THE DEVELOPMENT OF A STATE-SPACE MODEL OF AN AEROSPACE ENTERPRISE AND ITS EVALUATION AS A PRACTICAL MANAGEMENT TOOL

Thesis for the Degree of Ph. D.

MICHIGAN STATE UNIVERSITY

RENE V. ELICAÑO

1967

This is to certify that the

thesis entitled

THE DEVELOPMENT OF A STATE-SPACE MODEL
OF AN AEROSPACE ENTERPRISE AND ITS EVALUATION
AS A PRACTICAL MANAGEMENT TOOL

presented by

Rene V. Elicano

has been accepted towards fulfillment
of the requirements for

Ph.D. degree in Management

Richard F. Gonzalez
Major professor

Date November 17, 1967

O-169

ABSTRACT

THE DEVELOPMENT OF A STATE-SPACE MODEL OF AN AEROSPACE ENTERPRISE AND ITS EVALUATION AS A PRACTICAL MANAGEMENT TOOL

by Rene V. Elicaño

This study constructs a detailed mathematical model of the internal operations of an actual aerospace engineering and manufacturing enterprise. It uses a methodology originally developed for analyzing system models of electrical networks. This study evaluates the practical utility of this technique in assisting the business executive.

The methodology used offers a formal process for generating a model from a defined system structure and for analyzing the solution characteristics of the model. This approach involves the use of linear graphs, flow and unit cost variables and the development of a state model by reduction of the algebraic and difference equations that characterize the system to a minimal set. This model can be used for simulation and for stability and control analysis.

The model in this study is essentially a direct costing accounting type model. It follows product flow from receipt of order to shipment thru the internal sectors of the firm. It shows the flow of resources and overhead function services into the different product stages and imputes their cost

to the product.

Simulation based on the model is presented. The model is found to be stable and state controllable. The control strategy to reduce the state to zero is derived.

Considerable practical utility is found for both the methodology and the model. The methodology provides a convenient formal and generalized procedure for developing a model of the firm. This model of the firm can be quite detailed without sacrificing compactness. This makes feasible a real-time inquiry and simulation system whereby management can interrogate the model and get fast response time, without interfering with the production programs that the computer is processing simultaneously.

The model has many managerial uses such as forecasting costs, load, resource requirements or cash flow and analyzing total impact of alternative decisions.

The most promising facet of this methodology is its generalized analytical capabilities. As management pins down more of the true cause and effect relationships in the firm, the control strategy from the model will play an ever increasing role in guiding their key decisions.

THE DEVELOPMENT OF A STATE-SPACE MODEL OF AN AEROSPACE ENTERPRISE AND ITS EVALUATION AS A PRACTICAL MANAGEMENT TOOL

By

Rene V. Elicaño

A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Management

1967

ACKNOWLEDGMENTS

The author wishes to express his appreciation to the many persons who have assisted him directly or indirectly in making this study. Special thanks are due to Mr. T. R. Rao, Dr. Richard Gonzales, Dr. Thomas Farrell, and Dr. Herman Koenig for their invaluable advice and technical guidance.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF EXHIBITS

CHAPTER I

INTRODUCTION

Purpose of the Study

The purpose of this study is to attempt to construct a mathematical model of an actual aerospace engineering and manufacturing enterprise, using a methodology originally developed for analyzing system models of electrical networks. The study evaluates the practical utility of this technique in assisting the business executive.

It has been the dream of many executives to have at their disposal, a comprehensive and integrated computerized model of the entire firm, facilitating the planning and forecasting tasks, simulating the over-all impact of alternative decisions on a real-time basis, and utilizing sophisticated techniques for developing optimal policies for the total firm with its complex network of interacting relationships.

This study represents one phase of a long range program to provide the management of the firm being analyzed with such a model, tailored to its specific needs and characteristics and developed to the limits of technological and economic feasibility.

Although the model constructed here is strictly a prototype, every effort has been made not to compromise the accuracy and integrity of the model. Company-private data has been modified without detracting from the realism of the model.

Significance of the Study

The use of mathematical models as a managerial tool is nothing new. However, the majority of the practical applications have utilized either straightforward heuristic trial-and-error simulation or highly specialized analytical techniques. The approach evaluated in this study was chosen because of its potential value as a more generalized analytical tool for business models. If a realistic and comprehensive model of the internal operations of a firm can be built without being unwieldy, and if this model can be used not only to simulate the impact of alternative management decisions but also to determine analytically the optimum policies to attain specified objectives, then this methodology is of considerable practical utility to the executive.

As best as can be determined, this study represents the first attempt to apply this particular methodology to develop a detailed model of the internal operations of an actual firm. A previous study by Mr. Linn Soule,[1] applied this approach to a firm but the model was limited to a black-box cash-flow analysis which did not depict the inner workings of the firm. Most other socio-economic applications have been in areas other than models of the firm, such as a model of an educational institution.[2]

---

[1]B. Linn Soule, "A Discrete State Deterministic System Model for Analysis of the Firm," (Unpublished Ph.D. Thesis, Michigan State University, 1967).

[2]Herman E. Koenig et al, A Systems Approach to Higher Education, (East Lansing, Michigan: Michigan State University, 1966).

Scope of the Study

This study is concerned primarily with the generalized analytical capabilities of the methodology used, as opposed to heuristic simulation. This is not to imply that simulation is not applicable nor useful. On the contrary, heuristic simulation has proven to be the most widely applicable and most useful tool in the area of modeling. A recent unpublished survey[3] made by the IBM Corporation of its customers' applications of simulation, listed one hundred forty-eight different uses in twenty-seven separate areas. In this study, the term "heuristic simulation" is used in the sense of "cut-and-try" simulation with a model that tries to describe reality.

Heuristic simulation will certainly play a major role in the over-all modeling program of which this study is a part. However, since the field of simulation is well-developed and well-travelled, this study limits itself to demonstrating the feasibility of simulation as one of the facets of the methodology being evaluated.

Since the model constructed in this study is strictly a prototype, the degree of detail depicted has been simplified to the point where the size of the model is manageable but the realism of the model is not sacrificed. Contingent on favorable findings of the study, the model can be easily expanded to any desired degree of detail afterwards. Another limitation on the prototype model is the ready availability of certain data. Fortunately this has not been a major handicap due to the highly automated and sophisticated management

---

[3]IBM Corp., "Survey of Simulation Applications," (Unpublished Survey, White Plains, N.Y., 1966).

information and control system already existing in the firm being modeled.

The model constructed is symbolic and not iconic or analogue. In a symbolic model, the components of what is represented and their interrelationship are given by symbols. Iconic models such as photographs and sculptures "look like" what they represent. An analogue model represents one set of properties, such as distance on a graph representing time or cost.

The model constructed is dynamic instead of static in that it can generate time paths of model variables. It describes how the system changes over time. The model, at least the prototype version developed in this study, is an aggregate or macroscopic one.

It is deterministic. The introduction of probability and Monte Carlo simulation will come after this study. All equations are linear. Treatment of non-linear relationships and their evaluation thru simulation will also come afterwards.

Since the internal operations of the firm are being modeled, the approach taken is "structural" rather than "black box." The model is discrete. Hence the equations used are difference rather than differential equations. Difference equations parallel more closely the periodic batch-type reporting systems that characterize industry.

Methodology Used

The methodology used is composed of the group of analytical techniques described by Dr. Herman Koenig[4] of Michigan State University in his mimeographed notes for a course on "Systems Analysis for Social Scientists".

---

[4] Herman E. Koenig, "Systems Analysis for Social Scientists," (Unpublished Class Notes, Michigan State University, 1966). (Mimeographed)

These techniques were developed originally for the analysis of system models of electrical networks. This theory offers a formal process for generating a model of the system from a defined system structure and for analyzing the solution characteristics of the system model to simulate the physical behavior of the system as a function of a change in the structural features.

Like an electrical or mechanical system, a firm can also be conceptualized as a system of interacting components. These components are modular and are characterized by their behavior as measured at their interfaces with each other. Thus the components can be studied by themselves or at any desired subsystem level.

Linear graph theory provides a convenient way of representing system components and their interfaces by points (vertices) and connecting lines (edges). The behavioral characteristics of a component can be specified by a pair of complementary variables associated with each edge.

The X variable is called the propensity variable and is usually regarded as the "cause" or "result" of the flow or Y variable. In mechanical processes X and Y can be velocity and force respectively; in hydraulic processes, pressure and flow rate; in electrical processes, voltage and current; in a model of the firm, imputed cost and flow of units thru production.

The theory gives explicit procedures for developing a set of simultaneous algebraic and/or differential or difference equations showing the interdependence of a set of variables which characterize the observable behavior of the system. These equations are composed of the system's component

characteristic equations and the constraint equations from the interconnection pattern of the linear graph. The reduction of this set of equations to a minimal set of first ordered difference or differential equations produces the state model.

Solving the state model enables one to simulate its behavior over time. Analytical procedures are furnished for testing whether or not the system is stable, and whether or not any desired state or output can be reached by manipulating the control variables. Optimal control techniques are also available but will be reserved for future work beyond this study.

Organization of Study

Chapter II describes the business enterprise being modeled and various considerations in attempting to model it.

Chapter III narrates the construction of the model and problems encountered. The linear graph as well as the matrices that form the state model and output vector are developed.

Chapter IV treats the evaluation of the model. The derivation of parameters from empirical data is described. Heuristic simulation is demonstrated. Stability, state controllability, output controllability and control strategies are analyzed.

Chapter V presents the summary and conclusions of the study. An assessment is made of the practical utility of this methodology from the standpoint of the business executive. Plans for further development of the model are presented. The Bibliography follows Chapter V.

CHAPTER II

DESCRIPTION OF THE BUSINESS ENTERPRISE

Size

The business enterprise being modeled is the largest division of a highly diversified international corporation with yearly sales of $400 million. This division accounts for about a fifth of corporate sales and employs about four thousand people.

Product Line

Its products include reference and navigation systems for high performance aircraft, missile guidance systems, spacecraft controls and displays, etc. Its standard products can be categorized into gyroscopes, indicators, amplifiers, computers, and accessories. The orders received for these products can be divided into repeat business for old products and new business for products which require substantial engineering design effort. Spare parts orders contribute significantly to sales volume.

Besides the above hardware, pure engineering is also a major product line. Customers frequently fund research and development studies and prototype hardware. Some examples are lunar rendezvous simulation studies or development of a low cost inertial guidance system.

In order to check out the product, customers have to purchase aerospace ground equipment (AGE). Such ground support equipment may be used, say, by the maintenance crew on an aircraft carrier to check that the systems on

the jet aircraft are "go" prior to take-off. Like the standard product line, aerospace ground equipment orders can be classified into repeat and new business.

Users of the products need operating instructions, maintenance manuals, parts lists, and other technical data. The sale of this technical data constitutes an attractive block of business.

Finally, these products wear out or are damaged and have to be returned to the factory for repair or overhaul. These sales fall under the heading of Service, Repair and Overhaul (SRO).

Market Environment

The majority of the sales are made to the Government either directly to the Department of Defense and the National Aeronautics and Space Administration or indirectly through Air Frame Prime Contractors. Most business is won thru fixed-price competitive bidding. However, some contracts, especially those of a developmental nature, are based on other terms such as time-and-material, cost-plus-fixed-fee, cost-plus-incentive-fee, redeterminable price and fixed price with incentive provisions.

Organization

Figure 1 shows a functional organization chart. This organization is in contrast to the project oriented organization typical of the huge air frame prime contractors whose contracts are large enough to justify separate and

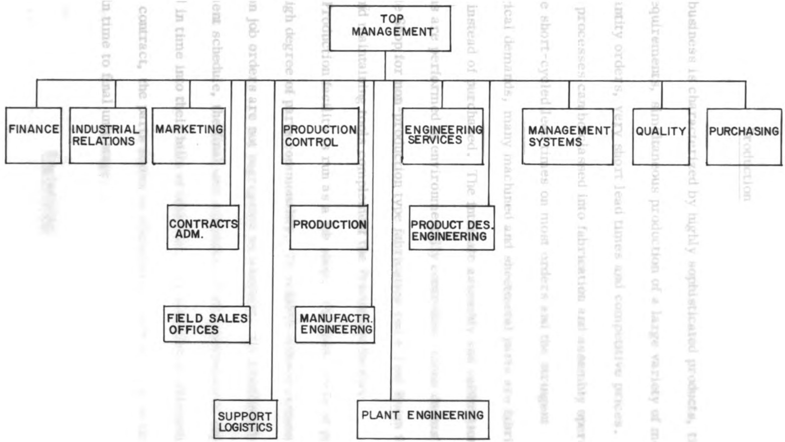

Figure 1. - Functional organization chart.

self-sufficient project organizations.

Production

The business is characterized by highly sophisticated products, tight quality requirements, simultaneous production of a large variety of models, small quantity orders, very short lead times and competitive prices. Manufacturing processes can be classed into fabrication and assembly operations. Due to the short-cycled lead times on most orders and the stringent technological demands, many machined and sheetmetal parts are fabricated in-house instead of purchased. The intricate assembly and calibration operations are performed in environmentally controlled "clean rooms". A Flexible Shop for non-production type fabrication and a Tool Room for making and maintaining tools complement the Production facility.

The Production facility is run as a job shop. The wide array of products enjoy a high degree of parts commonality. To achieve volume economies, production job orders are not segregated by contract. To establish the parts requirement schedule, the final unit schedules of all contracts are exploded backward in time into their bills of material. To forecast or determine status by contract, the parts status is allocated by contract as it is imploded forward in time to final unit stage.

Engineering

Like the Production function described previously, the Engineering function is an integral part of the basic line process of the firm. It can be looked upon as a production shop turning out design specifications and drawings for Manufacturing, and both software packages and prototype hardware for customers. The Engineering services group provides facilities such as a scientific computer, materials and environmental testing, and a model shop.

Besides line (direct labor) type engineering, other engineering activities include company-funded research and development, production support and sales support. Since the product technology often strains the state of the art, close support from Engineering is needed to manufacture the products to specifications, on schedule, and profitably. The highly technical nature of the product also requires the engineers and contracts personnel to work together in dealing with customers.

Contracts

The Contracts operations cover the Field Sales Offices, the Marketing function, the Contract Administration function and the Support Logistics function. The Field Sales Offices are the communication link with the customer. All marketing campaigns are planned and implemented by the Marketing group together with the sales support people from Engineering. Market Research and Analysis is also the responsibility of the Marketing Group. Once a sale is made, the Contracts Administration function processes the order and assumes responsibility for all the tasks related to that order till it is fulfilled. The Support Logistics function handles all sales of spare parts, repair and overhaul services and technical data. The actual production of the technical data is done within this group.

Manufacturing Service Functions

Production operations are supported by the Manufacturing Engineering, Production Control and Plant Engineering functions.

Manufacturing Engineering takes the design specifications and drawings from Engineering and develops the processing instructions and tooling to build the product. It generates all manufacturing cost estimates. It supports production by solving tooling, processing and other technical problems that interfere with production. Where design problems exist, the production support personnel from Engineering provide the necessary assistance. Cost reduction of production costs is also a prime responsibility of Manufacturing Engineering.

Production Control is responsible for the coordination and control of all manufacturing activities, from receipt of order to shipment of product. The procurement, manufacture and movement of all materials through the plant are controlled by a plan and schedule developed by Production Control. This function dispatches and expedites jobs, and assigns all priorities while reporting on work progress. Production Control is responsible for inventory control both from a planning and a physical control standpoint. It controls the incorporation of engineering changes on all products. By acting as the sole authorized source of all manufacturing commitments and other schedule information, it functions as a communications control center between Manufacturing and other operations. It forecasts, measures, and manipulates the manpower and facility workload and capacity so that schedules can be met with optimum efficiency and economy.

The Plant Engineering function is responsible for all plant layout, facility and building maintenance, equipment installation, and planning and supervision of brick-and-mortar activities.

Other Service Functions

The remaining service functions are Purchasing, Quality, Finance, Industrial Relations and Management Systems.

Purchasing is responsible for all outside procurement of materials, parts, supplies, equipment and services. It develops and maintains adequate and reliable sources of supply. Cost reduction of purchased material is a primary responsibility of this function.

The Quality function is responsible for the quality assurance of product at minimum cost through a continuing control function. It generates and implements the procedures necessary to see that all quality and reliability requirements are met. Other activities include measurement calibration and standards control, quality assurance testing, quality audits, failure reporting, and customer test correlation.

The Finance (or Accounting) function prepares budgets, develops financial objectives and prepares controls and reports for evaluating financial performance. Its activities include payroll and timekeeping, accounts receivable and payable, pricing, cost accounting, etc.

The Industrial Relations function is responsible for coordinating, administering and maintaining all company-employee relationships. This includes labor relations, recruiting, wage and salary administration, employee benefit programs, coordination of employee training and management development, security, plant protection and first aid.

The Management Systems function has a two-fold responsibility. First, it coordinates and controls all the firm's activities relative to major new product programs. Since the organization of the firm is functionally oriented, program-oriented control has to come from this activity. Second, Management Systems is responsible for all scientific management technology, systems analysis, programming, and computer operations. One of its prime objectives is the design and implementation of a highly computerized, integrated and sophisticated management information and control system. One step towards this is a long range program for the development of a model of the firm.

CHAPTER III

CONSTRUCTION OF THE MODEL

Alternative Formulations

A firm is such a complex multi-faceted entity that any attempt to achieve a truly comprehensive model is fraught with frustration.

To capture systematically the elements that characterize this firm, each sector was analyzed as to its function; the decisions it made; its inputs and outputs in terms of data, labor, material, facilities and funds; and the source and destination of each input and output. This mass of information represented the basic raw material to be molded into a model of the firm.

However, the shape of this model depends on its purpose. An all-purpose model would be so unmanageable that it would become a no-purpose model. One approach often visualized by the executive is to develop an integrated automated management information system which will not only perform the actual functions of the firm but also act as a model by processing upon request hypothetical instead of actual data. There is nothing conceptually wrong with this approach. Unfortunately the building of such a system is a long, tedious and sometimes economically unfeasible task. And the increased cost of a computer large and fast enough not only to process the programs using the actual operating data but also to simulate the firm with hypothetical data and provide management with the answers in an acceptable waiting time, will tend to discourage most firms from this approach to a model of the total firm.

The above approach has been used successfully for specific parts of the firm. For example, the firm being studied has used the regular machine load forecasting program to compute the impact of potential or hypothetical contracts. However to model the total firm usefully and economically, the model would have to limit itself to the truly significant factors and relationships.

Specific parts of a firm are easier to model than the total firm. Examples are job shop simulators for testing alternative sequencing rules, queuing models, inventory control models, marketing games, or manufacturing games.

Another approach to modeling a firm is to model specific aspects of the total firm. Some examples are a communications model, a decision-making model, a cash flow model, a paperwork flow model, an accounting model, etc.

Thus in specifying a model of the total firm, one must decide on the objectives of the model, the perspective to take, the important sectors, factors and relationships in the firm that should be included in the model and the degree of detail to portray.

Building the Model

For the prototype model in this study, it was decided to build what was essentially a direct costing accounting-type model. This model follows the flow of the product from receipt of order to shipment thru the various sectors that represent the basic line processes of the firm. The model depicts the flow of resources into the different product stages either directly or indirectly thru the service and other overhead functions. As these resources and services go into the product, their costs are imputed to the product cost. The two complementary variables specifying the behavioral characteristics of each component are the flow volume of the units and resources as the Y variable and the imputed cost of these flows as the X variable. To avoid repetition, the objectives of the prototype model will be deferred to the last chapter when its uses are discussed.

An initial attempt was made to build product price into the model. In other industries prices are determined after production by adding a fixed mark-up to costs or prices are set by market demand and supply or by a price leader, etc. In this industry pricing is done before the contract is won and competitive bidding is the predominant method of government procurement. Unfortunately competitive pricing data is one of the few areas where available empirical data leaves something to be desired. It was decided not to bog down the prototype model with the development of a bid strategy model. This can be done later on. Therefore product price is excluded from the model. It is felt that this does not jeopardize the usefulness of the prototype model provided sales input volume is expressed as a function of the previous period's product cost besides other factors. This relationship implies that lower costs will permit lower prices which will win more contracts. Furthermore, a cost model is quite pertinent to management's goal of measuring, forecasting, controlling and minimizing costs.

To keep the size of the model manageable, the product line is aggregated into seven categories, namely technical data, repair and overhaul work, new ground support equipment business, repeat ground support equipment business, engineering software and hardware, new standard product contracts and repeat standard product contracts. Subsequent models should expand the standard product category to the various product families.

The manufacturing processes are compressed into the two stages of fabrication and assembly.

Several factors are found which constitute the control variables used by management. These include company-funded research and development, the marketing budget, the sales support budget, the investment in facilities, inventory and equipment (especially automation equipment like computers), and investment in service function manpower. These can all be classed into cost reduction activities and sales promoting activities.

Twelve service and other overhead functions are built into the model. These are administration, flex shop and tool room, finance, industrial relations, plant engineering, management systems, production control, quality, manufacturing engineering, purchasing, contracts and engineering support.

Basic resources are reduced to labor, material and facilities. Since the enterprise under study is a division of a corporation, the funds it uses to pay for these resources are borrowed from the corporate office. A fixed charge is levied by the corporate office on its divisions to cover both interest and corporate overhead costs.

Figure 2 is the system graph of the model of the enterprise. The numbers in the circles are used to identify the edges of the graph. For example $Y_{6-25}(n)$ refers to the flow on the edge from the circle marked 6 to the circle marked 25. This is the number of units of standard product flowing out of the fabrication sector to the assembly sector during time n. $X_{6-25}(n)$ is the imputed cost of each of these units at that point in time. Table 1 lists the service and other overhead functions. In line with keeping down the size of this prototype model, the basic unit of time chosen is a quarter.

Table 1. - Service and other overhead functions.

A	B	Function
60	80	Administration
61	81	Flex Shop & Tool Room
62	82	Finance
63	83	Industrial Relations
64	84	Plant Engineering
65	85	Management Systems
66	86	Production Control
67	87	Quality
68	88	Manufacturing Engineering
69	89	Purchasing
70	90	Contracts
71	91	Engineering Support

Component and Constraint Equations

The following section shows the development of the component equations

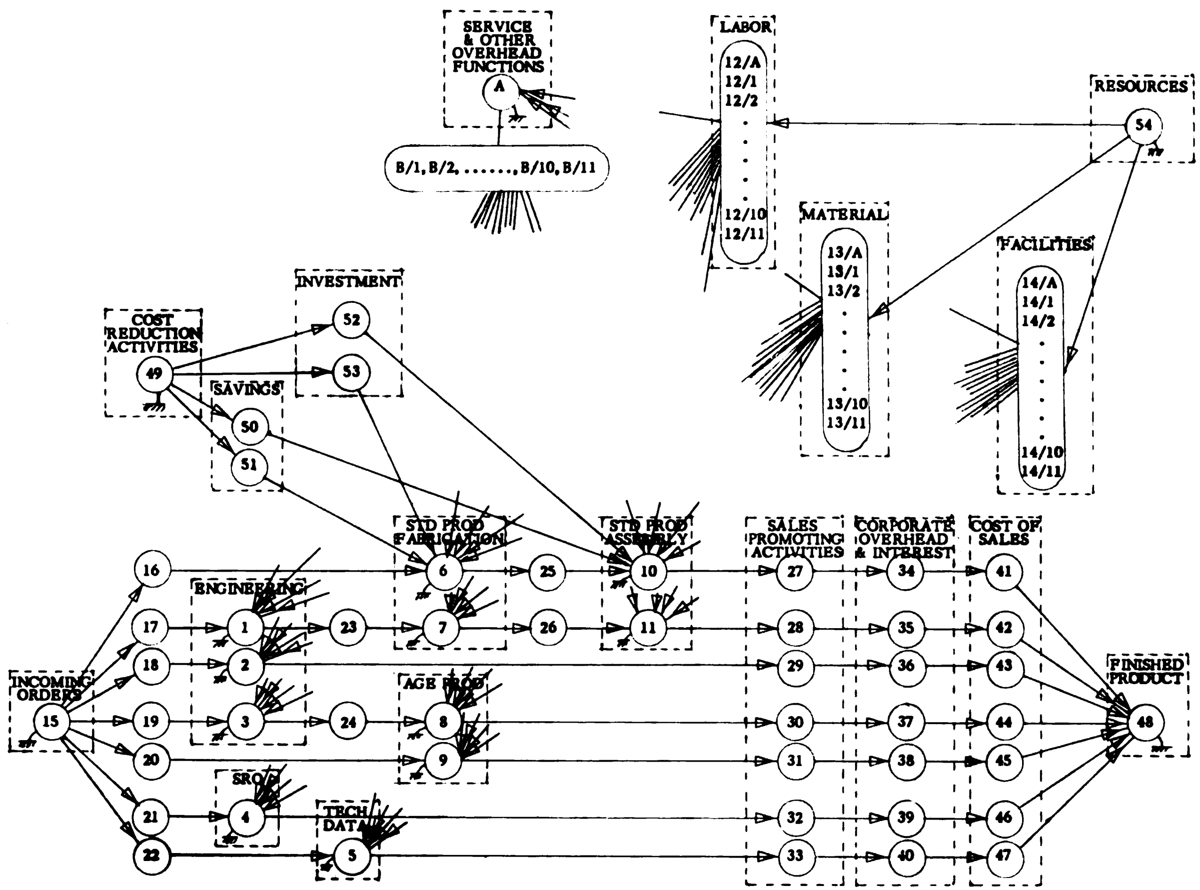

Figure 2. - System graph of the model of the enterprise.

describing the characteristic relationships between system components. Linear graph theory provides mechanical procedures for forming constraint equations. These are relationships which must exist between the system variables by virtue of their structural interconnections in the system linear graph. These mechanical procedures involve selecting a "proper tree" in the linear graph and developing "fundamental cut set and circuit equations" (which are the constraint equations). By reducing the component and constraint equations to a minimal set of first ordered difference equations, the state model is attained.

Table 2 lists the "edges" of the "proper tree" selected. In the following sections, all input unit costs are taken as positive numbers and output unit costs as negative numbers. This convention is necessary from the standpoints of both the methodology used and sound accounting practice.

Table 2. - Edges of proper tree.

16-6	3-24	
17-1	8-30	
18-2	9-31	
19-3	4-32	
20-9	5-33	
21-4	j-k	(j = 27 to 33; k = 34 to 40)
22-5	k-l	(k = 34 to 40; l = 41 to 47)
6-25	54-m	(m = 12/p, 13/p, 14/p)
10-27		(p = A/1 to A/11, 1 to 11)
1-23		(A = 60 to 71)

Table 2. - (continued)

7-26	A/q-B/q	(q=1 to 11)(A=60 to 71)(B=80 to 91)
11-28	49-s	(s = 50 to 53)
2-29		

Incoming Orders Sector

The per unit cost of incoming orders X_{15-i} (for i = 16 to 22) is expressed as a direct function of the previous period's per unit factory cost X_{m-k} (for m = 10, 11, 2, 8, 9, 4, 5 and k = 27 to 33). This indicates that the cost of acquiring business decreases as lower factory costs make this task easier.

The volume of incoming business Y_{15-i} (for i = 16 to 22), is determined partly by sales promoting activities, partly by the previous period's shipment volume Y_{j-48} (j = 41 to 47), partly by the previous period's cost of sales X_{j-48} (j = 41 to 47) and partly by some known trend F_j (j = 41 to 47). This trend allows for the basic expansion or contraction of the market.

The component and constraint equations are given below:

$X_{i-p}(n+1) = -K_{i-p}\, X_{m-k}(n)$

(i = 16 to 22; p = 6, 1, 2, 3, 9, 4, 5; m = 10, 11, 2, 8, 9, 4, 5;

k = 27 to 33) (1) to (7)

$Y_{15-i}(n+1) = \text{Known } F_j(n) + K_{k-1}L_{k-1}Z(n) + P_{j-48}Y_{j-48}(n) + Q_{j-48}X_{j-48}(n)$

(i = 16 to 22; j = 41 to 47; k = 27 to 33; 1 = 34 to 40) (8) to (14)

Cut Set Eqns:

$Y_{15-i}(n) = Y_{i-m}(n)$ (i = 16 to 22; m = 6, 1, 2, 3, 9, 4, 5) (15) to (21)

Circuit Eqns:

$X_{15-i}(n) = -X_{i-m}(n)$ (i = 16 to 22; m = 6, 1, 2, 3, 9, 4, 5) (22) to (28)

<u>Engineering Sector</u>

Most of the component equations for the engineering sector are quite similar to those for the other sectors in the basic line process, namely the Service, Repair and Overhaul Sector, the Technical Data Sector, the Aerospace Ground Support Equipment Sector, the Standard Product-Fabrication Sector and the Standard Product-Assembly Sector. Consequently the detailed explanation of the seven common forms of component equations will not be repeated in describing those sectors. Only equations unique to those sectors will be explained.

In this prototype model all the sectors use one time period for processing the product. Thus the volume of product flowing into the sector in a given time period is equal to the volume flowing out in the next time period. This first common form of component equation is illustrated by equations (74) to (76) for the Engineering Sector.

At each of these sectors, the support of the service and other overhead functions flows into the product. To simplify this prototype model, the number of service units flowing into the product is considered equal to the number of product units being serviced. Thus the X variable for these service functions is really their cost per unit of product. So the second common form merely equates flow of services to flow of products, as in equations (29) to (64).

Similarly labor, material and facilities are incorporated directly into the product. The heterogeneous nature of the products and processes make it necessary to equate the number of units of material and of facilities flowing into the product, to the number of product units. This third common form is illustrated by equations (68) to (73).

The flow of labor can be conveniently expressed in terms of manhours. Within the different functions, labor rates are reasonably uniform. Thus the fourth form of component equation specifies the number of labor hours per unit of product. (Equations 65 to 67)

The fifth form (Equations 77 to 112) breaks down the cost of the service functions per unit of product into its fixed and variable elements. The sixth form (Equations 113 to 115) does the same thing with the cost of facilities per unit of product.

The seventh form (Equations 116 to 118) expresses the cost of the product leaving the sector as the sum of product cost when it entered the sector and the costs of all resources and services that flowed into the product while it was in the sector.

$$Y_{B/i-i}(n) = Y_{j-i}(n) \; (B = 80 \text{ to } 91) \; (i = 1, 2, 3;\; j = 17, 18, 19) \qquad (29) \text{ to } (64)$$

$$Y_{12/i-i}(n) = K_{12/i-i} Y_{j-i}(n) \qquad (i = 1, 2, 3;\; j = 17, 18, 19) \qquad (65) \text{ to } (67)$$

$$Y_{r-i}(n) = Y_{j-i}(n) \; (r = 13/i, 14/i) \; (i = 1, 2, 3;\; j = 17, 18, 19) \qquad (68) \text{ to } (73)$$

$$Y_{j-i}(n) = Y_{i-k}(n+1) \; (i = 1, 2, 3;\; j = 17, 18, 19;\; k = 23, 29, 24) \qquad (74) \text{ to } (76)$$

$$X_{B/i-i}(n) = F_{B/i}(n) - K_{B/i-i} Y_{j-i}(n) (B = 80 \text{ to } 91) \; (i = 1, 2, 3;\; j = 17, 18, 19) \qquad (77) \text{ to } (112)$$

$$X_{14/i-i}(n) = F_{14/i}(n) - K_{14/i-i} Y_{j-i}(n) \; (i = 1, 2, 3;\; j = 17, 18, 19) \qquad (113) \text{ to } (115)$$

$$X_{i-k}(n+1) = -X_{j-i}(n) - \sum_{B=80}^{91} X_{B/i-i}(n) - \sum_{r=13/i}^{14/i} X_{r-i}(n) - K_{12/i-i} X_{12/i-i}(n)$$

$$(i = 1, 2, 3;\; j = 17, 18, 19;\; k = 23, 29, 24) \qquad (116) \text{ to } (118)$$

Cut Set Eqns:

$Y_{i-k}(n) = Y_{l-m}(n)$ (i = 1, 2, 3; k = 23, 29, 24; l = 23, 43, 24; m = 7, 48, 8)

(119) to (121)

Circuit Eqns:

$X_{i-k}(n) = X_{k-p}(n)$ (i = 1, 3; k = 23, 24; p = 7, 8) (122) to (123)

Service, Repair & Overhaul Sector & Technical Data Sector

$Y_{j-i}(n) = Y_{i-k}(n+1)$ (j = 21, 22; i = 4, 5; k = 32, 33) (124) to (125)

$Y_{B/i-i}(n) = Y_{j-i}(n)$ (B = 80 to 91) (i = 4, 5; j = 21, 22) (126) to (149)

$Y_{12/i-i}(n) = K_{12/i-i}Y_{j-i}(n)$ (i = 4, 5; j = 21, 22) (150) to (151)

$Y_{r-i}(n) = Y_{j-i}(n)$ (r = 13/i, 14/i) (i = 4, 5; j = 21, 22) (152) to (155)

$X_{B/i-i}(n) = F_{B/i}(n)-K_{B/i-i}Y_{j-i}(n)$ (B = 80 to 91) (i = 4, 5; j = 21, 22)

(156) to (179)

$X_{14/i-i}(n) = F_{14/i}(n)-K_{14/i-i}Y_{j-i}(n)$ (i = 4, 5; j = 21, 22) (180) to (181)

$$X_{i-k}(n+1) = -X_{j-i}(n)-\sum_{B=80}^{91} X_{B/i-i}(n)-\sum_{r=13/i}^{14/i} X_{r-i}(n)-K_{12/i-i}X_{12/i-i}(n)$$

(i = 4, 5; j = 21, 22; k = 32, 33) (182) to (183)

Cut Set Eqns:

$Y_{i-k}(n) = Y_{l-48}(n)$ (i = 4, 5; k = 32, 33; l = 46, 47) (184) to (185)

Aerospace Ground Support Equipment Sector

$Y_{j-i}(n) = Y_{i-k}(n+1)$ (j = 24, 20; i = 8, 9; k = 30, 31) (186) to (187)

$Y_{B/i-i}(n) = Y_{j-i}(n)$ (B = 80 to 91) (i = 8, 9; j = 24, 20) (188) to (211)

$Y_{12/i-i}(n) = K_{12/i-i}Y_{j-i}(n)$ (i = 8, 9; j = 24, 20) (212) to (213)

$Y_{r-i}(n) = Y_{j-i}(n)$ $(r = 13/i,\ 14/i)$ $(i = 8, 9;\ j = 24, 20)$ (214) to (217)

$X_{B/i-i}(n) = F_{B/i}(n) - K_{B/i-i}Y_{j-i}(n)$ $(B = 80 \text{ to } 91)$ $(i = 8, 9;\ j = 24, 20)$

(218) to (241)

$X_{14/i-i}(n) = F_{14/i}(n) - K_{14/i-i}Y_{j-i}(n)$ $(i = 8, 9;\ j = 24, 20)$ (242) to (243)

$$X_{i-k}(n+1) = -X_{j-i}(n) - \sum_{B=80}^{91} X_{B/i-i}(n) - \sum_{r=13/i}^{14/i} X_{r-i}(n) - K_{12/i-i}X_{12/i-i}(n)$$

$(i = 8, 9;\ j = 24, 20;\ k = 30, 31)$ (244) to (245)

Cut Set Eqns:

$Y_{i-k}(n) = Y_{l-48}(n)$ $(i = 8, 9;\ k = 30, 31;\ l = 44, 45)$ (246) to (247)

Standard Product-Fabrication Sector

$Y_{j-i}(n) = Y_{i-k}(n+1)$ $(j = 16, 23;\ i = 6, 7;\ k = 25, 26)$ (248) to (249)

$Y_{B/i-i}(n) = Y_{j-i}(n)$ $(B = 80 \text{ to } 91)$ $(i = 6, 7;\ j = 16, 23)$ (250) to (273)

$Y_{12/i-i}(n) = K_{12/i-i}Y_{j-i}(n)$ $(i = 6, 7;\ j = 16, 23)$ (274) to (275)

$Y_{r-i}(n) = Y_{j-i}(n)$ $(r = 13/i, 14/i)$ $(i = 6, 7;\ j = 16, 23)$ (276) to (279)

$X_{B/i-i}(n) = F_{B/i}(n) - K_{B/i-i}Y_{j-i}(n)$ $(B = 80 \text{ to } 91)$ $(i = 6, 7;\ j = 16, 23)$

(280) to (303)

$X_{14/i-i}(n) = F_{14/i}(n) - K_{14/i-i}Y_{j-i}(n)$ $(i = 6, 7;\ j = 16, 23)$ (304) to (305)

$$X_{7-26}(n+1) = -X_{23-7}(n) - \sum_{B=80}^{91} X_{B/7-7}(n) - \sum_{r=13/7}^{14/7} X_{r-7}(n) - K_{12/7-7}X_{12/7-7}(n)$$

(306)

$$X_{6-25}(n+1) = X_{16-6}(n) - \sum_{B=80}^{91} X_{B/6-6}(n) - \sum_{r=13/6}^{14/6} X_{r-6}(n) - K_{12/6-6}X_{12/6-6}(n)$$

(307)

Cut Set Eqns:

$Y_{i-k}(n) = Y_{k-m}(n)$ $(i = 6, 7;\ k = 25, 26;\ m = 10, 11)$ (308) to (309)

Circuit Eqns:

$X_{i-k}(n) = Y_{k-m}(n)$ (i = 6, 7; k = 25, 26; m = 10, 11) (310) to (311)

The equations (307) and (371) for the product cost leaving both this sector and the standard product-assembly sector differ from those in the preceding sectors in that the costs and savings of cost reduction activities are imputed into the "repeat" standard product business flowing thru the fabrication and assembly sectors. The cost reduction investment per product unit is segregated into its fixed and variable elements.

Standard Product-Assembly Sector

$Y_{j-i}(n) = Y_{i-k}(n+1)$ (j = 25, 26; i = 10, 11; k = 27, 28) (312) to (313)

$Y_{B/i-i}(n) = Y_{j-i}(n)$ (B = 80 to 91) (i = 10, 11; j = 25, 26) (314) to (337)

$Y_{12/i-i}(n) = K_{12/i-i}Y_{j-i}(n)$ (i = 10, 11; j = 25, 26) (338) to (339)

$Y_{r-i}(n) = Y_{j-i}(n)$ (r = 13/i, 14/i) (i = 10, 11; j = 25, 26) (340) to (343)

$X_{B/i-i}(n) = F_{B/i}(n)-K_{B/i-i}Y_{j-i}(n)$ (B = 80 to 91) (i = 10, 11; j = 25, 26) (344) to (367)

$X_{14/i-i}(n) = F_{14/i}(n)-K_{14/i-i}Y_{j-i}(n)$ (i = 10, 11; j = 25, 26) (368) to (369)

$$X_{11-28}(n+1) = -X_{26-11}(n)-\sum_{B=80}^{91} X_{B/11-11}(n)-\sum_{r=13/11}^{14/11} X_{r-11}(n) -K_{12/11-11}X_{12/11-11}(n) \quad (370)$$

$$X_{10-27}(n+1) = -X_{25-10}(n)-\sum_{B=80}^{91} X_{B/10-10}(n)-\sum_{r=13/10}^{14/10} X_{r-10}(n)-K_{12/10-10} X_{12/10-10}(n)+X_{5-10}(n)-X_{52-10}(n)K_{52-10}Y_{25-10}(n) \quad (371)$$

Cut Set Eqns:

$Y_{i-k}(n) = Y_{l-48}(n)$ (i = 10, 11; k = 27, 28; l = 41, 42) (372) to (373)

Sales Promoting Activities Sector

These activities are called sales promoting rather than sales promotion since the latter term is associated with advertising campaigns and the like. In the aerospace industry, advertising plays a relatively minor role. Sales promoting activities include visits to Defense agencies and airframe prime contractors by marketing and engineering personnel, technical presentations, and distribution of technical literature. Also included is the company-funded research and development which has a definite effect on sales volume.

The impact of these activities on sales has already been specified in the description of the Incoming Orders Sector.

These activities are one of the controls by which management can manipulate the system. An early version of the model used a separate control variable for each product line. To condense the model these variables have been reduced to one control variable Z(n) which is the total investment in sales promoting activities. The cost of Z(n) is allocated to each product line by the factor L_{i-k}.

In the next chapter, the procedure will be shown for solving for the values of Z(n) and the other control variables, which will enable the firm to attain any desired cost and sales volume objectives.

$X_{i-k}(n) = L_{i-k}Z(n)$ (i = 27 to 33; k = 34 to 40) (374) to (380)

where Z(n) = Unknown f(n) (control variable)

Cut Set Eqns:

$Y_{i-k}(n) = Y_{1-48}(n)$ (i = 27 to 33; k = 34 to 40; 1 = 41 to 47) (381) to (387)

$(X_{i-k}(n) \leq 0.1X_{1-48}(n))$

Corporate Overhead & Interest Sector

Since the enterprise modeled is a division of a firm whose corporate office provides all the funding required, X_{i-k} represents the overhead charge paid to the corporate office for interest, dividends and maintenance of that office. It is expressed as a function of the accumulated product cost.

$$X_{i-k}(n) = K_{i-k}(X_{j-i}(n)+X_{p-j}(n))$$

(i = 34 to 40; k = 41 to 47; j = 27 to 33; p = 10, 11, 2, 8, 9, 4, 5)

(K_{i-k} = 0.06) (388) to (394)

Cut Set Eqns:

$$Y_{i-k}(n) = Y_{k-48}(n)$$ (i = 34 to 40; k = 41 to 47) (395) to (401)

Cost of Sales Sector

Cost of sales is the grand total of all previous costs that went into the product including the corporate charge.

Circuit Eqns:

$$X_{p-j}(n)+X_{j-k}(n)+X_{k-i}(n) = -X_{i-48}(n)$$

(p = 10, 11, 2, 8, 9, 4, 5; j = 27 to 33; k = 34 to 40; i = 41 to 47)

(402) to (408)

Resources Sector

Labor cost per manhour and material cost per product unit are known functions of time.

$$X_{12/i-i}(n) = \text{Known } F_{12/i}(n)$$ (i = A/1 to A/11, 1 to 11)

(A = 60 to 71) (409) to (551)

$$X_{13/i-i}(n) = \text{Known } F_{13/i}(n)$$ (i = 1 to 11) (552) to (562)

Cut Set Eqns:

$Y_{54-r}(n) = Y_{r-i}(n)$ (r = 12/i, 13/i, 14/i) (i = A/1 to A/11, 1 to 11)

(A = 60 to 71) (563) to (991)

Circuit Eqns:

$X_{54-r}(n) = -X_{r-i}(n)$ (r = 12/i, 13/i, 14/i) (i = A/1 to A/11, 1 to 11)

(A = 60 to 71) (992) to (1420)

Service & Other Overhead Functions Sector

Equations (1421) to (1552) indicate the number of labor hours per unit of service. Equations (1553) to (1816) equate the units of measure of material and facilities flow and service flow. Equations (1817) to (1948) show what per cent material costs constitute of the total imputed service cost. Equations (1949) to (2080) merely state that the total imputed service cost is the sum of the costs of the labor, material and facilities used by the service function.

$Y_{12/i-i}(n) = K_{12/i-i}Y_{i-j}(n)$ (i = A/1 to A/11; j = B/1 to B/11)

(A = 60 to 71; B = 80 to 91) (1421) to (1552)

$Y_{r/i-i}(n) = Y_{i-j}(n)$ (r = 13, 14) (i = A/1 to A/11; j = B/1 to B/11)

(A = 60 to 71; B = 80 to 91) (1553) to (1816)

$X_{13/i-i}(n) = -K_{13/i-i}X_{i-j}(n)$ (i = A/1 to A/11; j = B/1 to B/11)

(A = 60 to 71; B = 80 to 91) (1817) to (1948)

$X_{i-j}(n) = -X_{13/i-i}(n)-X_{14/i-i}(n)-K_{12/i-i}X_{12/i-i}(n)$

(i = A/1 to A/11; j = B/1 to B/11)

(A = 60 to 71; B = 80 to 91) (1949) to (2080)

Cut Set Eqns:

$Y_{i-j}(n) = Y_{j-k}(n)$ (i = A/1 to A/11; j = B/1 to B/11; k = 1 to 11)

(A = 60 to 71; B = 80 to 91) (2081) to (2212)

Circuit Eqns:

$X_{i-j}(n) = -X_{j-k}(n)$ (i = A/1 to A/11; j = B/1 to B/11; k = 1 to 11)

(A = 60 to 71; B = 80 to 91) (2213) to (2344)

Cost Reduction Activities Sector

The cost reduction activities together with the sales promoting activities comprise the control vector of the model.

In the description of the Incoming Orders Sector, cost of sales are specified as having an inverse causal relationship with sales input volume (i.e., the lower the costs, the more contracts won). In the real world, this is not a stable absolute relationship because competitors are also striving to reduce costs. To make this prototype model realistic without expanding it to include a sector on competitive firms, a compensating simplification must be built into the model.

This simplification involves treating all company-funded research and development strictly as sales promoting activities and not as cost reduction activities. In other words the assumption is made that this research-and-engineering increases sales by developing new or improved products and enhancing in-house technical capabilities. However, its effect in reducing product cost is approximately just enough to maintain the current cost position relative to those of competitors. Furthermore this eliminates double-counting of its impact on sales.

Therefore cost reduction activities are limited in this model to the manufacturing engineering and purchasing efforts relative to the repeat standard product business flowing thru the fabrication and assembly sectors. The resulting savings are separated into fixed and variable elements.

$X_{49-i}(n)$ = Unknown f(n) (control variables)

(i = 52, 53) (2345) to (2346)

$X_{49-j}(n) = M_{49-j} - K_{49-j}(N_{49-j} - X_{49-i}(n))$

$(X_{49-j}(n) \leqslant M_{49-j})$

(j = 50, 51; i = 52, 53) (2347) to (2348)

Cut Set Eqns:

$Y_{49-i}(n) = Y_{i-j}(n)$ (i = 50 to 53; j = 10, 6, 10, 6) (2349) to (2352)

Circuit Eqns:

$X_{49-i}(n) = X_{i-j}(n)$ (i = 50 to 53; j = 10, 6, 10, 6) (2353) to (2356)

<u>Simplifying Variables</u>

To simplify the matrix entries, let:

$V_{m-i} = K_{m-i} + 1$

$D_{m-i} = L_{k-m} V_{m-i}$

$V_{j-k} = Q_{i-48} V_{m-i}$

$V_{k-m} = (K_{k-m} - V_{j-k}) L_{k-m}$ (i = 41 to 47; j = 10, 11, 2, 8, 9, 4, 5;

k = 27 to 33; m = 34 to 40)

$$V_{49-52} = (K_{49-50})^{-1}$$

$$V_{49-53} = (K_{49-51})^{-1}$$

$$V_{49-50} = K_{49-50} N_{49-50} - M_{49-50}$$

$$V_{49-51} = K_{49-51} N_{49-51} - M_{49-51}$$

$$F_{V/j}(n) = \sum_{B=80}^{91} F_{B/j}(n)$$

$$F_{r/j}(n) = F_{13/j}(n) + F_{14/j}(n) + K_{12/j-j} F_{12/j}(n) \qquad (j = 1 \text{ to } 11)$$

$$V_l = \sum_{B=80}^{91} K_{B/j-j} + K_{14/j-j} \qquad (l = 15\text{-}17 \text{ to } 15\text{-}22,\ 7\text{-}26,\ 1\text{-}23,\ 3\text{-}24;\ j = 1, 2, 3, 9, 4, 5, 11, 7, 8)$$

$$V_l = \sum_{B=80}^{91} K_{B/j-j} + K_{14/j-j} + K_{52-10} \qquad (l = 6\text{-}25,\ j = 10)$$

$$V_l = \sum_{B=80}^{91} K_{B/j-j} + K_{14/j-j} + K_{53-6} \qquad (l = 15\text{-}16,\ j = 6)$$

State Model and Output Vector

Table 3 lists the state, control and output variables of the model. The state variables are sufficient to specify any aspect of the system. They are the flow and imputed cost variables wherever a time lag occurs in the model. The other model variables can be expressed as linear functions of the state variables. The control variables are the sales promoting and cost reduction activities. The output variables are the final costs of product shipped.

Table 3. - State, control and output variables of model.

State Variables	=	$Y_{15-i}(n)$	($i = 16$ to 22)
		$Y_{j-48}(n)$	($j = 41$ to 47)
		$Y_{25-10}(n)$	
		$Y_{26-11}(n)$	
		$Y_{23-7}(n)$	
		$Y_{24-8}(n)$	
		$X_{6-25}(n)$	
		$X_{7-26}(n)$	
		$X_{1-23}(n)$	
		$X_{3-24}(n)$	
		$X_{k-l}(n)$	($k = 10, 11, 2, 8, 9, 4, 5$; $l = 27$ to 33)
		$X_{i-p}(n)$	($i = 16$ to 22; $p = 6, 1, 2, 3, 9, 4, 5$)
Control Variables		$Z(n)$	
		$X_{49-j}(n)$	($j = 52, 53$)
Output Variables	=	$X_{j-48}(n)$	($j = 41$ to 47)

By reducing the preceding equations to a minimal set of first ordered difference equations, the state model is derived. Exhibit 1 shows the state model. Its basic form is:

$$\psi(n+1) = P\,\psi(n) + QE(n) + SF(n)$$

where ψ is the state vector, P is the transition matrix, Q is the excitation matrix, E is the control vector, F is a vector of known functions of time and S is its coefficient matrix. Thus next period's state is expressed as a function of the current state, the control vector and known functions of time.

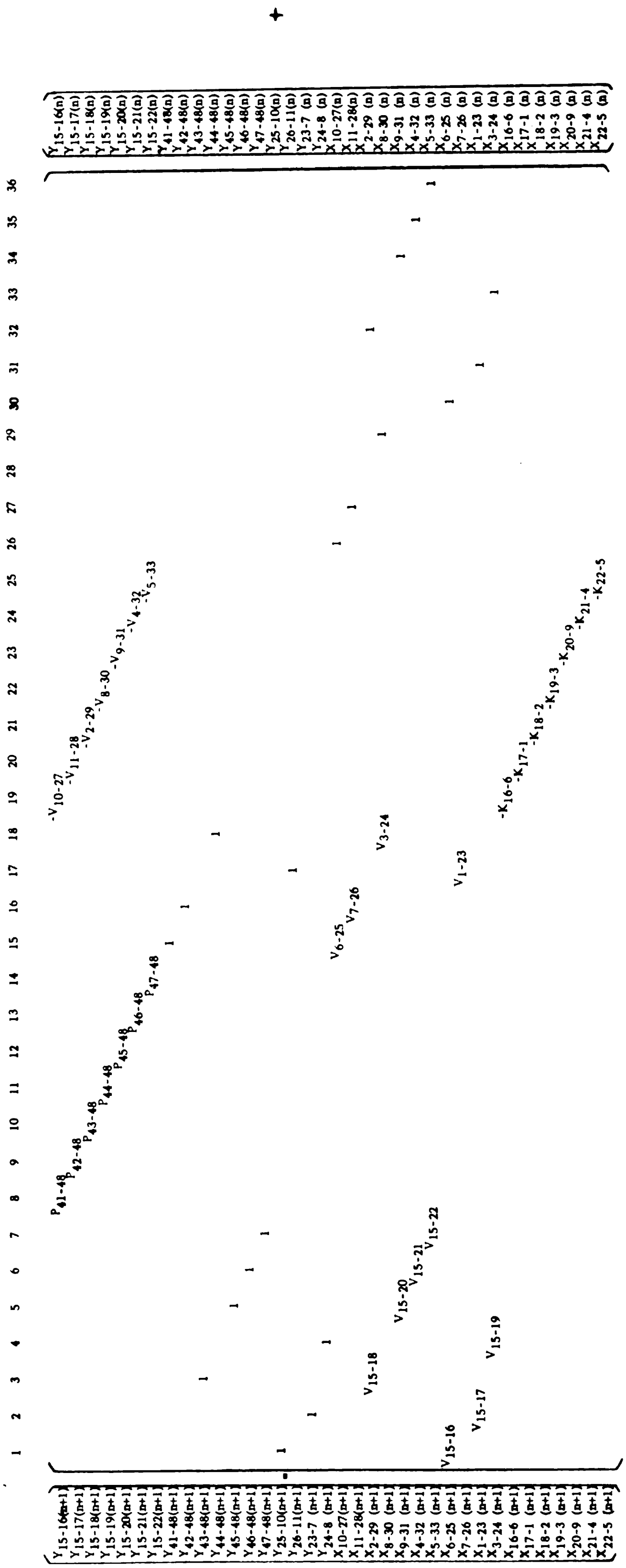

Exhibit 1. - State model.

$$
+ \begin{array}{ccc} 1 & 2 & 3 \end{array}
$$

$$
+ \begin{bmatrix}
V_{27-34} & & \\
V_{28-35} & & \\
V_{29-36} & & \\
V_{30-37} & & \\
V_{31-38} & & \\
V_{32-39} & & \\
V_{33-40} & & \\
\vdots & & \\
& -V_{49-52} & \\
& \vdots & \\
& & -V_{49-53} \\
& & \vdots
\end{bmatrix}
\begin{bmatrix}
Z_{(n)} \\
X_{49-52(n)} \\
X_{49-53(n)}
\end{bmatrix} +
$$

Exhibit 1. - (continued)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
1																														$F_{41(n)}$
	1																													$F_{42(n)}$
		1																												$F_{43(n)}$
			1																											$F_{44(n)}$
				1																										$F_{45(n)}$
					1																									$F_{46(n)}$
						1																								$F_{47(n)}$
																														$F_{v/1(n)}$
																														$F_{v/2(n)}$
																														$F_{v/3(n)}$
																														$F_{v/4(n)}$
																														$F_{v/5(n)}$
																														$F_{v/6(n)}$
																														$F_{v/7(n)}$
																														$F_{v/8(n)}$
																														$F_{v/9(n)}$
																														$F_{v/10(n)}$
																														$F_{v/11(n)}$
																-1											-1		V_{49-50}	$F_{r/1(n)}$
																	-1											-1		$F_{r/2(n)}$
								-1											-1											$F_{r/3(n)}$
														-1											-1					$F_{r/4(n)}$
															-1											-1				$F_{r/5(n)}$
										-1											-1									$F_{r/6(n)}$
											-1											-1								$F_{r/7(n)}$
												-1											-1						V_{49-51}	$F_{r/8(n)}$
													-1											-1						$F_{r/9(n)}$
							-1											-1												$F_{r/10(n)}$
										-1										-1										$F_{r/11(n)}$
																														$F_{1(n)}$

Exhibit 1. - (continued)

The output vector is shown in Exhibit 2. Its basic form is:

$$R(n) = M\psi(n) + NE(n)$$

where R(n) is the output vector expressed as a function of the state vector and the control vector.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36

$$\begin{bmatrix} X_{41-48(n)} \\ X_{42-48(n)} \\ X_{43-48(n)} \\ X_{44-48(n)} \\ X_{45-48(n)} \\ X_{46-48(n)} \\ X_{47-48(n)} \end{bmatrix} = \begin{bmatrix} &&&&&&&&&&&&&&&&&& -V_{34-41} &&&&&&&&&&&&&&&&& \\ &&&&&&&&&&&&&&&&&&& -V_{35-42} &&&&&&&&&&&&&&&& \\ &&&&&&&&&&&&&&&&&&&& -V_{36-43} &&&&&&&&&&&&&&& \\ &&&&&&&&&&&&&&&&&&&&& -V_{37-44} &&&&&&&&&&&&&& \\ &&&&&&&&&&&&&&&&&&&&&& -V_{38-45} &&&&&&&&&&&&& \\ &&&&&&&&&&&&&&&&&&&&&&& -V_{39-46} &&&&&&&&&&&& \\ &&&&&&&&&&&&&&&&&&&&&&&& -V_{40-47} &&&&&&&&&&& \end{bmatrix} \begin{bmatrix} Y_{15-16(n)} \\ Y_{15-17(n)} \\ Y_{15-18(n)} \\ Y_{15-19(n)} \\ Y_{15-20(n)} \\ Y_{15-21(n)} \\ Y_{15-22(n)} \\ Y_{41-48(n)} \\ Y_{42-48(n)} \\ Y_{43-48(n)} \\ Y_{44-48(n)} \\ Y_{45-48(n)} \\ Y_{46-48(n)} \\ Y_{47-48(n)} \\ Y_{25-10(n)} \\ Y_{26-11(n)} \\ Y_{23-7\ (n)} \\ Y_{24-8\ (n)} \\ X_{10-27(n)} \\ X_{11-28(n)} \\ X_{2-29\ (n)} \\ X_{8-30\ (n)} \\ X_{9-31\ (n)} \\ X_{4-32\ (n)} \\ X_{5-33\ (n)} \\ X_{6-25\ (n)} \\ X_{7-26\ (n)} \\ X_{1-23\ (n)} \\ X_{3-24\ (n)} \\ X_{16-6\ (n)} \\ X_{17-1\ (n)} \\ X_{18-2\ (n)} \\ X_{19-3\ (n)} \\ X_{20-9\ (n)} \\ X_{21-4\ (n)} \\ X_{22-5\ (n)} \end{bmatrix}$$

Exhibit 2. - Output vector.

$$
+ \quad
\begin{array}{c}
\begin{matrix} 1 & 2 & 3 \end{matrix} \\
\begin{bmatrix}
-D_{34-41} & & \\
-D_{35-42} & & \\
-D_{36-43} & & \\
-D_{37-44} & & \\
-D_{38-45} & & \\
-D_{39-46} & & \\
-D_{40-47} & &
\end{bmatrix}
\end{array}
\begin{bmatrix}
Z_{(n)} \\
X_{49-52(n)} \\
X_{49-53(n)}
\end{bmatrix}
$$

Exhibit 2. - (continued)

CHAPTER IV

ANALYSIS OF THE MODEL

Parametric Data

Ready availability of empirical data was one of the factors considered in the design of the model. All parametric data were estimated from actual empirical data. Tables 4 thru 9 list the parametric data. Exhibits 3 and 4 show the substitution of the parametric data into the state model and the output vector respectively.

Heuristic Simulation

In order to use the model for heuristic simulation, it is necessary to solve for $\psi(n)$. This solution is:

$$\psi(n) = P^n \psi(0) + \begin{bmatrix} P^{n-1}Q & P^{n-2}Q & \cdots & P^0Q \end{bmatrix} \begin{bmatrix} E(0) \\ E(1) \\ \cdot \\ \cdot \\ \cdot \\ E(n-1) \end{bmatrix} + G(n)$$

where $G(n) = \begin{bmatrix} P^{n-1}S & P^{n-2}S & \cdots & P^0S \end{bmatrix} \begin{bmatrix} F(0) \\ F(1) \\ \cdot \\ \cdot \\ \cdot \\ F(n-1) \end{bmatrix}$

Tables 10 and 11 give the initial state values and the control vector values required by the above formulas. Table 12 shows the results of the simulation from the initial state thru twelve periods of time.

Table 4. - Input output parameters.

j	Input Function $F_j(n) =$ a	+ b(n)	Rel. to Units Shipped P_{j-48}	Rel. to Cost of Sales Q_{j-48}
41	6900	172	.05	-.017
42	4300	107	.04	-.01
43	2800	70	.04	-.028
44	130	3	.02	-.001
45	1000	25	.02	-.01
46	500	12	.02	-.005
47	200	5	.02	-.002

k	l	Rel. to Sales Prom. K_{k-l}	% of Total Sales Prom. L_{k-l}
27	34	- 6.9	.3
28	35	- 3.8	.4
29	36	-11.2	.06
30	37	- .4	.06
31	38	- 4	.06
32	39	- 2	.06
33	40	- .8	.06

Input Cost Rel. to Factory Cost

$K_{i-p} = .001$ (i = 16 to 22; p = 6, 1, 2, 3, 9, 4, 5)

Table 5. - Resource parameters - Basic line processes - Direct resources.

i	Direct Labor Cost/Hr. $F_{12/i}(n) =$ a	+ b(n)	Direct Labor Hrs/Unit $K_{12/i-i}$	Direct Material Cost/Unit $F_{13/i}(n)$ a	+ b(n)	Direct Facility Cost/Unit $F_{14/i}(n)$ a	+ b(n)	$K_{14/i-i}$
1	5.00	.04	34.0	95.00	10.00	197.00	.25	.04
2	5.00	.04	20.6	65.00	7.00	25.70	.15	.004
3	4.87	.04	90.0	165.00	4.00	63.00	.50	.04
4	3.00	.02	100.0	300.00	15.00	26.00	.15	.023
5	3.65	.03	12.0	3.00	.04	5.50	.05	.014
6	2.96	.02	17.4	27.65	.26	635.00	4.00	.030
7	2.96	.02	17.4	27.65	.26	635.00	4.00	.030
8	2.95	.02	61.0	565.00	20.00	19.00	.14	.005
9	2.95	.02	61.0	565.00	20.00	19.00	.14	.005
10	3.00	.02	36.0	137.95	.92	1650.00	7.00	.130
11	3.00	.02	36.0	137.95	.92	1650.00	7.00	.130

Table 6. - Resource parameters - Basic line processes - Service and other overhead functions.

		Indirect Cost/Unit $F_{B/i}(n) =$			
i	B	a	+	b(n)	$K_{B/i-i}$
1	80	490		1.46	.08
	81	15		.11	.001
	82	75		.32	.01
	83	104		.18	.02
	84	31		.18	.003
	85	41		.28	.003
	86	94		.68	.006
	87	43		.30	.003
	88	55		.38	.004
	89	34		.25	.002
	90	38		.29	.002
	91	133		.90	.01
2	80	115		.95	.007
	81	10		.07	.001
	82	24		.21	.001
	83	14		.11	.001
	84	15		.12	.001
	85	21		.18	.001
	86	70		.44	.009
	87	22		.19	.001
	88	28		.25	.001
	89	19		.16	.001
	90	22		.19	.001
	91	78		.58	.007
3	80	426		3.43	.63
	81	39		.27	.09
	82	80		.76	.03
	83	47		.42	.04
	84	47		.43	.03
	85	85		.68	.13
	86	168		1.60	.06
	87	75		.71	.03
	88	95		.90	.04
	89	85		.59	.20
	90	103		.70	.25
	91	270		2.11	.45

Table 6. - (continued)

i	B	Indirect Cost/Unit $F_{B/i}(n) =$ a	+ b(n)	$K_{B/i-i}$
4	80	395	3.25	.14
	81	27	.26	.002
	82	87	.72	.03
	83	43	.40	.006
	84	51	.41	.02
	85	84	.64	.04
	86	167	1.52	.03
	87	72	.67	.01
	88	91	.86	.01
	89	59	.56	.005
	90	69	.66	.005
	91	208	2.00	.015
5	80	457	4.13	.23
	81	89	.33	.29
	82	123	.92	.16
	83	80	.51	.15
	84	54	.52	.008
	85	87	.81	.03
	86	234	1.93	.21
	87	106	.85	.11
	88	136	1.09	.14
	89	81	.71	.05
	90	98	.84	.07
	91	277	2.54	.12
6	80	20	.13	.001
	81	1	.01	--
	82	9	.02	.001
	83	8	.01	.001
	84	1	.01	--
	85	2	.02	--
	86	13	.06	.001
	87	9	.02	.001
	88	10	.03	.001
	89	2	.02	--
	90	2	.02	--
	91	15	.08	.001

Table 6. - (continued)

		Indirect Cost/Unit $F_{B/i}(n)$ =			
i	B	a	+	b(n)	$K_{B/i-i}$
7	80	20		.13	.001
	81	1		.01	--
	82	9		.02	.001
	83	8		.01	.001
	84	1		.01	--
	85	2		.02	--
	86	13		.06	.001
	87	9		.02	.001
	88	10		.03	.001
	89	2		.02	--
	90	2		.02	--
	91	15		.08	.001
8	80	252		1.66	.08
	81	14		.13	.001
	82	44		.37	.007
	83	23		.20	.003
	84	22		.21	.001
	85	34		.32	.002
	86	84		.77	.007
	87	37		.34	.003
	88	48		.44	.004
	89	29		.28	.001
	90	34		.33	.001
	91	104		1.02	.002
9	80	252		1.66	.08
	81	14		.13	.001
	82	44		.37	.007
	83	23		.20	.003
	84	22		.21	.001
	85	34		.32	.002
	86	84		.77	.007
	87	37		.34	.003
	88	48		.44	.004
	89	29		.28	.001
	90	34		.33	.001
	91	104		1.02	.002

Table 6. - (continued)

i	B	Indirect Cost/Unit $F_{B/i}(n) =$ a	+	b(n)	$K_{B/i-i}$
10	80	34		.27	.001
	81	2		.02	--
	82	13		.06	.001
	83	10		.03	.001
	84	3		.03	--
	85	5		.05	--
	86	20		.13	.001
	87	12		.05	.001
	88	14		.07	.001
	89	4		.04	--
	90	5		.05	--
	91	17		.17	--
11	80	34		.27	.001
	81	2		.02	--
	82	13		.06	.001
	83	10		.03	.001
	84	3		.03	--
	85	5		.05	--
	86	20		.13	.001
	87	12		.05	.001
	88	14		.07	.001
	89	4		.04	--
	90	5		.05	--
	91	17		.17	--

Table 7. - Service and other overhead functions - By type of resource.

i	A	Indirect Labor Cost/Hr $F_{12/i-i}(n)$ a	+ b(n)	Labor Hrs/Unit $K_{12/i-i}$	Material Cost/Unit $K_{13/i-i}$
1 to 11	60	4.70	.03	9.4	.32
	61	3.00	.02	1.5	.20
	62	2.80	.02	4.1	.21
	63	3.10	.02	1.3	.51
	64	3.00	.02	2.8	.07
	65	4.50	.03	1.4	.53
	66	2.80	.02	8.2	.24
	67	3.15	.02	3.5	.17
	68	4.70	.03	3.3	.15
	69	4.00	.03	1.9	.34
	70	4.10	.03	2.5	.26
	71	5.00	.04	5.2	.26

Table 8. - Cost reduction parameters.

K_{52-10}	.001
K_{53-6}	.001
K_{49-50}	.9
K_{49-51}	.9
N_{49-50}	-220
N_{49-51}	-165
M_{49-50}	-200
M_{49-51}	-150

Table 9. - Simplifying variables.

m	i	j	k	V_{m-i}	D_{m-i}	V_{j-k}	V_{k-m}
34	41	10	27	1.06	.318	-.018	-2.065
35	42	11	28	1.06	.424	-.011	-1.516
36	43	2	29	1.06	.064	-.030	- .670
37	44	8	30	1.06	.064	-.001	- .024
38	45	9	31	1.06	.064	-.011	- .239
39	46	4	32	1.06	.064	-.005	- .120
40	47	5	33	1.06	.064	-.002	- .048

		$F_{r/j}(n)$			$F_{v/j}(n)$	
j	l	a +	b(n)	V_l	a +	b(n)
1	15-16	462.00	11.61	.038	1153	5.33
2	15-17	193.70	7.97	.183	438	3.45
3	15-18	666.30	8.10	.036	1520	12.60
4	15-19	626.00	17.15	.202	1353	11.95
5	15-20	52.30	.45	.117	1822	15.18
6	15-21	714.15	4.61	.336	92	.43
7	15-22	714.15	4.61	1.582	92	.43
8	6-25	763.95	21.36	.137	725	6.07
9	7-26	763.95	21.36	.136	725	6.07
10	1-23	1895.95	8.64	.037	139	.97
11	3-24	1895.95	8.64	.117	139	.97

V_{49-52}	- .1
V_{49-53}	- .1
V_{49-50}	+2
V_{49-51}	+1.5

State (n+1)		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	State (n)
$Y_{15-16}(n+1)$									.05											.018																		$Y_{15-16}(n)$
$Y_{15-17}(n+1)$										.04											.011																	$Y_{15-17}(n)$
$Y_{15-18}(n+1)$											.04											.030																$Y_{15-18}(n)$
$Y_{15-19}(n+1)$												.02											.001															$Y_{15-19}(n)$
$Y_{15-20}(n+1)$													.02											.011														$Y_{15-20}(n)$
$Y_{15-21}(n+1)$														.02											.005													$Y_{15-21}(n)$
$Y_{15-22}(n+1)$															.02											.002												$Y_{15-22}(n)$
$Y_{41-48}(n+1)$																1																						$Y_{41-48}(n)$
$Y_{42-48}(n+1)$																	1																					$Y_{42-48}(n)$
$Y_{43-48}(n+1)$				1																																		$Y_{43-48}(n)$
$Y_{44-48}(n+1)$																			1																			$Y_{44-48}(n)$
$Y_{45-48}(n+1)$						1																																$Y_{45-48}(n)$
$Y_{46-48}(n+1)$							1																															$Y_{46-48}(n)$
$Y_{47-48}(n+1)$								1																														$Y_{47-48}(n)$
$Y_{25-10}(n+1)$	=	1																																				$Y_{25-10}(n)$
$Y_{26-11}(n+1)$																		1																				$Y_{26-11}(n)$
$Y_{23-7}(n+1)$			1																																			$Y_{23-7}(n)$
$Y_{24-8}(n+1)$				1																																		$Y_{24-8}(n)$
$X_{10-27}(n+1)$																.137											1											$X_{10-27}(n)$
$X_{11-28}(n+1)$																	.136											1										$X_{11-28}(n)$
$X_{2-29}(n+1)$				.036																													1					$X_{2-29}(n)$
$X_{8-30}(n+1)$																			.117											1								$X_{8-30}(n)$
$X_{9-31}(n+1)$						.117																													1			$X_{9-31}(n)$
$X_{4-32}(n+1)$							.336																													1		$X_{4-32}(n)$
$X_{5-33}(n+1)$								1.582																													1	$X_{5-33}(n)$
$X_{6-25}(n+1)$		.038																													1							$X_{6-25}(n)$
$X_{7-26}(n+1)$																		.037											1									$X_{7-26}(n)$
$X_{1-23}(n+1)$			.184																													1						$X_{1-23}(n)$
$X_{3-24}(n+1)$					.202																													1				$X_{3-24}(n)$
$X_{16-6}(n+1)$																				-.001																		$X_{16-6}(n)$
$X_{17-1}(n+1)$																					-.001																	$X_{17-1}(n)$
$X_{18-2}(n+1)$																						-.001																$X_{18-2}(n)$
$X_{19-3}(n+1)$																							-.001															$X_{19-3}(n)$
$X_{20-9}(n+1)$																								-.001														$X_{20-9}(n)$
$X_{21-4}(n+1)$																									-.001													$X_{21-4}(n)$
$X_{22-5}(n+1)$																										-.001												$X_{22-5}(n)$

+

Exhibit 3. - State model with parameter values.

+

1	2	3
-2.065		
-1.515		
-0.670		
-0.024		
-0.239		
-0.120		
-0.048		
	.1	
		.1

$$\begin{bmatrix} Z_{(n)} \\ X_{49-52(n)} \\ X_{49-53(n)} \end{bmatrix}$$

+

Exhibit 3. - (continued)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
1																														6900 + 172(n)
	1																													4300 + 107(n)
		1																												2800 + 70(n)
			1																											130 + 3(n)
				1																										1000 + 25(n)
					1																									500 + 12(n)
						1																								200 + 5(n)
																														1153 + 5.33(n)
																														438 + 3.45(n)
																														1520 + 12.60(n)
																														1353 + 11.95(n)
																														1822 + 15.18(n)
																														92 + 0.43(n)
																														92 + 0.43(n)
																														725 + 6.07(n)
																														725 + 6.07(n)
																														139 + 0.97(n)
																														139 + 0.97(n)
																-1											-1		+2	462.00 + 11.61(n)
																	-1											-1		193.70 + 7.97(n)
								-1											-1											666.30 + 8.10(n)
														-1											-1					626.00 + 17.15(n)
															-1											-1				52.30 + 0.45(n)
										-1											-1									714.15 + 4.61(n)
											-1											-1								714.15 + 4.61(n)
												-1											-1						+1.5	763.95 + 21.36(n)
													-1											-1						763.95 + 21.36(n)
							-1											-1												895.95 + 8.64(n)
									-1											-1										895.95 + 8.64(n)
																														1.00 + 0 (n)

Exhibit 3. - (continued)

$$\begin{bmatrix} X_{41-48(n)} \\ X_{42-48(n)} \\ X_{43-48(n)} \\ X_{44-48(n)} \\ X_{45-48(n)} \\ X_{46-48(n)} \\ X_{47-48(n)} \end{bmatrix} =$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
																		-1.06																	
																			-1.06																
																				-1.06															
																					-1.06														
																						-1.06													
																							-1.06												
																								-1.06											

$$\begin{bmatrix} Y_{15-16(n)} \\ Y_{15-17(n)} \\ Y_{15-18(n)} \\ Y_{15-19(n)} \\ Y_{15-20(n)} \\ Y_{15-21(n)} \\ Y_{15-22(n)} \\ Y_{41-48(n)} \\ Y_{42-48(n)} \\ Y_{43-48(n)} \\ Y_{44-48(n)} \\ Y_{45-48(n)} \\ Y_{46-48(n)} \\ Y_{47-48(n)} \\ Y_{25-10(n)} \\ Y_{26-11(n)} \\ Y_{23-7\ (n)} \\ Y_{24-8\ (n)} \\ X_{10-27(n)} \\ X_{11-28(n)} \\ X_{2-29\ (n)} \\ X_{8-30\ (n)} \\ X_{9-31\ (n)} \\ X_{4-32\ (n)} \\ X_{5-33\ (n)} \\ X_{6-25\ (n)} \\ X_{7-26\ (n)} \\ X_{1-23\ (n)} \\ X_{3-24\ (n)} \\ X_{16-6\ (n)} \\ X_{17-1\ (n)} \\ X_{18-2\ (n)} \\ X_{19-3\ (n)} \\ X_{20-9\ (n)} \\ X_{21-4\ (n)} \\ X_{22-5\ (n)} \end{bmatrix} +$$

Exhibit 4. - Output vector with parameter values.

$$
+ \quad
\begin{array}{c}
\begin{array}{ccc} 1 & 2 & 3 \end{array} \\
\begin{bmatrix}
-0.318 & & \\
-0.424 & & \\
-0.064 & & \\
-0.064 & & \\
-0.064 & & \\
-0.064 & & \\
-0.064 & &
\end{bmatrix}
\end{array}
\begin{bmatrix}
Z_{(n)} \\
X_{49-52(n)} \\
X_{49-53(n)}
\end{bmatrix}
$$

Exhibit 4. - (continued)

Table 10. - Initial state values.

$$\psi(0) = \begin{bmatrix} 5000 \\ 2200 \\ 1000 \\ 140 \\ 900 \\ 500 \\ 200 \\ 4500 \\ 2000 \\ 900 \\ 130 \\ 850 \\ 400 \\ 180 \\ 4700 \\ 2100 \\ 2150 \\ 135 \\ -4000 \\ -4500 \\ -1000 \\ -1000 \\ -1000 \\ -1000 \\ -1000 \\ -1700 \\ -1300 \\ -1200 \\ -400 \\ 4 \\ 4.5 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Table 11. - Given control vector values for heuristic simulation.

$$E(0) = \begin{matrix} -170 \\ -20 \\ -15 \end{matrix}$$

$$E(1) = \begin{matrix} -270 \\ -25 \\ -20 \end{matrix}$$

$$E(2) = \begin{matrix} -360 \\ -30 \\ -25 \end{matrix}$$

$$E(3) = \begin{matrix} -430 \\ -35 \\ -30 \end{matrix}$$

$$E(4) = \begin{matrix} -560 \\ -40 \\ -35 \end{matrix}$$

$$E(5) = \begin{matrix} -710 \\ -45 \\ -40 \end{matrix}$$

$$E(6) = \begin{matrix} -560 \\ -40 \\ -35 \end{matrix}$$

$$E(7) = \begin{matrix} -430 \\ -35 \\ -30 \end{matrix}$$

$$E(8) = \begin{matrix} -360 \\ -30 \\ -25 \end{matrix}$$

$$E(9) = \begin{matrix} -270 \\ -25 \\ -20 \end{matrix}$$

$$E(10) = \begin{matrix} -360 \\ -30 \\ -25 \end{matrix}$$

$$E(11) = \begin{matrix} -430 \\ -35 \\ -30 \end{matrix}$$

Table 12. - Simulated states from initial state thru twelve time periods.

	0	1	2	3	4	5	6
1	5000	7404	7809	8202	8646	9108	9610
2	2200	4588	4867	5105	5320	5729	6075
3	1000	2920	3073	3282	3405	3570	3745
4	140	136	140	144	148	154	161
5	900	1010	1034	1063	1090	1119	1148
6	500	523	545	569	589	617	648
7	200	210	219	228	237	248	261
8	4500	4700	5000	7404	7809	8202	8646
9	2000	2100	2150	2200	4588	4867	5105
10	900	1000	2920	3073	3282	3405	3570
11	130	135	140	136	140	144	148
12	850	900	1010	1034	1063	1090	1119
13	400	500	523	545	569	589	617
14	180	200	210	219	228	237	248
15	4700	5000	7404	7809	8202	8646	9108
16	2100	2150	2200	4588	4867	5105	5320
17	2150	2200	4588	4867	5105	5320	5729
18	135	140	136	140	144	148	154
19	-4000	-3091	-1972	-1568	-1515	-1463	-1403
20	-4500	-3049	-3679	-3691	-2872	-2806	-2753
21	-1000	- 595	- 537	- 543	- 546	- 553	- 558
22	-1000	-1873	-3656	-3706	-3751	-3796	-3843
23	-1000	-1383	-1397	-1421	-1444	-1468	-1492
24	-1000	-1810	-1831	-1852	-1873	-1895	-1915
25	-1000	-1557	-1557	-1557	-1558	-1560	-1558
26	-1700	- 612	- 526	- 518	- 510	- 500	- 488
27	-1300	-1926	-1935	-1431	-1392	-1362	-1337
28	-1200	-1206	- 783	- 750	- 723	- 700	- 643
29	- 400	-2157	-2179	-2198	-2216	-2236	-2256
30	4	4	3	2	2	2	1
31	5	4	3	4	4	3	3
32	1	1	1	1	1	1	1
33	1	1	2	4	4	4	4
34	1	1	1	1	1	1	1
35	1	1	2	2	2	2	2
36	1	1	2	2	2	2	2

Table 12. - (continued)

	7	8	9	10	11	12
1	9495	9423	9477	9457	9811	10130
2	5965	5884	5902	5888	6126	6335
3	3721	3711	3733	3742	3873	3990
4	161	161	162	163	168	172
5	1169	1192	1215	1238	1265	1292
6	642	639	652	653	676	696
7	259	258	259	260	269	278
8	9108	9610	9495	9423	9477	9457
9	5320	5729	6075	5965	5884	5902
10	3745	3721	3711	3733	3742	3873
11	154	161	161	161	162	163
12	1148	1169	1192	1215	1238	1265
13	648	642	639	652	653	676
14	261	259	258	259	260	269
15	9610	9495	9423	9477	9457	9811
16	5729	6075	5965	5884	5902	5888
17	6075	5965	5884	5902	5888	6126
18	161	161	161	162	163	168
19	-1337	-1264	-1298	-1326	-1332	-1352
20	-2708	-2596	-2504	-2577	-2638	-2664
21	- 563	- 574	- 586	- 596	- 607	- 613
22	-3889	-3935	-3984	-4032	-4079	-4127
23	-1515	-1540	-1565	-1589	-1614	-1637
24	-1934	-1964	-1994	-2019	-2048	-2069
25	-1553	-1572	-1589	-1602	-1616	-1617
26	- 474	- 483	- 492	- 494	- 501	- 493
27	-1270	-1216	-1263	-1303	-1322	-1347
28	- 596	- 634	- 666	- 679	- 699	- 672
29	-2276	-2297	-2318	-2338	-2359	-2379
30	1	1	1	1	1	1
31	3	3	3	3	3	3
32	1	1	1	1	1	1
33	4	4	4	4	4	4
34	1	2	2	2	2	2
35	2	2	2	2	2	2
36	2	2	2	2	2	2

As stated earlier, the treatment of simulation in this study is limited to a demonstration of its feasibility with the methodology being evaluated.

Stability Analysis

In lay terms, stability analysis seeks to ascertain whether the model by virtue of its internal characteristics will explode over time or will remain stable. Lyapunov[1] defined a stable system as one which, if disturbed by a small amount from an equilibrium state, would either return to that state or stay within some pre-assigned finite region near the equilibrium state. Using this definition, stability can be determined by solving for the eigenvalues of the minimal polynomial of the transition matrix P. The system is stable if the modulus of each eigenvalue of multiplicity one is less than or equal to one and the modulus of each eigenvalue of multiplicity two or greater is less than one. (One can also use with proper care the eigenvalues of the characteristic polynomial which are the same as those of the minimal polynomial but may have different multiplicity.)

Table 13 lists the coefficients of the characteristic polynomial of P. Table 14 lists the eigenvalues of P. Since the moduli of the eigenvalues are all less than one, the system is asymptotically stable.

---

[1] A. M. Lyapunov, "Le Probleme General de la Stabilite du Mouvement," Annals of Mathematical Studies, XVII (Princeton, N.J.: Princeton University Press, 1949).

Table 13. - Coefficients of characteristic polynomial of P.

1.	0.0	19.	$-0.1360869 \times 10^{-10}$
2.	0.1032110×10^{0}	20.	$-0.1812394 \times 10^{-11}$
3.	0.7146900×10^{-1}	21.	$0.3152086 \times 10^{-12}$
4.	0.3916918×10^{-1}	22.	$0.9166884 \times 10^{-15}$
5.	$-0.7376387 \times 10^{-2}$	23.	$-0.4320720 \times 10^{-14}$
6.	$-0.5312974 \times 10^{-2}$	24.	$0.2060674 \times 10^{-15}$
7.	$-0.2799382 \times 10^{-2}$	25.	$0.1587409 \times 10^{-16}$
8.	0.2978753×10^{-3}	26.	$-0.2072171 \times 10^{-17}$
9.	0.3107028×10^{-3}	27.	$0.1015041 \times 10^{-18}$
10.	0.3022769×10^{-4}	28.	$0.7953481 \times 10^{-20}$
11.	$-0.1416625 \times 10^{-4}$	29.	$0.1621995 \times 10^{-21}$
12.	$-0.3944258 \times 10^{-5}$	30.	$0.5053931 \times 10^{-22}$
13.	0.5427271×10^{-6}	31.	$0.1074626 \times 10^{-24}$
14.	0.1864992×10^{-6}	32.	$0.8071177 \times 10^{-25}$
15.	$-0.2013224 \times 10^{-7}$	33.	$-0.2377468 \times 10^{-28}$
16.	$-0.6039891 \times 10^{-8}$	34.	$0.8525294 \times 10^{-28}$
17.	0.5575385×10^{-9}	35.	$0.1769470 \times 10^{-28}$
18.	0.1444537×10^{-9}	36.	$-0.8107572 \times 10^{-30}$

Table 14. - Eigenvalues of P.

	REAL	IMAGINARY	MODULUS
1.	0.3198519×10^{-1}	0.0	9.3198520×10^{-1}
2.	0.3455048×10^{-1}	0.3762221×10^{-1}	0.5108001×10^{-1}
3.	0.3455048×10^{-1}	$-0.3762221 \times 10^{-1}$	0.5108001×10^{-1}
4.	0.2253598×10^{-2}	0.5944792×10^{-1}	0.5949062×10^{-1}
5.	0.2253598×10^{-2}	$-0.5944792 \times 10^{-1}$	0.5949062×10^{-1}
6.	$-0.2732943 \times 10^{-1}$	0.4771433×10^{-1}	0.5498686×10^{-1}
7.	$-0.2732943 \times 10^{-1}$	$-0.4771433 \times 10^{-1}$	0.5498686×10^{-1}
8.	$-0.8777088 \times 10^{-1}$	$-0.1478515 \times 10^{-15}$	0.8777093×10^{-1}
9.	-0.1061778×10^{0}	$0.2268977 \times 10^{-15}$	0.1061778×10^{0}
10.	-0.1431792×10^{0}	$0.1946615 \times 10^{-15}$	0.1431792×10^{0}
11.	-0.1504633×10^{0}	$-0.1385099 \times 10^{-2}$	0.1504698×10^{0}
12.	-0.1504633×10^{0}	0.1385099×10^{-2}	0.1504698×10^{0}
13.	$-0.5440717 \times 10^{-1}$	$-0.6100480 \times 10^{-14}$	0.5440717×10^{-1}
14.	0.5161469×10^{-1}	0.8587146×10^{-1}	0.1001897×10^{0}
15.	0.5161469×10^{-1}	$-0.8587146 \times 10^{-1}$	0.1001897×10^{0}
16.	0.4689423×10^{-1}	0.8579135×10^{-1}	0.9777130×10^{-1}
17.	0.4689423×10^{-1}	$-0.8579135 \times 10^{-1}$	0.9777130×10^{-1}
18.	0.1461591×10^{0}	0.1814614×10^{-2}	0.1461704×10^{0}

Table 14. - Eigenvalues of P. (continued)

	REAL	IMAGINARY	MODULUS
19.	0.1461591×10^{0}	$-0.1814614 \times 10^{-2}$	0.1461704×10^{0}
20.	0.1521912×10^{0}	$0.9834735 \times 10^{-11}$	0.1521912×10^{0}
21.	0.2026185×10^{0}	$-0.6848959 \times 10^{-13}$	0.2026186×10^{0}
22.	0.1228949×10^{0}	0.1228949×10^{0}	0.1737997×10^{0}
23.	0.1228949×10^{0}	-0.1228949×10^{0}	0.1737997×10^{0}
24.	-0.1228948×10^{0}	0.1228949×10^{0}	0.1737996×10^{0}
25.	-0.1228948×10^{0}	-0.1228949×10^{0}	0.1737996×10^{0}
26.	-0.1364051×10^{0}	0.2362606×10^{0}	0.2728103×10^{0}
27.	-0.1364051×10^{0}	-0.2362606×10^{0}	0.2728103×10^{0}
28.	-0.1879226×10^{0}	0.3254915×10^{0}	0.3758452×10^{0}
29.	-0.1879226×10^{0}	-0.3254915×10^{0}	0.3758452×10^{0}
30.	-0.2026187×10^{0}	$0.4475919 \times 10^{-15}$	0.2026187×10^{0}
31.	0.2728102×10^{0}	$0.3633910 \times 10^{-14}$	0.2728103×10^{0}
32.	0.3758452×10^{0}	$-0.1030937 \times 10^{-14}$	0.3758452×10^{0}
33.	0.4575791×10^{0}	$0.5503974 \times 10^{-16}$	0.4575791×10^{0}
34.	$0.1613792 \times 10^{-11}$	-0.4575791×10^{0}	0.4575791×10^{0}
35.	$0.1482293 \times 10^{-11}$	0.4575791×10^{0}	0.4575791×10^{0}
36.	$-0.1482293 \times 10^{-11}$	-0.4575791×10^{0}	0.4575791×10^{0}

Control Analysis

A system is said to be state controllable if by means of a series of control signals, it can be brought from any initial state to any desired state in a finite period of time. The number of control signals required is equal to the order of the system model, in this case thirty-six. Since there are three control variables, twelve time periods are required to reach a desired state.

To find out whether the system is state controllable, it is necessary to compute H.

$$H = \left[P^{11}Q \;\; P^{10}Q \; . \; . \; . \; P^{o}Q \right]$$

H in this case is a 36 x 36 matrix. If H is non-singular, the system is state controllable.

Table 15 lists the coefficients of the characteristic polynomial of H. Table 16 lists the eigenvalues of H. These results indicate that H is non-singular and hence the system is state controllable.

To find the values of the thirty-six control signals needed to reach a desired state, the following formula is used:

$$E_s = H^{-1}\left[\psi\text{ desired} - P^{12}\,\psi(o) - G(12)\right]$$

where E_s is the thirty-six control signals (twelve sets of three each) to bring the system to the desired state. The other variables have been defined previously. This formula is actually the formula for solving the state model, with the control vector on the left side of the equation.

A common way of demonstrating state controllability is to show that the system can be reduced from any initial state to a final state of zero (though the final state could just as easily be any desired state).

Table 15. - Coefficients of characteristic polynomial of H.

1.	$-0.8721831 \times 10^{-1}$	19.	$-0.4496994 \times 10^{-42}$
2.	0.2810221×10^{-2}	20.	$0.5456394 \times 10^{-42}$
3.	0.2870016×10^{-3}	21.	$-0.4993221 \times 10^{-43}$
4.	0.6446587×10^{-6}	22.	$0.5114684 \times 10^{-44}$
5.	0.5279475×10^{-8}	23.	$-0.4077664 \times 10^{-45}$
6.	$-0.2848693 \times 10^{-11}$	24.	$0.3406399 \times 10^{-46}$
7.	$-0.5126255 \times 10^{-14}$	25.	$-0.2600429 \times 10^{-47}$
8.	$0.2729302 \times 10^{-18}$	26.	$0.2024554 \times 10^{-48}$
9.	$-0.8353666 \times 10^{-23}$	27.	$-0.1513900 \times 10^{-49}$
10.	$-0.7212567 \times 10^{-26}$	28.	$0.1141644 \times 10^{-50}$
11.	$0.3169615 \times 10^{-30}$	29.	$-0.8439314 \times 10^{-52}$
12.	$-0.1121086 \times 10^{-31}$	30.	$0.6260988 \times 10^{-53}$
13.	$0.7454431 \times 10^{-33}$	31.	$-0.4596795 \times 10^{-54}$
14.	$-0.2619676 \times 10^{-34}$	32.	$0.3379925 \times 10^{-55}$
15.	$0.1675823 \times 10^{-35}$	33.	$-0.2471257 \times 10^{-56}$
16.	$-0.5235852 \times 10^{-37}$	34.	$0.1807917 \times 10^{-57}$
17.	$0.2995573 \times 10^{-38}$	35.	$-0.1318572 \times 10^{-58}$
18.	$-0.3366049 \times 10^{-40}$	36.	$0.9618951 \times 10^{-60}$

Table 16. - Eigenvalues of H.

	REAL	IMAGINARY	MODULUS
1.	0.9686768×10^{-1}	0.1662307×10^{0}	0.1923955×10^{0}
2.	0.9686768×10^{-1}	-0.1662307×10^{0}	0.1923955×10^{0}
3.	0.6419629×10^{-1}	0.1817901×10^{0}	0.1927922×10^{0}
4.	0.6419629×10^{-1}	-0.1817901×10^{0}	0.1927922×10^{0}
5.	0.2917200×10^{-1}	0.1910860×10^{0}	0.1932999×10^{0}
6.	0.2917200×10^{-1}	-0.1910860×10^{0}	0.1932999×10^{0}
7.	$-0.7076509 \times 10^{-2}$	0.1937758×10^{0}	0.1939050×10^{0}
8.	$-0.7076509 \times 10^{-2}$	-0.1937758×10^{0}	0.1939050×10^{0}
9.	$-0.4337440 \times 10^{-1}$	0.1896941×10^{0}	0.1945899×10^{0}
10.	$-0.4337440 \times 10^{-1}$	-0.1896941×10^{0}	0.1945899×10^{0}
11.	$-0.7851803 \times 10^{-1}$	0.1788594×10^{0}	0.1953350×10^{0}
12.	$-0.7851803 \times 10^{-1}$	-0.1788594×10^{0}	0.1953350×10^{0}
13.	-0.1112903×10^{0}	0.1614823×10^{0}	0.1961175×10^{0}
14.	-0.1112903×10^{0}	-0.1614823×10^{0}	0.1961175×10^{0}
15.	-0.1404754×10^{0}	0.1379801×10^{0}	0.1969057×10^{0}
16.	-0.1404754×10^{0}	-0.1379801×10^{0}	0.1969057×10^{0}
17.	-0.1648769×10^{0}	0.1090015×10^{0}	0.1976506×10^{0}
18.	-0.1648769×10^{0}	-0.1090015×10^{0}	0.1976506×10^{0}
19.	-0.1833575×10^{0}	0.7546777×10^{-1}	0.1982810×10^{0}
20.	-0.1833575×10^{0}	$-0.7546777 \times 10^{-1}$	0.1982810×10^{0}
21.	-0.1949226×10^{0}	0.3861605×10^{-1}	0.1987110×10^{0}
22.	-0.1949226×10^{0}	$-0.3861605 \times 10^{-1}$	0.1987110×10^{0}
23.	-0.1988646×10^{0}	$-0.3524585 \times 10^{-14}$	0.1988646×10^{0}
24.	0.1509723×10^{0}	0.1185385×10^{0}	0.1919480×10^{0}
25.	0.1509723×10^{0}	-0.1185385×10^{0}	0.1919480×10^{0}
26.	0.1705385×10^{0}	0.8792925×10^{-1}	0.1918722×10^{0}
27.	0.1705385×10^{0}	$-0.8792925 \times 10^{-1}$	0.1918722×10^{0}
28.	0.1840671×10^{0}	0.5410934×10^{-1}	0.1918555×10^{0}
29.	0.1840671×10^{0}	$-0.5410934 \times 10^{-1}$	0.1918555×10^{0}
30.	0.1909888×10^{0}	0.1826744×10^{-1}	0.1918604×10^{0}
31.	0.1909888×10^{0}	$-0.1826744 \times 10^{-1}$	0.1918604×10^{0}
32.	0.1261210×10^{0}	0.1449206×10^{0}	0.1921159×10^{0}
33.	0.1261210×10^{0}	-0.1449206×10^{0}	0.1921159×10^{0}
34.	-0.6650921×10^{0}	$-0.1929737 \times 10^{-10}$	0.6650921×10^{0}
35.	0.5650247×10^{0}	$-0.1114763 \times 10^{-11}$	0.5650247×10^{0}
36.	-0.5650247×10^{0}	$0.1114763 \times 10^{-11}$	0.5650247×10^{0}

Table 17 shows the twelve sets of control vector signals that will bring the state to zero.

Table 17. - Control signals to reduce state to zero.

	1	2	3
1	0.1398637×10^{9}	0.9657004×10^{6}	0.4602871×10^{7}
2	0.2353126×10^{6}	0.5304721×10^{5}	0.3534450×10^{6}
3	0.1527203×10^{6}	0.3557549×10^{9}	0.6364067×10^{7}
4	0.8184403×10^{6}	0.3478202×10^{7}	-0.1090784×10^{5}
5	0.2431166×10^{5}	0.1254539×10^{5}	-0.7918559×10^{7}
6	0.1793574×10^{10}	0.5466162×10^{8}	0.2431576×10^{4}
7	0.6201062×10^{8}	0.2293626×10^{7}	0.3894586×10^{5}
8	0.1160093×10^{7}	0.2311828×10^{5}	0.3908741×10^{5}
9	0.4612551×10^{5}	0.2330217×10^{6}	0.3834192×10^{9}
10	0.9635799×10^{7}	0.3343301×10^{5}	-0.4534569×10^{6}
11	-0.9012514×10^{4}	-0.4100174×10^{4}	-0.3637680×10^{4}
12	-0.8405845×10^{3}	-0.5680206×10^{4}	-0.1074879×10^{5}

The significance of reduction to state zero is purely mathematical since an executive would hardly want to drive his firm out of business. Since the model is based on relationships and empirical parameters which exist within a realistic range, manipulation of the model is meaningful if kept within this range. Thus control strategy should be concerned with the attainment of realistic sales and profit goals.

A prototype model such as the one in this study, with its simplification of factors and relationships, may generate impractical control strategies. Future work on more advanced and comprehensive models will cope with this problem.

The techniques of optimal control,which will be employed in future studies, give specific consideration to the magnitudes of the control variables imposed by the control strategy and the magnitudes of the state variables during the control interval.

Other variables which are linear functions of the state variables, such as total cost of sales, can be specified as the outputs which one may wish to bring to desired levels in a minimum amount of time thru a series of control signals. A system in which this is possible is said to be output controllable. To test for output controllability, one must compute K thus:

$$K = \begin{bmatrix} MPQ & MQ & N \end{bmatrix}$$

where M and N are the coefficient matrices of the state vector and control vector respectively in the expression for the output vector. P is the transition matrix and Q, the excitation matrix. If K is non-singular, the system is output controllable. The output control equation is a restatement of the formula for solving the output equation.

Unfortunately, with the simplified structure of the prototype model, K is singular and hence the system is not output controllable. This result is not particularly catastrophic. Since the model is state controllable and the output vector is a linear function of the state vector, management can still specify target costs of sales and find the control signals that will achieve them. As future versions of the model become more complex and comprehensive, the model will very probably become output controllable.

The intriguing area of optimal control is deferred to future studies since this would represent a major undertaking in itself.

CHAPTER V

SUMMARY AND CONCLUSIONS

Summary

This study constructs a detailed prototype mathematical model of the internal operations of an actual aerospace engineering and manufacturing enterprise. It uses a methodology originally developed for analyzing system models of electrical networks. This study evaluates the practical utility of this technique in assisting the business executive. It is concerned primarily with the generalized analytical capabilities of the methodology used.

The methodology used offers a formal process for generating a model from a defined system structure and for analyzing the solution characteristics of the model. This approach involves the use of linear graphs, flow and unit cost variables,and the development of a state model by reduction of the algebraic and difference equations that characterize the system to a minimal set. This model can be used for simulation and for stability and control analysis.

Chapter II gives a detailed description of the firm being modeled. It is essentially a large job shop with a wide range of highly technical precision products for use on high performance aircraft, missiles and spacecraft, sold mainly to meet Government defense or space needs.

Chapter III describes the construction of the model. It is essentially a direct costing accounting type model. This formulation is more salable to

management because its correspondence to the actual system can be readily understood. It also makes the model more useful for management by providing them with a structure with which they are familiar and comfortable. The model follows product flow from receipt of order to shipment thru the internal sectors of the firm. It shows the flow of resources and overhead function services into the different product stages and imputes their cost into the product.

Chapter IV discusses the analysis of the model. Simulation based on the model is presented. The model is found to be stable and state controllable. The control strategy to reduce the state to zero is derived.

Practical Utility to the Executive

Considerable practical utility is found for both the methodology and the model. Relative to other techniques, the methodology provides a convenient formal and generalized procedure for developing a model of the firm.

This model of the firm can be quite detailed without sacrificing compactness. This aspect is very pertinent. A model that requires considerable computer memory storage and processing time to run, will not be used frequently if at all by the executive. Its usefulness diminishes if the executive cannot get sufficiently rapid response time and if he knows that while the model is being processed, the regular production work on the computer is being jeopardized. Ideally the executive will want to interrogate and manipulate the model in conversational mode. Such a real-time inquiry and simulation system is feasible only if the model is compact enough to fit in a memory partition of a multi-programming computer and require an amount of computing that is not too voluminous to permit an acceptable response time.

The model has many managerial uses such as forecasting costs, load, resource requirements or cash flow and analyzing total impact of alternative decisions. The fact that the model automatically takes into account all the critical interactions of the firm, assists the executive in an area where he has traditionally felt impotent.

Since one can determine from the model each component cost and flow volume at any time period, this data can be translated easily to any accounting conventions, e.g., charging costs to a time period different from that in which they were incurred. Computer output from the model can be structured in the terms and formats of the management reports normally used by the executive.

The model is also useful for training purposes. It can be employed as a management game simulating the total firm or, if desired, just specific sectors.

The most promising facet of this methodology is its generalized analytical capabilities. As management pins down more of the true cause and effect relationships in the firm, the control strategy from the model will play an ever increasing role in guiding their key decisions.

Future Development of the Model

The route for future development of the model to take is toward greater detail, more variables, more sectors, more interrelationships, more realism and in general more complexity and sophistication.

Future models should introduce a finer breakdown of product lines. The next model should at least specify the major product families within the standard product line.

The basic unit of time should be reduced to a month. This will introduce a greater number of time lags into the process. This in turn can be handled by dummy sectors, or preferably by bringing in the various departments that make up each sector.

Modeling sectors external to the firm, such as competition or the labor market and linking these to the model of the firm, should provide interesting results.

More factors and interrelationships such as scrap and reject flows can be incorporated into the model. The introduction of price into the model will be difficult and require considerable data accumulation but it will enhance the realism and usefulness of the model significantly. More control variables can be used. An intensive search must be made for the true cause and effect relationships in the firm. Non-linear relationships such as the impact of research and development on transition coefficients can be evaluated by heuristic simulation.

A very promising approach for handling non-linear relationships thru linear analytical techniques is to divide the firm into profit centers wherever possible. Each of these profit centers can have independently derived imputed profits or losses which would then be brought together for the entire firm in the output vector or objective function.

The future development of the model should include the application of optimal control techniques. If successful, the model will become an invaluable tool for management. If price is part of the model, the objective function may be maximum profit.

All the above suggestions will entail considerable time and money as well as trade-offs in the size, response time and analyzability of the model.

However, this study has shown that the approach evaluated is of much practical utility to the executive and bears great promise as a generalized analytical tool for building and studying models of firms. Therefore the investment in future development is definitely justified.

BIBLIOGRAPHY

Books

An Introduction to Simulation by Computer - A Management Tool in Decision-Making. White Plains, N.Y.: IBM, 1965.

Bellman, R. Stability Theory of Differential Equations. McGraw-Hill Book Co., 1953.

Bonini, C. P. Simulation of Information and Decision Systems in the Firm. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1963.

Bowman, E. H. and Fetter, R. B. Analysis for Production and Operations Management. Homewood, Illinois: Richard D. Irwin, Inc., 1967.

Brown, R. G. and Nilsson, J. W. Introduction to Linear Systems Analysis. New York: John Wiley and Sons, Inc., 1962.

Buffa, E. S. Models for Production and Operations Management. New York: John Wiley & Sons, Inc., 1963.

Busacker, R. G. and Saaty, T. L. Finite Graphs and Networks - An Introduction with Applications. New York: McGraw-Hill Book Co., 1965.

Ellis, D. O. and Ludwig, F. J. Systems Philosophy, Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1962.

Forrester, Jay W. Industrial Dynamics. Cambridge: MIT Press, 1961.

Gantmacher, F. R. The Theory of Matrices, Vol I. New York: Chelsea Publishing Co., 1960.

General Purpose Systems Simulator III - White Plains, N.Y.: IBM, 1965.

Hall, A. D. A Methodology for Systems Engineering. Princeton, N.J.: VanNostrand, 1962.

Harary, F. and Norman, R. Z. Graph Theory as a Mathematical Model in Social Science. Ann Arbor, Mich.: University of Michigan Institute for Social Research, 1953.

Huggins, W. H., Flagle, C. D. and Roy, R. H. Operations Research and Systems Engineering. Baltimore, Md.: Johns Hopkins Press, 1960.

Koenig, H. E. and Blackwell, W. A. Electromechanical System Theory. McGraw-Hill Book Co., 1961.

Koenig, H. E. and others. A Systems Approach to Higher Education. East Lansing, Mich.: Michigan State University, 1966.

Koenig, H. E., Tokad, Y. and Kesavan, H. K. Analysis of Discrete Physical Systems. New York: McGraw-Hill Book Co., Inc., 1967.

LaSalle, J. and Lefschetz S. Stability by Lyapunov's Direct Method with Applications. New York: Academic Press, Inc., 1961.

Lorens, C. S. Flowgraphs for the Modeling and Analysis of Linear Systems. New York: McGraw-Hill Book Co., 1964.

McGuire, J. W. Interdisciplinary Studies in Business Behavior. Cincinnati, Ohio: South-Western Publishing Co., 1962.

McMillan, Claude and Gonzales, Richard F. Systems Analysis, A Computer Approach to Decision Models. Homewood, Illinois: Richard D. Irwin, Inc., 1965.

Malcolm, D. and Rowe, A. (ed.) Management Control Systems. New York: Wiley, 1960.

Proceedings-IBM Scientific Computing Symposium-Simulation Models and Gaming. White Plains, N.Y.: IBM, 1966.

Proceedings-IBM Scientific Symposium-Control Theory and Applications. White Plains, N.Y.: IBM, 1966.

Schwarz, R. J. and Friedland, B. Linear Systems. New York: McGraw-Hill Book Co., 1965.

Seely, S. Dynamic System Analysis. New York: Reinhold Publishing Co., 1964.

Stubberud, A. R. Analysis and Synthesis of Linear Time-Variable Systems. Berkeley, Calif.: University of California Press, 1964.

Zadeh, L. A. and Desoer, C. A. Linear System Theory. New York: McGraw-Hill Book Co., 1963.

Periodicals

Boulding, K. E. "General Systems Theory-The Skeleton of Science", Management Science, II: 197-208, April 1956.

Forrester, Jay W. "Industrial Dynamics-A Major Breakthrough for Decision Makers", Harvard Business Review, 36: 37-66, July-August 1958.

Gilbert, E. G. "Controllability and Observability in Multi-Variable Control Systems", Journal of the Society of Industrial Applied Mathematics, Series A: Control, I, 1963.

Katz, Abraham "An Industrial Dynamic Approach to the Management of Research and Development", IRE Transactions on Engineering Management, EM 6: 75-80, September 1959.

Koenig, H. E. and Tokad, Y. "A General Theorem for the Existence and Solution of State Models of Linear, Stationary Systems of Multi-Terminal Components", IEEE Transactions on Circuit Theory, CT-12: 4, December 1965.

Markowitz, H. M. "Simulating with Simscript", Management Science, 12: B-396-B-405, June 1966.

Unpublished Material

Bacon, C. M. "State-Space Design and Optimization of Linear Time-Invariant Systems." Unpublished Ph.D. Thesis, Michigan State University, 1964.

Ellis, J. B. "Description and Analysis of Socio-Economic Systems by Physical Systems Techniques." Unpublished Ph.D. Thesis, Michigan State University, 1965.

Koenig, H. E. "Systems Analysis for Social Scientists." Unpublished Class Notes, Michigan State University, 1966. (Mimeographed.)

Manetsch, T. J. "Simulation and Systems Analysis of the U.S. Softwood Plywood Industry." Unpublished Ph.D. Thesis, Department of Electrical Engineering, Oregon State University, 1964.

Rao, T. R. "Application of the Systems Theory Approach for the Development of a Mathematical Model of a Firm." Unpublished paper, Michigan State University, 1966. (Mimeographed.)

Soule, Linn "A Discrete State Deterministic System Model for Analysis of the Firm." Unpublished Ph.D. Thesis, Department of Management, Michigan State University, 1967.