A DIGITAL COMPUTER SOLUTION OF SYNCHRONOUS MACHINE DIFFERENTIAL EQUATIONS IN SOME POWER SYSTEM STABILITY PROBLEMS

By

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AN ABSTRACT

Submitted to the School of Advanced Graduate Studies of Michigan State University of Agriculture and Applied Science in partial fulfillment of the requirements for the degree of

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ABSTRACT

It is common practice to make simplifying assumptions in order that convenient or familiar means may be used in the study of synchronous machines. For the purpose of this thesis, these assumptions will be considered in two groups. With the first group of assumptions, there results the familiar system of non-linear differential equations interrelating voltages, currents, speed, and torque for the synchronous machine. This type of system of equations is not generally amenable to solution except by numerical methods. Rather than attempt to solve this formidable system of equations directly, engineers working in the area of power system stability use a second group of simplifying assumptions in order to solve the system of equations by convenient means. Using both groups of assumptions, the conventional method thus developed over the years is to solve part of the equations by modified steady state techniques and then apply numerical methods to the remaining equations in order to obtain a solution. Similar techniques are used by engineers in other areas where the transient operation of synchronous machines is of interest. The study of a synchronous motor with a periodically varying load or with impact loading is an important example of these areas.

Recently, much work has been done toward applying the digital computer to the study of the power system stability problem. However, the trend is toward programming the conventional techniques with their two sets of simplifying assumptions and empirical methods. In contrast, this thesis is a report on an investigation of the numerical solution of the complete system of synchronous machine non-linear differential equations without the second group of simplifying assumptions.

Two classes of power system stability problems are studied here and it is found that for the cases considered:

1. The solution can be carried out in detail without the second group of simplifying assumptions.

2. The solution can be carried out under transient conditions without developing or using extensive empirical relations that are necessary in using modified steady state techniques for the transient problems.

3. The form of the results reported here gives more stability information and gives the information more directly.

4. The direct solution of the complete set of synchronous machine equations rather than solution by conventional means, produces results that add to and verify accepted theory in some cases and contradict accepted theory in other cases.

A conclusion from the investigation reported here is that, for the class of problems considered in this thesis, methods developed before the advent of the digital computer should not be used as the basis for digital computer investigation. The more fundamental synchronous machine equations can now be solved directly.

The physical and mathematical structure for the cases considered here is the same as the structure in other areas involving transients in synchronous machines. Therefore the advantages of direct solution of the synchronous machine equations extend to areas other than power system stability.

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I. INTRODUCTION

The purpose of a study of a synchronous machine is either to obtain design information or to predict operating conditions for a machine. In either case, use is made of the relation between the machine parameters: inductances, resistances, inertia and the machine variables: voltages, currents, speed, torque. The design problem is the problem of obtaining the proper machine parameters for a desired range of the variables. The problem of predicting operating conditions is the problem of obtaining a subset of the variables for conditions imposed by specifying the machine parameters and the remaining variables.

To aid in the study of a synchronous machine, it is desirable to obtain a mathematical interrelation of the machine variables. Laboratory meter indications show the relationship among the variables to be so complicated that either exact mathematical interrelations cannot be found or the resulting mathematical system of equations cannot be solved. Some simplifying assumptions, which lead to a mathematical system of equations that can be solved, have become conventional. The system of equations resulting from the simplifying assumptions usually gives results that have acceptable correlation with laboratory meter indications over a limited range of the variables and for limited waveforms of the variables.

One of the problems encountered in obtaining a mathematical relation for the variables, is the irregular waveforms of the instrument indications. This problem is commonly treated by considering only the lower frequency harmonics. Often only the fundamental frequency terms are used in the representations.

Another problem is encountered in the fact that the apparent induc-

tance and resistance coefficients are not constants but are rather complicated functions of currents, power factor, field saturation, and other factors. There are clearly defined procedures for determining inductance and resistance numbers which are mean constants that partially take into account the dependence of the coefficients on current, power factor, and saturation.

The fundamental harmonic representation and constant inductance and resistance coefficients are taken here as the first of two groups of simplifying assumptions. With this first group of assumptions, there results the standard system of differential equations for synchronous machines non-linear in speed torque and current variables. In order to solve the synchronous machine equations by convenient or familiar means, it is conventional to make a second group of simplifying assumptions also. The second group of assumptions are: constant speed operation, steady state sinusoidal armature voltages and currents, constant direct current field. With both groups of assumptions, the equations reduce to an algebraic system of equations which are readily solved.

Relaxing any of these restrictions would yield pertinent information and give mathematical results which more closely correspond to laboratory observations of the meter indications. Much work has been done in the study of the effects of higher harmonics and in the study of the effects of saturation, current, and power factor on the inductance and resistance coefficients. Elaborate techniques have been developed to make steady state, constant speed techniques yield results correlating with observations under transient conditions. These techniques usually involve separating the equations into electrical and mechanical sets. The electrical equations are solved by modified steady state methods and the mechanical equations are solved as differential equations, usually by numerical methods. Little has been done, however, in the area of solving the complete set as simultaneous non-linear differential equations. The presently accepted standard techniques were developed before the advent of the digital computer, and numerical methods for solving merely the mechanical equation as a non-linear differential equation involve long and tedious calculations. With the modern high speed computers, however, there appeared, at the beginning of the research reported here, to be no reason why the complete set should not be solved as non-linear differential equations by numerical methods.

This thesis demonstrates the technique for solving the complete set of equations by numerical methods by applying the techniques to two classes of power system stability problems. Not only are the problems solved without making the assumptions of constant speed, constant field current, and sinusoidal armature voltages and currents, but for the classes of problems considered the solution process is simpler than the standard procedure. The form of the answers gives more stability information and gives the information more directly. Further, the greater generality of the techniques used in this thesis yields information which in some cases adds to or verifies the accepted theory and in other cases contradicts accepted theory. In particular, this additional information was obtained relative to the variation of the field current and the variation of the direct and quadrature components of armature currents under transient conditions of operation.

The techniques used here in solving the systems of equations for the two classes of power system stability problems can be considered in three parts: First, the variables are changed in order to obtain a convenient form for the coefficients. The variables used are closely related to the variables of A. Blondel's[1] two reaction analysis or the variables resulting from a transformation of variables illustrated by R. H. Park.[2] Secondly, the differential equations are manipulated into standard form to permit numerical solution. In order to assure a unique continuous solution, the form of the equations must be such that they can be put into the standard form. Thirdly, an existing digital computer library routine is used to solve the system of equations. The routine used is based on the fourth order Runge-Kutta formulas. In conjunction with the differential equation solving routine, a master and a function routine must be supplied. The master and the function routines depend on the particular problem to be solved and on the amount of data the computer is required to print out.

The greater simplicity of the technique of this thesis is evident, both in theory and application. The presently accepted standard techniques for solving the machine equations use the simplifying assumptions of constant speed and sinusoidal armature voltage and current variables in part of the equations. The constant speed assumption makes it possible to consider the equations in two separate groups. One group, called the electrical equations, is a system of linear differential equations containing only electrical variables. The other group, called the mechanical equations, or the swing equations, is made up of one equation for each machine under consideration. The swing equations are differential equations non-linear in speed, field current, and armature current. The further assumption of sinusoidal steady state armature voltages and currents and a change of armature variables to the familiar direct and quadrature components reduce the electrical equations to an algebraic system of equations. Under steady state conditions, if the power, power factor, speed, and armature voltages are specified, the algebraic system of electrical equations can be used to determine armature and field currents. Neglecting losses, the armature and field currents determined from the electrical equations satisfy the mechanical equations. But, under fault conditions, the electrical and mechanical variables are in a transient state and the simplified equations cannot be expected to yield a solution which correlates with a solution of the differential equations. To compensate for the difference in differential equation solution and a steady state solution, the standard solution proceeds as follows:

1. Initial armature currents, field currents, and angular positions are determined from the steady state electrical equations.

2. New reactances, called transient reactances, are used with the results of the steady state solution, previously obtained, to determine voltages and voltage angular positions. These voltages are called the voltages behind the transient reactances.

3. It is assumed that the voltages behind the transient reactances and the transient reactances of the machines do not change with the changes in speed and current to be considered.

4. The voltages behind the transient reactances and the transient reactances do change as fault and the fault removal change the network parameters, however.

5. Using initial conditions defined by previous steady state calculations and by the type of fault, the swing equations are solved by common numerical methods. For each increment of time assigned in the numerical process, corresponding increments of changes in phase angles for the voltages behind the transient reactances are determined. These changes in angles are incorporated into the electrical power terms in the swing equations when the next time increment is applied to the solution of the swing equations.

6. The solution is continued by adding increments of time until sufficient information is available on the phase angles of the voltages behind the transient reactances to determine if the system is stable* or unstable.

7. Since the electrical power is calculated in terms of transient reactance, voltage behind the transient reactance, and the phase angle of the voltage behind the transient reactance, different power formulas must be used for different states: steady state, fault on, fault removed.

8. If the changes in field currents are to be considered[8], the voltages behind the transient reactances must be modified for each increment of time considered.

The synchronous machine equations are solved directly in this thesis with only the first group of simplifying assumptions. Thus, the solution under transient conditions is obtained without formulation in terms of the so-called "transient reactance" and "field variation" effects and so

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*A system is considered unstable if any of the synchronous machines fall out of step with the system.

without defining processes in which these reactances are used to give results correlating with observations under transient conditions.

A common power system stability problem is that of determining how fast the circuit breaker must operate and remove a faulted line for the system to remain stable. Conventionally, the critical interval is obtained by a criterion which determines a critical rotor displacement as the variable. In order to determine the critical time, the numerical solution, by standard methods, must proceed up to the critical rotor displacement angle, although it need not be carried out until instability is indicated. An alternative method, illustrated by Kimbark[8], uses pre-calculated swing curves. The use of pre-calculated swing curves requires the steady state, constant speed, constant field current assumptions. The pre-calculated swing curves cannot be used for some limiting cases, and the pre-calculated swing curves do not represent conditions after the fault has been removed. The techniques of this thesis do not require the assumptions of either the criterion or the pre-calculated swing curve techniques. Further, the variables determined by the general solution presented here include both time and angle variables for the fault interval and for the interval after the fault has been removed. The switching time and the type of fault are controlled by the master routine in the computer program.

The results reported here of a typical problem indicate that the field current variation, commonly given considerable attention[8], is insignificantly small even under extreme conditions, whereas the commonly neglected change in speed is really a more dominant variation. The results of the numerical solution for the differential equations indicate that the direct and quadrature currents of Blondel's analysis or Park's transformation have characteristics much different under transient conditions than that usually assumed in standard steady state analysis and that usually assumed in modified steady state analysis applied to the power system stability problem.

II. THE ROUND ROTOR SYNCHRONOUS MACHINE EQUATIONS AND THE TRANSFORMATIONS OF THE VARIABLES

In this chapter the synchronous machine equations, obtained with only the first group of simplifying assumptions, are listed and all coefficients are defined. Next, the transformations are listed and the equations in the new variables are given.

The round rotor synchronous machine variables that are observed by means of meter indications are: the three-phase voltages of the machine armature, the three-phase currents of the machine armature, the field voltage and current, and the speed and torque at the shaft. With the usual simplifying assumptions that the interrelationship of the variables is represented with sufficient accuracy by considering only the fundamental harmonics and by considering the resistances and the inductances as constants, the mathematical interrelations of the variables are:

$$(2.1)a \qquad \begin{bmatrix} \mathcal{V}_s(t) \\ v_f(t) \end{bmatrix} = \begin{bmatrix} \mathcal{R}_{ss} + \frac{d}{dt}\mathcal{L}_{ss} & \frac{d}{dt}\mathcal{L}_{sr}(\phi) \\ \frac{d}{dt}\mathcal{L}_{rs}(\phi) & R_{ff} + \frac{d}{dt}L_{ff} \end{bmatrix} \begin{bmatrix} \mathcal{I}_s(t) \\ i_f(t) \end{bmatrix}$$

$$(2.1)b \qquad T(t) = -\frac{1}{2}\begin{bmatrix} \mathcal{I}_s'(t) & i_f(t) \end{bmatrix} \frac{\partial}{\partial \phi} \begin{bmatrix} \mathcal{L}_{ss} & \mathcal{L}_{sr}(\phi) \\ \mathcal{L}_{rs}(\phi) & L_f \end{bmatrix} \begin{bmatrix} \mathcal{I}_s(t) \\ i_f(t) \end{bmatrix} + (B + J\frac{d}{dt})\dot{\phi}$$

where $\mathcal{V}_s(t)$ and $\mathcal{I}_s(t)$ are column matrices representing the line to neutral voltages and currents of the three stator (armature) phases. Specifically,

$$\mathcal{V}_s(t) \begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix}, \qquad \mathcal{I}_s(t) = \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix}$$

The functions $v_f(t)$ and $i_f(t)$ represent the rotor (field) voltage and current respectively. The functions $T(t)$ and $\phi(t)$ are the shaft torque and position respectively.

The coefficient matrices $\mathcal{R}_{ss}$ and $\mathcal{L}_{ss}$ are of the form

$$\mathcal{R}_{ss} = \begin{bmatrix} R_A & 0 & 0 \\ 0 & R_B & 0 \\ 0 & 0 & R_C \end{bmatrix} = \begin{bmatrix} R_A & 0 & 0 \\ 0 & R_A & 0 \\ 0 & 0 & R_A \end{bmatrix}$$

$$\mathcal{L}_{ss} = \begin{bmatrix} L_{AA} & L_{BA} & L_{CA} \\ L_{AB} & L_{BB} & L_{CB} \\ L_{AC} & L_{BC} & L_{CC} \end{bmatrix} = \begin{bmatrix} L_{AA} & L_{AB} & L_{AB} \\ L_{AB} & L_{AA} & L_{AB} \\ L_{AB} & L_{AB} & L_{AA} \end{bmatrix}$$

where L_{AA}, L_{BB}, and L_{CC} are the self inductance constants for the respective stator phases and L_{AB}, L_{BA}, L_{AC}, L_{CA}, L_{BC}, and L_{CB} are the mutual inductance constants representing the coupling between stator phases.

The constants R_{ff} and L_{ff} represent the resistance and inductance coefficients of the rotor.

The coefficient matrix, $\mathcal{L}_{sr}(\phi)$, is of the form

$$\mathcal{L}_{sr}(\phi) = \mathcal{L}'_{rs}(\phi) = \begin{bmatrix} L_{sr} \cos\theta \\ L_{sr} \cos(\theta - 120^\circ) \\ L_{sr} \cos(\theta - 240^\circ) \end{bmatrix}$$

with $\theta = \frac{p}{2} \phi$

and p representing the number of poles of the machine and θ representing the displacement of the rotor in electrical angle measure.

The coefficients B and J represent the mechanical damping constant and polar moment of inertia respectively.

A change of stator variables greatly simplifies the interrelationship of (2.1). Variables closely related to those to be used here were developed by A. Blondel[1] in order to more accurately consider saliency effects of salient pole synchronous machines. R. H. Park[2] illustrated that the new variables can be considered as a set of variables resulting from a mathematical transformation of variables. Park's transformations on the stator variables are:

$$(2.2) \quad \begin{bmatrix} v_o(t) \\ v_d(t) \\ v_q(t) \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 2\cos\theta & 2\cos(\theta-120^\circ) & 2\cos(\theta+120^\circ) \\ -2\cos\theta & -2\sin(\theta-120^\circ) & -2\sin(\theta+120^\circ) \end{bmatrix} \begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix}$$

$$\begin{bmatrix} i_o(t) \\ i_d(t) \\ i_q(t) \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 2\cos\theta & 2\cos(\theta-120^\circ) & 2\cos(\theta+120^\circ) \\ -2\sin\theta & -2\sin(\theta-120^\circ) & -2\sin(\theta+120^\circ) \end{bmatrix} \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix}$$

It is a formidable task to apply the transformations of (2.2) to the equations of (2.1). This application of the transformations would be necessary, however, in order to rigorously define the coefficients in the resulting equations in terms of the observed relationships (2.1).

Koenig[3] has shown these transformations (2.2) to be the product of three non-singular transformations. The three component transformations applied to the stator voltage variables are:

$$(2.3)\quad \begin{bmatrix} v_s^o(t) \\ v_s^+(t) \\ v_s^-(t) \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{j2\pi/3} & e^{-j2\pi/3} \\ 1 & e^{-j2\pi/3} & e^{j2\pi/3} \end{bmatrix} \begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix}$$

$$(2.4)\quad \begin{bmatrix} v_s^o(t) \\ v_s^b(t) \\ \hat{v}_s^b(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-j\theta} & 0 \\ 0 & 0 & e^{j\theta} \end{bmatrix} \begin{bmatrix} v_s^o(t) \\ v_s^+(t) \\ v_s^-(t) \end{bmatrix}$$

where $\hat{v}_s^b(t)$ is the conjugate of $v_s^b(t)$.

$$(2.5)\quad \begin{bmatrix} v_s^o(t) \\ v_d(t) \\ v_q(t) \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & j & -1 \end{bmatrix} \begin{bmatrix} v_s^o(t) \\ v_s^b(t) \\ \hat{v}_s^b(t) \end{bmatrix}$$

The transformations defined in (2.3), (2.4), and (2.5) utilize symmetries and inherent characteristics of the coefficient matrices and with theorems developed by Koenig and Blackwell[4], these transformations are relatively easy to apply.

The transformation defined in (2.3) is the familiar <u>symmetrical component</u> transformation. Application of the transformation of (2.3) followed by the transformation in (2.4) defines the <u>backward sequence</u> variables. Application of the transformation in (2.5) to the backward sequence variables separates the real parts of the backward sequence complex variables from the imaginary parts of the backward sequence complex variables. The new armature real variables resulting from the three transformations are essentially the same as the direct-axis and quadrature-axis components of armature voltages.

Even though salient pole synchronous machines, for which these new variables were developed, are not considered in this investigation, the

new variables are in a convenient form for numerical solution of the differential equations.

After application of the transformations defined in (2.3), (2.4), and (2.5) for the voltage variables of (2.1) and the same transformations are applied to the current variables of (2.1), the machine equations become:

$$(2.6)\text{a} \quad \begin{bmatrix} v_s^o(t) \\ v_d(t) \\ v_q(t) \\ v_f(t) \end{bmatrix} = \begin{bmatrix} R_s^o+L_s^o \frac{d}{dt} & 0 & 0 & 0 \\ 0 & R_s^+ + L_s^+ \frac{d}{dt} & \dot{\theta} L_s^+ & \sqrt{2} L_{sr} \frac{d}{dt} \\ 0 & -\dot{\theta} L_s^+ & R_s^+ + L_s^+ \frac{d}{dt} & -\dot{\theta}\sqrt{2} L_{sr} \\ 0 & \sqrt{2} L_{sr} \frac{d}{dt} & 0 & R_f + L_f \frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_s^o(t) \\ i_d(t) \\ i_q(t) \\ i_f(t) \end{bmatrix}$$

$$(2.6)\text{b} \qquad T(t) = \frac{\sqrt{2}}{2} L_{sr} P i_q(t)\, i_f(t) \quad + \quad (B + J \frac{d}{dt})\, \dot{\phi}$$

where

$$R_s^o = R_{AA}$$

$$L_s^o = L_{AA} + 2L_{AB}$$

$$R_s^+ = R_{AA}$$

$$L_s^+ = L_{AA} - L_{AB}$$

The equations (2.6)a and (2.6)b make up the complete set of non-linear differential equations of the synchronous machine. The only simplifying assumptions required to obtain (2.6) were those required in the initial formulation of (2.1): only the fundamental frequency terms need to be considered, resistances and inductances can be represented by constants. In a numerical solution of these non-linear equations, the first question of concern is the existence of a solution. E. L. Ince[5] gives an existence theorem which applies to the problem considered here. Application of the theorem is given in Appendix A.

III. TWO POWER SYSTEM STABILITY PROBLEMS

The problems presented in this thesis serve the purposes of: illustrating the thesis technique for two classes of power system stability problems, comparing the thesis technique with the standard technique, comparing the results of the two methods. The comparison of the results is on the basis of the type and the amount of information available from the solutions and a quantitative comparison of the variables when agreement or discrepancy is evident.

This chapter states the two problems considered here. The assignment of the various machine parameters is discussed. The inductance, resistance, and inertia parameters are chosen as typical values for a particular class of machine rather than values for a particular machine. These general parameters were chosen to allow freedom in emphasizing the percent change in some variables by taking limiting values of some of the parameters for the class of machine considered. For example, in the first class of problem, the inertia constant was taken close to the lower limit for a turbogenerator and a severe short circuit was considered. Thus, the electromechanical transient components of all variables are emphasized.

The problems contained in this thesis are illustrative of two classes of power system stability problems. With minor changes in programming, the techniques illustrated here can be used in the study of a large variety of problems in either class.

3.1 THE FIRST CLASS OF PROBLEMS

To define the first class of problem, the term "infinite bus" must first be defined. "Infinite bus" is the term used to describe a set of terminals to which a machine is to be connected if that set of terminals

has the characteristic that the voltages of the terminals are independent of the currents over the range to be considered. The "infinite bus" concept is commonly used in considering the operation of an individual machine which is part of an interconnected system when the power capacity of the system is many times that of the individual machine.

The first class of problem considered here is the stability problem of an individual machine relative to an "infinite bus." Important examples of this class of problem include:

1. The determination of the circuit breaker operating time that would maintain machine stability if the breaker action removes the faulted section of the system but does not disconnect the machine from the system or bus.
2. The determination of the reclosure time of the circuit breaker, that would assure machine stability, after a temporary local fault has removed the machine from the bus.
3. The determination of the stability characteristics of an individual machine for local and less severe sustained faults.

When solved by the thesis technique, the three problems listed in this first group are mathematically identical. Only the engineering interpretations are different. The different situations are represented by specifying different variables or by changing constants at specified switching times. The first of the three problems is used in this thesis to illustrate the solution technique.

PROBLEM: Consider a generator connected to a power system where the power system can be represented as an "infinite bus." Under pre-fault conditions, the generator delivers 0.5 p.u. power at 0.85 power factor lag to the "infinite bus," at a bus voltage level of 1 p.u. volts. Consider a three-phase symmetric short circuit system fault near the generator such that during the fault the generator action can be described in terms of a short circuit at the bus. When the circuit breakers remove the faulted section of the system, the bus returns immediately to "infinite bus" characteristics. Determine how rapidly the breaker must operate and remove the faulted section in order that the generator remain stable.

The system parameters that must be specified to complete the statement of the problem are the coefficients in (2.6)a and (2.6)b.

In a typical steam turbine drive, the power output of the turbine is taken as a constant over the switching interval and so (2.6)b is usually modified by multiplying both sides by $\dot{\phi}$. If the B term is neglected and M is defined as

$$M = \dot{\phi}\, J$$

then (2.6)b becomes

$$P_{in} = \dot{\theta}\sqrt{2}\, L_{sr}\, i_f(t)\, i_q(t) + M\,\ddot{\theta} \tag{3.1}$$

where P_{in} is the power supplied by the prime mover and M is referred to as the momentum of the machine.

Rather than assign values from a particular machine, typical per unit values are assigned for a particular class of machine. Ranges for these typical per unit values as obtained from the literature[8,9], are listed in Table 3.1 along with the values used in the solution.

For a symmetric fault, the zero sequence voltage $v_s^o(t)$ and zero sequence current $i_s^o(t)$ are zero, so R_s^o and L_s^o numbers need not be assigned.

It can also be shown that parameters of transformers and lines which connect the generator to the bus can be included in R_s^+ and L_s^+ if the

transformers and the lines are represented as series inductance and resistance and if $v_d(t)$ and $v_q(t)$ are the bus voltage components in place of the generator terminal voltage components[4]. Further, from the form of the equations (2.6)a it is clear that R_s^+ and ωL_s^+ are the usual alternating current resistance per phase and direct axis synchronous reactance per phase respectively.

Typical per unit resistance for round rotor turbogenerators has a range of 0.003 to 0.008 p.u. A value of 0.005 is used and it is assumed to be the generator, transformer, and line resistances combined.

Typical per unit direct axis synchronous reactance for round rotor turbogenerators at 60 cycles, has a range of 0.95 to 1.45 p.u. A value of 1.2 is used and it is assumed that this value includes generator, transformer, and line reactances.

Following the conventional assignment, $\dot{\theta}\sqrt{2}L_{sr}\, i_f(t)$ is 1 p.u. for $\dot{\theta} = 377$ and $i_f(t) = 1$ p.u. field current. Thus, $\omega\sqrt{2}L_{sr} = 1$ p.u. Again, by convention, 1 p.u. $i_f(t)$ is obtained from 1 p.u. $v_f(t)$ and 1 p.u. R_f.

The range of the L_f/R_f for round rotor turbogenerator is 2.8 to 9.2. A value of 6 is used, then L_f is 6 p.u.

The momentum term is usually given in terms of H constants. H is related to the momentum by

$$M = \frac{GH}{\pi f}$$

where

G is the rating of the machine in megavolt-amperes

f is the frequency in cycles per second

M is the momentum in megawatts/electrical radian

For a round rotor turbogenerator, H has a range of 2.8 to 9.5 including the prime mover. The type of prime mover largely determines the particular value in this range. A value of H = 3 is used. The low inertia constant was chosen in order to emphasize the mechanical transient and make the stability problem more critical.

3.2 SOME REFINEMENTS ON THE ROTOR-STATOR COUPLING TERMS

Kimbark[8] has developed an equation for the purpose of considering the percent change in field as resulting from the combined effects of transients and regulator action. The change in field Kimbark represented in terms of the field circuit time constant and components of armature voltages. The regulator action is simulated by incrementing the armature voltage component that is due to field current. In a numerical solution of (2.6)a and (3.1) on the digital computer, the master routine can readily simulate regulator action by incrementing the field current variable.

In order to obtain more detailed and more accurate information on the field decrement, refinements in the rotor-stator coupling terms are considered as outlined in the following paragraphs.

The coefficients in equations (2.6)a and (3.1) are evaluated at the particular operating conditions under consideration. However, if a range of the variables is being considered, it is found that these coefficients vary. If the field decrement is to be studied, a more accurate representation of the rotor-stator (field-armature) relations is needed. The representation must take into account the particular operating conditions and in addition must take into account the effect of incremental changes about the operating point. Consider the magnetization curve of a synchronous machine as represented in Fig. (3.1).

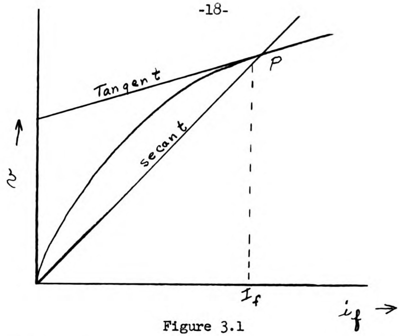


Figure 3.1

At any particular operating point, $i_f = I_f$, the voltage-current relation is given by the slope of the secant line through P. Thus, the $\dot{\theta}\sqrt{2}L_{sr}$ terms in (2.6)a and (3.1) can be assigned a value equal to the slope of this secant line. For incremental changes about I_f, however, the operating point moves along the tangent line. This refinement is incorporated into the term

$$v(t) = \dot{\theta}\sqrt{2}L_{sr}\, i_f(t)$$

by defining

$$i_f(t) = I_f + i_{f1}(t)$$

where

I_f is a constant or reference value of $i_f(t)$

i_{f1} is the time varying component of $i_f(t)$

If $\dot{\theta}\sqrt{2}L_{sr}$ is the slope of the secant line in Fig. (3.1) and $\dot{\theta}K$ is the slope of the tangent line at $i_f(t) = I_f$, then

$$v(t) = \dot{\theta}\sqrt{2}L_{sr}\, I_f + \dot{\theta}K\, i_{f1}(t)$$

All of the terms in (2.6)a and (3.1) which involve field and armature

coupling are appropriately modified. The machine equations become

$$(3.2)\text{a} \quad \begin{bmatrix} v_d(t) \\ v_q(t) + \dot{\theta}\sqrt{2}L_{sr}\, I_f \\ v_f - R_f\, I_f \end{bmatrix} = \begin{bmatrix} R_s^+ + L_s^+ \frac{d}{dt} & \dot{\theta}\, L_s^{+\delta} & K \frac{d}{dt} \\ -\dot{\theta}\, L_s^+ & R_s^+ + L_s^+ \frac{d}{dt} & -\dot{\theta}\, K \\ K \frac{d}{dt} & 0 & R_f + L_f \frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_d(t) \\ i_q(t) \\ i_{fl}(t) \end{bmatrix}$$

$$(3.2)\text{b} \qquad P_{in} = \dot{\theta}\sqrt{2}L_{sr}\, I_f\, i_q(t) + \dot{\theta}K\, i_{fl}(t)\, i_q(t) + M \frac{d^2}{dt^2}\theta$$

The value of the slope of the tangent line changes with a change in operating point and a given slope would be in error for a large increment of change in current. The use of the tangent slope is an improvement over the use of the secant slope if $i_{fl}(t)$ remains small.

A typical value of slope $\dot{\theta}K$ for a synchronous machine at a typical operating point is about 0.6 or 0.7 p.u. with $\dot{\theta} = 377$. This value of $\dot{\theta}K$ as compared with typical values of 1.2 to 1.8 for $\dot{\theta}\sqrt{2}L_{sr}$ shows the significance of the refinement on the field decrement effect. All terms involving field and armature coupling are appropriately modified before the problem is programmed.

3.3 THE SECOND CLASS OF PROBLEMS

The second class of problem considered here is the stability study for two interconnected machines. The significance of this class of problem lies in the fact that this problem represents an extension of the technique of the first problem toward a multi-machine system, and in the fact that with certain assumptions two interconnected systems fall into this class.

Examples of problems of this class include:

1. Two synchronous machines are connected by a transmission line. One machine is operating as a generator and the other as a

motor. For a sudden increase in load on the motor, determine whether the machines will fall out of step or not.

2. Two power systems, each represented by an equivalent synchronous machine, are interconnected by a transmission line. For a fault on the transmission line which is cleared in a specified time, determine whether the systems remain in synchronism or not.

3. Two power systems, each represented by an equivalent synchronous machine, are interconnected by a transmission line. System A supplies power over the line, and system B receives power from the line. For a sudden increase of demand in system B, or for loss of a generating unit due to local fault, system B requirements become $P + \Delta P$. Determine whether the two systems will remain in sychronism or not.

Under the assumptions that permit a system to be represented by an equivalent synchronous machine, the three problems are mathematically identical. The first of the three is used in this thesis to illustrate the technique.

PROBLEM: For the second problem of this thesis, consider a synchronous generator supplying 0.5 p.u. power over a transmission line to a synchronous motor receiving 0.5 p.u. power. For a sudden change in load on the motor from 0.5 p.u. to 0.6 p.u., will the machines stay in synchronism? Repeat for the case where the motor load changes from 0.5 p.u. to 0.8 p.u. power. Initially, let the voltage at the motor terminals be 1 p.u. volts and the motor power factor be 0.85 lag.

The machine parameters are assigned typical per unit values as in the first problem. The direct axis synchronous reactance of the motor is taken as 0.8 p.u. and the H constant of the motor is taken as 2. Let the motor phase resistance be 0.005 p.u. and let the $\dot{\theta}\sqrt{2}L_{sr}$ be 1 p.u. for both the motor and the generator. Assign the generator plus the transmission line the direct axis synchronous reactance of 1.2 p.u. per phase and a phase resistance of 0.005 p.u. Let the generator H constant be 6.

IV. SOLUTION OF A STABILITY PROBLEM FOR A SYNCHRONOUS MACHINE ON AN INFINITE BUS

This chapter presents the standard and the thesis techniques in general and for the particular problem. Results are presented and compared. Methods are compared and the extension of methods to other problems and other areas is discussed.

4.1 THE STEADY STATE INITIAL CONDITION CALCULATIONS

The pre-fault conditions are the constant speed, steady state, and constant field conditions under which equation (2.6)a reduces to

$$(4.1)\quad \begin{bmatrix} v_d \\ v_q \\ v_f \end{bmatrix} = \begin{bmatrix} R_s^+ & \omega L_s^+ & 0 \\ -\omega L_s^+ & R_s^+ & -\omega\sqrt{2}L_{sr} \\ 0 & 0 & R_f \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_f \end{bmatrix}$$

The pre-fault conditions given in the statement of the first problem are:

$|V|$ = 1 p.u. voltage at the bus.

P = 0.5 p.u. power at 0.85 power factor lag.

ωL_s^+ = 1.2 p.u. reactance

R_s^+ = 0.005 p.u. resistance

$|I|$ = .5882 p.u. current

In addition to equation (4.1), the following equations apply for steady state sinusoidal conditions:

$$v_d(t) = +|V| \sin \delta \qquad\qquad i_d(t) = -|I| \sin(\delta + \beta)$$

$$v_q(t) = -|V| \cos \delta \qquad\qquad i_q(t) = |I| \cos(\delta + \beta)$$

where

β is the power factor angle

δ is the phase angle between the bus voltage, V, and the field excitation voltage,

$$E_f = \omega \sqrt{2} L_{sr} I_f$$

Usually, this steady state problem is solved neglecting resistance and the various quantities are pictured on a vector diagram as in Fig. (4.1).

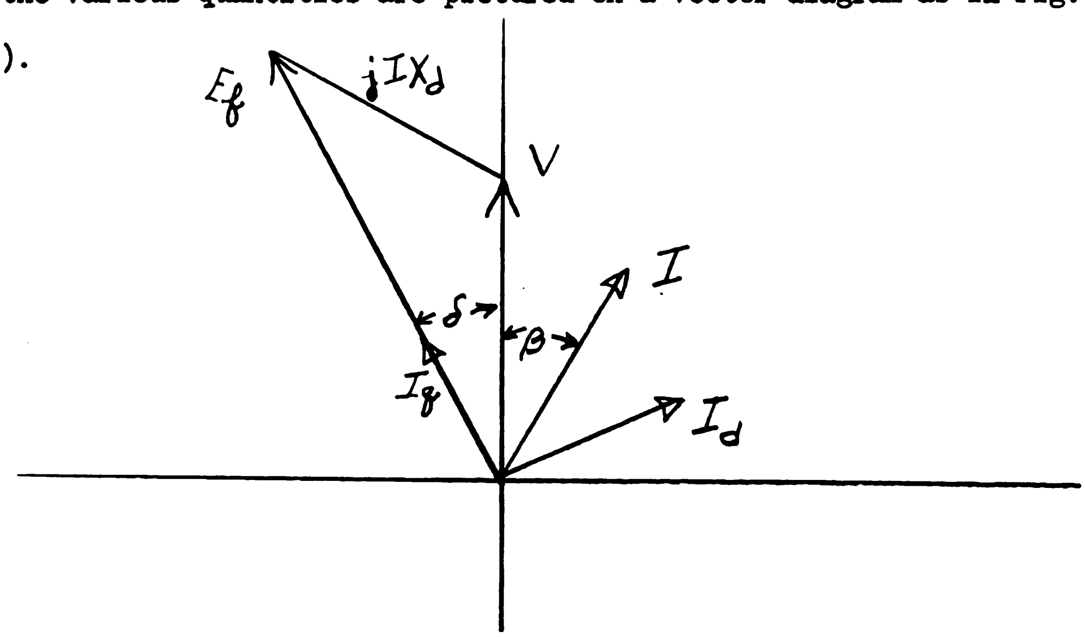


Figure 4.1

Solving (4.1), the results are:

E_f = 1.5 $\underline{/90 + 23.6^\circ}$ p.u. volts

δ = 23.6°

I_d = -0.496 p.u. current

I_q = 0.335 p.u. current

I_f = 1.5 p.u. since $E_f = I_f$ in p.u.

v_d = 0.40 p.u. volts

v_q = -0.916 p.u. volts

Further, since the pre-fault conditions are considered as constant synchronous speed, the initial values of θ and $\dot{\theta}$ are:

$$\dot{\theta} = 377 \text{ radians per second}$$

$$\theta = \omega t_o + \delta + \pi/2 = \delta + \pi/2 = 0.4116 \text{ radians}$$

In steady state terms, the angle δ, sometimes called the torque angle, is the angle represented in Fig. (4.1) as the phase angle between the bus voltage and the voltage generated in the armature due to the field current. In equations (2.1), and in the subsequent transformations, δ is the angle of deviation of the rotor from the no-load synchronous speed position. A synchronous machine is unstable if δ increases without limit. If a system is stable, the variation of δ is generally an oscillatory function of time. If a machine is stable for the first swing, it is classified as stable, for by the end of the first swing, regulator action and prime mover governor action will have begun and these actions tend to stabilize the machine.

It is generally accurate to assume that prime mover governor action does not take place until after the first swing of the oscillatory δ and does not enter into the stability calculations. Therefore, it is accurate to assume that the prime mover power input remains constant during the first swing. Some authors[7,8] have developed formulas to take into consideration generator field control by regulator action. The regulator action is not usually initiated before the fault has been on for 0.2 or 0.3 seconds, however.

4.2 NUMERICAL STUDY OF THE STABILITY PROBLEM OF A SYNCHRONOUS MACHINE ON AN "INFINITE BUS"

The mathematical statement of the problem to be solved here is given by:

(1) equations (3.2)a and (3.2)b,

(2) the initial conditions defined in section 4.1,

(3) the fault conditions,

and (4) the switching conditions.

The normal form of the system equations given in (3.2)a and (3.2)b as required for a numerical solution by the Runge-Kutta method is:

$$
(4.2) \quad
\begin{bmatrix}
\frac{d}{dt} i_d(t) \\[2em]
\frac{d}{dt} i_q(t) \\[1em]
\frac{d}{dt} i_{f1}(t) \\[1em]
\frac{d}{dt} \theta(t) \\[1em]
\frac{d}{dt} \dot{\theta}(t)
\end{bmatrix}
=
\begin{bmatrix}
\left[v_d(t) - R_s^+ i_d(t) - \dot{\theta} L_s^+ i_q(t)\right] \frac{1}{L_s^+} - \frac{K^2}{L_s^{+2}\left(L_f - \frac{K^2}{L_s^+}\right)} \left[v_d(t) - R_s^+ i_d(t) - \dot{\theta} L_s^+ i_q(t)\right] \\[1em]
\left[v_q(t) - R_s^+ i_q(t) - \dot{\theta}\sqrt{2} L_{sr} I_f + \dot{\theta} L_s^+ i_d(t) + \dot{\theta} K i_{f1}(t)\right] \frac{1}{L_s^+} \\[1em]
- \left[v_d(t) - R_s^+ i_d(t) - \dot{\theta} L_s^+ i_q(t)\right] \frac{K}{L_s^+\left[L_f - \frac{K^2}{L_s^+}\right]} - R_f i_{f1}(t)/L_f \\[1em]
\dot{\theta} \\[1em]
\left[P_{in} - \dot{\theta}\left(\sqrt{2} L_{sr} I_f + K i_{f1}(t)\right) i_q(t)\right] \frac{1}{M}
\end{bmatrix}
$$

where, from the transformations defined by equations (2.3), (2.4), and (2.5)

$$v_d(t) = |V| \sin \delta$$

$$v_q(t) = - |V| \cos \delta$$

and

$$\theta(t) = \omega t + \delta + \pi/2$$

The flow diagram for the computer program which was used to solve these equations is shown in Fig. (4.2).

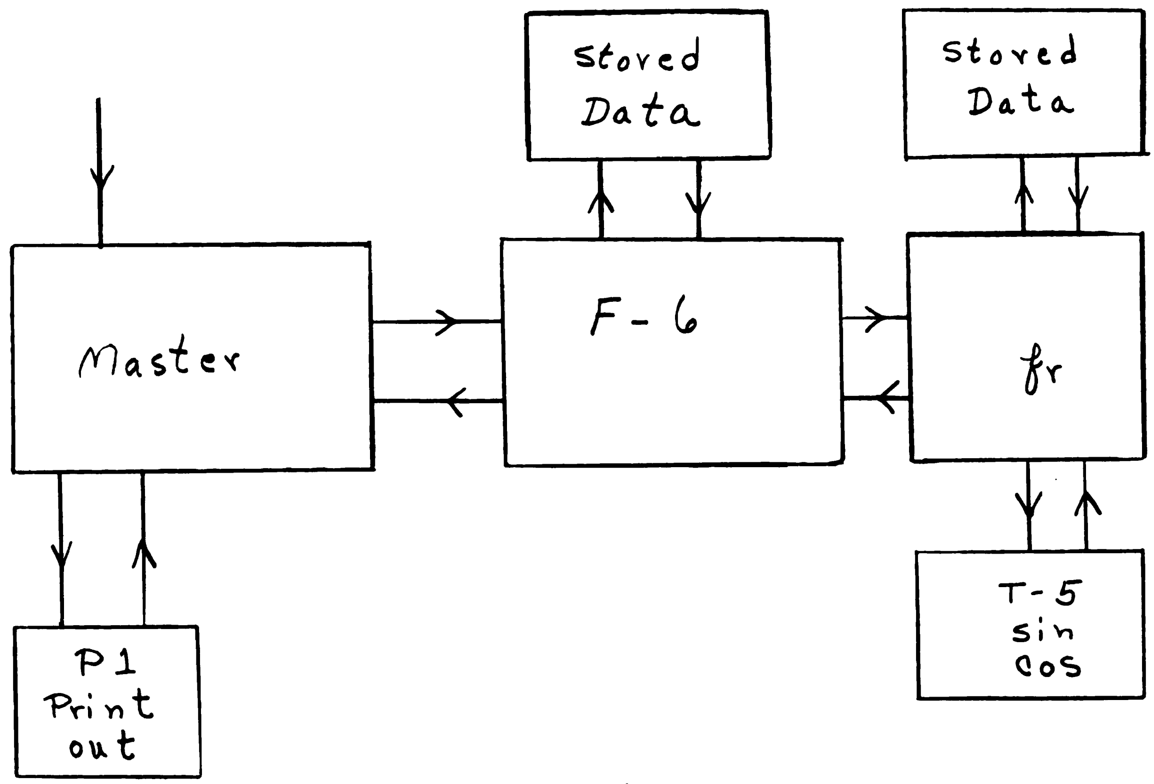


Figure 4.2

The digital computer library routine, F-6, was used to solve the differential equations. The F-6 routine was chosen because it uses the fourth order Runge-Kutta[10] formulas. The truncation error for the fourth order Runge-Kutta formulas is of the order of $(\Delta t)^5$, where Δt is the increment of time used for each step. Also, the F-6 routine limits the amount of error by an automatic control of the size of the increment. Both of these, the fourth order Runge-Kutta formulas and the increment control, tend to make the program slow, but both are valuable assets when the exact character of the variables being investigated is unknown.

The differential equation solving routine, the master routine, and the function routine are described in greater detail in Appendix B.

With the program described in Appendix B available, the solution to the first problem is obtained simply. Merely supply the program with the specified initial conditions, the parameters for the particular machine, and master routine counters.

The scaled initial values, scaled coefficients, and constants used in the problem are listed.

The numbers in memory location 3 through 8 as required by F-6.

In 3 the integer 30 indicates the location of the first of the sequence of initial conditions.

In 4 the integer 36 indicates the sum of entry 3 and n, the number of equations in the system.

In 5 the integer 13 indicates the scale factor on the f_r, 2^{-13}.

In 6 the integer 10 indicates the time increment, 2^{-10} sec.

In 7 the integer 36. The y_r will have their error held to about 1 in the 36 binary bit.

In 8 the integer 160 gives the memory location for the entry into the f_r routine.

In 10 the integer 100 gives the memory location of the first of a sequence of scaled coefficients and constants.

The initial values of all the variables are scaled by 2^{-13} and read in as follows:

Location	Variable	Scaled Value
30	t	0
31	$\mathcal{J} = \dot{\theta}$	0.04602
32	θ	0.0002420
33	$i_{f1}(t)$	0
34	$i_d(t)$	-0.000060546
35	$i_q(t)$	0.0000408825

The coefficients and constants for the f_r routine:

Location	Quantity	Scale	Scaled Value
100	$1/\sqrt{2}L_{sr}$	2^{-9}	0.73633
101	M	2^{0}	0.01592
102	K/L_s^+	2^{0}	0.500
103	$1/L_s^+$	2^{-9}	0.61360
104	$1/L_f$	2^{0}	0.1666
105	P/M	2^{-12}	0.0038540
106	V^*	2^{-9}	0
107	I_f	2^{-10}	0.0014648
108	K	2^{0}	0.0015915
109	V	2^{-12}	0.00024414
110	Work space		
111	Work space		
112	Work space		
113	Work space		
114	δ	2^{-3}	program
115	$\pi/2$	2^{-13}	0.000191747
116	$\delta/4\pi$	2^{0}	0.63662

For master routine, read the following counters to control read out and switching.

If the integer k is placed in the 29th word of the master routine, read out will occur after each (k+1)st calculation.

If the integer p is placed in the 36th word of the master, switching or fault removal occurs after p read outs.

If the integer q is placed in the 38th word of the master routine,

the program completes q read outs before stopping.

The master reads out t, $\dot{\theta}$, θ, $i_{f1}(t)$, δ, $i_d(t)$, $i_q(t)$. However, if zero is placed in the 33rd word of the master routine, only t, δ, and $\dot{\theta}$ will be read out.

The pre-set parameter in location 3 through 8 must be read in before F-6 is read in. All the other information can be placed at the end of the program. At the end of the read in, transfer control to the left side of the first word of the master routine and the computer gives the results. The critical switching time can be determined by altering the counter which is the 36th word of the master routine.

The solution was obtained for various switching times converging on a critical value. Some of the results appear in Graphs 4.1, 4.2, 4.3.

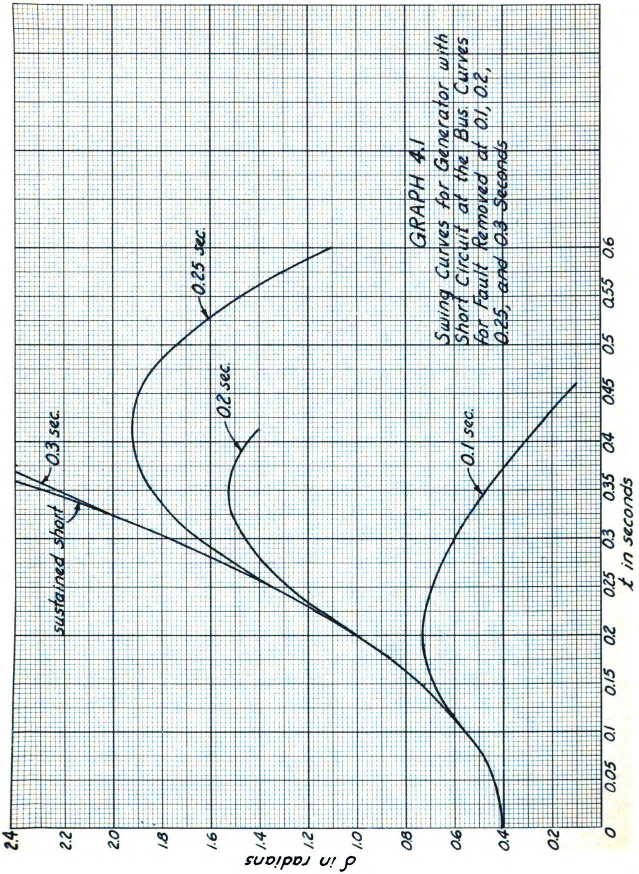
4.3 STANDARD PROCEDURE FOR SOLVING THE STABILITY PROBLEM OF A SYNCHRONOUS MACHINE ON AN "INFINITE BUS"

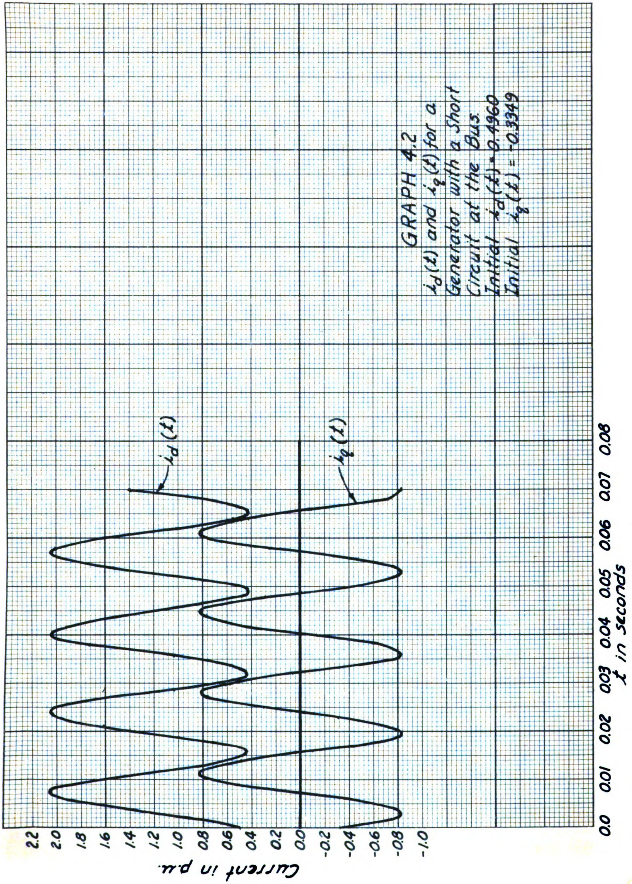
If a numerical solution is obtained for the system equations as in section (4.2), the technique is the same for all single machine stability problems. In sharp contrast, standard methods use a variety of techniques for various types of problems and also utilize additional assumptions and formulations to obtain results. In the interest of completeness and contrast, section (4.3) presents some of these standard techniques.

The standard procedure for the solution of the stability problem of a synchronous machine on an infinite bus is started with the steady state solution of section (4.1). The values and phase positions of the initial i_d, i_q, v_d, v_q, E_f and δ are determined.

Usually, the equation (3.1) is written in the form

$$P = \omega\sqrt{2}L_{sr}\, i_f(t)\, i_q(t) + M\frac{d^2}{dt^2}\phi \qquad (3.1)$$


GRAPH 4.1
Swing Curves for Generator with Short Circuit at the Bus. Curves for Fault Removed at 0.1, 0.2, 0.25, and 0.3 Seconds
0.25 sec.
0.2 sec.
0.3 sec.
0.1 sec.
sustained short
t in seconds
δ in radians
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4

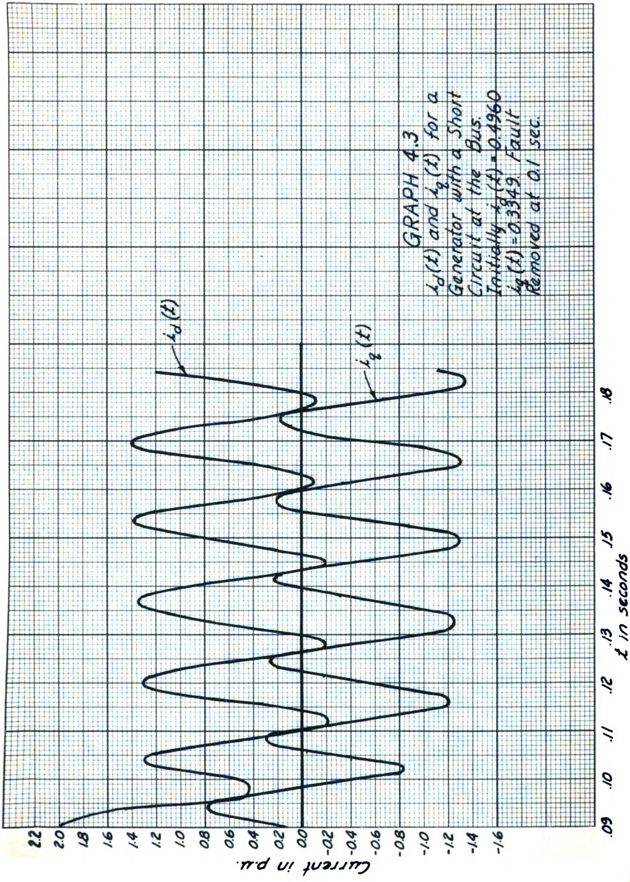


GRAPH 4.2

$i_d(t)$ and $i_q(t)$ for a Generator with a Short Circuit at the Bus.

Initial $i_d(t) = 0.4960$

Initial $i_q(t) = -0.3349$


GRAPH 4.3
$i_d(t)$ and $i_q(t)$ for a Generator with a Short Circuit at the Bus.
Initially $i_d(t) = 0.4960$
$i_q(t) = 0.3349$. Fault Removed at 0.1 sec.
$i_d(t)$
$i_q(t)$
Current in p.u.
t in seconds
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
-1.4
-1.6
.09
.10
.11
.12
.13
.14
.15
.16
.17
.18

By neglecting the resistance, the electrical power term can be written

$$(4.3) \qquad P_{elect} = \omega \sqrt{2} L_{sr}\, i_f(t)\, i_q(t) = \frac{|V||E_f|}{X_d} \sin\delta$$

where

$|V|$ is the p.u. bus voltage

$|E_f|$ is the p.u. voltage component due to the stator direct current field

δ is the torque angle

$X_d = \omega L_s^+$ the direct axis synchronous reactance

Standard methods, in order to use equation (4.1) with transient conditions, contain some modifications for equation (4.1) coefficients and contain some procedures for using equation (4.1). For a given steady state armature voltage, power, and power factor, equation (4.1) can be used to find v_d, v_q, i_d, i_q, as in section (4.1). Then equation (4.1) is modified by replacing ωL_s^+ by new reactances, X_{dT} and X_{qT}, called direct axis transient reactance and quadrature axis transient reactance, respectively. The steady state v_d, v_q, i_d, i_q with the new reactances define a new voltage called the voltage behind the transient reactance. Modified equation (4.1) becomes:

$$(4.4) \qquad \begin{aligned} v_d - R_s^+ i_d - X_{qT} i_q &= E_d \\ v_q - R_s^+ i_q + X_{dT} i_d &= E_q \end{aligned}$$

where

E_d and E_q are the direct and quadrature components respectively of the voltage behind the transient reactance.

Note that the equation (4.4) does not contain an $i_f(t)$ term. The current $i_f(t)$ was assumed constant in order to obtain (4.4).

The quantities in (4.4) are commonly pictured on a vector diagram as in Fig. 4.3.

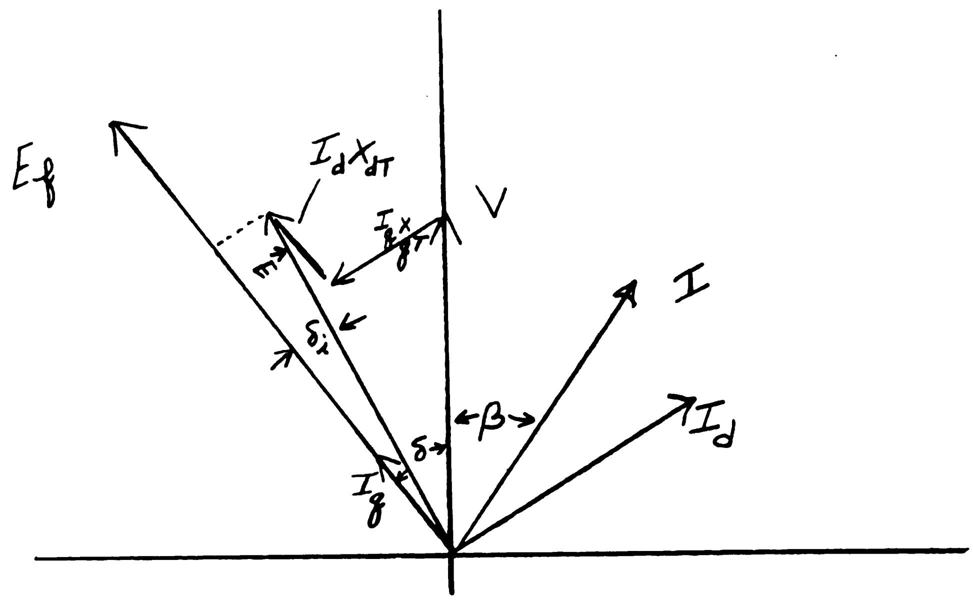


Figure 4.3

The voltage behind the transient reactance, E, defined by

$$E_q = E \cos \delta_i \qquad E_d = E \sin \delta_i$$

$$|E| = |E_q + j\, E_d|$$

is assumed to remain constant during the transient interval if the field decrement is neglected. During this interval if losses are neglected, equation (4.3) becomes

$$\text{(4.5)} \qquad \frac{|V||E_q|}{X_{dT}} \sin \delta = P_{elect}$$

Since $\theta = \omega t + \pi/2 + \delta$, equation (3.1) becomes

$$\text{(4.6)} \qquad P_{in} = \frac{|V||E_q|}{X_{dT}} \sin \delta + M \frac{d^2}{dt^2} \delta$$

Under the assumptions necessary to obtain equation (4.6), a solution to (4.6) will indicate stability or instability. Standard techniques for solving a stability problem use a numerical solution of (4.6) to determine stability characteristics.

For this particular problem of a short circuit at the bus, coefficients in equation (4.6) are obtained using steady state and modified steady state methods. Then, from the time of occurrence of the fault until the circuit breaker removes the fault, equation (4.6) is solved with V set equal to zero. From the time of the circuit breaker switching and continuing to the end of the calculations, V is returned to the infinite bus status. The solution is carried out until the value of the angle δ indicates stability or instability. Various circuit breaker operating times must be considered until the time is found for which the machine operation becomes unstable. This time is called the critical switching time and the corresponding angle is called the critical switching angle. For any switching time less than the critical value, the machine operation would be stable, and for any switching time greater than the critical value, the machine operation would be unstable.

Some important variations on the direct solution of equation (4.6) are now considered.

In order to avoid carrying out the numerical solution of equation (4.6) for various switching times until a critical switching time is determined, a criterion has been developed for determining the critical switching angle. Except in simple cases, the determination of this critical angle requires a trial and error graphical integration approach. Further, even with the critical angle known, the switching time must be determined by numerical solution of equation (4.6) out to the critical angle. For the particular problem of a short circuit at the bus as in this first thesis problem can be solved most readily by the equal area criterion. The

solution is presented in the following paragraphs.

Under the appropriate assumptions, equation (4.3) was obtained.

$$(4.3) \qquad P_{elect} = \frac{V\ E_f}{X_d} \sin \delta$$

A sketch of P_{elect} as a function of δ is given in Fig. (4.4). The horizontal line represents the initial power supplied to the generator, δ_o the initial torque angle, δ_c the switching angle. The angle, δ_c, is the critical switching angle when the cross-hatched area above the initial power line is equal to the cross-hatched area below the initial power line.

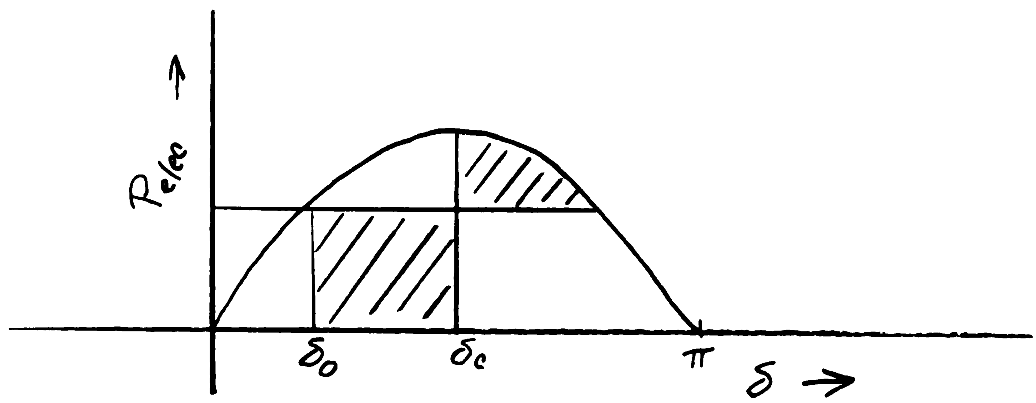


Figure 4.4

For the initial steady state condition

$$|V| = 1 \text{ p.u.}$$

$$|E_f| = 1.5 \text{ p.u.}$$

$$X_d = 1.2$$

$$P_{elect} = 0.5 \text{ p.u.}$$

$$\delta_o = 23.6^\circ = .4116 \text{ radians}$$

$$P_{elect} = \frac{(1)(1.5)}{1.2} \sin \delta = 1.25 \sin \delta$$

for δ_c the critical value

$$\int_{\delta_c}^{180^\circ - 23.6^\circ} 1.25 \sin \delta \, d\,\delta - .5\,(180^\circ - 23.6^\circ - \delta_c) = .5\,(\delta_c - 23.6^\circ)$$

then

$$\cos \delta_c = + 0.014$$
$$\delta_c = 89^\circ$$

Next, with $|V|$ equal to zero for short circuit at the bus, the critical δ_c is determined to occur at 0.28 seconds by numerical solution of equation (4.6).

A second variation from the direct solution of equation (4.6) becomes necessary if a study is to be made of the field decrement effect and/or if a study is to be made of the incrementing of the field by regulator action. Among the assumptions that were necessary to obtain equation (4.6) was the assumption that the field current remains constant. Kimbark[8] and Crary[7] show the development of a formula that alters E_q of equation (4.6) in order to consider these changes in field. The developed formula is:

$$(4.7) \qquad \frac{\Delta E_q}{\Delta t} = \frac{E_f(t + \frac{\Delta t}{2}) - \dot{\theta}\sqrt{2}L_{sr}\, i_f(t)}{T_f + \frac{\dot{\theta}\sqrt{2}L_{sr}\, i_f(t)}{E_q}\left(\frac{\Delta t}{2}\right)}$$

where

$E_f = \omega\sqrt{2}L_{sr}\, I_f$ the field excitation voltage in armature terms

$T_f = \frac{L_f}{R_f}$ the field circuit time constant (armature open)

Δt is the time increment considered

E_f is incremented to simulate regulator action and for a given E_f the formula determines the change in E_q or the change in field called the field decrement. For each increment of time, E_q of equation (4.6) is incremented by the amount indicated by (4.7) if the change in field effects are to be considered.

An example by Kimbark[8] uses equation (4.7) to determine the percent change in field for a machine operating with a short circuit at the bus.

The machine parameters for Kimbark's example placed the machine in the same class as the machine of the thesis problem. The results listed for Kimbark's example showed a decrease in field of about 23% in the first half second of sustained short circuit.

A third deviation from the direct solution of equation (4.6) occurs if the fault is a less severe fault than a symmetric short circuit at the bus. An example of a less severe fault is illustrated by the problem of a generator connected to a bus with a double circuit and the fault taking the form of a symmetric short circuit on one of the two circuits. In such a situation, equation (4.6) would have different E_q/X_{dT} values for pre-fault steady state, for the interval of the fault, and for the interval after the fault has been removed. Even the equal area criterion becomes complicated in this third case. That the equal area criterion is not so simple for this type of problem, and that the equal area criterion is not readily adaptable to digital computer solution, is demonstrated by a qualitative discussion of an example.

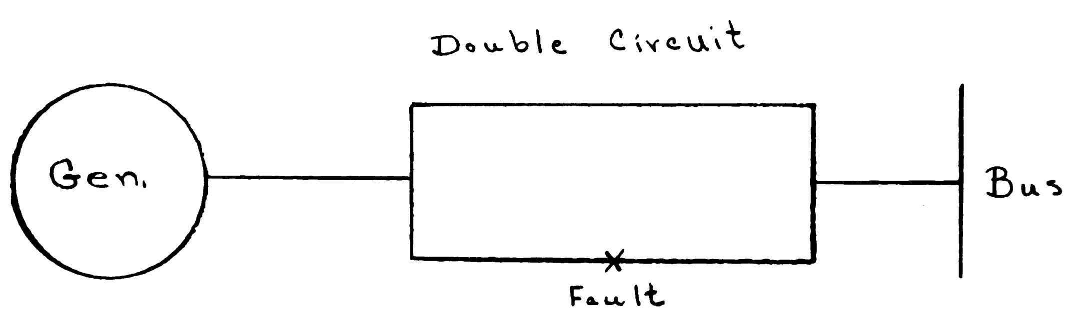


Figure 4.5

The diagram of Fig. (4.5) represents a generator connected to an infinite bus by two parallel lines. A fault occurs on one of the lines. If the faulted line is removed by circuit breaker action, the critical

switching angle can be determined using the equal area criterion as follows:

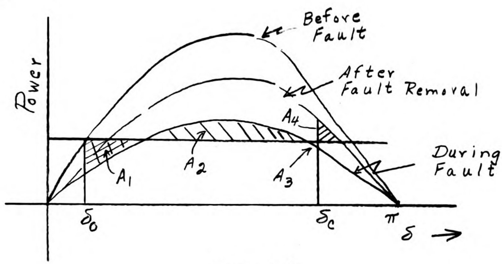


Figure 4.6

To apply the equal area criterion here, three different power angle curves are required as shown in Fig. (4.6). The three power angle curves are curves representing the relations in equation (4.3) for the X_d before, during, and after the fault.

The horizontal line represents the value of power being supplied to the generator by the prime mover at the initiation of the fault. The cross-hatched areas above the initial power line are taken as positive, and the cross-hatched areas below the initial power line are taken as negative. The angle, δ_o, is the initial torque angle and δ_c is the torque angle at the time the fault is removed. By the equal area criterion, δ_c is the critical switching angle when

$$A_1 + A_2 + A_3 + A_4 = 0$$

The critical δ_c is usually found by graphical trial and error means. The switching time is then found by numerical solution of equation (4.6) with E_q and X_{dT} determined for the faulted condition.

Pre-calculated swing curves[7,8] are sometimes used to determine the critical switching time. Pre-calculated curves cannot be applied to the open circuit and short circuit synchronous machines, however.

4.4 COMPARISON OF RESULTS AND METHODS

The program was run for switching times of 0.1, 0.2, 0.25 and 0.3 seconds. Graph 4.1 of the results indicates that the system was stable for switching at 0.25 seconds and the system was unstable for switching at 0.3 seconds. These results compare favorably with the 0.28 critical switching time obtained by the equal area criterion.

In contrast with Kimbark's[8] results of a change in field of about 23% in the first 0.5 seconds of sustained short circuit, the $i_{f1}(t)$ obtained by direct solution here had a maximum variation of about 0.1% of the steady field current I_f. This low order of magnitude for the change in the field current is as should be expected on the basis of the terms of equations of (4.2). In per unit quantities, the armature and field inductances are:

$$L_f = 5$$

$$L_s^+ = \frac{1.2}{377}$$

and in per unit quantities the voltage disturbances causing the changes in the field and armature currents are of the same order of magnitude. Both are approximately

$$\left[v_d(t) - \dot{\theta}\, L_s^+\, i_q(t)\right]$$

and

$$\frac{di_d(t)}{dt} = \left[v_d(t) - \dot{\theta}\, L_s^+\, i_q(t)\right] \frac{1}{L_s^+}$$

$$\frac{di_{f1}(t)}{dt} = \left[v_d(t) - \dot{\theta}\, L_s^+\, i_q(t)\right] \frac{K}{L_s^+\, I_f}$$

where K and L_s^+ are the same order of magnitude.

Kimbark[8] represents the so-called change in field flux by a ficticious voltage component in the armature circuit. This voltage component is derived from the same relationships that appear in the equations solved in this thesis. In the derivation the voltage component is related to the field current.

From the thesis solution, for the time interval of 0.5 seconds, the speed had increased by 4.5%. The commonly neglected change in speed appears to be a more significant variation than the field current decrement.

Very little more can be said in comparison of the results, since standard techniques usually give only δ as a function of time, but the $i_d(t)$ and the $i_q(t)$ of the computer solution shown on Graphs 4.2 and 4.3 exhibit characteristics distinctly different from the form commonly assumed for the $i_d(t)$ and $i_q(t)$ variables.

The approximate mathematical expression for these current variables obtained from the graph are:

$$i_q(t) = 0.82 \sin (\omega t + \Psi_1)$$

$$i_d(t) = 1.25 + 0.82 \sin (\omega t + \Psi_2)$$

A closer examination of the data showed that the ω terms were not constant at 377 radians/second, but were more nearly $\dot{\theta}$, the instantaneous angular velocity. It is commonly assumed that $i_d(t)$ and $i_q(t)$ have a slow variation with a period similar to the period of the mechanical oscillations. The period displayed here is around 0.0166 seconds, compared with a mechanical oscillation of around 1.0 second. It is common to assume that for the short circuit condition, $i_q(t)$ goes to zero immediately. The amplitude of the oscillations of $i_q(t)$ displayed in Graph 4.2 is around twice the initial value of $i_q(t)$. The currents were obtained for the conditions that the armature resistance was neglected and currents appear to have the form of sustained oscillations, but the armature circuit time constant is around one second so the decay would be slight in the time shown if the resistance had been considered.

A review of the mathematical relations for the equations solved substantiates the results obtained. For the equations (2.6)a and (3.1), with the assumption of constant speed and constant field current, the usual form of Park's equations are obtained:

$$\begin{aligned} v_d(t) &= (R_s^+ + L_s^+ \tfrac{d}{dt})\, i_d(t) + \dot{\theta}\, L_s^+\, i_q(t) \\ v_q(t) &= (R_s^+ + L_s^+ \tfrac{d}{dt})\, i_q(t) - \dot{\theta}\, L_s^+\, i_d(t) - \dot{\theta}\sqrt{2}L_{sr}\, I_f \end{aligned} \tag{4.8}$$

Neglect R_s^+ and for short circuit conditions, $v_d(t) = v_q(t) = 0$, so the equations (4.8) become:

$$\begin{aligned} \frac{d}{dt}\, i_d(t) &= -\, \dot{\theta}\, i_q(t) \\ \frac{d}{dt}\, i_q(t) &= +\, \dot{\theta}\, i_d(t) + \dot{\theta}\sqrt{2}L_{sr}\, I_f \end{aligned} \tag{4.9}$$

For the initial conditions and coefficients of the present problem, a Laplace transform solution of the two equations yields:

$$\begin{aligned} i_q(t) &= 0.82 \sin(\dot{\theta}t + \Psi_1) \\ i_d(t) &= 1.25 + 0.82 \sin(\dot{\theta}t + \Psi_2) \end{aligned} \tag{4.10}$$

With the resistance considered

$$\begin{aligned} i_q(t) &= 0.82\, e^{-0.8t} \sin(\dot{\theta}t + \Psi_1) \\ i_d(t) &= 1.25 + 0.82\, e^{-0.8t} \sin(\dot{\theta}t + \Psi_2) \end{aligned} \tag{4.11}$$

It is interesting to note that for the constant speed solution, the 0.82 coefficients of the sine terms and the 1.25 constant are independent of speed. A conclusion that for small changes in speed the $i_d(t)$ and $i_q(t)$ variables would have the same amplitudes and same forms is substantiated by the results on Graph 4.2.

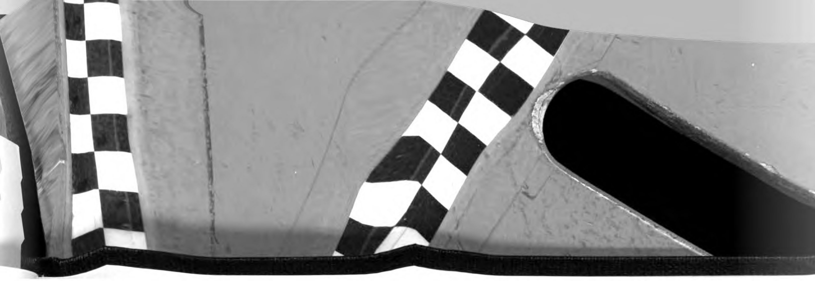
Admittedly, the average value of $i_q(t)$ is zero, and for a high inertia-torque ratio of a power generator, the variation of the $i_q(t)$ would add only a slight ripple to the swing curve δ as a function of time. This high inertia-torque ratio does not always exist, however, when the standard technique of considering a system as an equivalent machine is used. The high-inertia to torque ratio would not exist in small synchronous machines in control systems, so in such a system the standard assumptions would have questionable accuracy.

The form of $i_d(t)$ and $i_q(t)$ is of particular importance when solving the differential equations (2.6)a and (3.1) by numerical methods. If the $i_d(t)$ and $i_q(t)$ variables had a slow variation with a period of the order of one or two seconds, similar to the mechanical period, the time increment for the numerical solution could be taken as a value such as 0.05 seconds or some other value small compared with the period of variation of the variables. The correct form of the currents, $i_d(t)$ and $i_q(t)$, shows a period of around 0.0166 seconds and a time increment of 0.05 seconds, for the numerical solution could not be expected to give significant results.

Finally, in addition to comparing results of the standard and thesis techniques, consider the differences in principles and methods for the first class of problem.

In contrast with the standard technique, the thesis technique:

1. Solves differential equations as differential equations.



2. Does not require the development or use of the transient reactance concept.
3. By direct solution of equations (2.6)a and (3.1, gives values of armature current, field current, speed, and torque angle as a function of time. The standard method gives the torque angle as a function of time.
4. Does not drop field variation terms as in the development of the equation (4.6), so field variation effects can be considered without developing additional formulas.
5. Can solve a large class of problems without changing procedure. Changes in specified variables, specified parameters, and switching times are controlled by the master routine of the computer program.

V. TWO MACHINE SYSTEMS

The question arises as to the desirability of analyzing the single machine system and the two machine system as separate problems rather than as problems included in an n-machine formulation. A partial justification of this separate consideration is given in the following paragraphs.

There are many power system stability problems that fall into the one machine or into the two machine class of problem and give satisfactory correlation between calculated results and observation on the actual system. The study of the effect of machine parameters on stability, circuit breaker action for connecting or disconnecting individual machines, and many others, are single machine problems. After a preliminary investigation, it may be found that one machine and a system can be adequately represented by a two machine analysis as far as the individual machine is

concerned or two interconnected systems may be adequately represented as two equivalent machines in a two machine analysis. The case actually involving only two synchronous machines would arise only in isolated cases or in low power level control systems. The latter is not a power system stability problem.

The greater simplicity of the one machine or the two machine analysis methods makes these methods more desirable than a generalized form when these simpler methods give acceptable correlation with observed results from the physical system. The simpler forms occur because of the elementary patterns for the transmission line systems interconnecting the machines. The multi-machine formulation would need to include a network analysis program with the machine analysis program.

Whether standard or thesis methods of analysis are used, the two machine problem represents one step in the direction of a multi-machine analysis. Even if methods do not apply directly, information obtained by analyzing a two machine system should be helpful in formulating a multi-machine analysis.

Some problems which are conventionally considered as two machine problems are now listed.

1. Two machine representation is used in the study of an individual machine delivering power to a system over a long transmission line. As far as the individual machine variables are concerned, the distant system is adequately represented as an equivalent machine. Studies are commonly made of the effect on the individual machine of a fault and fault clearing time for a fault on the transmission line connecting the machine to the system. Studies are made of the effect

on the individual machine of an increase or decrease of the net generation of the system due to added loads, lightened loads, or faults on the system.

2. Two machine representation is used in the study of two systems which are interconnected by a transmission line. After some preliminary investigation, it may be found that the tie line stability is adequately represented when the two systems are considered as two equivalent machines. Studies are commonly made of the effect on the tie line stability of a fault and fault clearing time for a fault on the interconnecting transmission line. Studies are made of the effect on tie line stability of a net increase or decrease in generation of either system due to changes in load or local faults in either system.

3. The case of two interconnected synchronous machines would be rare in power system study; however, analysis techniques in terms of two machines are commonly presented and serve for the systems represented by two machines.

The problem worked in this chapter is that of two machines, and under appropriate assumptions does represent two systems. The stability problem considered is the problem caused by an increment of change in power of one of the machines. The transmission line fault is discussed but not illustrated by a numerical example. The standard techniques require a different approach to the two types of problems, while the thesis technique is basically the same for the two types of problems.

5.1 THE DIFFERENTIAL EQUATIONS OF THE TWO MACHINE SYSTEM

In this section, synchronous machine equations are combined, transformations are chosen and applied, and the resulting system of equations for a two machine system are listed.

Consider two synchronous machines interconnected by a transmission line system. Equations of the form of (2.1)a and (2.1)b represent the system as follows:

$$(5.1)\text{a} \quad \begin{bmatrix} \mathcal{V}_{s1}(t) \\ v_{f1}(t) \end{bmatrix} = \begin{bmatrix} \mathcal{R}_{ss1} + \frac{d}{dt}\mathcal{L}_{ss1} & \frac{d}{dt}\mathcal{L}_{sr1}(\phi_1) \\ \frac{d}{dt}\mathcal{L}_{rs1}(\phi_1) & R_{f1} + \frac{d}{dt} L_{f1} \end{bmatrix} \begin{bmatrix} \mathcal{I}_{s1}(t) \\ i_{f1}(t) \end{bmatrix}$$

$$(5.1)\text{b} \quad T_1(t) = -\frac{1}{2} \begin{bmatrix} \mathcal{I}'_{s1}(t) & i_{f1}(t) \end{bmatrix} \frac{\partial}{\partial \phi_1} \begin{bmatrix} \mathcal{L}_{ss1} & \mathcal{L}_{sr1}(\phi_1) \\ \mathcal{L}_{rs1}(\phi_1) & L_{f1} \end{bmatrix} \begin{bmatrix} \mathcal{I}_{s1}(t) \\ i_{f1}(t) \end{bmatrix} + (B_1 + J_1 \frac{d}{dt}) \dot{\phi}_1$$

$$(5.2)\text{a} \quad \begin{bmatrix} \mathcal{V}_{s2}(t) \\ v_{f2}(t) \end{bmatrix} = \begin{bmatrix} \mathcal{R}_{ss2} + \frac{d}{dt}\mathcal{L}_{ss2} & \frac{d}{dt}\mathcal{L}_{sr2}(\phi_2) \\ \frac{d}{dt}\mathcal{L}_{rs2}(\phi_2) & R_{f2} + \frac{d}{dt} L_{f2} \end{bmatrix} \begin{bmatrix} \mathcal{I}_{s2}(t) \\ i_{f2}(t) \end{bmatrix}$$

$$(5.2)\text{b} \quad T_2(t) = -\frac{1}{2} \begin{bmatrix} \mathcal{I}'_{s2}(t) & i_{f2}(t) \end{bmatrix} \frac{\partial}{\partial \phi_2} \begin{bmatrix} \mathcal{L}_{ss2} & \mathcal{L}_{sr2}(\phi_2) \\ \mathcal{L}_{rs2} & L_{f1} \end{bmatrix} \begin{bmatrix} \mathcal{I}_{s2}(t) \\ i_{f1}(t) \end{bmatrix} + (B_2 + J_2 \frac{d}{dt}) \dot{\phi}_2$$

where the coefficients and matrices are defined as in Chapter II. The (2) subscripts represent variables and coefficients for machine number (2), and, as in the first problem, let the variables and parameters with the (1) subscript represent machine (1) plus transmission system coefficients[4] and variables. For the specified connection pattern, by use of circuit equations and segregate equations, (5.3) and (5.4) and the application of the symmetrical component transformation of variables,

$$(5.3)\qquad \mathcal{V}_{s1}(t) = \begin{bmatrix} v_{a1}(t) \\ v_{b1}(t) \\ v_{c1}(t) \end{bmatrix} = \begin{bmatrix} v_{a2}(t) \\ v_{b2}(t) \\ v_{c2}(t) \end{bmatrix} = \mathcal{V}_{s2}(t)$$

$$(5.4)\qquad \mathcal{I}_{s1}(t) = \begin{bmatrix} i_{a1}(t) \\ i_{b1}(t) \\ i_{c1}(t) \end{bmatrix} = \begin{bmatrix} i_{a2}(t) \\ i_{b2}(t) \\ i_{c2}(t) \end{bmatrix} = -\mathcal{I}_{s2}(t)$$

(2.3), to the stator voltage and the stator current variables, there results

$$(5.5)\mathrm{a}\quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ v_{f1}(t) \\ v_{f2}(t) \end{bmatrix} = \begin{bmatrix} R_s^o+L_s^o\frac{d}{dt} & 0 & 0 & 0 & 0 \\ 0 & R_s^+ + L_s^+\frac{d}{dt} & 0 & \frac{d}{dt}L_{sr1}e^{j\theta_1} & -\frac{d}{dt}L_{sr2}e^{j\theta_2} \\ 0 & 0 & R_s^+ + L_s^+\frac{d}{dt} & \frac{d}{dt}L_{sr1}e^{-j\theta_1} & -\frac{d}{dt}L_{sr2}e^{-j\theta_2} \\ 0 & \frac{d}{dt}L_{sr1}e^{-j\theta_1} & \frac{d}{dt}L_{sr1}e^{j\theta_1} & R_{f1}+L_{f1}\frac{d}{dt} & 0 \\ 0 & -\frac{d}{dt}L_{sr2}e^{-j\theta_2} & -\frac{d}{dt}L_{sr2}e^{j\theta_2} & 0 & R_{f2}+L_{f2}\frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_1^o(t) \\ i_1^+(t) \\ i_1^-(t) \\ i_{f1}(t) \\ i_{f2}(t) \end{bmatrix}$$

$$(5.5)\mathrm{b}\quad \begin{aligned} T_1(t) &= (B_1 + J_1\tfrac{d}{dt})\dot{\phi}_1 - p_1 L_{sr1} i_{f1}\, \mathcal{I}_m\left\{i_1^+(t)\, e^{-j\theta_1}\right\} \\ T_2(t) &= (B_2 + J_2\tfrac{d}{dt})\dot{\phi}_2 - p_2 L_{sr2} i_{f2}\, \mathcal{I}_m\left\{-i_1^+(t)\, e^{-j\theta_2}\right\} \end{aligned}$$

where

$$R_s^o = R_{s1}^o + R_{s2}^o$$
$$R_s^+ = R_{s1}^+ + R_{s2}^+$$
$$L_s^o = L_{s1}^o + L_{s2}^o$$
$$L_s^+ = L_{s1}^+ + L_{s2}^+$$
$$\theta_1 = \omega t + \delta_1 + \pi/2$$
$$\theta_2 = \omega t + \delta_2 + \pi/2$$

and

$$\mathcal{I}_m = \left\{ i_1^+(t)\ e^{-j\theta_1} \right\} \quad \text{and} \quad \mathcal{I}_m \left\{ -i_1^+(t)\ e^{-j\theta_2} \right\}$$

means the imaginary part of the bracketed quantities.

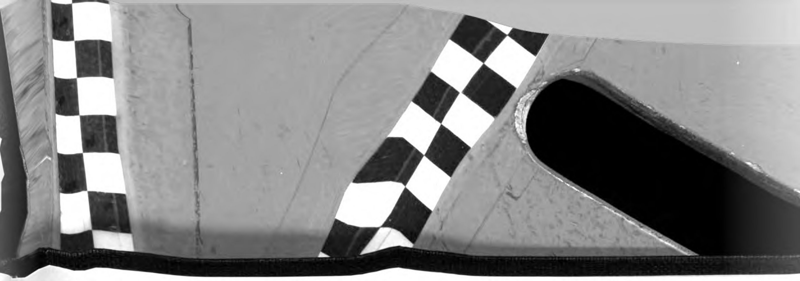
For the backward sequence transformation of the type represented by equations (2.4), a choice must be made. Some of the forms in the literature that reduce the system to an equivalent machine would seem to suggest a transformation in terms of θ, where

$$\theta = \omega t + \delta_1 + \delta_2 + \pi/2$$

This θ would be necessary for convenient reduction of the mechanical equations to a single equivalent in terms of

$$\delta_{12} = \delta_1 + \delta_2$$

The overall system of equations was simpler and the identity of the individual machines was retained, however, when either the θ_1 or θ_2 transformation was used. Thus, transformations of the form of equations (2.4) were applied to the stator variables for the theta equal to θ_1. Further, the transformations of the form of equations (2.5) were applied to separate the reals of the backwards sequence variables from the imaginaries of the backwards sequence variables, and the equations (5.6)a and (5.6)b were obtained.

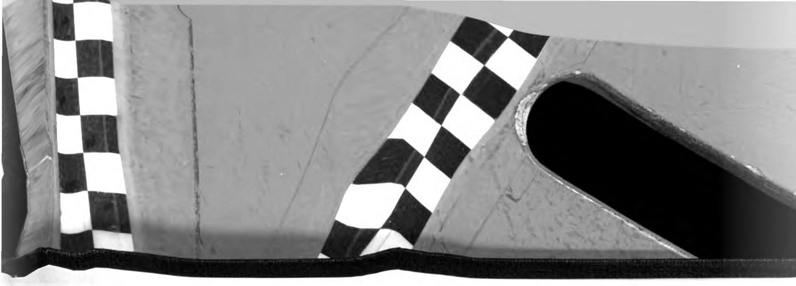


$$(5.6)a \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ v_{f1} \\ v_{f2} \end{bmatrix} = \begin{bmatrix} R_s^o+L_s^o\frac{d}{dt} & 0 & 0 & 0 & 0 \\ 0 & R_s^+ + L_s^+\frac{d}{dt} & \dot{\theta}_1 L_s^+ & \sqrt{2}L_{sr1}\frac{d}{dt} & -\sqrt{2}L_{sr2}\left[\cos(\delta_1-\delta_2)\frac{d}{dt} + \dot{\theta}_2 \sin(\delta_1-\delta_2)\right] \\ 0 & -\dot{\theta}_1 L_s^+ & R_s^+ + L_s^+\frac{d}{dt} & -\sqrt{2}L_{sr1}\dot{\theta}_1 & \sqrt{2}L_{sr2}\left[-\sin(\delta_1-\delta_2)\frac{d}{dt} + \dot{\theta}_2\cos(\delta_1-\delta_2)\right] \\ 0 & \sqrt{2}L_{sr1}\frac{d}{dt} & 0 & R_{f1}+L_{f1}\frac{d}{dt} & 0 \\ 0 & -\sqrt{2}L_{sr2}\left[\cos(\delta_1-\delta_2)\frac{d}{dt} - (\dot{\delta}_1-\dot{\delta}_2)\sin(\delta_1-\delta_2)\right. & \sqrt{2}L_{sr2}\left[\sin(\delta_1-\delta_2)\frac{d}{dt} + (\dot{\delta}_1-\dot{\delta}_2)\cos(\delta_1-\delta_2)\right] & 0 & R_{f2}+L_{f2}\frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_s^o(t) \\ i_d(t) \\ i_q(t) \\ i_{f1}(t) \\ i_{f2}(t) \end{bmatrix}$$

$$(5.6)b \quad \begin{aligned} T_1(t) &= (B_1+J_1\tfrac{d}{dt})\dot{\phi}_1 + \sqrt{2}/2\; L_{sr1}\, p_1\, i_{f1}(t)\, i_q(t) \\ T_2(t) &= (B_2+J_2\tfrac{d}{dt})\dot{\phi}_2 + \sqrt{2}/2\; L_{sr2}\, p\, i_{f2}(t)\,\left[i_q(t)\cos(\delta_1-\delta_2) - i_d(t)\sin(\delta_1-\delta_2)\right] \end{aligned}$$

Express the swing equations of (5.6)b in terms of power

$$(5.7) \quad \begin{aligned} P_1 &= \dot{\theta}_1\sqrt{2}L_{sr1}\, i_{f1}(t)\, i_q(t) + M_1\frac{d^2}{dt^2}\theta_1 \\ P_2 &= -\dot{\theta}_2\sqrt{2}L_{sr2}\, i_{f2}(t)\left[i_q(t)\cos(\delta_1-\delta_2) - i_d(t)\sin(\delta_1-\delta_2)\right] + M_2\frac{d^2}{dt^2}\theta_2 \end{aligned}$$



where

$$\theta_1 = \omega t + \delta_1 + \pi/2 \qquad \frac{d^2}{dt^2}\,\theta_1 = \ddot{\delta}_1$$

$$\theta_2 = \omega t + \delta_2 + \pi/2 \qquad \frac{d^2}{dt^2}\,\theta_2 = \ddot{\delta}_2$$

Equations (5.6)a and (5.7) are the differential equations of a two machine system. The thesis technique is the solution of these, (5.6)a and (5.7), directly by numerical methods on the digital computer. The standard technique makes some simplifying assumptions and uses a modified steady state method to obtain a solution to these equations.

5.2 STEADY STATE INITIAL CONDITIONS FOR THE TWO MACHINE PROBLEM

The initial conditions are usually specified as constant speed, constant direct current field, constant amplitude sinusoidal armature voltages, a constant average power, and constant power factor. If these initial conditions are imposed on the sets of equations (5.6)a and (5.7), the equations (5.6)a are independent of the set (5.7). From the results of the transformations on the steady state sinusoidal armature currents,

$$i_d(t) = -\left|I_m\right| \sin(\delta_1 + \beta)$$

$$i_q(t) = \left|I_m\right| \cos(\delta_1 + \beta)$$

where β is the power factor angle.

The results of the steady state solution to (5.6)a are commonly pictured on a vector diagram as in Fig. (5.1).

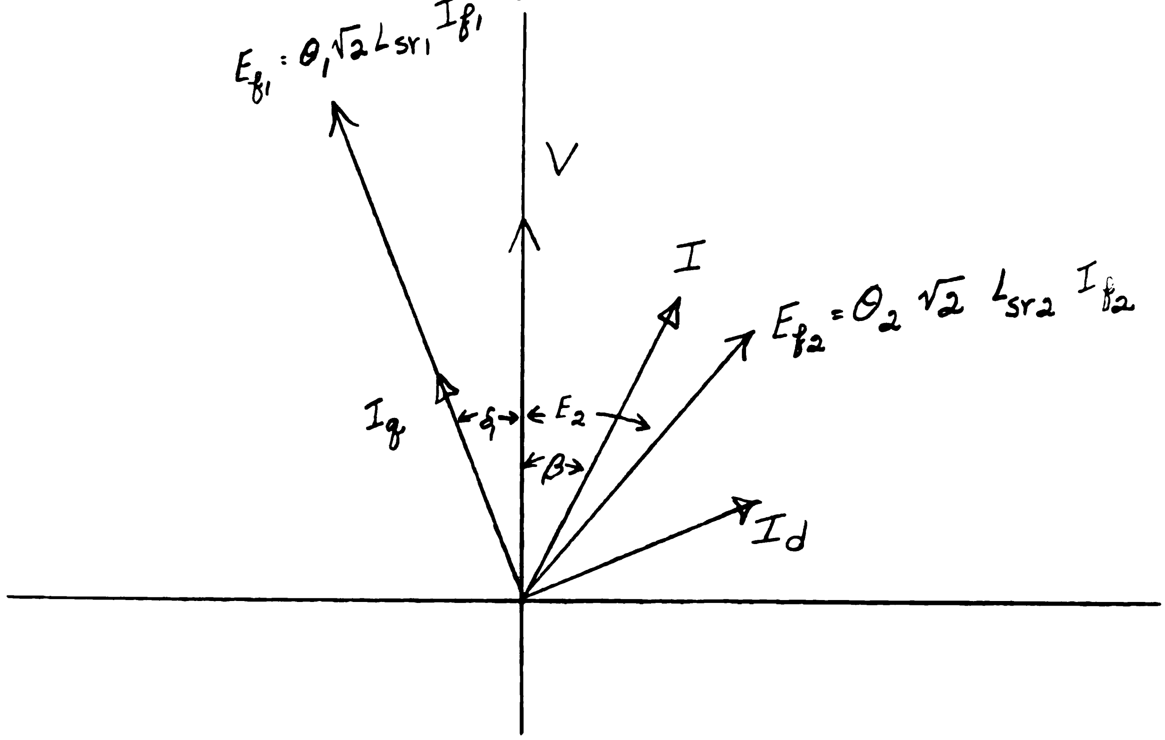


Figure 5.1

From the solution of (5.6)a the initial values are:

$i_d = -0.4840$ $\qquad$ $\delta_1 = 23.6^\circ = 0.4116$ radians

$i_q = 0.3350$ $\qquad$ $\delta_2 = -28^\circ = -0.4886$ radians

$i_{f1} = 1.5$

$i_{f2} = 0.852$

From the constant speed assumption:

$\dot{\theta}_1 = 377$ radians/second

$\dot{\theta}_2 = 377$ radians/second

$\delta_1 + \pi/2 = \theta_1 = 1.9823$ radians

$\delta_2 + \pi/2 = \theta_2 = 1.0821$ radians

The initial values obtained from (5.6)a satisfy the equations (5.7).

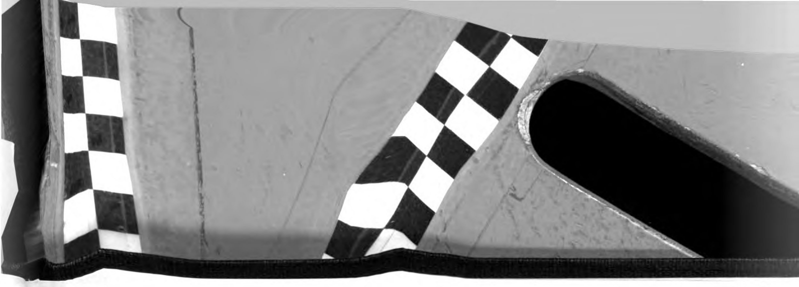
5.3 NUMERICAL STUDY OF THE STABILITY PROBLEM OF A TWO MACHINE SYSTEM

In this section the constant field current assumption is justified. The equations are put into standard form for numerical solution. The

computer program is discussed, and the use of the program is illustrated in solving the second problem.

Upon examination of the results of the first problem, it is found that the percent variation in the field current was less than 0.1 percent of the steady direct current. The system inertia was close to a minimum, and the fault one of the most severe likely to be encountered. As these results suggest, examination of terms in the differential equations shows certain terms to be negligible. Some of these terms will be discussed and eliminated as the two machine differential equations are put into standard form for solution.

$$
(5.13)\quad
\begin{bmatrix}
1 & 0 & +\sqrt{2}\,\frac{L_{sr_1}}{L_{f_1}} & -\sqrt{2}\,\frac{L_{sr2}}{L^{+}}\cos(\delta_1-\delta_2) & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -\sqrt{2}\,\frac{L_{sr2}}{L^{+}}\sin(\delta_1-\delta_2) & 0 & 0 & 0 & 0 \\
\frac{\sqrt{2}L_{sr_1}}{L_{f_1}} & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
\frac{\sqrt{2}L_{sr_2}}{L_{f_2}}\cos(\delta_1-\delta_2) & \sqrt{2}\,\frac{L_{sr_2}\sin(\delta_1-\delta_2)}{L_{f_2}} & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\frac{d}{dt}\, i_d(t) \\
\frac{d}{dt}\, i_q(t) \\
\frac{d}{dt}\, i_{f_1}(t) \\
\frac{d}{dt}\, i_{f_2}(t) \\
\dot{\theta}_1 \\
\ddot{\theta}_1 \\
\dot{\theta}_2 \\
\ddot{\theta}_2
\end{bmatrix}
=
$$



$$
\begin{bmatrix}
\left[-R_s^+ i_d - \dot{\theta}_1 L_s^+ i_q + \sqrt{2}L_{sr2}\dot{\theta}_2 \sin(\delta_1-\delta_2)\, i_{f2}\right] \frac{1}{L_s^+} \\
\left[-R_s^+ i_q + \dot{\theta}_1 L_s^+ i_d + \sqrt{2}L_{sr1}\dot{\theta}_1\, i_{f1} - \sqrt{2}L_{sr2}\, \dot{\theta}_2 \cos(\delta_1-\delta_2)\, i_{f2}\right] \frac{1}{L_s^+} \\
\left[v_{f1} - R_{f1}\, i_{f1}\right] \frac{1}{L_{f1}} \\
-\sqrt{2}L_{sr2}(\dot{\delta}_1-\dot{\delta}_2) \sin(\delta_1-\delta_2)\, i_d - \sqrt{2}L_{sr2}(\dot{\delta}_1-\dot{\delta}_2) \cos(\delta_1-\delta_2) i_q + v_{f2} - R_{f2} i_{f2}\Big] \frac{1}{L_{f2}} \\
\dot{\theta}_1 \\
\left[P_1 - \dot{\theta}_1 \sqrt{2}L_{sr1}\, i_{f1}\, i_q\right] \frac{1}{M_1} \\
\dot{\theta}_2 \\
\left\{P_2 + \dot{\theta}_2 \sqrt{2}L_{sr2}\, i_{f2} \left[i_q \cos(\delta_1-\delta_2) - i_d \sin(\delta_1-\delta_2)\right]\right\} \frac{1}{M_2}
\end{bmatrix}
$$

The 8 x 8 coefficient matrix of the first derivatives is of the form

$$
\begin{bmatrix}
\mathcal{B}_{11} & 0 \\
0 & \mathcal{B}_{22}
\end{bmatrix}
$$

where the $\mathcal{B}_{22}$ is a unit matrix.

The inverse is obtained when $\mathcal{B}_{11}^{-1}$ is obtained. The submatrix $\mathcal{B}_{11}$ is a 4 x 4 and the inverse will not be presented here but significant terms will be discussed.

The determinant of $\mathcal{B}_{11}$ is

$$
1 - \frac{2\, L_{sr2}^2}{L_{f2}\, L_s^+} - \frac{2\, L_{sr1}^2}{L_{f1}\, L_s^+} + \frac{(2L_{sr1}\, L_{sr2})^2}{L_s^{+2}\, L_{f1}\, L_{f2}} \sin^2(\delta_1-\delta_2)
$$

but

$$
\sqrt{2}L_{sr1} = \sqrt{2}L_{sr2} = \frac{1}{377} \text{ p.u.}
$$

$$
L_s^+ = \frac{2}{377} \text{ p.u.}
$$

and the order of magnitude of the field inductances would be about 6 p.u. for each L_{f1} and L_{f2}. Thus, the determinant of $\mathcal{B}_{11}$ is approximately unity.

Similarly, all the diagonal co-factor terms of the inverse are approximately unity. For illustration purposes, consider the 33 co-factor a_{33}.

$$a_{33} = 1 - \frac{2\,L_{sr_2}^2}{L_{f_2}\,L_s^+} = 1 - \frac{2\left(\frac{1}{377}\right)^2}{6 \times \frac{2}{377}} = 1 - \frac{1}{6 \times 377} = 1 - .00088 \approx 1$$

The off diagonal co-factor terms a_{12}, a_{13}, and a_{14} which reflect the right hand term into the $\frac{d}{dt}\, i_d(t)$ equation are:

$$a_{12} = -\frac{2\,L_{sr_2}^2}{L_{f_2}\,L_s^+} \sin\,(\delta_1 - \delta_2)\cos\,(\delta_1 - \delta_2)$$

$$a_{13} = \frac{\sqrt{2}L_{sr_1}}{L_s^+} - \frac{2\sqrt{2}L_{sr_1}\,L_{sr_2}^2}{L_{f_2}\,L_s^{+2}} \sin^2\,(\delta_1 - \delta_2)$$

$$a_{14} = \frac{\sqrt{2}L_{sr_2}}{L_s^+} \cos\,(\delta_1 - \delta_2)$$

The first four lines of the right hand side of (5.13) are of the form

$$\begin{bmatrix} A\,\frac{1}{L_s^+} \\ B\,\frac{1}{L_s^+} \\ C\,\frac{1}{L_{f_1}} \\ D\,\frac{1}{L_{f_2}} \end{bmatrix}$$

where B, C, D are of the same order of magnitude as A or smaller than A. Thus,

$$\frac{d}{dt}\, i_d(t) = A/L_s^+ + a_{12}\, {}^{B}/L_s^+ + a_{13}\, {}^{C}/L_{f_1} + a_{14}\, {}^{D}/L_{f_2}$$

where the last three terms are approximately $(.00044)({}^{A}/L_s^+)$.

A similar analysis applied to the $\frac{d}{dt}\, i_q(t)$ equation leads to similar results, so the equation (5.13) in standard form for solution with field variables as constants is as follows:

$$
(5.14)\qquad
\begin{bmatrix}
\frac{d}{dt} i_d \\
\frac{d}{dt} i_q \\
\dot\theta_1 \\
\ddot\theta_1 \\
\dot\theta_2 \\
\ddot\theta_2
\end{bmatrix}
=
\left[\begin{array}{l}
\left[-R_s^+ i_d - \dot\theta_1 L_s^+ \, i_q + \dot\theta_2 \sqrt{2}L_{sr2} \sin(\delta_1-\delta_2)\, i_{f2}\right] \frac{1}{L_s^+} \\
\left[-R_s^+ i_q + \dot\theta_1 L_s^+ \, i_d + \dot\theta_1 \sqrt{2}L_{sr1}\, i_{f1} - \dot\theta_2 \sqrt{2}L_{sr2} \cos(\delta_1-\delta_2) i_{f2}\right] \frac{1}{L_s^+} \\
\quad \dot\theta_1 \\
\left[P_1 - \dot\theta_1 \sqrt{2}L_{sr1}\, i_{f1}\, i_q\right] \frac{1}{M_1} \\
\quad \dot\theta_2 \\
P_2 + \dot\theta_2 \sqrt{2}L_{sr2}\, i_{f2} \left[i_q \cos(\delta_1-\delta_2) - i_d \sin(\delta_1-\delta_2)\right]\Big\} \frac{1}{M_2}
\end{array}\right]
$$

The equations (5.14) are the differential equations that describe the two machine system. Equations (5.14) are programmed as the closed f_r subroutine for the F-6 differential equation solving routine. The F-6 and the f_r routine for (5.14) under the control of a master routine can be used to solve all of the two machine stability problems discussed in this chapter. The master routine as in the first problem controls the amount of information extracted from the computer, simulates faults and fault removal by changing parameters in the f_r routine, and simulates changes in load by changing constants.

For the particular problem, the master routine was written to read out t, δ_1, δ_2, i_d and i_q. The change in load was affected on read in by determining initial conditions for $P_1 = -P_2 = 0.5$ p.u. power and reading in that $P_2 = -0.6$ p.u. at the beginning of the program. The frequency of the read out and the duration of the program are controlled by the master routine.

With the f_r routine written for a two machine system, and a master routine written for the type of stability problem under consideration, the second problem is solved when the proper initial conditions and system parameters are supplied to the computer with the program.

For the second problem, the scaled initial conditions and parameters are supplied to the computer as follows:

The numbers in memory location 3 through 8 as required by F-6:

In 3 The integer 20 indicates the location of the first of the sequence of initial conditions.

In 4 The integer 27 indicates the sum of entry 3 and n, the number of equations in the system.

In 5 The integer 13 indicates that the scale factor on the f_r is 2^{-13}.

In 6 The integer 10 indicates that the time increment is 2^{-10} sec.

In 7 The integer 36; The g_r will have their error held to about 1 in the 36^{th} binary bit.

In 8 The integer 160 gives the memory location for the entry into the f_r routine.

In 10 The integer 100 indicates the memory location of the first of a sequence of scaled coefficients and constants.

The initial values of all the variables are scaled by 2^{-13} and read in the following location.

Location	Variable	Scaled Value
20	t	0
21	$\dot{\theta}_2$	0.04602
22	θ_2	0.0001322
23	$\dot{\theta}_1$	0.04602
24	θ_1	0.0002420
25	i_q	0.00004088
26	i_d	- 0.0000590

The coefficients and constants for the f_r routine:

Location	Quantity	Scale	Scaled Value
100	P_1/M_1	2^{-13}	0.001917
101	$\frac{P_2 + \Delta P_2}{M_2}$	2^{-13}	- 0.006903
102	δ_1	2^{-13}	0.00005024
103	δ_2	2^{-13}	- 0.00005965
104	I_{f_1}	2^{-10}	0.0014648
105	I_{f2}	2^{-10}	0.0008320
106	M_1	2^0	0.03183
107	M_2	2^0	0.01061
108	$- R_s^+$	2^0	- 0.01
109	$1/L_s^+$	2^{-9}	0.368164
110	$\omega = 1/\sqrt{2}L_{sr_1} = \frac{1}{\sqrt{2}L_{sr_2}}$	2^{-9}	0.73632
111	$\pi/2$	2^{-13}	0.0001917
112	$2/\pi$	2^0	0.63662
113	L_s^+	2^0	0.005305

For the master routine, supply the following counters to control read out.

If the integer "k" is placed in the 30th word of the master routine, read out will occur after each (k+1)st calculation.

If the integer "p" is placed in the 36th word of the master routine, read out will occur "p" times before the program stops.

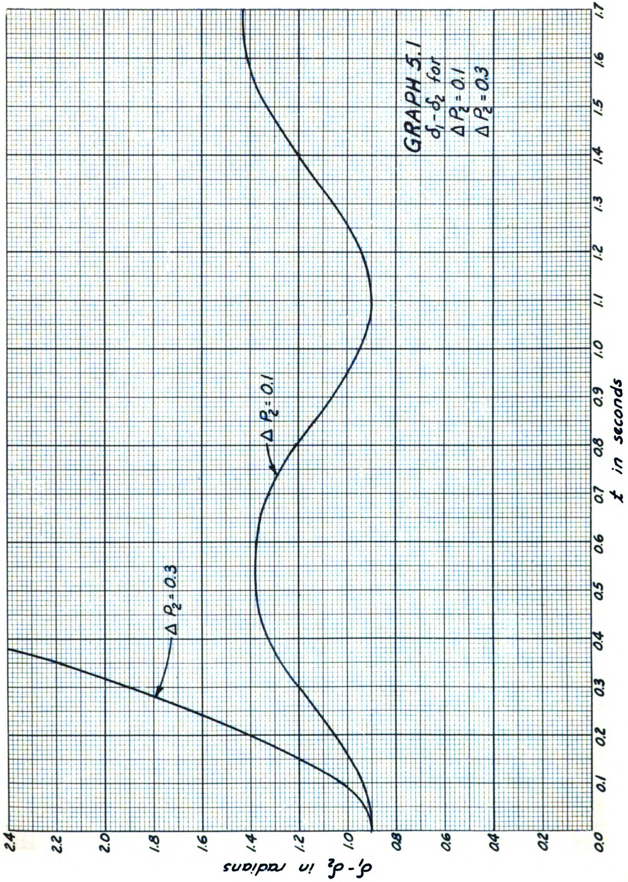
The pre-set parameters in memory locations 3 through 8 must be read in before F-6 is read in. All the other information can be placed at the end of the program tape. At the end of the read in, transfer control to the left side of the first word of the master routine and the computer gives the results.

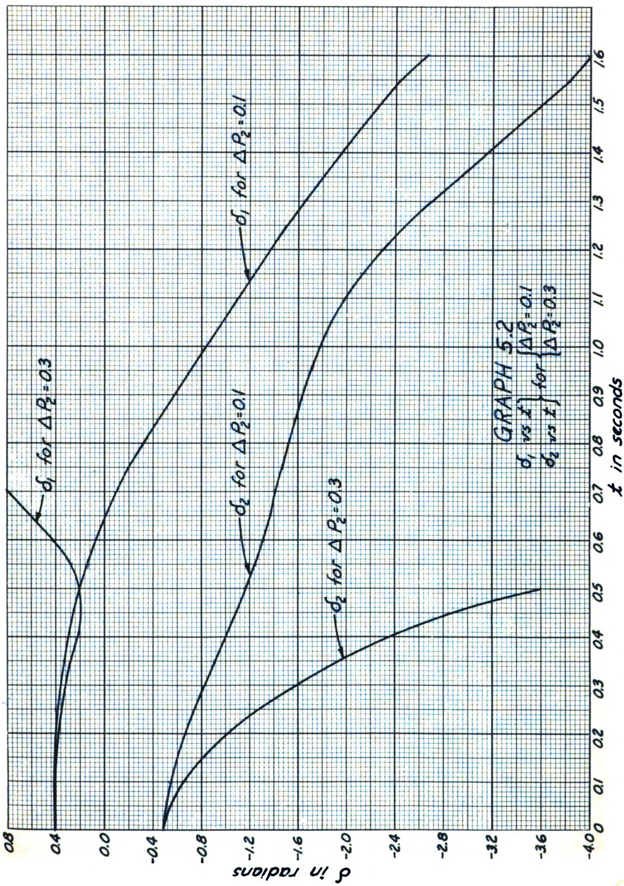
A common stability problem is the problem of determining the maximum ΔP_2 that can be permitted and still retain stability. The maximum ΔP_2 can be determined by running the program a number of times with different values of $P_2 + \Delta P_2$ in memory position 101. The assigned values of ΔP_2 can be made to converge on the critical value. The problem was run here for ΔP_2 as 0.1 p.u. and 0.3 p.u. or $P_2 + \Delta P_2$ as 0.6 and 0.8 p.u. power. The results are presented on Graphs (5.1), (5.2), (5.3) and (5.4).

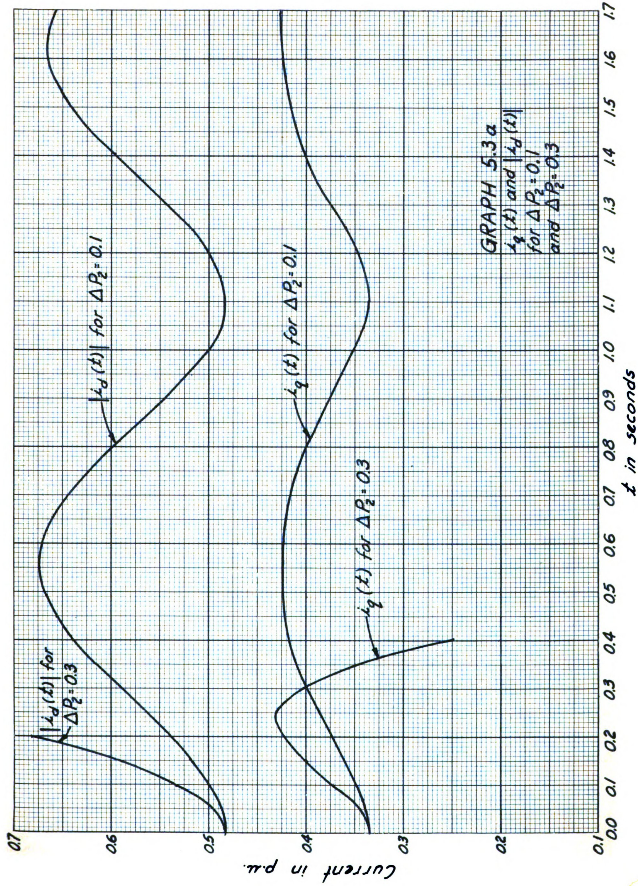
5.4 STANDARD SOLUTION FOR THE TWO MACHINE PROBLEM

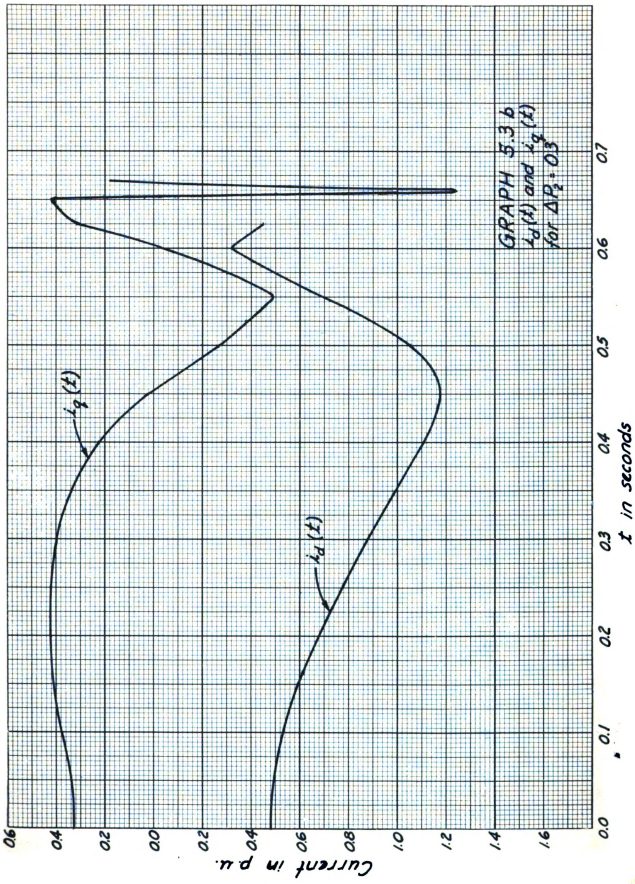
In the literature there are various approaches to the two machine problem. The approach varies somewhat with the particular problem. If it is desirable to consider system losses, the formulation is different from the formulation for the lossless case. For determining critical switching times, equal area criterion or pre-calculated swing curves may be used. For stability information with added load increments, a form of the equal area criterion may be used. All the standard techniques are based on the concept of transient reactances and voltages behind transient reactance, however.

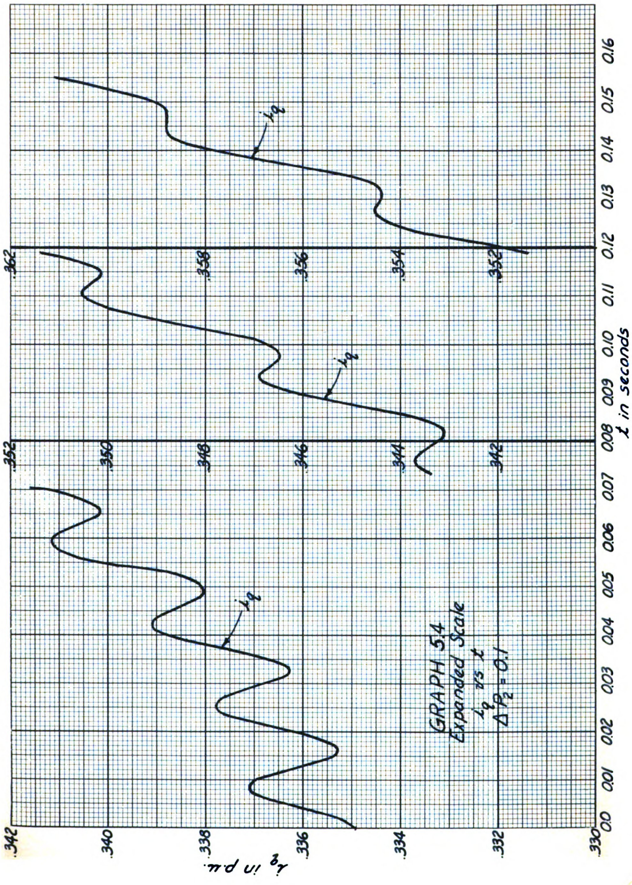
To obtain the voltages behind the transient reactances for the two machines, the steady state initial condition solution of section (5.2) is a necessary first step. For the second step, each machine is considered separately as in the case of individual machines of Chapter IV. From the initial condition, i_d, i_q, v_d, v_q, and machine transient reactances, a voltage behind transient reactance is found for each machine with equation (4.4). These voltages are designated here as E_1 and E_2 for machine (1) and machine (2) respectively. Next, if the machine and transmission line


GRAPH 5.1
δ1 - δ2 for
ΔP2 = 0.1
ΔP2 = 0.3
ΔP2 = 0.1
ΔP2 = 0.3
t in seconds
δ1 - δ2 in radians
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0 2.2 2.4


GRAPH 5.2
δ_1 vs t } for { $\Delta P_2 = 0.1$
δ_2 vs t } for { $\Delta P_2 = 0.3$
δ_1 for $\Delta P_2 = 0.1$
δ_1 for $\Delta P_2 = 0.3$
δ_2 for $\Delta P_2 = 0.1$
δ_2 for $\Delta P_2 = 0.3$
t in seconds
δ in radians
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6
0.8 0.4 0.0 -0.4 -0.8 -1.2 -1.6 -2.0 -2.4 -2.8 -3.2 -3.6 -4.0


GRAPH 5.3 a
$i_q(t)$ and $|i_d(t)|$
for $\Delta P_2 = 0.1$
and $\Delta P_2 = 0.3$
$|i_d(t)|$ for $\Delta P_2 = 0.1$
$i_q(t)$ for $\Delta P_2 = 0.1$
$i_q(t)$ for $\Delta P_2 = 0.3$
$|i_d(t)|$ for $\Delta P_2 = 0.3$
t in seconds
Current in p.u.
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7
0.1 0.2 0.3 0.4 0.5 0.6 0.7


GRAPH 5.3 b
$i_d(t)$ and $i_q(t)$
for $\Delta P_2 = 0.3$
$i_q(t)$
$i_d(t)$
Current in p.u.
t in seconds
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6


GRAPH 5.4
Expanded Scale
i_q vs t
$\Delta P_2 = 0.1$
i_q in p.u.
t in seconds
i_q
i_q
i_q
.342
.340
.338
.336
.334
.332
.330
.352
.350
.348
.346
.344
.342
.362
.358
.356
.354
.352
0.0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
0.11
0.12
0.13
0.14
0.15
0.16

losses are neglected, the system can be reduced to an equivalent single machine form.

$$M_o \frac{d^2 \delta_{12}}{dt^2} = \left[P_1 - \frac{|E_1||E_2|}{X_{12}} \sin \delta_{12} \right] \tag{5.8}$$

where

$$M_o = \frac{M_1 M_2}{M_1 + M_2}$$

δ_{12} is the angle between E_1 and E_2 initially, and the angle between machine rotors during the transient.

X_{12} is the sum of the transient reactances between points where E_1 and E_2 are theoretically measured.

Equation (5.8) has the same form as the swing equation for a single machine on an infinite bus. Under the assumptions that were necessary to obtain (5.8), the two machine problem can be solved by the standard method discussed in Chapter IV for the type of fault that would alter the transient reactance. If δ_{12} increases without limit, then the machines fall out of step. Equal area criterion can be used.

If losses are to be considered, the equivalent machine form of equation (5.8) cannot be obtained. A somewhat more complicated form results.

$$\frac{d^2 \delta_{12}}{dt^2} = \left[\frac{P_1 - T_{e1}}{M_1} - \frac{P_2 - T_{e2}}{M_2} \right] \tag{5.9}$$

where

$$(5.10) \quad \begin{aligned} T_{e1} &= \frac{|E_1|^2}{|\mathcal{Z}_{11}|} \sin \alpha_{11} + \frac{|E_1||E_2|}{|\mathcal{Z}_{12}|} \sin (\delta_{12} - \alpha_{12}) \\ T_{e2} &= \frac{|E_2|^2}{|\mathcal{Z}_{22}|} \sin \alpha_{22} + \frac{|E_1||E_2|}{|\mathcal{Z}_{12}|} \sin (\delta_{12} + \alpha_{12}) \end{aligned}$$

and where

$\mathcal{J}_{11}$ and $\mathcal{J}_{22}$ are driving point impedances (transient)

$\mathcal{J}_{12}$ a transfer impedance (transient)

α_{11}, α_{22}, α_{12} are the complements of the respective impedance angles.

For problems in determining critical switching times, the equal area criterion can be applied using (5.9), but the power angle curve is not a sinusoid and the graphical plot and analysis is tedious and subject to inaccuracies of graphical methods. If losses are neglected, the equivalent machine form of equation (5.8) is obtained and the critical switching time problem is the same as for a machine on an infinite bus if the equal area criterion is used.

For the particular problem of this chapter, the equations that permit consideration of changes in generation or load for machine (2) with losses considered, are:

$$(5.11) \qquad \frac{d^2 \delta_{12}}{dt^2} = \left[\frac{P_1 - T_{e1}}{M_1} + \frac{P_2 + \Delta P_2 - T_{e2}}{M_2}\right]$$

If losses are neglected

$$P_1 = - P_2 \qquad T_{e1} = -T_{e2}$$

then

$$(5.12) \qquad \frac{d^2 \delta_{12}}{dt^2} = \left[+ \frac{\Delta P_2}{M_2} + \frac{M_1 + M_2}{M_1 M_2} (P_1 - T_{e1})\right] = G(\delta_{12})$$

With losses, the right hand side of (5.12) is not a simple sinusoid as in the case of the individual machine on an infinite bus, but a plot of the right hand side of (5.11) as a function of δ_{12} gives a figure as in Fig. (5.2), where the δ_o is the initial value of δ_{12}.

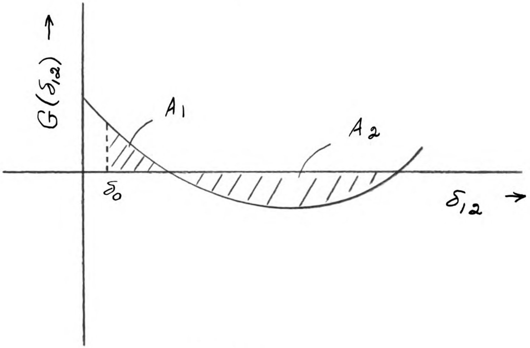


Figure 5.2

By the equal area criterion the system is stable if the increment of load is such that $A_1 < A_2$. For the machine of problem two neglecting losses:

$$H_1 = 6 \qquad H_2 = 2$$

$$\Delta P_2 = -0.1$$

$$f\ G(\delta_{12}) = \left[+ \frac{0.1}{2} + \frac{8}{12} (0.15 - \frac{1.5 \times .852}{2} \sin \delta_{12} \right]$$

$$= \left[- 0.05 + .333 - .426 \sin \delta_{12} \right]$$

$$= \quad .383 - .426 \sin \delta_{12}$$

But $\qquad f\ G(\delta_{12}) = 0 \quad \text{at} \quad \delta_{12} = 64^\circ$

$$A_1 = \int_{51.6}^{64^\circ} \left[.383 - .426 \sin \delta_{12} \right] \, d\,\delta_{12} = .0072$$

$$A_2 = \left| \int_{64^\circ}^{180-64^\circ} \left[.283 - .426 \sin \delta_{12} \right] \, d\,\delta_{12} \right| = .026$$

Thus, for the increment of load $\Delta P_2 = -0.1$ and the lossless assumptions,

the equal area criterion indicate stability.

For the increment of load $\Delta P_2 = -0.30$

$$\pi f\ G(\delta_{12}) = \left[\frac{0.3}{2} + \frac{8}{12}\left(0.5 - \frac{1.5 \times .852}{2} \sin \delta_{12}\right)\right]$$

$$\pi f\ G(\delta_{12}) = \left[.483 - .426 \sin \delta_{12}\right]$$

but the curve does not cross the $\pi f\ G(\delta_{12}) = 0$ axis, so $A_1 > A_2$ and the system would be unstable for $\Delta P_2 = -0.30$ p.u. power.

A common problem in stability studies is the problem of determining the maximum allowable ΔP_2 for a given initial steady state power for stability to be retained. A continuation of the above calculations would permit convergence on such a critical ΔP_2.

If losses are considered, the solution for the point at which the curve crosses the axis and the evaluation of areas would have to be performed graphically.

5.5 COMPARISON OF RESULTS AND METHODS

Graph (5.1) of the results, a plot of $\delta_1 - \delta_2$ as a function of time, illustrates that the system is stable for the case where ΔP_2 is 0.1 p.u. and that the system is unstable for the case where ΔP_2 is 0.3 p.u. These results correspond with the results of the standard solution.

Since the standard solution to a problem of this type simply indicates if a system is stable or unstable for a particular ΔP_2 and does not give information on δ_1, δ_2, $\delta_1 - \delta_2$, i_d, and i_q, no further comparison can be made. The additional information available from the thesis solution warrants some discussion, however.

The fact that δ_1, δ_2 and $\delta_1 - \delta_2$ are given as a function of time in the thesis solution, contributes information on how the individual machine or system contributes to the stability characteristic of the combination. In the standard approach this information is not available if the various criteria are used. When the individual δ's are desired, the standard techniques require the multi-machine approach and a numerical solution to two simultaneous swing equations of the form of equations (5.10). Also evident from the Graph (5.2) is the amount both machines fall behind the synchronous position even when the system is stable.

As in the first problem, the current variables $i_d(t)$ and $i_q(t)$ have a form much different than the form usually assumed for these variables. Graph (5.3) is a plot of the envelope of the $i_d(t)$ and $i_q(t)$ variation for both ΔP_2 values considered. Graph (5.4) is a plot which pictures the variation of $i_q(t)$ for the case of $\Delta P_2 = 0.1$. The oscillatory component of the current variables is not so pronounced as it was for the circuit fault, but the variation is large enough to be an important factor in choosing the program increment.

In any stability problem, an important consideration is the current variation in relation to the circuit breaker action or relay action. Standard solution for the two machine problem requires a separate solution for the currents when relay action is being considered. The solution presented here gives the currents in the direct and quadrature axis component forms. If desired, an inverse transformation to convert these to terminal variables could be included in the master routine.

In noting the differences in principles in standard solution and the solution presented here for the second problem, these differences are practically a repeat of those listed for the first problem.

In contrast with the standard techniques, the thesis techniques:

1. Solves differential equations as differential equations.
2. Does not require the development or use of the transient reactance concept.
3. By direct solution of equation (5.14), gives values of armature current, δ_1, and δ_2, as functions of time.
4. Can solve a large class of problems without changing procedure. The programmed equations (5.14) represent the system. Different stability problems are simulated by using the proper master routine.
5. Considers losses or neglects losses without a change in basic form.

VI. SUMMARY

The common simplifying assumptions were considered in two groups in this thesis. The first group of simplifying assumptions were the assumptions that fundamental harmonic representation is accepted and that the resistance and inductance coefficients can be represented by constants. For the first problem with the first group of assumptions, the synchronous machine on an infinite bus is represented by the non-linear differential equations (2.6)a and (3.1). For the second problem with the first group of assumptions, the two machine system is represented by the non-linear differential equations (5.6)a and (5.7). The technique of this thesis consists of solving these systems of non-linear differential equations with the use of the digital computer and numerical analysis methods. A second group of simplifying assumptions were necessary in order to obtain formulas used in standard stability techniques. These assumptions were constant speed, constant direct current field, and steady state armature

voltages and currents. The thesis solution is more general, in that the second group of assumptions are not necessary, and further, the identity of all the variables is maintained for more detailed study.

For the electromechanical transient of a stability study, the numerical solution of the equations (2.6)a, (3.1), (5.6)a and (5.7), gives the stability information directly. The standard solution of the electromechanical transient problem utilizes the assumptions that the electrical variables can be described by a steady state algebraic system of equations with the coefficients altered to values which apply under transient conditions. These transient parameters and fictitious armature voltage components are used in the numerical solution of the mechanical equation or the swing equation. The results of the computer solutions illustrated that of all the second group of assumptions, the assumption of the steady state character of the electrical variables was most radically out-of-line. Standard solutions of stability problems often use criteria based on the second group of assumptions and the further assumption that the system losses can be neglected.

In contrast with the solutions to similar problems in the literature, the field current variation was shown to be negligible in the results of the first problem. It can be concluded that the assumption of constant field current is a reasonable assumption; however, rather than eliminate the field current term as is done in standard solutions, it is convenient to leave the field current terms in the equations so that these terms can be incremented to simulate regulator action.

The change in speed was found to be a more pronounced variation than the variation in field current. The constant speed assumption did not appear to be unreasonable even for the low inertia and severe fault of the first problem, however.

The fact that the thesis solution gives the additional information about the field current, speed, and armature current components, while the standard solution does not give any of these directly, is certainly significant. The armature currents must be determined for relay and circuit breaker study.

The variation of $i_q(t)$ and $i_d(t)$ illustrated in the results of both problems was probably the most pronounced deviation from accepted theory. Clearly, both $i_d(t)$ and $i_q(t)$ have forms or components containing the form of

$$|I| \sin (\dot{\theta}t + \Psi)$$

In the standard derivation and use of the $i_d(t)$ and $i_q(t)$, it is common to start with Park's equations,

$$\text{(6.1)} \quad \begin{aligned} v_d(t) &= (R_s^+ + L_s^+ \tfrac{d}{dt})\, i_d(t) + \dot{\theta}\, L_s^+\, i_q(t) \\ v_q(t) &= (R_s^+ + L_s^+ \tfrac{d}{dt})\, i_q(t) - \dot{\theta}\, L_s^+\, i_d(t) - \dot{\theta}\sqrt{2}L_{sr}\, I_f \end{aligned}$$

Certainly it is not reasonable to assume the terms

$$\text{(6.2)} \quad \begin{aligned} &L_s^+ \tfrac{d}{dt}\, i_d(t) \\ &L_s^+ \tfrac{d}{dt}\, i_q(t) \end{aligned}$$

are zero or negligible for the current variations displayed in the results presented here. It can be seen, however, that neglecting the relations (6.2) in (6.1) and redefining

$$\text{(6.3)} \quad \begin{aligned} &\dot{\theta}\, L_s^+\, i_q(t) \\ &\dot{\theta}\, L_s^+\, i_d(t) \end{aligned}$$

in terms of transient reactances could lead to results similar to those obtained by direct solution of (6.1).

The form of the current variables played an important part in the determination of the proper time increment for the numerical solution. In particular in the first problem, the time increment had to be small compared with the period of the current variation, approximately 0.0166 seconds.

The standard techniques for both the one machine and the two machine problems require that different methods or criteria be used for different faults or different stability problems. If losses are to be considered, still a different form is needed. In sharp contrast, the thesis procedure is very flexible and is adapted readily to many types of stability problem. The f_r routine is programmed as required by the particular system under consideration. There are trivial differences in programming for the lossy and the lossless cases. With the system adequately described by programming f_r, all the various types of faults and stability problems can be solved by using a proper master routine.

The methods of solution used here and the advantages of these methods illustrated here extend to areas other than the areas of power system stability. The synchronous motor connected to an infinite bus has the same differential equations as the generator on the infinite bus. A master routine that controlled the P_{in} term could use the one machine program to study the synchronous motor for various periodic or impact loads. The control system problem of two synchros has the same differential equations as the two machine system. The terms sometimes neglected in power systems may not be negligible with control system apparatus, however.

The equations and programs presented here are for round rotor machines and for symmetric faults. The programs also cover only one and two machine systems. Further investigation aimed toward extending these methods into the areas of salient pole machines, non-symmetric faults, or multi-machine systems, appears to be indicated by the results presented here.

A conclusion based on the results of the investigation reported here is that, for the class of problems considered in this thesis, methods developed before the advent of the digital computer should not be used as the basis for digital computer investigation. The more fundamental synchronous machine differential equations can now be solved directly.

APPENDIX A

An Existence Theorem

E. L. Ince[5] gives an existence theorem which applies to the problem considered here. Appendix A is a restatement of that theorem.

The system of equations to which the existence theorem is to apply is a system of n ordinary differential equations in the n+1 variables.

$$x, y_1, y_2, \ldots\ldots\ldots, y_n.$$

The desired solutions are the n equalities

$$y_1 = g_1(x), \; y_2 = g_2(x), \; \ldots\ldots, \; y_n = g_n(x).$$

These equalities are to satisfy the conditions

$$y_1^o = g_1(x^o), \; y_2^o = g_2(x^o), \; \ldots\ldots\ldots\ldots, \; y_n^o = g_n(x^o)$$

where

$$x^o, y_1^o, y_2^o, \ldots\ldots\ldots\ldots, y_n^o$$

represent the initial values of the variables.

Further, for the theorem to apply, the differential equations must be expressible in the normal form:

$$\begin{aligned} \frac{dy_1}{dx} &= f_1(x, y_1, y_2, \ldots\ldots, y_n) \\ \frac{dy_2}{dx} &= f_2(x, y_1, y_2, \ldots\ldots, y_n) \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \frac{dy_n}{dx} &= f_n(x, y_1, y_2, \ldots\ldots, y_n) \end{aligned} \tag{A.1}$$

Equations involving higher order derivatives than first order are included by a change of variables. Specifically, consider, for example, the equation

$$\frac{d^2y}{dx^2} = f(x, y, \frac{dx}{dy}) \tag{A.2}$$

To obtain the required normal form, let

$$z = \frac{dy}{dx}$$

and write (3.2) as a pair of simultaneous equations.

$$\text{(A.3)} \qquad \frac{dz}{dx} = f(x, y, z) \qquad \frac{dy}{dx} = z$$

Theorem A.1: For a system of equations of the type of (3.1), let $(x^o, y_1^o, y_2^o, \ldots\ldots, y_n^o)$ be a set of real numbers assigned to the real variables $(x, y_1, y_2, \ldots\ldots, y_n)$. Let D be a domain defined by the inequalities:

$$\left|x-x^o\right| \leq a, \quad \left|y_1 - y_1^o\right| \leq b_1, \ \ldots\ldots\ldots, \quad \left|y_n - y_n^o\right| \leq b_n.$$

Let M be the greatest of the upper bounds of $f_1, f_2, \ldots\ldots, f_n$ for arguments restricted to D, and let h be the least of

$$a, b_1/M, b_2/M, \ldots\ldots, b_n/M.$$

If $f_1, f_2, \ldots\ldots, f_n$ are single valued and continuous in the (n+1) arguments when the arguments are restricted to D and if the Lipschitz conditions

$$\left|f_r(x, y_1, y_2, \ldots\ldots, y_n) - f(x, Y_1, Y_2, \ldots\ldots, Y_n)\right| < K_1\left|Y_1-y_1\right| + K_2\left|Y_2-y_2\right| + \ldots\ldots\ldots + K_n\left|Y_n-y_n\right|$$

apply for each $r = 1, 2, 3, \ldots\ldots\ldots, n$ when

$$(x, Y_1, Y_2, \ldots\ldots, Y_n) \text{ and } (x, y_1, y_2, \ldots\ldots, y_n)$$

are in D, then there exists a set of unique continuous solutions,

$$y_1 = g_1(x),\ y_2 = g_2(x),\ \ldots\ldots\ldots,\ y_n = g_n(x).$$

These solutions satisfy the differential equations for all x such that $\left|x - x^o\right| < h$, and reduce to:

$$y_1^o,\ y_2^o,\ \ldots\ldots,\ y_n^o \qquad \text{for} \quad x = x^o.$$

The types of terms occurring in the synchronous machine equations (5.7) and (6.14) are included in the following:

$$f_1 = A_1 \sin kx + A_2 y_1 y_4 + A_3 y_2$$
$$f_2 = B_1 \cos kx + B_2 y_4 y_3 + B_3 y_4 y_2 + B_4 y_1$$
$$f_3 = C_1 \sin kx + C_2 y_4 y_1$$
$$f_4 = D_1 y_5$$
$$f_5 = E_1 + E_2 y_4 y_3 y_1$$

Clearly, the elementary functional forms in (3.4), are single valued and continuous functions for finite (x^o, y_1^o, ..., y_5^o) and for finite (a, b_1, b_2, ..., b_5) defining the domain D of the theorem. Further, for these types of equations the Lipschitz conditions apply for the arguments in D.

The domain, D, of the existence theorem is, in general, very small relative to the range of the variables to be considered in any given problem. Different initial values of the variables and corresponding domains covering the range of the variables extend the existence theorem to a larger range of the variables.

Even with the assurance that a unique continuous solution exists for a desired range of variables, numerical approximation solutions are subject to limitations and errors of the particular numerical method used.

APPENDIX B

The Operation of the Digital Computer Program

The digital computer library routine, F-6, uses the fourth order Runge-Kutta formulas. The Runge-Kutta formulas, F-6, the differential equation routine, and the master are described in Appendix B.

For the system of differential equations in the normal form of equations (A.1), the r^{th} equation is the first derivative of the r^{th} variable:

$$\frac{d}{dt} y_r = f_r (t, y_1, ----, y_r, --- y_k)$$

For given values for all the variables at the beginning of the n^{th} increment of time, the Runge-Kutta formulas determine the value of the variable y_r at the end of the n^{th} increment or the beginning of the $(n+1)^{st}$ increment. Let the subscripts indicate the variable, the superscript the interval, and f_r the first derivative of the r^{th} variable, then by Runge-Kutta

$$y_r^{(n+1)} = y_r^n + \Delta y_r \tag{B.1}$$

where

$$\Delta y_r = \frac{1}{6} \Delta' y_r + \frac{1}{3} \Delta'' y_r + \frac{1}{3} \Delta''' y_r + \frac{1}{6} \Delta^{iv} y_r \tag{B.2}$$

and

$$\begin{aligned}
\Delta' y_r &= f_r \left[t^n, y_1^n, -----, y_r^n, ---, y_k^n \right] \Delta t \\
\Delta'' y_r &= f_r \left[t^n + \frac{\Delta t}{2}, y_1^n + \frac{\Delta' y_1}{2}, ---, y_r^n + \frac{\Delta' y_r}{2}, ---, y_k^n + \frac{\Delta' y_k}{2} \right] \Delta t \\
\Delta''' y_r &= f_r \left[t^n + \frac{\Delta t}{2}, y_1^n + \frac{\Delta'' y_1}{2}, ---, y_r^n + \frac{\Delta'' y_r}{2}, ---, y_k^n + \frac{\Delta'' y_k}{2} \right] \Delta t \\
\Delta^{iv} y_r &= f_r \left[t^n + \Delta t, y_1^n + \Delta''' y_1, ---, y_r^n + \Delta''' y_r, --- y_k^n + \Delta''' y_k \right] \Delta t
\end{aligned} \tag{B.3}$$

The f_r for this thesis problem are the expressions of equation (4.2).

The digital computer library routine, F-6, used here controls the amount of truncation error and the increment by comparing Δy_r of expression (B.2) with $\Delta''' y_r$ of equations (B.3). If these two approximations, Δy_r and

$\Delta'''y_r$, for the increment of the variable y_r differ by more, for any y_r, than an amount specified by the user, the program recomputes Δy_r with the interval reduced by half. If necessary, the routine continues decreasing the increment until the difference is within the specified limits.

The user of the F-6 routine must supply a closed subroutine which evaluates the f_r of formula (A.1). The F-6 routine referes to the f_r routine to evaluate $\Delta' y_r$, $\Delta'' y_r$, $\Delta''' y_r$, and $\Delta^{iv} y_r$. Scaling must be used so that all f_r remain less than one for the range of the variables considered.

In order to use F-6, the digital computer memory locations 3 through 8 must contain the following parameters during read in and operation of F-6.

Location 3 must contain the number "a", where "a" is the memory location of the first of the sequence of locations for the initial values of the variables. The t initial value is in "a" and the initial values of the other variables follow in sequence.

Location 4 must contain (a+n) where n is the number of equations including $^{dt}/dt = 1$.

Location 5 must contain the number "m". "m" is an integer and the f_r routine calculates scaled f_r, $\bar{f}_r$, such that

$$\bar{f}_r = 2^{-m} f_r < 1$$

Location 6 must contain the integer $\ell_o - m$. The quantity $2^{-\ell_o}$ is the specified scaled increment Δt. The program decreases this increment if necessary.

Location 7 must contain the integer "e" which is a number such that the integral part of $^3/_4$ e is equal to or less than (50 - m). The integer

"e" is used to specify the required accuracy of the calculations.

Location 8 must contain the integer "b" which is the memory location of the entry into the f_r routine.

When the master routine transfers control to F-6, F-6 uses the scaled initial values of the variables and the f_r routine to compute new scaled values of the variables. The initial values of the variables are replaced by the new values, then control is transferred back to the master.

The f_r routine is the program of the differential equations (A.1). The differential equations represent the system, thus the f_r adapts the program to the system. The f_r routine must be written as a subroutine, which, when entering, evaluates the scaled f_r and then returns control to F-6. The f_r routine takes the scaled variables from the computer memory locations containing the initial values required by the F-6 program and calculates the scaled f_r according to formulas (A.1). These f_r are placed in the memory positions a + n + r, where "a" through "a + n - 1" contain the initial values of the variables. The evaluation of the f_r for this particular problem requires the determination of $v_d(t)$ and $v_q(t)$ where

$$v_d(t) = |V| \sin \delta$$
$$v_q(t) = - |V| \cos \delta$$

In order to evaluate the trigonometric functions, the f_r program uses the digital computer library routine for sine and cosine evaluation. The f_r program was written so that it could be applied to any machine operating on an infinite bus. The integer "g" must be placed in the digital computer memory location 10. "g" is the location of the first of the scaled coefficients and constants of formula (A.1). The other coefficients are scaled and read in in a definite sequence. The scalings and the sequence are presented in the solution of the problem.

After each increment of time, F-6 returns control to the master routine. Thus, the master can be written so that it removes the scale factor, prints any of the desired variables, and prints out these variables as frequently as would be useful. At any desired time, the time determined by counting the increments, the master routine can simulate faults, circuit breaker action, or changes in load conditions. The master simulates these actions by changing coefficients or constants in the f_r routine at a specified time. For example, the short circuit at the bus in the first thesis problem is represented by the bus voltage being set equal to zero. At a time, determined by a counter in the master routine, the bus voltage is returned to the pre-fault value. The action of returning the bus voltage simulates circuit breaker action and the action is performed by the master routine.

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