

SIMULATION AND OPEN-LOOP CONTROL
OF A COCURRENT DRYER

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WAYNE HOWARD CLIFFORD
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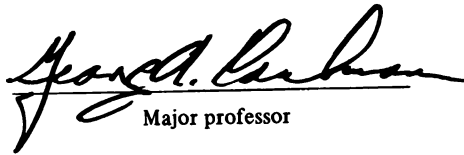
SIMULATION AND OPEN-LOOP CONTROL
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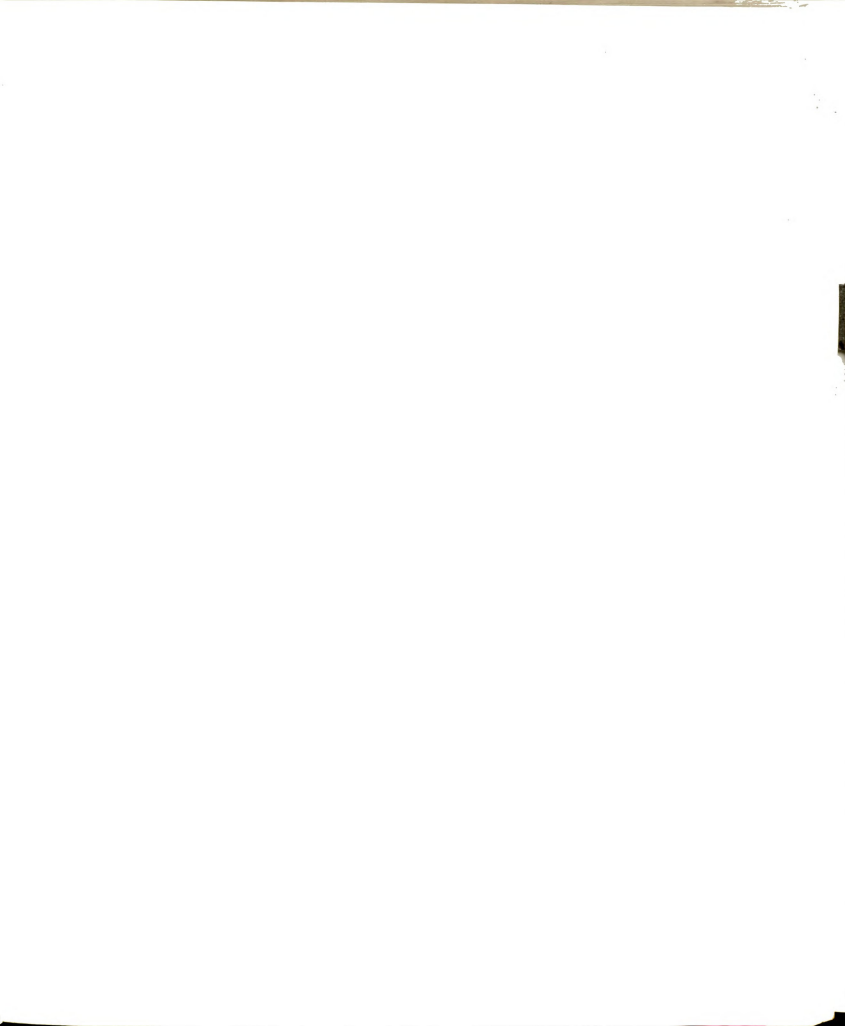
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ABSTRACT

SIMULATION AND OPEN-LOOP CONTROL OF A COCURRENT DRYER

By

Wayne Howard Clifford

The dynamic simulation of a cocurrent moving bed dryer for shelled corn involves at least three independent variables: time, distance along the bed, and whatever is used to model the moisture gradient within an individual corn kernel. The first problem is to simulate a thin layer of drying corn--the mathematical equivalent of simulating a single kernel. In this case the diffusivity of water in corn is available as a function of average moisture content and average temperature of the kernel. Also the Biot number is small enough so that temperature gradients within the kernel may be neglected. The model divides the kernel into three concentric regions with the same dry mass in each region. Geometric parameters in the model were fitted from experimental thin-layer drying data, using a hill-climbing parameter optimization program.

The parameter optimization requires that the model be solved many times, so a highly efficient integration

routine is important in order that the cost of the optimization not be prohibitive. A method is presented where a set of a few nonlinear differential equations is solved by linearizing and solving the linearized equations by Laplace transforms. For accuracy the linearization must be done at the center of the interval, but in practice it is done at the front and end and the average of the two is used. Iterations are needed since the values at the end of the interval are not known.

Once the geometric parameters have been found, the convective heat and mass transfer coefficients are needed. The steady-state equations corresponding to the thin layer model were derived and solved. This solution made use of the same technique as the thin-layer solution. Parameter optimization was again used, this time to find the convective heat and mass transfer coefficients.

The dynamic corn dryer model consists of 6 nonlinear partial differential equations. The backward difference approximation was used for the time partials, and then the equations were solved by the same technique used above. The dynamic solution was used to evaluate the net cost during the transition from one steady-state to another, corresponding to changing from one batch of corn to another at 5% higher moisture. The goal was to find the best input air temperature function during the transition. Several one- and two-parameter functions were

Wayne Howard Clifford

tested and optimized in terms of their own parameters.

Since the best control found was not significantly better than a single-step (or bang-bang) control, the latter would be the best to use under current normal operational conditions.

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By

Wayne Howard Clifford

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LIST OF SYMBOLS

A or A ₃	area of corn kernel in ft ² /ft ³ , 239.0 at ε = 0.5.
A ₁ , A ₂	area enclosing regions 1 or 2 in ft ² /ft ³
A _i '	ratio of area to Δr, ft
C _{pc} , C _{pa}	heat capacity of corn, air, Btu/lb of dry matter/°F
D	diffusivity of water inside the corn kernel, ft ² /hr
D _w	diffusivity of water in air, ft ² /hr
D _p	equivalent particle diameter, ft
G _c , G _a	mass flow rate of corn, air, lb of dry matter/hr/ft ²
H	absolute humidity of air, lb of water/lb of dry air
h	heat transfer coefficient, Btu/hr/ft ² /°F
h _m '	mass transfer coefficient based on moisture content differences, lb of water/hr/ft ²
h _{fg}	heat of vaporization of water, Btu/lb
k	thermal conductivity, Btu/hr/ft/°F
K _h	ratio of heat to mass transfer for coefficients, 16°F/2tu
K _G	proportionality constant in Equation 44
M	dry basis moisture fraction, lb of water/lb of dry corn
M ₁ , M ₂ , M ₃	average moisture content over region 1, 2, 3
M _e	moisture content in equilibrium with air
R _H	relative humidity

Δr	radial distance inside the corn kernel, ft
S	Laplace Transform variable
T_c, T_a	temperature of corn, air, °F
t	time, hours
U	overall mass transfer coefficient, lb of water/hr/ft ²
V	volume, ft ³
z	length along the dryer, ft
α_1, α_2	ratio of area to Δr for region 1, 2
ϵ	void fraction of the bed
ρ_c	dry basis particle density of corn, 72 lb/ft ³
ρ_a	density of air, lb of dry air/ft ³
μ	viscosity, lb/sec/ft

INTRODUCTION

As mankind moves toward the last quarter of the twentieth century, the message of R. Buckminster Fuller,⁷ ". . . the physical resources of earth can support all of a multiplying humanity at higher standards of living than anyone has ever experienced or dreamed," presents both an optimistic outlook for the future and the largest challenge ever to our technology: "So it is also part of the great message to humanity . . . that the world's problems cannot be solved by politics and can only be solved by a physical invention-and-design revolution."⁷

Mr. Fuller even mentions part of the method of approach. "We find that man is developing an increasing confidence in the way in which computers are resolving heretofore vexing and seemingly unsolvable problems."⁷

This work represents the author's concept of the next steps to be taken so that computers may be used to bring about one part of that invention-and-design revolution.

*Footnote numbers refer to Bibliography entries.

A cocurrent corn dryer was chosen for this research for several reasons. Corn is a very important raw material for various foods; therefore any method of increasing the supply, such as improving methods used in its preservation, has humanitarian as well as economic value. Most optimal control work has involved linear systems, so a nonlinear system was sought. In addition, relatively little control work has been done with distributed systems such as tubular reactors or moving bed dryers. Also, most optimal control uses the minimum time to reach steady state, a minimum amount of control action, or the time integral of the squared error from some desired value as the objective function, and all of these seem rather artificial. On the other hand, a straight economic objective function can easily and accurately be applied to a corn dryer. Finally, the strong interest in corn drying over the years has produced a wealth of basic technology.

The author feels that the following are important contributions in this thesis. First, a new thin layer model is presented based on a different way of looking at the corn kernel. Next, this model is used with some steady state data to find heat and mass transfer coefficients for the deep bed dryer. The corresponding dynamic deep bed dryer equations are derived and solved to find a practical optimum open loop control. In addition, a highly efficient method of solving sets of a few differential equations is presented and demonstrated.

LITERATURE REVIEW

The most significant factor in any simulation of corn drying is the method used to treat the drying of a single kernel of corn. Single kernel drying is also known as thin-layer drying since in a thin layer, each kernel will be exposed to the same drying air. Many reviews are available for such work, such as Bakker-Arkema and Lerew.¹ Two approaches are used in single particle models: single-lump models and moisture-gradient models.

In single-lump models the results of thin-layer drying tests are summarized in empirical expressions. Troeger and Hukill¹³ express the drying rate as a function of equilibrium moisture content (which is a function of air temperature and humidity), temperature, initial moisture content, current moisture content, and air velocity. Thompson et al.¹¹ use an empirical expression to calculate the time required for a certain amount of drying as an explicit function of all the same above parameters except air velocity. In each case the empirical constants found in the model have very little physical meaning.

A moisture gradient model with results like a single-lump model was used by Chittenden and Hustrulid.⁵

They used a perfectly spherical model with no surface resistance and a constant moisture diffusivity. Under those assumptions, a closed form solution can be found for moisture content versus time, involving an infinite series of exponentials.* Their main conclusion was that the diffusivity is not constant.

The work of Chu and Hustrulid⁶ represents an advancement in that the requirement of a constant diffusivity was dropped; however, the perfect sphere and no surface air resistance assumptions were kept. The diffusivity was then estimated from short-time drying results. This model is a pure internal moisture gradient type. However, it also uses a "dynamic equilibrium moisture content" and a "pseudo initial moisture content." While each of these is a plausible physical phenomenon and each is useful for making the data fit their model, they are both somewhat difficult to determine except in terms of a specific model. The "dynamic equilibrium moisture content" may arise from physical changes in the corn at different temperatures and moisture contents, perhaps related to the hysteresis effects found in repeated equilibrium moisture sorption isotherms. The "pseudo initial moisture content" is said to arise from

*This solution is then used in the same manner as the completely empirical models, but the constants have physical meaning, e.g. the equivalent spherical radius or diffusion coefficient.

the fact that the kernel is somewhat cooler than the drying air for the first part of the drying.

The work of Whitaker et al.,¹⁵ which does not involve corn directly, uses a variable diffusion coefficient on a spherical body with finite surface resistance. They divided the sphere into ten shells numerically and used a linear function of moisture content for the diffusion coefficient. This work was performed to prove the method before attempting to apply it to biological systems.

The most important work found on control of a cocurrent dryer for corn is that of Zachariah.¹⁶ He controlled the flow rate of corn in response to exit moisture content using four different control systems, and concluded that a proportional-integrated controller with on-off override would be best.

APPROACH TO THE PROBLEM

The goal of this work is to find a control scheme for a cocurrent corn dryer. In such a dryer, as is depicted in Figure 1, the wet corn and drying air enter at the same end. Although any configuration is possible, the flow generally is downward. An unloading mechanism is provided at the bottom to control the corn flow rate. Plug flow will be assumed for both corn and air, as well as perfect insulation at the sides, so there will be no gradients in planes orthogonal to the direction of flow and the cross-sectional shape will not be important.

The independent variables in a mathematical description of the dynamic operation of a deep bed corn dryer will be time and distance along the bed as well as whatever is used to model the moisture gradient inside the corn kernels. It is clear, then, that a thin-layer model must form the basis for any deep bed simulation. Once it is established, the extensions to a steady-state deep bed dryer and from there to the dynamic dryer are mostly straightforward.

The thin-layer model used here has three internal zones to determine the moisture gradient. Since each zone adds a differential equation to the steady-state

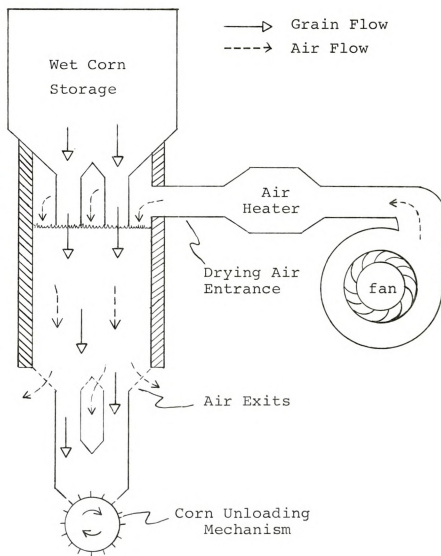


Figure 1.--Cocurrent corn dryer diagram.

simulation (and a partial differential equation to the dynamic simulation), the number of zones should be kept to an absolute minimum. It was decided that the diffusivity of Chu and Hustrulid⁶ would be used, but that it would be based on the average moisture content rather than the average between each pair of zones. The assumption of zero surface transport resistance was dropped, as was the spherical shape requirement. The resulting model has three shape parameters and the two outside transport coefficients, all of which must be determined by fitting experimental data.

The thermal conductivity in the kernel is high enough so the Biot number (hD_p/k , the ratio of internal to external resistance to heat transfer) is small. For this reason temperature gradients inside the kernel are neglected.

Since most chemical engineering processes are carried out in counter current fashion, a brief explanation of the use of a cocurrent dryer is in order. One benefit of cocurrent operation is the tempering effect of the end portion of the dryer, making control easier and the product more uniform. There are two drawbacks in countercurrent operation. The first is the tendency of a countercurrent dryer to heat the exit product. This can damage the starch so it cannot be used for certain applications. It also wastes energy since the corn should be

stored cool and so must be cooled in a subsequent operation and since the heat could be used to evaporate water. The other difficulty with countercurrent dryers is the tendency of the exit air to become saturated so that moisture is actually added to entering corn by condensation.

The method used here for solving sets of nonlinear ordinary differential equations is analogous to the improved Euler method, but the exact solution to the linearized set is used where a simple Euler step would be made. The set of nonlinear differential equations is linearized over short intervals and the exact solution to the linearized equations is used. The intervals, although short, are an order of magnitude longer than would be used for a corresponding numerical technique such as the predictor corrector. For good results the linearization must be done at the center of the interval. In practice it is done at both ends and the average value of each linearization coefficient is used. This requires that an estimate of the final values for the interval be available, so the method iterates until the values just calculated agree with their estimate. The value of this method is that it is both fast and accurate. The drawback is that it is complicated to program, requiring the "hand" calculation of the solution to the linearized equations. In addition to the thin-layer and dynamic dryer simulations described

here, a steady-state program was written and implemented. It ran about 50 times faster than the corresponding program using the Bulirsch-Stoer (4) method.

Control of a cocurrent distributed system is difficult due to the long time lags involved. For a corn dryer the air-side dynamics are very fast--generally 1000 or so times faster than the product dynamics--so that the air is very nearly at steady-state with respect to the instantaneous corn temperature and moisture distribution. Consider an attempt to read the exit corn moisture content continuously and use this to control the input air temperature. If the exit corn is too wet, the input air temperature must be raised. However this causes the corn near the entrance to dry faster, so the air near the exit becomes both cooler and more humid, making the exit corn wetter still. Lowering the input air temperature instead would be worse yet, since corn near the entrance would be dried less. When that corn reached the exit, the dryer would probably be incapable of recovering to properly dry it; certainly the control would be asking for still lower temperatures.

For the above reason an open loop control was used, where the input corn moisture content is used to control the input air temperature. It was assumed that a certain production rate is required, so corn flow is not manipulated. The input corn temperature has little

effect, since sensible heats are small compared to heats of vaporization. The air flow rate is determined by the fan and motor used and these are difficult to change. The only input variable left is the air temperature, which is easily controlled. The specific control problem is as follows: find the input air temperature as a function of time during the period when a new steady-state is achieved. It is thought that a given batch of corn would be fairly uniform in moisture content so a single input air temperature would achieve steady state drying for an economic optimum. The problem concerns the input air temperature over the time of transition from one batch to another; what is sought is the economic optimum.

The problem is well-posed once the simulation model is available. However, at this time optimal control theory has advanced only to the point where systems of order 10 or maybe 15 can be handled. The model would result in a system of 300 order if the dryer is divided into 50 length segments. In addition, this system is very highly nonlinear, while most of the optimal control work has been with linear systems. For this reason it is virtually impossible to apply current optimal control techniques; and, therefore, the results should not be termed an optimal control. The term "near-optimal" best describes the practical optimum which is actually found.

One possible solution would be to set up the input temperature profile as a parameter vector where each component is the temperature used in the numerical method at the time step in question. This would give a vector of about 150 elements. The excellent parameter optimization technique used in the thin-layer section takes about 50 function calls for each component, so 7500 calls would be required. Since the resulting program takes 5 minutes to carry out each approach to steady state when using the CDC 6500 computer, over 26 days straight would be required for this computation.

The more practical method used to find a near optimal control was to express the input temperature profile as a function of 2 parameters and find the optimum of those parameters.

NUMERICAL TREATMENT OF THE DIFFUSION
EQUATION IN APPROXIMATELY
SPHERICAL PARTICLES

The purpose of this section is to give the thin-layer method a stronger theoretical basis. Consider a particle which starts with a uniform moisture content and is treated in such a way that conditions at its surface vary only in time but are identical over the whole surface at any instant. Such a particle may be considered pseudo-spherical if there is a single point within, called the center, where the moisture gradients are zero in each direction ($\nabla M=0$), and if all net moisture transfer occurs in the radial direction. While corn may not fit these conditions exactly, one may assume that it comes close enough so that the results will apply.

The particle is divided into n^2 pyramid-shaped segments, each segment having angular widths $2\pi/n$, as is shown in Figure 2. Then each segment is divided into m zones in such a way that each zone within a segment contains the same amount of material. Note that n will tend to be large, while m is usually between 3 and 10. In what follows the first subscript refers to the segment,

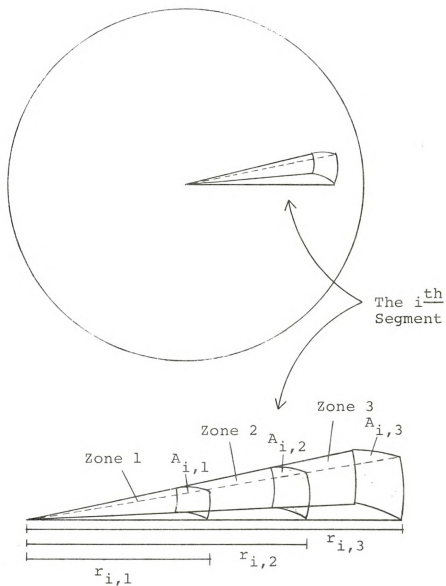


Figure 2.--Division of a spherical particle into pyramidal segments.

the second to the zone, so $r_{i,j}$ will be the outside radius of the j -th zone in the i -th segment, $\bar{M}_{i,j}$ will be the average moisture content, and $A_{i,j}$ will be the outside surface area of the same zone. As n is made larger the moisture moving between segments becomes negligible compared to that moving along a segment. Then each will behave as if it were part of a sphere of radius $r_{i,m}$. If D , the diffusivity, is independent of r then

$$\frac{\partial M_i}{\partial t} = \frac{D}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial M_i}{\partial r}) \quad (1)$$

The average moisture content of the j -th zone will be:

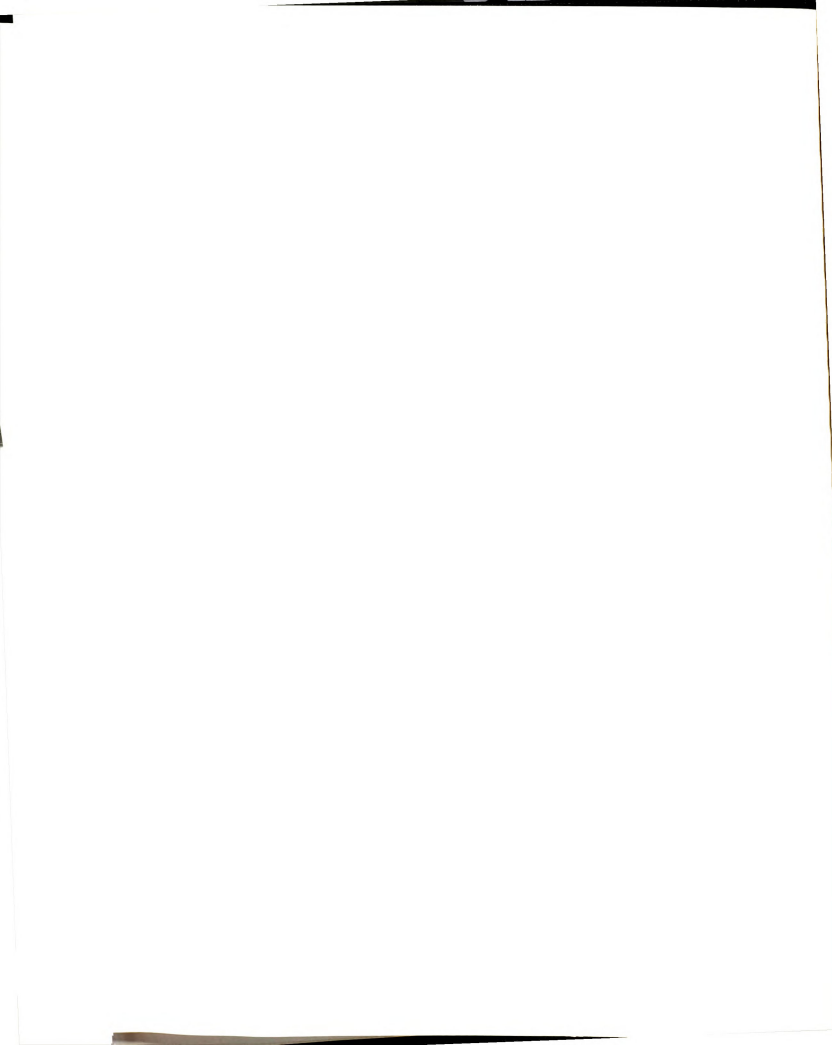
$$\bar{M}_{i,j} = \frac{1}{V_{i,j}} \int_{r_{i,j-1}}^{r_{i,j}} M_i \cdot \frac{4\pi r^2}{n^2} dr \quad (2)$$

where $V_{i,j} = \frac{4\pi}{3n^2} (r_{i,j}^3 - r_{i,j-1}^3)$, the volume of the j -th zone in the i -th segment. Of course, $r_{i,0} = 0$ for all i .

Next each side of equation 2 is differentiated with respect to time.

$$\frac{d\bar{M}_{i,j}}{dt} = \frac{1}{V_{i,j}} \int_{r_{i,j-1}}^{r_{i,j}} \frac{4\pi r^2}{n^2} \left(\frac{\partial M_i}{\partial t} \right) dr \quad (3)$$

But the $\partial M_i / \partial t$ is given by equation 1, so on substitution



Consider the case where there is a bulk outside moisture content, M_e , which is the equilibrium moisture content of the product at the temperature and humidity of the bulk air flowing past the product. In addition, there is a convective mass transfer coefficient, h'_m . The subscript m is used to denote a mass transfer coefficient and the prime is used to differentiate between this coefficient which is based on moisture content differences and h_m , the one more commonly used, which is based on humidity differences.

Then the rate of mass transfer per unit area is given by:

$$D\rho_C \left(\frac{\partial M_i}{\partial r} \right)_{r_{i,m}} = h'_m (M_e - M_{i,s}) = \frac{D\rho_C}{\Delta r_{i,m}} (M_{i,s} - M_{i,m}) \quad (9)$$

where $D\rho_C$ is the inside moisture conductivity and $M_{i,s}$ is the moisture content at the surface. After some algebraic manipulation, Equation 9 yields

$$\left(\frac{\partial M_i}{\partial r} \right)_{r_{i,m}} = \frac{h'_m}{D\rho_C + h'_m \Delta r_{i,m}} (M_e - \bar{M}_{i,m}) \quad (10)$$

where $\Delta r_{i,m}$ is the distance between the surface and the point where $M_i = \bar{M}_{i,m}$. Substituting this into Equation 5 results in

$$\frac{d\bar{M}_{i,j}}{dt} = \frac{1}{V_{i,j}} \int_{r_{i,j-1}}^{r_{i,j}} \frac{4\pi r^2}{n^2} \left\{ \frac{D}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \bar{M}_i}{\partial r} \right) dr \right\} \quad (4)$$

Carrying out the indicated integration

$$\frac{d\bar{M}_{i,j}}{dt} = \frac{4\pi D}{n^2 V_{i,j}} \left\{ \left(r^2 \frac{\partial \bar{M}_i}{\partial r} \right)_{r_{i,j}} - \left(r^2 \frac{\partial \bar{M}_i}{\partial r} \right)_{r_{i,j-1}} \right\} \quad (5)$$

Next $r_{i,j}'$ is defined as the radial center of mass of its zone:

$$r_{i,j}' = \left(\frac{r_{i,j}^3 + r_{i,j-1}^3}{2} \right)^{1/3} \quad (6)$$

Then $r_{i,j}'$ is the best available estimate of the point where $M_i = \bar{M}_{i,j}$, and is used to construct a divided difference form of the partial:

$$\left(\frac{\partial \bar{M}_i}{\partial r} \right)_{r_{i,j}} = \frac{\bar{M}_{i,j+1} - \bar{M}_{i,j}}{r_{i,j+1} - r_{i,j}'}, \quad j = 1, 2 \dots m-1 \quad (7)$$

Noting that $A_{i,j} = 4\pi r_{i,j}^2 / n^2$, and substituting into Equation 5 there results

$$\frac{d\bar{M}_{i,j}}{dt} = \frac{D}{V_{i,j}} \left\{ A_{i,j} \frac{\bar{M}_{i,j+1} - \bar{M}_{i,j}}{r_{i,j+1} - r_{i,j}'} - A_{i,j-1} \frac{\bar{M}_{i,j} - \bar{M}_{i,j-1}}{r_{i,j}' - r_{i,j-1}'} \right\} \quad (8)$$

for $j = 1, 2 \dots m-1$

Consider the case where there is a bulk outside moisture content, M_e , which is the equilibrium moisture content of the product at the temperature and humidity of the bulk air flowing past the product. In addition, there is a convective mass transfer coefficient, h'_m . The subscript m is used to denote a mass transfer coefficient and the prime is used to differentiate between this coefficient which is based on moisture content differences and h_m , the one more commonly used, which is based on humidity differences.

Then the rate of mass transfer per unit area is given by:

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where $D\rho_C$ is the inside moisture conductivity and $M_{i,s}$ is the moisture content at the surface. After some algebraic manipulation, Equation 9 yields

$$\left(\frac{\partial M_i}{\partial r} \right)_{r_{i,m}} = \frac{h'_m}{D\rho_C + h'_m \Delta r_{i,m}} (M_e - \bar{M}_{i,m}) \quad (10)$$

where $\Delta r_{i,m}$ is the distance between the surface and the point where $M_i = \bar{M}_{i,m}$. Substituting this into Equation 5 results in

$$\frac{d\bar{M}_{i,m}}{dt} = \frac{1}{V_{i,j}} \left\{ \frac{h_m^* DA_{i,m}}{D_{PC} + h_m^* \Delta x_{i,m}} (M_e - \bar{M}_{i,m}) - DA_{i,m-1} \frac{\bar{M}_{i,m} - \bar{M}_{i,m-1}}{r_{i,m} - r_{i,m-1}} \right\} \quad (11)$$

The quantities of interest will be the m average moisture contents throughout all n^2 j -th zones. This is denoted by dropping the first subscript.

$$\bar{M}_j = \frac{\sum_{i=1}^{n^2} V_{i,j} \cdot \bar{M}_{i,j}}{\sum_{i=1}^{n^2} V_{i,j}} \quad (12)$$

But since each segment was divided into m zones of equal

volume, $\sum_{i=1}^{n^2} V_{i,j} = V/m$, then

$$\frac{V}{m} \bar{M}_j = \sum_{i=1}^{n^2} V_{i,j} \cdot \bar{M}_{i,j} \quad (13)$$

and, by differentiating

$$\frac{V}{m} \frac{d\bar{M}_j}{dt} = \sum_{i=1}^{n^2} (V_{i,j} \frac{d\bar{M}_{i,j}}{dt}) \quad (14)$$

For $j = 1, 2 \dots m-1$, the result from Equation 8 is

$$\frac{d\bar{M}_j}{dt} = \frac{mD}{V} \{A'_j (M_{j+1} - M_j) - A'_{j-1} (M_j - M_{j-1})\} \quad (15)$$

where

$$A'_j = \frac{n^2}{\sum_{i=1}^n} \frac{A_{i,j}}{r_{i,j+1} - r_{i,j}}, \quad j = 1, 2, \dots, m-1. \quad (16)$$

And for $j = m$,

$$\frac{d\bar{M}_m}{dt} = \frac{m}{V} \cdot \left\{ \frac{Dh_m^2 A}{D\rho_C + h_m^2 \Delta r_m} (M_e - \bar{M}_m) - DA'_{m-1} (\bar{M}_m - \bar{M}_{m-1}) \right\} \quad (17)$$

where A is the total outside area and

$$\Delta r_m = \frac{A}{h_m^2 \sum_{i=1}^n \frac{A_{i,m}}{D\rho_C + h_m^2 \Delta r_{i,m}}} - \frac{D\rho_C}{h_m} \quad (18)$$

The result of the above is that the effects of particle geometry on diffusion are captured in m parameters, $A'_1, A'_2, \dots, A'_{m-1}$, and Δr_m . Of course, it is not practical to determine these parameters directly. Instead, they are estimated by fitting the model to experimental thin-layer drying data.

Another way of looking at this model of the corn kernel is as in Figure 3. Here the kernel is divided into three concentric layers with the same dry mass in each

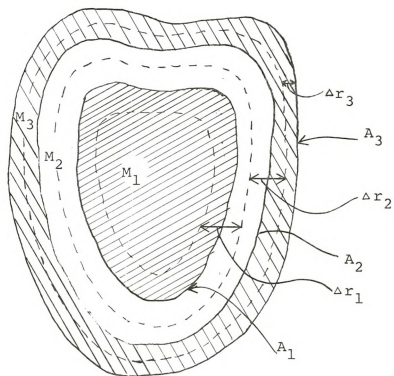


Figure 3.--Locating diagram for thin-layer model parameters.

layer, and with iso-moisture content surfaces between the layers. The solid lines in the figure denote these surfaces. Then within each layer there is another iso-moisture content surface where the moisture content is the mass average for that layer, denoted by dotted lines in the figure. The Δr values will be some kind of average separation between the average moisture content surfaces in the two layers. The same set of geometric parameters will result.

A THIN-LAYER MODEL FOR SHELLED CORN

Using the techniques of the previous section, the corn kernel is divided into three concentric spherical layers with the same dry mass in each layer. To aid in visualizing the process and for ease in applying the results to deep bed work, a calculation basis of a cubic foot of corn kernels with a void fraction of 0.5 is used.

Heat and mass balances inside the corn yield the following equations:

$$\frac{dM_1}{dt} = \frac{3A_1'}{V} D (M_2 - M_1) \quad (19)$$

$$\frac{dM_2}{dt} = \frac{3A_2'}{V} D (M_3 - M_2) - \frac{3A_1' D}{V} (M_2 - M_1) \quad (20)$$

$$\frac{dM_3}{dt} = \frac{3A h_m' D}{V(D\rho_c + h_m' \Delta r_3)} (M_e - M_3) - \frac{3A_2' D}{V} (M_3 - M_2) \quad (21)$$

$$\frac{dT_c}{dt} = \frac{h_{fg} A h_m' D}{C_{p_c} V (D\rho_c + h_m' \Delta r_3)} (M_e - M_3) - \frac{h A}{\rho_c C_{p_c} V} (T_c - T_a) \quad (22)$$

In the above set of equations the value of A/V is taken from Bakker-Arkema et al.,² namely 239 ft^{-1} . Chu and Hustrulid⁶ provide a formula for D as a function of $\bar{M}$ and T_c .

$$D = 1.629 \times 10^{-3} \text{ EXP } \{ (0.025 T_c + 6.008) \bar{M} - \frac{4523.4}{T_c + 459.7} \} \quad (23)$$

where T_c is in $^{\circ}\text{F}$, and $\bar{M}$ is the average dry basis moisture fraction. Three other quantities are assumed constant, $\rho = 72 \text{ lb of dry matter per cubic foot}$, $C_{p_c} = 0.6 \text{ Btu per } ^{\circ}\text{F per pound of dry matter}$, and $h_{fg} = 1040 \text{ Btu per pound of water}$. It is understood of course that the heat capacity is a function of moisture content. However, it was decided that the extra accuracy in using the function would not offset the difficulties encountered by the added nonlinearity in the system. For this reason all heat capacities here and in the deep bed sections are "stream" values, based on average moisture contents and pounds of dry matter in each stream. This value of C_{p_c} corresponds to $M = 0.2081$.

The unknowns are A_1'/V , A_2'/V , Δr_3 , h_m' , and h , the heat transfer coefficient. Although estimates of h_m' and h are available, the use of such estimates would mean that any error in the estimate would color the determination of the three geometric parameters.

The five unknowns were determined by fitting solutions of the differential equations to three sets of experimental data in a weighted least squares sense. The hill climbing method of parameter optimization of Rosenbrock⁹ was used to minimize the function

$$E = \sum t [\bar{M}_{\text{EXP}}(t) - \bar{M}_{\text{CALC}}(t)]^2 \quad (24)$$

The squared error at each point was multiplied (or weighted) by the time to that point because the moisture content changes fastest early in the drying and because errors in measuring time are relatively less significant at longer times.

Since each evaluation of the error represents 36 hours of system time (3 runs at 12 hours each) and a 5-parameter optimization normally requires over 200 trials, it is important that the solution be carried out as efficiently as possible. The equations are "almost" linear, so a Laplace transform may be used over short time intervals when the coefficients (diffusivity actually) are the average of the endpoints.

As may be seen in Figure 4, a front or starting point diffusivity is calculated from the known temperature and average moisture content at the start of the time interval. Next the moisture content at the end of the interval is estimated. This estimate need not be



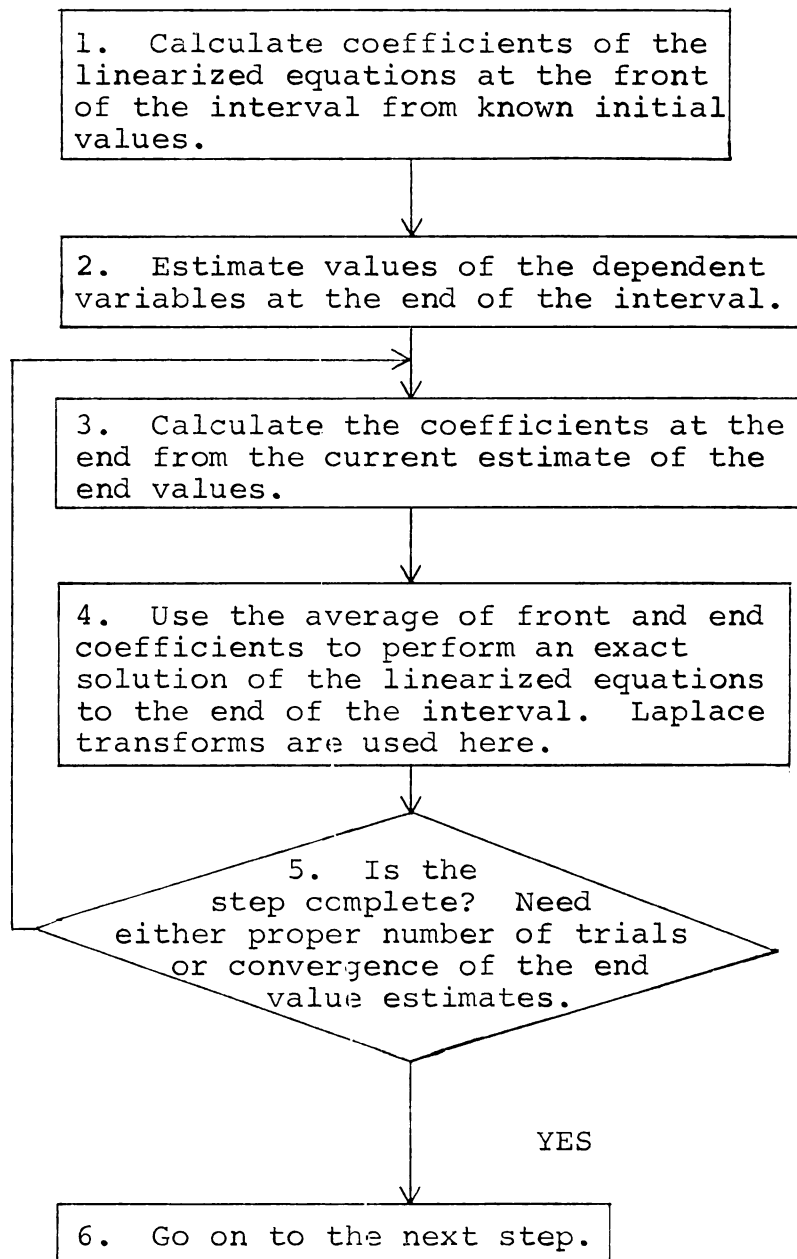


Figure 4. Flow chart of the method used for single-step integration for initial value problems.

particularly good, in fact, the values at the front may be used. The one used here was

$$M_{i, t+\Delta t} = M_{i, t} - 0.2 \Delta t (M_{i, t} - M_e). \quad (25)$$

The end diffusivity is calculated from this estimate and the coefficients are calculated from the average of the front and end diffusivities. Next, a single step is made. This provides much better values for the end moisture content and temperature, from which a much better end diffusivity is calculated. A new average diffusivity provides new coefficients so a second step may be made over the same interval. This loop is continued until it converges. For these runs the second and third trials always agreed within $\pm 10^{-5}$ moisture fraction.

The following substitutions were made to simplify the Laplace transform solution.

$$\begin{aligned} a &= \frac{3A_1 D}{V} & b &= \frac{3A_2 D}{V} & \alpha &= a + b \\ c &= \frac{3A}{V} \frac{D h_m'}{D\rho_c + h_m' \Delta r_3} & \beta &= c + b & (26) \\ d &= \frac{h_{fg}}{3C_{pc}} \cdot c & \delta &= \frac{h A}{C_{pc} V} \end{aligned}$$

So the equations are

$$\frac{dM_1}{dt} = a M_2 - a M_1 \quad (27)$$

$$\frac{dM_2}{dt} = b M_3 - \alpha M_2 + a M_1 \quad (28)$$

$$\frac{dM_3}{dt} = c M_e - \beta M_3 + b M_2 \quad (29)$$

$$\frac{dT}{dt} = d M_e - d M_3 - \delta T + \delta T_a \quad (30)$$

A bar over a variable will denote its Laplace transform, a superscript ° will denote value at the front of the interval, and S will be the transform variable. Then the transformed equations are

$$S\bar{M}_1 - M_1^\circ = a\bar{M}_2 - a\bar{M}_1 \quad (31)$$

$$S\bar{M}_2 - M_2^\circ = b\bar{M}_3 - \alpha\bar{M}_2 + a\bar{M}_1 \quad (32)$$

$$S\bar{M}_3 - M_3^\circ = \frac{cM_e}{S} - \beta\bar{M}_3 + b\bar{M}_2 \quad (33)$$

$$S\bar{T} - T^\circ = \frac{dM_e + \delta T_a}{S} - d\bar{M}_3 - S\bar{T} \quad (34)$$

Solving these (Cramer's rule works rather well)

$$\bar{M}_1 = \frac{S [(S+\alpha)(S+\beta) - b^2] M_1^0 + S(S+\beta) aM_2^0 + ab(SM_3^0 + CM_e)}{S [(S+a)(S+\alpha)(S+\beta) - b^2(S+a) - a^2(S+\beta)]} \quad (35)$$

$$\bar{M}_2 = \frac{aS(S+\beta) M_1^0 + S(S+a)(S+\beta) M_2^0 + b(S+a)(SM_3^0 + CM_e)}{S [(S+A)(S+\alpha)(S+\beta) - b^2(S+a) - a^2(S+\beta)]} \quad (36)$$

$$\bar{M}_3 = \frac{abSM_1^0 + bS(S+a) M_2^0 + [(S+a)(S+\alpha) - a^2] (SM_3^0 + CM_e)}{S [(S+a)(S+\alpha)(S+\beta) - b^2(S+a) - a^2(S+\beta)]} \quad (37)$$

$$T = \frac{-d \{abSM^0 + bS(S+a) M^0 + [(S+a)(S+\alpha) - a^2] (SM^0 + CM_e)\} + [(S+a)(S+\alpha)(S+\beta) - a^2(S+\beta) - b^2(S+a)] (ST^0 + dM_e + ST_\alpha)}{S(S+\delta) [(S+a)(S+\alpha)(S+\beta) - b^2(S+a) - a^2(S+\beta)]} \quad (38)$$

Writing the Heaviside expansion theorem in this notation,

If $U(S) = \frac{P(S)}{Q(S)}$ where $Q(S) = (S-a_1)(S-a_2) \dots (S-a_m)$

and $P(S)$ is a polynomial of degree $m-1$ or less

$$\text{Then } U(t) = \sum_{i=1}^m \frac{P(a_i)}{\left(\frac{dQ}{dS}\right)_{a_i}} e^{a_i t}$$

So the only difficulty in the solution was finding the roots of the polynomial

$$(S + a) (S + \alpha) (S + \beta) - b^2 (S + a) - a^2 (S + \beta).$$

It should be clear that these roots are all real and negative, so a special subroutine was written to solve a general cubic with all real roots.

The programs used to perform the integrations and the optimization are shown in Appendix A. Experimental thin layer drying data taken by D. G. Elzinga were thus fitted to the model. The results are shown in Table 1 and a representative set of results is plotted in Figure 5. The drying was carried out with air at 98°F and 50% relative humidity. The air flow rate was not determined, but was quite high. The resulting values were:

$$A_1'/V = 12.58 \times 10^4 \text{ ft}^{-2}$$

$$A_2'/V = 9.46 \times 10^4 \text{ ft}^{-2}$$

$$\Delta r_3 = 3.931 \times 10^{-3} \text{ ft}$$

$$h = 12.33 \text{ Btu/hr/ft}^2/\text{°F}$$

$$h_m' = 1.193 \text{ lb of water/hr/ft}^2.$$

There are two problems with the data of Elzinga. First, the whole set was taken with the same drying air temperature. Second, the air flow rate was not recorded. In order to give the model a broader foundation eleven

TABLE 1.--Thin-layer data* and simulation average moisture contents as a function of time.

Time, hours	Set 1		Set 2		Set 3	
	EXP	SIM	EXP	SIM	EXP	SIM
0	.3605	.3605	.3377	.3377	.3813	.3813
0.5	.2863	.2916	.2712	.2805	.2895	.3007
1	.2619	.2582	.2493	.2510	.2649	.2639
2	.2243	.2195	.2143	.2155	.2263	.2226
3	.2003	.1965	.1929	.1938	.2017	.1984
4	.1810	.1806	.1747	.1787	.1817	.1821
5	.1712	.1690	na	.1675	.1720	.1701
6	.1607	.1600	.1555	.1589	.1612	.1608
8	.1454	.1470	.1421	.1463	.1464	.1476
12	.1330	.1318	.1312	.1314	.1339	.1321

* The data were taken by Mr. Daniel G. Elzinga of the Michigan State University Department of Agricultural Engineering on September 11, 1969.

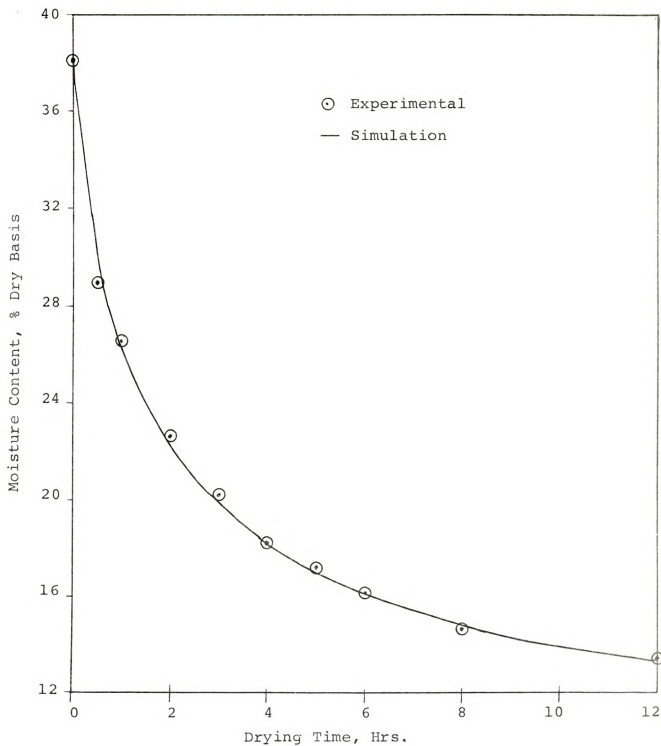


Figure 5.--Thin-layer moisture contents as a function of time for data set #3.

selected runs from the data of Troeger¹² were also used. Three of these were at about 90°F, five at about 125°F, and three more at about 160°F. All were at an air velocity of 160 ft/sec.

In checking the Troeger data with his thin layer model, errors of up to 1.6% were found. Also, his raw data could not be used since they were not taken at the same times for each run. Therefore, a correction factor was applied to the model assuming that the error would be a linear function of time between the data points.

The first action taken with this data was to fit it in the same manner as the Elzinga data, both grouped by temperature and with all eleven runs together. The results by temperature group varied a great deal from one group to the other, not only in geometric parameters, but also in the convective coefficients. For all eleven together the result was a very poor fit. Finally, the geometric parameters found from the Elzinga data were used and only the convective transfer coefficients were adjusted to fit the set of data at each temperature. This gave a better fit of the data.

The results of fitting the Troeger data are shown in Tables 2, 3, and 4, with selected runs plotted along with their raw data in Figure 6.

TABLE 2.--Thin-layer moisture contents as a function of time for low temperature Troeger data.*

Time, min.	Run #1404310		Run #1404410		Run #1504310	
	Sim	Troeger	Sim	Troeger	Sim	Troeger
0	.3458	.3548	.4276	.4276	.3481	.3481
10	.3462	.3418	.4128	.4074	.3409	.3350
20	.3393	.3309	.4031	.3939	.3348	.3239
30	.3330	.3217	.3950	.3825	.3292	.3158
40	.3273	.3146	.3876	.3729	.3240	.3091
50	.3219	.3080	.3807	.3645	.3191	.3033
70	.3116	.2961	.3677	.3496	.3099	.2928
90	.3021	.2857	.3555	.3364	.3012	.2848
110	.2931	.2768	.3440	.3245	.2930	.2766
130	.2845	.2691	.3329	.3139	.2853	.2694
150	.2764	.2614	.3224	.3043	.2780	.2629
200	.2578	.2452	.2984	.2831	.2613	.2490
250	.2415	.2314	.2772	.2653	.2467	.2375
300	.2272	.2200	.2586	.2503	.2339	.2274
350	.2145	.2104	.2422	.2375	.2228	.2187
400	.2034	.2014	.2278	.2253	.2130	.2114
500	.1850	.1870	.2039	.2065	.1970	.1987
600	.1706	.1757	.1854	.1909	.1846	.1889
700	.1591	.1667	.1709	.1790	.1750	.1811
800	.1500	.1594	.1594	.1701	.1675	.1748
900	.1427	.1535	.1502	.1626	.1615	.1696

*Using $h = 2.054$, $h_m' = .1002$.

TABLE 3.--Thin-layer moisture contents as a function of time for medium temperature Troeger data.*

Time, min.	Run #2304410		Run #2514310		Run #2514410		Run #2314430		Run #2314440	
	Sim	Troeger	Sim	Troeger	Sim	Troeger	Sim	Troeger	Sim	Troeger
0	.4183	.4183	.3612	.3612	.4246	.4246	.4004	.4004	.4003	.4003
4	.4103	.4090	.3571	.3570	.4173	.4177	.3934	.3903	.3933	.3904
8	.4031	.4001	.3528	.3531	.4106	.4111	.3869	.3805	.3868	.3808
12	.3964	.3916	.3483	.3468	.4044	.4047	.3806	.3712	.3806	.3717
16	.3900	.3838	.3439	.3408	.3984	.3985	.3747	.3622	.3746	.3629
20	.3838	.3766	.3395	.3351	.3926	.3925	.3689	.3553	.3688	.3551
28	.3721	.3632	.3309	.3243	.3816	.3814	.3578	.3424	.3577	.3419
36	.3609	.3513	.3226	.3157	.3711	.3720	.3473	.3305	.3472	.3298
44	.3502	.3409	.3147	.3086	.3610	.3632	.3372	.3196	.3371	.3186
52	.3400	.3312	.3070	.3021	.3514	.3551	.3275	.3095	.3275	.3084
60	.3302	.3219	.2997	.2956	.3422	.3476	.3183	.3001	.3182	.2990
80	.3075	.3018	.2829	.2811	.3208	.3303	.2968	.2781	.2968	.2778
100	.2870	.2816	.2678	.2678	.3017	.3145	.2775	.2587	.2775	.2592
120	.2686	.2652	.2543	.2562	.2845	.2994	.2601	.2428	.2601	.2435
140	.2521	.2501	.2423	.2457	.2693	.2857	.2446	.2302	.2445	.2308
160	.2373	.2361	.2315	.2360	.2557	.2734	.2306	.2173	.2305	.2184
200	.2120	.2123	.2133	.2193	.2327	.2522	.2066	.1966	.2066	.1977
240	.1915	.1934	.1987	.2054	.2143	.2345	.1871	.1791	.1871	.1808
280	.1747	.1777	.1868	.1942	.1995	.2186	.1711	.1652	.1711	.1672
320	.1609	.1650	.1772	.1844	.1875	.2057	.1578	.1536	.1578	.1561
360	.1494	.1548	.1692	.1762	.1777	.1949	.1468	.1438	.1467	.1467

*Using $h = 4.43$, $h'_m = .1955$

TABLE 4.--Thin-layer moisture contents as a function of time for high temperature Troeger data.*

Time, min.	Run #3204310		Run #3204410		Run #3404320	
	Sim	Troeger	Sim	Troeger	Sim	Troeger
0	.3630	.3630	.4314	.4314	.3665	.3665
2.33	.3591	.3535	.4244	.4201	.3629	.3600
4.67	.3541	.3443	.4166	.4092	.3581	.3538
7.0	.3483	.3354	.4084	.3986	.3527	.3478
9.33	.3422	.3272	.4002	.3882	.3468	.3421
11.67	.3359	.3197	.3921	.3782	.3408	.3367
16.33	.3235	.3053	.3764	.3606	.3288	.3266
21.0	.3115	.2917	.3615	.3441	.3172	.3156
25.67	.3002	.2798	.3474	.3286	.3063	.3045
30.33	.2895	.2703	.3341	.3150	.2959	.2941
35.0	.2794	.2614	.3215	.3033	.2862	.2844
46.67	.2565	.2420	.2931	.2773	.2641	.2626
58.33	.2367	.2259	.2684	.2563	.2451	.2439
70	.2195	.2115	.2471	.2386	.2286	.2293
81.67	.2045	.1995	.2285	.2234	.2143	.2169
93.33	.1913	.1890	.2124	.2097	.2018	.2044
116.67	.1695	.1696	.1859	.1867	.1812	.1856
140.0	.1523	.1565	.1652	.1698	.1651	.1699
163.33	.1384	.1448	.1488	.1555	.1522	.1575
186.67	.1269	.1355	.1356	.1439	.1417	.1471
210.0	.1174	.1275	.1246	.1338	.1331	.1382

*Using $h = 9.623$, $h_m' = 0.4188$.

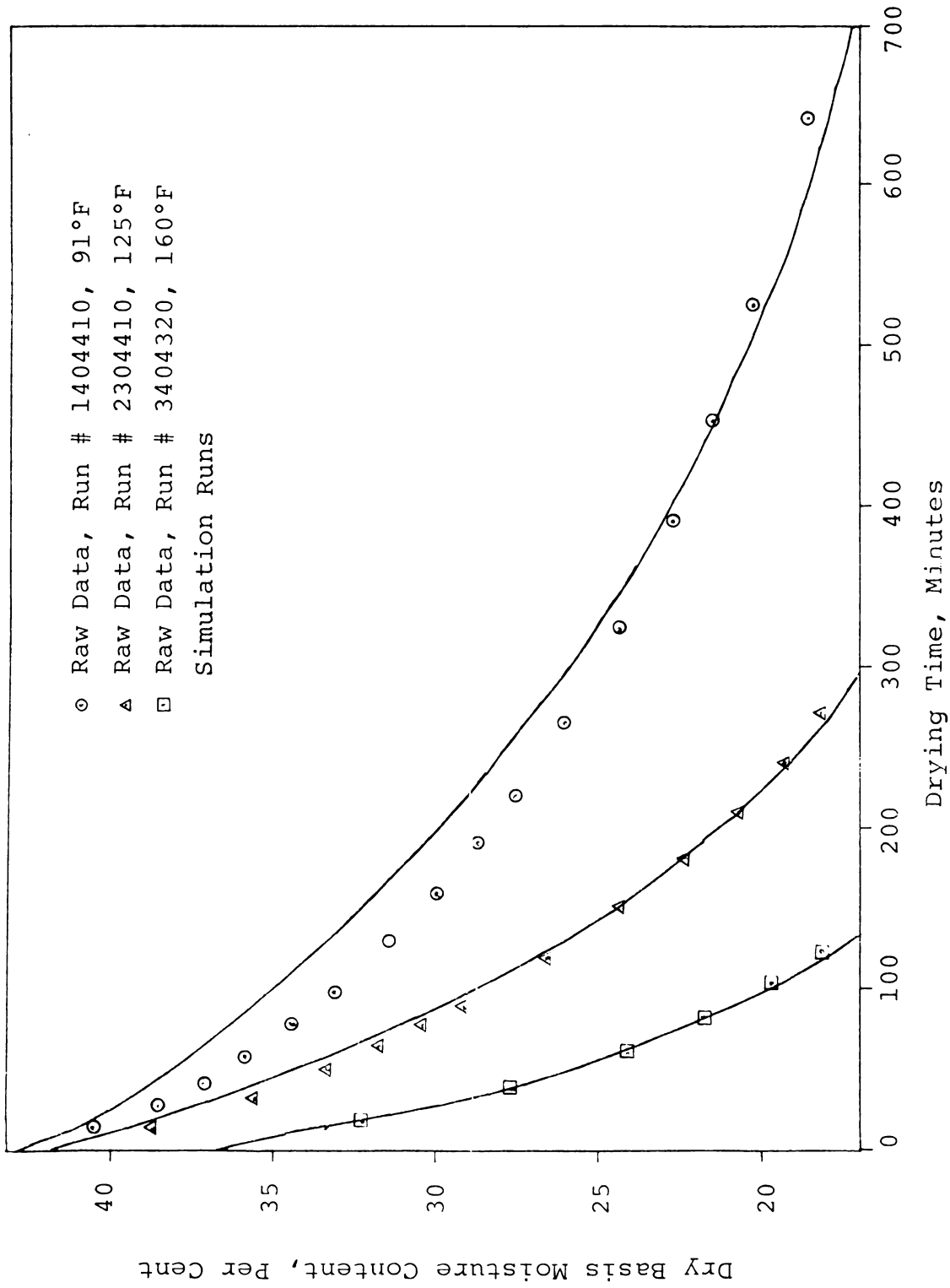


Figure 6.--Variation of moisture content with time for selected sets of Troeger data.

STEADY-STATE SIMULATION AND CONVECTIVE
HEAT AND MASS TRANSFER COEFFICIENTS
IN THE DEEP BED

Once a thin-layer model is established the dynamic equations for a deep bed may be derived assuming plug flow of both air and corn and no kernel to kernel heat or mass transfer. This is done in the next section. The steady-state equations are found by setting the time partial derivatives to zero and making the distance partial derivatives into total derivatives. This results in the set of equations shown in Table 5. Generally input conditions are known and the problem will be to determine temperatures, moisture contents and the humidity along the bed. For a cocurrent dryer the input conditions may be located at $z = 0$, making the simulation an initial value problem of six non-linear ordinary differential equations.

This set of equations is similar to the thin-layer set, except that air temperature and humidity equations are added. The same technique was used to solve them. Since it is illustrated in both the thin-layer section and the dynamic simulation section, it will not be repeated here. The program to do this is found in SUBROUTINE MBCDS in Appendix B.

TABLE 5.--Differential equations for a steady-state deep bed corn dryer.

$$\frac{dM_1}{dz} = -\frac{3 D \alpha_1 \rho_c}{G_c} (M_1 - M_2)$$

$$\frac{dM_2}{dz} = \frac{3 D \alpha_1 \rho_c}{G_c} (M_1 - M_2) - \frac{3 D \alpha_2 \rho_c}{G_c} (M_2 - M_3)$$

$$U = \frac{D \rho_c h'_m}{D \rho_c + h'_m \Delta r_3}, \quad M_e = \sqrt{\frac{\ln (1-R_H)}{-0.382 (T_a + 50)}}$$

$$\frac{dM_3}{dz} = \frac{3 D \alpha_2 \rho_c}{G_c} (M_2 - M_3) - \frac{3UA}{G_c} (M_3 - M_e)$$

$$\frac{dT_c}{dz} = \frac{h A}{G_c C_{pc}} (T_a - T_c) - \frac{UA h_{fg}}{G_c C_{pa}} (M_3 - M_e)$$

$$\frac{dH}{dz} = \frac{UA}{G_a} (M_3 - M_e)$$

$$\frac{dT_a}{dz} = -\frac{hA}{G_a C_{pa}} (T_a - T_c)$$

$$D = 1.629 \times 10^{-3} \text{ EXP } \{ (0.025T_c + 6.008)\bar{M} - \frac{4523.4}{T_c + 459.7} \}$$

$$R_H = \frac{H \cdot P_{TOT}}{(H + 0.622) \cdot P_{sat}}, \quad \bar{M} = \frac{M_1 + M_2 + M_3}{3}$$

The only things needed at this point are values for the convective heat and mass transfer coefficients in the deep bed. A great deal of work has been done on heat and mass transfer in packed beds, as may be seen in the review of Barker.³ The work which most closely parallels a corn dryer is by Gamson et al.⁸ and Wilke and Hougen,¹⁴ since they involve drying. There are two results from these papers. First, a relationship has been found for the heat transfer coefficient by dimensional analysis:

$$h = 1.064 C_p G \left(\frac{C_p \mu}{k}\right)^{-2/3} \left(\frac{D_p G}{\mu}\right)^{-0.41}, \frac{D_p G}{\mu} > 350 \quad (39)$$

$$h = 1.95 C_p G \left(\frac{C_p \mu}{k}\right)^{-2/3} \left(\frac{D_p G}{\mu}\right)^{-0.51}, \frac{D_p G}{\mu} < 350 \quad (40)$$

Using the following constants at 120°F

$$C_p = 0.24 \text{ Btu/lb}^\circ\text{F}$$

$$\mu = 0.0461 \text{ lb/hr ft}$$

$$D_p = 0.0322 \text{ ft, from Bakker et al. }^2$$

$$\frac{C_p \mu}{k} = 0.735$$

and for G in lb/hr ft^2 , these formulas yield



$$h = 0.363 G^{.59}, \quad G > 500 \quad (41)$$

$$h = 0.690 G^{.49}, \quad G < 500 \quad (42)$$

This last formula will be of most use, since G will usually be less than 500. Roughly speaking, the top formula corresponds to turbulent flow and the second to the transition region. If a continuous function is required, then a G of 615.78 should be used as the break point instead of 500. For G less than 60, corresponding to laminar flow, the heat transfer coefficient should be constant at its value for $G = 60$.

Under the assumption that the log mean partial pressure of air in the boundary layer, the mean molecular weight of the air water mixture and the Schmidt number $\frac{\mu}{\rho_a D_w}$ are all constant, the mass transfer coefficient should be directly proportional to the heat transfer coefficient, so

$$h'_m = K_h \cdot h \quad (43)$$

Then K_h may be estimated from the thin-layer results. The various values of K_h determined from the thin-layer data are shown in Table 6. In the absence of more direct estimates Equations 41, 42, and 43, along with a value of K_h from Table 6, could be used to estimate these coefficients for deep bed work.

To obtain better values of h and h'_m for deep bed work, however, it is advisable to use deep bed data directly.

Steady-state deep bed data were available from R. Gyax, so a program was written, using Rosenbrock's⁹ hill-climbing subroutine, to find the best values of K_h in Equation 43 and K_G in Equation 44:

$$h = K_G \cdot G^{0.49} \quad (44)$$

Here the objective function was based on the absolute value of the moisture content error and the temperature error, summed over all five runs,

$$\text{Objective} = 2 \times 10^4 \cdot \sum | \bar{M}_{\text{EXP}} - \bar{M}_{\text{SIM}} | + \sum | T_{a_{\text{EXP}}} - T_{a_{\text{SIM}}} | \quad (45)$$

and weighted so that a 0.1% error in moisture content counted as much as a 20°F error in temperature.

TABLE 6.--Values of the proportionality constant between convective coefficients for heat and mass transfer.

Data Source	h	h'_m	$K_h = h'_m/h$
Elzinga Data	12.33	1.193	0.0967
Troege--Low Temp.	2.054	0.1002	0.0488
Troege--Med. Temp.	4.430	0.1955	0.0441
Troege--High Temp.	9.623	0.4188	0.0435
Troege--Average	—	—	0.0455

The results of this work are shown in Table 7, along with the experimental data. The best values of K_h and K_G were found to be 0.0553 and 0.2803, respectively. The theoretical value of K_G in Equation 42 is 0.69. Since that

TABLE 7.--Steady-state dryer data and best fit simulation results.*

Run Number	6	8	11	14	15
G_c , Corn flow rate lb/hr/ft ² dry basis	193	216.5	144	115.4	140.8
G_a , Air flow rate lb/hr/ft ² dry basis	560	408	357	338	336
Entrance Conditions:					
T_c , Corn Temp °F	42.5	65	42	53	50
M, dry basis moisture fraction	.3141	.3141	.3403	.2697	.2697
T_a , Air Temp, °F	350	300	350	250	300
H, Abs. Humidity	.0047	.0045	.0043	.0048	.0048
Experimental Exit Conditions (2 ft of bed length)					
T_a , Air Temp, °F	134	107	112	102	106.5
M, Moisture Content	.2213	.2557	.2498	.2136	.2194
Simulation Exit Conditions:					
T_a , Air Temp, °F	141	113	117	113	122
M, Moisture Content	.2301	.2601	.2498	.2120	.2132

*For $K_G = .2803$ and $K_h = .0553$.

value refers to fixed beds of spheres, the results here may be considered to be in agreement with the theoretical values.

The final optimization program executed in about 85 seconds and used 25 trials. Since each trial needed 5 solutions of the steady-state equations, each solution for 2 feet of bed length took about 0.68 seconds of CDC 6500 running time.

DYNAMIC SIMULATION OF A COCURRENT
DRYER FOR SHELLLED CORN

Several assumptions are used in deriving the equations for a cocurrent corn dryer. First are plug flow of both corn and air, and perfect insulation at the dryer walls; the result of these is that there will be no radial gradients inside the bed. Also heat and mass transfer by conduction along the bed (z-axis) are assumed negligible.

Each region of the corn is treated as a continuum rather than as individual kernels. To do this the geometric parameters are expressed per cubic foot of dryer. Here α_1 is substituted for the A'_1/V of the thin-layer section. Since the thin-layer calculations were based on a value of ϵ of 0.5 they are

$$\alpha_1 = 12.58 \times 10^4 \text{ (2) } (1-\epsilon) \text{ ft}^{-2} \quad (46)$$

$$\alpha_2 = 9.46 \times 10^4 \text{ (2) } (1-\epsilon) \text{ ft}^{-2} \quad (47)$$

$$A = 239 \text{ (2) } (1-\epsilon) \text{ ft}^{-1} \quad (48)$$

A moisture balance on the innermost (M_1) region from z to $z + \Delta z$ is as follows

$$\begin{aligned} &(\text{FLOW INPUT AT } z) - (\text{FLOW OUTPUT AT } z + \Delta z) - \\ &(\text{OUTPUT BY DIFFUSION}) = (\text{ACCUMULATION RATE}) \end{aligned}$$

$$\begin{aligned}
& \frac{G_c}{3} \cdot M_1|_z - \frac{G_c}{3} M_1|_{z+\Delta z} - D \alpha_1 \rho_c \Delta z (M_1 - M_2) \\
& = \frac{\partial}{\partial t} \left(\frac{\Delta z \rho_c (1-\epsilon)}{3} M_1 \right) \quad (49)
\end{aligned}$$

Note that each term is in units of pounds of water per hour per square foot of bed cross-section. Each term is divided by Δz and the limit is taken as $\Delta z \rightarrow 0$:

$$\left(\frac{G_c}{3} \right) \lim_{\Delta z \rightarrow 0} \left\{ \frac{M_1|_{z+\Delta z} - M_1|_z}{\Delta z} \right\} - D \alpha_1 \rho_c (M_1 - M_2) = \frac{\rho_c}{3} (1-\epsilon) \frac{\partial M_1}{\partial t} \quad (50)$$

which gives

$$\frac{G_c}{3} \frac{\partial M_1}{\partial z} = - D \alpha_1 \rho_c (M_1 - M_2) - \frac{\rho_c}{3} (1-\epsilon) \frac{\partial M_1}{\partial t}. \quad (51)$$

In a similar manner, a moisture balance is made on the M_2 segment

$$\begin{aligned}
& \frac{G_c}{3} M_2|_z + D \alpha_1 \rho_c \Delta z (M_1 - M_2) - \frac{G_c}{3} M_2|_{z+\Delta z} - D \alpha_2 \rho_c \Delta z (M_2 - M_3) \\
& = \frac{d}{dt} \left(\Delta z \frac{\rho_c}{3} (1-\epsilon) M_2 \right), \quad (52)
\end{aligned}$$

which gives

$$\frac{G_c}{3} \frac{\partial M_2}{\partial z} = D \alpha_1 \rho_c (M_1 - M_2) - D \alpha_2 \rho_c (M_2 - M_3) - \frac{\rho_c (1-\epsilon)}{3} \frac{\partial M_2}{\partial t}. \quad (53)$$

The overall moisture transfer coefficient at the edge of the kernel, U , is defined as

$$U = \frac{D \rho_c h'_m}{D \rho_c + h'_m \Delta r_3} \quad (54)$$

Then a moisture balance on the outer segment is

$$\begin{aligned} \frac{G_c}{3} M_3|_z + D \alpha_2 \rho_c \Delta z (M_2 - M_3) - \frac{G_c}{3} m_3|_{z+\Delta z} - UA \Delta z (M_3 - M_e) \\ = \frac{d}{dt} \left(\Delta z \frac{\rho_c}{3} (1-\epsilon) M_3 \right), \end{aligned} \quad (55)$$

where M_e is the equilibrium moisture content of corn at the bulk air conditions. This gives

$$\frac{G_c}{3} \frac{\partial M_3}{\partial z} = D \alpha_2 \rho_c (M_2 - M_3) - UA (M_3 - M_e) - \frac{\rho_c (1-\epsilon)}{3} \frac{\partial M_3}{\partial t}. \quad (56)$$

Under the assumption that the temperature is essentially uniform inside a kernel, a heat balance is made on the corn. Here the units of each term are Btu's per hour per square foot.

$$\begin{aligned}
G_c C_{pc} T_c|_z + h A \Delta z (T_a - T_c) - G_c C_{pc} T_c|_{z+\Delta z} - U A h_{fg} \Delta z (M_3 - M_e) \\
= \frac{d}{dt} (C_{pc} \rho_c (1-\epsilon) \Delta z T_c)
\end{aligned} \quad (57)$$

which gives

$$C_{pc} G_c \frac{\partial T_c}{\partial z} = h A (T_a - T_c) - U A h_{fg} (M_3 - M_e) - C_{pc} \rho_c (1-\epsilon) \frac{\partial T_c}{\partial t} \quad (58)$$

Note here that the heat capacity of the corn is a dry basis "stream" value of 0.6 Btu/lb/°F. This corresponds to about 21% moisture content dry basis. The only value of the heat balance equation is to estimate the corn temperature needed in the expression for diffusivity of moisture in the kernel, so it was felt that the gain by having a linear equation would offset the loss in accuracy.

A moisture balance on the air is

$$G_a H|_z + U A \Delta z (M_3 - M_e) - G_a H|_{z+\Delta z} = \frac{d}{dt} (\rho_a \epsilon \Delta z H) \quad (59)$$

which gives

$$G_a \frac{\partial H}{\partial z} = U A (M_3 - M_e) - \rho_a \epsilon \frac{\partial H}{\partial t} \quad (60)$$

A heat balance on the air is

$$\begin{aligned}
 & G_a C_{pa} T_a|_z - G_a C_{pa} T_a|_{z+\Delta z} - h A \Delta z (T_a - T_c) \\
 & = \frac{d}{dt} (\rho_a \epsilon \Delta z C_{pa} T_a)
 \end{aligned} \quad (61)$$

which gives

$$G_a C_{pa} \frac{\partial T_a}{\partial z} = -h A (T_a - T_c) - \rho_a \epsilon C_{pa} \frac{\partial T_a}{\partial t} \quad (62)$$

In addition, the following relation from Thompson et al.¹² is used for M_e .

$$M_e = \sqrt{\frac{\ln(1 - R_H)}{-0.382(T_a + 50)}} \quad (63)$$

where R_H is the relative humidity,

$$R_H = \frac{H \cdot P_{TOT}}{P_{sat} (H + 0.622)}, \quad (64)$$

where P_{sat} is the saturation pressure of water at T_a , P_{TOT} is the atmospheric pressure, and H is the humidity in pounds of water per pound of dry air.

The complete set of equations for the dynamic model is shown in Table 8, where they are rearranged by solving for the z -partial. The strategy for solving the equations was to substitute a simple backward-difference formula for the time partial, for example

$$\frac{\partial M_1}{\partial t} = \frac{M_1 - M_1^P}{\Delta t} \quad (65)$$



TABLE 8.--Dynamic equations for a cocurrent corn dryer.

$$\frac{\partial M_1}{\partial z} = -\frac{3 D \alpha_1 \rho_c}{G_c} (M_1 - M_2) - \frac{\rho_c (1-\epsilon)}{G_c} \frac{\partial M_1}{\partial t}$$

$$\frac{\partial M_2}{\partial z} = \frac{3 D \alpha_1 \rho_c}{G_c} (M_1 - M_2) - \frac{3 D \alpha_2 \rho_c}{G_c} (M_2 - M_3) - \frac{\rho_c (1-\epsilon)}{G_c} \frac{\partial M_2}{\partial t}$$

$$U = \frac{D \rho_c h'_m}{D \rho_c + h'_m \Delta r_3}, \quad M_e = \sqrt{\frac{\ln (1-R_H)}{-0.382 (T_a + 50)}}$$

$$\frac{\partial M_3}{\partial z} = \frac{3 D \alpha_2 \rho_c}{G_c} (M_2 - M_3) - \frac{3UA}{G_c} (M_3 - M_e) - \frac{\rho_c (1-\epsilon)}{G_c} \frac{\partial M_3}{\partial t}$$

$$\frac{\partial T_c}{\partial z} = \frac{h A}{G_c C_{pc}} (T_a - T_c) - \frac{UA h_{fg}}{G_c C_{pa}} (M_3 - M_e) - \frac{\rho_c (1-\epsilon)}{G_c} \frac{\partial T_c}{\partial t}$$

$$\frac{\partial H}{\partial z} = \frac{UA}{G_a} (M_3 - M_e) - \frac{\rho_a \epsilon}{G_a} \frac{\partial H}{\partial t}$$

$$\frac{\partial T_a}{\partial z} = -\frac{hA}{G_a C_{pa}} (T_a - T_c) - \frac{\rho_a \epsilon}{G_a} \frac{\partial T_a}{\partial t}$$

$$D = 1.629 \times 10^{-3} \text{ EXP } \{ (0.025 T_c + 6.008) \bar{M} - \frac{4523.4}{T_c + 459.7} \}$$

$$R_H = \frac{H \cdot P_{TOT}}{(H + 0.622) \cdot P_{sat}}, \quad \bar{M} = \frac{M_1 + M_2 + M_3}{3}$$

where M_1 is the function of z in the current time step and M_1^P is that of the previous time. The result of substituting such a formula in each of the equations is a set of 6 nonlinear ordinary differential equations at each point in time. Those were solved by the same method used in the thin layer work, namely, linearizing followed by an exact (Laplace Transform) solution, where the linearization is done at the center of the z -step.

The previous time-step function was linearized into a slope and an intercept, so, for instance

$$M_1^P = M_1^{PZ} + z M_1^{PD} \quad (66)$$

where M_1^{PZ} is the intercept and M_1^{PD} is the slope. For convenience, each z -step was treated as if it were from 0 to Δz rather than from z to $z + \Delta z$. This made it easier to use Laplace transforms to find the exact solution to the linearized equations.

The method of choosing the slope and intercept of the previous time step is critical. As is shown in Figure 7, the dotted line between the actual values stored for the front and end of the interval overestimates the function everywhere in between. The use of that line is equivalent to adding an amount of moisture to the bed equivalent to the area between it and the curve. To estimate the actual value of the function, the quadratic was found which passes through the ends of the interval and is a least squares fit of the next two closest points.

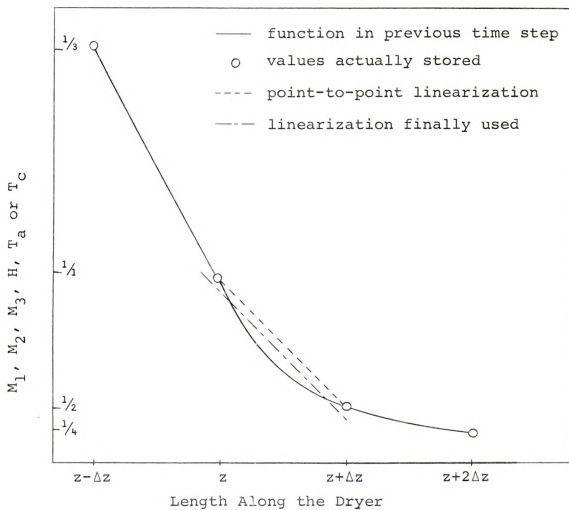


Figure 7.--The linearization used for the previous time step.

Find the line $y = Az^2 + Bz + C$ which coincides with points y_1 and y_2 in Figure 7, and where the z -axis is shifted so that y_1 is at $z = 0$. Then $C = y_1$ and $y_2 = Az^2 + Bz + y_1$. Solving for B , one gets

$$B = \frac{y_2 - y_1}{z_2} - Az_2 = S' - Az_2 \quad (67)$$

where

$$S' = \frac{y_2 - y_1}{z_2},$$

the slope. The least squares, L_s will be

$$L_s = \min_A \{ [y_3 - (Az_3^2 + Bz_3 + y_1)]^2 + [y_4 - (Az_4^2 + Bz_4 + y_1)]^2 \} \quad (68)$$

Working with the above equation

$$\begin{aligned} L_s = \min_A \{ & [y_3 - Az_3^2 - S'z_3 + Az_2z_3 - y_1]^2 \\ & + [y_4 - Az_4^2 - S'z_4 + Az_2z_4 - y_1]^2 \} \end{aligned} \quad (69)$$

$$\begin{aligned} L_s = \min_A \{ & [(y_3 - y_1 - S'z_3) + A(z_2z_3 - z_3^2)]^2 \\ & + [(y_4 - y_1 - S'z_4) + A(z_2z_4 - z_4^2)]^2 \} \end{aligned} \quad (70)$$

The minimum should be found by setting the derivative equal to zero

$$\begin{aligned}
0 = & 2[(y_3 - y_1 - S'z_3) + A(z_2z_3 - z_3^2)](z_2z_3 - z_3^2) \\
& + 2[(y_4 - y_1 - S'z_4) + A(z_2z_4 - z_4^2)](z_2z_4 - z_4^2)
\end{aligned} \quad (71)$$

Which gives

$$A = - \frac{(y_3 - y_1 - S'z_3)(z_2z_3 - z_3^2) + (y_4 - y_1 - S'z_4)(z_2z_4 - z_4^2)}{(z_2z_3 - z_3^2)^2 + (z_2z_4 - z_4^2)^2} \quad (72)$$

The point is to find the line $y = I + zS$, where I is the intercept and S is the slope, which most closely fits the curve $y = Az^2 + Bz + C$. One way to do this is to minimize the integral of the squared difference

$$\begin{aligned}
& \text{Min}_{I,S} \int_0^{z_2} [(Az^2 + Bz + C) - (I + zS)]^2 dz \quad (73) \\
= & \text{Min}_{I,S} \left[\int_0^{z_2} (Az^2 + Bz + C)^2 dz + \int_0^{z_2} \{ (I + zS)^2 - 2(Az^2 + Bz + C)(I + zS) \} dz \right] \quad (74)
\end{aligned}$$

Since the first integral involves neither I nor S , it will not be involved in the minimization

$$\begin{aligned}
= & \text{Min}_{I,S} \left[\int_0^{z_2} \{ -2ASz^3 + (S^2 - 2AI - 2BS)z^2 \right. \\
& \left. + (2IS - 2BI - 2CS)z + (I^2 - 2IC) \} dz \right] \quad (75)
\end{aligned}$$

$$= \min_{I,S} \left[-\frac{ASz_2^4}{2} + \frac{(S^2 - 2AI - 2BS)z_2^3}{3} + (IS - BI - CS)z_2^2 + (I^2 - 2IC)z_2 \right] \quad (76)$$

Setting the derivative with respect to S equal to zero

$$-\frac{Az_2^4}{2} + \frac{2S - 2B}{3}z_2^3 + Iz_2^2 - Cz_2^2 = 0 \quad (77)$$

This is simplified by multiplying by $6/z_2^2$

$$6I + 4Sz_2 = 4Bz_2 + 3Az_2^2 + 6C \quad (78)$$

$$\text{But } 3(Az_2^2 + Bz_2 + C) = 3y_2$$

$$\text{so } 6I + 4Sz_2 = Bz_2 + 3C + 3y_2 \quad (79)$$

Setting the derivative of equation 76 with respect to I equal to zero

$$\frac{-2Az_2^3}{3} + Sz_2^2 - Bz_2^2 + 2Iz_2 - 2Cz_2 = 0 \quad (80)$$

This is simplified by multiplying by $3/z_2$

$$6I + 3Sz_2 = 3Bz_2 + 2Az_2^2 + 6C \quad (81)$$

$$\text{But } 2(Az_2^2 + Bz_2 + C) = 2y_2$$

$$\text{so } 6I + 3Sz_2 = Bz_2 + 4C + 2y_2 \quad (82)$$



Subtracting equation 82 from equation 79

$$Sz_2 = y_2 - C \text{ but } C = y_1$$

so
$$S = \frac{y_2 - y_1}{z_2} = S'$$

Solving equation for I

$$6I = Bz_2 + 4y_1 + 2y_2 - 3S'z_2 \quad (83)$$

But $B = S' - Az_2$ from equation 65 so

$$6I = (S' - Az_2)z_2 + 4y_1 + 2y_2 - 3S'z_2 \quad (84)$$

$$6I = -Az_2^2 + 4y_1 + 2y_2 - 2S'z_2 \quad (85)$$

$$I = y_1 - \frac{1}{6} Az_2^2 \quad (86)$$

This means that the same slope is used but the intercept is moved down slightly, as shown by the broken line in Figure 7. In practice it was found that $Az_2^2/6$ tended to move the curve down too much, so a value of about $Az_2^2/8$ was actually used.

The presence of M_e in the H equation presents two problems. First it occupies a position where H is expected. Anytime a dependent variable is missing from its own equation there is great danger of numerical instability. In the method used here it means that an individual step might not converge. The second problem is that of finding

an expression for M_e in the linearized equations. In what follows M_e^I and H^I are initial values in the z -step interval and M_e^L and H^L are final values. One possible expression might be

$$M_e = C_2 + C_3 z \quad (87)$$

where $C_2 = M_e^I$ and $C_3 = \frac{M_e^L - M_e^I}{\Delta z}$.

This is not particularly good because the dependence on H is lost.

Another possible expression is

$$M_e = C_2 H + C_3 \quad (88)$$

where $C_2 = \frac{M_e^L - M_e^I}{H^L - H^I}$ and $C_3 = M_e^I - C_2 H^I$. This expression

works most of the time. However, there are situations where C_2 may be negative, since the air temperature may drop considerably over the interval. When the C_2 defined above is negative it artificially introduces a positive eigenvalue and makes the system very unstable. The expression finally used was

$$M_e = C_2 H \text{ where } C_2 = 0.5 \left\{ \frac{M_e^L}{H^L} + \frac{M_e^I}{H^I} \right\}. \quad (89)$$

The strong dependence on H makes for good system stability so the method used converges very quickly.

The substitutions used in the linearization are shown in Table 9, while the set of linearized equations is found in Table 10.

The Laplace transform is taken of each equation. Here a bar over a quantity will signify its Laplace transform, S is the transform independent variable, and a superscript $^{\circ}$ signifies the initial condition. Then for instance the first equation from Table 4 is

$$S\bar{M}_1 - M_1^{\circ} + -A_1\bar{M}_1 + B_1\bar{M}_2 + C_1M_1^{PZ}/S + C_1M_1^{PD}/S^2 \quad (90)$$

When all 6 equations are transformed in this manner there results:

$$\begin{bmatrix} S+A_1 & -B_1 & 0 & 0 & 0 & 0 \\ -B_1 & S+A_2 & -B_2 & 0 & 0 & 0 \\ 0 & -B_2 & S+A_3 & -B_3C_2 & 0 & C \\ 0 & 0 & -B_4 & S+A_4 & 0 & 0 \\ 0 & 0 & B_5 & -B_5C_2 & S+A_5 & -C_5 \\ 0 & 0 & 0 & 0 & -C_6 & S+A_6 \end{bmatrix} \begin{bmatrix} \bar{M}_1 \\ \bar{M}_2 \\ \bar{M}_3 \\ \bar{H} \\ \bar{T}_c \\ \bar{T}_a \end{bmatrix} = \frac{1}{S^2} \begin{bmatrix} S^2M_1^{\circ} + SC_1M_1^{PZ} + C_1M_1^{PD} \\ S^2M_2^{\circ} + SC_1M_2^{PZ} + C_1M_2^{PD} \\ S^2M_3^{\circ} + SC_1M_3^{PZ} + C_1M_3^{PD} \\ S^2H^{\circ} + SC_4H^{PZ} + C_4H^{PD} \\ S^2T_c^{\circ} + SC_1T_c^{PZ} + C_1T_c^{PD} \\ S^2T_a^{\circ} + SC_4T_a^{PZ} + C_4T_a^{PD} \end{bmatrix} \quad (91)$$

The solution of this set of 6 linear equations is straightforward, and is found in Appendix C. The Laplace

TABLE 9.--Substitutions used in the linearization of the cocurrent dryer dynamic equations.

$$B_1 = \frac{3 \alpha_1 \rho_c D}{G_c} \quad C_1 = \frac{\rho_c (1-\epsilon)}{G_c \Delta t} \quad A_1 = B_1 + C_1$$

$$B_2 = \frac{3 \alpha_2 \rho_c D}{G_c} \quad A_2 = B_1 + B_2 + C_1$$

$$C_2 = 0.5 \left\{ \frac{M_e^L}{H^L} + \frac{M_e^I}{H^I} \right\}$$

$$B_3 = \frac{3AU}{G_c} \quad A_3 = B_3 + B_2 + C_1$$

$$B_4 = \frac{UA}{G_a} \quad C_4 = \frac{\rho_a \epsilon}{G_a \Delta t} \quad A_4 = B_4 C_2 + C_4$$

$$B_5 = \frac{h_{fg} AU}{G_c C_{pc}} \quad C_5 = \frac{Ah}{G_c C_{pc}} \quad A_5 = C_5 + C_1$$

$$C_6 = \frac{hA}{G_a C_{pa}} \quad A_6 = C_4 + C_6$$

TABLE 10.--Linearized cocurrent dryer dynamic equations.

$$\frac{dM_1}{dz} = -A_1 M_1 + B_1 M_2 + C_1 M_1^{PZ} + z C_1 M_1^{PD}$$

$$\frac{dM_2}{dz} = B_1 M_1 - A_2 M_2 + B_2 M_3 + C_1 M_1^{PZ} + z C_1 M_2^{PD}$$

$$\frac{dM_3}{dz} = B_2 M_2 - A_3 M_3 + B_3 C_2 H + C_1 M_3^{PZ} + z C_1 M_3^{PD}$$

$$\frac{dH}{dz} = B_4 M_3 - A_4 H + C_4 H^{PZ} + z C_4 H^{PD}$$

$$\frac{dT_C}{dz} = C_5 T_a - A_5 T_C + B_5 C_2 H - B_5 M_3 + C_1 T_C^{PZ} + z C_1 T_C^{PD}$$

$$\frac{dT_a}{dz} = C_6 T_C - A_6 T_a + C_4 T_a^{PZ} + z C_4 T_a^{PD}$$

transform of each dependent variable is expressed as the ratio of two polynomials. In this case a double zero appears in the denominator, so a modified Heaviside expansion theorem is used:

$$\text{If } \bar{U} = \frac{P(S)}{S^2 Q(S)}, \text{ where } Q(S) = (S-a_1) \cdot (S-a_2) \cdots (S-a_m) \quad (92)$$

and $P(S)$ is a polynomial of degree $m + 1$ or less then

$$U(z) = \frac{P(0)}{Q(0)} \cdot z + \frac{Q(0)P'(0) - P(0)Q'(0)}{(Q(0))^2} + \sum_{i=1}^m \frac{P(a_i)}{a_i^2 Q'(a_i)} e^{a_i z} \quad (93)$$

where $P'(s) = \frac{dP}{ds}$ and $Q'(s) = \frac{dQ}{ds}$

The first four equations of the linearized set are clearly independent of the last two. This is because the only temperature involved in the mass transfer equations is T_c , the corn temperature, which is only involved in the diffusivity, D . This direct dependence is lost in the linearization.

Appendix D contains the resulting program.

It was checked for accuracy in two ways. First, the corresponding steady-state equations were solved using the Bulirsch-Stoer integration method,⁴ which has step-by-step error control and can be considered as the current standard. A program to do this was obtained from Mrs. Nancy Clark of Argonne National Laboratories and

rewritten for use on the IBM 1800. The program came to steady state with better than 5-place agreement with the standard. Second, the whole program was run with twice as large a time-step. In that case 4 to 5-place agreement with the original was found.

OPEN-LOOP NEAR-OPTIMAL
TEMPERATURE CONTROL

The control method one expects to use is a fundamental part of the design of a corn dryer. The concept used here is that corn flow rate is specified, perhaps by the number of bushels per day (or week) one expects to have to dry. For best use of electricity in the blower a single motor will be chosen and run at its most efficient operating point, so the air flow rate is specified indirectly. Both of these will then be constant at their specified values. Since input air humidity or input corn moisture content or temperature cannot be controlled, the only remaining variable available for manipulation is the input air temperature.

In this section a single control situation--step increase in input moisture content--was chosen. Then several control functions were tried to find the "best" one in a strictly economic sense. This "optimization" took 38 trials, and each trial took 5 minutes of running time on the CDC 6500. So it should be clear than an exhaustive search of various control situations--steps of various sizes from various starting points and both up and down--would be extremely expensive to run. This does, however, illustrate the method to use.

The problem chosen for the determination of a near-optimal control is the best input temperature as a function of time for a cocurrent dryer while changing steady-states from drying corn of 33% dry basis moisture content to 38%. The dryer is 1 foot long, the corn flow rate is 15 pounds of dry matter per hour per square foot, and the air flow rate is 200 pounds of dry air per hour per square foot. The input air has 0.014 pounds of water per pound of dry air, the input corn temperature is 72°F, and the desired exit moisture content is 18.3% dry basis. Under these conditions the initial input air temperature is 155.71°F and the final input air temperature is 176.02°F.

The objective or cost function has two components: First, corn of higher than the specified moisture content is less valuable. Second, drying past the specified moisture content means using too much heat. The penalty for insufficiently dried corn is given by Scott¹⁰ as one cent per bushel for each half per cent wet basis over 15.5%. Since there are 56 pounds of dry matter per bushel of corn, there would be 10.2722 pounds of water at 15.5%. At 16% wet basis a bushel would have 10.6667 pounds of water, so 0.3945 pounds of excess moisture costs 1 cent. So the cost of underdrying is 2.535 cents per pound of

excess water. The cost of overdrying was calculated assuming 90 cents per million Btu cost of heat (natural gas is the probable energy source) and 1040 Btu per pound of water evaporated. Then the differential cost of drying (meaning change in cost for small changes in amount of drying) is 0.0936 cents per pound of water dried. So the net cost of underdried corn is 2.441 cents per pound of excess water. Then the objective or penalty function will be

$$\frac{d(\text{penalty})}{dt} = \begin{cases} 2.441 (\bar{M} - \bar{M}_O), & \bar{M} > M_O \\ 0.0936 (M_O - \bar{M}), & \bar{M} \leq M_O \end{cases} \quad (94)$$

where M_O is the target moisture content. Note that since the air flow rate is constant the cost of blowing air through the bed will not depend on the exit moisture content.

The following function was used as a normalized input temperature:

$$y = \frac{T - 155.71}{176.02 - 155.71} \quad (95)$$

The first type of control tried was of the "bang-bang" or single-step type. Here y is zero until the switching time, t_s , is reached, after which $y = 1$. There are three reasons for choosing this type of control: (1) It is simple to implement. (2) Many control situations have such a control as the optimum. (3) It is easy to evaluate the temperature integral

$$I_T = \int_0^{\infty} (1-y) dt \quad (96)$$

for this control. The temperature integral will represent the total amount of heat energy applied to the system, so it is expected that I_T will be approximately the same at the optimum for various control functions.

The second type of control function used was an "S-curve" of two parameters:

$$y = \frac{B (B+1)}{e^{-at} + B} - B \quad (97)$$

Note that this function is zero at $t = 0$ and approaches 1.0 as t increases. The value of I_T for this function is

$$I_T = \frac{B+1}{a} \ln \left(\frac{B+1}{B} \right) \quad (98)$$

When the value of I_T is held constant the S-curve becomes a single-parameter function.

The results of the 5 bang-bang control trials are shown in Table 11, Runs 1-5. These were fitted to a quadratic least squares of the same type as in the linearization of the previous time-step on page 49. Here the two smallest values were fitted exactly and the others were weighted by one over the square of their X-distance from the average of the two lowest points. The result was that the best estimate of the minimum was at $I_T = 0.0361$.

TABLE 11.--Open-loop temperature control results, Runs 1-22.

Run #	Parameters		I_T	Penalty*
1	$t_s = 0.025$		.0125	7.522501E-4
2	$t_s = 0.05$		.0375	7.473014E-4
3	$t_s = 0.075$		.0625	7.449360E-4
4	$t_s = 0.1$		.0875	7.829919E-4
5	$t_s = 0.2$		.1875	1.230722E-3
6	$a = 40$	$B = 0.8370035$	.0361	7.390341E-4
7	$a = 30$	$B = 5.69979$	.0361	7.366338E-4
8	$a = 28.38209$	$B = 20$	.0361	7.361246E-4
9	$a = 27.83887$	$B = 100$	.0361	7.359437E-4
10	$a = 27.76997$	$B = 200$	.0361	7.359203E-4
11	$a = 27.71468$	$B = 1000$	.0361	7.359016E-4
12	$a = 27.70269$	$B = 7000$	.0361	7.358974E-4
13	$a = 28.57334$		.035	7.365738E-4
14	$a = 27.02884$		.037	7.353200E-4
15	$a = 25$		.04	7.332408E-4
16	$a = 20$		.05	7.277053E-4
17	$a = 15$		.0667	7.194253E-4
18	$a = 10$		.1	7.130775E-4
19	$a = 5$		.2	1.299459E-3
20	$a = 9$		.101	7.184637E-4
21	$a = 11$		.0909	7.131513E-4
22	$a = 10.23$		.09775	7.127858E-4

Runs 1-5 are single-step;

Runs 6-12 are S-curve;

Runs 13-22 are exponential.

*The penalty is in cent-hours per pound of dry corn.

Next I_T was held constant at 0.0361 and various pairs of a and B were run for the S-curve. This series appears as Runs 6-12 in Table 11. The penalty decreased as larger and larger values of B were used. It is clear that for very large values of B Equation 97 could become unstable, so this search was stopped at $B = 7000$. Then Equation 97 was rearranged to

$$y = B \left\{ \frac{B + 1}{e^{-at} + B} - 1 \right\} = \frac{B}{e^{-at} + B} \{1 - e^{-at}\}. \quad (99)$$

For very large B , this reduces to the following:

$$y = 1 - e^{-at}, \quad I_T = 1/a \quad (100)$$

so this exponential was tried. Runs 13-22 used this function; the optimum was estimated with the quadratic fit as above to be at $a = 10.23$.

The final set of functions used started out to be simple time lag, followed by the exponential of Equation 100. If t_e is the time lag this is

$$y = \begin{cases} 0, & t \leq t_e \\ 1 - e^{-a(t-t_e)}, & t > t_e \end{cases} \quad (101)$$

$$I_T = t_e + \frac{1}{a} \quad (102)$$

However, this equation actually went the wrong way, so instead an equation was tried which keeps y above some minimum value, y_m , and approaches one exponentially:

$$y = y_m + (1-y_m) (1-e^{-at}) \quad (103)$$

$$I_T = (1-y_m) \cdot \frac{1}{a} \quad (104)$$

This function was used in the final optimization. First I_T was held constant at 0.09775 as shown in Runs 22-28 in Table 12. Here the optimum was estimated at $a = 6.45$. Then a was held constant at 6.45 and I_T was varied for Runs 29-34. The optimum for that was estimated to be at $I_T = 0.076$. Finally, Runs 35 and 36 show that the best value of a was still near 6.45.

The last trial, Run 38, is very near the optimum for these functions. The input temperature for this optimum is shown in Figure 8. The exit moisture content as a function of time is shown in Figure 9 for this run and for the best single-step.

TABLE 12.--Open-loop temperature control results, Runs 22-38.

Run #	Parameters		I_T	Penalty *
22	A = 10.23	$y_m = 0$	0.09775	7.127858E-4
23	A = 9.5	$y_m = 0.071375$	0.09775	7.045864E-4
24	A = 8.0	$y_m = 0.2180$	0.09775	7.065060E-4
25	A = 5.0	$y_m = 0.5125$	0.09775	7.065060E-4
26	A = 7.38	$y_m = 0.278605$	0.09775	6.839579E-4
27	A = 5.7	$y_m = 0.442825$	0.09775	6.879560E-4
28	A = 6.45	$y_m = 0.3695125$	0.09775	6.814373E-4
29	A = 6.45	$y_m = 0.3550$	0.1	6.840006E-4
30	A = 6.45	$y_m = 0.38725$	0.095	6.787003E-4
31	A = 6.45	$y_m = 0.4840$	0.08	6.716968E-4
32	A = 6.45	$y_m = 0.6775$	0.05	6.834780E-4
33	A = 6.45	$y_m = 0.9355$	0.01	7.355592E-4
34	A = 6.45	$y_m = 0.525925$	0.0735	6.715480E-4
35	A = 7.0	$y_m = 0.4855$	0.0735	6.724805E-4
36	A = 6.0	$y_m = 0.5590$	0.0735	6.725858E-4
37	A = 4.0	$y_m = 0.7060$	0.0735	7.274961E-4
38	A = 6.45	$y_m = 0.5098$	0.06	6.710516E-4

These control functions are from Equations 100 and 101.

*The penalty is in cent-hours per pound of dry corn.

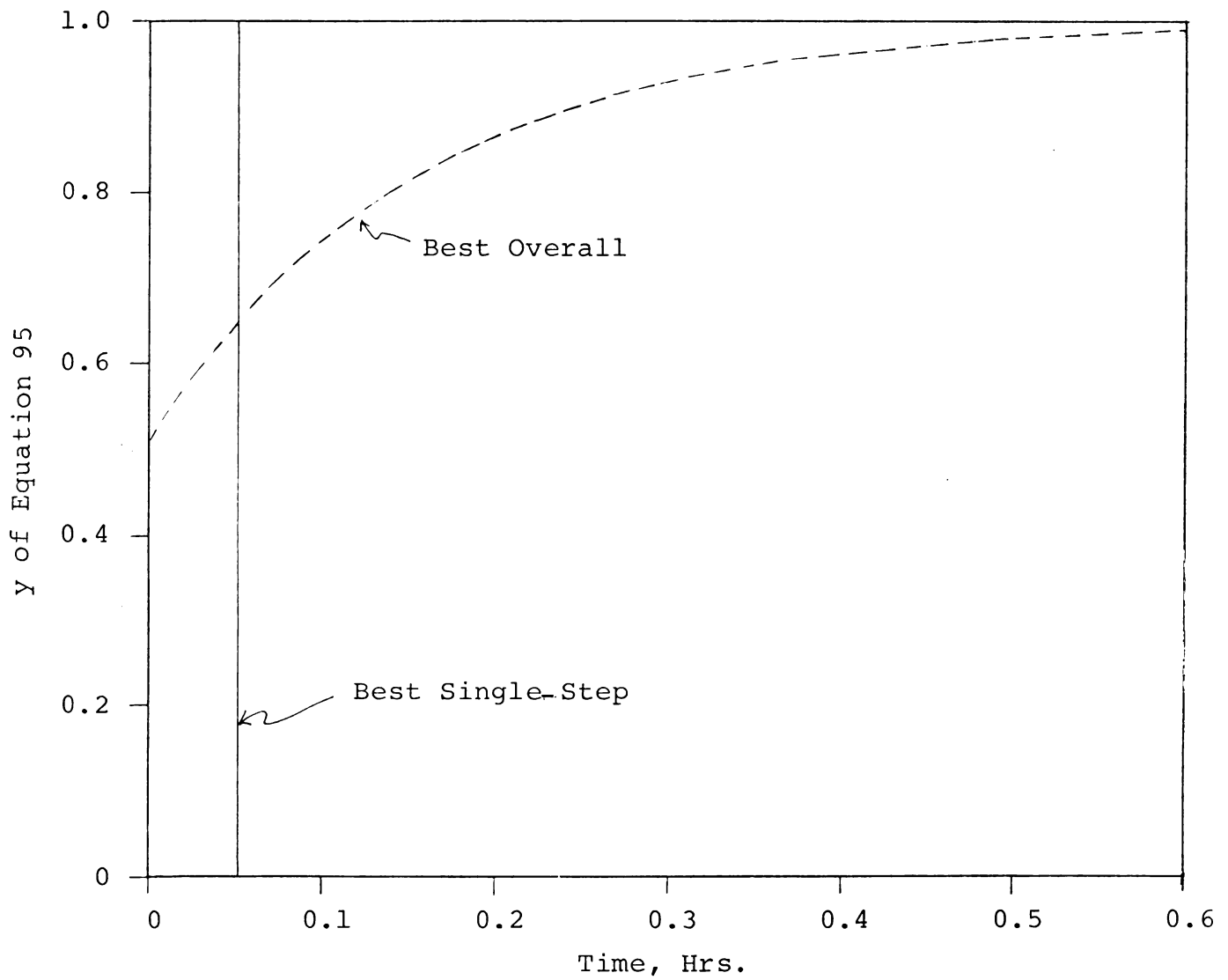


Figure 8.--Input temperature function for the best single-step and overall.

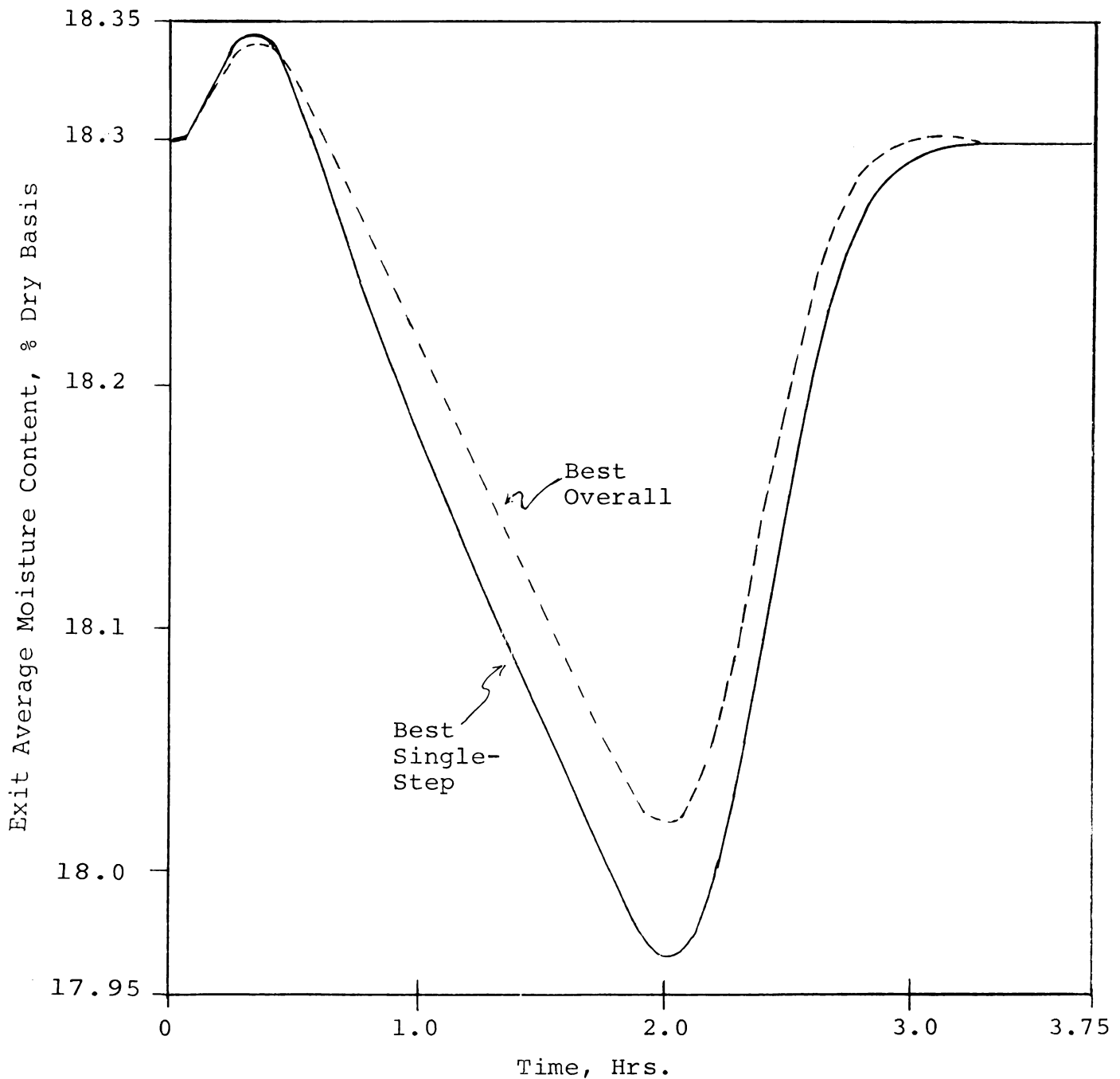


Figure 9.--Exit moisture contents in response to the best single-step and the best overall input temperature functions.

The cost penalty shown is in cent-hours per pound of dry corn. At a flow rate of 15 lb of dry corn per hour the net penalty at the optimum (6.710516×10^{-4} calculated penalty) is 0.010066 cents while for the best single step (calculated penalty 7.449360×10^{-4}) it is 0.011174 cents. Since this was for 3.75 hours operation, 56.25 pounds of dry corn were processed or 1.0045 bushels. Then the optimum here shows a net cost of 0.01002 cents per bushel while the best single step net cost is 0.0111 cents per bushel. In view of this, the best practical control probably is to change the input temperature set point as soon as the new moisture content corn reaches the bed.

To implement this control policy the operator (either human or automatic) must estimate the time when a new batch of corn will reach the drying zone. Additional single-step trials were run to test the sensitivity of the penalty to errors in this estimate. The results of these trials are shown in Table 13, and plotted along with Runs 1-5 in Figure 10. If a 50% increase in the penalty is allowed (to approximately 1.1×10^{-3}), there is a 25 minute band during which the step may be made, from about 14 minutes before to about 11 minutes after the new corn reaches the drying zone.

TABLE 13.--Additional single-step temperature control results.

Time of Step	Penalty
$t_s = -0.3$	1.226E-3
$t_3 = -0.2$	1.044E-3
$t_s = -0.1$	0.885E-3
$t_s = -0.05$	0.819E-3
$t_s = 0.0$	0.769E-3
$t_s = 0.3$	3.400E-3



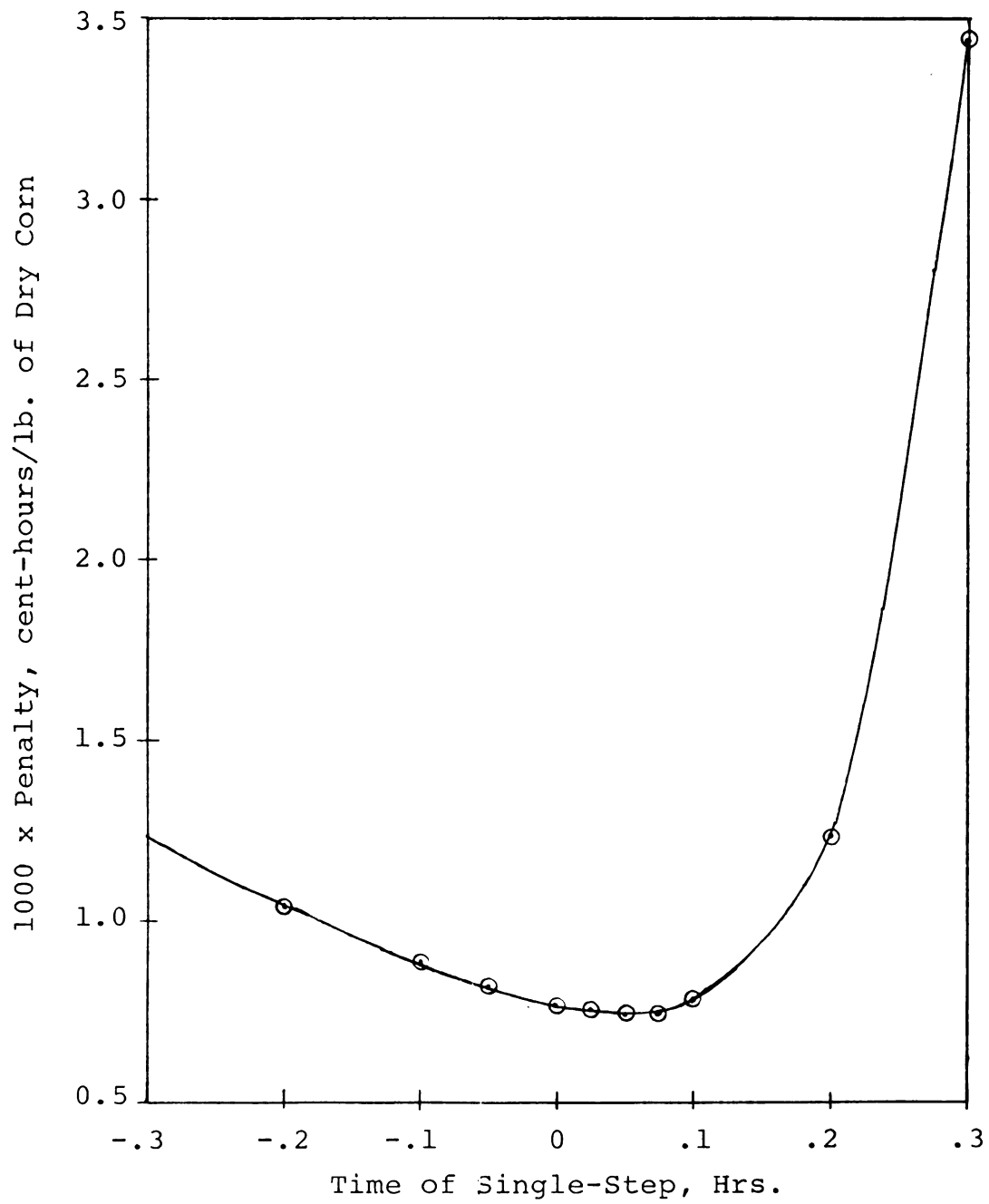


Figure 10.--Variation in penalty with time of single-step.

CONCLUSIONS

The quality of a corn dryer simulation is limited by its weakest link--the thin-layer model. Although a great deal of work has been done, this problem may be considered unsolved in that no model accurately predicts known thin-layer drying behavior. The author feels strongly that a good model must carry the internal moisture gradient, especially so for modeling nonstandard dryers such as cocurrent-countercurrent or intermittent air flow where the external history of the kernel is vastly different than in thin-layer experiments. On the other hand, if too many internal segments are used the model is expensive to run. The best number is probably in the range of three to five; at three some of the gradient is probably lost, and at five it would be very difficult to apply the integration method used here to the dynamic deep bed model.

Future thin-layer work along the lines of this model should be pointed toward finding a better estimate of the diffusivity, D . The one used here was determined with a model where the convective mass transfer coefficient, h'_m , was assumed to be infinite. From the thin-layer results here it appears that the D is too small at high moisture contents and too large at lower moisture contents.

The values of the convective heat and mass transfer coefficients were higher for the high temperature Troeger data than the low temperature data, so D is probably too small at higher temperatures also.

Convective heat and mass transfer coefficients are difficult to measure in the deep bed. The method used here--finding the set which best fits experimental data--is easy to apply if there is enough computer time available or the model runs efficiently so that many trials may be made. The result will, of course, depend on the thin-layer model upon which the steady-state model is based. Future work in this area should concentrate on obtaining more accurate and complete steady-state data, with several replicates or redundant measuring systems for each variable.

The control work done here demonstrates one of the good properties of the cocurrent dryer--it is relatively stable to changes in inputs. The control results make it clear that little would be gained in the use of a complicated open-loop control. The best choice might be to measure the incoming moisture content, look up the needed input air temperature, and reset the air temperature controller to that value. This could be either manual or automatic--even computerized. Future work along this line could make use of a more sophisticated objective function where sensible heats are accounted for as well as the heat of vaporization. Such work might also involve other sizes

of changes in input moisture content, a decrease instead of an increase, or multiple changes.

Other types of controls, probably corn flow rate manipulation, could also be simulated with this program. That control has the interesting property that the corn flow rate may always take its maximum value for the input air temperature being used.

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REFERENCES

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APPENDICES

APPENDIX A

THIN LAYER PROGRAM

APPENDIX A

THIN LAYER PROGRAM

This program is used to find A_1' , A_2' , Δr_3 , h_m' , and h from thin layer data by parameter optimization. The calling program, TL4F, is used to initialize the hill climbing search. It calls HCLMB, the subroutine which uses Rosenbrock's⁹ method. HCLMB calls DEL4Q, the subroutine to calculate the total error from the experimental data for a given set of the 5 parameters. The other subroutines are DIFF, which calculates the diffusivity of water in corn from Equation 23, and CUBIC, which finds the roots of a cubic equation.

Some of the variables in DEL4Q and their correspondence in Equation 26 are as follows:

AA = a	DE = d
ALF = α	QUE = $d \cdot M_e + \delta \cdot T_a$
BE = b	DEL = δ
SEA = C	DIFUS = D
BET = β	VOL = V
AREA = A	RHO = ρ
HFG = h_{fg}	CPP = C_{p_c}

In addition, the denominator polynomial

$$(s + a)(s + \alpha)(s + \beta) - b^2(s + a) - a^2(s + \beta)$$

is transformed into

$$s^3 + s^2 \cdot (BZ2) + s \cdot (BZW) + (BZZ)$$

where

$$BZ2 = a + \alpha + \beta$$

$$BZW = a(\alpha + \beta - a) + \alpha \cdot \beta - b^2$$

$$BZZ = a(\beta(\alpha - a) - b^2)$$


```

C      -INPUT VECTOR OF INITIAL VALUES OF THE INDEPENDENT
C      VARIABLES
C      XLB      -INPUT VECTOR OF LOWER BOUNDS OF THE INDEPENDENT
C      VARIABLES
C      XUB      -INPUT VECTOR OF UPPER BOUNDS OF THE INDEPENDENT
C      VARIABLES
C      FOX      -THE NAME OF AN EXTERNAL SUBROUTINE TO EVALUATE THE
C      OBJECTIVE FUNCTION. ITS PARAMETER LIST IS (F,X).
C      THE X-VECTOR MUST NOT BE CHANGED IN FOX.
C      FOX NEEDS AN EXTERNAL STATEMENT IN THE MAIN PROGRAM.
C
C      REMARKS
C      GENERALLY IT IS BEST TO SET EPS VERY LOW AND NTYPE=1, SO THE
C      PROGRAM WILL COMPUTE UNTIL NSTPS CALLS TO FOX ARE MADE. A
C      GOOD PRACTICAL VALUE OF NSTPS IS ABOUT 50*KP.
C      IT IS A GOOD IDEA TO OUTPUT F AND X FROM FOX, SINCE HCLMB
C      ONLY PRINTS THEM JUST PRIOR TO AN AXIS ROTATION, ABOUT
C      EVERY 5*KP CALLS TO FOX.
C
C      SAMPLE CALLING PROGRAM
C
C      PROGRAM MAIN
C      EXTERNAL FOF5X
C      DIMENSION V(25),D(5),E(5),N(5),AP(25)
C      DIMENSION XB(5),BT(5), XLB(5), XUB(5),X(5)
C      DATA XMIN,XMAX,XO/0.0,1.0,0.2/
C      KP=5
C      PCTMN=0.0
C      EPS=-100.0
C      NSTPS=250
C      NTYPE=1
C      DO 100 I=1,5
C      XLB(I)=XMIN
C      XUB(I)=XMAX
C      X(I)=XO
C      CALL HCLMB(KP,PCTMN,EPS,NSTPS,NTYPE,AP,BT
C
C      100
C

```

```

C      1D,E,N,V,X,XB,XLB,XUB,FOF5X)
C      END
C
C      SUBROUTINE FOF5X(F,X)
C      DIMENSION X(5)
C      F=THE FUNCTION TO BE MINIMIZED
C      PRINT 200, F, X
C      200  FORMAT(5X,6E15.7)
C      RETURN
C      END
C
C      REFERENCES
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C      GREATEST OR LEAST VALUE OF A FUNCTION,' COMPUTER JOURNAL,
C      VOL. 3, P. 175-184. (1960)
C
C      SUBROUTINE HCLMB(KP,PCTMN,EPS, NSTPS,NTYPE,AP,BT,D,E,N,V,X,XB,
C      1XLB,XUB,FOX)
C      DIMENSION AP(1),X(1),V(1),E(1),D(1),XB(1),BT(1)
C      DIMENSION XLB(1),XUB(1),N(1)
C      DATA M/3/
C      300  FORMAT(15H0STARTING POINT,/, (E16.8))
C      305  FORMAT(16H0INIT VALUE OF F,E16.8)
C      310  FORMAT(51H0 NO SUCC STEPS, NO UNSUCC STEPS, NO AXIS ROTATIONS)
C      315  FORMAT(/,19,11H SUCC STEPS,19,13H UNSUCC STEPS,19,10H ROTATIONS)
C      320  FORMAT(22H0 PRESENT VALUE OF F =,E15.8/,22H0 CORR INDEP VARIABLES
C      2,/, (E16.8))
C      325  FORMAT(///)
C      330  FORMAT(24H0 STOPPED-TOO MANY STEPS)
C      335  FORMAT(24H0 RELATIVE CRITERION MET)

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HCLMB060
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340  FORMAT(24H0 ABSOLUTE CRITERION MET)
      CALL FOX (FO,X)
      WRITE (M,300) (X(I), I=1, KP)
      WRITE (M,305) FO
      FFRST=FO
      C  PUT IDENTITY MATRIX IN V
        DO 105 K=1, KP
        DO 105 I=1, KP
        J=(K-1)*KP +I
        V(J)=0.0
        IF (I-K) 105,100,105
        V(J)=1.0
        100 CONTINUE
        105 WRITE (M,310)
            IC=0
            IU=0
            IS=0
            IQ=0
            C  STEP ALONG EACH AXIS UNTIL A MINIMUM IS BRACKETED
              DO 115 I=1, KP
                E(I)=0.1*X(I)
                D(I)=0.0
                115 N(I)=4
                120 K=1
                125 DO 130 I=1, KP
                  J=(K-1)*KP +I
                  130 XB(I)=X(I) + E(K)*V(J)
                    DO 140 I=1, KP
                      IF (XB(I)-XLB(I)) 145,135,135
                      135 IF (XUB(I)-XB(I)) 145,140,140
                      140 CONTINUE
                      CALL FOX (F,XB)
                      IF (F-FO) 155,155,145
                      145 E(K)= -0.5* E(K)
                        IU=IU +1
                        IF (N(K)-2) 170,150,170

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HCLMB130
HCLMB131




```

150 N(K)=0
    IQ=IQ +1
    GO TO 170
155 DO 160 I=1, KP
160 X(I)=XB(I)
    IS=IS +1
    D(K)= D(K) + E(K)
    E(K)= 3.0* E(K)
    FO=F
    IF (N(K) -3) 170,165,165
165 N(K)=2
170 IF (IQ-KP) 175,185,185
175 IF (K-KP) 180,120,120
180 K=K+1
    GO TO 125
C   OUTPUT PRESENT RESULTS
185 WRITE (M,315) IS, IU, IC
    WRITE (M,320) FO, (X(I), I=1, KP)
    WRITE (M,325)
C   CHECK IF TERMINATION IS REQUIRED
    IF (IS+IU-NSTPS) 195,190,190
190 WRITE (M,330)
    RETURN
195 IF (NTYPE-1) 200, 210,200
200 DIFF= FFRST-FO
    PCT= 100.0*DIFF/FO
    IF (PCT-PCTMN) 205,205,215
205 WRITE (M,335)
    RETURN
210 IF (FO-EPS) 212,212,215
212 WRITE (M,340)
    RETURN
C   ROTATE AXES (V-MATRIX) TO IMPROVE PER-STEP EFFICIENCY
215 FFRST=FO
    DO 220 I=1, KP
    J=(KP-1)*KP+I

```

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HCLMB167



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220  AP(J)=D(KP)*V(J)
    DO 225 JJ=2,KP
      J=KP+1-JJ
    DO 225 I=1,KP
      I2=J*KP+I
      I1=I2-KP
225  AP(I1)= AP(I2)+D(J)*V(I1)
    DO 260 K=1,KP
      KPR=K-1
    DO 230 I=1,KP
      J=KPR*KP+I
230  BT(I)=AP(J)
      IF(KPR) 235,250,235
235  DO 245 L=1,KPR
      W=0.0
    DO 240 I=1,KP
      I1=KPR*KP+I
      I2=(L-1)*KP+I
240  W=W+AP(I1)*V(I2)
    DO 245 I=1,KP
      I2=(L-1)*KP+I
245  BT(I)=BT(I) - W*V(I2)
250  VP=0.0
    DO 255 I=1,KP
      VP=VP+BT(I)*BT(I)
255  DENOM=SQRT(VP)
    DO 260 I=1,KP
      I1=KPR*KP+I
260  V(I1)=BT(I)/DENOM
      IC=IC+1
    GO BACK AND REPEAT
    GO TO 110
      END

```

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 HCLMB195
 HCLMB196
 HCLMB197
 HCLMB198
 HCLMB199
 HCLMB200


```

SUBROUTINE DEL4Q(ERROR,P)
  DIMENSION P(5),MQ(12),EM(3),EML(3),EMINI(3)
  DIMENSION EMC(12,3),EMD(12),EME(12),EMF(12)
  DIMENSION EX(5),Z(5),DM(3,4),DT(5)
  DIMENSION EMI(3)
  EQUIVALENCE (EMC(1,1),EMD(1))
  EQUIVALENCE (EMC(1,2),EME(1))
  EQUIVALENCE (EMC(1,3),EMF(1))
  DATA MQ/6*1.0,1.3*0.1/
  DATA EMD/26.19,22.43,20.03,18.10,17.12,16.07,0.,14.54,3*0.,13.3/
  DATA EME/24.93,21.43,19.29,17.47,0.,15.55,0.,14.21,3*0.,13.12/
  DATA EMF/26.49,22.63,20.17,18.17,17.2,16.12,0.,14.64,3*0.,13.39/
  DATA EMI/28.63, 27.12, 28.97/
  C   EMD, EME, EMF, AND EMI ARE VECTORS OF THE EXPERIMENTAL DATA
  DATA VOL,RHO,HFG,CPP,AREA/0.5,72.0,1040.0,0.6,239.0/
  DATA EMAIR,THAIR/0.1107,98.0/
  DATA EMINI/.3605,.3377,.3813/
  C   DATA SWITCH 8 OUTPUT AT EACH STEP
  C   DATA SWITCH 9 OUTPUT VALUES DURING ITERATIONS
  C   DATA SWITCH 10 OUTPUT VALUES FROM CUBIC
  DECOF=HFG/(3.0*CPP)
  DEL=P(5)*AREA/(RHO*CPP*VOL)
  Z(4)=0.0
  Z(5)=-DEL
  ERROR=0.0
  DO 220 JQ=1,3
    EMBAR=EMINI(JQ)
    DO 90 I=1,3
      090 EML(I)=EMBAR
      TPL=98.0
      TIME=0.0
    C   THIS COMPLETES INITIALIZATION, SO WE ARE READY TO INTEGRATE
    300 FORMAT(3X,F8.4,12H HRS, AVE = ,F12.8,4X,3F11.8,4X,F12.6,
      1 16H = TEMP AND ERR ,F15.10)
    305 FORMAT(1X,F15.10,16H = ERROR AT P = ,5E15.7)
    310 FORMAT(20X,16HCUBIC OUTPUT, IZ,13,5X,4HERR ,6E12.4)

```

```

315 FORMAT(40X,16HTRIAL VALUES, IK,I3,4F12.6)
C   THE PARAMETERS ARE AS FOLLOWS
C   P(1)=A1/(DEL-R1)
C   P(2)=A2/(DEL-R2)
C   P(3)=DEL-R3
C   P(4)=H(M), OUTSIDE MASS TRANS COEFF
C   P(5)=H(H), OUTSIDE HEAT TRANS COEFF
C   DO 210 KHR=1,12
C     JMAX=2
C     IF(KHR-7)95,105,105
095   JMAX=2*JMAX
C     IF(KHR-3)100,105,105
100   JMAX=3*JMAX
105   DELT=1.0/FLOAT(JMAX)
C     DO 200 J=1,JMAX
C     DOLD=DIFF(TPL,EMBAR)
C     IF(IDTSW(8))115,115,110
110   X=TIME + 0.5E-4
C     WRITE(3,300) X,EMBAR, EML, TPL, ERROR
C     NOW ESTIMATE VALUES AT THE END OF THE INTERVAL
115   TP=TPL
C     DO 120 I=1,3
120   EM(I)=EML(I) - 0.2*DELT*(EML(I)-EMAIR)
C     IK=-1
C     EMNA=0.333333333*(EM(1) + EM(2) + EM(3))
C     RETURN POINT FOR IK LOOP
125   DIFUS=0.5*(DOLD+DIFF(TP,EMNA))
C     IF(IDTSW(9))135,135,130
130   WRITE(3,315)IK,EM,TP
C     NOW CALCULATE COEFFICIENTS IN ORIGINAL DIFF EQUATION
135   AA=3.0*DIFUS/VOL
C     BE=P(2)*AA
C     SEA=AA*AREA*P(4)/(DIFUS*RHO+P(4)*P(3))
C     AA=P(1)*AA
C     ALF=AA+BE
C     BET=BE+SEA

```

```

DE=DECOF*SEA
QUE=DE*EMAIR+DEL*THAIR
C  NOW GET THE EIGENVALUE POLYNOMIAL COEFFICIENTS
    BZ2=AA+ALF+BET
    BZW=AA*(ALF+BET-AA)+ALF*BET-BE*BE
    BZZ=AA*( BET*(ALF-AA) - BE*BE)
    CALL CUBIC (Z(1),Z(2),Z(3),BZ2,BZW,BZZ,ERR,IZ)
    IF(100*IDTSW(10)-IZ)140,140,150
140 IF(ERR+Z(1)*5.0E-7)160,150,150
150 WRITE(3,310)IZ,ERR,Z
160 DO 180 I=1,5
    ZZ=Z(I)
    EX(I)=EXP(DELT*ZZ)
    ZA= ZZ + AA
    ZB=ZZ + ALF
    ZC=ZZ + BET
    EM3Z=ZZ*EML(3)+SEA*EMAIR
    ZQ=ZZ*(AA*EML(1)+ZA*EML(2))
    X=BE*ZQ+(ZA*ZB-AA*AA)*EM3Z
    DT(I)=-DE*X
    IF(I-3)175,175,165
165 FOS=((ZZ+BZ2)*ZZ+BZW)*ZZ+BZZ
    DT(I)=DT(I)+FOS*(ZZ*TPL+QUE)
    IF(I-5)175,170,170
170 DT(I)=DT(I)/(ZZ*FOS)
    GO TO 180
175 DENP=ZZ+ZZ+BZ2
    DENP=((DENP+DENP+BZ2)*ZZ+BZW+BZW)*ZZ+BZZ
    DT(I)=DT(I)/(DENP*(ZZ+DEL))
    DM(3,I)=X/DENP
    X=ZZ*((ZB*ZC-BE*BE)*EML(1)+ZC*AA*EML(2))+AA*BE*EM3Z
    DM(1,I)=X/DENP
    DM(2,I)=(ZC*ZQ+BE*ZA*EM3Z)/DENP
180 CONTINUE
    TP=DT(4)
    DO 185 I=1,3

```

```

EM(I)=DM(I,1)*EX(1)
TP=TP+DT(I)*EX(I)
DO 185 K=2,4
  185 EM(I)=EM(I)+DM(I,K)*EX(K)
TP=TP+DT(5)*EX(5)
EMNA=0.333333333*(EM(1)+EM(2)+EM(3))
C   NOW HAVE NEW ESTIMATE AT THE END OF THE TIME INTERVAL
  IK=IK+1
  IF(IK)125,125,190
  190 DO 195 I=1,3
  195 EML(I)=EM(I)
  TPL=TP
  EMBAR=EMNA
  TIME=TIME + DELT
  IF ( ABS(TIME-0.5) -1.0E-3 ) 197,197,200
  197 ERR=100.0*EMBAR-EMH(JQ)
  ERROR=ERROR + ERR*ERR*TIME
  200 CONTINUE
  IF(MQ(KHR)) 210,210,203
  203 IF(KHR-JQ-10) 205,210,205
  205 ERR=100.0*EMBAR-EMC(KHR,JQ)
  ERROR=ERROR + ERR*ERR*TIME
  210 CONTINUE
  IF(IDTSW(8))220,220,215
  215 X=TIME + 0.5E-4
  WRITE(3,300) X,EMBAR, EML, TPL, ERROR
  220 WRITE(3,305)ERROR,P
  RETURN
  END

```

```

C   SUBROUTINE CUBIC(Z1,Z2,Z3, C2, CW, CZ, ERR, I)
C   SOLVES THE EQUATION  $Z^3 + C2Z^2 + CWZ + CZ = 0$  WHEN ALL THE
C   ROOTS ARE REAL.
C   I IS EQUAL TO THE NUMBER OF ITERATIONS IN THE COMPUTATION OF THE
C   MIDDLE ROOT, Z2. IF I IS NEG, ALL ROOTS NOT REAL.

```






```

X=F/(DF-F*HD2F/DF)
J=J+1
XN=XN - X
IF(ABS(X) - 0.5E-8*ABS(XN)) 180,170,170
170 IF(J-15) 165,165,175
175 J=100*(IFG+4)
180 IFG=IFG + 1
I=I+J
IF(IFG) 125,150,155
155 Z3=XN
C NOW COMPUTE ERROR RETURN
ERR=ABS(Z1*(Z2+Z3) + Z2*Z3 - CW)
ERR=ERR + ABS(Z1+Z2+Z3+C2) + ABS(Z1*Z2*Z3 + CZ)
RETURN
END

```

APPENDIX B

STEADY-STATE PROGRAM

APPENDIX B

STEADY-STATE PROGRAM

The programs presented here are used to find h and h'_m for a steady-state corn dryer by fitting sets of experimental data. Three programs are shown here: WHC25 is a calling program for HCLMB, the optimization routine used. SSCAL is used to store the data, call MBCDS, calculate the error function, and return to HCLMB. MBCDS solves the 6 ordinary differential equations for the steady-state moving bed corn dryer. Three other routines are not shown: HCLMB and CUBIC are found in Appendix A. EMAIR is found in Appendix D. The variables correspond as much as possible to those of DCMCD in Appendix D. C2 and C3 in DCMCD correspond to Equation 88.


```

PROGRAM WHC25
EXTERNAL SSCAL
DIMENSION V(9), D(3), E(3), N(3), AP(9)
DIMENSION XB(3), BT(3), XLB(3), XUB(3), X(3)
DATA XLB / 0.0, 0.0, 0.2 /
DATA XUB / 10.0, 10.0, 0.8 /
FORMAT(15,5X,7F10.5)
PCTMN=1.0E-5
EPS=0.0
NTYPE=0
KP=2
READ(2,200) NSTPS, X
CALL HCLMB ( KP, PCTMN, EPS, NSTPS, NTYPE, AP, BT,
1 D, E, N, V, X, XB, XLB, XUB, SSCAL )
CALL EXIT
END

200

SUBROUTINE SSCAL (E,X)
DIMENSION X(3), TA7(10), EMZ7(10), GA7(10), GC7(10)
DIMENSION EML5(10), TPL5(10), I77(10), HA7(10), TC7(10)
COMMON TA, TC, GA, GC, HA, EMZ
COMMON IDTSW(12)
DATA L / 0 /
200 FORMAT(10X,6HERR = ,E15.8, 10X, 4HX = ,4F15.9)
210 FORMAT(10,F10.1,F11.5,F11.6,F11.1,2F11.5,3F10.1,F10.2)
220 FORMAT(6X,4HCODE,7X,2HTA,6X,5HM-I-X,6X,5HAIR-H,6X,5H T-C ,6X,
35HM-L-S,6X,5HM-L-X,7X,3HT-S,7X,3HT-X,8X,2HGA,8X,2HGC)
230 FORMAT(20X,/)
240 FORMAT(15I2)
250 FORMAT(15,5X,7F10.5)
260 FORMAT(10X,7F10.5)
ZMAX=2.0
IF(L) 90,70,90
70 L=5
XQ=0.0

```



```

      READ(2,240) NRUN, IDTSW
      WRITE(3,230)
      WRITE(3,220)
      DO 80 I=1,NRUN
      READ(2,250) I77(I), EMZ7(I), EML5(I), TA7(I), GA7(I), GC7(I)
      READ(2,260) TPL5(I), HA7(I), TC7(I)
      WRITE(3,210) I77(I), TA7(I), EMZ7(I), HA7(I), TC7(I), XQ
      2, EML5(I), XQ, TPL5(I), GA7(I), GC7(I)
      80 CONTINUE
      90 HHCOF=X(1)
      HMRAT=X(2)
      HHEXP=X(3)
      C DATA SWITCH 11 CAUSES THE RESULTS TO BE PRINTED
      IF(IDTSW(11)) 110,110,100
      100 WRITE(3,230)
      WRITE(3,220)
      110 E=0.0
      DO 150 I=1,NRUN
      TA=TA7(I)
      TC=TC7(I)
      HA=HA7(I)
      GA=GA7(I)
      GC=GC7(I)
      EMZ=EMZ7(I)
      CALL MBCDS ( ZMAX, HHCOF, HHEXP, HMRAT )
      IF(IDTSW(11)) 130,130,120
      120 WRITE(3,210)I77(I),TA7(I),EMZ7(I), HA, TC, EMZ,
      2 EML5(I),TA,TPL5(I), GA7(I), GC7(I)
      130 E=E + 0.01*ABS(TA-TPL5(I))
      E=E + 200.0*ABS(EMZ - EML5(I))
      150 CONTINUE
      WRITE(3,200) E, X
      RETURN
      END

```

```

SUBROUTINE MBCDS (ZMAX, HHCOF, HHEXP, HMRAT)

SUBROUTINES REQUIRED . . .
EMAIR(T,H) EQUILIB MOISTURE CONTENT CORR TO AIR AT T AND H
CUBIC COMPUTES THE ROOTS OF A 3-RD ORDER POLYNOMIAL

DATA SWITCH
2-4 EFFECT
7 ADD ITERATIONS ON INSIDE LOOP
8 START OVER
9 OUTPUT ALL VALUES (END OF EACH STEP)
10 OUTPUT TRIAL VALUES DURING STEPS
12 OUTPUT EIGENVALUES, RESULTS FROM QUART
    OUTPUT BOTTOM COEFFICIENTS

DIMENSION UZ(6), UL(6), R(4), IP2(4), TPW(4), TPZ(4)
DIMENSION TQ4(2), TQ3(2), TQ2(2), TQW(2), TQZ(2)
DIMENSION LNTH(5), MAX(5)
COMMON TA,TC,GA,GC,HA,EMZ
COMMON IDTSW(12)

DATA EPS,AFA1,AFA2,DR3 / 0.45,12.58E4,9.46E4,3.94E-3 /
DATA HFG, ARA3 / 1040.0, 239.0 /
DATA CPC, CPA, RHOC / 0.6, 0.24, 72.0 /
DATA MAX / 10,10,14,10,40 /
DATA LNTH / 1, 2, 5, 10, 20 /
DIFF(EM,T)=1.629E-3*EXP(EM*(0.025*T+6.008)-4523.4/(T+459.7))

FORMAT(1H2,/,20X,31HTHE STEADY-STATE RESULTS ARE... )
FORMAT(/,5X,8HPOSITION,12X,5HEMBAR,10X,4HM(1),10X,4HM(2),10X,
1 4HM(3),9X,8HHUMIDITY,5X,9HCORN-TEMP,6X,8HAIR-TEMP)
FORMAT(5X,F8.5,9X,4(F9.6,5X),F10.7,4X,F10.4,4X,F10.4)
FORMAT(4X,16HTRIAL FOR JTTR =,I3,2X,4(F9.6,5X),F10.7,2(4X,F10.4))
FORMAT(4H IER,I6,4HERR,E12.4,4H R=,6E15.7)
FORMAT(30X,4E18.8)

```

```

UZ(1)=EMZ
UZ(2)=EMZ
UZ(3)=EMZ
UZ(4)=HA
UZ(5)=TC
UZ(6)=TA
EMBAR=EMZ

C
  EPSP=2.0*(1.0-EPS)
  AREA=ARA3*EPSP
  ALFA1=AFA1*EPSP
  ALFA2=AFA2*EPSP
  B1CF=3.0*ALFA1*RHO C/GC
  B2CF=3.0*ALFA2*RHO C/GC
  B3CF=3.0*AREA/GC
  B4CF=AREA/GA
  B5CF=HFG*AREA/(GC*CPC)
C  PUT IN HMP AND HH AS FUNCTIONS OF GA HERE, IF DESIRED
  HH=HHCOF*GA**HHEXP
  HMP=HMRAT*HH
  EUCF=HMP*DR3/RHO C
  C5=AREA*HH/(GC*CPC)
  C6=AREA*HH/(GA*CPA)
  ALF7=C5 + C6
  R(4)=-ALF7
DO 145 I=1,6
  UL(I)=UZ(I)
C  NOW ENTER THE Z-MOVING LOOP
  Z=0.0
C  THIS IS THE ZERO DISTANCE VALUE OUTPUT
  IF(IDTSW(8)) 147,147,146
146 WRITE(3,420)
  WRITE(3,421)
  WRITE(3,425) Z,EMBAR,UL
147 DO 265 ISTEP=1,4
  DELZ=0.01*FLOAT(LNTH(ISTP))

```

```

C
LMAX=MAX(ISTP)
DO 265 JSTP=1,LMAX
  IF(ZMAX - (Z+DELZ+1.0E-4)) 155,160,160
155 DELZ=ZMAX - Z
160 CONTINUE
C
  C WE ALSO WANT THE DIFFUSIVITY AND EMAIR AT THE FRONT
  C2I=EMAIR(UZ(6),UZ(4))/UZ(4)
  DFH=DIFF(EMBAR,UZ(5))
  C ONE MAY ESTIMATE THE END VALUES HERE, IF DESIRED.
  C NO ACTION IS NEEDED IF THIS FIRST ESTIMATE IS TO BE THE
  C SAME AS THE FRONT OF THE INTERVAL.
  C ENTER THE ITERATION LOOP BETWEEN 165 AND 240
  J1TR=-(IDTSW(2) + IDTSW(3) + IDTSW(4) + 2 )
165 J1TR=J1TR + 1
  IF(J1TR) 170,250,250
170 DIF=0.5*(DFH+DIFF(EMBAR,UL(5)))
  B1=B1CF*DIF
  B2=B2CF*DIF
  EU=HMP*DIF/(DIF + EUCF)
  B3=B3CF*EU
  B4=B4CF*EU
  B5=B5CF*EU
C
  C3=0.0
  C2=0.5*(EMAIR(UL(6),UL(4))/UL(4) + C2I)
C
  ALF2=B1 + B2
  ALF3=B2 + B3
  ALF4=B4*C2
  B32=B3*C2
  AFWPF= B1 + ALF4
  AFWPT= B1 + ALF2
  AF34=ALF3*ALF4
  AF124=AFWPF + ALF2

```

```

AF123=AFWPT + ALF3
AF14= B1*ALF4
AF134=AFWPF + ALF3
AF3PF=ALF3 + ALF4
AF234=AF3PF + ALF2
B2S=B2*B2
AF128=B1*B2
AF348=B2*ALF4
C32U=B3*(C3 + C2*UZ(4))
SUM=UZ(1) + UZ(2) + UZ(3)
XQ=B2*(B3 + ALF4)
XQA=B1*AF3PF + XQ + AF348
XQB=B1*AF3PF + AF348
XQC=AF128 + ALF4*AFWPT
XQD=B3*AFWPT + AF128 + AF128 + AF128
B33=B3*C3
NOW COMPUTE THE BOTTOM COEFFICIENTS
BM3=AF123 + ALF4
BM2=AF128 + AF348 + AF3PF*(B1 + B1) + XQ
BMW=AF128*(B3 + ALF4 + ALF4 + ALF4)
IF(IDTSW(12)) 188,188,187
WRITE(3,445) BM3, BM2, BMW
187
C
C
C
C
188
TP2(1)=UZ(1)*AF234 + UZ(2)*B1
TPW(1)=UZ(1)*XQA + B1*(AF3PF*UZ(2) + B2*UZ(3))
TPZ(1)=AF128*(ALF4*SUM + C32U)
C
C
C
C
COMPUTE THE TOP COEFFICIENTS FOR M2
TP2(2)=UZ(1)*B1 + UZ(2)*AF134 + UZ(3)*B2
TPW(2)=UZ(1)*B1*AF3PF + UZ(2)*XQB + B2*(AFWPF*UZ(3) + C32U)
TPZ(2)=TPZ(1)
C

```



```

C      COMPUTE THE TOP COEFFICIENTS FOR M3
      TP2(3)=UZ(2)*B2 + AF124*UZ(3) + C32U
      TPW(3)=B2*(B1*UZ(1) + AFWPF*UZ(2)) + UZ(3)*XQC + AFWPT*C32U
      TPZ(3)=TPZ(1)

C
C      COMPUTE THE TOP COEFFICIENTS FOR H
C
      TP2(4)=B4*(UZ(3) - C3) + AF123*UZ(4)
      TPW(4)=B4*(B2*UZ(2) + AFWPT*UZ(3) - C3*(ALF2+ALF2)) + UZ(4)*XQD
      TPZ(4)=AF12B*(B3*UZ(4) + B4*(SUM-(C3+C3+C3)))

C      COMPUTE THE BOTTOM COEFFICIENTS FOR TC AND TA
C
      BN3=ALF7 + BM3
      BN2=ALF7*BM3 + BM2
      BNW=ALF7*BM2 + BMW
      BNZ=ALF7*BMW
      DZNQ=DELZ - BNW/BNZ

C      THE FOLLOWING SUBSTITUTIONS ARE USED.
C
      XQF=C6*UZ(5) + C5*UZ(6)
      XQE=XQF + B5*C3
      B536=B5*C3*C6
      THE TOP OF TC AND TA ARE NOW FILLED, IN THREE STEPS
      TQ3(2)=B5*(C2*UZ(4) - UZ(3))
      TQ2(2)=B5*(C2*TP2(4) - TP2(3))
      TQW(2)=B5*(C2*TPW(4) - TPW(3))
      TQZ(2)=B5*(C2*TPZ(4) - TPZ(3))

C      NOW DO THE SECOND STEP
C
      TQ4(1)=TQ3(2) + BM3*UZ(5) + XQE
      TQ3(1)=TQ2(2) + C6*TQ3(2) + BM2*UZ(5) + BM3*XQE + B536
      TQ2(1)=TQW(2) + C6*TQ2(2) + BMW*UZ(5) + BM2*XQE + BM3*B536
      TQW(1)=TQZ(2) + C6*TQW(2) + BMW*XQE + BM2*B536
      TQZ(1)=C6*TQZ(2) + BMW*B536

```

```

C      THE LAST STEP FILLS THE TOP OF TA.
      TQ4(2)=BM3*UZ(6) + XQF
      TQ3(2)=BM2*UZ(6) + BM3*XQF + B536 + C6*TQ3(2)
      TQ2(2)=BMW*UZ(6) + BM2*XQF + BM3*B536 + C6*TQ2(2)
      TQW(2)=BMW*XQF + BM2*B536 + C6*TQW(2)
      TQZ(2)=BMW*B536 + C6*TQZ(2)

C
C      NOW GET THE EIGENVALUES, ROOTS 1-4
      CALL CUBIC(R(1), R(2), R(3), BM3, BM2, BMW, ERR, IER)
      IF(IDTSW(10)) 190,190,200
190    IF(ABS(ERR)-1.0E-6*ABS(R(1))) 195,200,200
195    IF(IER) 200,200,205
200    WRITE(3,435) IER,ERR,R
205    IF(IDTSW(7)) 207,207,270
C
C      NOW THE END VALUES MUST BE INITIALIZED
      XQ=TPZ(1)/BMW
      UL(1)=XQ
      UL(2)=XQ
      UL(3)=XQ
      UL(4)=TPZ(4)/BMW
      UL(5)=(TQZ(1)*DZNQ + TQW(1))/BNZ
      UL(6)=(TQZ(2)*DZNQ + TQW(2))/BNZ

C
C      THIS COMPLETES THE INITIALIZATION
C      NOW COMPLETE FILLING UL
      DO 235 KI=1,4
      S=R(KI)
      S2=S*S
      EX=EXP(S*DELZ)
      XQ=S+S+S+BN3 + BN3
      XQA=BN2 + BN2
      XQA=XQA + XQA
      XQ=((XQ+XQ+BN3)*S + XQA )#S + BNW+BNW+BNW
      EXN=EX/(S*(XQ*S + BNZ + BNZ))
      IF(KI-4) 220,230,230

```



```

220 XQ=S + S + BM3
    XQ=((XQ + XQ + BM3)*S + BM2 + BM2)*S + BMW
    EX=EX/XQ
    DO 225 I=1,4
    XQ=TP2(I)*S2 + TPZ(I)
    XQ=XQ +(UZ(I)*S2 + TPW(I))*S
225  UL(I)=UL(I) + XQ*EX
230  DO 235 I=1,2
    IP=I+4
    XQ=(TQ4(I)*S2 + TQ2(I))*S2 + TQZ(I)
    XQ=XQ + ((UZ(IP)*S2 + TQ3(I))*S2 + TQW(I))*S
235  UL(IP)=UL(IP) + XQ*EXN
C
C   AT THIS POINT THE NEW ESTIMATE IS AVAILABLE.
    EMBAR=0.3333333333*(UL(1)+UL(2)+UL(3))
    IF(IDTSW(9)) 165,165,245
245  WRITE(3,430) J1TR,EMBAR,UL
    GO TO 165
C   AT THIS POINT THE Z-STEP IS COMPLETED
C   THEN GO BACK FOR ANOTHER STEP
250  CONTINUE
    DO 255 I=1,6
255  UZ(I)=UL(I)
    Z=Z + DELZ
    IF(IDTSW(8)) 262,262,260
260  XQ=Z + 0.5E-5
    WRITE(3,425) XQ,EMBAR,UL
262  IF(Z + 1.0E-4 - ZMAX) 265,265,270
265  CONTINUE
270  EMZ=EMBAR
    HA=UL(4)
    TC=UL(5)
    TA=UL(6)
    RETURN
    END

```

APPENDIX C

SINGLE-STEP SOLUTION FOR THE
DYNAMIC SIMULATION

APPENDIX C

SINGLE-STEP SOLUTION FOR THE DYNAMIC SIMULATION

The problem here is to solve Equation (91);
Cramer's Rule was used.

$$\begin{bmatrix} S+A_1 & -B_1 & 0 & 0 & 0 & 0 \\ -B_1 & S+A_2 & -B_2 & 0 & 0 & 0 \\ 0 & -B_2 & S+A_3 & -B_3C_2 & 0 & 0 \\ 0 & 0 & -B_4 & S+A_4 & 0 & 0 \\ 0 & 0 & B_5 & -B_5C_2 & S+A_5 & -C_5 \\ 0 & 0 & 0 & 0 & -C_6 & S+A_6 \end{bmatrix} \begin{bmatrix} \bar{M}_1 \\ \bar{M}_2 \\ \bar{M}_3 \\ \bar{H} \\ \bar{T}_c \\ \bar{T}_a \end{bmatrix} = \frac{1}{s^2} \begin{bmatrix} s^2M_1^o + SC_1M_1^{PZ} + C_1M_1^{PD} \\ s^2M_2^o + SC_1M_2^{PZ} + C_1M_2^{PD} \\ s^2M_3^o + SC_1M_3^{PZ} + C_1M_3^{PD} \\ s^2H^o + SC_4H^{PZ} + C_4H^{PD} \\ s^2T_c^o + SC_1T_c^{PZ} + C_1T_c^{PD} \\ s^2T_a^o + SC_4T_a^{PZ} + C_4T_a^{PD} \end{bmatrix} \quad (91)$$

The top four rows of this equation are independent,
so they can be solved by themselves for $\bar{M}_1$, $\bar{M}_2$, $\bar{M}_3$, and $\bar{H}$.

$$\begin{bmatrix} S+A_1 & -B_1 & 0 & 0 \\ -B_1 & S+A_2 & -B_2 & 0 \\ 0 & -B_2 & S+A_3 & -B_3C_2 \\ 0 & 0 & -B_4 & S+A_4 \end{bmatrix} \begin{bmatrix} \bar{M}_1 \\ \bar{M}_2 \\ \bar{M}_3 \\ \bar{H} \end{bmatrix} = \frac{1}{s^2} \begin{bmatrix} s^2M_1^o + SC_1M_1^{PZ} + C_1M_1^{PD} \\ s^2M_2^o + SC_1M_2^{PZ} + C_1M_2^{PD} \\ s^2M_3^o + SC_1M_3^{PZ} + C_1M_3^{PD} \\ s^2H^o + SC_4H^{PZ} + C_4H^{PD} \end{bmatrix}$$

Each will be expressed as the ratio of two polynomials in S . The denominator polynomial for each will be the same, the product of S^2 and the determinate of the left matrix.

$$S^2 \begin{vmatrix} S+A_1 & -B_1 & 0 & 0 \\ -B_1 & S+A_2 & -B_2 & 0 \\ 0 & -B_2 & S+A_3 & -B_3C_2 \\ 0 & 0 & -B_4 & S+A_4 \end{vmatrix} = S^2 (S+A_1) \begin{vmatrix} S+A_2 & -B_2 & 0 \\ -B_2 & S+A_3 & -B_3C_2 \\ 0 & -B_4 & S+A_4 \end{vmatrix} + B_1 S^2 \begin{vmatrix} -B_1 & 0 & 0 \\ -B_2 & S+A_3 & -B_3C_2 \\ 0 & -B_4 & S+A_4 \end{vmatrix}$$

$$= (S+A_1) \{ (S+A_2)(S+A_3)(S+A_4) - B_4B_3C_2(S+A_2) - B_2^2(S+A_4) \}$$

$$+ B_1 \{ -B_1(S+A_3)(S+A_4) + B_1B_4B_3C_2 \}$$

$$= (S+A_1)(S+A_2)(S+A_3)(S+A_4) - B_4B_3C_2(S+A_1)(S+A_2)$$

$$- B_2^2(S+A_1)(S+A_4) - B_1^2(S+A_3)(S+A_4) + B_1^2B_4B_3C_2$$

$$= \{ (S+A_1)(S+A_2) - B_1^2 \} \{ (S+A_3)(S+A_4) - B_4B_3C_2 \}$$

$$- B_2^2(S+A_1)(S+A_4)$$

$$\begin{aligned}
&= S^2 [S^4 + S^3 (A_1 + A_2 + A_3 + A_4) \\
&\quad + S^2 \{ (A_1 + A_2) (A_3 + A_4) + A_1 A_2 - B_1^2 + A_3 A_4 - B_4 B_3 C_2 - B_2^2 \} \\
&\quad + S \{ (A_1 A_2 - B_1^2) (A_3 + A_4) + (A_3 A_4 - B_4 B_3 C_2) - B_2^2 (A_1 + A_4) \} \\
&\quad + \{ (A_1 A_2 - B_1^2) (A_3 A_4 - B_4 B_3 C_2) - B_2^2 A_1 A_4 \}]
\end{aligned}$$

Then the denominator polynomial is

$$S^2 (S^4 + S^3 \cdot BM3 + S^2 \cdot BM2 + S \cdot BMW + BMZ) = \text{Denominator}$$

$$\text{where } BM3 = A_1 + A_2 + A_3 + A_4$$

$$BM2 = A_1 + A_2 - B_1^2 + A_3 + A_4 - B_4 B_3 C_2 - B_2^2 + A_1 A_2 + A_3 A_4$$

$$BMW = (A_1 A_2 - B_1^2) (A_3 + A_4) + (A_3 A_4 - B_4 B_3 C_2) (A_1 + A_2) - B_2^2 (A_1 + A_4)$$

$$BMZ = (A_1 A_2 - B_1^2) (A_3 A_4 - B_4 B_3 C_2) - B_2^2 A_1 A_4$$

As an example the numerator polynomial for $\bar{M}_1$ is as follows:

$$\begin{array}{l}
\text{Numerator} \\
\text{of } \bar{M}_1 =
\end{array}
\left| \begin{array}{lll}
S^2 M_1^\circ + SC_1 M_1^{PZ} + C_1 M_1^{PD} & -B_1 & 0 & 0 \\
S^2 M_2^\circ + SC_1 M_2^{PZ} + C_1 M_2^{PD} & S+A_2 & -B_2 & 0 \\
S^2 M_3^\circ + SC_1 M_3^{PZ} + C_1 M_3^{PD} & -B_2 & S+A_3 & -B_3 C_2 \\
S^2 H^\circ + SC_4 H^{PZ} + C_4 H^{PD} & 0 & -B_4 & S+A_4
\end{array} \right|$$

$$\begin{aligned}
&= (S^2 M_1^{\circ} + SC_1 M_1^{PZ} + C_1 M_1^{PD}) \begin{vmatrix} S+A_2 & -B_2 & 0 \\ -B_2 & S+A_3 & -B_3 C_2 \\ 0 & -B_4 & S+A_4 \end{vmatrix} \\
&\quad + B_1 \begin{vmatrix} S^2 M^{\circ} + SC_1 M^{PZ} + C_1 M^{PD} & -B_2 \\ S^2 M^{\circ} + SC_1 M^{PZ} + C_1 M^{PD} & S+A_3 & -B_3 C_2 \\ S^2 H^{\circ} + SC_4 H^{PZ} + C_4 H^{PD} & -B_4 & S+A_4 \end{vmatrix} \\
&= (S^2 M^{\circ} + SC_1 M^{PZ} + C_1 M^{PD}) \{ (S+A_2) (S+A_3) (S+A_4) - B_2^2 (S+A_4) - B_4 B_3 C_2 (S+A_2) \} \\
&\quad + (S^2 M_2^{\circ} + SC_1 M_2^{PZ} + C_1 M_2^{PD}) \cdot B_1 \cdot \{ (S+A_3) (S+A_4) - B_4 B_3 C_2 \} \\
&\quad + (S^2 M_3^{\circ} + SC_1 M_3^{PZ} + C_1 M_3^{PD}) \cdot B_1 \cdot B_2 (S+A_4) \\
&\quad + (S^2 H^{\circ} + SC_4 H^{PZ} + C_4 H^{PD}) \cdot B_1 \cdot B_2 B_3 C_2
\end{aligned}$$

This will be put in the form of a power series in S.

Here the coefficient subscript refers to the dependent variable.

$$= S^5 \cdot M_1^{\circ} + S^4 \cdot TP4(1) + S^3 \cdot TP3(1) + S^2 \cdot TP2(1) + S \cdot TPW(1) + TPZ(1)$$

$$TP4(1) = C_1 M_1^{PZ} + (A_2 + A_3 + A_4) \cdot M_1^{\circ} + B_1 \cdot M_2^{\circ}$$

$$\begin{aligned}
\text{TP3}(1) &= C_1 M_1^{\text{PD}} + (A_2 + A_3 + A_4) \cdot C_1 M_1^{\text{PZ}} \\
&\quad + M_1^{\circ} (A_3 A_4 - B_4 B_3 C_2 - B_2^2 + A_2 (A_3 + A_4)) \\
&\quad + B_1 (C_1 M_2^{\text{PZ}} + (A_3 + A_4) \cdot M_2^{\circ} + B_2 \cdot M_3^{\circ})
\end{aligned}$$

$$\begin{aligned}
\text{TP2}(1) &= (A_2 + A_3 + A_4) \cdot C_1 M_1^{\text{PD}} + (A_3 A_4 - B_4 B_3 C_2 - B_2^2 + A_2 (A_3 + A_4)) \cdot C_1 M_1^{\text{PZ}} \\
&\quad + (A_2 (A_3 \cdot A_4 - B_4 B_3 C_2) - B_2^2 \cdot A_4) \cdot M_1^{\circ} + B_1 \cdot (C_1 M_2^{\text{PD}} + (A_3 + A_4) \cdot C_1 M_2^{\text{PZ}} \\
&\quad + (A_3 \cdot A_4 - B_4 B_3 C_2) \cdot M_2^{\circ} + B_2 (C_1 M_3^{\text{PZ}} + A_4 M_3^{\circ} + B_3 C_2 \cdot H^{\circ}))
\end{aligned}$$

$$\begin{aligned}
\text{TPW}(1) &= (A_3 A_4 - B_4 B_3 C_2 - B_2^2 + A_2 (A_3 + A_4)) \cdot C_1 M_1^{\text{PD}} \\
&\quad + (A_2 (A_3 A_4 - B_4 B_3 C_2) - B_2^2 A_4) C_1 M_1^{\text{PZ}} \\
&\quad + B_1 \cdot ((A_3 + A_4) \cdot C_1 M_2^{\text{PD}} + (A_3 A_4 - B_4 B_3 C_2) \cdot C_1 M_2^{\text{PZ}} \\
&\quad + B_2 \cdot (C_1 M_3^{\text{PD}} + A_4 C_1 M_3^{\text{PZ}} + B_3 C_2 C_4 H^{\text{PZ}}))
\end{aligned}$$

$$\begin{aligned}
\text{TPZ}(1) &= (A_2 (A_3 A_4 - B_4 B_3 C_2) - B_2^2 A_4) \cdot C_1 M_1^{\text{PD}} \\
&\quad + B_1 ((A_3 A_4 - B_4 B_3 C_2) C_1 M_2^{\text{PD}} \\
&\quad + B_2 (A_4 \cdot C_1 M_3^{\text{PD}} + B_3 C_2 \cdot C_4 H^{\text{PD}}))
\end{aligned}$$

The other three numerator polynomials for $\bar{M}_2$, $\bar{M}_3$ and $\bar{H}$ are found in the same manner. Then the last two equations are solved in terms of $\bar{M}_3$ and $\bar{H}$. Again Cramer's Rule is used. The solution below is for $\bar{T}_c$.

$$\begin{bmatrix} S+A_5 & -C_5 \\ -C_6 & S+A_6 \end{bmatrix} \cdot \begin{bmatrix} T_c \\ T_a \end{bmatrix} = \frac{1}{S^2} \begin{bmatrix} S^2 T^\circ + SC_1 T_c^{PZ} + C_1 T_c^{PD} + S^2 (B_5 C_2 \bar{H} - B_5 \bar{M}_3) \\ S^2 T_a^\circ + SC_4 T_a^{PZ} + C_4 T_a^{PD} \end{bmatrix}$$

$$\bar{T}_c = \frac{(S+A_6) \{S^2 T_c^\circ + SC_1 T_c^{PZ} + C_1 T_c^{PD}\} + C_5 \{S^2 T_a^\circ + SC_4 T_a^{PZ} + C_4 T_a^{PD}\} + S^2 B_5 (C_2 \bar{H} - \bar{M}_3)}{S^2 (S^2 + (A_5+A_6) \cdot S + A_5 A_6 - C_5 C_6)}$$

The term involving $\bar{H}$ and $\bar{M}_3$ is as follows

$$S^2 (C_2 \bar{H} - \bar{M}_3) = \frac{S^5 (C_2 H^\circ - M_3^\circ) + S^4 (C_2 TP4(4) - TP4(3)) + \dots + (C_2 \cdot TPZ(4) - TPZ(3))}{S^4 \cdot S^3 \cdot BM3 + S^2 \cdot BM2 + S \cdot BMW + BMZ}$$

Then in order to make this substitution the top and bottom of the $\bar{T}_c$ equation are multiplied by the denominator above. Then the denominator of $\bar{T}_c$ and $\bar{T}_a$ is

$$\begin{aligned} \text{Denominator of } \bar{T}_c, \bar{T}_a &= S^2 \cdot (S^2 + (A_5+A_6) \cdot S + A_5 A_6 - C_5 C_6) \\ &\quad \cdot (S^4 + BM3 \cdot S^3 + BMZ \cdot S^2 + BMW \cdot S + BMZ) \end{aligned}$$

And the numerator of T_c may be found

$$\begin{aligned}
\text{Numerator of } \bar{T}_C = & ((S+A_6)(S^2 T_C^\circ + SC_1 T_C^{PZ} + C_1 T_C^{PD}) \\
& + C_5 (S^2 T_a^\circ + SC_4 T_a^{PZ} + C_4 T_a^{PD})) \\
& \cdot (S^4 + BM3 \cdot S^3 + BM2 \cdot S^2 + BMW \cdot S + BMZ) \\
& + B_5 S^5 (C_2 H^\circ - M_3^\circ) + S^4 (C_2 \cdot TP4(4) - TP4(3)) \\
& + \dots + (C_z \cdot TPZ(4) - TPZ(3))
\end{aligned}$$

The numerator is then put in a polynomial form

$$\begin{aligned}
\text{Numerator of } \bar{T}_C = & S^2 \cdot T_C^\circ + S^6 \cdot TQ6(1) + S^5 \cdot TW5(1) + S^4 \cdot TQ4(1) \\
& + S^3 \cdot TW3(1) + S^2 \cdot TW2(1) + S \cdot TQW(1) + TQZ(1)
\end{aligned}$$

where

$$TQ6(1) = TCW + BM3 \cdot T_C^\circ$$

$$TQ5(1) = TCT + TCW \cdot BM3 + BM2 \cdot T_C^\circ$$

$$TQ4(1) = TC3 + TCT \cdot BM3 + TCW \cdot BM2 + BMW \cdot T_C^\circ$$

$$TQ3(1) = TC3 \cdot BM3 + TCT \cdot BM2 + TCW \cdot BMW + BMZ \cdot T_C^\circ$$

$$TQ2(1) = TC3 \cdot BM2 + TCT \cdot BMW + TCW \cdot BMZ$$

$$TQW(1) = TC3 \cdot BMW + TCT \cdot BMZ$$

$$TQZ(1) = TC3 \cdot BMZ$$



and

$$TCW = A_6 \cdot T_C^o + C_5 \cdot T_a^o + C_1 T_C^{PZ}$$

$$TCT = A_6 \cdot C_1 T_C^{PZ} + C_1 T_C^{PD} + C_5 C_4 T_a^{PZ}$$

$$TC3 = A_6 \cdot C_1 T_C^{PD} + C_5 \cdot C_4 T_a^{PD}$$

The inverse Laplace transformation is as given in Equations 90 and 91. The value of M_1 at the end of a z-step is as follows:

$$M_1(\Delta Z) + \frac{TPZ(1)}{BMZ} \cdot \Delta Z + \frac{BMZ \cdot TPW(1) - TPZ(1) \cdot BMW}{(BMZ)^2}$$

$$+ \sum_{i=1}^4 \frac{(r_i^5 \cdot M_1^o + r_i^4 \cdot TP4(1) + r_i^3 \cdot TP3(1) + r_i^2 \cdot TP2(1) + r_i \cdot TPW(1) + TPZ(1))}{r_i^2 (4r_i^3 + 3r_i^2 \cdot BM3 + 2r_i \cdot BM2 + BMW)}$$

$$\text{EXP}(r_i \Delta Z)$$

where the r_i are the roots of the denominator polynomial.

APPENDIX D

COCURRENT DRYER DYNAMIC PROGRAM

APPENDIX D

COCURRENT DRYER DYNAMIC PROGRAM

The dynamic program consists of a large main program, DCMCD, and 5 subroutines. DCMCD does the main computation. It is controlled by the IDTSW vector and the first data card. It calls 4 subroutines: F4PT fits values in the previous time step 4 points at a time as shown in section 4. EMAIR calculates the equivalent moisture content of the air from its temperature and humidity. The relative humidity is limited to 0.9998 in this subroutine because higher values make the equivalent moisture content very large. QUARF gets the roots of a 4-th order polynomial; it uses QCUBE, a faster version of CUBIC in Appendix A, to estimate the roots of the derivative polynomial. ENTR supplies the values of the dependent variables at the entrance of the dryer. Since ENTR receives the exit conditions from the dryer, it could be used to implement a feedback controller or, as in this case, calculate a cost function.

The following are corresponding variables between the dynamic simulation section or Appendix C and DCMCD.

EPS = ϵ	RHOA = ρ_a
AFA1 = A_1^1/V	RHOC = ρ_c
AFA2 = A_2^1/V	ALF1 = A1
DR3 = Δr_3	ALF2 = A2
HFG = h_{fg}	ALF3 = A3
ARA3 = A_3	ALF4 = A4
CPC = C_{pc}	ALF5 = A5
CPA = C_{pa}	ALF6 = A6

The values below are the same in Appendix C and in DCMCD.

Here I = 1 is for M_1 , 2 for M_2 , 3 for M_3 , and 4 for H.

Also J = 1 is for T_c and J = 2 is for T_a .

BM3	TP4 (I)	TQ6 (J)
BM2	TP3 (I)	TQ5 (J)
BMW	TP2 (I)	TQ4 (J)
BMZ	TPW (I)	TQ3 (J)
	TPZ (I)	TQ2 (J)
		TQW (J)
		TQZ (J)

The U-matrix in DCMCD is large enough to hold a complete set of the 6 dependent variables along the dryer for one instant in time. At the start of a time-step it holds the values for the previous time-step, and it is filled with the present time-step values as the integration proceeds.

The step-size in the z -dimension is controlled by the DLZ-vector. Since the variables change much faster near the entrance to the dryer, Δz is made smaller there.


```
C  
C  
PROGRAM DCMCD(INPUT,OUTPUT, TAPE3=OUTPUT, TAPE2=INPUT)  
  
SUBROUTINES REQUIRED . . .  
F4PT FITS FOUR POINTS TO A COLLOCATION POLYNOMIAL.  
QCUBE ROOTS OF A 3-RD ORDER POLY, USED IN QUARF.  
QUARF ROOTS OF A 4-TH ORDER POLYNOMIAL  
EMAIR(T,H) EQUILIB MOISTURE CONTENT CORR TO AIR AT T AND H.  
ENTR(T,U) ENTERING U-VECTOR AS A FUNCTION OF TIME,T.  
  
EFFECT  
IDTSW ENTRY READ IN HEAT AND MASS TRANSFER COEFFS  
2 REVERSE SIGN OF ITERATION NUMBER CHANGE  
3 CHANGE ITERATION NUMBER, INCREASE BY 1.  
4 CHANGE ITERATION NUMBER, INCREASE BY 2.  
5 IGNORE 8-12 UNLESS JTM DIVISIBLE BY 2,  
6 IGNORE 8-12 UNLESS JTM DIVISIBLE BY 5.  
7 OUTPUT ALL VALUES (END OF EACH STEP)  
8 OUTPUT TRIAL VALUES DURING STEPS  
9 OUTPUT EIGENVALUES, RESULTS FROM QUART  
10 OUTPUT BOTTOM COEFFICIENTS  
11 OUTPUT U-MATRIX AT THE END OF TIME STEP  
12 OUTPUT CENTER AND END VALUES WITH TIME  
13  
  
THIS PROGRAM IS SET UP FOR 3 DIFFERENT INITIAL CONDITIONS.  
FOR IFLAG=0 THE INITIAL CONDITIONS ARE CONSTANT ALONG THE BED.  
FOR POSITIVE VALUES OF IFLAG THEY ARE LEFT AS IS (USED FOR  
MULTIPLE RUNS WHERE THE FINAL VALUE FOR ONE IS THE INITIAL  
VALUE FOR THE NEXT). FOR NEGATIVE IFLAG THE INITIAL CONDITIONS  
ARE READ FROM CARDS. IFLAG.GT. 100 TERMINATES THE PROGRAM.  
  
DIMENSION UZ(6), UL(6), TP4(4), TP3(4), TP2(4), TPW(4), TPZ(4)  
DIMENSION TQ6(2), TQ5(2), TQ4(2), TQ3(2), TQ2(2), TQW(2), TQZ(2)  
DIMENSION D(6,2), U(50,6), R(6), UP(6), DLZ(49)  
DIMENSION IDTSW(13)
```



```

DATA EPS,AFA1, AFA2, DR3/0.5,12.58E4, 9.46E4, 3.94E-3/
DATA HFG, ARA3 / 1040.0, 239.0 /
DATA CPC, CPA, RHOC, RHOA/0.6,0.24,72.0,0.075/
DATA DLZ/5#4.0E-3, 8#0.01, 36#0.025 /

C
DIFF(EM,T)=1.629E-3*EXP(EM*(0.025*T+6.008)-4523.4/(T+459.7))
EBR(X,Y,Z)=0.3333333333*(X+Y+Z)

C
FORMAT(2I10,5F12.5)
FORMAT(13I2)
FORMAT(8X,6F12.5)
FORMAT(1H0,/,20X,10HAT TIME = ,F9.5,25H HOURS THE RESULTS ARE....)
FORMAT(/,5X,8HPOSITION,12X,5HEMBAR,10X,4HM(1),10X,4HM(2),10X,
1 4HM(3),9X,8HHUMIDITY,5X,9HCORN-TEMP,6X,8HAIR-TEMP)
FORMAT(5X,F8.5,9X,4(F9.6,5X),F10.7,4X,F10.4,4X,F10.4)
FORMAT(3X,16H1R1AL FOR JTTR =,I3,2X,4(F9.6,5X),F10.7,2(4X,F10.4))
FORMAT(4H IER,I6,4HERR,E12.4,4H R=,6E15.7)
FORMAT(/,8H AT TIME,F9.5)
FORMAT(20X,6E15.7)
FORMAT(23X,4(F9.6,5X),F10.7,2(4X,F10.4))

C
CALL ENTR(-5000.0,UZ)
READ IN THE INTEGRATION DATA
C 100 READ(2,400) IFLAG,JTX,TINI,DELT,ZLNTH,GC,GA
WRITE(3,400) IFLAG,JTX,TINI,DELT,ZLNTH,GC,GA
C IFLAG IS AS ABOVE
C JTX IS THE NUMBER OF TIME STEPS
C TINI IS THE INITIAL TIME
C DELT IF THE TIME STEP IN HOURS
C ZLENTH IS THE TOTAL DRYER LENGTH
C GC IS THE CORN FLOW RATE
C GA IS THE AIR FLOW RATE
C BOTH FLOW RATES ARE IN POUNDS OF DRY MATTER
C PER SQUARE FOOT PER HOUR.
C READ(2,405) IDTSW
C WRITE(3,405) IDTSW

```



```

IF(IFLAG-100) 103,103,102
CALL EXIT
102 IF(IFLAG) 105,115,125
103 DO 110 I=1,50
105 READ(2,410) (U(I,J),J=1,6)
110 DO 112 I=1,6
112 UZ(I)=U(50,I)
GO TO 125
115 READ(2,410) UZ
DO 120 I=1,50
DO 120 J=1,6
120 U(I,J)=UZ(J)
125 EPS=2.0*(1.0-EPS)
ZH=0.5*ZLNTH
AREA=ARA3*EPSP
ALFA1=AFA1*EPSP
ALFA2=AFA2*EPSP
B1CF=3.0*ALFA1*RHOC/GC
B2CF=3.0*ALFA2*RHOC/GC
B3CF=3.0*AREA/GC
B4CF=AREA/GA
B5CF=HFG*AREA/(GC*CPC)
C PUT IN HMP AND HH AS FUNCTIONS OF GA HERE, IF DESIRED
IF(IDTSW(2)) 126,126,127
126 IF(GA-60.0) 575,575,580
575 HH=0.690*60.0*0.49
GO TO 595
580 IF(GA-615.78) 585,585,590
585 HH=0.690*GA**0.49
GO TO 595
590 HH=0.363*GA**0.59
595 HMP=0.0967*HH
GO TO 128
127 READ(2,410) HH,HMP
128 WRITE(3,410) HH,HMP
C

```

```

EUCF=HMP*DR3/RHOC
C5=AREA*HH/(GC*CPC)
C6=AREA*HH/(GA*CPA)
C1=RHOC*(1.0-EPS)/(GC*DELT)
C4=RHOA*EPS/(GA*DELT)
ALF5=C5 + C1
ALF6=C6 + C4
AF5P6=ALF5 + ALF6
AF56C=C1*ALF6 + C5*C4
XBR=-0.5*AF5P6
DX=SQRT(XBR*XBR - AF56C)
R(5)=XBR - DX
XQ=XBR + DX
129 XAW=(XQ + AF5P6)*XQ + AF56C
XAT=XQ + XQ + AF5P6
XBW=XAW/(XAT-XAW/XAT)
XQ=XQ - XBW
IF(ABS(XBW) - 0.5E-8*ABS(XQ)) 130,129,129
130 R(6)=XQ
C   OUTPUT INITIAL VALUES
IF(IDTSW(13)) 135,135,133
133 WRITE(3,440) TINI
EMBAR=EBR(U(30,1), U(30,2), U(30,3))
WRITE(3,425) ZH,EMBAR,(U(30,1),I=1,6)
EMBAR=EBR(U(50,1), U(50,2), U(50,3))
WRITE(3,425) ZLNTH,EMBAR, (U(50,1),I=1,6)
WRITE(3,425)
GO TO 138
135 IF(IDTSW(8)) 138,138,136
136 WRITE(3,420) TINI
WRITE(3,421)
IER=1
GO TO 268
C
138 DO 270 JTM=1,JTX
C   THIS IS THE TIME LOOP, AT EACH STEP THE U-MATRIX IS REFILLED.

```



```

ICHCK=0
IF(IDTSW(6)) 605,605,600
600 IF(JTM-(JTM/2)*2) 620,605,620
605 ICHCK=1
IF(IDTSW(7)) 620,620,610
610 IF(JTM-(JTM/5)*5) 615,620,615
615 ICHCK=0
620 TIME=TINI + FLOAT(JTM)*DELT
C
C NOTE THAT ON CALL TO ENTR THE UZ-VECTOR CONTAINS THE END
C VALUES FOR THE PREVIOUS TIME STEP. IN THIS WAY ENTR MAY
C BE USED TO CALCULATE AN OBJECTIVE FUNCTION BASED ON THE
C MATERIAL LEAVING THE DRYER.
C
CALL ENTR(TIME,UZ)
EMBAR=EBR(UZ(1),UZ(2),UZ(3))
DO 145 I=1,6
145 UL(I)=UZ(I)
C NOW ENTER THE Z-MOVING LOOP
Z=0.0
C THIS IS THE ZERO DISTANCE VALUE OUTPUT
IF(ICHCK) 147,147,625
625 IF(IDTSW(8)) 147,147,146
146 WRITE(3,420) TIME
WRITE(3,421)
WRITE(3,425) Z,EMBAR,UL
C
147 DO 265 JZ=1,49
C
DE LZ=DLZ(JZ)
C FIRST FILL IN THE D-MATRIX
DO 160 I=1,6
IF(JZ-1) 700,700,705
700 CALL F4PT( U(1,I), U(2,I), U(3,I), U(4,I), DELZ, DELZ+DLZ(2),
1 DELZ+DLZ(2)+DLZ(3), D(1,I), D(1,2) )
GO TO 720

```



```

705 IF(JZ-49) 710,715,715
710 CALL F4PT( U(JZ,1), U(JZ+1,1), U(JZ+2,1), UP(1), DELZ,
2 DELZ+DLZ(JZ+1), -DLZ(JZ-1), D(1,1), D(1,2) )
GO TO 720
715 CALL F4PT( U(49,1), U(50,1), UP(1), D(1,2), DELZ,
3 -DLZ(48), -(DLZ(48)+DLZ(47)), D(1,1), XQ )
D(1,2)=XQ
720 IF(IABS(I-5)-1) 150,155,150
150 D(1,1)=D(1,1)*C1
D(1,2)=D(1,2)*C1
GO TO 160
155 D(1,1)=D(1,1)*C4
D(1,2)=D(1,2)*C4
160 CONTINUE
C
C WE ALSO WANT THE DIFFUSIVITY AND EMAJR AT THE FRONT
C21=EMAIR(UZ(6),UZ(4))/UZ(4)
DFH=DIFF(EMBAR,UZ(5))
C
C ONE MAY ESTIMATE THE END VALUES HERE, IF DESIRED.
C NO ACTION IS NEEDED IF THIS FIRST ESTIMATE IS TO BE THE
C SAME AS THE FRONT OF THE INTERVAL.
C ENTER THE ITERATION LOOP BETWEEN 165 AND 240
JITR=-5*(IDTSW(3)+IDTSW(3)-1)*(IDTSW(5)+IDTSW(5)+IDTSW(4))
165 JITR=JITR + 1
IF(JITR) 170,250,250
170 DIF=0.5*(DFH+DIFF(EMBAR,UL(5)))
B1=B1CF*DIF
B2=B2CF*DIF
EU=HMP*DIF/(DIF + EUCF)
B3=B3CF*EU
B4=B4CF*EU
B5=B5CF*EU
C
C2=0.5*(C21 + EMAJR(UL(6),UL(4))/UL(4))
C

```

```

ALF1=B1 + C1
ALF2=ALF1 + B2
ALF3=B2 + B3 + C1
ALF4=B4+C2 + C4
C  ALL THE COEFFICIENTS ARE NOW AVAILABLE, BUT MAKE SOME SUBSTITUTIONS.
B32=B3*C2
AFWPF=ALF1 + ALF4
AFWPT=ALF1 + ALF2
AF14=ALF1*ALF4
AF34=ALF3*ALF4
AF124=AFWPF + ALF2
AF123=AFWPT + ALF3
AF134=AFWPF + ALF3
AF3PF=ALF3 + ALF4
AF234=AF3PF + ALF2
B2S=B2*B2
AF12B=C1*ALF2 + B1*(B2 + C1)
AF34B=(B2 + C1)*ALF4 + B3*C4
XQ=B3*C4 + C1*ALF4
XAW=AF34B + ALF1*AF3PF + B2*(B3 + C1 + ALF4)
XAT=AF34B*ALF1 + B2*XQ
XBW=ALF1*AF3PF + AF34B
XBT=ALF1*AF34B
XCW=AF12B + ALF4*AFWPT
XCT=ALF4*AF12B
XOW=AFWPT*(B3 + C1) + B2*(ALF1 + ALF1) + AF12B
XOT=(B3 + C1)*AF12B + B2*C1*(ALF1 + B1)
C  NOW COMPUTE THE BOTTOM COEFFICIENTS
BM3=AF123 + ALF4
BM2=AF3PF*(ALF1 + ALF1) + B2*(C1 + B3 + ALF4) + AF12B + AF34B
BMW=AF3PF*C1*(ALF1+B1) + AFWPT*XQ + B2*ALF1*(B3+C1+ALF4+ALF4)
BMZ=AF12B*XQ + B2*ALF4*C1*(ALF1 + B1)
IF(ICHCK) 188,188,630
630 IF(1DTSW(11)) 188,188,187
187 WRITE(3,445) BM3,BM2,BMW,BMZ,C2
C  COMPUTE THE TOP COEFFICIENTS FOR M1

```



```

188 TP4(1)=D(1,1) + AF234*UZ(1) + B1*UZ(2)
C
TP3(1)=D(1,2) + AF234*D(1,1) + XAW*UZ(1) + B1*( D(2,1) + AF3PF*
3 UZ(2) + B2*UZ(3))
C
TP2(1)=AF234*D(1,2) + XAW*D(1,1)+XAT*UZ(1) + B1*(D(2,2)+AF3PF*
2 D(2,1)+AF348*UZ(2)+B2*(D(3,1)+ALF4*UZ(3)+B32*UZ(4)))
C
TPW(1)=XAW*D(1,2)+XAT*D(1,1)+B1*(AF3PF*D(2,2)+AF348*D(2,1) +
1 B2*(D(3,2)+ALF4*D(3,1)+B32*D(4,1)))
C
TPZ(1)=XAT*D(1,2)+B1*(AF348*D(2,2)+B2*(ALF4*D(3,2)+B32*D(4,2)))
C COMPUTE THE TOP COEFFICIENTS FOR M2
TP4(2)=B1*UZ(1) + D(2,1) + AF134*UZ(2) + B2*UZ(3)
C
TP3(2)=B1*(D(1,1)+AF3PF*UZ(1))+D(2,2)+AF134*D(2,1) + XBW*UZ(2)
3 +B2*(D(3,1) + AFWPF*UZ(3) + B32*UZ(4))
C
TP2(2)=B1*(D(1,2)+AF3PF*D(1,1)+AF348*UZ(1))+AF134*D(2,2)+XBW *
2 D(2,1)+XBT*UZ(2)+B2*(D(3,2)+AFWPF*D(3,1)+AF14*UZ(3)+B32*
7 (D(4,1) + ALF1*UZ(4)))
C
TPW(2)=B1*(AF3PF*D(1,2)+AF348*D(1,1))+XBW*D(2,2)+XBT*D(2,1)+B2 *
1 (AFWPF*D(3,2)+AF14*D(3,1)+B32*(D(4,2)+ALF1*D(4,1)))
C
TPZ(2)=B1*AF348*D(1,2)+XBT*D(2,2)+B2*(AF14*D(3,2)+B32*ALF1*D(4,2))
C COMPUTE THE TOP COEFFICIENTS FOR M3
TP4(3)=B2*UZ(2) + D(3,1) + AF124*UZ(3) + B32*UZ(4)
C
TP3(3)=B2*(B1*UZ(1)+D(2,1)+AFWPF*UZ(2))+D(3,2) + AF124*D(3,1)
3 + XCW*UZ(3) + B32*(D(4,1) + AFWPT*UZ(4))
C
TP2(3)=B2*(B1*(D(1,1)+ALF4*UZ(1))+D(2,2)+AFWPF*D(2,1)+AF14*UZ(2))+
2 AF124*D(3,2) + XCW*D(3,1) + XCT*UZ(3) + B32*(D(4,2) +
7 AFWPT*D(4,1) + AF128*UZ(4))
C

```

TPW(3)=B2*(D(1,2)+ALF4*D(1,1))+AFWPT*D(2,2)+AF14*D(2,1) +XCW*
1D(3,2)+XCT*D(3,1)+B32*(AFWPT*D(4,2)+AF12B*D(4,1))

TPZ(3)=B2*(B1*ALF4*D(1,2)+AF14*D(2,2))+XCT*D(3,2)+B32*AF12B*D(4,2)
COMPUTE THE TOP COEFFICIENTS FOR H

TP4(4)=B4*UZ(3) + D(4,1) + AF123*UZ(4)

TP3(4)=B4*(B2*UZ(2) + D(3,1) + AFWPT*UZ(3)) + D(4,2) +
3 AF123*D(4,1) + XDW*UZ(4)

TP2(4)=B4*(B2*(B1*UZ(1)+D(2,1)+ALF1*UZ(2))+D(3,2)+AFWPT*D(3,1) +
2AF12B*UZ(3))+AF123*D(4,2)+XDW*D(4,1) + XDT*UZ(4)

TPW(4)=B4*(B2*(B1*D(1,1) + D(2,2) + ALF1*D(2,1)) + AFWPT*D(3,2)
1 + AF12B*D(3,1)) + XDW*D(4,2) + XDT*D(4,1)

TPZ(4)=B4*(B2*(B1*D(1,2)+ALF1*D(2,2))+AF12B*D(3,2))+XDT*D(4,2)

COMPUTE THE BOTTOM COEFFICIENTS FOR TC AND TA

BN5=BM3 + AF5P6

BN4=BM2 + AF5P6*BM3 + AF56C

BN3=BMW + AF5P6*BM2 + AF56C*BM3

BN2=BMZ + AF5P6*BMW + AF56C*BM2

BNW=AF5P6*BMZ + AF56C*BMW

BNZ=AF56C*BMZ

DZMQ=DELZ - BMW/BMZ

DZNO=DELZ - BNW/BNZ

THE FOLLOWING SUBSTITUTIONS ARE USED.

TCW=ALF6*UZ(5) + D(5,1) + C5*UZ(6)

TCT=ALF6*D(5,1) + D(5,2) + C5*D(6,1)

TC3=ALF6*D(5,2) + C5*D(6,2)

TAW=ALF5*UZ(6) + D(6,1) + C6*UZ(5)

TAT=ALF5*D(6,1) + D(6,2) + C6*D(5,1)

TA3=ALF5*D(6,2) + C6*D(5,2)

THE TOP OF TC AND TA ARE NOW FILLED, IN THREE STEPS

T05(2)=B5*(C2*UZ(4)-UZ(3))


```

TQ4(2)=B5*(C2*TP4(4))-TP4(3))
TQ3(2)=B5*(C2*TP3(4))-TP3(3))
TQ2(2)=B5*(C2*TP2(4))-TP2(3))
TQ1(2)=B5*(C2*TPW(4))-TPW(3))
TQZ(2)=B5*(C2*TPZ(4))-TPZ(3))
NOW DO THE SECOND STEP
TQ6(1)=TQ5(2) + BM3*UZ(5) + TCW
TQ5(1)=TQ4(2) + ALF6*TO5(2) + BM2*UZ(6) + BM3*TCW + TCT
TQ4(1)=TQ3(2) + ALF6*TO4(2) + BMW*UZ(5) + BM2*TCW + BM3*TCT + TC3
TQ3(1)=TQ2(2)+ALF6*TO3(2)+BMZ*UZ(5)+BMW*TCW+BM2*TCT+BM3*TC3
TQ2(1)=TQ1(2)+ALF6*TO2(2)+BMZ*TCW+BMW*TCI+BM2*TC3
TQ1(1)=TQZ(2) + ALF6*TO1(2) + BMZ*TCT + BMW*TC3
TQZ(1)=ALF6*TOZ(2) + BMZ*TC3
THE LAST STEP FILLS THE TOP OF TA.
TQ6(2)=BM3*UZ(6) + TAW
TQ5(2)=C6*TO5(2) + BM2*UZ(6) + BM3*TAW + TAT
TQ4(2)=C6*TO4(2) + BMW*UZ(6) + BM2*TAW + BM3*TAT + TA3
TQ3(2)=C6*TO3(2) + BMZ*UZ(6) + BMW*TAW + BM2*TAT + BM3*TA3
TQ2(2)=C6*TO2(2) + BMZ*TAW + BMW*TAT + BM2*TA3
TQ1(2)=C6*TO1(2) + BMZ*TAT + BMW*TA3
TQZ(2)=C6*TOZ(2) + BMZ*TA3
NOW GET THE EIGENVALUES, ROOTS 1-4
CALL QUARFIR(1),R(2),R(3),R(4),BM3,BM2,BMW,BMZ,ERR,IER)
IF(ICHCK) 190,190,635
635 IF(IDTSW(10)) 190,190,200
190 IF(IErr-8.0E-5*ABS(R(4))) 195,200,200
195 IF(IErr) 200,200,205
200 WRITE(3,435) IER,ERR,R
C NOW THE END VALUES MUST BE INITIALIZED
205 DO 215 I=1,4
    UL(I)=(TPZ(I)*DZMQ + TPW(I))/BMZ
    IF(I-3) 210,215,215
210 UL(I+4)=(TOZ(I)*DZMQ + TQW(I))/BNZ
215 CONTINUE
C THIS COMPLETES THE INITIALIZATION
C NOW COMPLETE FILLING UL

```

```

235 DO 235 KI=1,6
    S=R(KI)
    EX=EXP(S*DELZ)
    XQ=S + S + BN5
    XQ=XQ + XQ + BN5
    XQ=((XQ+XQ+BN5)*S + 6.0*BN4)*S + 5.0*BN3)*S + 4.0*BN2
    XQ=((XQ*S + BNW+BNW+BNW)*S + BNZ+BNZ)*S
    EXN=EX/XQ
    IF(KI-4) 220,220,230
220 XQ=S+S+BM3+BM3
    XQ=((XQ+XQ+BM3)*S + 4.0*BM2)*S + BMW+BMW+BMW)*S
    EX=EX/((XQ+BMZ+BMZ)*S)
    DO 225 I=1,4
    XQ=((UZ(I)*S + TP4(I))*S + TP3(I))*S + TP2(I)
    XQ=((XQ*S + TPW(I))*S + TPZ(I)
225 UL(I)=UL(I) + XQ*EX
230 DO 235 I=1,2
    IP=I+4
    XQ=((UZ(IP)*S + TQ6(I))*S + TQ5(I))*S + TQ4(I))*S
    XQ=((XQ + TQ3(I))*S + TQ2(I))*S + TQW(I))*S + TQZ(I)
235 UL(IP)=UL(IP) + XQ*EXN
    C AT THIS POINT THE NEW ESTIMATE IS AVAILABLE.
    EMBAR=EBR(UL(1),UL(2),UL(3))
    IF(ICHCK) 165,165,640
    IF(IDTSW(9)) 165,165,245
245 WRITE(3,430) J1TR,EMBAR,UL
    GO TO 165
    C AT THIS POINT THE Z-STEP IS COMPLETED
    C PUT UZ IN ITS PLACE IN THE U-MATRIX, AND PUT UL INTO UZ
    C THEN GO BACK FOR ANOTHER STEP
    C
250 DO 255 I=1,6
    D(I,2)=UP(I)
    UP(I)=U(JZ,I)
    U(JZ,I)=UZ(I)
255 UZ(I)=UL(I)

```



```

C
Z=Z + DELZ
IF(ICHCK) 265,265,645
IF(IDTSW(8)) 265,265,260
260 XQ=Z + 0.5E-5
WRITE(3,425) XQ,EMBAR,UL
265 CONTINUE
DO 266 I=1,6
U(50,I)=UL(I)
266 AT THIS POINT A TIME STEP HAS BEEN COMPLETED.
IF(IDTSW(13)) 649,649,648
648 WRITE(3,440) TIME
EMBAR=EBR(U(30,1), U(30,2), U(30,3))
WRITE(3,425) ZH,EMBAR,(U(30,I),I=1,6)
EMBAR=EBR(U(50,1), U(50,2), U(50,3))
WRITE(3,425) ZLNTH,EMBAR, (U(50,I),I=1,6)
WRITE(3,425)
649 IF(ICHCK) 270,270,650
650 IF(IDTSW(12)) 270,270,267
267 IER=0
WRITE(3,420) TIME
DO 269 I=1,50
EMBAR=EBR(U(I,1), U(I,2), U(I,3))
269 WRITE(3,450) EMBAR,(U(I,J),J=1,6)
IF(IER) 270,270,138
270 CONTINUE
GO TO 100
END

SUBROUTINE F4PT(Y1,Y2,Y3,Y4,X2,X3,X4,XINT,SLOPE)
SLOPE=(Y2-Y1)/X2
A=Y3-Y1-SLOPE*X3
C=X3*(X2-X3)
AA=Y4-Y1-SLOPE*X4
CC=X4*(X2-X4)

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```

C=-(A*C+AA*CC)/(C*C+CC*CC)
XINT=Y1-0.1255*C*X2*X2
RETURN
END

FUNCTION EMAIR(T,H)
  FUNCTION SUBROUTINE TO CALCULATE EQUILIBRIUM MOISTURE
  CONTENT, EM IN FRACTION, FROM TEMP,T, IN DEG F
  AND HUMIDITY,H, IN LB H2O/LB DRY AIR.
  H=0.622*PSAT(T)*RH(EM,T)/(PTOT-PSAT*RH)
  RH=1.0-EXP(-0.382*(T+50)*M**2)
  DATA PTOT,RHMAX / 0.976, 0.9998 /
  DATA E/2.302585093/
  TP=0.555555555*T + 255.3822
  RH=28.59051 - 8.2*ALOG(TP)/E + TP*2.4804E-3 - 3142.31/TP
  PSAT=EXP(E*RH)/1.01325
  RH=H*PTOT/(PSAT*(H + 0.622))
  IF(RH) 90,100,110
  90 WRITE(3,200)
  100 EMAIR=0.0
  RETURN
  110 IF(RH-RHMAX) 120,120,115
  115 WRITE(3,200)
  RH=RHMAX
  120 EMAIR=ALOG(1.0-RH)/(-0.382*(T+50.0))
  EMAIR=SQRT(EMAIR)
  RETURN
  200 FORMAT(1H+,118X,1H*)
  END

SUBROUTINE QUARF(R1,R2,R3,R4,C3,C2,CW,CZ,ERR,I)
  DIMENSION V(3)
  XN=CZ*CW/(CZ*C2 - CW*CW)
  I=0

```

```

100  GO TO 140
      R1=XN
      A2=C3 + R1
      AW=A2*R1 + C2
      AZ=AW*R1 + CW
      CALL QCUBE(V(1), V(2), V(3), A2,AW,AZ,ERR,I)
      IF(I) 110,110,120
110  RETURN
120  I=I + J
      DO 135 K=1,3
          XN=V(K)
      GO TO 140
130  I=I + J
135  V(K)=XN
      R2=V(1)
      R3=V(2)
      R4=V(3)
      XN=R1 + R2
      A2=R3 + R4
      AW=R1*R2
      AZ=R3*R4
      ERR=ABS(XN + A2 + C3) + ABS( AW + XN*A2 + AZ - C2)
      ERR=ERR + ABS(AW*A2 + XN*A2 + CW) + ABS(AW*A2 - CZ)
      RETURN
140  J=0
145  X=XN
      F=((X + C3)*X + C2)*X + CW)*X + CZ
      DF=X + X + C3
      HD=DF + X
      DF=((DF + DF + C3)*X + C2 + C2)*X + CW
      HD=(HD + HD + C3)*X + C2
      X=F/(DF - F*HD/DF)
      XN=XN - X
      J=J + 1
      IF(ABS(X) - 3.0E-9*ABS(XN)) 190,180,180
180  IF(J-25) 145,145,185

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```

185 J=100
190 IF(I) 100,100,130
    END

C SUBROUTINE QCUBE(Z1,Z2,Z3, C2, CW, CZ, ERR, I)
C SOLVES THE EQUATION Z**3 + C2*Z**2 + CW*Z + CZ = 0 WHEN ALL THE
C ROOTS ARE REAL.
C I IS EQUAL TO THE NUMBER OF ITERATIONS IN THE COMPUTATION OF THE
C MIDDLE ROOT, Z2. IF I IS NEG, ALL ROOTS NOT REAL.
C THE MIDDLE ROOT IS ISOLATED BETWEEN THE ZEROS OF THE DERIVATIVE.
C IT IS ESTIMATED BY INTERPOLATING BETWEEN THE MAX AND MIN, WITH THE
C INTERPOLATION CORRECTED BY 0.7 FOR FASTER CONVERGENCE.
C EQUIVALENCE (DX,XN)
C FOFX(X,A,B,C)= ((X+A)*X+B)*X+C

I=1
XBR=-0.333333333333*C2
DX=XBR*XBR - 0.333333333333*CW
IF(DX) 110,115,115
I=-I
110 RETURN
115 DX=SQRT(DX)
F=FOFX(XBR + DX, C2, CW, CZ)
DF=FOFX(XBR - DX, C2, CW, CZ)
DF=(F+DF)/(F-DF)
I=2
IF(ABS(DF)-1.0) 120,120,110
120 XN=XBR-0.7*DX*DF
165 X=XN
F=FOFX(X,C2,CW,CZ)
HD2F=X+X+X+C2
DF=(HD2F + C2)*X + CW
X=F/(DF-F*HD2F/DF)
XN=XN - X
125 IF(ABS(X) - 0.5E-5*ABS(XN)) 125,165,165
Z2=XN

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```

DX=C2 + Z2
XBR=-0.5*DX
DX=XBR*XBR - DX*Z2 - CW
IF(DX) 110,145,145
DX=SQRT(DX)
Z1= XBR + DX
Z3=XBR - DX
RETURN
END

145
SUBROUTINE ENTR(T,U)
DIMENSION U(6), V(5)
DATA A / 10.23 /
DATA V/3*0.38, 0.014, 72.0/
DATA TPP,TP,PTY,SH,SL/3*0.0, 2.441, 0.0936/
DATA EMZ/0.182995/
DATA TI,TL/ 155.71115, 176.0202/
IF(T+4500.0) 80,80,90
THIS IS A DUMMY CALL. IT IS USED WHEN
SOME INITIAL COMPUTATION IS NEEDED.
RETURN
80
90 XQ=0.333333333*(U(1)+U(2)+U(3)) - EMZ
IF(XQ) 105,110,100
100 PTY=PTY + (TP-TPP)*SH*XQ
GO TO 110
105 PTY=PTY - (TP-TPP)*SL*XQ
110 TPP=TP
TP=T
WRITE(3,205) PTY,TPP
205 FORMAT(20X,9HPENALTY =,E16.7,5X,7HAT TIME, F6.3, 6H HOURS)
DO 120 I=1,5
120 U(I)=V(I)
C THE INPUT TEMPERATURE FUNCTION IS PUT
C IN AT THIS POINT.
Y=1.0 - EXP(-A*T)

```


0.5	0.232218	0.215991	0.176132	0.0230930	108.5759	109.0516
0.525	0.229706	0.213868	0.175037	0.0232356	108.0759	108.5251
0.55	0.227316	0.211856	0.174018	0.02333705	107.6022	108.0272
0.575	0.225039	0.209947	0.173067	0.0234984	107.1527	107.5556
0.6	0.222867	0.208131	0.172179	0.0236197	106.7255	107.1080
0.625	0.220792	0.206402	0.171348	0.0237351	106.3189	106.6826
0.65	0.218808	0.204755	0.170569	0.0238450	105.9314	106.2777
0.675	0.216907	0.203181	0.169837	0.0239497	105.5617	105.8918
0.7	0.215086	0.201678	0.169149	0.0240496	105.2085	105.5236
0.725	0.213338	0.200240	0.168501	0.0241451	104.8708	105.1719
0.75	0.211660	0.198862	0.167889	0.0242364	104.5475	104.8356
0.775	0.210047	0.197542	0.167313	0.0243239	104.2379	104.5137
0.8	0.208496	0.196275	0.166768	0.0244077	103.9410	104.2052
0.825	0.207002	0.195058	0.166253	0.0244880	103.6560	103.9095
0.85	0.205563	0.193889	0.165765	0.0245652	103.3824	103.6257
0.875	0.204176	0.192765	0.165303	0.0246392	103.1194	103.3531
0.9	0.202838	0.191683	0.164866	0.0247104	102.8665	103.0912
0.925	0.201547	0.190641	0.164450	0.0247789	102.6232	102.8393
0.95	0.200300	0.189637	0.164056	0.0248449	102.3889	102.5969
0.975	0.199095	0.188670	0.163682	0.0249083	102.1631	102.3635
1.0	0.197930	0.187736	0.163326	0.0249695	101.9455	102.1386

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