THE IMPACT OF IMPUTED INTEREST RATES ON INVENTORY MANAGEMENT

Thesis for the Degree of D. B. A.

MICHIGAN STATE UNIVERSITY

SALVATORE D. COSTELLA

1967

THESIS

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D.B.A. degree in Accounting

Herbert E. Miller
Major professor

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ABSTRACT

THE IMPACT OF IMPUTED INTEREST RATES
ON
INVENTORY MANAGEMENT

by Salvatore D. Costella

Efficient management of investment in inventory can increase the rate of return on capital invested in the business. Since the beginning of the 1900's a "scientific approach" to the solution of inventory problems has been developed.

Scientific inventory management, using mathematical models, attempts to determine optimal inventory levels and minimum total inventory costs by balancing the opposing costs associated with inventory. The development of scientific inventory models and the subsequent application of these models to actual business situations has helped, to some extent, to solve the problem of having either too much or too little merchandise on hand.

The outstanding weakness in the application of scientific inventory management theory is that there is wide disagreement concerning the computations of some of the

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relevant inputs to be used in the models. The most controversial input concerns the value of the imputed interest charge for the investment tied up in inventory. It is generally agreed that an imputed interest charge is a relevant cost because the use of capital in inventory precludes the application of that capital to other uses. The specific rate to use in computing this opportunity cost is the subject of the dispute.

The relevant value of the capital cost for any firm, as developed in this research, is the value of the estimated rate of return that could be earned on potential capital investments for which funds are unavailable in the present capital budgeting decision making period. The minimum acceptable rate in any decision period is the firm's cost of capital for that period. The use of any other imputed interest rate value in the model will violate an underlying assumption of capital management theory--the long-run goal of maximization of long-run earnings of the present stockholders.

The sensitivity of total inventory costs to changes in the value of the imputed interest rate was determined for both of the dynamic inventory models for a variety of simulated cost structures. Since the costs associated with entory va

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inventory vary from firm to firm and from one decision making period to the next for any one firm, the sensitivity analysis was performed with numerous combinations of the different costs associated with inventory. The simulated cost structures used in this research were developed by varying the ordering cost, the stockout cost and the total carrying cost. The ordering cost was varied from forty percent to two hundred percent of per unit cost in twenty percent intervals. The stockout cost was varied from twenty percent to one hundred percent of per unit cost in ten percent intervals. The total carrying cost was varied from four percent to fifty-six percent of per unit cost in four percent intervals.

Total inventory costs were found to be sensitive to changes in total carrying costs for the entire range of ordering costs and stockout costs used in the simulated cost structures. The total inventory costs were most sensitive, for all simulated cost structures, where the imputed interest rate charge represented the most important element of total carrying costs.

Deviations in total inventory cost varied from less than one percent to more than fifteen percent for both the dynamic model under certainty and the dynamic model under nk for al

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risk for all of the simulated cost structures. For any specific cost structure and error factor in total carrying costs the sensitivity of the total inventory costs were greater for the dynamic model under risk than for the dynamic model under certainty.

Increases in profits and the long run earnings of a firm are possible by a reevaluation of the imputed interest rate value used in the scientific inventory models. The specific profit increases possible for any firm will depend on several factors. First is the importance of inventory relative to other assets. Second is the importance of the imputed interest rate relative to other carrying costs, and the difference in the rate presently used and the rate that would be used if the capital management decision theory criteria were employed. Third is the absolute dollar value of total inventory costs and the change in the firms' sales margin created by the inventory cost savings.

Although the percentage savings of total inventory costs might be small, perhaps one or two percent, two things must be remembered. First, a small percentage savings on large total inventory costs can represent a large absolute dollar amount. Second, small percentage savings of total inventory cost can result in much larger percentage increases net prof

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in net profit. The percentage increases in net profits resulting from the total inventory cost savings depends on the firm's sales margin.

For a firm working on a small sales margin the small percentage savings in cost of goods sold will effect the net profits more favorably than the same small percentage savings in cost of goods sold for the firm working on a larger sales margin.

Increases in profits through the reduction of total inventory costs are available to firms using scientific inventory control models if the firm is using imputed interest rate costs that are not in line with capital management decision theory. A reevaluation of this important element of inventory cost will result in increases in the long-run earnings of the firm's present stockholders.

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by

Salvatore D. Costella

A THESIS

Submitted to
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for the degree of

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ACKNOWLEDGMENTS

The completion of the formal requirements for an advanced degree, resulting in the receipt of a diploma by only one individual, is actually accomplished through the hard work and sacrifices of many people as well as through the efforts expended by the degree candidate.

The faculty of the Graduate School of Business at Michigan State University was a positive force in my decision to continue my formal education and in the final completion of the doctorate program. The faculty influenced my thinking and contributed to my learning outside the classroom as well as in formal classroom sessions.

A special note of thanks is due Dr. Herbert Miller, Chairman, Dr. Roland Robinson, and Dr. Frank Mossman for serving on my dissertation committee. Their patients and help, as well as their friendship, are deeply appreciated. A special note of thanks is also due Dr. Charles Gaa and Dr. Gardner Jones for their friendship and help in clearing up some fuzzy thinking on concepts incorporated in the dissertation. A special note of thanks is due Dr. James Don Edwards, Head of the Department of Accounting and Finance. His encouragement and special efforts on my behalf were of immeasurable help.

The general discussions and specific study sessions with my colleagues, the other graduate students, were also a great

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help in developing my knowledge of specific disciplines. The exchange of ideas and concepts and the expansion of these areas through discussions and sometimes rather loud verbal exchanges, not only reinforced concepts under study, but expanded them. Among the many graduate students with whom these discussions occurred, my thanks go especially to Carolyn Mueller. Miss Mueller's knowledge of the thesis topic and her outstanding ability as an editor helped to create a more readable and grammatically correct piece of work.

A special note of thanks is also due to Mrs. Lena Catalano for her diligence and perseverance in typing, at times from some very rough notes. Forgetting Sundays and holidays, Mrs. Catalano was able to make it much easier to meet deadlines. For any inconvenience to her family I am sorry, and to her I am very grateful.

A final note of thanks is to my family, especially my wife, who put up with my periods of discouragement and bad temper when things seemed bleakest. My wife's sacrifices of a normal social life, her employment and constant encouragement were, without a doubt, the major factor in my completion of the program. Not only did she plant the seed of an idea--to pursue a doctorate--but she cultivated the idea after it had become a fact, and cared for it until it bore the final fruit, the diploma. During this time she kept house, worked,

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and raised a family! I thank my children present during my student days, and expect that some day they will understand when they attempt their own approach toward their own goals in life.

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CHAPTER I

INTRODUCTION

The average American business firm has about one-third of its total capital invested in inventories of raw materials, work-in-process, and finished goods.[1] Proper management of the asset inventory is important because of the vital role it plays in the operations of the specific firm and because of economic and social implications it holds for the entire business community.

The Role of Inventory in the Firm

The reason for investing in inventories is the same as that for investing in any asset: to increase the rate of return on capital invested in the business. Therefore, any alternative investments must be at least as profitable as investment in inventory if one of the significant objectives of management--increasing the rate of return on capital--is to be achieved.

The need of business firms to hold inventories or the

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[1]National Industrial Conference Board, Inventory Management in Industry, Studies in Business Policy, No. 88 (New York: National Industrial Conference Board, 1958), p. 7.

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[3] John F.
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profitability of holding inventories instead of other assets is rooted primarily in two factors:[2] uncertainty and delay. Either of these factors alone is not sufficient cause to hold inventories: If goods were available immediately, uncertainty of demand by itself would create no need for inventories; similarly, if there were no uncertainty (with respect to both demand and delivery time), then delay by itself would create no need for inventories. Hence, it is the combination of uncertainty and delay which is the principal reason for maintaining inventories.[3]

Delay and uncertainty affect the profitability of the firm in many ways. The following areas particularly affected by delay and uncertainty are in the chain of events by which the firm provides its products to its customers:

1) Demand for the product

2) Production lead time

3) Delivery lead time of raw materials

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[2]The profits or losses from holding inventories that occur from changes in the price level are not included in this discussion. This is not meant to imply that this factor is not an important one. The inventory losses of the 1920-1921 depression and the inventory gains during World War II are accepted facts which do influence the thinking of businessmen. Speculation in inventories, however, is not easily included in the framework being developed and is therefore omitted.

[3]John F. Magee, Production Planning and Inventory Control (New York, Toronto, London: McGraw-Hill Book Company, Inc., 1958), p. 67.

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4) Variations in production rates

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The following is an example of the way in which uncertainty and delay affect the above areas and create the need for holding inventories. Variations in delivery lead time create uncertainty as to the time it will take to receive merchandise after placing an order. The differences in lead time could originate in variations in the transportation system used or in the supplier's production facilities. During periods of economic prosperity the supplier may be working at capacity and a long lead time will ensue. Conversely, during periods of slow economic activity the supplier will be able to meet new orders in a shorter time period. Thus, from the purchaser's point of view prompt delivery is not likely to occur precisely when it is most crucial, and vice versa.

Investment in inventories, like investment in any asset, will increase the firm's profitability only if managed wisely. Any asset has certain costs as well as profits associated with it. Inventory is no exception. Funds invested in inventory are unavailable for alternative investment; the economic value of perishable, seasonal, and style goods deteriorates rapidly over a short period of time; and handling,

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[4]Joseph Buchan and Ernest Koenigsberg, Scientific Inventory Management (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1963), p. 282.

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ordering and storage of inventory give rise to costs.

One writer, in discussing the advantages and disadvantages of holding inventories, states:

> Basically, inventories serve to decouple successive operations in the process of making a product and getting it to consumers. For example, inventories make it possible to make a product at a distance from customers or from raw-material supplies or to do two operations at a distance from one another (even if only across the plant or room!). Inventories make it unnecessary to gear production directly to consumption or, alternatively, to force consumption to adapt to the necessities of production. In these similar ways, inventories free one stage in the production-distribution process from the next, permitting each to operate more economically. The essential question is: At what point does the decoupling function of inventory stop earning enough advantage to justify the investment required?[5]

The Economic Implications of Inventory

Economists have long recognized the fact that fluctuations in aggregate inventory levels have adverse effects on the economy. In fact, many economists have claimed that a major factor contributing to the 1920-1921 depression was a buildup of excess stocks of goods within the economy. Businessmen have also recognized the importance of inventories as a factor causing fluctuations in business activity.

> the expansion of plant or hoarding of inventory in anticipation of a rise in prices is business

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[5]Magee, p. 17.

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[7] McGee,

> policy arrived at as a result of decision-making. Multiply such decisions by a hundred thousand similar ones and an industrial inflation is not only in the offing but actually in process. Reverse it at some later date and a deflation sets in.[6]

The Social Implications of Inventory

Inventories serve important social functions. First, inventories permit businessmen to meet variant customer demand at a reasonable cost. Inventories help the business man meet customer demands in periods of a "buyers' market" when the customer has alternative sources should the particular firm not have stock on hand to fill his order. Thus inventories are one factor which facilitates customer dominance in an economic system of highly integrated and relatively inflexible production units. The important role of inventories during a "buyers' market" can best be seen by comparing it to the typical situation when a "sellers' market" exists--frequently a period of national emergency--when inventories in general have been depleted, limiting the alternatives available for satisfaction of consumer wants.[7]

Second, inventories permit stabilized employment and increase utilization of skilled employees by absorbing

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[6] J. Martindell, The Scientific Appraisal of Management (New York: Harper Brothers, 1950), Chapter 10, p. 151.

[7] McGee, op.cit., p. 4.

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[9] *Ibid.*,

fluctuations in demand. Stability of employment and its effect on the well-being of the community is a recognized fact receiving full attention from various political and economic groups. If the image of the business firm as a positive social force is to be perpetuated, the firm must contribute to the welfare of the community by helping to stabilize employment.[8] Efficient inventory management as an integral part of business planning will help firms to achieve this result.[9]

The Problem Defined

Businessmen have been aware of the fact that it is possible to have either too much or too little merchandise on hand and have attempted in one way or another to achieve a balance between the advantages of holding large inventories and the advantages of holding small inventories. Since the beginning of the 1900's, a "scientific approach" to inventory management has been developed.

In the scientific approach to inventory management, mathematical models have been developed which attempt to

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[8] Production for stock is how businesses help to stabilize employment. Business firms do not do this out of any altruistic motives but only because it is to their self-interest to 1) maintain their skilled labor supply for when it is really needed, 2) avoid economic penalties imposed through legislation for failure to provide stable employment (supplementary unemployment taxes and unemployment insurance taxes) and, 3) to avoid personnel turnover costs.

[9] *Ibid*., p. 5.

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determine optimal inventory levels and minimum total inventory costs by balancing the opposing costs associated with inventory, given certain parameters. Basically these mathematical models are an application of differential calculus to inventory management. The opposing costs associated with inventory are the costs of acquiring inventory and the costs of carrying inventory. The former are fixed costs per lot, which means that the average cost for each item in a lot decreases as the lot size increases. The second type, the costs of carrying inventory, varies in total with the size of the investment in inventory; hence the per unit carrying cost will also vary with the size of the investment in inventory. This cost behavior is presented graphically in the familiar form of Illustration 1-1.

Sophisticated models have been developed for solving inventory problems under a wide range of conditions. This model development has, without a doubt, increased the number of inventory problems which can be solved by inventory theory. There is, however, a bleaker side to the development of inventory theory. The outstanding weakness in the application of the theory is that there is wide disagreement concerning the computations of some of the relevant data to be used in the models. Thus, even the simplest inventory model, the static model under certainty, provides vastly different

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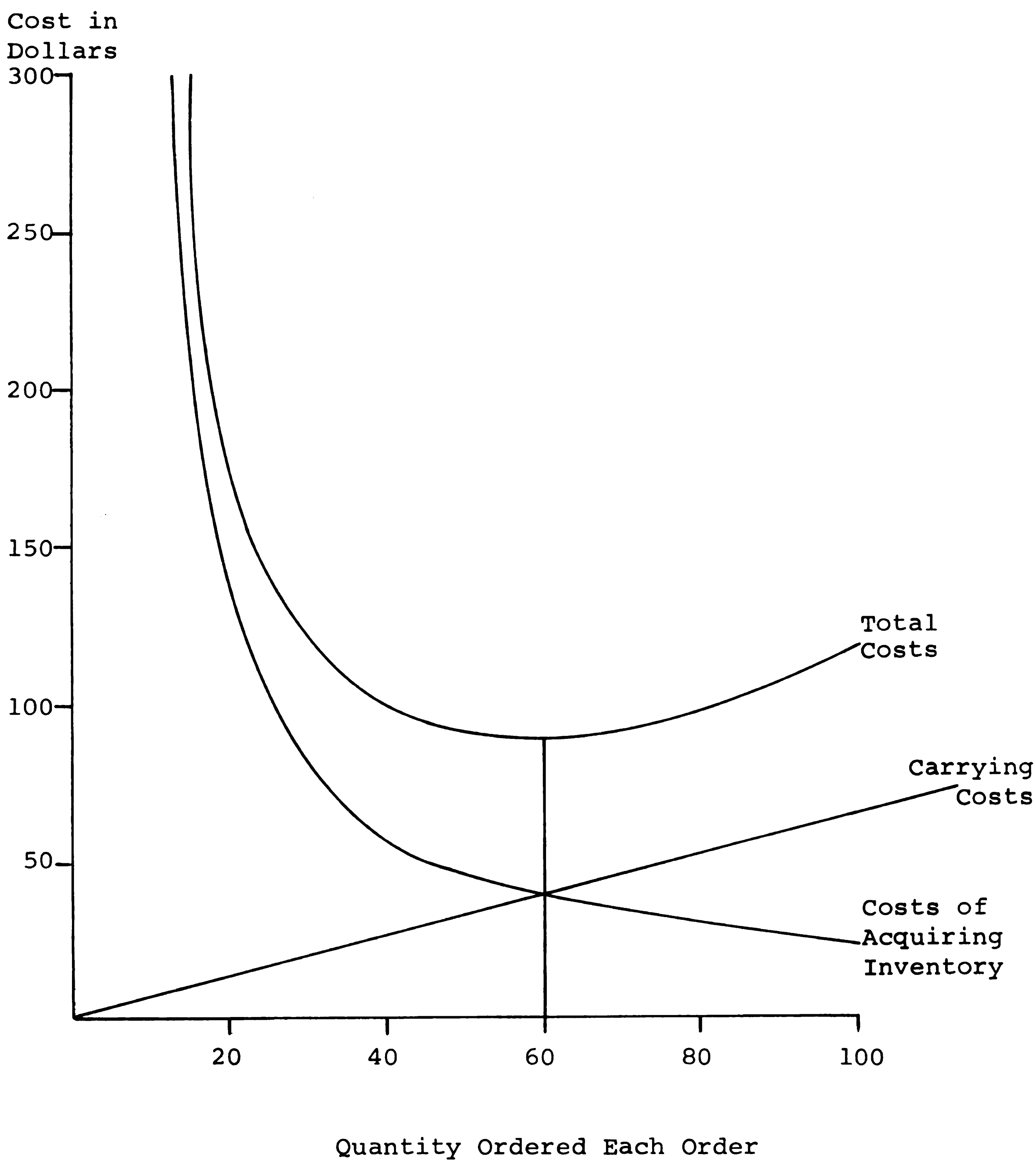
ILLUSTRATION 1-1

ANNUAL COST OF ORDERING AND HOLDING INVENTORY



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solutions depending upon the inputs considered relevant.

The most controversial input to be used in the model is the imputed interest on the investment in inventory. It is generally agreed that an imputed interest charge is a relevant cost because the use of capital in inventory precludes the application of that capital to other uses. The specific rate to use in computing this charge is subject to dispute, however.

One writer suggests the use of the rate paid on a short-term bank loan or the rate paid on a bond issue, if a short-term loan or a bond issue is outstanding.[10] At the other extreme is a suggestion of a high rate of thirty or thirty-five per cent for a firm which is not taking its purchase discounts. In general, economists have urged the use of high rates, while many firms use a low rate of somewhere around eight per cent for total carrying charges.[11]

The use of different imputed interest rates in the same model will result in different economic order quantities which will result in different average inventory levels which will, in turn, cause different total annual inventory costs.

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[10]Martin K. Starr and David W. Miller, Inventory Control: Theory and Practice (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1962), p. 11.

[11]Thomson M. Whitin, The Theory of Inventory Management (Princeton, N.J.: Princeton University Press, 1957), p. 239.

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Costs will, in turn, affect the profitability of the firm. The use of the wrong imputed interest rate in determining the "optimum inventory level" will result in a solution to the problem which is not, in reality, the optimum solution.[12]

Objectives of the Dissertation

As is obvious from the preceding discussion, there are few firms which need not maintain some inventories while for many firms inventory is a major element in the asset structure. Thus, proper inventory management is an important element of business concern. It is a broad field in which much work still needs to be done. This dissertation considers some of the important aspects of the inventory management problem, the specific objectives being:

1) to show that the imputed interest rate included in the model should be the opportunity cost of alternative investments available to the firm with a minimum cut-off rate which is equal to the firm's cost of capital;
2) to show the effect on the average inventory level and annual cost of inventory of using various imputed interest rates in a variety

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[12]The wrong imputed interest rate being defined as "an imputed interest rate that does not reflect the true cost of the investment tied up in inventory."

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It is not within the scope of this study to determine the appropriate interest rate or the cost of capital for any individual firm. Nor is the development of new inventory models to handle specific inventory problems within the scope of this dissertation.

Plan Of The Dissertation

This dissertation consists of six chapters. The first chapter is introductory in nature. Chapter Two is a short history concerning inventories, in which I review the historically changing attitudes of businessmen toward holding inventory, followed by a summary of the various inventory control systems used by businessmen to determine inventory levels.

There are two facets to inventory control under the scientific inventory management approach. One of these, the use of models in scientific inventory control, is the subject of Chapter Three. This chapter is divided into three sections: section one discusses the role of models in business; section two discusses the criteria for classifying various inventory models and defines terms used in scientific inventory control; section three discusses the three basic inventory models used in scientific inventory control systems.

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The second facet of scientific inventory control, the relevant data to be used as inputs in the models, is the subject of Chapter Four. The major emphasis of this chapter is on determination of the imputed interest rate portion of carrying cost. A review of capital management theory is presented and the interest rates suggested by the literature of scientific inventory control are evaluated in terms of acceptable capital management theory.

Chapter Five presents the methodology and effects of various imputed interest rates upon the average inventory level and upon the total carrying costs of inventory as determined by the models.

Chapter Six contains the summary and conclusions of the dissertation.

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CHAPTER II

BACKGROUND

Concepts of Wealth

Inventories of goods, including those in excess of necessary quantities to carry on the process of production and distribution, were considered signs of wealth prior to the industrial revolution. The wealth of a nation in this period was determined by the size of its flocks, herds, granaries, warehouses, and other commodities of high value and great durability which were not subject to loss of value through change of fashion. Gold and silver were included in the concept of merchandise of high value and durability and were not valued for their liquidity alone.

> The abuse or indefinite use of words, has in no one article of human reasoning caused greater confusion in ideas, than the calling wealth or riches by the name of money: --- Riches, in respect to a nation, are the universal plenty of all necessaries, as food, raiment, houses and furniture, provision for war, etc. Money, as gold or silver coin, are properly the medium of exchange, but by its quantity may become, and is an article of commerce itself; yet where it most abounds, as in Portugal, it makes but a small proportion of the riches of that country, though the country itself is extremely poor. And nothing is so erroneous, as to judge the

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[2]Ibid.

> riches of a country by the quantity of gold and silver in it.[1]

Any emphasis on the accumulation of precious metals in preference to other commodities was due to the use of these metals as a financial reserve, in liquid form, to meet emergencies. The medieval practice of accumulating precious metals as state treasures permitted the various kingdoms to meet unexpected emergency cash payments, since monetary transactions had become the normal state of affairs in place of barter, and public borrowing and taxation were not yet a regular source of revenue responsive to fiscal needs.[2]

After the industrial revolution money became more closely associated with wealth. Liquidity became an important objective of businessmen and they preferred to hold cash and securities rather than large inventories. Inventories in excess of those necessary to perform specific functions were viewed with alarm. Businessmen had become aware of the fact that it was possible to have either too much or too little merchandise on hand. An awareness of costs associated with holding inventory and their effect on the profitability of

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[1]Robert Wallace, "A View of the Internal Policy of Great Britain," 1764, p. 2. Quoted in Jacob Viner, Studies in the Theory of International Trade (New York: 1937), p. 21.

[2]Ibid., p. 22.

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an entity became widespread. Businessmen, according to one writer, looked upon inventory as the graveyard of American business.[3] Another said:

> Those who are overly aware of the cost of owning inventory tend to think of inventory as a hot potato--the longer it is held, the worse the burn.[4]

Methods Used To Control Inventory

As businessmen became cognizant of the adverse effects excess inventories could have upon profitability, they developed methods which would minimize the asset and still permit efficient operation of the firm. One of the earliest techniques developed, and also one of the crudest, was hand-to-mouth buying.

Hand-To-Mouth Buying

Hand-to-mouth buying reduces inventory levels because the businessman buys only in small lots for immediate requirements, and places no advance orders for goods. One author described it as "controlled buying, buying according to need, buying according to the doctrine of maximum use."[5]

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[3]Thomson M. Whitin, The Theory of Inventory Management (Princeton, N.J.: Princeton University Press, 1957), p. 4.

[4]Benjamin Melnitsky, Management of Industrial Inventory (New York: Conover-Most Publications, Inc., 1951), p. 114.

[5]Wheeler Sammons, Hand to Mouth Buying (Metropolitan Life Insurance Company, 1927), p. 101.

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Retailers in the dry goods industry were one of the first groups to use this device to minimize inventory levels. Several descriptions of this technique are found in the early literature, although it was not until 1868 that the term "hand-to-mouth buying" itself actually appeared.

> But at present we feel ourselves called upon to say that prudent merchants will husband their resources, be chary of their means, and extremely cautious in their commitments for some time to come.[6]

> Buyers keep in mind the panic of last spring and consequently purchase only for their immediate wants.[7]

> Dealers feel indisposed to place any orders for goods which are not needed for immediate requirements; and it is generally presumed that this hand-to-mouth policy will rule among the trade until the new cotton crop comes on the market and gives some greater stability to quotations.[8]

The use of hand-to-mouth buying was primarily during periods of sluggish business activity. This fact does not seem surprising, however. During periods of high business activity the advancing prices of most merchandise permit gains on sales of goods bought in earlier low-price periods.

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[6]Hunts' Merchants Magazine, 1839. Quoted in Leverett S. Lyons, Hand to Mouth Buying (Washington, D. C.: The Brookings Institution, 1929), p. 425.

[7]Hunts' Merchants Magazine, 1865. Quoted in Lyons, Ibid., p. 426.

[8]Commercial and Financial Chronicle, 1868. Quoted in Lyons, Ibid., p. 426.

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Conversely, during periods of falling prices businessmen want to minimize stock levels in order to minimize losses from selling high-cost goods at the now lower prevailing prices.

The losses from declining prices in times of recession, obsolescence of merchandise, and other costs of carrying excess inventories were a group of factors which gave rise to the use of hand-to-mouth buying. At the same time another group of factors, technological in nature, arose and helped smooth the way for effective use of the hand-to-mouth buying approach to inventory control. These technological factors--improved railroad service, express delivery service and more efficient communications systems--permitted speedier delivery of goods at lower rates, thus decreasing the lag time between the placing of orders and their subsequent delivery. The existence of overcapacity, in turn, caused suppliers to be willing to fill the hand-to-mouth orders (small lots, quick delivery, etc.).[9]

The use of the hand-to-mouth technique to control inventory levels was a crude device at best. The main purpose of this technique was to minimize inventory in stock while still meeting customer demand, within certain limits. Several specific systematic control systems were developed in this

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[9]Fred E. Clark, "An Analysis of the Causes of Hand to Mouth Buying," Harvard Business Review, VI (1927-1928), p. 395.

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period which helped to implement the hand-to-mouth device of minimizing inventories. Two of these systems still in common use today are the fixed ordering system and the periodic reordering system.

Fixed Order System[10]

Under a fixed order system an identical quantity of merchandise is ordered each time, but at irregular intervals. One of the oldest and best known examples of the fixed order approach is the two-bin system. It is still widely used by business today.

Under the two-bin system a specific type of merchandise is separated into two bins or piles. Initially all orders are filled from bin number one. When bin one is empty, the materials are reordered. Bin number two is used to meet demand until receipt of the merchandise from the supplier. Variations of the method exist: one approach is that upon receipt of the materials, bin number two is refilled and the balance is put into bin number one, which again becomes the "order-filling" bin. For items with a known demand and a known replenishment time, the two-bin system is fairly automatic and decreases the need for taking physical inventory.

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[10]The bulk of this material came from Whitin, op.cit., Chapter Two, and John F. McGee, "Guides to Inventory Policy II. Problems of Uncertainty," Harvard Business Review, March-April, 1956.

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Periodic Reordering System

Under the periodic reordering system, as contrasted to the fixed ordering system, the quantity of merchandise ordered at specific time intervals varies, but the orders are placed at regular intervals. This system is also popular today and is used extensively where perpetual inventory records are kept. Under this system orders are placed once each ordering cycle for a quantity of goods which will bring the inventory level up to some specified amount. A serious defect of this system is the instability of stock levels which might occur because the person ordering overcompensates for changes in the inventory level from one reordering period to the next. For example, a temporary increase in sales may trigger an excessively large order, while a temporary lag in sales may cause too small a quantity to be ordered, thus creating large fluctuations in the level of inventory.

Each of these basic systems has advantages and disadvantages. Which system is best depends on the type of merchandise being handled. In some cases a combination of these two systems might be best. One example of a hybrid between these two is the base stock system. Under this system a review is made periodically but orders are placed only if levels of stock have fallen to or below some specified level.

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The fixed order and periodic order systems are useful devices for controlling inventory levels when the usage rate for a product is known. However, for items with a variant demand a serious problem arises in applying either of the systems: How much merchandise should be on hand when the order is placed so that the number of stockouts can be kept within reasonable bounds?

Scientific Inventory Control

The advant of scientific inventory control brought a change in emphasis to the solving of inventory problems. Under most of the old systems, the attempt was to minimize the size of inventories in one manner or another without attempting to minimize the total costs of carrying and ordering goods. The scientific approach to inventory management has developed mathematical models which attempt to determine optimal inventory levels and minimum total inventory costs by balancing the opposing costs associated with inventory.

1915 to 1945

The most basic model developed was that which determines economic ordering quantities or economic production runs, under the assumption that the demand for the item is known. This formula determines the optimum quantity to order, optimum quantity being that which will minimize the costs associated the inve

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with the inventory, and is nothing more than the application of differential calculus to the opposing cost figures. This formula and a general description for its application was first published in 1915 by F. W. Harris.[11]

A decade or so after the appearance of the article by Mr. Harris in 1915 a flurry of articles, similar to or expanding slightly the ideas expressed earlier, appeared in the literature.[12] Some writers have advanced the theory that the inventory liquidation of the 1920-1921 depression was a major stimulant to the derivation of economic lot size formulae during the 1920's.[13] Although this may well have been the source of the inspiration which spurred on the writers, it did not seem to affect the businessman much; very few applications of these formulae were made in business at that time.[14]

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[11]F. W. Harris, Operations and Cost (New York: 1915), p. 48-52. Quoted in F. E. Raymond, Quantity and Economy in Manufacture (New York: 1931), p. 121.

[12]R. C. Davis, "Methods of Finding Minimum-Cost Quantity in Manufacturing," Manufacturing Industries, IX, 1925, pp. 353-356; Gordon Pennington, "Simple Formulas to Inventory Control," Manufacturing Industries, XIII, No. 3, 1927, pp. 199-203; G. F. Mellen, "Practical Lot Quantity Formulas," Management and Administration, X, 1925, p. 155; R. H. Wilson and W. A. Mueller, "A New Method of Stock Control," Harvard Business Review, V, 1926-1927, pp. 197-205; Benjamin Cooper, "How to Determine Economic Manufacturing Quantities," Industrial Management, XXII, No. 4, 1926, pp. 228-233.

[13]Whitin, op.cit., p. 6.

[14]William H. White, Inventory Investment and the Rate of Interest (Washington, D. C.: The Brookings Institution, No. 57), p. 37.

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In the 1930's and 1940's statisticians, mathematicians, economists, and students of management became interested in the problems of inventory control[15] and incorporated advanced techniques of their respective disciplines into the simple models of the earlier writers. The addition of new techniques to the basic models permitted greater application of the basic formulas to business problems, although businessmen still did not make wide use of the improved formulas.

1946 to Present Day

After the end of World War II a renewed interest in inventory theory developed. New dynamic models were developed in contrast to the static models which predominated before the war. The calculus of variations, incremental analysis, analogues of servo-theory, statistical decision theory, stochastic process analysis and mathematical analysis[16] were applied to inventory problems. The new sophisticated models greatly increased the number of business inventory problems which could

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[15]Delbert J. Duncan, "The Control of Stock Shortages in Department Stores," Harvard Business Review, XVI, 1937-1938, pp. 201-210; H. P. Dutton, "Inventory Control," Factory Management and Maintenance, XCIII, No. 8, 1935, pp. s77-s92, and XCIII, No. 6, 1935, pp. s45-s60; Churchill Eisenhart, Some Inventory Problems, National Bureau of Standards, Techniques of Statistical Inference, A2.2c, Lecture 1, 1-1-48, Hectographed notes; A. J. Nichols, "Probability Analysis in the Theory of Demand, Net Revenue, and Price," Journal of Political Economy, XLIX, 1941, pp. 637-661; A. J. Nichols, "Production and Probabilities of Cost," Quarterly Journal of Economics, LVII (November, 1942), pp. 69-89.

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be solved. Areas of inventory problems which had heretofore been considered strictly qualitative were now quantifiable and could be handled on a scientific basis through the application of the later mathematical models.

Several factors account for the development and subsequent growth of the scientific theory of inventory control and the renewed interest in the field after the end of World War II.

The first is the increased size of business establishments which has occurred since the early 1900's. Many of these larger firms operate on smaller profit margins. Smaller profit margins make the firm's profitability picture more sensitive to any inefficiencies in inventory management. Also, the increase in the size of the firm itself makes possibilities of substantial savings through inventory control particularly important[17] because of the increase in the absolute dollar value of such savings.

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[16]H. A. Simon, "On the Application of Servomechanism Theory in the Study of Production Control," Econometrica, April, 1952, pp. 247-268; H. J. Vassian, "Application of Discrete Variable Servo Theory to Inventory Control," Journal of the American Statistical Association, August, 1955, pp. 272-282; R. Bellman, "Some Applications of the Theory of Dynamic Programming," Journal of the Operations Research Society of America, August, 1954, pp. 275-288; R. Schlaifer, Probability and Statistics for Business Decisions (New York: McGraw-Hill Book Co., 1959); M. A. Geisler and H. W. Karr, "A Fruitful Application of Static Marginal Analysis," Management Science, July, 1956, pp. 313-326.

[17]Whitin, op.cit., p. 5.

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[18]Ibid.

[19]Ibid.

Second, the past sixty years has seen increases in both the amount of business training and in its sophistication. Business colleges and schools offering commercial training have had phenomenal growth over the past six decades. Higher education has also been recognized as an important facet in enhancing the mobility of people through the organizational hierarchy. The emphasis on increased education for businessmen has produced a more sophisticated business group cognizant of the possibilities for improving business decisions. In this same period the trade journals which communicate business problems and solutions have increased their circulation greatly. This wider circulation of business acumen to people better equipped to understand the problems has led to faster acceptance of new ideas.[18]

The third factor has been the influx of people from the deductive disciplines into the area of business administration. The trained scientist, in approaching business problems, has used the "scientific approach" which has helped to clarify the individual parts of complex problems.[19]

The fourth influence has been the use of return on investment as a measure of business performance and the pressure for maintaining return on capital that has been seen

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[18]Ibid.

[19]Ibid.

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since about the early part of 1949. The trend toward heavy fixed investment to reduce direct-labor cost and pressure for stabilized employment of workers requires more careful planning by business firms. Inventory control is an important element of business planning for two reasons. First, inventory control plays an important role in the functioning of many firms because of its tie-in to stable production. Second, inventory control is important because capital invested in inventory is not available for use in other areas of the firm.[20]

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[20]McGee, op.cit., p. 2.

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CHAPTER III

INVENTORY CONTROL MODELS

The advocates of scientific inventory control have developed numerous models in order to handle the many inventory problems with which they are confronted. Many of the models developed for specific cases are, in reality, merely modifications of one of the several basic inventory models.

The chapter consists of three sections. The first consists of a brief review of the general role of models in business; the second is a discussion of the criteria for classifying basic models. The final section will discuss the basic inventory models.

Section 1: Models In Business

The last few years have seen the emergence of the term "model" in many texts and professional journals in the field of business administration. The concept of models, however, is not new to the area of business. Models have been widely applied to business problems by both practitioners and academicians for many years, although their use may not have been recognized as such.

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understand reality by eliminating some of the complex facets of reality. The model is nothing more than an abstraction of the existing world in which certain real world phenomena are manipulated. The model is actually a double abstraction; the model itself is an abstraction from reality and the model deals only with a few of the many details to be found in the real world situation.

Historically a model was a physical representation of an object. It was generally accepted that the model was related in scale to the object which was being studied and, therefore, the analysis of a model was the analysis of physical dimensions. It is this type of model which is described by Webster.

> a) a small copy or imitation of an existing object, as a ship, building, etc., made to scale, or b) a preliminary representation of something, serving as the form which the final, usually larger, object is to be constructed . . .[1]

The simple models which were limited to dimensions of physical characteristics limited severely the number of business problems which could be analyzed using models. As more sophisticated models became necessary to handle a greater variety of business situations, additional variables, such as rates of change in processes, were added. Soon the factors of time, profit, uncertainty and/or satisfaction were added

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[1]<u>Webster's New Collegiate Dictionary</u> (Springfield, Mass.: G. & C. Merriam Company).

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to the models and the number of business problems which could be handled increased greatly.

The addition of characteristics other than physical dimensions to represent real world phenomena changed the nature of models so that a broader and more useful definition of models than that provided by Webster was developed. The broader definition states:

> A model is a representation of reality that attempts to explain the behavior of some aspect of it. Since it is an explicit representation of reality it is always less complex than the reality itself, but it has to be sufficiently complex to approximate those aspects of reality which are being investigated.[2]

The increased use of models for solving business problems has occurred because of the recognition of the advantages which models possess. The main advantages are:

1) The use of models has abetted the developments of some of the most successful prediction systems yet evolved. The continued use of models is based on the general idea of using a proven method to obtain required results.

2) The model provides a frame of reference for consideration of a problem. Even if the preliminary model is not a satisfactory predictive device the model may still prove to be beneficial in that it may provide clues to the

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[2]David W. Miller and Martin K. Starr, Executive Decisions and Operations Research (Englewood Cliffs, N. J.: Prentice-Hall, 1960), p. 115.

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development of a model which will more successfully describe, explain or predict real world phenomena.

3) The building of models brings the problem of abstraction into the open. Real life situations are complex; in constructing a model only pertinent details need be studied. If some level of abstraction were not used, a decision on a complex situation would not be possible. Even non-predictive models may help the model builder by clarifying his thinking to determine what factors are important for a prediction system.

4) The model will, in most cases, be the cheapest method of prediction.

There are also disadvantages to the use of models for solving business problems. They are:

1) The models are subject to the usual danger inherent in abstraction--gross oversimplification. The investment of time and money in developing and using models does not guarantee success or promise that real world events will correspond to the results derived from the simplified abstraction.

2) Quantitative models are limited by the mathematicians' ability to manipulate data.

3) The model may become the end rather than the means toward an end because of the model builders' attraction to the model.

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Types of Models

Precise classification of models is difficult because of the many dimensions and distinguishing characteristics which might be used as a classification criterion. One possible method of classification is by the degree of abstraction.

Under this classification approach, the lowest degree of abstraction is found in the concrete or physical model defined by Webster. This type of model may also have other characteristics of the object it represents besides shape. A model airplane, if it were a flying model, would be an extension of the simple concrete model in that it has a functional characteristic as well as the physical characteristics of a real airplane.

The next level of abstraction would be that represented by the verbal models. This type of model is the most commonly used but is subject to communication difficulties arising from problems of semantics. The terminology used in verbal models means different things to different people. In verbal models words are used to denote a specific class of objects, activities, or qualities. However, when two or more things can be called by the same name, an adjective must be used to distinguish between them.

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purposes. Mathematics treats large classes of items in a completely abstract fashion by a symbol such as the letter "X" as compared to verbal models which denote a class of objects, activities, or qualities with a word or phrase that may have different connotations to different individuals.

Mathematical models possess certain advantages over other types of models.

The transition of a verbal model into a mathematical form permits greater clarification of existing relationships and interactions because the connotative element of words is eliminated.

Mathematical models permit greater ease of communication. The mathematical model reduces the terminology of many disciplines into a common language.

Mathematical models facilitate manipulation and therefore will handle problems not capable of being handled by other types of models. Mathematical models permit quantification of many factors and increase the objectivity of the model more than is possible with other types of models.

Although this thesis will be concerned primarily with mathematical models, it must be understood that all models are useful. Any statement as to which type of model is "best" is meaningless. The standard for evaluating models is their utility--how well do they describe, explain, or predict some

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part of reality or, at the very minimum, how well do they advance thinking in developing a model which will help to predict reality. This is the only criterion which can be used to judge the value of various models. The model should not be an end in itself; by itself the model has no intrinsic value.[3]

Section 2: Classification Of Inventory Models

The complexity of any inventory problem and, therefore, the complexity of the model necessary to solve that problem is dependent upon the number of unknowns involved. The basic model to be used for any given situation and any changes to modify the chosen model will be dictated by the characteristics of the specific problem. A major concern in applying models is to develop models to fit the situation and not vice-versa.[4] Modification of a basic inventory model to fit the situation will provide solutions to most inventory problems to which the businessman seeks answers today.

Inventory models can be broadly classified into two basic groups: static models and dynamic models. For purposes of this research static models are defined to be those models designed to handle inventory problems in which only

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[3] Irwin P. Bross, Design for Decision (New York: The Macmillan Company, 1953), p. 172.

[4] Edward H. Bowman and Robert B. Fetter, Analysis for Production Management (rev. ed.; Homewood, Ill.: Richard D. Irwin, Inc., 1961), p. 276.

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one order is possible in the time period being considered. Conversely, dynamic models are those designed to handle inventory problems in which multiple ordering is possible within the time period.[5]

The static model group includes those situations in which the length of time necessary for delivery is long relative to the sales period. A purchase of seasonal goods from a foreign supplier is one example of this type of static problem. Another would be the ordering of seasonal merchandise in which production process time is too long to permit any effective reordering. The more "typical" static inventory situations refer to products whose economic value declines rapidly after a short period of time. At the end of the economic life of the item in question the value of the item drops sharply to a salvage value. Dealers in Christmas

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[5]Martin K. Starr and David W. Miller, Inventory Control: Theory and Practice (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1962); Harold Bierman, Jr., Lawrence E. Fouraker and Robert K. Jaedicke, Quantitative Analysis for Business Decisions (Homewood, Ill.: Richard D. Irwin, Inc., 1961). As mentioned in the previous section of this chapter many methods of model classification are possible. The grouping used in this thesis is similar to that found in the above books. Other possible methods of classifying models may be found in: Edward H. Bowman and Robert B. Fetter, Analysis for Production Management (rev. ed.; Homewood, Ill.: Richard D. Irwin, Inc., 1961); Robert B. Fetter and Winston C. Dallick, Decision Models for Inventory Management (Homewood, Ill.: Richard D. Irwin, Inc., 1961); Thomson M. Whitin, The Theory of Inventory Management (Princeton, N. J.: Princeton University Press, 1957).

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[7] *Ibid*

trees, bread, milk and other highly perishable products are examples of "typical" static inventory situations. Although the absolute number of inventory problems which are represented by the static group is small, the analysis of this group is useful for developing models to handle the more complex dynamic inventory problems.[6]

Dynamic inventory problems, as contrasted to static situations, include all other inventory problems in which items not sold in earlier periods are carried over to the next period. The quantity not sold in the first decision period will affect the quantity ordered in later periods and must be taken into account if the costs of carrying inventory are to be minimized.[7]

Each of these broad groups of inventory problems can be further classified by the amount of knowledge of demand which is known. A possible subgrouping of the major categories could be (1) demand under conditions of certainty, (2) demand under conditions of risk, and (3) demand under conditions of uncertainty, with these subcategories defined as follows:

(1) Demand under conditions of certainty refers to situations in which the requirements for the product are known for the period of time being considered. Inventory

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[6]Starr and Miller, <u>Ibid</u>., p. 19.

[7]<u>Ibid</u>., p. 78.

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problems with demand under conditions of certainty represent a small number of actual business problems. This "certainty" model, however, is useful in situations where the demand is known and is also a useful tool for approximation of other demand situations. One example of a problem of certainty of demand is the situation in which a factory has continuous production of some product for a set period of time. In order to maintain continuous production, inventories of production line inputs must be maintained.

(2) Demand under conditions of risk pertains to situations in which a probabilistic demand pattern for the item is available. This probabilistic pattern shows the frequency with which one can expect the demand for the item to occur on some random basis.

(3) Demand under conditions of uncertainty refers to a complete lack of knowledge concerning the demand for the item being stocked.

The distinction between demand under conditions of risk and demand under conditions of uncertainty is not clearly delineated in the literature. Decision theory assumes that conditions under risk include only those problems in which a complete probability distribution of demand is known. Uncertainty, on the other hand, is considered to include those situations in which no knowledge of the demand for the item s available.[8] Econ

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[8] Ibid., p.

[9] F. H. Knig
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is available.[8] Economists, however, classify uncertainty as uninsurable situations while risk is thought of as those areas in which a random probabilistic pattern exists.[9]

The dichotomy between risk and uncertainty which decision theorists use is unrealistic for two reasons. First, knowledge of demand is not represented only by the two extremes of a known probability distribution and no knowledge at all. There exists between these two extremes a host of intermediary situations in which partial information about demand is known. If some information concerning demand for the item is available, it is possible to construct a probabilistic demand pattern for the item. If the theory behind the development of the demand function is sound, the use of the constructed demand distribution should result in solutions to problems which are better than results which are obtained using subjective probabilities.

The second reason for concluding that the decisions theorists' dichotomy between risk and uncertainty is fallacious is related to the nature of economic data. Economic

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[8] Ibid., p. 152.

[9] F. H. Knight, Risk, Uncertainty and Profit (London: London School Reprints of Scarce Works, No. 16, 1938), quoted by Mark Alfondry-Alexander, An Inquiry into Some Models of Inventory Systems (Pittsburgh: University of Pittsburgh Press, 1962), p. 1.

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data are a continuous phenomenon with seasonal and secular trends. If past information pertaining to an economic good or a closely related economic good is available, some probability distribution of the item in question could be developed. Very few items will be stocked in which no past information concerning their use is available.

This research, then, will assume that the static and dynamic inventory problems can be sub-classified into (1) demand under condition of certainty or (2) demand under conditions of risk. Certainty of demand will include those situations in which the demand for the item is known. Demand under conditions of risk will include those situations in which a random probabilistic pattern of expected demand is available. Those situations in which demand under conditions of absolute and complete uncertainty, as defined in decision theory, will be omitted.

Section 3: The Basic Inventory Models

Economic Order Quantities--Purchases

The dynamic inventory model under conditions of certainty is used to determine the most economical size of purchases to make when the demand, also called the usage rate, and the lead time are known with certainty. Lead time is the time which elapses between placing the order and receipt of the merchandise. With

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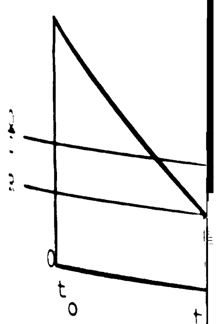
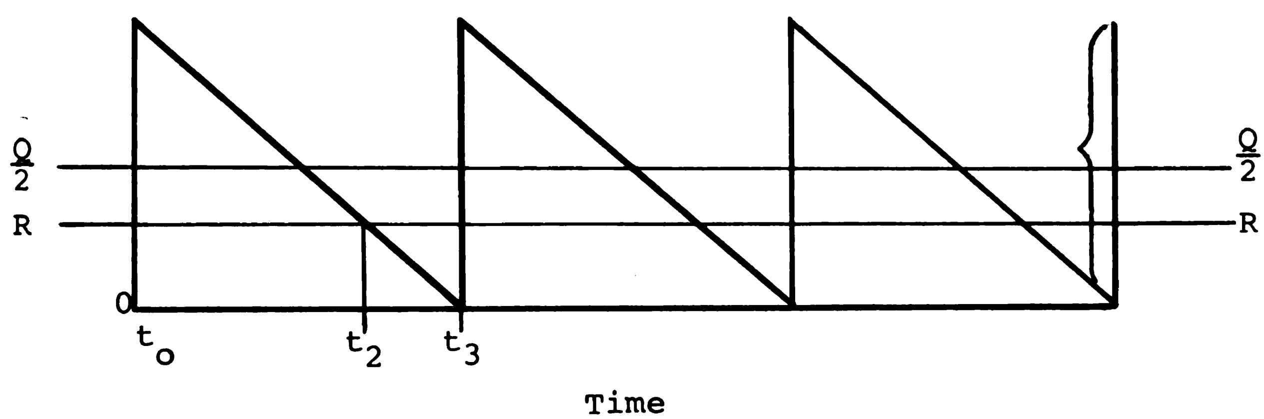


merchandise. With a known lead time and a known usage rate the reorder point, R, is also known. R, the reorder point, is the amount of inventory on hand when a new order is placed. The unknown in this situation is Q, the optimum order quantity, which is defined as that quantity which will minimize total inventory costs. Inventory behavior under conditions of certainty is portrayed graphically in Illustration 3-1.

ILLUSTRATION 3-1

INVENTORY BEHAVIOR WITH A KNOWN LEAD TIME
AND A KNOWN USAGE RATE



Q = quantity ordered

R = reorder point

t_2 to t_3 = lead time

RO = usage rate from time t_2 to t_3

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In the above situation a stockout is not possible because the lead time and the usage rate are known. The average inventory is equal to $\frac{Q}{2}$ or one-half the optimum order quantity. Determination of R, the reorder point, is straightforward. The reorder point is equal to the daily usage rate times the number of days in lead time. It is the quantity of inventory which should be on hand when the next order for merchandise is placed.

The model for determining Q in the above situation sets up total variable cost as a function of the opposing costs associated with carrying inventory. The opposing costs involved in this situation are the ordering costs and the carrying costs. These sets of costs must be balanced against one another in order to minimize the sum of the two cost groups.

With a known usage rate for the period two extreme ordering policies are possible. One extreme would be to place just one order for the total requirements of the period; the other extreme would be to place orders for one unit at a time as many times as necessary to acquire the quantity needed in the period. The first alternative would minimize ordering costs but would cause carrying costs to be at a maximum. The second alternative would result in exactly the reverse situation, minimum carrying costs but maximum ordering

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costs. Except in a very few cases the optimum order size will fall somewhere in between these two extremes.

Determination of the optimum order quantity and the number of orders to place in a given time period is readily determined by use of the basic calculus.[10] Set up total variable cost as a function of the opposing costs associated with carrying inventory.

$$T.V.C. = \frac{QC}{2} I + \frac{D}{Q} S$$

Where:

T.V.C. = total variable cost	D = annual demand
Q = quantity to order	C = unit cost
S = ordering cost per order	I = carrying cost per unit per year (as a % of cost)

Then $\frac{QC}{2}$ represents the average value of the inventory. $\frac{QC}{2} I$ is the annual carrying charge. $\frac{D}{Q}$ represents the number of orders placed per year. $\frac{D}{Q} S$ is the annual procurement cost.

Differentiating total variable cost with respect to Q, we get

$$\frac{\text{Change in cost}}{\text{Change in Q}} = \frac{d(T.V.C.)}{d(Q)} = \frac{IC}{2} - \frac{DS}{Q^2}$$

[10]The basic calculus must be used to determine the optimum quantity to order in the general case. When specific values of the various costs are known, it is possible to determine the optimum quantity to order by simple mathematical manipulation or by graphical interpretation. All three of these methods will be presented further along in the chapter.

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$$\frac{IC}{2} - \frac{DS}{Q^2} = 0$$

$$Q = \sqrt{\frac{2DS}{IC}}$$

Where Q represents the optimum quantity per order and $\frac{D}{Q}$ represents the number of orders per year which will minimize the costs associated with stocking merchandise.[12]

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[11]The slope of a curve at its minimum point must be zero. The first derivative of the curve gives the slope of the curve. Hence, by setting the first derivative equal to zero (finding the change in total variable cost for an infinitesimal change in the quantity) the value of Q can be found which satisfies the requirement that total variable cost is at a minimum.

[12]A similar model using total cost as the variable of interest would produce the same results. In this model total costs would be set up as a function of ordering costs, carrying costs and the cost of the items used in the period.

$$T.C. = \frac{QC}{2} I + \frac{D}{Q} S + CD$$

Where: T.C. = total cost; Q = quantity to order; S = order costs per order; D = yearly demand; C = unit cost; I = carrying cost per unit per time period as a % of cost. CD represents the cost of the items used per time period. Differentiating the total cost equation and setting the derivative equivalent to zero, we get

$$\frac{d(T.C.)}{d(Q)} = \frac{IC}{2} - \frac{DS}{Q^2} = 0$$

and

$$Q = \sqrt{\frac{2DS}{IC}}$$

This result is identical with that obtained when total variable cost is minimized. This occurs because the term CD is a constant (it is fixed for a given level of demand) and the first derivative of a constant is zero.

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Some of the assumptions underlying the model have already been mentioned. A complete list would include the following:

1) The demand or use rate is a known constant.

2) Procurement lead time is a known constant.

3) The cost price per unit is assumed to be constant. There are no provisions for quantity discounts in the model.[13]

4) Procurement costs vary directly in proportion to the number of orders placed--they have a linear relationship to the number of orders.

5) Carrying costs have a linear relationship to the value of the average inventory.[14]

A simple example will help to show how the optimum quantity to order can be derived by other methods as well as by the using of basic calculus. Assume the following:

Annual demand	100	(D = 100)
Per unit cost	\$3.00	(C = \$3.00)

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[13]A slight modification of the above formula is all that is necessary to develop a model which will include a continuous discount. The above model can also be modified to include a discontinuous discount. The modified model with the discontinuous discount, however, requires more mathematical manipulation to derive a solution than does the model for the continuous discount. The modified models are presented in Appendix One.

[14]Robert B. Fetter and Winston C. Dalleck, <u>Decision Models for Inventory Management</u> (Homewood, Ill.: Richard D. Irwin, Inc., 1961), p. 10; Clark W. Van Allan, Jr. and William E. Ritchie, "Economic Lot-Size and Inventory Control," <u>N.A.C.A. Bulletin</u>, XXXIV (February, 1953), p. 775.

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Ordering cost per order	\$1.00	(S = \$1.00)
Carrying costs per year	.05	(I = .05)

The costs associated with possible ordering quantities have been computed arithmetically in Table 3-1. Illustration 3-2 is a graphical representation of the variable cost data arrived at in Table 3-1. The graph of total variable costs shows that where the T.V.C. curve is at a minimum, it has a slope of zero (is parallel to the horizontal axis), beyond which point it begins to increase as the increasing carrying costs overpower the decreasing procurement costs. It follows that, in this case, the optimal order quantity is that at which procurement costs and carrying costs are equal.[15] The economic order quantity in this problem is thirty-seven. Use of the calculus method of solution results in a solution which is approximately the same.

$$\text{T.V.C.} = \frac{QC}{2} I + \frac{D}{Q} S$$

$$\text{T.V.C.} = \frac{Q3}{2} (.05) + \frac{100}{Q} (1)$$

$$\frac{d(\text{T.V.C.})}{d(Q)} = \frac{.05(3)}{2} - \frac{100(1)}{Q^2}$$

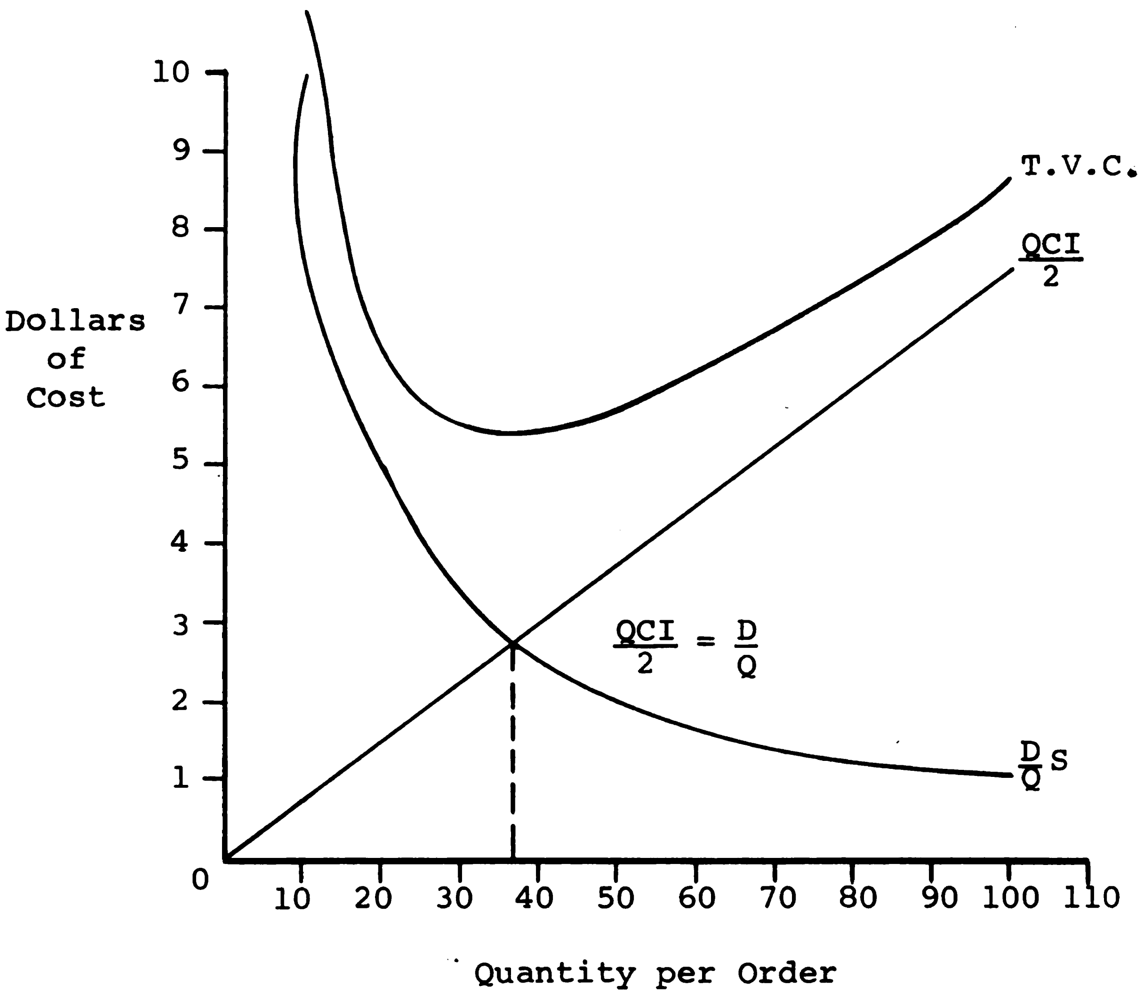
$$Q = \frac{2(100)(1)}{.05(3)} = \frac{200}{.15} =$$

$$Q = 1333.33 = 36.5$$

[15]The two costs are equal where they cross. This is the low point where total cost is at a minimum. This is, however, a special case. The low point on the sum line of a rectangular hyperbola and a straight line drawn through the origin is at the point where the latter two curves intersect. Other curves might not have this relationship.

ILLUSTRATION 3-2

GRAPHICAL ANALYSIS OF MINIMUM
TOTAL INVENTORY COSTS



TABL

(1) Order Quantity (Q)
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TABLE 3-1

TABLE OF EOQ COMPONENTS, TOTAL ANNUAL COST, AND TOTAL VARIABLE COST FOR VARIOUS ORDER QUANTITIES, GIVEN AN ANNUAL DEMAND OF 100 UNITS

(1) Order Quantity (Q)	(2) Cost Price x Demand (CD)	(3) Carrying Cost (QIC/2)	(4) Procure-ment Cost (DS/Q)	(5) Total Annual Cost (2+3+4)	(6) Total Variable Cost (3+4)
1	$300	$ .075	$100.00	$400.075	$100.075
2	300	.150	50.00	350.150	50.150
3	300	.225	33.33	333.555	33.555
4	300	.300	25.00	325.300	25.300
5	300	.375	20.00	320.375	20.375
6	300	.450	16.67	317.120	17.120
7	300	.525	14.29	314.815	14.815
8	300	.600	12.50	313.100	13.100
9	300	.675	11.11	311.785	11.785
10	300	.750	10.00	310.750	9.150
11	300	.825	9.09	309.915	9.150
12	300	.900	8.33	309.230	9.230
13	300	.975	7.69	308.665	8.665
14	300	1.050	7.14	308.190	8.190
15	300	1.125	6.66	307.785	7.785
16	300	1.200	6.25	307.450	7.450
17	300	1.275	5.88	307.155	7.155
18	300	1.350	5.56	306.910	6.910
19	300	1.425	5.26	306.685	6.685
20	300	1.500	5.00	306.500	6.500
21	300	1.575	4.76	306.335	6.335
22	300	1.650	4.55	306.200	6.200
23	300	1.725	4.35	306.075	6.075
24	300	1.800	4.17	305.970	5.970
25	300	1.875	4.00	305.875	5.875
26	300	1.950	3.85	305.800	5.800
27	300	2.025	3.70	305.725	5.725
28	300	2.100	3.57	305.670	5.670
29	300	2.175	3.45	305.625	5.625
30	300	2.250	3.33	305.580	5.580
31	300	2.325	3.23	305.555	5.555
32	300	2.400	3.13	305.530	5.530
33	300	2.475	3.03	305.505	5.505

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(1) Order Quantity (Q)	(2) Cost Price x Demand (CD)	(3) Carrying Cost (QIC/2)	(4) Procure-ment Cost (DS/Q)	(5) Total Annual Cost (2+3+4)	(6) Total Variable Cost (3+4)
34	$300	$2.550	$ 2.94	$305.490	$ 5.490
35	300	2.625	2.86	305.485	5.485
36	300	2.700	2.78	305.480	5.480
37	300	2.775	2.70	305.475	5.475
38	300	2.850	2.63	305.480	5.480
39	300	2.925	2.56	305.485	5.485
40	300	3.000	2.50	305.500	5.500
41	300	3.075	2.44	305.515	5.515
42	300	3.150	2.38	305.530	5.530
43	300	3.225	2.33	305.550	5.550
44	300	3.300	2.27	305.570	5.570
45	300	3.315	2.22	305.595	5.595
46	300	3.450	2.17	305.620	5.620
47	300	3.525	2.13	305.655	5.655
48	300	3.600	2.08	305.680	5.680
49	300	3.675	2.04	305.715	5.715
50	300	3.750	2.00	305.750	5.750
51	300	3.825	1.96	305.785	5.785
52	300	3.900	1.92	305.820	5.820
53	300	3.975	1.89	305.865	5.865
54	300	4.050	1.85	305.900	5.900
55	300	4.125	1.82	305.945	5.945
56	300	4.200	1.79	305.990	5.990
57	300	4.275	1.75	306.025	6.025
58	300	4.350	1.72	306.070	6.070
59	300	4.425	1.69	306.115	6.115
60	300	4.500	1.67	306.170	6.170
61	300	4.575	1.64	306.215	6.215
62	300	4.650	1.61	306.260	6.260
63	300	4.725	1.59	306.315	6.315
64	300	4.800	1.56	306.360	6.360
65	300	4.875	1.54	306.415	6.415
66	300	4.950	1.52	306.470	6.470
67	300	5.025	1.49	306.515	6.515
68	300	5.100	1.47	306.570	6.570
69	300	5.175	1.45	306.625	6.625

(1) Order Quantity (Q)
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Table 3-1 (con't)

(1) Order Quantity (Q)	(2) Cost Price x Demand (CD)	(3) Carrying Cost (QIC/2)	(4) Procure ment Cost (DS/Q)	(5) Total Annual Cost (2+3+4)	(6) Total Variable Cost (3+4)
70	300	5.250	1.44	306.690	6.690
71	300	5.325	1.41	306.735	6.735
72	300	5.400	1.39	306.790	6.790
73	300	5.475	1.37	306.845	6.845
74	300	5.550	1.35	306.900	6.900
75	300	5.625	1.33	306.955	6.955
76	300	5.700	1.32	307.020	7.020
77	300	5.775	1.30	307.075	7.075
78	300	5.850	1.28	307.130	7.130
79	300	5.925	1.27	307.195	7.195
80	300	6.000	1.25	307.250	7.250
81	300	6.075	1.23	307.305	7.305
82	300	6.150	1.22	307.370	7.370
83	300	6.225	1.20	307.425	7.425
84	300	6.300	1.19	307.490	7.490
85	300	6.375	1.18	307.555	7.555
86	300	6.450	1.16	307.610	7.610
87	300	6.525	1.15	307.675	7.675
88	300	6.600	1.14	307.740	7.740
89	300	6.675	1.12	307.795	7.795
90	300	6.750	1.11	307.860	7.860
91	300	6.825	1.10	307.925	7.925
92	300	6.900	1.09	307.990	7.990
93	300	6.975	1.08	308.055	8.055
94	300	7.050	1.06	308.110	8.110
95	300	7.125	1.05	308.175	8.175
96	300	7.200	1.04	308.240	8.240
97	300	7.275	1.03	308.305	8.305
98	300	7.350	1.02	308.370	8.370
99	300	7.475	1.01	308.485	8.485
100	300	7.500	1.00	308.500	8.500

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The reason for the slight difference is the indivisibility of the units which was implicitly assumed in developing Table 3-1. The calculus solution is the "mathematically correct" one but if the units are not perfectly divisible, the arithmetically determined solution is realistic.

Economic Order Quantities--Manufactured Lots

The model used to determine the optimum production (optimum size of the run) lot size in manufacturing processes under conditions of certainty is very similar to the model used for determining optimum size purchase quantities previously described. In fact, the model discussed above is a special case of the more general model developed by industrial engineers to determine the economic order quantity to produce. The model to determine the optimum lot size to produce balances set-up costs--the manufacturing counterpart of the procurement costs in the purchase lot size model--against the carrying costs associated with the inventory.

The production lot size model and the purchase lot size model are identical when it is assumed that production occurs instantaneously[16] or over a very short period of time relative to the usage rate. When the production rate is not

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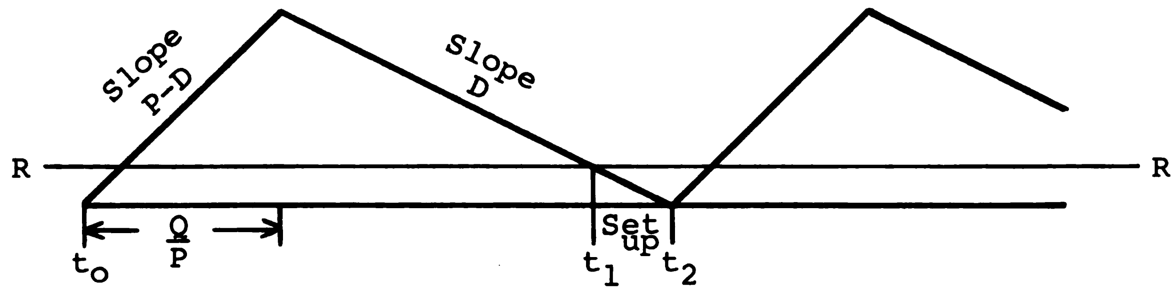
[16]Receipt of a shipment of purchased merchandise may be viewed as the equivalent of instantaneous production of such units in a manufacturing situation.

instantaneous or short relative to the usage rate, the inventory behavior under a production situation is quite different from that under a purchasing situation. Completed products enter inventory as they come from production at the production rate P. Inventory increases at the rate of (P-D), the production rate minus the usage rate when $P>D$, and reaches a maximum when the run is completed. At the completion of the production run the inventory is depleted at a rate of D. Set-up for the next run starts when the inventory reaches point R. After the period t_2-t_1 has elapsed the first item of product of the new run will enter inventory. At this point the inventory on hand will have been completely depleted. The behavior of inventory in this situation is portrayed graphically in Illustration 3-3.

ILLUSTRATION 3-3

INVENTORY BEHAVIOR OF A MANUFACTURED PRODUCT



P = the production rate
D = the demand rate
R = the start of set-up for next run

t_2-t_1 = set-up time

$\frac{Q}{P}$ = time of production run

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The relationship of the production rate to the usage rate has a definite effect on the optimum lot size to manufacture and on the maximum level of inventory. Average inventory varies with the length of the run (t). The inventory increases at the rate of (P-D). After t days, t being the length of the run in days, the inventory will reach a maximum value of (P-D)t. The average inventory is $\frac{(P-D)t}{2}$. The quantity produced, Q, is equal to the production rate times t, the length of the production run.

$$Q = Pt$$

or

$$\frac{Q}{P} = t$$

Average inventory

$$\frac{(P-D)t}{2} = \frac{(P-D)}{2} \cdot \frac{Q}{P} = \frac{Q(P-D)}{2P} = \frac{Q}{2} \cdot \frac{(P-D)}{P} = \frac{Q}{2} \left(1 - \frac{D}{P}\right)$$

As P changes in relation to D, demand for the item, the average inventory level changes. The average inventory under the assumption of instantaneous production (or purchase orders) is $\frac{Q}{2}$. The difference between the two average inventory formulas is $(1 - \frac{D}{P})$. As the production rate increases in relation to the usage rate the fraction $\frac{D}{P}$ becomes smaller. Hence, as P approaches infinity (instantaneous production) the value of $\frac{D}{P}$ approaches zero and the two average inventory formulas are almost identical. The effect of the production rate on the average inventory, $\frac{(P-D)t}{2}$, precludes the use of the special purchase-lot-si

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purchase-lot-size variation of the economic order quantity model for production problems because that model does not include P. However, with instantaneous production (or purchase orders) P does approach infinity and $(1 - \frac{D}{P})$ is almost equal to one.

Thus, the general formula for determining economic lot sizes may be stated as

$$T.V.C. = \frac{(P-D)Q}{2P} IC + \frac{D}{Q} S$$

Differentiating T.V.C. with respect to Q and setting the derivative equal to zero and solving for Q gives:

$$\frac{d(T.V.C.)}{d(Q)} = \frac{(P-D)}{2P} IC - \frac{DS}{Q^2} = 0$$

$$Q = \sqrt{\frac{2DS}{IC} \cdot \frac{P}{(P-D)}}$$

or

$$Q = \sqrt{\frac{2DS}{IC(1 - \frac{D}{P})}}$$

The basic assumptions for the general model are not changed, but must be modified to allow for possible changes in production techniques. The assumptions are:

1) The demand or use rate is a known constant.

2) The production rate is a known constant.

3) Production costs are constant:

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b) Manufacturing techniques remain the same no matter what the lot-size, throughout the range of lot-sizes studied.

c) Unit costs of input material are constant.

4) Carrying costs have a linear relationship to the value of the average inventory.[17]

A short example will help to illustrate the effect of the production rate on Q, the optimum quantity to produce, and also on $\frac{(P-D)t}{2}$, the average level of inventory. Assume the following:

Annual Demand is	365	(D = 365)
Per Unit cost is	\$3.00	(C = \$3.00)
Set-up costs per run	\$1.00	(S = \$1.00)
Carrying cost per year are	.20	(I = .20)

Under an assumption of instantaneous production (or purchase orders) the optimum lot size is

$$Q = \sqrt{\frac{2(365)(1)}{.6}} = 34.8 \text{ or } 35$$

When the length of the production run is short (the production rate is high), the same solution results. With a production rate of 365 units per day Q, for all practical purposes, is identical with the solution under the assumption of instantaneous production.

[17] Van Allan and Ritchie, op.cit., p. 775.

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$$Q = \sqrt{\frac{2(365)(1)}{.6(1-\frac{1}{365})}} = \sqrt{\frac{730}{.6(.9972)}} = 1221.75 = 34.8 \text{ or } 35$$

However, a different solution results when the length of time necessary to produce the item is long. If a production rate of 2 per day is assumed, the optimum lot-size run is:

$$Q = \sqrt{\frac{2(365)(1)}{.6(1 - \frac{1}{2})}} = \frac{730}{.6(.5)}$$

$$\frac{730}{.3} = 2433.37 = 49.3 \text{ or } 49$$

The average level of inventory is 17.5 units under the assumption of instantaneous production and 17.452 units when the rate of production is high but falls to 12.25 units when the production time for the item is long relative to the sales period.[18]

Models Which Incorporate Demand Under Risk

The assumptions of certainty of demand and lead time seriously restrict the applicability of the models previously

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[18]The average inventory figures were derived by substituting the various values assumed in each situation into the average inventory formulas, $\frac{(P-D)t}{2}$ or $\frac{Q}{2}$.

For purchase orders:
Average inventory is $\frac{Q}{2} = \frac{35}{2} = 17.5$ units
For a production rate equivalent to 365 per day:
Average inventory is $\frac{(P-D)t}{2} = \frac{365-1}{2}\ \frac{35}{365} = 17.452$ units
For a production rate equivalent to 2 per day:
Average inventory is $\frac{(P-D)t}{2} = \frac{(2-1)24.5}{2} = 12.25$ units

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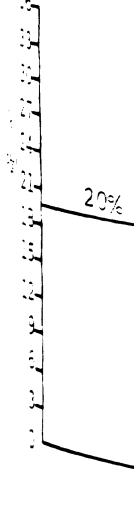
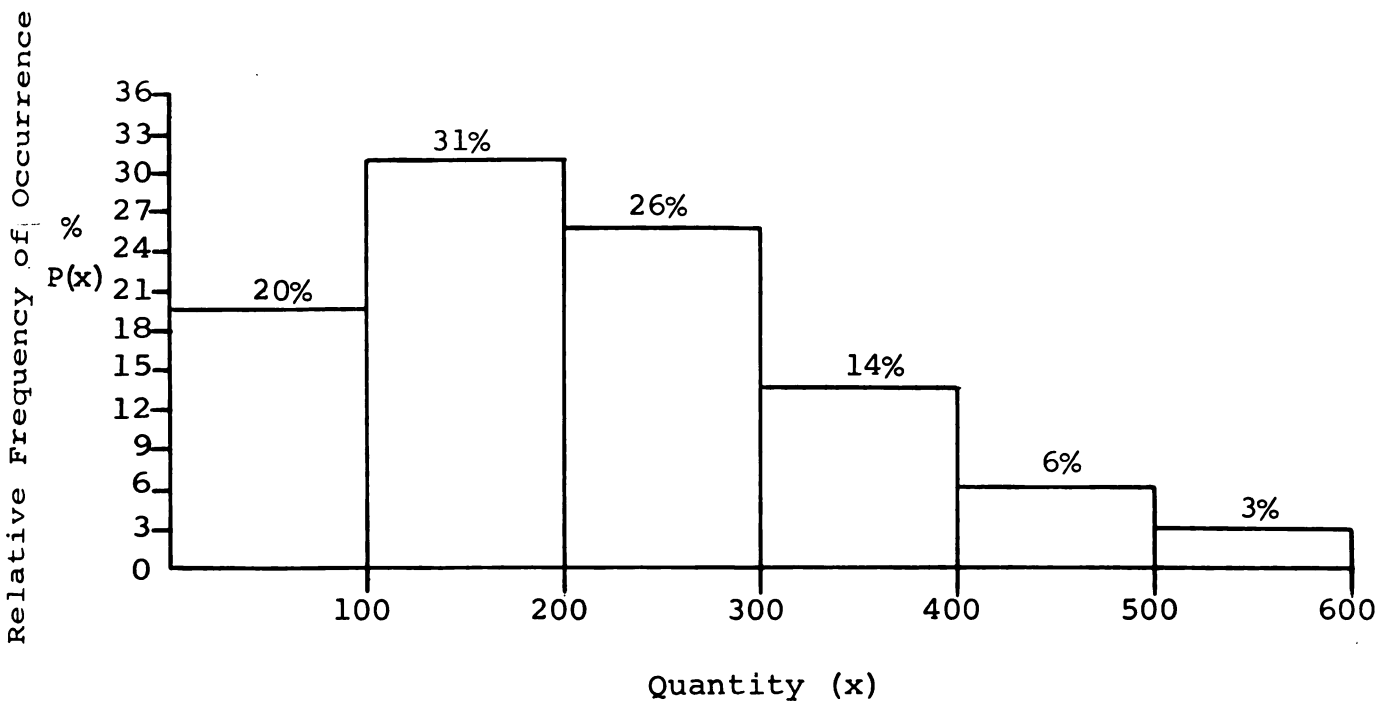


discussed. By including demand under conditions of risk in the model, a greater number of inventory problems can be analyzed.

Demand under conditions of risk, as previously defined, is the situation in which the demand variable employed in the model is <u>expected</u> demand determined on the basis of random probability applied to information, either complete or partial, derived from past behavior. The probabilistic pattern of demand is nothing more than a relative frequency distribution as shown in Illustration 3-4.

ILLUSTRATION 3-4

RELATIVE FREQUENCY DISTRIBUTION OF EXPECTED DEMAND



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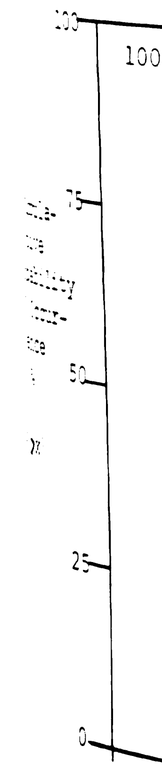
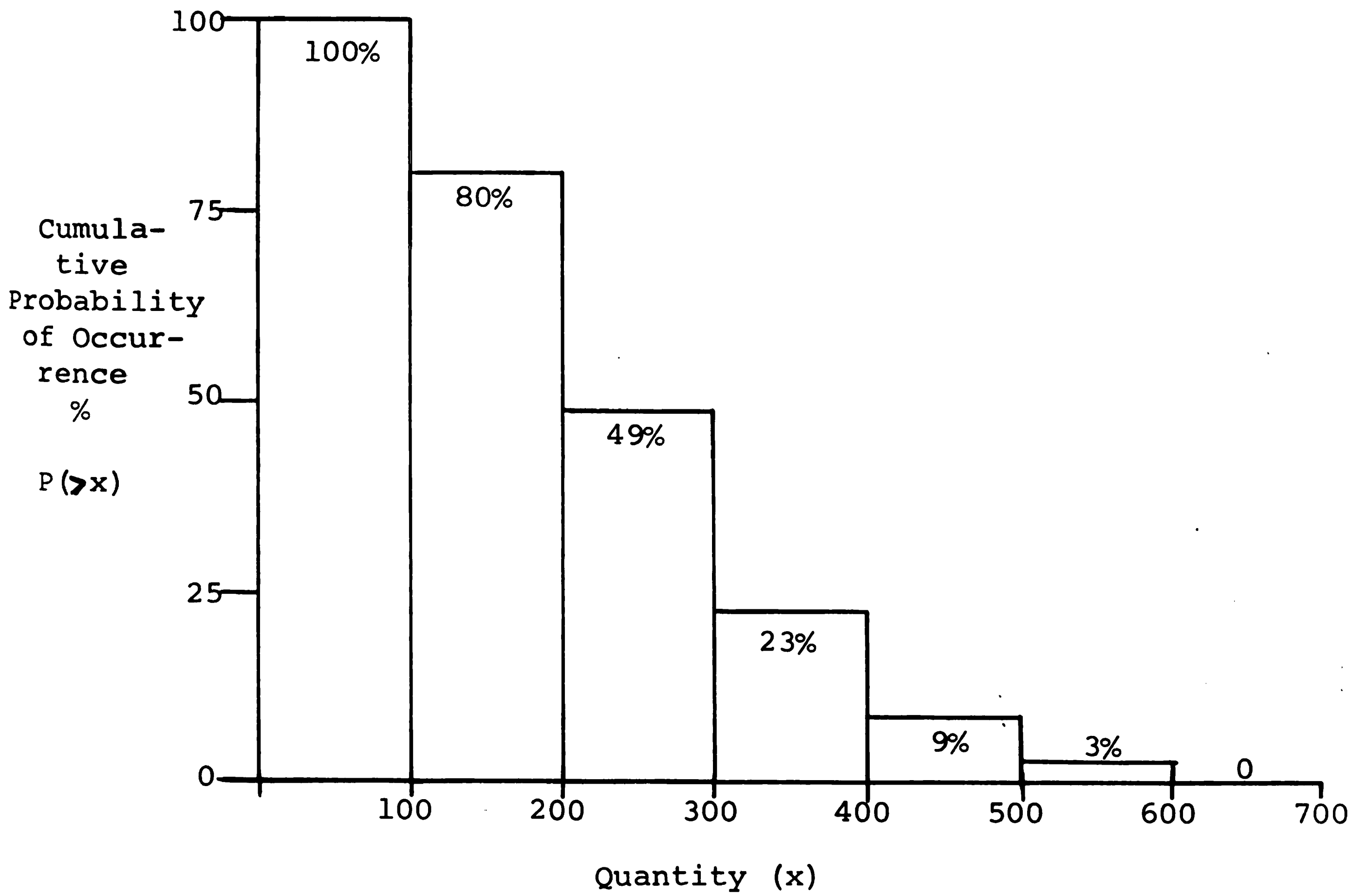


Illustration 3-4 shows that twenty percent of the time demand may be expected to be between zero and one hundred units, that thirty-one percent of the time the demand will range between one hundred and two hundred units, and so on. This probability distribution can also be presented as a cumulative distribution as in Illustration 3-5.

ILLUSTRATION 3-5

CUMULATIVE DISTRIBUTION OF EXPECTED DEMAND



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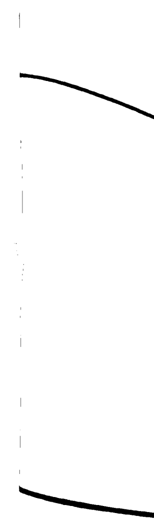

Static Models U

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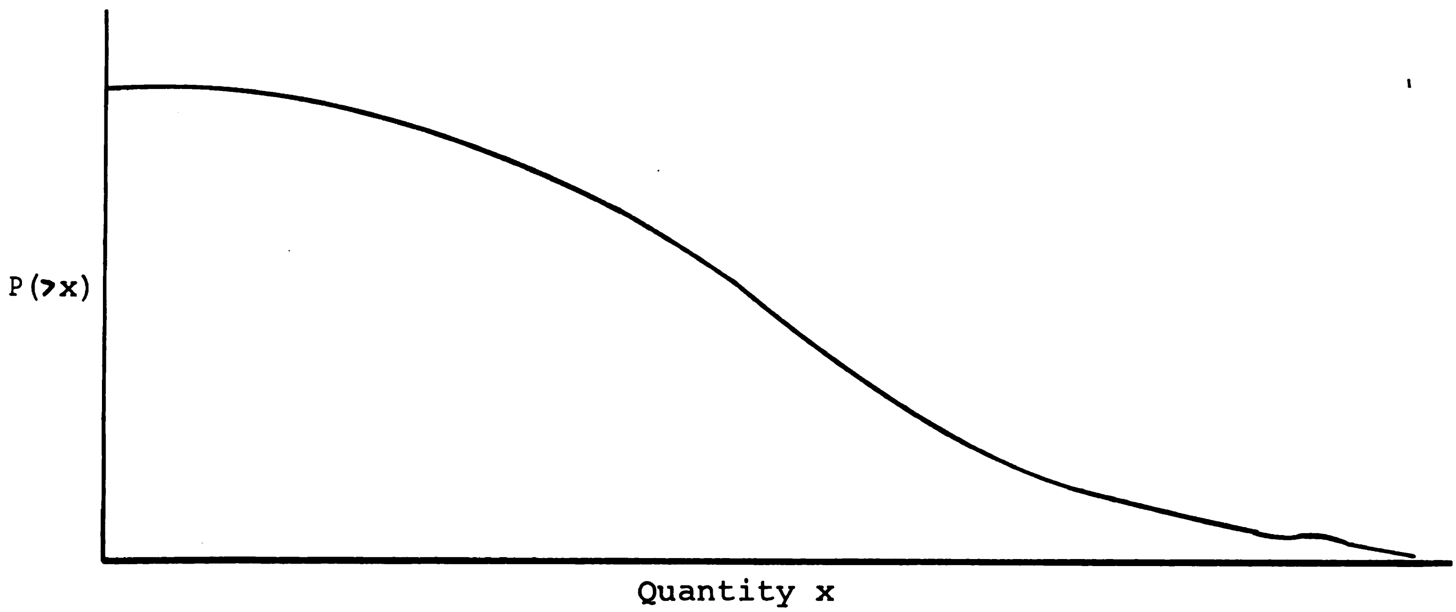
The cumulative distribution shows that the probability of selling between zero and one hundred units is one hundred percent; the probability of expected demand being between one hundred and two hundred units is eighty percent, and so on.

Illustration 3-6 presents the same information in another way. The stairstep pattern in Illustration 3-5 becomes a smooth curve as the intervals on the horizontal axis are made smaller and smaller until they approach zero.

ILLUSTRATION 3-6

CONTINUOUS CUMULATIVE PROBABILITY DISTRIBUTION



Static Models Under Risk

Static models are those models designed to handle inventory problems in which only one order is possible in the time period being cons

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period being considered. The static model under risk includes a varying demand pattern but does not allow for inventory carryover from one decision period to the next. Although the static model under risk has some limiting assumptions, its usefulness becomes obvious when combined with the dynamic model under certainty.

Given the distribution of expected demand, variable costs, and selling price of the item, optimum order quantity follows. Basically, the model optimizes expected net profit by determining the effect (on the expected net profit) of the expected marginal loss and expected marginal gain associated with each additional unit ordered.

Let:

p = the probability of selling at least one additional unit

(1-p) = the probability of not selling the additional unit or more[19]

MG = the incremental gain arising from the sale of each additional unit

ML = the incremental loss related to each additional unit bought but not sold

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[19]The sum of the probabilities of selling or not selling the unit must equal one. Hence, if p is the probability of selling the unit, then (1-p) must be the probability of not selling the unit.

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$p \cdot MG$ = the expected increase in profit or the expected marginal gain

$(1-p)ML$ = the expected decrease in profit or the expected marginal loss

Illustrations 3-5 and 3-6 show that there is a decreasing probability attached to selling each additional unit, given a constant selling price. Therefore, the value of p, the probability of selling each additional unit, decreases as the number of units ordered increases while (1-p), the probability of not selling the additional unit, increases as the size of the order increases. It follows that with a fixed selling price and constant costs, the expected marginal gain decreases while the expected marginal loss increases as the number of items ordered increases. Illustration 3-7 illustrates the relationship of expected marginal loss, expected marginal gain and expected net profit.

In Illustration 3-7 the optimum order size is X_o. Any other order size will result in lesser profits than will quantity X_o. Starting at a point to the left of X_o the expected gain from the additional unit ordered is greater than the expected loss; hence, ordering the additional unit will increase the net profits. Any units ordered to the right of X_o have an expected loss greater than the expected gain from he addit

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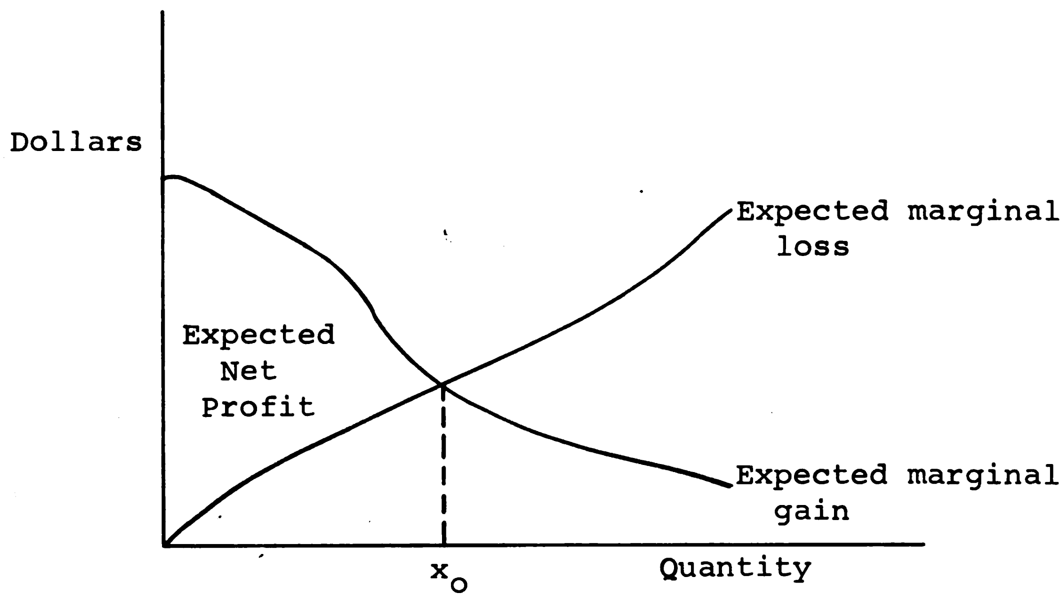
the additional unit and therefore reduce the net profits.[20]

ILLUSTRATION 3-7

EXPECTED MARGINAL GAIN, EXPECTED MARGINAL LOSS, AND EXPECTED NET PROFITS



The algebraic solution for optimizing net profit is:

Expected marginal loss = Expected marginal gain

or

$$p(MG) = (1-p)(ML)$$

$$p(MG) = (ML) - p(ML)$$

$$p(MG) + p(ML) = ML$$

$$p(MG + ML) = ML$$

$$p(x) = \frac{ML}{MG + ML}$$

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[20]Bierman, Fouraker, and Jaedicke, op.cit., p. 61.

where p(x) is the probability of selling the marginal unit at which the expected marginal loss equals the expected marginal gain[21] (the probability associated with selling the X_oth unit in Illustration 3-7).

A short example will help to illustrate the above model.

Given:

Sales price per unit	$7.50
Cost per unit	3.00
Carrying cost per unit	1.00
Scrap value of item not sold	.25

TABLE 3-2

EXPECTED DEMAND DISTRIBUTION

Demand	Probability of Selling	Cumulative Probability of Selling
0	.05	1.00
1	.15	.95
2	.20	.80
3	.40	.60
4	.15	.20
5	.05	.05

[21]A similar approach to determining the optimum quantity to order is found in Fetter and Dalleck, p. 12. Their model states that

$$c + p = r\left[P(x_i)\right]$$

$$\frac{c + p}{r} = P(x_i)^*$$

where c = cost of stocking the unit; p = price of the unit; $r\left[P(x_i)\right]$ = the expected profit on the i^{th} unit; $P(x_i)$ = the optimal probability in that the maximum profit occurs when the number of units corresponding to this cumulative demand probability is stocked. Either of these two models results in the same solution for any static inventory problem under risk.

Then

$$MG = \$7.50 - \$3.00 - \$1.00 = \$3.50$$

$$ML = \$3.00 + \$1.00 - \$\ .25 = \$3.75$$

$$p(x) = \frac{3.75}{3.50 + 3.75} = \frac{3.75}{7.25} = .517$$

Given the above conditions, the optimum size order is three units. (Actually, the optimum lies between three and four units, but because of the indivisibility of the units the optimal quantity in this situation is three units.)

The assumptions underlying this model are:

1) Constant selling price per unit.

2) Constant purchasing price per unit.

3) Constant carrying cost per unit.

4) Constant scrap value per unit.

5) No inventory carryover from one period to the next.

The Dynamic Model Under Risk

The number of real-world problems which can be solved by the dynamic model under certainty and the static model under risk is small. Most inventory problems are situations in which demand and lead time are not known with certainty and items not sold in one period are carried over to the next period. Hence, the restrictive assumptions of the previously discussed models exclude many actual business situations from those models. The dynamic model under risk removes the strictio

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restrictive assumptions of the dynamic model under certainty and the static model under risk and determines the economic lot size and reorder level which minimizes total variable cost for inventory problems with a varying usage rate during lead time and inventory carryover from one period to the next.[22]

The inclusion of varying usage rates during lead time complicates the analytical procedure for determining the optimum lot size to order or manufacture, but as pointed out by R. J. Oravec:

> . . . the existence of erratic period-to-period demand does not necessarily mean that this variation has no pattern which, when properly understood, can be usefully applied through recognized statistical procedures. Our multi-billion dollar insurance industry depends upon this fact.[23]

Including varying usage rates in the problem brings the possibility of stockouts into the analysis. The total variable cost of maintaining inventory now includes <u>three</u> components instead of two:

1) the annual carrying cost of inventory.

2) the annual ordering costs of inventory.

3) the annual cost associated with actual or potential shortages.

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[22]The dynamic model under risk is, in actuality, a hybrid of the dynamic model under certainty and the static model under risk.

[23]R. J. Oravec, "Statistical Inventory Management," <u>The Journal of Accountancy</u>, CX (December, 1956), p. 43.

One main difference between the dynamic model under certainty and the dynamic model under risk, then, "lies in the explicit recognition which is given to the cost implications occasioned by uncertainties of demand and replenishment lead times."[24]

Situations in which stockouts are possible require maintenance of a safety stock of inventory in order to limit stockouts to some acceptable level. (This assumes, of course, that stockouts represent a cost.) The safety stock is the quantity of inventory on hand when the new order is received. (Under conditions of certainty the inventory on hand when the new merchandise arrives is zero.) Maintenance of a safety stock increases the average inventory level and, therefore, the costs associated with carrying inventory. The average inventory is now $\frac{Q}{2}$ plus the average safety stock. Illustration 3-8 illustrates the inventory level under conditions of a constant use rate per day with a varying lead time. It is possible that usage during lead time might be great enough to completely deplete the safety stock. Although negative inventories do not have much meaning in many situations, they must be included in the analysis in order to determine the average safety stock because the carrying cost associated with the average inventory is a function of both $\frac{Q}{2}$ and the average safety stock.[25]

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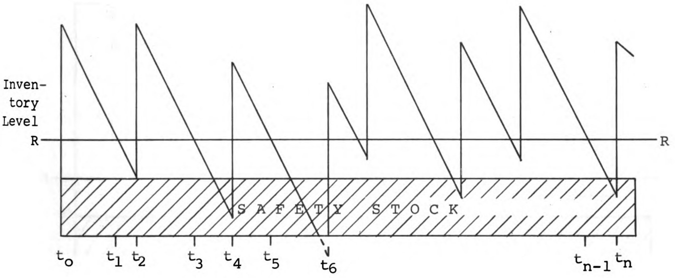
[24] _Ibid._, p. 45.

[25] See Appendix Two for the mathematical derivation of the average safety stock.

ILLUSTRATION 3-8

INVENTORY BEHAVIOR WITH A CONSTANT USE RATE AND VARYING LEAD TIME



R = reorder point
Q = economic lot size
$t_1 - t_2$, $t_3 - t_4$, ...$t_{n-1} - t_n$ = lead times of varying length

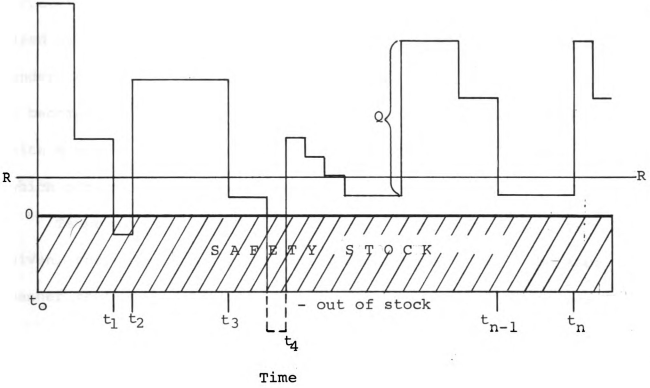
$\frac{Q}{2}$ + average safety stock = average inventory

Illustration 3-9 shows the inventory behavior in situations with both a varying lead time and a varying usage rate. Under these conditions the key variable in the determination of the safety stock is still the usage rate during lead time.[26]

[26]The dynamic model under risk includes those problems with a varying use rate, a varying lead time, or both. Given a relative frequency distribution of lead time and a relative frequency distribution of demand, a probability schedule of usage during lead time (the key variable) can be developed using probability theory. See Samuel Goldberg, Probability, An Introduction (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1960), p. 199.

ILLUSTRATION 3-9

INVENTORY BEHAVIOR WITH VARYING USAGE RATE AND VARYING LEAD TIME



R = reorder point
Q = economic lot size
$t_1 - t_2$, $t_3 - t_4$, ...$t_{n-1} - t_n$ = lead time of varying length

The dynamic model under certainty determines Q and R by finding the point at which ordering costs, $\frac{D}{Q}S$, and carrying cost, $\frac{ICQ}{2}$, are equal. Specifically, where Q is determined by:

$$\frac{ICQ}{2} = \frac{D}{Q}S$$

$$\frac{ICQ}{2} = \frac{2DS}{Q}$$

$$Q^2 = \frac{2DS}{IC}$$

$$Q = \sqrt{\frac{2DS}{IC}}$$

and R is calculated directly and independently by finding the product of the daily usage rate and the number of days in lead time. Basically, the same method is used to determine Q under conditions of risk.[27] Now, however, the computation of R becomes more complex inasmuch as variable cost associated with R must be included to find the optimum set of R and Q which minimizes total cost.

Temporarily making the simplifying assumption that Q is given, the determination of R is accomplished in much the same manner that Q is determined--by finding the R which minimizes total variable cost.

As R increases the average inventory increases. This occurs because the average safety stock (the inventory on hand when the order is received) is greater and, since carrying costs are a function of average inventory, it follows that carrying cost also increases as R increases. Conversely, as R increases the expected stockout cost--the cost represented by lost sales or slowdowns in the production process--is reduced because the probability of a stockout declines with an

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[27] See Appendix Two for the mathematical solution to dynamic inventory problems under risk.

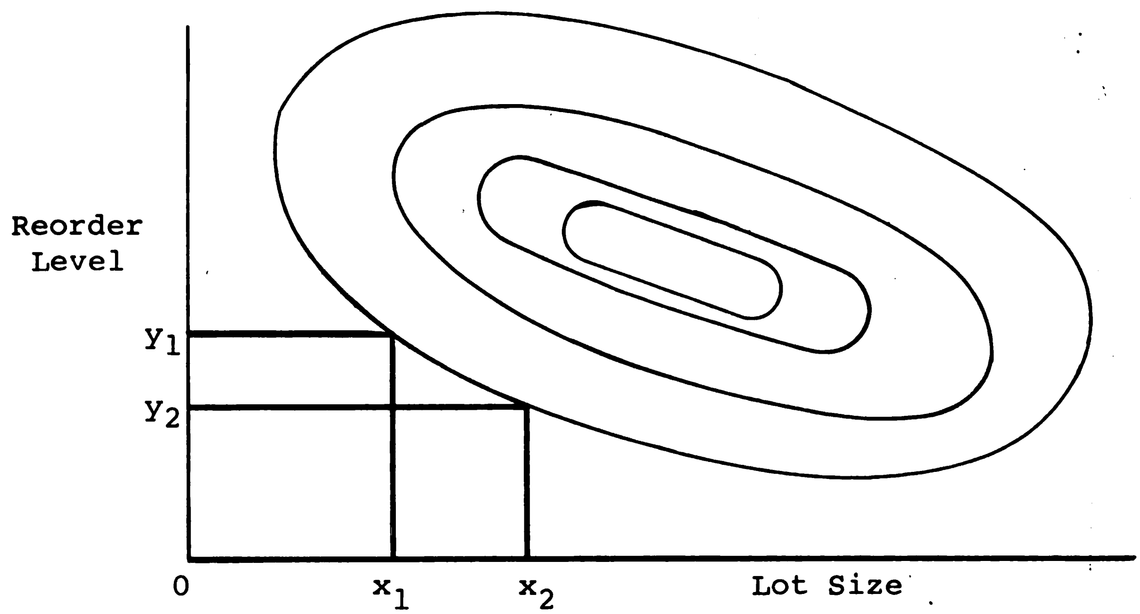
increase in safety stocks. At some R the increasing carrying cost equals the decreasing stockout cost. The optimum R is that quantity at which the carrying cost and stockout cost are equal.

However, it is now obvious that both the reorder point R and the economic order quantity Q affect the total variable cost. In addition, R and Q are interdependent: Q is dependent on R because it must provide for inventory during the reorder period; but R is related to Q because it is a function of the quantity ordered and usage. Hence, a series of approximations must be employed to find the unique set of R and Q which satisfy the requirement of minimizing total variable cost.

Because of the interdependence of R and Q, the dynamic model under risk represents a problem in three variables as contrasted to the two-variable problem of the dynamic inventory problem under certainty. One method of graphing the relationship between the total variable cost, the reorder point, and the economic lot size is to utilize a three dimensional figure. A simpler approach to expressing a three dimensional surface in a two dimensional figure is possible by use of contour lines or "isomers" employed by topographers. The contour line or isomer is a line on a plane surface joining all those points on the surface which have some quantity in common. The contour line approach to the three-variable dynamic model under risk is illustrated in Illustration 3-10.

ILLUSTRATION 3-10

THREE VARIABLE ISO-COST CURVES
TOTAL VARIABLE COST, LOT SIZE, AND REORDER POINT



Each curve in Illustration 3-10 represents a different total variable cost. Each of the iso-cost curves joins all the points on the surface which represent the same value of total variable cost. (Reorder point y_1 and order quantity x_1 will result in the same total variable cost as reorder point y_2 and order quantity x_2.) Each successive curve, reading from the origin up and to the right, represents a lower total variable cost.

The iso-cost curves which represent different total variable cost are identical to the iso-costs (or isoquants) used by economists to explain the principles which underlie costs, resource pricing and employment, resource allocation, and product distribution.[28] All iso-cost curves have the following characteristics:

1) Two iso-cost curves will not intersect.
2) For all rational combinations of R and Q the iso-cost slopes downward to the right.
3) Iso-cost curves are convex to the origin.[29]

Several methods[30] of determining R and Q for dynamic inventory problems under risk have been developed. One of these methods uses the iso-cost approach and chooses the value of R and Q which results in the minimum total variable cost from among the many possible combinations of R and Q.

Given alternative reorder levels, the mathematical expected value of stockout cost and the mathematical expected value of carrying cost of the safety stock are calculated for each given reorder level. The expected stockout cost is added to the ordering cost to determine the economic lot size appropriate to each reorder level. Then the expected carrying

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[28]Richard H. Leftwich, The Price System and Resource Allocation (New York: Rinehart and Company, Inc., 1955), p. 122.

[29]*Ibid.*, p. 124.

[30]See Appendix Two for the mathematical solution to dynamic inventory problems under risk.

cost for each safety stock level (which is a function of the reorder level) is added to the total variable cost for each combination of R and Q. The combination of R and Q which results in the minimum total variable cost is the optimum combination.[31]

A short example will help to illustrate the use of the reorder level-lot size method of solving the dynamic inventory problem under risk.

Assume that a firm has a continuous production process which will operate at capacity for the entire year. The major input to the production line is item Z which is purchased from a supplier in another part of the country. Relevant information concerning item Z is:

Cost per unit	$ 15.00	(C = $ 15.00)
Ordering cost per order	$ 10.00	(S = $ 10.00)
Stockout cost per day	$100.00	(O = $100.00)
Demand per year	73,000	(D = 73,000)
Carrying cost per year	.20	(I = .20)

The probability distribution of expected lead time is displayed in Table 3-3.

First, the expected stockout cost, O_E, and the expected carrying cost, I_C, associated with each possible reorder level

[31]Billy E. Goetz and Frederick B. Klein, Accounting in Action, Its Meaning for Management (Boston: Houghton Mifflin Company; Cambridge: The Riverside Press, 1960), p. 565.

TABLE 3-3

PROBABILITY DISTRIBUTION OF EXPECTED LEAD TIME

Days in Lead Time	Usage During Lead Time (Units)	Probability of Occurring
5	1000	.00
6	1200	.01
7	1400	.03
8	1600	.08
9	1800	.14
10	2000	.18
11	2200	.18
12	2400	.14
13	2600	.11
14	2800	.06
15	3000	.04
16	3200	.02
17	3400	.01
18	3600	.00

is calculated. These cost figures can be derived by use of a payoff matrix such as that in Table 3-4. The payoff matrix is an array in which the alternative reorder levels are represented by the rows; the possible lead times (the number of days it will take to receive the order) are represented by the columns. Each intersection of a row and column represents either the cost of carrying excess stock or the stockout cost associated with the combination of reorder level and possible lead time. The intersections of the rows and columns have the same number are zero because no stockout cost or excess carrying cost are incurred when the lead time equals the

TABLE 3-4

PAYOFF MATRIX

Probability of Lead Time Occurring	.01	.03	.08	.14	.18	.18	.14	.11	.06	.04	.02	.01
Lead Time	6	7	8	9	10	11	12	13	14	15	16	17
Reorder Level												
6	0	100	200	300	400	500	600	700	800	900	1000	1100
7	600	0	100	200	300	400	500	600	700	800	900	1000
8	1200	600	0	100	200	300	400	500	600	700	800	900
9	1800	1200	600	0	100	200	300	400	500	600	700	800
10	2400	1800	1200	600	0	100	200	300	400	500	600	700
11	3000	2400	1800	1200	600	0	100	200	300	400	500	600
12	3600	3000	2400	1800	1200	600	0	100	200	300	400	500
13	4200	3600	3000	2400	1800	1200	600	0	100	200	300	400
14	4800	4200	3600	3000	2400	1800	1200	600	0	100	200	300
15	5400	4800	4200	3600	3000	2400	1800	1200	600	0	100	200
16	6000	5400	4800	4200	3600	3000	2400	1800	1200	600	0	100
17	6600	6000	5400	4800	4200	3600	3000	2400	1800	1200	600	0

reorder level. Any row-column intersection to the right of the zero intersection represents stockout cost because actual lead time is greater than the lead time provided for by the reorder level. Conversely, any row-column intersection to the left of the zero intersection represents carrying cost because lead time is less than that lead time assumed in the reorder level.

In Table 3-4, the payoff matrix for the data in the given problem on page 70, the intersection of row eleven and column eight contains the value of eighteen hundred dollars. This intersection is to the left of the zero intersection in the row and, therefore, represents a carrying cost. If, on the average, an order is placed when eleven days' supply remains (2200 units), then the average inventory is six hundred units greater than necessary. Six hundred units at fifteen dollars each represent an investment in inventory of nine thousand dollars. With carrying cost of twenty percent a year the carrying cost of the excess stock, on the average, is eighteen hundred dollars (600 x $15.00 x .20 = $1800.00).

The values in the row-column intersections to the right of the zero intersections were calculated in a similar manner. The intersection of row ten, column fourteen, contains a figure of four hundred dollars. If, on the average, an order is placed when ten days' supply is left (2000 units)

and, on the average, delivery is not received until the fourteenth day, a shortage of four days' supply (800 units) results. Each day's stockout cost is one hundred dollars (or fifty cents a unit); four days of stockout result in stockout costs of four hundred dollars ($100.00 x 4 = $400.00).

It is important to realize that the carrying cost figures in Table 3-4 represent the annual cost associated with each reorder level-lead time intersection while the stockout cost of each reorder level-lead time intersection represents the stockout cost of only any one reorder period.

Table 3-5 is the expected value payoff matrix of the information shown in the payoff matrix (Table 3-4). Each value in the payoff matrix has been multiplied by the probability of its occurrence and entered in Table 3-5. For example, the expected value of row ten column twelve in the expected value payoff matrix is twenty-eight dollars. (Two hundred dollars, the value of the intersection of row ten and column twelve of Table 3-4, multiplied by .14, the probability of a lead time of fourteen days ($200.00 x .14 = $28.00). The sum of the expected cost for each reorder level (the rows in Tables 3-4 and 3-5 are shown at the sides of the expected value payoff matrix. The sum of all figures to the left of the zero in any row represents the expected carrying cost of each reorder level and the sum of the figures to the right of the zero in each

TABLE 3-5

EXPECTED VALUE PAYOFF MATRIX

Expected Carrying Cost	Lead Time / Reorder Level	6	7	8	9	10	11	12	13	14	15	16	17	Expected Stockout Cost
0	6	0	3	16	42	72	90	84	77	48	36	20	11	499
6	7	6	0	8	28	54	72	70	66	42	32	18	10	400
30	8	12	18	0	14	36	54	56	55	36	28	16	9	304
102	9	18	36	48	0	18	36	42	44	30	24	14	8	216
258	10	24	54	96	84	0	18	28	33	24	20	12	7	142
522	11	30	72	144	168	108	0	14	22	18	16	10	6	86
894	12	36	90	192	252	216	108	0	11	12	12	8	5	48
1350	13	42	108	240	336	324	216	84	0	6	8	6	4	24
1872	14	48	126	288	420	432	324	168	66	0	4	4	3	11
2430	15	54	144	336	504	540	432	252	132	36	0	2	2	4
3012	16	60	162	384	588	648	540	336	198	72	24	0	1	1
3606	17	66	180	432	672	756	648	420	264	108	48	12	0	0

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row represents the expected stockout cost for each possible reorder level.

As was pointed out above, the expected carrying costs shown in the intersections of the matrix represent the annual cost of carrying the safety stock for each reorder level. The expected stockout costs represent only the cost of any one reorder period. It follows that this is also true of the sum of the expected carrying cost and of the sum of the expected stockout cost. In order to get the annual expected stockout cost for each reorder level the expected value of stockout cost of one reorder period (the sum of the values to the right of the zero intersection of each row) must be multiplied by the number of times a year that the stockout cost can occur (the number of reorder periods) or $\frac{D}{Q}$.

Second, the economic order quantity associated with each possible reorder level is computed on the basis of three elements of variable cost: annual carrying cost of average inventory, annual ordering cost, and <u>annual</u> <u>expected</u> stockout cost.

$$\text{T.V.C.} = \frac{ICQ}{2} + \frac{D}{Q} S + \frac{D}{Q} O_E$$

$$\text{T.V.C.} = \frac{ICQ}{2} + \frac{D}{Q} (S + O_E)$$

where

T.V.C. = total variable cost

$\frac{ICQ}{2}$ = annual carrying cost of average inventory

$\frac{D}{Q} S$ = annual ordering cost

$\frac{D}{Q} O_E$ = annual expected stockout cost

Total variable cost is minimum where

$$\frac{ICQ}{2} = \frac{D}{Q} (S + O_E)$$

and

$$Q = \sqrt{\frac{2D(S + O_E)}{IC}}$$

(When $\frac{ICQ}{2}$ equals $\frac{D}{Q} (S + O_E)$ the slopes of the rising carrying cost and the falling ordering cost-expected stockout cost are equal. This is the minimum point of total variable cost.) For example, the economic order quantity associated with reorder level ten is

$$Q = \sqrt{\frac{2(73,000)(10 + 142)}{3}} = \sqrt{7,730,666} = 2780 \text{ units}$$

Third, compute the variable cost for the economic order quantity associated with each reorder level using the three elements of total variable cost used to determine Q in step two.

$$V.C. = \frac{ICQ}{2} + \frac{D}{Q} (S + O_E)$$

The variable cost for R of ten and Q of 2780 is

$$\frac{(.20)(15)(2780)}{2} + \frac{73,000(10+142)}{2780} = \$4,170.00 + \$3,991.37 = 8,161.37$$

Add to the variable cost calculated above the expected carrying cost associated with each reorder level. Thus the total variable cost for each reorder level-lot size combination is

$$T.V.C. = \frac{ICQ}{2} + \frac{D}{Q}(S + O_E) + I_E$$

where I_E is the expected carrying cost of each reorder level. The total variable cost for R of ten and Q of 2780 is

$4,170.00 + $3,991.37 + $258.00 = $8,419.37

Finally, the reorder level-lot size combination that results in the minimum total variable cost is chosen from among the possible combinations.

Table 3-6 shows the Q for each reorder level, the annual carrying cost of average inventory for each reorder level-lot size combination, the annual ordering cost-expected stockout cost for each reorder level-lot size combination, the annual expected carrying cost of safety stock of each reorder level, and the sum of these costs (total variable cost) for each reorder level-lot size combination.

It can be reaily seen from Table 3-6 that the R and Q which results in the minimum total variable cost for the example is: R = 14 days; Q = 1011 units.

TABLE 3-6

REORDER LEVEL, LOT SIZE, ANNUAL CARRYING COST, ANNUAL ORDERING-EXPECTED STOCKOUT COST, ANNUAL EXPECTED CARRYING COST, AND TOTAL VARIABLE COST

R	Q	$\frac{ICQ}{2}$	$\frac{D}{Q}(S+O_E)$	I_E	Total Variable Cost
6	4977	7,465.50	7,465.74	0	14,931.24
7	4467	6,700.50	6,700.25	6.00	13,406.75
8	3909	5,863.50	5,863.90	30.00	11,757.40
9	3316	4,974.50	4,975.27	102.00	10,051.27
10	2780	4,170.00	3,991.37	258.00	8,419.37
11	2161	3,241.50	3,242.94	522.00	7,006.44
12	1649	2,473.50	2,567.62	894.00	5,935.12
13	1286	1,929.00	1,930.02	1,350.00	5,209.02
14	1011	1,516.50	1,516.32	1,872.00	4,904.82
15	825	1,231.50	1,238.79	2,430.00	4,906.29
16	732	1,098.00	1,096.99	3,012.00	5,206.99
17	698	1,047.00	1,045.85	3,606.00	5,698.85

CHAPTER IV

THE INPUT DATA

The basic inventory models discussed in Chapter III are capable of handling inventory problems having varying lead time or varying demand or both, as well as accounting for inventory shortages or carryover from one decision period to the next. Slight modifications of these basic models to account for special characteristics of specific inventory situations increases greatly the range of problems for which the models are useful.

However, the realization of the full predictive potential inherent in the models depends on the quality of the input data. As was stated earlier, wide disagreement exists concerning some of the relevant costs to be used in the models. Basically, the models used in scientific inventory theory represent closed systems. The end result obtained from a closed system (in this case, the economic order quantity and reorder level) is determined directly by the values of the inputs into the system. Hence, the "optimum" inventory level of scientific management, like the end result of any closed system, is always relative. It is the "optimum" given the values of the various inputs.

This chapter is concerned with the relevant inputs to be used in the models discussed in Chapter III. Although certain fundamental concepts concerning the input data remain unsolved,[1] some other factors such as the definitions of the relevant costs, the marginal nature of the relevant costs, and the sources of the relevant cost figures related to the inventory model are generally agreed upon. The prime areas of disagreement lie in the determination of the values assignable to opportunity cost inputs--stockout cost and the imputed interest portion of carrying cost--which are necessary for effective operation of the models.

The areas of agreement will be covered first, followed by a discussion of the opportunity costs. The major emphasis of the discussion of the opportunity costs will be upon determination of the rate to use in computing the capital charge item in the carrying cost of inventory. The discussion of the interest rate will consist of three parts: first, a short review of present-day capital management theory; second, a review of the interest rates suggested by the literature with a classification of the rates into six categories; and third, an evaluation of each of the categories in terms of capital

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[1]Thomson M. Whitin, The Theory of Inventory Management (Princeton, N. J.: Princeton University Press, 1957), p. 239.

management theory.

Areas of Agreement

Definition of the Relevant Costs

First, agreement exists as to the general categories of costs which are relevant. Total inventory cost is the sum of three categories of costs: 1) the cost of acquiring inventory, 2) the cost of carrying inventory, and 3) stockout cost--the cost of demand lost or deferred owing to inventory shortage of finished goods and/or the cost of interrupted production due to raw material shortages.

The cost of acquiring inventory is the sum of the ordering cost, planning cost, and sometimes material handling cost of purchased items. For production problems it is the sum of the setup cost, planning cost and sometimes material handling cost. The cost of acquiring purchased inventory items or production setup cost is a fixed cost per lot of items which means that the average cost for each item in a lot decreases as the lot size increases.

The major components of carrying cost are imputed interest, personal property tax, insurance on the inventory, shelf wear, obsolescence, space costs and certain labor charges. Each of these individual costs vary in total with the size of investment in inventory; hence, the total carrying cost will also vary with the size of the investment in inventory.

The final cost, stockout cost, has two aspects. The lost sales or deferred sales cost is the loss of profit plus the loss of the contribution to overhead on the lost sale plus the loss of customer goodwill in the use of finished goods. The stockout costs for raw materials is the additional costs associated with slowdowns or stoppage of the production process. This cost can be either fixed or variable per unit and, therefore, the total stockout cost increases at a decreasing, increasing, or constant rate as the order lot size increases.

Marginal Nature of the Relevant Costs

The second factor which is generally agreed upon is that the costs to be used in the analysis of inventory problems are not exclusively the costs associated with inventory as determined by accountants. The difference between accounting costs and decision-making costs can be attributed to the basic objectives in developing each of these sets of costs.

> . . . the basic objective of accounting procedures is to provide a fair, consistent, and conservative valuation of assets and a flow of values in the business. . . . Contrasted with the principles and search for consistency underlying accounting costs, the definition of costs for production and inventory control may vary from time to time, depending on circumstances and on the length of time being planned.[2]

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[2]John F. Magee, Production Planning and Inventory Control (New York: McGraw-Hill Book Co., Inc., 1958), p. 25.

The relevant costs for decision-making purposes are based on the economists' familiar marginal cost concept. It is the extra cost associated with different inventory levels rather than the over-all average cost associated with different inventory levels that is relevant. Any fixed costs for a specific decision problem are sunk costs for that decision and can be ignored. One author, in referring to the relevant costs, said:

> 1) The costs used shall represent only those out-of-pocket expenditures or foregone opportunities for profit, and
> 2) The costs shall represent only those out-of-pocket expenditures or foregone opportunities for profit whose magnitudes is affected by the plan.[3]

The classification of costs into fixed and variable is dependent upon the period of time under consideration: In the short-run all costs are fixed; in the long-run all costs are variable. In classifying costs into fixed and variable for any one decision-making period, the analysis should consider the various effects upon the solution if the time period involved were different. The short-run analysis which is attempting to minimize costs of inventory should be consistent with the long-run objectives of inventory control. It is possible to have "optimum" results in the short-

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[3]*Ibid*., p. 27.

run which will not coincide with the long-run objectives of the firm.[4]

Sources of the Relevant Costs

The final area in which there is general agreement is the source of various cost figures. Some of the individual components of total cost used in the models may found within the normal accounting framework. On the other hand, some of the costs are strictly opportunity costs not normally found in the accounting records.

The costs which could be determined or at least approximated from the accounting records include: ordering costs, planning costs, personal property taxes, insurance on the inventory, shelf wear, obsolescence, space costs, and certain labor charges.

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[4]For example: storage costs associated with inventory can be fixed or variable for the decision period under consideration. If the storage space represents a fixed cost in the short-run, then it will not be included as a carrying cost. The storage cost could be fixed in the short-run because the building is owned or leased and no alternative uses of the storage space are available. This cost could, however, be variable over two decision periods because the building may be sold, the lease expires or other uses of the space are available. This change in cost from fixed to variable will require a change in the carrying costs and will result in different "optimum" inventory levels. If the time period under consideration is not included in the analysis, the long-run goal of profit maximization of the present stockholders will not be achieved.

The ordering costs, planning costs and other individual costs attributable to inventory will not, in most cases, be isolated in the accounting records in a single account. The accounting records classify the various expenses of the firm on a functional basis. For example, labor costs, insurance costs, taxes, etc., are allocated to cost of goods sold, selling expense, and administrative expense with each item shown as a subtotal within the major cost divisions. For inventory decision-making the portions of these account totals germane to the problem at hand must be ferreted out. Although many difficulties are involved in segregating the critical cost portions from the totals in the accounts, the important point is that these costs are, for the most part, available within the regular accounting framework.

Area Of Disagreement

The area of disagreement is concerned with the opportunity costs. The opportunity cost group includes the stockout cost and the imputed interest component of carrying cost, both of which are essential for obtaining meaningful solutions from the models.

Stockout Cost

Determination of the magnitude of stockout cost is extremely difficult because part of this cost is developed from information based on human behavior and is, therefore, highly conjectural in nature. The stockout cost for finished goods must reflect the attitudes and opinions of customers who cannot be served immediately (or within the time period expected for the product in question) as well as the loss of profit plus the loss of the contribution to overhead on the lost or deferred sale. Several questions pertaining to customer behavior must be evaluated? Will customers wait for the product in question or will they go elsewhere? If they go elsewhere now, will they return at another time? The answers to the above questions will depend on many factors such as: the availability of substitutes, the strength of brand image, the structure of the industry, the geographical location of the firm's outlets, to mention just a few of the possible influencing factors.

The determination of stockout cost from slowdowns in the production process does not represent as formidable a problem as stockout cost from lost or deferred sales of finished goods. The stockout costs from raw material shortages are, primarily, the unavoidable cost associated with a slowdown or stoppage in the work process. While these are by nature opportunity costs, the cost accounting records provide helpful data for meaningful measurement of these costs.

There are, however, some cost aspects of production slowdown due to stockouts which are as difficult to estimate as is the value of lost sales. For example, it is possible that layoffs and erratic work patterns induced by material shortages may result in workers seeking other employment and/or union demands for stabilized employment and guaranteed wages.

In addition, since every manufacturing firm also must sell its product as a finished good, those situations where production is closely geared to delivery of the final product, production slowdowns also give rise to lost sales costs.

The Method of Handling Stockout Cost

The problem of determining lost sales or deferred sales cost is widely recognized by people involved in scientific inventory control. Although precise development of this cost is impossible, a roundabout method of estimating this item has been developed.

Under this approach the cost associated with lost sales is equated to the estimated marginal cost of providing higher levels of customer service.

An example of that approach is used by John Magee and is as follows: In order to meet ninety percent of customer demand, about $67,000 of merchandise with an annual carrying cost of $13,500 must be held. To increase service to meet ninety-five percent of customer demand would increase carrying cost by about $1300, while to attain a ninety-nine percent customer service level would increase carrying cost an additional $2700. Given the estimates as to the cost of customer service, management must then make a decision as to the level of customer service which is desired.[5]

The above method of handling stockout cost obviously approaches but does not compute the real loss or real value associated with a stockout. It fails to measure the impact on present and future sales which result from a stockout. The area of determining the loss associated with stockouts requires much more work and is a fruitful area for future research.

The Imputed Interest Rate

The second major opportunity cost which must be determined for use in the models is the interest rate to be imputed against capital invested in inventory. It is generally agreed that this is a relevant cost because the use of capital in inventory precludes the application of that capital to other uses, but the question of what is the "correct" rate is still subject to dispute.

Since investment in inventory is really investment of capital in a business asset, it seems logical to assume that

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[5]Magee, op.cit., p. 74.

the desirability of such investment should be measured in terms of whether or not it is consistent with prudent capital management. For some reason many businessmen and some scientific inventory control personnel look upon inventory as a short-term investment which is relatively liquid. But is inventory really liquid? Much inventory carried in business is as permanent an investment as are machinery and buildings. A base stock of inventory must be maintained as long as the business is in operation. The base stock inventory has been called a "permanent current asset"[6] and can be liquidated only when the company is liquidated.[7] Thus, decisions pertaining to investment in inventory should be made in a manner consistent with good capital management.

A Review of Capital Management Theory

Capital management is an important facet of top management concern in the business world. It consists of three

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[6]Robert W. Johnson, Financial Management (Rev. ed.; Boston: Allyn and Bacon, Inc., 1962), p. 141.

[7]The base stock inventory or the permanent current asset concept can be applied even to merchandise of a highly seasonal nature if the base stock is defined as the minimum inventory required for a given level of demand in a given time period. As stated earlier modification of the basic inventory models takes into account such things as seasonal fluctuations. For a detailed inventory model which handles seasonal demand, see Mark B. Schupack, "Economic Lot Sizes with Seasonal Demand," Operations Research, VII (January-February, 1959), pp. 45-57.

decision areas: 1) the determination of the total amount of capital to be invested in the business, 2) the distribution of the total invested capital among various assets, and 3) the capital structure to be employed in financing the investments in total assets.

Although in the past many principles have been applied as bases for decisions relating to the management of business capital, there has been developed in recent years a "formulation of an explicit and defensible approach to capital allocation decisions . . . ."[8]

The present-day approach to capital allocation decisions is, basically, the economist's familiar marginal cost-marginal revenue approach to maximizing profits. It consists of three steps:

1) The firm's potential capital expenditures must be examined to determine the prospective rate of return from each project,

2) The cost of available sources of capital must be measured to determine the firm's supply schedule of funds, and

3) A comparison of the schedules developed in steps one and two above must be made to insure that each investment undertaken returns an amount greater than the cost of the funds invested in that project.[9]

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[8]Ezra Solomon, "Introduction," The Management of Corporate Capital, ed. Ezra Solomon (Glencoe, Ill.: Free Press of Glencoe,), p. 13.

[9]Ezra Solomon, "Measuring a Company's Cost of Capital," Journal of Business, XXVIII, No. 4 (October, 1955), p. 240.

The Underlying Assumption of Capital Management Theory

The underlying assumption of capital management theory is that the long-run goal of the firm is maximization of long-run earnings of the present stockholders. This assumption does not deny the possibility of other management goals. It merely assumes other operating goals to be subordinate to the overall goal of maximum long-run earnings of the present stockholders.

The assumption of profit maximization gives the decision-maker in an organization a central guide by which to govern his activities so as to accomplish the goal of efficient allocation of resources within the firm. Only when the firm's available funds are invested in a combination of assets which yield the maximum average rate of return is the goal of profit maximization realized.

The Firm's Demand Schedule for Funds

The competent capital manager must, as a first step, evaluate the firm's potential capital expenditures to determine the rate of return each of these projects promises to return.[10]

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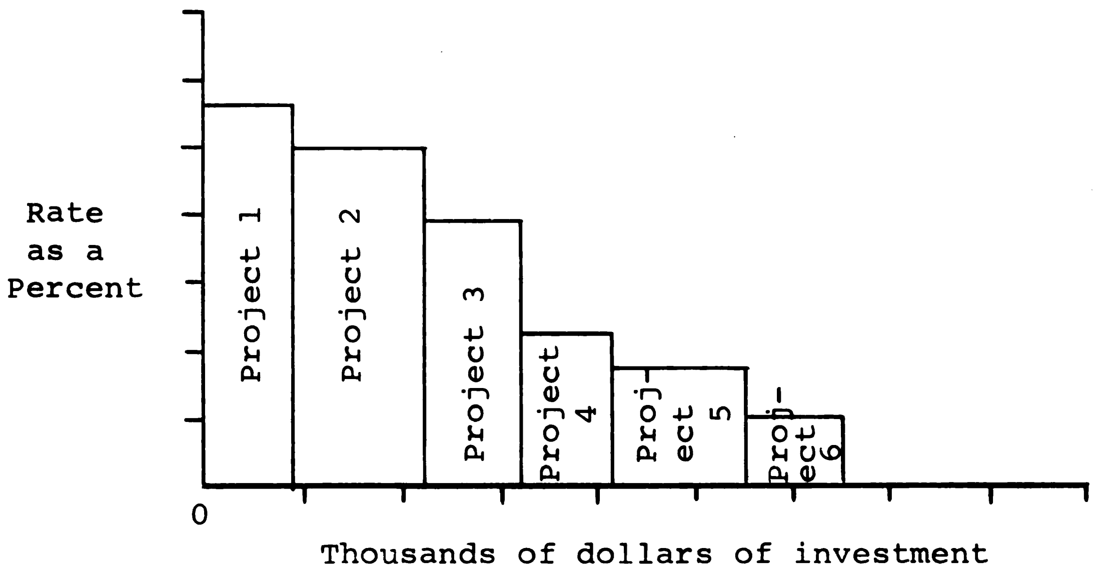
[10]Many systems for evaluating potential capital investments have been developed. Some of the more common approaches are: the payback, the average rate of return on average investment, and the present value or discounted cash flow method.

The present value or discounted cash flow approach to evaluating capital investments is advocated by most theorists as the "best" method. For a detailed explanation of the three

Then the projects will be ranked in descending order of their profitability. The resulting schedule is called the demand schedule[11] for capital and measures the opportunity for capital investment on the basis of prospective returns. A demand schedule is shown in Illustration 4-1.

ILLUSTRATION 4-1

THE DEMAND SCHEDULE FOR CAPITAL



methods and a comparison among the three methods, see: Myron J. Gordon, "The Payoff Period and The Rate of Profit," Journal of Business (October, 1955); and John G. McLean, "How to Evaluate New Capital Investments," Harvard Business Review, XXXVI, No. 6 (November-December, 1958), pp. 59-69.

[11]The term demand schedule is a misnomer. The usual demand schedule represents the quantity of goods which will be bought at various prices and represents an average, except when the demand schedule is perfectly elastic (a horizontal line). The demand schedule for funds is a marginal concept with decreasing marginal returns as capital investment increases.

The demand schedule for capital is a downward sloping curve which represents decreasing marginal returns as the level of capital investment increases.

Illustration 4-2 presents much the same information as in Illustration 4-1. It assumes that the potential projects can be made in infinitesimally small quantities. Thus, the stairstep pattern of Illustration I becomes a smooth curve as the intervals on the abscissa are made smaller and smaller until they approach zero.

ILLUSTRATION 4-2

THE DEMAND SCHEDULE AND AVERAGE RATE OF RETURN

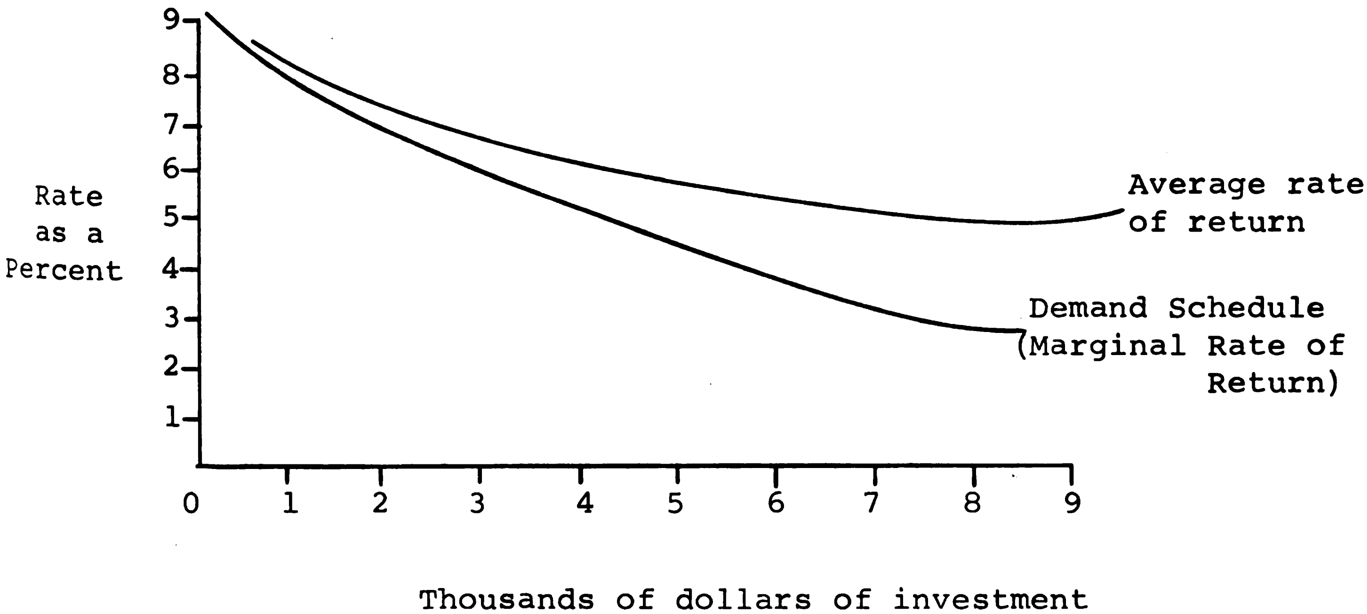


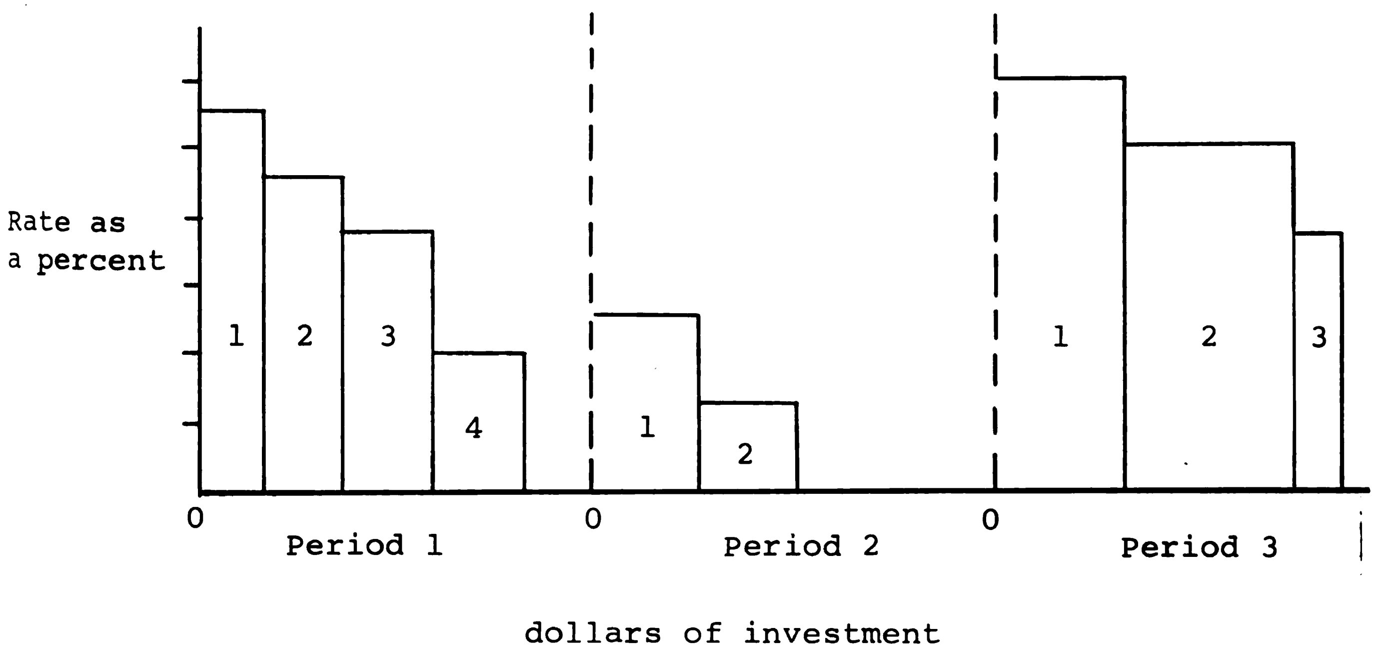
Illustration 4-2 shows the marginal rate of return (herein called the demand schedule) and the average rate of return at various levels of investment. As may be seen from Illustration 4-2, if the alternative investment promising the higher rate of return is undertaken before investing in any project with a lower return, the average rate of return will be decreasing as the level of investment increases. The first thousand dollars earns 8%, the second thousand dollars earns 7%. The average rate of return for a total investment of two thousand dollars is 7½%.

The demand schedule for capital is a concept which must be defined within a specific time period. Future capital projects depend upon technical progress and long-run demand for a firm's product. Projects which will be available five or ten years in the future are unknown now. If a comparative investment schedule were drawn up to show the investment opportunities for three decision-making periods, it might appear as shown in Illustration 4-3. Since the potential capital projects of the future period are unknown and past projects undertaken represent constraints, the only projects which are relevant for the demand schedule of funds are those that are available in the current decision period.

In order to attain the profit maximization goal, capital should be invested only in those projects which promise a

ILLUSTRATION 4-3

THE DEMAND SCHEDULES FOR THREE DECISION-MAKING PERIODS



return greater than the cost of the capital invested. Thus it is necessary to examine the firm's supply schedule of capital.

The Firm's Supply Schedule of Capital

The firm's supply schedule of capital shows the amounts of funds available to a firm at various costs. In order to develop this supply schedule of capital, the costs of additional capital from various sources--common stock, retained earnings and debt--must be measured.[12] In developing this schedule the present level of investment and the present capital structure are assumed given. The costs of capital to be developed is the cost of additional capital for investment.

The Cost of Common Stock

Many methods of computing the cost of common stock have been advanced. Some of the more common methods are:

1) the ratio of dividend payments to present market price

2) the ratio of present earnings to present market price

3) the ratio of expected earnings to present market price.

It is, however, generally agreed that the cost of common stock is best measured by the ratio of expected future earnings to the present market price of the stock.[13]

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[12]Two other sources of funds, depreciation and preferred stock, have not been included in the analysis.

Depreciation is not a source of funds but refers to funds recovered by the gradual liquidation of fixed assets. These funds are recovered only to the extent that depreciation charges are covered by cash income. Any funds recovered in the above manner should be reinvested if they can earn a rate equal to that being earned on retained earnings.

The cost of preferred stock would be measured by the ratio of promised dividend to sales price. This assumes that the stock will not be called. Preferred stock does not enjoy the tax benefit of other forms of debt and has not been used a great deal in the past few years as a means of financing.

[13]Alexander Barges, The Effect of Capital Structure on the Cost of Capital (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1963), p. 2.

The Cost of Retained Earnings

The cost of retained earnings--assuming no flotation costs, brokerage fees and personal income taxes--would be the same as the cost of common stock.

> If shareholders were all in the zero income tax bracket and underwriting costs and brokerage fees were also zero, the valid cost of capital funds derived internally from retained earnings would be clearly equal to the cost of externally derived equity funds.[14]

Earnings retained in the business should earn a rate equivalent to the ratio of expected earnings to market price. If the retained earnings cannot earn at least this rate, they should be distributed to the stockholders as cash dividends.

Foregoing the simplifying assumption of no flotation costs, brokerage fees and personal income taxes, the cost of retained earnings will be below the cost of common stock. Flotation costs, brokerage fees and personal income taxes reduce the amount of funds the company would have available for investment if instead of retaining the earnings it paid out all earnings as dividends and the stockholders in turn reinvested these dividends in more stock of the company.[15]

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[14] Solomon, op.cit., "Measuring . . .", p. 245.

[15] Assume, for example, that a stockholder is in the 20% tax bracket. Each dollar of dividends received will be taxed at 20%. Hence, each dollar of dividends received will increase the stockholder's investment cash by only eighty cents. If the stockholder is earning 10% on his investments, he will earn eight cents on the remaining eighty cents of his dollar of dividends. This results in an effective earning rate of only eight percent on the dollar of dividends received.

The Cost of Debt

Many firms obtain part of their capital from borrowed funds which have fixed interest and repayment provisions. However, the availability of these types of funds is contingent upon the use of equity capital to provide part of the funds necessary for total investment.

A problem arises in determining the cost of borrowed funds because the cost of the funds is not necessarily the coupon interest rate. The explicit interest rate is not the relevant rate because bonds sold at a premium or discount have an effective rate which is different from the coupon rate.

Another factor which affects the cost of debt is the corporate income tax. Interest payments on borrowed funds are a deductible expense in determining a corporation's taxable income. Therefore, the firm in the 50% tax bracket has a before-tax effective interest rate of 5%; the after-tax effective rate is 2½%.

Interdependence of the Cost of Equity and Debt

A further complication arises in computing both the cost of debt and equity because the cost of each is affected by the ratio of debt to net worth. It is possible that the cost of debt will rise as the debt to equity ratio increases. As the ratio of debt to net worth rises, the lender of the debt may require greater compensation for the additional risk which results with increases in contractual fixed cost payments by the firm.

The increase in debt relative to equity in the capital structure also may have an impact on the cost of equity funds in that the use of more fixed-cost capital increases the risk that common stockholders per share earnings will be reduced or will become more variable. This extra risk on common stock earnings is associated with a change in the capital structure and would not occur if only equity financing were used.[16]

In order to determine the effect of the interrelationship described above, the costs of debt and equity as defined earlier may be used only when working in terms of a given debt to net worth ratio. As soon as the ratio is disturbed, there may be an impact on either or both the cost of debt and the cost of the equity as measured above.

A Representative Illustration of the Firm's Supply Schedule of Capital

Assume that the cost of various sources of capital have been determined. When arranged in ascending order of cost

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[16]The use of a nominal cost debt instrument results in a leverage effect on common stock earnings after interest. When the firm has only common stock outstanding, the earnings per share vary as total earnings vary. When the firm has debt outstanding, the earnings per share will vary more from year to year. (This is not true in the case in which earnings are exactly equal to the effective interest paid on the debt. In this special case the earnings are zero.)

and presented graphically, the supply schedule for a firm might appear as that shown in Illustration 4-4.

ILLUSTRATION 4-4

THE SUPPLY SCHEDULE OF CAPITAL

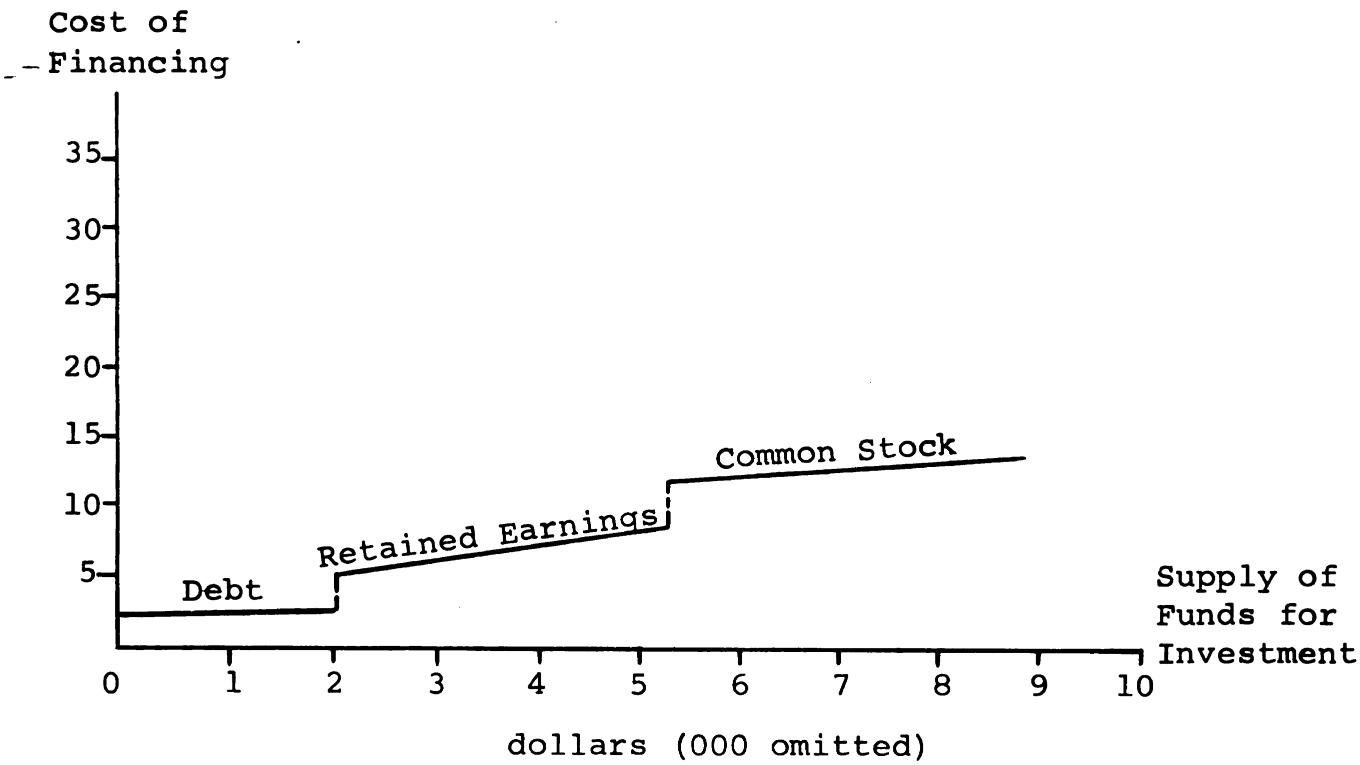

Illustration 4-4 assumes that $2,000 of debt can be borrowed at an effective after-tax rate of two and one-half percent. The maximum amount of earnings that could be retained is equal, of course, to the past period's net income. In most cases the amount of earnings retained in the business is some percentage of the past-period-earnings as determined by the firm's target payout ratio.[17] The portion of the graph

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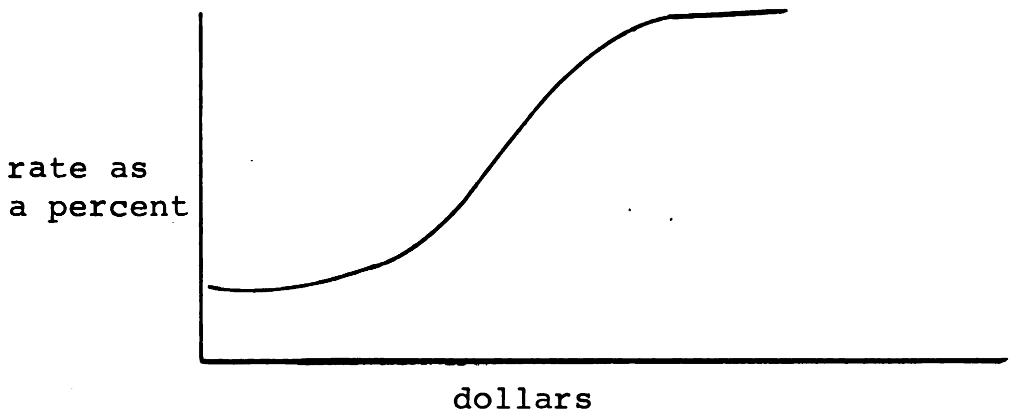
[17]John Lintner, "Distribution of Incomes of Corporations Among Dividends, Retained Earnings, and Taxes," American Economic Review (May, 1956), p. 102.

which shows the common stock has a higher cost than retained earnings because of flotation costs, brokerage fees and personal income taxes of the stockholders.

The graph in Illustration 4-4 can be smoothed out and would appear as the graph shown in Illustration 4-5.[18]

ILLUSTRATION 4-5

THE CONTINUOUS SUPPLY SCHEDULE OF CAPITAL



Source: Robert Lindsay and Arnold W. Sametz, Financial Management: An Analytical Approach (Homewood, Ill.: Richard D. Irwin, 1963), p. 160.

The supply schedule of capital, like the demand schedule for capital, is a concept which must be defined for a specific

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[18]The specific shape of the firm's supply schedule will depend upon the specific approach used to develop the schedule. The curve in Illustration 4-5 is a marginal approach which show the different costs of funds at different levels of investment.

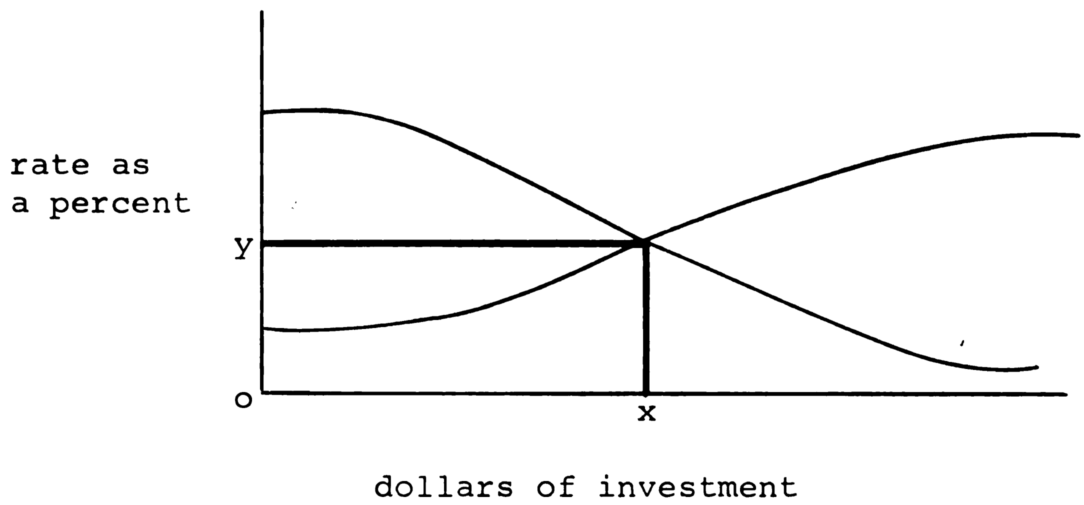
time period. The supply schedule of today is not necessarily the past or future supply schedule of capital. Estimates of the cost of various sources of capital involves the use of market prices for common stock and debt instruments as well as recognition of changes in the debt to net worth ratio. Because of these factors, the costs of various funds will vary from period to period.

Use of the Demand and Supply Schedules of Capital to Determine the Optimum Additional Level of Investment

In order to maximize the long-run earnings of the present stockholders, each investment in an asset must return an amount greater than the cost of capital invested in that asset. The optimal level of additional investment is that level at which the marginal cost of capital is equal to the marginal revenue from capital investment. Successive capital projects will be undertaken along the downward sloping array as long as the prospective yield from each potential project is higher than the cost of obtaining the funds required for its financing. The relationship of the two capital schedules is shown in Illustration 4-6 which merely superimposes the schedule in Illustration 4-2 upon the schedule in Illustration 4-5. The optimum level of investment to be undertaken by the firm is ox. The minimum acceptable rate for any project is oy percent, which is called the cut-off rate. If the level of

ILLUSTRATION 4-6

THE SUPPLY SCHEDULE AND DEMAND SCHEDULE OF CAPITAL



investment designated by x is undertaken, the long-run earnings of the present stockholders will be maximized.

It is important to note that in Illustration 4-6 the profit maximization goal stated above is attainable by the firm only if it undertakes capital expansion of that level designated by x dollars. If only one project were being considered, then the lowest cost of funds would be used to finance the specific project. This would, without a doubt, insure minimum cost of financing along with maximum rate of return. It would not, however, insure long-run maximum earnings. The profit maximization goal can be achieved only when all projects that yield net returns over cost of financing are adopted.

This is exactly the same as the economist's marginal cost-marginal revenue analysis which results in maximum profits when the marginal cost is equal to the marginal revenue. The marginal revenue for any one unit may be greater than the marginal cost for the same unit. However, profit maximization is attained only by going to the point where marginal cost is equal to the marginal revenue.

Using Capital Management Theory to Compute the Imputed Interest Rate for Use in the Inventory Models

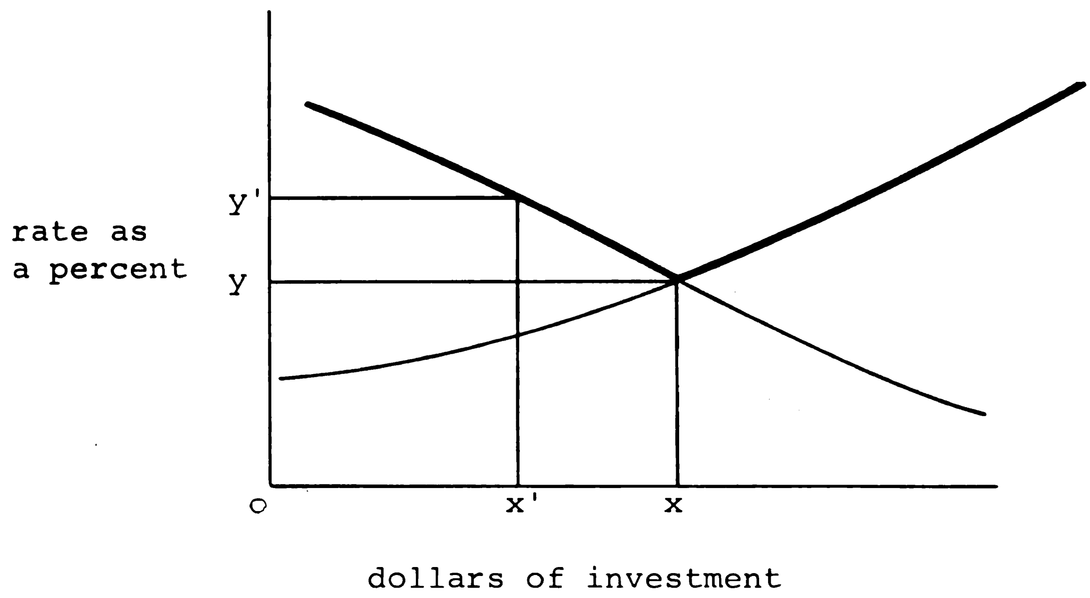
To be consistent with good capital management theory, the imputed interest used in the models should charge the inventory with a capital cost which is equal to the return that could be earned if the capital were applied to alternative uses, with the minimum charge to use being the cut-off cost of capital.

If, on the other hand, the firm does invest in projects up to the profit maximization point (investment level ox in Illustration 4-7), the appropriate rate to use in the models is Oy percent, the minimum acceptable return.

If the firm is <u>not</u> investing up to the optimum point (ox), then the proper rate to be charged against inventory is the rate returnable on the first project not undertaken. Assume that the firm undertakes expansion projects only to the level ox'. In this case the appropriate interest rate for the models is oy' percent. If funds were released from inventory, they could be invested in the project that would yield that rate immediately following y'. Hence, this is the opportunity cost of tying up funds in inventory.

ILLUSTRATION 4-7

THE RELEVANT INTEREST RATES FOR INVENTORY MODELS



Should the firm go beyond the optimum level of investment, say ox", the relevant interest rate becomes oy", the marginal cost of capital since it is higher than the available returns on alternative investments. The relevant interest rate for the inventory models at various levels of investment lies along the heavy black line in Illustration 4-7. The interest rate range developed above for use in the inventory models differs from the interest rates suggested by the literature on scientific inventory management. A quick review of the interest rates suggested by the literature shows the above to be true.

Interest Rates Suggested by the Literature

The literature on scientific inventory control contains numerous suggestions covering a wide range of rates for use in computing the interest charge included in carrying cost. This wide divergence of the suggested rates is caused by the differences in rationale, whether explicit or implicit, which lies behind the various rates suggested.

The following excerpts from the literature are not intended to include all discussions on imputed interest rates published since the inception of scientific inventory control. It is, rather, an attempt to provide a representative sampling of the arguments used to support the various interest rates.

The following excerpt by Billy Goetz in his managerial cost accounting text illustrates the multiplicity of rates suggested by a single author:

> If available cash discounts are being lost because of poor working capital position, the rate used for interest in figuring economic lot size may properly be as high as thirty or thirty-five percent. If the company is investing excess

> cash in government bonds paying one percent or less, the rate used in figuring economic lot size should be correspondingly low.[19]

A wide range of rates is also suggested in a recent work concerned specifically with scientific inventory control.

> . . . a company might be earning a return of ten percent on its total investment. If money released from inventory could also earn ten percent, then this is the cost of money tied up in inventory. Alternatively, many companies maintain short-term loans from banks. Money released from inventory could be used to retire some of these loans. In this case, the cost of the money in inventory might be measured by the interest rate paid for the short-term loans. Also, the money might be used to retire company bonds in which case the interest rate paid on the bonds would be used. At the very least the company could invest the money in government securities and earn the going rate on them. This rate, then, could be used as the cost of the money tied up in inventory.[20]

Another writer suggests a single basis by which to determine the appropriate rate for a given decision-making period to be modified by the amount of risk inherent in inventory. He states:

> . . .begin with the planned rate of return on all investments for the financial planning period ahead. . . . Then, this figure is adjusted upward or downward to account for the greater or lesser degree of business risk involved in inventories

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[19] Billy E. Goetz, Management Planning and Control: A Managerial Approach to Industrial Accounting (New York: McGraw-Hill Book Co., Inc., 1949), p. 211.

[20] Martin K. Starr and David W. Miller, Inventory Control: Theory and Practice (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1962), p. 10.

> versus other internal investment opportunities which the firm is considering.[21]

T. M. Whitin, in his definitive work on scientific inventory management, states:

> . . . capital tied up in inventories is unavailable for uses in the company and thus its use for inventory must be penalized at the rate given up. Thus, the actual charge should be the expected earnings on capital projects for which capital is not currently available.[22]

The following quotation illustrates the point of view of a business executive in discussing a newly installed inventory system in his company.

> We found ourselves borrowing a considerable amount of money in order to buy steel inventories. Our books showed me we're paying five and one-half percent interest on amounts up to $130,000 simply for the privilege of having a pile of steel in our yard.[23]

John F. Magee in his well-known series of articles on inventory management had this to say:

> Finally, it must be pointed out that the cost of the dollars invested in inventory may be underestimated if bank interest rate is used as the basis, ignoring the risk-bearing or entrepreneur's compensation. How many businessmen are actually

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[21] Howard L. Timms, Inventory Management of Purchased Materials (New York: National Association of Purchasing Agents, 1958), p. 56.

[22] Whitin, op.cit., p. 239.

[23] The Gledhill Inventory Story--A Cost of Possession, Case History, 50th Annual Meeting of the Steel Service Center Institute, p. 5.

satisfied with use of their company's capital funds which do not earn more than a lender's rate of return? In choosing a truly appropriate rate--the executive must answer some questions:

1) Where is the cash coming from?
2) What else could we do with the funds and what could we earn?
3) When can we get the investment back out, if ever?
4) How much risk of sales disappointment and obsolescence is really connected with this inventory?
5) How much of a return do we want, in view of what we could earn elsewhere or in view of the cost of money to us and the risk inventory investment entails?[24]

Classification of the Suggested Interest Rates

The numerous suggestions in the literature may be classified into six basic groups suggesting the following as the proper criteria for selecting the appropriate interest rate to charge against inventory.

1) The carrying charge on borrowed funds.
2) The rate of return being earned on a specific asset.
3) The average rate of return currently being earned on total assets.
4) The average rate of return for the planning period ahead.
5) The estimated rate of return on potential capital investments.
6) The cost of capital.

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[24] John F. Magee, "Guides to Inventory Policy: I. Functions and Lot Sizes," Harvard Business Review, XXXIV (January-February, 1956), p. 55.

An Evaluation of the Suggested Criteria for the Selection of the Interest Rate

By applying the yardstick of efficient capital management provided by current capital management theory, an evaluation of the six criteria listed above is possible.

Criterion 1. The Carrying Charge on Borrowed Funds

The carrying charge on borrowed funds criterion is an attempt to charge the inventory with an interest cost equal to the rate paid on funds borrowed from a bank or through a bond issue. The quotation on page 107 and the quotation from the business executive on page 109 are two which suggest this particular rate. The use of the carrying charge on borrowed funds is wrong for two reasons.

First, it associates a specific source of funds with a specific asset. Although the above relationship might well exist, it is irrelevant from the point of view of capital management. As was mentioned earlier, good capital management precludes considering only a single project or a small segment of the demand for capital. The overall picture of demand and supply of capital must be taken into account.

Second, while the carrying charge criteria does recognize a cost of capital invested in inventories, it uses the lowest-cost portion of the total capital structure and completely ignores the cost of equity, the most expensive capital.

Criterion 2. The Rate Being Earned on a Specific Asset

Those who suggest the rate of return being earned on a specific asset criterion use the rate of return presently being earned on a low-yield asset as a goal rate which other assets--including inventory--should earn. The specific asset most commonly suggested in the literature is government bonds and the rate used is the effective bond yield. It is logical that the proponents of this criterion suggest the yield on securities (governments or others) as the benchmark rate of return, inasmuch as securities represent one of the few assets which earn an explicit rate of return. This is not an acceptable criterion, however, for the following reasons.

First, if the firm is investing in government securities, the actual benefit of investment in this asset is not measured by the effective yield on the security. The firm invests in governments for numerous reasons. For example, to earn a small return on temporarily excess cash, perhaps to handle seasonal fluctuations or perhaps to accumulate cash for some specific capital expenditure or, the firm may hold an emergency reserve in the form of government securities to provide liquidity. In any of these cases, the benefits to the company are greater than the interest rate yield on the actual securities.

Second, the decision to invest in the governments or any other specific asset was made in prior periods. Past decisions, as mentioned earlier, represent constraints and no longer represent alternative opportunities for investment.

Criterion 3. The Average Rate of Return Currently Being Earned on Total Assets

The average rate of return currently being earned on total investment may be lower than, equal to, or greater than the present period's cost of capital. If the rate currently being earned on total investment is less than the present day cost of capital, it is too low a rate to be used because investment at this rate would not result in maximization of the present stockholders' long-run earnings. If the rate being earned on total investment is equal to the present day cost of capital, it will not charge the capital invested in inventory with a high enough cost if alternative investments with a higher return are available; hence, the long-run earnings of present day stockholders will not be maximized. If the rate being earned on total investment is higher than the cost of capital, it is still not an acceptable criterion. It must be compared with the rate which could be earned on alternative investments and in making this test it might be found to be too high or too low. In either case it precludes attainment of the goal of profit maximization of present stockholders long-run earnings.

Not only is the average rate of return on total investment an inadequate rate to apply to inventory decisions for the reasons stated above, but the rate used as the average is frequently too low because the effect of non-operating assets is often neglected.

The current rate of return on total investment will be too low if the firm has any sizable amount of non-operating assets such as government bonds, land held for future use, etc. Inclusion of these items and their earnings will tend to result in an average rate of return lower than the rate of return from operations. This is so because the present yield from the non-operating assets is low relative to the future benefits the company expects to obtain from holding these assets, as was pointed out in the discussion of the rate of return on a specific asset.

Also, the rate of return currently being earned on total investment is not an acceptable rate to use in computing the capital cost of inventory because it represents the results of past investment decisions when the cost of capital and the investment opportunities of the firm were different than in the present decision-making period.

Criterion 4. The Average Rate of Return for the Planning Period Ahead

The planned average rate of return for the coming decision period is not an acceptable criterion when viewed in the light of capital management theory.

The use of the planned average rate of return for the decision period ahead is an unacceptable rate because it is not an opportunity cost which represents the return on alternative uses of the capital invested in inventory. The demand schedule of investment opportunities is a negatively sloping function which shows the marginal rate of return to be decreasing as investment increases. After all investment opportunities with the highest marginal rate of return have been exhausted, succeeding investments will earn a lower marginal rate of return. The marginal rate of return is the rate expected to be earned on the last investment. By averaging in the lower rate of return on the last investment, the average return on total investment must drop, being somewhere between the old average and the marginal return on the new investment. Thus the marginal rate of return (the demand schedule) will always be below the average rate of return for the planning period ahead (see Illustration 4-2). Hence, the average rate of return for the planning period ahead may be too high a rate. It does not represent the alternative uses of the capital.

Criterion 5. The Estimated Rate of Return on Potential Capital Investments

The estimated rate of return on potential capital investments for which capital is presently unavailable is a partially acceptable rate to apply to inventory planning. It meets the requirement of capital management theory that the attractiveness of investments must be measured in terms of alternatives. There is only one fault with this rate, and that is that it sets no minimum acceptable return. Thus, it must be combined with the fifth criterion--the cost of capital--if it is to be consistent with good capital management.

Criterion 6. The Cost of Capital

The cost of capital criterion is, by itself, an acceptable rate with limitations. Of all the criteria suggested, it alone provides a meaningful minimum rate which, while not maximizing the long-run earnings of present day stockholders, does insure an improvement in their long-run earnings. The cost of capital criterion presents the most potential if used in conjunction with the fifth criterion--the estimated rate of return on potential capital investments. Cost of capital, by itself, is too low a rate, however, if the firm has not undertaken that level of investment where its demand and supply schedules for capital cross.

The Capital Management Rate

It is obvious, then, that the correct criterion to use in selecting the rate to impute a cost to the capital invested in inventory is a combination of the last two criteria discussed above, namely, the estimated rate of return on potential capital investments and the cost of capital. This combination will result in a rate which is fully consistent with capital management theory.

CHAPTER V

THE RESEARCH DESIGN

The value of the imputed interest rate that must be used in the inventory models developed under scientific inventory management theory, as described in detail in chapter four, should be developed within the framework of capital management decision theory. The imputed interest rate, although an opportunity cost, is a valid cost of carrying inventory. The imputed interest rate charge is included as a carrying cost of inventory in order to reflect the alternative earning power of the funds invested in inventory. The value of the imputed interest rate used in the inventory models must be equal to the estimated rate of return on potential capital investments for which funds are unavailable in the present decision making period. The minimum acceptable estimated rate of return or the capital budgeting cut-off point in any decision period is the firm's cost of capital. The use of an imputed interest rate not consistent with the above criteria will violate an underlying assumption of financial management theory: the long-run goal of maximization of long-run earnings of the present stockholders.

The values of the imputed interest rate to be used in the models, as suggested by the current literature, are representative of opportunity costs which are, in most cases, lower values than those that are in line with capital management decision theory. Utilizing values of the imputed interest rate suggested by the current literature results in capital cost carrying charges that do not represent the opportunity cost of the funds tied up in inventory and results in the use of imputed interest rate values that are lower than the firm's cost of capital in determining the organization's inventory levels. The use of these lower values of imputed interest or "erroneous" rates in the inventory models results in management decisions which will not maximize the long-run earnings of the firm.

This chapter, using the dynamic model under certainty and the dynamic model under risk, will answer the question: What is the effect upon total inventory costs when imputed interest rate values not consistent with capital management decision theory are used in the dynamic inventory models? We will determine the sensitivity of total inventory costs to different values of the imputed interest rate for a variety of simulated cost structures.

Sensitivity Analysis

Sensitivity analysis is concerned with the change, or lack thereof, in results that occur when data or assumptions upon which the results are based, are changed.[1] Sensitivity analysis will determine the elasticity of a dependent variable to changes in an independent variable. When more than one independent variable can effect the dependent variable, it is necessary to isolate the significant independent variable in order to insure that changes in the dependent variable result from the changes in the independent variable being tested.

For this analysis the dependent variable is total inventory cost and the independent variable of significance is the imputed interest rate portion of total carrying cost. Other independent variables that could effect total inventory costs are ordering cost, stockout cost and carrying costs exclusive of the imputed interest rate. For this research it will be assumed that all other inputs into the models, the other independent variables, have been developed within the limits set forth previously in chapter four and will remain constant as the value of the imputed interest rate, the independent variable of significance, changes.[2]

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[1] John C. G. Boot, "Sensitivity Analysis," Mississippi Valley Journal of Business and Economics, Vol. 1, No. 1 (Fall, 1955), p. 14.

[2] As is obvious from the following section on development of the simulated cost structures, the other independent

Variations in total inventory costs resulting from changes in the value of the imputed interest rate will then be recorded.

This chapter is divided into three sections. The first section deals with the development of the simulated cost structures used in the models. The second section is concerned with the methodology used to calculate the sensitivity of the total costs associated with inventory that results from changes in the imputed interest rate. The third section presents the results in summarized form as well as a discussion of the importance of the findings.

Section One: Developing the Simulated Cost Structures

The individual components of total inventory costs vary from industry to industry, from firm to firm within any industry, and from one decision period to the next for any specific firm. In order to show the sensitivity of

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variables are not held constant throughout the analysis. For any given cost structure of stockout cost, ordering cost and carrying cost exclusive of interest, only the values of the imputed interest rate are changed in order to determine the sensitivity of total inventory costs. To determine the sensitivity of total inventory costs to various values of the imputed interest rate for a variety of costs structures, it is necessary to vary some of the other imputs in developing the variety of cost structures. For any given cost structure, however, the only independent variable that changes is the value of the imputed interest rate.

total inventory costs to different values of the imputed interest rate for various combinations of the total inventory cost components it was necessary to simulate a variety of cost structures. The following factors were taken into consideration in developing the simulated cost structures used in the sensitivity analysis.

Developing the Ranges of the Various Costs

The components of total inventory costs that must be balanced by the inventory models are:

a) Carrying cost = I
b) Ordering cost = S
c) Stockout cost = O

The annual carrying cost and annual ordering cost are the opposing costs which must be balanced[3] by the dynamic inventory model under certainty if total inventory cost is to be

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[3]Ordering costs vary directly in proportion to the number of orders placed--they have a linear relationship to the number of orders placed. Carrying costs have a linear relationship to the value of the average inventory. The two costs are equal where they intersect. The poing of intersection of the two costs being balanced is also the minimum point of total inventory costs, the sum of ordering cost and carrying cost. The above relationship of the two curves is a special case. The low point on the sum line of a rectangular hyperbola (the ordering costs) and a straight line drawn through the origin of the axis (the carrying costs) is at a point where the latter two curves intersect. Other curves might not have this relationship. (Chapter three contains a detailed discussion of the component parts and the total costs associated with inventory.)

minimized.[4]

The Carrying Cost Range

Total inventory carrying cost, I, according to the literature, is estimated to be somewhere between ten percent and thirty-five percent per annum of the average dollar cost of inventory.[5] A representative breakdown between the extremes of this range into its various components is as follows:[6]

Item	Low End	High End
Interest Charge (Opportunity Cost)	4%	15%
Insurance	1	3
Taxes	1	3
Storage	0	3
Obsolescence	4	10
Total Carrying Cost.	10%	34%
Less: Interest	4	15
Carrying Cost Exclusive of Interest. .	6%	19%

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[4]Minimum total inventory cost refers to an optimum solution for a given demand schedule, usage rates, per unit price, ordering cost, carrying cost, and a set level of customer service.

[5]Johnson, op.cit., "Financial Management" place the total carrying cost between ten and twenty-five percent. (p. 594), Timms, op.cit., "Inventory Management of Purchased Materials" also places to the total carrying cost between ten and twenty-five percent. (p. 58). Goet op.cit., "Management Planning and Control: A Managerial Approach to Industrial Accounting" believes the range is wider and is somewhere between one percent and thirty-five percent. (p. 211-212).

[6]Evert Welch, Tested Scientific Inventory Control, Management Publishing Corporation, Greenwich, Conn., 1956, (p. 64).

According to the above breakdown the carrying cost range exclusive of the capital charge would be six percent to nineteen percent and the capital charge range would be four percent to fifteen percent. In order to determine the sensitivity of total inventory costs to various values of the imputed interest rate it is necessary that the various values of the imputed interest rate to be tested be combined with the entire range of carrying costs exclusive of the imputed interest rate charge.

For example, the annual total inventory cost variation that occurs when a six percent imputed interest rate value is used instead of a "correct" imputed interest rate value of twelve percent, as determined under capital management decision theory, will not be the same if the sum of total carrying costs are different. It is not the values of the "correct" and "incorrect" imputed interest rates alone that determine the shapes and locations on the graph of the total inventory cost curves being compared. It is the different total carrying costs of the various values of the imputed interest rate which determine the shapes and locations of the total cost curves being analyzed. If a six percent imputed interest rate charge and a twelve percent imputed interest rate charge are being compared the total cost curves of each imputed interest rate value will have a different shape and a different location when the carrying costs exclusive of interest are different.[7]

The typical range of values of the imputed interest rate charge, as shown in the above breakdown, would be between four percent and fifteen percent. If capital management decision theory criteria are used to determine the value of the capital charge the high end of the interest range shown above, fifteen percent, would be inadequate.[8] For example, assume that a firm were not taking its cash discounts on purchases because of a poor cash position. The value of the imputed interest rate to be used in the models in this case might be as high as thirty-six percent.[9]

In order to broaden the range of imputed interest rate values for this analysis the total carrying cost associated with inventory were varied from a low of four percent

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[7]The sensitivity of total inventory costs to different values of the imputed interest rates can only be determined by varying the value of total carrying cost. The development of an error factor of total carrying cost is discussed in detail in section two of this chapter.

[8]The low end of the interest range is also unreasonable when viewed in the light of capital management decision theory. Since the minimum acceptable imputed interest rate, the cut-off point, is the cost of capital it does not seem conceivable in this day and age that any figure less than ten percent would be representative of a firm's cost of capital.

[9]Goetz, op.cit., (p. 211).

to a high of fifty-six percent. The use of this broader range of carrying cost values allows for a greater number of combinations of the imputed interest rate and carrying cost values exclusive of the imputed interest rate. Under the broader range, higher values of the imputed interest rate that are in line with capital management decision theory can be combined with the typical range of carrying costs exclusive of interest. With the broader range of carrying cost values it is possible to analyze cost structures in which the carrying cost exclusive of interest can be at the high end of the typical range as shown above and can be combined with low, medium or high values of the imputed interest rate. Of course, cost structures in which the values of the carrying cost exclusive of interest are low can be combined with imputed interest rates that are low, medium or high as determined under capital management decision theory.

The Ordering Cost Range

Two problems arise in the development of ordering cost figures to be used in the models in practical solutions to inventory problems. Neither of these problems are encountered in the theoretical development of the inputs for the models.

The first problem concerns the sources of the ordering cost. The ordering costs attributed to any item of inventory can be determined from the accounting records. However, since the accounting records classify the various expenses of the firm on a functional basis the ordering costs associated with inventory will not, in most cases, be found in a single account.

Labor costs, supplies, etc. that are, in fact, ordering costs will be found in cost of goods sold, selling expenses and administrative expenses with the various item shown as subtotals within major cost divisions. For inventory decision-making the portions of these account totals applicable to the problem must be determined. Also, since only marginal costs are relevant for the inventory problems the ordering costs figures obtained from the accounting records must be further separated into fixed and variable for the present decision-making period.

The second problem which is closely associated with the first problem discussed above concerns the marginal nature of the ordering cost and the shape of the ordering cost function.

The equation to solve the dynamic inventory model under certainty is:

$$T.V.C. = \frac{QC}{2} I + \frac{D}{Q} S$$

Where:

T.V.C. =	total variable cost	D =	annual demand
Q =	quantity to order	C =	unit price
S =	ordering cost per order	I =	carrying cost per unit per year (as a % of cost)

S, the ordering cost per order used to solve the actual problem represents the marginal portion of the total ordering cost for the period and is always a specific numeral. This implies that the marginal cost of placing an order is a constant. The graphical presentation would show a straight line that increases at a constant rate with increases in the number of orders placed.

Although many marginal costs associated with placing an order are constant from one order to the next, cost of the purchase order, envelopes, mailing and certain supplies, there are other marginal costs which will vary from one order to the next. Hence, when we use one specific number in the model to represent the marginal cost of placing an order for that period we are, in fact, distorting slightly the inputs and, of course, the optimum order quantity which is the final result. The assumption of a constant ordering cost per order does not result in major deviations from the optimum solution, according to a present day author of scientific inventory management, and may be ignored in the practical applications of the inventory models.[10]

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[10]Timms, op.cit., (p. 48).

Changes in ordering cost at twenty percent intervals were programmed into both the dynamic model under certainty and the dynamic model under risk. The twenty percent interval variations of ordering cost were used for the total ranges of twenty percent of per unit cost at the low end to two hundred percent of per unit cost at the high end.[11]

Ordering costs are not usually expressed as a percent of per unit cost since the ordering cost does not, in most cases, have any relationship to the per unit price or the average dollar value of inventory.[12]

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[11]The twenty percent steps of ordering cost for the total range of twenty percent of per unit cost to two hundred percent of per unit cost results in nine different values of ordering cost. The nine different values of ordering cost were used in both of the dynamic models utilized in this analysis.

[12]Carrying cost, as contrasted to ordering cost, is always expressed as a percent of per unit cost or the average dollar value of inventory. The expression of carrying cost as a percent of per unit cost is realistic in that the components of carrying cost will vary directly with the size of the investment in inventory. For example, storage space charges will vary directly with the space used which depends on the physical size of the item in question and will be included as a marginal cost if alternative uses of the space are not possible. With a known cost of space and a known size of the item in question it is possible to convert space used as determined by item size to a common denominator of carrying cost for space charges into dollars. The capital cost charge, handling charges, variable taxes on inventory, and obsolescence all bear the same relationship to the average dollar value of inventory as do space charges and are rightly expressed as a percent of per unit cost or average dollar value of the inventory.

Conversely, the ordering cost is not directly related to the cost of the item. Ordering cost is the sum of the

As mentioned above, the ordering cost must be ferreted from the accounting records and from careful observation of the purchasing function. Also, the ordering cost used in the model must include only those portions of the cost which are marginal in nature for the time period under consideration.

Ordering costs, in fact all relevant costs used in the analysis, are expressed as a percent of per unit cost in order to generalize the final results. Hence, the variations in total inventory cost are expressed as a percentage change resulting from a percentage change in the independent variable, changes in total carrying costs. By generalizing the results, the per unit cost becomes irrelevant for any given demand schedule and usage rate. With a given demand distribution, the resulting average inventory percentage changes and the percentage changes of total inventory cost will be the same regardless of the per unit cost of the items in question. The absolute dollar savings for any company will be determined from the percentage changes in total inventory costs times the average dollar value of the inventory. The average dollar value of the inventory will depend, of course,

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supplies for placing an order, any variable cost of labor needed to complete the order and, perhaps, any additional marginal costs necessary to expedite the arrival of the merchandise into the firm.

on the per unit price of the inventory items.[13]

The Stockout Cost Range

Extreme difficulty is encountered in the development of stockout cost, as mentioned previously, for two reasons. First, part of this cost is developed from information based on human behavior and is therefore, highly conjectural in nature. Second, stockout cost is an opportunity cost not normally found in the accounting records.[14]

The stockout cost range substituted into the model was from a low of twenty percent of per unit cost to a high of one hundred percent of per unit cost. Ten percent intervals were used in covering the range of stockout costs.[15]

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[13]The analysis expresses the savings in total inventory costs as a percentage available from a percentage change in total carrying cost resulting from the use of imputed interest rates in line with capital management decision theory. For any specific firm, the actual numbers must be substituted into the models in order to develop the potential savings to the firm from a reevaluation of their inventory model inputs.

[14]The metal service industry is an example of one industry in which the stockout cost can be calculated directly. When items not in stock are ordered the supplier will purchase the out-of-stock-items from a competitor at a higher-than-mill price. The stockout costs consist then of the difference of purchase price and mill price plus the cost of picking the product up at a competitor's plant. See E. Martin Busic, Development and Application a Gamma-Based Inventory Management Theory. (Unpublished Doctoral Dissertation, Michigan State University, East Lansing, Michigan, 1965).

[15]Utilizing the range of stockout costs from ten percent of per unit cost to one hundred percent of per unit cost in ten percent intervals results in nine different values of stockout cost.

Stockout costs are not necessarily related directly to per unit cost. If the stockout cost represented only the loss of the gross profit margin or the contribution to overhead, then it could be directly related to per unit cost and could always be expressed in that manner. However, for the same reasons previously advanced concerning the expression of ordering cost as a percent of per unit cost, generalization of the results of the sensitivity analysis regardless of the per unit cost of an item, the stockout cost is expressed as a percent of per unit cost or of the average dollar value of inventory.

The low range of stockout cost used in the analysis might be representative of a firm in which a lost sale results in a loss of profits of only the gross profit on that specific sale without any subsequent loss of customer good will. In other words, the out-of-stock will not result in future lost sales from the customer. The high range of stockout costs, on the other hand, might represent a loss of the gross profit on the immediate sale as well as some amount of lost profits from future sales that will not be made because of the loss of customer goodwill which results from the out-of-stock.

Section Two: Methodology

Using the range of costs developed in section one to simulate a variety of cost structures a computer program was employed to solve the specific equations of the dynamic inventory models. For any given cost structure of ordering cost, stockout cost, carrying cost exclusive of the imputed interest rate charge and total inventory cost, the values of the imputed interest rates were varied in order to determine the sensitivity of total inventory cost to the imputed interest rate charge. Utilizing any specific ordering cost, stockout cost and carrying cost exclusive of interest it was possible to determine total inventory costs associated with inventory for different values of the imputed interest rate in order to show the sensitivity of total inventory costs to changes in the value of the imputed interest rate.

The Dynamic Model Under Certainty[16]

In order to simulate a variety of cost structures the

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[16]Most firms have demand and lead times that are not known with certainty and must, therefore, utilize the dynamic model under risk to solve their inventory problems. This analysis is concerned, primarily, with the dynamic model under risk but develops the steps used in the sensitivity analysis for the dynamic model under certainty for ease of discussion. The more complex analysis of the dynamic model under risk is identical to the analysis of the dynamic model under certainty except that the model under certainty has fewer variables which simplifies the discussion of the analysis. The sensitivity analysis is identical for both models except that the model under risk contains more variables.

equation for the dynamic model under certainty was solved using the following inputs:

Annual demand of 24,000 units with a constant usage.
Per unit cost of ten dollars.
Ordering cost varied from 40% to 200% of per unit cost.
Carrying cost varied from 4% to 56% of per unit cost.

Assuming ordering costs were 40% of per unit cost, or $4.00 per order, the total carrying cost was developed for carrying cost of 4%, 8%, 12%, etc., up to 56% of per unit cost. The same step was repeated for ordering cost of 60% of per unit cost, or $6.00, per order. Total ordering cost, total carrying cost, average inventory value and total inventory cost was developed for the entire range of carrying cost of 4% to 56% for each 20% interval of ordering cost from 20% of per unit cost to 200% of per unit cost.

Table 5-1[17] summarizes the output of the dynamic model under certainty for the values of total carrying cost from 4% to 56% in 4% intervals. Ordering costs are 40% of per unit cost or $4.00 per order for the simulated cost structure results displayed in Table 5-1.[18]

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[17]The numbers portrayed in Table 5-1 and in the similar Table 5-4 shown later can be developed using the equations explained in detail in chapter three by substituting the specific values of the dynamic model under certainty and the dynamic model under risk used in this chapter.

[18]Numerous combinations of cost structures are represented by the figures in Table 5-1. Comparisons of total costs for any of the carrying cost percentages in column one is possible with the carrying costs shown in columns

TABLE 5-1

ANNUAL INVENTORY COSTS AT SELECTED HOLDING RATES
ORDERING COST $4.00 or 40% OF PER UNIT COST

Column	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
	Holding Cost	Annual Ordering	Average Inventory	Inventory Holding Costs					Total Inventory Costs				
Row	%	Cost $	$	I=12%	I=20%	I=32%	I=44%	I=56%	I=12%	I=20%	I=32%	I=44%	I=56%
1	4	140	3,464	416	693	1,108	1,524	1,940	556	833	1,248	1,665	2,080
2	8	196	2,449	294	490	784	1,078	1,371	490	686	980	1,274	1,567
3	12	240	2,000	240	400	640	880	1,120	480*	640	880	1,120	1,360
4	16	276	1,732	208	346	554	762	970	484	622	830	1,038	1,246
5	20	308	1,549	186	310	496	682	867	494	618*	804	990	1,175
6	24	340	1,414	170	283	452	622	792	510	623	792	962	1,132
7	28	368	1,309	157	262	419	576	733	525	630	787	944	1,101
8	32	392	1,225	147	245	392	539	686	539	637	784*	931	1,078
9	36	416	1,155	139	231	370	508	647	555	647	780	924	1,063
10	40	440	1,095	131	219	350	482	613	571	659	790	927	1,053
11	44	460	1,044	125	209	334	459	585	585	669	794	919*	1,045
12	48	480	1,000	120	200	320	440	560	600	680	800	920	1,040
13	52	500	961	115	192	308	423	538	615	692	812	923	1,038
14	56	520	926	111	185	296	407	519	631	705	816	927	1,039*

Table 5-1 also includes some adjustments to total inventory cost that result when higher values of total carrying cost are used in the model instead of the total carrying cost shown in row one of Table 5-1. Tables identical to Table 5-1 were developed for each ordering cost represented by the 20% intervals of the entire range of ordering cost from 40% of per unit cost to 200% of per unit cost.

Column one of Table 5-1 is the range of total holding costs which were substituted into the model when per unit cost was $10.00 and ordering costs were assumed to be 40% of per unit cost. For each of the carrying costs shown in column one, the total annual ordering cost and average inventory dollars are shown.[19]

Column two is the total annual ordering cost, $\frac{D}{Q}S$, which occurs at each of the specific holding costs of column one. The annual ordering cost increases as the holding cost percentage increases. This is to be expected because the minimum point of total costs is at the point where total

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nine through thirteen. All of these comparisons in Table 5-1 are representative when ordering costs are 40% of per unit cost. The same number of comparisons are available for other ordering costs within the 40% to 200% range of ordering costs in Tables similar to 5-1.

[19]The total holding cost range of 4% to 56% was substituted into the models in four percent steps. The holding cost used was 4%, 8%, 12%. . . . .56%.

holding costs equal total ordering cost. Increases in the holding cost percentage shifts the holding cost curve upward and to the left and it is now equal to the ordering cost curve at higher cost levels resulting in lower economic order quantities.

Column three is the economic order quantity divided by two, $\frac{Q}{2}$ C, that results at each holding cost percentage. This average inventory level decreases as the holding cost percentage increases. Increase in holding cost shifts the total cost curve upward and to the left, resulting in lower economic order quantities as mentioned above.

Columns four through eight represent the holding costs, as a percent of average inventory dollar value, when it is assumed that 12%, 20%, 32%, 44%, and 56% represent the "true" holding costs as compared to the holding cost shown in column one. The $490.00 in row two and column five represents the costs of carrying an average dollar value inventory of $2,449 when the total carrying cost is 20%. The $490.00 is made up of two parts: the out-of-pocket costs of holding the inventory and the opportunity costs of tieing the funds up in inventory. The $490.00 shown in row two and column five is the carrying cost of the average dollar value of inventory as determined by the use of an eight percent total holding cost when it is assumed that a 20% total carrying cost should have been used. In other words, the average dollar value of inventory is determined by the use of 8% carrying cost in the model but the $490.00 of row two column five is the carrying cost at 20% of the average dollar value of inventory as determined by the 8% holding cost rate. ($2,449 x 20% = $490.00. Holding cost and carrying cost are synonomous.)

Columns nine through thirteen contain the "actual" total annual inventory costs that result if the "true" holding costs were 12%, 20%, 32%, 44%, and 56% respectively. The values of columns nine through thirteen represent the sum of the annual ordering cost, column two, and the holding cost at selected "true" or "correct" rates (columns four through eight). The $686.00 of row two and column ten is the ordering cost of $196.00 (row two column two) and the carrying cost of $490.00 at a 20% carrying cost (row two column five).

Assume that a firm uses a holding cost of eight percent.[20] Based upon inadequate knowledge concerning their opportunity cost, the use of eight percent results in annual

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[20]According to Whitin, many firms have used 8% to represent total carrying costs when the inventory models are used in actual business situations. Obviously, this figure is too low to agree with capital management decision theory.

ordering costs of $196.00 (49 orders of approximately 490 units at $4.00 per order). The average inventory is approximately 245 units or a dollar value of $2,450.00 when per unit cost is $10.00. The appropriate carrying costs of twenty percent based on more recent knowledge results in holding costs of $490.00. (Average inventory of $2,450.00 at twenty percent.) Total inventory costs of $686.00, representing the out-of-pocket costs of ordering and carrying inventory plus the opportunity cost at the higher imputed rate results rather than the $392.00 from the lower carrying cost rate of eight percent. (The $392.00 total inventory cost at eight percent is made up of $196.00 for carrying cost and $196.00 for ordering cost.)

The difference of $292.00 between the two total inventory costs at eight percent and 20% carrying costs is not the excess inventory costs resulting from the use of an eight percent carrying cost when 20% is more appropriate. Utilizing the 20% carrying cost rate instead of the eight percent rate increases the annual ordering cost. This is so because the economic order quantity is smaller requiring more orders per year. The 20% carrying cost rate results in annual ordering costs of $308.00 (row five column two) and $310.00 of holding costs (row five column five) for a total annual inventory cost of $618.00 (row five column ten).

It is the dollar difference between this adjusted total annual inventory cost at 20% (row five column ten) and the total annual inventory cost at eight percent (row two column ten) that represents the excess cost or deviation from optimum cost which results from the use of the lower imputed interest rate of eight percent for total carrying costs. Use of the eight percent imputed interest rate figure results in ordering costs of $196.00 and actual carrying costs, including the opportunity cost, of $490.00 for a total annual inventory cost of $686.00. The carrying cost of eight percent would be $196.00.[21] This includes something for the opportunity cost but is too low in that it does not represent the alternative earning power of the cost of the funds tied up in inventory. The difference between the adjusted carrying cost of $490.00 and the unadjusted carrying cost at eight percent of $196.00 is the extra value of the funds tied up in inventory as determined under capital management decision theory. The difference, $294.00, is equal to the dollar value of the average inventory, $2,499.00, times the increase in carrying cost between

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[21]As explained earlier, the dynamic model under certainty balances the opposing costs of ordering and carrying inventory. The point at which the two curves are equal represents the minimum total inventory costs. Thus, if ordering costs are $196.00, then carrying costs must also be $196.00.

the two rates, 12%. (.12 times \$2,449.00 = \$293.9 or about \$294.00.)

In the above example, the use of the "incorrect" carrying cost rate of eight percent instead of the 20% carrying cost rate resulted in an 11% deviation in total inventory costs from optimum. Total inventory costs are \$618.00 when the 20% rate is used (row five column ten). Adjusted total inventory costs are \$686.00 if the eight percent rate is used (row two column ten). The difference of \$68.00 in total inventory costs at the two rates is about an 11% deviation in total inventory costs from optimum. (\$68.00 ÷ \$618.00.)

The sensitivity of total inventory costs to various error factors in carrying costs were developed using the procedure outlined above. Table 5-2 is a summary of the percentage deviations of total annual inventory costs that result with various error factors in carrying costs. Table 5-2 is a summary of Table 5-1 and represents the results of only the dynamic model under certainty when ordering costs are equal to 40% of per unit cost. Identical analyses were performed on the output of the model for the ordering cost range of 40% to 100% of per unit cost. These results will be summarized later in the chapter.

TABLE 5-2

VARIATIONS IN TOTAL ANNUAL INVENTORY COSTS AT SELECTED ERROR FACTORS IN CARRYING COST

Error Factor in Carrying Cost	% Deviation in Total Inventory Costs From Optimum[22]
.88	.38
.82	.54
.80	.65
.77	.89
.75	1.02
.73	1.31
.74	1.34
.66	2.08
.64	2.72
.63	2.55
.60	3.56
.57	3.85
.55	4.68
.50	5.80
.45	7.73
.43	8.95
.40	11.00
.38	12.24
.36	12.95
.27	21.87

The Carrying Cost Error Factor

The deviations in total inventory costs are determined not by the error factor in imputed rates but by the error

[22]Optimum refers to the total inventory costs which would result, given individual ordering cost, individual stockout costs, carrying cost, a demand distribution, per unit price, a specific level of customer service, if the value of the imputed interest rate used in the model were in line with capital management decision theory.

factor in total carrying costs. The error factor is the dividend of the carrying cost rate used divided by the carrying cost rate that should have been used. In the case of the carrying cost error factor used in this analysis it is the carrying cost with the higher imputed interest rate as determined under capital management decision theory. Therefore, for this analysis the error factor will always be less than one and the smaller the numerical value of the error factor the greater the difference between the "correct" and the "incorrect" imputed interest being compared.

If the carrying cost is exclusively the imputed interest rate then the error factor in the imputed interest rate is the same as the error factor in total carrying cost. However, as the imputed interest rate becomes less important relative to the carrying costs the error factor in the imputed interest rate has less effect upon the error factor of total carrying costs. A firm with carrying costs exclusive of interest of four percent that uses a four percent imputed interest rate when a 12% value should be used will have a greater error factor in total carrying costs than a firm with an eight percent carrying cost exclusive of interest that uses a four percent imputed interest rate value when it should be using a 12% rate. Both firms have the error factor in the imputed interest rate of .33. However, the firm with a four percent carrying cost exclusive of interest has a carrying cost error factor of .5, while the firm with eight percent carrying cost exclusive of interest has a carrying cost error factor of .6. This means that the error factor in the imputed interest rate is of more importance in those situations in which the imputed interest rate is a significant element in total carrying costs.[23]

The columns of Table 5-1 showing total inventory costs at selected carrying costs represent numerous combinations of "correct" imputed interest rates combined with other

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[23]The error factor is the dividend of the value of the incorrect rate divided by the value of the correct rate.

$$\frac{\text{Incorrect rate}}{\text{Correct rate}} = \text{Error factor}$$

The error factors in the imputed interest rate for the example cited above are:

Firm #1	Firm #2
$\frac{4}{12} = .33$	$\frac{4}{12} = .33$

The error factors in the total carrying cost are:

$\frac{8}{16} = .5$	$\frac{12}{20} = .6$

For firm number one the carrying cost exclusive of interest of four percent is of less importance than the carrying cost exclusive of interest of eight percent of firm number two when combined with identical error factors of imputed interest of both firms. The numerical value of the error factor of total carrying cost of firm number one is smaller than the numerical value of the error factor of total carrying cost of firm number two. As mentioned previously, the smaller the error factor of total inventory costs the greater the sensitivity of total inventory costs.

carrying costs. A short example will illustrate this point.

Table 5-3 contains total inventory costs comparisons which result when a 12% carrying cost rate is used in the model instead of a 20% rate which reflects the "true" imputed interest rate as well as other carrying costs. The total inventory costs at these two rates is shown for three different levels of ordering costs.

TABLE 5-3

TOTAL INVENTORY COSTS AT SELECTED CARRYING COST ERROR FACTORS

Ordering Cost as a % of Per Unit Cost	Total Inventory Cost I = 12%	I = 20%	Different $	%
40%	$ 640	$ 618	$22	3.5%
100	1,012	780	32	3.0
200	1,434	1,393	41	2.9

Assume that a firm has carrying costs exclusive of interest of four percent and uses an imputed interest rate value of eight percent resulting in a combined total carrying costs of 12%. Assume also, that this firm should use an imputed interest rate value of 16% that would result in a total carrying cost of 20% to be used in the models. The total inventory costs that result when ordering costs are

40%, 100% and 200% of per unit cost are shown for both the "incorrect" total carrying cost of 12% and for the "correct" value of total carrying cost of 20%. The use of higher value of imputed interest as determined under capital management decision theory would result in a savings of 2.9% to 3.5% of total inventory cost.

The above results would be identical for a firm with holding costs exclusive of interest of six percent that uses an imputed interest rate of six percent when the "correct" imputed interest rate is 14%. For the 12% vs. 20% total carrying costs we see that there could be other possibilities of carrying costs exclusive of interest with "incorrect" imputed interest rate values and "correct" imputed interest rate values.

The Dynamic Model Under Risk

The simulated cost structures for the dynamic model under risk were developed using similar inputs to those used in the dynamic model under certainty. Since the dynamic model under risk solves inventory problems in which the demand is not known with certainty it was necessary to develop a probabilistic demand pattern and include stockout costs for potential out-of-stocks in simulating our variety of cost structures. The specific inputs into the dynamic model under risk were:

Per unit cost of $10.00.
Ordering cost varied from 40% to 200% of per unit cost.
Carrying cost varied from 4% to 56% of per unit cost.
Stockout cost varied from 20% to 100% of per unit cost.

The Demand Distribution

Past information concerning the demand for the product or the demand for a similar product is necessary in developing a demand distribution for use in the dynamic model under risk. With this past information, forecasts of future demand that allows for seasonal and secular trends are developed for use in the model.

Some of the more sophisticated techniques for forecasting future demand are based on weighted averages of past demand.[24] These techniques have proved successful for forecasting as determined by checks on the accuracy of the future forecasts made after the forecast period has passed.

For this analysis it was assumed that the demand for the product in question approximated a normal curve distribution.[25]

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[24] Valuable discussions of the uses of weighted averages, normal curves and potential forecasting errors can be found in Robert G. Brown's "Less Risk in Inventory Estimates", Harvard Business Review, July-August, 1959, p. 104-116, and International Business Machine, General Information Manual-Inventory Management Program and Control Techniques, 1965, p. 29-40.

[25] Numerous articles and books devoted to the develop-

Stockout Cost

As mentioned previously, the inclusion of a probabilistic demand requires the use of a stockout cost in the dynamic model under risk. Inclusion of the stockout cost effects the economic order quantities and reorder levels and, therefore, effects the sensitivity of the total inventory costs to different values of the imputed interest rate. The stockout cost ranges of 20% to 100% was varied in 10% steps for each level of the ordering cost range between 40% and 200%. In other words, assuming ordering costs of 40%, stockout cost of 20%, the entire range of carrying cost between 4% and 56% were substituted into the model. Then, holding ordering constant at 40%, the stockout cost was varied to 30% of per unit cost and the entire range of carrying cost of 4% to 56% were substituted into the model. This step was repeated for the entire range of stockout

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ment of forecasts of future demand have appeared in the past several years. One book, R. G. Brown's Statistical Forecasting for Inventory Control, McGraw-Hill Book Co., Inc., New York, 1959, includes some of the more advanced techniques of weighting past information to develop future forecasts as well as discussing the use of normal curves to approximate future demand. E. Martin Busic, Development and Application of a Gamma-Based Inventory Management Theory, unpublished Doctoral Dissertation, Michigan State University, East Lansing, Michigan has shown that certain industries might find that other statistical curves are more applicable than the normal curve in forecasting future demand.

cost and for the entire range of ordering cost. At each specific level of ordering cost and stockout cost, the entire range of carrying cost of 4% to 56% was substituted into the model.

For each of the above steps a table similar to Table 5-1 was prepared for the dynamic model under risk.[26] The tables for the dynamic model under risk also included a column for the stockout cost at each level of carrying cost. Table 5-4 shows the output of the dynamic model under risk when ordering costs are 40% of per unit cost and stockout costs are 20% of per unit cost.

The annual stockout cost increased as the carrying cost of column one increases. This is to be expected, however, since the use of higher carrying costs results in smaller inventory balances and greater stockout given a demand pattern.

Columns 5 through 9 show the carrying cost when it is assumed that 12%, 20%, 32%, 44%, and 56% are the "correct" carrying costs as versus the carrying cost shown in columns one. Columns 10 through 13 are the total inventory costs

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[26]At each level of ordering cost from 40% to 200% of per unit cost in 20% intervals there were nine different stockout costs. (The range was 20% to 100% in 10% steps.) For each of these 81 combinations of stockout cost and ordering cost, the carrying cost range of 4% to 56% was substituted into the model.

TABLE 5-4

ANNUAL COSTS AT SELECTED HOLDING COST RATES
Ordering Cost $4.00 or 40% of Per Unit Cost; Stockout Cost is 20% of Per Unit Cost

Holding Cost %	Annual Ordering Cost	Annual Stockout Cost	Average Inventory	Inventory Holding Costs D/Q (CI)					Total Inventory Costs D/Q (CI) + OCE (d R) + D/Q (S)				
				I=12%	I=20%	I=32%	I=44%	I=56%	I=12%	I=20%	I=32%	I=44%	I=56%
4	28	700	16,607	1,993	3,321	5,314	7,307	9,293	2,721	4,029	6,042	8,035	10,021
8	36	1,002	11,107	1,333	2,221	3,554	4,887	6,220	2,371	3,259	4,592	5,925	7,528
12	44	1,217	8,857	1,063	1,771	2,834	3,897	4,960	2,324*	3,032	4,095	5,158	6,221
16	52	1,415	7,357	883	1,471	2,354	3,237	4,119	2,350	2,938	3,821	4,704	5,586
20	60	1,549	6,607	793	1,321	2,114	2,907	3,700	2,402	2,930*	3,723	4,516	5,309
24	64	1,704	5,857	703	1,171	1,874	2,577	3,280	2,471	2,939	3,642	4,345	5,048
28	68	1,825	5,357	643	1,071	1,714	2,357	3,000	2,536	2,964	3,607	4,250	4,893
32	72	1,966	4,857	583	971	1,554	2,137	2,720	2,621	3,009	3,592*	4,175	4,758
36	80	2,129	4,357	523	871	1,394	1,917	2,440	2,732	2,080	3,603	4,126	4,649
40	89	2,222	4,107	493	821	1,314	1,807	2,300	2,799	3,127	3,620	4,113	4,606
44	88	2,323	3,857	463	771	1,234	1,697	2,160	2,874	3,182	3,645	4,108*	4,571
48	92	2,434	3,607	433	721	1,154	1,587	2,020	2,959	3,247	3,670	4,113	4,546
52	96	2,555	3,357	403	671	1,074	1,477	1,879	3,054	3,322	3,725	4,128	4,510
56	100	2,690	3,107	373	621	994	1,367	1,739	3,163	3,411	3,784	3,784	4,529*

that result when 12%, 20%, 32%, 44%, and 56% are assumed to be the "correct" carrying cost rather than the rates shown in column 1. Columns 10 through 14 include annual ordering cost and stockout cost at the rate shown in column one and the carrying cost rate shown in columns 4 through 8. (This is identical to the procedure used for the dynamic model under certainty except that the stockout cost is now included in the total inventory costs.)

The deviations in total inventory costs that results from the use of the "incorrect" rates and the assumed "correct" rates were then expressed as a percentage from the optimum. (The optimum again is assumed to be the total inventory cost that would result if the total carrying cost included the imputed interest rate as determined under capital management decision theory.) The deviations in total inventory costs from optimum for the two dynamic models along with a discussion of the significance of the deviations are included in the next section.

Section Three: The Sensitivity Analysis of the Simulated Cost Structures

The sensitivity of total inventory costs to changes in the values of the imputed interest rate are summarized in Tables 5-5 and 5-6. Table 5-5 is the summary of results for the dynamic model under certainty; Table 5-6 is the summary of results for the dynamic model under risk.[27]

TABLE 5-5

VARIATIONS OF TOTAL INVENTORY COST FROM OPTIMUM FOR SELECTED GROUPINGS OF THE ERROR FACTOR RANGE OF TOTAL CARRYING COST

The Dynamic Model Under Certainty	
Carrying Cost Error Factor	Percentage Variations of Total Inventory Cost From Optimum
From .76 to .99	Less than 1%
From .67 to .75	More than 1% but less than 2%
From .57 to .66	More than 2% but less than 4%
From .50 to .56	More than 4% but less than 6%
From .40 to .49	More than 6% but less than 11%
From .30 to .39	More than 11% but less than 15%
From .01 to .29	More than 15%

The percentage variations of total inventory cost from optimum in Table 5-5 and 5-6 represent the extra costs of maintaining inventory that a firm incurs when using a value of the imputed interest rate that is lower[28] than the value

[27]The groupings of the error factor range of .01 to .99 for the total carrying cost are identical for both the dynamic model under certainty and the dynamic model under risk. The groupings of the error factor range were developed on the basis of the percentage deviations of the total inventory costs for the dynamic model under certainty. The same groupings were set for the dynamic model under risk in order to compare the sensitivity of total inventory costs for both of the dynamic models.

[28]The analysis has concentrated on comparisons of values of the imputed interest rate in which the "incorrect" rate has been lower than the "correct" rate resulting in

TABLE 5-6

VARIATIONS OF TOTAL INVENTORY COST FROM OPTIMUM FOR SELECTED GROUPINGS OF THE ERROR FACTOR RANGE OF TOTAL CARRYING COST

The Dynamic Model Under Risk

Carrying Cost Error Factor	Percentage Variations of Total Inventory Cost From Optimum
From .76 to .99	From Less than 1% to 3%
From .67 to .75	From 1% to 4%
From .57 to .66	From 2% to 6%
From .50 to .56	From 5% to 7%
From .40 to .49	From 8% to 12%
From .30 to .39	From 11% to 17%
From .01 to .29	More than 15%

of the capital charge which would be used if capital management decision criteria were used. The percentage variation is the savings of total inventory costs that a firm would

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error factors with a fractional numerical value of less than one. If the value of the imputed interest rate is overstated, greater than the value of the imputed interest rate as determined under capital management decision theory, the numerical value of the error factor will be greater than one. (An error factor of one is not an error factor because the "incorrect" rate used and the "correct" rate are identical.)

An overstatement of the value of the imputed interest rate results in a smaller variation from optimum in total inventory cost than does the same value of an understatement of the imputed interest rate. In other words, for any given cost structure if the "correct" value of the imputed interest rate were twelve percent it would be better to overstate this cost by using sixteen percent than to understate this cost by using eight percent.

achieve if it reevaluated its imputed interest rate and used the value for this cost that was in line with capital management decision theory.

A comparison of Table 5-5 and Table 5-6 shows that the dynamic model under risk is more sensitive to error factors in the carrying cost than is the dynamic model under certainty. For error factors between .76 and .99 the maximum deviation from optimum total inventory costs for the dynamic model is less than 1%. For the dynamic model under risk for the same error factor range, .76 to .99, the deviation in total inventory costs from optimum can be as high as 3%. This greater sensitivity of the dynamic model under risk continues for all values of the carrying cost error factor groupings. In all cases, the deviation from optimum total inventory costs are greater for the dynamic model under risk than for the dynamic model under certainty.

The potential savings, as a percent of total inventory costs, that are available to a firm from a reassessment of their imputed interest cost depends on several factors.

First is the imputed interest rate value presently used and the imputed interest rate value that would be used if capital management decision theory was used.

Second is the relative importance of the imputed interest rate to the other carrying cost exclusive of the imputed interest rate.

The firms that will benefit most from a reevaluation of the imputed interest rate value used in the inventory models are those that presently use very low imputed interest rate values when the opportunity cost of capital is high and other carrying costs, relative to the imputed interest rate value, are low.

The use of a value below seven or eight percent because: 1) this is the rate that could be earned on government bonds, 2) this is the rate at which we can borrow, or 3) this is the rate presently being earned on other assets, when the real opportunity cost is twenty-five or thirty percent because projects are not being undertaken for lack of funds, represent some of the most serious violations of the use of imputed interest rate values in line with capital management decision theory. The correction of the use of these erroneous rates would be most beneficial to the firms concerned since the percentage savings of total inventory costs would be rather large.

Many firms will be able to reduce inventory costs by the larger percentages shown for the lower value groupings of error factors for another reason. The costs included for use in the models, as mentioned previously in detail, must always be the marginal cost portions of the total cost in question.

In the short run analysis or in any decision-making period many of the other costs associated with carrying inventory, such as storage cost, taxes, insurance and obsolescence, will be fixed for the period under consideration. On the other hand, the imputed interest rate cost is always marginal in nature. Hence, the imputed interest cost portion of total carrying cost will have a greater relative importance as a percent of total carrying cost than is first realized from a casual observation of the components of total carrying cost. The greater relative importance of this component of total carrying cost results in smaller numerical values of the error factors of total carrying cost, or greater potential savings of total inventory cost, for many firms.

Those firms with the above characteristics can attain substantial savings in total inventory cost and profit increases through a shift in the composition of their assets out of inventory into more profitable assets by the use of imputed interest rate values in line with capital management decision theory.

It should also be remembered that it is not only those firms with low numerical values of the carrying cost error factors that can benefit from a reevaluation of the imputed interest rate value used in the models.

For firms with a relatively high numerical value error factor, for example between .76 and .99, the deviation in total inventory costs from optimum is somewhere between 1% and 3% for the dynamic model under risk.

Although the percentage savings of total inventory cost appears small, the absolute dollar savings can be substantial and the percentage increase on profits can be greater than the percentage savings of inventory costs for those firms working on a small profit margin.

The percentage savings of total inventory cost for various error factors of total carrying costs, as shown in Table 5-5 and Table 5-6, is taken on the base of total cost and is expressed as:[29]

$$T.C. = 2DCSI$$

where

T.C. = total cost
D = annual demand
C = unit cost
S = ordering cost per order
I = carrying cost per year (as a % of per unit cost)

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29 where $T.V.C. = \frac{QC}{2} I + \frac{D}{Q} S$

$$Q = \frac{2DS}{IC}$$

(continued)

As is obvious from the formula, the total annual cost varies directly with the square root of demand times the per unit price. Because of this relationship, total cost varying directly with the square root of annual demand times per unit prices, we see that:

1) high demand items with high prices,
2) low demand items with high prices, and
3) high demand items with low prices

can result in a large annual total inventory cost. Even small percentage savings of the large total inventory costs can result in substantial absolute dollar amounts for a firm.

A small percentage decrease of two or three percent in total inventory cost can result in a much greater percentage increase in profits. The impact of the savings on profits will be greater for a firm working on a small profit

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then

$$T.V.C. = \frac{2DS}{IC} \cdot \frac{CI}{2} + \frac{DS}{\frac{2DS}{IC}}$$

$$T.V.C. = \frac{2DSC^2I^2}{IC2^2} + \frac{D^2S^2}{\frac{2DS}{IC}}$$

$$T.V.C. = \frac{DSCI}{2} + \frac{D^2S^2IC}{2DS}$$

$$T.V.C. = \frac{DSCI}{2} + \frac{DSCI}{2}$$

$$T.V.C. = \frac{2^2DSCI}{2}$$

$$T.V.C. = 2DSCI$$

margin than for a firm with a large profit margin. Decreasing total inventory cost by a small percentage could result in increasing profits in the same manner that taking advantage of purchase discounts can effect profits.

Assume, for example, that a firm has a cost of sales of purchased items of fifty cents and sells the item for one dollar. Assume also, that the fifty cent gross profit represents forty cents of other costs and ten cents of profits for a margain of .10. If the above merchandise were purchased under discount terms of 2/10 n/30, but the bill paid after ten days, the prompt payment of the bill would reduce the cost of the merchandise to forty-nine cents and increase the profits of each dollar to eleven cents and the margin to .11. A two percent reduction in the cost of the merchandise results in a ten percent increase in profits. A greater percentage increase in profits would have resulted if the firm's margain were even smaller. Conversely, if the margin were larger, the increase in profit would not have been as great. The small percentage savings of one to three percent of total inventory costs for error factors between .76 and .99 can effect profits and sales margins in the same manner as the purchase discount. Although the reduction in the cost of sales or other expenses might only be between one and three percent, it can effect the margin to a greater degree, depending on the size of the profit margin.

For firms with lower error factors in carrying cost (greater percentage savings of total inventory cost) the absolute dollar savings or their percentage increase in profits will be even greater. The absolute dollar savings will depend on the size and dollar value of the total inventory; the potential increase in profits will depend upon the firm's profit margin.

The savings to any firm in inventory costs will depend upon the dollar amount of inventory held by the firm. For those firms in which inventory is relatively unimportant this analysis is not very useful. Inventory management activity is a more vital activity in some types of businesses than in others. There are certain industries in which inventory management is the dominant element in the business operation. For those firms in which inventory management is an important activity, the use of imputed interest rates in line with capital management decision theory will prove most beneficial in maximizing profits.

The Requirements to Obtain the Savings in Total Inventory Cost

An effective inventory control system, one that minimizes the total cost associated with inventory for a given level of operations, must have an organizational structure that includes the following characteristics.

First, an alert management that is aware of the developments in research being performed presently to reduce current business problems. Not only must management be aware of the present day research but they must also be capable of developing a working organization that can benefit from the present day research by applying it to the firm's specific problems.

The working organization developed by an alert management must also include effective communication among the personnel involved. New ideas, old or new problems, and solutions to various problems must be communicated from one level of management to the next and among personnel at the same level in order to have an effective organization. Qualified personnel and the organization they belong to are useless if the problems, solutions and ideas that represent their operation do not circulate.

A well designed record keeping system, one broad enough to include other necessary information as well as the traditional accounting information, will be beneficial to management in developing figures necessary for the efficient operation of the firm. Required information that is both accurate and timely results in increased communication between the accountants and other members of the firm and is necessary for accurate decisions.

The specific organizational structure of the inventory control function is dictated by the type of firm involved. For example, if the firm is primarily a merchandising operation the overall responsibility of the inventory control system would most likely rest with a purchasing man. Conversely, in a production oriented organization in which a major portion of the inventory were raw materials the overall responsibility would probably rest with the production manager or a vice president in charge of production.

Regardless of the specific organizational structure and the background of the individual in charge the inputs required for the inventory models should be developed and frequently reevaluated by those persons most closely associated with the specific function which gives rise to the cost involved.

The accountant must develop, for each decision period, the inputs needed by collaboration with individuals closely associated with each function. The carrying costs other than the imputed interest charge can be developed from the accounting records and modified when necessary by discussions with the warehouse manager. The marginal ordering cost must be developed by observation and consultation with members of the purchasing department. The sales personnel must help in developing realistic estimates of future sales in the light of expected economic conditions. The financial people must develop the value of the imputed interest rate, as determined from the capital budgeting area, as well as an estimate of the firm's cost-of-capital when necessary. Stockout costs must also be developed for each decision making period by those people most aware of the impact an out-of-stock will have on the firm's overall operation.

The savings in total inventory cost, as shown in this research, are possible for firms using values of the imputed interest rate in the inventory models that are not in line with capital budgeting decision theory. The use of lower values of the imputed interest rate, as suggested by the current literature, results in total inventory costs that are not minimum and negates some of the advantages of using the inventory models.

In order to attain the potential savings of total inventory cost discussed in this research it is necessary that an effective organization be set up for the inventory control system. As is true of any cost savings operation in a firm, the overall responsibility for the system must be delegated to one individual who will insure the effective cooperation of needed personnel in attaining the predetermined goal of the firm--maximization of the long-run earnings of the present stockholders.

CHAPTER VI

SUMMARY

Businesses have maintained stocks of raw materials and finished goods for many years. The asset inventory is a significant portion of the total assets of many firms. The efficient management of inventory is an important element in the operation of the individual firm and also holds many economic and social implications when viewed in the aggregate.

Inventories of goods, even those in excess of necessary quantities to carry on the process of production and distribution, were maintained prior to the industrial revolution. This was so because inventories of goods, besides those of gold and silver, were viewed as forms of wealth. Hence, a man with large quantities of grain, flocks and herds, was considered to be very wealthy. Riches were considered to be all the necessaries such as raiments, houses, furniture and food. The use of gold and silver in these periods was only considered more favorably because of their more liquid nature than raiments, houses, furniture and food.

The concept of wealth has changed since the industrial revolution. With this change in the concept of wealth has come a new thinking on the maintenance of inventories. Now, the reason for investing in inventory is identical to the reason for investment in any asset; namely, to increase the rate of return on capital invested in the business.

Businessmen became aware of the fact that excessive investment in inventory, like excessive investment in any asset, would reduce the profitability of the firm. In order to reduce investment in inventory to optimal levels the businessmen developed several methods to control inventory levels. Some of the more common devices used were:

1) Hand-to-mouth buying
2) Fixed Order Systems
3) Periodic Reordering Systems
4) Scientific Inventory Methods

The scientific inventory methods were first published around the early 1900's. The basic concepts first published at that time have been refined and are now capable of handling almost any inventory problem a businessman might have. However, although the scientific inventory methods have been available since the early 1900's, they did not gain popular support by businessmen until after World War II, and are still only used by the relatively larger and more progressive business firms.

The scientific inventory method changed the emphasis of inventory control from attempting to minimize inventory levels to attempting to minimize the total costs associated with the inventory level. The minimization of total inventory costs is accomplished by using mathematical models which balance the costs of carrying inventory against the costs of holding inventory. (The costs used in the models are different than typical accounting costs and probably represent one of the reasons for the slow adoption of these models by business firms.

Using out-of-pocket costs, opportunity costs, probability distributions when necessary, and decisions by management as to customer service levels in the models, it is possible to minimize total inventory costs. The development of one of these opportunity costs, the cost of capital invested in inventory, has not been developed fully in the literature. It is the contention of this writer that the appropriate charge for this cost is the estimated rate of return on capital expenditures for which capital is unavailable with the minimum acceptable rate being equal to the firm's cost-of-capital. The use of any other rate for this cost will result in total inventory costs which are not optimal.

A sensitivity analysis of total inventory costs was determined by the models under alternative cost structures. The sensitivity analysis compared total inventory costs when the appropriate rate of interest, as suggested by the writer, was used rather than using the erroneous interest rates suggested by the literature.

The inventory models are closed systems in which the end result, re-order points and economic order quantities, depend on the value of the cost inputs. These factors determine total inventory cost. If the costs used in the models are not representative of the actual situation they are attempting to stimulate, the results derived from the models will not be optimal. Hence, using interest rates suggested by the literature in the scientific models does not result in minimum total inventory costs.

The degree of the error in the total inventory costs resulting from the use of "erroneous" imputed interest charges is dependent upon the significance of the imputed interest rate charge portion of total inventory costs.

Although the difference in the total inventory costs when the "correct" imputed interest rate is used and the "erroneous" imputed interest rate is used may not be great, there are two factors which must be taken into account. First, a small savings in total inventory costs might represent a large absolute dollar amount because of the significance of total inventory costs. Second, a small percentage savings in total inventory costs might have a much larger percentage increase in net profits for a firm working on a small profit margin. This

would be similar to the effect on net profit of taking a purchase discount not previously taken, one or two percent of cost of goods sold, when the firm has a very low sales margin.

The scientific inventory models, like most models used in business, can only produce answers as good as the inputs fed into the system. The more refined and researched the inputs the better the final output. This refinement of the inputs enhances the value of the models as well as increasing the value of the output of the models, resulting in the more efficient utilization of a firm's assets and, more important, increasing the long run earnings of present stockholders--the firm's major objective.

APPENDICES

APPENDIX ONE

DYNAMIC MODELS UNDER CERTAINTY WITH DISCOUNTS INCLUDED

The dynamic model under certainty sets up total variable cost as a function of the opposing costs associated with carrying inventory.

$$T.V.C. = \frac{QC}{2} I + \frac{D}{Q} S$$

where

T.V.C. = total variable cost	D = annual demand
Q = quantity to order	C = unit cost
S = ordering cost per order	I = carrying cost per unit per year (as a % of cost)

Differentiating T.V.C. with respect to Q, setting the derivative equal to zero and solving for Q, gives

$$Q = \sqrt{\frac{2DS}{IC}}$$

<u>The Continuous Discount</u>

Assume that the seller includes certain fixed costs, F, in his selling price. This assumption is not unrealistic since price differences due to quantity purchases arise as a result of fixed costs that have been incurred by the supplier.

The fixed cost, F, is included in the price of the product by the supplier although this fact may not be stated explicitly. The cost of a unit is, then, the sum of the fixed cost and a variable cost, designated as P in this example. The cost of Q units is (F + PQ) and the unit price is $(\frac{E}{Q} + P)$, which decreases as the quantity, Q, increases.

Then

$$T.V.C. = \frac{Q}{2}\left(\frac{F}{Q} + P\right) I + \left(\frac{D}{Q}\right)S + \frac{D}{Q}(F + PQ)$$

The first two terms are identical with those used in the model which does not include discounts. $\frac{D}{Q}(F + PQ)$, the annual cost of purchases, is included because it varies with the number of purchases made each year, $(\frac{D}{Q})$.

Rearranging terms

$$T.V.C. = \frac{IF}{2} + \frac{PQI}{2} + \frac{D}{Q}S + \frac{D}{Q}F + DP$$

Differentiating T.V.C. with respect to Q and setting the derivative equal to zero:

$$\frac{d(T.V.C.)}{d(Q)} = \frac{PI}{2} - \frac{D(S + F)}{Q^2} = 0$$

Then Q, the optimum quantity to order when a continuous discount is available is[1]

$$Q = \sqrt{\frac{2D(S + F)}{PI}}$$

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[1] Edward H. Bowman and Robert B. Fetter, _Analysis for Production Management_ (rev. ed.; Homewood, Ill.: Richard D. Irwin, Inc., 1961), p. 274; Thomson M. Whitin, _The Theory of Inventory Management_ (Princeton, N. J.: Princeton University Press, 1957), p. 34; Robert B. Fetter and Winston C. Dalleck, _Decision Models for Inventory Management_ (Homewood, Ill.: Richard D. Irwin, Inc., 1961), p. 36.

The Discontinuous Discount

Modification of the basic dynamic inventory model to include discontinuous discounts is performed in six operations.

First, compute total variable cost per annum when the economic lot size is purchased at the regular price.

$$T.V.C. = \frac{QC}{2} I + \frac{D}{Q} S$$

Second, compute total variable cost per annum if the quantity necessary to get the discount is purchased.

$$T.V.C._D = \frac{Q_D C_D}{2} I + \frac{D}{Q_D} S$$

Where

$T.V.C._D$ = annual total variable cost at the discounted price when the quantity necessary to get the discount is ordered

C_D = the unit cost at the discount price

Q_D = minimum quantity necessary to obtain the discount price

Third, compute the savings per annum that occurs when the discount price is paid for the merchandise.

$$A = D(C - C_D)$$

Where A = annual savings

Fourth, make the following test: If the annual savings plus the total variable cost per annum at the regular price less the total variable cost when the quantity necessary to obtain the discount is purchased is greater than zero, the discount should be taken.

Mathematically the test is expressed as follows: Take the discount if A + T.V.C. - $T.V.C._D > 0$. By manipulation of the formula we get A $>$ $T.V.C._D$ - T.V.C. The annual savings associated with buying the larger quantity must exceed the additional cost associated with a larger average inventory.

If the test in step four indicates it is advisable to take advantage of this price break, then two more steps are necessary.

Fifth, compute the optimum order size at the discount price.

$$T.V.C. = \frac{QC_D}{2} I + \frac{D}{Q} S$$

$$\frac{d(T.V.C.)}{d(Q)} = \frac{IC_D}{2} - \frac{DS}{Q^2}$$

$$Q_D^* = \sqrt{\frac{2DS}{IC_D}}$$

Q_D^* is the optimum order size at the discount price, C_D.

Sixth, begin with step one again to determine if it is advisable to take advantage of the next price break in the discount schedule.[2]

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[2]Fetter and Dalleck, p. 36; Whitin, p. 36.

APPENDIX TWO

MATHEMATICAL SOLUTION OF THE DYNAMIC MODEL UNDER RISK

The mathematical solution of the dynamic inventory problem under risk is similar to the solution of the dynamic inventory problem under certainty. One difference is the inclusion of stockout costs in the dynamic model under risk, necessitated by varying usage rates during lead time. Another difference between the dynamic model under certainty and the dynamic model under risk is the interdependence of R, the reorder point, and Q, the economic order quantity, which occurs when demand under conditions of risk is included in the analysis. Equations for R and Q are readily derived but cannot be solved simultaneously because R and Q are now interdependent variables. However, the reorder level and economic order quantity that minimize total variable cost can be determined by solving the equations for R and Q by iteration.

The total variable cost[1] for the dynamic model under risk consists of three components: carrying cost, ordering cost, and stockout cost.

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[1]The model assumes that items demanded but not available are lost sales. The model can be modified to permit back ordering of items demanded but not available.

$$T.V.C. = IC\left[\frac{Q}{2} + R - (\bar{d} - E(d>R)\right] + \frac{D}{Q}(S) + \frac{D}{Q}\left[OE(d>R)\right]$$

where

$$IC\left[\frac{Q}{2} + R - (\bar{d} - E(d>R)\right] = \text{annual carrying cost}$$

$$\frac{D}{Q} S = \text{annual ordering cost}$$

$$\frac{D}{Q}\left[OE\ (d>R)\right] = \text{annual expected stockout cost}$$

The first term of the equation, the annual carrying cost, is composed of the annual per unit carrying cost (IC) times the average inventory for the year $(\frac{Q}{2} + R - (\bar{d} - E(d>R))$. The average inventory is the sum of one half the economic order quantity plus the average safety stock. R is the number of units on hand when the order is placed. $\bar{d}$ is the average usage during lead time. If no stockouts were expected during any reorder period $(d \leq R)$ the average safety stock--defined as the stock on hand when an order is received--would be $R - \bar{d}$. However, when some stockouts are expected $(d > R)$ the average usage during lead time $(\bar{d})$ will be smaller than when stockouts do not occur by the expected number of units demanded but not available $E(d>R)$.[2] $E(d>R)$ is the weighted average of units demanded but not available and changes value for different values of R. The average safety stock is, therefore, the quantity of merchandise on hand when the order is

[2] $E(d>R)$ is the weighted average of the units demanded but not available. The method used to compute this figure is presented later in the appendix.

placed less the average usage during lead time. Average usage during lead time is the expected usage less the quantity of units demanded but not available.

Safety stock $= R - (\bar{d} - E(d>R)) = R - \bar{d} + E(d>R)$

The second term, $\frac{D}{Q}S$, represents the annual ordering cost. $\frac{D}{Q}$ is the number of orders placed per year. S is the cost of each order. It follows that the product of $\frac{D}{Q}$ and S is the annual ordering cost.

The third term represents the stockout costs per year ($\frac{D}{Q}\left[OE(d>R)\right]$). O is the stockout cost per unit demanded but not available. $E(d>R)$ is the expected number of units demanded per lead time that are not available. $OE(d>R)$ is the expected stockout cost for one lead time. $\frac{D}{Q}$ represents the number of lead time a year (the number of orders placed). Then, $\frac{D}{Q}OE(d>R)$ represents the annual expected stockout cost.

The Formula for Determining Q

The equation for total variable cost can be simplified by rearranging terms.

$$T.V.C. = IC\left[\frac{Q}{2} + R - (\bar{d} - E(d>R))\right] + \frac{D}{Q}S + \frac{D}{Q}\left[OE(d>R)\right]$$

$$T.V.C. = \frac{ICQ}{2} + ICR - IC\bar{d} + (IC)E(d>R) + \frac{D}{Q}\left[OE(d>R) + S\right]$$

$$T.V.C. = \frac{ICQ}{2} + \frac{D}{Q}(OE(d>R) + S) + IC\left[Q - \bar{d} + E(d>R)\right]$$

If R is assumed to be a constant independent of Q, the partial derivative[3] of total variable cost with respect to Q can be taken.

$$\frac{2(T.V.C.)}{2(Q)} = IC - \frac{2D}{Q^2}\left[S + OE(d > R)\right]$$

Setting the partial derivative equal to zero and solving for Q gives

$$Q = \sqrt{\frac{2D(S + OE(d > R)}{IC}}$$

The Formula for Determining R

R, the optimum reorder level, is determined by equating the opposing costs associated with changing the amount of stock on hand when an order is placed. As R increases (ordering earlier), the carrying cost associated with the increasing safety stock increases, while the probability of a stockout decreases because the safety stock increases and, it follows, that stockout cost decreases. The optimum R is that point where the rising carrying cost and the falling stockout cost are equal.

The cost of carrying one unit of inventory for one year is IC. The carrying cost of one unit for one reorder period is the annual cost (IC) divided by the number of reorder

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[3]The slope of a function of one variable is determined by the derivative of the function. Similarly, the slope of a function of several variables is called the partial derivative of the function when all variables but one are assumed to be constant.

periods per year $(\frac{D}{Q})$. In other words, increasing R by one unit increases the carrying costs for each reorder period by:

$$\frac{IC}{\frac{D}{Q}} = \frac{ICQ}{D}$$

The probability of a stockout decreases as R is increased. Therefore, the per unit stockout cost (O) times the probability of a stockout ($P(d>R)$) represents the decrease in expected stockout cost associated with a one unit increase in R. However, given demand greater than R, a further saving arises in that the safety stock is reduced. The reduction in the safety stock results in a reduction in carrying costs. The savings in carrying cost is the expected savings from the reduction in safety stock $P(d>R)\frac{ICQ}{D}$. The total saving or reduction in stockout cost at the margin resulting from a one unit increase in R is:

$$P(d>R)\frac{ICQ}{D} + OP(d>R)$$

or

$$\left[P(d>R)\right]\left[\frac{ICQ}{D} + O\right]$$

Instead of handling the probable reduction in carrying cost associated with reductions of safety stock as a part of the reduction in stockout costs, this item could be treated directly with carrying costs.[4]

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[4]The carrying cost which occurs with an increase in R of

The optimum reorder point is that level of inventory where the marginal carrying cost equals the marginal stock-out cost (marginal savings)[5] or where

$$\frac{ICQ}{D} = P(d>R)\left[\frac{ICQ}{D} + O\right]$$

$$P(d>R) = \frac{\frac{ICQ}{D}}{\frac{ICQ + OD}{D}}$$

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one unit $\left(\frac{ICQ}{D}\right)$ could be stated as the expected carrying cost of increasing R by one unit. Multiplying $\frac{ICQ}{D}$ by the probability of the item remaining in stock during the reorder period would give the expected carrying cost of increasing R by one unit. The probability of the item not being in stock is $P(d>R)$. $P(d>R)$ decreases as R increases. The probability of the item remaining in stock during the reorder period is $(1-P(d>R))$ and increases as R increases. (The sum of the probabilities of remaining in stock or not remaining in stock must equal one.) The expected carrying cost incurred by increasing R by one unit is

$$\left[1 - P(d>R)\right] \frac{ICQ}{D}$$

This must equal stockout costs which would consist only of $OP(d>R)$. It follows, then, that the optimum R is at that point where expected carrying costs equal expected stockout costs, or where

$$\left[1 - P(d>R)\frac{ICQ}{D}\right] = OP(d>R)$$

$$\frac{ICQ}{D} - P(d>R)\frac{ICQ}{D} = OP(d>R)$$

$$\frac{ICQ}{D} = P(d>R)\left[\frac{ICQ}{D} + O\right]$$

$$P(d>R) = \frac{\frac{ICQ}{D}}{\frac{ICQ}{D} + O}$$

$$P(d>R) = \frac{ICQ}{ICQ+OD}$$

[5]This result is identical with the solution of the static model under risk. However, it is only a partial solution because the interdependence of R and Q must be taken into account.

$$P(d > R) = \frac{ICQ}{ICQ + OD}$$

The above formula for R and the formula for Q

$$Q = \sqrt{\frac{2D(S + OE(d > R))}{IC}}$$

can be solved by the iterative process which contains the following steps.

1) Assume that E(d>R) is zero and compute Q, the economic order quantity.

2) Use the Q determined in step one and solve the equation for P(d>R) to arrive at an R.

3) Use the R computed in step two to determine E(d>R).

4) Insert the value of E(d>R) in the formula for Q to determine the new economic order quantity.

5) Repeat steps two, three and four until convergence occurs. This gives the optimum Q and R.[6]

Solving the problem previously[7] used to illustrate the reorder level-lot size method of determining R and Q for dynamic inventory problem by the mathematical method will help to clarify the method.

The relevant data for item 2 in the example are:

Cost per unit $15.00 (C = $ 15.00)

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[6]Robert B. Fetter and Winston C. Dalleck, Decision Models for Inventory Management (Homewood, Ill.: Richard D. Irwin, Inc., 1961), p. 17.

[7]See Chapter III, page 26.

Ordering cost per order	\$10.00	(S = \$ 10.00)
Stockout cost per day	\$100.00	(O = \$100.00)
Demand per year	73,000	(D = 73,000)
Carrying cost per year	.20	(I = .20)

(1) Solving for Q, assuming that $E(d>R)$ is zero, gives

$$Q = \sqrt{\frac{2(73,000)(10+0)}{3}} = 698 \text{ units.}$$

(2) Determining the value of $P(d>R)$ using Q equal to 698 units gives

$$P(d>R) = \frac{(.20)(15)(698)}{(.20)(15)(698)+.5(73,000)} = .0543$$

(3) The probability of demand being greater than R, $P(d>R)$, does not contain a value of .0543. The closest value is .07 and will be used. $P(d>R)$ of .07 is reorder level fourteen with a usage of 2800 units. With this information it is possible to compute the expected value of demand greater than that allowed for in the reorder level, $E(d>R)$.

If the order is placed when fourteen days of supply remains (2800 units), the recurrence of lead times of 15, 16, or 17 days would result in shortages. $E(d>R)$, the mean value of the expected number of units out of stock, is the sum of the expected values of 15-, 16-, or 17-day lead times occurring. It is computed by multiplying the number of units short for each of the possible lead times (15, 16, or 17 days) and multiplying the possible units short by the probability of this action occurring. The value of each of these lead times

PROBABILITY DISTRIBUTION OF EXPECTED LEAD TIME

Possible Reorder Levels R	Possible Demand During Lead Time	Probability of This Demand P(d)	Probability of Demand Being Greater than R, P(d>R)
5	1000	.00	1.00
6	1200	.01	.99
7	1400	.03	.96
8	1600	.08	.88
9	1800	.14	.74
10	2000	.18	.56
11	2200	.18	.38
12	2400	.14	.24
13	2600	.11	.13
14	2800	.06	.07
15	3000	.04	.03
16	3200	.02	.01
17	3400	.01	.00
18	3600	.00	.00

is then summed.

Lead Time in Days	Units Short P(d)	
15	(3000-2800) (.04) = (200) (.04)	= 8 units
16	(3200-2800) (.02) = (400) (.02)	= 8 units
17	(3400-2800) (.01) = (600) (.01)	= 6 units
	E(d>R) =	22 units

(4) Recompute Q using 22 units for E(d>R).

$$Q = \sqrt{\frac{2(73,000)(10 + .5(22)}{3}} = 1011 \text{ units}$$

(5) Recompute P(d>R).

$$P(d>R) = \frac{(.2)(15)(1011)}{(.2)(15)(1011) + .5(73,000)} = .0767$$

$P(d > R)$ results in a reorder level of 14 again. Hence, the solution is R of 14 days (2800 units on hand when order is placed) and the optimum quantity to order is 1011 units. This is the same result as developed by the reorder level-lot size method of solution.

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