This is to certify that the

thesis entitled

THE FIELD AND CARRIER WAVES INTERACTION
IN A
SEMI-INFINITE SEMICONDUCTOR

presented by

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has been accepted towards fulfillment
of the requirements for

Master's degree in Science

Jon C. Freeman
Major professor

Date August 2, 1982

O-7639

MSU is an Affirmative Action/Equal Opportunity Institution

THE FIELD AND CARRIER WAVES INTERACTION IN A SEMI-INFINITE SEMICONDUCTOR

By

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A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

MASTER OF SCIENCE

Department of Electrical Engineering

1982

ABSTRACT

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An analytical treatment of the transverse magnetic field waves and carrier waves is presented without the quasistatic approximation. The effects of diffusion and an external magnetic field are included. To evaluate the properties of the carriers at the surface, Sumi's stiff boundary model as well as the "ripple" boundary condition are examined. The resulting dispersion relations are evaluated numerically. Possible instabilities are analyzed, and gain shapes are studied under varying conditions. It is found that gain is possible theoretically under specific conditions.

ACKNOWLEDGMENT

The author wishes to thank his major professor, Dr. J.C. Freeman, for his guidance, helpful discussions, and advice.

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1. INTRODUCTION

The objective of this paper is to investigate analytically a device called the solid state traveling wave amplifier (STWA). This device is somewhat a solid-state analogue of the vacuum TWT where electromagnetic waves will be amplified if guided by a slow wave structure. The planar guiding structure is assumed to be a meander line. Amplification of waves is due to interaction of drifting carriers in semiconductor with the "external" slow waves.

Since the device is so similar to the TWT it seems reasonable to extend the three-wave theory for the TWT to semiconductors as well. Such an idea was first proposed by Pierce and Suhl [1] in 1955. Since that proposal, wave interactions between the slow EM waves and drifting carriers in a semiconductor have been studied extensively.

The STWA falls into the very broad category of wave instabilities in solids. The literature on this subject is quite extensive. Collectively, the theories of the STWA have been either one or two-dimensional analyses. The one-dimensional investigations follow very closely the successful theory developed by Pierce for the TWT. For reference purposes, the basic dispersion relation obtained by Pierce in the analysis of the TWT is given as follows:

$$(\gamma^2 - \gamma_1^2)(j\beta_e - \gamma)^2 = -j2\beta_e\gamma^2\gamma_1 C^3 \ .$$

Numerous modifications were made to accommodate space charge effects [2, 3]. Further works in this area are:

(1) In 1966, Solymar and Ash [4] furnished a one-dimensional analysis for an n-type STWA similar to the analysis for the vacuum TWT of Pierce.

(2) Fujisawa (1968, 1970) worked out a transmission line analogue and a kinetic power theorem for carrier waves in semiconductors [5]. Similar works in transmission lines analogues were done by B. Ho in 1970 [6].

(3) Coupled mode theory for STWA was given by Fujisawa and Ishikawa in 1969 [7].

However, it should be pointed out that though a one-dimensional analysis has been successful with TWT and AWA (acoustic-wave amplifier) and that it is much simpler than the many two-dimensional analyses used, it cannot be extended to the STWA without modification. One big difference between the TWT and the STWA lies in the fact that in the former the space charge waves are weakly damped and the carriers are inertia dominated. In the semiconductor, the stream is collision dominated and the normal modes are highly damped. Further discussion on these will be found in reference [8]. The two-dimensional models are basically field analyses and yield considerably more information about the device. In 1966, 1967 Sumi [9] published a two-dimensional analysis of the interactions between the surface TM waves on a semi-infinite semiconductor medium and the adjacent planar slow wave structure as an infinitely thin current sheet.

An extension of his work was done by Vural and Steele (1969) and Freeman (1972) [10, 11]. In their work, as distinct from Sumi's stiff boundary model with zero transverse current, they have taken into account the effect of the surface charge arising on the semiconductor by using the "ripple" boundary condition. Kino [12] and other authors have also applied the ripple boundary condition in the analysis of thin semiconductor slabs and wave instability in zero diffusion limit.

In these two-dimensional analyses, many researchers have predicted an extremely large gain of tens of decibels per mm length of circuit length. Yet the experimental results so far have been inconclusive though promising [13-15]; no net terminal gain has been achieved. This discrepancy between theoretical prediction and experimental result can be related to a simplification of model or imperfect experimental arrangement.

From the experimental side: Was gain unachievable solely due to the high transmission loss of the slow wave circuit? Or is the geometry of the meander line and its separation from the semiconductor a factor in achieving strong interaction between the drifting carrier and the slow waves?

Two major centers of difference can be noted from some of the above theories, as suggested by Freeman [16]. The first point of disagreement centers about the importance of diffusion forces in the model of the carrier stream. Understandably, the omission of diffusion simplified the mathematics considerably. Yet, it has

been shown that the effect of diffusion is important and must be included [17]. The other disagreement concerns the boundary conditions to be used at the surface of the semiconductor. This will ultimately determine the mechanism of coupling between the stream and the slow waves.

In the analysis to be presented here, diffusion will be included. Also, a new model for the boundary condition essential to the founding of a dispersion relationship will be established. The analysis is essentially a two-dimensional field analysis following closely that of Sumi, Freeman, and that of Okamoto and Mizushima [18] especially, since a static B field (magnetic field) is included in the analysis. A computer is used to aid in the solutions of the dispersion relations. Theoretical results were obtained and an interpretation of them is presented.

2. ANALYSIS OF THE STWA

2.1. Formulation of the Problem

In the analysis that follows here, we will consider a semi-infinite region which fills the lower half space $x \leq 0$. The meander line is represented by a current sheet at distance d above the surface of the semiconductor. The semiconductor is assumed to be uniform, isotropic, and homogeneous. Uniform, static electric and magnetic fields E_0 and B_0 are applied in the -z and +y directions, respectively, as shown in Figure 1.

The linearized forms of Maxwell's equations, along with Poisson's equation and the equation of motion, are used to describe the behavior of the carriers in the material. All the RF quantities are assumed to vary as exp. $j(\omega t - kz)$. Furthermore, we make the following assumptions:

(1) small signal assumption;

(2) the carrier stream is collision-dominated; $|\omega - \beta u_0| \ll \nu$;

(3) single type carriers; electron;

(4) the electrons drift in the +z direction under the influence of a constant drift field;

(5) no significant trapping, generation or recombination effect;

(6) the density $n_0(x)$ is uniform.

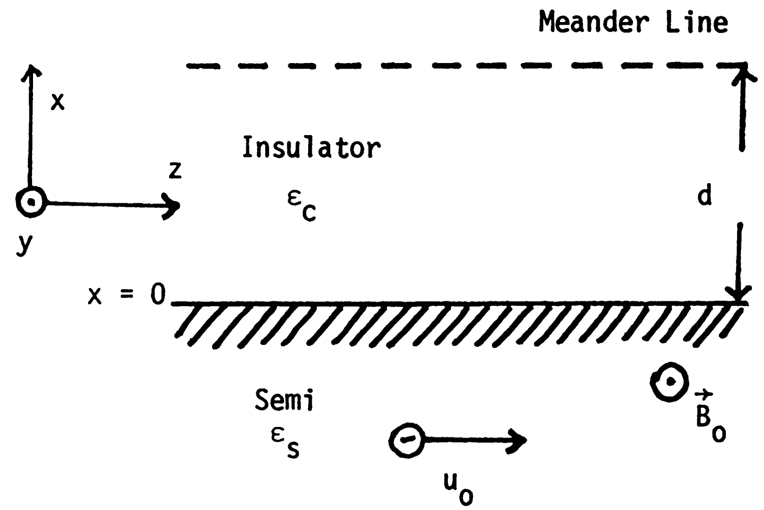


Figure 1. Semi-infinite semiconductor and meander line

The last assumption is a forced one. The transverse magnetostatic field $\vec{B}_o$ will cause an accumulation of electrons on the surface, the so-called Suhl effect. Thus, the density $n_o(x)$ might not be uniform. However, if a gate is applied on the far side of the meander line, we anticipate $n_o(x)$ to be approximately uniform.

The equations are as follows:

$$\nabla \times \vec{E} = -j\omega\mu_o\vec{H} \tag{1}$$

$$\nabla \times \vec{H} = \vec{J} + j\omega\varepsilon\vec{E} \tag{2}$$

$$\nabla \cdot \vec{D} = \rho \tag{3}$$

$$\vec{J} = -e(n_o\vec{v}_1 + n_1\vec{u}_o) \tag{4}$$

$$\frac{d\vec{v}}{dt} = -\frac{|e|}{m^*}\vec{E} - \frac{|e|}{m^*}\vec{v} \times \vec{B} - \nu\vec{v} - v_T^2\frac{\nabla\rho}{\rho} \tag{5}$$

where the d.c. drift velocity u_o is assumed to be in the +z direction; $\vec{u}_o = u_o\hat{a}_z$. m^* is the effective mass of the electron. The subscript "1" is used for RF quantity and "0" for d.c. quantity. It can be shown that

$$\frac{d\vec{v}}{dt} = j\bar{\omega}\vec{v}_1$$

where $\bar{\omega} = \omega - ku_o$ and $\vec{v}_1 = v_x\hat{a}_x + v_z\hat{a}_z$.

Now the term $\vec{v} \times \vec{B}$ is considered.

Since we are only assuming the TM mode; then $\vec{B}_1 = B_1\hat{a}_y$.

$$\vec{v} \times \vec{B} = \vec{v}_1 \times \vec{B}_o + \vec{u}_o \times \vec{B}_1 + \vec{u}_o \times \vec{B}_o + \vec{v}_1 \times \vec{B}_1$$

The third term on the r.h.s. is a d.c. quantity which we drop. We are assuming this pushes the carrier toward the surface which is the Suhl effect and is cancelled by the gate. The very last term on r.h.s. is dropped due to the small signal assumption. Thus, rewriting the equation of motion

$$j\bar{\omega}\vec{v}_1 = -\frac{|e|}{m^*}(\vec{E} + \vec{v}_1 \times \vec{B}_o + \vec{u}_o \times \vec{B}_1) - v_T^2 \frac{\nabla\rho}{\rho} - \nu\vec{v}_1 \qquad (6a)$$

Since $\frac{\nabla\rho}{\rho} \cong \frac{\nabla\rho_1}{\rho_o}$, and $\nu^* = \nu + j\bar{\omega}$

$$\mu^* = \frac{|e|}{m^*\nu^*}, \quad D^* = \frac{\nu_T^2}{\nu^*}$$

are the modified mobility, diffusion constants of the electron. With the understanding that these quantities are complex, we will drop the asterisk for simplicity in the development which follows. Rearranging equation (6a), we obtain the equation of motion.

$$\vec{v}_1 = -\mu[\vec{E} + \vec{v}_1 \times \vec{B}_o + \vec{u}_o \times \vec{B}_1] - D\frac{\nabla\rho_1}{\rho_o} \qquad (6b)$$

We now proceed to obtain the equation for the electric field in the semiconductor. Using equations (1) and (2)

$$\nabla \times \left(\frac{\nabla\times E}{j\mu_o\omega}\right) = j\varepsilon\omega\vec{E} + \vec{j}$$

$$\nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E} = \mu_o\varepsilon\omega^2\vec{E} - j\mu_o\omega\vec{J}$$

if $c^2 = (\varepsilon\mu_o)^{-1}$, $\omega_c = e\mu n_o/\varepsilon$, by using equation (4)

$$\nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E} = \frac{\omega^2}{c^2}\vec{E} - j\mu_o\omega[-|e|n_1\vec{u}_o - |e|n_o v_1]$$

thus

$$\vec{v}_1 = \frac{1}{j\mu_o\omega e n_o}\left[-\frac{\omega^2}{c^2}\vec{E} + \frac{j\mu\vec{u}_o}{c^2}\nabla\cdot\vec{E} + \nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E}\right] \tag{6c}$$

Substituting into the equation of motion, and also defining $\vec{\alpha} = -\mu B_o a_{\hat{y}}$, we obtain the following:

$$\left[\frac{-\omega^2}{c^2}\vec{E} + j\,\frac{\omega\vec{u}_o}{c^2}\nabla\cdot\vec{E} + \nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E}\right]$$

$$+ \alpha \times \left[\frac{-\omega^2}{c^2}\vec{E} + j\,\frac{\omega u_o}{c^2}\nabla\cdot\vec{E} + \nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E}\right]$$

$$= -i\,\frac{\omega\omega_c}{c^2}\vec{E} + \frac{\omega_c\vec{u}_o}{c^2}\times(\nabla\times\vec{E}) + i\,\frac{D\omega}{c^2}\nabla(\nabla\cdot\vec{E}) \tag{7}$$

Assuming the fields do not vary with the y-coordinate, that is, $\frac{\partial}{\partial y} = 0$, we can separate them into TE and TM waves. This assumption is justified if the semiconductor extends far enough in the y direction. Since the TE wave has only a y component of the electric field, we will study the TM mode here.

We assume the x variation to be $e^{\gamma x}$; thus, the coordinate dependence of all a.c. quantities is exp $(-jkz + \gamma x)$ where the time dependence is understood. The TM field is

$$\vec{E}(x,z) = (E_x a_x + E_z a_z) e^{\gamma x - jkz}$$

where E_x, E_z are complex amplitudes. Then

$$\nabla \cdot \vec{E} = [\gamma E_x - jkE_z] e^{\gamma x - jkz}$$

$$\nabla(\nabla \cdot \vec{E}) = [\gamma \hat{a}_x - jk\hat{a}_z][\gamma E_x - jkE_z] e^{\gamma x - jkz}$$

$$\nabla^2 \vec{E} = [\gamma^2 - k^2] \vec{E}$$

Substituting the above equation into equation (7) and equating the a_x and a_z components, we obtain

$$a_x: \quad E_x[j \frac{\omega D}{c^2} \gamma^2 - k^2 + \frac{\omega^2}{c^2} (1 - j \frac{\omega_c}{\omega}) + j \frac{\omega_c k u_o}{c^2} + j\alpha(k - \frac{\omega u_o}{c^2})\gamma]$$

$$= E_z[-jk(1 - \frac{j\omega D}{c^2})\gamma - \frac{\omega_c u_o}{c^2} \gamma - \alpha(\gamma^2 + \frac{\omega^2}{c^2} - \frac{\omega u_o k}{c^2})] \qquad (8a)$$

$$a_z: \quad E_x[\gamma(-jk - \frac{\omega D k}{c^2} + j \frac{\omega u_o}{c^2}) + \alpha(\frac{\omega^2}{c^2} - k^2)] =$$

$$E_z[\gamma^2 + \frac{\omega^2}{c^2} (1 - \frac{u_o k}{\omega}) - j \frac{\omega}{c^2} (\omega_c + k^2 D) - j\alpha k\gamma] \qquad (8b)$$

Now using the notation:

$$a = \frac{\omega^2}{c^2}\left(1 - \frac{u_o k}{\omega}\right) - j\,\frac{\omega}{c^2}\left(\omega_c + Dk^2\right)$$

$$b = -jk\left[1 - \frac{\omega u_o}{kc^2} - j\,\frac{\omega D}{c^2}\right]$$

$$G = j\,\frac{\omega D}{c^2}$$

$$d = k^2 - \frac{\omega^2}{c^2}\left(1 - j\,\frac{\omega_c}{\omega}\right)$$

$$e = -jk\left(1 - j\,\frac{\omega D}{c^2}\right)$$

$$f = j\alpha\left(k - \frac{\omega u_o}{c^2}\right)$$

$$g = \alpha\,\frac{\omega}{c^2}\,\bar{\omega}$$

$$\ell = \alpha\left(\frac{\omega^2}{c^2} - k^2\right)$$

$$h = j\alpha k$$

$$p = j\,\frac{\omega_c k u_o}{c^2}$$

$$q = -\frac{\omega_c u_o}{c^2}\,\gamma\ ,$$

then equations (8a) and (8b) become

$$[G\gamma^2 - d + p + f\gamma]E_x = [e\gamma - \alpha\gamma^2 - g + q)E_z \tag{9a}$$

$$[b\gamma + \ell]E_x = [\gamma^2 + a - h\gamma]E_z \, . \tag{9b}$$

2.2. The Stiff Boundary Case

If we drop the terms due to the magnetic fields, i.e., p, q and $\alpha = 0$, we have Sumi's case:

$$[G\gamma^2 - d]E_x = [e\gamma]E_z \tag{10a}$$

$$[b\gamma]E_x = [\gamma^2 + a]E_z \tag{10b}$$

In order to have a non-trivial solution, the determinant

$$\begin{vmatrix} G\gamma^2 - d & -e\gamma \\ b\gamma & -(\gamma^2 + a) \end{vmatrix} = 0$$

From Freeman's [19] evaluation, we know the roots are

$$\gamma_1^2 = d = k^2 - \frac{\omega^2}{c^2}\left(1 - j\,\frac{\omega_c}{\omega}\right)$$

$$\gamma_2^2 = -\frac{a}{G} = k^2 + \frac{\omega_c}{D} + j\,\frac{\bar{\omega}}{D}$$

For the geometry given, only positive roots are allowed. Thus, the components of the TM mode are:

$$E_z = A_1 e^{\gamma_1 x} + A_2 e^{\gamma_2 x}$$

$$E_x = B_1 e^{\gamma_1 x} + B_2 e^{\gamma_2 x}$$

The above case has been studied by Sumi and Freeman. Therefore, I will quote results from Freeman as reference,

$$B_1 = j \frac{k}{\gamma_1} A_1 \tag{11a}$$

$$B_2 = j \frac{\gamma_2 A_2}{\beta - \frac{\omega u_o}{c^2}} \tag{11b}$$

by utilizing equations (10a) and (10b).

Since for the TM mode H_z and $E_y = 0$, and we assume $\frac{\partial}{\partial y} = 0$, using equations (1), (11a), and (11b),

$$H_y = \frac{j\omega\varepsilon}{k} \left[\frac{k}{\gamma_1} \left(1 - j \frac{\omega c}{\omega}\right) A_1 e^{\gamma_1 x} + \frac{\gamma_2 u_o}{\omega} A_2 e^{\gamma_2 x}\right] \tag{12}$$

To find the dispersion relation for the entire system, we must match the fields of the semiconductor to those of the meander line. Several assumptions are made here:

(1) The insulator between the meander line and the semiconductor is perfect, thus there is no conduction current in this region.

(2) On the surface of the semiconductor, no real charge or current exists.

Due to these assumptions, we have Sumi's stiff-boundary case with the following boundary conditions:

(a) $v_{1x} = 0$ at the semiconductor surface, i.e., $x = 0$;

(b) the transverse admittances of the two regions are equal.

From condition (a), we obtain a relationship between A_1 and A_2

$$A_2 = -j \frac{\omega_c k^2}{\bar{\omega}\gamma_1\gamma_2} A_1 \tag{13}$$

With this relation we can express H_{y_s} and E_{z_s} in terms of A_1. At $x = 0$, the transverse admittance is

$$\frac{H_{y_s}}{E_{z_s}} = \frac{i\omega\varepsilon_s(1 - i\frac{\omega_c}{\omega})}{\gamma_1(1 - i\frac{k^2\omega c}{\gamma_1\gamma_2\bar{\omega}})}$$

Similarly, we can find an expression for that of the current sheet. This is done in detail by J. Freeman [20] and is also used by Yen [21].

$$\frac{H_{y_c}}{E_{z_c}} = \frac{i\omega\varepsilon_c}{\gamma_c} \frac{(1 + e^{-2\gamma_c d})k^2 - k_o^2}{k_o^2 - (1 - e^{-2\gamma_c d})k^2}$$

where

d = thickness of the insulator

γ_c = the circuit propagation constant

$k_o = \frac{\omega}{v_\phi}$, where v_ϕ is the phase velocity of the circuit.

With the assumption that

$$e^x \doteq 1 + x$$

and

$$\gamma_c^2 \doteq k^2$$

along with the approximations

$$\gamma_1 \doteq k$$

$$\gamma_2 \doteq \sqrt{\frac{\omega_c}{D}} ,$$

we obtain the final dispersion relation by using boundary condition (b). The result is as follows [22], with $K = \frac{\varepsilon_s}{\varepsilon_c}$.

$$[2du_o(1 - k) + j2d\sqrt{D\omega_c}]k^4 + \{2[d\omega(K - 1) - u_o]$$

$$- j2(K\omega_c d + \sqrt{D\omega_c})\}k^3 + 2\omega k^2 + [k_o^2 u_o(1 + K) + jk_o^2\sqrt{D\omega_c}]k$$

$$+ [-k_o^2\omega - Kk_o^2\omega + jKk_o^2\omega_c] = 0$$

This relationship is the stiff boundary case examined by Sumi and Freeman.

2.3. The Stiff Boundary Case with Static B_0 Field

Returning to equations (9a) and (9b), we now assume $|\vec{B}_0|>>|\vec{B}_1|$, and if $\vec{B}_0 = 0$, $\vec{B}_1$ is still not of significance in the motion. Thus, equations (9a) and (9b) lead to:

$$G\gamma^4 + (f - Gh + \alpha b)\gamma^3 + (aG - d - fh - eb + \alpha\ell)\gamma^3$$

$$+ (dh + af - e\ell + gb)\gamma - da + g\ell = 0 \ . \tag{14}$$

The coefficient of γ^3 vanishes, and the only non zero term in the coefficient of γ is much smaller in magnitude when compared to coefficients of other terms. Thus, equation (14) reduces to

$$\gamma^4 + \frac{(aG - d - fh - eb + \alpha\ell)}{G}\gamma^2 + \frac{-da + g\ell}{G} = 0 \ . \tag{15}$$

If we assume that the constant term is still a product of the roots, the above equation can be rearranged as a bi-quadratic with the roots perturbed from the stiff-boundary case.

Two possibilities exist here: either γ_1 or γ_2 is perturbed. After careful evaluation, it is found that γ_2 is perturbed. The solutions of γ^2 in equation (15) are given as follows:

$$\gamma_1^2 = d = k^2 - \frac{\omega^2}{c^2}\left(1 - j\frac{\omega_c}{\omega}\right)$$

$$\gamma_2^2 = \frac{-a + \frac{g\ell}{d}}{G} = k^2 + \frac{\omega_c}{D} + j\frac{\bar{\omega}}{D}(1 + \alpha^2) \ .$$

Utilizing equations (9a) and (9b) again, we obtain the relationship between A_1 and B_1, and A_2 and B_2:

$$B_1 \doteq \frac{-jk\gamma_1 - \alpha\gamma_1^2 - \alpha\frac{\omega\bar{\omega}}{c^2}}{\gamma_1[-\gamma_1 + j\alpha k(1-\beta)]} A_1 \tag{16a}$$

$$B_2 \doteq \frac{\gamma_2^2 - j\alpha k\gamma_2}{-jk(1-\beta)\gamma_2 + \alpha\left(\frac{\omega^2}{c^2} - k^2\right)} A_2 \tag{16b}$$

where $\beta = \frac{\omega u_o}{kc^2}$ and $j\frac{\omega D}{c^2}$ is dropped when compared to unity.

For reasons stated previously,

$$Hy = \frac{j\omega\varepsilon}{k}\left[\frac{-k\gamma_1\left(1 - j\frac{\omega_c}{\omega}\right) + j\alpha k^2}{(j\alpha k - \gamma_1^2)} A_1 e^{\gamma_1 x}\right.$$

$$\left. + \frac{j\gamma_2 k(j\alpha\omega - \gamma_2 u_o)}{jk\gamma_2 + \alpha k^2} A_2 e^{\gamma_2 x}\right] \tag{17}$$

Setting $v_{1x} = 0$ at $x = 0$, i.e., the transverse component of the RF current being zero, we obtain the relationship between A_1 and A_2 ,

$$A_1 = -j \frac{\gamma_1\gamma_2\bar{\omega}(j\alpha k - \gamma_1)}{k^2(\gamma_1\omega_c - \alpha k^2 u_o)} A_2 \tag{18}$$

It is comforting to note that in the absence of the magnetostatic field $\vec{B}_o$, $\alpha = 0$ the above equation reduces to the previous case as it should be. Substituting the above relationship into equation (17) yields

$$Hy = \frac{j\omega\varepsilon A_2}{k}\{\frac{j\gamma_2\bar{\omega}(\gamma_1 - j\alpha k)(1 - j \frac{\omega_c}{\omega})}{k[\gamma_1\omega_c - \alpha k^2 u_o]} e^{\gamma_1 x} + \frac{\gamma_2 u_o}{\omega} e^{\gamma_2 x}\} \tag{19}$$

Finally, we will match the fields at x = 0

$$\frac{H_{y_s}}{E_{z_s}} = \frac{H_{y_c}}{E_{z_c}} \tag{20}$$

where

$$\frac{H_{y_s}}{E_{z_s}} = \frac{j\omega\varepsilon_s[j\gamma_2\bar{\omega}(\gamma_1 - j\alpha k)(1 - j \frac{\omega c}{\omega}) + \frac{\gamma_2 u_o k^2(\gamma_1\omega_c - \alpha k^2 u_o)}{\omega k}]}{[k^2(\gamma_1\omega_c - \alpha k^2 u_o) + j\gamma_1\gamma_2 \bar{\omega}(\gamma_1 - j\alpha k)]}$$

and

$$\frac{H_{y_c}}{E_{z_c}} = \frac{j\omega\varepsilon_c(2\beta^2 - 2\beta^3 d - \beta_o^2}{(\beta\beta_o^2 - 2\beta^4 d)}$$

with

$$\gamma_1 \doteq k$$

$$\gamma_2 \doteq \frac{1}{\lambda_D}\left[1 + j\,\frac{\bar{\omega}(1+\alpha^2)}{2\omega_c^*}\right] \qquad \text{(see Appendix A)}$$

where

$$\lambda_D = \sqrt{\frac{D}{\omega_c}}$$

After a lengthy algebraic manipulation, we arrive at a 6-degree complex polynomial dispersion relation for the stiff-boundary case, with $\vec{B}_o = a_y B_o$. For convenience it is represented here by

$$a_1 k^6 + b_1 k^5 + c_1 k^4 + d_1 k^3 + e_1 k^2 + f_1 k^1 + g = 0 \; .$$

Coefficient of k^6:

$$a_1 = j\,\frac{2dK\phi u_o^2\alpha}{\omega}$$

Coefficient of k^5:

$$b_1 = [2d\alpha\lambda_D + 2u_o\phi d(1-K) + \frac{2du_oK\alpha}{\omega}(\phi\omega_c - 1) - j2\alpha\phi u_o d]$$

Coefficient of k^4:

$$c_1 = \{[2\alpha(d-\lambda_D) + 4d\phi\omega(K-1) - 4dK\alpha - 2u_o\phi - \frac{2d\omega_c\lambda_D}{u_o}]$$

$$+ j[2d(1-K) + 4\alpha\phi\omega d(1-K) + \frac{2dK\omega_c}{\omega}(\alpha - \omega\phi) + 2\alpha\phi u_o]\}$$

Coefficient of k^3:

$$d_1 = \{[\frac{2\phi\omega^2 d}{u_o}(1 - K) + \frac{2d\omega\alpha}{u_o}(K - 1) + \frac{2\omega_c}{u_o}(\lambda_D + Kd)$$

$$+ 2\omega\phi(2 + \frac{d\omega_c\alpha K}{u_o}) - 2\alpha] + j[\frac{2d\omega}{u_o}(K - 1)$$

$$+ \frac{2dK\omega\phi}{u_o}(\omega_c + \alpha\omega) - 2\alpha\phi\omega(2 + \frac{\omega d}{u_o}) - 2$$

$$- \frac{2dK\alpha\omega_c}{u_o} - \frac{k_o^2 K\phi u_o^2\alpha}{\omega}]\}$$

Coefficient of k^2:

$$e_1 = \{[k_o^2\phi u_o(1 + K) + \frac{k_o^2 K u_o\alpha}{\omega}(1 - \phi\omega_c) + \frac{2\omega}{u_o}(\alpha - \phi\omega)$$

$$+ k_o^2\alpha\lambda_D] + j[\frac{2\omega}{u_o}(1 + \alpha\phi\omega) - k_o^2 u_o\phi\alpha]\}$$

Coefficient of k^1:

$$f_1 = \{[\alpha k_o^2(1 + K) - 2k_o^2\omega\phi(1 + K) + k_o^2\omega_c(\frac{\lambda_D}{u_o} + 2K\phi\alpha)$$

$$+ j[k_o^2(1 + K) + 2\alpha\phi\omega k_o^2(1 + K) + k_o^2 K\omega_c(\frac{\alpha}{\omega} + \phi)]\}$$

Coefficient of k^0:

$$g_1 = \{[\frac{\phi\omega 2 k_o^2}{u_o}(1 + K) - \frac{\alpha\omega k_o^2}{u_o}(1 + K) - \frac{k_o^2 K\omega_c}{u_o}(1 + \phi\omega\alpha)]$$

$$+ j[\frac{k_o^2 K}{u_o}(\alpha\omega_c - \omega - \phi\omega\omega_c - \phi\omega^2\alpha) - \frac{\omega k_o^2}{u_o}(1 + \alpha\phi\omega)]\} \tag{21}$$

where $\phi = \dfrac{1 + \alpha^2}{2\omega_c}$

It is interesting to note that in the case with no magnetic field $\vec{B}_o$, and the relationship between A_1 and B_1 being

$$B_1 = j\frac{k}{\gamma_1}A_1 ,$$

the γ_1 mode is called the solensoidual wave since

$$\nabla\cdot\vec{E}(\gamma_1) = \frac{\rho}{\varepsilon_s} \equiv 0$$

$$\rho = 0 \text{ everywhere} .$$

However, with the addition of the $\vec{B}_o$ field, this is no longer true.

$$\nabla\cdot\vec{E}(\gamma_1) = \frac{\rho}{\varepsilon_s} \neq 0$$

Therefore $\rho \neq 0$.

The differences between this dispersion equation and the one by Sumi are (1) the existence of an external magnetostatic field $\vec{B}_o$ represented by α, and (2) the approximations for α_2 differ. When these significant differences are removed, the 6-degree polynomial here reduces to that of Sumi.

It is important to point out that the above dispersion relation, with or without the static magnetic field $\vec{B}_0$, is derived via the stiff boundary condition. However, it was point out by Dean and Robinson (1974) [23] that this condition is questionable when compared to experimental results and a free boundary condition must be used. Freeman in his dissertation used the ripple boundary condition, but only studied the dispersion relation for special cases.

We will attempt to find a dispersion relation of the system by adding a new constraint.

2.4. The Ripple Boundary Condition

It is apparent that in Sumi's development of the stiff boundary condition, he assumed the carriers only have motion parallel to the physical surface of the semiconductor at x = 0. He was able to accomplish this by using the two basic modes γ_1, γ_2 in such a way that the transverse component of the current is zero, i.e., $v_{1x} = 0$ at the physical surface. This constraint can be somewhat relaxed if we assume a ripple boundary.

Let us say the electrons have a transverse RF velocity and cause accumulation and depletion alternatively near the surface during each cycle. This change in carrier density results in an RF surface charge, ρ_s. As demonstrated by Collin [24], this surface charge results from two causes:

(1) the transverse conduction current, J_x;

(2) the charge being carried to a given point z due to the surface charge moving with the beam.

$$\frac{\partial \rho_s}{\partial t} = j\omega\rho_s = J_r - u_o \frac{\partial \rho_s}{\partial z} = J_r + jku_o\rho_s$$

Thus

$$\rho_s = \frac{Jr}{j\bar{\omega}} = \frac{-\rho_o v_x}{j\bar{\omega}} \tag{22}$$

Since this surface charge drifts along with the beam, it constitutes an RF surface current,

$$K_s = u_o\rho_s \tag{23}$$

We now proceed to model the system as such.

(1) Near the surface of the semiconductor there exists a perfect depletion region which extends about .5μ into the semiconductor.

(2) Since the surface of the semiconductor is rippled, standard boundary conditions cannot be applied. To remedy this we shall use the method described by Hahn [25], that is, to represent the ripple boundary by a static one with effective surface charge an current. At this static boundary, x = 0, which is not the physical surface of the semiconductor, but the unperturbed stream boundary.

(i) $E_{z_c} = E_{z_s}$ (tangential E must be continuous)

(ii) $\varepsilon_c E_{x_c} = \varepsilon_s E_{x_s} + \rho_s$ (normal E must be discontinuous by and effective charge ρ_s)

(iii) $H_{y_c} = H_{y_s} + K_s$ (tangential H must be discontinuous by K_s)

Conditions (i) and (iii) are useful for obtaining the dispersion relation.

(3) Both the solenoid (γ_1) and diffusion (γ_2) waves are highly damped and exist only on an extremely thin layer, with the former to within a skin depth and the latter to within a Debye length. Therefore at some depth from the surface

$$v_x(m\lambda_D) = 0 \qquad m = 1, 2, \ldots, 30 \tag{24}$$

(The geometry of the model is shown in Figure 2.)

Assumptions (2) and (3) are of interest for obtaining the dispersion relation. Applying equation (24), the relationship between A_1 and A_2

$$A_1 e^{\gamma_1 m\lambda_D} = -j \frac{\gamma_1\gamma_2\bar{\omega}(j\alpha k - \gamma_1)}{k^2(\gamma_1\omega_c - \alpha k^2 u_o)} A_2 e^{\gamma_2 m\lambda_D} \tag{25}$$

or rewriting,

$$\frac{A_1}{A_2} = -j \frac{\gamma_1\gamma_2\bar{\omega}(j\alpha k - \gamma_1)}{k^2(\gamma_1\omega_c - \alpha k^2 u_o)} e^{m\lambda_D(\gamma_2-\gamma_1)}$$

with the approximation (see Appendix B)

$$e^{m\lambda_D(\gamma_2-\gamma_1)} \doteq \frac{m^2\bar{\omega}}{2\omega_c}\left(j - \frac{m\bar{\omega}}{2\omega_c}\right) .$$

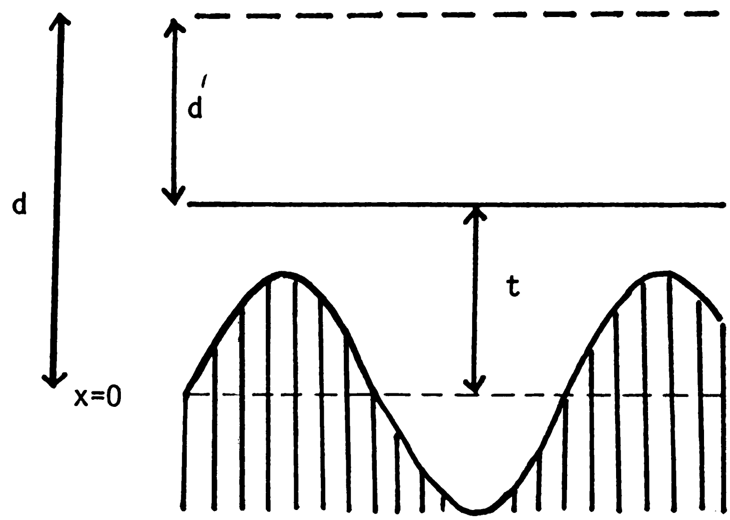


Figure 2. Rippled Beam Surface

Thus

$$\frac{A_1}{A_2} = -j\,\frac{\gamma_1\gamma_2\bar{\omega}(j\alpha k - \gamma_1)}{k^2(\gamma_1\omega_c - \alpha k^2 u_o)}\,\frac{m^2\bar{\omega}}{2\omega_c}\,(j - \frac{m\bar{\omega}}{2\omega_c}) \tag{26}$$

Since

$$\rho_s = \frac{-\rho_o v_x(0)}{j\bar{\omega}}$$

We must now evaluate v_x at $x = 0$.

$$v_x(0) = \frac{-j\mu c^2}{\omega\omega_c} A_2 \,\{[\eta_2(k^2 - \frac{\omega^2}{c^2}) - jk\gamma_2] + [\eta_1(k^2 - \frac{\omega^2}{c^2}) - jk\gamma_1] \cdot$$

$$\cdot\,[\frac{j\gamma_1\gamma_2\bar{\omega}(\gamma_1 - j\alpha k)}{k^2(\gamma_1\omega_c - \alpha k^2 u_o)}] \cdot [\frac{m^2\bar{\omega}}{2\omega_c}\,(j - \frac{m\bar{\omega}}{2\omega_c})]\} \tag{27}$$

with

$$\eta_1 = \frac{B_1}{A_1} \doteq \frac{-jk\gamma_1 - \alpha\gamma_1^2 - \alpha\frac{\omega\bar{\omega}}{c^2}}{\gamma_1[-\gamma_1 + j\alpha k(1 - \beta)]}$$

$$\eta_2 = \frac{B_2}{A_2} = \frac{\gamma_2^2 - j\alpha k\gamma_2}{-jk(1 - \beta)\gamma_2 + \alpha(\frac{\omega^2}{c^2} - k^2)}$$

The transverse admittances must match. Thus,

$$\frac{H_{y_s} + K_2}{E_{z_s}} = \frac{H_{y_c}}{E_{z_c}}$$

With $K_s = \rho_s u_o$.

The dispersion relation

$$H_{y_s} E_{z_c} + K_s E_{z_c} = E_{z_s} H_{y_c}$$

is at least thirteenth degree unless simplified. To avoid confusion and error, we will study the case where $\alpha = 0$; no magnetostatic field. With this simplification, the dispersion relation will reduce to sixth degree.

The quantities are

$$H_{y_c} = j\omega\varepsilon_c[(1 + e^{-2\gamma_c d})k^2 - k_o^2]$$

$$E_{z_c} = \gamma_c[k_o^2 - (1 - e^{-2\gamma_c d})k^2]$$

$$K_s = \frac{j\mu u_o c^2 \rho_o}{j\bar{\omega}\omega\omega_c} A_2 \{[\eta_2(k^2 - \frac{\omega^2}{c^2}) - jk\gamma_2] + [\eta_1(k^2 - \frac{\omega^2}{c^2}) - jk\gamma_1] \cdot$$

$$\cdot [\frac{j\gamma_1\gamma_2\bar{\omega}(\gamma_1 - j\alpha k)}{k^2(\gamma_1\omega_c - \alpha k^2 u_o)}] \cdot [\frac{m^2\bar{\omega}}{2\omega_c} (j - \frac{m\bar{\omega}}{2\omega_c})]\}$$

$$E_{z_s} = A_2\{1 + [\frac{j\gamma_1\gamma_2\bar{\omega}(\gamma_1 - j\alpha k)}{k^2(\gamma_1\omega_c - \alpha k^2 u_o)}] [\frac{m^2\bar{\omega}}{2\omega_c} (j - \frac{m\bar{\omega}}{2\omega_c})]\}$$

$$H_{y_s} = \frac{A_2}{\omega u_o} \{(k\eta_1 - j\gamma_1)[\frac{j\gamma_1\gamma_2\bar{\omega}(\gamma_1 - j\alpha k)}{k^2(\gamma_1\omega_c - \alpha k^2 u_o)}] [\frac{m^2\bar{\omega}}{2\omega_c} (j - \frac{m\bar{\omega}}{\omega_c})]$$

$$+ (k\eta_2 - j\gamma_2)\}$$

After a very lengthy algebraic manipulation, we arrive at the final dispersion relation

$$a_2 k^6 + b_2 k^5 + c_2 k^4 + d_2 k^3 + e_2 k^2 + f_2 k + g_2 = 0$$

where

$$a_2 = \frac{dmu_o^3}{2\omega_c} + j\,\frac{kmu_o^3 K}{2\omega_c}$$

$$b_2 = \{(-\frac{3dm\omega u_o^2}{2\omega_c} - u_o^2 dK) + j[du_o^2(1 + \frac{m}{2}) - \frac{u_o^2 mK}{2\omega_c}(u_o + 3d\omega)]\}$$

$$c_2 = \{[\frac{3dm\omega^2 u_o}{2\omega_c} + u_o^2 K + u_o d(2\omega K - \omega_c)] + j[-\frac{4du_o\omega_c^2}{m^2\omega\phi}$$

$$+ \frac{3u_o m\omega K}{2\omega_c}(u_o + d\omega) - d\omega u_o(2 + m)]\}$$

$$d_2 = \{[d\omega\omega_c - \omega K(2u_o + \omega d) - \frac{dm\omega^3}{2\omega_c} - \frac{k_o^2 mu_o^3}{4\omega_c}]$$

$$+ j[d\omega^2[1 + \frac{m}{2}] - \frac{\omega^2 mK}{2\omega_c}(3u_o + \omega d) + \frac{k_o^2 u_o^3 m}{4\omega_c}]\}$$

$$e_2 = \{[\omega^2 K - \frac{k_o^2 u_o^2}{2}(\frac{3m\omega}{2\omega_c} + K)] + j[\frac{m\omega^3 K}{2\omega_c} - \frac{k_o^2 u_o^2}{2}(\frac{m}{2} + 1 + \frac{3m\omega K}{2\omega_c})]\}$$

$$f_2 = \{[k_o^2 u_o(\omega_c + \omega K) - \frac{3k_o^2 m\omega^2 u_o}{4\omega_c}] + j[k_o^2 \omega u_o(1 + \frac{m}{2}]$$

$$+ \frac{2k_o^2 \omega_c^2 u_o}{m^2 \omega\phi} + \frac{3k_o^2 u_o m\omega^2 K}{4\omega_c}]\}$$

$$g_2 = \{[\frac{k_o^2 \omega^3 m}{4\omega_c} - \frac{k_o^2 m\omega}{2}(\omega_c + \omega K)] + j[-\frac{k_o^2 \omega^2}{2}(1 + \frac{m}{2}) - \frac{k_o^2 m\omega^3 K}{4\omega_c}]\}$$

We assume

$$\phi = 1 - k\lambda_D \cong 1 \ .$$

and ω_c is assumed to be real instead of complex; i.e.,

$$\nu >> \bar{\omega} \ .$$

3. EVALUATION OF THE DISPERSION RELATIONS

In this chapter, the propagation constant k is examined by solving the dispersion relations obtained in the previous chapter. In the past, many attempts to find k involved the use of the perturbation method. It is often assumed that the coupled modes differ from the normal modes by very small perturbations. There has been considerable disagreement in the literature concerning the validity of the perturbation approximations. With the advent of fast computers, numerical solution seems very convenient. This is the method we adopt in finding the roots for the dispersion relations.

3.1. Procedure Used in Evaluation of the Roots

The evaluation of the dispersion equation involves three parts: (1) finding all the roots for the dispersion under a specific set of parameters, (2) evaluation of possible growing root(s), and (3) examination of the behavior of the growing root(s) under varying conditions.

The first part is achieved through the use of a subroutine currently in the MSU computer library, called ZPOLY. This subroutine will find numerical solutions of complex polynomials up to 49th degree. We believe that this is far more accurate and convenient than the perturbation method.

In the second part, we are entering the rather confused area of instability criteria. One approach for physically interpreting

the solutions of the dispersion relation is based on the complex-mode method and the kinetic power theorem [26]. This method separates components of an interacting system into weakly interacting modes and studies them individually. As in the special case of AWA, one can decompose the system into a collision-dominant carrier stream and the sound waves. If the stream interacts with a sound wave with phase velocity less than the drift velocity of the electrons, it becomes "active"--it loses energy not because of collisions but due to interaction with the "passive" sound wave. If the energy flows of these two modes are in the same direction, we have a convective "spatial" instability. If the energy flows are in opposite directions, then a nonconvective (temporal) instability occurs. An example of this nonconvective instability is the BWO, where the energy flows in the two subsystems are oppositely directed, thus only oscillation can occur.

The above criteria set forth a necessary condition for convective instability to occur in the system. To determine which mode(s) is actually unstable due to the interaction, we will utilize the Bers and Briggs criteria for convective instability. This approach is based on the principal of causality and the initial value problem. It essentially studies the asymptotic response of the system to a signal bounded both in space and time [27]. In application, it offers a way to test the system for convective instabilities. The procedure applicable to our case is outlined here [28]:

(1) Solve the dispersion relation $D(\omega,k) = 0$ for real ω (i.e., setting $\omega_i = 0$) and find complex k ($k = k_r + jk_i$). The set of complex k gives forms of solutions for $z \geqslant 0$ and $z < 0$.

(2) For the given geometry, we are only considering roots that correspond to solutions that exist for $z > 0$. This means only roots with k_i greater than zero are possible candidates for instability. The sign of k_r is irrelevant.

(3) If k_i has a change of sign as we increase ω_i from zero to negative values, then a convective instability does occur and convects energy to the right, i.e., $z > 0$.

From the previous chapter, there are three dispersion equations as a result of various boundary conditions and assumptions. Sumi's general dispersion relation is a fourth-degree complex polynomial with coefficients composed of the following parameters:

(1) d is the distance between the plane of the meander line and the semiconductor surface. For the stiff boundary cases, this distance is just the thickness of the insulator. In the discussion to follow, d is to vary from 0 to 1μ.

(2) u_0 is the drift velocity of the electron beam. The coupling mechanism of the system is affected as we vary u_0. If the drift velocity is increased to a value near the phase velocity of the meander line, we hope to see a "growing root."

(3) ω_c is determined by the carrier density. (η_o is assumed to be able to vary from 10^{13} to 10^{14} cm^{-3}). Thus ω_c can vary from 10^{10} to 10^{11}.

(4) v_ϕ is the phase velocity of the "slow" wave. Due to physical limitation of the "slow" factor, it is allowed to vary from 10^6 cm/s to 10^7 cm/s.

(5) k_o is a measure of the "slow" factor of the line; $k_o = \omega/v_\phi$.

(6) D is the diffusion constant; for GaAs it is about 220 cm^2/s.

(7) K is the ratio

$$\frac{\varepsilon_s}{\varepsilon_c} \cong \frac{13}{4}$$

For the stiff boundary case with external transverse magnetic field, one more parameter is added. α is the product of the carrier mobility μ_m and the magnetic flux B_o. The electron mobility of GaAs is about 8500 cm^2(vs)$^{-1}$. The B_o field is allowed to vary from 0 to 10 KGauss.

In the final ripple boundary case, α is to be zero and the "depth" of the ripple is represented by the dimensionless m.

The above parameters will be varied within set limits and the movement of the root(s) will be plotted in the next section.

3.2. Evaluation of the Growing Roots

Recall we are looking for wave solutions of the form $\exp[j(\omega t - kz)]$ which represent waves traveling in the z direction. If k_i changes sign as we vary ω_i from zero to negative values, we have an amplifying wave. Contrary to conventional belief, the sign of k_r does not indicate the direction of wave propagation. For a more detailed discussion of this topic, see reference [27].

In Figure 3, movement of the roots

increasingly negative. Only one of the possible four roots is a "growing" root. As described in the previous section, this wave grows as $\exp(k_i z)$ as it propagates toward $z > 0$. The propagation constant of this root is near the value of the slowing parameter k_o. This requirement is necessary because only the normal modes that can be excited by the meander line are of interest.

With ω being purely real, we step the excitation frequency of the line in 1 GHz increments and note the change of k_i of the growing wave. The result is shown in Figure 4. The gain in dB per wavelength is expressed by

$$G(\frac{dB}{\lambda}) = 54.58|k_i|(\frac{v_\phi}{\omega}) .$$

Gain is possible over a range of 27 GHz, and limited by the carrier velocity u_o.

Interesting results occur when we vary the conduction frequency ω_c. It seems that ω_c must be near the operating frequency ω if gain is to occur. As shown in Figure 5, the gain increases with increasing ω_c up to a maximum, but decreases gradually after

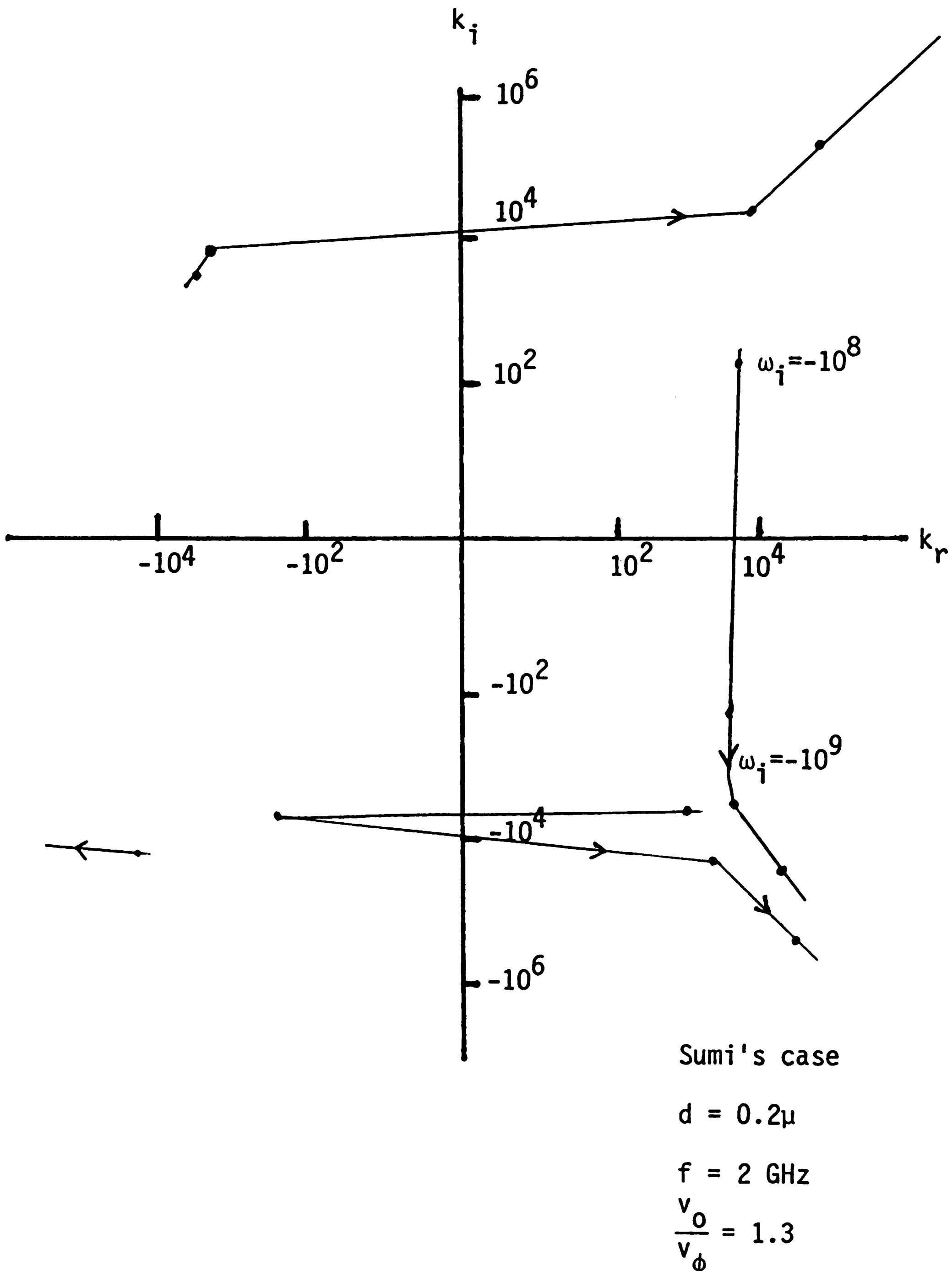


Figure 3. Growing Roots under the Bers and Briggs Criteria

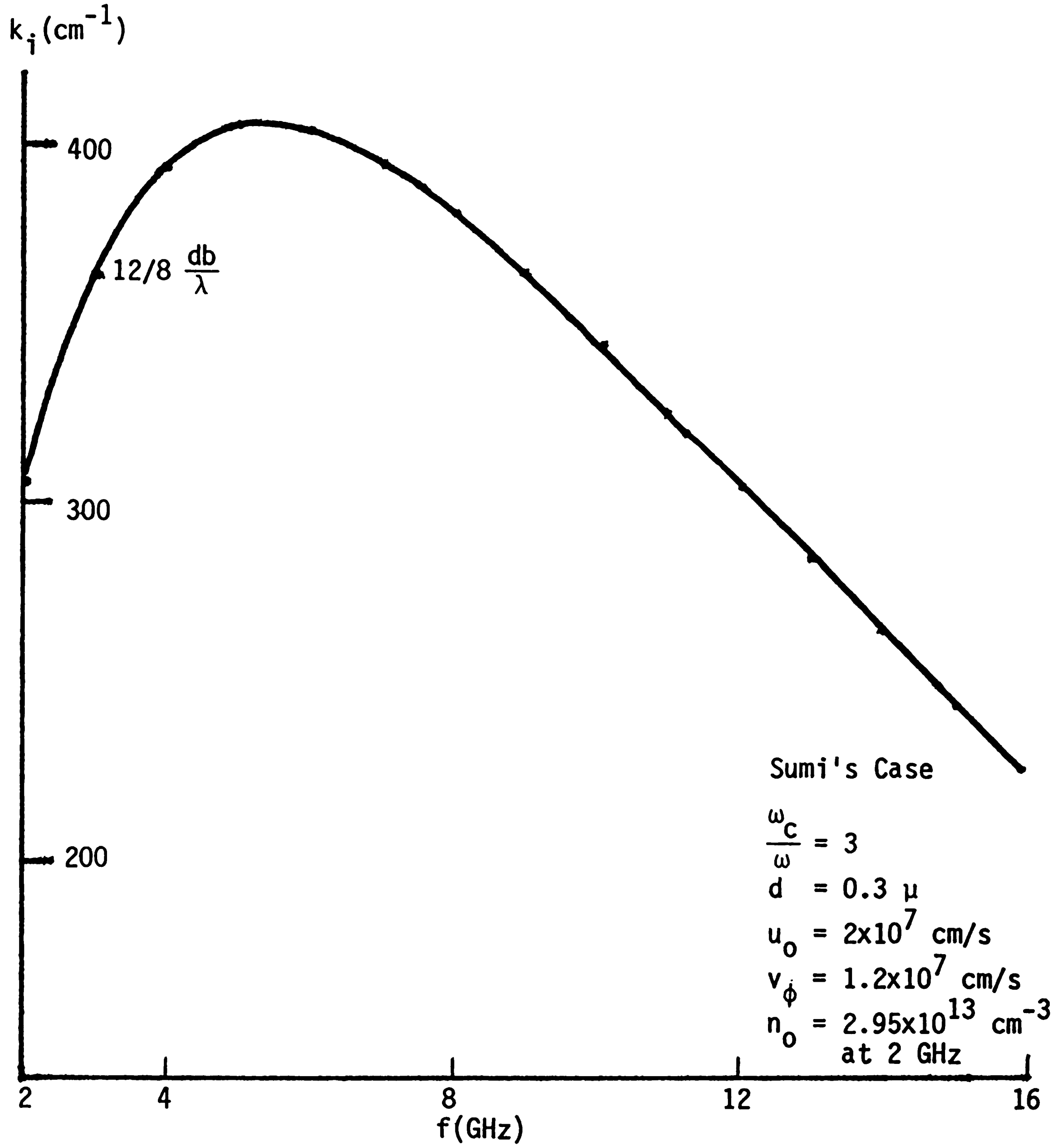


Figure 4. k_i vs. Operating Frequency

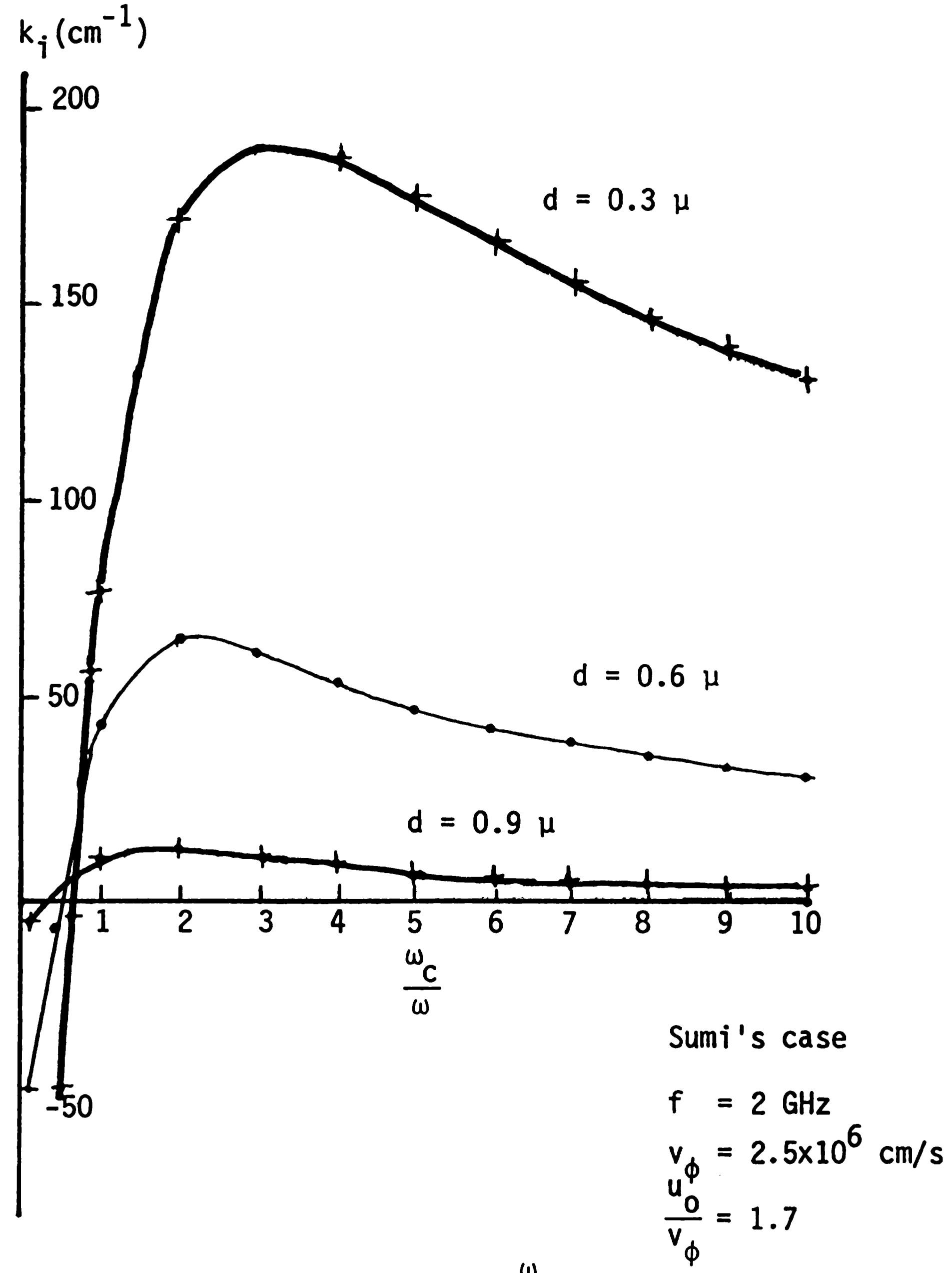


Figure 5. k_i vs. $\frac{\omega_c}{\omega}$

that. Thus the carrier density η_0 is an important factor in the coupling mechanism. Also, the threshold and the magnitude of the gain decreases in general as d increases. The reason for the decrease in gain magnitude is the less effective interaction between the circuit and the beam as the separation between them is increased.

In Figures 6 and 7, the general variation of the real and imaginary part of k is shown as a function of the drift velocity. Once again near-synchronism is a criterion for gain. Maximum gain occurs when drift velocity is around 1.5 times that of the phase velocity of the line. A closer comparison shows a surprising fact: greater separation does not show a reduction in gain. This result was observed by Freeman [29] in his theoretical analysis of the behavior of the roots. Figure 8 reveals the existence of an optimal distance of separation for the case we study.

Figure 9 shows that as the distance d increases k_r approaches k_0; the system decouples and only the normal mode(s) exists in the line.

The above illustrations conclude our investigation on the stiff boundary case pioneered by Sumi. We now turn to the case with the transverse magnetic field and the modification in γ_2.

As expected, the plot in Figure 10 resembles that of Figure 5 very closely. The only noted difference between the two is the lowering of the threshold for gain. This is understandable because if $\alpha = 0$, then the only contrast remaining is the modification of γ_2.

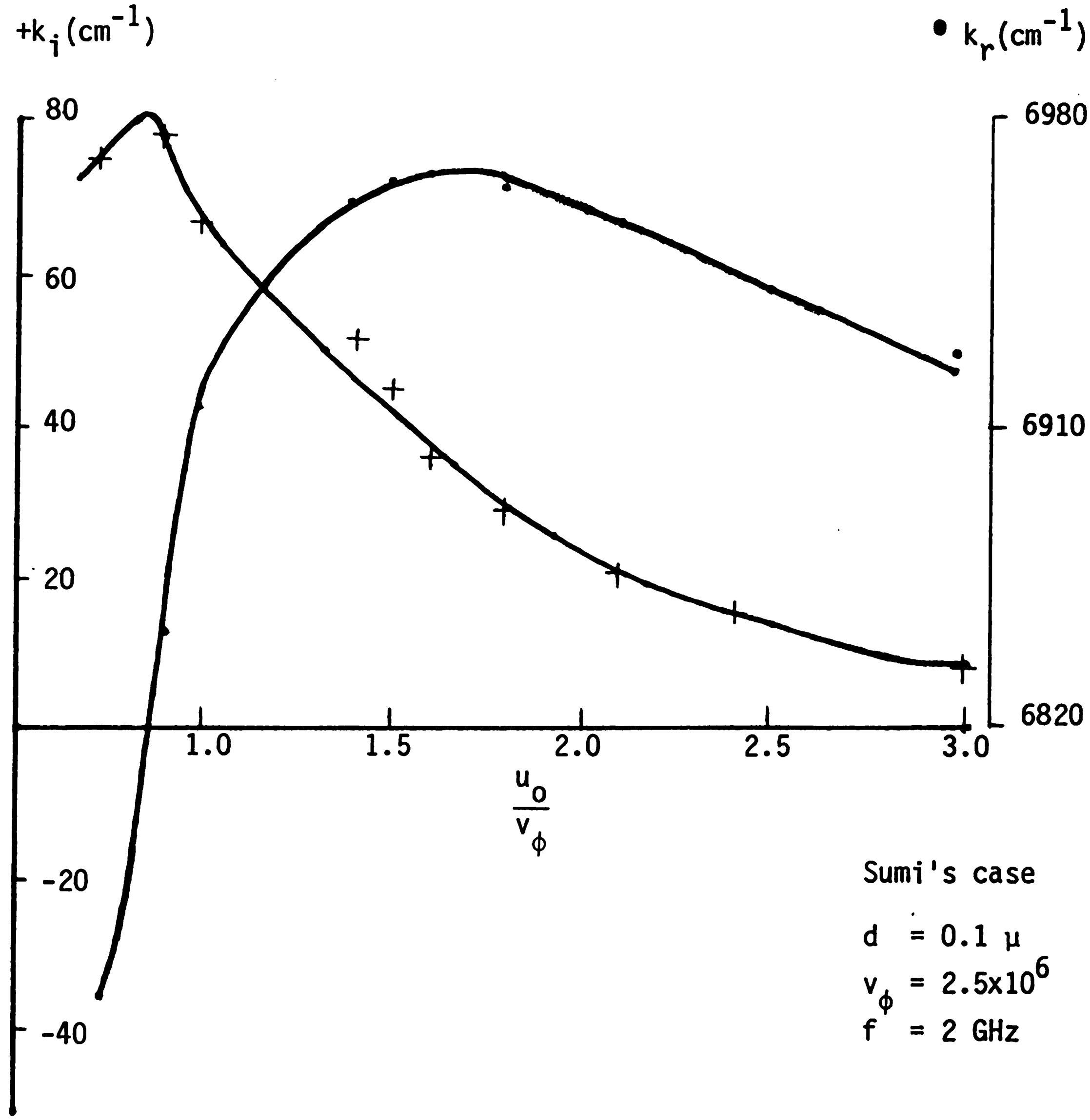


Figure 6. k vs. $\frac{u_o}{v_\phi}$ $(d = 0.1\ \mu)$

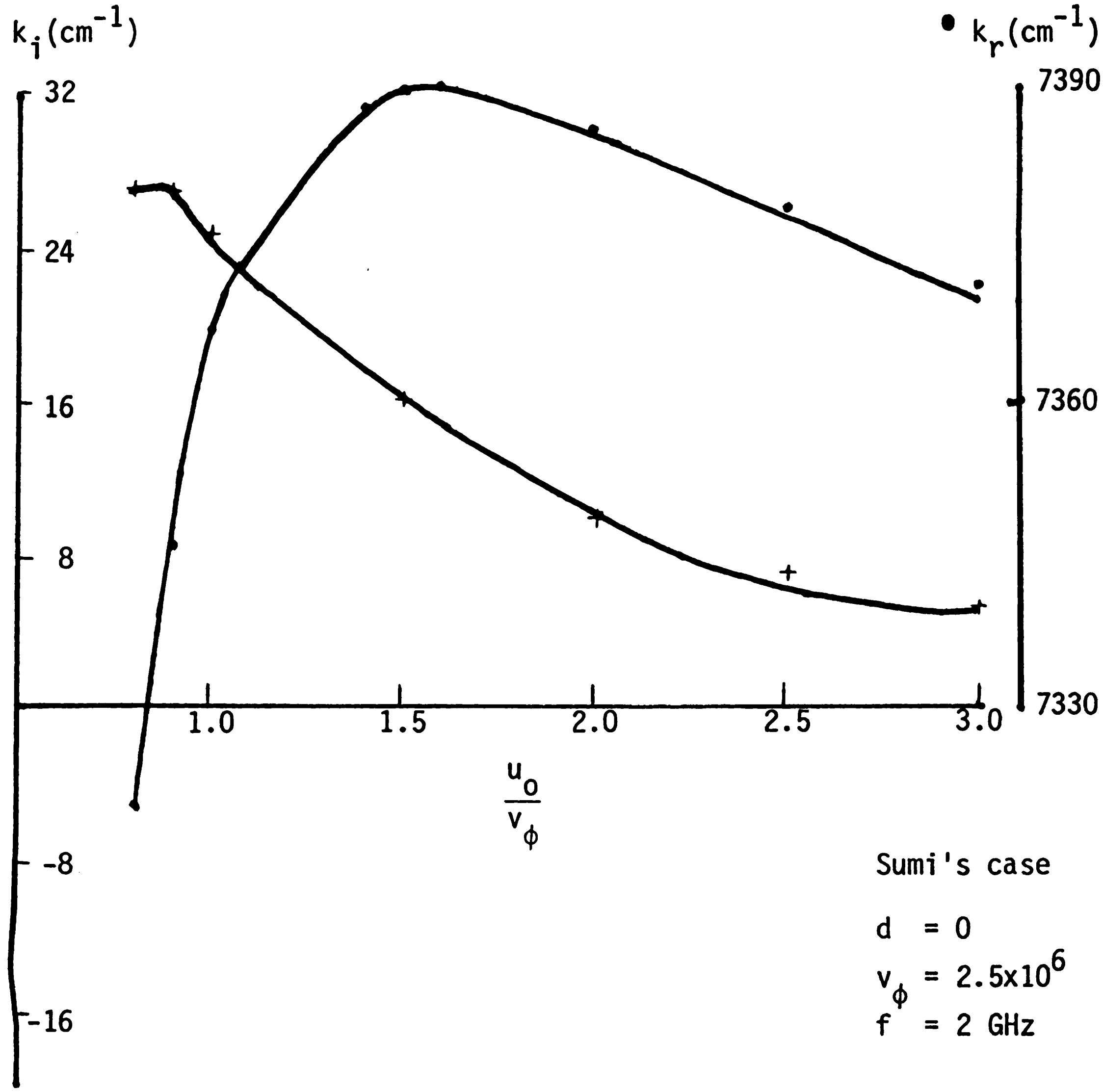


Figure 7. k vs. $\frac{u_o}{v_\phi}$ (d = 0)

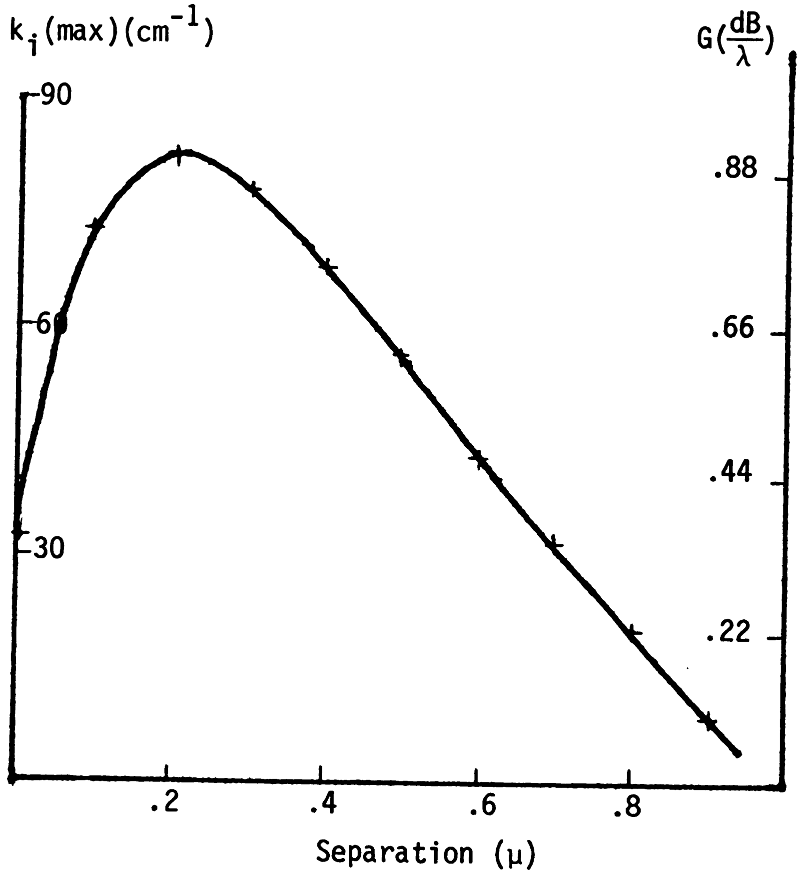
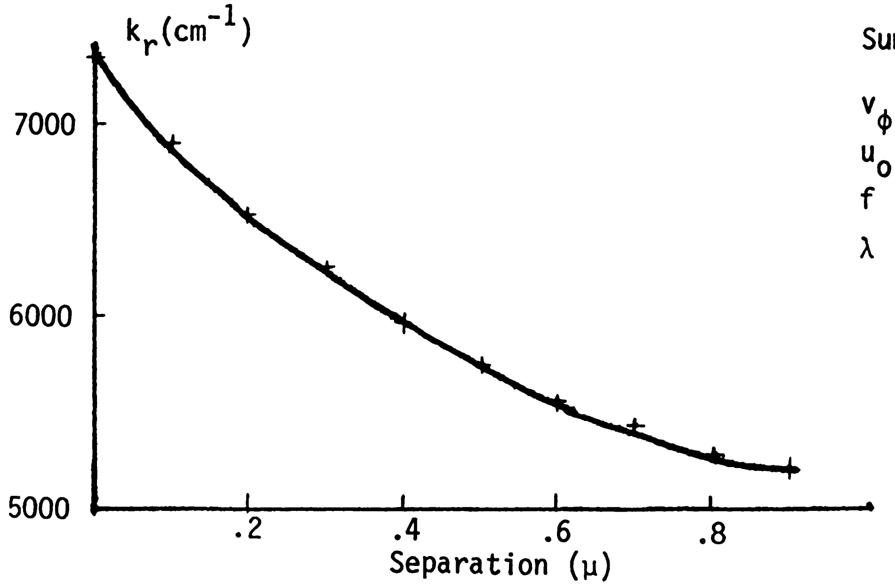


Figure 8. $k_i(max)$ vs. d



Sumi's case

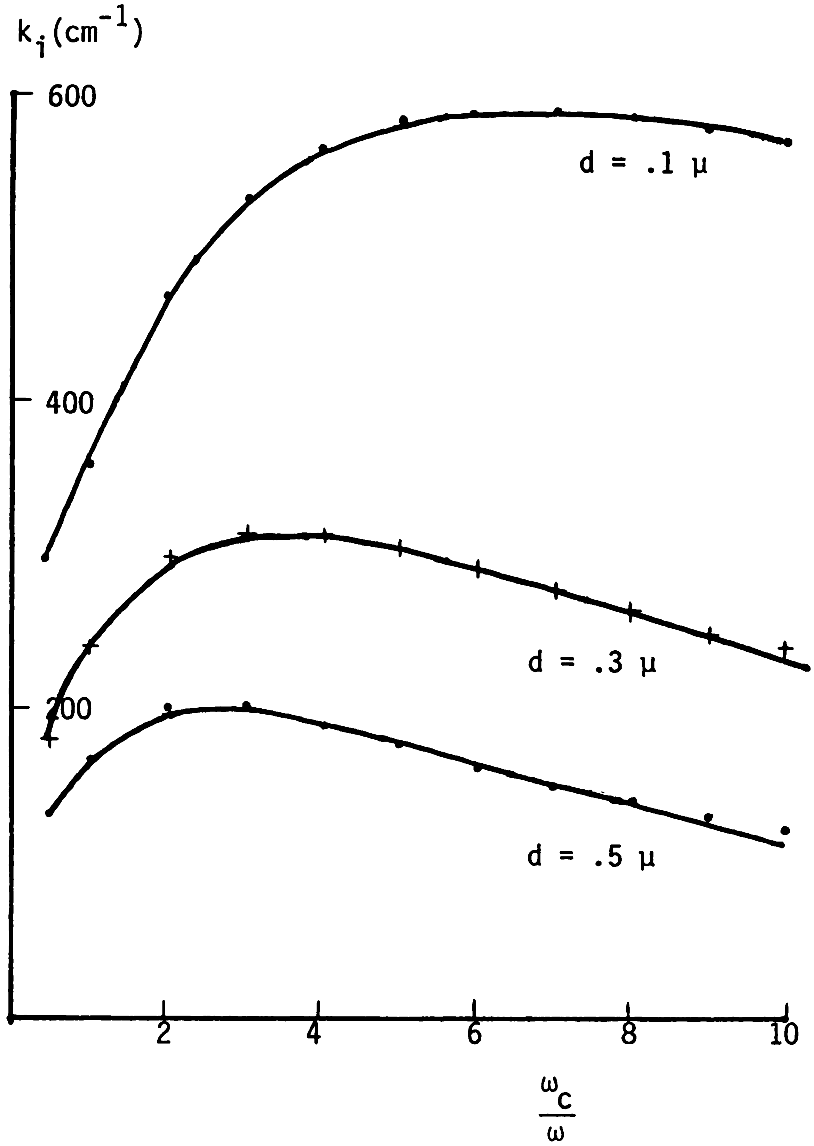
$v_\phi = 2.5x10^6$

$u_o = 5.0x10^6$

$f = 2$ GHZ

$\lambda \doteq 10.5\ \mu$

Figure 9. Wavenumber at maximum gain vs. d



$\alpha = 0$

$v_\phi = 5x10^6$

$\frac{u_o}{v_\phi} = 1.8$

$f = 2$ GHZ

Figure 10. k_i vs. $\frac{\omega_c}{\omega}$ of case II

The variation of the imaginary part of k as a function of α is shown in the next three figures. In Figure 11, k_i increases to a maximum value, then decreases gradually afterward as we increase the magnetic flux. However, in Figure 12, the variation of k_i does not follow the same trend as in the previous case. The magnitude of k_i actually decreases as we increase α, reaching a minimum, then rebounding to a less negative value. Figure 13 shows still another variation. Note that the conduction frequency ω_c is the only significant parameter varying in these cases. It seems as if the introduction of magnetic field will disturb the synchronism of the system. If the system is already at synchronism ($\frac{\omega_c}{\omega} = 1$), an increase of magnetic flux has the effect of "lowering" ω_c to below synchronism, thus the decrease in gain. However, if the system is much beyond the synchronous condition, the applied $\vec{B}_o$ field will bring the system into synchronism. This effect becomes more apparent if we examine the expression for ω_c:

$$\omega_c = \frac{n|e|\mu_m}{\varepsilon} \quad .$$

Due to the applied $\vec{B}_o$ field, the carrier mobility μ_m is reduced to

$$\mu_m^* = \mu_m \left(\frac{1}{1 + \alpha^2}\right) \quad .$$

Thus, the "effective" conduction frequency is

$$\omega_c^* = \omega_c\left(\frac{1}{1 + \alpha^2}\right) \quad .$$

Physically, this decrease in mobility probably reduces and limits the amplitude of the a.c. motion of the carrier.

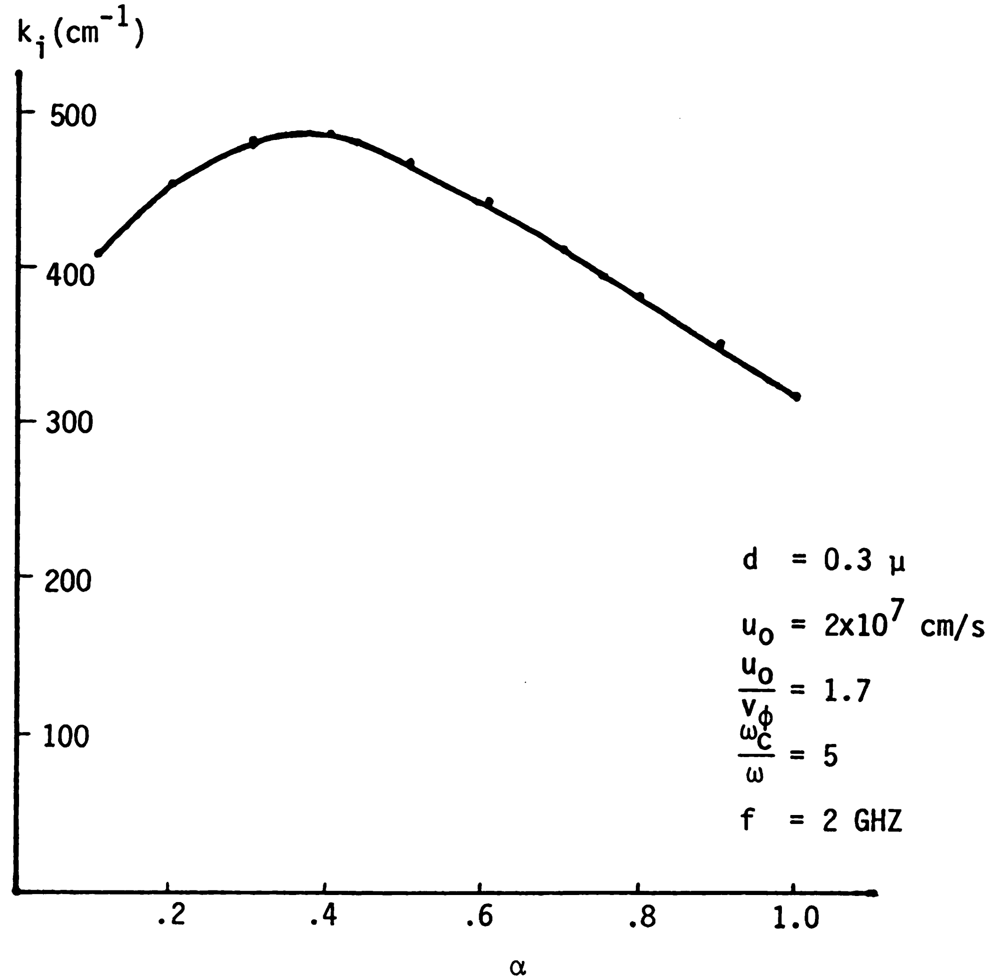


Figure 11. k_i vs. α ($\frac{\omega_c}{\omega} = 5$) of case II

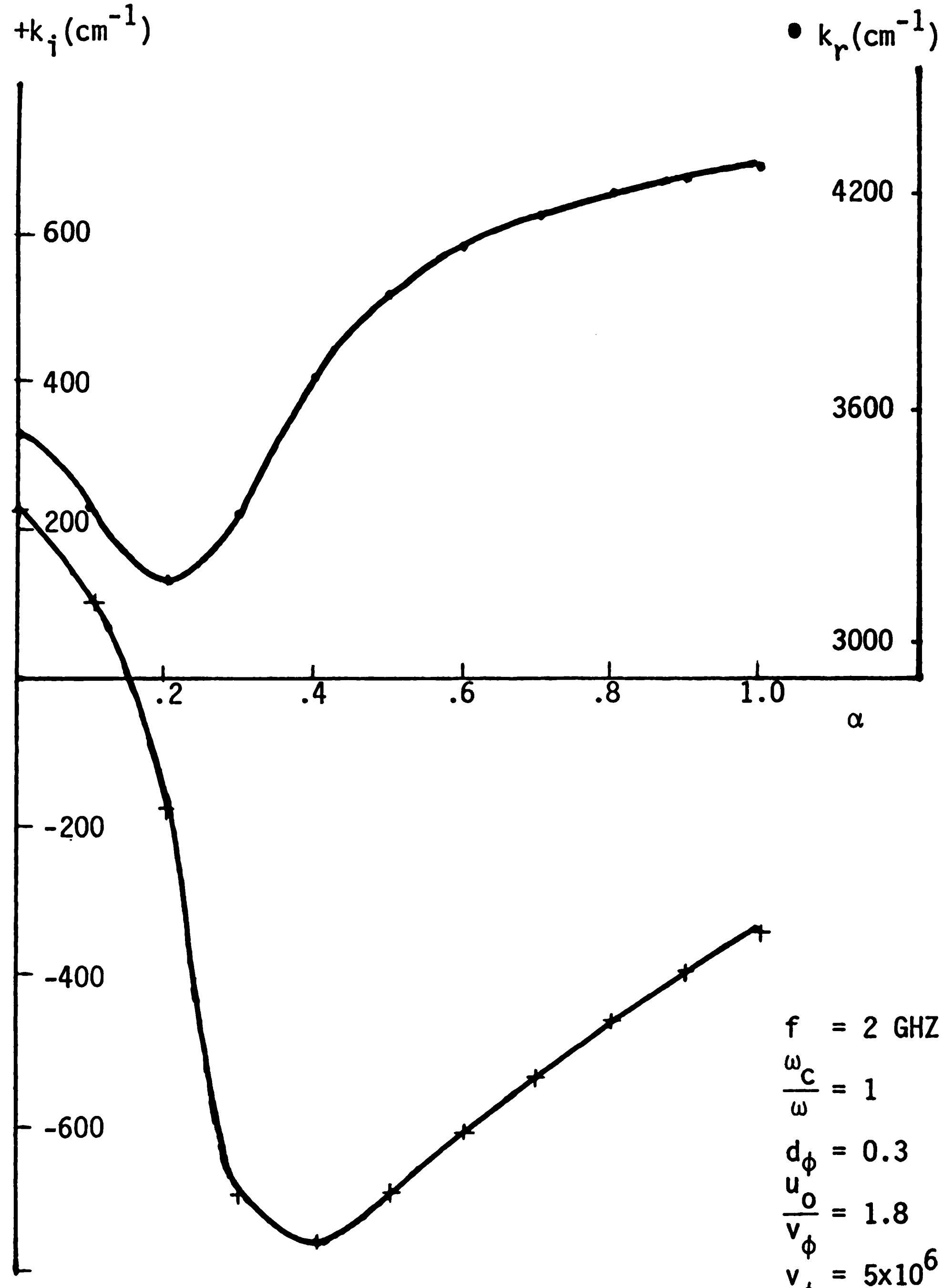
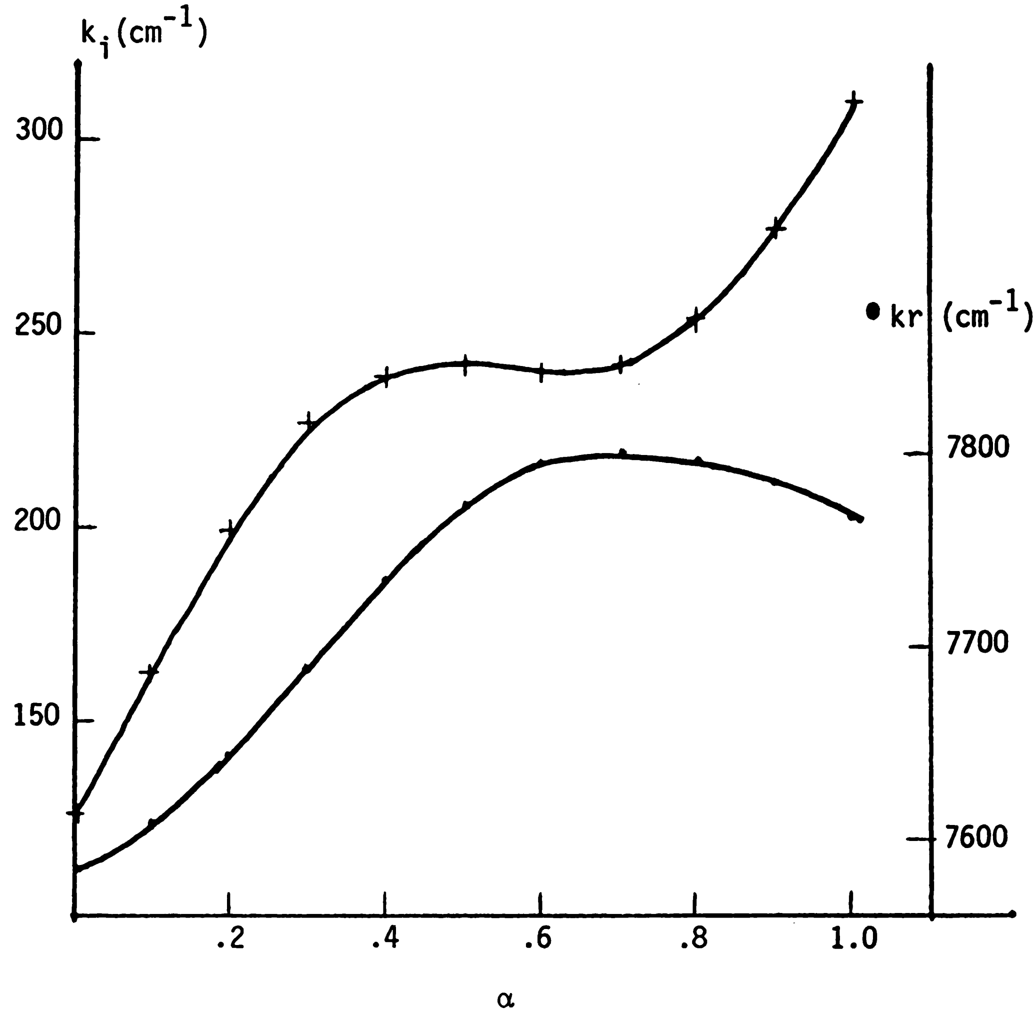


Figure 12. k vs. α ($\frac{\omega_c}{\omega}$ = 1) of case II



$f = 10$ GHZ

$u_o = 2 \times 10^7$ cm/s

$\frac{u_o}{v_\phi} = 1.7$

$\frac{\omega_c}{\omega} = 15$

$d = 0.3\ \mu$

Figure 13. k vs. α ($\frac{\omega_c}{\omega} = 15$) of case II

Quite different from the Sumi case, the imaginary part of k decreases monotonically as the distance d increases (as shown in Figure 14). One possible explanation could be the effect of the magnetic field on the "diffusion" wave γ_2. As α increases, this mode may become more "damped" and cancel any benefit from an optimal spacing. This is merely a speculation and the problem is open to discussion.

Figure 15 shows that as separation increases, k_r approaches the normal mode k_o.

Near-synchronism as a necessary condition for growth is demonstrated in Figure 16.

Figure 17 shows the growth rate as a function of operating frequency.

The above illustrations show some resemblance to the Sumi case as well as some marked differences. One thing to remember is that both cases assume a stiff boundary condition. It would be interesting to see how the "ripple" boundary case compares with these results.

As shown in Figure 18, the "window" for gain is much narrower and the decay much more abrupt than in the previous cases. Gain is possible only when ω_c is about 2ω to 4ω, though the gain can reach as high as 20 dB per wavelength. Figures 19 and 20 show the variation of k_i as a function of the ripple "depth" m. The gain increases as m increases. This result might not be as meaningful if we consider the physical viewpoint. Assuming the "ripple" is

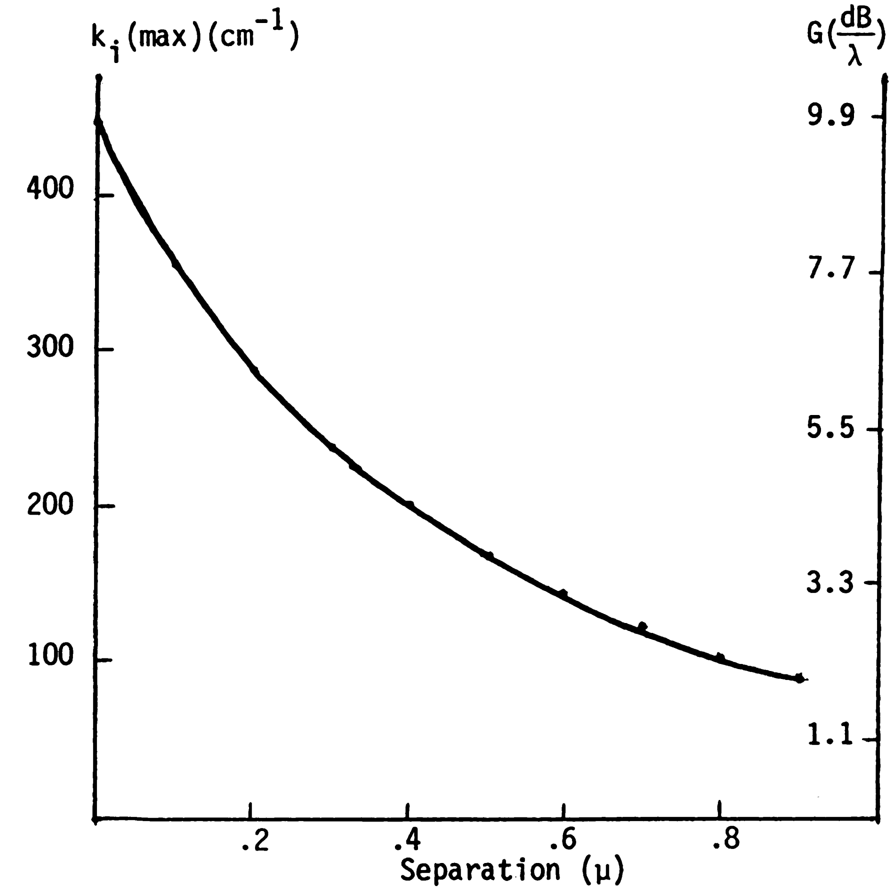


Figure 14. k_i vs. d (case II)

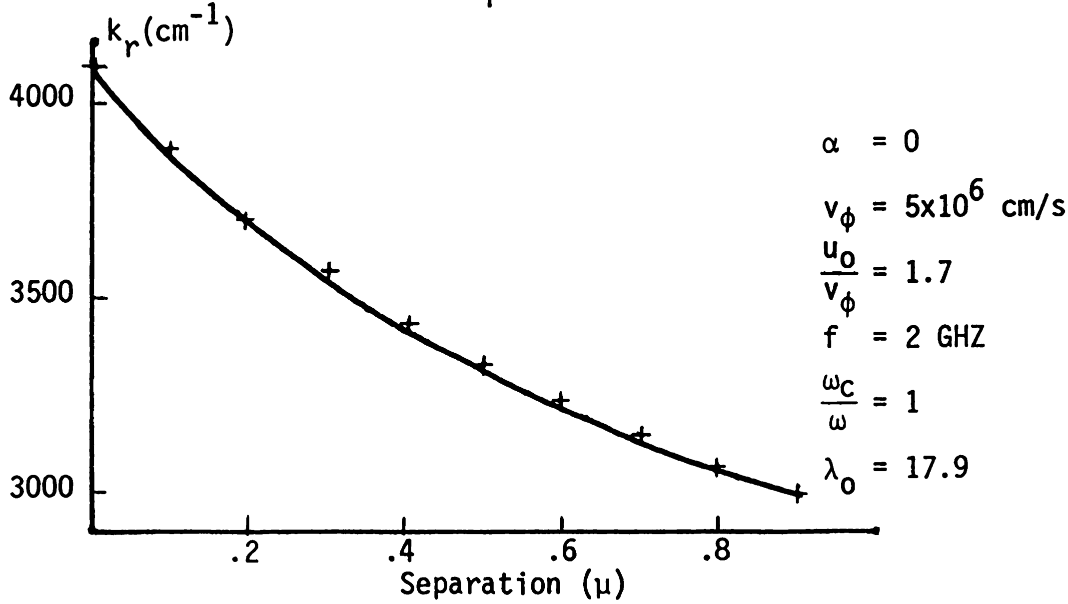


Figure 15. Wavenumber at maximum gain vs. d (case II)

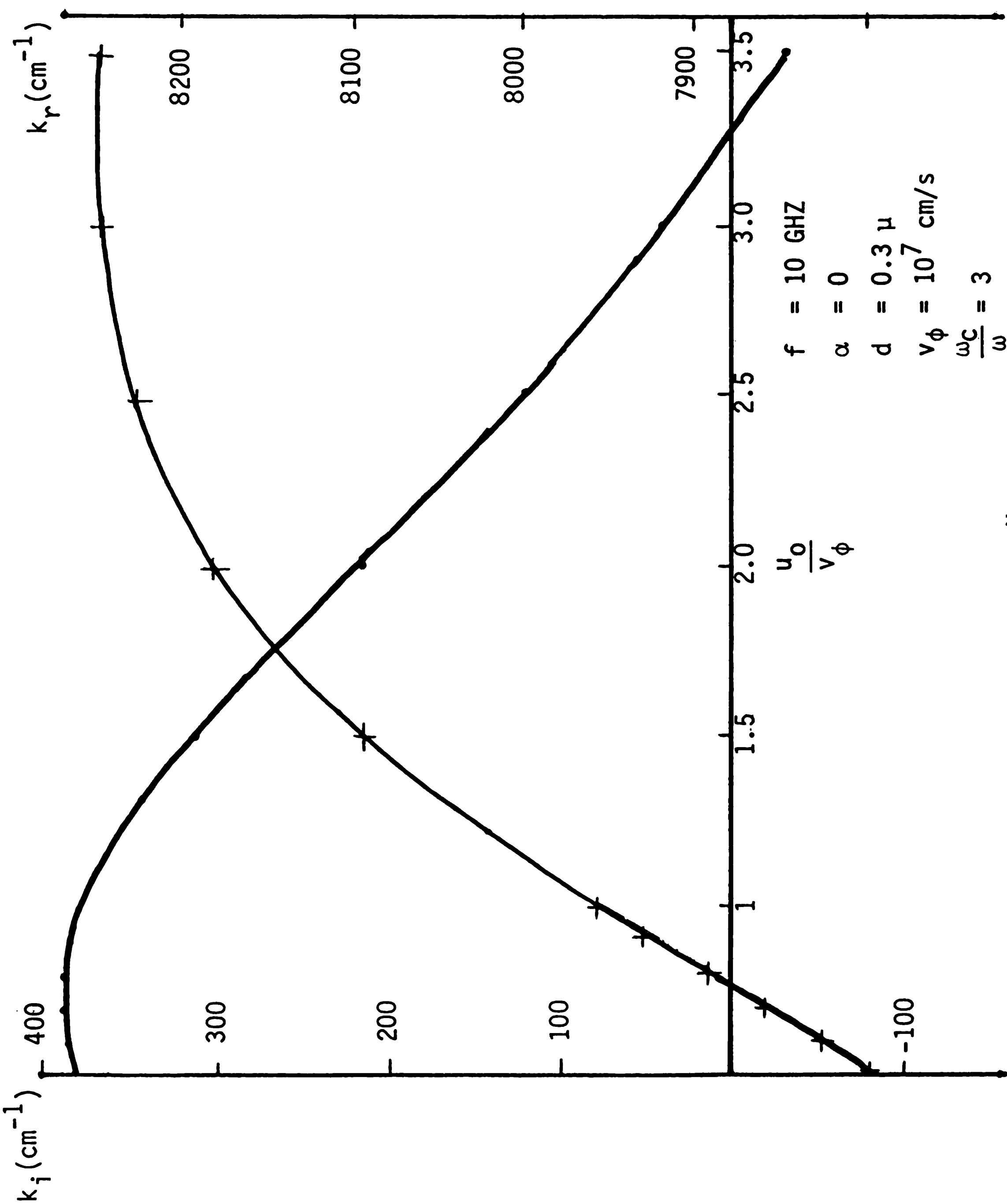


Figure 16. k vs. $\frac{u_o}{v_\phi}$ of case II

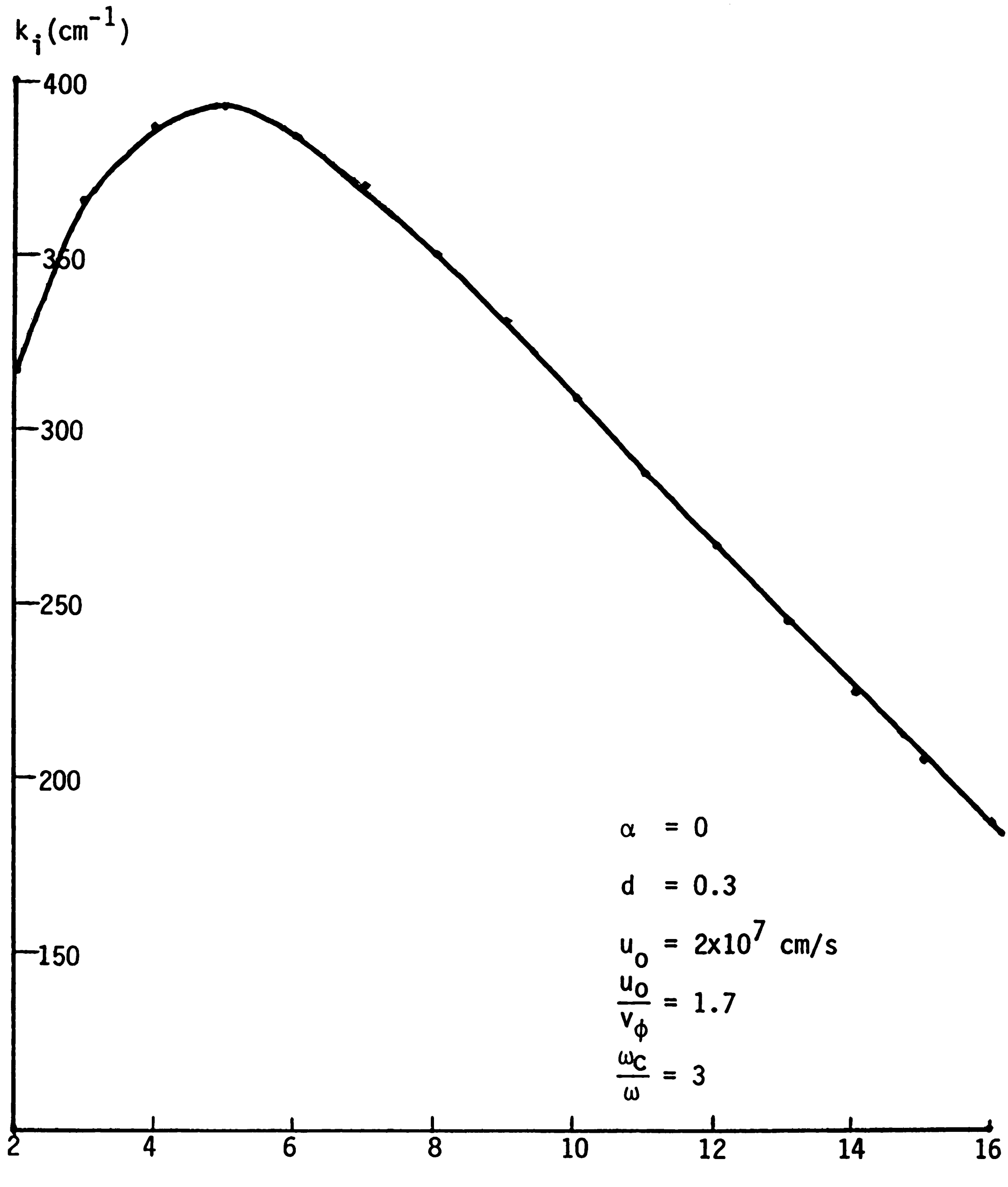


Figure 17. k_i vs. operating frequency (case II)

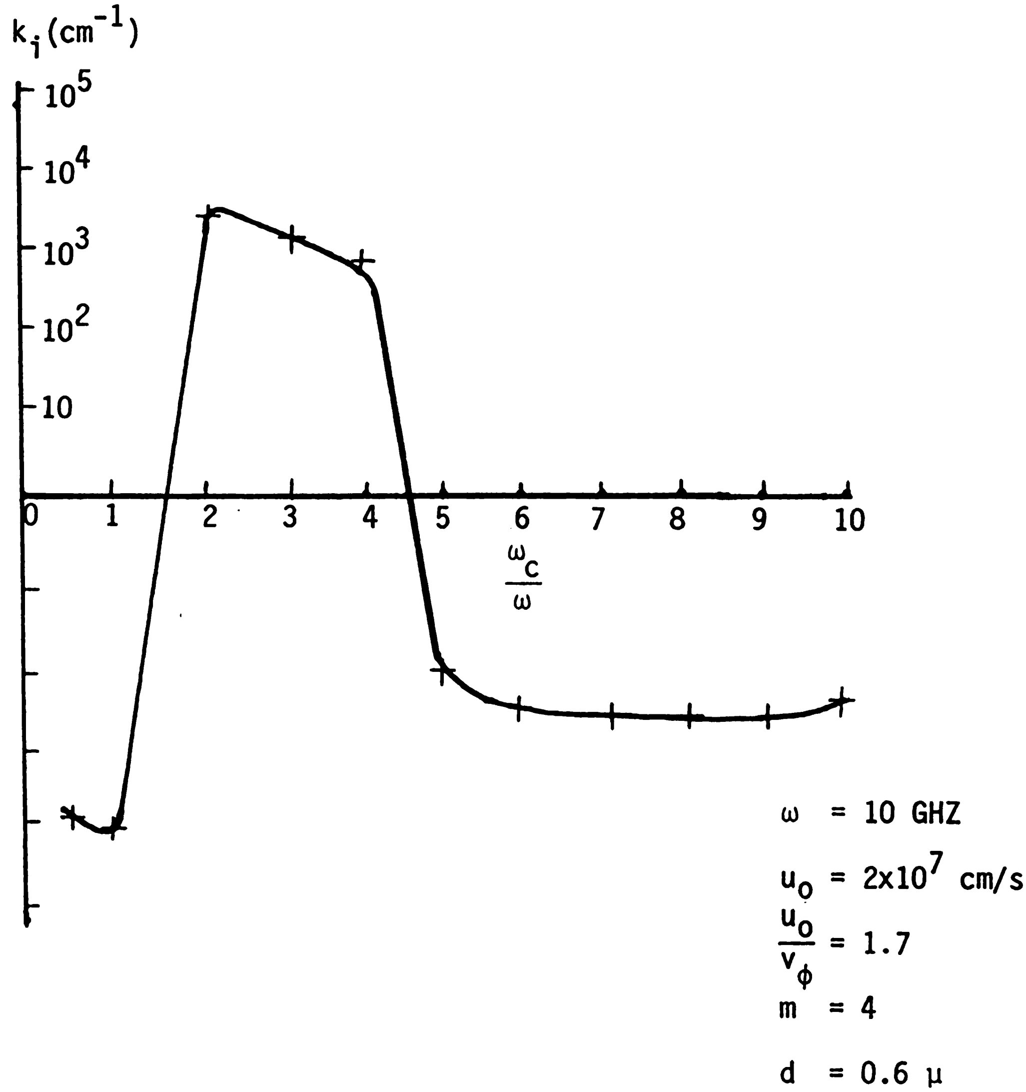
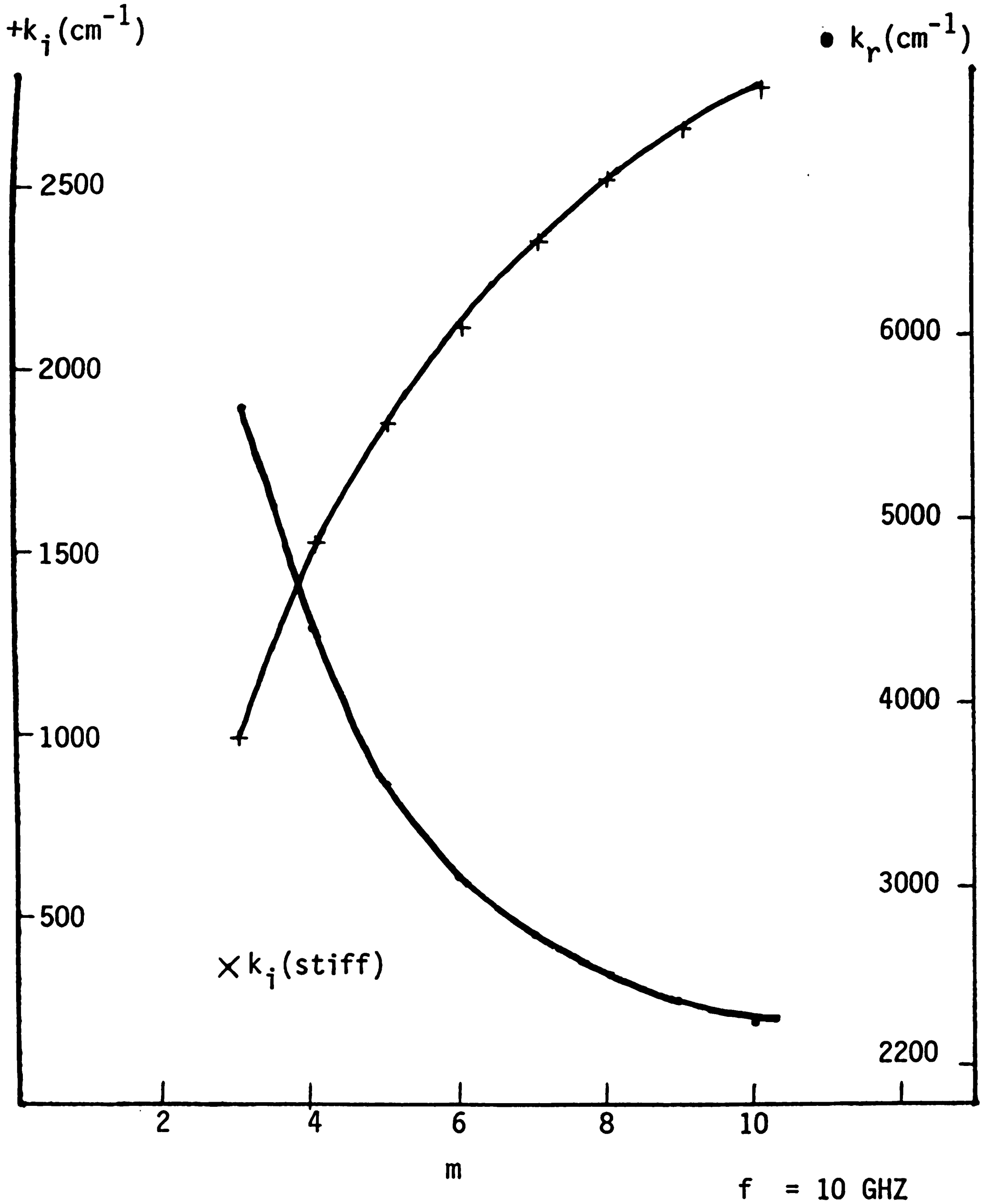


Figure 18. k_i vs. $\frac{\omega_c}{\omega}$ of the ripple boundary case



f = 10 GHZ

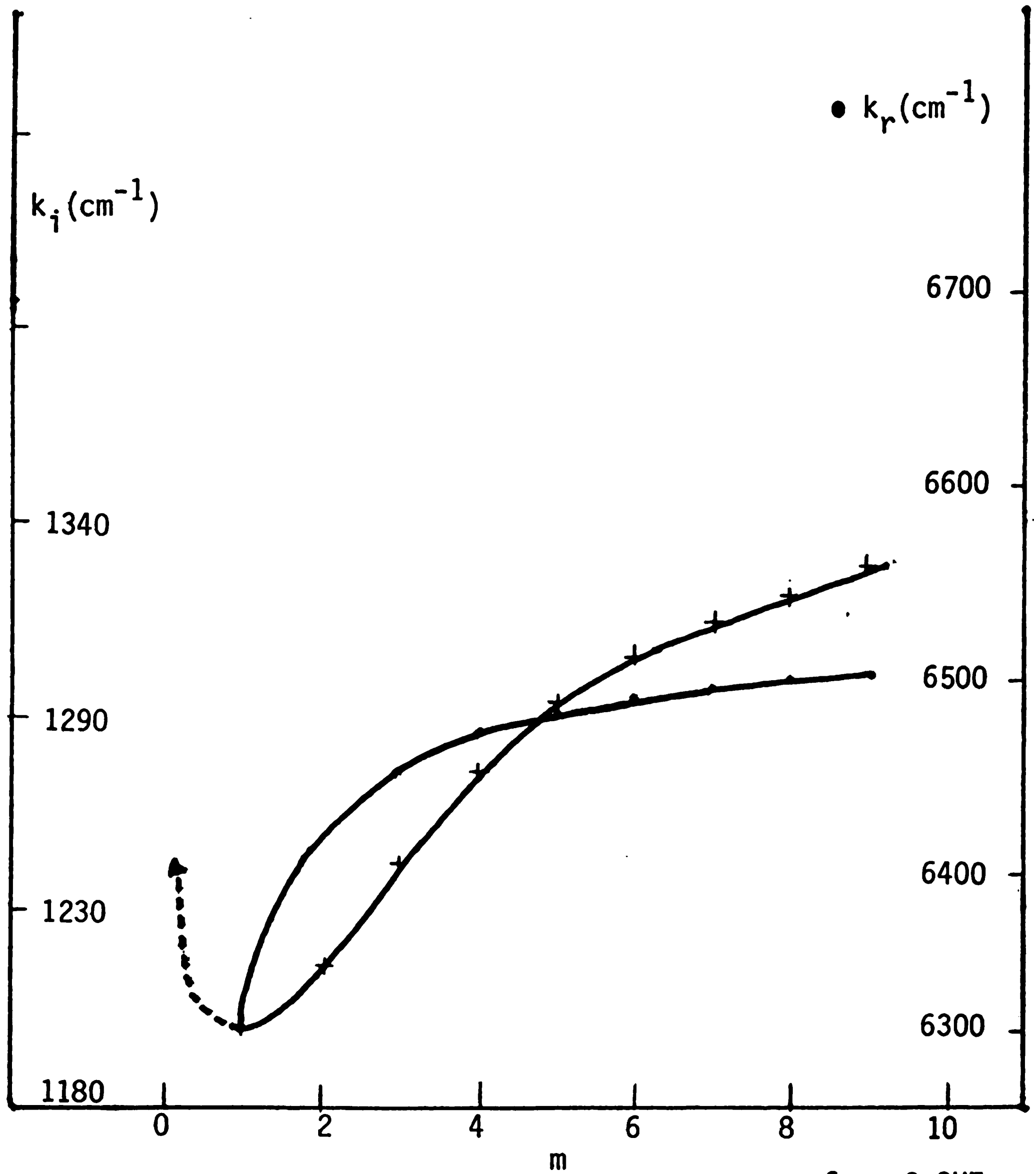
v_ϕ = 1.18x10^7 cm/s

u_o = 2x10^7 cm/s

d = 0.6 μ

$\frac{\omega_c}{\omega}$ = 3

Figure 19. k vs. the depth factor m (ripple boundary case)



f = 2 GHZ

d = 0.3 μ

v_ϕ = $5x10^6$ cm/s

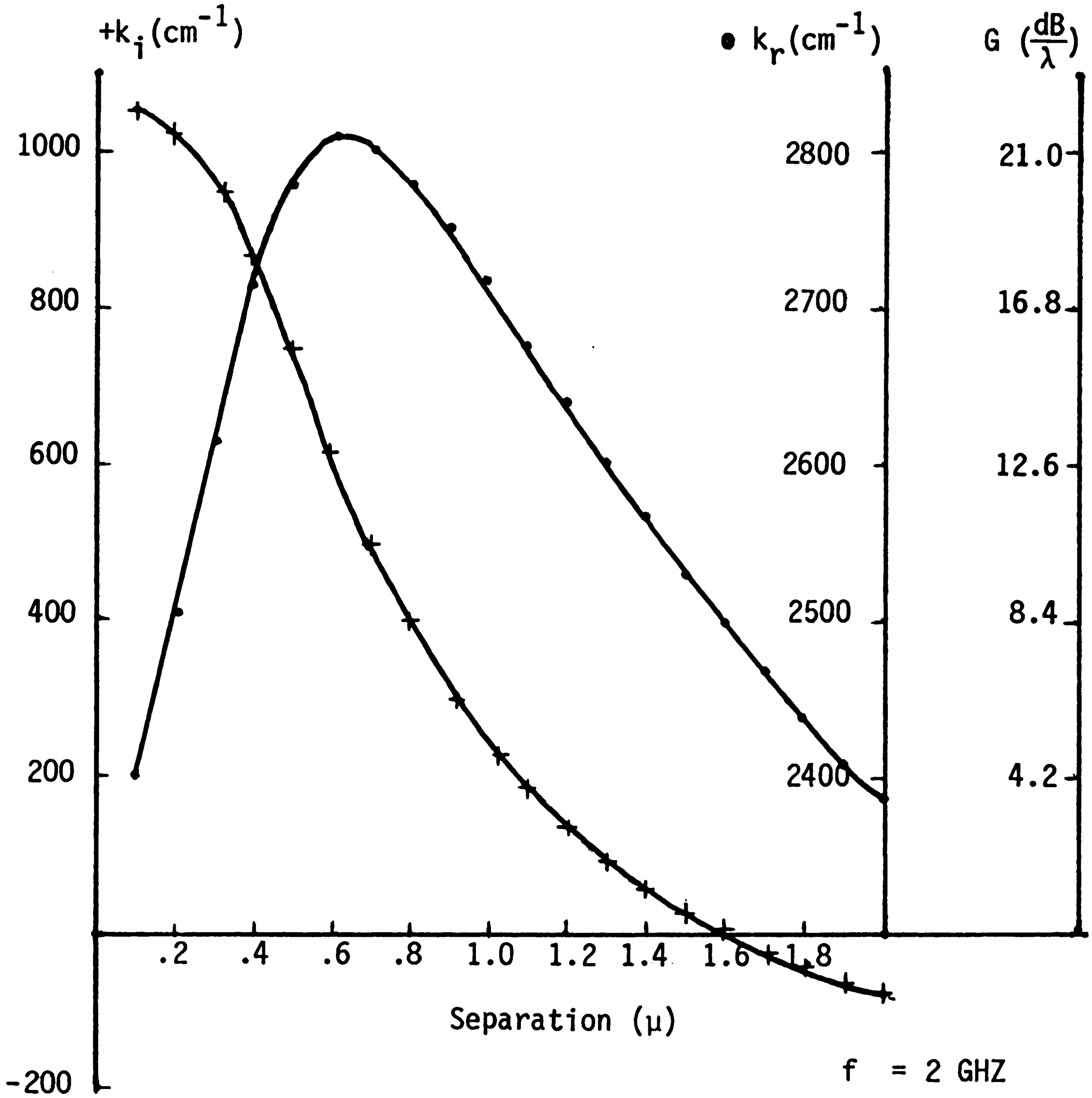
u_o = $9x10^6$ cm/s

Figure 20. k_i vs. m ($\frac{\omega_c}{\omega} = 1$), $\frac{\omega_c}{\omega} = 1$ (ripple boundary case)

symmetrical about the x = 0 line, the "ripple" might extend well into the insulator region if m is large. This would imply that carriers are injected into the insulator!

In Figure 21, k_i decreases if the separation is increased while k_r increases to an optimal value of 2800 cm^{-1} before decreasing.

In Figure 22, we confirm the necessity of near-sychronism for gain.



$f = 2$ GHZ

$m = 3$

$u_o = 2\times10^7$ cm/s

$v_\phi = 1.1\times10^7$ cm/s

$\frac{\omega_c}{\omega} = 1$

Figure 21. k vs. separation in the ripple boundary case

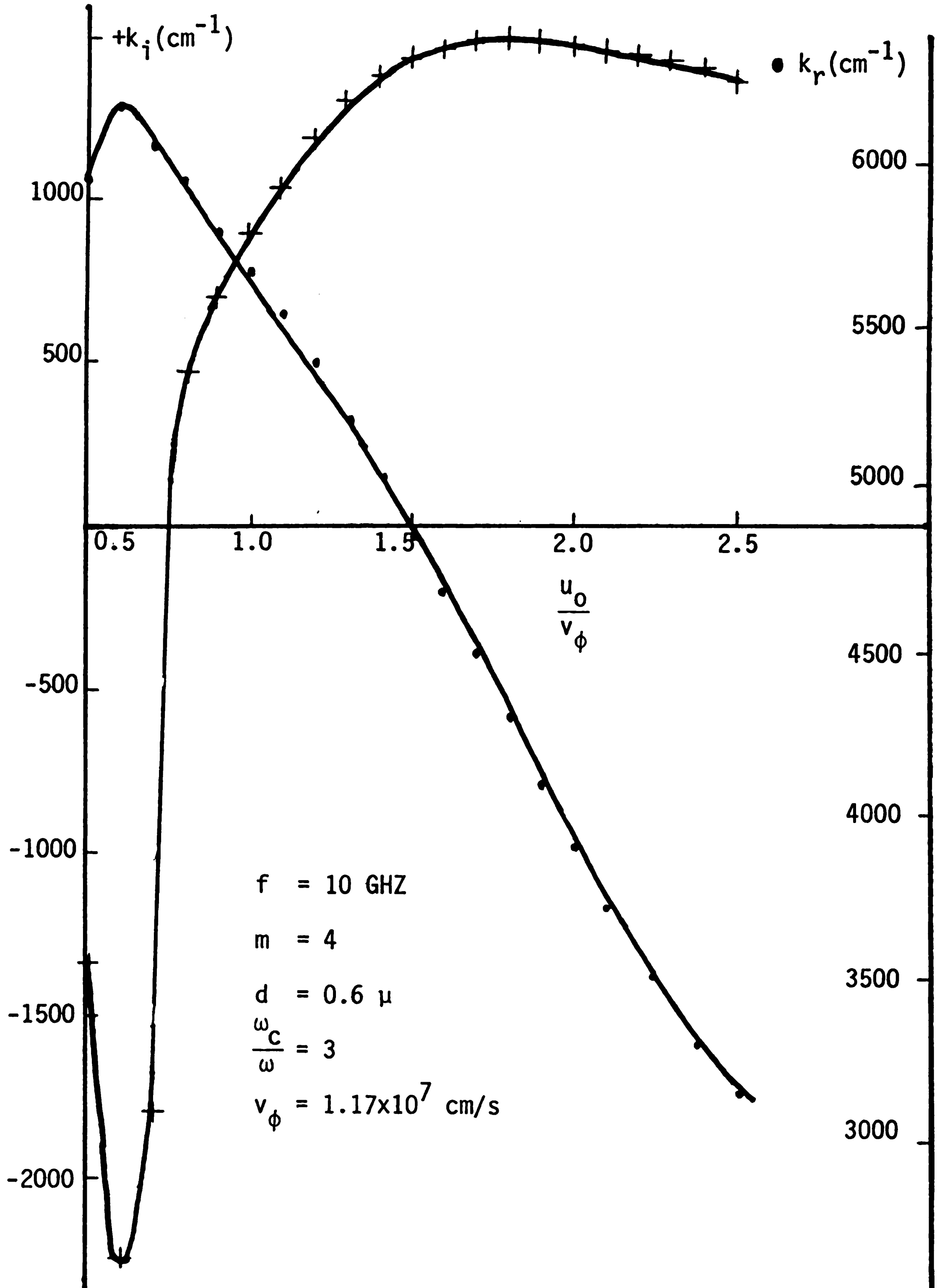


Figure 22. k vs. $\frac{u_o}{v_\phi}$ in the ripple boundary case

4. SUMMARY AND CONCLUSION

4.1. Summary

An analysis of the interaction between slow circuit waves guided by a meander line and drifting carrier in a semiconductor has been given. The system is decomposed into its components: the circuit and the carrier stream in the semiconductor. The investigation emphasizes the importance of the properties of the carriers at the surface and explores different models for its behavior. The theoretical treatment follows that of Sumi, Freeman, Okamoto, and Mizushima closely. Sumi's "stiff boundary" case is studied, and extended to include an external magnetic field. Secondly, the "rippled boundary" is applied and both modes are kept in our analysis. A new constraint is introduced to treat the ripple boundary condition. Evaluation of the dispersion relations is carried out numerically through the use of a Fortran subroutine. Possible convective instabilities are examined through criteria adopted from Bers and Briggs. Gain ranges and gain shapes are studied under varying conditions. Theoretical gain for STWA is predicted. Also, we find the frequency range for STWA operation to be around 27 GHz. This bandwidth is limited by the maximum carrier velocity obtainable in GaHs; 2×10^7 cm/sec. Theoretical gain for STWA is predicted.

4.2. Conclusion

The results from the different models are similarly encouraging. Amplification of signal due to a "collision dominated" carrier stream is possible theoretically if the conditions are proper. This conceptual problem can be viewed in light of the success of the AWA where the carrier stream is "collision dominated." Also, the interpretation of gain is supported by the Bers and Briggs criteria used consistently in the investigation.

The importance of synchronism in velocities and frequencies demonstrates itself in our study and cannot be understated. Gain is predicted whenever $\frac{u_o}{v_\phi}$ and $\frac{\omega_c}{\omega}$ are near unity throughout our investigation. Other factors which might enhance the possibility of gain include:

(1) the separation between circuit and semiconductor;

(2) the application of a transverse magnetic field to "tune" ω_c, thereby bringing $\frac{\omega_c}{\omega}$ toward unity when interaction occurs; and

(3) the carrier density of the semiconductor and ultimately the material itself.

In light of the results we gathered, I propose further investigation in the applicability of STWA.

APPENDICES

APPENDIX A

APPROXIMATION FOR THE DIFFUSION WAVE

The expression for γ_2 is

$$\gamma_2 = [k^2 + \frac{\omega_c + j(1 + \alpha^2)(\omega - ku_o)}{D}]^{\frac{1}{2}} .$$

Since

$$\frac{\omega_c}{D} >> k^2 \quad ,$$

then

$$\gamma_2 \doteq [\frac{\omega_c}{D} + j\,\frac{(1+\alpha^2)(\omega - ku_o)}{D}]^{\frac{1}{2}}$$

Assume k is purely real for the time being, and utilize the relationship

$$[x + jy]^{\frac{1}{2}} = r^{\frac{1}{2}}\ (\cos\frac{\theta}{2} + j \sin\frac{\theta}{2})$$

where

$$r = [x^2 + y^2]^{\frac{1}{2}},\ \cos\theta = \frac{x}{r} \text{ and } \sin\theta = \frac{y}{r} .$$

Then

$$r = [\frac{\omega_c^2}{D^r} + \frac{(1 + \alpha^2)^2(\omega - ku_o)^2}{D^2}]^{\frac{1}{2}} .$$

Assuming that in many cases,

$$\omega_c > (1 + \alpha^2)(\omega - ku_o) ,$$

then

$$r \doteq [\frac{\omega_c}{D}]^{\frac{1}{2}} .$$

Also,

$$\cos\frac{\theta}{2} = [\frac{1}{2}(1 + \cos\theta)]^{\frac{1}{2}}$$

$$= [\frac{1}{2}(1 + \frac{x}{r})]^{\frac{1}{2}}$$

$$= [\frac{4\omega_c^2 + (1 + \alpha^2)^2(\omega - ku_o)^2}{4\omega_c^2 + 2(1 + \alpha^2)^2(\omega - ku_o)}]^{\frac{1}{2}}$$

$$\cong 1$$

and

$$\sin\frac{\theta}{2} = [\frac{1}{2}(1 - \frac{x}{r})]^{\frac{1}{2}}$$

$$= [\frac{(1 + \alpha^2)^2(\omega - ku_o)^2}{4\omega_c^2 + 2(1 + \alpha^2)^2(\omega - ku_o)^2}]^{\frac{1}{2}}$$

$$\cong \frac{(1 + \alpha^2)(\omega - ku_o)}{2\omega_c} .$$

Thus,

$$\gamma_2 \cong [\frac{\omega_c}{D}]^{\frac{1}{2}}[1 + j\,\frac{(1 + \alpha^2)\bar{\omega}}{2\omega_c}] .$$

APPENDIX B

APPROXIMATION FOR exp $[m\lambda_D(\gamma_2 - \gamma_1)]$

We have

$$\gamma_2 \doteq \frac{1}{\lambda_D} [1 + j \frac{\bar{\omega}}{2\omega_c} (1 + \alpha^2)]$$

$$\gamma_1 \doteq k \ .$$

Therefore,

$$\exp [m\lambda_D(\gamma_2 - \gamma_1)] = \exp [m(1 - k\lambda_D)]\exp jm [\frac{\bar{\omega}(1 + \alpha^2)}{2\omega_c} \ .$$

Now,

$$\exp (x) \doteq 1 + x + \frac{x^2}{2!} + \ldots \ .$$

We will take a linear approximation for simplicity even though error will be introduced. Thus,

$$\exp [m\lambda_D(\gamma_2 - \gamma_1)] \doteq$$

$$[1 + m - mk\lambda_D]\{1 - [\frac{m\bar{\omega}(1 + \alpha^2)}{2\omega_c}]^2 + j[\frac{m\bar{\omega}(1 + \alpha^2)}{2\omega_c}]\}$$

Assume m to be no less than three;

$$\exp [m\lambda(\gamma_2 - \gamma_1)] \doteq \frac{m^2\bar{\omega}}{2\omega_c} (j - \frac{m\bar{\omega}}{2\omega_c}) \ .$$

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