A DEPOSIT-RUN MODEL OF MEMBER-BANK BORROWING

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ABSTRACT

A DEPOSIT-RUN MODEL OF MEMBER-BANK BORROWING

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The demand for member-bank borrowings from the Federal Reserve System is one of the links which connects the actions of the monetary authority to the stock of money. The purpose of this dissertation is to build and test a model that will identify and measure the variables that determine the volume of borrowings under uncertainty.

The theoretical structure used to derive the demand for borrowed reserves from the Federal Reserve System is based on a profit maximizing one-state optimal inventory model under uncertainty. Given a set of subjective probability distributions regarding reserve flows and interest rates, the bank derives the optimal quantity of reserves to hold that will minimize expected losses. The demand for borrowed reserves is then derived from the difference between the actual and desired stock of reserves. The model was tested with the monthly data for all member-banks from 1954 to 1967. All data on member-bank reserves, borrowings, and interest rates were taken from the various issues of the Federal Reserve Bulletin. The estimation procedure employed a distributed lag scheme to derive expected values for the independent arguments while a multiple regression analysis was used to determine the coefficients and stability of the demand function.

The empirical results show that a demand function for member-bank borrowings can be isolated from the monetary data. The most striking

and important aspect of these results is that the strength and significance of the interest rate variables have undergone a secular decline from 1954 to 1967. Evidence is provided that shows this behavior of the interest rate variables to be related to the rapid growth in federal funds trading as an alternative method of reserve adjustment.

From the viewpoint of monetary policy the evidence developed here supports the proposal that the non-price barriers to borrowing imposed by Regulation A be removed and that the Federal Reserve discount rate be tied to the Treasury bill yield. The enactment of these recommendations should slow or eliminate the current erosion of discounting by federal funds and should make discounting a more accurate and potent tool of monetary policy.

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Peter A. Formuzis, Jr.

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INTRODUCTION

The link between the monetary authority and the stock of money depends in large measure on the particular reaction of commercial banks to variables that affect their behavior. Economists' understanding of these links between Federal Reserve policy and bank behavior has quickened of late due to two major factors. The first has been the development of the theory of choice under uncertainty and the second is the widespread availability and use of the high-speed electronic computer.[1]

The theory of choice under uncertainty made its first entry into the theory of bank operation with an article by F. Y. Edgeworth on the mathematical theory of banking in 1888.[2] Edgeworth built a model, to be presented in more detail in Chapter 1, where the only uncertain factor facing the bank was the amount of daily cash deposits and withdrawals defined by a normally distributed density function. From this information Edgeworth was able to derive the bank's demand for cash given some probability of not being able to meet the cash demands of the bank's depositors. Little was done to extend Edgeworth's analysis into a more representative theory of bank behavior until after the publication of the seminal article on inventory policy under uncertainty by Kenneth Arrow, Theodore Harris, and Jacob Marchak, in 1951.[3]

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[1]Some of the relevant contributions on the theory of choice under uncertainty are J. Tobin, "Liquidity Preference as Behavior Toward Risk," Review of Economic Studies, XXV (February, 1958); H. Markowitz, Portfolio Selection (New York: John Wiley & Sons, 1959); D. Farrar, The Investment Decision Under Uncertainty, Ford Foundation Doctoral Dissertation (Prentice-Hall, 1962).

[2]F. Y. Edgeworth, "The Mathematical Theory of Banking," Journal of the Royal Statistical Society, LI (1888), pp. 113-27.

[3]K. Arrow, T. Harris, and J. Marchak, "Optimal Inventory Policy," Econometrica, XIX (1951), pp. 250-72.

Since 1951, a number of articles and books have emerged encorporating these developments into the theory of bank operation.[4] George Morrison has called the synthesis of these developments ". . . the existing formal theory of banking," and has stated its basic assumptions in the following way:

> (a) banks maximize expected profits (or minimize expected losses), (b) banks construct probability distributions of gains and losses from investment in assets, and (c) following the lead of Edgeworth, profit maximization takes place subject to a specified distribution of cash drains during the planning period.[5]

This existing formal theory of banking will be used as the base on which to build and test a model that will identify and measure the variables that determine the volume of member bank borrowing from the Federal Reserve under uncertainty. The theoretical model and method of estimation will be derived largely from the works of Samuel Karlin, Phillip Cagen, and Milton Friedman.[6] In brief, the model will view the amount of reserves that a bank has available for cash withdrawals and other clearing drains as the inventory of the bank. The hypothesis will then require the bank to choose a stock of reserves at the beginning of the

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[4]S. Karlin, "One State Inventory Models with Uncertainty," in K. Arrow, S. Karlin, and H. Scarf (eds.), Studies in the Mathematical Theory of Inventory and Production (Stanford, Calif.: Stanford University Press, 1958), pp. 109-34; R. C. Porter, "A Model of Bank Portfolio Selection," Yale Economic Essays, I (1961), pp. 322-59; D. Orr and W. G. Mellon, "Stochastic Reserve Losses and Expansion of Bank Credit," American Economic Review, LI (September, 1961), pp. 614-23; G. Morrison, Liquidity Preferences of Commercial Banks (Chicago: University of Chicago Press, 1966).

[5]G. Morrison, op. cit., p. 8.

[6]Karlin, op. cit., pp. 109-34; P. Cagen, "The Monetary Dynamics of Hyperinflation," in M. Friedman (ed.) Studies in the Quantity Theory of Money (Chicago: University of Chicago Press, 1958), pp. 25-117; M. Friedman, A Theory of the Consumption Function (Princeton: Princeton University Press, 1957).

period so as to minimize expected losses from holding reserves. If the existing stock of reserves at the beginning of the period is less than the optimum stock, the bank will equate actual and desired holding of reserves by either borrowing reserves from the Federal Reserve Bank, making some portfolio adjustment, or both.

The model will be tested with monthly time-series data for member banks from 1954-67. The statistical methods used will be a distributed lag hypothesis to estimate expected values for bank reserves and interest rates, and a multiple regression analysis to determine the coefficients and the stability of the demand function for borrowed reserves.

The plan of the dissertation is as follows. The first chapter will develop the theoretical model; the second chapter will develop the estimation procedure and describe the data to be used; the third chapter will present the empirical evidence supplied by the econometric model and its relation to other empirical work performed on member-bank borrowing; and the fourth chapter will summarize the model and the conclusions from the empirical tests.

CHAPTER I

THE THEORETICAL MODEL OF MEMBER-BANK BORROWING

F. Y. Edgeworth was the first economist to view a bank's demand for cash reserves to hold as a problem of holding an optimal stock of inventory under uncertainty.[1] In his model he assumed a large number of depositors, each of whose deposits in some part may be withdrawn during the next period. The amount withdrawn is a random variable X', normally distributed, with mean μ, and standard deviation σ. The bank then chooses some probability ρ of not being able to meet all cash withdrawals. In terms of a cumulative-distribution function, the probability of not being able to meet all cash withdrawals is the area to the left of any specified value on the density function. For the assumed density function, the normal deviate Z is given by $X\text{-}\mu/\sigma$ so that if X is the amount of cash on hand, then $X = \mu + \sigma Z$. The actual cash is a random variable $X\text{-}X'$ so that $X\text{-}X' = \mu\text{-}X' + Z\sigma$ whose expectation is $Z\sigma$. This quantity is then the safety allowance needed by the bank against cash withdrawals given the above specifications of the particular density function and the bank's chosen probability ρ of not being able to meet the withdrawals.[2]

Edgeworth's model is an incomplete specification of the actual inventory problem facing the bank in that it neglects other clearing drains and other cost and revenue considerations. We will now extend Edgeworth's model so as to obtain a more complete description of an hypothesis that derives the equilibrium quantity of reserves to hold in inventory against

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[1]F. Y. Edgeworth, _op. cit._, pp. 113-27.

[2]The following presentation of Edgeworth's mathematical banking model is taken from Arrow's chapter on the "Historical Background," in K. Arrow, S. Karlin, and H. Scarf, _op. cit._, p. 7.

cash withdrawals and clearing drains in an uncertain world. This development will depend on at least the four following assumptions. (1) The bank maximizes expected utility which is a function of expected profits; (2) there are only two earning assets which the bank can purchase. These are represented by loans and securities, where all loans and securities are homogeneous and have a single rate of return for each; (3) supplements to the actual stock of reserves at the beginning of the period can only come from the Federal Reserve through borrowing or through the sale of securities;[3] (4) the time horizon of the model is only one period.

The total amount of reserves available for cash withdrawals and clearing drains that the bank has at the beginning of the period is (y) which may have been supplemented by borrowing (B) from the Federal Reserve at a cost (c), where $c = a + r_d B$. (a) represents some fixed cost of borrowing and (r_d) can be taken as the Federal Reserve discount rate. The amount of reserves on hand before (B) was added to inventory was then $x = y - B$. During the period, demands on inventory can come from three sources. These are cash and deposit withdrawals, loan demand by the nonbank public, and the bank's own demand for securities. If the sum of these three demands is (z), then two possibilities arise. The first is if $y - z > 0$. In this case an opportunity cost will be incurred which depends on the loans and securities that would have been held if $y - z = 0$. In order to determine the appropriate opportunity cost, it is necessary to know what portion of $y - z > 0$ would have been in loans and what portion of $y - z > 0$ would have been in securities, assuming

[3]This assumption implies the absence of the Federal funds market as an additional source of reserves.

that their rates of return are different. This matter of portfolio choice will be handled by the introduction of a portfolio parameter (Q). (Q) is the proportion of reserves that would have been allocated toward loans. A simplifying assumption being made here is that the portfolio parameter (Q) is taken as datum and is constant for all relative and absolute levels of interest rates. This neglects changing portfolios between loans and securities but is justified on the basis of expositional simplicity and will neither be further explored or tested in the remainder of the dissertation. From the above information we will take the opportunity cost of not purchasing loans and securities as being represented by the term $r_m[Q(y-z)] + r_s[(1-Q)(y-z)]$, where (r_m) is the market rate of interest of loans and (r_s) is the interest rate on securities. Also if $y - z > 0$, some portion (k) of total demands (z) went toward earning assets while (1-k) took the form of cash withdrawals and certain clearing drains. The bank will then earn a positive gain (negative loss) on (kz) divided between loans and securities. This will be represented by the term $-r_m[Qkz] - r_s[(1-Q)kz]$.

The second possibility arises when $z - y > 0$. In this case, a penalty cost is incurred which is represented by the function $p(z-y)$. The penalty function (p) can be thought of as being composed of two parts. The first part is composed of the additional borrowing necessary from the Federal Reserve in order to meet some legal minimum of average reserves to be held over the accounting period. The second cost included in the penalty function is that some additional cost may be imposed by depositors who had expected a loan and had included this expectation as a part of the expected marginal revenue of demand deposit money and time deposit bonds. If we assume that individuals were in equilibrium at the old expected marginal revenues of cash, demand deposits, and time deposits, they cannot be in equilibrium any longer and thus will reduce their deposits in accord with the now lower marginal revenue of deposit money until new cash-demand deposit and cash-time deposit ratios are achieved at which individuals are again indifferent to additional increments of either type of money.[4] Also, if $z - y > 0$, a negative loss (positive gain) will be earned from the loans demanded and securities purchased. This will be represented by the term $-r_m[Qky] - r_s[(1-Q)ky]$.

If we combine the information in the two preceding paragraphs we can write the expected loss function of holding an inventory of reserves (y), where $\phi(z,r_m,r_s)$ is a continuous joint probability density function and where all functions are continuous and at least twice differentiable as

$$E[L(y)] = a + r_d(y-x) + \int_o^y\int_o^\infty\int_o^\infty\{r_m[Q(y-z)] + r_s[(1-Q)(y-z)]$$

$$- r_m[Qkz] - r_s[(1-Q)kz]\}\phi(z,r_m,r_s)dz\,dr_m\,dr_s + \int_y^\infty\int_o^\infty\int_o^\infty$$

$$\{P(z-y) - r_m[Qky] - r_s[(1-Q)ky]\}\phi(z,r_m,r_s)dz\,dr_m\,dr_s \qquad (1.1)$$

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[4]For an interesting analysis of this particular way of looking at the cash-deposit ratio, see B. P. Pesek and T. R. Saving, Money, Wealth, and Economic Theory (New York: The MacMillian Company, 1967), pp. 97-99.

The problem now is to find the optimum amount of borrowing $\hat{B}$ where $\hat{B} = \hat{y} - x$ for all (x) given, such that E[L(y)] is at a minimum for all $y > 0$. This requires that the first derivative of E[L(y)] vanish and second derivative be greater than zero.[5] The derivative of (1.1) set equal to zero is

$$\frac{d}{dy}E[l(y)] = r_d + \int_0^\infty\int_0^\infty\{r_m[Q(y-z)] + r_s[(1-Q)(y-z) - r_mQkz - r_s(1-Q)kz -$$

$$P(z-y) - r_mQky - r_s(1-Q)ky\}\phi(r_m,r_s,y)dr_m dr_s + \int_0^y\int_0^\infty\int_0^\infty\{r_mQ + r_s(1-Q)\}$$

$$\phi(r_m r_s z)dr_m dr_s dz + \int_y^\infty\int_0^\infty\int_0^\infty\{-k[r_mQ + r_s(1-Q)] - P\}\phi(r_m,r_s,z)dr_m dr_s dz = 0 \quad (1.2)$$

The terms in (1.2) represent the various expected marginal revenues and marginal costs from holding an inventory of reserves. If we expand and rearrange (1.2) by placing all the expected marginal cost terms on the left side of the equation and the expected marginal revenue terms on the right side of the equation we can state the profit maximizing condition in the more familiar "marginal cost equals marginal revenue" form. This rearrangement of terms is shown as

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[5]One must be cautious in taking the above derivative by use of the common form of the Fundamental Theorem of Calculus since (y) appears as a limit of integration. See Appendix A for this computation along with a discussion of certain internal relations of the function.

$$r_d + \int_0^{\infty}\int_0^{\infty}\{-r_s(1-Q)kz + r_m(1-Q)ky + r_s(1-Q)ky\}\phi(r_m,r_s,y)dr_m dr_s +$$

$$\int_0^{y}\int_0^{\infty}\int_0^{\infty}\{-r_mQk - r_s(1-Q)k\}\phi(r_m,r_s,z)dr_m dr_s dz + \int_y^{\infty}\int_0^{\infty}\int_0^{\infty}\{-kr_mQ - kr_s(1-Q)\}$$

$$\phi(r_m,r_s,z)dr_m dr_s dz = \int_0^{\infty}\int_0^{\infty}\{r_mQ(y-x) + r_s(1-Q)(y-z) - P(z-y)\}\phi(r_m,r_s,y)dr_m dr_s$$

$$+ \int_0^{y}\int_0^{\infty}\int_0^{\infty}\{-P\}\phi(r_m,r_s,z)dr_m dr_s dz + \int_y^{\infty}\int_0^{\infty}\int_0^{\infty}-P\phi(r_m,r_s,z)dr_m dr_s dz \qquad (1.3)$$

As long as the minimized value of (1.1) yields a value of the optimum amount of reserves ($\hat{y}$) such that $\hat{y} > x$, then $\hat{y} - x = \hat{B}$ which is the optimum amount of borrowing that will minimize the expected losses of the bank.

Thus far the model has been developed without providing empirical counterpart for the reserves variable (y) and (x). The (x) quantity of reserves is that quantity of reserves on hand at the beginning of the period before any borrowing has taken place that is available to meet both expected and unexpected clearing drains. As a first approximation, unborrowed reserves would serve as an appropriate variable. Using the following accounting identities between total reserves (R), borrowed reserves B, unborrowed reserves (R_u), required reserves R_R, and excess reserves R_E, we can write

$$R_u = R_R + R_E - B$$

$$R_u = R - B$$

R_u is the total amount of reserves supplied to banks which are not within the control of banks. R_u includes currency, the gold stock, Treasury balances, and the Federal Reserve holdings of securities. R_R, R_E, and B are factors which are under control of banks. However, given the nature of the deposit run model specified here, the sum of total unborrowed reserves is overstated since not all of required reserves is available to meet deposit runs. Only 83 1/2 and 88 cents are available along with 4 cents for time deposits. The unborrowed reserves figure has been reduced by a weighted average of these ratios so as to convert the reserves variables (x) and (y) into numbers that represent available reserves. However, in order to employ standard bank terminology, the reserves variable will be termed "unborrowed reserves."[6] Borrowing will be taken to be a function of those factors summarized in R_u, or the amount of unborrowed reserves.[7]

If we now return to the expected loss function we can say that borrowed reserves depend on unborrowed reserves, the market rate of interest on loans, the rate of interest on securities, the Federal Reserve discount rate, and the level of total deposits (D) since this will affect the size of total demands on reserves (z). If we now call out these variables and place them in a general function form, we have,

$$B = B(R_u, r_m, r_s, r_d, D) \qquad (1.4)$$

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[6]I am indebted to Thomas R. Saving for pointing out that total unborrowed reserves is not the appropriate independent variable.

[7]The amount of unborrowed reserves was taken to be one of the independent variables in two other pieces of research on member bank borrowing. See R. C. Turner, Member Bank Borrowing (Columbus: The Ohio State University, 1938); and S. M. Goldfield and E. J. Kane, op. cit.

and by assuming (1.4) to be homogeneous of degree one in R_u and D, we have

$$\lambda B = B(\lambda R_u, r_m, r_s, r_d, \lambda D) \tag{1.5}$$

and by setting $\lambda = 1/D$ we obtain

$$\frac{B}{D} = f(\frac{R_u}{D}, r_m, r_s, r_d) \tag{1.6}$$

The three interest rates in (1.6) will be interpreted in the following manner. The market rate of interest will be considered to represent the net yield on loans, or the marginal revenue on loans. In order for the bank to make a decision on whether to borrow in response to a given r_m, it must also have some measure of the relevant marginal cost, which in the case of borrowed reserves will be taken as the Federal Reserve discount rate. In order to take account of this profit maximizing behavior we will use the algebraic spread between the two rates rather than each rate separately, or $(r_m - r_d)$.[8]

The security interest rate will be considered as the yield on that asset that the bank views as the closest substitute for reserves in the sense that security sales represent a reasonable alternative for borrowing on the part of the bank. The particular securities involved are short-term bills since they are preferable to longer term bonds as a

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[8]A. J. Meigs has experimented with both the algebraic spread and the ratio of the two rates discussed here in his study of the demand for free reserves. His empirical evidence shows both the algebraic difference and the ratio to perform well, while such not being the case when the rates were used individually. See Meigs, _op. cit._, pp. 95-102.

method of obtaining reserves because the price variance is less on short-term bills than on long-term bonds. Just as in the case of loans, the bank in making a decision on whether to borrow or sell securities to obtain additional reserves must take into account the marginal cost of borrowing relative to the marginal cost of security sales in terms of interest returns lost so that the algebraic difference $(r_s - r_d)$ is the relevant variable.

If the algebraic difference terms $(r_m - r_d)$ and $(r_s - r_d)$ are now substituted for the three interest rates in (1.6) we obtain

$$\frac{B}{D} = g\ \frac{R_u}{D}, (r_m - r_d), (r_s - r_d) \qquad (1.7)$$

where the signs of the relevant partial derivatives are

$$\frac{\partial g}{\partial \frac{R_u}{D}} < 0, \quad \frac{\partial g}{\partial (r_m - r_d)} > 0, \quad \frac{\partial g}{\partial (r_s - r_d)} > 0 \qquad (1.8)$$

<u>THE UNCERTAINTY CHARACTERISTICS</u>

The general function (1.7) that describes the relationship between the volume of borrowing and its independent arguments was developed from a profit maximizing inventory model under uncertainty. However, the particular manner in which the presence of uncertainty affects (1.7) and in what way it differs from a more simple complete certainty formulation has not been made clear. In this section we will describe the development of a model under the condition of complete certainty and compare its implications with the implications from the uncertainty model. This will

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will also enable us to precisely formulate the manner in which uncertainty will be taken into account.

Under complete certainty the bank knows exactly the amount of reserves that will be forthcoming during the period, the exact quantity of the various demands on those reserves and the exact interest rates on loans, securities, and discounts that will prevail. This would require the optimal inventory relationship described in equation (1.1) to read

$$L(y) = a + r_d(y-x) + \int_o^y\{r_m[Q(y-x)] + r_s[(1-Q)(y-z)] - r_m Qkz$$

$$- r_s(1-Q)kz\}dz + \int_y^\infty\{P(z-y) - r_m Qky - r_s(1-Q)ky\}dz \qquad (1.9)$$

The significant difference between (1.1) and (1.9) is the absence in the latter of the density function $\phi(z,r_m,r_s)$ since all these variables are known with certainty.

Under these conditions a bank will borrow reserves for only two reasons.[9] The first is that the bank may have some desired rate of asset acquisition that cannot be achieved because its reserves at certain points of time are less than is desired even though when the entire period is taken into account, reserves are sufficient. This motive for borrowing is analogous to that of consumers selling bonds in order to make their actual consumption paths equal to their desired ones.

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[9]This discussion is mainly inspired from the analysis of consumer borrowing presented by M. Friedman, op. cit., pp. 7-19.

The second motive for borrowing derives from the profit maximizing behavior of the bank in taking advantage of discrepancies between market interest rates and the discount rate. This motive for borrowing is made up of two sub-hypotheses. The first sub-hypothesis is that the bank will raise the additional reserves by choosing the least-cost combination of security sales and borrowing; the second is that the bank borrows in order to purchase market income streams that possess yields greater than the discount rate. This second sub-hypothesis has been dubbed the "profit-theory" of borrowing and has been proscribed by the Federal Reserve's Regulation A. This provision and its effect on bank borrowing will be discussed more fully in Chapter III when the empirical evidence is presented.

The introduction of uncertainty into the loss function (1.9) will yield the expected loss function (1.1) where the variables no longer known with certainty are the demand for reserves, the market rate of interest, and the Treasury bill rate. These variables are described by the density function $\phi(z,r_m,r_s)$. This particular density function was chosen over the alternative approach of describing each density function separately, since one would not expect the separate density functions to be independent.

The major difference between the certainty and uncertainty formulations is that the bank will have to hold emergency reserves as a buffer inventory in order to meet unexpected clearing drains and cash withdrawals. It was this motive for holding reserves that was of prime interest to Edgeworth in his derivation of the safety allowance. Edgeworth assumed the density function of cash withdrawals to be normally distributed so that the distribution of the random variable could be fully specified by its mean and variance. In earlier work on the theory of uncertainty, Markowitz and Tobin have hypothesised that the variance can be viewed as a measure of risk with the degree of risk increasing as the variance increases.[10] With regard to the bank borrowing problem, let us suppose that the level of unborrowed reserves is constant but that the variance of those reserves becomes greater. This will increase the probability that the bank will find itself with insufficient reserves at some point during the period. In order to hold a constant probability of being able to meet total drains, the bank will have to increase the loss-minimizing quantity of reserves. Likewise, if the variance of reserves should decrease, the bank will find that the old loss minimizing quantity will decrease the probability of not being able to meet total drains and thereby will reduce the loss-minimizing quantity.

In order to take account of the effect of increases in the fluctuation of unborrowed reserves on the bank's demand for borrowed reserves, we will include the coefficient of variation (standard deviation divided by the mean) of unborrowed reserves as an independent variable in the demand function. The reason for choosing the coefficient of variation instead of the variance as a risk measure is that equal variances will not represent equal levels of risk if the expected value of unborrowed reserves should change. The coefficient of variation will standardize the risk variable for changing levels of unborrowed reserves.

The "emergency" motive seems, on an a priori basis, to be of significant value in the banking model since there exists a near zero marginal

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[10]H. Markowtiz, op. cit., J. Tobin, op. cit., pp. 65-86.

cost of deposit withdrawals thus reducing to a minimum the resistance of consumer portfolio switches from deposits to cash. The presence of uncertainty will tend to make the loss-minimizing quantity of reserves held greater than under perfect certainty since the presence of the buffer inventory motive is not offset by reductions in the other two motives.

The presence of uncertainty with regard to interest rates does not suggest whether the loss-minimizing quantity of reserves will be greater or less than under perfect certainty. It seems reasonable, however, on an a priori basis to consider the bank as holding expectations of interest rates as well as reserve flows and that it would be rather forced at a theoretical level of analysis to claim that interest rates are known with certainty while reserve flows are not.

It has been assumed that the bank holds subjective probability distributions of the flow of unborrowed reserves and the values of the various interest rates. This uncertainty notion implies that the bank holds some expectation of the value of each variable which may be different from their actual values. Unfortunately, the magnitudes that represent these expected values are not directly observable. The nature of the problem therefore is to conceptualize a hypothesis that will transform the actual observations in a manner consistent with the theoretical model so that they will be useful in interpreting the empirical evidence.

The same general problem described above was encountered by Milton Friedman and Philip Cagen in their studies of the consumption behavior and money demand in that the central problem was to take observable magnitudes and convert them into other magnitudes that more closely coincide with the theoretical model.[11] The Friedman approach which we will follow here, consists of separating any measured or actual observation into a permanent and a transitory component.[12] In terms of unborrowed reserves, the permanent component is derived from all those factors which the bank considers as determining the level of unborrowed reserves. Among those determinants would be the payment schedules of the depositing public, demand for loans, securities, cash, special economic conditions and the bank's expectation of Federal Reserve Open Market operations or other policy changes. This permanent component of unborrowed reserves would be analogous to the "expected" value of a probability distribution.[13] The transitory component is composed of all those factors which make the actual level of unborrowed reserves different from what was expected. It is composed of those "other" factors which the bank does not take into account because it is believed that they are largely unimportant or that they are so random that there is no way to forecast their presence or degree of severity. If we now apply the same procedure of separating all independent arguments in (1.7) into permanent and transitory components, we have

$$\frac{R_u}{D} = \frac{R_u}{D}_p + \frac{R_u}{D}_t \tag{1.10}$$

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[11]M. Friedman, op. cit.; P. Cagen, op. cit.

[12]M. Friedman, op. cit., pp. 21-22.

[13]Ibid., p. 21.

$$(r_m - r_d) = (r_m - r_d)_p + (r_m - r_d)_t \tag{1.11}$$

$$(r_s - r_d) = (r_s - r_d)_p + (r_s - r_d)_t \tag{1.12}$$

where the subscripts (p) and (t) indicate permanent and transitory components respectively. If we now substitute the permanent terms for each argument in place of its measured argument into (1.7) and add the coefficient of variation of unborrowed reserves V_{R_u} we have

$$\frac{B}{D} = h\left[\frac{R_u}{D}_p, (r_m - r_d)_p, (r_s - r_d)_p, V_{R_u} \right] \tag{1.13}$$

where the signs of the relevant partial derivatives are

$$\frac{\partial h}{\partial \frac{R_u}{D}_p} < 0, \quad \frac{\partial h}{\partial (r_m - r_d)_p} > 0, \quad \frac{\partial h}{\partial (r_s - r_d)_p} > 0, \quad \frac{\partial h}{\partial V_{R_u}} > 0 \tag{1.14}$$

The general demand function given by (1.13) represents the function to be fit to the time-series data by use of the estimation procedure set down in Chapter II.

THE TREATMENT OF VAULT CASH

The demand function developed here does not discriminate between reserves held as vault cash and reserves held as balances at the Federal Reserve Bank. In the expected loss function (1.1) it was assumed that some portion (1-k) of total demands (z) went for cash withdrawals. The aggregate density function $\phi(z,r_m,r_s)$ then includes the bank's estimate of these withdrawals. The portion of total demands on reserves (z) that come from cash withdrawals can be viewed as determining the bank's demand for vault cash while the reserve demands from loans, security purchases, and other clearing drains determine the demand for Reserve balances held at the Federal Reserve Bank. The procedure of aggregating these separate demands for reserves is clearly more appropriate after November 1960 than in the period previous to that date. This is so due to the fact that in July 1959 the Board of Governors was authorized to treat vault cash as a part of member bank reserves, whereas before vault cash could not be counted as reserves. In December 1959, the Federal Reserve allowed member banks to count a portion of vault cash as reserves. On August 25, 1960, and on September 1, 1960, the percentages of vault cash that counted toward reserves was increased, and in November 1960, all vault cash was counted as a part of member bank reserves.[14]

The addition to reserves of this change in the law was by no means negligible in that vault cash amounted to approximately 2.5 billion dollars, which was about 10 percent of the level of unborrowed reserves. With respect to member-bank borrowing, an important impact of this change was to make reserve balances at the Federal Reserve and vault cash perfect substitutes for one another with regard to the need to

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[14]M. Friedman and A. J. Schwartz, A Monetary History of the United States (Princeton University Press: Princeton, N. J., 1963), p. 447n.

meet the Federal Reserve requirements ratios on demand and time deposits, and thus their separate demand function more closely associated with one another.

The model has been constructed on the basis of total unborrowed reserves and thus for the earlier period the data are applicable only to that portion of borrowed reserves which was used to increase balances at the Federal Reserve. Some borrowing presumably was inspired by the need for additional vault cash for which the model is unable to estimate since the independent variable is void of vault cash. In order to compensate for this difference, we have added the figures for vault cash to unborrowed reserves so as to make an estimate for unborrowed reserves that will more closely coincide with the model.

THE DATA

All basic data, with the exception of vault cash estimates, were taken from the various issues of the Federal Reserve Bulletin from 1954 to 1967. All data, again with the exception of vault cash, represent daily average observations for one month periods for the aggregate of all member banks.

Daily average figures for vault cash did not exist before November 1958, and as a substitute we have taken the series developed by Milton Friedman and Anna Schwartz in their Monetary History of the United States.

The deposit level was taken as the sum of demand plus time deposits adjusted for interbank deposits.

The market rate of interest r_m was taken as the 4-6 month prime commercial paper rate which is an average of daily offering rates of dealers.

The security rate r_s, was taken as the market yield on U. S. Government 3-month bills.

The discount rate r_d, was taken as the rate applicable to advances and discounts under sections 13 and 13a which represent the rate charged if borrowing is secured by eligible paper or by U. S. Government obligations. With regard to changes in the discount rate, any change that occurred before the 15th of the month was recorded as applying to that month, while any change after the 15th was recorded as applying to the following month.

CHAPTER II

THE ESTIMATION PROCEDURE

The theoretical model developed in the last chapter must be made statistically operational if it is to be tested against the data on member-bank borrowing. This chapter will set down the method by which the bank arrives at particular values for "permanent" or "expected" unborrowed reserves, as well as some of the economic and psychological hypotheses that underlie the method.

DISTRIBUTED LAGS AND THE CONSTRUCTION OF EXPECTATIONS

Irving Fisher was the first to hypothesize that an effect of a change of one variable upon another may be produced during the passage of time in a manner such that the effect is divided up or distributed over that interval.[1] In Fisher's words:

> The reason for distributing the lag is that the full effect of each P' (a rate of change of a price level index item) is extremely unlikely to be felt at only one instant, such as seven months later, and not felt at any other time either earlier or later than this seven months . . . It is far more probable that the influence began at once, showing itself in the very next month . . . and that it then gradually increased to a maximum a few months later and thereafter tapered off indefinitely according to the probability distribution.[2]

Analytically, the general hypothesis suggested by Fisher can be written as

$$Y = \omega_o X_t + \omega_1 X_{t-1} + \omega_2 X_{t-2} + \ . \ . \ . \ + \omega_n X_{t-n} = \sum_{i=o}^{n} \omega_i X_{t-i} \qquad (2.1)$$

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[1]Irving Fisher, "Our Unstable Dollar and the So-Called Business Cycle," Journal of the American Statistical Association, Vol. XX, June, 1925, pp. 179-202.

[2]Ibid., p. 184.

where X_{t-i} is the measured value of the independent variable and ω_i is the appropriate weight for a particular point in the time horizon of n periods.[3] The particular economic and psychological rational of a distributed lag reaction as given in (2.1) with regard to bank behavior is two-fold in that it can be considered to be composed of psychological and institutional factors. A consideration of these factors will yield a basis about which a more formal and empirically determinable hypothesis can be constructed. Therefore, let us consider each of these factors separately.

The most straight forward method of computing an expected value of a variable would be to find the simple arithmetic mean of past levels of that variable. This method lacks appeal as a psychological proposition because it assumes that the memory path of the banker is such that it remains of undiminished intensity with respect to the passage of time until a certain point where it instantly vanishes! In terms of (2.1) this would imply that $\omega_o = \omega_1 = . . . = \omega_n$ which carries with it the same introspective objection that Fisher had with the case of only one ω_i different from zero. A more introspectively pleasing hypothesis put forward in the field of psychology is that of Jost's Law of Exponential Forgetting which states that if two observations are of equal strength but of different age, the older diminishes less with time.[4] A memory pattern compatible with Jost's Law would be some type of

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[3]This discussion is for the most part taken from the contributions of L. Koyck, Distributed Lags and Investment Analysis (North-Holland Publishing Co., 1954); M. Nerlove, "Distributed Lags and Demand Analysis for Agricultural and Other Commodities," Agriculture Handbook 141, U. S. Department of Agriculture, 1958.

[4]H. A. Simon, "A Note on Jost's Law and Exponential Forgetting," Psychometrika, XXXI (December, 1966), p. 505.

exponential decay function, say, of the form $e^{\beta t}$. If a banker's memory were of this exponential form, the affect on him of an observation would fade away as it recedes into the past.

Under institutional factors that would produce a distributed lag, one might consider such things as changes in the pace of economic activity, natural disasters, changes in tax rates, bank runs, etc. Given any of the above conditions it would be reasonable to assume that the effect of the particular situation would be greater the closer is the event to the present. An exponential decay pattern is also applicable here if we can suppose the dynamic adjustment rate of the system to be directly proportional to the degree of divergence of the system from equilibrium. As an example, suppose a series of bank runs produces an excess demand for excess reserves by some amount (E). Assuming the adjustment time to equilibrium where all excess demands are zero to be multi-period, let the degree of adjustment during period one be (A) where $0 < A < E$. The amount of excess reserves at the beginning of period 2 is now E-A and assuming the degree of adjustment to be proportional to the amount of excess demand, the second period adjustment will be less than A. This process continues with the pressure to adjust diminishing as the system moves to equilibrium.

Changes in Federal Reserve policy produced through open market operations is a logical candidate for a reaction to be described by a distributed lag. Changes in Open Market policy are not made public but rather are known only after the net purchases or net sales of securities have been made over a sufficiently long interval such that the bank can make a judgement about the policy. Since the precise opposite may be suggested by the evidence in any given week, the bank must form its expectation by taking into account past observations. Since Open

Market policy is reviewed every two weeks, the bank could be supposed to give observations that lie back in time less, but perhaps not zero, importance than observations that are closer to the present.

From the foregoing discussion it is clear that a distributed lag model will allow one to construct an expectation of a variable on the basis of actual levels of that variable in the past. In a similar manner an individual would alter his expectation by some degree if the actual value of that variable turned out to be different from what was expected. We can make this hypothesis analytically explicit by making the expected value of a variable depend on a weighted average of past values by use of an adjustment model that has been used by Cagen, Friedman, Morrison, Koyck, Nerlove, and others. This adjustment equation can be written in continuous form as

$$\frac{E(x)_t}{dt} = \beta[x_t - E(x)_t] \qquad (2.2)$$

where $E(x)_t$ is the expectation of x_t and β is a scalar. Equation (2.2) shows that the change in the expected value of a variable is proportionally related to the difference between the actual value of a variable and its expectation. β, the coefficient of expectation gives the speed of adjustment of expected to actual observations. An assumption to be made from this point on is that for a given time series of a particular length the value of β is constant. This however, will not prevent us from splitting up a time series in different ways to discover whether the value of β actually changes. Equation (2.2) is first-order linear differential equation whose solution with the constant term equal to zero is

$$E(x) = e^{\beta t} \int_{-T}^{t} \beta x_y e^{\beta y} dy \qquad (2.3)$$

The assumption that the constant term is zero implies that at some point in the past the actual values were constant. (2.3) can be expressed as

$$E(x) = \frac{\int_{-T}^{t} x_y e^{\beta y} dy}{\frac{e^{\beta t}}{\beta}} \qquad (2.4)$$

Equation (2.4) shows the expected value of a variable to be a weighted average of actual past observations with geometrically declining weights given by the term $e^{\beta y}$. Note that this weighting stream is compatible with that suggested by Jost's Law of memory decay mentioned earlier.

Another property of this particular method of forming the expectation of a variable is that the sum of the weights over the interval [-T,t] is equal to the denominatory of (2.4).[6]

Since only discontinuous data is available, a discrete approximation must be used in place of (2.4). A form developed by Cagen and also obtained by Nerlove is

$$E(x) = (1 - e^{-\beta}) \sum_{i=o}^{T} x_{t-i} e^{-\beta i} \qquad (2.5)$$

where 0 is current time. This approximation is derived from solving the differential equation given in (2.2) with the weighting pattern given by $e^{\beta t}$.[7] The weighting pattern $(1 - e^{-\beta})e^{-\beta i}$ shown in (2.5) will yield a different value for each predetermined β . The range of β values chosen for this study were $.1 < \beta < 1$ where each β differs by .05 from the one directly above it. The time horizon for each β was taken where the sum of the weights to three decimal places is unity.

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[6]This result is seen by the following:

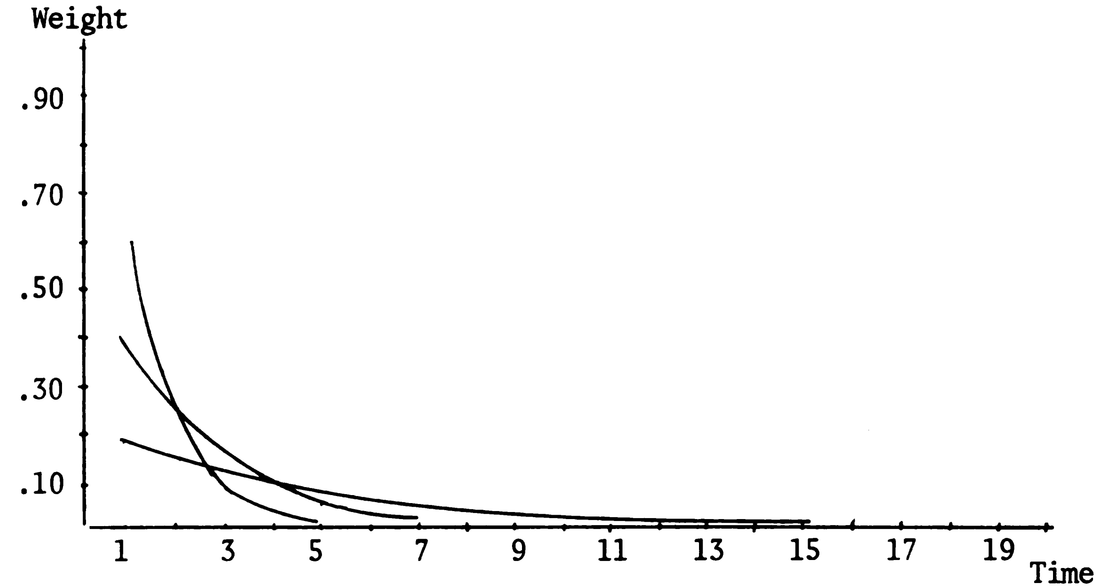
$$\int_{-T}^{t} e^{\beta y} dy = \frac{1}{\beta}\int_{-T}^{t} e^{y}(\beta dy) = \frac{1}{\beta} e^{\beta y}\Big|_{-T}^{t} = \frac{1}{\beta}[e^{\beta t} - e^{\beta(-T)}] =$$

$$\frac{1}{\beta}[e^{\beta t} - e^{\beta(-T+t-t)}] = \frac{1}{\beta}[e^{\beta t} - e^{-\beta(t+T)} e^{\beta t}] = \frac{e^{\beta t}}{\beta}(1 - e)^{-\beta(T+t)}$$

as long as -T is chosen so that the term $e^{-\beta(T+t)}$ can be neglected.

[7]P. Cagen, <u>op. cit.</u>, p. 39.

Some representative weighting streams for various β weights are shown in Figure 1.



EXPONENTIAL WEIGHTING STREAMS

Figure 1

The value of β selected for a particular variable will be that value of β that generates a series of permanent terms which minimizes the variance of the simple regression of borrowed reserves per deposits, B/D, on the permanent independent variable, i.e., maximized the R^2.[8] It will then be assumed that the value of β that maximized the R^2 will "best" represent the distributed lag weighting scheme of the bank. All time series of the independent variables will be converted into "best" permanent time series as explained above. Each "best" series will then be placed into a linear multiple regression to estimate the general demand function for borrowed reserves given by (1.13).

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[8]For an econometric development of this approach, see Ibid., pp. 92-93.

Unfortunately, the above procedure will produce an econometric problem of multicollinearity in that the two interest rate variables are highly intercorrelated. Since it is difficult, if not impossible, to estimate the separate effects of each variable when the degree of intercorrelation is high, we will report the empirical evidence in two forms. One form will include permanent unborrowed reserves and the market rate-discount rate differential and the other form will include permanent unborrowed reserves and the bill rate-discount rate differential.

CONTRUCTION OF THE COEFFICIENT OF VARIATION

The model requires that we have an estimate of the coefficient of variation of unborrowed reserves to coincide with each expected or permanent value of the variable. Since we have assumed a distributed lag reaction in forming the expectation of unborrowed reserves with the banker giving less importance to an observation as it recedes into the past, it is reasonable to assume that his memory of the variability of that observation also diminishes with the passage of time.

The variance of an observation about its expectation is given by

$$\sigma^2 = \frac{(R_u - R_u{}^P)_t^2 + (R_u - R_u{}^P)_{t-1}^2 + \cdot\ \cdot\ \cdot + (R_u - R_u{}^P)_{t-n}^2}{n} \qquad (2.6)$$

and the standard deviation as

$$\sigma = \sqrt{\frac{(R_u - R_u^p)_t^2 + (R_u - R_u^p)_{t-1}^2 + \cdot\cdot\cdot + (R_u - R_u^p)_{t-n}^2}{n}} \tag{2.7}$$

What is desired is to form an estimate of σ such that the bank "forgets" a particular variance as it recedes away. This requires that each root-mean-square deviation be weighted by the appropriate ω_i weight determined by the "best" β for its place in the relevant time horizon [-T,t]. This can be written symbolically as

$$\sigma_\omega = \sqrt{\frac{(R_u - R_u^p)_t^2\omega_t + (R_u - R_u^p)_{t-1}^2\omega_{t-1} + \cdot\cdot\cdot + (R_u - R_u^p)_{t-n}^2\omega_{t-n}}{n}} \tag{2.8}$$

and the coefficient of variation as

$$V = \frac{\sqrt{\frac{(R_u - R_u^p)_t^2\omega_t + (R_u - R_u^p)_{t-1}^2\omega_{t-1} + \cdot\cdot\cdot + (R_u - R_u^p)_{t-n}^2\omega_{t-n}}{n}}}{R_u^p}$$

In this form, the contribution of each coefficient of variation to the total variation at time (t) diminishes as the terms recede into the past. An assumption to be made here is that the same weighting pattern applies to the coefficient of variation as to the expected value of unborrowed reserves.

CHAPTER III

THE EMPIRICAL EVIDENCE

The empirical evidence reported in this chapter will be divided into three sections. The first section will discuss the reason for the time period chosen. The second section will describe the evidence relating to the demand function developed in the previous chapters. The third section will contrast and compare the theoretical model and empirical results developed and presented here with the results obtained by other researchers.

THE TIME PERIOD

The period chosen for the empirical test was from January, 1954 through December, 1966. In choosing a time period the goal was to restrict the data, within reasonable bounds, so as to most accurately represent the current functions and the time path and significance of the arguments. The only decision therefore was the particular beginning date. January, 1954 was chosen as the beginning date for three reasons: first, it is after the depression period of the 1930's; second, it is after the Federal Reserve--Treasury Accord in 1951; and third, it is after the excess profits tax of the early 1950's.

During the 1930's member banks, bitter and pessimistic, accumulated large quantities of excess reserves in response to their increase demand for liquidity as the Federal Reserve System, a potential producer of liquidity, lapsed into passivity. These large excess reserves along with the fact that the discount rate from 1934 through 1941 was seldom below the short-term bill rate and never below the 4-6 month prime commercial paper rate made the quantity of borrowed reserves negligible.

The System, during this period, felt that it was pursuing a policy of "monetary ease" due to a confusion of looking at the absolute rather than the relative position of the discount rate, a factor that we have corrected for by the use of the $(r_m - r_d)$ and $(r_s - r_d)$ terms. It may seem as if the experiences of the 1930's is in no way in conflict with the model developed here. However, such a view is likely to be erroneous since it was during the 1930's that the banks discovered the true meaning of unborrowed reserves. During the disasterous deposit runs of this decade banks found that only (1 - RR), where RR is the average reserve ratio, of unborrowed reserves were actually available to meet deposit runs.

In April, 1942, the Federal Open Market Committee announced that it would peg the yield of 90-day maturities at 3/8 of one percent by offering to purchase a sufficient quantity of bills so as to prevent their yield from rising. The bond support program was carried out for longer maturities as well with pegged yields ranging from 7/8 to 2.5 percent per year. These rates, which were a carry over from depression standards had the effect of removing the interest rate risk on bonds.

In late 1942, the discount rate was lowered to 1/2 of one percent on secured loans upon which Friedman-Schwartz have commented that ". . . if banks held such securities, it was generally cheaper for them to acquire any needed reserves by selling bills yielding 3/8 of one percent rather than by using them as collateral to borrow at 1/2 of one percent."[1] The effect of the bond support program during the war and into the post-war period when yields were attempting to rise, extended the disuse of

[1]M. Friedman and A. J. Schwartz, op. cit., p. 563.

the discount window which had begun in the 1930's. It was not until the Federal Reserve Accord in 1951 that discounting was again to occur in significant quantities.

During the post-accord period the enviornment with regard to member bank borrowing was again upset by the imposition of the excess profits tax in 1952 and 1953. The excess profits tax which was applicable between June 30, 1951 and January 1, 1954, allowed up to 75 percent of borrowed capital to be counted as a deduction from the tax base. Borrowed reserves fell into this category which could produce a tax saving equivalent to an after-tax yield of 2.7 percent on borrowed funds.[2] The effect of this on borrowings is reflected in the fact that borrowings rose to a peak during 1952 which has not been equaled since.

In light of these reasons the year 1954 has been chosen as the earliest date which marks a time span highly similar in relevant considerations to those which exist today. The only major change that took place during the 1954-67 period was the change in the role of vault cash in 1960 which we have made an attempt to compensate for by adding vault cash to member bank reserves.

THE REGRESSION RESULTS

Tables 1 and 2 record the multiple regression results for the demand function from 1954-60 and from 1960-67. The time period was split in this particular manner because 1960 was when vault cash was counted as reserves. Because the data before and after this change are not fully comparable, the data was separated at 1960 so as to make each time series consistently defined.

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[2]A. J. Meigs, op. cit., p. 73.

TABLE 1

MULTIPLE REGRESSION ESTIMATES OF THE DEMAND FUNCTIONS FOR BORROWED MEMBER BANK RESERVES 1954-60 AND 1960-67

No.	Period	Constant	$\frac{R_u}{D}p$	$(r_m - r_d)p$	$(r_s - r_d)p$	F	R^2
2499A	1954-60	0.00986	-.02563 (0.01557) .25	-.00262 (0.00199) .20	0.01017 (0.00225) .20	33	.76
2499B	1954-60	0.00176	-.00752 (0.01918) .25	0.00586 (0.00084) .20		24	.61
2499C	1954-60	0.00745	-.02021 (0.01519) .25		0.00737 (0.00075) .20	48	.76
2501A	1960-67	0.00924	-.09240 (0.00748) .30	0.00252 (0.00045) .15	-.00197 (0.00043) .30	128	.86
2501B	1960-67	0.01077	-.09243 (0.00918) .30	0.00019 (0.00023) .15		118	.79
2501C	1960-67	0.00943	-.08181 (0.00823) .30		0.00065 (0.00023) .30	136	.83

Standard errors appear below each coefficient.
Beta weights for the appropriate decay patterns appear below the standard errors.

TABLE 2

MULTIPLE REGRESSION ESTIMATES OF THE DEMAND FUNCTIONS FOR BORROWED MEMBER BANK RESERVES, 1960-67

No.	Period	Constant	$\left(\frac{R_u}{D}\right)_p$	$(r_m - r_d)_p$	$(r_s - r_d)_p$	V_{R_u}	F	R^2
2501AV	1960-67	0.00960	-.09150 (0.00764) .30	0.00244 (0.00047) .15	0.00189 (0.00045) .30	-.00037 (0.00057) .30	96	.86
2501BV	1960-67	0.00940	-.08046 (0.00814) .30	0.00063 (0.00022) .15		-.00106 (0.00062) .30	95	.83
2501CV	1960-67	0.01060	-.08974 (0.00917) .30		0.00022 (0.00022) .30	-.00117 (0.00069) .30	82	.81

Standard errors appear below each coefficient.
Beta weights for the appropriate decay patterns appear below the standard errors.

A comparison of the results from the two time periods is rather striking in the almost complete reversal of the significant variables. In the 1954-60 period, permanent unborrowed reserves, although having proper signs, failed to show statistical significance while the sign, strength and significance of each interest rate variable was outstanding. On the other hand, the 1960-67 results show a marked rise in the strength and significance of permanent unborrowed reserves and a large decline in the size of both interest rate variables with the $(r_m - r_d)$ term losing statistical significance and the $(r_s - r_d)$ term being significant at the .05 level rather than at .01 as during the 1954-60 period.

A possible explanation for this behavior may be centered around the nature of the Reserves data prior to 1960 and the growth of the Federal funds market as an alternative method of reserve adjustment. The nature of the vault cash data has been pointed out above and from its shortcomings, it is not surprising to find the earlier period performing poorly. The decline in the importance and significance of the interest rate variable however, is more interesting in that it reflects a changing attitude on the part of banks toward discounting.

The movement along the supply function for Federal funds in response to a higher Federal funds rate is reflected in the rapid growth of both dollar volume and number of participating banks in the Federal funds market. The Federal Reserve Bank of New York has estimated that the market has tripled since late 1956 and that a typical day's volume of purchases and sales approached $3 billion in early 1966.[3] The overall broadening of the market is further evidenced by the increased

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[3]Federal Reserve Bank of New York Monthly Review, Vol. 48, May 1966, p. 114.

participation of smaller banks in that they as sellers have increased their net sales of funds to the forty-six large banks included in the Federal Reserve Board's Federal Funds series from a daily average of about $250 million in late 1959 to approximately $500 million in 1962, and to over $1 billion in early 1966.[4]

This increased activity in the funds market reflects the general rise in interest rates during the 1960's and its encouragement for banks to find employment for their excess reserves for short periods, such as over-night. Two institutional factors which have aided in this task are the more active role of correspondent banks in informing country banks of the opportunities available in the funds market and the beginning of a substantial volume of odd-lot trading. Until recently, trading in Federal funds was confined to transactions of $1 million which precluded small banks from the market. The rise of odd-lot trading and more aggressive correspondent relations has caused the supply and demand functions for Federal funds to shift to the right and as a result has produced an important money market whose effects are bound to show up to an increasing extent in future empirical work in the field of money.

The results of a survey of the effect that trading in the Federal Funds Market had on the reserve adjustment practices of banks in the second Federal Reserve District, conducted by the Federal Reserve Bank of New York are shown in Table 3. This table shows how 189 banks ranked the effect of their funds activity on various aspects of their reserves and reserve adjustments. The interesting aspect of this table with regard to the observed decline in the coefficients of the interest rate

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[4]Ibid., pp. 114-115.

can be seen by the reduced reliance that banks place on Treasury bills for reserve adjustment.

The performance of the two interest rate variables relative to one another is interesting in that it shows the Treasury bill rate to be always more powerful and significant than the market rate as an independent variable. Under the assumptions of the roles of these rates in the inventory model developed in Chapter I, it may be said that banks place more importance on using the discount window as a method of reserve adjustment than as an outlet for additional reserves by which to acquire earning assets. The significance of the Treasury bill rate has also been confirmed in other pieces of empirical research on bank borrowing which will be presented in some detail in the next section.[5]

The distributed lag procedure for the construction of the bank's expectations regarding the independent variables has produced two results that are of interest. The first is the seemingly long view that the bank takes in forming its expectations, and the second is the upward drift of the β weight for unborrowed reserves between the two periods. This upward drift will be more clearly seen when we discuss the results of the two year intervals.

Cagen has defined $1/\beta$ to be the average length of the lag of current values to past values of a variable so that a $\beta = .25$ and a $\beta = .30$ imlies average lags of 4.0 and 3.3 months respectively.[6] These results are in agreement with the average lags found by Morrison in his study

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[5]M. E. Polakoff, "Reluctance Elasticity, Least-Cost, and Member-Bank Borrowing: A Suggested Integration," Journal of Finance, Vol. XV, (March 1960) p. 18; S. M. Goldfield and E. J. Kane, op. cit.

[6]P. Cagen, op. cit., p. 41.

TABLE 3

EFFECT OF FEDERAL FUNDS ACTIVITY ON RESERVE ADJUSTMENT PRACTICES

Type of Effect	Number of participating banks according to their ranking of effects.*				
	First	Second	Third	Fourth	Fifth
Reduction in excess reserves	116	42	10	2	0
Reduction in the use of Treasury bills and other short-term instruments	33	85	26	8	0
Reduction in borrowings from the Federal Reserve	28	27	38	8	0
Reduction in borrowing from other banks (other than Federal funds transactions)	1	6	19	16	0
Other	11	5	6	3	1

*Columns and rows may not add to 189, the total number of participating banks, since many banks ranked fewer than five effects.

Source: Federal Reserve Bank of New York Monthly Review, Vol. 48, May 1966, p. 114.

of liquidity preferences of commercial banks where β weights of .20 were not uncommon.[7] The shortening of the average memory period by one-half month in the twelve year period is consistent with the view that the development of the money markets has been able to increase the speed by which they make adjustments in their reserve positions. This is perhaps another piece of evidence on the influence of the Federal funds market.

Due to the nature of the reserves data prior to 1960 and its unsuccessful performance in the demand function, the coefficient of variation of unborrowed reserves has been omitted from the 1954-60 regressions. During the 1960-67 period, however, the results are disappointing, in that they display wrong signs and fail to achieve statistical significance.

Table 4 records the evidence from the data being split into two year periods beginning with 1957. 1957 was the earliest date for which expectations were able to be calculated given the size of the β weights involved. It should be noted that each two-year period of observations was allowed to derive its own β weight for all independent variables. In this way it is possible to examine more microscopically the particular changes in the bank's construction of its expectations.

One of the most outstanding pieces of empirical evidence that results from the two-year tests is the sharp rise in the β weight of unborrowed reserves during 1959-61. The value of β rose from .25 in 1957-59 to .95 in 1959-61 and then fell to .50 in 1961-63 and finally to .30 for the remaining four years. The sharp rise in β , implies a shortening

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[7]G. Morrison, <u>op. cit.</u>

TABLE 4

MULTIPLE REGRESSION ESTIMATES OF THE DEMAND FUNCTIONS FOR BORROWED MEMBER BANK RESERVES FOR TWO YEAR PERIODS

No.	Period	Constant	$\left(\frac{R_u}{D}\right)_p$	$(r_m - r_d)_p$	$(r_s - r_d)_p$	F	R^2
2300A	1957-59	0.04387	-.22534 (0.161131) .25	-.00862 (0.00250) .20	0.01766 (0.00300) .25	39	.85
2300B	1957-59	-.02487	0.17601 (0.23601) .25	0.00558 (0.00107) .20		16	.59
2300C	1957-59	0.00928	-.02961 (0.18608) .25		0.00769 (0.00098) .25	34	.77
2400A	1959-61	0.08067	-.65295 (0.07551) .95	-.00355 (0.00198) .25	0.00558 (0.00146) .20	110	.94
2400B	1959-61	0.08356	-.71600 (0.09454) .95	-.00326 (0.00109) .25		96	.90
2400C	1959-61	0.07706	-.64001 (0.07905) .95		0.00321 (0.00066) .20		.94

Standard errors appear below each coefficient.
Beta weights for the appropriate decay patterns appear below the standard errors.

TABLE 4 (Continued)

No.	Period	Constant	$\frac{R_u}{D}p$	$(r_m - r_d)p$	$(r_s - r_d)p$	V_{R_u}	F	R^2
2700A	1961-63	0.02381	-.22187 (0.16013) .50	0.00158 (0.00185) .30	-.00335 (0.00350) .25		2	.24
2700B	1961-63	0.00916	-.07609 (0.04881) .50	0.00006 (0.00070) .30			3	.21
2700C	1961-63	0.01310	-.11356 (0.09707) .50		-.00058 (0.00132) .25		3	.21
2700AV	1961-63	0.02576	-.24587 (0.13833) .50	0.00120 (0.00160) .30	-.00417 (0.00303) .25	0.00456 (0.00162) .50	4	.46
2700BV	1961-63	0.00758	-.06471 (0.04332) .50	-.00071 (0.00066) .30		0.00434 (0.00165) .50	5	.41
2700CV	1961-63	0.01706	-.15785 (0.08491) .50		-.00193 (0.00122) .25	0.00464 (0.00161) .50	5	.45

Standard errors appear below each coefficient.
Beta weights for the appropriate decay patterns appear below the standard errors.

TABLE 4 (Continued)

No.	Period	Constant	$\frac{R_u}{D}p$	$(r_m - r_d)p$	$(r_s - r_d)p$	V_{R_u}	F	R^2
2800A	1963-65	0.03055	-.28330 (0.06804) .35	-.00390 (0.00584) .20	-.00463 (0.00514) .25		9	.56
2800B	1963-65	0.03010	-.25833 (0.06185) .35	0.00851 (0.00279) .20			13	.55
2800C	1963-65	0.02863	-.28061 (0.06702) .35		0.00764 (0.00224) .25		13	.56
2800AV	1963-65	0.01979	-.14647 (0.07296) .35	-.00960 (0.00605) .20	0.00016 (0.00493) .25	-.00500 (0.00162) .35	7	.61
2800BV	1963-65	0.01981	-.14738 (0.06598) .35	0.00942 (0.00272) .20		-.00500 (0.00156) .35	10	.61
2800CV	1963-65	0.01660	-.15565 (0.07544) .35		0.00678 (0.00236) .25	-.00413 (0.00158) .35	8	.56

Standard errors appear below each coefficient.
Beta weights for the appropriate decay patterns appear below the standard errors.

TABLE 4 (Continued)

No.	Period	Constant	$\frac{R_u}{D}p$	$(r_m - r_d)p$	$(r_s - r_d)p$	V_{R_u}	F	R^2
2900A	1965-67	0.00687	-.05815	0.00157	-.00009		28	.81
			(0.04468)	(0.00032)	(0.00002)			
			.30	.35	.20			
2900B	1965-67	0.01372	-.12613	0.00044			19	.64
			(0.05672)	(0.00022)				
			.30	.35				
2900C	1965-67	0.02185	-.21072		0.00001		14	.58
			(0.04872)		(0.00016)			
			.30		.20			
2900AV	1965-67	0.00462	-.03311	0.00152	-.00067	-.00105	36	.88
			(0.03760)	(0.00025)	(0.00018)	(0.00030)		
			.30	.35	.20			
2900BV	1965-67	0.00845	-.07012	0.00074		-.00142	26	.79
			(0.04576)	(0.00019)		(0.00036)		
			.30	.35				
2900CV	1965-67	0.01886	-.17792		0.00021	-.00115	13	.66
			(0.04637)		(0.00017)	(0.00049)		
			.30		.20			

Standard errors appear below each coefficient
Beta weights for the appropriate decay patterns appear below the standard errors.

of the average memory period from 4 to 1.50 months. This rise coincides with the time period during which the Federal Reserve began to include approximately $2.5 billion as a part of member bank reserves.[8] This sharp change in the level of reserves had the effect of making "past" observations not as important in the construction of what the bank expected unborrowed reserves to be. In fact, with a β = .95, the bank only takes into account the previous six months, where the weights from current time back are .613, .237, .092, .036, .014, .005, .002. It is easy to see from this weighting pattern that any measured observation, more than one month away, receives little weight. This particular movement of the β weight is heartening in that it yields striking evidence regarding the appropriateness of the particular distributed lag scheme used here. This is so since one would expect on an a priori basis banks to discard or highly discount past estimates of unborrowed reserves during the 1959-61 period when vault cash began to be counted as a part of reserves.

The decline in the size and significance of the interest rate variables parallels that found for the larger period. The evidence, however, carries two other interesting points. The first is that during 1961-63 all interest rate variables showed a marked drop in the size of their coefficients, became non-significant and carried wrong signs. An examination of the period has failed to uncover any underlying reasons for the perverse nature of the interest rate variables for this period. The second piece of interesting evidence concerning interest rates is that the 1965-67 period coincided with the sharpest drop in the size of the

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[8]See p. 19.

interest rate coefficients. As it happened, this was precisely the period when the Federal funds market made its greatest gains which again points up the need for more research into this rapidly expanding money market.

For all periods, the coefficient of variation of unborrowed reserves was a disappointing variable. It appeared with wrong signs and significant coefficients much of the time. It was for this reason that the results were reported without that variable. The poor performance of this variable is due either to its not being important in the manner predicted or, more likely, it has not been specified in an appropriate manner.

A SURVEY OF RESEARCH ON MEMBER BANK BORROWING

Research on the determinants of member-bank borrowing falls into two rather distinct categories which are separated by the twenty year span which marks the end of discounting in the 1930's to the revival of discounting after the Accord. The early period literature was mainly concerned with the question of whether banks borrow in order to profit from interest rate differentials or whether they borrow only out of a "need." At bottom, this issue revolved around the role of the discount rate and its relation to other rates as an independent variable. The later period has been characterized first, by an integration of the "profit" and "need" theories into a more general hypothesis and second, by the placing of borrowing hypotheses within the traditional maximizing theory of the business firm.

The Early Period

The evolution of open market operations during the early 1920's placed the conception and role of discounting in a new light. It was discovered that System purchases of securities during 1922 was accompanied by a reduction in the amount of indebtedness of member banks of an approximately equal amount leaving total Reserve Bank Credit nearly unchanged. In 1923 the system reversed its open market policy with security sales only to find that member banks increased the amount of their borrowing by the amount of the reserve losses again leaving total Reserve Bank Credit unchanged. Two regressions computed by Brunner and Meltzer to estimate the magnitude and significance of compensating variations in discounts and advances A, for changes in the adjusted monetary base B^a during the 1920's are shown in Table 5.[9] The regressions were run on first differences between adjacent months or corresponding months of adjacent years with corresponding first differences in the adjusted monetary base. The near unity of the size of and significance of the coefficients indicates that the Federal Reserve's impression carried with it some empirical substance. If Open Market Sales and Purchases were to be offset by decreases and increases in member bank indebtedness to the Federal Reserve, how was the System able to make desired changes in member bank reserves? Further, it raised doubts that the volume of borrowing was at all responsive to the discount rate.

A new theory of credit control developed by W. Randolph Burgess and Winfield W. Riefler came to the rescue by showing how discounting could be an effective tool of monetary policy <u>even if</u> it did not make for changes total Reserve Bank Credit. This hypothesis, called the "Burgess-Riefler Doctrine," was based on the idea that the <u>ratio</u> of borrowed to

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[9]K. Brunner and A. H. Meltzer, "Genesis and Development of the Free Reserve Conception of Monetary Processes," in W. L. Smith and R. L. Teigen (eds.) <u>Readings in Money, National Income, and Stabilization Policy</u> (Irwin, Homewood, Ill.) 1965, p. 198.

TABLE 5

COMPENSATING VARIATION IN MEMBER BANK BORROWING FOR CHANGES IN THE MONETARY BASE FROM JANUARY, 1918 TO DECEMBER, 1929

$$\Delta A_{t,\ t-1} = \underset{(.0070)}{.0086} - \underset{(.0962)}{.8742} \Delta B^{a}_{t,\ t-1} \qquad R^2 = .3697$$

$$\Delta A_{t,\ t-12} = .1057 - 1.1729 \Delta B^{a}_{t,\ t-12} \qquad R^2 = .6059$$

Where

$$\Delta A_{t,\ t-1} = A_t - A_{t-1}; \qquad \Delta B^{a}_{t,\ t-1} = B^{a}_{t} - B^{a}_{t-1}$$

$$\Delta A_{t,\ t-12} = A_t - A_{t-12}; \qquad \Delta B^{a}_{t,\ t-12} = B^{a}_{t} - B^{a}_{t-12}$$

Source: K. Brunner and A. H. Meltzer, "Genesis and Development of the Free Reserve Conception of Monetary Processes," in W. L. Smith and R. L. Teigen (eds.) Readings in Money, National Income, and Stabilization Policy (Irwin, Homewood, Ill.) 1965, p. 198.

total reserves was the key variable rather than total reserves. If it could be shown that banks are reluctant to be in debt to the Federal Reserve, then it could be supposed that security sales matched by increased borrowings which would raise the ratio of borrowed to total reserves, would cause banks to restrict credit and attempt to reduce their indebtedness at the Fed thus influencing deposits. Likewise security purchases by the Fed would enable banks to reduce their borrowings thereby lowering the ratio of borrowed to total reserves, thus encouraging banks to expand credit.

The indoctrination of bankers into gaining an attitude toward a reluctance to borrow began in the early 1920's after the Federal Reserve's experience during the three year period following World War I where member bank borrowing exceeded the reserve balances of banks. The large quantities of borrowed reserves along with the practice of continuous borrowing that the Federal Reserve witnessed seemed objectionable since they considered themselves to be a "lender of last resort" rather than an inexhaustable source of funds. In that the discount rate was not set at a level which would make it a penalty rate, the Federal Reserve adopted the policy that continuous borrowing was not an appropriate use of the discount window which, furthermore, was considered to be a privilege granted by the Federal Reserve and not a right of member banks.

This changing attitude on the part of member banks marks the origin of the so-called "needs and reluctance" hypothesis as opposed to the more conventional "profit" hypothesis of the demand for one of the bank's factors of production. In Riefler's words:

> The most obvious theory is that member banks, on the whole, borrow at the Reserve Bank when it is profitable to do so and repay their indebtedness as soon as the operation proves costly. The cost of borrowing at the Reserve Banks, accordingly, is held to be the determining factor in the relation between Reserve Bank operations to money rates, and the discount rate policy adopted by the Reserve Banks to be the most important factor in making Reserve Bank policy effective in the money markets. At the other extreme, there is the theory that member banks borrow at Reserve Banks only in case of necessity and endeavor to repay their borrowing as soon as possible. According to this theory, the fact of borrowing in and of itself--the necessity imposed by circumstances on member banks for resorting to the resources of Reserve Banks--is a more important factor in the money market than the discount rate. . .[10]

From these two hypotheses, Riefler rejected the profit theory because of an observed divergence between the discount rate and short-term market rates. Again, in Riefler's words:

> If member bank borrowing has been governed primarily by motives of profit during this period, money rates would have

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[10]W. W. Riefler, op. cit., pp. 19-20.

> been dominated by the discount rates charged by the Reserve Banks. Particularly would this have been true of rates in the short-term open markets where member banks can lend freely and withdraw funds entirely on their own volition without regard to the results of their actions on future lending operations. . . If member banks borrowing had actually been governed by the profit motive. . . offers of additional funds in the short-term open markets would have been so plentiful whenever opportunity presented itself that rates in those markets could never have risen far above discount rates, so long as eligible paper continued available in ample supply. Nor could have rates in the short-term open markets have fallen much below discount rates so long as an appreciable volume of member bank borrowing at the Reserve Banks represented indebtedness incurred under the profit motive, since member banks would have withdrawn funds from the short-term open markets to repay indebtedness at the Reserve Banks whenever continued borrowing became unprofitable, and rates in those markets could not have fallen much below discount rates until member banks had liquidated a considerable portion of their indebtedness.[11]

The Riefler hypothesis was something less than convincing to a number of economists during the 1930's, notably to Charles O. Hardy, Seymour Harris, and Robert C. Turner.[12] The criticism of Hardy and Harris were more of a probing nature while Turner's represented a full-blown study complete with an operational hypothesis of member bank borrowing.

Hardy took the position that the proximate objective of Federal Reserve policy should be control over member bank reserves rather than the ratio of indebtedness to total reserves.[13] He arrived at this view from an impression that the Federal Reserve made the discount rate ineffective in controlling the volume of borrowing along with an empirical judgement that the inverse relationship between open market operations and opposite

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[11] Ibid. pp. 20-21.

[12] C. O. Hardy, Credit Policies of the Federal Reserve System (Washington: Brookings Institution, 1932); S. E. Harris, Twenty Years of Federal Reserve Policies, 2 Vols. (Cambridge Mass.: Harvard University Press, 1933); R. C. Turner, op. cit.

[13] C. O. Hardy, op. cit., pp. 228-32.

changes in indebtedness was not as close as Riefler supposed. Hardy carried his examination of the discount mechanism further by examining four separate situations where the Federal Reserve either attempted to increase or decrease the volume of credit. In each case, Hardy showed that changes in bank reserves were made in the desired direction. To be sure, borrowing moved in a direction to offset the change but not to a degree that nullified the open market operations.

Harris, like Hardy, noted that the inverse relationship between open market operations and borrowing was not near as close as was believed, even during the early 1920's when the Riefler doctrine was developed.[14] Harris was not willing to accept the hypothesis that banks did not borrow for profit. He felt that the ability of banks to borrow reduced their desired inventory of reserves held against possible cash withdrawals and clearing drains since additional reserve funds could be supplied on short notice. The reduction in desired inventory holdings was taken as a function of the relative costs and revenues from holding reserves relative to earning assets which implied that the discount rate would have an effect on the money stock.

Turner also was not persuaded by the view that the discount rate was ineffective in controlling the volume of borrowing.[15] As an alternative to the "need" theory of borrowing, he developed a "profit" theory of borrowing which was based on the assumption that borrowing and open market transactions were close substitutes as factors of production by which to make marginal adjustments in reserve positions. In Turner's words:

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[14]S. Harris, op. cit., pp. 175-85.

[15]R. Turner, op. cit.

> When faced with a deficit (or excess) in reserves, banks will adjust their reserve position either through increasing (or decreasing) borrowings, or decreasing (or increasing) open market loans, and the decision will rest upon the relative costliness of the two alternatives, to the effect that the volume of borrowing will be a function of the profitability of open market loans relative to the discount rate. (The original quotation is in italics.)[16]

Turner developed an operational model of the profit theory of borrowing by using the reserves identity

$$R_u = R_R + R_E - R_B \tag{3.1}$$

and the profit spread $(r_m - r_d)$. All the factors in R_u were taken as beyond the control of member banks while the terms on the right hand side of (3.1) were within the control of banks and depended on the relative costs and revenues associated with each. Turner did not feel that the profit theory implied that banks felt unconstrained with regard to the amount borrowed from the Federal Reserve. He felt that an

> . . . adequate theory includes the tradition against borrowing, which plays a role especially in setting limits to the volume of borrowing. These limits, however, are themselves influenced by the volume of borrowing, as well as business conditions, banking ethics, and financial organization. An adequate theory also includes as a factor the need for borrowing, defined as a need growing out of the demands of customers for working capital loans or for the maintenance of bank secondary reserves. To these must be added factors of custom and habit, of inadequate knowledge, of personal attitudes, and many others.[17]

Turner tested his hypothesis by running correlations between the amount of borrowing and the profit spread, $(r_m - r_d)$, during the time

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[16] Ibid., p. 96.

[17] Ibid., p. 156.

interval from 1922 to 1937. The correlations were computed for all member banks as well as for the member banks in each of the 12 Federal Reserve Districts. The results showed that while there were high correlations, there existed wide differences between the various Federal Reserve Districts and that there seemed to be a limit above which increases in the profit spread would not induce more borrowing. He also tested the relationship between borrowing and the profit spread lagged one and two months. In the more industrial Federal Reserve Districts of New York, Boston, and Chicago, no lag between changes in the profit spread and borrowing was discovered while in the more rural Districts, a lag of one or two months was found.

The Riefler hypothesis was not able to withstand the scrutiny of Hardy, Harris, Turner, and others in making the demand for borrowed reserves independent of the profit motive and thus conditioned by the tradition against borrowing. The prevailing opinion of the profession on this matter was aptly stated by Friedman and Schwartz when they wrote:

> The policy [i.e., the tradition against borrowing] was apparently effective in limiting the use of discounting, in the sense that it made the amount of discounting less under any given conditions than it otherwise would have been, though it did not, of course, eliminate a dependence between the amount of discounting and the profitability of discounting. In technical parlance, it shifted the demand curve for discounting to the left but did not render it perfectly inelastic and may not even have affected its elasticity.[18]

The Later Period

Chronologically, the article by Murray Polakoff clearly belongs to the collection of recent literature on member bank borrowing while in terms of content it belongs to the earlier literature as a reconciliation

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[18] M. Friedman and A. J. Schwarz, op. cit., pp. 268-269.

of the Riefler "need" theory and the Turner "profit" theory of borrowing.[19] Polakoff believes that from a theoretical view that both motives for borrowing can be distinguished and that there should be some way to integrate both motives into a single theory of borrowing capable of being tested against the empirical evidence.

Polakoff begins by hypothesizing a utility function for banks which contains as independent arguments the psychological cost of borrowing and the relative profitability of adjusting reserve positions by borrowing or through an alternative method. Figure 2 reveals the essence of Polakoff's model by postulating indifference curves between profits and the volume of borrowing for given values of the least cost spread K_i. As the least cost spread increases the profit constraint line K rotates upward indicating a greater profit at every level of borrowing. The equilibrium amount of borrowing is then given by

$$K_i = \frac{-MU_B}{MU_P} \tag{3.2}$$

where MU_B is the marginal utility of borrowing and MU_P is the marginal utility of profit. If there exists a reluctance to borrow as the least cost spread rises, borrowing should increase at a decreasing rate and perhaps even decrease in absolute amounts as, for example, when K increases from K_3 to K_4.

Polakoff tested his hypothesis by drawing a series of scatter charts of borrowing and the least cost spread for weekly and monthly data from

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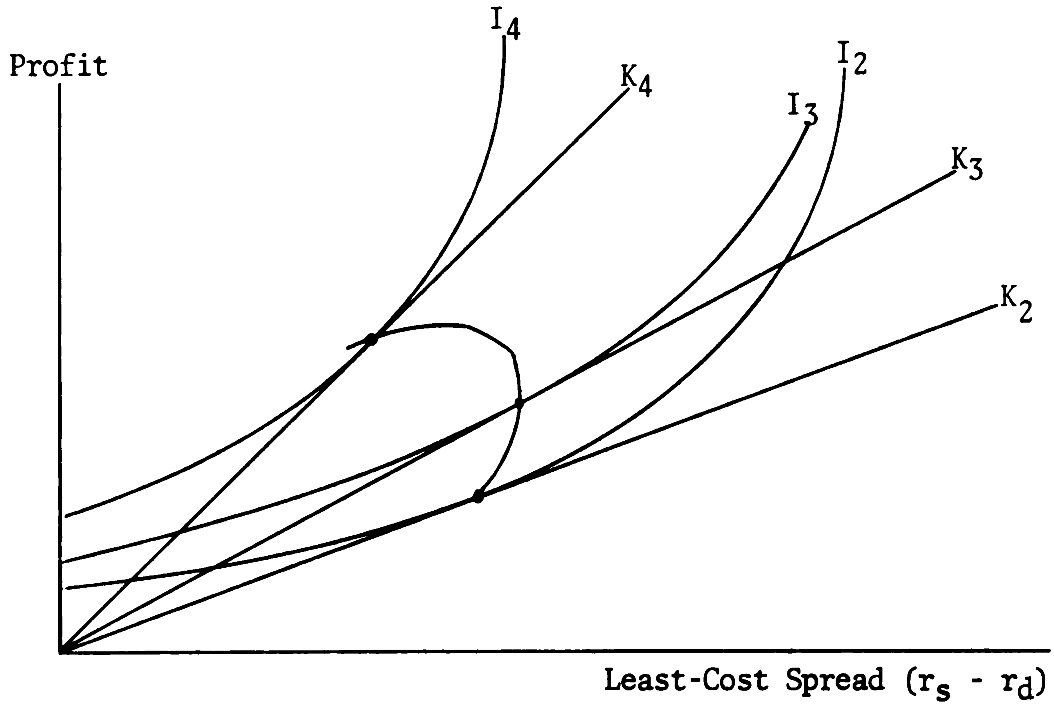
[19] M. E. Polakoff, "Reluctance Elasticity, Least Cost, and Member Bank Borrowing: A Suggested Integration," Journal of Finance, Vol. XV, March 1960, pp. 1-18.



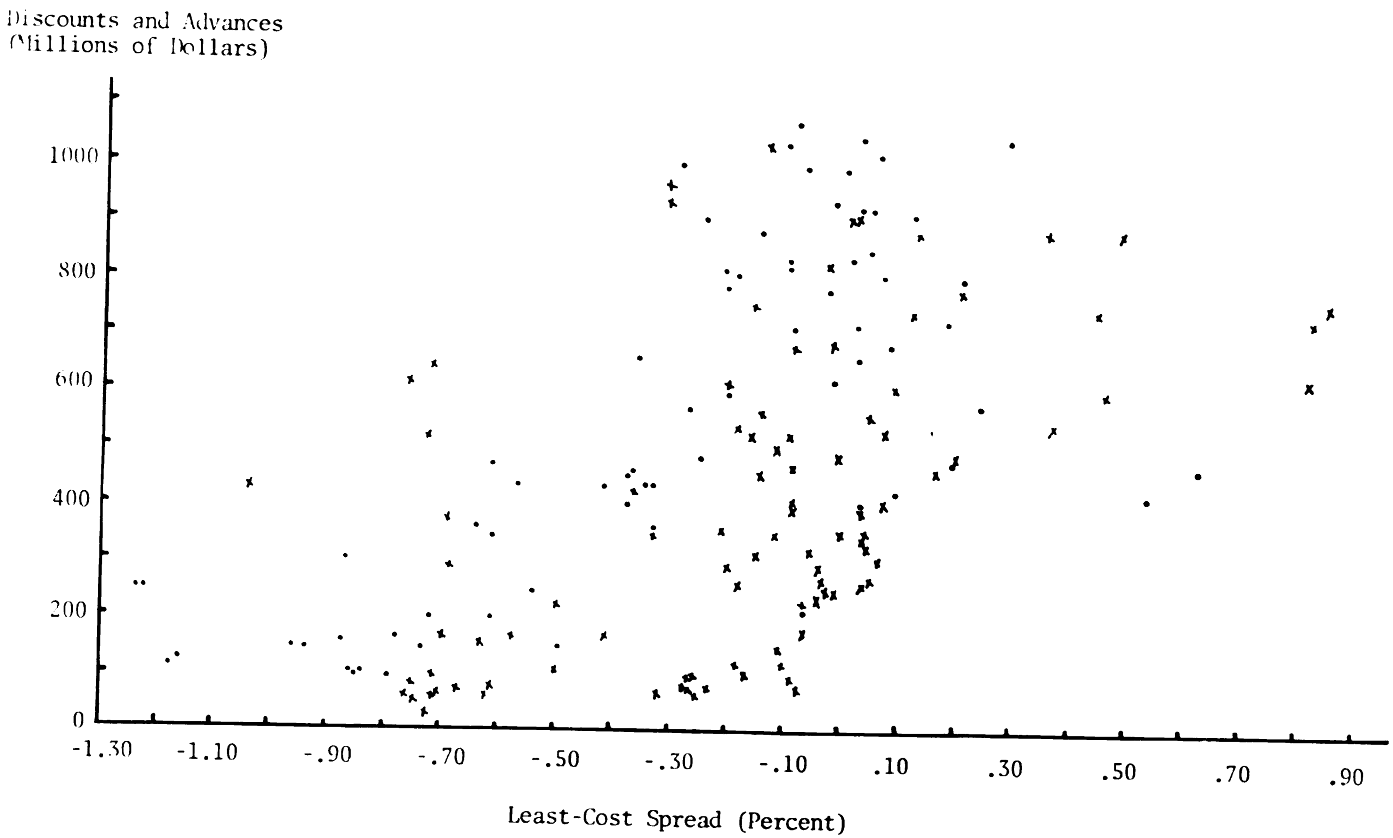
EXPANSION PATH FOR BORROWINGS

Figure 2

July, 1953 through December, 1958. The only statistical test employed was the so-called "eye-ball" test in that he drew his conclusions by inspection of the charts rather than with the aid of some standard statistical technique. He justified this on the ground that the borrowing-least cost spread combinations are too widely scattered for such a test and that his primary purpose was not to develop a demand function for estimation or prediction but rather to test the reluctance to borrow.

The scatter chart shown in Figure 3 is similar to one of the charts used by Polakoff. The only difference between Polakoff's chart and the one presented here is that the observations from May, 1958 to January, 1967 have been included so as to bring his data into line with the time span covered here. Polakoff's contention that there is a significant reluctance to borrow at high values of the least cost spread (say above .30) can be seen by the failure of these high spreads to be associated

MONTHLY AVERAGES OF DAILY FIGURES



Source: Federal Reserve Bulletin

RELATION OF MEMBER-BANK BORROWINGS TO LEAST-COST SPREAD, JULY, 1953, THROUGH DECEMBER, 1966

FIGURE 2

with relatively high levels of borrowing. There seems to be a rather consistent pattern, both from Polakoff's original data and the new data, that the reluctance to borrow is (a) strong, and (b) stable. I feel, however, that this conclusion cannot be supported upon a closer examination of the evidence. The separation of the early data from the later observations aids in bringing this point into relief. Notice that the later observations in general lie below their earlier period counterparts which implies that the demand for borrowed reserves has declined relative to any least cost spread (holding all else constant). If the reluctance to borrow were of equal intensity, the later observations for high least cost spreads should appear below their earlier period counterparts but as can be seen, they always lie above these observations. The implication from this is that the reluctance to borrow is diminishing in importance and while it has not completely disappeared, is probably of no real importance in determining the reserve positions of banks due to the presence of a widespread and well functioning Federal funds market.

The recent article by S. M. Goldfeld and E. J. Kane on the determinants of member bank borrowing is the only piece of published research that has as its goal the isolation of the demand function for borrowed reserves and thus is the only piece that bears similarity to this study.[20] Below we will contrast and compare in some detail the Goldfeld-Kane model to the model developed here and undertake an evaluation of their procedure.

The Goldfeld-Kane model is based on the following set of assumptions:

> At the beginning of each week, banks are assumed to face a specified need for new reserves (ΔR), a given rate of interest

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[20]S. M. Goldfeld and E. J. Kane, op. cit.

> on marketable securities (r_s), and a given discount rate (r_d). Each banker possesses only one decision variable: the total amount of borrowing (B) he will undertake over the upcoming period. Sales of securities ($-\Delta S$) plus borrowing must add up to (ΔR), and negative borrowing (at the discount rate) is not allowed. Finally, we assume that borrowing from previous periods expires automatically.
>
> Next, we postulate that bank portfolio managers determine each period's borrowing (and consequently adjust their security holdings) so as to strike a balance between the cost of raising whatever reserves they require (C) and the disutility which arises from increased debt to the Federal Reserve.[21]

From this set of assumptions they write the utility function to be maximized as depending on the cost of borrowing and the disutility of increasing debt to the Federal Reserve as

$$U = U(C,B) \tag{3.3}$$

where the relevant signs of the partial derivatives are

$$\frac{\partial U}{\partial C} < 0, \quad \frac{\partial U}{\partial B} < 0 \tag{3.4}$$

Since borrowing is a function of what they call the "reserve need" and takes place under the side constraint of cost minimization, they hypothesize a general demand function for borrowed reserves as

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[21] *Ibid.*, p. 500.

$$B = G(K_t, r_s, \Delta R) \tag{3.5}$$

where K_t is the least cost spread $(r_s - r_d)$. After pointing out that their theoretical model is static and that banks operate in a dynamic environment, they attempt to make the model dynamic by inclusion of the following two conditions. The first condition is that banks do not adjust their actual to their desired portfolios within a single period. Rather, the adjustment is only partially accomplished during the specified period and thus the borrowing need for any one period includes some borrowing for the adjustments only partially accomplished from previous periods. From this, they say that current borrowing is some function of borrowing lagged one or more periods. The second dynamic consideration comes from taking into account the permanent nature of a change in unborrowed reserves. In order to make this concept operational, they use the concept of separating any measured unborrowed observation into permanent and transitory components.

Permanent unborrowed reserves are important, claim Goldfeld and Kane, since "it represents the variable whereby banks incorporate their borrowing decisions into long-run developments. . . ."[22] They employ a distributed lag scheme by which they attempt to determine the bank's estimate of permanent unborrowed reserves. They write their estimate of permanent unborrowed reserves at time t, ${U^p}_t$, as

[22] Ibid., p. 507.

$$UP_t = \sum_{i=1}^{h} \omega_{t-i} \Delta U_{t-i} \qquad (3.6)$$

They point out that this permanent-transitory hypothesis

> is made operational by positing that permanent quantities are estimated as weighted averages of past observations, with the weights declining as the variables recede into the more distant past. . . and where the horizon [i,h] and the weights ω_{t-i} will have to be determined empirically.[23]

The general function that Goldfeld-Kane fit to the weekly time-series data on member bank borrowing is

$$B_t = B(K_t, B_{t-1}, B_{t-2}, \ldots; \sum_{i=1}^{h} \omega_{t-i} \Delta U_{t-i}) + V_t \qquad (3.7)$$

where B_t is the volume of borrowing at time t, K_t is the least cost spread of the Treasury bill rate minus the discount rate, ω_{t-i} is the distributed lag weight for each change in unborrowed reserves, ΔU_{t-i}, in the horizon [i,h] and V_t is the error term. The signs of the relevant partial derivatives are

$$\frac{\partial B}{\partial K_t} > 0, \quad \frac{\partial B}{\partial B_{t-1}} > 0, \quad \frac{\partial B}{\partial B_{t-2}} > 0, \quad \frac{\partial B}{\partial(\omega_{t-1} \Delta U_{t-1})} < 0 \qquad (3.8)$$

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[23] _Ibid._, pp. 506-7.

A representative sample of their regression results are given in Table 6. All variables display proper signed coefficients and all but two achieve statistical significance at the .05 level. Goldfeld and Kane suggest that these results provide "clear support" of the importance and significance of lagged values of borrowed reserves and unborrowed reserves, and the least cost spread.

Despite certain similarities in appearance, the Goldfeld-Kane model is much different in spirit as well as in the econometrics than the model developed and tested here. In particular one can view the model here as a "corrected" version of the Goldfeld-Kane model even though my model was developed independently of theirs. In what follows I intend to show: (1) that their model contains an error, the nature of which destroys the value of the model as a useful hypothesis; (2) that there is an error in their collection of the data which makes the data internally inconsistent; and (3) that their empirical procedure violates the general method of the distributed lag scheme that they suggest having borrowed from Milton Friedman and contains misspecifications which makes their empirical evidence suspect.

A major error in the construction of the Goldfeld-Kane model which led to equation (3.7) is the omission of the deposit level as an argument. As was pointed out in Chapter I, the relation between borrowing and the deposit level derives itself from the profit-maximizing behavior of a bank in choosing an optimal stock of reserves to hold during the period against cash withdrawals and clearing drains in an uncertain world. Since total demands on reserves can be reasonably assumed to vary with the volume of deposits, any change in the deposit level will change the safety allowance of reserves thereby affecting the amount of borrowing.

TABLE 6

GOLDFELD-KANE ESTIMATES OF THE DEMAND FUNCTION FOR BORROWED RESERVES, 1953-1963

Class	Const.	B_{t-1}	B_{t-2}	ΔU	ΔU_{t-1}	ΔU_{t-2}	$(r_s - r_d)$	R^2
New York	22.23 (3.78)	.684 (.043)	.093 (.043)	-.369 (.020)	-.059 (.026)	--	25.33 (5.92)	.722
Chicago	2.40 (1.10)	.775 (.042)	.183 (.042)	-.775 (.017)	-.143 (.037)	--	2.40 (1.77)	.939
Other Reserve City	16.53 (4.28)	.798 (.043)	.152 (.042)	-.440 (.019)	-.006 (.027)	--	18.93 (5.66)	.936
Country	10.98 (2.31)	.502 (.035)	.430 (.035)	-.125 (.008)	-.099 (.010)	-.097 (.008)	10.62	.886

Source: S. M. Goldfeld and E. J. Kane, op. cit., p. 509.

It does not seem reasonable to assume, as Goldfeld and Kane have implicitly done, that the demand function is homogeneous of degree zero in deposits just as it does not seem reasonable to assume that a commodity demand function is homogeneous of degree zero in population. In light of the fact that they used time series data over a period where the deposit level increased approximately 50 percent, an increase which is surely significant in determining the banks expectation of the size of cash withdrawals and clearing drains, it is important that their model be adjusted to account for this factor. If we assume that (3.7) is homogeneous of degree one in all nominal variables we can rewrite (3.7) as

$$\frac{B}{D} = f(K_t, (\frac{B}{D})_{t-1}, (\frac{B}{D})_{t-2}, \ldots; \sum_{i=1}^{h} \frac{\omega_{t-i} \Delta U_{t-i}}{D_{t-i}}) + V_t \qquad (3.9)$$

Goldfeld and Kane state that they used weekly data on reserves from July, 1953 through December, 1963 in order to calculate the unborrowed reserves value which appears as an independent variable in equation (3.7) As was pointed out in Chapter I, the time-series data for the period is not fully comparable, due to the fact that in December, 1959 member banks began to count a portion of their vault cash as reserves. Previously, it had been included in cash held by the public. This regulatory change added 2.5 billion dollars to reserves. Goldfeld and Kane make no mention of adjusting the pre-1960 data to account for this change causing one of their independent variables to become a non-homogeneous quantity, i.e., it does not represent the same variable. If they had used adjusted data, they would have used monthly rather than weekly data since only monthly estimates of vault cash are available prior to 1960.

The Goldfeld-Kane procedure of using lagged observations of borrowed reserves as independent variables on the ground that bank portfolios adjust slowly is inadequately specified. The misspecification of the lagged borrowing relation is that more than one borrowing term is present. The fault with this procedure is that the higher is the degree of correlation between B and B_{t-1} the more stable the function becomes and the more dependent is current borrowing on past levels of borrowing under the hypothesis. But as we proceed ahead in time one week, the B term becomes B_{t-1} and the B_{t-1} term becomes B_{t-2}. The movement of these B variables is important because it produces the result that the higher is the degree of correlation between B and B_{t-1} the higher will be the degree of correlation one period later between those same two variables when they both appear as independent variables which produces a higher degree of multicollinearity and thus destroys the empirical test. Also, their particular distributed lag specification is inadequate in that it is unable to provide any evidence on the existence of such a lag. That is, it cannot discriminate between the effect of serial correlation in the dependent variable that may arise, for example, through errors in measurement or misspecification, and the effect of the distributed lag. In that Goldfeld and Kane provide no information on this point, little can be concluded from the evidence they present.

As mentioned earlier, Goldfeld and Kane employ a distributed lag scheme by which they attempt to determine the bank's estimate of permanent unborrowed reserves. However, their method is of a different character and when analyzed will be found to display little or no relation to accepted distributed lag methods of forming expectations. In their article they make no mention of the actual weighting pattern used, the length of the memory horizon, or the lag of permanent to measured flows of unborrowed reserves. Another strange aspect of their procedure is that they did not sum the weighted stream of unborrowed reserves as is common in producing permanent estimates and as is required by equation (3.7) vis. the summation sign. Because their paper provided no explanation of these conundrums, I am indebted to Professor Kane, whom through personal correspondence provided an elaboration of their technique.

As I understand them, they did not estimate, as they implied, permanent unborrowed reserves in the manner used by Friedman but instead took past measured estimates of unborrowed reserves and converted them into permanent estimates by reducing and changing their values until "best" statistical results were obtained. They did not compute beforehand a stream of declining weights by which to weight each measured observation and then sum the observations to obtain a single estimate of the permanent level of unborrowed reserves. Obviously, there is some set of weights implicit in their procedure, although they made no effort to determine the actual weights used.

The Goldfeld-Kane method of using distributed lags to form an expectation of a variable is inappropriate in that it does not in any way take into account the method by which the expectation is formed. For example, the Fisher model employed a weighting pattern similar to the normal curve while the Friedman method suggests an exponentially decaying memory of the form $e^{\beta t}$. The crucial point of this in relation to Goldfeld and Kane's analysis is that the memory path must be hypothesized first in accord with some reasonable mode of psychological or economic behavior, such as Jost's Law of Exponential Forgetting, and then used to calculate permanent quantities. A permanent value is an "expected" value and it is necessary to show how that expectation is formed. Since Goldfeld and Kane did not specify a reasonable weighting scheme in advance but simply changed about the ω_{t-i} terms so as to obtain a good statistical fit, there is little possibility that the actual weight decline, let alone in an exponential manner, or are in some other way consistent with well defined psychological or economic behavior.

CHAPTER IV

A SUMMING UP AND FINAL REMARKS

SUMMARY

The theoretical structure used to derive the bank's demand for borrowed reserves from the Federal Reserve System is based on a profit maximizing one-state optimal inventory model under uncertainty. The bank is taken to operate as if the amount of reserves it has available to meet the sum of various demands on reserves is the reserves inventory of the bank. The bank begins the period with a given amount of reserves determined by the combined actions of the bank, the public, and the Federal Reserve, along with a set of subjective probability distributions regarding demands for reserves and interest rates. From this knowledge, the bank derives what it feels to be the quantity of reserves that minimizes expected losses, compares its actual stock of reserves with the optimal stock, and then makes the appropriate adjustment. This adjustment is made by either ordering additional reserves from the Federal Reserve, repaying borrowed reserves, buying or selling securities, or some combination of these methods. The final beginning-of-the-period equilibrium will be one in which the actual stock of reserves on hand will be equal to the desired stock.

The model was tested with the monthly time series data for all member banks from January, 1954 to January, 1967. The year 1954 was chosen as the beginning date because of certain historical and institutional factors that drastically affected the volume of discounts. In particular, it is after the nearly 20 year lapse of discounting that occurred during the depression of the 1930's and the pegged rate era that ended in 1951.

Also, it is after the excess profits tax imposed after the Korean War that exempted borrowed reserves which had a significant impact on the demand function. Except for the inclusion of vault cash into reserves during 1960, the time span used is highly similar in economic and institutional factors.

The estimation procedure employed a distributed lag scheme to derive expectations for unborrowed reserves, the market rate - discount rate spread, and the bill rate - discount rate spread. Banks were assumed to form these expectations on the basis of an exponential decay function which makes expectations depend on the present values of the variables and their past history with the values in the more distant past receiving less weight than current values. The expected values thus derived were placed into a linear multiple regression form to determine the coefficients and stability of the demand function.

The empirical results reveal a number of interesting aspects about the borrowing behavior of commercial banks, provides some information on commercial banks in general, and shows that a stable demand function can be obtained from monetary data. The regression for the whole period from 1954-67 is disappointing in that the permanent unborrowed reserves variable displays a wrong sign while being significant at the 1 percent level. However, when the period is split into the parts demarked by the inclusion of vault cash as reserves, proper signs are obtained for all coefficients although the permanent unborrowed reserves term is not significant for the 1954-60 period. This rather strange behavior of the permanent unborrowed reserves term may be due to the fact that the data used for vault cash were only "estimates" and thus not perfectly comparable, although certainly enough so as to justify its inclusion, as well as the confirmed change in the banks' demand for free reserves.[1]

The results from the two-year intervals shows a general rise in the coefficient of the permanent unborrowed reserves term, although both interest rate variables not only become weaker after 1961 but have a wrong sign and always fail to be significant. It is felt that this behavior of both the $(r_m - r_d)$ and $(r_s - r_d)$ variables reflects a change in the attitude of banks toward discounting relative to trading in Federal funds as a method of reserve adjustment. The rapid expansion of the Federal funds market, both in New York City and by large banks throughout the country that "make a market" in Federal funds with their correspondents, has given a large number of banks access to this alternative method of obtaining reserve balances.

The distributed lag expectation procedure used to compete permanent interest rates, in general, yielded an average memory period for unborrowed reserves and interest rates of approximately 4 to 5 months. The β weights for all variables increased slightly during the 13 year period which is consistent with the general notion that developments in the money market and other methods for making financial arrangements have increased the speed of banks' reaction to changes in their reserve positions and market interest rates. It is also heartening to note that the numerical values for the β weights during normal time spans are quite close to those obtained by George Morrison, who used the same weighting system, in his study of the liquidity preferences of commercial banks.[2]

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[1]G. Basevi, "Vault Cash and the Shift in the Desired Level of Free Reserves," Journal of Political Economy, LXIX (April, 1961), pp. 181-2.

[2]G. Morrison, op. cit.

FINAL REMARKS

Perhaps more than any other monetary instrument of the Federal Reserve, much controversy has surrounded the role of discount policy, both as a source of emergency reserves for member banks and as a weapon of monetary policy.[3] From the beginning, authorities have been concerned that the System would not be able to maintain control over the volume of borrowing as long as there existed an opportunity to profit from existing rate differentials. This had produced the feeling that the discount rate should be set as a "penalty rate" in the British sense of the term. The establishment of a British style penalty rate, however, does not seem possible in the United States given the prevailing institutional arrangements. In Britain, banks do not borrow directly from the Bank of England, but rather, make reserve adjustments with the discount houses who in turn borrow from the Bank at the going Bank rate. Because the discount houses specialize in holding only one or two assets rather than the wide array of assets with widely varying rates of return that banks in this country buy, the British Bank rate can be a true penalty rate while the discount rate cannot. It was from this hypothesis and institutional facts that the system envoked its "tradition against borrowing" as an effective non-price constraint to continuous and an ever expanding volume of borrowings along with a discount rate fixed at some level while market rates float.

The accumulating empirical evidence is ever more pointing in the direction that this policy is in error with regard to the role of the

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[3]M. Polakoff, "Federal Reserve Discount Policy and its Critics," in Banking and Monetary Studies, Dean Carson (ed.), (Homewood, Ill.: Richard D. Irwin, Inc.), pp. 190-212.

discount rate, the spread, and the influence of the tradition against borrowing. In particular, the work of Turner, Polakoff, Goldfeld, and Kane, and that performed here, all using different techniques, and different time periods give strong reason to believe that the rate spread is the important interest rate variable. While borrowings may be insensitive to the discount rate, it is not insensitive to changes in the discount rate relative to alternative methods of raising reserves. The result of a constant discount rate has been to make for an ever changing monetary policy as the market rate floated. While some may be impelled to argue that the authorities are learning and adjusting to this, in light of the studies mentioned and their own research efforts, the evidence seems to support exactly the opposite conclusion. Instead of changing the discount rate more frequently, they have changed it less frequently as this body of econometric evidence has accumulated. There were more than 4 times as many changes in the discount rate from 1954-61 as from 1961-67. Since 1960, there has never been more than a single change in the rate in any year, with one stretch of 35 months without a rate change. Of course, it might be argued that the least cost spread was moving in precisely the direction that the Fed wished and thus no change in the rate was necessary. Again, the evidence does not support this reasoning. Movements in the discount rate typically have been to wipe out growing or shrinking differentials. From 1954-59, discount rate changes were usually on the order of 1/4 of a percent while from 1959-67 they were always 1/2 of a percent, so that the Fed had to wait a longer period of time before having a sufficient differential to destroy. In any event, the less frequent changes in the discount rate have made discount policy as a credit-control weapon less stable in recent years.

Perhaps the most mistaken aspect of discount policy in recent years was the restatement and reinforcement of regulation A in 1955. The non-price barriers to borrowing incorporated in this regulation cannot be expected to be consistent with a strong and effective discount policy in the face of the rapid acceleration and growth of the Federal funds market. Banks, unwilling to subject themselves to increased inspection and criticism from the Fed because of borrowings, have turned in increasing numbers to Federal funds as an acceptable alternative. Preliminary evidence that banks are moving toward Federal funds and that this shift is related to regulation A is found by examining the reserve position of certain large banks and the position of the Federal funds rate. It is not unusual to find large city banks with zero Fed borrowings while being continuous purchasers of large amounts of Federal funds. This data, coupled with the fact that the funds rate has been consistently above the discount rate for the two years that this study was in progress is strong evidence in favor of the view that the tradition to borrow is significant and that banks are willing to sacrifice a certain monetary return to avoid the discount window. The effect of regulation A has been to drive banks toward alternative methods of obtaining reserves, has loosened the statistical relation between discounts and the least cost spread, and thereby has weakened considerably, if not destroyed altogether, the discount mechanism as a tool of monetary policy.

The empirical results, if they are anything like a reasonably accurate portrayal of bank attitudes toward discounting and if the influence of the Federal funds market has been judged appropriately, carry important implications for the future of the discount mechanism, both for its survival and its effect on the incentive for membership in the System.

A RECOMMENDATION

A strong case can be made for scrapping the discount mechanism altogether thereby letting the Federal funds market assume the full burden of reserve adjustments. The recommendation to be proposed here, however, is based on a personal opinion that Federal Reserve officials are of no mind, at least for the moment, to abandon discounting. If the present policy is continued, it becomes a simple matter of projection to invision the day when member banks completely abandon any significant borrowings from the Fed. It is feared that in an effort to maintain this institution, the authorities will impose restrictions on various forms of free market trading between banks causing reserve adjustments to be more costly, not to mention the costs that banks will incur if they try to devise an alternative system to replace the old. It is in an effort to prevent this unhappy state of affairs that the following recommendation is made.

1. The Federal Reserve should abandon its non-price barriers to borrowing as stated in regulation A. This will make the ability of a bank to borrow a right of membership in the System rather than a privilege granted by the Federal Reserve. The effect of this change will be to place discounting on an equal footing with Federal funds trading by equating their marginal costs. Banks, by combining their total borrowed reserves mix in a least cost fashion, will operate so as to guarantee an equality between the discount rate and the Federal funds rate.

2. The Federal Reserve should "tie" the discount rate to some market rate, preferably the 3-month Treasury bill rate. The 3-month bill rate is selected since the best available evidence has shown a stronger relation between borrowing and this market rate than other rates. The tying relationship between the discount rate and the bill rate should be such

that it is above the bill rate with the exact differential being left to the Fed. Since the goal of the tying relationship is to hold the least cost spread constant, it is apparent that the discount rate should be adjusted as frequently as possible. Perhaps the most workable solution would be to place the discount rate on any given day at some given percent over the closing yield of the previous day's bill rate, or

$$r_{d_t} = r_{s_{t-1}} + C \quad \text{for} \quad C \geq 0 \tag{4.1}$$

The exact value of the differential (C) will be left to the discretion of the Federal Reserve. The reason for choosing $C \geq 0$ is to prevent an unlimited volume of borrowing by producing a permanent profit differential. The reason for making (C) a discretionary variable for the Federal Reserve is twofold. First, since the discount rate cannot be a true penalty rate, the Fed must be able to control the appropriate amount of rate scalping. Second, it allows the discount rate to remain as a discretionary tool of monetary policy.

This recommendation, albeit simple, seems to provide for both the emergency valve and the credit control roles that the Federal Reserve has envisioned the discount mechanism to perform. Under this system the Federal Reserve is better off on both fronts. First, it will remain as an emergency source of reserves by nipping in the bud the current erosion of discounting by Federal funds and second, it will make discounting a more potent tool of monetary policy.

APPENDIX

APPENDIX

DERIVATIVE AND PROPERTIES OF THE EXPECTED LOSS FUNCTION

In this appendix the derivatives of the loss function will be computed along with a discussion of certain properties of the function. Let the expected loss function be written

$$E[L(Y)] = a + r_d(y-x) + \int_0^y\int_0^\infty\int_0^\infty G(y,r_m,r_s,z)\phi(z,r_m,r_s)dzdr_mdr_s$$

$$+ \int_y^\infty\int_0^\infty\int_0^\infty H(y,r_m,r_s,z)\phi(z,r_m,r_s)dzdr_mdr_s \qquad (1)$$

where

$$G(y,r_m,r_s,z) = r_m[Q(y-x)] + r_s[(1-Q)(y-z)] - r_mQkz - r_s(1-Q)kz \qquad (2)$$

and

$$H(y,r_m,r_s,z) = P(z-y) - r_mQky - r_s(1-Q)ky \qquad (3)$$

The partial derivatives of G and H with respect to y are given by

$$\frac{\partial G}{\partial Y}(y, r_m, r_s, z) = r_m Q + r_s(1-Q) \tag{4}$$

and

$$\frac{\partial H}{\partial Y}(y, r_m, r_s, z) = -P - r_m Qk - r_s(1-Q)k \tag{5}$$

$$= -k[\frac{\partial G}{\partial Y}(y, r_m, r_s, z)] - P \tag{6}$$

or, in brief

$$\frac{\partial H}{\partial Y} = -k(\frac{\partial G}{\partial Y}) - P \tag{7}$$

If k = 0, all demands on reserves are for non-earning assets so that an increase in the amount of reserves held (y) will reduce losses by the amount of the penalty function, or

$$\frac{\partial H}{\partial Y} = -P \tag{8}$$

If k = 1, all demands on reserves are for earning assets so that any increase in the amount of reserves held will reduce losses by the full value of the G function from adding the additional reserves, or $\frac{\partial G}{\partial Y}$, plus the value of the penalty function, or

$$\frac{\partial H}{\partial Y} = \frac{\partial G}{\partial Y} - P \tag{9}$$

Continuing with the computation of the derivative of the loss function, we note that the total derivative of E[L(Y)] with respect to (y) is

$$\frac{d}{dy}E[L(Y)] = r_d + \frac{d}{dy}\int_o^y\int_o^\infty\int_o^\infty G(y,r_m,r_s,z)\phi(z,r_m,r_s)dzdr_m dr_s$$

$$- \frac{d}{dy}\int_{-\infty}^y\int_o^\infty\int_o^\infty H(y,r_m,r_s,z)\phi(z,r_m,r_s)dzdr_m dr_s \tag{10}$$

Now the derivatives in (10) are given by

$$\frac{d}{dy}\int_o^y\int_o^\infty\int_o^\infty G(y,r_m,r_s,z)\phi(z,r_m,r_s)dzdr_m dr_s = \tag{11}$$

$$\int_o^\infty\int_o^\infty G(y,r_m,r_s,y)\phi(r_m,r_s,y)dr_m dr_s + \int_o^y\int_o^\infty\int_o^\infty \frac{\partial G}{\partial Y}(y,r_m,r_s,z)\phi(z,r_m,r_s)dzdr_m dr_s$$

and likewise,

$$\frac{d}{dy}\int_{-\infty}^{y}\int_{0}^{\infty}\int_{0}^{\infty}H(y,r_m,r_s,z)\phi(z,r_m,r_s)dzdr_mdr_s = \int_{0}^{\infty}\int_{0}^{\infty}H(y,r_m,r_s,y)\phi(r_m,r_s,y)dr_mdr_s$$

$$+ \int_{-\infty}^{y}\int_{0}^{\infty}\int_{0}^{\infty}\frac{\partial H}{\partial Y}(y,r_m,r_s,z)\phi(z,r_m,r_s)dzdr_mdr_s \qquad (12)$$

combining terms and using (7) we have the derivative as

$$\frac{d}{dy}E[L(Y)] = r_d + \int_{0}^{\infty}\int_{0}^{\infty}(G-H)(y,r_m,r_s,y)\phi(r_m,r_s,y)dr_mdr_s + \int_{0}^{y}\int_{0}^{\infty}\int_{0}^{\infty}\frac{\partial G}{\partial Y}(y,r_m,r_s,z)$$

$$\phi(z,r_m,r_s)dzdr_mdr_s + \int_{y}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}-k\frac{\partial G}{\partial Y}[(y,r_m,r_s,z) - P]\phi(z,r_m,r_s)dzdr_mdr_s \qquad (13)$$

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