MULTI-IMPEDANCE LOADING OF LINEAR ANTENNAS FOR IMPROVED TRANSMITTING AND RECEIVING CHARACTERISTICS

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This is to certify that the

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ABSTRACT

MULTI-IMPEDANCE LOADING
OF LINEAR ANTENNAS FOR IMPROVED
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This thesis presents a method by which the transmitting and receiving characteristics of a linear antenna may be improved or modified by the use of multi-loading.

The idea of modifying the antenna characteristics by impedance loading is not new. In recent years many researchers in the antenna area have studied the technique both theoretically and experimentally. Most works, however, are restricted to single or double loading. A few experimental studies dealing with multi-loading lack theoretical explanation. In this thesis, the general case of multi-loaded transmitting and receiving linear antennas is investigated based on a systematic and rigorous approach.

The main findings of this thesis are as follows. For a transmitting antenna if (1) the antenna current is specified at M + N points, (2) the radiation field is specified in M + N directions, (3) the input impedance is specified at M + N different frequencies, or (4) any linear combinations of case (1), (2), and (3), then M + N complex impedances can be found at M + N specified locations. For the case of pure reactive loading with M + N reactances, only (M + N)/2 values of the antenna characteristics can be specified. An interesting case of loadings which lead to the resonance or instability of a transmitting antenna is discussed. For a receiving antenna, M + N complex impedances can again be found at M + N specified locations if (1) the antenna current is specified at M + N points, (2) the scattered field is specified in M + N directions, or (3) the combination of cases (1) and (2) and if the central load is specified. The case of loading which will cause resonance or instability in a receiving antenna is also studied.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

This study will present a method by which the transmitting and receiving characteristics of a linear antenna may be improved or modified by the use of multi-loading.

The idea of modifying the antenna characteristics by impedance loading is not new. In recent years many researchers in the antenna area have studied this technique both the theoretically and experimentally. However, most works are restricted to single or double loading. A few experimental studies dealing with multi-loading lack theoretical explanation. In this study, the general case of multi-loaded transmitting and receiving linear antennas is investigated based on a systematic and rigorous approach.

In Chapter 2, the basic antenna characteristics are expressed in matrix form as functions of loading impedances and antenna parameters. In addition the desired antenna characteristics are expressed in the form of constraint equations. These constraint equations are combined with matrix equations to determine the solutions. In Chapters 3 and 4, antenna currents and field patterns for both short and long linear antennas are derived. Chapter 5 is devoted to numerical techniques and examples of transmitting antennas. Chapter 6 is concerned with the development of the matrix and constraint equations for a multi-loaded receiving antenna. In Chapter 7, the current and scattered field of an unloaded receiving antenna, both short and long, are presented. Finally, Chapter 8 presents additional numerical examples of a multi-loaded receiving antenna.

The main findings of this study are as follows. For a transmitting antenna if (1) the antenna current is specified at M + N points, (2) its radiation field is specified in M + N directions, (3) the input impedance is specified at M + N different frequencies, or (4) any linear combination of cases (1), (2), and (3), then M + N complex impedances can be found at M + N specified locations. For the case of pure reactive loading with M + N reactances, only (M + N)/2 values of the antenna characteristics can be specified. An interesting case of loadings which will lead to the resonance or instability of the antenna is discussed. For a receiving antenna, M + N complex impedances can again be found at M + N specified locations if (1) the antenna current is specified at M + N points, (2) the scattered field is specified in M + N directions, or (3) the combination of cases (1) and (2) and if the central load is specified. The case of loading which will cause resonance or instability in a receiving antenna is also studied.

CHAPTER II

MATHEMATICAL FORMULATION OF A MULTI-LOADED RADIATING ANTENNA

In this chapter the basic equations which mathematically describe a multi-loaded radiating antenna will be derived. The functional relationships between the driving voltage, the loading impedances, and the antenna dimensions will be developed. Finally constraint equations will be derived based on the required antenna performance.

2.1. Matrix Formulation

The problem of a multi-loaded antenna can be treated by considering each load as a voltage driver at an arbitrary driving point and applying the superposition principle. Consider a center-driven antenna of length 2h, and radius a, having N + M arbitrary loads along its axis, as shown in Figure 2.1. If each load is replaced by the voltage induced across it, the problem becomes equivalent to the antenna in Figure 2.2. This second antenna can easily be broken into M + N asymmetrically driven antennas as shown in Figure 2.3. Since the solution for the current on an asymmetrically driven antenna is a function of frequency, ω; radius, a; half length, h; and position of the driver, d_j, the current at

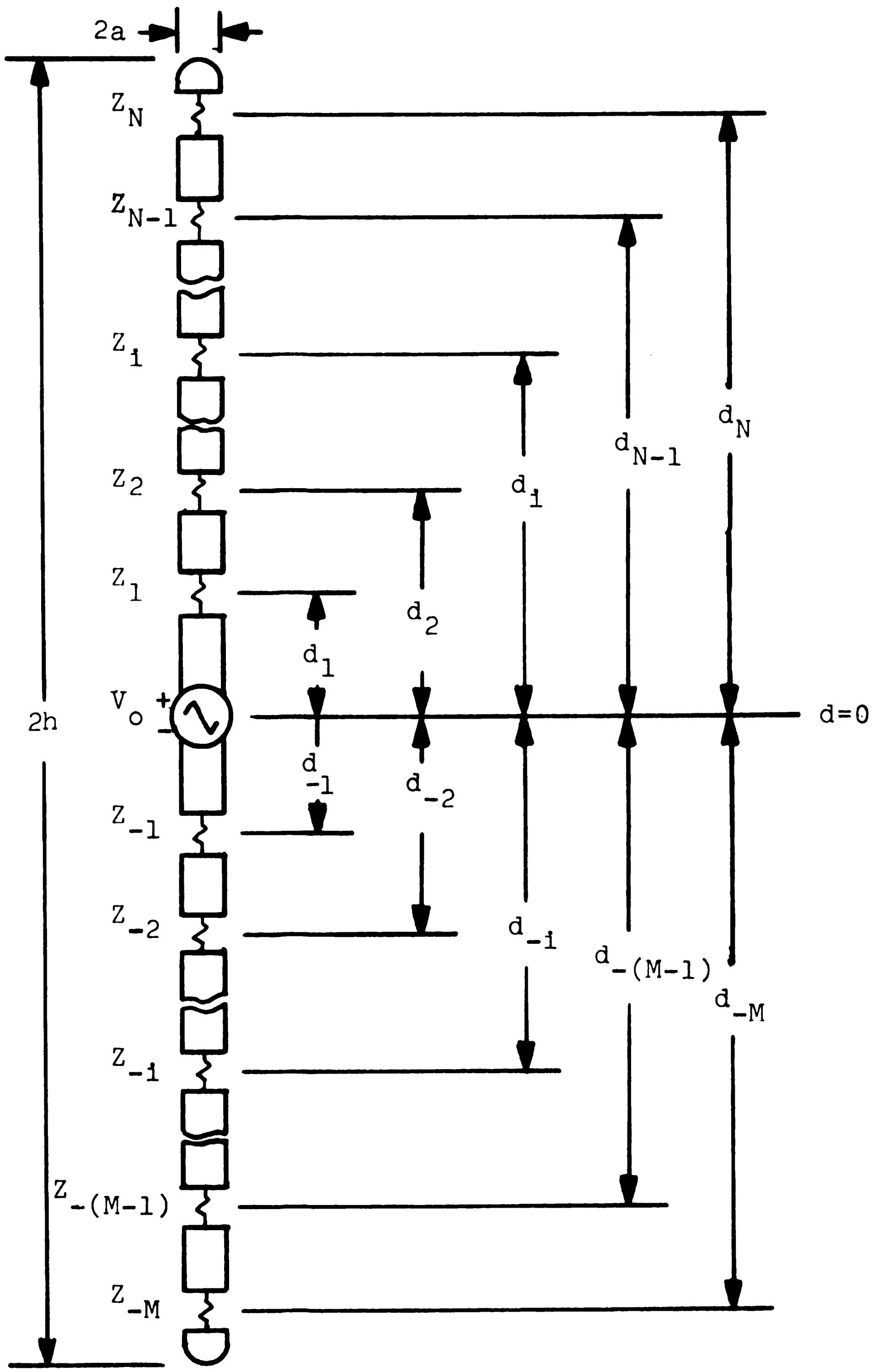


Figure 2.1 Multi-Loaded Transmitting Antenna

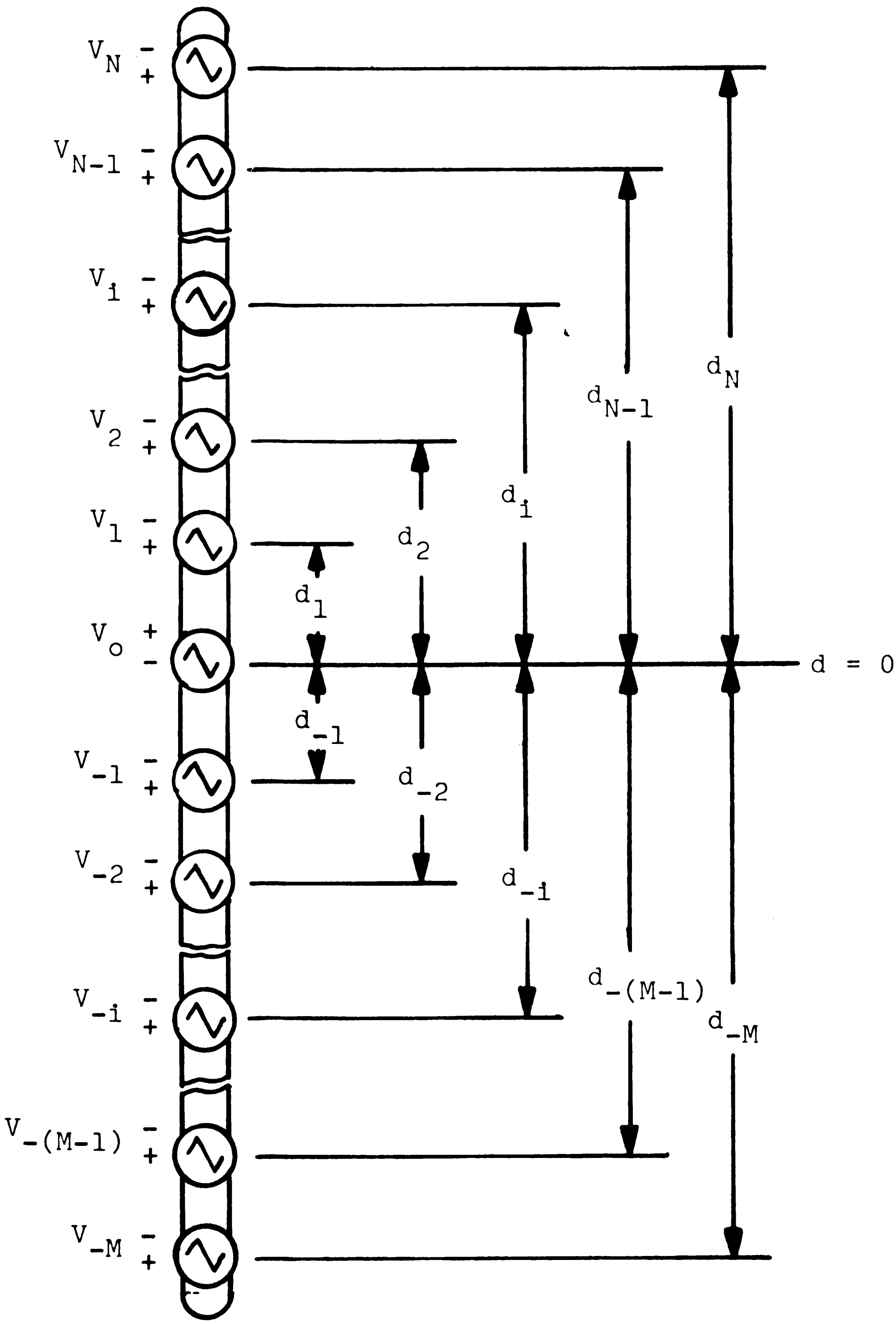


Figure 2.2 Voltage Equivalent Of A Multi-Loaded Transmitting Antenna

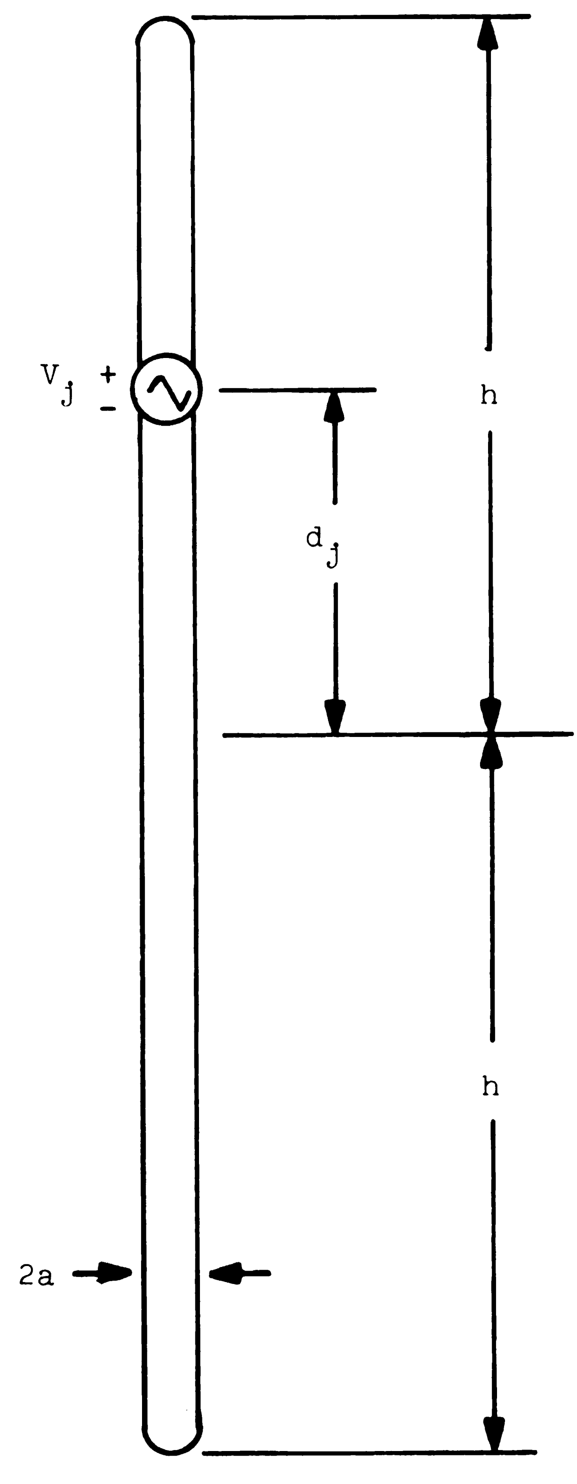


Figure 2.3 Asymmetrically Driven Antenna

any point can be expressed as

$$I_j(z) = V_j f(\omega,a,h,d_j,z) \quad j = -M \text{ to } N. \qquad (2.1)$$

To find the voltage V_j, at each load on the original antenna, the relation $V_j = -Z_j I_t(d_j)$ is used where Z_j is the impedance at d_j and $I_t(d_j)$ is the total current at d_j. By superposition,

$$I_t(d_j) = \sum_{i=-M}^{N} I_i(d_j).$$

Thus

$$V_j = -Z_j \cdot \sum_{i=-M}^{N} I_i(d_j).$$

Using Equation 2.1, the result is

$$V_j = -Z_j \sum_{i=-M}^{N} V_i f(\omega,a,h,d_i,d_j)$$

or

$$-Y_j V_j = \sum_{i=-M}^{N} V_i f(\omega,a,h,d_i,d_j)$$

where $Y_j = 1/Z_i$. Collecting all the terms on the right-hand side, the result is

$$0 = V_o f(\omega,a,h,o,d_j) + \sum_{\substack{i=-M \\ i\neq o,j}}^{N} V_i f(\omega,a,h,d_i,d_j)$$

$$+ V_j(f(\omega,a,h,d_j,d_j) + Y_j). \qquad (2.2)$$

If j runs from -M to N, the result is M + N equations of the form of Equation 2.2. Writing these in matrix form,

$$F \cdot V + GV_o = 0 \qquad (2.3)$$

where

$$F = \begin{bmatrix} f^d_{11} + Y_1 \ \ldots & f^d_{N1} & f^d_{-11} & \ldots\ldots & f^d_{-M1} \\ \vdots & \vdots & \vdots & & \vdots \\ f^d_{1N} \ \ldots & f^d_{NN} + Y_N & f^d_{-1N} & \ldots\ldots & f^d_{-MN} \\ f^d_{1-1} \ \ldots & f^d_{N-1} & f^d_{-1-1} + Y_{-1} & \ldots & f^d_{-M-1} \\ \vdots & \vdots & \vdots & & \vdots \\ f^d_{1-M} \ \ldots & f^d_{N-M} & f^d_{-1-M} & \ldots\ldots & f^d_{-M-M} + Y_{-M} \end{bmatrix}$$

$$G = \begin{bmatrix} f^d_{01} \\ \vdots \\ f^d_{0N} \\ f^d_{0-1} \\ \vdots \\ f^d_{0-M} \end{bmatrix}, \qquad V = \begin{bmatrix} V_1 \\ \vdots \\ V_N \\ V_{-1} \\ \vdots \\ V_{-M} \end{bmatrix}$$

and $f^d_{ik} = f(\omega,a,h,d_i,d_k)$.

Before proceeding, the number of unknowns in the M + N equations represented by 2.3 needs to be determined. There are M + N voltages, M + N admittances, and M + N distances plus a, h, and ω. Since a, h, and ω are usually determined by the problem being solved, they will be treated as known quantities. It should also be noted that the voltages and admittances are complex while the distances are real. Thus there are 5(M + N) real unknowns; 2(M + N) real voltages, 2(M + N) real conductances and susceptances, and M + N real distances. Since Equation 2.3 is also complex, it represents 2(M + N) real equations. Therefore, 3(M + N) real constraint equations must be introduced before Equation 2.3 can be solved.

A close examination of $f(\omega,a,h,d_i,d_k)$ will reveal that d_i and d_k can not be solved for explicitly. Thus the distances are taken as known variables, the same as a, h, and ω. This reduces the unknowns to 4(M + N) real, and the constraint equations to 2(M + N) real.

The number of unknowns will be reduced further if only symmetrical problems are considered, i.e., a monopole antenna on a ground plane. For this case $d_i = d_{-i}$, $V_i = V_{-i}$ $N = M$, and $Y_i = Y_{-i}$. Thus Equation 2.3 reduces to N complex equations of the form

$$F'V' + G'V_o = 0 \tag{2.4}$$

where

$$F' = \begin{bmatrix} f_{11}^{ds} + Y_1 & \cdots & f_{i1}^{ds} & \cdots & f_{N1}^{ds} \\ \vdots & & \vdots & & \vdots \\ f_{1i}^{ds} & \cdots & f_{ii}^{ds} + Y_i & \cdots & f_{Ni}^{ds} \\ \vdots & & \vdots & & \vdots \\ f_{1N}^{ds} & \cdots & f_{iN}^{ds} & \cdots & f_{NN}^{ds} + Y_N \end{bmatrix},$$

$$G' = \begin{bmatrix} f_{01}^{d} \\ \vdots \\ f_{0i}^{d} \\ \vdots \\ f_{0N}^{d} \end{bmatrix}, \qquad V' = \begin{bmatrix} V_1 \\ \vdots \\ V_i \\ \vdots \\ V_N \end{bmatrix} \qquad \text{and}$$

$$f_{ij}^{ds} = f_{ij}^{d} + f_{i-j}^{d},$$

and the number of unknowns in Equation 2.3 to 2N complex. Thus only N complex constraint equations are needed for a solution.

2.2 Constraint Equations

It has been shown that N + M complex constraint equations are needed to obtain a solution for the general case and N complex constraint equations for the symmetrical case. This number of equations is exactly equal to the number of unknown voltages or loading admittances. This leads to three types of constraint equations: those that determine the voltage, those that determine the loading admittances, or those that determine part of the loading admittances and part of the voltages. These three types of equations will now be examined.

2.2.1 Admittance Constraint

While the loading admittances can determine the current and radiation pattern of an antenna, they can not be explicitly related to the pattern on the current. This implies that the constraint equations determining the admittance must be based on other factors, i.e., available components, etc. This reduces these admittances from unknowns to known variables. Once the admittances are given, Equations 2.3 and 2.4 can be easily solved by inverting F and F' respectively, i.e.,

$$V = -F^{-1}GV_o \qquad (2.5a)$$

$$V' = -F'^{-1}G'V_o \qquad (2.5b)$$

provided F^{-1} and F'^{-1} exist. The case of the inverses not existing will be covered in sections 2.2.2 and 2.2.3.

Once V or V' is found, both the field pattern and the current distribution can be determined.

2.2.2 Resonance Loading

In section 2.2.1 it was assumed that F^{-1} and F'^{-1} existed. This section and the next will investigate the results if these inverses do not exist. For simplicity, the discussion will be limited to the symmetric case and N = 2. The results, however, can be extended to the general case.

For N = 2, F' and G' become

$$F' = \begin{bmatrix} f_{11}^{ds} + Y_1 & f_{21}^{ds} \\ f_{12}^{ds} & f_{22}^{ds} + Y_2 \end{bmatrix}$$

and

$$G' = \begin{bmatrix} f_{01}^{d} \\ f_{02}^{d} \end{bmatrix}$$

. If F' had an inverse, by Kramer's rule

$$V_1 = V_o \frac{\begin{vmatrix} f_{01}^{d} & f_{21}^{ds} \\ f_{02}^{d} & Y_2' \end{vmatrix}}{\begin{vmatrix} Y_1' & f_{21}^{ds} \\ f_{12}^{ds} & Y_2' \end{vmatrix}} \tag{2.6}$$

and

$$V_2 = V_o \frac{\begin{vmatrix} Y_1' & f_{01}^d \\ f_{12}^{ds} & f_{02}^d \end{vmatrix}}{\begin{vmatrix} Y_1' & f_{21}^{ds} \\ f_{12}^{ds} & Y_2' \end{vmatrix}} \tag{2.7}$$

where $Y_1' = f_{11}^{ds} + Y_1$ and $Y_2' = f_{22}^{ds} + Y_2$. If the determinants in the numerators of Equations 2.6 and 2.7 are not zero, then at any point where F'^{-1} does not exist, V_1 and V_2 must approach infinity. The input admittance, Y_o, is given by

$$V_oY_o = V_of_{00}^d + V_1f_{10}^{ds} + V_2f_{20}^{ds}. \tag{2.8}$$

Because $|F'| = 0$, V_1 and V_2 are related by either the first or second equation in the matrix 2.5b, thus solving the first equation for V_1

$$V_1 = -\frac{f_{21}^{ds}}{Y_1'} V_2 - \frac{f_{01}^d}{Y_1'} V_o. \tag{2.9}$$

Combining 2.8 and 2.9 and dividing by V_o, Y_o is given by

$$Y_o = f_{00}^d - \frac{f_{10}^{ds}f_{01}^d}{Y_1'} + \frac{V_2}{V_o}\left(f_{20}^{ds} - \frac{f_{21}^{ds}f_{10}^{ds}}{Y_1'}\right). \tag{2.10}$$

Providing the coefficient of V_2 is not zero, then Y_o must also approach infinity. The cases of the coefficient of V_2

equal to zero and the numerators of Equations 2.6 and 2.7 equal to zero will be covered in section 2.2.3.

To see the significance of V_1, V_2 and Y_o going to infinity, consider a simple L-C series circuit consisting of two inductors, L_1 and L_2, and two capacitors, C_1 and C_2. Let V_1 and V_2 be the voltage across L_1 and L_2 respectively and V_o be the driving voltage. The admittance at the driving point, Y_o' would be given by

$$Y_o' = 1/(j\omega(L_1 + L_2) - j(C_1 + C_2)/C_1C_2\omega).$$

When ω is adjusted so that the system is in resonance, the denominator of the expression for Y_o' goes to zero and Y_o' approaches infinity. In addition since V_o, L_1 and L_2 are assumed finite, the current in the system, and thus V_1 and V_2, must also approach infinity. Thus by analogy the antenna can be thought to be a series resonant circuit with distributed components.

2.2.3. Resonant Loading - Unstable Conditions

In Section 2.2.2 indeterminate conditions were deferred, these will now be discussed.

2.2.3.1 Numerator Of V_1 and V_2 Zero

If the determinant in the numerator of Equation 2.6 is also zero, then V_1 becomes indeterminate. To determine the value of V_1 a form of l'Hospital's rule will be used. Solving the numerator of V_1 for Y_2' gives

$$Y'_{2R} = f^{ds}_{21}\, f^{d}_{02}/f^{d}_{01}. \tag{2.11a}$$

Since the denominator of Equation 2.6 must also be zero,

$$Y'_{2R}Y'_{1R} = f^{ds}_{21}f^{ds}_{12}. \tag{2.12}$$

Using Equations 2.11a and 2.12 and solving for Y'_{1R} gives

$$Y'_{1R} = f^{ds}_{12}f^{d}_{01}/f^{d}_{02}\ . \tag{2.11b}$$

It should be noted that for $N = 2, Y'_{1R}$ also causes Equation 2.7 to become indeterminate. For $N > 2$, however, this is not true, up to $N - 2$ of the voltages can be indeterminate without all of them being indeterminate.

To find V_1 and V_2 let

$$Y'_2 = Y'_{2R} + \varepsilon \tag{2.13a}$$

$$Y'_1 = Y'_{1R} + \alpha\varepsilon \tag{2.13b}$$

where ε approaches zero and α is the tangent of the complex angle from which the point is approached. Substituting Equations 2.13a-b into Equations 2.6 and 2.7 and neglecting the second order ε terms gives

$$V_1 = -V_o f^{d}_{01}/(Y'_{1R} + \alpha Y'_{2R}) \tag{2.14}$$

$$V_2 = -V_o f^{d}_{02}/(Y'_{1R} + \alpha Y'_{2R}). \tag{2.15}$$

Since α can be any number in the complex plane, the values of both V_1 and V_2 are dependent on how the indeterminate point is approached. This is typical of an instability point.

2.2.3.2 Coefficient Of V_2 Zero

In Section 2.2.2, the second indeterminate condition was that if the coefficient of V_2 in Equation 2.10 equals zero, Equation 2.10 became indeterminate. Setting that coefficient to zero and solving for Y_1' gives

$$Y_{10}' = f_{21}^{ds} f_{10}^{ds} / f_{20}^{ds} \, . \tag{2.16}$$

Again since the denominator of Equation 2.6 is zero, from Equations 2.12 and 2.16 Y_{2C}' will be

$$Y_{2C}' = f_{12}^{ds} f_{20}^{ds} / f_{10}^{ds} \, . \tag{2.17}$$

Again letting

$$Y_2' = Y_{2C}' + \varepsilon \tag{2.18a}$$

$$Y_1' = Y_{1C}' + \alpha\varepsilon \tag{2.18b}$$

and neglecting second order ε terms

$$\frac{V_2}{V_o}(f_{20}^{ds} - f_{21}^{ds} f_{10}^{ds}/Y_1') = - \frac{f_{20}^{ds}}{Y_{1C}'(Y_{2C}'\alpha + Y_{1C}')}[Y_{1C}' f_{02}^{d} - f_{12}^{ds} f_{01}^{d}]. \tag{2.19}$$

Substituting Equation 2.19 into Equation 2.10 gives

$$Y_o = f_{00}^{d} - \frac{f_{01}^{d} f_{10}^{ds}}{Y_{1C}'} - \frac{f_{20}^{ds}}{Y_{1C}'(Y_{2C}'\alpha + Y_{1C}')}[f_{02}^{d} Y_{1C}' - f_{12}^{ds} f_{01}^{d}]. \tag{2.20}$$

Since α is again completely arbitrary, this is another instability point where the input impedance is indeterminate and loading voltages approach infinity. If, however, F' is a symmetric matrix (long antenna case), it can be shown that $Y'_{1R} = Y'_{1C}$ and $Y'_{2C} = Y'_{2R}$ and

$$Y_o = f^d_{00} - f^d_{01}/Y'_{1R} \ . \tag{2.21}$$

In addition, V_1 and V_2 become indeterminate as in the first case instead of infinite.

2.2.4 Voltage Constraint

The voltage can be specified in several ways. Since it is explicitly related to both the current distribution and the field pattern of an antenna, these types of relationships will now be developed.

2.2.4.1 Field Pattern Constraint

For an asymmetrically driven antenna, as shown in Figure 2.3, the field pattern can be shown to have the following form,

$$P_j(\Theta) = V_j g(\omega,a,h,d_j,\Theta).$$

Therefore by superposition, the field pattern of a multi-loaded antenna is given by

$$P_T(\Theta) = \sum_{i=-M}^{N} V_i g(\omega,a,h,d_i,\Theta)$$

which reduces for the symmetric antenna to

$$P_{T_s}(\Theta) = V_o g(\omega,a,h,o,\Theta) + \sum_{i=1}^{N} V_i g_s(\omega,a,h,d_i,\Theta)$$

where

$$g_s(\omega,a,h,d_i,\theta) = g(\omega,a,h,d_i,\theta) + g(\omega,a,h,d_i,\theta).$$

By specifying the field pattern at N + M angles in the general case and at N angles for the symmetric antenna, M + N or N complex equations are obtained. Thus the unknown voltages can be solved for either by matrix inversion or by Kramer's rule. Once the voltages are known, the admittances can be found by using Equation 2.3 in the form

$$F \cdot V = -GV_o. \tag{2.22}$$

Matrix F can be written as the sum of two matrices

$$F = A + Y \tag{2.23}$$

where

$$A = \begin{bmatrix} f^d_{11} & \cdots & f^d_{N1} & f^d_{-11} & \cdots & f^d_{-M1} \\ \vdots & & \vdots & \vdots & & \vdots \\ f^d_{1N} & \cdots & f^d_{NN} & f^d_{-1N} & \cdots & f^d_{-MN} \\ f^d_{1-1} & \cdots & f^d_{N-1} & f^d_{-1-1} & \cdots & f^d_{-M-1} \\ \vdots & & \vdots & \vdots & & \vdots \\ f^d_{1-M} & \cdots & f^d_{N-M} & f^d_{-1-M} & \cdots & f^d_{-M-M} \end{bmatrix}$$

and

$$Y = \begin{bmatrix} Y_1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ 0 & \cdots & Y_N & 0 & \cdots & 0 \\ 0 & \cdots & 0 & Y_{-1} & \cdots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & Y_{-M} \end{bmatrix}.$$

Combining Equations 2.22 and 2.23 and transposing AV to the other side of the equation results in

$$YV = -(GV_o + AV). \tag{2.24}$$

Examining the left hand side of Equation 2.24 it can be seen that YV is given by

$$YV = \begin{bmatrix} Y_1V_1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & & & \vdots \\ 0 & \cdots & Y_NV_N & 0 & \cdots & 0 \\ 0 & \cdots & 0 & Y_{-1}V_{-1} & \cdot & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & Y_{-N}V_{-N} \end{bmatrix}.$$

Thus the admittances are given by

$$Y_i = -(G_i V_o + \sum_{\substack{j=-M \\ j \neq 0}}^{N} A_{ij} V_j)/V_i \text{ for } i=-M \cdots N.$$

It should be noted that both the magnitude and the phase of the field must be specified before a solution can be obtained. Usually in this type of problem, the magnitude of the field is the only component of interest. Thus, there are many solutions of the problem which will satisfy the desired field pattern.

2.2.4.2 Current Constraint

From Equation 2.1, the current on an asymmetrically driven antenna is

$$I_j(z) = V_j f(\omega,a,h,d_j,z).$$

Thus by superposition the current at any point on a multi-loaded antenna would be

$$I_t(z) = \sum_{i=-M}^{N} V_i f(\omega,a,h,d_i,z). \tag{2.25}$$

If the current is again specified at N + M points, N + M complex equations for the N + M unknown voltages result. Using the same techniques as in Section 2.2.4.1, both the voltages and admittances can be found.

2.2.4.3. Input Admittance - Frequency Constraint

Until now it has been assumed that ω was fixed. A desirable capability would be to be able to specify the input and admittance at several frequencies. This can be done but it will lead to a set of non-linear simultaneous equations. To see this, consider Equations 2.25 and 2.3. If $I_t(0)$ is taken as V_oY_o, Equation 2.25 becomes

$$V_o(f(\omega,a,h,o,o) - Y_o) + \sum_{\substack{i=-M \\ i\neq 0}}^{N} V_i f(\omega,a,h,d_i,o) = 0 \,. \tag{2.26}$$

Rewriting Equation 2.3 and adding Equation 2.26 to it, results in

$$B\cdot V'' = 0 \tag{2.27}$$

where

$$B = \begin{bmatrix} f^d_{11}+Y_1 & \cdots & f^d_{N1} & f^d_{-11} & \cdots & f^d_{-M1} & f^d_{01} \\ \vdots & & \vdots & \vdots & & \vdots & \vdots \\ f^d_{1N} & \cdots & f^d_{NN}+Y_N & f^d_{-1N} & \cdots & f^d_{-MN} & f^d_{0N} \\ f^d_{1-1} & \cdots & f^d_{N-1} & f^d_{-1-1}+Y_{-1} & \cdots & f^d_{-M-1} & f^d_{0-1} \\ \vdots & & \vdots & \vdots & & \vdots & \vdots \\ f^d_{1-M} & \cdots & f^d_{N-M} & f^d_{-1-M} & \cdots & f^d_{-M-M}+Y_{-M} & f^d_{0-M} \\ f^d_{10} & \cdots & f^d_{N0} & f^d_{-10} & \cdots & f^d_{-M0} & f^d_{00}-Y_o \end{bmatrix}$$

and

$$V'' = \begin{bmatrix} V_1 \\ \vdots \\ V_N \\ V_{-1} \\ \vdots \\ V_{-M} \\ V_o \end{bmatrix}.$$

B is an M + N + 1 square matrix whose equations have a solution only when the determinant of B is zero. This gives one complex equation containing M + N complex unknowns, $Y_1, Y_2, \cdots Y_N, Y_{-1}, \cdots Y_{-M}$. If the input impedance is specified at M + N frequencies, a set of M + N complex equations for the M + N unknown admittances results. The form of these equations is

$$a \prod_{i=-M}^{N} Y_i + \sum_{j=-M}^{N} b_j \left(\prod_{i=-M}^{N} Y_i \right) / Y_j + \sum_{j=-M}^{N} \sum_{k=j+1}^{N} C_{jk} \left(\prod_{i=-M}^{N} Y_i \right) / Y_j Y_k \cdots$$

$$+ \sum_{i=-M}^{N} \alpha_i Y_i + \beta = 0. \tag{2.28}$$

It can be shown that Equation 2.28 has $2^{N + M}$ terms with $2^{N + M} - 1$ independent coefficients. It can be seen that for large N + M, Equation 2.28 becomes unmanageable.

In this development it has been tacitly assumed that the admittances are independent of frequency. While admittances which are independent of frequency over a finite bandwidth exist, they are generally a function of frequency. One method of partially circumventing this problem will be presented in Section 2.2.5.

2.2.5 Combined Admittance And Voltage Constraint

Until now the constraint equations have been conditions either on the voltage or on the admittances. The results have been somewhat less than desirable. If the loads are specified in a convenient form (i.e., capacitors, inductors, etc.), the current shape is arbitrary. If the current is specified, the load forms are arbitrary. It is desirable to be able to specify both the form of the admittance and the shape of the current.

2.2.5.1 Reactive Loading - Current Constraint

If for example all the admittances are required to be imaginary, the number of unknowns in Equation 2.3 is reduced from 4(M + N) real unknowns to 3(M + N) real unknowns with 2(M + N) real equations. Thus M + N real constraint equations are needed. If the current is specified at (M + N)/2 points, M + N even, both the current and admittance forms can be specified as desired. For M + N odd, the current is specified completely at (M + N)/2 points leaving one real constraint. The remaining constraint can be obtained by specifying either the real or imaginary part of the current at one additional point.

The solution for M + N even is as follows. Using Equation 2.25 at (M + N)/2 points, the matrix equation

$$C_1 V_1 = V_o D \tag{2.29}$$

where

$$
C_i = \left[\begin{array}{ccccccc}
f^z_{11} & \cdots & f^z_{k1} & f^z_{i1} & f^z_{k+2\,1} & \cdots \\
\vdots & & \vdots & \vdots & \vdots & \\
f^z_{1j} & \cdots & f^z_{kj} & f^z_{ij} & f^z_{k+2\,j} & \cdots \\
\vdots & & \vdots & \vdots & \vdots & \\
f^z_{1\frac{M+N}{2}} & \cdots & f^z_{k\frac{M+N}{2}} & f^z_{i\frac{M+N}{2}} & f^z_{k+2\frac{M+N}{2}} & \cdots
\end{array} \right.
$$

$$
\left. \begin{array}{cccccc}
\cdots & f^z_{i-1\,1} & f^z_{k+1\,1} & f^z_{i+1\,1} & \cdots & f^z_{-M1} \\
 & \vdots & \vdots & \vdots & & \vdots \\
\cdots & f^z_{i-1\,j} & f^z_{k+1\,j} & f^z_{i+1\,j} & \cdots & f^z_{-Mj} \\
 & \vdots & \vdots & \vdots & & \vdots \\
\cdots & f^z_{i-1\frac{M+N}{2}} & f^z_{k+1\frac{M+N}{2}} & f^z_{i+1\frac{M+N}{2}} & \cdots & f^z_{-M\frac{M+N}{2}}
\end{array} \right] ,
$$

$$
= \left(\frac{N + M}{2}\right) \text{ x } (M + N) \text{ matrix}
$$

$$D = \begin{bmatrix} X_1 - f^z_{01} \\ \vdots \\ X_j - f^z_{0j} \\ \vdots \\ X_{\frac{M+N}{2}} - f^z_{0\frac{M+N}{2}} \end{bmatrix}, \qquad V_i = \begin{bmatrix} V_1 \\ \vdots \\ V_k \\ V_i \\ V_{k+2} \\ \vdots \\ V_{i-1} \\ V_{k+1} \\ V_{i+1} \\ \vdots \\ V_{-M} \end{bmatrix},$$

$$k = -\left(\frac{M - N}{2}\right) \quad \text{for } M > N$$

$$k = \frac{N + M}{2} \quad \text{for } N \geq M$$

and $f^z_{ij} = f(\omega, a, h, d_i, z_j)$

results assuming $I_T(z_j) = V_o X_j$ where X_j is a specified value. To keep the resulting non-linear equations as simple as possible, Equations 2.29 and 2.3 are combined as follows:

$$\begin{bmatrix} E_i & G_i \\ C_i & -D \end{bmatrix} \begin{bmatrix} V_i \\ V_o \end{bmatrix} = 0 \qquad \begin{matrix} i = -M, -(M-1), \cdots, k + 1 \\ i \neq 0 \end{matrix} \tag{2.30}$$

where

$$E_i = \left[\begin{matrix} f^d_{11}+jB_1 & \cdots & f^d_{k1} & f^d_{i1} & f^d_{k+21} & \cdots \\ \vdots & & \vdots & \vdots & \vdots & \\ f^d_{1k} & \cdots & f^d_{kk}+jB_k & f^d_{ik} & f^d_{k+2k} & \cdots \\ f^d_{1i} & \cdots & f^d_{ki} & f^d_{ii}+jB_i & f^d_{k+2i} & \cdots \end{matrix}\right.$$

$$\left.\begin{matrix} \cdots & f^d_{i-11} & f^d_{k+11} & f^d_{i+1} & \cdots & f^d_{-M1} \\ & \vdots & \vdots & \vdots & & \vdots \\ \cdots & f^d_{i-1k} & f^d_{k+1k} & f^d_{i+1k} & \cdots & f^d_{-Mk} \\ \cdots & f^d_{i-1i} & f^d_{k+1i} & f^d_{i+1i} & \cdots & f^d_{-Mi} \end{matrix}\right]$$

$$= \left(\frac{M+N}{2}\right) \times (M+N) \text{ matrix}$$

$$G_i = \begin{bmatrix} f^d_{01} \\ \vdots \\ f^d_{0k} \\ f^d_{0i} \end{bmatrix} .$$

jB_i is the imaginary part of Y_i. Also it should be noted that the point z_j is not necessarily equal to the point d_j. Since $V_o \neq 0$, Equation 2.30 has a solution only if

$$\begin{vmatrix} E_i & G_i \\ C_i & -D \end{vmatrix} = 0 . \tag{2.31}$$

Since i indexes over $\frac{N + M}{2}$ terms, Equation 2.31 generates the $\frac{N + M}{2}$ equations needed to determine the N + M susceptances, B_i. The form of the equations generated by Equation 2.31 is like Equation 2.28 except the leading term is

$$B_i \cdot \prod_{\substack{j=1 \\ j\neq 0}}^{k} B_j \quad \text{instead of} \quad \prod_{\substack{j=-M \\ j\neq 0}}^{N} Y_j \ .$$

2.2.5.2 Reactive Loading - Frequency - Current Constraint

The above case still assumes ω to be fixed. For the case of several frequencies, let N_f be the number of frequencies and N_c be the number of current points. Then

$$\frac{N + M}{2} = N_f \cdot N_c \tag{2.32}$$

assuming M + N even and that the same number of current points is used for each frequency. This somewhat limits the choice of M, N, N_f and N_c.

Assuming Equation 2.32 is satisfied, for each frequency the generating equation would be

$$\begin{bmatrix} E_i' & G_i' \\ C_i' & -D' \end{bmatrix} \begin{bmatrix} V_i \\ V_0 \end{bmatrix} = 0 \qquad \begin{matrix} i=-M,-(M-1),\cdots,\ell + 1 \\ i\neq 0 \end{matrix} \tag{2.33}$$

where

$$\ell = -(M-N_c) \qquad \text{for } M > N_c$$

$$\ell = N + M - N_c \qquad \text{for } N_c \geq M$$

$$E_i' = \begin{bmatrix} f^d_{11}+jB_1 & \cdots & f^d_{\ell 1} & f^d_{i1} & f^d_{\ell+21} & \cdots & f^d_{i-11} & f^d_{\ell+11} & f^d_{i+11} & \cdots & f^d_{-M1} \\ \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ f^d_{1\ell} & \cdots & f^d_{\ell\ell}+jB_\ell & f^d_{i\ell} & f^d_{\ell+2\ell} & \cdots & f^d_{i-1\ell} & f^d_{\ell+1\ell} & f^d_{i-1\ell} & & f^d_{-M\ell} \\ f^d_{1i} & \cdots & f^d_{\ell i} & f^d_{ii}+jB_i & f^d_{\ell+2i} & \cdots & f^d_{i-1i} & f^d_{\ell+1i} & f^d_{i+1i} & & f^d_{-Mi} \end{bmatrix}$$

$$G_i' = \begin{bmatrix} f^d_{01} \\ \vdots \\ f^d_{0\ell} \\ f^d_{0i} \end{bmatrix} , \qquad D' = \begin{bmatrix} X_1 - f^z_{01} \\ \vdots \\ X_j - f^z_{0j} \\ \vdots \\ X_{N_c} - f^z_{0N_c} \end{bmatrix}$$

$$C_i' = \begin{bmatrix} f^z_{11} & \cdots & f^z_{\ell 1} & f^z_{i1} & f^z_{\ell+2\,1} & \cdots & f^z_{i-1\,1} & f^z_{\ell+1\,1} & f^z_{i+1\,1} & & f^z_{-M1} \\ \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ f^z_{1j} & \cdots & f^z_{\ell j} & f^z_{ij} & f^z_{\ell+2\,j} & \cdots & f^z_{i-1\,j} & f^z_{\ell+1\,j} & f^z_{i+1\,j} & \cdots & f^z_{-Mj} \\ \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ f^z_{1N_c} & \cdots & f^z_{\ell N_c} & f^z_{iN_c} & f^z_{\ell+2\,N_c} & \cdots & f^z_{i-1\,N_c} & f^z_{\ell+1\,N_c} & f^z_{i+1\,N_c} & & f^z_{-MN_c} \end{bmatrix}.$$

Since Equation 2.33 generates only N_c complex equations for each frequency, it takes N_f frequencies to have $\frac{M + N}{2}$ equations that are required.

In Equation 2.33 the B_i's are still independent of ω. Since the only elements which have pure reactive parts are capacitors or inductors, a logical frequency dependence for B_i would be either ω or $1/\omega$. Thus in Equation 2.33 the reactances could become either $B_i = \omega_j C_i$ or $B_i = -1/L_i\omega_j$ where $j = 1$ to N_f. In order to obtain a reasonable result, the solutions to Equation 2.33 would have to be restricted to positive values for C_1 and L_1. In general this restriction may not be satisfied.

CHAPTER III

CURRENT DISTRIBUTION ON A LOADED RADIATING ANTENNA

In Chapter 2 it was assumed that for an asymmetrically driven antenna the current distribution is known. This problem has been treated by many authors and is generally broken into two approaches based on the electrical length of the antenna. Current distributions for a short antenna and for a long antenna will be presented in this chapter. Since many antennas are monopoles and because of the formulation of the short antenna problem, only the asymmetrically driven short antenna current distribution will be presented. The asymmetrical case can be found in the literature.

3.1 Current Distribution On A Short Antenna

There are several types of approximate current distributions for a short symmetrically driven antenna. The one presented here will be based on King and Wu's theory (1) and is valid for $k_o h < 3\pi/2$ and $\Omega = 2\ln(2h/a) \geq 8$.

3.1.1. Vector Potential Of Symmetric Dipole

Consider the symmetrically driven antenna in Figure 3.1 The axial component of the vector field on the surface can be shown to satisfy the equation

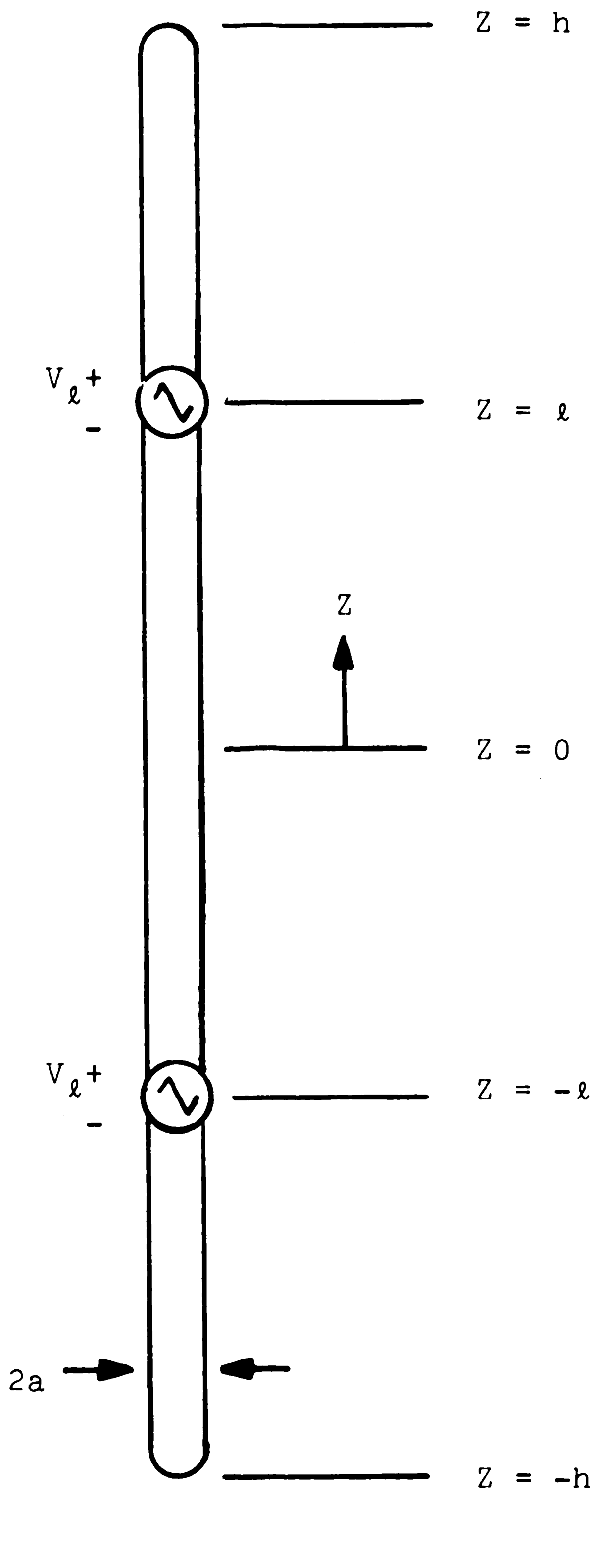


Figure 3.1 Symmetrically Driven Antenna

$$\left(\frac{d^2}{dz^2} + k_o^2\right)A_z(z) = -\frac{jk_o^2}{\omega} V_\ell(\delta(z - \ell) + \delta(z + \ell)). \qquad (3.1)$$

The homogeneous part of Equation 3.1 has solutions in three regions $h > z > \ell$, $\ell > z > -\ell$, and $-\ell > z > -h$ given by

$$A_z(z) = -j\sqrt{\varepsilon_o\mu_o}[C_1 \cos k_oz + C_2 \sin k_oz] \quad h > z > \ell \qquad (3.2a)$$

$$A_z(z) = -j\sqrt{\varepsilon_o\mu_o}C_3 \cos k_oz \quad \ell > z > -\ell \qquad (3.2b)$$

$$A_z(z) = -j\sqrt{\varepsilon_o\mu_o}[C_4 \cos k_oz + C_5 \sin k_oz] \quad -\ell > z > -h \qquad (3.2c)$$

where $-j\sqrt{\varepsilon_o\mu_o}$ is included to make the C's dimensional volts. From symmetry $A_z(z) = A_z(-z)$, thus $C_1 = C_4$ and $C_2 = -C_5$. Since $A_z(z)$ is continuous at $z = \underline{+}\ell$, from Equations 3.2a and 3.2b it can be shown

$$(C_1 - C_3) = -C_2 \tan k_o\ell. \qquad (3.3)$$

The scalar potential is given by $\phi(z) = \frac{j\omega}{k_o}\frac{\partial A_z(z)}{\partial z}$.

Thus

$$\phi(z) = -C_1 \sin k_oz + C_2 \cos k_oz \quad h > z > \ell \qquad (3.4a)$$

$$\phi(z) = -C_3 \sin k_oz \quad \ell > z > -\ell \qquad (3.4b)$$

$$\phi(z) = -C_1 \sin k_oz - C_2 \cos k_oz > -\ell > z \quad -h \qquad (3.4c)$$

which satisfy the symmetry condition $\phi(-z) = -\phi(z)$. The driving voltage is defined as

$$V_\ell = \phi(\ell^+) - \phi(\ell^-) = -(C_1 - C_3) \sin k_o\ell + C_2 \cos k_o\ell. \qquad (3.5)$$

Combining Equations 3.3 and 3.5 results in

$$C_2 = V_\ell \cos k_o\ell \ , \quad C_3 = C_1 + V_\ell \sin k_o\ell .$$

Therefore

$$A_z(z) = -j\sqrt{\varepsilon_o\mu_o}(C_1 \cos k_{oz} + (V_o/2)(\sin k_o|z+\ell| + \sin k_o|z-\ell|)). \tag{3.6}$$

All that remains to be evaluated is the constant C_1 which is done by requiring $I_z(h) = 0$. To this point, the development applies to either a short or long antenna.

3.1.2. Difference Equation

In King and Wu's method, since

$$A_z(z) = \frac{\mu_o}{4\Pi}\int_{-h}^{h} I(z')K(z,z')dz' \tag{3.7}$$

where

$$K(z,z') = \frac{e^{-jkr}}{r} \quad , \qquad r = \sqrt{(z'-z)^2 + a^2} \tag{3.8}$$

a difference equation of $A_z(z) - A_z(h)$ can be defined. Using Equation 3.6, it follows that

$$\int_{-h}^{h} I(z')K_d(z,z')dz' = -j\frac{4\Pi}{\zeta_o}[C_1 \cos k_oz + (V_o/2)(\sin k_o|z+\ell| + \sin k_o|z-\ell|) + U] \tag{3.9}$$

where

$$\zeta_o = \sqrt{\mu_o/\varepsilon_o} \quad \text{and}$$

$$U = \frac{-j\zeta_o}{4\Pi}\int_{-h}^{h} I(z')K(h,z')dz' . \tag{3.10}$$

The difference kernel is

$$K_d(z,z') = K(z,z') - K(h,z'). \tag{3.11}$$

Since the right hand side of Equation 3.9 is zero at $z = h$, the constant C_1 is

$$C_1 = -\left[\frac{U + V_\ell \cos k_o\ell \sin k_oh}{\cos k_oh}\right] \tag{3.12}$$

and Equation 3.9 becomes

$$\int_{-h}^{h} I(z')K_d(z,z')dz' = \frac{j4\pi}{\zeta_o \cos k_oh}[V_\ell(\cos k_o\ell \cos k_oz \sin k_oh$$

$$-(1/2)\cos k_oh\,[\sin k_o|z-\ell| + \sin k_o|z+\ell|]) + U(\cos k_oz - \cos k_oh)]. \tag{3.13}$$

3.1.3. Current Distribution

It can be shown that

$$\int_{-h}^{h} I_z(z')K_{dR}(z,z')dz' \sim I_z(z) \tag{3.14a}$$

$$\int_{-h}^{h} I_z(z')K_{dI}(z,z')dz' \sim \cos(1/2)k_oz - \cos(1/2)k_oh \tag{3.14b}$$

where

$$K_d(z,z') = K_{dR}(z,z') + jK_{dI}(z,z').$$

Thus Equations 3.13, 3.14a, and 3.14b suggest that the approximate form of the current distribution should be

$$I(z) = I_V(z) + I_u(z) + I_D(z) \tag{3.15}$$

where

$$I_V(z) = I_V(\cos k_o\ell \sin k_oh \cos k_oz - (1/2)\cos k_oh[\sin k_o|z+\ell|$$

$$+ \sin k_o|z-\ell|]) = I_VM_{oz} \tag{3.16a}$$

$$I_u(z) = I_u F_{oz} = I_u(\cos k_o z - \cos k_o h) \qquad (3.16b)$$

$$I_D(z) = I_D F_{oz}' = I_D(\cos(1/2)k_o z - \cos(1/2)k_o h). \qquad (3.16c)$$

Substituting Equation 3.15 into Equations 3.14a-b the following definitions can be made:

$$\int_{-h}^{h} I_V(z')K_{dR}(z,z')dz' = I_V(z)\psi_{dR}(z) \doteq I_V(z)\psi_{dR} \qquad (3.17a)$$

$$\int_{-h}^{h} I_V(z')K_{dI}(z,z')dz' \doteq I_D(z)\frac{I_V}{I_D}\psi_{dI}(z) \doteq I_D(z)\frac{I_V}{I_D}\psi_{dI} \qquad (3.17b)$$

$$\int_{-h}^{h} I_u(z')K_{dR}(z,z')dz' = I_u(z)\psi_{dUR}(z) \doteq I_u(z)\psi_{dUR} \qquad (3.17c)$$

$$\int_{-h}^{h} I_u(z')K_{dI})z,z')dz' \doteq I_D(z)\frac{I_u}{I_D}\psi_{dUI}(z) \doteq I_D(z)\frac{I_u}{I_D}\psi_{dUI} \qquad (3.17d)$$

$$\int_{-h}^{h} I_D(z')K_d(z,z')dz = I_D(z)\psi_{dD}(z) \doteq I_D(z)\psi_{dD}. \qquad (3.17e)$$

Noting that the integral on the right hand side of Equation 3.13 equals $4\pi\mu_o^{-1}[A_z(z) - A_z(h)]$ and using Equations 3.15 and 3.17a-e, the result is

$$4\pi\mu_o^{-1}[A_z(z) - A_z(h)] = I_V\psi_{dR}M_{oz} + I_u\psi_{dUR}F_{oz}$$
$$+ (I_D\psi_{dD} + jI_V\psi_{dI} + jI_u\psi_{dUI})F_{oz}'. \qquad (3.18)$$

If this expression is substituted into the original differential equation (3.1) in the form

$$4\pi\mu_o^{-1}[\frac{d^2}{dz^2} + k_o^2][A_z(z) - A_z(h)] = \frac{j4\pi k_o}{\zeta_o}[V_\ell[\delta(z-\ell) + \delta(z+\ell)]$$
$$-j\omega A_z(h)] \qquad (3.19)$$

then

$$-I_V\psi_{dR}k_o \cos k_oh[\delta(z-\ell) + \delta(z+\ell)] - I_u\psi_{dUR}k_o^2 \cos k_oh$$
$$+ k_o^2(I_D\psi_{dD} + jI_V\psi_{dI} + jI_u\psi_{dUI})((3/4)\cos(1/2)k_oz \cdot \qquad (3.20)$$
$$- \cos(1/2)k_oh) = -\frac{j4\pi k_o}{\zeta_o}(V_\ell[\delta(z-\ell) + \delta(z+\ell)] - j\omega A_z(h)).$$

The coefficients of the δ-function terms must be equal, so that

$$I_V = \frac{j4\pi V_\ell}{\zeta_o\psi_{dR} \cos k_oh} \qquad (3.21)$$

which leaves

$$I_u\psi_{dUR} \cos k_oh - 4\pi\mu_o^{-1}A_z(h) - (I_D\psi_{dD} + jI_V\psi_{dI} + jI_u\psi_{dUI})\cdot$$
$$((3/4)\cos k_oz - \cos(1/2)k_oh) = 0. \qquad (3.22)$$

Since Equation 3.22 must be true for all z, the coefficient of $((3/4)\cos k_oz - \cos(1/2)k_oh)$ must be zero.
Thus

$$I_D\psi_{dD} + jI_V\psi_{dI} + jI_u\psi_{dUI} = 0 \qquad (3.23a)$$

and

$$I_u\psi_{dUR} \cos k_oh = 4\pi\mu_o^{-1}A_z(h). \qquad (3.23b)$$

To find $A_z(h)$ Equations 3.15 and 3.7 are needed. Using the same type of approximation as used for Equations 3.17a-e, the result is

$$4\pi\mu_o^{-1}A_z(h) = I_V\psi_V(h) + I_u\psi_u(h) + I_D\psi_D(h) \qquad (3.24)$$

where

$$\psi_V(h) = \frac{\int_{-h}^{-\ell} \cos(\ell k_o) \sin k_o(h+z')K(h,z')dz' + \sin k_o(h-\ell)}{\int_{-\ell}^{\ell} \cos k_o z' K(h,z')dz' + \int_{\ell}^{h} \cos k_o \ell \sin k_o(h-z')K(h,z')dz'} \tag{3.25a}$$

$$\psi_u(h) = \int_{-h}^{h} (\cos k_o z' - \cos k_o h)K(h,z')dz' \tag{3.25b}$$

and

$$\psi_D(h) = \int_{-h}^{h} (\cos k_o z'/2 - \cos k_o h/2)\, K(h,z')dz'. \tag{3.25c}$$

Combining Equations 3.24 and 3.23b to form one equation and using Equation 3.23a, two constants, T_D and T_u, can be defined as follows:

$$T_D = \frac{I_D}{I_V} = \frac{-j[\psi_{dI}(\psi_{dUR}\cos k_o h - \psi_u(h)) + \psi_{dUI}\psi_u(h)]}{\psi_{dD}(\psi_{dUR}\cos k_o h - \psi_u(h)) + j\psi_D(h)\psi_{dUI}} \tag{3.26a}$$

$$T_u = \frac{I_u}{I_V} = \frac{\psi_{dD}\psi_V(h) - j\psi_{dI}\psi_D(h)}{\psi_{dD}(\psi_{dUR}\cos k_o h - \psi_u(h)) + j\psi_D(h)\psi_{dUI}}. \tag{3.26b}$$

Thus using Equations 3.26a, 3.26b, 3.21, and 3.15, the approximate current distribution along the antenna driven by identical voltage generators, V_ℓ, at $z = \pm\ell$ is

$$I_z(z) = \frac{j4\pi V_\ell f(z)}{\zeta_o \psi_{dR} \cos k_o h} \tag{3.27}$$

where

$$f(z) = M_{oz} + T_u F_{oz} + T_D F_{oz}' \tag{3.28}$$

with M_{oz}, F_{oz}, and F_{oz}' given in Equations 3.16a-c.

3.1.4. Evaluation Of Constants

All that remains to be specified is the ψ functions introduced in Equations 3.17a-e. Since the ψ functions represent the ratio of a component of the vector potential to the current, the desired ratio is obtained at the maximum of the appropriate current. For ψ_{dUR}, ψ_{dUI}, ψ_{dD}, and ψ_{dI} this maximum is at $z = 0$. Thus from Equations 3.17b-e

$$\psi_{dUR} = (1-\cos k_o h)^{-1} \int_{-h}^{h} (\cos k_o z' - \cos k_o h) K_{dR}(o,z')dz' \quad (3.29a)$$

$$\psi_{dD} = (1-\cos k_o h/2)^{-1} \int_{-h}^{h} (\cos k_o z'/2 - \cos k_o h/2) K_d(o,z')dz' \quad (3.29b)$$

$$\psi_{dUI} = (1-\cos k_o h/2)^{-1} \int_{-h}^{h} (\cos k_o z' - \cos k_o h) K_{dI}(o,z')dz' \quad (3.29c)$$

$$\begin{aligned} \psi_{dI} &= (1-\cos k_o h/2)^{-1} \int_{-h}^{-\ell} \cos k_o \ell \sin k_o (h+z') K_{dI}(o,z')dz' \\ &+ \sin k_o (h-\ell) \int_{-\ell}^{\ell} \cos k_o z' K_{dI}(o,z')dz' \\ &+ \int_{\ell}^{h} \cos k_o \ell \sin k_o (h-z') K_{dI}(o,z')dz'. \end{aligned} \quad (3.29d)$$

Since in the range $h \geq z \geq 0$, M_{oz} may have two relative maxima, the definition of ψ_{dR} is more complicated. The first maximum is at $z = h - \lambda/4$ when $h-\ell \geq \lambda/4$ and at $z = \ell$ when $h-\ell < \lambda/4$. The second is always at $z = 0$. The definition made by King and Wu is in terms of an average of the vector potentials.

Thus

$$\psi_{dR} = \frac{|M_{oo}\psi_{dR}(o)| + |M_{oz_1}\psi_{dR}(z_1)|}{|M_{oo}| + |M_{oz_1}|} \tag{3.30a}$$

where $z_1 = h-(\lambda/4)$, when $h-\ell > (\lambda/4)$, and $z_1 = \ell$ when $h-\ell < \lambda/4$ and where

$$\begin{aligned}\psi_{dR}(z) = (M_{oz})^{-1} & \int_{-h}^{-\ell} \cos k_o\ell \sin k_o(h+z')K_{dR}(z,z')dz' \\ & + \sin k_o(h-\ell) \int_{-\ell}^{\ell} \cos k_o z' K_{dR}(z,z')dz' \\ & + \int_{\ell}^{h} \cos k_o\ell \sin k_o(h-z')K_{dR}(z,z')dz'.\end{aligned} \tag{3.30b}$$

In particular, $M_{oo} = \sin k_o(h-\ell)$ and $M_{oz_1} = \cos k_o\ell$ when $z_1 = h - \lambda/4$, and $M_{oz_1} = \sin k_o(h-\ell) \cos k_o\ell$ when $z_1 = \ell$.

3.1.5. Special Cases

Two special cases need to be considered: $\ell = 0$ and $k_oh = \pi/2$. The first case is that of a center driven antenna. Since as $\ell \to 0$ the driving voltage becomes $2V_\ell$, thus V_o is equal to $2V_\ell$, and Equation 3.27 becomes

$$\begin{aligned} I_z(z) = \frac{j2\pi V_o}{\zeta_o\psi_{dR}\cos k_oh} & [\sin k_o(h-|z|) + T_u (\cos k_oz-\cos k_oh) \\ & + T_D(\cos k_oz/2 - \cos k_oh/2)]. \end{aligned} \tag{3.31}$$

The second case is that of a half wave antenna. The expression for $I_z(z)$ becomes indeterminate because of the $\cos k_oh$ in the denominator. An alternate form of $I_z(z)$ is thus needed when k_oh is near $\pi/2$.

If T_u and T_D are redefined as

$$T_u' = -\frac{T_u + \sin k_o h \cos k_o \ell}{\cos k_o h} \tag{3.32a}$$

$$T_D' = \frac{T_D}{\cos k_o h} \tag{3.32b}$$

which are finite at $k_o h = \Pi/2$, thus

$$I_z(z) = \frac{j4\Pi V_\ell}{\zeta_o \psi_{dR}} [M_{oz}' - T_u'(\cos k_o z - \cos k_o h)$$

$$+ T_D'(\cos 1/2\, k_o z - \cos k_o h/2)] \tag{3.33}$$

where (3.34)

$$M_{oz}' = \sin k_o h \cos k_o \ell - 1/2(\sin k_o|z + \ell| + \sin k_o|z-\ell|).$$

From Equations 3.26a-b and 3.32a-b,

$$T_u' = -\{[\psi_{dD}(\psi_V(h) - \psi_u(h)\sin k_o h \cos k_o \ell)$$

$$+ \psi_{dD}\psi_{dUR}\cos k_o h \sin k_o h \cos k_o \ell - j\psi_D(h)\cdot \tag{3.35}$$

$$(\psi_{dI} - \psi_{dUI}\sin k_o h \cos k_o \ell)]/[(\psi_{dD}(\psi_{dUR}\cos k_o h - \psi_u(h))$$

$$+ j\psi_{dUI}\psi_D(h)) \cos k_o h]\}$$

and

$$T_D' = \frac{-j[\psi_{dI}[\psi_{dUR}\cos k_o h - \psi_u(h)] + \psi_{dUI}\psi_V(h)]}{\psi_{dD}(\psi_{dUR}\cos k_o h - \psi_u(h)) + j\psi_{dUI}\psi_D(h) \cos k_o h}. \tag{3.36}$$

Noting that at $k_o h = \Pi/2$, $\psi_u(\Pi/4)\cos k_o \ell = \psi_V(\Pi/4)$, and $\psi_{dI} = \cos k_o \ell\ \psi_{dUI}$, Equations 3.35 and 3.36 reduce to

$$T_u' = \frac{\psi_{dD}\psi_{dUR} \cos k_o \ell}{\psi_{dD}\psi_u(\lambda/4) - j\psi_{dUI}\psi_D(\lambda/4)} \tag{3.37}$$

$$T_D' = \frac{j\psi_{dI}\psi_{dUR}}{\psi_{dD}\psi_u(\lambda/4) - j\psi_{dUI}\psi_D(\lambda/4)} \tag{3.38}$$

which are both finite.

3.2 Current Distribution On A Long Antenna

The approximate solution for a long asymmetrically driven antenna presented here will be based on papers by Wu (2) and Shen, Wu and King (3). The solution is valid for antennas whose shortest arm is $\geq .15\lambda$. To follow the original papers closely, the time dependence of $e^{-j\omega t}$ will be used.

3.2.1. Current On An Infinitely Long Antenna

Consider an infinitely long center-driven antenna with its driver at z = 0. The axial component of the vector field on the surface can be shown to satisfy

$$\left(\frac{\partial^2}{\partial z^2} + k_o^2\right)A_z(z) = \frac{+jk_o^2}{\omega} V_o\delta(z). \tag{3.39}$$

Thus

$$A(z) = \frac{\mu_o V_o}{2\zeta_o} e^{jk_o|z|} \qquad \text{for all } z. \tag{3.40}$$

But

$$A(z) = \frac{\mu_o}{4\pi} \int_{-\infty}^{\infty} dz' \; I_\infty(z')K(z-z') \tag{3.41}$$

where

$$K(z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\Theta[z^2 + (2z \sin \Theta/2)^2]^{-1/2} \exp[jk_o[z^2 + (2a \sin \Theta/2)^2]^{1/2}]. \tag{3.41a}$$

Combining Equations 3.40 and 3.41 gives

$$\frac{2\pi V_o}{\zeta_o} e^{jk_o|z|} = \int_{-\infty}^{\infty} dz' \; I_\infty(z')K(z-z'). \tag{3.42}$$

To solve Equation 3.42, let k_o have a small positive imaginary part which will eventually be allowed to approach zero and define the following Fourier transforms

$$\bar{I}_\infty(\zeta) = \int_{-\infty}^{\infty} dz \; I_\infty(z) e^{-j\zeta z} \tag{3.43a}$$

$$\bar{K}(\zeta) = \int_{-\infty}^{\infty} dz \; K(z) e^{-j\zeta z}. \tag{3.43b}$$

Applying the Fourier transforms to Equation 3.42 results in

$$\frac{2\pi V_o}{\zeta_o} \int_{-\infty}^{\infty} e^{jk_o|z|} e^{-j\zeta z} dz = \int_{-\infty}^{\infty} dz e^{-j\zeta z} \int_{-\infty}^{\infty} dz' \; I_\infty(z')K(z-z'). \tag{3.44}$$

Using the assumption k_o has a positive imaginary part, the left hand integral becomes

$$\int_{-\infty}^{\infty} e^{jk_o|z|} e^{-j\zeta z} dz = 2jk_o/(k_o^2 - \zeta^2). \tag{3.45}$$

Interchanging the order of integration of the right hand integral and using Equations 2.43a-b, the integral becomes

$$\int_{-\infty}^{\infty} dz e^{-k\zeta z} \int_{-\infty}^{\infty} dz' \; I_\infty(z')\bar{K}(z-z') = \bar{I}_\infty(\zeta)\bar{K}(\zeta). \tag{3.46}$$

ombining Equations 3.45, 3.46, and 3.44, the resulting orm for $\bar{I}_\infty(\zeta)$ is

$$\bar{I}_\infty(\zeta) = \frac{4\pi j V_o k_o}{\zeta_o(k_o^2 - \zeta^2)\bar{K}(\zeta)}. \tag{3.47}$$

To find $I_\infty(z)$, the inverse transform of Equation 3.47 is taken. Thus

$$I_\infty(z) = \frac{1}{2\Pi}\int_{C_o} \bar{I}_\infty(\zeta)e^{j\zeta z}d\zeta = \frac{jk_oV_o^2}{\zeta_o}\int_{C_o}\frac{e^{j\zeta z}}{(k_o^2-\zeta^2)\bar{K}(\zeta)}d\zeta \tag{3.48}$$

where C_o is the contour shown in Figure 3.2.

Contour C_o is chosen so that the inverse of Equation 3.45 results in the proper value. Also it can be shown that

$$\bar{K}(\zeta) = \Pi j J_o[a(k_o^2-\zeta^2)^{1/2}]H_o^{(1)}[a(k_o^2-\zeta^2)^{1/2}]. \tag{3.49}$$

Thus Equation 3.48 is completely defined, but is not in a usable form. If it is assumed (1) that the antenna is thin i.e., $a/\lambda << 1$ and (2) that the characteristic distance for variations of $A(z)$ is $>> a$, then Equation 3.49 can be approximated by

$$\bar{K}(\zeta) = 2\Omega_o + j\Pi - \ell n[(k_o^2-\zeta^2)/k_o^2] \tag{3.50}$$

where $\Omega_o = \ell n(2/k_oa) - \gamma$ and $\gamma = .57721566$.
Using Equations 3.50 and 3.48 and deforming C_o so that it is wrapped around the branch cut at $\zeta = k_o$, it can be shown that the major contribution to the integral comes from points near $\zeta = k_o$. Thus $I_\infty(z)$ can be approximated by

$$I_\infty(z) = \frac{V_oj}{\zeta_o}e^{jk_o|z|}\int_o^\infty \frac{e^{-2k_oz\eta}}{\eta}[(2C_\omega - \ell n\eta + j3\Pi/2)^{-1} -(2C_\omega - \ell n\eta - j\Pi/2)^{-1}]d\eta \tag{3.51}$$

where $C_\omega = \ell n(1/k_oa) - \gamma$.

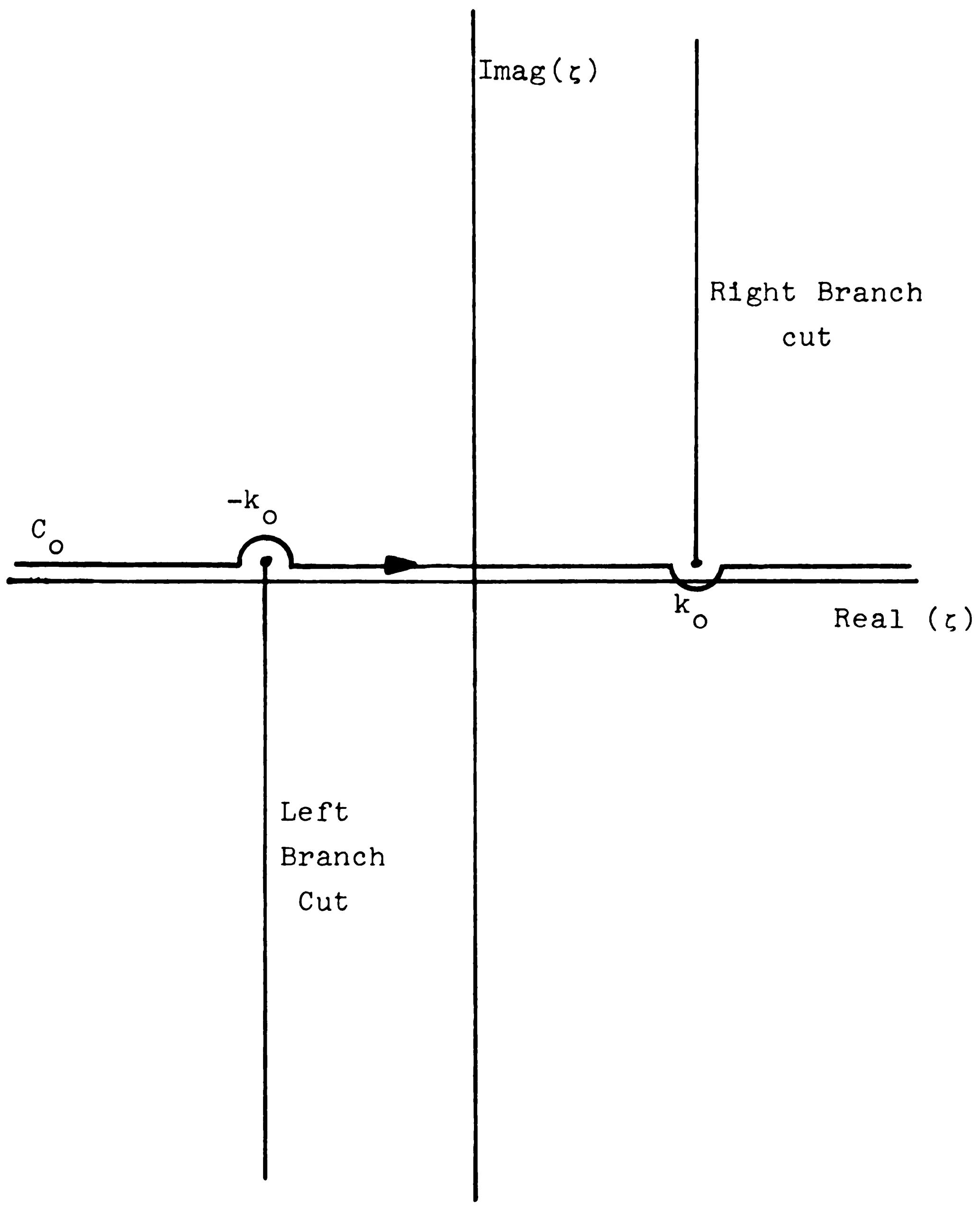


Figure 3.2 The ζ Plane And The Contour C_o

Because of $e^{-2k_o z\eta}$ in the integral, it can be shown that the upper limit can be replaced by $e^{-\gamma}/2k_o|z|$. Integrating Equation 3.51 and using the new upper limit the result is:

$$I_\infty(z) = V_o j \frac{e^{jk_o|z|}}{\zeta_o} \ln[1 - \frac{2\pi j}{2C_\omega + \ln 2k_o|z| + \gamma + j3\pi/2}] \qquad (3.52)$$

for large $k_o z$.

For small ka, the input conductance of the infinitely long antenna is approximately

$$G_\infty = \frac{Im}{\zeta_o} \ln(1 + \frac{j\pi}{C_\omega}) + \frac{\pi^2}{12}[C_\omega - \ln 2)^{-2} - (C_\omega - \ln 2 + \pi j)^{-2}]. \qquad (3.53)$$

To obtain a formula for $I_\infty(z)$ when $k_o z$ is small, the term $\ln 2k_o|z|$ must be replaced by $\ln(k_o|z| + ((k_o z)^2 + D'')^{1/2})$. To find D'' it is required that when $k_o z = 0$, the real part of Equation 3.52 divided by V_o and Equation 3.53 do not differ by more than $1/C_\omega{}^3$ in the limit as ka approaches zero. If this is done, $D'' = e^{-2\gamma}$. Thus for all $k_o z$,

$$I_\infty(z) = \frac{V_o j e^{jk_o|z|}}{\zeta_o} \ln[1 - 2\pi j/(2C_\omega + \ln(k_o|z| + \sqrt{(k_o z)^2 + e^{-2\gamma}} + \gamma + 1.5\,\pi j)]. \qquad (3.54)$$

3.2.2 Reflection Current At The End Of A Semi-Infinite Antenna

If i(z) is the normalized current on a semi-infinite antenna with one end at z = 0 and the other end at $z = \infty$ and is excited by $V = \infty$ at $z = \infty$ such that the amplitude of the incident current is unity, then i(z) for $k_o z$ not too small and C_o deformed around the branch cut at $\zeta = k_o$, can be shown to be

$$i(z) = \frac{k_o}{\pi j}[\bar{L}_+(+k_o)]^{-2} \int_{C_o'} \frac{e^{j\zeta z}}{(k_o^2 - \zeta^2)\bar{K}(\zeta)} d\zeta \tag{3.55a}$$

where

$$\ln \bar{L}_+(\zeta) = \frac{-1}{2\pi j} \int_{-\infty-\varepsilon}^{\infty-\varepsilon} d\zeta' \frac{\ln\bar{K}(\zeta')}{\zeta' - \zeta} . \tag{3.55b}$$

Comparing Equation 3.55 with 3.48 gives

$$i(z) = -RI_\infty(z)/V_o \tag{3.56}$$

for large kz, where

$$R = \frac{\zeta_o}{2\pi} [\bar{L}_+(k_o)]^{-2}.$$

The approximate value of $[\bar{L}_+(k_o)]^{-2}$ can be shown to be

$$[\bar{L}_+(k_o)]^{-2} = 2C_\omega + j\pi. \tag{3.57}$$

From Equation 3.56 it can be seen that -R is the ratio of the reflected current to the incident current and that the reflected current away from the end behaves just like the current of an infinitely long antenna.

3.2.3. Current On An Asymmetrical Finite Antenna

Section 3.2.2 suggests that the current on a long finite antenna may be expressed as the superposition of an outgoing wave, $I_\infty(z)$ and two reflected waves which are proportional to $I_\infty(h_1 + z)$ and $I_\infty(h_2 - z)$, respectively.

If the coordinates are chosen such that the ends are at $z = -h_1$ and $z = h_2$, and the driving point is at $z = 0$, then

$$I(z) = I_\infty(z) + C_d I_\infty(h_1 + z) + C_u I_\infty(h_2 - z). \tag{3.58}$$

If it is assumed that the reflection coefficient R at the end of an antenna is the same as that for a semi-infinitely long antenna, then C_d and C_u can be evaluated. Since at $z = -h_1$, R is the ratio of reflected to incident waves and since the amplitude of reflected waves is $C_d V_o$ and the amplitude of incident wave is $I_\infty(h_1) + C_u I_\infty(h_2 + h_1)$, the result is

$$-R = \frac{C_d V_o}{I_\infty(h_1) + C_u I_\infty(h_2 + h_1)} . \tag{3.59}$$

Similarly, at $z = h_2$

$$-R = \frac{C_u V_o}{I_\infty(h_2) + C_d I_\infty(h_2 + h_1)} . \tag{3.60}$$

Solving Equations 3.59 and 3.60,

$$C_d = -R\left[\frac{I_\infty(h_1)/V_o - R I_\infty(h_2)/V_o I_\infty(h_1 + h_2)/V_o}{1 - R^2[I_\infty(h_1 + h_2)]^2/V_o^2}\right] \tag{3.61}$$

$$C_u = -R\left[\frac{I_\infty(h_2)/V_o - R I_\infty(h_1)/V_o I_\infty(h_1 + h_2)/V_o}{1 - R^2[I_\infty(h_1 + h_2)]^2/V_o^2}\right] . \tag{3.62}$$

This completely defines Equation 3.58 and gives an approximate distribution for asymmetrically driven antennas. If a distribution is desired for the symmetrical antenna, it can be superimposed using Equation 3.58, $h_1 = h \pm \ell$, and $h_2 = h \mp \ell$.

CHAPTER IV

RADIATION PATTERNS OF SHORT AND LONG ANTENNAS

Since different current distributions were derived for the short and long antennas, different radiation pattern expressions will have to be found.

4.1. Radiation Pattern Of A Short Antenna

Because of the simple form of the current for a short antenna, the radiation pattern will be found by using Equation 3.27 and the usual far field approximations,

$$k_oR_o >> 1, \; R_o >> h, \quad R = R_o - z' \cos\Theta \quad \text{(phase terms)}$$

and $R = R_o$ (amplitude terms) where R_o is the distance from the center of the antenna to the field point. If this is done, it can be shown that the radiation pattern, $P_m(\Theta)$ is

$$P_m(\Theta) = \frac{V_m \zeta_o k_o \sin\Theta}{j \, \Pi} \int_{-h}^{h} I(z) e^{jk_o z \cos\Theta} dz \qquad (4.1)$$

where I(z) is given by Equation 3.27 or 3.33. Thus three integrals must be evaluated

$$\int_{-h}^{h} M_{oz} e^{jk_o z \cos\Theta} dz = I_1 \qquad (4.2)$$

$$\int_{-h}^{h} (\cos k_o z - \cos k_o h) e^{jk_o z \cos\Theta} dz = I_2 \tag{4.3}$$

$$\int_{-h}^{h} (\cos k_o z/2 - \cos k_o h/2) e^{jk_o z \cos\Theta} dz = I_3. \tag{4.4}$$

Both Equations 4.3 and 4.4 can be evaluated easily and can be shown to equal to

$$I_2 = \frac{\sin k_o h(1 + \cos\Theta)}{k_o(1 + \cos\Theta)} + \frac{\sin k_o h(1 - \cos\Theta)}{k_o(1 - \cos\Theta)} - \frac{2 \cos k_o h \sin(k_o h \cos\Theta)}{k_o \cos\Theta} \tag{4.5}$$

and

$$I_3 = \frac{\sin k_o h(1/2 + \cos\Theta)}{k_o(1/2 + \cos\Theta)} + \frac{\sin k_o h(1/2 - \cos\Theta)}{k_o(1/2 - \cos\Theta)} - \frac{2 \cos k_o h/2 \sin(k_o h \cos\Theta)}{k_o \cos\Theta}. \tag{4.6}$$

Using Equation 3.27, I_1 becomes

$$I_1 = \cos k_o \ell \sin k_o h \int_{-h}^{h} \cos k_o z e^{jzk_o \cos\Theta} dz - (1/2)\cos k_o h. \tag{4.7}$$
$$\left[\int_{-h}^{h} \sin k_o |z + \ell| e^{jzk_o \cos\Theta} dz + \int_{-h}^{h} \sin k_o |z-\ell| e^{jzk_o \cos\Theta} dz\right]$$

which can be shown to equal

$$I_1 = \cos k_o\ell \sin k_oh\left[\frac{\sin k_oh(1 + \cos\theta)}{k_o(1 + \cos\theta)} + \frac{\sin k_oh(1 - \cos\theta)}{k_o(1 - \cos\theta)}\right] - \cos k_oh \tag{4.8}$$

$$\left[\frac{\sin k_o\ell \sin k_o\ell(1 + \cos\theta)}{k_o(1 + \cos\theta)} + \frac{\sin k_o\ell \sin k_o\ell(1-\cos\theta)}{k_o(1 - \cos\theta)}\right.$$

$$- \cos k_o\ell\left(\frac{\cos k_oh(1 + \cos\theta)}{k_o(1 + \cos\theta)} - \frac{\cos k_o\ell(1 + \cos\theta)}{k_o(1 + \cos\theta)}\right.$$

$$\left.\left. + \frac{\cos k_oh(1 - \cos\theta)}{k_o(1 - \cos\theta)} - \frac{\cos k_o\ell(1 - \cos\theta)}{k_o(1 - \cos\theta)}\right)\right].$$

Reducing to a common denominator and simplifying, I_1, I_2 and I_3 are found to be

$$I_2 = \frac{2}{k_o\cos\theta\sin^2\theta}\left[\cos\theta\sin k_oh \cos\theta(k_oh \cos\theta) - \cos k_oh\cdot \sin(k_oh \cos\theta)\right] \tag{4.9}$$

$$I_3 = \frac{2}{k_o\cos\theta(1-4 \cos^2\theta)}\left[2 \cos\theta\sin k_oh/2 \cos (k_oh \cos\theta) - \cos k_oh/2 \sin(k_oh \cos\theta)\right] \tag{4.10}$$

$$I_1 = \frac{2}{k_o\sin^2\theta}\left[\cos k_o\ell \cos(k_oh \cos\theta) - \cos k_oh \cos (k_o\ell \cos\theta)\right]. \tag{4.11}$$

Replacing I_1, I_2 and I_3 in Equation 4.1, the result is

$$P_m(\Theta) = \frac{V_m \sin\Theta}{\psi_{dR}\cos k_o h}\Big[\frac{1}{\sin^2\Theta}(\cos k_o\ell \cos(k_o h \cos\Theta)$$

$$- \cos k_o h \cos(k_o\ell \cos\Theta)) + \frac{T_u}{\cos\Theta\sin^2\Theta}\cdot$$

$$(\cos\Theta\sin k_o h \cos(k_o h \cos\Theta) - \cos k_o h \sin(k_o h \cos\Theta)) \tag{4.12}$$

$$+ \frac{T_D}{\cos\Theta(1-4\cos^2\Theta)}(2\cos\Theta\sin(k_o h/2)\cos(k_o h \cos\Theta)$$

$$- \cos(k_o h/2)\sin(k_o h \cos\Theta))\Big].$$

If Equation 4.12 is inspected, it can be seen that Equation 4.12 is indeterminate if $\Theta = 0$, $\pi/2$, $\pi/3$, or $2\pi/3$ or if $k_o h = \pi/2$.

If $k_o h = \pi/2$ then Equation 3.33 must be used in Equation 4.1. Thus for $k_o h \approx \pi/2$, I_1 is

$$I_1 = \sin k_o h \cos k_o\ell \int_{-h}^{h} e^{jk_o z \cos\Theta} dz - 1/2 \int_{-h}^{h} (\sin k_o|z+\ell|$$

$$+ \sin k_o|z-\ell|)e^{jk_o z\cos\Theta} dz \tag{4.13}$$

which reduces to

$$I_1 = \frac{2\sin k_o h \cos k_o\ell}{k_o \cos\Theta}\sin(k_o h\cos\Theta) - \frac{2\cos(k_o\ell\cos\Theta)}{k_o\sin^2\Theta} \tag{4.14}$$

$$+ \frac{2\cos k_o\ell}{k_o\sin^2\Theta}(\cos k_o h \cos(k_o h\cos\Theta) + \cos\Theta\sin k_o h\cdot$$

$$\sin(k_o h\cos\Theta))$$

or

$$I_1 = \frac{2}{k_o \cos\theta \sin^2\theta}[\cos\theta\cos k_oh \cos(k_oh \cos\theta)$$

$$+ \cos k_o\ell \sin k_oh \sin(k_oh \cos\theta) \cos k_o\ell \tag{4.15}$$

$$+ \cos\theta \cos(k_o\ell \cos\theta)].$$

Thus $P_m(\theta)$ becomes

$$P_m(\theta) = \frac{V_m \sin\theta}{\psi_{dR}}[\frac{1}{\cos\theta\sin^2\theta}[\cos\theta\cos k_oh \cos k_o\ell \cos(k_oh \cos\theta)$$

$$+ \cos k_o\ell \sin k_oh \sin(k_oh \cos\theta) - \cos\theta\cos(k_o\ell \cos\theta)]$$

$$- \frac{T_u'}{\cos\theta\sin^2\theta}[\cos\theta\sin k_oh \cos(k_oh \cos\theta) - \cos k_oh$$

$$\sin(k_oh \cos\theta)] + \frac{T_D'}{\cos\theta(1-4 \cos^2\theta)}[2 \cos\theta\sin k_oh/2$$

$$\cos(k_oh \cos\theta) - \cos(k_oh/2)\sin(k_oh \cos\theta)]] \tag{4.16}$$

for $k_oh \approx \Pi/2$. It can be seen that Equation 4.16 is also indeterminate if $\theta = 0, \Pi/3, \Pi/2,$ or $2\Pi/3$. By appropriately taken the limit as $\theta \to 0$ it can be shown that $P_m(\theta) = 0$ for all k_oh.

At $\theta = \Pi/2$ the terms with the form

$$\lim_{\theta \to \Pi/2} \frac{\sin k_oh \cos\theta}{\cos\theta} = k_oh \tag{4.17}$$

become indeterminate.

Thus

$$P_\theta(\Pi/2) = \frac{V_m}{\psi_{dR}\cos k_o h}\,[\cos k_o \ell - \cos k_o h + T_u(\sin k_o h - k_o h \cos k_o h) + T_D(2 \sin k_o h/2 - k_o h \cos k_o h/2)] \qquad k_o h \neq \Pi/2 \tag{4.18}$$

and

$$P_m(\Pi/2) = \frac{V_m}{\psi_{dR}}[\cos k_o h \cos k_o \ell - 1 + k_o h \cos k_o \ell - T_u' \cdot (\sin k_o h - k_o h \cos k_o h) + T_D'(\sin k_o h/2 - k_o h \cos k_o h/2] \quad \text{for } k_o h = \Pi/2. \tag{4.19}$$

At $\Theta = \Pi/3$ or $2\Pi/3$ only the last term is indeterminate. It can be shown by l'Hospital's rule that the last term as $\Theta \to \Pi/3$ or $2\Pi/3$ reduces to $k_o h/2 - \frac{\sin k_o h}{2}$.

Thus $P_m(\Pi/3, 2\Pi/3)$ becomes

$$P_m(\Pi/3 \text{ or } 2\Pi/3) = \frac{V_m \sin \Pi/3}{\psi_{dR}}[\frac{1}{\sin^2 \Pi/3}(\cos k_o \ell \cos k_o h/2 - \cos k_o \ell/2 \cos k_o h) + \frac{T_u}{\sin^2 \Pi/3}(\sin k_o h \cos k_o h/2 - 2 \cos k_o h \sin k_o h/2) + T_D(k_o h/2 - \frac{\sin k_o h}{2}) \tag{4.20}$$

for $k_o h \neq \Pi/2$. By similarly replacing the last term of Equation 4.12 by $T_D'(k_o h/2 - \frac{\sin k_o h}{2})$, the expression for $P_m(\Pi/3, 2\Pi/3)_{k_o h = \Pi/2}$ can be obtained.

4.2. Radiation Pattern Of A Long Antenna

Because of the complex form of the current derived for a long antenna, a different approach is needed to find the radiation pattern for it. The approach to be used will follow that used by Wu (2) and Shen (4) and will be for a symmetrical antenna. The asymmetrical case can be derived using a similar argument.

Let a spherical coordinate system (r,Θ,ϕ) be set up such that the ends of the antenna are $(h, 0,\phi)$ and (h,Π,ϕ). The radiation pattern can be defined as

$$P_m(\Theta) = - \lim_{r\to\infty} E_\Theta(r,\Theta)re^{-jk_or} \tag{4.21a}$$

which reduces to

$$P_m(\Theta) = j\omega\mu_o(4\Pi)^{-1}\,\overline{I}_m(k\cos\Theta)\sin\Theta \tag{4.21b}$$

where

$$\overline{I}_m(\zeta) = \int_{-\infty}^{\infty} I(z)e^{-j\zeta z}dz$$

if the usual $ak_o << 1$ approximation is made. From Equations 3.41-3.47 it can be shown that

$$\overline{I}_m(\zeta) = \frac{4\Pi\ \mu_o^{-1}\ \overline{A}(\zeta)}{\overline{K}(\zeta)} \tag{4.22}$$

where $\overline{A}(\zeta) = \int_{-\infty}^{\infty} A_z(z)e^{-j\zeta z}dz$ and $\overline{K}(\zeta)$ is defined by Equation 3.43b. Substituting Equation 3.22 into Equation 3.21b results in

$$P_m(\Theta) = \frac{j\omega\overline{A}(k_o\cos\Theta)\sin\Theta}{\overline{K}(k_o\cos\Theta)} . \tag{4.23}$$

But from Equation 3.50

$$\overline{K}(\zeta) = 2\Omega_o + j\Pi - \ln[(k_o^2 - \zeta^2)/k_o^2]$$

thus

$$\overline{K}(k_o \cos\Theta) = 2(\Omega_1 - \ell n(\sin\Theta)] \quad (4.24a)$$

where

$$\Omega_1 = \Omega_o + j\pi/2. \quad (4.24b)$$

From Equation 3.6 and correcting for the $e^{j\omega t}$ instead of $e^{-j\omega t}$ time factor, $A_z(z)$ becomes

$$A_z(z) = -j\sqrt{\varepsilon\,\mu}\,[C_1 \cos k_o z - \frac{V_m}{2}(\sin k_o|z+\ell| + \sin k_o|z-\ell|)] \quad h > z > -h.$$

If it is assumed that the main contribution to the integrals comes from the vector potential on the antenna surface, then

$$\overline{A}(k \cos\Theta) \cong \int_{-h}^{h} -j\sqrt{\varepsilon_o\mu_o}\,[C_1 \cos k_o z - \frac{V_m}{2}(\sin k_o|z+\ell| + \sin k_o|z-\ell|)]e^{-jzk_o\cos\Theta}dz \quad (4.25)$$

which is very similar to Equation 4.7. Using the integrals evaluated to find Equation 4.8

$$\begin{aligned}\overline{A}(k_o \cos\Theta) = -j\frac{\sqrt{\varepsilon_o\mu_o}}{k_o}[C_1(&\frac{\sin k_o h(1-\cos\Theta)}{1-\cos\Theta} + \frac{\sin k_o h(1+\cos\Theta)}{1+\cos\Theta}) \\ - V_m(&\frac{\sin k_o\ell \sin k_o\ell(1-\cos\Theta)}{1-\cos\Theta} + \frac{\sin k_o\ell \sin k_o\ell(1+\cos\Theta)}{1+\cos\Theta} \\ &+ \frac{\cos k_o\ell \cos k_o\ell(1-\cos\Theta)}{1-\cos\Theta} + \frac{\cos k_o\ell \cos k_o\ell(1+\cos\Theta)}{1+\cos\Theta} \\ &- \frac{\cos k_o\ell \cos k_o h(1-\cos\Theta)}{1-\cos\Theta} - \frac{\cos k_o\ell \cos k_o h(1+\cos\Theta)}{1+\cos\Theta})]. \end{aligned} \quad (4.26)$$

All that remains is to evaluate C_1.

For a semi-finite antenna with one end at $z = 0$, Wu (2) has shown that

$$\int_{C_o} d\zeta\, \bar{L}_+(\zeta)[\bar{A}_-(\zeta) - A(0+)/j(\zeta+k_o)] = 0 \qquad (4.27)$$

where $\bar{A}_-(\zeta) = \int_o^\infty dz A_z(z) e^{-j\zeta z}$ and $\bar{L}_+(\zeta)$ is given by Equation 3.55b. If again it is assumed that $A_z(z) = 0$ for z outside the antenna, then the following functions can be defined:

$$T'(z') = \int_{C_o} d\zeta \bar{L}_+(\zeta)\left[\int_o^{z'} dz e^{-j\zeta z} \cos k_o z + \frac{j}{\zeta+k_o}\right] \qquad (4.28)$$

$$S'(z') = \int_{C_o} d\zeta \bar{L}_+(\zeta) \int_o^{z'} dz e^{-j\zeta z} \sin k_o z\,. \qquad (4.29)$$

Since it has been assumed that $z = 0$ is at the lower end of the antenna, $A_z(z)$ must be rewritten as

$$(4.30)$$

$$A_z(z) = -j\sqrt{\varepsilon_o\mu_o}[(C_1 \cos k_o h + V_m \cos k_o\ell \sin k_o h)\cos k_o z\cdot$$

$$H(2h - z) + (C_1 \sin k_o h - V_m \cos k_o\ell \cos k_o h)\sin k_o z\, H(2h-z)$$

$$-V_m(\sin k_o(h+\ell) \cos k_o z\, H(h+\ell-z) - \cos k_o(h+\ell)\sin k_o z H(h+\ell-z))$$

$$-V_m\, H(h-\ell-z)[\sin k_o(h-\ell) \cos k_o z - \cos k_o(h-\ell)\sin k_o z)]]$$

where $H(z) = 1 \quad z \geq 0$
$\qquad\qquad = 0 \quad z \leq 0$.

Substituting Equation 4.30 into Equation 4.27 and using Equation 4.28 and 4.29 the result is

$$(C_1 \cos k_o h + V_m \cos k_o \ell \sin k_o h)T'(2h)$$
$$+ (C_1 \sin k_o h - V_m \cos k_o \ell \cos k_o h)S'(2h)$$
$$- V_m(\sin k_o(h+\ell)T'(h+\ell) - \cos k_o(h+\ell)S'(h+\ell))$$
$$- V_m(\sin k_o(h-\ell)T'(h-\ell) - \cos k_o(h-\ell)S'(h-\ell)) = 0. \qquad (4.31)$$

Solving for C_1

$$C_1 = +V_m[\cos k_o\ell(\cos k_o hS'(2h) - \sin k_o hT'(2h))$$
$$+ \sin k_o(h+\ell)T'(h+\ell) - \cos k_o(h+\ell)S'(h+\ell)$$
$$+ \sin k_o(h-\ell)T'(h-\ell) - \cos k_o(h-\ell)S'(h-\ell)]/$$
$$[\cos k_o hT'(2h) + \sin k_o hS'(2h)]. \qquad (4.32)$$

Both $S'(z')$ and $T'(z')$ have been approximated by Shen (5) and are given by

$$S'(z') = [-y_1' + y\,(z') - y_3(z')]/2 \qquad (4.33)$$
$$T'(z') = [y_1' + y_2(z') + y_3(z')]/2j \qquad (4.34)$$

where

$$y_1' = \pi j/(\Omega_1 - \ell n2) \qquad (4.35)$$
$$y_2(z') = \ell n\left(\frac{\Omega_3(z')}{\Omega_2(z')}\right) + \frac{\gamma'}{2}\left[\frac{1}{\Omega_2^2(z')} - \frac{1}{\Omega_3^2(z')}\right] \qquad (4.36)$$
$$y_3(z') = \frac{je^{2jk_oz'}}{k_oz' + \sqrt{(k_oz)^2+(2/\pi)^2}} \times \left[\frac{1}{\Omega_2(z')} - \frac{1}{\Omega_3(z')}\right] \qquad (4.37)$$
$$\Omega_2(z') = 2\,\ell n(1/k_oa) + \ell n[|k_oz'| + \sqrt{(k_oz')^2e^{-2\gamma}}] - \gamma - j\pi/2 \qquad (4.38)$$

$$\Omega_3(z') = \Omega_2(z') + 2\pi j \tag{4.39}$$

$$\gamma' = 1.64493407$$

and Ω_1 is given by Equation 3.24b. Equation 3.32 can be simplified by using Euler's Theorem ($e^{j\theta} = \cos\theta + j\sin\theta$) and Equations 3.33 and 3.34. The result is

$$C_1 = -V_m[\cos k_o\ell(y_1' + e^{j2k_oh}\, y_2(2h) - y_3(2h)) + e^{jk_o\ell}(y_3(h+\ell) - y_2(h+\ell)e^{j2k_oh}) + e^{-jk_o\ell}[y_3(h-\ell) - y_2(h-\ell)e^{j2k_oh}]]/\Delta \tag{4.40}$$

where $\Delta = j(y_1' + y_2(2h)e^{j2k_oh} + y_3(2h))$.

Finally combining Equation 4.24a and a simplified Equation 4.26 into Equation 4.23

$$P_m(\theta) = \frac{\sin\theta\, V_m}{2(\ln(\sin\theta)-\Omega_1)}\Big[-\frac{C_1}{V_m}\Big(\frac{\sin k_oh(1-\cos\theta)}{1-\cos\theta} + \frac{\sin k_oh(1+\cos\theta)}{1+\cos\theta}\Big) + \frac{1}{(1+\cos\theta)}[\cos(k_o\ell\cos\theta) - \cos k_o\ell\cos k_oh(1+\cos\theta)] + \frac{1}{(1-\cos\theta)}[\cos(k_o\ell\cos\theta) - \cos k_o\ell\cos k_oh(1-\cos\theta)]\Big]. \tag{4.41}$$

It should be noted that like the current distribution, the pattern function is based on a time dependence of $e^{-j\omega t}$.

A comparison of Equations 4.40 and 4.41 with the results obtained by Shen (4) will reveal two differences. Instead of $\cos k_o\ell \cos k_oh(1\pm\cos\theta)$ in Equation 4.41, he has $\cos k_o(h\pm h\cos\theta + \ell)$. However, taking the inverse Fourier transform of his pattern function will not give the proper equation for the vector potential. Secondly, his C_1, which differs from the one here, will not reduce to Wu's (2) result when $\ell = 0$.

CHAPTER V

NUMERICAL SOLUTION OF THE SIMULTANEOUS NON-LINEAR EQUATIONS AND NUMERICAL EXAMPLES

In Section 2.2.4.3 it was found that in order to get a solution for the problem at hand, a set of non-linear equations of the form of Equation 2.28 i.e.,

$$a \prod_{i=-M}^{N} Y_i + \sum_{j=-M}^{N} b_j \left(\prod_{i=-M}^{N} Y_i \right)/Y_j + \sum_{j=-M}^{N} \sum_{k=j+1}^{N} c_{jk} \left(\prod_{i=-M}^{N} Y_i \right)/Y_k \cdot Y_k$$

$$\ldots + \sum_{i=-M}^{N} \alpha_i Y_i + \beta = 0 \qquad 2.28$$

must be solved. If $N = 1$ and $M = 0$, the solution is trival. For the symmetric case and $N = 2$, the analytical solution is straight forward. For N and M greater than 2, however, the analytical solution of the problem becomes extremely difficult. To circumvent this difficulty, a numerical method of solving these equations will be presented.

5.1 Method Of Minimization

A close examination of Equation 2.28 will reveal that if all but the ith variable are held constant, Equation 2.28 becomes linear in the ith variable. Thus the system of $N + M$ non-linear equations reduces to $N + M$ linear equations in the ith variable or $N + M$ equations and one unknown, Y_i. Except at a solution point, Y_i will not satisfy all $N + M$

equations simultaneously. If, however, a new function

$$FM = \sum_{j=-M}^{N} (a_j'Y_1 + b_j')^2 \qquad (5.1)$$

where $a_j'Y_1 + b_j'$ is one of the linearized M + N equations, is defined, a value for Y_1 can be found which will minimize FM. Since by the way FM is defined, it is either positive or zero, the least minimum of FM must be zero. If FM equals zero, then all the M + N equations are zero since each is added to FM as a square. Thus FM = 0 will occur if and only if FM is evaluated at a solution of the N + M non-linear equations.

Since FM is a quadratic in Y_1, the minimum value of FM occurs when $\frac{dFM}{dY_1} = 0$. Taking the derivative of FM with respect to Y_1 and setting it equal to zero gives

$$0 = \sum_{j=-M}^{N} 2a_j(a_j'Y_1 + b_j')$$

or

$$Y_1 = - \sum_{j=-M}^{N} a_j'b_j' / \sum_{j=-M}^{N} a_j'^2. \qquad (5.2)$$

If FM is not zero, the non-linear equations can be linearized with respect to another variable using the new found value of Y_1. Again define FM as before and find the value of the second variable which minimizes FM. Since the only variable which changed from the first iteration is Y_1 which made FM a minimum, this second minimum value of FM must be less than or equal to the first. If the variables are changed one at a time in the above procedure, FM will converge to some minimum value. If this value is zero, then the point is a solution to the desired equations. If it is not zero, then the point is one that minimizes the set of N + M functions. Because of the complex nature of the set of non-linear equations, several minimums and zeros may exist for a given problem. A procedure to check for other solutions or minimums will be given in Section 5.4.

5.2 Example Of Minimization Method - N = 2

To more clearly demonstrate the method outlined in the last section, consider the case of two real unknowns. For this case the non-linear equations would be

$$a_1XY + b_1X + c_1Y + d_1 = 0 \tag{5.3a}$$

$$a_2XY + b_2X + c_2Y + d_2 = 0 \tag{5.3b}$$

and FM would be

$$FM = (a_1XY + b_1X + c_1Y + d_1)^2 + (a_2XY + b_2X + c_2Y + d_2)^2. \tag{5.4}$$

Thus if Y is held constant,

$$a_j' = a_jY + b_j \tag{5.5a}$$

and

$$b_j' = c_jY + d_j \tag{5.5b}$$

and if X is held constant,

$$a_j' = a_jX + c_j \tag{5.5c}$$

and

$$b_j' = b_j X + d_j . \tag{5.5d}$$

From Equation 5.2 for constant Y

$$X = -\frac{[(a_1Y + b_1)(c_1Y + d_1) + (a_2Y + b_2)(c_2Y + d_2)]}{(a_1Y + b_1)^2 + (a_2Y + b_2)^2} \tag{5.6a}$$

and for constant X

$$Y = -\frac{[(a_1X + c_1)(b_1X + d_1) + (a_2X + c_2)(b_2X + d_2)]}{(a_1X + c_1)^2 + (a_2X + c_2)^2} . \tag{5.6b}$$

Thus given a Y, Equation 5.6a can be used to find a new X, and using the new X, Equation 5.6b can be used to find a new Y and so on. Finally the process will converge to an X and Y. Substituting this X and Y into Equation 5.4 will tell whether the point is a non-zero minimum or a solution.

5.3 Modified Newton's Method

While the method outlined in Section 5.1 will rapidly converge to the area of a solution or minimum, its rate of convergence slows as they are approached. Because of the nature of FM in the neighborhood of a minimum, the rate of convergence is adequate to locate the minimum. In the neighborhood of a solution, however, FM is very slowly changing, and the convergence becomes extremely slow. Thus a second method is needed to find the solution once the neighborhood of the solution has been reached. Because of its guaranteed convergence if one is within a small enough neighborhood of a solution, the method outlined by Robinson (6) will be presented here.

Let $\underline{f}(\underline{x})$ be a vector function of the vector $\underline{x}$ such that $\underline{f}(\underline{x}) = 0$. Define the norm of $\underline{x}$ to be

$$||\underline{x}||^2 = \underline{x}^T\underline{x} \tag{5.7}$$

and e^j to be the $j\underline{th}$ column of the identity matrix. Then if $\underline{x}^o$ is chosen close enough to $\underline{x}$, the following vector will converge to zero:

$$\Delta^{k+1} = \underline{x}^{k+1} - \underline{x}^k = -[F_k X_k^{-1}]^{-1}\underline{f}(\underline{x}^k) \tag{5.8}$$

where F_k and X_k are matrices defined by:

$$F_k = \underline{f}(\underline{x}^k + X_k e^j) - \underline{f}(\underline{x}^k) \tag{5.9}$$

and $X_k = ||\underline{x}^k - \underline{x}^{k-1}||\ H^k$. H^k is the matrix

$$H^k = \begin{bmatrix} \Delta_1^k\Delta_2^k & \cdots & \Delta_1^k\Delta_N^k & -\Delta_1^k \\ -(\Delta_1^k)^2 & & \vdots & \vdots \\ \vdots & & \vdots & \vdots \\ 0 & \cdots & \Delta_{N-1}^k\Delta_N^k & -\Delta_{N-1}^k \\ 0 & \cdots & -\sum_{j=1}^{N-1}(\Delta_j^k)^2 & -\Delta_N^k \end{bmatrix} D^k \tag{5.10}$$

where D^k is diagonal matrix chosen to normalize the columns in H^k, making H^k an orthonormal matrix. Carrying out the normalization, D^k becomes

$$D^k = \begin{bmatrix} d_1^k & \cdots & 0 & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & d_i^k & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & 0 & \cdots & d_N^k \end{bmatrix}$$

where $d_i^k = 1/[(\sum_{j=1}^{i} (\Delta_j^k)^2)^{1/2} \cdot (\sum_{j=1}^{i+1} (\Delta_j^k)^2)^{1/2}] \quad i \neq n,1$

$$d_N^k = 1/(\sum_{j=1}^{N} (\Delta_j^k)^2)^{1/2}$$

$$d_1^k = 1/\Delta_1^k \cdot (\sum_{j=1}^{N} (\Delta_j^k)^2)^{1/2}.$$

Since for the initial point Δ^k is not given, H^k can be taken as the unity matrix and Δ^k can be taken as $\underline{x}^o$ multiplied by some constant << 1.

5.4 Additional Techniques

To this point it has been tacitly assumed that the set of non-linear equations has only one solution or minimum. This, however, is not the case in general. If, for example, a set of non-linear equations has several solutions and several minimums, the solution or minimum found by the techniques outlined will depend entirely upon the starting point. Since the location of a good starting point is no better known than the location of the solution or minimum sought, an impasse seems to have been reached. The method used in this study to alleviate this problem was to try a point at random and find the solution or minimum associated with this point. Using this solution or minimum as the origin, look for new solutions or minimums on an N-dimensional sphere with a radius of at least an order of magnitude larger than the largest coordinate of the original solution or minimum. This process can be repeated until the desired solution or minimum is obtained or until the space has been thoroughly covered. In using this type of method two things should be noted: First, only $2(M + N) - 2L$ real variables need to be specified where L is the number of constraint equations since the minimization method uses the first $2(M + N) - 2L$ to find the remaining variables. Second, to insure checking all directions each new approach point should differ from the last by at least one sign change, i.e., for $N = 2$, start in a different quadrant each time.

5.5 Numerical Examples

In order to consolidate the theory and methods developed in the first three chapters, several numerical examples will now be presented. All of these examples will be for the symmetric case. The computer program used for the numerical computations is given in the Appendix.

5.5.1 Admittance Constraint

The requirements for the admittance constraint are that the loadings and their position be given. Once they are given the current distribution and field pattern for the antenna can be found. Figures 5.1 and 5.2 show the current distributions of two antennas which are subject to this constraint. Figure 5.1, which is an example of pure real loading, shows the very good agreement between the theory and Altshuler's (7) experimental results.

Figure 5.2, which is an example of pure reactive loading, compares two theoretical results, the theory of Nyquist and Chen (8) and the theory presented here. While the agreement is not as good as in the first case, the two distributions are quite similar. In addition, Nyquist and Chen's (8) results are based on a current distribution which has been forced to be a purely outward traveling wave. Thus it would seem likely that the actual solution would lie between the two curves.

While these two examples are for the $N = 1$ case, the agreement obtained is such, that it would be safe to assume the current distributions found by applying the admittance constraint to the general case would be quite adequate.

5.5.2 Field Pattern Constraints

The effectiveness of the field pattern constraint to control the pattern shape is demonstrated

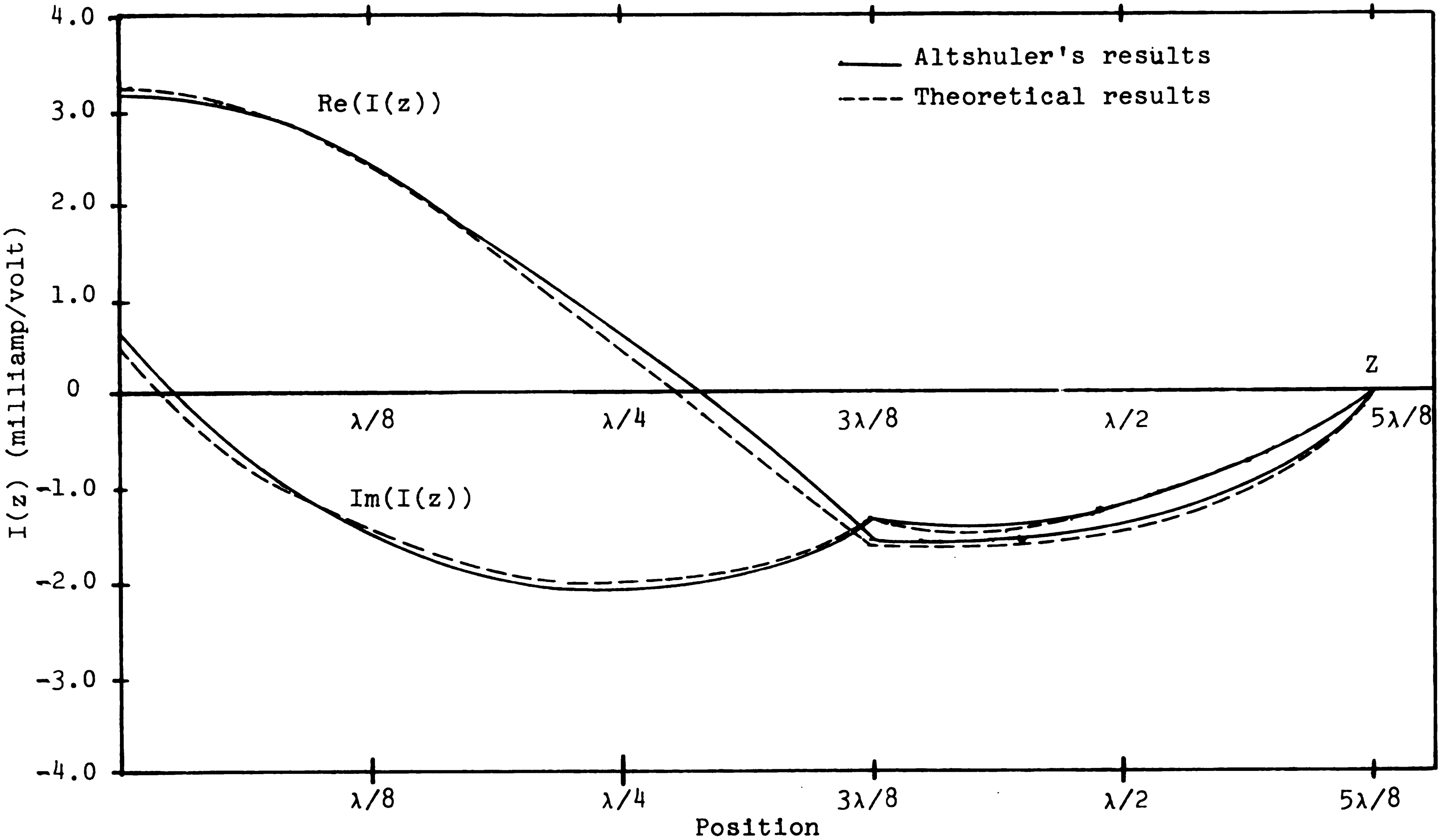


Figure 5.1 Current Distribution For A Transmitting Antenna Which Was Loaded With 240Ω At $d_1 = 3\lambda/8$ ($a = 0.00635\lambda$).

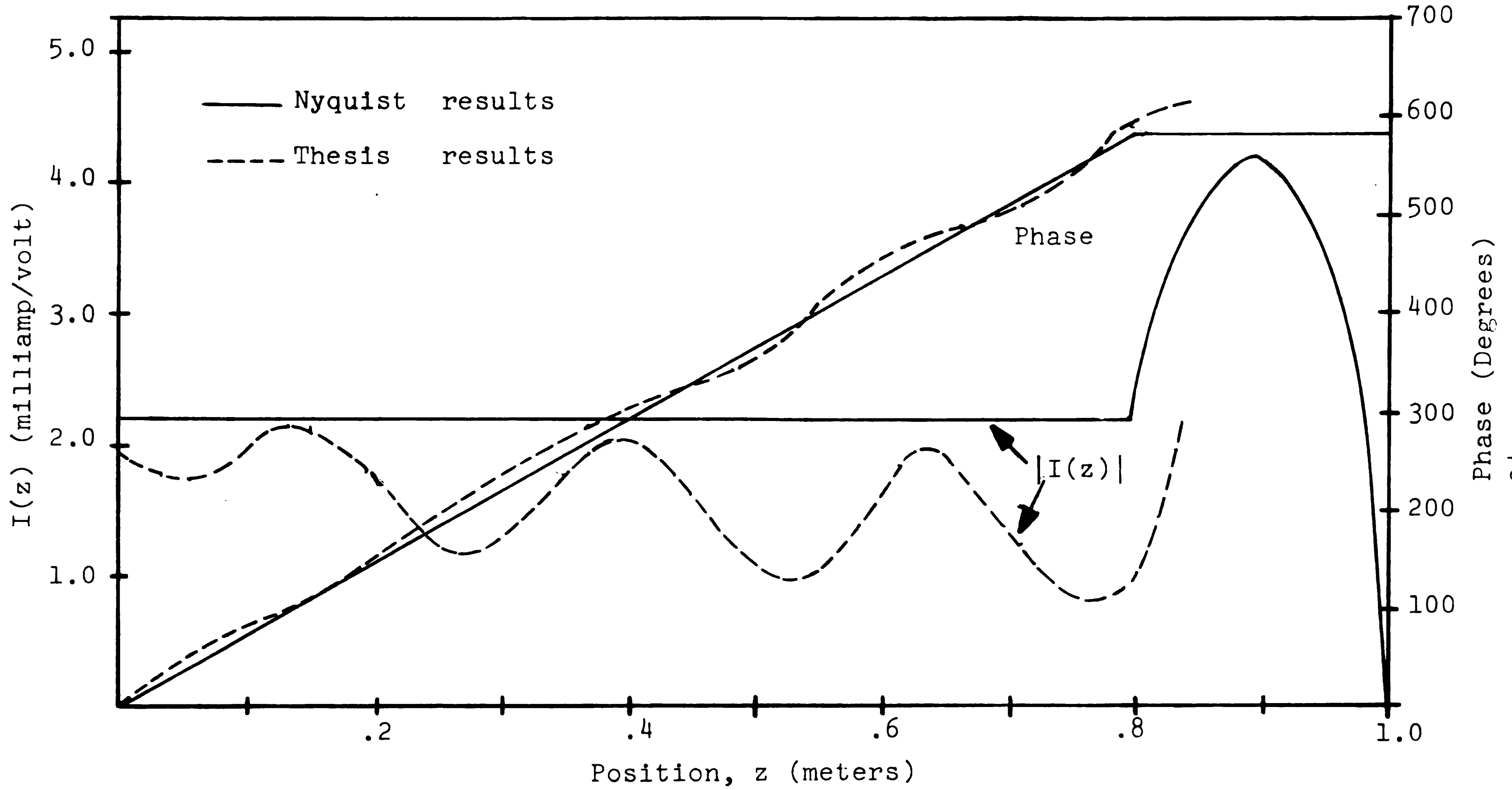


Figure 5.2 Current Distributions For A Transmitting Antenna Which Was Loaded With -j 363 Ohms At d = 0.792 meter (f = 600 MHz and a = 0.00318).

in Figures 5.3 and 5.4. Both examples are for an antenna of half length $.7\lambda$.

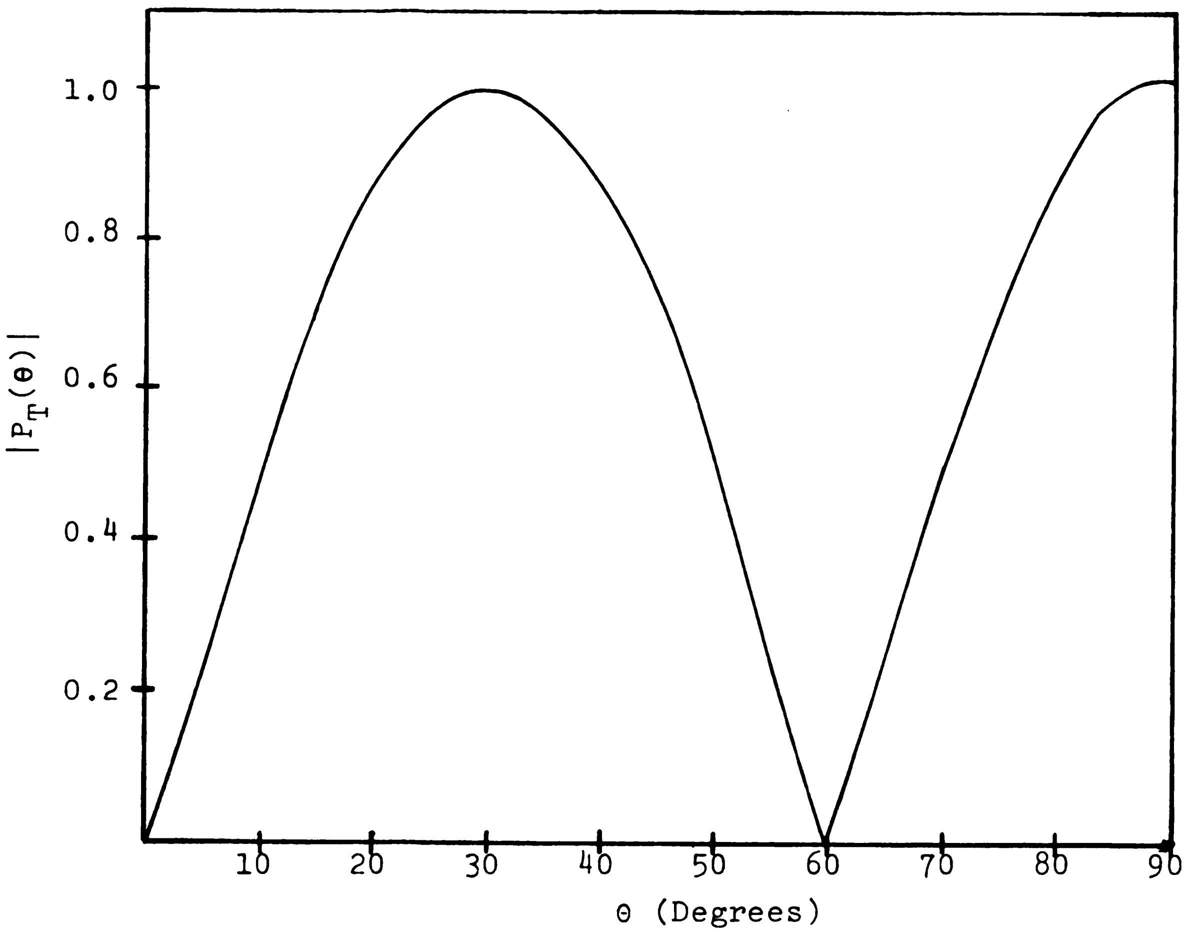
The desired field pattern in Figure 5.3 is sin 3Θ specified at nine points. The resulting pattern is excellent. There are two results of this solution, however, which are not desirable. First, six of the nine loading impedances have negative real parts. Second, the input impedance is extremely small.

The desired field pattern in Figure 5.4 is sin 2Θ specified at four points. Again the resulting pattern is good but not as good as the $N = 9$ case. In addition to some of the loading impedances having negative real parts, the input impedance also has a negative real part. Its magnitude, however, is four orders of magnitude larger than the $N = 9$ case. Thus while the field pattern constraint can adequately control the shape of the pattern, it leaves the other parameters of the antenna in less than desirable form.

5.5.3 Current Constraint

Figures 5.5 and 5.6 show the current distribution for two types of current constraints: 1) traveling wave and 2) decaying traveling wave, respectively. In both figures the current was specified at four points as indicated by the stars on the graphs.

Like the loading impedances for the field pattern constraint, most of the loading impedances for the current constraint have negative real parts. Thus in the two cases



$Z_o = 5.25 \times 10^{-5} + j5.03 \times 10^{-5}\Omega$

$Z_1 = 12.4 + j3.4 \times 10^3\Omega$

$Z_2 = 4.8 + j2.06 \times 10^3\Omega$

$Z_3 = 1.85 + j2.08 \times 10^3\Omega$

$Z_4 = -2.4 + j2.16 \times 10^3\Omega$

$Z_5 = -9.6 + j2.37 \times 10^3\Omega$

$Z_6 = -22.5 + j2.82 \times 10^3\Omega$

$Z_7 = -48 + j3.6 \times 10^3\Omega$

$Z_8 = -120 + j5.8 \times 10^3\Omega$

$Z_9 = -2.4 \times 10^3 + j4.04 \times 10^4\Omega$

$a = 0.00702\lambda$

$h = 0.7\lambda$

$d_1 = 0.06375\lambda$

$d_2 = 0.1275\lambda$

$d_3 = 0.19125\lambda$

$d_4 = 0.255\lambda$

$d_5 = 0.31875\lambda$

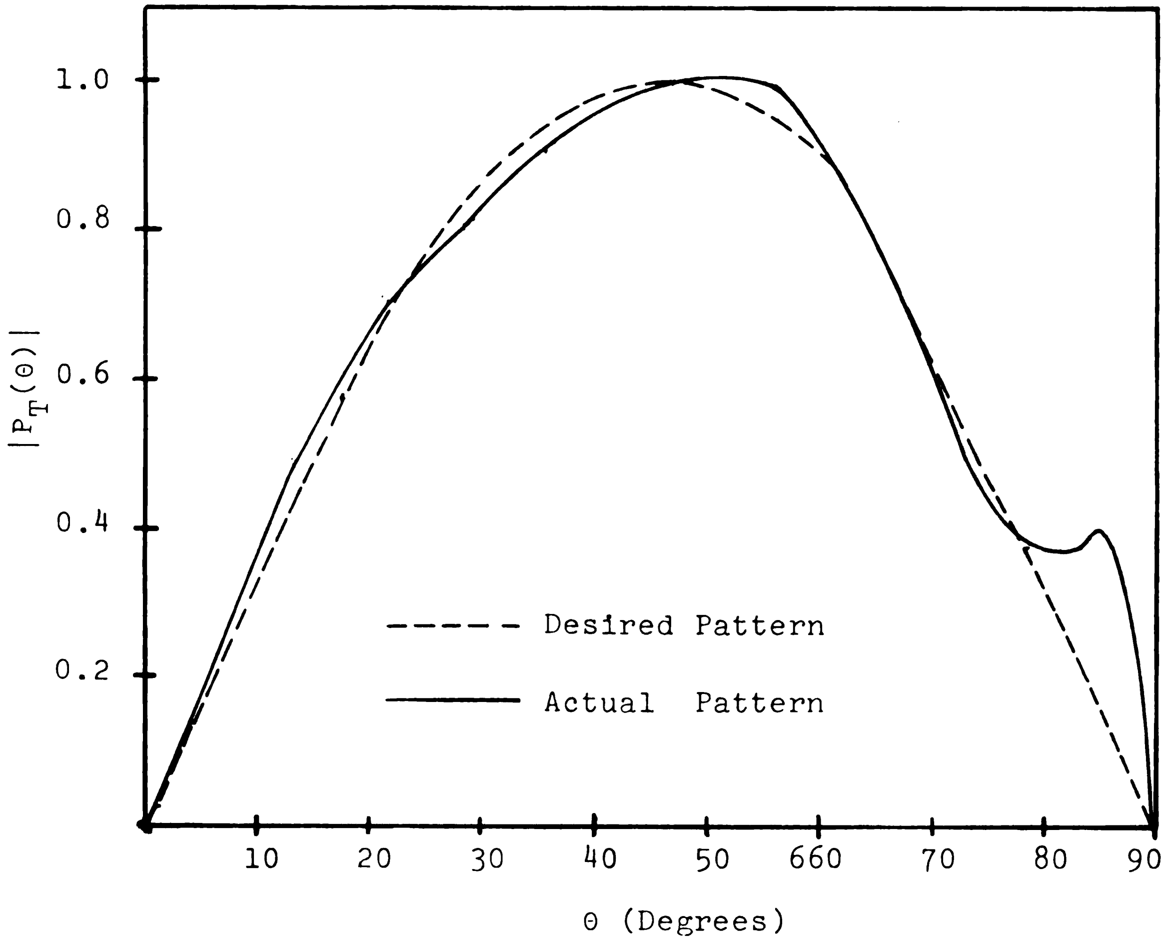
$d_6 = 0.3825\lambda$

$d_7 = 0.44625\lambda$

$d_8 = 0.51\lambda$

$d_9 = 0.57375\lambda$

Figure 5.3 Field Pattern For A Transmitting Antenna Loaded At Nine Points ($P_T(\Theta) = \sin(3\Theta)$).



$a = 0.00635\lambda$

$Z_o = -.194 + j.577\Omega$ $\qquad h = 0.7\lambda$

$Z_1 = -22.6 + j1.36 \times 10^3\Omega$ $\qquad d_1 = 0.1125\lambda$

$Z_2 = -.12 + j650\Omega$ $\qquad d_2 = 0.225\lambda$

$Z_3 = 6.03 + j1.16 \times 10^3\Omega$ $\qquad d_3 = 0.3375\lambda$

$Z_4 = -124.8 + j4.2 \times 10^3\Omega$ $\qquad d_4 = 0.45\lambda$

Figure 5.4 Field Pattern For A Transmitting Antenna Loaded At Four Points ($P_T(\Theta) = \sin(2\Theta)$).

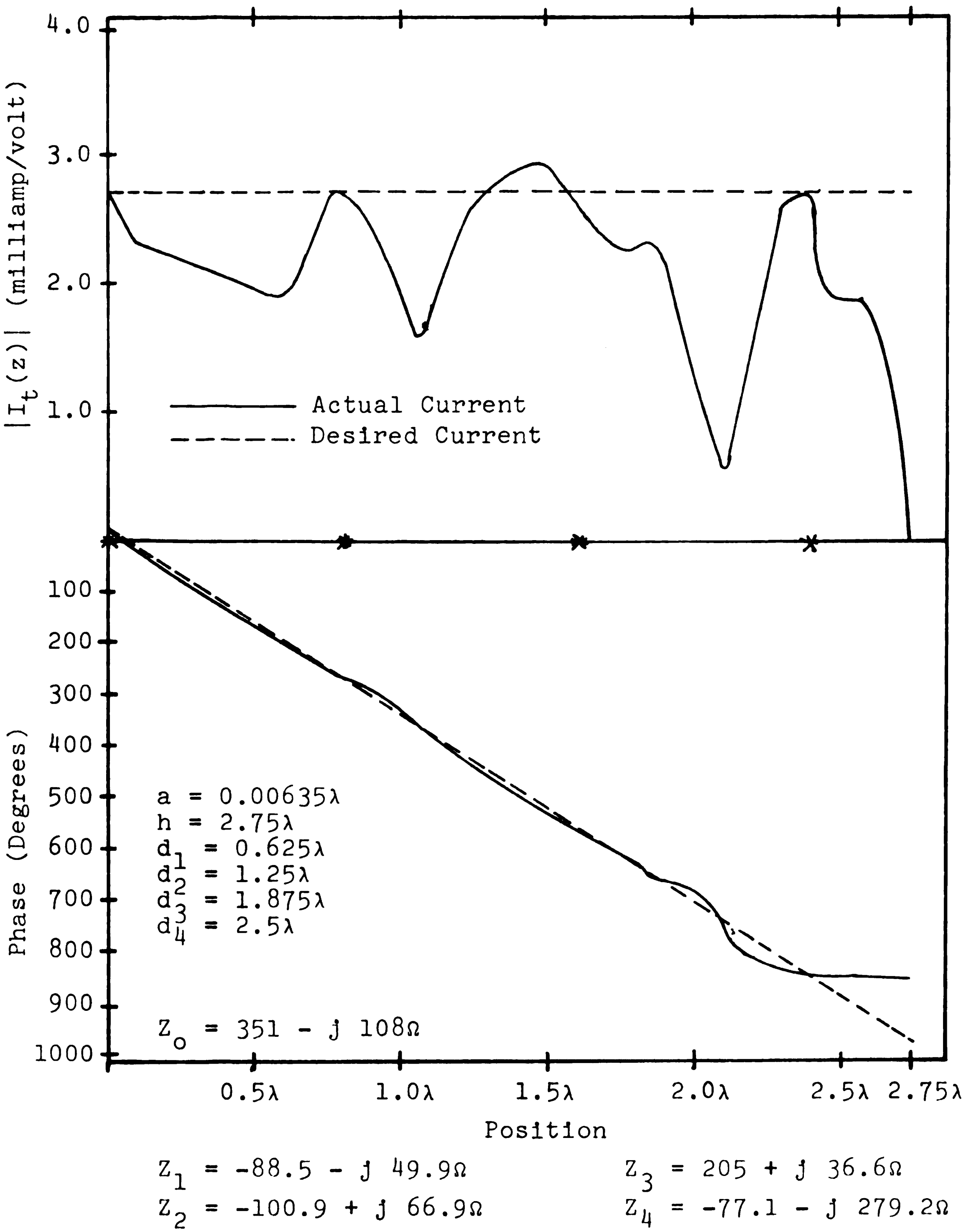
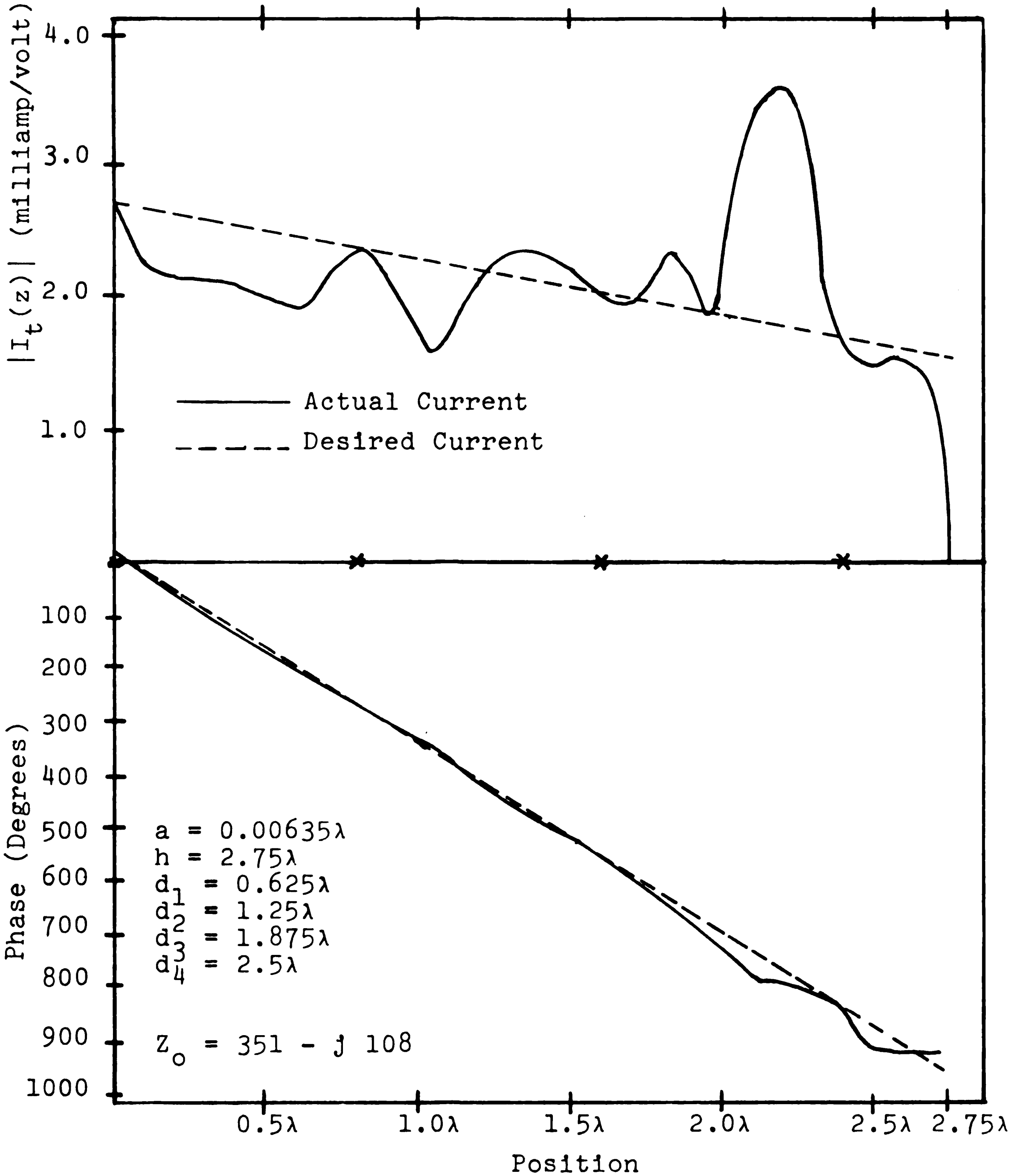


Figure 5.5 Current Distribution For A Transmitting Antenna On Which The Current Was Specified At Four Points (Traveling Wave).



$Z_1 = -41.9 - j\ 45.7\Omega$
$Z_2 = -53.8 + j\ 35.3\Omega$
$Z_3 = -53.5 + j187.6\Omega$
$Z_4 = 392.7 + j208\Omega$

Figure 5.6 Current Distribution For A Transmitting Antenna On Which The Current Distribution Was Specified At Four Points (Decaying Wave).

where the equations are linear, the resulting impedances are less than desirable.

The agreement between the desired currents and the actual currents, however, is good for the decaying traveling wave (Figure 5.6) but only fair for the traveling wave (Figure 5.5). Since pure traveling waves are hard to maintain for any length on an antenna (even the infinitely long unloaded antenna decays), the poor agreement in the traveling wave case should be expected. The good agreement in the decaying case should also be expected. Altshuler (7) in his experiments found that an antenna of the same half length loaded a quarter wavelength from the end with 240Ω produces a decaying traveling wave current.

5.5.4 Input Admittance - Frequency Constraint

Up to this point each example was the solution of linear equations having a unique solution. In this and the next two sections, the examples are solutions to sets of non-linear equations having several solutions. To demonstrate the multiple solutions of these equations, the input admittance-frequency constraint was applied to an antenna such that it would have an input impedance of 50Ω at four frequencies: 400, 500, 600, and 700 MHz. Five solutions to the problem are listed in Table 5.1. In addition to the frequency constraint, the imaginary part of the loading admittances were required to be frequency dependent. Thus all the loading impedances in Table 5.1

$h = 2.75m$	$a = 0.00635m$		
$d_1 = 1.0m$	$d_2 = 1.5m$	$d_3 = 2.0m$	$d_4 = 2.5m$
$R_1 = 1695\Omega$ $L_1 = 0.164\ \mu h$	$R_2 = -465\Omega$ $L_2 = 0.038\ \mu h$	$R_3 = -523\Omega$ $L_3 = 0.229\ \mu h$	$R_4 = -588\Omega$ $C_4 = 0.301\ pf$
$R_1 = 1715\Omega$ $L_1 = 0.766\ \mu h$	$R_2 = 161\Omega$ $L_2 = 0.268\ \mu h$	$R_3 = 181\Omega$ $L_3 = 0.268\ \mu h$	$R_4 = -595\Omega$ $L_4 = 0.215\ \mu h$
$R_1 = -4.3\Omega$ $L_1 = 0.138\ \mu h$	$R_2 = -133\Omega$ $L_2 = 0.144\ \mu h$	$R_3 = 79.4\Omega$ $L_3 = 0.592\ \mu h$	$R_4 = 2164\Omega$ $C_4 = 1.08\ pf$
$R_1 = -350\Omega$ $L_1 = 0.581\ \mu h$	$R_2 = 130\Omega$ $C_2 = 0.826\ pf$	$R_3 = -188\Omega$ $L_3 = 0.057\ \mu h$	$R_4 = -1166\Omega$ $L_4 = 0.299\ \mu h$
$R_1 = -4805\Omega$ $L_1 = 0.063\ \mu h$	$R_2 = 152\Omega$ $C_2 = 1.3\ pf$	$R_3 = -276\Omega$ $L_3 = 0.143\ \mu h$	$R_4 = -385\Omega$ $L_4 = 0.097\ \mu h$

Table 5.1 Solutions To Input Impedance-Frequency Constraint
(Four Frequency Case)

are parallel combinations of a resistor and either an inductor or capacitor.

Figures 5.7 and 5.8 show the behavior of the input impedance for the antenna as a function of frequency for two of the solutions. As can be seen, the input impedance is 50Ω only at the specified frequencies and varies widely for the other frequencies. In some regions of Figure 5.8 the real part even becomes negative.

A second example of the input admittance-frequency constraint is shown in Figures 5.9 and 5.10. The input impedance of 50Ω was specified at two frequencies, 800 and 1200 MHz, and the imaginary part of the loading admittances were made frequency dependent (Figure 5.9) and frequency independent (Figure 5.10). In both cases, the resulting input impedances as a function of frequency are quite similar. This indicates that whenever frequency dependent loading exists, frequency independent loading may also exist. In addition, unlike the first example, there are two regions, 800 to 900 MHz and 1200 to 1300 MHz, in which the input impedance is fairly constant indicating possible broadband operation, a desired goal.

In all the solutions to the input admittance-frequency constraint, however, most or all the real parts of the loading impedances were negative. Thus in the last three constraints considered, the resulting antenna which satisfies these constraints, has had one common factor,

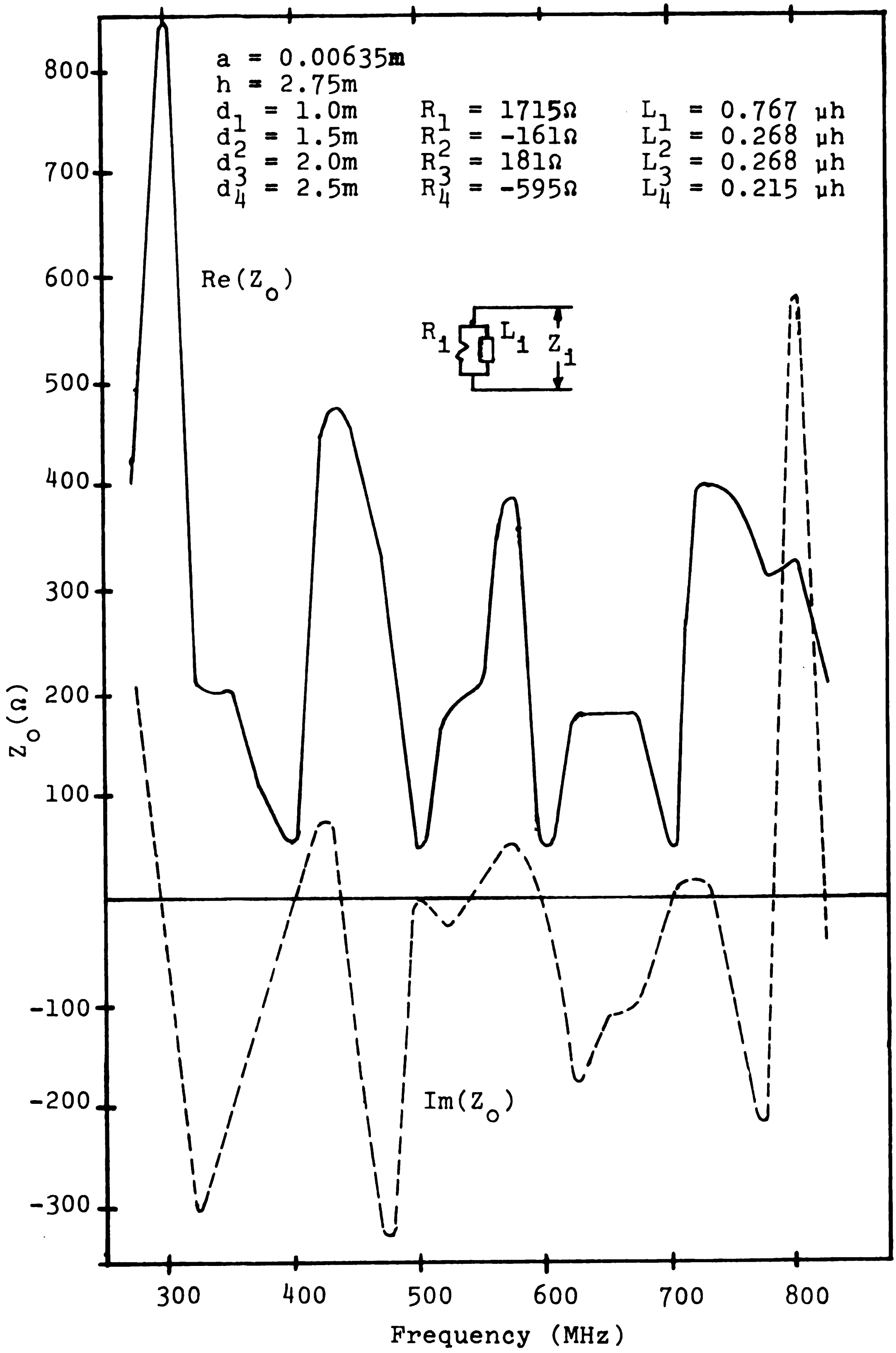


Figure 5.7 Input Impedance vs. Frequency For A Transmitting Antenna (Four Frequency Case - 50Ω - Solution I).

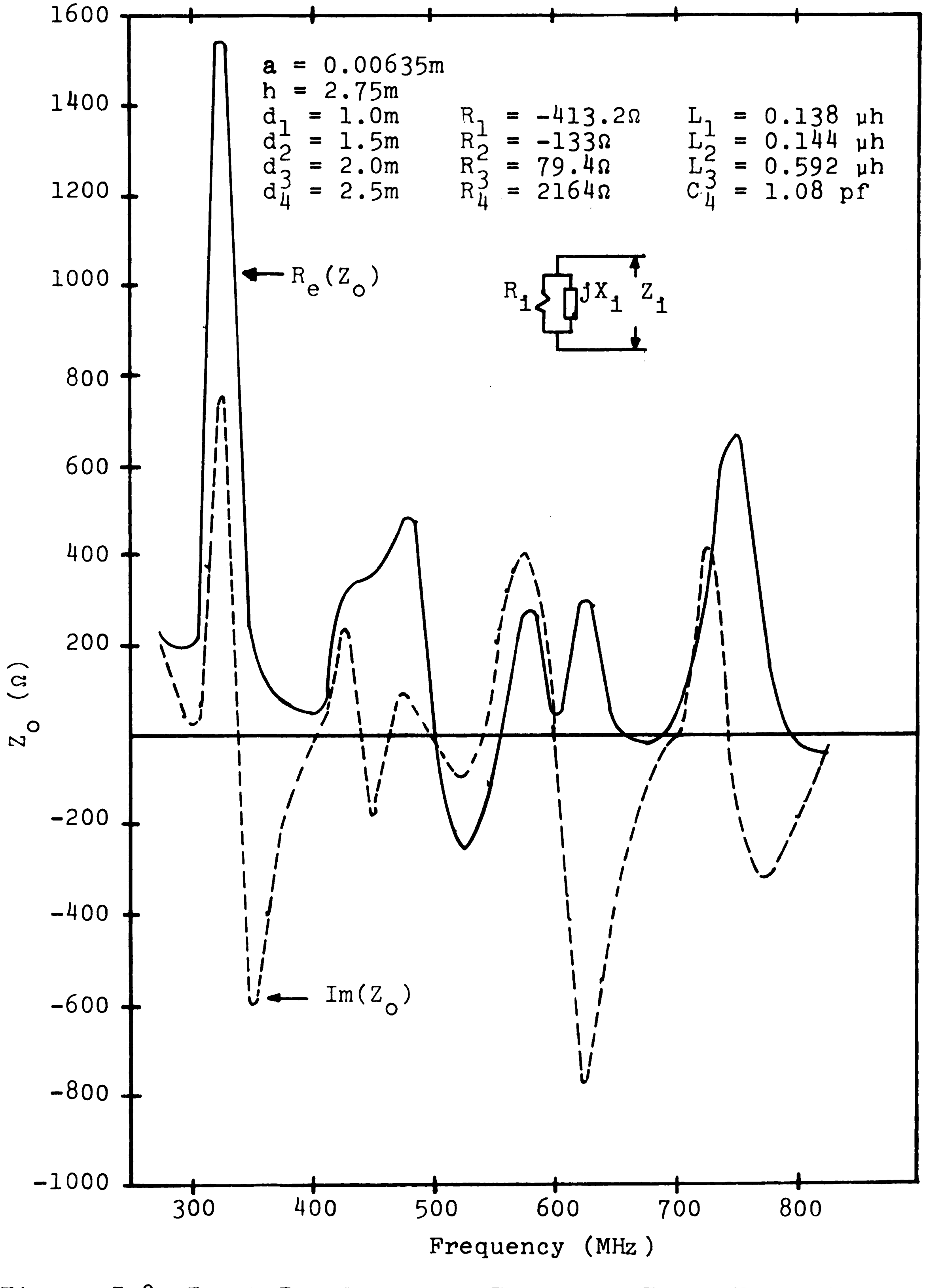


Figure 5.8 Input Impedance vs. Frequency For A Transmitting Antenna (Four Frequency Case - 50Ω - Solution II).

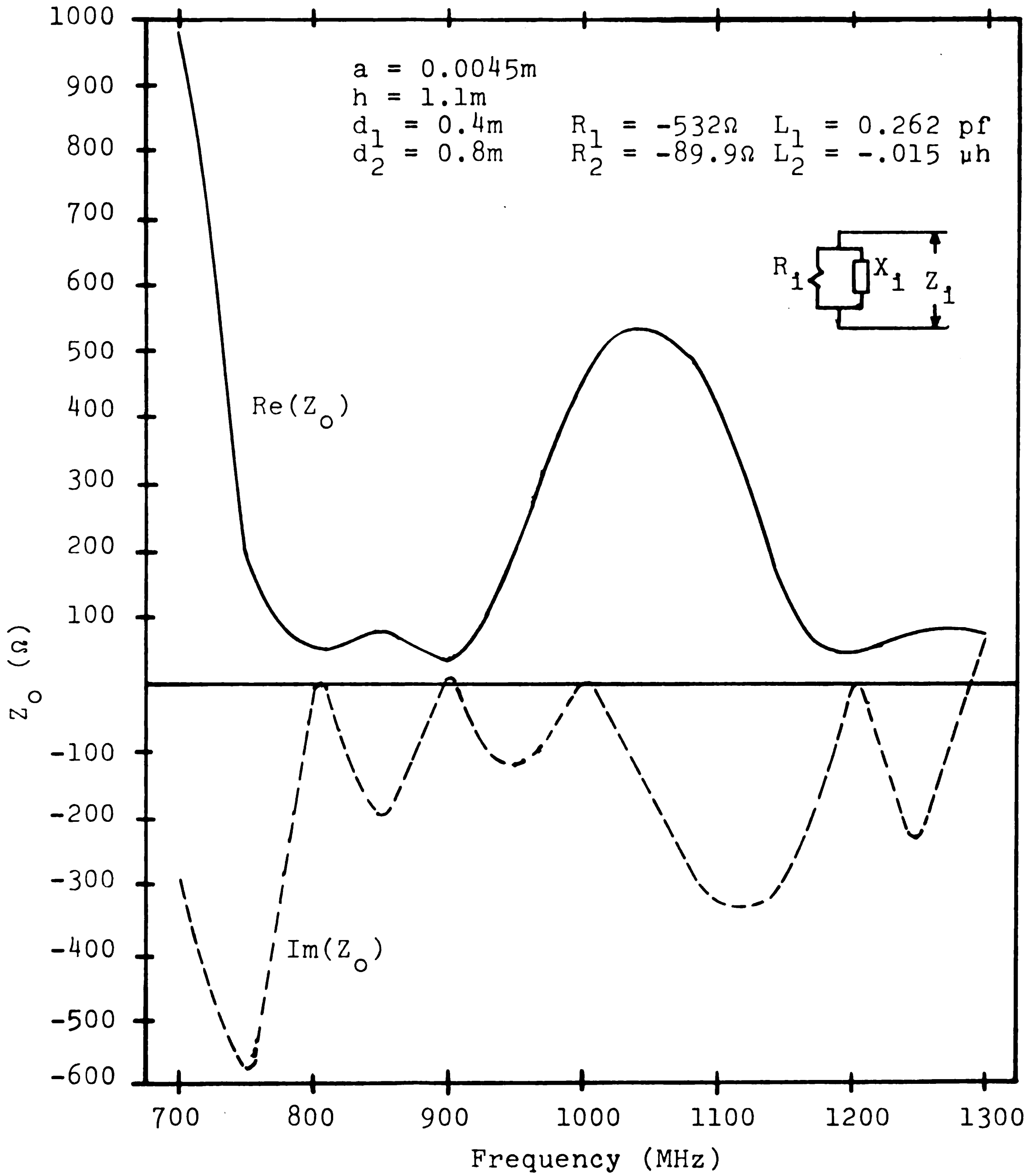


Figure 5.9 Input Impedance vs. Frequency For A Transmitting Antenna (Two Frequency Case - Frequency Dependent).

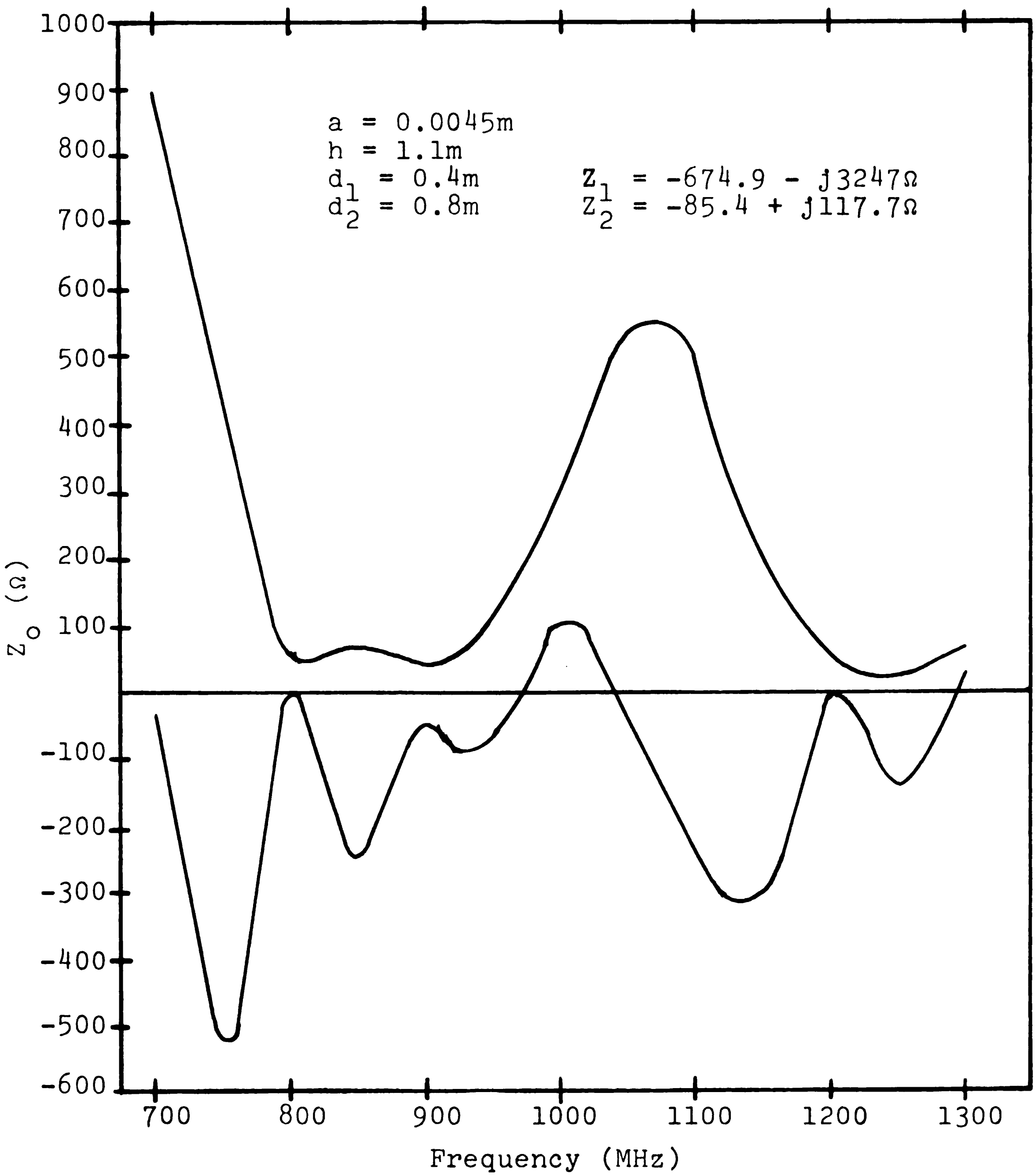


Figure 5.10 Input Impedance vs. Frequency For A Transmitting Antenna (Two Frequency Case - Frequency Independent).

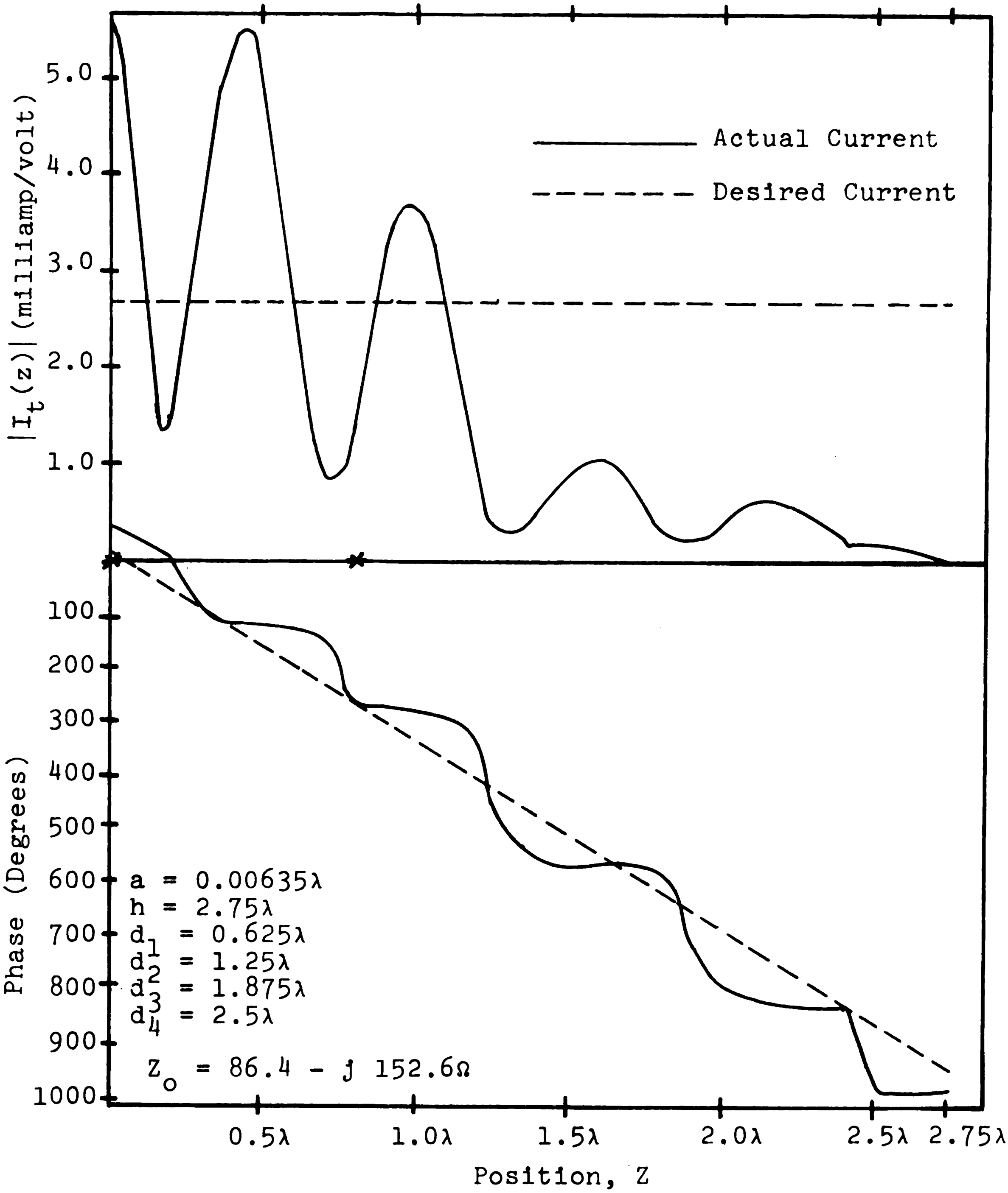
negative real loading impedances. The next two sections will show examples which do not have this problem.

5.5.5 Reactive Loading - Current Constraint

The simplest example of reactive loading with a current constraint is requiring an antenna to have a 50Ω input impedance at one frequency. This was done for an antenna of half length 1.1λ, radius $4.5 \times 10^{-3}\lambda$ and with loading at .4λ and .8λ. The resulting loadings were $Z_1 = j\ 45.3\Omega$ and $Z_2 = j\ 388\Omega$ and $Z_1 = j\ 189\Omega$ and $Z_2 = j\ 48.7\Omega$. Again there is more than one solution. However, for N = 2 the non-linear equations can be solved exactly and these are the only solutions.

A second example of this constraint is shown in Figure 5.11. Using the same conditions as the traveling wave example in Section 5.5.3 except the current was specified at two points, a reactive loading solution was sought. As can be seen, the current distribution does not satisfy the specified current points and is in fact a standing wave current. These loadings are a minimum of the set of non-linear equations rather than a solution. Because of the requirement that all the loads be reactive, no solution to the problem exists.

In the first example in this section, it was found that solutions exist for only a narrow band of input impedances. Thus while reactive loading removes the possibility of negative real loading impedances, it clearly restricts the types of problems that can be solved easily.



Z_1 = -j 224.4Ω Z_3 = j 988Ω
Z_2 = j 1698Ω Z_4 = j 589Ω

Figure 5.11 Current Distribution For A Transmitting Antenna On Which The Current Was Specified At Two Points (Reactive Loading).

5.5.6 Reactive Loading - Frequency and Current Constraint

Figures 5.12 and 5.13 show the frequency dependence of input impedance of two reactively loaded antennas. In both cases the input impedance was specified to be 50Ω at 60 and 70 MHz. Figure 5.12 is the case of frequency independent loading and Figure 5.13 is the case of frequency dependent loading.

Like the previous example of frequency dependent loading, the resulting input impedance curves are very similar. Unlike the previous example, the antenna described in Figure 5.13 can be easily built from available materials, four sets of inductors and a metal rod.

It should be noted that like the previous reactive loading example, the solution was not easily found. Several combinations of loading points, antenna lengths, and frequencies were tried before this particular example was found.

5.5.7 Resonance Loading

In Section 2.2.2 relations were derived for resonance loading and for the instabilities in that resonance. For a given set of antenna parameters and loading points, a family of resonance loadings exist. The instabilities, however, are unique for each set of parameters.

Figures 5.14 and 5.15 show typical values of Z_1 and Z_2 as a function of frequency for the two types of instabilities. As can be seen, the real part of either Z_1 or

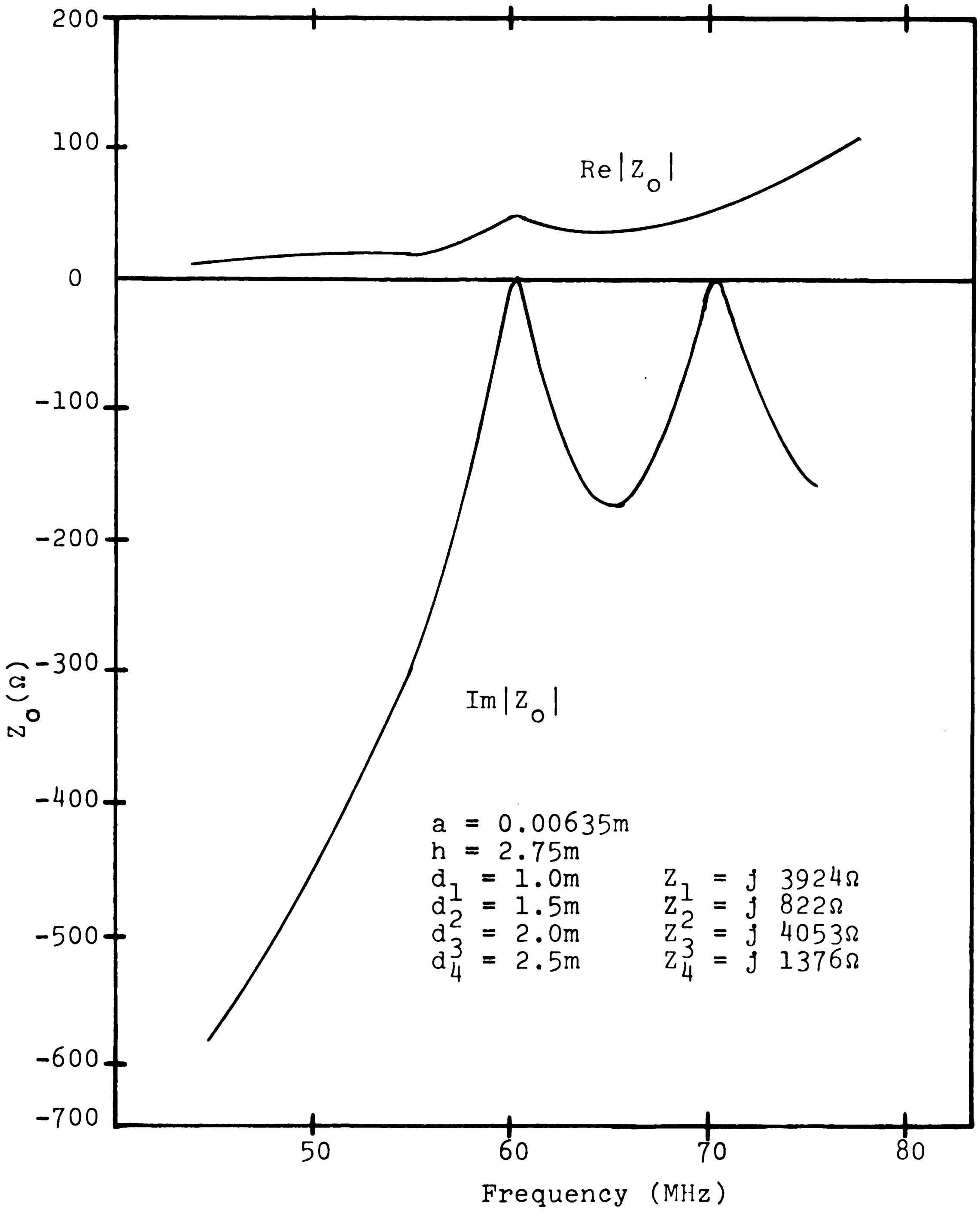


Figure 5.12 Input Impedance vs. Frequency For A Transmitting Antenna (Two Frequency - Reactive Loading - Frequency Independent).

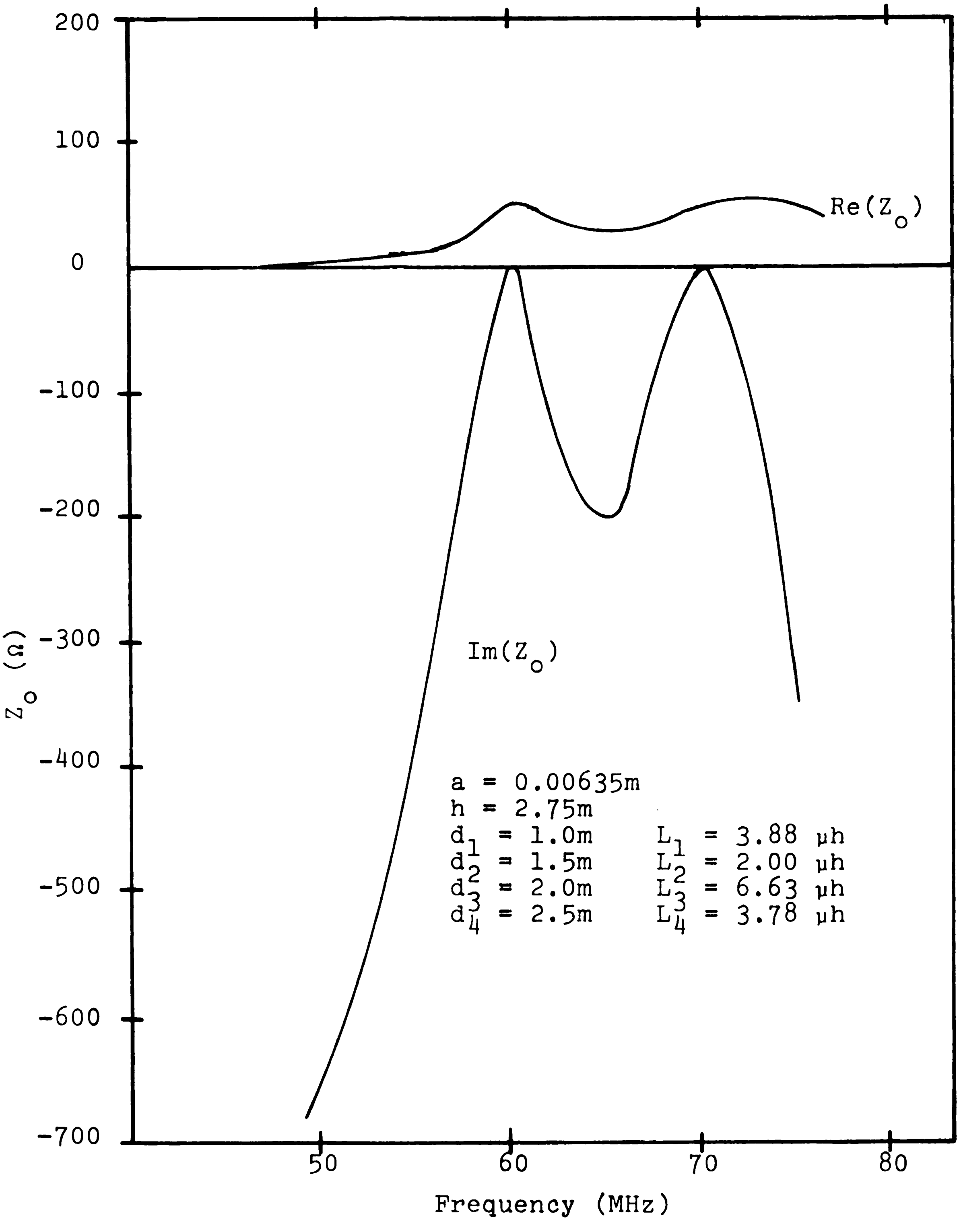


Figure 5.13 Input Impedance vs. Frequency For A Transmitting Antenna (Two Frequency - Reactive Loading - Frequency Dependent).

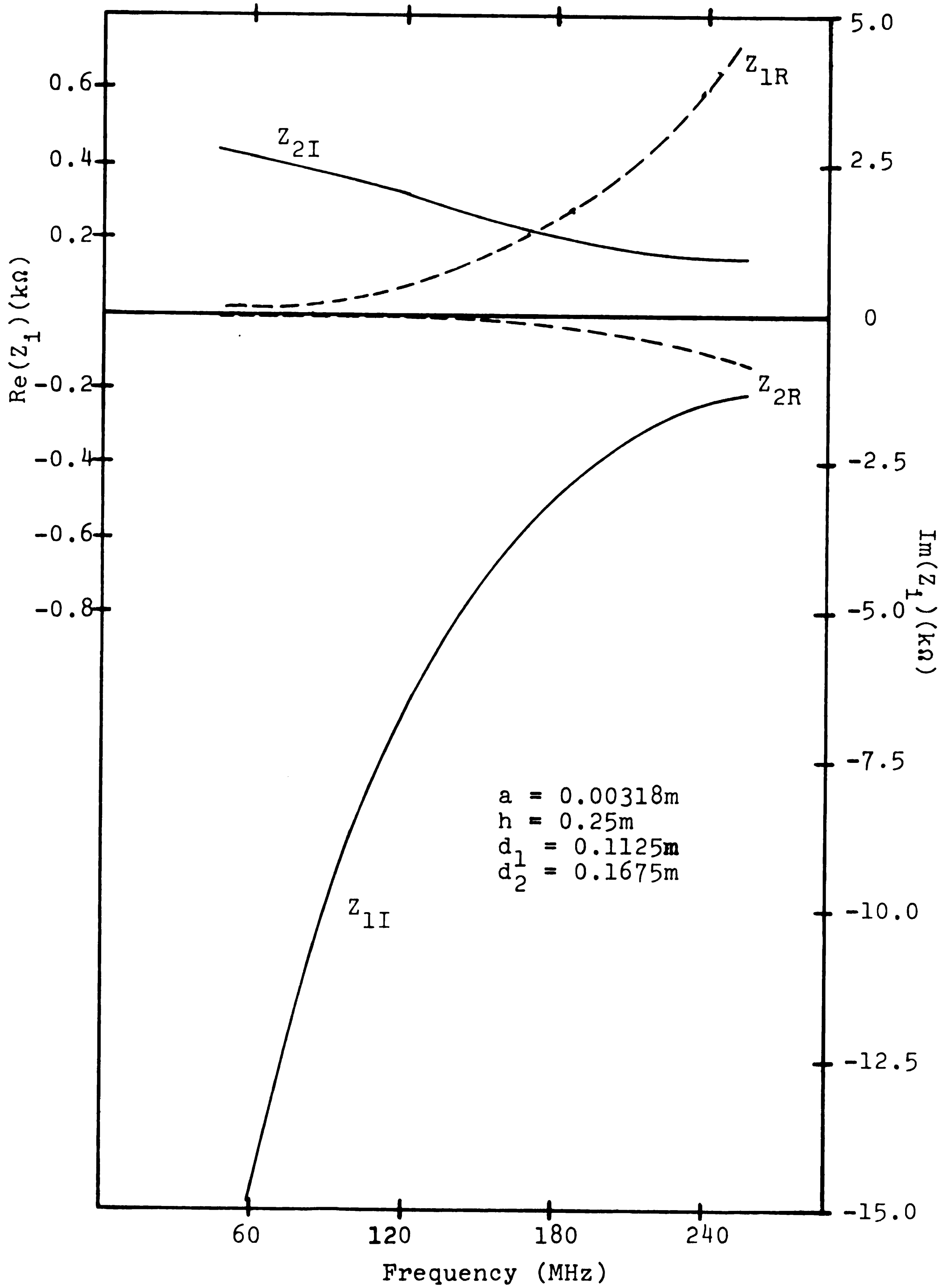


Figure 5.14 Loading Impedances vs. Frequency For A Transmitting Antenna To Maintain A Resonance Instability (Finite Voltage).

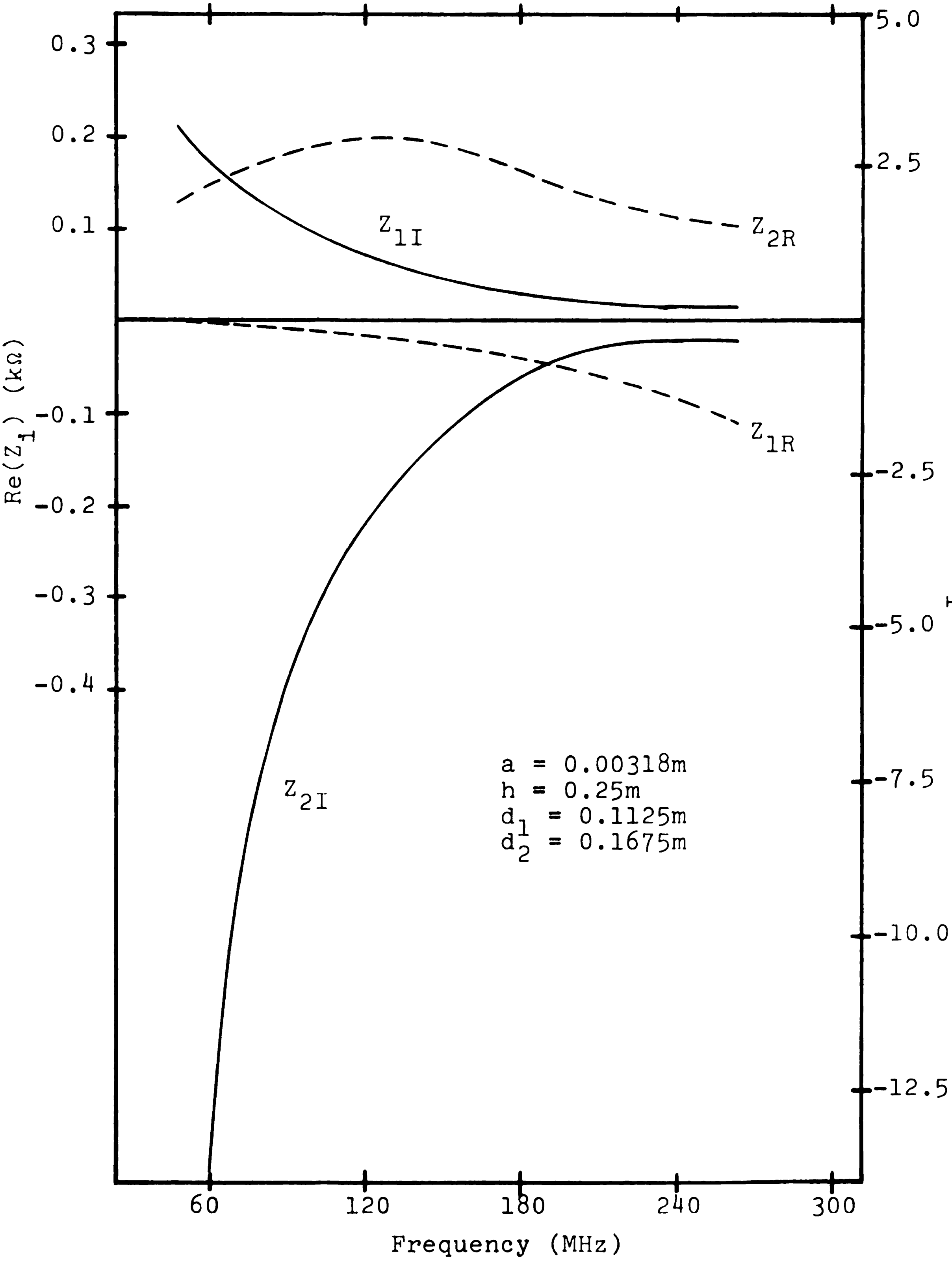


Figure 5.15 Loading Impedances vs. Frequency For A Transmitting Antenna To Maintain A Resonance Instability (Zero Coefficient).

Z_2 is negative in each case. Thus for the usual passively loaded antennas, these instabilities are of no consequence, but if active loading is employed they could become critical.

Figure 5.16 is a graphical solution of the resonance equation for f = 240 MHz. Because of the complex nature of the graph, a brief explanation is in order. Each circle on the line parallel to the real Z_2 axis, Z_{2R}, is a curve of constant real Z_1, Z_{1R}. Similarly each circle along the imaginary Z_2 axis, Z_{2I}, is a curve of constant imaginary Z_1, Z_{1I}. In addition to the circles there are two asymptotes, $Z_{1R} = -22.3$ and $Z_{1I} = 640$, which the circles approach. Since none of the constant Z_{1R} circles intersect except at the point $(-3.9, 1.24 \times 10^3)$, any other intersection on the graph gives the values of Z_1 and Z_2 for resonance, as shown in the circled example. The common intersection, however, is not a solution but equivalent to the condition of Y = 0 for the 2-dimensional hyperbola XY = 1.

Unlike the instabilities, the resonance loadings do not have to be active. Because the asymptotes are skewed with respect to the axes, there exists a small region in which the real parts of both Z_1 and Z_2 are positive. Thus the possibility exists of designing an antenna which has greatly enhanced radiation using only passive elements.

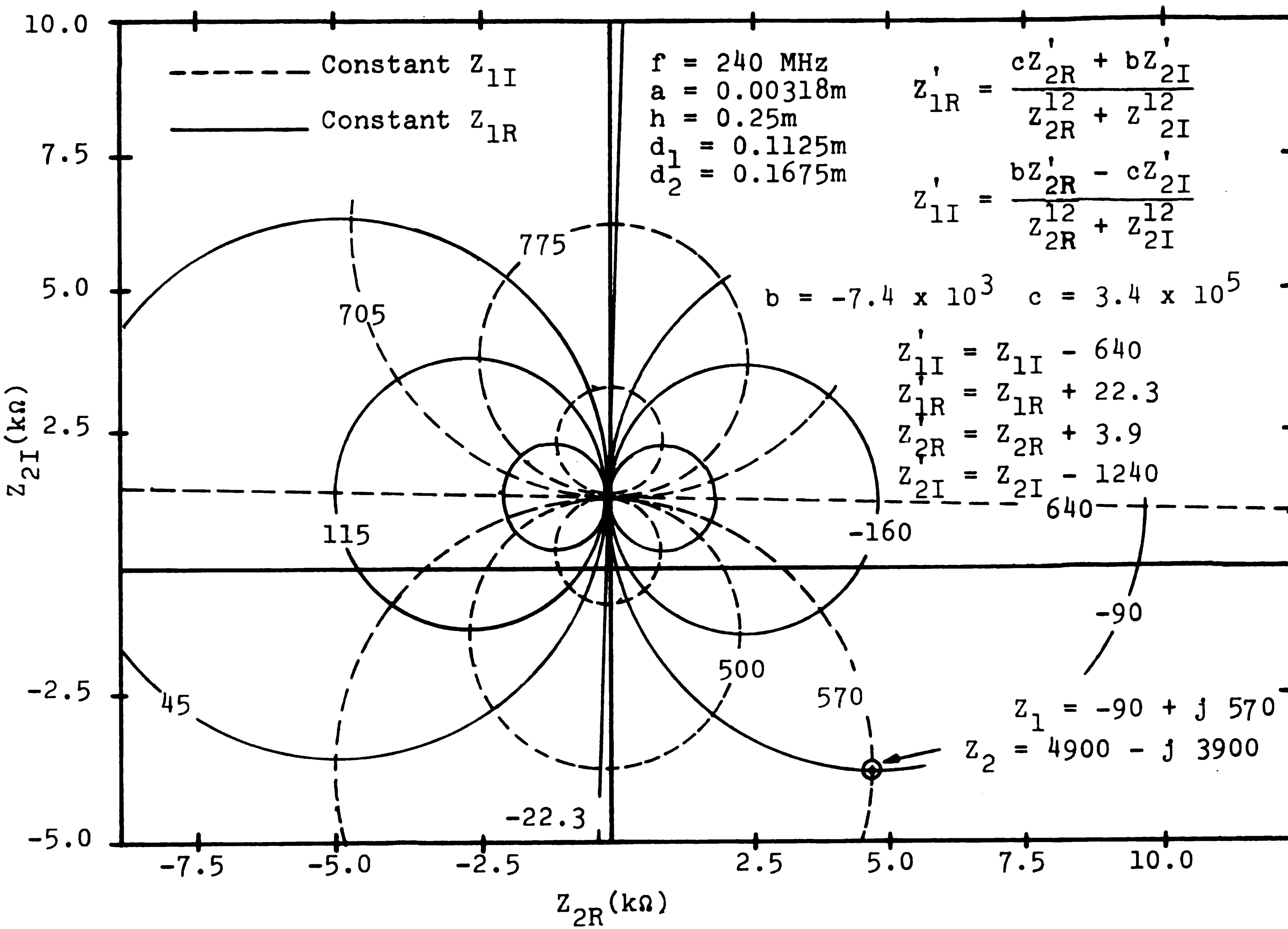


Figure 5.16 Graphical Solution Of Resonance Loading Condition For A Transmitting Antenna

CHAPTER VI

MATHEMATICAL FORMULATION OF A MULTI-LOADED RECEIVING ANTENNA

To this point only multi-loaded transmitting antennas have been considered. It will be shown that with the addition of one term, the theory can be extended to receiving antennas.

6.1 Matrix Formulation

Consider a multi-loaded receiving antenna of length 2h, radius a, and with N + M arbitrary loads and a central load illuminated by a normally incident electric field E_o as shown in Figure 6.1. Replacing the loads by the voltages across and separating the total antenna current into a component of current which is induced by the incident field on the corresponding unloaded antenna and a component of current which is excited by the voltages across the loads, the problem becomes equivalent to the antennas shown in Figure 6.2. It can be shown that

$$I_u(z) = E_o f_u'(z) \tag{6.1}$$

and from 1.1

$$I_t(z) = \sum_{i=-M}^{N} V_i \, f(\omega, a, h, d_i, z). \tag{6.2}$$

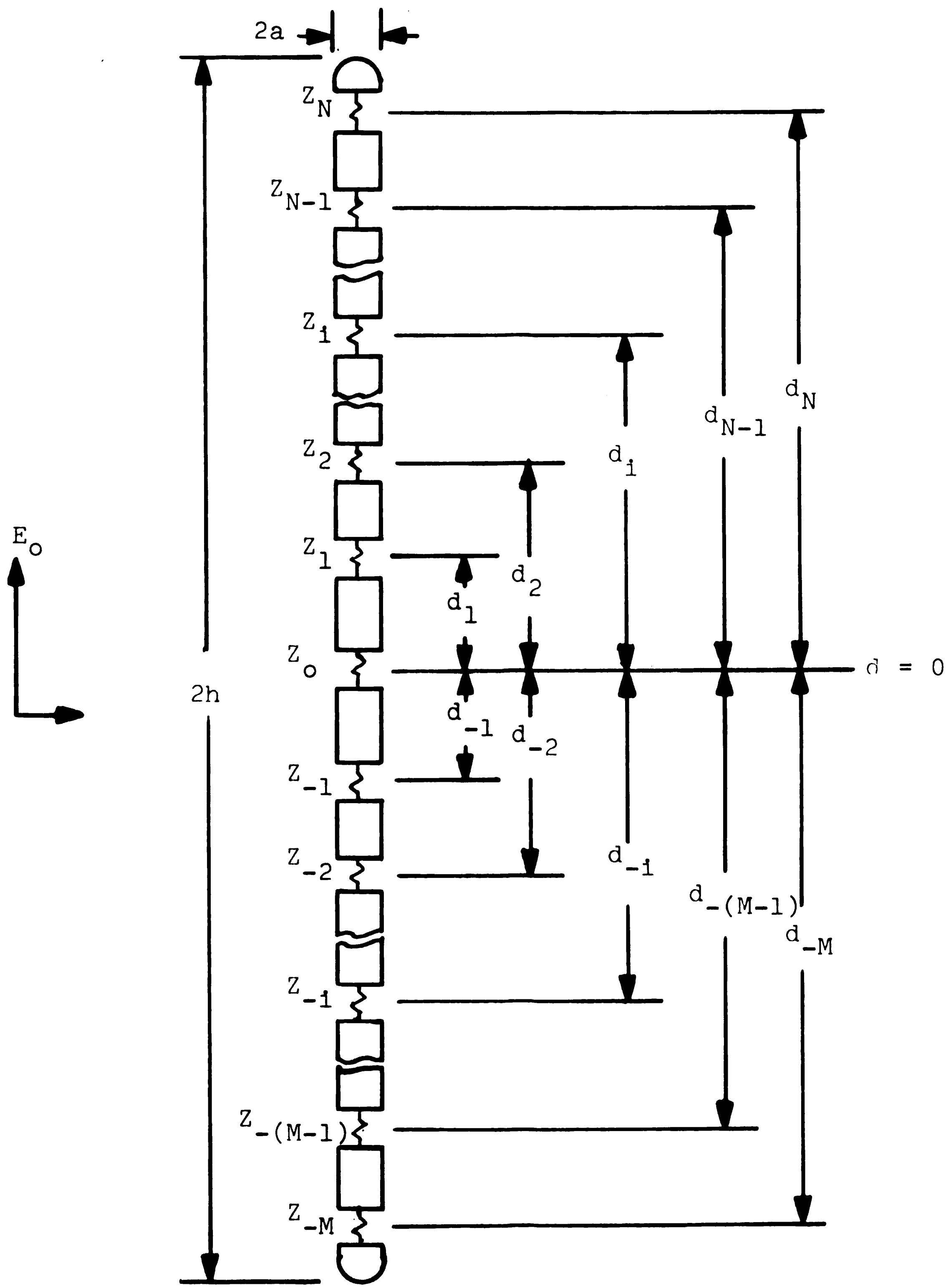


Figure 6.1 Multi-Loaded Receiving Antenna

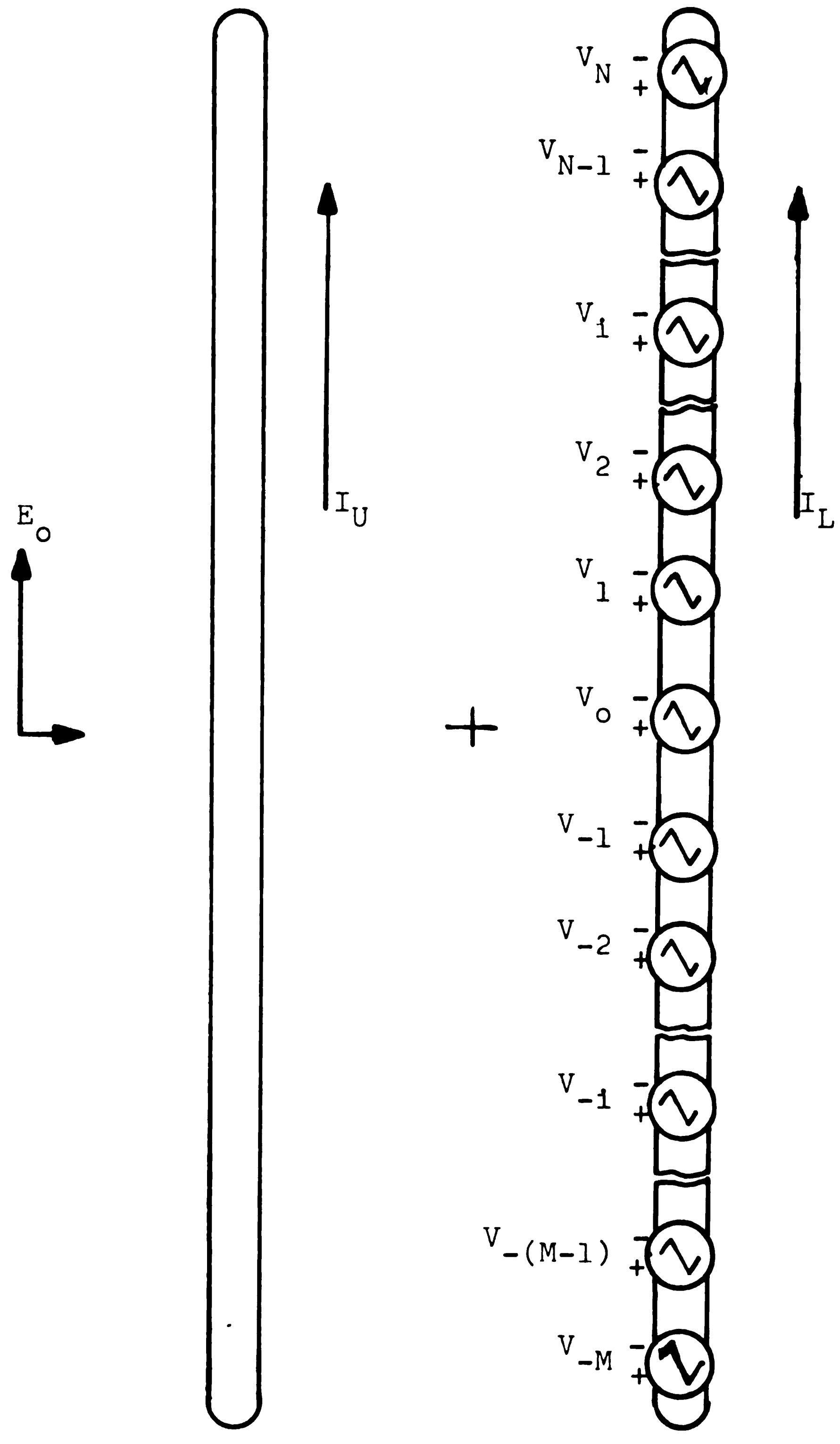


Figure 6.2 Voltage Equivalent Of A Multi-Loaded Receiving Antenna

Therefore the total current at any point is

$$I_t(z) = E_o f_u'(z) + \sum_{i=-M}^{N} V_i f(\omega, a, h, d_i, z). \tag{6.3}$$

Again to find the V_i's, the relation $Y_i V_i = -I_t(d_i)$ is used. Thus

$$E_o f_u'(d_j) + (Y_j + f^d_{jj})V_j + \sum_{\substack{i=-M \\ i \neq j}}^{N} V_i f^d_{ij} = 0. \tag{6.4}$$

Comparing Equation 6.4 to 2.2 it can be seen that Equation 6.4 differs only by the extra term $E_o f_u'(d_j)$ and that V_o is now an unknown rather than a known value. If j again runs from $-M$ to N, including $j = 0$, the result is $M + N + 1$ equations of the form of Equation 6.4. Writing these in matrix form,

$$F_R V_R + G_R E_o = 0 \tag{6.5}$$

where

$$F_R = \begin{bmatrix} f^d_{11} + Y_1 & \cdots & f^d_{N1} & f^d_{01} & f^d_{-11} & \cdots & f^d_{-M1} \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ f^d_{1N} & \cdots & f^d_{NN} + Y_N & f^d_{0N} & f^d_{-1N} & \cdots & f^d_{-MN} \\ f^d_{10} & \cdots & f^d_{N0} & f^d_{00}+Y_0 & f^d_{-10} & \cdots & f^d_{-M0} \\ f^d_{1-1} & \cdots & f^d_{N-1} & f^d_{0-1} & f^d_{-1-1}+Y_{-1} & \cdots & f^d_{-M-1} \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ f^d_{1-M} & \cdots & f^d_{N-M} & f^d_{0-M} & f^d_{-1-M} & \cdots & f^d_{-M-M}+Y_{-M} \end{bmatrix}$$

$$G_R = \begin{bmatrix} f_u'(d_1) \\ \cdot \\ \cdot \\ \cdot \\ f_u'(d_N) \\ f_u'(0) \\ f_u'(d_{-1}) \\ \cdot \\ \cdot \\ \cdot \\ f_u'(d_{-M}) \end{bmatrix} \quad \text{and } V_R = \begin{bmatrix} V_1 \\ \cdot \\ \cdot \\ \cdot \\ V_N \\ V_o \\ V_{-1} \\ \cdot \\ \cdot \\ \cdot \\ V_{-M} \end{bmatrix} .$$

Since F_R and V_R differ from F and V by the addition of the f_{10}^d's, V_o and Y_o, the number of complex unknowns for the receiving antenna is two greater than those for the transmitting antenna and the number of complex equations is one greater. Thus one additional complex constraint equation will be needed. Otherwise the analysis of required constraints follows that of the transmitting antenna.

6.2 Constraint Equations

Since the transmitting case requires N + M complex constraints, the receiving antenna needs N + M + 1 complex constraints. Also since the field pattern of the unloaded rod illuminated by an electric field can be shown to have the form,

$$P_u(\Theta) = E_o g_u(\omega,a,h,\Theta), \tag{6.6}$$

all the constraints outlined for a transmitting antenna can be applied to a receiving antenna.

6.2.1. Load Impedance Constraint

Since for most practical applications, it is desirable to specify the central load impedance, one constraint equation could be Y_o equals a constant. This constraint alters one of the equations in matrix 6.5,

$$(f^d_{00} + Y_o)V_o + \sum_{\substack{i=-M \\ i\neq 0}}^{N} f^d_{i0}V_i + E_o f'_u(0) = 0 \qquad (6.7)$$

where Y_o is now a known value, and reduces the number of unknowns by one. Thus there are 2M + 2N + 1 unknowns and N + M + 1 equations which reduces the number of additional constraint equations to M + N.

6.2.2. Scattered Field Constraint

It is desirable to be able to control the scattered field of a receiving antenna. Thus a possible constraint would be to specify the scattered field in M + N directions. Using Equation 6.6 and the first equation in Section 2.2.4.1., the equation for one direction would be

$$P_T(\Theta_j) = E_o g_u(\omega,a,h,\Theta_j) + \sum_{i=-M}^{N} V_i g_i(\omega,a,h,\Theta_j). \qquad (6.8)$$

Applying Equation 6.8 in M + N directions results in N + M equations. But there are M + N + 1 unknowns. The remaining equation can be generated by specifying one more direction, or it can be Equation 6.7. If one more direction is used the resulting equations are like those in Section 2.2.4.1. and the solution follows that outlined there.

If Equation 6.7 is used, the resulting set of equations in matrix form is

$$RV_R'' = -SE_o \tag{6.9}$$

where

$$V_R'' = \begin{bmatrix} V_{-M} \\ \vdots \\ V_o \\ \vdots \\ V_N \end{bmatrix}, \qquad S = \begin{bmatrix} g_u'(\theta_1) - P_T(\theta_1)/E_o \\ \vdots \\ g_u'(\theta_j) - P_T(\theta_j)/E_o \\ \vdots \\ g_u'(\theta_{M+N}) - P_T(\theta_{M+N})/E_o \\ f_u'(0) \end{bmatrix}$$

and

$$R = \begin{bmatrix} g_{-M}'(\theta_1) & \cdots & g_o'(\theta_1) & \cdots & g_N'(\theta_1) \\ \vdots & & \vdots & & \vdots \\ g_{-M}'(\theta_j) & & g_o'(\theta_j) & & g_N'(\theta_j) \\ \vdots & & \vdots & & \vdots \\ g_{-M}'(\theta_{M+N}) & & g_o'(\theta_{M+N}) & & g_N'(\theta_{M+N}) \\ f_{-M0}^d & & f_{00}^d + Y_o & & f_{N0}^d \end{bmatrix}$$

where $g'(\theta_j) = g(\omega,a,h,\theta_j)$. Multiplying Equation 6.9 by R^{-1} gives the solutions for all the V_i's.

Since one of the equations in matrix 6.5 has been used, care must be taken to find the Y_i's. If three new matrices Y, F_R', and G_R' are defined as

$$F_R' = \begin{bmatrix} f^d_{11} & \cdots & f^d_{N1} & f^d_{01} & f^d_{-11} & \cdots & f^d_{-M1} \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ f^d_{1N} & \cdots & f^d_{NN} & f^d_{0N} & f^d_{-1N} & \cdots & f^d_{-MN} \\ f^d_{1-1} & \cdots & f^d_{N-1} & f^d_{0-1} & f^d_{-1-1} & \cdots & f^d_{-M-1} \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ f^d_{1-M} & \cdots & f^d_{N-M} & f^d_{0-M} & f^d_{-1-M} & \cdots & f^d_{-M-M} \end{bmatrix}$$

$$Y = \begin{bmatrix} Y_1 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & Y_N & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & Y_{-1} & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & \cdots & Y_{-M} \end{bmatrix}$$

and

$$G_R' = \begin{bmatrix} f_u'(d_1) \\ \vdots \\ f_u'(d_N) \\ f_u'(d_{-1}) \\ \vdots \\ f_u'(d_{-M}) \end{bmatrix}$$

then matrix 6.5 less one row (Equation 6.7) may be written as

$$(F_R' + Y)V_R + G_R'E_o = 0 \; . \tag{6.10}$$

It should be noted that F_R' and Y are not square matrices but are M + N by M + N + 1 matrices. Thus they do not have inverses. To find the Y_i's the same procedure as outlined in Section 2.2.4.1. can be used, namely:

$$YV_R = -(G_R'E_o + F_R'V_R) \tag{6.11}$$

or

$$Y_i = -(G_R'E_o + \sum_{\substack{j=-M \\ j\neq 0}}^{N} F_{R_{ij}}' V_{R_j})/V_{R_i} \; . \tag{6.12}$$

6.2.3. Current Constraint

Since on a receiving antenna the current at the load is of primary concern, it would be desirable to specify the current at d = 0. This condition, however, only supplies one constraint equation and M + N + 1 are needed. Additional current points could be specified as was done in Section 2.2.4.2., but a more interesting case is to specify I(0) at several different frequencies.

From Equation 6.7 and $I(0) = -Y_oV_o$

$$f_{00}^zV_o + \sum_{\substack{i=-M \\ i\neq 0}}^{N} V_i f_{i0}^z + E_o f_u'(0) = I(0) \, . \tag{6.13}$$

But Equation 6.13 is not in a useable form. If $C_E(\omega)$ is defined as

$$C_E(\omega_1) = I_{\omega_1}(0)/E_o \tag{6.14}$$

then Equation 6.13 becomes

$$\sum_{\substack{i=-M \\ i\neq 0}}^{N} V_i f^z_{i0} + E_o[f'_u(0) - C_E(1 + f^z_{00}/Y_o)] = 0. \tag{6.15}$$

Applying $I(0) = -Y_o V_o$ and Equation 6.14 to Equation 6.10 and combining the results with Equation 6.15 gives

$$B_R \cdot V'_R = 0 \tag{6.16}$$

where

$$B_R = \begin{bmatrix}
f^d_{11}+Y_1 & \cdots & f^d_{N1} & f^d_{-11} & \cdots & f^d_{-M1} & (f'_u(d_1) - C_E f^d_{01}/Y_o) \\
\vdots & & \vdots & \vdots & & \vdots & \vdots \\
f^d_{1N} & \cdots & f^d_{NN}+Y_N & f^d_{-1N} & \cdots & f^d_{-MN} & (f'_u(d_N) - C_E f^d_{0N}/Y_o) \\
f^d_{1-1} & \cdots & f^d_{N-1} & f^d_{-1-1}+Y_{-1} & \cdots & f^d_{-M-1} & (f'_u(d_{-1}) - C_E f^d_{0-1}/Y_o) \\
\vdots & & \vdots & \vdots & & \vdots & \vdots \\
f^d_{1-M} & \cdots & f^d_{N-M} & f^d_{-1-M} & \cdots & f^d_{-M-M}+Y_{-M} & (f'_u(d_{-N}) - C_E f^d_{0-M}/Y_o) \\
f^z_{10} & \cdots & f^z_{N0} & f^z_{-10} & \cdots & f^z_{-M0} & (f'_u(0) - C_E(1 + f^z_{00}/Y_o)
\end{bmatrix}$$

and

$$V_R' = \begin{bmatrix} V_1 \\ \cdot \\ \cdot \\ \cdot \\ V_N \\ V_{-1} \\ \cdot \\ \cdot \\ \cdot \\ V_{-M} \\ E_o \end{bmatrix} .$$

Like Equation 2.27, Equation 6.16 has a solution only if the determinant of B_R is zero. This produces one equation with N + M + 1 unknowns. Since, however, Y_o is usually known, there are only N + M unknowns in Equation 6.16. Thus if I(0) is specified at N + M frequencies, the required N + M equations will be generated and all the admittances can be found.

6.2.4. Current And Scattered Field Constraint

The final type of constraint to be considered will be a combination of the current and scattered field constraints. Again the current constraint will be to specify the current at d = 0. If the scattered field is specified in L directions, then the resulting constraint equations in matrix form are:

$$C_{R_i} V_{R_i}' = 0 \qquad (6.17)$$

where

$$C_{R_i} = \left[\begin{array}{llllll} g_1'(\theta_1) & \cdots & g_k'(\theta_1) & g_i(\theta_1) & g_{k+2}(\theta_1) & \cdots \\ \vdots & & \vdots & \vdots & \vdots & \\ g_1'(\theta_L) & & g_k'(\theta_L) & g_i(\theta_L) & g_{k+2}(\theta_L) & \cdots \\ f^z_{10} & & f^z_{k0} & f^z_{i0} & f^z_{k+20} & \cdots \end{array}\right.$$

$$\begin{array}{lllll} \cdots & g_{i-1}(\theta_1) & g_{k+1}(\theta_1) & g_{i+1}(\theta_1) & \cdots \\ & \vdots & \vdots & \vdots & \\ \cdots & g_{i-1}(\theta_L) & g_{k+1}(\theta_L) & g_{i+1}(\theta_L) & \cdots \\ \cdots & f^d_{i-10} & f^d_{k+10} & f^d_{i+10} & \cdots \end{array}$$

$$\left.\begin{array}{lll} \cdots & g_{-M}(\theta_1) & (g_u(\theta_1) - (P_T(\theta_1)/E_o + C_E g_o(\theta_1)/Y_o) \\ & \vdots & \qquad\qquad\vdots \\ \cdots & g_{-M}(\theta_L) & (g_u(\theta_L) - (P_T(\theta_L)/E_o + C_E g_o(\theta_L)/Y_o) \\ \cdots & f^d_{-M0} & (f_u'(0) - C_E(1 + f^z_{00}/Y_o) \end{array}\right]$$

and

$$V'_{R_i} = \begin{bmatrix} V'_1 \\ \cdot \\ \cdot \\ \cdot \\ V_k \\ V_i \\ V_{k+2} \\ \cdot \\ \cdot \\ \cdot \\ V_{i-1} \\ V_{k+1} \\ V_{i+1} \\ \cdot \\ \cdot \\ \cdot \\ V_{-M} \\ E_o \end{bmatrix}.$$

Following the same method used to obtain Equation 2.30, define the matrix B_i as

$$B_i = \begin{bmatrix} f^d_{11}+Y_1 & \cdots & f^d_{k1} & f^d_{i1} & f^d_{k+21} & \cdots & f^d_{i-11} & f^d_{k+11} & f^d_{i+11} & \cdots & f^d_{-M1} & (f_u'(d_1) - C_E f^d_{01}/Y_o) \\ \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots \\ f^d_{1k} & & f^d_{kk}+Y_k & f^d_{ik} & f^d_{k+2k} & \cdots & f^d_{i-1k} & f^d_{k+1k} & f^d_{i+1k} & & f^d_{-Mk} & (f_u'(d_k) - C_E f^d_{0k}/Y_o) \\ f^d_{1i} & & f^d_{ki} & f^d_{ii}+Y_i & f^d_{k+2i} & \cdots & f^d_{i-1i} & f^d_{k+1i} & f^d_{i+1i} & & f^d_{-Mi} & (f_u'(d_i) - C_E f^d_{0i}/Y_o) \end{bmatrix}$$

where

$$k = -(M - L - 1) \qquad \text{for } M > L + 1$$

$$k = N + M - (L + 1) \qquad \text{for } L + 1 \geq M$$

and $i = k + 1, \ldots, -M$. B_i is a $M + N - L$ by $M + N + 1$ matrix and C_R is a $L + 1$ by $M + N + 1$ matrix. Thus if they are combined, the result is an $M + N + 1$ by $M + N + 1$ matrix

$$\begin{bmatrix} B_i \\ C_{R_i} \end{bmatrix} V'_{R_i} = 0 \qquad (6.18)$$

$$i = k + 1, \ldots, (-M)$$

whose determinant must be zero. Equation 6.18 generates $L + 1$ equations with $M + N$ unknowns. Thus Equation 6.18 must be used for P frequencies such that

$$(L+1)P = M + N \tag{6.19}$$

to obtain the required number of equations needed for a solution.

CHAPTER VII

CURRENT DISTRIBUTION AND SCATTERED FIELD PATTERN OF AN UNLOADED ROD

In Chapter 6 it was shown that the current and scattered field of an unloaded rod was needed to solve for the current and scattered field of a multi-loaded receiving antenna. In this chapter the induced current for both a short and long unloaded rod will be derived. The derivation will be for a normally incident wave. The derivation of other than normal incidence can be found in papers by King (9) and Yu and Shen (10).

7.1 Vector Potential Of An Unloaded Rod

For an unloaded rod of length 2h and radius a illuminated by a normally incident electric field, E_o, as shown in Figure 7.1, the vector potential is given by

$$\frac{d^2A_z}{dz^2} + \beta_o^2 A_z = -\frac{j\beta_o^2}{\omega} E_o \tag{7.1}$$

the solution of which is

$$A_z = -j\sqrt{\varepsilon_o\mu_o}\,[C_1 \cos k_o z + E_o/k_o] \quad -h \le z \le h. \tag{7.2}$$

Equation 7.2 is valid for both short and long antennas.

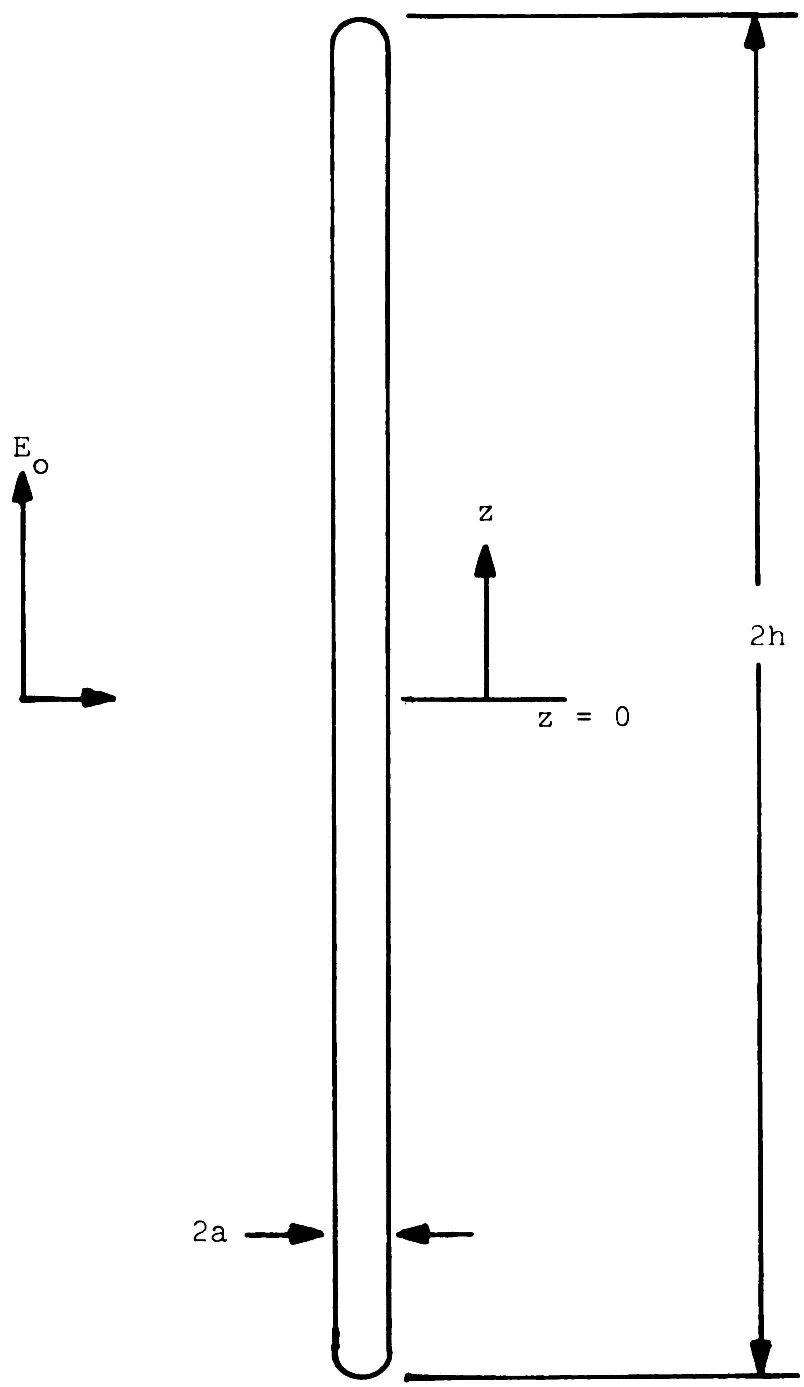


Figure 7.1 Unloaded Rod

7.2 Current Distribution On A Short Rod

For the short rod the procedure outlined in Section 3.1 will be used. From Equations 3.7 and 7.2, it follows that

$$\int_{-h}^{h} I_{rod}(z')K_d(z,z')dz' = -\frac{j4\pi}{\zeta_o}[C_1 \cos k_o z + \frac{E_o}{k_o} + U] \qquad (7.3)$$

where U is defined by Equation 3.10. Since at z = h the right hand side of Equation 7.3 is zero

$$C_1 = -[U + E_o/k_o]/\cos k_o h \qquad (7.4)$$

so that Equation 7.3 becomes

$$\int_{-h}^{h} I_{rod}(z')K_d(z,z')dz' = \frac{j4\pi}{\zeta_o \cos k_o h}[\cos k_o z - \cos k_o h]\cdot (U + E_o/k_o). \qquad (7.5)$$

Equations 7.5, 3.14a and 3.14b suggest that the approximate form of the current should be

$$I_{rod}(z) = I_{EI}(\cos k_o z - \cos k_o h) + I_{ER}(\cos k_o z/2 - \cos k_o h/2) \qquad (7.6)$$

Substituting Equation 7.6 into Equation 7.5 and using Equations 3.17c-e, the result is

$$I_{EI}[\psi_{dUR}(\cos k_o z - \cos k_o h) + j\psi_{dUI}(\cos k_o z/2 - \cos k_o h/2)] + I_{ER}\psi_{dD}(\cos k_o z/2 - \cos k_o h/2) = \frac{j4\pi}{\cos k_o h}[\cos k_o z - \cos k_o h]\cdot (U + E_1/k_o). \qquad (7.7)$$

Since Equation 7.7 is true for all z, then

$$j\psi_{dUI}I_{EI} + I_{ER}\psi_{dD} = 0 \qquad (7.8)$$

and

$$I_{EI}\psi_{dUR} = \frac{j4\pi}{\zeta_o \cos k_o h}(U + E_o/k_o). \tag{7.9}$$

All that remains is to find U. Substituting Equation 7.6 into Equation 3.10 and using Equations 3.25 b-c results in

$$U = -\frac{j\zeta_o}{4\pi}(I_{EI}\psi_u(h) + I_{ER}\psi_D(h)) . \tag{7.10}$$

Combining Equations 7.10 and 7.9 and using Equation 7.8, I_E, and I_{ER} can be found. Thus

$$I_{EI} = \frac{j\psi_{dD}}{\psi_{dD}(\psi_{dUR}\cos k_o h - \psi_u(h)) + j\psi_D(h)\psi_{dUI}}\,\frac{4\pi E_o}{\zeta_o k_o} \tag{7.11}$$

and

$$I_{ER} = \frac{\psi_{dUI}}{\psi_{dD}(\psi_{dUR}\cos k_o h - \psi_u(h)) + j\psi_D(h)\psi_{dUI}}\,\frac{4\pi E_o}{\zeta_o k_o}. \tag{7.12}$$

If new variables, T_{EI} and T_{ER}, are defined as

$$I_{EI} = T_{EI}\,\frac{4\pi}{\zeta_o}\,\frac{E_o}{k_o} \tag{7.13}$$

$$I_{ER} = T_{ER}\,\frac{4\pi}{\zeta_o}\,\frac{E_o}{k_o} \tag{7.14}$$

then Equation 7.6 becomes

$$I_{rod}(z) = \frac{4\pi}{\zeta_o}\frac{E_o}{k_o}[T_{EI}(\cos k_o z - \cos k_o h) + T_{ER}(\cos k_o z/2 - \cos k_o h/2)]. \tag{7.15}$$

7.3 Scattered Field Of A Short Unloaded Rod

Once $I_{rod}(z)$ has been found, the scattered field for a short rod can easily be found. Since $I_{rod}(z)$ has the very same form as the current of a transmitting antenna, with the use of Equations 4.1, 4.9 and 4.10 it can be shown that

$$P_{rod}(\theta) = -\frac{jE_o \sin\theta}{k_o \cos\theta}\Big[\frac{T_{EI}}{\sin^2\theta}[\cos\theta \sin k_o h \cos(k_o h \cos\theta) - \cos k_o h \sin(k_o h \cos\theta)] + \frac{T_{ER}}{(1-4\cos^2\theta)} \cdot [2\cos\theta \sin k_o h/2 \cos(k_o h \cos\theta) - \cos k_o h/2 \sin(k_o h \cos\theta)]. \tag{7.16}$$

Like Equation 4.12, Equation 7.16 is indeterminate if $\theta = 0$, $\pi/2$, $+ \pi/3$, or $2\pi/3$. For $\theta = 0$ it can again be shown that $P_{rod}(0) = 0$. If $\theta = \pi/2$, then Equation 4.17 is used and Equation 7.16 becomes

$$P_{rod}(\pi/2) = -\frac{jE_o}{k_o}[T_{EI}(\sin k_o h - k_o h \cos k_o h) + T_{ER}(2 \sin k_o h/2 - k_o h \cos k_o h/2)]. \tag{7.17}$$

For $\theta' = \pi/3$ or $2\pi/3$ only the T_{ER} term is indeterminate. It has been shown that it reduces to $k_o h/2 - \frac{\sin k_o h}{2}$.

Thus

$$P_{rod}(\theta') = -\frac{jE_o}{k_o\cos\theta'}\left[\frac{T_{EI}}{\sin\theta'}(\cos\theta'\sin k_oh\cos(k_oh\cos\theta')\right.$$

$$-\cos k_oh\sin(k_oh\cos\theta')) +$$

$$\left.+T_{ER}(k_oh/2-\frac{\sin k_oh}{2})\sin\theta'\right]. \qquad (7.18)$$

7.4 Current Distribution On A Long Rod

The approximate solution for the long rod will be based on a paper by Shen (11). Unlike Shen, only the normally incident wave will be considered.

7.4.1 Current On An Infinitely Long Rod

Applying Equation 7.1, altered for $e^{-j\omega t}$ time dependence, to an infinitely long rod, the equation's solution is

$$A_z(z) = j\frac{E_o}{\omega} \qquad \text{for all } z\ . \qquad (7.19)$$

Since

$$A_z(z) = \frac{\mu_o}{4\pi}\int_{-\infty}^{\infty}dz'I_\infty(z')K(z-z') \qquad (7.20)$$

where $K(z-z')$ is given by Equation 3.41a,

$$\frac{4\pi jE_o}{\zeta_o k_o} = \int_{-\infty}^{\infty}dz'I\infty_{rod}(z')K(z-z'). \qquad (7.21)$$

Applying the Fourier transform to Equation 7.21 and using Equation 3.46, the result is

$$\bar{I}\infty_{rod}(\zeta)\bar{K}(\zeta) = \frac{4\pi jE_o}{\zeta_o k_o}2\pi\delta(\zeta). \qquad (7.22)$$

Solving for $\bar{I}(\zeta)$ and taking the inverse transform,

$$I_{\infty rod}(z) = \frac{4\pi}{\zeta_o} \frac{jE_o}{k_o\ \bar{K}(0)} . \qquad (7.23)$$

But from Equation 3.50, $\bar{K}(0) = 2\Omega_o + j\pi$ thus

$$I_{\infty rod}(z) = \frac{4\pi}{\zeta_o} \frac{jE_o}{k_o} \frac{1}{(2\Omega_o + j\pi)} . \qquad (7.24)$$

7.4.2. Reflection At The End Of A Semi-Infinite Rod

If the semi-infinite rod is in the upper half plane with its end at z = 0, and if it is illuminated by a normal $\bar{E}$ field such that $I_{rod}(z) = 1$, the reflected current can be expressed as

$$i_r(z) = \frac{k_o}{2\pi j} \bar{L}_-(0) \int_{C_o} \frac{e^{j\zeta z}dz}{(k_o-\zeta)\zeta\bar{L}_-(\zeta)} . \qquad (7.25)$$

If again it is assumed that the contribution to the integral in Equation 7.25 comes mainly from $\zeta = k_o$ and that C_o can be deformed around the branch cut at $z = \zeta k_o$, Equation 7.25 becomes

$$i_r(z) = \frac{k_o}{2\pi j} \frac{\bar{L}_-(0)}{\bar{L}_+(k_o)} \int_{C_2} \frac{e^{j\zeta z}dz}{(k_o-\zeta)\zeta\bar{K}(\zeta)} \qquad (7.26)$$

which is approximately

$$i_r(z) = -R_S\ I_\infty(z)/V_o \qquad (7.27)$$

where

$$R_S = \frac{\zeta_o}{2\pi} \frac{\bar{L}_-(0)}{\bar{L}_+(k_o)} .$$

For large $k_o z$,

$$\frac{\bar{L}_-(0)}{\bar{L}_+(k_o)} = 2C_\omega + \ln 2 + j\pi \ . \qquad (7.28)$$

7.4.3. Current On A Finite Rod

To find the current on a finite rod the results of the last two sections and Section 3.2.2. must be used. Away from the end, the finite rod will seem like an infinitely long rod and this should support a current like $I_{\infty rod}(z)$. Near the ends $I_{\infty rod}(z)$ will produce a current like $I_\infty(z)/V_o$ from both Equations 7.27 and 3.56. Thus current on a finite rod should be of the form

$$I_{rec}(z) = I_{\infty rod}(z) + C_{R1}\frac{I_\infty(z+h)}{V_o} + C_{R2}\frac{I_\infty(z-h)}{V_o}. \qquad (7.29)$$

To find C_{R1} and C_{R2}, Equations 7.27 and 3.56 must be used. At $z = -h$, the incident current would be $I_{\infty rod}(-h) + C_{R2}\frac{I_\infty(2h)}{V_o}$ which would produce a reflected current at $z = -h$ of

$$I_{refl} = -R_S I_{\infty rod}(-h)\frac{I_\infty(0)}{V_o} - RC_{R2}\frac{I_\infty(2h)}{V_o}\frac{I_\infty(0)}{V_o}. \qquad (7.30)$$

But by definition $I_{ref} = C_{R1}\frac{I_\infty(0)}{V_o}$, thus

$$C_{R1} = -R_S I_{\infty rod}(-h) - RC_{R2}\frac{I_\infty(2h)}{V_o}. \qquad (7.31)$$

Similarly at $z = h$, $C_{R2} = -R_S\, I_{\infty rod}(h) - RC_{R1}\frac{I_\infty(2h)}{V_o}$. (7.32)

Solving Equations 7.31 and 7.32 for C_{R1} and C_{R2} yields:

$$C_{R1} = - \frac{R_S I\infty_{rod}(h)}{1 + RI_\infty(2h)/V_o} = C_{R2}. \tag{7.33}$$

Simplifying Equation 7.29, the current distribution for a finite long rod becomes

$$I_{rec}(z) = I\infty_{rod}(z)[1 - \frac{R_S}{1 + RI_\infty(2h)/V_o}(\frac{I\infty(z+h)}{V_o} + \frac{I\infty(z-h)}{V_o})] \tag{7.34}$$

since $I\infty_{rod}(z)$ is a constant.

7.5. Scattered Field Of A Long Rod

To find the scattered field of a long rod, the same type of method as used in Section 4.2 will be employed. From Equation 7.2 the vector potential, corrected for the $e^{-j\omega t}$ time factor, is

$$A_z = j\sqrt{\mu_o \varepsilon_o}[C_1 \cos k_o z + E_o/k_o] \quad - k \leq z \leq h$$

and from Equation 4.23

$$P_{rec}(\theta) = j\omega \frac{\bar{A}(k_o \cos\theta)}{\bar{\bar{K}}(k_o \cos\theta)} \sin\theta .$$

If again it is assumed that the main contribution to the integral is the vector potential on the antenna's surface, then

$$\bar{A}(k_o \cos\theta) \cong \int_{-h}^{h} j\sqrt{\mu_o \varepsilon_o}\, [C_1 \cos k_o z + E_o/k_o] e^{-jzk_o\cos\theta} dz \tag{7.35}$$

which using Equation 4.5 is

$$\bar{A}(k\cos\theta) = \frac{jC_1\sqrt{\mu_o\varepsilon_o}}{k_o}\left(\frac{\sin k_oh(1+\cos\theta)}{1+\cos\theta} + \frac{\sin k_oh(1-\cos\theta)}{1-\cos\theta}\right)$$

$$+ \frac{j2E_o\sqrt{\mu_o\varepsilon_o}}{k_o^{\,2}}\ \frac{\sin(k_oh\cos\theta)}{\cos\theta}\,. \qquad (7.36)$$

All that remains is to find C_1. This can be done by again using Equation 4.27. If $A_z(z) = 0$ for z outside of the antenna, the functions defined by Equations 4.28 and 4.29 and a third function

$$V'(z') = \int_o d\zeta\bar{L}_+(\zeta)\int_o^\infty dze^{-j\zeta z}(H(z'-z)-e^{-jk_oz}) \qquad (7.37)$$

are needed.

Rewriting $A_z(z)$ so that one end is at $z = 0$, $A_z(z)$ becomes

$$A_z(z) = j\sqrt{\mu_o\varepsilon_o}[C_1(\cos k_oh\cos k_oz + \sin k_oh\sin k_oz)$$

$$+ E_o/k_o]H(2h - z). \qquad (7.38)$$

Using Equations 4.28, 4.29, 4.27, and 7.37, the result is

$$C_1(\cos k_oh\,T'(2h) + \sin k_oh\,S'(2h) + E_o/k_o\,V'(2h) = 0 \qquad (7.39)$$

or

$$C_1 = -(E_o/k_o)2j\,V'(2h)/(y_1 + y_2(2h)e^{j2k_oh} + y_3(2h))e^{-jk_oh}$$

using Equations 4.33 and 4.34. Only $V'(2h)$ needs to be evaluated. It can be shown that $V'(z)$ is equivalent to $P_1(\theta_1 = \Pi/2)$ as defined by Yu and Shen (10).

Thus

$$V'(2h) = -2j\ y_3(2h) + 2\pi/(2C_\omega + \ell n2 + j\pi) \tag{7.40}$$

and

$$C_1 = +E_o/k_o 2e^{jk_o h}(-2j\ y_3(2h) + 2\pi/(2C_\omega + \ell n2 + j\pi))/\Delta \tag{7.41}$$

where Δ is defined by Equation 4.40. Finally combining Equations 7.36, 4.23 and 4.24a

$$P_{rec}(\Theta) = -(E_o/k_o)\frac{\sin\Theta}{(\Omega_1 - \ell n(\sin\Theta))}\left[\frac{C_1\ k_o}{2\ E_o}\left(\frac{\sin k_o h(1+\cos\Theta)}{1 + \cos\Theta} + \frac{\sin k_o h(1 - \cos\Theta)}{1 - \cos\Theta}\right) + \frac{\sin(k_o h\ \cos\Theta)}{\cos\Theta}\right]. \tag{7.42}$$

Equation 7.42 has two indeterminate points $\Theta = 0, \pi/2$. At $\Theta = 0$ it can be shown that $P_{rec}(\Theta) = 0$ as it should be. At $\Theta = \pi/2$ the last term approaches $k_o h$ and Equation 7.42 becomes

$$P_{rec}(\pi/2) = -(E_o/k_o\Omega_1)\left[\frac{C_1\ k_o}{2\ E_o}\ 2\ \sin k_o h + k_o h\right]. \tag{7.43}$$

CHAPTER VIII

ADDITIONAL NUMERICAL EXAMPLES

To this point all the numerical examples have been for transmitting antennas. This chapter will give some examples for symmetrically loaded receiving antennas.

8.1 Resonance Loading

In Section 2.2.2 the conditions for resonance loading on a transmitting antenna was derived. In Section 6.2 it was found that all the constraints of a transmitting antenna can be applied to a receiving antenna with only slight modifications. The modifications for resonance loading are that the second loading impedance is replaced by the central load impedance, Z_o; V_2 is replaced by V_o, V_o is replaced by E_θ, and the zero coefficient instability is dropped.

Figure 8.1 like Figure 5.14 shows the values of the load impedance as a function of frequency for the finite voltage instability. Figure 8.1 shows two things of interest: 1) the real part of central load is positive and 2) it is near 50 Ω, a typical impedance of a recieving antenna. Since for a receiving antenna, V_o is one of the unstable voltages, this instability point could prove to be extremely undesirable.

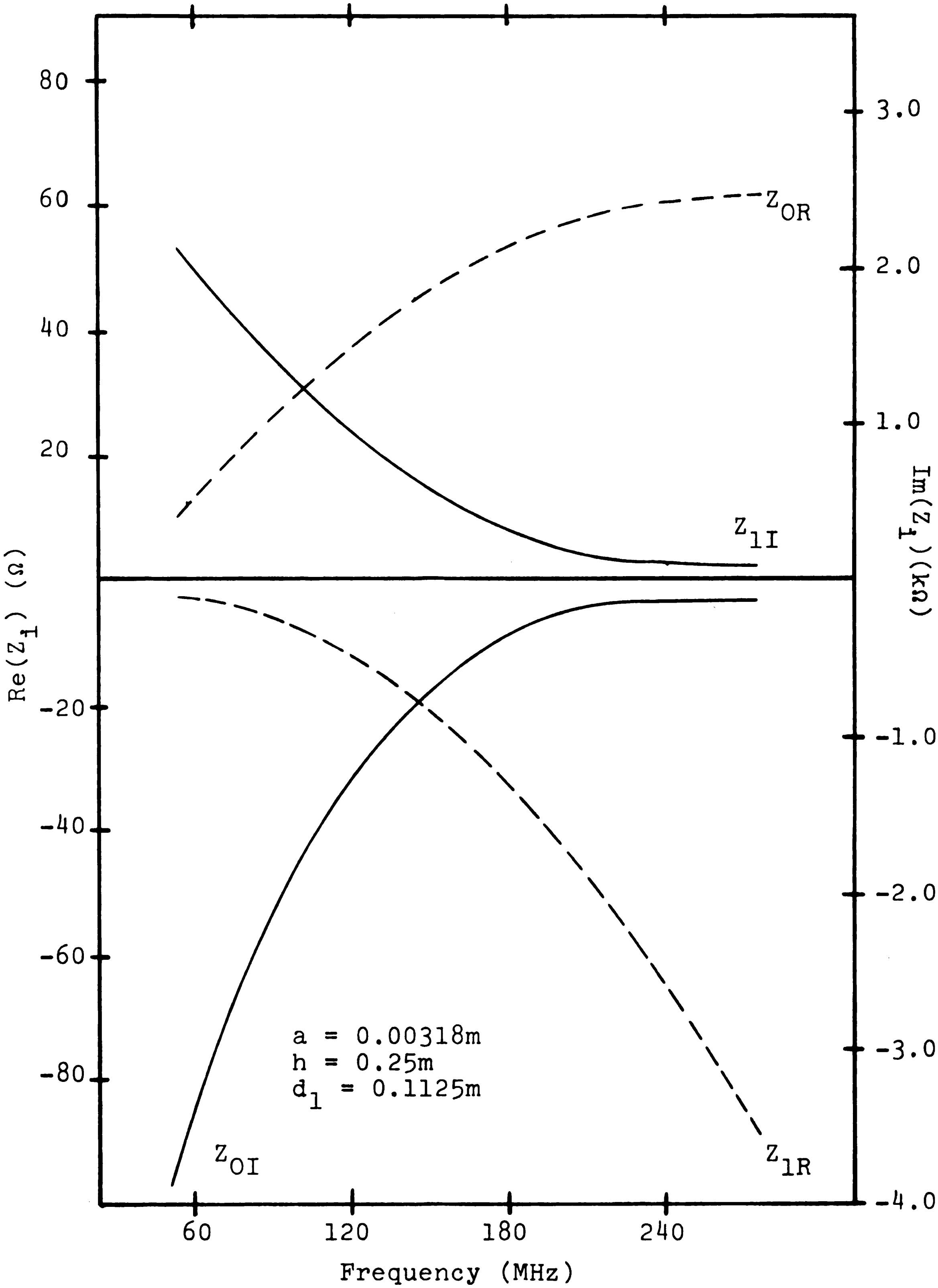


Figure 8.1 Load Impedance vs. Frequency For A Receiving Antenna To Maintain The Resonance Instability.

Figure 8.2 is constructed like Figure 5.16 and gives the other values of impedance required for resonance. Like Figure 5.16, it also has an area of passive loadings. Solving the equation given in Figure 8.2 for $Z_{1R} = 0$, it can be shown that resonance can occur if $Z_{0R} \leq 20\Omega$. Thus with the right reactive load at d_1, the antenna can be taken in and out of resonance by adjusting the central loading.

Unlike the transmitting case, the receiving antenna coefficients are dependent on the direction of the incident radiation. Thus the receiving antenna can also be taken in and out of resonance by varying the direction of the incident radiation. Since resonance enhances the voltage of the central load, this would indicate that an extremely sensitive and narrow beamed receiving antenna could be made.

8.2 Scattered Field Constraint

Figure 8.3 shows the desired (dashed line) and the resulting scattered field (solid line) for a receiving antenna of half length $.7\lambda$. Like the field pattern constraint for a transmitting antenna, Figure 5.4, the field was specified at four points. But unlike the transmitting case, the central impedance was fixed at 50Ω for the receiving antenna. This last constraint removes one of the problems with the transmitting case, uncontrolled input impedance. Thus the receiving scattered field contraint can produce the desired scattered pattern as well

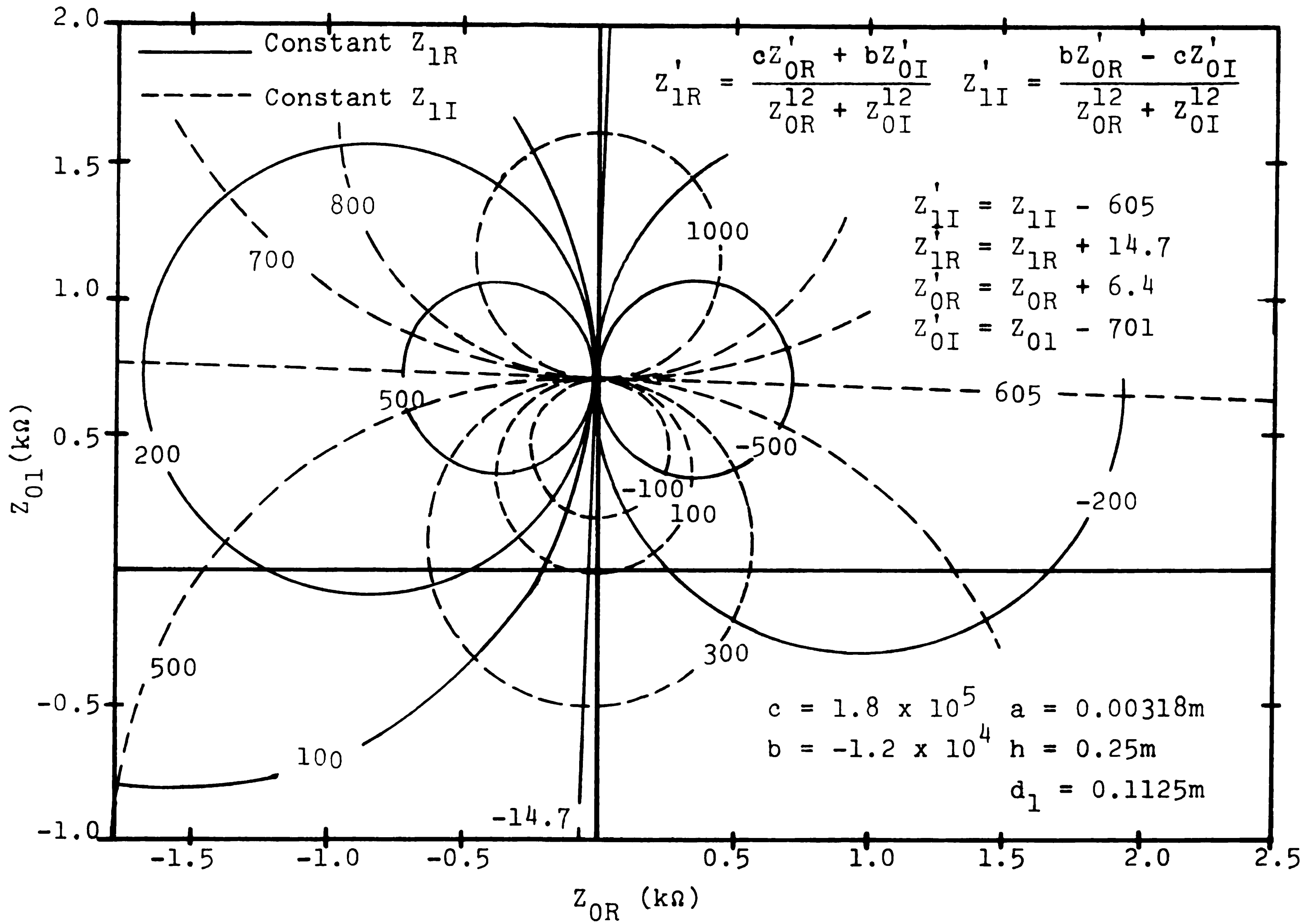
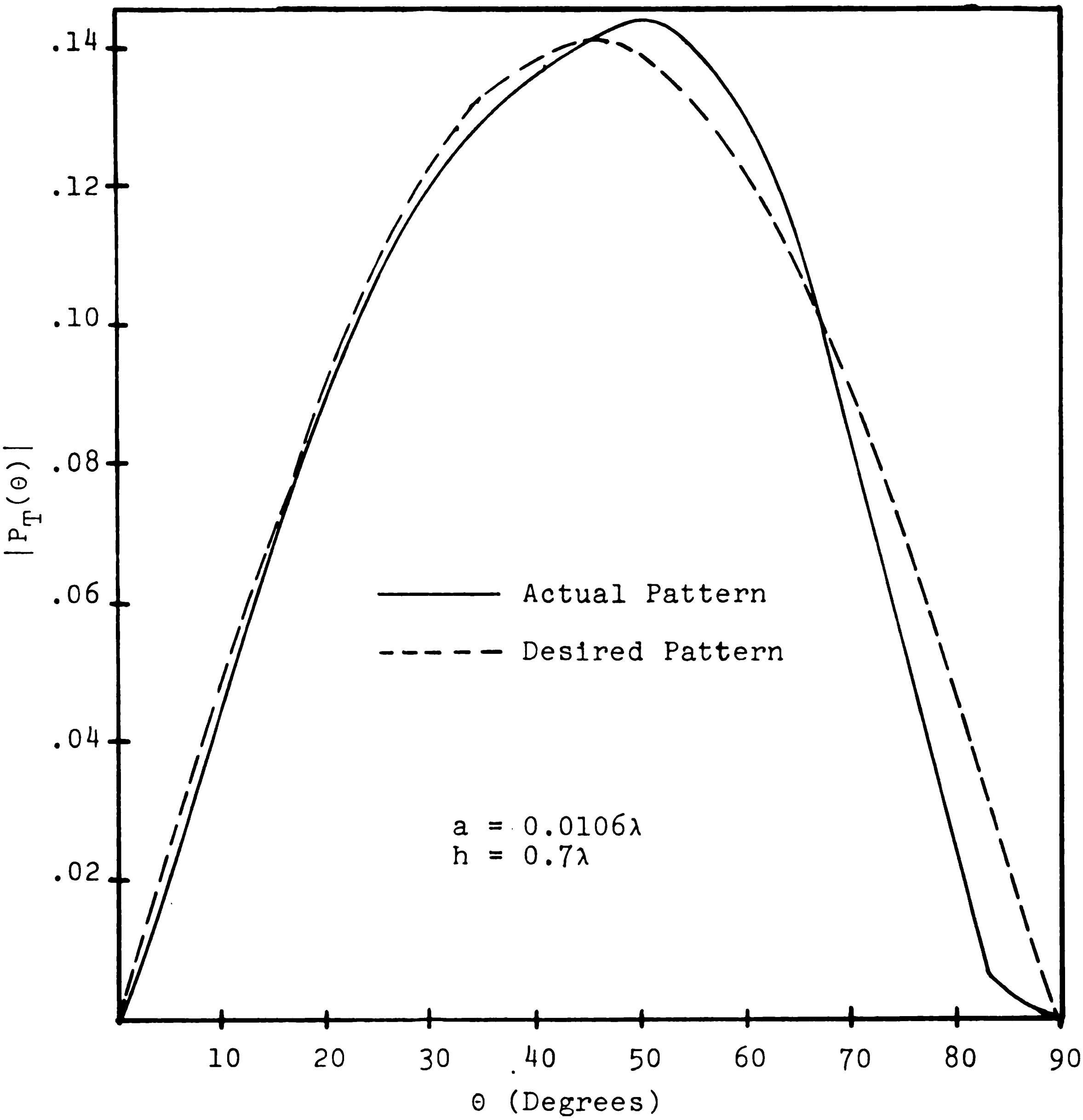


Figure 8.2 Graphical Solution Of Resonance Loading Condition For A Receiving Antenna



$Z_o = 50\Omega$
$Z_1 = -80.8 + j\ 311\Omega$ $\quad d_1 = 0.1\lambda$
$Z_2 = -37.0 + j\ 502\Omega$ $\quad d_2 = 0.29\lambda$
$Z_3 = 16.6 + j\ 628\Omega$ $\quad d_3 = 0.40\lambda$
$Z_4 = 168.9 + j\ 582\Omega$ $\quad d_4 = 0.55\lambda$

Figure 8.3 Field Pattern For A Receiving Antenna Specified At Three Points ($P_T(\Theta) = \sin 2\Theta$).

as maintaining a reasonable central impedance. However, like the transmitting case, it also has negative real loading impedances.

8.3 Current Constraint

Unlike the transmitting current constraint, the receiving current constraint is designed to control only current at the central loads and the value of the central load. Figures 8.4, 8.5, 8.6, and 8.7 show the current distribution for an antenna of half length 2.75 meters at four frequencies: 300, 320, 340 and 360 MHz. The specified currents and the resulting currents are given on the figures. As can be seen, the currents outside the neighborhood of the central load vary widely. Thus good control of the central current and load is obtained by this constraint. Therefore the objective of constructing a frequency rejecting receiving antenna with constant central loading can be achieved by this constraint if active loading is acceptable.

8.4 Current And Scattered Field Constraint

The final example of the versatility of the methods outlined in this thesis will be current and scattered field constraint. Figures 8.8 and 8.9 show the scattered fields and current distributions, respectively, for a receiving antenna for which the backscattered field was specified to be zero at two frequencies, 300 and 400 MHz, and the central current was specified to be 10 ma-m/V at 300 MHz and

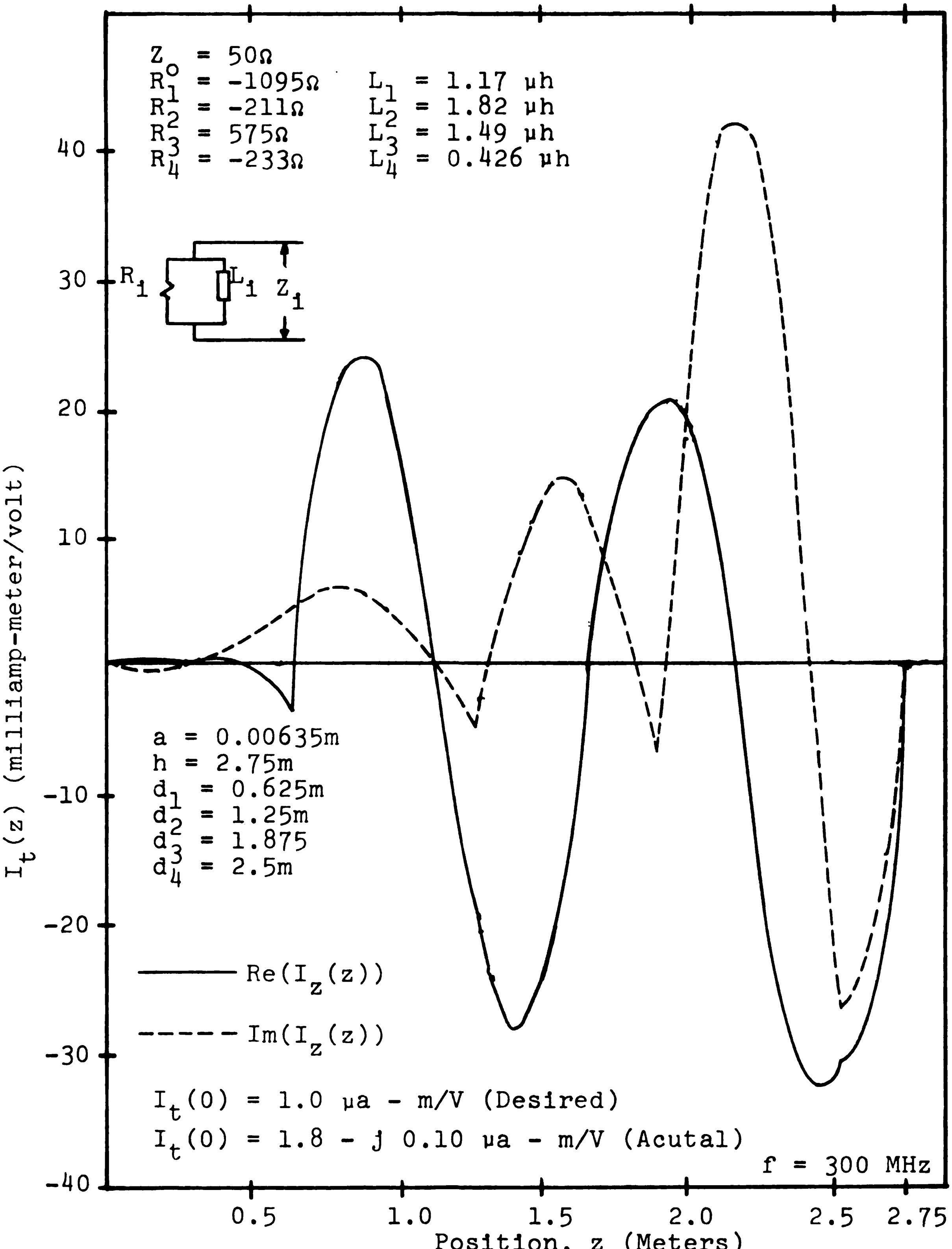


Figure 8.4 Current Distribution For A Receiving Antenna For Which The Central Load And Current Were Specified At Four Frequencies (f = 300 MHz).

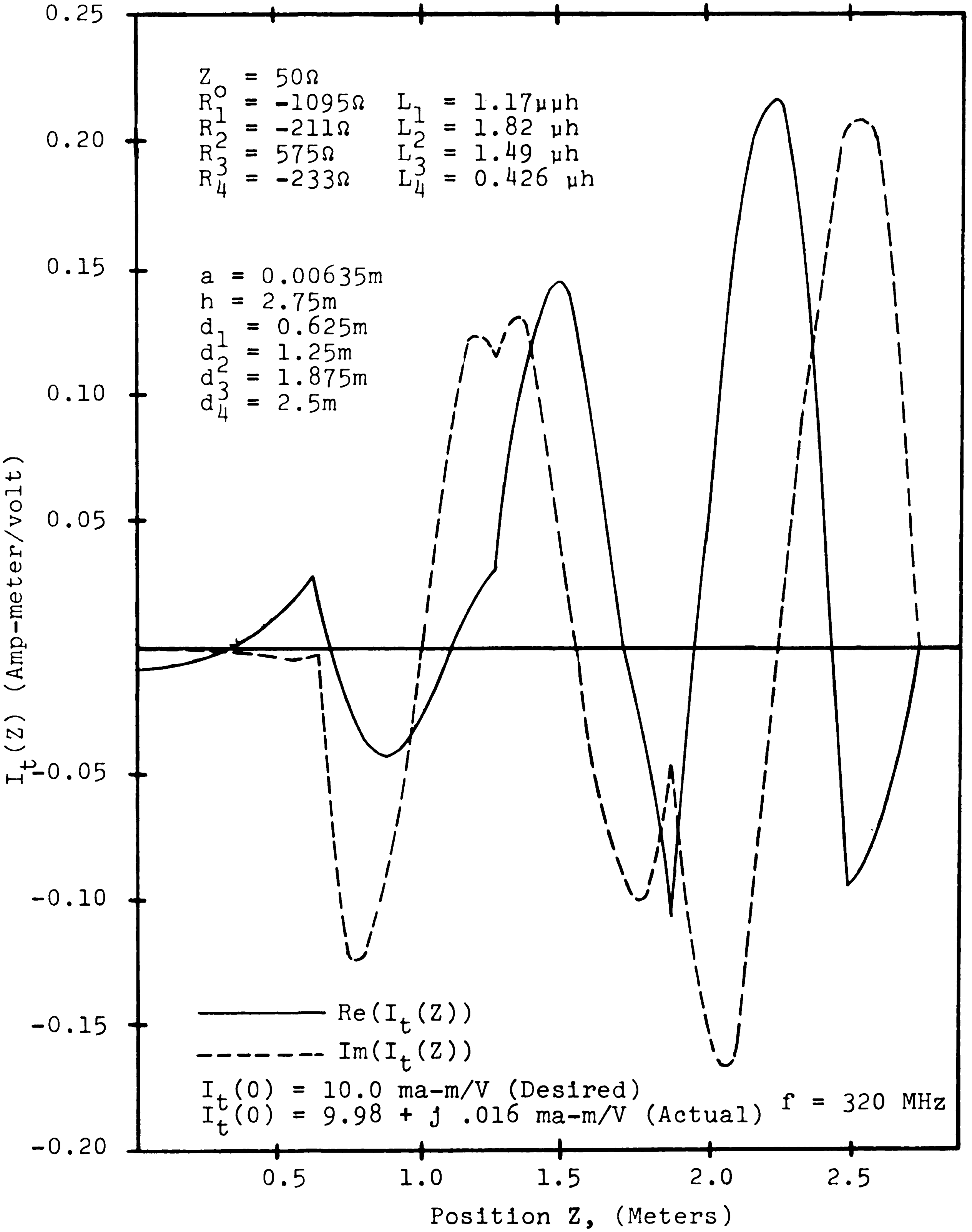


Figure 8.5 Current Distribution For A Receiving Antenna For Which The Central Load And Current Were Specified At Four Frequencies (f = 320 MHz).

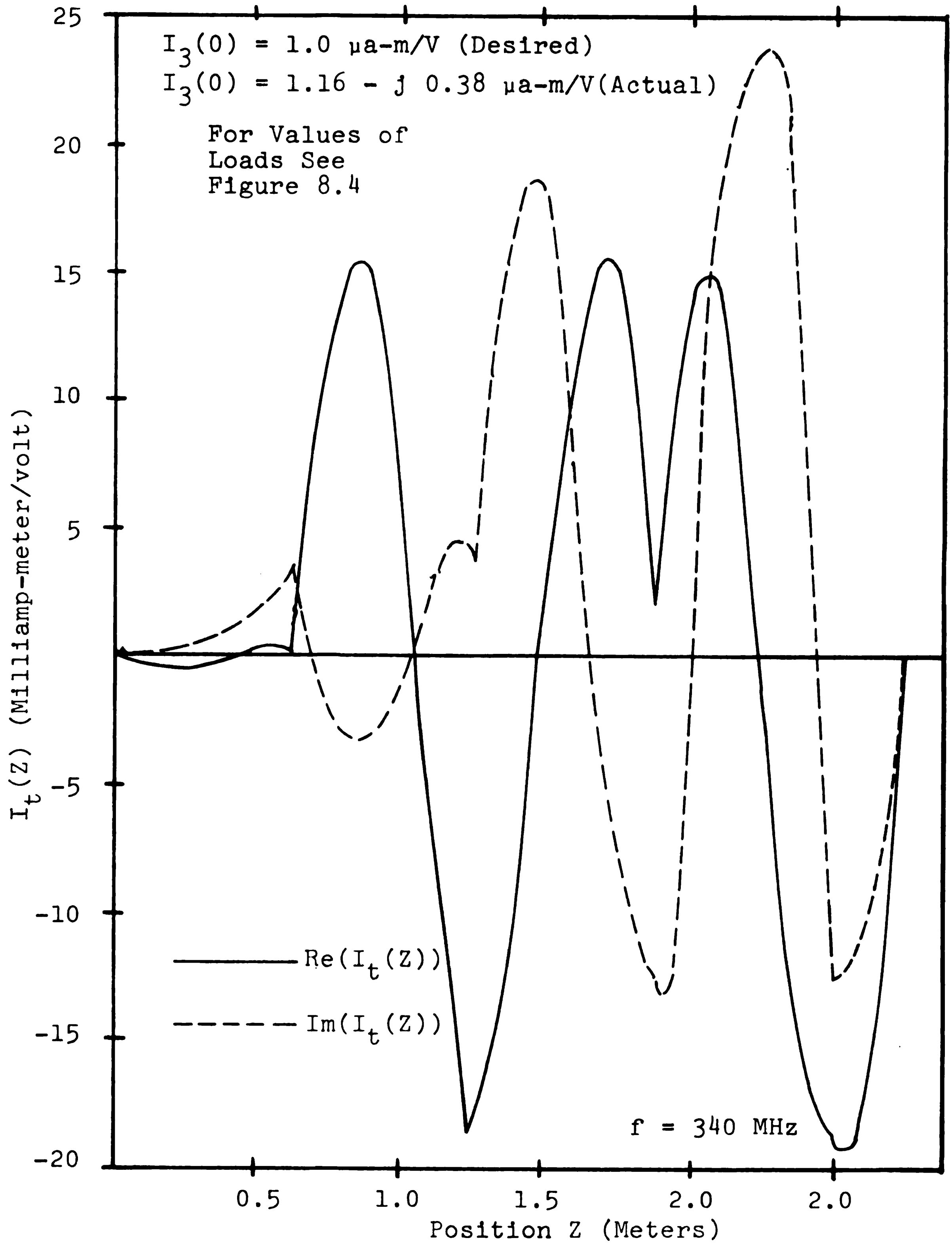


Figure 8.6 Current Distribution For A Receiving Antenna For Which The Central Load And Current Were Specified At Four Frequencies (f = 340 MHz).

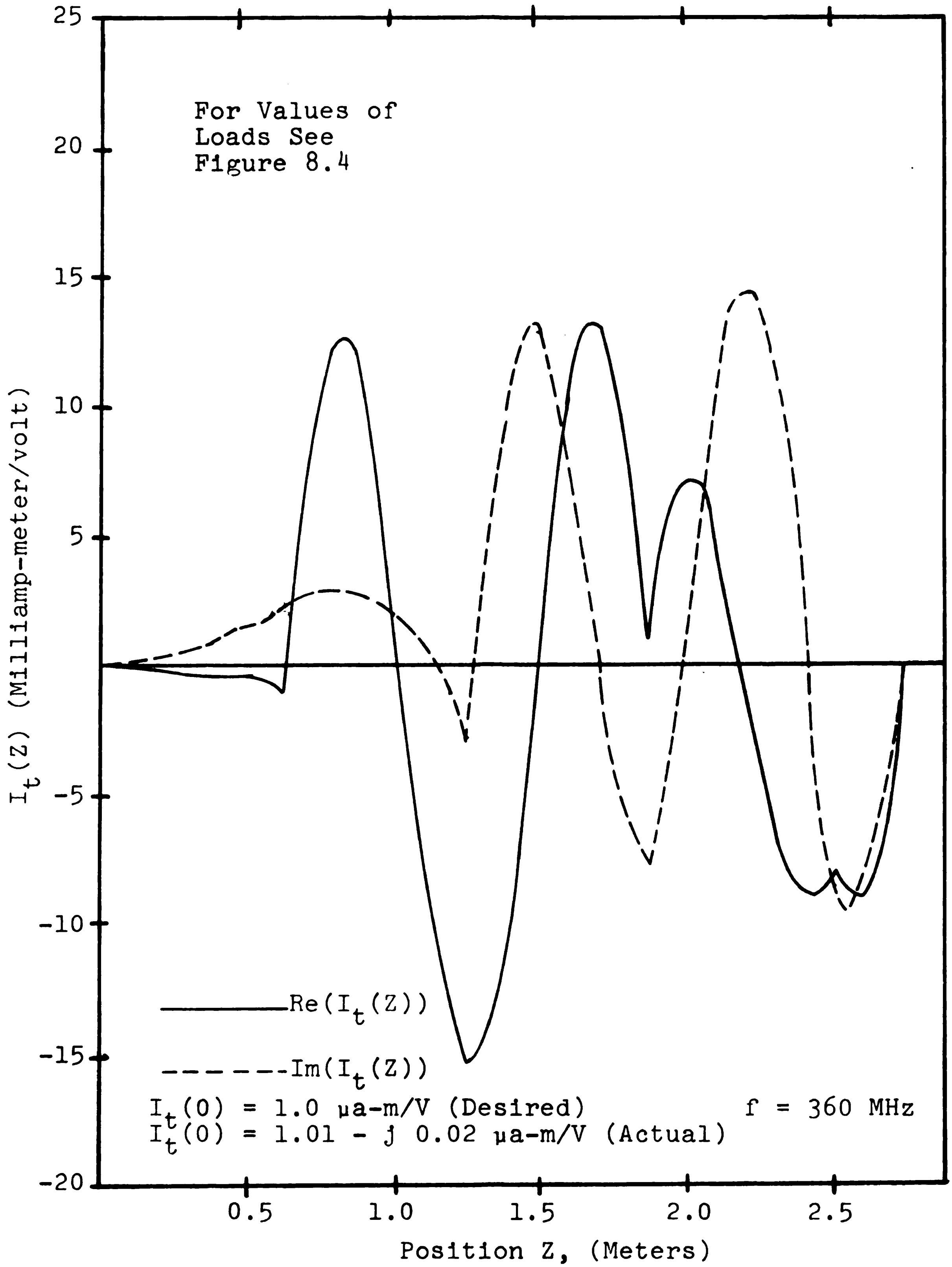


Figure 8.7 Current Distribution For A Receiving Antenna For Which The Central Load And Current Were Specified At Four Frequencies (f = 360 MHz).

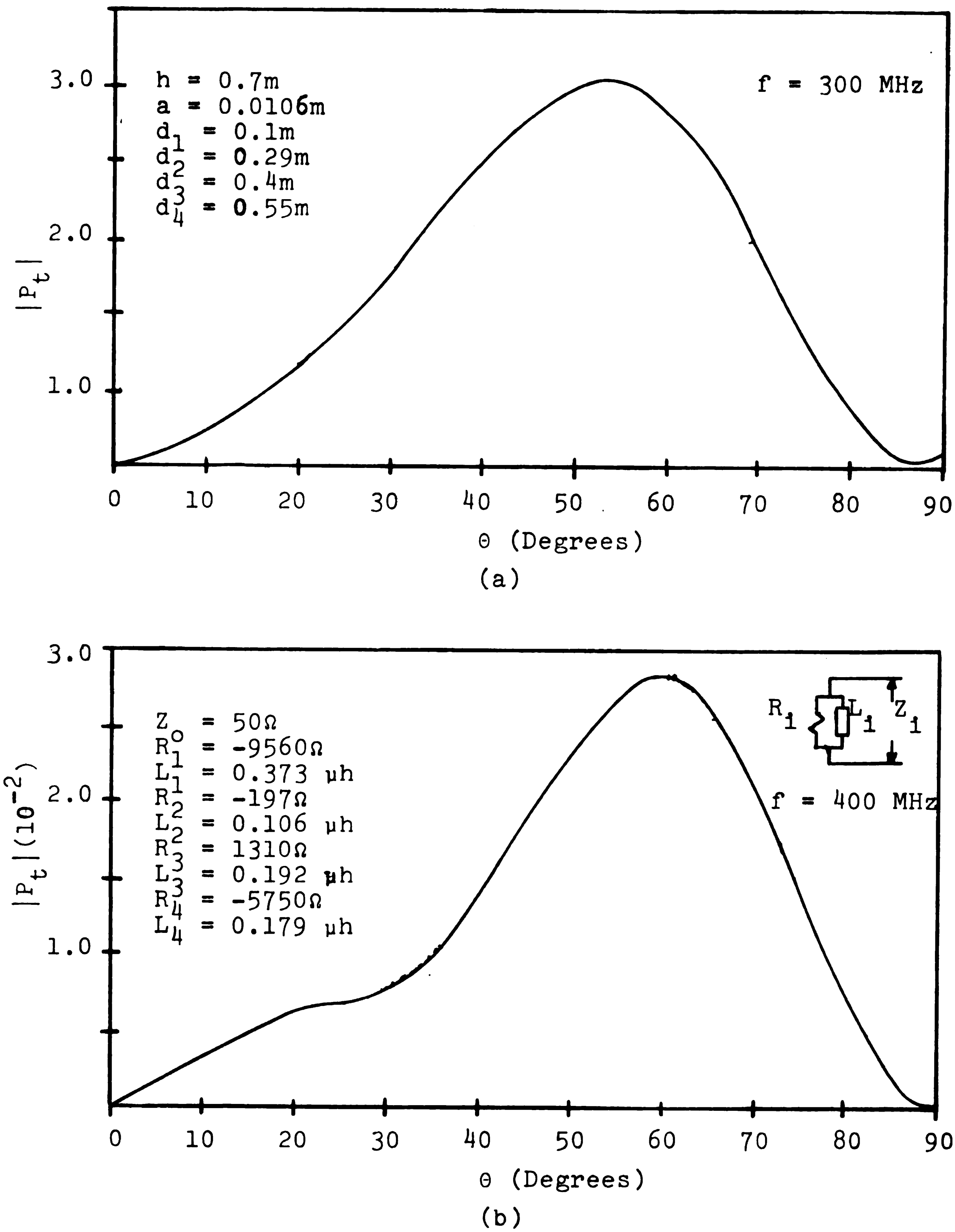


Figure 8.8 Scattered Fields For A Receiving Antenna For Which The Current And Radiation Pattern Are Specified At Two Frequencies: (a) f = 300 MHz (b) f = 400 MHz.

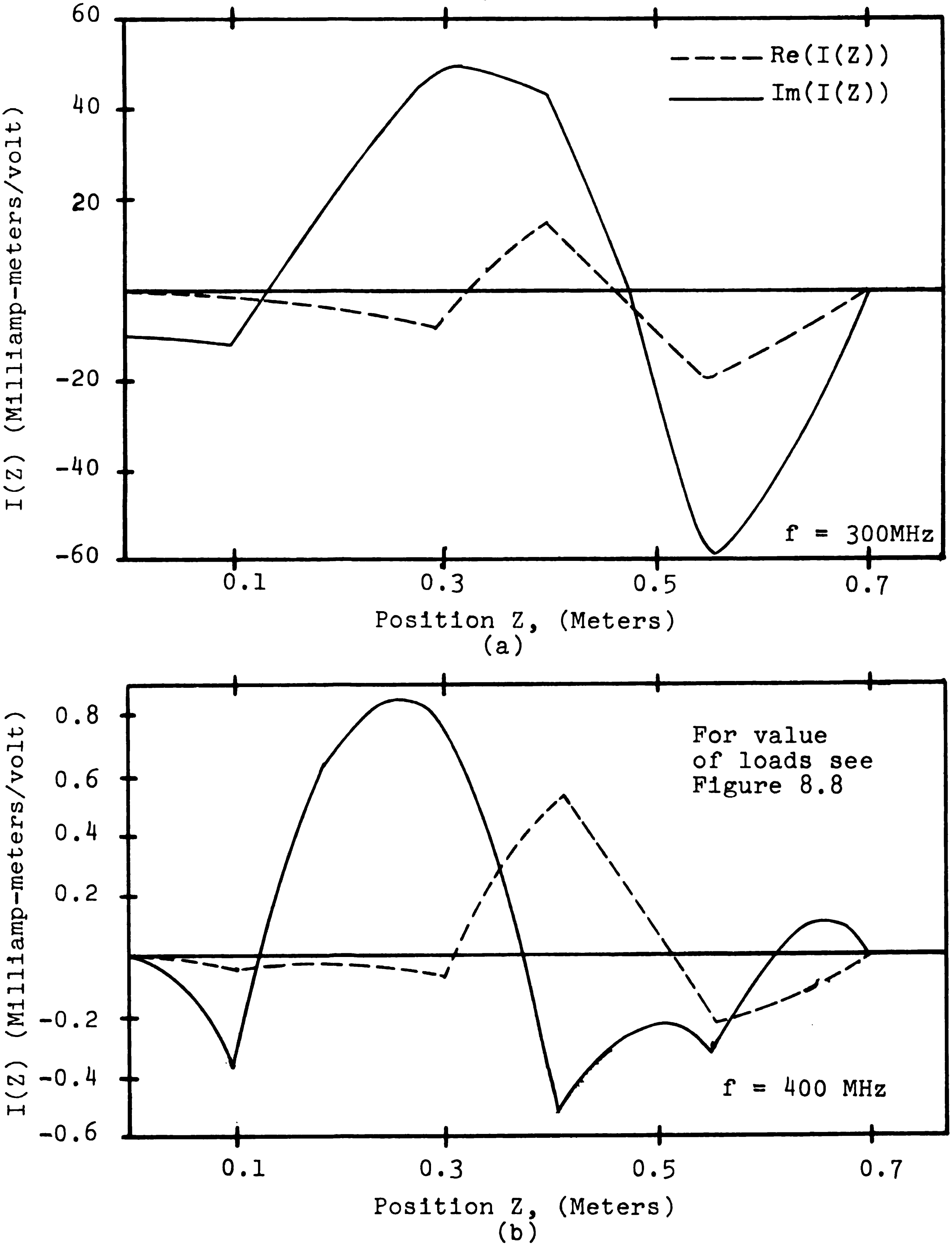


Figure 8.9 Current Distributions For A Receiving Antenna For Which Current And Radiation Pattern Are Specified At Two Frequencies: a) f = 300 MHz And b) f = 400MHz

0.01 μa-m/V at 400 MHz. Again with the use of active loading, the desired conditions were obtained. Thus if active loading is allowed, the current distribution, the scattered fields and the central load can be controlled as a function of frequency. Since these are all the important characteristics of a receiving antenna, the objective of improving the receiving characteristics of a linear antenna has indeed been achieved.

CHAPTER IX

CONCLUSIONS

In the preceeding chapters a method has been developed to improve the transmitting and receiving characteristics of a linear antenna. Matrix equations to describe multi-loaded transmitting and receiving antennas were derived. Further constraint equations were developed which related the loading impedances to the desired antenna characteristics and the characteristics to the loading impedances.

In particular, relations were developed for a transmitting antenna such that:

1) M + N loading impedances can be specified and the current and field pattern could be found,
2) M + N current points can be specified and the M + N complex loading impedances could be found,
3) M + N field pattern points can be specified and the M + N complex loading impedances could be found,
4) the input impedance can be specified at M + N frequencies and the M + N complex loading impedances could be found,
5) any of the above can be specified at (M + N)/2 and the M + N reactive loading impedances could be found,

6) resonance loading can be achieved and no characteristics could be found.

In addition to the above relations, constraints were developed for a receiving antenna such that the central load impedance could be specified and relations 2) and 3) from above be applied.

The first three relations are linear and presented no problems in obtaining a solution. The remaining relations were non-linear and may or may not have had one or more solutions. Since in general these non-linear equations could not be solved analytically, two numerical methods were employed: 1) a minimization method for finding the neighborhood of a solution and 2) a linear method for finding the solution.

Examples of each of the constraints were presented. As a result, it was found that in each case of complex loading, the desired characteristic could be obtained but only by active loading. When reactive loading was required, it was found that if a solution existed, the results were as good as the case of active loading. However, the requirement of reactive loading severely limited the chances for a solution.

The most interesting result of the study, was resonance loading of a receiving antenna. It was found that a single reactive load and the proper adjustment of the central load impedance could create an antenna with high gain and a very narrow beamwidth. In this resonance loading, however, it was found that certain values of loadings would cause the antenna characteristics to become unstable.

Thus multi-loaded antennas can achieve improvements in receiving and transmitting characteristics normally requiring extensive arrays.

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APPENDIX

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      PROGRAM COFTS (INPUT,OUTPUT)
C  COFTS CAN SOLVE MULTI-LOADED SHORT AND LONG ANTENNA PROBLEMS FOR
C  FIELD PATTERN CONSTRAINTS VARYING AS YE*SIN(ZZ*THE) AND CURRENT
C  CONSTRAINTS VARYING AS YE*EXP(ZY*X).  THE PARAMETERS WHICH CONTROL
C  THE PROGRAM ARE AS FOLLOWS-  NT,NUMBER OF CONSTRAINT EQUATIONS,
C  NH, NUMBER OF FREQUENCIES, N, NUMBER OF LOADING POINTS, ICO,
C  FREQUENCY DEPENDENCE OF THE ADMITTANCES, ICO=0 FOR FREQUENCY INDEPENDENCE
C  AND ONE FOR DEPENDENCE, IA, IB,IC, AND ID RERUN THE ORIGINAL PROBLEM
C  CHANGING THE LOAD LOCATIONS, FREQUENCY, LENGTH, YO, ZE, ZZ, OR CL
C  RESPECTIVELY, M01, CURRENT DISTRIBUTION, ZERO FOR LONG, ONE FOR SHORT,
C  J01=1 GIVES A CURRENT PLOT, ZERO NONE, J02=1 IS THE CASE OF GIVEN LOADS,
C  ZERO OF UNKNOWN LOADS, K01=1 GIVES A PATTERN PLOT, ZERO NONE, ICP=0 IS
C  FOR A PATTERN CONSTRAINT, ONE FOR A CURRENT CONSTRAINT,  N01 IS THE NUMBER
C  OF POINTS IN THE CURRENT PLOT, NTR=0 IS THE TRANSMITTING CASE, ONE THE
C  RECEIVING CASE, KCR=1 IS THE (RECEIVING CASE) CURRENT CONSTRAINT, ZERO
C  OTHER CONSTRAINTS, AND I=0 CALLS THE MINIMIZATION METHOD IN YNONL AND
C  ONE CALLS THE LINEARIZED METHOD.
      DIMENSION AA(9,9),AAA(9,9,5),FX(8),HD(9),W(5),A(5,32),BOL(9),
     1 DDV(9,5),AAO(9,5),CL(16),BB(5,8),FC(3,9,5),FFC(3)
      DOUBLE PRECISION FX
      COMPLEX CHIVS,CHIUS,CHIDS,CHIDDS
      COMPLEX FT,AI,ZP,ZE,YS,AA,BB,DET,AAA,DDV,AAO,FC,FFC,CF,A,ZY
      COMPLEX TE,ZCR
      DIMENSION TE(2,5),ZCR(5)
   11 FORMAT(9E12.4,/9E12.4,//9E12.4,/9E12.4,//9E12.4,/9E12.4,//9E12.4,/
     1   9E12.4,//9E12.4,/9E12.4,//9E12.4,/9E12.4,//9E12.4,/9E12.4,//
     2  9E12.4,/9E12.4,//9E12.4,/9E12.4,//9E12.4,/9E12.4,//
     3  9E12.4,/9E12.4,//9E12.4,/9E12.4,//9E12.4,/9E12.4,//
     4  9E12.4,/9E12.4,//9E12.4,/9E12.4,//9E12.4,/9E12.4,//
     5  9E12.4,/9E12.4,//9E12.4,/9E12.4,//9E12.4,/9E12.4,//
     6  9E12.4,/9E12.4,//9E12.4,/9E12.4,//9E12.4,/9E12.4)
   71 FORMAT(6I2)
C  READ PARAMETERS
      READ 71,NT,ICO,IA,IB,IC,ID
      READ 71,N,NH,M01,J01,J02,K01
      READ 71,ICP,N01,NTR,KCR,I
      AN=I
```

```
      FX(1)=AN
      TN=NT
      NM=2*N
      NN=N+1
      NRT=NT+KCR
      NCP=N-NT+1-KCR
      NPC=NCP-1
      KT=NN+1
      KR=NN+NTR
      AN=N
      CC=3.0E+8
      N2=2**NCP
      AI=CMPLX(0.0,1.0)
      PI=3.14159
      BO=1.0
      HD(1)=0.0
      DO 60 IGA=1,IA
C   READ LOAD POINTS
      DO 31 I=2,NN
      READ 30,F
   31 HD(I)=F
      DO 60 IGB=1,IB
C   READ FREQUENCIES
      DO 32 I=1,NH
      READ 30,F
   32 W(I)=2.0*PI*F*1.0E+6
      DO 60 IGC=1,IC
C   READ LENGTH, RADIUS, AND INPUT IMEDANCE EXCEPT FOR THE CASE OF THE
C   CURRENT CONSTRAINT (TRANSMITTING ANTENNA) THEN IT IS THE ATTENUATION FACTOR
      READ 30,BOW,AOW,YO
      BOL(KT)=-BOW
C   READ SPACING OF CURRENT CONSTRAINT POINTS
      READ 30,DHZ
C   READ RECEIVING ANTENNA CURRENT AT Z=0.
      DO 230 I=1,NH
      READ 30,ZP
  230 ZCR(I)=ZP
```

```
C  READ ZE,COMPLEX MAGNITUDE OF THE CONSTRAINTS, ZZ,PHASE FACTOR (CURRENT
C  CONSTRAINT) OR FREQUENCY (PATTERN CONSTRAINT).
      READ 30,ZE,ZZ
      ZY=CMPLX(YO,ZZ)
      DO 33 IT=1,NH
      BOA=AOW*W(IT)/CC
      BOH=BOW*W(IT)/CC
      DO 34 I=1,NN
   34 BOL(I)=HD(I)*W(IT)/CC
      PRINT 49,BOL,BOH,BOA
      BB(IT,NN) =1.0
      IF(M01) 83,83,82
C  CALCULATION SHORT CURRENT CONSTANTS
   82 CONTINUE
      CH=COS(BOH)
      CALL COF (BOA,BOH,CHIUS,CHIDS,CHDURS,CHDUIS,CHIDDS)
      PRINT 11,CHIUS,CHIDS,CHDURS,CHDUIS,CHIDDS
      CF=CHIDDS*(CHDURS*CH-CHIUS)+AI*CHDUIS*CHIDS
      TE(1,IT)=BOW*AI*CHIDDS/(BOH*30.0*CF)
      TE(2,IT)=BOW*CHDUIS/(BOH*30.0*CF)
      FFC(4)=TE(1,IT)
      FFC(5)=TE(2,IT)
      DO 80 I=1,NN
      X=BOL(I)
      CX=COS(X)
      CALL COFD(BOA,BOH,X,CHIVS,CHIDIS,CHIDRS)
      PRINT 11,CHIVS,CHIDIS,CHIDRS
      FC(3,I,IT)=CHIDRS+0.0*AI
      IF (ABS(BOH-PI/2.0).LE.0.01) GO TO 81
      FC(1,I,IT)=-AI*(CHIDIS*(CHDURS*CH-CHIUS)+CHDUIS*CHIVS)/CF
      FC(2,I,IT)=(CHIDDS*CHIVS-AI*CHIDIS*CHIDS)/CF
      GO TO 80
   81 FC(1,I,IT)=AI*CHIDIS*CHDURS/(CHIDDS*CHIUS-AI*CHIDIS*CHIDS)
      FC(2,I,IT)=FC(1,I,IT)*CHIDDS/(AI*CHIDIS)*CX
   80 CONTINUE
      IF(IT.NE.NH) GO TO 84
      PRINT 11,FC
```

```
      PRINT 11,TE
      GO TO 84
   83 FFC(1)=1.0
      FFC(2)=1.0
      FFC(3)=1.0
   84 CONTINUE
      IF(NPC.LE.-1) GO TO 74
      IF(J02.EQ.1) GO TO 74
      IF(ICP.EQ.1) GO TO 87
C  PATTERN CONSTRAINT
      DO 51 J=1,KR
      BOZ=BOL(J)
      IF (M01.EQ.0) GO TO 85
      IF (J.EQ.KT) GO TO 85
      FFC(1)=FC(1,J,IT)
      FFC(2)=FC(2,J,IT)
      FFC(3)=FC(3,J,IT)
   85 CONTINUE
      DO 51 I=1,NT
      NNT=NCP+I
      X=I
      THE=X/TN*PI/2.0
      CALL FPAT(BOH,BOA,BOZ,THE,FFC,M01,FT)
      IF (J.EQ.1) GO TO 52
      IF(J.EQ.KT) GO TO 201
      NNJ=J-1
      AA(NNT,NNJ)=FT
      GO TO 51
   52 CONTINUE
      IF(NTR.EQ.1) GO TO 200
      AA(NNT,NN)=FT-ZE*SIN(THE*ZZ)
      GO TO 51
  200 AA(NNT,NN)=FT
      GO TO 51
  201 AA(NNT,KT)=FT-ZE*SIN(THE*ZZ)
      IF(KCR.EQ.1)AA(NNT,NN)=AA(NNT,KT)-AA(NNT,NN)*ZCR(IT)/YO
   51 CONTINUE
```

```
      GO TO 74
   87 DHW=DHZ*W(IT)/CC
      IF(NTR.EQ.1) GO TO 74
C  CURRENT CONSTRAINT
      DO 88 J=1,NN
      BOD=BOL(J)
      IF (M01.EQ.0) GO TO 72
      FFC(1)=FC(1,J,IT)
      FFC(2)=FC(2,J,IT)
      FFC(3)=FC(3,J,IT)
   72 CONTINUE
      DO 88 I=1,NT
      C=I-1
      NNT=NN-I+1
      X=C*DHW
      CALL CUR(BOH,BOA,BOD,X,FFC,M01,YS)
      IF(J.EQ.1) GO TO 89
      NNJ=J-1
      AA(NNT,NNJ)=YS
      GO TO 88
   89 AA(NNT,NN)=YS-ZE*CEXP(ZY*X)
   88 CONTINUE
   74 CONTINUE
C  LOAD EQUATIONS
      DO 5 K=1,NN
      I=K-1
      IF (I.EQ.0) I=1
      X=BOL(K)
      DO 4 JJ=1,KR
      J=JJ-1
      BOD=BOL(JJ)
      IF (M01.EQ.0) GO TO 73
      IF(JJ.EQ.KT) GO TO 73
      FFC(1)=FC(1,JJ,IT)
      FFC(2)=FC(2,JJ,IT)
      FFC(3)=FC(3,JJ,IT)
   73 CONTINUE
```

```
      CALL CUR(BOH,BOA,BOD,X,FFC,M01,YS)
      IF(JJ.EQ.1) GO TO 6
      IF(JJ.EQ.KT) GO TO 6
      AAA(I,J,IT)=YS
      GO TO 4
    6 IF(NTR.EQ.1) GO TO 203
      DDV(I,IT)=-YS
      IF(K-1) 15,15,4
   15 DDV(NN,IT)=DDV(1,IT)
      GO TO 4
  203 JF=I
      IF(K.EQ.1) JF=NN
      IF(JJ.EQ.KT) GO TO 204
      AAA(JF,NN,IT)=YS
      GO TO 4
  204 DDV(JF,IT)=-YS
    4 CONTINUE
      IF (K-1) 12,12,5
   12 DO 14 L=1,N
      IF(NTR.EQ.1) GO TO 206
      AAO(L,IT)=AAA(1,L,IT)
      GO TO 14
  206 AAA(NN,L,IT)=AAA(1,L,IT)
   14 CONTINUE
    5 CONTINUE
      IF(NPC.LE.-1) GO TO 33
C  COMPUTE COEFFICIENTS FOR  NON-LINEAR EQUATIONS
      IF (NPC.EQ.0) GO TO 92
      DO 53 I=1,NPC
      DO 53 J=1,N
      AA(I,J)=AAA(I,J,IT)
      IF(NTR.EQ.1) GO TO 205
      AA(I,NN)=-DDV(I,IT)
      GO TO 53
  205 IF(KCR.EQ.0)GO TO 207
      AA(I,NN)=-DDV(I,IT)-AAA(I,NN,IT)*ZCR(IT)/YO
      GO TO 53
```

```
207 AA(I,NN)=AAA(I,NN,IT)
    AA(I,KT)=-DDV(I,IT)
 53 CONTINUE
 92 CONTINUE
    IF(J02.EQ.1) GO TO 33
    JF=NN
    IF(NTR.EQ.0) GO TO 208
    IF(KCR.EQ.0) JF=KT
    DO 209 I=1,N
209 AA(JF,I)=AAA(NN,I,IT)
    IF(KCR.EQ.0)GO TO 210
    AA(NN,NN)=-DDV(NN,IT)-ZCR(IT)*(1.0+AAA(NN,NN,IT)/YO)
    GO TO 208
210 AA(KT,NN)=AAA(NN,NN,IT)+YO
    AA(KT,KT)=-DDV(NN,IT)
208 CONTINUE
    DO 54 I=1,NRT
    NTP=I+NCP-1
    DO  55 J=1,N
 55 AA(NCP,J)=AAA(NTP,J,IT)
    IF(NTR.EQ.1) GO TO 211
    AA(NCP,NN)=-DDV(NTP,IT)
    GO TO 222
211 IF(KCR.EQ.0)AA(NCP,NN)=AAA(NTP,NN,IT)
    IF(KCR.EQ.1)AA(NCP,NN)=-DDV(NTP,IT)-ZCR(IT)*AAA(NTP,NN,IT)/YO
    IF(KCR.EQ.0) AA(NCP,KT)=-DDV(NTP,IT)
222 CONTINUE
    IF(NCP.EQ.NTP) GO TO 150
    DO 151 J=1,JF
    ZP=AA(J,NCP)
    AA(J,NCP)=AA(J,NTP)
151 AA(J,NTP)=ZP
150 CONTINUE
    IIT=(IT-1)*NRT+I
    CALL COEP(JF,N2,NCP,IIT,AA,A)
    IF(NCP.EQ.NTP) GO TO 54
    DO 152 J=1,JF
```

```
      ZP=AA(J,NCP)
      AA(J,NCP)=AA(J,NTP)
  152 AA(J,NTP)=ZP
   54 CONTINUE
   33 CONTINUE
      IF(NPC.LE.-1) GO TO 500
      PRINT 11,A
      DO 60  IG=1,ID
C  READ TRIAL OR KNOWN LOAD ADMITTANCES
      ICON=ICO
      DO 35 I=1,NM
      READ 30,F,FJ
      JF=FJ
      CLC=10.0**JF
   35 CL(I)=F*CLC
      PRINT 49,CL
      IF(J02.EQ.1) GO TO 90
C  FIND ADMITTANCES
      CALL YNONL(N,N2,NH,NRT,A,CL,W,FX,ICON)
      PRINT 49,CL
   90 CONTINUE
      DO 42 K=1,NH
      WK=W(K)
      FFC(4)=TE(1,K)
      FFC(5)=TE(2,K)
      BOA=AOW*W(K)/CC
      BOH=BOW*W(K)/CC
      DO 91 I=1,NN
   91 BOL(I)=HD(I)*WK/CC
C  COMPUTE VOLTAGES
      DO 43 I=1,N
      DO 43 J=1,N
   43 AA(I,J)=AAA(I,J,K)
      DO 37 I=1,N
      J=2*I
      IF (ICO) 38,38,39
   38 AA(I,I)=AA(I,I)+CMPLX(CL(J-1),CL(J))
```

```
      GO TO 37
   39 IF(CL(J)) 41,40,40
   40 AA(I,I)=AA(I,I)+CMPLX(CL(J-1),(CL(J)*W(K)))
      GO TO 37
   41 AA(I,I)=AA(I,I)+CMPLX(CL(J-1),(CL(J)/W(K)))
   37 CONTINUE
      IF(NTR.EQ.1) GO TO 212
      DO 65 I=1,N
   65 AA(I,NN)=DDV(I,K)
      JF=N
      GO TO 213
  212 IF(KCR.EQ.1) GO TO 214
      JF=NN
      DO 215 I=1,NN
      AA(NN,I)=AAA(NN,I,K)
      AA(I,NN)=AAA(I,NN,K)
  215 AA(I,KT)=DDV(I,K)
      AA(NN,NN)=AA(NN,NN)+YO
      GO TO 213
  214 DO 223 I=1,N
  223 AA(I,NN)=DDV(I,K)+ZCR(K)*AAA(I,NN,K)/YO
      JF=N
  213 CALL CMATPAC(-1,9,10,AA,JF,1,DET,1.0E-50)
      IT=JF+1
      DO 66 I=1,JF
   66 BB(K,I)=AA(I,IT)
      PRINT 400
C  COMPUTE INPUT IMPEDANCE
  400 FORMAT(1X,27HINPUT AND LOAD ADMITTANCES.//)
      IF(NTR.EQ.0) GO TO 300
      IF(JF.EQ.N) BB(K,NN)=-ZCR(K)/YO
  300 CONTINUE
      JF=N+NTR
      DO 22I=1,NN
      ZP=CMPLX(0.0,0.0)
      DO 21 J=1,JF
      IF(NTR.EQ.1) GO TO 216
```

```
      IF (I.EQ.1) GO TO 23
      ZP=AAA((I-1),J,K)*BB(K,J)+ZP
      GO TO 21
   23 ZP=AAO(J,K)*BB(K,J)+ZP
      GO TO 21
  216 ZP=AAA(I,J,K)*BB(K,J)+ZP
   21 CONTINUE
      IF(CABS(BB(K,I)).EQ.0.0) BB(K,I)=1.0E-50
      IF(NTR.EQ.1) GO TO 217
      IF (I.EQ.1) GO TO 50
      ZP=(DDV((I-1),K)-ZP)/BB(K,(I-1))
      GO TO 8
  217 ZP=(DDV(I,K)-ZP)/BB(K,I)
      GO TO 8
   50 ZP=(DDV(NN,K)-ZP)/BO
    8 PRINT 7,WK,ZP
      YS=1.0/ZP
      PRINT 7,YS
      ZA=CABS(ZP)
      ZB=REAL(ZP)
      IF(ABS(ZB).LT.1.0E-50) ZB=1.0E-50
      ZC=AIMAG(ZP)
      ZN=ATAN2(ZC,ZB)*180./PI
   26 PRINT 7,ZA,ZN
   22 CONTINUE
      IF(K01.NE.1)GO TO 79
      PRINT 401
C  FIELD PATTERN PLOT
  401 FORMAT(1X,19HFIELD PATTERN PLOT.//)
      DO 9 I=1,18
      ZP=CMPLX(0.0,0.0)
      X=I
      THE =X*PI/36.0
      DO 1 J=1,KR
      BOZ=BOL(J)
      IF (M01.EQ.0) GO TO 86
```

```
      IF (J.EQ.KT) GO TO 86
      FFC(1)=FC(1,J,K)
      FFC(2)=FC(2,J,K)
      FFC(3)=FC(3,J,K)
   86 CONTINUE
      CALL FPAT(BOH,BOA,BOZ,THE,FFC,M01,FT)
      IF(J.EQ.KT) GO TO 219
      IF (J.EQ.1) GOTO 2
      NNJ=J-1
      ZP=FT*BB(K,NNJ)+ZP
      GO TO 1
    2 CONTINUE
      IF(NTR.EQ.1) GO TO 218
      ZP=FT*BO+ZP
      GO TO 1
  218 ZP=FT*BB(K,NN)+ZP
      GO TO1
  219 ZP=FT+ZP
    1 CONTINUE
      THE =THE*180.0/PI
      PRINT 7,THE,ZP
      ZC=AIMAG(ZP)
      ZB=REAL(ZP)
      ZA=CABS(ZP)
      ZN=ATAN2(ZC,ZB)*180./PI
      PRINT 7,ZA,ZN
    9 CONTINUE
   79 IF(J01.NE.1) GO TO 42
      PRINT 402
C CURRENT DISTRIBUTION PLOT
  402 FORMAT(1X,21HCURRENT DISTRIBUTION.//)
      A01=N01
      DHW=(BOH-.50)/A01
      DO 75 I=1,N01
      C=I-1
      ZP=CMPLX(0.0,0.0)
      X=C*DHW
      DO 78 J=1,KR
```

```
      BOD=BOL(J)
      IF (M01.EQ.0) GO TO 76
      IF (J.EQ.KT) GO TO 76
      FFC(1)=FC(1,J,K)
      FFC(2)=FC(2,J,K)
      FFC(3)=FC(3,J,K)
   76 CONTINUE
      CALL CUR(BOH,BOA,BOD,X,FFC,M01,YS)
      IF (J.EQ.1) GOTO 77
      IF(J.EQ.KT) GO TO 220
      NNJ=J-1
      ZP=YS*BB(K,NNJ)+ZP
      GO TO 78
   77 CONTINUE
      IF(NTR.EQ.1) GO TO 221
      ZP=YS*BO+ZP
      GO TO 78
  221 ZP=YS*BB(K,NN)+ZP
      GO TO 78
  220 ZP=YS+ZP
   78 CONTINUE
      PRINT 7,X,ZP
      ZA=CABS(ZP)
      ZB=REAL(ZP)
      ZC=AIMAG(ZP)
      ZN=ATAN2(ZC,ZB)*180./PI
      PRINT 7,ZA,ZN
   75 CONTINUE
    7 FORMAT(3E14.6)
   49 FORMAT(9E12.4,/7E12.4,I6/)
   30 FORMAT(3F10.5)
   42 CONTINUE
      PRINT 11,BB
      PRINT 7,ZE,ZZ
      PRINT 49,YO,ZCR
      PRINT 70,FX
      PRINT 49,HD,BOW,AOW,W,ICON
```

```
   70 FORMAT(8D16.6)
   60 CONTINUE
      GO TO 501
  500 CONTINUE
      PRINT 502
  502 FORMAT(1X,26HFINITE VOLTAGE INSTABILITY)
      ZP=AAA(1,2,1)*DDV(1,1)/DDV(2,1)- AAA(1,1,1)
      ZP=1.0/ZP
      ZE=AAA(2,1,1)*DDV(2,1)/DDV(1,1)- AAA(2,2,1)
      ZE=1.0/ZE
      PRINT 11,ZP,ZE
      PRINT 503
503   FORMAT(1X,16HZERO INSTABILITY)
      X=0.0
      BOD=BOL(1)
      CALL CUR(BOH,BOA,BOD,X,FFC,M01,YS)
      ZE=YS
      BOD=BOL(2)
      CALL CUR(BOH,BOA,BOD,X,FFC,M01,YS)
      ZP=AAA(2,1,1)*ZE/YS-AAA(1,1,1)
      ZP=1.0/ZP
      ZE=AAA(1,2,1)*YS/ZE-AAA(2,2,1)
      ZE=1.0/ZE
      PRINT 11,ZP,ZE
      PRINT 504
  504 FORMAT(1X,47HB**2                          A(1,1)              A(2,2))
      YS=AAA(2,2,1)
      ZE=AAA(1,1,1)
      ZP=AAA(2,1,1)*AAA(1,2,1)
      PRINT 11,ZP,ZE,YS
  501 CONTINUE
      END
```

```
      SUBROUTINE CMATPAC(IJOB,II,JJ,A,N,M,DET,EP)                       CMA00001
      TYPE COMPLEX A,B,DET,CONST,S                                      CMA00002
      DIMENSION A(II,JJ)
   30 FORMAT(1X,42HTHE DETERMINANT OF THE SYSTEM EQUALS ZERO./          CMA00004
     11X,36HTHE PROGRAM CANNOT HANDLE THIS CASE.//)                     CMA00005
C CMATPAC IS A COMPLEX MATRIX INVERSION PACKAGE FOR ALL RANKS OF MATRICES
      IF(N.EQ.1) GO TO 40
      DET=1.                                                            CMA00006
      NP1=N+1                                                           CMA00007
      NPM=N+M                                                           CMA00008
      NM1=N-1                                                           CMA00009
      IF(IJOB) 2,1,2                                                    CMA00010
    1 DO 3 I=1,N                                                        CMA00011
      NPI=N+I                                                           CMA00012
      A(I,NPI)=1.                                                       CMA00013
      IP1=I+1                                                           CMA00014
      DO 3 J=IP1,N                                                      CMA00015
      NPJ=N+J                                                           CMA00016
      A(I,NPJ)=0.                                                       CMA00017
    3 A(J,NPI)=0.                                                       CMA00018
    2 DO 4 J=1,NM1                                                      CMA00019
      C=CABS(A(J,J))
      JP1=J+1                                                           CMA00021
      DO 5 I=JP1,N                                                      CMA00022
      D=CABS(A(I,J))
      IF(C-D) 6,5,5                                                     CMA00024
    6 DET=-DET                                                          CMA00025
      DO 7 K=J,NPM                                                      CMA00026
      B=A(I,K)                                                          CMA00027
      A(I,K)=A(J,K)                                                     CMA00028
    7 A(J,K)=B                                                          CMA00029
      C=D                                                               CMA00030
    5 CONTINUE                                                          CMA00031
      IF(CABS(A(J,J))-EP) 14,15,15
   15 DO 4 I=JP1,N                                                      CMA00033
```

```
      CONST=A(I,J)/A(J,J)                                 CMA00034
      DO 4 K=JP1,NPM                                      CMA00035
    4 A(I,K)=A(I,K)-CONST*A(J,K)                          CMA00036
      IF(CABS(A(N,N))-EP) 14,18,18
   14 DET=0.                                              CMA00038
      IF(IJOB) 16,16,17                                   CMA00039
   16 PRINT 30                                            CMA00040
      DO 25 I=1,N
      DO 25 J=NP1,NPM
       A(I,J)=0.0
   25 CONTINUE
   17 RETURN                                              CMA00041
   18 DO 11 I=1,N                                         CMA00042
   11 DET=DET*A(I,I)                                      CMA00043
      IF(IJOB) 10,10,17                                   CMA00044
   10 DO 12 I=1,N                                         CMA00045
      K=N-I+1                                             CMA00046
      KP1=K+1                                             CMA00047
      DO 12 L=NP1,NPM                                     CMA00048
      S=0.                                                CMA00049
      IF(N-KP1) 12,19,19                                  CMA00050
   19 DO 13 J=KP1,N                                       CMA00051
   13 S=S+A(K,J)*A(J,L)                                   CMA00052
   12 A(K,L)=(A(K,L)-S)/A(K,K)                            CMA00053
      RETURN                                              CMA00054
   40 DET=A(1,1)
      IF(IJOB.EQ.0) A(1,2)=1.0/A(1,1)
      IF(IJOB.EQ.-1) A(1,2)=A(1,2)/A(1,1)
      RETURN
      END                                                 CMA00055
```

```
      FUNCTION EIZ(Z,CW)
C   EIZ COMPUTES THE CURRENT ON AN INFINITELY LONG ANTENNA
      COMPLEX EIZ,AI,E3,E4
      GAMMA=.57721566
      AI=CMPLX(0.0,1.0)
      PI=3.14159
      Q=ABS(Z)
      E=2.0*CW+ALOG(Q+SQRT(Q*Q+EXP(-2.0*GAMMA)))+GAMMA
      E3=CEXP(AI*Q)
      E4=CLOG(1.0-AI*2.0*PI/(E+AI*1.5*PI))
      EIZ=AI*E3*E4/(PI*120.0)
      RETURN
      END


      SUBROUTINE COEP(NP,N2,N,K,AA,A)
C   COEP COMPUTES THE COEFFICIENTS FOR THE NON-LINEAR EQUATIONS FROM THE
C   MATRIX AA.  A IS THE MATRIX OF THE COEFFICIENTS.  THE INTEGER VECTOR IB
C   IS CONSTRUCTED TO GIVE THE PROPER NUMBER AND ORDER OF THE ROWS AND
C   COLUMNS OF THE MATRIX AA TO FIND THE DESIRED COEFFICIENT IN A.
      COMPLEX AI,AM,AA,BB,DET,AC,A
      DIMENSION AA(9,9),A(5,32),BB(9,9),IB(9)
      NN=N+1
      NNP=NP-N-1
      IA=1
C   COMPUTE A(K,1)
      DO 11 I=NN,NP
      II=I-N
      DO 11 J=NN,NP
      JJ=J-N
   11 BB(II,JJ)=AA(I,J)
      CALL CMATPAC(1,9,9,BB,II,0,DET,1.0E-50)
      A(K,1)=DET
```

```
C   CONSTRUCT IB
    1 NN=NN-1
      M=N-NN+2
      MA=2
      IB(2)=N+1
      IB(1)=NN
      GO TO 2
    4 J1=1
    7 J2=J1+1
      IF((IB(J1)+1).NE.IB(J2)) GO TO 6
      J1=J2
      GO TO 7
    6  IF (J2.EQ.MA) GO TO 8
      M1=MA
    9 M2=M1-1
      IF ((IB(M2)+1).NE.IB(M1)) GO TO 10
      M1=M2
      GO TO 9
   10 IF (M1.EQ.MA) GO TO 8
      MA=M1+1
      IB(M1)=IB(M1)-1
      IB(MA)=N+1
      GO TO 2
    8 MM=MA
      MA=MA+1
      IB(MA)=N+1
      IB(MM)=N
    2 IA=IA+1
      IF(NNP.EQ.0) GO TO 13
      DO 12 I=1,NNP
      JJ=I+MA
   12 IB(JJ)=N+1+I
      GO TO 14
   13 JJ=MA
   14 CONTINUE
C   FILL MATRIX BB
      DO 3 I=1,JJ
```

```
      II=IB(I)
      DO 3 J=1,JJ
      IJ=IB(J)
    3 BB(I,J)=AA(II,IJ)
C   COMPUTE A(K,I)
      CALL CMATPAC(1,9,9,BB,JJ,0,DET,1.0E-50)
      A(K,IA)=DET
C   TEST FOR LAST COEFFICIENT
      IF (IA.GE.N2) GO TO 5
      IF (MA.EQ.M) GO TO 1
      GO TO 4
    5 CONTINUE
      RETURN
      END


      SUBROUTINE MATPAC(IJOB,II,JJ,AA,N,M,DET,EP)
C DOUBLE PRECISION MATRIX INVERSION PACKAGE
      DIMENSION A(8,16)
      DIMENSION AA(II,JJ)
      DOUBLE PRECISION A,AA,B,S,DET,CONST
   30 FORMAT(1X,42HTHE DETERMINANT OF THE SYSTEM EQUALS ZERO./              CMA00004
     11X,36HTHE PROGRAM CANNOT HANDLE THIS CASE.//)                         CMA00005
      DO 25 I=1,N
      DO 25 J=1,N
      A(I,J)=AA(I,J)
   25 CONTINUE
      DET=1.                                                                CMA00006
      NP1=N+1                                                               CMA00007
      NPM=N+M                                                               CMA00008
      NM1=N-1                                                               CMA00009
      IF(IJOB) 2,1,2                                                        CMA00010
    1 DO 3 I=1,N                                                            CMA00011
      NPI=N+I                                                               CMA00012
```

```
      A(I,NPI)=1.                                                       CMA00013
      IP1=I+1                                                           CMA00014
      IF(IP1.GT.N) GO TO 3
      DO 3 J=IP1,N                                                      CMA00015
      NPJ=N+J                                                           CMA00016
      A(I,NPJ)=0.                                                       CMA00017
      A(J,NPI)=0.
    3 CONTINUE
    2 DO 4 J=1,NM1                                                      CMA00019
      C=DABS(A(J,J))
      JP1=J+1                                                           CMA00021
      DO 5 I=JP1,N                                                      CMA00022
      D=DABS(A(I,J))
      IF(C-D) 6,5,5                                                     CMA00024
    6 DET=-DET                                                          CMA00025
      DO 7 K=J,NPM                                                      CMA00026
      B=A(I,K)                                                          CMA00027
      A(I,K)=A(J,K)                                                     CMA00028
    7 A(J,K)=B                                                          CMA00029
      C=D                                                               CMA00030
    5 CONTINUE                                                          CMA00031
      IF (DABS(A(J,J))-EP) 14,15,15
   15 DO 4 I=JP1,N                                                      CMA00033
      CONST=A(I,J)/A(J,J)                                               CMA00034
      DO 4 K=JP1,NPM                                                    CMA00035
    4 A(I,K)=A(I,K)-CONST*A(J,K)                                        CMA00036
      NM1=N-1
      NPM=N+M
      NP1=N+1
      IF (DABS(A(N,N))-EP) 14,18,18
   14 DET=0.                                                            CMA00038
      IF(IJOB) 16,16,17                                                 CMA00039
   16 PRINT 30                                                          CMA00040
      GO TO 31
   17 RETURN                                                            CMA00041
   18 DO 11 I=1,N                                                       CMA00042
   11 DET=DET*A(I,I)                                                    CMA00043
      IF(IJOB) 10,10,17                                                 CMA00044
   10 DO 12 I=1,N                                                       CMA00045
      K=N-I+1                                                           CMA00046
```

```
      KP1=K+1                                                           CMA00047
      DO 12 L=NP1,NPM                                                   CMA00048
      S=0.                                                              CMA00049
      IF(N-KP1) 12,19,19                                                CMA00050
   19 DO 13 J=KP1,N                                                     CMA00051
   13 S=S+A(K,J)*A(J,L)                                                 CMA00052
   12 A(K,L)=(A(K,L)-S)/A(K,K)                                          CMA00053
      DO 26 I=1,N
      DO 26 J=1,M
      IJ=J+N
   26 AA(I,J)=A(I,IJ)
      RETURN                                                            CMA00054
C  IF INVERSE DOES NOT EXIST, AA IS REDUCED TO A UNITY MATRIX
   31 DO 32 I=1,N
      DO 32 J=1,N
   32 AA(I,J)=0.0
      DO 33 I=1,N
   33 AA(I,I)=1.0
      RETURN
      END                                                               CMA00055


      SUBROUTINE FPAT(BOH,BOA,BOZ,THE,FC,M,FT)
C  FPAT COMPUTES THE FIELD PATTERN IN THE DIRECTION THE FOR AN ANTENNA OF LENGTH
C  2*BOH, RADIUS BOA, AND LOADED AT BOD.  THE PARAMETER M CONTROLS THE TYPE OF
C  DISTRIBUTION, M=0 FOR THE LONG WIRE AND M=1 FOR THE SHORTWIRE. FT IS
C  THE VALUE OF THE FIELD PATTERN.
      DIMENSION FC(5)
      COMPLEX FC,Z1,AH,BH,DT,FT,CFS,CF,AI
      SH=SIN(BOH)
      CH=COS(BOH)
      CZ=COS(BOZ)
      ST=SIN(THE)
      CT=COS(THE)
```

```
      CHC=COS(BOH*CT)
      SHC=SIN(BOH*CT)
      CZC=COS(BOZ*CT)
      PI=3.14159
      AI=CMPLX(0.0,1.0)
      IF (M.EQ.1) GO TO 51
C   LONG WIRE PATTERN
      GAMMA=.57721566
      Z1=ALOG(2.0/BOA)-GAMMA+AI*PI/2.0
      CF=CFS(BOA,BOH,BOZ)
      AS=ALOG(ST)
C   TEST FOR UNLOADED WIRE
      IF(BOZ.LT.0.0) GO TO 3
      DT=ST*(AS-Z1)
      AH=2.0*CF*SH-CH*CZ
      BH=2.0*CF*CH+SH*CZ
      FT=(AH*CHC-BH*SHC*CT+CZC)/DT
      FT=CONJG(FT)
      IF (BOZ.EQ.0.0) GO TO 52
      RETURN
   52 FT=FT/2.0
      RETURN
C   UNLOADED WIRE
    3 IF(ABS(CT).LE.0.0001) GO TO 4
      CH2=SHC/CT
      GO TO 5
C   DISTRIBUTION FOR THE=PI/2
    4 CH2=BOH
    5 FT=BOZ/BOH*ST/(Z1-AS)*(CH2+CF*(SH*CHC-CT*SHC*CH)*2.0/ST**2)
      FT=CONJG(FT)
      RETURN
   51 CONTINUE
C   SHORT WIRE PATTERN
      SH2=SIN(BOH/2.)
      CH2=COS(BOH/2.)
C   TEST FOR UNLOADED WIRE
      IF(BOZ.LT.0.0) GO TO 1
```

```
      IF (ABS(BOH-PI/2.0).LE.0.01) GO TO 41
      AT=(CZ*CHC/CH-CZC)/ST
      CZ=1.0
      IF (ABS(CT).LE.0.0001) GO TO 43
      AF2=(SH*CHC-CH*SHC/CT)*CZ/(ST*CH)
      IF ((ABS(CT)-.5).LE.0.0001) GO TO 44
      AF1=(2.0*SH2*CHC-CH2*SHC/CT)*ST*CZ/(CH*(1.0-4.0*CT**2))
      GO TO 42
   44 CONTINUE
C  DISTRIBUTION FOR THE=PI/3 OR 2*PI/3
      AF1=ST*CZ*(BOH-SH)/(2.0*CH)
      GO TO 42
   43 CONTINUE
C  DISTRIBUTION FOR THE=PI/2
      AF1=(2.0*SH2*CHC-CH2*BOH)*ST*CZ/(CH*(1.0-4.0*CT**2))
      AF2=(SH*CHC-CH*BOH)*CZ/(ST*CH)
      GO TO 42
   41 CONTINUE
C  DISTRIBUTION FOR BOH=PI/2
      IF (ABS(CT).LE.0.0001) GO TO 45
      AT=(CZ*CHC*CH-CZC+SH*SHC*CZ/CT)/ST
      CZ=1.0
      AF2=(SH*CHC-CH*SHC/CT)*CZ/(-ST)
      IF ((ABS(CT)-.5).LE.0.0001) GO TO 46
      AF1=(2.0*SH2*CHC-CH2*SHC/CT)*ST*CZ/(1.0-4.0*CT**2)
      GO TO 42
C  DISTRIBUTION FOR THE=PI/3 OR 2*PI/3
   46 CONTINUE
      AF1=ST*CZ*(BOH-SH)/2.0
      GO TO 42
C  DISTRIBUTION FOR THE=PI/2
   45 CONTINUE
      AT=(CZ*CHC*CH-CZC+SH*BOH)/ST
      CZ=1.0
      AF2=(SH*CHC-CH*BOH)*CZ/(-ST)
      AF1=(2.0*SH2*CHC-CH2*BOH)*ST*CZ/(1.0-4.0*CT**2)
   42 CONTINUE
```

```
      FT=(AT+FC(2)*AF2+FC(1)*AF1)/FC(3)
      IF (BOZ.EQ.0.0) GO TO 50
      RETURN
   50 FT=FT/2.0
      RETURN
C  UNLOADED WIRE
    1 IF (ABS(CT).LE.0.0001) GO TO 6
      AF2=(SH*CHC-CH*SHC/CT)/ST
      IF((ABS(CT)-.5).LE.0.0001) GO TO 7
      AF1=(2.0*SH2*CHC-CH2*SHC/CT)*ST/(1.0-4.0*CT**2)
      GO TO 2
    7 AF1=ST*(BOH-SH)/2.0
      GO TO 2
    6 AF1=(2.0*SH2*CHC-BOH*CH2)*ST/(1.0-4.0*CT**2)
      AF2=(SH*CHC-CH*BOH)/ST
    2 FT=30.0*(-AI)*(FC(4)*AF2+FC(5)*AF1)
      RETURN
      END


      SUBROUTINE CUR(BOH,BOA,BOD,X,FC,M,YS)
C  CUR COMPUTES THE CURRENT AT THE POINT X FOR AN ANTENNA OF LENGTH 2*BOH,
C  RADIUS BOA, AND LOADED AT BOD.  THE PARAMETER M CONTROLS THE  TYPE OF
C  DISTRIBUTION, M=0 FOR THE LONG WIRE AND M=1 FOR THE SHORT WIRE.  YS IS THE
C  VALUE OF THE CURRENT
      DIMENSION FC(5)
      COMPLEX E1Z,EI0,R,CD,CU,E2,E1,E2H,EZ,EZ1,EZ2,DE,YS,FC,AI
      AI=CMPLX(0.0,1.0)
      PI=3.14159
      GAMMA=.57721566
      IF (M.EQ.1) GO TO 41
C  LONG WIRE CURRENT
      CW=ALOG(1.0/BOA)-GAMMA
      R=60.0*(2.0*CW+AI*PI)
```

```
C   TEST FOR UNLOADED WIRE
      IF(BOD.LT.0.0) GO TO 2
      Z=X-BOD
      BOH1=BOH+BOD
      BOH2=BOH-BOD
      E2H=EIZ(2.0*BOH,CW)
      E1=EIZ(BOH1,CW)
      E2=EIZ(BOH2,CW)
      EZ=EIZ(Z,CW)
      EZ1=EIZ((BOH1+Z),CW)
      EZ2=EIZ((BOH2-Z),CW)
      DE=R*E2H
      DE=1.0-DE*DE
      CU=-R*(E2-R*E1*E2H)/DE
      CD=-R*(E1-R*E2*E2H)/DE
      EI0=EZ+CD*EZ1+CU*EZ2
      IF (BOD.EQ.0.0) GO TO 6
      YS=CONJG(EI0)
      BOD=-BOD
      Z=X-BOD
      BOH1=BOH+BOD
      BOH2=BOH-BOD
      E1=EIZ(BOH1,CW)
      E2=EIZ(BOH2,CW)
      EZ=EIZ(Z,CW)
      EZ1=EIZ((BOH1+Z),CW)
      EZ2=EIZ((BOH2-Z),CW)
      CU=-R*(E2-R*E1*E2H)/DE
      CD=-R*(E1-R*E2*E2H)/DE
      EI0=EZ+CD*EZ1+CU*EZ2
      EI0=CONJG(EI0)
      YS=EI0+YS
      BOD=-BOD
      RETURN
    6 YS=CONJG(EI0)
      RETURN
```

```
C   UNLOADED WIRE
  2 EZ=AI*(-BOD)/(30.0*BOH*(2.0*CW+ALOG(4.0)+AI*PI))
    E2H=EIZ(2.0*BOH,CW)
    E1=EIZ((BOH-X),CW)
    E2=EIZ((BOH+X),CW)
    DE=60.0*(2.0*CW+ALOG(2.0)+AI*PI)
    YS=EZ*(1.0-DE*(E1+E2)/(1.0+R*E2H))
    YS=CONJG(YS)
    RETURN
C   SHORT WIRE CURRENT
 41 CONTINUE
    SH2=SIN(BOH/2.)
    CH2=COS(BOH/2.)
    SH=SIN(BOH)
    CH=COS(BOH)
    CD=COS(BOD)
    CX=COS(X)
C   TEST FOR UNLOADED WIRE
    IF(BOD.LT.0.0) GO TO 1
C   TEST FOR BOH=PI/2
    IF (ABS(BOH-PI/2.0).LE.0.01) GO TO 77
    EI=CD*SH*CX-CH*(SIN(ABS(X+BOD))+SIN(ABS(X-BOD)))/2.0
    GO TO 78
 77 EI=CD*SH-(SIN(ABS(X+BOD))+SIN(ABS(X-BOD)))/2.0
 78 CONTINUE
    BOH1=CX-CH
    BOH2=COS(X/2.)-CH2
    IF (ABS(BOH-PI/2.0).LE.0.01) GO TO 27
    IF (BOD.EQ.0.0) GO TO 17
    YS=AI*(EI+FC(2)*BOH1+FC(1)*BOH2)/(30.*FC(3)*CH)
    RETURN
 17 YS=AI*(EI+FC(2)*BOH1+FC(1)*BOH2)/(60.*FC(3)*CH)
    RETURN
 27 IF (BOD.EQ.0.0) GO TO 18
    YS=AI*(EI-FC(2)*BOH1+FC(1)*BOH2)/(30.*FC(3))
    RETURN
 18 YS=AI*(EI-FC(2)*BOH1+FC(1)*BOH2)/(60.*FC(3))
```

```
      RETURN
C  UNLOADED WIRE
    1 YS=FC(4)*(CX-CH)+FC(5)*(COS(X/2.)-CH2)
      RETURN
      END



      SUBROUTINE COFD(BOA,BOH,BOD,CHIVS,CHIDIS,CHIDRS)
C  COFD EVALUATES THE INTEGRALS CHI-V, CHI-DI, AND CHI-DR DEFINED BY KING.
      COMPLEX VALINT,CHIVS,CHIDI,CHIDR
      TYPE REAL MOZIS
      PI=6.28319
      CALL GENINTG (BOA,-BOH,-BOD,BOH,-1,VALINT)
      CHIVS=VALINT*SIN(BOH)*COS(BOD)
      CALL GENINTG(BOA,-BOH,-BOD,BOH,1,VALINT)
      CHIVS=CHIVS+VALINT*COS(BOH)*COS(BOD)
      CALL GENINTG(BOA,-BOD,BOD,BOH,-1,VALINT)
       CHIVS=CHIVS+VALINT*SIN(BOH-BOD)
      CALL GENINTG(BOA,BOD,BOH,BOH,-1,VALINT)
      CHIVS=CHIVS+VALINT*SIN(BOH)*COS(BOD)
      CALL GENINTG(BOA,BOD,BOH,BOH,1,VALINT)
      CHIVS=CHIVS-VALINT*COS(BOH)*COS(BOD)
      CALL GENINTG(BOA,-BOH,-BOD,0,-1,VALINT)
      CHIDI=VALINT*SIN(BOH)*COS(BOD)
      CALL GENINTG(BOA,-BOH,-BOD,0,1,VALINT)
      CHIDI=CHIDI+VALINT*COS(BOH)*COS(BOD)
      CALL GENINTG(BOA,-BOD,BOD,0,-1,VALINT)
      CHIDI=CHIDI+VALINT*SIN(BOH-BOD)
      CALL GENINTG(BOA,BOD,BOH,0,-1,VALINT)
      CHIDI=CHIDI+VALINT*SIN(BOH)*COS(BOD)
      CALL GENINTG(BOA,BOD,BOH,0,1,VALINT)
      CHIDI=CHIDI-VALINT*COS(BOH)*COS(BOD)
      CHIDIS=AIMAG(CHIDI-CHIVS)/(1-COS(BOH/2.0))
      CHDRSO=REAL(CHIDI-CHIVS)
```

```
      IF (BOH-BOD-PI/4.0)2,3,3
    2 BOZ=BOD
      GO TO 4
    3 BOZ=BOH-PI/4.0
    4 CALL GENINTG(BOA,-BOH,-BOD,BOZ,-1,VALINT)
      CHIDR=VALINT*SIN(BOH)*COS(BOD)
      CALL GENINTG(BOA,-BOH,-BOD,BOZ,1,VALINT)
      CHIDR=CHIDR+VALINT*COS(BOH)*COS(BOD)
      CALL GENINTG(BOA,-BOD,BOD,BOZ,-1,VALINT)
      CHIDR=CHIDR+VALINT*SIN(BOH-BOD)
      CALL GENINTG(BOA,BOD,BOH,BOZ,-1,VALINT)
      CHIDR=CHIDR+VALINT*SIN(BOH)*COS(BOD)
      CALL GENINTG(BOA,BOD,BOH,BOZ,1,VALINT)
      CHIDR=CHIDR-VALINT*COS(BOH)*COS(BOD)
      CHDSZ=REAL(CHIDR-CHIVS)
      IF (BOH-BOD-PI/4.0) 5,6,6
    5 MOZIS=SIN(BOH-BOD)*COS(BOD)
      GO TO 7
    6 MOZIS=COS(BOD)
    7 A=SIN(BOH-BOD)
      CHIDRS=(ABS(CHDRSO)+ABS(CHDSZ))/(ABS(MOZIS)+ABS(A))
      RETURN
      END


      SUBROUTINE COF (BOA,BOH,CHIUS,CHIDS,CHDURS,CHDUIS,CHIDDS)
C  COF EVALUATES THE INTEGRALS CHI-U, CHI-UR, CHI-UI,AND CHI-DD DEFINED
C  IN KING≠S METHOD.
      COMPLEX VALINT,CHIUS,CHIDS,CHIDDS,CHIDU
      CALL GENINTG(BOA,-BOH,BOH,BOH,-1,VALINT)
      CHIUS=VALINT
      CALL GENINTG(BOA,-BOH,BOH,BOH,0,VALINT)
      CHIUS=CHIUS-VALINT*COS(BOH)
      CHIDS=-VALINT*COS(BOH/2.0)
```

```
      CALL GENINTG(BOA,-BOH,BOH,BOH,2,VALINT)
      CHIDS=CHIDS+VALINT
      CALL GENINTG(BOA,-BOH,BOH,0,-1,VALINT)
      CHIDU=VALINT
      CALL GENINTG(BOA,-BOH,BOH,0,0,VALINT)
      CHIDU=CHIDU-VALINT*COS(BOH)
      CHDURS=REAL(CHIDU-CHIUS)/(1-COS(BOH))
      CHDUIS=AIMAG(CHIDU-CHIUS)/(1-COS(BOH/2.0))
      CHIDDS=-VALINT*COS(BOH/2.0)
      CALL GENINTG(BOA,-BOH,BOH,0,2,VALINT)
      CHIDDS=(VALINT+CHIDDS-CHIDS)/(1-COS(BOH/2.0))
      RETURN
      END



      SUBROUTINE GENINTG (BOA,WER,BOH,BOZ,M,VALINT)
C     GENINTG EVALUATES INTEGRALS OF THE FORM F(X)*EXP(-J*R)/R INTEGRATED
C     FROM WER TO BOH AND EVALUATED AT BOZ WHERE R=SQRT((X-BOZ)**2+BOA**2).
C     F(X) IS CONTROLLED BY M.  FOR M=-1, F(X)=COS(X), FOR M=0, F(X)=1.0, FOR
C     M=1, F(X)=SIN(X). FOR M=2, F(X)=COS(X/2), FOR M=3, F(X)=SIN(X/2), AND FOR
C     M=4, F(X)=X.  VALINT IS THE VALUE OF THE INTEGRAL.
      COMPLEX VALINT
      A=BOH-BOZ
      B=WER-BOZ
      D=BOH+BOZ
      U=BOA*SQRT(3.0)/2.0
      X=A-SQRT(A**2+BOA**2)/2.0
      W=A+SQRT(A**2+BOA**2)/2.0
      Y=D+SQRT(D**2+BOA**2)/2.0
      Z=D-SQRT(D**2+BOA**2)/2.0
      IF (M) 1,2,3
    1 VALINT=COS(BOZ)*(CCI(BOA,A)-CCI(BOA,B))+SIN(BOZ)*(CSO(BOA,B)-CSO
     1  (BOA,A))+CMPLX(0.0,1.0)*(COS(BOZ)*(SCO(BOA,B)-SCO(BOA,A))
     2  +SIN(BOZ)*(SSO(BOA,A)-SSO(BOA,B)))
```

```
      RETURN
    2 VALINT=ALOG((SQRT(A**2+BOA**2)+A)/(SQRT(B**2+BOA**2)+B))
     1  -C(WER,BOH,BOZ,BOA)-CMPLX(0.0,1.0)*S(WER,BOH,BOZ,BOA)
      RETURN
    3 IF (M-2) 4,5,6
    4 VALINT=SIN(BOZ)*(CCI(BOA,A)-CCI(BOA,B))-COS(BOZ)*(CSO(BOA,B)-CSO
     1  (BOA,A))+CMPLX(0.0,1.0)*(SIN(BOZ)*(SCO(BOA,B)-SCO(BOA,A))
     2  -COS(BOZ)*(SSO(BOA,A)-SSO(BOA,B)))
      RETURN
    5 VALINT=ALOG((D+SQRT(D**2+BOA**2))*(A+SQRT(A**2+BOA**2))/BOA**2)*
     1  COS(BOZ/2.0)-CEXP(CMPLX(0.0,1.0)*BOZ/2.0)/2.0*
     2 (C(0.0,X,0.0,U)+C(0.0,Y,0.0,U)+CMPLX(0.0,1.0)*(S(0.0,X,0.0,U)
     3  +S(0.0,Y,0.0,U)))-CEXP(CMPLX(0.0,-1.0)*BOZ/2.0)/2.0*
     4 (C(0.0,Z,0.0,U)+C(0.0,W,0.0,U)+CMPLX(0.0,1.0)*(S(0.0,Z,0.0,U)
     5  +S(0.0,W,0.0,U)))
      RETURN
    6 IF (M-3) 7,7,8
    7 VALINT=ALOG((D+SQRT(D**2+BOA**2))*(A+SQRT(A**2+BOA**2))/BOA**2)*
     1  SIN(BOZ/2.0)+CMPLX(0.0,1.0)*CEXP(CMPLX(0.0,1.0)*BOZ/2.0)/2.0*
     2 (C(0.0,X,0.0,U)+C(0.0,Y,0.0,U)+CMPLX(0.0,1.0)*(S(0.0,X,0.0,U)+S(0
     3.0,Y,0.0,U)))-CMPLX(0.0,1.0)*CEXP(CMPLX(0.0,-1.0)*BOZ/2.0)/2.0*
     4 (C(0.0,Z,0.0,U)+C(0.0,W,0.0,U)+CMPLX(0.0,1.0)*(S(0.0,Z,0.0,U)
     5  +S(0.0,W,0.0,U)))
      RETURN
    8 VALINT=CMPLX(0.0,1.0)*CEXP(CMPLX(0.0,-1.0)*SQRT(A**2+BOA**2))
     1       -CMPLX(0.0,1.0)*CEXP(CMPLX(0.0,-1.0)*SQRT(B**2+BOA**2))
     2 +BOZ*(ALOG((SQRT(A**2+BOA**2)+A)/(SQRT(B**2+BOA**2)+B))
     3  -C(WER,BOH,BOZ,BOA)-CMPLX(0.0,1.0)*S(WER,BOH,BOZ,BOA))
      RETURN
      END
```

```
      FUNCTION SCO(A,U)
C     CALCULATES THE INTEGRAL OF SIN(W)*COS(U)/W FROM 0 TO U WHERE
C     W=SQRT(U**2+A**2).
      B=SQRT(A**2+U**2)+U
      D=SQRT(A**2+U**2)-U
      SCO=(S(0.0,B,0.0,0.0)-S(0.0,D,0.0,0.0))/2.0
      RETURN
      END


      FUNCTION CCI(A,U)
C     CALCULATES THE INTEGRAL OF COS(W)*COS(U)/W FROM 0 TO U WHERE
C     W=SQRT(U**2+A**2).
      D=SQRT(A**2+U**2)-U
      B=SQRT(A**2+U**2)+U
      CCI=(ALOG(B/D)-C(0.0,B,0.0,0.0)+C(0.0,D,0.0,0.0))/2.0
      RETURN
      END


      FUNCTION CSO(A,U)
C     CALCULATES THE INTEGRAL OF COS(W)*SIN(U)/W FROM 0 TO U WHERE
C     W=SQRT(U**2+A**2).
      B=SQRT(A**2+U**2)+U
      D=SQRT(A**2+U**2)-U
      CSO=(S(0.0,B,0.0,0.0)+S(0.0,D,0.0,0.0))/2.-S(0.0,A,0.0,0.0)
      RETURN
      END
```

```
      FUNCTION SSO(A,U)
C   CALCULATES THE INTEGRAL OF SIN(W)*SIN(U)/W FROM 0 TO U WHERE
C   W=SQRT(U**2+A**2).
      B=SQRT(A**2+U**2)+U
      D=SQRT(A**2+U**2)-U
      SSO=(C(0.0,B,0.0,0.0)+C(0.0,D,0.0,0.0))/2.-C(0.0,A,0.0,0.0)
      RETURN
      END



      FUNCTION C(WER,BOH,BOZ,BOA)
C   MODIFIED COSINE INTERGAL - TERM BY TERM SERIES INTEGRATION.
      C=0.0
      X1=BOH-BOZ
      X2=WER-BOZ
      X3=X1**2+BOA**2
      X4=X2**2+BOA**2
      X5=X1/SQRT(X3)
      IF (BOA.EQ.0.0) X6=1.0
      IF (BOA.NE.0.0) X6=X2/SQRT(X4)
      IF (BOA) 1,2,1
    1 C=-ALOG((SQRT(X3)+X1)/(SQRT(X4)+X2))*CA(0,BOA)
    2 IF (BOA.EQ.0.0) B=1.0
      IF (BOA.NE.0.0) B=1.0+CA(1,BOA)
      D1=-X3*X5/4.0*B
      D2=X6*X4/4.0*B
      C=C-D1-D2
      N=1
    3 A=N
      IF (BOA.EQ.0.0) E=1.0
      IF (BOA.NE.0.0) E=1.0+CA((N+1),BOA)
      D1=-X3*D1*A*E/B*2.0/((2.0*A+2.0)**2*(2.0*A+1.0))
      D2=-X4*D2*A*E/B*2.0/((2.0*A+2.0)**2*(2.0*A+1.0))
      C=C-D1-D2
```

```
      N=N+1
      B=E
      IF(ABS(D1+D2).LE.1.0E-10) GO TO 4
      GO TO 3
    4 RETURN
      END



      FUNCTION S(WER,BOH,HOZ,BOA)
C  MODIFIED SINE INTERGAL - TERM BY TERM SERIES INTEGRATION.
      N=0
      X1=BOH-BOZ
      X2=WER-BOZ
      X3=X1**2+BOA**2
      X4=X2**2+BOA**2
      IF (BOA.EQ.0.0) B=1.0
      IF (BOA.NE.0.0) B=SA(0,BOA)
      D1=X1*B
      D2=X2*B
      S=D1-D2
    3 A=N
      IF (BOA.EQ.0.0) E=1.0
      IF (BOA.NE.0.0) E=SA((N+1),BOA)
      D1=-D1*X3*E/B*(2.0*A+1.0)/((2.0*A+3.0)**2*(2.0*A+2.0))
      D2=-D2*X4*E/B*(2.0*A+1.0)/((2.0*A+3.0)**2*(2.0*A+2.0))
      S=S+D1-D2
      B=E
      N=N+1
      IF(ABS(D1-D2).LE.1.0E-10) GO TO 4
      GO TO 3
    4 RETURN
      END
```

```
      FUNCTION SA(M,BOA)
C   COEFFICIENT FOR   SINE INTERGAL INTEGRATION.
      A=BOA**2
      B=M
      SB=1.0
      SA=1.0
      D=0.0
    1 SB=-A*SB/(2.0*(B+D)+3.0)**2
      D=D+1.0
      SA=SA+SB
      IF(ABS(SB).LE.1.0E-10) GO TO 2
      GO TO 1
    2 RETURN
      END
```

```
      FUNCTION CA(M,BOA)
C   COEFFICIENT FOR COSINE INTERGAL INTEGRATION.
      A=(BOA/2.0)**2
      B=M
      CB=1.0
      CA=0.0
      D=0.0
    1 CB=-A*CB/(B+D+1.0)**2
      D=D+1.0
      CA=CA+CB
      IF(ABS(CB).LE.1.0E-10) GO TO 2
      GO TO 1
    2 RETURN
      END
```

```
      FUNCTION Y2(BOA,BOZ)
C   LONG ANTENNA FIELD PATTERN FUNCTION Y2.
      COMPLEX Y2,Z2,Z3,ZZ2,ZZ3
      GP=1.64493407
      ZZ3=Z3(BOA,BOZ)
      ZZ2=Z2(BOA,BOZ)
      Y2=CLOG(ZZ3/ZZ2)
      Y2=Y2+GP/2.0*(1.0/ZZ2**2-1.0/ZZ3**2)
      RETURN
      END
```

```
      FUNCTION Z2(BOA,BOZ)
C   LONG ANTENNA FIELD PATTERN FUNCTION Z2.
      COMPLEX Z2,AI
      GAM=0.57721566
      PI=3.14159
      AI=CMPLX(0.0,1.0)
       Z=2.0*ALOG(1.0/BOA)+ALOG(BOZ+SQRT(BOZ**2+EXP(-2.0*GAM)))-GAM
      Z2=Z-AI*PI/2.0
      RETURN
      END
```

```
      FUNCTION Y3(BOA,BOZ)
C   LONG ANTENNA FIELD PATTERN FUNCTION Y3.
      COMPLEX Y3,AI,Z2,Z3,ZZ2,ZZ3
      PI=3.14159
      AI=CMPLX(0.0,1.0)
      CE=CEXP(2.0*AI*BOZ)
      B=BOZ+SQRT(BOZ**2+(2.0/PI)**2)
```

```
      ZZ2=Z2(BOA,BOZ)
      ZZ3=Z3(BOA,BOZ)
      Y3=AI*CE/B*(1.0/ZZ2-1.0/ZZ3)
      RETURN
      END


      FUNCTION Z3(BOA,BOZ)
C   LONG ANTENNA FIELD PATTERN FUNCTION Z3.
      COMPLEX  Z3,Z2,AI
      AI=CMPLX(0.0,1.0)
      PI=3.14159
      Z3=Z2(BOA,BOZ)
      Z3=Z3+2.0*PI*AI
      RETURN
      END


      FUNCTION CFS(BOA,BOH,BOZ)
      COMPLEX CFS,AI,Z1,Y1,DS,Z2,Z3,Y2,Y3
      COMPLEX CE,CZP,CZM,YH2,YH3,Y2P,Y2M,Y3P,Y3M
C   CFS CALCULATES THE COEFFICIENT FOR THE PARTICLAR SOLUTION OF THE LONG
C   ANTENNA FIELD PATTERN.
      PI=3.14159
      GAM=0.57721566
      AI=CMPLX(0.0,1.0)
      Z1=ALOG(2.0/BOA)-GAM+AI*PI/2.0
       A=ALOG(2.0)
      Y1=PI*AI/(Z1-A)
      CE=CEXP(-2.0*AI*BOH)
C   TEST FOR THE LONG UNLOADEDD WIRE.
```

```
      IF(BOZ.LT.0.0) GO TO 1
      CZP=CEXP(+AI*BOZ)
      CZM=CEXP(-AI*BOZ)
      CS=COS(BOZ)
      YH2=Y2(BOA,2.0*BOH)
      YH3=Y3(BOA,2.0*BOH)
      Y2P=Y2(BOA,(BOH+BOZ))
      Y3P=Y3(BOA,(BOH+BOZ))
      Y3M=Y3(BOA,(BOH-BOZ))
      Y2M=Y2(BOA,(BOH-BOZ))
      DS=2.0*AI*(YH2+Y1*CE+YH3*CE)
      CFS=(CS*(Y1*CE+YH2-YH3*CE)+(Y3P*CE-Y2P)*CZP+(Y3M*CE-Y2M)*CZM)/DS
      RETURN
    1 Y2M=2.0*Z1-A
      Y3M=CEXP(-AI*BOH)
      YH2=Y2(BOA,2.0*BOH)
      YH3=Y3(BOA,2.0*BOH)
      DS=AI*(YH2+Y1*CE+YH3*CE)
      CFS=Y3M*(-2.0*AI*YH3+2.0*PI/Y2M)/DS
      RETURN
      END



      SUBROUTINE H(N,M,DEL,X)
C   H FINDS BOTH THE X AND X INVERSE MATRICES FOR THE LINEARIZED METHOD.
      DIMENSION D(8),DEL(8),X(8,16)
      NM=N-1
      NP=N+1
      A=DEL(1)**2
      D(1)=1.0/(SQRT(A+DEL(2)**2)*DEL(1))
      IF(NM.LT.2) GO TO 6
      DO 1 I=2,NM
      A=DEL(I)**2+A
      D(I)=1.0/SQRT(A*(A+DEL(I+1)**2))
```

```
    1 CONTINUE
    6 CONTINUE
      D(N)=1.0/SQRT(A+DEL(N)**2)
C   X MATRIX
      DO 2 I=1,NM
      DO 2 J=I,NM
      X(I,J)=D(J)/D(N)*DEL(I)*DEL(J+1)
    2 CONTINUE
      DO 3 I=1,N
      X(I,N)=-DEL(I)
    3 CONTINUE
      A=0.0
      DO 4 I=1,NM
      J=I+1
      A=DEL(I)**2+A
      X(J,I)=-A*D(I)/D(N)
    4 CONTINUE
      IF (N.LT.3) GO TO 8
      DO 5 I=3,N
      I2=I-2
      DO 5 J=1,I2
      X(I,J)=0.0
    5 CONTINUE
    8 CONTINUE
C   X INVERSE MATRIX
      DO 7  I=1,N
      DO  7 J=NP,M
      JJ=J-N
    7 X(I,J)=X(JJ,I)*D(N)*D(N)
      RETURN
      END
```

```
      SUBROUTINE YNONL(N,N2,NH,NT,A,CL,W,FX,ICON)
C     YNONL SOLVES N NON-LINEAR EQUATIONS WITH N UNKNOWNS.
      DIMENSION XK(8,8),X(8,16),XKM(10),XKN(10),AN(32),F(8,8),DEL(8)
      DIMENSION A(5,32),CL(16),W(5),WM(5),FX(8),FE(2)
      DIMENSION JC(32)
      DOUBLE PRECISION XK,XKM,XKN,FE,F,B,FX,WCP
      COMPLEX A,AN
      NCP=(N-NT+1)*2
      NPC=(N-NT)*2
C     TEST FOR N2=2
      IF (NPC.EQ.0) GO TO 92
      NR=0
C     TEST FOR REAL PART OF ADMITTANCES EQUAL TO ZERO.
      IF((NH*NT).EQ.N/2) NR=1
      IF(NR.EQ.0) N=N*2
      IF(NR.NE.1) GO TO 75
C     SET REAL PART EQUAL TO ZERO
      DO 76 I=1,N
      K=2*I-1
   76 CL(K)=0.0
   75 CONTINUE
      NS=1+NR
      NI=N*NS
      NS2=NS*2
      NCM=NCP-1
      JNI=NI/2
      ICO=ICON
      NP=N+1
      NM=2*N
      IC=1
      ICON=1
C     TEST FOR FREQUENCY DEPENDENCE
      IF (ICO) 70,70,71
C     FREQUENCY  NORMALIZATION
   70 DO 72 I=1,NH
   72 WM(I)=1.0
      GO TO 73
```

```
   71 CONTINUE
      WM(1)=1.0
      IF (NH.LE.1) GO TO 46
      DO 45 I=2,NH
   45 WM(I)=W(I)/W(1)
   73 CONTINUE
   46 CONTINUE
      NE=N/2
      E=CABS(A(1,N2))
      IF(NE.EQ.1) GO TO 85
      DO 41 I=2,NE
      D=CABS(A(I,N2))
      IF (D.LT.E) E=D
   41 CONTINUE
   85 CONTINUE
      E=E*1.0E-06
   43 CONTINUE
C  MINIMIZATION METHOD
      FXI=FX(1)
      IF(FX(1).EQ.1.0) GO TO 220
      D=200.1
      ICON=ICON+1
      DO 204 JI=NT,JNI
      IF(JI.EQ.NT) INS=NCP/2
      IF(JI.GT.NT) INS=JI-NT
C  LINEARIZATION INTEGERS JC
      CALL TRI(N2,INS,JC)
      AS=0.0
      AB=0.0
      ABR=0.0
      B=0.0
      DO 200 K=1,NT
      NHH=NPC+2*K
      DO 210 KK=1,NH
      WC=WM(KK)
      NW=(KK-1)*NT+K
      DO 201 J=2,NPC,2
```

```
      II=J-1
      IF(CL(J)) 202,203,203
  202 XKM(J)=CL(J)/WC
      GO TO 201
  203 XKM(J)=CL(J)*WC
  201 XKM(II)=CL(II)
      IF(CL(NHH)) 205,206,206
  205 XKM(NCP)=CL(NHH)/WC
      GO TO 207
  206 XKM(NCP)=CL(NHH)*WC
  207 XKM(NCM)=CL(NHH-1)
      DO 208 J=1,N2
      JM=JC(J)
  208 AN(J)=A(NW,JM)
      XKN(1)=XKM(1)
      XKN(2)=XKM(2)
      J=2*INS
      JJ=J-1
C  ORDERING OF UNKNOWNS
      XKM(1)=XKM(JJ)
      XKM(2)=XKM(J)
      XKM(JJ)=XKN(1)
      XKM(J)=XKN(2)
      FE(1)=10.0
      FE(2)=NR
C  CALCULATION OF LINEARIZATION COEFFICIENTS
      CALL FESR(AN,XKM,NCP,N2,FE)
      B=FE(1)**2+FE(2)**2+B
      AS=AS+REAL(AN(1))
      AB=AB+AIMAG(AN(1))
      IF(NR.EQ.1) GO TO 210
      ABR=ABR+REAL(AN(2))
  210 CONTINUE
      IF(AS.LT.1.0E-50) AS=1.0E-50
      IF(INS.EQ.NCP/2) GO TO 209
      GO TO 200
```

```
C  CALCULATION OF UNKNOWNS
  209 C=CL(NHH)
      CL(NHH)=-AB/AS
      IF(ABS(C/(CL(NHH)-C)).LT.D)D=ABS(C/(CL(NHH)-C))
      IF(NR.EQ.1) GO TO 200
      NHH=NHH-1
      C=CL(NHH)
      CL(NHH)=-ABR/AS
      IF(ABS(C/(CL(NHH)-C)).LT.D)D=ABS(C/(CL(NHH)-C))
  200 CONTINUE
      IF(INS.EQ.NCP/2) GO TO 204
      INS=INS*2
      C=CL(INS)
      CL(INS)=-AB/AS
      IF(ABS(C/(CL(INS)-C)).LT.D)D=ABS(C/(CL(INS)-C))
      IF(NR.EQ.1) GO TO 204
      INS=INS-1
      C=CL(INS)
      CL(INS)=-ABR/AS
      IF(ABS(C/(CL(INS)-C)).LT.D)D=ABS(C/(CL(INS)-C))
  204 CONTINUE
      PRINT 51,CL
      G=B
      G=SQRT(G)
      C=E*1.0E+05
      PRINT 16,G,FE
C  TEST FOR CONVERGENCE
      IF(G.LT.C) GO TO 220
      IF(ICON.GT.100) GO TO 40
      IF(D.LT.200.0) GO TO 43
  220 CONTINUE
C  MODIFIED NEWTON-RAP. METHOD
C  CALCULATION OF INITIAL X MATRIX
      DO 10 I=1,N
      X(I,I)=1.0E-06
      IP=I+1
      IF (IP.GT.N) GO TO 10
```

```
      DO 10 J=IP,N
      X(I,J)=0.0
      X(J,I)=0.0
   10 CONTINUE
      DO 50 I=1,N
      DO 50 J=NP,NM
      JJ=J-N
   50 X(I,J)=X(JJ,I)
      DO 52 I=1,N
      JJ=I+N
   52 X(I,JJ)=1.0/X(I,JJ)
   20 CONTINUE
      DO 1 I=1,N
      K=I*(1+NR)
      DO 1 J=1,N
      XK(I,J)=CL(K)+X(I,J)
    1 CONTINUE
      DO 2  KK=1,NH
      WC=WM(KK)
      WCP=WC
      DO 2   K=1,NT
      NW=(KK-1)*NT+K
      JJ=NW*2
      JM=JJ-1
      NHH=NPC+2*K
      DO 30  I=2,NPC,2
      II=I-1
      IF (CL(I)) 31,32,32
   31 XKM(I)=CL(I)/WC
      GO TO 30
   32 XKM(I)=CL(I)*WC
   30 XKM(II)=CL(II)
      IF(CL(NHH)) 61,62,62
   61 XKM(NCP)=CL(NHH)/WC
      GO TO 65
   62 XKM(NCP)=CL(NHH)*WC
   65 XKM(NCM)=CL(NHH-1)
```

```
      IF (ICON.EQ.1) PRINT 16,WC,XKM
      DO 4 I=1,N2
      AN(I)=A(NW,I)
    4 CONTINUE
      FE(1)=0.0
C   CALCULATION OF FUNCTIONS AT THE POINT
      CALL FESR(AN,XKM,NCP,N2,FE)
      FX(JM)=FE(1)
      FX(JJ)=FE(2)
      DO 2 J=1,N
      DO 3   I=2,NPC,2
      II=I-1
      INS=I/NS
      IF(CL(I)) 33,34,34
   33 XKN(I)=XK(INS,J)/WCP
      GO TO 100
   34 XKN(I)=XK(INS,J)*WCP
  100 IF(NR.EQ.1) XKN(II)=0.0
    3 IF(NR.EQ.0) XKN(II)=XK((INS-1),J)
      NNS=NHH/NS
      IF(CL(NHH)) 63,64,64
   63 XKN(NCP)=XK(NNS,J)/WCP
      GO TO 66
   64 XKN(NCP)=XK(NNS,J)*WCP
   66 IF(NR.EQ.1) XKN(NCM)=0.0
      IF(NR.EQ.0) XKN(NCM)=XK((NNS-1),J)
      FE(1)=0.0
C   CALCULATION OF F MATRIX
      CALL FESR(AN,XKN,NCP,N2,FE)
      F(JM,J)=FE(1)-FX(JM)
      F(JJ,J)=FE(2)-FX(JJ)
    2 CONTINUE
C   CALCULATION OF  INVERSE JACOBIAN
      DO 5 I=1,N
      DO 5 J=1,N
      B=0.0
      JJ=J+N
```

```
      DO 6 K=1,N
      B=F(I,K)*X(K,JJ)+B
    6 CONTINUE
      XK(1,J)=B
    5 CONTINUE
      CALL MATPAC(0,8,8,XK,N,N,DET,1.0E-150)
      IF(ICON.EQ.5) PRINT 51,X,F,XK
C     CALCULATION OF DIFFERENCE VECTOR
      DO 7 I=1,N
      B=0.0
      DO 8 J=1,N
      B=-XK(I,J)*FX(J)+B
    8 CONTINUE
      DEL(I)=SNGL(B)
    7 CONTINUE
C     CALCULATION OF NEW VARIABLES
      DO 9 I=NS,NI,NS
      J=I/NS
      CL(I)=CL(I)+DEL(J)
    9 CONTINUE
      PRINT 51,DEL
C     TEST FOR CONVERGENCE
      IF (ABS(DEL(1)).LT.1.0E-50) DEL(1)=1.0E-50
      C=CL(NS)/DEL(1)
      C=C-1
      C=ABS(C)
      DO 17 I=NS2,NI,NS
      J=I/NS
      IF (ABS(DEL(J)).LT.1.0E-50) DEL(J)=1.0E-50
      D=CL(I)/DEL(J)
      D=D-1
      D=ABS(D)
   17 IF (D.LT.C) C=D
      IF (C.GT.1.0E+9) GO TO 11
      IF(IC.LT. 5) GO TO 38
      PRINT 12,FX,DEL
      PRINT 51,CL
```

```
      PRINT 12,XKM
      IC=0
   38 CONTINUE
      IC=IC+1
   12 FORMAT(8D16.6,//8E16.8,/8E16.6/)
   16 FORMAT(E14.4,/10D12.4/)
   51 FORMAT(8E16.6,/8E16.6,/8E16.6,/8E16.6,/8E16.6,/8E16.6,/8E16.6,/
     1       8E16.6,/8E16.6,/8E16.6,/8E16.6,/8E16.6,/8E16.6,/8E16.6,/
     2      8E16.6,/8E16.6,//8D16.6,/8D16.6,/8D16.6,/8D16.6,/8D16.6,/
     3       8D16.6,/8D16.6,/8D16.6,/8D16.6,/8D16.6,/8D16.6,/8D16.6,/
     4       8D16.6,/8D16.6,/8D16.6,/8D16.6)
      ICON=ICON+1
      IF (ICON.GT.100) GO TO 40
      IF(FXI.EQ.1.0) GO TO 400
C  TEST FOR DIVERGENCE
      IF(C.LT.0.2) GO TO 300
  400 CONTINUE
C  CALCULATION OF NEW X MATRIX
      CALL H(N,NM,DEL,X)
      GO TO 20
   11 CONTINUE
      PRINT 51,X,F,XK
      PRINT 51,CL
      PRINT 12,FX,DEL
      ICON=ICON+1
      IF (ICON.GT.100) GO TO 40
      IC=IC+1
C  TEST FOR A SOLUTION
      G=DABS(FX(1))
      DO 42 I=2,N
      D=DABS(FX(I))
   42 IF(D.LT.G) G=D
      IF (G.LT.E) GO TO 40
      GO TO 43
   40 CONTINUE
      IF (ICO.EQ.0) GO TO 74
C  DENORMALIZATION OF VARIABLES IN THE  FREQUENCY DEPENDENT  CASE
      IF(NR.EQ.0) NM=NM/2
```

```
      DO 44 I=2,NM,2
      IF (CL(I)) 47,48,48
   47 CL(I)=CL(I)*W(1)
      GO TO 44
   48 CL(I)=CL(I)/W(1)
   44 CONTINUE
   74 CONTINUE
      IF(NR.EQ.0) N=N/2
      RETURN
C     CALCULATION OF  ADMITTANCES IF N2=2
   92 DO 93 I=1,N
      J=2*I
      JJ=J-1
      CL(J)=AIMAG(-A(I,2)/A(I,1))
   93 CL(JJ)=REAL(-A(I,2)/A(I,1))
      RETURN
  300 DO 301 I=NS,NI,NS
      J=I/NS
  301 CL(I)=CL(I)-DEL(J)
      GO TO 40
      END


      SUBROUTINE FESR(AN,X,N,N2,FE)
C     FESR CALCULATES THE VALUE OF THE NON-LINEAR FUNCTION AT THE VECTOR POINT X.
      DIMENSION B(32,2),X(10),AN(32),FE(2)
      COMPLEX AN
      DOUBLE PRECISION FE,B,X
      DOUBLE PRECISION GOOF
      J=N
      M=N2
      DO 1 I=1,M
      B(I,1)=REAL(AN(I))
    1 B(I,2)=AIMAG(AN(I))
```

```
      M=M/2
    2 DO 3 I=1,M
      K=2*I-1
       GOOF=B(K,1)*X(J-1)-B(K,2)*X(J)+B((K+1),1)
      B(I,2)=B(K,1)*X(J)+B(K,2)*X(J-1)  +B((K+1),2)
    3 B(I,1)=GOOF
      J=J-2
      M=M/2
      IF(J.EQ.2) GO TO 5
      IF (J) 4,4,2
    4 FE(1)=B(1,1)
      FE(2)=B(1,2)
      RETURN
C  TEST  FOR LINEARIZATION
    5 IF(FE(1).NE.10.0) GO TO 2
C  TEST FOR REAL PART OF IMPEDANCE ZERO
      IF(FE(2).EQ.1.0) GO TO 6
C  CALCULATE COEFFICIENTS FOR THE NON-LINEAR FUNCTION, LINEARIZED IN THE
C  I TH VARIABLE.
      A=B(1,1)**2+B(1,2)**2
      C=B(1,1)*B(2,1)+B(1,2)*B(2,2)
      D=B(2,2)*B(1,1)-B(2,1)*B(1,2)
      AN(1)=CMPLX(A,D)
      AN(2)=CMPLX(C,A)
      GO TO 2
    6 A=B(1,1)**2+B(1,2)**2
      C=B(2,2)*B(1,1)-B(2,1)*B(1,2)
      AN(1)=CMPLX(A,C)
      GO TO 2
      END
```

```
      SUBROUTINE TRI(N2,I,IC)
C   TRI PRODUCES INTEGERS FROM 1 TO N2 SO THAT THE COEFFICIENTS OF THE NON-
C   LINEAR EQUATIONS ARE ORDERED TO MAKE THEM LINEAR IN THE I TH VARIABLE.
      DIMENSION IC(32)
      J=N2/2**I
      K=N2/2
      DO 2 L=1,N2
    2 IC(L)=L
      IF(K.EQ.J) GO TO 3
      I2=2**I/4
      DO 1 M=1,I2
      MA=J+J*(M-1)*2
      MB=K+J*(M-1)*2
      DO 1 N=1,J
      NB=MA+N
      NC=MB+N
      NA=IC(NB)
      IC(NB)=IC(NC)
    1 IC(NC)=NA
    3 RETURN
      END
```