THE APPLICATION OF SYSTEMS ENGINEERING TECHNIQUES TO URBAN TRAFFIC FORECASTING

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ABSTRACT

THE APPLICATION OF SYSTEMS
ENGINEERING TECHNIQUES TO
URBAN TRAFFIC FORECASTING

By William Louis Grecco

The research reported here shows the application of systems engineering techniques to the problems of urban traffic forecasting. The traffic forecasting model contains three parts; trip generation, trip distribution and trip assignment. Only trip distribution is analyzed here. The problem resolves itself into determining exactly what proportion of the trips (for each purpose) originating in one zone, shall have destinations in each of the other zones. Specifically, the work trip distribution system was chosen for detailed analysis.

An extensive review was made of the known techniques of traffic forecasting, prior to presenting a new systems engineering model. To deal quantitatively with the interaction of the components of a system, each component must be describable mathematically and must be incorporated into the system in accordance with the requirements of linear graph theory.

A summary of the requirements and procedures of linear graph theory is presented to provide a brief insight into its use. The analysis of the problem was based on these requirements, which are:

1. The basic component must be describable mathematically by relating two valid measurements on the components.
2. When the components are arranged in a systems graph; one of the measurements taken on the component, which is noted as "x", must sum to zero when the summation is made around a circuit; and the other measurement "y" must sum to zero at the vertices of the systems graph.
3. The x measurement must be related to the y measurement through a linear or nonlinear function.

The results of the synthesis and analysis processes when applied to the system and its components can be stated as postulates below:

1. The best components of those evaluated for the system are:
   a. Residential zone components
   b. Employment zone components
   c. Route components
2. The most logical measurements established and tested on the components are:
   a. Residential Zone
      (1) y is the actual number of work trips generated
      (2a) x is the pressure in terms of desire to generate work trips.
      (2b) x is the pressure in terms of money available to generate work trips
   b. Employment zone

(1) y is the actual number of work trips attracted

(2) R is the relative attraction of each employment zone

c. Route system

(1) R is a resistance value expressed as a friction to travel due to travel time

(2) R is a resistance value expressed as a friction to travel due to travel costs

3. A single terminal equation form relating x and y was used for all components

$$R = \frac{x}{y}$$

The equation form for the individual values of R, x and y from items one and two above is expressed as a function of its parameters and is generally more complex.

4. In order to evaluate the possible use of the postulates noted above, four illustrative solutions are presented. The results, when compared with other models, indicate that the systems approach, using linear graph theory can offer a useful tool for predicting trip distribution in an urban area.

This research has established an initial evaluation of a systems engineering approach to traffic forecasting. An analysis process was used to identify the problem and the units and a synthesis was employed to disclose a number of possible solutions. Further analysis and testing is necessary to fully evaluate and interpret what is presented here.

THE APPLICATION OF SYSTEMS

ENGINEERING TECHNIQUES TO

URBAN TRAFFIC FORECASTING

By

William Louis Grecco

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INTRODUCTION

An advancement is made in a new engineering science when the engineer has, through proper formulation, established a mathematical approach to the problems. In any of the relatively new engineering sciences, preliminary emphasis must, through necessity, be with the qualitative and judgement decisions.

How do we proceed from the qualitative to the quantitative, from an engineering judgement decision to one based on a mathematical formulation of the problems? Here, it is possible to observe what others have done. They collected data and studied it. When it was possible to control such collection, they hypothesized and made a design of experiment which would better insure that data collection would not be too little and prove valueless or too much and be costly. From studying the properly collected data, the hypothesis can be verified or altered and a postulation drawn. In sequence, these postulates are compared with empirical data and a re-evaluation made to provide a better experimental design on which to base new data collection. The process will go on until some degree of confidence is reached. Even then, continued research can be expected to show up shortcomings, since many of the long accepted theories have required drastic revisions.

As more factors are included and the problem becomes more complex, a stage is reached when a rigorous solution is impossible without the aid of mathematics.

The trend in many areas of traffic engineering is toward a more mathematical and theoretical approach. In relation to the other sciences, traffic engineering is yet in its infancy, but it is growing up. This fact is evidenced by the writings, of Herman, Schneider, Howe, Bevis, and others. One has only to refer to the bibliography of the special report by F. A. Haight (1) to conclude that traffic engineering is on the verge of a break-through. Even with the variable item of individual human choice or behavior, many phases of the traffic problem will evolve to a scientific level comparable to that of the physical sciences. The traffic engineering profession will probably have to settle for somewhat less replicability than the physical sciences because only two of their ingredients are physical, the vehicle and the facility, whereas the third, the user, presents different and still unsolved problems. This does not imply that the user problem is insurmountable. Although the individual has shown immunity to prediction, groups of many such individuals have shown that patterns can be observed.

The bulk of the work by the traffic theorists has been in the areas of car-following theory, queuing and waiting line theory, and traffic simulation. In the area of theoretical origin and destination studies, there have been attempts at formulation utilizing such techniques as the gravity model, the electro-static model, the opportunity model and linear programming.

The work reported in this thesis deals with development of methodology for solving traffic flow in a road network by a mathematical model. Preliminary theoretical testing of systems analysis as a technique for theoretical origin and destination studies has been accomplished during the study. The theoretical results obtained with this technique are shown to compare very closely with those available with other techniques. In the analysis of systems as a means for achieving a simple systematic procedure for formulating the system equations, the theory of oriented linear graphs, developed as an abstract mathematical topic, is utilized.

Origin and destination (O and D) survey techniques have improved since their initiation less than twenty years ago. They have become more comprehensive in nature and have focused more specifically on the problem to be solved. The primary purpose of this type of survey is to provide the information necessary for present and future planning of an urban major arterial system. The arterial system is so strongly related to the urban growth of the area that it is essential to the development of a comprehensive master plan.

Data collection strictly for research has found funds to be quite minimal. Analysis must be made on data collected in the process of the comprehensive O and D survey. Although these studies placed primary emphasis on the job to be accomplished, they have produced much data from which travel patterns can be synthesized. The travel patterns can then be expressed mathematically as traffic models.

The comprehensive type origin and destination survey, as performed recently in the metropolitan areas of Detroit and Chicago, has proved successful. When this type of survey is followed by a continuing program, the data remains current, to provide the basis for future planning. The chief deterrent to the use of this type of solution is the cost involved. The Chicago Area Transportation Study had a cost estimate of over $2\frac{1}{4}$ million, which didn't include an appropriation necessary for the continuing study.

The justification for the work that follows is:

1. The natural development of a new science requires it,
2. The travel patterns from many comprehensive surveys are available for analysis,
3. There is the economic consideration.

In summary, this dissertation will review what has been accomplished by others and will then present a new approach to urban traffic forecasting.

BACKGROUND

HISTORY OF TRAFFIC FLOW STUDIES

As early as 1914, a consulting firm had done an O and D study of workers in Chicago. (2) During the early 1930's, improvements made to the main rural routes had established definite traffic flows. It was realized that future growth of the nation's highway system should be on a scientific rather than a haphazard basis. (3) Up to this time, the greater portion of highway funds had been used on the main rural routes. Although these routes, as well as the urban streets, were quite inadequate, the question of where the limited money available should be spent could not be answered from the available data.

The Federal-Aid Act of 1934 authorized an expenditure not exceeding one and one-half per cent ($1\frac{1}{2}\%$) of the Federal-aid funds apportioned to each state for the making of surveys, and engineering investigations on future construction projects. During the following years, O and D studies were undertaken primarily to determine the status of the rural system. These early studies showed that the intra urban travel was much more significant than had been realized. Rural traffic increased proportionately as the highway approached the city or town. Prior to this analysis, it was erroneously assumed that a large part of this traffic was bound to destinations beyond the city and that if this traffic would bypass the city, then the traffic congestion of the city would be relieved. (3)

The early O and D surveys were designed to determine where the traffic originates and where it is destined. There are many common methods (4) of making this type of study, of which the following are the most important:

1. Moving vehicle driver interview
2. Moving vehicle driver postal cards
3. Moving vehicle license plates
4. Parked vehicle license plates
5. Tag on vehicle
6. Dwelling unit interview
7. Motor vehicle owner mail questionnaire
8. Transit terminal passenger questionnaire
9. Transit route passenger questionnaire

The following discusses two approaches to traffic planning. One evaluates the street network strictly by volume count and the second utilizes data from O and D studies. The definition of traffic planning used herein, can be established through context. The two approaches have specialized application and should not be used interchangeably.

The first approach, which we will label as an objective approach, has application to an urban street network, which has proper location throughout or cannot justifiably be changed or when only a small portion of the system is to be studied. This approach is simply a matter of evaluating an urban network, street by street. Each street's adequacy is determined on the basis of the volume it carries in comparison with its capacity. The whole system is then rated on the basis of how the individual parts collectively meet this criteria. In order to plan the location and capacity of future streets in the network, it is not sufficient to know the number of vehicles using the street during a given period of time. The vehicular ccunts reveal only the existing volume and is not necessarily, indicative of the preferred route of the drivers. It is not possible to determine where a new expressway should be located and what its design capacity should be, merely by observing traffic volumes at selected locations. The results of this type study would give us wider streets in the future at the same location. Only through a comprehensive knowledge of existing trip origins and destinations can the proper location and adequacy of proposed facilities be determined. This, then, is the reason for the general decrease in the use of the first approach and an introduction into the second.

The second approach, that of desired flow, is so titled because the evaluation of the network is made on the basis of how closely the street network satisfies the desired flow. The early simplified type of O and D survey was the start of this method, and the first step toward satisfying the traffic engineer's need to know more about traffic flow characteristics. There has been a definite correlation between the advances made in predicting and evaluating traffic flow and the amount of pertinent data asked by the O and D survey. Early studies asked from where did you come and where are you going. The number of questions has increased over the years until the more recent surveys ask not only where, but how (mode of travel), why (purpose of trip), and when (peak movements), and many questions on parameters which affect traffic movement. As techniques improved, there was added to this travel inventory, a study of land use inventory and an inventory of existing transportation facilities.

Today, the methods for carrying out O and D surveys range from the simple external cordon survey in small communities, to the comprehensive home interview transportation study for the larger cities and metropolitan areas. Since 1953, notable large-scale comprehensive transportation studies have been undertaken in several large metropolitan areas. Special study staffs made up of a team of specialists from the various professions and disciplines were used throughout the work. These studies were generally a cooperative effort encompassing all levels of government; city, county, state and federal. The Detroit and Chicago studies have been completed. Studies are underway in Pittsburgh and Philadelphia. Although there have been other studies, these provide the most recent and comprehensive approach.

LITERATURE REVIEW

Prior to 1950, most of the literature of this field discussed the techniques of actually conducting origin and destination surveys. Those interested in these early works, are referred to the review of literature and bibliography of R. E. Barkley (5). Significant contributions published after 1950, are presented here. Some of these deal with the broad aspect of traffic forecasting rather than with the specific area of urban traffic forecasting.

Urban traffic planning requires a knowledge of future travel patterns. For this reason traffic engineers have devoted much time and effort in the development of procedures for traffic forecasting. The procedures proposed to date are reviewed under two general categories; that of trend forecasting and theoretical forecasting.

Methods of Trend Forecasting

The Eno publication by Schmidt and Campbell (6) contains an extensive discussion of trend forecasting. This discussion is briefly

summarized here and supplemented by the writings of others. Schmidt and Campbell subdivide trend forecasting into two procedural types; mechanical and analytical.

Mechanical Methods

The mechanical methods assume that future travel patterns will be continuations of past experience, and then, simply project the composite past trend forward.

Ratio.--The growth of the city is estimated on the basis of the ratio of its past growth to past state growth and then projected in terms of estimated state growth.

Correlation Index.--Several growth trends have been correlated somewhat to traffic growth, included are gasoline consumption, national income and the gross national product. Correlation indexes might serve for the composite growth of travel in the United States or in a whole state inasmuch as data may be obtained relative to income, production and gasoline consumption on a national or state-wide basis. Such data are usually not obtainable nor are they as applicable, for estimating on a one-project or local basis.

Analogy.--This method requires that a situation be found which historically parallels the subject situation in several attributes and has adequate records of traffic behavior growth whereas the subject situation doesn't have these records. The trend in behavior and growth of traffic may be appropriated as a pattern for the subject situation and used as a guide in projecting future growth. While analogy can provide a pattern and helpful guide in estimating, it should be used with care.

Projection of Composite Trends.--The record of traffic volumes are plotted and the curve is extended. This extension should reflect an informed judgement based on foreseeable changes in competition created by new facilities, changes in mode of travel, changes in land use, or approach of the saturation point in land use or in capacity of facilities.

Generalized Growth Formulas.--Formulas have been used to forecast future traffic volumes in accordance with various concepts of growth such as:

1. Straight line formula where the future volume equals the base year volume plus the number of years times the annual growth increment.
2. Compound interest curve formula where the base year volume is multiplied by the quantity of one plus i, the quantity to the nth power. The yearly percentage increase is i and n equals the number of years.
3. General growth law which came from the biological sciences and has been applied to population and new industries as well as to traffic.

This general growth concept assumes a slow but constantly accelerating rate in the early years, then a period of rapid and steady growth followed by a decelerating rate until the curve continues on with minimum or no further growth when the saturation point is reached.

Analytical Methods

The analytical method recognizes the fact that traffic growth is a product not only of time but of certain varying internal forces and external stimuli that operate to affect the rate of growth of each contributing factor to the composite growth. Some of these forces, referred to as operative factors, are level of economy, extent and state of improvement of the highway system or subject project, changes in competition, purpose of travel, change in land use, promotion of travel, decentralization of homes and industry and consolidation of schools, and tradition and habit. There are two general types of analytical methods:

Projection of Component Trends.--The choice of this method presupposes that historical data are available for each of the following categories:

1. Change in population (growth and distribution)
2. Change in persons--vehicle ratio (by vehicle type)
3. Change in average vehicle use (by vehicle type).

Upon the completion of the projection of these three individual determinants through the forecast period, the estimate of traffic for any particular future year can be made by determining the ratio of the future year to the base year. The ratios of each of the categories are then multiplied together to form a single index for the future year. In forecasting passenger car travel, a variation of the above method, substitute driver population for population and licensed drivers per car instead of persons per car.

Expansion of Existing Patterns.--A knowledge of present interchanges is required, in order to predict future travel patterns by this method. Although there are several methods detailed here, they are only variations of a basic growth factor method.

Uniform Factor Method.--This is the simplest approach to expanding trips. An average growth factor is determined for the entire area ($\overline{G}$) and this is used to multiply existing or base year zone to zone movements (t_{ij}). The expansion formula to produce future trips between zones i and j (T_{ij}) can be stated mathematically as:

$$T_{ij} = t_{ij} \cdot \overline{G}$$

This method assumes only small variations of growth in the individual zones from the average area growth. When this does not hold true, the method overestimates on the zones of lower growth and underestimates the zones with growth values higher than the average.

Average Factor Method.--The Minnesota State Highway Department, in cooperation with the U. S. Bureau of Public Roads, developed this method in 1953. A traffic growth factor (G_i) for each zone within the area is computed as a ratio of traffic resulting from the anticipated land use development to the traffic currently generated. The future zonal interchange is found by multiplying the present interchange by an average of the growth factors for the origin zone and the destination zone as follows:

$$T_{ij} = t_{ij} \cdot \frac{G_i + G_j}{2}$$

This method shows a variation between the sum of the trips destined to a zone and the total trips originating at the zone. The variation can be eliminated by a series of iterations using a correction term (K):

$$K_{ij} = \frac{1}{2} \quad (F_i + F_j)$$

Where F_i is the ratio of actual trip ends in zone i divided by the sum of all calculated trips to zone i, and similarly for zone j or F_j.

$$T'_{ij} = T_{ij} \cdot K_{ij}$$

These approximations are continued until the value K_{ij} approaches unity. Brokke and Mertz (7) discuss this method and point out that:

> One of the inherent disadvantages of the average factor method is that the calculated trips into zones with higher-than-average growth factors generally total less than the predicted number of trips. Conversely the calculated trips into zones with lower-than-average growth factors total more than the predicted total of trips. This systematic bias of the predicted values could result in an inordinate number of approximations and may affect the accuracy of the method.

Fratar Method.--This method, which was the first to successfully use successive approximations in distributing future zonal trips, was developed by Thomas F. Fratar (8) in connection with forecasts made for Cleveland, Ohio. The distribution of trips from any zone of origin is considered to be proportional to the present trips out of that zone modified by the growth factor for the destination zone. The volume of zone to zone movement is computed for the zone of origin and similarly for the destination zone. The results are then averaged for the first approximation. Using variables previously defined, the above can be described mathematically for the zone of origin (i) as:

$$T_{ij} = \frac{t_{ij} \cdot G_j \cdot \sum_{j=1}^{m} t_{ij} \cdot G_i}{\sum_{j=1}^{m} (t_{ij} \cdot G_j)}$$

This expression for the zone of destination (j) can be stated as:

$$T_{ji} = \frac{t_{ji} \cdot G_i \cdot \sum_{i=1}^{n} t_{ji} \cdot G_j}{\sum_{i=1}^{n} (t_{ji} \cdot G_i)}$$

Then:

$$T'_{ij} = \frac{T_{ij} + T_{ji}}{2}$$

Similar to the average factor method, the calculated trip ends in a zone will probably disagree with the predicted volumes so it requires the use of a correction term.

$$K_{ij} = F_i \cdot F_j \cdot \frac{1}{2} \left[\frac{\sum_{j=1}^{m} t'_{ij}}{\sum_{j=1}^{m} (t'_{ij} \cdot G'_j)} + \frac{\sum_{i=1}^{n} t'_{ij}}{\sum_{i=1}^{n} (t'_{ij} \cdot G'_i)} \right]$$

Then:

$$T'_{ij} = T_{ij} \cdot K_{ij}$$

The new values are signified by the prime mark and the process is repeated until the growth factor (G) approaches unity. This method, according to Fratar (9) is somewhat similar to the Hardy Cross method of moment distribution in that the attractiveness factor used in the above equations is comparable to the stiffness factor used in moment distribution.

The Detroit Method.--This method was established as part of the Detroit Metropolitan Area Traffic Study (10). Proper consideration was given to the previously discussed method before a procedure was evolved.

According to Bevis (11), before a model could be used to predict zonal interchanges, certain criteria had to be established. These criteria are:

1. All trips within the system should be duly considered because certain factors cause people to substitute certain types of trips for other types. This substitution may be intra-zonal for inter-zonal trips, short for long trips, motorcar driver for transit passenger trips, or any of many other possible substitutions;

2. The sum of the future interchange volumes terminating at any zone must equal the postulated number of trips generated by that zone;

3. Results must be internally consistent. That is, the predicted trip volumes should be independent of the grouping or partitioning of zones into different size units except as this affects the number of trip termini for the zones in question.

To establish future interzonal volumes, the existing volume is multiplied by the ratio of the product of growth factors at both origin and destination to the area growth factor as follows:

$$T_{ij} = t_{ij} \cdot \frac{G_i \cdot G_j}{\overline{G}}$$

When estimating future intrazonal trips, the equation for zone i becomes:

$$T_{ii} = t_{ii} \cdot \frac{G_i^2}{\overline{G}}$$

In the case of interzonal trip volumes, trip balance was achieved by successive iterations similar to the previous techniques.

In discussing the techniques used in Detroit, Bevis emphasized the factors which govern trip distribution. These are (1) a measure of the relative attractiveness of the zonal interchange (2) the amount of friction for the interchange relative to all other possible interchanges.

The equation for interzonal volumes clearly relates item one from above with the expression $(G_i \cdot G_j / \overline{G})$. A measure of the friction effect is recognized as the cause of the difference between the actual interchange and the probable interchange. Since they assumed equal indices of friction for the future and the present, the equation was modified and the friction term dropped out. This last assumption is consistent with the previously discussed techniques.

Theoretical Forecasting Methods

As far back as 1954, F. C. Ikle (12), an urban sociologist, pointed out the need for a concentrated effort toward the development of theoretical logic concerning the flow of traffic. His work, which will be discussed more fully later, sets the framework for this development. He cautioned traffic engineers then, that much research was needed to define the theoretical factors which influence traffic flow. The problem of theoretical formulation can be categorized as (1) trip generation, (2) trip attraction and distribution, and (3) trip purpose. In the light of the research which will be reported, it seemed advisable to separate, wherever possible, into these divisions.

Trip Generation

In the trend forecasting methods cited previously, it was possible to use the travel patterns established by any O and D survey and to expand them to a future date by one of the techniques cited in the above section. Most of those methods required some procedure which could also be used for those areas which were at that time undeveloped and which showed zero trips at the time of the O and D survey. If

these O and D surveys could be used to establish a synthetic pattern, where none existed, it seems possible that further synthesis of the O and D data might establish these predictable patterns into a group of tools for forecasting traffic based upon a number of easily measured parameters. Among the studies reported here, there has been little agreement on where the trip is actually generated. Whether a shopping trip is generated in the residential where it begins or by the commercial area where it ends, has been a point of disagreement. In the light of this, it was difficult at times to record the literature in the proper division. The divisions used were residential, commercial and industrial; and rather than deviate from this, it was necessary to review the same paper several times.

Residential Generation.--Many of the previous studies emphasize the home as a generator of trips. About 82 per cent of all urban area trips are made either to or from the home, or some 41 per cent originate in residential areas. Previous studies show a high level of correlation between vehicle ownership and the number of daily trips. Patterns of trip generation has been presented by the following writings.

Schmidt (6).--On the basis of a tabulation by the Bureau of Public Roads, it was estimated the average total daily trips per dwelling unit was 5.61 or 8.94 trips per vehicle and 1.76 trips per person. These values were found from an analysis of 49 cities with varying populations. Although the cities in the highest population group showed the lowest number of trips per dwelling unit and those with the lowest population had the highest value, the population groups between these extremes

showed no significant trend. Since 41 per cent of all trips originate in the residential areas, the previously stated values of trips per dwelling or person would be reduced accordingly. When the number of trips per person by all modes was plotted against a population-vehicle ratio, the patterns were quite similar. (See Fig. 1)

Using O and D surveys of some 36 cities, Schmidt established a correlation between total daily trips from a residential area per dwelling unit and the vehicles per dwelling unit that was nearly linear. (See Fig. 2.) This curve, from the Eno Report, seems most significant for the estimation of trips from residential zones, and trip purpose, but needs to be up dated.

Wynn (13).--An extensive analysis of the traffic study conducted in Washington, in 1956, is reported by Wynn. The report presents a family of curves which relate the most significant parameters to trip production. Trips made by the persons in each zone have been related to: income levels; car ownership; distance from the metropolitan center; and a degree of isolation in the outer fringe area. As average income level increases, the number of trips increased from a low of 1.35 trips per person per day to a high of 2.45 trips per person. The number of work trips shows the least correlation to income level. In the areas of high car ownership, there was a higher rate of trips per person, but Wynn's study shows that car ownership is a much less significant predictor than level of income. Because, as the range of car ownership narrows, the effect of ownership tends to become constant. As the distance outward from the center of the city increases, the number of trips increases. A curve representing this relationship was presented

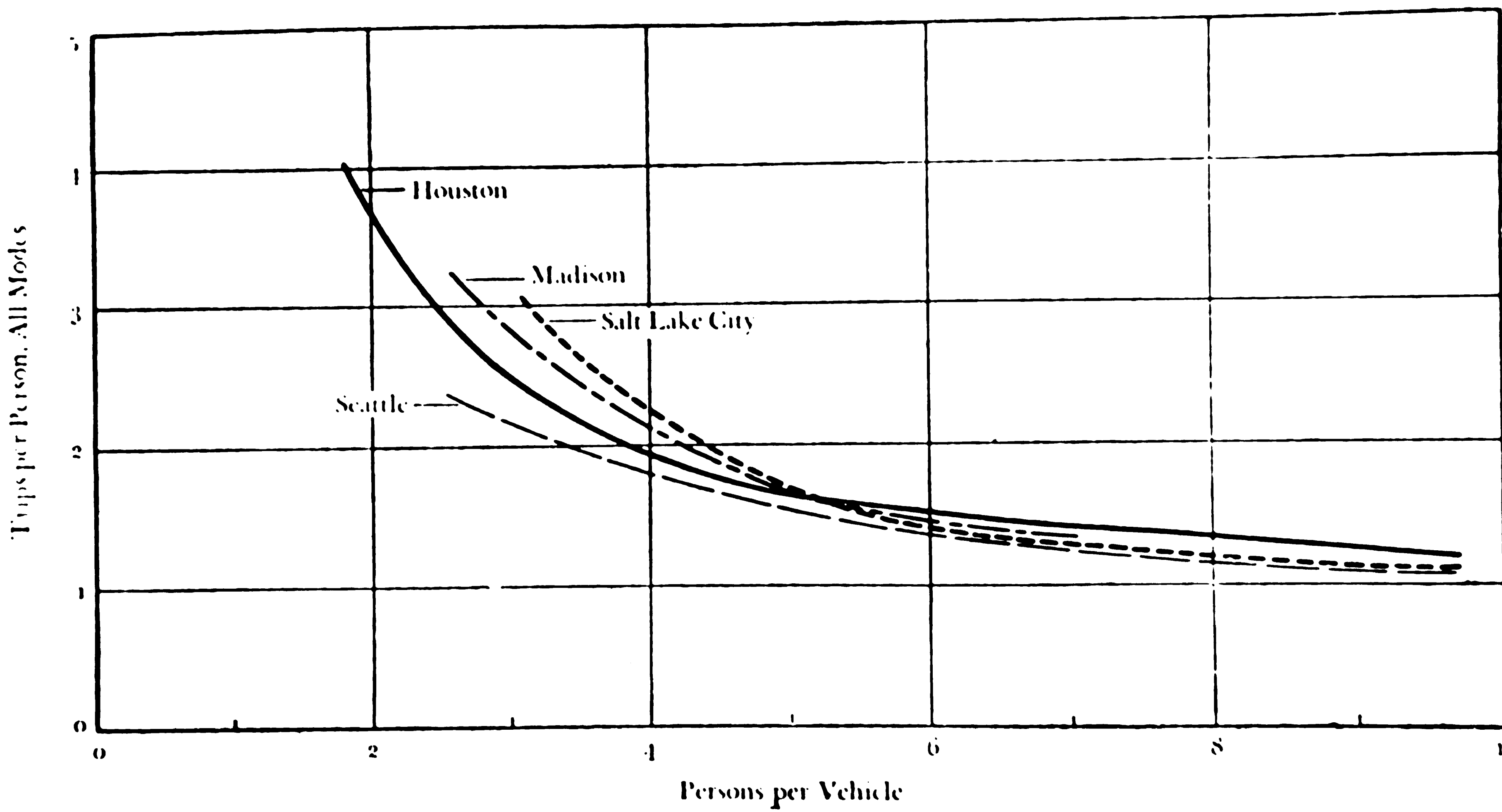


Figure 1. Trips per person vs. persons per vehicle. 4 cities
Source: Reference 6

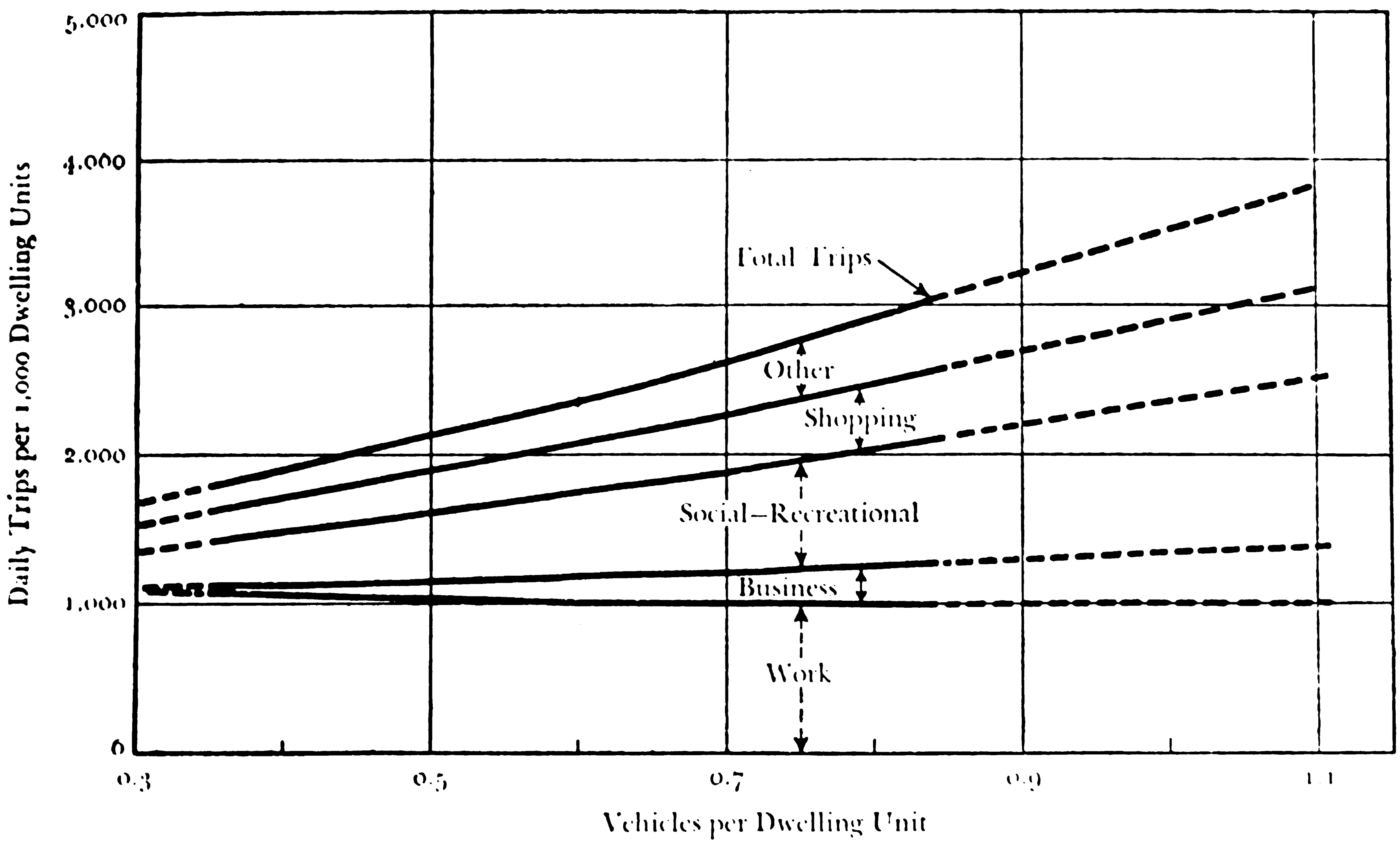


Figure 2. Distribution of trips from dwelling units by purpose of trip.
Source: Reference 6

in his study, although it was felt that the distance factor was interrelated with income levels. The fourth variable, that of isolation, was only significant in the outer suburban areas where the intensity of land use was quite low. The lack of trip opportunities within some short distance can discourage trips, therefore, the isolation factor has a negative effect on trip production. This can eliminate the errors which generally occur in the outlying districts at some large distance from the city's center.

Hall (14, 15).--The analysis of O and D data, gathered on two residential subdivisions in San Diego, developed relationships between land use and traffic generation. Origin factors per dwelling unit were established to develop weekday origins for the horizon year. The horizon year is defined as a stage on development and not by date. This factor includes commercial origins generated in the zone when the acreage in commercial use is not over one per cent of the gross useable land. For residential areas these factors are:

1. Areas with less than 1.0 vehicle/dwelling unit - 2.7 origins per dwelling unit
2. Areas with 1.0 to 1.5 vehicle/dwelling unit - 3.2 origins per dwelling unit
3. Areas with over 1.5 vehicle/dwelling unit - 3.7 origins per dwelling unit.

Carroll (16, 17).--Of the metropolitan transportation studies completed to date, the two for which J. D. Carroll, Jr. served as study director

are most noteworthy. Since the reports are quite extensive, only a brief resume of their presentation on residential traffic generation will be presented here.

Several methods were used to estimate future trips for the Chicago study. The future year land use forecast was used by assuming that the study year generation rates per acre would hold constant for the future year. The present land use rates were computed for each distance ring. A preliminary estimate was found by multiplying the trip rates per acre for the study year by the future estimates of acreage of land in the various land use categories.

Trips were next evaluated on the basis of future residential population. Since it has been established that trips per family is a function of car ownership and population density, it was necessary to predict these parameters for the future year. From these projections, it was possible to establish the future year level of car ownership at the individual zones. Zone forecasts of population density were established as part of the future land use forecasts. The trips per dwelling unit (Y_1) for each zone were estimated on the basis of car ownership (X_1 - cars per 100 dwelling units) and net residential density (X_2 - dwelling units per 10 acres). The equation used was:

$$Y_1 = 682.84 + 3.8109X_1 - 0.1939 \ \text{Log } X_2$$

The total future year estimate of person trips was based upon the latter method. As a population based forecast, this estimate reflects more directly the needs of people to travel. Travel satisfies that basic requirement of humans for work, shopping, recreation and etc.

The formula was applied to all future year zones to establish a total estimate of person trips per average weekday. These total trips were then allocated to each of the major land uses. In distributing the trips by land use, Figure 3 was used as it relates the per cent of trips for each purpose to the number of trips per family. Trips from each zone were then summarized for each land use. The number of origins on the residential land of a given zone can be established from Figure 3. For Chicago, in the year 1980, the sum of the estimates averaged 55.8 per cent of all trips going to residential land.

The procedures for estimating total trips described above were developed in the Detroit study. Volume One of the Detroit report shows the effect of the four variables on trip making per dwelling unit. These are shown as Figures 4, 5, 6 and 7. The relationships are also presented in equation form through the use of multiple correlation techniques as Table 1.

Keefer (18).--The problem of trip generation is one of correlating current trip ends with individual zone variables, so that future trip origin forecasts can be made. Keefer notes that there are alternate approaches available and the transportation study staff must find that combination of explanative methods which yields the best over-all prediction of trip making. It seems unlikely that the trip generating rates for a residential zone at a certain distance from the CBD will remain stable for some future year. This requires that trip generation rates must be adjusted in accordance with certain parameters such as income, residential density and car ownership. Preliminary work on the

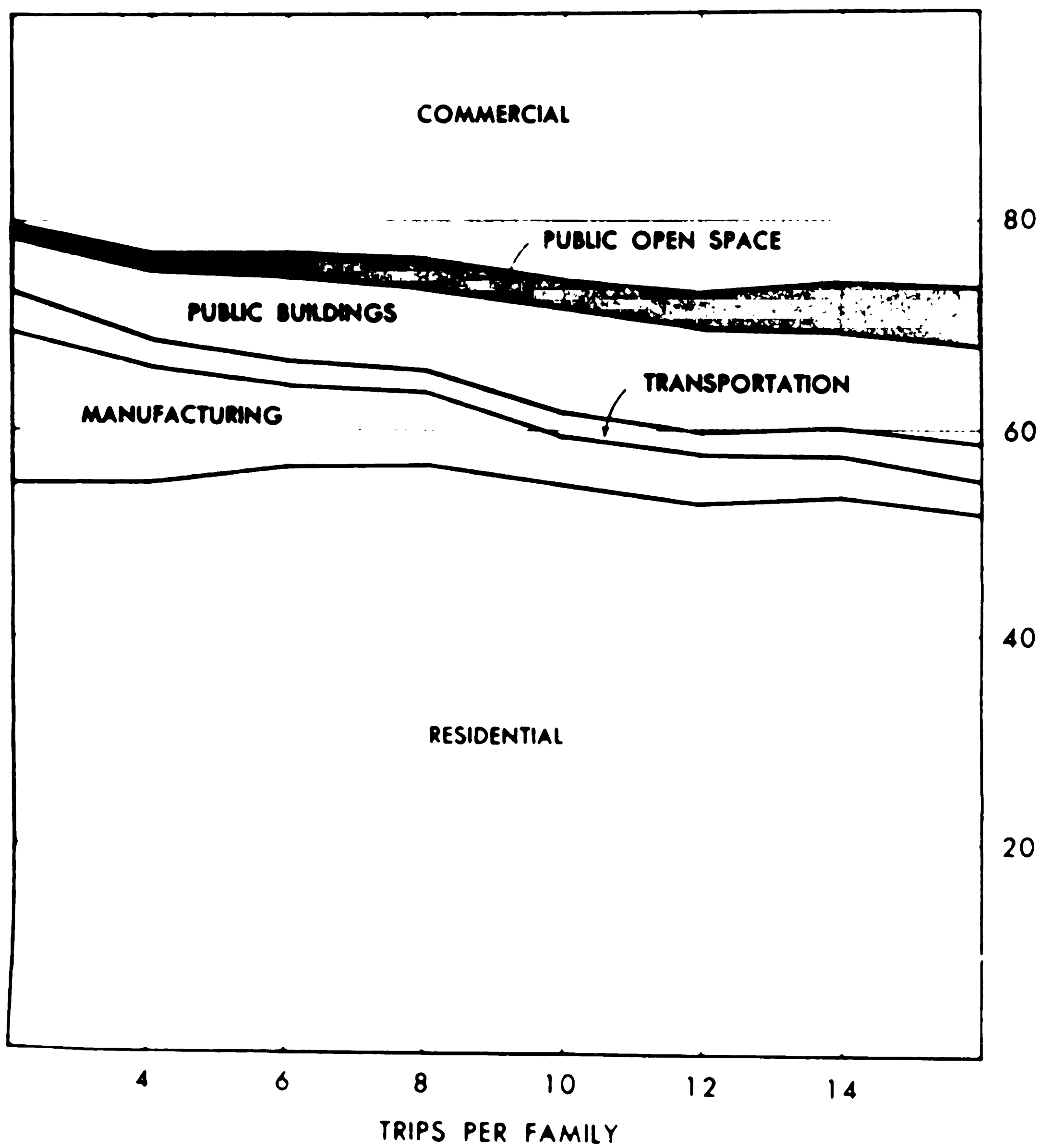


Figure 3. Percentage distribution of weekday person trips by land use related to trip making per family.

Source: Reference 16

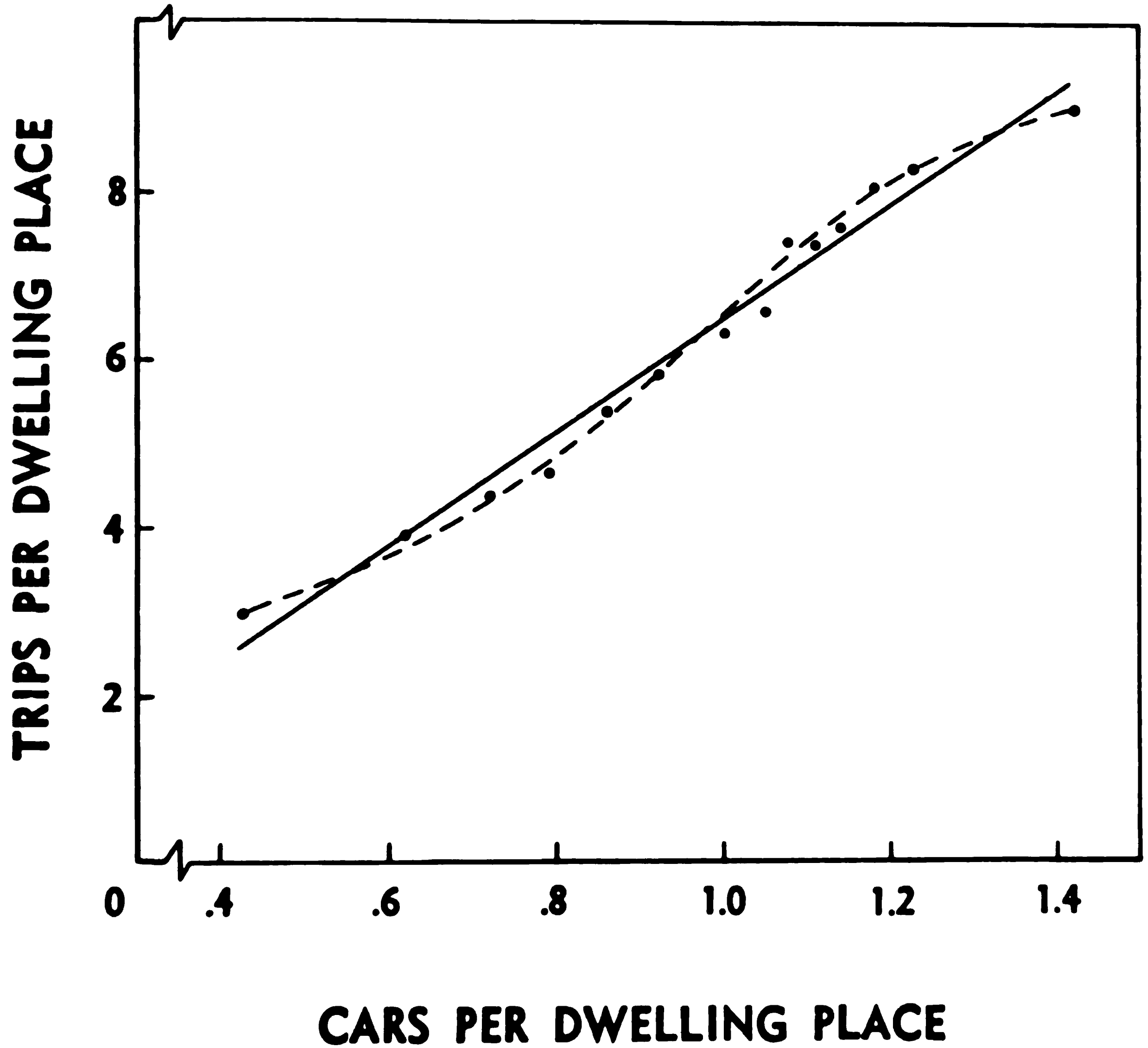


Figure 4. Effect of cars per dwelling place on trip making per dwelling unit

Source: Reference 17

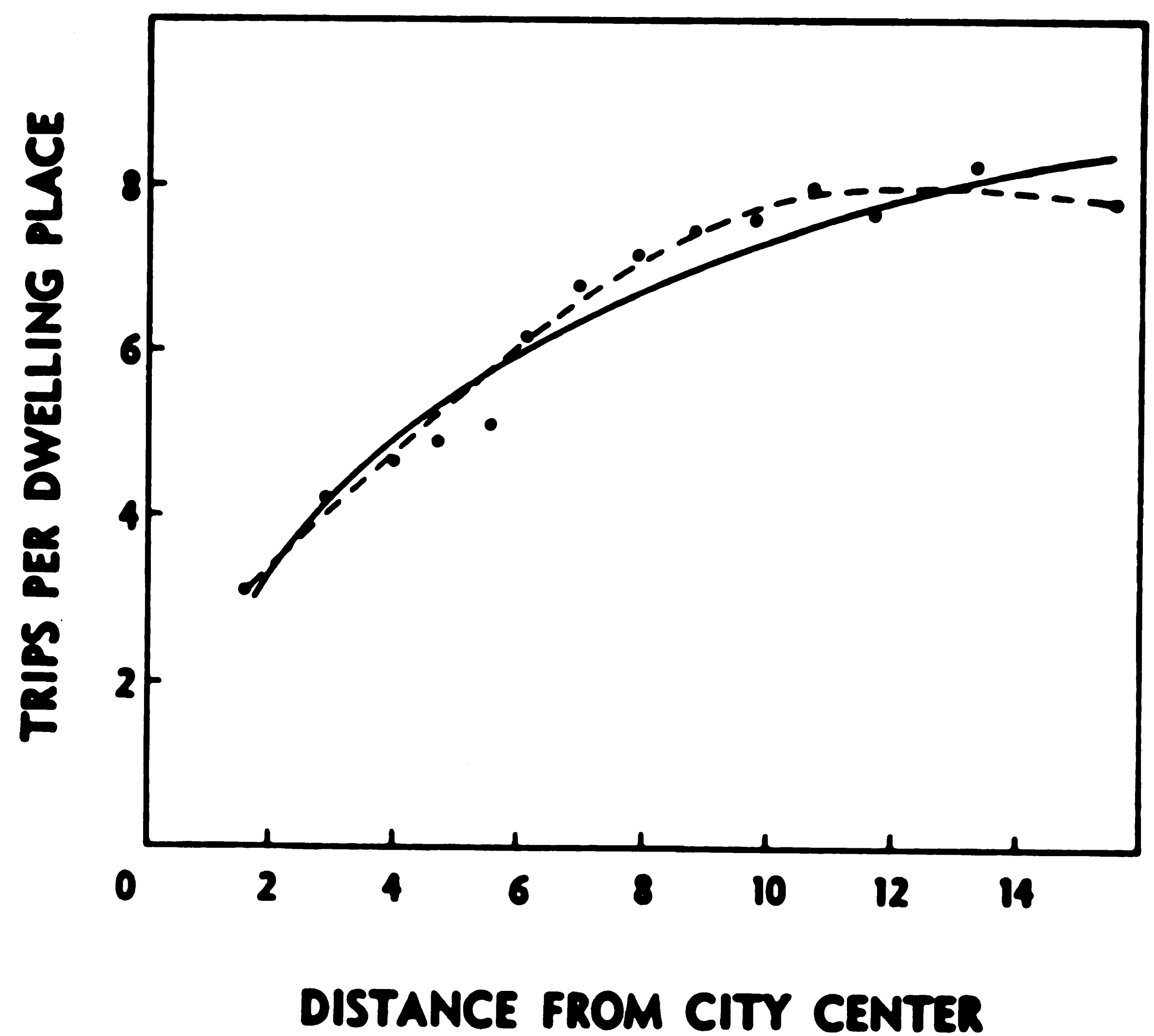


Figure 5. Effect of distance from city center on trip making per dwelling unit
Source: Reference 17

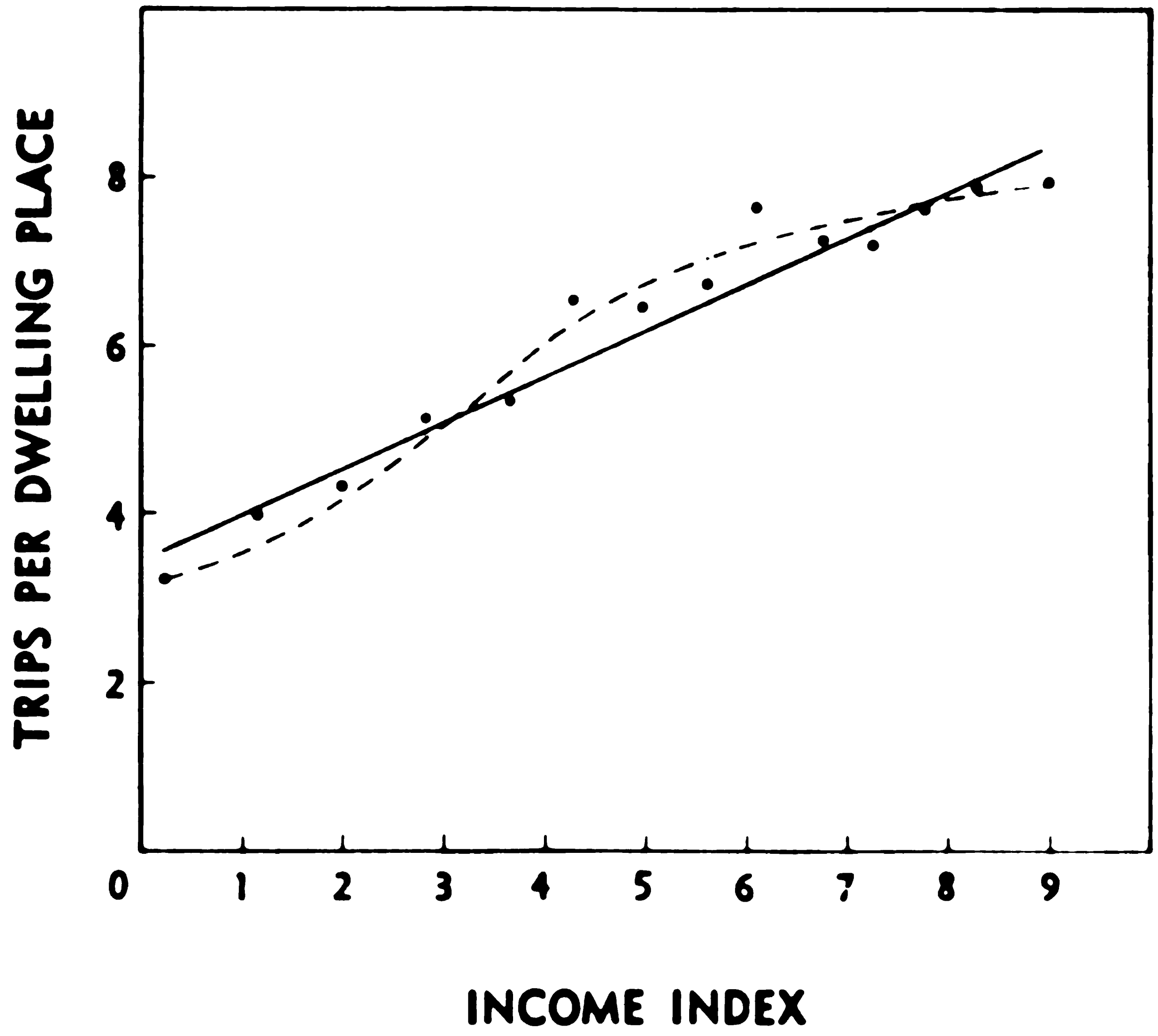


Figure 6. Effect of income index on trip making per dwelling unit

Source: Reference 17

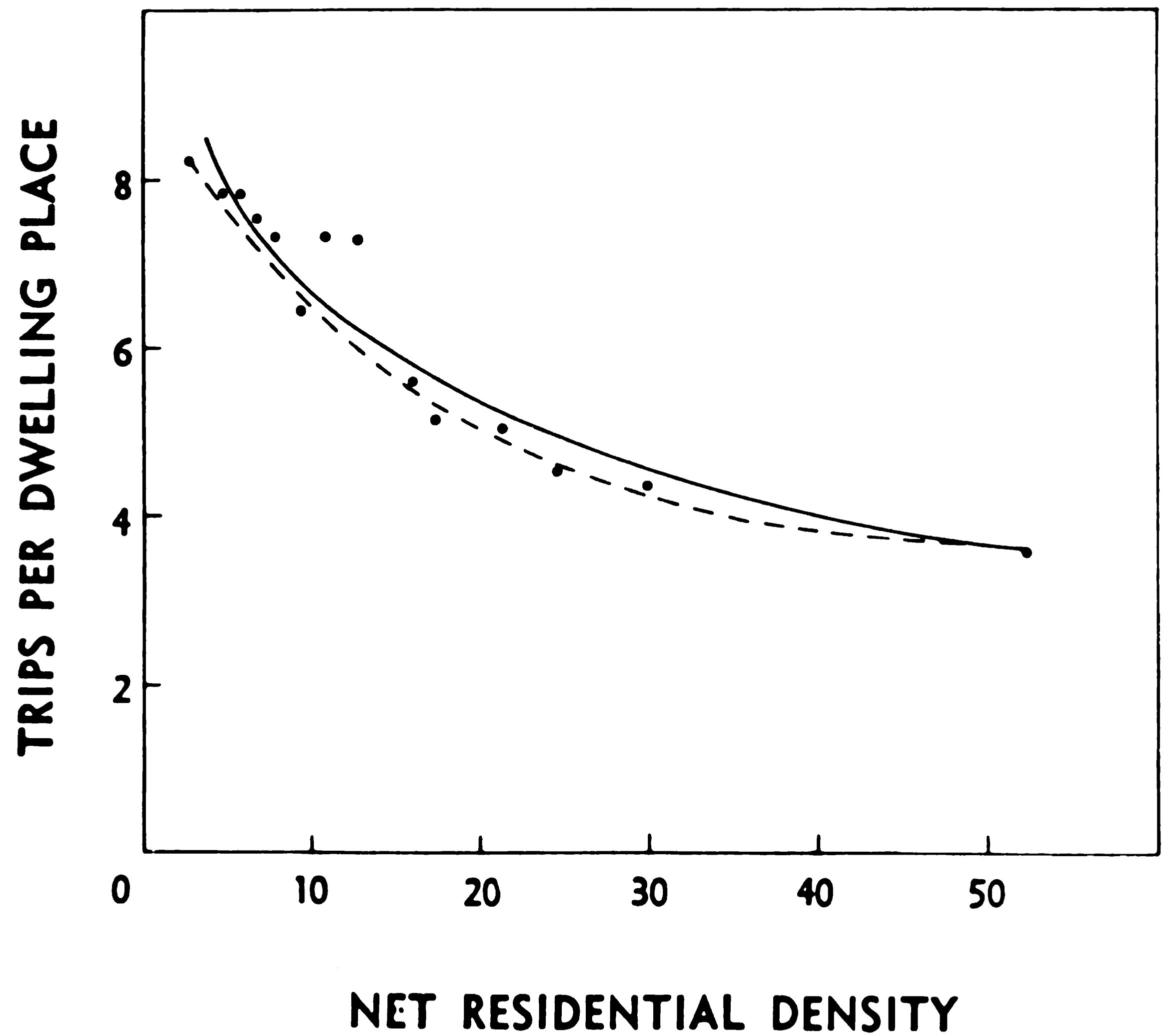


Figure 7. Effect of net residential density on trip making per dwelling unit

Source: Reference 17

Table 1. Measures of correlation between trips per dwelling place and distance, density, income and car ownership

Symbol	Independent Variables	Correlation Coefficients	Std. Error of Estimate	Coef. of Variation (%)	Estimating Equation
X_1	Car Ownership[1]	0.889	±0.88	±13.9	$Y_i = -.047 + 6.59\ X_1$
X_2	Net Residential Density[2]	-.748	±1.27	±20.1	$Y_i = 15.07 - 4.23\ \text{Log}\ X_2$
X_3	Distance[3]	0.812	±1.12	±17.7	$Y_i = -4.33 + 5.84\ \text{Log}\ X_3$
X_4	Income Index[4]	0.725	±1.32	±20.9	$Y_i = 3.69 + .526\ X_4$
	X_1 and X_3	0.907	±0.81	±12.8	$Y_i = -2.22 + 2.10\ \text{Log}\ X_3 + 4.88\ X_1$
	X_2 and X_3	0.832	±1.07	±16.9	$Y_i = 1.87 + 4.26\ \text{Log}\ X_3 - 1.60\ \text{Log}\ X_2$
	X_2, X_3 and X_4	0.882	±0.91	±14.2	$Y_i = 0.957 + 3.686\ \text{Log}\ X_3 + 0.195\ X_4 - 1.124\ \text{Log}\ X_2$
	X_1, X_2, X_3 and X_4	0.908	±0.80	±13.0	$Y_i = -0.1958 + 1.7288\ \text{Log}\ X_3 + 0.0008\ X_4 - 0.5464\ \text{Log}\ X_2 + 4.6480\ X_1$

1. Cars Owned divided by dwelling place in zone.
2. Dwelling places in tenths per net acre in residential use in zone.
3. Straight line distance in tenths of mile from zone to City Hall.
4. Income index developed by dividing 1950 U.S. census tract median income into deciles and assigning an index to each tract.

SOURCE: Reference 17.

Pittsburgh study developed a predicting equation which showed only a $\pm$ 5 per cent error between actual and predicted values.

$$Y_1 = .5275 + 2.07\ X_1 - 0.000078\ X_2$$

where Y_1 = trips per person from residentially used land

X_1 = automobiles per person

X_2 = Persons per million square feet of residential land used

Sharpe, et al (19).--The O and D studies made for Washington, D. C. were most significant of the limited number of repeated studies completed to date. Because of this, there has been much written on the analysis of this data. The report presents the results of an analysis of the effect that differences in population, car ownership, income per household, distance from the CBD and population per net residential acre had on the number of person trips attracted to and generated by residential land. Estimating equations were developed for predicting future traffic potential in terms of total residents trips and these equations were tested by comparing actual 1955 trips with the estimated values from the equations. In this study it was established that the use of all four variables combined did not significantly increase the accuracy of predicting trips over that which was obtained, using only automobile ownership and population density combined. The most reliable single predictor was found to be automobile ownership and very little additional accuracy was gained by combining it with population density. This report presents three general methods for estimating future traffic potential of residential areas, assuming that the travel patterns

of the Washington metropolitan area are not unlike the travel patterns of other cities. The specific procedures of each method are detailed below (19):

1. Method one--using the tabular data from an O and D study.

   a. Compute the residents trips per person for each zone in the area for which population and trip data are given.
   b. Compute car ownership (passenger cars owned per 100 persons) for each zone.
   c. Letting x = cars per 100 persons, y = trips per person, and n = number of zones, solve the following simultaneous equations for a and b:

$$\sum y = an + b\sum x$$
$$\sum xy = a\sum x + b\sum x^2$$

   d. Determine estimating equation

$$y = a + bx$$

   This equation provides an estimate of the trips per person corresponding to the car ownership in a particular area for an over-all citywide car ownership ($\Sigma x/n$) existing at the time of the survey. To apply this relationship to a future time, it is necessary to compute the parallel curve for an over-all car ownership estimated for the future data ($\Sigma x'/n$). This is done by assuming "y" and "b" remain constant and solving for the new "a" in the following equation:

$$\bar{y} = a' + b\bar{x}'$$

   The trips per person for each area can then be calculated by substituting in the new equation ($y = a' + bx'$) for each estimated value of car ownership in the study areas.

   Although this has not been tested, instead of using car ownership data, it should be possible to utilize these techniques by substituting population density, income, distance from the CBD, or a combination of these variables.

2. Method two--Using the tabular data from an O and D study.

   a. Compute the ratio of residents trips per person for each zone in the area.
   b. Estimate the future population of each zone.
   c. Multiply a times b for each respective zone.
   d. In zones for which prior trip data are not available, estimates can be made by comparison with zones having similar characteristics.

3. Method three--Using the tabular data from an O and D study.

    a. Determine the number of trips and the cars owned for each zone in the area.
    b. Estimate the future cars owned in each zone.
    c. Multiply the number of trips by the ratio of future to present cars owned.

The application of the most desirable method of estimating the potential trip generation in residential areas is dependent upon the availability and reliability of correlative data.

Mertz and Hamner (20).--This study was undertaken to determine the effect of automobile ownership, population density, distance from the CBD, and income per household on the number of vehicular trips residents made in Washington, D. C., on an average weekday in 1948.

It was found that the use of all four variables combined did not produce a significant increase in the accuracy of predicting trips over that which was obtained using automobile ownership and population density combined. Furthermore, automobile ownership was found to be the most reliable single predictor with very little additional accuracy gained by combining it with population density. The results of this analysis are shown in Table 2.

Commercial Generation.--As was noted in the discussion of Carroll's (16) work, there are two philosophies of trip generation. Trips can be generated on the basis of the acreage of a land use or on the population of the area. Those who follow the latter scheme discuss commercial attraction rather than generation and this will be presented later. Trip generation for commercial land use should be separated into CBD trips and trips to off-center commercial areas (6).

Table 2. Correlation of residents' daily trips per dwelling unit with various combinations of independent variables, as measured by a study of 95 census tracts in Washington, D. C. on an average weekday in 1948

Independent Variables	Trips per Dwelling Unit (Dependent Variable) Correlation Coefficient	Std. Error of Estimate	Predicting Equation
Automobile ownership & population density	0.835	0.87	$Y_1 = 3.80 + 3.79\ X_1 - 0.0033\ X_2$
Automobile Ownership	0.827	0.89	$Y_1 = 2.88 + 4.60\ X_1$
Population density and Income	0.764	1.02	$Y_1 = 5.49 - 0.0089\ X_2 + 0.227\ X_4$
Population Density	-.718	1.10	$Y_1 = 7.22 - 0.013\ X_2$
Income	0.655	1.20	$Y_1 = 3.07 + 0.44\ X_4$
Distance from CBD	0.575	1.30	$Y_1 = 3.55 + 0.74\ X_3$

Note: Y_1 is residents' trips per dwelling unit

X_1 is auto ownership (cars per dwelling unit)

X_2 is population density

X_3 is distance from CBD

X_4 is income

Source: Reference 20.

Schmidt (6).--The number of trips to the CBD will depend on many factors. Some of these are the population of the city and its metropolitan area, how the population is distributed at varying distances from the CBD, workers in the CBD, car ownership, availability of public transit, goods and services offered and competition from other areas.

Wynn (21).--An estimate of the internal auto driver trips generated by the CBD can be obtained from the use of three charts presented by Wynn. One must know the population of the metropolitan area, the car ownership and population of the zone, and the distance from the CBD. The charts, noted here as Figures 8, 9 and 10 are used in sequence to provide an estimate of trips per day. Given a zone i, at a specified distance from the CBD and located in a metropolitan area of some population value, Figure 8 will give an estimate of the trips generated. Figure 9 uses the distance value and car ownership to provide a second value which is added to the first. Figure 10 relates the percentage of the metropolitan area population within designated radius distances of the CBD to trips generated. This value is subtracted from the sum of the previous two in order to establish the number of trips per car per day generated by the CBD.

Wynn recommends that zones be chosen of equal size or population and the population and car ownership be carefully determined. The centroid of the population distribution is established for each zone and the shortest distance between that centroid and the center of the CBD determined, as measured along existing streets. The population vehicle ratio and population compactness for each zone is determined before the curves are used to make the estimate.

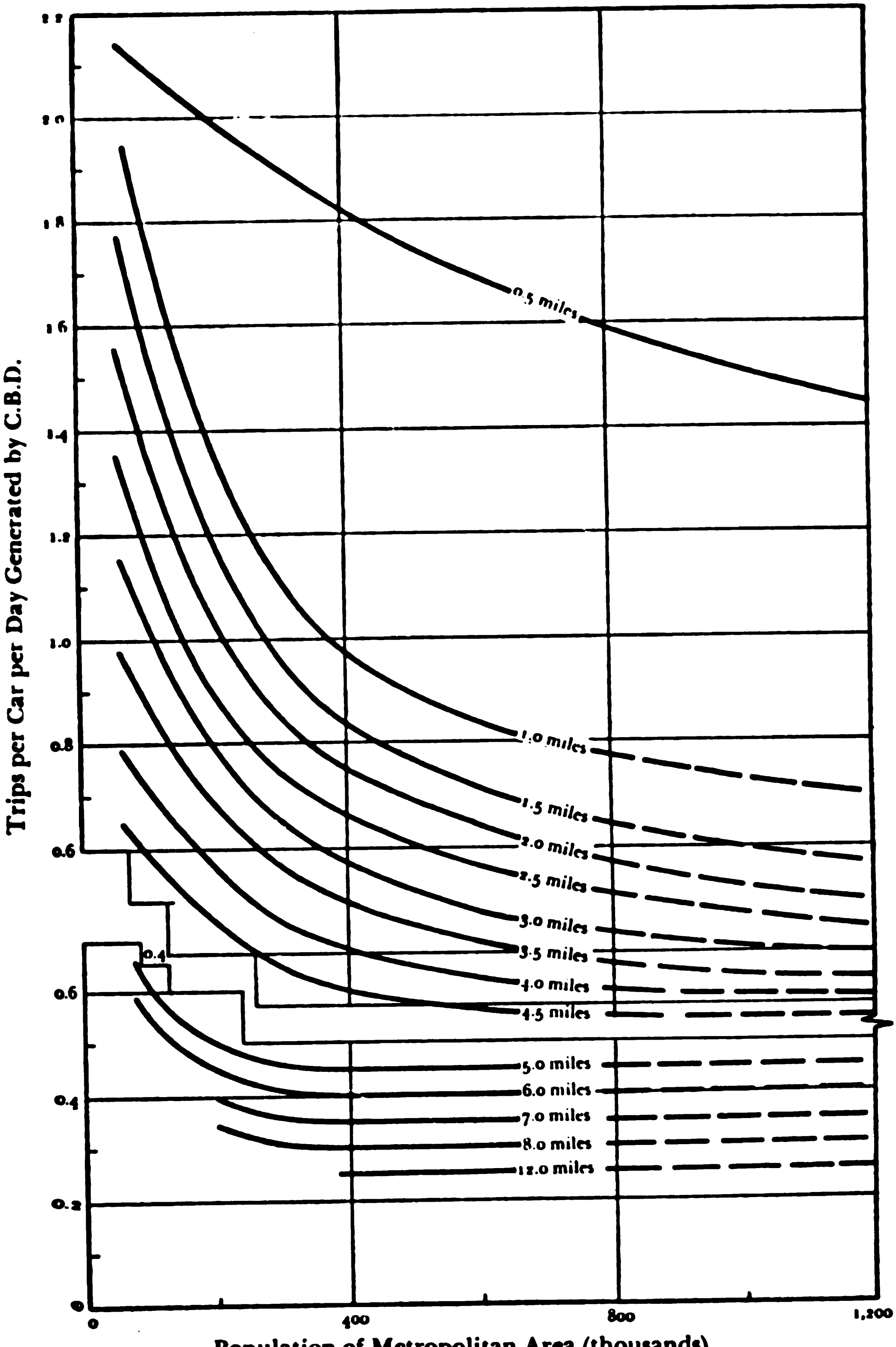


Figure 8. Auto driver trips generated by the CBD. Measure of city size for urban areas, 50,000 to 600,000 population.
Source: Reference 21

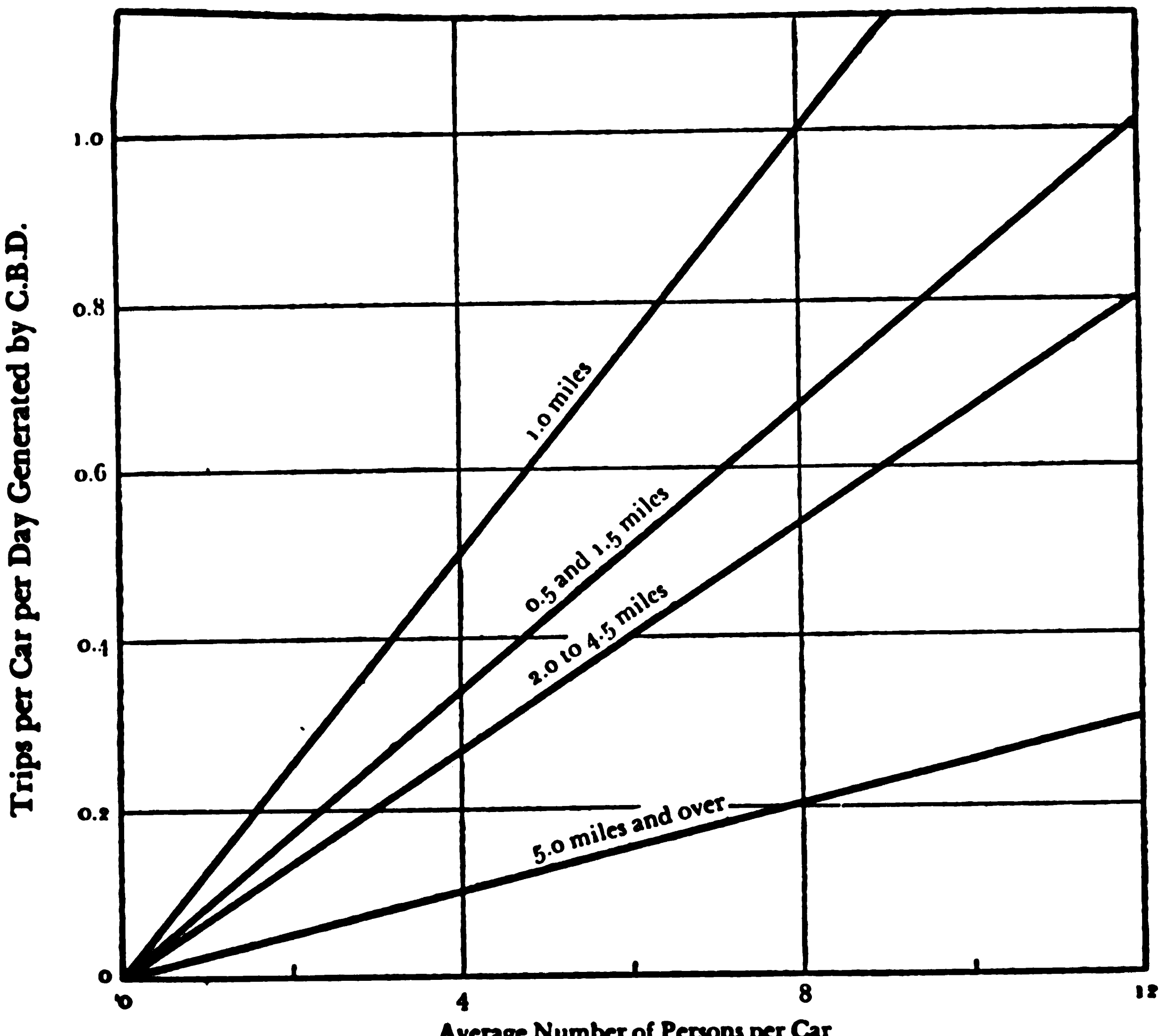


Figure 9. Auto driver trips generated by the CBD. Measure of population vehicle ownership ratio. Source: Reference 21

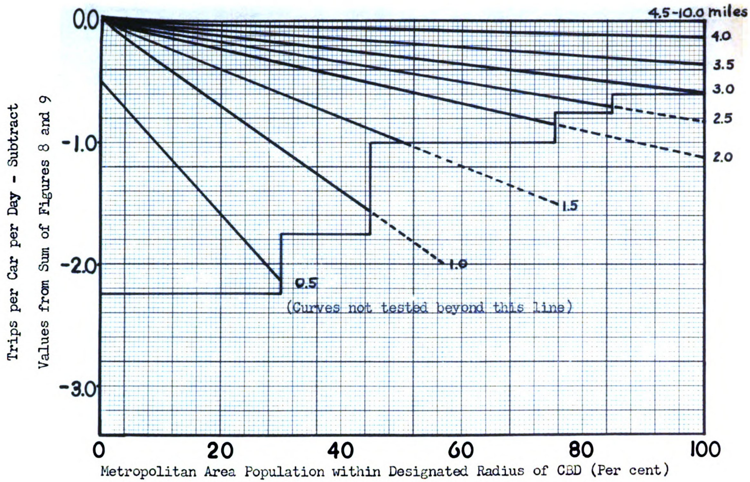


Figure 10. Auto driver trips generated by CBD - Measure of city compactness

Source: Reference 21

Hall (14).--Listed below are the factors developed for weekday origins in San Diego. These are expressed as origins per net acre.

1. Commercial (Factor to be applied to the commercial acreage in excess of one per cent of the gross usable land)

   a. Mature trend areas and strip development - 215 origins
   b. Community and regional shopping centers - 275 origins

2. CBD

   a. Central core - 700 origins
   b. Professional district - 550 origins
   c. Mixed commercial district - 250 origins
   d. Industrial areas - 180 origins
   e. Bay front district - 150 origins
   f. Apartment district - 200 origins
   g. Schools - 60 origins

Harper and Edwards (22).--This report was an attempt to correlate the generation of person trips with the amount of various floor areas present within a CBD. Three broad categories of floor area usage was adopted and coded X_1 - Retail floor area, X_2 - Service floor area, and X_3 - Manufacturing-Warehouse floor area, and each is expressed in 1000 square feet of space in use. Similar equations were evolved for Detroit, Philadelphia and Baltimore and the authors feel that these equations could be valid estimates of one true equation which would work for all cities, but no effort was made to predict this equation. The equation for Detroit will be the only one shown here:

$$Y \text{ (the person destinations)} = 13.918\, X_1 + 4.613\, X_2 + 1.717\, X_3 - 2280$$

<u>Industrial or Work Trip Generation</u>.--Studies have shown that from thirty to fifty per cent of all daily trips, are made to and from work. Couple this with the fact that these trips are usually made during the periods of peak flow, one then realizes the importance of work trips to urban

travel. A more thorough understanding of work trip movements can do much to improve the techniques of urban planning (23). A later section will discuss the factors through which a work area attracts trips. In contrast to this, a few articles will be presented, which deal with the industrial area as a trip generator.

Wynn (21).--Work trips were segregated from trips made for other purposes because work trips were more completely reported in the home interviews than other trips. Analysis of studies made, showed that the labor force made up about forty per cent of the residents of each zone and this was distributed throughout most of the area proportionally to the population distribution. Wynn, then proposed that if the same proportion of the labor force in each city can be expected to work each day, then it seems logical that work trips would be made in direct proportion to the population. The results of his investigation on twenty cities are shown in Figure 11.

Hall (14).--The following factors are used for developing weekday origins for industrial areas. The values are expressed as origins per net acre.

1. Aircraft
    a. Without parking - 180
    b. With some parking - 110
    c. Parking provided - 60
2. Modern industrial parks - 65
3. Research parks - 35
4. Distributional industrial area - 85
5. Bay front and shipbuilding - 25
6. Large warehousing - 50
7. Open storage and materials plants - 10

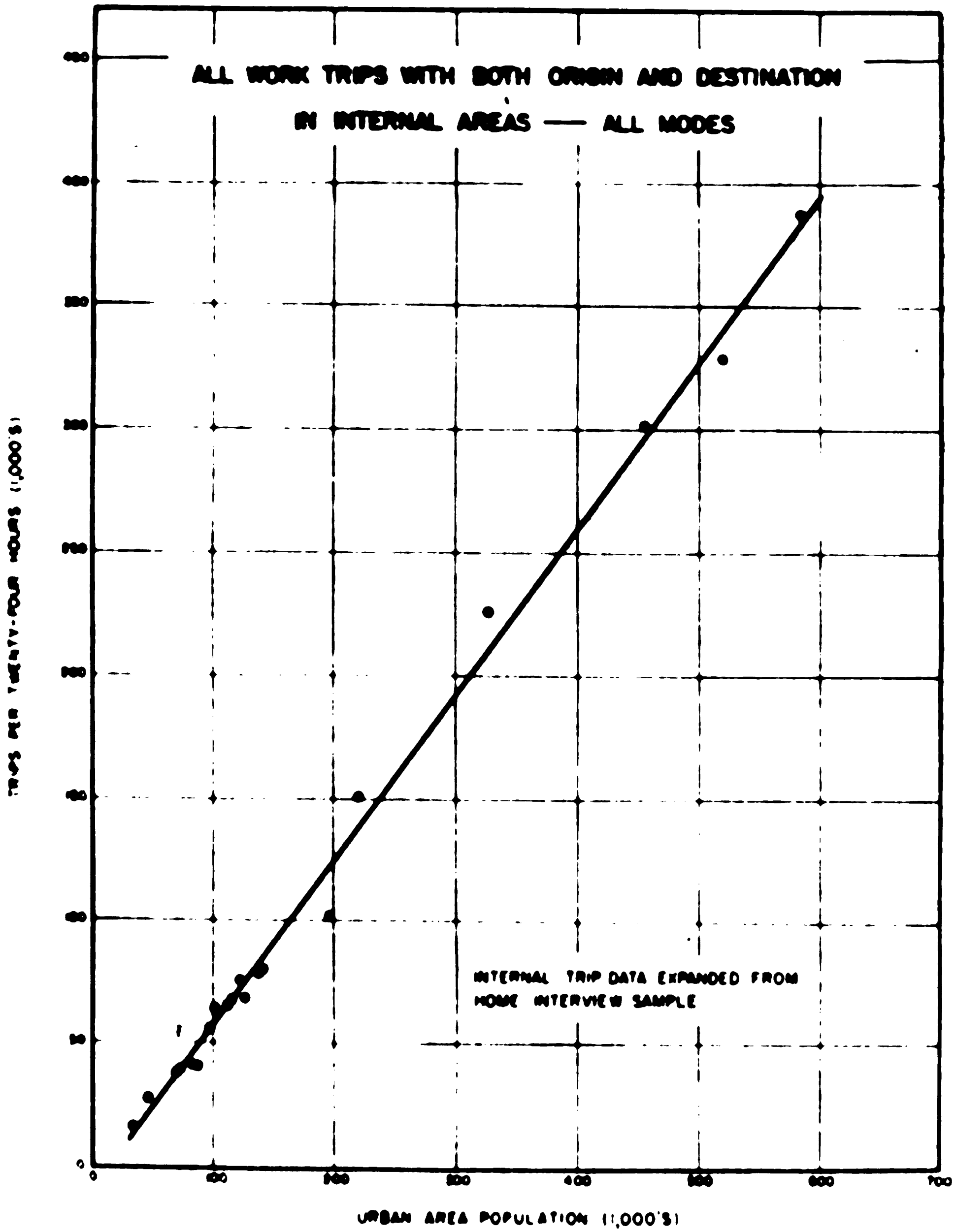


Figure 11. The relationship between total work trips and urban area population. Source: Reference 21

Lapin (24, 25).--The importance of work trips to the overall urban travel pattern is noted because of the relative volume of work trips and their possible use as a basis in predicting total urban trip generation. Lapin points out that work trips are more completely measurable. If a multiplier could be found to predict total patterns from work trips, tremendous cost savings could be realized in data collection.

Regular patterns were found for the proportional distributions by mileage or time rings of residence origins of workers about employment centers. The longer the distance from an employment center, the smaller the number of workers that will commute to it. This commuting pattern is also dependent upon the size (number of workers) of the employment zone. Measurable characteristics of the origin zone (population density and distance from the CBD) are related to the work-trip distribution pattern.

Lapin presents a work-trip analysis model of the future and describes the procedure in the following steps (25):

1. Prepare origin and terminal trip-end data for each area unit, based upon current information.
2. Develop inter-relationships of area units through gravity formulations, utilizing varying constants, weights for individuals, and varying exponents for systems of trips, location of area units, etc. Where functions are excessively complex, apply graphical solutions.
3. Modify trip-end and trip-exchange data in terms of independent forecast measures.
4. Test work-trip production and interchange findings by means of graphical distribution analysis.
5. Test total trip-production and distribution findings by means of derived relationships between work-trip systems and other systems of travel.

Lapin suggests that graphical analysis can lead the way toward generalization in algebraic formulation. Where a consistency of pattern

is indicated, and where a logical basis exists for selection of a particular type of equation, then further and more detailed study is probably warranted.

Trip Purpose

It should be apparent from the literature review so far that travel patterns within a particular urban area vary. One of the basic reasons for these variations is trip purpose. Because of this variation, trip distribution models, which are presented in the following section, generally are solved for a specific trip purpose. Each model uses a distinct set of parameters which reduces the variability. Furthermore, trip purposes can be associated with a particular land use and the present trend of forecasting future trip origins is predicted upon future land uses. It seems essential that a search be made to establish patterns of trip purpose from O and D data.

A most comprehensive analysis was made by the Bureau of Public Roads and was reported by Curran and Stegmaier (26). Their report is summarized in the following figure and tables with some additional data included by this writer. Table 3 shows the percentage of person trips from all origins to specific trip purposes. The number of person trips from home only by trip purpose is given in Table 4 (55). Figure 12 shows the varying percentages of person trips from each origin to each destination according to purpose. Percentage values within the chart represent the individual per cent of all trips.

Table 3. Percentage of person trips from all origins classified according to purpose

O and D Study	Work & Business	Social & Recreation	Shopping	Miscel-laneous	Home
4 Cities* Population over 1,000,000	28.9	10.7	6.3	14.5	39.6
6 Cities* Population 500,000-1,000,000	28.9	10.9	7.8	9.5	42.9
3 Cities* Population 250,000-500,000	28.4	13.8	8.0	10.4	39.4
20 Cities* Population 150,000-250,000	25.4	14.3	8.9	11.3	40.1
12 Cities* 100,000-150,000	26.5	14.8	7.9	10.2	40.6
5 Cities* Population Less than 50,000	24.0	17.5	8.8	11.9	37.8
All 50 Cities* Average of 6 groups above	27.9	12.0	7.5	11.8	40.8
Chicago Area Trans-portation Study (1956)	30.8	12.8	5.5	7.4	43.5
Detroit Metropoli-tan Transportation Study (1953)	28.6	12.1	8.2	11.6	39.5

Sources: Reference 26 *
Reference 17
Reference 56

Table 4. Home-based trips by urban residents in study areas according to purpose

Urban Area	Home-Based Trips as % of all Linked Trips	Work	Business	Shopping	Social Recreational	School	Other	All Purpose	Total Home-Based Trips per Dwelling Unit
Chicago	86.8	37.5	9.7	18.9	22.8	4.0	7.1	100.0	5.17
Detroit	87.0	41.6	8.6	13.9	20.1	6.3	9.5	100.0	4.67
Washington	91.6	43.1	9.6	14.2	12.5	9.4	11.2	100.0	4.23
Pittsburgh	87.0	37.7	21.6	14.9	13.8	12.0	- -	100.0	4.21
St. Louis	91.3	37.5	8.1	17.3	21.5	6.4	9.2	100.0	4.90
Houston	91.0	33.1	8.9	17.3	18.6	10.8	11.3	100.0	5.51
Kansas City	88.2	33.4	8.8	17.2	22.7	6.0	11.9	100.0	5.14
Phoenix	85.3	25.2	10.2	19.7	20.0	11.6	13.3	100.0	4.76
Nashville	85.5	30.3	8.5	16.9	23.9	7.4	13.0	100.0	5.48
Ft. Lauderdale	86.5	27.9	15.3	24.0	22.9	0.9	9.0	100.0	2.82
Charlotte	83.9	32.2	8.0	15.6	23.8	6.6	13.8	100.0	5.56
Reno	86.5	29.2	12.7	18.1	26.3	0.5	13.2	100.0	4.88
Average %	87.6	34.0	10.8	17.4	20.8	6.8	10.2	100.0	4.78

Source: Reference 55

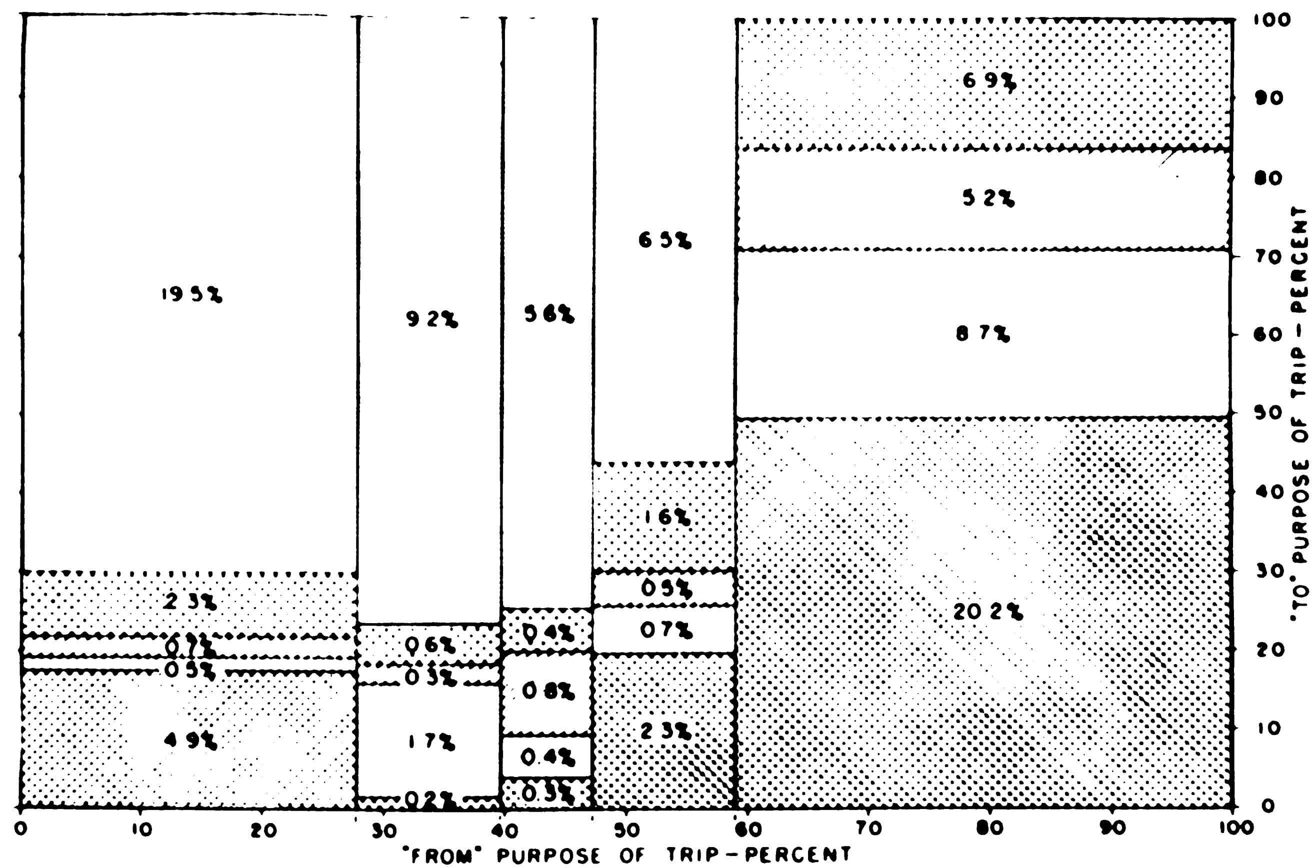


Figure 12. Percentage distribution of trips from each purpose to each purpose.
Source: Reference 26

Trip Attraction and Distribution

It seems feasible from the preceeding sections that, given the amount and location of future land uses and population, the number of trips generated in each area can be calculated. These generation rates would come from an analysis of many past origin and destination surveys which contained inventories of trips and of land uses. It also seems possible that future trips can be classified according to purpose, time of travel and mode of travel. So that given the trips generated in an area and knowing their division by purpose, one needs only to know how these various trips are attracted to available destinations in order to be able to draw a desired line flow for an urban area. The following sections will be a brief presentation of the methods used or proposed to date.

Gravity Model.--A discussion of the early attempts to develop travel formulas was reviewed by Schmidt (6). The earliest reference to Newton's law of gravitation for traffic attraction was in a paper published in 1930 by a Swedish investigator, H. N. Pallin. The references that follow show modifications made to the general formula by Pallin; his equation can be stated:

$$V = K \frac{P_1 \cdot P_2}{D^x}$$

Where:

V is the number of trips (persons or vehicles)

P_1 is the population (or vehicles registered) of area 1

P_2 is the same for area 2.

D is the distance between areas 1 and 2.

x is some power of D, usually 2.

K is a constant used to adjust the variation in dimension

Variations of this basic equation, which do not warrant greater amplification, will be stated briefly here. Schmidt (6) discusses a formula developed by W. R. Bellis for computing inter-area traffic volume in the state of New Jersey.

$$V = \frac{R_1 \quad f_2}{T_2}$$

Where:

V is the volume of traffic from area 1 to area 2

R_1 is the number of motor vehicles registered in area 1.

f_2 is the force of attraction which area 2 exerts on area 1.

T is the total elapsed time to travel from area 1 to area 2.

A modification of the gravity model was used by Willa Mylroie (27) to evaluate intercity travel desire. Her formula for the intercity travel desire factor (F) is shown below using previously defined terms.

$$F = \frac{\sqrt{P_1 \cdot P_2}}{D^2}$$

These equations are sometimes referred to as the P/D relationship. A comprehensive discussion of the relationship of traffic to population and distance can be found in the article by Ikle (12). The effect of population size upon the frequency of trips can be best shown with a social or visit trip between zones. Ikle reasons that since such a trip involves a person from a zone who visits a person in another zone, that the greater the number of possible pairs, the more likely or frequent the trip. The number of possible pairs between zones of populations P_1 and P_2 is the product P_1 times P_2. He explains the distance effect as the greater the distance, the greater the cost and time of traveling; the greater the distance, the less likely is an actual relationship, between a potential pair of people, that might lead to a trip. This type of equation, which relates population and distance, can be expressed as:

$$H_{ij} = K \cdot \frac{P_i \cdot P_j}{D_{ij}}$$

Where: K is a constant used to adjust the different dimensions.

The leaders in proposing this hypothesis were John Q. Stewart (28) and George K. Zipf (29). They applied this formulation to telephone calls, letters, and bank checks as well as traffic. Inherent in their logic was the fact that the distance factor could not assume any other but the inverse linear function. Ikle's work led to a reformulation of the hypothesis with distance expressed to an exponent b.

$$H_{ij} = K \cdot \frac{P_i \cdot P_j}{D_{ij}^{\ b}}$$

Voorhees (30, 31, 32, 33).--Strongest advocate of the gravity model is Voorhees, as evidenced by his many articles on the subject. His model shows that trip destinations reflect a competition between existing land uses which is dependent upon the attraction of the zone and the travel time to it. His basic equation is:

$$T_{ij} = T_i \frac{M_j \,/\, (D_{ij})^x}{\sum_{1}^{j} (M_j \,/\, (D_{ij})^x)}$$

Where:

T_{ij} is number of trips from zone i to zone j.

T_i is number of trips from zone i

M_j is some measure which reflects the attraction of zone j for certain type trips

D_{ij} is the travel time from zone i to zone j.

x is the empirically determined exponent

In Table 5, which follows, Voorhees (30) lists his early concepts of attraction units and exponents for various purposes.

The gravity model can be applied as a single model for all trips or separate models can be developed for each trip purpose. In applying the model to peak hour patterns, Voorhees (31) used a trip purpose model. By knowing the percentage of trips made for each purpose that occur during the peak hour, one can establish peak flows through the use of separate models. An analysis of work trip patterns based on a sample of O and D data showed close agreement on the empirically established x values. The values determined by method of least squares are: Baltimore (1946) - 0.647, Wichita (1955) - 0.680, South Bend (1947) - 0.703, Oklahoma City (1948) - 0.745, Fort Wayne (1946) - 0.805 and Philadelphia (1947) - 0.805. This range in exponent values would have less than a ten per cent variation in trip estimates. Voorhees feels that the correlation between cities is significant.

A more recent application of the gravity model is also reported by Voorhees (32). In arriving at trip frequencies for Baltimore, it was found that commercial and social trips were a function of car ownership. For every 1,000 cars garaged in a residential area, there would be 900 commercial and 700 social auto trips originated in that area.

Table 5. The apparent factors affecting the destinations of trips starting from a residential area

Purpose of Trip	Unit to express "size of attractor"	Effect of "distance factor"
Work	No. of workers employed	$D^{\frac{1}{2}}$
Social	Dwelling units	D^3
Shopping Convenience Goods	Floor area in foods & drugs	D^3
Shopping Goods	Floor area in apparel	D^2
Business	(1)	(1)
Recreational	(1)	(1)
Other	(1)	(1)

(1) In light of existing research it is recommended that these trips be considered as shopping goods trips.

Source: Reference 30

Work trips were related to employment rather than car ownership. The total number of employed persons in each work area was used as an indication of the attractor size for work trips. For commercial trips, the retail employment for each zone was used. The number of people living in each area was chosen to indicate the attractor size for social trips. The influence of travel time on trips for various purposes is shown as a series of factors given in Table 6.

Table 6. Effect of travel time on frequency of travel

Travel Time in Minutes	Relative Frequency of Trips by Type Work	Social	Commercial	Non-Home-Based
2	4.00	5.00	8.0	8.0
3	2.86	3.33	7.0	7.0
4	2.28	2.50	6.0	6.0
5	1.90	2.00	4.0	4.0
6	1.60	1.62	2.7	2.7
7	1.40	1.42	2.0	2.0
8	1.21	1.25	1.5	1.5
9	1.11	1.11	1.2	1.2
10	1.00	1.00	1.0	1.0
11	0.93	.91	.80	.80
12	.86	.83	.68	.68
13	.80	.77	.57	.57
14	.75	.71	.50	.44
15	.70	.67	.44	.40
16	.66	.62	.40	.35
17	.62	.59	.35	.32
18	.59	.55	.32	.28
19	.56	.52	.28	.25
20	.53	.50	.25	.22
21	.50	.46	.23	.19
22	.47	.43	.21	.16
23	.44	.40	.20	.13
24	.41	.37	.18	.10
25	.39	.34	.16	.08
26	.36	.32	.15	.06
27	.33	.30	.14	.04
28	.31	.28	.13	.02
29	.27	.26	.12	.01
30	.25	.25	.11	-
31	.23	.23	.10	-
32	.21	.21	.10	-
33	.19	.19	.09	-
34	.18	.18	.08	-
35	.17	.17	.08	-
36	.16	.16	.07	-
37	.15	.15	.07	-
38	.14	.14	.07	-
39	.13	.13	.07	-
40	.12	.12	.07	-
41	.11	.11	.07	-
42	.10	.10	.06	-
43	.09	.09	.06	-
44	.08	.08	.06	-
45	.07	.07	.05	-
46	.06	.06	.05	-
47	.05	.05	.04	-
48	.05	.05	.04	-

Effect of travel time on frequency of travel (continued)

Travel Time in Minutes	Relative Frequency of Trips by Type Work	Social	Commercial	Non-Home-Based
49	.04	.04	.04	-
50	.04	.04	.03	-
51	.03	.03	.03	-
52	.03	.03	.03	-
53	.02	.02	.02	-
54	.02	.02	.02	-
55	.02	.02	.02	-
56 - 60	.01	.01	.01	-

Source: Reference 32

Since car ownership is very important to the prediction of trips for this formulation, it was essential that a method be selected to forecast car ownership. The method selected was based on a study by the Bureau of Public Roads. This study showed that the type of residential area and the income of the household influenced the number of cars per household. It also showed an increase in car ownership up to income levels of $8,000 to $10,000 per year but then leveled off beyond this range. Therefore, there was a ceiling for car ownership for various types of residential areas. The ceilings for car ownership are shown in Table 7.

In his latest writings on this subject, Voorhees (33) has revised the estimate of trip production from what was noted earlier. He now estimates that for each 1,000 cars in a residential area there will be about 1,600 commercial trips. The number of trips for a specific purpose, except work trips, increases directly with car ownership. Work trips reach a ceiling at approximately 400 cars per 1,000 persons due to the limitation in the size of the labor force.

Table 7. Projected car ownership per household

Baltimore: 1980

Residence Type	Autos per Household
Single Family	
new area	1.6
old area	1.0
Two Family	
new area	1.2
old area	0.9
Row House	
good transit and poor parking	0.4
good transit and good parking	0.6
poor transit and good parking	1.0
High Rise	
good transit and poor parking	0.2
good transit and good parking	0.4
poor transit and good parking	0.6

Source: Reference 32

Best use of the gravity model can be made when trip models are separated by purpose. In the Baltimore and Hartford studies, four trip purposes were used: (1) home-based work trips, (2) home-based commercial trips, (3) home-based social trips, (4) non-home based trips. In both studies, the value M_j from the gravity model was expressed in terms of employment in dealing with work trips and commercial trips; population was used for social trips and for non-home based trips; a factor that equals the population plus twenty-five times the retail employment for the zone.

The gravity model was used in seven cities in Iowa. The trips were divided by purpose as follows: (1) home-based work trips, (2) other home-based trips, (3) non-home based trips. The M_j value used for the other home-based trips equaled the population plus twenty-five times the retail employment plus employment for each zone. The value for non-home-based trips is computed in the same way.

Friction Factor Method (34, 35).--The friction factor method was developed by W. B. Calland (34), in process of forecasting future traffic volumes for San Diego. After carefully examining all the known methods, they decided to use a model based on the gravitational principle, where the attraction of the zones could be related to the trips generated and modified by the "friction" incurred in traveling. This theory can be stated in equation form as:

$$T_{ij} = T_i \cdot T_j \cdot f_{ij}$$

Where:

T_{ij} is the volume of trips from zone i to zone j

T_i is the trip origins in zone i

T_j is the trip origins in zone j

f_{ij} is the factor which denotes relative friction in traveling from zone i to zone j

The problem was to predict the values for f_{ij} for each possible zonal interchange. Preliminary investigation revealed that it would be necessary to separate the trip with respect to the CBD, cordon line and other areas. The cordon line is a hypothetical line which delimits the metropolitan area. Trips were divided into the following categories:

1. Trips with either origin or destination outside the cordon
2. Trips with both origin and destination within the CBD
3. Trips with either origin or destination within the CBD
4. Other interzonal trips

The friction factors f for each category were calculated from data available from a 1952-1953 O and D survey. These values were found by using the equation:

$$f_{ij} = \frac{T_i \cdot T_j}{T_{ij}}$$

The friction values were then plotted against a distance value for each of the above categories. This method requires a separation of trips into the various categories before future trip movements can be computed. The curve for other interzonal trips (category four) can be stated mathematically as $f = K \cdot d^{-1.45}$ where k is a correction factor. In the use of any gravity type model, one generally finds first an unbalance between the total trips originating in a zone and the sum of all the distributed trips from that zone. The unbalance is corrected by using the ratio of these two values and is called K.

The friction factor curves developed for San Diego were tested in Sacramento by D. A. Merchant (35). Following a similar procedure, Merchant found variations in the curves and by comparison: $f = K \cdot d^{-0.72}$. Unlike the San Diego study, friction factor curves were obtained for only the interzonal trips and trips from the CBD to all other zones. A second set of these curves were obtained using a modified approach. In this approach the trip generation volumes for T_i and T_j included all trips regardless of whether they had a destination outside of the cordon. Merchant feels that the modified approach is much simpler and more accurate. He also states that the curves obtained in San Diego are not applicable to Sacramento.

BPR Model.--A discussion of this model is presented by Lynch (36). In discussing past and present O and D studies, Lynch points out the uniqueness of the Chicago study. This study was the first to make a detailed classification of land uses at trip ends. The collection of this information serves two important purposes. First, it is important to determine the number of trips attracted to each type of land use and to relate the trips to some measurable parameters such as population, automobile ownership, employment, dollar sales or area. Secondly, the effect of distance or travel time on trip interchange for various purposes can be studied. These are the factors which are important to any interarea travel formula such as the gravity model. The formula used by the Bureau of Public Roads in evaluating the data collected for the Washington Survey is shown as follows:

$$T_{ij} = \left(T_i \frac{A_j}{\Sigma A} + T_j \frac{A_i}{\Sigma A}\right)\left(\frac{K}{D_{ij}^x}\right)$$

in which:

T_{ij} is the number of primary trips between zones i and j, that is trips with one end at home

T_i is number of primary trips produced in zone i by residents of zone i

T_j is number of primary trips produced in zone j by residents of zone j

A_i is number of primary trips attracted to zone i by non-residents plus intrazone trips by the residents

A_j is number of primary trips attracted to zone j by non-residents plus intrazone trips by the residents

ΣA is the number of primary trips attracted to all zones

D_{ij} is the travel time distance between zones i and j.

K is a constant

x is the empirical exponent

Before the equation can be applied, values of K and x must be established for each purpose by utilizing existing O and D data. As in other modifications of the gravity model, it can be expected that the volume of trips attracted to and from each zone will differ. This unbalance of trips will then be neutralized by some method such as the successive approximation method.

Electrostatic Model.--The only writing on this type of model has been done by Robert T. Howe (37, 38, 39). In his theory, he considers people as electrons, so that a residential zone is assumed to be a distribution of negative charges. Employment zones are represented as positive charges. The probability of movement between these two zones is predicted on the basis of electrostatic field theory.

Preliminary application of this theory has been made in the area of work trip prediction. Although the model shown below seems quite similar to the gravity models presented previously, Howe emphasizes that his model was arrived at theoretically by assuming that the movement of people can be represented by that of electrons in a field of positive charges. The equation can be stated:

$$V_{P_i\,Q_j} = \frac{\frac{Q_j}{R_{ij}} \cdot P_i}{\sum_{j=1}^{m} \frac{Q_j}{R_{ij}}} \quad (i = 1, 2, --, n)$$

In Which:

$V_{P_i\,Q_j}$ is defined as the probability of directional movement from i to j.

P_i is the number of workers living in zone i

Q_j is the number of jobs available at center j

R_{ij} is the straightline distance from i to j if the field contains no physical bariers. Where such barriers exist, R would have to be the straight line distance from i to the point of passage across the barrier plus that from the point of passage to j.

Similar to the gravity model, this model does not take into account the total number of workers at each employment zone and therefore, usually overassigns or underassigns workers to the various job centers. A balanced assignment can be achieved through the use of the correction equations noted below.

Corrections in this computation are based on the following equations:

Step a. Multiply first assignment by the appropriate correction factor C_j for

$$j = 1, 2, ----, m$$

Step b.

$$C_j = \frac{Q_j}{\sum_{i=1}^{n} V_{P_i} Q_j}$$

Step c. Multiply the second assignment by the appropriate correction factor C_i for $i = 1, 2, ---- n$

Step d.

$$C_i = \frac{P_i}{\sum_{j=1}^{m} V_{P_i} Q_j}$$

Step e. Repeat steps a through d for each successive assignment.

Howe applied this model to Lafayette, Indiana and Cincinnati, Ohio, with only limited success in Lafayette. At the time of the latest report, the test of the theory on Cincinnati was not completed.

Interactance Model.--The scope of the paper by Wynn and Linder (40) is limited to the discussion of current travel patterns for three metropolitan areas. These travel patterns were developed by the use of interactance formulas derived from existing O and D data.

An analysis of the O and D data showed that 80 to 90 per cent of all person-trips reported in home interviews either begin or end at home. Trips in this study have been reclassified by purpose, according to the purpose at the non-home terminus. Work trips were defined as those trips between work and home. Trips between home and personal business, medical-dental, shopping and eat-meal purposes are grouped as commercial trips. Social trips and school trips are as usually defined. Trips between work and commercial generators, with neither terminus at home, are classified as miscellaneous trips.

Several advantages are gained by analyzing and projecting person trips separately for each of these basic purposes. It is obvious that trip production for each purpose will be related to different land use variables, thereby greatly systematizing the procedure of estimating trip ends. Of even greater significance, however, is the distinct difference in distribution patterns for each of the various purpose categories. For example, work trips are longer than trips for the other purposes.

The most significant factors related to trip generation were median family income, vehicle ownership, population density and relative decentralization. Despite the variability observed between populations of different characteristics in certain trip categories, such as social and school trips, they felt that consideration of the four variables in conjunction with each other tends to give a balanced picture of the factors influencing trip production by residents. Trip production at non-home termini showed even greater variability.

The problem resolves itself into determining exactly what proportion of the trips (for each purpose) originating in Zone A shall have destination in each of the other zones.

In order to estimate the number of shopping trips made by Zone A residents to the retail outlets in Zone B, consideration must be given to:

1. Total number of home-based shopping trips made by residents of Zone A.
2. Total number of trips made for shopping purposes to Zone B by residents of all zones.
3. Total number of shopping trips made to all other zones.
4. Travel time between Zones A and B.

The interactance formulas were derived from data collected in O and D studies for Charlotte, North Carolina (41), St. Louis (42), and Kansas City (43). Population statistics and trip length studies were used to develop a series of curves relating the average off-peak driving time and the relative number of trip attractions to travel desires for each purpose between zones. Although the slopes of the curves for each purpose varied considerably, they all showed an inverse relationship between trip production and travel time. Work trips had the slowest rate of decrease as driving time increased. Social trips tended to be somewhat longer and not so sensitive to increased travel time. Commercial and miscellaneous trips decreased quite rapidly, although not as rapidly as school trips.

The interactance curves presented are those established for Kansas City (43). Figure 13 represents the relative attraction of jobs at different distances from a zone as trip rates. The curve for each ring was separated from the others by raising or lowering the curve. Therefore, the actual values obtained from the curve must be adjusted in order to be meaningful. This is done by taking a residential zone at some specific ring and using the travel times to each employment zone, in order to establish the relative values of work trips from the curve. The actual number of work trips generated in the zone (the control total) is divided by the sum of the relative estimates from the curves and this provides a correction factor. The correction factor is applied to each curve estimate to give the number of work interchanges. Work trips are also distributed by the curve in Figure 14 as trips to and from employment zones. Two separate estimates are provided for work trips between any two zones. Since it is unlikely that the computed averages will balance, an iteration procedure must be used. Figures 15 and 16 are shown for distributing commercial trips. Social trips, school trips and miscellaneous trips are distributed in accordance with Figures 17, 18, 19, 20 and 21. This method establishes travel characteristics from current O and D data and then applies these to future land use estimates in order to synthesize future travel patterns.

Opportunity Model.--This model was developed in the Chicago study for predicting travel between zones. The basic premise for this model is that: a trip has a tendency to remain as short as possible, subject to the probability of finding an acceptable destination. The probability of any specific trip origin stopping at any randomly chosen destination

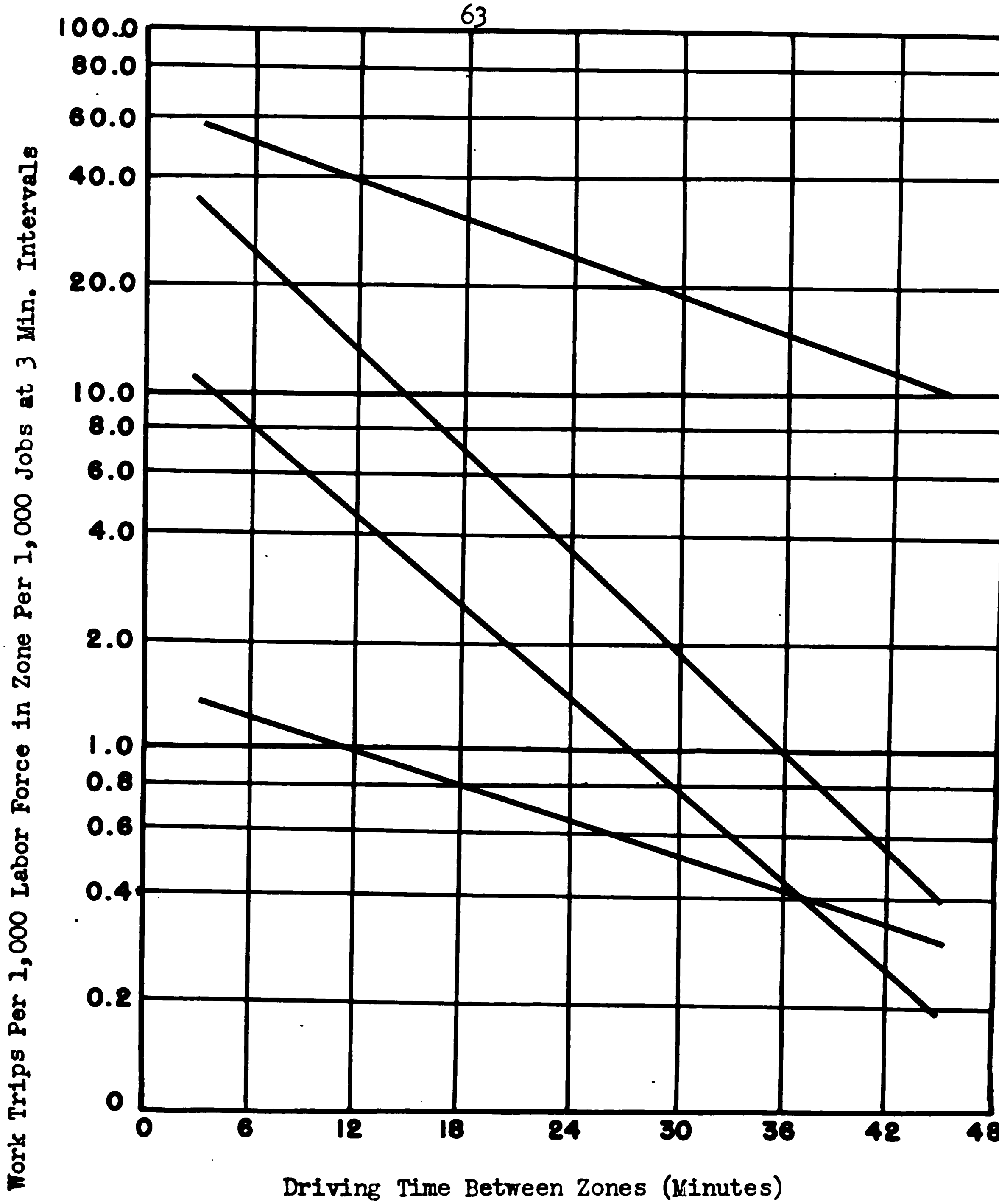


Figure 13. Distribution of Work Trips Between Zones (Trips to and from Home-All Modes)
Source: Reference 43

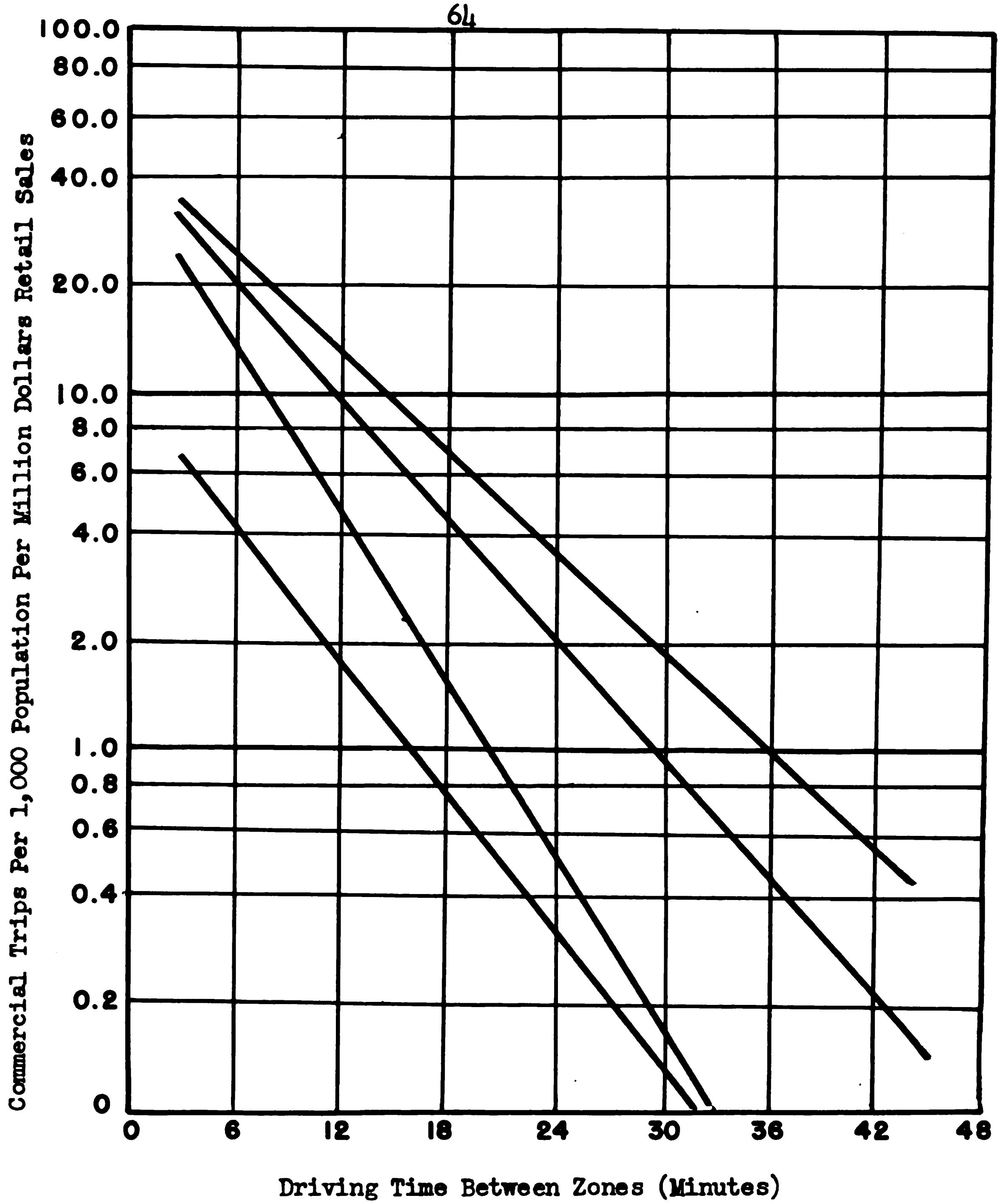


Figure 14. Distribution of Commercial Trips Between Zones (Trips to and from Home-All Modes)
Source: Reference 43

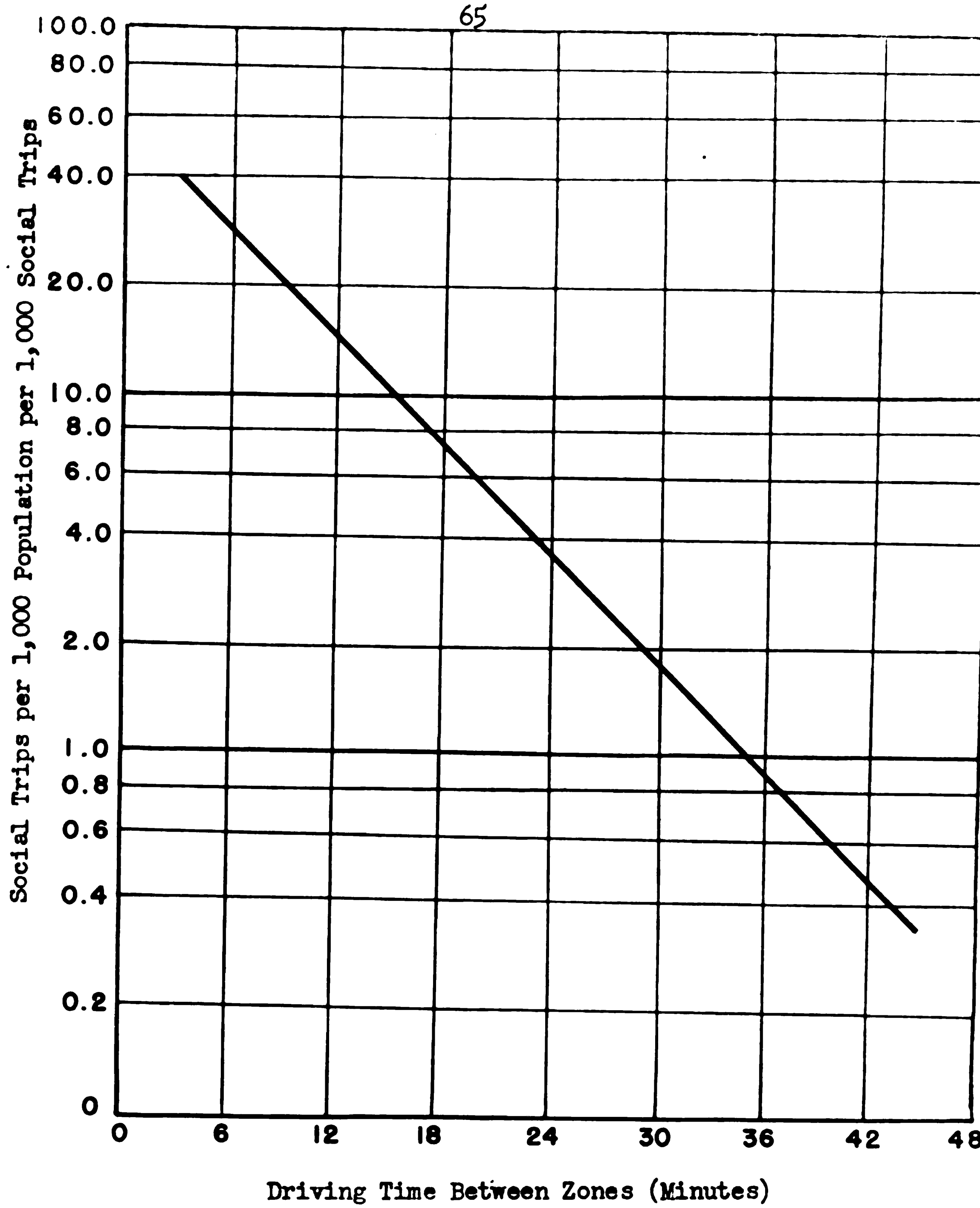


Figure 15. Distribution of Social Trips Between Zones
(Trips to and from Home-All Modes-All Rings)
Source: Reference 43

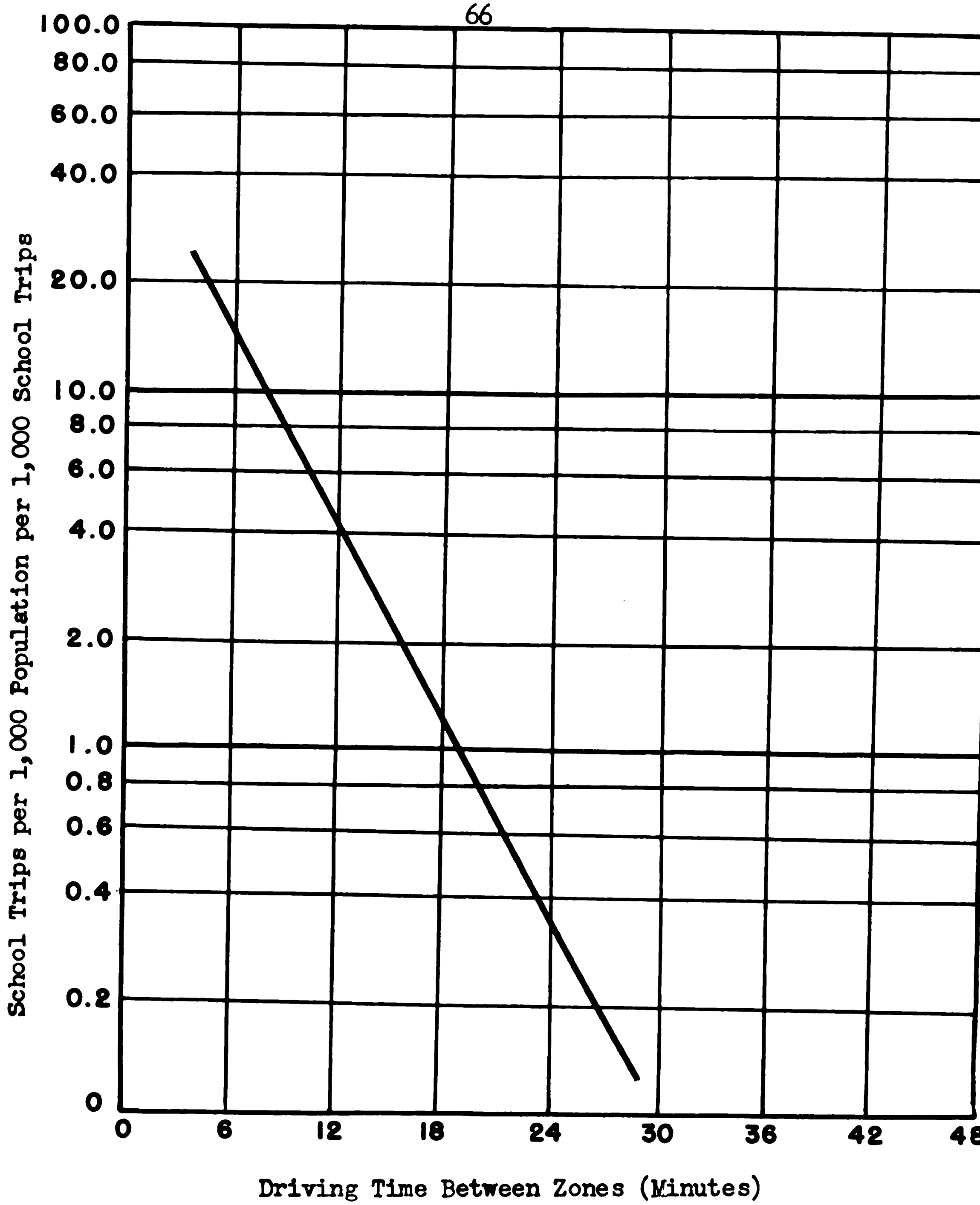


Figure 16. Distribution of School Trips Between Zones (Trips to and from Home-All Modes)
Source: Reference 43

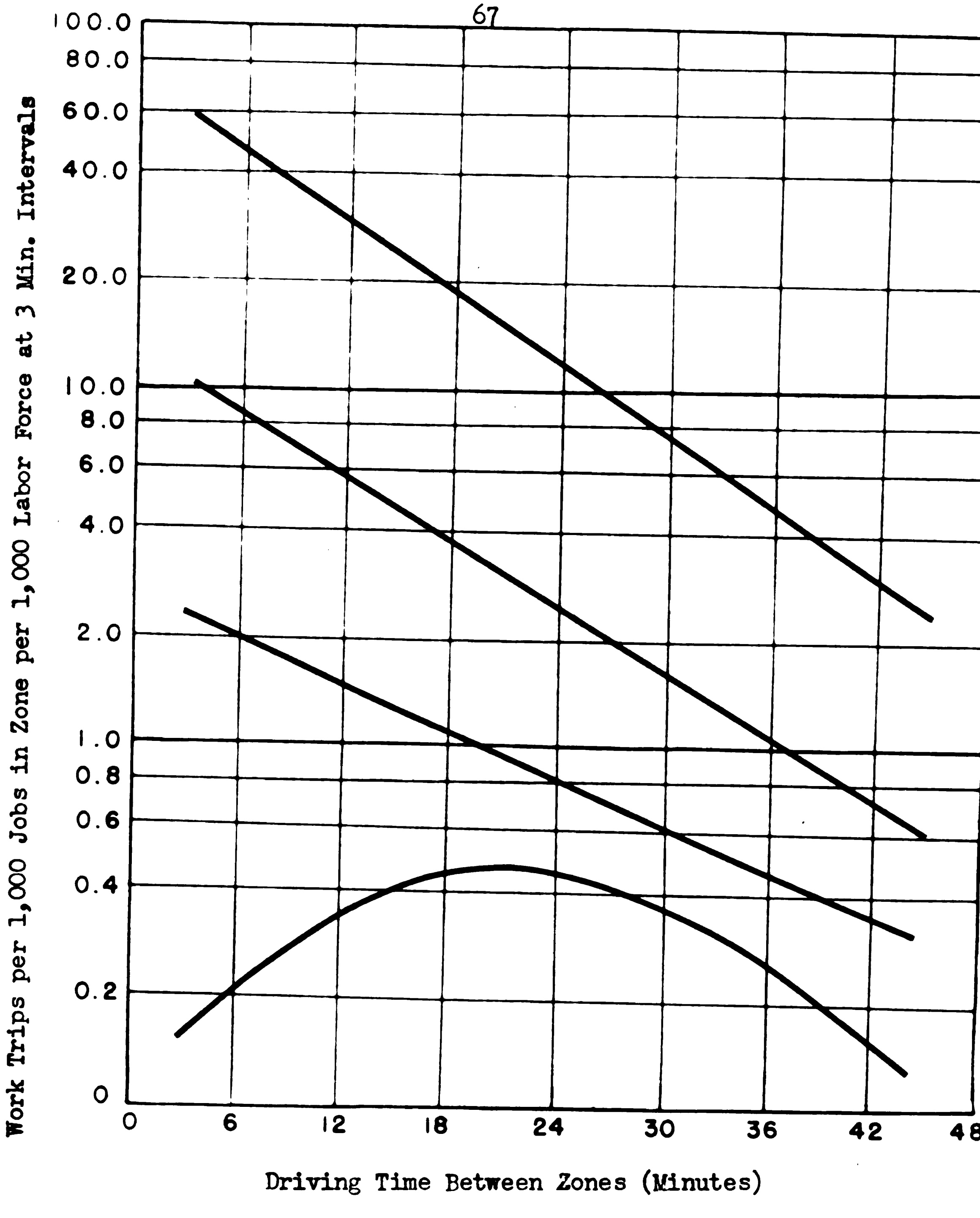


Figure 17. Distribution of Work Trips Between Zones
(Trips to and from Employment-All Modes)
Source: Reference 43

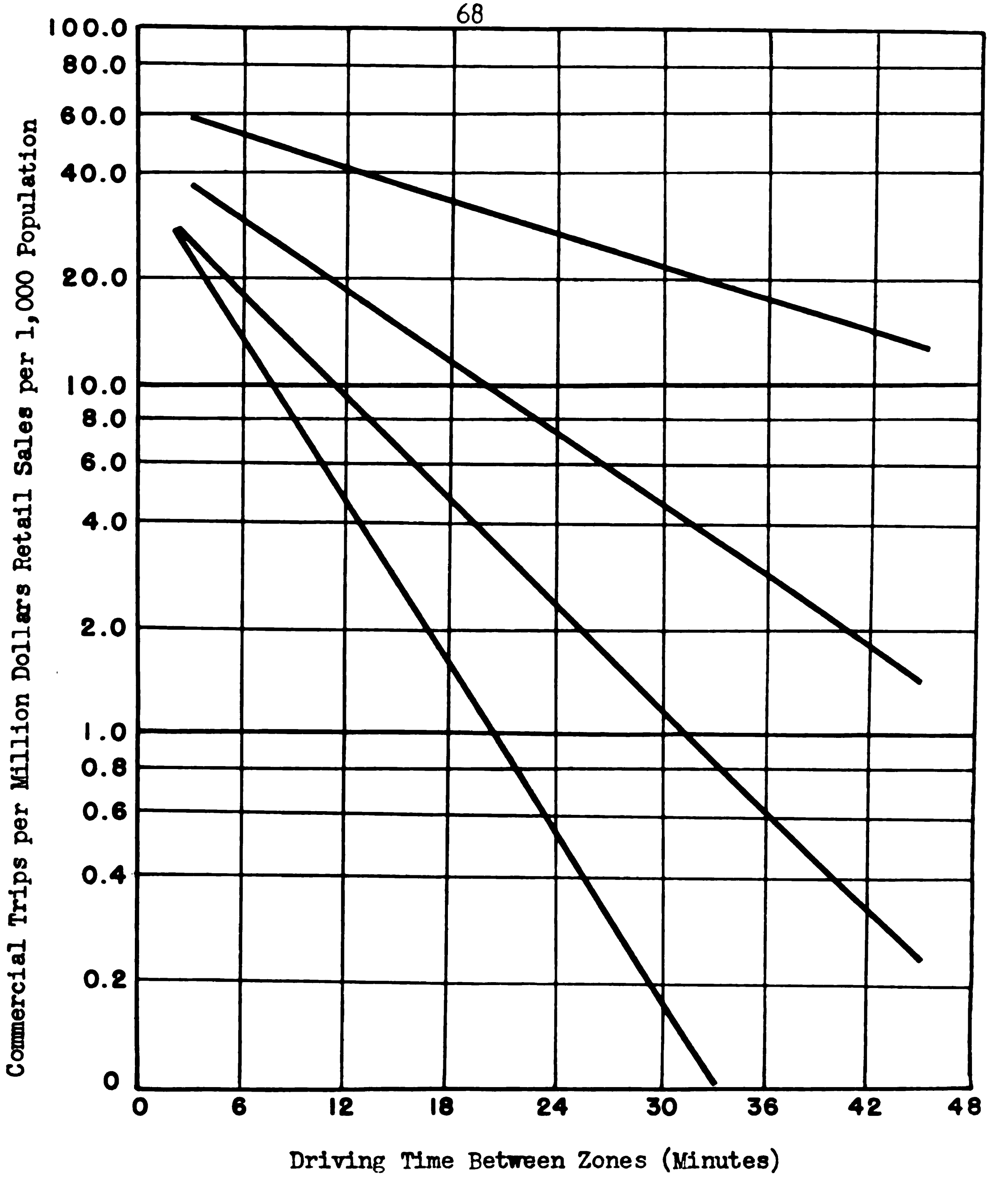


Figure 18. Distribution of Commercial Trips Between Zones (Trips to and from Commercial Purposes-All Modes) Source: Reference 43

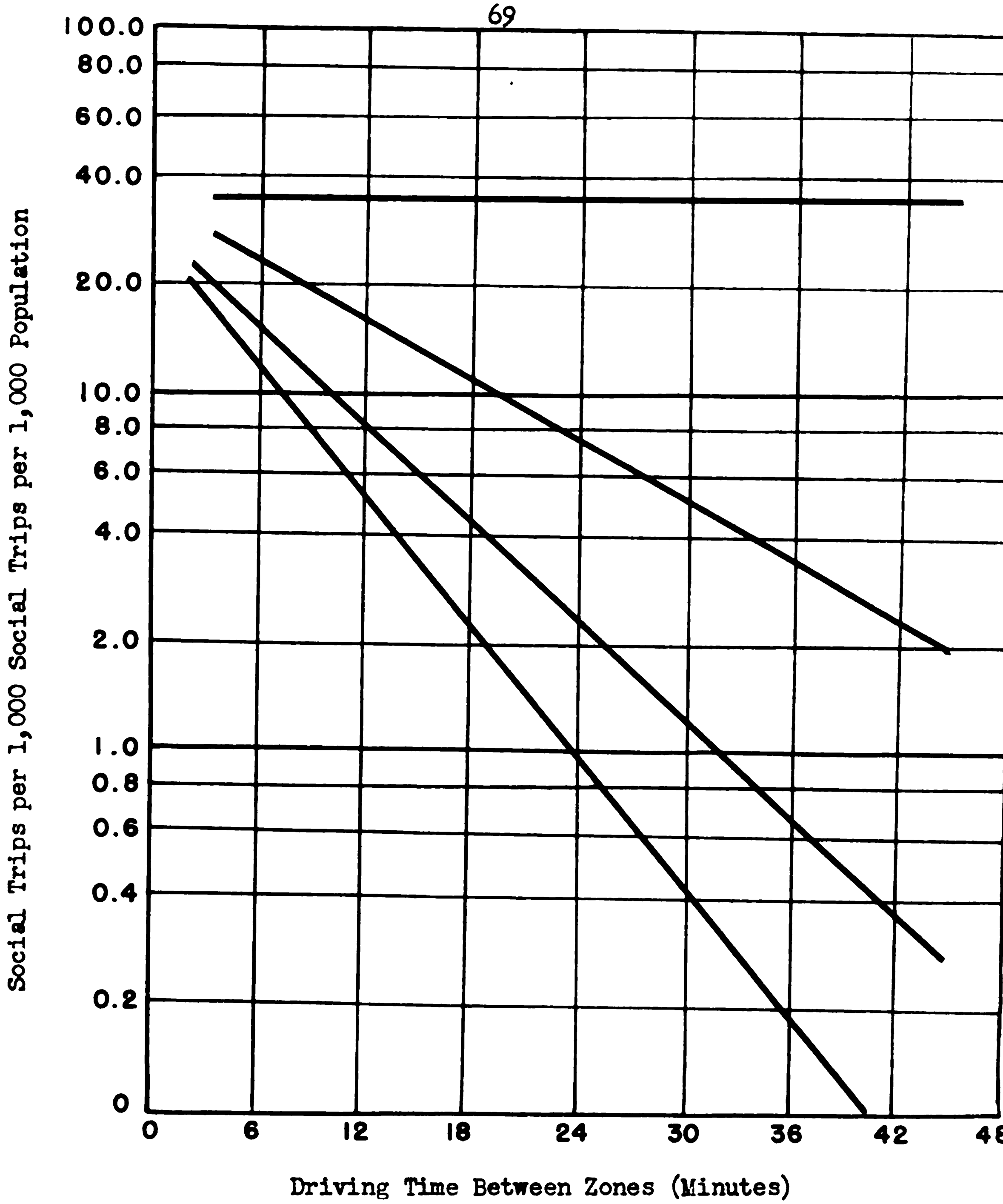


Figure 19. Distribution of Social Trips Between Zones
(Trips to and from Social Purposes-All Modes)
Source: Reference 43

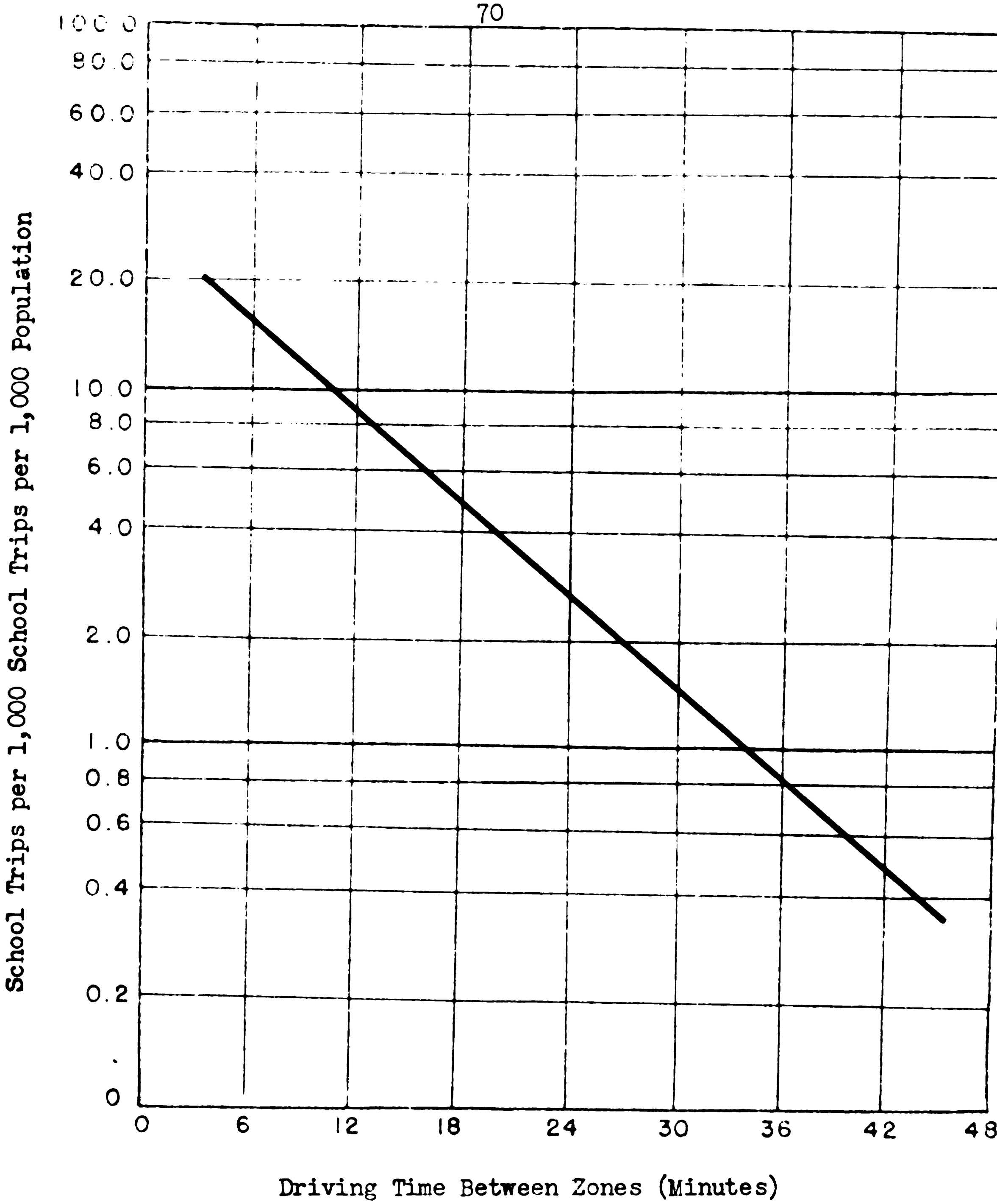


Figure 20, Distribution of School Trips Between Zones (Trips to and from School-All Modes)
Source: Reference 43

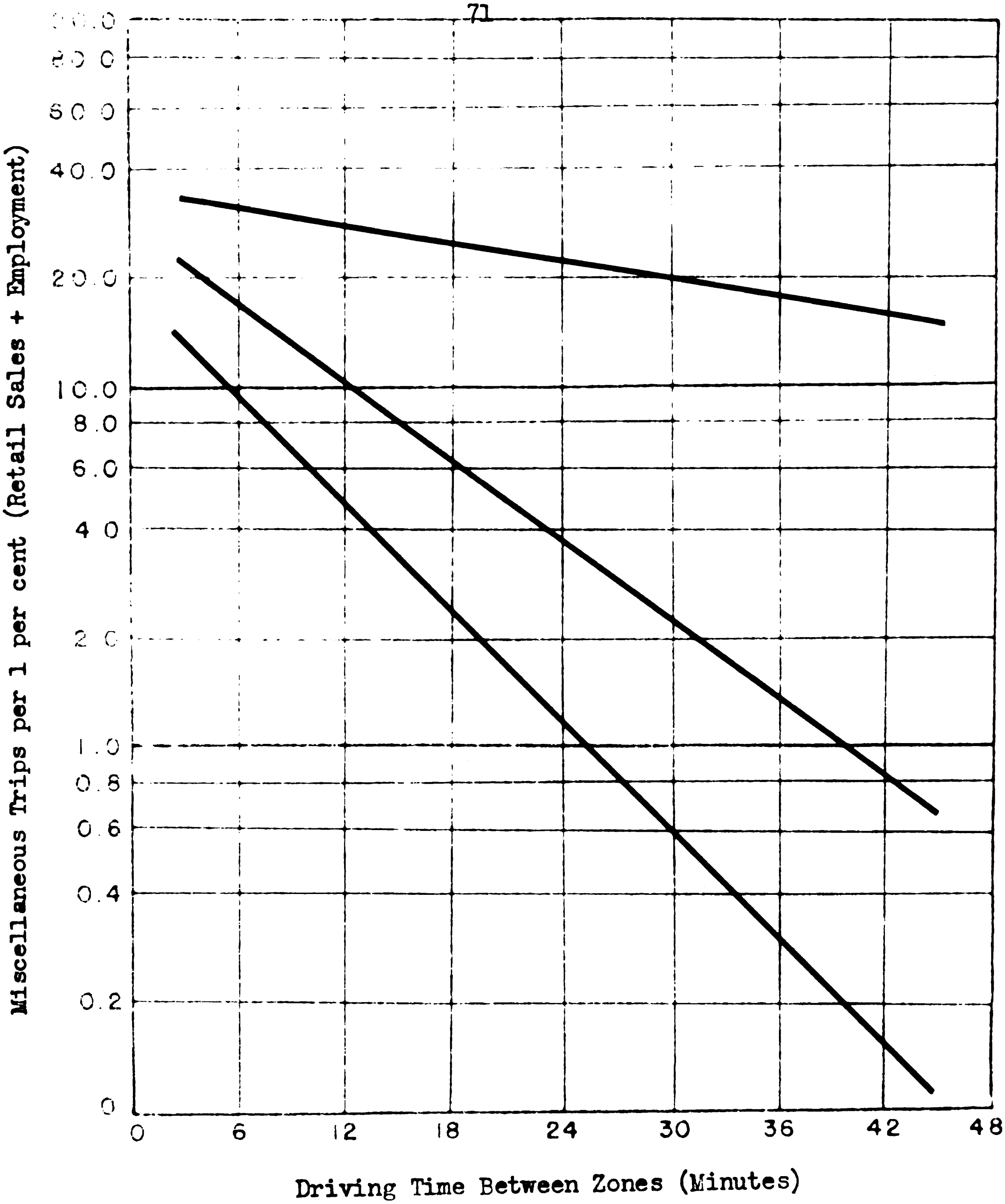


Figure 21. Miscellaneous Trips by Internal Residents-All Modes
(Trip Rate vs. Auto Driving Time)
Source: Reference 43

spot is designated as L. The chance that this trip will get to this specific destination point is dependent on the number of available like destinations (opportunities) encountered sooner. The probability of getting to this destination can be expressed as $(1 - L)^V$ where V is the number of opportunities closer. The probability of stopping at this specific destination then becomes $L\,(1-L)^V$. A complete derivation of the equation can be found in Carroll (16) and Schneider (44). The final form of the mathematical formula is:

$$V_{ij} = \sum_{1}^{s} V_i\,(s)\left[e^{-L(s)\,V(s)} - e^{-L(s)(V(s)\;+\;V_{j(s)})}\right]$$

Where:

V_{ij} is trips from zone i to j.

V_{is} is trip origins in zone i for sub-population s.

V_s is all destinations closer in time to zone i than zone j for sub-populations s.

L_s is the probability per destination of the acceptability of destination at the zone under consideration. (L_s is a constant for the sub-population s.)

In order to use the above equation, it is necessary to order all possible destinations by travel time from the zone and secondly, to establish values of L. It was found that a single L value would not suffice and that a stratification of trips was necessary. Short trips with the higher L values were grouped for a single L value. Longer trips, which had lower L values were then subdivided further into groups of long residential trips and long non-residential trips. Within these sub-populations, a constant value for L could be determined.

In discussing this model, Schneider (45), notes that it shares two flaws with the gravity model: there is usually an unbalanced number of trips to a zone in comparison to the number generated, and the problem of establishing proper L values for future prediction has been difficult.

This same model is being used presently in the Pittsburgh study. Witheford (46), makes a limited comparison of the opportunity model with the gravity model. He concludes that the opportunity model is much better at describing trip characteristic and at simulating the distribution of trips. In contrast to the techniques used in Chicago for forecasting future trip distribution, Witheford (47), notes that Pittsburgh was at that time using the same value of L for present and future trips.

Linear Programming Method.--Linear programming is a mathematical technique which maximizes or minimizes the linear functions of varying parameters, subject to certain constraints. The use of linear programming for traffic problems has been limited to date. The potential of this technique is recognized. A paper by W. R. Blunden (48), describes the procedure briefly and discusses some possible applications in the field of transportation and traffic problems. For a more complete discussion of the solution procedures, the readers are directed to the work of Charnes (49). Linear programming was used by E. L. Killin (50) for estimating the traffic flow into a highway interchange. Using a 50-mile radius area, an estimate of the four approach volumes was found. Volumes were established from the use of such factors as population and distance. A constraint was established from the fact that the volume of the traffic approaching and leaving the interchange in any direction must be equal to the sum of the traffic movement volumes on the interchange serving this approach.

The use of linear programming, which is of most interest here, is reported by Bevis (51). The research which was conducted by the Chicago Area Transportation Study, indicate that a combination of the gravity model and a linear programming technique provides a sound basis for forecasting. In the Chicago model, the following constraints were used:

1. $\sum_{1}^{i} t_i = \sum_{1}^{j} t_{ij}$

2. $\sum_{1}^{j} t_j = \sum_{1}^{i} t_{ij}$

3. $t_{ij} \geq 0$

Where:

t_i is the trips originating in zone i

t_j is the trips originating in zone j

t_{ij} is the interchange of trips between zones i and j.

The capacity measure, which is the upper limit of interchange between two zones, was found first by calculating a potential constant and multiplying this by the ratio of average intrazonal travel friction for the two zones and the travel friction between zones. To establish the value of the potential constant, it was necessary to establish values empirically for x, an exponent, and C, a constant.

$$T_{i-i} = C \left(\sum_{1}^{i} t_i \cdot \sum_{1}^{j} t_{ii} \right)^x$$

Where:

T_{i-i} is number of intrazonal trips for zone i.

$\sum_{1}^{i} t_i$ is number of trips originating at zone i.

$\sum_{1}^{j} t_{ij}$ is number of trips attracted to zone i.

From empirical data, values are established for C and x from the above equation. Next:

$$P_{ij} = \sqrt{C} \left[\left(\sum_{1}^{i} t_i \right)^x \cdot \left(\sum_{1}^{j} t_{ij} \right)^x \right]$$

Where: P_{ij} is the potential constant for trips between zones i and j.

Thus:

$$L_{ij} = P_{ij} \left[\frac{F_i + F_j}{2} \Big/ F_{ij} \right]$$

Where:

L_{ij} is the upper bound of any zonal interchange between zones i and j.

F_i is the travel friction for zone i measured as a straight-line distance

F_j is the travel friction for zone j measured as a straight-line distance

F_{ij} is the travel friction between zones i and j measured as a straight-line distance.

For this model, trips were divided into residential and non-residential trips. The equations used for predicting the potential constants are:

For residential: $P_i = 0.108 \left(\sum_{1}^{i} t_i\right)^{0.753}$

For non-residential: $P_i = 0.144 \left(\sum_{1}^{i} t_i\right)^{0.524}$

To determine the potential constant for residential to residential movements, the respective potentials are multiplied together. A similar procedure is followed for the non-residential linkage potentials. Multiplying the residential and non-residential potentials by 3.11 will provide a residential to non-residential potential. This model, when checked on a sample of zone to zone movement, had a correlation value of 0.89 as compared with a gravity model of only 0.78. Bevis (51) states that the tests reveal that this model provides a satisfactory and reliable procedure for forecasting traffic.

The Multiple Regression Method.--The research for the development of this method was done at the California Division of Highways under the supervision of Sam Osofsky (52). This type of solution became possible with the availability of a medium size computer and personnel trained in the necessary mathematics and statistics.

A fundamental assumption of this method, as well as others, is that the parameters which establish travel variations today will remain valid for future trips. Osofsky presented several corollaries based upon the travel patterns of present traffic.

1. Trips will increase proportionately to the population of the zone and will decrease as distances between zones increase.
2. Trip volumes between zones will have the following relationships when other parameters are held constant:
   a. Trips increase proportionately to car ownership (See Figure 22)
   b. Trips increase as the number of employed persons increases (See Figure 23)

c. Trips increase in accordance with an increase in commercial acreage. (See Figure 24)

3. Selection of best parameters for use depends on:

a. Can it be measured or estimated now and for the future.
b. Will it consistently reduce errors of estimation.
c. Will the factors used establish a design year estimate of trip volumes that is both reasonable and acceptable.

These factors were used to establish the feasibility of the regression method. The steps involved in his multiple regression method are:

a. Select an equation form determined from experience and theory. Using trip interchange and independent variable values from the survey data, obtain a set of coefficients relating a specific single zone to all zones, including itself. Repeat this process treating each zone as a specific zone. A different set of coefficients (a_1, a_2, a_3, a_4, and a_0) for each zone and each cordon station is determined.

b. Estimate the theoretical trips at the survey period by using the coefficients and the independent variable values of Step a. Then analyze the differences between theoretical and survey trips. Step a will be repeated if the analysis warrants modification in equation form.

c. Estimate the independent variables at the design year.

d. Use the estimated independent variables at the design year with the coefficients of Step a to calculate the theoretical trips at the design year.

In this regression method, a single equation form was used for all zone and cordon stations. A set of regression coefficients was calculated for each zone and for each cordon station. For each movement there were two estimates, which were then averaged. Mr. Osofsky also noted that a single overall equation was possible with one set of regression coefficients for all intra- and inter-zonal movements and another set for cordon station to zone trips. Applications of this method in California showed variations in the equation form most applicable to each area. The equation forms are noted below:

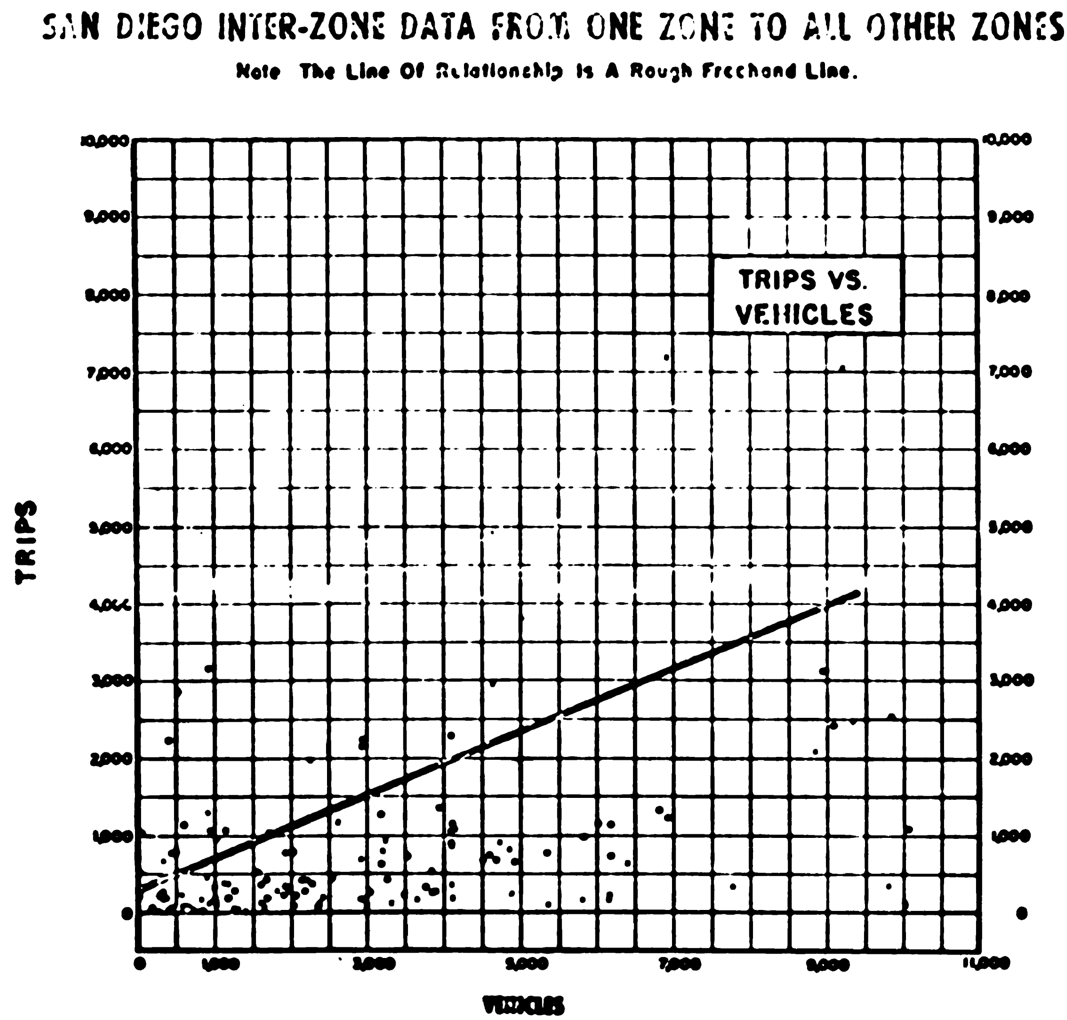


Figure 22. The Relationship of Trips made to vehicles owned in San Diego, California
Source: Reference 52

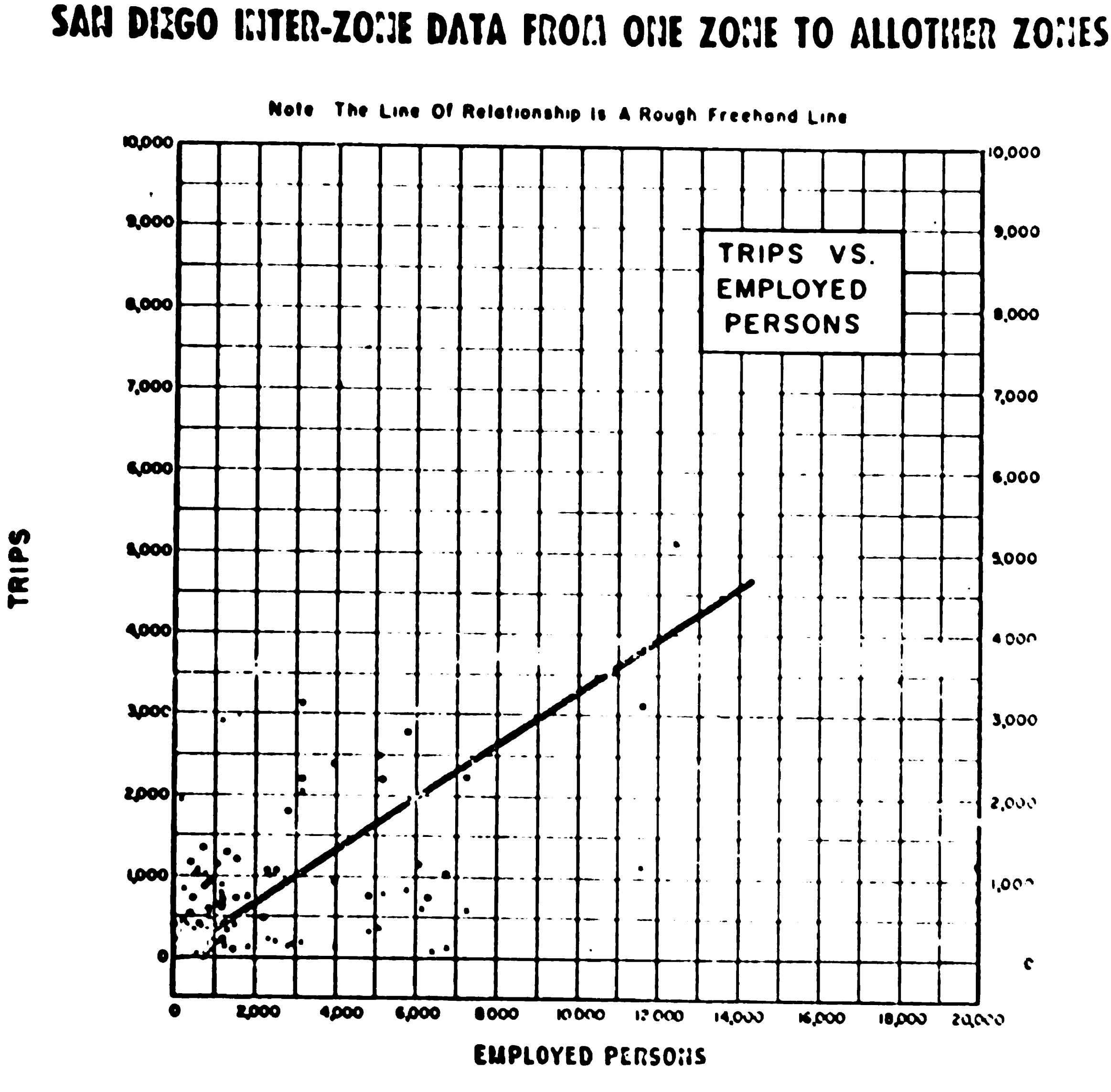


Figure 23. The Relationship of Trip made to the number of employed persons in San Diego, California

Source: Reference 52

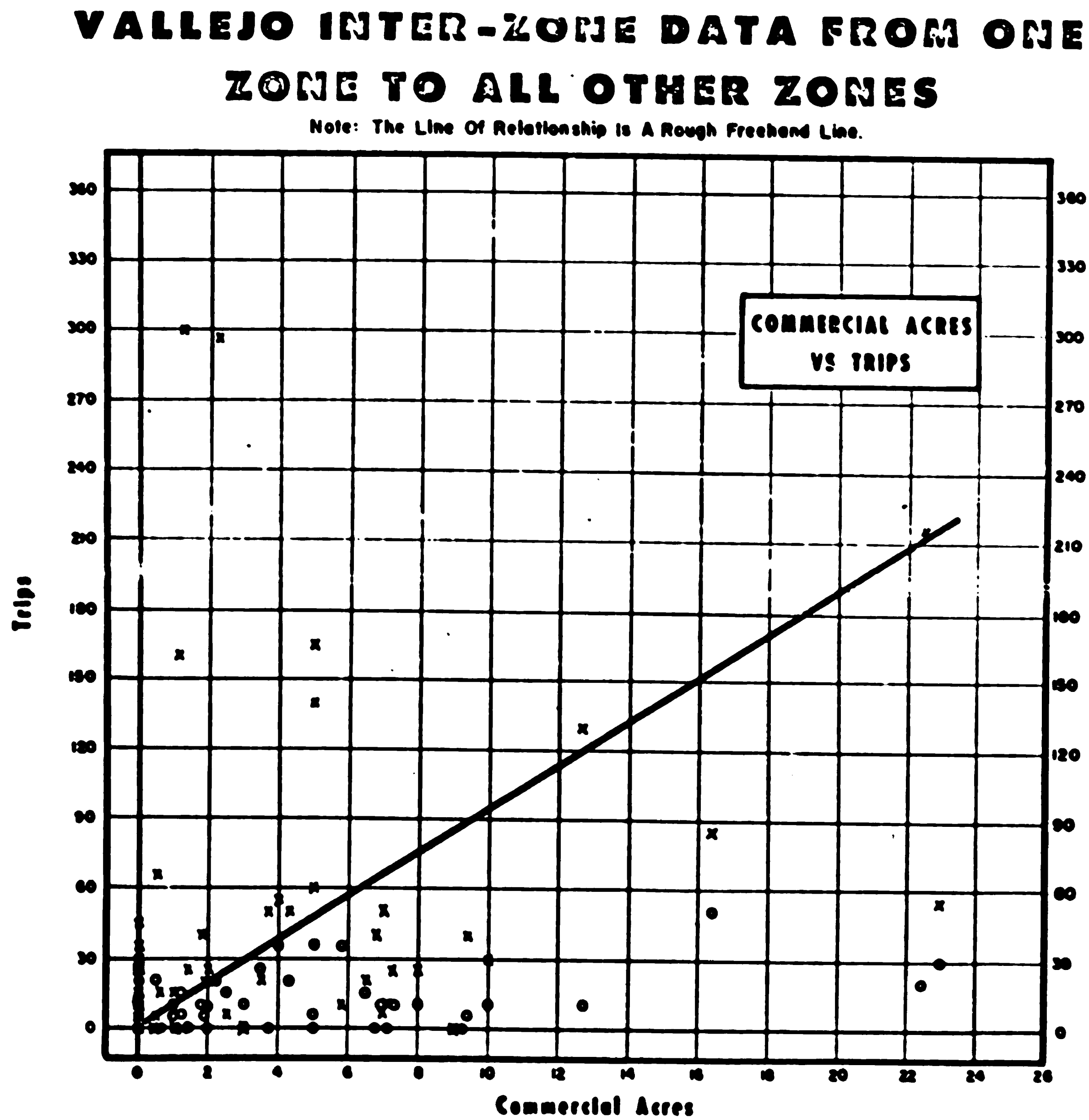


Figure 24. The Relationship of Trips made to the number of commercial areas for Vallejo, California
Source: Reference 52

Medesto Equation Form:

$$Y = A_o + A_1 \frac{P}{(1 + D)^2} + A_2 \frac{E^2}{(1 + D)^2} + A_3 \frac{V}{(1 + D)^2} + A_4 \frac{L}{(1 + D)^2}$$

San Diego Equation Form

$$Y = A_o + A_1 \frac{P^2}{D^{1.5}} + A_2 \frac{E^2}{D^{1.5}} + A_3 \frac{V}{D^{1.5}} + A_4 \frac{L}{D^{1.5}}$$

Vallejo Equation Form:

$$Y = A_o + A_1 \frac{(P\,V)^{\frac{1}{2}}}{(1+D)^2} + A_2 \frac{E^2}{(1+D)^2} + A_3 \frac{C}{(1+D)^2} + A_4 \frac{I}{(1+D)^2}$$

Where the variables are noted by the following symbols:

Y = Trips between zones or cordon station and zone

D = Straight-line Distance between zones or from station to zone

P = Population in each zone

E = Employed persons in each zone

V = Vehicles owned by persons in each zone

C = Commercial acreage in each zone

I = Industrial acreage in each zone

L = Land use index of each zone

There are several limitations to multiple regression method that must be considered. The effect of individual parameters on an equation form must almost be empirical when the net regression effect of the factors is not completely developed or understood. Also an equation form developed for the present data may not adequately predict trip interchanges for a design year. With this method it is possible to compute negative trip values, which then must be set to zero. Results from the California research indicate that it is not advisable to use these estimating equations developed in one area in another area.

SYSTEMS THEORY

INTRODUCTION

The pertinent material presented here has been abstracted from two of the prominent texts in the field, Seshu and Reed (53), and Koenig and Blackwell (54). For a complete discussion of the details of the subject, the reader is referred to the above books and their bibliographies.

"Systems engineering" has moved to the fore as one of the many new and sophisticated terms of this age. There are those who believe that it is the same old engineering process with a new title. Others believe that it is both new and different because of its multi-disciplinary approach and of its emphasis on the system, as opposed to the component approach. This difficulty is not aided by the ambiguity of the word: system. System, on the one hand, is used to mean a collection of rules for procedure and on the other hand, it can be a collection of similar or interrelated things. The definition implied in this work for systems engineering is a combination of both designations. A system is defined as an orderly arrangement of interrelated elements acting together to achieve a specific purpose. Thus, a system must have an avowed purpose, and must be free of extraneous or mathematically redundant parts and must have the elements or components joined in an orderly fashion. Systems engineering is the art by which one formulates a concept of the system by the application of certain principles.

System engineering problems require the use of some type of mathematical techniques to achieve solutions. Techniques are applied in accordance with the problem to be solved. Among the techniques applied to various problems are: 1. game theory, 2. linear programming, 3. queuing theory, 4. search theory, and 5. linear graph theory.

Although this report refers to the application of systems engineering techniques, it can be more specifically stated as the application of linear graph theory to urban traffic forecasting.

OPERATIONAL TECHNIQUES

Problems of systems design can be separated into two parts, one of analysis and the other of synthesis. Analysis is primarily concerned with the breakdown of systems into components before synthesis can progress. Each part or component (hardware) is examined in order to establish a mathematical description. Then the components are combined to form the system and further analysis made to determine the system equations.

Synthesis is the combining of the components into the system to achieve a unique function. Ideas are the food of the synthesis process and are assumed valid until disproved by the analysis process. This is in the area of creativeness since generally there is no unique combination of parts to give a required system performance. The choice of components and their mode of interconnection is made largely on the basis of past experience with systems having similar properties, but this experience must not bias the synthesis process. The problem of choosing the right combination of certain types of components which will optimize the system, is greatly facilitated by vision, qualitative understanding and past experience. After this selection, there remains the inevitable problem of finding a suitable set of numerical values for the component dimensions or parameters and of proof by analysis, that the proposed system will work. The engineer's ability to explore the consequences of a systems design resulting from his imagination is only limited by his familiarity with mathematical analysis techniques. The use of high speed computers enables the engineer to explore hundreds of ideas in the time normally required for one.

The crux of synthesis is ideation. Analysis techniques, although necessary can only operate within the framework of ideas created in the synthesis process. Familiarity with available components (hardware) and their characteristics, coupled with experience, should be helpful in stimulating ideas, but there is no assurance that something will result. Synthesis is then simply a matter of combining facts, principles or laws uniquely into a whole idea which will accomplish the results or solve the systems problem. The technique resolves itself into one of synthesis and analysis, with repetitions until an acceptable solution is found.

ANALYSIS BY LINEAR GRAPH

Although the techniques of linear graph theory were developed primarily in electrical network analysis, during the past several years, this fundamental discipline of analysis has been applied usefully to many areas, such as mechanical, hydraulic, and heat-transfer systems.

The main purpose of this thesis is to apply linear graph theory to the problem of urban traffic forecasting.

Linear graph theory is an orderly technique for formulating the mathematical characteristics of a physical system. The steps in the solution of a physical system by linear graph can be illustrated by the block diagram of Figure 25.

The discussion which follows is detailed enough to present the reader with an insight into the techniques; yet brief enough to force his inquiry into the references previously cited. Because of the elemental nature of the work that follows, discussion will be limited to systems made up of two terminal components. For the computation of the system characteristics, two steps are necessary, namely:

1. To establish a mathematical model of the relevant physical characteristics of the system components expressed in terms of measurements.
2. To establish in mathematical form and in terms of measurements from a knowledge of the component characteristics and their mode of interconnection, the characteristics of the system, i.e., a mathematical model of the system.

Terminal Characteristics

The analysis of any physical system requires a mathematical description of each component part, as well as, a mathematical description of how the components are joined to form the system. Collectively, these mathematical equations provide what is referred to as system equations. In the mathematical analysis of any given type of physical system (electrical, mechanical, thermal, etc.) the tie between the mathematics and the system is generally accomplished through the use of two basic measurements; the "across" or x and the "through" or y measurements. The x and y measurements used to date are:

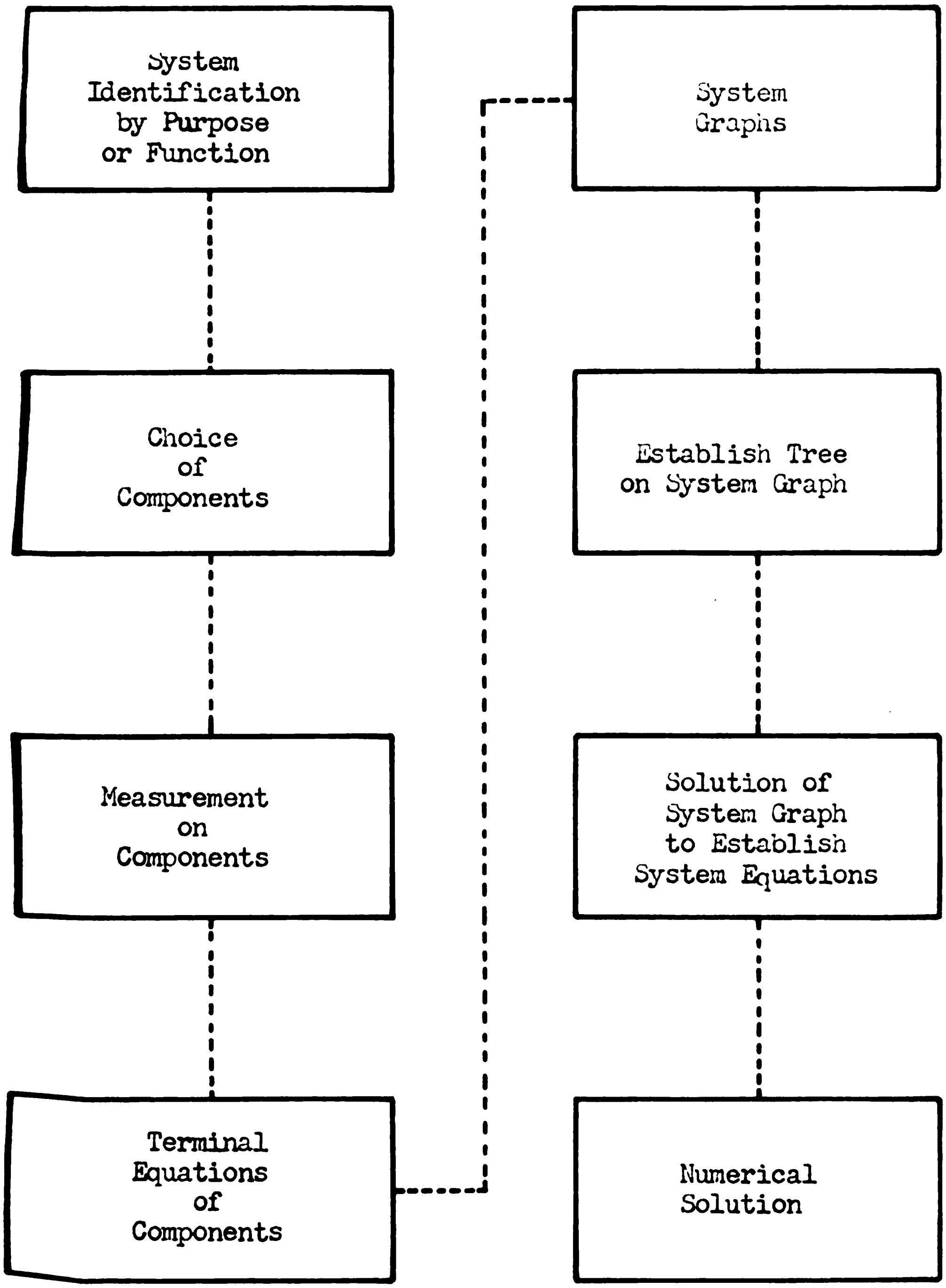


Figure 25. Steps in the solution of a physical system by linear graph theory

1. Electrical-- x is voltage and y is current flow
2. Mechanical Translation-- x is displacement and y is force
3. Thermal Systems-- x is temperature and y is heat flow
4. Hydraulics-- x is pressure and y is flow

The terminal characteristics of the component is completely described by the equation which relates the x and y measurements or:

$$y = G x$$

This equation, which is referred to as the terminal equation, plus the terminal graph of the component, forms the terminal representation of the component. Components are described mathematically by relating the two measurements x and y on the component in isolation from other components. This infers that the terminal equations of the components are independent of the system in which they are used. These measurements must be such that one is a "through" (or series) measurement called y, which when summed at the vertices must equal zero, and the other is an "across" (or parallel) measurement, called x, which when summed around the circuit must equal zero. The selection of the proper x and y measurements for the traffic problem is primarily a synthesis problem.

System Graph

Since the aim of this section is to show how linear graph theory serves to establish a mathematical description of the system graph, both of these terms warrant more description. A system graph is a collection of component terminal graphs obtained by uniting the vertices of the terminal graphs in a one to one correspondence with the union of the physical components. When the fundamental operational concept of the linear graph is adopted, the system graph follows directly from the prescribed manner in which the components of the system are connected. If the characteristics of the system components can be determined and they must be if the system is to be analyzed, there is never any question as to the form of the system graph.

A linear graph is a collection of oriented elements which provide the basis for mathematically describing the system. Since certain terminology might be unfamiliar to some readers, brief definitions will be given along with the theorems presented.

Definitions

1. An oriented element or edge is an oriented line segment with its distinct ends.
2. A vertex is an endpoint of an element or edge.
3. An oriented linear graph is a collection of oriented elements, no two of which have a point in common that is not a vertex.
4. A subgraph is a subset of the elements of a graph and actually a graph in itself.
5. A circuit is a loop or closed path, where the vertices have two and only two elements incident thereto.
6. A complement of a subgraph contains all the elements remaining when the subgraph is removed.

Postulates

Kirchoff's loop and node equations, although established for electrical networks, exhibit the same fundamental properties as those found for other physical systems. That is, the "across" variables x sum to zero around the circuits of the system graph, when element orientation is considered. Also, the "across" variables y sum to zero at the vertices of the system graph when due consideration is given to orientation. Systems made up of electrical, mechanical, thermal or hydraulic components, each have these fundamental properties in common. It seems logical to assume these same fundamental conditions can be solved by this formulation. These hypotheses are defined as postulates.

The Vertex Postulate (54).--Let the System Graph of a physical system contain "e" oriented elements or edges and let $y_i\,(t)$ represent the fundamental through variable of the i^{th} element, then at the v^{th} vertex of the graph

$$\sum_{i=1}^{e} a_i\, y_i\,(t) = 0$$

Where:

$a_i = 0$ if the i^{th} element is not incident at the v^{th} vertex

$a_i = 1$ if the i^{th} element is oriented away from the v^{th} vertex

$a_i = -1$ if the i^{th} element is oriented toward the v^{th} vertex

The Circuit Postulate (54).--Let the system graph of a physical system contain "e" oriented elements or edges and let $x_i\,(t)$ represent the fundamental across variable of the i^{th} element then for the v^{th} circuit

$$\sum_{i=1}^{e} b_i\, x_i\,(t) = 0$$

Where

$b_i = 0$ if the i^{th} element is not included in the v^{th} circuit

$b_i = 1$ if the orientation of the i^{th} element is the same as the orientation chosen for the v^{th} circuit

$b_i = -1$ if the orientation of the i^{th} element is opposite to that of the v^{th} circuit.

Regardless of the complexity of any given system, it can be solved by the combined use of the terminal equations, and the circuit and vertex equations. The equations established by the above will not be all independent equations. In order to select the minimum number of independent equations for the system analysis, the fundamental cut-set and fundamental circuit equations are used in lieu of the above postulates. These equations can be developed by using the following techniques.

The Tree and its Complement

The vertex postulate implies that one equation on the through variables y can be written at each vertex of the system graph. The circuit postulate implies that one equation on the across variables x can be written for each circuit. Since these equations are not each independent, it is necessary to establish a tree of the graph which will provide for a convenient set of independent equations.

A tree can be defined as a subgraph which contains all the vertices of the graph but no circuits. The elements in the tree are referred to as branches. The elements of the complement of the tree are called chords. The number of branches in a tree of a connected subgraph of v vertices equals v-1. The number of chords for a graph having e elements and v vertices equals $e - v + 1$. Illustrations of chords and trees will be shown in a later section in connection with their actual use.

Fundamental Circuit Equations

In accordance with the above, a tree is selected from the elements of the system graph. Since there are many such trees, the one actually used in the analysis is called the formulation tree. The number of independent circuit equations will be equal to the number of chords. The fundamental circuit equations are developed by including one and exactly one chord for each circuit written in accordance with the circuit postulate. A position orientation is assigned to the direction of the chord used. When the across variables are separated into branches and chords, the equation takes on this form:

$$\begin{bmatrix} B_{11} & U \end{bmatrix} \begin{bmatrix} X_b \\ X_c \end{bmatrix} = 0 \qquad (1)$$

Where:

B_{11} is a coefficient matrix.

X_b is the column matrix of the branches

X_c is the column matrix of the chords

A completion of the matrix product gives the across variable of the chords as an explicit function of the chords.

$$U\, X_c = -B_{11}\, X_b$$

Another important requirement of the fundamental circuit equations is that the specified across variables should be included in the branches of the tree.

Fundamental Cut-set Equations

A convenient set of independent equations will be established if when applying the vertex postulate exactly, one tree branch is included. A positive orientation is then assigned to this branch. Specified through variables are placed in the chord system. Symbolically, the fundamental cut-set equations take this form:

$$\begin{bmatrix} U & A_{11} \end{bmatrix} \begin{bmatrix} Y_b \\ Y_c \end{bmatrix} = 0 \qquad (3)$$

Where:

A_{11} = is a coefficient matrix.

Y_b = is the column matrix of the branches

Y_c = is the column matrix of the chords

The across variable of the branches can be expressed explicitly

$$U\ Y_b = -A_{11}\ Y_c \qquad (4)$$

Most texts on linear graph theory show a general property of the cut-set and circuit matrices. This will simply be stated here without proof.

$$A_{11} = -B_{11}^T \quad \text{or} \quad B_{11} = -A_{11}^T \qquad (5)$$

Where:

The superscript T indicates the transpose of the matrix.

This shows that either set of the system equation is necessary but not both. All of the through variables of the system graph can be expressed as linear combinations of the through variables of the chord system by using the transpose of the fundamental circuit matrix, or;

$$\begin{bmatrix} Y_b \\ Y_c \end{bmatrix} = \begin{bmatrix} B_{11}^T & Y_c \end{bmatrix} \qquad (6)$$

Likewise for the across variables:

$$\begin{bmatrix} X_b \\ X_c \end{bmatrix} = \begin{bmatrix} A_{11}^T X_b \end{bmatrix} \qquad (7)$$

Formulation

The basis for the analysis of a system is to establish the terminal equations, the cut-set equations and the circuit equations. The formulation presented here requires that the across variables, for which values are specified, be placed in the branches, X_{b-1}. Also specified through variables are included as chords Y_{c-2}. Two procedures are available to the systems analyst, depending upon the form of the terminal equations and the number of independent equations afforded by each procedure. Since specific examples are presented in a later section, the procedure will be outlined in symbolic form.

General Mesh Form

This procedure is used when the terminal equations have the across variables expressed as explicit functions of the through variables. Where R represents the coefficient matrix, the form is:

$$\begin{bmatrix} X_{b-2} \\ X_{c-1} \end{bmatrix} = \begin{bmatrix} R_{b-2} \\ R_{c-1} \end{bmatrix} \begin{bmatrix} Y_{b-2} \\ Y_{c-1} \end{bmatrix} \qquad (8)$$

When the tree is selected in the manner previously described, the fundamental circuit equations can be written. In symbolic form, these are:

$$\begin{bmatrix} B_{11} & B_{12} & U & 0 \\ B_{21} & B_{22} & 0 & U \end{bmatrix} \begin{bmatrix} X_{b-1} \\ X_{b-2} \\ X_{c-1} \\ X_{c-2} \end{bmatrix} = 0 \qquad (9)$$

Using the property expressed by equation (5), the cut-set equations can be expressed as:

$$\begin{bmatrix} U & 0 & -B_{11}^T & -B_{12}^T \\ 0 & U & -B_{21}^T & -B_{22}^T \end{bmatrix} \begin{bmatrix} Y_{b-1} \\ Y_{b-2} \\ Y_{c-1} \\ Y_{c-2} \end{bmatrix} = 0 \qquad (10)$$

The matrix products of equation (9) gives the following:

$$\begin{bmatrix} B_{11} \\ B_{21} \end{bmatrix} \begin{bmatrix} X_{b-1} \end{bmatrix} + \begin{bmatrix} B_{12} & U \\ B_{22} & 0 \end{bmatrix} \begin{bmatrix} X_{b-2} \\ X_{c-1} \end{bmatrix} + \begin{bmatrix} 0 \\ U \end{bmatrix} \begin{bmatrix} X_{c-2} \end{bmatrix} = 0 \qquad (11)$$

The terminal equations (8) which are explicit in the across variable are substituted into the expanded circuit equations (11).

$$\begin{bmatrix} B_{11} \\ B_{12} \end{bmatrix} \begin{bmatrix} X_{b-1} \end{bmatrix} + \begin{bmatrix} B_{12} & U \\ B_{22} & 0 \end{bmatrix} \cdot \begin{bmatrix} R_{b-2} & 0 \\ 0 & R_{c-1} \end{bmatrix} \begin{bmatrix} Y_{b-2} \\ Y_{c-1} \end{bmatrix} +$$

$$\begin{bmatrix} 0 \\ U \end{bmatrix} \begin{bmatrix} X_{c-2} \end{bmatrix} = 0 \qquad (12)$$

One of the key advantages of this type of analysis is the possibility of replacing certain unknown variables in an equation with a relation of known values. In the substitution below, the unknown Y_{b-2} are replaced by the known values Y_{c-2}.

$$\begin{bmatrix} Y_{b-2} \\ Y_{c-2} \end{bmatrix} = \begin{bmatrix} B_{12}^T & B_{22}^T \\ U & 0 \end{bmatrix} \begin{bmatrix} Y_{c-1} \\ Y_{c-2} \end{bmatrix}$$

which, when inserted into equation (12), gives the main equation:

$$\begin{bmatrix} B_{11} \\ B_{21} \end{bmatrix} \begin{bmatrix} X_{b-1} \end{bmatrix} + \begin{bmatrix} B_{12} & U \\ B_{22} & 0 \end{bmatrix} \begin{bmatrix} R_{b-2} & 0 \\ 0 & R_{c-1} \end{bmatrix} \begin{bmatrix} B_{12}^T & B_{22}^T \\ U & 0 \end{bmatrix} \begin{bmatrix} Y_{c-1} \\ Y_{c-2} \end{bmatrix} +$$

$$\begin{bmatrix} 0 \\ U \end{bmatrix} \begin{bmatrix} X_{c-2} \end{bmatrix} = 0 \qquad (13)$$

The bottom line of equations, in this main expression, deals only with known values and does not contribute to the solution. The general mesh form or circuit equation can be written using the top equation from above.

$$\left[B_{11} \cdot X_{b-1}\right] + \left[B_{12} \cdot R_{b-2} \cdot B_{12}^T + U \cdot R_{c-1}\right] \left[Y_{c-1}\right] +$$

$$\left[B_{12} \cdot R_{b-2} \cdot B_{22}^T\right] \left[Y_{c-2}\right] = 0 \qquad (14)$$

General Branch Form

The derivation of the general branch form equations follows the form outlined above so that only the equation will be stated. The terminal equations have the through variables expressed as explicit functions of the across variables, where G represents the coefficient matrix.

$$\begin{bmatrix} Y_{b-2} \\ Y_{c-1} \end{bmatrix} = \begin{bmatrix} G_{b-2} \\ G_{c-1} \end{bmatrix} \begin{bmatrix} X_{b-2} \\ X_{c-1} \end{bmatrix} \qquad (15)$$

The fundamental cut-set equations are written:

$$\begin{bmatrix} U & 0 & A_{11} & A_{12} \\ 0 & U & A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} Y_{b-1} \\ Y_{b-2} \\ Y_{c-1} \\ Y_{c-2} \end{bmatrix} = 0 \qquad (16)$$

The results of a similar derivation gives a general branch form equation:

$$\left[A_{21} \cdot G_{c-1} \cdot A_{11}^{T}\right] \left[X_{b-1}\right] + \left[G_{b-2} + A_{21} \cdot G_{c-1} \cdot A_{21}^{T}\right] \left[X_{b-2}\right] + \left[A_{22}\right] \left[Y_{c-2}\right] = 0 \qquad (17)$$

The two general equations presented, the mesh form and the branch form, can be used where the system is made up of two-terminal components. Further, the elements with specified across variables (X_{b-1}) must be included as branches of the tree and those elements with specified through variables (Y_{c-2}) must be included as chords. Besides the type of terminal equations given, the number of independent equations developed by each of the general form equations could influence the choice made. The number of independent equations by the mesh form will be equal to the number of elements in the branch set (X_{b-2}). The number of elements in the chord set (Y_{c-1}) equals the number of independent equations by the branch form.

By proper substitution and matrix multiplications, equations (14) or (17) will result in a system of simultaneous equations, which can be solved for numerical answers.

DISCUSSION OF THE APPLICATION POSSIBILITIES OF SYSTEMS THEORY TO TRAFFIC FORECASTING

GENERAL

The work presented here is one of the earliest attempts to apply the techniques of systems theory to traffic problems. While there are many problems in the traffic field, for which a system solution might be attempted, this work is concentrated in the area of traffic forecasting. The techniques of systems engineering are specifically applied to forecasting future work trip distribution. The walk to work trip and the modal split of work trips are not considered in this research, since they are both refinements which can come later.

The work trip distribution system was selected for this study. Work trips provided the largest single group of trips by purpose. Also they generally occur at the time of peak travel and this increases their importance to the traffic forecastor.

The application possibilities of systems theory to the work trip distribution system will be researched under the following steps as outlined in Figure 25. See page 87

1. System identification by purpose or function
2. Choice of components
3. Measurements on components

4. Terminal equations of components

5. Systems graphs

6. Solution of systems graphs to establish systems equations

7. Numerical solution and computer use

SYSTEM IDENTIFICATION

The object of this section is to establish the definition of the system by purpose and structure and the problem statement.

The structural make-up of this system includes: people, routes (or street system), vehicles, and employment opportunities (or jobs). There are two basic functions of a transportation system. The system analysis presented here deals only with the movement of persons and not goods. The problem can be stated as follows, given:

1. Several residential areas composed of people, some of whom are workers,
2. Several employment zones containing jobs,
3. Routes or street systems joining these zones.

The object is to determine:

1. The number of daily work trips originating in each residential zone,
2. The number of daily work trips destined to each employment zone,
3. The number of daily work trips between each of the residential zones and each of the employment zones.

The system will be limited to trips made within the urban area.

CHOICE OF COMPONENTS

The components established as the best of those surveyed are:

1. Residential and employment zones, similar to those generally established in O and D studies.
2. Route components which include the various types of streets and intersections used in traveling from an origin to a destination.

The choice of components relies on the structure and purpose of the system studied. Selection is made on the basis that components must be:

1. Conceptual
2. Definitive
3. Quantitatively descriptive
4. Small enough to give objectivity to the system
5. Large enough to hold the computational work of the system analysis within manageable proportions.

The components for this system must come from the constituents of the structure. Since it was previously established that our structure consists of people, vehicles, routes and jobs, selection must start with an evaluation of these.

Residential Area

A person can be defined and visualized. But as an individual, he defies the prediction of a quantitative measurement which can describe his action. Even if the person met all other criteria, the magnitude of the problem would be such as to preclude its use.

The choice of a single vehicle as a component would meet with the same objections as above. Furthermore, vehicles do not move without the control and the motivation of people. Since both of the basic elements fail to meet the criteria, some logical component which includes these two might be the proper one.

A dwelling unit or a family would combine vehicles and people. This has reduced to some extent, the number of individual components, but the matter of prediction still remains. One could then follow this same summation of dwelling units to all lots in a block and finally grouping the blocks into zones. Each increment of size tends to reduce the volume of components and increase the possibility of prediction. Care must be exercised that the unit chosen is not so large as to destroy the purpose of the system. For example, urban work trip movements would be meaningless when the single component represents the complete urban area.

A zone, similar or identical to those used in origin and destination surveys, seems to be the best component evaluated. These zones should be so defined that the traffic characteristics within the zones are as homogeneous as possible.

Route or Street System

The next basic element is the street which is both definitive, conceptual, and subject to quantitative measurements. But variations in the characteristic of each street component compounds the problem of analysis. Then too, if each street is selected as a component, each intersection would also have to be included. For a system whose purpose was the assignment of vehicles to the individual streets, these might then be proper components. Since the purpose here is to predict the desired line of flow, from the point of residence to the place of employment, the best unit is one which represents the best route from origin to destination so that only a single component is required.

Employment Area

The employment opportunity or job completes the structure of our system. To use a single job as a component of the systems would mean that it is important to know from which residential zone that particular job-holder has come. This would involve the probability that some particular residential zone would have persons willing and able to fill this job and the probability that such a person would take this job instead of a similar job in another employment zone. Individual jobs as well as small employment organizations would be difficult to evaluate and provide an extremely large number of components. Organizations in the same area should be grouped into employments. The extent and inclusion of these organizations in each zone should provide homogeneity with respect to certain parameters to be discussed later.

The best components seem to be the residential zone of the origin and destination study, the best route system from origin to destination and the employment zone with certain homogeneous characteristics. These zones must be re-evaluated with regard to the next requirement of systems theory which follows.

MEASUREMENTS ON COMPONENTS

As a result of the following analysis it was concluded that the best measurement for the through variable y, would be the flow of work trips from, to, or through the component. The measurement for the across variable x, would be a pressure type measurement related to a measure of desire or trip motivation and on another basis to a measure of income and consumption assigned to trip making.

The components selected previously must meet the following requirements, if the techniques of linear graph theory are to be used in the systems solution. The requirements are:

1. The basic component must be describable mathematically by relating two valid measurements on the components
2. When the components are arranged in a systems graph; one of the measurements taken on the component, which is noted as x, must sum to zero when the summation is made around a circuit; and the other measurement y must sum to zero at the vertices of the systems graph
3. The x measurement must be related to the y measurement through a linear or nonlinear function.

If previously selected components cannot meet these requirements, new units must be established on the basis of the preceeding criteria.

<u>The Y Measurement</u>

The dimensions of the y measurement must be consistent from component to component so that the sum of the y measurements can be truly zero. The same rule holds for the x measurements.

The most logical y measurement for a traffic system of this type appears to be flow. This flow would represent the movement of persons, vehicles or both. In the systems approach to electrical, thermal and hydraulic systems, the y measurement represents flow of current, heat and water or fluids, respectively. It seemed reasonable to assume that the y measurement for the traffic system also represent flow since the flow of vehicles will satisfy the criteria, that the algebraic sum of y's at a vertex must equal zero.

The X Measurement

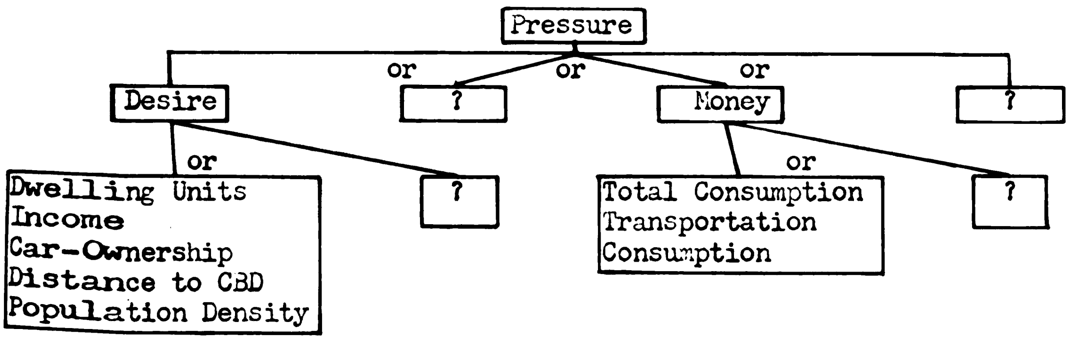
In the fields of electrical, thermal and hydraulic systems, where y was a measure of flow, the x measurement was a type of pressure differential which causes flow. The establishment of the units for the x measurement is not so obvious in the traffic system. There are no physical components on which one might use a voltmeter, piezometer or thermometer to record the pressure differential. In the previously cited systems and in criteria three noted above in the introduction to this section, it is stated that the measurement x is related to y. Flow then, is a function of the pressure differential. In the traffic system, it is not possible to measure this pressure differential directly but it can be evaluated in terms of its effect on the flow of trips which can be easily measured. The development of the x measurement follows this reasoning process, that:

1. There is some basis for the variation in the flow of trips from several residential zones,
2. For the sake of a title, this basis is called pressure, here,
3. The "pressure" term, which is meaningless in traffic terminology, can best be described as a function of certain factors which explain the variations in the y values.
4. Two factors are used here separately to approximate the pressure term. One factor might be labeled "desire" and the other "money".

5. The above factors, though still vague terms, can be related to more specific parameters which are subject to actual physical measurement.

The procedures and extent of the research into the x measurement can be shown more clearly by the following Figure 26.

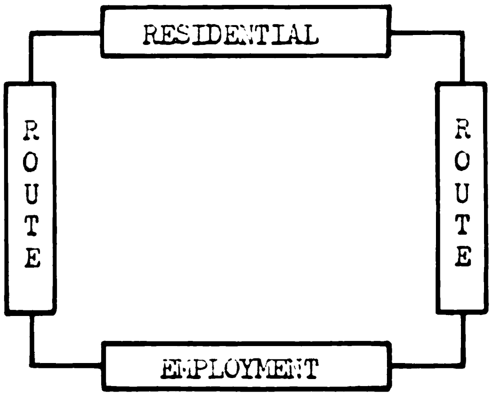
Figure 26. The organization of research on the x measurement



Eventual acceptance of the method will depend on how well it can predict changes in the flow of trips and on how easily the final parameter can be measured.

Desire

A measure of desire or motivation can be related to the pressure which induces flow. The logic of this assumption can be checked by referring to sketch below. A schematic representation of a simple circuit is shown, from a residential zone component, through a route component away from the zone, on through an employment component, and the return route component for a fixed period of time - 24 hours.



Considering only the single purpose work trips which are made along the circuit, the desire or pressure produced or generated in the residential zone must be precisely enough to overcome the pressure loss at each of route components and through the employment zone. The sum of the pressure measurements will vanish around the circuit and, in so doing, will produce y trips. Some routes and employment zones will require larger pressure differentials than others. If a residential zone has a specific x value available, the circuit with the least R or resistance value will produce the greatest flow of trips to utilize the available x value. The pressure value, like that used in hydraulic systems, is equal in all directions from the residential zone. A discussion of the quantitatively descriptive parameters used to evaluate the term of desire is presented in the next section.

Money

From the fact that travel costs money, it was assumed that the number of trips made could be a function of the total amount of money available for transportation and the expenditure required per trip. The use of money to estimate the flow of trips has been used by others but this is the first use of money in a systems approach. The amount of money consumed yearly for transportation purposes had steadily increased. The approach used here assumes that the cost of travel will be minimized in terms of time and money. The measurement on x, although not equal to the money value, is related to it. A detailed explanation of the relationship between transportation consumption and the pressure x will be presented in the next section.

TERMINAL EQUATIONS OF COMPONENTS

The terminal equations for all components have been assumed to be of this simple form:

$$x = R\,y$$

Each term of the above equation is generally a more complex expression. The following postulates concerning these terms have been established:

1. The y value shall be the flow of work trips. The y value might be specified for the residential and employment components.
2. The x value shall be a measure of pressure which can be related to a function of desire in one case and of money spent in another. The x value is specified only for the residential components.
3. The R value shall be a measure of resistance to flow or reciprocal of attraction which can be specified for the route and employment components. For the employment component, R is a function of the jobs available. The alternates presented for the establishment of R on the route component are:

a. R is a function of trip frequency

b. R is a function of the ratio of probable trips to actual trips (Normalized)

c. R is a function of the ratio of probable trips to actual trips (Unnormalized)

d. R is a function of the cost of travel through the route component.

The discussion of the postulates is presented under each of the components for which it was developed.

Residential Zone Component

Of the three possible measurements noted above, only y and x are utilized for residential zones.

The Y Measurement--Flow of Work Trips

The y measurement which represents the flow of work trips from the residential zone can be established on the basis of the following:

1. From the present O and D study, an equation will be established which related trips per dwelling unit and the independent variables as

$$Y_i = -A + B\alpha - C \operatorname{Log} \delta + D \operatorname{Log} \gamma + E\beta$$

Where:

Y_i is the number of work trips per dwelling unit

α is the income index (dimensionless)

β is the car ownership per dwelling place

γ is the straight line distance from zone to CBD in miles

δ is the net residential density per net residential acre.

2. From future estimates of the parameters, α, β, γ, and δ in the above equation, the future trips per dwelling units Y_i' will be computed.

3. The total number of future work trips per residential zone y_i' will be determined from Y_i' modified by future estimates of the number of dwelling units N, and the percentage of trips to work K_w.

$$y_i' = N\,K_w\,Y_i'$$

The X Measurement--Desire

The x measurement which represents the pressure for work trips from the residential zone will be established as a function of desire on the following basis:

1. From existing O and D data for income index α, car ownership β, distance to CBD γ, and population density δ, an equation will be established for Y, resident trips per dwelling unit.

$$Y = -A + B\alpha - C \operatorname{Log}\delta + D \operatorname{Log}\gamma + E\beta$$

2. This equation is maximized with limiting values of all parameters as forecasted in the whole area for the future year. This maximum value is related to a measure of the theoretical pressure as desire, X_T

$$X_T = F \cdot Y$$

Where: F is a factor used to adjust for the difference in units If Y is developed for all trips, then the pressure or desire for work trips per dwelling units will be found by using the previously defined factor Kw.

3. Using maximizing input values for all parameters except income, one can establish the change in X, ΔX due to particular value of income α.

$$\Delta X = X_W - X_{\alpha:\beta,\gamma,\delta}.$$

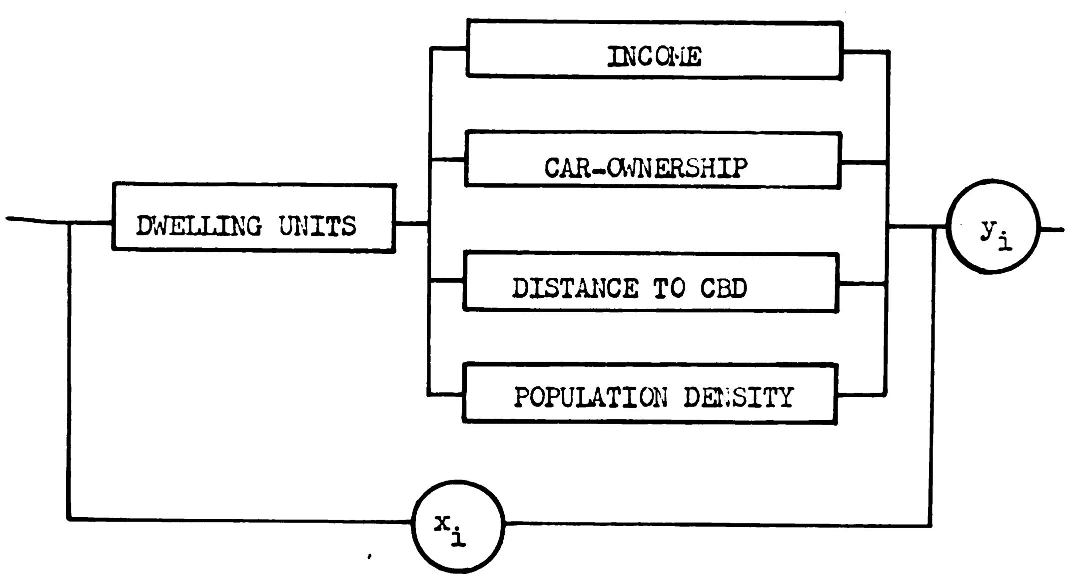
The maximum change will be found for some limiting value α, then R_α the resistance value can be found by equating:

$$R_\alpha = \frac{\Delta X_\alpha}{\Delta X_\alpha (\text{max.})}$$

A relationship for R_α versus the income α is shown in Figure 27, which isolates the effect of income alone on the pressure to make trips.

Similar techniques will be used to establish values for R_β, R_γ and R_δ as shown in Figures 28, 29 and 30.

4. A subsystem of the zone parameters will be solved in order to establish each value of x_i. Schematically, this can be shown:



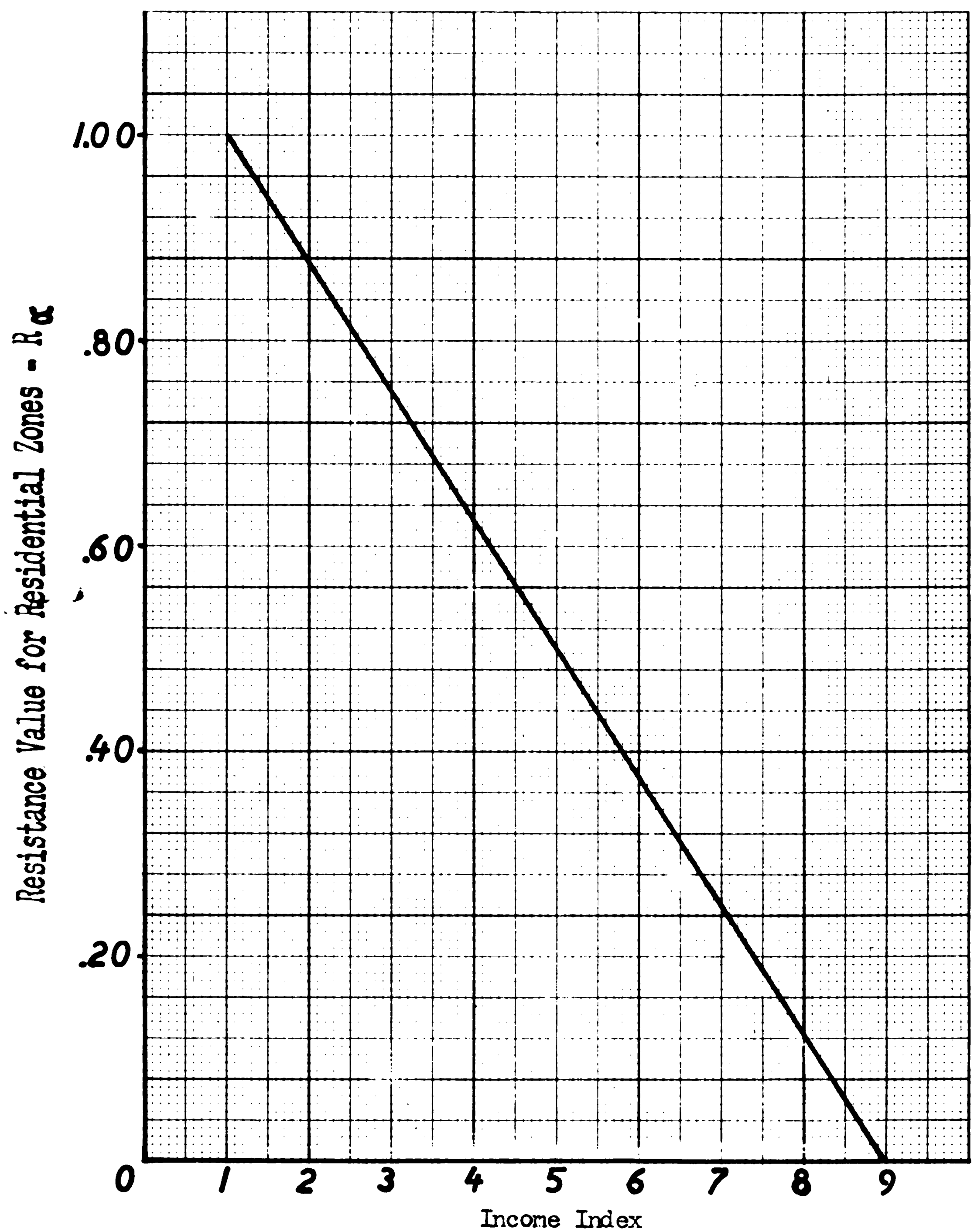


Figure 27. Relationship between resistance on residential zones and income index.

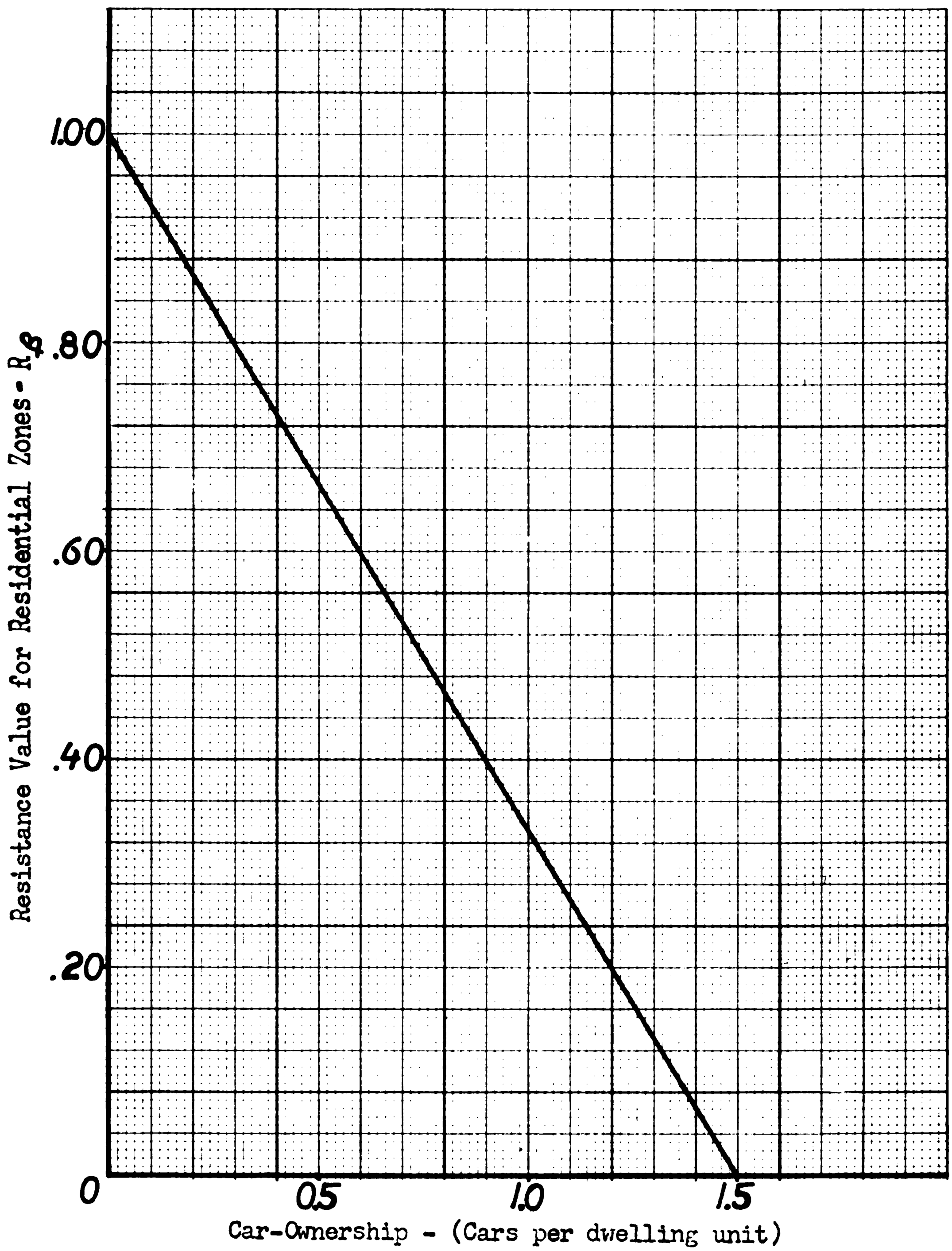


Figure 28. Relationship between resistance on residential zones and car-ownership

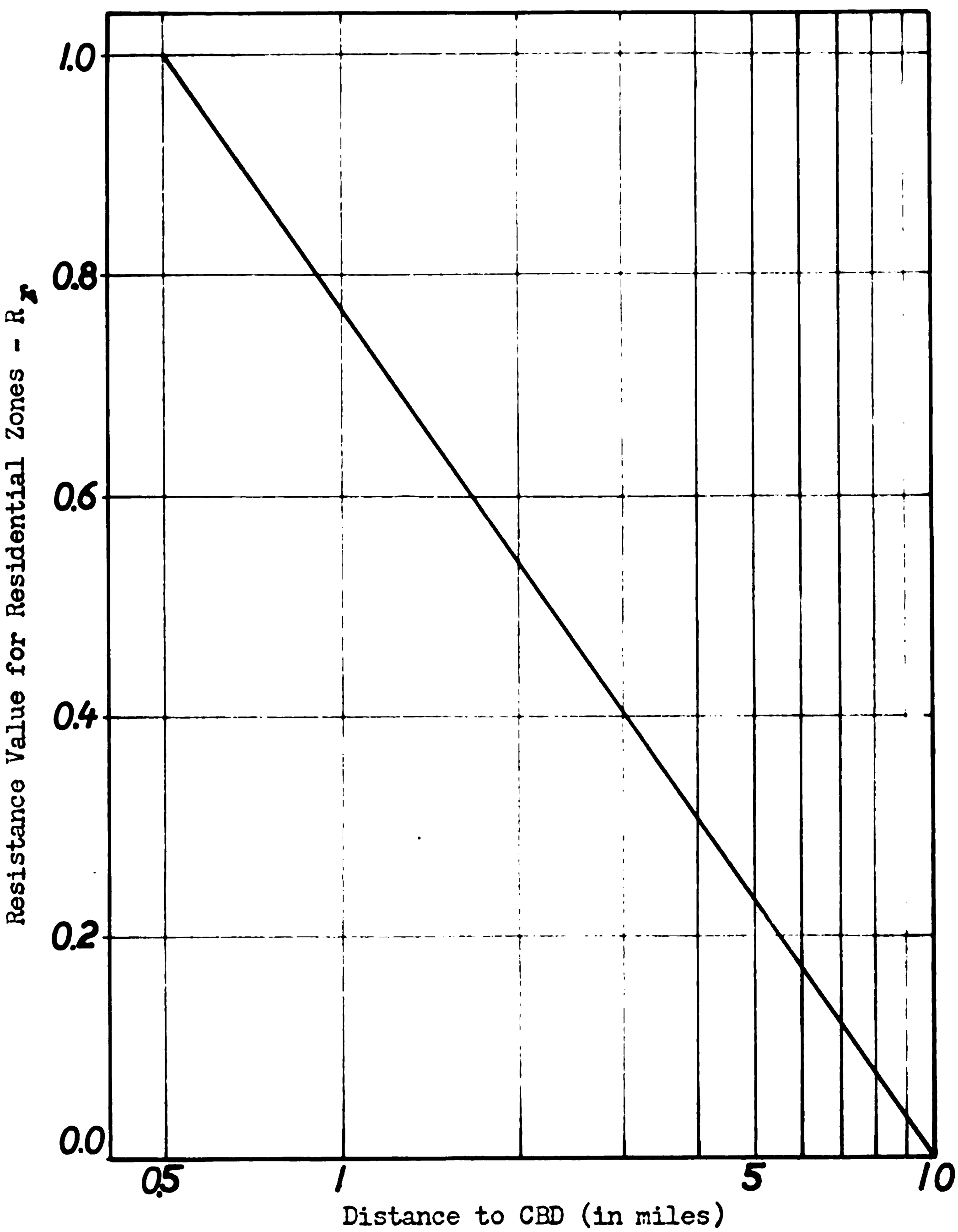


Figure 29. Relationship between resistance on residential zones and the distance to the CBD

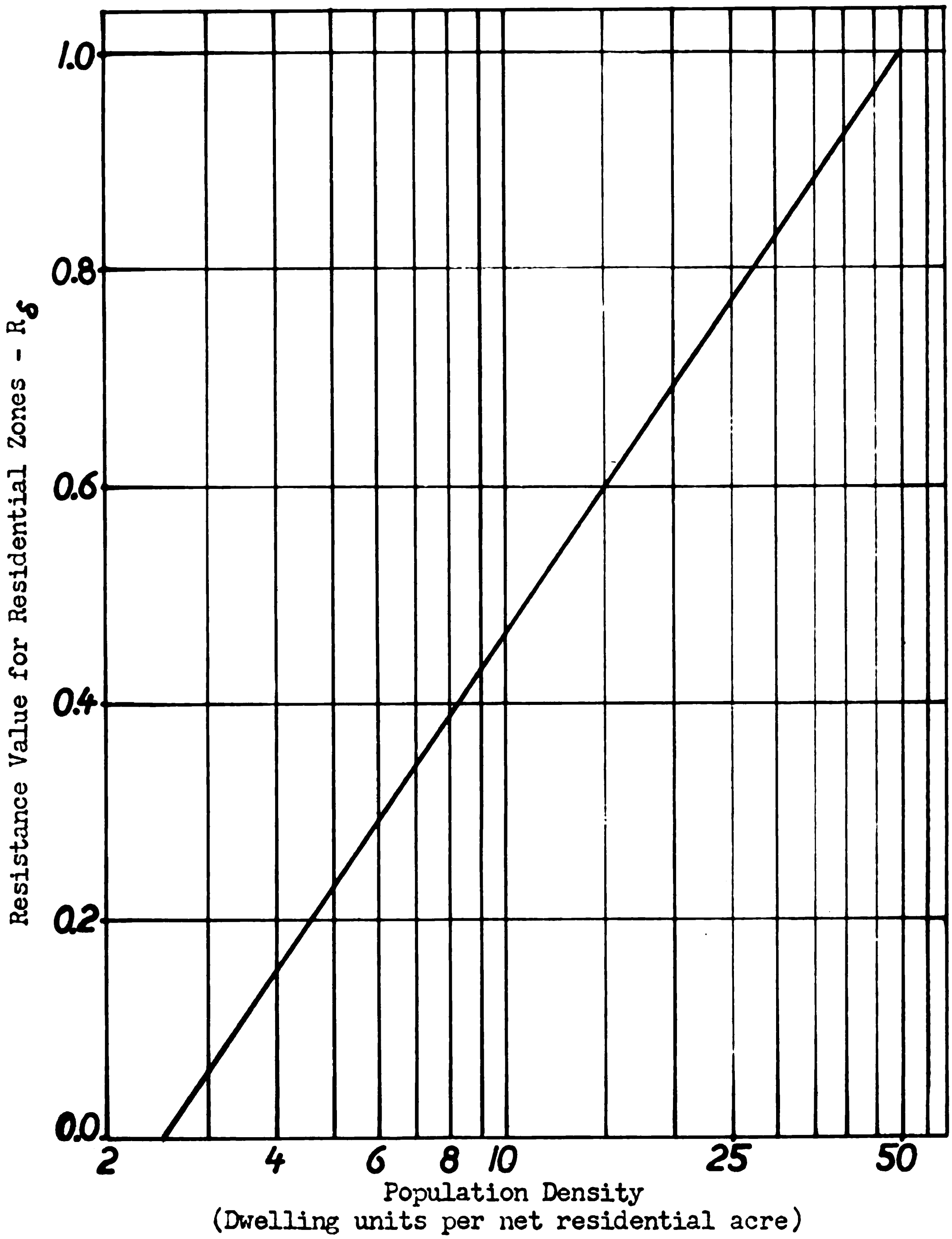
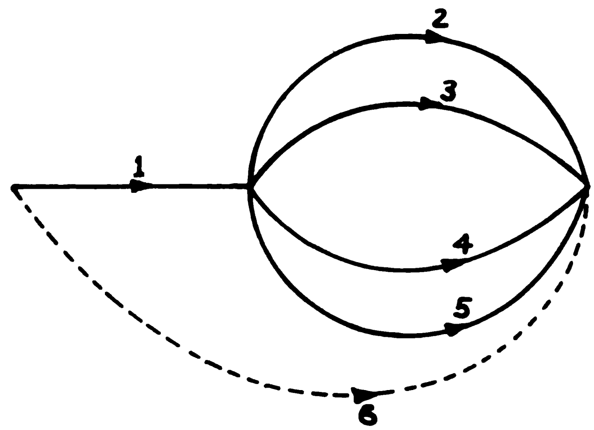


Figure 30. Relationship between resistance on residential zones and population density

As a system graph:



Where:

$x_1 = N(X_W)$

R_α , R_β , R_γ and R_δ are found from previous curves.

The subsystem can then be solved for x_6 in terms of the values, x_1, R_α , R_β , R_γ and R_δ . The element number 6 can be used in lieu of the subgraph.

$$x_i = x_6 = x_1 + \frac{1}{\sum R^{-1}} y_6$$

The X Measurement--Money

The x measurement which represents the pressure for work trips from the residential zone will be established as a function of money or consumption allotted to the making of work trips. The consumption for work trips is found by first determining income and the percentage of income consumed. A portion of the total consumption per family is spent for transportation costs. This amount is distributed over the trips for each purpose, in order to establish the actual amount allotted for work trips.

The x measurement will be established on the basis of the following:

1. The average amount of money allotted yearly to the cost of work trips made from a residential zone i is referred to as C_i.

$$x_i = K \frac{C_i}{250}$$

Where:

K is a constant determined to adjust for the differences in dimensions; 250 is the number of working days per year.

The value C_i can be estimated in the following manner:

a. Determine from a previously established plot of income versus consumption, an estimate of the future year consumption Z_i based on an estimate of the future year mean income I for the zone. (See Figure 31)

$$Z_i = f\, I_i$$

b. Establish from previous data, a relationship between transportation consumption Z_i^t and total consumption Z . (See Figure 32) Based on observations of trends, the yearly change in this ratio can be established:

$$E_f = E_b + n \Delta E$$

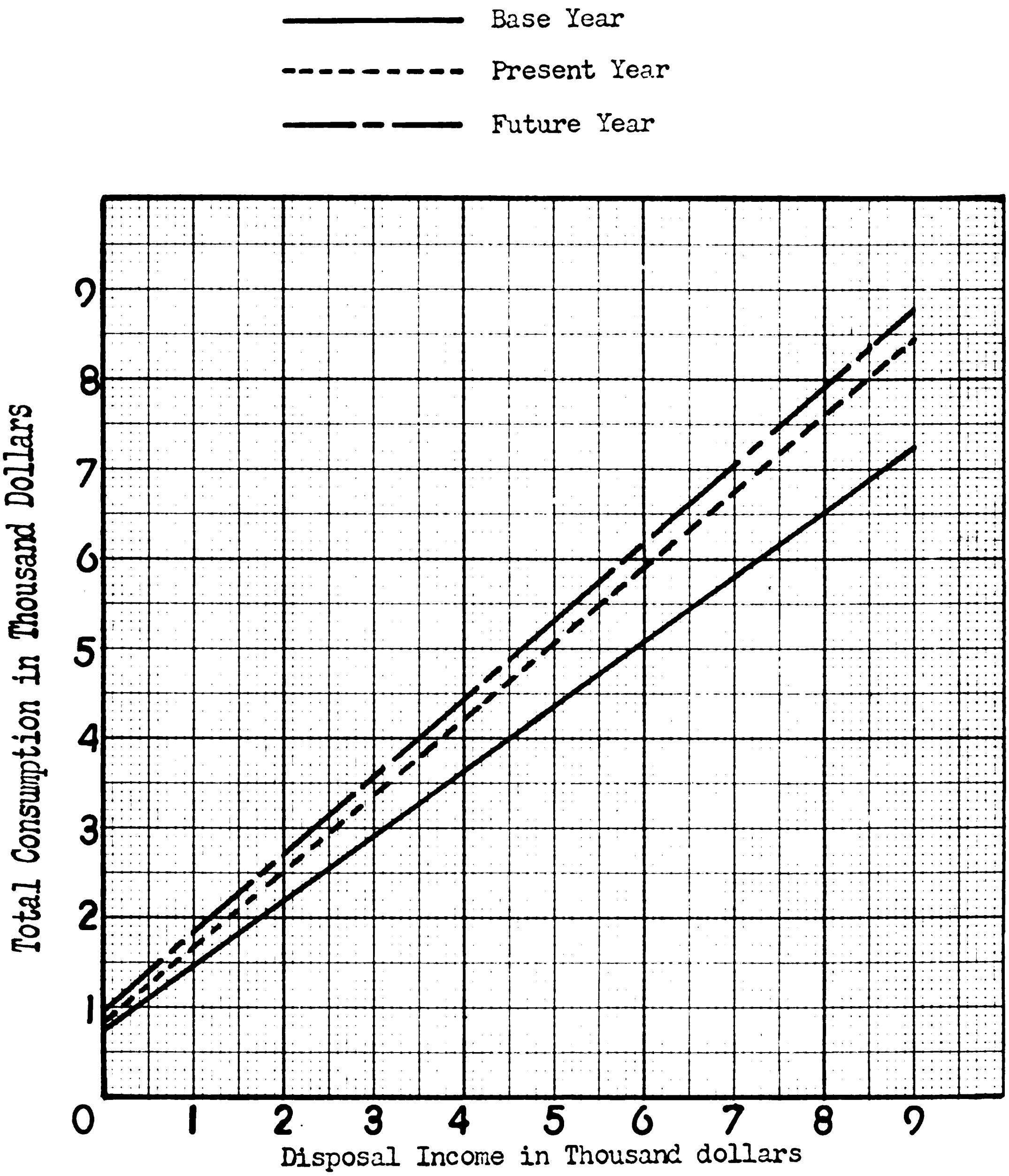


Figure 31. Relationship of consumption to disposable income.

Source: Reference 59

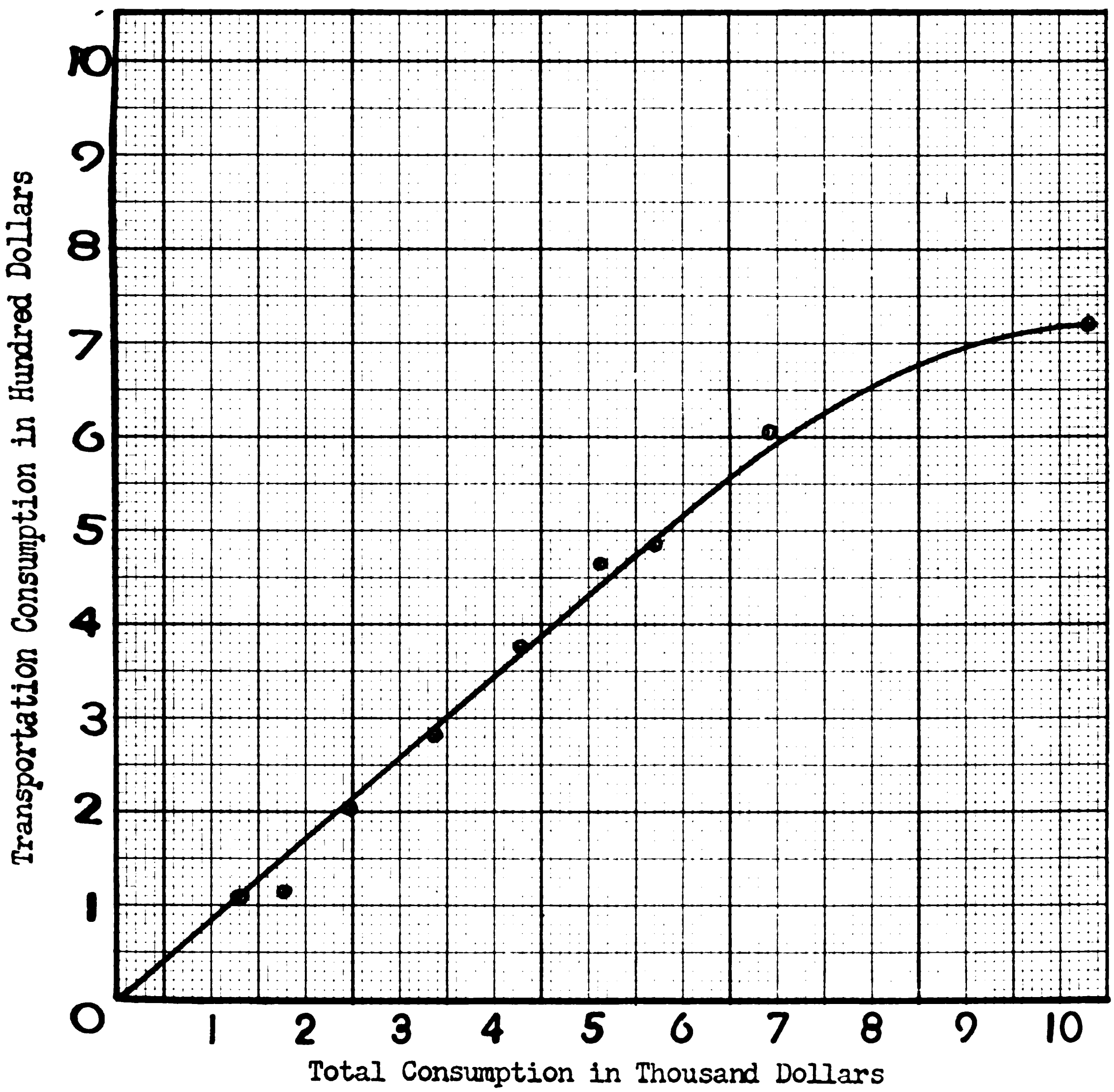


Figure 32. Relationship of transportation Consumption to total consumption

Source: Reference 60

Where:

E_f is the ratio of future amount spent on transportation per total consumption for the future year.

E_b is the same for the base year

ΔE is the yearly change

n is the number of years from base year to future year.

The amount of expenditure spent on transportation for the future year can be found.

$$z_i^t = E_f \cdot Z$$

c. The z_i^t represents the total amount spent for transportation per family in zone i. The amount spent for transportation per zone for the future year (Z^t) will z_i^t times (N_f) the number of families in zone i,

$$Z^t = N_f \cdot z_i^t$$

d. Since the primary concern is home based work trips, it is necessary to determine the amount spent for these, where K_h is the percentage of all trips that are home-based.

$$Z_i^t = K_h \cdot Z^t$$

e. The amount of money spent for trips to work can be found by first establishing the ratio of the total mileage for work trips to the total mileage traveled for all trip purposes. The ratio K_w is equal to:

$$K_w = \frac{k_w \cdot m_w}{M_{total}\text{(All purposes)}}$$

Where:

$$M_{total} = (k_w \cdot m_w) + (k_s \cdot m_s) + (k_{sc} \cdot m_{sc}) +$$
$$(k_{sr} \cdot m_{sr}) + (k_{cm} \cdot m_{cm}) + (k_b \cdot m_b)$$

Where:

k_w is the percentage of trips to work

m_w is the average trip length for work

k_s is the percentage of trips to shopping

m_s is the average trip length to shopping

k_{sc} is the percentage of trips to school

m_{sc} is the average trip length to school

k_{sr} is the percentage of trips to social-recreation

m_{sr} is the average trip length to social-recreation

k_{em} is the percentage of trips to eat meals

m_{em} is the average trip length to eat meals

k_b is the percentage of trips to business

m_b is the average trip length to business

The input values for the above equation can be established from Tables 4 and 8.

Table 8. The average trip length for home-based trips by purpose

Purpose for Home-based Trips	Average Trip Length in Miles
Work	5.56
Shopping	3.15
School	2.97
Social-Recreation	4.27
Eat Meals	3.44
Business	3.71

Source: Reference 57

Then:

$$Z_i^{wt} = K_w \cdot Z_i^t$$

By definition:

$$C_i = Z_i^{wt}$$

Employment Zone Component

The discussion of employment zones will use the flow of trips or y measurement and the resistance R, as a reciprocal of the attraction.

The Y Measurement--Flow of Work Trips

The number of existing work trips destined to an employment zone j, can be found from the data of the O and D study. A relationship of work trips to some other parameter can be determined such as trips per acre, per job, per labor force or other standard. It might be necessary to establish a predicting equation on the basis of multiple correlating parameters. The parameters which provide the highest correlation would then be estimated for the future year, so that a growth factor is established for each zone. The number of work trips to zone j at some future year Y_j' would be solved by this equation:

$$Y_j' = G_j \cdot Y_j$$

The R Measurement--Reciprocal of Attraction.

From an analysis made of the existing O and D data, the parameter which relates to work trips most highly will be determined. For the illustrative problem that follows, it was assumed that this might be the number of jobs Q_j, estimated for zone j for the future year. The resistance value R_j can then be equated to:

$$R_j = \frac{Q_j^{-1}}{\sum_{j=1}^{m} Q_j^{-1}}$$

A similar resistance value could be found by using the number of work trips Y_j' established above in the same equation form.

$$R_j = \frac{Y_j'^{-1}}{\sum_{j=1}^{m} Y_j'^{-1}}$$

This type of manipulation contributes nothing to the method of solution. If the flow of trips is known for any component of the system, it should be used as such.

Route Components

The only one of the three possible measurements that can be used for the route component is the parameter R. For a route component, as in a piece of hydraulic pipe, there is no mechanics for generating pressure or flow. It then seems logical if a long period of time be chosen, and further, if the arterial routes chosen as components provide no overnight parking then there will not be any storage in the component. If a 24 hour period is chosen from 4:00 A.M. to 4:00 A.M., then during this period the flow into the component equals the flow out of the component.

The R Measurement

The flow of trips through a route component can be determined from a knowledge of either the pressure drop through the component or the friction encountered throughout the component. If pressure measurements for the components are in terms of either desire or money, it is difficult to vary the amount of this pressure in order to note the effect on the flow of trips. Friction can be related to such measurements as travel time or distance and these can be easily varied and measured while the relationship to flow of trips is recorded.

Several methods are proposed to establish the measure of friction through the route component which relates the pressure x to the flow y.

In several of the procedures that follow, it was assumed that the best single measurement that could quantitatively assign a value to the resistance term would be travel time. Contemporary writers on this subject have used, besides travel times, travel distances and straight line distances from origin to destination. Of the three parameters, the straight line distance is most easily measured, but gives the poorest indication of the friction or resistance encountered. Travel distances are a little more difficult to measure, but give a better estimate of the resistance. Travel distances can be the same for alternate routes where one has most of its travel on minor routes and the other is arterials and expressways. The friction established by distance alone would not show that the friction on the minor routes is much larger than on the other.

Trip Frequency.--The values used for the R are not assumed to have general applicability to all urban areas. From the data of an O and D, a curve is plotted which relates trip frequency to travel time. It was assumed that the reason for the high trip frequency was the trip encountered little resistance. A resistance scale from zero to one was then applied to this frequency curve as shown in Figure 33.

Probable vs. Actual (M_{ij}^{-1}).--The curve used here is established using the data from an O and D study. The number of trips from each of the residential zones to each of the employment zones Y_{ij} is determined. The probable number of interchanges P_{ij}, are calculated by assuming equal travel times from each residential zone to all of the employment zones. A ratio M_{ij}^{-1}, is determined where:

$$M_{ij}^{-1} = \frac{P_{ij}}{Y_{ij}}$$

The value of M_{ij}^{-1} was then plotted against travel time in minutes d_{ij}. and the following equation established:

$$M_{ij}^{-1} = 0.151 + 0.0327\, d_{ij}$$

The plot of these values, shown in Figure 34, can be interpreted as a straight line. It seems reasonable to assume that the differences

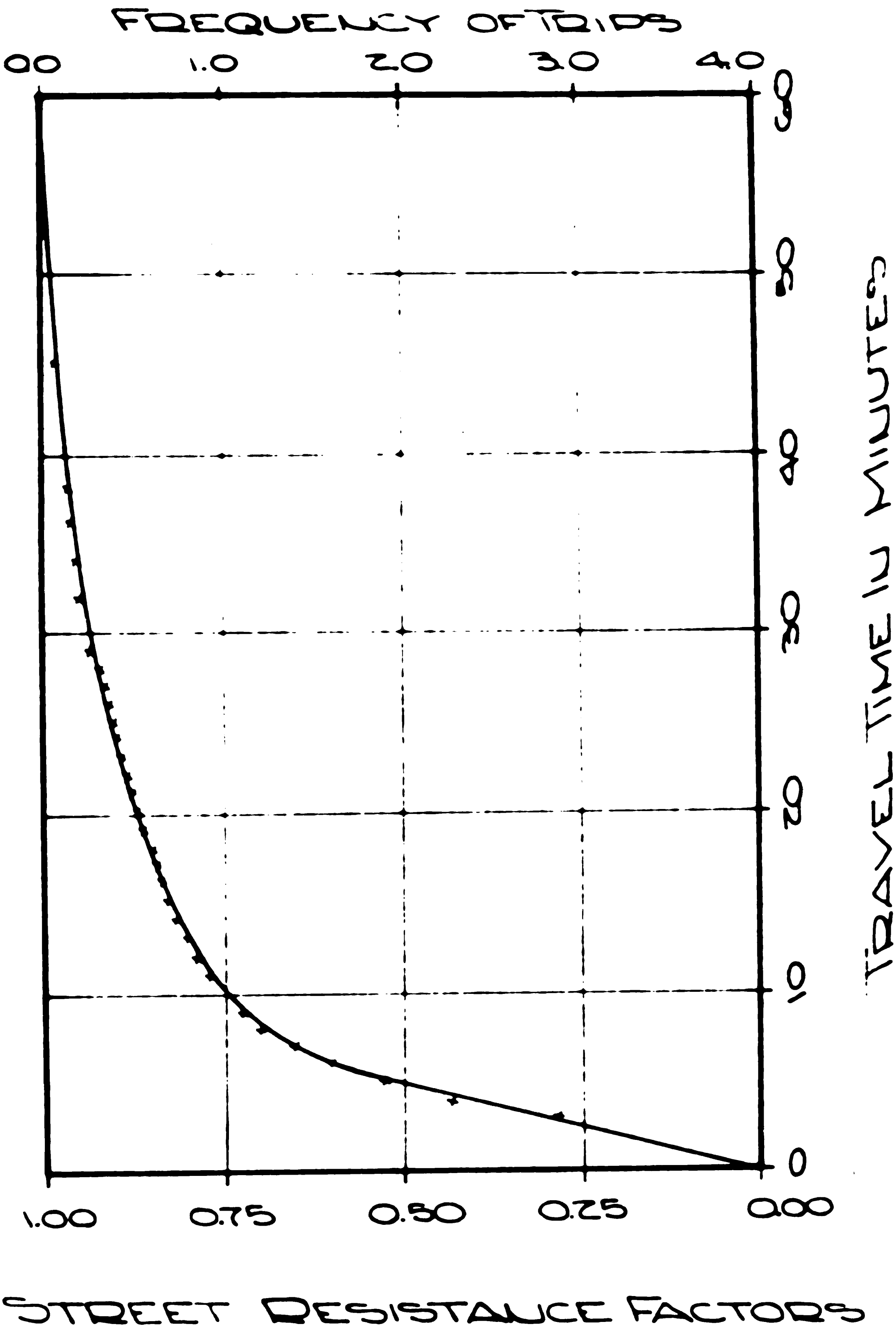


Figure 33. The relationship between travel time and the work trip resistance factors developed from trip frequency. Source: Reference 30

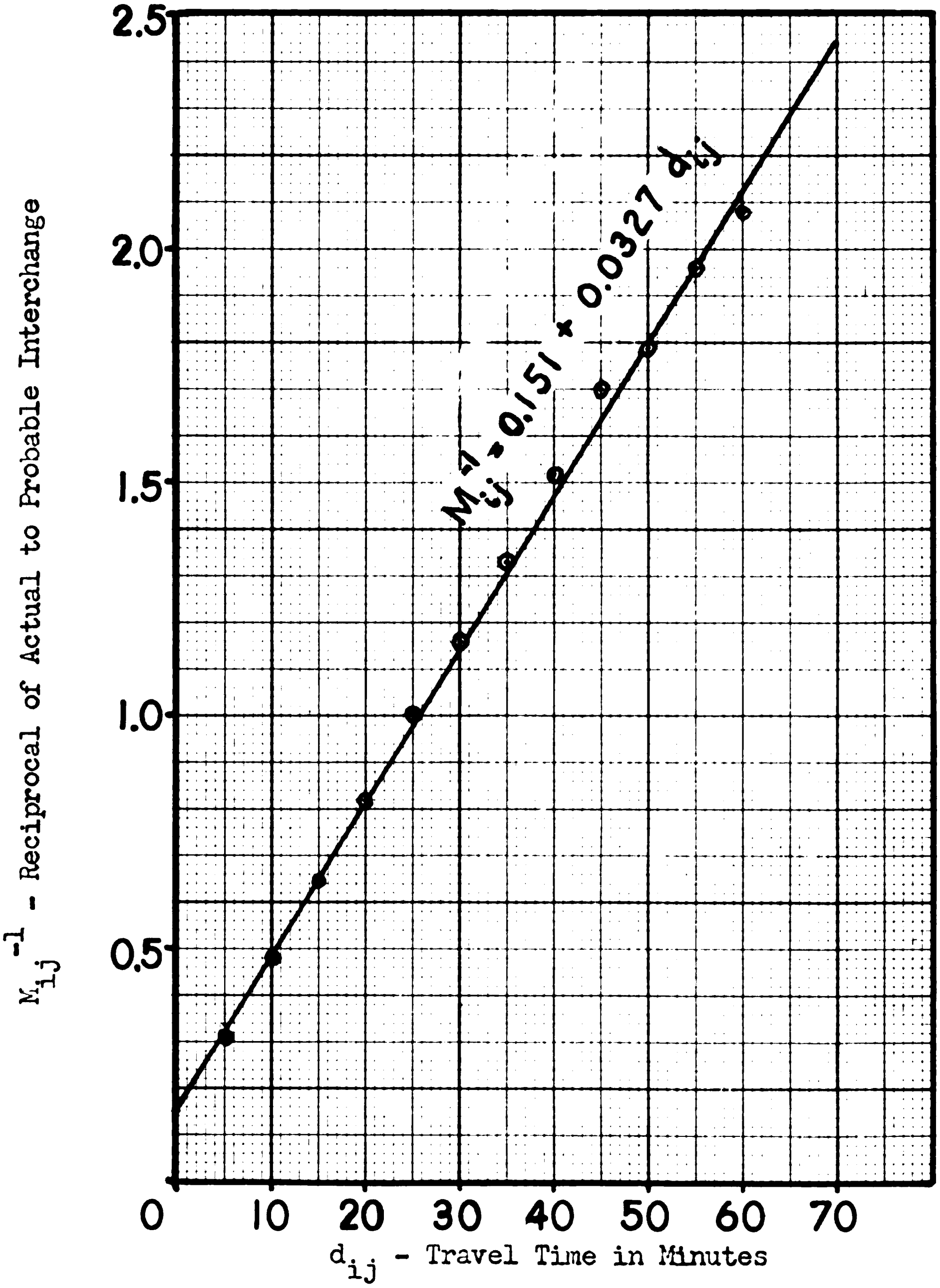


Figure 34. Relationship of the travel time, d_{ij} to the reciprocal of the actual trips per unit of probability interchange, M_{ij}^{-1}.

between the actual interchanges and the probable values would be related to travel time.

Normalized M_{ij}^{-1} .--This approach follows logically after the above since the values M_{ij}^{-1} were used. But given a specific origin from which trips were made to a number of destinations, it was reasoned that the person making such trips would analyze the travel times to the alternate destinations and that trips would be made accordingly. The values of M_{ij}^{-1} were established from the previously cited curve Figure 34. The value of R_{ij} was then calculated from the values and this equation.

$$R_{ij} = \frac{M_{ij}^{-1}}{\sum_{j=1}^{m} M_{ij}^{-1}}$$

Money Value The following approach to R is the only one which does not use travel time alone as the controlling factor. This technique follows from an early attempt to establish an x_{ij} or pressure value on the route component. The subscripts ij will be used to designate the route component from a zone of residence i to a zone of employment j. The pressure drop along route ij was called x_{ij} and was related to the total cost of making the trip per vehicle C_{ij}. The factor K was used to adjust the equation dimensionally and Y_{ij} represents the actual flow.

$$x_{ij} = K \cdot C_{ij} \cdot y_{ij}$$

The value of C_k would be equal to the sum of the products of vehicle costs per mile times the number of miles and travel time costs per minute times the number of minutes for the k th route.

$$C_k = c_v \cdot m_k + c_t \cdot d_{ij}$$

Where:

c_v is the total operating cost per vehicle mile

m_k is the length of the route in miles

c_t is the time costs per vehicle mile per minute of travel, assuming an occupancy of one person per vehicle

d_{ij} is the travel time per vehicle per route excluding terminal time.

Since the value to be solved y_{ij} for the component is also the same value y_{ij} required in the system solution, the equation can be further reduced:

$$R_{ij} = \frac{x_{ij}}{y_{ij}}$$

$$x_{ij} = K \cdot C_{ij} \cdot y_{ij}$$

$$\frac{x_{ij}}{y_{ij}} = K \cdot C_{ij}$$

$$R_{ij} = K \cdot C_{ij}$$

Evaluation

Since there are four possible treatments to the value R and many varieties of x and y measurements, a complete design would require many illustrative solutions. It seemed advisable to present each method at least once but to conclude which might be the best and to use it for most solutions. The resistance value established on the basis of money, logically requires that the pressure or x value is also evaluated in terms of money. This limits the use of this method and it is only shown once.

The trip frequency method precludes that the maximum value of R is limited to 1.00 and this has no basis for acceptance. The attraction factor of the zones of employment may actually be confounded in the frequency number unless the values recorded from the O and D are from a single destination zone to all points of origin. To then combine these values with those from another employment or destination zone would require proper weighting of values to establish an average.

The use of the M_{ij}^{-1} curve provides the best estimate of the resistance value. The trips from each residential zone are distributed to each employment zone on the basis of the attraction of the zone itself since all travel times are set equal. The difference between this probable value and the actual interchanges can only be related to the resistance of travel time. The question then becomes one of whether it is proper to normalize. This can best be answered by knowing how the system solution functions. Referring to a single residential zone, it seems proper that the individual trip maker only evaluates those routes to destinations available to him and it is proper to normalize. The system solution doesn't function in this manner. If it did make the distribution zone by zone it would probably over assign trips to some destinations and under assign trips to others.

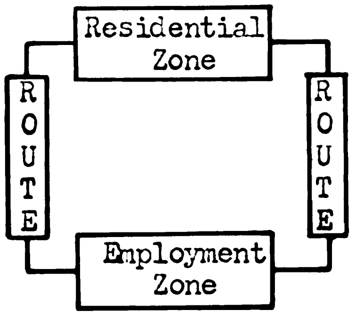
This is an inherent fault with the gravity model. The systems approach distributes trips to a zone of destination in accordance with capacity of that zone for trips and on the basis of the relative travel times of all zones of residence. A problem would arise when one zone of residence, Zone A, had relatively small but equal travel times to an employment zone, and another residential zone, Zone B, had its lowest travel time much higher than those cited for Zone A, but it also had some high travel times. Normalizing would produce a higher trip interchange on the one route with the shortest travel time from Zone B than on any of the routes from Zone A, even though the travel times from Zone A are much smaller.

SYSTEMS GRAPHS

This section can be abbreviated along with the others that follow, because the techniques are precisely the same, regardless of the type of system analyzed. The specific details of constructing the systems graph are presented in Chapter three.

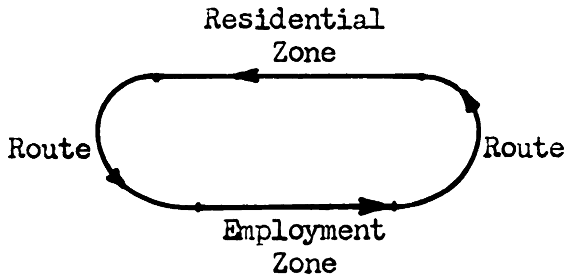
The systems graph is a collection of the component terminal graphs obtained by uniting the vertices of the terminal graphs in a one to one correspondence with the components in the physical system.

If the systems analyst follows the above definition, there is little choice in the form of the system graph. The possible variations will be demonstrated using the following system of physical components.

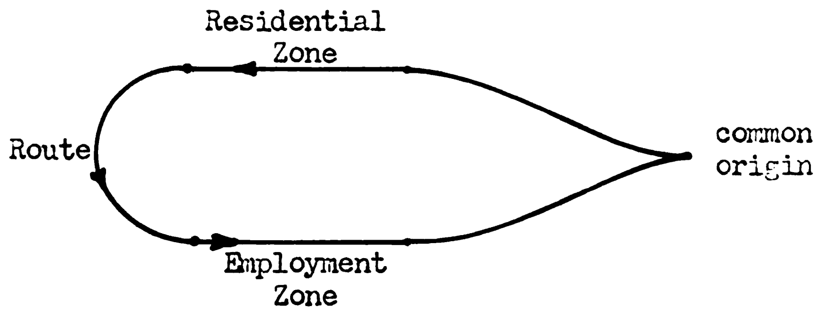


The choice of graphs for the schematic above are:

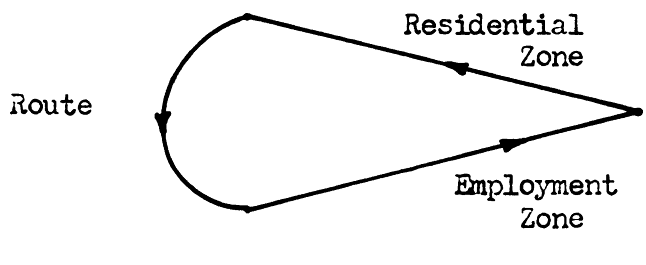
1.



2.



3.



Choice one presents the clearest relationship between the system graph and the actual physical system. It is the only choice of those presented which can be used when the characteristics of the two routes are dissimilar. For example, this occurs when the trip made has one direction during the off-peak period of traffic flow while the other direction is made during the peak period. If the routes are balanced, this choice presents a much larger matrix solution than required by the other choices. This type of system graph is illustrated in Figure 35.

The second choice illustrates the point that measurements are meaningless except when established to some reference. This choice graphically shows that reference while in the other two choices the reference is implied.

The most efficient mathematical solution comes from choice three. This graph can only be used when the route characteristics are exactly the same. Its relationship to the physical system is not so evident, but this is not important to the experienced analyst. This type of system graph is shown in Figure 36.

The operations performed on the system graph are presented in the following section.

SOLUTION OF SYSTEM GRAPHS TO ESTABLISH THE SYSTEM EQUATIONS

The operational procedures presented in this section are precisely the same regardless of the type of physical system analyzed. The rules to be followed are given in greater detail in section three and only briefly reviewed here.

1. A tree is selected from the elements of the system graph such that it will contain all the vertices of the graph but

establish no circuits. Generally there are many possible trees that can be formed on a system graph. The one which is used is called the formulation tree.

2. The system graph is divided into two subgraphs; one of which is the tree and called branches, and the other, which is the complement of the tree called chords.

3. Those elements for which the x value is specified are made a part of the branch subgraph (b-1) and those elements which have specified y values are placed in the chord set (c-2).

4. By using the system graph with the formulation tree identified, one can write either the fundamental circuit equations or the fundamental cut-set equations.

5. The fundamental circuit equations are used when the across variables x are expressed as explicit functions of the through variables y. The number of independent equations equals the number of chords since the expression for each circuit includes one and only one chord.

6. The fundamental cut-set equations are used when the through variables y are expressed as explicit functions of the across variable x. The number of independent equations equals the number of branches in the tree since each vertex equation written includes one and only one branch.

The solution of the illustrative problems which follow in the next chapter are solved by the use of either the general mesh form equation or the general branch form equation. These equations incorporate the terminal equations along with the cut-set and circuit equations. Although the form of these equations is presented in chapter three, they will be listed here for reference.

1. General mesh form

$$\left[B_{11} \cdot X_{b-1}\right] + \left[B_{12} \cdot R_{b-2} \cdot B_{12}^{T} + U \cdot R_{c-1}\right] \left[Y_{c-1}\right] + \left[B_{12} \cdot R_{b-2} \cdot B_{22}^{T}\right] \left[Y_{c-2}\right] = 0 \qquad (14)$$

2. General branch form

$$\left[A_{21} \cdot G_{c-1} \cdot A_{11}^{T}\right] \left[X_{b-1}\right] + \left[G_{b-2} + A_{21} \cdot G_{c-1} \cdot A_{21}^{T}\right] \left[X_{b-2}\right] + \left[A_{22} \cdot Y_{c-2}\right] = 0 \qquad (17)$$

NUMERICAL SOLUTION AND COMPUTER USE

The illustrative solutions of theoretical systems which follow in the next chapter show by operation the numerical solution of the mesh and branch formulation. The triple matrix products of the either equations, 14 or 17, are solved and the proper values of R or G are substituted into the final matrix form.

The end result is a system of simultaneous equations too difficult for long hand solution. A computer program for the Michigan State University Mistic computer was used to solve all but the most simple matrix.

ILLUSTRATIVE SOLUTIONS OF THEORETICAL SYSTEMS USING POSTULATES DEVELOPED IN PART IV

ILLUSTRATIVE PROBLEM ONE

Problem Statement

Given Information

For the hypothetical city "Owenshall", the following information is available from an O and D study.

1. The present interzonal trips - t_{ij} .
2. The present travel times between zones d_{ij} .
3. Present and future evaluation of employment zones to establish relative attraction values on the basis of number of jobs and/or other parameters.
4. Probability interchange established on the basis of the relative attraction of employment zones, assuming equal travel times - P_{ij} .
5. The actual trips per unit of probability interchange.

$$M_{ij} = y_{ij} / P_{ij}$$

6. Curve plot or equation for $1 / M_{ij}$ versus d_{ij} (Figure 34).

7. Equation which relates trips per dwelling unit and independent variables as:

$$Y_i = -A + B\alpha - C \operatorname{Log} \delta + D \operatorname{Log} \gamma + E \beta$$

where:

β is car ownership per dwelling place

δ is net residential density (dwelling units per net residential acre)

γ is straight line distance from zone to CBD in tenths of mile.

α is income index

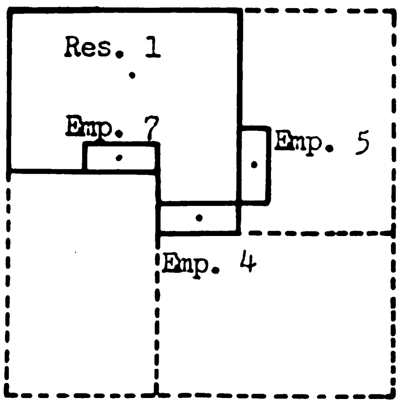
8. Distribution of trips by purpose. K_W is the work trip factor.

9. Existing land use data.

To Find

The problem is to forecast future interzonal work trips from one zone of residence to each of three employment zones.

Schematic of the Physical System



Input Values for System Solution and How Established

The following input values are necessary for solution of this sample problem by linear graph theory:

1. The number of work trips y_i' for residential zone i for the future year is required. This will be established from a prediction of future land uses and the equation developed in item 7 from existing data. The estimate of future land uses presumes that estimates of the independent variables will also be made. The prime values in the equation shown indicate future year estimates:

$$Y_i' = -A + B\,\alpha' - C \operatorname{Log} \delta' + D \operatorname{Log} \gamma' + E\,\beta'$$

The work trips y_i' can be found by multiplying Y_i' by the factor K_w for work trips.

$$y_i' = Y_i' K_w$$

2. The relative attraction value of each employment zone is established as a resistance value R_j.

$$R_j = \frac{Q_j^{-1}}{\sum_{j=1}^{m} Q_j^{-1}}$$

where:

Q_j is number of jobs in employment zone j

3A and B. The value of R_{ij} for each route system from zone i to j is established from an estimate of future travel times d_{ij}. The values of d_{ij} are used in connection with the curve or equation stated in item 6 above. In order to develop values of M_{ij}^{-1}, this equation will be used:

$$\frac{1}{M_{ij}} = 0.151 + 0.0327 \; d_{ij}$$

Then, for alternate A, the value of R_{ij} is normalized by the equation:

$$R_{ij} = \frac{M_{ij}^{-1}}{\sum_{j=1}^{m} M_{ij}^{-1}}$$

The input values for items 1 to 3A above are shown in Table 9.

A complete long hand solution is presented for problem using the input values 1 to 3A.

For alternate 3B, the resistance R_{ij} is not normalized.

$$R_{ij} = M_{ij}^{-1} .$$

The input values for this solution are shown in Table 10 and only the results of the solution are presented in Table 11.

<u>Terminal Representation of the Component</u>

1. The residential zone has a specified y or through driver

$$\xleftarrow[26]{\hspace{6em}}$$ Specified y

2. The employment zones and route components are given by the equation form where:

$$R = \frac{x}{y} \qquad\qquad 2$$

The terminal equations for these components can be shown in general matrix form.

$$\begin{bmatrix} R_{b-2} & 0 \\ 0 & R_{c-1} \end{bmatrix} \begin{bmatrix} Y_{b-2} \\ Y_{c-1} \end{bmatrix} = \begin{bmatrix} X_{b-2} \\ X_{c-1} \end{bmatrix}$$

Specifically for these components

$$\begin{bmatrix} R_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & R_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & R_{28} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & R_{29} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_{16} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_{18} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_{11} \\ y_{12} \\ y_{28} \\ y_{29} \\ y_{16} \\ y_{18} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_{11} \\ x_{12} \\ x_{28} \\ x_{29} \\ x_{16} \\ x_{18} \end{bmatrix}$$

The values of R are shown in Table 9.

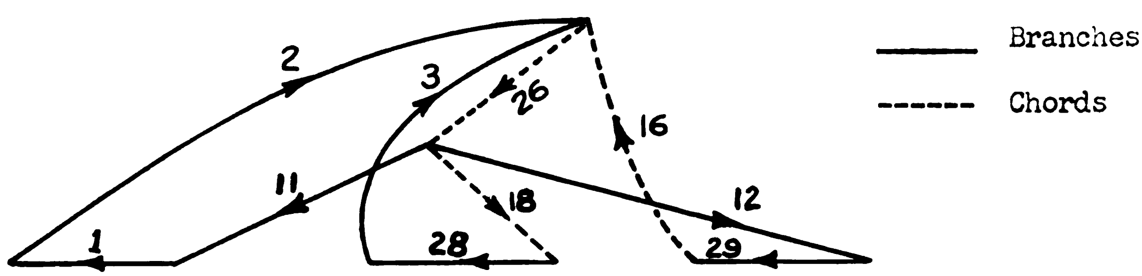
Table 9. Component information necessary to solve the linear graph

Element No.	Component	Travel Time (minutes)	Resistance Factors	Jobs	Work Trips
1	Employment (I-7)	--	0.3846	3000	
2	Street	10	0.3055		
3	Street	10	0.3055		
11	Street	10	0.3055		
12	Street	14	0.3890		
16	Street	14	0.3890		
18	Street	10	0.3055		
26	Residential (R-1)	--	- ----		4000
28	Employment (I-4)	--	0.3846	3000	
29	Employment (I-5)	--	0.2308	5000	

Table 10. Component information necessary to solve the linear graph

Element No.	Component	Travel Time (minutes)	Resistance Factors	Jobs	Work Trips
1	Employment (I-7)	--	0.3846	3000	
2	Street	10	0.4780		
3	Street	10	0.4780		
11	Street	10	0.4780		
12	Street	14	0.6088		
16	Street	14	0.6088		
18	Street	10	0.4780		
26	Residential (R-1)	--	- ----		4000
28	Employment (I-4)	--	0.3846	3000	
29	Employment (I-5)	--	0.2308	5000	

System Graph and Tree



Circuit Equations

The circuit equation for this example can be shown in general form as:

$$\begin{bmatrix} B_{11} & B_{12} & U & 0 \\ B_{21} & B_{22} & 0 & U \end{bmatrix} \begin{bmatrix} X_{b-1} \\ X_{b-2} \\ X_{c-1} \\ X_{c-2} \end{bmatrix} = 0$$

The equations developed from following the procedures outline in part three, provides the equation form noted:

$$\left[\begin{array}{ccccccc|cc|c} -1 & -1 & 0 & -1 & 1 & 0 & 1 & 1 & 0 & 0 \\ -1 & -1 & 1 & -1 & 0 & 1 & 0 & 0 & 1 & 0 \\ \hline 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{c} \left[\begin{array}{c} X_1 \\ X_2 \\ X_3 \\ X_{11} \\ X_{12} \\ X_{28} \\ X_{29} \\ \hline X_{16} \\ X_{18} \\ \hline X_{26} \end{array}\right] \end{array} \begin{array}{c} X_{b-2} \\ \\ \\ X_{c-1} \\ \\ X_{c-2} \end{array} = 0$$

<u>Substitution in General Mesh Form</u>

The next step is to make a substitution in the general mesh form equation using the values of the coefficient matrices. The general mesh equation is:

$$\left[B_{11} \cdot X_{b-1}\right] + \left[B_{12}\, R_{b-2}\, B_{12}^{T} + U\, R_{c-1}\right] \left[Y_{c-1}\right] +$$

$$\left[B_{12}\, R_{b-2}\, B_{22}^{T}\right] \left[Y_{c-2}\right] = 0$$

It will be noted that the first term is non-existent in this sample because no X_{b-1} values exist in this problem. The resulting equation is then:

$$\left[B_{12}\, R_{b-2}\, B_{12}^{T} + U\, R_{c-1}\right] \left[Y_{c-1}\right] + \left[B_{12}\, R_{b-2}\, B_{22}^{T}\right] \left[Y_{c-2}\right] = 0$$

The result of this first term becomes:

$$\begin{bmatrix} (R_1 + R_2 + R_{11} + R_{12} + R_{29} + R_{16}) & (R_1 + R_2 \quad R_{11}) \\ (R_1 + R_2 + R_{11}) & (R_1 + R_2 + R_3 + R_{11} + R_{28} + R_{18}) \end{bmatrix} \begin{bmatrix} y_{16} \\ y_{18} \end{bmatrix}$$

The results of the second term becomes:

$$\begin{bmatrix} -(R_1 + R_2 + R_{11}) \\ -(R_1 + R_2 + R_{11}) \end{bmatrix} \begin{bmatrix} y_{26} \end{bmatrix}$$

<u>Final Solution</u>

Substituting the values of R into the above equation gives the two independent equations:

$$\begin{bmatrix} 2.0044 & 0.9956 \\ 0.9956 & 1.9912 \end{bmatrix} \begin{bmatrix} y_{16} \\ y_{18} \end{bmatrix} + \begin{bmatrix} -0.9956 \\ -0.9956 \end{bmatrix} \begin{bmatrix} 4000 \end{bmatrix} = 0$$

$$2.0044\ y_{16} + 0.9956\ y_{18} = 3982.40 \qquad (1)$$

$$0.9956\ y_{16} + 1.9912\ y_{18} = 3982.40 \qquad (2)$$

By multiplying equation two, above, by 2.0044/0.9956 and subtracting equation two from one, gives an expression for y_{18} as:

$$3.0132\ y_{18} = 4035.21$$

Then:

$$y_{18} = 1,339.17 \text{ work trips}$$

The computed value can be substituted in the above equations in order to solve for the value y_{16}

$$y_{16} = 1{,}321.66 \text{ work trips}$$

The number of trips to employment zone (I-7) or element number one can be now found as y_{11} .

$$y_{11} = 1{,}339.17 \text{ work trips}$$

Discussion of the results

The flow of work trips to each employment zone is shown in Table 11. The distribution of trips, when equal travel times were assumed, reflects the relative attraction of the employment zones only. The values of this distribution are the most extreme. When the travel times are related to the probable versus actual ratio, the trip distribution values are brought closer together, since the highest employment attractor also had the highest route resistance. This is in complete agreement with the basic premise that work trip interchange is directly proportional to the size of the attractor and inversely proportional to the travel time.

When the R values were normalized, the effect was a closer agreement on the trip distribution to all zones. This tends to lessen the effect of travel time on trip distribution.

Table 11. Results of illustrative problem one showing trips to employment zones

Work Trips To Zone No.	All Rs Equal	R_{ij} Normalized	R_{ij}-Not Normalized
I - 7	1,090.91	1,339.17	1,367.22
I - 4	1,090.91	1,339.17	1,367.22
I - 5	1,818.18	1,321.66	1,265.56

ILLUSTRATIVE PROBLEM TWO

Problem Statement

Given Information

For the hypothetical city "Sparty", a current O and D survey is available. From this comprehensive survey, the following information can be taken:

1. The present interzonal trips - t_{ij} .
2. The present travel times between zones - d_{ij} .
3. Present and future evaluation of employment zones to establish relative attraction values on the basis of number of jobs and/or other parameters.
4. Probability interchange established on the basis of the relative attraction of employment zones, assuming equal travel times - P_{ij}

5. The actual trips per unit of probability interchange.

$$M_{ij} = Y_{ij} / P_{ij}$$

6. Curve plot or equation for $1 / M_{ij}$ versus d_{ij} (Figure 34)
7. Curve plot which relates frequency of trip and travel time to route resistance R_{ij} (Figure 33)
8. Equation which relate trips per dwelling unit and independent variables as:

$$Y_i = -A + B\alpha - C \operatorname{Log} \delta + D \operatorname{Log} \gamma + E \beta$$

Where:

β is car ownership per dwelling place

δ is net residential density (dwelling units per net residential acre)

γ is straight line distance from zone to CBD in tenths of mile.

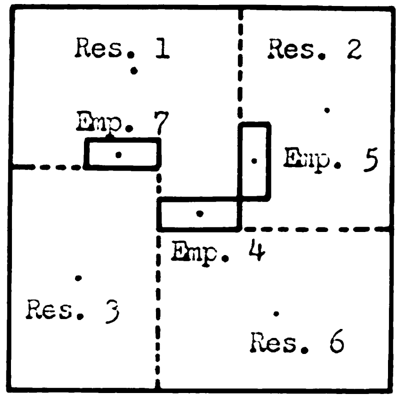
α is income index

9. Distribution of trips by purpose. K_w is work trip factor.
10. Existing land use data
11. Future land use forecasts

To Find

The problem is to forecast future interzonal work trip movements when specified y's or through drivers for the residential and employment zones and resistance values for the route components are given.

Schematic of the Physical System



Input Values for System Solution and How Established

The following input values are necessary for solution of this sample problem by linear graph theory:

1. The number of work trips Y_i' from each residential zone i for the future year will be determined by methods previously defined in sample number one.

2. The number of trips at each employment zone j can be established from knowing the present number of trips per acre, per job, per labor force or other standard and the predicted future land use. The ratio of future over existing values for any of the standards chosen will be called a growth factor G_j.

$$Y_j' = Y_j \cdot G_j$$

3A. The input values for the route component are found the same way as in problem one. (Normalized M_{ij}^{-1}). These values are shown in Table 12.

3B. The input values for the route component are found the same way as in problem one. (Not Normalized M_{ij}^{-1}). These values are shown in Table 13.

3C. The input values for the route component is established from Figure 33, which is a plot of frequency of trips versus travel time. A resistance scale of R_{ij} was related to trip frequency. These values are shown in Table 14.

Terminal Representation of the Components

1. Residential Zones and Employment zones are used as through drivers

$$\xleftarrow[26]{\qquad\qquad} \qquad \text{Specified y's}$$

2. Route Components are given in equation form where:

$$R = \frac{x}{y} \qquad\qquad \xleftarrow[2]{\qquad\qquad}$$

Systems Graph and Tree

With these components, the system graph can now be drawn in accordance with the following rules:

1. Components are joined in the system graph according to the manner in which the components are combined in the physical system.
2. The direction of flow is indicated by the direction of the line representing the components.
3. A "tree" is selected. This tree is a subgraph of the system graph containing all vertices but no circuits. The tree is used in formulating the systems equations.

4. Specified desire, "x" values, are placed in the branches of the trees (b-1) (there are no specified x values here).
5. Specified flow, "y" are placed in the chord set (c-2).

Figure 35 shows the development of a system graph and Figure 37 shows the system graph and a formulation tree for alternate 3C. The complete solution of this alternate can be found in a paper by Grecco and Breuning (58). Only the results are presented here in Table 16. A more simplified system graph was used for alternates 3A and 3B (Figure 36).

Circuit Equations

In order to achieve a numerical solution of the system by this method, a set of equations must be written in accordance with the circuit postulate of linear graph theory.

1. The general equation for the circuit postulate is:

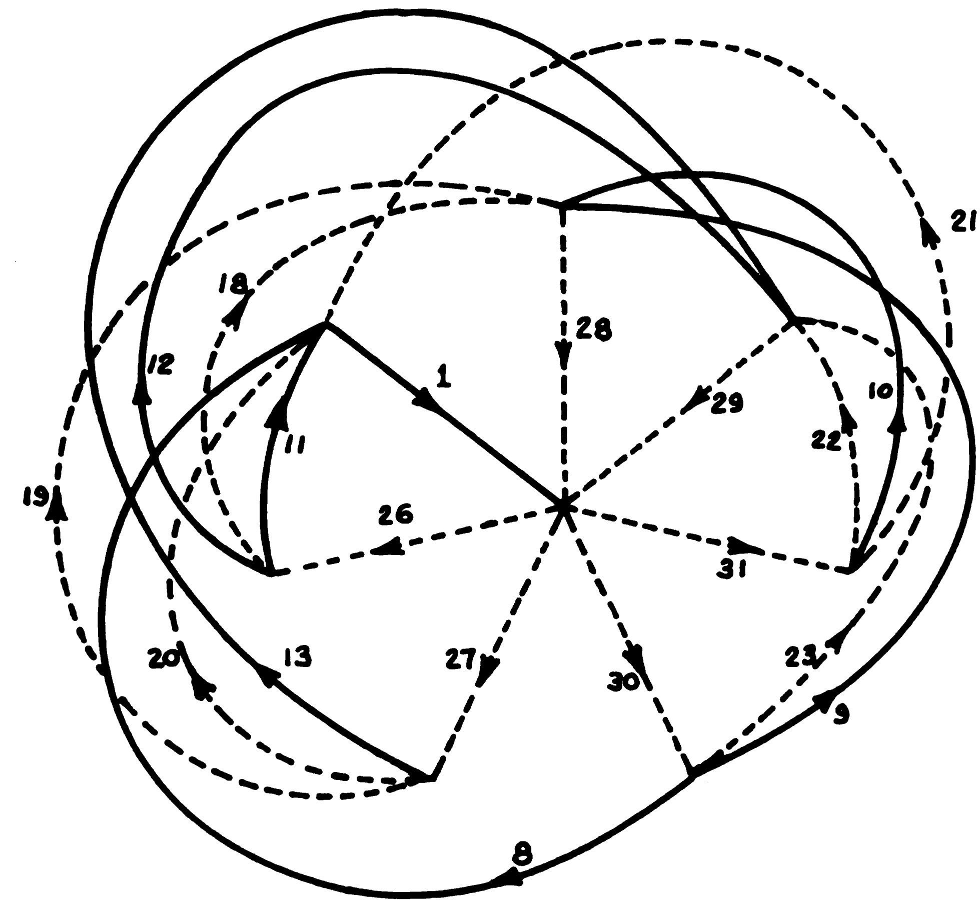
$$\sum_{i=1}^{e} b_i x_i = 0$$

Where:

b_i is 0 if the i^{th} element is not included in the k^{th} circuit

b_i is 1 if the orientation of the i^{th} element is the same as the orientation for the k^{th} circuit

b_i is -1 if the orientation of the i^{th} element is opposite to the orientation of the k^{th} circuit.



Branches (b-2) Elements (1, 8, 9, 10, 11, 12 & 13)

Chords (c-1) Elements (18 to 23)

Chords (c-2) Elements (26 to 31)

Figure 36. System graph and tree

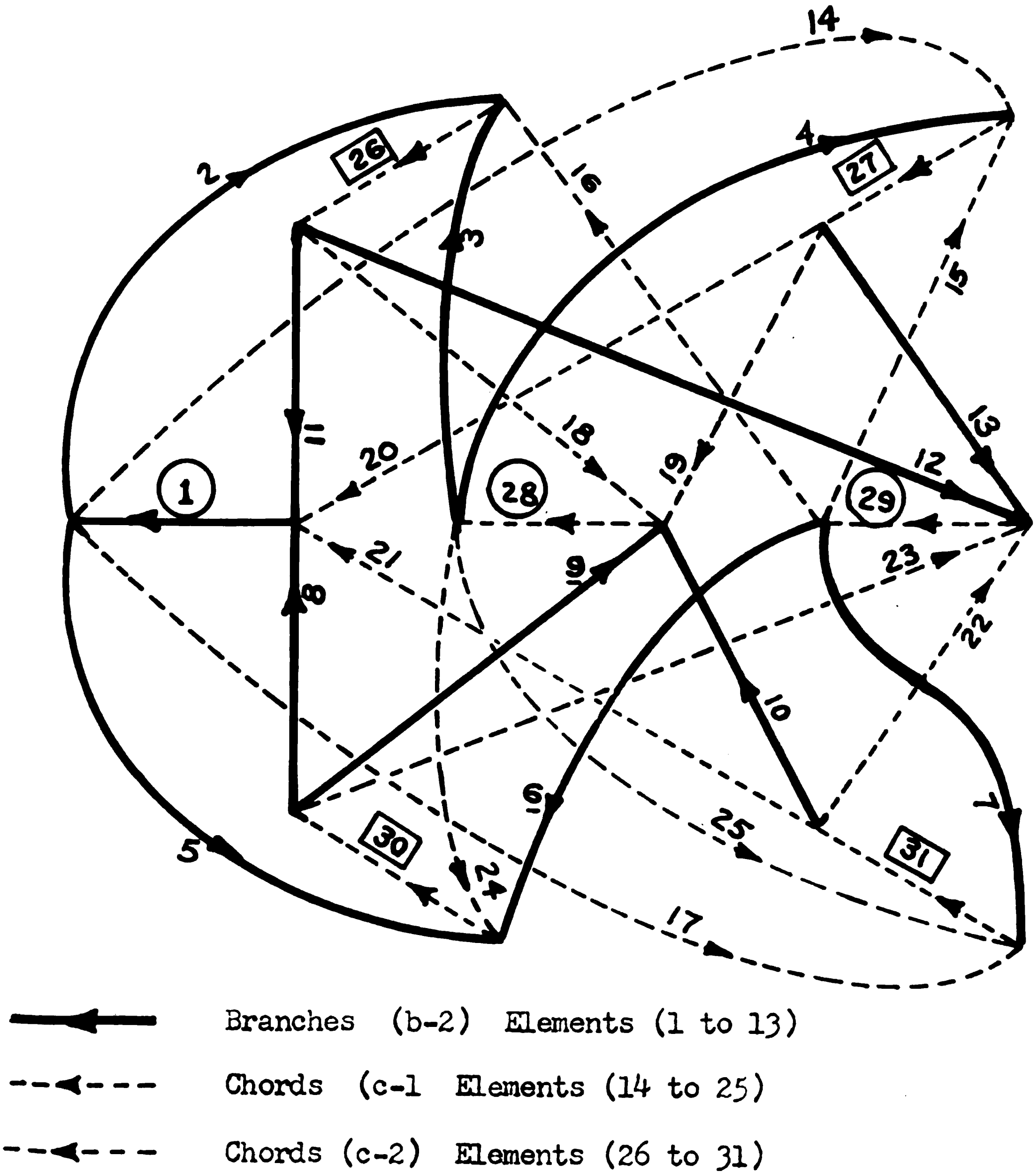


Figure 37. System graph and tree

2. Each circuit will have one and only one chord, and will be written in such a sequence that a unit matrix results for the entries c-1 and c-2.

3. Write equations using chords (c-1) first and chords (c-2) last.

4. Arrange the x's in the column matrix in the following order:

$$X_{b-1},\ X_{b-2},\ X_{c-1} \text{ and } X_{c-2} .$$

5. The resulting matrix product is shown in Table 15.

Substitution in General Mesh Form

The general mesh form or circuit equation can be written:

$$\left[B_{11} \cdot X_{b-1}\right] + \left[B_{12} \cdot R_{b-2} \cdot B_{12}^{T} + U \cdot R_{c-1}\right] \left[Y_{c-1}\right] +$$

$$\left[B_{12} \cdot R_{b-2} \cdot B_{22}^{T}\right]\left[Y_{c-2}\right] + \left[0 \cdot X_{c-2}\right] = 0$$

Note that the first term is non-existent in this case, because no X_{b-1} values exist in this problem and, since the last term is multiplied by zero, it vanishes. The resulting equation for this problem is:

$$\left[B_{12} \cdot R_{b-2} \cdot B_{12}^{T} + R_{c-1}\right] \left[Y_{c-1}\right] + \left[B_{12} \cdot R_{b-2} \cdot B_{22}^{T}\right] \left[Y_{c-2}\right] = 0$$

Table 15. Resulting matrix product

$$\left[\begin{array}{ccccccc|cccccc|cccccc}
0 & 1 & -1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & & & & & & \\
0 & 1 & -1 & 0 & -1 & 1 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & & & & & & \\
0 & 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & & & & 0 & & \\
0 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & & & & & & \\
0 & -1 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & & & & & & \\
0 & -1 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & & & & & & \\
\hline
1 & 0 & 0 & 0 & 1 & 0 & 0 & & & & & & & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & -1 & 1 & & & & & & & 0 & 1 & 0 & 0 & 0 & 0 \\
-1 & -1 & 1 & 0 & 0 & 0 & 0 & & & 0 & & & & 0 & 0 & 1 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & -1 & 1 & 0 & & & & & & & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & & & & & & & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 1 & -1 & 0 & 0 & 0 & 0 & & & & & & & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]
\begin{bmatrix} x_1 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{18} \\ x_{19} \\ x_{20} \\ x_{21} \\ x_{22} \\ x_{23} \\ x_{26} \\ x_{27} \\ x_{28} \\ x_{29} \\ x_{30} \\ x_{31} \end{bmatrix}$$

b-2 c-1 c-2

The result of the first term from the above general equation becomes the symmetric matrix shown below:

$$
\begin{bmatrix}
(R_8+R_9+R_{11}+R_{18}) & (R_8+R_9+R_{11} & (R_{11}) & -(R_8+R_9) & -(R_8+R_9+R_{11}) & -(R_8+R_{11}) \\
 & (R_8+R_9+R_{11}+R_{12}+R_{13}+R_{19}) & (R_{11}+R_{12}+R_{13}) & -(R_8+R_9) & -(R_8+R_9+R_{11}+R_{12}) & -(R_8+R_{11}+R_{12} \\
 & & (R_{11}+R_{12}+R_{13}+R_{20}) & 0 & -(R_{11}+R_{12}) & -(R_{11}+R_{12}) \\
 & & & (R_8+R_9+R_{10}+R_{21}) & (R_8+R_9+R_{10}) & (R_8) \\
 & & & & (R_8+R_9+R_{10}+R_{11}+R_{12}+R_{22}) & (R_8+R_{11}+R_{12}) \\
 & & & & & (R_8+R_{11}+R_{12}+R_{23})
\end{bmatrix}
\begin{bmatrix}
y_{18} \\ y_{19} \\ y_{20} \\ y_{21} \\ y_{22} \\ y_{23}
\end{bmatrix}
$$

The resulting product of the second term is a non-symmetrical matrix:

$$\begin{bmatrix} -(R_{11}) & -(R_{11}) & -(R_8+R_9) & (R_{11}) & R_8 & (R_8+R_9) \\ -(R_{11}) & -(R_{11}+R_{12}+R_{13}) & -(R_8+R_9) & (R_{11}+R_{12}) & R_8 & (R_8+R_9) \\ -(R_{11}) & -(R_{11}+R_{12}+R_{13}) & 0 & (R_{11}+R_{12}) & 0 & 0 \\ 0 & 0 & (R_8+R_9) & 0 & -(R_8) & -(R_8+R_9+R_{10}) \\ (R_{11}) & (R_{11}+R_{12}) & (R_8+R_9) & -(R_{11}+R_{12}) & -(R_8) & -(R_8+R_9+R_{10}) \\ (R_{11}) & (R_{11}+R_{12}) & (R_8) & -(R_{11}+R_{12}) & -(R_8) & -(R_8) \end{bmatrix} \begin{bmatrix} y_{26} \\ y_{27} \\ y_{28} \\ y_{29} \\ y_{30} \\ y_{31} \end{bmatrix}$$

Table 12. Component information necessary to solve the linear graph of alternate 3A

Element No.	Component	Travel Time (Minutes)	Resistance Factor	Y (Work Trips)
1	Employment (I-7)	--	- ----	?
2	Street	10	0.3055	
3	Street	10	.3055	
4	Street	14	.3890	
5	Street	10	.2665	
6	Street	14	.3394	
7	Street	10	.2527	
8	Street	10	.2665	
9	Street	17	.3941	
10	Street	20	.4255	
11	Street	10	.3055	
12	Street	14	.3890	
13	Street	10	.3055	
14	Street	17	.3055	
15	Street	10	.3055	
16	Street	14	.3890	
17	Street	14	.3218	
18	Street	10	.3055	
19	Street	14	.3890	
20	Street	17	.3055	
21	Street	14	.3218	
22	Street	10	.2527	
23	Street	14	.3394	
24	Street	17	.3941	
25	Street	20	0.4255	
26	Residential (R-1)	--	- ----	4000
27	Residential (R-2)	--	- ----	3000
28	Employment (I-4)	--	- ----	3000
29	Employment (I-5)	--	- ----	5000
30	Residential (R-3)	--	- ----	2000
31	Residential (R-6)	--	- ----	2000

Table 13. Component information necessary to solve the linear graph of alternate 3B

Element No.	Component	Travel Time (Minutes)	Resistance Factor	Y (Work Trips)
1	Employment (I-7)	--	- ----	?
2	Street	10	0.4780	
3	Street	10	.4780	
4	Street	14	.6088	
5	Street	10	.4780	
6	Street	14	.6088	
7	Street	10	.4780	
8	Street	10	.4780	
9	Street	17	.7069	
10	Street	20	.8050	
11	Street	10	.4780	
12	Street	14	.6088	
13	Street	10	.4780	
14	Street	17	.4780	
15	Street	10	.4780	
16	Street	14	.6088	
17	Street	14	.6088	
18	Street	10	.4780	
19	Street	14	.6088	
20	Street	17	.4780	
21	Street	14	.6088	
22	Street	10	.4780	
23	Street	14	.6088	
24	Street	17	.7069	
25	Street	20	0.8050	
26	Residential (R-1)	--	- ----	4000
27	Residential (R-2)	--	- ----	3000
28	Employment (I-4)	--	- ----	3000
29	Employment (I-5)	--	- ----	5000
30	Residential (R-3)	--	- ----	2000
31	Residential (R-6)	--	- ----	2000

Table 14. Component information necessary to solve the linear graph of alternate 3C

Element No.	Component	Travel Time (Minutes)	Resistance Factor	Y (Work Trips)
1	Employment (I-7)	--	- ----	?
2	Street	10	.7500	
3	Street	10	.7500	
4	Street	14	.8125	
5	Street	10	.7500	
6	Street	14	.8125	
7	Street	10	.7500	
8	Street	10	.7500	
9	Street	17	.8450	
10	Street	20	.8675	
11	Street	10	.7500	
12	Street	14	.8125	
13	Street	10	.7500	
14	Street	17	.8450	
15	Street	10	.7500	
16	Street	14	.8125	
17	Street	14	.8125	
18	Street	10	.7500	
19	Street	14	.8125	
20	Street	17	.8450	
21	Street	14	.8125	
22	Street	10	.7500	
23	Street	14	.8125	
24	Street	17	.8450	
25	Street	20	.8675	
26	Residential (R-1)	--	----	4000
27	Residential (R-2)	--	----	3000
28	Employment (I-4)	--	----	3000
29	Employment (I-5)	--	----	5000
30	Residential (R-3)	--	----	2000
31	Residential (R-6)	--	----	2000

Final Solution on Alternate 3A

A substitution of the proper R values from Table 12 into the above gives the following independent equations:

$$\begin{bmatrix} 1.2716 & 0.9661 & 0.3055 & -0.6606 & -0.9661 & -0.5720 \\ 0.9661 & 2.0496 & 1.0000 & -0.6606 & -1.3551 & -0.9610 \\ 0.3055 & 1.0000 & 1.3055 & 0 & -0.6945 & -0.6945 \\ -0.6606 & -0.6606 & 0 & 1.4079 & 1.0861 & 0.2665 \\ -0.9661 & -1.3551 & -0.6945 & 1.0861 & 2.0333 & 0.9610 \\ -0.5720 & -0.9610 & -0.6945 & 0.2665 & 0.9610 & 1.3004 \end{bmatrix} \begin{bmatrix} y_{18} \\ y_{19} \\ y_{20} \\ y_{21} \\ y_{22} \\ y_{23} \end{bmatrix} +$$

$$\begin{bmatrix} -0.3055 & -0.3055 & 0.6606 & 0.3055 & 0.2665 & 0.6606 \\ -0.3055 & -1.0000 & 0.6606 & 0.6945 & 0.2665 & 0.6606 \\ -0.3055 & -1.0000 & 0 & 0.6945 & 0 & 0 \\ 0 & 0 & 0.6606 & 0 & -0.2665 & 1.0861 \\ 0.3055 & 0.6945 & 0.6606 & -0.6945 & -0.2665 & 1.0861 \\ 0.3055 & 0.6945 & 0.2665 & -0.6945 & -0.2665 & -0.2665 \end{bmatrix} \begin{bmatrix} 4000 \\ 3000 \\ 3000 \\ 5000 \\ 2000 \\ 2000 \end{bmatrix} = 0$$

The results of solving the above six equations on the computer are shown in Table 16.

Final Solution on Alternate 3B

A substitution of the proper R values from Table 13 into the the preceeding R matrix gives the following independent equations:

$$
\begin{bmatrix}
+2.1409 & +1.6629 & +0.4780 & -1.1849 & -1.6629 & -0.9560 \\
+1.6629 & +3.3585 & +1.5648 & -1.1849 & -2.2717 & -1.5648 \\
+0.4780 & +1.5648 & +2.0423 & 0.0000 & -1.0868 & -1.0868 \\
-1.1849 & -1.1849 & 0.0000 & +2.5987 & +1.9899 & +0.4780 \\
-1.6629 & -2.2717 & -1.0868 & +1.9899 & +3.5547 & +1.5648 \\
-0.9560 & -1.5648 & -1.0868 & +0.4780 & +1.5648 & +2.1736
\end{bmatrix}
\begin{bmatrix} y_{18} \\ y_{19} \\ y_{20} \\ y_{21} \\ y_{22} \\ y_{23} \end{bmatrix}
+
$$

$$
\begin{bmatrix}
-0.4780 & -0.4780 & -1.1849 & +0.4780 & +0.4780 & +1.1849 \\
-0.4780 & -1.5648 & -1.1849 & +1.0868 & +0.4780 & +1.1849 \\
-0.4780 & -1.5648 & 0.0000 & +1.0868 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & +1.1849 & 0.0000 & -0.4780 & -1.9899 \\
+0.4780 & +1.0868 & +1.1849 & -1.0868 & -0.4780 & -1.9899 \\
+0.4780 & +1.0868 & +0.4780 & -1.0868 & -0.4780 & -0.4780
\end{bmatrix}
\begin{bmatrix} 4000 \\ 3000 \\ 3000 \\ 5000 \\ 2000 \\ 2000 \end{bmatrix}
= 0
$$

The results of solving the above six equations on the computer are shown in Table 16.

Discussion of the results

This problem was used primarily to establish a basis for computing the resistance value for the route component. On the basis of the results, the resistance value R_{ij} will be determined from the M_{ij}^{-1} curve (Figure 34) and the values will not be normalized. Normalizing the R_{ij} values has the effect of analyzing one section of system at a time. The results of the solution by the three approaches studied are shown in Table 16. For most interchanges, the not-normalized M_{ij}^{-1} results are somewhere in between the other and all results are quite close.

A comparison of the not-normalized M_{ij}^{-1} results is made with solutions of the same problem by the gravity model and the electrostatic model. When the systems approach results are compared with either model, the agreement is as close as when the two models are compared. The comparison between the two models and systems engineering is shown in Table 17.

Table 16. Work trips from each residential zone to each employment zone by the three alternate methods of establishing route resistance

Work Trips	R_{ij} as a function of: M_{ij}^{-1} Normalized	M_{ij}^{-1} Not Normalized	Trip Frequency
From Zone 26			
to Zone 1	1221.78	1203.24	1199.10
28	1360.18	1353.85	1215.40
29	1418.04	1442.91	1585.50
From Zone 27			
to Zone 1	828.43	806.85	778.87
28	759.30	751.76	825.08
29	1412.27	1441.39	1396.05
From Zone 30			
to Zone 1	555.43	568.77	536.13
28	482.90	486.45	490.33
29	961.67	944.78	973.53
From Zone 31			
to Zone 1	394.36	421.14	485.90
28	397.62	407.94	469.19
29	1208.02	1170.92	1044.91

Table 17. Comparison of systems engineering approach using not normalized M_{ij}^{-1} and the gravity and electrostatic models for work trips from each residential zone to each employment zone

Work Trips	Gravity Model	Electrostatic Model	System Engineering
From Zone 26			
to Zone 1	1161.5	1217.64	1203.24
28	1231.9	1359.77	1353.85
29	1620.5	1423.53	1442.91
From Zone 27			
to Zone 1	695.0	583.67	806.85
28	812.1	791.63	751.76
29	1494.1	1624.22	1441.39
From Zone 30			
to Zone 1	623.5	707.76	568.77
28	507.2	464.87	486.45
29	869.5	827.44	944.78
From Zone 31			
to Zone 1	519.9	490.93	421.14
28	461.4	383.72	407.94
29	1014.7	1124.81	1170.92

ILLUSTRATIVE PROBLEM THREE

Problem Statement

Given Information

The differences between problems three and two should be noted. Problem three will use the same y_j for the employment and only the $R_{ij} = M_{ij}^{-1}$, not normalized for the route components. The main difference is that problem three uses specified x values in terms of desire instead of specified y's for the residential components.

For the hypothetical city "Red Cedar", the following information is established from a current O and D survey and a special sampling survey.

1. The present interzonal work trips - t_{ij} .
2. The present travel times between zones - d_{ij} .
3. Present and future evaluation of employment zones to establish relative attraction values on the basis of number of jobs and/or other parameters.
4. Probability interchange established on the basis of the relative attraction of employment zones, assuming equal travel times - P_{ij}
5. The actual trips per unit of probability interchange.

$$M_{ij} = Y_{ij} / P_{ij}$$

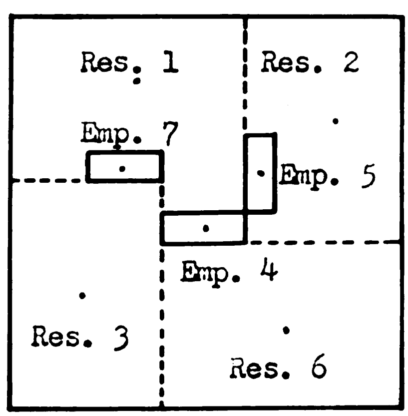
6. Curve plot or equation for $1/M_{ij}$ versus d_{ij} (Figure 34)
7. Present estimates of income, car-ownership, distance to CBD and population density.
8. Forecast of these parameters for each zone i for the future year.

9. Forecast of area wide limits or ceilings on these parameters for the future year.
10. Distribution of trips by purpose where K_W is the work trip factor.
11. Existing land use data
12. Future land use forecasts

To Find

The problem is: forecast future interzonal work trip movements when specified x's or pressure drivers are given for the residential zones, specified y's or through drivers are given for the employment zones and resistance values are given for the route components.

Schematic of the Physical System



Input Values for System Solution and How Established

The following input values are necessary for solution of this sample problem by linear graph theory:

1. The value of desire or pressure x_i for each residential zone i is required. The values will be established in the following manner.

a. From existing O and D data for income index α, car-ownership β, distance to the CBD r, and population density δ, an equation will be established for Y, resident trips per dwelling unit.

For this example:

$$Y = -0.1958 + 0.0008\,\alpha + 4.6480\,\beta + 1.7288 \text{ Log } r - 0.5464 \text{ Log } \delta$$

This equation will then be maximized with limiting values of all parameters as forecasted in the whole area for the future year. The limiting values used to maximize the preceeding equation are:

Income Index - 9

Car-ownership - 1.5 (cars per dwelling unit)

Distance to CBD - 100 (tenths of miles)

Population Density - 2 5 (dwelling units, in tenths, per net acre)

This maximum value of Y will be defined as the theoretical pressure or desire, X_T

$$X_T = Y \qquad (2)$$

The pressure or desire for work trips per dwelling unit will be found by using the previously defined factor K_W.

$$X_W = X_T K_W \qquad (3)$$

b. Using maximizing inputs for equation (1) on all parameters except income α, will develop a relationship between discrete values of income and $Y_{\alpha:\beta,r,\delta}$. A curve plot or equation will then be established for r_α.

The decrease in X from the maximum value will be related to the coefficient r_α.

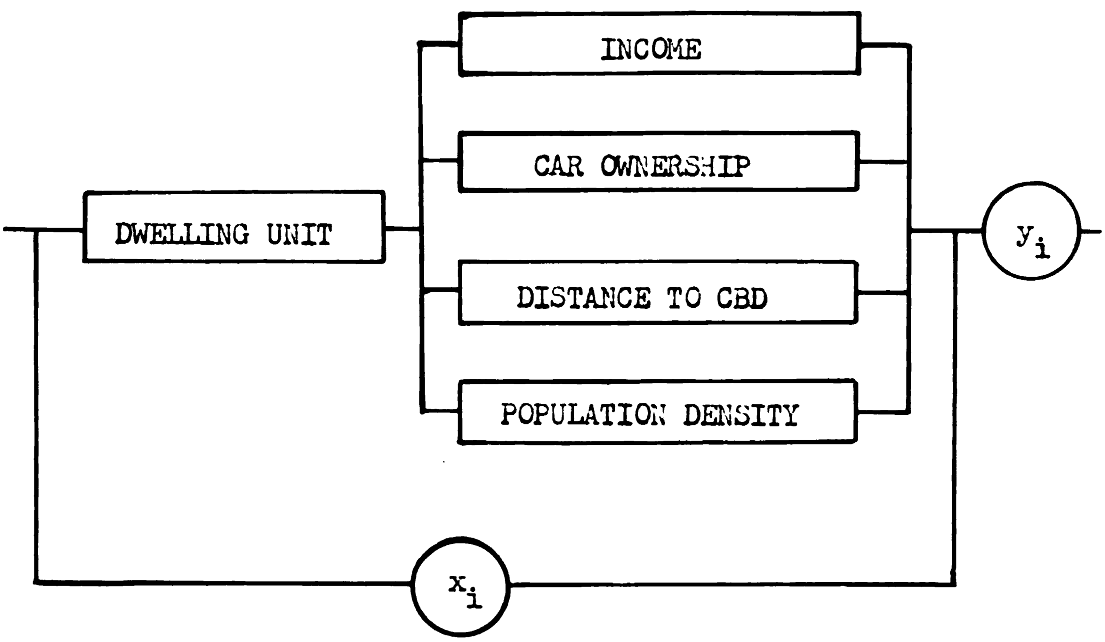
$$\left[X - Y_{\alpha:\beta,\gamma,\delta}\right] = \left[r_{\alpha} \cdot X\right] \tag{4}$$

$$r_{\alpha} = \frac{\left[X - Y_{\alpha:\beta,\gamma,\delta}\right]}{X} \tag{5}$$

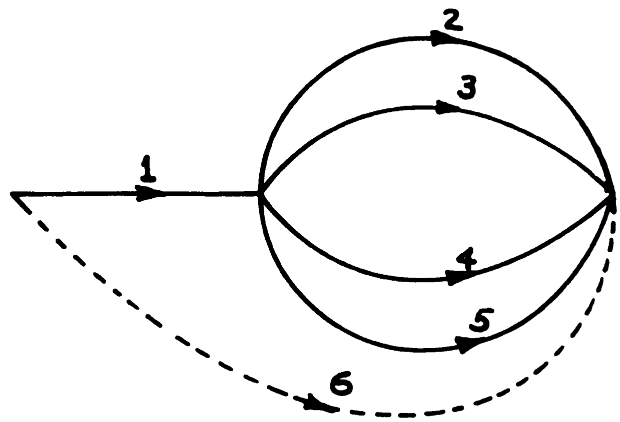
The value of R_{α} will be found by normalizing r_{α}. R_{α}, shown in Figure 27, should serve to isolate the effect of income on pressure and trips made.

c. Similar techniques will be used to establish curves or equations for R_{β}, R_{γ}, and R_{δ}. (See Figures 28, 29 and 30)

d. A subsystem of the zone parameters will be solved in order to find each value X_i. Schematically this can be shown:



As a system graph



Where:

$$X_1 = N(X_w)$$

R_α, R_β, R_γ and R_δ are found from previous curves or equations. The subsystem can then be solved for X_6, in terms of the values x_1, R_α, R_β, R_γ and R_δ. The element number six can be used in lieu of the subgraph.

$$x_i = x_6 = x_1 + \frac{1}{\sum R^{-1}} y_6$$

The summary of solution of input x values and the $(\sum G_i)^{-1}$ are shown in Table 19.

2. Y_j values for each employment zone will be determined by methods previously defined in sample number two.
3. The input values for the route components will be R_{ij}^{-1} or G_{ij} values as shown in Table 18.

<u>Terminal Representation of the Component</u>

1. Residential Zones i will have a computed x value

$\xleftarrow{\quad 26 \quad}$ Specified x's

2. Employment zones j will have specified y's or through drivers.

Specified y's

3. Route Components are given in equation form where

$$R = \frac{x}{y}$$

Systems Graph and Tree

The system graph and tree are shown in Figure 38. The specified flows or y's at the employment zones are placed in the chord set, while the residential zones are placed in the branches. The subgraph shown just previous to this section has been replaced by its equivalent element in the system graph.

Formulation in the Cut-set Equations

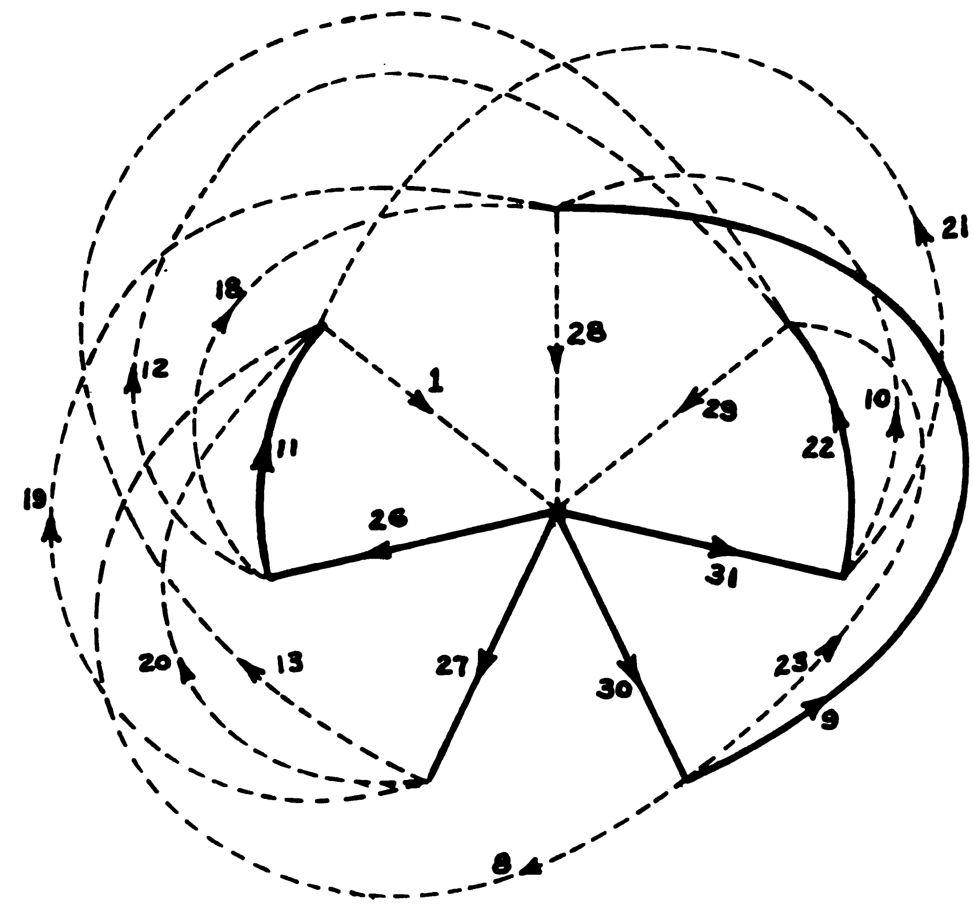
Symbolically the cut-set equation can be shown without the Y_{b-1} term since there are no specified $\dot{x}$ variables.

$$\begin{bmatrix} u & A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} Y_{b-2} \\ Y_{c-1} \\ Y_{c-2} \end{bmatrix} = 0$$

By multiplying the matrix through,

$$\begin{bmatrix} u & A_{21} \end{bmatrix} \begin{bmatrix} Y_{b-2} \\ Y_{c-1} \end{bmatrix} + A_{22} \cdot Y_{c-2} = 0$$

The Y values can be established as an explicit function of the X values and the x, term is the specified x value from the previously noted subgraph.



Branches (b-2) (26, 27, 30, 31, 9, 11 & 22)

Chords (c-1) (8, 10, 12, 13, 18, 19, 20, 21 & 23)

Chords (c-2) (1, 28 & 29)

Figure 38. System graph and tree

$$\begin{bmatrix} Y_{b-2} \\ Y_{c-1} \end{bmatrix} = \begin{bmatrix} G_{b-2} \\ G_{c-1} \end{bmatrix} \begin{bmatrix} X_{b-2} \\ X_{c-1} \end{bmatrix} - \begin{bmatrix} G_{b-2} \\ 0 \end{bmatrix} \begin{bmatrix} X_1 \\ 0 \end{bmatrix}$$

The above value can be substituted into the preceeding equation.

$$\begin{bmatrix} U & A_{21} \end{bmatrix} \begin{bmatrix} G_{b-2} \\ G_{c-1} \end{bmatrix} \begin{bmatrix} X_{b-2} \\ X_{c-1} \end{bmatrix} - \begin{bmatrix} U & A_{21} \end{bmatrix} \begin{bmatrix} G_{b-2} \\ 0 \end{bmatrix} \begin{bmatrix} x_1 \\ 0 \end{bmatrix} + A_{22}\, Y_{c-2} = 0$$

The X_{c-1} term can then be expressed in terms of the X_{b-2}.

$$X_{c-1} = A_{21}^{\,T}\, X_{b-2}$$

The final equation form becomes:

$$\left[G_{b-2} + A_{21} \cdot G_{c-1} \cdot A_{21}^{\,T}\right]\left[X_{b-2}\right] - \left[A_{21} \cdot G_{b-2} \cdot x_1\right] + A_{22}\, Y_{c-2} = 0$$

Final Solution

The details of the final solution have been omitted from this sample, as the procedure is the same as that illustrated in problem two.

Discussion of the results

The work trip interchanges for this problem are shown in Table 20. The results cannot be compared with those from other problems. The value of desire, as a measurement of the pressure, x, was found by maximizing the multiple regression equation for Y, given in the section on input values. The specified x value for work trips for all residential zones was found to be 0.734 per dwelling unit. The final x values for each residential zone varied according to the number of dwelling units and the resistance factors. The resistance factors reflect the reduction from the theoretical pressure due to the limitations imposed by the parameters of income, car-ownership, and etc.

The trips, from each residential zone, computed in this problem is compared with the trips generated by each zone, using the multiple regression equation for Y with specific parameter values. The comparison, found in Table 20, shows that the total trips by the Y equation is only 7,803 while the stated input trips to the employment zones total 8,000. Other techniques would probably increase the flow from each zone proportionately, without due regard to the total system's effect. The linear graph solution provides not only for a balanced flow of 8,000, but for a reduction from the estimate of Y in the residential zone R-6 (element 31), due to its larger travel resistance.

Table 18. Component information necessary to solve the linear graph

Element No.	Component	Travel Time (minutes)	R_{ij}^{-1} or G_{ij}	Y (Work Trips)
1	Employment (I-7)	--	- ---	2000
8	Street	10	2.092	
9	Street	17	1.414	
10	Street	20	1.242	
11	Street	10	2.092	
12	Street	14	1.642	
13	Street	10	2.092	
18	Street	10	2.092	
19	Street	14	1.642	
20	Street	17	2.092	
21	Street	14	1.642	
22	Street	10	2.092	
23	Street	14	1.642	
26	Residential (R-1)	--	- ---	----
27	Residential (R-2)	--	- ---	----
28	Employment (I-4)	--	- ---	2000
29	Employment (I-5)	--	- ---	4000
30	Residential (R-3)	--	- ---	----
31	Residential (R-6)	--	- ---	----

Table 19. Summary of solution of the input x values and the $(\sum G_i)^{-1}$ for the subgraph of the residential zones

	Res. 1 (26)	Res. 2 (27)	Res. 3 (30)	Res. 4 (31)
No. of Dwelling Units	1050	1010	818	715
Income Index	8	7	5	6
R_{α}	0.118	0.250	0.500	0.368
Car-ownership	1.3	1.1	0.8	.9
R_{β}	0.133	0.266	0.466	0.400
Distance to CBD	8.0	3.0	2.0	4.0
R_{γ}	0.075	0.401	0.538	0.306
Population Density	10	20	30	20
R_{δ}	0.462	0.694	0.828	0.694
Factor K_x	0.22	0.22	0.22	0.22
Factor K_w	0.348	0.348	0.348	0.348
x_1 (total)	-773	-742	-600	-525
$(\sum G_i)^{-1}$ or R_e	0.0317	0.0855	0.1387	0.1007

Table 20. The results of illustrative problem three

	Work Trips			
	Systems Engineering		Multiple Regression	
		Total		Total
From Zone 26		3003		2940
to Zone 1	873		--	
28	965		--	
29	1165		--	
From Zone 27		2333		2180
to Zone 1	590		--	
28	536		--	
29	1207		--	
From Zone 30		1340		1310
to Zone 1	322		--	
28	281		--	
29	737		--	
From Zone 31		1324		1373
to Zone 1	215		--	
28	218		--	
29	891		--	
	Totals	8000		7803

ILLUSTRATIVE PROBLEM FOUR

Problem Statement

Given Information

The basic difference in problem four, from those which precede, is that the x values for the residential zones are specified by a pressure in terms of money. Two alternatives for establishing the R_{ij} for the routes are used. One is the same as used in problem three and the other is an R_{ij} based on the total cost of traveling the route.

For the hypothetical city "Grand River", the following information is established from a current O and D survey and a special sampling survey.

1. The present interzonal trips - t_{ij}.
2. The present travel times between zones - d_{ij}
3. Present and future evaluation of employment zones to establish relative attraction values on the basis of number of jobs and/or other parameters.
4. Probability interchange established on the basis of the relative attraction of employment zones assuming equal travel times - P_{ij}.
5. The actual trips per unit of probability interchange.

$$M_{ij} = Y_{ij} / P_{ij}$$

6. Curve plot or equation for $1 / M_{ij}$ versus d_{ij} (Figure 34)
7. Present and future estimates of income, consumption and their relationship. (Figure 31).

8. Relationship for transportation consumption, present and future by yearly change.

9. Families per zone present and future.

10. Percentage of trips for each purpose and their mean length. Tables 4 and 8.

11. Existing land use data.

12. Future land use forecasts.

13. Additional requirements for alternate solution B on route component.

    a. length of each route in miles

    b. average speed over each route component

    c. average vehicle costs per mile. Table 21.

    d. average time costs per hour.

The problem is to forecast future interzonal work trip movements when: x values for the residential zones are specified on the basis of money available for work trips, y values are specified for the employment zones as work trips, and the route components have specified resistance value R established first on the basis of travel time and secondly by the money spent on traveling the route.

Schematic of the Physical System

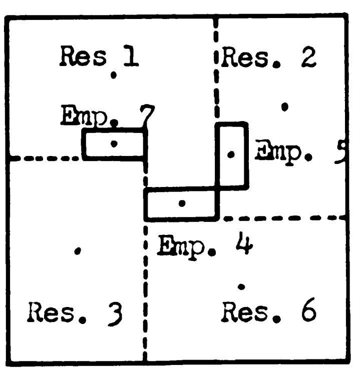


Table 21. Estimated cost of operating a motor vehicle

Item	Cents Per Mile	Per Cent of Total
Costs Excluding Taxes		
Depreciation	2.54	26.0
Repairs, Maintenance	1.72	17.6
Replacement Tires and Tubes	.18	1.8
Accessories	.14	1.4
Gasoline (Except Tax)	1.45	14.9
Oil	.19	2.0
Insurance	1.29	13.2
Garaging, Parking, Tolls, etc.	1.08	11.1
Sub-Total	8.59	88.0
Taxes and Fees		
Gasoline	.70	7.2
Registration	.10	1.0
Titling and Property	.10	1.0
Oil	.01	0.1
Auto, Tires, Parts, etc	.26	2.7
Sub-Total	1.17	12.0
TOTAL OPERATING COST	9.76	100.0

Source: Reference 55

Input Values and How Established

The necessary input values for the solution of this sample problem by linear graph theory are:

1. The amount of money in dollars to be spent for the total work trips made from the residential zone i. This will be referred to as C_i and x_i will be found by multiplying C_i by K, a constant determined to adjust for the differences in dimensions and the number, 250, represents the working days per year

$$x_i = K C_i / 250$$

The value C_i can be estimated in the following manner:

a. Determine from a previously established plot of income versus consumption, an estimate of the future year consumption based on an estimate of future year mean income I for zone i.

b. Establish from previous data a relationship between expenditures on transportation Z_i^t and total consumption Z. Based on observations of trends, the yearly change in this ratio can be established.

$$E_f = E_b + n \Delta E$$

where:

E_f is the ratio of future amount spent on transportation per total consumption for future year

E_b is the same for the base year

ΔE is the yearly change

n is the number of years from base year to future year.

The amount of expenditure spent on transportation for the future year can be found.

$$z_i^t = E_f \cdot Z$$

c. The z_i^t represents the total amount spent for transportation per family in zone i. The amount spent for transportation per zone for the future year (Z^t) will z_i^t times (N_f) the number of families in zone i,

$$Z^t = N_f \cdot z_i^t$$

d. Since the primary concern is home based work trips, it is necessary to determine the amount spent for these, where K_h is the percentage of all trips that are home-based.

$$Z_i^t = K_h \cdot Z^t$$

e. The amount of money spent for trips to work can be found by first establishing the ratio of the total mileage for work trips to the total mileage traveled for all trip purposes. The ratio K_w is equal to:

$$K_w = \frac{k_w \cdot m_w}{M_{total}\ \text{(All purposes)}}$$

where:

$$M_{total} = (k_w \cdot m_w) + (k_s \cdot m_s) + (k_{sc} \cdot m_{sc}) + (k_{sr} \cdot m_{sr}) + (k_{em}\ m_{em}) + (k_b \cdot m_b)$$

where:

k_w is the percentage of trips to work

m_w is the average trip length for work

k_s is the percentage of trips to shopping

m_s is the average trip length to shopping

k_{sc} is the percentage of trips to school

m_{sc} is the average trip length to school

k_{sr} is the percentage of trips to social-recreation

m_{sr} is the average trip length to social-recreation

k_{em} is the percentage of trips to eat meals

m_{em} is the average trip length to eat meals

k_b is the percentage of trips to business

m_b is the average trip length to business

then:

$$Z_i^{wt} = K_w \cdot Z_i^t$$

allow:

$$C_i = Z_i^{wt}$$

A summary of the solution leading to values of x by the above procedures is shown in Table 22.

2. The input y_i values for each employment zone will be determined by methods previously defined in sample number two.

3A. The input values for the route component will be R_{ij} as defined in the previous sample number two, alternate B, as shown in Table 13.

Table 22. Summary of solution to x input values for residential zones

	Residential Zones			
	Res. 1 (26)	Res. 2 (27)	Res. 3 (30)	Res. 6 (31)
Present Year				
Average Disposable Income per dwelling unit ($)	5300	6300	4300	4300
Total Consumption - Z ($) (Figure 31)	5300	6150	4400	4400
Transportation Consumption - Z^t ($) (Figure 32)	455	530	375	375
Ratio $Z^t / Z = E_b$	.0859	.0862	.0852	.0852
Future Year Estimates				
Average Disposable Income per dwelling Unit ($)	5700	6500	4800	4600
Total Consumption - Z ($) (Figure 31)	5900	6600	5100	4900
Ratio - Z^t / Z $(E_f = E_b + n\,\Delta E)$	0.1002	0.1005	.0995	.0995
Transportation Consumption - Z^t ($) (Figure 32)	591	663	507	488
Per cent for Work Trip - K_w	0.505	0.505	0.505	0.505
Work trip Consumption $-Z^{wt}$ ($)	283	335	256	246
Number of Dwelling Units	1400	900	780	820
Total Work Trip Consumption ($,000)	396	302	200	202
$x_i = K(C_i / 250)$ (assume K = .25)	396	302	200	202

3B. A x_k input value might also be computed for an alternate solution on the route component. The value C_k might logically be established as the money spent per vehicle on the route component k. The total value of x for the k^{th} route component would be equal to

$$x_k = K \cdot C_k \cdot y_k$$

where:

K is a constant for all route components to adjust for the differences in dimensions

y_k is the number of vehicles on the k^{th} route for work trips from zone i to j.

The value C_k would be equal to the sum of the products of vehicle cost per mile times the number of miles and travel time costs per minute times the number of minutes for the k^{th} route

$$C_k = c_v \cdot m_k + c_t \cdot d_{ij}$$

where:

c_v is the total operating cost per vehicle mile

m_k is the length of the route in miles

c_t is the time cost per vehicle per minute assuming an occupancy of one person per vehicle.

d_{ij} is the travel time per vehicle per route excluding terminal time.

A summary of the solution leading to values of R_{ij} for the route components on the basis of vehicle and time costs is shown as Table 23.

Table 23. Summary of the solution of R_{ij} by the sum of the vehicle and time costs

Route No. (Element)	Length (m_k)	Vehicle Cost ¢ ($c_v \cdot m_k$)	Travel Time (minutes) d_{ij}	Time Cost ($d_{ij} \cdot c_t$)	Total Cost ¢ C_k	Resistance $R_{ij} = K \cdot C_k$
8	3.3	32.5	10	22.5	55.0	0.3850
9	6.8	66.4	17	38.2	104.6	0.7322
10	8.0	78.1	20	45.0	123.1	0.8617
11	3.3	32.5	10	22.5	55.0	0.3850
12	4.7	45.8	14	31.5	77.3	0.5411
13	3.3	32.5	10	22.5	55.0	0.3850
18	3.3	32.5	10	22.5	55.0	0.3850
19	4.7	45.8	14	31.5	77.3	0.5411
20	3.3	32.5	10	22.5	55.0	0.3850
21	4.7	45.8	14	31.5	77.3	0.5411
22	3.3	32.5	10	22.5	55.0	0.3850
23	5.6	54.6	14	31.5	86.1	0.6027

Note: Vehicle Cost per mile assumed at 9.76¢ (Reference 55)

Time Cost per hour assumed at $1.35 (Reference 55)

K is assumed to be 7×10^{-3}

Since the value to be solved y_k for the component is also the same value provided by the system solution, the equations can be further reduced

$$R_{ij} = R_k = \frac{x_k}{y_k}$$

$$\frac{x_k}{y_k} = K \cdot C_k$$

$$R_{ij} = K \cdot C_k$$

This last equation makes it evident that for route components, the parameters, which established x, are confounded in y and that specified x's are not possible. Specified R values of the route components can be determined on the basis of money.

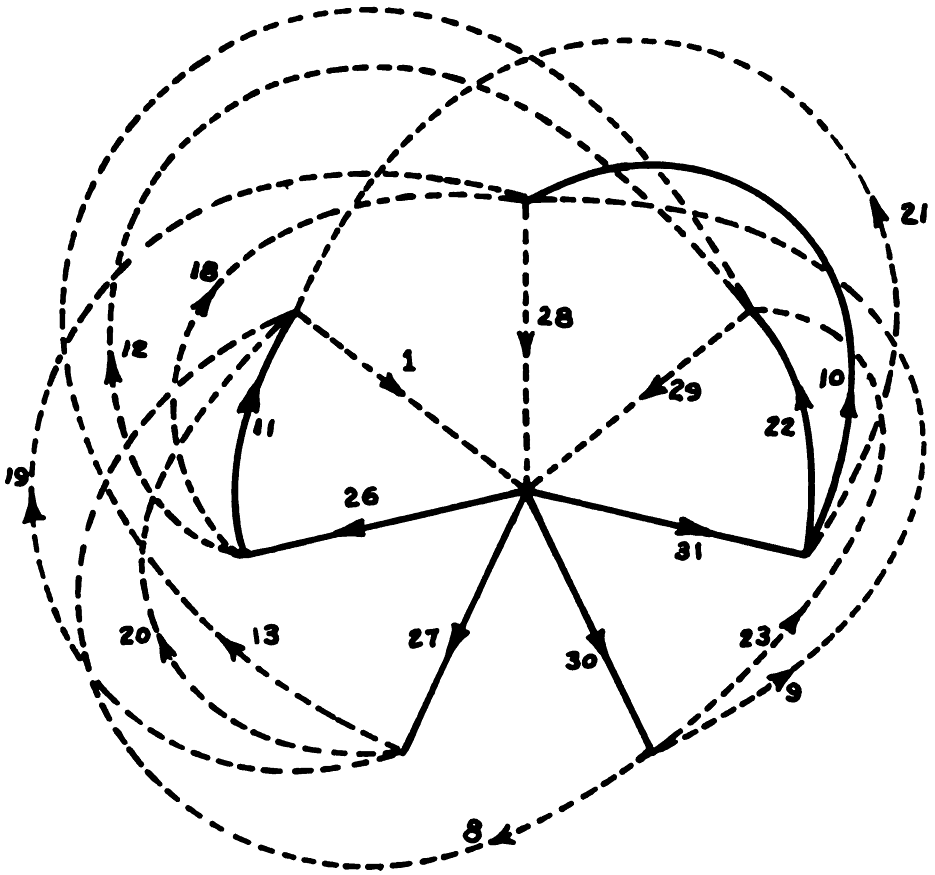
System Graph and Tree

Figure 39 shows the system graph and tree used for both alternate solutions. The specified x values on the residential components were placed in the branches (b-1) and the specified y's from the employment zones were placed in the chord set (c-2).

Since the solutions follow the procedures for the mesh form equations, illustrated in problem two, they were omitted to reduce repetition: The results are presented in the next section.

Discussion of the results

In both alternates of problem four, the specified work trips for the employment zones I-7, I-4 and I-5, are 3000, 3000 and 5000, respectively. Even though these are the same destination values as used



Branches (b-1) Elements (26, 27, 30 & 31)

Branches (b-2) Elements (10, 10 & 22)

Chords (c-1) Elements (8, 9, 12, 13, 18, 19, 20, 21 & 23)

Chords (c-2) Elements (1, 28 & 29)

Figure 39. System graph and tree

in problem two, there is no basis for the comparison of the results with problem two. The specified x values were determined as a function of the money spent for work trips. An adjustment in the number of dwelling units and the constant k could have established flows, from the residential zones, similar to those of problem two. Residential zone R-2 (element 27) has similar origin flows, 3093 vs. 3000. The flows from zone R-2 to each employment zone are in closer agreement than the other interchanges. Even if the two zones had identical origin flows, the trip distributions would vary due to the system's effect. The system solution weighs the interrelation of all the component parts in order to establish a solution of interchanges.

The results of problems 4A and B are shown in Table 24. There is also little basis for a comparison between these alternates since the variation was established on purpose. The speed for the several routes of equal travel time was varied in order to give different travel distances. Since the route resistance values of alternate B are a function of travel time and distance, the solution reflects the differences in speed. The travel resistance for the residential zone R-3 (element 30) to the employment zones I-4 and I-5 (elements 28 and 29) was specifically increased by assuming higher travel speeds. The effects of these increased resistance values from residential zone R-3 (element 30) are noted below:

1. The flow of trips to the employment zones I-4 and I-5 (elements 28 and 29) are reduced because of the increased resistance factors.
2. The flow of trips to employment zone I-7 (element 1) are also reduced because the total of the resistance value from zone R-3 has increased.

3. The trips originating in zone R-4 are also logically reduced and the reduction is absorbed by the other residential zones.

Table 24. The results of illustrative problem four, alternates A and B

	Work Trips			
	Alternate A		Alternate B	
		Totals		Totals
From Zone 26		3542		3764
to Zone 1	1037		1108	
28	1202		1366	
29	1303		1290	
From Zone 27		3092		3229
to Zone 1	840		863	
28	789		798	
29	1463		1568	
From Zone 30		2144		1883
to Zone 1	627		599	
28	536		* 451	
29	981		* 833	
From Zone 31		2222		2124
to Zone 1	496		430	
28	473		* 385	
29	1253		1309	
	TOTALS	11000		11000

Note: The asterisk denotes those interchanges for which the route resistances were increased.

SUMMARY

Traditionally the urban traffic forecasting model has been separated into three distinct parts; trip generation, trip distribution and trip assignment. The research has concentrated primarily on trip distribution with little work on generation and none on trip assignment.

A work trip distribution system was chosen for analysis because work trips constitute the largest percentage of all urban travel and occur generally at times of peak flow. Two hypotheses were presented; first, the work trip distribution system could meet the requirements of a system solution by linear graph theory, secondly, the results of the system solution would provide acceptable trip interchanges which would compare well with other models. The research was separated into a study of components and the system.

COMPONENTS

The most pertinent findings, obtained from researching the components, were in their selection, measurements and terminal equations.

Selection

A work trip distribution system contains workers, jobs and some facility to bring the workers to the jobs. From the analysis, it was determined that the best components would be the residential zone component, the route component and the employment zone component.

Measurements

Two measurements are necessary on each component; where one, labeled y, would sum to zero at vertices of the system graph and the other, labeled x, would sum to zero around the circuits of the system graph. Analogous to the other fields of system analysis, it can be established that the proper y measurement should be the flow of work trips. The x measurement was assumed to be a pressure-type measurement which is a causative influence on the flow and diminishes around the circuit.

The fact that the amount of travel varies from person to person is generally accepted. It might be reasoned that the variation can be tied to a set of circumstances which influence travel. In an effort to be more specific, two possible measurements were proposed which could be related to trip making. It was hypothesized that the trip interchange was a function of demand, as expressed in a need or willingness to travel. This x measurement was designated as desire. As the amount of desire increased, the number of trips would also increase. This relationship, between desire and trips, would also be influenced by the relative attraction of the trip and the friction or deterrent factors. Based on the amount of desire available for trip making, it seems logical that a number of trips would be made, subject to the constraints imposed by the attraction of the destination and friction incurred on the route of travel.

The second expression for the pressure-type x measurement was money. The amount of money available for travel could be determined for each residential zone. The flow of trips would then be large enough to use up the money or desire available.

Terminal Equation

A basic equation which utilizes the above postulates can be stated. The flow of trips equal the pressure measurement divided by the resistance encountered or $y = x / R$. This can also be related to a relative attraction term G or $Y = G x$.

For expediency in the formulation process, the basic equation was retained, even though values for x and R or G could not be directly measured. The x measurement in terms of desire was related to parameters such as income, car ownership, distance to the CBD and population density. The x measurement in terms of money was related to disposable income, transportation consumption and specifically the consumption utilized for the making of work trips.

For the employment zones, the flow or y measurement was used throughout the research. This served as a control volume that would properly adjust the magnitudes of the trip interchange.

The route components were related to the friction term R. The best single predictor of R for the route components seems to be travel time. When the pressure measurement x was related to money, it seemed advisable to express R as a function of travel costs in terms of time and vehicle costs.

SYSTEM SOLUTION

Four illustrative solutions were presented to test on a theoretical basis, the application of the postulates in a system solution. A full discussion of the results of each illustrative solution was presented in the previous chapter. In summary, it was shown that:

1. To achieve the true system solution which interrelates the characteristics of all the component parts, one cannot validly sum the solutions made on separate subsystems.
2. The most basic solution of future zonal interchanges occurred when the estimated trips from each residential zone y_i and the estimated trips to each employment zone y_j were specified. The R value on the route components was established as a function of an empirically established ratio of probable to actual trip interchanges. The results of this analysis compare reasonably well with those obtained from solutions of the gravity and electrostatic models.
3. The use of the pressure measurement in terms of desire provided reasonable interchanges and the trips origins at each residential zone closely approximated those found by a multiple regression equation. Balanced flows at the residential zones and employment zones are assured by the systems approach. In contrast, all other models used to date require balancing through an iteration procedure.
4. The use of money as a function of the x measurement for residential zones and the R measurement on the route component provided results which were balanced and reasonable. No effort was made to use zones with characteristics similar to those of the other illustrations but the variations in zonal interchanges reflect quite closely the differences in characteristics.
5. All system solutions are adaptable to changing parameter values for research purposes.

CONCLUSIONS

Early advances in the fields of science have been chiefly those of identifying the units from which a complex system is developed. The progress made toward system synthesis is contingent upon the validity of the analysis process.

The work presented here is an initial attempt to apply the techniques of systems engineering to urban traffic forecasting. The primary concern of this study was the analysis of the components. The synthesis of the system and the solutions obtained were of only secondary importance.

The justification, for further research into the application of systems engineering techniques to urban traffic forecasting, can be evaluated on the basis of the following conclusions.

The systems engineering model for urban traffic forecasting has the following advantages.

1. Models of this type have many advantages. The parameters which influence travel patterns can be better understood through testing and evaluation in a mathematical model. A procedure for keeping the model up to date can be devised which will make periodic tests and adjustments.
2. The complex interaction of persons, vehicles, facilities and jobs in the work trip distribution system cannot be simply

stated in equation form without the use of a formulation technique such as linear graph theory.

3. Through the use of systems theory, it was possible to establish a mathematical model of the relevant physical characteristics of the system components in terms of measurements.

4. A mathematical model of each system can be formulated in terms of the characteristics of the components and their mode of interconnection.

5. The systems approach provides for a balanced flow between inputs and outputs of the system. Other models reviewed generally require an iterative process to produce this balance. The iteration procedures used are aimed primarily to achieve the balance of flows. The true system's effect or the interaction of the component parts is placed second in importance to the balance of flow.

6. The system engineering model is much more flexible than iteration type models for establishing parameter values from empirical data.

7. The eventual goal of the traffic forecastor is a theoretical model which can be used in any urban area, independent of a previous O and D study. The models proposed by others have not had much success in their application to other areas. Though it has yet to be substantiated, it is proposed that a general model, which predicts the pressure for the flow of trips from any residential zone, can be established by the techniques of systems engineering.

Several areas of future research should be noted. The concepts presented must be tested with empirical data in order to reaffirm or adjust the hypotheses. The solution of the final system equations from the circuit or cut-set equations and the terminal equations is quite time consuming. A computer program is planned in the near future which will eliminate the hand work presently required.

Another criticism of the system approach is the large number of possible interchanges that accompany an increase in the number of components. This problem will be partially solved by the acquisition of a new high speed, large storage computer by Michigan State University. Preliminary investigations have indicated that the high volumes of interchanges could be handled through the use of subgraphs. Trip distribution could be made first, on a neighborhood basis and then, subgraphs would be used to distribute trips to the individual zones. This area needs further research to fully evaluate its potential use.

Future research should also be attempted in the area of trip assignment so that a single model might be used which would determine the generation, distribution and assignment of trips for urban traffic forecasting.

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