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AN EXAMINATION OF SAFETY STOCK POLICIES IN
MULTI-ECHELONED DISTRIBUTION SYSTEMS

By

Robert Lorin Cook

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

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Department of Marketing and Transportation
Administration

1980

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ABSTRACT

AN EXAMINATION OF SAFETY STOCK POLICIES IN MULTI-ECHELONED DISTRIBUTION SYSTEMS

By

Robert Lorin Cook

An accurate determination of channel system safety stock requirements is desirable as one aspect of customer service policy. Safety stock is located throughout a distribution system to buffer uncertainties due to demand and lead time performance.

An accurate determination of safety stock requirements depends on two factors: (1) accuracy of the technique used to formulate safety stock policy; and (2) understanding the combined effects of safety stock and uncertainty on customer service. There has been little prior research which studies the customer service effects of safety stock levels and locations, particularly in multi-echeloned distribution systems. The objective of this research was to measure both the accuracy of a statistical approach commonly used for setting safety stocks and effects of alternate safety stock policies and uncertainty levels on customer service performance in single and multi-echeloned channel systems.

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Two series of dynamic simulations were performed using a dynamic simulation model of a distribution channel. Numerous simulations were made using alternate combinations of safety stock policy and uncertainty level for both single and multi-echeloned channel systems. The customer service results were examined using a two-part analysis. The first part compared the simulated customer service results of both channel system designs to the customer service predicted by a statistical approach commonly used for setting safety stocks. The second part investigated the individual and combined effects of alternate safety stock policies and uncertainty levels on the simulated customer service results of each channel system. The following conclusions for safety stock planning and control were drawn from the research results. First, the statistical approach, for the most part, should not be used for setting safety stocks in single or multi-echeloned channel systems. In single echelon systems, the statistical approach did not accurately predict simulated customer service. Lead time uncertainty, as opposed to demand uncertainty, had the greatest effect on the relative accuracy of the statistical approach. In multi-echeloned systems, the statistical approach was also found to be inaccurate due, in part, to a failure to consider the interrelationship of inventory performance at sequential locations. The statistical approach does not consider safety stock location within the channel.

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Safety stock level and location was found to have a significant effect on customer service in both single and multi-echeloned channel systems. In single echelon systems, the initial increments of safety stock provided the most significant increases in customer service, regardless of uncertainty level. In multi-echeloned systems, safety stock positioning had significant impacts on customer service. Safety stocks introduced at the lowest channel echelon resulted in higher customer service than safety stocks introduced at the second echelon. This difference in customer service increased as uncertainty level increased. Lower echelon safety stock additions increased customer service slowly, while additional increments of safety stock at the second echelon rapidly decreased in effectiveness. A second echelon positioning policy increased the probability that retail replenishment orders were filled, but second echelon safety stocks did not buffer lead time uncertainty between the two echelons.

Partial postponement of safety stocks at the second echelon resulted in higher customer service than either absolute postponement or absolute speculation because inventory is necessary at each echelon since inventory performance at sequential locations is interrelated. This higher customer service was accomplished without increasing

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safety stock levels and carrying costs. Thus, a total channel approach to setting safety stock policy results in higher customer service performance for a given amount of safety stock.

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CHAPTER I

INTRODUCTION

General Research Situation

The formulation of a customer service strategy is an important part of physical distribution performance. Overall, this strategy requires both a specified level of inventory availability and a desired speed and consistency of delivery. A critical aspect of a customer service strategy is inventory placement. Only by having the correct quantity of inventory at the right place and when demanded can a prescribed customer service level be realized.

Two types of uncertainty are experienced in the placement of inventory throughout a distribution channel. The first uncertainty is demand which may be greater than that forecast. The second is delay in replenishing inventory to a stocking location.

Managers attempt to overcome the effects of both types of uncertainty by using safety stocks. When demand exceeds forecast or when replenishment is delayed, safety stock may be required to fill orders. In both situations, safety stock provides a degree of protection against

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stock-outs. A stock-out occurs when inventory is unavailable to satisfy an order. Thus, because the safety stock level influences inventory availability, determination of such levels is a major aspect of customer service strategy.

Numerous techniques have been developed to assist in the selection of safety stock levels.¹ These techniques range from judgment and simple mathematics, to complex statistical formulations, and to various forms of simulation. The most commonly used techniques for determining safety stock level involve statistical probability. Selected statistical techniques provide methods for estimating both demand and lead time uncertainty in a single formulation.²

Inventory is stocked at one or more stages (echelons) of a physical distribution system. When inventory is stocked at only one stage, the system is defined as single echelon. The structure of most physical distribution systems, however, consists of two or more stocking stages; for example, products stocked at distribution centers and retail locations. When two or more stocking stages exist, the system is defined as multi-echeloned.

A multi-echeloned channel structure adds complexity to planning inventory level and positioning because performance at sequential locations is interrelated.³ Thus, decisions regarding the level and positioning of safety

stocks can be expected to influence performance at all locations in a multi-echeloned distribution channel.

Statistical techniques used for establishing safety stock level have not taken into consideration the inter-relation of echelons in multi-echeloned situations. With minor exceptions, no research has been done supporting the validity of using identical statistical techniques regardless of the echeloned structure.⁴ The validity of using probabilistic statistical techniques to set safety stocks is questionable, due to the interactions of demand, lead time, multiple order cycles, and multiple echelons.

Determination of the level and positioning of safety stocks, without consideration of the above noted interactions, could result in higher or lower than planned customer service. In the first instance, inventory carrying cost increases because of excessive safety stocks. In the second instance, sales are lost unless orders can be backordered.

The attainment of a prescribed customer service level depends on two factors: (1) the accuracy of the technique used to formulate the safety stock policy; and (2) understanding the relationship of alternative safety stock policies and uncertainty levels on customer service performance. The objective of this research was to seek a better understanding of each of these two factors in both a single and multi-echeloned inventory system.

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Statistical Technique Accuracy

The first factor deals with the accuracy of a statistical approach used to determine safety stocks. This approach involves two formulations. A convolution formulation and a service level formulation are jointly used to determine the safety stock level required to realize planned customer service performance. Customer service performance is defined as a fill rate (quantity sold in units divided by quantity demanded).

The convolution formulation combines demand and lead time uncertainties. Specifically, the formulation mathematically combines the variance about the average daily demand during lead time with the variance about the average lead time. The formula is as follows:

$$\sigma_c = \sqrt{\bar{t}\sigma_d^2 + \bar{d}^2\sigma_t^2}$$

where:

σ_c = combined standard deviation of demand during lead time (units);

$\bar{t}$ = average lead time (days);

σ_d^2 = variance of demand over lead time (units²/day);

$\bar{d}$ = average demand per day (units); and

σ_t^2 = variance of lead time (days²).⁵

The product is the combined standard deviation of demand during lead time in units. This measure can be used to determine the probability of a stock-out during replenishment.⁶ For example, given a situation where normal demand and lead time distributions are assumed, the policy of having a safety stock level equal to one standard deviation would mean that stock-outs would occur, on the average, during only 15.87 percent of the lead times.⁷

This measure of customer service performance does not directly yield fill rate. The actual quantity fill rate, over time, depends on the number of replenishment orders. The number of replenishment orders required over a planning period is directly related to order quantity. The conversion of the probability of a stock-out, during a given replenishment cycle, to a quantity fill rate, over time, is accomplished by the service level formulation introduced by Robert Brown.⁸ The formula is:

$$SL = 1 - \frac{(K) \sigma_c}{OQ}$$

where:

SL = service level as a quantity fill rate;

K = safety factor or protection provided by
(S_K) x (σ_c) units of safety stock;

S_K = the number of combined standard deviations
of safety stock placed at a location;

σ_c = combined standard deviation of demand over lead time in units; and

OQ = replenishment order quantity.⁹

The resulting output is the quantity fill rate expressed as a percentage. The accuracy of this statistical approach can be tested by making a statistical comparison of customer service levels predicted by that approach to those obtained from dynamic simulation.

Relationship--Safety Stock Policy and Uncertainty

The second factor deals with an evaluation of safety stock policies and combined demand and lead time uncertainties. Both the safety stock policy employed, and the uncertainty level experienced, have impacts on channel system customer service performance. Thus, to achieve a prescribed customer service level, it is desirable to understand the relationship between safety stock policy and uncertainty.

In a single echelon channel system, the safety stock policy consists of setting a safety stock level. The determination of a specific safety stock level must be closely related to the combined demand and lead time variability, since the function of safety stock is to buffer uncertainty. Thus, a joint probability distribution of demand over lead time must be estimated and related to

safety stock level and customer service performance. Recent research has indicated that demand and lead time variability may interact causing a "cancellation effect" in total variability.¹⁰ This interaction of demand and lead time variabilities reduces the accuracy of current techniques used in estimating the impact of uncertainty on channel system performance. However, the impact of uncertainty on channel system performance must be precisely defined, since both over and underestimation of safety stock requirements penalize the firm.

In a multi-echeloned channel system, the safety stock policy consists of setting a safety stock level and position. An additional safety stock policy consideration is that inventory performance of echeloned locations is interrelated. For example, a wholesaler who derives demand from a retailer may set a safety stock level based on the combined variability of demand over lead time. The retailer may set safety stock level in the same manner. Thus, by setting safety stocks independently, both have set aside stock to cover the same variability in demand. The result is excess safety stock in the channel. Therefore, safety stock level and position must be considered simultaneously when setting safety stocks in multi-echeloned systems.

The relationship between safety stock policy and uncertainty and its impact on customer service performance must be evaluated, and then represented in safety stock

techniques. Customer service levels resulting from dynamic simulations were analyzed statistically in evaluating the relative impacts of safety stock policy and uncertainty on customer service performance.

Detailed Problem Statement

Two specific problems are examined in studying the two factors upon which desired customer service performance depend.

Problem I

The first problem is a comparison of predicted and simulated customer service performance in a single echelon channel system. It is divided into two parts so that each part can be compared to that of Problem II.

Part A. Part A determines the accuracy of a statistical approach for determining customer service levels under selected safety stock policies and uncertainty levels.

Part B. Part B evaluates the relative impact of the safety stock level employed on customer service performance under selected uncertainty levels.

Problem II

The second problem is a comparison of predicted and simulated customer service performance in a multi-echeloned channel system. It also is divided into two parts.

Part A. Part A determines the accuracy of a statistical approach for determining customer service levels under selected safety stock policies and uncertainty levels.

Part B. Part B evaluates the relative impact of the safety stock level employed and echelon positioning on customer service performance under selected uncertainty levels.

Research Procedure

Ideally, each problem could be researched by performing a series of controlled experiments using actual distribution channels. The researcher could then observe channel system performance related to changes in safety stock policy under different uncertainty situations. Direct experimentation is not feasible because of an inability to control relevant variables. Therefore, this particular research problem was solved using a channel system model.

A model is a representation of an object, system, or idea in some form other than that of the entity itself.¹¹ Two broad model categories exist for analysis of physical distribution systems--mathematical programming and simulation.¹² A mathematical model describes the system, its components, and their interactions in quantitative terms. In order to develop and use a mathematical model, all

system relationships must be completely understood and quantified. This can be a difficult task. As Closs states:

Although it is not difficult to quantify the relationship between total transportation costs and the elements contributing to total cost, it is very difficult to establish with any degree of certainty a quantifiable relationship between customer service and resultant sales.¹³

Another major weakness of employing mathematical modeling in distribution research is the inability to treat day-to-day dynamics of distribution system operations.

Simulation is an alternative form of modeling that has been successfully employed to replicate physical distribution systems.¹⁴ One definition of simulation is:

the process of designing a model of a real system and conducting experiments with this model for the purpose of either understanding the behavior of the system or of evaluating various strategies (within the limits imposed by a criterion or set of criteria) for the operation of the system.¹⁵

Computer simulations are designed as static or dynamic. Static simulation models measure the state of variables and relationship at a given point in time. Dynamic simulation measures the state of the variables and relationships over time. Dynamics permit the researcher to investigate time-dependent relationships between both the parameters and variables of the system simulated. Simulations may also be deterministic or stochastic. In deterministic simulation models, the relationships

between model variables are constant. Models, in which at least one relationship is represented by a probability function, are stochastic.¹⁶ Two significant stochastic variables found in simulation models are demand and lead time. Therefore, a dynamic/stochastic simulator is suitable for experimentation of safety stock policies.

The specific simulation model employed in this research is the Simulated Product Sales Forecasting (SPSF) model.¹⁷ The model is capable of measuring both channel system cost and service performance; it incorporates the ability to simultaneously handle several channels of both single and multi-echeloned structure; it is dynamic and stochastic; and it replicates both spatial and temporal systems' dimensions.

Two series of simulations were required by the research designs. They combined all possible combinations of three safety stock policies and nine uncertainty levels. The series of simulations pertaining to Problem I replicated twenty-seven different single echelon channel systems. The series of simulations pertaining to Problem II replicated eighty-one different multi-echeloned channel systems. The increase in simulations for Problem II stems from the echelon-positioning of three safety stock levels.

For Problems I and II the customer service results are analyzed for each experimental simulation. In Part A,

of both Problems I and II, the formulated and simulated customer service levels are compared statistically. In Part B of the same problems, the simulated customer service levels are analyzed to determine the individual and combined effects of the experimental treatments on customer service performance. In both problems, the statistical analysis consists of analysis of variance, post hoc multiple comparison procedures, and t-tests.

Dissertation Outline

This dissertation is presented in five chapters. Chapter II reviews the nature of safety stock determination, specific safety stock techniques, and the literature pertaining to multi-echeloned inventory control. Chapter III discusses the hypotheses tested, the research methodology utilized, and statistical analysis procedures. Chapter IV reports the results of the analyses and reviews the general hypotheses. Chapter V presents conclusions for safety stock planning and control and suggests areas for future research.

FOOTNOTES--INTRODUCTION

¹For a review of safety stock techniques, refer to Chapter II pp. 37-46 below.

²Robert B. Fetter and Winston C. Dalleck, Decision Models for Inventory Management (Homewood, Ill.: Richard D. Irwin Inc., 1971), pp. 50-52; and Robert G. Brown, Decision Rules for Inventory Management (New York: Holt, Rinehart & Winston, Inc., 1967), pp. 108-109.

³There are numerous sources, including: Jay Forrester, Industrial Dynamics (New York: John Wesley & Sons, Inc., 1961); and Andrew J. Clark, "An Informal Survey of Multi-Echelon Inventory Theory," Naval Research Logistics Quarterly, December 1972, pp. 621-650. For additional discussion, see Chapter II of this dissertation, pp. 47-60.

⁴Refer to Table 2.2, Chapter II, pp. 61-62 below.

⁵For a more complete derivation of this formula, as applied to inventory control, see Fetter and Dalleck, Decision Models for Inventory Management, pp. 50-52.

⁶For a discussion of using standard deviation as a measure for setting the level of safety stocks, see Chapter II, pp. 43-45 below.

⁷Since stock-outs only occur when demand is above the expected level, a one-tailed distribution is utilized. One standard deviation above the mean for a one-tailed normal distribution includes 84.13 percent of the area under the normal curve.

⁸Brown, Statistical Forecasting for Inventory Control, pp. 107-119.

⁹Ibid., p. 109.

¹⁰George D. Wagenheim, "The Performance of a Physical Distribution Channel System under Various Conditions of Lead Time Uncertainty: A Simulation Experiment" (Ph.D. dissertation, Michigan State University, 1974).

¹¹Robert E. Shannon, Systems Simulation--The Art and Science (Englewood Cliffs, N.J.: Prentice Hall, Inc., 1975), p. 4.

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¹² Robert G. House and Jeffrey J. Karrenbauer, "Logistics System Modeling," International Journal of Physical Distribution and Materials Management, May 1978, pp. 194-196.

¹³ David Joseph Closs, "Simulated Product Sales Forecasting: Mathematical Model, Computer Implementation, and Validation" (Ph.D. dissertation, Michigan State University, 1978).

¹⁴ Donald J. Bowersox et al., Dynamic Simulation of Physical Distribution Systems (East Lansing, Mich.: Bureau of Business Research, Michigan State University, 1973).

¹⁵ Shannon, Systems Simulation--The Art and Science p. 4.

¹⁶ Thomas H. Naylor et al., Computer Simulation Techniques (New York: John Wiley & Sons, Inc., 1966), p. 17.

¹⁷ Donald J. Bowersox, David J. Closs, John T. Mentzer, Jr., and Jeffrey R. Sims, Simulated Product Sales Forecasting--Documentation (East Lansing, Mich.: Graduate School of Business Administration Research Bureau, Michigan State University, 1978).

CHAPTER II

LITERATURE REVIEW

Introduction

Chapter II provides a review of the nature of safety stock requirements determination and an examination of the literature concerning inventory control in multi-echeloned physical distribution systems. The literature review is organized into three sections. The first section discusses the nature of safety stock, and reviews the major factors affecting the determination of safety stock requirements. The second section provides an overview of safety stock techniques. Section three reviews inventory control literature pertaining to multi-echeloned channel systems.

Nature of Safety Stock Determination

Safety stock, also referred to as buffer or protective stock, is locational inventory. Safety stock is placed in certain locations in an attempt to buffer the uncertainty that is inherent in the forecasts of customer demand and lead time performance of the firm.

Figures 2.1 and 2.2 illustrate the two types of uncertainty.¹ In Figure 2.1, demand occurs in excess of

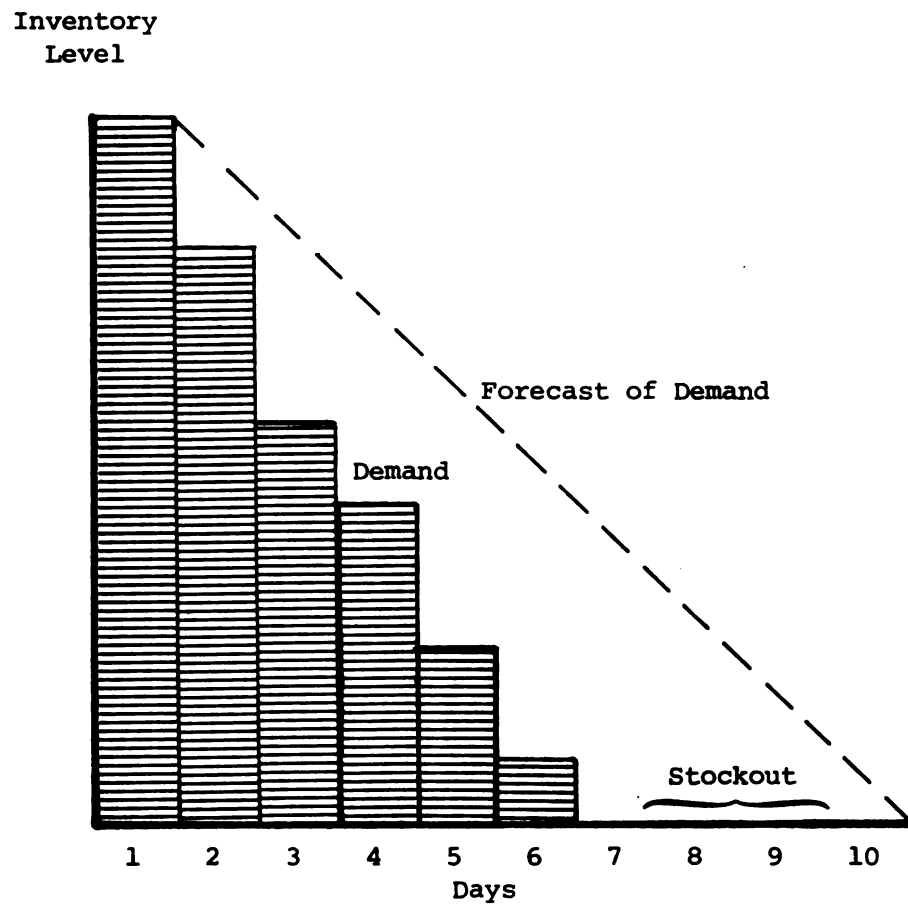


Figure 2.1 Demand Uncertainty.

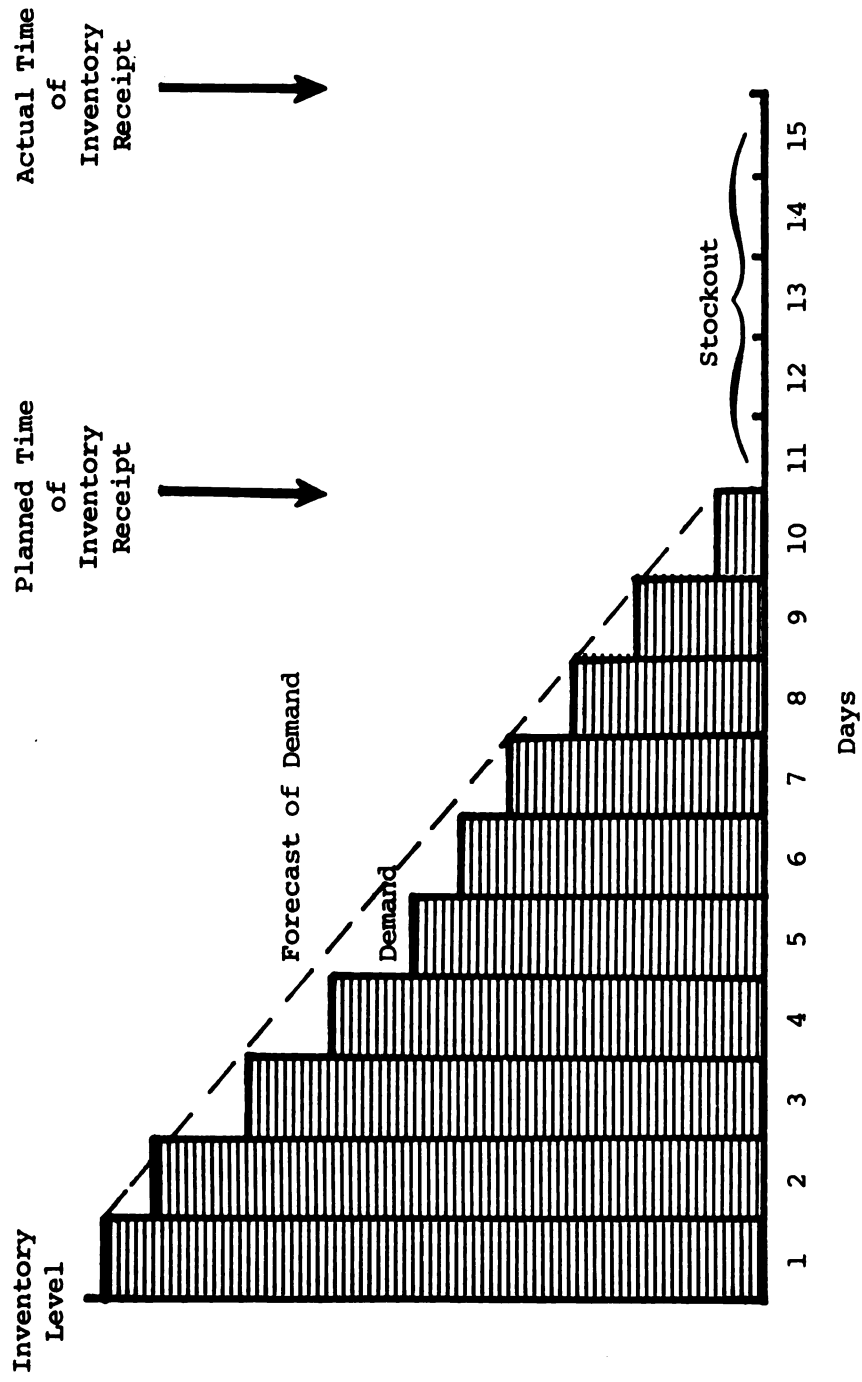


Figure 2.2 Lead Time Uncertainty.

forecasted demand during the period. This results in an out-of-stock condition at the end of the sixth day. Customer orders received by the retailer during the next four days will not be filled. Figure 2.2 depicts the second type of uncertainty: time delays in excess of the planned time for replenishing inventory stocks between two locations. As Figure 2.2 illustrates, replenishment of inventory stocks is planned to occur just as on-hand inventory reaches zero. However, because of unexpected delays, actual replenishment occurs five days later. Thus, an out-of-stock condition exists for the five-day period. Both types of uncertainty may result in inventory shortages. Safety stock, strategically placed, provides the firm with a specified degree of protection against inventory shortages. Two prerequisites to the successful determination of channel system safety stock requirements are: (1) the identification of the principal factors affecting safety stock; and (2) the choice of a safety stock technique which accurately reflects the uncertainty faced by the firm. The principal factors that affect a safety stock calculation are discussed in the remainder of this section.

When managers seek to deploy safety stocks successfully, a question that must be answered is: "On what does the level of safety stock required at each location depend?"

There is no simple answer to this question. To a large extent, the answer depends on the specific channel system characteristics and the customer service policies of the firm.

An examination of the general literature on safety stocks revealed an extensive number of factors which may affect the determination of safety stock requirements.

Plossl and Wight stated:

The amount of reserve stock required is a function consisting principally of the following elements: the ability to forecast demand accurately; the length of the lead time; the ability to forecast or control lead time; the size of the order quantity; and the service level desired.²

Prichard and Eagle identified the key factors as: level of performance, including reorder point determination, and service level desired; lead time demand; standard deviation of lead time demand; and shortage measures, such as shortage costs.³ Welsh defined the key factors as follows: accuracy of lead time estimate, probability of variations in lead time, accuracy of estimate of usage, cost of carrying "safety" stock, cost of "stockouts," and probability of variations in usage.⁴

A review of each of the numerous factors and their specific functional relationships with safety stock is a large undertaking. To provide a manageable discussion, the key factors from the literature have been grouped into categories. These categories are: demand, lead time,

forecasting, order quantity-costs, reorder point, customer service policy, and system structure. A discussion of the impact of these factors on the determination of safety stock requirements follows.

Demand

The decision as to how much safety stock to establish is partly dependent on demand uncertainty. For this reason, managers are interested in accurately determining the nature of the demand pattern. Specifically, the distribution and level of uncertainty provide a description of the demand pattern which helps managers set safety stock policies.⁵ A positive relationship exists between the level of uncertainty around average demand during the replenishment cycle and safety stock requirements. For example, an increase in the variance of demand during the replenishment cycle requires an increase in safety stocks, if it is assumed all other factors such as desired customer service are held constant.

Conversely, safety stock levels may affect demand. Ben Schwartz proposed the idea of Perturbed Demand and suggested the following formula:⁶

$$\lambda = \lambda_0 / (1 + \alpha I)$$

where:

- λ_0 = the expected demand rate that would prevail with no stockouts;
- α = the relative number of stockouts (percentage of stockouts);
- I = a constant parameter of the model related to customer response; and
- λ = the customer demand altered by previous stockout occurrences.

Schwartz stated:

The effect of a stockout is not to impose a cost ("penalty cost") against the firm at the time of the incident, but rather to modify the demand pattern in the future from that which otherwise would have occurred. This is a manifestation of loss of good will, in which the customer alters his future course of action due to meeting a stockout, rather than causing the firm some immediate, vaguely defined, loss.⁷

Thus, demand and safety stock are interdependent.

The practice of backordering has an impact on safety stock requirements. A customer's order becomes a backorder when it cannot be filled on request and the customer is willing to wait until delivery can be made.⁸ Backorders represent a sale made when inventory is zero. Thus, backorders are closely related to stockouts. Stockouts could occur at the time of order placement, at any time until the order is filled, or not at all. This can significantly affect a manager's determination of safety stock needs. For example, in a system where all orders

that occur after inventory depletion are backordered, and eventually filled, stockouts are impossible. In such an instance, safety stocks are not required. There is, in essence, a reciprocal relationship between backordering and safety stock. An increase in customer willingness to wait for orders not filled on request reduces the need for safety stocks.

Lead Time

The inventory replenishment cycle has been identified as a combination of order communication, order processing, order shipment, and update.⁹ When the replenishment cycle duration is longer than planned, stockouts may occur. For this reason, managers are interested in accurately determining the distribution and level of uncertainty of the replenishment cycle. Once the lead time pattern is determined, managers can set safety stock levels in an attempt to reduce the effects of uncertainty. Thus, lead time uncertainty and safety stock requirements have a positive relationship.

Vinson used a computerized cost-minimization model to study the importance of lead time unreliability (variability of lead time from mean lead time) in inventory management. Using different combinations of stockout cost, demand variability, mean lead time, and variability of lead time around the mean, Vinson observed changes in optimum safety stock and inventory costs.

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The conclusions of Vinson's study were, in part,

1. For a particular mean lead time, as the unreliability of lead time *increases*, optimum safety stock *increases* substantially, other variables remaining constant.
2. In general, to the extent that the two influences are mutually independent, variability of lead time around the mean is much more important than variability in the mean itself in explaining stockouts, stockout cost, and in setting optimum safety stocks.
3. Unreliability of lead time is a considerably more important factor in explaining optimum safety stocks than is unreliability in demand, where the two factors operate independently of one another.
4. Managers and inventory theorists should be particularly careful not to ignore lead time unreliability under the following conditions: when mean lead time is relatively short; when stockout cost is relatively high; when variability in demand is relatively small; and when lead time unreliability is relatively great.¹⁰

Thus, the level of variability of lead time is an important factor in the determination of safety stock requirements.

Forecasting

Managers design a forecasting system to provide a firm with accurate estimates of future demands and lead times. When forecasts are accurate, forecast errors are low. The result is a smaller safety stock requirement.¹¹ As Packer stated in a study of adaptive forecasting applied to inventory control: "It appears logical to establish the safety stock as a function of the success attained in

forecasting demand during a lead time."¹² There is an inverse relationship between the accuracy of forecasting and safety stock requirements.

Order Quantity-Cost

A major consideration in inventory control is the determination of the order quantity. The size of the order quantity is a factor which must be considered in setting safety stocks. For instance, a decrease in the order size will result in an increased number of smaller orders to obtain a given quantity. Since stockouts can only occur at the end of an order cycle, the increase in order frequency increases exposure to stockouts.¹³ The result is that an increase in safety stock may be necessary to maintain the customer service level that existed prior to the decrease in order size.

In general, order quantity determination is a *cost* based determination. Therefore, costs affect the determination of safety stock requirements.

The classical inventory order quantity model is the Economic Order Quantity (EOQ) which was first derived in 1915 by F. W. Harris.¹⁴ The formula often is referred to in the literature as the "Wilson Formula." The EOQ is the optimal balance of ordering costs and inventory carrying (maintenance) costs on an annual basis. The costs generally associated with ordering and inventory carrying costs are:

Ordering costs--order preparation; order communication; update activities; and managerial supervision.¹⁵

Carrying costs--cost of capital; inventory service costs such as taxes and insurance; storage space costs; and inventory risk costs including damage, pilferage, obsolescence, and relocation costs.¹⁶

Figure 2.3 illustrates the basic relationship.¹⁷ The point at which the ordering cost curve and carrying cost curve intersect indicates the number of orders which should be placed per year to minimize the combined costs of ordering and maintaining inventory.

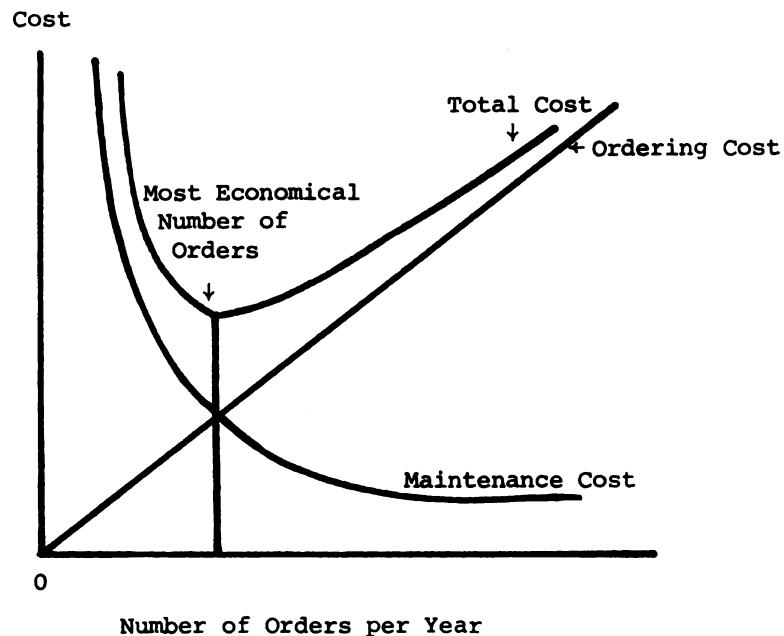


Figure 2.3 Economic Order Quantity.

A formulation of EOQ is:

$$EOQ = \sqrt{\frac{2C_o S}{C_m U}}$$

where:

C_o = cost per order;

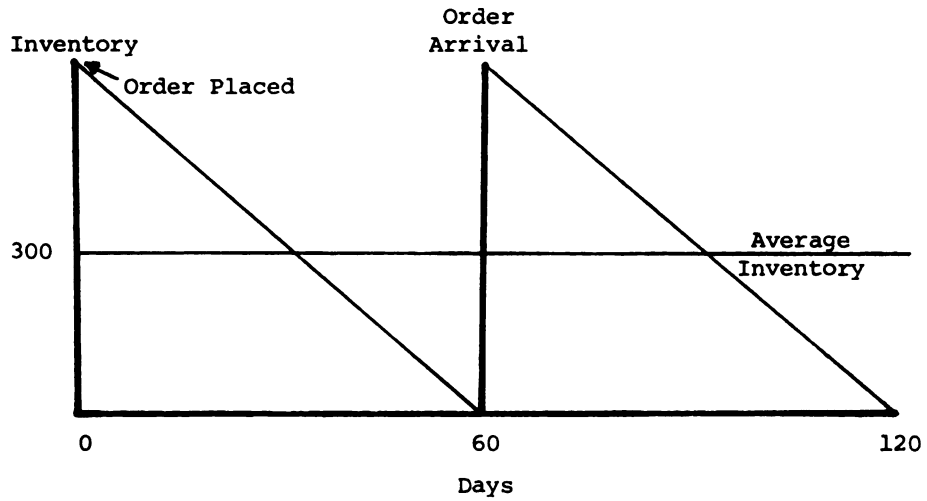
C_m = cost of maintenance per year;

S = annual sales volume, units; and

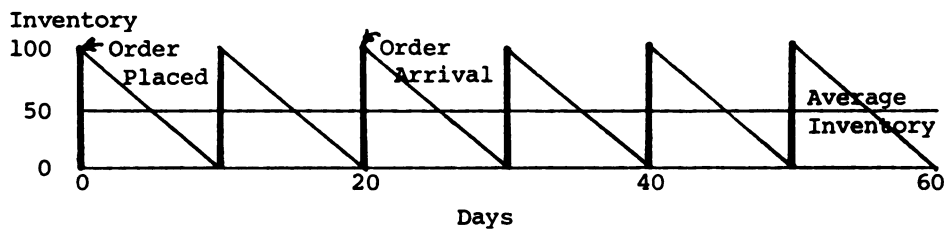
U = cost per unit.

As shown in Figure 2.3, order costs have a positive slope, and increase as the number of orders increase. Order costs increase as safety stock requirements rise.

Inventory carrying costs have an inverse relationship with safety stocks. This can be explained as follows: As the number of orders increase, the size of each order decreases for a given quantity. The average inventory is defined as one-half the order quantity. Since the order quantity is declining, average inventory is also declining and so are the associated costs. At the same time, since more orders are being issued, stockout exposure is increasing and so are safety stock requirements. Figure 2.4 illustrates the relationship.¹⁸ As order size decreases, average inventory decreases, but the number of stockout exposures increases.



Example: Order 600



Example: Order 100

Figure 2.4 Illustration of Variable Order Quantity and Average Inventory.

After the appearance of the basic EOQ model, attempts were made to expand the scope of the formulation. Among the variables added to the model were: manufacturing costs, interest rates, cost of capital, shortage costs, transportation costs, backorder costs, and provision for quantity discounts. Each of these variables had its own impact on the final EOQ and subsequently, on safety stock requirements.

Several authors have devised order quantity formulas which include a stockout cost variable.¹⁹ Essentially, their attempts are to devise a formula for ordering, which balances the cost of holding inventory with the cost of stocking out. However, stockout cost is extremely difficult to quantify. It may include the cost of backordering and the cost of lost sales.²⁰ Vinson described the relationship between stockout costs and safety stock requirements as follows:

Other things remaining the same, an increase in the unit stockout cost triggers a buildup of safety stock at *all* degrees of lead time unreliability. That is, magnitude of stockout cost and the degree of lead time unreliability appear to be of roughly equal importance in accounting for optimum safety stocks.²¹

A positive relationship exists between a unit's stockout cost (worth) and its safety stock requirements. The more important a particular product is to customers, the more likely it will be that sufficient safety stocks will be

maintained for that particular product. However, the overall relationship between stockout cost and safety stocks is inverse. Increases in safety stocks reduce the total number of stockouts that occur, and therefore, total stockout costs decline.²²

Buffa provided an example of the use of stockout costs in an inventory model. He uses a goal programming model to set safety stocks in a fixed interval, variable order quantity inventory system. The values of the safety stocks are determined subject to two types of constraints relating to the limited resources available for maintaining safety stocks. The first constraint is capital resources measured in wholesale costs per unit. The second constraint is space available for storing the safety stock (measured in cubic feet of warehouse space). Costs resulting from the use of the model are contrasted with costs resulting from an alternative method based solely on a forecasting model. Excess resources are required when safety stock levels, determined by the forecasting model, would utilize more than the available capital and space. In the goal programming model, a tradeoff occurs between the potential for greater stockout costs versus the costs of acquiring excess resources, in the form of borrowing additional funds or renting additional space. Buffa concluded in setting safety stocks that the goal programming model was less costly than the forecast model.²³

Other distribution costs affect the amount of safety stock placed at certain locations in a channel system. For example, a manager faced with a short-run demand or lead time uncertainty may decide to use premium transport (expediting) instead of safety stock to reduce the uncertainty. The decision as to which method to employ in a given situation can be based on a tradeoff of the relative costs of the two choices. The tradeoff would be between the cost of carrying extra inventory as safety stock and the cost of premium transport, assuming customer service levels are comparable for the two choices. Thus, safety stock requirements can depend, in part, on the relative cost of premium transport.

Recently, Whybark used a heuristic procedure for jointly determining the inventory reorder points, order quantities, and transportation alternatives that provide minimum total transportation and inventory costs. He stated that the interaction between inventory parameters and selecting transport alternatives suggests that the decisions should be made simultaneously. Whybark's paper suggested a way to make this joint decision. The solution required choosing the transportation alternative, order quantity, and reorder point that will minimize total costs.²⁴

Reorder Point

A reorder system designates "when" to order a replenishment order quantity. Two basic methods are used to determine order points: fixed reorder quantity method and fixed reorder cycle system.²⁵ These two methods have a different impact on safety stock.

The fixed reorder quantity system is based on an order point which triggers orders at a predetermined quantity. The determination of the quantity may be based on an EOQ formulation. When inventory falls to the reorder point, the EOQ is ordered.

The fixed reorder cycle system is based on a fixed review period. When a certain number of days pass, the inventory is reviewed and an amount is ordered which will bring inventory on-hand back up to a predetermined level. In comparing the two systems, the fixed reorder cycle system requires larger safety stocks since protection against stockout must cover a longer period (review period plus lead time), instead of only the lead time, as in the fixed reorder quantity system.²⁶

Reorder point systems incorporating features of both the fixed order quantity and fixed order cycle systems are called "optional replenishment" systems. The most common of these systems is the Buffa optional replenishment model, also referred to as the (S,s) or min-max model.²⁷

The optional replenishment model incorporates a fixed order point with a fixed order period. If the fixed order point is not reached at the time of the review, no order is placed. The optional replenishment model allows orders to be placed in economic order quantities, and also reduces costs resulting from the frequent placement of small orders. However, in cases where stockout costs are significant, the amount of safety stock under an (S,s) system may have to be increased to guard against the possibility that the level on hand at the time of review may be slightly above the order point. This raises the necessary coverage to two order intervals, plus the order cycle.²⁸

Customer Service Policy

When managers seek to implement a customer service policy successfully, they must carefully consider inventory availability in the channel system. Customer service standards usually are stated in terms of percentage of orders filled or percentage of quantity filled, or conversely, the percentage of stockouts allowed.

Safety stocks are used to help managers attain a desired customer service level. By placing safety stocks of a specified amount at certain locations, managers may be able to accurately control the level of service of individual products or product categories. Many firms use an ABC analysis to sort products into separate service level

categories. This analysis involves the sorting of inventories into categories so that managers may apply separate inventory policies to each category. Generally, profit contribution margin or percentage of total sales are used as a basis for categorizing products.

The relationship between customer service level and amount of safety stock is positive. An increase in the size of safety stocks results in a higher customer service level. However, the incremental gain in service level realized by increasing safety stocks diminishes as safety stocks increase. In other words, service level increases at a decreasing rate for each additional unit of safety stock.

System Structure

Structural aspects of a channel system must be considered when managers are determining safety stock requirements. The number of stock locations in a channel system, and the manner in which the locations are linked together, both have impacts on safety stocks.

Camp, in a doctoral dissertation, provided a statement of the relationship between number of locations and safety stock levels in a channel system. Camp introduced a heuristic model to test the effects of variable lead times on a single echelon physical distribution system. The model allowed the researcher to observe the customer

service and total cost performance of alternative configurations of warehouses, given some variation in lead time.

Safety stock was incorporated into the model and was quantified on a deterministic basis, as warehouse mode selected determined safety stocks. The research showed that the safety stock curve increased but at a decreasing rate with the number of warehouses. In addition, the level and shape of the safety stock curve further depends on the mode selected and the locations of warehouses.²⁹

Figure 2.5 illustrates the safety stock curve. The curve flattens as more locations are added; the actual shape of the curve is the result of two impacts. First, the addition of stock locations reduces the planned duration of the replenishment cycle and therefore, reduces the variability in lead time demand. The result is a reduction in uncertainty and safety stock requirements. At the same time, a second impact, the addition of replenishment cycles to the system increases uncertainty and the need for new safety stock.³⁰ The second impact is greater and thus, as the number of locations increases the amount of safety stock increases.

Linkage patterns can have an impact on locational safety stock requirements in a channel system. An example

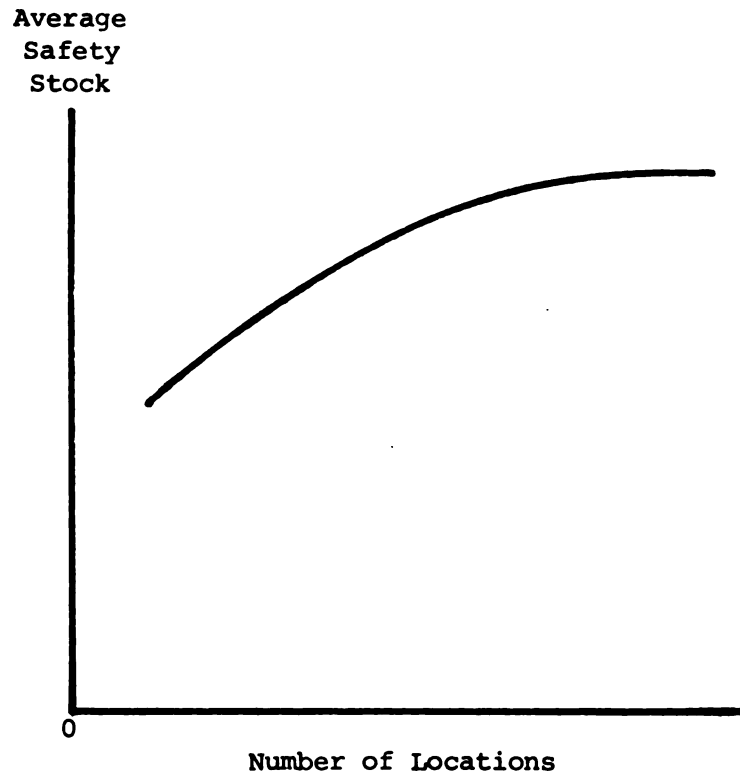


Figure 2.5 Relationship Between Safety Stock and Locations.

of the impact may be obtained by assuming a channel system with the structure illustrated in Figure 2.6.

The manufacturing plant replenishes two retail locations A and B. Safety stocks are held by the manufacturer for both retail locations. Assuming that the coefficient of variation is identical on all links, and that Retailer A can hold the safety stocks for both retailers; then allowing cross-shipping from Retailer A to Retailer B would reduce the absolute amount of safety

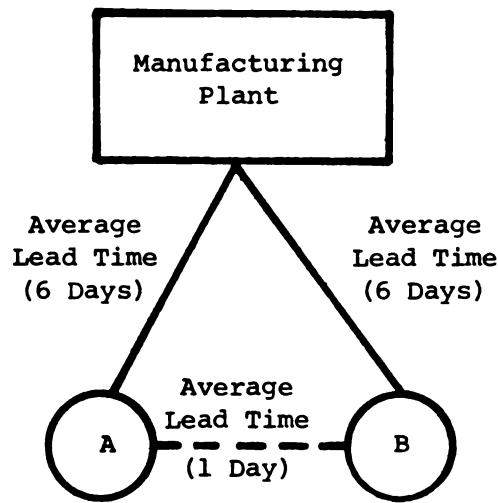


Figure 2.6 Channel System Structure.

stocks required for Retailer B. The absolute size of the lead time demand variance would be reduced. Therefore, the linkage pattern is an important determinant of safety stocks required in the channel system.

The first section of the literature review contained a discussion of the major categories of factors that have impact on the determination of safety stock requirements in channel systems. In addition, an explanation of the nature of the impact was provided.

The numerous impacts on safety stocks were dealt with independently in this section. However, when a manager determines safety stock level and positioning in a channel system, these impacts must be considered simultaneously.

Many techniques and formulations have been created to reflect the simultaneous and interdependent impact of several factors on safety stock requirements. A review of these techniques is presented next.

Safety Stock Techniques

A review of the general inventory theory and control literature reveals a plethora of formulations for estimating inventory requirements under all conditions. A review of every technique is beyond the purpose of this research. This section presents a review of techniques and procedures used in the determination of safety stock requirements in channel systems. For purposes of review, the techniques discussed are grouped into four categories: intuitive techniques; mathematical formulations; statistical techniques; and simulation procedures. Techniques reviewed in each category are intended as samples of the complexity and sophistication currently available.

Intuitive

Intuitive techniques are non-mathematical determinations of safety stock requirements. Management judgment or executive opinion forecasts are representative of this type.

Larson described an intuitive technique for planning safety stocks, and his *Fixed Period Quantities Method*

described it in the following manner: "Set safety stock *arbitrarily* at 50 units for one month, 75 units for the next month and so on."³¹ The "method" is very quick and simple to use. However, as Plossl and Wight noted, intuitive techniques usually fail to take into account all the relevant factors.³²

Mathematical Formulations

Mathematical formulas used to estimate safety stock requirements cover a wide range of sophistication. The formulations are as simplistic as unweighted averages and complex enough to involve eight to ten factors.

A simple technique is the *Percentage of Average Demand Method*.³³ Larson depicted this method as follows:

Percentage of Average Demand Method

Data:	Period	1	2	3
	Demand	5	15	25

Step 1: Calculate Average Demand: $\frac{5 + 15 + 25}{3} = 15$

Step 2: Set safety stock = 1/3 of average demand =
1/3 x 15 = 5 units.

The advantage of the technique is its ease in use. Its major deficiencies are the equal weighting of period demands, and the fact that the method ignores lead time uncertainty.

Welsh derived a safety stock formula which he believes contains all the factors that enter into a safety stock decision. The derivation follows:

Example:

Safety stock is a function of usage and that function relates to the square root of demand, times the average demand. Since we are looking at performance over the estimated lead time period, that factor becomes:

$$\sqrt{\text{Demand per Lead Time}} \times \text{Average Demand}$$

A factor (P) is developed which approximates the difference in protection needed as safety stock relates to order frequency and is added to the formula. A constant factor (K) also is added as a calibrating factor to bring the theory into relative position with actual experience. The formula then becomes:

$$(K) \times (P) \times \sqrt{\text{Demand per Lead Time}} \times \text{Average Demand}$$

Because short lead times have a greater relative tendency to vary than do the longer ones, the formula was multiplied by $\frac{1}{\text{lead time}}$ which yields:³⁴

$$\text{Safety Stock} = (K) \times (P) \times \sqrt{\text{Monthly Usage}} \times \sqrt{\text{Average Demand}}$$

Smith considered the problem of finding optimal stocking policies for an inventory system with poisson demands, arbitrary replacement time distributions, and

emergency handling of shortages at a premium cost.³⁵

He developed an approximate stocking level formula which has accuracy over a wide range of parameter values. The formula is:

$$S^* = \lambda\gamma + \alpha \sqrt{\lambda\gamma}$$

where:

S^* = optimal stock level;

$\lambda\gamma$ = expected units demand over lead time; and

$\alpha \sqrt{\lambda\gamma}$ = safety stock.

In the safety stock formula:

$$\alpha = (2 \log [1 + L/H\gamma])^{\frac{1}{2}};$$

L = penalty cost of stockout;

H = holding cost;

γ = mean lead time; and

λ = demand rate.³⁶

This formula acts as a safety stock which increases with demand rate, shortage penalty, and mean lead time, and decreases with holding cost. Smith's formulation is representative of the numerous safety stock techniques that are built for very specific conditions.

Mathematical formulas provide managers with an easy method for approximating the complex relationships that

exist in an inventory model and a quantifiable solution to the determination of safety stock requirements. A major drawback of these techniques is the inability to reflect accurately a combined variation of factors such as demand and lead time variabilities.

Statistical Techniques

The most commonly used technique for setting safety stock level is an application of statistical probability. Since a solution requires a combined analysis of demand and lead time uncertainty, it is common to use frequency distribution to represent the data patterns under analysis. The most commonly used statistical pattern is the normal distribution.

A random variable (X) is said to have a normal distribution if its probability density function is given by:

$$f(X; \mu; \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left[-\frac{1}{2\sigma^2} (X - \mu)^2\right]}$$

The normal distribution parameters are μ , the mean, and σ^2 , the variance.³⁷

A normal distribution is characterized by a symmetrical bell shaped curve, as illustrated in Figure 2.7.

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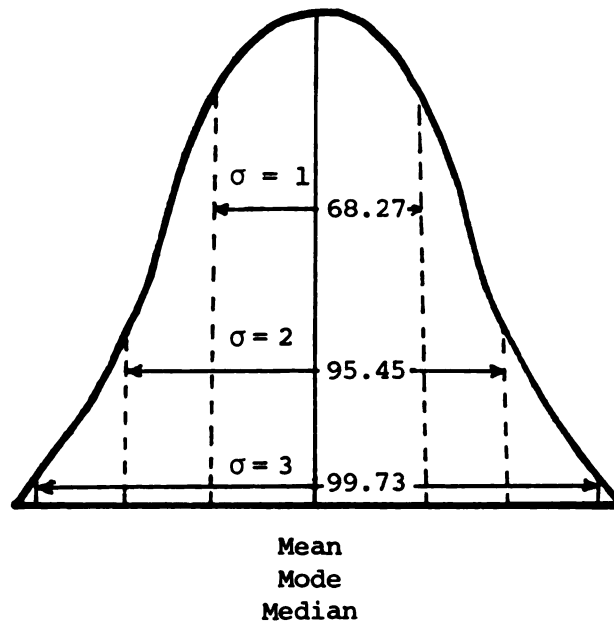


Figure 2.7 Normal Distribution.

The essential characteristic of normal distribution is that the three measures of central tendency most commonly used in statistical analysis are identical. The mean, or average; the median, or middle value; and the mode, or the most frequently observed value, all have the same numerical value.

The basis for prediction using a normal distribution is the standard deviation of observations around the measures of central tendency. This standard deviation is a measure of the dispersion of events under the areas of the normal curve. With $\pm$ one standard deviation from the mean, 68.27 percent of all events occur. With $\pm$ two

standard deviations, 95.45 percent of all events are included. Within $\pm$ three standard deviations, 99.73 percent of all events occur.

The standard deviation is the measure most widely used to set safety stock level. It provides a means of estimating the safety stock required to provide a specified degree of protection against uncertainty.

The formula for standard deviation is:

$$\sigma = \sqrt{\frac{\sum fd^2}{N}}$$

where:

σ = standard deviation;

f = frequency of event;

d = deviation of event from mean; and

N = size of sample.

Managers only worry about events such as excessive demand or longer than average lead times that exceed the mean value. Therefore 50 percent of the time, when variations are below the mean, safety stock is not required. Thus, a level of safety stock which should cover 95 percent of all events included in a frequency distribution will actually cover 97.72 percent of the relevant events. Table 2.1 provides a list of safety stock protection at various levels of standard deviation under a normal

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Table 2.1 Safety Stock Protection at Various Levels of Standard Deviation

<u>Safety Stock Set at</u> <u>Standard Deviation</u>		<u>Degree of</u> <u>Protection</u>
1.0		84.13
1.3		90.32
1.6		94.52
2.0		97.72
2.3		98.93
2.6		99.53
3.0		99.86

distribution. Brown provided an example of the use of standard deviation in a safety stock formulation. Brown's formula looks at the marginal cost of providing additional inventory versus the marginal service gained. The procedure is as follows:

$$\text{Step 1: } f(k) = \frac{Q}{M} (1 - P) \quad \text{Solve for } f(k)$$

where:

$f(k)$ = service function;

Q = number of months between successive reviews;

M = standard deviation of forecast errors, expressed in months' supply; and

P = desired service percentage, expressed as a decimal.

Step 2: Relate the $f(k)$ value to a safety factor (standard deviation).³⁸

The standard deviation measure has been utilized widely by managers to measure demand and/or lead time uncertainty. The technique is adaptable to situations where the combined variations of demand and lead time must be estimated.

A statistical convolution formulation, detailed in Chapter I, is used to relate the standard deviation of the frequency distribution for demand with that of lead time. Through the statistical convolution formulation, managers are able to determine safety stock requirements, while taking into account the joint impact of environmental and operational uncertainty.³⁹

A more rigorous method of combining the variation in demand with the variation in lead time is *Numerical Compounding*. The Exact Numerical Method of compounding two variables involves the expansion of a multinomial of the type:

$$p(tm) = [p(X_1) + p(X_2) + \dots p(X_n)]^{tm}$$

where:

t = lead time;

X = demand per period;

m = number of lead time periods;

n = number of demand periods; and

$p()$ = probability of event in parentheses.⁴⁰

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An expansion of this equation determines the exact probabilities for all possible demands during all lead times. These probabilities are then used to set safety stocks by setting the desired level of protection corresponding to a probability of stockout.⁴¹ While this method provides an exact answer, it is extremely lengthy and requires the use of a computer for all but the smallest problems.

In summary, the most frequently used techniques for determination of safety stock requirements are statistical techniques. Specifically, the standard deviation influences the safety stock level. The technique is adaptable to various types of distributions and situations where joint probabilities exist.

Simulation

A simulation technique used for compounding environmental and operational uncertainty is Monte Carlo Simulation.⁴² The procedure consists of assigning probabilities to each event in a manner that reflects the frequency with which the event would be expected to occur over a large sampling of events. Then a lead time is randomly selected and subsequently demands are randomly selected for each day of the lead time. By repeating this procedure a large number of times, the relationship between the two uncertainties is simulated, or approximated. All

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situations in which the simulated average demand exceeds the expected average demand during lead time are kept for purposes of formulating a safety stock policy. All situations where the average demand during lead time is less than or equal to expected demand may be eliminated from considerations, since no safety stock is required. Monte Carlo Simulation is especially useful when either demand or lead time do not have a normal or poisson distribution.

This section has reviewed the techniques used to determine safety stock requirements in channel systems. The next section provides an overview of the inventory control literature relating to multi-echeloned physical distribution systems.

Multi-Echeloned System Inventory Control Literature

In broad terms, multi-echeloned system inventory theory is concerned with a variety of inventory problems involving two or more interrelated stocking locations.

A multi-echeloned inventory system can be portrayed as a directed network wherein the nodes represent the various activities or facilities in the system and the linkages represent flows of goods. If the network has, at most, one incoming link for each node, and flows are acyclic (no loops to the network), it is called an "arborescence" or inverted tree structure.⁴³ Figure 2.8

depicts an arborescence. The various stages of the system are commonly identified as echelons and the problems investigated are defined as multi-echeloned.

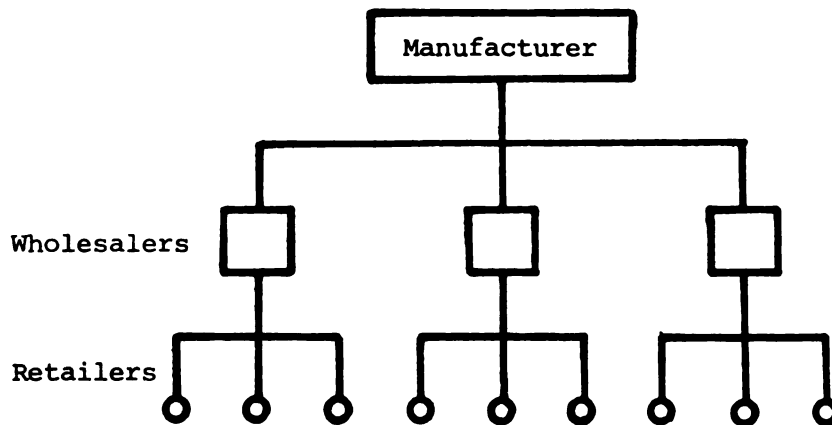


Figure 2.8 An Arborescence.

Two special kinds of arborescence structures are commonly used in the literature. As illustrated in Figure 2.9, these are (1) the series structure consisting of two or more activities (stock locations) with each supplying only one other (lower echelon) activity, and (2) the parallel structure, consisting of a number of activities experiencing independent external demands.

Two broad categories of analysis dominate the multi-echeloned system inventory literature--Formal mathematical techniques such as Dynamic Programming, which is the most often used mathematical technique, and simulation. The

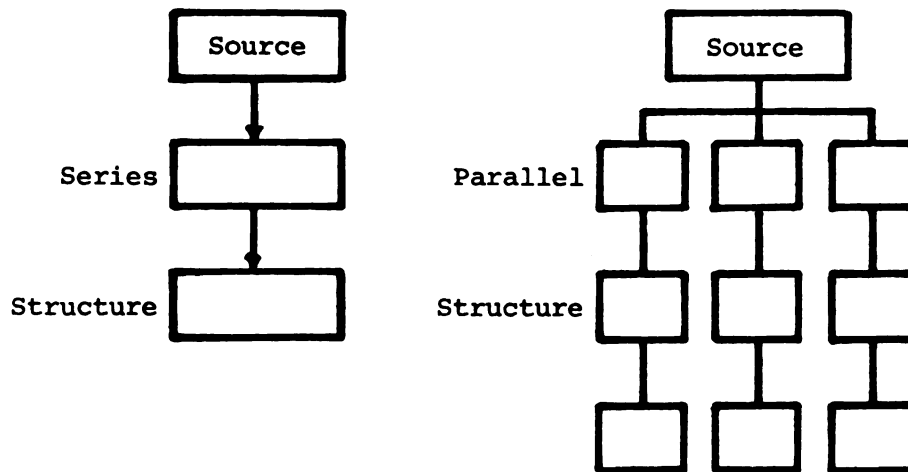


Figure 2.9 Series and Parallel Structured Multi-Echeloned Systems.

literature in each category will be reviewed, with emphasis on the works that pertain to this research.

Mathematical

Bessler and Veinott⁴⁴ extended Veinott's "Dynamic Process Analysis Technique"⁴⁵ to a multi-echeloned problem. The multi-echeloned structure used was an arborescence with n facilities each stocking a single product. Customer demand was satisfied by the stock at each location, but excess demand was successively passed up the channel to the next echelon until it was satisfied. Backlogging was allowed, but only at the top of the channel, at the top echelon. Given these policies, the one period cost function was established from which optimal stock levels were derived.

Then, the optimum policy was obtained by solving N n -dimensional minimization problems, where (N) was the number of periods and (n) was the number of stocking locations.

Upper and lower bounds for optimal stock levels were calculated for each stocking location. Finally, Bessler and Veinott presented an algorithm for computing approximations of the optimal stock levels based on the values obtained for the lower bound of the stock level.

Since excess demand at all echelons (except the top one) was passed up the channel immediately, no safety stock policy was considered for these echelons. Since backlogging occurred at the top echelon, no safety stock was required. Service level was 100 percent over time.

Clark and Scarf⁴⁶ used a dynamic programming approach to solve the cost minimizing problem for a single product at an arbitrary number of stocking locations. The stocking locations were arranged in a series structure.⁴⁷

Implied shortage costs were generated at each echelon and passed up the channel to become part of the cost function of the echelon above. Thus, the optimal policy for inventory was first established at the lowest echelon. The shortage costs were obtained and passed on. The process was then repeated at the next echelon. This continued until all echelons had an optimal policy.

One of the features of this model was an allowance of backorders at all echelons. Service level was 100 percent in the long run. Safety stock policy was not considered.

Hochstaedeter⁴⁸ extended the Clark-Scarf⁴⁹ formulation adopting the same cost-demand structure. However, Hochstaedeter permitted fixed reorder costs at the lower echelons as well as at the top echelon. He established upper and lower bounds for optimal system costs, with each set of bounds yielding (s, S) type ordering policies for each echelon. The calculation of shortage costs was similar to that of Clark and Scarf, and only the functional form was changed. As with Clark and Scarf, the backorder policy eliminated safety stock from consideration.

Sherbroke,⁵⁰ using the Stationary Process Approach pioneered by R. L. Love,⁵¹ analyzed the multi-product problem involving inventories of recoverable (repairable) items in a parallel structure. At each stocking location, Sherbroke attempted to establish stock objectives which would minimize the sum of expected backorders on all recoverable items, given budget constraints. Investment dollars were allocated across product using a marginal analysis, and priority was given to those items which incrementally could have the largest reduction in system backorders. Safety stock was not considered.

Simon⁵² refined and extended portions of Sherbroke's model. As before, a parallel structure of bases supported by depots was considered, and where repair of a failed item could occur at any stocking location. The model allowed system losses to occur at a specified rate. However, Simon had $(S-1, S)$ or one-for-one replenishment policy for bases, and (s, S) or optional replenishment policies for depots. Simon derived exact expressions for stationary distributions of stock on hand, stock in repair and backlogged demand at each facility and indicated how these expressions could be used for optimization of system backorders and/or costs. Safety stocks were not considered in this model.

Tan derived optimal policies for a multi-echeloned inventory problem with periodic ordering.⁵³ A central warehouse supplied two facilities with a product. The warehouse could allocate the stock to the facilities in each period, but could reorder from an exogenous source only after every "T" period. Given this problem, Tan solved it for the optimal allocation policy. Safety stocks were not considered.

Simulations

Aggerwal and Dhavale measured the performance of a multi-product, multi-echeloned distribution system in terms of five important management criteria: average

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investment in inventories, average shortage costs per year, average reorder costs, average annual inventory carrying costs, and the total number of reorders per year.⁵⁴ Three levels of demand, lead time, and inventory costs were utilized. There were $3 \times 3 \times 3 = 27$ factor level combinations. Three simulations were run for each factor level combination. This resulted in $3 \times 27 = 81$ *values* for each of the five management criteria.

Within cell variances were calculated by using an analysis of variance. Thus, the authors could examine the percentage of variance in the five performance measures caused by the factors (demand, lead time, costs, and their interactions).

One finding was that annual shortage costs of the system were most sensitive to lead times, less sensitive to demand and least sensitive to cost factors.⁵⁵ Additional findings were: mean demand significantly affected inventory investment in the same direction; inventory carrying costs were affected most by mean demand; as lead time increased, the number of reorders decreased. The model made no provision for safety stock.

Ballou's doctoral dissertation involved a computer simulation of an inventory system consisting of three firms linked together as a multi-echeloned physical distribution system.⁵⁶ With this simulation, various inventory policies

were tested. The major effects of any particular inventory policy on the individual firm as well as the cost tradeoffs within the system were determined.

Of the inventory models used, the comprehensive model was the most sophisticated.⁵⁷ Although the comprehensive model was a deterministic inventory model, with backorder costs and quantity discounts, it did include a number of stochastic variables. These variables included demand, lead time, and inventory level. The inventory level depended on stochastic demand, stochastic lead time, and back-order costs. Other less sophisticated inventory models also were utilized and compared with the comprehensive model.

Research conclusions were:

1. When highly predictable conditions prevailed, a less sophisticated model yielded the best profits.
2. Backorder cost was a significant factor, and for the highest profits to be achieved, the model selected had to include this factor.
3. A single stage model with backorder costs could be used effectively when backorder costs were high.
4. Transportation and inventory carrying costs had little affect on model selection within a reasonable range of values.
5. Higher profits could be achieved if the inventory problem was viewed from a total systems approach rather than a single firm approach.

Since there was an assumption that *all* orders eventually would be filled (100 percent backorders, with a fixed backorder cost), the service level was 100 percent. Safety stock was not considered in the design.⁵⁸

Forrester developed a multiple stock location, multi-echeloned model to study repercussions from disturbances occurring primarily at the retail level.⁵⁹ The model used one product and did not consider costs. Delays in the flow of information and of goods were used to characterize the dynamics of the model. Inventory was held at each stage of the system and reorders were made to the echelon above, when necessary.

Forrester succeeded in showing that small disturbances or changes occurring at the retail level are amplified by a multi-echeloned inventory and ordering structure. Forrester's work dramatically pointed out the temporal aspects of inventory. Safety stock policies were not a part of this design.

Gross and Soriano studied a military overseas resupply system, where air transport replaced the reductions in on-shelf inventory and safety stocks in a multi-echeloned distribution system.⁶⁰ Curves depicting the relationship between safety stock level and various (demand and lead time combinations) were developed. The results of the study were: first, the higher the initial service level, the greater the

safety stock reduction when mean lead time was reduced. Second, changes in system performance were more sensitive to changes in mean lead time and variation and less sensitive to variation in demand and distribution shape. The absolute value of the mean demand had no significant affect on inventory. Third, the standard deviation of demand over lead time indicated what the reduced safety stock levels should be when lead time was reduced and also measured the effects on (on-shelf) inventory. The demand over lead time formula was:

$$\sigma/ux = \sqrt{u_t^2 V_x^2 + u_x^2 V_t^2}$$

where:

σ/ux = combined variance;

u_t = mean lead time;

V_x = coefficient of demand variation; and

V_t = coefficient of lead time variation.

Fourth, safety stock and on-shelf inventory levels seemed to be insensitive to changes in the operating inventory level ($S - s$). Fifth, changing the time between inventory reviews affects system performance. This research showed that *mean* lead time does significantly effect safety stocks in multi-echeloned systems. Discussion of the actual techniques used to calculate safety stock was omitted.

Sims used a dynamic computer simulation model to measure the combined effects of demand and lead time uncertainty on the performance of a physical distribution system.⁶¹ Sims analyzed performance under four time series forecasting techniques, in combination with various levels of demand and lead time uncertainty. Performance measures were in terms of sales, service, and cost. The total discrepancy between sales and *demand* was separated into two types of errors: (a) forecasting error and (b) operating error.

Sims found that increases in variation of demand resulted in increased stockouts and reduced profit, regardless of the level of operating uncertainty or forecast technique used. Also, increases in operating uncertainty resulted in increased stockouts and reduced profit across all combinations of forecast techniques and demand patterns, except one. In addition, variations in forecast accuracy lead to variations in channel performance across all demand and operating uncertainty combinations; complexity of technique was inversely related to accuracy; and the effects of increases in demand and operating uncertainty tended to cancel each other.

Two conclusions were cited: defining forecast error as the difference between sales and forecast is incorrect. Such a procedure generates future forecasts based on past

levels of *operating* as well as forecast error. Therefore, more consistent system performance is achieved using simpler forecast techniques which are not affected as easily by operating uncertainty. Safety stock was not considered in this research design, as it would have worked against the main purpose of the research.

Speh examined the performance of a multi-echeloned physical distribution system under various conditions of demand uncertainty. Performance measures were cost and customer service capability. The experimental factors were probability distribution, mean, and variance of demand. Six probability distributions of daily demand, three levels of demand variance, and two levels of average demand per day were used:

Speh's major conclusions were:

1. Total cost was not measurably affected by various demand uncertainties, although demand uncertainties did cause significantly higher stockouts to occur.
2. Exponential and normal demand distributions created the greatest impact on system performance.
3. Demand uncertainty affected demand stocked out.
4. In general, the amount of demand stocked out was more sensitive to demand uncertainty than total costs. Total cost did not vary with changes in variances or distribution.
5. The more symmetrical distributions, the lower demand variances, and the high average demand

level appeared to create cost and service impacts at the wholesaler and manufacturer levels of the system. The less symmetrical distributions, the higher variances and low average demand level led to a higher incidence of stockouts at the retail level.⁶²

Safety stocks were not carried at any level since demand per day and lead time were fixed. Thus, no safety stock technique was employed.

Wagenheim examined the performance of a multi-echeloned physical distribution system under various conditions of lead time uncertainty.⁶³ Performance measures were cost and customer service capability. The probability distribution, average duration, and variance of lead time were the experimental factors. Eighteen different combinations of the experimental factors were examined, with the aid of a dynamic simulation. Wagenheim concluded: "For all three experimental factors, uncertain lead time resulted in higher costs and lower service level than the deterministic lead time." In addition, all probability distributions had similar effects on the system, except exponential. The symmetrical distributions (normal and gamma) had the least effect on cost and service while the skewed distributions (exponential and erlang) had the greatest effect. A major finding was that higher variance and longer lead time impacts the system cost and services negatively. A tentative conclusion was that lead time uncertainty is more

critical than demand uncertainty in the physical distribution system. No safety stocks were employed in this research because the objectives of the research were such that daily demand and lead time are fixed.⁶⁴

The literature review showed that many of the factors affecting safety stock requirements determination have been studied and included in various statistical safety stock formulations. However, one aspect which has not been fully explored is the interdependent nature of the factors impacting on safety stock requirements. Since current safety stock formulations may not accurately replicate the interdependence impact of factors upon safety stock requirements, their use in attaining a prescribed customer service is questionable. Additionally, the applicability of current statistical safety stock formulations to multi-echeloned physical distribution situations has not been examined. Table 2.2 illustrates the lack of consideration of safety stock requirements determination in multi-echeloned systems.

Table 2.2 Summary of Safety Stock Factors in Multi-Echeloned Case

Author	Demand	Lead-Time	Forecast Method	Order Quantity	Costs	R.O.P.	Service Level
Aggerwal Dhaval ⁶⁵	Independent varied (hi-med-low) mean	Independent varied (hi-med-low) mean			Carrying average shortage, average order investment	(s,S)	
Ballou ⁶⁶ (Comprehensive Model)	Exogenous normal distribution level--hi-med-constant 100% backorder one product	Exogenous normal distribution level--hi-med-constant	Demand forecast	30 day moving average	Order carrying transport backorder (fixed) manufacturer	Fixed	100%
Bessler- Veinott ⁶⁷	Exogenous variable backorder at top echelon	Immediate (zero)	Historical trend		Minimize--proportional order cost, storage, transport		100% (over time)
Clark- Scarf ⁶⁸	Independent variable backorder--all echelons	Constant			Shipping--linear, holding--convex, shortage--convex	Low echelons (S-1,s), top echelons (s,S)	100% (over time)
Forrester ⁶⁹	Stochastic generation constant	Variable			None		
Gross and Soriano ⁷⁰	Monte Carlo generated poisson or normal "lump sum withdrawal"	Monte Carlo normal/constant, uniform/exponential				Periodic (s,S)	Various
Hochstaed- eter ⁷¹	Independent backorder all echelons	Integer multiples of review periods			Shipping--linear, holding--linear, shortage--linear	(s,S)	100% (over time)
Sherbroke ⁷²	Bayesian generation poisson distribution				Backorder	Initial (S-1,S) continuous review, no reorders	
Simon ⁷³	Independent poisson	Independent Deterministic			Backorder	Base--(S-1,S) Depot (s,S)	
Sims ⁷⁴	Increase and decrease trend, normal distribution	Erlang distribution level, zero-hi-low	1. Exponential smoothing 2. adaptive smoothing 3. WTD moving ave. 4. self-adaptive 5. perfect	10 Days of forecasted sales	Handling and inventory variable	10 Days (fixed)	Not set

Table 2.2--Continued

Author	Demand	Lead-Time	Forecast Method	Order Quantity	Costs	R.O.P.	Service Level
Speh ⁷⁵	Distribution fixed negative binomial, exponential, poisson, normal, log normal, gamma, mean--25, 75, variance--.10, .30, .50, coefficient	Fixed time and distance 10 days		EOQ	Total cost contribution margin thruput warehouse inventory		Not set
Tan ⁷⁶	Stochastic all backlogged	Zero, no trans-shipments			Holding, shortage, convex, order, allocation, linear		
Wagenheim ⁷⁷	Deterministic	Deterministic vs normal, log normal, gamma, poisson exponential erlang, level 4, 7, coefficient of variation -.18, .375		EOQ	Total cost CM, order transport inventory	Perpetual daily inventory	Not set
Author	Safety Stock Technique	Mathematical/ Simulation	Static/Dynamic	Deterministic/ Stochastic			
Aggerwal ⁷⁸	Not considered	S		S			
Ballou ⁷⁹	Not considered	S	D	D/S			
Bessler Veinott ⁸⁰	Not considered	M	S	D			
Clark & Scarf ⁸¹	Not considered	M	S	D			
Forrester ⁸²	Not considered	S	D	S			
Gross & Soriano ⁸³	Various levels used (standard deviation of demand over lead-time)	S	D	S			
Hochstaedeter ⁸⁴	Not considered	M	S	D			
Sherbroke ⁸⁵	Not considered	M	S	D			
Simon ⁸⁶	Not considered	M	S	D			
Sims ⁸⁷	Not considered	S	D	S			
Speh ⁸⁸	Not considered	S	D	S			
Tan ⁸⁹	Not considered	M	S	D			
Waggenheim ⁹⁰	Not considered	S	D	S			

FOOTNOTES--CHAPTER II

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CHAPTER III

HYPOTHESES AND RESEARCH METHODOLOGY

Introduction

The objective of this research is to measure both the accuracy of a statistical safety stock technique and the relative impact of alternative safety stock policies and uncertainty levels on customer service performance in single and multi-echeloned channel systems. This chapter identifies the specific hypotheses and outlines the research methodology required to investigate these issues.

Chapter III is divided into three sections. The first and second sections present the hypotheses and research methodology dealing with the single and multi-echeloned problems, respectively. The third section describes the statistical analysis employed in this research.

Problem I: Single Echelon Systems

Hypotheses

Part A. In a comparison of customer service predicted by a statistical approach and observed from a dynamic simulation, no difference exists:

1. Independent of the safety stock policy employed;
2. Independent of the uncertainty level experienced;
3. Independent of the combination of safety stock policy employed and uncertainty level experienced.

Part B. In a comparison among customer service performance observed from dynamic simulations, no significant difference exists:

1. Independent of the safety stock level employed;
2. Independent of the uncertainty level experienced;
3. Independent of the combination of safety stock level employed and uncertainty level experienced.

Methodology

The hypotheses for Part A and Part B of Problem I were tested by using dynamic simulations of single echelon channel systems. To create these channel simulations, the Simulated Product Sales Forecasting (SPSF) Model was employed.¹ A detailed description of the SPSF Model is provided in the Appendix.

System configuration. The simulated system configuration consisted of one echelon, containing a single retail facility, with inventory stocking capability. Figure 3.1 illustrates the system configuration. A single plant facility and retailer are linked by order communication and transportation.

Single Echelon

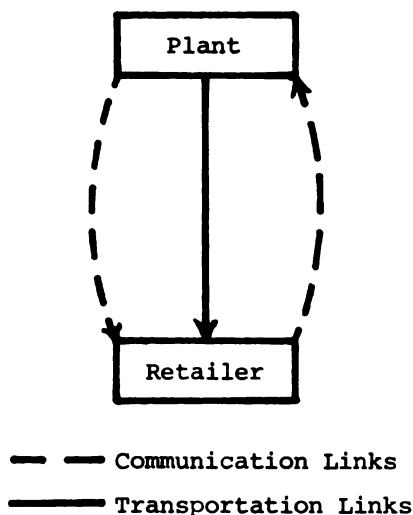


Figure 3.1 Representative Channel System
Configuration: Problem I

Ten identical products were handled simultaneously by this channel system. The products had identical physical and economic characteristics. The use of multiple products in each channel system simulation allowed more observations per simulation.

Uncertainty. Channel system uncertainty was introduced through stochastic demand and lead time variables. The simulation of channel systems with alternative uncertainty levels was accomplished by changing the standard deviation of one or both variables while holding their mean values constant.

Demand was generated stochastically by the ORDGEN program, a part of the SPSF Demand module.² Specifically, the ORDGEN program generated orders containing some or all of the ten products for each simulated day. Within a given channel system, the demand distribution of each product had the same mean value and standard deviation, but they were generated on an independent basis.

Throughout the research of Problem I, the demand distribution and mean value were held constant. Demand was normally distributed with a mean value of daily sales equal to fifty units per day.

Three demand patterns were utilized to represent low, medium, and high demand uncertainty. Specifically, the levels were defined as "low," "medium," and "high," and respectively correspond to a coefficient of variation of .1, .3, and .6. The specific levels of the coefficient of variation were selected arbitrarily, but they were set so that differences that might exist due to variability in demand could be measured.

Lead time, referred to as replenishment cycle duration, is the amount of time required from order placement to order fulfillment. The major time components are operational in nature and include: order communication time, order processing time, and transit time.

Lead times for Problem I were generated stochastically from an erlang distribution. The erlang distribution is a special case of the gamma probability family. The density function, expected value, variance, and standard deviation of the erlang distribution are the same as for the gamma distribution.³

In choosing a lead time distribution for this research, several criteria had to be met.⁴ First, the distribution chosen had to contribute to the generality of the research by being representative of actual lead time distributions. Second, the lead time distribution chosen had to be both limited to non-negative values greater than some arbitrary minimum, and skewed to the right, exhibiting a greater range of possible values above the mean value than below it. The first of these requirements results from the fact that there is some minimum amount of time which must elapse between the placement and the receipt of an order. Orders are not filled instantaneously. The second requirement results from the fact that while there is some required minimum amount of time which must elapse before the completion of the order cycle, there is not a maximum amount before which the cycle must be completed. Third, the distribution used had to be able to vary the size of the standard deviation about a single mean.

The normal distribution was not employed as the lead time distribution because it failed to satisfy the second criterion. The distribution is not skewed to the right. In addition, the normal distribution would not be applicable when the distribution exhibited a high variability around a low mean value since some of the events, which would have to occur, would be negative.

The poisson distribution was not utilized as the lead time distribution because it did not meet the third criterion. The mean value could not be held constant while the standard deviation varied.

The erlang distribution was chosen because it met all three criteria. Specifically, the distribution can be very representative of lead time distribution since it is applicable in situations where a series of service operations such as order communications, order processing, and order shipment are present. The distribution can be skewed to the right, and applied only to non-negative random variables. Finally, the erlang distribution is flexible enough to assume several shapes.

The mean value of the lead time distribution was held constant at ten days. Three patterns of variability around the mean value were employed to represent "low," "medium," and "high" lead time uncertainty. The low, medium, and high variability were represented by

coefficients of variation of .1, .3, and .6, respectively. As with demand, the specific levels of coefficient of variation were selected arbitrarily, but were set so that differences that might exist due to variability in lead time, could be measured.

Nine uncertainty levels were produced using the combinations of three demand and three lead time patterns. Table 3.1 lists the uncertainty levels utilized in the research of Problem I.

Table 3.1 Uncertainty Levels: Problem 1

Number of Uncertainty Levels	Uncertainty Level Abbreviation	Lead Time Level	Demand Level
1	L1D1	Low (.1)*	Low (.1)*
2	L1D2	Low (.1)	Medium (.3)
3	L1D3	Low (.1)	High (.6)
4	L2D1	Medium (.3)	Low (.1)
5	L2D2	Medium (.3)	Medium (.3)
6	L2D3	Medium (.3)	High (.6)
7	L3D1	High (.6)	Low (.1)
8	L3D2	High (.6)	Medium (.3)
9	L3D3	High (.6)	High (.6)

*Coefficient of variation.

Inventory policies. Demand for products was placed initially against the retailer in the form of daily orders from customers. These orders were filled with inventory held by the retailer. In the event that the order could not be filled in total, a partial shipment was sent to the customer exhausting retail supply. The unfilled part of the order was treated as demand permanently lost and recorded as a stockout. Thus, backorders were not a part of this research design. In fact, backordering would have worked directly against the objective of this research.

The retailers' source of inventory replenishment was the plant which produced an infinite inventory. The retailers' reorder policy was a fixed order quantity policy.⁵ Thus, inventory level was reviewed daily and when the sum of the on-hand and on-order inventory was less than the specified order point, a replenishment order was initiated. In accordance with this policy, the reorder point (ROP) was set at a quantity equal to the average demand at retail during the lead time between plant and retail locations, plus safety stocks. The reorder point varied depending on the safety stock policy employed. The order quantity was set at an amount equal to the average demand at retail during the replenishment cycle between the plant and retailer. Thus, order quantity was set at 500 units

(50 units per day average demand for ten days average lead time).

Safety stock policy considerations for single echelon channel systems consist of setting a safety stock level. Three safety stock levels were chosen to examine Problem I of this research. Amounts of safety stock equal to 0, .5, and 1.0 combined standard deviations of demand over lead time were utilized. These standard deviations were chosen arbitrarily, but were set so that differences which might result from safety stock policies could be measured. Safety stocks were held only by the retailer. Table 3.2 summarizes the inventory policies used to research Problem I.

Table 3.2 Inventory Policies: Problem I

<u>Indirect System:</u>	All customer demand draws on the retailer	
<u>Manufacturer:</u>		
Production policy:	Infinite inventory	
Inventory policy:	No stock maintained at plant	
<u>Retailer:</u>		
Replenishment policy:	Source	Plant replenishes retailer
	Reorder point	500 units plus safety stock
	Order quantity	500 units (constant)
	Review period	each simulation day
Safety stock policy:	Stocking location: Retailer only	
	Safety stock levels (combined standard deviations) 0, 0.5, 1.0	

Simulations. In testing the hypotheses of Parts A and B of Problem I, twenty-seven channel systems were simulated. Each simulation represented one of the possible combinations of the three safety stock policies and nine uncertainty levels. Table 3.3 displays these simulations.

Table 3.3 Simulations: Problem I

Uncertainty Levels	Safety Stock Policies		
	1	2	3
	(0)	(0.5)	(1.0)
1. L1D1	(1)	(2)	(3)
2. L1D2	(4)	(5)	(6)
3. L1D3	(7)	(8)	(9)
4. L2D1	(10)	(11)	(12)
5. L2D2	(13)	(14)	(15)
6. L2D3	(16)	(17)	(18)
7. L3D1	(19)	(20)	(21)
8. L3D2	(22)	(23)	(24)
9. L3D3	(25)	(26)	(27)

Each simulation was 500 days in length, with customer service levels for each of the ten products being generated every sixty days, after an initial twenty-day period. Of the nine periods, only the customer service results from periods six and eight were included in the sample to be analyzed.

Prior to performing the set of simulations, the stability of the simulation was tested. The test measure was customer service levels. A simulation was performed in which ten customer service levels were generated every sixty days for a 300 day period after an initial twenty-day period. The simulated channel system contained no safety stock and experienced the highest uncertainty level (L3D3) used in the research on Problem I. An analysis of variance was performed on the customer service results generated for periods three through six. The result was an F value of 1.709. The significance level of the F value was .183. Thus, the customer service results of the four periods were not significantly different, even at $\alpha = .10$. The choice of sample periods was made arbitrarily from the stable periods. However, non-contiguous periods were selected to avoid interdependence within the sampling data.

Thus, simulation output consisted of twenty sample observations: one for each of ten products observed over two separate time periods. The total sample resulting from all simulations was 540 customer service levels. Table 3.4 details the output recorded.

In addition to the simulated output, 540 predicted customer service levels were required to test the hypotheses for Part A of Problem I. These customer service levels were

Table 3.4 Simulation Output: Problem I

Total sample size	540
Simulations	27
Observations per simulation	20
Time periods sampled per simulation	2
Products per simulation	10
Response variable: (customer service level)	1

predicted using the statistical approach detailed in Chapter I. The next section describes the hypotheses and methodology for Problem II.

Problem II: Multi-Echeloned Systems

Hypotheses

Part A. In a comparison of customer service predicted by a statistical approach and observed from a dynamic simulation, no difference exists:

1. Independent of safety stock policy employed;
2. Independent of the uncertainty level experienced;
3. Independent of the combination of safety stock policy employed and uncertainty level experienced.

Part B. In a comparison among customer service performance observed from dynamic simulations, no significant difference exists:

1. Independent of safety stock level employed;
2. Independent of safety stock positioning employed;
3. Independent of safety stock policy employed;
4. Independent of uncertainty level experienced;
5. Independent of safety stock policy employed and uncertainty level experienced.

Methodology

In testing the hypotheses for Parts A and B of Problem II, dynamic simulations of multi-echeloned channel systems were required. As with Problem I, the Simulated Product Sales Forecasting (SPSF) Model was employed.

System configuration. The simulated system configuration consisted of two echelons, each containing a single facility with inventory stocking capability. Figure 3.2 illustrates the system configuration. A single plant facility, distribution center, and retailer are linked by order communication and transportation. Ten identical products were handled simultaneously by this channel system. The products had identical physical and economic characteristics. As in Problem I, the use of multiple products in each simulation allowed more observations per simulation.

Uncertainty. The demand and lead time uncertainty levels employed in Problem II were identical to those utilized in Problem I. Specifically, the method of generation, the distribution shapes, mean values,

Multi-Echeloned

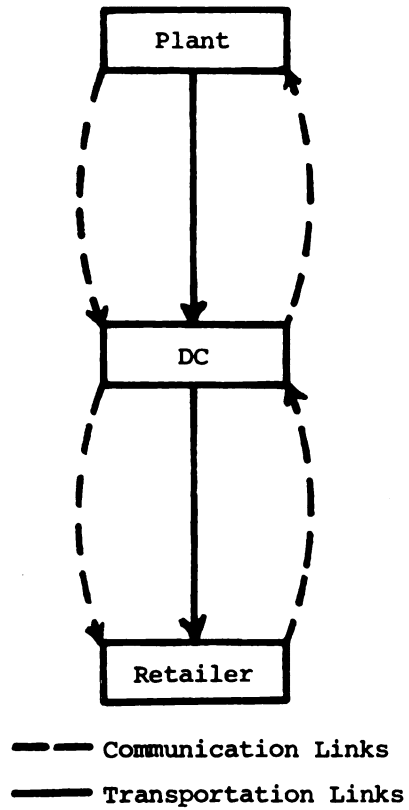


Figure 3.2 Representative Channel System
Configuration: Problem II.

variability levels, and the reasons for these choices all remained the same. Thus, nine uncertainty levels were produced, using the combinations of three demand and three lead time patterns. Table 3.1, which illustrated the uncertainty levels utilized in the research of Problem I, is also applicable to Problem II.

Inventory policies. Initially, demand for products was placed against the retailer in the form of daily customer orders. These orders were filled with inventory held by the retailer. In the event that an order could not be filled in total, a partial shipment was sent to the customer exhausting retail supply. The unfilled part of the order was treated as a stockout.

The retailer's source of inventory replenishment was the distribution center. The retailer used a fixed order quantity reorder policy. Therefore, the reorder point was set at a quantity equal to the average demand at retail during the lead time between the distribution center and retailer plus safety stock. The order quantity was set at an amount equal to the average demand at retail during the replenishment cycle between the distribution center and retailer or 500 units.

Demand placed against the distribution center was in the form of periodic orders from the retailer. As with the retailer, the distribution center filled orders with

its locational inventory. When an order could not be filled in total, a partial shipment was sent exhausting supply at the distribution center. The remainder of the order was recorded as a stockout. Thus, backordering was not a part of the research of Problem II.

The distribution center issued replenishment orders to the plant periodically. The distribution center reorder policy and order quantity were the same as those of the retailer. All orders placed against the plant were filled from an "infinite" inventory.

Safety stock policy considerations for multi-echeloned channel systems consisted of setting a safety stock level and choosing echelon position. Three safety stock levels in each of two echelon positions were chosen to examine Problem II of this research. Amounts of safety stock equal to 0, .5, and 1.0 combined standard deviations of demand over lead time uncertainty were utilized. These amounts were chosen arbitrarily, but were set so that differences that might result from safety stock policies could be measured. Safety stocks were held by the retailer and/or distribution center. Thus, nine possible combinations of safety stock level and echelon positioning existed. Table 3.5 summarizes the inventory policies used to research Problem II.

Table 3.5 Inventory Policies: Problem II

<u>Indirect System:</u>	All customer demand draws on the retailer	
<u>Manufacturer:</u>		
Production policy:	Infinite inventory	
Inventory policy:	No stock maintained at plant	
<u>Distribution Center:</u>		
Backorder policy:	No backorders	
Replenishment policy:	Source	Plant replenishes DC
	Reorder point	500 units plus safety stock
	Order quantity	500 units (constant)
	Review period	each simulation day
Safety stock policy:	Safety stock levels (combined standard deviations) 0, 0.5, 1.0	
<u>Retailer:</u>		
Backorder policy:	No backorders	
Replenishment policy:	Source	DC replenishes retailer
	Reorder point	500 units plus safety stock
	Order quantity	500 units (constant)
	Review period	each simulation day
Safety stock policy:	Safety stock levels (combined standard deviations) 0, 0.5, 1.0	

Simulations. In testing the hypotheses of Parts A and B of Problem II, eighty-one channel systems were simulated. Each simulation represented one of the possible combinations of nine safety stock policies and nine uncertainty levels. Table 3.6 displays the simulations that were performed. Each simulation was 500 days in length, with customer service levels for each of ten products being generated every sixty days, after an initial twenty-day period. As with Problem I, only the customer service results from periods six and eight were included in the sample analyzed.

Before the set of multi-echeloned channel simulations were performed, the stability of the simulation was tested. Customer service levels were utilized as the test measure. A simulation was performed in which ten customer service levels were generated every sixty days for a 300 day period after an initial twenty-day period. The channel system simulated contained no safety stock and experienced the highest uncertainty level (L3D3) used in Problem II. An analysis of variance was performed on the customer service results generated for periods three through six. The result was an F value of 1.499 with a significance level of .231. Thus, the periods were not significantly different. Non-contiguous periods were selected to avoid interdependence within the sampling data.

Table 3.6 Simulations: Problem II

	Safety Stock Policies								
	1	2	3	4	5	6	7	8	9
Uncertainty Levels	(0,0)	(0,.5)	(0,1)	(.5,0)	(.5,.5)	(.5,1)	(1,0)	(1,.5)	(1,1)
1. L1D1	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
2. L1D2	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
3. L1D3	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
4. L2D1	(28)	(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)
5. L2D2	(37)	(38)	(39)	(40)	(41)	(42)	(43)	(44)	(45)
6. L2D3	(46)	(47)	(48)	(49)	(50)	(51)	(52)	(53)	(54)
7. L3D1	(55)	(56)	(57)	(58)	(59)	(60)	(61)	(62)	(63)
8. L3D2	(64)	(65)	(66)	(67)	(68)	(69)	(70)	(71)	(72)
9. L3D3	(73)	(74)	(75)	(76)	(77)	(78)	(79)	(80)	(81)

Thus, simulation output consisted of twenty sample observations, one for each of ten products observed over two separate time periods. The total sample which resulted from all simulations was 1,620 customer service levels. Table 3.7 details the output recorded from the simulations performed for Problem II.

Table 3.7 Simulation Output: Problem II

Total sample size	1,620
Simulations	81
Observations per simulation	20
Time periods sampled per simulation . . .	2
Products per simulation	10
Response variable: (customer service levels)	1

Additionally, 1,620 predicted customer service levels were required to test the hypotheses in Part A of Problem II. These customer service levels were predicted using the statistical approach described in Chapter I of the research. The next section describes the statistical analysis employed in this research.

Statistical Analysis: Problems I and II

In testing the hypotheses of this research, analysis of variance techniques were employed. Specifically, the techniques were the F-test, Tukey's Method of Multiple Comparisons and Scheffé's Multiple Comparison Method.⁶ Additionally, t-tests were employed.

In Part A of Problems I and II, two-factor analysis of variance was used to compare statistically, the customer service predicted by a statistical approach with that observed from dynamic simulation. The dependent variable was the difference in customer service levels. The two independent variables were safety stock policy and uncertainty. Table 3.8 summarizes the experimental factors, their levels, and the number of treatments utilized. The results of the two-factor analyses of variance (ANOVA) were reviewed in order to present the simultaneous impact of the two main effects on the response (dependent) variable. When a significant two-way interaction was indicated, inferences from the results of the two-way ANOVA were limited to interpretations of the two-way interaction. This was required due to the fact that the calculations and corresponding F-tests made for all lower order interactions and main effects could be accomplished by collapsing the data over the one remaining independent variable. This would cause no difficulty in interpretation,

Table 3.8 ANOVA: Part A

Two-Factor ANOVA: Problem I

Factor A: Safety Stock Policy Factor B: Uncertainty

Levels0
.5
1.0LevelsL1D1
L1D2
L1D3
L2D1
L2D2
L2D3
L3D1
L3D2
L3D3

Treatments: (27) Full Factorial Design

Two-Factor ANOVA: Problem II

Factor A: Safety Stock Policy Factor B: Uncertainty

Levels(0,0)
(0,.5)
(0,1.0)
(.5,0)
(.5,.5)
(.5,1.0)
(1.0,0)
(1.0,.5)
(1.0,1.0)LevelsL1D1
L1D2
L1D3
L2D1
L2D2
L2D3
L3D1
L3D2
L3D3

Treatments: (81) Full Factorial Design

if the two-way interaction was insignificant, and it would signify that the effects of the two independent variables are additive. However, if the two-way interaction was significant, this would mean that the effects of the one-way interactions were different across the levels of the second variable (over which the data were collapsed). Thus, each of the one-way interactions could only be interpreted by specifying a particular level of the second variable. When one or more significant F-tests were found, Tukey's or Scheffé's multiple comparison techniques were used to identify the exact nature of the difference.

In Part B of Problems I and II, two-factor analysis of variance was used to measure the relative impact of safety stock policies employed and uncertainty levels experienced on customer service observed from dynamic simulations. The dependent variable was customer service level. The independent variables, factor levels, and treatments were identical to those in Part A of Problems I and II. The results of both two-factor (ANOVA's) were analyzed as those of Part A of Problems I and II had been analyzed.

This chapter specified the hypotheses of this research and the methodology employed to investigate each. It also reviewed the statistical analysis used to test the hypotheses.

FOOTNOTES--CHAPTER III

¹Donald J. Bowersox, David J. Closs, John T. Mentzer, Jr., and Jeffrey R. Sims, Simulated Product Sales Forecasting-Documentation (East Lansing, Mich.: Graduate School of Business Administration Research Bureau, Michigan State University, 1978).

²Ibid., p. 170, Appendix of this research.

³E. Martin Basic, "Development and Application of a Gamma Based Inventory Management Theory" (Ph.D. dissertation, Michigan State University, 1965).

⁴G. Wagenheim, "The Performance of a Physical Distribution Channel Under Various Conditions of Lead-Time Uncertainty" (Ph.D. dissertation, Michigan State University, 1974), p. 83.

⁵Elwood S. Buffa, Production-Inventory Systems (Homewood, Ill.: Richard D. Irwin, Inc., 1968), p. 161.

⁶John Neter and William Wasserman, Applied Linear Statistical Models (Homewood, Ill.: Richard D. Irwin, Inc., 1974).

CHAPTER IV

RESULTS OF ANALYSIS

Introduction

This chapter presents the research results. Specifically, it shows the results of both Problems I and II. The analyses of variance performed on the simulation results are discussed. The post hoc tests, results, and analyses are presented.

Table 4.1 lists the additional abbreviations employed to present the results. The abbreviations were employed to simplify and clarify the results.

This chapter is divided into two sections, each containing two parts. Section one reports the results of Parts A and B of Problem I. Section two reports the results of Parts A and B of Problem II.

Results: Problem I

Part A

The general hypothesis is that in a comparison of customer service, predicted by a statistical approach and that observed from a dynamic simulation, no difference exists. The response variable was the mean difference

Table 4.1 Abbreviations

Safety Stock Policy Abbreviations:

<u>Policy</u>	<u>Abbreviation</u>
Distribution Center	DC
Retailer	R
0 combined standard deviations of safety stock .	1
.5 combined standard deviations of safety stock .	2
1.0 combined standard deviations of safety stock .	3

Safety Stock Level Abbreviations:

<u>Level</u>	<u>Abbreviation</u>
0 combined standard deviations of safety stock .	SSL-0
.5 combined standard deviations of safety stock .	SSL-.5
1.0 combined standard deviations of safety stock .	SSL-1.0
1.5 combined standard deviations of safety stock .	SSL-1.5
2.0 combined standard deviations of safety stock .	SSL-2.0

Safety Stock Positioning Policy Abbreviations:

<u>Positioning Policy</u>	<u>Abbreviation</u>
All safety stock held at distribution center	(DC)
All safety stock held at retailer	(R)
Some safety stock held at both distribution center and retailer	(DC-R)

in customer service level recorded as a quantity fill rate. Table 4.2 presents the mean differences in customer service level resulting from comparisons of the predicted and simulated mean values of customer service levels for each treatment. A positive mean difference indicated that the simulated mean value of customer service was higher than predicted. A negative mean difference indicated the reverse. The grand mean difference for the 540 pairs was 3.02 percentage points. A t-test resulted in a t-value of 9.41, which was significant at a value of $\alpha = .001$. Thus, the simulated customer service levels were significantly higher than predicted. T-tests performed on the treatment mean differences in Table 4.2 resulted in significant differences at $\alpha = .05$ for seventeen of the twenty-seven treatments. As shown, twelve of the significant mean differences were positive values and five were negative. Individually, the simulated mean values of customer service level were significantly higher than predicted at high uncertainty levels. Conversely, the simulated mean values of customer service level that were significantly lower than predicted occurred at low uncertainty levels. Figures 4.1 through 4.3 illustrate these differences graphically.

Table 4.2 Mean Differences in Simulated and
Predicted Customer Service Levels

Uncertainty Level	Safety Stock Policy		
	R1	R2	R3
L1D1	-2.0*	-0.3	-1.1*
L1D2	-5.6*	-1.0	-1.6*
L1D3	-1.6	-0.6	-0.7
L2D1	-5.6*	-0.7	0.4
L2D2	2.6	2.3*	1.4*
L2D3	-0.1	2.4*	1.6
L3D1	13.6*	11.0*	5.7*
L3D2	14.4*	8.7*	7.4*
L3D3	13.7*	9.9*	7.4*

*Significant difference.

The two-way analysis of variance performed on the mean differences is presented in Table 4.3. The two factors were safety stock policy (A) and uncertainty (B), and the response variable was the mean difference in customer service levels. The two-way interaction was significant. Thus, the combined impact of the factors on the response variable was significant. Since the significance of a two-way interaction bars direct interpretation of factor effects, Tukey's and Scheffé's tests were used to investigate each of the remaining hypotheses in Part A of Problem I.

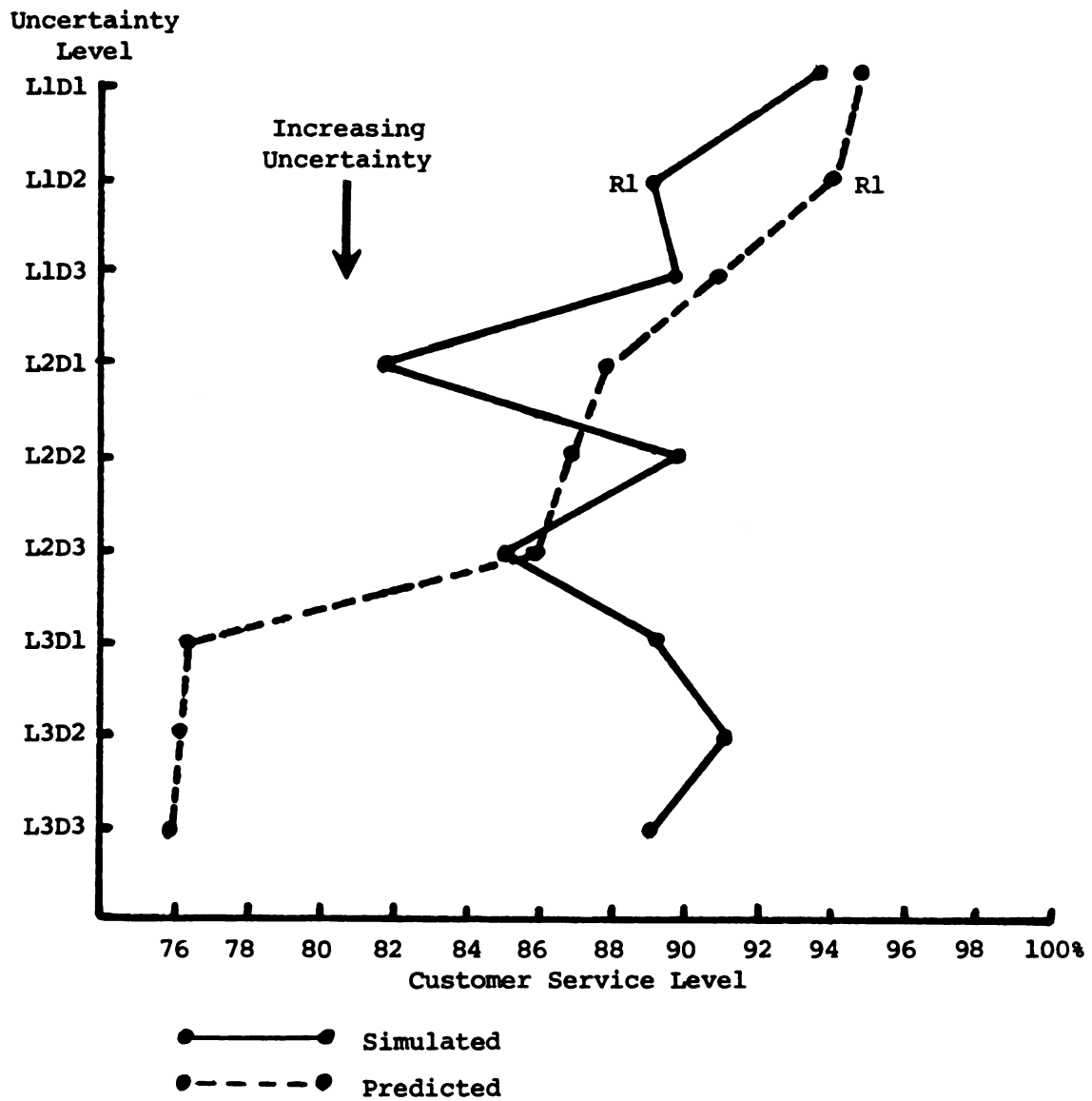


Figure 4.1 Mean Values of Simulated and Predicted Customer Service Levels for Safety Stock Policy R1 Under All Uncertainty Levels.

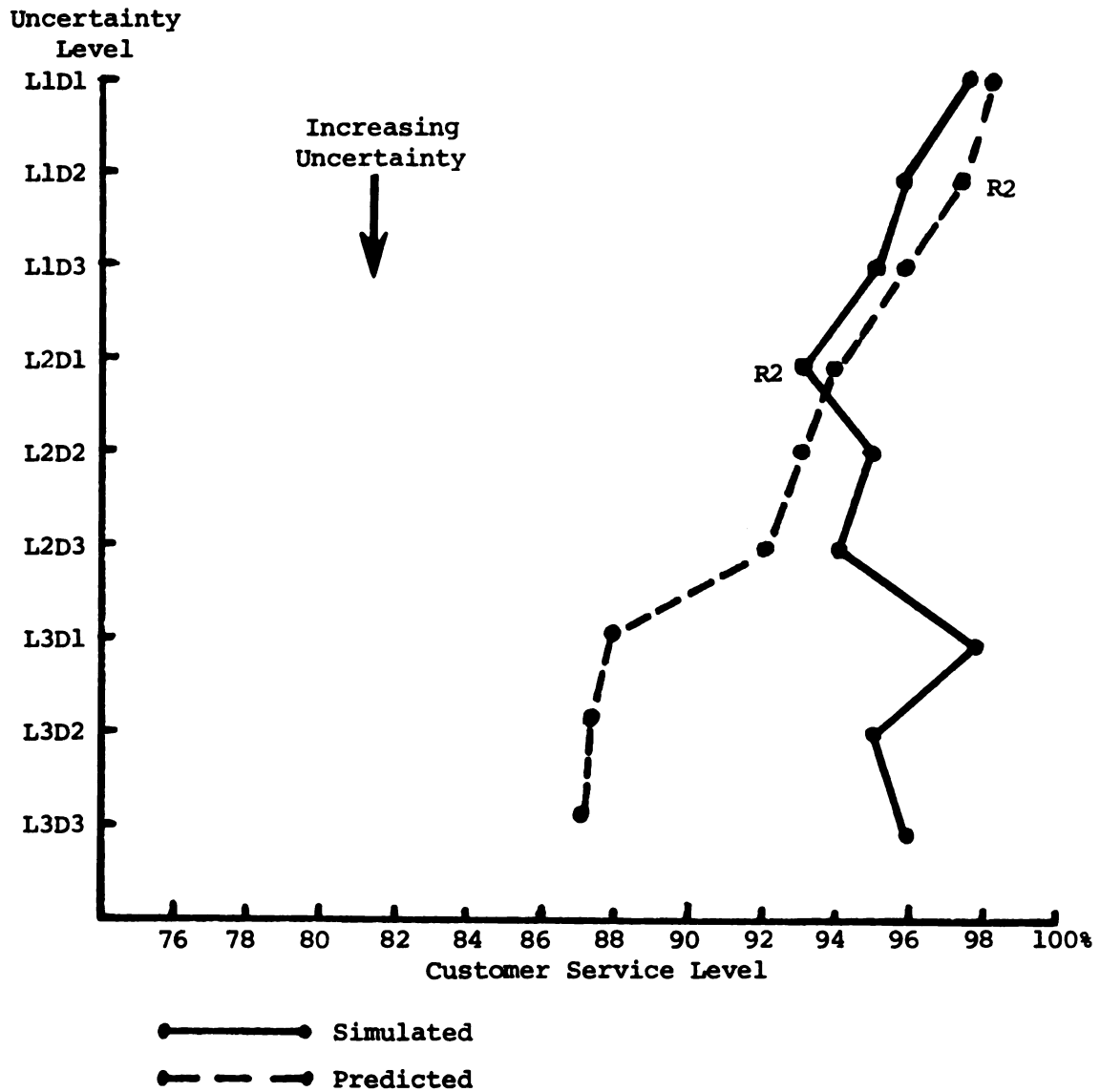


Figure 4.2 Mean Values of Simulated and Predicted Customer Service Levels for Safety Stock Policy R2 Under All Uncertainty Levels.

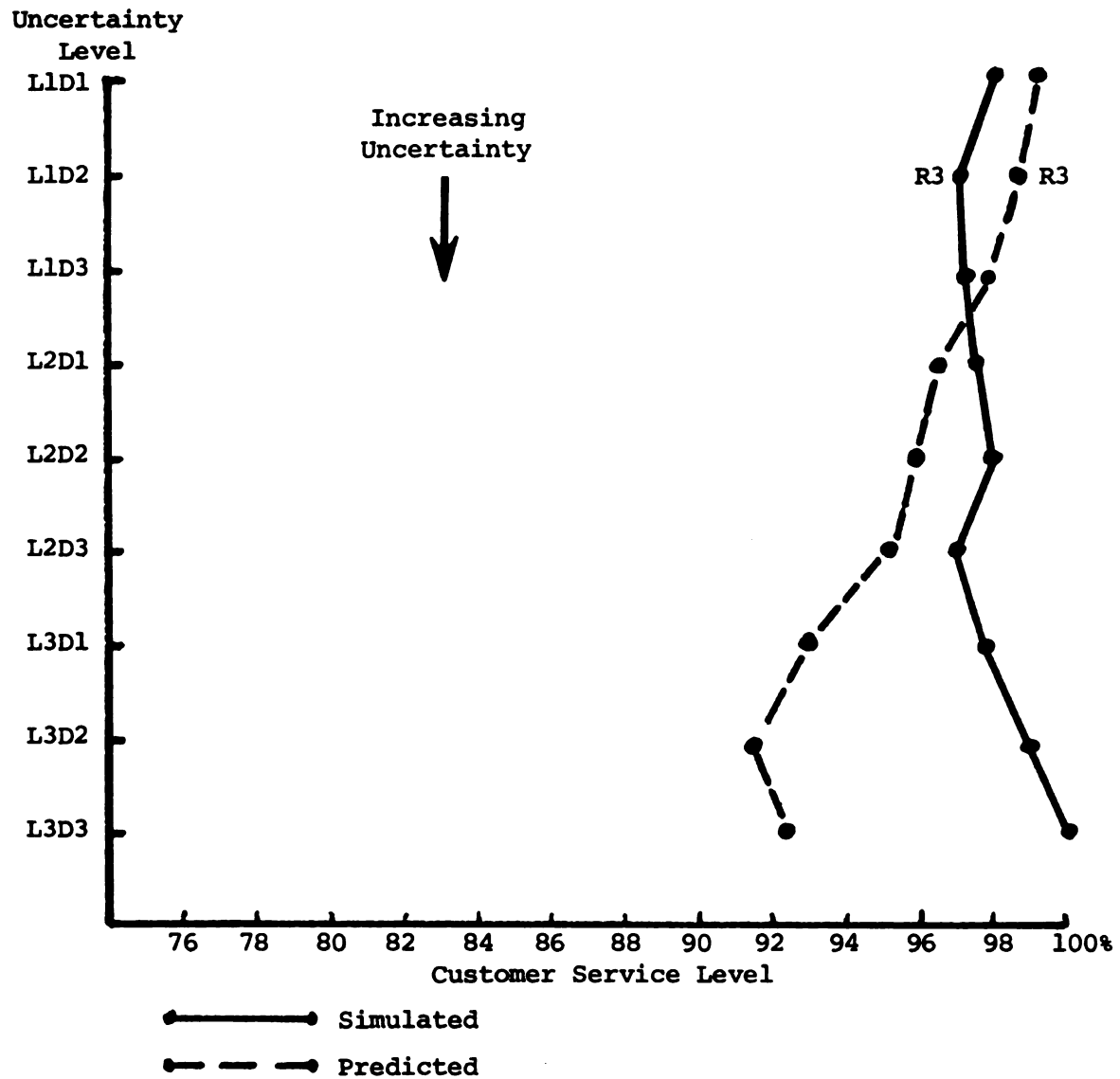


Figure 4.3 Mean Values of Simulated and Predicted Customer Service Levels for Safety Stock Policy R3 Under All Uncertainty Levels.

Table 4.3 Analysis of Variance: Mean Difference in Simulated and Predicted Customer Service Levels

Source of Variation	Degrees of Freedom	Mean Square	F	Critical F $\alpha = .05$
Main Effects:				
A	2	.008	3.244*	3.00
B	8	.187	75.901*	1.94
Two-Way Interaction:				
AB	16	.014	5.668*	1.67
Residual	513	.002		
Total	539	.006		

*Significant.

Hypothesis 1:

In a comparison of customer service predicted by a statistical approach and observed from dynamic simulation, no difference exists: *independent of the safety stock policy employed.*

Figure 4.4 displays the mean difference in customer service level for each safety stock policy employed (R1, R2, R3) under all uncertainty levels. Tukey's multiple comparison technique was used for comparisons of the mean differences in customer service between safety stock policies at each uncertainty level. The critical value at $\alpha = .05$ was 5.81.

Of the twenty-seven comparisons made, four resulted in significant differences. Comparisons of the mean

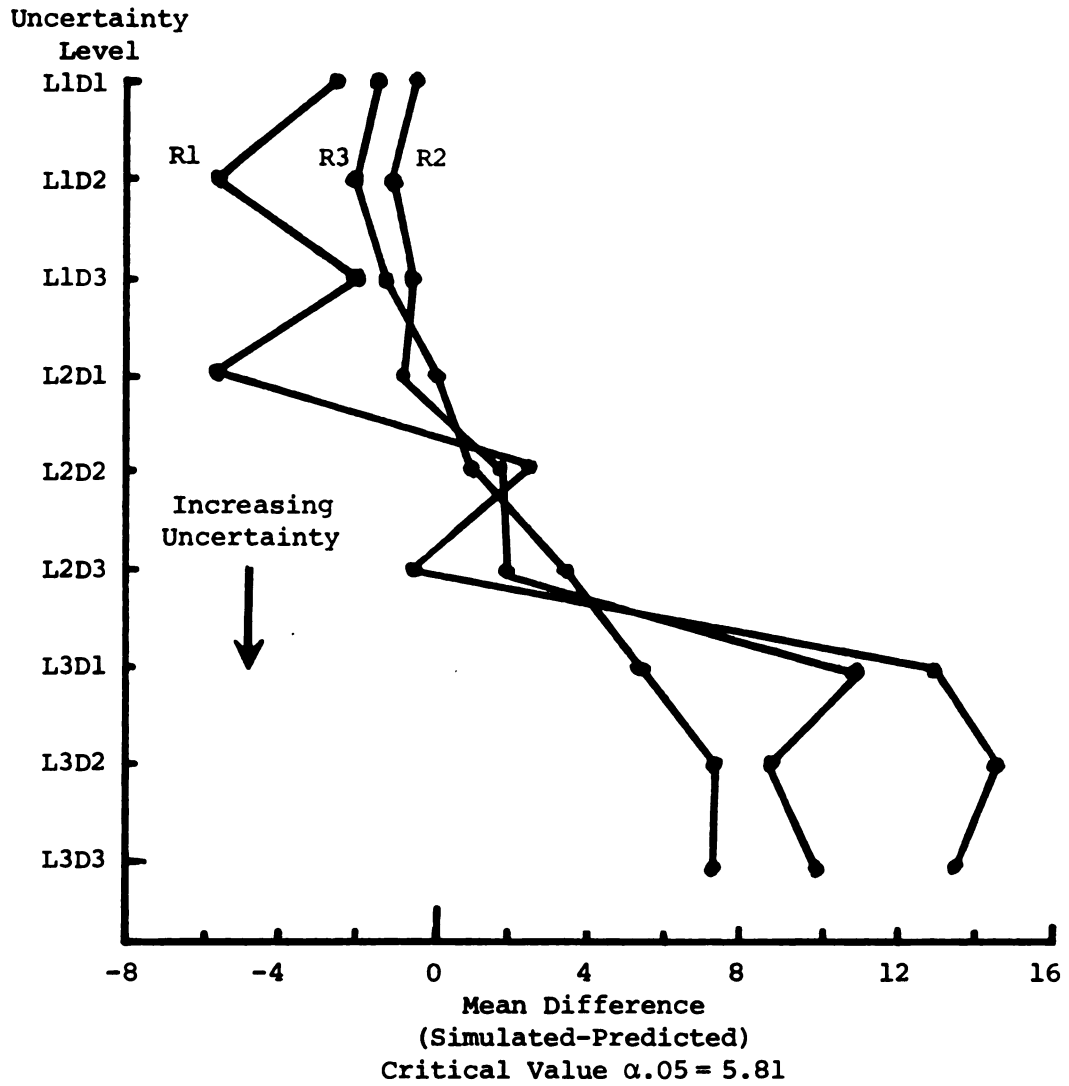


Figure 4.4 Mean Difference in Customer Service for Each Safety Stock Policy Under All Uncertainty Levels.

differences in customer service for safety stock policies R1 and R3 resulted in significant differences at uncertainty levels L2D1, L3D1, L3D2, and L3D3. No other significant differences were found. Thus, Hypothesis 1 was rejected.

The safety stock policy employed had an impact on the difference between simulated and predicted customer service levels in a single echelon channel system. Specifically, a safety stock increase from 0 to 1.0 standard deviations significantly reduced the mean difference at high uncertainty levels. However, the simulated mean difference still was significantly higher than predicted at these high uncertainty levels.

Hypothesis 2:

In a comparison of customer service predicted by a statistical approach and observed from a dynamic simulation, no difference exists:
independent of the uncertainty level experienced.

Figure 4.5 illustrates the mean difference in customer service level for each uncertainty level experienced under all safety stock policies. Tukey's multiple comparison technique was used for comparisons of the mean differences in customer service between uncertainty levels under each safety stock policy. The critical value at $\alpha = .05$ was 5.81. Comparisons of the mean differences in customer service between each of the three highest uncertainty levels and each of the other uncertainty levels

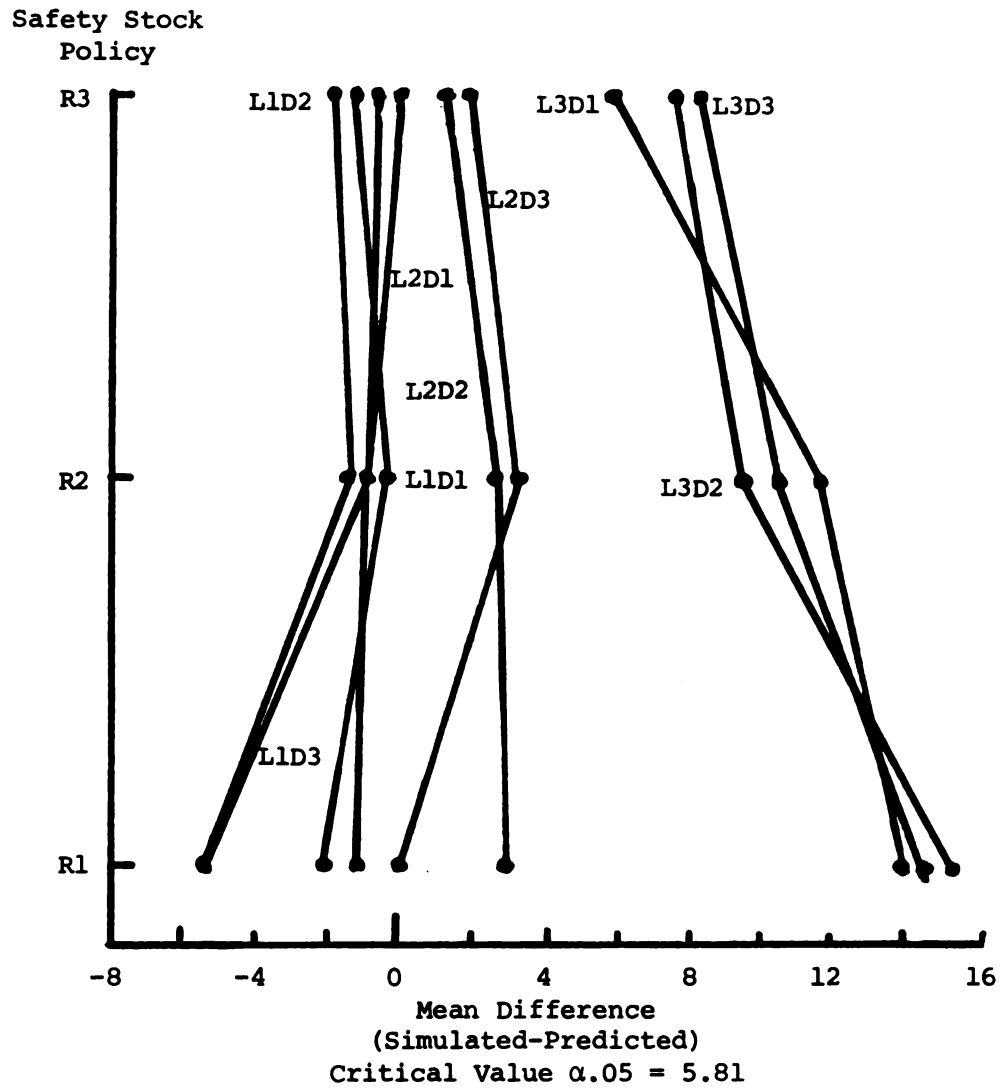


Figure 4.5 Mean Difference in Customer Service for Each Uncertainty Level Under All Safety Stock Policies.

resulted in significant differences under safety stock policy R1. In addition, comparisons of the mean differences of L2D2, with L2D1 and L1D2 resulted in significant differences.

Comparisons of the mean differences in customer service between each of the three highest uncertainty levels and each of the other uncertainty levels resulted in significant differences under safety stock policy R2. No other significant differences were found.

Comparisons of the mean differences in customer service between the two highest uncertainty levels and each of the lowest six uncertainty levels resulted in significant differences under safety stock policy R3. Additionally, comparisons of the mean difference in customer service between L3D1 and the three lowest uncertainty levels resulted in significant differences. Thus, Hypothesis 2 was rejected.

The uncertainty level had a significant impact on the mean difference between simulated and predicted customer service in a single echelon system. Two findings stand out. First, as uncertainty level increased, the simulated customer service level moved from being lower than predicted to higher than predicted. Second, at very high uncertainty levels, the mean difference was significantly different from lower uncertainty levels. Uncertainty

level had a significant impact on the nature of the mean difference (positive or negative) and on the degree of difference.

Hypothesis 3:

In a comparison of customer service predicted by a statistical approach and observed from a dynamic simulation, no difference exists: *independent of the combination of safety stock policy employed and uncertainty level experienced.*

Table 4.2 displayed the treatment mean differences of simulated and predicted mean values of customer service levels. The range was from -5.64 to 14.42. The analysis of variance confirmed that the two-way interaction was significant at a level of $\alpha = .001$. Accordingly, Hypothesis 3 was rejected.

The combined impact of safety stock policy employed and uncertainty level experienced had a significant impact on the mean difference between simulated and predicted customer service in a single echelon system. The uncertainty level experienced had a relatively greater impact than did the safety stock policy employed. When no safety stock was in the system, the statistical approach overestimated customer service, at low uncertainty levels and underestimated customer service at high uncertainty levels. The pattern was the same when safety stock was introduced into the system. However, the degree of error in predicting customer service was reduced relatively,

although the error remains significant at low and high uncertainty levels.

Part B

The general hypothesis is that in a comparison among customer service performance observed from dynamic simulations, no significant difference exists. The response variable representing customer service performance is customer service level. Table 4.4 presents mean values of the customer service levels resulting from the set of simulations.

The two-way analysis of variance performed on the simulation results is presented in Table 4.5. The factors were safety stock level (A) and uncertainty (B), and the response variable was customer service level. The two-way interaction was highly significant. Thus, the combined impact of the factors on the response variable was significant. The F values in Table 4.5 are all significant at a level of $\alpha = .005$. Since the significance of a two-way interaction bars direct interpretation of factor effects, Tukey's and Scheffé's tests were used to investigate each of the remaining hypotheses in Part B of Problem I.

Hypothesis 1:

In a comparison among customer service performance observed from dynamic simulations, no significant difference exists: *independent of safety stock level employed.*

Table 4.4 Mean Values of Simulated Customer Service Levels

Uncertainty Levels		Safety Stock Levels		
		(0)	(.5)	(1.0)
1	L1D1	93.8*	97.7	98.1
2	L1D2	89.0	96.4	97.0
3	L1D3	90.0	94.8	97.3
4	L2D1	82.4	93.2	97.0
5	L2D2	90.0	95.7	97.8
6	L2D3	85.5	94.8	97.2
7	L3D1	90.0	97.6	98.1
8	L3D2	90.8	95.3	99.2
9	L3D3	88.7	95.9	99.6

*Rounded to one decimal place.

Table 4.5 Analysis of Variance: Simulated Product Customer Service Levels

Source of Variation	Degrees of Freedom	Mean Square	F	Critical F $\alpha = .05$
Main Effects:				
A	2	.397	160.825*	3.00
B	8	.016	6.679*	1.94
Two-Way Interaction:				
AB	16	.054	2.176*	1.67
Residual	513	.002		
Total	539	.004		

*Significant.

Figure 4.6 displays the mean values of customer service levels for each safety stock level employed (SSL-0, SSL-.5, SSL-1.0) under all uncertainty levels. Tukey's multiple comparison technique was used for safety stock level comparisons at each uncertainty level. The critical value at $\alpha = .05$ was 5.81.

Of the twenty-seven comparisons made, thirteen resulted in significant differences. Comparisons of customer service levels for safety stock levels SSL-0 and SSL-1.0 resulted in significant differences at all uncertainty levels except the lowest level, L1D1. The customer service level under SSL-0 was lower in each instance. Comparisons of customer service levels for safety stock level SSL-0 and SSL-.5 resulted in five significant differences and three differences which were close to the critical value. In each instance, the customer service level under SSL-0 was lower than that under SSL-.5. No significant difference was found at the lowest uncertainty level. Comparisons of customer service levels for safety stock levels SSL-.5 and SSL-1.0 resulted in no significant differences. The largest difference was 3.9 percentage points. Thus, Hypothesis 1 was rejected.

Safety stock level had a significant impact on customer service performance. Specifically, customer service levels increased as safety stock level increased, but at a decreasing rate. The customer service level

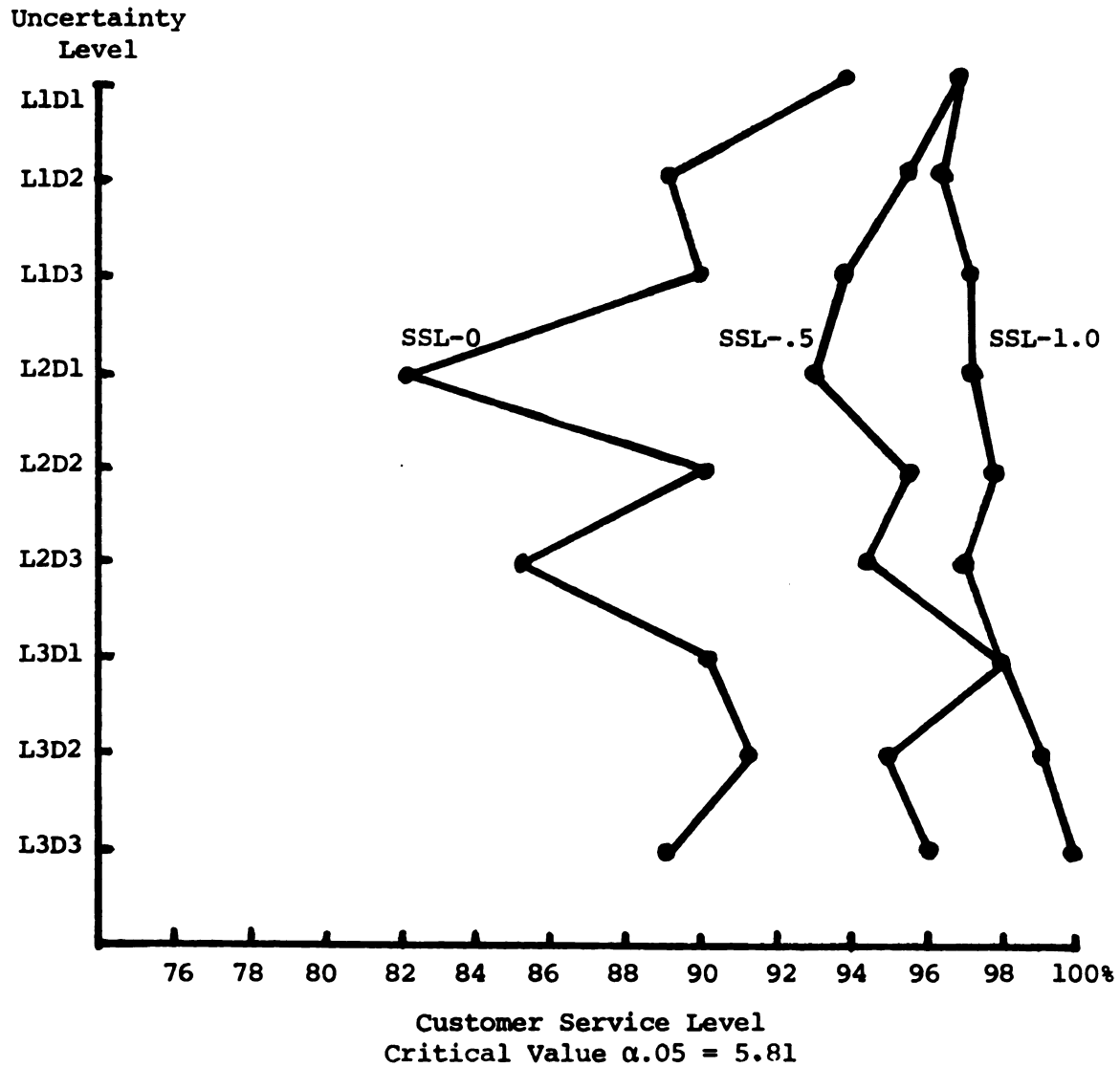


Figure 4.6 Mean Values of Customer Service Level for Each Safety Stock Level Under All Uncertainty Levels.

difference between SSL-0 and SSL-.5 was greater than that between SSL-.5 and SSL-1.0 at all uncertainty levels. Additionally, nine of the fourteen non-significant findings had differences of between 3.6 and 5.7 percentage points. Although these differences were not large enough to be statistically significant, they could have managerial significance.

Hypothesis 2:

In a comparison among customer service performance observed from dynamic simulations, no significant difference exists: *independent of uncertainty level experienced.*

Figure 4.7 presents the mean values of customer service levels for each uncertainty level experienced under all safety stock levels. Tukey's multiple comparison technique was used for uncertainty level comparisons at each safety stock level. The critical value at $\alpha = .05$ was 5.81.

Of the 108 pairwise comparisons made, nine resulted in significant differences. All of the significant differences were at safety stock level SSL-0. The customer service level under L2D1 was significantly lower than that under all other uncertainty levels except L2D3. The customer service level under L2D3 was significantly lower than under L1D1. At higher safety stock levels, no significant differences were found by changing uncertainty levels. Thus, Hypothesis 2 was rejected.

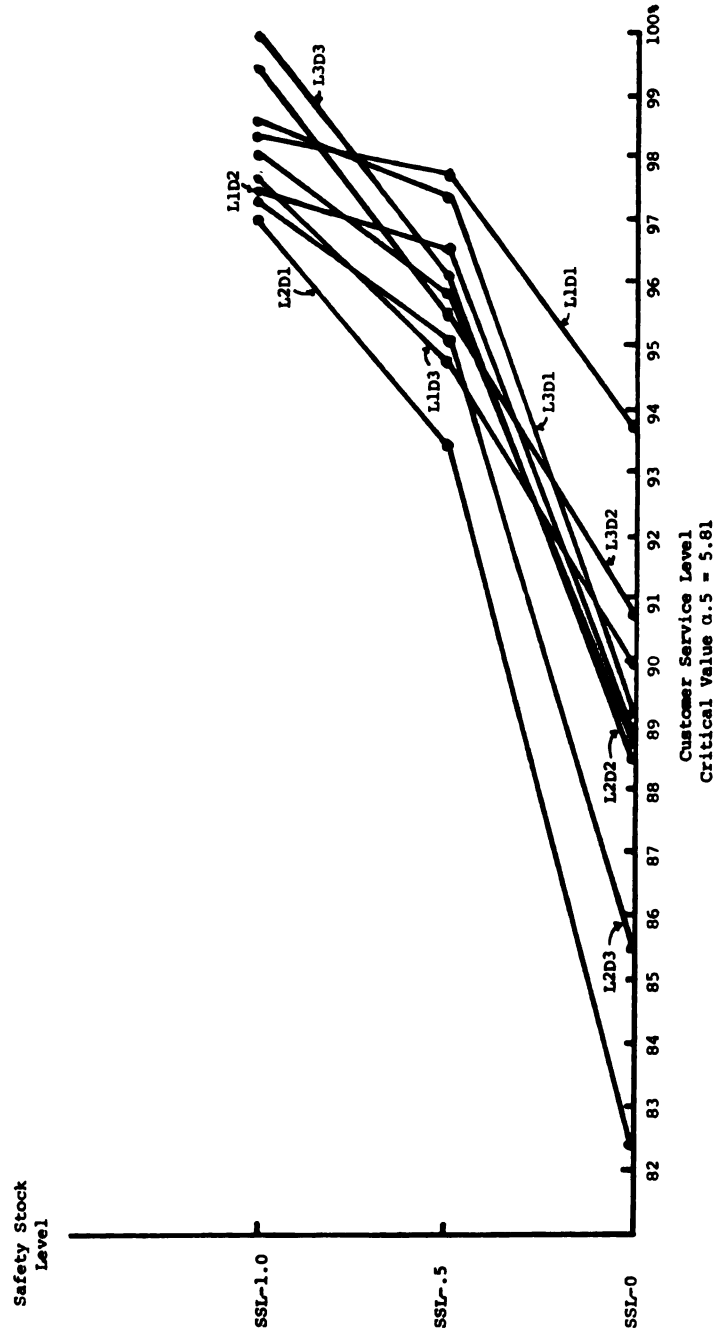


Figure 4.7 Mean Values of Customer Service Level for Each Uncertainty Level Under All Safety Stock Policies.

The uncertainty level experienced did impact customer service performance in a single echelon system. Specifically, the impact was significant when there was no safety stock in the system. The impact of uncertainty level on customer service performance decreased as safety stock level increased. Additionally, increasing the variations in combined uncertainties of demand over lead time did not always result in decreasing customer service levels as shown in Table 4.4. This is due to the random interactions of demand and lead time over many contiguous order cycles.

Hypothesis 3:

In a comparison among customer service performance observed from dynamic simulations, no significant difference exists: *independent of the combinations of safety stock level employed and uncertainty level experienced.*

The analysis of variance revealed that the main effects of both factors and the two-way interaction were highly significant. The mean values of customer service varied from a minimum of 82.4 percent to a maximum of 99.6 percent across all treatments. Thus, Hypothesis 3 was rejected. The combined impact of safety stock level and uncertainty level on customer service performance was significant.

Results: Problem IIPart A

The general hypothesis is that in a comparison of customer service, predicted by a statistical approach and that observed from a dynamic simulation, no difference exists. The response variable was the mean difference in service level recorded as a quantity fill rate. Table 4.6 presents the mean differences in customer service resulting from comparisons of the predicted and simulated mean values of customer service levels for each treatment. A positive mean difference indicated that the simulated mean value of customer service was higher than predicted. A negative mean difference indicated the reverse. The grand mean difference for the 1,620 pairs of comparisons was 1.20 percentage points. A t-test resulted in a t-value of 5.36, which was significant at a value of $\alpha = .001$. Thus, the simulated product customer service levels were significantly higher than predicted. T-tests performed on the treatment mean differences in Table 4.6 resulted in significant differences at $\alpha = .05$ for sixty of the eighty-one treatments. As shown, thirty-three of the significant mean differences were positive values and twenty-nine were negative. Specifically, the simulated mean values of customer service level were significantly higher than predicted at high uncertainty levels, regardless of the safety stock

Table 4.6 Mean Differences in Simulated and Predicted Customer Service Levels

Uncertainty Level	Safety Stock Policy								
	DC1R1	DC1R2	DC1R3	DC2R1	DC2R2	DC2R3	DC3R1	DC3R2	DC3R3
1 L1D1	-13.5*	-5.7*	-14.9*	-3.5	-1.9*	-2.2*	-3.5*	-1.9*	-2.2*
2 L1D2	-7.9*	-8.5*	-7.7*	-6.5	-0.9	0.3	-6.5*	-0.9	0.3
3 L1D3	-3.5*	-5.2*	-5.3*	-3.2*	0.6	0.5	-3.2*	0.6	0.5
4 L2D1	-7.8*	-5.7*	-3.4*	4.1*	0.8	0.9	4.1*	-0.8	0.9
5 L2D2	-9.6*	-3.7*	-8.1*	-1.8	2.8*	1.9*	-1.8	2.8*	1.9*
6 L2D3	-10.6*	-6.6	-7.0*	-0.7	0.6	-0.3	-0.7	0.6	-0.3
7 L3D1	10.1*	9.0*	6.5*	6.9*	9.0*	6.0*	6.9*	9.0*	6.0*
8 L3D2	12.7*	9.1*	7.8*	11.7*	9.1*	7.4*	11.7*	9.1*	7.4*
9 L3D3	10.4*	12.0*	6.8*	13.4*	12.0*	6.5*	13.4*	12.0*	6.5*

*Significant difference.

policy employed. Conversely, the simulated mean values of customer service level were significantly lower than predicted at lower levels of uncertainty. Figures 4.8 through 4.10 illustrate these differences graphically.

The two-way analysis of variance performed on the mean differences is presented in Table 4.7. The two factors were safety stock policy (A) and uncertainty (B), and the response variable was the mean difference in customer service levels. The two-way interaction was significant. Thus, the combined impact of the factors on the response variable was significant. Since the significance of the two-way interaction bars direct interpretation of factor effects, Tukey's and Scheffé's tests were used to investigate each of the remaining hypotheses in Part A of Problem II.

Table 4.7 Analysis of Variance: Mean Difference in Simulated and Predicted Customer Service Levels

Source of Variation	Degrees of Freedom	Mean Square	F	Critical F $\alpha = .05$
Main Effects:				
A	8	.105	28.990*	1.94
B	8	.693	190.525*	1.94
Two-Way Interaction				
AB	64	.016	4.480*	1.30
Residual	1,539	.004		
Total	1,619	.008		

*Significant.

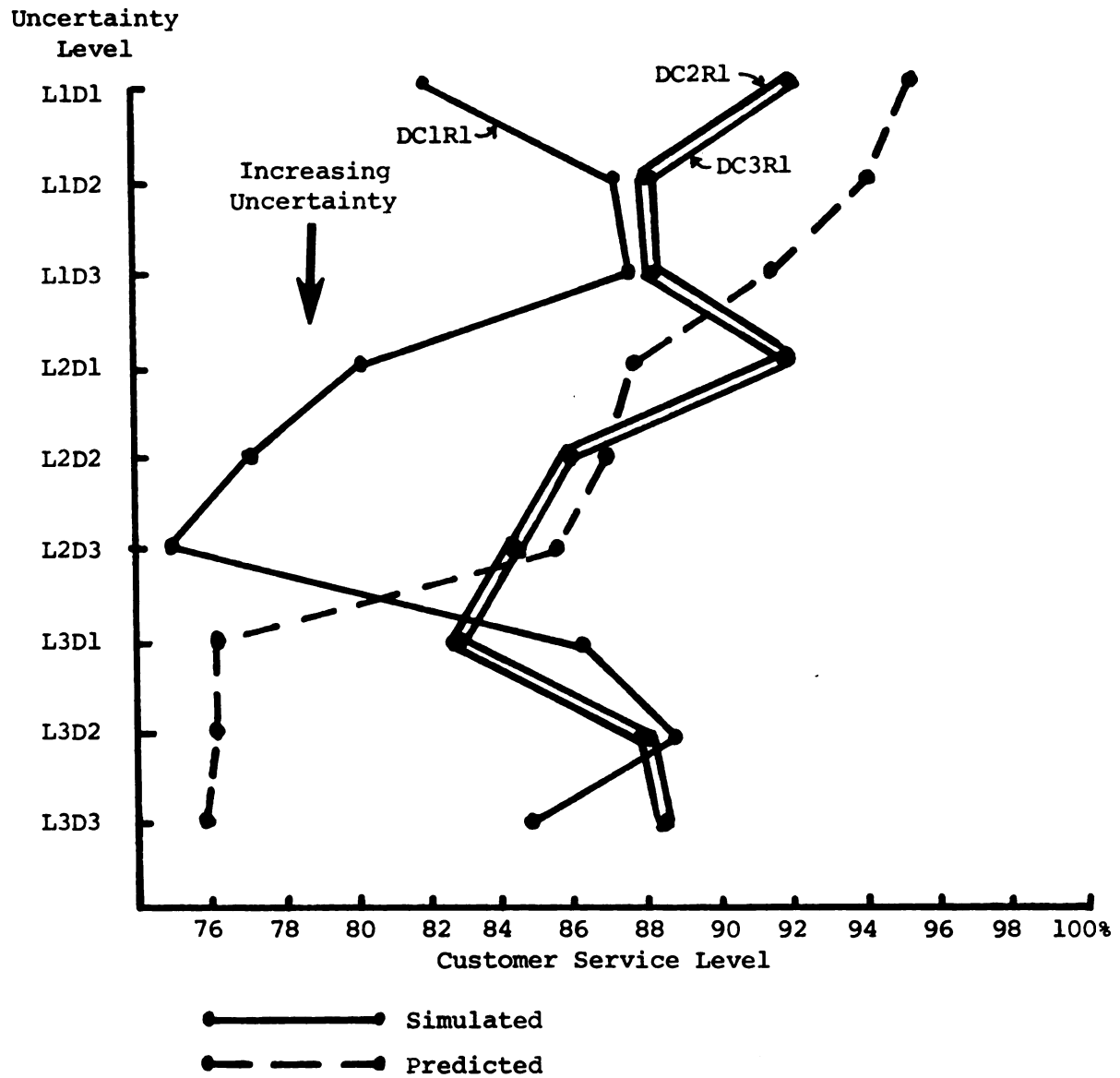


Figure 4.8 Simulated and Predicted Mean Values of Customer Service Level for Safety Stock Policies With R1.

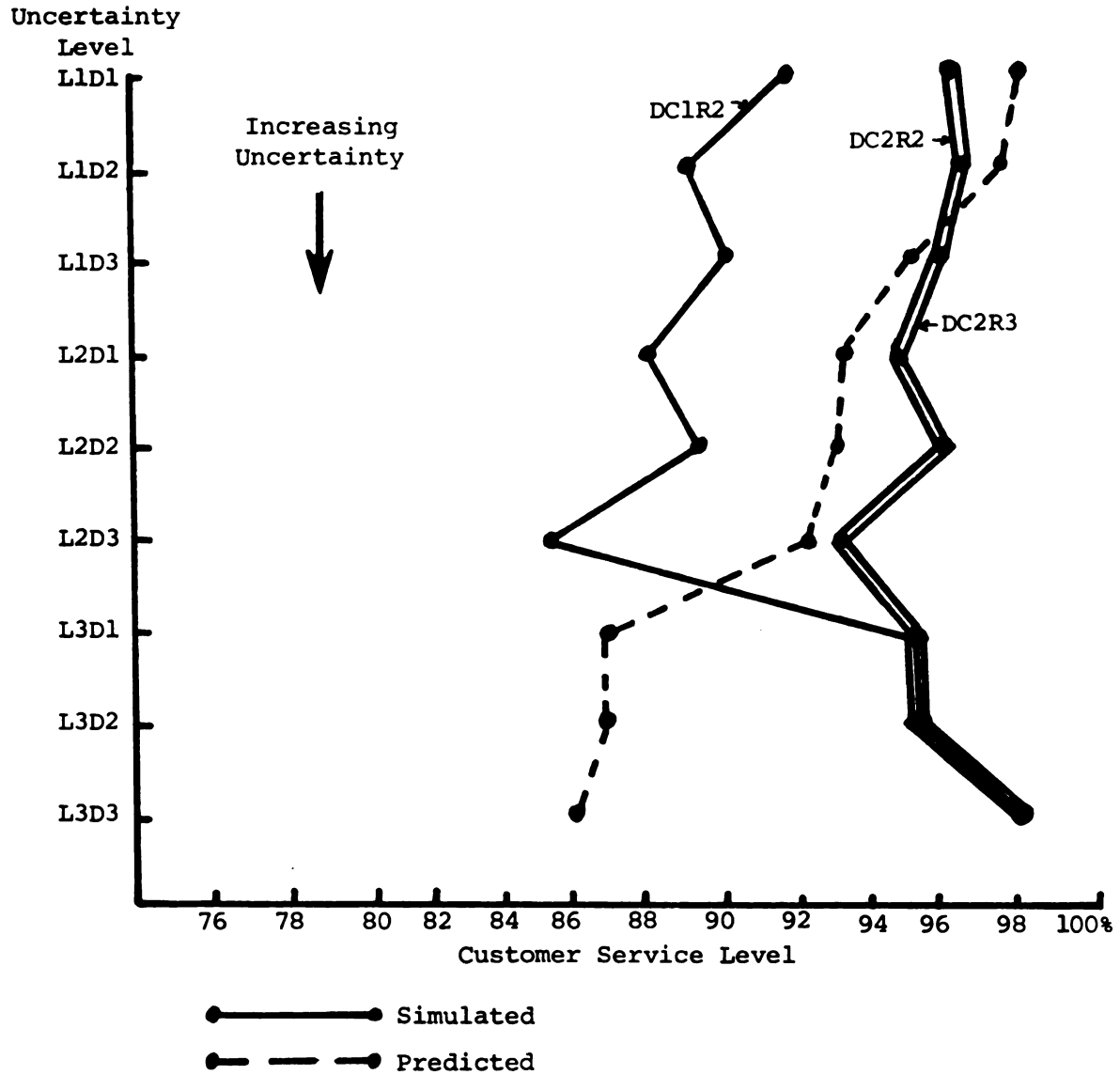


Figure 4.9 Simulated and Predicted Mean Values of Customer Service Level for Safety Stock Policies With R2.

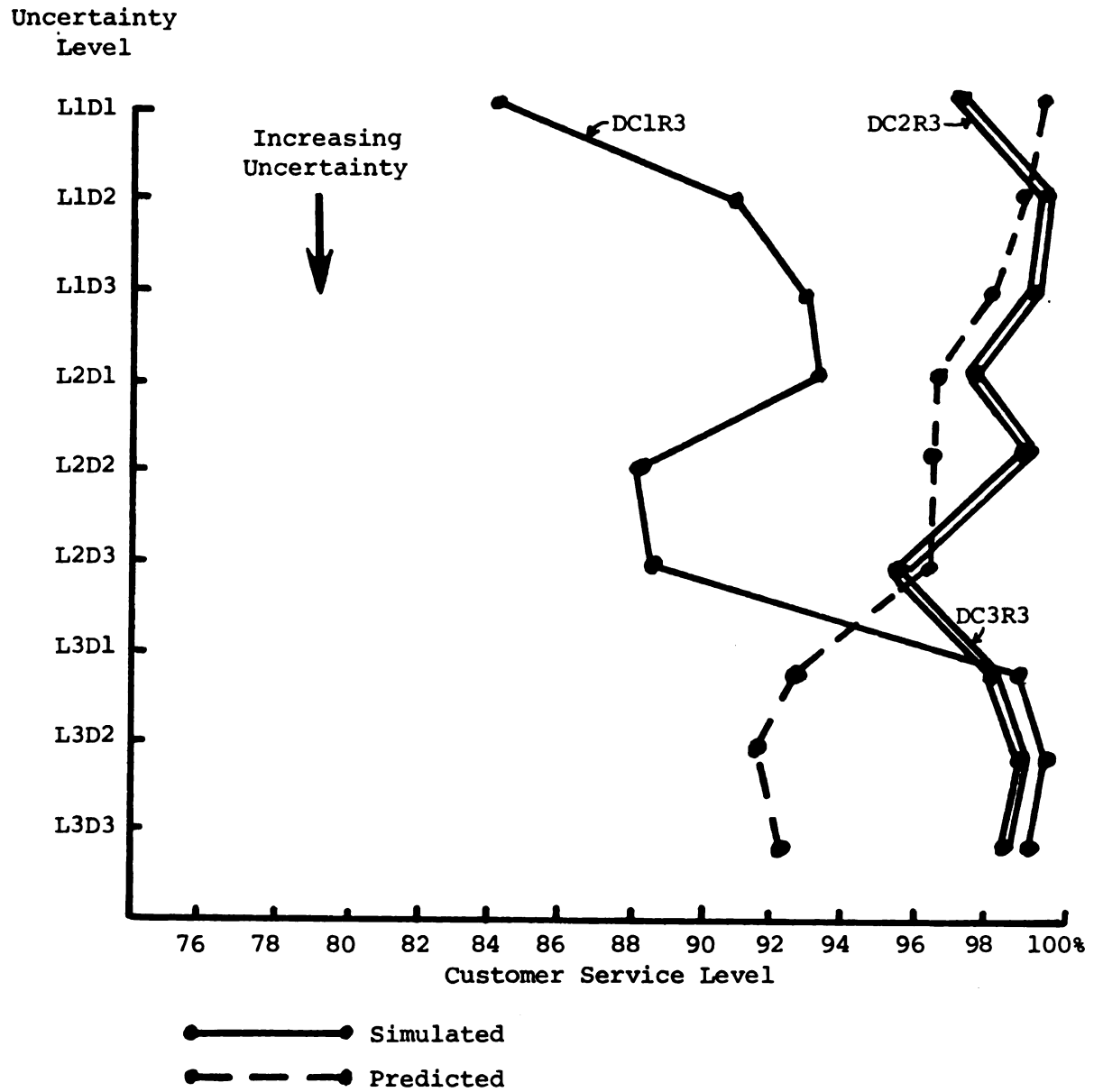


Figure 4.10 Simulated and Predicted Mean Values of Customer Service Level for Safety Stock Policies With R3.

Hypothesis 1:

In a comparison of customer service predicted by a statistical approach and observed from a dynamic simulation, no difference exists:
independent of safety stock policy employed.

Figure 4.11 displays the mean difference in customer service level for each safety stock policy employed under all uncertainty levels. Scheffé's multiple comparison techniques ($\alpha = .05$) were used for comparisons of the mean differences in customer service between safety stock policies at each uncertainty level. Comparisons of the mean difference in customer service between safety stock policies at each uncertainty level resulted in the following significant differences: DC1R1 and DC1R3 were different from the rest of the safety stock policies at uncertainty level L1D1; DC1R1, DC1R2, and DC1R3 were different from DC2R3 and DC3R3 at uncertainty level L1D2; DC1R1 and DC1R2 were different from the rest of the safety stock policies except DC1R3, at uncertainty level L2D1; DC1R1 and DC1R3 were different from the rest of the safety stock policies at uncertainty level L2D2; DC1R1 and DC1R3 were different from the rest of the safety stock policies at uncertainty level L2D3. No significant differences were found at uncertainty levels L1D3, L3D1, L3D2, or L3D3. Thus, Hypothesis 1 was rejected.

The safety stock policy employed did impact the difference between simulated and predicted customer service levels in a multi-echeloned channel system. Specifically,

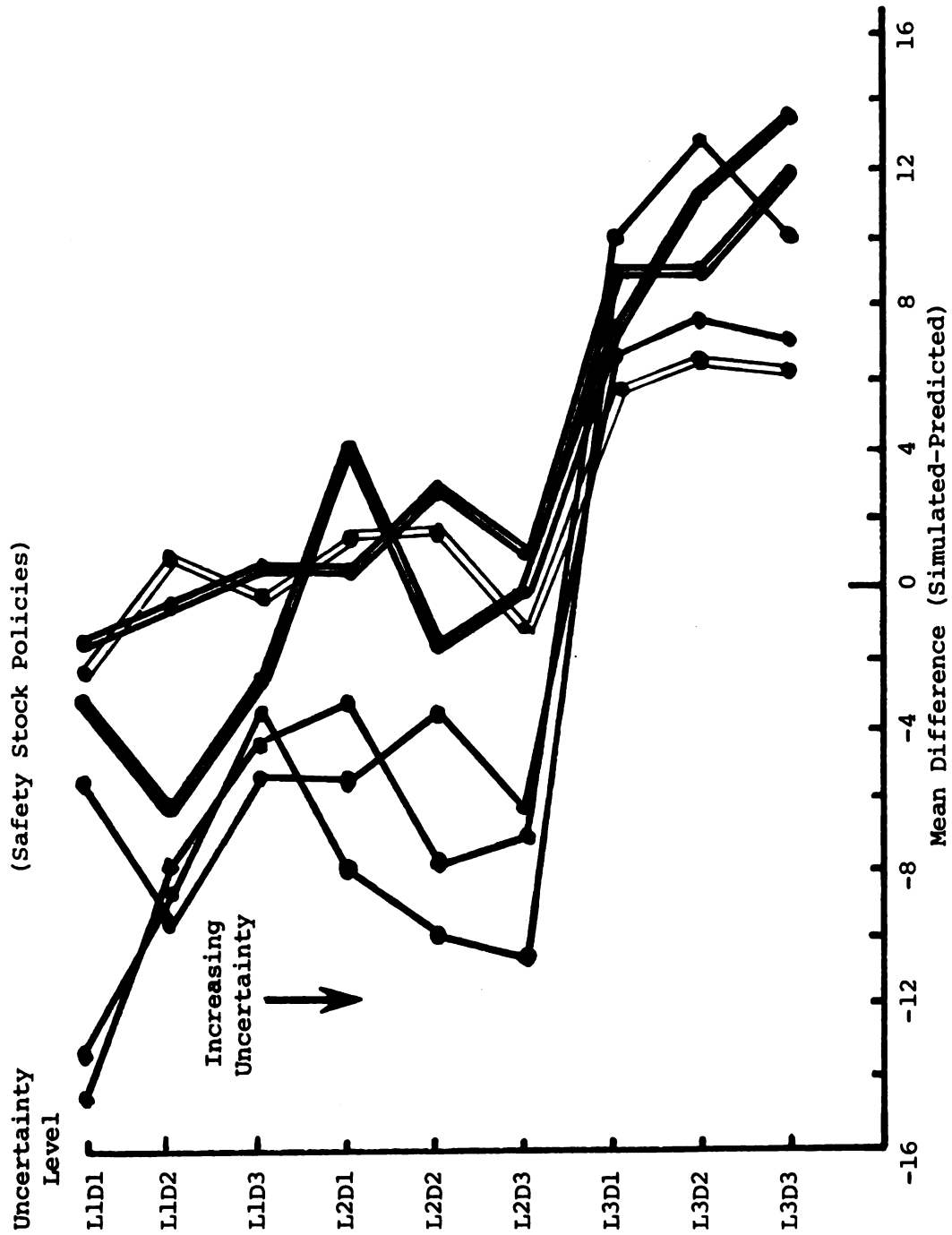


Figure 4.11 Mean Difference in Customer Service Level for Each Safety Stock Policy Under All Uncertainty Levels.

safety stock policies having an (R) retailer positioning policy (DC1R1, DC1R2, DC1R3) and employed at low uncertainty levels resulted in significantly larger negative differences than safety stock policies having a (DC-R) positioning policy. Safety stock policies having an (R) retailer positioning policy and employed at intermediate uncertainty levels resulted in significantly larger negative differences than all other safety stock policies. The safety stock policy employed at high uncertainty levels did not have a significant impact on the mean differences. However, using safety stock policies having a (DC) distribution center positioning policy resulted in larger (seven percentage points) positive differences than safety stock policies using 1.0 standard deviations of safety stock at retail.

Hypothesis 2:

In a comparison of customer service predicted by a statistical approach and observed from a dynamic simulation, no difference exists:
independent of uncertainty level experienced.

Figure 4.12 illustrates the mean difference in customer service level for each uncertainty level experienced under all safety stock policies. Scheffé's multiple comparison procedures were used ($\alpha = .05$) for comparisons of the mean difference in customer service between uncertainty levels under each safety stock policy. Comparisons of the mean difference in customer service

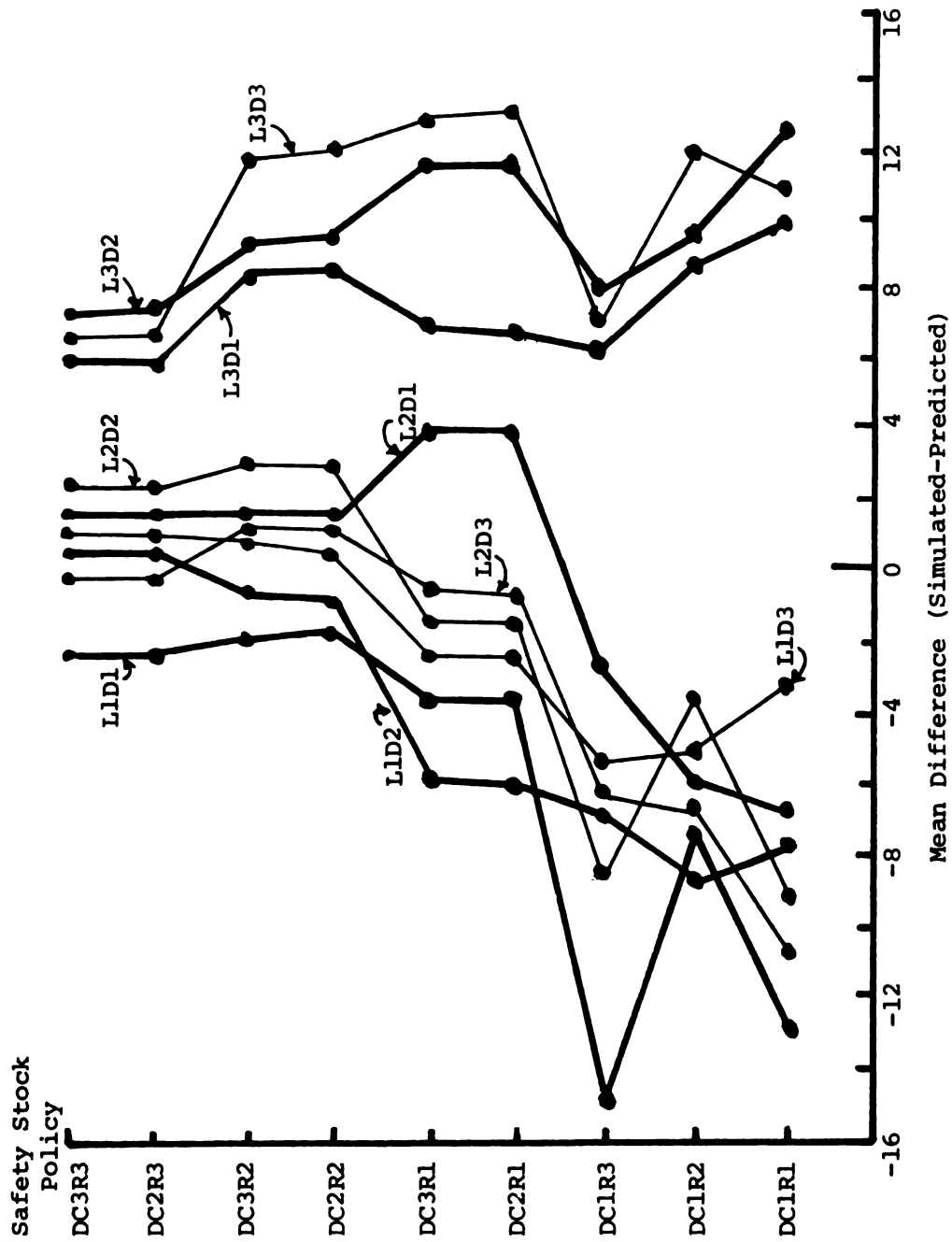


Figure 4.12 Mean Difference in Customer Service Level for Each Uncertainty Level Under All Safety Stock Policies.

between uncertainty levels under each safety stock policy resulted in the following significant differences: the three highest uncertainty levels were significantly different from the other six uncertainty levels under safety stock policies DC1R1, DC1R2, DC1R3, DC2R2, and DC3R2. Additionally, the three highest uncertainty levels were different from all other uncertainty levels, except L2D1, under safety stock policies DC2R1 and DC3R1. The three highest uncertainty levels were different from only uncertainty level L1D1 under safety stock policies DC2R3 and DC3R3. Uncertainty level L1D1 was different from L1D3 under safety stock policies DC1R1 and DC1R3. Thus, Hypothesis 2 was rejected.

The uncertainty level had a significant impact on the mean difference between simulated and predicted customer service in a multi-echeloned channel system. Specifically, at low uncertainty levels, simulated customer service levels were significantly lower than predicted. At high uncertainty levels, simulated customer service levels were significantly higher than predicted. Thus, the uncertainty level experienced had a significant impact on whether the statistical approach over or underestimated customer service.

Hypothesis 3:

In a comparison of customer service predicted by a statistical approach and observed from a dynamic simulation, no difference exists: *independent of the safety stock policy employed and the uncertainty level experienced.*

Table 4.6 displayed the treatment mean differences of simulated and predicted mean values of customer service. The range was from -14.92 to 13.36. The analysis of variance confirmed that the two-way interaction was significant at a level of $\alpha = .001$. Thus, Hypothesis 3 was rejected.

The combined impact of safety stock policy employed and uncertainty level experienced had a significant impact on the mean difference between simulated and predicted customer service in a multi-echeloned channel system.

The statistical approach overestimated customer service at low uncertainty levels, and particularly when the safety stock policy employed had no safety stock at the distribution center. The statistical approach underestimated customer service at high uncertainty levels across all safety stock policies. The underestimation was particularly high when the safety stock policy employed had no safety stock at the retail stage.

Part B

The general hypothesis is that in a comparison among customer service performance observed from dynamic simulations, no significant difference exists. The

response variable which represents customer service performance is customer service level. Table 4.8 presents mean values of the customer service levels resulting from the set of simulations performed for Problem II.

The two-way analysis of variance performed on the simulation results is presented in Table 4.9. The factors were safety stock policy (level and positioning) (A) and uncertainty (B).

A review of the fifth column of Table 4.9 indicated that the two-way interaction was highly significant. The F value of 4.259 clearly exceeded the critical F value (.95, 64,1539) of 1.320. The two-way interaction was confirmed graphically in Figure 4.13. The lines were not parallel and therefore the two factors A and B were not additive. Since the significance of a two-way interaction bars any direct interpretation of the factors, Tukey's and Scheffé's tests were used to investigate each of the remaining hypotheses in Part B of Problem II.

Hypothesis 1:

In a comparison among customer service performance observed from dynamic simulations, no significant difference exists: *independent of safety stock level employed.*

The impact of safety stock level on customer service performance was isolated by holding constant both the safety stock positioning policy and uncertainty level.

Table 4.8 Mean Values of Simulated Customer Service Levels

Uncertainty Level	Safety Stock Policy											
	DC1R1	DC1R2	DC1R3	DC2R1	DC2R2	DC2R3	DC3R1	DC3R2	DC3R3			
1 L1D1	82.3*	92.3	84.3	92.3	96.1	97.0	92.3	96.1	97.0			
2 L1D2	86.7	89.0	90.0	88.1	96.6	98.9	88.1	96.6	98.9			
3 L1D3	88.1	90.2	92.7	88.4	96.0	98.5	88.4	96.0	98.5			
4 L2D1	80.3	88.1	93.2	92.1	94.6	97.5	92.1	94.6	97.5			
5 L2D2	77.9	89.7	88.3	85.6	96.2	98.3	85.6	96.2	98.3			
6 L2D3	75.0	85.8	88.6	84.9	93.0	95.3	84.9	93.0	95.3			
7 L3D1	86.5	95.6	98.9	83.3	95.6	98.4	83.3	95.6	98.4			
8 L3D1	89.1	95.7	99.6	88.1	95.7	99.2	88.1	95.7	99.2			
9 L3D3	85.4	98.0	99.0	88.4	98.0	98.7	88.4	98.0	98.7			

*Rounded to one decimal place.

Table 4.9 Analysis of Variance: Simulated Customer Service Levels

Source of Variation	Degrees of Freedom	Mean Square	F	Critical F $\alpha = .05$
Main Effects:				
A	8	.463	127.213*	1.940
B	8	.066	18.016*	1.940
Two-Way Interactions:				
AB	64	.015	4.259*	1.320
Residual	1,539	.004		
Total	1,619	.007		

*Significant.

Figures 4.14 through 4.16 display the mean values of customer service levels for the safety stock levels employed under each combination of safety stock positioning policy and uncertainty level. Tukey's multiple comparison technique was used for safety stock level comparisons under each set of conditions. The critical value at $\alpha = .05$ was 7.99.

Figure 4.14 presents the mean values of customer service levels for each safety stock level employed (SSL-0, SSL-.5, SSL-1.0) under a distribution center safety stock positioning policy. Comparisons of customer service levels for safety stock levels SSL-0 and SSL-.5 resulted in significant differences at four of the nine uncertainty levels: L1D1, L2D1, L2D2, and L2D3. The customer service level for

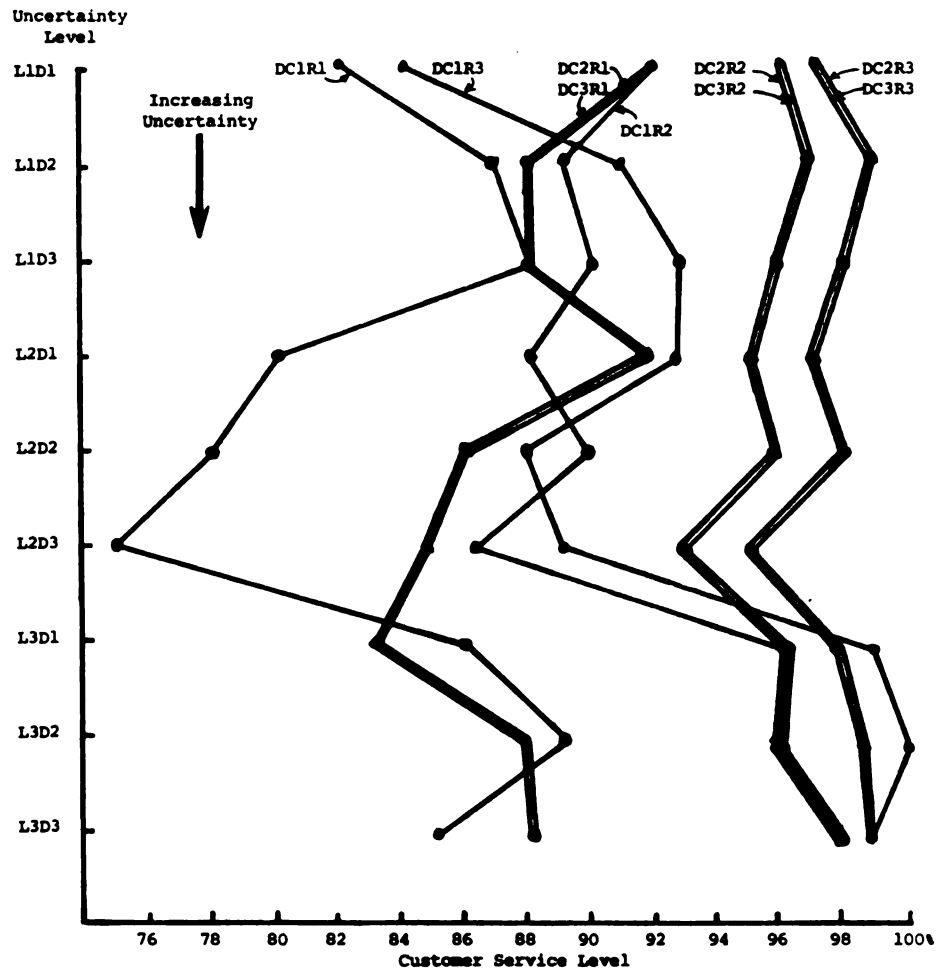


Figure 4.13 Mean Values of Customer Service Level for Each Safety Stock Policy Under All Uncertainty Levels.

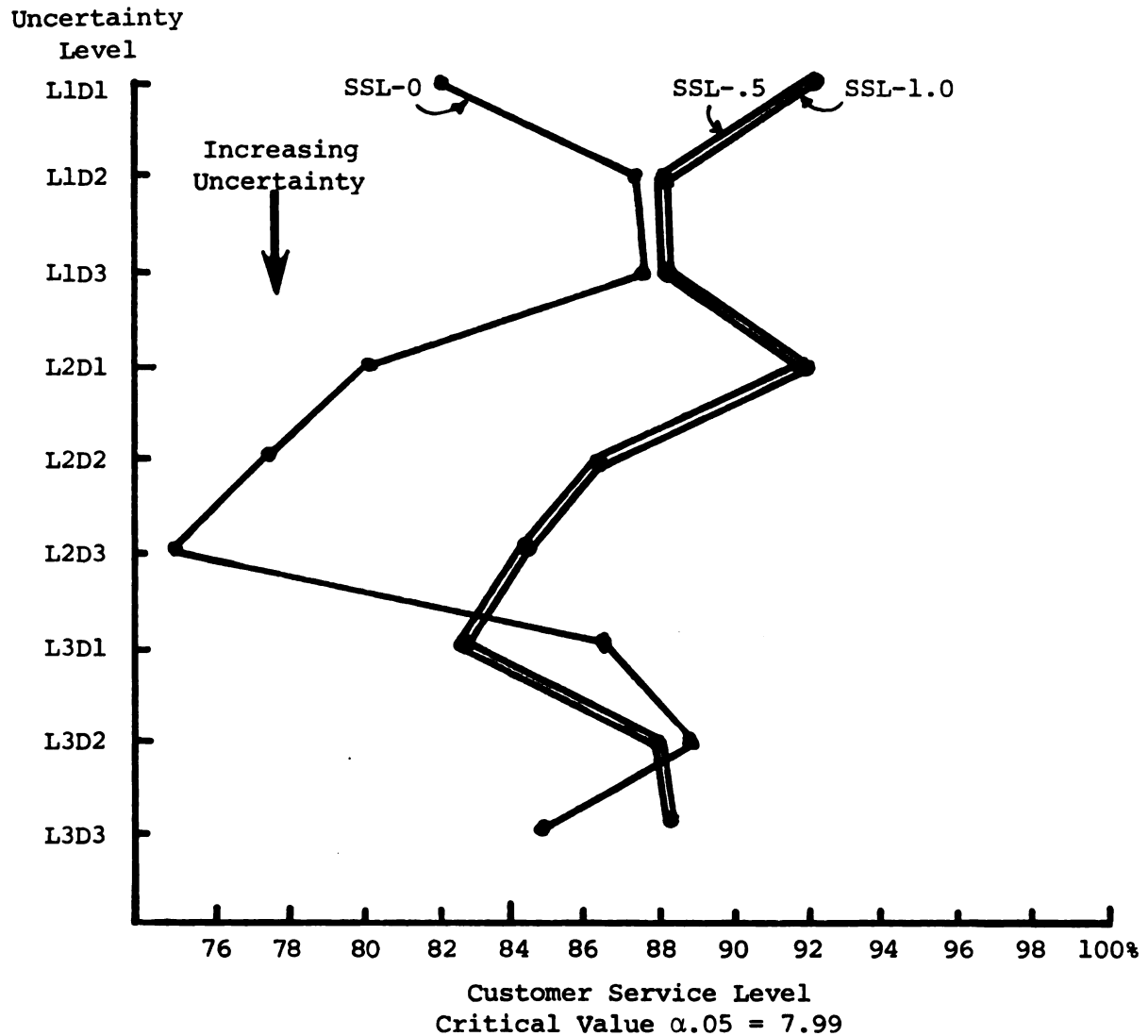


Figure 4.14 Mean Values of Customer Service Levels for Each Safety Stock Level Under a Distribution Center Safety Stock Positioning Policy.

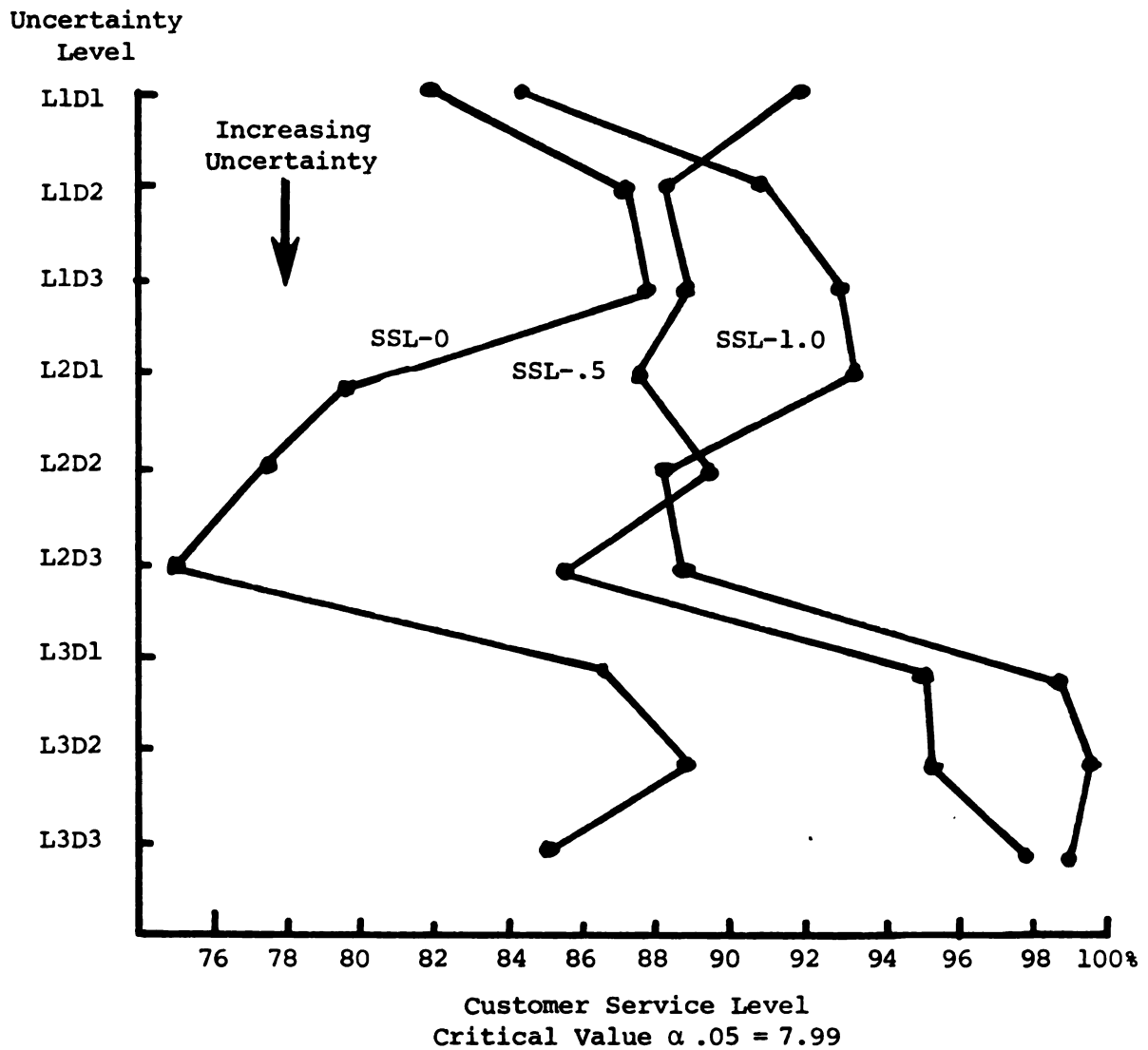


Figure 4.15 Mean Values of Customer Service Levels for Each Safety Stock Level Under a Retailer Safety Stock Positioning Policy.

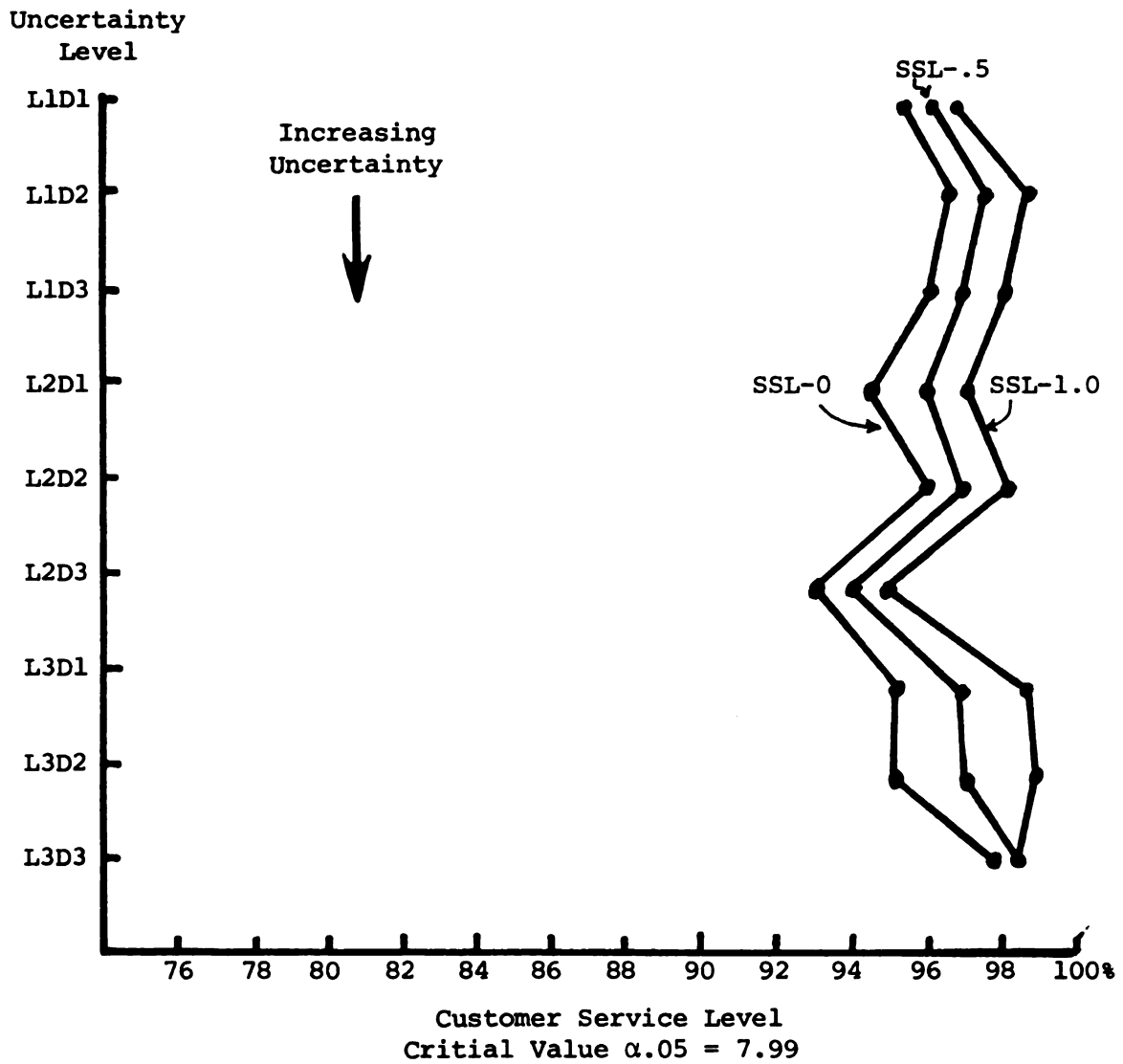


Figure 4.16 Mean Values of Customer Service Levels for Each Safety Stock Level Under a Distribution Center-Retailer Safety Stock Positioning Policy.

safety stock SSL-0 was lower in each instance. The same results appear when comparisons were made between safety stock levels SSL-0 and SSL-1.0. No differences were found in comparing customer service levels for safety stock levels SSL-.5 and SSL-1.0.

Figure 4.15 presents the mean values of customer service levels for each safety stock level employed (SSL-0, SSL-.5, SSL-1.0) under a retailer safety stock positioning policy. Comparisons of customer service levels for safety stock levels SSL-0 and SSL-.5 resulted in significant differences at all uncertainty levels except L1D2 and L1D3. The customer service levels for safety stock level SSL-0 were lower in each instance. Comparisons between safety stock levels SSL-0 and SSL-1.0 yielded the same results. Comparisons of customer service levels for safety stock levels SSL-.5 and SSL-1.0 resulted in only one significant difference. The customer service level under safety stock level SSL-.5 was significantly lower than that under SSL-1.0 at uncertainty level L1D1.

Figure 4.16 shows the mean values of customer service levels for each safety stock level employed (SSL-1.0, SSL-1.5, SSL-2.0) under a distribution center-retailer safety stock positioning policy. Comparisons between the customer service levels of the three safety stock levels resulted in no significant differences.

Hypothesis 1 was rejected. The safety stock level employed had a significant impact on customer service performance in multi-echeloned channel systems. Table 4.10 summarizes the significant differences. Twenty-two of the twenty-three significant differences result from comparing a safety stock level of zero with that of .5 or 1.0 standard deviations. At safety stock levels of SSL-.5 and higher, increases in safety stock equaling .5 or 1.0 standard deviations had no significant impact on customer service levels, regardless of the safety stock positioning policy employed and/or the uncertainty level experienced.

Thus, customer service level increased as safety stock level increased, but at a decreasing rate. Increases in safety stock level from .5 to 1.0 standard deviations resulted in customer service level increases ranging from zero to four percentage points. Increases in safety stock level from 1.0 to 2.0 standard deviations resulted in customer service level increases ranging from one to three percentage points. In most instances, the larger increases (3 to 4 percent) occurred at higher uncertainty levels.

Table 4.10 Summary of Safety Stock Level Comparisons

Uncertainty Level	(DC) Positioning Policy			(R) Positioning Policy			(DC-R) Positioning Policy		
	SSL			SSL			SSL		
	0-.5	0-1.0	.5-1.0	0-.5	0-1.0	.5-1.0	1.0-1.5	1.0-2.0	1.5-2.0
1 L1D1	*	*	..	*	*	*	..	..	..
2 L1D2	..	..	..	..	..	..	..	..	..
3 L1D3	..	..	..	..	..	..	..	..	..
4 L2D1	*	*	..	*	*	..	..	..	..
5 L2D2	*	*	..	*	*	..	..	..	..
6 L2D3	*	*	..	*	*	..	..	..	..
7 L3D1	..	..	..	*	*	..	..	..	..
8 L3D2	..	..	..	*	*	..	..	..	..
9 L3D3	..	..	..	*	*	..	..	..	..

*Significant.

Hypothesis 2:

In a comparison among customer service performance observed from dynamic simulations, no significant difference exists: *independent of safety stock positioning employed.*

The impact of safety stock positioning on customer service performance was isolated by holding constant both the safety stock level and the uncertainty level. Figures 4.17 and 4.18 display the mean values of customer service levels for the safety stock positioning policy employed under each combination of safety stock level and uncertainty level. Tukey's multiple comparison technique was used for safety stock positioning comparisons under each set of conditions. The critical value at $\alpha = .05$ was 7.99.

Figure 4.17 presents the mean values of customer service levels for each safety stock positioning policy employed [(DC), (R)] under safety stock level SSL-.5. Comparisons of customer service levels for safety stock positioning policies (DC) and (R) resulted in significant differences at the three highest uncertainty levels, L3D1, L3D2, and L3D3. In each instance, the customer service level under safety stock positioning policy (R) was higher than that under safety stock positioning policy (DC). The average significant difference was 9.9 percentage points.

Figure 4.18 presents the mean values of customer service levels for each safety stock positioning policy employed [(DC), (R), (DC-R)] under safety stock level

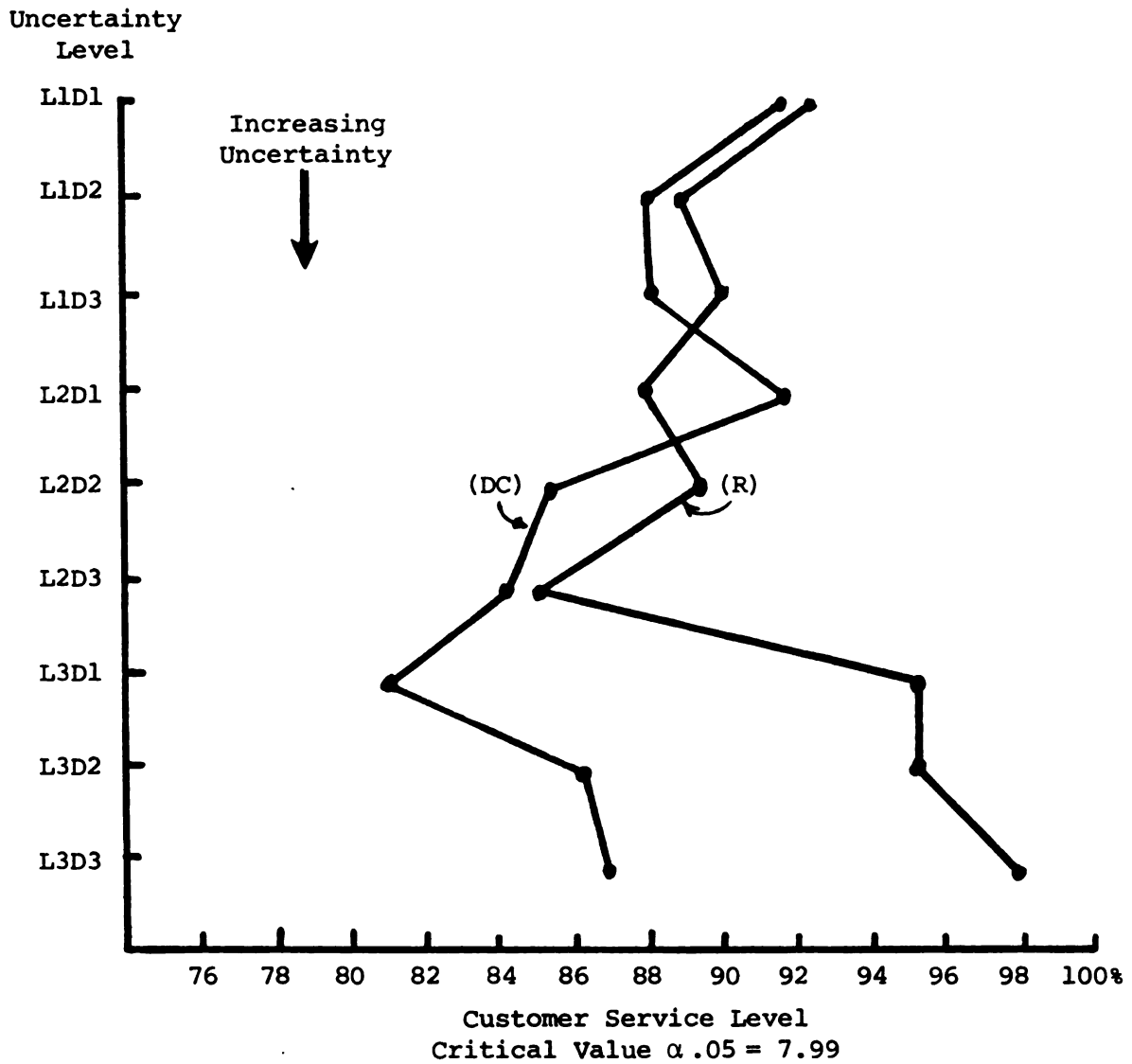


Figure 4.17 Mean Values of Customer Service Levels for the Safety Stock Positioning Policies Under Safety Stock Level SSL-.5.

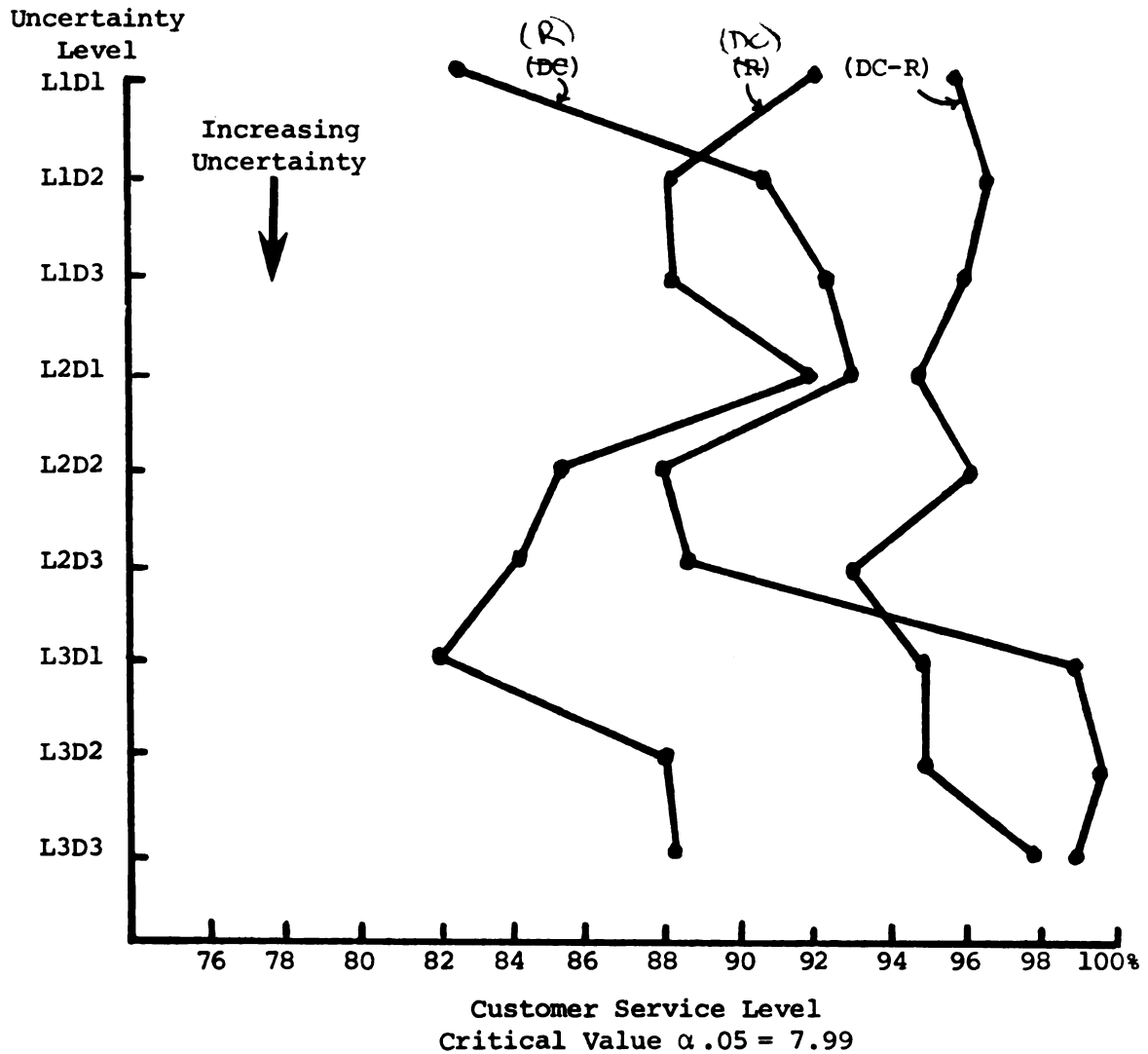


Figure 4.18 Mean Values of Customer Service Levels for the Safety Stock Positioning Policies Under Safety Stock Level SSL-1.0.

SSL-1.0. Comparisons of customer service levels for safety stock positioning policies (DC) and (R) resulted in significant differences at the lowest (L1D1) and three highest uncertainty levels, L3D1, L3D2, and L3D3. At uncertainty level L1D1, the customer service level under safety stock positioning policy (R) was significantly lower than that under safety stock positioning policy (DC). At the three highest uncertainty levels, the customer service level under safety stock positioning policy (R) was significantly higher than that under safety stock positioning policy (DC). The average significant difference was 12.6 percentage points.

Comparisons of customer service levels for safety stock positioning policies (DC) and (DC-R) resulted in significant differences at all uncertainty levels except L1D1 and L2D1. In each instance, the customer service level under safety stock positioning policy (DC-R) was significantly higher than that under safety stock positioning policy (DC). The average significant difference was 9.2 percentage points.

Comparisons of customer service levels for safety stock positioning policies (R) and (DC-R) resulted in significant differences at uncertainty levels of L1D1, L2D2, and L2D3. The difference at uncertainty level L1D2 was close to the critical value. In each instance, the

customer service level under safety stock positioning policy (DC-R) was significantly higher than that under safety stock positioning policy (R). Thus, Hypothesis 2 was rejected.

The safety stock positioning policy employed significantly impacts customer service performance in multi-echeloned channel systems. Specifically positioning a safety stock level of .5 standard deviations at the retailer rather than the distribution center, resulted in significantly higher customer service levels when employed under conditions of high uncertainty. When positioning a safety stock level of 1.0 standard deviations in a multi-echeloned system experiencing low or intermediate uncertainty levels, a safety stock positioning policy where the stock is equally divided between distribution center and retailer resulted in significantly higher customer service than those that concentrated the full amount of safety stock at either the distribution center or retailer. At higher uncertainty levels, positioning the full amount of safety stock at the retailer, resulted in customer service levels of one to four percentage points higher than a positioning policy that divided the stock between distribution center and retailer. However, either of these safety stock positioning policies resulted in significantly higher customer service levels than a

policy of concentrating the full amount of safety stock at the distribution center.

Hypothesis 3:

In a comparison among customer service performance observed from dynamic simulations, no significant differences exist: *independent of safety stock policy employed.*

The safety stock policy in a multi-echeloned channel system consists of setting a safety stock level and position. It has been determined that both safety stock level and position (independently) impact customer service performance significantly. A determination of the impact of safety stock policy on customer service level requires an examination of the combined impact of safety stock level and position on customer service. Figures 4.19 through 4.21 present the mean values of customer service levels resulting from safety stock policies employed at each uncertainty level. These figures illustrate the impact of different combinations of safety stock level and positioning on customer service. As shown, the safety stock policy employed did significantly effect customer service at all uncertainty levels. The nine safety stock policies employed in Problem II resulted in a wide range of customer service levels. Within a given uncertainty level, the minimum range of customer service levels was ten percentage points.

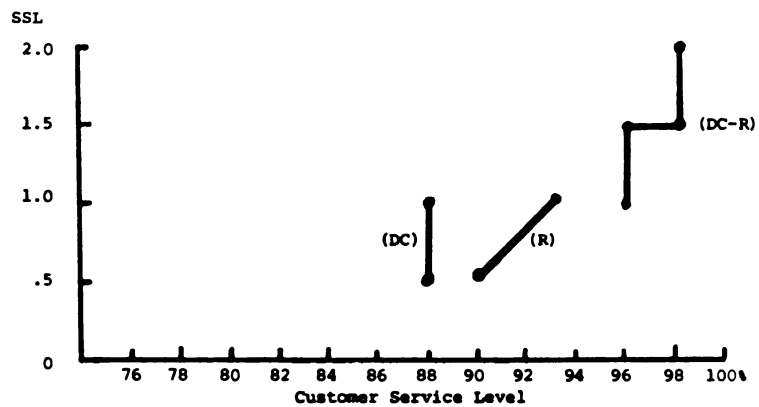
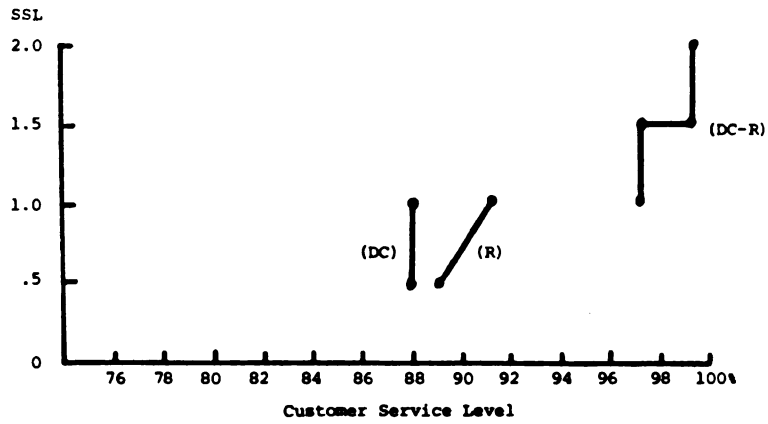


Figure 4.19 Customer Service of Safety Stock Policies Under Low Uncertainty Levels L1D1, L1D2, L1D3.

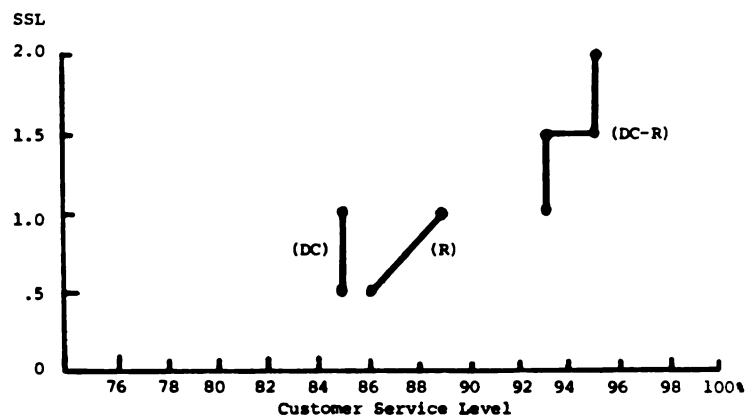
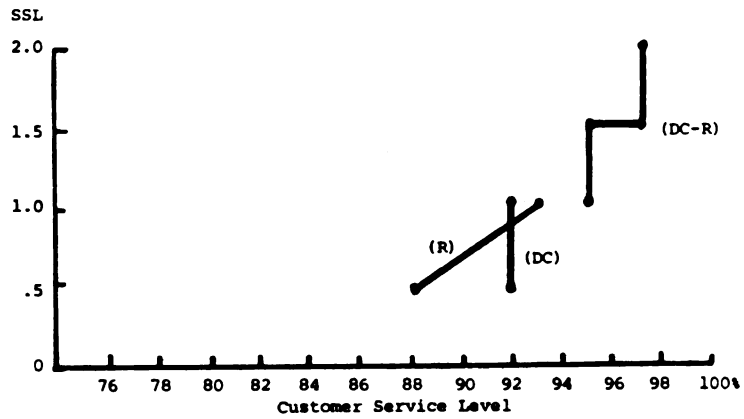


Figure 4.20 Customer Service of Safety Stock Policies Under Intermediate Uncertainty Levels, L2D1, L2D2, L2D3.

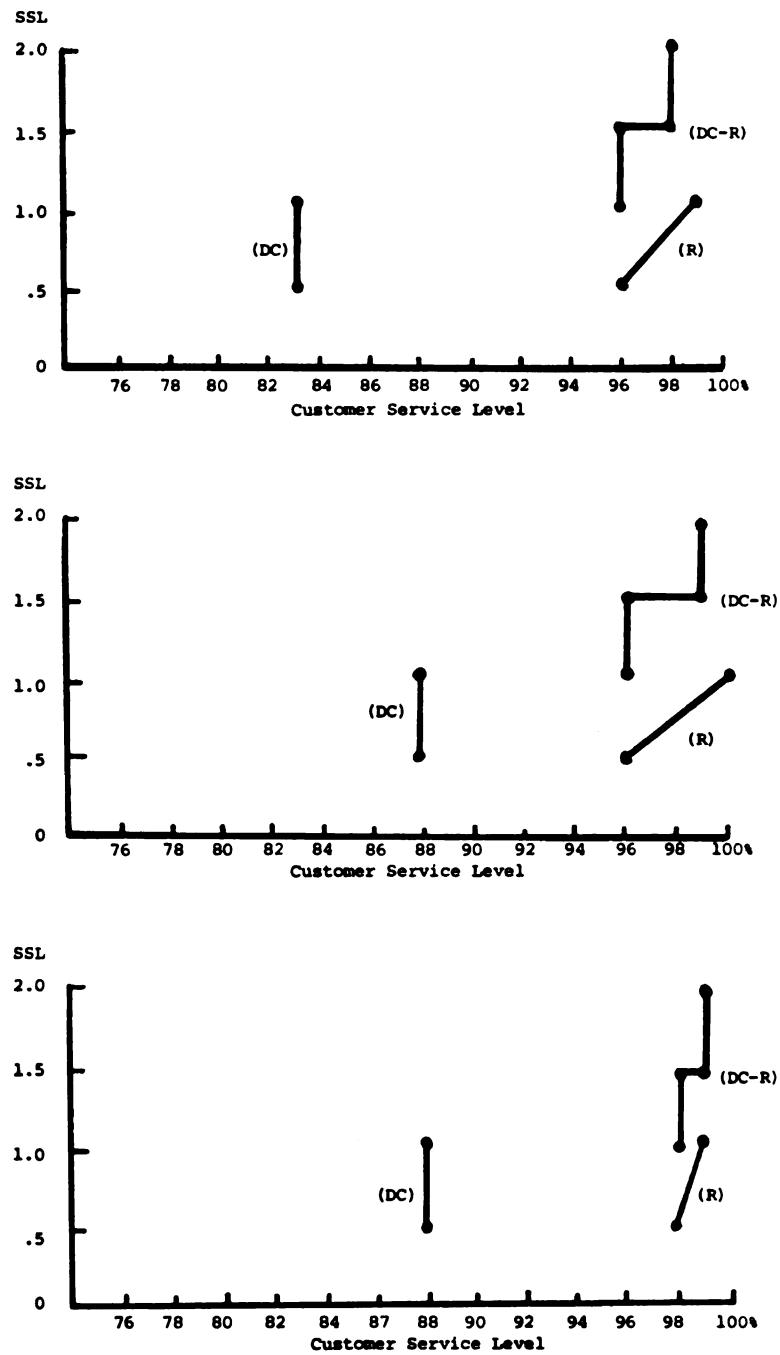


Figure 4.21 Customer Service of Safety Stock Policies Under High Uncertainty Levels, L3D1, L3D2, L3D3.

Two findings stand out. First, safety stock positioning had a greater influence on the resulting customer service levels than did the safety stock level. The significant differences in positioning policy were much larger than those for safety stock level. Second, in general, the safety stock positioning policies can be ordered, from the highest customer service to the lowest, as follows: (DC-R) followed by (R) followed by (DC). An exception to this order occurs at very high uncertainty levels, where (R) and (DC-R) may change places. In effect, this shows that more safety stock at retail locations results in higher customer service levels when high uncertainty levels are experienced. Accordingly, Hypothesis 3 was rejected. The safety stock employed had a significant impact on customer service performance in a multi-echeloned channel system.

Hypothesis 4:

In a comparison among customer service performance observed from dynamic simulations, no significant difference exists: *independent of uncertainty level experienced.*

Figure 4.22 shows the mean values of customer service level for each uncertainty level experienced across all safety stock policies. Tukey's multiple comparison technique was used for uncertainty level comparisons. The critical value of F at $\alpha = .05$ was 7.99. As illustrated in Figure 4.22, the uncertainty level experienced under safety

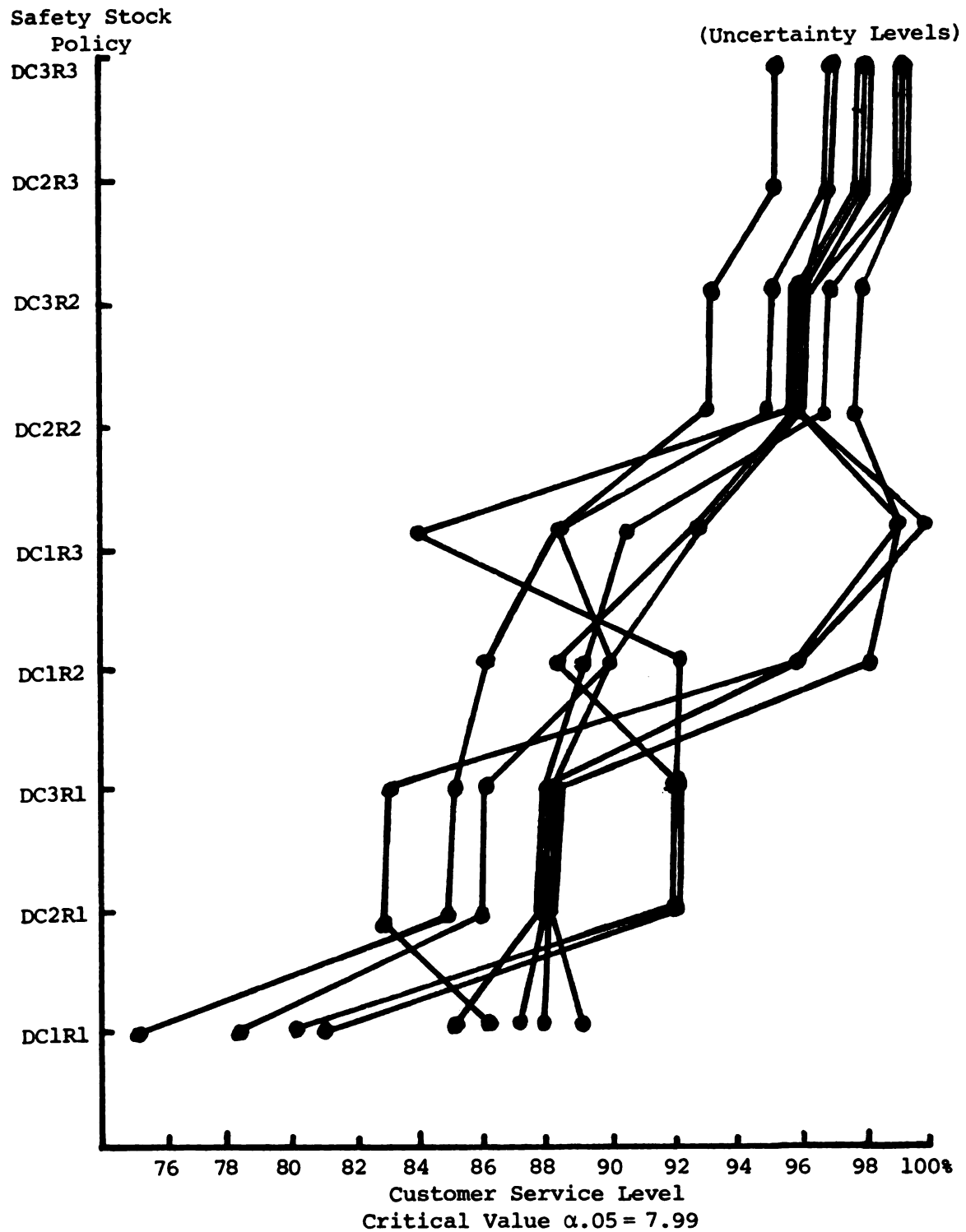


Figure 4.22 Mean Values of Customer Service Level for Each Uncertainty Level Across All Safety Stock Policies.

stock policies, D2R2, D3R2, D2R3, and D3R3, did not significantly impact customer service levels. These four safety stock policies were those which employed safety stocks at both the distribution center and retailer. Comparisons of uncertainty levels under each of the other safety stock policies resulted in significant differences. Thus, Hypothesis 4 was rejected.

The uncertainty level experienced had a significant impact on customer service performance in a multi-echeloned channel system. The analysis of variance (Table 4.9) confirmed this conclusion. The main effect of factor (B) was significant. The F value was significant at a level of $\alpha = .001$.

Two findings were especially significant. First, the impact of the uncertainty level on customer service performance diminished as the system safety stock level increased. Specifically, the range of the customer service levels was fourteen percentage points at SSL-0; twelve and nine percentage points at SSL-.5; sixteen, nine, and five percentage points at SSL-1.0; five and four percentage points at SSL-1.5; and four percentage points at SSL-2.0. Second, an increase in uncertainty level did not always result in a decrease in customer service level. While the basic relationship between uncertainty level and customer service level was inverse, 37 percent of the increases in

uncertainty level resulted in increases in customer service. Although these increases were not statistically significant, they were increases, not decreases. In addition, 5 percent of the increases in uncertainty level resulted in significant increases in customer service. Thus, the direction customer service would take was not fully predictable.

Hypothesis 5:

In a comparison among customer service performance observed from simulations, no significant difference exists: *independent of the combinations of safety stock policy employed and uncertainty level experienced.*

Table 4.8 shows the wide range of customer service levels resulting from combinations of safety stock policy employed and uncertainty level experienced. The values range from 75 percent to 99.6 percent. The analysis of variance confirmed that the two-way interaction was significant at $\alpha = .001$. Thus, Hypothesis 5 was rejected. The combined impact of safety stock policy employed and uncertainty level experienced on customer service performance in a multi-echeloned channel system was significant.

CHAPTER V

CONCLUSIONS

Introduction

This chapter presents the research conclusions, relates the conclusions to the present body of knowledge for distribution inventory planning and suggests areas for future research. The first section of the chapter discusses the conclusions derived from the hypotheses in Parts A and B of both research problems. The second section acknowledges the research limitations and offers areas for future research.

Conclusions for Safety Stock Planning and Control

The objective of this research was to measure both the accuracy of a statistical approach and the relative impact of alternative safety stock policies and uncertainty levels on customer service performance in single and multi-echeloned channel systems. The measures are discussed first for single echelon channel systems followed by a discussion of the conclusions drawn for multi-echeloned channel systems.

Problem I: Conclusions

Safety stock policy effects. Safety stock policy has a significant effect on the difference between predicted and simulated customer service. Specifically, higher safety stocks decrease the difference between predicted and simulated customer service. This behavior can be explained by examining the safety stock policy-customer service relationship.

Safety stock policy has a significant effect on customer service. A safety stock increase results in a customer service increase, but at a decreasing rate.¹ Specifically, the introduction of safety stocks into the channel system resulted in an average customer service increase of seven percentage points, while an addition to safety stocks, equal in size to the introductory amount, resulted in an average customer service increase of only two percentage points. Thus, given the safety stock policy-customer service relationship and the fact that customer service has an upper limit, the potential margin for error in predicting customer service diminishes as the upper limit is approached. So, managers should not use the statistical approach unless the desired customer service is close to the upper limit.

Uncertainty effects. Uncertainty has a significant effect on the difference between predicted and simulated

customer service. Specifically, higher uncertainty increases the absolute difference between predicted and simulated customer service. At high uncertainty levels, the differences were never below six percentage points. Thus, managers should not use the statistical approach in channel systems experiencing high uncertainty levels.

A second conclusion relating to uncertainty effects is that of the two types of uncertainty, lead time had the greatest effect on the difference between predicted and simulated customer service. The effect was in terms of degree of difference and direction of difference. Predicted customer service was higher than simulated customer service when low lead time variances were experienced and lower than simulated customer service when high lead time variances were experienced. Thus, managers who use the statistical approach under low lead time variance conditions, will fall short of their desired customer service goal. This may be a significant problem for managers who are facing highly competitive market situations where high customer service goals must be attained. Under market conditions such as this, managers would not want to fall short of their planned customer service goal since stockout costs are high compared to inventory carrying costs. So, use of the statistical approach under low lead time variance conditions is not recommended when managers must attain a desired customer service goal.

Since the statistical approach underestimates uncertainty effects on customer service at low uncertainty levels and overestimates uncertainty effects on customer service at high uncertainty levels, this seems to imply that the statistical approach is overreacting to changes in uncertainty level. It is possible that within the statistical approach, the effect of lead time variance on customer service has been misrepresented. In addition, part of the difference may be explained by examining the relationship between uncertainty and customer service.

Uncertainty has a significant effect on customer service. Specifically, increasing uncertainty generally resulted in decreasing customer service. Although this was expected, the change in customer service was lower than expected based on tables compiled by T. A. Burgin and A. R. Wild from Pearson's *Tables on the Incomplete Gamma Function*.² This may be explained, in part, by the interaction of demand and lead time variabilities. Because demand and lead time variabilities are not additive, a sort of "cancellation effect" may cause customer service to be higher than expected.³ For example, demand variability may cause the average daily demand to exceed the expected or mean value of daily demand over a specific lead time. However, at the same time, lead time variability may result in this specific lead time being shorter than the mean value of

lead time. Therefore, the shortened lead time "cancels" some or all of the negative effects that excess demand might have had on customer service. The implication is that the statistical approach does not accurately replicate the uncertainty-customer service relationship because, in part, it does not replicate demand-lead time interactions.

Safety stock policy and uncertainty effects (combined). Safety stock policy and uncertainty have a significant combined effect on the difference between predicted and simulated customer service. Figure 5.1 shows the feasibility of using the statistical approach under alternative combinations of safety stock policy and uncertainty in a single echelon channel system. The conditions under which the statistical approach can be accurately used are represented in Figure 5.1 by blank squares. It should be noted that in the two blank squares where uncertainty is low, use of the statistical approach results in customer service that falls short of the prescribed customer service. Thus, managers who face highly competitive market situations may find the statistical approach unusable under these conditions as well. It is apparent that the statistical approach, for the most part, does not accurately predict customer service in single echelon channel systems and should, therefore, not be used.

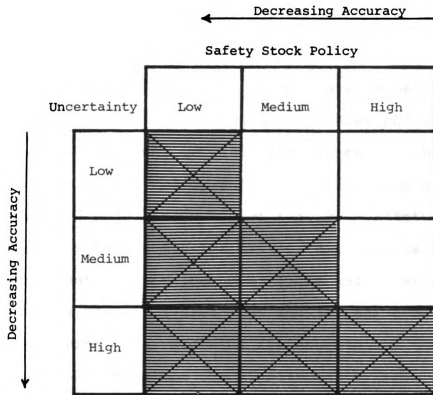


Figure 5.1 Statistical Approach-Application in Single Echelon Channel Systems.

Currently, there are no similar alternative statistical techniques available for managers to use in setting safety stocks. Thus, either a new statistical approach must be devised or the original statistical approach must be modified. In either case, a better understanding of the effects of safety stock policy and uncertainty on customer service performance will be useful.

Safety stock policy and uncertainty have a significant combined effect on customer service. Safety stock policy changes have a greater effect than uncertainty changes on customer service. The first safety stocks placed in the channel system have the largest effect on customer service, regardless of the uncertainty level. A partial explanation for the significant effects of safety stocks on customer service, even at low uncertainty levels, is that the placement of the safety stocks is at retail. Because the safety stock is at the point of demand, it can be fully used, if needed. Managers, who attempt to devise a new statistical approach or modify the existing statistical approach, may want to consider this combined effect as well as the independent effects of safety stock policy and uncertainty on customer service.

Problem II: Conclusions

Safety stock policy effects. Safety stock positioning policies have a significant effect on the difference between predicted and simulated customer service in a

multi-echeloned channel system. The use of a distribution center-retailer positioning policy rather than a distribution center or retailer positioning policy results in a significant decrease in the difference between predicted and simulated customer service. Specifically, a distribution center-retailer positioning policy under low or intermediate uncertainty conditions results in a minor difference between predicted and simulated customer service, while distribution center or retailer policies under the same conditions results in simulated customer service that is significantly lower than predicted. The average difference is five percentage points. Thus, managers can use the statistical approach when safety stocks are positioned at both echelons, except at high uncertainty levels. However, managers should not use the statistical approach when all safety stocks are positioned at only one of the two echelons. To further understand why the statistical approach should not be used under absolute postponement or absolute speculation positioning policies, the relationship between safety stock policy and customer service was examined.

Safety stock policy has a significant effect on customer service in a multi-echeloned channel system. Both safety stock positioning and safety stock level have significant effects on customer service, but safety stock positioning effects are greater. A distribution

center-retailer positioning policy results in significantly higher customer service than a retailer positioning policy, except at high uncertainty levels where this changes. The distribution center-retailer positioning policy was six percentage points higher at low and intermediate uncertainty levels and three percentage points higher overall. Thus, the principle of postponement is verified.⁴ The reason that this partial postponement positioning policy results in higher customer service than absolute speculation is that safety stocks are held at both the distribution center and retailer. This is important because inventory performance at sequential locations is interrelated. Total system performance is dependent, in part, on the inventory performance at all stocking locations. Thus, inventory performance at retail depends on the inventory performance at the distribution center and the lead time characteristics between the distribution center and retailer. The amount of safety stock at the distribution center directly affects the ability of the distribution center to fill retail replenishment orders, which, in turn, affects the retailers' inventory performance and ultimately customer service performance. Thus, managers can use partial postponement to increase customer service significantly in multi-echeloned channel systems, except at very high uncertainty levels.

A related conclusion is that a distribution center-retailer positioning policy results in significantly higher customer service than a distribution center positioning policy. The average difference is eight percentage points. Thus, absolute postponement suffers from the same limitation as absolute speculation in that one echelon will have poor inventory performance which will affect total system performance negatively. Thus, managers can attain higher customer service by positioning safety stocks at all interrelated stocking locations in the channel rather than positioning safety stocks at one location. The exception is the placement of all safety stock at retail, when the channel is experiencing high uncertainty levels.

In more complex channel systems, where a distribution center supplies multiple retail locations, a comparison of the distribution center-retailer positioning policy and retailer positioning policy would result in an even higher difference in customer service. In this situation, postponement of some retail safety stock to the distribution center by each retailer improves distribution center inventory performance which, in turn, impacts total system performance. An additional benefit is that the distribution center can combine retail variations in demand which, in effect, reduces uncertainty. The effect of safety stock postponement on customer service would be higher in channels

where multiple retail locations were supplied by a distribution center.

The previous point reemphasizes the importance of a total channel concept. Employment of a channel-wide inventory policy rather than independent wholesaler and retailer inventory policies can result in increased channel system customer service performance.

Safety stocks introduced at retail result in significantly higher customer service than if placed at the distribution center. This difference increases as uncertainty increases. Thus, the best single location for introducing safety stocks in the channel is at retail.

Additional safety stocks, at retail, will increase customer service slowly, while additional safety stocks at the distribution center become useless quickly. Thus, managers who have safety stocks at both echelons and want to increase customer service, should place the additional safety stock at retail.

The examination of the safety stock policy-customer service relationship clearly identifies why the statistical approach failed in channels with absolute postponement or absolute speculation positioning policies. Simulated customer service was significantly lower than predicted because the statistical approach did not account for the inter-related inventory performance of sequential stocking

locations. Thus, managers employing absolute speculation or postponement safety stock positioning policies should not use the statistical approach.

Uncertainty effects. The uncertainty level has a significant effect on the difference between predicted and simulated customer service. Specifically, at low uncertainty levels, the simulated customer service is significantly lower than predicted, while at high uncertainty levels the simulated customer service is significantly higher than predicted. At high uncertainty levels, the statistical approach error averages nine percentage points. To explain these conclusions, the uncertainty-customer service relationship was examined.

Uncertainty has a significant effect on customer service. In general, an increase in uncertainty level resulted in a decrease in customer service. However, in many instances, an increase in uncertainty resulted in customer service that was higher or lower than expected. This may be explained by the interaction of demand and lead time variabilities and the interaction of uncertainties at sequential echelons. Thus, the statistical approach does not accurately represent the uncertainty-customer service relationship because, in part, it fails to consider the interaction of uncertainties of sequential echelons.

Safety stock policy and uncertainty effects

(combined). Safety stock policy and uncertainty have a significant combined effect on the difference between predicted and simulated customer service. Figure 5.2 shows the feasibility of using the statistical approach under alternative combinations of safety stock policy and uncertainty in a multi-echeloned channel system. The condition under which the statistical approach can be accurately used is represented in Figure 5.2 by a blank square. It is apparent that the statistical approach does not accurately predict customer service in multi-echeloned channel systems, and, for the most part, should not be used. There are no alternative statistical safety stock techniques that consider the effects of interaction of sequential echelons. Thus, managers can either devise a new statistical approach or attempt to modify the existing statistical approach. In either case, a better understanding of the effects of safety stock policy and uncertainty on customer service performance in a multi-echeloned channel system will be beneficial.

Safety stock policy and uncertainty have a significant combined effect on customer service. Safety stock positioning policy changes have a greater impact than uncertainty changes on customer service. Customer service under the distribution center-retailer positioning policy remained stable as uncertainty increased. This occurred,

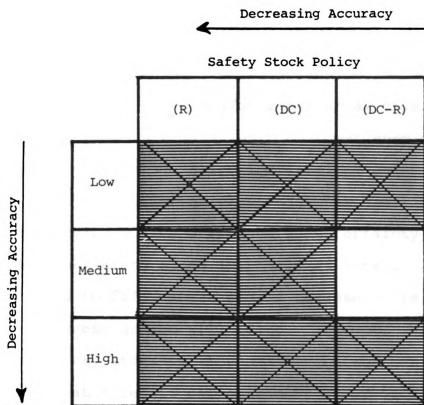


Figure 5.2 Statistical Approach-Application in Multi-Echeloned Channel Systems.

in part, because safety stocks were held at each echelon to act as a buffer against uncertainty. Thus, inventory performance at all sequential locations was adequate. The implication is that managers can achieve relative stability in customer service if safety stocks are held at all echelons. This positioning policy can be beneficial to managers who face rapid and/or substantial changes in demand or lead time uncertainties. Customer service under the retailer positioning policy increased as uncertainty increased, while customer service under the distribution center positioning policy decreased as uncertainty increased. When all of the safety stock is placed at retail, it can be fully utilized to fill excess orders because it is at the point where excess demand will occur. However, when all of the system safety stock is at the distribution center, the probability that a retail replenishment order is filled increases, but whether or not the replenishment order reaches the retailer when needed, depends on the lead time characteristics between the distribution center and retailer. As uncertainty increases, fewer of the filled replenishment orders reach the retailer when required. Thus, as uncertainty increases, the effect on customer service of a retail positioning policy versus a distribution center positioning policy widens. A retailer positioning policy is more desirable at high uncertainty levels.

Managers who devise new statistical approaches or modify the existing statistical approach for use in multi-echeloned channel systems, should consider the effects of safety stock positioning on customer service performance. Because inventory performance at sequential locations is interrelated, the safety stock policy-customer service relationship of single and multi-echeloned channel systems differs. Thus, identical statistical approaches should not be used to set safety stocks for both single and multi-echeloned channel systems.

Summary

The statistical approach, for the most part, should not be used for setting safety stocks in either single or multi-echeloned channel systems. New statistical approaches should be devised or the existing statistical approach modified so that the relationship between the combined factors and customer service are more closely replicated. Using identical statistical approaches for setting safety stocks in single and multi-echeloned channel systems should be avoided because the safety stock policy-customer service relationship in single and multi-echeloned channel systems differs and safety stock policy effects on customer service are significant.

Safety stock policy has a more significant effect than uncertainty on customer service in both single and

multi-echeloned channel systems. In single echelon systems, introductory amounts of channel system safety stock are the most effective in increasing customer service. In multi-echeloned systems, higher customer service can be achieved by positioning some of a given amount of safety stock at each echelon rather than by positioning all of the safety stock at one echelon. Partial postponement results in higher customer service than either absolute speculation or absolute postponement because inventory performance at sequential locations is interrelated. Therefore, a total channel approach to setting safety stock requirements increases customer service.

Research Considerations

This section discusses the research limitations and suggests areas for future research. The section contains two parts. The first part states research limitations and the second part discusses future research endeavors which might be undertaken.

Research Limitations

As is typical of all simulation studies, the generalizability of this research is constrained to the extent that the model employed replicates actual distribution systems. However, the SPSF testing environment, which was used in this research, has been subjected to extensive evaluation.⁵

The research was limited as to the types of network designs employed. Only a parallel structure of both single and multi-echeloned distribution systems was used, so cross-shipments were not accommodated in the network design, and any interactions of the inventory performance between multiple locations at the same echelon were omitted. Although many channel systems have cross-shipment capability, an examination of this added complexity was beyond the scope of this research effort.

An additional limitation was the number of levels employed for each of the independent factors. Ideally, numerous demand, lead time, and safety stock policies would be employed. However, cost and time constraints made this impractical.

A further constraint upon the research was that several other factors which have an effect on inventory policy decisions in channel systems either were held constant or assumed to be constant. Thus, reorder policies such as reorder system and order quantity were held constant. Again, cost and time constraints made the addition of these considerations impractical.

Future Research

The statistical approach did not accurately predict customer service in single echelon or multi-echeloned channel systems. This occurred because the statistical approach

did not accurately reflect the relationship between the combined factor effects and customer service. Thus, an area for future research would involve the reformulation of the statistical approach for use in both single and multi-echeloned channel systems.

Factors which have an effect on safety stock policy and ultimately customer service performance were not considered by this research. These factors were held constant to allow measurement of the impact of the experimental factors. Such factors, including order quantity, might be allowed to vary, to measure their relative effect on the accuracy of the statistical approach and customer service. Answering such questions represents research areas which would more fully explain the relationship between safety stock policies and channel system performance.

Another path that must be followed in the future is the expansion of the research to include inventory policies outside of physical distribution that may impact channel system effectiveness and efficiency. For example, the inventory policies of vendors who supply manufacturers in the channel as well as the raw material or in-process inventory policies of manufacturers could be included in the research design. System effectiveness and efficiency could be compared, through consideration of alternate safety stock policies, such as the form in which safety stocks are

held, as raw material, in-process inventory, or as finished goods. This wider view is consistent with the systems concept and should be explored if a desired channel system effectiveness and efficiency is to be attained.

FOOTNOTES--CHAPTER V

¹This relationship is described in many current production control and logistics textbooks.

²T. A. Burgin and A. R. Wild, "Stock Control-Experience and Usable Theory," Operational Research Quarterly 18 (): 48-49.

³George D. Wagenheim, "The Performance of a Physical Distribution Channel System Under Various Conditions of Lead Time Uncertainty: A Simulated Experiment" (Ph.D. dissertation, Michigan State University, 1974).

⁴Wroe Alderson, Marketing Behavior and Executive Action (Homewood, Ill.: Richard D. Irwin, Inc., 1957), p. 425.

⁵David Joseph Closs, "Simulated Product Sales Forecasting: Mathematical Model, Computer Implementation, and Validation" (Ph.D. dissertation, Michigan State University, 1978).

APPENDIX

APPENDIX

SPSF MODEL

This appendix provides an overview of the Simulated Product Sales Forecasting Model and its analytical capabilities. Specifically, the Appendix contains a brief overview of the model; a discussion of the general model design; and a detailed review of the four model modules.

SPSF Model

SPSF is a computer simulation model that provides a forecast testing environment by combining the attributes of market area demand simulation, dynamic operational simulation, and statistical sales forecasting. The SPSF model is capable of rendering a sales forecast while simultaneously creating customer orders and replicating the physical distribution process of providing timely inventory to satisfy customer order requirements. Thus, through the combined capabilities of two types of simulation and a statistical forecasting package, a complete time-sequenced picture of events leading to forecast deficiency is captured and fully documented. Such a documentation provides the basis for subsequent sensitivity analysis. A prime feature of the

SPSF model is that it provides a testing environment for controlled experimentation.

SPSF Model Design

The SPSF testing environment consists of four separate modules which are capable of being used simultaneously or independently. The four modules are: (1) Demand Module, (2) Forecast Module, (3) Operations Simulator Module, and (4) Analysis Module. Each module is reviewed briefly in terms of its function within the overall SPSF model.

The first module is the Demand Module which provides a methodology for creating potential sales. The purpose of this module is to produce synthetic orders from a geographical market area. Thus, the Demand Module is employed to quantify pattern, level, and dispersion of product orders over a time period. The design approach of this module is important to the SPSF testing environment as it provides the primary data for forecasting. Four alternative demand generating procedures are included in the Demand Module.

The second module is the Forecast Module in which various forecasting techniques may be used to forecast the demand generated by the Demand Module. Sales forecasts are used to establish inventory levels. Thus, the Forecast Module provides the interface between the firm's operations and its environment.

The demand and forecast, generated by their respective modules, are fed into an Operations Module which replicates a specific physical distribution system. The Operations Module is capable of replicating inventory availability and movements based upon a variety of different replenishment policies. It replicates the performance of the test operating system across the time horizon of the forecast period. The operations simulator is designed as a generalized stochastic model capable of simulating the performance of a distribution system with either a single or multi-echeloned network structure. To obtain maximum realism, it is designed on a dynamic basis wherein the state of the model at any given point in time is dependent upon the performance of the system in preceding periods and, in addition, forms the basis for operating performance in future periods. For example, the Operations Module adjusts inventory levels on a time dependent basis to replicate both receipts and shipments over time. Thus, the Operations Module can explore management problems in areas such as operation policy, network structure design, and inventory policy.

The fourth module is the Analysis Module which is primarily concerned with the reporting of the interactive effects of the Demand, Forecast, and Operation Modules. The module provides management status, system activity, and cost computation reports.

Figure A.1 provides an overview of the SPSF Testing Environment general design and the foundation for a more detailed discussion of the four modules. As a matter of terminology, the term "simulator" is used to refer to the combined use of the Operations, Forecast, and Demand Modules. The term "SPSF Testing Environment" is used to refer to the above defined simulator plus the Analysis Module.

SPSF Module Review

Demand Module

The Demand Module generates customer orders on a daily basis. Through this module, various types of demand environments can be created to facilitate observations of system performance.

Three alternatives are available for demand generation in the SPSF Testing Environment: (1) the direct input of actual orders from sales history; (2) the use of Monte Carlo processes or probability distributions to create individual orders; and (3) determination of an aggregate sales figure, such as the industry sales for a market area which is reduced to individual product orders.

The first method of demand generation, directly inputting product orders is termed Procedure One in the Demand Module. The use of orders as sales history is the most realistic and simple method of demand replication.

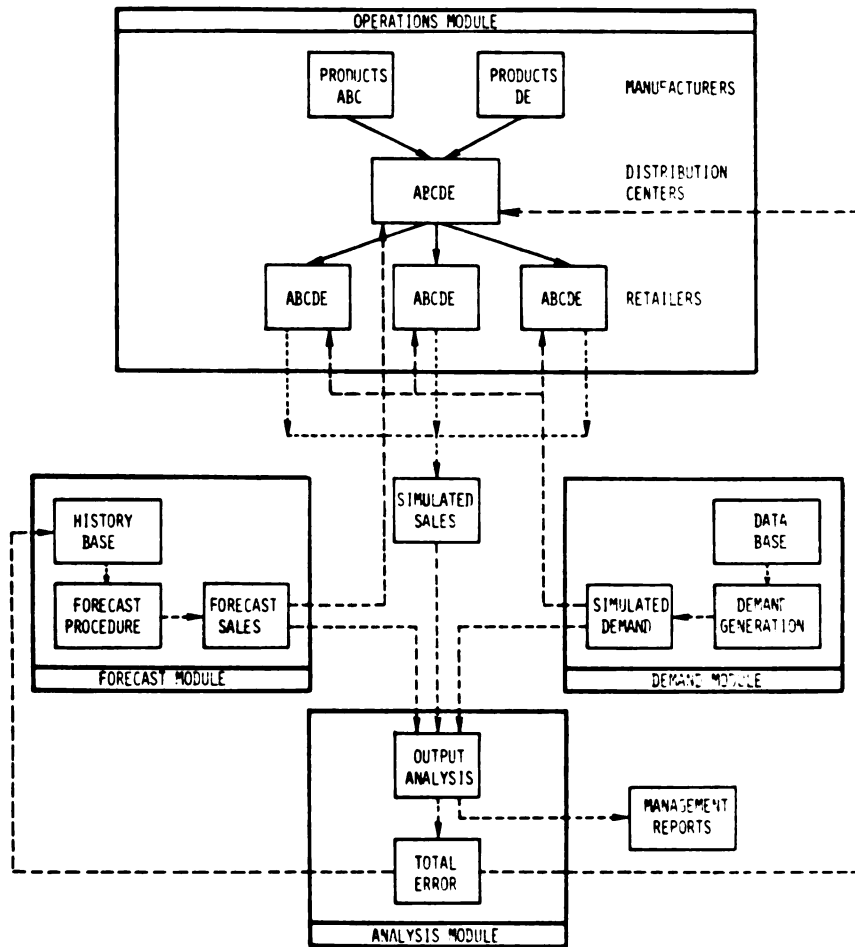


Figure A.1 SPSF Testing Environment--General Design.

However, this procedure has the shortcomings of requiring more data than alternative methods of demand generation and an inability to create experimental test conditions.

Procedure Two employs statistical procedures such as normal, log-normal, erlang, and poisson distributions to generate total daily sales. Although easy to implement, this approach offers the researcher no assured approximation to market reality. Even if the probability distribution used is statistically fitted according to historical sales, the parameters of the model are static.

Once the periodic sales level is identified, it is reduced to daily orders for use by the Operations Module. The general procedure is to select orders randomly from a predetermined sample order file until daily demand is complete. Pseudo orders can be constructed to permit experimental conditions or to replicate events such as new product introduction.

Procedure Three utilizes the correlation between historical demand and other economic indicators to generate an estimate for the periodic sales level. There are numerous economic indices which can be correlated to sales. Indices which may be considered include population, Gross National Product, net income, and new housing starts. The choice of indices depends on the specific situation. In Procedure Three, firm sales is determined using multiple

linear regression. The effect of various factors upon firm sales is measured using the method of least squares. Given a number of factors as independent variables, a linear regression equation is used to arrive at firm sales for a particular month. Daily sales are generated by multiplying this monthly sales level by a daily sales factor based on an expected mean and variance for the month in question.

Procedure Four uses an alternate correlation procedure. In Procedure Four, industry demand is generated within the market area by inputting independent variables into a correlation equation of the general form:

$$IS = a + b_1X_1 + b_2X_2 + b_3X_3 + \dots + b_nX_n$$

where:

IS = industry sales for the period in question;

a = the vertical axis intercept;

X_{1-n} = the independent variable influencing industry sales; and

b_{1-n} = the respective factors of the independent variables.

This form of generalized equation permits inclusion of any independent variables deemed relevant to the determination of market area industry sales. In addition, it allows specification of different variables for different product situations.

The regression formulation provides an estimate of industry demand which is decomposed to monthly estimates. Next, specific market share is determined using the approach proposed by Kotler.¹ This method uses a ratio of the firm's "market effort" to the industry "marketing effort" to arrive at specific market share.

The result of the regression analysis for Procedures Three and Four is a monthly or weekly sales estimate. Given this figure, daily sales are determined from a normal distribution using the expected mean and variance of daily sales for the month under analysis. Given a value for daily sales, Procedure Two is used to arrive at individual product orders.

Use one of the four procedures in the Demand Module allows the modeler to simulate the demand environment desired. The generated daily orders become a part of the input into the Operations Module.

Forecast Module

The Forecast Module provides the firm's interface between the demand environment and the operationalization of its policies and actions.² Incorporation of the forecast into the simulation is accomplished in the following manner.

Given a product, the first forecasting option is to generate independent forecasts for each location in the

operation structure. Each forecast is generated from an analysis of historical sales patterns (throughput) at each location.

The second option generates forecasts at the echelon level serving the final customer. A forecast is made for several periods in advance. This forecast is then exploded back to the source locations which replenish the customer service location. Upper echelon forecasts depend on the estimated lead time from the location in question to the market. The estimates are computed for all products in an event subroutine.

The SPSF Model is equipped with four forecasting techniques for time series analysis. The techniques are:

1. Brown's Simple Exponential Smoothing Model;
2. Trigg and Leach's Adaptive Smoothing Model;
3. P. R. Winters' Exponentially Weighted Moving Averages; and
4. Robert's and Reed's Self-Adaptive Forecasting Technique.

Brown's Simple Exponential Smoothing Model is the most basic technique provided in the module.³ This technique applies a static smoothing constant (α) to the previous period's sales and forecast by the formula:

$$\text{Forecast}_1 = \alpha(\text{Sales}_0) + (1-\alpha)(\text{Forecast}_0).$$

It is easy to use, but contains no forecast error adaptability or specific trend, cyclical or seasonal elements which limits its capability.

Trigg and Leach's Adaptive Smoothing Model is adaptive.⁴ The technique operates with a smoothing constant (α) set equal to the absolute value of a tracking signal. The tracking signal is a measure relating the degree of forecast error during the most recent period to that in the preceding periods. When error becomes large, the tracking signal approaches a value of one. As the value of the tracking signal increases so does the corresponding value of α , allowing the forecast to adapt more quickly to changes in demand. As this adaptation leads to decreases in the error, the tracking signal and the value of α will decline.

P. R. Winters' Exponentially Weighted Moving Average incorporates trend and seasonality.⁵ This technique utilizes a smoothing constant and smoothed estimates of trend and seasonality. The combination of these factors yields a sales forecast.

Since the Winters' technique incorporates trend and seasonal variations, it represents a more sophisticated approach than either Brown's or Trigg and Leach's technique. Winters' technique has the deficiency of not adapting smoothing constants to changes in the level of forecast error.

Robert's and Reed's Self-Adaptive Forecasting Technique is the most complex technique furnished by the Forecast Module.⁶ Level, trend, and seasonal factors are all considered by the adjustment of smoothing constant values.

The techniques of the Forecast Module are used by the modeler in an attempt to forecast the demand generated by the Demand Module. The forecasts generated by the Forecast Module become a part of the input to the Operations Module.

Operations Module

The Operations Module is a dynamic simulator that integrates input from the Demand and Forecast Modules. This permits both time and function-dependent observations to be replicated and observed. The simulator models the individual activities of physical distribution process which might include order processing, order shipping, production, transportation, and inventory management. The dynamic characteristic of actual operations is achieved through activity interaction.

In the linkage of events, dynamic realism is achieved through the inclusion of probabilistic time variables. Typical components requiring probability variables are communication, order processing, and transit time.

The design of the Operations Module provides substantial flexibility in permitting multiple plants, distribution centers, echelons, and products. The Operations Module has the capability to simulate physical distribution systems with up to fifteen locations and ten individual products or product groups. In addition, the location may be arranged in any number of echelons from manufacturing plants to final customers. These capabilities allow the Operations Module to replicate the typical physical distribution configuration used to replenish inventory to a specific market area.

Figures A.2 and A.3 illustrate the structural flexibility of the Operations Module. Figure A.2 presents a simplified distribution structure consisting of inventory stocking locations at manufacturer, distribution center, and retailer. The product flows from manufacturing plant to a distribution center and then to the customer. Policies capable of alternative testing are illustrated on the right side of Figure A.2. Figure A.3 illustrates a more complex example of a physical distribution structure consisting of four echelons and twelve locations. The complexity of the network exists because of the multiple source destinations.

These examples illustrate only two of the many system structures which can be designed using the Operations Module. The specific network design employed in this research is developed in Chapter III.

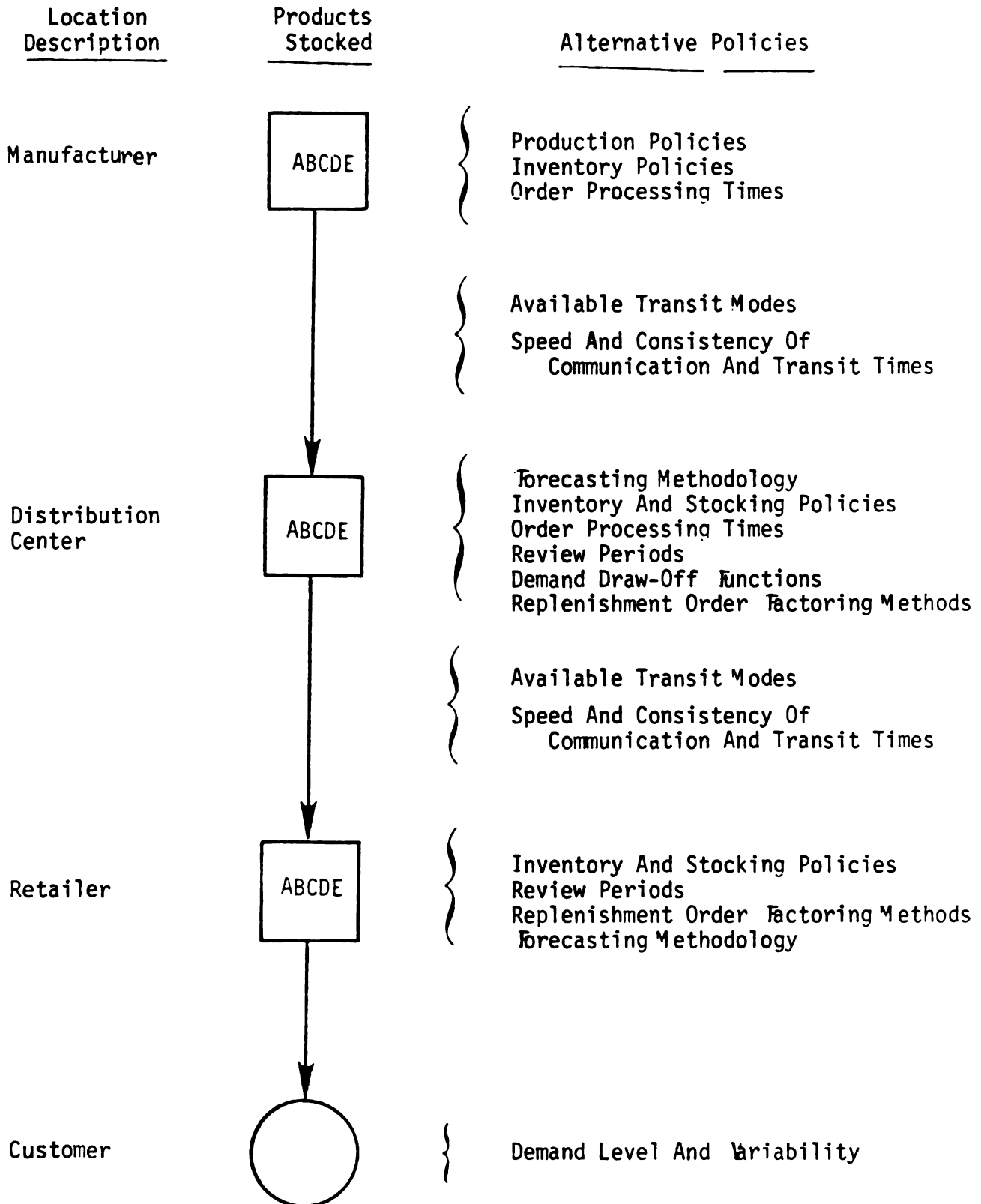


Figure A.2 Example of Simplified Distribution Structure.

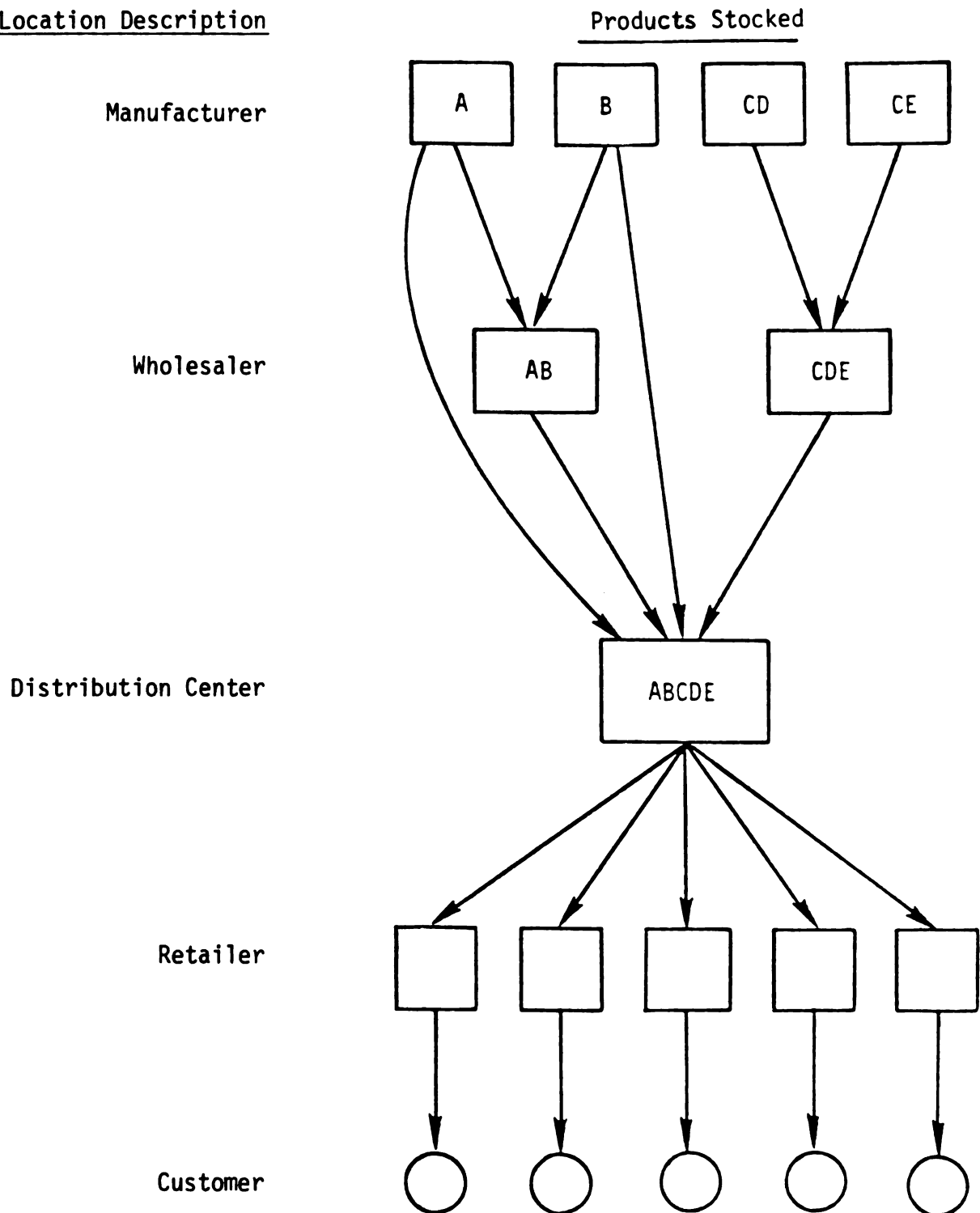
Location Description

Figure A.3 Example of Complex Distribution Structure.

Analysis Module

The Analysis Module of the SPSF testing environment calculates the levels of cost and service variables for each simulation. It consists of two distinct routines, the Cost Generator and the Report Generator.

The Cost Generator computes the costs incurred by simulated distribution operations. Cost elements include transportation, warehousing, storage, inventory, ordering, and backorder costs.

The Report Generator prepares reports on system performance. The reports may be divided into four categories. The categories of reports are sales and physical levels, costs, service, and error. In addition, run summary reports are provided. Each category is divided into records which can be selected independently. The categories and their records are listed in Table A.1. These reports provide a means of analyzing operations and correcting them.⁷

Table A.1 Report Categories and Records

<u>Category</u>	<u>Records</u>
Sales and physical levels	• System customer sales • System shipments • System receipts • System inventory report • Replenishment sales • Replenishment sales and in-transit inventory • Replenishment volume: weight shipped • Product inventory report • Product sales report (units) • System bleeding report • Product bleeding report
Costs	• System cost • Replenishment volume: cost of shipping
Service	• System service measure • System percentage of orders by quantity met • System backorder recovery and thousands of dollars filled within days • System backorder recovery: tens of units filled within days • Replenishment order cycle time summary
Error	• Operating error report • Forecast error report
Managerial summary	• Run summary: sales • Run summary: service • Run summary: inventory and backorders • Run summary: costs

FOOTNOTES--APPENDIX

¹Philip Kotler, Marketing Decision Making: A Model Building Approach (New York: Holt, Rinehart & Winston, 1971).

²David Joseph Closs, "Simulated Product Sales Forecasting: Mathematical Model, Computer Implementation, and Validation" (Ph.D. dissertation, Michigan State University, 1978).

³R. G. Brown and Richard F. Meyer, "The Fundamental Theorem of Exponential Smoothing," Operations Research 9 (September-October 1961): 673-685.

⁴D. W. Trigg and A. G. Leach, "Exponential Smoothing with an Adaptive Response Rate," Operations Research Quarterly 18 (March 1967): 53-59.

⁵P. R. Winters, "Forecasting Sales by Exponentially Weighted Moving Averages," Management Science 6 (April 1960): 324-342.

⁶Stephen D. Roberts and Ruddell Reed, Jr., "The Development of a Self-Adaptive Forecasting Technique," AIIE Transactions 1 (December 1969): 314-322.

⁷For the complete documentation of the SPSF Testing Environment, see Donald J. Bowersox et al., Simulated Product Sales Forecasting: Managerial Documentation (East Lansing: Bureau of Business Research, Michigan State University, 1978).

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