0}$. As such, the mean and variance of D, (μ_D, σ_D^2) can be derived as follows:

$$\mu_D = \mu_L + (i-1)\mu_{L0} = \lambda(p_1)[E(L) + (i-1)L_0]$$

$$\sigma_D^2 = \sigma_L^2 + (i-1)^2\sigma_{L0}^2 = \lambda(p_1)^2 Var(L) + \sigma_d^2 E(L) + (i-1)^2 L_0 \sigma_d^2$$

$$= \lambda(p_1)^2 Var(L) + \sigma_d^2 [E(L) + (i-1)^2 L_0] \qquad (4.9)$$

Now we have the mean and variance of the demand calculated, we can then approximate the solution by using the normalization technique. Similar to solutions in corollary 4.5, we have the following:

Corollary 4.7 $x^* = y(p_1) + \mu_D + z_4^* \sigma_D$, where $z_4^* = \Phi^{-1}(\frac{s_m - \Delta}{h_m + s_m})$

To facilitate the task of solving the optimal ending inventory when ND>NW, we develop an heuristic algorithm to find the optimal solution x^*. See the Appendix of chapter 4 for the details.

4.5.2.2 NW>ND

Let $\bar{L}_0$=NW-ND. During $\bar{L}_0$, delivery of materials is possible but manufacturer has no demand. However, random variable L needs to be considered in the non-ND hours, the NW does not prevent the material's arrival at the carrier's local depot. To avoid changing the definition of the random variable L, we assume that there is there is pseudo demand

$D(p_1)$ per unit time during $\bar{L}_0$. Total pseudo demand during $\bar{L}_0$ is labeled as $D_{\bar{L}_0}$.
Assuming the component's arrival time falls on the *i-th* day after the price-break's effective date, the total demand prior to the arrival of the new shipment of materials with lower price can be expressed as $D = D_L - \sum_0^{i-1} D_{\bar{L}0i}$. Note that if the arrival time falls on the *i-th* $\bar{L}_0$, it will automatically be considered as arriving on the *(i+1)-th* day, which is the earliest possible day for the OEM to receive the materials. D_L has $\mu_L = \lambda(p_1)E(L)$ and $\sigma_L^2 = \lambda(p_1)^2 Var(L) + \sigma_\varepsilon^2 E(L)$. For each $D_{\bar{L}0i}$, $\mu_{\bar{L}0i} = \mu_{\bar{L}0} = \lambda(p_1)\bar{L}_0$ and $\sigma_{\bar{L}0i}^2 = \sigma_{\bar{L}0}^2 = \bar{L}_0\sigma_\varepsilon^2$. So based on the newsvendor problem framework, we have

$$P(D_L - \sum_0^{i-1} D_{\bar{L}0i} \le x^*) = \frac{s_m - \Delta}{h_m + s_m} \tag{4.10}$$

We express (4.10) differently from all previous models in this section due to the fact that $D_{\bar{L}_0}$ is part of D_L and they should NOT be independent of each other. The expressions of μ_D and σ_D^2 can be obtained, but they are somewhat complex and the derivation is tedious. As a result, we omit the expression of μ_D and σ_D^2 and present only the heuristic algorithm for solving x^* in (4.10), the details of which can be found in the appendix of this chapter.

In summary, we have developed the OEM's model in response to the price break, considering the lead-time demand as well as the mismatch of schedules. The delivery uncertainty in our OEM's model has two facets: the variation of arrival time and the match/mismatch of the operating schedules of the carrier and the OEM. Price difference would lead the OEM to lower the ending inventory, however, the delivery uncertainty would bring the inventory up. Our model can assist the OEM to understand the impact of these two forces on the optimal ending inventory decision more thoroughly and thus identify the optimal ending policy when there is a price break announced. In the next section we present a numerical example to demonstrate the usefulness of our models and discuss the managerial implications.

4.6. Numerical Experiments

We present the numerical experiments and results of for the buyer, supplier and the OEM in the supplier chain. For the convenience of analysis, we consider a base dataset for the numerical experiments as follows: a=2000, b=2, w_m=30, c_k=5, c_m=15, h=10, h_m=4, s=15, s_m=5, s_B=5, μ_ε=20, σ_ε=10, unless otherwise specified.

Our analysis mainly focuses on the impact of price sensitivity, supplier's production cost, characteristics of the random demand error on the optimal retail and component prices, and capacity planning and stocking policies. Additionally, we investigate how the magnitude of price difference, combined with the consideration of the lead-time uncertainty, affects the OEM's optimal ending inventory decisions.

4.6.1 On the price sensitivity

Demand is a key factor that affects inventory levels and capacity planning. Our research considers price-sensitivity, stochastic demand. As such, in the first experiment, we vary the magnitude of price sensitivity in the demand function to see how it affects the pricing and inventory/capacity decisions. Figure 4.1 presents the resulton the retail and component pricing decisions. We can observe that both the optimal retail and transaction prices decrease more rapidly when price sensitivity is low. Nonetheless, both prices continue to decrease as price sensitivity increases. When price sensitivity is high, the price needs to be lowered, as price affects the demand to a greater extent than it does when price sensitivity is low. Additionally, we incorporate two separate experiments in which supplier's production cost is set at difference values. The results confirm that both retail and component prices need to be higher when production cost is higher.

Figure 4.2 shows how the price sensitivity and supplier's production affect the optimal capacity planning and stocking policies for the supplier, the buyer, and the OEM. All of the three policies have a downward slope with respect to the price sensitivity, with each varying in the same direction as the supplier's production cost. Notice that in each level of the production cost, the supplier has the highest capacity, followed by the OEM and the buyer in this specific experimental setting. If we decrease the supplier's stockoutcost, the optimal capacity will decrease accordingly. On the other hand, one can observe that the set of lines at high production cost (c_k =100) has a steeper slope. This suggests that higher production cost accelerates the effects of higher price sensitivity on lowering the optimal capacity and stocking policies in the supply chain. Notice that higher production cost results in higher retail price and component price as shown in Figure 4.1.

The corresponding profits for each party in this supply chain are presented in Figure 4.3. In our experiment, the supplier earns the highest profit, followed by the buyer. The OEM only earns the production service fee, thus has more stable profits across the selected range of price sensitivity. As the price sensitivity increases, the supplier's and buyer's profits go down, and the difference of all the profits decreases as the same time.

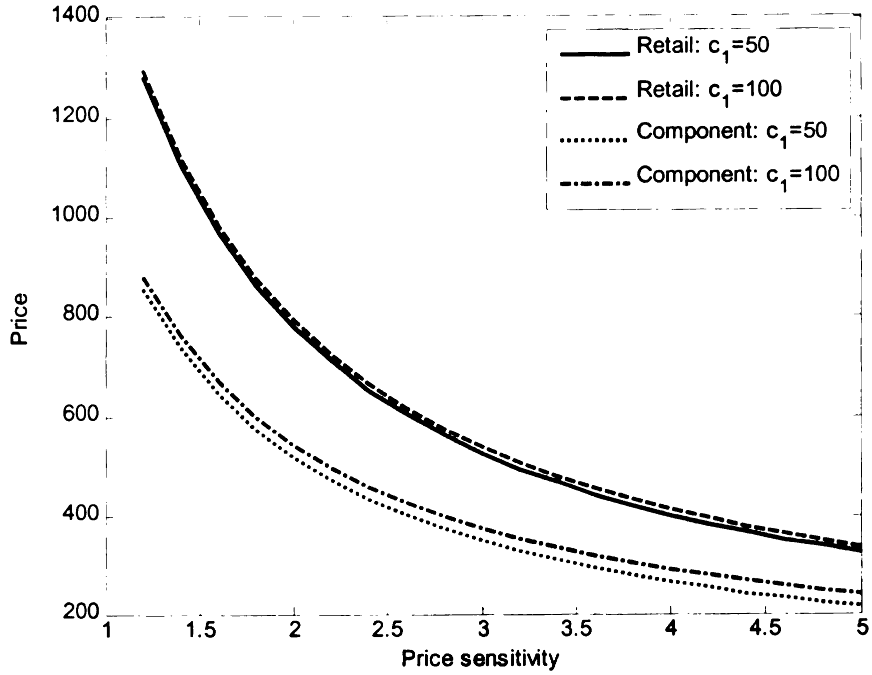

Figure 4.1 Optimal pricing: varying price sensitivity

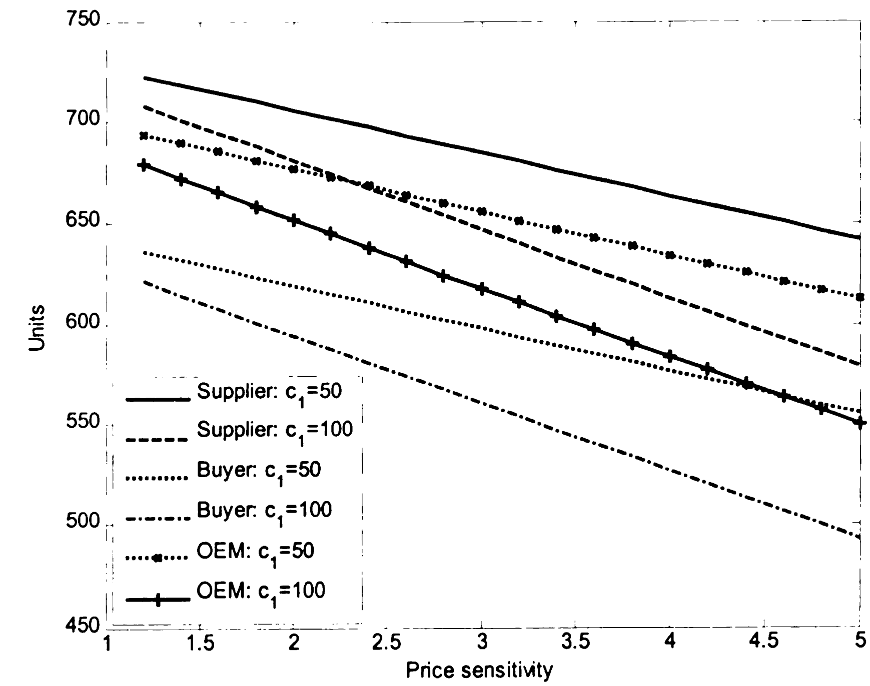

Figure 4.2 Optimal capacity and stocking policies: varying price sensitivity

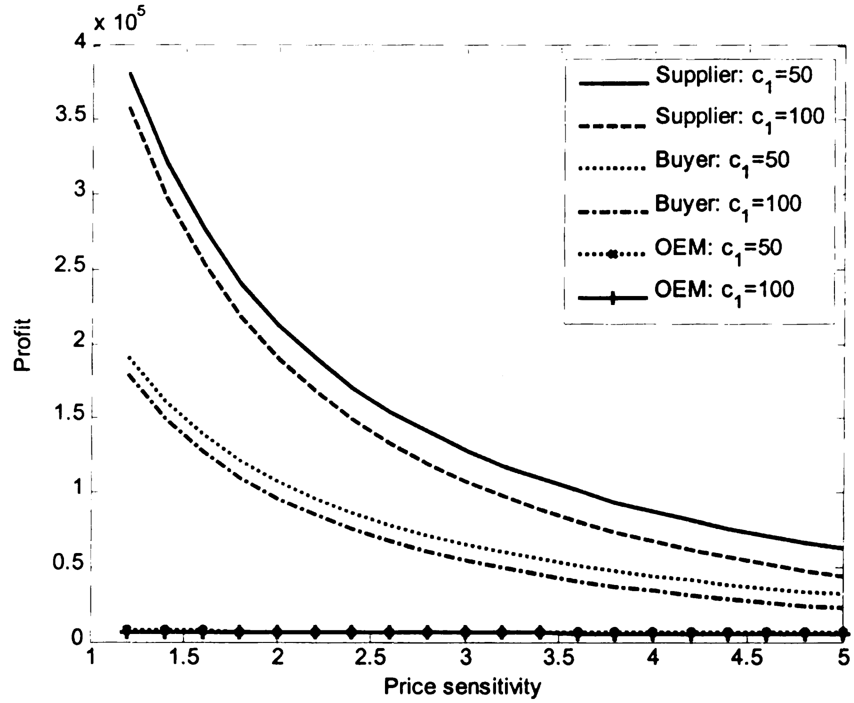

Figure 4.3 Optimal Profits: varying price sensitivity

4.6.2 On the characteristics of the random demand error

In this experiment, we examine the effects of the stochastic element of the demand function on the optimal pricing and stocking/capacity policies. We fix the price sensitivity b at 2 , and the supplier's production cost c_k at 100. The solution format of the optimal capacity and stocking policies lead us to expect a positive linear relationship. Interestingly, we found the buyer's stocking decision does not follow that pattern. A further investigation reveals the root cause is when h=10 and s_B=5, the z_1^* value in $I_1^* = y(p_1) + \mu_\varepsilon + z_1^*\sigma_\varepsilon$ is actually negative. So when $y(p_1) + \mu_\varepsilon$ is not large, I_1^* may decrease as σ_ε increases, which was somewhat unexpected before conducting this experiment. Overall, the results resemble that in Figure 4.2 in that the supplier's capacity is largest when the buyer's stocking level is the lowest.

Figure 4.5 indicates that both the retail and component pricing decisions are not affected by the size of the standard deviation of the demand error, so is not the profit level. Although the random demand error is included in all the profit functions, it gets eliminated during the normalization process for the buyer and the OEM, thus disappearing from the final expressions of estimated profits. However, it is not the case for the supplier, due to the lack of the expected stockout cost in the supplier's profit function. So one can observe that in Figure 4.5, the buyer's and the OEM's profits do not vary with σ_ε while the supplier's profit functions decrease as σ_ε increases.

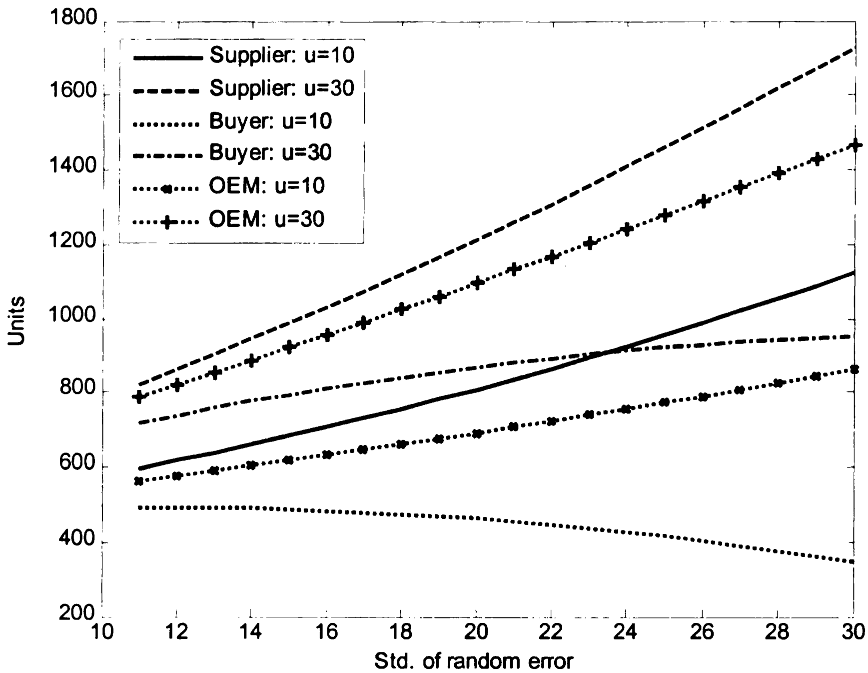

Figure 4.4 Optimal capacity/stocking policies: varying σ_ε at two levels of μ_ε

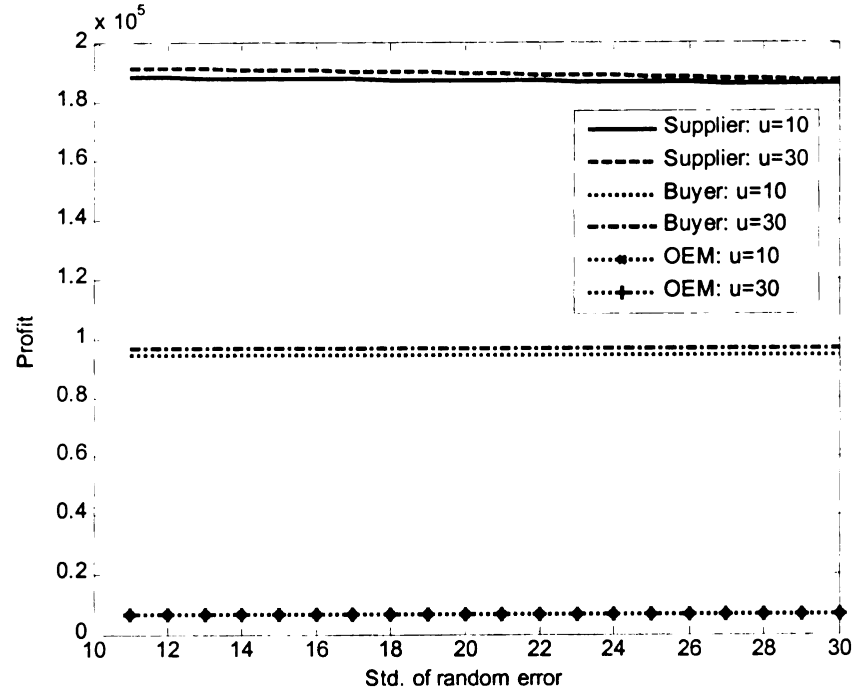

Figure 4.5 Optimal profits: varying σ_ε at two levels of μ_ε

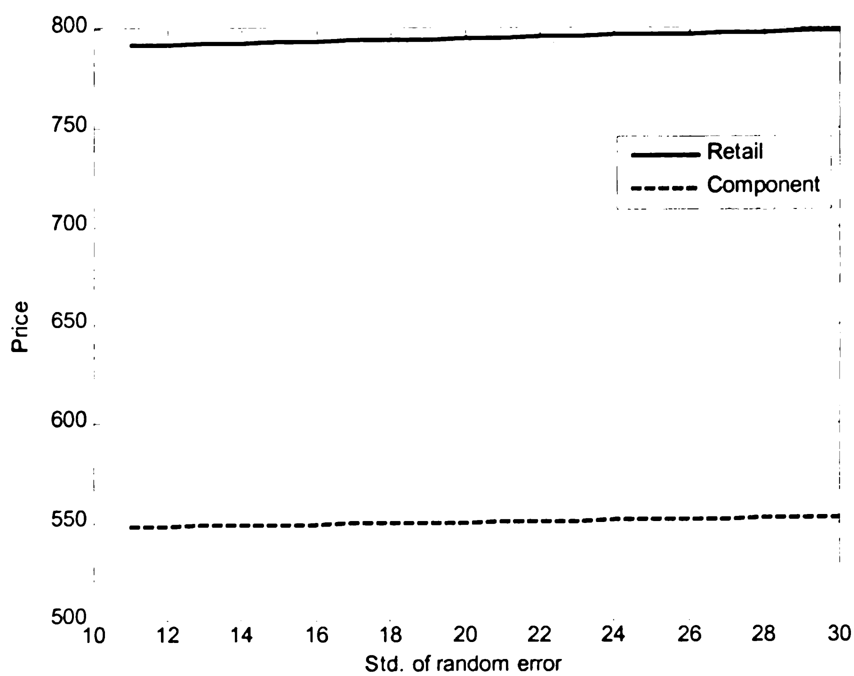

Figure 4.6 Optimal pricing: varying σ_ε

On the other hand, μ_ε does affect both pricing decisions as it is included in $\lambda(p_1)$ that appears in both optimal pricing schemes. Figure 4.6 presents the relationship between the optimal pricing decisions and the mean of the random demand error (μ_ε). One can observe that the effect of μ_ε on the optimal prices is relatively minor compared with that from price sensitivity, as indicated in Figure 4.1.

4.6.3 On the OEM's ending inventory during price breaks

In this experiment, we examine the effects of the price discount on the ending inventory decisions prior to a price break. We introduce the lead-time uncertainty into the experiment and set the mean of the arrival time distribution $E(L)$=0.5 and the variance $Var(L)$=0.5. If the "unit time" demand function represents daily demand function, arrival time of the component from the supplier at the OEM site is in average 0.5 days after the price break, with a standard deviation of 0.25. All the other parameters follow the same value utilized in the previous experiments.

Figure 4.7 shows the effect of the price difference on the ending inventory decisions. As one can see, the ending inventory decreases as the price difference increases. This is because the understocking cost decreases and the holding cost increases tremendously when price difference is considered as a portion of "benefit" for not holding inventory. From the solution of the optimal ending inventory shown in corollary 4.5, we can see clearly that when the price difference approaches towards the original understocking cost (s_m=15), the z_4^* value in corollary 4.5 will be approaching negative infinity so that the optimal ending inventory x^* becomes negative, in which case we set x^*=0. So in our experiments, one can observe that as the price difference nears 15, the ending inventory solution moves towards zero. In this case, the price difference is so significant that it balancesoff the impact of the original understocking cost. Thus, the OEM should not keep any inventory right before the price break.

In Figure 4.7 we also compare the ending inventory decisions between the two cases when we consider stochastic lead-time demand (LTD) and when we consider only stochastic demand. We can observe that when lead-time uncertainty is also considered, the inventory level will be much higher than in the regular demand cases. This is due to the fact that incorporating the lead-time uncertainty results in a very large LTD variance σ_L^2, which directly leads to a high optimal ending inventory value x^*. However, when we only consider stochastic demand, we can clearly see that price markdown results in a lower-than-regular inventory level. As the price difference reaches 15, the OEM should decide not to keep any inventory.

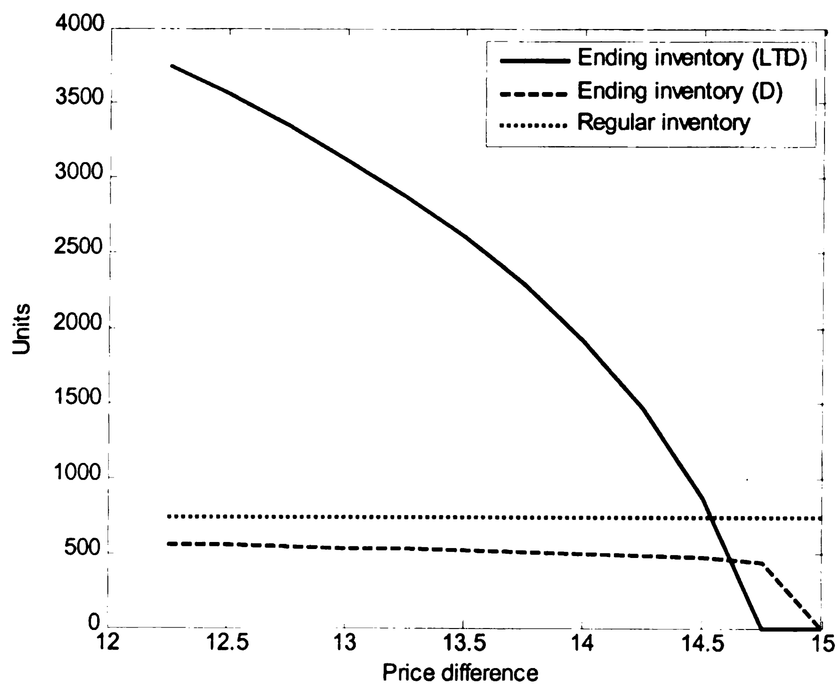

Figure 4.7 Optimal stocking policies *LTD* vs. *D*: varying price difference

Finally, Figure 4.8 presents the expected profits at price break. Recall that the ending inventory decision is to cope with the price difference as well as the arrival uncertainly of the very first shipment of new components after price break becomes effective. Our results suggest that, if without considering the lead time uncertainty, lowering inventory as indicated in Figure 4.7 will be able to achieve higher profit for the OEM. The difference of the expected profits increases as the price discount increases. However, the potential profit is relatively insignificant compared with the expected profit for OEM if lead-time uncertainty is known to exist. When the price difference is small, the saving per unit is not worth therisk and cost of running out of inventory due to the variation of new material's arrival time. As such, experiment results convey one signal to the OEM: if delivery uncertainty is a concern and the price break does not result in a significantamount of price difference, the OEM will be better-off simply having higher ending inventory right before the price break. But if the material's arrival time is very stable and predictable, the OEM may want to consider to taking the lower inventory approach.

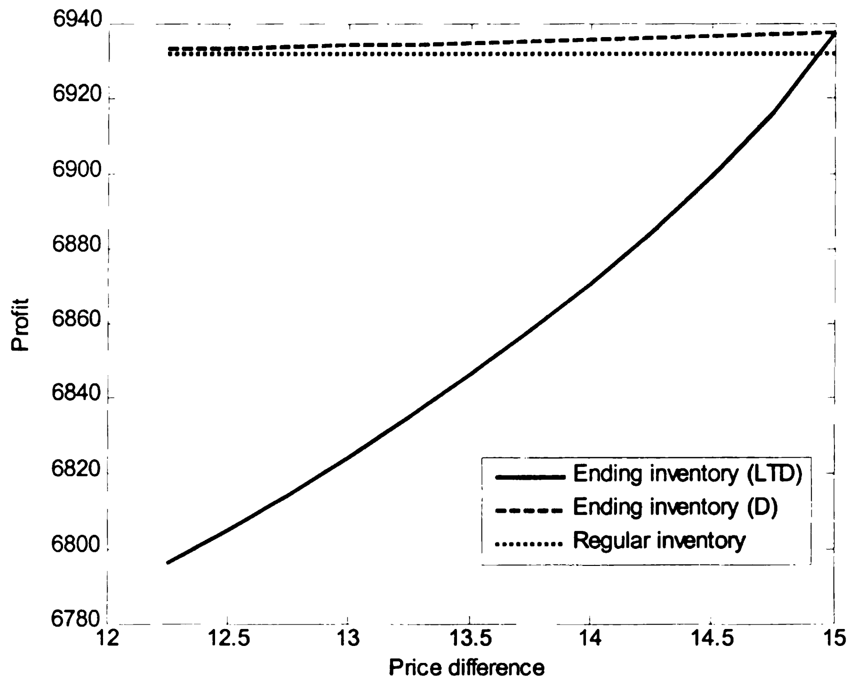

Figure 4.8 OEM profits *LTD* v.s. *D*: varying price difference

4.6.4 Discussion

The treatment of incorporating both lead-time and demand uncertainties seems to create a very large gap up from not considering the lead-time uncertainty. Combing both uncertainties make the inventory decision very sensitive to the price difference. As one can observe from Figure 4.7, the curve of the LTD ending inventory has a much steeper slope than the other ending inventory curve that considers only demand uncertainty. Intuitively, one would try to reduce the inventory level when there is a price break to come. We expect that lead time variation will recommend the OEM to increase stocking level. However, the experiment result indicates that the resulting optimal solutions could be actually several times higher if the price difference is small. Of course, factors such as the choice of demand and lead-time distributions matter. However, our results cast doubts onwhether the treatment of lead-time demand is truly capable of describing the combined effect of both uncertainties. As such, we recommend that firms should use the lead-time demand to make inventory decisions with discretion. A comparison between using lead-time demand and using only the stochastic demand is a must.

Note that the computational experiments are based on the selected demand functions as well as on random error distribution. Despite the advantage of easy implementation and analyses, the linear demand function may not be ideal for capturing the true demand pattern. An example is that the optimal retail pricing and the component pricing which seem to be set much higher than the corresponding production cost and OEM's service charge in our experiments. Additionally, the selection of the parameters of lead-time uncertainty also results in higher-than-expected ending inventory decisions as mentioned. The demand function and the lead time distribution are key to acquiring these optimal decisions. The purpose of these experiments is to demonstrate the effectiveness of the model's capabilities in facilitating managers' decisions. We nonetheless note the importance of being able to identify the demand and lead-time random patterns so that one can take full advantage of the development of this research.

4.7 Concluding Remarks and Discussion

We investigate a problem involving price discount and corresponding inventory decisions in a decentralized supply chain that consists of a supplier, an OEM and a buyer (retailer). The supplier offers a new, lower component price to the buyer, due to innovation of manufacturing technologies that allow lower production cost. Such a price markdown will lead buyer to reduce his retail price of the final product. On the other hand, demand is price-sensitive; the reduced price will induce higher demand, thus affecting all agents' capacity planning and stocking policies in this supply chain. We develop a supply chain model that can identify the optimal pricing scheme for both the component and the retail prices, as well as the optimal capacity reservation and stocking level for the supplier, buyer and the OEM.

Our models allow the entire supply chain to make decisions on capacity planning, component pricing, stocking policies, and the retail pricing. We show how these decisions are affected by the demand patterns such as price sensitivity and parameters of random demand error. We also found that increasing the standard deviation of the random demand error does not necessarily lead to increase of stocking level as one would predict. Furthermore, we develop models and solution algorithms specifically to facility the OEM's ending inventory decision making right before the price break. We found that when the new price is not much lower than the old price and if the material's arrival time is known to be varying, the OEM should keep higher inventory than usual to cope with the lead-time uncertainty. But when the price difference is large and more importantly, the material's arrival is highly predictable, the OEM will benefit from lowering the inventory, thus enjoy the benefit of profit gain by cost saving.

The main contribution is that we develop a full supply chain model in the presence of price discount. Past supply chain contract research primarily focused on a two-echelon supply chain. Our research incorporates the OEM's problem and develops a three-echelon supply chain model. Furthermore, we specifically develop models to describe the effects of the price discount on the inventory decisions beyond the price break point for the OEM. We incorporate the demand uncertainty and consider the mismatch of schedules for the OEM to more accurately predict such impact. We demonstrate that lead-time uncertainty can substantially affect the ending inventory decisions via numerical examples.

An immediate extension is to incorporate quantity-sensitive discount scheme into this model. In this study, we only consider the price markdown scheme and the price-sensitivity demand. Inclusion of quantity-dependent discount contracts can possibly bring more fruitful insights to supply chain managers and facilitate their decision making. Furthermore, there is need to consider also the "time" factor as part of the demand pattern. Demand will be simulated by lower price. However, after a period of time, demand may decrease even when price does not change. A good example is from the PC market or cell phone industry. When a product is launched at a lower price, the demand is normally strong. Once end-users expect newer, better product to be available in the future, demand for the same product starts declining. Thus, a demand function that can capture both the price and time elements will be most suitable for the problems studied in our models. Furthermore, the decisions in capacity planning and inventory stocking policies, though all linked to the same demand function, are decided individually. When gaps in these decisions exist, it's easy to see that the agent that has the lowest capacity or inventory level will become a bottleneck for other agents should demand be higher than its capacity or stocking level. As such, an "integrated" decision making in capacity planning and stocking policies should be desirable. Finally, we omit to consider the fact that the discount timing may not be aligned across the entire supply chain. For example, the supplier offers a price markdown, however, the buyer does not take any action until two weeks later. The model will be more complete if the synchronization of the discount schedules at each echelon of the supply chain is considered. Future research needs to investigate these issues.

Appendix of Chapter 4

Algorithms

(1) ND>NW

Step 1. Let i=1, &

$$x^* = \mu_D + z_4^* \sigma_D = \lambda(p_1)E(L) + \Phi^{-1}(\frac{s_m - \Delta}{h_m + s_m})[\lambda(p_1)^2 Var(L) + \sigma_d^2 E(L)].$$

Calculate the mean and variance of lead-time demand when L=24-ND:

$$\mu_{D1} = \lambda(p_1)E(L) = \lambda(p_1)(24 - ND)$$

$$\sigma_{D1}^2 = \sigma_L^2 = \lambda(p_1)^2 Var(L) + \sigma_d^2 E(L) = \sigma_d^2 (24 - ND)$$

Check if $x^* \le \mu_{D1} + z_4^* \sigma_{D1}$?

(a) If true, x^* is the optimal solution. Go to Step 3.

(b) Otherwise, proceed toStep 2, because it contradicts the assumption that shipment falls in day 1.

Step 2. Let i=i+1, then

$$x^* = \mu_D + z_4^* \sigma_D = \lambda(p_1)[E(L) + (i-1)L_0]$$

$$+ \Phi^{-1}(\frac{s_m - \Delta}{h_m + s_m})\sqrt{\lambda(p_1)^2 Var(L) + \sigma_d^2 [E(L) + (i-1)^2 L_0]}$$

Calculate the mean and variance of lead-time demand when $L = i$(24-ND):

$$\mu_{Di} = \mu_L + (i-1)\mu_{L0} = \lambda(p_1)[E(L) + (i-1)L_0]$$

$$= \lambda(p_1)[i(24 - ND) + (i-1)L_0]$$

$$\sigma_{Di}^2 = \sigma_L^2 + (i-1)^2 \sigma_{L0}^2 = \lambda(p_1)^2 Var(L) + \sigma_d^2 E(L) + (i-1)^2 L_0 \sigma_d^2$$

$$= \sigma_d^2 [i(24 - ND) + (i-1)^2 L_0]$$

Check if $x^* \le \mu_{Di} + z_4^* \sigma_{Di}$?

(a) If true, x^* is the optimal solution. Go to Step 3.

(b) Otherwise, goto Step 2.

Step 3. Stop.

(2) ND<NW

Step 1. Let i=1 &

$$x_L = \mu_D + z_4^* \sigma_D = \lambda(p_1)E(L) + \Phi^{-1}(\frac{s_m - \Delta}{h_m + s_m})[\lambda(p_1)^2 Var(L) + \sigma_\varepsilon^2 E(L)]$$

Calculate the mean and variance of lead-time demand when L'=24-NW:

$$\mu_{D1} = \lambda(p_1)E(L') = \lambda(p_1)(24 - NW)$$

$$\sigma_{D1}^2 = \lambda(p_1)^2 Var(L') + \sigma_\varepsilon^2 E(L') = \sigma_\varepsilon^2 (24 - NW)$$

Check if $x_L \le \mu_{D1} + z_4^* \sigma_{D1}$?

(a) If true, $x^* = x_L$ is the optimal solution. Go to Step 3.

(b) Otherwise, proceed toStep 2, because it contradicts the assumption that shipment falls in day 1.

Step 2. Let i=i+1,

Calculate the mean and variance of lead-time demand when $L' = 24i - NW$

$$\mu_{Di} = \lambda(p_1)E(L') = \lambda(p_1)(24i - NW)$$

$$\sigma_{Di}^2 = \lambda(p_1)^2 Var(L') + \sigma_\varepsilon^2 E(L') = \sigma_\varepsilon^2 (24i - NW)$$

Check if $x_L \le \mu_{Di} + z_4^* \sigma_{Di}$?

(a) If true, $x^* = x_L - (i-1)\mu_{\bar{L}0}$ is the optimal solution. Go to Step 3.

(b) Otherwise, go to Step 2.

Step 3. Stop.

CHAPTER 5 CONCLUDING REMARKS

This dissertation studied the quantity flexibility contract and price markdown scheme in three types of supply chains. Chapter 2 designed a new quantity flexibility contract with price discount incentives, from the supplier's perspective, in a one-buyer-one-supplier supply chain. Chapter 3 investigated the competition between the quantity flexibility contract and the price discount scheme from the buyer's perspective in a two-competing-supplier, one-buyer supply chain. Chapter 4 extended the price markdown scheme to a three-echelon supply chain and analyzed pricing decisions and inventory policies for each agent in the supply chain. We summarize the contributions of each of these chapters and revisit directions for future research.

5.1 Summary of Contributions

5.1.1 Chapter 2

In this research we found that QF*i* contract combined with the QF contract can create the most benefit for this 1-1 decentralized supply chain. We showed that QF*i* creates Pareto improvement from the QF contract under certain conditions. When executed, the QF*i* contract enables both agents to enhance profits from where they are at under the QF contract.

Our results indicated that QF*i* is most appropriate when the discount is only slightly off from the QF contractual price. As such, we suggest that the supplier should not "overdo" the discount. The QF*i* contract enables agents in a supply chain to estimate the "worth" of flexibility offered. We presented conditions where the QF*i* contract outperforms the traditional QF contract and where the QF contract has an advantage.

We showed that the QFi contract is able to achieve supply chain coordination. The coordination pricing schemes do not require the knowledge of demand distribution as the QF contract does. The coordination can be achieved, without loss of generality, at the very component price that is agreed upon by the two parties under the QF contract. As such, achieving 100% supply chain efficiency needs not to radically change the QF contract price. The QFi contract can work equally well as the QF contract from the system point of view. It provides an option for the supply chain to re-distribution the inventory risk burden.

5.1.2 Chapter 3

This research considered competing suppliers in the presence of the quantity flexibility contract and the price discount scheme. We specified the optimal decisions for the buyer in allocating the forecast and purchase orders to the two suppliers. The development of this research contributed to the literature by adding knowledge to supply chain contract problems involved with two competing sources of supply.

Our analysis suggested that the 1-1 QF supply chain is favored over the 2-1 QF supply chain from the system's perspective. We found that in general, the system profit in 1-1 QF supply chain always outperforms a 2-1 QF supply chain. Although having an alternative source of supply is always beneficial to the buyer, our finding suggested that supply chain deficiency occurs in the presence of multiple suppliers

The analysis identified areas where flexibility will be favored and areas where the discount price is winning the competition. Our results provided insights to suppliers in that it enabled them to better initiate business strategies and production plans to cope with competition. Interestingly, we found that the buyer should simply use either supplier in most cases. Areas where the buyer utilizing both suppliers simultaneously creating higher profits are somewhat limited when compared to simply using a single supplier.

5.1.3 Chapter 4

We developed a complete supply chain model in the presence of price discount in this research. We developed an approach to specify the optimal pricing decisions, capability planning and inventory stocking policies affected by the new, lower pricing. Specifically, we analyzed the effects of the price discount and the delivery uncertainties on the inventory decision beyond the price break point for the OEM. We found that lead-time uncertainty can change the inventory decisions tremendously if it is considered.

We found that the effects of higher price sensitivity on lowering the optimal capacity and stocking policies will be fueled by higher production cost. Our results also suggest that the impact of random error on inventory decisions is not as significant as one would expect. Numerical experiments lead us to conclude that unless the price difference is large, the benefit of lowering ending inventory to cope with price discount effects is piecemeal.

The main contribution is that we develop a full supply chain model in the presence of price discount. Past supply chain contract research primarily focuses a two-echelon supply chain

5.2 Future Research Directions

The three essays in this dissertation all raise interesting questions that provides future research directions. First, in essay 1, the QFi contract allows the buyer to choose freely the desirable combination of forecast under the QF mechanism and firm order at the discount price. However, if the price difference is large enough, the buyer may only execute the discount scheme of the QF*i* contract. An extension that is worth pursing may involve imposing a minimum required quantity on the QF mechanism before the buyer can consider firm order at the discount price. Another potential topic is to incorporatea price-sensitive demand function and examine the QF*i* contract performance when demand varies with price. Finally, it will be interesting to examine how well the QFi contract performs in more complex supply chains or in a multi-period setting.

Second, essay 2 looks at the competition between flexibility and price. Other factors, such as quality performance, logistics capability, cost sharing, promotional efforts, or service agreements to name a few, can also be incorporated into a contractual setting. Future research can borrow our developments in essay 2 and study the tradeoffs between these competing forces. We considered two competing suppliers in essay 2, one can expand the problem to consider more buyers and/or more competing suppliers, thus allowing the analysis and results to more closely resemble the real-world setting and provide fruitful managerial insights. Moreover, further investigation of the competition between flexibility and price discount in the long run is also warranted.

Finally, in essay 3, we considered a price markdown scheme. An immediate extension is to incorporate a more complex quantity discount scheme into the model and examine how a different price discount scheme can affect the supply chain decisions. On the other hand, in our model the supplier takes the leading role. It is possible that the buyer is the one who retains all the power. Future research can look at this problem and analyze how firms will behave differently when the buyer takes charge of the pricing negotiation. Furthermore, an integrated capacity planning and stocking policy is more desirable from the system point of view. This can provide a benchmark for evaluating the performance of the decentralized supply chain. Finally, when the discount timing of the supplier and buyer is not synchronized, price discount may affect inventory stocking policies in a different manner. Further investigation that considers the desynchronizing discount schedules along a supply chain is worth pursuing.

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