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INDETERMINATE STRUCTURES
ANALYZED BY MECHANICAL METHOD

Thesis for the Degree of M. S.
MICHIGAN STATE COLLEGE
Peng Chung-Whe-Wen Tu
1949

This is to certify that the
thesis entitled

INDETERMINATE STRUCTURES
ANALYZED BY MECHANICAL METHOD
presented by

Peng Chung

has been accepted towards fulfillment
of the requirements for

M.S. degree in C.E.


Major professor

Date May 27, 1949

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THESIS

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I. INTRODUCTION

The application of mechanical methods to the analysis of engineering structures is relatively new and has been becoming increasingly important in the treatment of the more complex structural problems. The purely mathematical methods, though perfectly correct in theory, are in certain cases difficult in application, involving tedious arithmetical operations in which the opportunities for error in quantity and sign are very numerous. Therefore, the mechanical method has been used as an advantageous short cut instead of mathematical calculation or employed for checking the reactions and moments in statically indeterminate structures.

There are two general types of mechanical methods used to solve structural problems:

- (a) The direct method: The test is made by loading the model in the same manner as the actual loaded structure. The required result is then obtained from the deformation of the model.
- (b) The indirect method: Contrasted to direct methods, there are some indirect methods of model analysis in which the model is not loaded as the actual loading on the prototype. Usually they involve first finding the influence lines for the model. These influence lines may then be used to compute the required stresses for the actual loading conditions.

The indirect method is the one of more importance on account of its convenience and is used in this thesis.

Recently there have been devised various instruments and apparatus to carry out the mechanical analysis. The best known devices are the Torg's deformer, Lloyd deformer, Gottschall deformer, constant indicator and constant deformer. It should be explained that such apparatus may be used for convenience and in more refined research field. Yet it is still possible to make small scale and less expensive experiments to obtain the indirect results.

II. CHOICE OF THE MODELS

In this thesis we select some statically indeterminate structures and make models for testing. We compare the test results with those we get from calculation model. Both the tests and the calculations are shown in this thesis. We feel that there is a wide field for ingenuity in devising similar small scale models and it is valuable to make further development of this branch of study in structural problems.

The problems which we select for the purpose of testing involve various kinds of statically indeterminate structures. They are continuous beams, unsymmetrical bent, rigid frame and fixed arch. By these problems we show that any indeterminate structure can be solved by our simple devices without special instruments.

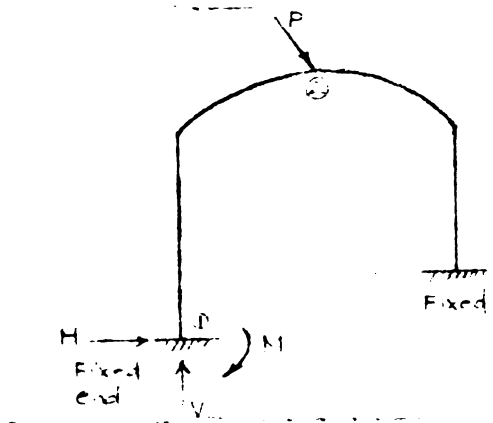
the stability of the system.

The theory of structural analysis can be well understood by observing the static stability of a system, which are the following elements:

Suppose a rigid structure, such as a bar, whether it be a solid or a fluid element, to be in equilibrium under any number of forces or couples applied by $F_1, F_2, \dots, F_n; M_1, M_2, \dots, M_n$, and let the displacements of these forces and couples in their line of application be $\delta_1, \delta_2, \dots, \delta_n; \theta_1, \theta_2, \dots, \theta_n$. If these forces and couples are replaced by a second system in equilibrium represented by $F'_1, F'_2, \dots, F'_n; M'_1, M'_2, \dots, M'_n$, acting in each case in the same direction as the original acting values of the first system, and if the forces and couples in this second system be $\delta'_1, \delta'_2, \dots, \delta'_n; \theta'_1, \theta'_2, \dots, \theta'_n$, then static stability is obtained when the

$$P_1 \delta'_1 + P_2 \delta'_2 + \dots + P_n \delta'_n + \dots + M'_1 \theta'_1 + \dots + M'_n \theta'_n = \\ P'_1 \delta_1 + P'_2 \delta_2 + \dots + P'_n \delta_n + \dots + M'_1 \theta_1 + \dots + M'_n \theta_n$$

In the system, let us consider the analysis of a bar of length L fixed at one end and free at the other.

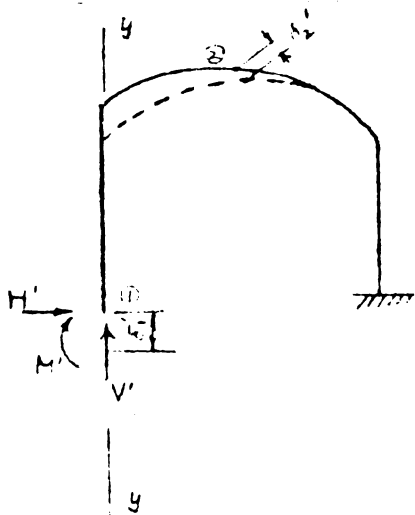


(1). The bar is fixed at the left end and free at the right end. The displacement of point (P) in the direction of the force P is δ_1 . Let the forces and couples in this system be $F_1, F_2, \dots, F_n; M_1, M_2, \dots, M_n$.

System I -- Unrestrained Structure

	at (1)			at (2)
Force	V	1	0	0
Displacement	0	0	0	δ_2

Consider an element of the frame at a unit load P applied at the joint. Joint (1) is externally restrained, a force δ'_1 is applied at (1) so that there will be no displacement and any reaction at the



joint (1). A reaction δ'_1 will be developed at (1) by not only the vertical force P but also a horizontal reaction δ'_1 to prevent movement out of the line $P-V$, and a reaction H' to prevent rotation. Since no rotation and no horizontal or vertical movement is permitted at (1) by the restraint,

therefore, the corresponding displacement along the same load line P due to δ'_1 is called δ'_2 , due to joint (1), the degree of freedom.

System II -- Displaced Structure

	at (1)			at (2)
Force	V'	δ'_1	H'	0
Displacement	δ'_2	0	0	δ'_3

Since the above analysis is correct we will obtain by combining systems I and II:

$$+\delta'_1 + +\delta'_2 = 0 \quad \text{or} \quad V = -\frac{\delta'_2}{\delta'_1} \dots (1)$$

Equation (1) states: the ratio of the load to the reaction in the case of the ratio of any arbitrary movement of the end (1) in the direction of the reaction to the possible movement of the load point in the direction of the load.

Then, if a point of the structure subjected to any movement of some in point of the end (1) displaced purely vertically at (1) by a known amount δ'_1 , it is only necessary to know the resulting displacement δ'_2 of point (2) in the direction of that assumed load to get the value of η . It should be noticed that no load is actually applied to the beam and the only movement is related to the displacement. The relative sign in the equation indicates that the defl. of δ'_2 of the load point (2) due to the assumed defl. of δ'_1 is in the opposite direction to the motion of the load.

The same method of finding η may be used for finding the horizontal value η , if instead of a vertical displacement, purely horizontal displacement is introduced. By the same method we get

$$\eta = -1 \frac{\delta''_2}{\delta''_1} \dots (2)$$

δ''_2 is the displacement of the load point corresponding to the purely horizontal displacement δ''_1 .

In case of determining the deflection η at (2) we give a pure rotation θ to the end (1) and assume the corresponding defl. amount δ'''_1 in the load direction at point (1), then by the same method as on 1. find the deflection

the reaction force shall be a function

$$R = - \gamma \frac{\delta^2}{\theta} \dots (5)$$

Equation (5) states:

The value of the load at the instant t , R , must be the value of any arbitrary function $f(t)$ of the error to the reaction, measured at the load point in the direction of the load.

The above three equations are the basic principles of all kinds of tests at mechanical systems and lead us to establish the following three principal features to be observed in a test cell to wit, they are:

- a. A well made set of rules of conduct of any test
- b. A means for producing and controlling the collection of the point to be analyzed
- c. A method of recording resulting collections at the load point

In our clinic, the details described with the above three points will be described in the following chapters.

24. PREPARATION OF THE MODEL

The material employed for making the cylindrical model is the transparent plastic cellulose acetate of density 1.18, 0.4 inch on 4 sheets of the thickness, 0.0015, and the case with inside 1.5 mm in radius. The prepared points to be followed for the use of making are:

1. Larger blocks should be selected to reduce the percentage of wastage of model. Both the ends for the model should be parallel to the outer surface of the model surface and be proportional to the size of the prototype. The thickness of the block should be the same value.
2. The length, width and the thickness of the material should be approximately that the model is within one sixth inch for 1/16 inch.
3. Since the diameter is of uniform thickness, the thickness of the block of the model will vary as the ratio of the two diameters. Hence the transverse dimensions along the model should vary on the same ratio of the radius of inside I of the cylinder and dimensions in the diameter in order to have the model behavior faithfully as the prototype.
4. Mark the specific use of lines and concentricity on the cylindrical block.
5. After the model has been cut down from the sheet, it should be dried in a dry place and protected by enclosing it in the transparent plastic as far as possible.

1. The statement of the child to the fact-
finder, the child shall be treated as a witness.
The child shall be given a fair and full trial in an
electronic session.

V. CONCLUSIONS

1. When the thickness of the substrate is 0.01 mm, the film is 0.1 mm thick and is 0.1 mm thick, a small amount of film is also found on the surface.
2. When the film is 0.1 mm thick, the film is 0.1 mm thick.
3. When the film is 0.1 mm thick, the film is 0.1 mm thick.
4. When the film is 0.1 mm thick, the film is 0.1 mm thick.
5. When the film is 0.1 mm thick, the film is 0.1 mm thick.
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9. When the film is 0.1 mm thick, the film is 0.1 mm thick.
10. When the film is 0.1 mm thick, the film is 0.1 mm thick.

1. The first table is a 4x4 grid with the following data:

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1	2	3	4
2	3	4	5
3	4	5	6
4	5	6	7

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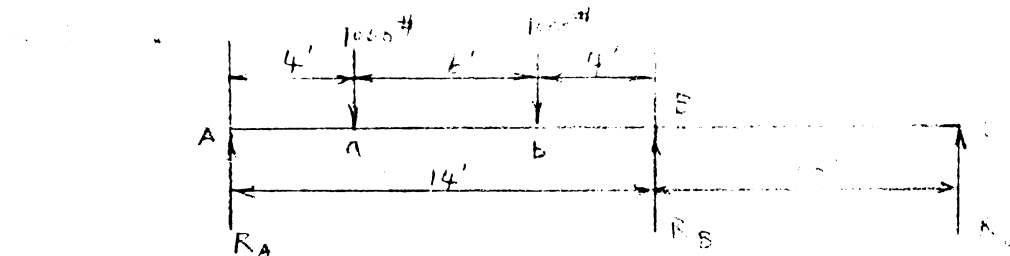
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2	3	4	5
3	4	5	6
4	5	6	7

2. The second table is a 4x4 grid with the following data:

1. The first table is a 4x4 grid with the following data:	2. The second table is a 4x4 grid with the following data:	3. The third table is a 4x4 grid with the following data:	4. The fourth table is a 4x4 grid with the following data:
1	2	3	4
2	3	4	5
3	4	5	6
4	5	6	7

2. Moment Distribution

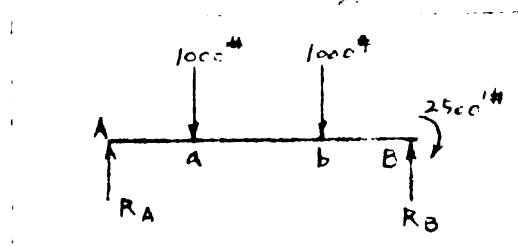
2.1. Continuous Beams and the Moment Distribution



$$\sum M_A = 0 \Rightarrow 1000(4) + 1000(14) - R_B(14) - R_C(20) = 0$$

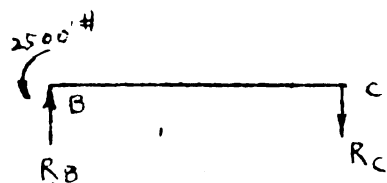
$$1000(4) + 1000(14) - R_B(14) - R_C(20) = 0$$

$$1000(4) + 1000(14) - R_B(14) - R_C(20) = 0$$



$$\sum M_B = 0 \Rightarrow 1000(10) + 1000(4) - R_A(14) = 0$$

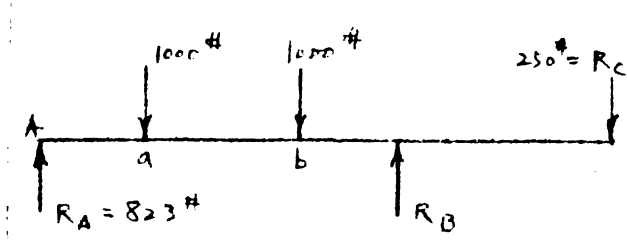
$$1000(10) + 1000(4) - R_A(14) = 0 \Rightarrow R_A = 823 \text{ lb}$$



2.2. Moment Distribution

$$R_C = 250 \text{ lb}$$

2.3. Moment Distribution



$$R_C = 250$$

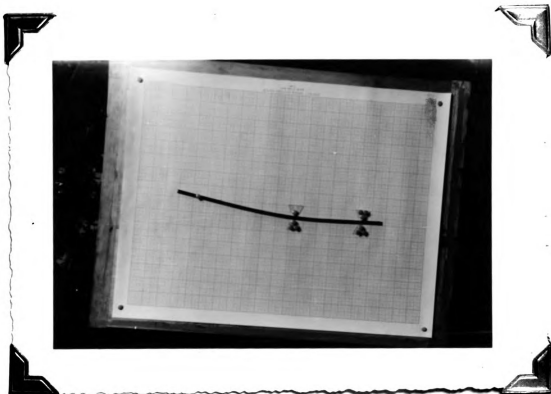
2.4. Moment Distribution

$$\sum M_A = 0 \Rightarrow 1000(4) + 1000(14) - R_B(14) - R_C(20) = 0$$

$$1000(4) + 1000(14) - R_B(14) - R_C(20) = 0$$

$$1000(4) + 1000(14) - R_B(14) - R_C(20) = 0$$

Problem 1



Deformation of model to find vertical
reaction at left end

1000#. Vertical distance from end of beam to

the center of the beam is 14'.

Use of the following:

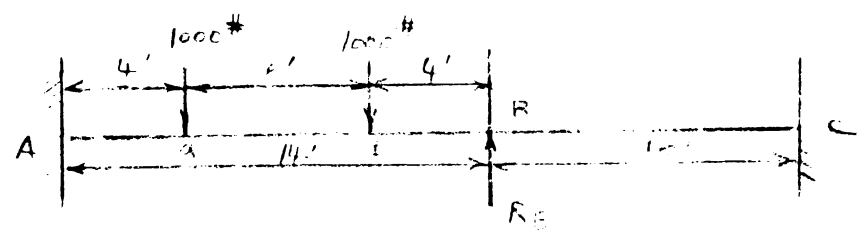


Diagram 10

1. The reactions R_A , R_B , and R_C are to be found.

- 2. The reactions R_A , R_B , and R_C are to be found.

The reactions R_A , R_B , and R_C are to be found. The reactions R_A , R_B , and R_C are to be found. The reactions R_A , R_B , and R_C are to be found.

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- 3. The reactions R_A , R_B , and R_C are to be found.

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at this moment is not united to the drawing board, so that we can move it vertically upward or downward. The orientation is shown in Figure 4b.

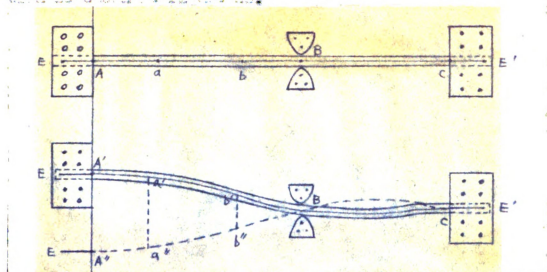


Figure 4b

To measure purely vertical movement and only the point A is moved along vertical line through it, but the line is allowed to slide horizontally after the completion of the movement. The beam is drawn at its starting position (as Figure 4b shows) and the position of load points a, b transferred down onto the cross-section paper. In order to obtain the greatest possible accuracy, the deflections are displaced in equal amount in opposite directions from the normal position, so that in the resulting table the reading at A is the amount $A'A''$, reading at a and b are the amount of $a'a''$ and $b'b''$ respectively. They are measured with 1/100 in. unit scale.

Reaction at C is found as exactly the same procedure. Reading at B is similar; the double support reaction is taken by fixing both ends and giving vertical movement to that support as did in problem 1.

To find the fixed end moment, first cut the points E and E' on the center line outside of A and B respectively. The distance of E and E' are both exactly one inch. Then both ends A and B are fitted into the grooves and the end A could be rotated around the fixed shaft which goes through hole A , while end B is completely fixed. (See Figure 10).

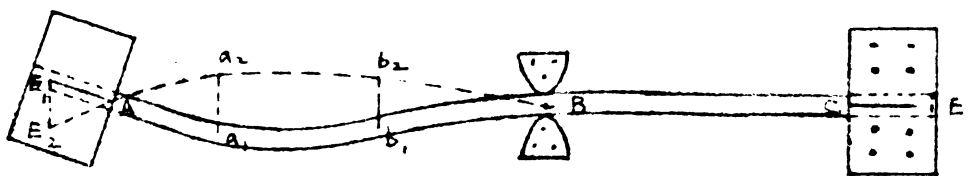


Figure 10

Since the distance of AE equals to 1 inch the total rotating displacement in inch at E will be the angular value in radians of the deflection at E .

a_1, a_2, b_1, b_2 are the corresponding vertical displacements measured in inch. It should be noted that the moment at A due to load at a or b is actually $\frac{a_1 a_2}{2EI} \times 1000$ or $\frac{b_1 b_2}{2EI} \times 1000$, for $a_1 a_2, b_1 b_2$ are not in reduced scale — same as the model — and should be transformed back to get their actual deformation. The actual value of $a_1 a_2$ will be

$$\frac{a_1 a_2}{\frac{1}{12} \times 1} = 1' = a_1 a_2 \quad (\text{in ft})$$

and that of $b_1 b_2$ will be

$$\frac{b_1 b_2}{\frac{1}{12} \times 1} = 1' = b_1 b_2 \quad (\text{in ft})$$

$$\text{Hence } M_A = \frac{a_1 a_2}{2EI} \times 1000 + \frac{b_1 b_2}{2EI} \times 1000 = 1000 \quad (\text{in ft. lbs.})$$

For R_1

No. of test	Reading at A (in.)	Reading at load point (in.)		R_1 lbs.
		a	b	
1	<u>11.1</u> 60	<u>7.0</u> 60	<u>10.3</u> 60	1119
2	<u>12.8</u> 60	<u>5.6</u> 60	<u>11.7</u> 60	1125
3	<u>11.5</u> 60	<u>6.3</u> 60	<u>11.1</u> 60	1132
Average				1125 ↑

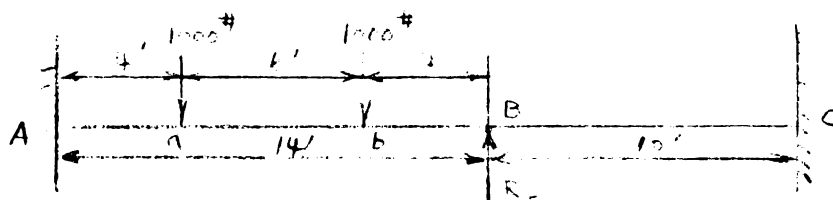
For R_2

No. of test	Reading at C (in.)	Reading at load point (in.)		R_2 lbs.
		a	b	
1	<u>45.3</u> 60	<u>4.1</u> 60	<u>7.6</u> 60	258
2	<u>60.2</u> 60	<u>5.7</u> 60	<u>10.1</u> 60	262
3	<u>75.5</u> 60	<u>7.0</u> 60	<u>12.3</u> 60	256
Average				259 ↑

4. Comparison of results

	R_1 lbs.	R_2 lbs.	R_3 lbs.	M_A ft.-lbs.	M_C ft.-lbs.
Calculated	1120 ↑	1122 ↑	250 ↑	-3452	+823
Average of Experiments	1124 ↑	1125 ↑	259 ↑	-3369	+818

3. Bending Moment



$$M_{0.0} + 2(10)(0+14) + R_B(14) = -\frac{1000(14^2 - 4^2)}{14} - \frac{2000(14^2 - 10^2)}{14}$$

$$\text{or } 28 M_A + 14 R_B = -\frac{1000}{14} [4(14^2 - 4^2) + 10(14^2 - 10^2)]$$

$$= -\frac{1000(112)}{14} = -120,000 \quad \dots (1)$$

$$M_B(14) + 2(10)(14+10) + R_C(10) = -120,000$$

$$\text{or } 14M_B + 40M_C + 10R_C = -120,000 \quad \dots (2)$$

$$10M_C + 3M_C(10+0) = 0 \quad = 0$$

$$\text{or } 10M_C + 30M_C = 0 \quad = 0$$

$$M_C = -\frac{0}{40} \quad \dots (3)$$

Substitute (3) into (2).

$$14M_B + 40M_C + 10R_C = -120,000$$

$$\text{or } 14M_B + 40M_C = -120,000 \quad \dots (4)$$

$$10M_B + 14M_C = -120,000 \quad \dots (5)$$

$$\text{From (1)} \quad 7M_C = -120,000$$

$$(4) \times 2 - (5)$$

$$M_B = -1007 \text{ ft.-lbs.}$$

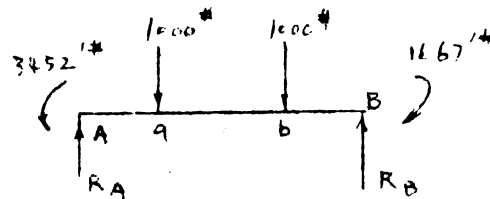
$$\text{From (5)} \quad 2M_B + M_C = -8371$$

$$\therefore M_A = -\frac{8371 + 1007}{2}$$

$$= -\frac{9378}{2} = -4689 \text{ ft.-lbs.}$$

$$M_C = -\frac{1007}{2} = -503.5 \text{ ft.-lbs.}$$

To find R_A



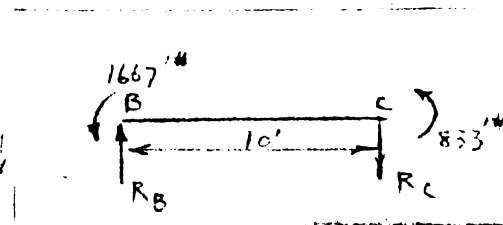
$$\sum M = 0$$

$$3452 + 1000(4+10) - 1667 - R_B(14) = 0$$

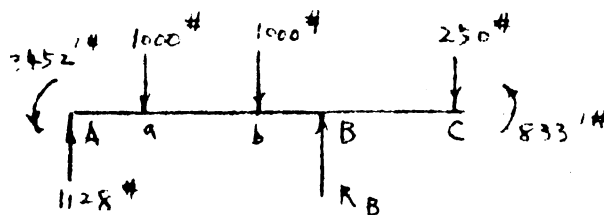
$$\therefore R_B = \frac{1667 + 14000 - 3452}{14} = 1128 \text{ lbs.} \uparrow$$

To find R_B

$$R_B = \frac{1667 + 14000 - 3452}{14} = 1128 \text{ lbs.} \uparrow$$



To find R_C

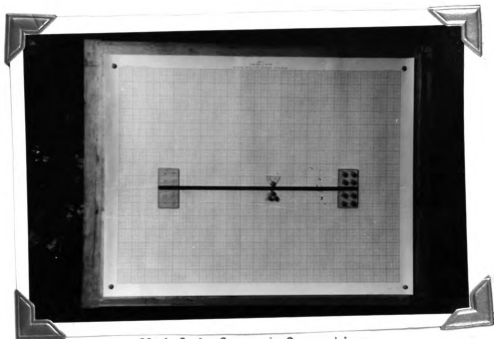


$$\sum M = 0$$

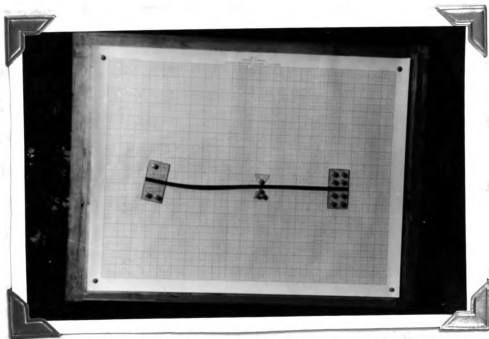
$$1128(14) + 833 + 3452 - 1000(4+10) - R_B(14) = 0$$

$$\therefore R_B = \frac{14000 + 833 + 3452 - 14000}{14} = 1128 \text{ lbs.} \uparrow$$

Problem 2

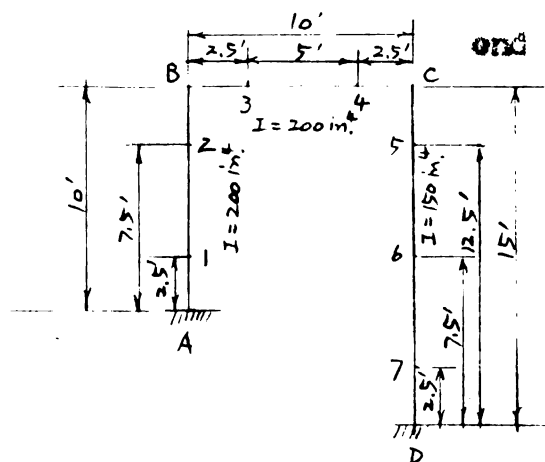


Model before deformation



Deformation of model to find left end moment

Problem 1. Given a 1:12 unsymmetrical bent with different moments of inertia in its members as shown in the figure below. Find the influence lines of the fix



1. Dimensional Reduction

To reduce the model for the of the members in the proportion $1' = 1'$. Then the members AB and CD which are 10' long in the actual problem will have a length of 10' in the model. The third member BC is reduced from 15' into 15' by the same ratio.

The moments of inertia of the members in the model should be proportional to those of the prototype. With uniform thickness, the width of a member varies as the cubic root of its moment of inertia. To select the width of the model member in such a proper value as to facilitate the test, the reduction of the width dimensions is shown in the following table:

Member	Moment of inertia I	$\sqrt[3]{I}$	Proportional Constant	Width in the model
AB	200	5.848	1/20	0.292"
BC	200	5.848	1/20	0.292"
CD	150	5.313	1/20	0.266"

The final dimension of the model is shown in full scale at plate II.

To fulfill the fin end condition, the portions beyond the ends A and B are enlarged into $1.5" \times 1.5"$ squares. Four holes are drilled in each enlarged square so that we can mount the model on the testing table by pulling drawing pins through these holes.

3. Instrumentation

To the influence^{lines} of H_x and H_y are required, we divide the beam into seven 5 - ft. intervals. The middle point of each interval is marked on the ends of the member, as points 1, 2, 3, 4, 5, 6 and 7. These points 1, 2, 3, 4, 5, 6, 7 and 8, 9, 10, 11 are punched through with small needle.

cross-section paper is used to ease the set up of the model and the measurement of the deflections.

The model is oriented on the cross-section paper. In order to get H_x , the enlarged portion of end A is mounted to the testing table to obtain the final condition. The hole at point A is attached to the table by a needle so that the model can rotate freely about A.

A small rotation to left side about A is introduced first, and the deformed positions of points 1, 2, 3, 4, 5, 6, 7 and 8 are marked on the cross-section paper by sharp needle point through the holes at each points. Then the model is released to its normal position and a

rotation to the right side of 1 of the same amount is applied. Again the several deflected positions of points 1, 2, 3, 4, 5, 6 and 7 are marked.

The amount of rotation can be measured by dividing the arc described by the end line. This can be done conveniently by measuring from 1 a distance 1" on the line prolonged from the end 12 on the enlarged part. The two deflected positions are marked on cross-section paper also. As the radius of rotation of this point about 1 is 1" exactly, the horizontal distance in inches of this point between its two extreme positions has the same value as the angle of rotation introduced in radians, and this amount will be called "rotation at 1".

As the loads at points 3 and 4 are applied vertically, the deflections at such two points should be measured vertically. All other deflections are horizontal.

For 13, the end 1 is fixed and the other end 2 is forced to rotate. The procedure of test is exactly the same as we have described above for 12.

3. Experiment

Three different amounts of circular rotation have been introduced at end 1 in sequence, and the same number of test is made at end 2.

In account of the proportion of length dimension of the model to the actual portion is $1' = 1'$, so we can calculate U_1 when 1 lb. load applied at point 1 by the

following formula:

$$M_1 \text{ due to } I \text{ at point } 1 = \frac{\text{rotation at point } 1}{\text{rotation at } I}$$

The rotation at I , as we mentioned before, is the angular rotation at I in radians. So if we measure the deflection at point 1 in unit of inches, the M_1 computed from this formula will be in the unit of lb - in.

The test results are shown in two tables which follow:

M_1 (not in 3 are in inches, A_1 in ft.-lbs.)

		Test Number			Average ft.-lbs.
		1	2	3	
Reading at A		18 48	18.2 48	20.4 48	
1	Reading	22.2 48	22.1 48	20.8 48	
	M_1	-2.056	-2.068	-2.006	-2.039
2	Reading	1 23.2 48	1 31.2 48	1 35.1 48	
	M_1	-4.667	-4.161	-4.074	-4.110
3	Reading	1 22.1 48	1 32.2 48	1 32.2 48	
	M_1	-4.381	-4.466	-4.366	-4.350
3	Reading	0.3 48	0.4 48	0.3 48	
	M_1	-0.017	-0.021	-0.015	-0.013
4	Reading	1.3 48	2.8 48	2.7 48	
	M_1	-0.181	-0.148	-0.132	-0.154
5	Reading	1 22.4 48	1 36.2 48	1 41.4 48	
	M_1	-4.300	-4.460	-4.388	-4.361
5	Reading	1 18.2 48	1 25.7 48	1 25.4 48	
	M_1	-3.733	-3.968	-3.704	-3.609
6	Reading	24.8 48	37.7 48	38.2 48	
	M_1	-1.233	-1.224	-1.226	-1.251
7	Reading	4.8 48	5.8 48	5.7 48	
	M_1	-0.227	-0.206	-0.272	-0.204

H_2 (readings in inches, H_3 in ft. - lbs.)

Test Number		Test Number			Average ft.-lbs.
		1	2	3	
Reading at D		16.0 48	17.1 48	20.7 48	
1	Reading	2.8 48	4.1 48	4.7 48	
	H_2	+0.211	+0.330	+0.227	+0.226
2	Reading	22.1 48	22.4 48	23.1 48	
	H_2	+1.336	+1.300	+1.337	+1.329
3	Reading	34.2 48	32.2 48	30.8 48	
	H_2	+1.911	+1.605	+1.925	+1.809
3	Reading	5.0 48	5.5 48	5.7 48	
	H_2	+0.553	+0.321	+0.536	+0.327
4	Reading	5.7 48	5.3 48	5.5 48	
	H_2	+0.317	+0.310	+0.314	+0.314
5	Reading	34.2 48	32.4 48	30.0 48	
	H_2	+1.933	+1.624	+1.932	+1.830
5	Reading	43.8 48	41.4 48	1 2.1 48	
	H_2	+2.453	+2.421	+2.430	+2.425
6	Reading	1 7.2 48	1 4.4 48	1 16.5 48	
	H_2	+3.067	+3.064	+3.111	+3.061
7	Reading	33.2 48	31.8 48	30.8 48	
	H_2	+1.883	+1.840	+1.874	+1.872

4. Influence of the Initial Conditions

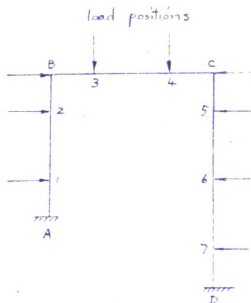
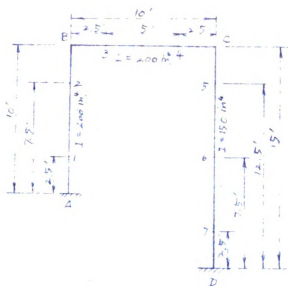
The influence lines obtained from both methods are plotted on the same scale for comparison. The value of these values may be seen directly.

The influence line from the results are plotted in solid lines while the influence line from the influence line is plotted in dashed lines.

5. Conclusion

To solve this problem satisfactorily by means of a computer program it is necessary to use the method.

Column analogy method



Column properties

Member	AB	BC	CD	Σ
Length (L)	10'	10'	15'	
I	200	200	150	
Relative value $\frac{1}{EI}$	3	3	4	
$A = \frac{L}{EI}$	30	30	60	120
x (from A)	0	5	10	
Ax	0	150	600	750
y (from A)	5	10	2.5	
Ay	150	300	150	600
x_0 (from c.g.)	-3.25	-1.25	+3.75	
y_0 (from c.g.)	0	+5	+2.5	
$I_{x-x} = Ay_0^2$	0	75.0	375	
$+ I_{xy}$	250	0	1125	2500
$I_{yy} = Ax_0^2$	1171.9	46.9	843.7	
$+ I_{yy}$	0	2500	0	2312.5
$I_{x-y} = Ax_0y_0$	0	-187.5	-562.5	-750.0
$+ I_{x'y}$	0	0	0	



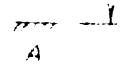
$$\bar{x} = \frac{\sum x}{n} = \frac{700}{100} = 7.0'$$

$$\bar{y} = \frac{\sum y}{n} = \frac{100}{100} = 1.0'$$

$$\bar{z} = \bar{z}_s = -f$$

x

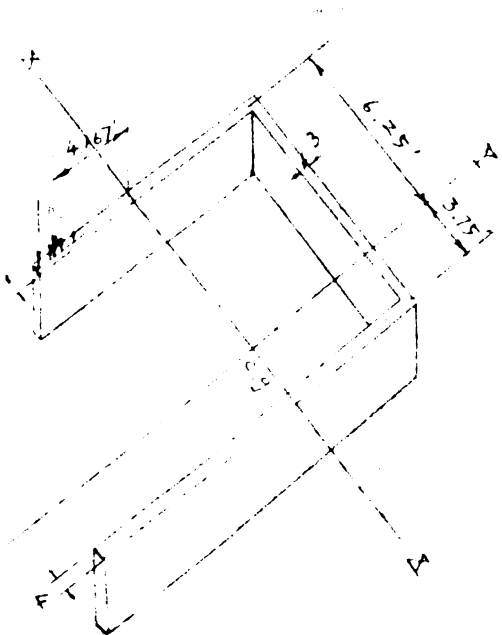
y'



D

$$f = \frac{1}{n} \left[\frac{\sum x^2}{\sum x} - \frac{(\sum x)^2}{n^2} \right] = \frac{1}{100} \left[\frac{5000}{100} - \frac{(700)^2}{100^2} \right]$$

2. 10' at point 2



$$\bar{x} = \frac{\sum x}{n} = \frac{1000}{100} = 10.0'$$

$$\bar{y} = \frac{\sum y}{n} = \frac{100}{100} = 1.0'$$

$$\bar{z} = \frac{\sum z}{n} = \frac{100}{100} = 1.0'$$

$$\bar{z} = \bar{z}_s = -f$$

$$= -1.0 - \left[\frac{9.0}{10.0} + \frac{20.0(10.0) - 20.0(1.0)(-1.0)}{100(10.0) - 100} \right] (10.0)$$

$$+ \frac{20.0(10.0) - 20.0(1.0)(-1.0)}{100(10.0) - 100} (-1.0)$$

$$= -1.0 - \left[-0.9 + \frac{20.0(10.0) - 20.0(1.0)(-1.0)}{100(10.0) - 100} (-1.0) \right]$$

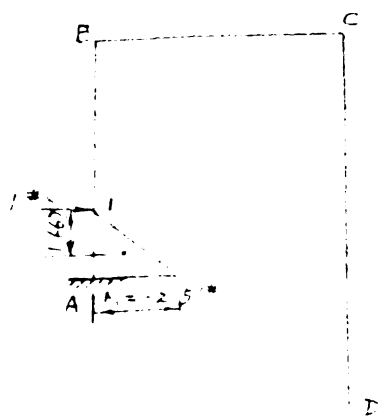
$$+ \frac{20.0(10.0) - 20.0(1.0)(-1.0)}{100(10.0) - 100} (-1.0)$$

$$= -1.0 - \left[-0.9 + (0.0227)(10.0) + (0.0227)(-1.0) \right]$$

$$= -1.0 + 0.9 + 0.227 - 0.0227 = -0.0957'$$

$$= 0 - \left[-0.9 + (0.0227)(10.0) + (0.0227)(-1.0) \right]$$

$$= +0.9 - 0.227 + 0.0227 = +0.6957'$$



cut at end D.

1.5 load at point B

$$P = -04.575$$

$$M_{B-A} = 010.039$$

$$M_{A-B} = 007.344$$

$$\begin{aligned} \Delta_A &= -7.5 - \left[\frac{-04.575}{120} + \frac{010.039(-120) - 010.039(-120)}{510750} (-6.25) \right. \\ &\quad \left. + \frac{010.039(-120) - 010.039(-120)}{510750} (-5) \right] \\ &= -7.5 + 0.703 + 0.03 = \frac{110.039(-120) + 010.039(-120)}{510750} \\ &= -7.5 + 0.703 + 1.709 + 0.045 = -4.103' \\ \Delta_D &= 0 - 0.703 + 3.75 = 0.003 + (-10) = 0.109 = +1.022' \end{aligned}$$

1.5 load at point B

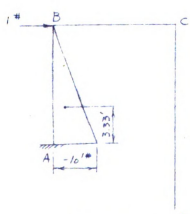
$$P = -10 \times 20 \times 3 = -150$$

$$M_{B-A} = (-1.07) \times (-150) = +160.5$$

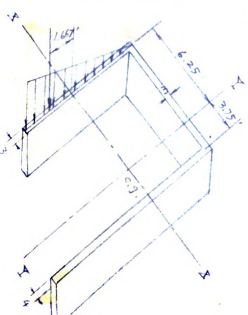
$$M_{A-B} = (-150) \times (-0.45) = +67.5$$

$$\begin{aligned} \Delta_A &= -10 - \left[\frac{-150}{120} + \frac{0160.5(-120) - (067.5)(-120)}{510750} (-6.25) \right. \\ &\quad \left. + \frac{(067.5)(-120) - (0160.5)(-120)}{510750} (-5) \right] \\ &= -10 - \left[-1.25 + \frac{0160.5(-120) + 008100}{510750} (-6.25) \right. \\ &\quad \left. + \frac{008100(-120) + 019200}{510750} (-5) \right] \\ &= -10 + 1.25 + 1.35 = 0.405 + 5 = 0.045 \\ &= -10 + 1.25 + 3.031 + 1.030 \\ &= -4.409' \end{aligned}$$

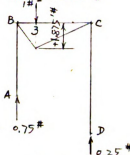
$$\begin{aligned} \Delta_D &= 1.25 - 0.405 = 3.75 - 0.045(-10) \\ &= 1.25 - 1.029 + 0.40 = +1.021' \end{aligned}$$



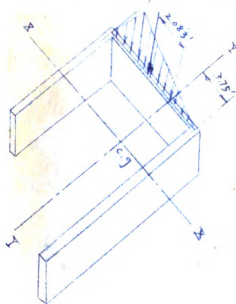
cut at end D



1" at point 3

But at both ends
A and D

$$\begin{aligned}
 H_A &= C - \left[\frac{(30.125)}{120} + \frac{(-22.221)(0.75) - (1/2)(0.625)(-1.75)}{5216750} (-1.25) \right. \\
 &\quad \left. + \frac{(140.625)(0.75) - (22.221)(-1.75)}{5216750} (-5) \right] \\
 &= -0.254 + \frac{40000}{5216750} (-1.25) - \frac{22221}{5216750} (-5) \\
 &= -0.254 + 0.00765(1.25) + 0.00505 \\
 &= -0.254 + 0.010 + 0.005 \\
 &= -0.014"/\#
 \end{aligned}$$



$$\begin{aligned}
 H_D &= -0.254 + 0.00765(1.75) \\
 &= -0.254 + 0.0054 + 0.005 \\
 &= +0.005"/\#
 \end{aligned}$$

1" at point 4

$$V = \frac{1.5711 \times 12 \times 1}{2} = +30.125$$

$$H_{D-R} = 140.625$$

$$H_{y-y} = -11.733$$

$$\begin{aligned}
 H_A &= C - \left[\frac{(30.125)}{120} + \frac{(-11.733)(0.75) - (1/2)(0.625)(-1.75)}{5216750} (-1.25) \right. \\
 &\quad \left. + \frac{(140.625)(0.75) - (-11.733)(-1.75)}{5216750} (-5) \right] \\
 &= -0.254 - \frac{75140.00}{5216750} (-1.25) - \frac{22222}{5216750} (-5)
 \end{aligned}$$

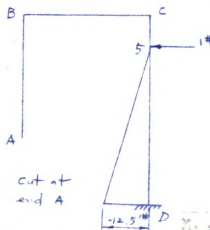
$$\begin{aligned}
 &= -0.054 + 0.0140(0.75) + 0.000000 \\
 &= -0.054 + 0.0105 + 0.0000 = -0.0435 \\
 M_D &= -0.054 + 0.0140(0.75) + 0.0000 = -0.0435 \\
 &= -0.054 + 0.0105 + 0.0000 = -0.0435
 \end{aligned}$$

1° at point C same as 1° at point B, but with the signs of M_B and M_D changed

$$M_B = +4.400 / \text{ft}$$

$$M_D = -1.001 / \text{ft}$$

1° at point D



$$M_S \text{ at A} = 0 \quad M_S \text{ at D} = -12.5$$

$$P = -12.5 \times 12.5 = 4 = -12.5 \times 12.5 = -312.5$$

$$M_{B-D} = +1000.000$$

$$M_{D-B} = -1271.075$$

$$M_A = 0 - \left[\frac{-12.5}{12.5} + \frac{(-12.5 \times 12.5)(12.5) - (-12.5 \times 12.5)(-12.5)}{2 \times 12.5 \times 12.5} (-12.5) \right]$$

$$+ \frac{(-12.5 \times 12.5)(12.5 \times 12.5) - (-12.5 \times 12.5)(-12.5)}{2 \times 12.5 \times 12.5} (-12.5) \Big]$$

$$= 0 - \left[-2.504 - \frac{121271.5}{2 \times 12.5 \times 12.5} (-12.5) \right]$$

$$+ \frac{121271.5}{2 \times 12.5 \times 12.5} (-12.5)$$

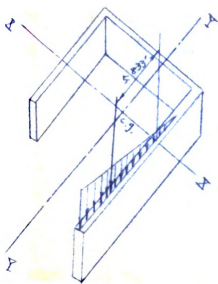
$$= +2.504 + 0.00000000 + 0.000 = 5$$

$$= +2.504 - 1.0703 + 3.100 = +3.533 / \text{ft}$$

$$M_D = -12.5 + 2.504 + 0.0000 = 3.75 + 0.000000$$

$$= -12.5 + 2.504 + 1.1250 + 0.00$$

$$= +1.125 / \text{ft}$$



2. at point 6

$$H_S \text{ at } A = 0 \quad H_S \text{ at } D = -7.5$$

$$P = -\left[\frac{1}{2} \times 7.5 \times 4\right] = -112.5$$

$$H_{max} = -112.5 + (-7.5) = +105.0$$

$$H_{p-y} = -112.5 + 0.75 = -111.75$$

$$M_1 = 0 - \left[\frac{-112.5}{2} + \frac{(-112.5)(-7.5)(1+2)}{3 \times 4} \right] (-1.0)$$

$$+ \left[\frac{105.0(1+2)}{3 \times 4} + \frac{(-7.5)(-1)}{2} \right] (-1)$$

$$= 0 - \left[-112.5 + \frac{1}{2} \times \frac{105.0}{2} (-1.0) + \frac{1}{2} \times \frac{105.0}{2} (-1.0) \right]$$

$$= 0 - \left[-112.5 + 0.000 (1.0) + 0.000 (1.0) \right]$$

$$= 0 + 112.5 + 0.000 + 0.000 = +112.5 \text{ /-}$$

$$H_D = -7.5 + 112.5 + 0.000 (0.75) + 0.000 (1.0)$$

$$= -7.5 + 112.5 + 0.000 + 0.000 = +105.0 \text{ /-}$$

2. at point 7

$$H_S \text{ at } A = 0 \quad H_S \text{ at } D = -1.5$$

$$P = -\left[\frac{1}{2} \times 1.5 \times 1\right] = -0.75$$

$$H_{max} = -0.75 + (-1.5) = +10.75$$

$$H_{p-y} = -0.75 + 0.75 = -0.00$$

$$M_1 = 0 - \left[\frac{-0.75}{2} + \frac{(-0.75)(-1.5)(1+2)}{3 \times 1} \right] (-1.0)$$

$$+ \left[\frac{10.75(1+2)}{3 \times 1} + \frac{(-1.5)(-1)}{2} \right] (-1)$$

$$= 0 - \left[-0.375 + \frac{10.75 \times 3}{2} (-1.0) + \frac{10.75}{2} (-1.0) \right]$$

$$+ \left[\frac{10.75 \times 3}{2} + \frac{1.5 \times 1}{2} (-1) \right]$$

$$= 0 + 0.375 + 0.000 (-1.0) + 0.000 (-1.0)$$

$$= 0 + 0.375 + 0.000 + 0.000 = +0.375 \text{ /-}$$

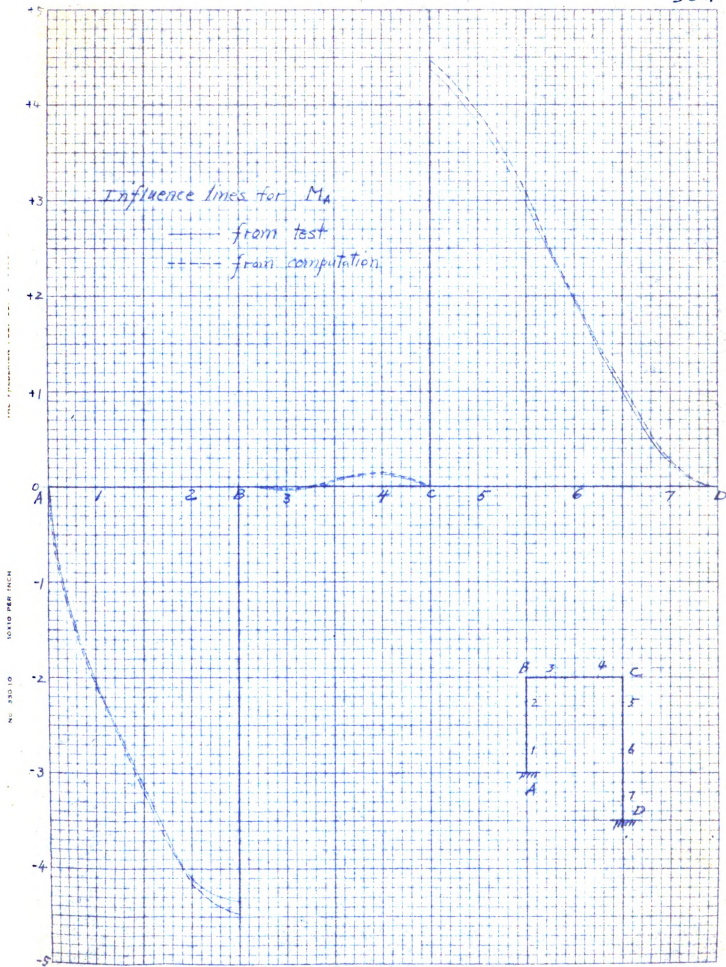
$$D = -1.1 + 0.24 + 0.025 \cdot 5 = 0.075 - 0.044 (-20)$$

$$= -1.1 + 0.24 + 0.024 + 0.17 = -0.49 \quad 1\#$$

Year	1970	1971
1	-1.100	-1.100
2	-1.100	-1.100
3	-1.100	-1.100
4	-1.100	-1.100
5	-1.100	-1.100
6	-1.100	-1.100
7	-1.100	-1.100
8	-1.100	-1.100
9	-1.100	-1.100
10	-1.100	-1.100

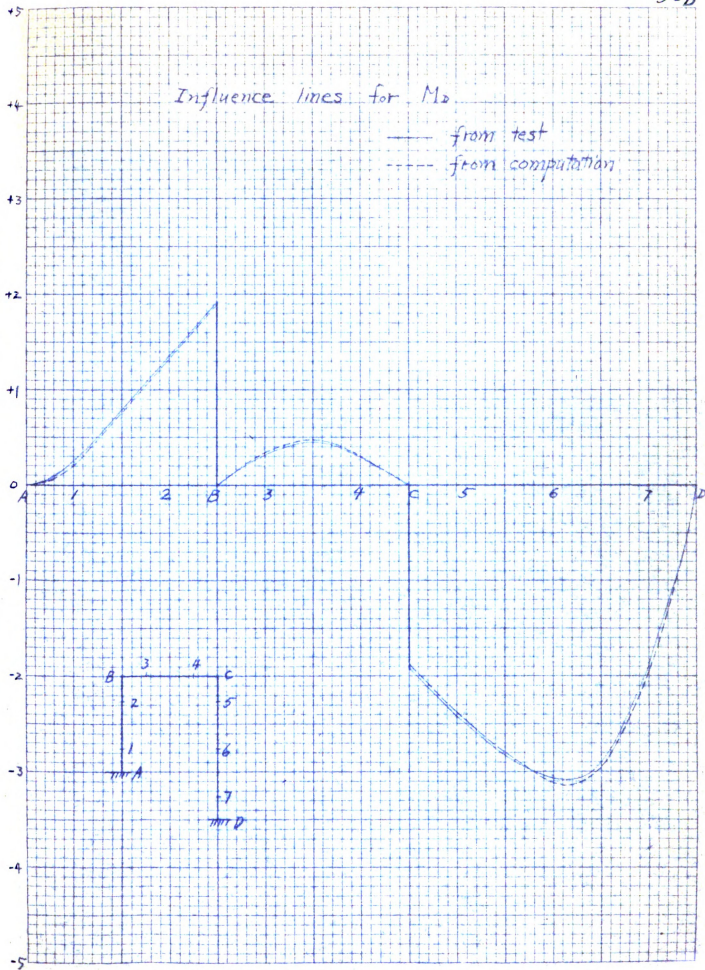
Influence lines for M_A

— from test
 - - - from computation

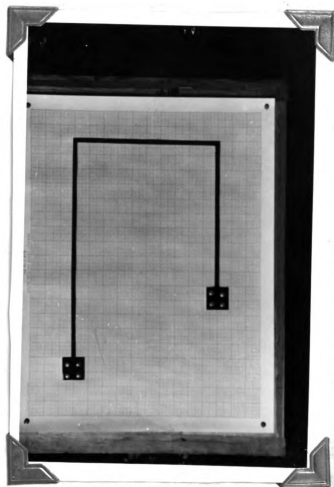


Influence lines for M_b

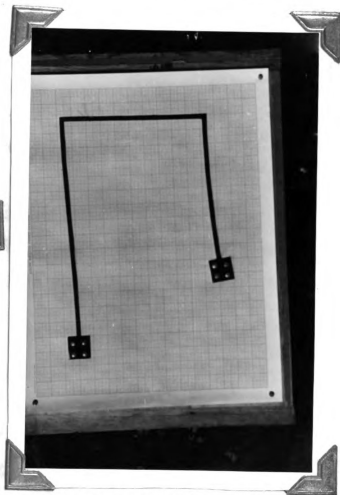
— from test
 --- from computation



Problem 3



Model before deformation

Deformation of model to find
right end moment

[illegible][illegible]

The total length of the 11 sections is 210.5' -
 divided by 20.0' in the ratio of 10.5' = 21. The length of the
 section is 20.5', which is rounded down to the next full length
 20.0' by 0.5' = 10. (20.5' ÷ 20.0' = 21.25')

In the original design, it was intended the time representation of the prototype in the model of inside of the wheel is proportional to that of the actual structure at the corresponding point. Since the extent of rotation of the wheel in the time is variable from point to point, the wheel should have variable compression also.

Let t be the width of the central structure at a certain point which gives a value of I to this cross-section, and t^2 and I^2 be the corresponding values of the 1st and 2nd moments of the area of the solid.

• k (a constant)

643
643

where b and b' are the minimum and the maximum values of the function ϕ in the interval $[a, a']$.

$$\frac{b^t}{b^t} = \frac{b^t}{b} \cdot \frac{t^0}{t} = k$$

Since $\frac{b'}{b}$ is constant, $\frac{t'}{t}$ is constant also. We can find the constant value by using the fact that $\frac{t'}{t} = 1$ at $t = 0$.

$$\frac{t'}{t} = \frac{b}{b'} k$$
$$\frac{t'}{t} = \sqrt[3]{\frac{b}{b'}} = c$$

where c is a constant.

Now if t is small, then, in order to find the value of $\frac{t'}{t}$ at $t = 0$, we can use the fact that $\frac{t'}{t} = 1$ at $t = 0$. We can also use the fact that $\frac{t'}{t} = c$ at $t = 0$. We can find the value of c by using the fact that $\frac{t'}{t} = 1$ at $t = 0$.

Now if t is large, then, in order to find the value of $\frac{t'}{t}$ at $t = \infty$, we can use the fact that $\frac{t'}{t} = c$ at $t = \infty$. We can also use the fact that $\frac{t'}{t} = 1$ at $t = 0$. We can find the value of c by using the fact that $\frac{t'}{t} = 1$ at $t = 0$.

Now if t is small, then, in order to find the value of $\frac{t'}{t}$ at $t = 0$, we can use the fact that $\frac{t'}{t} = 1$ at $t = 0$. We can also use the fact that $\frac{t'}{t} = c$ at $t = 0$. We can find the value of c by using the fact that $\frac{t'}{t} = 1$ at $t = 0$.

Now if t is large, then, in order to find the value of $\frac{t'}{t}$ at $t = \infty$, we can use the fact that $\frac{t'}{t} = c$ at $t = \infty$. We can also use the fact that $\frac{t'}{t} = 1$ at $t = 0$. We can find the value of c by using the fact that $\frac{t'}{t} = 1$ at $t = 0$.

L and l are the lengths of the L and l segments respectively. θ is the angle between the L and l segments. θ is the angle between the L and l segments.

R

R is the radius of the circle. R is the radius of the circle.

12

(Readings are in inches, HL in lbs.)

		Test Number			Average (lbs.)
Reading		1	2	3	
at A		11.0	11.2	11.4	
		30	30	30	
1L	Reading	10.0	11.3	12.3	
	HL	30	30	30	0.806
2L	Reading	11.2	11.4	12.1	
	HL	0.702	0.703	0.703	0.702
3L	Reading	10.2	11.7	12.7	
	HL	0.304	0.307	0.307	0.306
4L	Reading	11.0	11.8	11.2	
	HL	0.500	0.500	0.504	0.501
5L	Reading	11.7	11.7	11.1	
	HL	0.102	0.102	0.097	0.100
6L	Reading	11.2	11.1	10.5	
	HL	0.202	0.203	0.195	0.200
7L	Reading	10.5	10.2	10.5	
	HL	0.203	0.206	0.206	0.203
8L	Reading	10.3	10.7	10.3	
	HL	0.273	0.274	0.261	0.271
9L	Reading	11.0	10.7	10.0	
	HL	0.228	0.228	0.222	0.227
10L	Reading	11.0	10.7	10.7	
	HL	0.304	0.302	0.301	0.301

To be continued

IL (Continued)

		Test Number			Average (lbm.)
Reading at A		1	2	3	
1R	Reading	21.6	21.2	21.4	
	IL	0.107	0.112	0.103	0.107
2R	Reading	21.0	20.8	20.2	
	IL	0.316	0.312	0.304	0.310
3R	Reading	21.8	21.8	20.5	
	IL	0.316	0.321	0.309	0.315
4R	Reading	21.0	20.5	21.2	
	IL	0.412	0.405	0.409	0.413
5R	Reading	21.7	21.0	21.3	
	IL	0.103	0.108	0.099	0.103
6R	Reading	21.2	21.0	20.0	
	IL	0.301	0.195	0.303	0.199
7R	Reading	21.5	21.5	21.2	
	IL	0.395	0.392	0.396	0.397
8R	Reading	21.1	20.2	20.1	
	IL	0.307	0.304	0.306	0.309
9R	Reading	21.0	21.3	21.2	
	IL	0.440	0.432	0.435	0.451
10R	Reading	21.2	21.5	21.0	
	IL	0.405	0.424	0.407	0.409

(Readings are in inches, kg in 294.)

Reading at B		Test Number			Average (Inch.)
		1	2	3	
1L	Reading	22.1	22.0	22.1	0.100
	kg	0.107	0.107	0.108	
2L	Reading	22.2	22.2	22.2	0.210
	kg	0.210	0.210	0.211	
3L	Reading	27.0	27.2	27.2	0.315
	kg	0.315	0.316	0.318	
4L	Reading	22.2	22.0	22.0	0.418
	kg	0.418	0.416	0.407	
5L	Reading	9.2	12.0	9.0	0.102
	kg	0.096	0.103	0.104	
6L	Reading	22.0	22.5	22.2	0.102
	kg	0.107	0.108	0.108	
7L	Reading	22.5	22.0	22.0	0.322
	kg	0.308	0.300	0.300	
8L	Reading	22.0	22.1	22.2	0.300
	kg	0.374	0.384	0.382	
9L	Reading	27.0	27.2	26.5	0.424
	kg	0.433	0.438	0.427	
10L	Reading	22.0	22.0	22.1	0.425
	kg	0.428	0.420	0.425	

(to be continued)

1. (Continued)

B					
1					
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4. Results of Analysis

Since the χ^2 test is only qualitative, χ^2 and ν have no definite significance. In this case, the χ^2 value of 7.1, the corresponding value of ν is 2, and the probability of χ^2 is 0.03. The third column of the table shows the χ^2 and ν are listed in Table 4.

Table 4 shows the results of the χ^2 test. The results of the χ^2 test are listed in Table 4. The results of the χ^2 test are listed in Table 4.

Table 4

Sample Size	χ^2	ν	P
10	0.000	1	0.100
20	0.000	2	0.100
30	0.000	3	0.100
40	0.000	4	0.100
50	0.000	5	0.100
60	0.000	6	0.100
70	0.000	7	0.100
80	0.000	8	0.100
90	0.000	9	0.100
100	0.000	10	0.100

5. Discussion

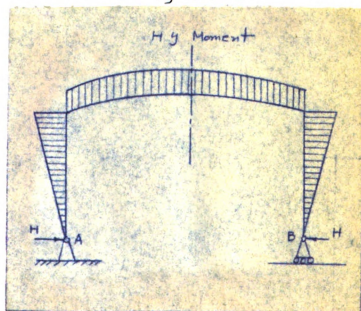
The results of the χ^2 test are listed in Table 4. The results of the χ^2 test are listed in Table 4. The results of the χ^2 test are listed in Table 4.

The results of the χ^2 test are listed in Table 4. The results of the χ^2 test are listed in Table 4. The results of the χ^2 test are listed in Table 4.

Dimensions of the bridge as shown in
plate III

pt	C ₁	X	C	Y	ds/I
1	4.3	4.3	3.67	1.59	0.704
2	4.3	6.6	3.50	3.67	1.304
3	4.3	11.9	4.53	6.76	0.636
4	4.3	17.2	4.67	8.49	0.706
5	4.3	18.3	4.00	9.33	0.607
6	4.3	18.3	3.15	9.00	1.654
7	4.3	19.9	2.50	1.90	3.368
8	4.3	20.4	2.00	0.67	6.410
9	4.3	20.6	1.67	0.39	11.016
10	4.3	21.0	1.00	0.30	15.307

Fig. 106 cont



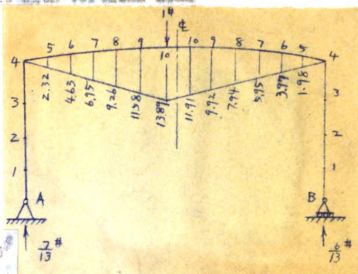
n	$\frac{1}{n}$	$\frac{1}{n^2}$	$\frac{1}{n^3}$	$\frac{1}{n^4}$	$\frac{1}{n^5}$
1	1.0	1.000	1.000	1.000	1.000
2	0.5	0.250	0.125	0.062	0.031
3	0.333	0.111	0.037	0.012	0.004
4	0.250	0.062	0.016	0.004	0.001
5	0.200	0.040	0.008	0.002	0.000
6	0.167	0.028	0.004	0.001	0.000
7	0.143	0.020	0.003	0.000	0.000
8	0.125	0.016	0.002	0.000	0.000
9	0.111	0.012	0.001	0.000	0.000
10	0.100	0.010	0.001	0.000	0.000

$\Sigma = 1.000$

max moment with vertical load

1" at point 10

$$V_L = 7/13 \quad V_R = 6/13$$



St	7	Dist I	St	$\frac{1}{13} \frac{A}{13}$
5L	15.3	0.007	2.32	34.75
6L	19.2	1.054	4.65	107.03
7L	19.9	3.308	6.95	407.51
8L	20.4	6.418	9.26	1212.39
9L	20.8	11.026	11.50	2357.75
10L	21.0	15.307	13.89	4408.40
10R	21.0	15.307	11.91	3848.45
9R	20.8	11.026	9.00	2075.05
8R	20.4	6.418	7.04	1030.56
7R	19.9	3.308	5.05	302.69
6R	19.2	1.054	3.07	125.07
5R	15.3	0.007	1.50	22.04

$$\Sigma = 1000.11$$

$$H = \frac{1000.11}{1212.39} = 0.82507$$

$$V_L = 0.49507 \rightarrow \quad V_R = 0.49507 \leftarrow$$

1. 3. 10. 10. 10.

$$v_1 = \frac{0.7}{13} \quad v_2 = \frac{2}{13}$$

x_i	x_i	$-x_i$	x_i	x_i^2
51	10.7	0.007	0.00	50.00
61	10.0	1.000	0.00	20.00
72	10.0	0.000	7.00	50.00
81	10.7	1.000	10.00	2007.00
92	10.0	11.000	20.00	2000.00
101	11.0	10.000	11.00	3707.00
101	11.0	10.000	0.00	3707.00
91	10.0	11.000	0.00	2000.00
82	10.0	0.000	0.00	00.00
72	10.0	0.000	0.00	00.00
61	10.0	1.000	0.00	00.00
51	10.0	0.000	0.00	00.00

 $\Sigma 2007.00$

$$x = \frac{\Sigma x_i}{n} = \frac{2007.00}{13} = 154.38$$

$$x_1 = 0.4230 \rightarrow$$

$$x_2 = 0.4230 \leftarrow$$

2/ 01.11.19

$$x_1 = \frac{1}{15}$$

$$x_2 = \frac{1}{15}$$

16	17	18/1	19	$\frac{1}{15} \cdot 5$
01	10.3	6.667	0.10	11.06
02	10.3	2.222	0.05	105.05
03	10.3	3.333	0.03	107.03
04	10.3	7.778	11.11	1001.33
05	10.3	11.111	10.70	1007.44
101	11.0	11.537	7.05	1000.37
102	11.0	11.117	7.04	1001.36
1	10.3	11.013	7.01	1110.34
6	10.3	7.011	0.10	111.61
7	10.3	2.005	0.07	111.54
01	10.3	1.134	1.05	04.16
01	20.3	0.667	1.01	10.34

Σ 1113.00

$$x = \frac{1113.00}{1113.00} = 0.9999$$

$$x = 0.9999 \rightarrow$$

$$x = 0.9999 \leftarrow$$

1. β at point 7

$$v_1 = \frac{10}{15} \text{ m/s}$$

$$v_2 = \frac{10}{15} \text{ m/s}$$

s	r	v/Δ	π	$u \frac{d\pi}{ds} r$
50	10.0	0.667	3.01	40.00
60	10.0	1.000	6.28	120.00
70	10.0	1.000	9.42	200.00
80	10.0	0.667	9.42	120.00
90	0.0	11.000	7.85	1100.00
100	10.0	11.000	0.00	1100.00
110	10.0	11.000	0.00	1100.00
120	10.0	11.000	3.14	1100.00
130	10.0	1.000	6.28	200.00
140	10.0	1.000	9.42	200.00
150	10.0	1.000	9.42	200.00
160	10.0	1.000	9.42	200.00
170	10.0	1.000	9.42	200.00
180	10.0	1.000	9.42	200.00
190	10.0	1.000	9.42	200.00
200	10.0	1.000	9.42	200.00
210	10.0	1.000	9.42	200.00
220	10.0	1.000	9.42	200.00
230	10.0	1.000	9.42	200.00
240	10.0	1.000	9.42	200.00
250	10.0	1.000	9.42	200.00
260	10.0	1.000	9.42	200.00
270	10.0	1.000	9.42	200.00
280	10.0	1.000	9.42	200.00
290	10.0	1.000	9.42	200.00
300	10.0	1.000	9.42	200.00
310	10.0	1.000	9.42	200.00
320	10.0	1.000	9.42	200.00
330	10.0	1.000	9.42	200.00
340	10.0	1.000	9.42	200.00
350	10.0	1.000	9.42	200.00
360	10.0	1.000	9.42	200.00
370	10.0	1.000	9.42	200.00
380	10.0	1.000	9.42	200.00
390	10.0	1.000	9.42	200.00
400	10.0	1.000	9.42	200.00
410	10.0	1.000	9.42	200.00
420	10.0	1.000	9.42	200.00
430	10.0	1.000	9.42	200.00
440	10.0	1.000	9.42	200.00
450	10.0	1.000	9.42	200.00
460	10.0	1.000	9.42	200.00
470	10.0	1.000	9.42	200.00
480	10.0	1.000	9.42	200.00
490	10.0	1.000	9.42	200.00
500	10.0	1.000	9.42	200.00

$$\Sigma 1100.00$$

$$v = \frac{2000.00}{2000.00} = 1.000 \text{ m/s}$$

$$v_1 = 0.667 \text{ m/s} \rightarrow$$

$$v_2 = 0.667 \text{ m/s} \leftarrow$$

1. $\mu = 10.1111$

$$C_1 = \frac{21}{13}$$

$$C_2 = \frac{19}{13}$$

y_i	f	y_i/f	y_i	y_i^2/f
34	10.5	9.007	3.04	51.73
51	10.4	11.54	7.10	122.10
74	10.0	11.070	3.00	457.70
81	10.4	11.118	3.05	771.00
91	10.0	11.000	3.00	1121.00
101	10.0	11.007	3.00	1344.24
100	10.0	11.007	2.07	1041.15
91	10.0	11.000	1.01	730.10
61	10.4	11.110	3.00	540.00
71	10.0	11.000	1.00	150.00
81	10.4	11.104	1.04	42.00
97	10.0	9.007	0.00	0.07

$$\Sigma 5771.55$$

$$\mu = \frac{100 \times 5771.55}{10000} = 0.5771$$

$$\mu_1 = 0.5771 \rightarrow$$

$$\mu_2 = 0.5771 \leftarrow$$

1" at point 1

$$E = \frac{30 \times 10^6}{12}$$

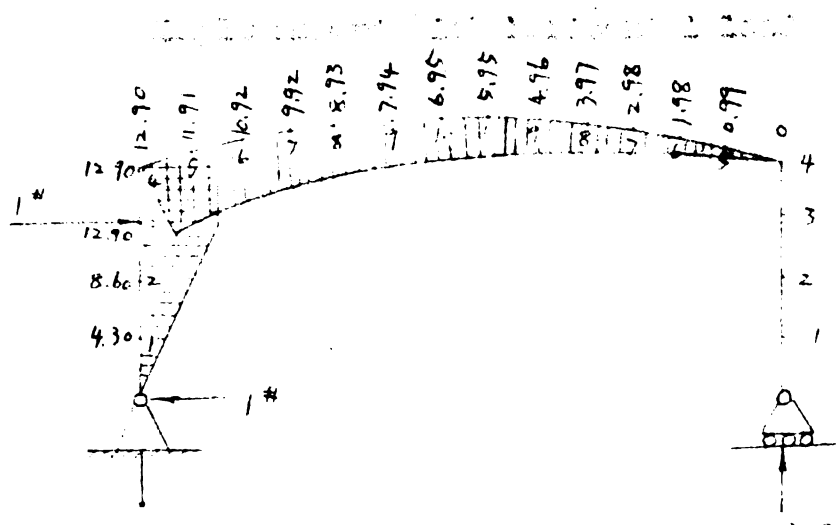
$$I = \frac{12^4}{12}$$

x	y	$\frac{y}{x}$	$\frac{y}{x^2}$	$\frac{y}{x^3}$
1	10.3	0.007	1.07	10.3
2	11.0	1.001	3.04	110.00
3	11.0	1.000	1.01	117.00
4	11.1	0.000	0.00	100.10
5	10.0	11.000	0.00	87.50
10	11.0	15.000	0.00	70.00
15	12.0	12.000	1.00	50.07
20	20.0	11.000	1.00	57.01
25	10.1	0.000	1.01	100.00
30	13.0	0.000	0.00	0.00
35	12.1	1.000	0.00	0.00
40	10.3	0.007	0.00	4.00

$$\Sigma 510.02$$

$$E = \frac{30 \times 10^6}{12} = 2,500,000$$

$$H_L = 1.218 \rightarrow H = 0.2010 \rightarrow$$



1. 1. 1. 1. 1. 1.

$$V_1 = 0.0000 \quad V_2 = 0.0000$$

no	y	dx/T	u	u dx y
11	4.3	2.704	8.00	50.00
21	6.5	1.004	8.00	30.00
31	11.0	0.000	10.00	100.00
41	17.1	0.000	10.00	100.00
51	18.3	0.007	11.01	175.00
61	19.0	1.004	10.00	240.00
71	19.0	1.000	9.00	150.00
81	20.4	0.000	8.00	120.00
91	20.0	11.000	7.00	200.00
101	21.0	10.000	6.00	200.00
111	21.0	10.000	5.00	100.00
121	20.0	11.000	4.00	120.00
131	20.4	0.000	3.00	60.00
141	19.0	1.000	2.00	20.00
151	18.3	0.007	0.00	0.00

$$\Sigma 1000.00$$

$$V_1 = \frac{1000.00}{1000.00} = 0.0000$$

$$V_2 = 1 - 0.0000 = 0.0000 \quad V_3 = 0.0000$$

2. $\bar{y}$ and $\bar{x}$

$$\bar{y} = 0.2559 \quad \bar{x} = 0.2559$$

x	y	xy	x^2	y^2
11	4.5	49.50	121	20.25
21	6.5	136.50	441	42.25
31	11.0	341.00	961	121.00
41	17.0	697.00	1681	289.00
51	22.0	1122.00	2601	484.00
61	25.0	1525.00	3721	625.00
71	22.0	1562.00	5041	484.00
81	20.4	1652.40	6561	416.16
91	20.0	1820.00	8281	400.00
101	22.0	2222.00	10201	484.00
111	22.0	2442.00	12321	484.00
121	20.0	2420.00	14641	400.00
131	20.4	2680.40	17361	416.16
141	21.0	2961.00	19801	441.00
151	20.0	3020.00	22801	400.00
161	12.0	1932.00	25921	144.00
171	12.0	2052.00	29241	144.00
181	12.0	2172.00	32641	144.00
191	12.0	2280.00	36481	144.00
201	12.0	2412.00	40401	144.00
211	12.0	2532.00	44521	144.00
221	12.0	2652.00	48841	144.00
231	12.0	2772.00	53361	144.00
241	12.0	2892.00	58081	144.00
251	12.0	3012.00	62901	144.00
261	12.0	3132.00	67921	144.00
271	12.0	3252.00	73141	144.00
281	12.0	3372.00	78561	144.00
291	12.0	3492.00	84181	144.00
301	12.0	3612.00	90001	144.00
311	12.0	3732.00	96021	144.00
321	12.0	3852.00	102241	144.00
331	12.0	3972.00	108661	144.00
341	12.0	4092.00	115281	144.00
351	12.0	4212.00	122101	144.00
361	12.0	4332.00	129121	144.00
371	12.0	4452.00	136341	144.00
381	12.0	4572.00	143761	144.00
391	12.0	4692.00	151381	144.00
401	12.0	4812.00	159201	144.00
411	12.0	4932.00	167221	144.00
421	12.0	5052.00	175441	144.00
431	12.0	5172.00	183861	144.00
441	12.0	5292.00	192481	144.00
451	12.0	5412.00	201301	144.00
461	12.0	5532.00	210321	144.00
471	12.0	5652.00	219541	144.00
481	12.0	5772.00	228961	144.00
491	12.0	5892.00	238581	144.00
501	12.0	6012.00	248401	144.00
511	12.0	6132.00	258421	144.00
521	12.0	6252.00	268641	144.00
531	12.0	6372.00	279061	144.00
541	12.0	6492.00	289681	144.00
551	12.0	6612.00	300501	144.00
561	12.0	6732.00	311521	144.00
571	12.0	6852.00	322741	144.00
581	12.0	6972.00	334161	144.00
591	12.0	7092.00	345781	144.00
601	12.0	7212.00	357601	144.00
611	12.0	7332.00	369621	144.00
621	12.0	7452.00	381841	144.00
631	12.0	7572.00	394261	144.00
641	12.0	7692.00	406881	144.00
651	12.0	7812.00	419701	144.00
661	12.0	7932.00	432721	144.00
671	12.0	8052.00	445941	144.00
681	12.0	8172.00	459361	144.00
691	12.0	8292.00	472981	144.00
701	12.0	8412.00	486801	144.00
711	12.0	8532.00	500821	144.00
721	12.0	8652.00	515041	144.00
731	12.0	8772.00	529461	144.00
741	12.0	8892.00	544081	144.00
751	12.0	9012.00	558901	144.00
761	12.0	9132.00	573921	144.00
771	12.0	9252.00	589141	144.00
781	12.0	9372.00	604561	144.00
791	12.0	9492.00	620181	144.00
801	12.0	9612.00	636001	144.00
811	12.0	9732.00	652021	144.00
821	12.0	9852.00	668241	144.00
831	12.0	9972.00	684661	144.00
841	12.0	10092.00	701281	144.00
851	12.0	10212.00	718101	144.00
861	12.0	10332.00	735121	144.00
871	12.0	10452.00	752341	144.00
881	12.0	10572.00	769761	144.00
891	12.0	10692.00	787381	144.00
901	12.0	10812.00	805201	144.00
911	12.0	10932.00	823221	144.00
921	12.0	11052.00	841441	144.00
931	12.0	11172.00	859861	144.00
941	12.0	11292.00	878481	144.00
951	12.0	11412.00	897301	144.00
961	12.0	11532.00	916321	144.00
971	12.0	11652.00	935541	144.00
981	12.0	11772.00	954961	144.00
991	12.0	11892.00	974581	144.00
1001	12.0	12012.00	994401	144.00

$$\Sigma 7115.00$$

$$= \frac{7115.00}{100} = 71.15$$

$$x_2 = 1 - 0.2885 = 0.7115 \leftarrow$$

$$x_1 = 0.2885 \leftarrow$$

1. 1st year 1.1

$$P_2 = 0.0719 \quad P_3 = 0.0719$$

x	y	x/y	z	$x/y \cdot z$
15	4.5	3.754	4.50	34.80
25	6.5	2.308	4.50	44.52
35	11.0	0.773	4.50	34.80
45	17.0	0.706	4.50	31.73
55	17.5	0.687	3.17	22.03
65	15.0	1.778	3.17	22.50
75	21.0	1.710	3.17	22.50
85	22.5	1.733	3.17	22.50
95	30.0	11.000	3.17	34.80
105	11.0	17.727	3.17	74.55
115	11.0	11.735	1.00	63.07
125	30.0	11.000	1.00	37.54
135	11.0	17.727	1.00	171.01
145	11.0	11.735	0.00	63.07
155	11.0	11.735	0.00	33.06
165	10.5	0.687	0.00	4.07

$$\Sigma 1183.02$$

$$P = \frac{1183.02}{3637.25} = 0.3277$$

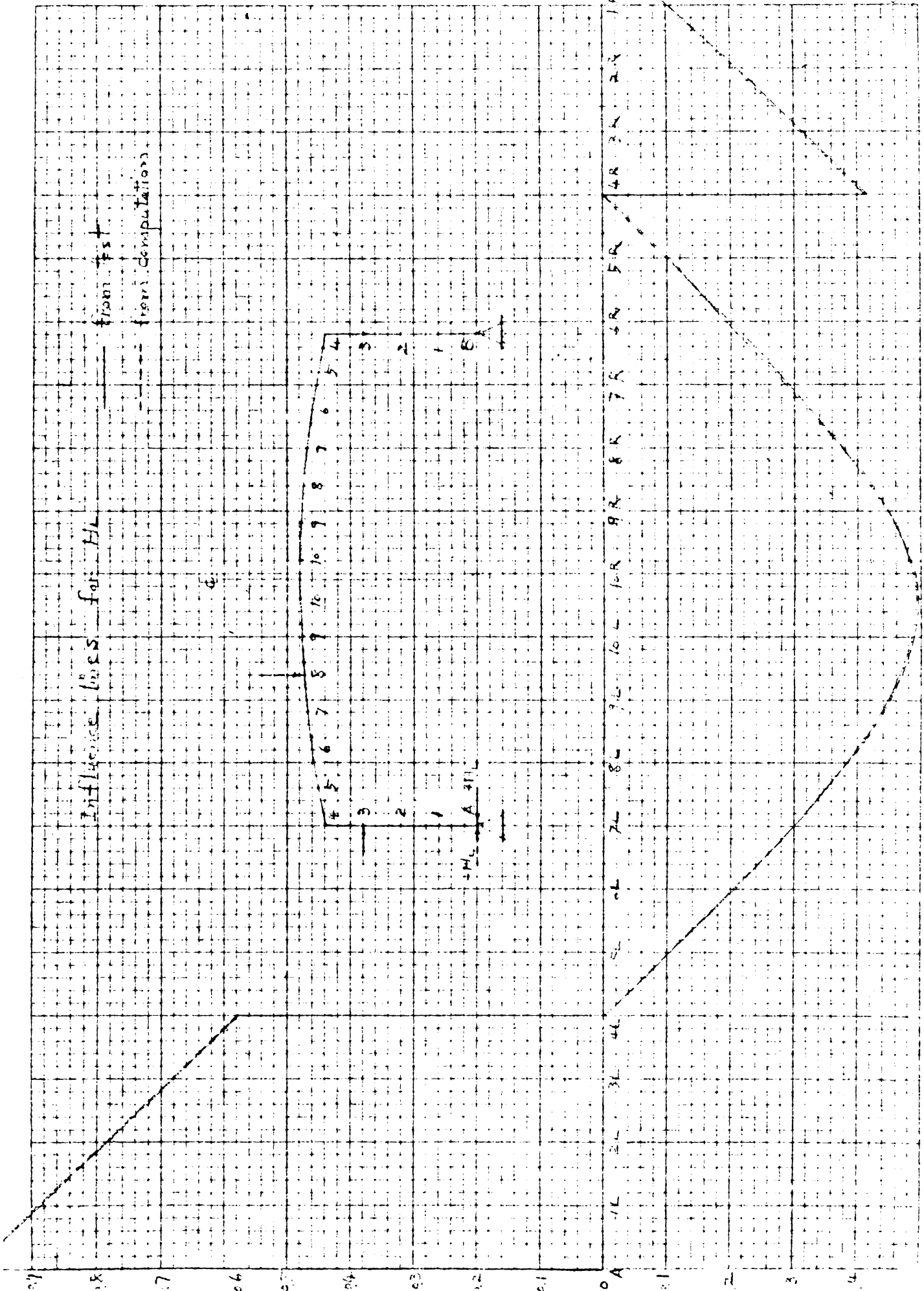
$$P_1 = 1 - 0.3277 = 0.6723 \leftarrow$$

$$P_2 = 0.3277 \leftarrow$$

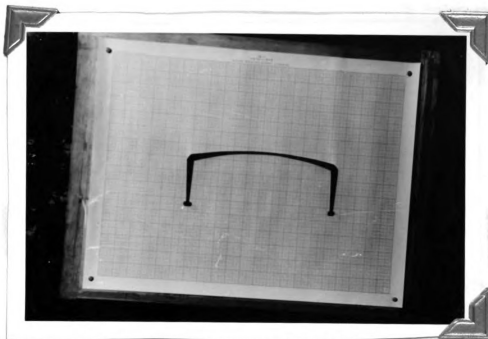
position	on line of	position	on line of
10	0.0000	10	0.0000
11	0.0000	11	0.0000
12	0.0000	12	0.0000
13	0.0000	13	0.0000
14	0.0000	14	0.0000
15	0.0000	15	0.0000
16	0.0000	16	0.0000
17	0.0000	17	0.0000
18	0.0000	18	0.0000
19	0.0000	19	0.0000
20	0.0000	20	0.0000

Influence Lines for HL

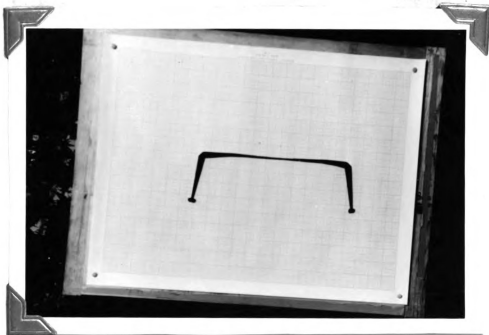
from test
from computation



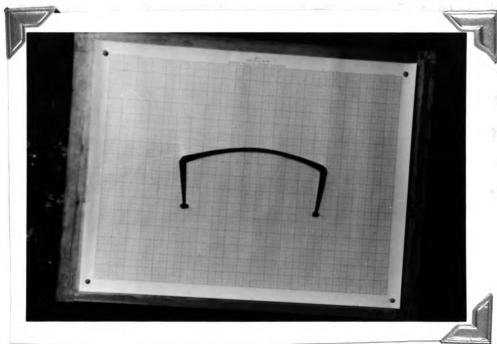
Problem 4



Model before deformation



Deformation of model to find right end thrust
(outward horizontal movement)



Deformation of model to find right end thrust
(inward horizontal movement)

Point	I	I'	$\log I'$	$1/3 \log$	$\sqrt[3]{I'}$	F	Thickness of model (in inches)
center	0.47000	470.00	2.67245	0.89081	7.812	1/30	0.1303
1	0.51401	514.01	2.71125	0.90375	8.111	1/30	0.1333
2	0.57700	577.00	2.76070	0.92023	8.537	1/30	0.1380
3	0.70000	700.00	2.84509	0.94836	8.912	1/30	0.1465
4	0.84100	841.00	2.92474	0.97491	9.299	1/30	0.1553
5	1.00000	1000.00	3.00000	1.00000	10.000	1/30	0.1664
6	1.08400	1084.00	3.03480	1.01160	10.343	1/30	0.1700
7	0.74007	4745.37	3.67610	1.22537	14.082	1/30	0.2335
8	4.00000	4000.00	3.60206	1.20042	15.849	1/30	0.2505
9	0.50001	1001.01	3.00043	1.00014	17.320	1/30	0.2665
10	0.30000	3000.00	3.47712	1.15904	21.544	1/30	0.3411
Spring	10.1007	1010.07	3.00430	1.00143	21.544	1/30	0.3410

In the above table, I is the square of the radius of the original arch, I' is the square radius of the flat at each section of the original arch, the factor F is so chosen that no final deformation thickness that distance will make the model either too stiff or too flexible.

The final end conditions are fulfilled by the original portions beyond the spring. A hole is drilled on them to hold the lower plate which, in turn, will fix two ends to immovably on the lower base owing to it. Small holes are drilled along the axis of the arch on every load point, such as $P_1, P_2, \dots, P_5, P_6, P_7, \dots, P_{10}$, so that their deformations may be transferred down on the base-section

paper. It is at the center of each of the circles the possibility of a rotating plane rotation about it. There are two holes in the paper and drilled on the external portions beyond the center of each of the circles. The distance between the two holes is exactly one inch. The purpose of the hole in the center is to facilitate the measurement of angular rotations. All the circles of the paper are in plane VI.

B. MEASUREMENTS

The first circle of the paper is drilled into the center of the circle. The distance between the two holes is exactly one inch. The purpose of the hole in the center is to facilitate the measurement of angular rotations. All the circles of the paper are in plane VI.

The second circle of the paper is drilled into the center of the circle. The distance between the two holes is exactly one inch. The purpose of the hole in the center is to facilitate the measurement of angular rotations. All the circles of the paper are in plane VI.

The third circle of the paper is drilled into the center of the circle. The distance between the two holes is exactly one inch. The purpose of the hole in the center is to facilitate the measurement of angular rotations. All the circles of the paper are in plane VI.

is not only in the large and small in the end without
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2. Construction

The construction of the arch is the same as that of the arch.

P_0 (122 PROHIBITIONED TWO IN THE UNIT OF 1/100)

		Test Number			Average Value of P_0 (lbs.)
		1	2	3	
Reading at spring		22.6	10.7	11.8	
A _L	Reading	45.0	16.7	21.2	
	P_0	1.200	1.748	1.796	1.612
A _R	Reading	45.0	16.7	21.0	
	P_0	1.200	1.748	1.779	
B _L	Reading	27.5	16.4	18.5	
	P_0	1.657	1.532	1.368	1.575
B _R	Reading	27.4	16.2	18.0	
	P_0	1.652	1.512	1.536	
C _L	Reading	29.0	15.1	15.1	
	P_0	1.282	1.223	1.379	1.254
C _R	Reading	29.5	12.8	14.0	
	P_0	1.303	1.106	1.230	
D _L	Reading	19.5	8.8	10.0	
	P_0	0.863	0.824	0.847	0.851
D _R	Reading	20.0	9.2	9.8	
	P_0	0.084	0.859	0.830	
E _L	Reading	12.0	5.2	6.0	
	P_0	0.531	0.406	0.508	0.510
E _R	Reading	12.0	5.5	5.8	
	P_0	0.531	0.514	0.492	
F _L	Reading	5.5	2.4	3.0	
	P_0	0.245	0.224	0.254	0.246
F _R	Reading	5.8	2.7	2.9	
	P_0	0.256	0.252	0.246	
G _L	Reading	1.6	0.8	0.9	
	P_0	0.071	0.075	0.075	0.075
G _R	Reading	1.5	0.7	1.0	
	P_0	0.066	0.065	0.085	

I_2 (left) (all loadline errors are in the unit of 1/100")

		Test Number			Avg. value of I_2 (left) (St.-lbs.)
		1	2	3	
Reading at spring (left)		0.0	3.1	7.9	
C _L	Reading	9.0	3.5	8.6	
	I_2	-5.62	-5.65	-5.45	-5.57
F _L	Reading	15.5	5.3	12.7	
	I_2	-8.45	-8.39	-8.04	-8.29
P _L	Reading	15.0	5.6	15.0	
	I_2	-9.38	-9.36	-9.42	-9.41
D _L	Reading	13.2	5.2	13.1	
	I_2	-8.26	-8.29	-8.39	-8.31
G _L	Reading	8.5	3.4	6.0	
	I_2	-5.32	-5.48	-5.97	-5.46
H _L	Reading	2.3	0.7	2.2	
	I_2	-1.44	-1.13	-1.39	-1.32
A _L	Reading	4.6	1.8	4.2	
	I_2	+2.88	+2.80	+3.66	+2.81
Ag	Reading	8.5	3.7	8.7	
	I_2	+5.31	+5.97	+5.50	+5.59
Bg	Reading	11.0	4.4	10.8	
	I_2	+6.88	+7.10	+6.84	+6.94
Gg	Reading	9.8	4.1	10.5	
	I_2	+6.12	+6.61	+6.65	+6.46
Dg	Reading	8.0	3.1	8.1	
	I_2	+5.00	+5.00	+5.12	+5.04
Eg	Reading	5.1	2.0	5.0	
	I_2	+3.18	+3.22	+3.16	+3.19
Fg	Reading	2.5	1.5	2.5	
	I_2	+1.56	+1.77	+1.58	+1.64
Gg	Reading	0.0	0.5	0.9	
	I_2	+0.20	+0.40	+0.57	+0.52

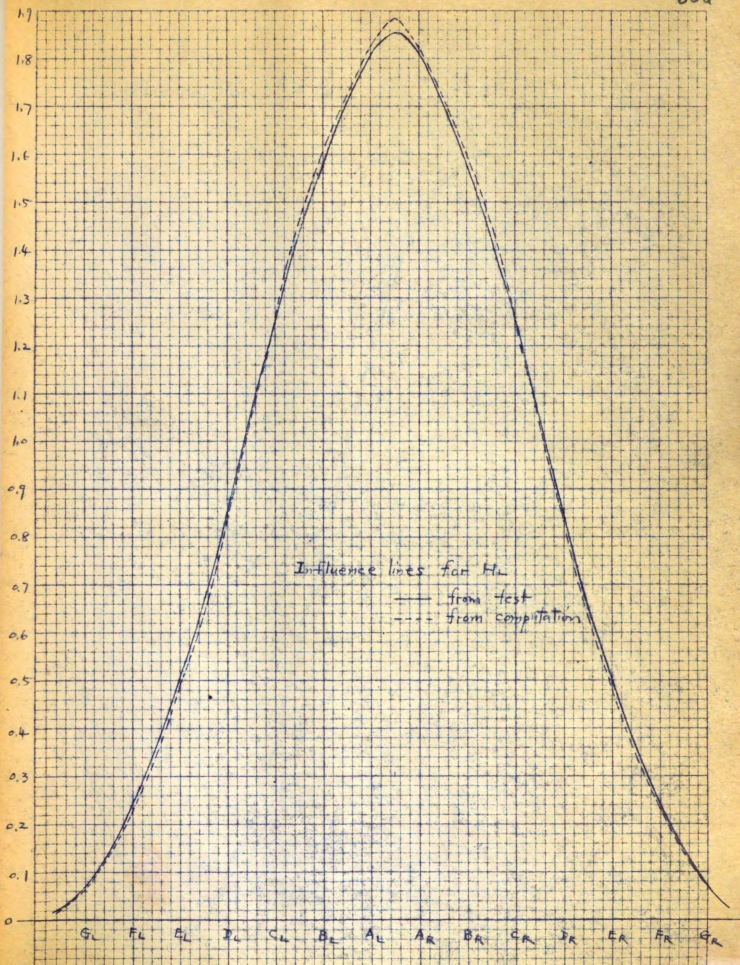
4. Comparison of results

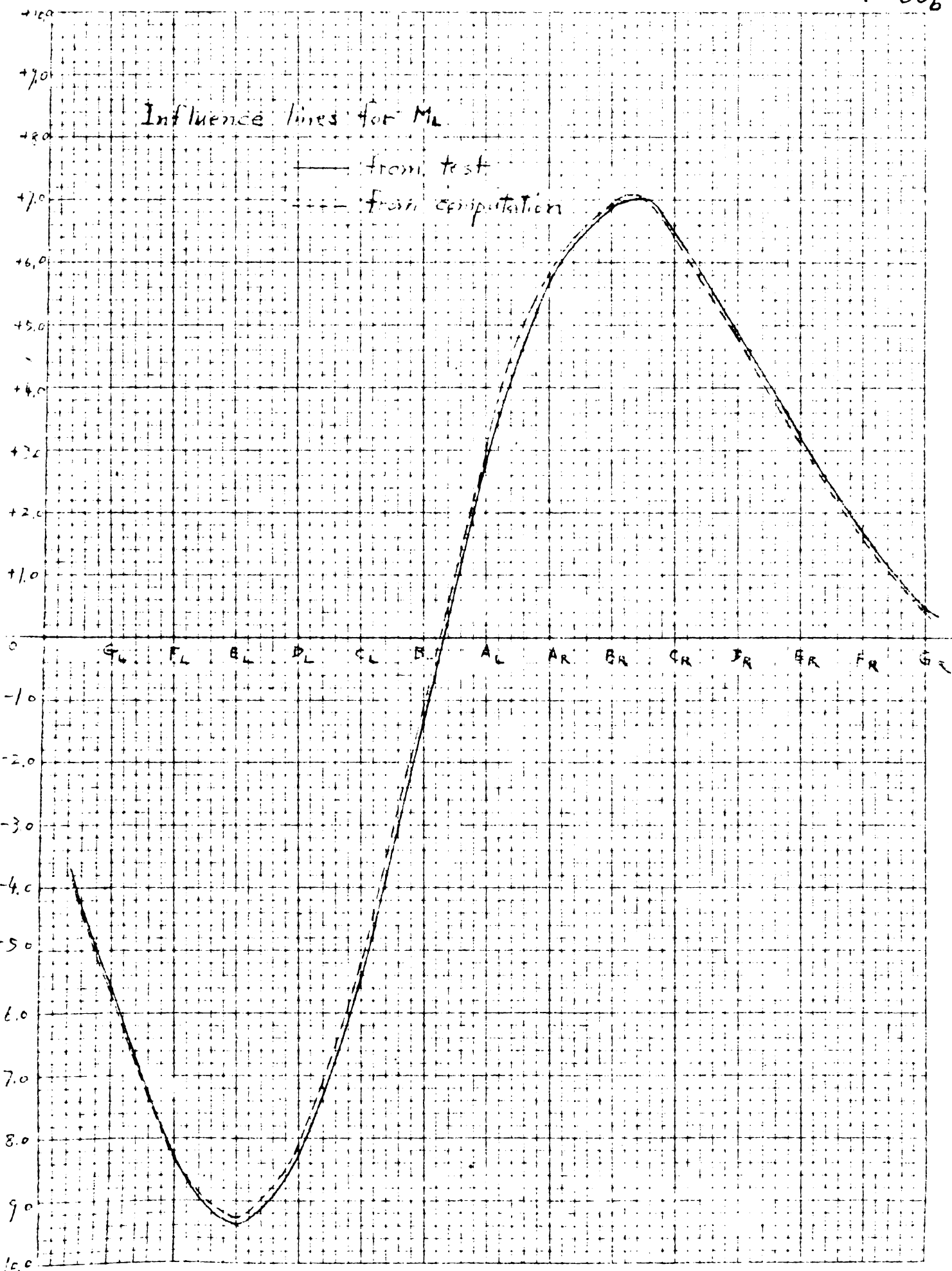
The final average values for the impact of the 1994 spring are calculated below:

Table 1 (continued)

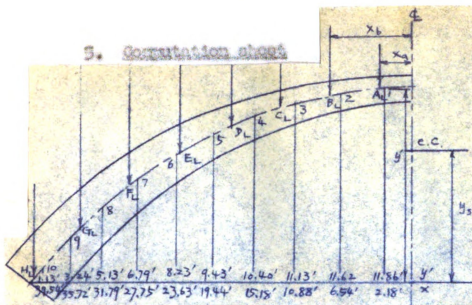
Local location	1994 (1993)	Local location	1994 (1993)
Δ_1	+1.50	Δ_1	+5.00
Δ_2	-2.50	Δ_2	+1.00
Δ_3	-5.40	Δ_3	+1.54
Δ_4	-3.20	Δ_4	+1.07
Δ_5	-0.57	Δ_5	+3.32
Δ_6	-0.00	Δ_6	+1.70
Δ_7	-0.50	Δ_7	+1.50

The influence lines obtained for the 1994 results are plotted on the graphs of the original data.





5. Computation sheet



$H_0 = 3.0754$
 $H_1 = 3.0291$
 $H_2 = 13.3315$
 $H_3 = 10.7346$
 $H_4 = 24.0073$
 $H_5 = 32.4461$
 $H_6 = 34.7920$
 $H_7 = 40.1455$

sec	y'	t _n	$\frac{t_n^3}{12}$	0.0453 $(t_n - 0.25)^2$	I	1/I	$\frac{t_n^4}{4}$
1	11.0517	1.71	0.41970	0.00773	0.51431	1.9445	33.0626
2	11.6183	1.78	0.46993	0.10731	0.57719	1.7305	30.1205
3	11.1320	1.91	0.58066	0.12321	0.70007	1.4217	15.7300
4	10.4610	2.11	0.78303	0.15005	0.94133	1.0634	11.6457
5	9.4350	2.34	1.00774	0.20006	1.20730	0.7900	7.4403
6	8.2220	2.66	1.30042	0.25302	1.63444	0.5451	4.4058
7	6.7900	3.06	2.30772	0.32165	2.74037	0.3637	2.4607
8	5.1807	3.51	3.00363	0.40376	4.09039	0.2445	1.2534
9	3.3203	3.99	5.03343	0.50030	5.60031	0.1713	0.5581
10	1.1333	4.53	7.74564	0.63050	8.58562	0.1165	0.1320
						0.5043	35.3313

$$x_0 = \frac{\sum \frac{t_n^4}{4}}{\sum \frac{t_n^3}{12}} = \frac{31.7313}{8.3563} = 3.8098$$

$$A = t_n + (n-1) \frac{(2.40)}{12} = t_n + \frac{2.40(n-1)}{12} =$$

$$t_n + 0.19333$$

$$I = \frac{1}{12} t_n^3 + 0.19333 \left(\frac{t_n^3}{3} - \frac{t_n^2}{2} \right)$$

$$= \frac{t_n^3}{12} + 0.04533 (t_n - 0.25)^2$$

Table 3-11-104

no.	x	y	z	$2/z$	y^2/z	x^2/z
1	1.12115	+1.5135	1.0135	0.9862	4.7131	0.01140
2	6.55224	+1.5022	1.1025	0.9063	1.0075	74.10000
3	10.97750	+0.0343	0.0325	0.7787	0.0005	127.50402
4	15.12012	+0.1040	1.0313	0.4331	0.0117	224.00775
5	19.45750	+0.0030	0.5035	0.3985	0.0017	387.00000
6	23.95221	-0.0072	1.0755	0.3117	1.5054	564.15553
7	27.71315	-0.0052	0.2455	0.8055	4.4725	800.13000
8	31.79017	-0.1055	0.1055	0.8705	0.0034	107.01175
9	35.71050	-7.0075	4.1755	0.7555	0.5010	127.01105
10	39.94455	-0.1550	4.7155	0.2110	0.0714	152.14127
Σ				5.7502	11.0015	2027.00041

TABLE 3-11-105

no.	$z \cdot (1.0135)$ $-(1.0135)$	$(1.0135)^2$	$(1.0135)^2$	$(1.0135)^2$
1	5.0005	1.0135	45.0	0.0
2	0.1011	11.4001	101.1	0.7
3	10.0045	15.1004	101.7	1.4
4	10.7401	13.1057	105.0	-11.4
5	00.0007	11.4032	100.0	-20.6
6	11.0737	9.1020	107.1	-32.0
7	01.1103	7.1107	101.1	-36.0
8	13.0401	5.0001	100.1	-40.2
9	17.0102	1.0011	101.1	-12.4
Σ		80.4505	1751.1	-153.4



TABLE 10-10 at $\bar{X}_1$

no.	$\frac{(X_1 - \bar{X}_1)}{s_1}$	$(X_1 - \bar{X}_1) \frac{1}{f}$	$(X_1 - \bar{X}_1) \frac{f}{f}$	$(X_1 - \bar{X}_1) \frac{f}{f}$
3	0.0004	4.0006	40.0	3.4
4	7.2526	7.2570	112.5	0.0
5	11.2574	8.2570	175.0	-7.0
6	15.4050	8.0000	161.0	-17.0
7	19.7140	7.2572	112.1	-23.2
8	23.0775	5.0001	100.0	-30.0
9	27.0704	4.7577	170.5	-33.7
10	31.0256	1.0002	111.2	-32.2
		Σ 50.0004	1004.4	-343.0

TABLE 10-11 at $\bar{X}_2$

4	1.7500	1.0010	80.0	0.0
5	6.0540	4.7577	90.0	-4.1
6	10.2500	5.0075	130.0	-11.0
7	14.5712	5.0070	140.1	-10.5
8	18.4040	4.0000	140.0	-13.3
9	22.3400	3.0000	137.0	-17.0
10	26.1247	1.0015	110.1	-21.0
		Σ 30.0005	720.0	-115.0

TABLE 10-12 at $\bar{X}_3$

5	0.7019	0.0035	20.8	-0.3
6	4.0075	2.0000	60.1	-3.5
7	9.0100	3.0000	61.0	-11.5
8	13.0512	3.1000	101.4	-10.5
9	16.0070	2.0077	100.0	-20.7
10	20.0100	1.0000	50.0	-20.0
		Σ 15.0001	400.0	-70.0

Unit Load at F_L

Loc.	$\frac{M_L - (M_L + M_R)}{2(L_1 - L_2)}$	$(M_L + M_R) \frac{1}{I}$	$(M_L - M_R) \frac{x}{I}$	$(M_L + M_R) \frac{z}{I}$
7	3.6158	1.3333	37.0	-4.7
8	7.6594	1.8823	59.8	-9.7
9	11.5352	2.0052	71.6	-14.3
10	15.4073	<u>1.6004</u>	<u>73.2</u>	<u>-16.5</u>
		Σ 7.0212	239.6	-45.1

Unit Load at F_L

8	2.3466	0.5737	18.2	-3.0
9	6.2824	1.0827	36.7	-7.6
10	10.1045	<u>1.1712</u>	<u>44.5</u>	<u>-10.8</u>
		Σ 2.8333	103.4	-21.4

Unit Load at G_L

9	0.9297	0.1602	5.7	-1.1
10	4.7318	<u>0.5335</u>	<u>21.2</u>	<u>-5.1</u>
		Σ 0.7137	27.6	-6.2

$$H_0 = \frac{\sum \frac{(M_L + M_R)}{I}}{2 \sum \frac{1}{I}} ; \quad V_0 = \frac{\sum \frac{(M_L - M_R)x}{I}}{\sum \frac{M_L^2}{I}} ;$$

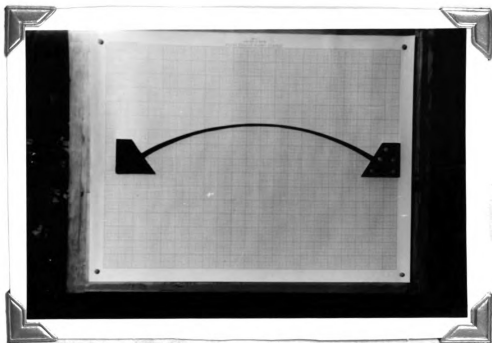
$$H_0 = \frac{\sum \frac{(M_L + M_R)z}{I}}{2 \sum \frac{M_L^2}{I} + 2 \sum \frac{1}{I}}$$

Influence Table

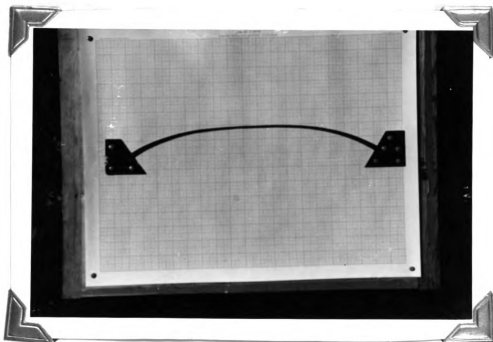
Unit Load at

	δ_1	δ_2	δ_3	δ_4
M_0	4.9157	3.0146	1.7833	0.8978
R_0	1.8832	1.6945	1.3519	0.9880
$V_0 = V_R$	0.4319	0.3045	0.1973	0.1151
V_L	0.3581	0.6955	0.8057	0.8349
$10.296M_0$	16.7717	16.5200	12.8897	8.0544
$41.411V_0$	17.8854	12.6089	8.1702	4.7878
$\sum M_0 + 10.296M_0 + 41.411V_0$	41.5738	39.1488	22.7831	14.4994
$-RL$	-33.7346	-33.3019	-28.0291	-22.6754
M_0 left	2.8392	-1.2391	-5.2459	-2.1770
M_0 right	3.8030	0.9263	6.4428	4.5632
	L	δ_L	δ_L	
M_0	0.4187	0.1979	0.0426	
R_0	0.3832	0.2388	0.0592	
$V_0 = V_R$	0.0301	0.0255	0.0018	
V_L	0.9409	0.9749	0.9932	
$10.296M_0$	3.1812	2.4584	0.7123	
$41.411V_0$	2.4473	1.0351	0.2819	
$\sum M_0 + 10.296M_0 + 41.411V_0$	0.0472	2.5153	1.0340	
$-RL$	-17.3497	-11.9709	-5.6182	
M_0 left	-9.2765	-8.2074	-5.5814	
M_0 right	3.1526	1.5713	0.4730	

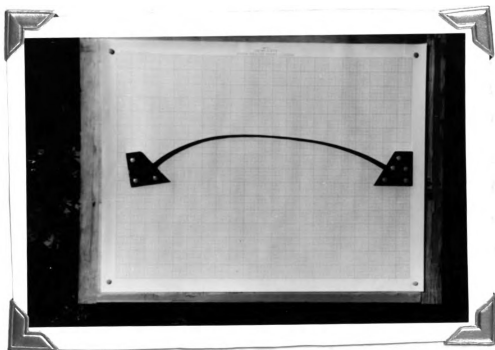
Problem 5



Model before deformation



Deformation of model to find left end thrust



Deformation of model to find left end moment

JAN 30 1964

VII. Summary

It was from the book "Methods of Change" by Hollander and Ploner that we got the idea of so-called cyclical analysis of stress analysis and we were so impressed by the fundamental principles involved in this method that we were very anxious to introduce experiment with our animals to see its practical value. Since this subject is relatively new in the structure engineering field, so reference books are available, and we were prevented from trying to do some actual experiment, simply because we did not realize it is possible to apply this method to the most general structural problem without any high cost instrument. Also as the book "The Experimental Study of Structures" by L.S. Liggett, also brought our idea of utilizing the fundamental principles to actual problems in a relatively simple way without the need in general and in the case of some, due to the fact that the method is simple.

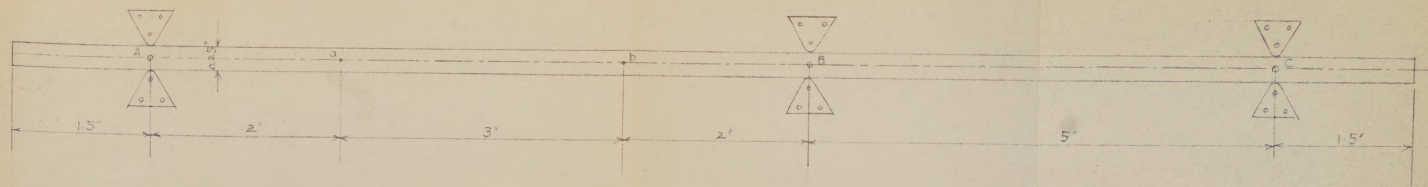
During every step of our work, we have to make, cutting, measuring, and loading of the model, we met more or less trouble and find every possible mistake and then satisfactory. The details described in this book are the final methods adopted, some of them have been changed or improved several times from the very beginning to the final stages.

There are certainly some unfavorable factors entering in this method which will affect the final results, such as

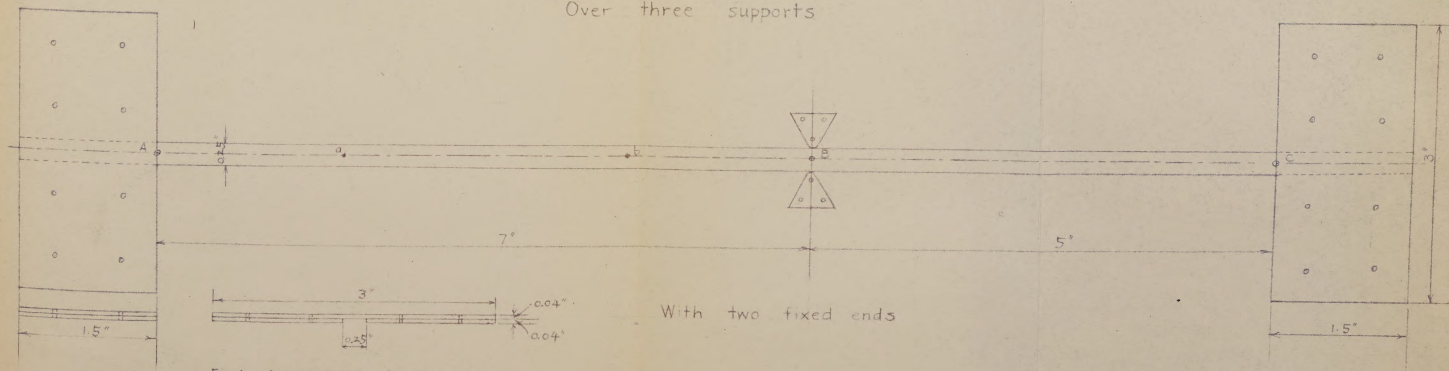
the intensity of noise, the level, is affected in orientation and in its tendency to be constant, and causes a considerable error in measurement. The least squares gives a definite amount of error while the other two give varying amounts depending upon each and every test made.

As shown, by testing, that each is more sensitive to every introduced moment, and the closer and greater of any error may be very large. This can be seen from the fact that results which vary each slightly with each other at different settings of α etc. These indirect measurements indicate that one is not justified for analyzing this type of problem.

In general of this, the values obtained by this method are sufficiently close to those obtained by other methods and may be applied to real and in this condition, problems or used freely to obtain a quick check on the calculations of a very important structural design.



Over three supports



With two fixed ends

End view of the fix end device

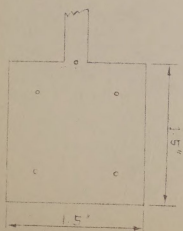
CONTINUOUS BEAMS
MODEL DIMENSIONS

FIX END BENT

MODEL DIMENSIONS

LENGTH: 1" = 1'

Enlargement at
Column Ends for
Fixed Condition



Scale: $1'' = 4'$

RIGID FRAME BRIDGE

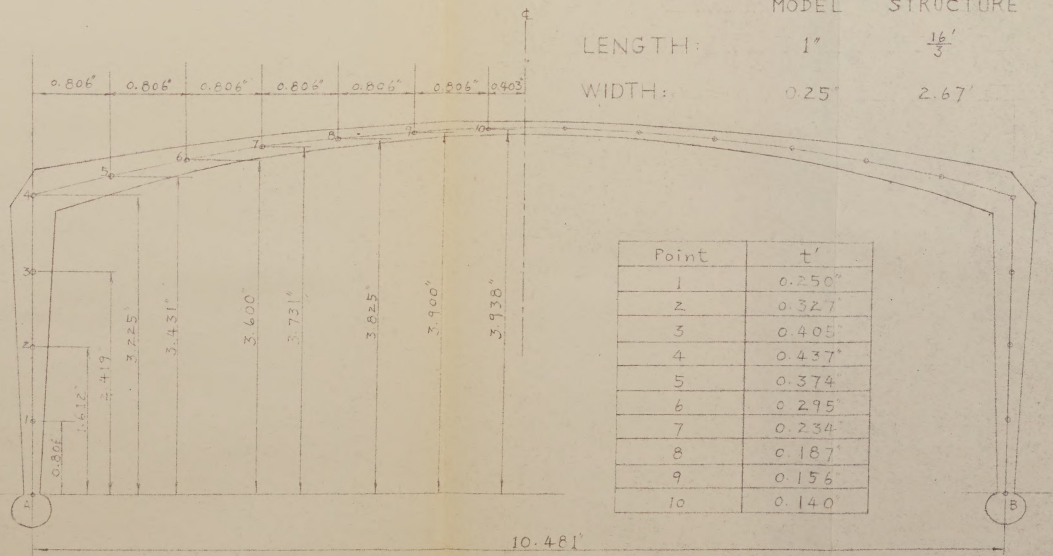
MODEL DIMENSION

PROPORTION

MODEL STRUCTURE

LENGTH: 1' $\frac{16'}{3}$

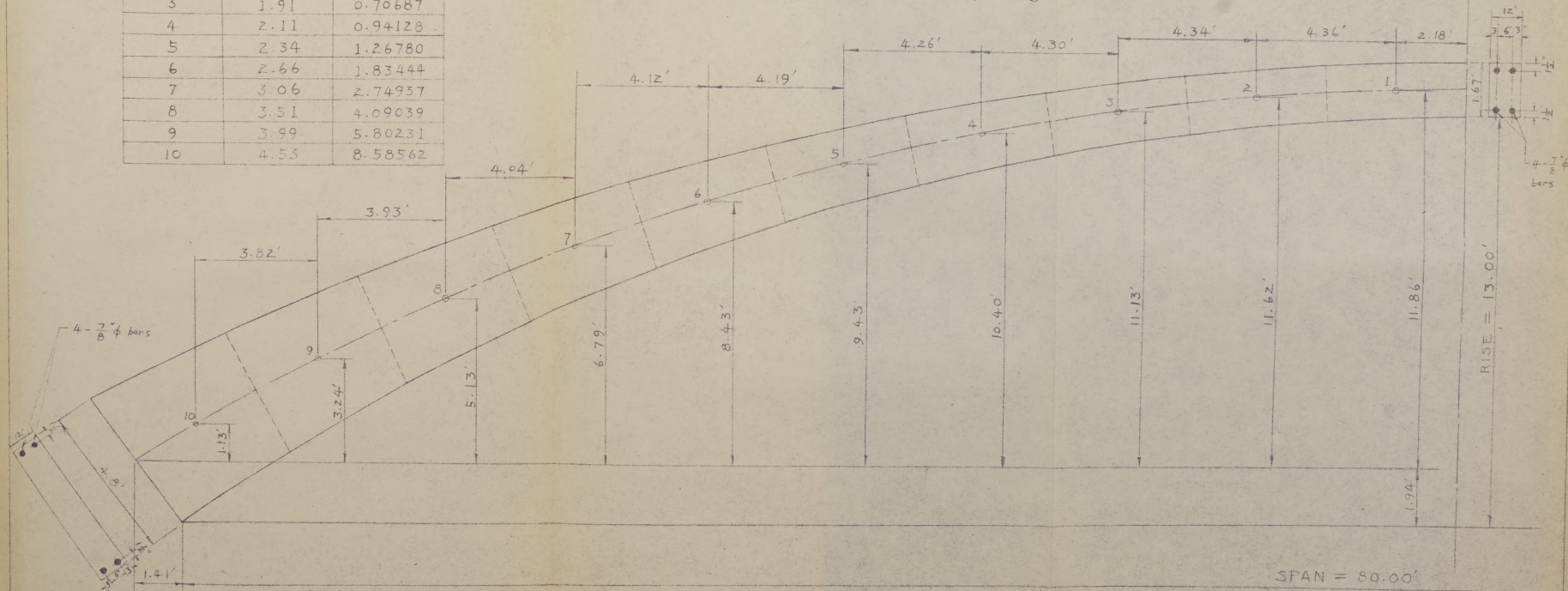
WIDTH: 0.25' 2.67'



FIX END ARCH

Scale : 1" = 2.5'

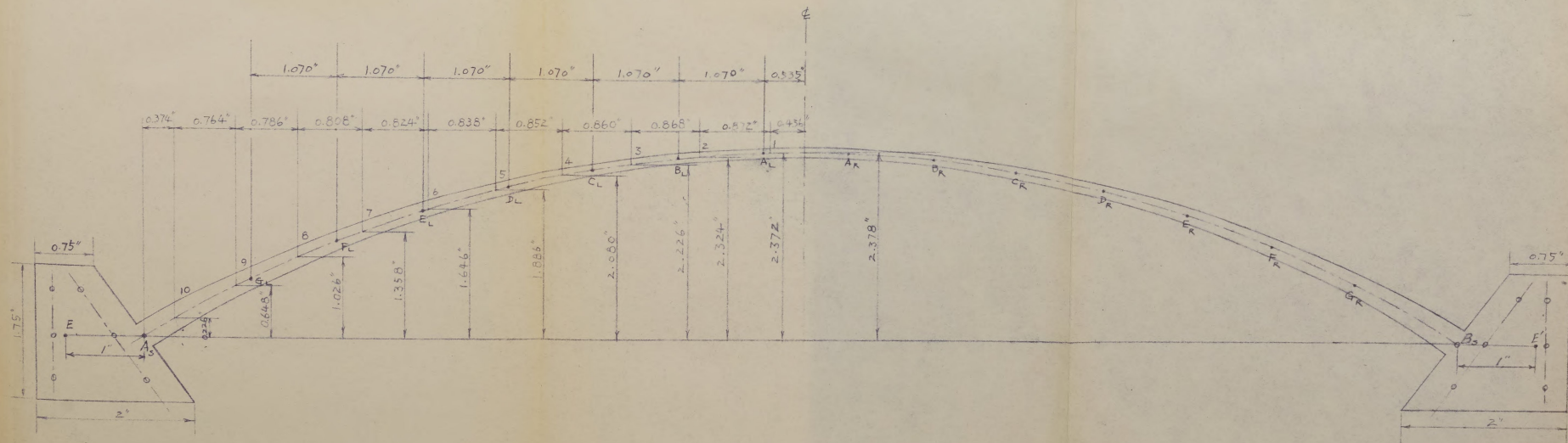
Point	t (ft)	I (ft ⁴)
1	1.71	0.51431
2	1.78	0.57719
3	1.91	0.70687
4	2.11	0.94128
5	2.34	1.26780
6	2.66	1.83444
7	3.06	2.74957
8	3.51	4.09039
9	3.99	5.80231
10	4.53	8.58562

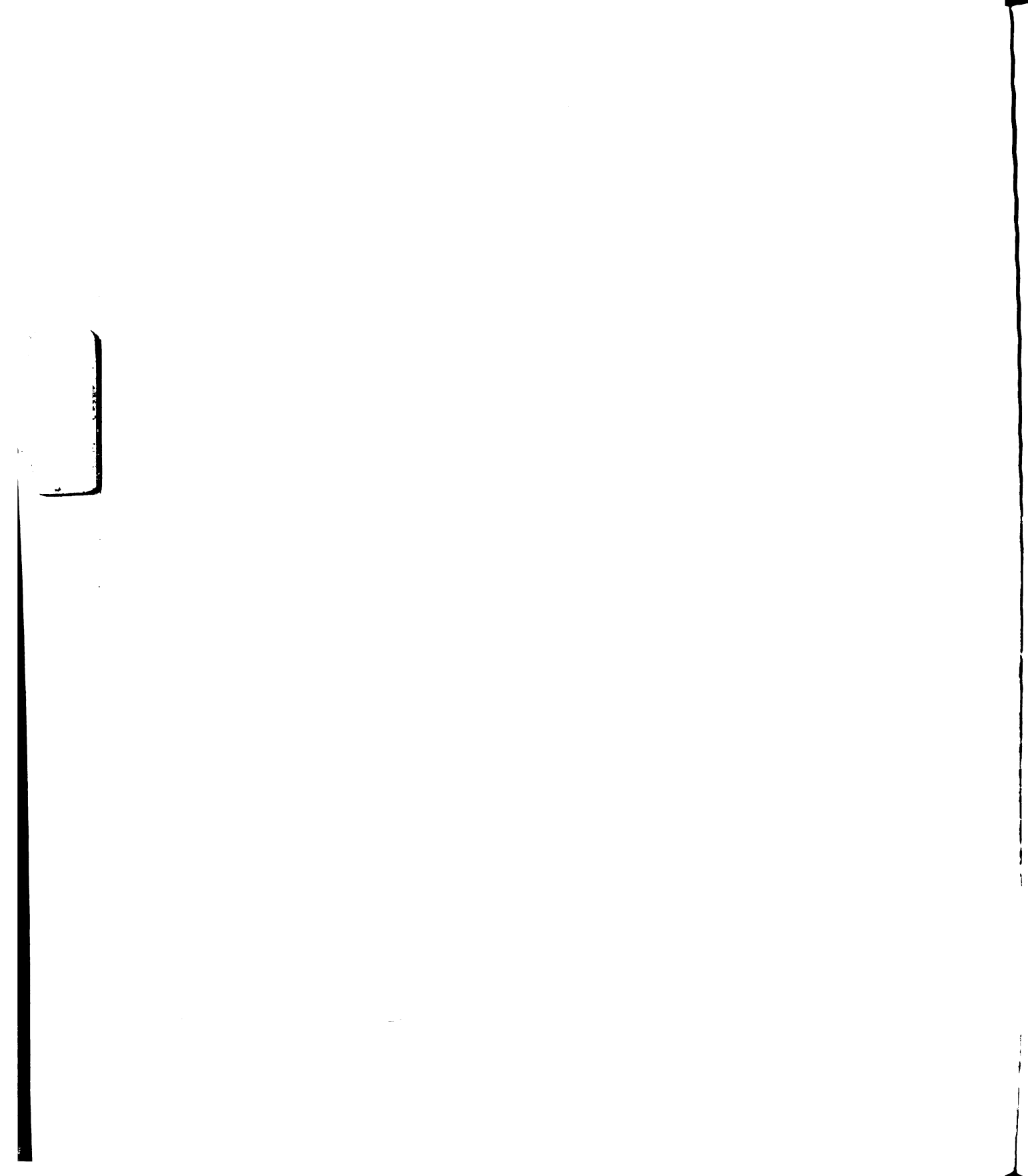


FIX END ARCH

MODEL DIMENSIONS

Point	Crown	1	2	3	4	5	6	7	8	9	10	Springing
Thickness	0.130"	0.134"	0.139"	0.145"	0.163"	0.180"	0.204"	0.234"	0.267"	0.299"	0.341"	0.361"





ROOM USE ONLY.

Jul 3 '52

Jul 3 '52

ROOM USE ONLY.

Oct 11 '54