ANALYSIS OF
CONCRETE SKEW RIGID FRAMES

Thesis for the Degree of M. S.

MICHIGAN STATE COLLEGE

Atru M. Chowdiah

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THESIS

This is to certify that the

thesis entitled

ANALYSIS OF CONCRETE SKEW RIGID FRAMES

presented by

Atru M. Chowdiah

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ANALYSIS OF

CONCRETE SKEW RIGID FRAMES

By

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INTRODUCTION

The first paper on the subject entitled "Analysis of the Stresses in the Ring of a Concrete Skew Arch" by J. C. Rathbun was published in A.S.C.E. Transactions, 1924, p. 611. At that time Professor Rathbun's theory was questioned and the A.S.C.E. appointed a a special Committee on Concrete and Reinforced Concrete Arches to give a practical test of the theory. Professor G. E. Beggs of Princeton, N. J., was engaged in making deformeter analysis on models of skew arches. At the same time, Professor Rathbun was making mathematical analysis of the structures of the same proportions. After the analysis was completed individually without consultation, results were compared and were found to be in close agreement, and so Professor Rathbun's theory was accepted.

In 1925 Professor Rathbun extended his theory of skew rigid arches to skew rigid frames and an adaptation of his theory was made by Mr. A.G. Hayden and is presented in a book "The Rigid-Frame Bridge" by A. G. Hayden, John Wiley & Sons, Inc., 1940, 2nd Edition. In the presentation made by Mr. Hayden, it is necessary to derive and solve 4 simultaneous equations.

Later Mr. Hodges presented a paper "Simplified Analysis of Skewed Reinforced Concrete Frames and Arches" and it is published in A.S.C.E. Transactions (1944), p. 913. Mr. Hodges has simplified the work by reducing the solution of four simultaneous equations to solution of two simultaneous equations.

For this thesis, Mr. Hodges' paper is the principal reference although references are made to other papers and one example has been worked out to illustrate the application of the theory.

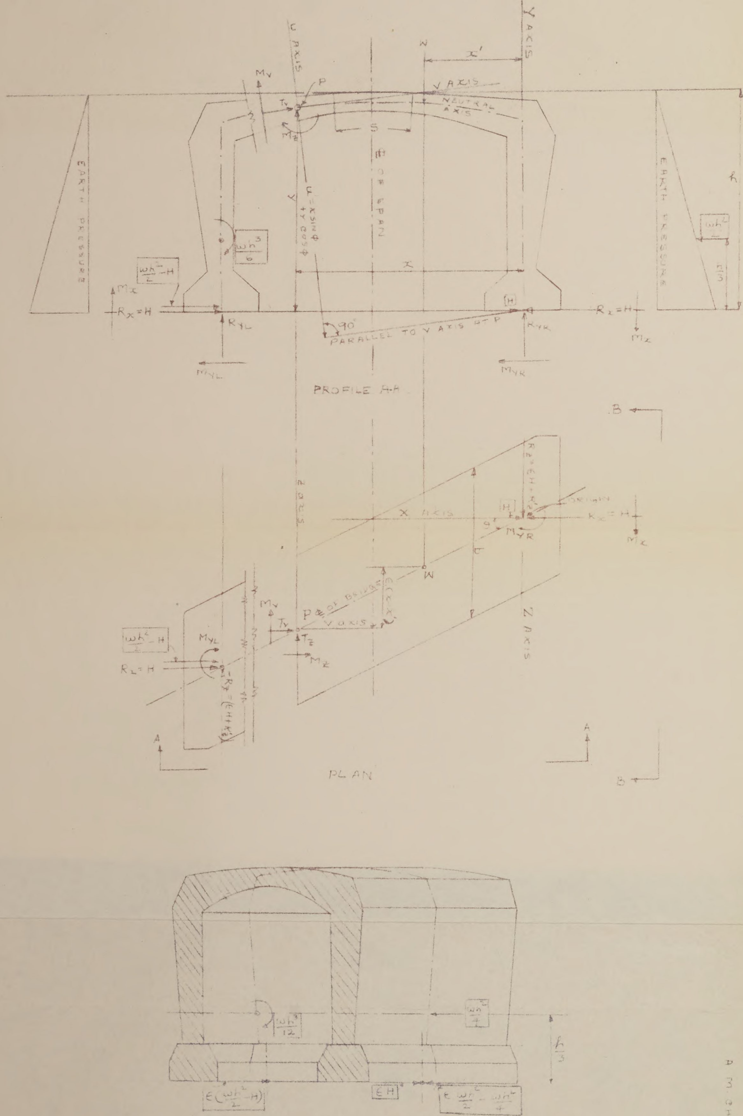
The analysis has been made for assumptions that rotation may occur about any axis. There is no justification to assume complete fixity in

any direction since the degree of rotation required to change the effect of complete fixity is small. On the other hand, any assumption of complete unrestricted rotation about any axis is not correct. Therefore, the frame is analyzed for:

Assumption 1 - rotation about Z axis.

Assumption 2 - rotation about Z and X axes.

Assumption 3 - rotation about Z, X and Y axes.



FORCES AND REACTIONS FOR VERTICAL LOADING AND EARTH PRESSURE. QUANTITIES IN RECTAN-GLES ARE EXTERNAL FORCES DUE TO EARTH PRESSURE.

FIG 1 & 2.

A. M. CHOWDHRY

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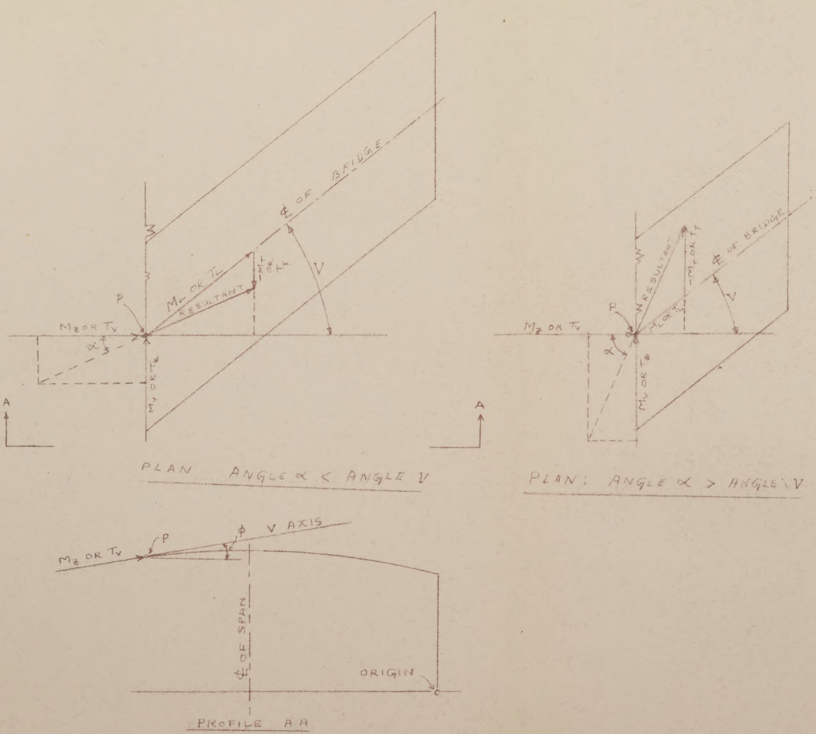


FIG 3. TRANSFORMATION OF INTERNAL STRESSES AT P.

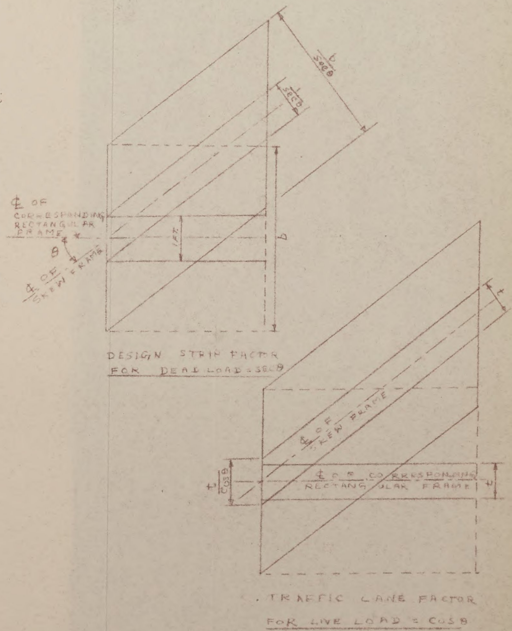
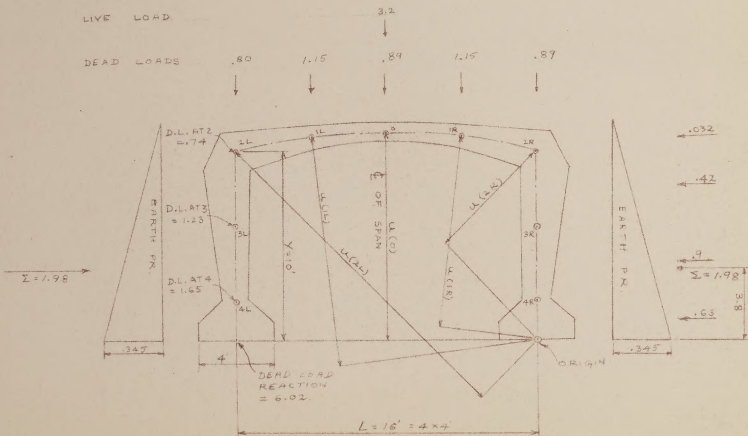
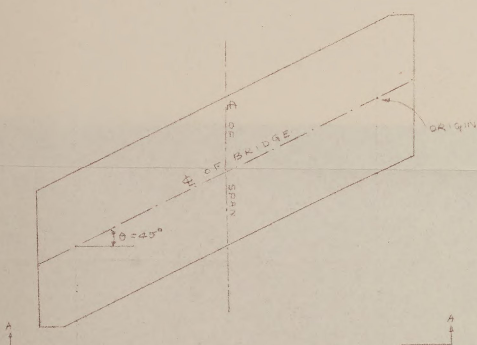


FIG.4. SKEW CORRECTIONS



PROFILE A-A

SCALE : 1" = 5'



PLAN

NOT TO SCALE

FIG 5.

DIMENSIONS AND LOADINGS FOR THE PROBLEM.
LOADS ARE IN KIPS AND FOR 1' WIDTH OF BRIDGE.

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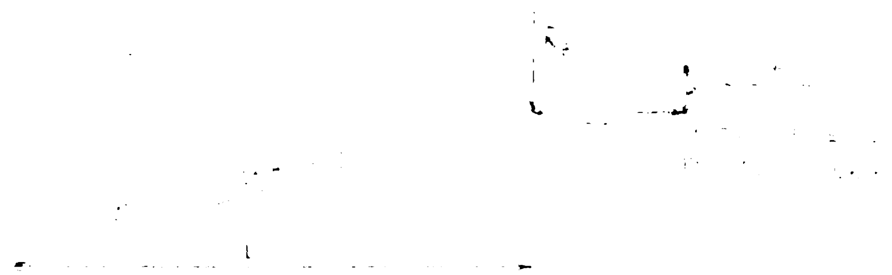

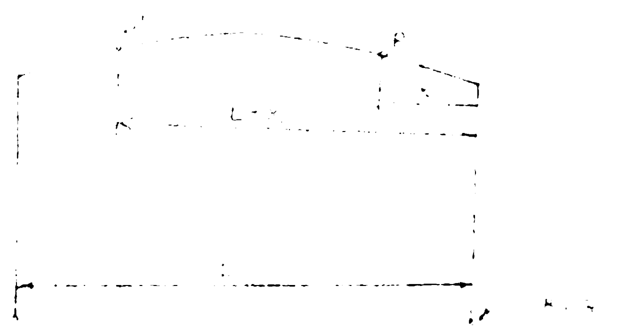

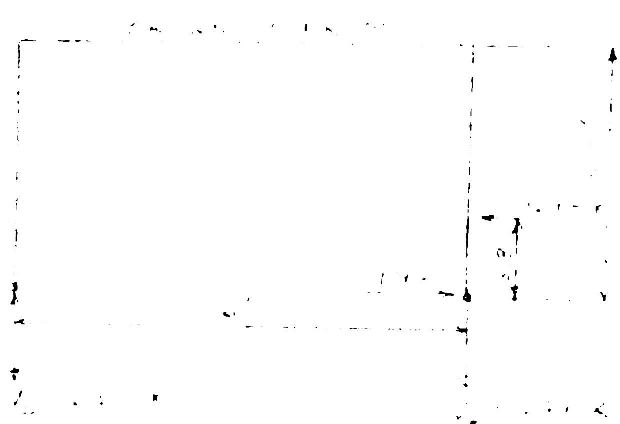

OUTLINE FOR DETERMINATION OF REACTIONS

The skew frame, if restrained against translation and rotation in all three principal directions, would have six reaction components. Let the principal axes be designated by X, Y, and Z. The X axis is horizontal and is perpendicular to the axis of the abutments, Y axis is vertical, and Z axis is parallel to the axis of the abutment.

The reaction components at the right footing are R_x, M_x, R_{yr}, M_{yr}, R_z, M_z. The usual assumption for footings on earth foundations is that there can be free rotation about the Z axis. So $M_z = 0$. R_{yr} is obtained by statics and will be the same as the reaction for a simply supported beam over a span = the span of the corresponding rectangular frame.

The reactions to be determined are R_x, M_x, M_{yr}, R_z.

It has been proven before that the component H of a two hinged rectangular rigid frame is so nearly equal to the R_x of a two hinged skewed rigid frame of equal right span, that the difference is negligible for purposes of design. This assumption has greatly reduced the amount of labor in the solution of the problem. The reaction components H and its equilibriant ←H are the most important in the design.

The equilibrant ←H is the principal part of the R_z component. The remaining part of R_z which will be designated R^1_z, the reactions M_x and M_y constitute a separate group which depend on the torsional elastic deformation, so we have to solve three simultaneous equations for the three unknowns. However, M_y turns out to be a function of R^1_z so we need to solve only two simultaneous equations for the remaining redundants.

The solution of two separate elastic systems is certainly less likely to be of error than a combined elastic system, and so it is definitely an advantage.

DETAILS OF ANALYSIS

The forces acting on the structure are shown in Figure 1 for vertical loading and in Figure 2 for balanced earth pressure. Unbalanced earth pressure is omitted due to the fact that it is not usually considered in practical design.

Let us determine the reaction components at the point of origin at the right support. The solution of the reaction components is done in two stages.

Stage 1: This consists of computing the horizontal component H of the reaction at the footing of a two-hinged rectangular rigid frame (designated as the "corresponding rectangular frame".) which is the projection of the skew frame on a plane parallel to the XY plane for loads on one foot width of the bridge. The H is equal to the R_x of the skew for vertical loading and earth pressure and $H(1 + \zeta^2) = R_x$ in case of temperature change.

Stage 2: This comprises of setting up equations and solving for the redundants R_z^1, M_x, M_{yr} for the structure in equilibrium under the forces H, ζH, R_y, R_z, M_x, and M_{yr} acting at the origin.

The equations are set up to satisfy the following conditions:

There can be no displacement along any of the axes. There can be no rotation about any axis except for

Assumption 1 - about the Z axis.

Assumption 2 - about Z and X axes.

Assumption 3 - about X and Y axes, and Z axis.

The internal stresses acting at any point P are represented as the reactions exerted by the portion to the left of P on the portion to the right of P. The reactions whose directions are unknown are shown in Figure 1 in their assumed positive directions. The subscript on each moment term indicates the axis about which the moment occurs.

Geometric Relations and Definitions.

The coordinate axes, the applied loads, the reaction components, the components of the internal stresses acting at any point P lie in a plane parallel to or perpendicular to the XY plane. The profiles shown in Figures 1 and 2 are projections of the skew frame on a plane parallel to the XY plane. The point P is on the neutral axis and at the center of any subdivision S. The internal stress components at P are along the axes u, v, and z axes with origin at P. The angle Φ is the slope of the neutral axis (v axis) at P. The quantity u is the component parallel to the u axis of the distance between P and the origin O. Also:

c = coefficient of linear expansion for concrete and reinforcing steel.

t^o = temperature rise or fall, in degrees Fahrenheit.

w = equivalent fluid pressure of earth, in pounds per square foot per foot of depth.

M_o = simple span moment on the corresponding rectangular frame, for vertical loading.

$$K = \frac{\text{Modulus of elasticity for concrete under axial stress}}{\text{Modulus of elasticity for concrete in shear}}$$

$$= \frac{E}{G}$$

= 2.67 (assumed)

F = factor of torsion (corresponding to the moment of inertia I in case of flexure)

$$= \frac{bt^3}{3.58}$$

H = horizontal thrust in the corresponding rectangular frame for the particular loading.

h = height of the structure from bottom of footing to the highest point of the structure (crown).

$(^s$ = the summation sign Σ

$($ = $($ = tan θ

Derivation of Equations for the Redundants $R^1{}_z$, M_{yr}, M_{yl}, M_{xr}.

Three sets of equations are derived to satisfy the three different assumptions. Each set of equations is derived for vertical loading, balanced earth pressure and temperature change.

The assumption is made that the effects of all thrusts and shears are negligible in considering deflections. In the following equations, the effect of only the torsional moment M_v at all points of the structure is considered.

Let

M_v = moment at P about the v-axis due to all forces acting.

$m_{v(z)}$ = moment at P about the v-axis, due to a unit force acting along and in the direction of $R^1{}_z$.

$m_{v(ox)}$ = moment at P about the v-axis due to a unit moment acting along and in the direction of M_x.

$m_{v(oy)}$ = moment at P about the v-axis due to a unit moment acting along and in the direction of M_y.

The equation for no deflection along the Z axis is:

$$\frac{s}{G} \Sigma^{s} \frac{M_v m_{v(x)}}{F} = 0 \qquad (1a)$$

The equation for no rotation about X-axis is:

$$\frac{s}{G} \Sigma^{s} \frac{M_v m_{v(ox)}}{F} = 0 \qquad (1b)$$

The equation for no rotation about Y-axis is:

$$\frac{s}{G} \Sigma^{s} \frac{M_v m_{v(oy)}}{F} = 0 \qquad (1c)$$

Assumption 1:

The expression for M_v for vertical loading from Figure 1 is:

$$M_v = R_x (-x \sin \Phi - R_z u + M_x \cos \Phi - M_{yr} \sin \Phi + R_{yr} (-x \cos \Phi$$

$$- W (-(x-x^1) \cos \Phi$$

$$= R_x (-x \sin \Phi - R_z u + M_x \cos \Phi$$

$$-M_{yr} \sin \Phi + M_o (-\cos \Phi \qquad (2)$$

But for vertical loading $R_x = H$

Also

$$m_{v(z)} = -u \tag{3a}$$

$$m_{v(ox)} = \cos \Phi \tag{3b}$$

$$m_{v(oy)} = -\sin \Phi \tag{3c}$$

Due to symmetry $\int^{s} \frac{\sin \Phi \cos \Phi}{F} = 0$ (3e)

and $\int^{s} \frac{y \sin \Phi \cos \Phi}{F} = 0$ (3f)

$$R_z = \epsilon H + R^1_z \tag{3g}$$

From Figure 1:

$$u = x \sin \Phi + y \cos \Phi \tag{3h}$$

$$\therefore \quad x = \frac{u - y \cos \Phi}{\sin \Phi} \tag{3k}$$

Substituting Eqs. 2 and 3 in Eq. 1a, we have for vertical loading,

$$-H \epsilon \int^{s} \frac{x\,u \sin \Phi}{F} + R^1_z \int^{s} \frac{u^2}{F} + \epsilon H \int^{s} \frac{u^2}{F}$$

$$-M_x \int^{s} \frac{u \cos \Phi}{F} + M_{yr} \int^{s} \frac{u \sin \Phi}{F}$$

$$-M_o \epsilon \int^{s} \frac{u \cos \Phi}{F} = 0$$

$$\therefore \quad -H \epsilon \int^{s} \frac{u^2}{F} + H \epsilon \int^{s} \frac{y\,u \cos \Phi}{F} + \epsilon H \int^{s} \frac{u^2}{F}$$

$$-M_x \int^{s} \frac{u}{F} \cos \Phi + M_{yr} \int^{s} \frac{x \sin^2 \Phi}{F}$$

$$-M_o \epsilon \int^{s} \frac{u \cos \Phi}{F} + R^1_z \int^{s} \frac{u^2}{F} = 0$$

$$\therefore \quad R^1_z \int^{s} \frac{u^2}{F} - M_x \int^{s} \frac{u}{F} \cos \Phi + M_{yr} \int^{s} \frac{x \sin^2 \Phi}{F}$$

$$= \epsilon \left\{ \int^{s} \frac{M_o\, u \cos \Phi}{F} - H \int^{s} \frac{u\, y \cos \Phi}{F} \right\} \tag{4a}$$

By similar substitution of Eqs. 2 and 3, in Eq. (1b) we have for vertical loading

$$\int^{s} H \frac{\epsilon\; x \sin \Phi \cos \Phi}{F} - \int^{s} \frac{\epsilon H\, u \cos \Phi}{F}$$

$$- \int^s \frac{R^1_g \, u \cos \Phi}{F} + \int^s \frac{M_x \cos^2 \Phi}{F} + \int^s \frac{M_o \, \epsilon \cos^2 \Phi}{F}$$

$$= 0$$

$$\therefore \int^s \frac{H \, \epsilon \, x \sin \Phi \cos \Phi}{F} - \int^s \frac{\epsilon \, H \, x \sin \Phi \cos \Phi}{F}$$

$$- \int^s \frac{\epsilon \, H \, y \cos^2 \Phi}{F} - \int^s \frac{R^1_g \, u \cos \Phi}{F}$$

$$+ \int^s \frac{M_x \cos^2 \Phi}{F} + \int^s \frac{M_o \, \epsilon \cos^2 \Phi}{F}$$

$$= 0$$

$$\therefore R^1_g \int^s \frac{u \cos \Phi}{F} - M_x \int^s \frac{\cos^2 \Phi}{F}$$

$$= \epsilon \left\{ \int^s \frac{M_o \cos^2 \Phi}{F} - H \int^s \frac{y \cos^2 \Phi}{F} \right\} \qquad (4b)$$

By similar substitution of Eqs. 2 and 3 in Eq. (1c), we have for vertical loading

$$- \int^s \frac{H \, \epsilon \, x \sin^2 \Phi}{F} + \int^s \frac{R_g \, u \sin \Phi}{F}$$

$$- \int^s \frac{M_x \sin \Phi \cos \Phi}{F} + \int^s \frac{M_{yr} \sin^2 \Phi}{F}$$

$$- \int^s \frac{M_o \, \epsilon \sin \Phi \cos \Phi}{F}$$

$$= 0$$

In the expression on the L.H.S., the 3rd term becomes 0 and substituting $u = x \sin \Phi + y \cos \Phi$ in the 2nd and 5th terms, we have

$$- \int^s \frac{H \, \epsilon \, x \sin^2 \Phi}{F} - \int^s \frac{\epsilon \, H \, x \sin^2 \Phi}{F}$$

$$+ \int^s \frac{\epsilon \, H \, Y \sin \Phi \cos \Phi}{F} + \int^s \frac{R^1_g \, x \sin^2 \Phi}{F}$$

$$+ \int^s \frac{R^1_g \, y \sin \Phi \cos \Phi}{F}$$

$$= 0$$

$$R^1_{z} \quad \int^s \frac{x \sin^2 \Phi}{F} + M_{yr} \quad \int^s \frac{\sin^2 \Phi}{F} = \leftarrow \quad \int^s \frac{M_o \sin \Phi \cos \Phi}{F} \quad (4c)$$

Equations 4 are modified for application to earth pressure and temperature change as follows:

1. Balanced earth pressure

   In Eq. 2

$$Rx = \frac{wh^2}{2} - H$$

$$R_{yr} = 0$$

$$W = 0$$

$$\therefore M_o = 0$$

In equations 4, M_o term is omitted and H is replaced by $\frac{wh^2}{2} - H$.

2. Temperature rise:

   The increase in span = C t^o L

   Deflection of right end in direction of R_z = $- \leftarrow ct^o L$

(See Figure 6.)

Equation 1a represents the deflection of the right footing in the direction of R_z

$\therefore$ (1a) becomes

$$\frac{s}{G} \quad \int^s \frac{M_v m_{v(z)}}{F} - \leftarrow ct^o L = 0$$

multi plying by the constant $\frac{G}{s}$

$$\int^s \frac{M_v m_{v(z)}}{F} - \frac{G}{s} \leftarrow ct^o L = 0$$

$$\int^s \frac{M_v m_{v(z)}}{F} - \frac{E}{Ks} \leftarrow ct^o L = 0$$

Also $W = 0$: $R_{yr} = 0$ $\therefore$ $M_o = 0$

and $R_x = H\ (1 + (-^2)$

$\therefore$ In Eq. 4a M_o term is omitted, $H\ (1 + (-^2)$ is substituted for H and the term $\frac{-E}{Ks}$ (- ct°L is added to the L.H.S. (or $\frac{E}{Ks}$ (- ct°L can be added to the R.H.S.)

In Eqs. (4b) and (4c) omit the M_o term and replace H by $H\ (1 + (-^2)$.

(See Figure 7.)

For a symmetrical frame as shown in the Figure, there are pairs of points like P and P' in such a way that

$$\sin^2 \Phi \text{ for } P = \sin^2 \Phi \text{ for } P'$$

and P is at a distance x from origin and P' is at a distance (L-x) from origin.

$\therefore$ For the 2 points, $\Sigma \frac{x \sin^2 \Phi}{F} = (x+L-x) \frac{\sin^2 \Phi}{F} = L \frac{\sin^2 \Phi}{F}$

$\therefore$ For the whole frame, $\Sigma \frac{x \sin^2 \Phi}{F} = L\ \Sigma \frac{\sin^2 \Phi}{F}$

where $\Sigma \frac{\sin^2 \Phi}{F}$ is the sum of $\frac{\sin^2 \Phi}{F}$ for half the number of points on the frame. For sake of convenience, if $\Sigma \frac{\sin^2 \Phi}{F}$ has to be the sum of $\frac{\sin^2 \Phi}{F}$ for all the points on the frame, then

$$\Sigma \frac{x \sin^2 \Phi}{F} = \frac{L}{2}\ \Sigma \frac{\sin^2 \Phi}{F}$$

<u>For symmetrical loading</u> in Eq. (4c) Mo term becomes zero.

$$\therefore R^1_s\ \frac{L}{2}\ \Sigma \frac{\sin^2 \Phi}{F} + M_{yr}\ \Sigma \frac{\sin^2 \Phi}{F} = 0$$

$$\therefore M_{yr} = -\ R^1_s\ \frac{L}{2}$$

Also R^1_z and M_x at right support = R^1_z and M_x at left support

Taking moments about left support

$(\Sigma H + R^1_z)L + M_{yl} + M_{yr} - H\,\Sigma L = 0; \therefore\ M_{yl} + M_{yr} = -R^1_z\,L$

$\therefore\ M_{yr} = M_{yl} = -R^1_z\,\frac{L}{2}$

<u>For unsymmetrical loading</u>

$$M_{yr} = \frac{\Sigma \int^s \frac{M_o \sin\Phi\cos\Phi}{F} - R^1_z\,\frac{L}{2} \int^s \frac{\sin^2\Phi}{F}}{\int^s \frac{\sin^2\Phi}{F}}$$

$$= \frac{\Sigma \int^s \frac{M_o \sin\Phi\cos\Phi}{F}}{\int^s \frac{\sin^2\Phi}{F}} - R^1_z\,\frac{L}{2}$$

$$= C - R^1_z\,\frac{L}{2}$$

where $C = \dfrac{\Sigma \int^s \frac{M_o \sin\Phi\cos\Phi}{F}}{\int^s \frac{\sin^2\Phi}{F}}$

but $M_{yr} + M_{yl} = -R^1_z\,L$

$\therefore\ M_{yl} = -C - R^1_z\,\frac{L}{2}$

Now supposing the unsymmetrical loading is made symmetrical by adding equal loading placed symmetrically with the unsymmetrical loading, then M_{yr} due to additional loading $= -C - \frac{R^1_z\,L}{2}$

M_{yl} " " " " $= C - R^1_z\,\frac{L}{2}$

$\therefore$ The total M_{yr} due to the combined balanced loading $= -R^1_z\,L$

and M_{yl} $= -R^1_z\,L$.

Designating the M_{yr} and M_{yl} for the combined balanced loading by M_y, we have

$$M_y = -R^1_z\,L$$

$$\therefore\ R^1_z\,\frac{L}{2} = -\frac{M_y}{2}$$

$\therefore$ For unsymmetrical loading

$$M_{yr} = C + \frac{M_y}{2}$$

$$M_{yl} = -C + \frac{M_y}{2}$$

where $M_y = M_{yr}$ or M_{yl} for balanced loading.

M_x and R^1_z for unsymmetrical loading are half of the respective quantities for symmetrical loading. So in case of unsymmetrical loading also it is only necessary to solve only 2 equations in which quantities of the balanced loading are used, and to apply the correction discussed.

Making the aforesaid substitutions, L.H.S. of Eq. (4a)for aymmetrical loading

$$= R^1_z \int^s \frac{u^2}{F} - M_x \int^s \frac{u \cos \Phi}{F} - \frac{L}{2} R^1_z \int^s \frac{x \sin^2 \Phi}{F}$$

$$= R^1_z \left\{ \int^s \frac{u^2}{F} - \frac{L^2}{4} \int^s \frac{\sin^2 \Phi}{F} \right\} - M_x \int^s \frac{u \cos \Phi}{F}$$

$\therefore$ Eq. (4a) becomes

$$R^1_z \left\{ \int^s \frac{u^2}{F} - \frac{L^2}{4} \int^s \frac{\sin^2 \Phi}{F} \right\} - M_x \int^s \frac{u \cos \Phi}{F}$$

for symmetrical vertical loading,

$$= \epsilon \left\{ \int^s \frac{M_o u \cos \Phi}{F} - H \int^s \frac{uy \cos \Phi}{F} \right\} \qquad (5a\text{-}1)$$

for balanced earth pressure,

$$= -\epsilon \left\{ \frac{wh^2}{2} - H \right\} \int^s \frac{uy \cos \Phi}{F} \qquad (5a\text{-}2)$$

and for temperature rise,

$$= \epsilon \left\{ \frac{Ect^o L}{Ks} - H (1+\epsilon^2) \int^s \frac{uy \cos \Phi}{F} \right\} \qquad (5a\text{-}3)$$

Eq. (4b) becomes

$$R^1_z \int^s \frac{u \cos \Phi}{F} - M_x \int^s \frac{\cos^2 \Phi}{F}$$

for symmetrical vertical loading,

$$= - \epsilon \left\{ \int^s \frac{M_o \cos^2 \Phi}{F} - H \int^s \frac{y \cos^2 \Phi}{F} \right\} \qquad (5b\text{-}1)$$

for balanced earth pressure,

$$= - \epsilon \left\{ \frac{wh^2}{2} - H \right\} \int^s \frac{y \cos^2 \Phi}{F} \qquad (5b\text{-}2)$$

and for temperature rise,

$$= - \epsilon H (1+\epsilon^2) \int^s \frac{y \cos^2 \Phi}{F} \qquad (5b\text{-}3)$$

For symmetrical loading,

$$M_{yr} = M_{yl} = - R^1_z \frac{L}{2} \qquad (5c)$$

For unsymmetrical vertical loading,

$$M_{yr} = \frac{M_y}{2} + C \quad \text{and} \quad M_{yl} = \frac{M_y}{2} - C \qquad (5d)$$

where M_y is M_{yl} or M_{yr} for balanced vertical loading.

$$\text{and C is } = \frac{\epsilon \int^s \frac{Mo \sin \Phi \cos \Phi}{F}}{\int^s \frac{\sin^2 \Phi}{F}}$$

in which Mo are values for actual vertical unsymmetrical loading. Also, R^1_z and M_x for unsymmetrical loading are equal to one half of R^1_z and M_x for balanced vertical loading.

Eqs. 5 constitute a set from which R^1_z, M_{yr}, M_{yl}, and M_x are determined for Assumption 1.

Assumption 2.

Since $M_x = 0$, it is only necessary to solve one equation, say Eq. 5a.

For vertical loads and temperature rise in the L.H.S. of Eq. 5a, omit the M_x term.

For balanced earth pressure loading, we assume that the frictional resistance of the concrete surface also acts.

Assuming the coefficient of friction of earth on concrete = .5, the total frictional force = $\frac{wh^2}{4}$ acting at a distance of $\frac{h}{3}$ above the bottom of the footing on either faces.

$\therefore$ The moment at the right end = $\frac{wh^3}{12}$

$\therefore$ The contribution towards M_v due to $\frac{wh^3}{12}$ is $\frac{wh^3}{12}$ cos Φ

$\therefore$ In the expression for M_v in Eq. 2, omit M_x-term and add the term

$$\frac{wh^3}{12} \cos \Phi$$

$\therefore$ The R.H.S. of Eq.(5a) will have an additional term = $\frac{wh^3}{12} \int^s \frac{u \cos \Phi}{F}$

$\therefore$ Eq. (5a) becomes:

$$R^1{}_z \left(\int^s \frac{u^2}{F} - \frac{L^2}{4} \int^s \frac{\sin^2 \Phi}{F} \right)$$

$$= - \int H \int^s \frac{uy \cos \Phi}{F} + \frac{wh^3}{12} \int^s \frac{u \cos \Phi}{F}$$

<u>Assumption 3</u>.

Since $M_x = 0$ and $M_y = 0$, in the expression for M_v, the M_x-term and M_y-term are omitted and add $\frac{wh^3}{12}$ cos Φ.

$\therefore$ Eq.(5a) becomes:

$$R^1{}_z \int^s \frac{u^2}{F}$$

$$= - \int \left\{ \frac{wh^2}{2} - H \right\} \int^s \frac{uy \cos \Phi}{F} + \frac{wh^3}{12} \int^s \frac{u \cos \Phi}{F}$$

<u>Internal Stresses at a Point</u>.

The stresses at Point P are represented by M_v, T_v, M_z, and T_z. M_z and T_v are determined from the analysis of the corresponding rectangular frame. $T_z = R_z$, and M_z is computed after solving for the redundant reactions and substituting the values in Eq. 2.

But for design purposes, the stresses have to be transformed in directions parallel and perpendicular to the skew.

Let M_L = direct moment

T_L = axial thrust

M_T = torsional moment

T_T = shear

The transformation is shown in Figure 3 and $M_L = M_s$ Sec. V

$$T_L = T_v \text{ Sec. } V$$

$$M_T = M_s \text{ Tan } V - M_v$$

$$T_T = T_v \text{ Tan } V - T_s$$

where V = projection of skew angle θ on the vs plane

The M_L and T_L control the cross section of the structure and the longitudinal reinforcement and they are entirely independent of M_s and T_v; M_T and T_T determine the transverse reinforcement necessary.

This concludes the analysis of the skew rigid frame.

EXAMPLE

Analyze a rigid frame 16' being span of corresponding rectangular frame, skew angle = 45° for w = 30 lb. per square foot per foot of depth, and live loading H-20-44-no impact, and temperature change $t = \pm 45^{\circ}$ F. for the frame shown in the Figure 5.

Solution.

Part 1.

Analysis of the corresponding rectangular frame.-

The frame is divided into equal parts with s = 4'

The dead loads assumed acting at the center of the subdivisions are shown in the Figure 5. The earth pressure acting is also shown in Figure 5.

The origin of coordinate system is chosen at the bottom of right footing.

By the formula $H = \dfrac{\Sigma s\ M \frac{Y}{I}}{\Sigma s\ \frac{y^2}{I}}$

where H = horizontal thrust

M = simple span bending moment

Y = Y coordinate of any point

I = moment of inertia of the section at the point whose coordinate is Y.

Tables are self-explanatory and Tables 1 to 5 show the complete analysis for obtaining the moments and normal thrusts at the required points of the frame for various loads.

Some definitions.

In Table 1:

M = simple span moment at a point in KIP-FT.

H = horizontal component of reaction at the origin in KIPS

t = depth at any section in ft.

I = moment of inertia.

M. F. = moment factor which is the influence line ordinate for the moment at the particular point.

Tables 2 and 3:

M.F. = moment factor obtained from Table 1.

M = (M.F.) x load in KIPS

= Kip ft.

N.F. = normal thrust factor

N = normal thrust in KIPS

Table 4:

V = vertical reaction at the hinges due to the earth pressure.

Part 2.

This consists of tabulating the maximum moments and thrusts which control the cross-sectional areas and longitudinal reinforcement after applying the necessary skew corrections. This is indicated in Tables 6 and 7.

Part 3.

This comprises of analysis of the frame for torsional moments and shearing forces acting in the transverse direction. This is carried through in Tables 8 to 15.

DEAD LOADS FOR THE FRAME

POINT	WT. COMPUTATIONS		WT. IN KIPS
4	$= 4 \times 1$	$= 4$	
	$\frac{(4+1\frac{1}{2})}{2} \times 1$	$= 2\ 3/4$	
	$\frac{(1\frac{3}{4}+1\frac{1}{2})}{2} \times 2$	$= \frac{3\frac{1}{4}}{11}$	1.65
3	$=\frac{(2.35+1.75)}{2} \times 4$	$= 8.2$	1.23
2	$=\frac{(2.35+2.5)}{2} \times 1$	$= 2.42$	
	$\frac{2.5 \times 2}{2}$	$= 2.5$	
		$= 4.92$	.737)
2	$= 2.5 \times \frac{2}{2}$	$= 2.5$	)=1.54
	$= 1.7 \times \frac{1.5}{2}$	$= 1.28$	
		$= 3.78 \times 1.414 = 5.35$	.802)
1	$= \frac{1.7+1.3}{2} \times 2$	$= 3$	
	$= \frac{1.3+1.1}{2} \times 2$	$= 2.4$	
		$= 5.4 \times 1.414 = 7.65$	1.15
0	$= 1.05 \times 4$	$= 4.2 \times 1.414 = 5.95$	.892

TABLE I

SOLUTION FOR THE RECTANGULAR FRAME

Point	t	t^3	$I = \frac{t^3}{12}$	$\frac{1}{F} = \frac{3.58}{t^3}$	Y	Y^2	$\frac{Y^2}{I}$	$\frac{Y}{I}$	Influence Load at IR				Influence Load at 0			
									Mom	$M\frac{Y}{I}$	HY	Total MF	M	$M\frac{Y}{I}$	HY	Total MF
4L	1.75	5.35	.445	0.67	2	4	9.0	4.5			- .32	- .32			- .52	- .52
3L	2.0	8.0	.667	0.448	6	36	54.0	9.0			- .96	- .96			-1.57	-1.57
2L	2.5	15.6	1.3	0.2295	10	100	76.9	7.7			-1.6	- 1.6			-2.6	-2.6
1L	1.4	2.74	.228	1.31	10.75	116	509.0	47.25	1	47.25	-1.72	- .72	2	94.5	-2.8	- .8
0	1.0	1.0	.083	3.58	11.0	121	1460.0	132.8	2	265.6	-1.76	+ .24	4	531.2	-2.87	+1.13
1R	1.4	2.74	.228					47.25	3	141.75	-1.72	+1.28	2	94.5	-2.8	- .8
2R	2.5	15.6	1.3					7.7			-1.6	-1.6			-2.6	-2.6
3R	2.0	8.0	.667					9.0			- .96	- .96			-1.57	-1.57
4R	1.75	5.35	.445					4.5			- .32	- .32			- .52	- .52
						(s- - -	2758			455				720		

$H = \frac{455}{2758} = .16$ $\qquad$ $H = \frac{720}{2758} = .261$

TABLE 2

DEAD LOAD MOMENTS

Points		Load Points			Total D.L.M.
		1R	0	1L	
	Load	1.15K	.89K	1.15K	
4L	MF	- .32	- .52	- .32	
	M	- .37	- .46	- .37	-1.20
3L	MF	- .96	-1.57	- .96	
	M	-1.11	-1.4	-1.11	-3.62
2L	MF	-1.6	-2.6	-1.6	
	M	-1.84	-2.3	-1.84	-5.98
1L	MF	- .72	- .8	+1.28	
	M	- .83	- .71	+1.47	- .07
0	MF	+ .24	+1.13	+ .24	
	M	+ .28	+1.00	+ .28	+1.56
1R	MF	+1.28	- .8	- .72	
	M	+1.47	- .71	-.83	- .07
2R	MF	-1.6	-2.6	-1.6	
	M	-1.84	-2.3	-1.84	-5.98
3R	MF	- .96	-1.57	- .96	
	M	-1.11	-1.4	-1.11	-3.62
4R	MF	- .32	- .52	- .32	
	M	- .37	- .46	- .37	-1.20

<u>Dead Load Thrusts</u>

Point 4 $N = V = 6.02$

" 3 $N = 6.02 - 1.65 = 4.37$

Points 1L, 0, 1R $= (1.15+1.15).16 + (.89 \times .261) = .6$

Point 2 $= (4.37-1.23).71 + .6 \times .71 = 2.65$

TABLE 3
CONCENTRATED LIVE LOAD, MOMENTS AND THRUSTS
(C.L.L.M., C.L.L.N.)
C.L.L. = 3.2 KIPS

Load Point	Point 4R				Point 3R			
	MF	M	NF	N	MF	M	NF	N
IR	-.32	-1.02	.75	2.4	- .96	-3.07	.75	2.4
0	-.52	-1.67	.5	1.6	-1.57	-5.03	.5	1.6
IL	-.32	-1.02	.25	.8	- .96	-3.07	.25	.8

Load Pt.	Point 2R				Point 1R				Point 0			
	MF	M	NF	N	MF	M	NF	N	MF	M	NF	N
1R	-1.6	-5.13	.65	2.08	+1.28	+4.1	.16	.51	+ .24	+ .77	.16	.51
0	-2.6	-8.32	.54	1.73	- .8	-2.56	.261	.83	+1.13	+3.62	.261	.83
1L	-1.6	-5.13	.29	.93	- .72	-2.3	.16	.51	+ .24	+ .77	.16	.51

Normal thrust for Point 2R

For load at 1R V factor = .75

" H factor = .16

∴ N factor = .71 (.75+.16) = .645

For load at 0 V factor = .5

" H factor = .261

" N factor = .71(.5+.26) = .54

For load at 1L V factor = .25

" H factor = .16

" N factor = .71(.25+.16) = .29

LIVE LOAD

We assume that only one truck can pass and for maximum moment only one concentrated load of 3.2 KIPS is placed at the position where it gives the maximum moment.

Temperature Stresses

C = coefficient of thermal expansion = 6.5×10^{-6}

t = temperature change in degrees Fahrenheit

= $\pm 45^{\circ}$

L = span length = 16'

E = modulus of elasticity of concrete

= 144 x 2,000,000 = 288,000,000 p.s.f.

s = length of axis divisions = 4'

$$H = \frac{ECtL}{s\left(\sum \frac{y^2}{I}\right)}$$

$$H = \frac{288 \times 10^6 \times 6.5 \times 10^{-6} \times 45 \times 16}{4 \times 2758}$$

= 122 lb. = .122 K.

Points	y	Hy	Normal Thrust
4	2	∓.244	± 0
3	6	∓.732	± 0
2	10	∓1.22	±.086
1	10.75	∓1.31	±.122
0	11.0	∓1.34	±.122

(The earth pressure is calculated as varying from 0 at the top to a max. at the bottom of the footing increasing at the rate of 30 lb. per ft. ht.)

TABLE 4

EARTH PRESSURE

(See Figure 8.)

Point	Vert. Dist. From 0	Vert. Dist. Between	Earth Prs. Right. Hor. Loads: Loads (KIPS)	Sum of Loads	Inc. of Mom.	M_A	Hor. Dist. From 0 = x	Mom. B = Vx	Earth Pr. Right. Hor. & Vert. Loads: Sub.Total $M_A + M_B$	y	$\frac{y}{I}$	$\frac{My}{I}$	Hy	Total Mom.
H_R	0		1.98											
		1		1.98	1.98									
E_4	1		.63			+1.98								
		1		1.35	1.35									
4R	2					+3.33	0		+3.33	2	4.5	15.0	- .78	+2.55
		2		1.35	2.70									
E_3	4		.90			+6.03								
		2		.45	.90									
3R	6					+6.93	0		+6.93	6	9.0	62.4	-2.33	+4.60
		2		.45	.90									
E_2	8		.42			+7.83								
		2		.03	.06									
2R	10					+7.89	0		+7.89	10	7.7	60.7	-3.88	+4.01
		.5		.03	.01									
E_1	10.5		.03			+7.90								
		.25		0	0									
R_1	10.75					+7.90	4	-1.9	+6.00	10.75	47.25	284.0	-4.17	+1.83
		.25			0									
0	11.0					+7.90	8	-3.8	+4.10	11.0	132.8	544.0	-4.27	- .17
1L						+7.90	12	-5.7	+2.20	10.75	47.25	104.0	-4.17	-1.97
2L						+7.90	16	-7.6	+0	10.0	7.7	0	-3.88	-3.88
3L						+7.90	16	-7.6	+0	6.0	9.0	0	-2.33	-2.33
4L						+7.90	16	-7.6	+0	4.0	4.5	0	- .78	- .78
											(s	1070		

Discrepancy in quantities of (M_A+M_B) for 2L, 3L, 4L is due to the fact that the individual forces are assumed to act at 1/2 height of trapezium.

Normal thrust N at (2R)= Right $-(.71 x .48)+(.71 x .39) = -.064$

Left $+(.71 x .48)+(.71 x .39) = +.617$

TABLE 5

SUMMARY OF MOMENTS (M)
AND THRUSTS (N) RESULTING FROM VARIOUS LOADS
IN THE CORRESPONDING RECTANGULAR FRAME

Loading	Point 4R		Point 3R		Point 2R		Point 1R		Point 0	
	M	N	M	N	M	N	M	N	M	N
Dead	-1.20	6.02	-3.62	4.37	-5.98	2.65	- .07	.6	+1.56	.6
Earth P. Right	+2.55	-.48	+4.60	-.48	+4.01	-.06	+1.83	.39	- .17	.39
Earth P. Left	- .78	+.48	-2.33	+.48	-3.88	+.62	-1.97	.39	- .17	.39
Sub-Total	+ .57	6.02	-1.35	4.37	-5.85	3.21	- .21	1.38	+1.22	1.38
Live Load										
0	-1.67	1.6	-5.03	1.6	-8.32	1.73	-2.56	.83	+3.62	.83
1R	-1.02	2.4	-3.07	2.4	-5.13	2.08	+4.1	.51	+.77	.51
1L	-1.02	.8	-3.07	.8	-5.13	.93	-2.3	.51	+ .77	.51
Temperature	∓.24	0	∓.73	0	∓1.22	±.09	∓1.31	±.12	∓1.34	±.12

TABLE 6

VALUES OF FUNCTIONS OF THE ANGLE V

$\cos\theta = .707$; $\theta = 45^{\circ}$; $\varepsilon = 1$; $\varepsilon^2 = 1$; $\sec\theta = 1.414$; $\sec^2\theta = 2$;

Function	Point o	Point 1R	Point 2R	Point 3R	Point 4R
$\cos\Phi$	1.00	0.99	0.97	0	0
$\tan V = \varepsilon\cos\Phi$	1.00	0.99	0.97	0	0
V	45°	44°.8	44°.18	0	0
Sec V	1.414	1.412	1.395	1.0	1.0
$\sec\theta \sec V$	2.00	2.00	1.972	1.414	1.414

TABLE 7

QUANTITIES IN TABLE 5 CORRECTED FOR SKEW

Loads	Step*	Point 4R		Point 3R		Point 2R		Point 1R		Point 0	
		M_s	T_v	M_s	T_v	M_s	T_v	M_s	T_v	M_s	T_v
Dead load plus earth pressure		+ .57	6.02	-1.35	4.37	-5.85	3.21	- .21	1.38	+1.22	1.38
Live Load											
0	1	-1.18	1.13	-3.56	1.13	-5.88	1.22	-1.81	.59	+2.56	.59
1R	1	- .72	1.70	-2.17	1.7	-3.63	1.47	+2.9	.36	+ .54	.36
1L	1	- .72	.57	-2.17	.57	-3.63	.66	-1.63	.36	+.54	.36
Temp.											
Rise) Fall)	2	∓.48	0	∓1.46	0	∓2.44	±.18	∓2.62	±.24	∓2.68	±.24
Totals Max.	3	+.95 -1.09	6.02 7.15	-6.37	5.5	-14.17	4.61	-4.64 +5.31	2.21 1.50	+6.46	1.73
Corrected	3	+1.34 -1.54	8.51 10.10	-9.00	7.78	-20.0	6.52	-6.55 +7.50	3.12 2.12	+9.14	2.44

*Note: Step 1: Live load quantities multiplied by cos θ (.71)

Step 2: Temperature load quantities multiplied by $\sec^2 \theta$ (2)

Step 3: Max. moments & thrusts summed up and multiplied by sec θ sec V (Table 6)

TABLE 8

SOME CONSTANTS

Point	$\frac{1}{F}$	y	u	u^2	sin Φ	$\sin^2$ Φ	cos Φ	$\cos^2$ Φ	ucos Φ	y $\cos^2$ Φ
4R	0.67	2.0	0	0	-1	1	0	0	0	0
3R	0.45	6.0	0	0	-1	1	0	0	0	0
2R	0.23	10.0	7.0	49.0	-.71	0.501	0.71	0.501	4.97	5.01
1R	1.31	10.75	10.2	104.0	-.122	0.0149	0.99	0.98	10.1	10.55
0	3.58	11.0	11.0	121.0	0	0	1.00	1.00	11.0	11.0
1L	1.31	10.75	12.2	148	+.122	0.0149	0.99	0.98	12.1	10.55
2L	0.23	10.0	18.4	338	+.71	0.501	0.71	0.501	13.1	5.01
3L	0.45	6.0	16.0	256	+1	1.0	0	0	0	0
4L	0.67	2.0	16.0	256	+1	1.0	0	0	0	0

Point	$\frac{\cos^2 Φ}{F}$	$\frac{u \cos Φ}{F}$	sin Φ cos Φ	$\frac{\sin Φ \cos Φ}{F}$
1R	1.285	13.2	-.121	-.1585
0	3.58	39.4	0	0
1L	1.285	15.85	+.121	+.1585

Table 8 (Continued)

Point	uy cos Φ	$\frac{u^2}{F}$	$\frac{\sin^2 \Phi}{F}$	$\frac{\cos^2 \Phi}{F}$	$\frac{u \cos \Phi}{F}$	$\frac{y \cos^2 \Phi}{F}$	$\frac{uy\cos\Phi}{F}$
4R	0	0	0.67	0	0	0	0
3R	0	0	0.45	0	0	0	0
2R	49.7	11.30	0.115	0.115	1.145	1.15	11.45
1R	108.6	136.20	0.0195	1.285	13.22	13.82	142.20
0	121.0	433.00	0	3.58	39.40	39.40	433.00
1L	130.0	194.0	0.0195	1.285	15.85	13.82	170.40
2L	131.0	77.7	0.115	0.115	3.015	1.15	30.15
3L	0	115.1	0.45	0	0	0	0
4L	0	171.5	0.67	0	0	0	0
	(^{s}	1138.8	2.509	6.38	72.630	69.34	787.20

TABLE 9

LOADING TERM SUMMATIONS FOR EQUATIONS

Point	Dead Load			Live Load at Point 0			Live Load at 1R			Live Load at 1R and 1L		
	Mo.	$\frac{Mo\ u\cos\theta}{F}$	$\frac{Mo\cos^2\theta}{F}$	Mo.	$\frac{Mo\ u\cos\theta}{F}$	$\frac{Mo\cos^2\theta}{F}$	Mo.	$\frac{\sin\theta\cos\theta}{F}$	$\frac{Mo\sin\theta\cos\theta}{F}$	Mo.	$\frac{Mo\ u\cos\theta}{F}$	$\frac{Mo\cos^2\theta}{F}$
1R	6.38	84.2	8.2	6.4	84.5	8.22	9.6	-.159	-.153	12.8	169	16.4
0	8.16	322.0	29.2	12.8	504.0	45.8	6.4	0	0	12.8	504	45.8
1L	6.38	101.0	8.2	6.4	101.5	8.22	3.2	+.159	+.51	12.8	203	16.4
Σ_s		507.2	45.6		690.0	62.2			-1.02		876	78.6

Quantities for direct substitution in Equations. Assumption 1.

The following values of H are obtained for the rectangular frame.

Dead Load	Balanced Earth Pressure	Temp. Rise	Live Load Point 0	Live Load Point 1R	Live Load Points 1R & 1L
.6	.78	.12	.83	.51	1.02

Substitutions for the left hand sides of the equations are:

Eq.	Expression	Solution	Substitution
5a	$(^{s} \frac{u^2}{F} - \frac{L^2}{4} (^{s} \frac{\sin^2 \Phi}{F}$	1139-64x2.51 =(1139-158)	981
5a and 5b	$(^{s} \frac{u \cos \Phi}{F}$		72.6
5b	$(^{s} \frac{\cos^2 \Phi}{F}$		6.38

Quantities for the right hand sides of the equations are listed in Table 6.

For assumptions 2 and 3, quantities for direct substitution:

Balanced earth pressure substitution, in the right hand sides of Equations 5a and 5b for assumptions 2 and 3 are:

$$- ((\frac{wh^2}{2} - H) \quad (^{s} \frac{uy \cos \Phi}{F} + \frac{wh^3}{12} \quad (^{s} \frac{u \cos \Phi}{F}$$

$$= -1(1.98-1.2)\ 787.2 \quad + \frac{30x11.5^3}{12} \times 72.6$$

$$= -1.2 \times 787.2 \quad + 3.77 \times 72.6$$

$$= -945 \quad + 274$$

$$= -671$$

The substitution for left hand side (Assumption 3) is $(^{s} \frac{u^2}{F} = 1139$.

The M_y correction for Assumption 2 is same as shown in Table 6 for Assumption 1. With the above mentioned exceptions, all substitutions for the R.H.S. of Eqs. 5a for Assumptions 2 and 3 are the same as the corresponding substitutions in Eqs. 5a for Assumption 1.

TABLE 10

SUBSTITUTIONS FOR THE R.H.S. OF EQ. 5a, ASSUMPTION 1

Loading	Expression	Computation	Substi- tution
Dead Load:	$\epsilon \left\{ \left(\sum \frac{M_{ou}\cos\Phi}{F} - H \left(\sum \frac{uy\cos\Phi}{F} \right\} \right.\right.$	1(507.2-.6x787.2) =(507.2-472)	35
Balanced earth pressure	$-\epsilon \left(\frac{wh^2}{2} - H\right) X \left(\sum \frac{uy\cos\Phi}{F} \right)$	-1(1.98-.78)787.2	-945
Temperature rise.	$\epsilon \left[\frac{Ect^{o}L}{Ks} - (1+\epsilon^2) H \left(\sum \frac{uy\cos\Phi}{F} \right] \right.$ $= 1 \left[\frac{.288x10^6x6.5x10^{-6}x45x16}{2.67x4} \right.$ $- (1+1^2)(.122 \times 787.2)$ $= -(126.4-192)$		-66
Live Load: 0	$\left\{ \epsilon \left(\left(\sum \frac{M_{ou}\cos\Phi}{F} - H \left(\sum \frac{uy\cos\Phi}{F} \right) \cos\theta \right\} \right.\right.$	1(690-.83x787.2) x.707 =(690-653).707 = 37x.707	26.2
Live load 1R (balanced)	"	1(876-1.02x787.2) x.707 =(876-803)x.707 = 73x.707	51.6

TABLE 11

SUBSTITUTIONS FOR THE R.H.S. OF EQUATIONS 5b: ASSUMPTION 1.

Loading	Expression	Computation	Substitution
Dead Load:	$\epsilon \left\{ \left(^s \frac{Mo \cos^2 \Phi}{F}\right. \right.$	1(45.6–.6x69.34)	
	$\left. -H \left(^s \frac{y \cos^2 \Phi}{F}\right. \right\}$	=(45.6–41.6)	4.0
Balanced earth pressure	$-\epsilon \left(\frac{wh^2}{2} - H\right) \left(^s \frac{y\cos^2 \Phi}{F}\right.$	-1(1.98–.78)69.34	
		=–(1.2)69.34	-83.0
Temperature rise.	$-\epsilon (1+\epsilon^2) H \left(^s \frac{y\cos^2 \Phi}{F}\right.$	-1(2).122(69.34)	-16.9
Live Load:	$\epsilon \left\{ \frac{Mo \cos^2 \Phi}{F} \right.$	1(62.2–.83x69.34) x.707	
0	$- H \left(^s \frac{y \cos^2 \Phi}{F}\right) \cos \theta$	= (62.2–57.5).707	
		= (4.7).707	3.32
Live load R1 (balanced)	"	1(78.6–1.02x69.34) x.707	
		=(78.6–70.7)x.707	5.6
M_y correction C (Eq.5e) for Live Load at 1R	$\epsilon \left(^s \frac{Mo \sin \Phi \cos \Phi}{F}\right. \div \left(^w \frac{\sin^2 \Phi}{F}\right.$	$\frac{1 \times (-1.02)}{2.509}$	-.406

TABLE 12

SOLUTION OF EQUATIONS FOR VARIOUS ASSUMED FOOTING CONDITIONS: ASSUMPTION 1

Eq.	Left Hand Sides	Dead Load	Balanced earth Pressure	Temp. Rise	Live Load 0	Live Load 1R Balanced	Live Load 1R
5a	981 R^1_s −72.63 M_x=	+35	−945	−66	+26.2	+51.6	
5b	72.63 R^1_s − 6.38 M_x=	+4.0	−83.0	−16.9	+3.32	+ 5.6	
	Dividing throughout by the coefficients of M_x.						
5a	13.52 R^1_s − M_x =	+.482	−13.01	−.91	+.361	+.711	
5b	11.4 R^1_s − M_x =	+.627	−13.01	−2.65	+.52	+.878	
5a−5b	2.12 R^1_s=	−.145	+ 0	+1.74	−.16	−.167	
	R^1_s =	−.0684	+ .0	+ .82	−.0755	−.0788	−.0394
5c	$M_y = -\frac{L}{2} R^1_s$						
	=−8 R^1_s =	+.547	0	−6.56	+.604	+.63	+.315(Av.)
	For live load at 1R.						
5d	M_{yR} = .315 −.406 = −.091						
5d	M_{yL} = .315 + .406 = +.721						
	−11.4 R^1_s =	+.78	0	−9.34	+.86	+.896	
	Adding −11.4 R^1_s to 5b						
	$-M_x$ =	+1.407	−13.01	−11.99	+1.38	+1.774	+.887
	M_x =	−1.407	+13.01	+11.99	−1.38	−1.774	−.887

TABLE 12 (Continued)

SOLUTION OF EQUATIONS, FOR ASSUMPTION 2.

(b). ROTATION ABOUT x AXIS AND z AXIS

Eq.	L.H.S.s	Dead Load	Balanced E. Press.	Temp. Rise	Live Load 0	Live Load 1R Balanced	Live Load 1R
5a	$981\ R^1_z$ =	+ 35	-671	- 66	+26.2	+51.6	
	R^1_z =	+.0357	-.684	-.0673	+.0268	+.0526	+.026
5c	$M_y = -8R^1_z$ =	-.286	+5.47	+.538	-.214	-.42	-.21(Av.)
	For live load at 1R						
5d	$M_{yR} = -.21 - .406 = -.616$						
5d	$M_{yL} = -.21 + .406 = +.196$						
	(c). SOLUTION OF EQUATIONS FOR ASSUMPTION 3 ROTATION ABOUT x-AXES, y-AXES, and z-AXES						
5a	$1138.8\ R^1_z$ =	+35	-671	-66	+26.2	+51.6	
	R^1_z =	+.0308	-.591	-.058	+.023	+.0454	+.0227(Av.)

TABLE 13

TABULATION OF REACTIONS

No.	Load	R_x	μR_x	Assumption 1				Assumption 2			Assumption 3	
				R^1_s	M_{yr}	M_x	R_z	R^1_s	M_{yr}	R_z	R^1_s	R_s
1	Dead Load	.6	.6	-.0684	+.547	-1.407	+.53	+.0357	-.286	+ .64	+.0308	+ .63
2	Earth Pr.	1.2	1.2	0	0	+13.01	+1.2	-.684	+5.47	+ .62	-.591	+ .61
3	Total	1.8	1.8	-.068	+.547	+11.6	+1.73	-.648	+5.18	+1.26	-.56	+1.24
	Live Load:											
4	0	.586	.586	-.0755	+.604	-1.38	+ .51	+.0268	-.214	+.613	+.023	+.609
5	1R	.36	.36	-.0394	-.091	-.887	+ .32	+.026	-.616	+.39	+.023	+.38
6	1L	.36	.36	-.0394	+.721	-.887	+ .32	+.026	+.196	+.39	+.023	+.38
	Temperature											
7	Rise-Fall	±.244	±.244	±.82	∓6.56	±11.99	±1.06	∓.0673	±.538	±.177	∓.058	±.186
	Summations: Dead Load + Earth Pressure + Temperature											
8	Rise	+2.04			-6.01	+23.6	+2.79		+5.72	+1.44		+1.43
9	Fall	+1.56			+7.11	- 0.4	+ .67		+4.64	+1.08		+1.05

Note: Values of R_x for dead load = H

for live load = H $\cos\theta$

for earth pressure = $\frac{wh^2}{2} - H$

for temperature rise = $(1 + \mu^2)$ H

TABLE 14

Computations for M_v

Load	(- Mo	Assumption 1						Assumption 3				
		R_x(- x sin Φ −	$R_s u$ +	M_x cos Φ −	M_{yR} sin Φ +	M_o(- cos Φ =	M_v	R_x (- x sin Φ −	R_s u +	($\frac{1}{12}wh^3$=3.77) $\frac{1}{12}wh^3$ cos Φ +	M_o (- cos Φ =	M_v
(a) Point 0: x = 8; sin Φ = 0; cos Φ = 1; u = 11.												
Dead Load	8.16		-19.0	+11.6		+8.16	+ .76		-13.62	+3.77	+8.16	-1.69
Live Load Point 0	9.06		-5.61	-1.38		+9.06	+2.07		-6.70		+9.06	+2.36
Point 1	4.52		-3.52	-.887		+4.52	+ .11		-4.06		+4.52	+ .46
Temperature Rise) Fall)			∓11.66	±11.99			±.33		∓2.04			∓2.04
(b) Point 1R: x = 4; sin Φ = -.122; cos Φ = .99; u = 10.2; (- x sin Φ = -.488.												
Dead Load	6.38	-.88	-17.65	+11.5	+.0667	+6.32	- .64	-.88	-12.66	+3.73	+6.32	-3.49
Live Load Point 0	4.52	-.286	- 5.2	-1.37	+.0736	+4.47	-2.31	-.286	- 6.21		+4.47	-2.03
Point 1R	6.78	-.176	- 3.26	-.878	-.0111	+6.71	+2.38	-.176	- 3.88		+6.71	+2.65
Point 1L	2.26	-.176	-3.26	-.878	+.0879	+2.24	-1.98	-.176	- 3.88		+2.24	-1.82
Temperature Rise) Fall)		∓.119	∓10.8	±11.89	∓.80		±.17	∓.119	∓1.90			∓2.02

Table 14 (Continued)

Computations for M_v

Load	γM_o	Assumption 1: $R_x(\gamma x \sin\Phi)$ −	$R_z u$ +	$M_x \cos\Phi$ −	$M_{yr} \sin\Phi$ +	$M_o(\gamma \cos\Phi)$ =	M_v	Assumption 3: $R_x(\gamma x \sin\Phi)$ −	$R_z u$ +	$\frac{1}{12}\gamma h^3 \cos\Phi$ +	$M_o(\gamma \cos\Phi)$ =	M_v
(c) Point 2R: $x = 0$; $\sin\Phi = -.71$; $\cos\Phi = .71$; $u = 7$. $\gamma x \sin\Phi = 0$.												
Dead Load	-	-	-12.1	+8.24	+.388	-	-3.47	-	-8.70	+2.68	-	-6.02
Live Load Point 0	-	-	-3.57	- .98	+.429	-	-4.12	-	-4.26	-	-	-4.26
Point 1R	-	-	-2.24	- .63	-.0645	-	-2.93	-	-2.66	-	-	-2.66
Point 1L	-	-	-2.24	- .63	+.512	-	-2.36	-	-2.66	-	-	-2.66
Temperature Rise) Fall)	-	-	∓7.42	±8.52	∓4.66	-	∓3.56	-	∓1.30	-	-	∓1.30

(d) Assumption 2 Points 3R and 4R

$x = 0$; $\sin\Phi = -1$; $\cos\Phi = 0$; $u = 0$.

Evidently $M_v = -M_{yr}\sin\Phi = M_{yr}$

M_v can be taken directly from Table 13.

Notes: M_v for points 0, 1R, 2R, are calculated for Assumptions 1 and 3. For Assumption 2 - $M_y \sin\Phi$ is added.

Since Φ values are small for 0 & 1R, and since 2R is not a critical section because of increased depth, Assumption 2 is confined to points 3R and 4R.

Values of M_o are given in Table 9.

Values of R_x, R_z, M_x and M_{yr} are given in Table 13.

TABLE 15

TRANSVERSE TORSIONAL MOMENTS M_T AND SHEARS T_T

Loading	M_z	M_z tan V	T_v	T_v tan V	Assumption 1				Assumption 3			
					M_v	M_T	T_z	T_T	M_v	M_T	T_z	T_T
(a) Point 0: Tan V = 1												
Dead Load plus earth pr.	+1.22	+1.22	+1.38	+1.38	+ .76	+ .46	+1.73	- .35	-1.69	+2.91	+1.24	+ .14
Live Load Point 0	+2.56	+2.56	+ .59	+ .59	+2.07	+ .49	+ .51	+ .08	+2.36	+ .20	+.609	-.019
Point 1	+ .54	+ .54	+ .36	+ .36	+ .11	+ .43	+ .32	+ .04	+ .46	+ .08	+ .38	- .02
Temperature Rise)	-2.68	-2.68	+ .24	+ .24	+ .33	-3.01	+1.06	- .82	-2.04	- .64	+.186	+ .05
Fall)	+	+	-	-	-	+	-	+	+	+	-	-
Summation						+3.96		+ .55		+3.75		+ .07
(b) Point 1R: Tan V = 0.99												
Dead Load plus earth pr.	- .21	-.208	+1.38	+1.367	- .64	+ .43	+1.73	- .36	-3.49	+3.28	+1.24	+ .13
Live Load Point 0	-1.81	-1.791	+ .59	+ .584	-2.31	+ .52	+ .51	+ .074	-2.03	+ .24	+ .609	- .025
Point 1R	+2.9	+2.870	+ .36	+ .356	+2.38	+ .49	+ .32	+ .036	+2.65	+ .22	+ .38	- .024
Point 1L	-1.63	-1.614	+ .36	+.356	-1.98	+ .37	+ .32	+ .036	-1.82	+ .21	+ .38	- .024
Temperature Rise)	-2.62	-2.593	+ .24	+ .238	+ .17	-2.76	+ 1.06	- .82	-2.02	- .57	+ .186	+ .052
Fall)	+	+	-	-	-	+	-	+	+	+	-	-
Summation						+3.71		+.534		+4.09		+.053

Table 15 (Continued)

Loading	M_s	M_s tan V	T_v	T_v tan V	Assumption 1				Assumption 3			
					M_v	M_T	T_s	T_T	M_v	M_T	T_s	T_T
(c) Point 2R: Tan V = .97												
Dead Load plus earth pr.	-5.85	-5.67	+3.21	+3.115	-3.47	-2.20	+1.73	+1.385	-6.02	+ .35	+1.24	+1.875
Live Load Point 0	-5.88	-5.70	+1.22	+1.183	-4.12	-1.58	+ .51	+ .673	-4.26	-1.44	+ .609	+ .574
Point 1R	-3.63	-3.52	+1.47	+1.425	-2.93	- .59	+ .32	+1.105	-2.66	- .86	+ .38	+1.045
Point 1L	-3.63	-3.52	+ .66	+ .640	-2.36	-1.16	+ .32	+ .32	-2.66	- .86	+ .38	+ .26
Temperature Rise) Fall)	-2.44 +	-2.365 +	+.18 -	+ .175 -	-3.56 +	+1.195 -	+1.06 -	- .885 +	-1.30 +	-1.065 +	+ .186 -	- .011 +
Summation						-4.98		+2.94		-2.16		+2.438
(d) Points 3R, 4R, Tan V = 0					Assumption 1				Assumption 2			
Dead Load plus earth pr.					+.547	-.547	+1.73	-1.73	+5.18	-5.18	+1.26	-1.26
Live Load Point 0					+.604	-.604	+ .51	- .51	- .214	+ .214	+ .613	- .613
Point 1R					-.091	+.091	+ .32	- .32	- .616	+ .616	+ .39	- .39
Point 1L					+.721	-.721	+ .32	- .32	+ .196	- .196	+ .39	- .39
Temperature Rise) Fall)					-6.56 +	+6.56 -	+1.06 -	-1.06 +	+ .538 -	- .538 +	+ .177 -	- .177 +
Summation						-7.83		- .99		-5.91		-1.83

Notes for Table 15:

$$M_T = M_z \tan V - M_v$$

$$T_T = T_v \tan V - T_z$$

Values of M_z and T_v are given in Table 7, $T_z = R_z$ in Table 13, M_v in Table 14.

CONCLUSION

A look at Table 15 in our example indicates that M_T and T_T for Assumption 1 are greater than the corresponding quantities for any other assumption. So it appears that values of M_T and T_T have to be computed for all the three assumptions for any skew rigid frame, to determine the worst condition.

Also, the M_T due to temperature change, is not less than .74 of the total M_T for Assumption 1 and is not more than .17 of the M_T for Assumption 2 or 3 for all points except for Point 2R. Thus torsional moment due to temperature change constitutes the principal part of the final torsional moment in Assumption 1, and constitutes a minor part of the final torsional moment for Assumptions 2 and 3.

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