STATIC AND DYNAMIC MODULUS OF ELASTICITY OF HARDBOARD

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THESIS

ABSTRACT

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by James Gibson Bair Jr.

The purpose of this study was to determine the relationship between static and dynamic modulus of elasticity of standard S-1-S hardboard and how these moduli vary with increasing moisture content. It was also decided to determine (1) if the effects of shear and rotatory inertia could be neglected in calculating the dynamic M. E. and (2) the behavior of the logarithmic decrement with increasing moisture content.

The dynamic and static modulus of elasticity and the logarithmic decrement were determined at each of five moisture content levels. The dynamic M. E. was determined from the fundamental frequency and the first five overtones for each moisture content level.

It was concluded that the dynamic M. E., calculated from any of the six modes of vibration is valid, that is, that the effects of shear and rotatory inertia are negligible and that the dynamic and static M. E. can be highly correlated. Due to creep, the static M. E. drops off at a more rapid rate than the dynamic M. E.

with increasing moisture content. It was also observed that the logarithmic decrement increases with increasing moisture content as a result of greater plastic deformation in the hardboard at higher moisture contents.

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By

James Gibson Bair Jr.

A THESIS

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CHAPTER I

INTRODUCTION

Hardboard is a wood based product, manufactured in sheet form from wood fibers. The properties of hardboard are functions of the raw material and process variables. Commercial standards have been adopted which specify maximum and minimum values for these properties, regardless of the raw material or process used.

In the present commercial standards the modulus of rupture is used as an indicator of quality, whereas the modulus of elasticity* is not required to be determined. This is due to two reasons (1) the modulus of rupture is determined by a simple test, whereas the determination of M.E. is more difficult and relatively time consuming and (2) hardboard is not considered an engineering material. However, it is because of (1) that the bending strength of hardboard is specified in terms of the modulus of rupture.

One important disadvantage of the modulus of rupture is that the material under consideration must be stressed

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* Hereafter the modulus of elasticity will be designated by M.E. or E.

beyond the elastic limit to obtain this property, hence the result is not as accurate an indicator of the elastic properties as is the M.E.

Defects or discontinuities in the material will have more effect on the modulus of rupture than they will on the M.E.

Since the M.E. is a better indicator of the elastic properties of a material, it would be highly desirable to obtain this property by a rapid, simple test.

Determination of the M.E. by vibrational methods could be such a test, and it is for this reason that this study was initiated.

This vibrational method of testing or dynamic testing, as it is often called, has two important advantages. (1) The method can be easily carried out and the M.E. calculated easily under certain conditions and (2) this method of testing is non-destructive and this allows the same samples to be tested under different conditions, for example varying moisture content or temperature, and eliminates "between groups" variation.

The most important disadvantage is that the effects of shear and rotatory inertia may cause the calculated M.E. to be low. It is quite difficult to compensate for these effects if they are appreciable.

These effects are due to too large a depth/length ratio and if this ratio is such that the effects of shear and rotatory inertia are small enough to be neglected, then the M.E. can be obtained easily.

In general, the dynamic M.E. does not equal the static M.E. because in the short time periods involved in the dynamic test, time dependent properties such as creep and relaxation do not affect the M.E. as they do in the conventional static test. Because of this and the fact that the dynamic test does not measure M.E. directly, but rather the frequency of the vibrating beam, statistical and mathematical relationships are necessary to correlate the dynamic M.E. to the static M.E.

Another property of interest is the logarithmic decrement which indicates the internal friction of the material. The greater the internal friction, the greater is the plastic deformation.

Specifically, the purpose of this study was to (1) determine the effect of shear and rotatory inertia with increasing depth/length ratio and increasing moisture content (2) to establish the relationship between dynamic M.E. and static M.E. as a function of the moisture content and (3) to determine the logarithmic decrement as a function of the moisture content.

CHAPTER II

THEORY OF DYNAMIC TESTING

Transverse Vibration

The classical differential equation describing the transverse bending of a uniform beam is developed in the following manner (see references 1, 2, 3, 4): In Figure (1A) the element is compressed by a length $r\theta$, from the original length δ_x as a result of the bending of the beam.

Therefore

$$\varepsilon = \frac{r\theta}{\delta x}, \tag{1}$$

where ε = strain in the element

and since

$$F = \sigma A, \tag{2}$$

where F = force necessary to compress the element

σ = stress

A = cross sectional area of the element

and

$$\sigma = E\varepsilon \tag{3}$$

the force on the element may be written

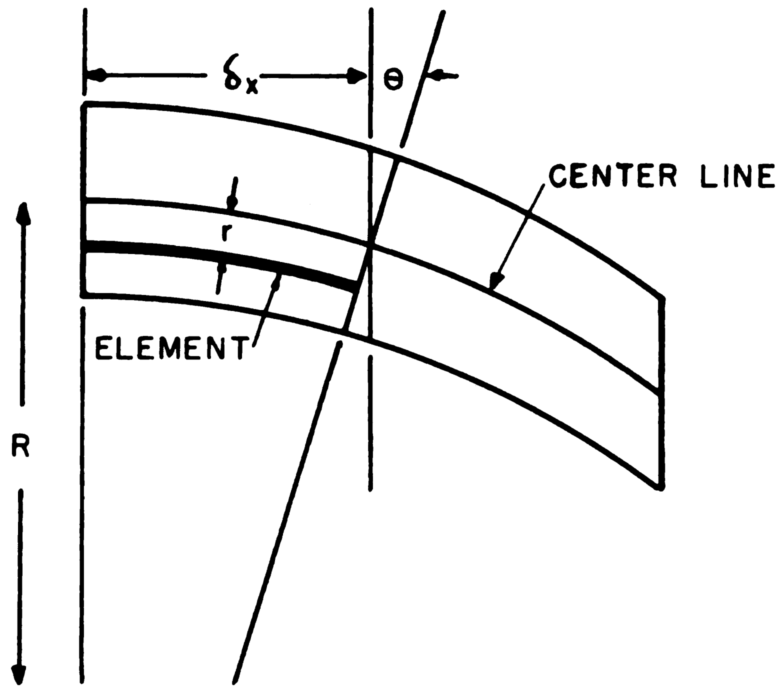
$$F = EA\,\frac{r\theta}{\delta_x}\,. \tag{4}$$

The moment of this force about the center line is

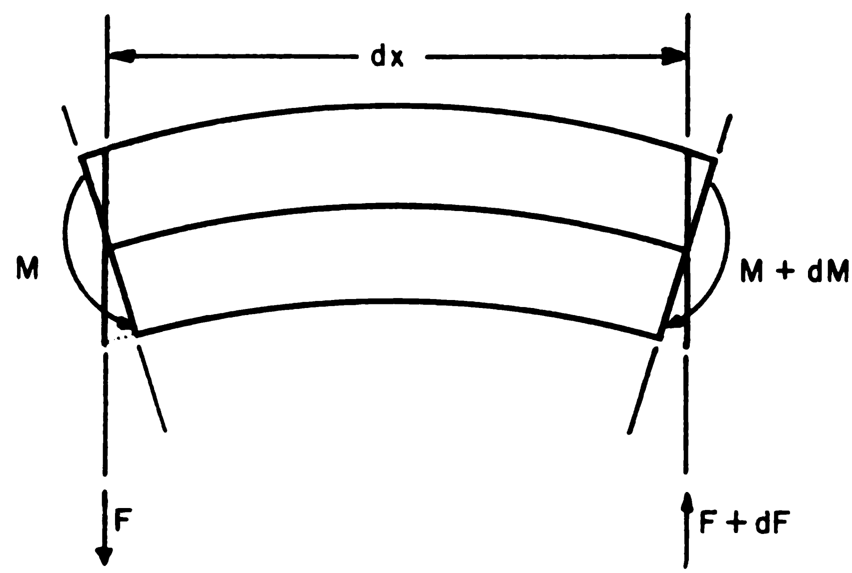
$$M = EA\,\frac{r\theta}{\delta_x}\,r \tag{5}$$

Fig. 1.--A. Section of bar deformed during transverse vibration to show compression of an element at distance r from the center line. δ_x is the original length of the element.

B. Section of bar showing moment M and force F acting on one end and the resultant force F+ df and opposing moment M + dM. acting on the other end. The length of the elemental section is dx.



A



B

and the total moment of all elements about the center line is

$$M = \frac{E\theta}{dx} \int r^2 A \quad . \tag{6}$$

The radius of gyration is defined by

$$k^2 = \frac{1}{S} \int r^2 A,$$

where k = radius of gyration

S = total cross sectional area of the bar.

For a rectangular cross section

$$k = \frac{t}{12}$$

where t = thickness of beam

Therefore

$$M = \frac{E\theta S k^2}{dx} \tag{8}$$

For small deflections

$$\theta = \frac{-d\left(\frac{dy}{dx}\right) dx}{dx} = -dx\left(\frac{d^2y}{dx^2}\right) \tag{9}$$

or

$$\frac{\theta}{dx} = \frac{-d^2y}{dx^2} \tag{10}$$

By substituting (10) into (8) the following is obtained:

$$M = -ESk^2 \frac{d^2y}{dx^2} \tag{11}$$

The bending moment is a function of x, when x is measured along the bar. The difference in the bending moments at the two ends of an element of the bar is equal to the shearing force times the length of the element (see Fig. 1B) or

$$dM = Fdx \tag{12}$$

Therefore $F = \frac{dM}{dx}$ (13)

Taking the derivitive of (11) with respect to x and substituting in (13) we have:

$$F = -ESk^2 \frac{d^3y}{dx^3} \qquad (14)$$

Since F is also a function of x there is a resultant vertical force

$$dF = \frac{dF}{dx} \delta_x \qquad (15)$$

acting on the element. This force is equal to the elements acceleration times its mass.

$$\frac{dF}{dx} \delta_x = \rho S \delta_x \frac{d^2y}{dt^2}, \qquad (16)$$

where ρ = density

t = time.

Taking the derivitive of (14) with respect to x, we have

$$\frac{dF}{dx} = -ESk^2 \frac{d^4y}{dx^4} \qquad (17)$$

By substituting (17) into (16) we obtain

$$-ESk^2 \frac{d^4y}{dx^4} \delta_x = \rho S \delta_x \frac{d^2y}{dt^2} \qquad (18)$$

which may be written

$$\frac{Ek^2}{\rho} \frac{d^4y}{dx^4} + \frac{d^2y}{dt^2} = 0 \qquad (19)$$

which is the classical differential equation desired. In order to solve equation (19) the following solution is assumed:

$$y = a \cos nt,$$

where y = deflection at time t and distance X from the origin

a = maximum amplitude of vibration for any given x

n = angular velocity

t = time

Then

$$\frac{d^2y}{dt^2} = -n^2 a \cos nt \tag{21}$$

and by substituting (20) into (21) we obtain

$$\frac{d^2y}{dt^2} = -n^2y \quad . \tag{22}$$

Since y in equation (20) is a function of x and t, (a) is assumed to be a function of x, as cos nt is clearly a function of t.

Let

$$\frac{d\ (a \cos nt)}{dx} = \frac{d^4y}{dx^4} \tag{23}$$

Then by substituting (22) into (19) we obtain

$$\frac{d^4y}{dx^4} = \frac{\rho n^2}{Ek^2}\ y. \tag{24}$$

Define the quantity m

$$m = \frac{\rho n^2}{EK^2} \tag{25}$$

so that

$$\frac{d^4y}{dx^4} = m^4y \tag{26}$$

Equation (26) in operator form is

$$(D^4 - m^4)y = 0 \tag{27}$$

with the characteristic equation

$$D^4 = m^4 \tag{28}$$

and the roots are

$$\begin{aligned} D_1 &= m \\ D_2 &= -m \\ D_3 &= im \\ D_4 &= -im \end{aligned} \tag{29}$$

Then

$$y = c_1 e^{mx} + c_2 e^{-mx} + c_3 e^{imx} + c_4 e^{-imx} \quad (30)$$

$$y = c_1(\cosh mx + \sinh mx) + c_2(\cosh mx - \sinh mx)$$
$$+ c_3(\cos mx + i \sin mx) + c_4(\cos mx - i \sin mx) \quad (31)$$

Expanding and collecting terms this reduces to

$$y = A \cosh mx + B \sinh mx + C \cos mx + D \sin mx, \quad (32)$$

where
$$A = c_1 + c_2$$
$$B = c_1 - c_2$$
$$C = c_3 + c_4$$
$$D = ic_3 - ic_4$$

This is the deflection of the bar as a function of x only and is equal to (a) of equation (20). Considering y as a function of x and t both

$$y = [A \cosh mx + B \sinh mx + C \cos mx + D \sin mx] \cos nt \quad (33)$$

Boundary Conditions for a free-free beam

Let

$x = 0$ at $1/2\ l$

$\frac{d^2y}{dx^2} = 0$ at $x = \pm 1/2\ l$ (no curvature at ends)

$\frac{d^3y}{dx^3} = 0$ at $x = \pm 1/2\ l$ (no shear forces at ends)

The solution (33) includes both symmetrical and assymmetrical vibrations of the bar. If only the symmetrical vibrations are considered (33) becomes

$$y = [A \cosh mx + C \cos mx] \cos nt \quad (34)$$

From the assumed boundary conditions four equations are obtained to give a value of m which satisfies

$$\tan 1/2\, ml = \tanh 1/2\, ml \tag{35}$$

The roots of (35) are obtained graphically and

$$x = \frac{ml}{2} = (s-1/4)\pi + \beta \tag{36}$$

where s = 1,2,3. . . (modes of vibration)

β = small quantity only appreciable at s = 1.

Then, from (25)

$$m = \frac{2x}{l} = \frac{2(s-1/4)}{l}\pi = \sqrt[4]{\frac{\rho n^2}{Ek^2}} \tag{37}$$

and finally the frequency of vibration is

$$f_r = \frac{n}{2\pi} \quad \frac{\pi(4s-1)^2}{8} \quad \frac{k}{l^2}\sqrt{\frac{E}{\rho}} \tag{38}$$

A similar method may be employed for the asymmetric frequencies and these are found to be proportional to (4s + 1). Let

$$\frac{(4s - 1)^2\pi}{8} = \frac{[1/2\,(2p + 1)\,\pi\,]^2}{2\pi} \tag{39}$$

where p = s

and define

$$m_p = 1/2\,(2p + 1)\pi \tag{40}$$

so that (38) becomes

$$f_r = \frac{km^2}{2\pi l^2}\sqrt{\frac{E}{\rho}} \tag{41}$$

As mentioned on the previous page, these equations do not take into account the effects of the shear force or rotatory inertia which act on the bar.

As the depth/length ratio of the beam increases, these effects become more pronounced. This ratic increases with higher overtones causing the observed frequencies to be too low which in turn cause the calculated M.E. to be too low. The differential equation which takes into account these two factors is (4)

$$\frac{Ek^2}{\rho}\frac{d^4y}{dx^4} + \frac{d^2y}{dt^2} - k\left(1 + \frac{sE}{G}\right)\frac{d^4y}{dx^2dt^2} + \frac{\rho s k^2}{G}\frac{d^4y}{dt^4} = 0, \quad (42)$$

where G = shear modulus

s = shear deflection coefficient to allow for the fact that the shear stress is not uniform over the cross section.

Although there have been several solutions to this equation the following develops the relationship into a rather simple transcendental equation which, though it is approximate, gives the effect of shear and rotatory inertia (13). The theoretical frequency f_g is defined as

$$f_g = \frac{f_r}{\sqrt{T}} \quad (43)$$

where f_r is from equation (41)

T = the expression below

$$T = 1 + \frac{k^2}{l}2[m^2F^2(m) + 6mF\ (m)] + \frac{k^2sE}{l^2G}[m^2F^2(m) - 2mF(m)]$$

$$- \frac{4\pi^2\rho s k^2 fg^2}{G} \quad (44)$$

Equation (42) may be simplified by the following expressions:

$$m^2F^2(m) + 6mF(m) = F_1m \tag{45}$$

$$m^2F^2(m) - 2mF(m) = F_2m \tag{46}$$

Equation (44) then becomes

$$T = 1 + \frac{k^2}{l^2} F_1(m) + \frac{k^2sE}{l^2G} F_2(m) - \frac{4\pi^2\rho sk^2fg^2}{G} \tag{47}$$

Combining (41) and (43) the following is obtained:

$$f_g = \frac{k^2m^4E}{4\pi^2l^4\rho} T \tag{48}$$

Substituting (48) into (47)

$$T = 1 + \frac{k^2}{l^2}\left(F_1m + \frac{sE}{G} F_2\ m[1 - \frac{m^4k^2}{F_2(m)\ l^2T}\ ]\right) \tag{49}$$

The last term in (49) is an approximation, but its numerical value is small in comparison with 1 and T is very close to 1. Hence a good approximation of T can be obtained by setting the right hand side of the equation (47) equal to 1. The T obtained can then be inserted into the right hand side of (47) to improve the first estimate. This method may be repeated to obtain an even better estimate of T.

Logarithmic Decrement

This is defined as the logarithm of the ratio of the amplitudes of two consecutive cycles in the free vibrating system of a cantilever beam or as the logarithm of the ratio of the amplitudes of two cycles, n oscillations appart and divided by n or

$$\delta = \ln \frac{x_1}{x_2} = \frac{1}{n} \ln \frac{x_1}{x_n}$$

where δ = logarithmic decrement

x_1 = amplitude of first cycle

x_2 = amplitude of second cycle

x_n = amplitude of nth cycle

The logarithmic decrement is a measure of the loss of energy in a vibrating beam. With each cycle of the vibration a certain amount of the energy of the beam is lost as heat energy due to internal friction. Consider a purely elastic cantilevered bar which has been given a certain amount of energy to set it in motion. As the bar passes through the rest position, all the energy is of the kinetic form, but is continually converted to potential energy until it is all of this type and the bar will stop in a position of maximum deflection. It will immediately begin to move in the opposite direction and convert all the potential energy to kinetic energy by the time it passes through the rest position again. The bar will go to a position of maximum deflection in the other direction, then reverse and again pass through the rest position. This procedure will continue indefinitely. However, if the material is not purely elastic, but exhibits some plastic deformation also, part of the energy is lost as heat energy in causing the plastic deformation, hence there is less energy after each cycle and the amplitude of the free vibration falls off with time or is "damped."

CHAPTER III

EXPERIMENTAL PROCEDURE

Design of Experiment

The experiment was designed so that one group* of specimens would be tested at each of the five moisture content levels for the dynamic M.E., then logarithmic decrement, and finally for the static M.E. This helped minimize variation between the two test methods, for the values determined here were used in the comparison of the two methods. The values obtained here were also used to determine the effect of moisture content on these properties.

Another group was tested only by the dynamic method at each moisture content level to minimize variation from one level to the next. The results of the dynamic M.E. of both groups at each moisture content level were combined to determine the effect of shear and rotatory inertia at the overtones. Table 1 summarizes the design.

Samples

The samples used in this experiment were cut from one 4' x 8' standard commercial screenback hardboard as

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*Each group represents ten samples

TABLE 1

Summary of Design of Experiment

Level	Moisture Content Nominal	% Actual	Dynamic and Static M.E. Logarithmic Decrement	Dynamic M.E. Only	Dynamic M.E. from Overtones
1	0	.68	Group 1	Group 13	Groups 1 & 13
2	4	3.52	Group 2	Group 13	Groups 2 & 13
3	8	7.34	Group 3	Group 13	Groups 3 & 13
4	12	11.80	Group 4	Group 13	Groups 4 & 13
5	20	20.44	Group 5	Group 13	Groups 5 & 13

indicated in Fig. 2. The samples were grouped at random into fifteen groups. Groups 1 to 5 and 13 were used as indicated in Table 1, the remaining groups are intended for future experiments.

Conditioning

For moisture content level 1 all samples were placed over phosphorous pentoxide, P_2O_5, in two large covered aquariums. Each of the aquariums contained a fan for air circulation and a light bulb to maintain the temperature somewhat above room temperature to speed attainment of equilibrium. The temperature in the tanks was approximately 80°F. For moisture content levels 2 to 5, the samples were placed in a controlled relative humidity cabinet which was properly adjusted for each moisture content level as the testing proceded. In the conditioning cabinet the temperature was maintained at approximately 100°F.

Dynamic Modulus of Elasticity

After reaching equilibrium, a small piece of steel shim stock .5" x .5" .002" was bonded to each end of the sample on the smooth side, using Duco Household Cement.

In the determination of equation (41) the beam was considered to be free-free or unsupported. Since supports are necessary in order to test the beam they must be placed in such a position so as to cause a minimum of damping. The nodes, or positions of zero amplitude, are chosen as the points of support, for at these points only, will the damping due to the supports be a minimum. The distances from the end of the sample to the nodes are listed in Table 2. In all cases the supports were placed at the nodes nearest the ends of the specimen. The supports were fine taut wires.

The test set up is shown in Fig. 3 and in Fig. 4 a block diagram is given. Following the diagram, the test

Fig. 2.--Plan of how samples were cut from 4' x 8' sheet of hardboard. Each sample was 1" x 20".

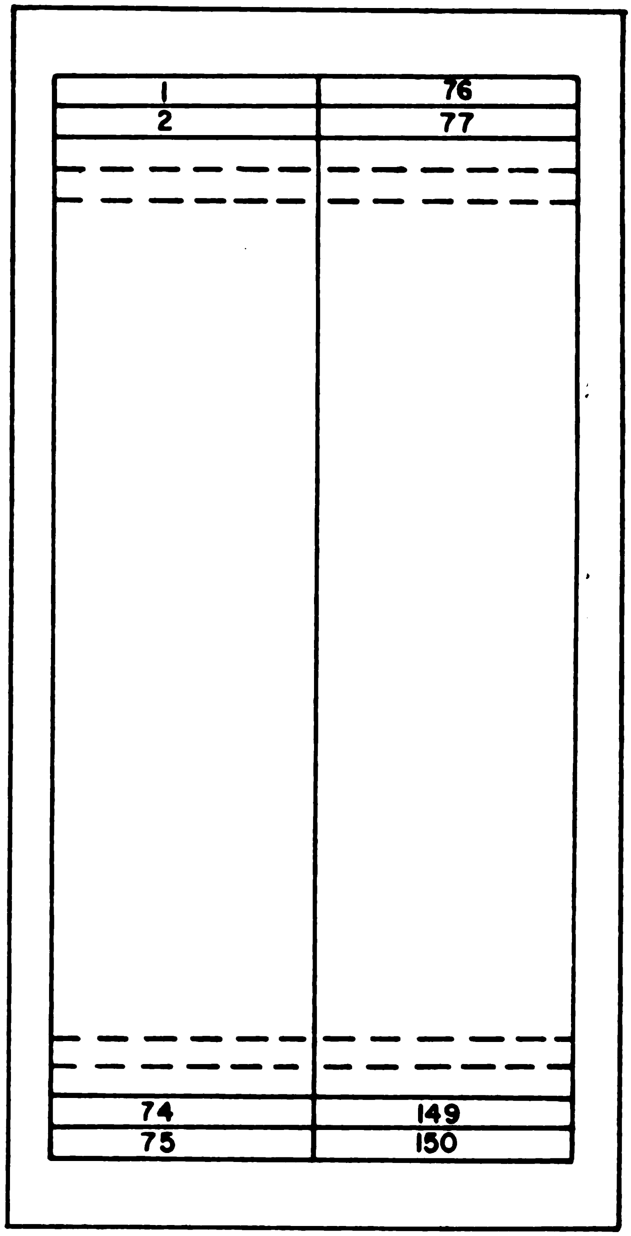

1
76
2
77
74
149
75
150

Fig. 3.--Equipment used for dynamic testing, showing sample in place cn wire supports, positioned for obtaining the fundamental frequency.

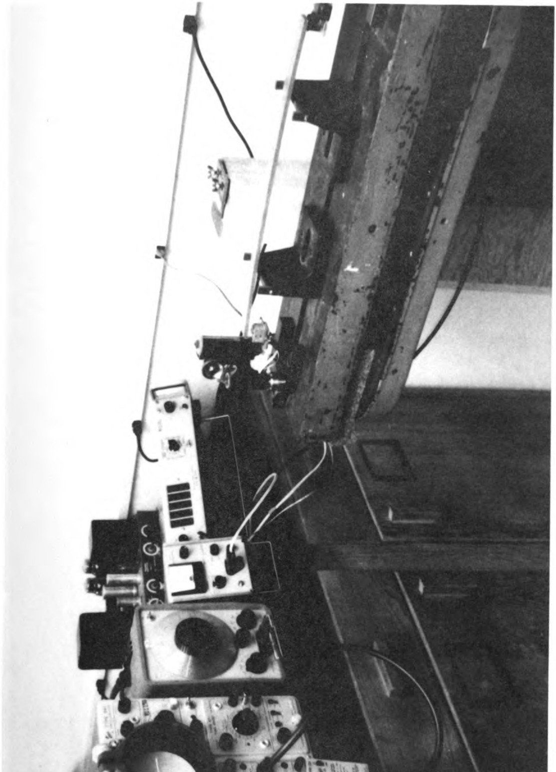


Fig. 4.--Block diagrams of instrumentation for static and dynamic tests.

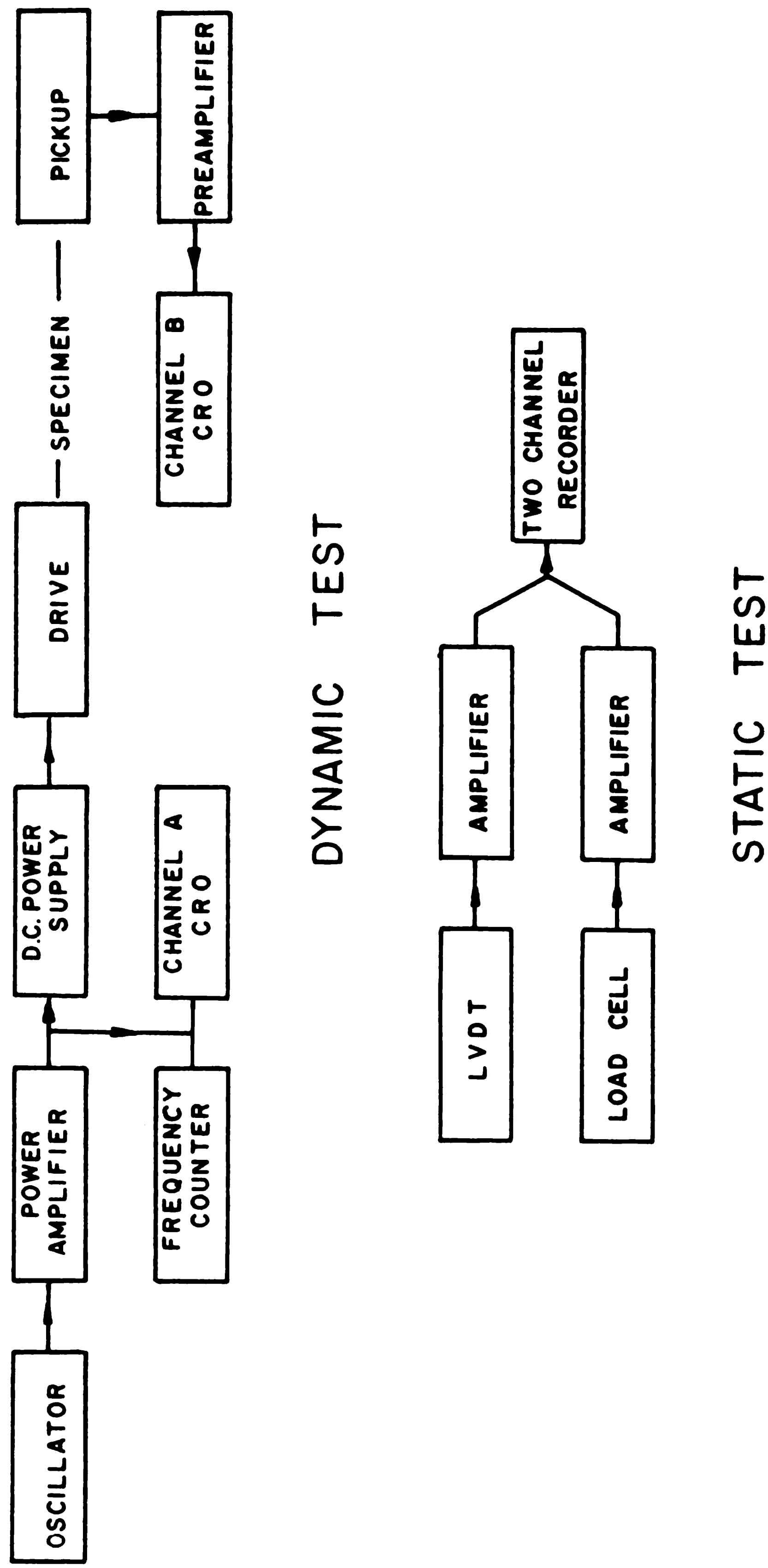

OSCILLATOR
POWER AMPLIFIER
FREQUENCY COUNTER
D.C. POWER SUPPLY
CHANNEL A CRO
DRIVE
SPECIMEN
PICKUP
PREAMPLIFIER
CHANNEL B CRO
DYNAMIC TEST
LVDT
AMPLIFIER
LOAD CELL
AMPLIFIER
TWO CHANNEL RECORDER
STATIC TEST

TABLE 2

Positions of Nodes

Mode of Vibration	Distance of nodes from one end in terms of l (length of beam)				
Fundamental frequency	.2242	.7758			
1st overtone	.1321	.5000	.3679		
2nd overtone	.0944	.3558	.6442	.4056	
3rd overtone	.0734	.2770	.5000	.7230	.9266

operation is as follows: The desired frequency is generated by the oscillator, amplified and fed into the drive coil, which is directly under the steel shim stock on the sample causing the sample to vibrate at the given frequency. The period of the frequency is monitored by the frequency counter and may be visually observed on channel A of the oscilloscope. The pickup or receiving coil is directly under the shim stock at the other end of the beam. This picks up the oscillations of the beam and sends the signal to channel B of the oscilloscope. At resonant frequency and at each overtone the vibrations reach a maximum which is easily determined by visual means on the oscilloscope. The period was recorded at each important node of vibration.

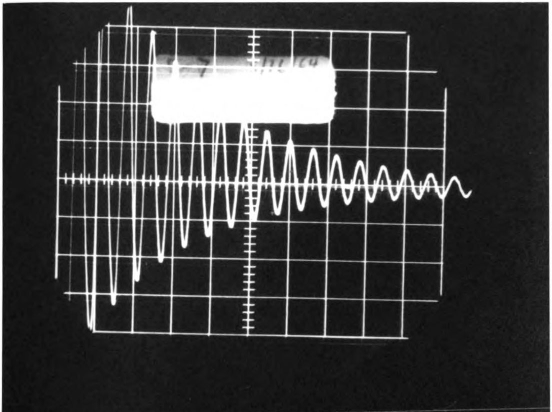
Logarithmic decrement

This property was obtained from the free vibration of the sample clamped as a cantilever beam. The sample was set into oscillation by a light tap. The oscillations were picked up by the receiving coil and traced on the oscilloscope where they were photographed with a polaroid camera mounted on the oscilloscope. (Not shown in Fig. 3) The right side of the dynamic test set up in Fig. 4 was used to obtain this property. Fig. 5 is an example of the trace obtained from which the logarithmic decrement was obtained.

Static Modulus of Elasticity

The static bending tests were carried out according to A.S.T.M. standard. The span used was 11.04", the same distance as between the nodes for the fundamental frequency. This was done so that any defects in the sample would have the same effect on either the dynamic or static test. The test set up is shown in Fig. 6 and in Fig. 4 the block diagram is given. The Linear Variable Differential Transformer (LVDT) measures the deflection which is recorded on channel I of the recorder and the load cell measures the load applied, which is recorded on channel 2 of the recorder. From the two traces obtained on the dual channel recorder (see Fig. 7 for example of chart) a load versus deflection curve was plotted to obtain the slope for the formula:

Fig. 5.--A representative photograph of the decay of free vibrations of the cantilevered beam used to obtain the logarithmic decrement.



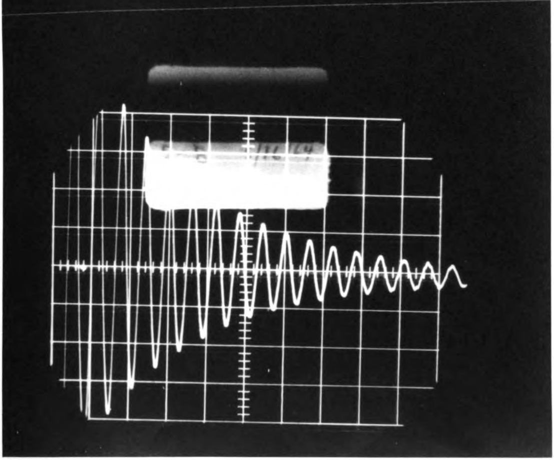


Fig. 6.--Equipment used for static testing, with sample in place in the compression cage of the testing machine.

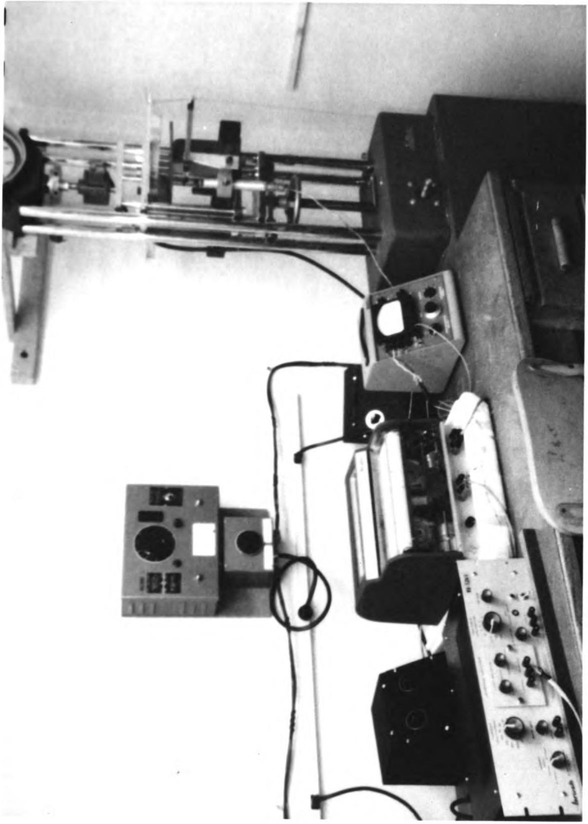
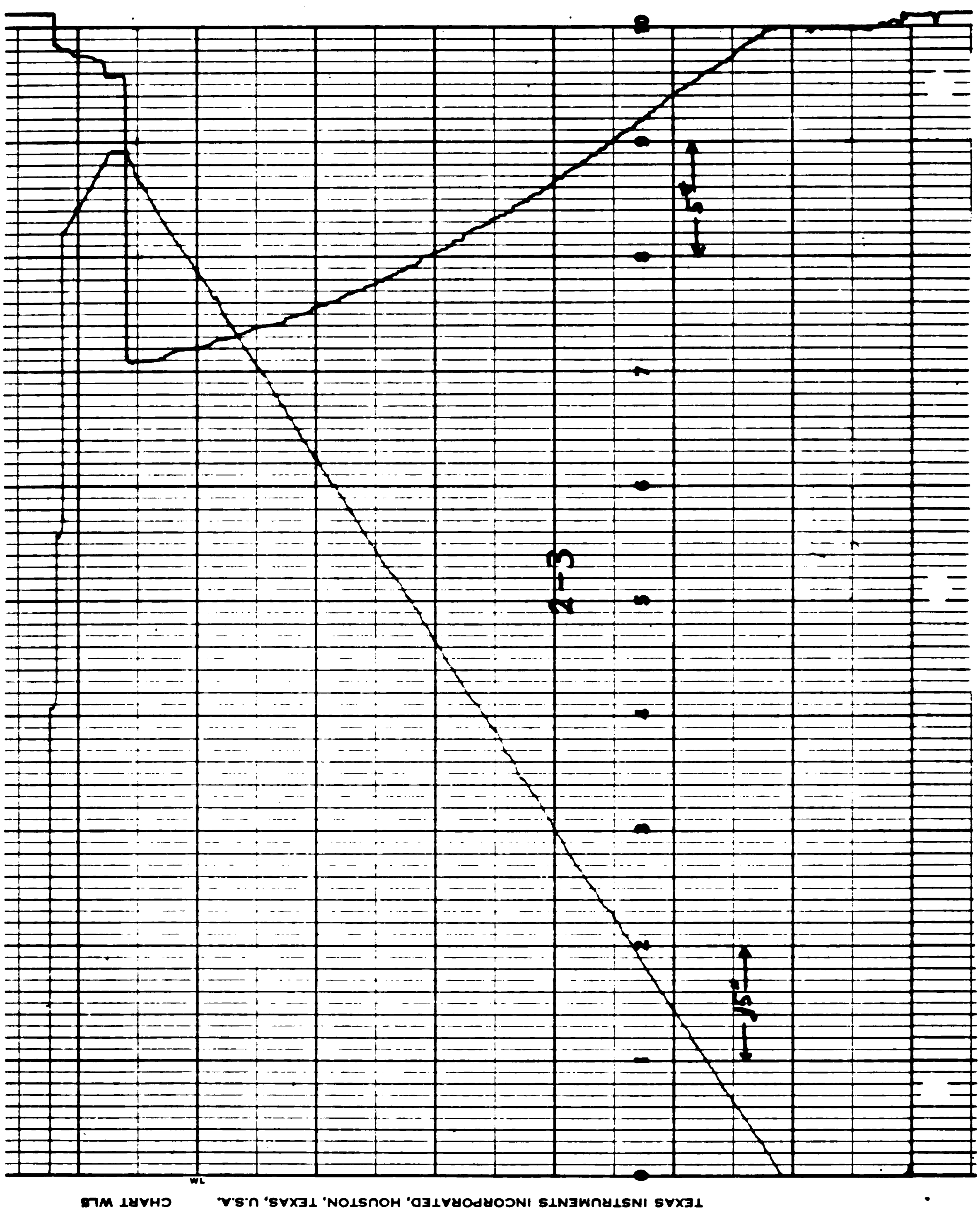


Fig. 7.--A representative section of the chart obtained from the dual channel recorder used for the static tests. The trace starting at the left represents the deflection and the trace starting at the right the represents the load.


2-3
TEXAS INSTRUMENTS INCORPORATED, HOUSTON, TEXAS, U.S.A.
CHART WLB

$$E = \frac{P_1}{Y_1} \frac{l^3}{4bh^3}$$

where P_1 = load at any point within proportional limit

Y_1 = deflection at P_1

l = Span

b = width of sample

h = thickness of sample

CHAPTER IV

RESULTS

Shear and Rotatory Inertia Effects

Table 3 tabulates the dynamic M.E. from the fundamental frequency and first five overtones for each moisture content level. These values are uncorrected for shear and rotatory inertia, and are shown graphically in Fig. 8. Each of the moisture content levels is represented with a straight, horizontal line through the overall mean for each level. In all cases the dynamic M.E. from the fundamental frequency is lower than the overall mean for each level. Each point represents the average of 20 tests.

Dynamic and Static Modulus of Elasticity

The results of the comparison are tabulated in Table 4 and presented graphically in Fig. 9. This figure presents the regressions of dynamic and static M.E. on moisture content. Each point represents the average of 10 tests, and at each moisture content level the points on both regression lines were obtained from the same samples. The correlation coefficients for the dynamic and static tests are .986 and .981, respectively. The statistical procedure

TABLE 3

Dynamic M.E. Uncorrected for Shear and Rotatory Inertia

Level	Moisture Content	Fundamental Frequency	Overtone				
			First	Second	Third	Fourth	Fifty
1	.68	711,000	715,000	714,000	715,000	711,000	717,000
2	3.52	673,000	685,000	683,000	685,000	681,000	679,000
3	7.34	593,000	606,000	607,000	610,000	607,000	604,000
4	11.80	498,000	507,000	503,000	501,000	498,000	495,000
5	20.44	332,000	351,000	349,000	351,000	346,000	344,000

Fig. 8.--Dynamic M.E. obtained from the fundamental frequency and first 5 overtones, uncorrected for shear and rotatory inertia for all levels of moisture content on hardboard. [The samples are 1" wide x 20" long and vary in thickness from .215 to .240 inches approximately, depending on the moisture content.]

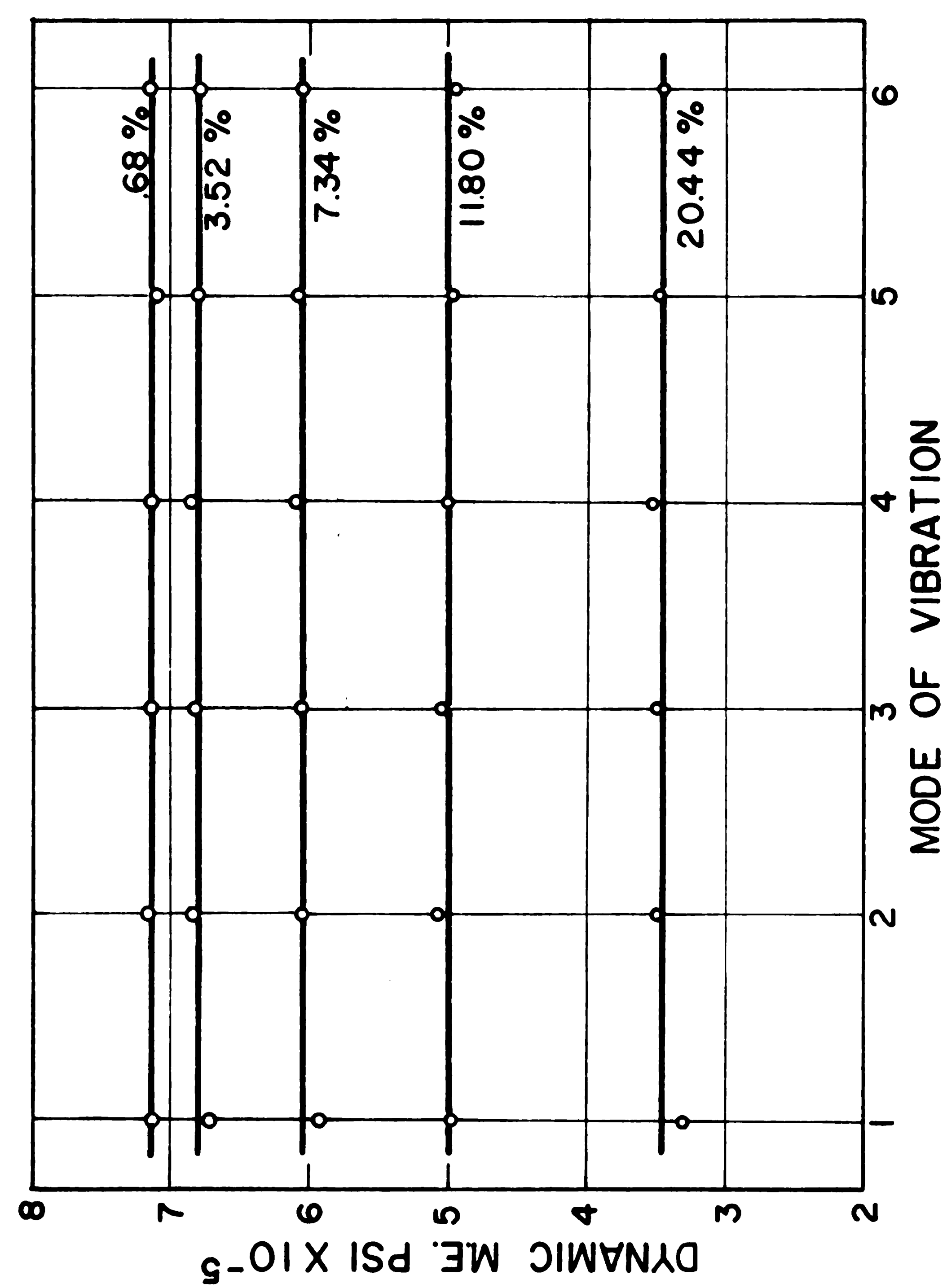

.68 %
3.52 %
7.34 %
11.80 %
20.44 %
1
2
3
4
5
6
MODE OF VIBRATION
2
3
4
5
6
7
8
DYNAMIC M.E. PSI X 10^-5

Fig. 9.--Regressions of dynamic and static M.E. on moisture content. The same samples were used to obtain both lines in order to reduce variation.

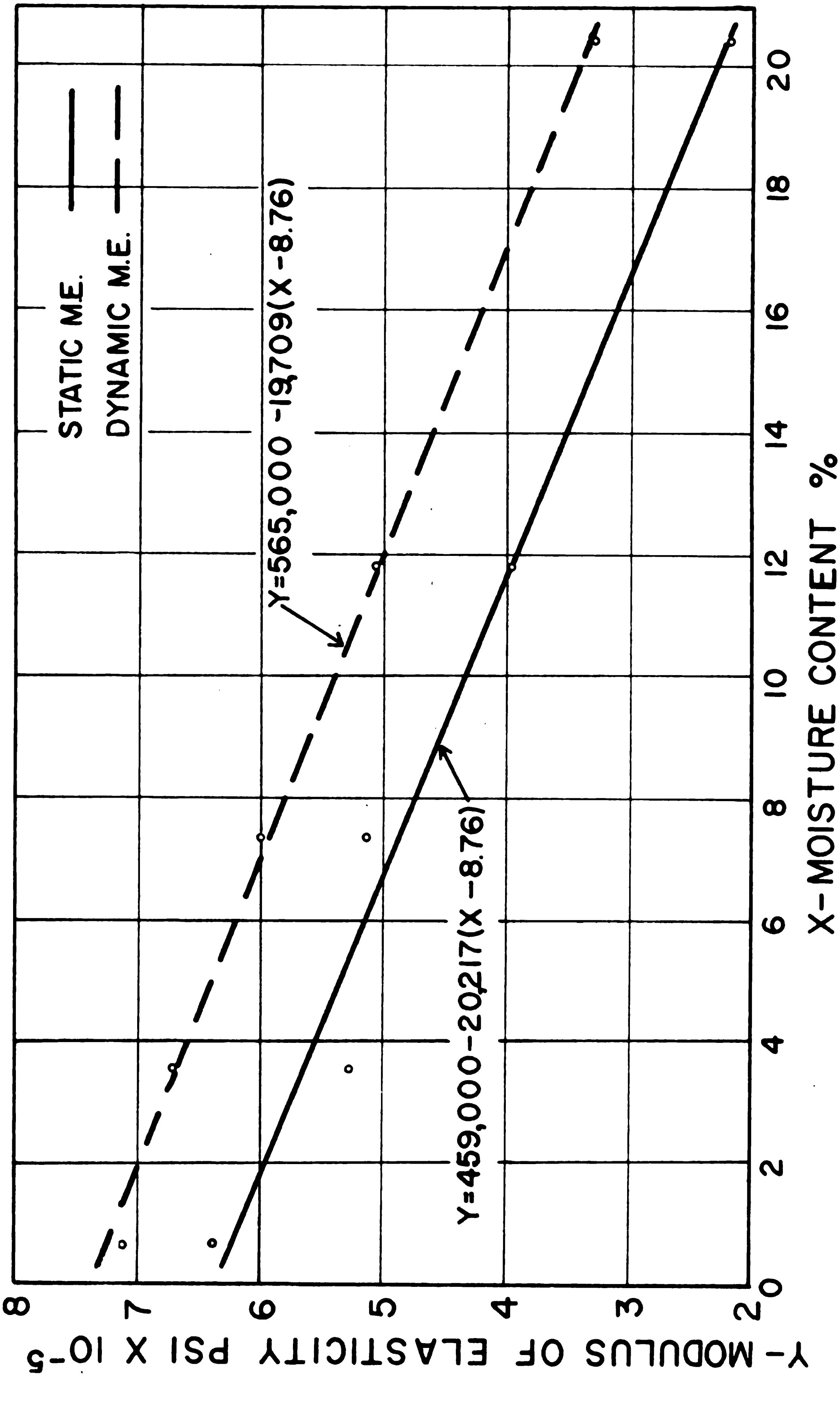

STATIC M.E.
DYNAMIC M.E.
Y=565,000 -19,709(X -8.76)
Y=459,000-20,217(X -8.76)
X-MOISTURE CONTENT %
Y-MODULUS OF ELASTICITY PSI X 10-5

TABLE 4

Results of Static and Dynamic Tests
and Logarithmic Decrement at 5
Moisture Content Levels

Level	Moisture Content %	Static M.E. P.S.I.	Dynamic M.E. P.S.I.	Logarithmic Decrement
1	.68	637,000	714,000	.0702
2	3.52	530,000	671,000	.0785
3	7.34	512,000	602,000	.0952
4	11.80	395,000	506,000	.1349
5	20.44	221,000	330,000	.1929

used for obtaining the regression lines and correlation coefficients is given in Appendix 1.

The two regression lines are related by the equation

$$E_D = E_S + 101,250 + 508(x)$$

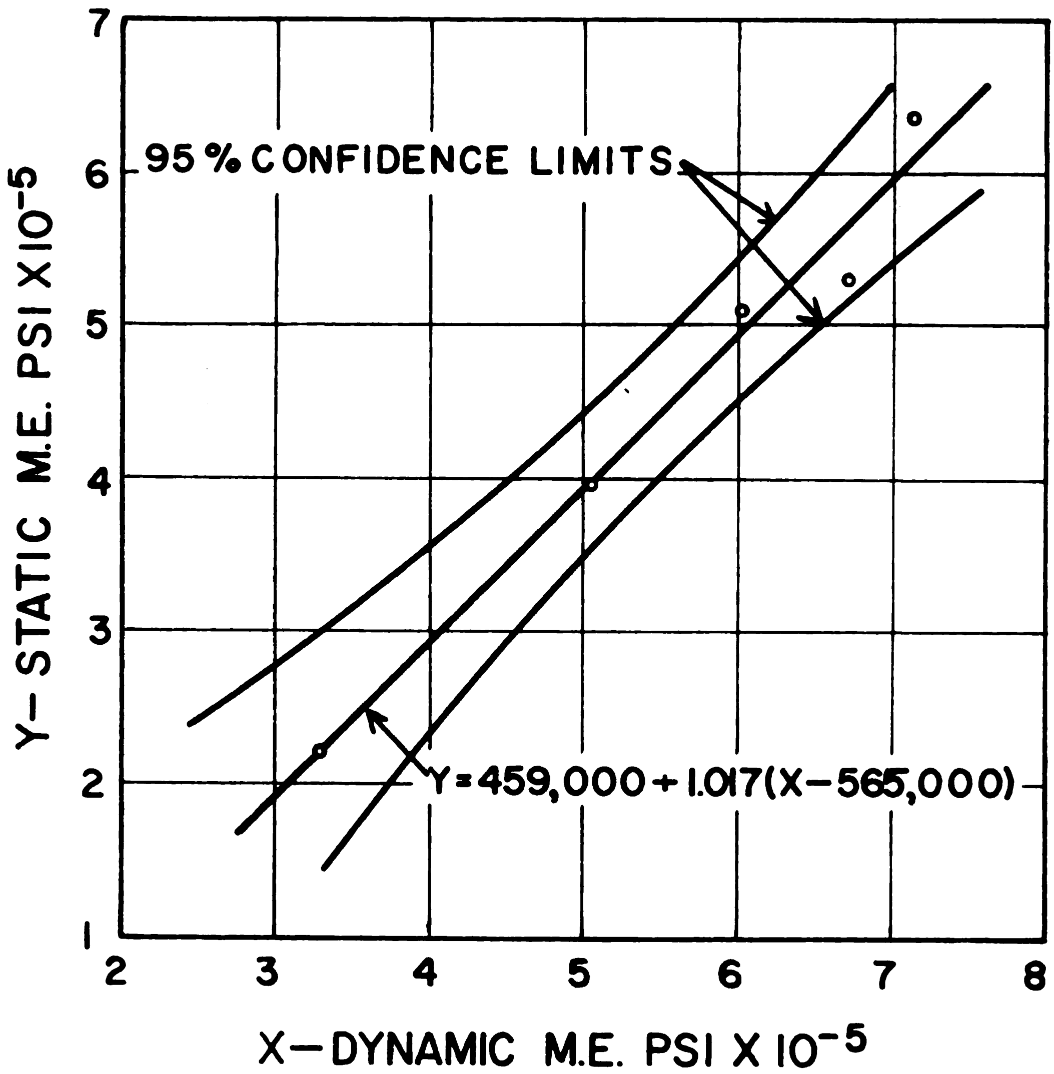
where E_D = Dynamic M.E.

E_S = Static M.E.

x = Moisture Content

As can be seen from this equation or from examination of the slopes of the two lines, the M.E. values diverge as the moisture content increases. Also the relative difference increases considerably with increasing moisture content.

Fig. 10 is the regression of static M.E. on dynamic M.E. with 95% confidence limits for the regression (see Appendix 1). The correlation coefficient is .981.

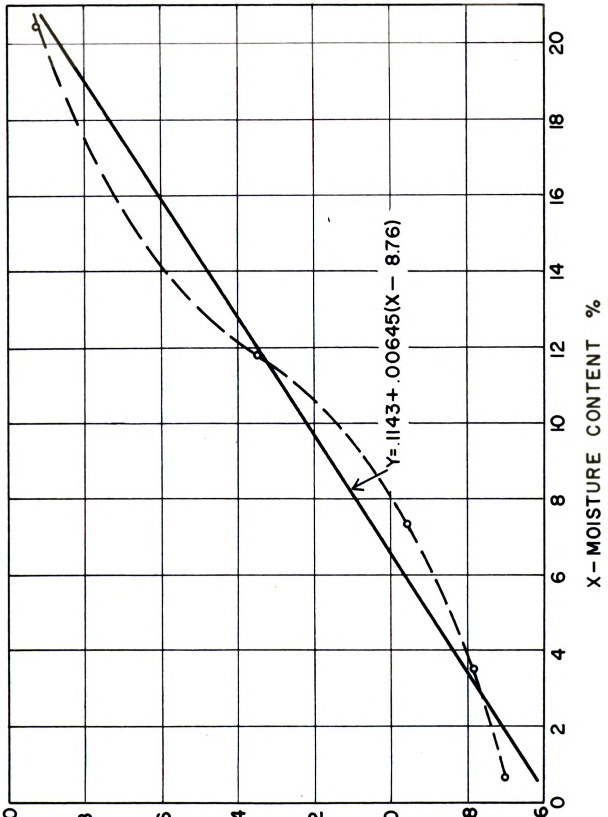

95 % CONFIDENCE LIMITS
Y = 459,000 + 1.017(X − 565,000)
Y − STATIC M.E. PSI X 10⁻⁵
X − DYNAMIC M.E. PSI X 10⁻⁵
1
2
3
4
5
6
7
8

For a given dynamic M.E. value the expected static value will fall between the limits intersected by the vertical lines through the dynamic value 95 out of 100 times.

Logarithmic Decrement

The results are tabulated in Table 4 for each moisture content level and in Fig. 11 the regression of logarithmic decrement on moisture content is given. The correlation coefficient is .984. Each point represents an average of 10 tests. A dashed line through the data points is included and will be discussed later.

Fig. 11.--Regression of logarithmic decrement on moisture content. Dashed line indicates a curve bearing some resemblence to results of solid wood.



Y=.1143+.00645(X − 8.76)
Y-LOGARITHMIC DECREMENT
X-MOISTURE CONTENT %
.20
.18
.16
.14
.12
.10
.08
.06
0
2
4
6
8
10
12
14
16
18
20

CHAPTER V

DISCUSSION

Shear and Rotatory Inertia Effects

When the dynamic M.E. was calculated without correcting for shear or rotatory inertia (apparent M.E.) and plotted against the mode of vibration, straight horizontal lines were obtained, with the average of 20 dynamic tests falling close to this line (see Fig. 8). This demonstrates that the effect of shear and rotatory inertia is negligible. It can be seen that even at the 5th overtone the depth/length ratio, which is directly related to shear and rotatory inertia, is such that these effects are still negligible. Using spruce and oak specimens 8 x 10 x 200 mm Kollmann (5) found a considerable drop in M.E. obtained from overtones as a result of shear and rotatory inertia (see Table 5 and Fig. 12). Here the depth/length ratio is much higher than that of the hardboard samples used in this experiment.

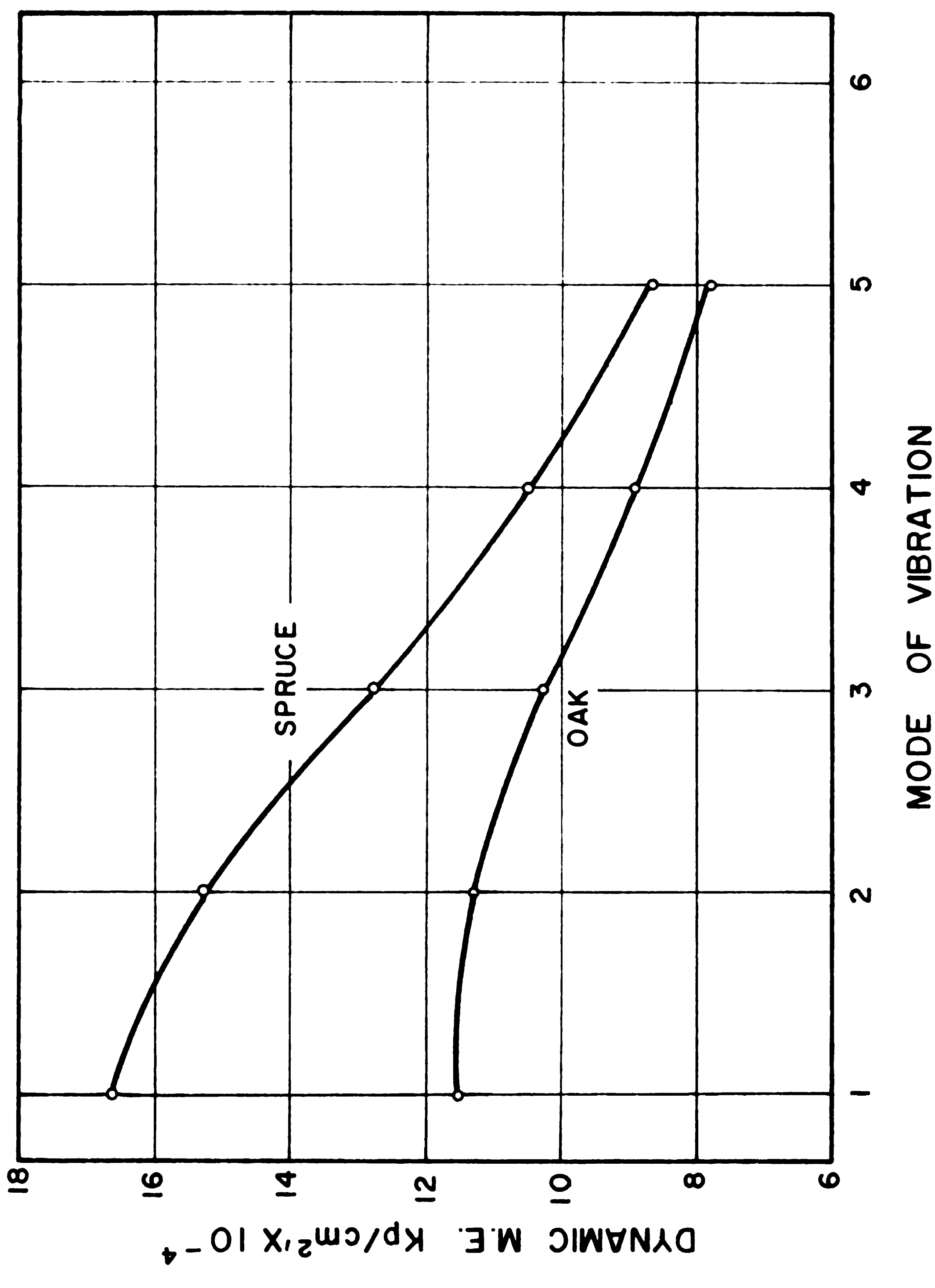
The slight increase in the average M.E. obtained from the overtones as compared to that obtained from the fundamental frequency is attributed to small defects or irregularities in the density along the sample. For

TABLE 5

Comparison of Dynamic and Static M.E.

		Spruce		Oak	
	Dimension	Average	Coefficient of Variation %	Average	Coefficient of Variation %
Specific Weight	p/cm	.501	2.3	.653	6.4
Dynamic M.E. uncorrected E_D	K_p/cm^2	161,800	3.6	115,800	11.7
Dynamic M.E. corrected for shear E_{DC}	K_p/cm^2	178,800	3.6	123,000	11.7
Static M.E. uncorrected E_S	K_p/cm^2	151,400	4.0	110,500	10.4
Static M.E. corrected for shear E_{SC}	K_p/cm^2	171,000	4.0	118,400	10.4
$\frac{E_{DC}}{E_{SC}}$		1.04	2.9	1.03	2.5
$\frac{E_D}{E_S}$		1.07	2.9	1.05	2.5

Fig. 12.--Dynamic M.E. obtained from the fundamental frequency and the first 4 overtones, uncorrected for shear and rotatory inertia for Spruce and Oak specimens 8 x 10 x 200mm, as reported by Kollmann (5).



DYNAMIC M.E. Kp/cm²·X 10⁻⁴
18
16
14
12
10
8
6
SPRUCE
OAK
1
2
3
4
5
6
MODE OF VIBRATION

example, a defect at the center would have considerable effect on the M.E. determined at the fundamental frequency but almost no effect on the frequency of the first overtone where a node occurs at the center. The overall result is that as the frequency increases the effect of defects is less significant. In comparing the results on spruce and oak obtained by Kollmann (Fig. 12) this effect can explain the difference in the two curves between the first and second nodes of vibration. Spruce is quite homogeneous and there is little lowering of the M.E. at the fundamental due to defects or at the first overtone there is not much compensation for defects, hence the curve is quite steep. In oak, however, the wood is not so homogeneous and the presence of defects and irregularities lowers the dynamic M.E. at the fundamental frequency and at the first overtone, the drop in M.E. due to shear or rotatory inertia is partially compensated for and the curve has little slope up to this point. Beyond the first overtone the shear and rotatory inertia effects dominate.

Dynamic and Static Modulus of Elasticity

The difference in the regression lines for dyanmic and static M.E. on moisture content is attributed to creep. Harris (6) defines creep as "a continuing deformation with time, at a constant stress" and relaxation as "the reduction in stress, over a period of time, at constant strain." In the tests for M.E. both stress and strain are increasing with time so that a combination of the two results. Moslemi (7) has shown that for hardboard, creep is a function of time and level of stress. Both of these factors are important here, (1) the time of the static test is many times greater than the time for the dynamic test, the order of magnitude being approximately 9,000 to 1, (2) the level of stress is increasing during the static test, whereas the stresses involved in the dynamic test are very small, due to the small deflections.

Hence, from what actually amounts to a loss of load during the static test, the static M.E. is lower than the dynamic M.E.

The difference in the slopes of the two regression lines is also attributed to creep, since creep increases with moisture content (7). This accounts for the large relative change from dry to wet conditions, for the creep in the static tests has much more effect than in the dynamic test, where the time is so small that time dependent properties such as creep and relaxation cannot have much effect. Therefore, the static M.E. falls off at a more rapid rate than the dynamic M.E.

The negative slopes in Fig. 9 are due to the weakening of the fibers by swelling. [As described above, the regression of static M.E. on moisture content is more negative than the regression of dynamic M.E. on moisture content as a result of increased creep at the higher moisture contents.]

The slope of the regression line in Fig. 10 is greater than 1 since the difference between the two M.E.'s is less at the higher values of M.E. (lower moisture content) as can be seen from the equation

$$E_D + E_S + 101, 250 + 508(x)$$

The line lies completely below a 45° line through the points $E_D = E_S$ since the dynamic M.E. is consistently higher than the static M.E.

The 95% confidence limits are curved outward from the overall mean of all M.E. values. The reason for this is that in estimating the regression line of the form

$$Y = a + b\ (X-\bar{X})$$

where $a = \bar{Y}$

b = slope

if the estimate of b is in error, the slope will change and the greater the distance from the overall mean, the greater is the change of the line, hence the error is greater at the extremes and the confidence limits curve outward.

Logarithmic decrement

The dashed line in Fig. 11 is included because it has some resemblence to the curves obtained by Kollman

(5) and Pentoney (8). Kollmanns curves are reporduced in Fig. 13. These are for solid wood, and show a definite minimum at 8%, which indicates that at this moisture content plastic deformation is at a minimum. Moslemi (7) obtained similar results for hardboard.

The results obtained in this study indicate that plastic deformation increases with moisture content in a fairly linear fashion, and this is supported by the fact that the static M.E. was linear with moisture content. If plastic deformation is greatest at high and low M.C. and at a minimum at 8% M.C. the expected difference in dynamic and static M.E. would be greater at high and low M.C. but there was no evidence of this obtained.

Nomogram

The nomogram in Fig. 14 is a graphical solution to the equation for dynamic M.E., without corrections for shear or rotatory inertia. See Appendix 2 for some details of the construction.

The nomogram has an accuracy of $\pm$ 2000 psi as constructed and is quite simple to use, as can be seen from following the example in Fig. 14.

However, the chart could be simplified with some loss in accuracy, by assuming that the dimensions of all samples are within a certain tolerance. For example, if

Fig. 13.--Logarithmic decrement for Oak and Spruce as reported by Killmann (5).

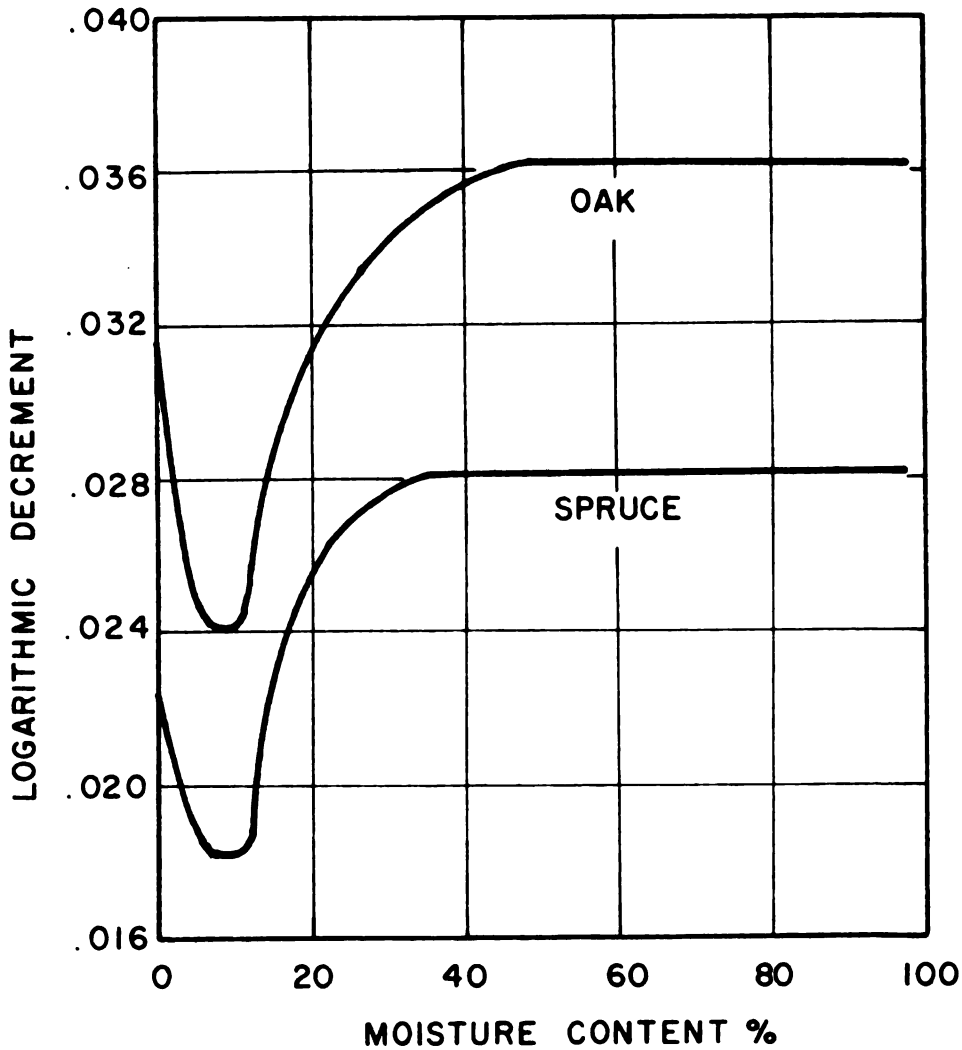

.040
.036
.032
.028
.024
.020
.016
LOGARITHMIC DECREMENT
OAK
SPRUCE
0
20
40
60
80
100
MOISTURE CONTENT %

Fig. 14.--The nomogram prepared here is a graphical solution to the equation

$$\frac{E}{5.4 \times 10^{-6}} = \frac{fr^2 l^3 W}{h^3 b}$$

Where E = dynamic M.E.
fr = fundamental frequency
l = length of sample, inches
h = thickness of sample, inches
b = width of sample inches
W = weight of sample grams

The equation may be reduced to log form and written as the following set of equations

$$3\log l - 3\log h = A$$
$$A - \log b = B$$
$$B + 2\log fr = C$$
$$C + \log W = \log\left(\frac{E}{5.4 \times 10^{-6}}\right)$$

Example: Let l = 20" h = .25" b = 1" fr = 50 cps and W = 75 gms.

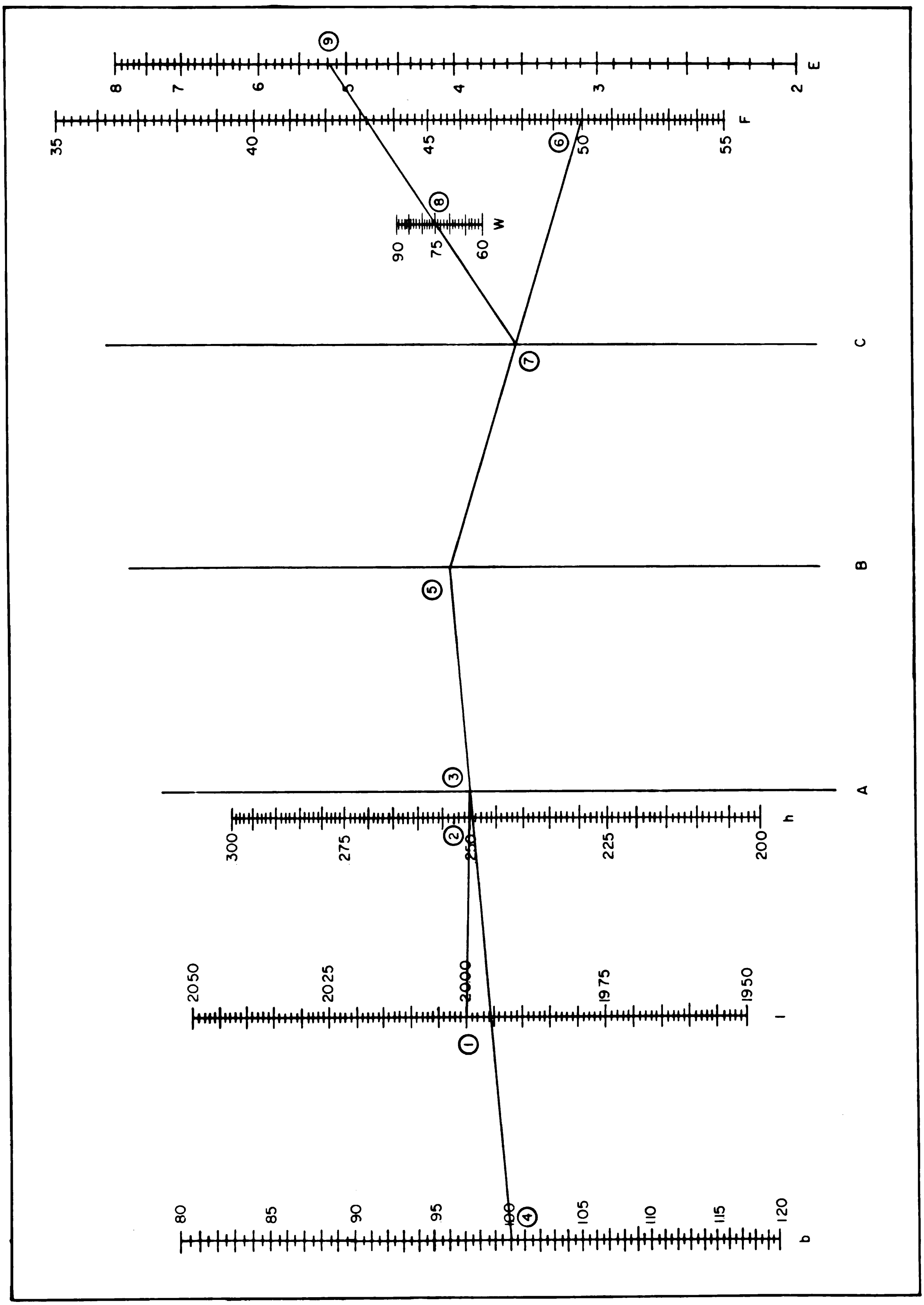
E is determined by

1. Construct a line through l = 20" (1) and h = .25" (2) to obtain a point (3) on reference line A.

2. Construct a line through the point on A (3) and through b = 1" (4) to obtain a point (5) on reference line B.

3. Construct a line through the point on B (5) and through fr = 50 cps (6) to obtain a point (7) on reference line C.

4. Construct a line through the point on C (7) and through W = 75 gms (8) to obtain point (9) the desired dynamic M.E.


b
80
85
90
95
100
105
110
115
120
I
2050
2025
2000
1975
1950
h
300
275
250
225
200
A
B
C
W
90
75
60
F
35
40
45
50
55
E
8
7
6
5
4
3
2
1
2
3
4
5
6
7
8
9

the length is assumed to be 20.00 ± .05" and the width 1 ± .01" the chart could be reduced to four graduated lines and one reference line. The result will then be approximately within the range of ± 6000 psi.

CHAPTER VI

CONCLUSIONS

From the results obtained it may be concluded that the dynamic test for modulus of elasticity is a valid measure of this property and that the M.E. can be obtained without applying a correction for shear or rotatory inertia if the depth/length ratio is small enough. If the overtones are more easily obtained, the dynamic M.E. may be calculated using these values.

The dynamic M.E. can be correlated with the static M.E. to obtain the more conventional values for indications of the quality.

Since the dynamic test is nondestructive, it proves very useful in research, where the same specimens may be tested under different conditions, but would also be very useful in quality control work. The dynamic test combined with a simplified nomogram or <u>even</u> with the same type nomogram as prepared here could give the dynamic M.E. in a very short time, which would be highly desirable in quality control work.

It may also be concluded that creep is of major importance in testing hardboard. Creep caused the static

M.E. to fall below the dynamic M.E., with the difference increasing with increasing moisture content.

Plastic deformation, which increases with increasing moisture content, causes the logarithmic decrement to increase with the moisture content.

APPENDICES

APPENDIX 1

STATISTICAL PROCEDURES

Regression Line and Correlation Coefficients.--Let Y be the dependent variable and X the independent variable. From the data obtain the following statistics:

$$\Sigma X_i$$

$$\Sigma X_i^2$$

$$\Sigma Y_i$$

$$\Sigma Y_i^2$$

$$\Sigma X_i Y_i$$

From these statistics the following are obtained:

$$S_{xy} = \frac{\Sigma X_i Y_i - \frac{(\Sigma X_i)\ (\Sigma Y_i)}{n}}{n - 1}$$

$$S_x = \frac{\Sigma X_i^2 - \frac{(\Sigma X_i)^2}{n}}{n - 1}$$

$$S_y = \frac{\Sigma Y_i^2 - \frac{(\Sigma Y_i)^2}{n}}{n - 1}$$

The regression equation of Y on X is

$$Y = a + b\ (X - \bar{X}),$$

Where $a = \bar{Y} = \frac{\Sigma Y_i}{n}$

$b = \frac{S_{xy}}{S_{x2}}$ (which is the slope)

$\bar{X} = \frac{\Sigma X_i}{n}$

The correlation coefficient is

$$r = \frac{S_{xy}}{S_x S_y}$$

<u>95% Confidence Limits.</u>--The interval (see (9)) at any point on the regression line is

$$Y - t_{.05}\, S_{\hat{y}} \leqslant \mu \leqslant Y + t_{.05}\, S_{\hat{y}}$$

Where $Y = a + b\,(X - \bar{X})$

$t_{.05}$ = 2.010 for 49 degrees of freedom

$S_{\hat{y}}$ = Sample standard error of $\hat{Y}$

$$S_{\hat{y}} = \sqrt{\Sigma y^2 - \frac{(\Sigma xy)^2}{\Sigma x^2}} \quad \sqrt{\frac{1}{n} + \frac{x^2}{\Sigma x^2}}$$

Where $y = Y - \bar{Y}$

$x = X - \bar{X}$

APENDIX 2

NOMOGRAM

The nomogram of Fig. 14 was constructed using the method of Giet (10). The equation for the dynamic M.E. is

$$E = fr^2 \rho \frac{1^4}{i^2} (11.44 \times 10^{-7}),$$

Where $\rho = \frac{W}{lbh}$

$i = \frac{h^2}{12}$

fr = frequency

W = weight of sample

l = lenghth of sample

b = width of sample

h = thickness of sample

The equation may be written

$$\frac{E}{5.4 \times 10^{-6}} = \frac{fr^2 l^3 W}{h^3 b}$$

and in log form is

$$\log(E/5.4 \times 10^{-6}) = 2\log fr + 3\log l + \log W - 3\log h - \log b$$

This may be further simplified as a system of four equations

$$3\log l - 3\log h = A$$

$$A - \log b = B$$

$$B + 2\log fr = C$$

$$C + \log W - \log (E/5.4 \times 10^{-6}) = 0$$

From these four equations the length and graduations of the reference lines were established as well as the distance between the lines.

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BIBLIOGRAPHY

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