VEHICULAR MECHANICS OF A TRACTOR
OPERATING ON YIELDING SOILS

by

Arnold G. Berlage

AN ABSTRACT

Submitted to the Colleges of Agriculture and Engineering
of Michigan State University of Agriculture and
Applied Science in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE IN AGRICULTURAL ENGINEERING

Department of Agricultural Engineering

1962

Approved by Wesley F. Buchele

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ABSTRACT

A tractor was instrumented to record dynamic data (rear axle torque, weight transfer, rolling resistance of front wheel and slip) at various constant drawbar pulls. These data were applied to a set of dynamic vehicular equations proposed by Buchele (1961) in order to test their validity.

Soil values were measured with a Bevameter at the time the above tests were run. These values were applied to another set of vehicular equations (Buchele 1961) to predict the performance of the vehicle operating under various soil conditions.

Indications of weight transfer and rolling resistance were obtained by attaching strain gages directly to the pertinent members of the vehicle thus making the transducers an integral part of the vehicle. Actual drawbar pull was measured by using a strain gage dynamometer between the drawbar and load cable when lifting a constant dead weight. Rear axle torque was obtained by means of a specially designed compression ring torque transducer. Construction and operation of the torquemeter are described.

Excitations from the strain gage bridges were amplified and recorded. The recorded data were processed and the experimental results were plotted for comparison.

Experimental data were substituted into the theoretical equations and the weight transfer calculated. The calculated weight transfer closely agreed with the experimentally determined weight transfer data. Weight transfer was shown to vary directly as drawbar pull while front wheel rolling resistance varied inversely as the pull.

Soil values were obtained with a Bevameter following the procedure suggested by Stong (1960). The prediction of vehicle performance by substituting these soil values into the vehicular mechanics equation cannot be justified by this study.

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INTRODUCTION

The state of the design of mobile equipment is such that if further progress is to be made, the empirical approach must be replaced by a theoretical approach to secure optimum design. The high cost of constructing a single full scale experimental model to test empirical designs makes such a practice unfeasible. Decreased production lead time necessitates a rapid design analysis.

During the past decade the problems of land locomotion and the methods required for their solution have gained wide interest. The ability of the soil to support a given vehicle and the ability of that vehicle to transport or tow a given load is determined by the soil strength. Vehicle stability is also related to the soil strength.

Bekker (1955) presented a means (the soil value system) for classifying a given soil in relation to its "strength". Since the development of this system, an empirical approach to the mobility problem has been possible. A theoretical approach is now desired.

A relationship between the soil values and vehicle design papameters would enable a designer to analyze in a theoretical manner many possible solutions to a given design problem. Vehicle performance and/or stability could be predicted for each given situation. Such a

theoretical approach through vehicular mechanics would reduce the expense, time and material which are encountered in the "experimental model" type of approach. The result would be an optimum design produced with a minimum of expense.

Buchele (1961) developed such a set of vehicular mechanics equations which related soil strength to vehicle performance. The resulting equations provide a means of utilizing soil value information for the prediction of vehicular performance under various soil conditions.

The purpose of this investigation is to test the proposed theoretical dynamic vehicular equations and the application of the soil value system to wheeled vehicles operating on yielding soils. The results as determined from the equations were compared with experimental data obtained with the use of strain gage transducers.

The required experimental values were:

1. Rear axle torque
2. Drawbar pull
3. Weight transfer
4. Front wheel rolling resistance

The soil values were determined from experimental data obtained with a Bevameter and consisted of the following:

K_c, the modulus of coh sion

K_ϕ, the modulus of deformation

n , the terrain coefficient

ϕ , the angle of internal friction

C , the soil cohesion

Additional test data determined were:

1. Soil bulk density
2. Soil moisture
3. Theoretical forward travel of vehicle
4. Actual forward travel of vehicle

LITERATURE REVIEW

The application of strain gage techniques for the design of various transducers has become an important phase of many research analyses. A convenient test system consists of a strain gage transducer, amplifier, and direct writing oscillograph.

Lockery (1959) lists the available torque-measuring devices in two classes as follows:

1. Angular-twist types:
   - (a) Optical
   - (b) Electrical:
     - (i) Variable capacitance
     - (ii) Variable coupling
     - (iii) Variable reluctance
     - (iv) Frequency sensitive
     - (v) Phase sensitive
2. Surface stress or strain types:
   - (a) Variable permeability
   - (b) Photostress type
   - (c) SR-4 strain gage

Of the above devices, the SR-4 strain gage is most applicable for determination of dynamic torque measurements on a drive axle.

The conventional method of determining the driving torque of a powered shaft consists of applying strain gages along 45° helices on the shaft surface. A brush-slip ring system is required to collect the strain indications of the rotating shaft. Perry and Lissner (1955) suggest three general methods for combatting the problems inherent in slip ring systems.

Hayes (1961) obtained an indication of rear axle torque by applying strain gages along the 45° helices of a reduced section of the power shaft ahead of the final drive. A brush-slip ring assembly was used to transmit the excitation and signal voltages.

The strain gage and slip ring method of determining rear axle torque was used by Davis (1961). Four gages were attached to each rear axle shaft, and the slip ring assembly was used to connect the gages to a terminal box.

Trabbic (1959) obtained rear axle torque by applying strain gages on the axle but did not use a slip ring collector. The test runs were relatively short; the strain gage lead cable wrapped around the axle and unwrapped as the tractor was backed to the starting point. The wheel was jacked up and turned until the additional rotation due to slip was unwound.

Both Hayes and Davis obtained vertical weight transfer by attaching strain gages to the rear axle housings in the area of maximum loading moment. The gages were placed at the same location on the centerline of both right fa

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and left axle housings.

Newbury (1961) obtained an indication of front wheel rolling resistance by redesigning the junction between front wheel spindle and axle. The strain gage transducer was essentially a flexible paralleogram joining the spindle and axle.

Walters and Jensen (1954) measured front wheel rolling resistance on both a row crop and general purpose tractor. The technique used to check rolling resistance on the row crop tractor consisted of placing strain gages on a cantilever member which was located so as to resist the fore and aft movement of the steering spindle. The steering linkage of the general purpose tractor was rebuilt to provide center point steering. Morehouse proving rings were installed in the tie rod to either side of the center steering point. The strain gaged proving rings provided an indication of the rolling resistance force.

Jensen (1954) reports that drawbar pull was satisfactorily obtained by using a steel ring with four strain gages applied so that two were in tension while two were in compression. The torquemeter described by Jensen consisted of a strain gaged shaft with a brush and slip ring collector.

The drawbar dynamometer described by Clyde (1955) consisted of two linked beams pivoted on clevises. Strain gages were applied at points of maximum bending. The circuit used was a modification of that used by Jensen.

Trabbic (1959) obtained the actual distance traveled per test run by attaching a microswitch to the pulley support of the dead weight loading frame so that the switch was activated twice per revolution of the pulley over which the load cable passed. The theoretical distance traveled was indicated by a microswitch operating against the lug bars of the rear tire. The drawbar load placed on the tractor consisted of various amounts of cast iron weights lifted over a pulley arrangement.

APPARATUS AND INSTRUMENTATION

The test vehicle (a Massey Ferguson 50 tractor) was instrumented by attaching strain gages and strain gage transducers to the vehicle to obtain the dynamic values of the rear axle torque, drawbar pull, rear axle weight change or weight transfer, and front wheel rolling resistance.

Rear Axle Torque

The literature survey indicated that the most suitable method currently available for the indication of rear axle torque consisted of four SR-4 strain gages attached (on 45° helices) to a reduced section of the axle shaft. The excitation and signal voltages were transmitted through a brush-slip ring assembly. Because of the problems inherent in slip ring systems and in the turning down and drilling of a rear axle shaft, a more reliable method of obtaining the axle torque was desired.

Torquemeter design

A compression ring torque transducer (Figure 1) was designed. The design requirements for the instrument were as follows:

(1) Easily attached to either left or right rear axle

(2) No moving or sliding electrical connections

(3) Able to withstand field service

(4) Easily removed strain sensitive component so that the test vehicle would be capable of serving multiple requirements

The basic principle of the Buchele hydrostatic torquemeter (1951) was used in the final design of the transducer which consists of one rotating and one stationary assembly. Since the wheel mounting flange of the test vehicle is an integral part of the axle shaft, the rotating assembly had to include a component which would attach to the mounting flange (Figure 2).

This member consists of an 11 1/2 in. diameter steel plate, 1 in. thick, press fit and welded on a 2 1/2 in. diameter shaft 14 1/2 in. long. The plate has a bolting circle consisting of eight bolt holes with recessed lug nut seats which matches the bolt circle of either axle flange. Thus the plate and stub axle assembly is attached to the flange by the existing lug bolts in the flange. Also in the plate are eight conical, hardened seats equally spaced around an 8 7/8 in. circle concentric with the shaft center. The stub axle shaft is threaded to receive a 2 1/2 in. national fine thread nut.

A second component of the rotating assembly consists of a matching steel plate welded concentrically to a hub which provides the centering guide for the wheel and

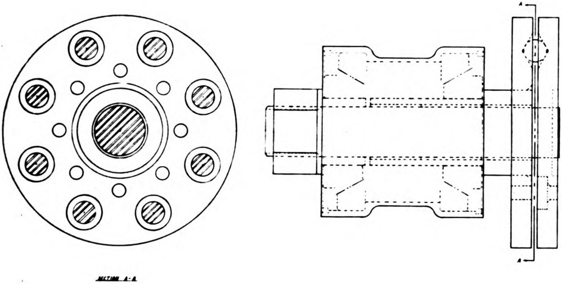

Figure 1. Compression ring torque transducer

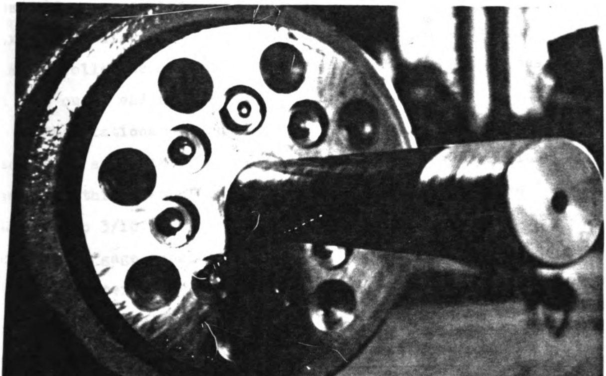

Figure 2. Torquemeter inner plate assembly

the seat for one of the Timken roller bearings (Figure 3).

The eight lug bolts required to attach the wheel to the transducer are pressed into the lug bolt holes. Bronze bushings are pressed into each end of the hub and the entire hub assembly is positioned on the 2 1/2 in. diameter shaft so that the eight conical seats in each plate are exactly opposite each other.

Eight 1 1/8 in. diameter hardened steel ball bearings are placed in the matching seats and separate the two plates by approximately 1/4 in. A rubber dust sealing band is placed around the plates to protect the balls and seats from excessive abrasive wear which might occur if dirt were allowed to enter between the plates. A second Timken roller bearing was pressed on a collar which rides on the outer end of the shaft.

The stationary assembly (the compression ring) was made from a length of 8 1/2 in. diameter tubing 8 in. long. The wall thickness of the three inch center section was reduced to 3/16 in. to provide increased sensitivity for strain gage application.

The outer races of the two Timken bearings were pressed into the machined ends of the tubing. Dust shields made of circular galvanized sheet steel pieces were placed at each end of the compression ring to prevent foreign material from entering the bearings. A protective sheet steel cover was placed around the reduced section of the tubing to

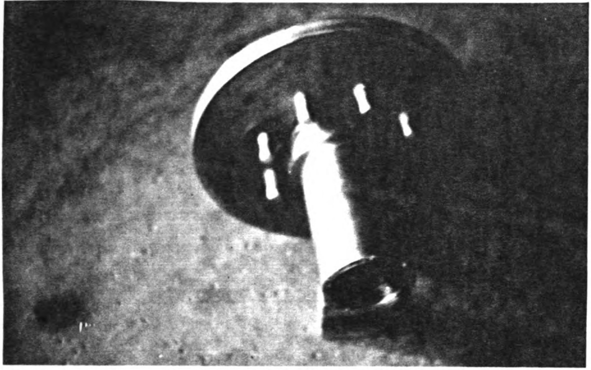

Figure 3. Torquemeter outer plate assembly

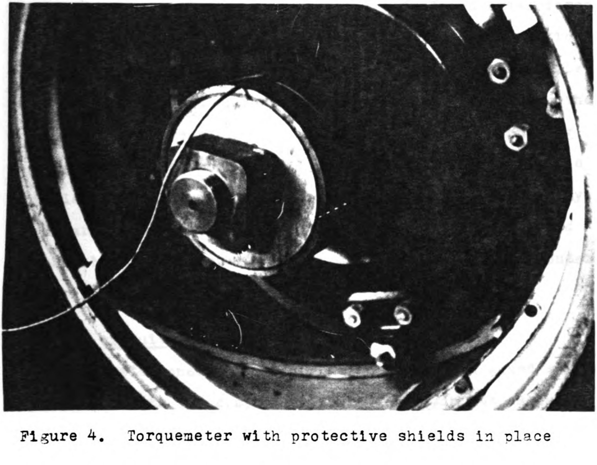

Figure 4. Torquemeter with protective shields in place

protect the strain gages (Figure 4).

A clamping ring was placed around the nut end of the compression ring. Two rods were fastened to the clamp and anchored to a support to resist the tendency to rotate caused by bearing friction. The support extended from the redesigned foot rest and provided a convenient attaching point for the strain gage cables (Figure 5).

Operational analysis of the torquemeter

When the final drive of the test vehicle is engaged, the axle flange and inner plate assembly of the torquemeter rotate as a unit. The outer plate and hub assembly, with vehicle wheel attached, receive tangential and axial force components from the inner plate through the caming action of the eight steel balls. This action tends to force the outer plate around and outward thus exerting torque on the wheel and compression on the stationary compression ring.

The compression ring riding on the Timken high angle roller bearings, resists the axial force components. Any outward movement of the outer plate and hub assembly and inner bearing is restricted by the 2 1/2 in. nut and outer bearing on the shaft of the inner plate assembly.

Bridge circuit

The stationary compression ring is strain gaged so as to indicate the compressive (axial) loads but cancel bending and torsional loads. Four type A-5, SR-4, strain

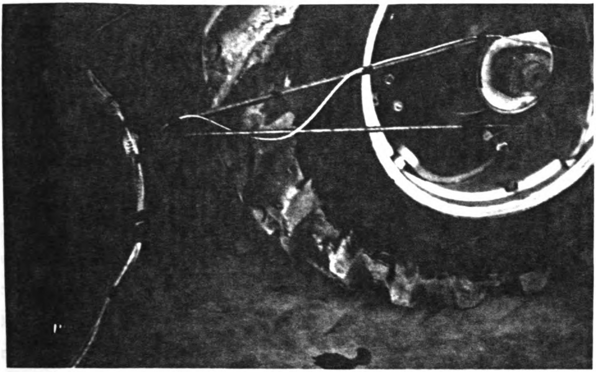

Figure 5. Torquemeter clamping ring and support

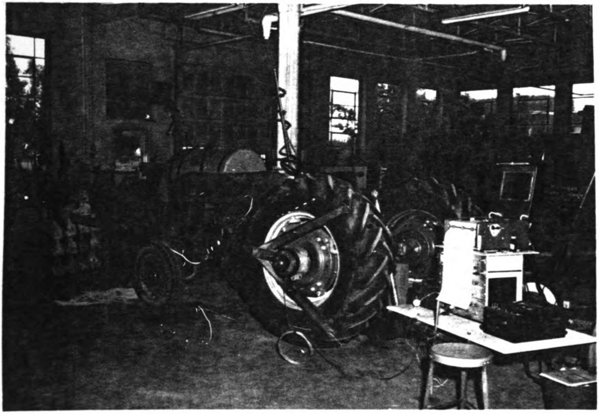

Figure 6. Static calibration of torquemeter

gages were applied to the outer surface of the reduced section.

Two of the gages were placed with their axes parallel to the centerline of the tubing. These primary sensing elements were temperature compensated by placing the other two gages in a Poisson arrangement, that is with their axes perpendicular to the centerline. The pair of gages in each arrangement were placed diametrically opposite each other, and because of their arrangement in the Wheatstone bridge, forces other than compression and tension (which is not possible with this design) were canceled. A schematic drawing of the bridge circuit is shown in Figure 7.

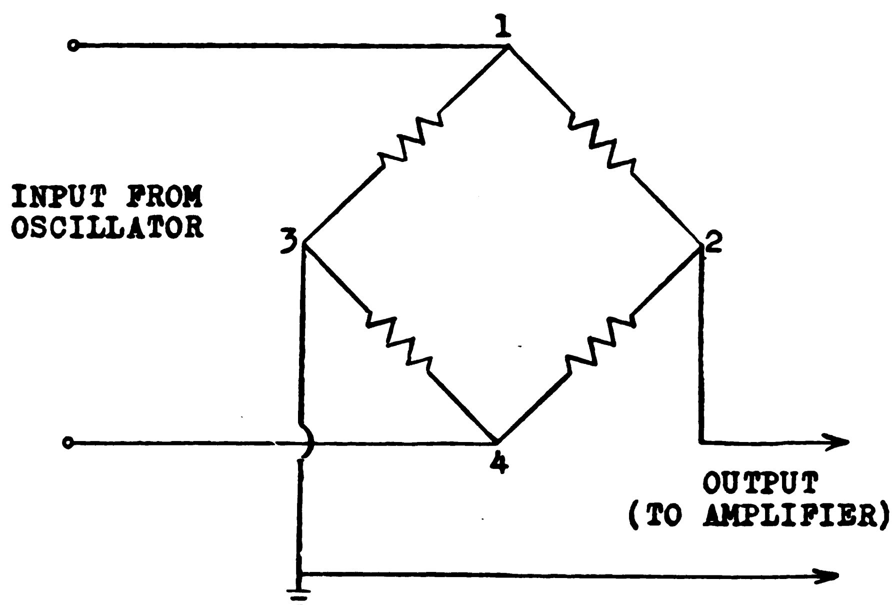

Figure 7. Strain gage bridge circuit

A layer of Petrosene-A wax was applied over each gage for water proofing. The strain gage cable was securely fastened to the stationary ring so the strain gages would not be injured if tension were accidentally applied to the cable.

Calibration

The torquemeter was first calibrated statically by placing known weights on a fabricated rectangular bar at a distance of ten feet from the wheel center. The bar was fastened rigidly to the wheel which was not in contact with the floor. The axle flange and inner plate assembly were locked in place by the brake. As weights were added to the calibration bar, the known torque caused the outer plate assembly and wheel to apply a compressive force to the strain gaged stationary compression ring. Figure 6 shows the static calibration arrangement.

A dynamic calibration was made and compared with the static torque calibration. One end of a series of three tire chains was fastened to the rim of the wheel attached to the torquemeter. A rigid member with a center pull point was welded between the side links at the opposite end of the chain.

The torque sensing wheel was raised off the ground and the vehicle was anchored to the rear. The chains were arranged in a straight line directly ahead of the tire and fastened to a load cable. When torque was applied to the wheel, the chains began to wrap around the tire thereby applying a known load at a known radius. The calibration arrangement is shown in Figure 8.

The resulting calibration curves are shown in Figure 10. Linearity is excellent for torque values above 900 lb-ft. The transducer is capable of operation in more than one torque range simply by machining additional compression rings with various wall thicknesses. The rings can be interchanged merely by removing the 2 1/2 in. nut and the outer bearing. When the compression ring and both bearings are replaced with a spacer ring, the vehicle may be used for other operations (Figure 9).

Drawbar Pull

A constant drawbar pull was desired to facilitate experimental data analysis. The minimum length of test run was to be twenty feet.

The two requirements were satisfied by erecting a thirty five foot power pole with a special crossarm mounted at the top. The top of the pole was notched to receive two channel iron crossarms, one on each side. A five inch pulley was placed between and at each end of the crossarms. The assembly was braced and bolted to the pole. The pulleys were placed an equal distance to either side of the pole center so that no unbalanced moment would

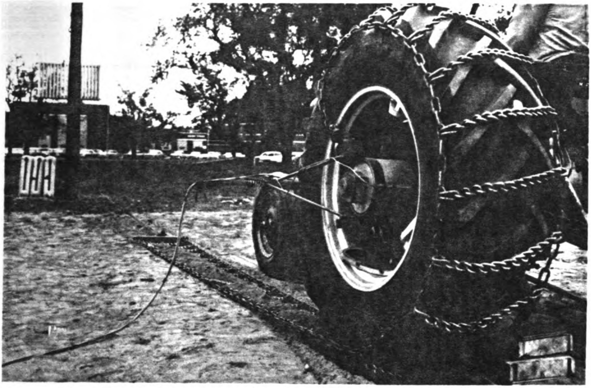

Figure 8. Dynamic calibration of torquemeter

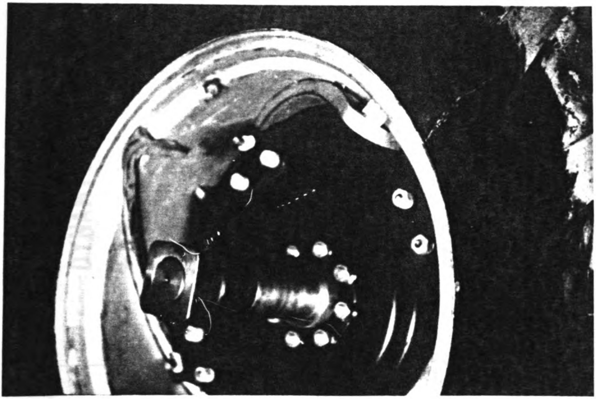

Figure 9. Torquemeter with sensing element removed

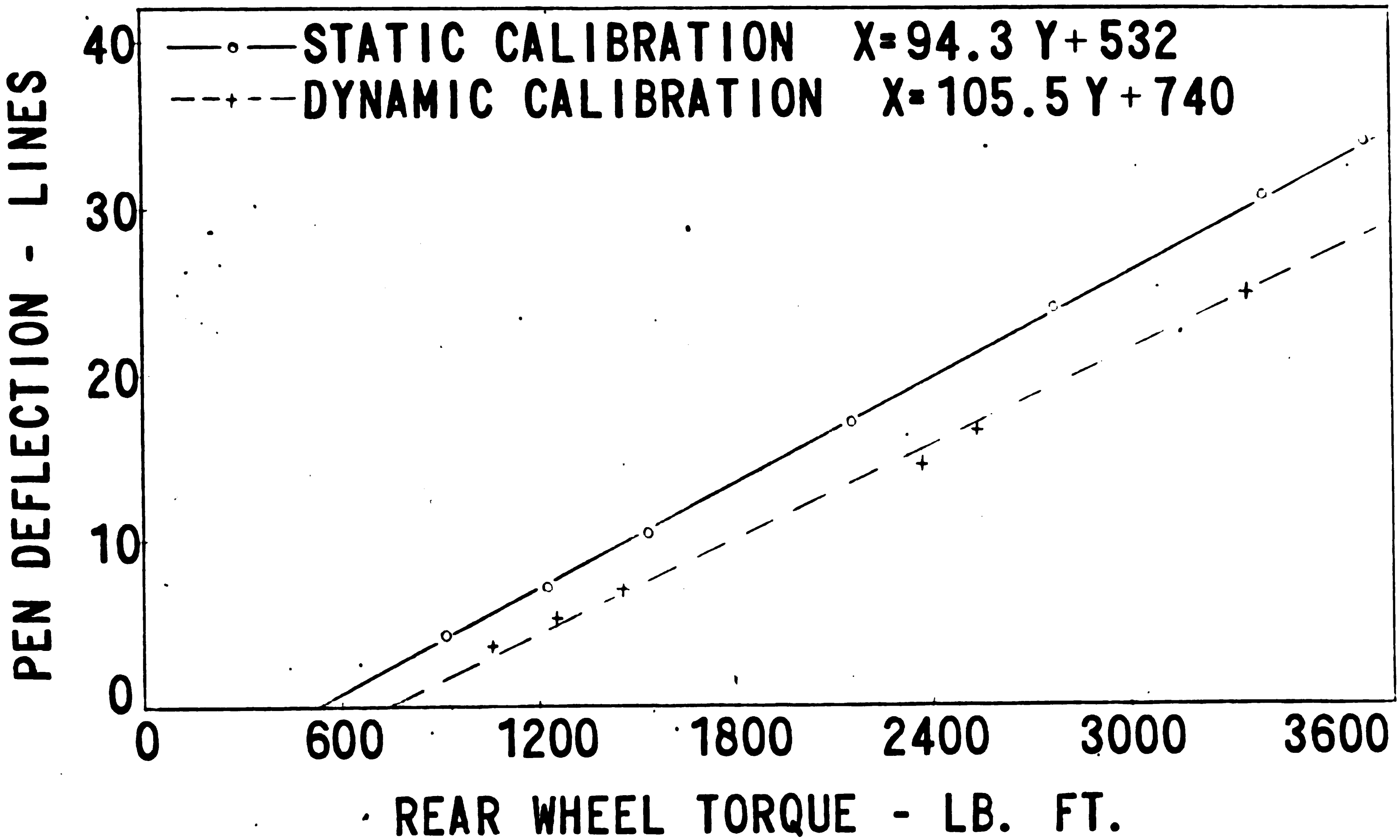

Figure 10. Static and dynamic calibration curves for compression ring torque transducer

act about this center.

A 3/8 in. cable passed over the crossarm pulleys and under an identical pulley which was anchored to the base of the pole. The bottom of the lower pulley was positioned at the height of the tractor pull points. Since the lower links of the three-point hitch were floating, the load cable was level during test runs. One end of the cable was connected to the drawbar and the other end was hooked to the desired number of weights.

The weights consisted of four, large, discarded steam-radiators. A strain gaged ring drawbar dynamometer was connected between the drawbar and the cable prior to the actual test runs (Figure 11). The dynamometer had previously been calibrated for lines of oscillograph pen deflection versus a known load as shown in Figure 12, and the resulting calibration curve is shown in Figure 13.

Each load was pulled through the test run and its drawbar pull, as indicated by the dynamometer, was recorded. All successive runs were made without the dynamometer. Figure 14 shows the loading arrangement. Table I gives the individual static weight for each of the four weights. The dynamic (static weight plus pulley friction) drawbar pull values of the individual weights as well as the pull of the combinations used in the test runs are given in Table II.

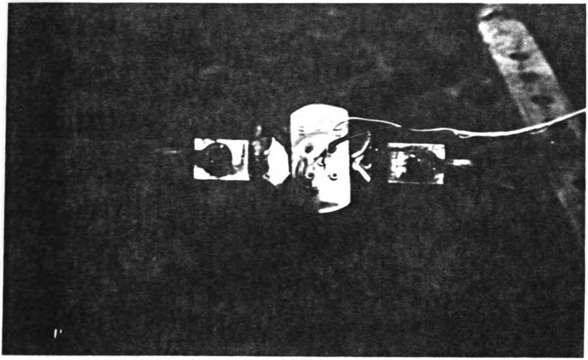

Figure 11. Strain gaged ring drawbar dynamometer

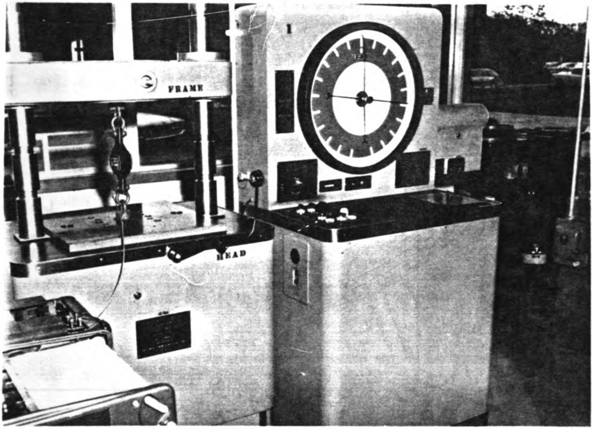

Figure 12. Calibration arrangement for drawbar dynamometer

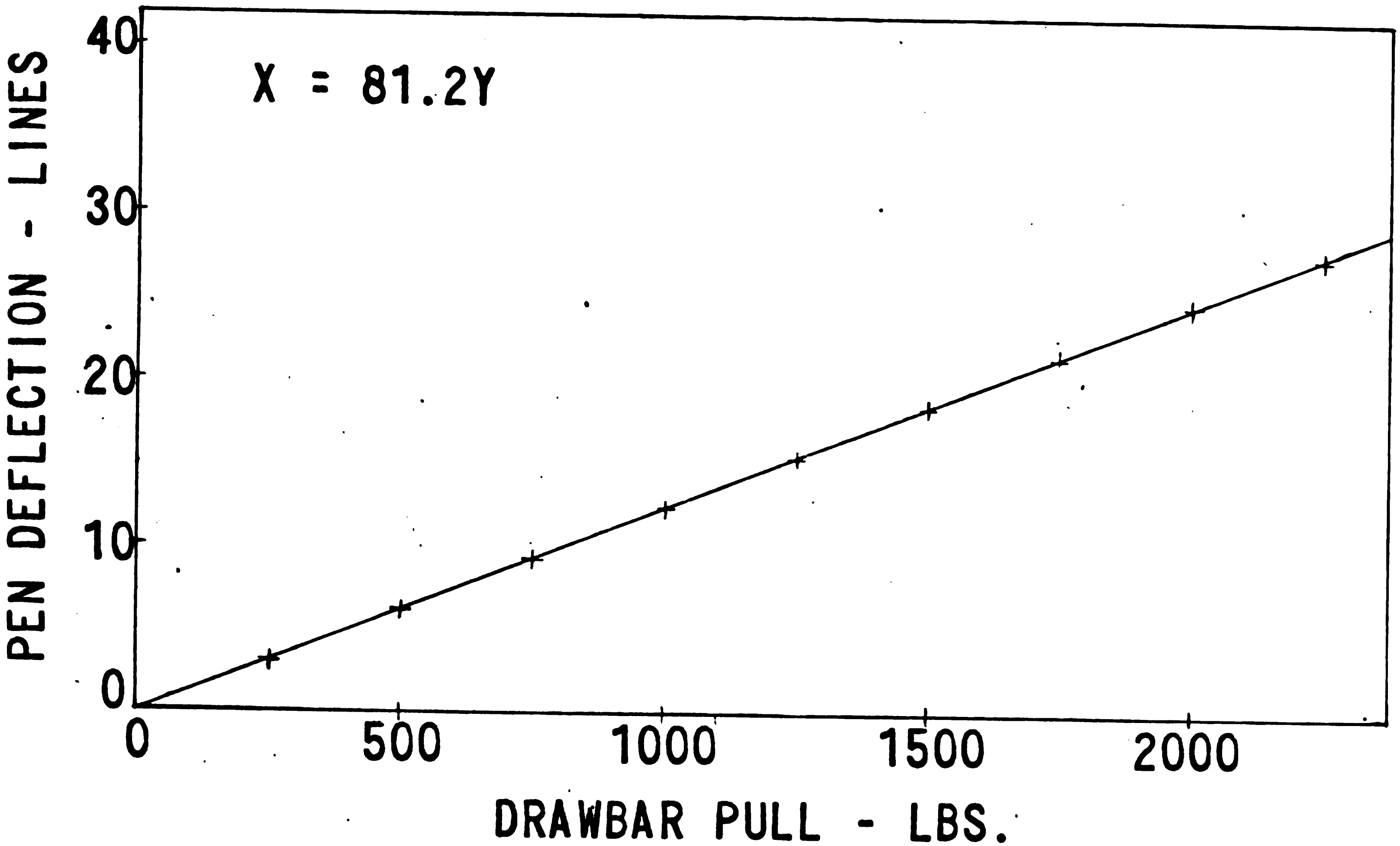

Figure 13. Calibration curve for drawbar dynamometer

Figure 14. Loading arrangement for a constant drawbar pull

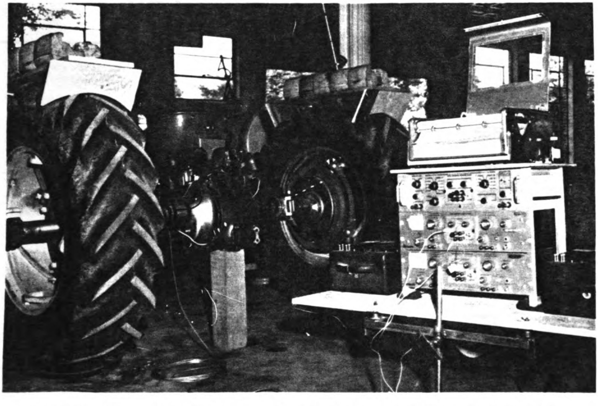

Figure 15. Calibration of weight transfer transducer

TABLE I

WEIGHTS USED FOR DRAWBAR PULL

Identification Number	Pounds Static Weight
4	397
5	469
6	475.5
7	544

TABLE II

DRAWBAR PULL WITH WEIGHTS ON LOAD CABLE

Identification Number	Pounds Drawbar Pull
4	481
5	576
6	585
7	666
4,6	1082
4,7	1161
5,6	1193
4,5,6	1688
4,6,7	1761
5,6,7	1851
4,5,6,7	2289

Weight Transfer

Weight transfer was determined by recording the change in rear axle weight. This increase over the static weight was obtained by applying four SR-4 strain gages to the rear axle housings. The gages were placed on the center-line of the axle housings and as close to the differential housing as possible for maximum cantilever action. Two gages were applied directly opposite each other on both housings, and all four gages were connected into a bridge and arranged for maximum sensitivity.

The calibration arrangement is shown in Figure 15. A single support was placed under the center of the differential housing so that the rear wheels were not in contact with the floor. Weight platforms were constructed to fit the tire contour and, with rear wheels locked, known weights were placed on the platforms. Figure 16 shows the rear axle calibration curve.

Front Wheel Rolling Resistance

An indication of rolling resistance was obtained from the front wheel spindle. A short length near the center of the spindle was reduced in diameter so as to increase the strain sensitivity in torsion (Figure 17). Four type A-7 SR-4 strain gages were applied in a torsion arrangement (diametrically opposite along 45° helices). Since the point of contact between the tire and the ground

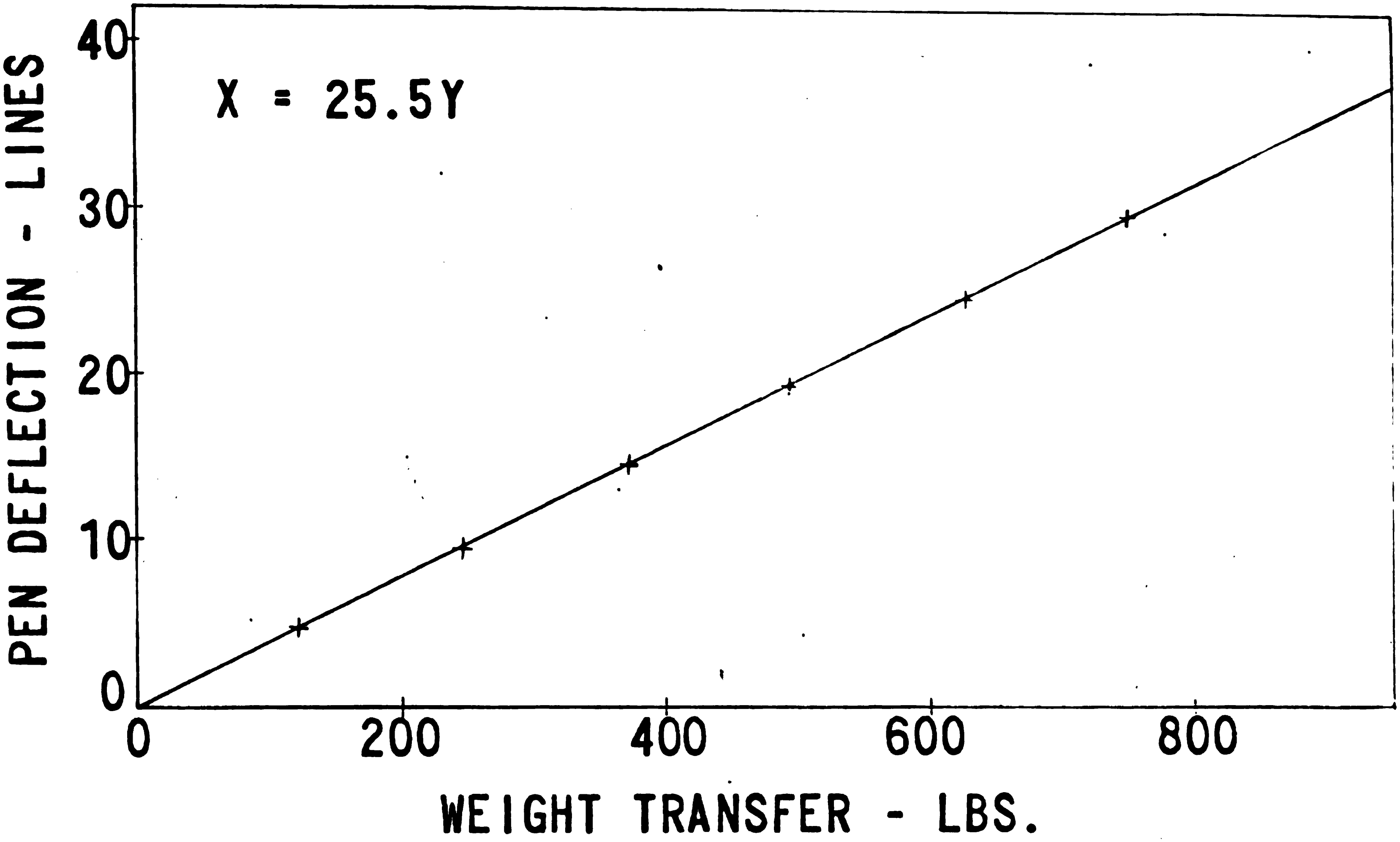

Figure 16. Calibration curve for rear axle weight transfer transducer

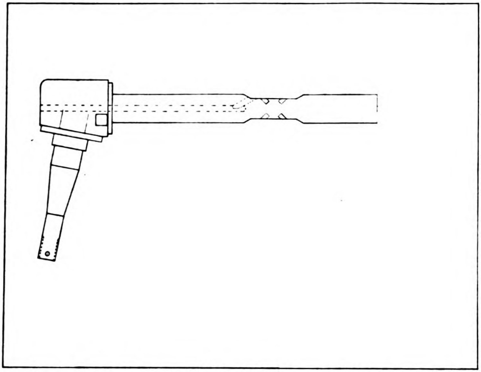

Figure 17. Rolling resistance transducer

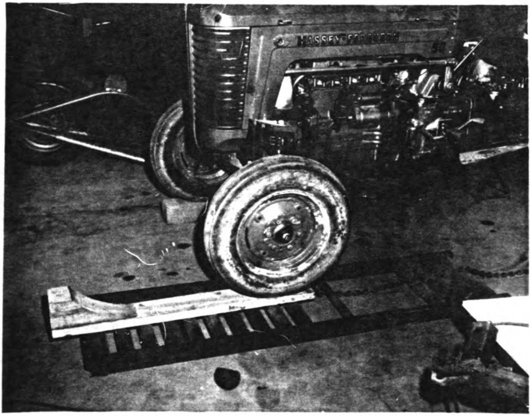

Figure 18. Calibration of rolling resistance transducer

is outside of the spindle center line, the rolling resistance places the spindle in torsion. The resulting torsional strain was calibrated as shown in Figure 18.

The tire was placed on a smooth board which was supported by a series of rollers. With the test vehicle level and with the front wheels subjected to their normal static load, the board was drawn between the rollers and tire. After rolling approximately 18 in., a cleat, shaped to fit the tire curvature, contacted the tire. The cleat was to simulate rolling resistance in a yielding soil and afford a means of applying an increasing torsional strain to the spindle. Figure 19a shows the resulting calibration curve.

Amplification and Recording

Shielded cables (four conductor) were used to connect the bridge circuit of each transducer to a Brush amplifier. The individual cables were taped together and directed from the vehicle to the amplifiers along the torquemeter support (Figure 5). Coil springs were attached to the end of the support to assure maximum cable flexibility and reduce the possibility of cable damage should the cables become entangled with the rear wheel. The amplifier outputs were connected to a portable Brush direct writing oscillograph for recording of the dynamic loads applied to each transducer. The instruments are shown in Figure 20.

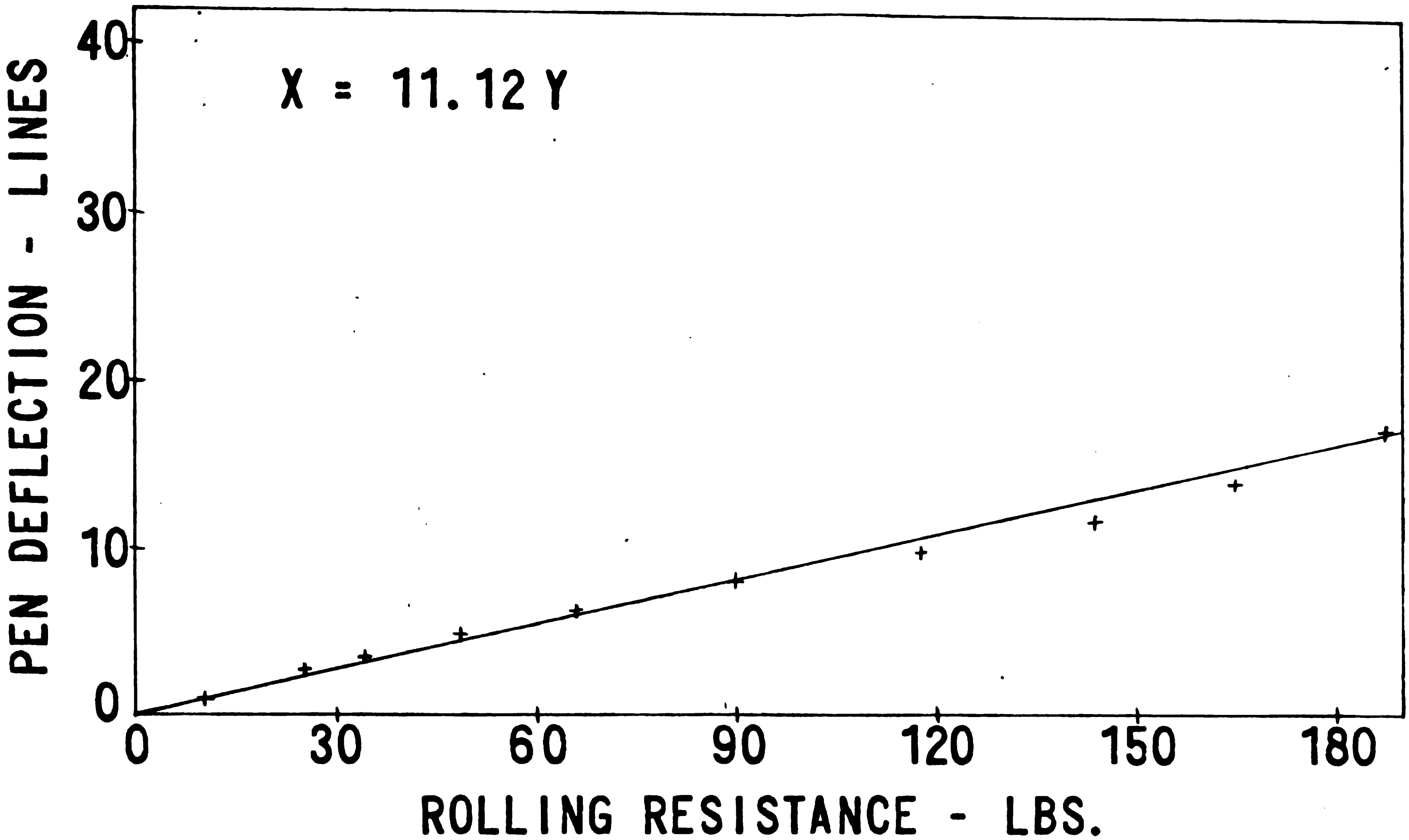

Figure 19a. Calibration curve for front wheel rolling resistance transducer

Soil Values

The equations proposed by Buchele (1962) are listed here as follows (see Figure 19b):

Dynamic weight transfer when soil conditions are limiting (wheels may spin):

$$W_D = \frac{R_{21}X_{21} + R_{24}Y_{24} - P_2Y_2 - R_{22}Y_{22} - R_{12}Y_{12}}{X_{15}} \qquad (1)$$

Dynamic weight transfer when engine torque is limiting (engine may stall):

$$W_D = \frac{TRE_2 - P_2Y_2 - R_{12}Y_{12}}{X_{15}} \qquad (2)$$

where:

W_D = Dynamic weight transfer

R_{21} = Vertical soil reaction on rear tires

X_{21} = Horizontal distance to R_{21}

R_{24} = Horizontal soil reaction to tire lug

Y_{24} = Vertical distance to R_{24}

P_2 = Drawbar pull

Y_2 = Vertical distance to P_2

R_{22} = Rolling resistance at rear wheels

Y_{22} = Vertical distance to R_{22}

R_{12} = Rolling resistance at front wheels

Y_{12} = Vertical distance to R_{12}

X_{15} = Tractor wheel base

TRE_2 = Rear axle torque

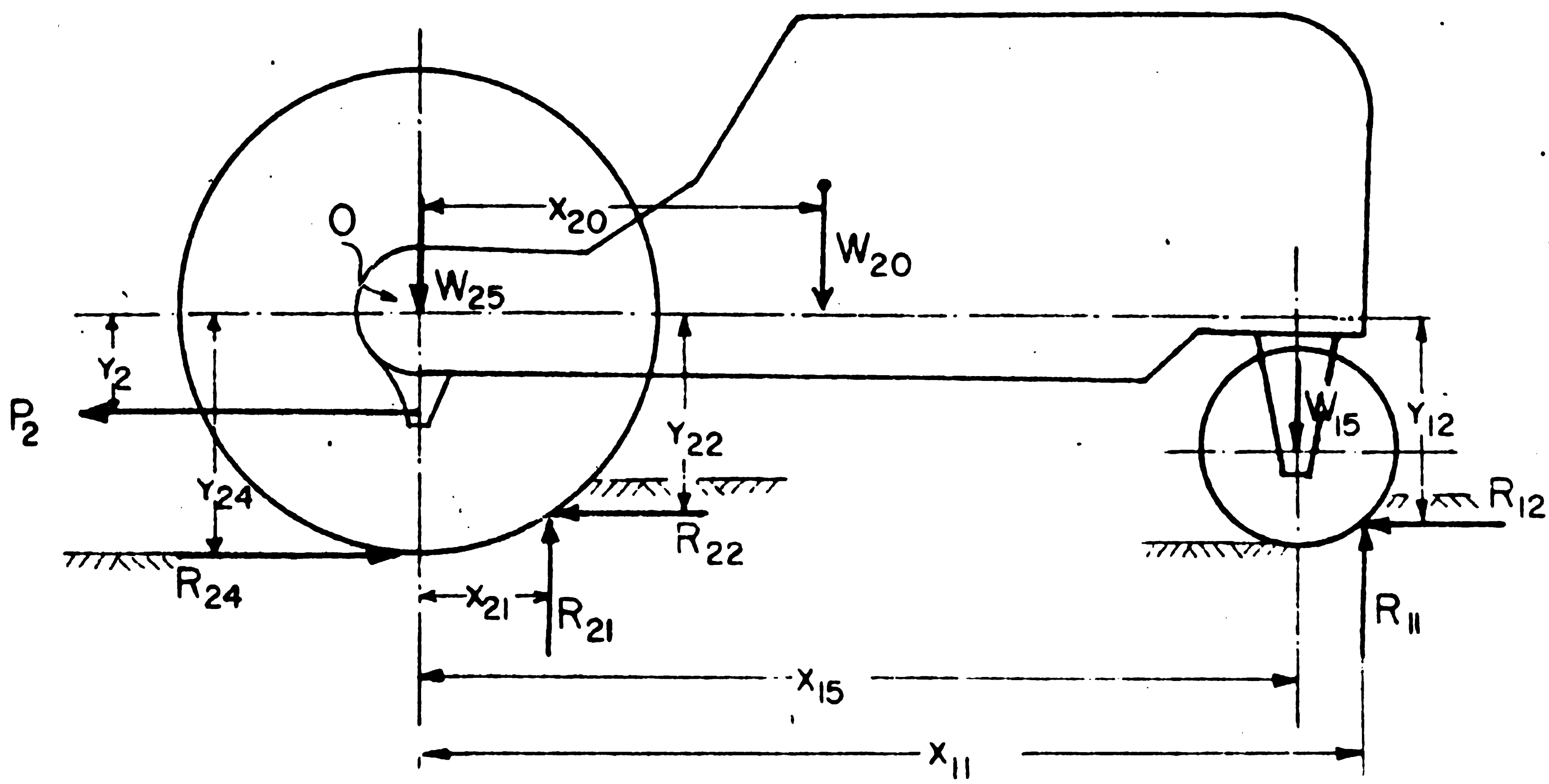

Figure 19b. Free body diagram of tractor

When the soil values are substituted into equations 1 and 2 respectively, the following equations are formed:

$$W_D = \frac{(W_{2s}+W_D)X_{21} + \left[b_2L_2C + (W_{2s}+W_D)\tan\phi\right]Y_{24} - P_2Y_2}{X_{15}}$$

$$\frac{-\dfrac{Y_{22}}{(n+1)(K_c+b_1K_\phi)^{1/n}}\left[\left(\dfrac{W_{2s}+W_D}{L_2}\right)^{\frac{n+1}{n}} + \left(\dfrac{W_{1s}-W_D}{L_1}\right)^{\frac{n+1}{n}}\right]}{X_{15}} \qquad (3)$$

$$W_D = \frac{TRE_2 - P_2Y_2 - \dfrac{Y_{12}}{(n+1)(K_c+b_1K_\phi)^{1/n}}\left(\dfrac{W_{1s}-W_D}{L_1}\right)^{\frac{n+1}{n}}}{X_{15}} \qquad (4)$$

where:

W_{2s} = Static weight on rear wheels

W_{1s} = Static weight on front wheels

b_2 = Width of rear wheel contact patch

b_1 = Width of front wheel contact patch

L_2 = Length of rear wheel contact patch

L_1 = Length of front wheel contact patch

C, K_ϕ, K_c, ϕ and n = Soil values defined in the introduction

Values of C, K_ϕ, K_c, ϕ and n were obtained through the use of a Bevameter (Figure 21); an apparatus which records the force required to penetrate the soil with various diameter probes as well as the torque required to shear the soil when various amounts of normal pressure are applied. Soil moisture and bulk density were obtained

Figure 20. Major Professor and author observing instruments

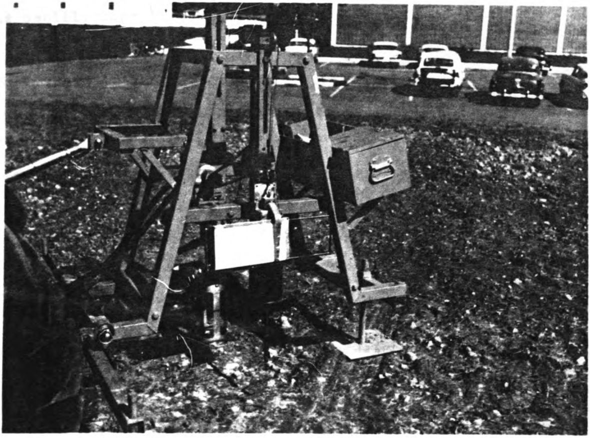

Figure 21. Bevameter setup for recording the shear force

from soil samples taken with a Uhland sampler.

Slip

In order that vehicle slip could be determined, the actual and theoretical distance traveled was obtained through the use of microswitches.

Theoretical travel

One switch was placed directly above the power-take-off shaft and activated by a bolt which extended through the shaft (Figure 22). The switch made contact twice during each shaft revolution. It was electrically connected in the oscillograph left event marker circuit.

The operating lever for the power-take-off shaft was placed in the "ground drive" position. In this position the shaft speed was determined by the angular velocity of the vehicle drive wheels rather than by the engine speed. The ratio between shaft and drive wheel rotations was found to be 8.72 to 1. The forward travel per wheel revolution was found to be 157 inches.

Actual travel

A second switch was fastened to the lower pulley bracket as shown in Figure 23. The switch was closed twice per revolution by cams welded to the pulley. The switch was electrically connected in the oscillograph right event marker circuit.

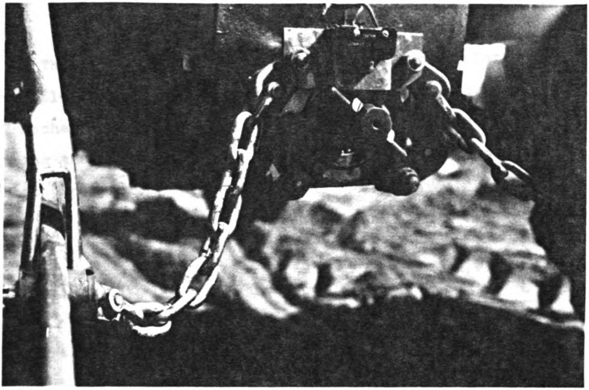

Figure 22. Microswitch indicating theoretical travel

Figure 23. Microswitch indicating actual travel

The length of cable displaced per pulley revolution was calculated and then verified experimentally to be 17 inches. The actual travel was then obtained by determining the number of event marks on the right side of the chart paper for all or any part of the test run.

EXPERIMENTAL PROCEDURE

The instrumented test vehicle was operated on a plot approximately 15 ft. by 40 ft. The soil was a typical agricultural soil classified as a Miami fine sandy loam.

A test series consisted of four runs, each with a different drawbar pull. Prior to each series of runs the soil was prepared by a rototilling operation followed by a firming operation accomplished with a roller. After preparing the soil, Bevameter recordings were made and soil samples were taken.

The wheel spacing of the front wheels was 49 in; that of the rear wheels was 72 in. With this arrangement, both front and rear wheels traveled in undisturbed soil.

Each test run was made with a known constant drawbar pull. The variables recorded during each run were rear axle torque, weight transfer, front wheel rolling resistance, theoretical forward travel and actual forward travel.

After the amplifiers were warm, the bridge of each transducer was balanced and electrically calibrated to insure identical amplifier sensitivity for all succeeding runs. The first few trial runs indicated the necessity of limiting the engine speed to a maximum of 900 revolutions per minute. Mechanical vibrations were transmitted to the rear axle housing at engine speeds above this maximum limit, and the resulting oscillograph pen oscillation precluded the accurate recording of the weight transfer.

The front wheel rolling resistance transducer was balanced and the amplifier sensitivity checked prior to each run. This procedure was followed after the wheel had been jacked up and was free of any soil contact. The vehicle operator was in position when the wheel was returned to the soil. The torquemeter and rear axle bridge balance and amplifier sensitivity were then checked with operator in place.

The function of the operator was to start the engine, engage the transmission in its lowest forward gear, adjust the engine throttle and engage the clutch causing the vehicle to travel forward at a constant velocity. Following each test the vehicle was repositioned for the subsequent run. At that time the steering wheel was placed in the position necessary to guide the vehicle through undisturbed soil without manipulating the steering mechanism during the run. Such manipulation would have been indicated as a change in rolling resistance. Immediately after the weights were freely suspended on the cable and the vehicle was traveling at a constant velocity, the drawbar pull was found to be constant and all variables thus recorded were a function of the constant pull.

RESULTS AND DISCUSSION

Experimental data were substituted into the vehicular equations, and an error analysis of the results was conducted.

Analysis of Dynamic Vehicular Mechanics Equations

Experimental values obtained from the transducers described in the previous section were substituted into equations 2 and 4. Equation 2 consists of force values exclusively while equation 4 involves soil values in combination with force values.

Results involving experimental force values

The dynamic torque value was extremely uniform for each test run. Weight transfer and the percent slip (calculated from the theoretical and actual forward travel) varied only slightly during a run.

Rolling resistance values obtained from the oscillograph chart were determined by counting the lines of deflection from the "zero" line about which the bridge was balanced. An initial pen deflection, which was not repeatable, occurred when the static front end weight acted on the wheel when lowered after the bridge was balanced. This initial deflection, when it occurred, appeared as if a rolling resistance force were acting on the wheel with the vehicle in reverse motion.

The zero line, rather than the line of initial deflection, was used as the base from which the number of lines deflection were counted because it was found, by a series of no-load forward runs on concrete, that regardless of the initial deflection the rolling resistance on concrete was always the same number of lines from the zero line. The initial pen deflection varied due to the different amount of torsional strain induced in the spindle when the static weight was placed on the wheel after balancing. It was theorized that this variation occurred because of the relative difference in the position of the spindle each time the wheel was lowered. When the above procedure was followed in the analysis of data, the resulting indications of rolling resistance were repeatable for a given drawbar pull.

Figure 24 shows a section of oscillograph chart paper with the recording of theoretical travel, weight transfer, torque, rolling resistance, and actual travel for a constant drawbar pull of 2289 lbs. An average value for lines of pen deflection over a given chart length was determined for the weight transfer, torque and rolling resistance values. Theoretical and actual travel were computed for the same chart length.

The experimental values of the above terms were obtained from the recorded pen deflections by means of

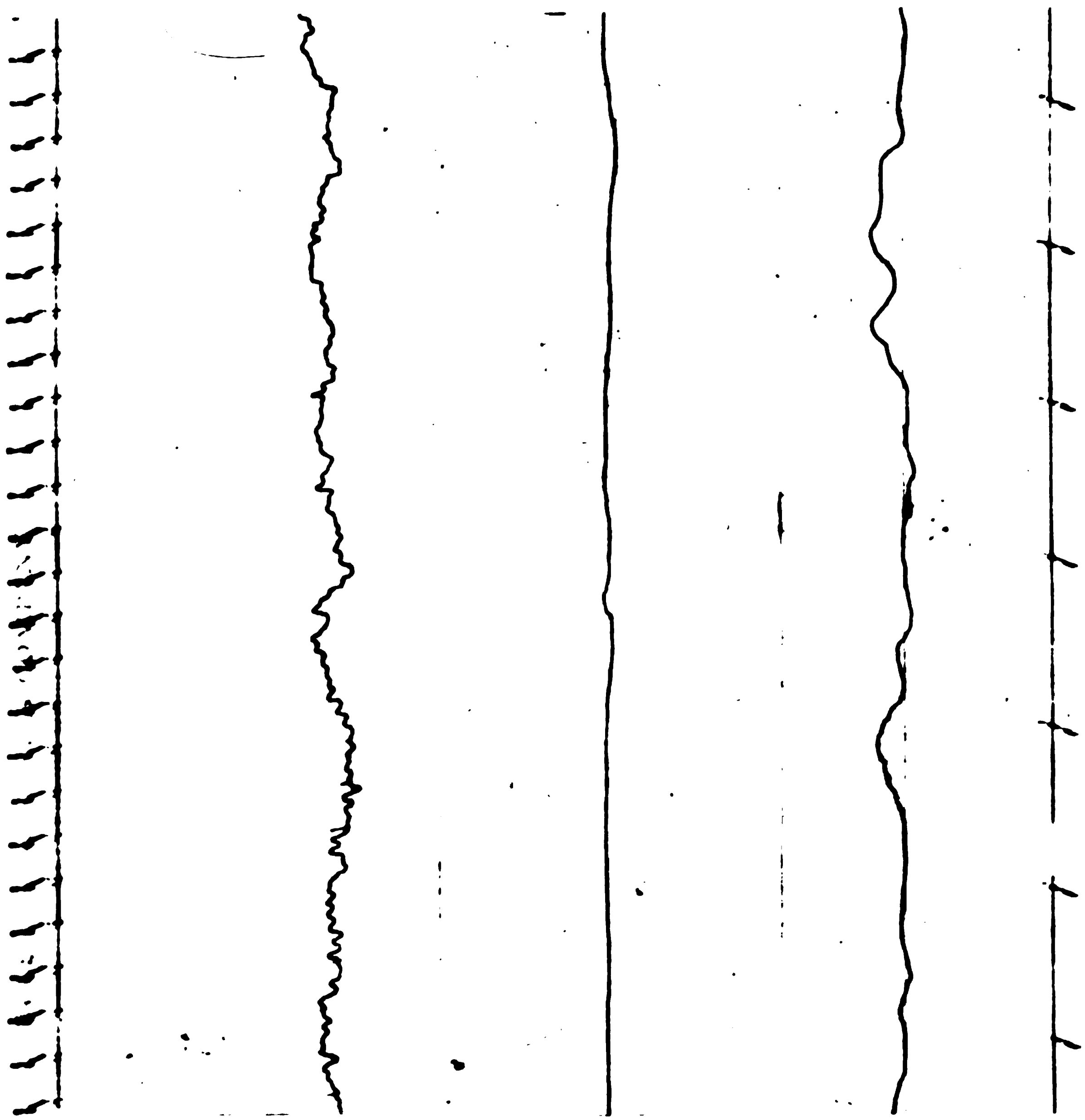

Figure 24. Oscillograph chart paper showing direct writing ink recording of (from left to right) theoretical travel, weight transfer, torque, rolling resistance and actual travel

the calibration curves obtained for each transducer. The dynamic calibration curve of Figure 10 was used to obtain the experimental torque values.

The classical vehicular equations as presented by Barger (1952) express the change in soil reactions R_1 (vertical component of soil reaction against traction wheels) and R_2 (vertical component of soil reaction against front wheels) as:

$$\text{Weight transfer} = \frac{PY_1}{X_1} \tag{5}$$

where:

P = Drawbar pull

Y_1 = Vertical distance between pull point and the point of reaction between soil and tire

X_1 = Horizontal distance between centers of front and rear axle (wheel base)

Table III compares weight transfer for four drawbar pull values. The experimental values obtained by the weight transfer transducer are greater than the values obtained from equations 2 and 5 in all cases.

Weight transfer values obtained from equation 5 were determined using the experimental values of drawbar pull. The terms Y_1 and X_1 were measured on the test vehicle. Results obtained from equation 2 involve the experimental values of torque, drawbar pull and rolling resistance. The measured values of Y_2, Y_{12}, and X_{15} were used.

TABLE III

WEIGHT TRANSFER VALUES FOR A GIVEN DRAWBAR PULL

Drawbar Pull	Weight Transfer: Experimental	Equation 2	Revised Equation 2	Equation 5
	Pounds			
666	319	255	275	164
1082	403	343	361	266
1688	590	536	549	415
2289	832	807	816	563

In the derivation of equation 2 it was assumed that the front wheel rolling resistance force, R_{12}, acted horizontally at the point of reaction between the soil and wheel (Figure 19b). If instead it is assumed that R_{12} acts at the center of the front wheel (Figure 25) and Y_{12} is the vertical distance between rear wheel and front wheel centers, the computed weight transfer is then in closer agreement with the experimental weight transfer values obtained from the rear axle transducer.

Figure 26 shows a comparison of weight transfer vs. drawbar pull curves for values obtained from the rear axle transducer, from equations 2, 5 and from revised

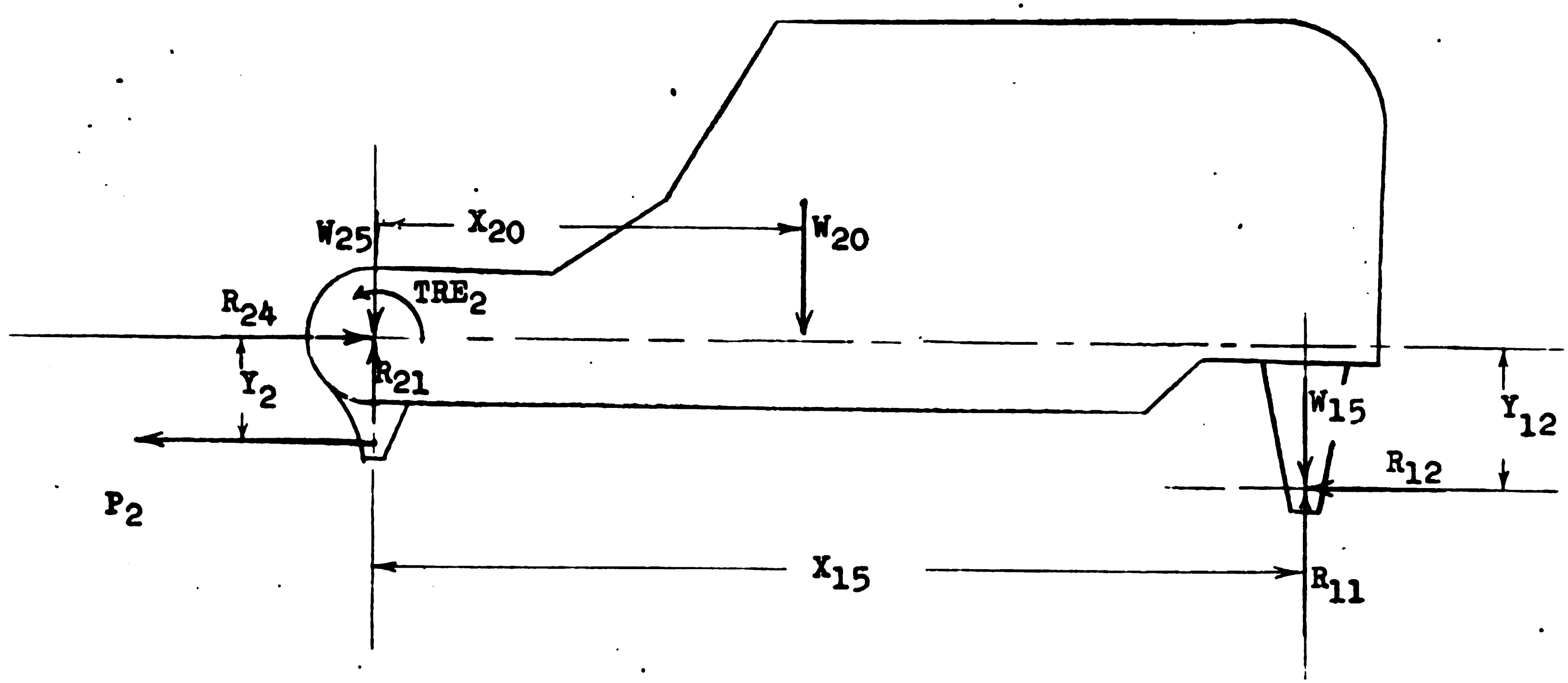

Figure 25. Free body diagram of frame

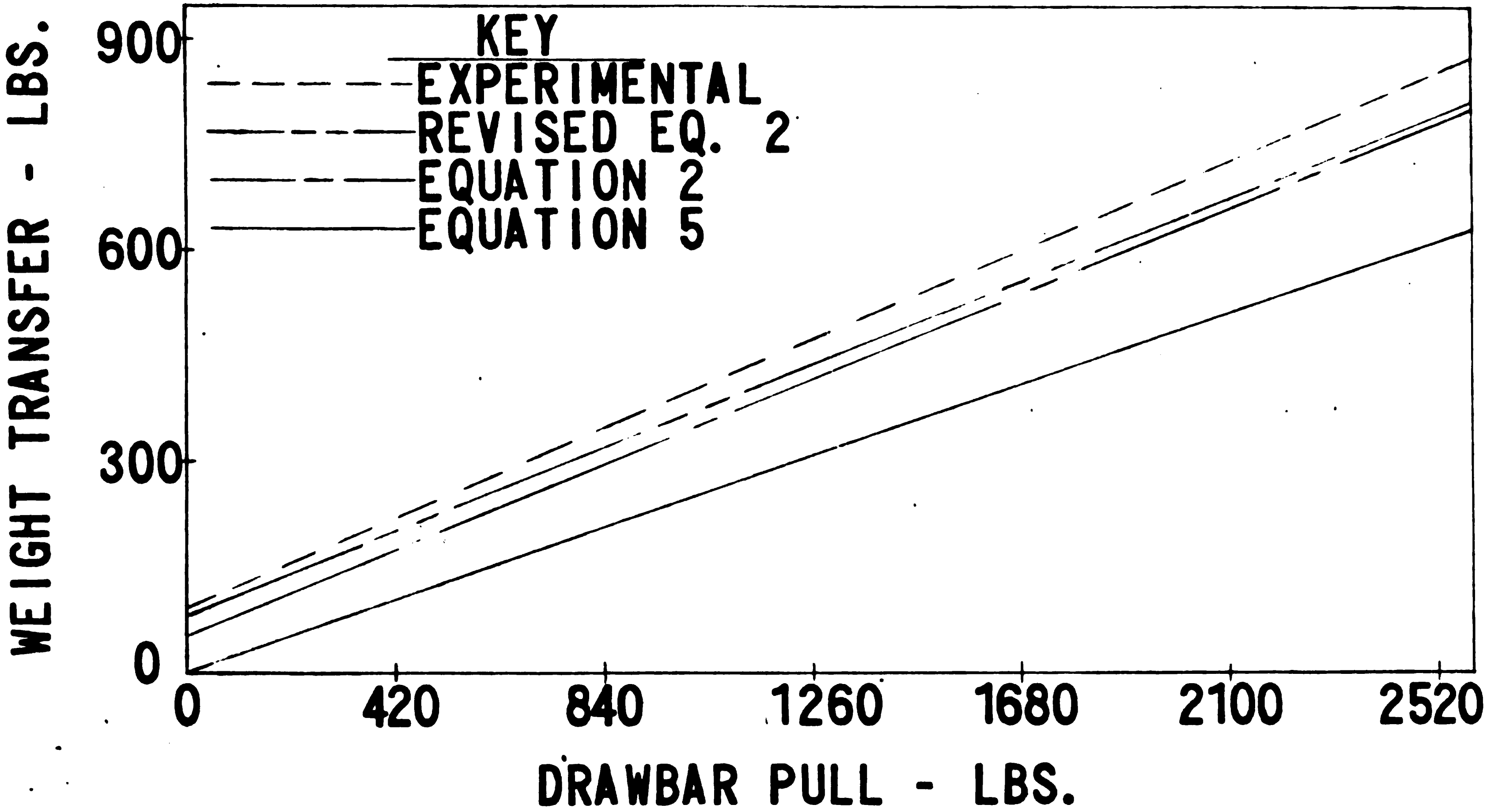

Figure 26. Weight transfer curves comparing the experimental value with the theoretical values obtained from equation 2, 5 and revised equation 2

equation 2 (with R_{12} acting at the front wheel center). The linear regression method presented by Dixon (1957) was used to determine the best straight line available from the experimental data. These data are on file in the Agricultural Engineering Department, Michigan State University, East Lansing, Michigan.

It is evident that the results obtained from equation 2 and from revised equation 2 are in much closer agreement with the weight transfer obtained experimentally than the values obtained from equation 5. An error analysis was conducted for the various weight transfer values given in Table IV. The error equation is as follows:

$$\% \text{ error} = 100 \left(1 - \frac{\text{calculated weight transfer}}{\text{experimental weight transfer}}\right) \qquad (6)$$

TABLE IV

WEIGHT TRANSFER ERROR OBTAINED FROM THEORETICAL EQUATIONS

Experimental	Percent Error		
Weight Transfer	Equation 2	Revised Equation 2	Equation 5
319	20.1	13.8	48.6
403	14.9	10.4	34.0
590	9.2	6.9	29.7
832	3.0	1.9	32.3

The error becomes less as weight transfer increases, but revised equation 2 provides the most accurate results.

The curve for weight transfer as obtained from the revised equation 2 approaches the experimental weight transfer curve much closer at low drawbar pull values than at higher values. This occurs because the term, $\frac{R_{12}Y_{12}}{X_{15}}$, is much smaller in the revised equation; therefore the resulting difference between results obtained from the revised equation and results obtained from the original equation is greater when the rolling resistance values are large. The assumption here is that rolling resistance values vary inversely with drawbar pull.

The above assumption appears valid since it would be expected that the front wheel rolling resistance should decrease as drawbar pull increases due to the transfer of weight from the front to the rear drive wheels. This was verified experimentally and Figure 27 shows the resulting curve of rolling resistance vs. drawbar pull.

Figure 28 shows the results obtained when slip and torque values are plotted versus drawbar pull. The variation does not appear to be linear. Both terms increase more rapidly at the higher values of drawbar pull.

The variation of slip with the coefficient of traction is shown in Figure 29. Coefficient of traction is defined as the ratio of drawbar pull to dynamic rear axle weight.

Integration of soil values with experimental results

The Bevameter recordings taken prior to each series of test runs were processed and their data used to determine

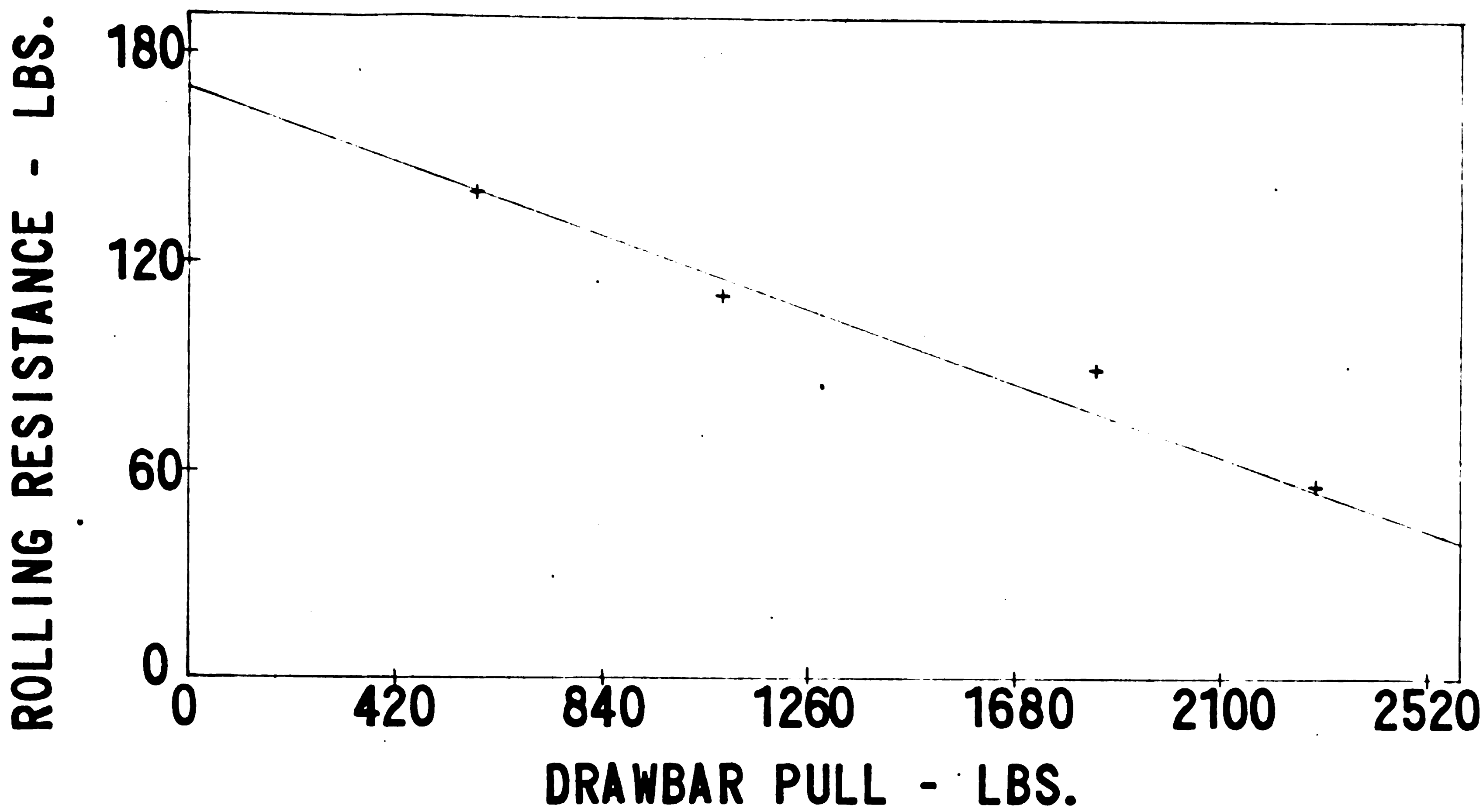

Figure 27. Variation of rolling resistance with drawbar pull

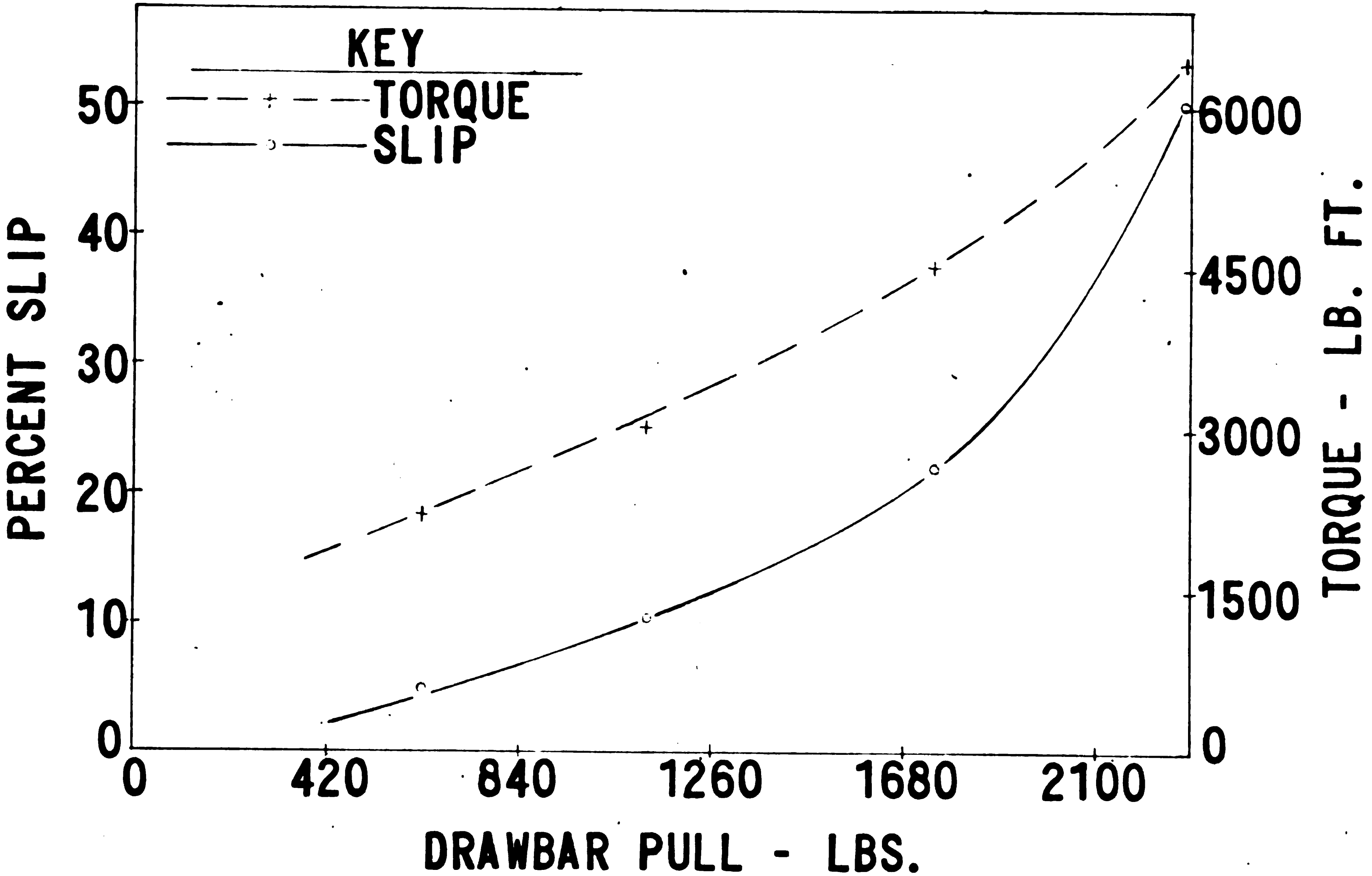

Figure 28. Variation of slip and torque with drawbar pull

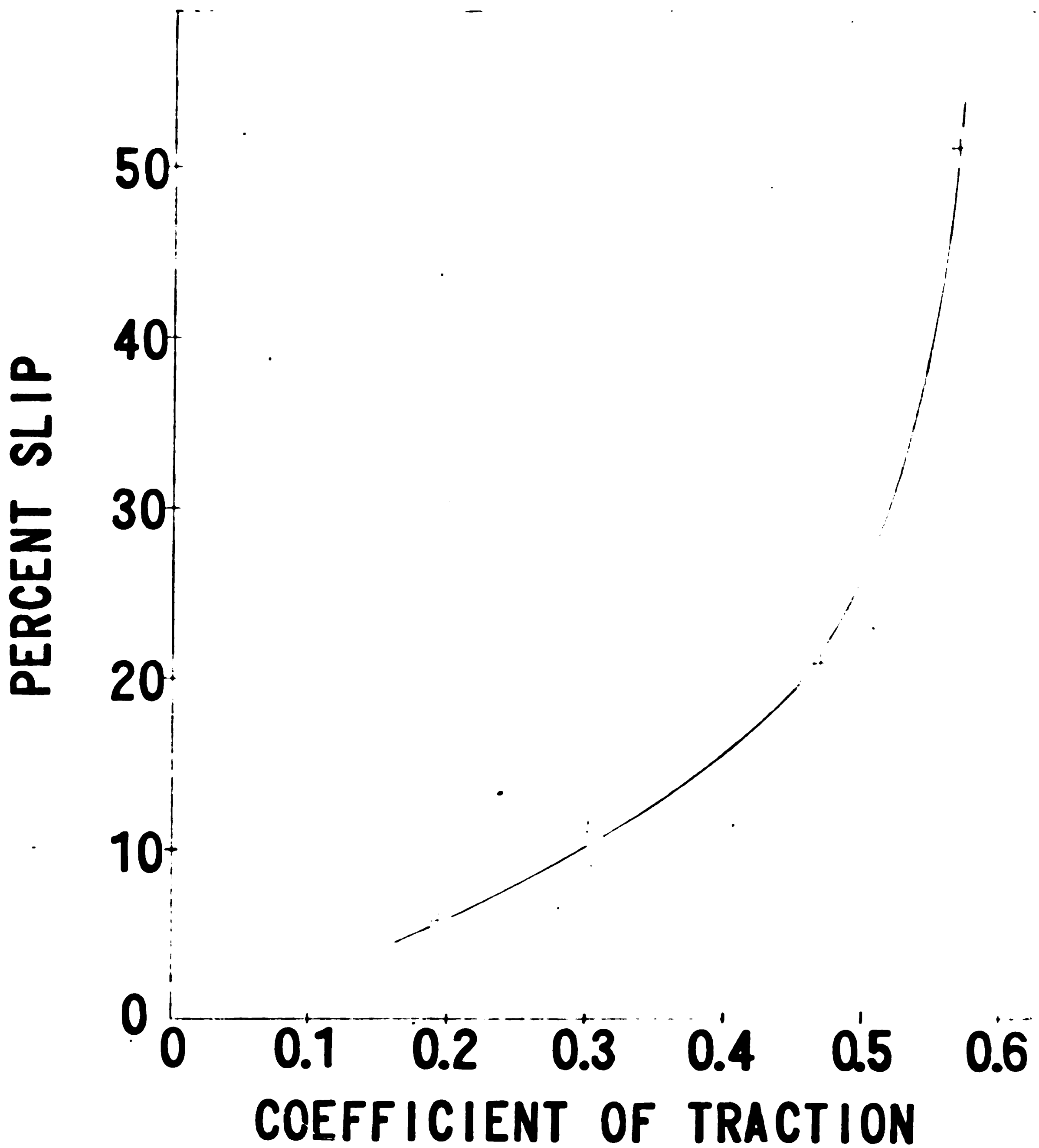

Figure 29. Variation of slip with coefficient of traction

the Bekker soil parameters. The tractor was weighed on portable highway scales to determine the front and rear static weights; 1360 lbs. and 3180 lbs. respectively. Values for front and rear wheel contact patch lengths, L_1 and L_2, and front and rear wheel contact patch widths, b_1 and b_2, were estimated for calculation purposes.

A wide distribution of soil values was obtained from the Bevameter data. The scatter of values (Table V) did not appear reasonable in view of the fact that the bulk density and soil moisture values varied only slightly between test runs.

TABLE V

EXPERIMENTAL SOIL VALUES

Test Date	K_c	K_ϕ	n	C	ϕ degrees	Bulk Density gm/cc	Percent Moisture
9-12	-33.5	75.4	1.3	2.2	26.8	1.30	11.8
9-14	14.7	17.2	0.7	0.9	50.4	1.36	16.2
9-15	-7.5	37.0	1.1	0.7	36.0	1.30	12.0
9-17	85.5	-66.0	1.3	0.9	25.0	1.28	11.7
9-20	10.8	42.6	0.7	1.0	26.9	1.35	12.3
9-21	-87.3	142.4	0.6	0.9	32.0	1.30	11.2
9-24	70.8	-33.8	0.8	1.3	33.6	1.30	11.5

The most reasonable soil values were substituted into equations 3 and 4 along with an experimental drawbar pull value and its associated weight transfer and torque values.

The values of b_1 and b_2 were set equal to the sum of the front and rear wheel contact patch widths respectively.

When L_1 and L_2 were given values equal to the assumed patch length of one wheel only, it was impossible to obtain an equality within the range of weight transfer values. When the value of L_1 was increased to the sum of both front wheel patch lengths and L_2 increased to the sum of both rear wheel patch lengths and with all other procedures and values identical, it was necessary to reduce the weight transfer value in equation 4 but increase this value in equation 3 in order to achieve an equality by successive approximations. In these cases the resulting values of weight transfer were unrealistic.

APPLICATION OF RESULTS

A further analysis of the soil value system is required before the soil value quantities may be relied upon to produce accurate and repeatable results. When this is achieved, equations similar to 3 and 4 will make it possible to utilize soil value information, along with certain vehicle design parameters, for the prediction of vehicle stability and performance.

The results obtained from equation 2, in both its original and revised form, indicate that the theoretical approach utilized in its derivation can be used in further developing the dynamic vehicular mechanics equations. A further improvement in the soil value system and a complete verification of the dynamic vehicular equations could allow future designers to predict vehicle performance over a given soil if the specific soil values were known.

The vehicular equations can be rearranged into whichever mathematical form presents itself most readily to the solution of the equation for a particular term. Stability and mobility problems may then be initially analyzed without the luxury of experimental models.

An example of the application of revised equation 2 to a design problem follows:

Given:	Static front end weight	= 1350 lbs.
	Maximum allowable weight transfer (W_D)	= 1000 lbs.
	Maximum rear axle torque (TRE_2)	= 7500 lb-ft.
	Minimum front wheel rolling resistance (R_{12} is assumed from past experience under similar conditions)	= 50 lbs.
	Wheel base (X_{15})	= 6.5 ft.
	Rear wheel radius (r_2)	= 2 ft.
	Front wheel radius (r_1)	= 1 ft.
	$r_2 - r_1 = Y_{12}$	= 1 ft.
	Vertical distance between pull points and rear wheel center (Y_2)	= 0.3 ft.

Required: Maximum drawbar pull

Solution: Apply revised equation 2

$$W_D = \frac{TRE_2 - P_2Y_2 - R_{12}Y_{12}}{X_{15}}$$

Solve for P_2

$$P_2 = \frac{TRE_2 - W_DX_{15} - R_{12}Y_{12}}{Y_2} \qquad (7)$$

Substitute given terms into equation 7

$$P_2 = \frac{7500 - (1000)(6.5) - (50)(1)}{0.3}$$

$$P_2 = 3166.7$$

SUMMARY

Theoretical dynamic vehicular mechanics equations proposed by Buchele were studied. After determining which parameters were required for an experimental analysis of the equations, a test vehicle was instrumented for the purpose of obtaining dynamic values of drawbar pull, rear axle torque, weight transfer and rolling resistance.

The above values were determined through the use of strain gage transducers. Two of the transducers were integral parts of the test vehicle while two were separate detachable instruments. Each transducer was calibrated in terms of the required parameter.

Test runs were made on a Miami fine sandy loam soil. Drawbar pull was the controlled variable. Several runs were made for each drawbar pull and the average values of the recorded data were substituted into the theoretical equation.

A Bevameter was used to record the forces required to (1) shear the soil and (2) penetrate the soil surface. These data were processed to obtain soil values.

The experimental values of torque, drawbar pull and rolling resistance were substituted into the theoretical equation. The resulting calculated weight transfer values were compared with experimental weight transfer values

obtained from the rear axle transducer.

Curves plotted from the experimental results show a close agreement between the actual and calculated weight transfer. The weight transfer obtained from the classical equation is definitely in disagreement with the actual values.

It was shown that weight transfer varied directly as drawbar pull. Torque and percent slip increased gradually over the lower range of drawbar pull. A more rapid increase was observed for higher values of drawbar pull. Front wheel rolling resistance decreased linearly as drawbar pull increased.

The experimental soil values, when substituted into the theoretical equations, yielded unacceptable results. The weight transfer values required to achieve an equality within the equation were obviously not comparable with the actual values obtained experimentally.

CONCLUSIONS

1. The dynamic values of rear axle torque can be obtained without applying strain gages to the rotating shaft.
2. The relationship between rear axle torque and drawbar pull varied in a non-linear manner.
3. The rolling resistance varied inversely with drawbar pull.
4. The percent slip increased for higher values of drawbar pull.
5. Weight transfer values varied linearly with drawbar pull.
6. The results of the classical weight transfer equation did not agree with the experimental weight transfer values.
7. Weight transfer values obtained from the theoretical dynamic vehicular mechanics equation were comparable with experimental weight transfer.
8. The wide variation of the soil values did not reflect the apparent similarity of soil conditions existing for each series of tests.
9. The substitution of reasonable soil values and vehicle dimensions into the theoretical equations yielded impractical values of weight transfer.

10. A further analysis of the interrelation between the forces applied by soil reactions and the soil values is necessary if soil characteristics are to be utilized for the purpose of vehicle design.

RECOMMENDATIONS FOR FUTURE INVESTIGATIONS

1. Reduce the compression ring wall thickness slightly to obtain greater sensitivity for low torque values.
2. Verify the weight transfer obtained from the rear axle transducer by redesigning the tractor front axle assembly into a vertical force transducer.
3. Integrate a rolling resistance transducer into the above redesigned front axle to verify the rolling resistance.
4. Obtain experimental results from several test runs with the drawbar locked at various heights.
5. Utilize a greater number of drawbar pull values to assure a more accurate statistical analysis of the experimental terms.
6. Derive more applicable soil value relationships for substitution into the theoretical dynamic vehicular equations.

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