ECONOMIC ANALYSIS OF FERTILIZER INPUT-OUTPUT DATA FROM THE CAUCA VALLEY, COLOMBIA

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By

HERNAN BERTOLOTTO

AN ABSTRACT

Submitted to the College of Agriculture of
Michigan State University of Agriculture
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APPROVED Glenn L. Johnson

ABSTRACT

Current fertilizer recommendations generally reflect inadequate attention to economic considerations.

Farmers are being supplied with fertilizer information from which, implicitly or explicitly, the conclusion is being drawn that the most adequate level of fertilization is the one at which maximum yields per acre are attained. This is seldom consistent with the more important concern of maximizing profits. Profits are increased only so long as the cost of adding fertilizer inputs is less than the added return derived from their use.

The experimental work to determine fertilizer input-crop output relationships and to provide information for making more dependable recommendations to farmers was conducted cooperatively by the Colombia Project of Michigan State University, the Facultad de Agronomia of the Universidad Nacional de Colombia at Palmira (Colombia) and a Colombian farmer, Senor Edgardo Patino, during 1957.

The crops studied are corn and beans. The variable nutrients studied are nitrogen, phosphorous and potassium.

The analysis of the data produced by these experiments permits more adequate analysis of fertilization rates and of the recommendations which are given to farmers.

The analysis of these data are based on the concept of a continuous mathematical production function. Yield responses to different fertilizer nutrients are dependent upon the levels of the variable nutrient inputs. The economic optima are determined where the marginal value productivity of a nutrient input is equal to the cost of adding another unit of such input of fertilizer.

Two two-variable functions were fitted to the experimental data for corn Patino Lower and corn Patino Upper Field. After applying various statistical tests it was decided that a cross product production function of the form:

$$Yo = a + bN + cP + dN^2 + eP^2 + fNP$$

where Yo is yield and N and P are per acre applications of nitrogen and phosphoric acid was considered a better representation of the functional relationships involved than a square root equation which was the alternative production function fitted to the data.

For corn Patino Lower Field data, nitrogen was found to exert the predominant influence on yield, even though response was also obtained from the applications of phosphoric acid.

For corn Patino Upper Field data, only phosphoric acid applications were found to influence yield.

Three three-variable functions were fitted to the experimental data for bean Patino 1957/58. After applying statistical tests a Cobb-Douglas function of the form:

$$Yc = aX_1^{b1} \; X_2^{b2} \; X_3^{b3}$$

was considered a more appropriate fit for this set of experimental data, than the square root and cross product production functions the alternative equations fitted to the data.

The economic optima conditions are based both in the physical functional relationships and in the price conditions for fertilizer inputs and product output existing at a given moment.

If the price relationships involved change, a new optimum amount of fertilizer inputs and nutrient combinations to apply become profitable as determined by the new nutrient-crop price ratio existing after the change.

This study shows that further experimental work in corn, beans and other crops is needed in the Cauca Valley. The experimental design used in this study has proved useful to obtain the kind of experimental data needed to make sound recommendations to farmers.

In view of present agricultural development projects under way in Colombia, this kind of research work may be the best way to promote an efficient reallocation of resources and can make an important contribution to increased productive capacity of the Colombian agriculture.

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CHAPTER 1

THE INTERRELATIONSHIPS OF AGRONOMIC AND ECONOMIC CONCEPTS IN FERTILIZER RESEARCH

<u>The Type of Information Needed by Farmers</u>.-

One aspect of fertilizer research deals with the presence or absence of response in crop yields to fertilizer applications.

However, once responses have been found to exist, the farmer needs to consider fertilizer along with other resources and practices in his farm management decisions.

These decisions can be made most efficiently if fertilizer information is provided in the form of incremental response data.

Incremental response data show the successive additions to yield resulting from successive fertilizer applications. Accordingly, once research has shown that crop yields do respond to fertilizer, the next steps in research are to estimate:

(a) the incremental yields forthcoming from different rates of fertilizer applications under specified crop and soil conditions, and

(b) the economic optimum quantity of fertilizer, considering crop and fertilizer prices and production costs.

Farmers can be divided into two groups: those who

have ample capital and those who have limited capital. They are seldom interested in maximizing yields per acre, and not even the farmer with unlimited capital is interested in maximum per acre yields; he is interested in higher acre yields only to the extent that greater production adds more to returns than to costs.

The extent to which higher yields increase profits depend on:

(a) the rate at which inputs are transformed into crops, and

(b) the price ratio.

Maximum profits come when the crop/fertilizer transformation ratio is equal to the fertilizer/crop price ratio; the transformation ratio declines with heavier fertilization rates under diminishing returns. The slope of the response function represents the incremental or marginal yield due to small increases in fertilizer use. The farmer with limited capital needs this information in determining how much fertilizer to apply.

For instance[1], suppose that a farmer with limited capital can earn $2.50 return on funds spent elsewhere in his business (such as tractor fuel, crop seed, or hog supplement).

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[1] Heady, E. O., "Methodological Problems in Fertilizer Use", (in) Baum, E. L., Heady, E. O., Blackmore, J., "Methodological Procedures in the Economic Analysis of Fertilizer Use Data", Ames, Iowa State College Press, 1956, Chapter 1, pp. 3.

He is given information showing that one discrete level of fertilization, 30 pounds of nitrogen, will increase oat yield by 17 bushels. With oats at 70 cents per bushel and nitrogen application costing 18 cents per pound, the total return is $11.90 and the total cost is $5.40, a net of $6.50.

However, the return per dollar spent on fertilizer (11.90:$5.40) is only $2.20 and the farmer will allocate his scarce funds where he can get $2.50.

Suppose, however, that the farmer is given even three points from a response function showing: the first 10 pounds of nitrogen has a marginal yield of 10 bushels; the second 10 pounds has a marginal yield of 5 bushels, and the third 10 pounds has a marginal yield of 2 bushels.

With a unit costing $1.80, the first 10 pounds returns $3.89 per dollar invested in fertilizer, and the second returns $1.95.

Hence, since the farmer can realize only $2.50 elsewhere in his business, he now is encouraged to invest in at least 10 pounds of nitrogen.

Obviously, then, knowledge of the response function, coupled with information on the economics of fertilizer use, can encourage a greater investment in this resource on the great majority of farms with limited capital.

The method of research and form of presentation, when the findings and recommendations are in terms of "one discrete level" can lead the farmer to use no fertilizer when fertilization actually represents a profitable investment within his situation of limited capital.

Knowledge of the response function is equally important for the farmer with unlimited capital.

It is known that the optimum or most profitable level of fertilization for these farmers is defined in equation (1)

$$\frac{dY}{dF} = \frac{Pf}{Py}$$

where the term to the left of the equality is the marginal yield or response and the term to the right is the price ratio (price per unit of fertilizer divided by the price per unit of yield).

The marginal yield is the derivative of yield with respect to nutrient; it is the slope of the response function for any particular input level. This is the type of information basic for making recommendations to farmers who seek to maximize profits when unlimited capital is available to them.

Economic-Agronomic Integration in Fertilizer Research.- As Johnson[1] points out, "fertilization research should be

[1]Johnson, G.L., "Interdisciplinary Considerations in Designing Experiments to Study the Profitability of Fertilizer Use", in Baum, E.L., Heady, E.O., Blackmore, J., Op. cit., p. 22.

looked at from the point of view of an agriculturist rather than from the confined viewpoints of the farm management specialist, the soil specialist, the marketing specialist, the mathematical statistician, or the specialist in leguminous nitrogen fixation".

Agronomists and economists recognize that fertilizer recommendations should be based on data and principles drawn from both sciences.

First, it is necessary that agronomic findings be available for application of the relevant economic principles, stating where and how much fertilizer should be used. The economic principle is, of course, quite sterile without the response data to go with it.

However, agronomic data alone do not provide the basis for efficient fertilizer use.

Some of the reasons for the lack of integration of economic and agronomic principles in fertilizer research in the past and at the present time in many cases and countries, as Heady[1] points out are the following:

(a) lack of training of agricultural economists in mathematical economics and statistical techniques to develop the kind of estimates for economic analysis.

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[1]Baum, E.L., Heady, E.O., "Over-all Economic Considerations in Fertilizer Use", (in) Baum, E.L., Heady, E.O., Pesek, J.T., Hildreth, C.G., "Fertilizer Innovations and Resource Use", Ames, Iowa State College Press, 1957, pp.128.

(b) overspecialization in land-grant colleges and other research institutions has not always encouraged sufficient cooperative work.

(c) improved statistical techniques for handling multi-variable fertilizer experiments have been emphasized only recently.

(d) in many areas of the United States, fertilizer became an important factor of production only recently.

(e) the reluctance of agronomists to consider economic optima studies as a part of their research program due, in part, to a lack of understanding of the mathematical procedures used by the economists.

(f) agronomists[1] had been interested largely in variance-type studies to establish response and relate it to soil characteristics rather than to determine the marginal quantities and the optimum use of fertilizer.

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[1]Heady, E.O., Pesek, J.T., "A Fertilizer Production Surface with Specifications of Economic Optima for Corn Grown on Calcareous Ida Silt Loam", Journal of Farm Economics, Volume 36, August 1954, pp. 466.

CHAPTER 2

THE PRODUCTION FUNCTION AND THE CONCEPT OF MAXIMIZATION

The Production Function in Fertilizer Research.-

As has been pointed out by Johnson[1], the typical experiment design investigates a production function of the form of equation (2)

(2) $$Y = f(N,P,K \,/\, X_4 \ldots\ldots X_n) + u$$

where Y is yield in bushels per acre; N, P and K are the usually investigated independent variables or levels of fertilization; $X_4 \ldots\ldots X_n$ include variables "fixed" at a certain level (cultivations, insect control, rotations, Ph levels, varieties, drainage, soil uniformity, etc.).

U is the variation in yield not explained by the experimental variations in the independent variables.

The Concept of Maximization.-

Production functions express the functional relationship between resource inputs and product output.

The conventional procedure in a production function

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[1] Johnson, G.L., "Planning Agronomic-Economic Research in View of Results to Date", (in) Baum, E.L., Heady, E.O., Pesek, J.T., Hildreth, C.G., "Fertilizer Innovations and Resource Use, The Iowa State College Press, Ames, Iowa, 1957, Chapter 19, pp. 219.

study is to predict the total output curve or surface as an estimating (or regression) equation.

Maximization concepts help locate such economic optimum as the quantity of Y to produce maximum profit and the least cost combination of fertilizer F_1 and F_2 to use in producing that amount of product output Y, and also how these optima shift with price changes.[1]

In a function such as equation (3)

$$g\ (Y_c\ \ P_c, F_1\ \ P_{f1}, F_2\ \ P_{f2}/X_d \ldots\ldots X_m) = \pi \tag{3}$$

in which

Y_c = yield of crop

P_c = price of crop

F_1 = fertilizer input

F_2 = fertilizer input

P_{f1} = price of fertilizer input F_1

P_{f2} = price of fertilizer input F_2

$X_d \ldots\ldots X_m$ = inputs fixed at specified conditions.

π = profit

when F_2 is constant

$$\frac{d\pi}{dF_1} = 0 \tag{4}$$

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[1] Johnson, G. L., "Interdisciplinary Considerations in Designing Experiments to Study the Profitability of Fertilizer Use", (in) Baum, E. L., Heady, E. O., Blackmore, J., op. cit., Chapter 2, p. 27.

defines the most profitable amount of F_1 to use with the constant amount of F_2. Under ordinary competitive conditions, the condition for maximizing profits is defined as in equation (5)

(5) $$\frac{d\pi}{dF_1} = \frac{dYo}{dF_1} \cdot P_{yo} - P_{f1} = 0$$

If the most profitable combination of F_1 and F_2 in producing a given amount of Yo is desired, equation (6) defines the least cost combination of F_1 and F_2 to use in producing the amount of Yo under consideration:

(6) $$\frac{\frac{dYo}{dF_1}}{\frac{dYo}{dF_2}} = \frac{P_{f1}}{P_{f2}}$$

As the condition defined in equation (5) is, for F_1 and F_2 respectively, the slope of the production function defined in equation (3), these conditions permit determination of the most profitable (least-cost) combination of F_1 and F_2 to use in obtaining a given yield (where Yo is held constant).

When it is desired to determine the most profitable amounts of F_1 and F_2 to use and of Yo to produce, the derivatives for profit with respect to F_1 and F_2 are set equal to zero, and solved simultaneously for F_1 and F_2.

Having secured F_1 and F_2 in this way, the values are substituted in equation (3) and solved for Yo.

CHAPTER 3

STATISTICAL MEASUREMENTS

Introduction.-

When the values assumed by one variable (Y) depend on (i.e., are a function of) the values taken by one or more other variables ($X_1, X_2 \ldots\ldots X_n$) a functional relationship is defined.

Y is called the "dependent variable" and the variables on which Y depends are referred to as "independent variables".

Correlation analysis gives a measure of how the dependent variable changes with a given change in the variable or variables on which it depends.

This measure is an "estimating equation" which makes possible estimates of the dependent variable from the independent variable or variables.

Correlation analysis also provides a measure of the accuracy of such estimates - "the standard error of estimate"; and finally, "the coefficient of correlation" tells the degree of correlation.

When the analysis is limited to two variables the method is called "simple correlation" but it is often necessary to include the influence of several independent variables to explain the variation in the dependent variable and this is known as "multiple correlation".

Graphic Analysis of Functional Relationships.-

A method of plotting the data and a method peculiarly suited to the analysis of functional relationships is that of the scatter diagram.

Here one variable is scaled on the X axis, the second on the Y axis, and the paired values of the two variables are plotted on these scales.

Because of the characteristic of the data, the tendency of the points is to scatter diagonally across the diagram from the lower left hand to the upper right.

When two variables are plotted in the scatter diagram, one ordinarily should be characterized as independent (in the present study it is fertilizer input) and the other as dependent variable (being output of crop yield).

The independent variable is the one upon which the variation in the second seems to depend. The independent variable is ordinarily plotted on the X axis and the dependent variable on the Y axis.

The Estimating Equation.-

Mathematically computed lines may be passed through the data, which are called "lines of average relationship", or "regression lines", because they reveal the typical change in the dependent variable Y which has accompanied a given change in the independent variable or variables.

This average relationship may be determined mathematically by the method of least squares.

In computing the equation for an estimating equation, an "a" value must be secured which will be the value of the regression line at its origin, and a "b" value, which will describe the average change in Y with a given change in X.

When these two values are obtained, a complete description of a regression line is secured, the mathematical characteristic of which may be described in equation (7) for functional relationships with one independent variable.

(7)

$$Yc = a + bX$$

To obtain the "a" value, the following equation is used:

(8)

$$a = \frac{\Sigma X^2 \, \Sigma Y - \Sigma X \, \Sigma XY}{N \, \Sigma X^2 - (\Sigma X)^2}$$

The formula for "b", showing the average change in Y for a given change in X is given in (9)

(9)

$$b = \frac{N \, \Sigma XY - \Sigma X \, \Sigma Y}{N \, \Sigma X^2 - (\Sigma X)^2}$$

If two or more independent variables are used to explain the changes in a dependent variable, it becomes possible to measure the influence of each of these (X_2 and X_3, for example) when the influence of the other (X_2 and X_3) is considered.

The Standard Error of Estimate.-

Error must be expected in all estimates made from regression equations. If there are variables which have been ignored in computing the regression equation and they are important, the estimates made may be very poor and the actual observations may scatter widely about the regression line.

Since the departure of the observations from the line of regression is due to such "other" factors as have been suggested, these deviations are known as "residuals". They are residuals in the sense, then, that after X has been used to explain the variation of Y there may remain a residual variation which is due to a large number of forces which has been ignored in the correlation.

If, on the other hand, the residuals in simple correlation are not due to a mass of other influences, but can be explained by the introduction of one or more additional independent variables, the analysis should be converted to a problem in multiple correlation.

The new function with several independent variables may provide a better explanation of the variation in Y, and one evidence of this will be a reduction in the residuals.

The next step in the analysis will be to measure the residual variation. If it proves to be small, the forces ignored in the function as stated have little influence on Y.

In order to gain some idea of the adequacy of the regression equation as an explanation of the variations in Y, it is necessary to have a mathematical device which will measure the scatter of the points around the regression line. If the regression line is a good fit and the actual data plot close to it, there is indication that the values of Y are related to those of X in the manner described by the regression equation.

In this case, the derived mathematical measure of the residual variation should give a low value.

Should the points scatter widely from the regression line, the use of that estimating equation as an explanation of the variations in Y must be questioned and any estimate of Y based on its functional relationship to X must be expected to be inaccurate.

In this case, the mathematical measure of the residuals would have a relatively large value.

The measure of the scatter of the points is known as the "standard"error of estimate". If Yc is used to symbolize the computed values of Y, and Y to symbolize the actual values of Y, this process of calculating the standard error of estimate is as follows in equation (10), where Sy symbolizes the standard error of estimate:

(10)
$$Sy = \sqrt{\frac{\Sigma(Y-Yc)^2}{N}}$$

The interpretation of the standard error of estimate is similar to that of the standard deviation. It may be said that approximately 68% of the points in the scatter diagram will be within the range of the regression line plus and minus one standard error.

Measurement of Correlation.-

The estimating equation reveals the change in the dependent variable which typically accompanies a given change in X.

The scatter of points around a regression line gives a first visual impression of the extent to which the independent variable, or variables, actually succeed in explaining the variation in the dependent variable, and whether useful estimates of it can be made from those relations. The standard error of estimate gives a measure of this scatter.

Now, it is desired to obtain one summary figure which will indicate the "extent" to which two or more variables are correlated.

This should be a pure number, so that the units in which the values are quoted will not affect it.

It should have known limits so that it may be readily interpreted. The "coefficient of correlation" (symbolized by "r" for simple correlation and by "R" for multiple correlation) is such a measure.

Variation Explained - The Coefficient of Determination.- The procedure in correlation analysis is to compute the per cent of the variation of Y which is explained by the independent variables.

Traditionally, this result is obtained by first computing the per cent variation of Y which is not explained by the independent variable or variables.

(a) The Variation of Y-The Value of σy^2: Since the variation of Y can be defined as the standard deviation squared, the calculation is made substituting the values for Y, Y^2 and N in the formula (11):

(11)
$$\sigma = \sqrt{\frac{X^2}{N} - \frac{(\ X)^2}{N}}$$

(b) The Variation of Y not explained by X-The Value of Sy^2:

Just as the variation of Y is the standard deviation squared, the variation of Y not explained by X is the standard error of estimate squared (Sy^2). If the variation of Y were completely explained by X, all observations of Y would fall on the regression line and the value of Sy and Sy^2 would be zero.

The larger the value of Sy and Sy^2 the less perfect the explanation of the variation of Y by the independent variable or variables, and the more important in determining Y are other factors, which were not included in the function.

Therefore, the larger Sy^2 the greater the per cent of the variation of Y not explained by X.

(12)
$$Sy^2 = \frac{\Sigma Y^2 - [a\,\Sigma(Y) + b\,\Sigma(XY)]}{N}$$

(c) <u>The Per Cent of the Variation of Y Explained by X.</u>-

The variation not explained by X is divided by the variation of Y, that is, Sy^2/y^2 and by subtraction then from one results the per cent of the variation of Y explained by the independent variable or variables, which is one of the most useful results in correlation analysis, and is known as the "coefficient of determination", as shown in (13)

(13)
$$\text{coefficient of determination} = 1 - \frac{Sy^2}{\sigma y^2}$$

The per cent, $Sy^2/\sigma y^2$ is known as the "coefficient of non determination" because it represents the per cent of the variation of Y not explained by the independent variable.

<u>The Coefficient of Correlation.</u>-

When the standard error of

estimate is zero there is perfect correlation. If the standard error of estimate is zero, actual and estimated values of Y are identical.

That is, the regression equation provides a perfect fit to the actual values of Y, and the variation of Y is completely explained by the independent variable and completely dependent upon it.

In that case the value of $Sy = 0$ and the ratio $Sy^2/\sigma y^2$ will be zero indicating zero per cent of the variation explained by the independent variable or variables.

If the standard error is equal to, or nearly equal to the standard deviation, there is no correlation. Under this condition the value of the ratio $Sy^2/\sigma y^2$ would be one, or approximately one, the per cent of the variation unexplained would be near 100 per cent and the variation explained would be near zero. In this case, the coefficient of determination will approximate zero.

The "coefficient of correlation" is based on the coefficient of determination, that is, the per cent of the variation explained by the independent variable. Thus, when the standard error of estimate is zero and the explained variation is therefore 100 per cent, the coefficient of correlation has a value of one.

If the variation unexplained (Sy^2) should be as large should be as large as the total variation σy^2 then the unexplained variation is 100 per cent, the explained variation is zero, and the coefficient of correlation is also zero.

The coefficient of correlation is a pure number not influenced by the units in which the data are quoted and it is computed as in equation (14)

(14)

$$r = \sqrt{1 - \frac{Sy^2}{\sigma y^2}}$$

When more than one independent variable is used to explain the variation of the dependent variable, "R" is used to symbolize the coefficient of multiple correlation, being conceptually the same as the coefficient of correlation.

Tests of Correlation Results.-

Simply because a high value of R is obtained in a correlation analysis, it cannot be assumed that a valid, reliable, and suitable function has been established.

There are some further tests that should be made to establish the reliability of the results from correlation analysis.

(a) Validity Test.-

The validity test consists of a critical appraisal of results to assure the researcher that the relations assumed in the function are correct; that the relationships do not violate canons of reasonableness; that the observed results are consistent among themselves and that the terms included actually reflect the variables they are intended to represent.

The validity test rests partly on theory, partly on experimentation with other alternative functions, and partly on the comparison of the attained results with those achieved in other similar studies.

This test is important because when the relations revealed are not valid, in the sense the term is used here, they likely will not be stable over a period of time either, so very poor estimates may result. Furthermore, wrong answers may be suggested for analytical and operational problems and easy satisfaction with high R's values may discourage further research which would produce much better results.

(b) Reliability Test.- Another reason to take a second critical look at correlation results is that they may lack reliability.

The most obvious case is when random samples are used.

If many such samples were taken from the same population, one can be sure that the regression coefficient, for example, would differ from sample to sample; that a sampling distribution could be constructed and the standard error of the regression coefficients estimated.

Of course, Sy (the standard error of estimate) and σy must also be expected to differ somewhat, from sample to sample, so that "R" which depends on them, will also have a sampling distribution and its own standard error.

Tests of significance of the regression coefficient can be made under a null hypothesis. The hypothesis is that the population value of the regression coefficients are zero and that the estimated value is due to sampling error or other chance elements in the experiment.

As Tintner[1] points out, the logic of the tests of significance consists in determining what is the probability that certain deviations from a postulated hypothesis (called the null hypothesis) could have arisen by chance. If this probability is small, then, the chances are that the null hypothesis does not hold.

What is needed to test this hypothesis is a critical ratio subject to probability distribution. For this test,

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[1]Tintner, G., "Significance Tests in Production Function Research", (in) Heady, E.O., Johnson, G.L., Hardin, L.S., Op. cit., Chapter 14, pp. 128.

the standard error of the sampling distribution of the regression coefficients is as follows:

(15)

$$\text{regression coefficient} = \frac{Sy^2}{\sigma_x^2 \ (n-2)}$$

The critical ratio is the difference between the hypothetical regression coefficient zero and the observed value of regression coefficient over the standard error of the regression coefficient as shown below in (16)

(16)

$$t = \frac{\text{regression coefficient}}{\text{standard error of regression coefficient}}$$

CHAPTER 4

EXPERIMENTAL WORK AND SOURCE OF DATA

The experimental work to determine fertilizer input-crop output relationships was conducted cooperatively by the Colombia Project of Michigan State University, the Facultad de Agronomia of the Universidad Nacional de Colombia at Palmira (Colombia) and a Colombian farmer, Senor Edgardo Patino.

Doctor Leonard Kyle and Mr. Gerald Trant from the Agricultural Economics Department; Dr. Kirk Lawton from the Soil Department, at Michigan State University, and members of the staff from the Michigan Agricultural Experiment Station, participated in the design of the experiment and collection of the experimental data.

<u>The Corn Data</u>.- The corn experiments were developed at Senor Patino's farm near Florida (Colombia) on a well drained clay loam soil.

The experiments included all three of the primary plant nutrients, nitrogen, phosphoric acid and potash, the first two in varying combinations and the last generally constant.

The corn experiments were conducted in two fields which will be called Patino Lower Field and Patino Upper Field respectively.

Both experiments had the same design and in both the same treatment levels of fertilizer were used.

Eight treatment levels were included for nitrogen, seven treatment levels for phosphoric acid, with potash generally constant.

These treatment levels measured in pounds per acre are:

N	-	0	20	40	60	80	100	120	140
P	-	0	20	40	60	80	100	120	

Except for the zero treatment levels, potash was held constant at 60 pounds per acre.

The levels of fertilization for corn are shown in detail in Table 1, and the design of the experiment in Table 2.

The Bean Data.-

The objective of this experiment conducted also in Patino's farm from October 1957 to January 1958, was to evaluate the response of beans to different combinations of fertilizer inputs.

Five treatment levels were included for nitrogen, five treatment levels for phosphoric acid, with potash generally constant.

These treatment levels measured in pounds per acre are:

N	-	0	20	40	60	80
P	-	0	25	50	75	100

Except for the zero treatment levels, potash was held constant at 40 pounds per acre.

TABLE 1

LEVELS OF FERTILIZATION FOR THE CORN EXPERIMENTS, PATINO LOWER AND PATINO UPPER FIELD, 1957.

Plant Nutrients (Pounds per Acre)		
N	P_2O_5	K_2O
0	0	0
0	0	60
0	20	60
0	40	60
0	60	60
0	80	60
0	100	60
0	120	60
20	0	60
20	20	60
20	40	60
20	60	60
20	80	60
20	100	60
20	120	60
40	0	60
40	20	60
40	40	60
40	60	60
40	80	60
40	120	60
60	0	60
60	20	60
60	40	60

Plant Nutrients (Pounds per Acre)		
N	P_2O_5	K_2O
60	60	60
60	100	60
60	120	60
80	0	60
80	20	60
80	40	60
80	80	60
80	120	60
100	0	60
100	20	60
100	60	60
100	100	60
100	120	60
120	0	60
120	40	60
120	80	60
120	120	60
140	0	60
140	20	60
140	40	60
140	60	60
140	80	60
140	100	60
140	120	60

TABLE 2

EXPERIMENTAL DESIGN OF THE CORN EXPERIMENTS, PATINO LOWER AND PATINO UPPER FIELD, 1957. *

Pounds of P O per Acre	Pounds of K O per Acre	Pounds of Nitrogen per Acre 0	20	40	60	80	100	120	140
0	0	X							
0	60	X	X	X	X	X	X	X	X
20	60	X	X	X	X	X	X		X
40	60	X	X	X	X	X		X	X
60	60	X	X	X	X		X		X
80	60	X	X	X		X		X	X
100	60	X	X		X		X		X
120	60	X	X	X	X	X	X	X	X

* Each "X" represents an experimental plot.

The fertilizer carriers used were, nitrogen as ammonium sulfate (20-0-0); phosphorous as concentrate superphosphate (0-46-0); and potash as potassium chloride.

Individual plots were two rows by 40 feet in size, with the rows 24 inches apart. There were two replicates of each treatment and the total area covered was approximately 0.2 acre or 52 x 160 feet.

The fertilizer was to be applied as a banded side-dressing to the side and below the seedling rows in furrows, when the beans were 2 to 3 inches tall. The beans were planted approximately October 21st and harvested January 14, 1958.

The levels of fertilization for the bean experiment are presented in detail in Table 3 and the design of the bean experiment in Table 4.

TABLE 3

LEVELS OF FERTILIZATION FOR THE BEAN EXPERIMENT, PATINO 1957/58.

Plant Nutrients

(Pounds per Acre)

N	P_2O_5	K_2O	N	P_2O_5	K_2O
0	0	0	40	0	0
0	0	40	40	0	40
0	25	0	40	25	40
0	25	40	40	50	40
0	50	40	40	75	40
0	75	40	40	100	40
0	100	40			
20	0	0	60	0	0
20	0	40	60	0	40
20	25	40	60	25	40
20	50	40	60	50	40
20	75	40	60	75	40
20	100	40	60	100	40
			80	0	40

TABLE 4

EXPERIMENTAL DESIGN FOR THE BEAN EXPERIMENT, PATINO 1957/58. *

Pounds of P_2O_5 per Acre	Pounds of K_2O per Acre	Pounds of Nitrogen per Acre 0	20	40	60	80
0	0	X	X	X	X	
0	40	X	X	X	X	X
25	0	X				
25	40	X	X	X	X	
50	40	X	X	X	X	
75	40	X	X	X	X	
100	40	X	X	X	X	

* Each "X" represents an experimental plot.

CHAPTER 5

DERIVATION OF PRODUCTION FUNCTIONS FROM YIELD DATA

After collection of yield observations, the next step in the analysis is that of estimating production functions, input-output or response coefficients.

Production Functions Fitted to the Data.-

The production functions[1] fitted to the data included:

(a) a square root equation of the form of equation (17)

$$(17) \quad Yo = a + bN + cP + d\sqrt{N} + e\sqrt{P} + f\sqrt{NP}$$

(b) a cross product production function of the form of equation (18)

$$(18) \quad Yo = a + bN + cP + dN^2 + eP^2 + fNP$$

(c) a Cobb-Douglas production function of the form of equation (19)

$$(19) \quad Yo = a\, X_1^{b1}\, X_2^{b2}\, X_3^{b3}$$

Previous Experimental Results Using these Functions.-

(a) Square Root Production Function.-

This function has been

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[1] The author is indebted both to Bernard Hoffnar of the Agricultural Economics Department and to the personnel of the Statistical pool of the same Department, at Michigan State University, for fitting the alternative production functions.

used by Heady[1] in fitting yield data for corn on calcareous Ida silt loam soil in western Iowa in 1952, and the results proved to be satisfactory with 91 per cent of the yield variance explained by the varying quantities of the two nutrients used, nitrogen and phosphoric acid.

This function was selected as being the most efficient for predicting the production surface, yield isoquants, and marginal quantities for corn.

Heady, working on red clover[2] and alfalfa[3] on Webster and Nicollet loam in north-central Iowa, explained 64 per cent and 77 per cent respectively of the variance present in the data using the square root production function as the predicting equation.

Knetsch working on corn[4] experiments on a Kalamazoo sandy loam soil during 1955 in Kalamazoo and Calhoun counties and using all three of the nutrients N, P_2O_5 and K_2O, fitted this function to the data. It proved to fit

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[1] Heady, E.O., Pesek, J.T., Brown, W.G., "Crop Response Surfaces and Economic Optima in Fertilizer Use", Agricultural Experiment Station, Iowa State College, Research Bulletin 424, March 1955, Ames, Iowa, p. 304.

[2] Ibid., p. 312.

[3] Ibid., p. 317.

[4] Knetsch, J.L., "Methodological Procedures and Applications for Incorporating Economic Considerations into Fertilizer Recommendations", Unpublished Master of Science, Thesis, Dept. of Agricultural Economics, Michigan State University, p. 47.

the data adequately for low applications of nitrogen but failed to fit at high levels, 41 per cent of the variance being explained by this equation.

This function is used by some researchers[1] to fit yield data when extremely large marginal products over small inputs are followed by a long range of small and fairly constant marginal products, i.e., in cases where a steep curve is expected at the outset, followed by a flat in the middle.

The square root equation allows interaction of the nutrients in the production process and also allows the inputs:

(a) to be substitutes only for small inputs.

(b) to be both substitutes and complements for higher levels of output, and

(c) to be only complements at maximum yield levels.[2]

(b) Cross Product Production Function.—

This production function has been used in previous studies with variable results.

Heady fitted a cross product equation to corn data from calcareous Ida silt loam soil in western Iowa from

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[1] Heady, E.O., Johnson, G.L., Hardin, L.S., *op. cit.*, Chapter 1, pp. 10-12.

[2] *Ibid.*, pp. 10-12.

[3] Heady, E.O., Pesek, J.T., Brown, W.G., *op. cit.*, p. 304.

experiments conducted in 1952, and it was considered a second best fitting with 83 per cent of the variance in yield explained by variable quantities of the two nutrients nitrogen and phosphoric acid.

Heady fitted this function to red clover[1] yield data from experiments conducted on Webster and Nicollet loam in north-central Iowa in 1952, and it was considered a second best fitting with 58 per cent of the yield variance explained.

Heady, working on alfalfa[2] fitted this function to yield data from Webster and Nicollet loan in north-central Iowa in 1952 and, as in the previous cases it was considered the second best fitting equation with 66 per cent of the variance explained.

Knetsch fitted the cross product production function to corn[3] data from Kalamazoo and Calhoun counties for experiments conducted on a Kalamazoo sandy loam soil during 1955. It gave a poor fit as indicated by only 29 per cent of the variance explained by the fitting equation.

Sundquist, working on oat[4] yield data from experiments

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[1]*Ibid.*, p. 312.

[2]*Ibid.*, p. 317.

[3]Knetsch, J.L., *op. cit.*, p. 47.

[4]Sundquist, W.B., "An Economic Analysis of Some Controlled Fertilizer Input-Output Experiments in Michigan", Unpublished Ph.D. Thesis, Dept. of Agricultural Economics, Michigan State University, 1957, p. 51.

conducted in Kalamazoo and Calhoun counties on a Kalamazoo sandy loam soil during 1955, explained only 48 per cent of the variance in yield using the cross product production function.

Sundquist, working on wheat[1] yield data from experiments conducted in Kalamazoo and Calhoun counties on a Kalamazoo sandy loam soil during 1955, explained only 41 per cent of the variance present in the data using this function.

Sundquist, working on corn[2] data from experiments conducted in Kalamazoo and Calhoun counties on a Kalamazoo sandy loam soil during 1955, fitted this equation; it proved to be highly unsatisfactory with only 8 per cent of the variance explained by the cross product production function.

Sundquist fitted this function to continuous corn[3] yield data from Tuscola county on a Wisner clay loam soil for experiments conducted during 1955 and it resulted in only 13 per cent of the variance explained.

Sundquist, working on beans[4] fitted this equation to data from Gratiot county for experiments conducted during 1955 on a Simms loam soil; 42 per cent of the variance present in the data was explained.

Sundquist, working on potato[5] data fitted this equation

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[1] *Ibid.*, p. 65.

[2] *Ibid.*, p. 78.

[3] *Ibid.*, p. 80.

[4] *Ibid.*, p. 83.

[5] *Ibid.*, p. 92.

for experimental data from a Houghton muck soil on the Experiment Station muck farm of the Michigan Agricultural Experiment Station near East Lansing; the regression equation was capable of explaining only 25 per cent of the variance present in the data.

The cross product production function allows interaction of the nutrients in the production process and being polynomial with first and second degree terms can show both diminishing marginal and diminishing total yields.

The cross product production function has greater flexibility than the Cobb-Douglas function and also allows, like the square root production function, the inputs:

(a) to be substitutes only for small inputs

(b) to be both substitutes and complements for higher levels of output, and

(c) to be only complements at maximum yields.

For marginal products of "medium" magnitude for small resource inputs, followed by an early maximum in total product, it is desirable to try this equation.

(c) <u>Cobb-Douglas Production Function</u>.-

This function assumes that the percentage increase in yield is constant and equal to "b" for all increments of fertilizer.

This function allows the yield to increase at either a diminishing, constant, or increasing rate, although the response curve can be represented by only one of these and never by a combination.

Therefore, if more than one stage of production seems to be present in the data, a Cobb-Douglas function is not adequate, since it can approximate only one stage at a time.

Another disadvantage of this function is that it takes on a value of zero whenever any input is zero. This disadvantage has been solved in this analysis by the addition of one-tenth[1] of a unit to all zero fertilizer levels. This introduces an upward bias in the predicted yield but overcomes the problem of having $Yc = 0$ when any of the treatments is zero.

On the other hand, the Cobb-Douglas function is easy to fit and work with. However, in view of the asymptotic nature of the function, considerable care must be used when drawing inferences from the extreme ranges of the data.[2]

This function has been used in fitting yield data in previous research work. Trant fitted this function to to corn[3] yield data in the Cauca Valley (Colombia), using

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[1]That is, 2 pounds for nitrogen; 2.5 pounds for phosphorous; and 4 pounds for potash.

[2]An explanation of the computation of the predicted yields, high profit point, and marginal productivities using the Cobb-Douglas function is presented in Appendix A.

[3]Trant, G.I., "Implications of Calculated Economic Optima in the Cauca Valley, Colombia, S.A.", Journal of Farm Economics, Volume XL, N. 1, Feb. 1958, pp. 1[illegible]3-133.

nitrogen and phosphorous and potassium combined in a 1:1 proportion and obtained 56 per cent of the variance explained in the first experiment; 60 per cent in a first replication of the same experiment; and 45 per cent in a second replication. In a second experiment on corn, using phosphorous and potassium as the two independent variables, no statistical evidence of correlation between the independent variables and the dependent variable was found; the coefficient of multiple determination had a value of zero.

Trant, working on sugar cane[1] in the Cauca Valley (Colombia), and using nitrogen as one independent variable and phosphorous and potassium combined in a 1:1 proportion as the other independent variable fitted the Cobb-Douglas function to data on newly planted cane. He found no statistical evidence of correlation between the independent and dependent variables; the coefficient of multiple determination had a value of zero. In fourth cutting cane, as a second experiment, the Cobb-Douglas function was capable of explaining only 18 per cent of the variance present in the data.

Knetsch fitted this function to corn[2] yield data from

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[1] *Ibid.*

[2] Knetsch, J.L., *op. cit.*, p. 48.

experiments conducted during 1955 in Kalamazoo and Calhoun counties on a Kalamazoo sandy loam soil but it was not considered the best representation of the data and it was rejected because of undesirable characteristics when the function and the data were plotted in natural numbers, although 51 per cent of the variance was explained by this production function.

Choosing the "Best" Fitting Function.- The selection of the best functional form is a problem of considerable importance.

As Mason[1] states, the problem of choosing the "best" function is not soluble from a single set of rules.

Evidence presented by Mason[2] seems to give no indication of preference of one function over the other from a strict statistical point of view. For using as a "prediction equation" one would seem to work as well as another and the criteria for choosing among them would rest on simplicity of computation considerations.

By the use of least squares procedures the value of the constants for the equation may be computed. These procedures give the best fit for the particular form of

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[1] Mason, David D., "Functional Models and Experimental Designs for Characterizing Response Curves and Surfaces", (in) Baum, E.L., Heady, E.O., Blackmore, J., *op. cit.*, Chapter 5, p. 80.

[2] Mason, David D., "Statistical Problems of Joint Research", *Journal of Farm Economics*, Vol. 39, May 1957, p.376.

functional model, in the sense of describing a curve from which the mean of the squares of the deviations of the individual points from that curve is a minimum.

However, the estimating equation is used as a "production function" to estimate optimum rates or levels of fertilization under various price relationships, which is the ultimate end in estimating these functional relationships, and as it is shown by Mason[1] considerably more variation exists among the estimate optima for the various functions than between the estimated values of yield.

It cannot be claimed that any of the functions represent fundamental biological laws of growth, although one may rationalize the form of a particular function in a particular situation.

It is likely that the best fitting function of the fertilizer production function varies by crop, year, soil, or other variables.[2]

Direct statistical tests (analysis of variance quantities and F tests) are available for determining whether a significant reduction in variance is obtained by including one more or less terms in equations such as the cross product

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[1] Ibid., p. 376.

[2] Heady, E.O., "Methodological Problems in Fertilizer Use", (in) Baum, E.L., Heady, E.O., Blackmore, J., op. cit., Chapter 1, pp. 6.

or square root functions.

However, as Heady[1] points out, "direct" tests are not available for choosing between such widely different functions as the Cobb-Douglas, Spillman, cross product or square root equations.

Testing the significance of coefficients for individual variables in an equation which contains more than one variable for a given plant nutrient is a practice of limited usefulness.

The related variables in an equation such as N, N^2, P, P^2, etc., are obviously highly correlated. Estimates of individual parameters may be subjected to large standard errors reflecting these high intercorrelations and it might be concluded that since individual parameters are not statistically significant, no significant effects are present.

This line of analysis may be misleading. If the aggregate effect of all variables could be tested for significance, the test might indicate a significant aggregate effect for the nutrient.

One way to approach the problem of selecting the

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[1] Heady, E.P., "Technical Considerations in Estimating Production Functions", (in) Heady, E.O., Johnson, G.L., Hardin, L.S., *op. cit.*, Chapter 1, p. 13.

best function has been suggested by Johnson[1].

It consists in testing the degree to which the alternative functions individually meet the usual assumptions with respect to the distribution of unexplained residuals on which objective statistical tests are based, and rejecting all functions that fail to pass this test.

If a choice between those functions which do meet the assumptions is necessary, objective tests are used as a second step in the selection of the best function.

If such tests also fail to reveal statistically differences between the alternative functions, less objective criteria, such as the experimenter's judgment and expert opinion could be applied, but only if necessary.

Analysis of the Data.- The results from fitting the above equations to the yield data are presented in this section.

The basic statistics relating to the functions fitted are given in Table 8 for Patino Lower Field corn data; in Table 9 for Patino Upper Field corn data, and in Table 13 for Patino bean data.

The meaning and significance of these statistics are

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[1] Johnson, Glenn L., "Discussion: Economic Implications of Agricultural Experiments", Journal of Farm Economics, Volume 39, May 1957, pp. 395-398.

explained in detail in Chapter 3.

Analysis of the Patino Lower Field Corn Data 1957.-

(a) Square Root Production Function.-

The first formulation of the functional relationship which was attempted for the data was a square root function of the form of equation (17) above.

This formulation containing the estimated parameters is shown in equation (20). Values listed below the estimated parameters and included in parentheses are standard errors of the respective parameters. N and P represent per acre applications of nitrogen and phosphoric acid respectively and yield is measured in bushels per acre.

$$(20) \quad Ye = 42.734948 + \underset{(.064078601)}{.095727036}\,N - \underset{(.073615241)}{.376305107}\,P$$

$$- \underset{(.923413823)}{.313685120}\sqrt{N} + \underset{(.980684688)}{5.095714910}\sqrt{P} - \underset{(.066703825)}{.043078163}\sqrt{NP}$$

The coefficient of multiple correlation for this equation was .656. The coefficient of multiple determination indicated that 43 per cent of the variance present in the yield data was explained by this regression equation.

(b) <u>Cross Product Production Function</u>.-

Because of the large amount of variance not associated with the estimating equation, a second formulation of the functional relationship was attempted of the type of equation (18) above.

The equation with the estimated parameters is shown in equation (21):

(21)

$$Y_c = 42.925175 + \underset{(.068813811)}{.083559729\,N} + \underset{(.075882981)}{.548996724\,P} + \underset{(.000422583)}{.000102622\,N^2} -$$

$$- \underset{(.000559148)}{.0040675289\,P^2} - \underset{(.00041177279)}{.00047068339\,NP}$$

The coefficient of multiple correlation for this equation was .747. The coefficient of multiple determination indicated that about 56 per cent of the variance in the yield data was associated with the regression equation.

Based on the statistical measurements derived, the cross product equation was considered a more appropriate formulation than the square root equation to represent the functional relationship between varied quantities of input nutrients and crop yield output.

<u>Production Surface Estimates</u>.-

One set of production surface estimates is presented in Table 6. These quantities showing the total per acre yield of corn for various rates of N and

TABLE 6

VALUES OF "R" AND "R^2" FOR TWO VARIABLE NUTRIENTS AND VALUES OF "t" FOR INDIVIDUAL REGRESSION COEFFICIENTS, FOR CORN PATINO LOWER FIELD 1957.

Equation	Value of "R"	Value of "R^2"	Value of "t" for coefficients in order listed in equation.
Square root	.656	.430	N : 1.49 P : 5.11 $\sqrt{N}$: .34 $\sqrt{P}$: 5.20 $\sqrt{NP}$: .65
Cross product	.747	.558	N : .96 P : 7.24 N^2: .24 P^2: 7.27 NP : 1.14

P_2O_5 are the counterpart of a production surface; they represent specific points on the surface for specific points in the nutrient input plane.

While not all of the treatments in every cell in Table 2 were included in the experiment, their yields can now be predicted. Each column in Table 6 is the counterpart of a vertical slice through the production surface parallel with the P_2O_5 axis and yields correspond to points on a single-variable input output curve. P_2O_5 is the variable while nitrogen is fixed in the amount shown in the top of the column.

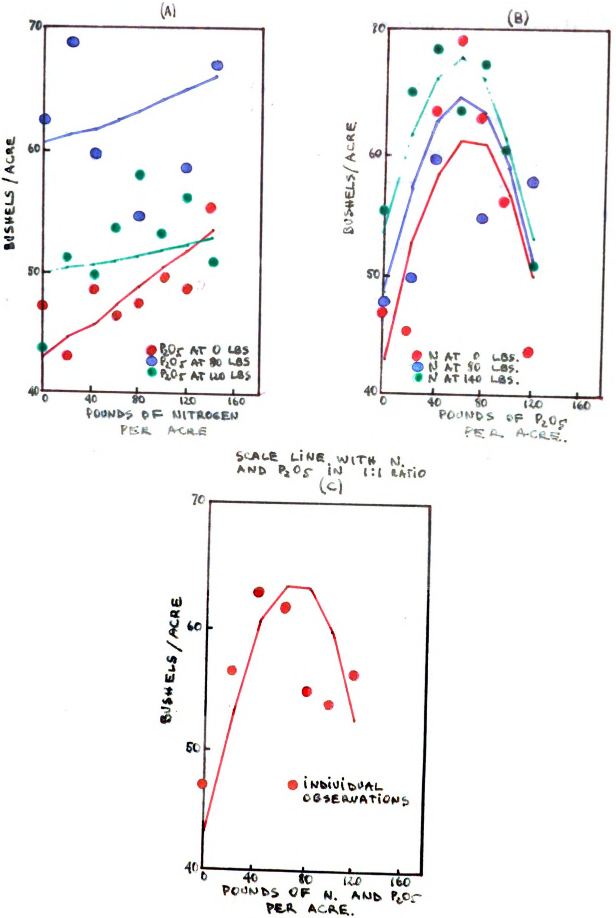
The rows represent the same thing with nitrogen variable and P_2O_5 fixed. It is obvious that interaction exists, the productivity of any input of one nutrient depends on the quantity of the other with which it is combined.

Figures 1.a and 1.b illustrate the effect of one nutrient on the productivity of the other. They are response curves for one variable nutrient with the other one fixed in the quantity indicated. It can be seen from these figures that nitrogen exerts the predominant effect on yields.

In Figure 1.a with P_2O_5 fixed and nitrogen variable,

FIGURE 1

CORN PATINO LOWER FIELD 1957



increasing yields for nitrogen applications are obtained, being the yields higher and higher until P_2O_5 is fixed at the 60 pounds per acre level (not shown in Figure). If P_2O_5 is fixed at a higher than 60 pounds per acre level, increasing yields for nitrogen are obtained, but the yields are lower than those obtained with a 60 pounds per acre level. In other words, too much P_2O_5 is being combined with nitrogen and diminishing returns for P_2O_5 are present.

In Figure 1.b with N fixed and P_2O_5 variable, it can be seen that when nitrogen is being fixed at higher and higher levels, higher yields are encountered until P_2O_5, the nutrient variable in this case reach the 60 pounds per acre level. Afterwards, diminishing returns and diminishing total yields are present.

Economic Optima.-

As was shown in detail in Chapter 2, the purpose of deriving functional relationships between fertilizer nutrient input and crop yield output was to provide the basis for making more efficient recommendations in the economic use of fertilizer specifying:

(a) the combination of nutrients to give least cost for the particular yield, and

(b) the amount of the nutrients to apply to maximize profits.

TABLE 6

TOTAL CALCULATED CORN YIELDS FROM SPECIFIED RATES OF APPLICATION OF NITROGEN AND PHOSPHORIC ACID, PREDICTED FROM THE CROSS PRODUCT EQUATION FOR CORN PATINO LOWER FIELD 1957. *

Pounds of P_2O_5 per Acre	Pounds of Nitrogen per Acre							
	0	20	40	60	80	100	120	140
0	42.9	44.2	45.6	47.1	48.7	50.3	52.0	53.8
20	52.2	53.4	54.6	55.9	57.2	58.7	60.2	61.8
40	58.3	59.2	60.3	61.4	62.6	63.8	65.1	66.6
60	61.1	61.8	62.7	63.6	64.6	65.7	66.8	68.1
80	60.7	61.2	61.9	62.6	63.4	64.3	65.2	66.3
100	56.9	57.3	57.8	58.3	58.9	59.6	60.4	61.3
120	50.0	50.2	50.4	50.8	51.2	51.7	52.3	53.0

* Yields are given in bushels per acre.

TABLE 7

TOTAL PREDICTED AND OBSERVED YIELDS FOR CORN PATINO LOWER FIELD, 1957.

Treatment (Pounds per Acre) N	P_2O_5	K_2O	Predicted Yield (Bu./Acre)	Observed Yield (Bu./Acre)
0	0	60	42.9	47.1
0	20	60	52.2	44.9
0	40	60	58.3	63.4
0	60	60	61.1	69.4
0	80	60	60.7	62.9
0	100	60	56.9	56.8
0	120	60	50.0	43.5
20	0	60	44.2	42.5
20	20	60	53.4	56.2
20	40	60	59.2	45.3
20	60	60	61.8	55.8
20	80	60	61.2	59.3
20	100	60	57.3	54.3
20	120	60	50.2	50.7
40	0	60	45.6	48.1
40	20	60	54.6	47.8
40	40	60	60.3	63.2
40	60	60	62.7	64.2
40	80	60	61.9	59.8
40	100	60	57.8	——
40	120	60	50.4	49.8
60	0	60	47.1	46.3
60	20	60	55.9	58.7
60	40	60	61.4	72.3
60	60	60	63.6	61.8
60	80	60	62.6	——
60	100	60	58.3	61.1
60	120	60	50.8	53.8

TABLE 7 , Continued.

80	0	60	48.7	47.2
80	20	60	57.2	43.8
80	40	60	62.6	59.2
80	60	60	64.6	----
80	80	60	63.4	54.5
80	100	60	58.9	----
80	120	60	61.2	57.9
100	0	60	50.3	49.7
100	20	60	52.7	63.4
100	40	60	63.8	----
100	60	60	65.7	73.7
100	80	60	64.3	----
100	100	60	59.6	63.8
100	120	60	61.7	52.9
120	0	60	62.0	48.1
120	20	60	60.2	----
120	40	60	65.1	72.0
120	60	60	66.8	----
120	80	60	65.2	58.6
120	100	60	60.4	----
120	120	60	62.3	55.9
140	0	60	63.8	65.1
140	20	60	61.8	64.8
140	40	60	63.6	63.8
140	60	60	68.1	65.6
140	80	60	66.3	67.0
140	100	60	61.3	59.9
140	120	60	63.0	61.0

These optima are attained when the partial derivatives (the marginal product) for both nutrients (in this case N and P) are equal to the nutrient/crop price ratio.

The marginal productivities of nitrogen and phosphoric acid are presented in Table 8, with the marginal productivity for nitrogen at the top of each pair and the marginal productivity of phosphoric acid at the bottom.

These marginal productivities are computed from equation (22) for nitrogen and from equation (23) for phosphoric acid.

$$(22)\quad \frac{dY}{dN} = .063569729 + 2(.000102622)N - .000470683390P$$

$$(23)\quad \frac{dY}{dP} = .546996724 - 2(.004067526900)P - .000470683390N$$

In computing the optimum amount of fertilizer nutrient input to use, the following prices were used:

Corn : \$1.45/bu.

N : \$ .16/lb.

P : \$ .10/lb.

To find the amount of nutrients to apply to maximize profits, equations (22) and (23) are solved simultaneously[1]

[1]The equations solved were as follows:

(a) For Nitrogen:

$$\underbrace{2(.000102622)}_{a_1}N - \underbrace{.000470683390}_{b_1}P = .1103 - \underbrace{.063569729}_{c_1}$$

(b) For Phosphorous:

$$\underbrace{-2(.004067526900)}_{b_2}N - \underbrace{.000470683390}_{a_2}P = .069 - \underbrace{.546996724}_{c_2}$$

using equations (24) and (25):

(24) $$N = \frac{(c_1\ b_2) - (c_2\ b_1)}{(a_1\ b_2) - (b_1\ a_2)}$$

(25) $$P = \frac{(a_1\ c_2) - (a_2\ c_1)}{(a_1\ b_2) - (b_1\ a_2)}$$

The nutrient/crop price ratio for nitrogen was found to be .1103 and the nutrient/crop price ratio for phosphoric acid was found to be .069. Using the above price ratios, the optimum amount of nitrogen to use was found to be 320.34 pounds per acre, which is an extrapolation well beyond the range of recorded experimental observations which does not indicate, by any means, an economic optimum point from which actual fertilizer recommendations can be made.

However, this characteristic indicates that further experimental work is needed with higher fertilizer levels than those studied in this particular experiment and for which no information is available.

The optimum amount of phosphoric acid to use was found to be 40.28 pounds per acre.

If these values with the above qualifications are substituted in equation (21) a predicted yield of 89.08 bushels per acre is found but, again, this figure represents a theoretical solution based on the above value for nitrogen whose accuracy cannot be determined.

Analysis of the Patino Upper Field Corn Data 1957.-

(a) Square Root Production Function.-

The first formulation of the functional relationship which was attempted for the data was a square root function of the form of equation (17) above.

This formulation containing the estimated parameters is shown in equation (27). N and P represent per acre applications of nitrogen and phosphoric acid respectively and yield is measured in bushels per acre.

$$(27)\quad Ye = 30.107083 - \underset{(.0308850610)}{.053411999 2N} - \underset{(.445073578)}{.0873990431P} + \underset{(.035481598)}{.714140548\sqrt{N}} + \underset{(.472677395)}{1.455248559\sqrt{P}} - \underset{(.0321503848)}{.0224733044\sqrt{NP}}$$

The basic statistics for this function are shown in Table 9. The coefficient of multiple correlation for this equation was .470 and the coefficient of multiple determination indicated that only 22 per cent of the variance present in the yield data was explained by this formulation.

TABLE 8

MARGINAL PRODUCTIVITIES OF NITROGEN AND PHOSPHORIC ACID IN THE PRODUCTION OF CORN FOR INPUTS INDICATED (NITROGEN AT TOP OF EACH PAIR AND PHOSPHORIC ACID AT BOTTOM), FOR CORN RATING LOWER FIELD 1957. *

Pounds of P_2O_5 per Acre	Pounds of Nitrogen per Acre							
	0	20	40	60	80	100	120	140
0	0 0	.067 0	.071 0	.075 0	.079 0	.084 0	.088 0	.092 0
20	0 .384	.058 -.374	.062 .365	.066 .356	.070 .346	.074 .337	.078 .327	.082 .318
40	0 .221	.048 .212	.052 .202	.057 .193	.061 .183	.065 .174	.069 .165	.073 .155
60	0 .058	.039 .049	.043 .040	.047 .030	.051 .021	.055 .011	.059 .002	.064 -.007
80	0 -.103	.030 -.113	.034 -.122	.038 -.132	.042 -.141	.046 -.150	.050 -.160	.054 -.179
100	0 -.266	.020 -.275	.024 -.285	.028 -.294	.032 -.304	.037 -.313	.041 -.322	.045 -.332
120	0 -.429	.011 -.438	.015 -.448	.019 -.457	.023 -.466	.027 -.476	.031 -.485	.035 -.495

* Marginal productivities are given in bushels per acre.

(b) Cross Product Production Function.-

Because of the large amount of variance not associated with the functional relationship above explained, a second formulation was attempted of the type of equation (18) above.

This equation and the estimated parameters are presented in equation (28):

$$(28) \quad Yo = 31.338108 + \underset{(.0355700598)}{.0567480394N} + \underset{(.040420165)}{.150470573P} - \underset{(.0002259778)}{.00044996016N^2} - \underset{(.0002990206)}{.00103468418P^2} - \underset{(.000220207)}{.0000691438lNP}$$

The coefficient of multiple correlation for this equation was .505. The coefficient of multiple determination indicated that about 26 per cent of the variance present in the data was explained by this equation.

Based on the statistical measurements derived neither of the two equations was considered to fit the data adequately. Nonetheless, the cross product functional expression gave a slightly better fit than the square root production function.

Production Surface Estimates.-

The cross product production function was used to compute the predicted yields for specific points on the production surface.

TABLE 9

VALUES OF "R" AND "R^2" FOR TWO VARIABLE NUTRIENTS AND VALUES OF "t" FOR INDIVIDUAL REGRESSION COEFFICIENTS FOR CORN PATINO UPPER FIELD 1957.

Equation	Value of "R"	Value of "R^2"	Value of "t" For Coefficients in Order Listed in Equation.
Square root	.470	.221	N : 1.73
			P : 2.46
			$\sqrt{N}$: 1.60
			$\sqrt{P}$: 3.08
			$\sqrt{NP}$: .70
Cross product	.505	.255	N : 1.60
			P : 3.72
			N^2 : 1.99
			P^2 : 3.46
			NP : .31

As was the case for corn Patino Lower Field 1957, each column in Table 10 represents a vertical slice through the production surface parallel with the P_2O_5 axis, and each row represents the same thing with nitrogen variable and P_2O_5 fixed.

Figures 2.a and 2.b illustrate the effect of one nutrient on the productivity of the other. They are response curves for one variable nutrient with the other fixed in the quantity indicated.

It can be seen from Table 10 and from Figures 2.a and 2.b that both nitrogen and phosphoric acid have a negligible effect on crop yields and that diminishing total yields are rapidly encountered.

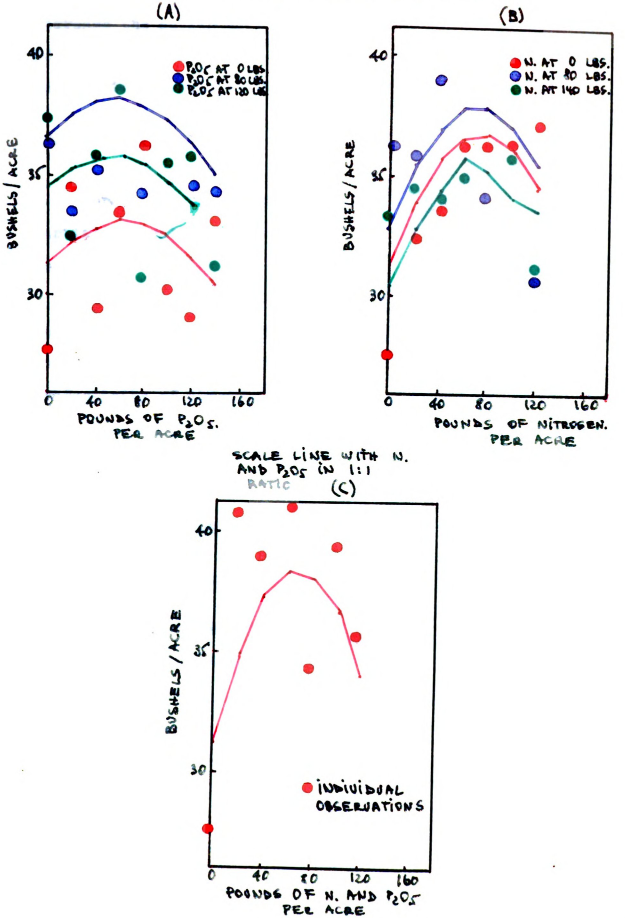
In Figure 2.a with P_2O_5 fixed and nitrogen variable, the level of the crop yield depends to a certain extent on the amount of P_2O_5 present in the soil. Up to the 80 pounds per acre level of P_2O_5 slightly higher levels of yields result as a consequence of the interaction between nitrogen and phosphoric acid.

Beyond that level, too much P_2O_5 is being combined with the specified level of nitrogen and yields are at a lower level.

In Figure 2.b in which nitrogen is being held constant and P_2O_5 varied, higher levels at which nitrogen is fixed

FIGURE 2

CORN PATINO UPPER FIELD 1957



increase the action of P_2O_5. However, beyond the level of 60 pounds per acre of nitrogen this element is redundant and the total yield is decreased.

Economic Optima.-

As was shown for the Patino Lower Field 1957 corn data, the economic optimum is attained when the partial derivatives for both nutrients are equal to the nutrient/crop price ratio.

The marginal productivities of nitrogen and phosphoric acid are presented in Table 12, with the marginal productivity of nitrogen at the top of each pair and the marginal productivity of phosphoric acid at the bottom.

These marginal productivities are computed from equation (29) for nitrogen and from equation (30) for phosphoric acid.

$$(29) \quad \frac{dY}{dN} = .056748099 - 2(.000449960160)N - .000069143810P$$

$$(30) \quad \frac{dY}{dP} = .150470579200 - 2(.001034684180)P - .000069143810N$$

In computing the amount of nutrients to apply to maximize profits, the following prices were used:

Corn : \$1.45/bu.

N : \$ .16/lb.

P : \$ .10/lb.

TABLE 10

TOTAL CALCULATED CORN YIELDS FROM SPECIFIED RATES OF APPLICATION OF NITROGEN AND PHOSPHORIC ACID, PREDICTED FROM THE CROSS PRODUCT EQUATION FOR CORN PATINO UPPER FIELD, 1957.*

Pounds of P_2O_5 per Acre	Pounds of Nitrogen per Acre							
	0	20	40	60	80	100	120	140
0	31.3	32.3	32.9	33.1	33.0	32.6	31.7	30.6
20	33.9	34.9	35.4	35.6	35.5	35.0	34.1	32.9
40	35.7	36.6	37.1	37.3	37.1	36.6	35.7	34.4
60	36.6	37.5	38.0	38.2	38.0	37.4	36.5	35.8
80	36.8	37.6	38.1	38.2	38.0	37.4	36.4	35.1
100	36.0	36.9	37.3	37.4	37.1	36.5	35.6	34.2
120	34.6	35.3	35.7	36.3	35.5	34.9	33.8	33.6

* Yields are given in bushels per acre.

TABLE 11

TOTAL PREDICTED AND OBSERVED YIELDS FOR CORN PATINO UPPER FIELD 1957.

Treatment (Pounds per Acre)* N	P_2O_5	K_2O	Predicted Yield (Bu./Acre)	Observed Yield (Bu./Acre)
0	0	0	31.3	27.6
0	20	60	33.9	32.5
0	40	60	35.7	33.6
0	60	60	36.6	36.3
0	80	60	36.8	36.3
0	100	60	36.0	36.1
0	120	60	34.5	37.2
20	0	60	32.3	34.5
20	20	60	34.9	40.7
20	40	60	36.6	33.2
20	60	60	37.5	33.2
20	80	60	37.6	33.4
20	100	60	36.9	40.6
20	120	60	35.3	32.4
40	0	60	32.9	29.3
40	20	60	35.4	32.8
40	40	60	37.1	39.0
40	60	60	38.0	41.6
40	80	60	38.1	35.2
40	100	60	37.3	----
40	120	60	35.7	35.6
60	0	60	33.1	33.4
60	20	60	35.6	35.0
60	40	60	37.3	40.4
60	60	60	38.2	41.6
60	80	60	38.2	----
60	100	60	37.4	37.1
60	120	60	35.8	38.4

TABLE 11, Continued.

80	0	60	33.0	36.1
80	20	60	35.5	35.8
80	40	60	37.1	39.0
80	60	60	38.0	----
80	80	60	38.0	34.1
80	100	60	37.1	----
80	120	60	38.5	30.7
100	0	60	32.5	30.2
100	20	60	35.0	33.4
100	40	60	36.6	----
100	60	60	37.4	41.9
100	80	60	37.4	----
100	100	60	36.5	39.3
100	120	60	34.9	35.5
120	0	60	31.7	29.1
120	20	60	34.1	----
120	40	60	35.7	29.8
120	60	60	36.5	----
120	80	60	36.4	34.5
120	100	60	35.5	----
120	120	60	33.8	35.8
140	0	60	30.5	33.2
140	20	60	32.9	34.5
140	40	60	34.4	34.2
140	60	60	35.2	35.0
140	80	60	35.1	34.3
140	100	60	34.2	35.0
140	120	60	33.6	31.1

To find the amount of nutrients to apply to maximize profits, equations (29) and (30) are solved simultaneously[1] using equations (24) and (25) as before.

Using .1103 as the nutrient/crop price ratio for nitrogen and .069 as the nutrient/crop price ratio for phosphorous, the optimum amount of nitrogen to use is a negative quantity -62.70 pounds per acre, and the optimum amount of phosphoric acid to use is 41.46 pounds per acre.

Substituting these quantities in equation (28), the predicted yield is found to be 30.65 bushels per acre.

This solution for the profit maximizing point is valid only if the price conditions specified above are true. If the given prices of fertilizer nutrient inputs and/or crop change, so change the profit maximizing level of fertilizer inputs and its combinations to use and the amount of crop yield to produce.

Analysis of the Patino Bean Data 1957/58.-

(a) Square Root Production Function.-

The first formulation

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[1]The equations solved were as follows:

(a) For Nitrogen:

$$-2(\underset{a_1}{.000449960160})N - \underset{b_1}{.000069143810}P = .1103 - \underset{c_1}{.056748099}$$

(b) For Phosphorous:

$$-2(\underset{b_2}{.001034684180})N - \underset{a_2}{.000069143810}P = .069 - \underset{c_2}{.150470579200}$$

TABLE 12

MARGINAL PRODUCTIVITIES OF NITROGEN AND PHOSPHORIC ACID IN THE PRODUCTION OF CORN FOR INPUTS INDICATED (NITROGEN AT TOP OF EACH PAIR AND PHOSPHORIC ACID AT BOTTOM), FOR CORN PATINO UPPER FIELD, 1957. *

Pounds of P_2O_5 per Acre	Pounds of Nitrogen per Acre							
	0	20	40	60	80	100	120	140
0	0 0	.036 0	.020 0	.002 0	-.015 0	-.033 0	-.051 0	-.069 0
20	0 .109	.037 .107	.019 .106	.001 .104	-.016 .103	-.034 .102	-.052 .100	-.070 .099
40	0 .067	.035 .066	.017 .064	.000 .063	-.018 .062	-.036 .060	-.054 .059	-.072 .058
60	0 .026	.034 .024	.016 .023	-.001 .022	-.019 .020	-.037 .019	-.055 .018	-.073 .016
80	0 -.015	.033 -.016	.015 -.017	-.002 -.019	-.020 -.020	-.038 -.021	-.056 -.023	-.074 -.024
100	0 -.056	.031 -.057	.013 -.059	-.004 -.060	-.022 -.061	-.040 -.063	-.058 -.064	-.076 -.066
120	0 -.097	.030 -.099	.012 -.100	-.005 -.102	-.023 -.103	-.041 -.104	-.059 -.106	-.077 -.107

* Marginal productivities are given in bushels per acre.

of the functional relationship which was fitted to the bean yield data was a square root function of the form of equation (17) above.

This formulation containing the estimated parameters is shown in equation (31). N and P represent per acre applications of nitrogen and phosphoric acid respectively and yield is measured in bushels per acre.

$$(31)\quad Y_c = 1.73957525 - \underset{(.005960145)}{.0066676964 9N} - \underset{(.004124697)}{.001851434 13P} - \underset{(.003029390)}{.002160281K} + \underset{(.047112116)}{.1826605968}\sqrt{N} + \underset{(.046056907)}{.0423035974}\sqrt{P}$$

The coefficient of multiple correlation for this equation was .806 and the coefficient of multiple determination was .650, indicating that 65 per cent of the variance present in the yield data was explained by the independent variables.

The basic statistics for this and the following equations fitted for this set of yield data are presented in Table 13.

(b) <u>Cross Product Production Function</u>.— A second formulation of the functional relationship was attempted using a cross product production function of the form of equation (18) above.

TABLE 13

VALUES OF "R" AND "R^2" FOR THREE VARIABLE NUTRIENTS AND VALUES OF "t" FOR INDIVIDUAL REGRESSION COEFFICIENTS FOR BEAN RATING 1957/58.

Equation	Value	Value	Value of "t" For Coefficients in Order Listed in Equation.
Square root	.806	.650	N : 1.12
			P : .45
			K : .71
			$\sqrt{N}$: 3.88
			$\sqrt{P}$: 1.06
Cross product	.816	.665	N : 6.97
			P : 1.12
			K : .39
			N^2 : 4.24
			P^2 : .70
Cobb-Douglas	.849	.720	N : 11.38
			P : 1.98
			K : .46

This equation with the estimated parameters is shown in equation (32).

$$(32) \quad Yc = 1.760349605 + \underset{(.005203363)}{.03625854304N} + \underset{(.004119018)}{.00462240615P} +$$

$$+ \underset{(.002976016)}{.00115231883K} - \underset{(.000075872)}{.00032173563N^2} - \underset{(.000040272)}{.0000781833P^2}$$

The coefficient of multiple correlation for this equation was .816 and the coefficient of multiple determination was .665, indicating that about 66 per cent of the variance was explained by the regression equation.

(c) Cobb-Douglas Production Function.-

A third formulation of the functional relationship was attempted using a Cobb-Douglas production function of the form of equation (19) above.

This formulation with the estimated parameters is shown in equation (33) and in its logarithmic form in equation (34).

$$(33) \quad Yc = 1.758 \; N^{.105337643} \; P^{.017355082} \; K^{-.005592104}$$

$$(34) \quad \log Yc = .244929 + \underset{(.009360381)}{.105337643 \log N} + \underset{(.009027808)}{.017355082 \log P} -$$

$$- \underset{(.012126062)}{.005592104 \log K}$$

The coefficient of multiple correlation for this equation was .849 and the coefficient of multiple determination was .720 indicating that 72 per cent of the variance present in the data was explained by the independent variables of the regression equation.

Based on the statistical measurements derived, the Cobb-Douglas production function was considered a more appropriate formulation than the two previously derived equations, to represent the functional relationship between varied quantities of input nutrients and crop yield output.

Production surface estimates.- A set of production surface estimates is presented in Table 14. These quantities showing the total per acre yield of beans for various rates of N and P_2O_5 represent specific points on the surface for specific points in the nutrient input plane.

Each column in Table 14 corresponds to a vertical slice through the production surface parallel with the P_2O_5 axis, and they show the yields when nitrogen is fixed at a given level and P_2O_5 is being varied.

The rows represent the same thing with nitrogen variable and P_2O_5 fixed in the amount specified.

In Table 15 the total predicted and observed yields for beans are compared to show how accurately this function is capable of predicting them from their functional relationship.

Economic optima.-

A detailed explanation on how to work with the Cobb-Douglas function has been presented in Appendix A, so these concepts will not be repeated here.

In computing the profit maximization point the following prices have been used:

Beans	:	$5.76/bu.
N	:	$ .16/lb.
P	:	$ .10/lb.
K	:	$ .06/lb.

The optimum amount of nitrogen to use was found to be 56.9 pounds per acre; the optimum amount of phosphoric acid 15.0 pounds per acre; and the optimum amount of potash was a negative quantity of -8.1 pounds per acre.

The estimated yield was found to be 14.3 bushels per acre.

The calculated marginal productivity of a pound of nitrogen in the production of beans at various rates of application of nitrogen are presented in Table 16, for selected beans prices.

As has been explained, profit maximization concepts must include price relationships and when these price conditions change, so change the amount and composition of fertilizer nutrients to apply and of product output to produce.

It can be seen, that when beans are priced at $ 5.75 per bushel, with the above indicated fertilizer prices, the optimum level of application of nitrogen is at the level of the 60 pounds per acre level when the condition

$$MVP_n - P_n = 0$$

is satisfied.

The same analysis has been done in Table 17, in which the marginal productivities of a pound of phosphoric acid in the production of beans at various rates of application of phosphoric acid, are presented.

With the above specified price conditions, the optimum level of application of phosphoric acid is at the level of the 15 pounds per acre.

TABLE 14

TOTAL CALCULATED BEAN YIELD FROM SPECIFIED RATES OF APPLICATION OF NITROGEN AND PHOSPHORIC ACID, PREDICTED FROM THE COBB-DOUGLAS EQUATION FOR BEAN PATINO, 1957/58. *

Pounds of P_2O_5 per Acre	Pounds of Nitrogen per Acre				
	0	20	40	60	80
0	11.4	14.5	15.6	16.3	16.8
25	11.9	15.1	16.2	17.0	17.5
50	12.0	15.3	16.4	17.2	17.7
75	12.1	15.4	16.6	17.3	17.8
100	12.1	15.5	16.6	17.4	17.9

* Yields are given in bushels per acre.

TABLE 15

TOTAL PREDICTED AND OBSERVED YIELDS FOR BEAN PATINO 1957/58.

Treatment (Pounds per Acre) N	P_2O_5	K_2O	Predicted Yield (Bu./Acre)	Observed Yield (Bu./Acre)
0	0	0	11.4	10.6
0	25	40	11.9	11.2
0	50	40	12.0	10.9
0	75	40	12.1	11.2
0	100	40	12.1	11.5
20	0	40	14.5	15.1
20	25	40	15.1	16.4
20	50	40	15.3	13.6
20	75	40	15.4	14.2
20	100	40	15.5	14.6
40	0	40	15.6	14.6
40	25	40	16.2	16.1
40	50	40	16.4	19.4
40	75	40	16.6	15.7
40	100	40	16.6	17.0
60	0	40	16.3	15.1
60	25	40	17.0	15.7
60	50	40	17.2	18.2
60	75	40	17.3	17.6
60	100	40	17.4	19.1
80	0	40	16.8	15.1
80	25	40	17.5	----
80	50	40	17.7	----
80	75	40	17.8	----
80	100	40	17.9	----

TABLE 16

CALCULATED MARGINAL PRODUCTIVITIES OF A POUND OF NITROGEN IN THE PRODUCTION OF BEANS AT VARIOUS RATES OF APPLICATION OF NITROGEN AND SELECTED BEAN PRICES, FOR PATINO, 1957/58.

Pounds of N per Acre		Price of Beans (Dollars / Bushel)				
		$ 5.25	$ 5.50	$ 5.75	$ 6.00	$ 6.25
	MPP *	MVP	MVP	MVP	MVP	MVP
1	1.51	7.93	8.31	8.68	9.06	9.44
2	.75	3.94	4.13	4.31	4.50	4.69
3	.50	2.63	2.75	2.88	3.00	3.13
4	.38	2.00	2.09	2.19	2.28	2.38
5	.30	1.58	1.65	1.73	1.80	1.88
6	.25	1.31	1.38	1.44	1.50	1.56
7	.22	1.16	1.21	1.27	1.32	1.38
8	.19	1.00	1.05	1.09	1.14	1.19
9	.17	.89	.94	.98	1.02	1.06
10	.15	.79	.83	.86	.90	.94
20	.08	.42	.44	.46	.48	.50
40	.04	.21	.22	.23	.24	.25
60	.03	.16	.17	.17	.18	.19
80	.02	.11	.11	.12	.12	.13

* Bushels per acre.

TABLE 17

CALCULATED MARGINAL PRODUCTIVITIES OF A POUND OF PHOSPHORIC ACID IN THE PRODUCTION OF BEANS AT VARIOUS RATES OF APPLICATION OF PHOSPHORIC ACID AND SELECTED BEAN PRICES, FOR PATINO, 1957/58.

Pounds of P_2O_5 per Acre		Price of Beans (Dollars / Bushel)				
		$ 5.25	$ 5.50	$ 5.75	$ 6.00	$ 6.25
	MPP *	MVP	MVP	MVP	MVP	MVP
1	.25	1.31	1.38	1.44	1.50	1.56
2	.12	.63	.66	.69	.72	.75
3	.08	.42	.44	.46	.48	.50
4	.06	.32	.33	.35	.36	.38
5	.05	.26	.27	.29	.30	.31
6	.04	.21	.22	.23	.24	.25
7	.04	.21	.22	.23	.24	.25
8	.03	.16	.17	.17	.18	.19
9	.03	.16	.17	.17	.18	.19
10	.03	.16	.17	.17	.18	.19
25	.01	.05	.06	.06	.06	.06
50	.005	.03	.03	.03	.03	.03
75	.003	.02	.02	.02	.02	.02
100	.002	.01	.01	.01	.01	.01

* Bushels per acre.

CHAPTER 6

EVALUATION, CONCLUSIONS AND IMPLICATIONS

Evaluation.-

Experimental Plots and Commercial Farms.-

Comparing the conditions faced by the researcher working with experimental plots and the situation faced by the operator on a commercial farm, it can be seen that, even though, the number of independent variables usually included in experimental work is very small and many problems encountered by the farmers (rotations, etc.) cannot be solved entirely within the framework of a single experiment, the information obtained from it can be combined and improved by subsequent research leading to better and broader knowledge of the functional relationships involved.

The elements considered "fixed" in experimental work such as the recommended practices, are also controllable by the farmer.

The troublesome element seems to lie on the difference of levels at which non-controllable, non-studied variables are fixed in the experimental work and on the farm.

Each individual experiment field has certain unique characteristics associated with it, which are the determinant

factors when the results from the experimental field are trying to be generalized for a large number of farms.

Thus, the problem, is to try to reduce the variance in the experimental results to conform closer to those on the farm, enabling the economic optimum conditions to be defined more accurately.

This problem, on the other hand, is aggravated by the desire of the researcher of minimizing within field variability choosing the location of the experiment in such a way, that generally the levels at which these uncontrolled variables are fixed in the experimental work and the levels at which they are fixed on the commercial farms are pushed still farther apart.

Variance.-

The unstudied and uncontrolled variables causing large amounts of unexplained between-plot variance may be important elements when determining:

(a) the appropriate mathematical function to fit, and

(b) the optima located on a selected function.

One cause of this unexplained variance is believed to be the small size of the plots usually used in experimental work. However, as has been suggested, the variance present in yield data obtained from small experimental plots might be higher than the amount of variance experienced by the operator on a commercial farm in which larger areas are involved.

This is a very important point to consider, because the accuracy of the recommendations made to farmers depends to a great extent on how representative of the conditions on the average farm are the estimates secured from experimental work.

As to ways to handle this problem, the following have been suggested:

(a) Increase in the Size of the Experimental Plots.- Larger plots should be used in experimental work. When the causes of the variance are randomly distributed throughout the experimental area, the use of larger plots will be indicated, but, to the extent that the causes of variance are not randomly distributed but are correlated between adjacent small plots, replications of plots become relatively more effective than larger plots in reducing variance.

(b) Investigation and Measurement of the Causes of Unexplained Variance.- The causes of unexplained variance might be investigated and measured and included in the study as independent variables.

However, obstacles are encountered at present to study accurately this point because appropriate methods of measurement are still not very well developed.

Validity of Experimental Results over Time.- Yield data from one year experimental work has been analyzed in previous chapters only.

As has been pointed out previously, the uncontrolled and unstudied variables present in the experimental work influence the results in such a way that, based on one year's data, generalizations cannot and must not be made trying to extend the analysis to future years.

The U element analyzed above is likely to change year after year and predictions based on such unstable ground will have a large percentage of probabilities to be wrong.

Problems of residual fertility accumulation and depletion and rotation effects become important when long run conclusions are to be drawn from the experimental data and long run decisions are to be made on commercial farms.

Conclusions.- Three sets of yield data were analyzed in the present study.

The first one, yield data for corn Patino Lower Field 1957 included nitrogen and phosphoric acid as the independent variables, with potash generally held constant at 60 pounds per acre.

Two production functions of the type of equations (17) and (18) above were fitted to the data. Statistical measurements indicated that the cross product type of production function fitted the data better than the square root production function.

As it was seen before, the economic optima conditions are based both in the physical functional relationships and in the price conditions for fertilizer inputs and product output existing at that time.

If the price relationships involved change, a new optimum amount of fertilizer inputs and nutrient combinations to apply become profitable as determined by the new nutrient/crop price ratio existing after the change.

This point is generally overlooked in the present fertilizer recommendations which are being given to farmers.

It is the principal reason why a new approach integrating agronomic and economic concepts is being used in the design of fertilizer experiments which allows the location of points at which maximum profits from a given application of fertilizer nutrients are possible. At the same time, recommendations to farmers are made in terms of maximum profits in which changing price conditions of fertilizer and/or crops are considered, that is, these recommendations are made more realistic approaching situations which are usually faced by the farmers in the planning of their fertilizer programs.

A significant response to both nitrogen and phosphoric acid was found to exist for this set of data.

The economic optima point was computed. With respect to nitrogen it was found to be located outside the range of experimental observations. Therefore, the figure of 320.24 pounds per acre of nitrogen is an extrapolation and cannot be used for actual fertilizer recommendation purposes. However, it does indicate that further research using higher fertilizer treatment levels would be very useful to complement the experimental resultspresented here.

The optimum amount of phosphcrous to use was found to be 40.23 pounds per acre which is within the experimental range observed.

The same two functions above indicated were fitted to the second set of data for corn Patino Upper Field 1957. This yield data included nitrogen and phosphoric acid as the two independent variables, with potash generally constant at 60 pounds per acre.

The two functions indicated gave a very poor fit of the yield data, with the cross product production function considered to be a slightly better representation of the functional relationships involved. The most profitable amount of fertilizer input to use was computed using the cross product equation and was found to be a negative quantity of -62.70 pounds of nitrogen per acre and 41.46 pounds of phosphoric acid per acre.

The predicted yield computed for the profit maximization point was 30.65 bushels per acre.

A significamt response was recorded only for phosphoric acid with no response on yield recorded for nitrogen.

The third set of data for bean Patino 1957/58 was fitted with the same two production functions previously cited and with a Cobb-Douglas function of the type of equation (13) above.

Based on the statistical measurements derived, the Cobb-Douglas function was considered a more appropriate fit for this set of yield data.

Economic optima quantities of fertilizer inputs to apply were computed using this equation and the results showed that 56.9 pounds of nitrogen per acre ought to be used for profits to be at a maximum, indicating a significant response to this element.

A slight response to phosphoric acid was recorded indicated by an optimum amount of 15.0 pounds per acre to be used.

The estimated yield computed using this functional relationship was found to be 14.3 bushels per acre.

The same concepts on how the optimum amount of nutrients to use are dependent on the fertilizer/crop price relationship existing at the time as they were explained above for the corn experiments are valid in this case and they must be taken into consideration.

Implications.-

Significance of Results for the Researcher.-

It has been remarked[1] "that the only time an experiment can be properly designed is after it has been completed".

One sometimes finds, after a set of experiments have

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[1]Box, G.E.P., Hunter, J.S., "The Exploration and Exploitation of Response Surfaces", (as cited in) Mason, David D., "Statistical Problems of Joint Research", Journal of Farm Economics, Volume 39, May 1957, p. 371.

been made, that one or more important variables have been overlooked or that more could have been learned if the factors could have been varied over different ranges.

As has been the case in the present study, additional information would have been extremely useful if higher levels of fertilization would have been included for Corn Patino Lower Field 1957, in which the economic optimum point estimate was well beyond the levels within which experimental observations were recorded.

Thus, the results from the above analysis have indicated the direction in which further research work with fertilizer may be profitably carried out in corn and bean crops.

These results seem to suggest the need for additional experimental work in the Cauca Valley, Colombia, for corn and beans and other important crops which over the years would provide useful and dependable information for making fertilizer recommendations to farmers. As has been shown in this study, agronomic and economic concepts are closely interrelated in fertilizer research.

However, economic considerations are still neglected in the current fertilizer recommendations that are given to farmers. In Colombia, as elsewhere, this is also true and it is necessary that researchers planning future fertilizer research work be able and willing to recognize these interrelationships and to include them in their studies. It is not an easy task to obtain cooperation, and in many cases to be willing to cooperate in research work which seems outside one's area of specialization or interests. This is particularly true in an environment in which efforts toward this end have not been made yet. Nevertheless, this cooperation is necessary and probably badly needed in countries like Colombia, in which agricultural development projects are under way and where capital and trained technicians are usually in short supply.

A fertilizer research project designed to provide experimental data when plant nutrient inputs are varied over different ranges and from which economic optima estimates can be located, certainly represents a considerable improvement over experimental designs from which marginal productivities and economic optimum cannot be determined.

As a matter of fact, more information is provided by designs of the type considered herein both of agronomic and economic interest.

The ratio of useful information to expenditures is probably higher for agronomic-economic than for purely agronomic designs.

Significance of results for the Farmer.-

The analysis of experimental work presented in this study is rather limited in scope with respect to soil conditions, crops, and growing seasons.

For the reasons stated elsewhere, seldom are any recommendations made upon the basis of a single experiment such as this one.

Additional work is needed to support or deny the optimum plant nutrient treatment estimates presented here and before reliable recommendations can be made to farmers for rational planning of their fertilizer programs.

Historically, this information has not been available to farmers for the very simple reason that fertilizer research has been conducted independently from any economic considerations.

The farmers are being supplied with fertilizer information in which, implicitly or explicitly, the conclusion is being drawn that the most adequate level of fertilization is the one at which maximum yields per acre are attained.

As has been shown, maximum yields per acre and maximum profits from a given application of fertilizer are seldom located at the same level.

Therefore, the Colombian farmer in the Cauca Valley and, it is suspected, elsewhere along and across the main agricultural regions of South America, ought to be supplied with fertilizer information in which recommendations are made to maximize profit instead of yields.

When resources are so far out of adjustment as other similar studies in the Cauca Valley show they are[1] bits of information provided by partial studies and preliminary surveys of the fertilizer problems of a given area, are perhaps the best way and the most economic one to promote a reallocation of those resources even though more refined and elaborated studies may prove to be useful afterwards.

If reliable information could be secured with respect to the returns the Colombian farmer in the Cauca Valley is earning on inputs other than fertilizer in his business, a comparison of their marginal productivities would provide an additional tool of decision-making to farmers for whom limited capital is an important consideration.

As surveys of the Colombian agriculture show a rate of increase in the agricultural production higher than that accomplished during the past years will be needed to keep pace with the increasing population and improving levels

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[1]Trant, G. I., op. cit.

of living.

Colombian farmers will certainly be required to increase the productivity of their farms.

Fertilizer, as well as other forms of capital and "know how" representing technical improvements in agriculture can make an important contribution toward that end.

APPENDIX A

(A) The Cobb-Douglas equation is of the form of equation (1)

(1) $$Yc = a \cdot N^{b1} \cdot P^{b2} \cdot K^{b3}$$

in which Yc is the predicted yield, the term "a" is a constant, and b1, b2, b3 are the regression coefficients and the elasticities of the dependent variable with respect to each dependent variable, that is, the percentage change in the dependent variable associated with one per cent change in the dependent variable.

The Cobb-Douglas function becomes easier to manage in logarithmic form such as equation (2)

(2) $$\log. Yc = \log. a + b1 \log. N + b2 \log. P + b3 \log. K$$

(B) The marginal physical productivity in this function is defined in the equation (3)

(3) $$\frac{dY}{dN} = b1 \cdot \frac{Yc}{N}$$

in which N takes different values according to the treatment levels specified. In the same way it is possible to compute the marginal productivities for P and K.

From equation (3), equation (4) is derived indicating the condition of higher profit point:

(4) $$b1 \cdot \frac{Yc}{N} - P_n = 0$$

in which P_n is the price of (i.e) nitrogen.

The same condition can be derived for P and K. The expression (4) is used now to derive another equation, solving for N (or P or K) such as (5)

$$(5) \quad N = \frac{b1}{P_n} \cdot Yo$$

The expression $\frac{b1}{P_n}$ is a constant K_1 (K_2 for P, and K_3 for K) and now the equation for yield can be expressed as in (6):

$$(6) \quad Yo = a \cdot (K_1 \cdot Yo)^{b1} \cdot (K_2 \cdot Yo)^{b2} \cdot (K_3 \cdot Yo)^{b3}$$

and in logarithmic forms as in expression (7):

$$(7) \quad \log.Yo = \log.a + b1 \log.K_1 + b1 \log.Yo + b2 \log.K_2 + b2 \log.Yo + b3 \log.K_3 + b3 \log.Yo$$

or in a more abbreviate form such as (8):

$$(8) \quad \log.Yo = \frac{\log.a + b1 \log.K_1 + b2 \log.K_2 + b3 \log.K_3}{1-b1-b2-b3}$$

(C) To work with the Cobb-Douglas equation, constants K_1, K_2, and K_3, are first computed, being necessary to know the prices of N, P, and K, from equation (5).

Then an estimate of yields (Yo) can be made using equation (8) and converting the logarithm into a natural number.

Now, the optimum quantities of N, P, and K, can be estimated from equation (5) substituting the appropriate values in it.

Finally, estimated marginal physical productivities can be computed substituting the corresponding values in equation (3), and at the high profit point the equality (4) must be true.

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