This is to certify that the
dissertation entitled

AN INVESTIGATION INTO THE PROFILE OF GHANAIAN HIGH SCHOOL MATHEMATICS TEACHERS' KNOWLEDGE FOR TEACHING ALGEBRA AND ITS RELATIONSHIP WITH STUDENT PERFORMANCE

presented by

ERIC MAGNUS WILMOT

has been accepted towards fulfillment
of the requirements for the

Ph.D. degree in Department of Teacher Education

Joan Ferrini-Mundy
Major Professor's Signature

5/3/08
Date

AN INVESTIGATION INTO THE PROFILE OF GHANAIAN HIGH SCHOOL MATHEMATICS TEACHERS' KNOWLEDGE FOR TEACHING ALGEBRA AND ITS RELATIONSHIP WITH STUDENT PERFORMANCE

By

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A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Teacher Education

2008

ABSTRACT

AN INVESTIGATION INTO THE PROFILE OF GHANAIAN HIGH SCHOOL MATHEMATICS TEACHERS' KNOWLEDGE FOR TEACHING ALGEBRA AND ITS RELATIONSHIP WITH STUDENT PERFORMANCE

By

Eric Magnus Wilmot

Available literature on teaching and student performance is replete with evidence that the teacher is one of the most important factors that influence student performance, especially in the developing world. In spite of this, there is widespread disagreement among researchers about which aspect of teachers' subject matter knowledge best relates to student performance. Several studies that have attempted to establish this link have relied on proxy measures of teacher knowledge (e.g., the number of university courses taken). In addition, various conceptualizations of teacher knowledge have presented it as a domain neutral domain. Consequently, there is the need for re-conceptualization of teacher knowledge in ways that is both domain specific and lends itself to some form of direct measurement instead of by proxy. One of the ground-breaking works in this direction is currently being done by researchers in the Knowledge of Algebra for Teaching (KAT) project at Michigan State University.

Through analyses of research literature and videos of teaching, researchers in the KAT project have hypothesized that there are three types of knowledge for teaching high school algebra: knowledge of school algebra, advanced knowledge of mathematics, and teaching knowledge. In addition, the

KAT project is currently developing and validating instruments to measure their conceptualization. This study used the KAT project's conceptualization as the framework. Rather than rely on proxy measures of teacher knowledge, the study also adapted the instruments developed by the KAT project.

This study investigated whether the KAT conceptualizations could be corroborated in Ghana. It also examined the differences in the knowledge for teaching algebra among in-service and prospective high school mathematics teachers in Ghana. In addition, this study investigated how Ghanaian high school teachers' knowledge for teaching algebra is related to the performance of their students.

In all, 1565 high school elective mathematics seniors from eight public schools in Ghana, as well as 38 mathematics teachers from these schools participated in the study. In addition, 301 university seniors comprising 132 mathematics, 44 statistics and 125 mathematics education majors participated in the study. Factor Analysis of data from this study did not corroborate the three hypothesized types of knowledge. In addition, Analysis of Variance (ANOVA) conducted on the data revealed that in-service teachers performed best on both instruments used in the study. Finally, Linear Regression performed revealed that student performance is positively related to teachers' advance knowledge. However, the relationship was found not to be significant. In the light of the findings, recommendations for further research have been made.

DEDICATION

This dissertation is dedicated to my loving spouse, Vida, and my wonderful children, Dennis, Vanessa, and Shirley-Anne for their sacrifice, support and prayers. It is also dedicated to my late mother whose sweat and inspiration laid the foundation for every aspect of my life. Mother, I believe it is not a goodbye but a goodnight because we shall meet in the morning.

ACKNOWLEDGEMENTS

I would like to thank family members and relatives whose support has made this happen. My loving wife and children provided a serene atmosphere that made it possible for me to do most of my readings and writings at home while enjoying their company. I cannot thank Vida, my loving wife enough for extensively proof-reading my writings throughout the program. I am grateful too for the support and advice of my elder brother Sam and his wife Krista.

This work would not have been possible without the financial support of a number of people and institutions. Though not a direct financier, I am grateful to the National Science Foundation for funding the Knowing Mathematics for Teaching Algebra (KAT) Project (NSF REC-0337595), at the Division of Science and Mathematics Education, Michigan State University. The KAT project provided the needed framework for this dissertation. Thus, this material is based upon work supported, in part, by the National Science Foundation, award number REC-0337595. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the National Science Foundation. I am grateful to the Principal Investigators on this project for allowing me to use their instruments and other project materials for this dissertation at such an early stage of the project.

I am also grateful to the University of Cape Coast, Ghana, for sponsoring aspects of my fieldwork and especially to the Department of Science and Mathematics Education for providing free accommodations during the fieldwork.

I am particularly thankful to the Graduate School and the Teacher Education Department at Michigan State University, as well as Professors Jack Schwille and Joan Ferrini-Mundy for the financial support for my data collection data.

I am especially indebted to Prof. Jack Schwille for working with me as my advisor throughout the time it took me to complete this doctoral course of study. He was very supportive of my career goals and he worked actively to provide me with the professional guidance to pursue those goals. I am grateful for his time and efforts.

My dissertation directors, Professors Joan Ferrini-Mundy and Sharon Senk, and members of my Dissertation Committee, Professors Jack Schwille, Mark Reckase and Natasha Speer, have generously given their time and expertise to improve this dissertation. I am grateful for the extensive personal and professional guidance I received from each member of my Dissertation Committee.

My special appreciation and thanks also goes to the many friends and colleagues who assisted, advised, and supported my research and writing efforts over the years. In particular, I express my gratitude to J. Y. Appiah for the invaluable support he gave me during my fieldwork. I am thankful to J. Y. for visiting schools with me and helping supervise students and teachers as they completed forms for this dissertation. I appreciate, as well, the invaluable family resources he placed at my disposal, including a car that helped me get so much done in a short period. I am also grateful to my friends, Hilda, Irini and Violeta, fellow graduate assistants on the KAT project, who spent considerable time and energy to score responses to the open-ended items and give me the needed second opinions to shape my data. In addition, I express my gratitude to Hilda and another graduate student, Xin, for the introducing me to SPSS. The introductory sessions with them were the key to my ability to run my analyses on time. I also sincerely thank Dr. Steve Sharra, a close friend of mine, who read the very first draft and gave me feedback on my writing. In a special way, I express my gratefulness to Dr. and Mrs. Addai whose friendship and generosity supported my family throughout our stay in Lansing.

The journey to my doctoral degree started with the encouragement and support of two special friends, Dr. Shirley Miske and Anne Schneller. I cannot end without expressing my sincerest gratitude and thanks for their encouragement and support beginning with my application into the doctoral program and through to the end. But for Shirley and Anne I would have pursued this program in a different university. Therefore, I owe the two of them the unique experience and learning I have enjoyed here at Michigan State University. I am particularly grateful to Shirley for regularly coming down from Minnesota to visit and make sure all was well with my family. While in these doctoral studies, opportunities Shirley gave me to work with her on some of her projects were valuable, both financially and for my professional experience. Thanks for being there for my family and me and for helping to get us this far.

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CHAPTER 1: INTRODUCTION AND RATIONALE

This chapter opens with a discussion of the rationale for focusing this study on high school teachers' knowledge for teaching algebra and its relationship to student performance. Thereafter, because the setting of the study is in Ghana, a country different from the United States of America (U.S.), background discussions about school mathematics, as well as about high school mathematics teachers (both in-service and prospective) in Ghana have been presented in turn. After this introductory material, the purpose of the study and the significance of the study are presented. The chapter ends with the research questions that guided the study, followed by an overview of how the entire dissertation is organized.

1.1 Rationale for the Focus of this Study

The momentum for the "Standards Reform" in mathematics education in the US began in the early 1980s as educators started responding to a "back-to-basics" cry, a reaction to the "new mathematics" of the 1960s and 1970s (Van de Walle, 2007). The origin and impetus of this reform can be traced to the low level of students' performance and stratification of mathematics instruction in U.S. schools (see the National Commission on Excellence in Education's "A Nation at Risk" report of 1983; Clune 1998).

According to Usiskin (1995), however, advocates of the Standards Reform did not only aim at improving the quality of teaching and learning in schools, they began challenging the notion that only some could learn algebra. The standards reformers argued that because of the foundation algebra provides for other mathematics courses and its application to other fields, every student in the US has to be given the opportunity to study algebra (Usiskin, 1995). Senk and Thompson (2003) have argued that such "algebra for all" calls were consistent with the call for increased mathematics requirement for all U.S. schools, highlighted in the "A Nation at Risk" report. The argument has been that, without the opportunity to study school algebra, it would be almost impossible to raise the mathematics performance of many high school students in the US. In addition, without studying algebra, some students would be denied access to certain careers.

A look at the National Council of Teachers of Mathematics' (NCTM) *Standards* (NCTM, 2000) indicate that the NCTM with regards to "algebra for all" seems to favor teaching for algebraic thinking in all of school mathematics from the early primary grades. Though specific courses in algebra are not completely discouraged, the emphasis on algebra thinking in all of school mathematics is quite different from specific course(s) in a specific grade that have traditionally been emphasized prior to the standards reform era. This position of the NCTM is seen in its emphasis on the need to help all students develop reasoning and problem-solving skills built upon exploration, modeling, describing/conjecturing, explaining and generalizing, which are basic to the formation of algebraic reasoning (NCTM, 2000). It is this shift from focusing only on single course(s) in algebra to content strands that are strengthened each year by weaving with other domains of school mathematics that seem to be emphasized in the "algebra for all" call.

As Roberts (1991) appropriately puts it, "the student who closes the door on school algebra (and so on all mathematics) closes the door on much more" (Roberts, 1991, p. 15a cited in Chazan, 1996, p.456). Moses (1995) and Moses and Cobb (2001) have aptly referred to algebra as "the new civil right" and have argued that because algebra opens the door to productive careers it serves as a precious resource for providing equity to all students. In other words, in the US it is largely accepted by mathematicians and mathematics educators that the egalitarian vision that schools should "provide each student with an opportunity for social, political and economic equality" (Cusick, 1983, p. 1) is achievable if all students are allowed to study algebra in school.

Strength has been provided to this "algebra for all" notion by studies such as that of Gamoran and Hannigan (2000). Gamoran and Hannigan (2000) studied the impact of high school algebra on 12,500 students who differed in their mathematics skills prior to entering high school and found that the students improved in their mathematics achievement after taking algebra. In the Gamoran and Hannigan (2000) study, though the regression analysis showed differential benefit across ability groups with students who had high prior achievement benefiting more, the results showed that algebra is useful to all students. Now in the U.S., almost every high school student studies algebra before graduating (Dossey & Usiskin, 2000).

According to the *Nation's Report Card: Mathematics 2007*, the report of the 2007 National Assessment of Education Progress (NAEP) results, after about twelve years of various states in the US responding to the call for "algebra for all" there is evidence from the 2007 results of the NAEP that this is yielding positive effects in the US. The mathematics report of the 2007 NAEP revealed that among fourth- and eighth-graders of all ethnic, racial and income groups generally improved in performance over earlier years. For instance, among fourth-graders Black students improved by 35 points over their 1990 scores while the gains White and Hispanic students were 28 and 27 points respectively. Even among American Indian/Alaska Native students who showed no significant improvements over their 2005 average score, in 2007 there was a 4 percent increase in the number that performed at or above proficient level over the 2005 level. The NAEP 2007 report also showed improved performance of White, Black and Hispanic eighth-grade students as well as students across all income levels. Compared with the 1990 scores White, Black, and Hispanic students improved by 21, 23, and 19 points respectively while Asian/Pacific Islander students improved by 22 points. The only group of eighth-graders who showed no significant gain in scores over previous years was the American Indian/Alaska Native students.

In addition, the 2007 NAEP results showed that the achievement gap between students of color and their white peers as well as between students from low-income families and others is narrowing. For example, among eighth-graders, the Black-White achievement gap narrowed by eight points while the Hispanic-White gap narrowed by four points from the 1996 outcomes. Within the same period, the 2007 NAEP results for fourth-graders showed that though improved achievement by Black students narrowed the White-Black gap by eight points over the 1996 outcomes, the reduction was significantly lower than the 2005 gap. Similarly, reduction in the White-Hispanic gap in 2007 was not significantly different from the 2005 or 1990 outcomes. Thus, by opening the door of algebra to all students not only through specific course(s) in algebra but also integrating algebraic reasoning in all domains of school mathematics, many more doors (i.e., improved mathematics achievement for all and the closure of the achievement gap among different racial groups of students) are being opened to all.

After attaining this noble ideal of "algebra for all", the next step is to, perhaps, begin thinking about issues of how to ascertain the type of knowledge teachers need or have for teaching high school algebra. Whereas it is true to say that teachers' subject matter knowledge is not the only factor that influences instructional practice, it can also be said to be one of the major factors that influenced teachers' decisions. As Even (1989) puts it, "a teacher who has a solid mathematical knowledge,. . ., is more capable of helping his/her students achieve meaningful learning" (p. 4). Such considerations provide the rationale for focusing this study on high school teachers' knowledge for teaching algebra and its relationship to student performance.

1.2 Background of Work Informing this Study

The question of what teachers need to know about their subjects to help them promote powerful and flexible knowledge and understanding in students has been difficult to answer. In addition, this difficulty has further been complicated by the different conceptualizations so far given of teachers' knowledge (Kennedy 1991). For instance, Cochran and Jones (1998) identified the following four components of it:

1) Content knowledge: facts, concepts and procedures within a discipline
2) Substantive knowledge: knowledge of explanatory frameworks in a field
3) Syntactic knowledge: knowledge of ways in which new knowledge is brought to the field, as well as;
4) Beliefs about the subject matter.

The argument by Thompson (1984) that teachers' beliefs, views and preferences about mathematics are some of the factors that play a significant role in shaping their instructional behavior has also implied that the type of mathematical knowledge teachers draw upon in teaching could be influenced by their beliefs, views and preferences about the subject. In addition, Shulman (1986b) also introduced the following seven types of knowledge as the knowledge base for teaching:

1) Content knowledge
2) General pedagogical knowledge (e.g. classroom management)
3) Curriculum knowledge (school "stuff," "tools of the trade," materials, tests, etc.)
4) Pedagogical content knowledge (interaction between content and pedagogy)
5) Knowledge of learners and their characteristics

6) Knowledge of educational contexts (workings of the classroom, school financing, etc.)

7) Knowledge of educational ends, purposes, and values

At the centerpiece of Shulman's (1986b) conceptualization is the idea of "pedagogical content knowledge" (PCK) because it distinguishes the teacher as professional from content experts, and those who simply understand kids. Throwing more light on his conceptualization of PCK and its importance so far as the knowledge base for teaching is concerned, Shulman (1987) argued that,

> The key to distinguishing the knowledge base of teaching lies at the intersection of content and pedagogy, in the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students. (p. 15)

Thus, Shulman argues that good teaching is contextualized and interwoven with good understanding of an academic discipline. In addition, to ensure teacher competence, there is the need to focus attention on developing the knowledge base of teaching which supports such a complex view of teaching.

Ma (1999) also introduced the idea of *profound understanding of fundamental mathematics* (PUFM). According to Ma, teaching and learning which demonstrates PUFM is "connected, has multiple perspectives, includes the basic ideas of mathematics, and has a longitudinal coherence" (Ma, 1999, p.122). This means that teachers with PUFM are able to explain the why of the mathematics they teach and come up with multiple and flexible ways to teach. In this way, Ma's conceptualization of PUFM can be said to be related to Shulman's PCK in the sense that they both suggest that a teacher who demonstrates any of the two types of knowledge, he/she should have a deep, broad, and thorough understanding of mathematics and the ability to teach it in flexible ways. Earlier, Leinhardt and Smith (1985) had proposed two types of teacher knowledge. These are "lesson structure knowledge" (LSK) and "subject matter knowledge" (SMK). Lesson structure knowledge comprised smooth planning and organizing of lessons and providing clear explanations. Leinhardt and Smith's (1985) conceptualized subject matter knowledge to consist of concepts, algorithmic operations, connections among different algorithms and knowledge of the types of errors students make.

Recently, in their work on elementary school mathematics teachers' knowledge, Deborah Ball and her colleagues have also contributed to the idea of the type of knowledge needed by teachers by introducing the idea of mathematical knowledge for teaching (Ball & Bass, 2000; Hill, Ball & Schilling, 2004; Hill, Rowan & Ball, 2005). Relying on existing theories about teacher knowledge, Ball and her colleagues have developed and administered survey-based questions based on teaching mathematics at the elementary school level. Through factor analyses of their data, they have proposed, among other things, the idea of "specialized knowledge of content". As they put it,

> In addition to a general factor, specific factors representing knowledge of content in number and operations, knowledge of students. . . [there is also] a specialized knowledge of content (SKC) made up of several items:

representing numbers and operations, analyzing unusual procedures or algorithms and providing explanations for rules. (Hill, Ball & Schilling, 2004, pp 27-28)

Not only have there been such conceptualizations of the knowledge base for teaching, a number of instruments have been developed to measure teachers' knowledge of the content of school mathematics, as well as issues related to pedagogy. In the US, for instance, to be certified to teach mathematics, various states require pre-service teachers to pass a mathematics test. An example of this is PRAXIS, a teachers' licensing examination developed by Educational Testing Service (ETS), which is currently used by over 30 states. Such efforts are meant to ensure that mathematics teachers have a good knowledge of the mathematics students are required to learn in school. In spite of this, the quality of achievement of K-12 students in mathematics continues to be of national concern. Consequently, the RAND Mathematics Study Panel (2003) made a number of recommendations for improving teachers' mathematical knowledge for teaching. These include the need for further clarification of the knowledge demands of teaching mathematics, and a deeper understanding of ways to provide opportunities for prospective and practicing teachers to acquire this kind of knowledge. In addition, the RAND Mathematics Study Panel (2003) recommended the development of instruments for assessing the mathematical knowledge for teaching across grade levels and mathematical domains. The RAND panel also singled out algebra as an important area of focus in all these efforts.

As already explained, the development of items to measure teachers' knowledge for teaching mathematics at the elementary school level by Ball and her colleagues can be said to be in line with the issues raised by the RAND Mathematics Study Panel. At the secondary school level, and also in line with RAND panel recommendations, Ferrini-Mundy and her colleagues on the Knowledge of Algebra for Teaching (KAT) project have begun conceptualizing and framing questions about mathematical knowledge for teaching algebra (see for instance, Ferrini-Mundy, Burrill, Floden, & Sandow, 2003; Ferrini-Mundy, Senk, & McCrory, 2005). In their conceptualization, The KAT project has hypothesized that mathematical knowledge for teaching algebra consists of three types of knowledge. These are, 1) knowledge of school algebra (referred to later in this dissertation as "school knowledge" as used by the KAT project team), 2) advanced knowledge (i.e., knowledge of the content of other mathematics domains different from algebra) and 3) teaching knowledge. These three categories are explained in the conceptual framework section of chapter two of this dissertation. In addition to their conceptualization, the KAT project is developing items and designing instruments to measure knowledge for teaching algebra at the secondary school level.

Through several pilots of the instruments they are developing, the KAT project is using empirical data from pre-service and in-service teachers in different settings across the country to validate their framework for knowledge for teaching algebra. In addition, KAT is studying the status and variation of knowledge for teaching algebra among pre-service and in-service teachers drawn from across the US. The importance of the KAT work can be seen in the fact that, after two decades of various conceptualizations of the knowledge base for teaching, the KAT project is a specific attempt at focusing on conceptualizing the knowledge required for teaching in one specific domain of mathematics at the secondary school level (i.e., algebra). Another thing that makes the KAT project unique is the work it is doing towards developing tools for assessing mathematical knowledge for teaching algebra. The KAT conceptualization has the potential of becoming a good framework for other researchers who may be interested in conceptualizing knowledge demands for teaching other domains of mathematics especially at the secondary school level. In addition, the KAT instrument could serve as a potential tool for accessing and improving knowledge of pre-service teachers or for professional development of in-service high school algebra teachers. The KAT studies, which include investigating the status and variations of knowledge for teaching algebra among pre-service and in-service teachers, also have the potential of providing empirical data on what needs to be done nationally to improve teachers' knowledge.

This dissertation study is based largely on the work being done by the KAT project, but conducted in a different context and setting. It used the instruments that are being developed by the KAT project to examine issues of knowledge for teaching algebra among prospective and in-service high school mathematics teachers in Ghana. This dissertation study also extended the KAT work by focusing on the relationship between high school teachers' knowledge and the performance of the students in the Ghanaian context. As discussed under the section titled "significance of the study", examining these issues in the Ghanaian context is important for a number of reasons. For instance, since Ghana is a country with a slightly different system of secondary education and teacher certification programs than what exists in most U.S. states, investigating issues of teacher knowledge in Ghana, especially using KAT instrument and framework, is a good first step to examining how the KAT conceptualizations could be corroborated not only internationally but also across systems whose pre-service teacher education is different from the US. Such a corroboration or otherwise of the KAT conceptualization is useful not only for the team of researchers in the KAT project at Michigan State University, but also the entire mathematics education research community worldwide.

In the next three sections, the contexts of education in Ghana, school mathematics and the background of senior secondary school (high school) mathematics teachers in Ghana are discussed.

1.3 The Context of Education in Ghana

At the time of this dissertation study, the structure of formal education in Ghana comprised six years of primary school, three years of junior secondary school (JSS), and three years of senior secondary school (SSS) prior to entry into various forms of tertiary institutions. This structure has been in place since the 1987 Educational Reform in Ghana. Basic school education (i.e., primary and junior secondary school education) was tuition-free and compulsory to all children of school-going age in Ghana. The content of the curriculum of all public primary, junior secondary and senior secondary schools in Ghana is centrally controlled by the Ministry of Education (MOE) and the Ghana Education Service[1] (GES). This means that for each of the school subjects, every student in Ghana's public schools studies the same content (except in situations where students are permitted to select different electives at the SSS level). In addition, the GES controls the school calendar and mandates timetabling, re-opening and closing dates for all public schools up to the SSS level. All public primary schools, JSS and SSS in Ghana run a trimester system each school year.

Students enter class one (i.e., the first grade) at age six and the medium of instruction is English[2] except when the subject of study is Ghanaian Language and Culture. School subjects offered at the primary school include Mathematics, English language, Ghanaian language, Integrated Science, Agriculture and Environmental Studies, Vocational Skills, Life Skills, French (where a teacher is available), and Religious and Moral Education (RME).

At the end of the JSS, students write a national examination, the Basic Education Certificate Examination, which is also used for selection into senior secondary schools. At the time of the dissertation study, the following subjects were offered at the JSS level: Mathematics, English language, Ghanaian language, General Science, Agricultural Science, Social Studies, Environmental Studies, Vocational Skills, Life Skills, French (where a teacher is available),

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[1] In Ghana, the Ministry of Education is responsible for enacting policies while the Ghana Education Service is the body that implements educational polices.

[2] Though Ghana is a multi-lingual and multi-ethnic country with over forty mutually unintelligible indigenous languages spoken within her borders, English, the language of its colonizers has historically become the official language of the country. In 2002, the early exit school language policy was abolished for an all-English policy.

Music and Art[3], and Religious and Moral Education (RME). Grades obtained in various subjects at the Basic Education Certificate Examinations (BECE) are the aggregate of their scores at the BECE (70%) and students' continuous assessment scores throughout their primary and JSS education (30%).

After JSS, students may enter SSS into specific programs that prepare them for university or other specialized tertiary institutions (e.g. the nurses' training college, post-secondary teachers' training colleges, etc). The programs offered include General Arts, General Science, Business, Technical, and Vocational programs. Not all senior secondary schools in the country offer all programs. Consequently, every student is given a special SSS entrance form that lists every senior secondary school in the country and the programs available in those schools. Counselors at the JSS collaborate with students and their parents to help students to select three schools and programs that best suit the student's ability. Once a student's grades qualifies him/her into the SSS, assignment to specific schools and programs is made randomly using a special computer software at the headquarters of the Ghana Education Service based on the quality of grades obtained at the BECE in specific subjects considered fundamental in the program chosen by the student.

At the senior secondary school level, students usually study a combination of three (in some cases, four) elective subjects and a number of core subjects. For instance a student in the General Science program could select Elective Mathematics, Physics, Chemistry and Biology as his/her electives. A General Arts student, on the other hand may select any three or four from subjects such

[3] Music and Art ware not examined at the BECE

as Economics, Geography, Elective Mathematics, and Literature in English, French for his/her electives. In addition to the elective subjects, there are core subjects, which every student studies at the SSS level. These include Core Mathematics, Core English language, Ghanaian language and Culture, Integrated Science, Social Studies, Life Skills, Religious and Moral Education (RME).

1.4 School Mathematics in Ghana

In U.S. junior high and high schools, separate courses in algebra (e.g., Algebra I, Algebra II etc.) could be offered to students. However, in Ghana only one integrated mathematics course is offered at the JSS (the equivalent of seventh to ninth grade) to all students. This mathematics course is a national curriculum, and is therefore, offered to all students in the public school system for the entire three years of the JSS education. The Teaching Syllabus for Junior Secondary Schools (Ministry of Education, 2001) lists the major content areas covered on page (iv) as

- Number
- Investigations with Numbers
- Shape and Space "Geometry"
- Estimation and Measurement
- Introduction to the Set Theory
- Algebra
- Collecting and Handling Data

In addition to these content areas, problem solving which does not appear as a topic in itself is emphasized throughout the syllabus. Furthermore, these topics are not covered in succession. They have been broken down into smaller content pieces, called units (and sub-units) and have been sequenced in a spiral manner. The various units are arranged in such a manner that the topics taught in the early grades are not covered in complete detail but are returned to repeatedly throughout the years and developed further, with increasing detail and depth, as students progress through the grade levels. In *The Process of Education,* Bruner (1960) made a case for this type of sequencing when he said, "A curriculum as it develops should revisit the basic ideas repeatedly, building upon them until the student has grasped the full formal apparatus that goes with them" (p. 13).

At the time of this study, there were two types of mathematics programs offered in Ghana at the Senior Secondary School (SSS) level (the equivalent of grades 10 to 12 in the US). These were Core Mathematics and Elective Mathematics. As already discussed, in the public school system, every SSS student took Core Mathematics for the entire three years of SSS education. Elective Mathematics, on the other hand, was selected by students who require further mathematics content preparation beyond the core mathematics coverage. For instance, Elective Mathematics was an automatic elective course for students in the Science and Technical programs. Other students in the General Arts and Business programs could also select Elective Mathematics. Like the mathematics course at the JSS level, both of the mathematics courses in SSS were, at the time of this dissertation study, also integrated mathematics programs with their content sequenced in a spiral manner. In addition, like all other school subjects, the Ghana Education Service centrally controlled the syllabi for both Core Mathematics and Elective Mathematics. Being a national curriculum, the content of each of these mathematics courses is also the same for all public schools in Ghana.

According to the Ministry of Education (2003a), the major content areas to be covered in Core Mathematics comprise the following:

- Number and Numeration
- Algebra
- Mensuration[4]
- Plane Geometry
- Trigonometry
- Statistics and Probability
- Vectors and Transformation in a Plane
- Investigations and Problem-solving
- Use of Calculators and Computers

"Investigations and problem solving together with the use of calculators are not topics by themselves in the syllabus but nearly all topics include activities involving them" (Ministry of Education, 2003a, p. iii). Core Mathematics is offered for ten class periods of 40 minutes a week for the two terms of the first year.

[4] Mensuration, in the syllabus, comprises finding surface area and volume of solids such as cuboids, prisms cylinders, rings, pipes, cones, spheres, pyramids and calculation of distances between two points on the same latitude or longitude

Thereafter, Core Mathematics is supposed to be offered six periods a week for the third term of the first year and the remaining two years of SSS.

In contrast, the syllabus for Elective Mathematics contains the following content:

- Algebra
- Logic
- Coordinate Geometry
- Trigonometry
- Calculus
- Linear Transformation
- Vectors
- Mechanics
- Statistics
- Probability

The time allotted to Elective Mathematics in the SSS timetable is a minimum of seven class periods of 40 minutes each per week (Ministry of Education, 2003b). Appendix II contains an extraction of the content of algebra in both of these mathematics courses at the SSS level in Ghana at time of the study.

In spite of the two distinct approaches to offering algebra to all students in both Ghana and the U.S., there continues to be national outcry over the performance of students on algebra in national and international assessments. For instance, in Ghana, students' progression from the SSS level to the university and other tertiary levels of the education system is through a national examination, the Senior Secondary School Certificate Examinations (SSSCE). This examination has been in place in Ghana since the National Educational Reform of 1987. Following similar educational reforms in Anglophone West Africa, beginning May 2006, this SSSCE was changed into the West African School Certificate Examinations (WASCE). All high school leavers in Anglophone West Africa now take the WASCE for selection into universities and other tertiary institutions. The sad situation is that, since 1993, several reports by the Chief Examiners of the SSSCE have highlighted students' poor handling of some of the problems on algebra. For instance, in 2004, the Elective Mathematics Paper 2[5] of the SSSCE had the following as one of the questions,

> Express $3x^2 - 6x + 10$ in the form of $a(x-b)^2 + c$ where *a, b* and *c* are integers. Hence state the minimum value of $3x^2 - 6x + 10$ and the value of x for which it occurs (WAEC, 2004).

The chief examiners' report that year acknowledged that most of the candidates attempted the question. However, the reported highlighted the fact that students performed poorly on it because many of them either could not complete the square or resorted to calculus to find the minimum value, a method that was not accepted.

Due to the foundational nature of algebra, there is the need to reverse this trend of students' poor performance on algebra in Ghana. Though in Ghana no studies have been conducted to examine the reasons for this poor performance, worldwide, a number of studies on student performance in mathematics have

[5] At the SSSCE, candidates write two papers at separate times during the examination period. The first paper consists of multiple choice questions while the second paper has open ended questions.

revealed that one of the factors that can improve student achievement in school mathematics is teachers' knowledge (see for example, Harbison & Hanushek, 1992; Mullens, Murnane & Willett, 1996; Hill, Rowan & Ball, 2005). Informed decisions about the type of improvement needed in the knowledge base of teachers could be made if data about the nature of teachers' knowledge and which aspects of it best relate to student performance were available. In the case of Ghana, the integrated nature of each of the two mathematics curricula means that a good knowledge of the entire content of school mathematics in some integrated way is essential for teacher effectiveness. However, to study teacher knowledge relative to the coverage of entire mathematics syllabus (either core or elective) is too broad to cover in a single study. Therefore, given the foundational nature of algebra and the issue of poor performance in this area this study is limited to algebra.

It is in the light of this that this study was designed to investigate the profile of knowledge that Ghanaian senior secondary school mathematics teachers and prospective teachers have for teaching the content of algebra in the SSS syllabus, and how this knowledge is related to student performance.

1.5 Background of SSS Mathematics Teachers in Ghana

Across the spectrum of literature of teacher competence and student performance are studies that project the value added by teacher education. These studies show that graduates of teacher education programs feel better prepared for their job and can positively affect their students' achievement than those who enter teaching without adequate teacher education background (see

Darling-Hammond, 2003; Laczko-Kerr & Berliner, 2003; Kennedy, Ahn & Choi, 2006). These issues of pedagogical preparation, subject matter preparation and experience are especially important in the case of Ghana due to the manner in which teachers are recruited for the senior secondary school level. The population of senior secondary school mathematics teachers in Ghana can be categorized into four main groups based on their courses of study at the university as shown in the table below.

Table 1.5.1 Background of SSS Mathematics Teachers in Ghana[6]

Type of Bachelors Degree	Explanation of Academic Background
B.Sc. (Math), Diploma in Education	Graduated before 1991 from the University of Cape Coast (UCC)
B.Ed. (Math)	Graduated from UCC's Faculty of Education from 1991 or from University of Education, Winneba
B.Sc. (Math) or related field and Post Graduate Certificate/Diploma	Graduated in a non-education faculty/college from any university of Ghana but after entering the teaching field returned to pursue either a certificate or diploma in education at UCC
B.Sc. (Math) or related field	Graduated in a non-education faculty/college from any university of Ghana and has been teaching thereafter. But has not yet returned to pursue either a certificate or diploma in education at UCC

As shown in Table 1 above, first, there are those who graduated from the University of Cape Coast (UCC) before 1991. This group pursued four-year bachelor degrees with a major or minor emphasis in mathematics. In addition,

[6] Percentage of teachers of Ghana's teaching force in each category was not available at the time of the studies.

they pursued a three-year university diploma in education alongside their degree programs.

From 1991, the arrangement whereby a student at UCC could pursue both a bachelor degree in mathematics and a diploma in education together was discontinued. Students who gained admission into UCC from 1991 and were interested in degrees in education or the teaching profession were admitted into the faculty of education to pursue a bachelor degree in education with emphasis in mathematics (i.e., B.Ed. (Math)). The University of Education, Winneba (UEW), also in Ghana, now offers a similar B. Ed. (Math) degree with two tracks: secondary education and basic education. Graduates from these B.Ed. (Math) programs constitute the second group of secondary mathematics teachers in Table 1.5.1.

The third group comprises teachers who graduated from any of the country's universities with bachelor degrees in mathematics or in a related discipline. Every year, Ghana loses a number of its teachers to developed countries like the UK, US etc., as well as to other African countries. To fill the vacuum left because of this exodus, the National Service Secretariat in Ghana posts graduates of related content areas to teach mathematics in the senior secondary schools. Many of these graduates eventually remain as mathematics teachers without any pre-service teacher education background. The University of Cape Coast (UCC) offers a summer sandwich diploma program in mathematics education (until 1992, this was a certificate program) for this third group of teachers over a minimum of two summers. Between the two summers, participants of this sandwich program work with mentor teachers in the schools where they teach while faculty from the university pay occasional visits to oversee their progress.

The fourth group, like those in the third group graduated with Bachelor of Science degrees in mathematics or a related field without any emphasis in mathematics education. These may have entered the teaching field by either being posted through the National Service Secretariat or having applied to teach on their own. However, since entering the teaching field, this group of teachers may not have been able to take advantage of UCC's summer sandwich program in education. They could therefore be teaching without going through any college-level teacher preparation program.

Distinguishing between these groups is necessary because they may have gone through different types of coursework in college. For instance, teachers in Ghana who pursued bachelor degrees in mathematics take such courses as abstract algebra, linear algebra, real analysis, complex analysis, advanced calculus and ordinary differential equations prior to graduating. In contrast, students in the B. Ed (Math) program are prepared in mathematics content courses, mathematics education as well as foundations of education courses. At the University of Cape Coast in Ghana, for instance, mathematics education students take a minimum of 30 credits of mathematics content courses with their counterparts in the mathematics departments. These comprise six credits of university-based mathematics courses per semester from level 100 to 300 and the first semester of the 400 level. In addition, B.Ed. (Math) students take other mathematics content courses targeted at the senior school curriculum, such as algebra and trigonometry, advanced algebra, analytic geometry and calculus, statistics and probability, vectors and mechanics at the Department of Science and Mathematics Education. In addition, B.Ed. (Math) students take a number of mathematics education courses, such as The *Nature of Mathematics*, *Psychological Basis of Mathematics Instruction*, *Curriculum Studies in Mathematic Education*, and *Assessment in Mathematics Education*. Other mathematics education courses are, *Methods of Teaching Secondary Mathematics, Problem-solving in Mathematics, Introduction to Research Methods in Mathematics Education*, and a small research project that culminates into the writing of what is referred to as a "long essay". Mathematics education students also take foundations of education courses such as *History of Education in Ghana, Social Foundations of Education, Psychology of Human Development, Psychology of Human Learning, Guidance and Counseling, Educational Measurement and Evaluation, Educational Administration,* and *Sociology of Education in Ghana*. In addition, during the third year of the undergraduate teacher education program, mathematics education students undertake microteaching sessions, under the guidance of experienced professors. Thereafter, these pre-service teachers are attached to schools for one semester of supervised internships (referred to as *Off-Campus Teaching Practice*), with the help of experienced mentors. During the internship, pre-service teachers get the opportunity to observe lessons and take lead roles in teaching under the mentorship of collaborating teachers in their subject areas while faculty from the university occasionally visit to assess their performance and progress.

Thus, it is obvious that mathematics teachers at the secondary school level in Ghana have varied backgrounds and experience. Such variation in background preparation could mean that the four different group of teachers discussed above could have different profiles of knowledge. With such variation, the question that arises is the extent to which each aspect of teachers' knowledge reflects their program of study at the university, their experience or a combination of the two.

In the light of the possibility of this difference in background and, therefore, knowledge base, this study was set up to investigate the knowledge for teaching algebra of each category of teachers. The same reason provides the rationale for extending the study beyond the profile of teachers' knowledge to exploring how teachers' knowledge relates to student performance.

1.6 Purpose of the Study

This dissertation study had two main objectives. As already explained, the study is based on the KAT project's three conceptualizations of knowledge for teaching mathematics: school knowledge, advanced knowledge and teaching knowledge. However, due to the varied paths to teaching of Ghanaian senior secondary mathematics teachers, it is hypothesized that depending on their prior university coursework and experience the four different types of teachers identified in the previous section would have different levels of each of the three knowledge types proposed by the KAT project. Consequently, one aim of this study was to answer the question, "How do the different types of teachers (both prospective and in-service) who participated in the study differ in the profile of knowledge they have?" To answer this, three different populations were identified. These were:

1) In-service senior secondary school mathematics teachers; a working hypothesis in this study was that this population could comprise teachers in each of the four groups discussed in the previous section;
2) Pre-service secondary school mathematics teachers in colleges of education in Ghana. It was hypothesized that this population's knowledge profile would define the profile of the in-service teachers with B.Ed. (Math) degrees prior to entering the teaching profession; and,
3) Undergraduate final-year students in mathematics or related subjects in Ghana's universities who, through the national service scheme, had the potential of being posted to senior secondary schools to teach after graduation. It was hypothesized that this population's knowledge profile would define the profile of the in-service teachers with non-education degrees prior to entering the teaching profession.

Since it is expected that their background could lead teachers and prospective teachers to answer the questions on the instrument used for this study differently, the study could explore whether there are any differences in their knowledge base and how, if any, their knowledge differed. In addition, the in-service teachers in the aforementioned four categories (see Table 1.5.1) would be compared.

Second, of the in-service teachers selected, the study aimed at examining the performance of their students and finding out how, if any, students' performance and their teachers' knowledge are related. In particular, the study aimed at investigating the extent to which, if any, the different types of teachers' knowledge for teaching algebra, hypothesized in the theoretical framework, relate to their students' performance.

1.7 Research Questions

The study was guided by the following research questions,

1) To what extent does Ghanaian pre-service and in-service secondary mathematics teachers' knowledge for teaching algebra corroborate the three categories of knowledge hypothesized in the KAT framework: knowledge of school algebra (or school knowledge), advanced knowledge and teaching knowledge?
2) How does the knowledge for teaching algebra differ among the different categories of secondary school mathematics teachers and potential teachers in Ghana? As already explained, the categories of teachers aimed at here are in-service teachers, pre-service teachers in colleges of education in Ghana, and undergraduate final-year students in mathematics or related subjects. In addition, among the in-service teachers, the aforementioned four categories (see Table 1.5.1) would be compared.
3) What is the relationship between the performance of in-service secondary school mathematics teachers in Ghana and the students of their classes?

1.8 Significance of the Study

As already discussed, this study is an attempt to examine issues of in-service and pre-service teacher knowledge for teaching algebra at the high school level. These are the issues of concern to the KAT project at Michigan State University and the entire mathematics education research community worldwide. This study is an attempt to use the KAT instruments to measure high school mathematics teachers' knowledge in a setting outside the US.

Research of this type, which studies issues of teacher knowledge, has never been done in Ghana. Therefore, as far as Ghana is concerned, this is a groundbreaking study. It is hoped that the findings of this study will contribute to a better understanding of the knowledge Ghanaian teachers and potential teachers have for teaching algebra. The findings could also contribute to understanding how teachers' knowledge relates to student learning specifically in Ghana. Such an understanding could be helpful in future curriculum planning in mathematics at the SSS level, as well as pre-service teacher education. It could also be useful in discussions about professional development of the Ghana's senior secondary school mathematics teachers.

Also, in one sense, this study was an attempt to give an international dimension to the work that has been started by the KAT project. Ghana was a good international site for three main reasons. First, unlike the US where standards-based curricula are sometimes used, in Ghana, schools are still using syllabi that directly break down objectives for the content of school mathematics at each grade level. Second, unlike most U.S. schools[7], where separate mathematics courses in algebra, geometry, calculus etc. are offered, in Ghana, the core and elective mathematics courses offered to all high school students are integrated and the content sequenced in a spiral manner. Third, the syllabi in Ghana, unlike the US, are centrally controlled by the Ghana Education Service. These differences in curricular arrangement between most U.S. schools and schools in countries that have arrangements similar to those in Ghana could lead to questions of generalizability of findings from the KAT project across different curricula and settings. Therefore, extending aspects of the KAT study in Ghana, as was done in this study, is essential in investigating whether KAT's framework will work in countries with different curricular arrangement such as Ghana.

In addition, the extension made in this study of relating teachers' knowledge to student performance has the potential to contribute to the discussion of which type of knowledge relates to improved instruction and student learning. For researchers interested in finding ways of improving teacher competence, findings from this study about which of the three hypothesized knowledge types (i.e., knowledge of school algebra, advanced knowledge and teaching knowledge) best relates student performance, as well as how the three combined relate to student performance may be important. Replications of a study like this could lead to decisions about what type of knowledge to

[7] According to Reys, Dingman, Nevels & Teuscher (2007), six states in the U. S. (Florida, Georgia, Indiana, New York, North Carolina and Tennessee) provide standards organized for integrated high school mathematics courses. There are also states, such as Minnesota, with individual schools (e.g., Wayzata High School in Plymouth, MN, which is personally known to me) providing integrated mathematics courses using the Core Plus Mathematics Project (CPMP).

emphasize in pre-service teacher education, as well as professional development programs.

1.9 Organization of Chapters

This section presents the organization of the chapters of this dissertation in the order they are presented.

The first chapter provides the rationale for this study. In addition, because the setting of the study is in Ghana, a country different from the United States of America (US), background information about education in Ghana, the nature of high school mathematics curriculum, as well an overview of the background of high school mathematics teachers in Ghana is presented. The research questions that guided the study are then presented. The chapter ends with the purpose of the study and its significance.

The second chapter provides a review of literature relevant to the study. The review is organized around issues that have influenced research on teachers' knowledge and teaching. It has been extended to cover studies involving teachers' knowledge and their practice, as well as teacher knowledge and student performance. The theoretical framework used in the study is presented after these reviews.

In Chapter 3, the research designs used in this study are presented. An account of the procedures used in the study has also been given, as well as a description of the research instruments. The chapter ends with brief discussion of how data for this study were analyzed.

Chapter 4 presents results and analysis of the study. This chapter has been organized in line with the research questions. In this way, analyses of data related to a specific research question are conducted and the results presented before moving on to another research question. To help make sense of the statistical analyses, conclusions drawn are presented at the end of each set of analyses and results. These conclusions have been limited to issues related to each of the research questions.

Chapter 5 summarizes the study and discusses conclusions drawn from the analyses and results. The chapter ends with recommendations for further research in the area.

CHAPTER 2: REVIEW OF RELEVANT LITERATURE

In this chapter, literature about teacher knowledge is reviewed. The review draws out some of the issues that have influenced research on teachers' knowledge and teaching. It has also been extended to cover studies involving teachers' knowledge and their practice, as well as teacher knowledge and student performance. Further more, the research studies reviewed in this chapter are mostly those that are grounded in the cognitive perspective (as opposed to, for example, the socio-cultural perspective). This reliance on studies framed by cognitive perspectives is a result of the fact that the domains of knowledge hypothesized in the theoretical framework, which guided this dissertation study, emphasize the cognitive dimensions (i.e., on what is in teachers' heads and which they apply to their work) of teachers knowledge for teaching algebra. At the time of this study, the knowledge domains hypothesized in the theoretical framework, were in their early stages of conceptualization. As a result, the possibility or otherwise of the social dimensions of teachers' knowledge had not yet been the focus of study; hence the reliance on studies that emphasize the cognitive domains of teacher knowledge in the review of literature for this study.

2.1 The Issue of Teacher Knowledge

For many years, researchers have debated the issue of which school factors influence student achievement. According to Duthilleul & Allen (2005), this debate was started in the US when the report entitled *Equality of Educational Opportunity* by Coleman et al., (1966) "concluded that family background characteristics and community level variables accounted for more variance in student achievement than school resource variables like . . . teacher characteristics" (p.3). Coleman and his colleagues analyzed data from about 600,000 students and 60,000 teachers in more than 4,000 schools and concluded that only about 10 percent of the variance in student achievement could be explained by school factors. In the intense debate that ensued thereafter, viewpoints emerged that questioned whether schools matter in student learning. Such negative findings and views about schools and teachers in particular could be what underlined early attempts at conceptualizing the knowledge base for teaching that was spearheaded by Shulman (1986b, and 1987). As Strom (1991) puts it, "at one level, concern about the knowledge base focuses on improving the respect and status accorded teaching, thereby making it a more rewarding career" (p. 1). The idea was that for teaching to be respected as a profession that influences learning outcomes, a case needed to be made that it involved a wise application of a specialized body of knowledge. It is to this end that Shulman (1986b) introduced the idea of "pedagogical content knowledge" as a type of knowledge, which comprises such things as how students understand, and how to use resources effectively to present ideas in ways that make them more accessible to different types of students.

Substantial research has been conducted on the issue of the knowledge base for teaching since the time Shulman (1986b, 1987) put forward his framework. Some of these have led to further conceptualizations. A good example of this is the study by Liping Ma. In *Knowing and Teaching Elementary*

Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States, Ma (1999) presents the results and analyzes of interviews with 23 elementary schoolteachers from the U.S. and 72 from China. Through the analyses of the interview data, Ma (1999) introduces a different conceptualization of the knowledge base for teaching mathematics, which she calls, "profound understanding of fundamental mathematics" (PUFM). Ma's (1999) PUFM involves a type of knowledge more than content knowledge; it also has to do with communicating the subject matter of school mathematics to students. In this regard, with the exception of the fact that Shulman's (1986) pedagogical content knowledge (PCK) is a generalized form of knowledge (possibly not restricted to a particular subject matter like mathematics), Ma's (1999) conceptualization can be said to show some resemblance to Shulman's (1986b) PCK. At least both seem to involve a complex combination of content knowledge and pedagogical knowledge.

Not only did the 1966 *Equality of Educational Opportunity* report by Coleman and his colleagues influence the need for providing a framework aimed at professionalizing teaching, it also fueled studies into the effects of instruction on student learning. In the *Handbook of Research on Teaching*, Brophy and Good (1986) reviewed several studies conducted in the 1970s, and drew the following conclusion, "The myth that teachers do not make a difference in student learning has been refuted" (p. 370). In fact, currently there is wide-ranging agreement, from numerous studies that the availability and effectiveness of teachers contribute in no small ways to student learning outcomes (see for example, Jordan, Mendro, and Weerasinghe, 1997; Sanders and Rivers, 1996; Wright, Horn, and Sanders, 1997). The Wright, Horn and Sanders (1997) study for instance is one of the latest studies to reveal that the most important factor affecting student learning is the teacher. After analyzing the achievement scores of more than 100,000 students, Wright and his colleagues concluded, "Effective teachers appear to be effective with students of all achievement levels, regardless of the level of heterogeneity in their classrooms. If the teacher is ineffective, students under the teacher's tutelage will show inadequate progress academically despite how similar or different they are regarding their academic achievement (Wright et al., 1997, p. 63).

In the face of this renewed confidence in the teacher as one of the most important factors that influence student performance, one issue that remains is the question of which aspects of the teacher's knowledge best relate to student performance. This issue is important to research into meaningful ways of school improvement; an importance further enhanced by current not so good performance in mathematics of US and Ghanaian high school students' on international comparative studies (e.g. TIMMS, 2003). For instance, though eighth graders in the US performed above average in the 2003 Trends in International Mathematics and Science Study (TIMMS, 2003), they were outperformed by ten of the twelve OECD-member countries. In the case of Ghana participating for the first time in the TIMMS assessment, Ghanaian eighth graders did not only perform below average, they were the second from last among the 44 participating countries. Studying which aspects of teacher knowledge best relates to student performance warrants new conceptualization of the knowledge needed to teach various content of school mathematics. Such a new conceptualization would need to lend itself to being assessed both qualitatively and quantitatively and on a large scale. The current study sought to do exactly this. The study relied on the conceptualization of teachers' knowledge for teaching algebra provided by the Knowledge of Algebra for Teaching (KAT) project and investigated whether data obtained by adapting the KAT instrument on Ghanaian teachers would corroborate the KAT conceptualization. In addition, the study sought to examine how the different types of teachers' knowledge hypothesized by the KAT project relates to student performance.

2.1.1 Teachers' Knowledge and their Teaching Practice

Lampert (1990) and Ball (1993) used their own classrooms to examine the relationship between possession of subject matter knowledge in mathematics and the teaching of mathematics. Their work confirms the argument by Shulman and Quinlan (1996) that, "excellent teachers transform their own content knowledge into pedagogical representations that connect with their prior knowledge and dispositions of the learner" (p. 409). While it is important not to diminish the significance of subject matter competency of teachers, it is also important to consider how much of it helps teachers to transform their knowledge in ways described by Shulman and Quinlan (1996). In addition, it is also important to consider the role of experience in facilitating this transformation.

Research on teaching is replete with attempts at examining how teachers transform their knowledge into their teaching practice (Peterson & Clark, 1978;

Leinhardt, & Greeno, 1986; Shulman, 1986, 1987; Wilson, Shulman & Richert, 1987; Leinhardt, 1988). For example, Leinhardt & Greeno (1986) described a skilled teacher as one who has "a complex structure composed of interrelated sets of organized actions [called schemata…] which are applied flexibly with little cognitive effort in circumstances that arise in the classroom"(p.75). This perspective of a skilled teacher emphasizes the strong belief in knowledge transformation into teacher actions. However, there has been wide variation on what aspects of teacher knowledge to focus on in research.

For instance, available literature shows that initial research into how teachers translate their knowledge into their teaching practice was in the form of process-product research, which started in the US from the 1920 onwards (see reviews of these early types of studies by Brophy & Good (1986), Gage (1978) and Doyle (1977)). The rationale behind the design of this early research into teaching was the presumption that a direct connection could be established between teacher actions in the classroom and student achievement. Consequently, in studying teaching, process-product researchers coded teacher actions and related them to student behaviors (outcomes) that were measured. Coding teacher actions was an indirect attempt at breaking down which aspects of teachers' knowledge are transformed into their teaching practice.

In the mid 1970s, this early approach into studying teaching received four main kinds of criticisms, including its over-reliance on correlational methods and the types of teacher actions that were coded. A complete review of all these critiques have been summarized and evaluated by Gage and Needels (1989).

Their review grouped the criticisms of process-product research on teaching into four main categories. The first of these categories are those criticisms that have attacked the conceptualization of process-product research, including the conception of causality implied in this research paradigm. The second category of criticisms is those that focused on the methods used by process-product researchers (i.e., pre-determined coding categories, the need for experimental methods, the role of mediation and the quality of the outcome measures). Third are critiques concerned with the predictive power of process-product research. The fourth and last of Gage and Needels' (1989) categories are criticisms that have focused on the use of meta-analysis and the conversion of findings into rules for teaching by process-product researchers.

Based on these critiques, several researchers proposed a modification in the process-product design (see for example, Berliner, 1979; Peterson & Swing, 1982). For instance, according to Berliner (1979), he and his colleagues in the Beginning Teacher Evaluation Study (BTES) study introduced a variable, which they called Academic Learning Time (ALT) in their modification of the process-product research design. The BTES program insisted that this variable does not only serve as the link between teacher behavior and student achievement but is also an important operationally behavioral indicator of student learning. ALT therefore became the research variable of interest to the BTES program. One aspect of ATL was what Berliner and his colleagues referred to as "engaged time"; the actual time students spend on tasks provided by the teacher in learning a particular content. Their argument was that if the time is spent on materials that are too difficult for the student, he/she could not acquire any extra concepts, skills and operations that are needed for effective performance at that grade level. On the other hand, engaging students over long periods on many easy tasks will also not improve academic learning. Unfortunately, in keeping the focus on how teachers transform their knowledge into their teaching practice, the BTES program failed to show what type of knowledge teachers use in judging the difficulty level of the tasks they give to their students, especially in heterogeneous situations where students come with varied competencies. Their ALT construct also failed to indicate how teachers are able to decide when to move to new materials.

Later, other researchers brought the mental life of the teacher to the center of teaching research (see for example, Peterson and Clark, 1978; Putnam, 1987). Focusing on the mental life of teachers suggests that researchers in this area hoped that the type of knowledge teachers transform into their practice could be seen in the thought process of teachers before, during after teaching. The rationale for this line of research is that experienced teachers' knowledge about teaching is organized into packages of question and explanations that make it possible for them to enhance student learning and overcome student misconceptions about subject matter they teach (Putnam, 1987; Shulman, 1987). Putnam (1987) refers to these packages as "curriculum scripts" and argues it is these curriculum scripts, which shape teachers' agenda for teaching and not students' prior knowledge (or intuitive knowledge of students). The "curriculum scripts" that experience teachers possess enable them to adopt flexible and interactive approaches to teaching and enhance their efficiency. To researchers with this perspective, focusing on the mental life of the teacher before, during and after teaching is one way of accessing what aspects of their knowledge they transform into teaching.

The work by Shulman and his colleagues threw the brightest light on how teacher knowledge could influence teaching (see Shulman, 1986, 1987; Wilson, Shulman & Richert, 1987). Shulman's (1986) conceptualizations of "content knowledge" and "pedagogical content knowledge" and the distinction between them brought the attention of researchers in several content domains to issues involving the type of knowledge teachers need about content for teaching, different from what an ordinary adult may have (see for example, Ball, 1988; Wilson & Winneburg, 1988; Grossman, 1990).

All the aforementioned research paradigms produced mostly qualitative information about teachers' knowledge and its influence on their teaching practice. At the time of this study, work by Ferrini-Mundy and her colleagues on the KAT project appears to be groundbreaking work aimed at developing measures of knowledge of mathematics for teaching algebra at the high school level. Past approaches reviewed above either ended in conceptualization of the knowledge for teaching or with measures of teacher knowledge and actions that influence their teaching practice. Even when attempts were made to link to student performance, as was done in the BTES study, the construct conceptualized (e.g., the ATL construct) was mostly limited to teacher actions.

These later constructs, ATL, for example, failed to show which type of knowledge teachers use to judge the difficulty level of the tasks they give to their students.

The approach by the KAT project is an improvement over past approaches in a number of ways. For instance, after the initial conceptualizations, researchers in the KAT program are designing items and reliable and valid instruments to measure knowledge hypothesized in their framework in large scale settings for teaching algebra among pre-service and in-service secondary school mathematics teachers. These measures being developed by the KAT project have been adapted in this study.

2.1.2 Teachers' Knowledge and Student Performance

Traditionally, education researchers have felt that somehow there has to be a relationship between teacher knowledge and student achievement. In particular, many have felt that teachers' subject matter knowledge needed to be related to student achievement. Even those who feel that a teacher's actions in the classroom have to be related to his/her subject matter knowledge also feel that quality pedagogical representations have to be related to improved student achievement. The argument by Shulman and Quinlan (1996) that, "excellent teachers transform their own content knowledge into pedagogical representations that connect with their prior knowledge and dispositions of the learner" (p. 409) summarizes this suspicion that teachers' content knowledge would lead to quality teaching practices and, perhaps, indirectly to student achievement. It is therefore, not surprising the volume of critique directed at process-product researchers when this was not made an explicit focus of their study (see reviews by Brophy & Good, 1986; Gage, 1978; Doyle, 1977). Consequently, at the time when the Coleman et al., (1966) report sparked a wave of research aimed at correlating factors such as students' family background and socio-economic status, etc (Hanushek, 1981; Greenwald, Hedges & Laine, 1996), some researchers still focused on the relationship between teacher knowledge and student performance. Proxy measures of teacher knowledge such as performance on certification examinations or other forms of examinations were used as variables to examine relationships with student achievement (see for example, Hanushek 1972; Boardman, Davis & Sanday, 1977; Strauss & Sawyer, 1986; Ferguson 1991; Harbison & Hanushek, 1992; Tatto, Neilsen, Cummings, Kularatna & Dharmadasa, 1993; Mullens, Murnane & Willett, 1996; Rowan, Chiang, & Miller, 1997). A number of studies have shown that secondary school students' achievement in mathematics is related to their mathematics teachers' knowledge. In the developing world, teachers' subject matter knowledge was found to be a better predictor of student achievement than other home-based factors by Harbison and Hanushek (1992) in Brazil, as well as Mullens et al. (1996) in Belize.

Though this was a good development, and several studies found a positive correlation between teacher knowledge and student achievement in mathematics, there were a number of disagreements among researchers as to what aspects of teachers' knowledge are related to student achievement and how much of it. Rowan, Chiang, & Miller (1997), for example, revealed that teachers' knowledge of subject matter has a direct effect on students' achievement in mathematics. This study also found that that the size of the effect of teachers' subject matter knowledge depends on the average levels of ability of students in a school. Others have found, however, that students taught by teachers with advanced degrees in mathematics performed poorer compared with students taught by teachers without advanced degrees in mathematics (see Monk, 1994; Rowan, Correnti & Miller, 2002). For instance, Monk's (1994) study revealed that the taking of advanced mathematics courses beyond five by teachers produces virtually no returns in terms of the impact on the achievement of their students. This finding pointed to the possible existence of the economists' law of diminishing returns on the number of advanced mathematics courses taken by teachers and the impact on their students' achievement.

In addition, there were questions about how much of the relationship between teacher knowledge and student achievement was due to experience. For instance, the study by Rowan et al. (2002) pointed more, at the elementary school level, to teachers' years of experience than subject matter competency as perhaps the most consistent predictor of students' achievement.

In addition to these, there are also studies that seem to project the value added by teacher education to teacher competence. It is well documented that even with strong subject matter background, teachers without subject matter pedagogical preparation could not engage their students in deep thinking about the subject matter of mathematics but only teach the way they were taught (Darling-Hammond, 1999 & 1991; Fergusson & Womack, 1993; Grossman, 1989; Shulman, 1987). For instance, in a three-year study, Ferguson and

Womack (1993) found that course work in teacher education better predicted teacher effectiveness than measures of content expertise alone. This study found that education courses were better predictors of teaching success (i.e., student achievement) than teachers' grade point average (GPA) prior to entering teacher education programs. Monk (1994) found similar results about science education coursework and student achievement. Review of studies by Darling-Hammond (1999), which show that students of certified teachers score higher on standardized mathematics test than those of uncertified teachers, lend support to the value of teacher education in teaching success. Earlier, Darling-Hammond (1991) had argued in favor of the efficacy of subject-specific methods courses for those preparing to teach. She cited several studies to support her conclusion that teachers admitted to the profession through quick-entry alternative routes had difficulty with pedagogical content knowledge and curriculum development (see also Darling-Hammond, 2003; Laczko-Kerr & Berliner, 2003 and Kennedy, Ahn & Choi, 2006). The paper by Kennedy, Ahn & Choi, (2006) in particular provides a synthesis of research on the relationship between teachers' educational background and the mathematics achievement of their students. Their synthesis concluded that "additional teacher courses in content, content pedagogy, and pedagogy, all benefit students" (p. 36). Consequently, Kennedy et al. (2006) recommended, that instead of debating the relative merits of the various domains, an intensive study in both content and teacher education domains could be one way of improving the curriculum of pre-service teachers.

Unlike the situation in the US, not many studies correlating the value added by teacher education or teachers subject matter knowledge to student achievement have been examined in Africa. The few studies that have examined similar issues in Africa have found that the relationship depends on the SES status of the school. For instance, datasets collected by the Southern African Consortium for Monitoring Educational Quality (SACMEQ) between 2000 and 2002 have led Duthilleul and Allen (2005) to examine the relative contribution of teacher's education, subject matter competency and pedagogical practices to sixth graders' mathematics achievement in Namibia. Duthilleul and Allen (2005) found that the relative contributions of each of these factors depend on the socio-economic status of the school. According to Duthilleul and Allen (2005), in low SES schools, effective teachers had a high level of subject matter competency, while in high SES schools teacher training continued to be associated with effective teaching. These findings support the notion that effective teachers should not only have a sufficient level of subject matter competency but should also receive adequate teacher training in order to develop effective pedagogical practices and contribute to student achievement.

The lack of agreement among researchers as to which aspect of teacher knowledge and how much of it, as well as how much experience contributes to influencing student achievement point to the need for better measures of teacher knowledge. Rather than rely on proxy measures, such measures need to be related to conceptualization of teachers' knowledge for teaching mathematics. It is in the light of this that the conceptualization of knowledge for teaching algebra at the high school level by the KAT project has been used as a framework for this study. In the next section, this conceptual framework is discussed, as well as how the conceptualizations fit into the present study.

2.2 Conceptual Framework

The Knowledge of Algebra for Teaching (KAT) project currently in progress at Michigan State University is working on conceptualization and validation of the type of knowledge used by teachers in teaching algebra. Through analyses of research literature, recommendations by professional organizations and videos of teaching, researchers in this project have hypothesized that the knowledge used by teachers in teaching school algebra consists of three types. These are "knowledge of school algebra" (referred to in short as "school knowledge"), "advanced knowledge of mathematics" (also referred to as "advanced knowledge"), and "teaching knowledge". These three types of knowledge, discussed below, constitute the theoretical frame of algebra knowledge for teaching that guided this dissertation study.

2.2.1 Knowledge of School Algebra

The KAT project defines "Knowledge of School Algebra" (or simply "School Knowledge") as the knowledge of mathematics in the intended curriculum of middle school and high school. This is the content of school algebra that teachers are expected to help students discover or learn in their algebra classes. In the US, the big ideas of this type of knowledge are described in documents such as the National Council of Teachers of Mathematics (NCTM)'s

Principles and Standards for School Mathematics (NCTM, 2000) while the specific grade-level algebra content is described in the various states' standards, textbooks and other instructional materials used in the schools. In their work, researchers in the KAT project delimited this type of knowledge by reviewing content standards of ten different states in the US (Appendix I lists content areas generated by the KAT project from this review). At the SSS level in Ghana, the content of this type of knowledge is included in both the Core and Elective Mathematics Syllabuses. The content of this domain of mathematics, school algebra, at the high school level in Ghana has been presented in Appendix II (from Core Mathematics) and Appendix III (for Elective Mathematics), after a review of the Core and Elective Mathematics syllabi from Ghana. This type of knowledge is considered important by the KAT project because unless teachers understand the grade-level algebra content they are to teach, they would find it difficult to influence student learning. Since students are expected to learn their school algebra, it sounds reasonable to hypothesize that for teachers to influence students learning, they (teachers) need to understand the content of school algebra themselves.

2.2.2 Advanced Knowledge of Mathematics

According to the KAT project, *Advanced Knowledge of Mathematics* (or simply "Advanced Knowledge") "includes other mathematical knowledge, in particular college level mathematics, which gives a teacher perspective on the trajectory and growth of mathematical ideas beyond school algebra" (Ferrini-Mundy, Senk and McCrory, 2005, p.1).

The KAT project lists areas like calculus, linear algebra, number theory, abstract algebra, complex numbers and mathematical modeling as some of these general areas (see Ferrini-Mundy et al., 2005). In addition, in the conceptualization of advanced knowledge, members of the KAT project acknowledge that "knowing alternate definitions, extensions and generalizations of familiar theorems, and a wide variety of applications of high school mathematics are also characteristics of an advanced perspective of mathematics" (Ferrini-Mundy et al., 2005, p. 1). Thus, it can be argued that having an advanced perspective of mathematics affords teachers with a deep or profound understanding of school algebra. Figure I below is a diagram that exemplifies how the possession of an advanced perspective of mathematics could make it possible for a teacher to make connections across topics while unpacking the complexity of a mathematics content to make that content more comprehensible.

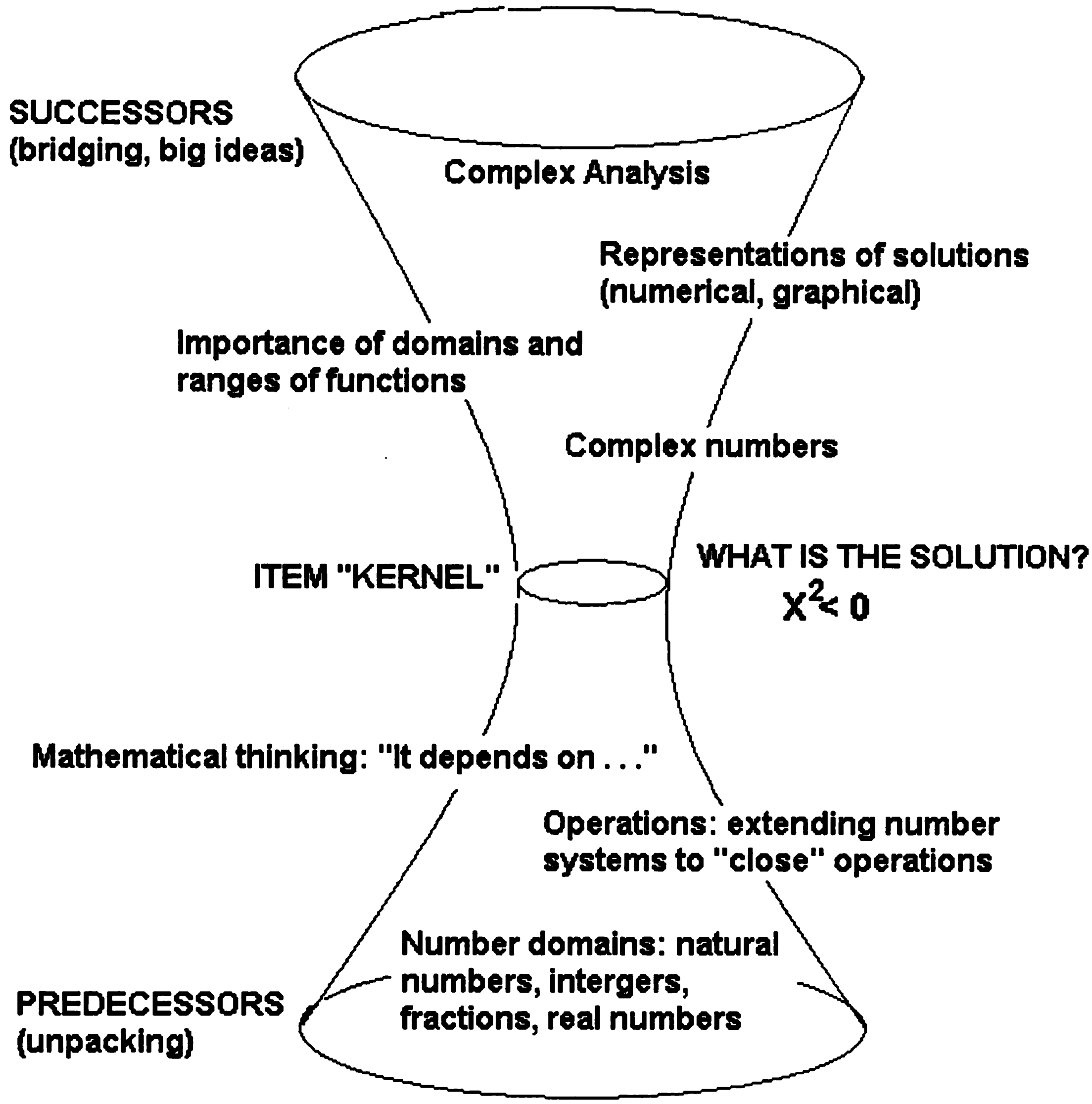

Figure 2.2.1 Conceptual Representation of "Advanced Knowledge"

Figure 2.2.1 above shows an example of a specific content of school algebra, the content indicated by the item kernel, "what is the solution of $x^2 < 0$?" In class, the KAT project hypothesizes that a teacher's task, when this item is the focus, is unpacking content preceding the content of focus. In addition, teachers would also need to engage in bridging the big ideas that come after the item of interest. As shown in the figure, predecessor content that need to be unpacked by the teacher includes content such as sets of numbers, operations on numbers and engage in getting students to mathematically think broadly about conditions on which such solutions would depend. Also shown in the figure are examples of related content that go beyond the content of school algebra as complex numbers, domain and range of functions, and various forms of representations of solutions. As already indicated, the KAT project considers "advanced knowledge" important because possessing it affords teachers with a deep or profound understanding of school algebra. In addition, it is hoped that teachers who possess this type of knowledge would have a good knowledge of the trajectory of the content of school mathematics. This knowledge in turn could help teachers to engage in bridging (making connections across topics), trimming (removing complexity while retaining integrity and decompressing (unpacking complexity to make content more comprehensible) of the content of school algebra to students; processes that could be vital to effective teaching.

2.2.3 Teaching Knowledge

The third category of knowledge in the KAT framework is the teaching knowledge. In the KAT framework, this knowledge is described as "knowledge specific to teaching algebra that may not be taught in advanced mathematics courses. It includes such things as what makes a particular concept difficult to learn and what misconceptions lead to specific mathematical errors. It also includes mathematics needed to identify mathematical goals, within and across lessons, to choose among algebraic tasks or texts, to select what to emphasize with curricular trajectories in mind and to enact other tasks of teaching" (Ferrini-

Mundy, McCrory, Senk & Marcus, 2005, p.2). Thus, this is the type of knowledge that teachers have and which they use in the teaching the subject matter of school algebra (see Appendix III for the tasks of teaching from the KAT framework). This point is made by the KAT researchers when they say that, "the knowledge referred to here may fall into the category of pedagogical content knowledge or it may be pure mathematical content applied to teaching" (Ferrini-Mundy et al., 2005, p.1). In addition, since this type of knowledge may not be taught in advanced mathematics courses, it may not necessarily be available to mathematicians. Consequently, this is the knowledge that could differentiate an engineer or a mathematician from an algebra teacher.

2.3 Relationship between the Three Types of Knowledge

The KAT project conceptualizes that their hypothesized three types of algebra knowledge for teaching, School Knowledge, Advanced Knowledge and Teaching Knowledge are not hierarchical in nature. Neither do they exist in a continuum with well-definable boundaries. Rather, their boundaries are blurry in the sense that they are interwoven in many ways. A schematic diagram of this conceptualization is presented in Figure 2.3.1 below. In chapter 4, data from this dissertation study has been used to confirm this conceptualization.

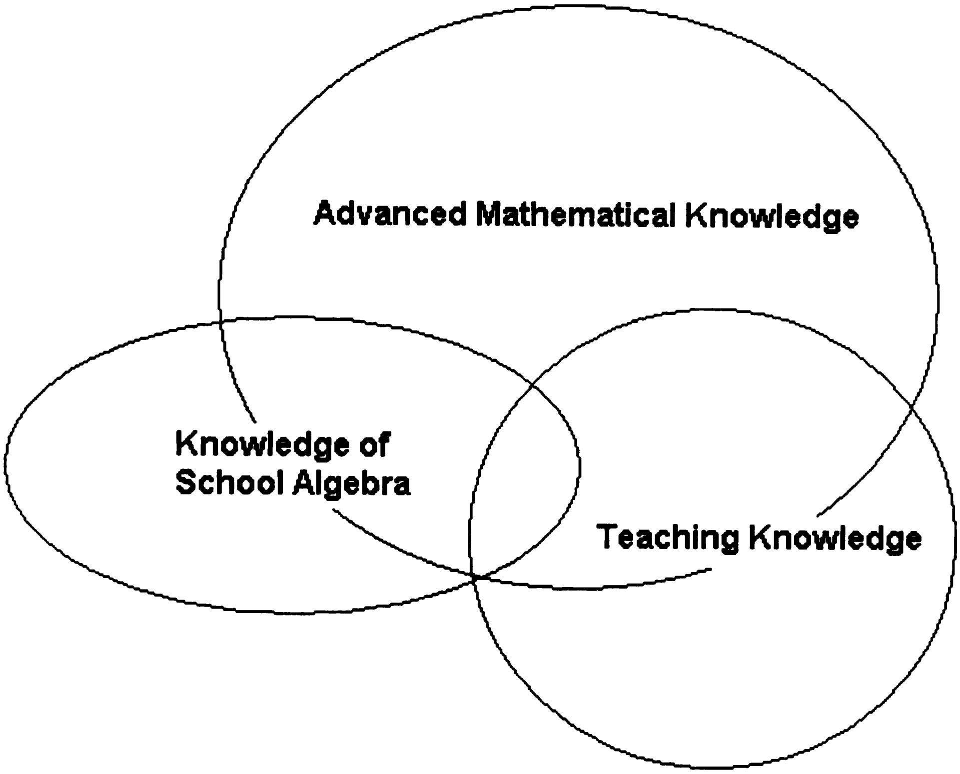

Figure 2.3.1 Conceptual Representation of the Three Types of Knowledge.

2.4 The KAT Project's Item Development Matrix

As already discussed, one aim of the KAT project is to develop items and design reliable and valid instruments to measure knowledge in large-scale settings for teaching algebra among pre-service and in-service secondary school mathematics teachers. To achieve this, two other constructs were conceptualized by the KAT project in addition to the construct of the "algebra knowledge for teaching" (which comprised the three types of knowledge hypothesized). The other two construct were defined as "Algebra Content" and "Domains of Mathematical Knowledge". The three constructs where presented in an item development matrix that is cuboid in nature (or in three-dimensional space) as shown in Figure 2.4.1 below.

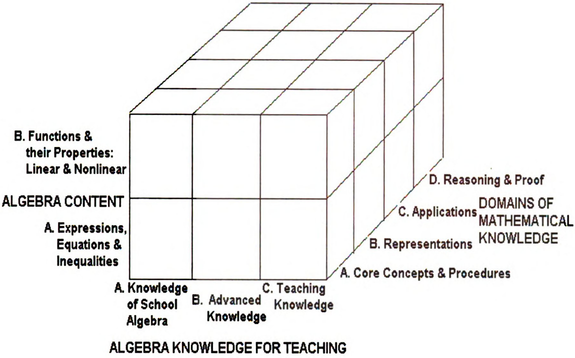

Figure 2.4.1 The KAT Project's Item Development Matrix

As shown in Figure 2.4.1, the three types of algebra knowledge for teaching are pictured on the x-axis, the "algebra content" is pictured on the y-axis and "domains of mathematical knowledge" are pictured on the z-axis.

In the sub-sections that follow, the content of the constructs of "algebra content" and "domains of mathematical knowledge" are briefly explained.

2.4.1 The Y-Axis: Algebra Content

Though this dimension could have many categories, the KAT project limited this to two areas or topics that are considered central mathematically, included across K-12 curricula for school algebra (traditional as well as reform) and which have been the focus of research on student learning in algebra.

2.4.1.1 Expressions, Equations, and Inequalities

Researchers in the KAT project decided to formulate items on expressions, equations and inequalities for a number of reasons. First, review of the content of school algebra in various states indicated that at the heart of high school algebra is the algebra of polynomial and rational expressions, equations, and inequalities. It was therefore considered that the ability of students to work with these concepts is integral to their success in algebra and all higher-level courses. Second, research has shown that students could have difficulties with concepts involved in working with expression, equations and inequalities. For instance, available literature is replete with students' difficulties in understanding differences between expressions and equations as well as the many uses of the equal sign (see, for example, Wagner & Kieran, 1989; Kieran, 1993; Bednarz, Kieran & Lee, 1996). Third, according to Nathan and Koedinger (2000), whereas students performed better on the word problems than on the symbolic equations, teachers have consistently rated solving symbolic equations easier than solution of similar word problems. Consequently, in designing valid and reliable instruments for assessing teachers' knowledge, researchers in the KAT project have felt the need to formulate items that could be categorized as involving expressions, equations and inequalities.

2.4.1.2 Functions and their Properties: Linear and Non-linear.

Function is the second topic of this study. The KAT project defines a function broadly as "as a relationship between two sets of objects, usually numbers where every number in the first set is related to exactly one number in the second set. The first set (domain) and second set (range) may be infinite or finite and consist of real or complex numbers" (Ferrini-Mundy et al., 2005, p. 3). The need to include this topic in the content domains is based in part of it being considered a central mathematical idea. Another rationale for focusing on this topic follows from the literature on the possible misconceptions of functions to pre-service teachers (Even, 1993). In addition, like expressions, equations and inequalities, concepts of functions are fundamental for higher-level mathematics.

2.4.2 The Z-Axis: Domains of Mathematical Knowledge

2.4.2.1 Core Concepts and Procedures

According to the conceptualizations by researchers in the KAT project, core concepts and procedures consists of knowledge of the concepts, definitions, axioms, theorems, and algorithms, as well as mathematical language, notation, and conventions. This is the type of knowledge Shulman (1986) calls declarative or substantive knowledge.

2.4.2.2 Representation

The term representation in the KAT framework is taken in this general sense as Kaput (1985) talks as "involving some kind of relationship between symbol and referent…" (P. 383). Thus, the KAT framework defines representation as the various forms used to describe or picture mathematical concepts and procedures. It includes number lines, tables, graphs, area models, and matrices.

2.4.2.3 *Applications*

"By applications, we mean the representation of a real world situation by a mathematical one" (Ferrini-Mundy, McCrory, Senk & Marcus, 2005 p.5). Because of the application of algebra to many real situations, it is important that teachers be able to link the algebraic concepts they teach with real situations; hence the focus on applications.

2.4.2.4 *Reasoning and Proof*

According to the KAT project, "reasoning and proof includes knowledge of the specialized vocabulary of reasoning, the ability to find examples and counterexamples of statements, and the ability to use analogies or geometric arguments to justify statements, and the ability to use various proof techniques within an axiomatic system to make convincing arguments. ... [It also includes] the ability to judge the reasonableness of conjecture" (Ferrini-Mundy, McCrory, Senk & Marcus, 2005 p. 5). This conceptualization is consistent with the components of reasoning and proof emphasized by NCTM (2000) *Principles and Standards for School Mathematics*:

- Make and investigate mathematical conjectures;
- Develop and evaluate mathematical arguments and proofs;
- Select and use various types of reasoning and methods of proof (NCTM, 2000, p. 342)

This domain has been focused by the KAT project because of the need for teachers to also be able to help students understand mathematical justifications, and to be able to construct additional justifications that make sense to them.

2.5 Importance of this Conceptual Framework to this Study

The focus of this study is in part influenced by conceptualizations of content knowledge, curriculum knowledge, pedagogical knowledge, and pedagogical content knowledge put forward Shulman and his colleagues (Shulman, 1986b; Wilson, Shulman & Richert, 1987). My perspective on the conceptualizations of content knowledge, curriculum and pedagogical knowledge is that effective mathematics teachers, especially at the high school level, use them in ways that blend these three types of knowledge into a somewhat new form of knowledge. My personal intellectual work has gotten me to think of the teacher knowledge in terms of connected or overlapping packages of knowledge (see Ma, 1999) or curriculum scripts, to use the words of Putnam (1987). In my perspective, these connected packages of knowledge are a blend knowledge of content of the subject matter they teach, knowledge of other content in the school curriculum and their relationship, as well as why particular representations could be problematic or easy to some students. To this end, the concept of pedagogical content knowledge, which Shulman defines to include representations of specific content together with why the learning of that content is easy or difficult for students, have resonated well with my personal perspective. Unfortunately, until now, earlier researchers who have relied on Shulman's conceptualizations have only concerned themselves with teacher knowledge qualitatively.

The KAT conceptualization has illuminated the perspectives I had and contributed to my learning about teaching in a number of ways. First, their emphasis on both school knowledge and advanced knowledge confirm for me the fact that teachers need to know not only the content they are teaching, but also content of other areas of their subject (and sometimes beyond their subject) that are connected to it. This perspective is inherent in the KAT project's construct of "advanced knowledge" covered content beyond the content of school algebra that afford teachers with a deep or profound understanding of school algebra. Second, the argument by Ferrini-Mundy and her colleagues that the boundary between their three conceptualizations is blurry connects well with my perspective of connected or overlapping nature of components of teachers' knowledge. Third, attempts by the KAT project to develop measures of the profile of knowledge for teaching algebra using specific school algebra curriculum and ideas about the tasks involved in teaching is a departure from the qualitative measures of earlier researchers. Finally, this conceptualization by KAT has implications for my thinking about the types of knowledge teachers need for teaching other content of high school mathematics curriculum.

It is my hope that this conceptual framework will not only help me to focus on which of these three types of knowledge hypothesized by KAT is best related to student performance, but also how they are transformed into Ghanaian high school mathematics teachers' teaching.

CHAPTER 3: METHODS

This chapter opens with a discussion of research designs used in this study. This discussion is linked to the research questions that guided the study. This is followed by description of the target and accessible populations. After this, the selection of participants for this study is discussed. In addition, a description of the instruments used in collecting data for the study is presented. Thereafter, a detailed account of the procedures used in data collection is presented. The chapter ends with a look ahead into how data for this study were analyzed.

3.1 Research Design

This study adopted two types of designs based on the nature of the research questions used in the study. For instance, the first and second research question involves four different populations based on their college-level coursework and teaching experience. These were 1) pre-service mathematics education students in the final year of their college preparation program, 2) final year mathematics major students (considered possible prospective teachers), 3) final year statistics major students (also considered possible prospective teachers) and 4) in-service teachers from the participating senior secondary schools. For these different populations, the quality of their school knowledge, advanced knowledge and teaching knowledge was what was assessed using the adapted KAT instruments. The three hypothesized knowledge types were considered as three discrete factors of knowledge for each person. Therefore, a factorial design was used for the first and second research questions.

The third research question was aimed at investigating the relationship between in-service teachers and their students who participated in the study. Thus, this research question emphasizes causal analysis as the main research goal and multivariate linear regression as the main statistical tool. Thus, the design used with respective to the investigation into the issues raised in the third research question was measuring the relation between two variables and was therefore conducted using a correlational design.

3.2 Target Populations

The target population from which participants of the study were selected comprised three main populations. These were, 1) university seniors pursuing mathematics education, mathematics and other mathematics related programs in the country's universities, 2) in-service mathematics teachers from senior secondary schools (i.e., high schools) in Ghana, and their 3) high school seniors taking elective mathematics.

The decision to include each of these subgroups was based on the aims of the study and hypotheses made about the type of knowledge each subgroup would have by the time of the study. For instance, one aim of the study was to investigate the profile of knowledge (i.e., the nature of and possible differences in the level or quality of knowledge) of prospective and in-service of senior secondary school mathematics teachers. Based on this objective, it became necessary to focus on both in-service and prospective teachers. Consequently, there was the need to target teachers who were at the time of the study teaching mathematics (either Core Mathematics or Elective Mathematics) in the senior secondary schools. Since some of these in-service mathematics teachers would have been teaching for a long time, it was hypothesized that some of them would have forgotten aspects of the content of their university coursework. In addition, it was hypothesized that, depending on the number of years of teaching experience, some in-service teachers will have a high level of teaching knowledge while for others the level could be low.

In addition, it was also necessary to focus on university students who could be teaching soon after their programs in order to find out what their profile of knowledge looked like before they entered the teaching field. In Ghana, this group of pre-service teachers could be said to comprise not only mathematics education students, but also students majoring in mathematics or a related area. This is because, to fill vacancies left in the teaching field following the mass exodus of Ghanaian teachers, the National Service Secretariat, every year, posts both mathematics education majors and mathematics (or related area) majors to the senior secondary schools to teach mathematics. Therefore, pre-service teachers were thought of as comprising final year students of mathematic education, mathematics and related discipline. It was hoped that by the time of the study, final year students in the selected universities would be close to completing their respective college coursework and would consequently have acquired the advanced knowledge, which most teachers at the senior secondary schools covered while at college. Based on this, it was hypothesized that the mathematics majors in this group will have high knowledge of school algebra, high advanced knowledge of mathematics but low teaching knowledge.

Similarly, it was hypothesized that the mathematics education majors in this group will also have high knowledge of school algebra, mixture of high and low advanced knowledge of mathematics but a mixture of low and high teaching knowledge because of their teacher certification program.

A summary of the hypothesized level of knowledge of the target population of in-service teachers and the final year university students is presented in Table 3.2.1 below.

Table 3.2.1 Rationale for Inclusion of University Students and Teachers

Target Population	School Knowledge	Advanced Knowledge	Teaching Knowledge
Final year mathematics majors	High	High	Low
Final year students majoring in areas related to math	High	Mixture	Low
Final year mathematics education majors	High	Mixture	Mixture
In-service high school mathematics teachers	High	Mixture	High

Another aim of the study was to examine the relationship between in-service teachers' knowledge for teaching algebra and the performance of their students in algebra. Prior to the conduct of fieldwork for this study, the content of algebra in both the Core and Elective Mathematics syllabuses was reviewed and extracted (see Appendix II). It was the initial review that provided the rationale for deciding that Elective Mathematics students in SSS 3 in Ghana were the right group of students who would have had the best opportunity to cover what the student instrument was testing.

3.3 Accessible Population

Because of financial and other practical constraints, not everyone in the target populations described in Section 3.2 above could be contacted. A decision had to be made to narrow the selection to a smaller number of institutions. For instance, at the time of this study, there were about twenty universities in Ghana. Of these, six were public institutions and the remaining fifteen were private institutions- mostly established by various religious missions. However, only three of the public universities had mathematics departments that offered degrees in mathematics. These were the University of Ghana in Accra, Kwame Nkrumah University of Science and Technology in Kumasi, and University of Cape Coast in Cape Coast. In addition, only two public universities, University of Cape Coast and University of Education, Winneba, offered degree programs in mathematics education. Thus, in all, four of the country's universities hosted departments from which final year undergraduate students in mathematics, mathematics education or a related field could be selected. The accessible population of university students (earlier referred to as pre-service senior secondary school mathematics teachers) therefore consisted of students from the relevant departments in these four universities. Overall, it was estimated that a minimum of 100 students could be recruited in each of the five aforementioned departments to participate in the study.

The location of the aforementioned four universities had implications for the senior secondary schools to be selected for this study. To reduce cost and facilitate fieldwork, a decision was taken to limit the selection of senior secondary schools to schools located within and around cities in which the aforementioned universities were located. Thus, the accessible population of senior secondary school mathematics teachers and their students were limited those in schools located at Accra, Kumasi, Cape Coast, and Takoradi. Takoradi was added to the list of towns from which to select senior secondary schools because of its proximity and easy assess from Cape Coast, as well as, the concentration of schools there. Overall, it was estimated that about 200 in-service teachers and 2000 students could be accessed from the senior secondary schools.

3.4 Selection of Subjects

The original plan was to conduct the study between May 2006 and July 2006. As is the practice in Ghana, selection of subjects began with the seeking of permission of the heads of departments or institutions. Since the researcher was located in the US, the beginning of this process of securing such permission was done with telephone contacts with heads of mathematics and mathematics education departments in the aforementioned universities, as well as, headmasters/headmistresses of the target senior secondary schools. During these initial telephone conversations, the purpose of the study was explained to them and a time for face-to-face meetings with them was set. The plan was to get approval from the heads of departments and institutions for the study to be conducted in their institutions and to allow access to the students and teachers who would eventually decide to agree to participate or opt out of the study.

These initial telephone discussions were therefore aimed at facilitating planned visits to each of the research sites originally in May 2006. Unfortunately, approval for the commencement of the study from Michigan State University's Social Science, Behavioral Science and Education Institutional Review Board (SIRB) was delayed due to issues that needed to be clarified about the initial application sent to the Review Board. By the time approval was obtained, senior secondary school teachers in Ghana were on strike over working conditions. Consequently, fieldwork for the study was done in November and December 2006.

3.4.1 Selection of the University Students for the Study

In all, 301 students from three departments in three of the target universities agreed to participate in the study. These students were from the Departments of Mathematics at University of Ghana and Kwame Nkrumah University of Science and Technology, and the Department of Mathematics and Statistics at University of Cape Coast. Also included were students from the Department of Science and Mathematics Education at University of Cape Coast. Table 3.4.1 shows the distribution of the participating university students by their major area of specialization instead of their universities. This style of presenting this aspect of data collected has been adopted to maintain confidentiality of participants

Table 3.4.1 Number of Participating University Students by Major Area

Major Area of Specialization	N
Mathematics	132
Statistics	44
Mathematics Education	125

Selection of participants from these universities started with meetings with heads of departments who had agreed during earlier phone discussions to permit the study to be conducted in their institutions. These meetings took place before the commencement of the project and gave the heads of departments the opportunity to further discuss the project and ask questions. At the initial meeting in each university, the head of department was given two copies of the consent forms. One copy was for his/her records and the other copy was signed by him/her and collected back. The meeting also provided opportunities for a preliminary meeting with the students who participated in the study.

At the meeting with the students, they were informed about the project and the voluntary nature of students' participation was emphasized. In this way, they were given the opportunity to decide, without any form of coercion, to participate or not to participate in the study after any questions participants had were addressed. The meeting also discussed how the privacy of participants was going to be protected, as well as the risks and benefits to be derived in participating in the study. In addition, steps taken in the study to protect or minimize any risks involved in participating in the study were discussed. Participants were also provided with the researcher's contact information, as well as those of relevant personnel at MSU who they could contact should they have any further questions.

Those who agreed to participate in the project were then given the written consent forms on the day of administration of the instruments. There were no criteria to exclude any student in the mathematics and mathematics education departments of any of the participating universities. Instead, any student in these departments who agreed to participate was given the chance to do so and none was paid to participate.

3.4.2 Selection of In-service Teachers for the Study

A similar approach was taken in the senior secondary schools to select in-service teachers who participated in the study. Through earlier telephone calls to heads of institutions about the study, eight headmasters/headmistresses agreed to allow the study to be conducted in their schools. These consisted of five schools in Cape Coast and three in Takoradi. The Cape Coast schools comprised Mfantsipim School, Wesley Girls' High School, St. Augustine's Secondary School, Adisadel College and Holy Child School. Ghana Secondary Technical School, Fijai secondary School and Archbishop Porter Girls' Secondary School were the schools in Takoradi that participated in the study.

On arrival in Ghana, initial visits were made to these eight schools to meet with headmasters or headmistresses in charge to further discuss the purpose of the study and arrange to meet with the mathematics teachers. At the meeting with teachers, the purpose of the study was explained to them. In addition, steps taken to protect their privacy were discussed, as well as, the risks and benefits involved with their participation. Teachers were also given the opportunity to ask questions and the written consent was shown to them. As was done in the universities, teachers who agreed to be participants were also provided with contact information of the researcher, as well as, those of relevant personnel here at MSU who they could contact should they have any further questions. At the end of the meeting, a day was agreed upon for administration of the instrument. Participating teachers only signed consent forms on the day of administration of the instrument.

With in-service teachers too, there were no plans to exclude any mathematics teacher in any of the participating senior secondary schools from the study. Participation was purely voluntary and the teachers' instrument was administered to all teachers who agreed to participate in the study. Unfortunately, only 38 teachers from the eight schools participated in the study. Many of them complained that their strike action had caused them to miss too many class hours and they needed to make up for the lost hours. As was the case of the university students, none of the in-service teachers was paid to participate in the study.

Table 3.4.2 below shows the breakdown of in-service teachers by school who participated in the study. To ensure confidentiality, participating schools have been identified by codes not traceable to the schools.

Table 3.4.2 Number of Participating In-service Teachers by School

School Code	Number of Classes	Number of Participating Teachers	Number of Classes whose Teachers participated
A	3	6	3
B	2	4	0
C	8	13	2
D	8	1	0
E	7	2	0
G	6	6	2
H	7	6	3
J	3	0	0
Total	44	38	10

As Table 3.4.2 indicates, with the exception of school J, where no in-service teacher agreed to participate in the study, there was at least one teacher from each of the participating school in the study. In addition, each of the participating senior secondary schools had multiple classes per grade level. Unfortunately, not all the teachers who, at the time of the study, were assigned to teach the specific participating classes agreed to participate in the study. Only four out of the eight participating schools had classes whose assigned teachers participated in the study. These were schools with codes A, C, G and H. School A was the only school in which teachers of all the participating classes also participated in the study. In School C, teachers from only two out of the eight participating classes took part in the study. In Schools G and H, the number of participating teachers who were assigned to participating student classes was two out of 6 and three out of seven respectively. In the remaining four schools, with codes B, D, E and J, none of the teachers who participated in the study was assigned to any of the participating student classes.

In the sections that follow, the demographic data of the in-service teachers who participated in the study are discussed. Table 3.4.3, below, presents the college level mathematics and mathematics education courses taken by the in-service teachers who participated in the study.

A cursory look at Table 3.4.3 shows that though, a greater percentage of teachers whose classes were tested than the entire 38 teachers in the sample said they had taken each of the courses, the trend of courses by the two groups was similar.

Table 3.4.3 Courses Taken by In-service Teachers in the Sample

Courses Taken	Percent of Sample, N = 38	Percent of Sample Whose Classes were Tested, N = 10
Calculus	73.7	90
Linear Algebra	71.1	90
Abstract Algebra	63.2	70
Advanced Geometry / Topology	47.4	50
Real/Complex Analysis	13.2	20
Differential Equations	68.4	80
Number Theory/Discrete Math	36.8	40
Methods of Teaching Math	53.3	70
Psychology of Learning Math	53.3	70
Assessment of Math Education	47.4	50

In addition, of the mathematics and mathematics education courses taken, only two (i.e., Calculus and Linear Algebra) had been taken by more than 70 percent of the 38 participating teachers. Among the 10 teachers whose classes were tested in the study, the number of mathematics courses that had been taken by 70 percent or more increased to six, including two mathematics education courses. Table 3.4.4 below presents the teaching experience of the participating in-service teachers.

Table 3.4.4 Teaching Experience of Participating In-service Teachers

Number of Years Teaching	Core Mathematics		Elective Mathematics	
	Percent of Sample, N=38	Percent of Sample whose Classes were Tested, N=10	Percent of Sample, N=38	Percent of Sample whose Classes were Tested, N=10
0	7.9	20	10.5	–
1-2	15.8	–	15.8	10
3-6	28.9	30	21.1	30
7-10	15.8	20	18.4	30
10+	18.4	10	21.1	10
Missing Cases	13.2	20	13.2	20

From Table 3.4.4 it can be seen that only 7.6% and 10.5% respectively of the participating Core Mathematics and Elective Mathematics teachers indicated that they were in their first year of teaching at the time of the study. Of the sample whose classes were tested, 20% of the Core Mathematics teachers (i.e., 2 out of 10) were in their first year of teaching. None of the Elective Mathematics teachers whose classes were tested was in the first year of their teaching profession.

Also, about 63 and 60 percent respectively of the teachers in the sample and those whose classes were tested had taught Core Mathematics for a minimum of 3 years. For Elective Mathematics, these percentages changed slightly to 60 and 70 respectively.

The next table, Table 3.4.5, presents the nature of teaching certification held by the participating in-service teachers.

Table 3.4.5 Teaching Certificates of In-service Teachers in the Sample

Type of Certification	Percent of Sample, N = 38	Percent of Sample Whose Classes were Tested, N = 10
Bachelor of Education (Math)	50.0	40
Diploma in Education	10.5	20
Post Graduate Diploma/Certificate in Education	7.9	10
Other	10.5	10
Missing Cases	21.1	20

A quick look at Table 3.4.5 shows that the largest proportion of in-service teachers in the sample, as well as those whose classes were tested had Bachelor of Education (Math) certificates. The proportions were 50 and 40 percent respectively. As already explained in section 1.5, these were teachers who undertook degree programs in mathematics education and graduated not earlier than 1991. At the time of the study, only two universities in Ghana, the University of Cape Coast and University of Education, Winneba offered such degree programs.

Only 10.5 % of the teachers in the sample and 20% of those whose classes were tested indicated that they had Diploma in Education certifications. These were teachers who graduated from the University of Cape Coast not later than 1991. They pursued four-year Bachelor of Science degrees in mathematics or a related field and while on that program spent the second to the fourth year pursuing the education diploma alongside. This arrangement was mandatory for every student at the University of Cape Coast those years and UCC was the only university that offered such programs. Finally, the 7.9% and 10% respectively of the sample and those whose classes were tested had obtained Post Graduate Diploma/Certificate in Education. At the time of the study, this program was run on sandwich basis only at the University of Cape Coast during the university's regular summer break. It was opened to teachers who enter the teaching field with bachelor's degrees in mathematics or a related field other than education.

3.4.3 Selection of Senior Secondary School Students for the Study

The process of selecting the participating senior secondary school seniors from the aforementioned eight senior secondary schools, however, followed a slightly different approach. This was because in the senior secondary schools the majority of the students were between age seventeen and eighteen.

Consequently, the consent process needed to be different since parental consent was also needed for the students.

The senior secondary schools who agreed to participate in the study were boarding schools. This system made it possible for parents across the country to send their children or wards to those schools. Consequently, it was difficult to get access to each parent for his or her consent at the time of the study. Permission was therefore sought from the Institutional Review Board at MSU to use parental notification in place of the required parental consent. This was done by mailing written parental notification of the study to the parents of all final year Elective Mathematics students through the heads of the participating institutions. This was done about two months prior to the conduct of the study to make it possible for parents to discuss participation in the study with their wards or children ahead of the study. In spite of this parental notification, a meeting was conducted with the students during the initial visit to each school. At this meeting, the purpose of the study was explained to the students and questions they had were addressed. In addition, steps taken to protect their privacy were discussed, as well as the risks and benefits involved with their participation. Students who agreed to be participants were also provided with contact information of the researcher, as well as those of relevant personnel at MSU who they could contact should they have any further questions. Students who agreed to participate were given the opportunity to provide a signed assent of their agreement to participate in the study on the day of administration of the instrument.

In addition, there were no criteria to exclude any final year elective mathematics student in any of the participating senior secondary schools. The student instrument was administered to all students who agreed to participate in the study. In all, 1565 students from the eight senior secondary schools participated in the study.

The break down of the participating senior secondary school students in the eight schools by class is shown in Table 3.4.6 below. In Table 3.4.6, the participating senior secondary schools are identified by codes consistent with those used earlier in Table 3.4.2 above for the in-service teachers.

As shown in Table 3.4.6, above each of the participating schools had multiple classes per grade level. In all, the participating students came from 42 classes across eight schools. However, as explained in the previous section, teachers assigned to only ten out of these forty-two classes participated in the study.

Table 3.4.6 Participating High School Students by School and Class

School	Class	Number of students	Subtotal
	A1	48	
A	A2	45	134
	A3	41	
	B1	33	
B	B2	42	75
	C1	19	
	C2	26	
	C3	08	
C	C4	28	
	C5	25	207
	C6	42	
	C7	29	
	C8	30	
	D1	23	
	D2	37	
	D3	24	
	D4	25	
D	D5	32	218
	D6	42	
	D7	35	
	E1	31	
	E2	52	
	E3	42	
E	E4	35	
	E5	75	322
	E6	87	
	G1	46	
	G2	42	
	G3	45	
G	G4	17	230
	G5	35	
	G6	45	
	H1	39	
	H2	52	
H	H3	54	
	H4	13	280
	H5	24	
	H6	49	
	H7	49	
	J1	41	
J	J2	17	99
	J3	40	
Total		1565	1565

The breakdown of students in each of the classes whose teachers took part of the study is shown in Table 3.4.7 below.

Table 3.4.7 Classes with Teachers Who Participated in the Study

School Code	Class Code	Number of Students	Sub-total
A	A1	48	
	A2	45	134
	A3	41	
C	C1	19	45
	C2	26	
G	G1	46	88
	G2	42	
H	H1	39	140
	H2	52	
	H7	49	
Total		407	407

3.5 Instrumentation

The study involved administering mathematics assessment instruments adapted from the Knowledge of Algebra for Teaching (KAT) project at Michigan State University (MSU) to participants of the study. The adaptations involved changing the contexts and wording of questions in the KAT instrument to reflect Ghanaian contexts. For example an item that originally read,

> "At a storewide sale, shirts cost \$8 each and pants cost \$12 each. If S is the number of shirts and P is the number of pants bought, which of the following is a meaning for the expression 8S + 12P?"

was adapted into,

> "At a storewide sale, shirts cost ¢80000 each and a pair of trousers cost ¢120000 each. If S is the number of shirts and P is the number of trousers bought, which of the following is a meaning for the expression 80000S + 120000P?"

In this way, not only was the US currency changed into the Ghanaian currency, the prices of the items were also changed to reflect market values in Ghana at the time of the study. In addition, variations in names commonly used for the commodities used in the item were also changed to reflect the right contexts in Ghana. For example, in the adaptation made in the item used above, "pants" was changed into "trousers" as is commonly called in Ghana.

Similar changes were made in the survey questions to reflect Ghanaian contexts. For example, one of the original survey questions in the KAT instrument asked teachers to select algebra courses they have taught in the past as shown below,

> Which of the following algebra courses have you taught in the last five years? Check all that apply.
>
> - □ Pre-algebra
> - □ Remedial algebra
> - □ First year algebra
> - □ Second year algebra
> - □ Advanced algebra
> - □ Algebra in an integrated program
> - □ Other (please specify) ____________________
> - □ I have never taught any kind of algebra courses

This question was changed to focus on the type of mathematics (core or elective) and the grade level in the following way,

> Which of the following algebra courses have you taught in the last five years? Check all that apply.
>
> - □ Core Mathematics in SSS 1
> - □ Core Mathematics in SSS 2
> - □ Core Mathematics in SSS 3
> - □ Elective Mathematics in SSS 1
> - □ Elective Mathematics in SSS 2
> - □ Elective Mathematics in SSS 3
> - □ Other (please specify) ____________________

The process of adapting the items to suit Ghanaian contexts occurred at two levels. First, the researcher used his experience of the Ghanaian school and examination system to change the wording and contexts of the original KAT instrument appropriately. Next, the resultant versions of the instrument were given to two mathematics education professors in Ghana to look at and further modify appropriately, before they were administered. In all, only two content items on Form 1 were in contexts that were not applicable in Ghana. These two items were the only ones whose contexts were modified into Ghanaian contexts. One of the items was the example given in the preceding paragraph, in which the word "pants" was changed into "trousers" and the prices changed from dollars into the then Ghanaian currency. The other item only had the following phrase in it, "In a first year algebra class" modified into "In a first year elective mathematics class". In this way, the mathematics needed to answer the adapted questions, as well as their levels of difficulty were kept the same as they were in the original KAT instrument. The other aspects of that item were maintained. None of the items on Form was based on contexts that were not applicable to Ghanaian contexts. As a result, no item on Form 2 was modified. Thus, the instruments administered in this study were comparable with the instrument administered in the US by the KAT project.

Two main types of assessment instruments were administered. One of these was administered to the participating university students and in-service teachers. There were two main versions of this instrument; Form 1 and Form 2.

The other instrument was administered to final year senior secondary school students taking elective mathematics. Each instrument consisted of two sections: - a first section of survey questions and a second section of content items to be answered by participants. None of the instruments required the use of any identifiers like names, gender, school, identity numbers or anything that could be traced to participants.

The first section of the instrument administered to the participating university students and in-service mathematics teachers contained ten survey questions about participants' prior university coursework, their major and minor areas of specialization and their years of experience as teachers. The second section of each version of the instrument contained seventeen multiple-choice and three open-ended content items. These items were based on the algebra in the senior secondary school syllabus (i.e., school knowledge items), related advanced mathematics items, and items based on the tasks of teaching (i.e., teaching knowledge items).

Table 3.5.1 below presents the complete categorizations and distribution of the items among the three hypothesized types of knowledge in the framework that guided the study. In the categorizations of items used, the first number represented the type of knowledge in the domain of algebra knowledge for teaching (see the horizontal or x-axis of Figure 2.5.1). Among these first numbers, X = 1 represents "schools knowledge", X = 2 represents" advanced knowledge", while X = 3 represents "teaching knowledge". The second and third numbers represents the domains on y-axis (algebra domain) and the z-axis (domains of mathematical knowledge) respectively. Y = 1 represents content in "expressions, equations and inequalities" and Y = 2 is an indication that the content is item based on "functions. For the third number in the categorization, Z = 1, 2, 3, and 4 respectively indicates that the item was categorized to be based on core concepts and procedures, representations, applications, and reasoning and proof (see Fig 3 for these domains). For instance, the first item on Form 1 was categorization (1 2 3). This means that it was a school knowledge item based on function and their applications.

Table 3.5.1 Categorization of Items on Forms 1 and Form 2

Form 1		Form 2	
Item #	**Categorization**	**Item #**	**Categorization**
1	123	1	112
2	321	2	321
3	111	3	113
4	211	4	312
5	321	5	212
6	122	6	121
7	312	7	221
8	213	8	322
9	221	9	113
10	314	10	311
11	313	11	321
12	224	12	122
13*	224	13*	224
14*	121	14*	121
15*	324	15*	324
16*	211	16*	211
17	112	17	211
18*	311	18*	311
19	121	19	124
20	224	20	214

* Items in both forms

As shown in Table 3.5.1 above, on Form 1, six of the 20 content items were classified as assessing "school knowledge". The remaining 14 items were split exactly between "advanced knowledge" and "teaching knowledge". On Form 2, there were seven "school knowledge" items, six "advanced knowledge" and seven "teaching knowledge" items. In addition, between the two versions of this instrument were five common items (four multiple choice and one open-ended). Each of these versions of the instrument was completed by participants in no more than 60 minutes.

The first section of the instrument administered to the participating senior secondary students was a four-item questionnaire about participants' parents/guardians' educational background, and the extra resources (both human and material) available to them for learning their school mathematics. The second section of this instrument consisted of 12 of the 13 school knowledge items on the teachers'/university students' instrument. These comprised ten multiple-choice and two open-ended content items based on the algebra in the secondary school curriculum. The secondary students' instrument was also completed in no more than 60 minutes.

Samples of the content items on these instruments are shown in the "Public Released Items of the KAT Project" in Appendix IX. None of the actual instruments (i.e., Form 1, Form 2, or the high School students' Form) could be displayed in the appendix. The KAT project that owns the copyright was still in progress at the time of the writing of this study and so their items had not been made public.

Furthermore, it could not be guaranteed that by the time of the study each participating senior secondary school would have covered all the content being examined on the student instrument. As a result, an Opportunity to Learn Form (see Appendix X for a sample) was also created to give teachers of the participating classes the chance to indicate the extent to which students in their classes have had the opportunity to cover the mathematics required to answer each of the items on the student instrument. Using the KAT project public released items, an example of the opportunity to learn questions that followed a content item is as follows;

3. A student solved the equation, $3(n - 7) = 4 - n$ and obtained the solution $n = 2.75$.

   What might the student have done wrong?

Your students' opportunity to learn the mathematics in the question above:

Please, indicate whether or not your students had the opportunity to learn the mathematics needed to answer the question above.

A. Yes, I taught it because it is part of the required curriculum

B. Yes, I taught it, even though it is not part of the required curriculum

C. No, it is not part of the required curriculum so I did not teach it

D. No, even though it was part of the required curriculum, I did not/have not taught it.

The actual Opportunity to Learn form used in the instruments was based on the instrument adapted for study. However, because the KAT project was still in progress at the time of the writing of this dissertation and the actual items had not been made public the form could not be displayed.

3.6 Procedure

Between April 24 and May 5, 2006, telephone contacts were made with the universities that participated in the study as well as with heads of senior secondary schools in Accra, Kumasi, Cape Coast and Takoradi. During these discussions, the rationale of the study was discussed and verbal approval obtained to use the institutions as sites for the study. However, it was not until August 17, 2006, after approval for the study was obtained from Institutional Review Board (IRB) at MSU, that official invitation letters and parental notification for the senior secondary schools were mailed. The invitation letters were addressed to the heads of departments in the universities that had agreed to participate, the headmasters and headmistresses of the eight senior secondary schools as well as the heads of the mathematics departments in these secondary schools. Unfortunately, no follow-up for actual fieldwork could be done because teachers in senior secondary schools in Ghana were on strike by the time the IRB approval was obtained. However, continued telephone calls were made to keep options for the study open until the strike action was over on November 8, 2006 following a court order on October 31, 2006. The period between October 31, 2006 and November 10, 2006 was used to complete flight arrangements (i.e., renegotiating for ticket and transit visa issuance) for the study. In addition, between October 31, 2006 and November 10, 2006, final phone calls were made to heads of the participating departments and secondary schools. This provided opportunities for a timeline to be agreed upon for initial meetings and data collection.

Following these developments, the study was conducted towards the end of the fall semester of the 2006/2007 academic year. Fieldwork for this study began on November 13, 2006. The first two weeks, November 13 to 24, were spent visiting with heads of the participating departments in the three universities and the eight participating senior secondary schools. As already discussed, these initial meetings were aimed at further discussing the purpose of the study and agreeing on dates and times for administration of the instruments. At these meetings, each head of department or institution was given two copies of the consent forms. One copy of this form was signed by the head and collected while the other was meant for their records. In addition, each headmaster or headmistress of the participating senior secondary school was given the chance to complete the headmasters/headmistress' questionnaire during these initial meetings. The meeting also provided opportunities for preliminary meetings with the teachers and students who participated in the study. In addition, on November 13, 2006, the adapted instruments for the study were given to two Ghanaian experts (i.e., mathematics education professors) for review. By Friday, November 24, 2006, this review was completed and meetings had been held with each of the experts to discuss their points of view. It also made it possible for their suggestions to be incorporated during the weekend and for the instruments to be printed.

As already discussed, this study involved administering the instrument adapted from the KAT project to final year mathematics and mathematics education students in three universities, as well as teachers and final year elective mathematics students in eight senior secondary schools in Ghana.

Administration of the instruments commenced from the senior secondary schools. Data collection was done in the 3 participating schools in Takoradi on November 27 and 28 2006. The five schools in Cape Coast were done from November 29 to December 1, 2006. In each school, the student instrument was administered after the normal school hours so as not to disrupt classes. This was also possible because of the boarding facilities available in the schools. In Ghana, senior secondary schools usually have big halls where the entire school usually meets for morning devotion, entertainment and other whole school programs. These same rooms are usually converted into examination centers for the conduct of national examinations for the final year students each year. It was originally envisaged that all the students who agreed to participate in the study could be brought together in each of these rooms in the various schools so the instrument could be administered in one room. Unfortunately, it became clear during the initial meetings with the headmasters and headmistresses that there was insufficient time to get tables and chairs arranged in these rooms and make them usable as was envisaged. Consequently, participating students in each school were brought together in their own classrooms where they were given 60 minutes to complete the instruments. To make this possible, provision was made to travel with six graduate students from the University of Cape Coast (UCC) to help supervise the students. In addition, one lecturer from UCC, who is a friend, also traveled with us to help with supervision of students during the administration of the instruments. In the same way, the in-service teachers in each of the schools who agreed to participate were brought together to complete the instruments at a sitting lasting no more than 60 minutes. The class teachers completed the OTL forms during the time when the sessions with students were in progress.

Administration of the instruments in the universities was done in a slightly different manner. The universities were writing their end-of-semester examinations in December. These examinations notwithstanding, the heads of the participating departments had agreed with the participating students to incorporate the administration of the instruments with the examinations. Consequently, opportunity was provided during the examination weeks, from December 4 to December 8, 2006 for students who agreed to participate to come together to complete the instruments. This made it possible for participants in each department of the three universities to complete the instrument at a sitting lasting no more than 60 minutes. During the administration of the instruments, available forms, Form 1 and Form 2, were distributed in such a way that as students in a particular row completed Form 1, those in the next adjacent row completed Form 2. This was done for all the university students except with the statistics majors who were all given Form 1 because of their relatively small number. At the University of Ghana, the instrument was completed on December 4, 2006 and at Kwame Nkrumah University of Science and Technology on

December 6, 2006. At the University of Cape Coast it was completed on December 7, and 8 respectively by mathematics (and statistics) majors and their mathematics education counterparts.

3.7 Scoring of Content Items

For the content items on both the high school students' form and the teachers' forms, responses to the open-ended items were scored on a four-point scale. The exact scoring rubrics used could not be presented in the appendix because the KAT project, whose instruments was used in the study was still in progress at the time of the writing of this dissertation and the actual items had not been made public. The following, however describe the general spirit of the rubrics used for scoring the open-ended items:

Score of 4: All steps of the solution have carefully been laid. There does not have to be a reason for each step but each step follows reasonably from the one before. The solution can be shown as a model solution to any audience.

Score of 3: All steps of the solution have carefully been laid but there are minor errors.

Score of 2: There is an evidence of a chain of reasoning but some major conceptual mistake was made or there is an evidence of a chain of reasoning but the solution is not complete.

Score of 1: There is at least one correct statement.

Score of 0: Something mathematical is said but is not valuable for the question.

Score 777: Nothing mathematical is said (e.g. "no clue", "I don't Know")

Score 999: Blank

For each question, specific explanations have been given to make the scoring less difficult. For instance, the difference between what constitutes a minor error and a major conceptual error is made for each question. In addition, the KAT project had anchor papers for each score that could be used to resolve issues of disagreement.

To ensure reliable scoring, three doctoral students who have worked as graduate assistants on the KAT project, and were familiar with the rubrics used provided a second scoring independent of the scoring of the researcher. Papers on which there were variations in scoring were then identified and meetings arranged to discuss the differences and settle on agreed scores. Second opinion on the score for teachers' Form 1 and Form 2 were given by two different graduate assistants. Tables 3.7.1, below, summarizes the agreements among the scores. As shown in Table 3.7.1, below the agreement in scoring the two teachers' forms was more that 96% for each of the items. Similar high percentages in agreement were obtained in the scoring of the senior secondary school students' form.

Table 3.7.1 Agreements in Scoring Open-ended Items on Forms 1 & 2

Outcome of Scoring	Form 1			Form 2		
	Item 18	Item 19	Item 20	Item 18	Item 19	Item 20
Number Scored	150	150	150	189	189	189
Number of Agreements	150	147	145	186	188	185
Percentage Agreement	100	98	96.67	98.41	99.47	97.88

As shown in Table 3.7.2 below, the percentage agreement between the scores given to the two open-ended items on the student form by two scorers was between 98% and slightly over 90%.

Table 3.7.2 Agreements in Scoring the SSS students' Open-ended Items

Outcome of Scoring	Item 11	Item 12
Number scored	1565	1565
Number of agreements	1533	1540
Percentage agreement	99.04	98.85

For the purposes of analyses, the four-point scale of the open-ended items was reduced to a two-point scale by dividing each open-ended score by two. This was done to avoid putting too much weight on the open-ended items since responses to the multiple-choice items were scored as zero (0) for wrong responses and one (1) for right responses.

3.8 Mode of Data Analysis

Data for this study is both qualitative and quantitative in nature. The qualitative data comes from participants' responses to the demographic survey questions while the quantitative data comes from participants' responses to content items on the instruments used. The responses to the demographic survey were intended to be used to analyze the background of participants. In the case of the in-service teachers, the plan was to use these data to distribute them into the clusters shown in Table 1.5.1 using factor analysis performed on the data. Unfortunately, since very few in-service teachers participated in the study, this initial factor analyses was not performed.

Since the purpose of the analysis of data collected in the study is to help answer the research question, data analysis was done by research question.

3.8.1 Analysis of Data Related to the First Research Question

The first research question that guided this study was,

> "To what extent does Ghanaian pre-service and in-service secondary mathematics teachers' knowledge for teaching algebra corroborate the three categories of knowledge hypothesized in the KAT framework?"

To answer this question data from the university students and in-service teachers were used. Factor analysis was performed on each item in the instrument using the full set of data across both groups. The factor loading for each item was then analyzed to determine whether conclusions could be based on three factors. The decision to look for three factors is because the theoretical framework used for this study had <u>three</u> conceptualizations of teachers' knowledge: school knowledge, advanced knowledge and teaching knowledge. In addition, the nature of the loading of the items was also examined to confirm the extent to which items originally categorized as assessing the same type of knowledge loaded-together on separate factors. This was done to determine whether the three types of knowledge hypothesized by the KAT project came out distinctively and whether a differentiation could be made among the three different types of knowledge.

3.8.2 Analysis of Data Related to the Second Research Question

The second research question was,

"How does the knowledge for teaching algebra differ among the different categories of secondary school mathematics teachers and potential teachers in Ghana?"

Data for this research question came from the scores obtained by in-service teachers and university students on each of Forms 1 and Form 2. The university students who participated in the study comprised students majoring in three different areas; mathematics, mathematic education and statistics. Originally, two types of analyses were envisaged to answer this research question. The first involved investigating the difference in knowledge among the different categories of in-service teachers discussed in Section 1.5 (see Table 1.5.1) as well as how differences in their teaching experience affect their knowledge. The second involved comparing the four broad groups of teachers (in-service teachers, final year university students majoring in mathematics, mathematics education and statistics). However, because of the small number of in-service teachers who participated in the study the first type of analysis could not be performed. As a result, Analysis of Variance (ANOVA) was performed on the sub-total scores on the school knowledge items, advanced knowledge items and teaching knowledge items by in-service teachers and the various groups of prospective teachers (i.e., final year university students majoring in mathematics, mathematics education and statistics) on Forms 1 and 2 together. This was useful in establishing how the three types of knowledge (school knowledge, advanced knowledge and teaching knowledge) differed among the different categories of secondary mathematics teachers and potential teachers who participated in the study.

The outcomes from the analyses have been presented in Chapter 4.

3.8.3 Analysis of Data Related to the Third Research Question

The third question that guided this study was,

> "What is the relationship between the performance of in-service secondary school mathematics teachers in Ghana and the students of their classes?"

Data from the participating senior secondary school students and their corresponding in-service teachers were be used to analyze this question. Linear-regression was the main analyses done to answer this research question. The rational for this study is provided in the Section 4.4 (the rationale for using linear regression is discussed in this section).

3.8.4 Data not Used in the Analyses

Because of the small number of participants, especially the in-service teachers (38 in all and 10 for the high school classes used) and the statistics majors (44 in all), the demographic data of the university students and in-service teachers could not be used as effectively as planned in the analyses. Those data could, for instance, have been useful in dividing the in-service teachers into groups depending on the number of years they have taught, the mathematics courses and grade levels they have taught, and their teachers' certification. It could also have been used to group the entire data into the number of algebra courses taken. Analysis of Variance could have been performed on the subgroups that could have been generated to answer aspects of the research question. However, the number of participants in these subgroups would have been too small for any meaningful analysis.

Another data type that was not used was data generated using the Opportunity to Learn (OTL) forms. In all, 19 teachers completed these OTL forms. Only six of these were for the ten classes with teachers who participated in the study. Using this data therefore would have further reduced the number of classes used for the linear regression in Section 4.4.3 (i.e., Chapter 4, Section 3.2). A decision was therefore taken not to use these data.

CHAPTER 4: ANALYSIS AND RESULTS

In this chapter, the analyses of data and the results are presented. As already discussed in chapter three, the present study adapted the instruments generated form the KAT project for use in Ghana. The adaptation maintained the mathematics content of the items but only changed item contexts from the US contexts to Ghanaian contexts. This chapter has, therefore, been organized to open with a discussion of some of the item statistics, difficulty levels of the items, the point-biserial coefficients, and instrument reliability, generated from this study. However, instead of discussing these item statistics in isolation, a comparison is made with data collected from the KAT project as at the time of the study. After that, the analyses relevant to the research questions that guided this study have been presented. These analyses have been linked to the research questions in succession. In this way, beginning with the first research question that guided the study, data, analyses and results related to a specific research question are presented before attention is directed to the next research question.

4.1 Comparing Data from This Study and the KAT Validation Study

4.1.1 Item Difficulty Levels and Point-biserial Correlations

In this section, the present study has been referred to as the Ghana study. The difficulty levels presented in this section are the proportion of correct responses obtained from participants in the two studies (Ghana and KAT) respectively. As already explained in Chapter 3, multiple-choice items (items 1 to 17) on each of the two forms were given a score of 1 or 0 for right or wrong responses respectively. The open-ended items (items 18 to 20) were scored on a 4-point scale (i.e., scores from 0 to 4). The score obtained were later rescaled to a 2-point scale (i.e., scores from 0 to 2). On each of the two forms, the difficulty levels and point-biserial coefficients presented were obtained by further rescaling the open-ended scores to 0 to 1.

The item difficulty levels presented in this section are the proportions of responses to the item that were correct. In this way, a low proportion for an item is indicative of the item being more difficult than another item with a higher proportion.

The point-biserial correlation coefficients presented in this section are the item-total correlations between performance of participants on the individual items and their performance on the entire instrument administered in the study. In other words, the point biserial coefficients reflect how well items were "discriminating" between low- and high-performing participants. Ideally, high point-biserial coefficient of an item means that participants providing correct responses to that item are participants who obtained high total scores and low coefficients would mean that there is no correlation between score on an item and score on the test.

As discussed in Chapter 3, 449 participants in the KAT project and 150 from Ghana completed Form 1 while Form 2 was completed by 392 and 189 participants respectively in the KAT and Ghana studies. Table 4.1.1 presents results of a paired-sample *t*-test performed to compare the difficulty level of items on Forms 1 and 2 to the two samples.

Table 4.1.1 T-Test of the Difference in Difficulty of the Two Forms

Form	Mean Difficulty Ghana	Mean Difficulty KAT	Variance Ghana	Variance KAT	Pearson Correlation	df	*t*-stat	p-values
1	.4086	.5509	.0778	.0325	.7897	19	-3.62	.0018**
2	.4022	.5450	.0681	.0316	.4867	19	-2.71	.0138**

** Significant difference in difficulty level

From Table 4.1.1 each of the two forms was significantly more difficult to the sample from Ghana than the KAT sample in the US. The means and variances of the two samples on Forms 1 and 2 indicate that the items were more difficult to the sample who participated in this study (i.e., the sample from Ghana) than their US counterparts. In addition, the correlation coefficients computed for the two forms (see Table 4.1.1) reveals that the difficulty levels of the items on Form 1 to the Ghanaian sample were more strongly correlated to the levels on the US sample than the items on Form 2.

To examine the difficulty of each of the individual items, Table 4.1.2[8] was used. Table 4.1.2 presents the difficulty levels of items on Form 1 from the Ghana study and from the KAT project. A cursory look at this table reveals that, in general, majority of the items on Form 1 were more difficult for participants in the Ghana study than participants in the KAT study. Only six out of twenty items appeared slightly less difficult to the Ghana participants than the KAT participants. These were items 1, 2, 3, 11, 15 and 17. On Form 1, item 3, with difficulty .889 for KAT participants and .933 for the Ghana participants was the least difficult to participants in both studies.

[8] Values of difficulty levels, point-biserial coefficients and Cronbach's alpha of the KAT project were taken from the KAT project's draft summer 2007 technical report of their validation study.

Table 4.1.2 Difficulty Level and Rank of Items on Forms 1 and 2

Form	Item ID	Ghana Study Difficulty	Ghana Study Rank	KAT Study Difficulty	KAT Study Rank
	1	.880	3	.771	2
	2	.653	5	.760	3
	3	.933	1	.889	1
	4	.147	17	.314	20
	5	.347	10	.394	14
	6	.247	14	.510	11
	7	.487	6	.731	6
	8	.060	20	.356	18
	9	.340	11	.379	15
1	10	.427	8	.751	4
	11	.753	4	.708	7
	12	.200	16	.368	12
	13	.130	18	.560	9
	14	.201	15	.584	8
	15	.457	7	.339	19
	16	.298	12	.546	10
	17	.913	2	.748	5
	18**	.270	13	.482	12
	19**	.362	9	.451	13
	20**	.067	19	.377	16
	1	.989	1	.684	5
	2	.979	2	.620	8
	3	.328	9	.625	7
	4	.296	12	.635	6
	5	.169	18	.235	20
	6	.640	5	.758	3
	7	.222	15	.490	14
	8	.275	13	.513	13
	9	.730	3	.847	1
2	10	.682	4	.745	4
	11	.217	16	.783	2
	12	.164	19	.306	18
	13	.130	20	.560	10
	14	.201	17	.584	9
	15	.457	6	.339	16
	16	.298	11	.546	11
	17	.381	8	.281	19
	18**	.270	14	.482	15
	19**	.314	7	.529	12
	20**	.302	10	.317	17

** Open-ended items

In addition, four of the top five least difficult items to participants in the KAT study (i.e., items 3, 17, 1, and 2), were also in the top five least difficult items in this study. However, while item 4 (with difficulty .314) was the most difficult item for participants in the KAT study, the most difficult item for the Ghana study was item 8 (with difficulty .060). Also, four of the five most difficult items to participants in the KAT study (i.e., items 20, 12, 8, and 4), were also among the five most difficult items in the Ghana study.

Also, Table 4.1.2 reveals that, the items on Form 2 were more difficult to the Ghanaian sample than the U.S. On Form 2, as with Form 1, the Ghana participants performed better than the KAT participants in the only six out of twenty items. These were items 1, 2, 6, 10, 15 and 17. This is further discussed in chapter 5. In addition, item 9, with difficulty .847, and item 1, with difficulty level of .989 were the least difficult items for KAT participants and Ghana participants respectively. In addition, as with Form 1, four of the top five least difficult items to participants in the KAT study (i.e., items 1, 6, 9, and 10), were also in the top five least difficult items in this study. However, while item 5 (with difficulty .235) was the most difficult item for participants in the KAT study, the most difficult item for the Ghana study was item 13 (with difficulty .130). Also, unlike the trend in Form 1, only two of the five most difficult items to participants in the KAT study (i.e., items 5 and 12), were also among the five most difficult items in the Ghana study. These are further discussed in chapter 5.

Another statistic of importance is the point-biserial coefficients. Table 4.1.3 presents the point-biserial coefficients of items on the two forms from the Ghana study and from the KAT project.

Table 4.1.3 Point-biserial Coefficients of Items on Forms 1 and 2

Item ID	Form 1		Form 2	
	Ghana	KAT	Ghana	KAT
1	.083	.315	-.024	.421
2	.044	.397	.062	.330
3	.154	.341	.221	.389
4	.145	.234	-.062	.366
5	.149	.312	.244	.439
6	.013	.376	.139	.415
7	.183	.435	.385	.426
8	.173	.468	.206	.288
9	-.003	.313	.151	.195
10	.174	.378	.313	.229
11	.284	.285	.216	.363
12	-.002	.310	.410	.439
13	.155	.546	.242	.510
14	.070	.328	.278	.298
15	-.002	.254	.111	.342
16	.272	.596	-.131	.565
17	.286	.534	.129	.445
18**	.426	.610	.442	.577
19**	.392	.684	.384	.628
20**	.285	.611	.499	.603

** Open-ended items

A cursory look at Table 4.1.3 reveals that, compared with data from the KAT study, the point-biserial coefficients obtained in this study were smaller for the items on both forms. To determine whether the average discriminations of the two samples on the two forms were significantly different or not, a paired-samples *t*-test was conducted.

The output of this *t*-test is presented in Table 4.1.4. Table 4.1.4 reveals that there is a significant difference in the point-biserial coefficients of the two samples on each of Forms 1 and 2.

Table 4.1.4 T-Test of the Difference in the Point-biserial Coefficients

Form	Mean of Discriminations		Variance of Discriminations		Pearson Correlation	df	*t*-stat	p-values
	Ghana	KAT	Ghana	KAT				
1	.1641	.4164	.0165	.0184	.7406	19	-11.62	.000**
2	.2108	.4134	.0288	.0143	.2683	19	-5.05	.000**

** Significant difference in difficulty level

From Table 4.1.4 the coefficients obtained from the sample in Ghana are statistically smaller than those obtained from the U.S. sample on both forms. It was therefore concluded that the items on the two instruments could, worked significantly better in discriminating between high and low performing candidates in the KAT project's sample than their counterparts in Ghana sample.

4.1.2. Reliability for the Ghana Study and the KAT Study

The following table, Table 4.1.5, presents a comparison of the internal inconsistencies of the data from this study and the KAT Validation study conducted at the time of this study as calculated using Cronbach's Alpha. In Table 4.1.5, the internal consistencies of the total scores on each of Form 1 and Form 2 are presented.

Table 4.1.5 Reliability for Form 1 and Form 2[9]

Form	Cronbach's Alpha		Mean Score		Std. Deviation	
	Ghana	KAT	Ghana	KAT	Ghana	KAT
1	.521	.837	0.4086	0.5509	0.2789	0.1802
2	.643	.842	0.4022	0.5440	0.2610	0.1777

Since Forms 1 and 2 are instruments not only with the same number of items but also designed to measure the same construct, Cronbach's alpha can be interpreted as the correlation between scores obtained on the two forms. In this way, instrument reliability as measured by Cronbach's alpha can be seen as a measure or form of the inter-item correlation or an estimate of the proportion of variance that is consistent in the set scores on each of Forms 1 and 2. As a result, a high reliability would mean that the items on the instrument are measuring the same underlying construct well. Sometimes Cronbach's alpha is reduced because of a small standard deviation. As a result, in comparing the Cronbach's alpha obtained in this study (i.e., using the sample from Ghana) to value obtained from the KAT project's sample, it is necessary to also look at the standard deviations from the two samples. From Table 4.1.5, the Cronbach's alpha obtained on each of the forms from this study was smaller than the corresponding values obtained from KAT's sample at the time of this study. Compared with the KAT project, the small reliabilities values obtained in this project is worthy of note because of the higher standard deviations obtained from this study on each of the two forms.

9 Scores for the open-ended items range from 0 to 2 while those for the multiple choice items range from 0 to 1.

According to Nunnally and Bernstein (1994), though a reliability coefficient of .70 is sufficient in exploratory research, a value of .80 or higher should be required in basic research. Applying this cut-off criterion, it is clear that the coefficients obtained in this study were too small (see Table 4.1.5 above). In other words, whereas the instruments performed quite well in terms of reliability in measuring the underlying construct of knowledge for teaching algebra among the US participants (refer to coefficients obtained from the KAT data in Table 4.1.5 above), in this study conducted in Ghana, they did not perform well in measuring this construct.

Another interpretation of the reliability coefficient is to consider the nature of the construct being measured by the instrument. In general, if the construct being measured (in this case, knowledge for teaching high school algebra) is multi-dimensional in nature, reliabilities measured by Cronbach's alpha could be low. From this perspective, the low reliability coefficients obtained in the present study could be an indication that the instrument could be measuring a multi-dimensional type of knowledge. When this happens, a factor analysis could reveal the items which load strongest on which dimensions or factors. This provides one of the bases for the factor analysis presented in the next section to answer research question one.

4.2 Research Question One

The first research question that guided this study was, "To what extent does Ghanaian pre-service and in-service secondary mathematics teachers' knowledge for teaching algebra corroborate the three categories of knowledge hypothesized in the KAT framework?"

To answer this question, data from the university students and in-service teachers were used. Altogether, 150 and 189 participants respectively completed Form 1 and Form 2 (refer to Table 4.3.1 for descriptive statistics on each form). Exploratory factor analysis was performed on the data for each instrument separately (i.e., Form 1 and Form 2). Factor analysis was chosen because it helps, among other things, to examine the number of variables, called factors, which could be used to either completely or to a large extent explain the variation in scores in the data collected. In the conceptual framework, three types of knowledge had been hypothesized. However, in this study, no prior assumption was made about the truth or otherwise of this hypothesis. In other words, no specific decision was made earlier in this study about the exact number and nature of the underlying factor structure (i.e., of the type of knowledge measured by the instrument). Consequently, Exploratory Factor Analysis was used. The idea was to allow as many factors as items on each of the instruments to be extracted so that a decision could be made, based on the factor loadings, as to the number of factors that could be retained to explain the pattern of relationship among scores in the data. It helped to answer the question of whether there is enough evidence to conclude that three factors could be distinguished, a number corresponding to the types of knowledge hypothesized in the theoretical framework. In addition, exploratory factor analysis helped to determine whether the factors that emerged could be described, using the three types of knowledge hypothesized in the framework. The extraction method used was principal component analysis and the rotation method used was *Oblimin* with Kaiser Normalization. Oblimin rotation was used because of its ability to allow the factors extracted to be correlated.

The number of variables or factors needed to explain the variation in the data could have been modeled by using structural equation modeling (SEM) and factor analysis could have been incorporated in SEM to confirm these variables. However, SEM could not be used in this analysis because the 339 scores collected from the university students and in-service teachers in this study were far less than the minimum required for a very good SEM[10] analysis. Therefore Factor Analysis[11], as a stand-alone test, was the best for the small sample size (150 subjects or participants for Form 1 and 189 for Form 2) obtained in this aspect of the study. The analysis was initially done to retain three factors and later with two and eight factors. Three reasons accounted for this. First, as the next sections reveal, the strength of factor loadings in the initial analysis (using three factors) was not effective in allowing distinct labeling of the factors. Second, a scree-plot was used to confirm the three factors. However, the use of the scree-test is usually criticized for being subjective as different researchers could focus on different points on the curve as the points where the steepness

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[10] To use SEM, Pedhazur (1997) has argued the subject to variable ratio must be at least 30:1. Comfrey & Lee (1992) have also argued that to use SEM, "the adequacy of sample size might be evaluated very roughly on the following scale: 50 – very poor; 100 – poor; 200 – fair; 300 – good; 500 – very good; 1000 or more – excellent" (p. 217).

[11] According to Gorsuch (1983) and Hatcher (1994), in Exploratory Factor Analysis, a subject to item ratio of at least 5:1 is recommended while Nunnally (1978) argues for a ratio of 10:1.

smoothens. As a result, two such contentious points were focused on for analysis. As will be seen in the next section, the scree-plot appeared to have its elbow between 2 and 3 factors, suggesting that two factors needed to be retained. Consequently, the analysis was done with 2 factors as well. Third, the Kaiser-criterion applied in the test also pointed to the retention of 8 factors. The idea was to determine which of the three dimensions, 3, 2, and 8, gave the best interpretation of the factors.

To do this interpretation, loadings of absolute value above 0.30 were considered strong enough to be indicative of the nature of the factor. Therefore, item loadings on the various factors were compared and the ones with loadings 0.30 and above used. Also, since cross loading (i.e., loading of 0.30 or above on more than one factor) is indicative that an item cannot be uniquely assigned to any of the factors, such items were removed and not used to determine the nature of that factor (see Guadagnoli and Velicer, 1988). Finally, factors with fewer than three items were considered unstable and not labeled (see Costello and Osborne, 2005). In the next two sub-sections, results of the factor analysis are presented and discussed beginning with Form 1.

4.2.1 Factor Analysis on Form 1

4.2.1.1 Factor Analysis of Form 1 with Three Factors

The first step in the factor analysis was the examination of the number of factors needed to explain the variation in scores on the various items on Form 1. The table below, Table 4.2.1 shows how items loaded on various factors and the variance explained by all possible factor loadings when Factor Analysis was done to retain three factors. This table, Table 4.2.1, shows results of the number of possible factors that could be extracted from the data to explain the variation among the scores and their corresponding eigenvalues. The eigenvalues give an indication of the strength level of each of the extracted factors. Consequently, the eigenvalues could be used to decide on the required number of factors needed to represent the relationships in the data.

Table 4.2.1 Total Variance Explained by Each of the Factors of Form 1[12]

Factor	Initial Eigenvalues		
	Total	% of Variance	Cum. %
1	2.526	12.628	12.628
2	1.649	8.244	20.872
3	1.468	7.342	28.214
4	1.368	6.838	35.052
5	1.304	6.520	41.573
6	1.196	5.978	47.551
7	1.137	5.686	53.236
8	1.056	5.281	58.517
9	.992	4.962	63.479
10	.940	4.700	68.179
11	.882	4.412	72.591
12	.785	3.924	76.516
13	.705	3.523	80.039
14	.699	3.495	83.534
15	.638	3.192	86.726
16	.610	3.048	89.774
17	.596	2.981	92.755
18	.498	2.492	95.248
19	.482	2.409	97.656
20	.469	2.344	100.000

[12] Extraction Method: Principal Component Analysis.

In Table 4.2.1, a low eigenvalue for a given factor implies that factor's contribution to the explanation of variances in the variables is small and may be ignored. Consequently, in this analysis, the Kaiser-criterion (also referred to as the K-1 rule) of retaining only the factors with eigenvalues greater than 1.0 was initially used. Based on the initial eigenvalues, it is clear that by the K-1 rule, eight main factors could be retained. Together these eight explain about 58.5% of the variance. However, since the theoretical framework guiding this study hypothesizes three main knowledge types, the eight factors revealed by the Kaiser criterion did not make much sense in this analysis. Consequently, the scree-test plot was used for further check. Essentially, scree-test plots are graphs of the factors (as shown in Table 4.2.1 above) on the horizontal axis against the corresponding eigenvalues on the vertical axis. On this graph, as the number of factors increases (i.e., as one moves from left to right along the horizontal axis), the eigenvalues decrease (refer to this also from Table 4.2.1). However, the change in slope of the graph resulting from these variations is usually not constant but decreases as the number of factors increases. Conventionally, the steepness of the slope of various sections of the graph is examined and the x-coordinate of the point beyond which the variation in slope begins to be uniform is chosen as the needed number of factors.

The graph below, Figure 4.2.1, shows the plot needed for the scree-test. It can be seen from Fig. 4.2.1 that from between factor numbers 2 and 3 there is a major variation in the slope of this graph. The variation in the steepness of the graph reduces relatively more beyond factor 2 or 3. Hence, from the scree-test, only the first two or three factors were retained for further analysis. Thus, from the factor analysis, one inference was that two or three factors were needed. In other words, whereas the factor analysis extracted 20 factors for examination (because of the 20 content items on the research instrument), it can be inferred from the scree plot that the number of factors needed to explain the variation in scores in the data is either two or three. This subjective interpretation of the scree-plot was what provided the rationale for the analysis with not only 3

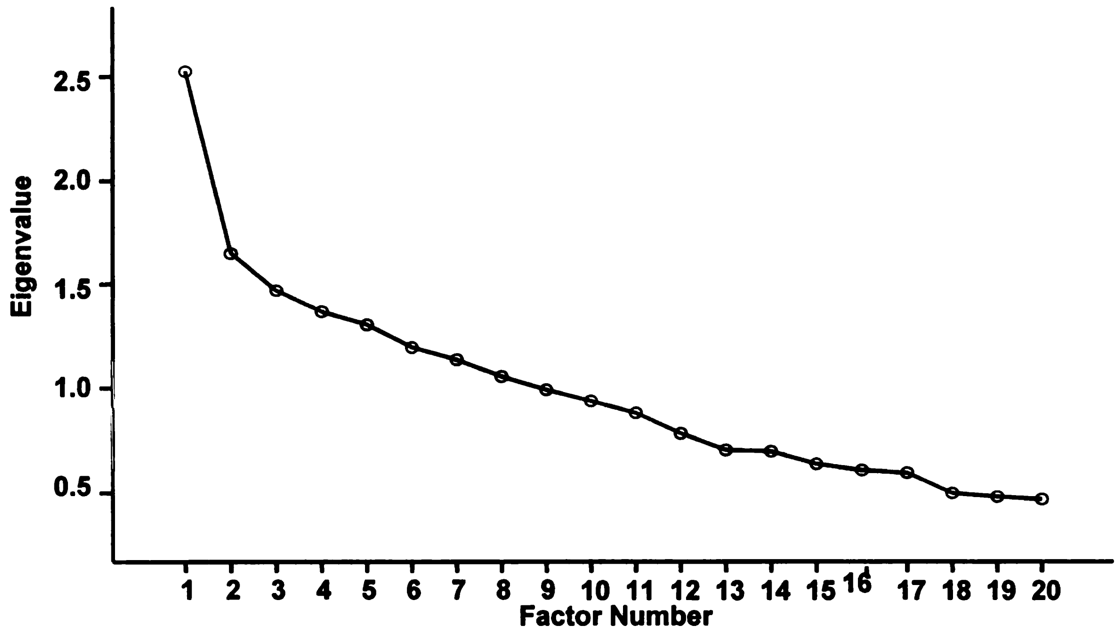

Figure 4.2.1 Graph of Factor Loadings for Items on Form 1

Focusing first on three factors, it can be seen from Table 4.2.2 that these three factors together explain about 28.2% percent of the variation in scores (see Table 4.2.1).

The next item from the SPSS output was Table 4.2.2, a table of communalities shown below. This communalities table shows how much (the percent) of the variance in the scores of the items has been accounted for or explained by all the extracted factors. The "communality" column shows the percent of variance in any given variable that is explained by the extracted factors. In general, high values of variances in the "communality" column of the table would mean that the items had a lot of characteristics in common and low values would mean that the items have little in common with each other.

Table 4.2.2 Communalities of Form 1[13]

Item ID	Communality
1	.185
2	.273
3	.282
4	.113
5	.247
6	.398
7	.379
8	.165
9	.145
10	.310
11	.344
12	.276
13	.233
14	.125
15	.198
16	.419
17	.203
18	.504
19	.476
20	.365

[13] Extraction Method: Principal Component Analysis. The communalities shown in this table is from the output of the Factor Analysis run to retain three factors.

In general, communalities need to be interpreted with respect to the interpretability of the factors. As a result, an item or variable with communality as high as 0.82, for instance, may tell much unless the factor on which that item is loaded is interpretable. On the other hand, communality as low as 0.28 could be meaningful if the item contributes to the interpretation of the factor on which it loads. In other words, the most important thing to focus on is not the communalities alone but the extent to which the items contribute in the interpretation of the factors though frequently this contribution is greater when communalities are high.

A cursory look at Table 4.2.2 reveals that in general, the variance in each item that was accounted for by the extracted factors was very low. It can be seen from Table 4.2.2 that twelve out of the 20 items had less than 30% of their variance explained by the other factors extracted. It can be seen from Table 4.2.2 that nineteen out of the 20 items had less than 50% of their variance explained by the other factors extracted. In addition six of the items had communalities below 0.2 meaning less than 20% of their variances are explained by the extracted factors. Thus, though test items generally have low reliabilities which may result in low communalities, the low communalities in Table 4.2.2 are worth noting because, as would be seen in the sections that follow, the pattern of item loadings on the factors did not help to interpret the factors.

A rotation matrix was used to help interpret how the items were loaded on the three retained factors. Generally, each loading on a table such as this corresponds to the correlation between the item and the factor. Conventionally for each item or variable the table is examined for the factor on which the variable loads (i.e., where the correlation is greatest) by using the absolute values of the loadings. As already explained, in an attempt to label the three extracted factors, the criterion of considering loadings of absolute value greater than 0.3 as illustrative of the potential nature of the factor was used. The idea was to see whether a case could be made, using the pattern of the strength of loading to uniquely label each of the three retained factors using one of the three hypothesized knowledge types in the theoretical framework that guided the study. Then by examining the types of items or variables loading strongly on a particular factor, an attempt was made to define or label the factor. Table 4.2.3 below presents the how the items on Form 1 were loaded on the three factors retained.

Table 4.2.3, reveals that four items (items 3, 16, 18, and 19) cross loaded on more than one factor. Consequently, they were not assigned to any of the three retained factors. Four other items (items 5, 9, 12, and15) uniquely loaded strongly on Factor 1. Of these two each were classified by the members of the KAT project as measuring advanced knowledge (items 9 and 12), and teaching knowledge (items 5 and 15). Five items (items 1, 2, 6, 8, and 20) were uniquely loaded on Factor 2. Two of these (items 1 and 6), according to the KAT classification, were school knowledge items, another two (items 8 and 20) were advanced knowledge items, and the remaining one (items 2) was a teaching knowledge item.

Table 4.2.3 Item Loadings on the Three Retained Factors of Form 1[14]

Item ID	Factor 1	Factor 2	Factor 3
1	-.109	-.419	.014
2	-.120	-.514	.051
3**	.375	.007	.396
4	.148	-.043	.311
5	.440	-.035	.257
6	.015	.521	.284
7	-.235	-.060	.553
8	-.128	-.305	.272
9	-.352	-.158	-.016
10	.268	.209	.432
11	.091	-.080	.584
12	-.507	-.128	.069
13	-.163	-.092	.443
14	-.266	-.098	.209
15	.426	-.067	-.056
16**	.363	-.448	.349
17	.017	-.217	.419
18**	.345	-.604	.149
19**	.444	-.481	.262
20	-.113	-.579	.216

** Items with cross-loadings

Six items (4, 7, 10, 11, 13, and 17) uniquely loaded on Factor 3. According to the classification by the KAT project (refer to Table 3.5.1), two of these were advanced knowledge (items 4, 13) items, three (items 7, 10, and 11) were teaching knowledge items and the remaining one was a school knowledge item (i.e., item17).

From the foregoing discussion, items measuring the same type of knowledge, at least as classified by KAT as, did not load selectively on the

[14] Extraction Method: Principal Component Analysis.
Rotation Method: Oblimin with Kaiser Normalization.

respective factors. It was, therefore, not possible to label any of the factors exclusively using the three hypothesized knowledge.

The nature of the factor loadings was also analyzed using the other two dimensions (Algebra Content and Domain of Mathematical Knowledge). The idea was to test whether, instead of the types of knowledge these items were categorized (according to KAT) to be measuring, a case could be made about the nature of the item loadings using their attributes in the other two domains in the KAT framework (see Figure 2.4.2). Of these two domains, only the Algebra Content dimension showed some promise on one of those factors. On Factor 1, all the four strongly loaded items (items 5, 9, 12 and 15) were originally classified as items based on the content of "Functions" (see Figure 2.4.2). Factors 2 and 3 had items loading strongly on both categories of "Algebra Content" in the KAT framework. For instance, of the six items loading strongly on Factor 3, five had prior to this study been classified by members of the KAT project as based on the content of "Expressions, Equations and Inequalities" (items 4, 7, 10, 11, and 17) while the remaining one had been classified as based on "Functions" (item 13). Also, of the five items that loaded strongly and uniquely on Factor 2, four (items 1, 2, 6, and 20) were based on "Functions", while the remaining one, item 8, was based on the content of "Expressions, Equations and Inequalities". Thus, since items measuring only one of the two kinds of algebra content did not load exclusively on any of the three factors, it was not possible to label the factors using any of the two content of algebra categories as developed in the KAT project.

Similarly, analyzing the nature of item loadings on the three retained factors in terms of the third dimension of the items as defined in the KAT project, the domain of mathematical knowledge, did not help to uniquely label any of the factors. For instance, of the four strongly loading items of Factor 1, two (items 5 and 9) were classified by the KAT project as measuring core concepts and procedures and the other two (items 12 and 15) as measuring reasoning and proof. On Factors 2 and 3, items measuring each of the four domains of mathematical knowledge were strongly loaded. On Factor 2, one item, item 2, was based on core concepts and procedures, two items (items 1 and 8) were based on applications, one item (item 6) on representation and another one (item 20) on reasoning and proof were strongly loaded. Finally, on Factor 3 one item each was based on core concepts and procedures (item 4), and applications (item 11), while two each were based on representations (items7 and 17) and reasoning and proof (items 10 and 13).

Thus, none of the three dimensions of the KAT framework could be used as the basis for labeling the three retained factors. Based on the possibility of interpreting the elbow shown of the scree-plot in Figure 4.2.1 as corresponding to two factors, factor analysis was done with two dimensions. In the next sub-section, this analysis is presented.

4.2.1.2 Factor Analysis of Form 1 with Two Factors

The total variance explained using two dimensions (i.e., when the factor analysis was run to retain two factors are presented in Table 4.2.1.

A cursory look at this table shows that only about 20.9% of the total variance is explained by the two retained factors. Also as in the earlier analysis, applying the Kaiser-criterion suggests that eight factors could be retained (explaining 58.5% of the variance).

The factor loadings were also examined using the two factors. Table 4.2.4 below how the items on Form loaded on two factors. As revealed by Table 4.2.4, using a minimum strength of 0.3 as illustrative of the potential nature of the factors, the factors could not be labeled using any of the three types of knowledge hypothesized in the KAT project.

Table 4.2.4 Item Loadings on the Two Retained Factors of Form 1

	Factor	
Item ID	1	2
1	.392	-.101
2	.486	-.088
3	.095	.517
4	.098	.330
5	.132	.423
6**	-.475	.326
7	.077	.352
8	.304	.127
9	.095	-.204
10	-.115	.527
11	.153	.530
12	.050	-.201
13	.109	.288
14	.073	.039
15	.128	.144
16**	.532	.405
17	.257	.335
18	.659	.203
19**	.568	.364
20	.568	.046

** Items with cross-loadings.

As can be seen from Table 4.2.4, there were cross loadings of items 6, 16, and 19. They were, as a result, not uniquely assigned to any of the factors. For the items that strongly loaded on only one of the two factors, five (items 1, 2, 8, 18, and 20) loaded on Factor 1. Of these, two each had been categorized in the KAT project as measuring advanced knowledge (items 8 and 20) and teaching knowledge (items 2 and 18), while the remaining one, item 1, was categorized in the KAT project as a school knowledge item. On Factor 2, seven items (items 3, 4, 5, 7, 10, 11, and 17) loaded strongly. These were distributed as follows using the KAT categories: two school knowledge (items 3 and 17), one advanced knowledge (item 4) and four teaching knowledge items (5, 7, 10, 11). Thus, on each of the two factors, items earlier categorized to be measuring all the three hypothesized knowledge were loaded. In other words, the nature of the loadings on the two retained factors did not permit a differentiation in the nature of the factors on the basis of the three hypothesized knowledge categories.

The items loading were also analyzed with respect to the classification of the items in terms of the algebra content (i.e., "Expressions, Equations and Inequalities" or "Functions") they were measuring. In this regard, it was observed that the items that loaded strongly on Factor 1 comprised two "Expressions, Equations and Inequalities" items (items 8 and 18) and three items (1, 2, and 20) based on "Functions". Then on Factor 2 there were six "Expressions, Equations and Inequalities" items (items 3, 4, 7, 10, 11, and 17) and one item, item 5, on "Functions". Thus, both types of content loaded together on each of the two retained factors. Consequently, the factors could not be defined in terms of the two algebra content areas on which the items were based in the KAT project.

The third dimension of the KAT framework, the domain of mathematical knowledge, was also used to analyze the nature of item loadings on the two factors. The items did not load on the factors in any unique manner even on the basis of this third domain. For instance, of the five items that strongly loaded on Factor 1, two each (items 2 and 18 and items1 and 8), were respectively classified as measuring core concepts and procedures, and reasoning and proof. The remaining one, item 20, measured application. On Factor 2, there were three items on core concepts and procedures (items 3, 4, and 5), two on representation (items 7 and 17) and one each on application (item 11) and reasoning and proof (item 10). As a result of this non-unique nature of the factor loadings, it was not possible to distinctively label the two factors in terms of the domain of mathematical knowledge as hypothesized in KAT.

Thus, attempts at using all the three dimensions of the KAT framework to label each of the two retained factors were not successful. Consequently, the next level of analysis was done using eight dimensions. As has already been explained the basis of using eight factors was the outcome of applying the Kaiser-criterion. This analysis is discussed in the next subsection.

4.2.1.3 Factors Analysis of Form 1 with Eight Factors

Table 4.2.1, below, presents, the variance explained when the analysis was run with eight factors. A cursory look at this table shows that when eight factors are the focus, about 58.5% of the total variance is explained.

Next, the factor loadings were examined using eight factors. As in the earlier analyses, a minimum strength of 0.3 was used as illustrative of the potential nature of the factors. Also, consistent with the earlier analyses, in an attempt to interpret the factors, items with cross loadings could not be uniquely assigned to any of the factors. Table 4.2.5 below presents how the items on Form 1 loaded on the eight retained factors.

Table 4.2.5 Item Loadings on the Eight Retained Factors of Form 1

Item ID	Factors							
	1	2	3	4	5	6	7	8
1**	.182	-.126	-.120	.578	.073	.351	.066	.049
2	.282	-.040	-.100	.054	-.117	.100	.706	.168
3	.165	.164	.242	-.061	.044	.170	.059	-.724
4	.086	.283	.145	.596	.069	-.287	.171	-.268
5**	.388	.418	-.103	-.248	-.233	-.072	-.077	-.065
6**	-.250	.650	.019	-.025	.049	-.035	-.353	.145
7	.097	.095	.757	-.119	-.063	.100	-.010	.071
8	.103	.080	.190	.068	-.816	.063	.056	-.037
9	-.133	-.173	.143	.602	-.254	.014	-.266	.210
10	.050	.611	.040	-.076	-.051	.006	.069	-.263
11	.247	.602	.169	.084	.089	.190	.278	.000
12	-.028	.000	.190	-.066	.130	.162	.189	.702
13	.026	.015	.650	.262	-.074	-.132	.102	-.166
14	-.106	.119	.000	.004	.020	.788	.010	-.007
15**	.378	-.012	-.168	.050	-.014	.023	-.613	-.020
16	.719	.096	.189	-.059	.002	.008	.105	.008
17**	.392	.105	.384	.127	.462	.316	-.056	.141
18**	.622	-.011	-.136	.332	-.180	.070	.218	-.164
19	.695	.016	.063	.071	-.056	.069	.042	-.188
20**	.319	-.096	.051	.053	-.487	.540	.172	-.025

** Items with cross loadings

To simplify the discussion with these eight factors, the strongly loaded items on each of the factors have been extracted and presented with their KAT categorization (in parenthesis) in Table 4.2.6 below (without items with cross loadings).

As shown in Table 4.2.6, three factors, Factors 5, 6, and 7 had only one item each loading strongly on them. In addition, four factors, Factor 1, 3, 4, and 8 had only two items each strongly loading on each of them respectively. Again, one or two item loadings were considered too small to be indicative of the nature of these factors.

Table 4.2.6 Form 1 Item Loadings and Categorization on the Eight Factors

Factors	Number of Items	Item ID and Categorization (in parenthesis)		
1	2	16 (2, 1, 1)	19 (1, 2, 1)	
2	3	5 (3, 2, 1)	10 (3, 1, 4)	11 (3, 1, 3)
3	2	7 (3, 1, 2)	13 (2, 2, 4)	
4	2	4 (2, 1, 1)	9 (2, 2, 1)	
5	1	8 (3, 2, 1)		
6	1	14 (1, 2, 1)		
7	1	2 (3, 2, 1)		
8	2	3 (1, 1, 1)	12 (2, 2, 4)	

As a result, seven of the eight retained factors (i.e., Factors 1, 3, 4, 5, 6, 7, and 8) were considered too unstable to be correctly interpreted in this study. The only factor on which the minimum of three items loaded was Factor 2. These three items were items 5, 10, and 11. All three items were teaching knowledge items. They did not have any other attribute in common. Thus, only one of the eight retained factors could be labeled, if necessary in terms of one of the types of knowledge hypothesized in the study.

Thus, none of the three factor analyses helped to label all the factors retained on Form 1. This result is further discussed in chapter five. Similar analyses were done on Form 2. The next section presents these analyses.

4.2.2 Factor Analysis on Form 2

As was done with Form 1, exploratory factor analysis using the principal component analysis method of extraction was conducted on participants' score on each of the items on Form 2. The aim was to examine whether variations in participants' score on the 20 items on the research instrument (i.e., Form 2) correspond to multiple underlying factors or to three factors (e.g. the three types of knowledge for teaching algebra; school knowledge, advanced knowledge and teaching knowledge) as hypothesized in the theoretical framework that guided this study.

As was done with Form 1, factor analysis was performed with three, two and eight components. The procedure started with an examination of the number of factors needed to explain the pattern of relationships among these items on Form 2. Table 4.2.7 below shows the possible number of factors initially extracted by the factor analysis. The eigenvalues obtained were the same for each of the three analyses. In addition, values of the "Extraction sum of squares" obtained for three and two components were respectively contained within the first three and the first two rows of the eight component output. As a result, the output with eight factors is what has been presented in Table 4.2.7.

From Table 4.2.7, three retained factors account for about 31.3% of the variation in scores while two and eight factors account for about 23.7% and

60.3% of the variance respectively. Also, as Table 4.2.7 indicates, using the common rule of thumb of the Kaiser-criterion for dropping all factors with eigenvalues less than 1.0 from the analysis, 8 factors needed to be retained. However, since the Kaiser-criterion always has the tendency to overestimate the number of true factors needed to explain a given data set (see (Lance, Butts, and Michels, 2006), the scree-test was used for confirmation.

Table 4.2.7 Total Variance Explained by Each of the Factors of Form 2[15]

Factor	Initial Eigenvalues		
	Total	% of Variance	Cum. %
1	3.186	15.929	15.929
2	1.560	7.798	23.727
3	1.513	7.565	31.292
4	1.363	6.815	38.106
5	1.227	6.136	44.342
6	1.154	5.768	50.010
7	1.061	5.306	55.316
8	1.001	5.003	60.319
9	.940	4.698	65.017
10	.903	4.515	69.532
11	.853	4.363	73.795
12	.785	3.924	77.719
13	.759	3.794	81.513
14	.673	3.365	84.878
15	.642	3.211	88.089
16	.550	2.750	90.840
17	.539	2.693	93.533
18	.466	2.330	95.863
19	.442	2.209	98.072
20	.386	1.928	100.000

[15] Extraction Method: Principal Component Analysis.

Figure 4.2.2 below shows the scree plot of the extracted factors. It can be inferred from Figure 4.2.2 that the variation in the slope of the graph greatly reduces beyond factor number 2 or 3. As a result of the possible of settling on the factor number of 2 or 3 as the point beyond which a gentle variation in slope occurs, the analysis was done with both factors 2 and 3 to see which better explained the pattern of relationships among these items.

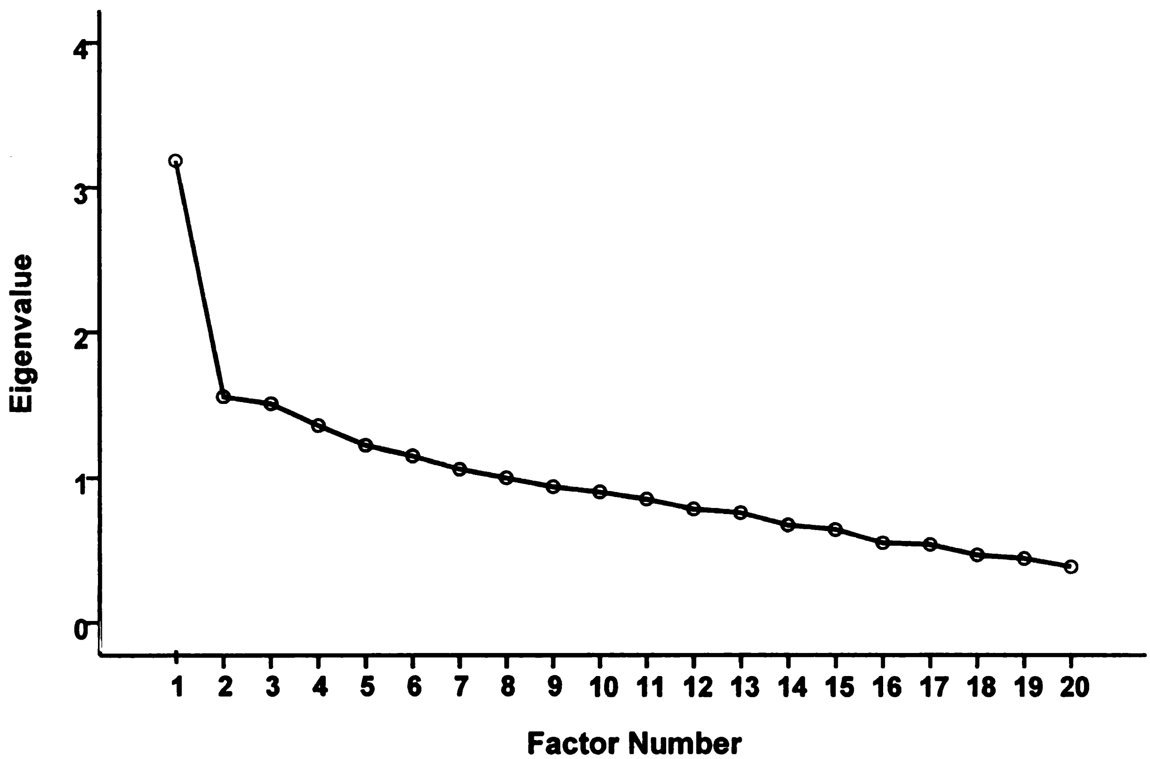

Figure 4.2.2 Graph of Factor Loadings for Items on Form 2

The next table from the SPSS output, Table 4.2.8, shows how much of the variation in the score of each item was explained by the extracted factors. As discussed earlier, values in the "communality" column show the percent of variance in any given variable that is explained by the other extracted factors.

Table 4.2.8 Communalities of Form 2[16]

Item ID	Communality
1	.293
2	.039
3	.330
4	.110
5	.352
6	.226
7	.358
8	.146
9	.076
10	.228
11	.264
12	.459
13	.292
14	.380
15	.522
16	.428
17	.319
18	.598
19	.407
20	.431

A cursory look at Table 4.2.8 reveals that in general, the variance in each item that was accounted for by the extracted factors was very low. It can be seen from Table 4.2.8 that eighteen out of the 20 items on Form 2 had less than 50% of their variance explained by the other factors extracted. In general, communalities need to be interpreted with respect to the interpretability of the factors. As a result, an item or variable with communality as high as 0.82, for instance, may tell much unless the factor on which that item is loaded is

[16] Extraction Method: Principal Component Analysis.

interpretable. Whether items have high or low communalities is important only when the nature of factor loadings helps to interpret the factors (see section 4.2.3 for further discussion of these communality values). In the next section the analysis done with three components is discussed.

4.2.2.1 Factors Analysis of Form 2 with Three Factors

Table 4.2.9 below presents how the items loaded on three retained factors.

Table 4.2.9 Item Loadings on the Three Retained Factors of Form 2[17]

	Factor		
Item ID	1	2	3
1	.058	.445	.266
2	.094	.184	-.037
3**	.450	.370	.077
4	-.304	.062	-.044
5	.582	.106	.018
6	.005	.048	-.471
7	.585	.035	-.204
8	.379	.055	-.102
9	.119	.255	-.089
10	.190	.091	-.461
11	.473	.062	-.270
12**	.630	.121	-.342
13**	.084	.382	-.413
14	.608	.179	-.102
15	-.039	.705	-.130
16	.033	-.644	.045
17	.069	-.200	-.508
18**	.506	.212	-.646
19**	-.018	.316	-.563
20	.280	.037	-.628

** Items with cross loading

[17] Extraction Method: Principal Component Analysis.
Rotation Method: Oblimin with Kaiser Normalization.

From Table 4.2.9, it can be seen that five items (items 3, 12, 13, 18, and 19) cross loaded on more than one factors. They were therefore not assigned to any of the retained factors and not used in interpreting any of the factors. In terms of the strength of their loadings, six items (items 4, 5, 7, 8, 11, and 14) loaded on Factor 1, three items (items 1, 15, and 16) loaded on Factor 2 and four items (items 6, 10, 17, and 20) loaded on Factor 3. The KAT categorizations of the items that loaded on each of the three factors are shown in parenthesis besides the item ID below (refer to the explanations accompanying Table 3.5.1 for explanations of the KAT categorizations);

Factor 1- items 4(312), 5 (212), 7(221), 8(322), 11(321), and 14(121)

Factor 2- items 1(112), 15(324), and 16(211)

Factor 3- items 6(121), 10(311), 17(211), and 20(214)

As can be seen from these listings, items originally categorized as measuring each of the three types of knowledge hypothesized in the KAT framework loaded on each of the three factors. Similarly, some of the items originally categorized to be based on each of the two types of algebra content loaded on each of the factors. In addition, none of the factors was uniquely loaded with items based on one aspect of the KAT domain of mathematical knowledge. As a result, none of the three factors could be labeled using any of the three dimensions from the KAT framework. The next section presents results of the factor analysis performed using two components.

4.2.2.2 *Factor Analysis of Form 2 with Two Factors*

The scree-plot obtained for this analysis was the same as the one obtained in the previous section (see Figure 4.2.2). Consequently, no new scree-plot is presented in this section. To interpret the two factors retained the factor loadings were essential. Table 4.2.10 below presents these factor loadings when two components or factors were retained.

Table 4.2.10 Item Loading on the Two Retained Factors of Form 2[18]

	Factor	
Item ID	1	2
1	-.137	.322
2	.077	.171
3	.274	.262
4	-.217	.112
5	.431	.018
6	.254	.196
7	.560	.022
8	.343	.034
9	.118	.251
10	.387	.209
11	.506	.083
12	.661	.141
13	.254	.479
14	.510	.121
15	-.022	.705
16	.057	-.619
17	.345	-.032
18**	.721	.340
19	.262	.478
20	.552	.200

** Item with cross loadings

As will be observed from Table 4.2.10, only one item, item 18, loaded strongly on both factors. For that reason, item 18 was the only strongly loaded

[18] Extraction Method: Principal Component Analysis.
Rotation Method: Oblimin with Kaiser Normalization.

item that could be assigned to any of the two retained factors and used in the interpretation of the factors. After taking out item 18, nine items (items 5, 7, 8, 10, 11, 12, 14, 17, and 20) loaded strongly on Factor 1 and four items (items 13, 15, 16, and 19) on Factor 2.

Analysis of the item attributes indicated that the items on each of the two retained factors did not load uniquely on the basis of any of the three dimensions (domain of algebra knowledge, algebra content domain, or domain of mathematical knowledge) of the items (refer to Table 3.5.1 for their categorization). The two factors could therefore not be labeled in terms of aspects of any of the three dimensions. In the next section, the analysis using eight components is presented.

4.2.2.3 Factor Analysis of Form 2 with Eight Factors

Table 4.2.11 below presents the strengths of the factor loadings of each of the items on the eight retained factors. A cursory look at Table 4.2.11 shows that nine items loaded on more than one of the factors. These were items 4, 5, 6, 7, 10, 11, 13, 17, and 18. They were, therefore, excluded in the attempt at labeling the factors.

After taking out these nine cross loading items, items that strongly loaded on each of the factors were items 19 and 20 on Factor 1, items 15 and 16 on Factor 2, items 3 and 7 on Factor 3 and items 12 and 14 on Factor 4. The rest were item 8 on Factor 5, item 2 on Factor 6, item 9 on Factor 7 and item 1 on Factor 8.

Table 4.2.11 Item Loading on the Two Retained Factors of Form 2[19]

Item	Factor							
ID	1	2	3	4	5	6	7	8
1	-.038	.212	.144	.100	.062	-.149	.042	-.741
2	-.021	.206	-.075	-.038	.121	.739	-.040	.029
3	.081	.182	.723	-.279	.090	-.074	-.032	.050
4**	-.146	.216	.140	.258	.113	-.389	.021	.546
5**	.184	-.218	.540	-.238	.111	-.060	.379	-.172
6**	.380	-.023	-.028	.200	.533	.121	.066	.209
7**	.230	-.211	.356	-.225	.280	.344	.523	.133
8	-.013	-.192	.184	-.120	.721	.207	.081	-.151
9	.006	.189	-.072	-.040	-.008	-.082	.804	-.035
10**	.132	.106	-.192	-.411	.475	-.289	.285	.169
11**	.522	-.178	.200	-.156	-.010	.390	.313	-.151
12	.279	.043	.147	-.772	.093	.167	.204	.098
13**	.282	.321	.049	-.090	.566	-.068	-.053	.042
14	.124	.025	.269	-.692	.115	.054	.056	-.158
15	.090	.768	.114	-.035	-.011	.100	.208	.101
16	-.052	-.644	-.002	.099	-.077	-.025	.042	.273
17**	.213	-.022	-.491	-.481	.052	-.134	.183	.273
18**	.693	.041	.129	-.417	.329	.156	.197	.049
19	.701	.231	-.004	.096	.227	-.148	-.066	-.069
20	.738	-.033	-.014	-.281	.043	-.011	.097	.099

** Items with cross loadings

Thus, none of the eight retained factors had at least three items uniquely loading onto it. Consequently, it was that there were too few items on each factor to use for any reliable labeling of the factors.

4.2.3 Conclusions Related to Research Question One

To answer the first research question, factor analysis was initially done using three and two components based on the possibilities of selecting two or three as elbows of the scree-plot. A third analysis was also run using eight components based on interpretation of the eigenvalues obtained by applying the

[19] Extraction Method: Principal Component Analysis.
Rotation Method: Oblimin with Kaiser Normalization.

Kaiser-criterion. However, on both forms, factor analysis using eight factors was not pursued because after taking out the cross-loading items, the factors had insufficient items (less than three) left on them to make any meaningful interpretation.

In general, factor analysis run on the data from the two forms revealed that the items had small communality values. On Form 1 and Form 2, only one and two items respectively had at least 50 percent of its variance explained by the extracted factors. Though test items generally have low reliabilities which may result in low communalities, the low communalities obtained in the factor analysis in this study are worth noting especially because the pattern of item loadings on the factors did not help to interpret the factors.

Also, when the analysis was done with three factors, the three factors could explain only about 28% and 31% respectively of the variation in scores on Form 1 and Form 2. When the analysis was done with two factors, the proportion of the variation in scores that could be explained dropped to about 21% and 24% respectively.

In addition, the nature of the factor loadings, after the cross loading items were removed for any of the analysis run using 2 and 3 factors did not permit interpretation of the retained factors. The analyses revealed that items originally categorized as measuring any of the three types of knowledge hypothesized in KAT (i.e., school knowledge, advanced knowledge, and teaching knowledge) did not uniquely load any of the retained factors. In other words, items meant to assess different types of knowledge loaded together. Even when the item loadings were analyzed in terms of the other two dimensions of the KAT Item Development Matrix, (i.e., "algebra content" or "domain of mathematical knowledge"), the nature of the loadings did not provide evidence for labeling the factors. These results are further discussed in the last chapter of this dissertation.

4.3 Research Question Two

The second research question was,

> "How does the profile of knowledge for teaching algebra differ among the different categories of secondary school mathematics teachers and potential teachers in Ghana?"

As already explained in chapter three, data for this research question came from the scores obtained by in-service teachers and university students on Forms 1 and Form 2. The university students who participated in the study comprised students majoring in three different areas; mathematics, mathematics education and statistics. Thus, together with the in-service teachers there were four groups whose sub-scores on items classified as assessing the three types of knowledge specified in the KAT project needed to be compared. Consequently, it was initially decided that Analysis of Variance (ANOVA) performed on the sub-total scores on the school knowledge items, advanced knowledge items and teaching knowledge items (as well as the total scores) by the various categories of in-service and prospective teachers on Forms 1 and 2 together was the best analysis needed to answer this research question. However because the planned dimensional structure of the instruments was not supported in the factor analysis, ANOVA could not be performed on the sub-total scores on the three types of knowledge hypothesized by KAT. The analysis was therefore limited to only the total scores of participants on Forms 1 and 2.

Before merging the data on Forms 1 and 2, it was necessary to test to see whether there was any significant difference between the performances of participants who completed Form 1 and Form 2. The two groups were assumed to be independent samples. Consequently, Independent Samples *t*-test was performed on their total mean scores on Form 1 and From 2. This preliminary *t*-test was essential because the results had implications for whether there was any justification to proceed with ANOVA on the combined forms or on the individual forms separately. For instance, if the *t*-test test had revealed significant differences between the two groups' performances, then there could have been no basis to consider them as equivalent groups and thereby run ANOVA on the combined forms. The preliminary *t*-test revealed no significant differences in performance on Form 1 and Form 2. This was what provided the basis for proceeding with ANOVA on the combined Form 1 and Form 2 scores as explained in the preceding paragraph. The results of the preliminary t-test are presented in the next section.

4.3.1 Preliminary T-Test on Form 1 and Form 2

Table 4.3.1 below shows descriptive statistics of the performance of these participants on each of the items on the two forms. As shown in Table 4.3.1, altogether, 150 and 189 participants respectively completed Form 1 and Form 2.

A cursory look at the table reveals that in general, performance of participants on both forms was quite low. For instance on Form 1, participants were able to score an average of more than 60 percent of the maximum allowable points (see mean scores for items 1, 2, 3, 11, and 17) only on five items (out of the twenty items) .

Table 4.3.1 Descriptive Statistics of Form 1 and Form 2 by Item[20]

	Form 1 (N = 150)		Form 2 (N = 189)	
Item ID	Mean	Std. Deviation	Mean	Std. Deviation
1	.880	.326	.989	.103
2	.653	.477	.979	.144
3	.933	.250	.328	.471
4	.147	.355	.296	.458
5	.347	.477	.169	.376
6	.247	.432	.640	.481
7	.487	.501	.222	.417
8	.060	.238	.275	.448
9	.340	.475	.730	.445
10	.427	.496	.682	.467
11	.753	.432	.217	.413
12	.200	.401	.164	.371
13	.147	.355	.116	.322
14	.173	.380	.222	.417
15	.493	.502	.429	.496
16	.540	.500	.106	.308
17	.913	.282	.381	.487
18	.510	.554	.563	.540
19	.723	.462	.627	.644
20	.133	.404	.603	.571

Participants scored above 50% of the maximum score on only six of the items on Form-- items 1, 2, 3, 11, 16, and 17 (items 18 and 19 were open-ended items so their respective .510 and .723 mean scores were out of the scaled down score of 2 points and are therefore less than 50% of the maximum score for each of those items). On Form 2, the performance appears even slightly lower. As in

[20] For the multiple choice items (items 1-17) participants either scored 1 for a correct response or 0 for a wrong response. The open-ended items (items 18 to 20) were originally scored on a 0 to 4 scale and then scaled down to 0 to 2 for this analysis.

Form 1, though, participants were able to score an average of more than 50 percent of the maximum allowable points on only five items (items 1, 2, 6, 9 and 10). Again, items 18, 19, and 20 on Form 2 were open ended items with maximum score of 2 points each. Thus the .563, .627, and .603 respectively shown as the mean scores were less than 50% of the allowable maximum score in each case.

Table 4.3.2 below presents the performance of participants on items that were originally categorized in the KAT project as measuring each of the three types of knowledge.

Table 4.3.2 Descriptive Statistics of Forms1 and 2 by Knowledge Type

	Form 1 (N = 150)		Form 2 (N = 189)	
Type of Knowledge	Mean	Std. Deviation	Mean	Std. Deviation
School Knowledge	3.870	.965	3.701	1.394
Advanced Knowledge	1.567	1.187	1.598	1.232
Teaching Knowledge	3.670	1.503	3.442	1.372

From Table 4.3.2 performance of participants was highest on both forms on items originally categorized to be measuring school knowledge. On both forms, their worst performance was on the subtest of advanced knowledge items.

The total mean performance on the two forms is summarized in Table 4.3.3 below.

Table 4.3.3 Group Statistics of Form 1 and Form 2

Form used	N	Mean	Std. Deviation	Std. Error Mean
1	150	9.107	2.670	.218
2	189	8.741	3.135	.228

From Table 4.3.3 participants who completed Form 1 performed slightly better than those who completed Form 2. To test whether the difference in performance on the two forms was statistically significant or not, independent samples t-test was run on the total mean scores at 5 percent level of significance. Table 4.3.4 presents the results of this test.

Table 4.3.4 Independent Samples T-Test[21]

	Levene's Test for Equality of Variances		t-test for Equality of Means				
	F	Sig.	t	df	Sig. (2-tailed)	Mean Diff.	Std. Error
Equal variances assumed	2.758	.098	1.139	337	.256	.366	.321
Equal variances not assumed			1.160	335.291	.247	.366	.315

From Table 4.3.4, it is clear that the group variances and the mean scores on Form 1 and Form 2 were not significantly different. Thus, the two groups that completed Form 1 and Form 2 could be considered equivalent group. As already explained the subtotal scores on the school knowledge, advanced knowledge and teaching knowledge items of participants could not be compared because the dimensional structure of the instruments was not supported in the factor analysis. As a result, in an attempt to answer the second research question, participants' scores on the two forms were combined and ANOVA was performed on their total score. These analyses and results are shown below.

[21] The *t*-test performed was two-tailed at 5% level of significance

4.3.2 ANOVA on Total Scores of Forms 1 & 2

Table 4.3.5 shows the descriptive statistics obtained from the data analysis of scores from the Form 1 and Form 2 combined.

Table 4.3.5 Descriptive Statistics of Total Scores by Major Area

Major	N	Mean	Std. Dev.	Std. Error	95% Confidence Interval for Mean Lower Bound	95% Confidence Interval for Mean Upper Bound	Min	Max
Math	132	9.140	2.711	.236	8.670	9.607	3.00	17.50
Math Ed	125	7.676	2.030	.181	7.317	8.035	2.50	12.50
Statistics	44	7.977	2.277	.343	7.284	8.670	.50	11.00
In-service	38	13.184	2.822	.458	12.257	14.212	7.50	19.50
Overall	339	8.903	2.940	.160	8.589	9.217	.50	19.50

In Table 4.3.5 above, minimum and maximum scores have been calculated as mixed numbers (numbers with whole number and fractional components). For example, the minimum score of the in-service teachers has been given as 7.50. Though none of the items had a possible score of 0.5, the fractions were the result of the scaling down of the open-ended items' score from a maximum of 4 (used during the scoring) to 2 (for the analyses). In this way, a participant who scores only 7 multiple choice items and receives a score of 1 on only one open-ended item will have his/her total score reduced from 8 to 7.50 after halving the open-ended score during the analyses of data. Similar, minimum and maximum scores with fractional components are presented in Table 4.3.9 for the same reason.

A cursory look at Table 4.3.5 suggests that the mean the total scores on the two combined forms may be different for each of these fours groups of in-service and prospective teachers. As shown in Table 4.3.5 above, in this study, the group that performed best on the total score of the combined forms was the in-service teachers. On the other hand, the group that scored least was the mathematics education majors. These relative mean scores of participants majoring in the different domains is presented graphically in Figure 4.3.1 below.

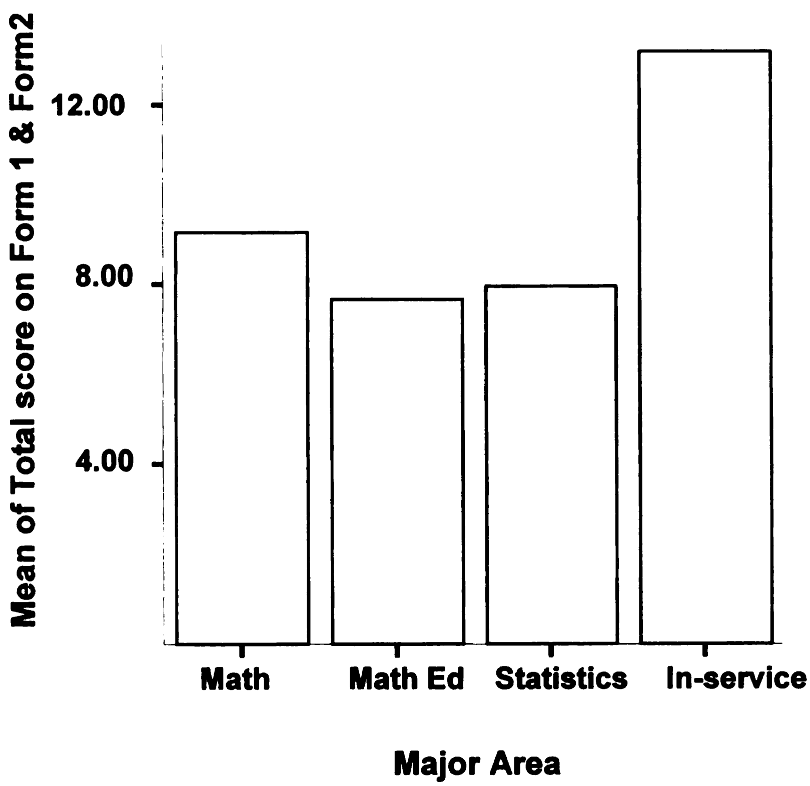

Figure 4.3.1 Mean Total Score on Form 1 & 2 by Major Area

One assumption implicit in Analysis of Variance is that the groups under consideration have approximately equal variances. That is, even if the group means are different, the dispersion of each group's scores about the group mean will be approximately equal (i.e., there will be no significant difference in the variances of the various group's score). In this study, Levene's test was used to test this assumption. In general, if in the output table of Levene's Test, the value under the column marked "Sig." is less than .05 then an inference is made that the group variances are significantly different at the .05 level (i.e., the group variances are not approximately equal). On the other hand, if the value under "Sig." is greater than .05, then the group variances are not significantly different (i.e., the group variances are approximately equal). The Table 4.3.6 below presents the results of the Levene's test on the combined Form 1 and Form 2.

Table 4.3.6 Test of Homogeneity of Variance in the Total Scores

Levene Statistic	df1	df2	Sig.
3.343	3	335	.019

From the outcome of the homogeneity test, it was observed that the value under Sig. is less than .05. Thus, the group variances are significantly different. This means that the group variances are not approximately equal.

To examine whether the group means of the total scores on the two forms combined were significantly different or not, the ANOVA table was used. The results of this test are presented in Table 4.3.7 below.

Table 4.3.7 ANOVA of the Mean Differences in Total Score

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	929.815	3	309.938	52.150	.000
Within Groups	1990.973	335	5.943		
Total	2920.788	338			

Table 4.3.7 above shows that there is a large F-ratio (i.e., 52.150) for the Between-Groups variance. In addition, there is an associated F probability (.000), smaller than the 0.05 significance level. Thus, there seems to be adequate evidence that there is a difference overall among the total mean scores for at least one pair of these four groups.

All ANOVA tables, such as Table 4.3.7 above, are able to reveal the possibility of at least one pair of groups differing significantly on their mean scores or the difference between their means. ANOVA tables do not, however, reveal the actual pairs of groups whose means differ significantly. To investigate which particular pairs of groups have significant between mean differences, Tukey's HSD Tests was used at the 5% level of significance. Tukey's HSD Test is useful because it presents results of multiple comparisons among all the possible pairings of the groups of focus, which in this case are final year university students majoring in mathematics, mathematics education and statistics majors, as well as in-service high school mathematics teachers in Ghana. The result of this multiple comparison is presented in Table 4.3.8 below.

Table 4.3.8 shows that at the 5% level of significance, the in-service teachers had significantly higher scores on the combined forms than any group of university students. In addition, the mathematics majors had higher score than the statistics and mathematics education majors. However, there was no significant difference between the total mean score of the statistics and mathematics majors on the combined forms.

Table 4.3.8 Multiple Comparisons of Differences in Total Scores

(I) Major Area	(J) Major Area	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Upper Bound	Lower Bound
Math	Math Ed	1.464*	.304	.000	.679	2.250
	Statistics	1.163*	.424	.033	.067	2.259
	In-service	-4.044*	.449	.000	-5.202	-2.885
Math Ed	Math	-1.464*	.304	.000	-2.250	-.679
	Statistics	-.301	.427	.895	-1.405	.802
	In-service	-5.508*	.452	.000	-6.674	-4.442
Statistics	Math	-1.163*	.424	.033	-2.259	-.067
	Math Ed	.301	.427	.895	-.802	1.405
	In-service	-5.207*	.540	.000	-6.601	-3.813
In-service	Math	4.044*	.449	.000	2.885	5.203
	Math Ed	5.508*	.452	.000	4.442	6.674
	Statistics	5.207*	.540	.000	3.813	6.601

* The mean difference is significant at the .05 level.

4.3.3 Conclusions Related to Research Question Two

A number of conclusions can be drawn from the foregoing analysis and results. First, a look at the raw total mean score of participants who completed Form 1 and Form 2 (see Table 4.3.2) shows that the mean score on Form 1(9.107) was slightly higher that the mean score on Form 2 (8.741). The observed difference suggests that perhaps the items on Form 2 were of a slightly higher difficulty level than those on Form 1 (except that data from this study does not provide evidence that the difficulty level is significant).

Second, though different participants completed Form 1 and Form 2, the *t*-test conducted revealed that overall the performance of those who completed Form 1 was not significantly different statistically from the performance of Form 2 participants. In each of the participating universities, students majoring in the same area who agreed to take part in the study completed the forms at one sitting. At this sitting, the number of available Forms 1 and 2 were randomly distributed to participants. A similar approach was used to distribute the forms to the in-service high school mathematics teachers who agreed to participate in the study. The result of no significant difference in performance between those who completed Form 1 and Form 2 was therefore not surprising following the random manner of distribution of the two forms.

Third, results of the Analysis of Variance (ANOVA) discussed in the preceding section reveal that in Ghana, the sub-group of in-service teachers performed best on the two forms. The analysis of variance performed on the data revealed that in-service high school teachers in Ghana performed significantly better than each sub-group of prospective mathematics teachers majoring in mathematics, mathematics education and statistics from the country's universities. It was therefore concluded that knowledge for teaching algebra of in-service high school mathematics teachers is significantly different from that of prospective teachers.

Among university students in Ghana, ANOVA revealed differences in the performances among students in the different majors. In general the mathematics majors performed significantly better than their counterparts majoring in statistics and mathematics education. Between the statistics and mathematics education students, the statistics majors did slightly better than the mathematics education students. However, this difference was not significance at the .05 level. These results are further discussed in the last chapter of this dissertation.

4.4 Research Question Three

The third question that guided this study was,

> "What is the relationship between the performance of in-service secondary school mathematics teachers in Ghana and the students of their classes?"

As already discussed in chapter three, data from the participating senior secondary school students and their corresponding in-service teachers were to be used to analyze this question. Essentially, this research question asks for the relationship between student performance (on the high school students' form) and the performance of their teachers on the school knowledge, advanced knowledge, and teaching knowledge items (of teachers' Form 1 and Form 2). Analysis of the variables that could be extracted from this research question reveals one dependent variable (students score) and three predictor (or independent) variables. Structural Equation Modeling (SEM) and multiple regression, because they can each be used to investigate the relationship between a dependent variable and a number of independent variables, were thought of as the possible analytical methods to use to analyze data for this research question. Prior to commencing with fieldwork for this study, SEM was the preferred method of analysis of data for this third research question because of its inherent advantages over linear regression. For instance, SEM makes room for measurement errors and estimates them (Byrne, 2001, pp. 3-4) while in multiple regression it is assumed that the variables are devoid of any measurement error. However, SEM could not be used for this analysis because the number of in-service teachers who participated in the study was too small.

Multiple regression could also not be performed using the three types of knowledge hypothesized by KAT because the factor analysis used to test the first research question did not support the dimensional structure in the KAT framework. Consequently, the teachers' scores were combined and linear regression was performed between the mean total teachers' score and the mean class scores of the students in their classes.

As already discussed in Chapter 3, of the 42 senior secondary school classes who participated in the study (refer to Tables 3.4.7 and 3.4.8), teachers of only ten of them also participated. Consequently, the linear regression was performed using only the ten participating classes whose teachers also participated in the study. These were the classes with codes A1, A2, A3, C1, C2, G1, G2, H1, H2 and H7. This analysis is presented in the next section.

4.4.1 Linear Regression

A Linear regression model involving the mean class score for the ten classes and the mean scores of their teachers' total score was estimated and tested to answer the third research question. Consequently, for the ten classes of interest, the dependent variable to be used in the model was the mean score for of all ten classes of focus. The independent variable was the mean total score of teachers of the ten classes. The model tested was

$$Y = \beta_0 + \beta_1 X + e, \quad \text{where,}$$

Y = the dependent variable representing the mean class score,

X = the independent variable representing the mean of the total knowledge score for teachers of the classes used

β_0 = the intercept (mean class score when all independent variables are equal 0).

β_1 = the slope for predictor X_1, where the slope is the change in mean class score for one unit increase in mean teacher total score, holding all other independent variables constant.

e = the error for the class score (e's are assumed to be independently and identically normal distributed with a mean of 0 and variance, σ^2)

Table 4.4.1 shows the mean scores for students and teachers of these ten classes used to perform the linear regression.

Table 4.4.1 Mean Class Scores and Teachers' Total Scores

Class ID	Mean Teachers' Total Score	Mean Class Score
A1	13.50	3.65
A2	12.00	4.40
A3	7.50	3.23
C1	16.00	5.68
C2	19.00	4.39
G1	16.00	5.00
G2	10.00	4.39
H1	14.50	3.50
H2	13.50	2.66
H7	13.00	3.42
Mean	13.50	4.03
Std Dev	3.24	0.91

Table 4.4.2 shows the correlation coefficients for the linear model.

Table 4.4.2 Linear Regression Model Summary[22]

Multiple R	R Square	Adjusted R Square	Std. Error
.428	.183	.081	.868

[22] Predictors: (Constant), Mean teachers' total score
Dependent Variable: Mean class score

The "R-square" value in the model is the coefficient of determination. It is indicative of how much of the variability in the dependent variable (mean class score) is explained by the model. Table 4.4.3 shows that 18.3% of the variability in the mean score of the ten classes of focus (i.e., A1, A2, A3, C1, C2, G1, G2, H1, H2, and H7) could be explained by the model.

The "Multiple R" in Table 4.4.2 is the square root of R-square. It is the correlation between the predictor variable and the dependent variable. Table 4.4.2 shows that in the linear regression model the correlation between the mean teachers' total score and the mean class score is .428.

In general, the "Adjusted R-square" value in the model is the modification or correction made in the R-Square as the number of predictor variables is increased. It takes into account the number of predictor variable and the sample size, and is therefore adjusted based on the number of degrees of freedom. In reality, what happens is that increasing the number of predictor variables only increases the possibility that the estimated regression model may not be significantly better than what could be obtained by chance. Therefore, when there are many predictor variables present, it is the adjusted R-square values that need to be used to report the percentage of the variability in the dependent variable explained by the predictors present. As a result, it becomes more relevant as a diagnostic tool when used in multiple regression. Since the linear regression model does not have many predictor variables, attention was not focused on the adjusted R- square value in Table 4.4.2.

In linear regression models, there are always differences between the observed values and the values predictable from the model. These differences are called residuals. In general, the "Standard Error of the Estimate" in the model summary is a measure of the standard deviation of the residuals. In the model that was estimated from this study, the standard error of the estimate was .868. Compared with the standard deviation of the dependent variable of 0.91 (see Table 4.4.1), this standard error of estimate from the model is much smaller.

To estimate this linear regression model, values of coefficients of the independent variables and the constant term were used. Table 4.4.3, presents these coefficients.

Table 4.4.3 Coefficients of the Linear Regression Model

Variable	Unstandardized Coefficients				95% Confidence Interval	
	B	Std. Error	t	p-value	Lower	Upper
Constant	2.419	1.236	1.957	.086	-.431	5.269
Tchr Total	.119	.089	1.338	.218	-.086	.325

Usually, the linear regression output (shown in Table 4.4.3) gives the needed coefficients for the model (refer to the column marked "B" under the "Unstandardized Coefficients"). Therefore, using Table 4.4.3, the linear regression model was estimated as

$$\boldsymbol{Y = 2.419 + .119X + e}, \quad \text{where,}$$

$Y =$ the dependent variable representing the mean class score,

$X =$ the independent variable representing the mean of the total knowledge score for teachers of the classes used, *2.419* = the intercept (mean class score when the independent variable is zero),

.119 = the slope for predictor X, where the slope is the change in mean class score for one unit increase in mean teacher total score, holding all other independent variables constant, and

e = the error for the class score (*e*'s are assumed to be independently and identically normal distributed with a mean of 0 and variance σ^2).

Figure 4.4.1 presents the scatter plot with the linear regression line.

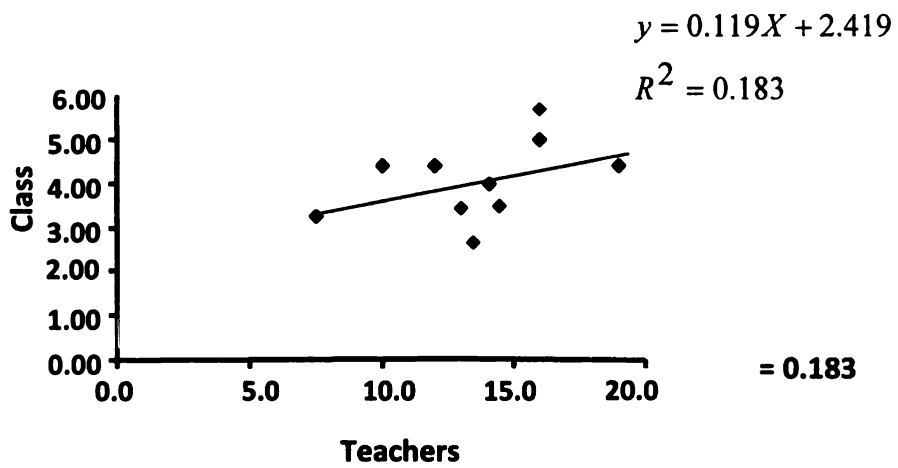

Figure 4.4.1 Scatter Plot with the Regression Line

Next the ANOVA table presented in Table 4.4.4 was used to ascertain whether the effect of the predictor was significant.

Table 4.4.4 ANOVA Test of Significance of the Predictor in the Model

	Sum of Squares	df	Mean Square	F	Significance
Regression	1.349	1	1.349	1.791	.218
Residual	6.025	8	.753		
Total	7.374	9			

The model was tested at 5% level of significance. From Table 4.4.4, it was clear that predictor variable, mean teachers' knowledge, was not a significant predictor of the mean class score. This is further discussed in the next chapter.

4.4.2 Conclusions Related to Research Question Three

After testing for no violation in the assumptions inherent in linear regression, the final model was estimated to be of the form,

$$Y = 2.419 + .119X + e, \quad \text{where,}$$

Y = the dependent variable representing the mean class score,

X = the independent variable representing the mean of the total knowledge score for teachers of the classes used,

2.419 = the intercept (mean class score when the independent variable is zero),

.119 = the slope for predictor X, where the slope is the change in mean class score for one unit increase in mean teacher total score, holding all other independent variables constant, and

e = the error for the class score (e's are assumed to be independently and identically normal distributed with a mean of 0 and variance σ^2).

Thus, in terms of answering the third research question it was concluded that a linear relationship existed between the performance of in-service teachers who participated in the study and the students of their classes. This relationship was estimated by the equation above.

Though the coefficient of mean teachers' total score in the model is positive, the ANOVA test of significance shows that the effect is not statistically significant. This is also shown by the p-value of the independent variable. In theory, the p-value of the independent variable is indicative of whether the independent variable has statistically significant predictability. As shown in Table 4.4.3, the p-value (.218) shows that the mean teachers' total knowledge has no statistically significant predictability. These results are further discussed in the last chapter of this dissertation.

CHAPTER 5: SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

This chapter summarizes the study and discusses conclusions drawn from the analyses and results. In the discussion of the conclusions, the limitations of the study have been highlighted since they have implications for the extent to which the conclusions of the study can be generalized. In addition, because different statistical analyses were done with respect to each research question, conclusions from this study have been organized in line with the research questions. This style of organization was adopted because it helped avoid mixing up conclusions from the different analyses and results. In addition, it helped to avoid rushing into conclusions beyond the results of the analyses. The chapter ends with recommendations for further research in the area.

5.1 Summary of the Study

This study was premised on the fact that the teacher is one of the most important factors that influence student performance, especially in the developing world (see for example, Mullens et al., 1996; Sanders and Rivers, 1996; Jordan, Mendro, and Weerasinghe, 1997; Wright, Horn, and Sanders, 1997). The study acknowledges that though literature on teaching and student performance is replete with evidence that supports this premise, there is widespread disagreement among researchers about the exact nature of the link between teachers' subject matter knowledge and student performance. The lack of agreement between the findings of, for instance, Monk (1994) and Rowan, et al.

(2002), on one side, and Harbison and Hanushek (1992) and Mullens et al. (1996), on another side, exemplifies the disagreement.

In the face of the many different general conceptualizations of teacher knowledge, this study also acknowledges the need for re-conceptualization of teacher knowledge in ways that is not only domain specific, even within coverage of school mathematics, but also lends itself to some form of direct measurement instead of by proxy. This is because it is unclear whether the type of knowledge teachers need for teaching calculus, for instance, would be the same as that needed to teach a different domain like algebra. In addition, because of the many different proxy measures that have so far been used by various researchers, the study concedes the need for improved ways of measuring the different components or types of teacher knowledge that could result from such re-conceptualizations.

As discussed in the second chapter of this dissertation, these considerations underlay the pivotal role of the work of the Knowledge of Algebra for Teaching (KAT) project at Michigan State University to this study. As already discussed, through analyses of research literature, recommendations by professional organizations and videos of teaching, researchers in the KAT project have hypothesized that the knowledge used by teachers in teaching school algebra consists of three types: "knowledge of school algebra" (referred to in short as "school knowledge"), "advanced knowledge of mathematics" (also referred to as "advanced knowledge"), and "teaching knowledge". In addition, the

KAT project is currently developing instruments to measure and validate their conceptualization (refer to Ferrini-Mundy et. al., 2003 and 2005).

This study used the KAT conceptualization as the framework for studying Ghanaian high school teachers' knowledge for teaching algebra. Rather than rely on proxy measures of teacher knowledge, the study also adapted the instruments developed by the KAT project for its data collection in Ghana.

The aim of the study was three-fold. First, though the KAT project was at its initial stages, at the time of this study, the present study was an attempt at investigating whether the KAT conceptualizations could be corroborated in settings outside the US such as Ghana. As explained in chapter one, Ghana becomes a good international setting for this exploration because of the differences in the type of mathematics offered its high schools, as well as the background of the country's high school mathematics teachers. This is important because of the integrated mathematics curriculum in Ghana, as well as differences in the background of teachers in Ghana's high schools as against what prevails in many US schools.

Second, the study aimed at investigating the differences, if any, in the level or quality of the three hypothesized knowledge types in KAT by Ghanaian high school teachers of different backgrounds. As explained in chapter one, the importance of this aspect of the study could be seen in the differences in background of prospective, as well as in-service high school mathematics teachers. In Ghana, university students majoring in mathematics education, mathematics and other related areas could be posted to teach high school mathematics by the National Service Secretariat and many of them could remain in such capacity until they retire. In addition, not all the non-mathematics education graduates could take advantage of summer sandwich programs run by the University of Cape Coast to obtain a Diploma in Education, one of the certificates considered as a teaching certificate in Ghana. As a result, the prior university coursework of high school mathematics teachers in Ghana could vary as could their professional development experience; hence the need to examine differences in the profile of their knowledge for teaching algebra.

Third, this study also aimed at investigating how Ghanaian high school mathematics teachers' knowledge for teaching algebra is related to the performance of their students. As can be seen from some of the literature reviewed in the first two chapters of this dissertation, attempts at using proxy measures of teacher knowledge to estimate this type of relationship have so far produced mixed results. In the face of these mixed results, it was important to find out the type of results the KAT project's domain specific conceptualization and instruments could provide to the larger research community worldwide.

Participants of the study came from three main types of populations. These were, 1) high school seniors studying elective mathematics in senior secondary schools in Ghana, 2) in-service senior secondary school mathematics teachers from the participating high schools, and 3) prospective senior secondary school mathematics teachers comprising final year students with majors in mathematics, statistics and mathematics education from three public universities in Ghana. In all, 1565 high school seniors offering elective mathematics from eight public schools and thirty-eight mathematics teachers from the participating high schools participated in the study. In addition, 301 university students comprising 132 mathematics majors, 44 statistics majors and 125 mathematics education majors participated in the study. Their participation was voluntary and no one was paid to participate.

As already explained, the study involved administering assessment instruments adapted from the KAT project at Michigan State University to participants. The adaptation involved changing the contexts and wording of questions to reflect Ghanaian contexts. Two adapted instruments, Form 1 and Form 2, from the KAT project were administered to the prospective and in-service teachers. Each instrument consisted of two sections. The first section of each of these two instruments was a survey questionnaire that was focused on finding out the college level courses participants have taken, the grade level and mathematics courses that they have been teaching or have taught before (and for how many years), and the type of teaching certification they have, if any. The second section consisted of twenty content items based on the content of algebra in the high school curriculum this content, related advanced mathematics content, as well as items based on the tasks of teaching. In the case of Ghana, making contextual adaptations to a few of the items to suit Ghanaian contexts helped to make the instrument more appropriate because analysis of the senior secondary school mathematics curriculum at the time of the study revealed that the high school algebra content was consistent with the algebra covered in the core and elective mathematics curricula. As a result, at each research site, every participant (in-service and prospective teachers) completed only one of the two forms. At the universities, participants in a particular department or major completed their forms at a sitting and in no more than 60 minutes. A similar approach was adopted in each senior secondary school. In all, 150 participants completed Form 1 and 189 completed Form 2.

The participating final year senior secondary school students, on the other hand, completed one common instrument. This instrument was also an adaptation of the KAT instruments. It also consisted of two sections, a survey questionnaire and a set of assessment items. The survey questionnaire section of the instrument contained questions about the educational background of students' parents/guardians, whether they have extra resources for learning mathematics apart from what they receive at school, and whether their parents hire the services of other teachers to provide extra tutorials for them. The second section of content items comprised items from the KAT instruments that were based on the algebra content in the senior secondary school core and elective mathematics curriculum. In each of the participating schools, students in the elective mathematics classes in the third year (seniors) who agreed to participate were brought together to complete the instrument at one sitting lasting no more than 60 minutes.

To protect participants' privacy, no identifiers like participants' names, their gender, identity numbers or anything that could be traced to them was required to complete any of the instruments. In addition, participants' performance on the instrument was not shared with their academic departments or institutions.

Analyses and discussion of the results of the study were also devoid of the names of the participating institutions. Where names were needed for the purpose of analyses, pseudo names or codes not traceable to any institution or individual have been used. Finally, analyses of data have not been done to compare performance of participants from the different institutions. Instead, in the universities students in a specific major were grouped together and the analysis was done to compare them by their major area. For instance, students majoring in mathematics in this study came from more than one university. Students majoring in the same area from the different participating universities were combined and no attempt was made to compare them as separate units from their individual universities. In the same way, the participating high schools were not compared but only used in the linear regression to build the linear regression model.

Factor analyses was done on the performance of the prospective and in-service teachers on each item of Form 1 and Form 2 to find out whether a case could be made for the three types of knowledge: school knowledge, advanced knowledge and teaching knowledge, hypothesized in the theory framework. The aim was to test whether it was reasonable to talk about three possible knowledge types (i.e., of three factor loadings and the manner of loadings on them) to explain the variation of knowledge used in completing the instruments. To investigate the differences, if any, in the level or quality of knowledge of by Ghanaian high school teachers of different backgrounds, Analysis of Variance (ANOVA) was performed using the mean of the total scores of participants on the items on Forms 1 and 2 combined. The results of an earlier t-test that confirmed that participants did not perform significantly different on the two forms and this provided the justification for combining both forms for the Analysis of Variance. Finally, linear regression was used to estimate a model of the relationship between the performance of senior secondary school students who participated in the study and their assigned mathematics teachers' knowledge. Due to the fact that not every participating class' teacher agreed to participate in the study, only those classes whose teachers also participated were used for the linear regression.

5.2 Conclusions from the Study

5.2.1 Limitations of the Study

Before proceeding with the conclusions from this study, it is worthwhile to note that this study is one of the first attempts, especially in Ghana, at investigating issues related to high school mathematics teachers' knowledge and its relationship with student performance. However, the study is fraught with a number of limitations that could have implications on the degree to which the study's outcomes could be generalized.

First, due to limited financial resources for carrying out the study, it was not possible to select the participating senior secondary schools (i.e., high schools) randomly from all the high schools in Ghana. Instead, schools were selected from towns like Cape Coast and Takoradi that are not too far from each other and had large concentration of schools. At the time of the study, there were over 500 public senior secondary schools in Ghana. Therefore the number, 8, that participated was less than two percent of the schools in Ghana. Traditionally, the schools used are among the top class of high schools in terms of their students' performance in national examinations. As a result, they do not reflect the full spectrum of high schools in Ghana.

Second, as was the case in the selection of high schools, there was no random sampling of in-service and pre-service teachers for this study. In-service teachers from the participating high schools who voluntarily agreed to participate were allowed to participate. In the same way, pre-service teachers majoring in mathematics education from only one of the two universities where mathematics education programs are offered participated. The mathematics and statistics majors who participated also did so voluntarily and were not selected randomly.

Third, the strike action, at the beginning of the 2006/2007 academic year, by members of the National Association of Graduate Teachers in Ghana limited the number of days that could be spent on the fieldwork. It also limited the number of high schools that could be used in the study. The same constraints limited the number of university students and in-service teachers that participated in the study. For instance, there are students pursuing degrees in engineering and other related science and mathematics degrees who could enter the teaching profession through initial National Service postings and other means but few such students could be used in the study. Thus, the sample of university students did not cover the full range of all such related mathematics majors. This has implications for the generalizability of the results.

Fourth, the limited time available to the in-service teachers to complete the school term, after their strike action, did not make it possible for many of them who had originally agreed to participate to do so during the fieldwork. The small number of participating in-service teachers made it impossible for certain type of analysis to be performed. It also limited the number of cases used for the linear regression. Consequently, caution need to be exercised in generalizing from the model estimated in this study.

Fifth, as will be seen from the discussions of the results of the study, there is a possibility that the KAT framework is either not enough to explain the data collected or not all the items are measuring what they had been categorized as measuring in the Ghanaian context.

5.2.2 How the Instrument Performed on the Sample from Ghana

From the results of the study, a number of conclusions could be drawn about how the performance of the participating in-service and pre-service teachers in Ghana compares with their counterparts in the US who had, as at the time of the study, completed the two instruments for the KAT project. First, in terms of the difficulty levels of the items, some of the items that were most difficult and least difficult items to the US participants were also the most difficult and least difficult respectively to the Ghanaian participants. For instance, on Form 1, four of the five least difficult items to participants in the KAT study (i.e., items 3, 17, 1, and 2), were also in the five least difficult items in this study. Similarly, on Form 2, four of the five least difficult items to participants in the KAT study (i.e., items 1, 6, 9, and 10), were also in the five least difficult items in this study. However, whereas on Form 1, four of the five most difficult items to participants in the KAT study (i.e., items 4, 8, 12, and 20) were also among the five most difficult items in the Ghana study, on Form 2, only two (items 5 and 12) of the five most difficult items to the KAT participants were also among the five most difficult items to the Ghana participants.

Second, in general, the KAT participants performed significantly better than the Ghanaian participants in this study on both Forms 1 and 2. A majority of the items, 14 out of twenty, on each of Form 1 and Form 2 (i.e., 12 out of the total of 40 items on the two Forms) were more difficult for participants in Ghana than their counterparts in the US who had as at the time of the study completed the two forms for the KAT project. The only items that appeared slightly less difficult to the Ghana participants than the KAT participants were items 1, 2, 3, 11, 15 and 17 on Form 1 and items 1, 2, 6, 10,15, and 17 on Form 2. Of these only item 17 on Form 2 was an advanced knowledge item. The rest were either school knowledge or teaching knowledge items.

Three possible explanations could be given for the difference in performance between the US participants and their Ghanaian counterparts. These are; 1) differences in curricular emphasis, 2) possible affordances of handheld technological devices which the US participants use and which their Ghanaian participants did not have, and 3) familiarity to the nature of type of questions on the KAT forms.

5.2.2.1 *Differences in Curricular Emphasis*

As already mentioned, one possible explanation of the differences in performance of the US participants and their Ghanaian counterparts on items such as those cited above would the differences in the curricula emphasis between US schools and those in Ghana.

To illustrate this point, let us begin by taking a look at the issue of proof. What constitutes a proof varies from course to course. For instance, in Ghana, informal proofs are accepted in the core mathematics course at the high school level. Therefore, a Ghanaian teacher who teaches only core mathematics may take a good informal proof as a valid proof especially when the proof is presented to him as a students' work. This could possibly explain why only about 13% of the Ghanaian participants answered item number 13 (one of the common items in Forms 1 and 2) correctly versus 56% of their US counterparts. In this question different proofs of a statement by three high school students and participants were asked to determine which of the constituted valid proofs. Since this was a multiuple-choice item, Ghanaian core mathematics teachers who conceptualize the students being referred to as similar to those who could be in their classes were likely to selected the option that included the informal proof as the correct answer. In another item (i.e., item 19 of Form 2), a mathematical statement was given and participants were asked to determine if it was true and justify their answers. Compared with about 53% of the U.S. participants, only 31% of Ghanaians answered this questions right because many of the Ghanaian participants gave a number of correct examples as their justification and earned only 1 out of the maximum 4 points.

Another area where differences in curricula emphasis could have caused the difference in performance is the approaches projected in textbooks or the mathematics books available to teachers in the two countries. Presently in the US textbooks promote graphical approaches to dealing with functions more than textbooks in Ghana. For instance, in their precalculus book, Stewart, Redlin and Watson (2006) use graphs of standard functions such as $f(x) = x^2$, $g(x) = \frac{1}{x}$ and so on, and the idea of transformations to lead students into drawing graphs of functions such as $p(x) = (x+4)^2$ and $h(x) = \frac{2}{x+2}$. On the other hand, because of the fact that national examinations tend to emphasize analytical methods more than graphical approaches in dealing with functions, Ghanaian textbooks also tend to emphasize analytical methods more than such graphical solutions. Consequently, Ghanaian students and teachers would most likely graph the functions *p(x) and h(x)* by finding the intercepts, turning points, asymptotes and the behavior of the curves at the critical points. Such an approach, though effective, is not economical especially for "timed tests" such as the KAT instruments. The point being made is that with such graphical emphasis in most of the U.S. books, the KAT participants may have developed the ability to sketch graphs and use them to answer questions faster than their Ghanaian counterparts. That could explain why only about 20% of the Ghanaian participants answered item 14 of Form 2 correctly while 58% of U.S. participants got it right. In that question[23], an equation involving two distinct expressions under radical signs on each side of the equal sign was given and participants were asked to determine how many solutions the equation has. Using the graphs of the requisite standard functions and appropriate transformations, U. S. participants can save more time answering this question than their Ghanaian counterparts who could lose on other questions because of too much time spent on this one.

5.2.2.2 Possible Affordances of Handheld Technological devices

To explain the graphical calculator affordance it will be good to take a look at two multiple-choice questions on the KAT form that both groups of participants answered. One of the questions gave participants a logarithmic function and asked students to determine which of three other logarithmic functions have the same graph as the original function. Only about 15% of the Ghanaian participants answered this question correctly compared with about 31% of the US participants (refer to Table 4.1.2).

For the first of this question, from known properties of logarithmic functions, it is easy to see that the function that one of three options was the same as the given function. The question therefore reduces to making a decision about the other two options. However, by simply graphing these other options on the calculator together with original function one can decide between these two faster than graphing them from first principles without the calculator. In this case,

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[23] The KAT project had not released the items on their instruments for public consumption. Consequently, instead of quoting the actual questions, the two items being referred to are described generally.

any of the U.S. participants who had access to graphing calculators and had the opportunity to use them in completing the KAT forms were more likely to solve problems such as this faster than their Ghanaian counterparts[24].

On another item, of participants in Ghana who completed both Forms 1 and 2, only 28% answered this item correctly compared with about 58% of the US participants (see Table 4.1.2). In the question, an equation involving functions with the radical sign on both sides of the equal sign was given and participants were asked to solve it. By drawing the function defined by the expression on the left hand side and that on the right hand side of the equal sign on the same axes, using the graphing calculator, it is considerably easier to answer this question than to draw the same graphs without the calculator. Typical Ghanaian teachers and students who do not use the graphing calculator are more likely to solve this second problem analytically by squaring both sides and grouping like terms, a process that could be more time consuming that the graphical solution. This way, the Ghanaian participants could spend much more time on questions such as these to the detriment of the other questions.

In a synthesis of peer-reviewed, published research on the impact of graphing calculators on student performance, Burrill, Allison, Breaux, Kastberg, Leatham, and Sanchez (2003), concluded that, "overall, … the use of handheld technology [in the form of graphing calculators] had a positive impact on student performance" (p. 38). Such positive impacts, according to Burrill et al. (2003), are

[24] At the time of the study, graphing calculators were not permitted in Ghanaian high schools. As a result, not only were such devices not available for teachers to use, teachers did not develop the competence to use them. Teachers in Ghana were expected to lead students to graph functions from first principles.

possible when "calculator-friendly tasks [are used than when] parallel tasks that removed the calculator advantage were presented" (p. 41). In completing the instruments, the KAT project did not discourage the use of handheld technology in completing the instruments. As a result, any of the US teachers in the KAT sample who had their graphing calculators available and used them would perform better on the calculator-friendly tasks such as the two items described than the Ghanaian teachers who, not only, do not have access to them but are also not exposed to using them even for instruction.

Even for the US teachers who may not have had graphing calculators available when completing the KAT forms, it is possible that their sense about the way the graphs might look could be more fully developed than their Ghanaian counterparts because of their experience with graphing calculators and other types of software than enhance graphing abilities. Ghanaian participants in this study, on the other hand, had not been exposed to the use of graphing calculators either in school or in their teaching practice[25]. The affordances of the graphing calculator technology could contribute to the lower performance of the Ghanaian sample. The point being made is that given two people of identical knowledge, one in the US sample and the other in the Ghana sample, the US participant who uses the graphing calculator could work faster and have a higher chance of going through the items than the Ghana participants within the 60 minutes allowed. In addition, for some of the questions, drawing a quick graph on

[25] At the time of this study, senior secondary schools in Ghana were not allowed to use graphing calculators in their national examinations. As a result, teachers themselves do not learn to use it for teaching and are not allowed to use them in their classes. It came as no surprise when during the study none of the participating in-service or pre-service teachers in Ghana was seen using any graphing calculator.

the calculator could improve one's chances of getting it right. Under these conditions, therefore, higher scores from the US participants should not come as a surprise.

5.2.2.3 *Differences in the Level of Familiarity to Types of Questions*

A third possible explanation, as already mentioned, lies in the differences in the level of familiarity with the types of questions on the instruments used in the study. After subjecting the items on the KAT instruments to review by two mathematics educators in Ghana, they both agreed that the content being measured by the KAT instrument is covered in the mathematics curriculum of Ghanaian high schools. However, the items developed here in the US are not necessarily the type of items the sample in Ghana are used to in their curriculum. Some of the Ghanaian in-service teachers made anecdotal remarks to this effect (i.e., their unfamiliarity with some of the items on the instruments) during informal discussions after the administration of the instruments in Ghana. Those remarks confirmed for me similar remarks made by a section of mathematicians and mathematics educators with whom I shared a table with during one of the MSU item development workshops about an item I had formulated. One of the items I had formulated for our group's discussion did not see the light of the day because, as I was told, "it was not the typical question teachers in the US were exposed to". Therefore, the possibility of teachers in the US being familiar with some of the items could improve their chances of performing better than the teachers in Ghana.

Third, compared with data from the KAT study, on both forms, the point-biserial coefficients obtained in this study were significantly smaller for the items of both forms. It can therefore be concluded that the items on the instruments could not discriminate as well between high and low performing candidates in Ghana as they did among the US participants in KAT at the time of this study. As has already been mentioned, there was no random selection of teachers for this study. Therefore, the sample used cannot be said to be representative of high school mathematics teachers in Ghana. In addition, since there was a small variation in teaching experience and teaching certification among the in-service teachers (see Tables 3.4.5 and 3.4.6), the small number of in-service teachers used in this study (38 out of the total of 339) may not have been large enough to bring much variation in knowledge of the teachers. The possible lack of enough variation in knowledge of the small number of participating in-service teachers in Ghana is important especially because of the centrally controlled national curriculum used in the country. The teachers in Ghana, at the time of the study, were all teaching the same topics and the same textbooks. In addition, Ghanaian high school mathematics teachers collaborate among themselves through in-school departmental meetings, as well as workshops organized at annual regional and biennial conferences of the Mathematical Association of Ghana. All these factors may have been responsible for reducing any differences in knowledge that existed among these few participating teachers (i.e., 38 in-service teachers from only eight schools of comparable performance) before their teaching career started. These may explain the inability of the KAT instrument to discriminate much between low and high performers in Ghana.

Fourth, the reliability coefficients obtained in this study were too small compared with coefficients obtained from the KAT data. This means that the instruments performed quite well in measuring the underlying construct of knowledge for teaching algebra in the US but not well in this study (see reliability coefficients in the Summer 2007 draft technical report for the KAT validation study). Another interpretation of the reliability coefficient is to consider the nature of the construct being measured by the instrument. In general, if the construct being measured (in this case, knowledge for teaching high school algebra) is multi-dimensional in nature, reliabilities measured by Cronbach's alpha could be low. From this perspective, the low reliability coefficients obtained in the present study are an indication that the instrument could be measuring a multi-dimensional type of knowledge. When this happens, a factor analysis could reveal the items which load strongest on which dimensions or factors. This provides one of the bases for the factor analysis performed to answer research question one.

5.2.3 How Data from this Study Corroborated the KAT Framework

Related to the first research question, factor analysis was initially done using three components. Then based on the possibilities of selecting two or three as elbows of the scree-plot, the analysis was repeated using two components. A third analysis was also run using eight components based on interpretation of the eigenvalues obtained by applying the Kaiser-criterion. The idea was to use the nature of the factor loadings to determine which of these analyses best explained the retained factors and therefore, provided evidence of the extent to which the data on teachers' performance corroborated the three-dimensional conceptualization of knowledge for teaching algebra in the theoretical framework.

However, on both forms, the eight-component factor analysis revealed that only one of the retained factors had three items loading uniquely on it. The rest had one or two items loading uniquely on them. Therefore, using the Costello and Osborne (2005) recommendation not to interpret factors when less than three items are uniquely loaded, the eight-component factor analysis was not pursued much further because no meaningful interpretation could be drawn with the resultant small unique factor loadings.

With the three-component factor analysis, the three factors could explain only about 28% and 31% respectively of the variation in scores on Form 1 and Form 2. When the analysis was done with two factors, the proportion of the variation in scores that could be explained dropped to about 21% and 24% respectively. In addition, there were several cross-loading of items even with the two- and three-component factor analysis. For instance, on the three-component factor analysis, four items on Form 1 and five items on Form 2 cross-loaded on more than one factor. The two-component analysis also resulted in three items on Form 1 and one item on Form 2 cross-loading on both factors.

In addition, on both the two- and three-component analyses, items originally categorized as measuring any of the three types of knowledge (school knowledge, advanced knowledge, and teaching knowledge) did not uniquely load on any of the retained factors. In other words, items meant to assess different types of knowledge loaded together on the retained factors. Consequently, after the cross-loading items were removed, the nature of the factor loadings did not permit interpretation of the retained factors from both the two- and three-component analyses. Even when the item loadings were analyzed in terms of the other two dimensions of the KAT Item Development Matrix, (i.e., "algebra content" and "domain of mathematical knowledge"), the nature of the loadings did not provide evidence for labeling the factors.

These results imply that data from this study could not corroborate the KAT framework. In other words, the profile of knowledge of the participating pre-service and in-service high school mathematics teachers in Ghana did not corroborate the three categories of knowledge, *knowledge of school algebra, advanced knowledge and teaching knowledge*, hypothesized in the KAT framework.

As already discussed in the preceding chapter, the low reliability figures obtained for the instruments pointed to the possibility that the construct measured by the instrument-- knowledge used by teachers for teaching school algebra-- was multi-dimensional in nature. It was therefore expected that factor analysis would result in item loadings that could help identify the components. The inability of the items of the same nature, according to the KAT categorizations, loading uniquely on any particular factor in all the analysis could be explained in a number of possible ways.

First, as has already been mentioned, there was no random selection of teachers for this study. Therefore, the sample used could not be considered as representative of the broad variations in knowledge of senior secondary school mathematics teachers in Ghana. In addition, the small number of in-service teachers used in this study (38 out of the total of 339) may not have been sufficient to bring enough variations in knowledge not only among the in-service teachers, but also between the in-service and pre-service teachers necessary to be detected by the research instrument.

Second, related to the issue of sample size used for the factor analysis are the low communalities resulting from this study, which together may have influenced results of this study. Hogarty, Hines, Kromey, Ferron, and Mumford (2005) have shown that, in factor analysis, sample size is less likely to influence the quality of factor solutions when communalities are high than when there are low communalities. In the case of this study, very low communalities were obtained. On both forms no item had communality of 0.6 or above (see Tables 4.2.2 and 4.2.10. These low communalities, coupled with the relatively small sample sizes of 150 and 189 for Forms 1 and 2 respectively (compared with 449 for Form 1 and 392 for Form 2 in the KAT project) may together have contributed to the poor quality of factor solutions obtained in this study.

Third, there may be the need to take another look at the categorization of the items. This is because, if the underlying construct being elicited by the item does not fit the type of knowledge it has been categorized to be measuring, then the items will load differently than expected. The fact that items categorized to be measuring one of the three types of knowledge (school knowledge, advanced knowledge and teaching knowledge) are not loading uniquely on any of the factors could, thus, point to possible errors in some of the original categorizations. This possibility of wrong categorization of some of the items may have accounted for some of the cross-loadings that resulted from the factor analysis. The other possibility could be that the KAT framework is not sufficient in explaining the type of knowledge needed to teach algebra at the high school level (i.e., the hypothesis of the three distinct dimensions may not be accurate).

In the next two sub-sections two other possible explanations to how data from this study failed to support the dimensional structure hypothesized in the KAT framework. The first of these provides a critique of the KAT framework and uses the critique to offer a possible explanation about why the KAT framework was not supported by data from this study. After this, an alternative explanation is given based on how it is possible for a different curriculum context, such as Ghana's could lead to different conceptualization.

5.2.3.1 Critique of KAT Framework and Instruments

As discussed in chapter two, the KAT project's conceptualization of teachers' knowledge is important in the sense that it attempts to focus on looking at the knowledge required for teaching in one specific domain of mathematics at the secondary school level (i.e., algebra). In addition, the work by researchers in the KAT project towards re-conceptualization of the knowledge used by teachers in teaching algebra at the high school level and the development of tools for assessing this type of knowledge has the potential of being valuable to the mathematics education community worldwide. However, in trying to work with the KAT framework and their instruments in Ghana, some shortcomings or potential issues have come to light, which suggests the possibility that the framework is not sufficient in explaining the type of knowledge needed to teach algebra at the high school level.

For instance, in the KAT framework knowledge of mathematics content that precedes algebra (e.g., the content of arithmetic), so far as the trajectory of content of school mathematics, has not been separated from the mathematics content that comes after school algebra. Both types of content, content preceding and content beyond high school algebra, seem to have been put together in what the project calls advanced knowledge (refer to the KAT project's conceptual representation of advanced knowledge in Figure 2.2.1). It is generally true that, to be effective, teachers would need to engage in unpacking of content preceding the content of focus, as well as in bridging of big ideas that come after the content of interest. Unfortunately, the KAT framework does not cater separately for knowledge of school mathematics preceding algebra (e.g., arithmetic and number sense) and the knowledge that comes after algebra. Findings from the present study, for instance, points to teachers' advanced knowledge as the form of knowledge that is most likely to positively relate to students' performance (see section 5.2.5). Unfortunately, even in the light of such finding, it is impossible to discuss which aspect of the KAT project's advanced knowledge is more helpful to teachers. This issue is important because even if the KAT framework, in its present form, becomes corroborated through large scale research, it would still be difficult to be explicit as to whether the advanced knowledge teachers need to teach algebra well is the aspect of knowledge of school mathematics that precedes algebra or the knowledge that comes after algebra or whether they both contribute significantly. Separating these two types of knowledge in the framework will, therefore, help to better understand the aspect of advanced knowledge teachers need to teach algebra well.

In addition, though the KAT framework acknowledges that the boundary between the three types of knowledge emphasized in the KAT framework (i.e., knowledge of school algebra, advanced knowledge and teaching knowledge) is blurry, the discussion of the three types of knowledge in the framework is silent on any conceptualization on the type of knowledge that results from the complex interaction of the three types of knowledge in flexible ways by teachers. In practice, teachers use curriculum scripts (Putnam, 1987) or complex structure composed of interrelated sets of organized actions, which Leinhhardt and Greeno (1986) have termed schemata. This notion of how teachers transform their knowledge into pedagogical representations that connect with their prior knowledge and dispositions of the learner could be seen as a multifaceted combination of content knowledge (either school or advanced knowledge or both) and teaching knowledge consistent with Ma's (1999) conceptualization of PUFM. Unfortunately, the KAT framework is silent on the type of knowledge produced from the interaction or combination of two or more of their hypothesized three types of knowledge. If the framework had taken this into account, the cross-loadings, as well as the loading of items of different categorizations on a common factor in this study could have been explained. This, in turn, could have improved the analyses of the knowledge of teachers in this study's sample using the KAT framework. In the face of this, elaboration in the KAT framework is recommended to incorporate the type of knowledge formed by interactions among the present three hypothesized knowledge.

Further more, even in its present form, the KAT item development matrix has 24 different cells. However, the items on the KAT instruments do not span all the 24 categories. In addition, for the statistical procedures such as factor analysis, performed to retain three to eight factors, the twenty items on each of the KAT instruments seems to be too small. Gorsuch (1983) prefers six variables per factor but suggests a minimum of four variables per factor except in situations where the factors have been exceptionally defined in previous research. In addition, the possibility of cross-loadings in factor analysis that could reduce the number of items loading on each of the factors even when only three factors are retained. Consequently, on the basis of the theoretical framework, it seems that twenty items per instrument such as the KAT instruments may affect the analysis. Further more, a close look at Table 3.5.1 reveals that though the KAT item development matrix has 24 cells on each of Form 1 and Form 2, the items fitted only 15 cells. In other words, assuming that each cell has a unique attribute, the 20 items are showing only 15 of such attributes. This number, 15, further affects the analysis (e.g., the mode of factor loadings) more than if there were 20 uniquely categorized items. The implications of all these is that since, the development of items by the KAT project was in part based on their framework, any downside of the framework could mean that the instruments do not have the right types of items or the full range of items necessary for good factor solutions. It is important, for future work using the KAT framework, even as it exists now, to not only consider increasing the number of items but also ensuring that items that fit each of the cells are incorporated. Due to the possibility of cross-loadings in Factor Analysis, as was observed in this study, increasing the number of items on the forms could ensure sufficient numbers of unique loading items on the factors retained and improve the chances of being able to label the factors. In addition, ensuring that each of the 24 cells in the item development matrix has an item on the instrument would help produce a form with a wider variation of items and possibly improve the quality of factor loadings as well as the item communalities.

5.2.3.2 Effect of Curriculum on Conceptualization of Teacher Knowledge

In Ghana Algebra courses are therefore not offered as separate courses from other aspects of mathematics at the high school level. Instead, both the core mathematics and elective mathematics offered at the senior secondary school level are forms of integrated mathematics (see discussion of this in section four of chapter one). Elective mathematics teachers, for example, teach all the branches of mathematics to students in an integrated manner (refer to section 1.4 for major content areas covered in both core and elective mathematics syllabi). The curriculum for both Core and Elective Mathematics is a form of spiral curriculum. To use the words of Bruner (1960), the high school mathematics curriculum "as it develops [from the tenth grade is designed] to revisit the basic ideas repeatedly, building upon them [until the twelfth grade]" (p.13). By this spiral arrangement of topics, elective mathematics teachers, for example teach mathematics content that covers not only algebra but also geometry, trigonometry, vectors, mechanics, statistics and probability.

As a result, items on topics such as the calculus in the elective mathematics syllabus that would be categorized as measuring KAT's advanced knowledge could measure school knowledge in the Ghanaian context. A good example of such an item is the one shown below[26], which appeared on one of the elective mathematics papers of the 2004 senior secondary school certificate examinations in Ghana.

a) The gradient function of $y = px^2 + qx + r$ is $y = 8x + 4$. The function has a minimum value of 1. Find the values of *p, q,* and *r.*

b) Find the value of *c* for which $y = 3x + c$ is perpendicular to the tangent of $y = x^2 - 1$

Such a question, if used on one of the KAT instruments would be categorized as advanced knowledge. However, in Ghana, it is meant for twelfth graders and would therefore be categorized as a school knowledge item in the Ghanaian context.

This issue is important because in Ghanaian high schools a particular mathematics teacher is expected to teach the entire content of any one of the two mathematics courses assigned to him or her. Unless a teacher transfer from a school most schools, especially those who participated in this study will keep

[26] This was question number 12 of the Elective Mathematics Paper 2 of the SSSCE for twelve graders in Ghana in 2004

one teacher with a class from the ninth through the twelfth grade; an arrangement that makes teachers informally accountable for the success of their students at the SSSCE. As a result of this arrangement, teachers are given the opportunity to teach across the entire mathematics syllabus are supported to do so through collaborations with each other at the departmental, professional development programs sponsored occasionally by the Ghana Education Service, and workshops provided by the Mathematical Association of Ghana (MAG) at the district, regional and national levels.

Consequently, to Ghanaian teachers, many of the advanced knowledge items may appear to them as would KAT's school knowledge items. This may cause items categorized as school knowledge by KAT to load together with advanced knowledge items. And this could possibly explain why the KAT framework did not work well Ghana.

The foregoing implies that in order to conceptualize teachers' knowledge for teaching algebra for a country such as Ghana in which the curriculum and experience of teachers is different from their counterparts in the US, it is necessary to take into account such contextual differences. This will ensure that findings from studies using such conceptualizations would be directly applicable in the local (i.e., the country's) context by curriculum developers, teacher educators and other stakeholders of education. It will also contribute to a more open discussion among mathematics educators and researchers worldwide about the need to be cautious about the best way to go to scale worldwide with ideas, conceptualizations, theories etc. developed in different contexts.

5.2.4 Differences Between Prospective and In-service Teachers

Related to the second research question, a number of conclusions can be drawn. First, though different participants completed Form 1 and Form 2, initial *t*-test conducted revealed that, overall, the performance of those who completed Form 1 was not significantly different from the performance of Form 2 participants, at the .05 level of significance. The implication of this is that should there be the need to consider using only one of these forms, especially in situations where large sample sizes could not be guaranteed, anyone one of them would work just fine.

Second, results of the analysis of variance (ANOVA) discussed in the preceding section reveal that in Ghana, the sub-group of in-service teachers performed best on the two forms. The analysis of variance performed on the data revealed that in-service high school teachers in Ghana performed significantly better than each sub-group of prospective mathematics teachers majoring in mathematics, mathematics education and statistics from the country's universities. It was therefore concluded that knowledge for teaching algebra of in-service high school mathematics teachers is significantly different from that of prospective teachers (i.e., the three categories of university seniors majoring in mathematics, mathematics education and statistics).

This finding from the study is consistent with the argument by Sherin (2002) that in the course of teaching new curriculum, especially reform oriented curricula, teachers adapt their knowledge and in the process develop new content and pedagogical content knowledge in order to cope with the demands of the new curriculum. In Ghana, in-service teachers are required to teach an integrated mathematics curriculum. While in the university, these teachers took mathematics courses on different aspects of mathematics not in an integrated manner. To be successful in teaching the integrated mathematics curriculum at the high school level, in-service teachers in Ghana have to adapt their knowledge. They are helped in most cases by new textbooks developed and revised since the 1987 educational reforms embarked on by the country and the school-level collaborations that exist among teachers in high schools in the country. Since the 1987 reform new curriculum materials, including textbooks have periodically been introduced to help teachers adapt their knowledge and cope with the demands of the new materials and changes in content. In addition, the Mathematical Association of Ghana organizes annual conferences at the regional and district level, as well as biennial conferences at the national level. At these conferences teachers and mathematics educators share their research and best practices and provision is made for professional development on areas where teachers have earlier indicated they need help with. With these changes and activities constantly going on, teachers could be engaging in various forms of adaptation that may have improved their content and teaching knowledge beyond the level at which they were when they were in college. It is therefore not surprising that in-service teachers outperformed all categories of university seniors in this study.

Among university students in Ghana, ANOVA revealed differences in the performances among students in the different majors. In general the mathematics majors performed significantly better than their counterparts majoring in statistics and mathematics education. It is unclear whether these differences existed before the different categories of students entered the university or whether the differences are the result of the type of courses they have taken at the university level. It is hoped, as recommended in Section 5.3 that future research will take this into account in the design. Between the statistics and mathematics education students, the statistics majors did slightly better than the mathematics education students. However, this difference was not significance at the .05 level.

5.2.5 Relationship Between Teacher and Student Performances

In this study, a linear model was estimated for the relationship between Ghanaian high school mathematics teachers' knowledge and the performance of their teachers. This relationship was estimated by the equation below.

$$\boldsymbol{Y = 2.419 + .119X + e}, \quad \text{where,}$$

Y = the dependent variable representing the mean class score,

X = the independent variable representing the mean of the total teachers' knowledge (i.e., the mean score of teachers whose classes participated in the study),

2.419 = the intercept (i.e., the mean class score when the independent variable and other confounding variables are zero,

.119 = the slope for Predictor X, where the slope is the mean change in mean class score for one unit increase in the mean teachers' total score, holding all other independent variables constant, and

e = the error term

Thus, in terms of answering the third research question it was concluded that a linear relationship existed between the performance of in-service teachers who participated in the study and the students of their classes. This relationship was estimated by the equation above.

The positive coefficient of *X*, the mean teachers' total score, in the regression model implies that, granted that all possible confounding variables have been included in the model, a unit increase in mean total score of the participating in-service teachers would correspond to an increase in mean class score of their students (by a factor of .119). As was discussed in Chapter 4, this coefficient of .119 is not statistically significance as would be expected in theory. Studies in the developing world have established that teachers' subject matter knowledge is a better predictor of student achievement than other home-based factors (see for instance, Harbison and Hanushek, 1992; Mullens et al., 1996). However, the findings from this study suggests that in terms of teaching high school algebra in Ghana, for the schools that participated in the study, teachers knowledge does not significantly affect students' performance. Two possible explanations could be given to explain the non-significant coefficient of teacher knowledge obtained in this study.

The first possible explanation has to do with the small number of teachers who participated in the study as well as the type of students these schools attract. Apart from the fact the number of sample points (i.e., the ten teachers and their classes) used for the regression analysis, few schools that participated in this study were not selected randomly. In addition, at the time of the study, the participating high schools were among the best performing and most resourced schools in Ghana. In Ghana, high schools are schools of choice and students across the country compete at the Basic Education Certificate Examinations for admission in the traditionally good performing high schools. This makes it possible for the best performing high schools to attract the cream of students across the country each year. As a result, there is the possibility that other factors such as student ability and school resources, not provided in the model, could be latent in the model. Such hidden factors may have acted as confounding variables that have introduced systematic errors in the model. It is the possibility of such systematic errors that may have affected the degree of significance of the coefficient of the predictor variable (teacher knowledge). Hence, caution needs to be exercised in interpreting the coefficients or making generalizations with this model over schools that are not as high performing as the ones used in the study. As recommended in the next section, making room for random sampling of schools and increasing the sample size in future studies could lead to the building of a better or more stable model.

The second possible explanation is to consider this finding in the light of literature on the influence of teacher knowledge on student performance in algebra (see for instance, Begle, 1972; Copeland and Doyle, 1973, and Eisenberg, 1977). Begle (1972), for instance, found no significant correlation between teacher knowledge and student performance in algebra. Begle (1972) explained his finding by arguing that perhaps there is a lower bound of teacher knowledge below which there is a relationship between teacher knowledge and student performance. And that this lower bound could be so low that in algebra there may be no need to worry about the influence of teacher knowledge on student performance. One of criticisms leveled against the Begle (1972) study was that he used teachers who were participants of the National Science Foundation Institute and could therefore be considered highly motivated teachers. However, the fact that the study by Eisenberg (1977) that took care of this weakness also produced findings similar to Begle (1972) means that Begle's interpretation could not be easily discarded.

Applying Begle's (1972) interpretation on the traditionally high performing nature of students from the schools used in this study, it can be argued that perhaps when very high performing high school students are the focus of the study, such students could be so motivated that teachers' knowledge may have to be very high for its relationship with student performance to be significant. In other words, for high performing students such as those used in this study, instead of a lower bound below which teacher knowledge can significantly influence student performance, it may be necessary to think of *a form of threshold level of teacher knowledge, above which an increase in teacher knowledge will cause corresponding significant improvement in student* *performance*. This is because such high performing and highly motivated students may be able to understand enough of school algebra on their own with the availability of curriculum materials (for example, their mathematics textbooks). This last point is particularly significant because literature on student performance in developing countries is replete with the fact that school resources, such as textbooks, are some of the biggest predictors of student performance in the developing world, after family inputs have been taken care of (see Hanushek, 1997).

The positive constant term, 2.419, implies that controlling the effect of teachers' knowledge the mean class score of students in the participating school would be positive. In the participating high schools in Ghana parents, through the Parent Teacher Associations (PTAs), pay for teachers to provide extra tuition for the students beyond what is stipulated on the academic time tables by the Ghana Education Service. This improves the performance of the students generally and especially on the national examinations used for selection into the country's universities. As a result of this, children learn a great deal of the content of their school subjects, including the algebra in the mathematics curriculum outside of class (and sometimes independent of their teachers). This, together with the originally strong academic background of the students prior to their admission into these high schools could explain the high positive constant term in the model.

5.3 Recommendations for Further Study

The conclusions drawn from this study have far-reaching implications for further studies into teacher knowledge and student performance especially in Ghana. The following options are recommended for further studies:

First, universities in Ghana have other mathematics-related programs from which students have exited into the field of teaching either through initial national service postings or sometimes due to the limited job opportunities available in the country. For instance, currently there are high school mathematics teachers who majored in various engineering programs or even in economics (with mathematics as a minor area of emphasis). This study did not involve students from the full spectrum of all the possible programs. The results of the study therefore apply to the selected major areas of mathematics, statistics and mathematics education and it is not known whether university students majoring in these other areas would compare differently with in-service teachers or with any of the other groups who took part in this study. Because research in the developing world has found teachers' subject matter knowledge as a better predictor of student performance than students' home-based factors (see Harbison and Hanushek, 1992; Mullens et. al., 1996), further research is needed to include all these groups. This is important because knowing how all the possible groups compare would help to provide a framework for investigating what experiences in their programs of study could be responsible for the trend observed. This, in turn would be useful for professional development of teachers and for improving pre-service mathematics teacher education in Ghana.

Second, the limitations imposed on this study due to the small number, especially of the participating in-service teachers call for the need to get more in-service teachers involved in further studies such as this. Extending fieldwork to cover a period of at least one year is one recommended option. This extension could offset the effects of any unexpected disruption in the school calendar as happened when teachers went on strike at the time the study was to take off. Also, spending this extended time in fieldwork could improve the chances of involving a larger sample of in-service teachers in any future studies. Another option involves seeking enough funding to pay participants to get more of them involved. Taking such steps to increase the participation of more in-service teachers could help investigate how the number of years of teaching experience could improve or expand teachers' knowledge.

Third, in theory, it would be expected that the interaction of school knowledge, advanced knowledge and teaching knowledge should produce significant effects on mean class score. Unfortunately, because of the limited number of cases that were available for use in estimating the regression model in this study structural equation modeling could not be employed in estimating a regression model that included the interaction terms. In addition, the findings of the factor analysis could not permit multiple regression to be performed as an alternative model. Consequently, linear regression with a single predictor (the mean of the teachers' total scores) was used in the analysis. Further research involving a larger number of schools and teachers is needed to increase the number of cases and study the effects of such interaction terms.

In addition, future studies may need to make modification in the KAT framework to possibly split KAT's advanced knowledge into knowledge of mathematics content that precedes school algebra and knowledge that comes after school algebra. That way, items based, for instance, on concepts in arithmetic could load on a factor different from those items on content such as calculus. This modification is necessary because in its present form, if KAT's advanced knowledge is found to be significantly positively related to student performance it will still be difficult to know what aspect of advanced knowledge is being referred to.

It is further recommended that such re-conceptualization be done to include the types of knowledge formed as a result of the interaction of the individual knowledge types since in practice teachers do not only use KAT's hypothesized knowledge types in isolation but also rely on a complex interaction of any two or more of them. Further more, if factor analyses in such future studies are able to interpret the extracted factors using the KAT framework or a modified framework that takes into account other types of teacher knowledge (e.g., knowledge generated from the interaction of the main hypothesized knowledge types), such studies would contribute to the discussion of which aspect of teacher knowledge is best related to student achievement. In this regard, adapting an instrument such as the KAT instruments or modifying them in the light of new conceptualizations is helpful in moving away from the use of proxy measures to more direct measures of teacher knowledge. In this way, the model that is eventually estimated will help answer the question of which aspect of teacher knowledge is the best predictor of student performance at least in Ghana. Such a contribution is also necessary for curriculum policy maker and implementers who are responsible for the content of teacher education curriculum in Ghana. It is also useful for deciding the focus of in-service program meant aimed at improving high school mathematics teacher knowledge for improved student performance.

The case for further research using a larger number of schools and teachers is strengthened by the fact that the coefficient of regression in the model was found not to be significant. As already explained, taking this step in future research could help build a more stable model or confirm a model of the form developed in this study. Findings of such studies have the potential to contribute to the discussion on the relationship between teacher knowledge and student performance especially in developing countries such as Ghana. As already discussed, so far there is widespread disagreement among researchers about the exact relationship between teacher knowledge and student achievement. Though part of the disagreement could be due to the proxy measures so far used to represent teacher knowledge, even among those who argue for positive effect of teacher knowledge, it is unclear which aspect of teacher knowledge shows what effect. Further research that uses a larger number of schools and teachers will not only help build a more stable model, it can also improve the generalizability of the results across all senior secondary mathematics teachers in Ghana.

In addition, Shulman and Quinlan (1996) have argued that is teachers' ability to transform their content knowledge into pedagogical actions that connect to their students' dispositions could make them effective in their. In this regard, it is recommended that future studies should include classroom observations and interviews of teachers about how they prepare for their work and why they responded in specific ways in class to student difficulties and questions. Including these observations and interviews in future studies could help contribute to the discussion of how teachers transform their subject matter knowledge into effective pedagogical practices.

One aspect of the KAT instruments adapted in this study is that the school knowledge items were developed with US school curriculum in mind. Therefore, extracting the school knowledge items and administering the resulting student instrument to high school students without taking into consideration what students have had the opportunity to cover in their curriculum could result in false conclusions. This issue is important especially depending on the grade level of the students and the time of the school year the study is conducted. For this reason, initial decisions regarding this study involved administering opportunity-to-learn forms so students' performance could be assessed only on the items they have had the opportunity to learn. Unfortunately, this could not be done, as several teachers did not agree to complete it. It is therefore recommended that further studies build in ways of including such opportunity-to-learn measures.

Finally, this study has found that among university seniors, there are marked differences in the level of the three types of algebra knowledge for teaching. It is not clear whether the differences in knowledge found in this study were the result of the differences in their university coursework or the differences in performance prior to entering the different programs at the university level. A longitudinal study that helps students' entry knowledge levels to be determined and the changes in them as they progress in their programs is needed to assess the effect of their experiences in the growth in their knowledge base. Findings from such a study could be useful in decisions about curriculum development, especially for the mathematics education students who are actually being prepared for the classroom, as well as for professional development of high school mathematics teachers in Ghana.

APPENDICES

Appendix I Content of School Algebra Used by the KAT Project

Functions and their Properties: Linear and Nonlinear
A. Analyze or interpret linear relationships expressed in symbols, graphs, tables, diagrams, or written descriptions.
B. Generalize a pattern appearing in a numerical sequence or table or graph using words or symbols. Analyze or create patterns, sequences or functions given a rule.
C. Understand relations and functions, and select, convert flexibly among, and use various representations for them.
D. Express the function in general terms (either recursively or explicitly), given a table, verbal description, or some terms of a sequence.
E. Recognize, describe, or extend numerical and geometric patterns using tables, graphs, words, or symbols
F. Identify or analyze distinguishing properties of linear and non-linear including: quadratic, inverse (y = k/x) logarithmic, power, radical, polynomial or exponential functions from tables, graphs, or equations.
G. Understand and perform functional transformations such as arithmetically combining, composing, and inverting commonly used functions
H. Solve problems involving functional concepts such as composition, defining the inverse function and performing arithmetic operations of functions.
I. Describe and analyze functions of one variable by investigating domain and range, rate of change, intercepts, zeros, and local and global behavior.
J. Recognize and analyze the general forms of linear, quadratic, inverse, or exponential functions (e.g., in $y = ax + b$, recognize the roles of a and b). In other words, analyze functions with parameters as to their behavior and how they change as parameters change.
K. Identify or represent functional relationships in meaningful contexts including proportional, and common nonlinear (e.g., compound interest, bacterial growth) in tables, graphs, words, or symbols.
L. Express linear and exponential functions in recursive and explicit form given a table or verbal description
M. Use an algebraic model of a situation to make inferences or predictions.
N. Given a real-world situation, use a linear, quadratic, inverse, logarithmic, power, radical, polynomial or exponential function to fit the situation (e.g., half-life bacterial growth) and/or use the function to solve problems.
O. Approximate and interpret rates of change from symbolic, graphical and numerical data.
P. Solve systems of linear or nonlinear systems of equations by graphical methods or using functional concepts.

Appendix I continued

Equations, expressions, and inequalities
A. Write algebraic expressions, equations, or inequalities to represent a situation.
B. Perform basic operations, using appropriate tools, on linear, polynomial, logarithmic, exponential and rational expressions (including grouping and order of multiple operations involving basic operations, exponents, roots, simplifying, and expanding).
C. Solve linear, rational, or quadratic exponential, polynomial, logarithmic equations.
D. Solve linear, rational, quadratic or polynomial inequalities.
E. Use, evaluate, and solve problems involving formulas. Formulas include both common (e.g. relationship between a circle's circumference and diameter (C = pi d), distance and time under constant speed) and more advanced (e.g., the volumes and surface areas of three dimensional solids; or such formulas as: $A = P(1 + r)^t$, $A = Pe^{rt}$).
F. Analyze situations or solve problems using linear, quadratic equations, or inequalities symbolically or graphically.
G. Solve analyze, represent or interpret systems of equations or inequalities

Appendix II Algebra in the Core Mathematics Syllabus in Ghana

Functions and their Properties: Linear and Nonlinear
A. Recognize relation between two sets and establish the relation between the two sets by mapping, distinguish between various types of relations,
B. Identify functions from relations, draw graphs of sets of points lying on a line
C. Solve quadratic functions by factorization and graphical method, solve simultaneous equations in two variables, one linear one quadratic by graphical methods only, find solution of analogous equations from a given equation, use graphs of quadratic function to solve problems, determine maximum and minimum points.
Equations, expressions and inequalities
A. Express statements in mathematical symbols, formulate algebraic expressions from given situations, evaluate algebraic expression for a given value, perform operations on simple algebraic fractions with monomial and binomial denominators
B. Perform simple operations on algebraic expression, multiply two binomial expressions, change the subject of a relation, factorize algebraic expression of various forms
C. Apply difference of two squares to solve problems
D. Find the solution set of linear equations in one variable, illustrate the solution set of inequalities in one variable on the number line, illustrate graphically the regions corresponding to inequalities on two variables
E. Translate word problems into mathematical sentences, find solution set of simultaneous linear equations algebraically and graphically, graphical solution of inequalities in two variables, translate practical non-linear programming situations into inequalities and solve graphically
F. Solve quadratic equations by factorization and graphical method, find lines of symmetry from quadratic graphs, solve simultaneous equations in two variables, one linear one quadratic by graphical methods only, find solution of analogous equations from a given equation, solve equations involving exponents, use logarithms and antilogarithms to evaluate expressions
G. Use, evaluates, and solves problems involving formulas. Formulas include both common (e.g. relationship between a circle's circumference and diameter; finding area of a ring, surface area of a cone, a pyramid and other regular solids; and volume of a pipe and other regular solids) and more advanced (e.g., computing compound interests, depreciation).
H. Solve problems involving direct, indirect, partial variations and joint variation including their application to real life situations
I. Solve and explain problems involving income tax, value added tax and custom duties; calculate monthly amount payable to social security fund on a given income

Appendix III Algebra in the Elective Mathematics Syllabus in Ghana

Functions and their Properties: Linear and Nonlinear
A. Mappings and Functions (recognize the difference between functions and relations; determine inverse of one-to-one functions, composite functions)
B. Polynomial functions (recognize linear, quadratic and other polynomial functions; sketch graphs of quadratic functions; use graphs of quadratic functions to solve problems; determine parallel and perpendicular lines;
C. Rational functions (recognize a rational function; carry out the four basic operations on rational functions; resolve rational functions into partial fractions
D. Binomial theorem (write down the binomial expansion of expressions with positive integral indices; application of binomial theorem)
E. Logarithmic and exponential functions (recognize a logarithmic function, use the laws of logarithms, draw graphs of logarithmic functions, reducing relations involving exponents into linear forms using logarithms and draw and use such linear graphs to predict/estimate the value of the dependent variable given the value of the independent variable and vice versa)
F. Linear transformation and matrices (use linear transformations to find image and object points, find the inverse of a linear transformation, find the composition of two linear transformations, recognize the identity transformation, reflections in the x- and y-axes, and in the line y=x, rotation about the origin and enlargement from a point with a scale factor as some special linear transformations; state the matrix representing a linear transformation, recognize some special function (e.g. identity matrix, triangular matrices), perform algebraic operations on matrices, find the determinant of a matrix, properties of determinants, and apply determinants to find area of triangles and quadrilaterals, solve 2 and 3 simultaneous liner equations, find inverse of a matrix)
Equations, expressions and inequalities
A. Use the method of completing the square to solve quadratic equations, apply the concepts of surds to solve equations; use the sum and product of the roots of quadratic to solve equations, factor and remainder theorems and their application to cubic expressions and the solution of cubic equations, perform algebraic operations on polynomial
B. Binary operations on numbers and sets(closure, commutative, associative, and distributive properties; identity elements and inverse of an element)
C. Sequences and series (recognize a linear sequence including finite and infinite sequences, find the expression for the general term of the sequence, obtain the finite sum of a linear sequence; recognize an exponential sequence, find the expression for the general term of the sequence, obtain the finite sum of a exponential sequence, find the limit of the sum of an exponential sequence)
D. Inequalities in 2 variables (draw graphs of linear inequalities in two variables, graphical solution of two simultaneous linear inequalities in 2 variables, analytical method to find solutions to simultaneous linear inequalities)

Appendix IV Tasks of Teaching Defined by the KAT Project

KAT Label	Full Explanation
Analyzing students' mathematical work and thinking	• Teacher's mental ideas about what students have in their heads. Listening and interpreting students' explanations. • Determining the mathematical validity of a student strategy, solution or conjecture. • Teachers' models, representations, or understandings of the ideas, mental representations, thought processes, or mathematical reasoning that students may exhibit or express in some way. • Interpreting and responding to students' questions. • Teachers' attempts to understand comments, reasoning, or processes that students have made. • Evaluating students' work from a mathematical point of view. This includes challenging students with the need for using sound mathematical reasoning / argumentation and what counts as a sound argument. • Making sense of student thinking even when it is incorrect. • Teachers generating questions, problems, or other activities that will lead students to a deeper understanding of mathematics. • Deciding whether a surprising idea that a student provides is worth capitalizing on and exploring. • Solving mathematical problems that were not anticipated. Such problems may emerge in the class from students' observations or questions; sometimes they are problems in textbooks.
Designing, modifying and selecting mathematical tasks	• Making the task accessible to a range of learners. • Modifying a task to fit the needs of particular groups of learners while preserving the mathematical intentions of the tasks. • Appraising and assessing the mathematical elements of the task. • Teachers generating questions, problems, or other activities that will lead to a deeper understanding of mathematics.
Establishing and revising mathematical goals for students	• Consider policy documents. • Deciding the central ideas in a given domain, what should be emphasized. • Deciding how to approach particular topics. • Teachers' understanding and knowledge of how school math curriculum develops both across the years and within one year to build upon and further develop specific math concepts and their relation to other math concepts.

Appendix IV continued

Accessing and using tools and resources for teaching	• Evaluating the appropriateness of instructional materials e.g. technology applications, concrete materials etc for a given math task and a given set of students. • Determine how to integrate or coordinate ideas from across a set of resources. • Organizing the use of instructional materials so as to support student learning of math.
Explaining mathematical ideas and solving mathematical problems	• Teachers' attempts to communicate or explain math concepts, definitions, or ideas from school math to students. • Teachers' attempts with students to make connections between math concepts. • Presenting and encouraging multiple representations of math concepts. • Teachers' attempts to explain deep mathematical ideas in a mathematically accurate and appropriate manner so that students may be able to understand. This goes beyond simple definitions or formulas to the "Big Ideas" that may lie behind them. • Putting content in more elementary form that is still intellectually respectable. • Teachers find themselves doing math problems publicly in their classrooms.
Building and supporting mathematical community and discourse	• Deciding what a class will hold as mathematical assumptions, axioms, or starting points for justification and argument. Such decisions involve trimming. • Helping students learn to interact with one another mathematically, respond appropriately to each others' claims and arguments. • Collectively build a body of math knowledge for the classroom. • Teachers' attempts to understand or discern students' conceptions or misconceptions behind questions or comments that do not immediately make good mathematical sense. • Teachers' attempts to locate students' questions/comments in mathematical space. • Designing instruction and instructional responses to maximize student interest and motivation.

Appendix V Invitation/Permission Letters

Permission from heads of institutions/departments

Eric M. Wilmot
118 Erickson Hall
Teacher Education Department
Michigan State University
East Lansing, MI 48824

Tel: Home (in US): 517-394-0561
Cell (While in Ghana): 0246-226-244
Email: wilmoter@msu.edu

July 12, 2006

Dear Sir/Madam,

This letter is to seek permission and your approval for the conduct of a research project in your institution or department. The study is being conducted using final year students at the mathematics and mathematics education departments of the University of Ghana, Kwame Nkrumah University of Science and Technology, University of Cape Coast and the University of Education. Also participating are mathematics teachers in senior secondary schools in Accra, Kumasi, Takoradi and Cape Coast, all in Ghana and the final year elective mathematics students in the participating schools. This research project is being conducted for a PhD dissertation by a Ghanaian doctoral student at the Michigan State University in the US. It is being conducted between October and December 2006. The purpose of the study is to investigate the profile of knowledge for teaching algebra of potential Ghanaian senior secondary school (SSS) mathematics teachers, as well as, that of in-service SSS mathematics teachers in the selected senior secondary schools in Ghana.

The study aims at investigating how previous university coursework in mathematics and/or mathematics education, as well as previous teaching experiences, if any, of participants is related to the knowledge used in teaching algebra in the SSS core and the elective mathematics syllabus. In addition, the study seeks to explore the possible relationship between teachers' knowledge for teaching algebra and the achievement of their students on the algebra in the SSS mathematics curriculum.

To achieve the objectives of the study, a survey and an assessment instrument will be administered to participants who agree to participate in the study. This instrument has been adapted from the Knowing Mathematics for Teaching Algebra project currently underway at the Michigan State University in the US. It has been designed to take not more than one hour to complete in a single session. The aspect of the instrument for you consists of items based on the algebra in the SSS mathematics syllabus; some advanced knowledge items, as well as, items related to the task of teaching. In fact you have the right to accept or not to accept your institution's participation in the study.

All the data collected from the study will be confidential, to the maximum extent allowable by law. It will be kept in a safe place, under lock and key. To ensure confidentiality of the students and teachers who agree to participate in the research, no identifiers like your names, or any personal data that could be traced to participants will be required of them in completing the instrument. All the names of the university or institution in which you are currently enrolled or are teaching will be changed in the dissertation write-up and any subsequent publications. Finally, because the study is not part of your usual course assessment, your responses will not be shared with any lecturer in your academic department or institution; neither will your performance be compared with those in the other universities or institutions for the purpose of making any claims about the different universities or schools participating in the study. Only the researcher will have access to the data generated from the study.

All the data collected from the study will be confidential, to the maximum extent allowable by law. It will be kept in a safe place, under lock and key. To assure confidentiality of the students' who agree to participate in the research, no identifiers like their names, or any personal data will be required of them in completing the instrument. All the names of the university in which they are currently enrolled will be changed in the dissertation write-up and any subsequent publications. Finally, because the study is not part of your students' or teachers' usual assessment, their responses will not be shared with anyone in your academic department or institution. Neither will their performance be compared with those in the other institutions. Only the researcher and the professors supervising this dissertation study will have access to the data generated from the study.

There are no known risks associated with participation in this study. Teachers and students who agree to participate in the study may benefit from working through and reflecting on some of the questions related to mathematics teaching and learning. For teachers, this reflection will alert them to possible areas they need to research further in order to improve their teaching. For the students, such a reflection is likely to expose areas they need further reading on.

It is hoped that the finding of this study will contribute to a better understanding of the knowledge Ghanaian teachers and potential teachers have for teaching algebra at the SSS level and how it relates to improved instruction and student learning in Ghana. Such an understanding could be helpful in future curriculum planning in mathematics at the SSS level, as well as, pre-service teacher education and professional development of the nation's senior secondary school mathematics teachers.

You have the opportunity to ask questions and express concerns to me, the researcher, about your students' and/or teachers' participation in the research, as an ongoing process. You will be provided with a statement of any significant new findings developed during the course of the research that may relate to your students' and/or teachers' willingness to continue participating. You will be given a copy of your signed consent form.

There might be occasions when you might need to consult me, my dissertation directors, or the Director of Human Research Protections at the Michigan State University. Such occasions might be when you are concerned about the confidential nature of your child/ward's participation in the research or that of his/her institution.

If you have any questions about the research or research related inquiries, you may contact me directly or by phone: (517)394-0561, or e-mail: wilmoter@msu.edu or regular mail: 118 Erickson Hall, East Lansing, MI 48910.

Alternatively, if you have questions about the research or research related inquiries you may contact – anonymously, if you wish, my dissertation directors whose names, contact addresses, telephone numbers and e-mail addresses are shown below:

Prof. Joan Ferrini-Mundy
Assoc. Dean & Dir, Sci & Math Ed
211 N Kedzie Hall
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East Lansing MI 48824-1031
Phone: (517) 432-1490
Email: jferrini@msu.edu

Prof. Sharon L. Senk
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East Lansing MI 48824-1027
Phone: (517) 353-4691
Email: senk@msu.edu

If you have any questions or concerns regarding your rights as a study participant, or are dissatisfied at any time with any aspect of the study, you may contact – anonymously, if you wish – Dr. Peter Vasilenko, Director of human research protections by phone: (517)-355-2180, fax: (517)-432-4503, email: irb@msu.edu, or regular mail: 202 Olds Hall, Michigan State University, East Lansing, MI 48824

In signing below, you grant permission for students' written work to be included in the research. The participation of students of your department in the study is completely voluntary. You reserve the right to refuse your institution's participation in this project. If you give consent now, you still reserve the right to discontinue your institution's participation in the study at any time, without giving reasons.

Please, feel free to contact me directly if you have any concerns or questions.

Sincerely,

Eric M Wilmot
Teacher Education Department
Michigan State University

Please put your signature in the spaces provided below to indicate your acceptance to permit the researcher to conduct the study in your department/institution.

Your signature below indicates your voluntary agreement to allow your institution/department to participate in this study: Name & Signature: Date:

Invitation of university students and in-service teachers

Eric M. Wilmot
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Michigan State University
East Lansing, MI 48824

Tel: Home (in US): 517-394-0561
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Email: wilmoter@msu.edu

July 12, 2006

Dear Sir/Madam,

This letter is to invite you to participate in a research project, which is being conducted using final year students at the mathematics and mathematics education departments of the University of Ghana, Kwame Nkrumah University of Science and Technology, University of Cape Coast and the University of Education. Also participating are mathematics teachers in senior secondary schools in Accra, Kumasi, Takoradi and Cape Coast, all in Ghana and the final year elective mathematics students in the participating schools. The project is being conducted for a PhD dissertation and is being conducted between May, 2006 and June 2006.

The purpose of the study is to investigate the profile of knowledge for teaching algebra of potential Ghanaian senior secondary school (SSS) mathematics teachers, as well as, that of in-service SSS mathematics teachers in the selected senior secondary schools in Ghana. The study aims at investigating how previous university coursework in mathematics and/or mathematics education, as well as previous teaching experiences, if any, of participants is related to the knowledge used in teaching algebra in the SSS core and the elective mathematics syllabus. In addition, the study seeks to explore the possible relation ship between teachers' knowledge for teaching algebra and the achievement of their students on the algebra in the SSS mathematics curriculum. To achieve the objectives of the study, therefore, a survey and an assessment instrument will be administered to participants who voluntarily agree to participate in the study. This instrument has been designed to take not more than one hour to complete in a single session. The aspect of the instrument for you will consist of items based on the algebra in the SSS mathematics syllabus, as well as, some advanced knowledge items and items related to the task of teaching.
It is hoped that the findings of this study will contribute to a better understanding of the knowledge Ghanaian teachers and potential teachers have for teaching

algebra at the SSS level and how it relates to improved instruction and student learning in Ghana. Such an understanding could be helpful in future curriculum planning in mathematics at the SSS level, as well as, pre-service teacher education and professional development of the nation's senior secondary school mathematics teachers.

Eligibility and criteria
You are being invited to participate in this study because you meet any one of the following criteria:

You are a final year student of good academic standing in the mathematics department or in mathematics education of your university and have the potential of being able to teach mathematics at the SSS level should you be posted to one of the senior secondary schools for your national service.

You have in the past taught or are now teaching Core or Elective Mathematics in one of the senior secondary schools in Accra, Kumasi, Takoradi or Cape Coast, all in Ghana.

Due to the fact that your participation in the study is voluntary, you may decline to participate in the research or withdraw from the research anytime you wish, without providing reasons for doing so. In the event of this happening, there will be no penalty for your withdrawal.

How your privacy will be protected
All the data collected from the study will be confidential, to the maximum extent allowable by law. It will be kept in a safe place, under lock and key. To assure confidentiality of your participation in the research, no identifiers like your name, the name of your institution or any personal data traceable to you will be required of you in completing the instrument. In addition, all names, including that of the university or school you are currently enrolled in, or the school in which you have taught or are currently teaching will be changed in the dissertation write-up and any subsequent publications. Finally, because the study will be conducted under the auspices of your school or academic department, permission will be sought from the head of your school or department. However, your performance will not be shared with your academic department or institution. Only the researcher will have access to the data generated from the study.

Risks and benefits
There are no known risks associated with participation in this study. Teachers and students who agree to voluntarily participate in the study may benefit from working through and reflecting on some of the questions related to mathematics teaching and learning. For teachers, this reflection will alert them to possible areas they need to research further in order to improve their teaching. For the students, such a reflection is likely to expose areas they need further reading on. The privacy and confidentiality measures being taken by the researcher are

aimed at protecting you from possible consequences as a result of your responses.

You will have the opportunity to ask questions and express concerns to me, the researcher, about your participation in the research, as an ongoing process. You will be provided with a statement of any significant new findings developed during the course of the research that may relate to your willingness to continue participating. You will be given a consent form to sign to indicate your voluntary agreement to participate in the study. And you will be given a copy of your signed consent form for your records.

There might be occasions when you might need to consult me, my dissertation directors, or the Director of Human Research Protections at the Michigan State University. Such occasions might be when you are concerned about the confidential nature of your child/ward's participation in the research or that of his/her institution.

If you have any questions about the research or research related inquiries, you may contact me directly or by phone: (517)394-0561, or e-mail: wilmoter@msu.edu or regular mail: 118 Erickson Hall, East Lansing, MI 48910.

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Yours sincerely,

Eric Wilmot

Appendix VI Consent Form

Consent Form for University Students and In-Service Teachers

This research project involves final year students at the mathematics and mathematics education departments of the University of Ghana, Kwame Nkrumah University of Science and Technology, University of Cape Coast and the University of Education. Also participating are mathematics teachers in senior secondary schools in Accra, Kumasi, Takoradi and Cape Coast, all in Ghana and the final year elective mathematics students in the participating schools. The study is being conducted between October and December 2006.

The purpose of the study is to investigate the profile of knowledge for teaching algebra of potential Ghanaian senior secondary school (SSS) mathematics teachers, as well as, that of in-service SSS mathematics teachers in the selected senior secondary schools in Ghana. The study aims at investigating how previous university coursework in mathematics and/or mathematics education, as well as previous teaching experiences, if any, of participants is related to the knowledge used in teaching algebra in the SSS core and the elective mathematics syllabus. In addition, the study seeks to explore the possible relationship between teachers' knowledge for teaching algebra and the achievement of their students on the algebra in the SSS mathematics curriculum. To achieve the objectives of the study, the following survey and assessment instrument have been designed to be administered to participants who agree to participate in the study. This instrument has been adapted from the Knowing Mathematics for Teaching Algebra project currently underway at the Michigan State University in the US. It has been designed to take not more than one hour to complete in a single session. The aspect of the instrument for you consists of items based on the algebra in the SSS mathematics syllabus, as well as, some advanced knowledge items and items related to the task of teaching.
All the data collected from the study will be confidential, to the maximum extent allowable by law. It will be kept in a safe place, under lock and key. To assure confidentiality of your participation in the research, no identifiers like your name, the name of your institution or any personal data traceable to you will be required of you in completing the instrument. In addition, all names, including that of the university or school you are currently enrolled in, or the school in which you have taught or are currently teaching will be changed in the dissertation write-up and any subsequent publications. Finally, because the study will be conducted under the auspices of your school or academic department, permission will be sought from the head of your school or department. However, your performance will not be shared with your academic department or institution. Only the researcher and the professors supervising this dissertation study will have access to the data generated from the study.

There are no known risks associated with participation in this study. Teachers and students who agree to participate in the study may benefit from working through and reflecting on some of the questions related to mathematics teaching and learning. For teachers, this reflection will alert them to possible areas they need to research further in order to improve their teaching. For the students, such a reflection is likely to expose areas they need further reading on. The privacy and confidentiality measures being taken by the researcher are aimed at protecting you from possible consequences as a result of your responses.

It is hoped that the findings of this study will contribute to a better understanding of the knowledge Ghanaian teachers and potential teachers have for teaching algebra at the SSS level and how it relates to improved instruction and student learning in Ghana. Such an understanding could be helpful in future curriculum planning in mathematics at the SSS level, as well as, pre-service teacher education and professional development of the nation's senior secondary school mathematics teachers.

The research project is being conducted for a PhD dissertation by a Ghanaian doctoral student at the Michigan State University in the US. Due to the fact that your participation in the study is voluntary, you may decline to participate in the research or withdraw from the research anytime you wish, without providing reasons for doing so. In the event of this happening, there will be no penalty for your withdrawal. If you voluntarily agree to participate in the research, , you will have the opportunity to ask questions and express concerns to me, the researcher, about your participation in the research, in the course of the study, as an ongoing process. You will be provided with a statement of any significant new findings developed during the course of the research that may relate to your willingness to continue participating. You will be given a copy of your signed consent form.

There might be occasions when you might need to consult me, my dissertation directors, or the Director of Human Research Protections at the Michigan State University. Such occasions might be when you are concerned about the confidential nature of your child/ward's participation in the research or that of his/her institution.

If you have any questions about the research or research related inquiries you may contact me directly or by phone: (517)394-0561, or e-mail: wilmoter@msu.edu or regular mail: 118 Erickson Hall, East Lansing, MI 48910.

Alternatively, if you have questions about the research or research related inquiries you may contact – anonymously, if you wish, my dissertation directors whose names, contact addresses, telephone numbers and e-mail addresses are shown below:

Prof. Joan Ferrini-Mundy
Assoc. Dean & Dir, Sci & Math Ed
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East Lansing MI 48824-1031
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Prof. Sharon L. Senk
D320 Wells Hall
Mathematics Department
Michigan State University
East Lansing MI 48824-1027
Phone: (517) 353-4691
Email: senk@msu.edu

If you have any questions or concerns regarding your rights as a study participant, or are dissatisfied at any time with any aspect of the study, you may contact – anonymously, if you wish – Dr. Peter Vasilenko, Director of Human Research Protections by phone: (517)-355-2180, fax: (517)-432-4503, email: irb@msu.edu, or regular mail: 202 Olds Hall, Michigan State University, East Lansing, MI 48824

Please put your signature in the spaces provided below to indicate your acceptance to participate in the research. The absence of a signature indicates your decision not to participate in the study.

Your signature below indicates your voluntary agreement to participate in this study:	
Name & Signature:	Date:

Appendix VII High School Students' Assent Form

Secondary School Students' Assent Form

The purpose of this study is to investigate the profile of knowledge for teaching algebra of potential Ghanaian senior secondary school (SSS) mathematics teachers, as well as that of in-service SSS mathematics teachers in the selected senior secondary schools in Ghana. The study aims at investigating how previous university coursework in mathematics and/or mathematics education, as well as previous teaching experiences, if any, of participants is related to the knowledge used in teaching algebra in the SSS core and the elective mathematics syllabus. In addition, the study seeks to explore the possible relationship between teachers' knowledge for teaching algebra and the achievement of their students on the algebra in the SSS mathematics curriculum.

The following survey and assessment instrument have been designed to take not more than one hour to complete in a single session. The content items are based on the algebra in the SSS mathematics syllabus. It is not compulsory that you participate. In fact, you have the right to accept or not to accept to participate in the study.

All the data collected from the study will be confidential, to the maximum extent allowable by law. It will be kept in a safe place, under lock and key. To ensure your confidentiality, no identifiers like your name, or any personal data is needed to complete the instrument. All the names of the institution in which you are currently enrolled will be changed in the dissertation write-up and any subsequent publications. The study is not part of your usual school assessment. As a result, your responses and performance will not be shared with anyone in your institution, including your teachers and headmaster/headmistress. Neither will your performance be compared with students in other institutions. Only the researcher and the professors supervising this dissertation study will have access to the data generated from the study.

There are no known risks associated with your participation. There will be no monetary or other types of compensation for your participation. Since the items are all based on your mathematics curriculum, you may benefit from working and reflecting on the questions. Such a reflection is likely to expose areas you may need further reading on, as well as, improve your confidence on areas you have mastered in your curriculum.

It is hoped that your participation in this study will contribute to a better understanding of the knowledge Ghanaian teachers and potential teachers have for teaching algebra at the SSS level and how it relates to improved instruction and student learning in Ghana. Such an understanding could be helpful in future curriculum planning in mathematics at the SSS level, as well as, pre-service teacher education and professional development of the nation's senior secondary school mathematics teachers.

This research project is being conducted for a PhD dissertation by a Ghanaian doctoral student at the Michigan State University in the US. It is being conducted between October and December 2006. Your parents have been notified about your participation in this study. In spite of that parental notification, your personal agreement to participate is still essential. Due to the fact that your participation in the study is voluntary, you have the right to decline to participate in the research or may withdraw from the research anytime you wish, without providing reasons for doing so. In the event of this happening, there will be no penalty for your withdrawal.

If you voluntarily agree to participate in the research, please put your signature in the space provided below to indicate your agreement to participate in the study.

Yours signature below indicates your voluntary agreement to participate in this study. Name & Signature: Date:

Appendix VIII Parental Notification for High School Students

Eric M. Wilmot
118 Erickson Hall
Teacher Education Department
Michigan State University
East Lansing, MI 48824

Tel: Home (in US): 517-394-0561
Cell (While in Ghana): 0246-226-244
Email: wilmoter@msu.edu
July 12, 2006

Dear Sir/Madam,

This letter is to inform you of the conduct of a research project in your child/ward's senior secondary school. The study is being conducted using final year students at the mathematics and mathematics education departments of the University of Ghana, Kwame Nkrumah University of Science and Technology, University of Cape Coast and the University of Education. Also participating are mathematics teachers in senior secondary schools in Accra, Kumasi, Takoradi and Cape Coast, all in Ghana and the final year elective mathematics students in the participating schools.

The purpose of the study is to investigate the profile of knowledge for teaching algebra of potential Ghanaian senior secondary school (SSS) mathematics teachers, as well as, that of in-service SSS mathematics teachers in the selected senior secondary schools in Ghana. The study aims at investigating how previous university coursework in mathematics and/or mathematics education, as well as previous teaching experiences, if any, of participants is related to the knowledge used in teaching algebra in the SSS core and the elective mathematics syllabus. In addition, the study seeks to explore the possible relationship between teachers' knowledge for teaching algebra and the achievement of their students on the algebra in the SSS mathematics curriculum.

To achieve the objectives of the study, therefore, a survey and an assessment instrument will be administered to participants who agree to participate in the study. This instrument has been adapted from the Knowing Mathematics for Teaching Algebra project currently underway at the Michigan State University in the US. It has been designed to take not more than one hour to complete in a single session. The aspect of the instrument for your child or ward consists of items based on the algebra in the SSS mathematics syllabus. In addition, there is a second part of the study that will involve observing and recording lessons of selected number of teachers. It is not compulsory that you allow your child to participate. In fact you have the right to accept or not to accept your child's participation in the study.

Apart from this notification, your child will also be given the opportunity to decide to participate in the study or not. Your child's participation is not compulsory. The study is not part of his/her usual assessment in school so his/her decision not to participate will not cause her to lose anything at school.

All the data collected from the study will be confidential, to the maximum extent allowable by law. It will be kept in a safe place, under lock and key. To ensure confidentiality of the students and teachers who agree to participate in the research, no identifiers like their names, or any personal data will be required of them in completing the instrument. All the names of the institution in which they are currently enrolled or are teaching will be changed in the dissertation write-up and any subsequent publications. Finally, because the study is not part of your child's usual course assessment, his/her responses will not be shared with anyone in their institution. Neither will his/her performance be compared with those in the other institutions. Only the researcher and the professors supervising this dissertation study will have access to the data generated from the study.

There are no known risks associated with your child/ward's participation in this study. There will be no monetary or other types of compensation for your participation. Your child/ward may benefit from working through and reflecting on the questions which are all related to mathematics (s) he is learning. Such a reflection is likely to expose areas (s)he may need further reading on as well as improve his/her confidence on the mathematics (s)he is currently learning. The privacy and confidentiality measures being taken by the researcher are aimed at protecting your child/ward from possible consequences as a result of his/her responses.

It is hoped that the findings of this study will contribute to a better understanding of the knowledge Ghanaian teachers and potential teachers have for teaching algebra at the SSS level and how it relates to improved instruction and student learning in Ghana. Such an understanding could be helpful in future curriculum planning in mathematics at the SSS level, as well as, pre-service teacher education and professional development of the nation's senior secondary school mathematics teachers.

This research project is being conducted for a PhD dissertation by a Ghanaian doctoral student at the Michigan State University in the US. It is being conducted between October and December 2006. Due to the fact that your child's participation in the study is voluntary, you may decline to agree that he/she participates in the research or ask him/her to withdraw from the research anytime you wish, without providing reasons for doing so. In the event of this happening, there will be no penalty for his/her withdrawal.

During the course of the project, you have the opportunity to ask questions and express concerns to me, the researcher, about your child/ward's participation in the research, as an ongoing process. You will be provided with a statement of any significant new findings developed during the course of the research that may relate to your willingness to allow your child/ward to continue participating.

There might be occasions when you might need to consult me, my dissertation directors, or the Director of Human Research Protections at the Michigan State University. Such occasions might be when you are concerned about the confidential nature of your child/ward's participation in the research or that of his/her institution.

If you have any questions about the research or research related inquiries you may contact me directly or by phone: (517)394-0561, or e-mail: wilmoter@msu.edu or regular mail: 118 Erickson Hall, East Lansing, MI 48910.

Alternatively, if you have questions about the research or research related inquiries you may contact – anonymously, if you wish, my dissertation directors whose names, contact addresses, telephone numbers and e-mail addresses are shown below:

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Email: senk@msu.edu

If you have any questions or concerns regarding your child's rights as a study participant, or are dissatisfied at any time with any aspect of the study, you may contact – anonymously, if you wish – Dr. Peter Vasilenko, Director of human research protections by phone: (517)-355-2180, fax: (517)-432-4503, email: irb@msu.edu, or regular mail: 202 Olds Hall, Michigan State University, East Lansing, MI 48824

Sincerely,

Eric M Wilmot
Michigan State University

Appendix IX Public Released Items of the KAT Project

1. Which of the following situations can be modeled using an exponential function?

   i. The height *h* of a ball *t* seconds after it is thrown into the air.

   ii. The population *P* of a community after *t* years with an increase of *n* people annually.

   iii. The value *V* of a car after *t* years if it depreciates *d%* per year.

A. i only

B. ii only

C. iii only

D. i and ii only

E. ii and iii only

2. For which of the following sets S is the following statement true?

   For all a and b in S, if ab = 0, then either a = 0 or b = 0.

   i. the set of real numbers

   i. the set of complex numbers

   iii. the set of integers mod 6

iv. the set of integers mod 5

v. the set of 2x2 matrices with real number entries

A. i only

B. i and ii only

C. i, ii and iv only

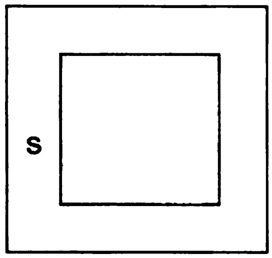
D. i, ii, iii and iv only

E. i, ii, iii, iv, and v

3. A student solved the equation, $3(n - 7) = 4 - n$ and obtained the solution $n = 2.75$.

   What might the student have done wrong?

4. Hot tubs and swimming pools are sometimes surrounded by borders of tiles. The drawing at the right shows a square hot tub with sides of length s feet. This tub is surrounded by a border of 1 foot by 1 foot square tiles.



How many 1-foot square tiles will be needed for the border of this pool?

a. Paul wrote the following expression:

$$2s + 2(s+2)$$

Explain how Paul might have come up with his expression.

b. Bill found the following expression:

$$(s+2)^2 - s^2$$

Explain how Bill might have found his expression.

c. How would you convince the students in your class that the two expressions above are equivalent?

Appendix X Sample Opportunity to Learn Questions

1. Which of the following situations can be modeled using an exponential function?

    i. The height *h* of a ball *t* seconds after it is thrown into the air.

    ii. The population *P* of a community after *t* years with an increase of *n* people annually.

    iii. The value *V* of a car after *t* years if it depreciates *d%* per year.

A. i only

B. ii only

C. iii only

D. i and ii only

E. ii and iii only

Your students' opportunity to learn the mathematics in the question above:

Please, indicate whether or not your students had the opportunity to learn the mathematics needed to answer the question above.

A. Yes, I taught it because it is part of the required curriculum

B. Yes, I taught it, even though it is not part of the required curriculum

C. No, it is not part of the required curriculum so I did not teach it

D. No, even though it was part of the required curriculum, I did not/have not taught it.

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