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ABSTRACT

A THEORETICAL AND EXPERIMENTAL ANALYSIS OF ANTENNAS IN BOUNDED MEDIA WITH APPLICATION TO ELECTROMAGNETIC PROBING

By

Carl Edward Grove

A theoretical and experimental investigation is made of coupled linear antennas in rectangular waveguide filled with a homogeneous, linear, isotropic medium. It is assumed that the antennas do not span the waveguide, the waveguide has arbitrary terminations and the medium is known in terms of its macroscopic electrical parameters. The integral equations for the unknown antenna currents are found by applying modal analysis and the Lorentz reciprocity relation to the electric field boundary condition on the antenna. First a delta function source model is assumed with both the thin-wire and thin-strip approximations made for the antenna. The integral equations are then solved numerically by the method of moments and the thin-strip approximation is shown to exhibit improved convergence properties over the conventional thin-wire approximation. The modal series in this method of moments solution are related to sampling theory for estimating optimum numerical truncation points. Besides the delta function model, the source is also modeled as a TEM mode coaxial aperture (for a single antenna only) in a parallel development. The coaxial contribution

to the fields in the waveguide is found as Bessel function series and the numerical solution is used in comparing with the delta-gap model infinite input susceptance. An experimental system is described for verifying the theory at L-band frequencies for semi-infinite waveguide and for the cavity configuration with air and dissipative media. Although verification is obtained by systematically varying all possible experimental parameters, emphasis is placed on the single antenna in a cavity. From the extensive data it is found that new and unique antenna current distributions exist which relate primarily to cavity resonance coupling (and not simply to the typical antenna height resonance). Finally, experimental results for distilled water, salt water solution and a sample of sewage treatment plant effluent give support to the fundamental proposition of this research, namely: that the antenna-waveguide-media system can be effectively used as a tool for environmental media monitoring.

A THEORETICAL AND EXPERIMENTAL ANALYSIS OF ANTENNAS
IN BOUNDED MEDIA WITH APPLICATION TO
ELECTROMAGNETIC PROBING

By

Carl Edward Grove

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Electrical Engineering and Systems Science

1975

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From the beginning of my graduate education to the conclusion of my research I have continually needed encouragement, guidance, criticism, and advice. For this and for the lasting influence it has had on my personal and technical development, I will be forever grateful for the wise and willing counsel of Dr. David Fisher.

From conception to completion the work for this dissertation has required not only my own devotion but also that of an experienced advisor. I am thankful for the time and effort of Dr. Marvin Siegel who ably served in this capacity.

Also deserving of much credit for this research is Dr. Dennis Nyquist. His availability and patience for my endless stream of questions, as well as his knowledge of both the theoretical and experimental aspects of this problem, have become a benchmark for my own development.

I am also indebted to Dr. Ken-Mu Chen and Dr. Robert Ball whose time spent on the advisory committee helped form both the purpose and perspective of this dissertation.

Finally, I am anxious to recognize my wife, Carole, who has not only sustained me spiritually and physically through this endeavor, but whose hours of graphing data and copying manuscript have been invaluable in the compiling of this work.

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OUTLINE

	Page
PREFACE	
I. INTRODUCTION	1
1.1 Background	2
1.2 Discussion of the Problem and Related Works	4
1.3 Method of Investigation	9
II. THEORETICAL DEVELOPMENT	13
2.1 The Model	13
2.2 Boundary Conditions; Slice Generator Approximation	17
2.3 Electric Field in the Waveguide	20
2.4 Integral Equations; Reduction to Hallen-Type Equations	34
2.5 Cylindrical Antenna Approximations	38
2.5.1 Thin-Wire Approximation	39
2.5.2 Strip Approximations	40
2.6 Coaxial Gap Solution	48
2.7 Conclusion; Comparison with Literature	61
III. NUMERICAL METHODS	66
3.1 Matrix Equation Representation	67
3.1.1 Method of Subsections	69
3.1.2 Method of Point Matching	70
3.1.3 Galerkin's Method	72
3.1.4 Matrix Equations for the Strip and Coaxial Gap Solutions	76
3.1.5 Numerical Determination of Charge Distribution	82
3.2 Convergence Properties	83
3.3 Comparison of the Solutions	100
IV. EXPERIMENTAL INVESTIGATIONS	109
4.1 Description of Apparatus	110
4.2 Experimental Methods	128

	Page
V. NUMERICAL AND EXPERIMENTAL RESULTS	140
5.1 Correlation of Theoretical and Experimental Input Admittance	143
5.1.1 Infinite Driving Point Susceptance	143
5.1.2 Terminal Zone Correction Network	146
5.2 Single Antenna	156
5.2.1 Waveguide with Matched Termination	157
5.2.2 Air Filled Cavity; Current Distribution	172
5.2.3 Air Filled Cavity; Charge Distribution	215
5.2.4 Air Filled Cavity; Input Admittance	220
5.2.5 Cavity Filled with Dissipative Media; Current Distribution	252
5.2.6 Cavity Filled with Dissipative Media; Input Admittance	267
5.3 Coupled Antennas	275
5.3.1 Waveguide with Matched Terminations	278
5.3.2 Air Filled Cavity; Current Distribution	282
5.3.3 Air Filled Cavity; Admittance	294
5.3.4 Cavity Filled with Dissipative Media; Current and Admittance	299
5.4 Special Applications to Sewage Plant Effluent	306
VI. CONCLUSION; RECOMMENDATIONS	329
BIBLIOGRAPHY	337
GENERAL REFERENCES	340
APPENDICES	
I DESCRIPTION AND DEFINITION OF NOTATION	342
II EVALUATION OF DEFINITE INTEGRALS	349

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101 Input A0
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103 Input A0

LIST OF TABLES

Table		Page
5.1	Input Admittance as a Function of Antenna Height in a Semi-Infinite Waveguide ($d_1 = 2.53\lambda_g$, 1.05 Ghz)	169
5.2	Input Admittance as a Function of Antenna Height in a Semi-Infinite Waveguide ($d_1 = 0.251\lambda_g$, 1.3 Ghz)	170
5.3	Input Admittance as a Function of Antenna Location in a Semi-Infinite Waveguide ($h = 6$ cm, 1.05 Ghz)	171
5.4	Resonances of Rectangular Cavity ($a = 16.51$ cm, $b = 8.255$ cm, $l = 60.96$ cm)	177
5.5	Resonances of Rectangular Cavity ($a = 22.6$ cm, $b = 8.255$ cm, $l = 60.96$ cm)	180
5.6	Input Admittance as a Function of Frequency Near a Cavity Resonance ($a = 16.51$ cm, $d_1 = 0.5l$)	223
5.7	Input Admittance as a Function of Frequency Near a Cavity Resonance ($a = 22.6$ cm, $d_1 = 0.5l$)	225
5.8	Input Admittance as a Function of Frequency 0.6 - 2.2 Ghz ($a = 16.51$ cm, $d_1 = 0.5l$)	227
5.9	Input Admittance as a Function of Frequency 0.85 - 1.9 Ghz ($a = 22.6$ cm, $d_1 = 0.5l$)	229
5.10	Input Admittance as a Function of Frequency 0.6 - 2.2 Ghz ($a = 16.51$ cm, $d_1 = 0.29l$)	231
5.11	Input Admittance as a Function of Antenna Location ($a = 16.51$ cm, 1.2 Ghz)	236
5.12	Input Admittance as a Function of Antenna Location ($a = 22.6$ cm, 1.2 Ghz)	238
5.13	Input Admittance as a Function of Antenna Location ($a = 16.51$ cm, 1.53 Ghz)	240
5.14	Input Admittance as a Function of Antenna Location ($a = 16.51$ cm, 1.4 Ghz)	242

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Table		Page
5.15	Input Admittance as a Function of Antenna Height ($d_1 = 0.238$, 1.05 Ghz)	246
5.16	Input Admittance as a Function of Antenna Height ($d_1 = 0.133$, 1.1 Ghz, $a = 16.51$ cm)	247
5.17	Input Admittance as a Function of Antenna Height ($d_1 = 0.133$, 1.1 Ghz, $a = 22.6$ cm)	248
5.18	Input Admittance as a Function of Antenna Height ($d_1 = 0.133$, 1.2 Ghz)	249
5.19	Input Admittance as a Function of Antenna Height ($d_1 = 0.133$, 1.3 Ghz)	250
5.20	Input Admittance as a Function of Antenna Height ($d_1 = 0.500$, 1.53 Ghz)	251
5.21	Input Admittance in Highly Dissipative Media Comparison of Theory with Experimental Literature	269
5.22	Input Admittance in Distilled Water at Different Frequencies	270
5.23	Input Admittance in Distilled Water at Different Locations	271
5.24	Input Admittance in Distilled Water for Different Heights	272
5.25	Input Admittance for Distilled and Salt Water at 350 MHz	273
5.26	Input Admittance for Distilled and Salt Water at 1.0 GHz	274
5.27	Admittance for Coupled Antennas in Semi- Infinite Waveguide	281
5.28	Input Admittance for Coupled Antennas as a Function of Separation ($f = 1.3$ GHz)	295
5.29	Input Admittance for Coupled Antennas as a Function of h_1 (1.2 GHz)	296
5.30	Input Admittance for Coupled Antennas as a Function of h_1 (1.3 GHz)	297

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Table		Page
5.31	Input Admittance for Coupled Antennas as a Function of h_2 (1.3 Ghz)	298
5.32	Input Admittance for Coupled Antennas in Dissipative Media at 350 MHz	304
5.33	Input Admittance for Coupled Antennas in Dissipative Media at 1.0 GHz	306
5.34	Analysis of Sewage Sample	313
5.35	Admittance for Single Antenna in Sewage (350 MHz)	320
5.36	Admittance as a Function of Height of Single Antenna in Sewage (1.0 Ghz)	321
5.37	Admittance as Function of Location for Single Antenna in Sewage (1.0 Ghz)	322
5.38	Admittance as Function of Height of Single Antenna in Sewage (1.2 Ghz)	323
5.39	Theoretical Admittance for Coupled Antennas in Sewage at 350 MHz	326

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29. Rectang
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LIST OF FIGURES

Figure	Page
1.1 Cutaway View of Antenna-Waveguide System	6
2.1 Physical Structure and Geometry for the Antennas in the Waveguide	14
2.2 Geometry for Slice Generator Approximation	18
2.3 Geometry for Application of the Lorentz Reciprocity Relation	23
2.4 Coaxial Gap Excitation	49
2.5 Geometry for Finding Gap Induced Fields Using the Lorentz Reciprocity Relation	53
3.1 Pulse Function for Expansion of Antenna Current	68
3.2 Comparison of Galerkin's Method and Point Matching	75
3.3 Comparison of Off-Diagonal to Diagonal Elements of $[S]$ Matrix	88
3.4 Convergence as a Function of m	90
3.5 Convergence as a Function of n for the Thin Wire Approximation	94
3.6 Convergence as a Function of n for the Transverse Strip Approximation	97
3.7(a) Comparison of Various Approximations	102
3.7(b) Comparison of Various Approximations	103
3.7(c) Comparison of Various Approximations	104
3.7(d) Comparison of Various Approximations	105
3.7(e) Comparison of Various Approximations	106
4.1 Rectangular Waveguide Section in Cavity Configuration with Antenna Measurement System in Place	112

Page

- 1. End View
Antenna
- 2. Impedance
- 3. Slotted A
- 4. Slotway V
- 5. Second Ar
- 6. Total Ene
Measureme
- 7. Represent
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Infinite
- 17. Current o
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- 18. Theoretic
Infinite
Location
- 19. Current o
Density ρ
 $d_1 = 0.02$

Figure		Page
4.2	End View of Rectangular Waveguide Section with Antenna	112
4.3	Impedance Sensing Block	115
4.4	Slotted Antenna with Current and Charge Probes	117
4.5	Cutaway View of Slotted Antenna with Probes	117
4.6	Second Antenna(s) for Coupled Antenna Measurements	118
4.7	Total Experimental Set-Up for Current Distribution Measurements	118
4.8	Representative Measurements of Conductivity	122
4.9	Glass Tubing Properties	125
4.10	Representative Measurements of Dielectric Constant	127
4.11	Effect of Charge Probe on Current Measurements	131
4.12	Effect of Probe Orientation on Current Measurements	133
5.1	Effect of Infinite Susceptance on Numerical Solution	145
5.2	Lumped Equivalent Terminal Zone Networks	148
5.3	Functional Behavior of Terminal Zone Correction Network	148
5.4	Correction of Measured Data via Terminal Zone Network	152
5.5	Current Distribution for Antenna in Semi-Infinite Waveguide ($h = 5$ cm, $f = 1.05$ Ghz)	159
5.6	Current Distribution for Antenna in Semi-Infinite Waveguide ($h = 5$ cm, $f = 1.3$ Ghz)	161
5.7	Theoretical Current Distribution for Semi-Infinite Waveguide Configuration with Antenna Location as the Parameter	164
5.8	Current Distribution as a Function of Frequency Near a Cavity Resonance ($a = 16.51$ cm, $d_1 = 0.5\lambda$)	178

Page

11 Current :
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12 Current :
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1.3 - 1.4

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26 Theoretic
Function
(a = 16.5

27 Theoretic
Function
(a = 21.6

Figure		Page
5.9	Current Distribution as a Function of Frequency Near a Cavity Resonance ($a = 27.6$ cm, $d_1 = 0.5l$)	181
5.10	Current Distribution as a Function of Frequency 0.6 - 2.2 GHz ($a = 16.51$ cm, $d_1 = 0.5l$)	183
5.11	Current Distribution as a Function of Frequency 1.1 - 1.9 GHz ($a = 22.6$ cm, $d_1 = 0.5l$)	186
5.12	Current Distribution as a Function of Frequency 0.6 - 2.2 GHz ($a = 16.51$ cm, $d_1 = 0.290l$)	188
5.13	Current Distribution as a Function of Antenna Location (1.2 GHz)	197
5.14	Current Distribution as a Function of Antenna Location (1.4 GHz)	199
5.15	Current Distribution as a Function of Antenna Location (1.53 GHz)	201
5.16	Current Distribution as a Function of Antenna Height ($a = 16.51$ cm, 1.05 GHz)	206
5.17	Current Distribution as a Function of Antenna Height ($a = 16.51$ cm, 1.1 GHz)	207
5.18	Current Distribution as a Function of Antenna Height ($a = 22.6$ cm, 1.1 GHz)	208
5.19	Current Distribution as a Function of Antenna Height ($a = 16.51$ cm, 1.2 GHz)	210
5.20	Current Distribution as a Function of Antenna Height ($a = 16.51$ cm, 1.3 GHz)	212
5.21	Current Distribution as a Function of Antenna Height ($a = 16.51$ cm, 1.53 GHz)	214
5.22	Charge Distribution as a Function of Frequency ($d_1 = 0.5$, $h = 6.25$ cm)	217
5.23	Charge Distribution as a Function of Frequency ($d_1 = 0.29$, $h = 6.25$ cm)	218
5.24	Theoretical Input Admittance (Susceptance) as a Function of Frequency Near a Cavity Resonance ($a = 16.51$ cm, $d_1 = 0.5l$)	224
5.25	Theoretical Input Admittance (Susceptance) as a Function of Frequency Near a Cavity Resonance ($a = 22.6$ cm, $d_1 = 0.5l$)	226

101 Theoretic
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Figure		Page
5.26	Theoretical Input Admittance as a Function of Frequency 0.3 - 2.2 Ghz ($a = 16.51$ cm, $d_1 = 0.5\lambda$)	228
5.27	Theoretical Input Admittance as a Function of Frequency 0.85 - 1.9 Ghz ($a = 22.6$ cm, $d_1 = 0.5\lambda$)	230
5.28	Theoretical Input Admittance as a Function of Frequency 0.6 - 2.2 Ghz ($a = 16.51$ cm, $d_1 = 0.29\lambda$)	233
5.29	Theoretical Input Admittance as a Function of Antenna Location ($a = 16.51$ cm, 1.2 Ghz)	237
5.30	Theoretical Input Admittance as a Function of Antenna Location ($a = 22.6$ cm, 1.2 Ghz)	239
5.31	Theoretical Input Admittance as a Function of Antenna Location ($a = 16.51$ cm, 1.53 Ghz)	241
5.32	Theoretical Input Admittance as a Function of Antenna Location ($a = 16.51$ cm, 1.4 Ghz)	243
5.33	Current Distribution in Highly Dissipative Media Comparison of Theory with Experimental Literature	256
5.34	Current Distribution in Distilled Water at Different Frequencies	259
5.35	Current Distribution in Distilled Water at Different Locations	261
5.36	Current Distribution in Distilled Water for Different Heights	262
5.37	Current Distribution for Distilled and Salt Water at 350 MHz	264
5.38	Current Distribution for Distilled and Salt Water at 1.0 Ghz	266
5.39	Driven Antenna Current for Coupled Antennas in Semi-Infinite Waveguide ($f = 1.05; 1.3$ Ghz)	280
5.40	Driven Antenna Current for Coupled Antennas as a Function of Separation ($f = 1.3$ Ghz)	285
5.41	Driven Antenna Current for Coupled Antennas as a Function of h_1 (1.2 Ghz)	288
5.42	Driven Antenna Current for Coupled Antennas as a Function of h_1 (1.3 Ghz)	290

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Figure		Page
5.43	Driven Antenna Current for Coupled Antennas as a Function of h_2 (1.3 Ghz)	292
5.44	Driven Antenna Current for Coupled Antennas in Dissipative Media (350 MHz)	302
5.45	Driven Antenna Current for Coupled Antennas in Dissipative Media (1.0 Ghz)	303
5.46	Current for Single Antenna in Sewage (350 MHz)	314
5.47	Current as Function of Height for Single Antenna in Sewage (1.0 Ghz)	315
5.48	Current as Function of Location for Single Antenna in Sewage (1.0 Ghz)	317
5.49	Current as Function of Height for Single Antenna in Sewage (1.2 Ghz)	318
5.50	Current for Single Antenna in Sewage at 1.6 Ghz	319
5.51	Theoretical Driven Antenna Current for Coupled Antennas in Sewage (350 MHz, $h_1 = 5$ cm)	324
5.52	Experimental Driven Antenna Current for Coupled Antennas in Sewage (350 MHz, $h_1 = 6$ cm)	325
5.53	Received Voltage versus Separation for Coupled Antennas in Cavity Filled with Dissipative Media	327
II.1	Transformation to Polar Coordinates for Gap Regions	357

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PREFACE

In the formative stages this research was aimed at a direct extension of the recent developments of antenna theory in dissipative media. The result was to be a comprehensive theoretical and experimental investigation into the area of electromagnetic probing with direct application to ecological media. It was hoped that extensive experimental work on clean and polluted air, and fresh water, and water containing organic and inorganic matter would provide the foundation for effective use of antennas as environmental measuring tools. However, as was so accurately stated by George Bernard Shaw, "Science is always wrong. It never solves a problem without creating ten more." Thus it was that as the research developed, it became obvious that a well defined, closed, bounded, and portable system was needed to meet the requirements of applicability. The final system chosen was that of coupled antennas in a rectangular section of waveguide or rectangular cavity. This system satisfied the desire to exploit the correlation between antenna characteristics and properties of a medium, as well as offering an attractive theoretical, experimental and practical solution to the problem. In order to use this system, however, an extensive amount of effort was needed to formulate the general theoretical problem. Therefore, this report primarily addresses itself to the development of this theory and the subsequent

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experimental verification of the results. Because of the lack of adequate literature on this problem, this development in and of itself provides a significant contribution to the field of electromagnetic theory. Although some representative cases of specific dissipative media are considered briefly, time has not permitted the type of comprehensive study first sought. This type of study is still badly needed, and it is hoped that the results presented in this report may be used as the basis for such an effort in the future.

C. Edward Grove
Holt, Michigan
December 22, 1974

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CHAPTER 1

INTRODUCTION

When the cry went up to "Save the Environment", there was a call to the sciences to measure, monitor and quantify what man had produced. In the field of electromagnetic theory, this stimulated renewed interest in investigating the possibilities of using electromagnetic theory to develop new information gathering and diagnostic tools for use by the environmentalists, ecologists, biologists and chemists. The approach has been to apply fundamental field principles and well understood microwave systems to obtain meaningful relationships between measurable electromagnetic phenomena and the environmental quantities of interest. With this approach the electromagnetic engineer has been able to obtain useful, reliable and even pertinent data concerning the environment; the most dramatic example is in the rapidly expanding area of remote sensing. The possibility of useful results has also been found in the area of antenna and probe theory. By applying the fundamental principles of electromagnetic wave propagation and the theory of antenna interaction with a general medium, unique relationships have been found which demonstrate the feasibility of using antennas as measuring tools. The research described in this report addresses itself to the further development of the theory of antennas as diagnostic tools by investigating a microwave system which can be directly applied to problems of environmental interest.

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1.1 Background.

Basic investigation into fundamental probe and antenna theory has previously been done for the purpose of analyzing and predicting both the qualitative and quantitative composition of the earth and the ionosphere (see Grant and West (1965) and Ament (1964)^{*}). Extensive theoretical analyses of electromagnetic propagation in conducting media have been carried out (see Brekhovskikh (1960) and Wait (1962)) which are applicable to antenna-media interaction in induction studies. Further theoretical and experimental work on probes in various types of conducting and stratified media has indicated the ability to obtain meaningful correlation between measurements and media characteristics (see Ament (1964), Iizuka (1963, 1964, 1965), Iizuka and King (1962), and Biggs (1968)). Experimental investigations attempting to verify the validity of several of the approximate formulas for the electromagnetic field of infinitesimal dipoles in a dissipative half-space include Saran and Held (1960) for fresh water, Kraichman (1960) for a tank containing salt solution and Blair (1963) for scale model tank measurements. Experimental checks have been made in situ for ocean and fresh water environments which verify a general theory (see Siegel (1970)).

Studies involving water and dissipative media must necessarily involve electrochemical considerations. Water alone involves complicated electromagnetic and chemical interactions as well as

* The references by author and date in this section are given in the general references following the bibliography.

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diffusion and electrical potentials (see Golterman (1969)) which may be useful in predicting and interpreting various experimental phenomena. Existing methods of electrochemical analysis include the conductometric, electrophoretic, electrolytic, potentiometric and polarographic methods (see Muller (1956)). Of these methods the last three involve electron exchanges at an electrode which is directly applicable to antenna probing investigations. Another area of importance in electrochemistry is the method of high-frequency conductance (see Blaedel (1950) and Johnson and Enke (1970)). This method directly utilizes the microwave frequencies to monitor chemical changes in solutions below the range where spectroscopy is useful and would be useful in identifying specific phenomena relative to antenna probing in water solutions.

Electromagnetic measurements also have application in air quality studies (see Sauter and Chilton [3]). The major areas of concern are particulate matter (presently detected optically in the visible and infrared region), irritants and oxidants such as SO_2 , NO_2 and ozone compounds, and systemic poisons such as CO and H_2S (presently measured by chemical reaction in solution). Measurements of these pollutants could best be performed in situ near the source (smoke stacks, etc.) where concentrations are on the order of five parts per thousand. Thus far, the little application of microwave analysis that has been done shows promise particularly for compounds which have molecular absorption spectra in the lower frequencies. Antenna impedance, current, and charge measurements correlated to the macroscopic properties of air in the presence of various pollutants of controlled concentrations could be useful

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in monitoring the output from industrial exhausts. This would thus involve the same instrumentation as in water studies and the general theory could be applied to predict and interpret the antenna measurements made.

With the above mentioned areas of concern providing the background and impetus, the electromagnetic engineer is capable of extending theoretical analyses of antenna measurements to the area of ecological monitoring. The research described in this report draws on this background in an effort to extend the use of antennas as diagnostic probes.

1.2 Discussion of the Problem and Related Works.

The prime motivation for this research is in the recent extension of the theory of antennas in the presence of arbitrary, general, dissipative media. This theory relates the terminal characteristics; e.g., impedance or admittance parameters, and the current and charge distributions on single or coupled antennas to the constitutive electrical parameters; i.e., the electrical conductivity, permittivity and permeability of the medium surrounding the antenna. Much of this theory for infinite (or semi-infinite) media is presented in the book by King and Harrison [18]. Much of the important preliminary work is found in the works of King and Iizuka [12, 13, 14, 19] and of King, Scott, and Mishra [21, 27, 35, 36]. In particular, King and Scott ([21] and [35]) have formulated and tested the theory of the single antenna while Mishra, King and Scott ([27] and [36]) have treated the coupled antenna case. These latest works clearly demonstrate the possibility of

relating the characteristics of an antenna to the electrical properties of a medium. Extensions of these theories to the direct application of antenna probing techniques is most recently reported by Rogers and Peden [30] in their studies of deep polar ice.

Perhaps the most obvious drawback in all the above mentioned developments on antennas in arbitrary media is the need for an (at least effectively) infinite or semi-infinite environment. This is, of course, very attractive theoretically and can be closely approximated experimentally, but is unfortunately restrictive when it is desired to apply the antenna probing technique to a generally localized environment with boundaries that cannot be easily characterized or modeled. The most attractive alternative to this problem is to consider the antenna and media of interest in well defined, fixed surroundings. Thus, it is this consideration that forms the starting point for the research presented here.

By approaching the antenna probing problem primarily from an applications viewpoint, the selection of an appropriate effective system was the first consideration. The system chosen to meet the requirements of theoretical, experimental and practical viability is that of an array of antennas in a section of rectangular waveguide as depicted in Figure 1.1. This system has the potential of being portable and easily adaptable for use as a continuous flow-through monitor or discrete sampling system for application to the study of fluids. It is experimentally convenient since the waveguide and antenna size can be chosen physically large for easy construction while the results remain valid for any size system by proper scaling. The theoretical analysis of this system is also

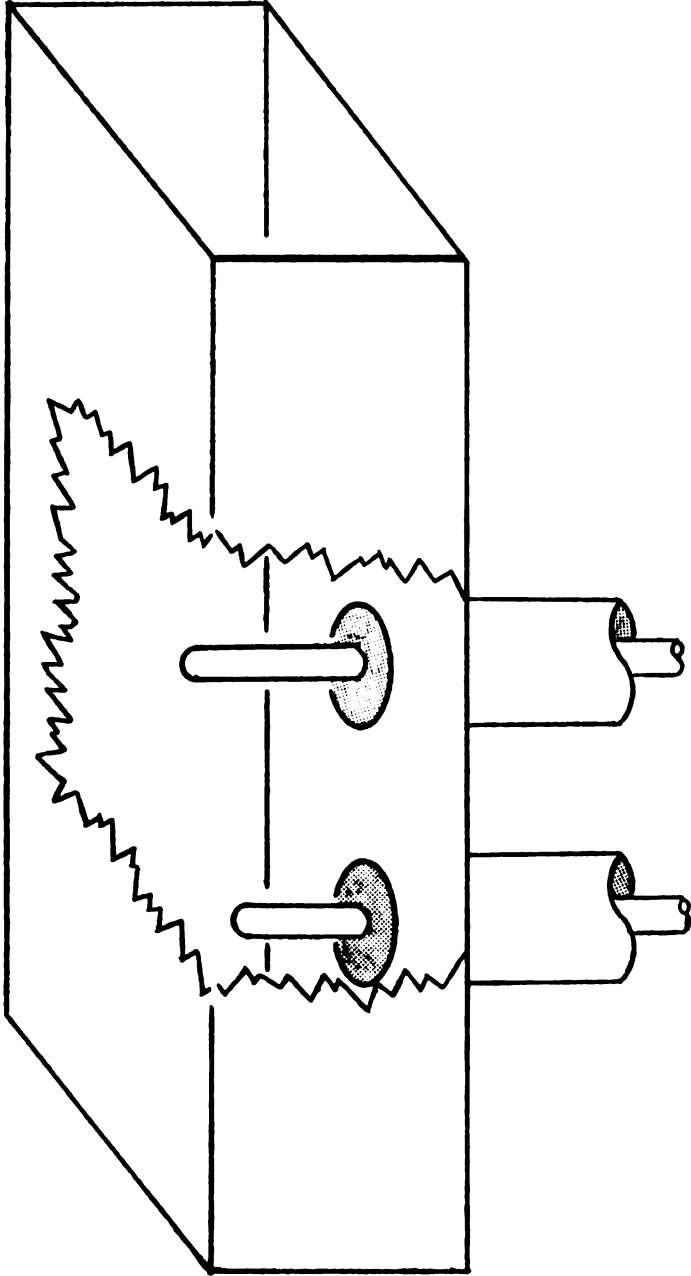


FIGURE 1.1

Cutaway View of Antenna-Waveguide System

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attractive because the theories of both the waveguide and the antenna are well understood and the basic techniques used to solve each problem separately can be applied to yield a tractable solution for the combination. Unfortunately, an adequate general theory for an array of antennas in a rectangular waveguide is not available in the literature. Therefore, it is to the development of this theory that the research reported here is primarily directed.

To approach the theoretical problem an understanding of the solution sought must be made clear. Since it is desired to expand on the previously mentioned conclusions of antennas as diagnostic probes, the system chosen must be approached as an antenna problem. That is, the antenna characteristics are to be viewed as a function of the waveguide boundaries and the electrical parameters of the medium filling the waveguide. When approached in this manner, the desired result is seen to be the solution of the antenna terminal characteristics and antenna current and charge distribution in terms of the waveguide dimensions and terminations, and the conductivity, permittivity and permeability of the medium.

The fundamental solution for the antenna characteristics is the current distribution on the antenna. From this the charge distribution is readily obtained by application of the continuity equation; the terminal characteristics (e.g. self and mutual admittances) are available from the known (specified) antenna driving voltages and the derived value of input current by Ohm's law. Therefore, the theoretical problem is basically to develop expressions for the current distributions on the antennas. It is this requirement that makes the present literature on the theory of antennas in waveguides inadequate for this problem.

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In the literature available on the theory of antennas in waveguides, consideration is either given to very specialized cases of the problem considered here or assumptions are made on the antenna, waveguide, current distribution or medium which are unacceptable for the solution sought here. Thus it is, in the book by Lewin [24] where only a single antenna is considered and the antenna current distribution has an assumed approximate form. Similarly in the paper by Eisenhart and Khan [7], only a single antenna is handled. Their paper also considers only antennas which span the waveguide (i.e., posts). (It may be noted that this condition leads to a simpler theoretical solution for the antenna current than does the case of an antenna which does not span the guide.) Rao [29] has adequately solved the case of coupled antennas but again the antennas span the waveguide (which is taken to be a parallel plate waveguide rather than a rectangular waveguide). Finally, in the monograph by Lewin [25], again only the case of a post is considered. It is characteristic of much of the above literature that the theories are developed from the viewpoint of the waveguide terminals rather than from the antenna or post terminals; i.e., they are regarded as waveguide problems rather than antenna problems. This restricts the application of these works to the problem undertaken in this research. It goes without saying, however, that each of the above articles is helpful in providing verification of the general theory through special cases as well as being of assistance through various aspects of the development.

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1.3 Method of Investigation

It has been established that the antenna-waveguide-medium system is to be approached from the antenna viewpoint and that the fundamental solution is the antenna current distribution. This implies the derivation of an integral equation (or set of integral equations for coupled antennas) for a suitable theoretical model of the antenna. In Chapter 2 these integral equations are developed based on the electric field boundary condition at the antenna surface. The electric field is found as a superposition of fields induced by the unknown currents on each antenna. These unknown fields are first expressed as modal expansions in terms of the rectangular waveguide modes. The modal expansion coefficients are then found as functions of the unknown current distributions on each antenna by application of the Lorentz reciprocity theorem in the waveguide volume. The resulting induced electric field involves integrals of the unknown current volume density functions. The antenna excitation is first modeled as a slice generator or delta-gap source and application of the electric field boundary condition results in the desired integral equations for the unknown current distributions. These equations are then reduced from the delta function form to Hallen-type equations. The unknown volume current density functions are finally converted to one dimensional current distribution functions by two different approximations on the cylindrical antennas: the thin-wire approximation and the infinitely-thin strip approximation. The resulting one dimensional set of coupled integral equations is then in suitable form for numerical solution.

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To improve the approximate antenna model consider first, the problem is approached again as described above but with the antenna excitation modeled as the physical coaxial gap instead of as a slice generator. Because of the increased complexity this is only done for a single antenna and the cylindrical antenna is handled only with the thin-wire approximation. No reduction to a Hallen-type equation is necessary and the integral equations are directly solvable by numerical methods.

The numerical methods for the solution of the integral equations developed in Chapter 2 are treated in Chapter 3. Discretized equations in matrix form are found by the moment method using the method of subsections and point matching. Comparison is then made between this point matching solution and the results of applying Galerkin's method. No significant improvement in the solution is found, thus supporting use of the simpler point matching results. The solutions of each approximate form of the integral equations developed in Chapter 2 are also compared with respect to their convergence properties. It is shown that the thin strip approximation exhibits improved convergence over the thin-wire approximation. The delta-gap and coaxial gap solutions are found to give comparable results for the current distributions. Also, criteria are developed for proper truncation of the two infinite series modal summations which improve the numerical evaluation process.

In Chapter 4 the apparatus for experimental investigation of the antenna-waveguide-media system is described. Attention is paid to the details of construction and quirks of operation for the components used to measure current distribution and antenna input

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admittance (the impedance sensing block). Data is presented which illustrates several sources of experimental error. Methods are also described for the independent determination of dielectric constant and (low-frequency) conductivity of the dissipative media considered.

The presentation of theoretical results and the comparison with experimental data is contained in Chapter 5. The problems of correlating theoretical and experimental input admittance are treated in general in Section 5.1. Evidence of the infinite susceptance introduced by the delta gap theory and the problems associated with applying conventional terminal zone correction networks are described. New ways of approaching the determination of a correction network for the case of the antenna in a bounded region are presented along with typical results for several physical and electrical configurations. The experimental admittance results given in the following sections, however, are not corrected for terminal zone effects to avoid obscuring the original data.

The results for current distribution and input admittance are primarily divided into the cases of the single antenna and of two coupled antennas with the single antenna in an air filled cavity receiving the most emphasis. Comparison of the results predicted by the theory and experimental work available in the literature for highly dissipative media is also included. Results of experimental measurements on distilled water and one salt solution are compared with the theory for a limited range of parameters. Finally, the results of investigations on one sample of effluent from a sewage treatment plant is included in Section 5.6.

In general the results show excellent agreement between theory and experiment for the current distributions over all variations of the physical and electrical configurations. Discrepancies between input admittance values exist, but, in general, are expected. The overall agreement substantiates the ability of the theory to predict the measurable data over the range of parameters of interest.

Of particular interest are the current distribution and input admittance found for the single antenna in the air filled cavity. Variations in current and admittance are strongly coupled to the resonant frequencies of the cavity with the current distribution showing dramatic variation from that expected in an unbounded (free space) region. It is anticipated that advantage can be taken of this unique feature for implementation of the antenna-cavity-media system as a diagnostic tool.

In Chapter 6 the results are summarized and conclusions drawn on the applicability of this system for monitoring the electrical characteristics of media over the lower microwave frequencies. Indication is also made of areas of this research which show promise for further investigation. Recommendations are made for expanding, continuing, and complementing the findings reported here along with possible areas for improving the analysis.

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CHAPTER 2

THEORETICAL DEVELOPMENT

2.1 The Model

The system under investigation is that of N linear antennas in a rectangular waveguide. A solution for the antenna characteristic (in particular, the current distributions) is sought for the boundary value problem with an arbitrary medium filling the waveguide. This chapter contains the details of the theoretical development of the necessary integral equations and the reduction of the integral equations to one dimensional forms suitable for numerical solution.

The physical structure and geometry of the general problem are shown in Figure 2.1. The antennas form a linear array with coplanar axes located in the center of the broad wall of the waveguide and parallel to the $\hat{y}$ axis. They are cylindrical antennas formed by extending the center conductor of coaxial transmission lines beyond the inside surface of the waveguide wall and are capped with hemispherical surfaces. A rectangular coordinate system is used with the origin located in one corner of the waveguide such that the center of one antenna is at $z = 0$. The waveguide has interior transverse dimensions a and b in the $\hat{x}$ and $\hat{y}$ directions, respectively, and longitudinal dimension $d_1 + d_2$ in the $\hat{z}$ direction. For generality, the waveguide is terminated at the planes $z = -d_1$ and $z = +d_2$ in arbitrary complex reflection

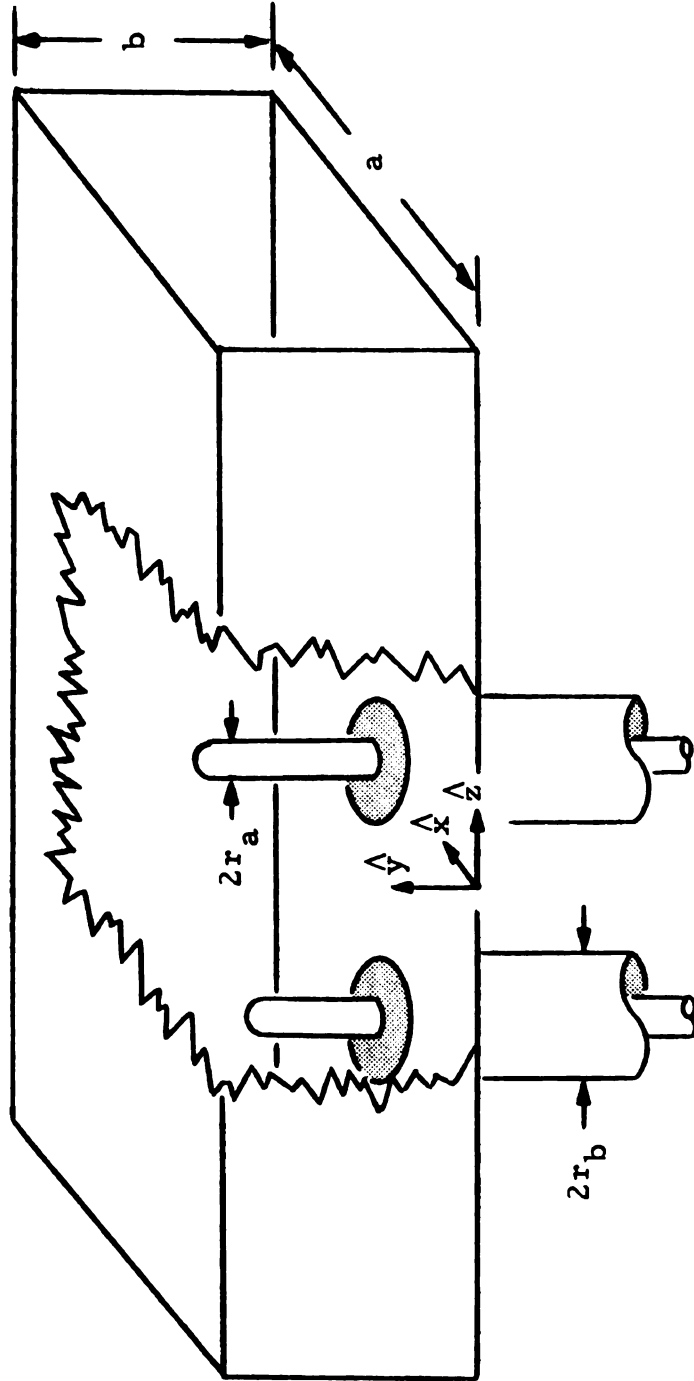


FIGURE 2.1

Physical Structure and Geometry for the Antennas in the Waveguide

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coefficients Γ_{1L} and Γ_{2L} , respectively. The medium filling the waveguide is characterized by the constitutive electrical parameters: namely, permittivity, $\epsilon_r \epsilon_0$; conductivity, σ ; and permeability, μ . For this development, the conductivity and permittivity are combined into an effective complex permittivity as

$$\epsilon = \epsilon_r \epsilon_0 - j \sigma / \omega \quad .$$

As stated in Chapter 1, the most fundamental results to be obtained are the current distributions on the N antennas for known excitations or loads at the antenna terminals. These antenna currents are a direct consequence of the electric field boundary condition on the antenna surface and thus it is on this boundary condition that the development is based. The interrelationships of the other field components in the waveguide are based on the boundary conditions on the interior waveguide walls. That these boundary conditions are met is tacitly implied in the modal expansion technique (only fields satisfying these boundary conditions are permitted). Therefore, explicit application of the electric field boundary condition on the antenna surfaces serves to satisfy all boundary conditions and results in expressions for the unknown current distributions.

A knowledge of the assumptions and approximations made in this development serves to define the degree of generality and the limits of applicability of the results. Therefore, the various assumptions and approximations are listed here for reference. The details and their implications will be described more fully as they

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- 1) The medium filling the waveguide is assumed to be linear, isotropic, and homogeneous with complex permittivity, ϵ , and permeability, μ .
- 2) The waveguide terminations are assumed to be independent of waveguide mode and produce no mode coupling.
- 3) All conductors are assumed to be perfectly conducting.
- 4) The antenna terminals are handled in two ways:
 - (a) primarily, the coaxial gap in the waveguide wall is removed and the slice generator or delta-gap approximation is used.
 - (b) For the special case of a single antenna, the theory is developed for the coaxial gap excitation assuming a coaxial TEM field in the gap (Section 2.6).
- 5) In no case does the antenna span the guide; i.e., they do not touch the wall opposite from their terminals ($h < b$).
- 6) The antenna radii are assumed to be electrically small and not comparable with the waveguide dimensions ($r_a < \lambda_0$, $r_a \ll a$).
- 7) Each antenna radius and height is assumed to satisfy $r_a < h$.
- 8) The unknown antenna currents are approximated as having only a $\hat{y}$ or axial component.
- 9) The cylindrical antennas are approximated in three ways:
 - (a) thin-wire approximation (a line current is assumed to flow on the axis).
 - (b) transverse strip approximation (the cylinder is represented by an infinitely thin strip lying in the (x,y)

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- (c) longitudinal strip approximation (same as (b) lying in the (y,z) waveguide plane).

2.2 Boundary Conditions; Slice Generator Approximation

To solve the general problem of N antennas, the slice generator or delta-gap approximation is made for each antenna terminal. For this approximation, the physical coaxial gaps are replaced by conducting surfaces and a time harmonic $(e^{j\omega t})$ voltage of radian frequency ω is maintained across an infinitesimal gap between the waveguide wall and the antenna. The specific geometry is shown in Figure 2.2. Each antenna has radius r_a and is centrally located in the broad wall at $x = a/2$. The general antenna is located at longitudinal position $z = z_j$; is excited by a voltage V_j , referenced to the waveguide wall; has total height $y = h_j$; and carries an unknown current $J_{yj}(x,y,z)$ in the $\hat{y}$ direction.

The assumption here that the currents have only a $\hat{y}$ component implies that the sufficient boundary condition on the antenna surfaces is the continuity of the $\hat{y}$ component of the electric field. The N boundary conditions for all N antennas are then:

$$E_{yj}^i(x,y,z) = E_{yj}^e(x,y,z) \quad j = 1, 2, \dots, N \quad (2.1)$$

for locations (x,y,z) on each antenna surface. The impressed electric field (superscript e) is the field maintained directly by external sources; i.e., all ideal generators and the fields set up by internal system losses (imperfect conductors). The induced field (superscript i) is the "radiation" field which results (via



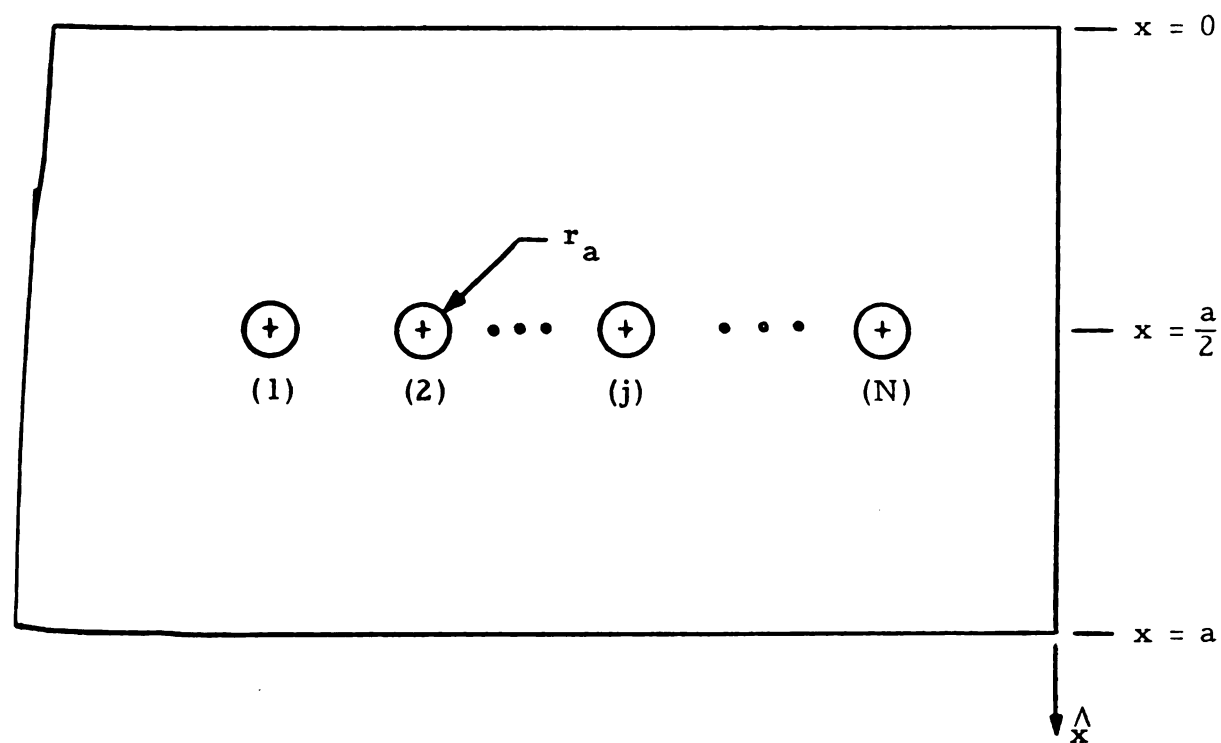
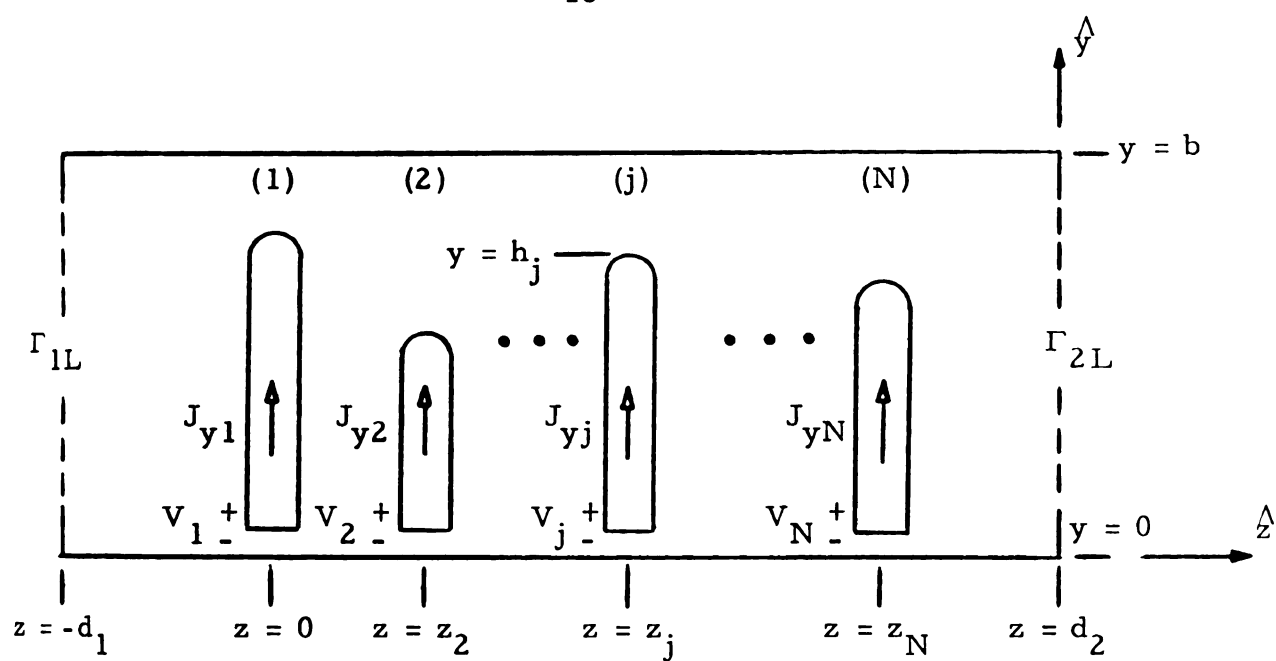


FIGURE 2.2

Geometry for Slice Generator Approximation

Maxwell's equations) from the external source fields. Since it is assumed that the antennas are perfect conductors, the impressed electric field is identically zero along the antenna surface. (This assumption can easily be relaxed by approximating a finite conductivity with a surface impedance as noted in Section II.4 of King [17].) The impressed electric field in the infinitesimal gap at $y = 0$ is non-zero and may be handled mathematically by employing the Dirac delta function (see Appendix I, page 273 of Papoulis [28]). This yields the desired properties of an infinite electric field for the zero width gap which when integrated across the gap gives the finite applied voltage. Some care must be exercised, however, in using the delta function to represent the infinitesimal gap electric field when only the range $y \geq 0$ is considered. In other words, the delta function centered at $y = 0$, $\delta(y)$, does not appear appropriate. The most unambiguous approach is to consider first a small but non-zero gap width and then apply the solution in the limit as the width approaches zero (see Section 2.4). By assuming the non-zero, but vanishing small, gap width to be τ and the voltage applied across this gap to be V , a proper expression for the gap electric field is

$$E(y) = -V \delta(y - \tau/2) .$$

The **negative** line integral of this electric field from the lower to the **higher** potential ($+\hat{y}$ direction, Figure 2.2) then yields the **applied** voltage as it must:

$$-\int_0^{\tau} E(y) dy = \int_0^{\tau} V \delta(y - \tau/2) dy = V .$$

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Note that had the delta function centered at $y = 0$, $\delta(y)$, been used, the integration from the lower limit of $y = 0$ (the boundary of the valid range of y) would have lead to an ambiguity in evaluating the integral; i.e., the integration would not have been formally "across" the delta function. Using this infinitesimal gap width τ for each of the N antennas, the impressed electric field for the j -th antenna then becomes:

$$E_{yj}^e(x,y,z) = -V_j \delta(y - \tau/2) \quad (2.2)$$

$$j = 1, 2, \dots, N \quad \begin{cases} 0 \leq y \leq h_j \\ (x,z) \text{ on } j\text{-th antenna surface} \end{cases}$$

With the impressed electric field specified, it now remains to determine the induced electric field for application of the boundary condition, Equation (2.1).

2.3 Electric Field in the Waveguide

The field maintained by any general current in the waveguide may be expressed as a superposition integral of the current and a dyadic Green's function (see Chapter 2 of Collin [2]). The dyadic Green's function is the solution to the inhomogeneous vector Helmholtz equation in the presence of a unit intensity, point localized, current source and is subject to the boundary conditions that the electric field to be derived from the Green's function has zero tangential component on the interior waveguide walls. Stated mathematically, the induced field for a general current $\vec{J}(\vec{r}')$ may be written as

$$\vec{E}(\vec{r}) = j\omega\mu \int_V \vec{J}(\vec{r}') \cdot \vec{\bar{G}}(\vec{r}|\vec{r}') dV$$

where $\vec{\bar{G}}$ is the solution to

$$\nabla \times \nabla \times \vec{\bar{G}} - k_0^2 \vec{\bar{G}} = -\vec{\bar{U}} \delta(\vec{r} - \vec{r}')$$

under mixed Neumann and Dirichlet boundary conditions. Here, $\vec{\bar{U}}$ is a unit dyadic, $\delta(\vec{r} - \vec{r}')$ is the three dimensional Dirac delta function, and $k_0^2 = \omega^2 \mu \epsilon$ is the propagation constant of the medium filling the waveguide. In the above, as well as throughout this development, the primed coordinates locate source points and the unprimed coordinates locate field points.

In order to apply the boundary conditions of Equation (2.1), it is necessary to determine a formulation for the $\hat{y}$ -component of induced electric field in the waveguide, E_y^i , with the N unknown antenna currents considered as sources. Since the antenna currents are assumed to have only $\hat{y}$ -components and since only the $\hat{y}$ -component of electric field is required, the vector superposition integral for the Green's function technique becomes a scalar integral and only the $\hat{y}\hat{y}$ -component of the dyadic Green's function is required. By expressing the total electric field as a linear superposition of the fields maintained by each antenna current independently, the following expression may be written down:

$$E_y^i(x, y, z) = \sum_{i=1}^N \int_{V_i} J_{yi}(x', y', z') G_{yy}(x, y, z | x', y', z') dV' \quad (2.3)$$

where the factor $j\omega\mu$ has now been included in the unknown function G_{yy} . Again, the primed coordinates locate source points (in this case on the i -th antenna); whereas, the unprimed coordinates locate

field points (on the j -th antenna). Application of Equation (2.3) now requires determination of the appropriate Green's function for the interior of the waveguide. However, rather than seek the Green's function specifically, the induced electric field will be found directly by another method, and, based on Equation (2.3), the Green's function will be identified. This approach avoids the possibility of misusing a Green's function derived for free space in the assumed general medium filling the waveguide which can have non-zero conductivity. (The problems related to applying the free space Green's function in a current carrying region are discussed in a paper by Van Bladel [39].)

The method used to determine the general induced electric field is to apply the modal analysis technique to an appropriate formulation of the Lorentz reciprocity relation. This development and the notation used are similar to that in Section 4.10 of Collin [3] or Section 5.6 of Collin [2]. The Lorentz reciprocity relation for electric current sources in a volume V enclosed by a surface S is

$$\oint_S (\vec{E}_1 \times \vec{H}_2 - \vec{E}_2 \times \vec{H}_1) \cdot \hat{n} \, dS = \int_V (\vec{E}_2 \cdot \vec{J}_1 - \vec{E}_1 \cdot \vec{J}_2) \, dV \quad (2.4)$$

$\hat{n}$ = outward normal to S

where $(\vec{E}_1, \vec{H}_1)$ and $(\vec{E}_2, \vec{H}_2)$ are independent solutions to Maxwell's equations with externally applied source currents $\vec{J}_1$ and $\vec{J}_2$, respectively. Application of Equation (2.4) will be made in a volume V_i (containing the unknown current on the i -th antenna) within the waveguide as shown in Figure 2.3. In Equation (2.4) the choice

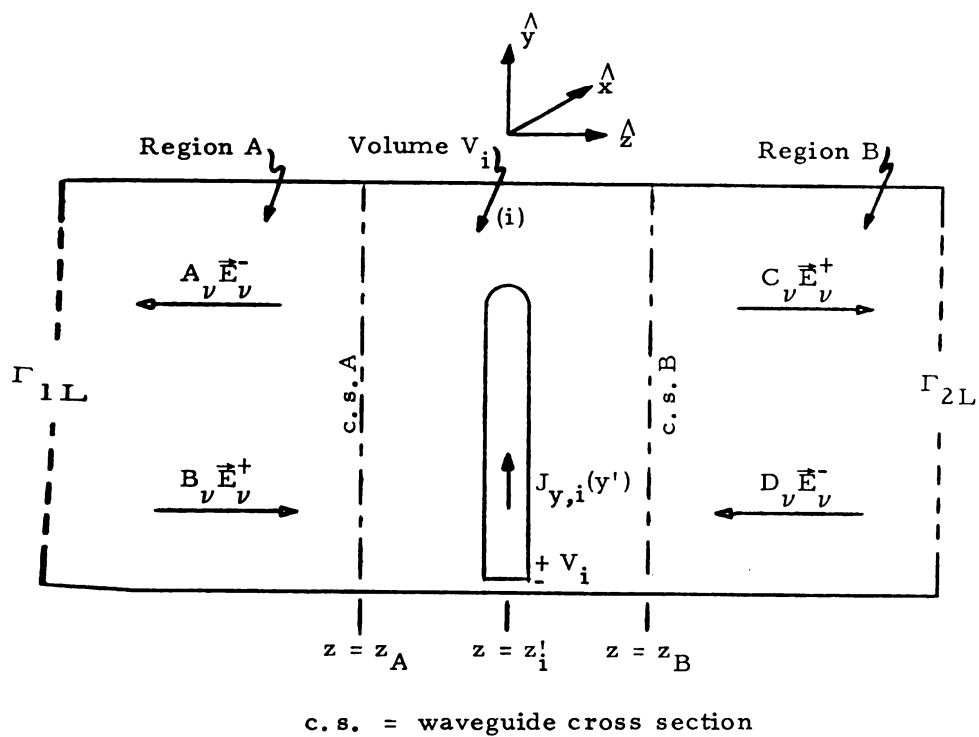


FIGURE 2.3

Geometry for Application of the Lorentz Reciprocity Relation

$$\vec{J}_2(x,y,z) = 0$$

is made such that the corresponding fields $(\vec{E}_2, \vec{H}_2)$ are any fields which can exist in a source free region in the waveguide; the particular choice for $(\vec{E}_2, \vec{H}_2)$ will be made later. The current $\vec{J}_1$ in Equation (2.4) is taken to be the unknown current distribution on the i -th antenna:

$$\vec{J}_1 = J_{yi}(x',y',z')\hat{y} ,$$

such that the corresponding fields $(\vec{E}_1, H_1)$ are the desired induced fields in the waveguide.

The solutions to Maxwell's equations in a source free (externally applied) region inside the waveguide are the complete set of orthogonal waveguide modes. Employing the notation used by Collin [2, 3], it is expedient to separate the transverse (x,y) and longitudinal (z) dependence of the eigenmode fields, and to suppress the assumed harmonic time dependence factor $(e^{j\omega t})$. The waveguide fields are expressed as

$$\begin{aligned} \vec{E}_v^\pm(x,y,z) &= \vec{e}_{t,v}(x,y)e^{\pm\gamma_v z} \pm \hat{z} e_{z,v}(x,y)e^{\pm\gamma_v z} \\ \vec{H}_v^\pm(x,y,z) &= \pm \vec{h}_{t,v}(x,y)e^{\pm\gamma_v z} + \hat{z} h_{z,v}(x,y)e^{\pm\gamma_v z} \end{aligned}$$

where the transverse (t) and longitudinal (z) vector field components are also distinguished. The upper case letters (E,H) are used to represent the total x,y , and z functional dependence; whereas, the lower case letters $(e_{u,v}, h_{u,v})$ are used to represent

only the spatial dependence of the field on the two transverse plane coordinates x and y . (The duplicate use of the lower case e as the exponential function should be unambiguous since this will always have an exponent associated with it; whereas, the field will always have a subscript indicating the component and/or the mode index.) The upper series of signs in the above expressions (E^+, H^+) refers to fields propagating in the positive $\hat{z}$ direction $(e^{-\gamma z})$, and the lower series (E^-, H^-) refers to propagation in the negative $\hat{z}$ direction $(e^{+\gamma z})$. The indices for the eigenmodes are represented by the single subscript v as a short-hand notation implying all combinations of indices (n, m) for TE and/or TM modes. When the distinction between modes is necessary, fields for TE modes are denoted by a superscript h and fields for TM modes by a superscript e .

The mode function solutions for the fields in the waveguide involve an arbitrary amplitude constant which may be evaluated by assuming a value for the amount of power carried by each mode in the waveguide. By specifying the power in each mode, the mode fields become normalized and form an orthonormal set of basis functions for the interior waveguide region. The choice made for the normalization condition is arbitrary as long as it is used consistently throughout the development. In this development it is convenient to use the following normalization condition:

$$\int_{c.s.} (\vec{e}_v \times \vec{h}_{v'}) \cdot \hat{z} \, dS = \begin{cases} 1/2 & v = v' \\ 0 & v \neq v' \end{cases}$$

where $c.s.$ denotes a waveguide cross section. The normalized

modes for this development are given in Equations (2.5) (see, for example, Section 3.6 of Collin [3] for their derivation).

for TE modes:

$$\begin{aligned} e_{x,v}^h(x,y) &= \sqrt{\frac{Z_v^h \epsilon_{on} \epsilon_{om}}{2 ab}} \frac{k_y}{k_c} \cos(k_x x) \sin(k_y y) \\ e_{y,v}^h(x,y) &= \sqrt{\frac{Z_v^h \epsilon_{on} \epsilon_{om}}{2 ab}} \frac{k_x}{k_c} \sin(k_x x) \cos(k_y y) \end{aligned} \quad (2.5a)$$

$$h_{x,v}^h(x,y) = -\frac{1}{Z_v^h} e_{y,v}^h(x,y)$$

$$h_{y,v}^h(x,y) = \frac{1}{Z_v^h} e_{x,v}^h(x,y)$$

$$h_{z,v}^h(x,y) = \sqrt{\frac{\epsilon_{on} \epsilon_{om}}{Z_v^h 2 ab}} \frac{k_c}{\gamma_v} \cos(k_x x) \cos(k_y y)$$

for TM modes:

$$\begin{aligned} e_{x,v}^e(x,y) &= -\sqrt{\frac{2 Z_v^e}{ab}} \frac{k_x}{k_c} \cos(k_x x) \sin(k_y y) \\ e_{y,v}^e(x,y) &= -\sqrt{\frac{2 Z_v^e}{ab}} \frac{k_y}{k_c} \sin(k_x x) \cos(k_y y) \\ e_{z,v}^e(x,y) &= +\sqrt{\frac{2 Z_v^e}{ab}} \frac{k_c}{\gamma_v} \sin(k_x x) \sin(k_y y) \end{aligned} \quad (2.5b)$$

$$h_{x,v}^e(x,y) = -\frac{1}{Z_v^e} e_{y,v}^e(x,y)$$

$$h_{y,v}^e(x,y) = \frac{1}{Z_v^e} e_{x,v}^e(x,y)$$

where

$$Z_v^e = -j \frac{\gamma_v}{\omega \epsilon},$$

$$Z_v^h = +j \frac{\omega \mu}{\gamma_v},$$

and

$$\epsilon_{on} = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases} .$$

All other notations are described in Appendix I.

The complete expressions for either a TE or a TM mode field (in a source free region) propagating in either the $\pm \hat{z}$ direction can now be expressed as

$$\begin{aligned} \vec{E}_v^{\pm} &= (\vec{e}_{t,v} \pm \hat{z} e_{z,v}) e^{\pm \gamma_v z} \\ \vec{H}_v^{\pm} &= (\pm \vec{h}_{t,v} + \hat{z} h_{z,v}) e^{\pm \gamma_v z} \end{aligned} \quad (2.6)$$

where, for example,

$$\vec{e}_{t,v} = \hat{x} e_{x,v} + \hat{y} e_{y,v} .$$

Because the fields of Equation (2.6) form a complete orthonormal set of basis functions over the interior waveguide region, any waveguide field can be expanded in an infinite series of these functions with unknown expansion coefficients. Thus, for example, any electric field propagating in the $-\hat{z}$ direction may be expressed as

$$\vec{E}(x,y,z) = \sum_v A_v \vec{E}_v^- .$$

Referring now to Figure (2.3), the general fields outside the volume of interest are thus:

in region A:

$$\begin{aligned}\vec{E}_A &= \sum_v A_v \vec{E}_v^- + B_v \vec{E}_v^+ \\ \vec{H}_A &= \sum_v A_v \vec{H}_v^- + B_v \vec{H}_v^+\end{aligned} \quad z \leq z_A$$

in region B:

$$\begin{aligned}\vec{E}_B &= \sum_v C_v \vec{E}_v^+ + D_v \vec{E}_v^- \\ \vec{H}_B &= \sum_v C_v \vec{H}_v^+ + D_v \vec{H}_v^-\end{aligned} \quad z \geq z_B$$

The expansion coefficients can be interrelated via the waveguide termination reflection coefficients as

$$B_v = A_v \Gamma_{1v}$$

$$D_v = C_v \Gamma_{2v}$$

where

$$\begin{aligned}\Gamma_{1v} &= \Gamma_{1L} e^{-2\gamma_v d_1} \\ \Gamma_{2v} &= \Gamma_{2L} e^{-2\gamma_v d_2}.\end{aligned}$$

Therefore, the general fields become

$$\begin{aligned}\vec{E}_A &= \sum_v A_v (\vec{E}_v^- + \Gamma_{1v} \vec{E}_v^+) \\ \vec{H}_A &= \sum_v A_v (\vec{H}_v^- + \Gamma_{1v} \vec{H}_v^+)\end{aligned} \quad z \leq z_A \quad (2.7a)$$

and

$$\vec{E}_B = \sum_{\nu} C_{\nu} (\vec{E}_{\nu}^{+} + \Gamma_{2\nu} \vec{E}_{\nu}^{-}) \quad z \geq z_B \quad (2.7b)$$

$$\vec{H}_B = \sum_{\nu} C_{\nu} (\vec{H}_{\nu}^{+} + \Gamma_{2\nu} \vec{H}_{\nu}^{-})$$

The coefficients A_{ν} and C_{ν} are now determined from the reciprocity relation, Equation (2.4), in terms of the unknown current source $J_{yi}(x', y', z')$. In Equation (2.4), let

$$\vec{J}_1(x', y', z') = \hat{y} J_{yi}(x', y', z'),$$

such that

$$(\vec{E}_1, \vec{H}_1) = \begin{cases} (\vec{E}_A, \vec{H}_A) & \dots z \leq z_A & \text{(Equation (2.7a))} \\ (\vec{E}_B, \vec{H}_B) & \dots z \geq z_B & \text{(Equation (2.7b))} \end{cases}$$

To determine the coefficients A_{ν} , let

$$\vec{J}_2(x', y', z') = 0$$

and choose

$$\vec{E}_2 = \vec{E}_{\nu}^{+} + \Gamma_{2\nu} \vec{E}_{\nu}^{-}$$

$$\vec{H}_2 = \vec{H}_{\nu}^{+} + \Gamma_{2\nu} \vec{H}_{\nu}^{-}$$

where this choice of linear combination of mode fields is a solution to Maxwell's equations in a source free region. Using the above, the reciprocity relation becomes:

$$\begin{aligned} & \int_{c.s.A} [\vec{E}_A \times (\vec{H}_{\nu}^{+} + \Gamma_{2\nu} \vec{H}_{\nu}^{-}) - (\vec{E}_{\nu}^{+} + \Gamma_{2\nu} \vec{E}_{\nu}^{-}) \times \vec{H}_A] \cdot (-\hat{z}) dS + \\ & + \int_{c.s.B} [\vec{E}_B \times (\vec{H}_{\nu}^{+} + \Gamma_{2\nu} \vec{H}_{\nu}^{-}) - (\vec{E}_{\nu}^{+} + \Gamma_{2\nu} \vec{E}_{\nu}^{-}) \times \vec{H}_B] \cdot \hat{z} dS = \\ & = \int_{V_i} (\vec{E}_{\nu}^{+} + \Gamma_{2\nu} \vec{E}_{\nu}^{-}) \cdot \hat{y} J_{yi} dV' , \end{aligned}$$

since the surface integrals are identically zero on the perfectly conducting waveguide walls (e.g., $(\vec{E}_A \times \vec{H}_v) \cdot \hat{n} = (\hat{n} \times \vec{E}_A) \cdot \vec{H}_v$ and $\hat{n} \times \vec{E} \equiv 0$ on the waveguide walls). This equation may be simplified by performing the following steps:

- (a) substitute Equations (2.7) into the reciprocity relation
- (b) apply orthogonality; e.g.,

$$\int_{c.s.} (\vec{E}_v \times \vec{H}_{v'}) \cdot d\vec{S} = 0 \quad \text{for } v \neq v'$$

- (c) expand cross products and cancel appropriate terms
- (d) expand mode fields according to Equation (2.6)
- (e) collect and cancel appropriate terms
- (f) invoke the assumed normalization condition; i.e.,

$$\int_{c.s} (\vec{e}_v \times \vec{h}_v) \cdot \hat{z} dS = 1/2 \quad .$$

Thus, after substituting and applying orthogonality, there results,

$$\begin{aligned} & - \int_{c.s.A} A_v [(\vec{E}_v^- + \Gamma_{1v} \vec{E}_v^+) \times (\vec{H}_v^+ + \Gamma_{2v} \vec{H}_v^-) - \\ & \quad - (\vec{E}_v^+ + \Gamma_{2v} \vec{E}_v^-) \times (\vec{H}_v^- + \Gamma_{1v} \vec{H}_v^+)] \cdot \hat{z} dS \\ & + \int_{c.s.B} C_v [(\vec{E}_v^+ + \Gamma_{2v} \vec{E}_v^-) \times (\vec{H}_v^+ + \Gamma_{2v} \vec{H}_v^-) - \\ & \quad - (\vec{E}_v^+ + \Gamma_{2v} \vec{E}_v^-) \times (\vec{H}_v^+ + \Gamma_{2v} \vec{H}_v^-)] \cdot \hat{z} dS = \\ & = \int_{V'} (\vec{E}_v^+ + \Gamma_{2v} \vec{E}_v^-) \cdot \hat{y} J_{yi} dV' \quad . \end{aligned}$$

The integrand in the second integral is identically zero (hence the choice for $(\vec{E}_2, \vec{H}_2)$ to solve for A_v). When the cross products in the first integral are expanded, the terms with the single coefficient Γ_{1v} or Γ_{2v} cancel. When the remaining fields

are expanded according to Equations (2.6), the exponential z dependence factors are seen to cancel. Since the integrand involves the scalar product with $\hat{z}$, only the $\hat{z}$ -component of the cross products contribute to the integral so that the terms involving $\hat{z} e_{z,v}$ and $\hat{z} h_{z,v}$ have no contribution. With the above the equation becomes:

$$\begin{aligned} -2A_v \int_{c.s.A} [(\vec{e}_{t,v} \times \vec{h}_{t,v}) - \Gamma_{1v} \Gamma_{2v} (\vec{e}_{t,v} \times \vec{h}_{t,v})] \cdot \hat{z} dS = \\ = \int_{V'} (\vec{E}_v^+ + \Gamma_{2v} \vec{E}_v^-) \cdot \hat{y} J_{yi} dV' . \end{aligned}$$

Invoking the assumed normalization then yields the desired result:

$$A_v = \frac{1}{\Gamma_{1v} \Gamma_{2v} - 1} \int_{V'} e_{y,v}(x', y') [e^{-\gamma_v z'} + \Gamma_{2v} e^{+\gamma_v z'}] J_{yi}(x', y', z') dV' \quad (2.8)$$

which is different for TE or TM modes accordingly as e_{yv}^e and e_{yv}^h are different.

To determine the coefficients C_v , choose $\vec{J}_1$ and $(\vec{E}_1, \vec{H}_1)$ as above and again let

$$\vec{J}_2(x', y', z') = 0,$$

but now choose

$$\begin{aligned} \vec{E}_2 &= \vec{E}_v^- + \Gamma_{1v} \vec{E}_v^+ \\ \vec{H}_2 &= \vec{H}_v^- + \Gamma_{1v} \vec{H}_v^+ . \end{aligned}$$

Following steps (a) through (f) above, the result for C_v is:

$$C_v = \frac{1}{\Gamma_{1v} \Gamma_{2v} - 1} \int_{V'} e_{y,v}(x', y') [e^{+\gamma_v z'} + \Gamma_{1v} e^{-\gamma_v z'}] J_{yi}(x', y', z') dV' \quad (2.9)$$

which is again TE and TM mode dependent.

It should be observed that, in the course of the simplification (step (d) above), the expansion coefficients lose any dependence on the particular choice of waveguide cross-section planes at z_A and z_B . This choice may thus be taken as close to the i -th antenna as desired.

These expansion coefficients may now be substituted into the general expansions for the fields in a waveguide, Equations (2.7a,b), to yield expressions for the fields induced in the waveguide by the unknown current on the i -th antenna. At this point, however, it is most expeditious to recognize that only the $\hat{y}$ -component of the induced electric field is required for application of the boundary conditions, Equation (2.1). From Equations (2.7) then,

$$E_{y,A}(x,y,z) = \sum_v A_{y,v} e_{y,v}(x,y) (e^{+\gamma_v z} + \Gamma_{1,v} e^{-\gamma_v z}) \quad z \leq z_A \quad (2.10a)$$

$$E_{y,B}(x,y,z) = \sum_v C_{y,v} e_{y,v}(x,y) (e^{-\gamma_v z} + \Gamma_{2,v} e^{+\gamma_v z}) \quad z \geq z_B \quad (2.10b)$$

Upon substitution of A_v and C_v from Equations (2.8) and (2.9), it may be seen (after some algebra) that Equations (2.10a) and (2.10b) can be combined into one equation (unconditional in z) by appropriate use of absolute values in the exponential terms. Thus, the field induced by the i -th antenna current is:

$$E_y^i(x,y,z) \big|_{(i)} = \sum_v \frac{1}{\Gamma_{1,v} \Gamma_{2,v} - 1} \int_{V_i} J_{yi}(x',y',z') e_{y,v}(x,y) (e^{-\gamma_v |z-z'|} + \Gamma_{1,v} e^{-\gamma_v (z+z')} + \Gamma_{2,v} e^{+\gamma_v (z+z')} + \Gamma_{1,v} \Gamma_{2,v} e^{+\gamma_v |z-z'|}) dV' \quad (2.11)$$

Equation (2.11) may now be expanded by applying Equation (2.5) for

$e_{y,v}$. The distinction must here be made between TE and TM modes.

The integrand contains the term

$$e_{y,v}^e(x',y')e_{y,v}^e(x,y) = e_{y,v}^e(x',y')e_{y,v}^e(x,y) + e_{y,v}^h(x',y')e_{y,v}^h(x,y) .$$

Using Equations (2.5), this becomes

$$\left[\frac{2 Z_v^e k_y^2}{ab k_c^2} + \frac{Z_v^h \epsilon_{on} \epsilon_{om} k_x^2}{2 ab k_c^2} \right] \sin(k_x x) \sin(k_x x') \cos(k_y y) \cos(k_y y')$$

which reduces to

$$\left[\frac{j \epsilon_{om}}{ab \omega \epsilon \gamma_v} (k_o^2 - k_y^2) \right] \sin(k_x x) \sin(k_x x') \cos(k_y y) \cos(k_y y') .$$

Further, the order of volume integration and modal summation in Equation (2.11) may be interchanged. Finally, the total $\hat{y}$ -component of induced electric field in the waveguide is found by a linear superposition of the fields from each of the N antennas.

The complete result is:

$$E_y^i(x,y,z) = \sum_{i=1}^N \int_V J_{yi}(x',y',z') \sum_v \frac{j \epsilon_{om}}{ab \omega \epsilon \gamma_v} (k_o^2 - k_y^2) \sin(k_x x) \sin(k_x x') \cos(k_y y) \cos(k_y y') f_v(z|z') dV' \quad (2.12)$$

where

$$f_v(z|z') = \frac{e^{-\gamma_v |z-z'|} + \Gamma_{1v} e^{-\gamma_v (z+z')} + \Gamma_{2v} e^{+\gamma_v (z+z')} + \Gamma_{1v} \Gamma_{2v} e^{+\gamma_v |z-z'|}}{\Gamma_{1v} \Gamma_{2v} - 1} \quad (2.13)$$

By comparing Equation (2.12) with Equation (2.3), the function G_{yy}

(in this report referred to as the Green's function) is identified as:

$$G_{yy}(x,y,z|x',y',z') = \sum_n \sum_m \frac{j \epsilon_{om}}{ab\omega\epsilon\gamma_{nm}} (k_o^2 - k_y^2) \sin(k_x x) \sin(k_x x') \cos(k_y y) \cos(k_y y') f_{nm}(z|z') \quad (2.14)$$

such that

$$E_y^i(x,y,z) = \sum_{i=1}^N \int_{V_i} J_{yi}(x',y',z') G_{yy}(x,y,z|x',y',z') dV'. \quad (2.15)$$

Equations (2.12) through (2.15) now completely define the $\hat{y}$ -component of the induced electric field in the waveguide in terms of the N unknown antenna current distributions. The equations necessary to solve for the current distributions are obtained in the next section.

2.4 Integral Equations; Reduction to Hallen-Type Equations.

Having thus far obtained expressions for the impressed and induced electric fields in the waveguide, it now remains to apply the boundary conditions on each antenna surface and solve the resulting integral equations for the unknown current. A straightforward solution is not attempted, however, for two reasons. First, the fact that the antennas do not span the waveguide implies that one set of boundary conditions does not apply across the entire waveguide cross-section. This makes an analytical solution unwieldy, and thus a numerical solution is sought. A second reason, with this in mind, is that the assumed mathematical delta function excitation cannot be implemented numerically. Therefore, the integral equations are expanded into equivalent integro-differential equations which are separable. These integro-differential equations are readily reduced to Hallen-type equations by solving the indicated differential equation. The result is then solved numerically in the next chapter.

The boundary conditions on the surfaces of the N antennas are given in Equation (2.1):

$$E_{yj}^i(x,y,z) = E_{yj}^e(x,y,z) \quad j = 1, 2, \dots, N \quad (2.1)$$

Substitution of Equations (2.2) and (2.15) for the impressed and induced electric fields yields

$$\sum_{i=1}^N \int_{V_i} J_{yi}(x', y', z') G_{yy}(x, y, z | x', y', z') dV' = -V_j \delta(y - \tau/2) \quad (2.16)$$

$$\dots j = 1, 2, \dots, N \quad \begin{cases} 0 \leq y \leq h_j \\ (x, y) \text{ on } j\text{-th antenna surface} \end{cases}$$

where G_{yy} is defined in Equation (2.14). Equation (2.16) is the desired system of N coupled integral equations in the unknown currents for the assumed slice generator approximation.

To convert these pure integral equations to integro-differential equations, the following observation is made:

$$(k_o^2 + \frac{\partial^2}{\partial y^2}) \cos(k_y y) = (k_o^2 - k_y^2) \cos(k_y y) \quad (2.17)$$

where $k_y = m\pi/b$. The right hand side of Equation (2.17) is recognized as a factor in the Green's function, Equation (2.14), and the equivalent differential operator is substituted. Since the derivative with respect to y is no longer a function of mode index m (k_y) and is also independent of the volume integration over (primed) source variables, it may be taken outside the integral and summation so that the integral equations (2.16) become:

$$\begin{aligned}
 (k_o^2 + \frac{\partial^2}{\partial y^2}) \sum_{i=1}^N \int_{V_i} J_{yi}(x', y', z') G_{yy}^1(x, y, z | x', y', z') dV' = -V_j \delta(y - \tau/2) \\
 \dots j = 1, 2, \dots, N \quad \begin{cases} 0 \leq y \leq h_j \\ (x, z) \text{ on } j\text{-th antenna surface} \end{cases}
 \end{aligned} \tag{2.18a}$$

where the new "Green's function" or kernel is:

$$G_{yy}^1(x, y, z | x', y', z') = \sum_n \sum_m \frac{j \epsilon_{om}}{ab\omega\epsilon\gamma_{nm}} \sin(k_x x) \sin(k_x x') \cos(k_y y) \cos(k_y y') f_{nm}(z | z') .$$

This is now a system of N coupled integro-differential equations.

By letting

$$B_j(y) = \sum_{i=1}^N \int_{V_i} J_{yi}(x', y', z') G_{yy}^1(x, y, z | x', y', z') dV' , \tag{2.19}$$

Equations (2.18a) become:

$$(k_o^2 + \frac{\partial^2}{\partial y^2}) B_j(y) = -V_j \delta(y - \tau/2) \quad j = 1, 2, \dots, N \tag{2.18b}$$

The solution to these differential equations is (a similar equation is solved in Section 3.2 of King and Harrison [7])

$$B_j(y) = C_j \cos(k_o y) - \frac{V_j}{2k_o} \sin(k_o |y - \tau/2|) \tag{2.20}$$

where the symmetry property

$$B_j(y) = B_j(-y)$$

has been invoked (as seen from the cosine dependence of G_{yy}^1 and hence $B_j(y)$). Equating the right hand sides of Equations (2.19) and (2.20) yields pure integral equations for the unknown currents:

$$\sum_{i=1}^N \int_{V'_i} J_{yi} G_{yy}^1 dV' = C_j \cos(k_o y) - \frac{V_j}{2k_o} \sin(k_o |y - \tau/2|)$$

$$\dots j = 1, 2, \dots, N \quad \begin{cases} 0 \leq y \leq h \\ (x, z) \text{ on } j\text{-th antenna surface} \end{cases}$$

These integral equations have been derived for the slice generator approximation with a finite gap width τ . As stated in the discussion leading up to Equation (2.2), the zero width gap, which is desired, is found by taking the limit as τ approaches zero. It is necessary to delay this limiting process until this point so that the ambiguous nature of a delta function centered at $y = 0$ is avoided in the solution of the differential equations (2.18b). Performing the limit as τ approaches zero is straightforward. Also, since the integral equations are valid only for $y \geq 0$, the absolute value may be removed to give:

$$\sum_{i=1}^N \int_{V'_i} J_{yi}(x', y', z') G_{yy}^1(x, y, z | x', y', z') dV' = C_j \cos(k_o y) - \frac{V_j}{2k_o} \sin(k_o y)$$

$$j = 1, 2, \dots, N \quad \begin{cases} 0 \leq y \leq h_j \\ (x, z) \text{ on } j\text{-th antenna surface} \end{cases} \quad (2.21)$$

Equation (2.21) is the desired system of N coupled Hallen-type integral equations in $2N$ unknowns; namely, N current density functions and N new unknowns C_j which result from the solution of the differential equations. There are also available N new boundary conditions which render the equation solvable. These result from the fact that the antenna current is zero at the end of each antenna; i.e.,

$$J_{yi}(x, h_i, z) \equiv 0 \quad \begin{cases} i = 1, 2, \dots, N \\ (x, z) \text{ on } i\text{-th antenna surface} \end{cases} \quad (2.22)$$

It should be emphasized that the vanishing of the current will only be true for antennas for which $h_i < b$, and the foregoing reduction to Hallen-type equations would be pointless for the problem of posts which span the waveguide.

2.5 Cylindrical Antenna Approximations.

A rigorous solution for the antenna current, under the assumption of a perfectly conducting antenna, would yield a cylindrical sheet of surface current flowing on the surface of the antenna. The complexity involved, especially numerically, in obtaining an accurate description of the current in three dimensions is obvious. In this thesis, therefore, approximations are made for the cylindrical antenna to simplify the numerical solution in rectangular coordinates. First, the thin-wire approximation is employed as in standard linear antenna theory [17]. It is found, however, that this approximation yields a divergent series in the numerical solution (see Chapter 3) and is not useable. Therefore, two other approaches are taken in which the cylinder is approximated by an equivalent strip. This approach is similar to that of Eisenhart and Khan [7] for a post spanning a waveguide, except that a more realistic representation is made of the strip surface current density. In this section, the three approximations are described and the integral equations corresponding to each approximation are obtained.

2.5.1 Thin-Wire Approximation

The simplest approximation, and the one most often used in antenna theory, is the thin wire approximation. This approximation assumes that the antenna radius is small compared to the wavelength; i.e., $r_a \ll \lambda$. Under this condition the current on the antenna is approximated as a line current flowing on the antenna axis. For the case in hand, the thin-wire approximation implies

$$J_{yi}(x', y', z') = I_i(y') \delta(x' - a/2) \delta(z' - z_i) \quad 0 \leq y' \leq h_i.$$

The boundary condition on tangential electric field on the antenna is still maintained at the cylindrical surface. However, for simplicity of numerical implementation, the boundary condition is only satisfied along lines on the antenna surface. The lines chosen are defined by:

$$\begin{aligned} x &= a/2 \pm r_a \\ 0 &\leq y \leq h_j \\ z &= z_j. \end{aligned}$$

Under this approximation, the integral equation (2.21) becomes

(using superscript tw for thin-wire)

$$\sum_{i=1}^N \int_{y'=0}^{h_i} I_i(y') G_{ij}^{tw}(y|y') dy' = C_j \cos(k_o y) - \frac{V}{2k_o} \sin(k_o y) \quad (2.23a)$$

$$0 \leq y \leq h_j \quad j = 1, 2, \dots, N$$

where the triple volume integration was reduced to the line integral by straightforward evaluation of the x and z integration over the specified delta functions. The resulting kernel for the integral

equations (thin-wire approximation) is thus

$$G_{ij}^{tw}(y|y') = \sum_{\substack{n \\ \text{odd}}}^{\infty} \sum_{m}^{\infty} \frac{j \epsilon_{om}}{ab\omega\epsilon\gamma_{nm}} \cos(k_x r_a) \cos(k_y y) \cos(k_y y') f_{nm}(z_j|z_i) \quad (2.23b)$$

where

$$f_v(z_j|z_i) = \frac{e^{-\gamma_v|z_j-z_i|} + \Gamma_{iv} e^{-\gamma_v(z_j+z_i)} + \Gamma_{2v} e^{+\gamma_v(z_j+z_i)} + \Gamma_{1v}\Gamma_{2v} e^{+\gamma_v|z_j-z_i|}}{\Gamma_{1v}\Gamma_{2v} - 1} \quad (2.23c)$$

and v represents both n and m . The appropriate boundary conditions from Equation (2.22) are

$$I_i(h_i) \equiv 0 \quad i = 1, 2, \dots, N \quad (2.23d)$$

Equation (2.23a) is now a system of N coupled integral equations in one dimension which is suitable for numerical solution. The details of the solution are discussed further in Chapter 3.

2.5.2 Strip Approximations

A second approximation made for the cylindrical antenna is the strip approximation. Several references are available stating the equivalence between the strip and the cylinder through a conformal mapping (in particular, see page 3-6 of Jasik [15] and section I.7 of King [17]). For the infinitely thin strip, the strip width, w , is related to the cylinder radius by

$$w = 4 r_a \quad (2.24)$$

The total axial ($\hat{y}$) current flowing on the strip must satisfy the relation

$$I(y') = \int_{S'} J_y(x', y', z') dS'$$

where S' is the cross-section of the strip. It will be assumed that the current is separable into x, y , and z dependences such that

$$J_y(x', y', z') = I(y') f(x', z') .$$

This is only approximate for the antenna of finite length, but is a realistic approximation under the previous assumption of $r_a < h$. From the above, it is obvious that the current cross-sectional dependence must satisfy the relation

$$\int_{S'} f(x', z') dx' dz' = 1 . \quad (2.25)$$

Thus, by determining the functional dependence that satisfies Equation (2.25), the problem is again reduced to a one dimensional one, as in the thin wire approximation. As will be seen, the function $f(x', y')$ is easily obtained and results in a straightforward reduction of the problem. If this had not been the case, the (more rigorous) two dimensional problem could have been approached numerically. The degree of numerical complexity, however, would greatly increase and this in itself encourages the approximate approach.

For the antenna in the waveguide, the lack of "rotational" symmetry with respect to the antenna axis requires that the orientation of the strip be specified. This thesis considers two cases:

- i) the strip in a transverse or (x, y) plane, and ii) the strip in a longitudinal or (y, z) plane.

i) Transverse Strip Approximation

For the strip oriented in a transverse waveguide plane, the current in the strip is a surface current and the cross-sectional dependence may be expressed in terms of the Dirac delta function as

$$f(x', y') = g(x') \delta(z' - z_i) ,$$

for the i -th antenna. The condition (2.25) then becomes

$$\int_{a/2-w/2}^{a/2+w/2} g(x') dx' = 1 \quad (2.26)$$

for the antenna located at $x = a/2$ on the waveguide wall. An appropriate guess for the transverse variation of current density, $g(x')$, is deduced through the following reasoning. The $\hat{y}$ -component of current on the strip is proportional to the $\hat{x}$ -component of magnetic field (by the boundary condition for tangential magnetic field) and thus $g(x')$ will be proportional to the variation of H_x with x . This variation is obtained from the "physical dual" or dual complementary problem consisting of a slot in an infinite plane (see Chapter 13 of King and Harrison [18]). In the complementary slot, the dual quantity of interest is the $\hat{x}$ -component of electric field, so that $g(x')$ is proportional to the variation of E_x with x . The appropriate distribution to use for E_x in a slot is stated in Galejs [8] (derivable by a conformal mapping technique using a Schwartz transformation), the result being

$$g(x') = \frac{1}{\pi \sqrt{(2r_a)^2 - (x' - a/2)^2}} \quad (2.27)$$

for the geometry of the case in hand. This is seen to satisfy Equation (2.26) and thus the condition (2.25).

It should be noted here that in the strip approximation made by Eisenhart and Khan [7], the x distribution is chosen as constant across the strip. They subsequently find the resulting input impedance in error and consequently reduce the equivalent strip width to obtain agreement. Based on the foregoing arguments, one would expect the error in impedance to be caused by the choice of current distribution rather than the strip width. Of course, the equivalent strip width is truly an approximation for the cylinder in a waveguide since the arguments of complementarity only hold in an infinite unbounded system. However, this approximation is expected to be valid as long as the antenna radius is small compared with the waveguide dimensions.

To proceed with the transverse strip approximation, the current distribution on the i -th strip will be taken to be

$$J_{yi}(x', y', z') = I_i(y') \frac{\delta(z' - z_i)}{\pi \sqrt{(2r_a)^2 - (x' - a/2)^2}} \quad (2.28)$$

$$\text{for } 0 \leq y' \leq h_i ; a/2 - 2r_a \leq x' \leq a/2 + 2r_a .$$

The j -th antenna surface, along which the integral equations must be satisfied, is described by the line

$$\begin{aligned} x &= a/2 \\ 0 &\leq y \leq h_j \\ z &= z_i \end{aligned} \quad (2.29)$$

That only a line is necessary, results from the assumption of an analytical form for the x distribution of current; this being

tantamount to satisfying the boundary condition across the entire strip width. Substitution of Equations (2.28) and (2.29) into the integral equations (2.21) gives

$$\sum_{i=1}^N \int_{V_i} I_i(y') \frac{\delta(z' - z_i)}{\pi \sqrt{(2r_a)^2 - (x' - a/2)^2}} G_{yy}^1(a/2, y, z_j | x', y', z') dV' =$$

$$= C_j \cos(k_o y) - \frac{V_i}{2k_o} \sin(k_o y) \quad \begin{cases} 0 \leq y \leq h_j \\ j = 1, 2, \dots, N \end{cases}$$

where $G_{yy}^1(x, y, z | x', y', z')$ is given in Equation (2.18). The volume integration may be performed successively, the z' integration begin straightforward. Integration over the x' variable requires evaluation of

$$\int_{a/2-2r_a}^{a/2+2r_a} \frac{\sin(k_x x')}{\sqrt{(2r_a)^2 - (x' - a/2)^2}} dx'.$$

By a change of variables, $u = x' - a/2$, this becomes

$$(-1)^{\frac{n-1}{2}} \int_{-2r_a}^{2r_a} \frac{\cos(k_x u)}{\sqrt{(2r_a)^2 - u^2}} du \quad \text{for odd } n$$

$$0 \quad \text{for even } n$$

or, by equation (23) in Galejs [8],

$$(-1)^{\frac{n-1}{2}} \pi J_0(2k_x r_a) \quad \text{odd } n$$

where J_0 is the zeroth order Bessel Function. With this, the integral equations reduce to (using superscript st for transverse strip):

$$\sum_{i=1}^N \int_{y'=0}^{h_i} I_i(y') G_{ij}^{st}(y|y') dy' = C_j \cos(k_o y) - \frac{V_j}{2k_o} \sin(k_o y) \quad (2.30a)$$

$$0 \leq y \leq h_j \quad j = 1, 2, \dots, N$$

The kernel for the transverse strip approximation is

$$G_{ij}^{st}(y|y') = \sum_{\substack{n \\ \text{odd}}}^{\infty} \sum_m^{\infty} \frac{j \epsilon_{om}}{ab\omega\epsilon\gamma_{nm}} \phi_o(2k_x r_a) \cos(k_y y) \cos(k_y y') f_{nm}(z_j|z_i) \quad (2.30b)$$

where $f_{nm}(z_j|z_i)$ is given in Equation (2.23c). The system of integral equations (2.30) is now in final form for the numerical evaluation to be done in Chapter 3.

ii) Longitudinal Strip Approximation

For the strip oriented in a longitudinal waveguide plane, the analysis proceeds as above. In this case, however, the current distribution on the i -th strip is, by analogy,

$$J_{yi}(x', y', z') = I_i(y') \frac{\delta(x' - a/2)}{\pi \sqrt{(2r_a)^2 - (z' - z_i)^2}} \quad (2.31)$$

$$\text{for } 0 \leq y' \leq h_i ; z_i - 2r_a \leq z' \leq z_i + 2r_a .$$

The j -th antenna surface along which the integral equations are to be satisfied is, as before, the line described by

$$x = a/2$$

$$0 \leq y \leq h_j$$

$$z = z_i$$

The integral equations (2.21) become

$$\sum_{i=1}^N \int_{V'_i} \frac{\delta(x' - a/2)}{\pi \sqrt{(2r_a)^2 - (z' - z_i)^2}} G_{yy}^1(a/2, y, z_j | x', y', z') dV' =$$

$$= C_j \cos(k_o y) - \frac{V_j}{2k_o} \sin(k_o y) \quad \begin{cases} 0 \leq y \leq h_j \\ j = 1, 2, \dots, N \end{cases} \quad (2.32)$$

where $G_{yy}^1(x, y, z | x', y', z')$ is given in Equation (2.18). Again, the separable kernel allows the triple volume integral to be performed successively. Here, however, the x' integration is straightforward and the z' integration is the more complicated. Integration over z' requires evaluation of the following integral

$$\int_{z_i - 2r_a}^{z_i + 2r_a} \frac{f_v(z_j | z')}{\pi \sqrt{(2r_a)^2 - (z' - z_i)^2}} dz' = F_v(z_j | z_i) .$$

The details of this integration are given in Appendix II; the results are, from Equations (II.15a) and (II.16b),

$$F_v(z_j | z_i) = \mathcal{J}_0(2\gamma_v r_a) f_v(z_j | z_i) \quad i \neq j \quad (2.33a)$$

$$F_v(z_i | z_i) = \mathcal{J}_0(2\gamma_v r_a) f_v(z_i | z_i) - \mathcal{L}_0(2\gamma_v r_a) \quad i = j \quad (2.33b)$$

where

$\mathcal{J}_0(2\gamma_v r_a)$ is the Modified Bessel function of zeroth order and complex argument.

$\mathcal{L}_0(2\gamma_v r_a)$ is the Modified Struve function of zeroth order and complex argument.

and

$f_v(z_j | z_i)$ is given in Equation (2.23c).

It is worthwhile to recall here that the subscripts (i, j) refer to the source point antenna and the field point antenna,

respectively. Therefore, the condition $i \neq j$ implies the use of the field excited by the current on one antenna to satisfy the boundary condition on a different antenna; whereas, the condition $i = j$ implies that the source current and field matching points are on the same antenna.

When the above integrations are applied to Equations (2.32) the resulting system of integral equations is (using $s\ell$ for longitudinal strip):

$$\sum_{i=1}^N \int_{y'=0}^{h_j} I_i(y') G_{ij}^{s\ell}(y|y') dy' = C_j \cos(k_o y) - \frac{V_i}{2k_o} \sin(k_o y) \quad (2.34a)$$

$$\begin{cases} 0 \leq y \leq h_j \\ j = 1, 2, \dots, N \end{cases}$$

The kernel for the longitudinal strip approximation is

$$G_{ij}^{s\ell}(y|y') = \sum_{\substack{n \\ \text{odd}}}^{\infty} \sum_m^{\infty} \frac{j \epsilon_{om}}{ab\omega\epsilon\gamma_{nm}} \cos(k_y y) \cos(k_y y') F_{nm}(z_j|z_i) \quad (2.34b)$$

with $F_{nm}(z_j|z_i)$ given in Equations (2.33). This system of equations is now suitable for numerical evaluation.

To summarize Section 2.5, the three dimensional set of N coupled integral equations which were derived in Section 2.4 have been reduced to one dimensional equations via three approximate approaches: Equations (2.23) result from the thin-wire approximation; Equations (2.30) result from the strip approximation applied in a transverse waveguide plane; Equations (2.34) result from the strip approximation applied in a longitudinal waveguide plane. It also may be recalled that all of the above expressions are based

on the delta-gap or slice generator approximation for the antenna terminals. In the next section, the slice generator approximation will be removed and the exact representation of the physical coaxial antenna terminals will be made.

2.6 Coaxial Gap Solution.

In the foregoing sections of this chapter, the physical problem of N antennas coupled to the interior of a waveguide through coaxial ports was analyzed using the slice generator approximation for the antenna terminals. It is possible, however, to approach the problem without this approximation by applying the techniques of the previous section. The complexity of this problem necessitated the consideration of the special case of a single antenna and its coaxial gap.

The specific geometry for the coaxial gap solution is shown in Figure 2.4. As can be seen, the problem is the same as that solved previously except for the addition of the annular region made by the coaxial transmission line in the waveguide wall. This annular region, S_g , is defined by an inner radius r_a and an outer radius r_b , centered on the broad wall of the waveguide at $x = a/2$. A time harmonic ($e^{j\omega t}$) voltage of magnitude V_0 is maintained across the coaxial transmission line, referenced to the outer conductor and waveguide wall. The electric field that exists in the coaxial gap is assumed to be the fundamental TEM coaxial field; namely,

$$\vec{E}_g(r) = \frac{V_0}{r \ln(r_b/r_a)} \hat{r} . \quad (2.35)$$

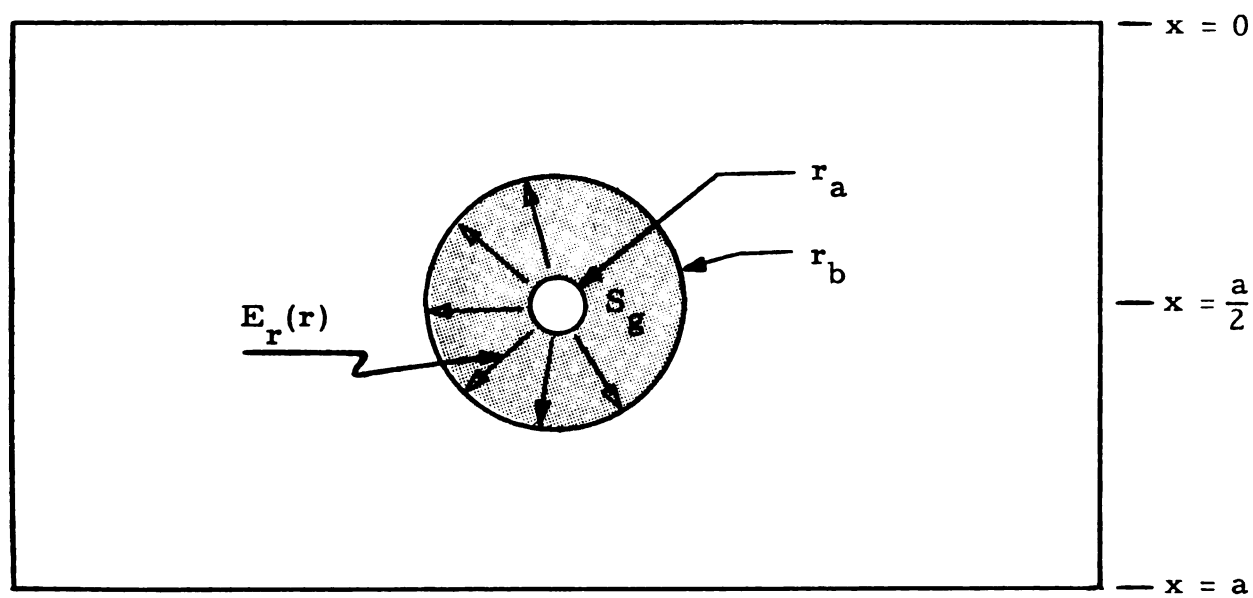
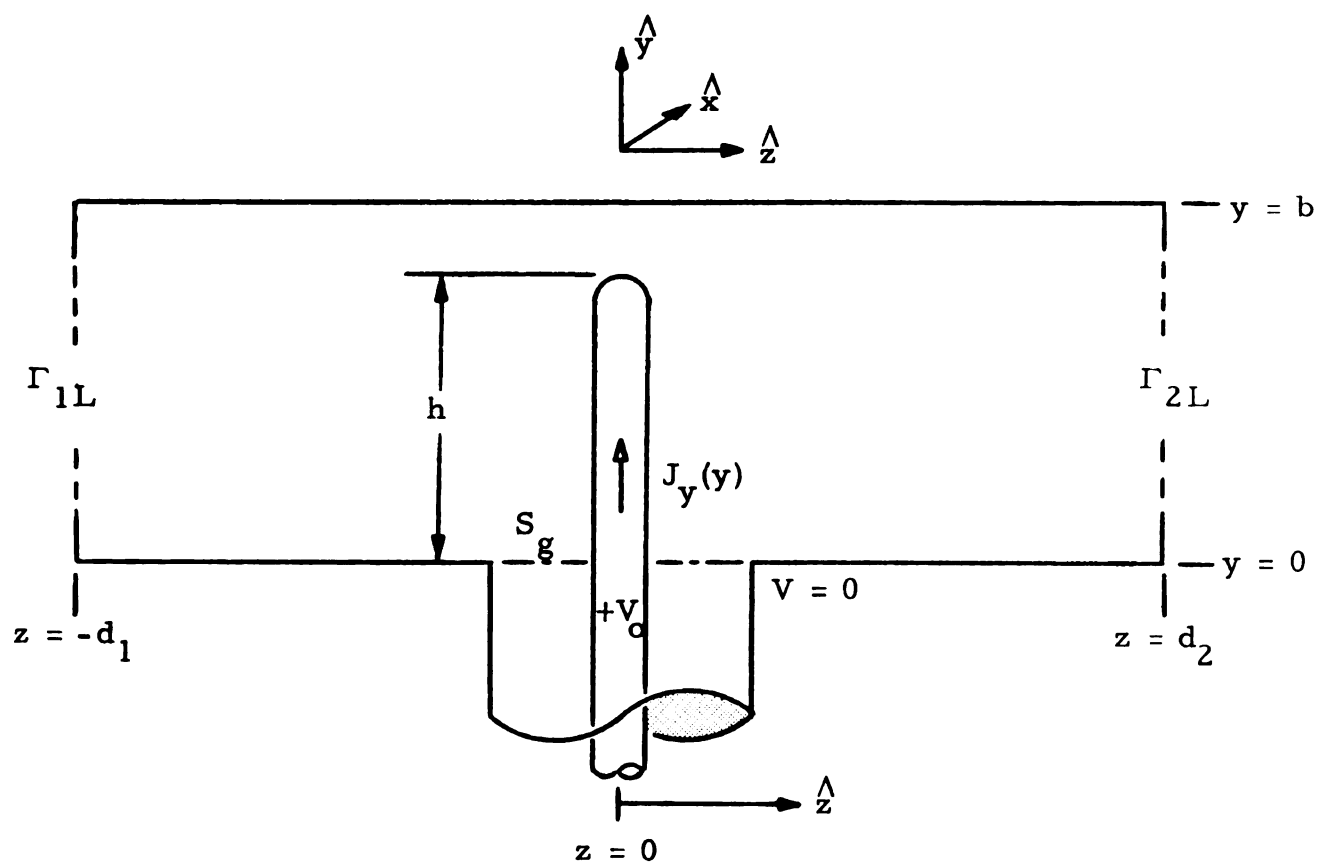


FIGURE 2.4

Coaxial Gap Excitation

In other words, the higher order coaxial field modes, which are excited by the abrupt junction between the coaxial line and the waveguide, have been ignored.

As already stated, the approach to this problem is the same as in the slice generated approximation; that is, integral equations are developed based on applying the boundary conditions; where the fields used in applying the boundary conditions are expressed as modal expansions via the Lorentz reciprocity relation.

Boundary conditions

To begin, the boundary conditions to be satisfied for the continuity of tangential electric field are: as before,

$$E_y^i(x,y,z) = E_y^e(x,y,z) \quad \dots \text{ on antenna surface} \quad (2.36a)$$

and the new condition for tangential electric field in the coaxial gap:

$$-\hat{y} \times \vec{E}^i(x,y,z) = -\hat{y} \times \vec{E}^e(x,y,z) \quad \dots \text{ in coaxial gap} \quad (2.36b)$$

Impressed fields

The impressed electric fields, from Figure 2.4, are seen to be:

On the conducting antenna

$$E_y^e(x,y,z) \equiv 0 \quad (2.37a)$$

for $0 \leq y \leq h$; (x,z) on antenna surface

and in the coaxial gap,

$$\begin{aligned} \vec{E}^e(x,y,z) &= \vec{E}_g(x,y,z) \\ \text{for } (x,y,z) &\text{ on } S_g \end{aligned} \quad (2.37b)$$

Induced fields

The induced field required in Equation (2.36a) is the linear superposition of the field induced by the current on the antenna with the coaxial gap removed (replaced by conductor), and the field induced by the electric field in the gap with the antenna removed. The former is exactly as found in Section 2.3 for the case $N = 1$ and $V = 0$; whereas, the latter is the field specified by (and resulting from) the boundary condition (2.36b). Using superscript J for the current induced field and superscript G for the gap induced field, the total induced field becomes, by superposition,

$$E_y^i(x,y,z) = E_y^J(x,y,z) + E_y^G(x,y,z) . \quad (2.38)$$

The current induced field, from Equation (2.15), is

$$E_y^J(x,y,z) = \int_V J_y(x',y',z') G_{yy}(x,y,z|x',y',z') dV' \quad (2.39a)$$

for $0 \leq y \leq h$, (x,y) on the antenna surface

where

$$\begin{aligned} G_{yy} = \sum_n \sum_m \frac{j \epsilon_{om}}{ab\omega\epsilon\gamma_{nm}} (k_o^2 - k_y^2) \sin(k_x x) \sin(k_x x') \\ \cos(k_y y) \cos(k_y y') f_{nm}(z|z') \end{aligned} \quad (2.39b)$$

and, $f_{nm}(z|z')$ is given in Equation (2.13).

Gap induced field

As before, the gap induced field is to be found as a modal expansion, where the coefficients are determined by application of the Lorentz reciprocity relation. In this case there are no physical current sources, so the reciprocity relation becomes:

$$\oint_S (\vec{E}_1 \times \vec{H}_2 - \vec{E}_2 \times \vec{H}_1) \cdot \hat{n} \, dS = 0 \quad (2.40)$$

The problem is shown in Figure 2.5 (a) where

$$S = (\text{c.s.A}) + (\text{c.s.B}) + S_g + (\text{waveguide walls}).$$

The analysis proceeds in the same manner as in Section 2.3, but before beginning, some forethought is required. It should first be recalled, that the induced electric field is sought in order to satisfy the boundary condition (2.36a) on the antenna surface. In other words, the field must be evaluated on the inner circle of radius r_a in Figure 2.5 (a). It is also recalled, however, that the reciprocity relation only yields results which apply on or outside the chosen surface; i.e., the results are valid for the regions $z \leq -r_b$ and $z \geq +r_b$ for the problem of Figure 2.5 (a). Because of the mixing of cartesian and cylindrical coordinates, the analytical evaluation of the integral over S_g is difficult unless care is taken in selecting the volume for the reciprocity relation. Also, the total coaxial gap must obviously be accounted for in obtaining expressions for the induced field. For these reasons, it is best to consider the two independent problems shown in Figures 2.5 (b) and (c), which constitute the whole problem by superposition. In this way, the antenna surface (inner circle) is included on the

General Problem

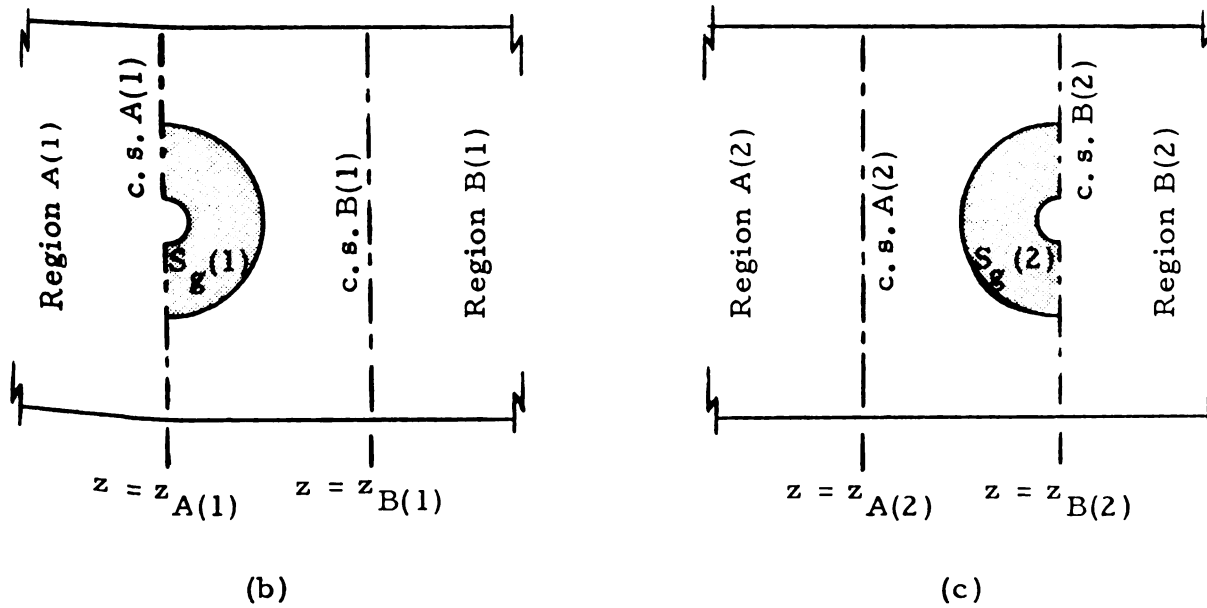


FIGURE 2.5

Geometry for Finding Gap Induced Fields Using the Lorentz Reciprocity Relation

surface enclosing the source for each case and the results can be used to satisfy the boundary condition (2.36a).

Considering first the case shown in Figure 2.5 (b), the modal expansions for the waveguide fields are:

in Region A(1),

$$\begin{aligned}\vec{E}_{A(1)} &= \sum_{\nu} A_{\nu}^{(1)} (\vec{E}_{\nu}^{-} + \Gamma_{1\nu} \vec{E}_{\nu}^{+}) \\ \vec{H}_{A(1)} &= \sum_{\nu} A_{\nu}^{(1)} (\vec{H}_{\nu}^{-} + \Gamma_{1\nu} \vec{H}_{\nu}^{+})\end{aligned}\quad \begin{aligned}z &\leq (\text{c.s. A}(1))\end{aligned}\quad (2.41a)$$

and in Region B(1),

$$\begin{aligned}\vec{E}_{B(1)} &= \sum_{\nu} C_{\nu}^{(1)} (\vec{E}_{\nu}^{+} + \Gamma_{2\nu} \vec{E}_{\nu}^{-}) \\ \vec{H}_{B(1)} &= \sum_{\nu} C_{\nu}^{(1)} (\vec{H}_{\nu}^{+} + \Gamma_{2\nu} \vec{H}_{\nu}^{-})\end{aligned}\quad \begin{aligned}z &\geq (\text{c.s. B}(1))\end{aligned}\quad (2.41b)$$

These fields are used as $(\vec{E}_1, \vec{H}_1)$ in Equation (2.40), while $(\vec{E}_2, \vec{H}_2)$ are chosen as fields for a source free region as in Section 2.3. The surface integral in Equation (2.40) is then iterated over the waveguide walls, the cross-sections, and $S_g(1)$. On the waveguide walls, the tangential electric field is zero so that $(\vec{E} \times \vec{H}) \cdot \hat{n} = (\hat{n} \times \vec{E}) \cdot \vec{H} = 0$, and these integrals disappear. On the cross-sections, the steps (a) through (f) in Section 2.3 (preceeding Equation (2.8)) are again employed to reduce and evaluate the integrals. Thus, for example, when solving for $A_{\nu}^{(1)}$, the choice is made that $(\vec{E}_2, \vec{H}_2) = (\vec{E}_{\nu}^{+} + \Gamma_{2\nu} \vec{E}_{\nu}^{-}, \vec{H}_{\nu}^{+} + \Gamma_{2\nu} \vec{H}_{\nu}^{-})$, and $(\vec{E}_1, \vec{H}_1)$ are taken from Equation (2.41). With this choice, the integral over cross section B(1) is identically zero (as in Section 2.3) so that

$$\begin{aligned}
\int_{\substack{\text{c.s. A}(1) + \\ \text{c.s. B}(1)}} (\vec{E}_1 \times \vec{H}_2 - \vec{E}_2 \times \vec{H}_1) \cdot \hat{n} \, dS &= \int_{\text{c.s. A}(1)} (\vec{E}_1 \times \vec{H}_2 - \vec{E}_2 \times \vec{H}_1) \cdot \hat{n} \, dS + 0 \\
&= A_v^{(1)} (1 - \Gamma_{1v} \Gamma_{2v}) (2 \int_{\text{c.s.}} (\vec{e}_v \times \vec{h}_v) \cdot \hat{n} \, dS) \\
&= A_v^{(1)} (1 - \Gamma_{1v} \Gamma_{2v})
\end{aligned}$$

In the remaining surface integral over $S_g(1)$, the following observations are made:

$$\begin{aligned}
(\vec{E}_1 \times \vec{H}_2) \cdot \hat{n} &= (\hat{n} \times \vec{E}_1) \cdot \vec{H}_2 \quad \dots \text{ by vector identity} \\
&= (\hat{n} \times \vec{E}^i) \cdot \vec{H}_2 \quad \dots \text{ by choice of } \vec{E}_1 \text{ as induced field} \\
&= (\hat{n} \times \vec{E}^e) \cdot \vec{H}_2 \quad \dots \text{ by Equation (2.36b)} \\
&= (\hat{n} \times \vec{E}_g) \cdot \vec{H}_2 \quad \dots \text{ by Equation (2.37b)}
\end{aligned}$$

and

$$\begin{aligned}
(\vec{E}_2 \times \vec{H}_1) \cdot \hat{n} &= (\hat{n} \times \vec{E}_2) \cdot \vec{H}_1 \quad \dots \text{ by vector identity} \\
&= 0
\end{aligned}$$

since $\vec{E}_2$ is chosen as a linear combination of waveguide modes (for source free region) and $\hat{n} \times \vec{E}_v = 0$ for $y = 0$. Thus,

$$\int_{S_g(1)} (\vec{E}_1 \times \vec{H}_2 - \vec{E}_2 \times \vec{H}_1) \cdot \hat{n} \, dS = \int_{S_g(1)} (\hat{y} \times \vec{E}_g) \cdot \vec{H}_1 \, dS$$

After combining the above steps, the reciprocity relation as stated in Equation (2.40) yields:

$$A_v^{(1)} = \frac{1}{\Gamma_{1v} \Gamma_{2v} - 1} \int_{S_g(1)} (\hat{y} \times \vec{E}_g) \cdot (\vec{H}_v^+ + \Gamma_{2v} \vec{H}_v^-) \, dS \quad (2.42a)$$

and

$$c_v^{(1)} = \frac{1}{\Gamma_{1v}\Gamma_{2v}-1} \int_{S_g(1)} (\hat{y} \times \vec{E}_g) \cdot (\vec{H}_v^- + \Gamma_{1v}\vec{H}_v^+) dS \quad (2.42b)$$

Following the foregoing procedure for the problem of Figure 2.5 (c) yields

$$A_v^{(2)} = \frac{1}{\Gamma_{1v}\Gamma_{2v}-1} \int_{S_g(2)} (\hat{y} \times \vec{E}_g) \cdot (\vec{H}_v^+ + \Gamma_{2v}\vec{H}_v^-) dS \quad (2.43a)$$

$$c_v^{(2)} = \frac{1}{\Gamma_{1v}\Gamma_{2v}-1} \int_{S_g(2)} (\hat{y} \times \vec{E}_g) \cdot (\vec{H}_v^+ + \Gamma_{2v}\vec{H}_v^-) dS \quad (2.43b)$$

where, for example, the following expansion was made

$$\vec{E}_{B(2)} = \sum_v c_v^{(2)} (\vec{E}_v^+ + \Gamma_{2v}\vec{E}_v^-) \quad z \geq (\text{c.s.B}(2)) \quad (2.44)$$

With the expansion coefficients determined in Equations (2.42) and (2.43), in terms of the known gap field, the problem is essentially completed since substitution into the modal expansions defines all the desired field quantities. The field which is sought for the boundary condition is seen to be the superposition of the electric fields on cross-section c.s.A(1) and on cross-section c.s.B(2). Thus,

$$\vec{E}^G = \vec{E}_{A(1)} + \vec{E}_{B(2)} \quad \text{for } z = z_i (= 0) \quad (2.45)$$

Since these are all that is required, further attention need only be focused on the coefficients $A_v^{(1)}$ and $c_v^{(2)}$.

As can be seen, the coefficients are dependent upon whether TE or TM modes are considered. It is best to make the distinction at this time, such that, from Equation (2.41a),

$$\vec{E}_{A(1)} = \sum_{\nu} A_{\nu}^e (\vec{E}_{\nu}^{e-} + \Gamma_{1\nu} \vec{E}_{\nu}^{e+}) + \sum_{\nu} A_{\nu}^h (\vec{E}_{\nu}^{h-} + \Gamma_{1\nu} \vec{E}_{\nu}^{h+})$$

and from Equation (2.44),

$$\vec{E}_{B(2)} = \sum_{\nu} C_{\nu}^e (\vec{E}_{\nu}^{e+} + \Gamma_{2\nu} \vec{E}_{\nu}^{e-}) + \sum_{\nu} C_{\nu}^h (\vec{E}_{\nu}^{h+} + \Gamma_{2\nu} \vec{E}_{\nu}^{h-}) .$$

Since only the $\hat{y}$ -component of these fields is required for the boundary condition, the fields of interest are,

$$\begin{aligned} E_{y,A(1)} = & \sum_{\nu} A_{\nu}^e e_{y,\nu}^e(x,y) [e^{+\gamma_{\nu}z} + \Gamma_{1\nu} e^{-\gamma_{\nu}z}] + \\ & + \sum_{\nu} A_{\nu}^h e_{y,\nu}^h(x,y) [e^{+\gamma_{\nu}z} + \Gamma_{1\nu} e^{-\gamma_{\nu}z}] \end{aligned} \quad (2.46a)$$

and

$$\begin{aligned} E_{y,B(2)} = & \sum_{\nu} C_{\nu}^e e_{y,\nu}^e(x,y) [e^{-\gamma_{\nu}z} + \Gamma_{2\nu} e^{+\gamma_{\nu}z}] + \\ & + \sum_{\nu} C_{\nu}^h e_{y,\nu}^h(x,y) [e^{-\gamma_{\nu}z} + \Gamma_{2\nu} e^{+\gamma_{\nu}z}] \end{aligned} \quad (2.46b)$$

In Equations (2.46) the expressions given in Equation (2.6) have been used to separate the (x,y) and z dependences. Expressions for $e_{y,\nu}(x,y)$ are given in Equations (2.5). The coefficients in Equations (2.46) may be expressed (from Equations (2.42a) and (2.43b)) as

$$A_{\nu}^e = \frac{1}{\Gamma_{1\nu}\Gamma_{2\nu}-1} [\mathcal{F}_{(1)}^{e+} + \Gamma_{2\nu} \mathcal{F}_{(1)}^{e-}] \quad (2.47a)$$

$$A_{\nu}^h = \frac{1}{\Gamma_{1\nu}\Gamma_{2\nu}-1} [\mathcal{F}_{(1)}^{h+} + \Gamma_{2\nu} \mathcal{F}_{(1)}^{h-}] \quad (2.47b)$$

$$C_{\nu}^e = \frac{1}{\Gamma_{1\nu}\Gamma_{2\nu}-1} [\mathcal{F}_{(2)}^{e-} + \Gamma_{2\nu} \mathcal{F}_{(2)}^{e+}] \quad (2.47c)$$

$$C_{\nu}^h = \frac{1}{\Gamma_{1\nu}\Gamma_{2\nu}-1} [\mathcal{F}_{(2)}^{h-} + \Gamma_{2\nu} \mathcal{F}_{(2)}^{h+}] \quad (2.47d)$$

where

$$\mathfrak{F}_{(1)}^{(e)\pm} = \int_{S_g(1)} (\hat{y} \times \vec{E}_g) \cdot \vec{H}_v^{(e)\pm} dS \quad (2.48a)$$

$$\mathfrak{F}_{(2)}^{(e)\pm} = \int_{S_g(2)} (\hat{y} \times \vec{E}_g) \cdot \vec{H}_v^{(e)\pm} dS \quad (2.48b)$$

These eight integrals are evaluated in Appendix II. As can be seen from Equations (II.31), (II.41), (II.43) and (II.44), the expressions for the integrals are extremely lengthy. Subsequent substitution into the expressions for the coefficients, Equations (2.47), and then the expressions for the electric field, Equations (2.46), requires a great deal of algebra, albeit straightforward. The final result for the $\hat{y}$ -component of the gap induced electric field is, from Equation (2.45),

$$\begin{aligned} E_y^G = \sum_v \frac{V_0}{\ln(r_b/r_a)} (-1)^{\frac{n+1}{2}} \frac{\epsilon_{0m}}{ab\gamma_v} \sin(k_x x) \cos(k_y y) \frac{1}{\Gamma_{1v}\Gamma_{2v}-1} \times \\ \times \{ [(1 + \Gamma_{1v}\Gamma_{2v}) \cosh(\gamma_v z) + \Gamma_{1v} e^{-\gamma_v z} + \Gamma_{2v} e^{+\gamma_v z}] \pi \text{DBES}(0) + \\ + j4(1 - \Gamma_{1v}\Gamma_{2v}) \cosh(\gamma_v z) \sum_{\substack{p \\ \text{odd}}}^{\infty} \text{DBES}(p) \cos(p\psi)/p \} \\ \text{for } z = z_i (= 0) \text{ and } n = \text{odd} \end{aligned} \quad (2.49)$$

where

$$\text{DBES}(p) = \mathcal{J}_p(r_b \xi) - \mathcal{J}_p(r_a \xi)$$

$$\xi^2 = k_x^2 - \gamma_v^2$$

$$\psi = \arctan(k_x / j\gamma_v) \quad .$$

Other notation is described in Appendix I.

Now that the gap induced field is explicitly evaluated, the total problem undertaken in this section is complete. Substitution of Equations (2.49) and (2.39) into Equation (2.38) gives the y -component of induced electric field. The boundary condition (2.36a) with Equation (2.37a) then represents an expression which may be solved for the unknown current distribution. That is,

$$E_y^J(x,y,z) + E_y^G(x,y,z) = 0 \quad (2.50)$$

is the desired integral equation for the coaxial gap problem.

In the slice generator approximation, it was found convenient to reduce the integral equation to a Hallen type by creating an integro-differential equation. This reduction is not necessary here because all functions are non-singular (there is no delta function). In order to obtain a numerical solution, however, it is best to approximate the current distribution, as before, in order to reduce the problem to one dimension. It can be seen that the strip approximations made for the slice generator problem are not acceptable here because the coaxial gap would also need to be transformed. Therefore, the most straightforward solution is found by making the thin-wire approximation.

To apply the thin-wire approximation to the coaxial gap problem, the current is again taken to be

$$J_y(x',y',z') = I(y')\delta(x' - a/2)\delta(z') \quad 0 \leq y' \leq h$$

where the one antenna considered is taken at the origin ($z' = 0$) of coordinates. This approximation is now applied to the integral expression for the current induced field, E_y^J , given in Equation

(2.39), and the x' and z' integration performed. Next, the integral equation represented by Equation (2.50) is satisfied on the cylindrical antenna surface along lines defined by

$$\begin{aligned} x &= a/2 \pm r_a \\ 0 &\leq y \leq h \\ z &= 0 \end{aligned}$$

With these steps, the integral equation becomes (from Equation (2.50)):

$$E_y^J(a/2 \pm r_a, y, 0) + E_y^G(a/2 \pm r_a, y, 0) = 0 \quad (2.51a)$$

or

$$E_y^J(y) = -E_y^G(y) \quad (2.51b)$$

where, from Equation (2.39) after integration, the current induced field is

$$E_y^J(y) = \int_{y'=0}^h I(y') G_{yy}(y|y') dy' \quad 0 \leq y \leq h \quad (2.52a)$$

with

$$\begin{aligned} G_{yy}(y|y') &= \sum_{\substack{n \\ \text{odd}}} \sum_m \frac{j \epsilon_{om}}{ab\omega\epsilon\gamma_{nm}} (k_o^2 - k_y^2) (-1)^{\frac{n-1}{2}} \cos(k_x r_a) (-1)^{\frac{n-1}{2}} \\ &\quad \cos(k_y y) \cos(k_y y') f_{nm}(0|0) \end{aligned} \quad (2.52b)$$

and, from Equation (2.49), the gap induced field is

$$\begin{aligned} -E_y^G(y) &= \sum_v \frac{V_o}{\ln(r_b/r_a)} \frac{\epsilon_{om}}{ab\gamma_v} \cos(k_x r_a) \cos(k_y y) \frac{2}{\Gamma_1 \sqrt{2} \gamma_v - 1} \times \\ &\times \left\{ (1 + \Gamma_{1v} + \Gamma_{2v} + \Gamma_{1v} \Gamma_{2v}) \pi \text{DBES}(0) + j4(1 - \Gamma_{1v} \Gamma_{2v}) \sum_{\substack{p \\ \text{odd}}}^{\infty} \text{DBES}(p) \cos(p\psi)/p \right\} \end{aligned} \quad (2.53)$$

Equation (2.51b) is now the integral equation to be solved for the unknown current distribution. It is in suitable one dimensional form for numerical evaluation, which is discussed in Chapter 3.

2.7 Conclusion; Comparison with Literature.

As was stated in Chapter 1, the problem approached in this research has not been solved in the published literature. Nonetheless, there have been similar problems solved which represent important special cases of the theory presented here. Thus, by giving proper consideration to the approaches of other authors, it is possible to obtain support for this general theory through comparison with their results. For example, in his book Advanced Theory of Waveguides (see Chapter 4 of [24]) Lewin approaches the tuned post (i.e. the post does not span the waveguide) from the viewpoint of the waveguide and derives an integral expression in terms of the antenna current for the component of electric field in the waveguide which is parallel to the antenna or post. Since in deriving this expression Lewin has made assumptions equivalent to the thin wire approximation considered in this report (see expression 4.3 of [24]), it is possible to compare Lewin's result to the expression for induced field obtained in Chapter 2: namely, Equation (2.12). Since in Equation (2.12) the thin wire approximation has not been made, this must be done before comparing to Lewin's expression (4.3). Thus using

$$J_{yi}(x', y', z') = I_i(y') \delta(x' - a/2) \delta(z' - z_i) \quad \text{for } 0 \leq y' \leq h$$

in Equation (2.12) and integrating over the specified delta functions,

the induced electric field in the waveguide is found. Also, since Lewin considers the special case of an infinite waveguide, the expression (2.12) must further be specialized by considering zero reflection coefficients:

$$\Gamma_{1L} = \Gamma_{2L} = 0 .$$

When this is done, Equation (2.12) becomes (with the additional simplification of a single antenna, $N = 1$, located at $z_i = 0$)

$$E_y^i(x, y, z) = \sum_{n, m} \frac{-j \epsilon_{om}}{ab\omega\epsilon\gamma_{nm}} (k_o^2 - k_y^2) \sin(k_x x) \sin(k_x a/2) \cos(k_y y) e^{-\gamma_{nm}|z|} \int_{y'=0}^h I(y') \cos(k_y y') dy' . \quad (2.54)$$

This is seen to be identical to Lewin's expression (4.5) for the case of a post centered in a broad wall of the waveguide (i.e. set $d = a/2$ in his result). (Note that Lewin has chosen to interchange the roles of x and y coordinates and uses a different notation for Neumann's number, ϵ_{om} .) This equivalence between the two theories lends support to the results found in this report.

Further comparisons may be obtained from the developments of Eisenhart and Khan [7]. In their paper they considered the single post and approached it as an equivalent strip. Their final expression for induced electric field cannot be directly compared to the general theory presented here because of its dependence upon the assumption of a current of uniform density across the strip, in contrast to the inverse square root edge singularity form developed in this report (Equation 2.27). However, various aspects of their development are comparable; in particular, their Green's

function and their consideration of a terminated waveguide. The Green's function is given in their expression (5) and is obtained from the work of Tai [38]. It is seen to be identical to the Green's function which is independently derived in this report, Equation (2.14), except for the coefficient and the exclusion of the factor accounting for waveguide terminations (i.e., $\Gamma_{1L} = \Gamma_{2L} = 0$ in Equation (2.14)). The difference in coefficients results from the term $j\omega\mu$ being included in Equation (2.14) (as described following Equation 2.3)). Removing this term from Equation (2.14) gives exactly the Green's function shown in (5) of Eisenhart and Khan [7]; namely,

$$G_{yy}(\vec{r}|\vec{r}') = \sum_{n,m} \frac{\epsilon_{om}}{abk_o^2 \gamma_{nm}} (k_o^2 - k_y^2) e^{-\gamma_{nm}|z-z'|} \sin(k_x x) \sin(k_x x') \cos(k_y y) \cos(k_y y') . \quad (2.55)$$

To include the case of arbitrary waveguide terminations, Eisenhart and Khan have stated that this Green's function may be multiplied by a factor, denoted by τ , which takes the terminating boundary conditions into account. This factor (from expression (30) of [7]) may be written in the notation used in this report as

$$\tau = \frac{1 + \Gamma_{1,nm} + \Gamma_{2,nm} + \Gamma_{1,nm} \Gamma_{2,nm}}{1 - \Gamma_{1,nm} \Gamma_{2,nm}} \quad (2.56)$$

where $\Gamma_{1,nm}$ is defined in Appendix I. This is the same as the factor $f_v(0|0)$ given in Equation (2.13) (except for a minus sign which is incorporated in the expression G_{yy} of Equation (2.14)). (Recall that v is used to represent both n and m .) In other

words, the factor τ of Eisenhart and Khan agrees with the results of this report in the case of

$$z = z' = 0 .$$

This condition implies that the source point, z' , and field point, z , are at the same location; i.e., on the post (taken as the origin of coordinates). This is, of course, all that is required for the single post problem and thus supports the results of this report's general theory in the special case. Since Eisenhart and Khan were only concerned with this special case, there is no difficulty in their use of this factor τ . However, it is extremely important to realize that this factor τ is not appropriate for arbitrary source and field points, z' and z , in the waveguide. In this respect the statements of Eisenhart and Khan must be modified, since they imply that the general Green's function for a waveguide with arbitrary terminations is obtainable by multiplying the Green's function for an infinite waveguide by this factor τ . Since τ is independent of coordinates, their implication is that the fields reflected from each termination experience the same amount of phase change regardless of the location of the field point. This is seen to be obviously incorrect when the field point is located arbitrarily with respect to the center of coordinates and the source point. A simple discussion to extend the results of Eisenhart and Khan is not possible, but it is seen that the theory developed in this report includes the proper phase shift factors in the term $f_v(z|z')$ shown in Equation (2.13). It may therefore be concluded that the Green's function for a waveguide with arbitrary terminations is not

directly obtainable from the infinite waveguide Green's function by a simple multiplying factor. The proper "Green's function" is given in Equation (2.14). It bears repeating at this time that the term "Green's function" is used loosely in referring to G_{yy} of Equation (2.14). As the reader should be well aware, the factor $j\omega\mu$ is included in G_{yy} and this must be considered before using G_{yy} in the sense of a general Green's function in another context.

The theoretical development for the problem of N coupled antennas in a rectangular waveguide has been presented in this chapter. Although it has been necessary for conciseness and clarity to omit much of the algebraic details, the resulting expressions have been shown to be valid in comparison with similar specialized cases available in the literature. Further verification comes from the agreement among the results from each of the approaches considered (see Section 3.3) and from the ability of the theory to predict published experimental results of other authors as well as the experimental results obtained in this research effort (see Chapter 5).

CHAPTER 3

NUMERICAL METHODS

In Chapter 2, the integral equations for the current distributions on N antennas in a rectangular waveguide were developed. Three different representations for the slice generator model and one representation for the coaxial gap model were obtained in approximate one dimensional forms suitable for solution by numerical methods. Although analytical solutions could be sought in the manner of Hallen or of King and Middleton (see King [17]) the results essential to this report can most readily be obtained numerically. As will be shown in this chapter, the solutions obtained from the single antenna coaxial gap theory indicate the validity of using the slice generator model for comparison with experimental results. Also discussed in this chapter are the reasons for selecting the transverse strip approximation in the slice generator model as the best approach.

Section 3.1 contains the techniques used to obtain the numerical solution while Section 3.2 discusses the convergence properties of each representation. The last section presents some representative numerical results which demonstrate the agreement between the theories. The numerical results obtained in this report are computed using the Fortran Extended Language on a Control Data 6500 Computing System.

3.1 Matrix Equation Representation.

The integral equations obtained in Chapter 2 are solved numerically by the method of moments. For the problems in hand, this is a two step process: first, the unknown current is approximated by the method of subsections (see section 1-5 of Harrington [11]); second, the equations are satisfied discretely by the point matching method (see section 1-4 of Harrington [11]). The results of this procedure is a matrix equation in the unknown current distributions which is easily implemented for computer solution. This process is the same regardless of the form of the integral equation and thus is described in detail only for the thin-wire case (the other approximations will be treated analogously).

Equations (2.23) for the thin-wire approximation are repeated here for reference.

$$\sum_{i=1}^N \int_{y'=0}^{h_i} I_i(y') G_{ij}^{tw}(y|y') dy' = C_j \cos(k_o y) - \frac{V_i}{2k_o} \sin(k_o y) \quad (3.1a)$$

$$0 \leq y \leq h_j \quad j = 1, 2, \dots, N$$

where

$$G_{ij}^{tw}(y|y') = \sum_n \sum_{\substack{m \\ \text{odd}}} \frac{j \epsilon_{om}}{ab \omega \epsilon \gamma_{nm}} \cos(k_x r_a) \cos(k_y y) \cos(k_y y') f_{nm}(z_j|z_i) \quad (3.1b)$$

and

$$f_v(z_j|z_i) = \frac{e^{-\gamma_v |z_j - z_i|} + \Gamma_{1v} e^{-\gamma_v (z_j + z_i)} + \Gamma_{2v} e^{+\gamma_v (z_j + z_i)} + \Gamma_{1v} \Gamma_{2v} e^{+\gamma_v |z_j - z_i|}}{\Gamma_{1v} \Gamma_{2v} - 1} \quad (3.1c)$$

$$u_{i,p}(y') = \begin{cases} 1 & y'_{p-1} \leq y' < y'_p \\ 0 & \text{elsewhere} \end{cases}$$

$$y'_p = p(\Delta y)_i$$

$$y'_q = (q - \frac{1}{2})(\Delta y)_i$$

$$(\Delta y)_i = h_i/K_i$$

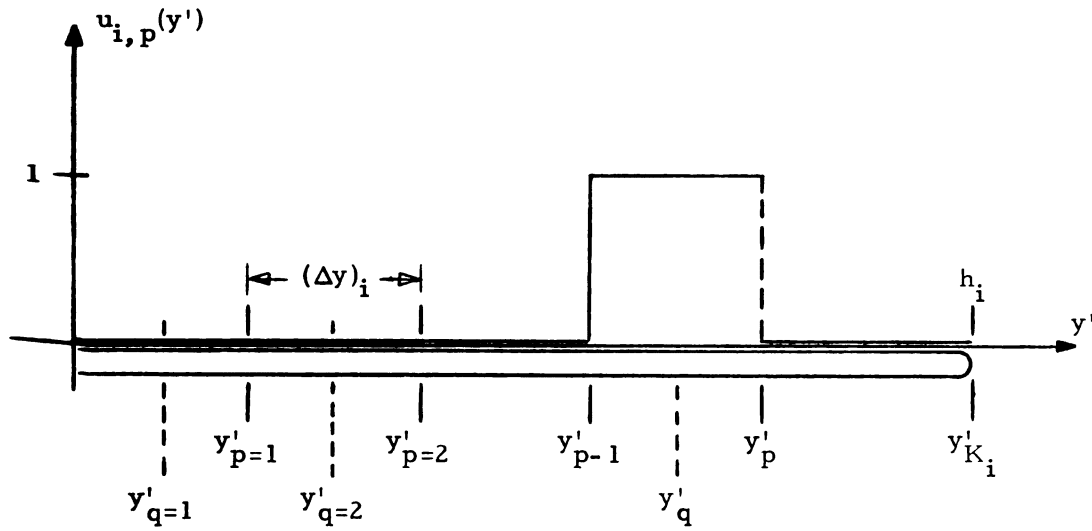


FIGURE 3.1

Pulse Function for Expansion of Antenna Current

3.1.1 Method of Subsections.

The first step in converting the above system of equations (3.1) to a matrix equation is to expand the current in a set of basis functions by the method of subsections. The i^{th} antenna is subdivided into K_i equal subsections and the functional dependence of the current is specified in each subinterval by the basis functions. The set of basis functions used here are the pulse functions shown in Figure 3.1. The current is then approximated as

$$I_i(y') = \sum_{p=1}^{K_i} \alpha_{i,p} u_{i,p}(y') \quad (3.2)$$

where the pulse functions $u_{i,p}$ are

$$u_{i,p}(y') = \begin{cases} 1 & y'_{p-1} \leq y' < y'_p \\ 0 & \text{elsewhere} \end{cases} \quad (3.3)$$

and the subsections are defined by

$$\begin{aligned} y'_p &= p(\Delta y)_i \\ (\Delta y)_i &= h_i/K_i \end{aligned} \quad (3.4)$$

as shown in Figure 3.1. The current is therefore assumed constant over each interval and the unknown current distribution becomes a set of K_i unknown amplitudes $\alpha_{i,p}$.

When the current of Equation (3.2) is substituted into Equation (3.1) the indicated integration becomes simply:

$$\int_{y'=0}^{h_i} \cos(k_y y') u_{i,p}(y') dy' = \int_{y'_{p-1}}^{y'_p} \cos(k_y y') dy' \quad (3.5)$$

For conciseness, let

$$\int_{y_{p-1}}^{y_p} \cos(k_y y') dy' = (T_m)_{i,p} \quad (3.6a)$$

where, using a trigonometric identity,

$$(T_m)_{i,p} = \begin{cases} \frac{2}{k_y} \sin(k_y (\Delta y)_i / 2) \cos(k_y (\Delta y)_i (p - \frac{1}{2})) & m \neq 0 \\ (\Delta y)_i & m = 0 \end{cases} \quad (3.6b)$$

where the subscripts indicate the p-th subsection on the i-th antenna and emphasize that the result is dependent only on mode index m (k_y). With the method of subsections then, the system of integral equations become a system of algebraic transcendental equations:

$$\sum_{i=1}^N \sum_{p=1}^{K_j} \alpha_{i,p} \sum_{\substack{n \\ \text{odd}}}^{\infty} \sum_m^{\infty} \frac{j \epsilon_{om}}{ab \omega \epsilon \gamma_{nm}} \cos(k_x r_a) f_{nm}(z_j | z_i) (T_m)_{i,p} \cos(k_y y) =$$

$$= C_j \cos(k_o y) - \frac{V_j}{2k_o} \sin(k_o y) \quad (3.7)$$

$$0 \leq y \leq h_j \quad j = 1, 2, \dots, N$$

The condition that the current be zero at the end of each antenna becomes the set of conditions

$$\alpha_{i,K_i} \equiv 0 \quad i = 1, 2, \dots, N \quad (3.8)$$

3.1.2 Method of Point Matching

This system of algebraic equations is now converted to a matrix equation by the method of point matching. Instead of satisfying the equations continuously in the variable y, an

approximate solution is found by matching the two sides of the equation at discrete points in the interval $0 \leq y \leq h_j$. This corresponds to satisfying the boundary condition at a discrete set of field points on the j -th antenna. In order to obtain a consistent set of equation, the number of matching points must equal the number of subsections on the same antenna. The midpoint of each subsection is chosen, so that for the j -th antenna, the matching points are (see Figure 3.1):

$$y_{j,q} = (q - \frac{1}{2})(\Delta y)_j \quad q = 1, 2, \dots, K_j \quad j = 1, 2, \dots, N \quad (3.9)$$

with

$$(\Delta y)_j = h_j / K_j .$$

Applying this matching to the set of equations (3.7) yields

$$\begin{aligned} \sum_{i=1}^N \sum_{p=1}^{K_i} \alpha_{i,p} \sum_{\substack{n \\ \text{odd}}}^{\infty} \sum_m^{\infty} \frac{j \epsilon_{om}}{ab \omega \epsilon \gamma_{nm}} \cos(k_x r_a) f_{nm}(z_j | z_i) (T_m)_{i,p} \cos(k_y y_{j,q}) = \\ = C_j \cos(k_o y_{j,q}) - \frac{V_i}{2k_o} \sin(k_o y_{j,q}) \end{aligned} \quad (3.10)$$

$$q = 1, 2, \dots, K_j \quad j = 1, 2, \dots, N$$

This is a matrix equation which can be written as

$$\sum_{i=1}^N \sum_{p=1}^{K_i-1} (S_{ji}^{tw})_{pq} \alpha_{i,p} - C_j \cos(k_o y_{j,q}) = \frac{-V_i}{2k_o} \sin(k_o y_{j,q}) \quad (3.11a)$$

$$q = 1, 2, \dots, K_j \quad j = 1, 2, \dots, N$$

where the boundary condition (3.8) has been used. The term

$(S_{ji}^{tw})_{pq}$ is the (p,q) -th submatrix element for each antenna

combination (j,i) with the thin-wire approximation, and is given by

$$(S_{ji}^{tw})_{pq} = \sum_{n \text{ odd}}^{\infty} \sum_m^{\infty} \frac{j\epsilon_{om}}{ab\omega\epsilon\gamma_{nm}} \cos(k_x r_a) \cos(k_y y_{j,q}) f_{nm}(z_j|z_i) (T_m)_{i,p} \quad (3.11b)$$

where $(T_m)_{i,p}$ is given in Equation (3.6), and $f_{nm}(z_j|z_i)$ is given in Equation (3.1c). This matrix equation can now be solved by computing each matrix element numerically, and applying standard (complex) matrix inversion routines. Since $(S_{ji}^{tw})_{pq}$ involves two infinite series, consideration must be given to the accuracy and convergence properties for truncating the sums. This is discussed in Section 3.2.

3.1.3 Galerkin's Method.

In the previous section the matrix equation was formulated by the method of point matching. In the terminology of the method of moments this is equivalent to a choice of "weighting functions" or "testing functions" which are Dirac delta functions. Although this point matching is particularly easy to implement numerically and to visualize physically, a more exact solution may be obtained by a different choice of testing functions. In particular, the choice of testing functions identical to the basis functions chosen for the method of subsection is known as Galerkin's method [11]. In using testing functions to formulate the matrix equation, the system of equations (resulting from an initial application of the method of subsections) is multiplied by the set of testing functions and the result integrated over the defined range of the

equations. In general these integrals are difficult to evaluate analytically, but for the case in hand involving simple trigonometric functions, particularly simple analytical expressions are obtained.

To apply Galerkin's method to the equations obtained for the thin-wire approximation (Equation 3.7), the testing functions are chosen as pulse functions as in Equation (3.3), i.e.;

$$u_{j,q}(y) = \begin{cases} 1 & y_{q-1} \leq y < y_q \\ 0 & \text{elsewhere} \end{cases} \quad (3.12)$$

where again the testing (or field matching) is performed on the j -th antenna and the j -th antenna segmenting is denoted by the subscript q . Multiplying Equation (3.7) by $u_{j,q}(y)$ from Equation (3.12) and integrating from $y = 0$ to $y = h$ gives

$$\begin{aligned} & \int_{y=0}^h \sum_{i=1}^N \sum_{p=1}^{K_i} \alpha_{i,p} \sum_{\substack{n \\ \text{odd}}}^{\infty} \sum_{m=1}^{\infty} \frac{j e_{om}}{ab \omega \epsilon \gamma_{nm}} \cos(k_x r_a) f_{nm}(z_j | z_i) (T_m)_{i,p} \\ & \quad \cos(k_y y) u_{j,q}(y) dy = \\ & = C_j \int_{y=0}^h \cos(k_o y) u_{j,q}(y) dy - \frac{V}{2k_o} \int_{y=0}^h \sin(k_o y) u_{j,q}(y) dy \end{aligned} \quad (3.13)$$

$$\text{for } q = 1, 2, \dots, K_j ; j = 1, 2, \dots, N .$$

From the left-hand side of Equation (3.13) the integral is

$$\int_0^h \cos(k_y y) u_{j,q}(y) dy = \int_{y_{q-1}}^{y_q} \cos(k_y y) dy \quad (3.14)$$

In the notation of Equations (3.5) and (3.6) this may be expressed as

$$\int_{y_{q-1}}^{y_q} \cos(k_y y) dy = \begin{cases} \frac{2}{k_y} \sin(k_y (\Delta y)_j / 2) \cos(k_y (\Delta y)_j (q - \frac{1}{2})) & \dots m \neq 0 \\ (\Delta y)_j & \dots m = 0 \end{cases}$$

$$= (T_m)_{j,q} \quad (3.15)$$

Again this notation indicates the result $(T_m)_{j,q}$ is for the q -th subsection on the j -th antenna and is a function of the mode index $m(k_y)$ only.

The right-hand side of Equation (3.13) involves the integral

$$\int_{y=0}^h \cos(k_o y) u_{j,q}(y) dy = \frac{2}{k_o} \sin(k_o (\Delta y)_j / 2) \cos(k_y (\Delta y)_j (q - \frac{1}{2}))$$

$$= (T)_{j,q} \quad (3.16)$$

which is dependent on the wavenumber of the medium (k_o) and the q -th subsection on the j -th antenna. The last integral on the right-hand side is denoted as

$$\int_{y=0}^h \sin(k_o y) u_{j,q}(y) dy = \frac{2}{k_o} \sin(k_o (\Delta y)_j / 2) \sin(k_o (\Delta y)_j (q - \frac{1}{2}))$$

$$= (T')_{j,q} \quad (3.17)$$

Substituting the results $(T_m)_{j,q}$, $(T)_{j,q}$, and $(T')_{j,q}$ given in (3.15), (3.16) and (3.17), respectively, back into the Equation (3.13) gives

$$\sum_{i=1}^N \sum_{p=1}^{K_i} \alpha_{i,p} \sum_{\substack{n \\ \text{odd}}}^{\infty} \sum_m^{\infty} \frac{j \epsilon_{om}}{ab \omega \epsilon \gamma_{nm}} \cos(k_x r_a) f_{nm}(z_j | z_i) (T_m)_{i,p} (T_m)_{j,q} =$$

$$= C_j (T)_{j,q} - \frac{V_i}{2j_o} (T')_{j,q} \quad (3.18)$$

for $q = 1, 2, \dots, K_j$; $j = 1, 2, \dots, N$.

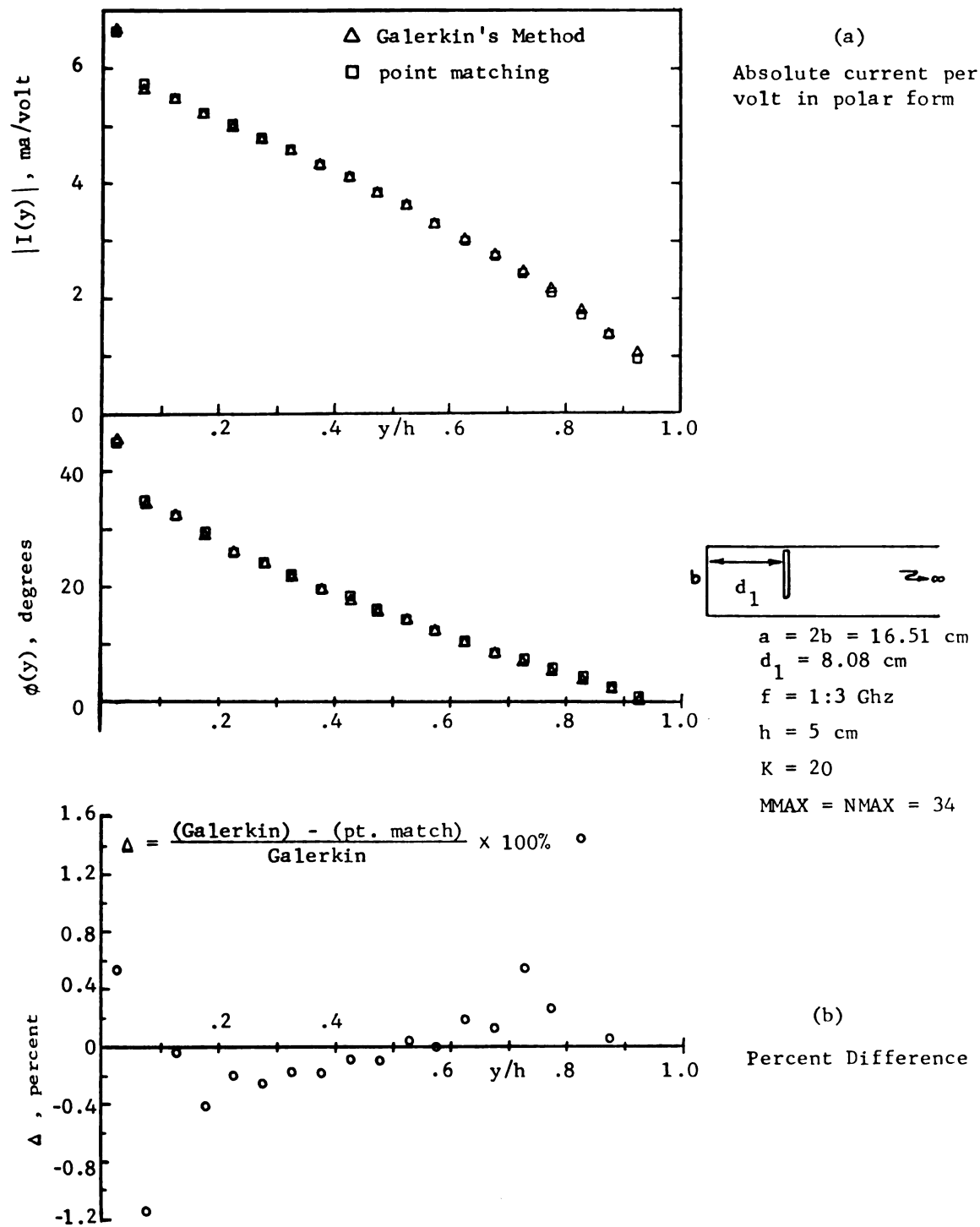


FIGURE 3.2

Comparison of Galerkin's Method and Point Matching

Equation (3.18) is now the matrix equation desired for the unknown current distribution amplitudes $\alpha_{i,p}$ which results from applying Galerkin's method. In order to investigate the degree of improvement in the numerical solution, several cases were solved both with the result given in Equation (3.11) from the point matching approach and Equation (3.18) from the Galerkin's method approach. Despite the differences in form between Equations (3.11) and (3.18), the results in all cases were practically identical. An example of the computed current distribution for a single antenna-waveguide configuration is shown in Figure (3.2a) with the percent difference shown in Figure (3.2b). (Note the highly expanded scale.) Obviously, for this problem the point matching method is as accurate as required. Since point matching also offers the greatest intuitive understanding of the numerical process, it is used throughout the remainder of this report.

3.1.4 Matrix Equations for the Strip and Coaxial Gap Solutions.

In the previous subsections the details of applying the method of moments to the thin-wire approximation integral equation are presented. The same methods may be applied in an analogous manner to the integral equations that have been developed for the other approximations to obtain matrix equations suitable for numerical solution. The similarity of forms between the various equations, and the fact that only basic trigonometric or delta function integrals are involved makes it possible to write the results down by direct comparison with the steps of Sections 3.1.1 and 3.1.2.

For the transverse strip (superscript "st") approximation

from Equation (2.30); namely,

$$\sum_{i=1}^N \int_{y'=0}^{h_i} I_i(y') G_{ij}^{st}(y|y') dy' = C_j \cos(k_o y) - \frac{V}{2k_o} \sin(k_o y)$$

$$0 \leq y \leq h_j \quad j = 1, 2, \dots, N$$

with

$$G_{ij}^{st}(y|y') = \sum_{n \text{ odd}}^{\infty} \sum_m^{\infty} \frac{j \epsilon_{om}}{ab \omega \epsilon \gamma_{nm}} \mathcal{J}_o(2k_x r_a) \cos(k_y y) \cos(k_y y') f_{nm}(z_j | z_i) .$$

By the method of subsections as in Section 3.1.1 this becomes

$$\sum_{i=1}^N \sum_{p=1}^{K_i-1} \alpha_{i,p} \sum_{n \text{ odd}}^{\infty} \sum_m^{\infty} \frac{j \epsilon_{om}}{ab \omega \epsilon \gamma_{nm}} \mathcal{J}_o(2k_x r_a) f_{nm}(z_j | z_i) (T_m)_{i,p} \cos(k_y y) =$$

$$= C_j \cos(k_o y) - \frac{V}{2k_o} \sin(k_o y) \quad 0 \leq y \leq h_j \quad j = 1, 2, \dots, N$$

(where the boundary condition (3.8) has been used). Now, by point

matching as in Section 3.1.2, this becomes

$$\sum_{i=1}^N \sum_{p=1}^{K_i-1} (S_{ji}^{st})_{pq} \alpha_{i,p} - C_j \cos(k_o y_{j,q}) = \frac{-V}{2k_o} \sin(k_o y_{j,q}) \quad (3.19a)$$

$$q = 1, 2, \dots, K_j \quad j = 1, 2, \dots, N$$

where

$$(S_{ji}^{st})_{pq} = \sum_{n \text{ odd}}^{\infty} \sum_m^{\infty} \frac{j \epsilon_{om}}{ab \omega \epsilon \gamma_{nm}} \mathcal{J}_o(2k_x r_a) \cos(k_y y_{j,q}) f_{nm}(z_j | z_i) (T_m)_{i,p} \quad (3.19b)$$

with $(T_m)_{i,p}$ and $f_{nm}(z_j | z_i)$ given in Equations (3.6) and (3.1c), respectively.

It should be noted that the only difference between the thin-wire approximation (Equation (3.11)) and the transverse strip approximation above is the replacement of $\cos(k_x r_a)$ by $J_0(2k_x r_a)$. This will be seen to be significant in relation to the convergence of the series described in Section 3.2. The numerical solution of the matrix equation found above is, of course, straightforward through the use of standard matrix manipulation techniques.

For the longitudinal strip (superscript "sl") approximation from Equation (2.34); namely,

$$\sum_{i=1}^N \int_{y'=0}^{h_j} I_i(y') G_{ij}^{sl}(y|y') dy' = C_j \cos(k_o y) - \frac{V}{2k_o} \sin(k_o y)$$

$$0 \leq y \leq h_j \quad j = 1, 2, \dots, N$$

with

$$G_{ij}^{sl}(y|y') = \sum_{\substack{n \\ \text{odd}}}^{\infty} \sum_m^{\infty} \frac{j \epsilon_{om}}{ab \omega \epsilon \gamma_{nm}} \cos(k_y y) \cos(k_y y') F_{nm}(z_j | z_i)$$

and

$$F_{nm}(z_j | z_i) = \begin{cases} J_0(\zeta_{nm}) f_{nm}(z_j | z_i) & i \neq j \\ J_0(\zeta_{nm}) f_{nm}(z_1 | z_1) - \mathcal{L}_0(\zeta_{nm}) & i = j \end{cases}$$

where $\zeta_{nm} = 2\gamma_{nm} r_a$. By the method of subsections as in Section

3.1.1 this becomes

$$\begin{aligned} \sum_{i=1}^N \sum_{p=1}^{K_i-1} \alpha_{i,p} \sum_{\substack{n \\ \text{odd}}}^{\infty} \sum_m^{\infty} \frac{j \epsilon_{om}}{ab \omega \epsilon \gamma_{nm}} (T_m)_{i,p} \cos(k_y y) F_{nm}(z_j | z_i) = \\ = C_j \cos(k_o y) - \frac{V}{2k_o} \sin(k_o y) \quad 0 \leq y \leq h_j \quad j = 1, 2, \dots, N \end{aligned}$$

(again boundary condition (3.8) has been used). Using point matching as in Section 3.1.2 this becomes

$$\sum_{i=1}^N \sum_{p=1}^{K_i-1} (S_{ji}^{sl})_{pq} \alpha_{i,p} - C_j \cos(k_o y_{j,q}) = \frac{-V_j}{2k_o} \sin(k_o y_{j,q}) \quad (3.20a)$$

$$q = 1, 2, \dots, K_j \quad j = 1, 2, \dots, N$$

where

$$(S_{ji}^{sl})_{pq} = \sum_{\substack{n \\ \text{odd}}}^{\infty} \sum_m^{\infty} \frac{j \epsilon_{om}}{ab \omega \epsilon \gamma_{nm}} \cos(k_y y_{j,q}) F_{nm}(z_j | z_i) (T_m)_{i,p} \quad (3.20b)$$

and where, again,

$$F_{nm}(z_j | z_i) = \begin{cases} \mathcal{J}_o(\zeta_{nm}) f_{nm}(z_j | z_i) & i = j \\ \mathcal{J}_o(\zeta_{nm}) f_{nm}(z_i | z_i) - \mathcal{L}_o(\zeta_{nm}) & i \neq j \end{cases} \quad (3.21)$$

with $\zeta_{nm} = 2\gamma_{nm} r_a$ and $(T_m)_{i,p}$, $f_{nm}(z_j | z_i)$ are given in Equations (3.6) and (3.1c), respectively.

The form of this matrix equation (3.20) differs from the other approximate representations in the factor $F_{nm}(z_j | z_i)$. Qualitatively, however, the behavior of the modified Bessel function $\mathcal{J}_o(\zeta_{nm})$ is similar to the Bessel function $\mathcal{J}_o(2k_x r_a)$. Solving Equation (3.20) numerically is again a straightforward matrix problem. Computation of the modified Bessel function $\mathcal{J}_o(\zeta_{nm})$ is generally available as a complex subroutine. The modified Struve function $\mathcal{L}_o$, however, is not but is very easily and rapidly computed using the series representation:

$$\mathcal{L}_0(\zeta_{nm}) = \sum_{\mu=1}^{\infty} \frac{2(\zeta_{nm})^{2\mu-1}}{\pi(1 \cdot 3 \cdot 5 \cdots (2\mu-1))^2} .$$

The matrix equation representation of the coaxial gap integral equation is also obtained by the method of moments. Since the thin wire approximation is made in the coaxial gap model, the procedure is exactly the same as described in Sections 3.1.1 and 3.1.2 (except that only one antenna is considered). The integral equation, represented by Equations (2.51) through (2.53) in Chapter 2, becomes the following system of equations:

$$E_y^J(y_q) = -E_y^G(y_q) \quad q = 1, 2, \dots, K \quad (3.22a)$$

where, using superscript cg for coaxial gap,

$$E_y^J(y_q) = \sum_{p=1}^K S_{pq}^{cg} \alpha_p \quad q = 1, 2, \dots, K \quad (3.22b)$$

with

$$\begin{aligned} S_{pq}^{cg} &= (p, q)\text{-th matrix element} \\ &= \sum_{\substack{n \\ \text{odd}}}^{\infty} \sum_{m}^{\infty} \frac{j\epsilon_{om}}{ab\omega\epsilon\gamma_v} (k_o^2 - k_y^2) \cos(k_x r_a) \cos(k_y y_q) \\ &\quad f_{nm}(0|0)(T_m)_{1,p} . \end{aligned} \quad (3.22c)$$

The factors $(T_m)_{1,p}$ and $f_{nm}(0|0)$ are given in Equations (3.6) and (3.1c), respectively, for the case of $i = 1$ (the single antenna considered). The right-hand side of the matrix equation (3.22a) is the one dimensional matrix given by (from Equation (2.53))

$$-E_y^G(y_q) = \sum_{\substack{n \\ \text{odd}}} \sum_m R_{nm} \cos(k_y y_q) \quad q = 1, 2, \dots, K \quad (3.22d)$$

where

$$R_v = \frac{V_0}{\ln(r_o/r_a)} \frac{\epsilon_{om}}{ab\gamma_v} \cos(k_x r_a) \{ 2\pi f_v(0|0) \text{DBES}(0) - j 8 \sum_{\substack{\mu \\ \text{odd}}} \text{DBES}(\mu) \cos(\mu \Psi) / \mu \} \quad (3.22e)$$

$f_v(0|0)$ is given in Equation (3.1c), and v represents both n and m . The symbols used are defined following Equation (2.49) and in Appendix I. Combining the above results the matrix equation for the single antenna coaxial gap problem may be written as:

$$\sum_{p=1}^K S_{pq}^{cg} \alpha_p = \sum_{\substack{n \\ \text{odd}}} \sum_m R_{nm} \cos(k_y y_q) \quad q = 1, 2, \dots, K \quad (3.23)$$

with S_{pq}^{cg} and R_{nm} defined in Equations (3.22c) and (3.22e), respectively.

In summary, any of the four matrix equations (3.11), (3.19), (3.20) and (3.23) may be used to calculate the current distribution on the antennas. The results will be a set of K_i complex amplitudes, $\alpha_{i,p}$, for each of the $i = 1, 2, \dots, N$ antennas ($N = 1$ for the coaxial gap problem Equation (3.23)). Although the amplitudes are assumed constant over each subsection, they are the correct amplitudes only at the center of each subsection $(y_{i,q})$ where point matching was applied. The real and imaginary parts of the $\alpha_{i,p}$ correspond to the in-phase and quadrature components of current on the antenna (referenced to the driving voltage). The input currents are taken

to be the amplitudes calculated on the first subsections, $\alpha_{i,1}$. From these input currents and the known applied voltages, all pertinent characteristics of the system (as viewed from the antenna terminals) may be determined. Finally, it should be pointed out that the dependence of the current on the electrical parameters of the medium is contained explicitly in the matrix representation. In other words, the complex permittivity and the permeability must first be calculated for the specified dielectric constant, conductivity, and relative permeability of the medium of interest. The numerical results are presented in Chapter 5.

3.1.5 Numerical Determination of Charge Distribution.

Up to this point, all work has been aimed at determination of the antenna current distribution. Also of interest, however, is the distribution of charge on the antenna. It has not been necessary to be concerned with the charge density in the theoretical development since it can be determined from the current via the continuity equation. Although the charge density provides no independent information theoretically, the current and charge can be measured independently in an experiment. Therefore, it is desirable to calculate the charge distribution for use as an alternate experimental check.

For a $\hat{y}$ component of current with only y dependence, the continuity equation is

$$\frac{dI(y)}{dy} + j\omega Q(y) = 0 . \quad (3.24)$$

This gives the charge distribution, Q , referenced to the input

current. It is desired, however, to obtain the charge referenced, in phase, to the input voltage (denoted as $\bar{Q}$). This is just

$$\bar{Q}(y) = [-j Q(y)] = \frac{1}{\omega} \frac{dI(y)}{dy} . \quad (3.25)$$

Numerically, Equation (3.25) is evaluated by the method of finite differences; that is, charge at a point between two adjacent subsections on the antenna is approximately proportional to the slope of the (straight) line connecting the current amplitudes at the center of each subsection. Therefore,

$$\bar{Q}_{i,p} = \frac{1}{\omega} \frac{(\alpha_{i,p+1} - \alpha_{i,p})}{(\Delta y)_i} \quad p = 1, 2, \dots, K_i \quad (3.26)$$

Results obtained for the charge distribution are presented in Chapter 5.

3.2 Convergence Properties.

The matrix elements for each of the representations found in Section 3.1 involve two infinite summations over the waveguide mode indices m and n . For numerical evaluation, these summations are truncated at M_{MAX} and N_{MAX} , where these values are determined by the desired accuracy and the convergence properties of the series. In this section the nature of these series will be discussed and the criteria derived which determine the optimum values for M_{MAX} and N_{MAX} . Because of the complicated nature of the series and the coupling that exists between the effects of indices m and n , much of the analysis is necessarily qualitative.

Under suitable approximations, however, the criteria for truncation are found and it is possible to compare the results to the work of other authors.

The series involved in the matrix equation for the thin-wire approximation is given in Equation (3.11b) and is repeated here for reference.

$$(S_{j,i}^{tw})_{pq} = \sum_{\substack{n \\ \text{odd}}}^{\infty} \sum_m^{\infty} \frac{j\epsilon_{om}}{ab\omega\epsilon\gamma_{nm}} \cos(k_x r_a) \cos(k_y y_{j,q}) \\ (T_m)_{i,p} f_{nm}(z_j|z_i) \quad (3.27)$$

It is simplest to first dispense with the reflection coefficient factor, $f_{nm}(z_j|z_i)$ for large m and n . As given in Equation (3.1c) this factor is (again using ν as a shorthand notation for both m and n)

$$f_{\nu}(z_j|z_i) = \frac{e^{-\gamma_{\nu}|z_j-z_i|} + \Gamma_{1\nu} e^{-\gamma_{\nu}(z_j+z_i)} + \Gamma_{2\nu} e^{+\gamma_{\nu}(z_j+z_i)} + \Gamma_{1\nu}\Gamma_{2\nu} e^{+\gamma_{\nu}|z_j-z_i|}}{\Gamma_{1\nu}\Gamma_{2\nu} - 1} \quad (3.28)$$

When m or n is large, the propagation constant γ_{nm} is a large positive real quantity

$$\gamma_{nm} = \sqrt{(n\pi/a)^2 + (m\pi/b)^2 - k_o^2} \quad (3.29)$$

which describes the attenuation of evanescent mode fields. Recalling that the mode dependent reflection coefficients are given by

$$\Gamma_{1,nm} = \Gamma_{1L} e^{-2\gamma_{nm} d_1}$$

and

$$\Gamma_{2,nm} = \Gamma_{2L} e^{-2\gamma_{nm} d_2}$$

where d_1 and d_2 are positive distances locating the waveguide terminations, it can be shown that the reflection coefficient factor of Equation (3.28) becomes

$$f_{nm}(z_j | z_i) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \quad \text{for large } m \text{ or } n \quad (3.30)$$

In words this statement says that the highly evanescent modes are not affected by the waveguide terminations, thus making the self coupling ($i = j$) terms independent of the reflection coefficients, and that the higher order modes have very little contribution to the mutual coupling between two antennas. Therefore, the convergence of the terms involving mutual coupling ($i \neq j$) is insured as long as the attenuation of the evanescent modes is large enough for the given separation of the antennas. For example, for antennas separated by a distance s , the (n,m) -th mode propagates axially in the waveguide as

$$e^{-\gamma_{nm} s}.$$

For large m , say, this is approximated by (from Equation (3.29))

$$e^{-\gamma_{nm} s} \approx e^{-\frac{\pi m}{b} s}.$$

Thus, even for very closely spaced antennas having $s = b/10$, modes having $m = 10$ are attenuated as $e^{-\pi}$ between the antennas and the coupling is less than 5% of that for non-attenuated modes. Of

course, m and n are not independent in determining attenuation and neither m nor n need be as large as required if they were independent.

For the case of the self coupling terms involving $i = j$ the series in Equation (3.27) may be simplified using the result of Equations (3.30), (3.6) and (3.9). The resulting (m,n) -th term of the series is then

$$[S_{pq}]_{mn} = C \frac{1}{k_y \gamma_{nm}} \cos(k_x r_a) \cos(k_y \Delta y (q - \frac{1}{2})) \sin(k_y \Delta y / 2) \cos(k_y \Delta y (p - \frac{1}{2})) \quad (3.31)$$

... for m and n large .

Recalling that

$$k_x = n\pi/a \quad ; \quad k_y = m\pi/b$$

$$\Delta y = h/K \quad (3.32)$$

$$\gamma_{nm} = k_x^2 + k_y^2 - k_o^2$$

it can be seen that the convergence of the terms involving m in Equation (3.31) depends upon the particular (p,q) -th matrix element considered. It is therefore useful to first study the relative significance of each matrix element to determine which combinations (p,q) will dominate. In order to regain the total perspective of the problem it is helpful at this point to recall that the matrix defined by the elements $(S_{ii}^{tw})_{pq}$ is the discretized form of the kernel of the integral equation and that the p -th elements refer to the source points (from the current subsectioning) while the

q-th elements refer to the field points (from the point matching). Therefore the relative significance of each element (p,q) is related to the peaking properties which the kernel exhibits as a function of separation between the source point and the field point. An example of the results for one case of a coarsely segmented antenna ($K = 5$) is depicted in Figure 3.3, where the relative magnitudes of the off-diagonal elements are compared to the normalized diagonal elements. As can be seen, the diagonal ($p = q$) elements dominate and significant peaking does occur when source point and field point coincide as expected from conventional antenna theory. Based on this result, it is obvious that the convergence of the series for diagonal elements is the most critical and the significant terms in Equation (3.28) become

$$(S_{pp}^{tw})_{mn} = C \frac{1}{\gamma_{nm} k_y} \cos(k_x r_a) \cos^2(k_y \Delta y (p - \frac{1}{2})) \sin(k_y \Delta y / 2) \quad (3.33)$$

... for m and n large.

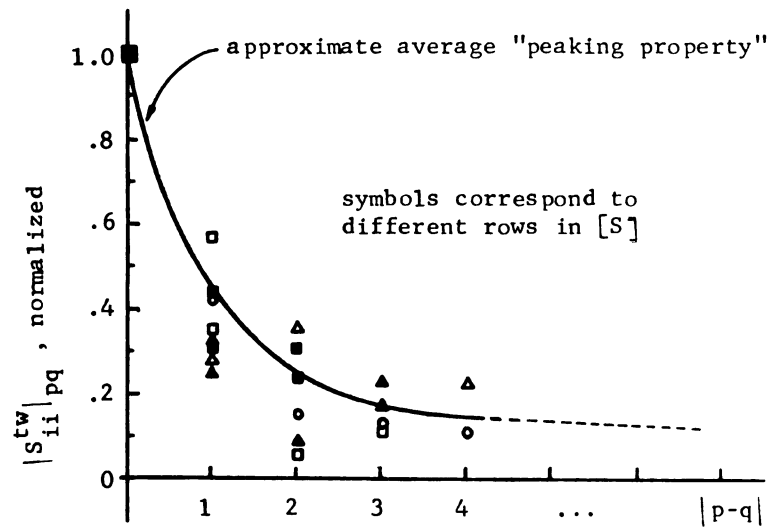
To study the convergence of the sum of terms given in Equation (3.33) as a function of m only, the following approximation can be made:

$$\gamma_{mn} \approx k_y, \text{ for large } m,$$

so that Equation (3.33) becomes

$$(S_{pp}^{tw})_{mn} \approx C_n \frac{\cos^2(k_y \Delta y (p - \frac{1}{2})) \sin(k_y \Delta y / 2)}{k_y^2} \quad (3.34)$$

The result of summing these terms as a function of m is shown in Figure 3.4 for the values of the parameters given by:



$$S_{pq}^{tw} = \sum_{\substack{n \\ \text{odd}}}^{NMAX} \sum_m^{MMAX} C(1/k_y \gamma_{nm}) \cos(k_x r_a) \cos(k_y \Delta y (q - \frac{1}{2})) \sin(k_y \Delta y / 2) \cos(k_y \Delta y (p - \frac{1}{2}))$$

$$\text{for } h = b/2 \quad K = 5 \quad r_a = a/20$$

Normalized to diagonal element in each row.

FIGURE 3.3

Comparison of Off-Diagonal to Diagonal Elements of [S] Matrix

$$\begin{aligned}
 \text{antenna height: } h &= b/2 \\
 \text{number of sections: } K &= 5 \\
 \text{test points } p &= 1, 2, 3, 4, 5
 \end{aligned}
 \tag{3.35}$$

The curves shown in Figure 3.4 indicate the best and worst cases for $p = 1$ and $p = 5$, respectively. As can be seen, the minimum truncation point for the series is at $m = 10$. By truncating the series at this point, errors for the example of Figures 3.4 are 5% or better. Generalizing the results of this example, the criterion for truncation of the series as a function of m is:

$$M_{MAX} = \frac{bK}{h} \tag{3.36}$$

Some rather gratifying observations can be made about this particular choice of truncation point by elaborating on its implications. For example, the $\hat{y}$ -component of electric field in the waveguide for this M_{MAX} -th mode goes through one-half "wavelength" variation in the y direction in the distance (see Equation 2.5)

$$b/M_{MAX} = h/K .$$

Since h/K represents the interval between locations chosen for point matching (according to Equation (3.9)) - that is, the spatial period of the field matching - and at $m = M_{MAX}$ the field has a spatial period of twice this amount, then the criterion of Equation (3.36) corresponds to the use of a "sampling" frequency which is twice that of the field being "sampled". For those readers acquainted with sampling theory this observation is seen to be equivalent to the fundamental theorem for sampling (Nyquist criterion)

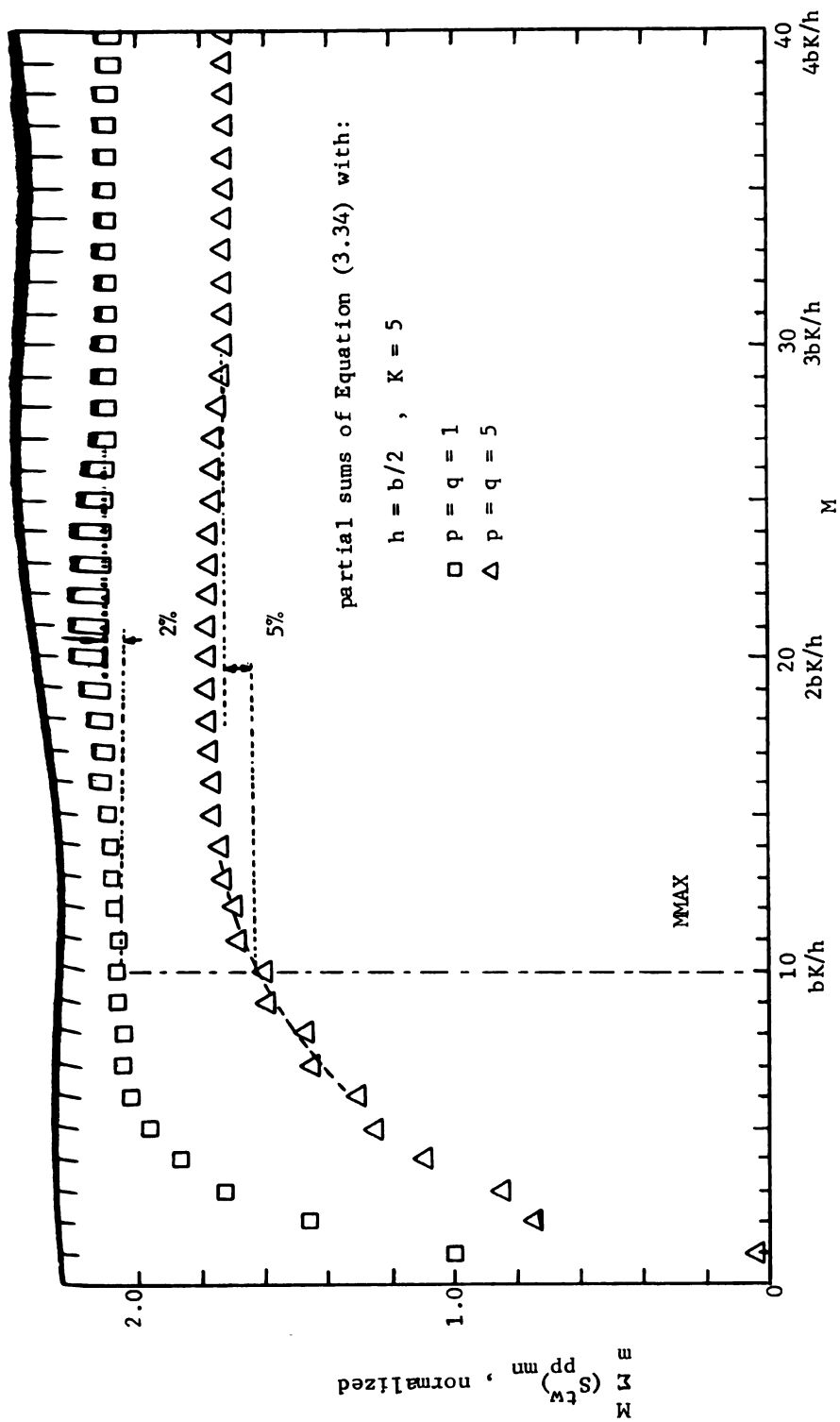


FIGURE 3.4

Convergence as a Function of m

which states that the minimum sampling frequency must be twice that of the signal being sampled in order to accurately portray or reconstruct the signal. This apparent connection between the method of moments in modal analysis and sampling theory provides an interesting point for further study for those familiar with sampling theory and techniques. For the moment, however, it serves to supply some helpful intuitive insight into the choice of truncation point.

Further observations concerning the criterion given in Equation (3.36) can be made by comparing this to the results obtained by other authors concerned with convergence of solutions in modal analysis problems. For example, in the paper by Eisenhart and Khan [7] the convergence of an admittance function is studied for a gap (having an assumed constant impressed electric field across it) in a post spanning a waveguide. They determined that convergence was reached by taking the maximum mode field $m = M_{MAX}$ given by

$$M_{MAX} = b/g ,$$

where in their case g is the gap width or interval over which the waveguide field must be matched to the impressed field. A correspondence can thus be drawn to the case studied here where the interval over which the fields are matched is given by h/K . In this context then the criterion of Eisenhart and Khan and the one developed here are seen to be equivalent.

Another pertinent example in the literature may be found in the paper by Lee, et al. [22] where mode matching across a waveguide iris leads to similar convergence phenomenon. They determine that convergence is reached when the number of modes

considered in the incident waveguide (MMAX) is related to the number of modes chosen in the iris (say, K) by the expression

$$MMAX = \frac{b}{h} K$$

where b is the incident waveguide dimension and h is the iris dimension. This criterion is obviously the same as that chosen in Equation (3.36) if the antenna subsectioning is viewed as a modal expansion (which indeed it is since the current is expanded in a set of basis functions).

A further point of interest which is observed by Lee, et al. [22], is that the number of waveguide modes, MMAX, should be selected first and that the number of iris modes should then be determined by Equation (3.36); i.e.,

$$K = \frac{h}{b} MMAX$$

since in their problem the solution sought diverged if MMAX (from Equation (3.36)) was greatly exceeded. It is certainly not obvious that this type of relative convergence would be expected, and it is not known if the solution of this report also exhibits this phenomenon, but it is a point well taken that the criterion of Equation (3.36) should be satisfied as nearly as possible.

Having studied the convergence of the series given in Equation (3.31) as a function of mode index m, it now remains to investigate the series as a function of mode index n to determine a suitable truncation point NMAX. In this light, Equation (3.31) may be written as

$$(S_{pp}^{tw})_{mn} = C_m \frac{\cos(k_x r_a)}{\gamma_{mn}} \quad (3.37)$$

It is obvious that m and n are not independent because of their relation to γ_{mn} . Assuming m and n are large enough to dominate the term k_o^2 and assuming waveguide dimensions given by $a = 2b$, then

$$\gamma_{mn} \approx \pi/a \sqrt{n^2 + 4m^2}$$

With M_{MAX} already determined above, Equation (3.37) becomes

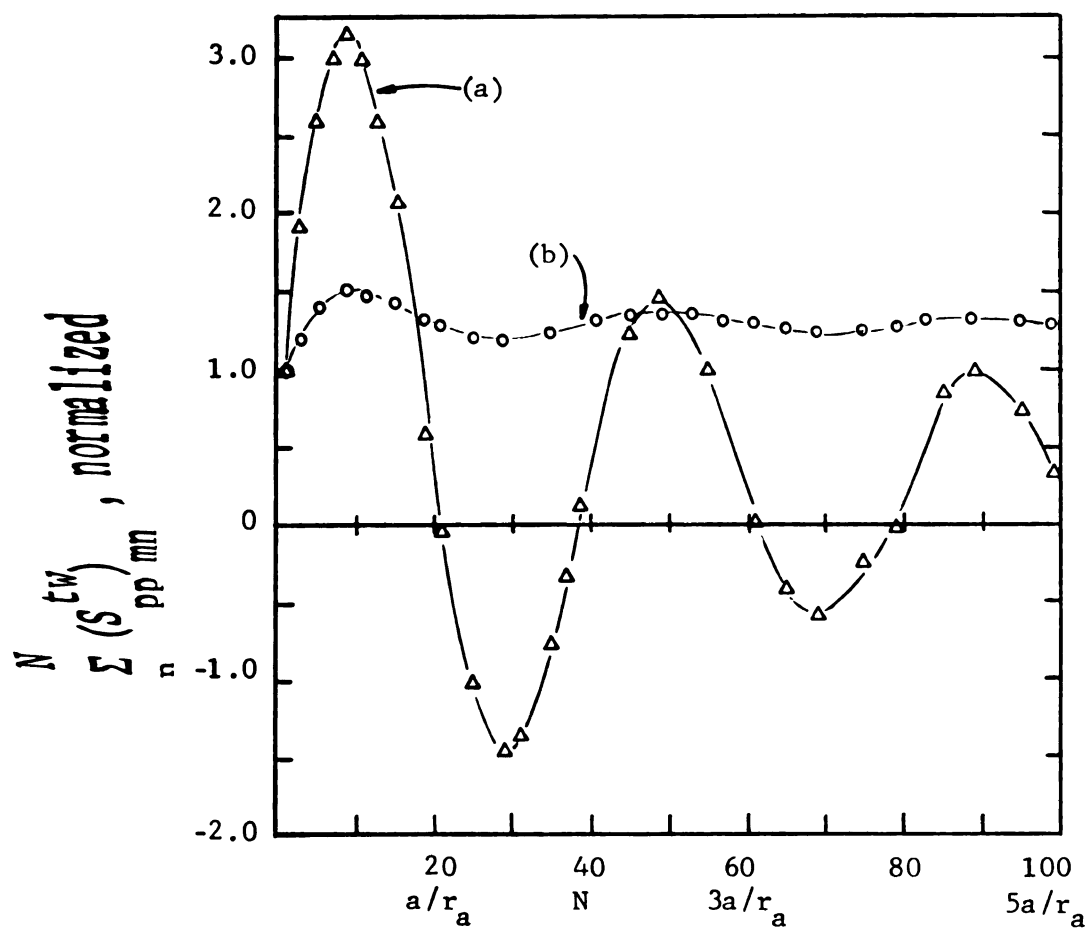
$$(S_{pp}^{tw})_{M_{MAX},n} \approx C_{M_{MAX}} \frac{\cos(n\pi r_a/a)}{\sqrt{n^2 + 4(M_{MAX})^2}} \quad (3.38)$$

This may now be studied for the example case of Figure 3.4, where $M_{MAX} = 10$. If it is assumed, for the example, that $r_a = a/20$, the result of summing terms of the form given in Equation (3.38) is as shown in Figure 3.5, curve (a). Obviously convergence is not reached even when summing terms as large as $N = 100$ and the result of premature truncation could be totally erroneous. Part of this apparent problem stems from the fact that m and n have been considered independently. In fact, if the convergence as a function of n is considered for the m -th terms of the series in which m is much less than n , then Equation (3.37) becomes

$$(S_{pp}^{tw})_{mn} \approx C_m \frac{\cos(n\pi r_a/a)}{n} \quad (3.39)$$

The convergence of these terms is shown in curve (b) of Figure 3.5.

(Note: Equation (3.39) decays as $(n)^{-1}$ while Equation 3.39 decays as $(n^2 + 4(M_{MAX})^2)^{-\frac{1}{2}}$). In this case convergence is reached to



curve (a) - partial sum of Equation (3.38)

curve (b) - partial sum of Equation (3.39)

FIGURE 3.5

Convergence as a Function of n for
the Thin Wire Approximation

within about 15% error for $N_{MAX} = 10$. The actual case which exists when m and n are considered together would be some combination of the two extremes described above. Since it was shown that as m increases, the terms in Equation (3.33) approach zero, then in Equations (3.38) or (3.39) the coefficients C_m will also approach zero. Therefore, even though the factor involving n oscillates wildly, its contribution to the double summation over m and n should become less significant as required for convergence. Nonetheless, the results shown in Figure (3.5) indicate that considerable care should be exercised in using the results obtained for almost any truncated numerical summation. Fortunately, this potential problem may be overcome, since a comparison of the results for the thin-wire approximation studied above and for the strip approximation shows that the convergence problems for the mode index n are unique to the thin wire approximation formulation. As pointed out before, the only difference between the thin-wire results and the transverse strip results (Equations (3.11) and (3.19)) is the replacement of the Bessel function, $J_0(2k_x r_a)$, for the cosine function, $\cos(k_x r_a)$. Therefore, the convergence for the m summation is identical for both approximations, but the Bessel function will provide an additional convergence factor for the n summation. This is apparent in the asymptotic form of the Bessel function for large argument [26]:

$$J_0(2k_x r_a) \approx \frac{\cos(2k_x r_a - \pi/4)}{\sqrt{\pi k_x r_a}}$$

$$\dots \text{ for } 2k_x r_a > 10 \text{ or } n > 1.6(a/r_a),$$

where the $n^{-1/2}$ dependence provides the required convergence. As

was done above in Equation (3.37), the n -dependence of the matrix elements for the transverse strip approximation may be obtained from Equation (3.19b). The (m,n) -th term in the double summation may be written as

$$(S_{pp}^{st})_{mn} = C_m \frac{j_o(2k_x r_a)}{\gamma_{mn}} \quad (3.40)$$

With the M_{MAX} given by Equation (3.36) this becomes

$$(S_{pp}^{st})_{mn} = C_{M_{MAX}} \frac{j_o(2k_x r_a)}{\sqrt{n^2 + 4(M_{MAX})^2}} \quad (3.41a)$$

for $a = 2b$. The convergence of the sum of these terms is shown in curve (a) of Figure (3.6) for the same parameters as studied in the example of Figure 3.4. Obviously the series is much more rapidly convergent than when the pure cosine term is present. The convergence is even more dramatic when the terms involving $m < n$ are considered such that Equation (3.40) becomes

$$(S_{pp}^{st})_{mn} \approx C_m \frac{j_o(2k_x r_a)}{n} \quad (3.41b)$$

The series of these terms is shown summed in curve (b) of Figure (3.6). As before, the true convergence of the series involving mode index n lies between the two extremes shown in Figure (3.6) depending on the relative size of index m (k_y). As shown in Figure (3.6) the smallest value of n at which the series can safely be truncated is given by

$$N_{MAX} = \frac{a}{2r_a} \quad (3.42)$$

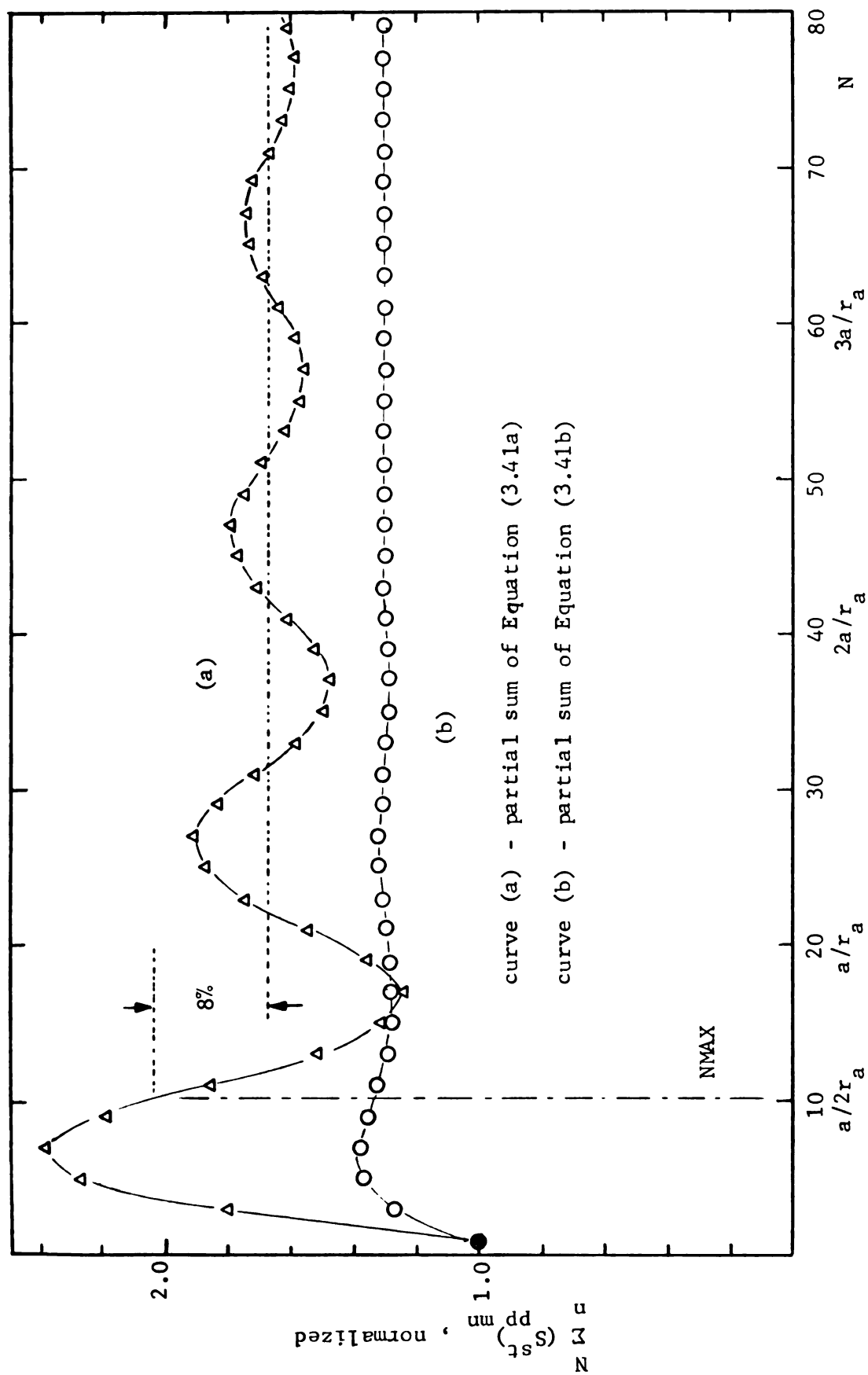


FIGURE 3.6

Convergence as a Function of n for the
Transverse Strip Approximation

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At this point errors less than 8% can be expected and with the additional convergence of the series for index m this error will be substantially reduced. Therefore, although the choice of an appropriate truncation point for the thin-wire approximation was difficult, the transverse strip approximation has lead to a satisfactory result given in Equation (3.42).

As with the criterion developed for N_{MAX} in Equation (3.36), some reflection on the implication of the criterion for N_{MAX} offers intuitive appeal for its choice. For the $\hat{y}$ component of electric field considered, the N_{MAX} -th mode exhibits one-half wavelength variation in the x direction in the distance

$$a/N_{MAX} = 2r_a .$$

Since this represents the diameter of the cylindrical antenna or the interval of field matching, this criterion can again be related to the Nuquist criterion for sampling as before. In general, of course, this connection with sampling theory is not definitive (as it now stands) but the insight is beneficial.

The N_{MAX} criterion of Equation (3.42) can also be compared to the results of other authors concerning similar problems in the literature. In particular, Eisenhart and Khan [7] in analyzing the thin post spanning a rectangular waveguide determined that their impedance function series would be satisfactorily convergent to 1% for a truncation point given by

$$N_{MAX} = a/2r_a$$

which is identical to Equation (3.42) above. In actual use they

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decided that half of this value was sufficient due to the stationary nature of the impedance formulation with respect to approximations made on the currents. It is not apparent from the above discussion that a similar easing of the criterion given in Equation (3.42) is desirable and so this is not done here. The use of Equation (3.42) throughout the numerical solution is, however, substantiated through this comparison with the literature.

Quantitative studies of the convergence of the other approximate forms for the integral equations, namely, the longitudinal strip approximation (Equation 3.20) and the coaxial gap solution (Equation 3.23) is not presented here. However, the results would be qualitatively comparable to the above discussion. By comparing Equation (3.20) with Equation (3.19), one sees that the longitudinal strip approximation exhibits the same dependence on mode index m as the transverse strip and a similarly convergent dependence on mode index n (i.e., both the ordinary Bessel function, J_0 , and the modified Bessel function, I_0 , decay for increasing arguments). Also, by comparing Equation (3.23) with Equation (3.11), one sees that the coaxial gap solution should have convergence properties similar to the thin wire approximation. The comparison of the results for the four approximations is done in Section 3.3 where the criterion of Equation (3.36) and Equation (3.42) are used for all solutions.

In summary, the discussion of the convergence properties of the various solutions has lead to the following criterion for truncating the numerical evaluation of the series in each formulation:

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$$\begin{aligned} M_{MAX} &= \frac{bK}{h} \\ N_{MAX} &= \frac{a}{2r_a} \end{aligned} \tag{3.43}$$

It has also been shown that the thin-wire approximation leads to a series which converges very slowly compared to the series for the strip approximation. This fact is the primary reason for not using the thin-wire approximation for determining the final results, and is an important point to investigate whenever the thin-wire approximation is made for antennas in a modal analysis type solution.

3.3 Comparison of the Solutions.

In this section the four approximate forms of the integral equation for the antenna current are compared. The examples shown are representative of the excellent agreement for the current distributions among the four solutions despite the differences in functional form. Because of this basic agreement in current distribution, it is concluded that the transverse strip approximation with its computationally simpler form and attractive convergence properties already discussed is the most desirable representation of the solution. The values computed for the antenna input admittance (note, the values are related to the overall scaling of the current distributions) show varying degrees of agreement among the four representations. The differences were found to depend upon the particular configuration and frequency studied with no significant pattern being detected. These discrepancies of admittance are included in the following examples where current distribution along

the antenna is presented in terms of milliamperes per volt of excitation and the results are not normalized. The discussion involving attempts at absolute admittance correlation is presented in Chapter 5 where experimental results will also be included. Observations concerning the infinite susceptance resulting from the delta function excitation will also be given in Section 5.1. The examples considered are shown in Figure 3.7(a) through Figure 3.7(e). They are all for the waveguide in the air filled cavity configuration (i.e., short circuit terminations). As can be seen the form of the current distributions are nearly identical in all cases except that of Figure 3.7(d) where the thin-wire approximation gives a current which is somewhat shifted toward the driving point. This is thought to be related to the convergence problems associated with this solution. No significance should be attached to the differences in the absolute magnitudes of the solutions shown. Even though the differences are large in some cases, this does not detract from the overall validity considering the dramatic differences in the mathematical forms and convergence properties of the four approximations. It can also be seen from Figure (3.7) that excellent agreement is obtained between the transverse and longitudinal strip approximations and that apparently no significant information is lost by choosing the transverse strip representation over the longitudinal strip for the overall problem solution. For different electrical and physical configurations, one approximation may be closer to the true currents and fields than the others. Thus, the results shown here indicate the range of variation the various approximations have on the solution. Since the purpose of

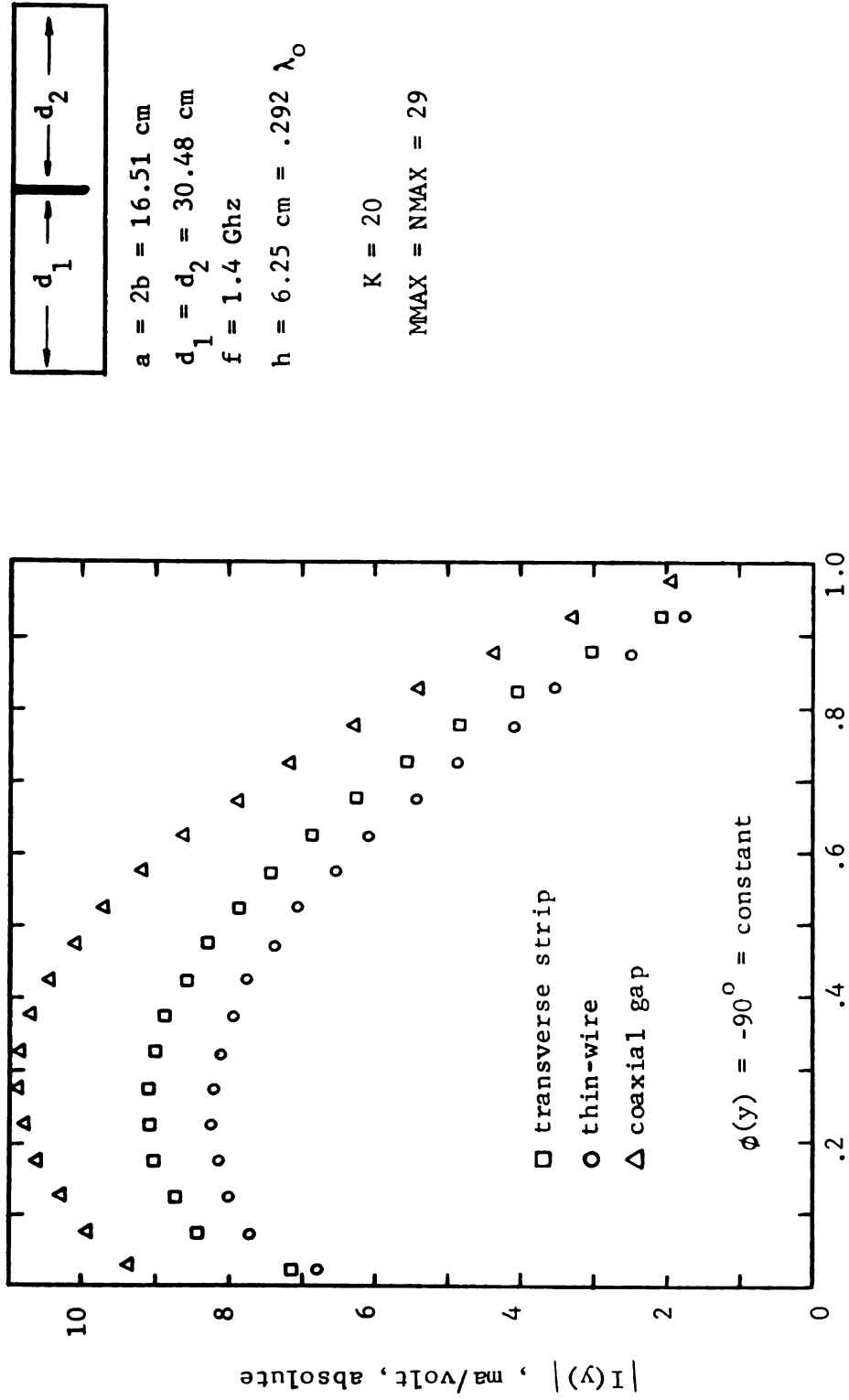


FIGURE 3.7(a)

Comparison of Various Approximations

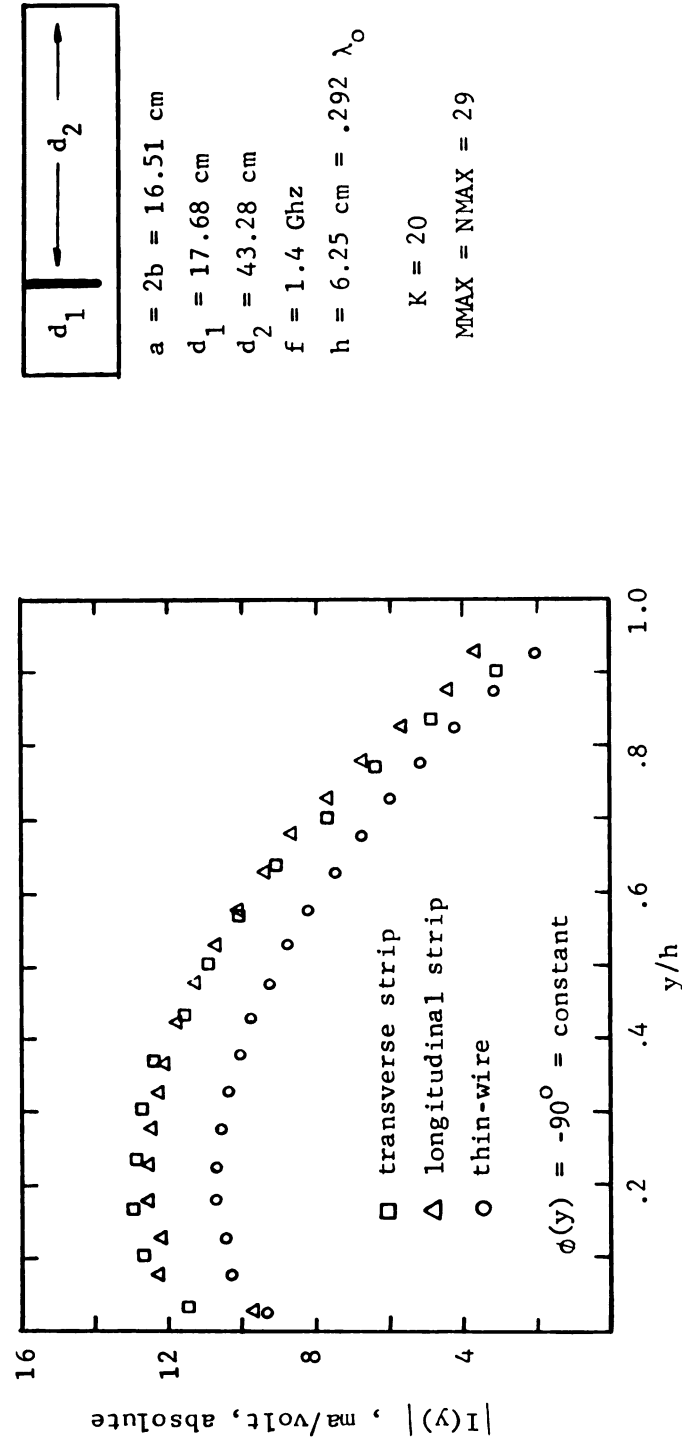


FIGURE 3.7(b)

Comparison of Various Approximations

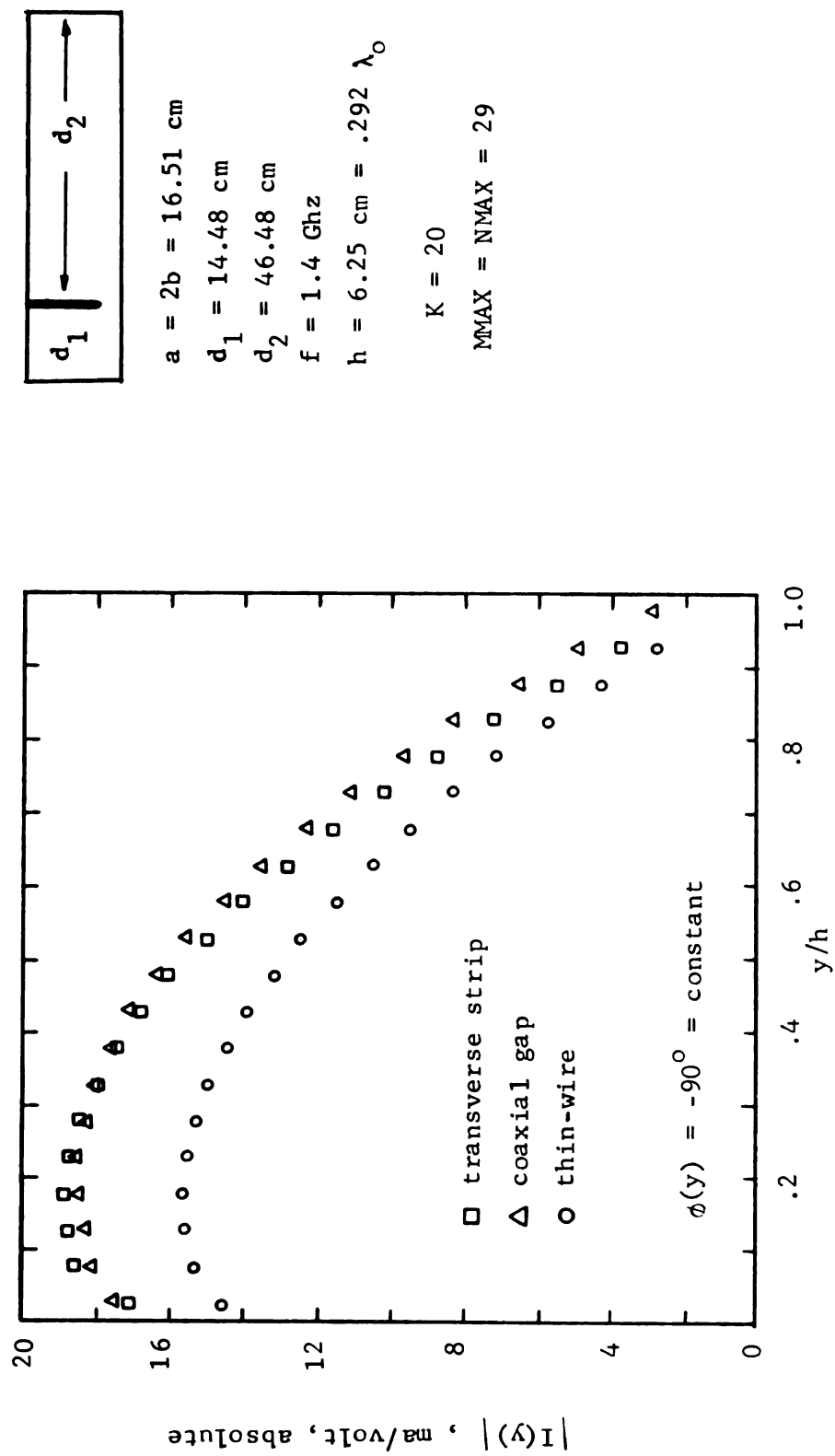
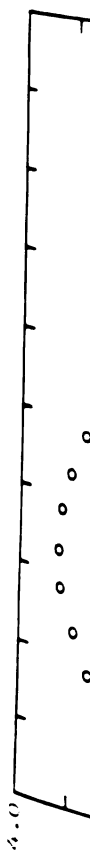
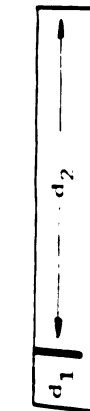


FIGURE 3.7(c)

Comparison of Various Approximations



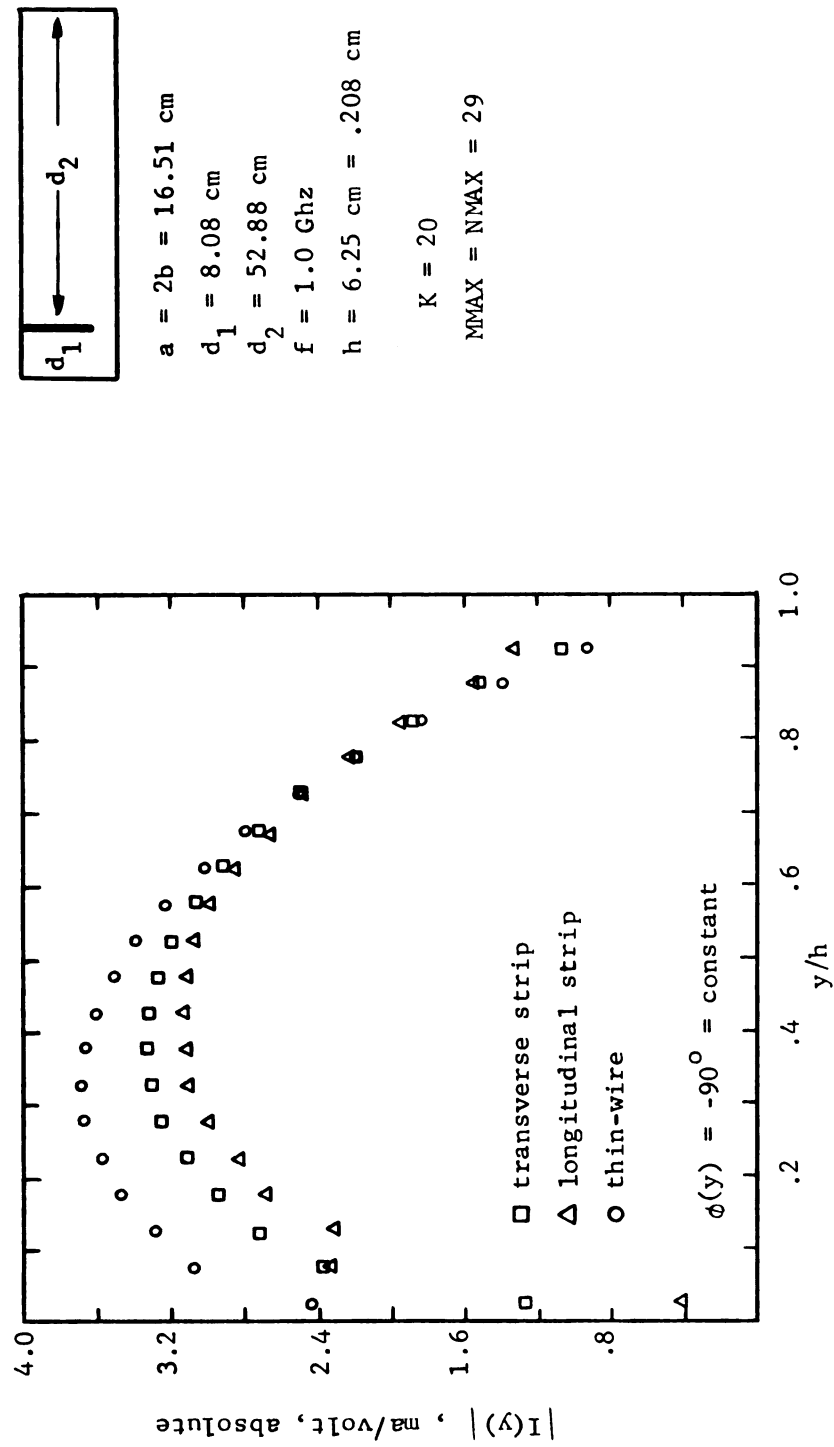


FIGURE 3.7 (d)

Comparison of Various Approximations



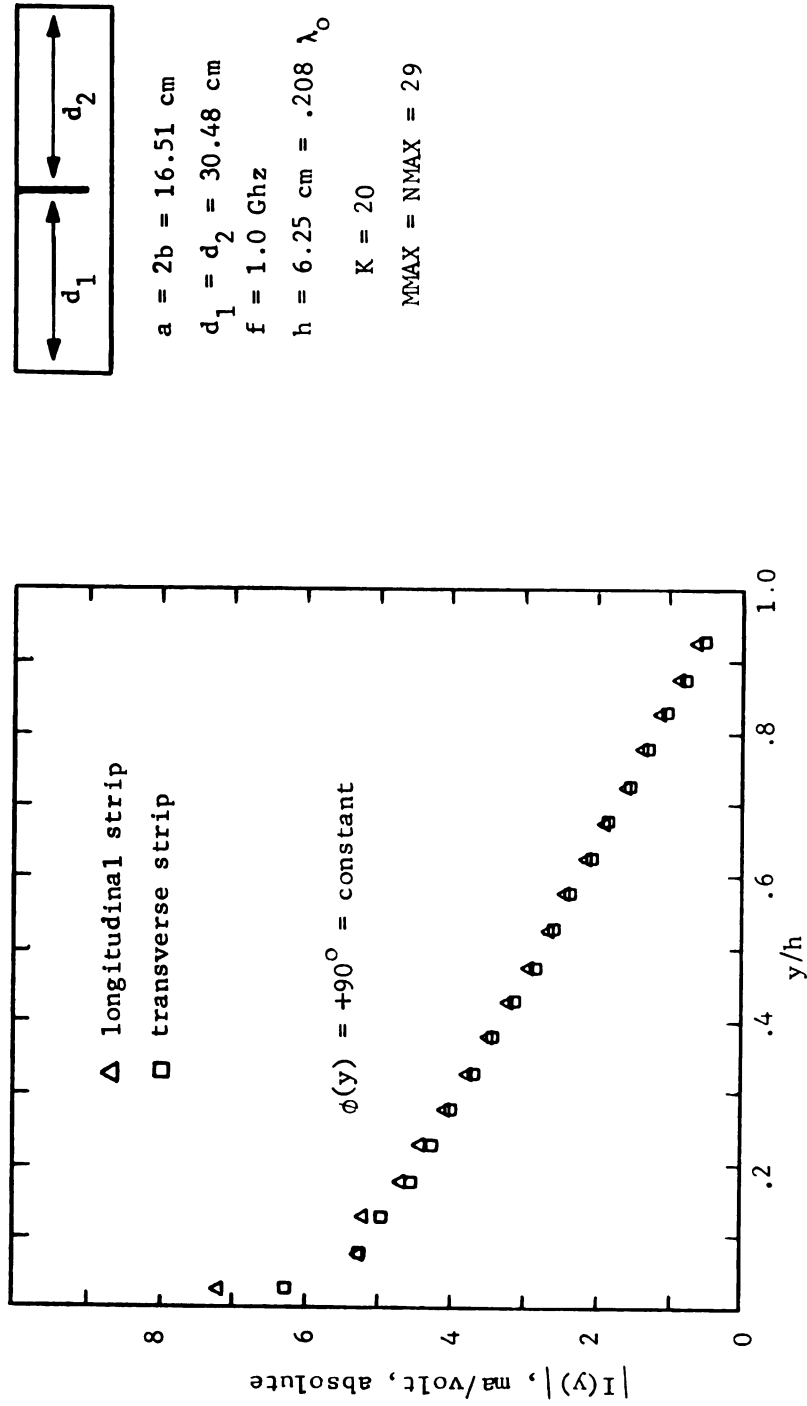


FIGURE 3.7(e)

Comparison of Various Approximations

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Figure (3.7) is to show the comparable results obtained from the various approximations of the integral equations, a discussion of the nature of the current distributions shown is reserved until Chapter 5. One remark is appropriate, however, concerning the phase angle of the current. With the current separated into in-phase and quadrature components referenced to the driving voltage as

$$I(y) = I''(y) + j I'(y) \quad (3.44)$$

where $I(y)$ is in units of amperes per volt of excitation, the phase angle of the current is defined as

$$\phi(y) = \arctan(I'/I'') \quad (3.45)$$

so that the current may be written in polar form

$$I(y) = |I(y)| e^{j\phi(y)} .$$

Since the configuration chosen for all curves in Figure (3.7) is the antenna in a (ideal) cavity, there is no in-phase component of current (no power can be radiated by the antenna). Therefore, the phase angle will be constant along the antenna and equal to $+90^\circ$ or -90° . This result is the same for all approximations and each curve.

The conclusion which may be drawn from the above discussion is that all approximations lead to essentially the same form of current distribution. Therefore, the transverse strip approximation is primarily used throughout the remainder of this report to describe the theoretical results. The coaxial gap solution, however, is not entirely abandoned at this point because of the different

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value of input admittance it will yield owing to the difference in assumed form of excitation. When appropriate, the coaxial gap solution will be compared with the transverse strip approximation (with delta function excitation) and with the observed experimental values in Chapter 5.

CHAPTER 4

EXPERIMENTAL INVESTIGATIONS

The theoretical and numerical developments presented in Chapters 2 and 3 provide the analysis necessary to predict the performance of the chosen rectangular waveguide and antenna system. In order to support this analysis, an experimental system was constructed of sufficient flexibility to adequately test the theory. This chapter describes the experimental system and the methods used to obtain the data.

In designing the experimental set-up, the choice was made to perform the experiments around the L-band frequencies; i.e., 0.9 to 2.4 GHz, for several reasons. First, the vector voltmeter which was chosen as the primary measuring tool had a maximum operating frequency of 2.4 GHz. Second, the physical size of the L-band waveguide (inside dimensions = 6.5" x 3.25") meant that the field probes could be large enough to easily construct and handle without greatly perturbing the fields. Finally with an eye toward direct application, it was noted that Sauter and Chilton [31] reported in their 1970 study that research emphasis is needed at the low end of the microwave spectrum for use in air quality monitoring. It goes without saying, however, that the choice made for experimentally testing the theory places no restriction on its application at other frequencies.

Other considerations in designing the experimental system included the decision to use at most two (2) antennas and to primarily use the waveguide with short circuit terminations. In designing the two antennas, only one was constructed to provide access for measurement of current and charge distribution and measurement of input impedance. This antenna was also constructed to be continuously variable in length. The second antenna was made in interchangeable discrete lengths and could be either driven or loaded as desired. The decision to use short circuit terminations on the waveguide (i.e. to primarily test the case of a rectangular cavity) was made not only because of the experimental simplicity, but also because this case represented the most critical condition for testing the theory. (Again, no restriction is placed on the validity of the theory.) The rectangular cavity is also attractive from the applications viewpoint, since it offers a well-defined, closed system whose behavior is independent of the environment surrounding the apparatus.

4.1 Description of Apparatus.

Historically, measurements of antenna characteristics has required somewhat elaborate systems of microwave components for accurate determination of both amplitude and phase of pertinent signals (see section 8.8 of King, et al. [20]), each component and interconnection increasing the possibility of error. At present, however, the state of the art in commercial vector voltmeters (VVM) has greatly reduced the need for discrete components, since the VVM performs both absolute magnitude and relative phase

measurements for two microwave signals over a broad frequency range. The experimental work done in this thesis takes advantage of this simplicity by using a VVM as the heart of the measurement system. This not only offers experimental advantages, but also makes the total system convenient for direct practical application. The vector voltmeter chosen for this work was the PRD 2020 (PRD Electronics, Inc.) primarily because of the useable frequency range of 1.5 MHz to 2.4 GHz. This allows operation up to the L-band (approximately 0.9 to 1.8 GHz) which has both the experimental and practical advantages noted earlier. With the VVM selected, the remaining apparatus was designed to take maximum advantage of its use: the waveguide was constructed such that the dominant mode (TE_{10}) would propagate for the L-band frequencies, and an impedance sensing block (to be described later) was employed to simplify the impedance determinations.

The rectangular waveguide, as shown in Figures 4.1 and 4.2, is made of one-half inch (top only) and one-quarter inch brass plate. The narrow inside dimension (theoretical "a") is 3.25 inches, which is the standard for rectangular L-band waveguides. For experimental flexibility, the wide dimension (theoretical "b") was made adjustable in three steps of 6.5 inches, 7.7 inches, and 8.9 inches corresponding to a cut-off frequency for the dominant $TE_{1,0}$ mode of 0.908 GHz (L-band), 0.767 GHz, and 0.664 GHz. The waveguide length corresponding to the theoretical dimension $d_1 + d_2$ was set at two feet or approximately 0.6096 meters. The flanges on the waveguide section were adaptable to the standard L-band flange. As shown in Figure 4.1, the usual configuration was to short circuit

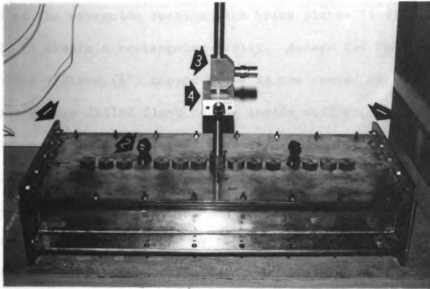


FIGURE 4.1

Rectangular Waveguide Section in Cavity Configuration
with Antenna Measurement System in Place
(Numbers described in text)

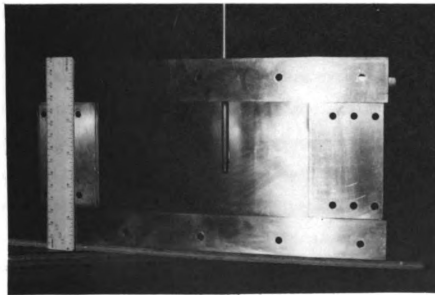


FIGURE 4.2

End View of Rectangular Waveguide Section with Antenna

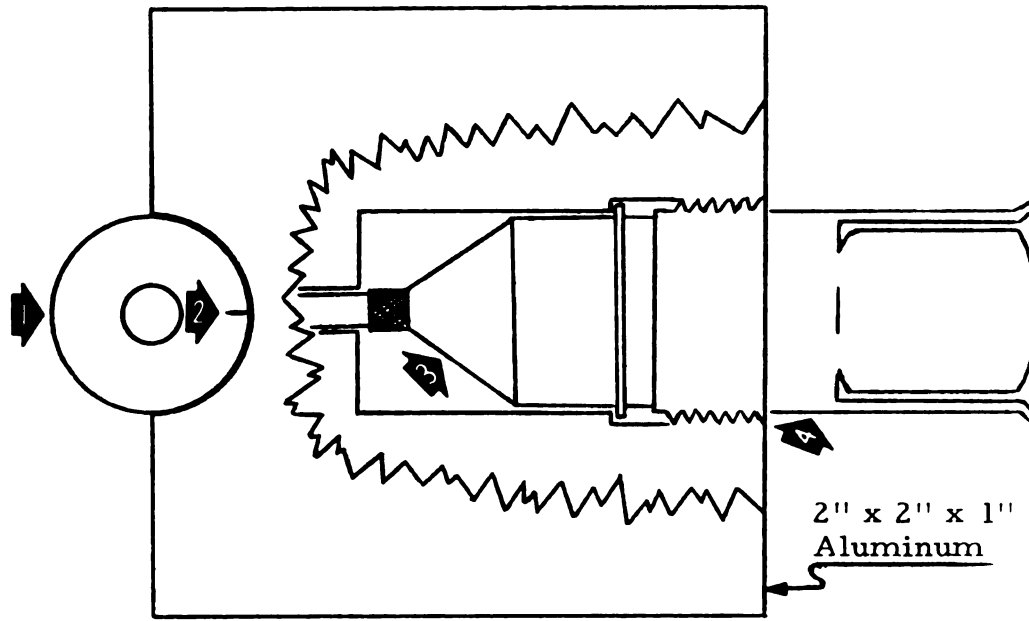
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the ends of the waveguide section with brass plates (indicated as number 1) to create a rectangular cavity. Access for the antennas was made via fifteen (15) tapped holes in the center of the broad wall. These were filled flush to the inside wall with threaded plugs when not in use. The hole spacing was 3.2 centimeters with the center hole (hole number 8) being equidistant from the end flanges. Although the holes restricted the flexibility of axial location, this was felt to be outweighed by the lack of internal field perturbation which a continuous slot would introduce (especially at frequency where higher order modes could exist). Also shown in Figure 4.1 is the antenna measurement system placed in the center hole (hole number 8). This antenna feed system is made of air filled coaxial transmission line having 0.25 inches (O.D.) diameter brass tubing as a center conductor and 0.5625 inch (I.D.) diameter brass tubing as an outer conductor (standard General Radio rigid coax outer conductor tubing was used). This configuration yields a characteristic impedance of 48.66 ohms. The antenna protrudes into the waveguide as a continuous extension of the center conductor beyond the inside surface of the waveguide wall.

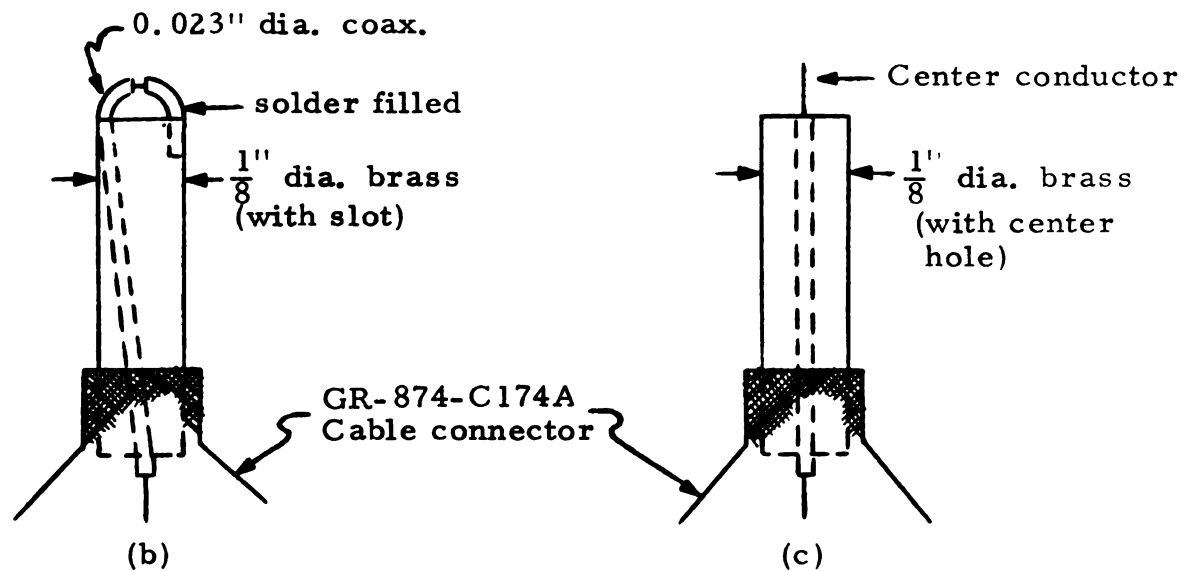
Other parts of the antenna measurement system shown in Figure 4.1 are the sliding tee (number 3) and the impedance sensing block (number 4). The sliding tee is used to excite the antenna while allowing the center conductor to slide, thus allowing a continuous variation of antenna height inside the waveguide. Above the sliding tee is an adjustable short circuited section of coaxial line which allows one degree of matching to increase power transfer from the generator to the antenna. (Obtaining maximum power transfer by

completely matching with a double stub is not done because of the increased complexity required.) The shorting plunger was taken from a General Radio adjustable stub. The impedance sensing block (number 4) is used to probe the voltage and current at the fixed physical location on the coaxial line, in this case 12.55 centimeters (physically) from the end (the inside wall of the waveguide; i.e., the antenna terminal plane). The construction, operation and effectiveness of the impedance sensing block was also reported in Scott [35]. The idea is basically simple and its effectiveness is based on the use of a vector voltmeter. A sketch of the block and probes is shown in Figure 4.3. Sensing of the coaxial field is accomplished by a short electric probe and a small magnetic probe which protrude through the outer conductor of the coaxial line. The electric probe is sensitive to radial electric field and thus has a voltage induced across its terminals which is proportional to the voltage (amplitude and phase) at that location on the line. The magnetic probe is a small shielded, loaded loop which is sensitive to circumferential magnetic field and thus has a voltage induced across its terminals which is proportional to the current (amplitude and phase) at that location on the line. The theory and properties of these probes is discussed in section 8.6 of King, et. al. [20]. The probes are constructed of 0.023 inch (O.D.) solid copper jacketed coax (obtained from Uniform Tubes, Inc., Collegeville, Pa.), and are of physical dimensions much smaller than a wavelength so that negligible line loading occurs. Determination of impedance using these probes is done by using the vector voltmeter to measure the absolute magnitude and relative phase between the



(a)

Cutaway View of Half the Block Showing: (1) Coaxial Line,
(2) Probe, (3) and (4) General Radio Connector



Current Probe Mount

Voltage Probe Mount

FIGURE 4.3

Impedance Sensing Block

voltages induced in the probes. The ratio of these voltages is then proportional to the impedance at that point on the coaxial line. In order to obtain absolute impedance, it is necessary to compensate for the proportionality constants in amplitude and phase for each probe. Although this can be done analytically for the probes (for example as done by Scott [35]), it is much simpler and more reliable to obtain the proper compensation for the entire probing system experimentally. The method used to obtain the calibration constant for the impedance sensing block is described in the next section.

As mentioned above, the antenna for the antenna measurement system is the extension of the center conductor of the coaxial air line beyond the inside surface of the waveguide wall. This antenna is constructed so that current and charge distribution measurements can be made, as shown in Figure 4.4. At the terminal plane the center conductor is supported by a one-eighth inch thick styrofoam wafer as indicated by the number 1 in the figure. This is the only dielectric support necessary in the entire system because both the sliding tee and sliding short circuit act to center the conductor. The quarter inch diameter brass tubing is slotted axially to accept the probes for current and charge distribution measurements. The probes are mounted in the key of a hollow brass plug which is attached to a small stainless steel tubing push rod, both sliding freely inside the antenna tube (see Figure 4.5). The charge probe (number 2 in Figure 4.4) is constructed as the extension of the center conductor of miniature solid dielectric coaxial line. Since this probe is mounted radially (normal to the antenna surface), the voltage induced in it by the electric field is proportional to the charge



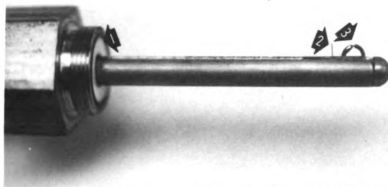


FIGURE 4.4
Slotted Antenna with Current and Charge Probes

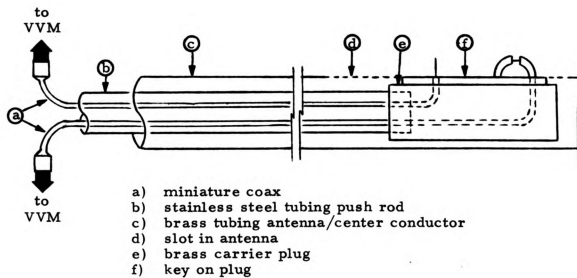


FIGURE 4.5
Cutaway View of Slotted Antenna with Probes



FIGURE 4.6

Second Antenna(s) for Coupled Antenna Measurements

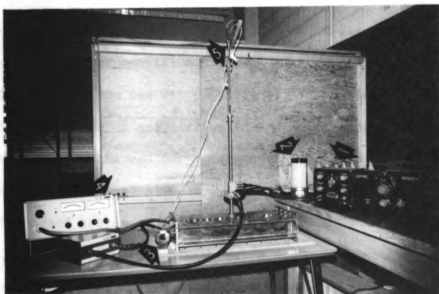


FIGURE 4.7

Total Experimental Set-Up for Current Distribution Measurements
(Numbers are described in the text)

at that point on the antenna (by the boundary condition for normal electric field). The current probe (number 3 in Figure 4.4) is a shielded, loaded, circular loop which has a voltage induced in it by the circumferential (antenna) magnetic field. This voltage, by the boundary condition for tangential magnetic field, is proportional to the current at that point on the antenna. The miniature coaxial lines (0.034 inch diameter, solid copper jacketed) of which the probes are made run continuously from the probes to the free end of the center conductor (beyond the short circuited section of line) where subminiature coaxial connectors are attached. At this point, the vector voltmeter is used to measure the voltage induced in the probes as they are moved axially along the antenna surface (see Section 4.2).

When it is desired to experimentally test the case of two antennas in the waveguide, the antenna shown in Figure 4.6 is used. This antenna is connected directly to a General Radio type connector via a short section of coaxial air line. Five antennas were made in lengths ranging from one (1) to three (3) inches in half-inch steps. They are made of one quarter inch diameter solid brass rod and are threaded on the end for attaching to the coaxial line center conductor at the terminal plane.

The total set up used to obtain experimental data is shown in Figure 4.7. The microwave generator used for L-band frequencies is the General Radio type 1218 B shown by arrow number 1 in the photograph. The generator is connected through a 10db attenuator (to minimize loading effects) to a cavity type frequency meter indicated by arrow number 2. Although not visible in the photograph, this is

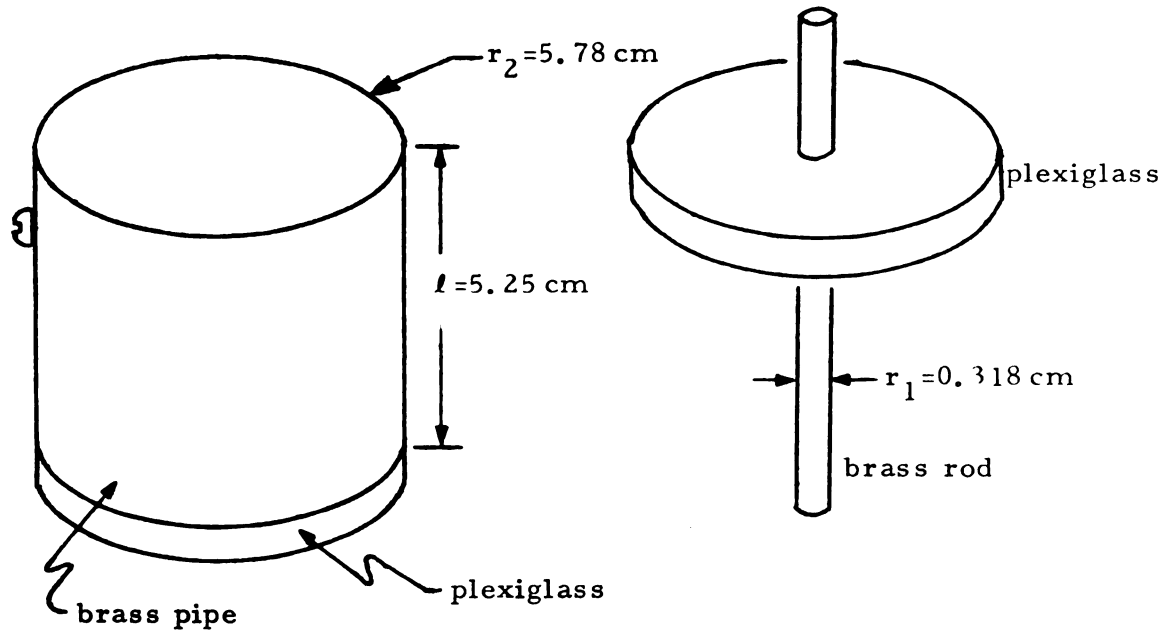
followed by a 2 GHz low pass filter (for work in the L-band) which drives the antenna measurement system and waveguide described in the preceding paragraphs. In the photograph of Figure 4.7, connections are made for current and charge distribution measurements on the antenna in the waveguide. The solid miniature coax and subminiature connectors at the top of the antenna measurement system (arrow number 5) are connected to flexible miniature coax which goes to the coaxial switch (arrow number 3). This switch is used to select either the current probe or charge probe for connection to the vector voltmeter. The vector voltmeter, indicated by arrow number 4, has the signal processing and metering unit with the controls and a sampling head which converts the two applied signals to an intermediate frequency. The sampling head is connected through a ten foot cable to the main unit. The reference channel input for the VVM is shown in the photograph connected (arbitrarily) to the voltage probe of the impedance sensing block. This offers a convenient reference for making relative phase measurements along the antenna.

The experimental set-up shown in Figure 4.7 is used for measurements with the air filled or empty cavity. For measurements with water filling the cavity a wooden box large enough to hold the waveguide section is lined with plastic sheeting and filled with the water. The water level is kept even with the top of the waveguide to minimize the forcing of water into the antenna driving line. Air is removed from the waveguide by venting through the antenna access holes. This method of studying fluids is of course cumbersome, but would not be necessary had the waveguide been constructed water-tight. This, however, would have meant either sacrificing experimental

flexibility, or increasing the complexity of the waveguide design, and neither alternative was acceptable. For practical application to fluid monitoring, the waveguide or cavity size would be chosen and constructed water tight, and ports could be cut and covered with fine wire screening to allow for filling or continuous flow studies. The screening would provide gross filtering while maintaining, at least approximately, the conducting surface at the cavity wall.

In order to test the theory for the case of a dissipative medium the values of relative dielectric constant, ϵ_r , and conductivity, σ , for the medium must be known. For water, a great deal of literature is available (e.g. Dorsey [5], Grant [10], Saxton [32, 33], and Lepley [23]) based primarily on the Debye equation. However, it is best to determine the properties independently for the specific water samples used. In this report the conductivity and permittivity are measured at low frequencies and high frequencies respectively, much as was done by Scott [35].

Measurement of conductivity is performed using the low frequency test cell shown in Figure 4.8. The test cell, a cylindrical capacitor, is filled with the sample and the impedance of the capacitor measured on a low frequency bridge (e.g. GR type 1606-A) at frequencies in the vicinity of 1 MHz. This impedance is a function of the complex dielectric constant of the medium filling the capacitor as well as the physical dimensions. The impedance can be obtained theoretically by solving the problem via Maxwell's equations (Scott, [35]). The same result, based on electrostatics with fringing fields neglected -- a good assumption with water having $\epsilon_r \gg 1$ -- can be obtained from the formula for a capacitor filled



$$Z = R + jX$$

$$\sigma = \frac{K + \sqrt{K^2 - 4(R\omega\epsilon_r\epsilon_o)^2}}{2R} \doteq \frac{K}{R} \quad \text{v/m}$$

$$K = \frac{1}{2\pi l} \ln \left(\frac{r_2}{r_1} \right) = 8.799$$

(a)

Coaxial Capacitor

Salinity (NaCl)	Frequency MHz	Conductivity, σ , millimhos/meter
0.5 gm/liter	0.5	97.7
	1.0	97.5
	2.0	96.3
1.0 gm/liter	1.0	196.

(b)

Representative Measurements of Conductivity

FIGURE 4.8

with a pure dielectric medium (i.e., by calculating the "complex" capacitance for the complex dielectric constant case). Thus, from electrostatics, the capacitance of a coaxial capacitor of length l is

$$C = \frac{2\pi\epsilon l}{\ln(r_b/r_a)} \quad (4.1)$$

where r_b is the inner radius of the outer conductor and r_a is the radius of the inner conductor. For a complex dielectric constant

$$\epsilon = \epsilon_r \epsilon_0 - j \sigma/\omega$$

the corresponding "complex" capacitance is

$$\bar{C} = (1/K)(\epsilon_r \epsilon_0 - j \sigma/\omega) \quad (4.2)$$

where

$$K = \ln(r_b/r_a)/2\pi l \quad (4.3)$$

is a constant dependent upon the physical dimensions of the capacitor.

The impedance of this complex capacitance is then simply

$$\begin{aligned} Z &= 1/j \omega \bar{C} \\ &= K/\sigma + j\omega \epsilon_r \epsilon_0 \\ &= R + jX \end{aligned}$$

where

$$\begin{aligned} R &= K\sigma/(\sigma^2 + (\omega \epsilon_r \epsilon_0)^2) \\ X &= -\omega\epsilon K/(\sigma^2 + (\omega \epsilon_r \epsilon_0)^2) . \end{aligned}$$

Solving for σ in terms of ϵ_r and R gives

$$\sigma = \frac{K + \sqrt{K^2 - 4(R \omega \epsilon_r \epsilon_o)^2}}{2R} . \quad (4.4)$$

For the test cell used in this study the constant is (see Figure 4.8 for the dimensions):

$$K = 8.8 .$$

For all the media used in this study, the measured value of R from the low frequency bridge was less than 200 ohms. Therefore at frequencies below 1 MHz, the factor

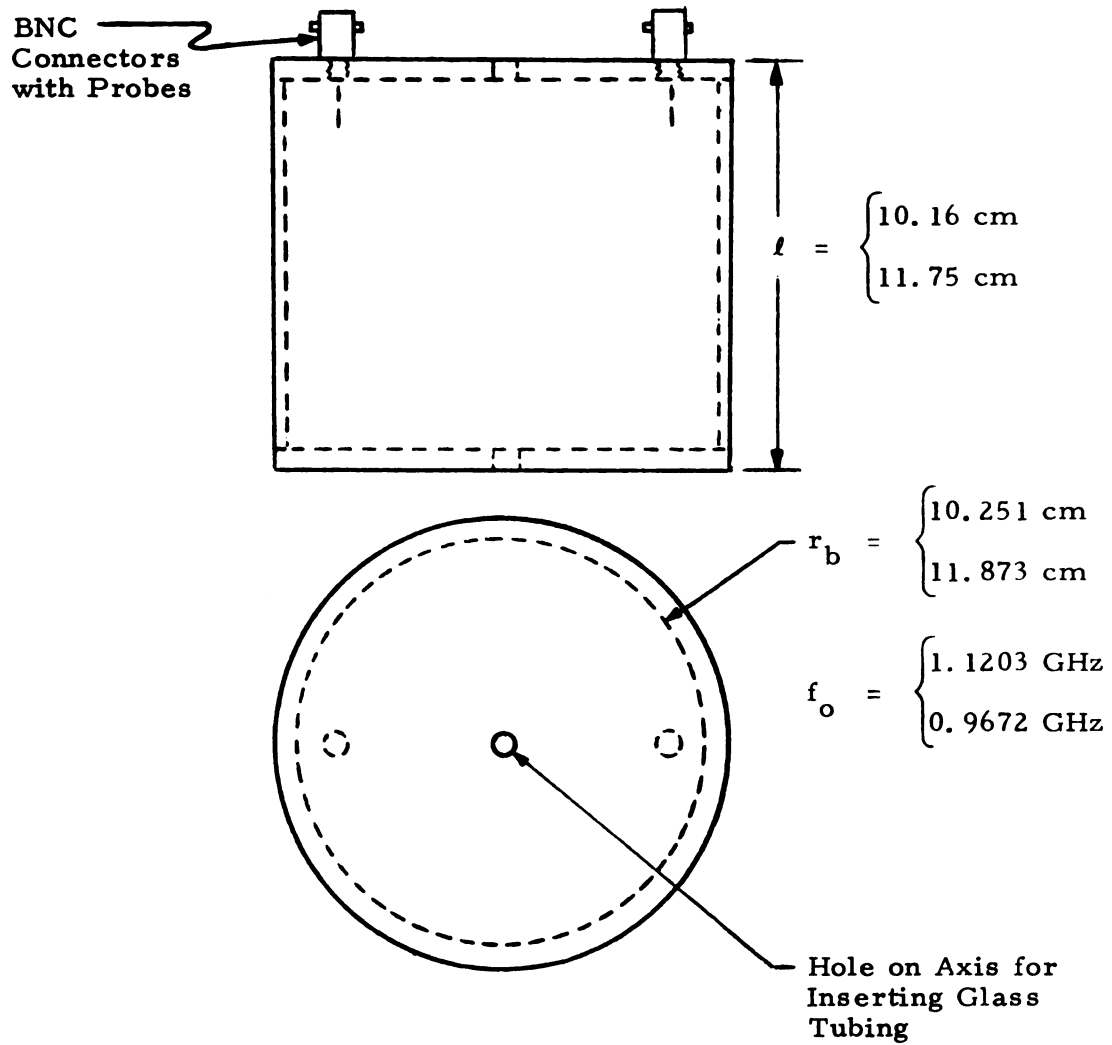
$$4(R \omega \epsilon_r \epsilon_o)^2 \leq 3.2 \ll K^2$$

for an assumed $\epsilon_r = 81$, and the conductivity is approximately given by

$$\sigma = K/R \quad (4.5)$$

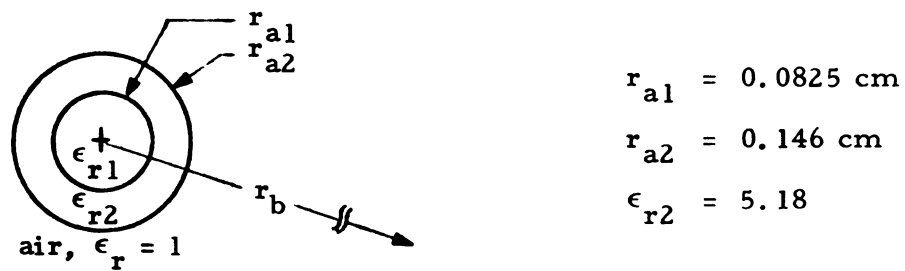
which is insensitive to errors in the assumed value of ϵ_r . The approximation, Equation (4.5), is of course not used, but only serves to demonstrate that this method of determining conductivity is effective even without a precise knowledge of the dielectric constant. The results obtained for two concentrations of sodium chloride in distilled water are tabulated in Figure 4.8. Based on available literature (see Saxton [32]), it is assumed that these conductivities are constant up to the L-band frequencies of interest.

The measurement of relative dielectric constant is accomplished in this study by the cavity perturbation method. Two cylindrical cavities were available as shown in Figure 4.9 (a) for absorption type measurements. Based upon perturbation theory for a cylindrical cavity excited in the TM_{010} mode, it can be shown (see Section 6.8



(a)

Cylindrical Cavity



(b)

Glass Tubing Properties

FIGURE 4.9

of Johnson [16]) that for a dielectric rod (dielectric constant = ϵ_r) located on the cavity axis the shift in resonant frequency, δf , is

$$\frac{\delta f}{f_o} = - \frac{(\epsilon_r - 1)(r_a/r_b)^2}{0.54} \quad (4.6)$$

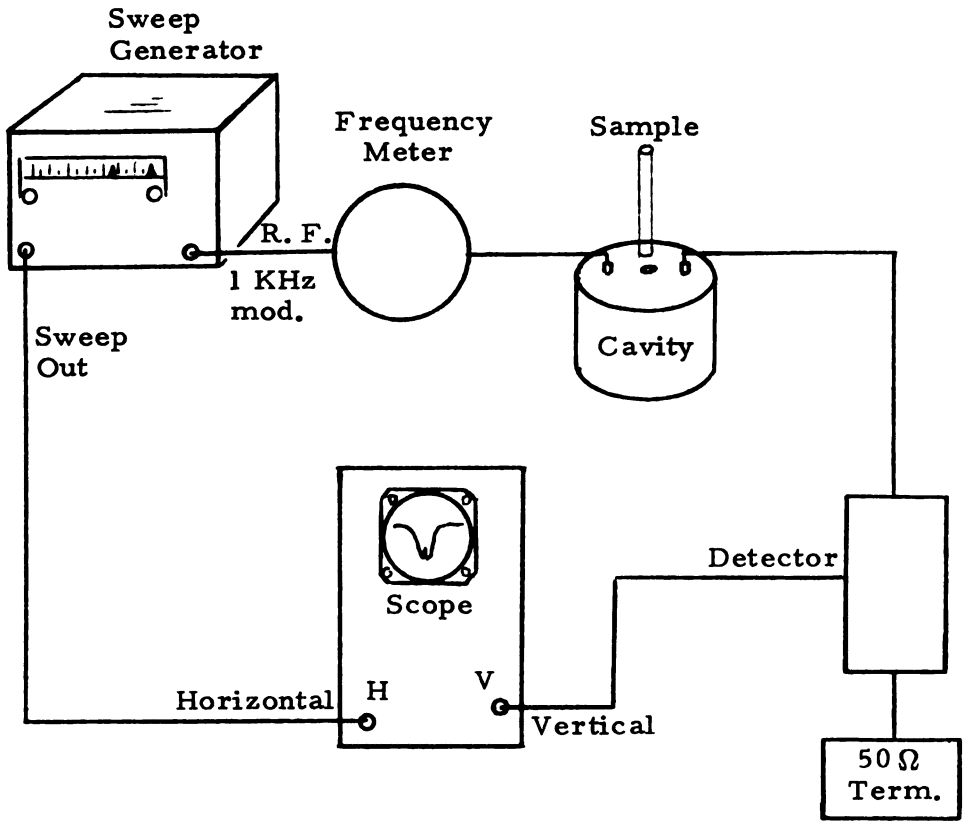
where r_b is the radius of the cavity and r_a is the radius of the dielectric rod and f_o is the unperturbed cavity resonance. It is desired in this study to place a water sample held in small glass tubing on the axis of the cavities and thus the theory must be modified to account for the double dielectric (glass-water) rod. This can be done by following the method in section 6.8 of Johnson [16]. The result for tubing of inner radius r_{a1} and outer radius r_{a2} made of material having a dielectric constant ϵ_{r2} and filled with a material having dielectric constant ϵ_{r1} (see Figure 4.9 (b)) is

$$\frac{\delta f}{f_o} = \frac{(\epsilon_{r1} - 1)r_{a1}^2 + (\epsilon_{r2} - 1)(r_{a2}^2 - r_{a1}^2)}{0.54 r_b^2} \quad (4.7)$$

To use Equation (4.7), the experimental set up shown in Figure 4.10 (a) is used. First, the unperturbed resonant frequency of the cavity, f_o , is measured. Then the empty glass tubing is placed on the axis of the cavity and the shift in resonant frequency δf_g (g for glass alone) is measured. Since in this case $\epsilon_{r1} = 1$, Equation (4.7) becomes

$$\frac{\delta f_g}{f_o} = \frac{(\epsilon_{r2} - 1)(r_{a2}^2 - r_{a1}^2)}{0.54 r_b^2} \quad (4.8)$$

which can be solved for ϵ_{r2} . For the glass tubing used in this experiment the value $\epsilon_{r2} = 5.2$ was found. It is not necessary



(a)

Circuit Diagram for Permittivity Measurements

Salinity (NaCl)	Frequency f_o , GHz	Dielectric Const. ϵ_r
Pure water	0.9672	83.5
	1.1203	82.9
0.5 gm/liter	0.9672	93.4
	1.1203	85.7
1.0 gm/liter	0.9672	99.3

(b)

Representative Measurements of Dielectric Constant

FIGURE 4.10

to solve Equation (4.8), however, since combining with Equation (4.7) yields:

$$\frac{\delta f_{gw}}{f_o} - \frac{\delta f_g}{f_o} = \frac{(\epsilon_{r1} - 1)r_{a1}^2}{0.54 r_b^2} . \quad (4.9)$$

Thus by measuring the resonant frequency shift for both glass and water, δf_{gw} , and for glass alone, δf_g , the dielectric constant of the water (ϵ_{r1}) is given by Equation (4.9). The results obtained for two concentrations of sodium chloride in distilled water are shown in Figure 4.10 (b). Since these results are obtained near 1 GHz, they are assumed constant over the entire L-band frequencies of interest in the remainder of this report. The appropriate values of permittivity as well as conductivity for each medium studied will be presented with the other experimental results in Chapter 5.

4.2 Experimental Methods.

As stated in Section 4.1 the use of a vector voltmeter to make antenna measurements is a relatively new procedure. For this reason some time will be spent in this section briefly describing the methods used to obtain the data. Also, by describing these procedures, account may be taken of the possible introduction of error from the procedures alone, so that, if it is deemed necessary in the future, proper refinements may be made toward obtaining results with a high degree of precision. Even though the most precise methods are not used, it is found that the precision is more than adequate to verify the theory and to indicate the feasibility of application.

Since the vector voltmeter reads directly the absolute magnitude and relative phase between two signals, most of the pro-

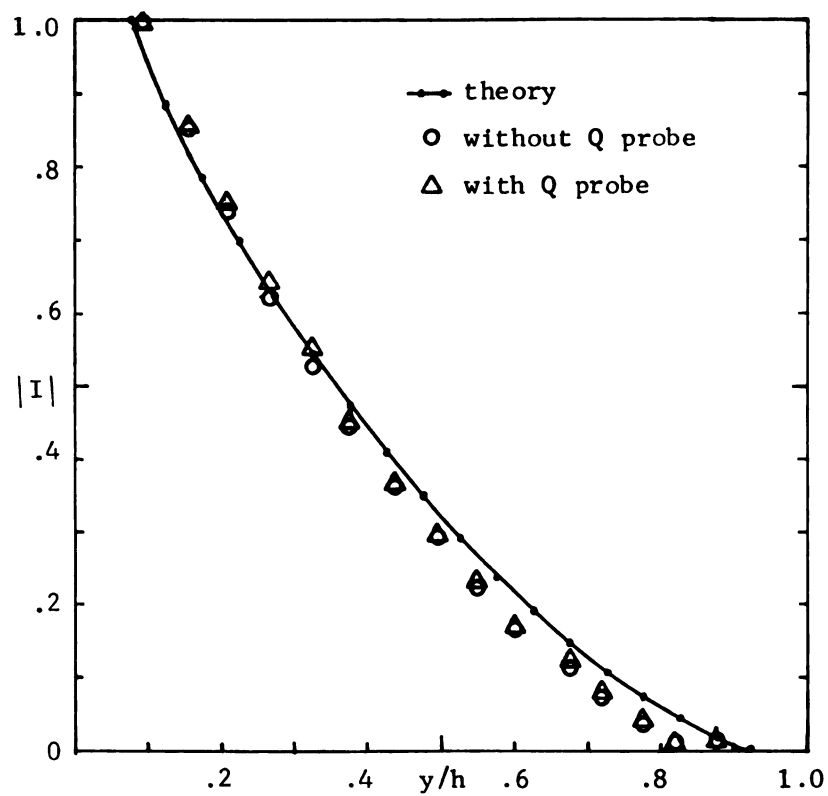
cedures necessary to obtain useable data center around selecting the proper signals and calibrating the system to convert the VVM readings from relative to absolute. As with all antenna measurement, the data for current and charge distribution need only be relative, in both amplitude and phase, to the input current or applied voltage at the antenna terminals. Therefore, no account need be taken of the phase shift or transmission line losses in the connecting lines between the antenna current and charge probes and the VVM (these being constant for a fixed frequency). A constant phase reference must be used, however, for each probe as it is moved axially along the antenna. In the set-up used here (as shown in Figure 4.7), the reference is taken to be either the voltage or current probe (whichever is larger) at the impedance block. This signal is maintained at a constant level throughout the measurement to assure constant driving condition and its phase is, of course, independent of location of the probes on the antenna. By subtracting off the phase measured at the driving point from the phase at all other locations on the antenna the data is then referenced to the current at the input terminals.

Regarding the current and charge probes on the antenna, the question may be raised as to the effect of having both probes on the antenna simultaneously. In answer, it is first realized that the current and charge distribution are inter-related via the continuity equation and not independent. Thus, measuring both is a duplication of effort except that they may be used as checks for each other. In designing these probes, it was realized that the current distribution was to be the fundamental result and so this

probe was built and some representative current distribution measurements made. Then the charge probe was added and the measurements repeated. The results showed that the charge probe had no appreciable effect on the data. This data is presented in Figure 4.11.

As can be seen, only the data near the driving point is changed and since current probe measurements at this point are always unreliable due to unavoidable loading, nothing is lost. On the other hand, the current probe is expected to have an appreciable effect on the charge probe measurements owing to the relative size and proximity (no comparative study was made) and thus the charge probe data may be inaccurate. Therefore, the current probe is primarily used to obtain data and the charge probe is occasionally used as a check.

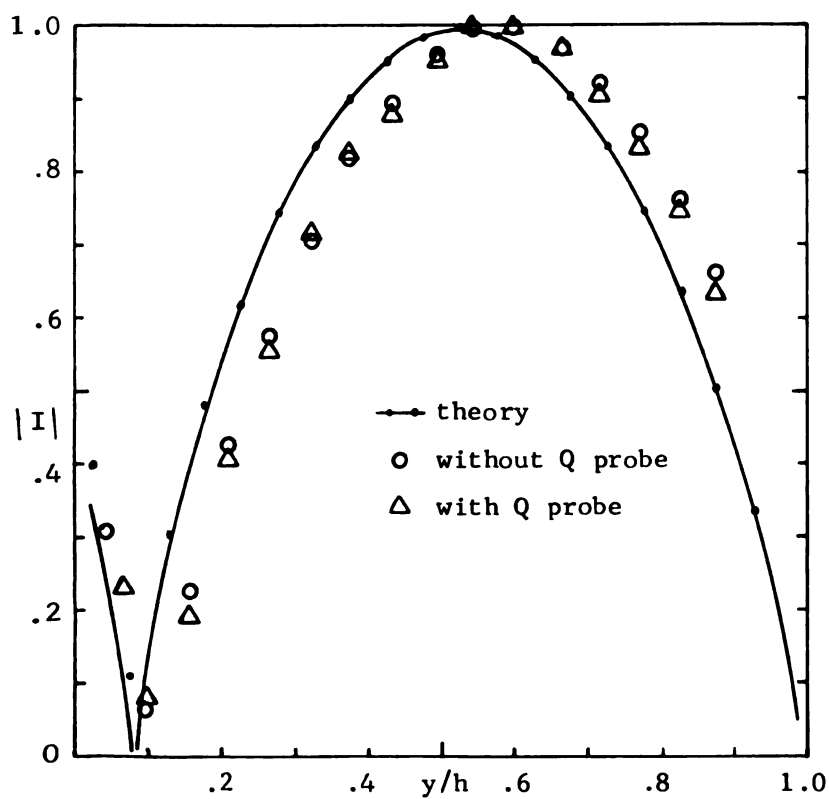
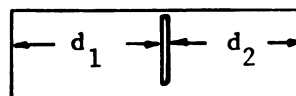
One further consideration needs to be made concerning the current distribution measurements; namely, the orientation of the current sensing probe. Unlike antennas in an infinite medium, the fields surrounding the antenna in the waveguide are not rotationally symmetric. Because of the physical size of the current probe (radius = 0.25 cm), it is sensitive to the magnetic field a short ways from the antenna surface and thus is excited by fields other than those at the antenna boundary. Even though this distance is much less than a wavelength, the fields are rapidly varying near the antenna, since besides satisfying the antenna boundary condition, the fields must also satisfy the waveguide boundary conditions. For example, consider the case of the current probe lying in the (y,z) plane; i.e., the longitudinal plane at the center of the waveguide. In this orientation the current loop is excited by magnetic fields which are $\hat{x}$ directed -- normal to the plane of the



(a)
 $f = 1.2 \text{ GHz}$

FIGURE 4.11

Effect of Charge Probe
 on Current Measurements

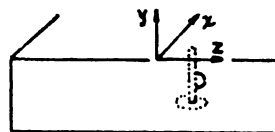


$a = 2b = 16.51 \text{ cm}$
 $d_1 = d_2 = 30.48 \text{ cm}$
 $h = 6.25 \text{ cm}$

(b)
 $f = 1.5 \text{ GHz}$

loop. By looking at the distribution of the $\hat{x}$ -component of the waveguide mode magnetic field (see Equations (2.5a)), one sees that

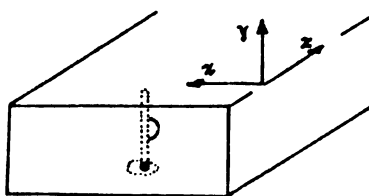
$$h_{x,nm}^{e,h} \propto \sin(n\pi x/a)$$



which is a maximum at the center of the waveguide ($x = a/2$) for all $n = \text{odd modes}$ (the only ones excited according to the theory). Thus, for example, near the end of the antenna, where the current is approaching zero, the magnetic field on the antenna surface must be small; yet a short distance away from the antenna this component of magnetic field must approach the maximum for the waveguide mode. These rapidly varying fields will obviously adversely affect any attempt to relate the induced probe voltage to the tangential magnetic field or current. On the other hand, consider the case of the current probe lying in the (x,y) or transverse waveguide plane. In this orientation the loop is excited by the $\hat{z}$ -component of magnetic field. The waveguide mode field, again from Equation (2.5a), is distributed as

$$h_{z,nm}^h \propto \cos(n\pi x/a)$$

$$h_{z,nm}^e \equiv 0.$$



Therefore, all modes of odd order n go through a minimum at the center of the waveguide ($x = a/2$). In the x,y plane, then, the current loop voltage is primarily induced by the magnetic field nearest the antenna surface and more nearly represents the current on the antenna. Figure 4.12 shows some representative results for the probe in the two orientations. In either orientation, the probe-size will introduce errors, but, based on the above arguments and the results of Figure 4.12, these errors are expected to be

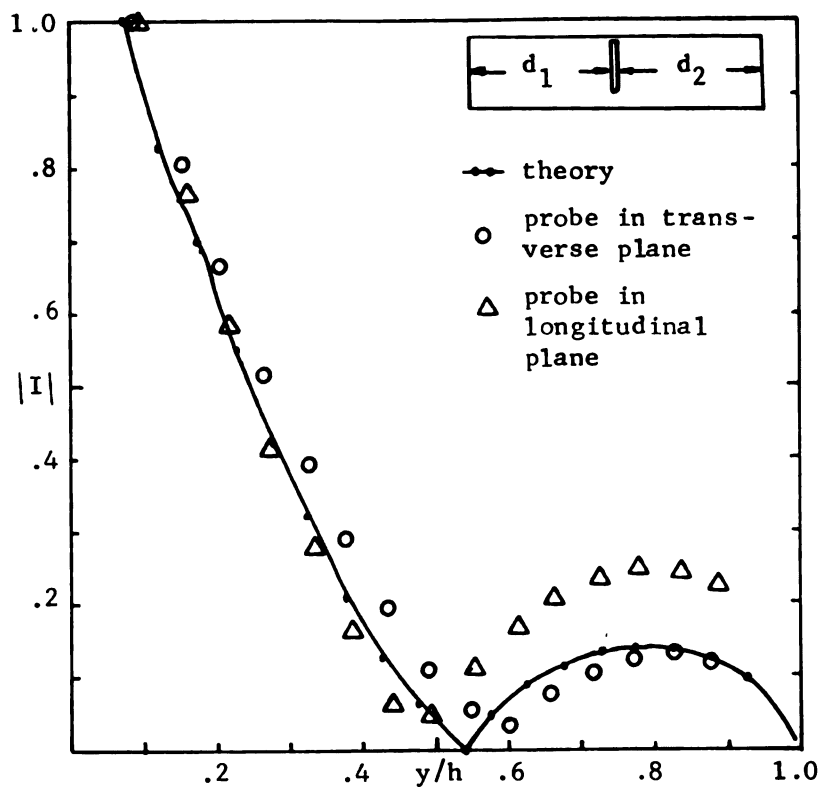
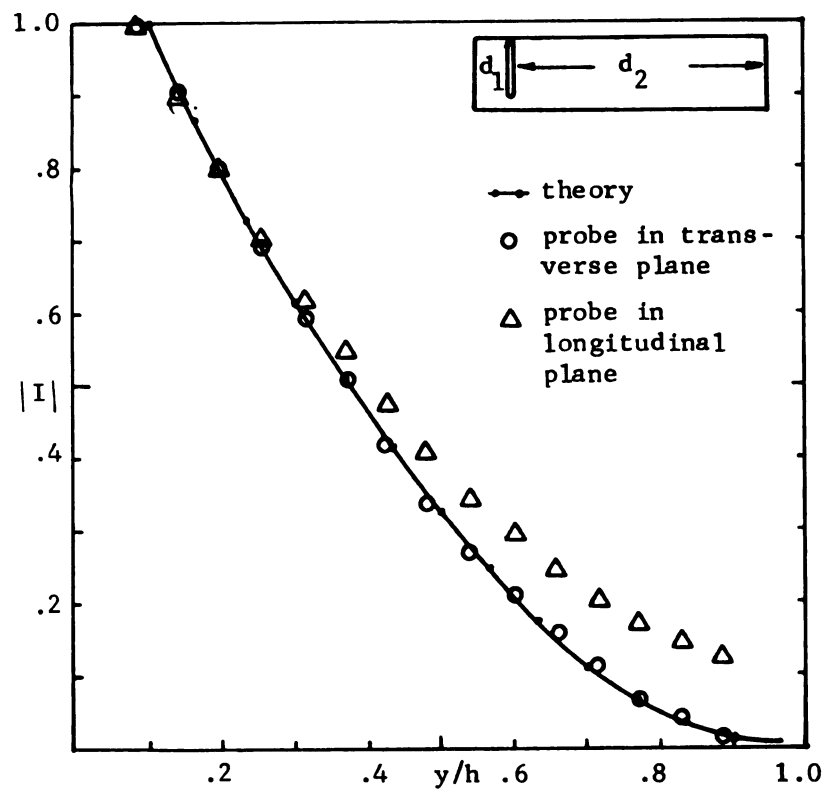


FIGURE 4.12

Effect of Probe Orientation on Current Measurements



minimized for the second case. For this reason, all the experimental data presented in the next chapter is taken with the current probe oriented in a transverse waveguide plane.

With the current and charge distribution data taken relative to the input or terminal values, the absolute antenna characteristics are determined from a knowledge of the absolute input admittance. This requires a calibration of the impedance sensing block and the interconnecting cables. Based on the theory of the probes used, the ratio of the current to voltage signals is proportional to the magnitude of the admittance, and the phase angle between these signals is within an additive constant of the phase of the admittance. Since it is nearly impossible to theoretically account for the losses and phase shift introduced by the probes and probe mounts, the system is most easily calibrated experimentally. Because the impedance sensing block is not located at the antenna terminals, it is also desirable to accurately determine the equivalent electrical length through which the impedance data must be transformed; this is different from the physical length because of the dielectric support wafer and the physical size of the probes. Both the effective length and the calibration constant are determined in the following way. At a fixed frequency, a short circuit is attached at the antenna terminals (the end of the outer conductor), and the phase angle of the susceptance ($\pm jB$) at the impedance measuring block determined (exact length is not required unless it is near a multiple of a quarter wavelength). Using the vector voltmeter phase offset and zero adjustments, the phase angle between the current and voltage

signals is set to the proper value ($\pm 90^\circ$). This calibrates the phase angle measurements. The current and voltage signals are recorded, and from transmission line theory, the following equation holds:

$$K_z \frac{|V_1|}{|I_1|} = Z_c \tan (\beta \ell) , \quad (4.10)$$

where K_z is the unknown impedance amplitude calibration constant and ℓ is the unknown length between the antenna terminals and the measuring point. Z_c is the characteristic impedance and β is the free space propagation constant. The existence of the dielectric wafer is explicitly ignored in writing Equation (4.10) so that its effect can be included in the effective length and calibration constant. Next, the short is removed and a fixed length (say, d) of outer conductor is attached to the antenna terminals, thus extending the length of line along the antenna a fixed distance, d . (This is facilitated by having the threads on the antenna support plug compatible with General Radio parts so that a G-R nut is used to secure the extension.) The short circuit is again attached and the above process repeated. The phase angle remains calibrated and the transmission line equation gives:

$$K_z \frac{|V_2|}{|I_2|} = Z_c \tan (\beta (\ell + d)) , \quad (4.11)$$

where V_2 and I_2 are the new voltage and current readings from the impedance sensing block. Equations (4.10) and (4.11) are now two equations in the two unknowns K_z and ℓ . Dividing the equations eliminates the unknown K_z and gives a transcendental equation

to be solved for l , which is very easily and rapidly solved with a calculator since an approximate value of l is known (physical length). The resulting value of l then yields the calibration constant via Equation (4.10) or (4.11). The two step process described above now calibrates the impedance sensing block for obtaining absolute complex admittance data. That is, the VVM phase is left untouched and the data is taken for the desired experimental conditions. The measured voltage and current are multiplied by the calibration constant and the resulting complex admittance is translated to the antenna terminals through the effective length l via the transmission line equation. Although this procedure is seemingly lengthy, the bulk of the work is handled easily by a computer or calculator and there are very few places for experimental error (in fact, fewer than in the slotted line method). It is also a characteristic of this specific set-up that the short circuit method is necessary. In a future arrangement it would be best to remove the length as an unknown by proper compensation of the support wafer. Then only the calibration constant need be determined and this is easily done by applying a matched load at the antenna terminals and taking one set of readings.

One drawback of this impedance system is that it is inherently frequency dependent and thus must be calibrated for each frequency used. This information, however, can be determined independent of the experiments and stored for retrieval and use by a computing system. (This drawback, unfortunately, prohibits performing direct swept frequency experiments, but to my knowledge, this problem is shared with other techniques.) As an example of the frequency

dependence, it was found that over the frequency range of 1.0 GHz to 1.4 GHz, the effective length changed by no more than $\pm 0.4\%$ (around 12.7 cm) and was repeatable to within $\pm 0.1\%$. Owing to the rapid variation of the tangent function, however, no attempt was made to standardize this variable, thus avoiding the introduction of unnecessary error. The calibration constant was much more frequency dependent, having more than 10% variation, repeatable to within 1.0%, over this frequency range.

One further comment concerning the use of this impedance measuring system and its inherent errors in this application should be made. When the waveguide is in the cavity configuration, the antenna input admittance is very nearly a pure susceptance. (The theoretical lossless cavity results are, of course, exactly so.) This implies that the voltage standing wave ratio in the antenna driving coaxial line is infinite. As a result, for some conditions the voltage to the current ratio at the impedance measuring block terminal plane is nearly infinite or zero at a given frequency. This would then yield a signal below the noise level of the VVM from either the current or voltage probe and no data could be taken. (The same problem could, of course, arise when calibrating the probe using a short circuit as a reference, as discussed previously.) Coupled with this problem is the nature of the transmission line equation for transforming impedance. Because the transformed impedance as a function of electrical length has a steep slope near the infinities, small errors in impedance measurement (or in length) can lead to large errors in transformed antenna input impedance near the frequencies where this tangential infinity occurs. For

example, if the antenna input impedance (or admittance) is near infinity, a small error in the electrical length through which the measured ratio of $|V|/|I|$ is transformed could lead to large errors in the transformed experimental data even though the raw experimental data was accurate. This source of error is unavoidable, but it should be noted that this problem arises because the antenna impedance being measured is purely reactive. For antenna impedances which involve a comparable resistive component (a waveguide with matched terminations or a cavity filled with a dissipative medium), the standing wave ratio in the line is not infinite and accurate results are obtainable at all frequencies.

A potential source of error which arises in the measurement of phase should also be mentioned. It was found that the flexible coaxial line used to connect the impedance measuring block to the VVM introduced variations in the measured phase angle. As the cable was moved, the phase angle was found to vary by less than $\pm 2^\circ$. Thus as the location of the antenna in the waveguide was changed, there was a slight change in the phase angle reference from the value determined during calibration. At a fixed location, however, no variation was found. Thus, in the cavity configuration, the measured phase angle was found to be within $\pm 2^\circ$ of the 90° phase reference determined during calibration and this variation was ignored. Although one may be concerned about the effect of wall losses (or cavity Q) causing an impedance which has a small resistive component, the experimental system used showed this to be a negligible effect within the accuracy described above. (In fact, the theoretical solution was altered to include the approximate

formulation of wall losses as described by Collin [2], and the phase angle of the antenna input impedance changed by less than 0.5° from pure reactive.) Therefore, the phase angle measurements are assumed to be accurate to better than 2° for this experimental system.

CHAPTER 5

NUMERICAL AND EXPERIMENTAL RESULTS

The previous chapters have dealt with the theoretical development, numerical solution, and the experimental techniques for the study of linear thin wire antennas in a rectangular waveguide. The characteristics of the antenna, namely, the current distribution and input admittance, have been viewed as a function of the physical configuration and the electrical properties of the medium filling the waveguide. In order to fully and optimally exploit the relationship which exists between the antenna characteristics and the properties of the medium, it is necessary to understand the operation of this antenna-waveguide-medium system as an independent function of the physical configuration alone. This chapter, therefore, is devoted primarily to the establishing of this relationship through the comparison of theoretical and experimental results for various configurations. In order to obtain meaningful correlation between theory and experiment, however, consideration must be given to the basic differences between the theoretical and experimental models. In particular, there exists well known (and extensively studied [17, 18, 20]) discrepancies in antenna input admittance. The problems associated with these discrepancies in relation to the antenna-waveguide system are discussed in Section 5.1.

To effectively understand the operation of the antenna-waveguide system, the correlation between experimental and theoretical results is divided into the case of the single antenna (Section 5.2) and the case of two coupled antennas (Section 5.3). In each case the parameters of operating frequency, physical dimensions, and properties of the medium are varied in turn. Also, for each configuration the antenna current distribution and input admittance are treated separately.

The current distribution is always presented in normalized form. The normalization point is taken as the point of maximum current. When this point is at the input, the normalization is done at the second numerical current solution point; i.e., $\alpha_{i,2}$ from the theory. This was necessary to avoid introducing discrepancies in the nature of the distributions which arise from the delta-gap solution, as described in Section 5.1.1. This also was close to the nearest point at which experimental data was able to be obtained thus facilitating the normalization of measured current. Where possible, the current on the first numerical subsection (i.e., $\alpha_{i,1}$) is also included in the figures. In general, only the delta gap theory with the transverse strip approximation is used and presented. However some theoretical results from the coaxial gap model are also included for comparison.

The variation in the phase of the current is also presented with the relative experimental data being normalized to the same second current point as discussed above. In the case of the cavity configuration, however, the phase of the current is

a constant $\pm 90^\circ$ along the antenna with exception of possible 180° phase reversals occurring at points of current zero crossings.

Therefore, the cavity phase data is given as the constant theoretical value obtained. The experimental phase measurements were found to differ by no more than 3° from constant with the greatest variation occurring at the end of the antenna. This was therefore felt to be primarily caused by experimental errors resulting from variations in the flexible coaxial line leading to the probe as discussed at the end of Chapter 4. Additionally, since the antenna surface impedance introduces losses along the antenna, some variation from the idealized theoretical results is expected. At the point of phase reversal the experimental data showed a somewhat gradual change, but because of the small magnitude of the current at this point (zero), the VVM was unable to measure the relative phase values. The reversal was very rapid, though, and occurred in a distance less than $0.05h$ (the antenna height) for this cavity case.

The measured and calculated admittances for each configuration studied is presented following the current distributions. In the theory the admittance is taken to be the value of the current on the first subsection ($\alpha_{i,1}$) for a one volt excitation. This is different from the actual input point by one-half the subsection length ($0.5 \Delta y$) which is taken generally as $0.025h$ (20 segments). Since the delta gap theory and coaxial gap theories differ most significantly in this value of input admittance, both results are presented where they have been obtained. The experimental data is given in unaltered form from the value obtained by transforming down the driving transmission line from the measuring

terminal plane. Discussion of the use of impedance matching networks is given in Section 5.1.2 but, for reasons to be discussed, no matching is applied to the data.

5.1 Correlation of Theoretical and Experimental Input Admittance.

In the theoretical model of the antenna developed in Chapter 2 the electromagnetic source exciting the antenna is approximated either as an ideal delta function generator or as a coaxial gap supporting a perfect TEM mode field. The experimental system described in Chapter 4, on the other hand, is obviously different than either approximation. This difference is known to result in disagreement between predicted and measured input admittance. Much effort has been spent in trying to account and correct for this difference, especially concerning the infinite susceptance predicted by the delta function generator (e.g., see Section 3.10 of King and Harrison [18] or Duncan [6]) and concerning the development of terminal zone correction networks (e.g., see Chapter 8 of King, Mack and Sandler [20]; Sections II.1 through II.10 of King [17]; or Chang [1]). In the following paragraphs these areas of interest are discussed as they relate to the results obtained in this study.

5.1.1 Infinite Driving Point Susceptance.

The delta function generator model for the antenna source leads to an infinite input susceptance, since the zero width source region implies an infinite capacitance and, hence, an infinite driving point current. As pointed out in Section 3.2 of King and

Harrison [18], this current singularity is logarithmic in nature and extends only for a very short range along the antenna. An approximate order of magnitude value for this effective distance is given as

$$h e^{-1/k_o r_a} \quad (5.1)$$

which for a one-quarter inch diameter antenna at 1.4 ghz gives $(2 \times 10^{-5})h$. King and Harrison go on to point out that the removal of this infinity from the solution is essentially accomplished as long as the antenna current is found as an expansion of continuous functions and terms of extremely high order are not included. Unfortunately, the solution for the current by the numerical methods of Chapter 3 is not obtained from continuous functions and, in fact, the step discontinuity presented by the pulse functions necessarily includes effects of infinitely high order. Therefore, the solutions obtained for the delta function generator model in this report must include the effect of this infinite susceptance. It is very difficult to theoretically predict the contribution of this singularity, but its effect can be seen in the numerical results. An example of this is shown on a much expanded scale in Figure 5.1, where the smooth extrapolation of the calculated current amplitude at points along the antenna obviously does not coincide with the calculated value of the input current at the first point on the antenna. It is interesting to compare this result to the solution obtained with the approximate coaxial gap excitation. In this case there is no infinite input

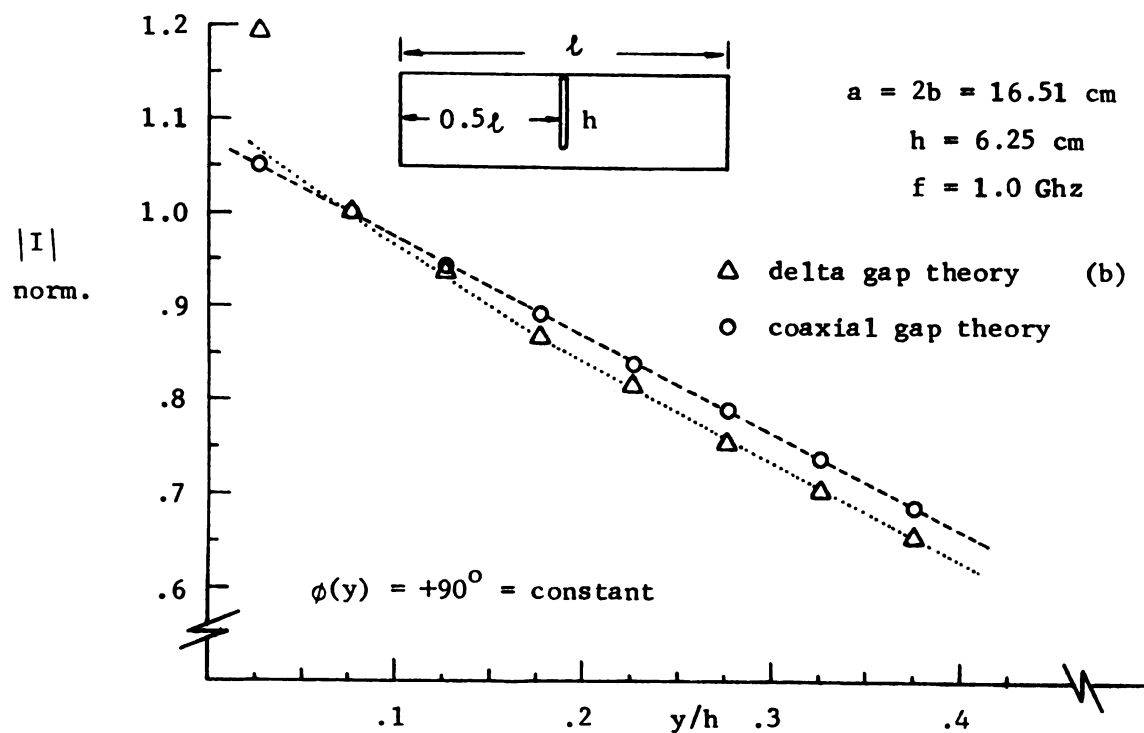
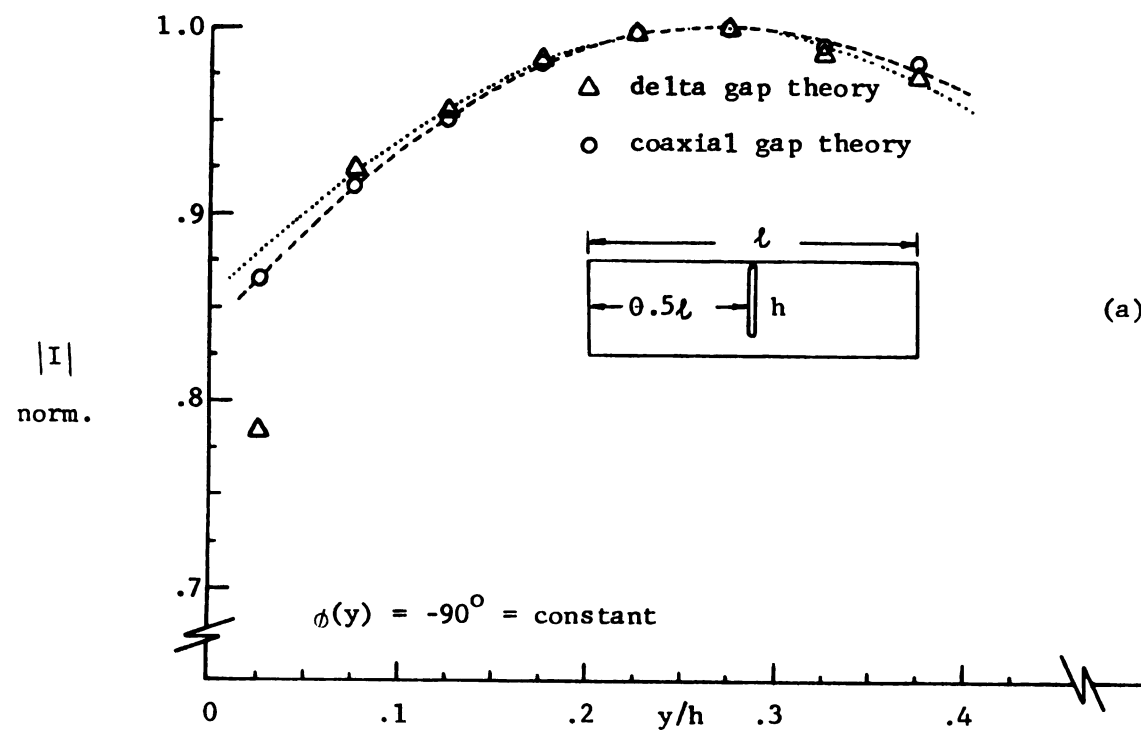


FIGURE 5.1

Effect of Infinite Susceptance on Numerical Solution

susceptance and (see Figure 5.1) the calculated input current agrees well with the continuous extrapolated value. It may be noted that, as expected, the error in input current from this capacitance singularity is in the direction of increased positive susceptance (note the phase of the current referred to the driving voltage in each figure).

Since it is difficult to theoretically predict and correct for this inherent error in driving point susceptance, and since any attempt to compensate for the effect by extrapolation would be imprecise, the numerical results in this chapter for the delta function generator model always include this effect.

5.1.2 Terminal Zone Correction Network.

One technique which may be applied to obtain satisfactory correlation between predicted and measured antenna input admittance involves the determination of a terminal zone correction network. The terminal zone network is a lumped element approximation which attempts to account for the coupling between the transmission line and the combination of the antenna and terminating structure. There has been considerable effort spent in the past to obtain analytical expressions for these terminal zone effects (e.g., see King [17] and Wu [40]) with results which yield sufficiently accurate correction to the observed differences between measured and predicted admittance. As pointed out by King, et. al. (Chapter 8 of

[20]), however, the simplest procedure for determining these networks (once the theory has substantiated their existence) is by a direct comparison of the experimental and theoretical admittances. This reduces the problem to one of network analysis and the solutions are much more readily obtained.

The lumped equivalent terminal zone network is shown in Figure 5.2 in two equivalent representations. The circuit relationships between calculated admittance (impedance), Y_L (Z_L), and observed or apparent admittance (impedance), Y_a (Z_a), are simply:

for the circuit of Figure 5.2(a),

$$Y_a = \frac{1}{Z_L + j\omega L_T} + j\omega C_T \quad (5.2a)$$

and for Figure 5.2(b),

$$Z_a = \frac{1}{Y_L + j\omega C_T} + j\omega L_T \quad (5.2b)$$

For application to an experimental system which has the antenna as the load on the coaxial transmission line, the physical significance attached to the lumped capacitor is basically the effect of terminating the transmission line and the effect of the terminating structure, whereas the lumped inductor includes the effect of the coupling between transmission line and antenna. To determine the appropriate values of L_T and C_T from Equations (5.2) the measured and theoretical values of admittance (in general, complex) for one antenna configuration lead to two easily solved equations from the real and imaginary parts of Equations (5.2a or b). The computed L_T and C_T are then applied to data from similar configurations and are

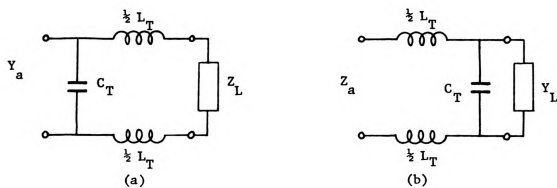


FIGURE 5.2

Lumped Equivalent Terminal Zone Networks

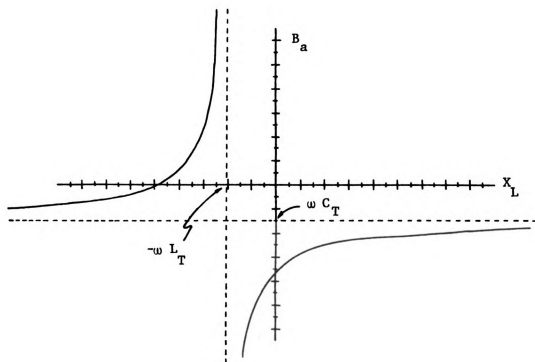


FIGURE 5.3

Functional Behavior of Terminal Zone Correction Network

assumed frequency independent. In general, changing the physical or electrical properties of the configuration is expected to change the values of L_T and C_T [40].

The antenna-waveguide system considered in this report offers some unique variations from the typical antenna driven over an infinite ground plane. In particular, since the corrective network elements are dependent upon the transmission line terminating structure, one expects the network to be different for each waveguide configuration. Furthermore, since the waveguide fields (and hence the transmission line coupling to the waveguide) are highly frequency sensitive, the network is expected to be frequency dependent. For the special case of the rectangular cavity, which is extensively studied here, both of these effects become enhanced because of the existence of resonances. The one physical parameter which is not expected to cause drastic variation in the correction network is the antenna height (although for the cavity or antenna near resonance this could also prove an important influence on coupling effects). With these problems in mind, the task of determining a useful corrective network appears formidable. Nonetheless, a great deal more can be learned about the implication of the terminal zone network through initial attempts here to apply the theory as it now stands.

The necessity of investigating a terminal zone network is based on discrepancies on the order of a factor of two between observed and predicted antenna input admittance. Although seemingly large, similar differences have been reported (see Section 3.11 of King and Harrison [18]) for which a corrective network

provided satisfactory agreement. It is the purpose of this section to study the possibility of the network providing similar corrections and to give some representative results. Since a great deal of insight into a problem is gained by the proper approach, some time will be spent discussing the methods used to obtain the corrective networks. Because of the complicated dependence of the network on the configuration studied in each case, the results presented in the later sections of this chapter are not corrected for terminal zone effects in order to avoid obfuscating the experimental data.

To apply the Equations (5.2) for the case of the antenna in a rectangular cavity,* it is noted that the antenna input admittance is a pure susceptance, since there can be no real power radiated by the antenna for the assumed ideal case. This means that two sets of values for observed and calculated admittance must be used to determine L_T and C_T . Based on the arguments of the preceding paragraphs the best parameter to use as an independent variable in the sets of values is the antenna height. Using the network of Figure 5.1(a) as an example, Equation (5.2a) becomes

$$B_a(h) = \frac{-1}{X_L(h) + X_l} + B_c \quad (5.3a)$$

where

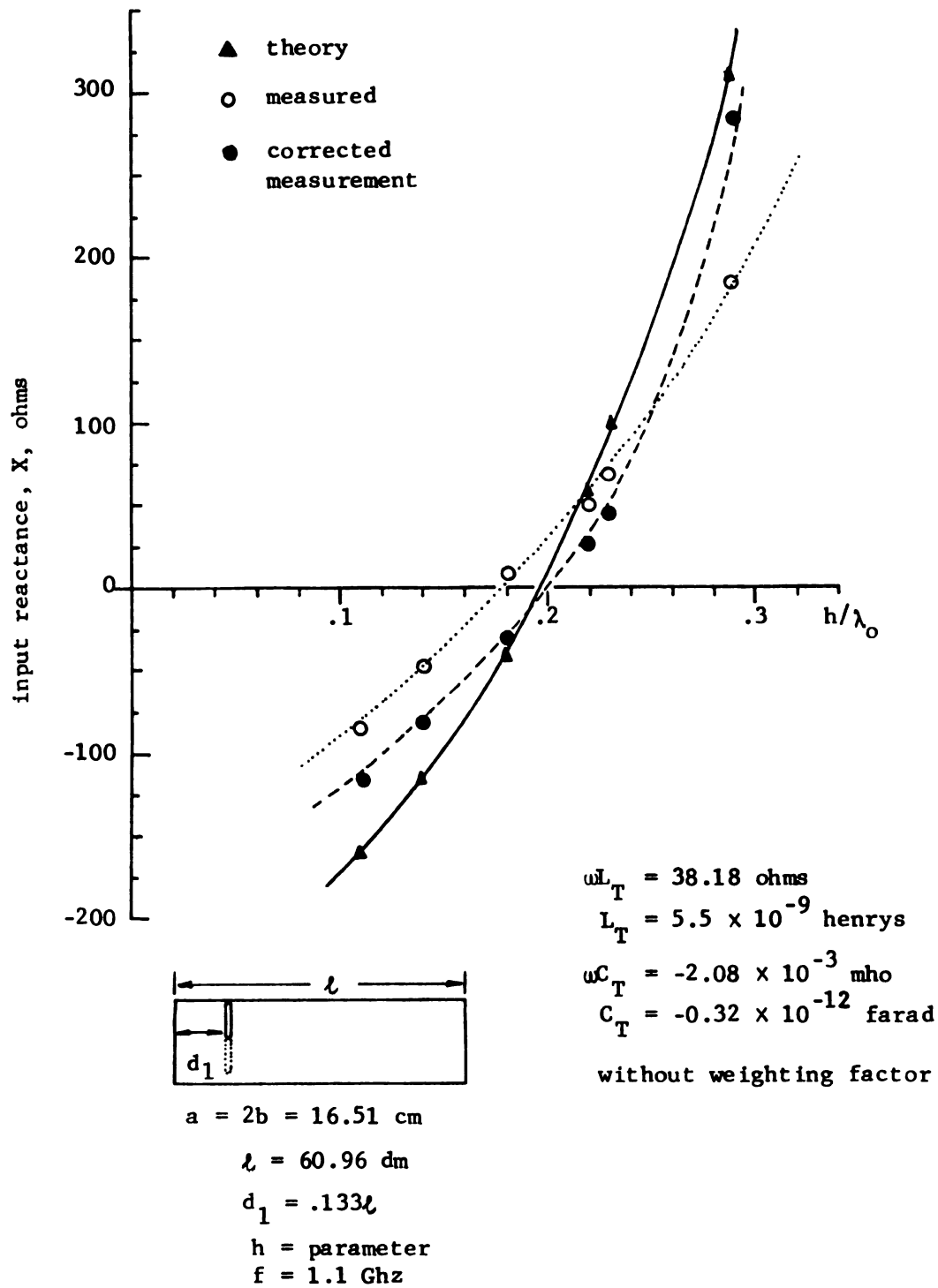
$$\begin{aligned} X_l &= \omega L_T \\ B_c &= \omega C_T \end{aligned} \quad (5.3b)$$

* The author is indebted to Dr. J.A. Resh, Department of Electrical Engineering and Systems Science, Michigan State University, for his helpful discussions concerning this problem.

If the apparent susceptance is viewed as a function of the theoretical reactance, then Equation (5.3a) results in the curve shown in Figure 5.3. In view of this, by graphing a set of data as B_a versus X_L (for fixed frequency) one can obtain approximate values for the terminal zone inductor and capacitor. Solving for X_L (L_T) and B_c (C_T) from Equation (5.3a) is seen to be equivalent to selecting the hyperbola which is a best fit to the data given and any curve fitting criterion can be used to optimize the results.

As an example of this process, the results from three combinations of physical configuration and frequency are shown in Figure 5.4. The inductor and capacitor are determined by a least square minimization method via an interactive routine on a Hewlett Packard model 9821A calculator. Because of the nature of the data the problem was very nearly ill-conditioned, and it was found that a weighted squared error criterion was often desirable. A weighting factor equal to the inverse square of the slope of the $B_a(h)$ versus $X_L(h)$ curve (i.e., $(dB_a/dX_L)^{-2}$) was therefore used to reduce the effects of the critical asymptotic points. As is seen in Figure (5.4) the corrective network reduced discrepancies from the order of a factor of two to differences of 30% or less.

Better agreement was not attained in every case primarily because of the possibility of experimental errors and the fact that the antenna height could be a factor in the coupling and perturbation of the waveguide modes and the transmission line fields. Indeed, as shown in Figure 5.4(c), when the antenna input admittance does not pass through a zero or an infinity as the height is varied, the corrective network is seen to give highly satisfactory results.



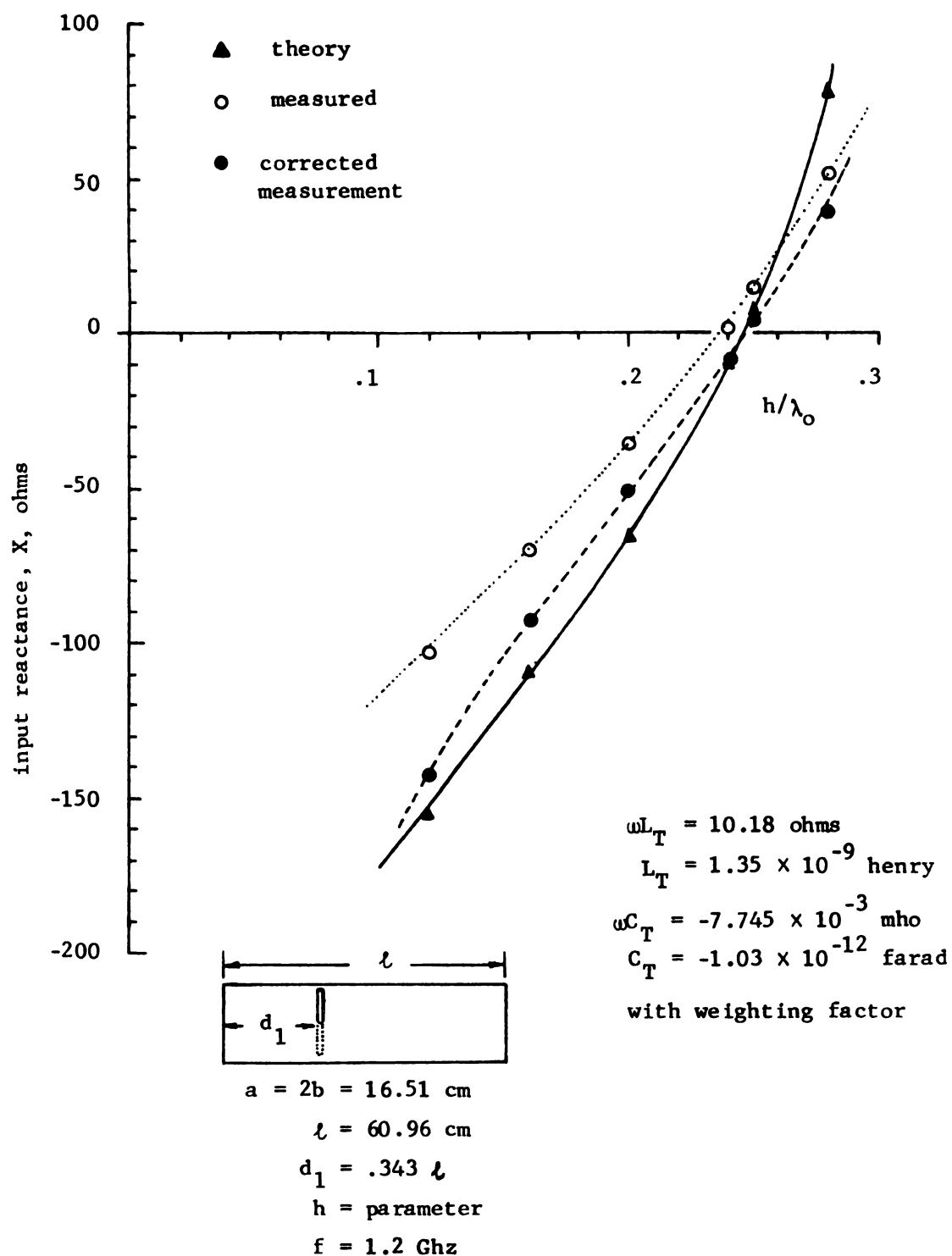
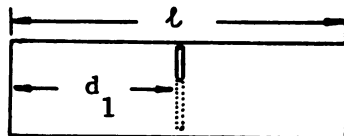
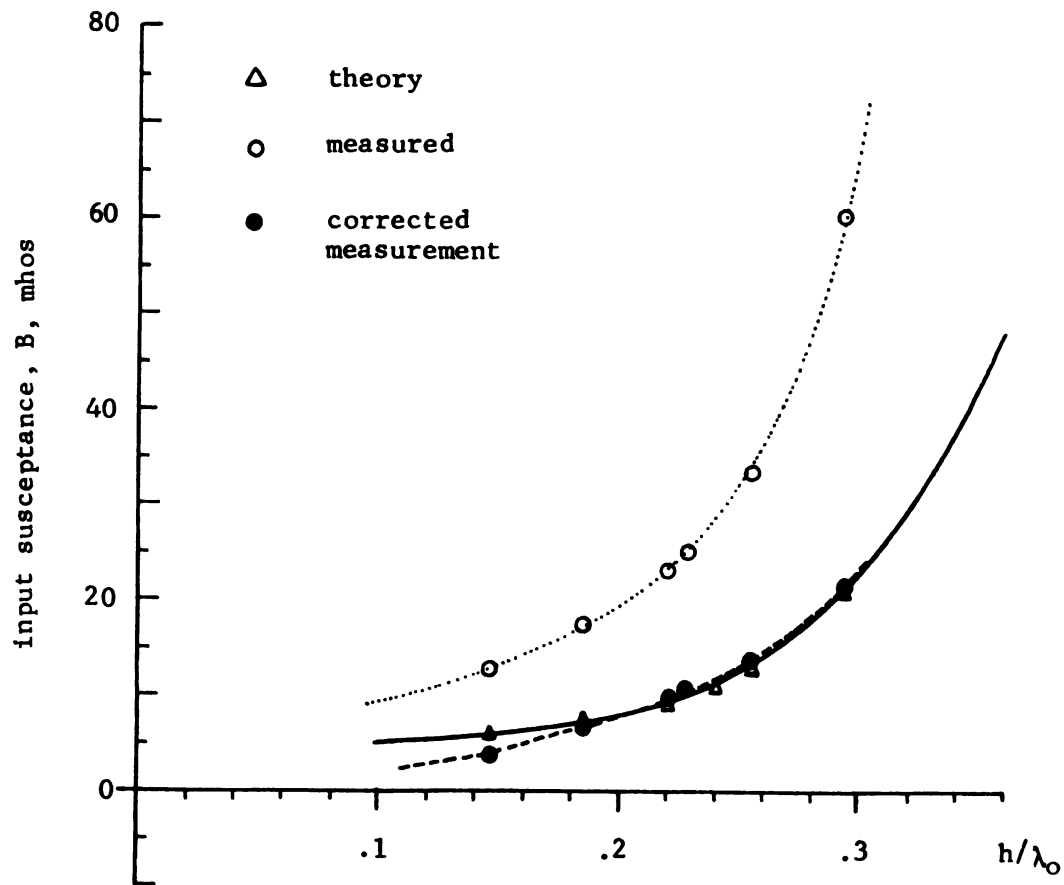


FIGURE 5.4(b)

Correction of Measured Data via Terminal Zone Network



$$a = 22.6 \text{ cm}$$

$$b = 8.255 \text{ cm}$$

$$l = 60.96 \text{ cm}$$

$$d_1 = 0.5l$$

h = parameter

$f = 1.1 \text{ GHz}$

$$\omega L_T = 29.28 \text{ ohm}$$

$$L_T = 4.25 \times 10^{-9} \text{ henry}$$

$$\omega C_T = -0.168 \times 10^{-3} \text{ mho}$$

$$C_T = -0.0244 \times 10^{-12} \text{ farad}$$

without weighting factor

FIGURE 5.4(c)

Correction of Measured Data via Terminal Zone Network

A further consideration as a possible source of error is the variation in accuracy of the numerical results. Since the finite series evaluation of the matrix elements is a function of the antenna height (see criteria for truncation in Equation (3.43)), the accuracy of the numerical results will be somewhat dependent on height. Also, for a fixed number of segments on the antenna ($K = 20$), the distance to the first matching point is a function of height. This will result in varying degrees of divergence of the input current (input admittance) resulting from the infinite delta-gap susceptance as discussed in Section 5.1.1. Each of these effects indicates that the problem of admittance correlation is by no means a simple one for this antenna-waveguide-medium system and that a correction network is expected to yield a variable range of agreement.

Another observation from the results for the correction network shown in Figure 5.4 is the variation in values obtained for the inductor and capacitor. For comparison, it may be noted that for the same antenna above an infinite half plane the corrective network for a coaxial transmission line (see Section II.38 of King [17]) consists only of a capacitor which, for the dimension used in this problem, has a value of -0.95×10^{-13} farads. This variation clearly indicates the dependence of the corrective network on the physical and electrical configuration. As the antenna is placed in different locations and as the frequency is changed, the electric and magnetic fields which will interact with the coaxial transmission line may be changed drastically. This will result in varying degrees of electric and magnetic perturbation in the terminal

zone and consequently require variation in the corrective network to compensate. More insight into the complicated nature of the variation in coupling which can occur as the configuration is changed can be best obtained from the collective results presented in the following sections.

5.2 Single Antenna.

By first studying the case of the single antenna, a great deal of insight can be gained about the operation of the antenna in the waveguide. A common configuration of an antenna in a waveguide is the coaxial to rectangular waveguide coupler - the waveguide is shorted at one arm and the other is assumed matched to a guided system. This configuration is studied first with results compared to the usual assumptions made in the classical analysis of this waveguide system. In the cavity configuration, the current distribution and input admittance are presented first as a function of frequency from below the first cavity resonance to about 2.0 Ghz. Correlation is made between the nature of the current and admittance, and the resonant frequencies. The results are then viewed as a function of location where the effects of coupling to the cavity modes are observed. And, finally, the antenna characteristics are presented as a function of antenna height which is seen to be the least significant factor of the three in determining current distribution and input admittance. In the final section the results of applying the theory to dissipative media filling the waveguide or cavity are compared both to pertinent data available in the literature and to experiments performed here.

5.2.1 Waveguide with Matched Termination.

The coaxial to waveguide coupler has been studied in many classical texts (see Collin [2, 3] and Lewin [24]) with the primary purpose of determining the input impedance to the antenna or probe. The treatment is basically similar in approach to that used in this report except the current distribution on the antenna is taken to have an assumed form:

$$I(y) = I_0 \sin k_0(h - y) \quad 0 \leq y \leq h \quad (5.4)$$

that is, the approximate distribution expected on an antenna in unbounded space. The resulting input impedance is assumed to be fairly accurate because of the stationary nature of the impedance formulation and the form of the approximate impedance gives a useful starting point for design considerations. By using the assumed current of Equation 5.4 the input resistance to the antenna is found to be

$$R_{in} = \frac{2 Z_{1,0}^h}{ab k_0^2} \sin^2(\beta_{1,0} d_1) \tan^2(k_0 h/2) \quad (5.5)$$

when $TE_{1,0}$ mode propagation exists, where k_0 is the free space propagation constant.

This configuration of the semi-infinite waveguide has been studied with the theory developed in this report. Experimental measurements were made by filling one end of the waveguide section described in Chapter 4 with microwave absorber (visible in the right background of Figure 4.7), since a matched L-band load was not available. The other end of the waveguide was covered with the

brass shorting plate and the antenna was located at approximately one-quarter of a guided wavelength from this short for each operating frequency. A test was run to determine the degree of matching provided by the microwave absorber by using a second short antenna ($h = \frac{1}{2}$ inch, $r_a = 1/8$ inch) to probe the field between the driven antenna and the absorber. These measurements indicated an approximate standing wave ratio of

$$\text{VSWR} \approx 1.15 \text{ at } 1.3 \text{ GHz}.$$

Since part of this mismatch could possibly be attributed to the perturbation caused by the probing antenna, this result indicates that a good match was obtained.

Current Distributions

In Figure 5.5 and 5.6 the theoretical and experimental current distributions for different antenna heights at 1.05 GHz and 1.3 GHz are compared. The distribution of the current is seen to show excellent agreement. The discrepancies in phase at 1.05 GHz are most probably caused by decreased matching by the absorber, since this frequency is relatively near the cutoff frequency for the waveguide ($f_c = 0.909$ GHz) thus causing a long guided wavelength ($\lambda_g = 57.1$ cm). Also included in these figures is the shifted sinusoidal distribution of Equation (5.4). This distribution is seen to be fairly representative of the character of the current, but of course assumes that the current is everywhere in-phase with the driving voltage. The theoretical and experimental results, on the other hand, indicate that the in-phase and quadrature components of current are comparable.

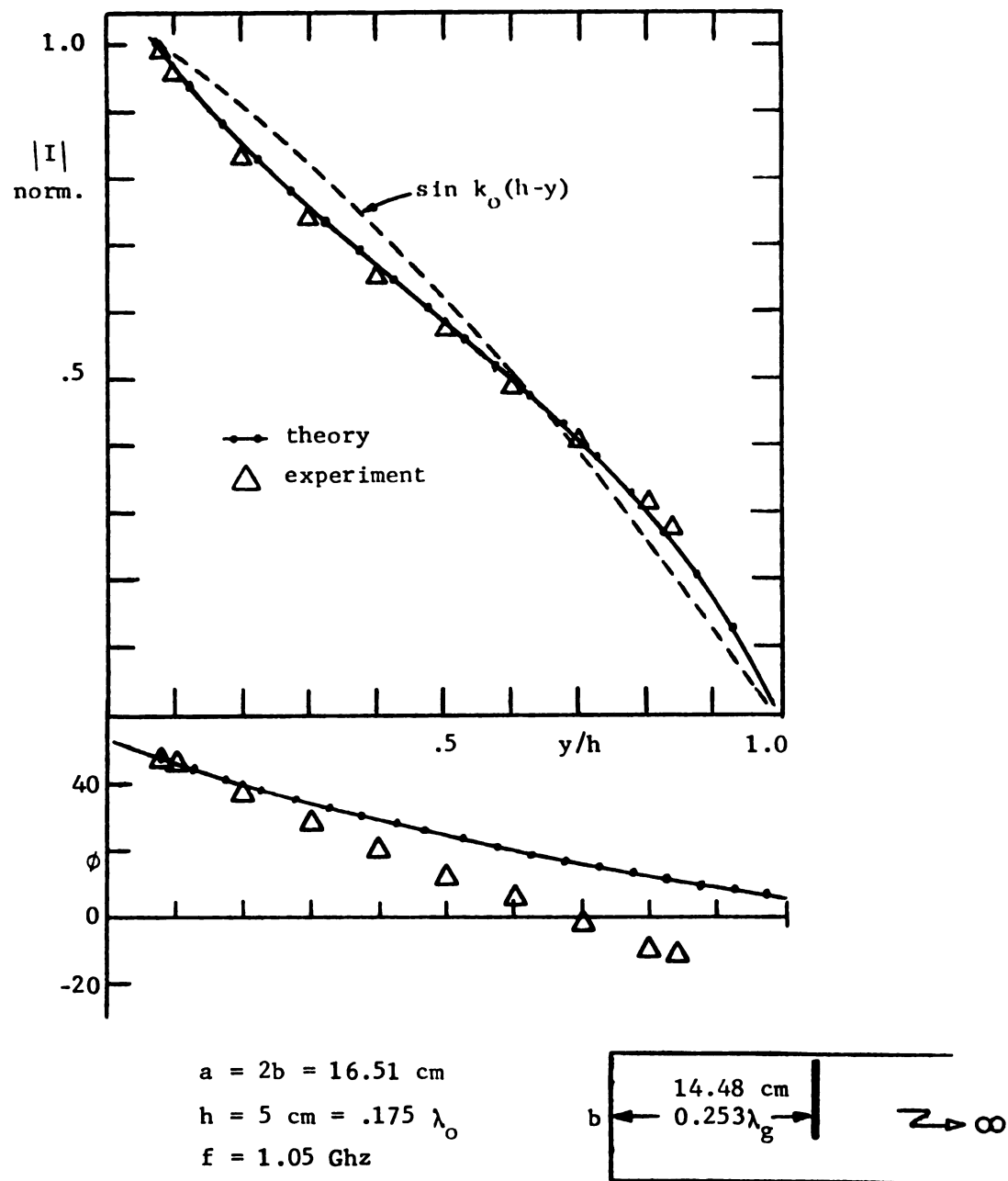


FIGURE 5.5(a)

Current Distribution for Antenna in Semi-Infinite Waveguide
 ($h = 5 \text{ cm}$, $f = 1.05 \text{ GHz}$)

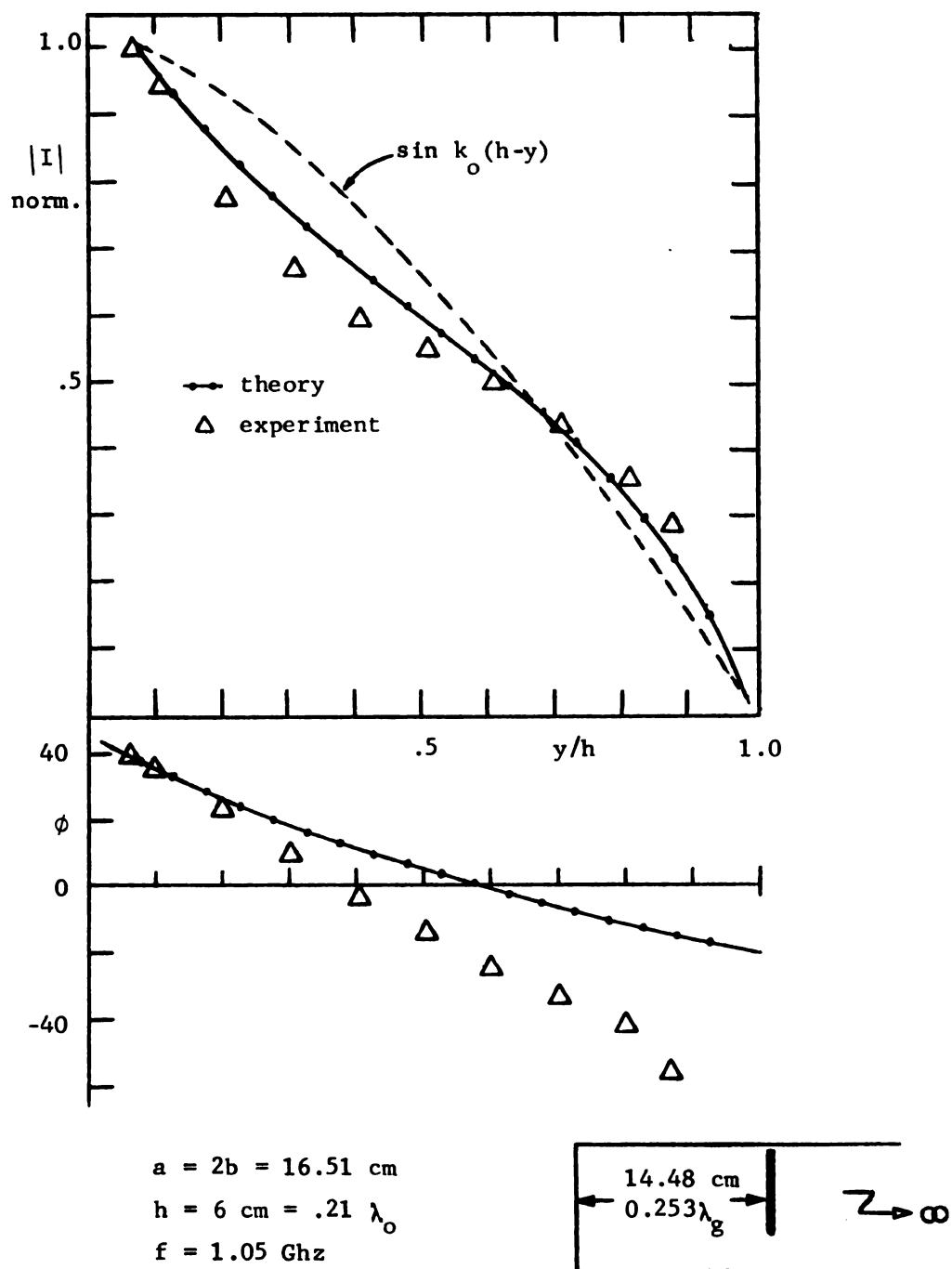


FIGURE 5.5(b)

Current Distribution for Antenna in Semi-Infinite Waveguide

(h = 6 cm, f = 1.05 GHz)

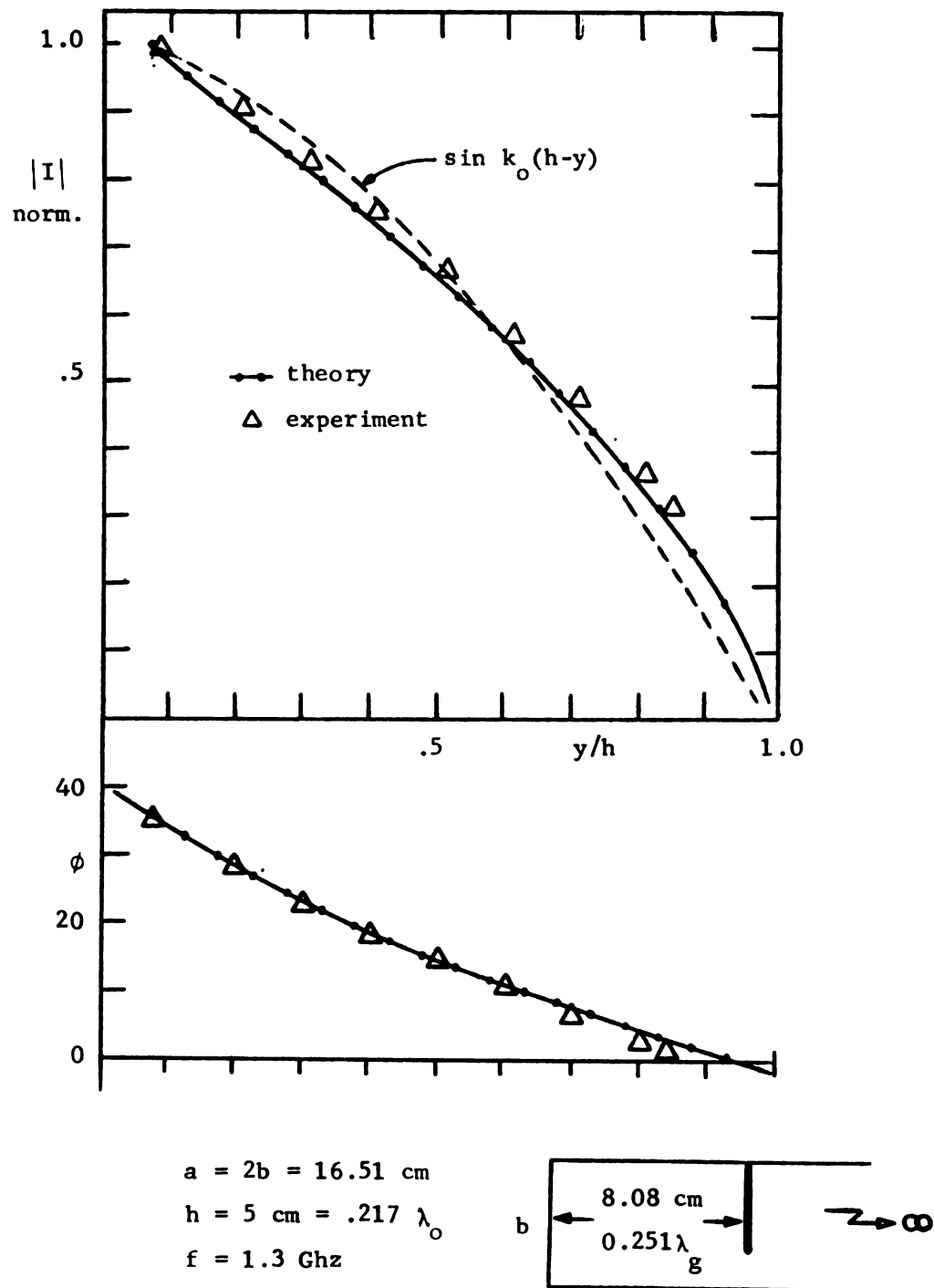


FIGURE 5.6(a)

Current Distribution for Antenna in Semi-Infinite Waveguide

(h = 5 cm, f = 1.3 GHz)

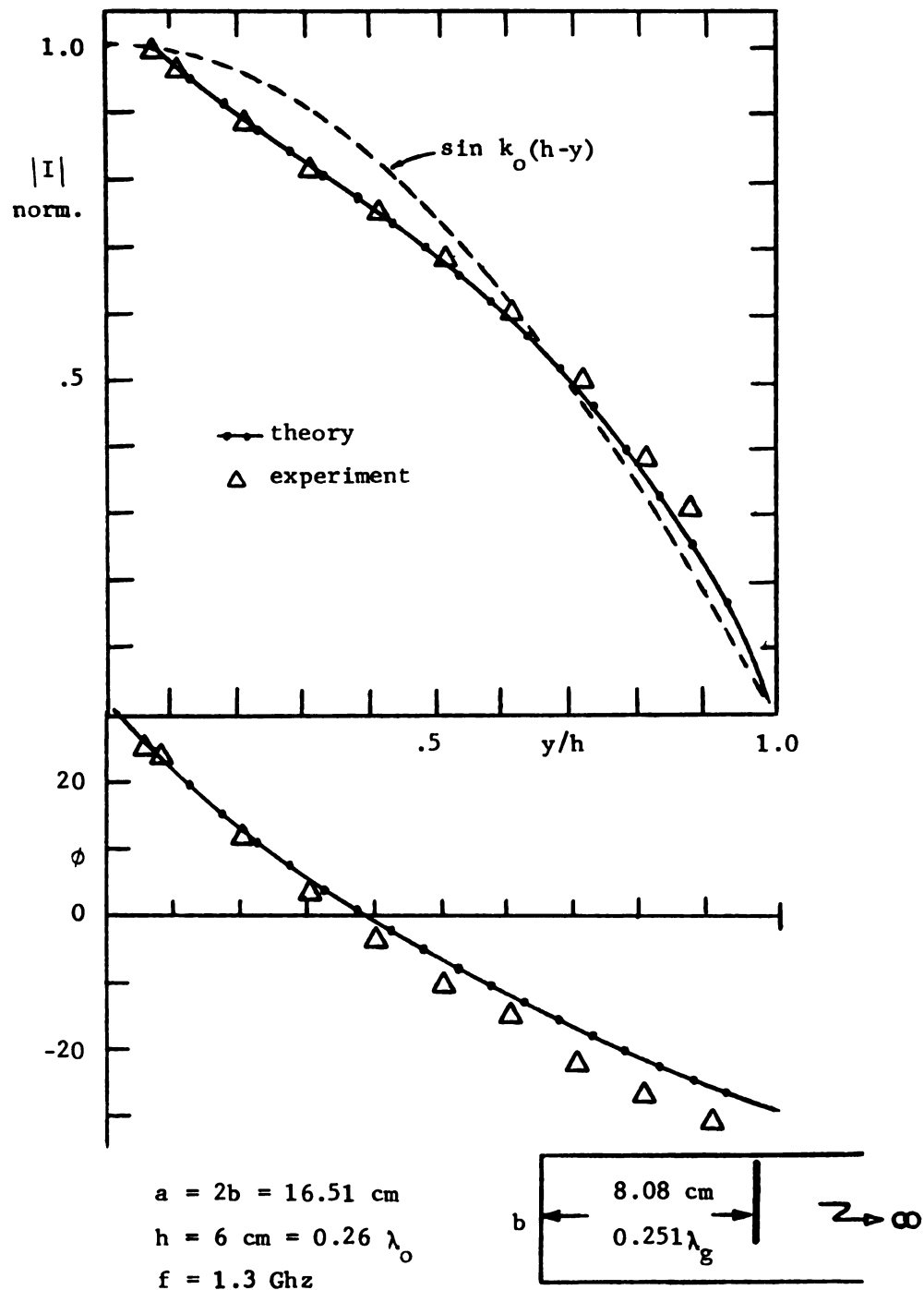


FIGURE 5.6(b)

Current Distribution for Antenna in Semi Infinite Waveguide

(h = 6 cm, f = 1.3 GHz)

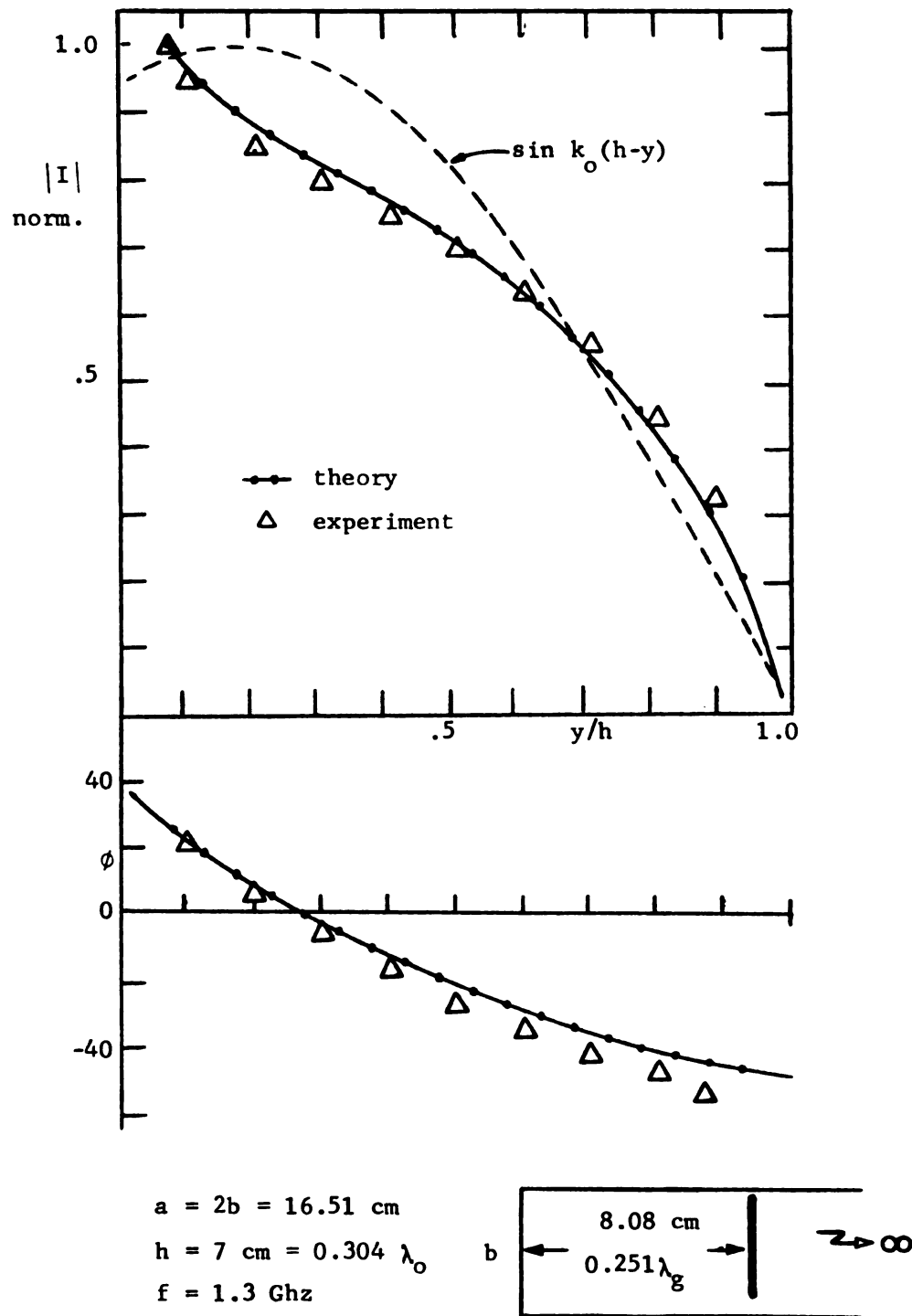


FIGURE 5.6(c)

Current Distribution for Antenna in Semi Infinite Waveguide

(h = 7 cm, f = 1.3 GHz)

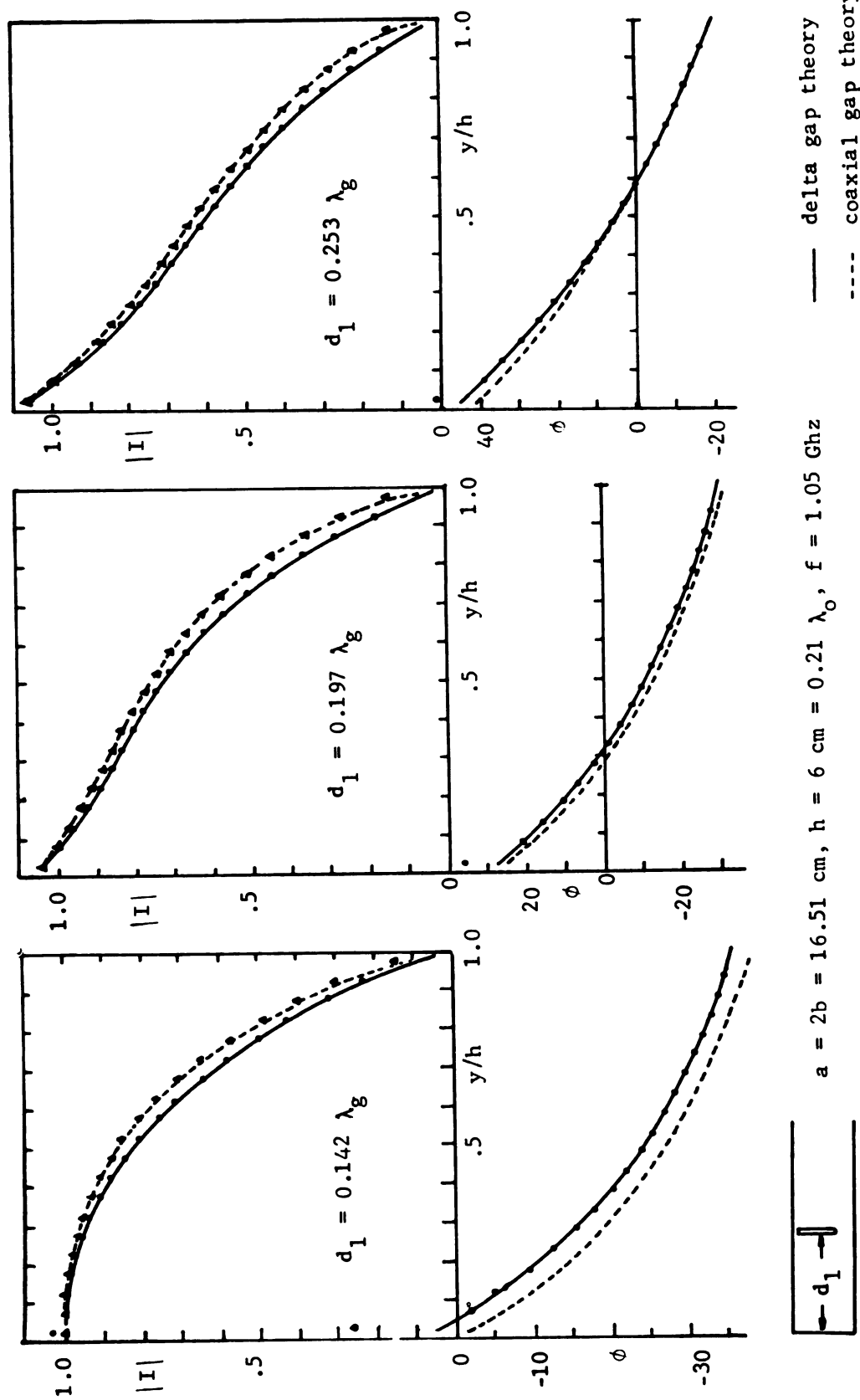


FIGURE 5.7

Theoretical Current Distribution for Semi-Infinite Waveguide Configuration with
Antenna Location as the Parameter

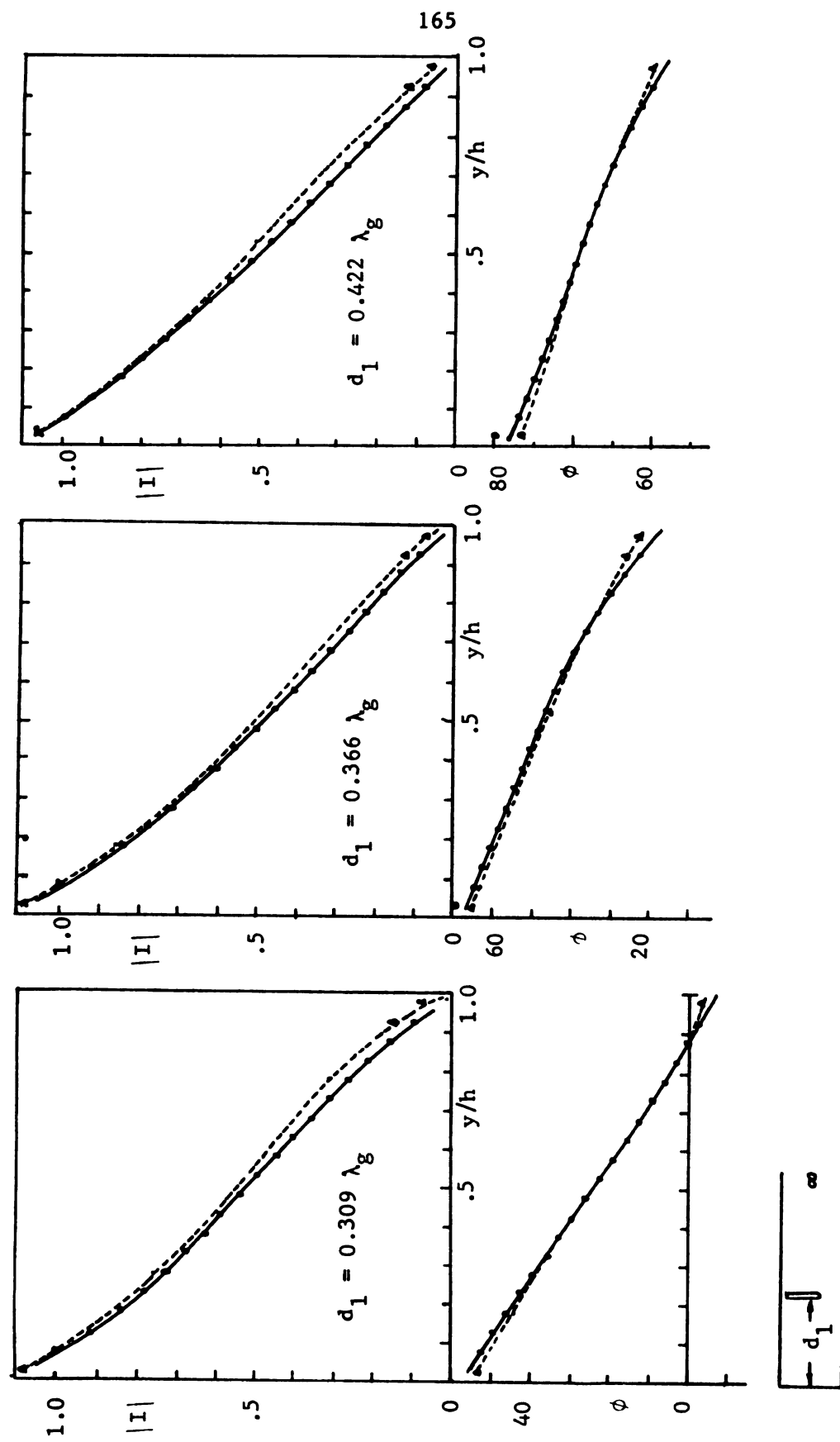


FIGURE 5.7 (cont'd)

— delta gap theory, ---- coaxial gap theory

If the distribution of the in-phase and quadrature components are derived from these figures, one finds that the in-phase component behaves very nearly the same as the shifted sine function while the quadrature component decays as an inverted sinusoid from the driving point to the end of the antenna. This is not shown here, since it will be most evident in the results for the cavity configuration in the following sections.

In Figure 5.7 the effect of varying the antenna location is shown with comparison between the delta-gap and the coaxial gap theory. (No experiments were run for this case.) The current is seen to change from having a large in-phase component (small phase angle) at locations near the short to having a large quadrature component (large phase angle) as the location approaches one-half a guided wavelength from the short. Since a large in-phase component implies larger radiated than stored energy, one would expect the locations closer than $\lambda_g/4$ to be the better for consideration in a coaxial to waveguide coupler. In fact the result at $d_1 = 0.142 \lambda_g$ having nearly zero phase angle at the input would provide the best conditions for radiating energy into the waveguide.

Input Admittance

The antenna input admittance for the cases considered above are given in Table 5.1 through Table 5.3. Also included is the resistance value computed from Equation (5.5). There is some difficulty in comparing resistance values, since errors in the imaginary part of the admittance from the theory (e.g. the delta gap infinite susceptance noted in an earlier section) become combined with the real

part of the impedance upon inversion of Y . With this in mind, the theoretical input resistance compared with that computed from the approximate formula of Equation (5.5) show fairly good agreement, at least in overall behavior as a function of antenna height or location.

Agreement at 1.05 Ghz in Table 5.1 between theoretical and experimental admittance is very good in view of the difficulties noted in Section 5.1. However, there is reason to suspect that at this frequency the experimental "matched" boundary is failing and thus this agreement may be anomolous.

In Table 5.2 at 1.3 Ghz, where the experimental set-up is expected to be fairly accurate, there is seen to be acceptable agreement in the real part of the admittance between the coaxial gap theory and the experiment. The delta gap theory, on the other hand, shows lack of agreement in both parts of the admittance. This indicates that the coaxial gap provides a better model than the delta gap for this semi-infinite waveguide configuration. However, since both theories give comparable susceptance values, one sees that neither model has adequately included the effect of stored energy near the antenna as compared with the experiment. This is the effect for which a corrective network of the type described in Section 5.1.2 is classically used as compensation for the approximations. Because of the scarcity of data, no network was developed for this case.

The results of varying the antenna location in Table 5.3 follow much the same agreement as noted above. However, the agreement in susceptance between the two theories becomes worse as the antenna is moved away from the point of maximum $\hat{y}$ -component of

electric field ($d_1 = 0.25 \lambda_g$). Again this effect results from the different approximations on the antenna driving field and the way each approximation ignores effects of stored energy.

No attempt is made to discuss the comparison in impedance values given in the tables because of the effect of mixing of information in the real and imaginary parts. That is, since the fundamental theoretical solution is the current for an assumed fixed applied voltage, the admittance results are the best means of correlating theoretical phenomenon to the assumptions made.

TABLE 5.1

Input Admittance as a Function of Antenna Height in a
Semi-Infinite Waveguide

($d_1 = .253 \lambda_g$, 1.05 GHz)



Y, millimhos

h	5 cm = $.175\lambda_0$	6 cm = $.21\lambda_0$	7 cm = $.246\lambda_0$
Theoretical (Delta gap model)	$2.6 + j3.94$	$2.44 + j3.02$	$2.0 + j2.56$
Theoretical (Coaxial gap model)	$3.92 + j4.25$	$3.73 + j3.06$	$3.03 + j2.56$
Experimental	$4.71 + j4.66$	$3.93 + j4.26$	$3 + j4$

Input Impedance , Z_{in} , ohms

h	5 cm	6 cm	7 cm
Theoretical (Delta gap model)	$117 - j177$	$162 - j200$	$189 - j242$
Theoretical (Coaxial gap model)	$117 - j127$	$160 - j131$	$192 - j162$
Experimental	$107 - j104$	$117 - j127$	$119 - j160$
Approximate R_{in} Eqn. (5.5)	85.6	13.7	214.2

TABLE 5.2

Input Admittance as a Function of Antenna Height in a
Semi-Infinite Waveguide
($d_1 = 0.251\lambda_g$, 1.3 GHz)



Y, millimhos

h	5 cm = $.217\lambda_o$	6 cm = $.26\lambda_o$	7 cm = $.304\lambda_o$
Theoretical (Delta gap model)	$4.69 + j4.73$	$4.0 + j3.11$	$3.06 + j2.61$
Theoretical (Coaxial gap model)	$7.33 + j4.75$	$6.05 + j2.47$	$4.50 + j2.16$
Experimental	$8.9 + j8.43$	$7.7 + j6.05$	$7.17 + j5.1$

Input Impedance , Z_{in} , ohms

h	5 cm = $.217\lambda_o$	6 cm = $.26\lambda_o$	7 cm = $.304\lambda_o$
Theoretical (Delta gap model)	$105.5 - j106.4$	$155.6 - j121$	$189.2 - j161$
Theoretical (Coaxial gap model)	$96.1 - j62.2$	$142 - j57.7$	$180 - j86.7$
Experimental	$59.3 - j56.2$	$80 - j63$	$92.5 - j65.5$
Approximate R_{in} Eqn (5.5)	68.42	118	206.6

TABLE 5.3

Input Admittance as a Function of Antenna Location in a
Semi-Infinite Waveguide
($h = 6$ cm, 1.05 Ghz)

$$\begin{array}{l}
 a = 2b = 16.51 \text{ cm} \\
 d_1 = \text{parameter}
 \end{array}
 \qquad
 \begin{array}{l}
 h = 6 \text{ cm} \\
 f = 1.05 \text{ Ghz}
 \end{array}
 \qquad
 \begin{array}{c}
 \boxed{
 \begin{array}{c}
 \text{---} d_1 \text{---} \\
 \text{---} h \text{---}
 \end{array}
 }
 \qquad
 \infty
 \end{array}$$

d_1	Y_{in} , millimhos		Z_{in} , ohms		Approximate R_{in} Eqn. (5.5)
	Delta gap Theory	Coaxial gap Theory	Delta gap Theory	Coaxial gap Theory	
.142 λ_g	4.05 + j0.95	5.78 - j0.115	234 - j54.9	172 + j3.42	83
.199 λ_g	3.0 + j2.21	4.47 + j1.88	216 - j160	190 - j80	123
.253 λ_g	2.44 + j3.02	3.73 + j3.06	161 - j200	155 - j132	137
.311 λ_g	2.01 + j3.72	3.14 + j4.31	118 - j207.8	110.6 - j151	118.6
.368 λ_g	1.59 + j4.51	2.55 + j5.60	69.4 - j197.2	67.6 - j148	76.1
.424 λ_g	1.06 + j5.69	1.77 + j7.56	31.7 - j169.8	29.5 - j125	30

5.2.2. Air Filled Cavity; Current Distribution

The antenna-waveguide-media system in the cavity configuration has been discussed previously as the most attractive choice for application to probing of the medium. It is also an interesting configuration theoretically because of the inter-relationship and superposition of the two resonant systems: the cavity and the antenna. Although use of an antenna to excite a cavity is common, very little has been done to determine the characteristics of the antenna in this configuration. The theoretical and numerical solution of this problem discussed in the previous chapter accurately predicts the antenna current distribution with excellent experimental agreement as will be seen in the results which follow. The complete understanding of the characteristics of this system is gained when the nature of the input admittance is considered. This is done in Section 5.2.4.

The case considered is a cavity of fixed length ($l = 24$ inches), fixed height ($b = 3.25$ inches), and two widths ($a = 6.5$ inches, 8.5 inches). The antenna is centered on the broad wall (at $x = a/2$) and is moveable axially (z -direction) in discrete steps. The antenna height is also a parameter. The following results are presented first as a function of frequency, then as a function of antenna location, and finally as a function of antenna height, with the other parameters fixed in each case.

Current Distribution with Frequency as the Parameter

The dominant factor in determining the antenna current distribution is the resonant nature of the cavity. As the frequency is varied, the existence of the cavity resonances results in a pattern

of current distributions which is dependent upon the proximity of the operating frequency to the antenna-cavity resonances. For the ideal empty cavity considered here with the narrowest width ($a = 16.51$ cm) the first several resonances up to about 2.0 Ghz are given in Table 5.4. The location of the antenna in the cavity determines which resonances are excited and thus is also important in describing the nature of the current distribution. When the antenna is centered longitudinally (equidistant from the waveguide short circuits at $d_1 = 0.5\lambda$), only the resonances having odd numbered longitudinal modes (" λ " = 1,3,5,...) are excited. The variation in current distribution in the vicinity of one of these resonances ($f_{1,0,3} = 1.171$ Ghz) is shown in Figure 5.8 for a fixed antenna height. As the frequency is varied through this resonance the current distribution changes through a complete "cycle." Starting from essentially a triangular distribution well below the resonance (and well above the next lower resonance) the distribution becomes similar to a quarter-wave cosine distribution as the resonance is approached. At the resonance the distribution is like a half-wave sinusoid and as the frequency goes above resonance, this distribution shifts outward along the antenna until it again appears essentially triangular. This "cycle" is repeated through each cavity resonance which is excited.

Since resonance is defined in terms of admittance or impedance at the input terminal plane (e.g., when the susceptance or reactance is zero), it is not evident from the current distribution alone when the resonant condition is achieved. This will be examined in detail in Section 5.2.4, but since the current is purely in phase --

quadrature with the driving voltage, resonance may be expected to be the point at which the current is zero at the driving point. Thus resonance is seen to be just below 1.15 Ghz for the case shown in Figure 5.8. This is, of course, different than the ideal resonance at 1.171 Ghz, since the antenna perturbs the cavity and is a resonant system itself.

For the cavity in a wider configuration ($a = 22.6$ cm) the ideal empty cavity resonances are given in Table 5.5. The current distributions at frequencies near the $f_{1,0,3}$ resonance are shown in Figure 5.9. Again the same repetitive pattern of current distributions is seen to exist as the frequency is changed from below to above the resonance which is excited, and again the resonant condition occurs below the ideal empty cavity resonance.

In both of the above cases, the experimental current distributions at frequencies where data was taken is seen to compare extremely well with the theoretical results. The slight differences which are shown near the resonance points are readily accepted as the failure of the theoretical model to precisely describe the perturbation of the ideal cavity resonances by the presences of the antenna. Indeed, the model assumes a strip approximation for the antenna which will obviously perturb the cavity differently than the physical cylindrical antenna. Because of the rapidly changing nature of the current over a relatively narrow frequency range the agreement which is shown is very satisfactory.

The current distributions for the entire frequency range between 0.6 and 2.0 Ghz is shown in Figures 5.10 and 5.11 for the above two cases of cavity width ($a = 16.51$ cm and $a = 22.6$ cm,

respectively). The agreement between theory and experiment is again excellent and the nature of the distributions is seen to be dependent upon the relative proximity of an excited cavity resonance.

One discrepancy between theory and experiment occurred at 1.2 Ghz in the wide ($a = 22.6$ cm) configuration shown in Figure 5.11. The measured current distribution did not agree with the theoretical result at 1.2 Ghz but is shown to agree with the theory at 2.4 Ghz. This is a consequence of the experimental set-up alone. When making experimental measurements, the sliding short circuit above the tee in the antenna driving system (see Figure 4.1 and 4.7) was adjusted to give maximum signal from the current measuring probe near the antenna input plane. It was possible, however, to erroneously tune to the first harmonic of the source frequency if the full range of the sliding short was not investigated. This in turn de-tuned the fundamental frequency with the result being primarily dependent on the first harmonic frequency. It was possible to detect this error before making measurements by noting the phase angle between voltage and current probes at the impedance sensing block. Since the block was calibrated at 1.2 Ghz and a phase angle reference was obtained, the measured impedance should be very nearly $\pm 90^\circ$ with respect to the reference. Since only in the most unlikely of circumstances is this phase angle reference the same for both the fundamental and the first harmonic, the measured impedance phase angle for the first harmonic is significantly different from the expected $\pm 90^\circ$. Although this was overlooked in the single case of 1.2 Ghz in Figure 5.11, care was taken to insure tuning to the fundamental in all other cases.

Another indication of experimental difficulty regarding tuning to the first harmonic is the deviation of the measured current distribution from the repetitive pattern shown in Figures 5.8 and 5.9. That is, the current distribution at frequencies above the resonance determined by mode index " n " = 2 has an inflection point instead of the straight line or triangular characteristic evident at the lower frequencies. In the case of Figure 5.11 at 1.2 Ghz this inflection point was evident and thus experimental difficulty was suspected.

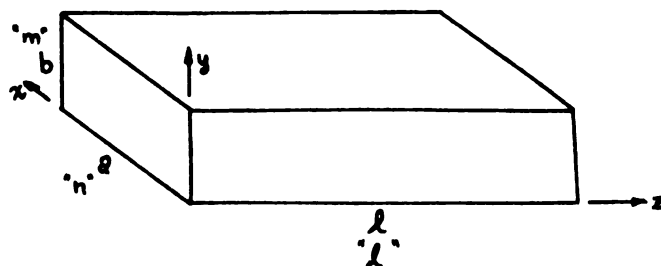
In Figure 5.12 the current distributions are shown as a function of frequency again for the case of the narrow cavity ($a = 16.51$ cm). However, the antenna is not centered longitudinally and thus couples to (or excites) all cavity resonance given in Table 5.4. In this case the repetitive pattern of the current distribution occurs twice as often as the frequency is changed from below the first resonance to 2.2 Ghz. Agreement between theory and experiment is again very good with acceptable differences accountable by the approximations of the theoretical model as discussed previously.

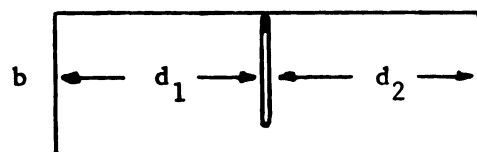
TABLE 5.4

Resonances of Rectangular Cavity

(a = 16.51 cm, b = 8.255 cm, l = 60.96 cm)

n	m	l	f_{res} , Ghz
1	0	1	.941
1	0	2	1.033
1	0	3	1.171
1	0	4	1.339
1	0	5	1.529
1	0	6	1.734
1	0	7	1.947
1	1	1	2.046
1	0	8	2.168




 l

$$a = 2b = 16.51 \text{ cm}$$

$$l = 60.96 \text{ cm}$$

$$d_1 = d_2 = 0.5l$$

$$h = 6.25 \text{ cm}$$

$$(f_{\text{res}})_{1,0,3} = 1.171 \text{ GHz}$$

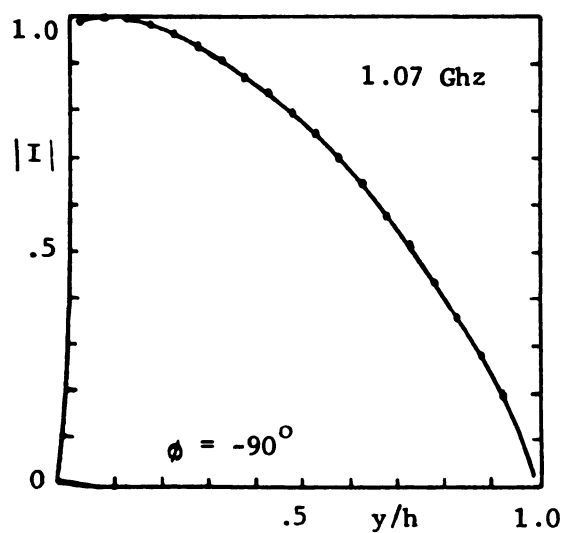
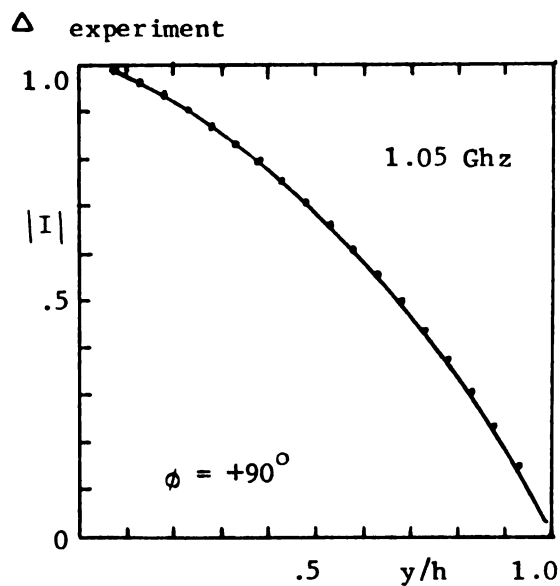
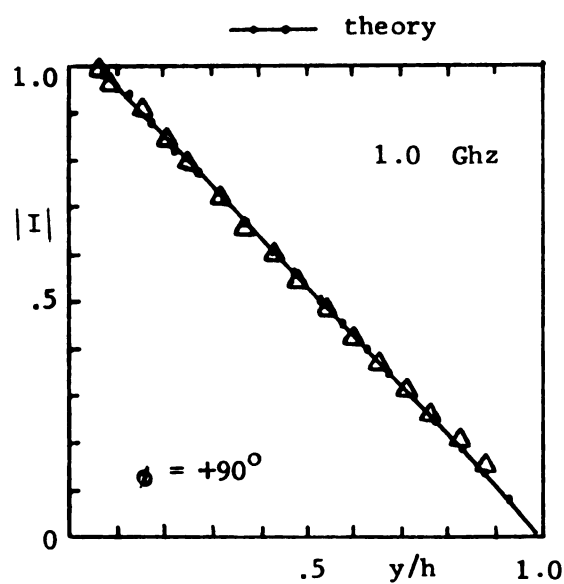


FIGURE 5.8

Current Distribution as a Function
of Frequency Near a Cavity
Resonance

$$(a = 16.51 \text{ cm}, d_1 = 0.5l)$$

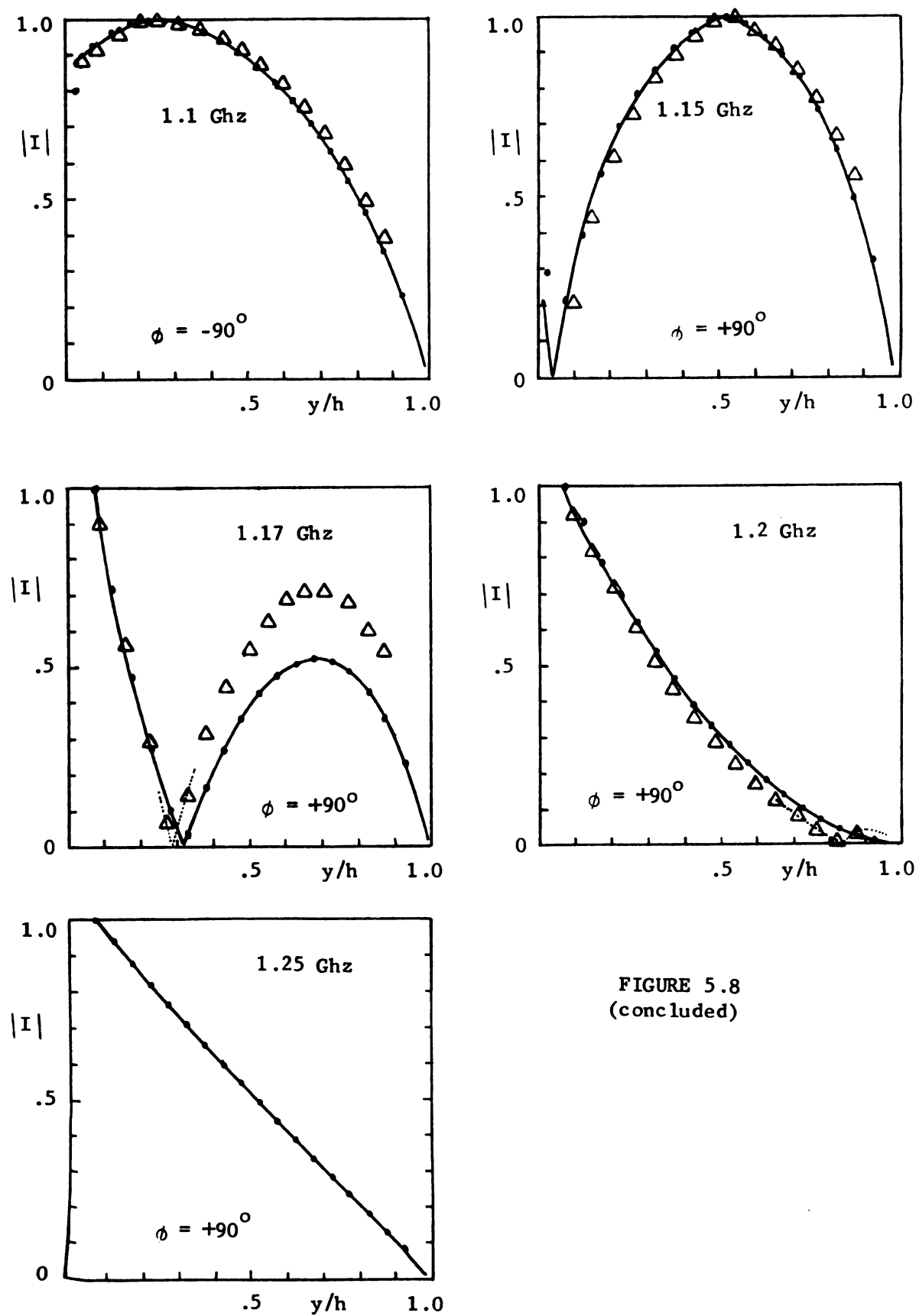


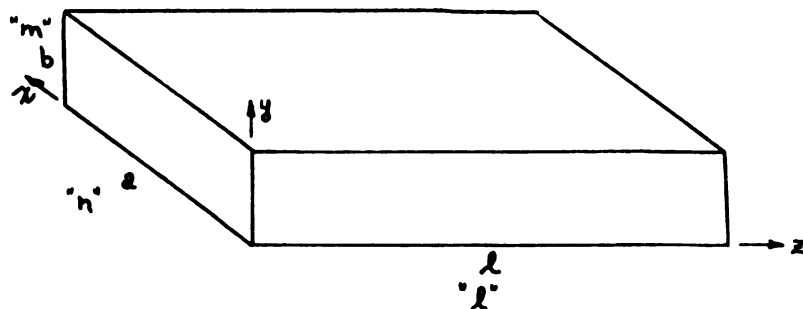
FIGURE 5.8
(concluded)

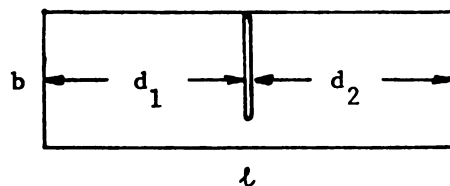
TABLE 5.5

Resonances of Rectangular Cavity

(a = 22.6 cm, b = 8.255 cm, l = 60.96 cm)

n	m	l	f_{res} , Ghz
1	0	1	.708
1	0	2	.826
1	0	3	.992
1	0	4	1.187
2	0	1	1.350
1	0	5	1.397
2	0	2	1.416
2	0	3	1.519
1	0	6	1.618
2	0	4	1.652
2	0	5	1.809
1	0	7	1.845
1	1	1	1.950
2	0	6	1.985
1	1	2	1.996
3	0	1	2.006





$$a = 22.6 \text{ cm}$$

$$b = 8.255 \text{ cm}$$

$$l = 60.96 \text{ cm}$$

$$d_1 = d_2 = 0.5l$$

$$h = 6.25 \text{ cm}$$

$$(f_{\text{res}})_{1,0,3} = 0.992 \text{ GHz}$$

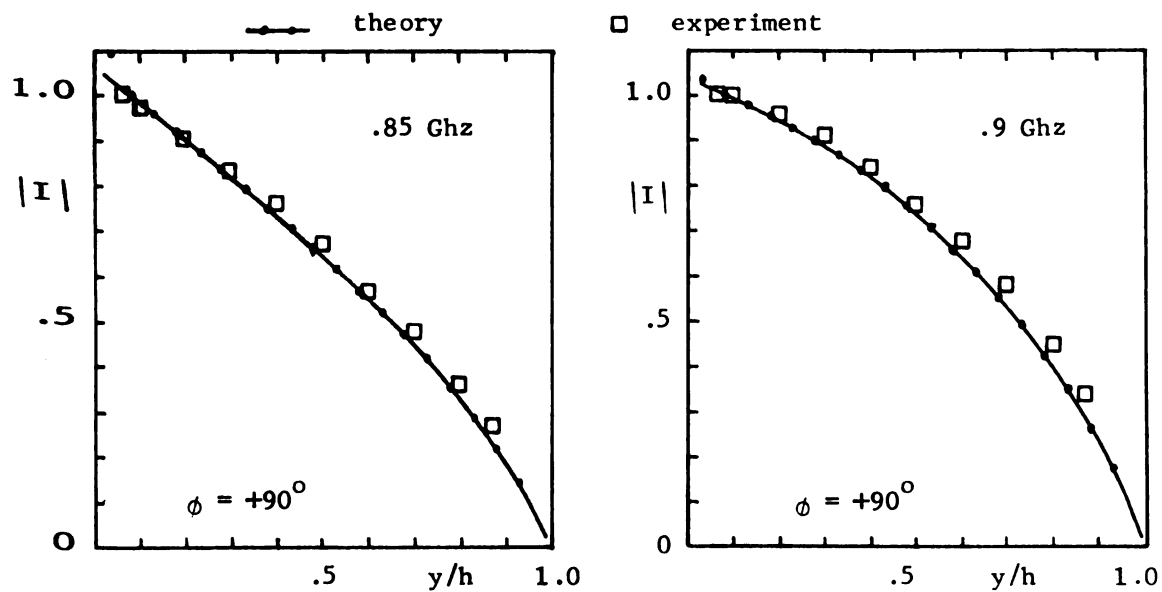


FIGURE 5.9

Current Distribution as a Function of Frequency Near a
Cavity Resonance
($a = 27.6 \text{ cm}$, $d_1 = 0.5l$)

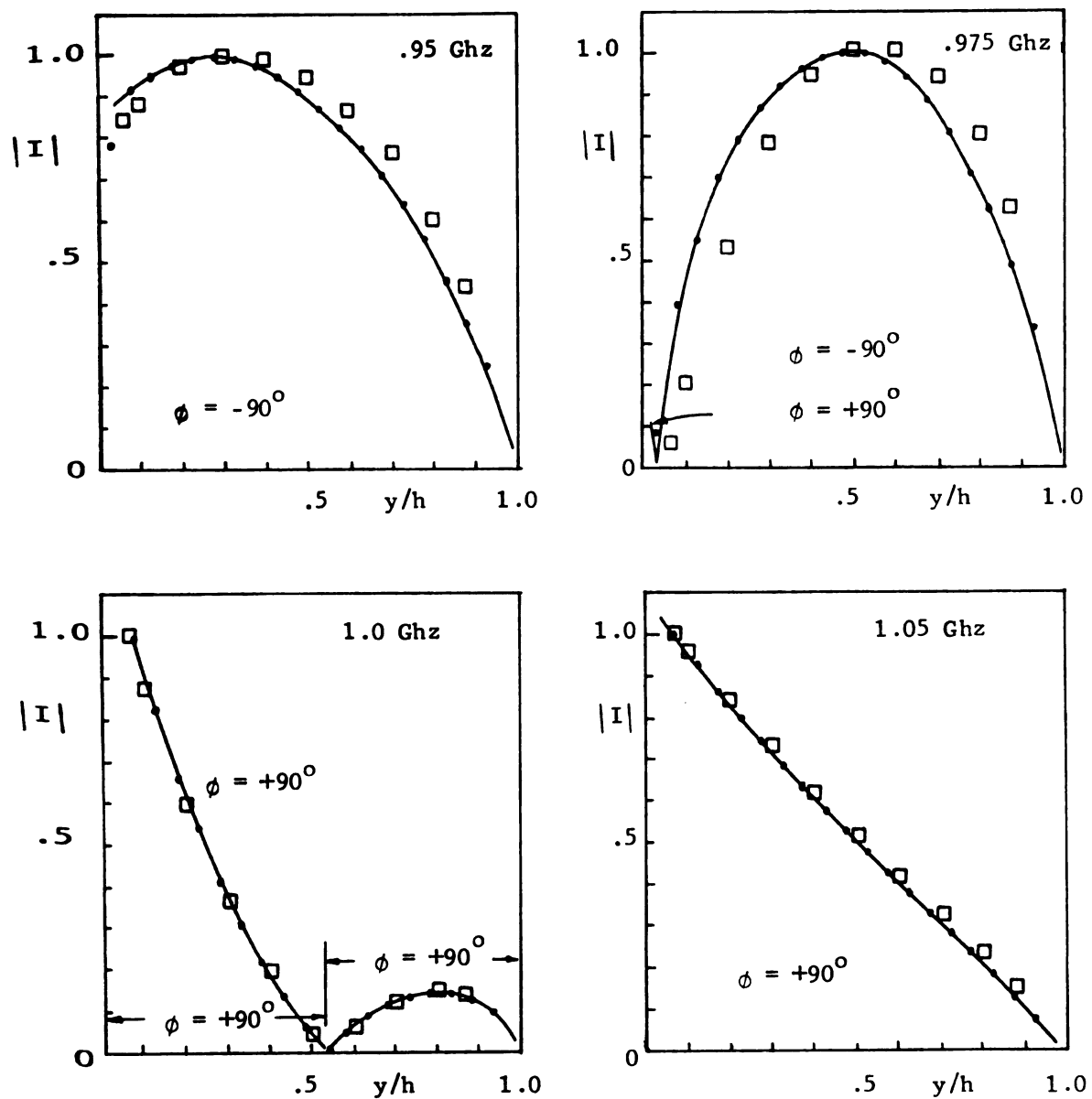
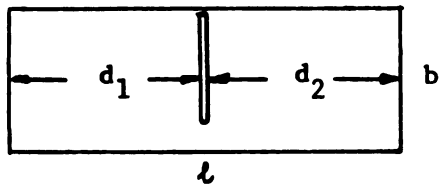


FIGURE 5.9
(concluded)



$$a = 2b = 16.51 \text{ cm}$$

$$l = 60.96 \text{ cm}$$

$$d_1 = d_2 = 0.5l$$

$$h = 6.25 \text{ cm}$$

$$f = \text{parameter}$$

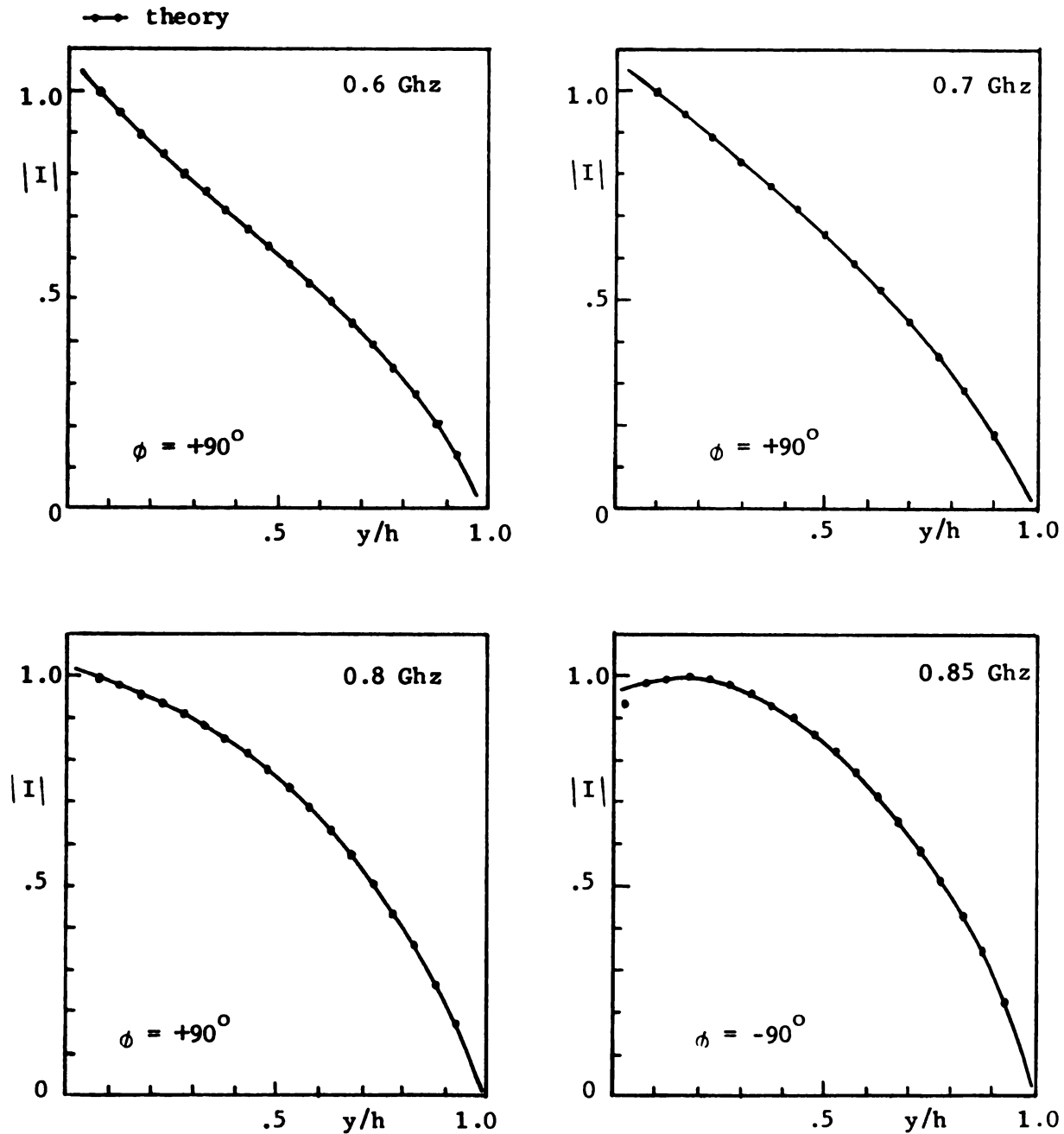
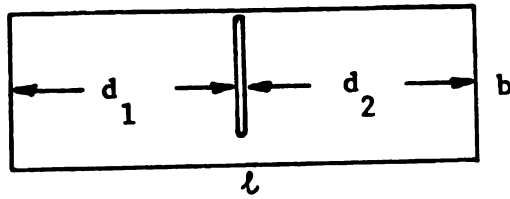


FIGURE 5.10

Current Distribution as a Function of Frequency 0.6 - 2.2 Ghz

(a = 16.51 cm , $d_1 = 0.5l$)



$$a = 2b = 16.51 \text{ cm}$$

$$l = 60.96 \text{ cm}$$

$$d_1 = d_2 = 0.5l$$

$$h = 6.25 \text{ cm}$$

$$f = \text{parameter}$$

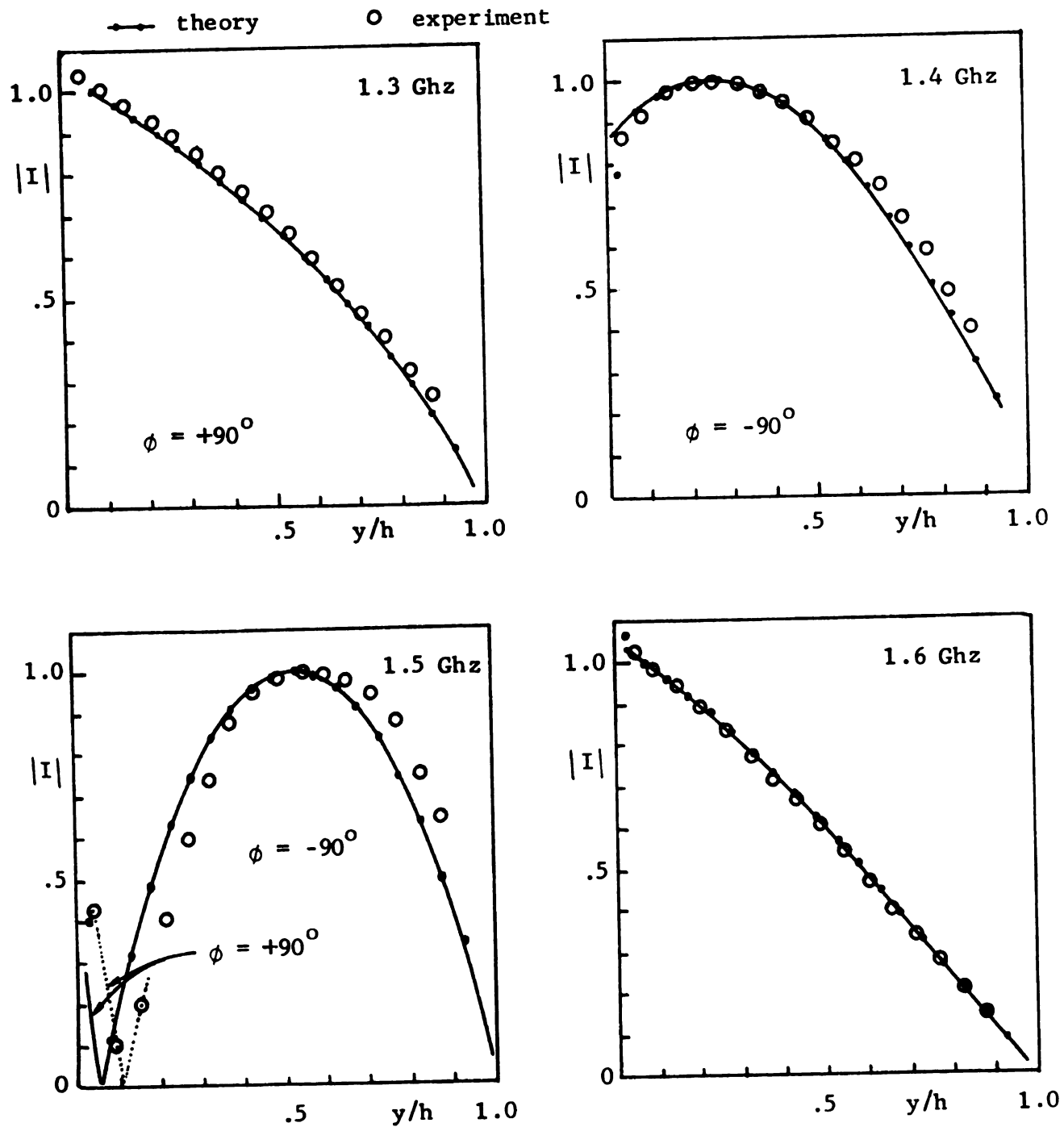


FIGURE 5.10
(continued)

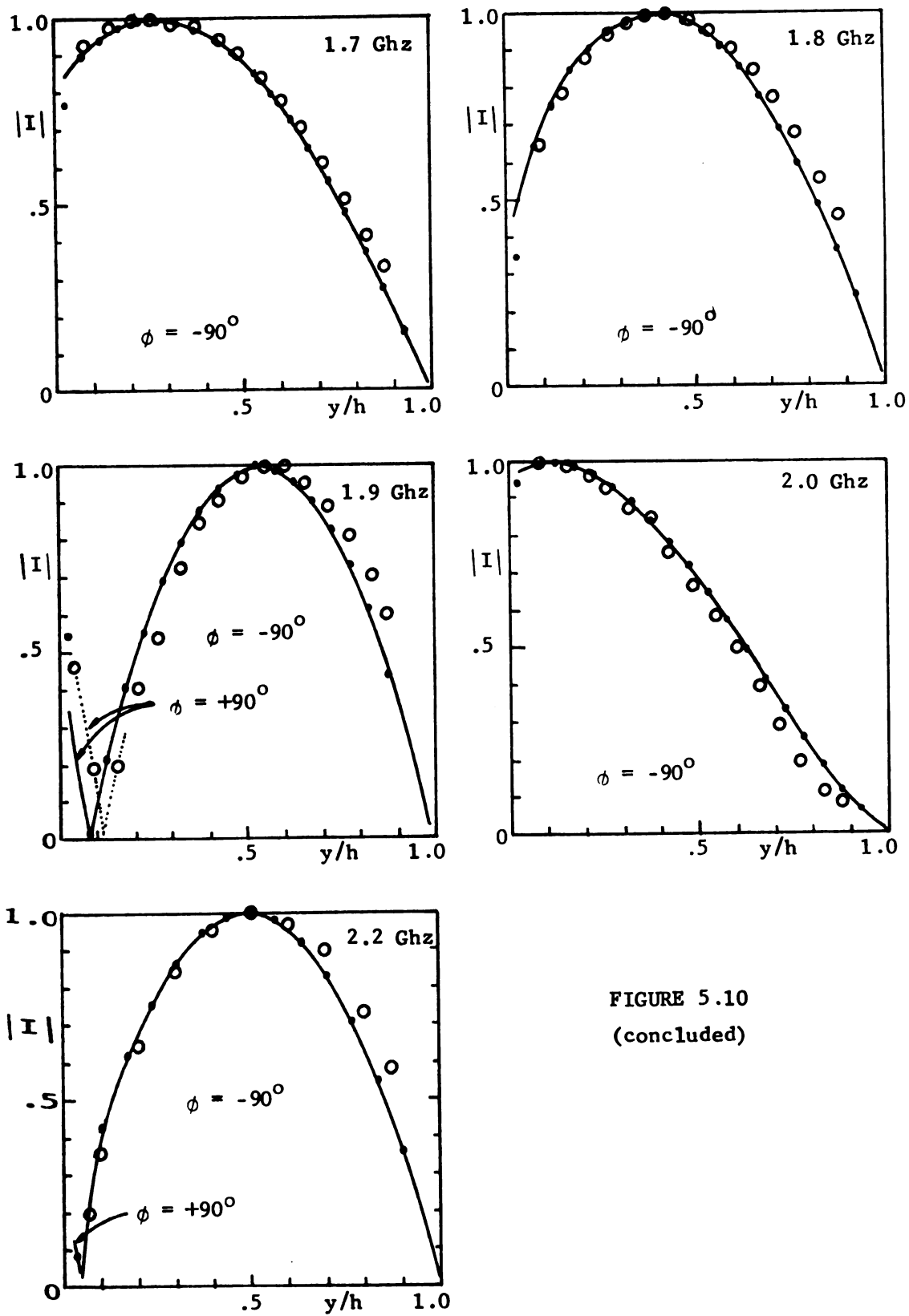
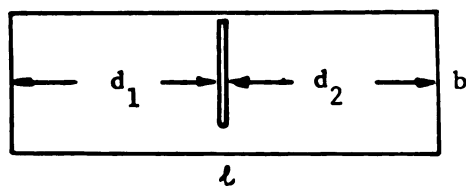


FIGURE 5.10
(concluded)



$a = 22.6 \text{ cm}$
 $b = 8.255 \text{ cm}$
 $l = 60.96 \text{ cm}$
 $d_1 = d_2 = 0.5l$
 $h = 6.25 \text{ cm}$
 $f = \text{parameter}$

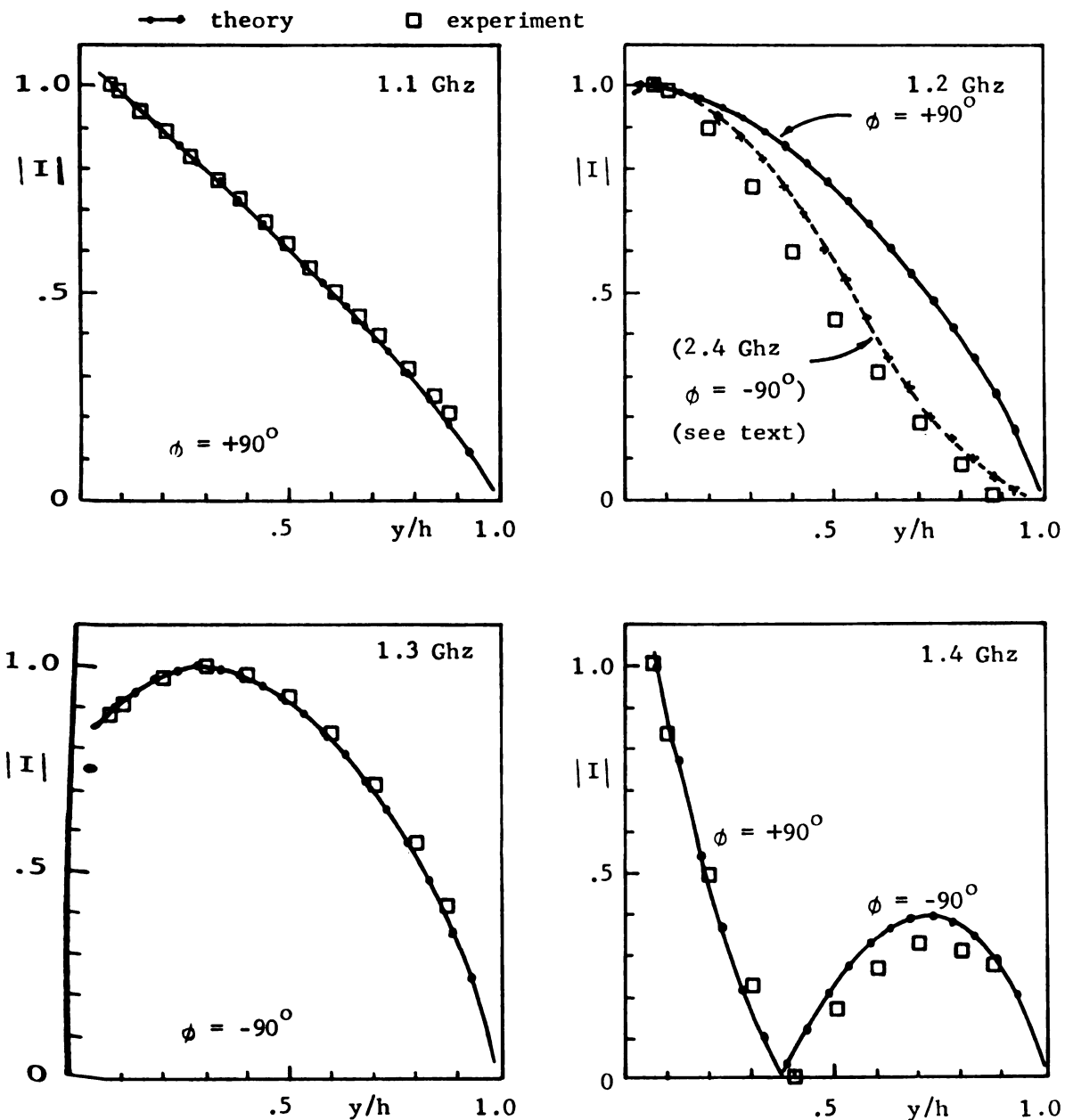


FIGURE 5.11

Current Distribution as a Function of Frequency, 1.1 - 1.9 GHz

(a = 22.6 cm, $d_1 = 0.5l$)

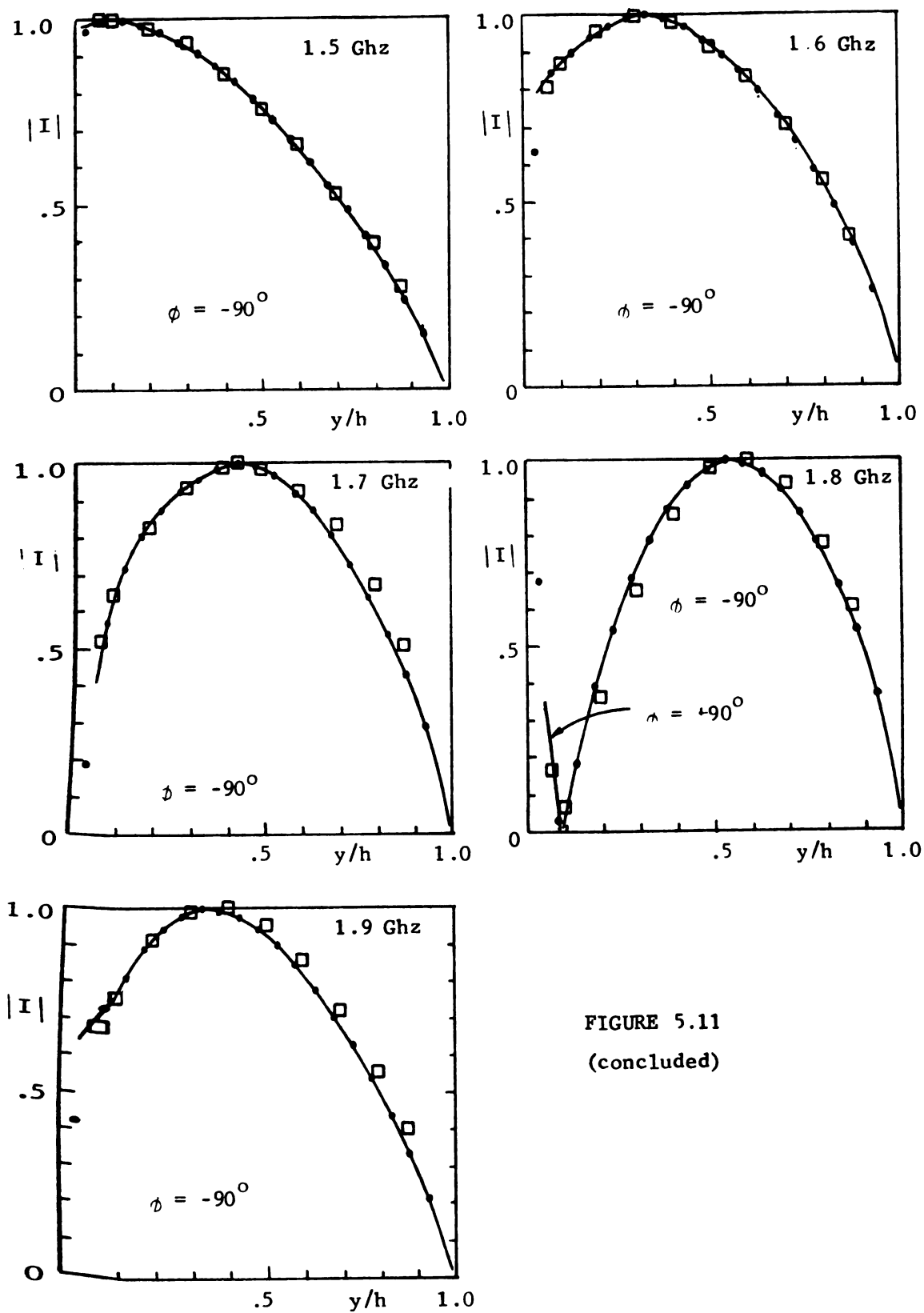
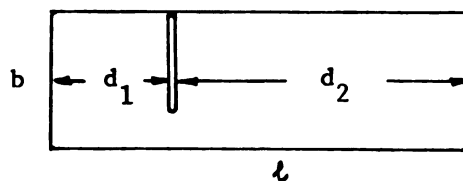


FIGURE 5.11
(concluded)



$$a = 2b = 16.51 \text{ cm}$$

$$l = 60.96 \text{ cm}$$

$$d_1 = 0.290l$$

$$h = 6.25 \text{ cm}$$

$$f = \text{parameter}$$

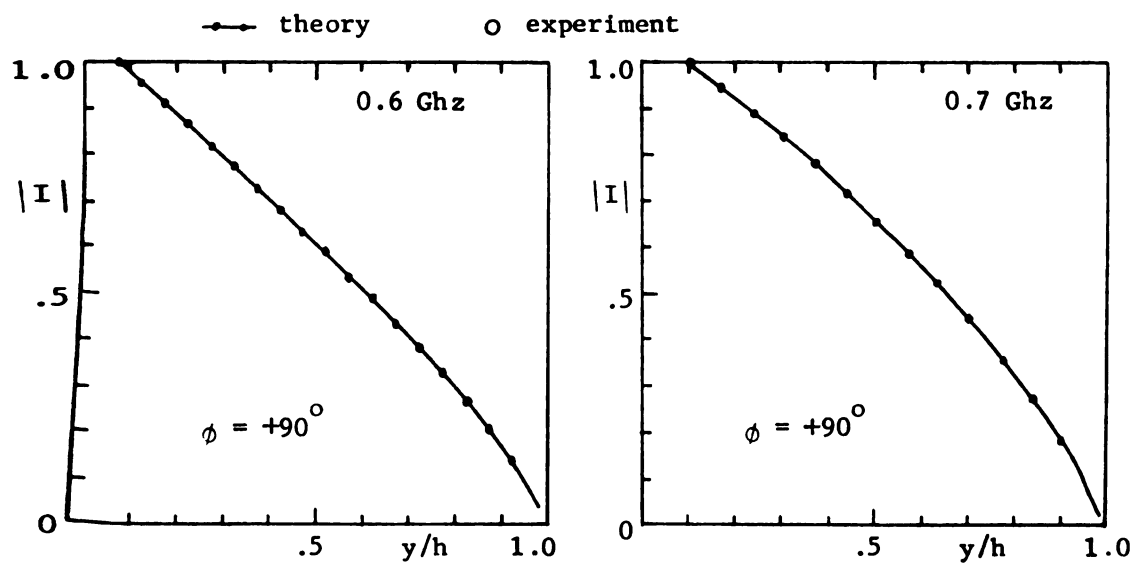


FIGURE 5.12

Current Distribution as a Function of Frequency, 0.6 - 2.2 GHz

($a = 16.51 \text{ cm}$, $d_1 = 0.290l$)

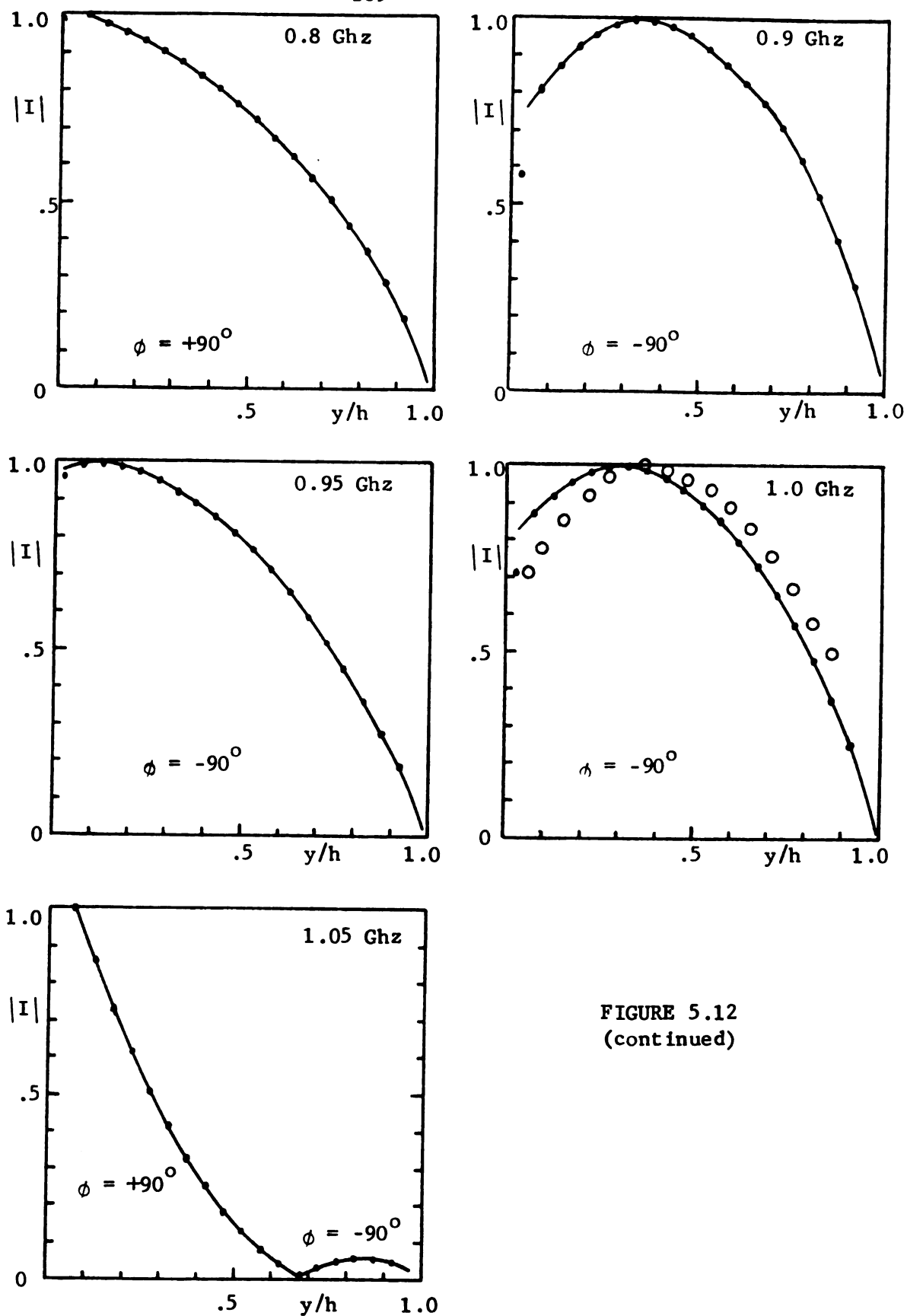


FIGURE 5.12
(continued)

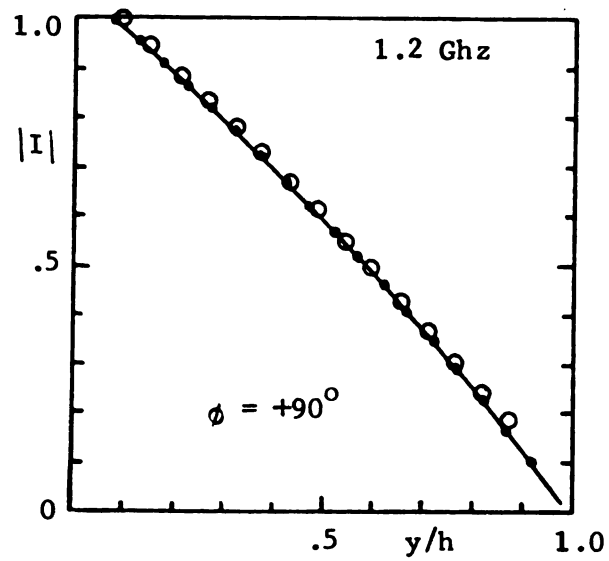
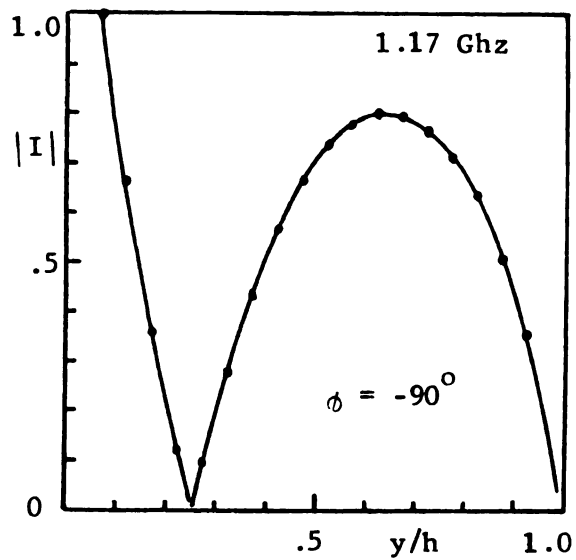
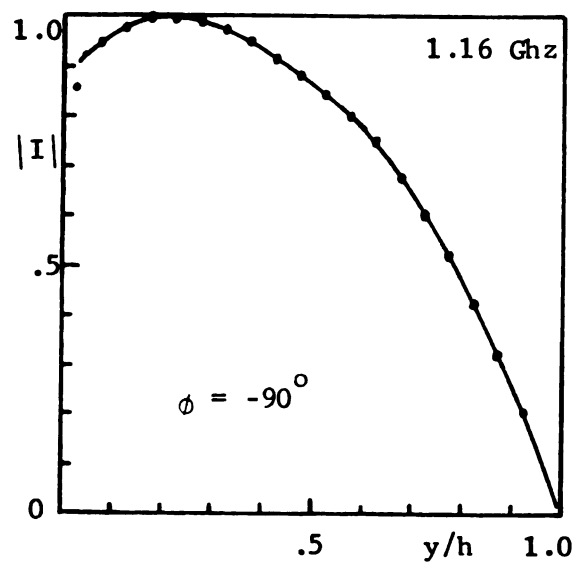
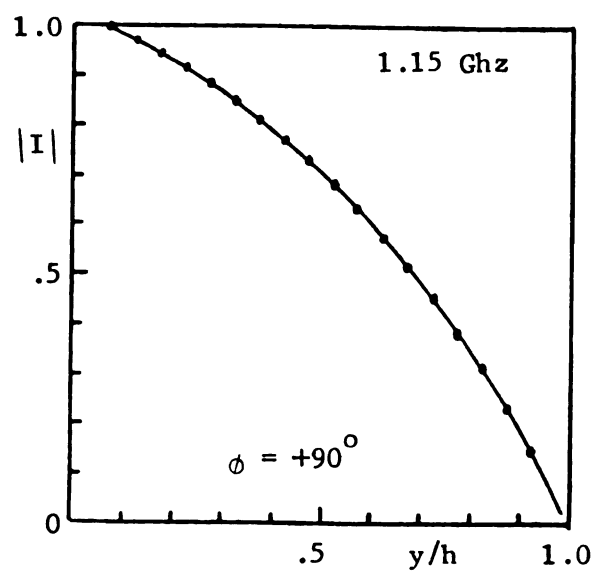
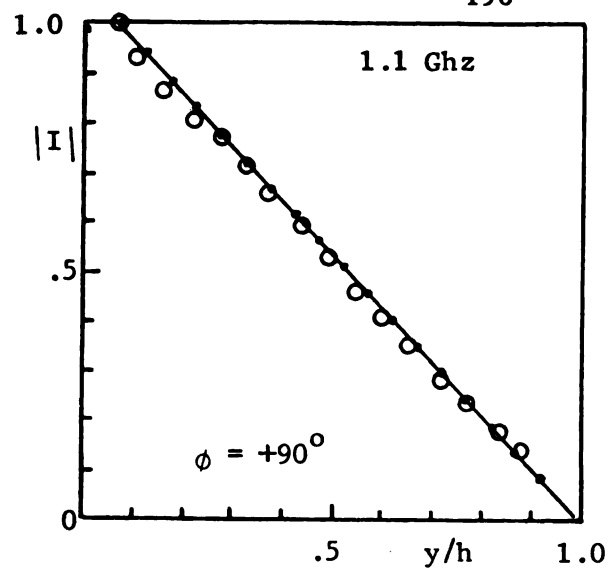


FIGURE 5.12
(continued)

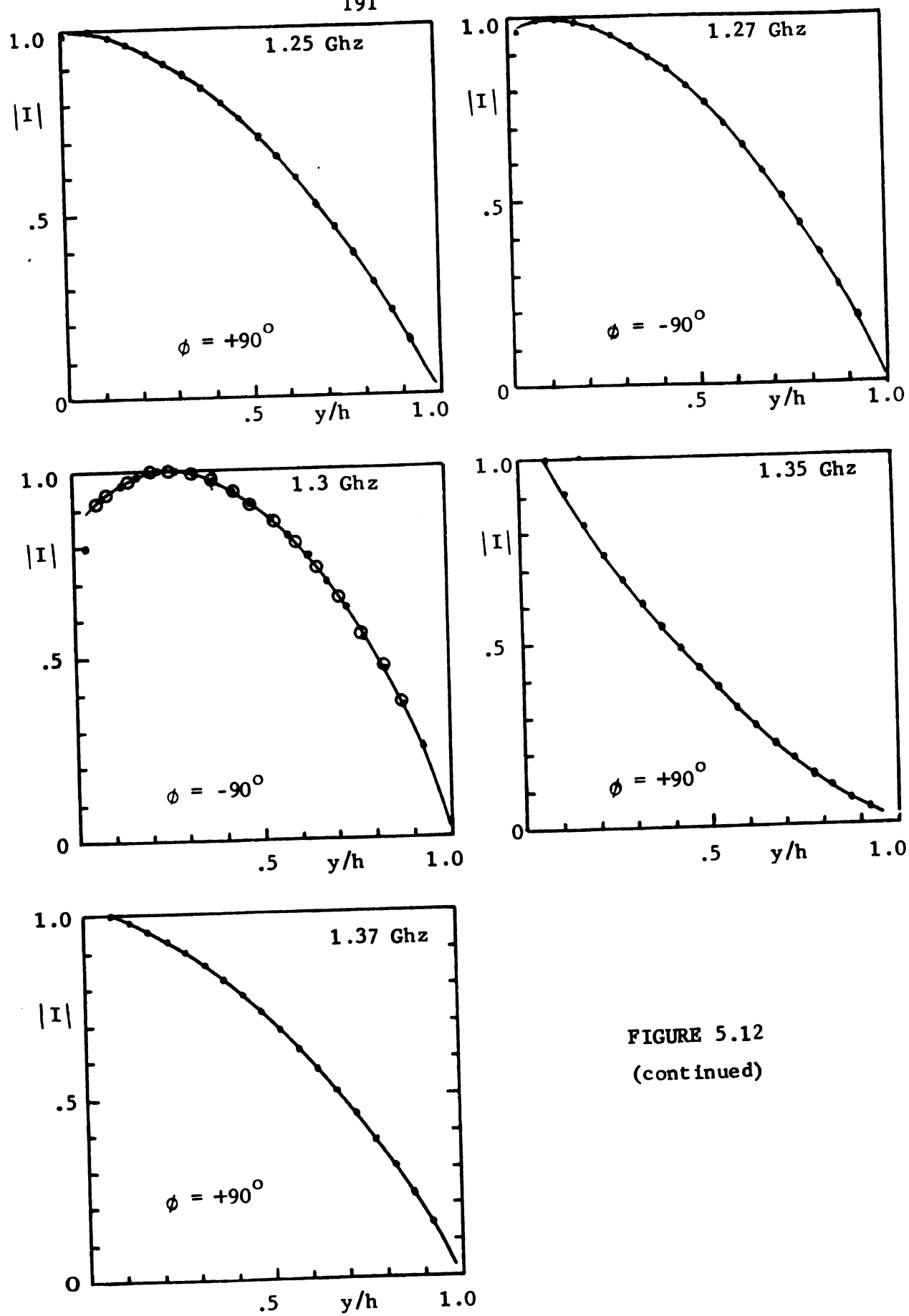


FIGURE 5.12
(continued)

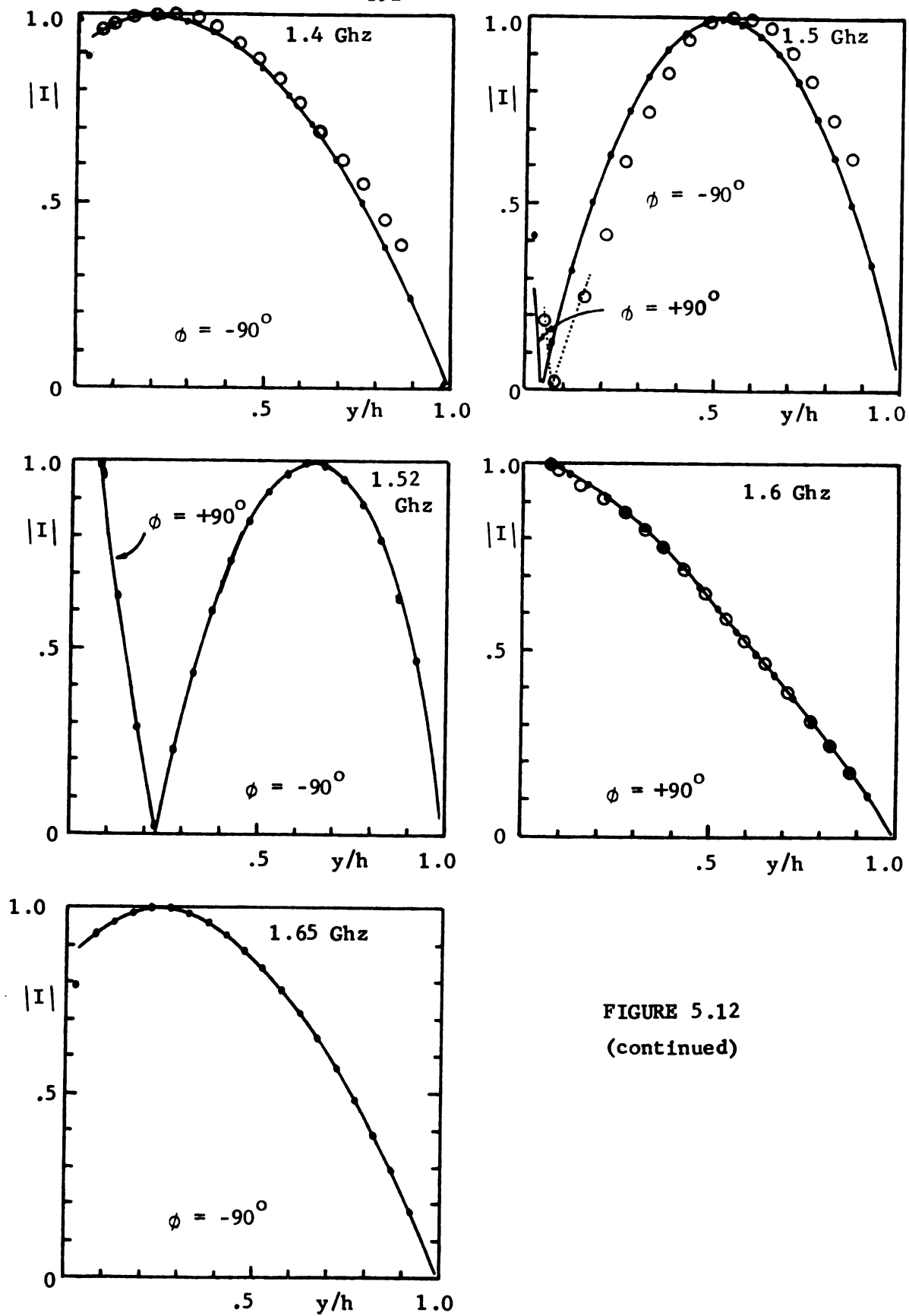


FIGURE 5.12
(continued)

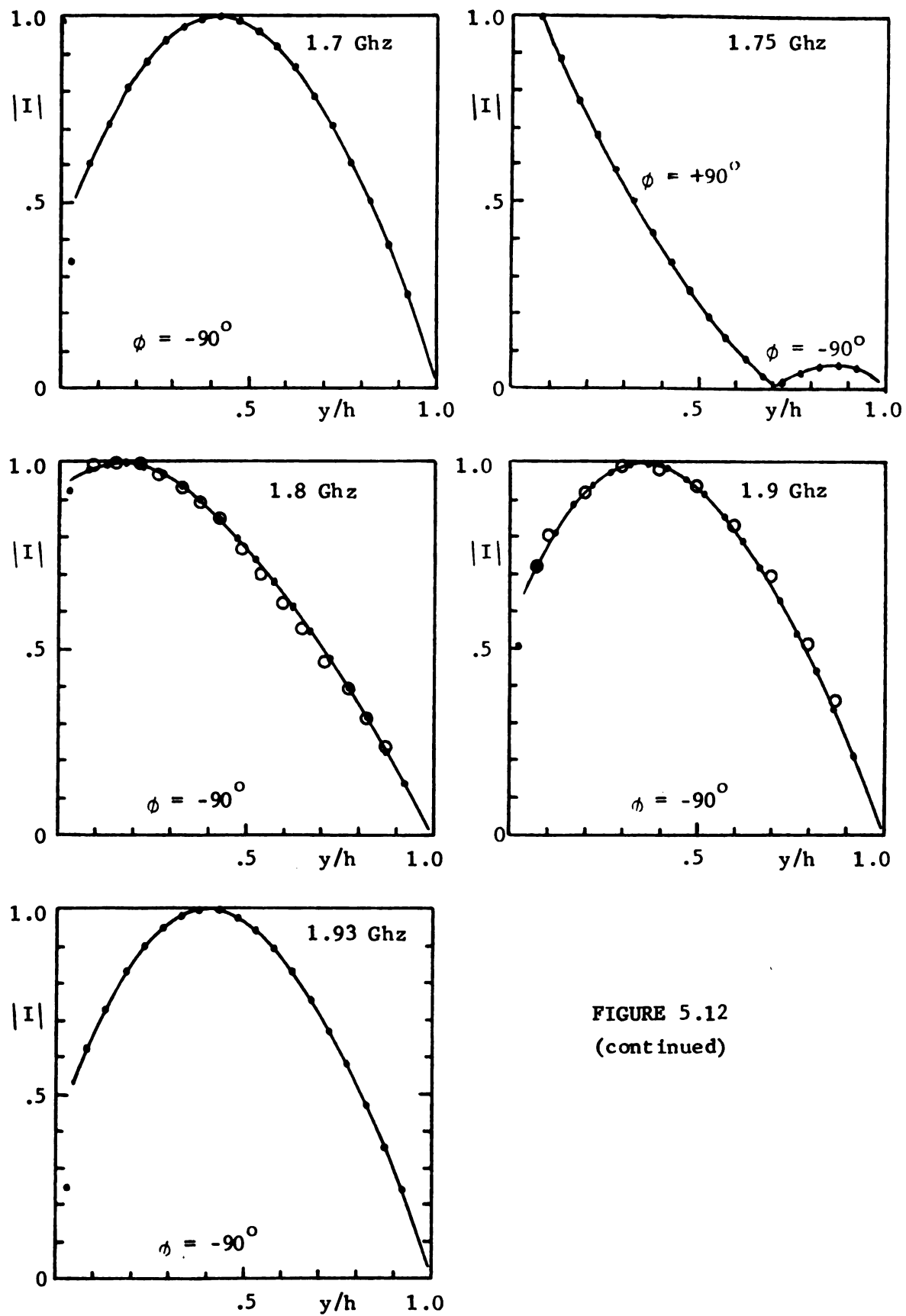


FIGURE 5.12
(continued)

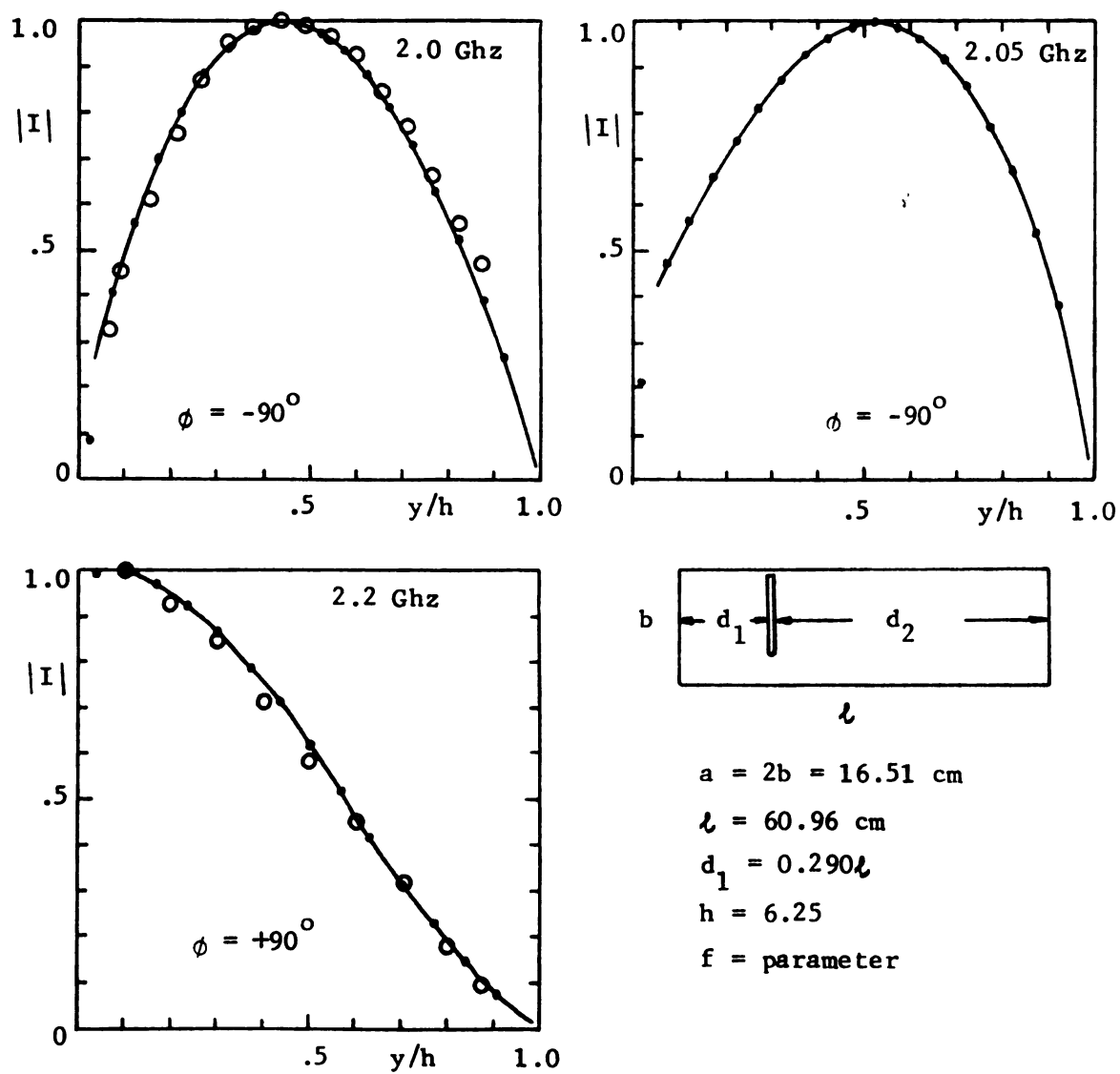


FIGURE 5.12
(concluded)

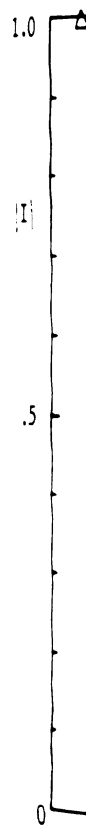
Current Distribution as a Function of Antenna Location

In the preceding paragraphs the antenna current distribution was seen to be strongly dependent upon the relationship between the operating frequency and the cavity resonant frequencies. The location of the antenna in the cavity determined the degree of coupling to each cavity resonance. In Figures 5.13 through 5.15 the current distributions on an antenna at various axial cavity locations is shown for fixed operating frequency and antenna height. As can be seen, the general nature of the distribution (i.e., triangular, quarter-wave cosinusoid, shifted sinusoid, etc.) is determined by which cavity resonance is excited for a given location. That is, for example, in Figure 5.13 at 1.2 Ghz, two basic distributions are exhibited which for convenience may be called "concave" (with a decreasing slope along the antenna) and "convex" (with an increasing slope along the antenna). These distributions are qualitatively related to the degree of coupling to different cavity resonances for each location. That is, the resonance described by mode index " ℓ " = 3 ($f_{1,0,3} = 1.171$ Ghz for the empty cavity) has zero $\hat{y}$ -component (parallel to the antenna) of electric field at the axial locations $z = 0.333\lambda$ and $z = 0.667\lambda$. Thus at the locations $d_1 = 0.290\lambda$, $d_1 = 0.343\lambda$ and $d_1 = 0.396\lambda$ there is reduced coupling to this $f_{1,0,3}$ resonance and the nature of the current depends on the coupling to the nearby " ℓ " = even resonances $f_{1,0,2}$ and $f_{1,0,4}$. At the other locations considered in Figure 5.13, however, there is stronger coupling to the $f_{1,0,3}$ resonance and the current distribution depends on the proximity of the 1.2 Ghz operating frequency to the system $f_{1,0,3}$ resonance. This is

obviously a simplified view but serves to describe general overall behavior. There are, of course, differences between all the distributions which reflect the degree of coupling to each resonance and the relative nearness of the operating frequency to each resonant frequency.

This qualitative description of the nature of the current distribution applies to the results shown in Figures 5.14 and 5.15 as well. In each case the coupling to the odd or even numbered " λ " mode index resonances is determined by the antenna location and the current distribution can be identified with a different part of the "cycle" described at the beginning of this section.

In Figure 5.15 the operating frequency is very nearly equal to the empty cavity resonance $f_{1,0,5} = 1.529 \text{ GHz}$. (The antenna-cavity system resonance is below this frequency.) In this case, the locations $d_1 = 0.185\lambda$ and $d_1 = 0.396\lambda$ do not couple to the " λ " = 5 resonance (which has $E_y = 0$ at $z = 0.2\lambda$ and $z = 0.4\lambda$) resulting in different current distributions. Greater discrepancies between theory and experiment are seen in this case for basically two reasons: first, the differences between model and experiment and thus between theoretical system resonance and physical system resonance; and second, the large magnetic fields which exist in the cavity near resonance thus increasing the current probe errors (discussed in relation to Figure 4.12). These probing errors are accountable for the apparent shift in zero-crossing of the experimental current distributions, since the shift is dependent upon location in qualitatively the same manner as the cavity magnetic field.



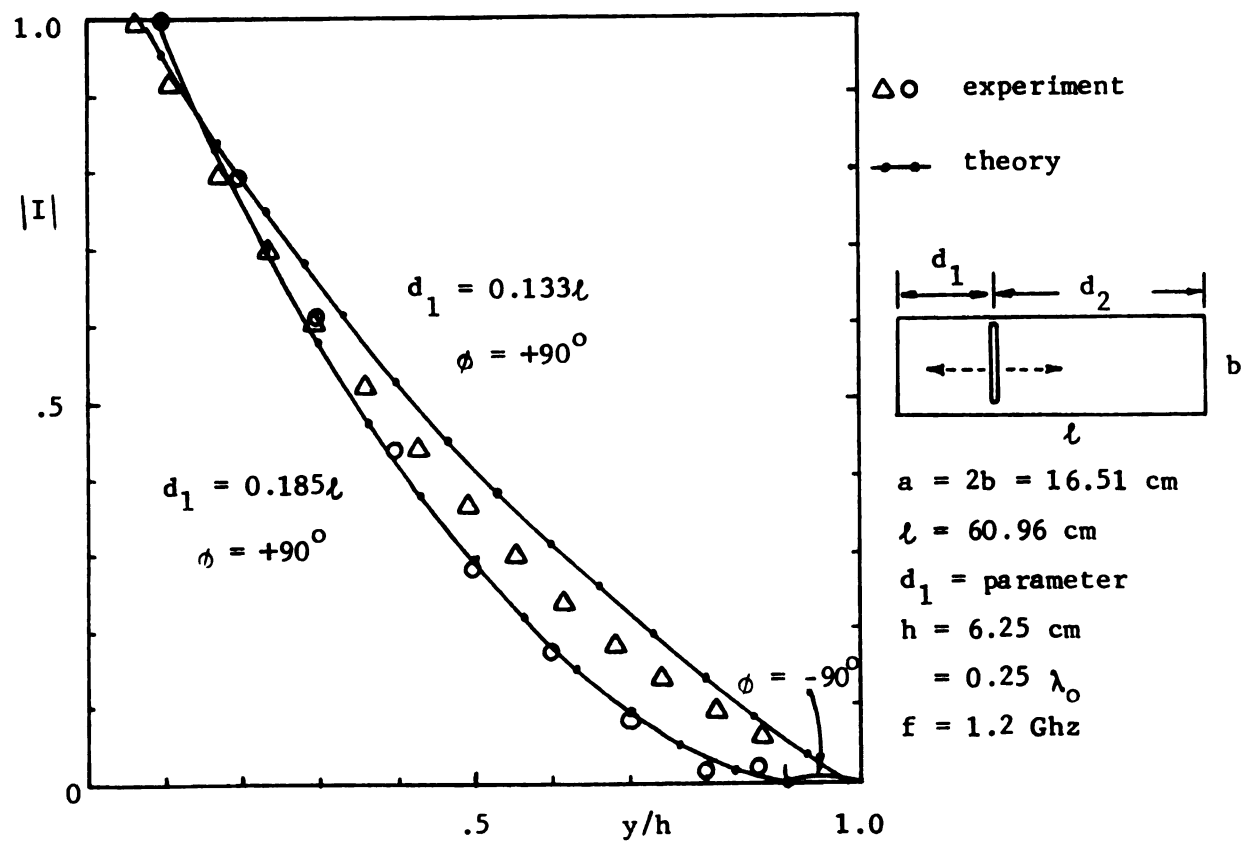
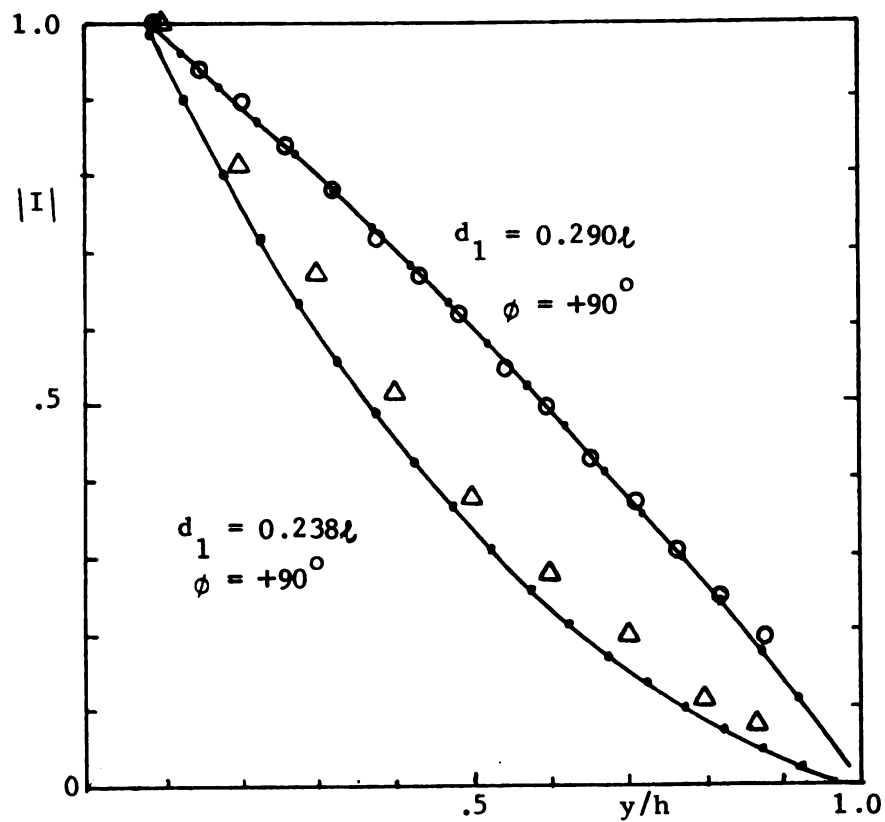


FIGURE 5.13

Current Distribution
as a Function of
Antenna Location
(1.2 GHz)



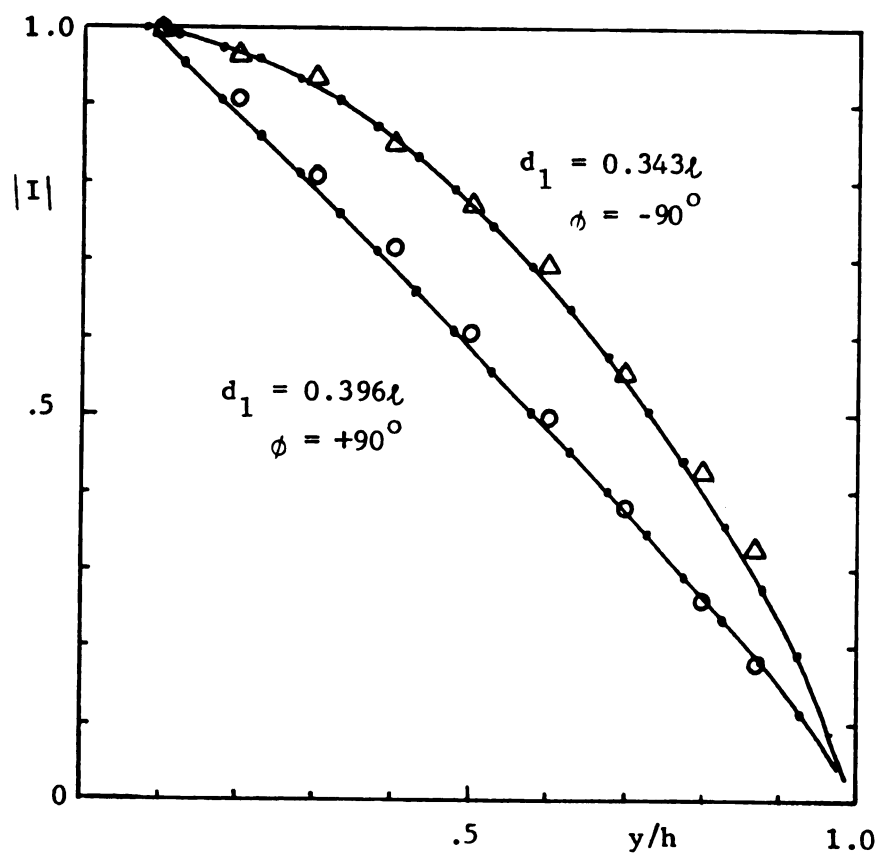
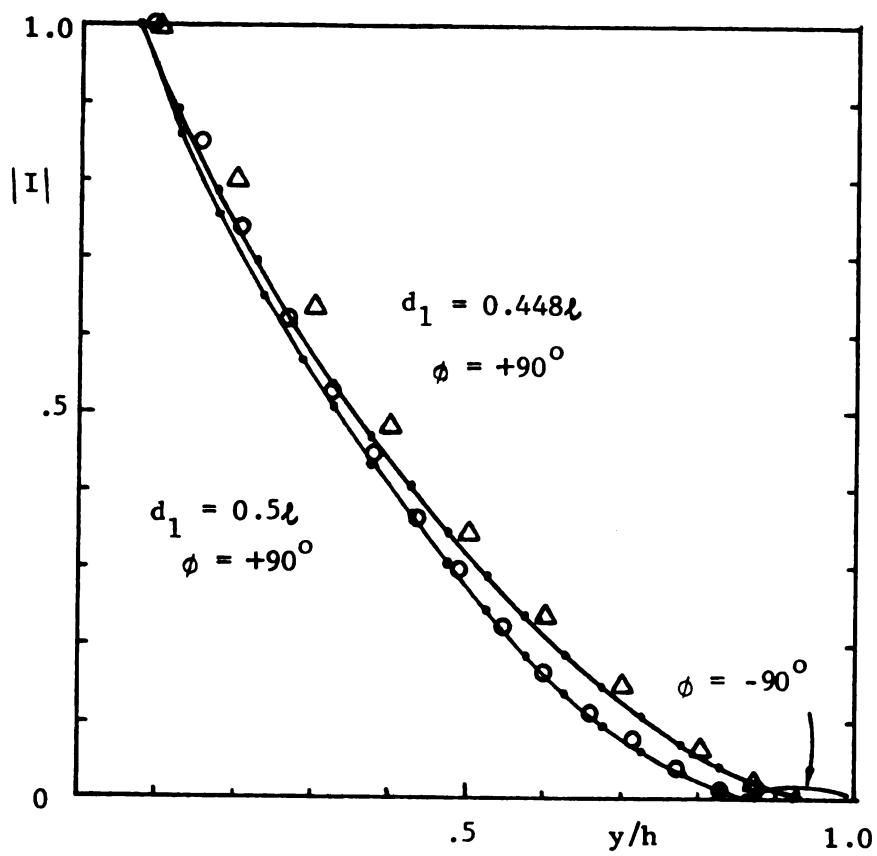
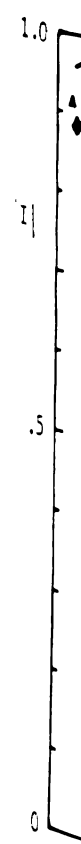
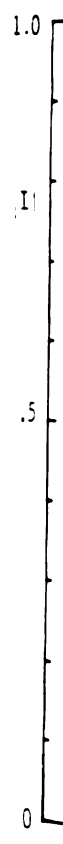


FIGURE 5.13
(concluded)





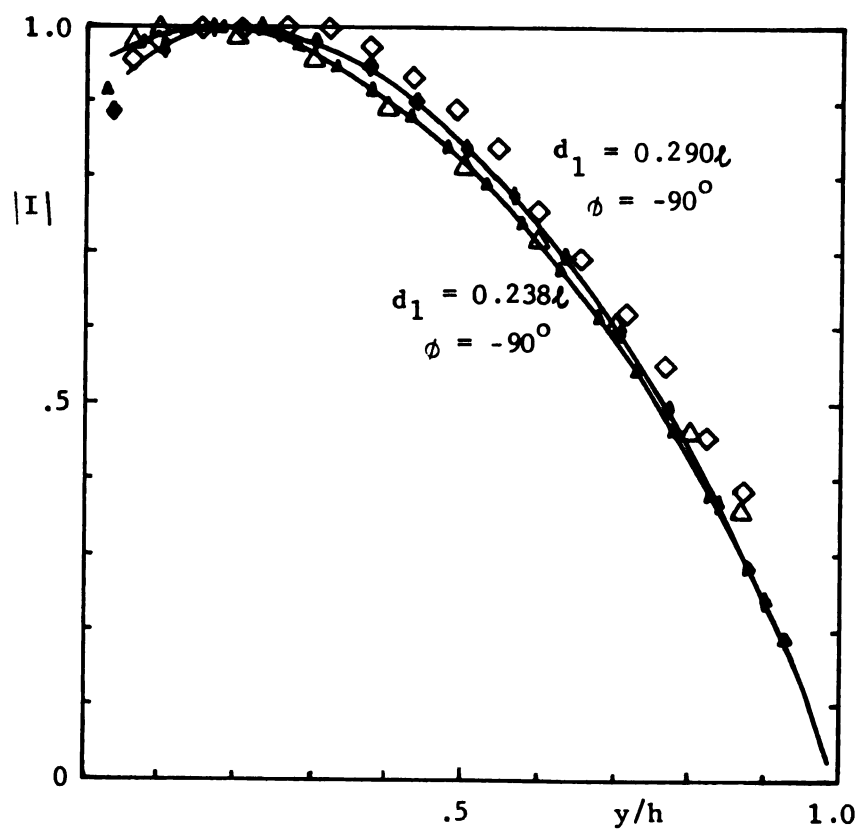
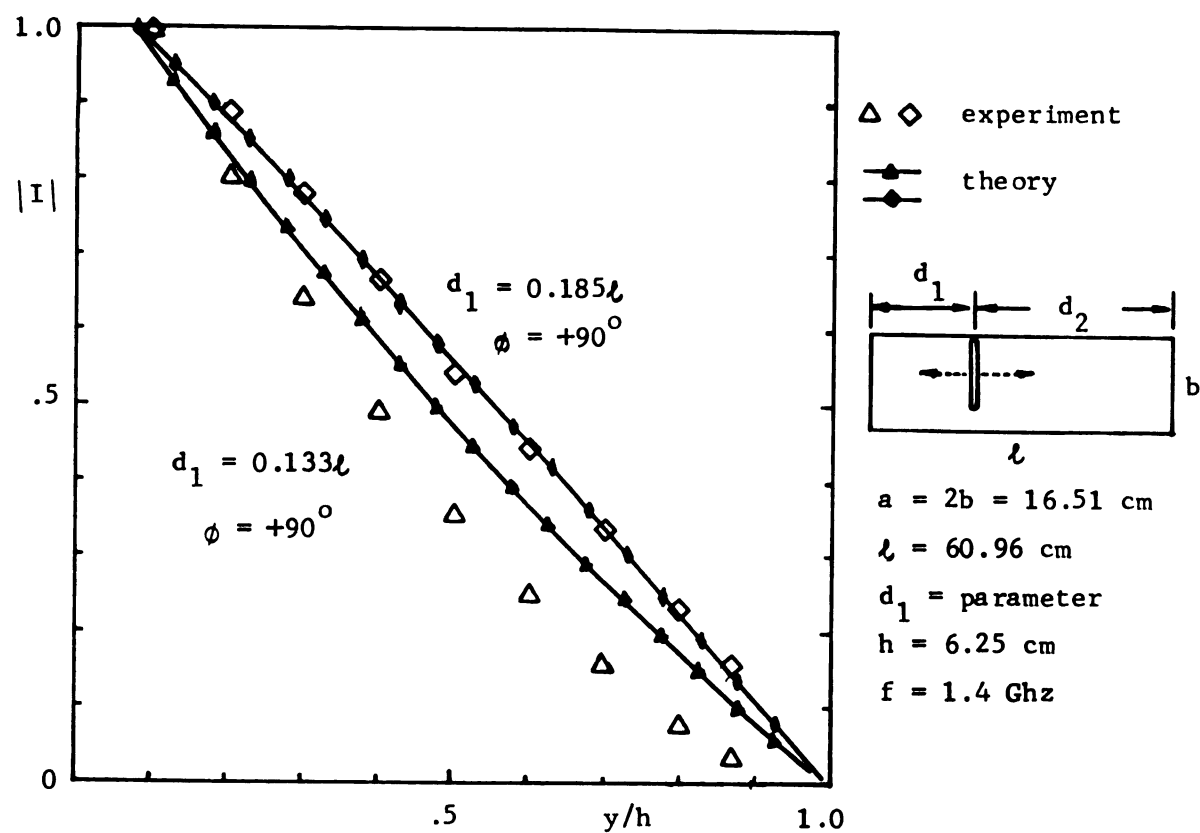
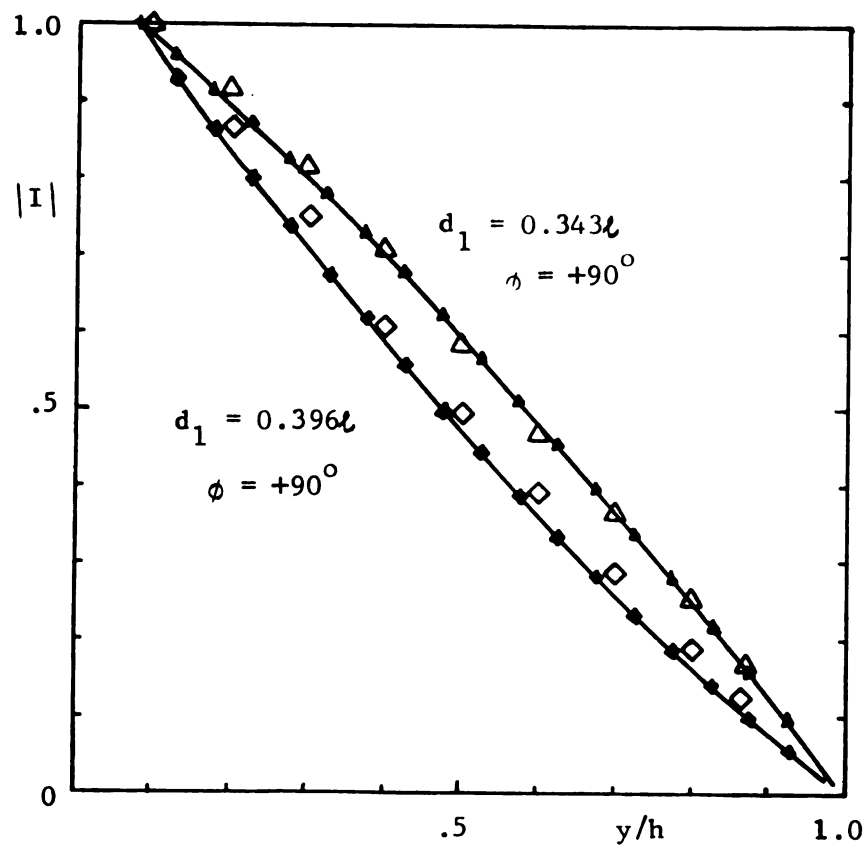
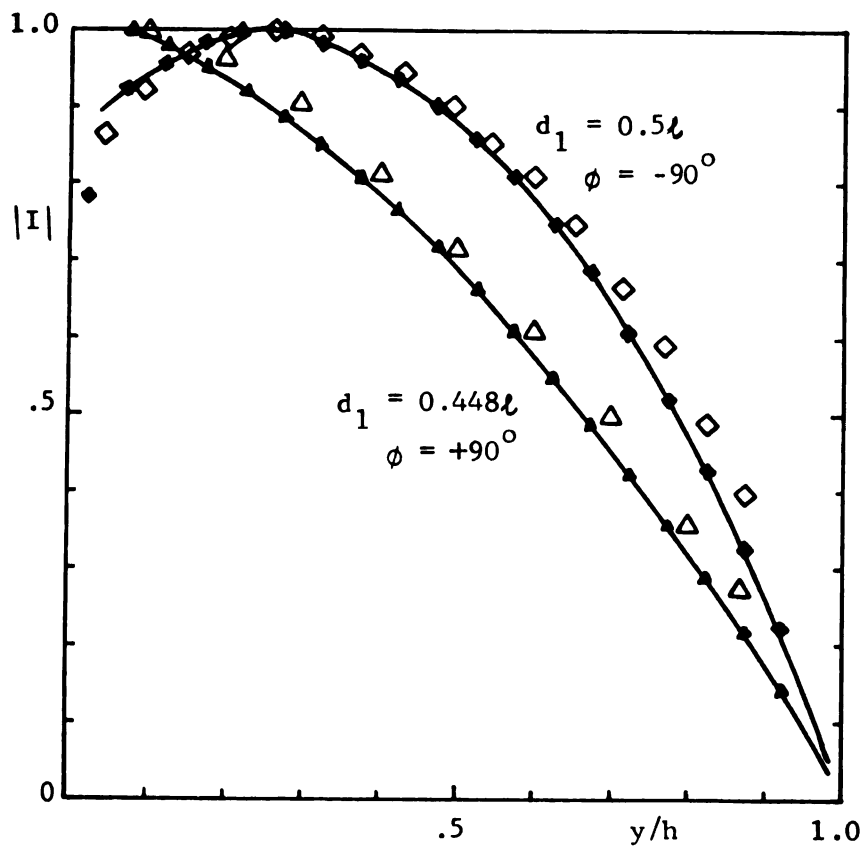
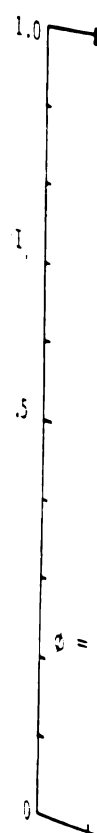
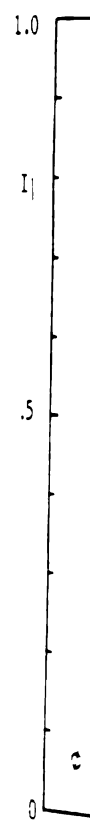


FIGURE 5.14
 Current Distribution
 as a Function of
 Antenna Location
 (1.4 GHz)

FIGURE 5.14
(concluded)



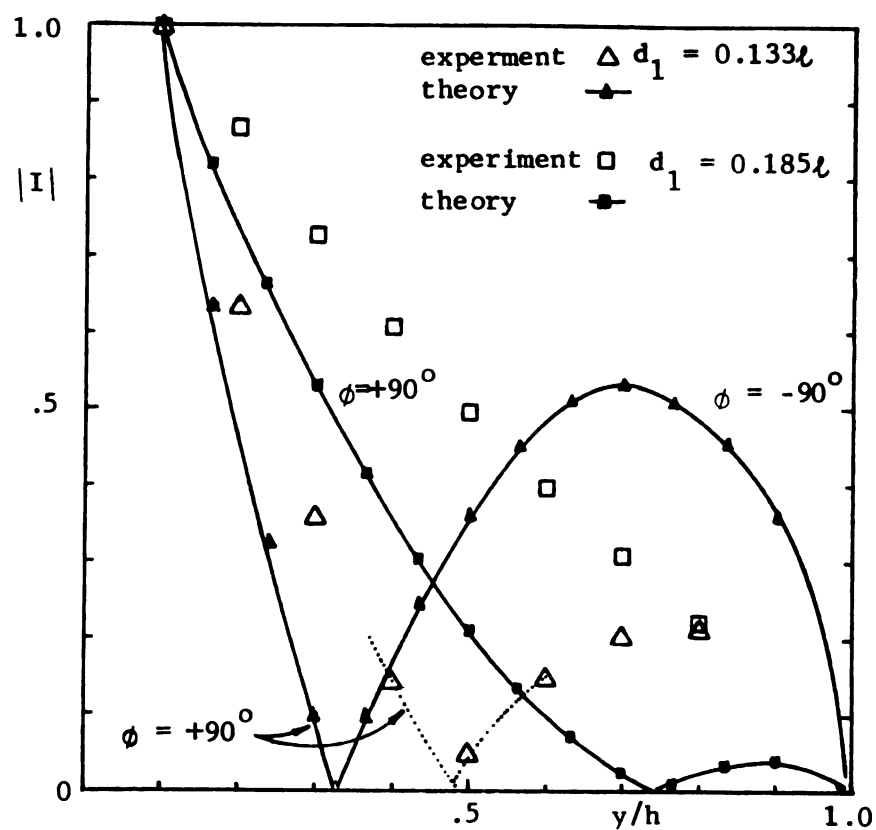
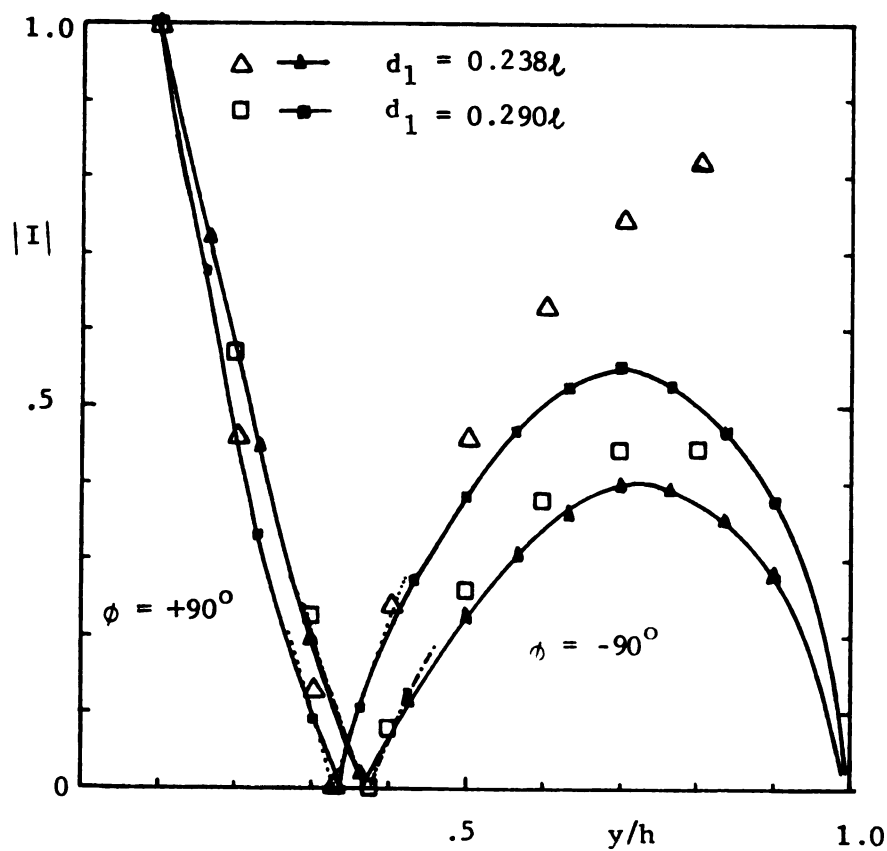
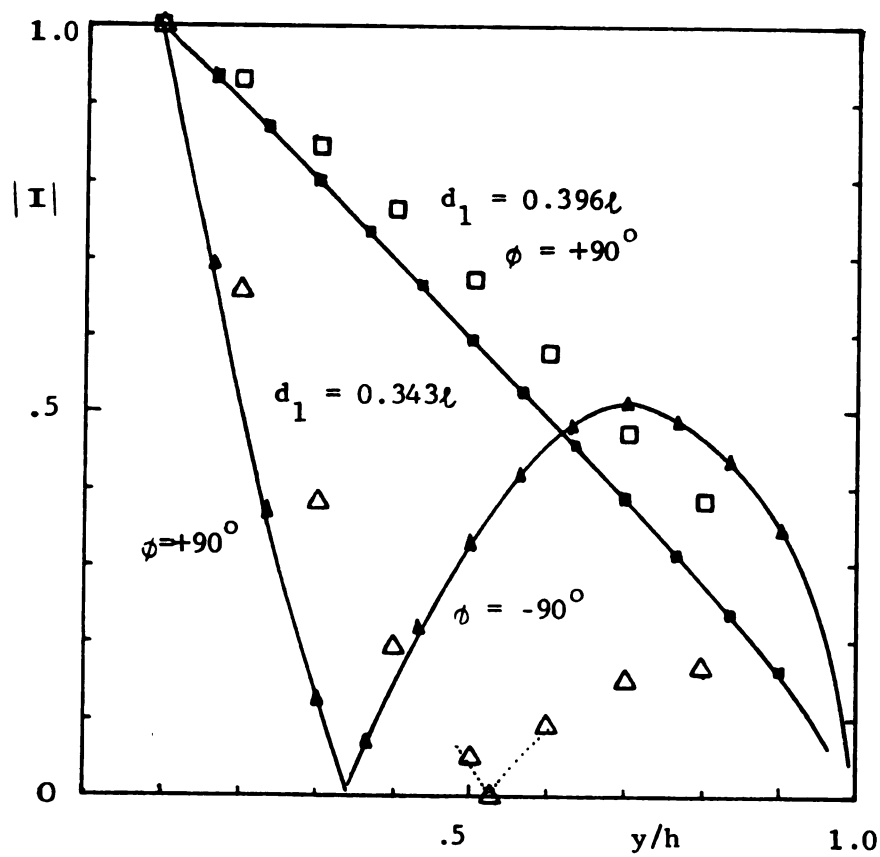
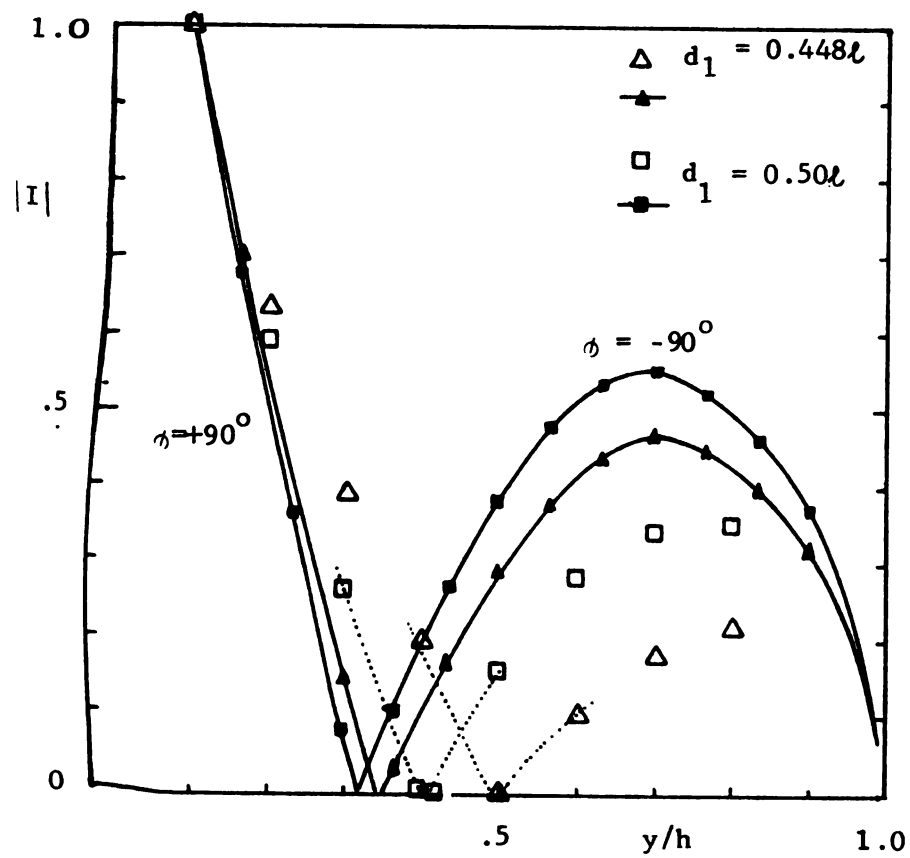


FIGURE 5.15

Current Distribution
as a Function of
Antenna Location
(1.53 GHz)



FIGURE 5.15
(concluded)

Current Distribution with Antenna Height as the Parameter

The final independent parameter considered in the antenna-cavity system with air as the medium is the antenna height. The height is, of course, of primary importance in studies involving antennas in unbounded space but is found to be of secondary importance in this bounded medium configuration. The current distributions as a function of antenna height at various fixed frequencies and fixed axial locations in the cavity are shown in Figures 5.16 through 5.21. Agreement between theory and experiment is excellent in all cases except at 1.53 Ghz in Figure 5.21 which is near a cavity resonant frequency and suffers the theoretical and experimental errors already discussed in relation to Figure 5.15.

In general, the antenna height is given only in physical centimeters because the current distribution is not dependent upon the height as much as on the cavity resonant characteristics. As can be seen in the figures, the general shape (concave, convex, quarter wave cosine, etc.) of the current distribution is not affected by antenna height but instead the height determines "how much" of the shape is included on the antenna. This is, of course, qualitatively similar to the unbounded case, but here the unusual changes in the nature of the current distribution as a function of cavity resonances predominates.

As an example of the above discussion, consider Figure 5.16 where antenna heights are shown in electrical free space wavelengths. The nature of the current is obviously different than the analogous unbounded case. The interesting point in this figure is that the current zero crossing point (or point of phase reversal) remains

physically at approximately the same location on the antenna: (at $y \approx 4$ cm.) as the height is varied. This physical location falls very near the midpoint between top and bottom waveguide walls (i.e. at $b/2$). This behavior then implies some type of coupling to a waveguide mode having mode index " m " = 1 rather than the dominant " m " = 0. Although this correlation is not precise, it implies that there is strong influence at this frequency from these higher evanescent " m " = 1 modes.

In Figures 5.17 and 5.18 the current distributions appear more like that expected in unbounded space but are still, of course, related to the cavity resonances and operating frequency. The effect of varying height is in general similar to that expected. Note, however, that at 1.2 Ghz (Figure 5.19) the effect of varying antenna height is seen to have very little effect on the nature of the current distribution, while at 1.3 Ghz (Figure 5.20) the antenna height has more effect on the current than would be expected in **unbounded space**. Finally at 1.53 Ghz (figure 5.21) the theoretical current again appears relatively independent of antenna height.

It is interesting to compare the results shown in Figures 5.21 and 5.16. In contrast to the current at 1.05 Ghz discussed above, the physical location (y , in centimeters) of the current zero crossing at 1.53 Ghz does not stay fixed as the height is varied. Instead, this phase reversal point moves so that it occurs at approximately the same normalized distance (y/h) along the antenna, and there seems to be very little correlation between evanescent cavity mode distributions and the current distribution.

In general, the effect of varying the antenna height on the current distribution depends upon the operating frequency and nearness of cavity resonances. There is apparently no unique relationship between antenna electrical length and current distribution; it is difficult to describe resonance or anti-resonance behaviour of the antenna from the height alone.

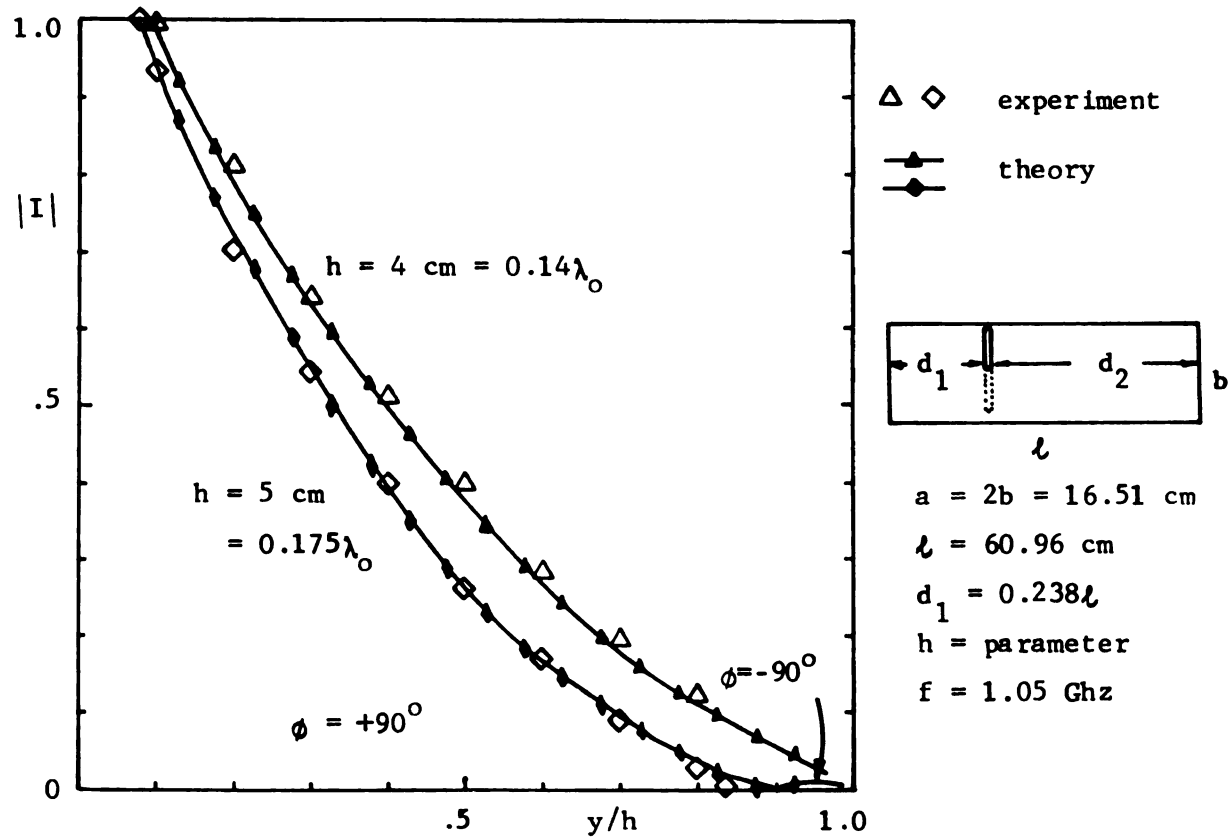
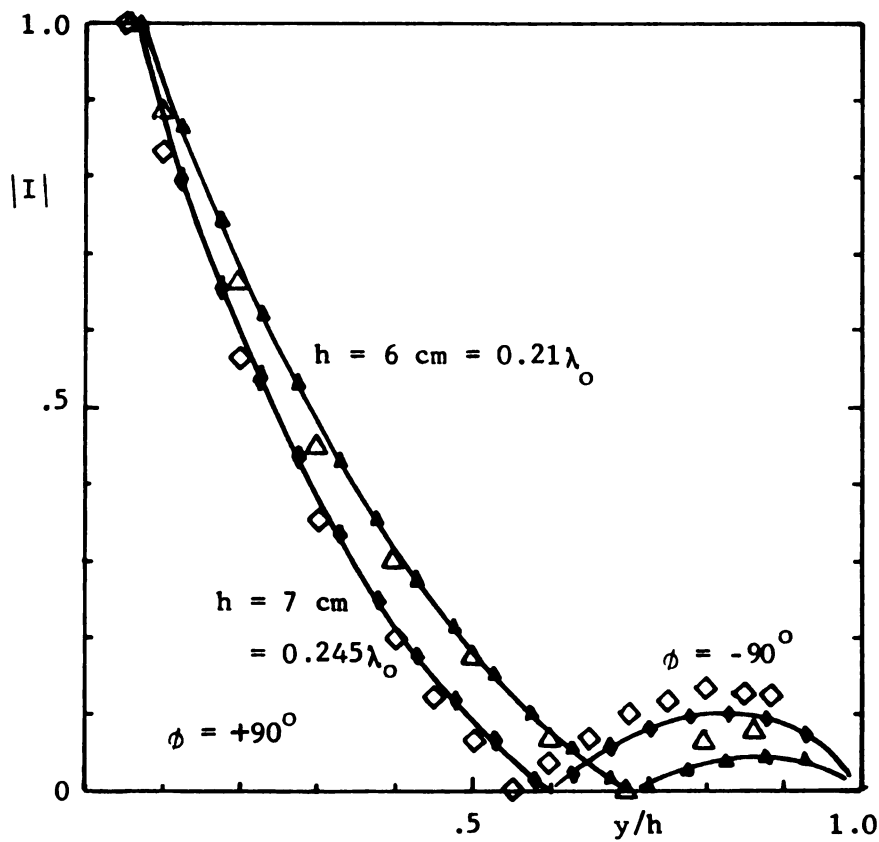


FIGURE 5.16

Current Distribution
 as a Function of
 Antenna Height
 ($a = 16.51 \text{ cm}$,
 1.05 GHz)



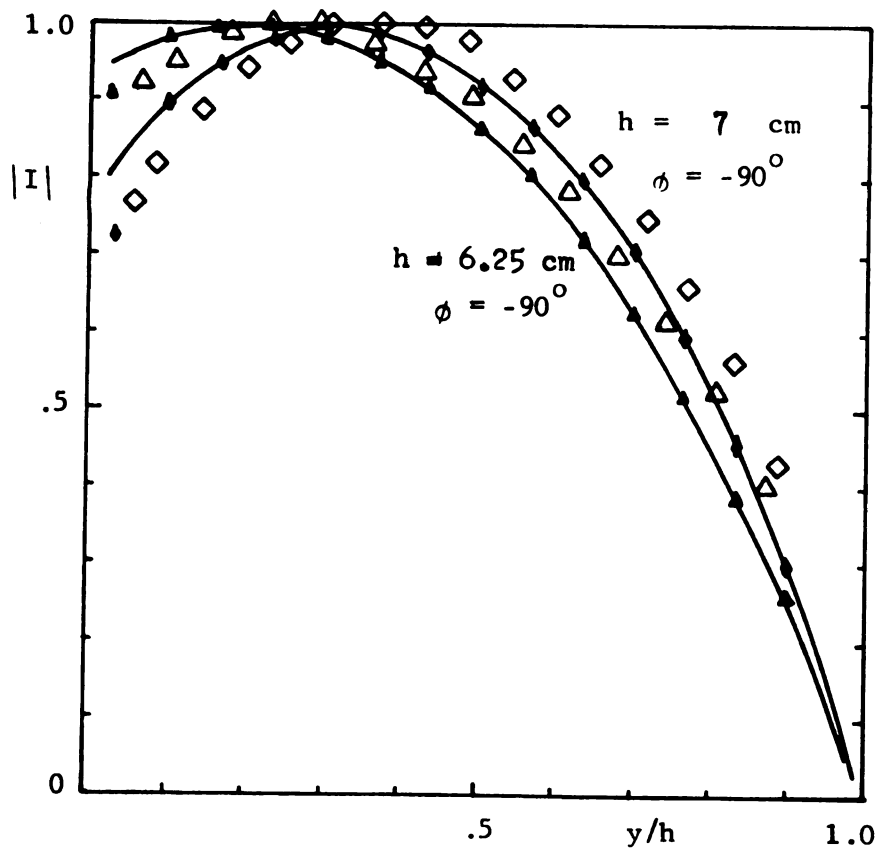
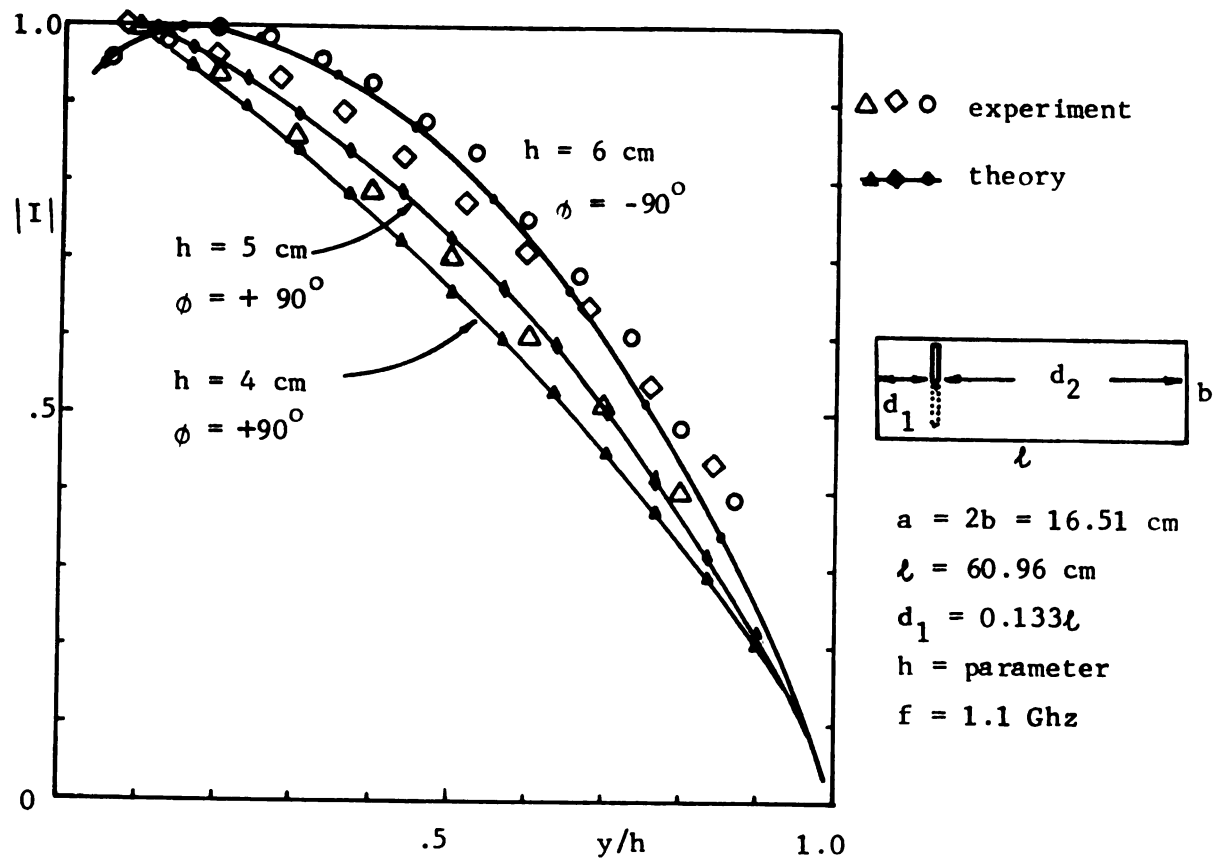
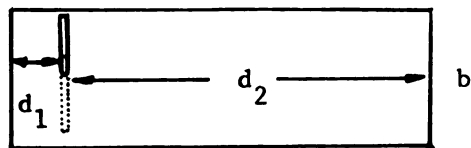


FIGURE 5.17

Current Distribution
 as a Function of
 Antenna Height
 ($a = 16.51 \text{ cm}$,
 1.1 GHz)



l
 $a = 22.6 \text{ cm}$
 $b = 8.255 \text{ cm}$
 $l = 60.96 \text{ cm}$
 $d_1 = 0.133l$
 $h = \text{parameter}$
 $f = 1.1 \text{ GHz}$

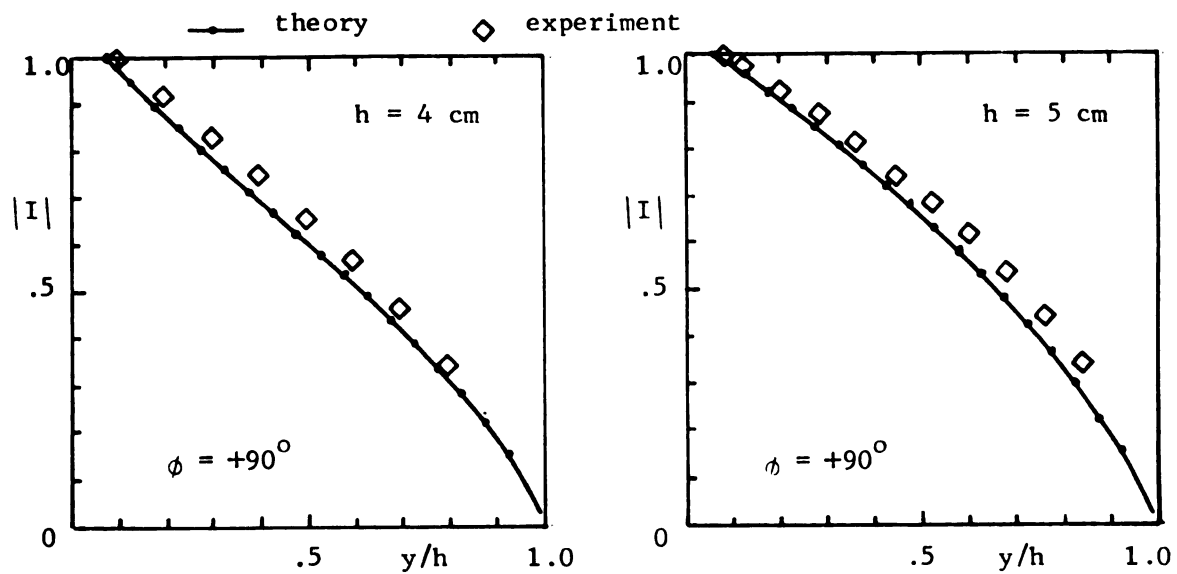
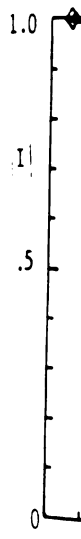


FIGURE 5.18

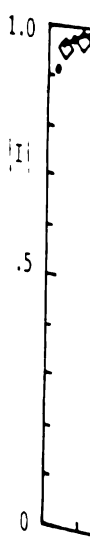
Current Distribution as a Function of Antenna Height

(a = 22.6 cm, 1.1 GHz)

1



1



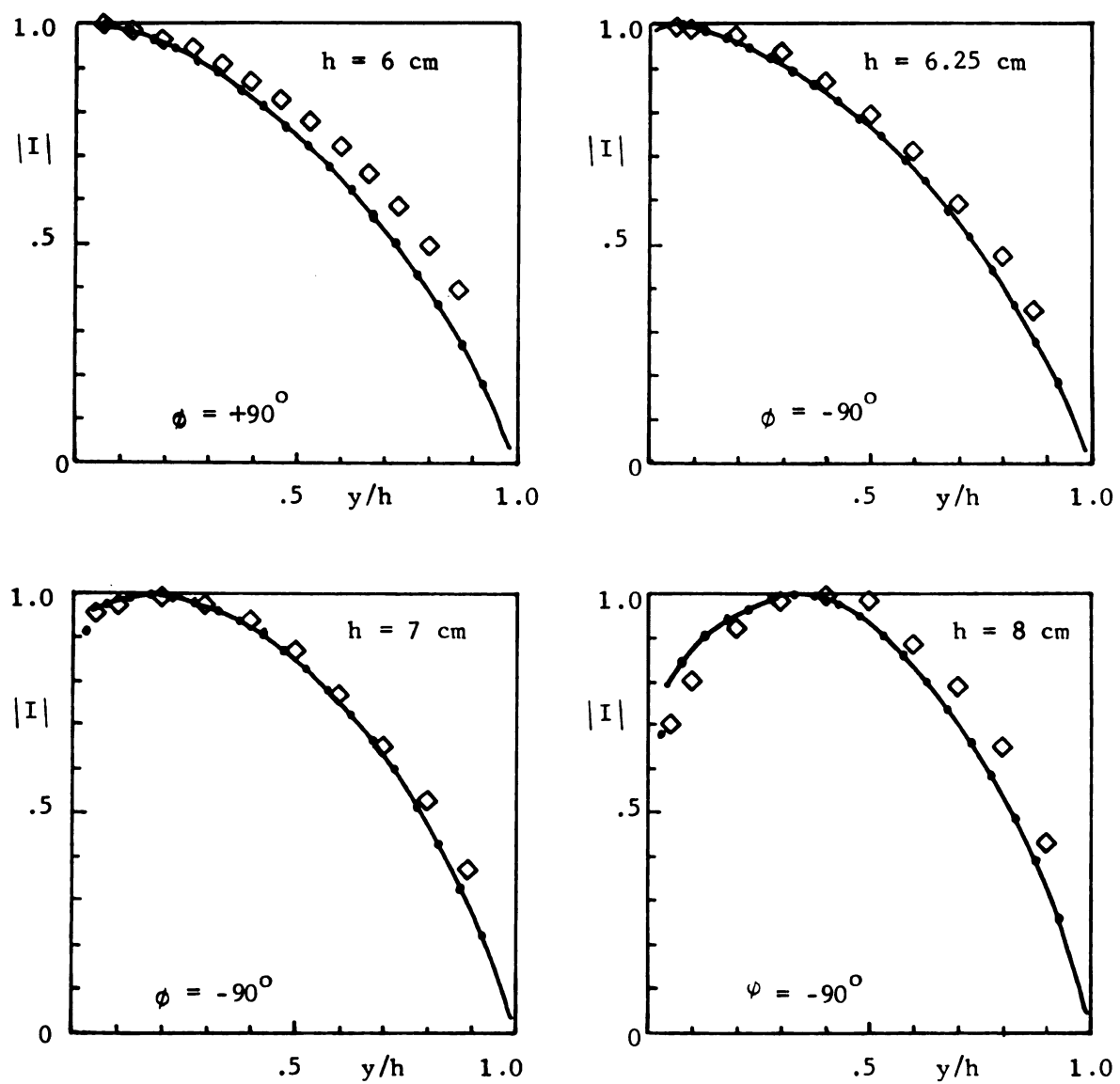
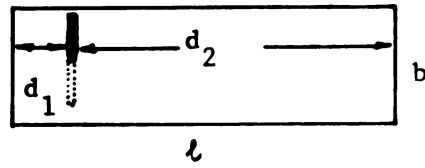


FIGURE 5.18
(concluded)



$$a = 2b = 16.51 \text{ cm}$$

$$l = 60.96 \text{ cm}$$

$$d_1 = 0.133l$$

$$h = \text{parameter}$$

$$f = 1.2 \text{ GHz}$$

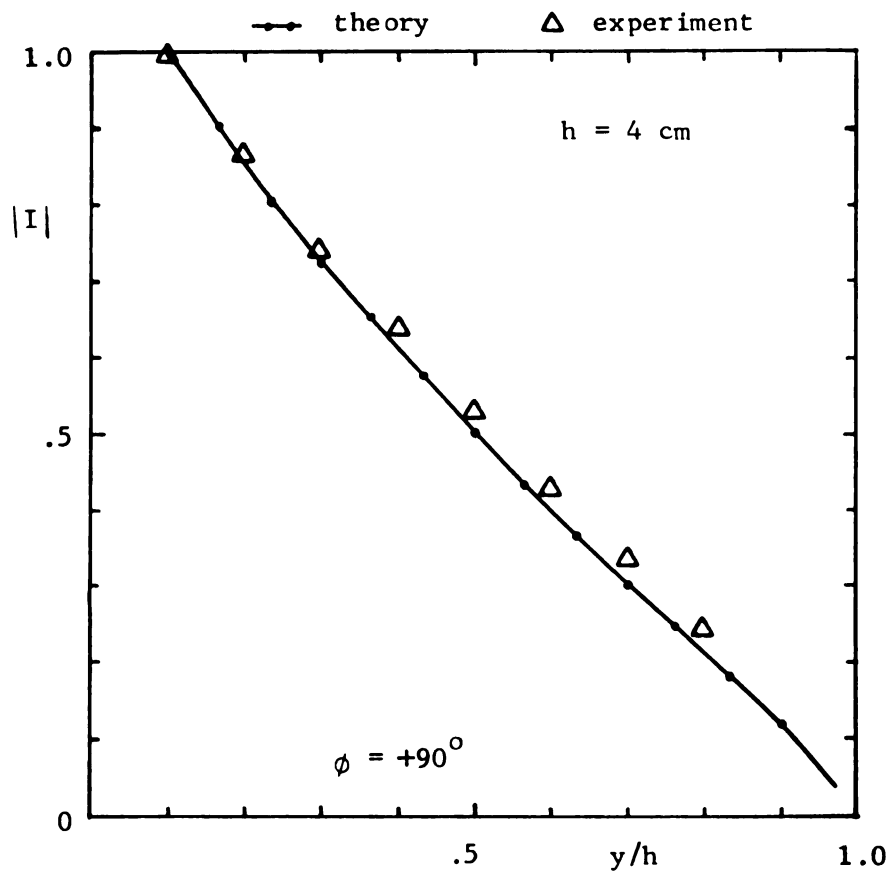


FIGURE 5.19

Current Distribution as a Function of Antenna Height
 ($a = 16.51 \text{ cm}$, 1.2 GHz)

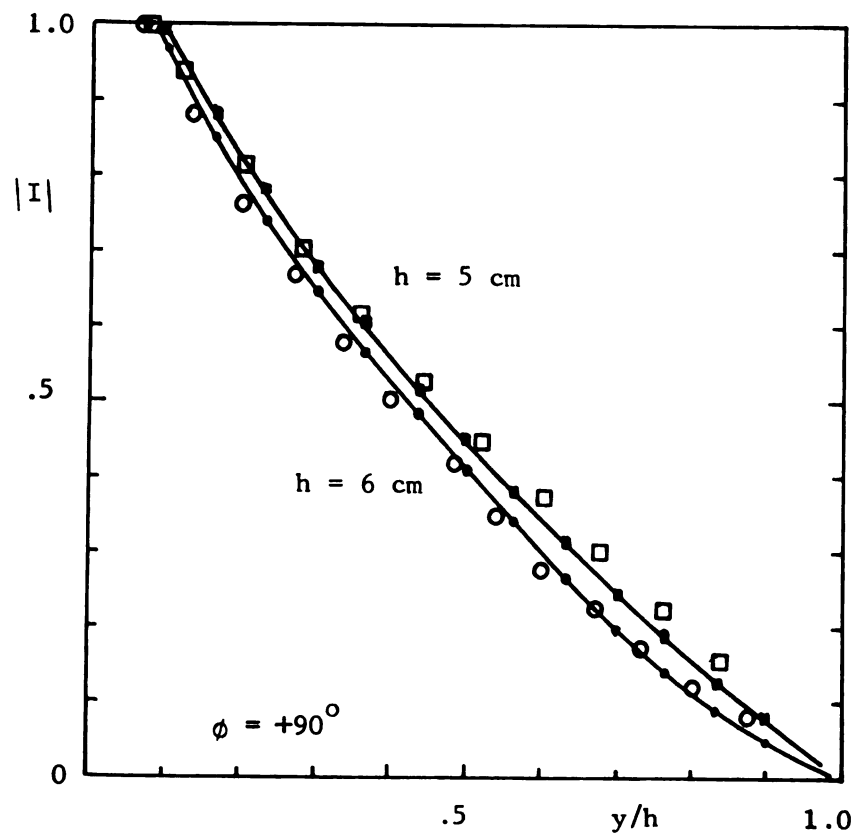
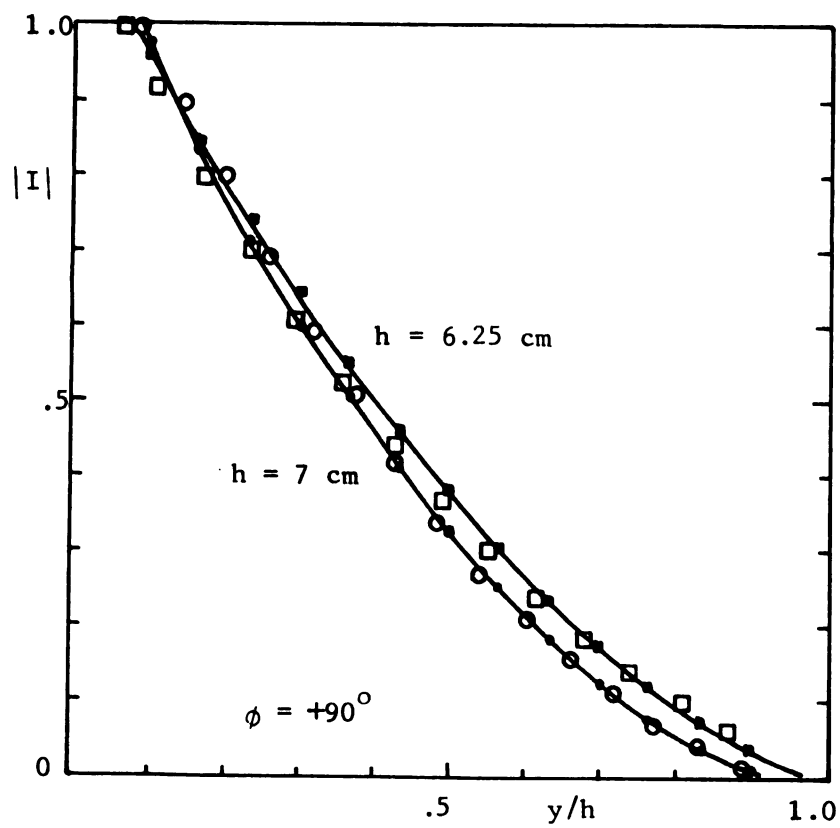
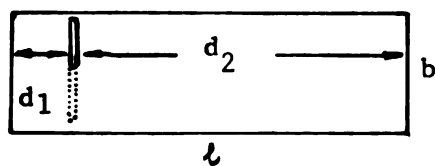


FIGURE 5.19
(concluded)





$$a = 2b = 16.51 \text{ cm}$$

$$l = 60.96 \text{ cm}$$

$$d_1 = 0.133l$$

h = parameter

$$f = 1.3 \text{ GHz}$$

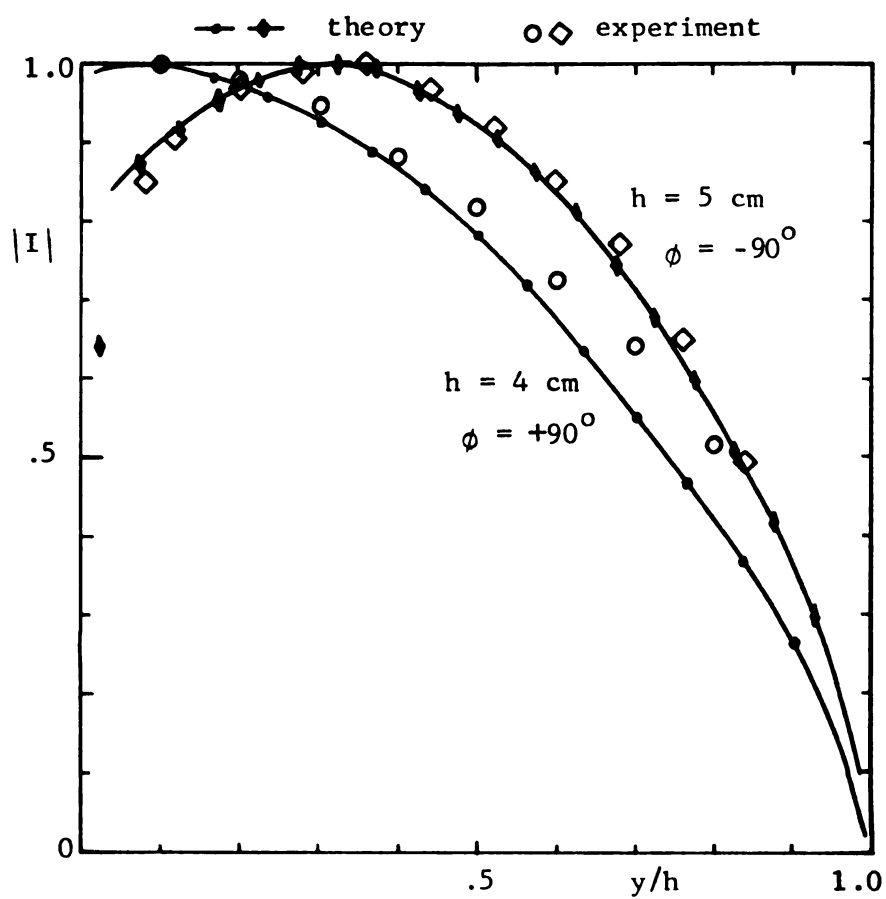


FIGURE 5.20

Current Distribution as a Function of Antenna Height
($a = 16.51 \text{ cm}$, 1.3 GHz)

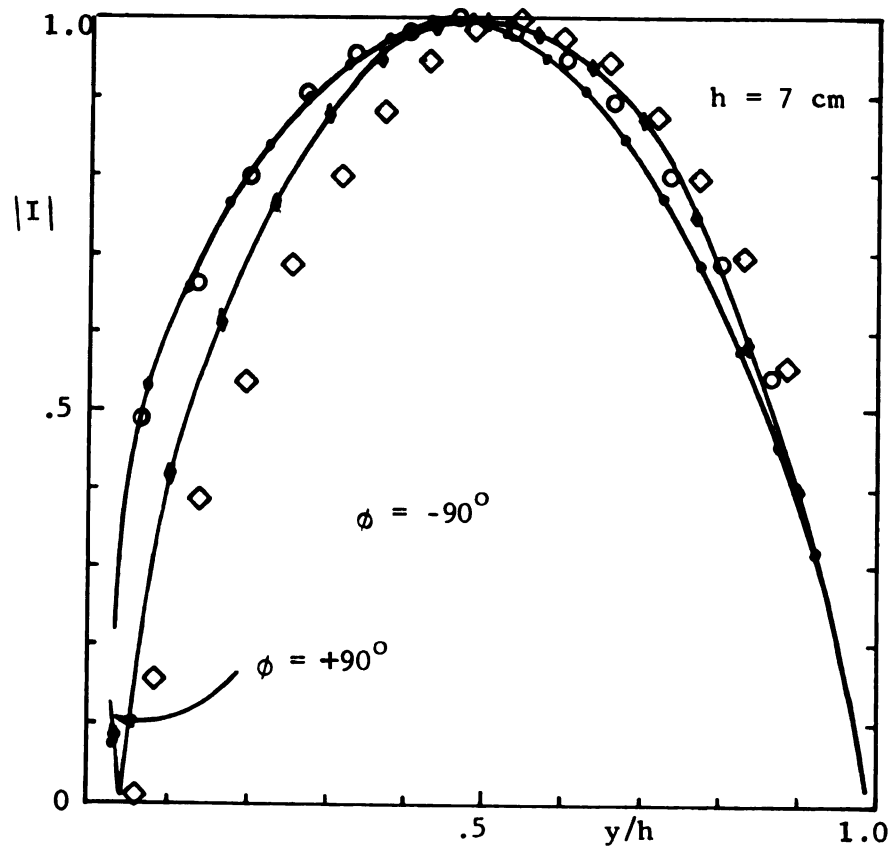
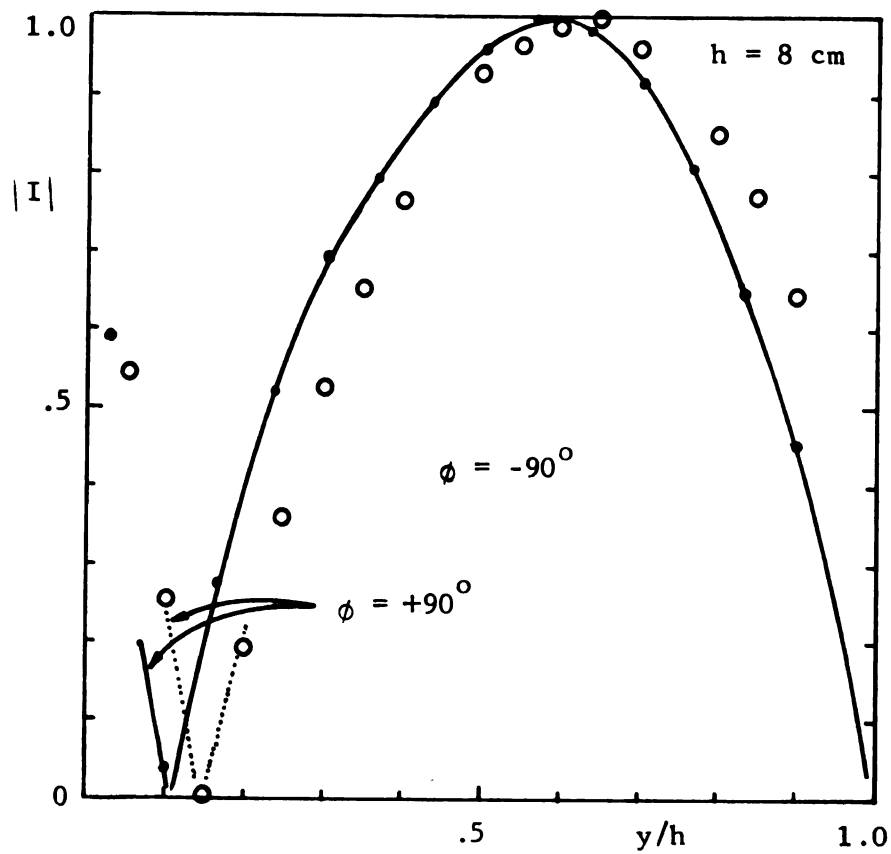


FIGURE 5.20
(concluded)



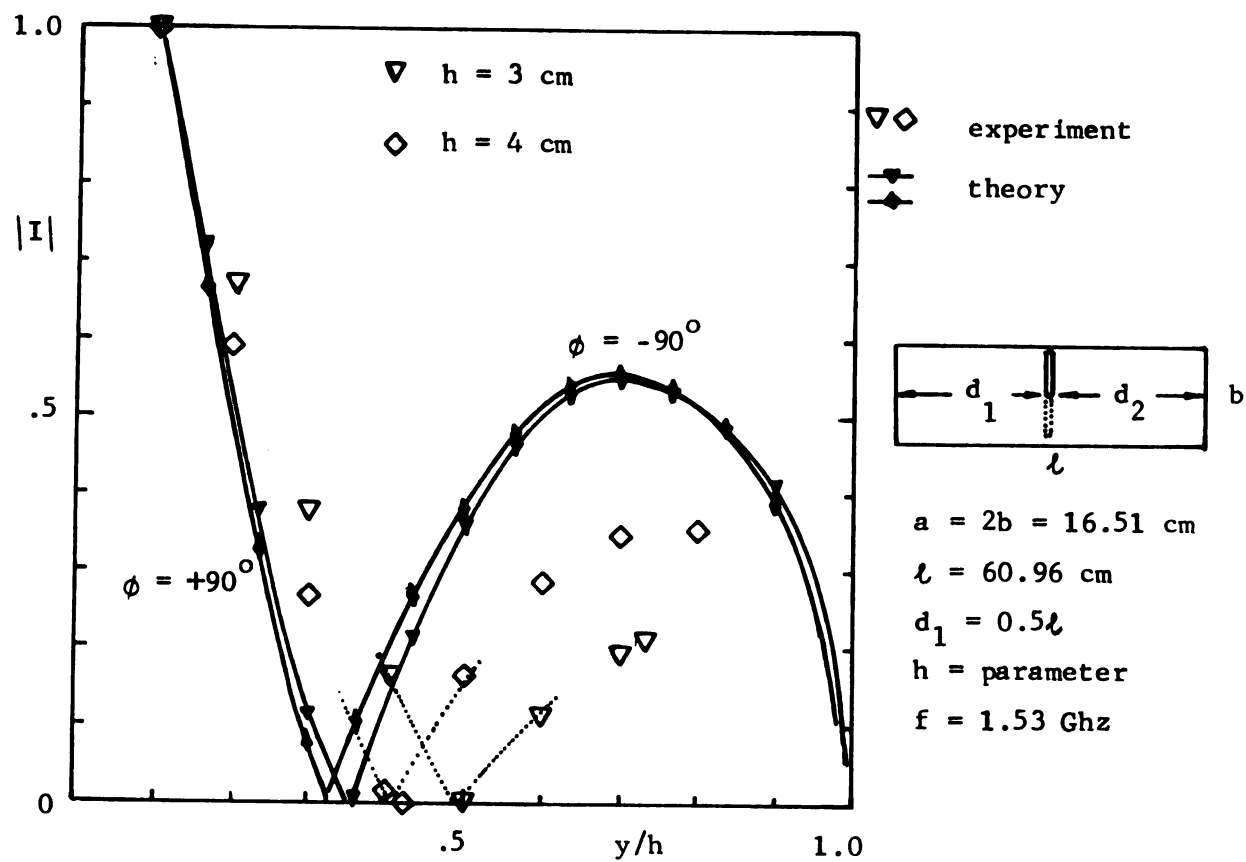
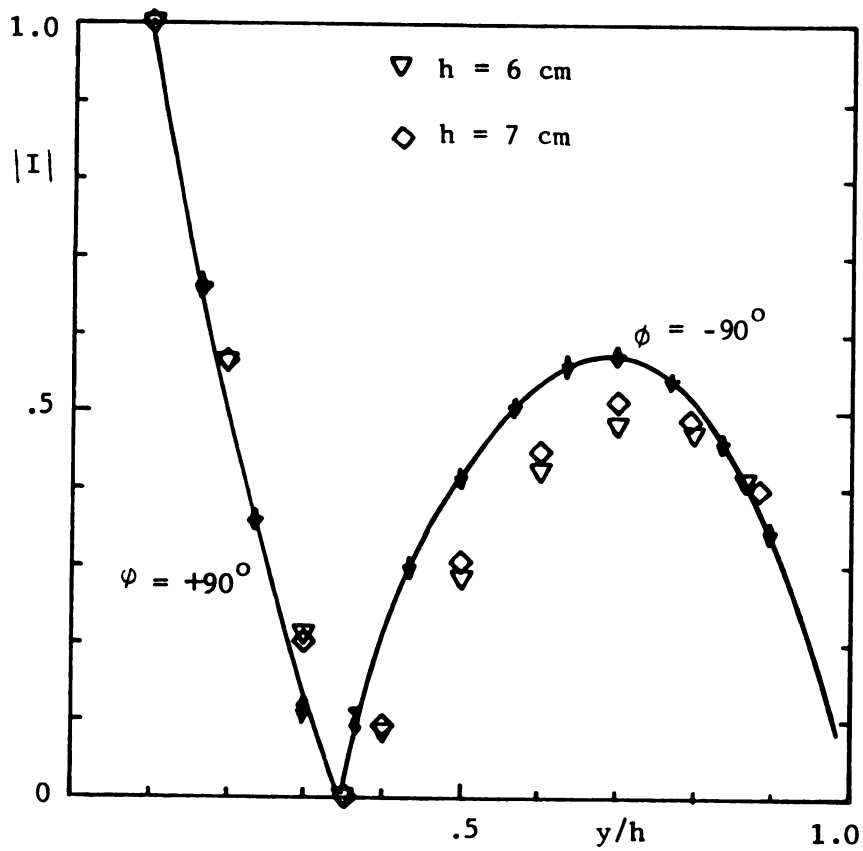


FIGURE 5.21



Current Distribution
as a Function of
Antenna Height
($a = 16.51 \text{ cm}$,
 1.53 GHz)

5.2.3 Air Filled Cavity; Charge Distribution

The distribution of charge on the antenna is determined numerically as the derivative of the current based on the continuity equation (see Section 3.1.5). Experimental methods of measuring relative charge distribution are discussed in relation to Figure 4.5 and in Section 4.2. Since the current distribution is taken to be the fundamental result, very few experimental charge distributions are presented here. In Figures 5.22 and 5.23 the results for several different frequencies with two different physical configurations are shown. One experimental factor which is unique to this charge distribution data is that the dielectric support wafer at the antenna terminal plane was made of nylon (replaced by styrofoam for the current and admittance measurements). This results in large experimental charge at the input terminals because of the build up of charge on the dielectric. Therefore, the experimental charge measurements are not normalized to the theoretical point nearest the input plane but instead to the points

$$y/h \approx 0.3 \quad \text{or} \quad y/h \approx 0.5$$

depending on which has the greatest magnitude of the charge. The normalization point chosen is shown in the figures. As can be seen, the overall agreement between theory and experiment is very good. Many of the differences are directly related to the errors found in the current distributions.

One observation concerning the numerically determined charge distribution is the oscillation in magnitude which is seen, for example, in Figure 5.22 at 1.5 Ghz. This is a result of the truncation

of the series in the theoretical solution. When the series is truncated prematurely by just one or two modes, this oscillation appears. This observation was subsequently used as a qualitative measure of the appropriateness of the chosen truncation point and adjustments were made if the oscillations were large. It is perhaps interesting to note that these oscillations occur only toward the tip of the antenna -- the worse the truncation point, the closer the oscillations came to the driving point.

Another interesting point concerning the theoretical charge is the values predicted at the driving point (the first numerical subsection) and at the tip (the last subsection). These are included in Figures 5.22 and 5.23 where possible. The jump in magnitude at the driving point is a result of the anomolous rise in current predicted by the delta-gap model as discussed in Section 5.1.1. The rise in magnitude of charge at the tip has been observed by others (see King and Harrison [18]). It was not physically possible to experimentally verify this charge build up on the antenna tip because of the presence of the current probe (see Figure 4.5).

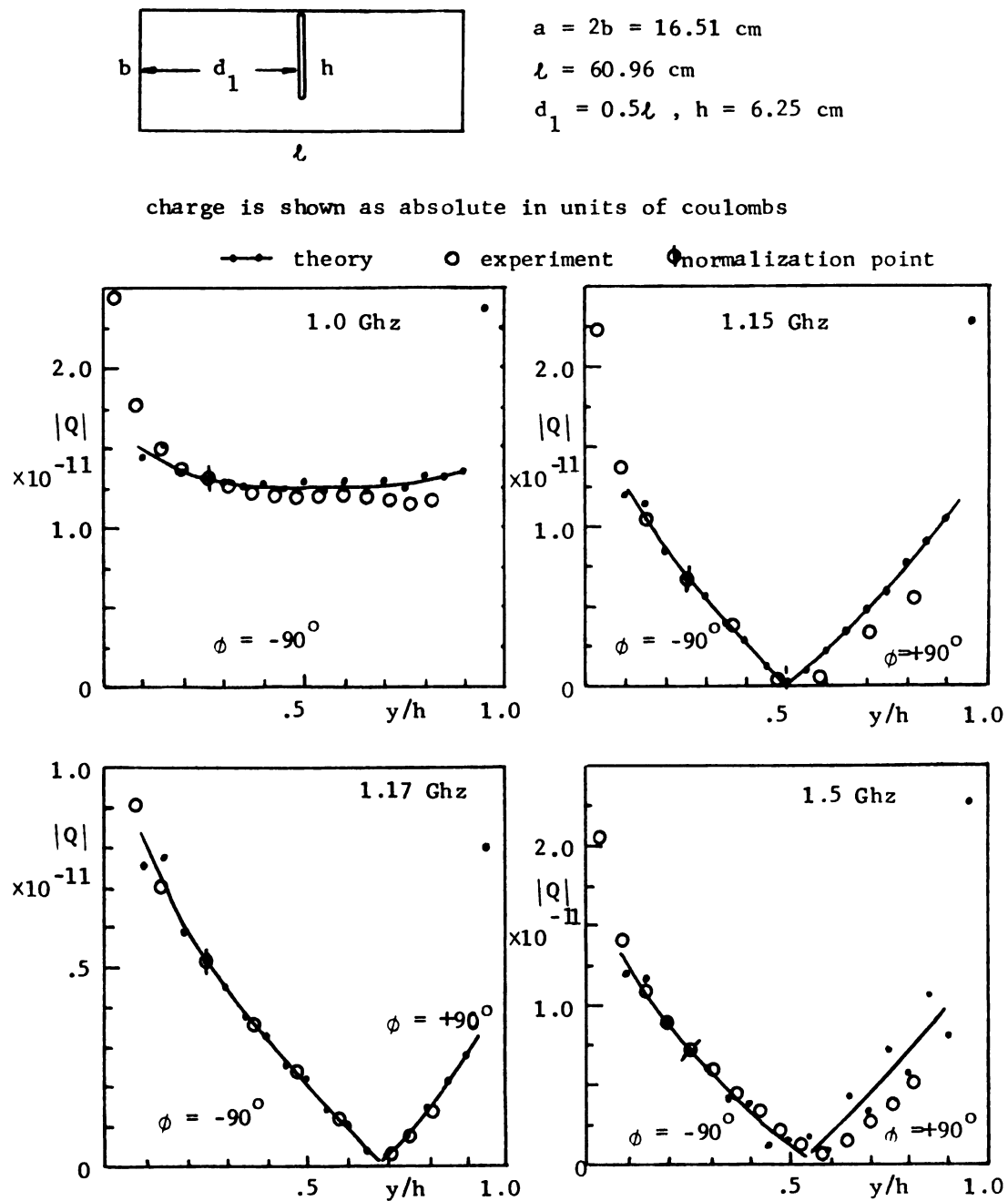


FIGURE 5.22
 Charge Distribution as a Function of Frequency
 ($d_1 = 0.5l, h = 6.25 \text{ cm}$)

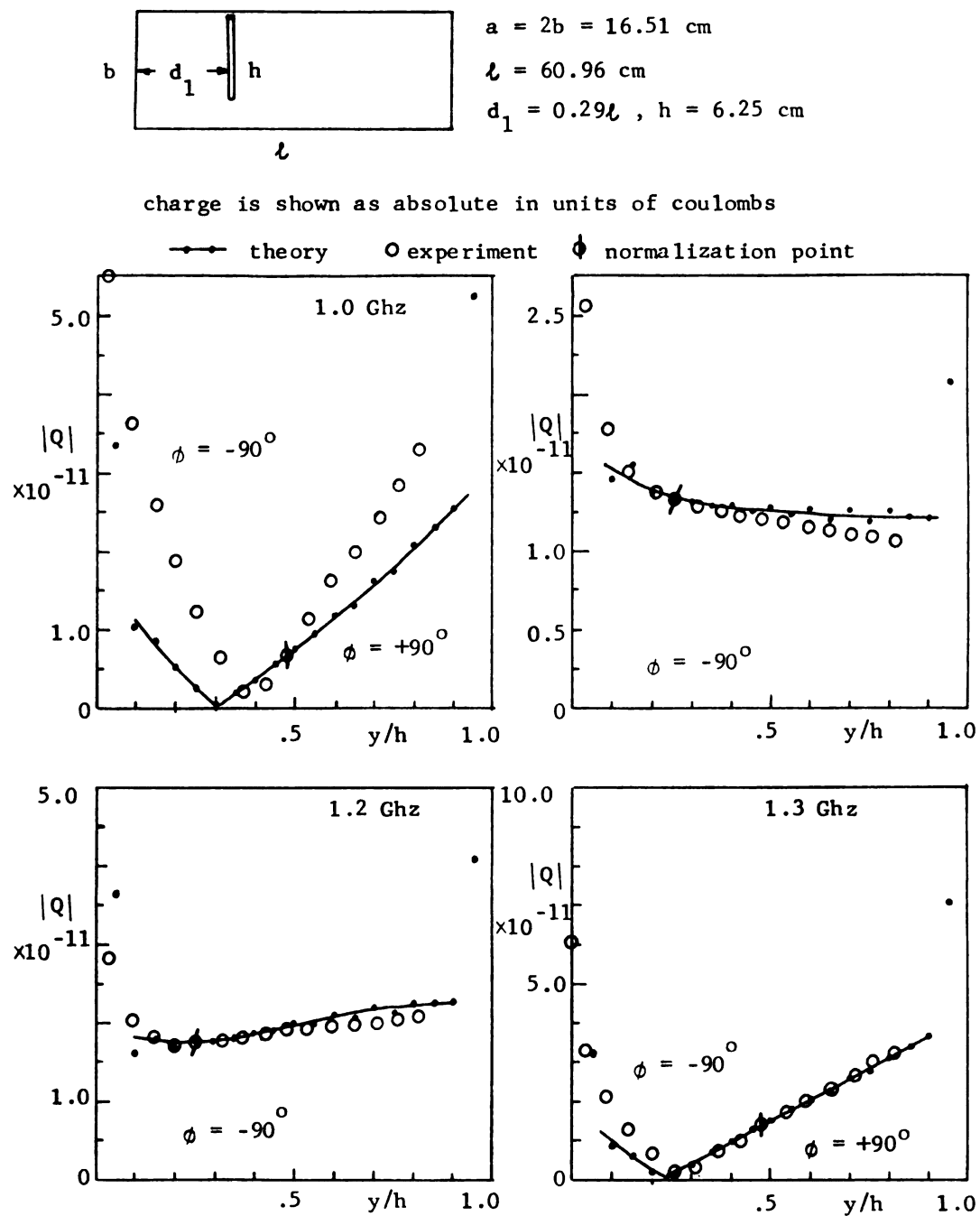


FIGURE 5.23

Charge Distribution as a Function of Frequency
 ($d_1 = 0.29l, h = 6.25 \text{ cm}$)

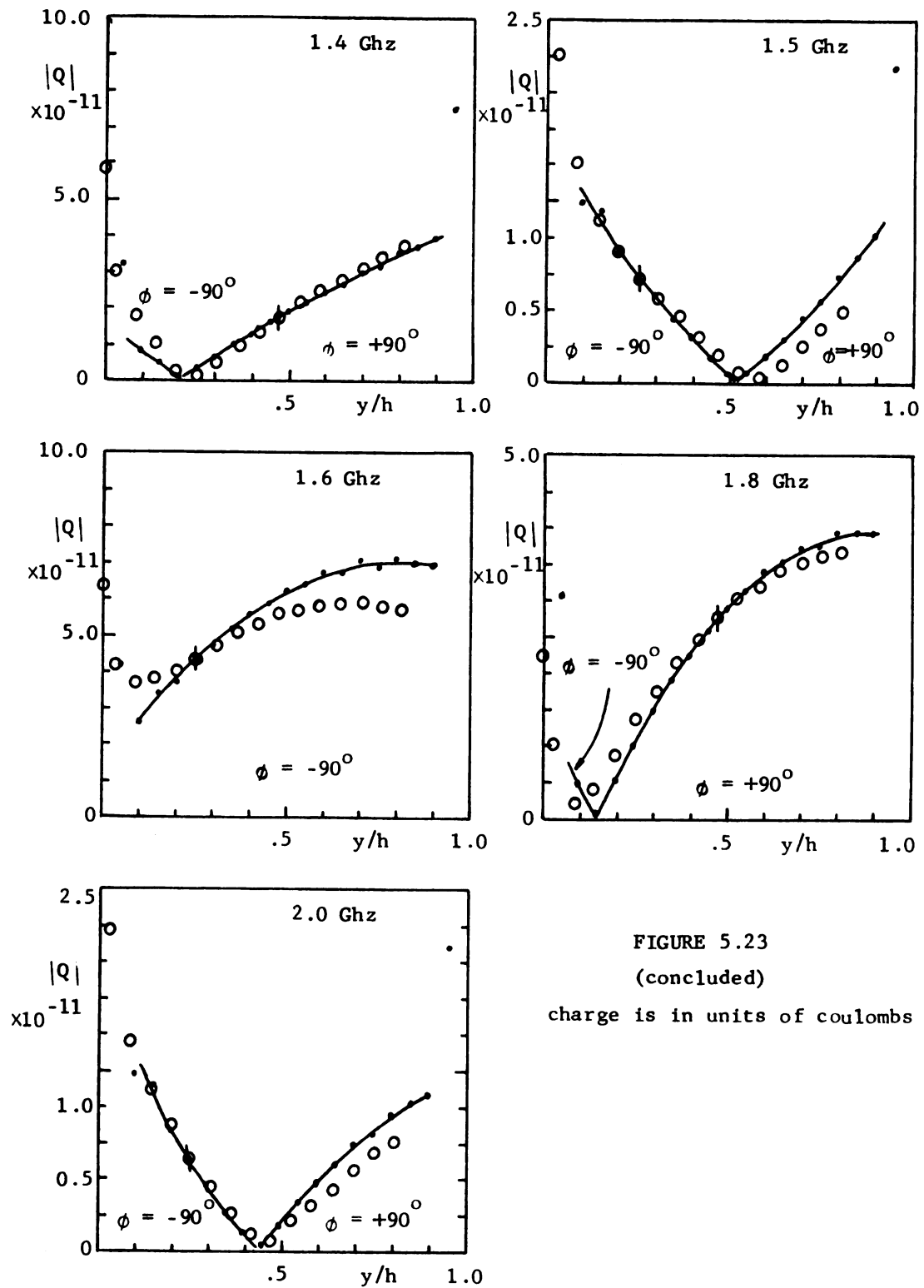


FIGURE 5.23

(concluded)

charge is in units of coulombs

5.2.4 Air Filled Cavity; Input Admittance

The input admittance of a single antenna in an air filled cavity for all the cases considered in Section 5.2.2 is presented in this section. The experimental data is in unaltered and uncompensated form for comparison with the theoretically predicted values. The comparative tables are followed by corresponding curves showing the theoretical admittance for each case. Where possible, the discrete theoretical points are connected by approximate lines to indicate the qualitative dependence predicted. These dashed lines are meant only to aid in visualizing overall behavior so that various phenomena such as resonance and anti-resonance can be readily identified. Extrapolation of the discrete theoretical results to form these curves is based on the known or expected continuity of the solution and such fundamental results as the Foster reactance theorem.

Since the cases considered here are only for the cavity configuration and a single antenna, there is no theoretical mechanism for loss (all conductors are assumed perfect). Therefore, the input admittance will be purely susceptive. Although conduction losses are present in the experimental system, these were not detectable within the $\pm 2^\circ$ accuracy of the admittance measuring system used.

Input Admittance with Frequency as the Parameter

As was stated in Section 5.2.2, the dominant factor in determining the behavior of the antenna-cavity-media system is the resonant nature of the cavity. The "cyclic" nature of the current distribution which is observed as the frequency is swept through a

cavity resonance is also seen in the input admittance. This data is shown in Tables 5.6 and 5.7. In the corresponding Figures 5.24 and 5.25 the theoretical admittance is plotted along with the appropriate approximate current distributions. In these figures the "cycle" in the shape of the current distribution is seen to have a unique relationship to the resonances and anti-resonances of the system. If the system is visualized as an equivalent parallel (lossless) LC resonant network then resonance is defined as the point at which the susceptance is zero and anti-resonance as the point at which the susceptance passes through infinity. This definition is chosen, since the system resonance falls very near the theoretical empty cavity resonance considered in each of the figures. As can be seen in Figures 5.24 and 5.25 the current distribution at anti-resonance is similar to a quarter wave cosinusoidal distribution, while at resonance it is similar to a half-wave sinusoid.

The input admittance over the frequencies between 0.6 Ghz and 2.2 Ghz for a cavity of width $a = 16.51$ cm is shown in Table 5.8 and Figure 5.26. In Figure 5.26 the resonant nature of the admittance is evident. Comparison with the current distributions presented previously in Figure 5.10 shows the correlation between the shape of the current and the cyclic nature of the admittance as depicted in the composite Figure 5.24. The theoretical empty cavity resonances from Table 5.4 are also included in Figure 5.26. Since the case shown is for a centered ($d_1 = 0.5\ell$) antenna, only the odd " ℓ " modes are excited and these are shown as solid triangles in the figure. The even " ℓ " mode resonances are included as open triangles.

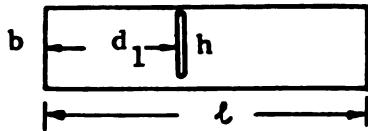
In Table 5.9 and Figure 5.27 similar results are presented for a centered antenna ($d_1 = 0.5\ell$) but for a wider cavity ($a = 22.6$ cm). The same correlation between cavity resonances and zeros of the input admittance is shown. As in all the admittance data (see Section 5.1), the theoretical and experimental data show notable discrepancies in magnitude. However, the behavior as a function of frequency is qualitatively comparable.

The input admittance with frequency as a parameter is lastly considered for the case of the antenna off-center ($d_1 = 0.29\ell$) in Table 5.10 and Figure 5.28. Here the antenna couples to all cavity resonances and the cyclic nature of the impedance is seen to repeat for every " ℓ " mode resonance. Again magnitude discrepancies between theory and experiment are found but in view of the rapidly varying nature of the admittance, the agreement in overall behavior is very satisfactory.

TABLE 5.6

Input Admittance as a Function of Frequency Near a Cavity Resonance

($a = 16.51$ cm , $d_1 = 0.5\ell$)



$a = 2b = 16.51$ cm, $\ell = 60.96$ cm

$d_1 = 0.5\ell$, $h = 6.25$ cm

current distribution in Figure 5.8

frequency, Ghz	Input Admittance, millimhos	
	Theory	Experiment
1.0	+ j5.6	+ j13
1.05	+ j21.3	-
1.07	- j121.4	-
1.1	- j3.26	- j10.1
1.15	+ j0.44	-
1.17	+ j1.25	-
1.2	+ j4.45	+ j7.3
1.25	+ j8.6	-

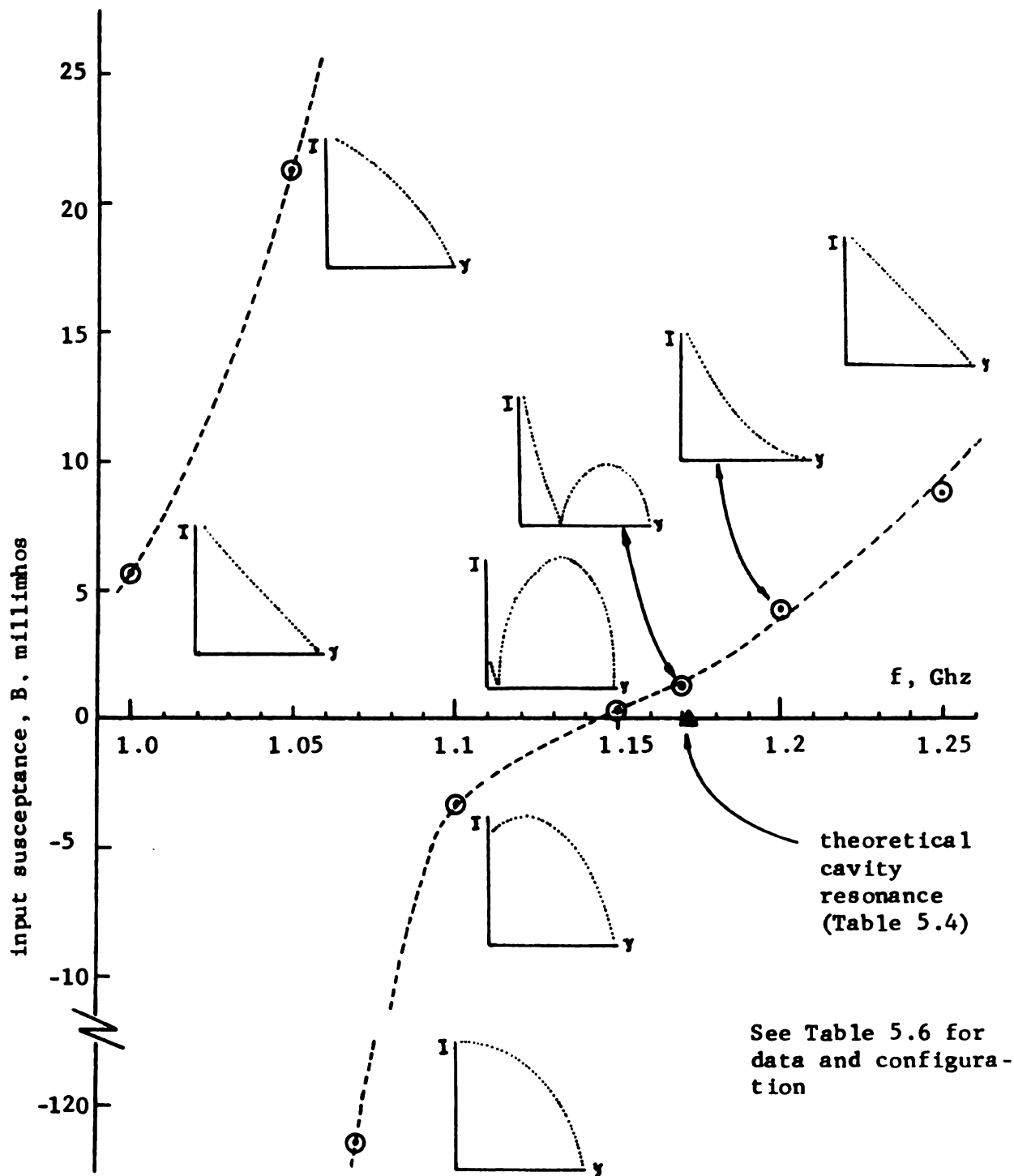


FIGURE 5.24

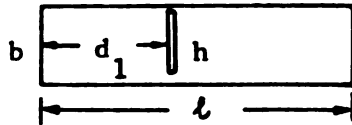
Theoretical Input Admittance (Susceptance) as a Function of Frequency Near a Cavity Resonance

($a = 16.51$ cm, $d_1 = 0.5\ell$)

TABLE 5.7

Input Admittance as a Function of Frequency Near a Cavity Resonance

$$(a = 22.6 \text{ cm}, d_1 = 0.5\ell)$$



$$a = 22.6 \text{ cm}, b = 8.255 \text{ cm}, \ell = 60.96 \text{ cm}$$

$$d_1 = 0.5\ell, h = 6.25 \text{ cm}$$

current distribution in Figure 5.9

frequency, Ghz	Input Admittance, millimhos	
	Theory	Experiment
.85	+ j8.45	+ j19.4
.9	+ j21.88	+ j105
.95	- j5.15	- j8.26
.975	+ j.165	+ j.856
1.0	+ j2.65	+ j5.13
1.05	+ j6.08	+ j13.7

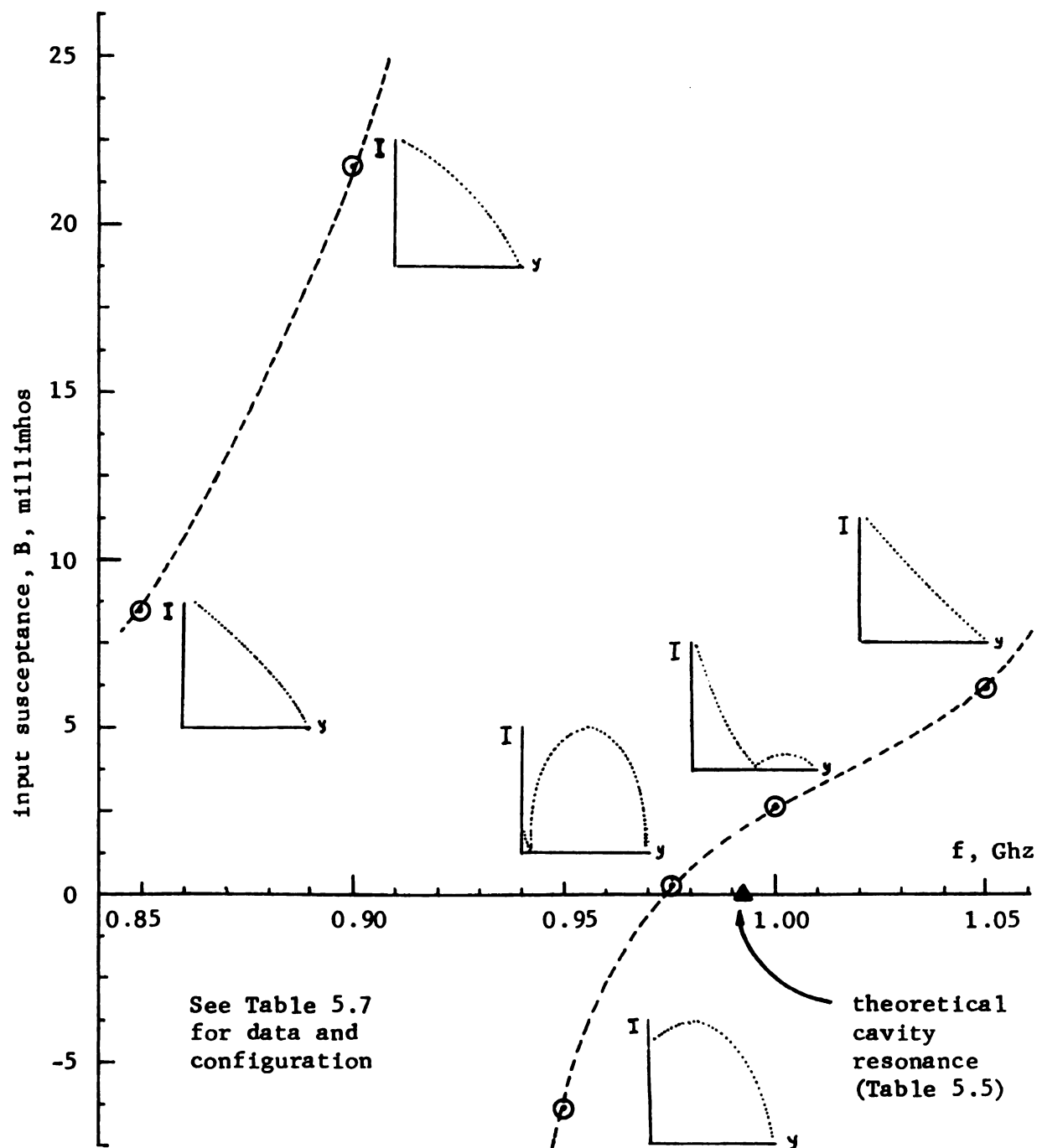


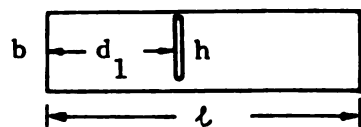
FIGURE 5.25

Theoretical Input Admittance (Susceptance) as a Function of Frequency Near a Cavity Resonance

($a = 22.6$ cm , $d_1 = 0.5\lambda$)

TABLE 5.8

Input Admittance as a Function of Frequency 0.6 - 2.2 Ghz

(a = 16.51 cm, $d_1 = 0.5\ell$)a = 2b = 16.51 cm, $\ell = 60.96$ cm $d_1 = 0.5\ell$, h = 6.25 cmcurrent distribution shown in Figure 5.10
resonances in Table 5.4

frequency, Ghz	Input Admittance, millimhos	
	Theory	Experiment
.6	+ j4.73	-
.7	+ j7.5	+ j18.9
.8	+ j31.65	-
.85	- j12.67	-
...	... see Table 5.6 ...	
1.3	+ j24.6	+ j44
1.4	- j6.8	- j10.8
1.5	+ j.77	+ j2.79
1.6	+ j18.7	+ j28
1.7	- j6	- j17.3
1.8	- j1.4	- j2.83
1.9	+ j1.27	+ j3.64
2.0	- j18.6	-
2.2	+ j.187	-

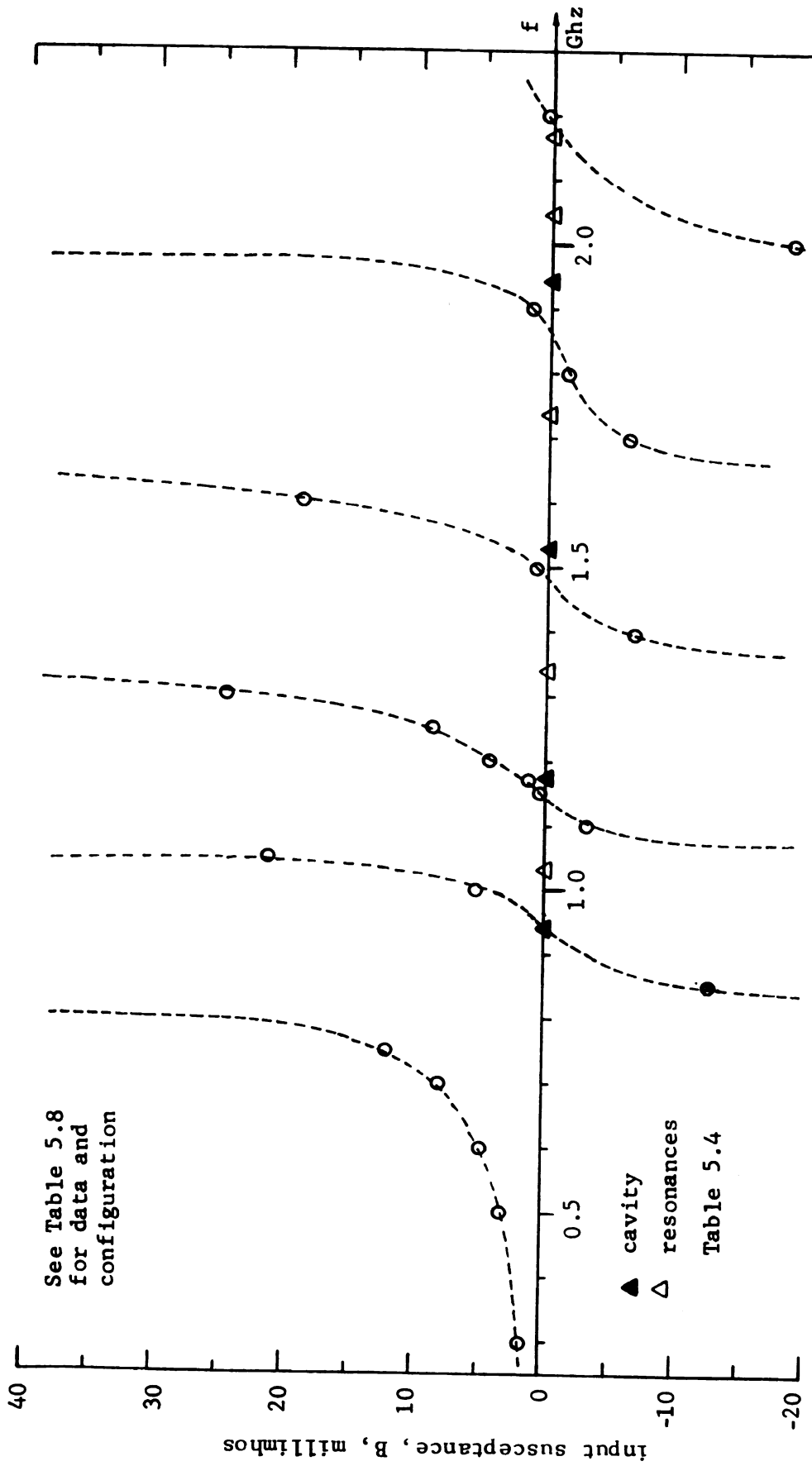
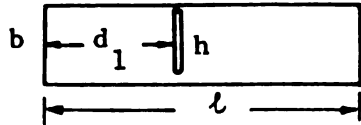


FIGURE 5.26

Theoretical Input Admittance as a Function of Frequency 0.3 - 2.2 Ghz
($a = 16.51$ cm, $d_1 = 0.5\lambda$)

TABLE 5.9

Input Admittance as a Function of Frequency 0.85 - 1.9 Ghz

(a = 22.6 cm , $d_1 = 0.5\ell$)

$a = 22.6$ cm, $b = 8.255$ cm, $\ell = 60.96$ cm
 $d_1 = 0.5\ell$, $h = 6.25$ cm

current distributions in Figure 5.11

frequency, Ghz	Input Admittance, millimhos	
	Theory	Experiment
0.85	+ j8.45	+ j19.4
0.90	+ j21.88	+ j105
0.95	- j5.15	- j8.26
0.975	+ j0.165	+ j0.86
1.0	+ j2.65	+ j5.13
1.05	+ j6.08	+ j13.7
1.1	+ j10.17	-
1.2	+ j427	-
1.3	- j5.52	- j10
1.4	+ j3.22	+ j8.06
1.5	- j49.5	- j102.5
1.6	- j4.33	- j5.52
1.7	- j0.78	- j1.90
1.8	+ j1.74	+ j2.61
1.9	- j2.41	- j5.07

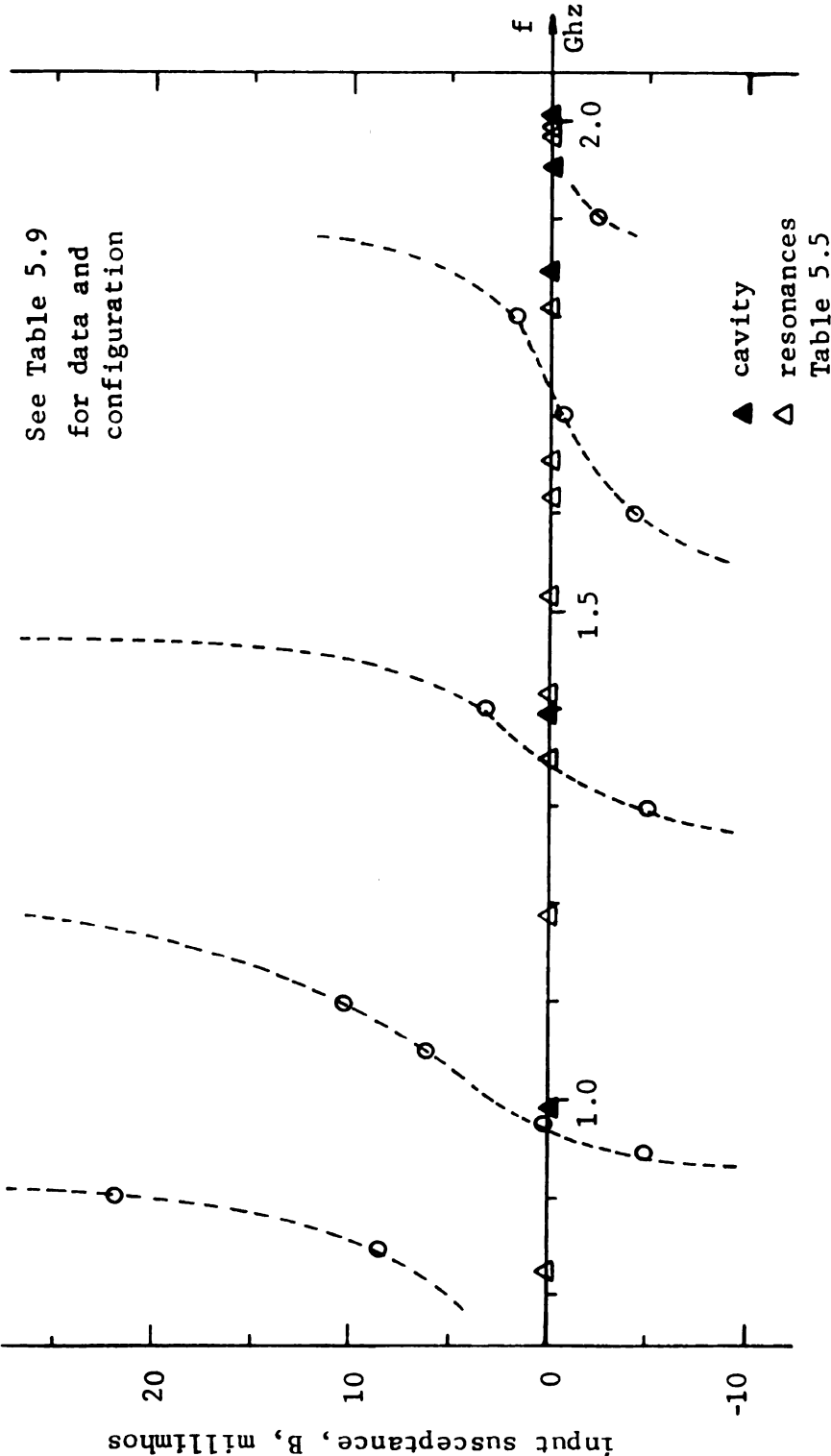
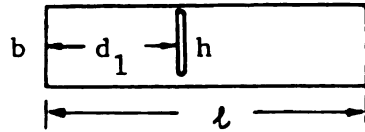


FIGURE 5.27

Theoretical Input Admittance as a Function of Frequency 0.85 - 1.9 Ghz
($a = 22.6$ cm, $d_1 = 0.5\lambda$)

TABLE 5.10

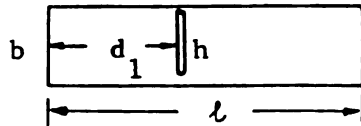
Input Admittance as a Function of Frequency 0.6 - 2.2 Ghz

(a = 16.51 cm, $d_1 = 0.29\ell$) $a = 2b = 16.51$ cm, $\ell = 60.96$ cm $d_1 = 0.29\ell$, $h = 6.25$ cm

current distributions in Figure 5.12

frequency, Ghz	Input Admittance, millimhos	
	Theory	Experiment
0.60	+ j4.72	-
0.70	+ j6.70	+ j18.0
0.80	+ j26.8	-
0.90	- j2.14	-
0.95	+ j2.95	-
1.00	- j2.90	- j3.05
1.05	+ j3.24	-
1.10	+ j5.98	+ j12.9
1.15	+ j57.0	-
1.16	- j7.22	-
1.17	+ j1.7	-
1.20	+ j9.7	+ j20.4
1.25	+ j289	-
1.27	- j33.2	-
1.30	- j7.43	- j11.5
1.35	+ j6.84	-
1.37	+ j53	-

(continued)

TABLE 5.10
(concluded)

$$a = 2b = 16.51 \text{ cm}, \quad l = 60.96 \text{ cm}$$

$$d_1 = 0.29l, \quad h = 6.25 \text{ cm}$$

frequency, Ghz	Input Admittance, millimhos	
	Theory	Experiment
1.4	- j9.42	- j16.9
1.5	+ j0.78	+ j3.14
1.52	+ j2.75	-
1.6	+ j35.5	+ j41.4
1.65	- j8.35	-
1.7	- j0.92	- j1.05
1.75	+ j7.28	-
1.8	- j16	- j69.0
1.9	- j1.8	- j5.05
1.93	- j1.16	-
2.0	+ j0.34	-
2.05	- j1.59	-
2.2	- j91.0	-

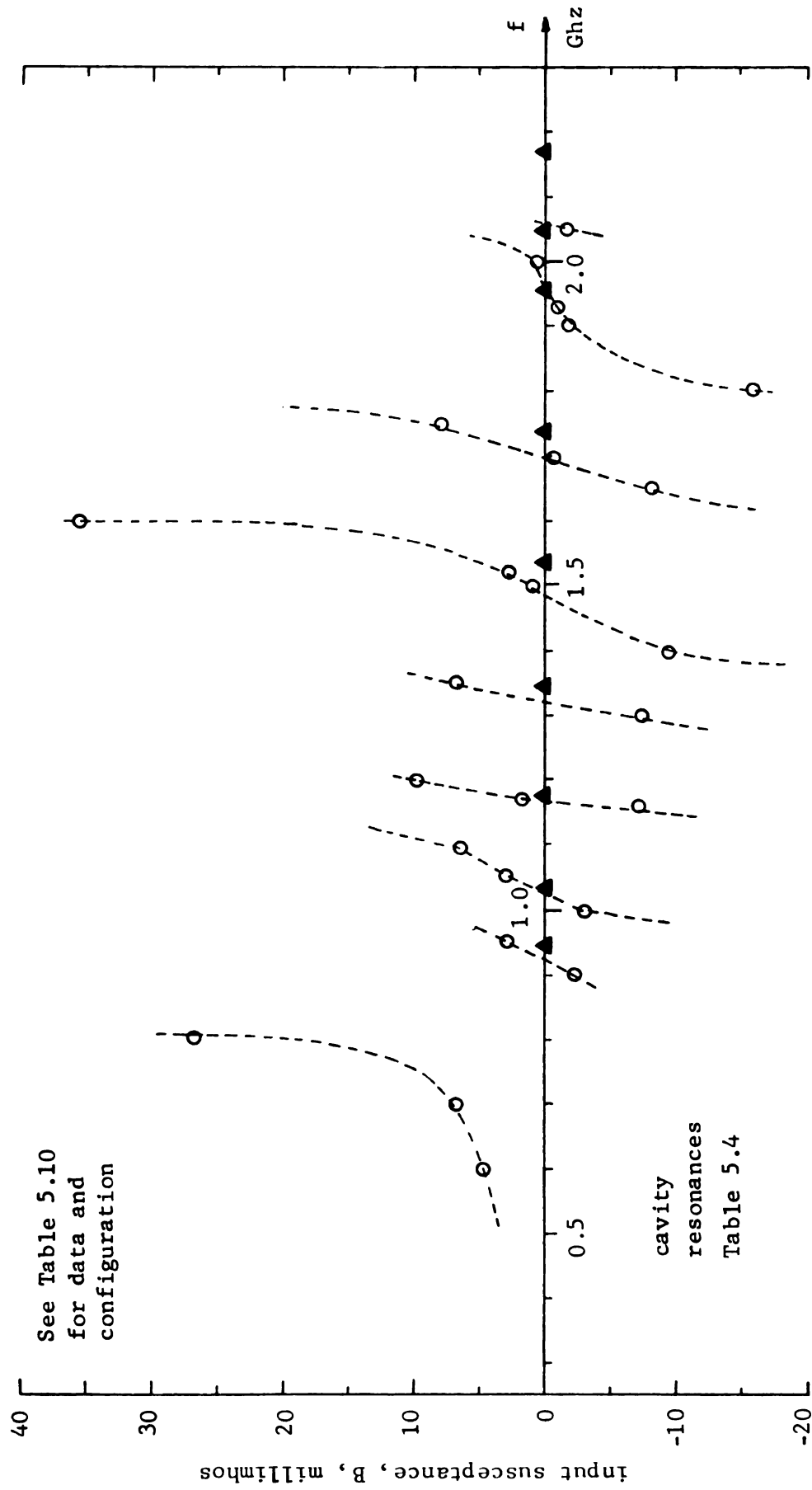


FIGURE 5.28

Theoretical Input Admittance as a Function of Frequency 0.6 - 2.2 GHz

(a = 16.51 cm, $d_1 = 0.29\lambda$)

Input Admittance with Antenna Location as the Parameter

As the location of the antenna in the cavity is varied, the degree of coupling to cavity resonances differs. This (as shown in Section 5.2.2) affects the general nature of the current distribution. The input admittance also varies and the coupling effect is more obvious. For the same cases discussed in relation to the current distribution the theoretical and experimental input admittance values are compared in Tables 5.11 through 5.14. As in other cases there is a sizeable difference in magnitude but both theory and experiment display the same behavior as the location is varied. The theoretical values are plotted for each case considered in Figures 5.29 through 5.32. The lines drawn in the figures are only approximate to indicate the general behavior of the admittance. The indication that the susceptance passes through infinity in some cases (e.g. Figure 5.29) is based upon the continuity and smoothness expected of the admittance or impedance.

The shape of the admittance as a function of location in the cavity can be related easily to the distribution of fields in the cavity. In Figure 5.29, for example, the operating frequency of 1.2 Ghz is very near the ideal cavity resonance $f_{1,0,3} = 1.171$ Ghz. This resonance exhibits zeros in the $\hat{y}$ -component of electric field at locations $z/\ell = 0.333$ and $z/\ell = 0.667$. These locations also correspond to points of maximum magnetic field. The input admittance shown in Figure 5.29 readily demonstrates the change from electric (capacitive) to magnetic (inductive) coupling as the location is varied. The magnitude of the admittance is related to the degree of coupling; the lower values of admittance correspond to the locations of maximum coupling.

The cavity fields (steady state) at frequencies off resonance are ideally zero for the case of the perfectly conducting boundaries considered here, but the dominant evanescent fringing fields are apparently related to the nearby resonances.

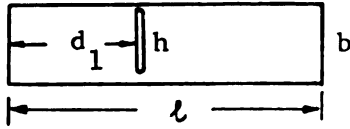
The change from capacitive to inductive is not demonstrated in all cases considered, since the value of admittance depends on the electrical properties of the antenna as well as the cavity. However, in all the figures there is a direct correlation to the distribution of fields for the closest resonance.

One observation that can be made from these results is that the sensitivity to variations in location is related to the nearness of the operating frequency to the resonant frequencies of the cavity-antenna system. Near a resonance (e.g., Figure 5.31) the admittance is relatively insensitive to location over portions of the cavity, whereas farther from a resonance (e.g., Figure 5.32) the value of input admittance varies widely with antenna location. These differences in sensitivity can be exploited to advantage in designing an antenna-cavity system as a tool. These relationships are also important for consideration of coupled antenna systems.

TABLE 5.11

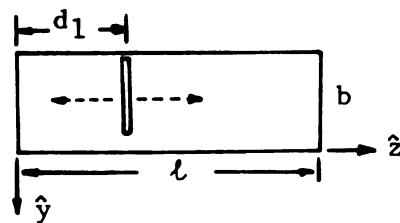
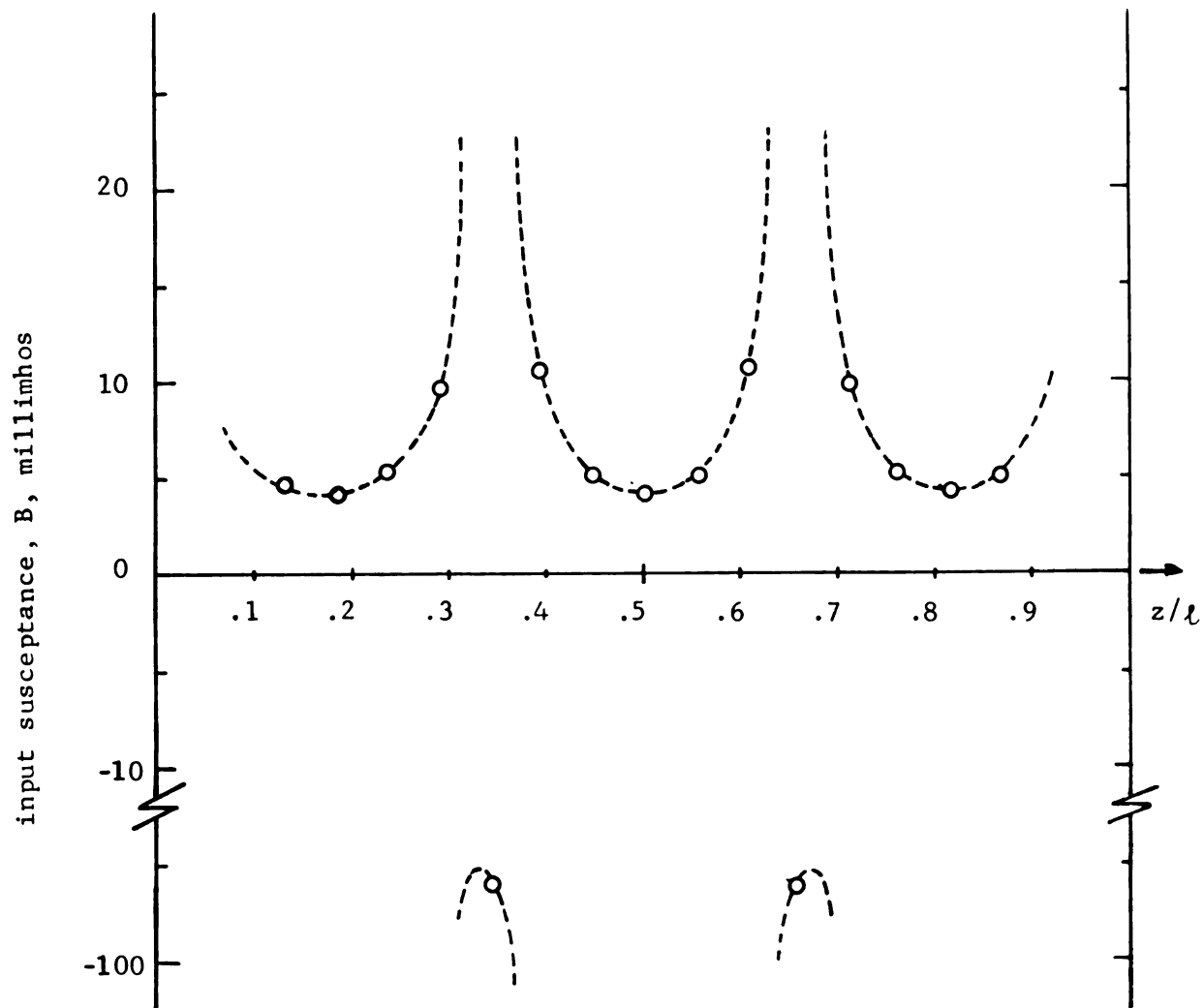
Input Admittance as a Function of Antenna Location

(a = 16.51 cm, 1.2 GHz)

 $a = 2b = 16.51 \text{ cm}$, $l = 60.96 \text{ cm}$ $d_1 = \text{parameter}$, $h = 6.25 \text{ cm}$ $f = 1.2 \text{ GHz}$

current distributions are in Figure 5.13

d_1	Input Admittance, millimhos	
	Theory	Experiment
$0.133l$	$+ j4.82$	$+ j9.9$
$0.185l$	$+ j4.24$	$+ j7.94$
$0.238l$	$+ j5.38$	$+ j8.62$
$0.290l$	$+ j9.7$	$+ j20.4$
$0.343l$	$- j96.1$	$- j125$
$0.396l$	$+ j10.75$	$+ j20.2$
$0.448l$	$+ j5.15$	$+ j9.1$
$0.500l$	$+ j4.46$	$+ j7.3$



$$a = 2b = 16.51 \text{ cm}, \quad l = 60.96 \text{ cm}$$

$$d_1 = \text{parameter}, \quad h = 6.25 \text{ cm}$$

$$f = 1.2 \text{ GHz}$$

data in Table 5.11

FIGURE 5.29

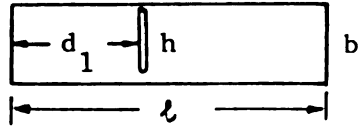
Theoretical Input Admittance as a Function of Antenna Location

($a = 16.51 \text{ cm}$, 1.2 GHz)

TABLE 5.12

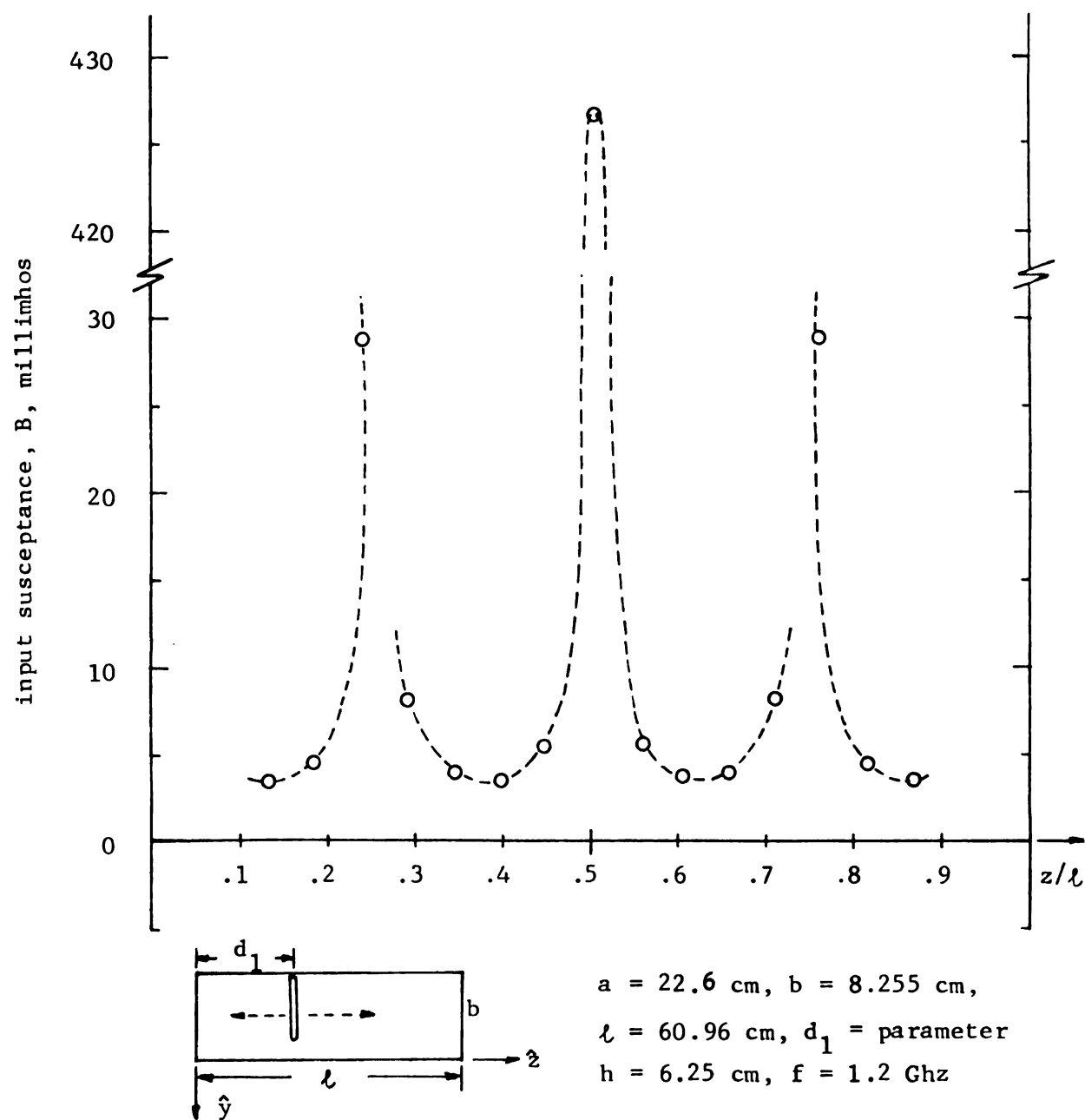
Input Admittance as a Function of Antenna Location

(a = 22.6 cm , 1.2 Ghz)

a = 22.6 cm , b = 8.255 cm, l = 60.96 cm d_1 = parameter, h = 6.25 cm

f = 1.2 Ghz

d_1	Input Admittance, millimhos	
	Theory	Experiment
0.133 l	+ j3.65	+ j7.25
0.185 l	+ j4.45	+ j9.8
0.238 l	+ j27.7	+ j74.0
0.290 l	+ j8.25	+ j16.5
0.343 l	+ j3.92	+ j8.34
0.396 l	+ j3.73	+ j7.75
0.448 l	+ j5.48	+ j11.5
0.500 l	+ j427	- j357



data in Table 5.12

Figure 5.30

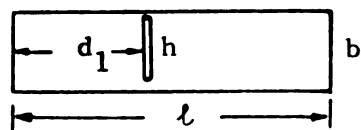
Theoretical Input Admittance as a Function of Antenna Location

($a = 22.6$ cm , 1.2 GHz)

TABLE 5.13

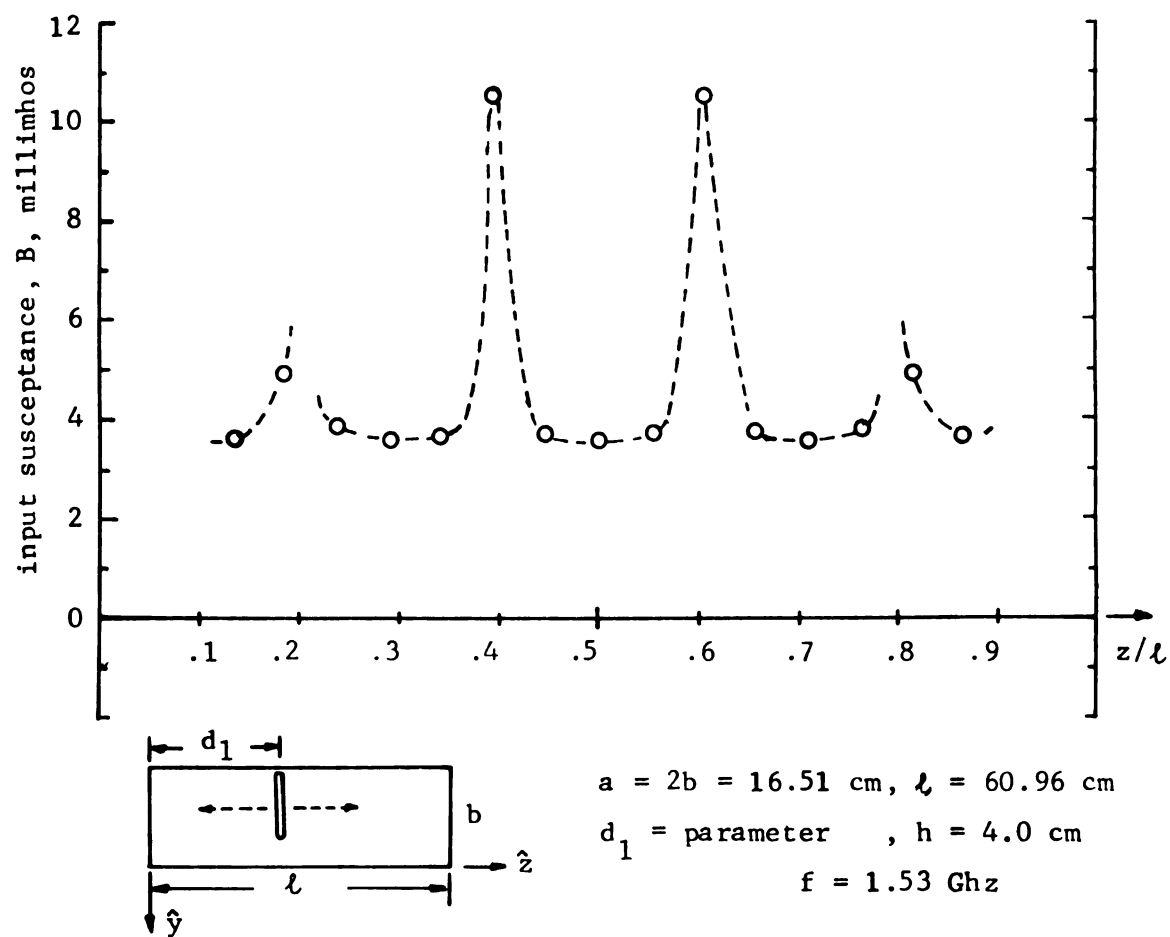
Input Admittance as a Function of Antenna Location

(a = 16.51 cm , 1.53 Ghz)

 $a = 2b = 16.51 \text{ cm}$, $l = 60.96 \text{ cm}$ $d_1 = \text{parameter}$, $h = 4.0 \text{ cm}$ $f = 1.53 \text{ Ghz}$

current distributions in Figure 5.15

d_1	Input Admittance, millimhos	
	Theory	Experiment
$0.133l$	$+ j3.64$	$+ j4.83$
$0.185l$	$+ j4.93$	$+ j12.4$
$0.238l$	$+ j3.80$	$+ j6.85$
$0.290l$	$+ j3.61$	$+ j5.1$
$0.343l$	$+ j3.66$	$+ j5.4$
$0.396l$	$+ j10.58$	$+ j27.8$
$0.448l$	$+ j3.71$	$+ j6.07$
$0.500l$	$+ j3.61$	$+ j5.13$



data in Table 5.13

FIGURE 5.31

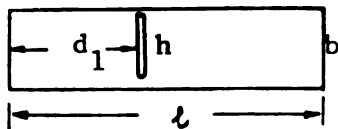
Theoretical Input Admittance as a Function of Antenna Location

($a = 16.51 \text{ cm}, 1.53 \text{ GHz}$)

TABLE 5.14

Input Admittance as a Function of Antenna Location

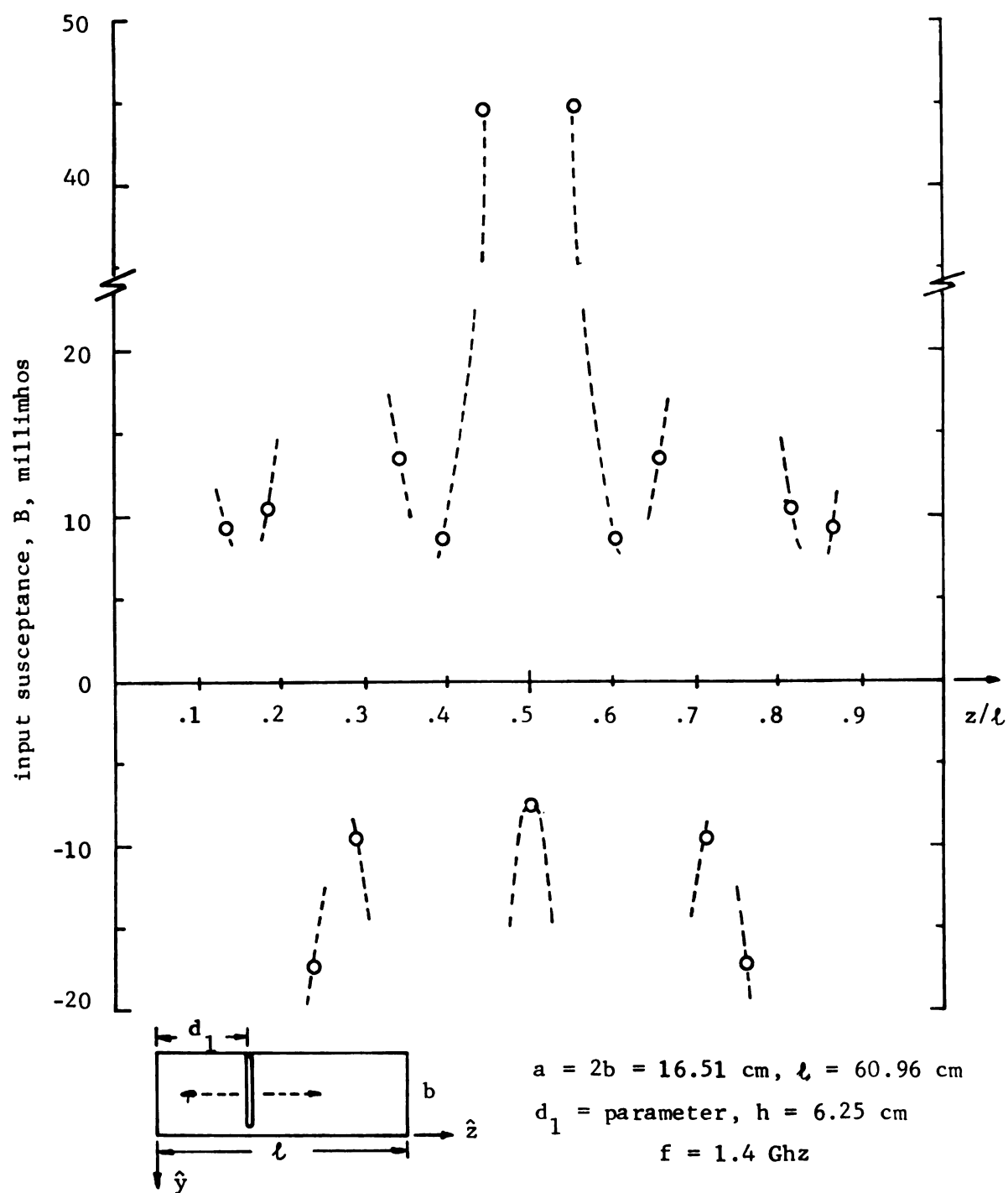
(a = 16.51 cm , 1.4 Ghz)

 $a = 2b = 16.51 \text{ cm}$, $l = 60.96 \text{ cm}$ $d_1 = \text{parameter}$, $h = 6.25 \text{ cm}$ $f = 1.4 \text{ Ghz}$

current distributions in Figure 5.14

d_1	Input Admittance, millimhos	
	Theory	Experiment
$0.133l$	$+ j9.24$	$+ j18$
$0.185l$	$+ j10.5$	$+ j19.8$
$0.238l$	$- j17.3$	$- j27.4$
	$- j17.6 *$	
$0.290l$	$- j9.42$	$- j16.9$
$0.343l$	$+ j13.5$	$+ j25.8$
$0.396l$	$+ j8.62$	$+ j16.5$
	$+ j12.6 *$	
$0.448l$	$+ j44.9$	$+ j171$
	$- j20.4 *$	
$0.500l$	$- j7.16$	$- j12.2$
	$- j9.41 *$	

* Coaxial-Gap theory



data in Table 5.14

FIGURE 5.32

Theoretical Input Admittance as a Function of Antenna Location

($a = 16.51 \text{ cm}, 1.4 \text{ GHz}$)

Input Admittance with Antenna Height as a Parameter

The comparison between theoretical and experimental input admittance as a function of antenna height is given in Tables 5.15 through 5.20. (Because of the small amount of data in each case, the theoretical results have not been plotted.) As usual there is a lack of agreement in absolute magnitude of admittance but good agreement in behavior as the height is varied. Some attempts at correcting the magnitude discrepancies are given in Section 5.1.

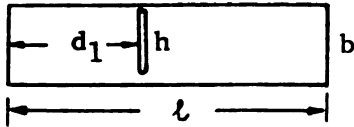
In Table 5.15 both the delta-gap and coaxial gap theories are included for comparison. The coaxial gap theory obviously represents the relatively constant nature of the experimental admittance better than the delta gap theory. The variation in the delta-gap theory results from the change in size of the sampling interval in the numerical solution. That is, as the height is increased, the sampling interval gets longer ($\Delta y = h/K$, $K = \text{constant}$). Attempts were made at maintaining a more constant sampling interval by changing the number of sections as the height changed, but this introduced variations in convergence properties which were less readily predicted and recognized. Nonetheless, the similarity in value of the coaxial gap and delta-gap theories is, of course, satisfactory.

In general the greatest insight into the effect of varying height is gained by comparing the results for admittance with the current distribution. The occurrence of resonances and anti-resonances is then more obvious. It may be noted that there are conditions when the admittance is either insensitive to changes in height or strongly a function of height. As with the sensitivity to location discussed previously this change of sensitivity can be used to advantage depending on the properties desired.

It is also interesting to note that in the case when the current distribution is independent of height (at 1.53 Ghz) the admittance increases with height (compare Table 5.20 and Figure 5.21), whereas when the input admittance is independent of height (at 1.05 Ghz) the current distribution changes to maintain a constant physical location for the zero crossing (see Table 5.15 and Figure 5.16).

TABLE 5.15

Input Admittance as a Function of Antenna Height

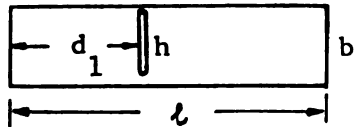
 $(d_1 = 0.238\ell, 1.05 \text{ GHz})$  $a = 2b = 16.51 \text{ cm}, \ell = 60.96 \text{ cm}$ $d_1 = 0.238\ell, h = \text{parameter}$ $f = 1.05 \text{ GHz}$

current distributions in Figure 5.16

h, cm	Input Admittance, millimhos		
	Delta-Gap Theory	Coaxial-Gap Theory	Experiment
4	+ j3.56	+ j3.63	+ j5.73
5	+ j3.56	+ j3.68	+ j5.73
6	+ j3.33	+ j3.63	+ j5.73
7	+ j3.18	+ j3.68	+ j5.73

TABLE 5.16

Input Admittance as a Function of Antenna Height

 $(d_1 = 0.133\ell, 1.1 \text{ GHz}, a = 16.51 \text{ cm})$ 

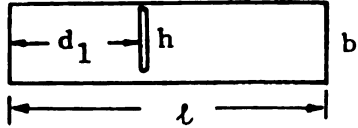
$a = 2b = 16.51 \text{ cm}$, $\ell = 60.96 \text{ cm}$
 $d_1 = 0.133\ell$, $h = \text{parameter}$
 $f = 1.1 \text{ GHz}$

current distributions in Figure 5.17

h, cm	Input Admittance, millimhos	
	Theory	Experiment
4	+ j8.61	+ j21
5	+ j22.2	- j116
6	- j17.6	- j19
6.25	- j10	- j15
7	- j3.95	- j5.4

TABLE 5.17

Input Admittance as a Function of Antenna Height

 $(d_1 = 0.133\ell, 1.1 \text{ GHz}, a = 22.6 \text{ cm})$  $a = 22.6 \text{ cm}, b = 8.255 \text{ cm}, \ell = 60.96 \text{ cm}$ $d_1 = 0.133\ell, h = \text{parameter}$ $f = 1.1 \text{ GHz}$

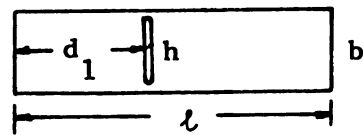
current distributions in Figure 5.18

h, cm	Input Admittance, millimhos	
	Theory	Experiment
4	+ j7.41	+ j15.5
5	+ j11.7	+ j29
6	+ j53.7	+ j740
6.25	- j3660	-
7	- j14.3	- j24.9
8	- j4.13	- j7.38

TABLE 5.18

Input Admittance as a Function of Antenna Height

($d_1 = 0.133\ell$, 1.2 Ghz)



$a = 2b = 16.51 \text{ cm}, \ell = 60.96 \text{ cm}$

$d_1 = 0.133\ell$, $h = \text{parameter}$

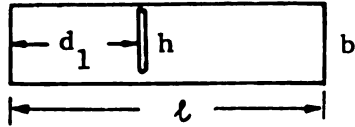
$f = 1.2 \text{ Ghz}$

current distributions in Figure 5.19

Input Admittance, millimhos		
h, cm	Theory	Experiment
4	+ j5.11	+ j9.35
5	+ j4.93	+ j9.80
6	+ j4.84	+ j9.90
6.25	+ j4.82	+ j9.90
7	+ j4.77	+ j9.75

TABLE 5.19

Input Admittance as a Function of Antenna Height

 $(d_1 = 0.133\ell, 1.3 \text{ GHz})$ 

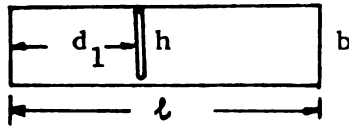
$a = 2b = 16.51 \text{ cm}$, $\ell = 60.96 \text{ cm}$
 $d_1 = 0.133\ell$, $h = \text{parameter}$
 $f = 1.3 \text{ GHz}$

current distributions in Figure 5.20

h, cm	Input Admittance, millimhos	
	Theory	Experiment
4	+ j172	- j152
5	- j5.50	- j7.88
6	- j1.30	- j1.22
7	+ j0.20	+ j1.87
8	+ j1.18	+ j3.36

TABLE 5.20

Input Admittance as a Function of Antenna Height

 $(d_1 = 0.500\lambda, 1.53 \text{ GHz})$  $a = 2b = 16.51 \text{ cm}, \lambda = 60.96 \text{ cm}$ $d_1 = 0.500\lambda, h = \text{parameter}$ $f = 1.53 \text{ GHz}$

current distributions in Figure 5.21

h, cm	Input Admittance, millimhos	
	Theory	Experiment
3	+ j2.90	+ j4.69
4	+ j3.02	+ j5.14
6	+ j3.48	+ j6.8
7	+ j3.83	-

5.2.5 Cavity Filled with Dissipative Media; Current Distribution

To study the effect of dissipative media on the theory as well as the experimental system, three cases are considered. First, the theory is tested against some special cases of measured current distributions available in the literature. Second, the theory is compared with experimental measurements taken on distilled water. The third case involves experimental measurements in sodium chloride solution.

For comparing the results of the theory with experimental data published in the literature, it is necessary to use results for highly dissipative media. If the losses are high enough, the cavity or waveguide walls are essentially shielded from the antenna and measurements taken in semi-infinite media will be comparable to the theoretical results for the waveguide. In the report published by Scott [35] such highly dissipative data are available. By proper choice of operating frequency, conductivity and permittivity the attenuation characteristics of the media studied by Scott can be duplicated in the theory presented here. The comparison of the measurements by Scott to the theory is shown in Figure 5.33 for three values of dissipation and is seen to be excellent. The theoretical curves were all obtained at 1.2 Ghz with appropriate choices of conductivity and relative permittivity.

The comparison of the theoretical and experimental results for distilled water are shown in Figures 5.34 through 5.36. The value used in the theory for relative permittivity of 83.5 is that determined near 1.0 Ghz by the cavity perturbation method described in Section 4.1. The value of conductivity is approximate because

measurement of conductivity is only done at low frequency. Since the conductivity of distilled water is a strong function of frequency, the low frequency data (not presented in this report) are not useable. The value of 0.2 mhos per meter was therefore taken as approximate from the literature. (See, for example, Lepley [23] for typical values of water conductivity as a function of frequency.) The theoretical curves shown in Figures 5.34 through 5.36 are therefore only for reference and should not be taken as absolute measures of the experimental results.

In Figure 5.34 the current distribution is shown for three frequencies. A constant value of conductivity was used for reference in the theoretical results at all frequencies even though the experimental data clearly indicates an increasing conductivity. Both the delta-gap and coaxial-gap theories are presented in this figure to show the effect of conductivity on the different antenna excitation models. Both theories indicate approximately the same effective wavelength and attenuation of the current along the antenna (as expected) but a phase displacement is evident in the coaxial model. This difference is only evident at 1.2 Ghz in the phase data, however. The phase of the current distribution (i.e., maxima and minima) of the experimental data appears to roughly fall between the delta-gap and coaxial-gap theories.

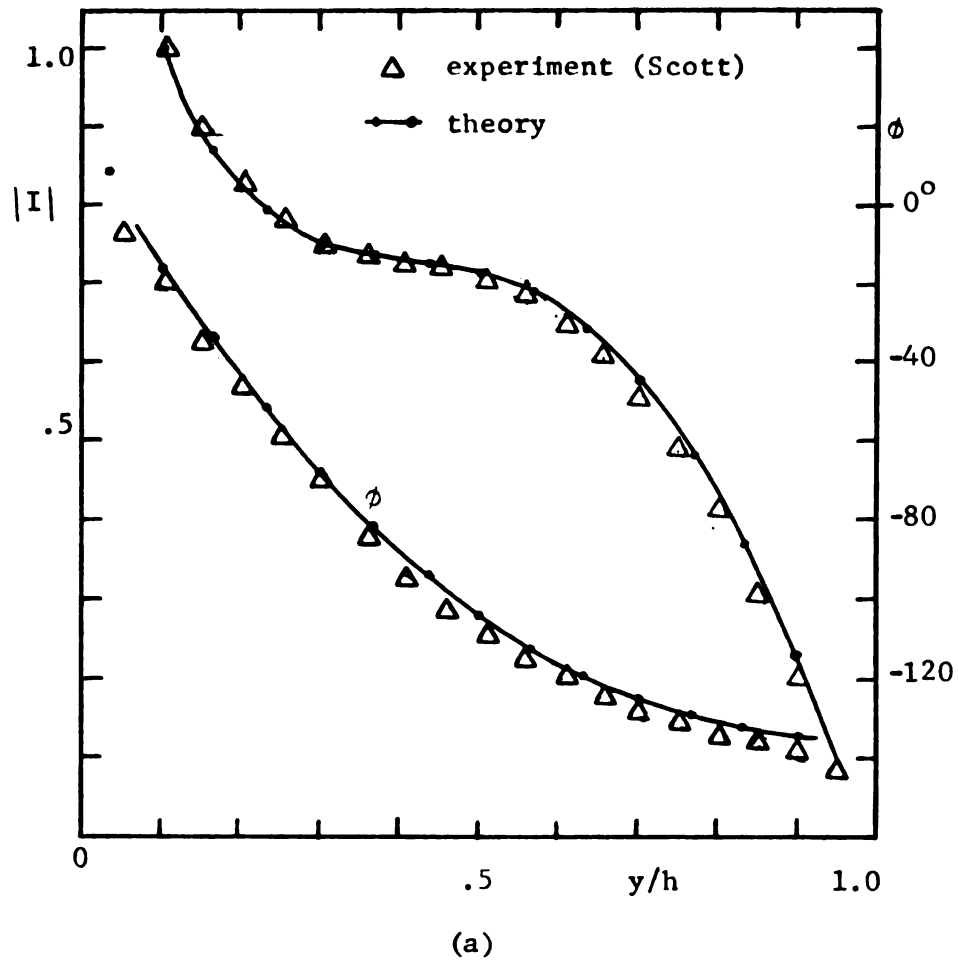
The effect of changing the antenna location in the cavity (Figure 5.35) is seen to be nearly negligible. This indicates that the losses for these frequencies are sufficiently high to mask the presence of some of the conducting boundaries, if not all.

The study of experimental results in a sodium chloride solution is shown in Figures 5.37 and 5.38. Again the theory is only used at a constant conductivity of 0.2 mhos/meter for reference. The effect of lower frequency is obvious at 350 MHz in Figure 5.37 where the distilled water gave much less attenuation than the salt water. This change in conductivity changes the effective dielectric constant and hence the effective wavelength, thus accounting for the phase shift between distilled water and salt water experimental data. The effect of changing location is much more pronounced at this low frequency for the distilled water case indicating the effect of the cavity boundaries.

At the higher frequency (1.0 GHz) in Figure 5.38 the conductivity of distilled water and the salt solution are much more comparable. This is the well-known dipolar relaxation effect [23] of the water which is sufficiently pronounced at 1.0 GHz to overshadow the ionic conductivity of the salt solution.

The overall results for dissipative media indicate that meaningful results can be obtained both experimentally and theoretically. It is evident from the excellent agreement with Scott's results that a sufficiently accurate measure of the effective conductivity at the desired operating frequency would yield theoretical results which would compare extremely well with the experiment. Because of the high dissipation exhibited at the L-band frequencies, it is seen that no use can be made of the cavity resonance coupling effects seen in the previous sections for the empty cavity. It appears that these effects would become useful

at frequencies where the lower order water filled cavity resonances occur (i.e. scaled by the square root of the effective dielectric constant) and where the dissipation is reduced.



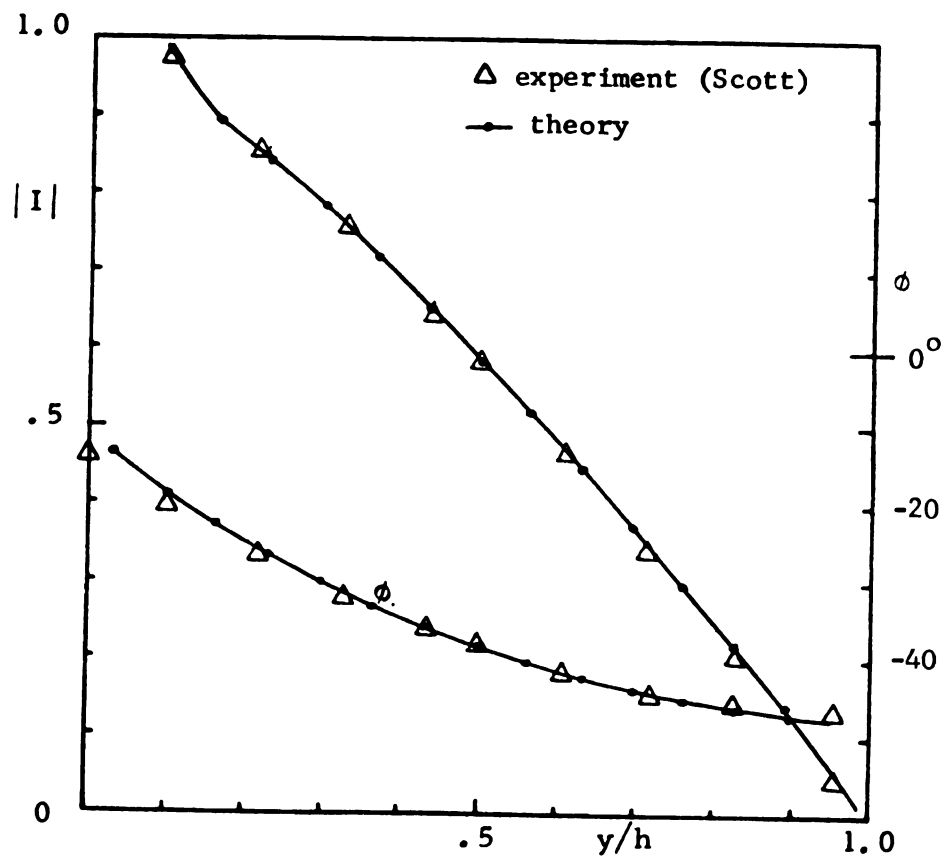
$$\alpha/\beta = 0.3$$

$$\beta h = 3.427$$

FIGURE 5.33

Current Distribution in Highly Dissipative Media

Comparison of Theory with Experimental Literature



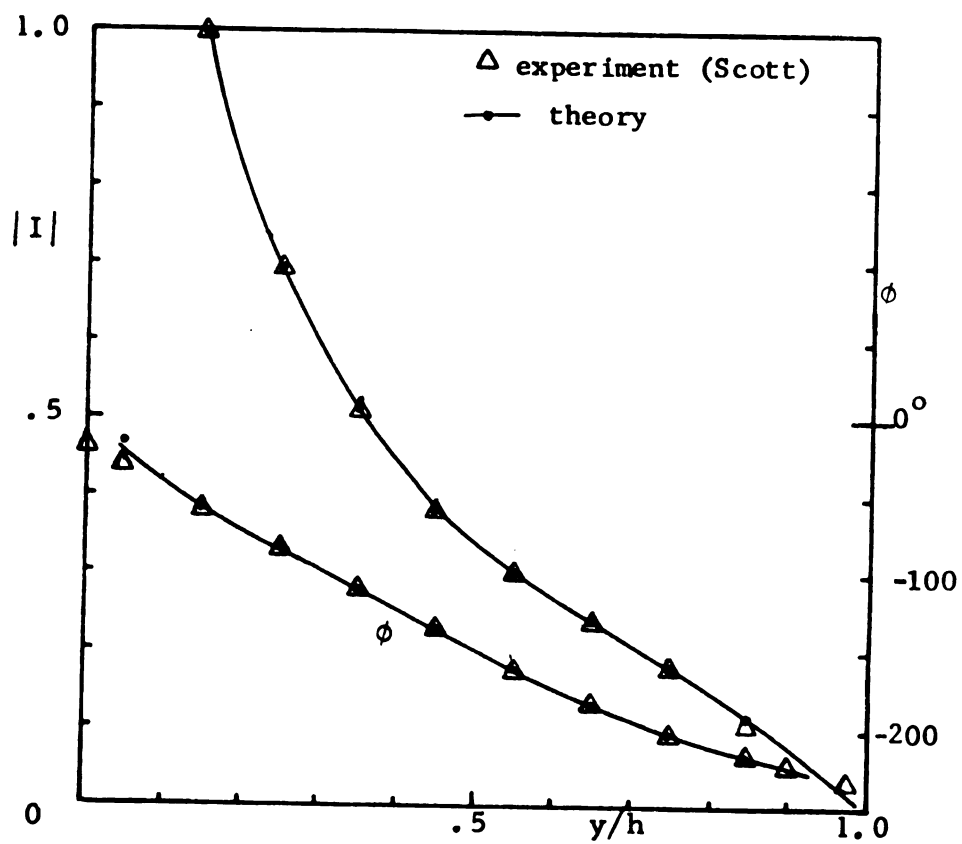
(b)

$$\alpha/\beta = 0.592$$

$$\beta h = 1.324$$

FIGURE 5.33

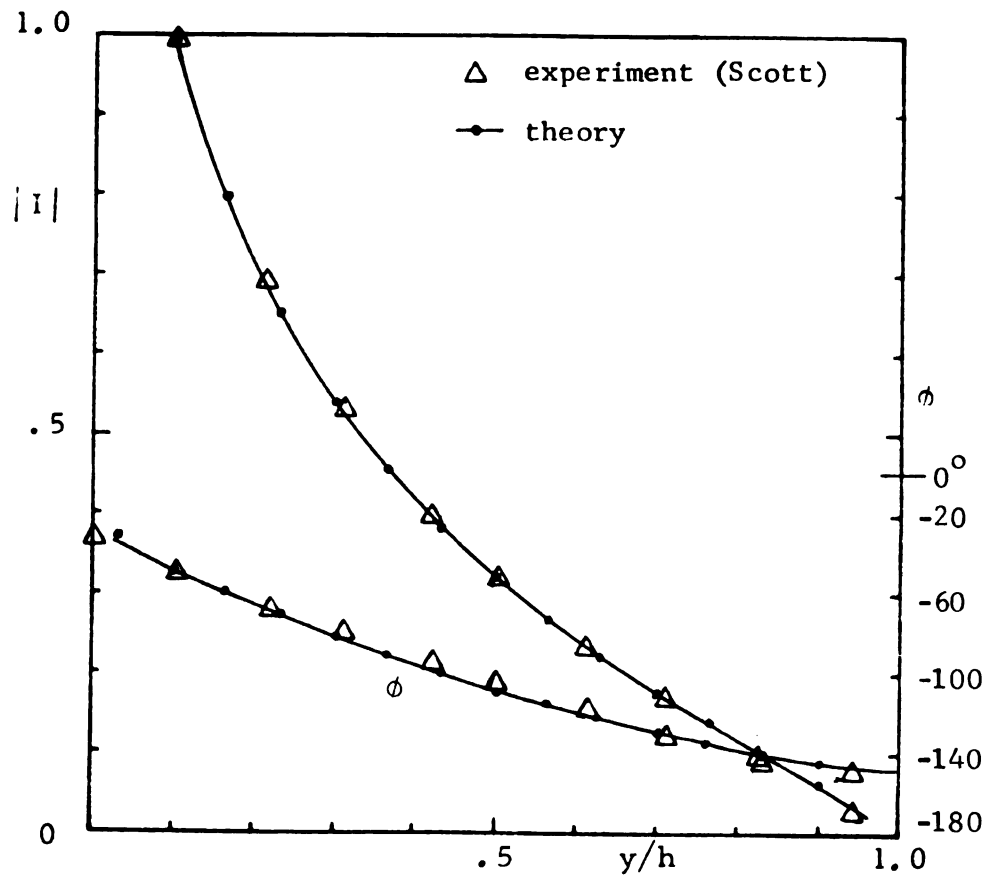
(Cont 'd)



(c)

$$\alpha/\beta = 0.592$$

$$\beta h = 4.487$$



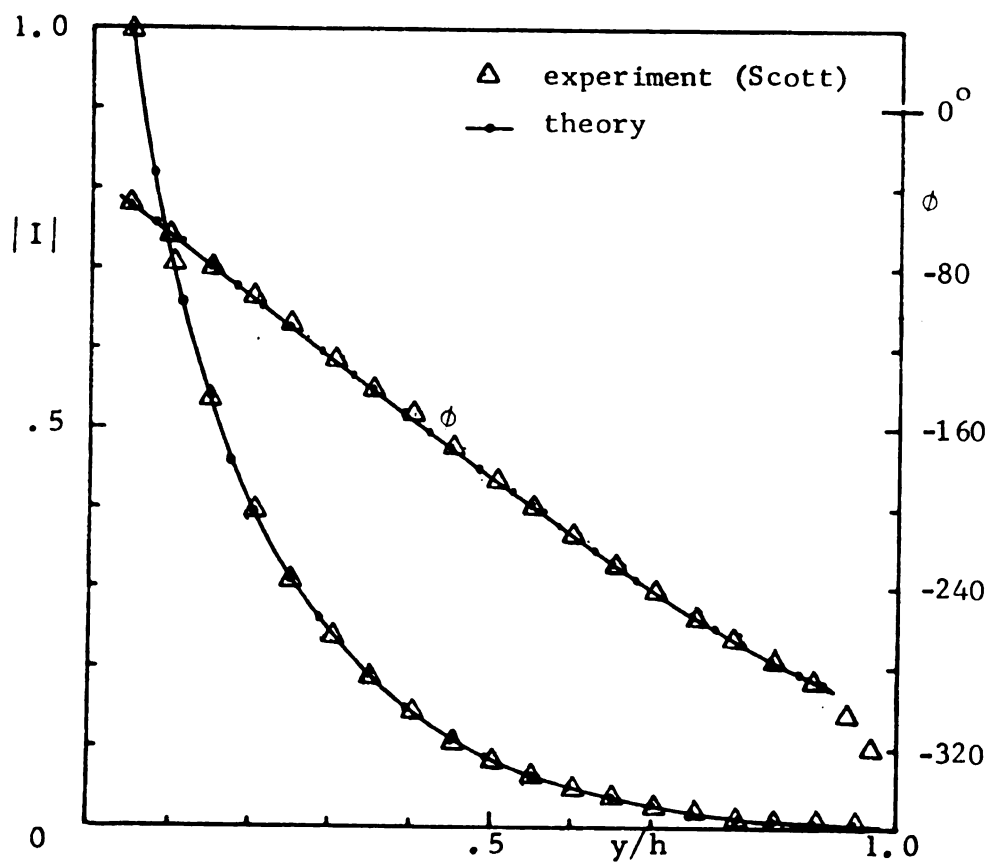
(d)

$$\alpha/\beta = 0.97$$

$$\beta h = 2.564$$

FIGURE 5.33

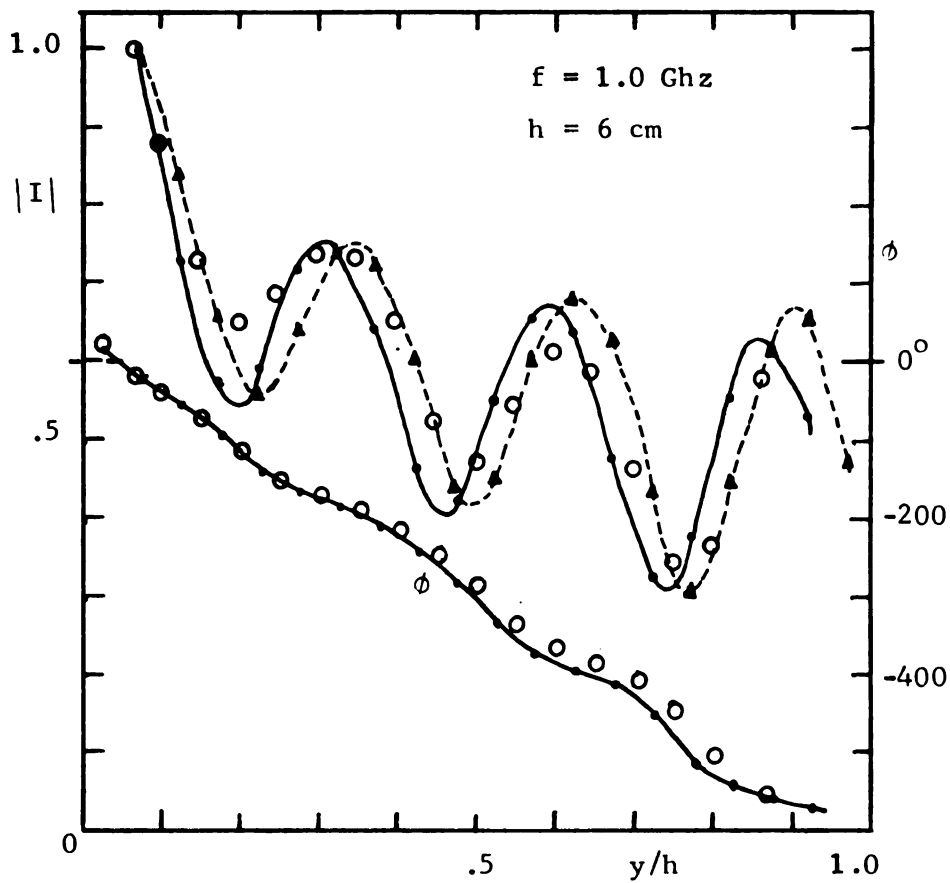
(Concluded)



(e)

$$\alpha/\beta = 0.97$$

$$\beta h = 4.986$$



○ experiment, distilled water

—●— delta-gap theory, $\sigma = 0.2$ mho/m
 -▲- coax gap $\epsilon_r = 83.5$

cavity

configuration: $a = 2b = 16.51$ cm, $l = 60.96$ cm

$d_1 = 0.5l$, $h = 6$ cm, $f = \text{parameter}$

FIGURE 5.34

Current Distribution in Distilled Water at Different Frequencies

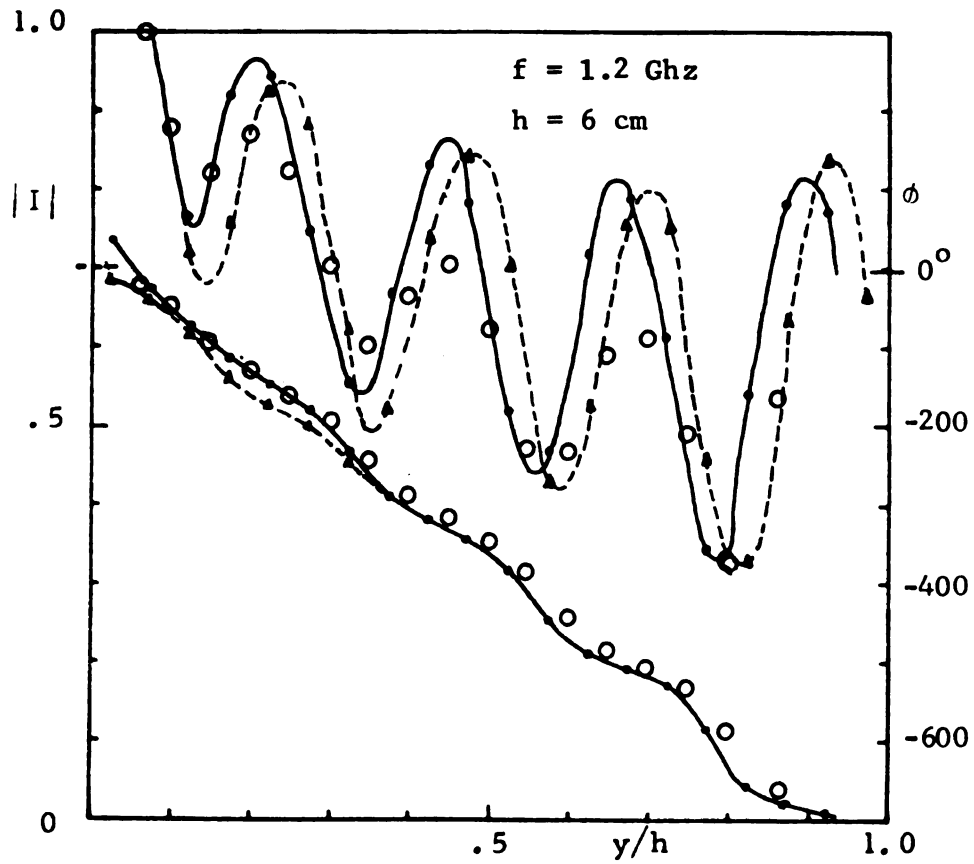
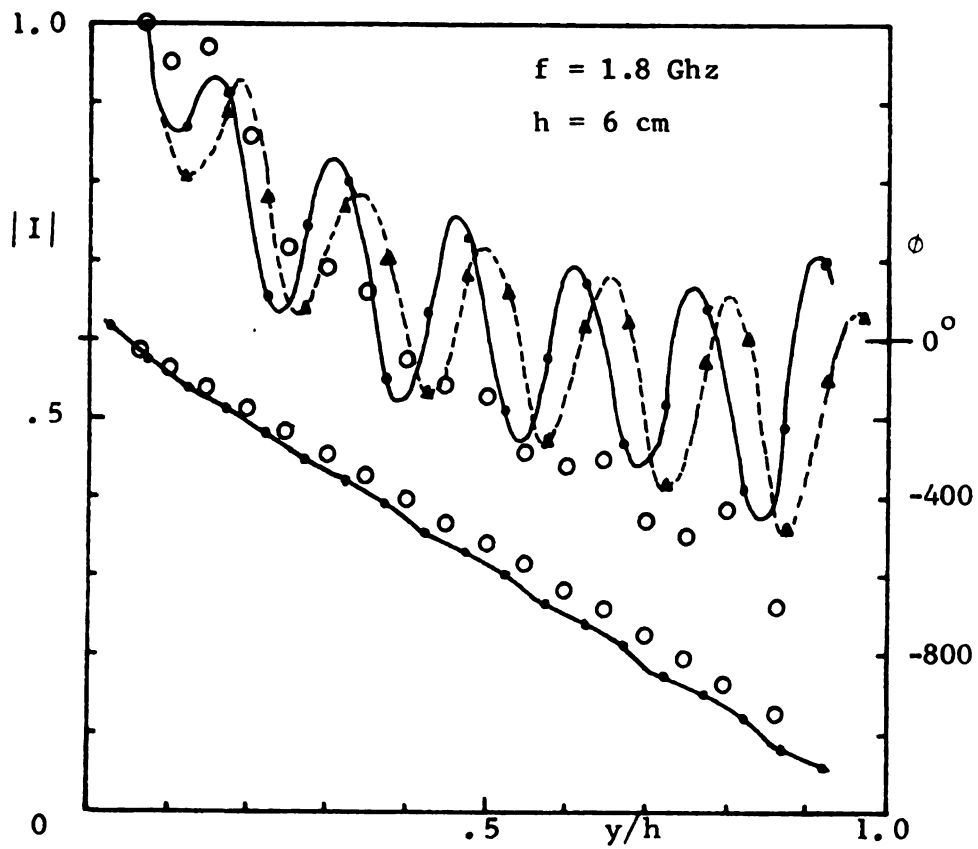


FIGURE 5.34
(Concluded)



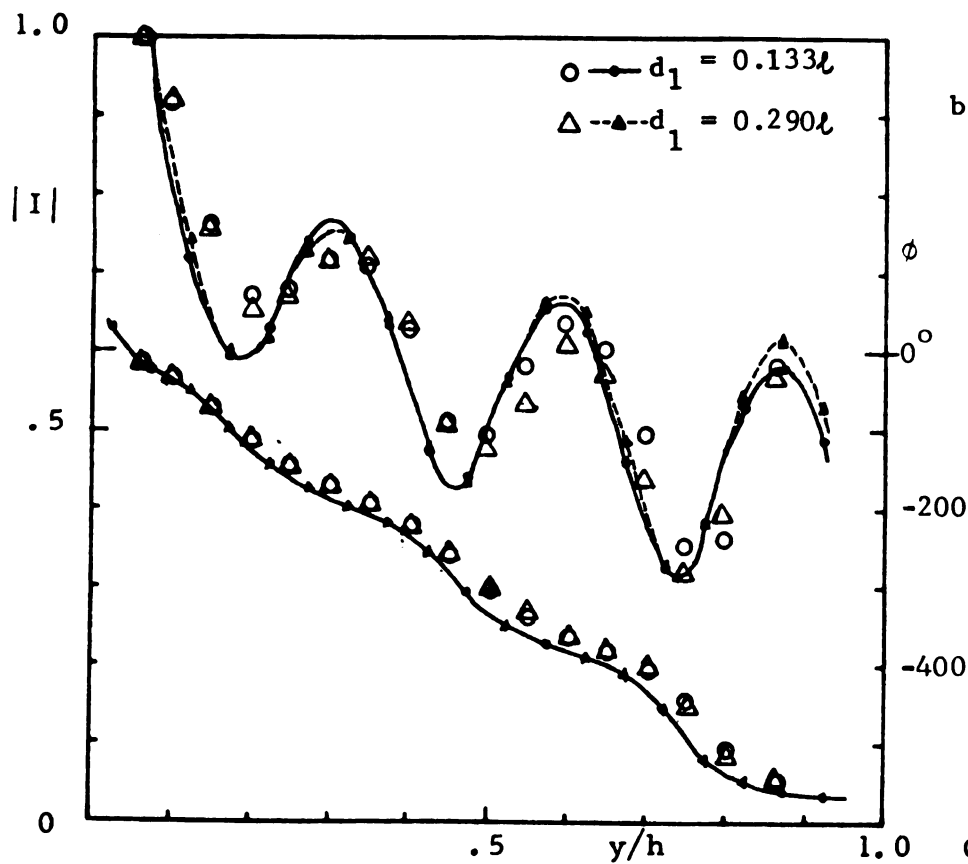
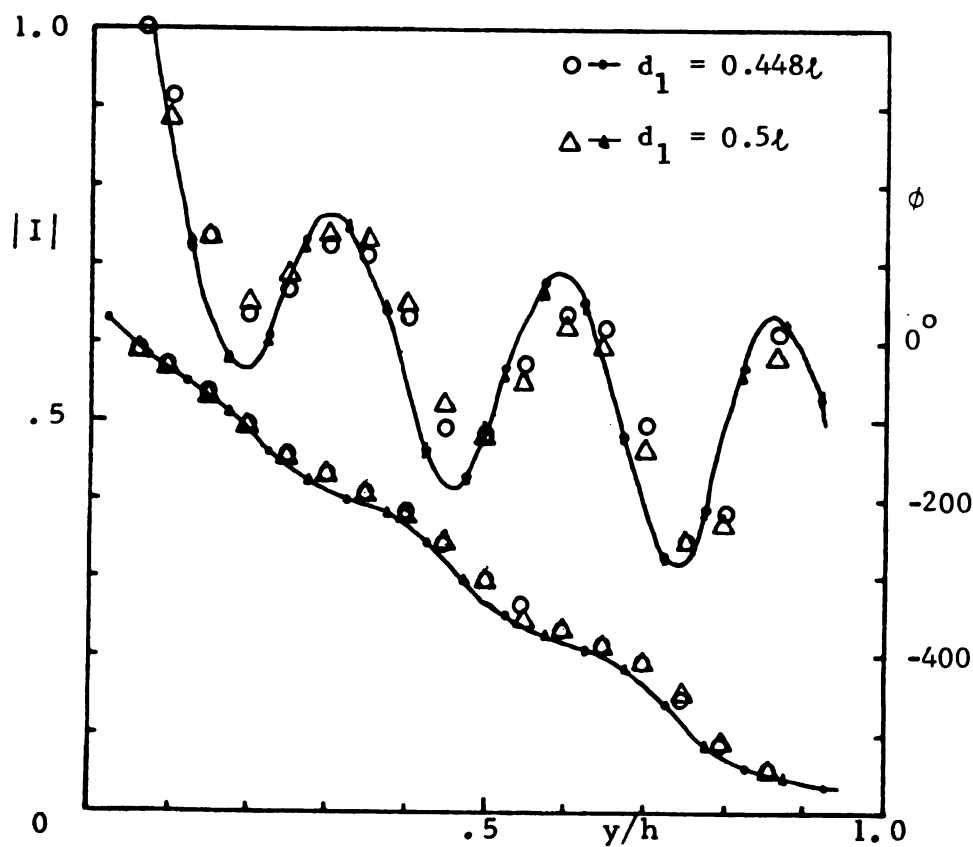


FIGURE 5.35

Current Distribution
in Distilled Water
at Different Loca-
tions



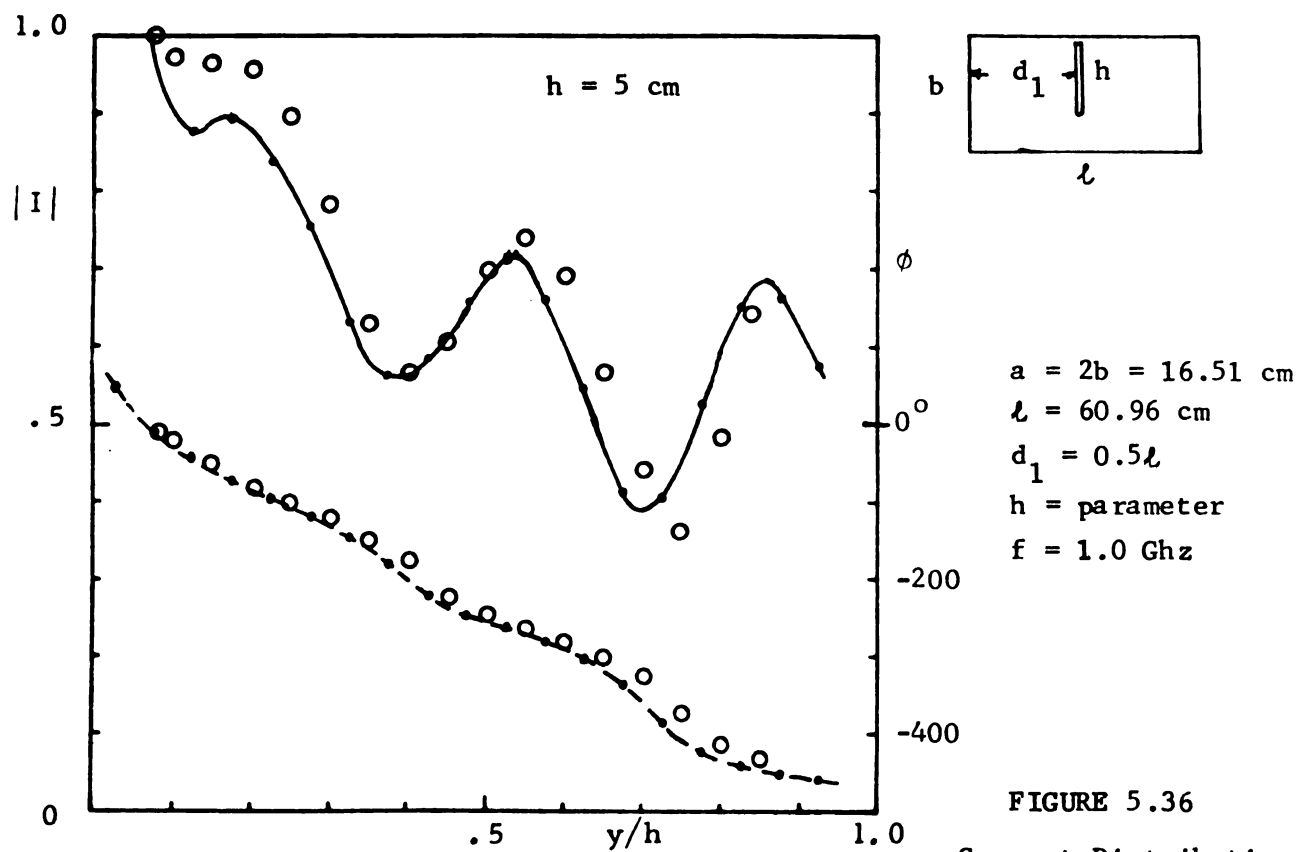
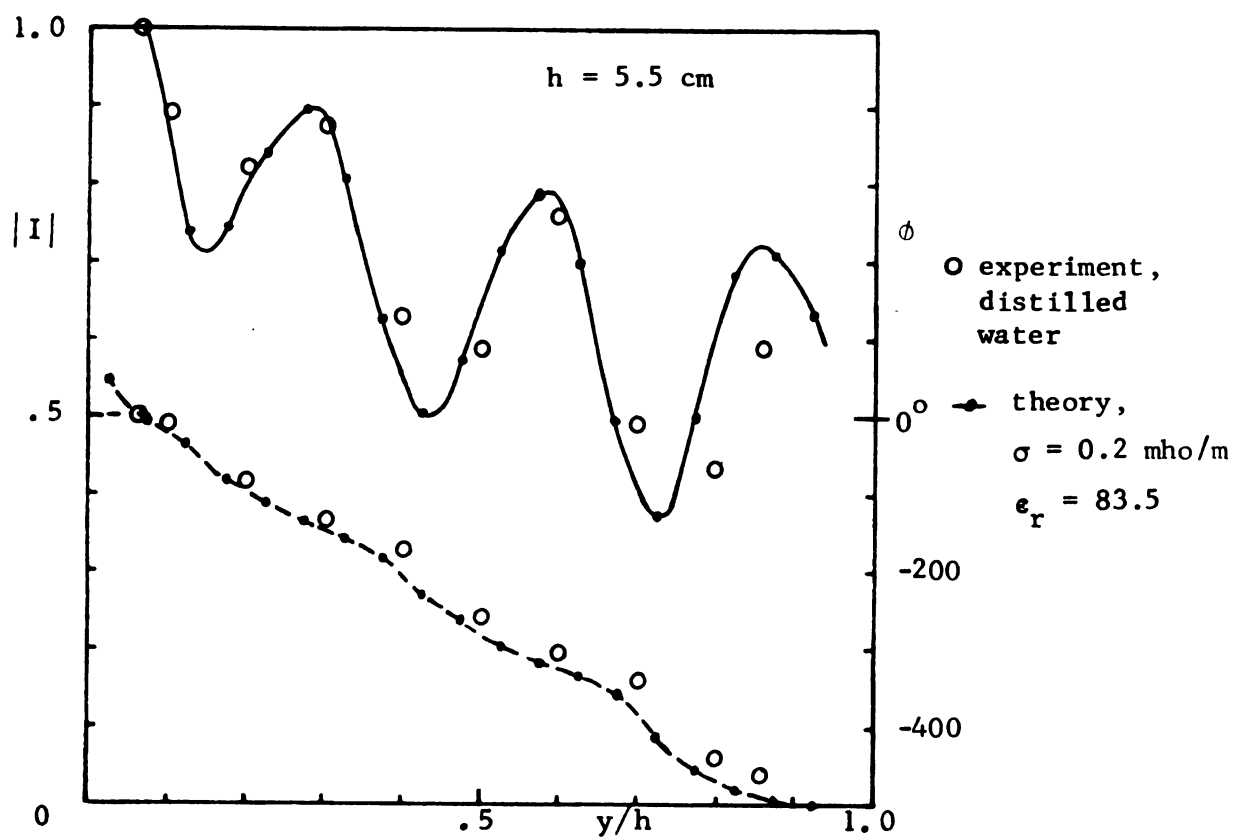


FIGURE 5.36

Current Distribution
in Distilled Water
for Different Heights



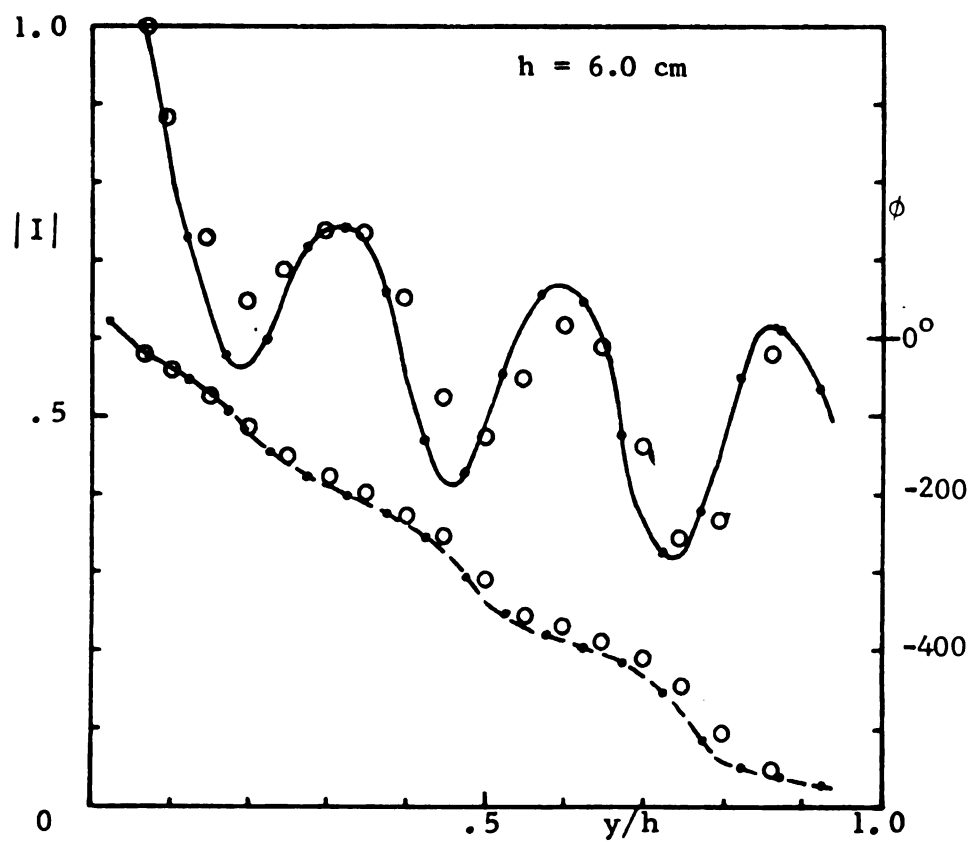
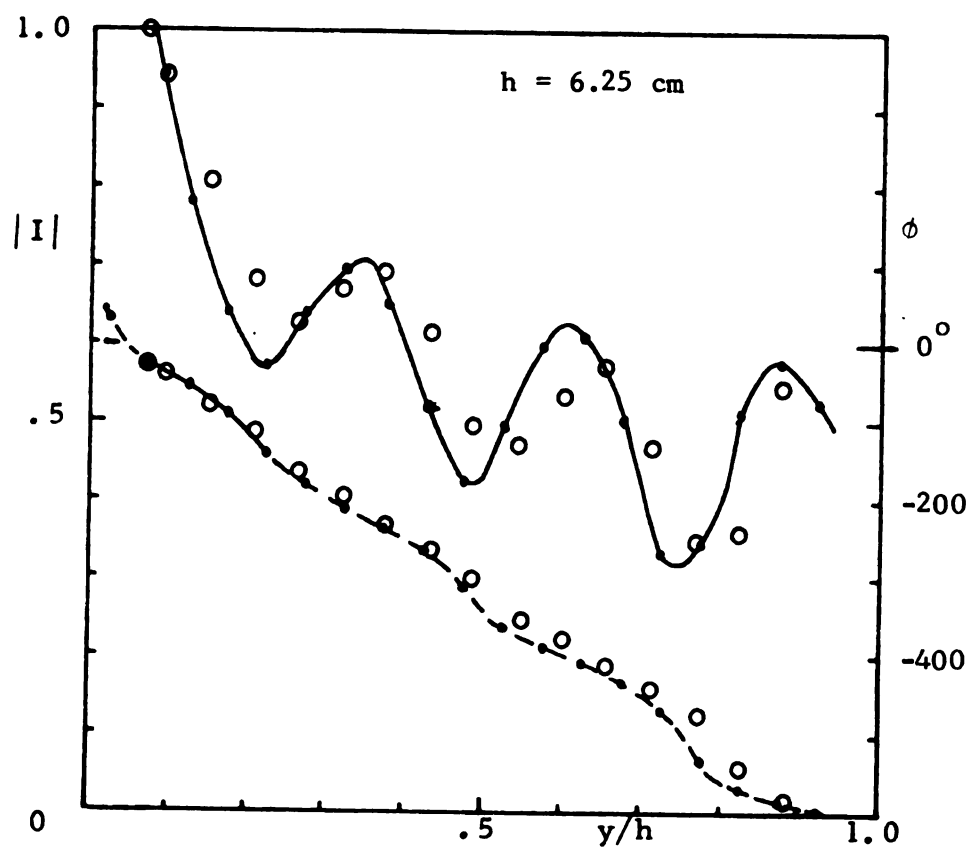
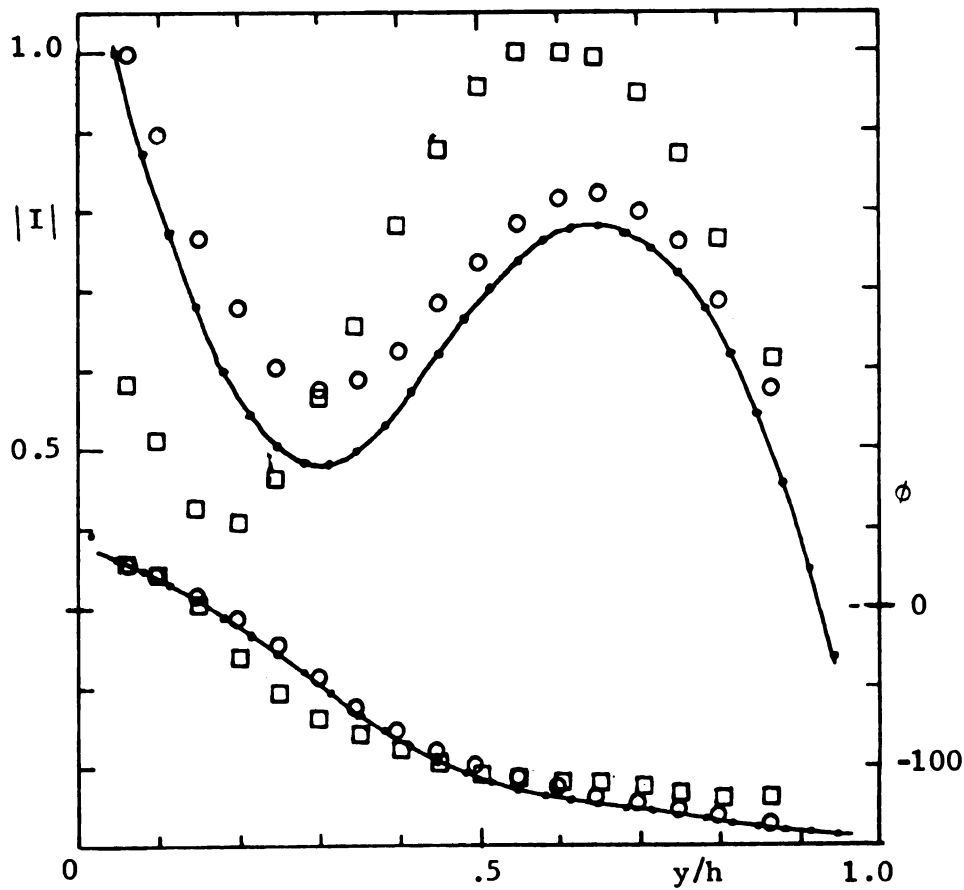


FIGURE 5.36
(Concluded)





○ experiment, 1 gm/liter NaCl solution

□ experiment, distilled water

—●— theory, $\sigma = 0.2$ mho/m, $\epsilon_r = 83.5$

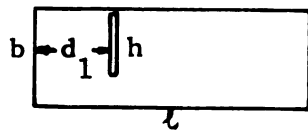
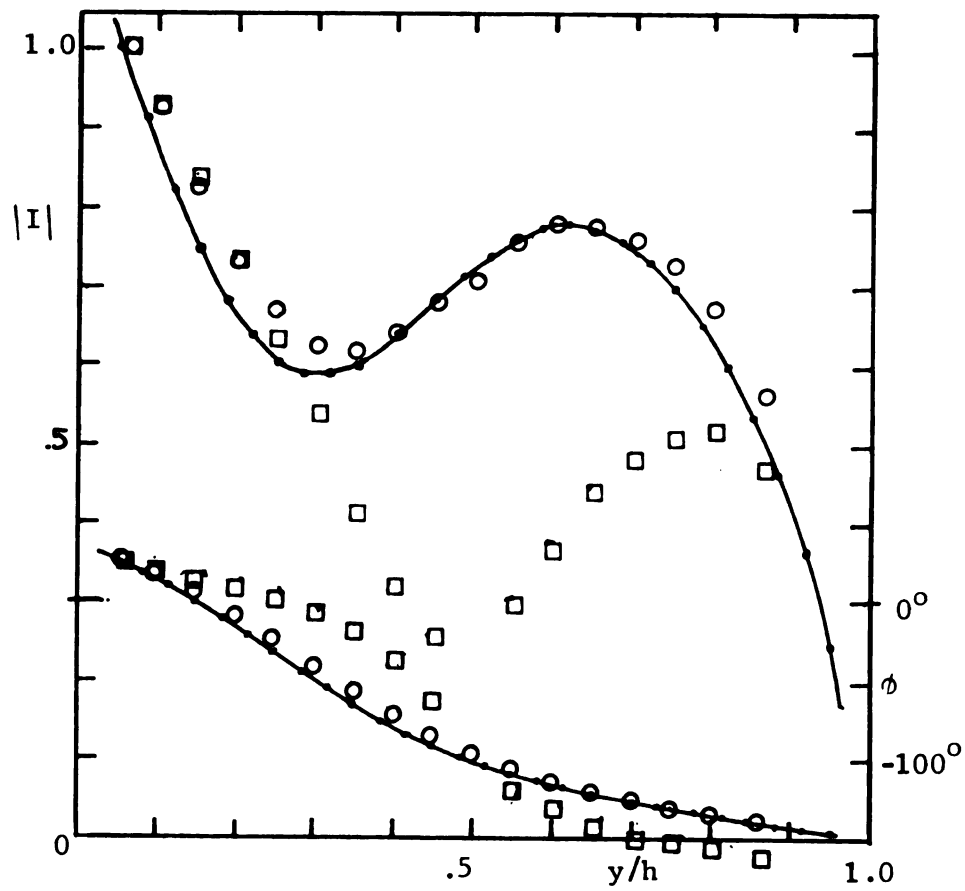
cavity

configuration: $a = 2b = 16.51$ cm, $\ell = 60.96$ cm

$d_1 = 0.5\ell$, $h = 6$ cm, $f = 350$ MHz

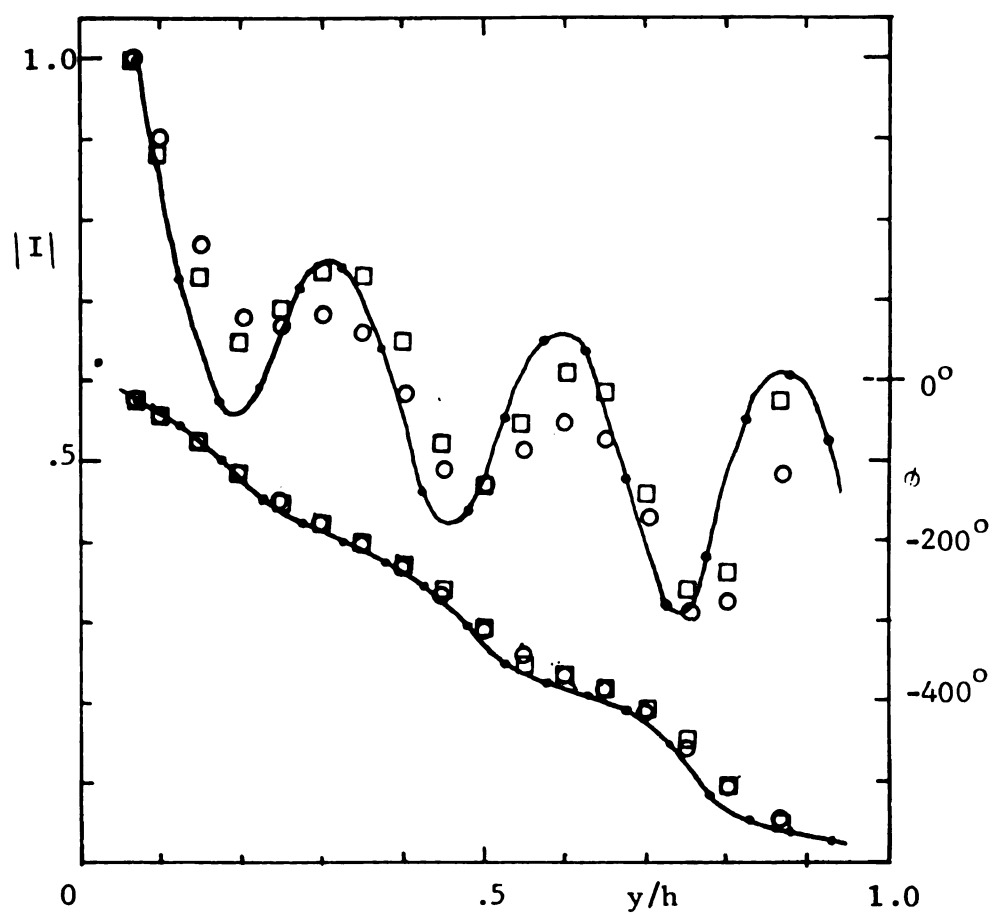
FIGURE 5.37

Current Distribution for Distilled and Salt Water at
350 MHz



$$d_1 = 0.133\ell$$

FIGURE 5.37
(Continued)



○ experiment, 1 gm/liter salt solution

□ experiment, distilled water

—●— theory, $\sigma = 0.2$ mho/m, $\epsilon_r = 83.5$

cavity

configuration: $a = 2b = 16.51$ cm, $l = 60.96$ cm

$d_1 = 0.5l$, $h = 6$ cm, $f = 1.0$ GHz

FIGURE 5.38

Current Distribution for Distilled and Salt Water at
1.0 GHz

5.2.6 Cavity Filled with Dissipative Media; Input Admittance

The input admittance for each case considered in the previous section is given in Tables 5.21 through 5.26. As in other comparisons of experimental and theoretical admittance the lack of agreement is basically accounted for by the inadequacy of the excitation models. It should also be remembered that the theoretical results are determined only for a fixed approximate conductivity and thus should not be compared exactly to the experimental data.

The comparison of the theory with Scott's results [35] in Table 5.21 shows the same degree of difference that the results for the air filled cavity do. This tends to support the theoretical results for dissipative media even though no attempt is made at determining a terminal zone correction network.

The comparison between theory and experiment in Tables 5.22 through 5.26 is poor in all cases. However, the differences between coaxial-gap and delta-gap theories also is significant and indicates the sensitivity of the models to the properties of the medium in the dissipative case. The determination of proper terminal zone correction networks for the case of abrupt transitions from air filled coaxial line to dissipative media is not yet well understood and thus the seriousness of the differences seen in these results cannot be effectively evaluated. For the high frequency data in Tables 5.22, 5.23 and 5.24 the sign on the input susceptance is seen to change sign between the coaxial-gap and delta-gap theories. This seems to qualitatively indicate that a correction network would need to compensate to a large degree in the susceptive component and

results comparable to the experimental data are not unreasonable.

The low frequency data in Table 5.25 again shows results which differ on the order of the air cavity results and thus are felt to be acceptable. The differences between experimental data for the sodium chloride solution and distilled water indicates the sensitivity of the input admittance to changes in the conductivity of the medium. At 1.0 GHz (Table 5.26), on the other hand, where the effective conductivities of both media are comparable, the experimental results agree well. The difference can easily be related to the probable higher conductivity of the ionic NaCl solution.

TABLE 5.21

Input Admittance in Highly Dissipative Media
 Comparison of Theory with Experimental Literature
 (see Figure 5.33 for current)

	input admittance, millimhos	
	Theory	Scott, exp.
$\alpha/\beta = 0.3$ $\beta h = 3.427$	$21.55 + j3.25$	$27 + j2.48$
$\alpha/\beta = 0.592$ $\beta h = 1.324$	$13.0 - j2.46$	$22.1 - j5.88$
$\alpha/\beta = 0.592$ $\beta h = 4.487$	$16.4 - j4.6 \quad *$	$17.6 - j6.0$
$\alpha/\beta = 0.97$ $\beta h = 2.564$	$18.7 - j11.6$	$35.8 - j21.9$
$\alpha/\beta = 0.97$ $\beta h = 4.986$	$19.0 - j11.5$	$35.2 - j22.1$

* The ratio h/r_a was not equal in this case.

TABLE 5.22

Input Admittance in Distilled Water
at Different Frequencies

theory based on $\sigma = 0.2$ mho/m, $\epsilon_r = 83.5$

current shown in Figure 5.34

configuration: $a = 2b = 16.51$ cm, $l = 60.96$ cm

$d_1 = 0.5l$, $h = 6$ cm, $f =$ parameter

input admittance, millimhos

frequency	theory	experiment
1.0 Ghz	$93.8 + j52.6$ (delta)	$23.8 - j1.9$
	$85.5 - j29.8$ (coax)	
1.2 Ghz	$103 + j85$ (delta)	$162 + j7.8$
	$118 - j35$ (coax)	
1.8 Ghz	$131 + j87$ (delta)	$183 + j16$
	$82 - j74$ (coax)	

TABLE 5.23

Input Admittance in Distilled Water
at Different Locations

current shown in Figure 5.35

theory based on $\sigma = 0.2$ mho/m, $\epsilon_r = 83.5$

configuration: $a = 2b = 16.51$ cm, $l = 60.96$ cm

d_1 = parameter, $h = 6$ cm, $f = 1.0$ Ghz

input admittance, millimhos

d_1	theory	experiment
0.133 l	96.1 + j58.1 (delta) 90.7 - j26.2 (coax)	23.2 - j4.1
0.290 l	92 + j53.5 (delta) 84.4 - j28.2 (coax)	25.6 - j6.3
0.448 l	93.6 + j53.0 (delta) 85.3 - j29.7 (coax)	24.3 - j3.3
0.500 l	93 + j51.3 (delta) 85.5 - j29.8 (coax)	23.8 - j1.9

TABLE 5.24

Input Admittance in Distilled Water
for Different Heights

current shown in Figure 5.36

theory based on $\sigma = 0.2 \text{ mho/m}$, $\epsilon_r = 83.5$

configuration: $a = 2b = 16.51 \text{ cm}$, $\ell = 60.96 \text{ cm}$

$d_1 = 0.5\ell$, $h = \text{parameter}$, $f = 1.0 \text{ GHz}$

input admittance, millimhos

h, cm	theory	experiment
5.0	$77.8 + j90.5$ (delta)	$21.9 - j0.3$
	$81.9 + j11.8$ (coax)	
5.5	$96.4 + j87.8$ (delta)	$21.7 - j0.7$
	$123.6 - j7.02$ (coax)	
6.0	$93.0 + j51.3$ (delta)	$23.8 - j1.9$
	$85.5 - j29.8$ (coax)	
6.25	$81.0 + j48.6$ (delta)	$24.4 - j0.8$
	$75.5 - j17.9$ (coax)	

TABLE 5.25

Input Admittance for Distilled and
Salt Water at 350 MHz

current shown in Figure 5.37

theory based on $\sigma = 0.2$ mho/m, $\epsilon_r = 83.5$

configuration: $a = 2b = 16.51$ cm, $l = 60.96$ cm

d_1 = parameter, $h = 6$ cm, $f = 350$ MHz

input admittance, millimhos

d_1	theory (delta)	experiment	
		NaCl 1 gm/l	Distilled
0.133l	49.5 + j49.3	108 + j75.7	118 + j76.5
0.500l	50 + j56.5	93.9 + j85.7	55.7 + j89.2

TABLE 5.26

Input Admittance for Distilled and
Salt Water at 1.0 GHz

current shown in Figure 5.38

theory based on $\sigma = 0.2$ mho/m, $\epsilon_r = 83.5$

configuration: $a = 2b = 16.51$ cm, $l = 60.96$ cm

$d_1 = 0.5l$, $h = 6$ cm, $f = 1.0$ GHz

input admittance, millimhos

theory	experiment	
	NaCl 1 gm/l	Distilled
93.6 + j52.7 (delta)	21.7 - j0.8	23.6 - j3.2
85.5 - j29.8 (coax)		

5.3 Coupled Antennas

In order to take full advantage of the antenna-waveguide-media system for potential application, an understanding must be had of the functioning of two coupled antennas in the waveguide or cavity. This section presents these results. Because the results of the single antenna presented in Section 5.2 show the coupling that exists between the antenna and the cavity, only a few representative results of varying pertinent parameters will be presented for the two-antenna case. Each physical and electrical configuration considered in this section has its parallel in the single antenna case of the previous section and it is instructive to make the comparison to qualitatively evaluate the effect of the second parasitic antenna.

In all the cases presented in this section, only the results of the delta gap theory are included, since the coaxial gap theory, as developed to date, is capable of only a single antenna solution. The current distributions shown are only those on the driven antenna (always designated #1) with the parasitic antenna (#2) loaded with a standard calibrated 50 ohm termination. As well as calculating the current (and admittance) for the loaded case, Y_{50} , the theory also determines the admittance parameters, $[Y]$, for this two port system. The theoretical admittance matrix is given in the tables of input admittance.

The experimental measurements of admittance are three fold, with all measurements taken on the driven antenna. First, the input admittance is measured with the 50 ohm load on the parasitic antenna (Y_{50}). Second, the admittance is measured with the parasitic antenna electrically short-circuited (Y_{sc}). The admittance is finally

measured for an open-circuited parasite (Y_{oc}). These three measurements are then sufficient to determine the admittance parameters for the system. Although, as noted in King [17], these measurements are not the most desirable (since the calculation of the admittance parameters involves differences of large nearly equal numbers), they are the simplest to make with the experimental system at hand. The short-circuit and open-circuit conditions for the parasitic antenna were obtained with a sliding short attached to the second antenna's General Radio connector. The existence of the short or open condition was first determined from auxiliary measurements taken on a slotted line driving the connector-sliding short combination from the antenna terminal plane. Because of the problems already noted in the measurement of admittance (Section 5.1), the experimental data has not been converted to admittance parameters to avoid compounding errors. For comparison purposes, it is seen that $Y_{11} = Y_{sc}$, $Z_{11} = Z_{oc}$ and, of course, the 50 ohm loaded cases are equivalent.

For calculation purposes it should be noted that, for the cavity case, the admittance parameters are all purely imaginary. Thus, measurement of Y_{sc} (Y_{11}) and Y_{50} ($G_{50} + jB_{50}$) is sufficient to determine the three unknown admittance parameters: $Y_{11} = jB_{11}$, $Y_{12} = Y_{21} = jB_{12}$, and $Y_{22} = jB_{22}$. The necessary relationships are readily determined from the admittance parameter two port network equations. Since only purely imaginary results are desired, the solution of the resulting cubic equation for B_{12} or B_{22} in terms of G_{50} , B_{50} , and B_{sc} is readily solved by algebraic methods.

In general, the purpose of the coupled antenna measurements in this report is to test the theory and demonstrate the operation of

the experimental system for potential application. The results for two antennas in a semi-infinite waveguide are presented first. In Sections 5.3.2 and 5.3.3 the results for an air filled cavity are shown for independent variations in antenna separation and the two antenna heights. For the case of dissipative media, the theory is compared to measurements in distilled water and in sodium chloride solution with the results presented in Section 5.3.4.

5.3.1 Waveguide with Matched Terminations

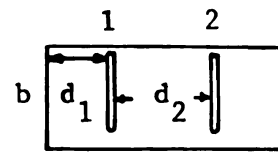
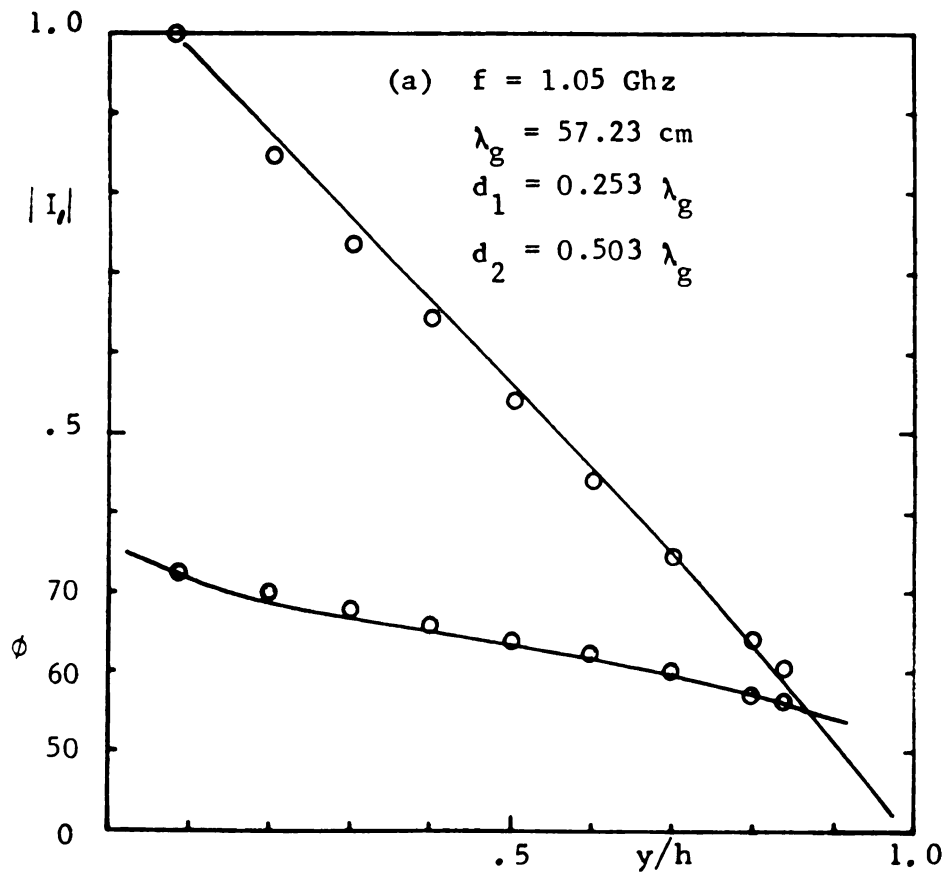
For the case of the semi-infinite waveguide studied and discussed for the single antenna in Section 5.2.1, two representative configurations were tested with coupled antennas. The experimental system again used microwave absorber in the open end of the short-circuited section of waveguide. The theory assumes a perfect short at one arm and a perfect match at the other arm. The driven antenna is located (as in the single antenna case) at approximately one-quarter of a guided wavelength from the short circuit. The second parasitic antenna is then placed approximately one-half of a guided wavelength from antenna number 1.

The theoretical and experimental current distributions on the driven antenna at 1.05 and 1.3 GHz are shown in Figure 5.39 (a) and (b). These results can be compared to the same single antenna results in Figures 5.5(a) and 5.6(a), respectively. In the coupled case the current is more nearly triangular and the phase of the current is more nearly quadrature than in the single case. This indicates that the parasitic antenna is acting to increase the stored energy in the vicinity of the antennas while decreasing the energy radiated out of the open end of the waveguide. (This would also account for the better agreement between theory and experiment, since the crude experimental matched termination is not as important as in the single antenna case.)

The input admittance for these two cases are given in Table 5.27. Comparing to Tables 5.1 and 5.2 shows a decrease in input conductance and hence a decrease in radiated power for the case of the 50 ohm loaded parasite. The agreement between theory and

experiment (Y_{sc} to Y_{11} , Y_{50} to Y_{50}) again is roughly acceptable within the usual factor of two for this system. The mutual resistance is seen to be negative which is expected for antennas at one-half wavelength spacing in free-space.

These few results for the semi-infinite waveguide serve as a first-order indication of the operation of the experimental apparatus and theoretical model for the coupled antenna configuration. Since this type system was not envisioned as part of the applications aspect, no further data is presented on the semi-infinite waveguide at this time.



$$a = 2b = 16.51 \text{ cm}$$

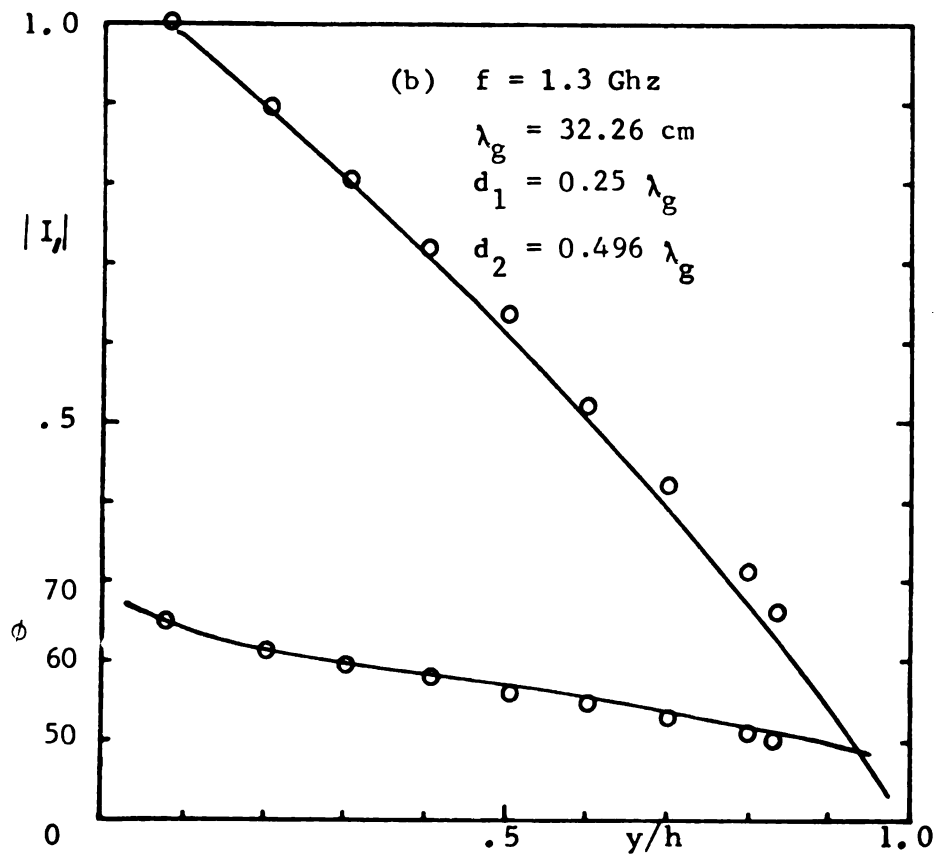
$$h_1 = 5 \text{ cm} = .175 \lambda_o$$

$$h_2 = 5.08 \text{ cm} = .178 \lambda_o$$

Ant. #1: driven

Ant. #2: 50 ohm load

FIGURE 5.39



Driven Antenna Current
 for Coupled Antennas in
 Semi-Infinite Waveguide
 ($f = 1.05; 1.3 \text{ GHz}$)

TABLE 5.27

Admittance for Coupled Antennas in
Semi-Infinite Waveguide

For configuration of Figure 5.39(a): $f = 1.05$ Ghz

Theory:

$$[Y] = \begin{bmatrix} 0.84 + j4.59 & -0.84 + j2.35 \\ -0.84 + j2.35 & 0.84 + j4.53 \end{bmatrix} \text{ millimhos}$$

$$Y_{50} = 1.35 + j4.75 \quad \text{m-mhos}$$

Experiment:

$$Y_{50} = 2.84 + j10.25 \quad \text{m-mhos}$$

$$Y_{sc} = 0.944 + j10.37 \quad \text{m-mhos}$$

$$Y_{oc} = 3.4 + j6.12 \quad \text{m-mhos}$$

For configuration of Figure 5.39(b): $f = 1.3$ Ghz

Theory:

$$[Y] = \begin{bmatrix} 1.21 + j7.80 & -1.32 + j5.22 \\ -1.32 + j5.22 & 1.55 + j7.75 \end{bmatrix} \text{ millimhos}$$

$$Y_{50} = 3.31 + j7.64 \quad \text{m-mhos}$$

Experiment:

$$Y_{50} = 13.3 + j26.2 \quad \text{m-mhos}$$

$$Y_{sc} = 2.43 + j18.7 \quad \text{m-mhos}$$

$$Y_{oc} = 8.03 + j9.58 \quad \text{m-mhos}$$

5.3.2 Air Filled Cavity; Current Distribution

Some representative results for coupled antennas in the air filled cavity are presented in this section and the next. The cavity, as always, is specified by dimensions a , b , and l for width, height and length, respectively. The current distributions for the case of the parasitic antenna loaded with 50 ohms are presented in Figures 5.40 through 5.43.

Figure 5.40 shows the excellent agreement between theory (solid line) and experiment (circles) for various separations (d_2) of the driven antenna (#1) and the parasitic antenna (#2). Since the antennas are contained in a theoretically lossless cavity, the only mechanism for dissipation is the coupling of excitation energy into the 50 ohm load on the parasitic antenna. Therefore, a qualitative measure of the coupling between the antennas is the amount of phase variation of the current compared to the pure quadrature current that would exist in the unloaded case. (For comparison, the single active antenna in the same configuration is given in Figure 5.20.) The change in shape shown for the current in Figure 5.40 as the separation is varied is primarily due to the perturbation of the cavity resonance caused by the location of the parasitic antenna. As the resonant frequency is shifted relative to the operating frequency (and effective Q lowered) the current distribution is expected to change characteristics in a manner similar to that discussed for the single antenna (Section 5.2.2). Thus the tendency toward a triangular distribution for the larger separations results from the system resonant frequency being moved from above to below the operating frequency (see, for example, Figure 5.23).

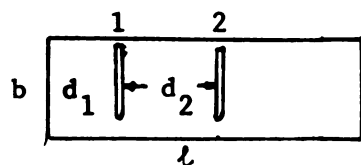
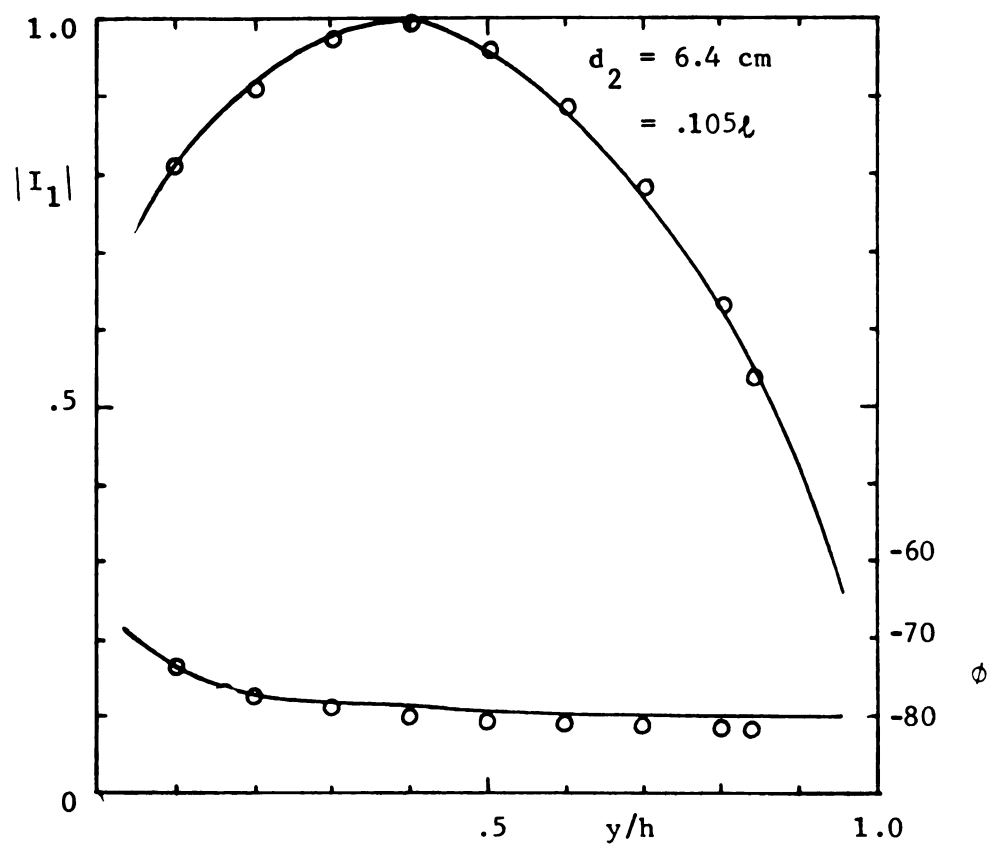
The variation in current distribution on the active antenna for changing active antenna heights is shown in Figures 5.41 and 5.42. In each case the antennas are located symmetrically in the cavity at locations as close as experimentally possible to the shorting end plates. The comparable single antenna results for the operating frequency of 1.2 Ghz are given in Figures 5.19. Comparing Figure 5.19 to Figure 5.41 indicates a greater sensitivity of the current to height changes for the coupled antenna case. This is again readily accounted for by the shift in cavity resonance such that small changes in antenna length result in larger variation in the resonant nature of the system. The shift of resonance is evident, since the quadrature component of current in the coupled case is inductive (-90°) compared to the capacitive nature for the single antenna thus indicating a shift of the resonant frequency past the operating frequency.

For an operating frequency of 1.3 Ghz the results are given in Figure 5.42 (the comparable single case is in Figure 5.20). The differences between single and coupled antenna results is again directly relateable to the shift in resonant frequency. At either 1.2 Ghz or 1.3 Ghz the in-phase component of current is some measure of the coupling to the parasitic antenna and 50 ohm load as discussed in relation to Figure 5.40.

For variations in the height of the parasitic antenna, the active antenna current is given in Figure 5.43. Comparing again to the single antenna results of Figure 5.20, it is seen that for the 1" parasite the current is very similar to the single case and has only a small in-phase component (small coupling) as expected. As

the height of the parasite is increased the coupling becomes greater, the cavity resonance is increasingly shifted, and the characteristic shape and phase of the current on the driven antenna changes.

The overall agreement between theory and experiment for various changes in parameters of two coupled antennas in the cavity is excellent. Differences that are seen are primarily accountable by the approximation made in the theory for antenna shape which ultimately affects the effective shift in cavity resonances. As has been shown, the coupling between antennas is most sensitive to variations in separation and heights in a manner directly related to the coupling between each antenna and the cavity resonances. The variations in coupling will be more evident in the admittance results presented in the next section.



$$a = 2b = 16.51 \text{ cm}; l = 60.96 \text{ cm}$$

$$h_1 = 5 \text{ cm}; h_2 = 5.08 \text{ cm}$$

$$d_1 = 8.08 \text{ cm} = .133l; d_2 = \text{parameter}$$

$$f = 1.3 \text{ GHz}$$

Ant. #1 driven

Ant. #2 50Ω load

FIGURE 5.40

Driven Antenna Current for Coupled Antennas as a Function of Separation ($f = 1.3 \text{ GHz}$)

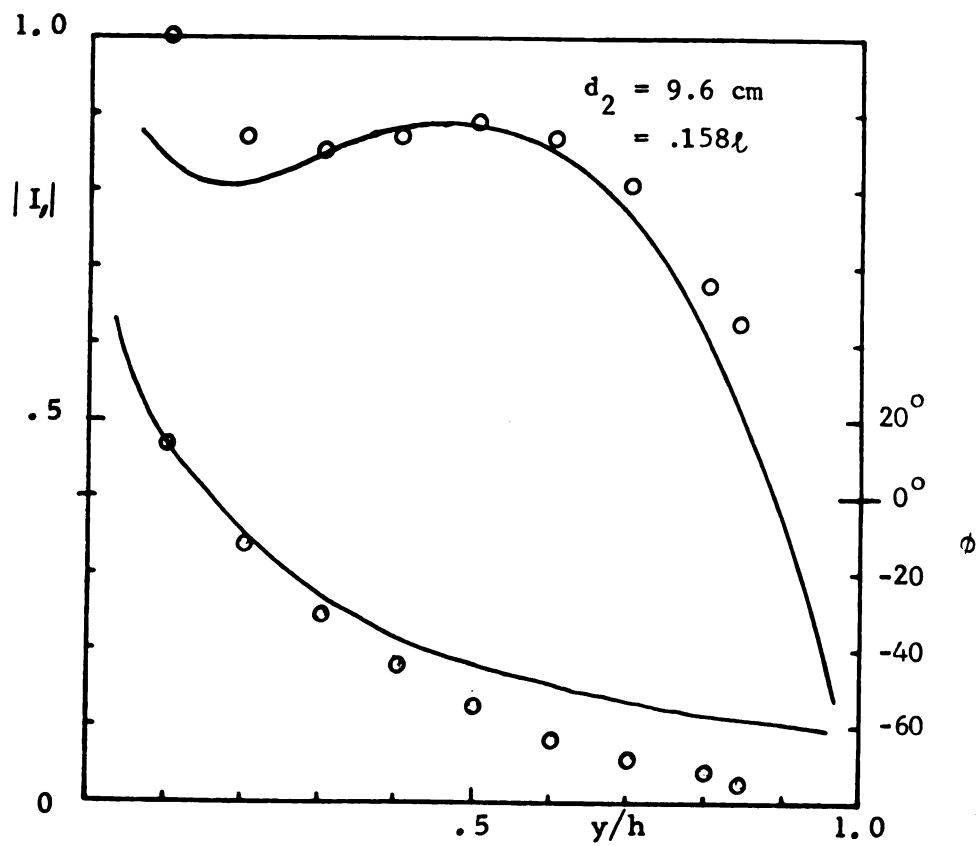
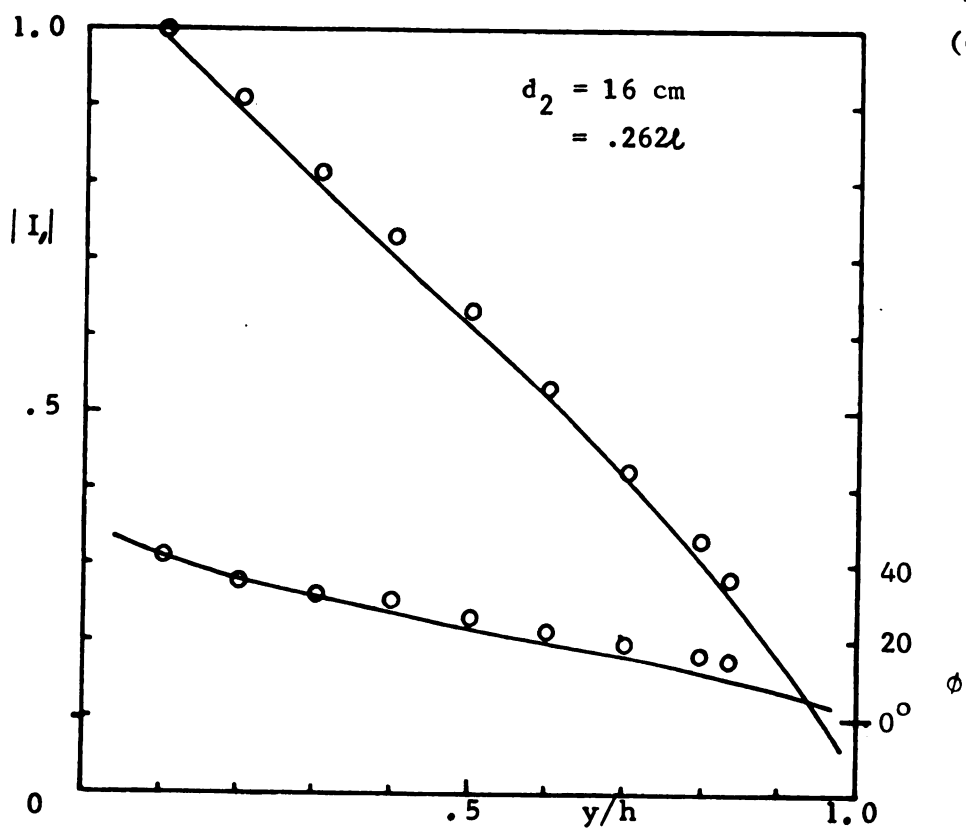


FIGURE 5.40
(continued)



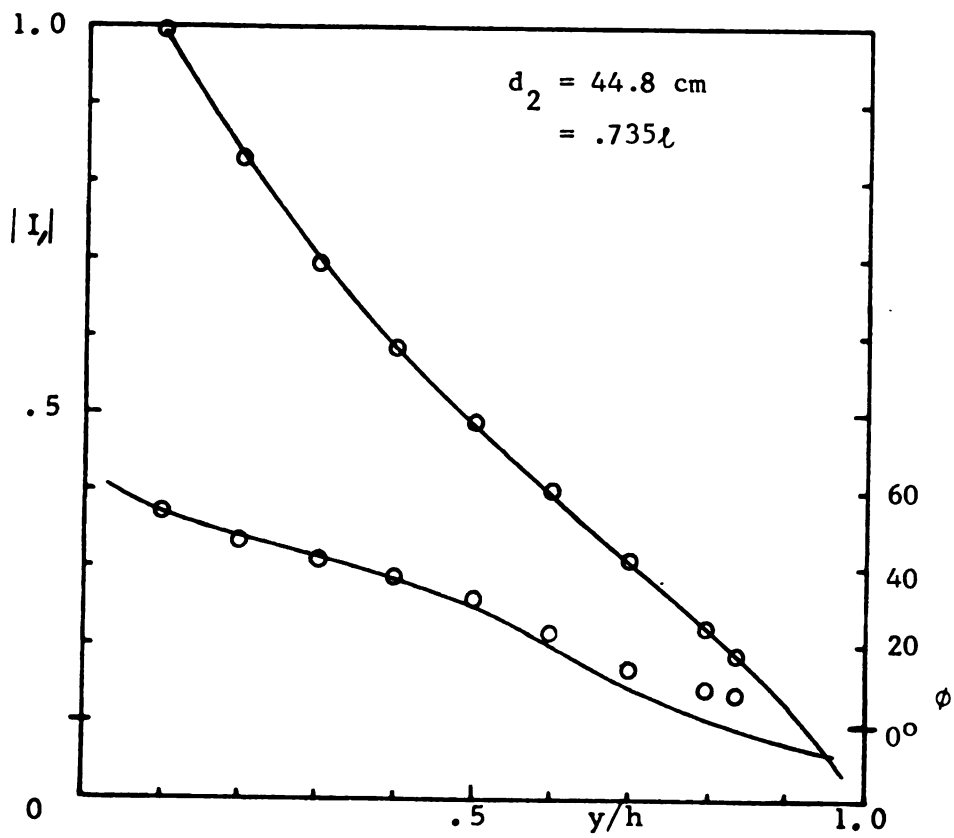
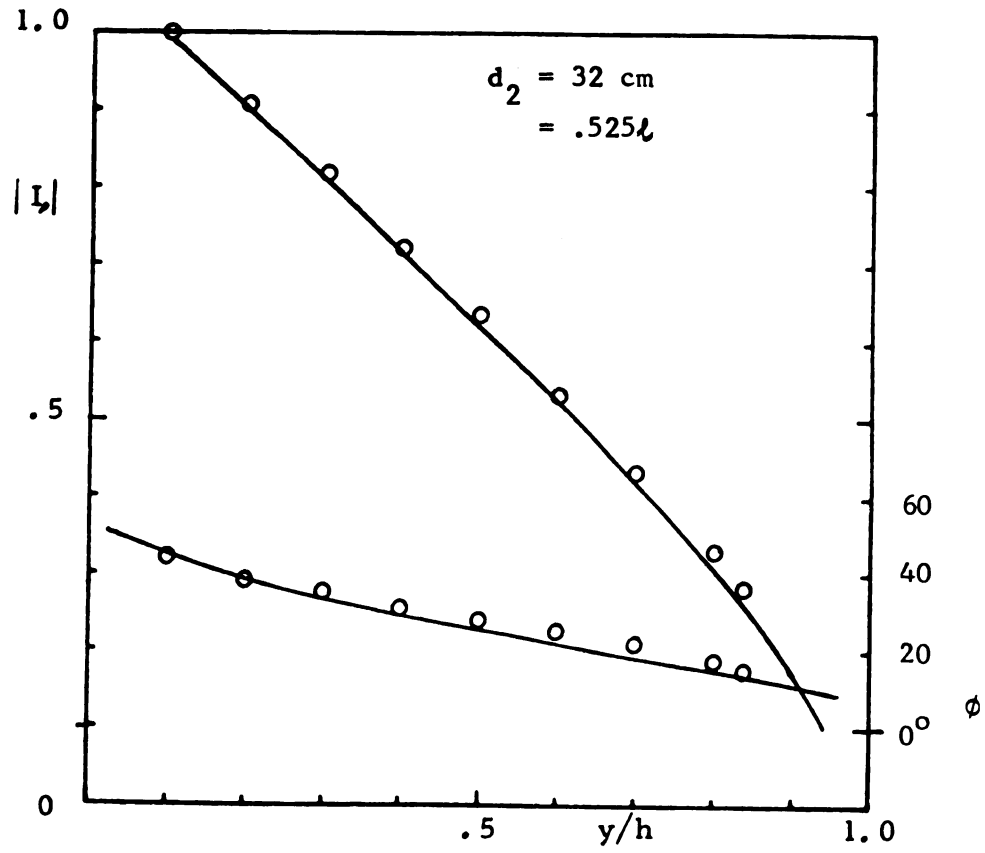


FIGURE 5.40
(continued)

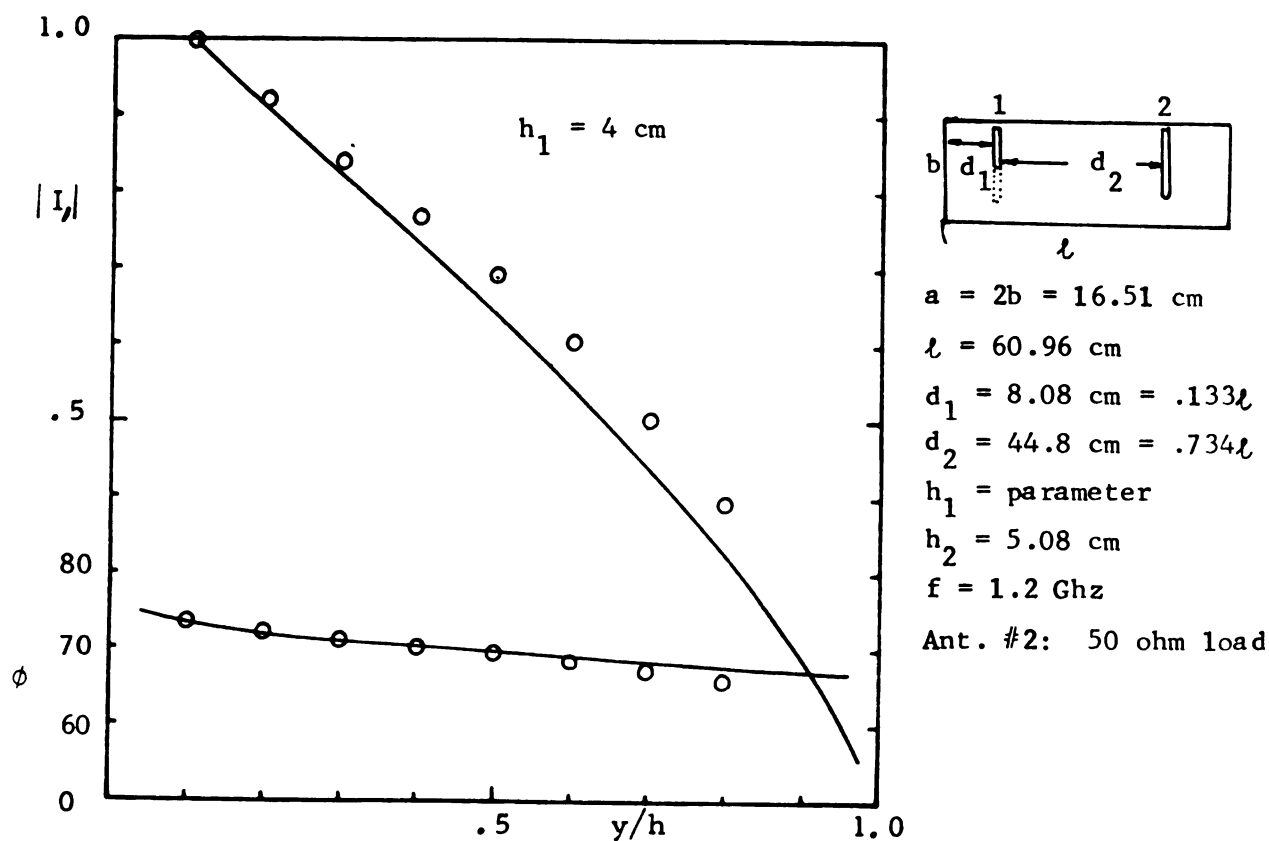
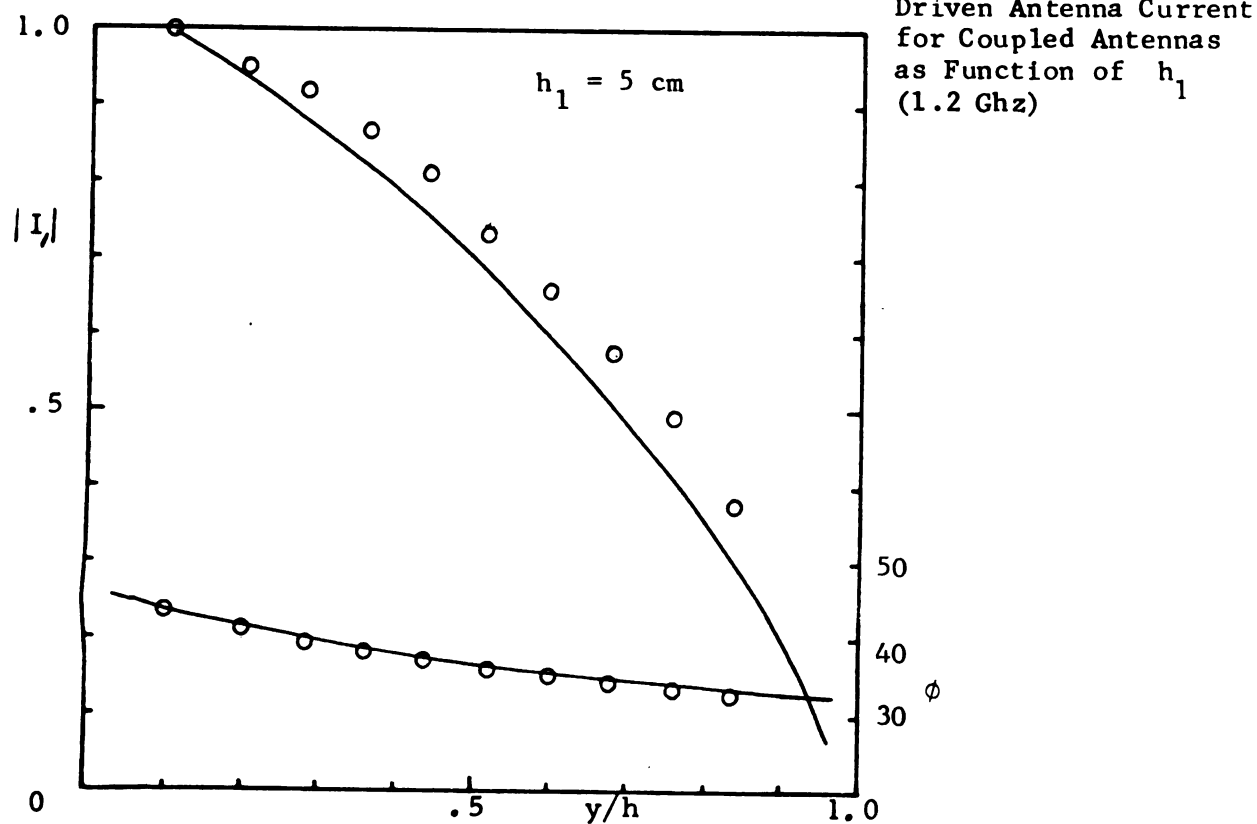
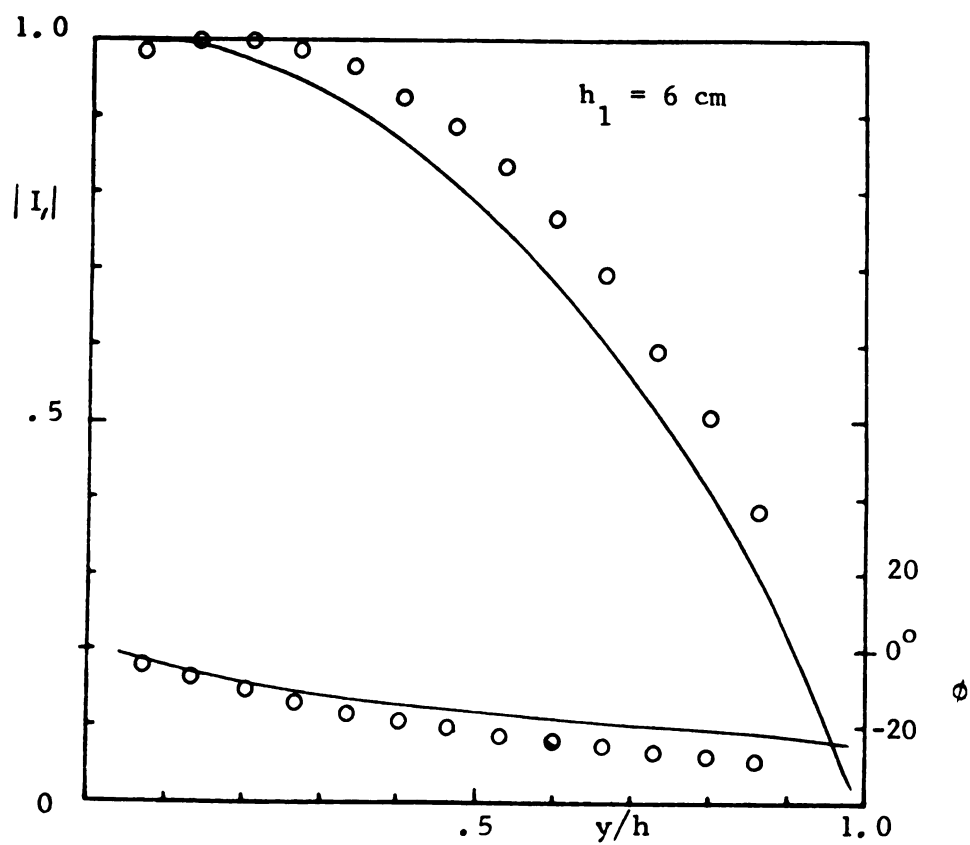
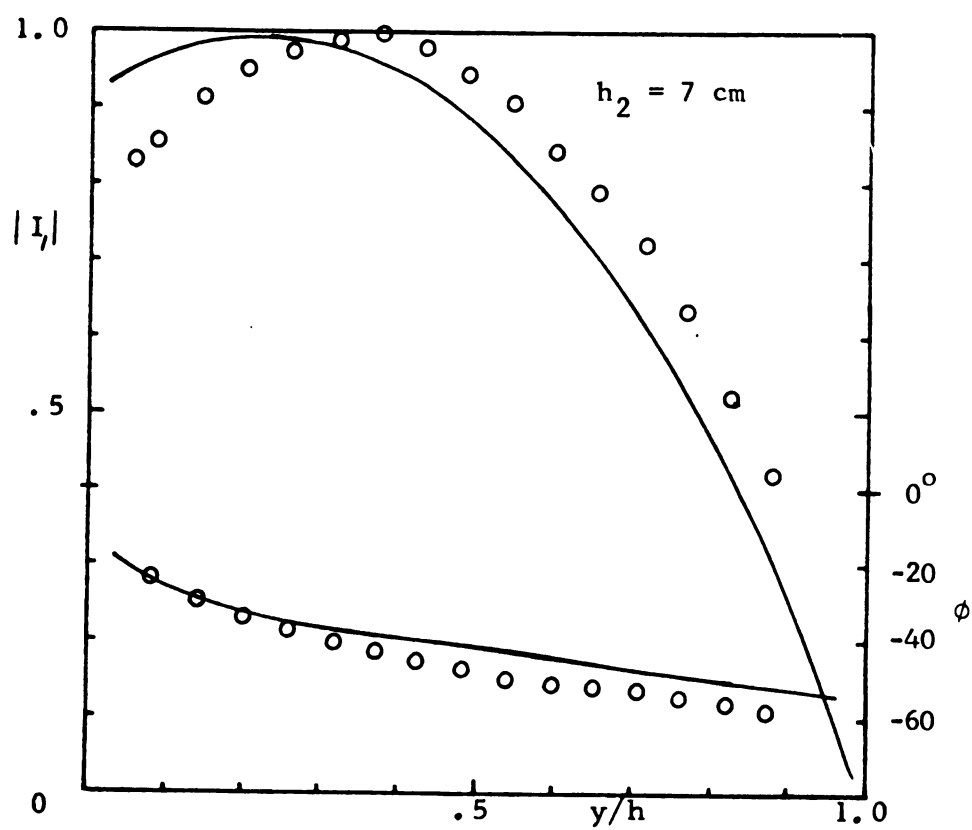


FIGURE 5.41

Driven Antenna Current
for Coupled Antennas
as Function of h_1
(1.2 GHz)

FIGURE 5.41
(continued)

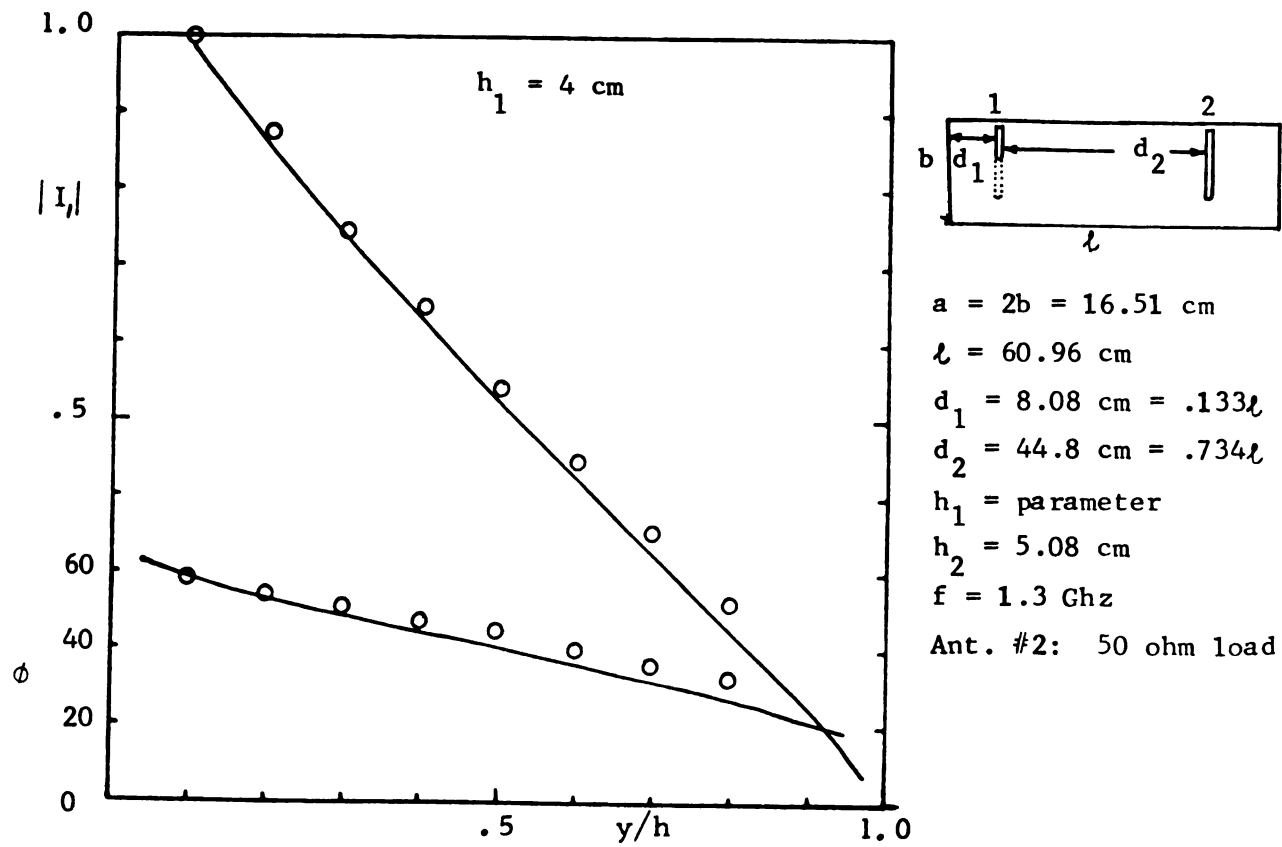
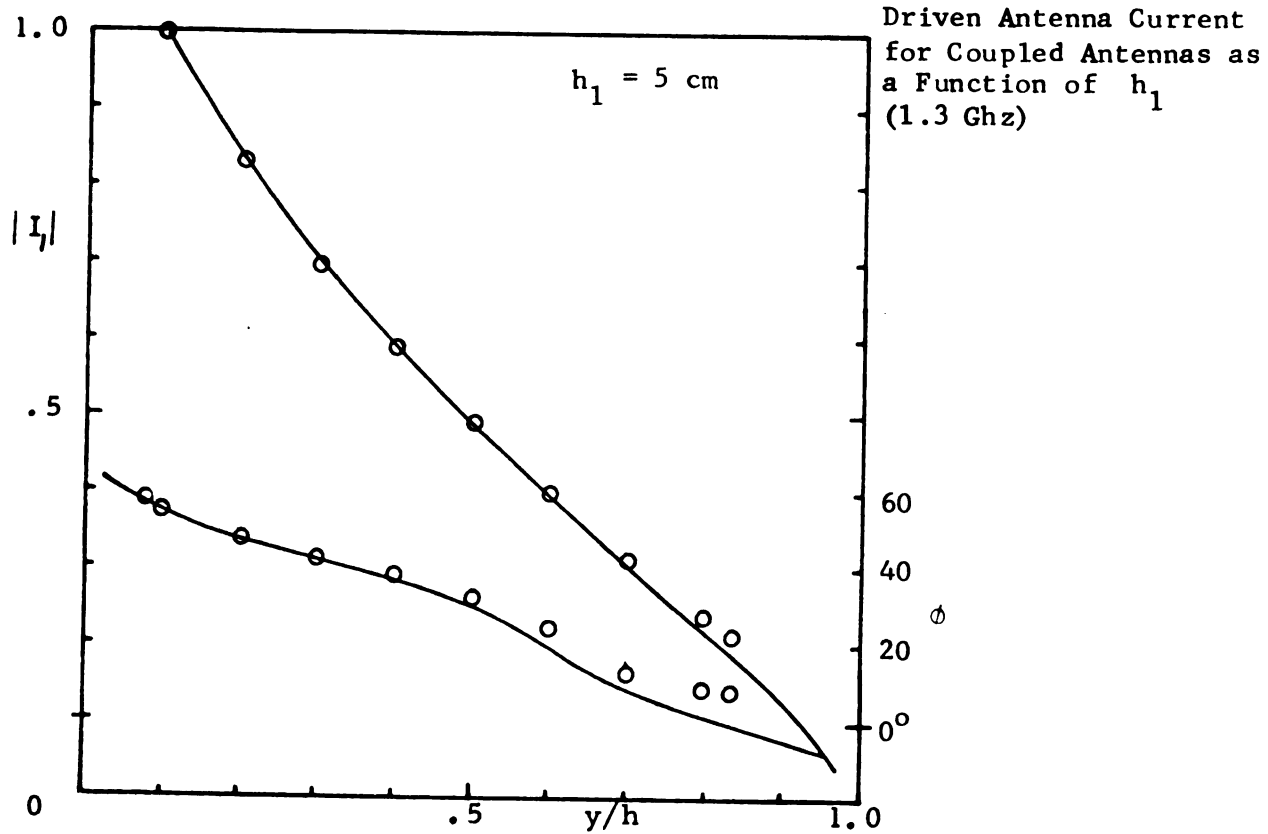


FIGURE 5.42



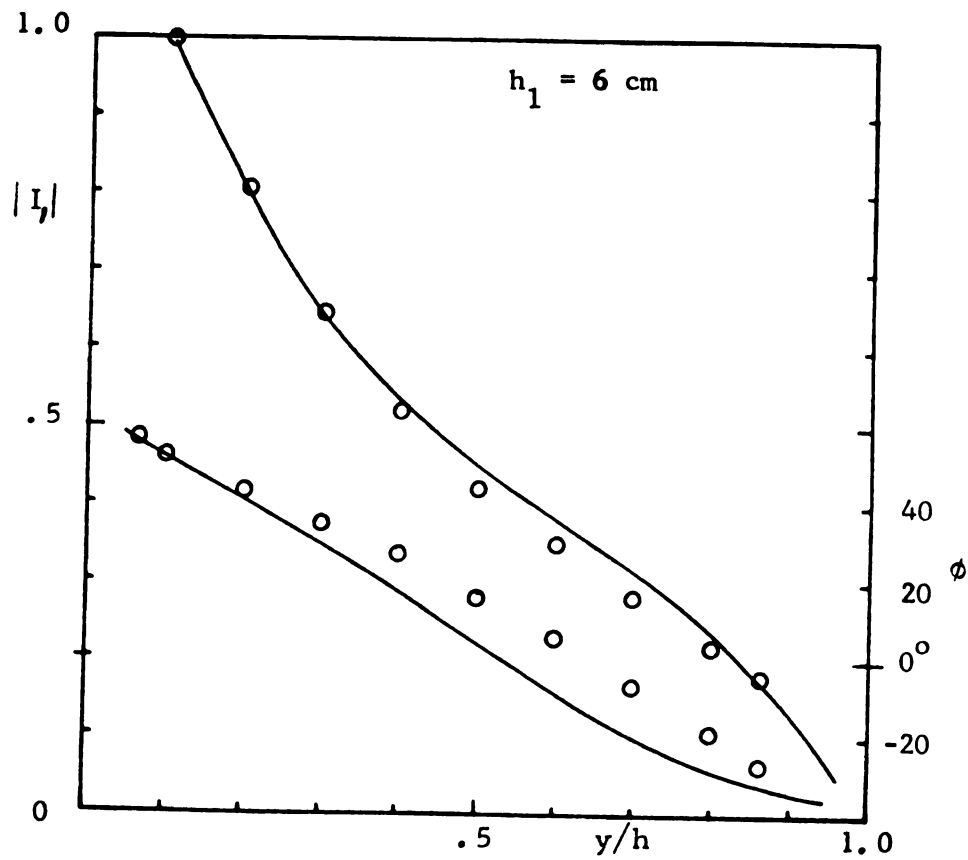
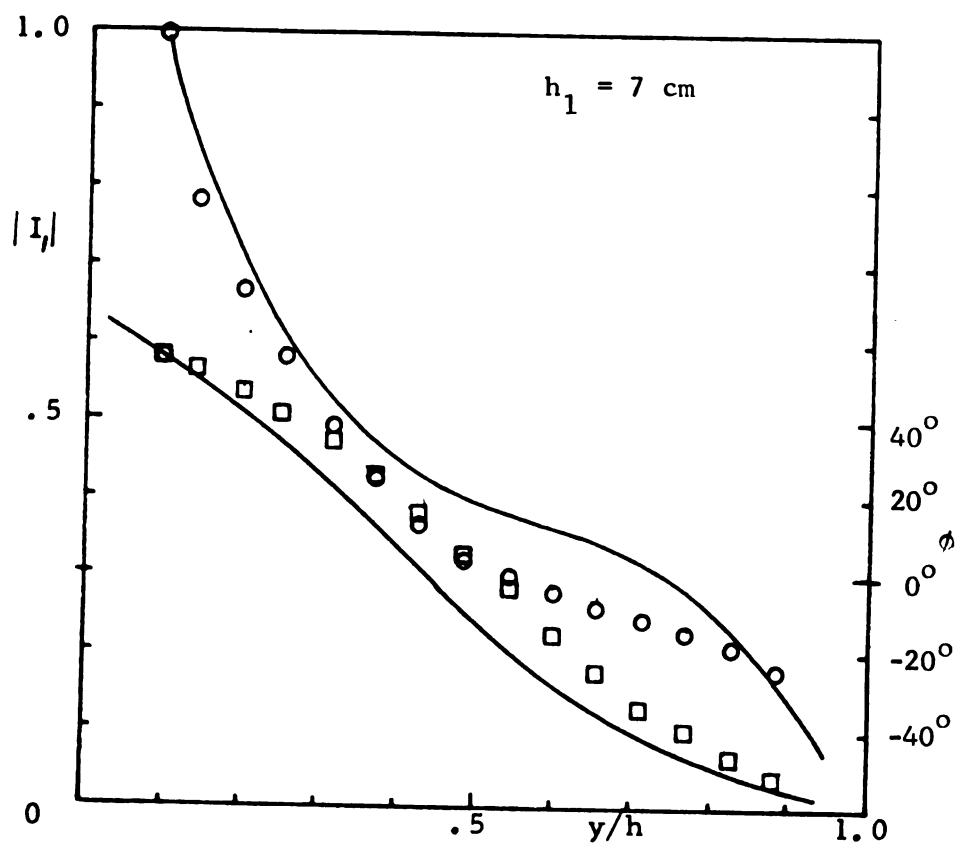


FIGURE 5.42
(continued)



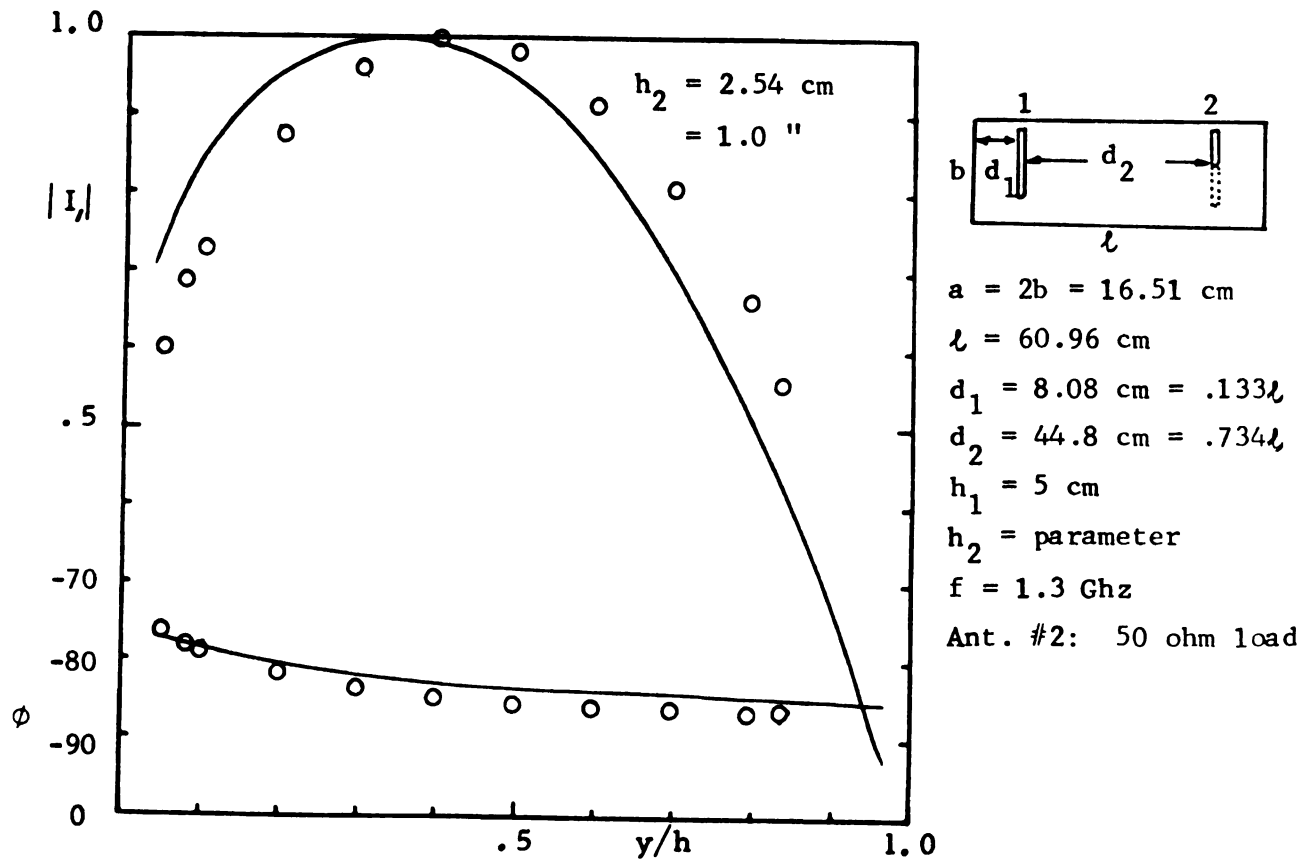
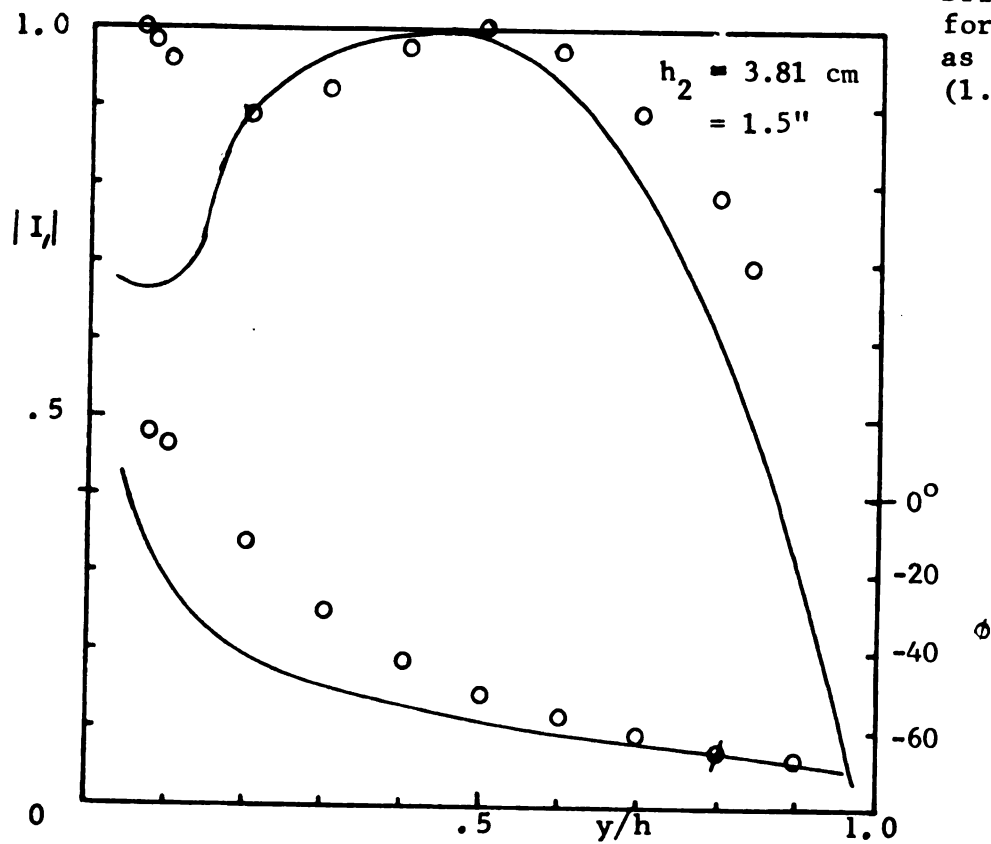


FIGURE 5.43

Driven Antenna Current
for Coupled Antennas
as Function of h_2
(1.3 GHz)



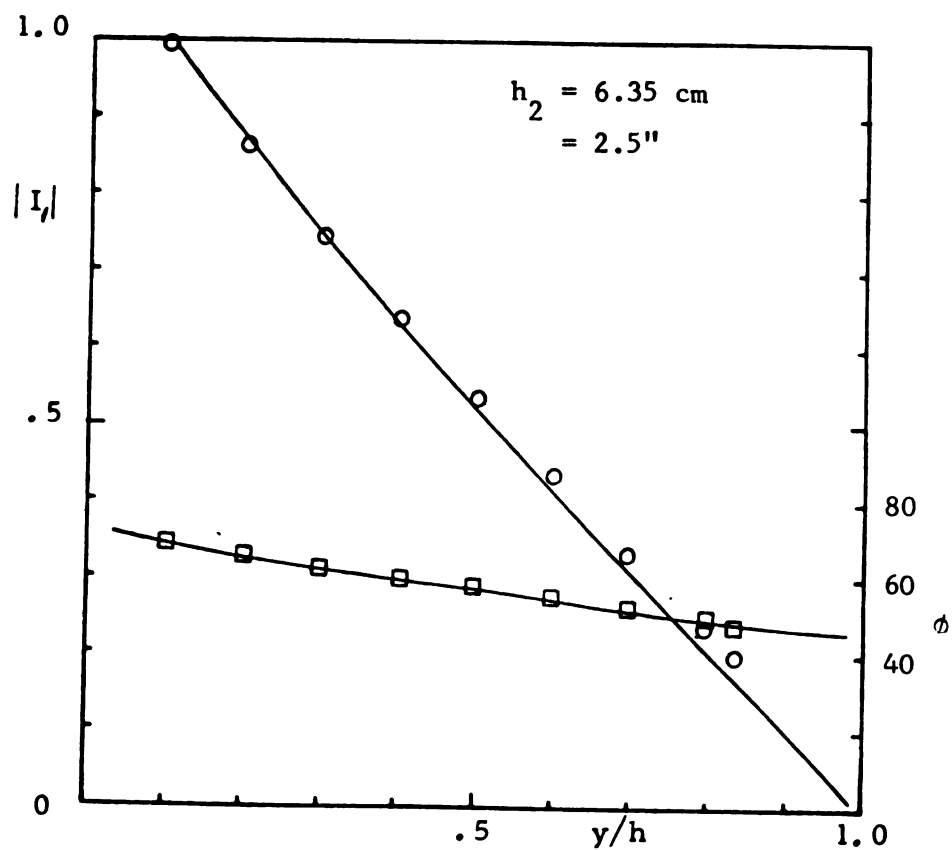
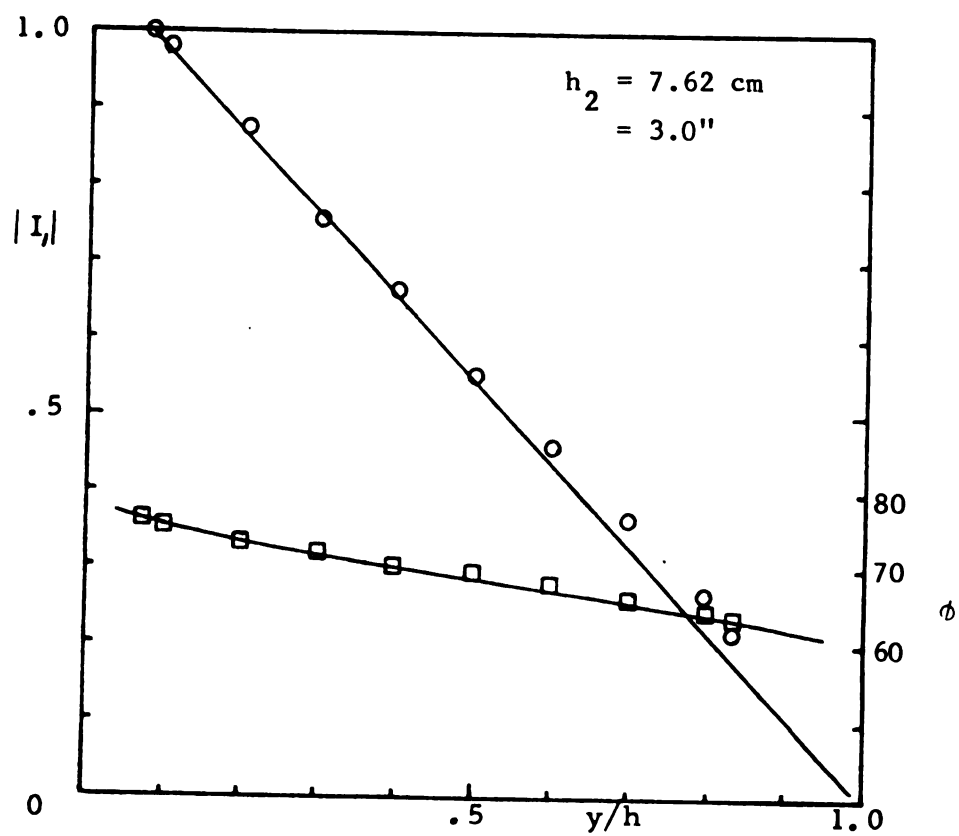


FIGURE 5.43
(continued)



5.3.3 Air Filled Cavity; Admittance

The values of measured and calculated admittance for each case considered in Section 5.3.2 are presented in Tables 5.28 through 5.31 of this section. As stated earlier, the experimental data are given in unaltered form because of the uncertainty of a terminal zone network and the problems of calculating the admittance parameters as differences of large numbers. The raw data is given for general comparison and future reference.

Since the antenna-cavity-media system is reciprocal for all cases considered here (isotropic media) the theoretical admittance matrix shown in the tables is symmetric ($Y_{12} = Y_{21}$). Although the two port admittance parameters are sufficient to characterize the system, the driven antenna input admittance for the 50 ohm load case is also included to provide a check with the experiment. It belabors the point to discuss again differences between the experimental and theoretical values shown in the tables, but it is meaningful to note that the overall tracking as a function of the various parameters is maintained.

From the small sampling of cases given for the coupled antennas little can be concluded in a definitive manner. However, it is important to observe that the mutual admittance is highly dependent upon both location and height of the antennas. (The relative insensitivity shown in Table 5.30 at 1.3 Ghz is readily related to the insensitivity of the cavity resonances to changes in height.) Both this and the variation shown in self admittances indicate at least qualitatively that the coupled antenna system could also be used to advantage in detecting changes in a medium contained in the cavity.

The results for dissipative media follow in the next section.

TABLE 5.28

Input Admittance for Coupled Antennas
as a Function of Separation ($f = 1.3$ GHz)

configuration of Figure 5.40

Theory: units: millimhos

d_2	Y_{11}	$Y_{12} = Y_{21}$	Y_{22}	Y_{50}
$.105\lambda = 6.4$ cm	$-j1.82$	$+j4.15$	$+j7.52$	$1.10 - j2.65$
$.158\lambda = 9.6$ cm	$+j2.32$	$+j4.17$	$+j4.72$	$1.42 + j1.65$
$.262\lambda = 16.0$ cm	$+j7.52$	$+j7.12$	$+j6.98$	$3.90 + j4.79$
$.525\lambda = 32.0$ cm	$+j7.44$	$-j7.44$	$+j6.79$	$3.78 + j4.87$
$.735\lambda = 44.8$ cm	$+j4.79$	$+j4.69$	$+j4.61$	$1.81 + j3.95$

Experiment: units: millimhos

d_2 cm	Y_{sc}	Y_{oc}	Y_{50}
6.4	$-j2.3$	--	--
9.6	$+j5.5$	$-j3.5$	$2.3 + j2.4$
16.0	$+j18.2$	$+j21.7$	$8.8 + j10.9$
32.0	$+j14.2$	$-j2.7$	$7.3 + j8.4$
44.8	$+j10.8$	$-j0.4$	$3.8 + j8.9$

TABLE 5.29

Input Admittance for Coupled Antennas
as a Function of h_1 (1.2 GHz)

for configuration of Figure 5.41

Theory: millimhos

h_1 cm	Y_{11}	$Y_{12} = Y_{21}$	Y_{22}	Y_{50}
4	+ j9.97	- j6.25	+ j9.97	1.96 + j8.01
5	+ j40.9	- j3.73	+ j40.8	7.89 + j8.72
6	- j11.6	+ j15.7	- j12.8	9.34 + j0.3
7	- j3.58	+ j7.78	- j5.14	4.78 - j1.12

Experiment: millimhos

h_1 cm	Y_{sc}	Y_{oc}	Y_{50}
4	- j90	+ j12.9	22.1 + j10.3
5	+ j24.4	+ j11.3	6.1 + j16.8
6	- j10.8	+ j14.9	12.5 - j2.7
7	- j3.3	+ j15.4	6.2 - j1.2

TABLE 5.30

Input Admittance for Coupled Antennas

as a Function of h_1 (1.3 Ghz)

for configuration of Figure 5.42

Theory: millimhos

h_1 cm	Y_{11}	$Y_{12} = Y_{21}$	Y_{22}	Y_{50}
4	+ j4.10	+ j4.25	+ j1.28	1.77 + j3.87
5	+ j4.63	+ j4.67	+ j4.45	1.82 + j3.82
6	+ j3.88	+ j4.64	+ j5.58	1.64 + j2.97
7	+ j3.78	+ j4.45	+ j6.69	1.36 + j2.87

Experiment: millimhos

h_1 cm	Y_{sc}	Y_{oc}	Y_{50}
4	+ j8.0	- j8.8	2.7 + j7.2
5	+ j8.5	- j1.5	3.3 + j7.0
6	+ j8.6	+ j1.1	3.2 + j6.5
7	+ j8.6	+ j2.5	2.5 + j6.6

TABLE 5.31

Input Admittance for Coupled Antennas

as a Function of h_2 (1.3 Ghz)

for configuration of Figure 5.43

Theory: millimhos

h_2 in	Y_{11}	$Y_{12} = Y_{21}$	Y_{22}	Y_{50}
1	- j2.84	+ j3.04	+ j4.29	0.78 - j3.17
1.5	- j0.69	+ j4.22	+ j3.97	1.53 + j0.07
2	+ j4.63	+ j4.67	+ j4.45	1.82 + j3.82
2.5	+ j6.71	+ j4.61	+ j4.44	1.78 + j5.92
3	+ j7.08	+ j4.16	+ j3.62	1.53 + j6.53

Experiment: millimhos

h_2 in	Y_{sc}	Y_{oc}	Y_{50}
1	- j3.0	- j7.0	1.1 - j3.4
1.5	+ j3.1	- j4.6	2.6 + j1.9
2	+ j8.5	- j1.5	3.3 + j7.0
2.5	+ j11.5	+ j1.7	3.2 + j9.9
3	+ j13.3	+ j5.5	2.9 + j12.0

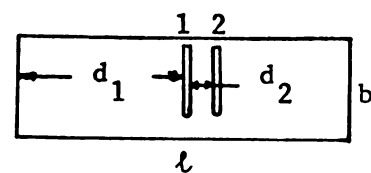
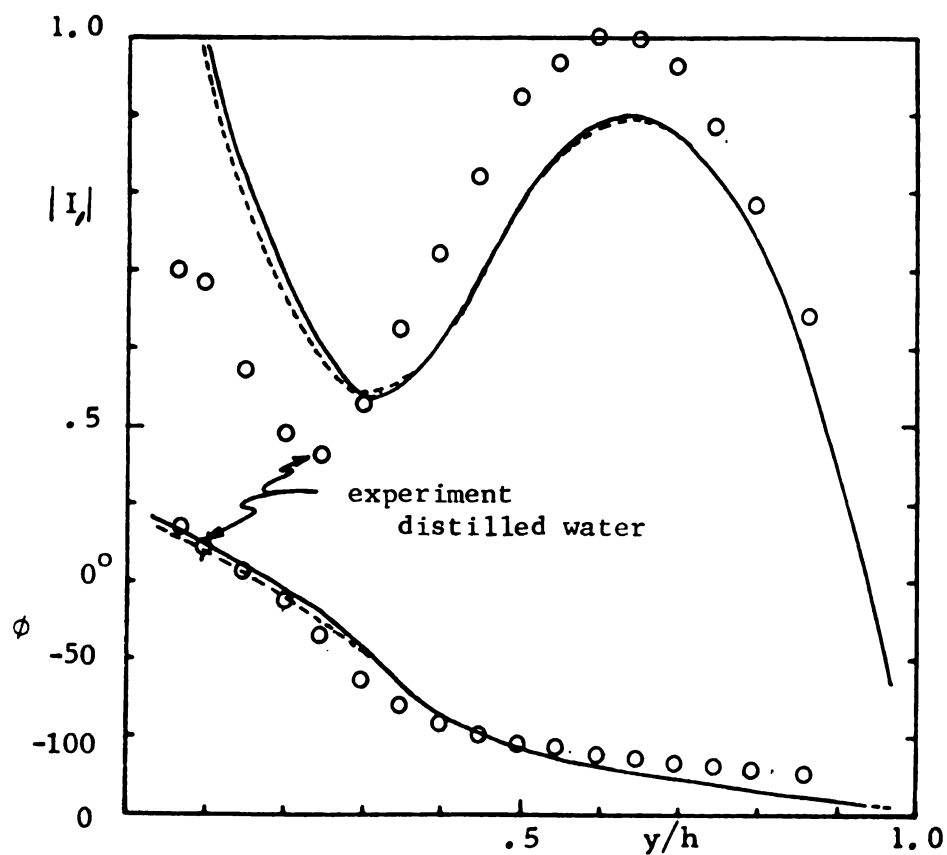
5.3.4 Cavity Filled with Dissipative Media; Current and Admittance

For coupled antennas in a dissipative medium, experimental measurements were taken on distilled water and sodium chloride solution at two frequencies. The results in Figure 5.44 show the 350 MHz case with the driven antenna (#1) centered in the cavity and the parasite (#2) as near as experimentally possible (3.2 cm). The experimental results are compared in both curves to the theoretical current based on a dielectric constant of 83.5 and a conductivity of 0.2 mhos/meter. These values were used even though the conductivity value does not represent that for distilled water at the low frequencies where dipolar relaxation effects are dominant. The experimental results for distilled water show the lower conductivity, since the dynamic range of the current is larger and the phase variation along the antenna is more rapid than predicted from the theory. The sodium chloride solution result in the lower curve shows a higher conductivity and loss than the 0.2 mho/meter theoretical results, probably because of the combination of ionic effects (measured at low frequency to be 0.2 mhos/meter) and the beginning of the dipolar effects. Also shown in Figure 5.44 as a dashed line is the theoretical result for the same configuration except with the parasite removed (single antenna results, Figure 5.37). Apparently the conductivity of 0.2 mhos/meter used in the theory is somewhat unique in the particular configuration such that little difference occurs between the single and coupled cases. A greater difference is at least expected and comparison with the experimental results in Figure 5.37 shows that, indeed, an effect is seen.

Similar results for distilled water and salt solution at 1.0 GHz are given in Figure 5.45. At this high frequency the differences between distilled water and salt solution become less distinct because of the dominance of the dipolar relaxation. The difference between the single antenna (dashed lines) and coupled antennas is quite pronounced. Although the experiment appears to be comparable with the single results, it is important to remember that the theoretical values used for dielectric constant and conductivity are not expected to be exact at this frequency and are used only as a reference.

The input admittance and theoretical admittance parameters for the 350 MHz and 1.0 GHz cases are given in Tables 5.32 and 5.33, respectively. As shown, the experimental results for both frequencies are the same regardless of the loading of the parasitic antenna (where sc, oc, and 50 indicate short circuit, open circuit and the 50 ohm loaded parasite measurements, respectively). Based on the theoretical results (differing as usual by the approximate factor of two) for the same measurements (e.g. $Y_{11} = Y_{sc}$), this roughly indicates the sensitivity of the admittance measuring methods. The fact that the coupling term ($Y_{12} = Y_{21}$) is small shows that the medium is responsible for the majority of the loss and approximately shields the parasite from the driven antenna at the chosen separations of 3.2 cm (approximately $0.35 \lambda_w$ at 350 MHz; $1 \lambda_w$ at 1.0 GHz). The experimental results at 1.0 GHz in Table 5.33 are markedly different from the theory in the susceptance and it is suspected that an error was made in recording the reference phase angle during calibration for these measurements.

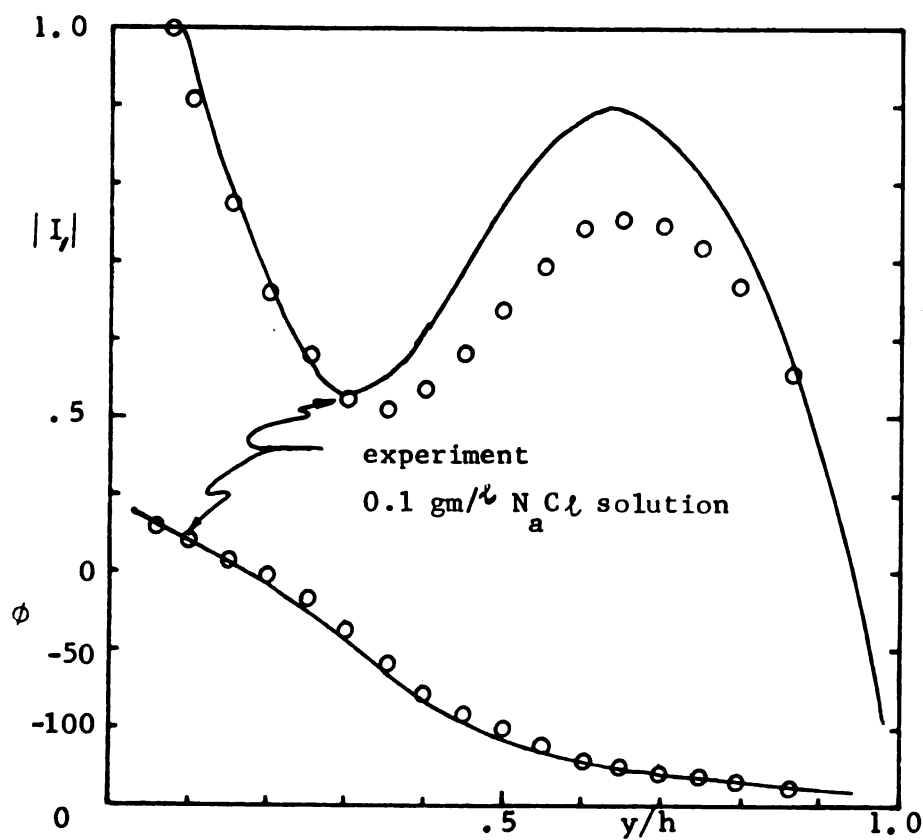
These few examples investigated for coupled antennas in dissipative media are obviously not complete. They do, however, serve to show the ability of the theory to predict results comparable to experiment as well as qualitatively representing the experimental accuracy and results expected. This therefore concludes the study of coupled antennas in known media. The next and final section considers the results from measurements on a sample of sewage treatment plant effluent.



$a = 2b = 16.51 \text{ cm}$
 $l = 60.96 \text{ cm}$
 $h_1 = 6.0 \text{ cm}$
 $h_2 = 6.35 \text{ cm}$
 $d_1 = 0.5l$
 $d_2 = 0.052l$
 $f = 350 \text{ MHz}$

FIGURE 5.44

Driven Antenna Current
for Coupled Antennas
in Dissipative Media
(350 MHz)



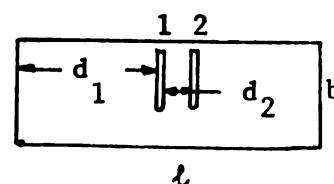
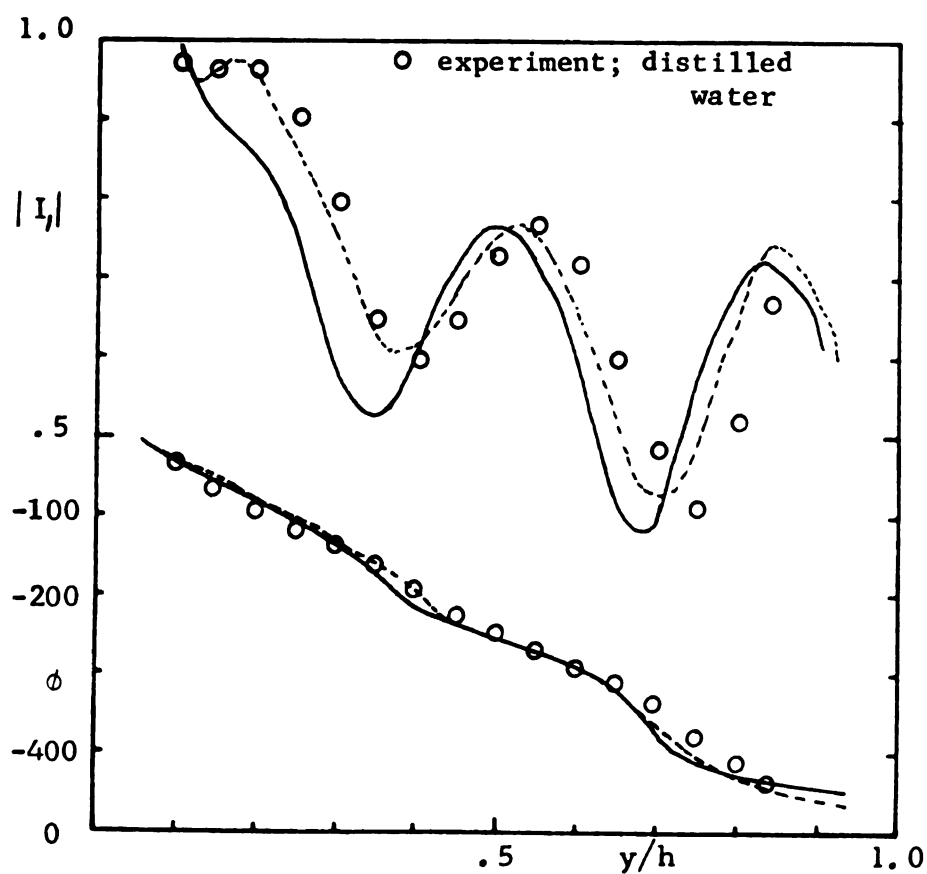
Theory, both curves:

$$\epsilon_r = 83.5$$

$$\sigma = 0.2 \text{ mho/m}$$

— coupled antennas

--- Ant. #1 alone



$$a = 2b = 16.51 \text{ cm}$$

$$l = 60.96 \text{ cm}$$

$$h_1 = 5.0 \text{ cm}$$

$$h_2 = 5.08 \text{ cm}$$

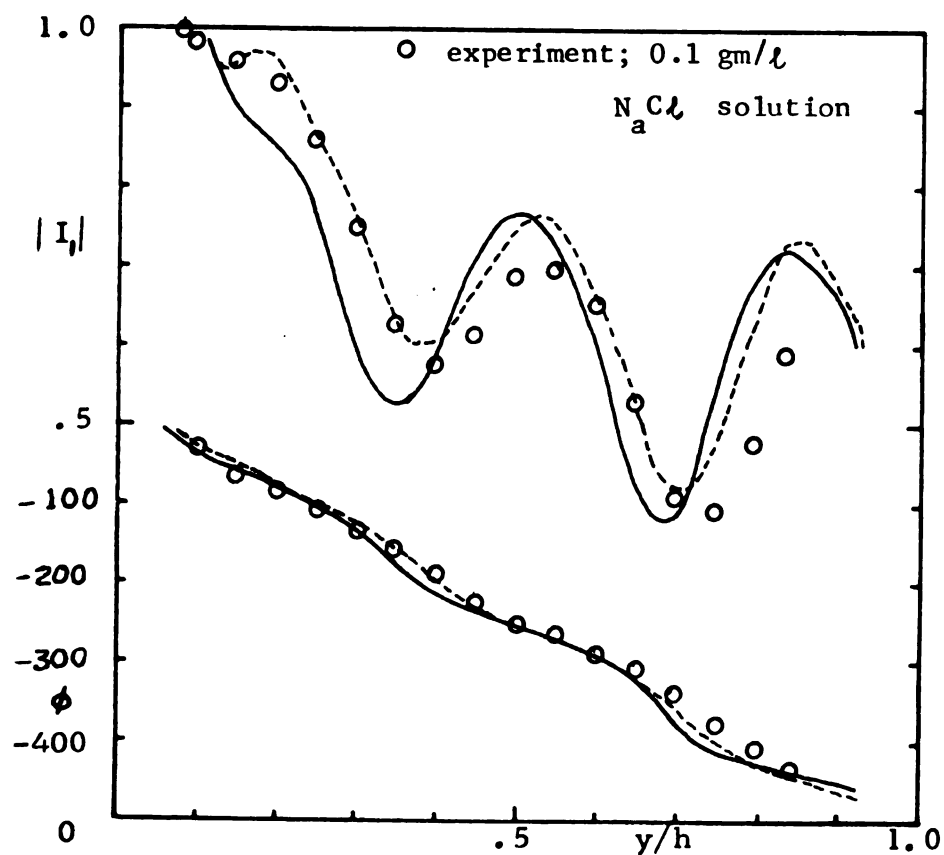
$$d_1 = 0.5l$$

$$d_2 = 0.052l$$

$$f = 1.0 \text{ GHz}$$

FIGURE 5.45

Driven Antenna Current
for Coupled Antennas in
Dissipative Media
(1.0 GHz)



Theory, both curves:

$$\epsilon_r = 83.5$$

$$\sigma = 0.2 \text{ mho/m}$$

— coupled antennas

--- Ant. #1 alone

TABLE 5.32

Input Admittance for Coupled Antennas
in Dissipative Media at 350 MHz

configuration of Figure 5.44

Theory $\sigma = 0.2 \text{ mhos/m}$, $\epsilon_r = 83.5$

$$[Y] = \begin{bmatrix} 50 + j51 & -6.0 - j7.7 \\ -6.0 - j7.7 & 58 + j44 \end{bmatrix} \text{ millimhos}$$

$$Y_{50} = 50 + j50 \text{ millimhos}$$

Experiment

Distilled water

$$Y_{sc} = Y_{oc} = Y_{50} = 74.6 + j104 \text{ millimhos}$$

NaCl solution, 1 gm/liter

$$Y_{sc} = Y_{oc} = Y_{50} = 103 + j92 \text{ millimhos}$$

Table 5.33

Input Admittance for Coupled Antennas
in Dissipative Media at 1.0 GHz

configuration of Figure 5.45

Theory $\sigma = 0.2 \text{ mhos/m}$, $\epsilon_r = 83.5$

$$[Y] = \begin{bmatrix} 70 + j57 & -2.5 - j16 \\ -2.5 - j16 & 69 + j60 \end{bmatrix} \quad \text{millimhos}$$

$$Y_{50} = 71 + j55 \quad \text{millimhos}$$

Experiment (error expected in recording data)

Distilled water

$$Y_{sc} = Y_{oc} = Y_{50} = 23.1 - j2.4 \quad \text{millimhos}$$

NaCl solution, 1 gm/liter

$$Y_{sc} = Y_{oc} = Y_{50} = 21.5 - j0.4 \quad \text{millimhos}$$

5.4 Special Applications to Sewage Plant Effluent

In the preceding section, the operation of the antenna-waveguide-media system was studied for dissipative media by using solutions of controlled amounts of pure sodium chloride. The question may legitimately be raised about the usefulness of the chosen concentration in predicting performance in a natural environment medium. To approach the question, this section presents the results of studying one extreme medium: the effluent from a sewage treatment plant. This case study represents one example of the type of application oriented study which is necessary for this system, although it is of course far short of the comprehensive experimental and theoretical correlation that is required.

The chosen sample for this one case was obtained from the first of a series of large ponds used to store the raw effluent from the final stage of the East Lansing, Michigan, sewage treatment plant.* Since very little natural breakdown of the sewage occurs in the first pond, this sample closely represents the undiluted output of a typical municipal waste water treatment plant. The chemical analysis of the sample taken is given in Table 5.34 along with the measured low frequency conductivity. When compared to the results given in Figure 4.8 it is seen that the sewage sample corresponds to a salinity (of NaCl) of 0.5 grams per liter in low frequency conductance. The 1.0 gram per liter concentration used in the previous section yielded twice the conductivity of the sewage.

* These storage ponds are part of the ongoing research under the Institute of Water Research, Michigan State University, whose assistance I gratefully acknowledge.

The sewage sample, therefore, shows that the dissipative media were realistically representative. (Of course, much more highly dissipative media are possible, for example, if sea water is considered with a typical salinity of 35 grams per liter and conductivity of 4 mhos per meter.)

Single antenna in cavity; current

The results for the antenna current distribution for sewage filling the experimental rectangular cavity are given for four frequencies (0.35, 1.0, 1.2, 1.6 Ghz) in Figures 5.46 through 5.50. In all but the 1.6 Ghz case the theoretical results are included for two combinations of permittivity and conductivity:

$$\begin{aligned}\epsilon_r &= 83.0 \\ \sigma &= 0.1 \text{ mho/m}\end{aligned}$$

and

$$\begin{aligned}\epsilon_r &= 83.5 \\ \sigma &= 0.2 \text{ mho/m} .\end{aligned}$$

The former combination represents the measured low frequency conductivity of the sewage sample (the permittivity was chosen arbitrarily), while the latter combination is that used for the comparison with the sodium chloride solution in Sections 5.2.5-6 and 5.3.4.

The results at 350 MHz in Figure 5.46 best indicate the type of results expected. The experimental data appears to be bounded by the high and low conductivity theories. It is difficult to readily obtain quantitative explanations for the results, since the effect of the media on the interaction between antenna and

cavity is complex.

This interaction effect may also explain the differences seen at 1.0 GHz in Figure 5.47 and 5.48. In that data as the height of the antenna is increased from 5.0 cm, the agreement between theory and experiment worsens. The interesting point of this example is that the theory exhibits greater dynamic range variation ("peak-to-peak" level) as the height of the antenna is increased. This is not immediately obvious, but one could also imagine that proper changes in dielectric constant could result in better agreement, since the cavity resonance structure would then be changed very rapidly (whereas a two-fold change in conductivity shows little effect). In Figure 5.48 it is seen that as the antenna location is changed, very little change occurs in either theory or experiment (realize that the theoretical curves are rough hand drawn representations of the twenty point numerical solution and are therefore irregular in smoothness). This is indicative of the small coupling between cavity and antenna at the high frequency and high effective dissipation.

The unexpected variation of the current with antenna height found at 1.0 GHz is again seen at 1.2 GHz in Figure 5.49. Again it is reasonable to expect that changes in dielectric constant could bring the theoretical result more in line with the experiment. Unfortunately, time and computing cost precluded further investigation into this.

The final case presented for the single antenna is at 1.6 GHz in Figure 5.50. At this higher frequency the attenuation is so large that the phase shift along the antenna has become nearly

linear and the current has been smoothed to the point that very little information other than rate of decay can be obtained from the data. This effect is seen even for the low conductivity theory, although it is more evident for the experimental results of the effective (high frequency) higher conductivity sewage plant effluent.

Single antenna in cavity; admittance

The input admittance for the single antenna in sewage plant effluent for the configurations presented in the preceding paragraphs is given in Tables 5.35 through 5.38. In all cases only the theoretical results comparing the solutions for the two different conductivities are compared. No experimental data is presented because of error in the phase angle calibration of the admittance measuring system. A length of line of unknown length (and phase shift) was inserted inadvertently in one cable between admittance measuring block and vector voltmeter. The phase calibration was not changed, however, and thus the conversion of the data from the measuring plane to the antenna terminal plane was impossible.

From the tables one can see that at 350 MHz the difference in conductivity has an effect on the level of input admittance and the coupling to the cavity is evident by the sensitivity to changes in location. At the $\mathcal{L}$ -band frequencies, however, the differences are considerably reduced, although a sensitivity is still noticeable. The theory of course does not account for dipolar relaxation effects, but the higher frequency increases the electrical size of the cavity thus increasing the electrical distance between antenna and walls. The admittances at 1.6 GHz are given in Figure 5.50 instead of in a table.

Coupled Antennas

Very little data was taken for coupled antennas in sewage. The high loss and low coupling that was observed for salt solution at the higher frequencies (above 1.0 GHz) indicated that little is gained from experimental measurements of current distribution for the coupled case. Therefore, only a few cases at 350 MHz are presented in Figures 5.51 and 5.52 and Table 5.39. The theoretical and experimental cases in the two figures differ by the height of the driven antenna, but the effect of varying separation is seen to be the same for both sets of data.

The input admittance parameters in Table 5.39 show that there is very little coupling but that the coupling definitely varies as the separation (d_2) varies. The self-admittance terms remain relatively fixed, however, which indicates that the cavity-antenna coupling is not a strong function of the location of the antennas. The value of input admittance when the parasite is loaded with 50 ohms (Y_{50}) is seen to differ very slightly from the driven antenna self-admittance. The losses of the medium are thus nearly masking the effect of second antenna loading from the active antenna.

Received Voltage

One possible use of coupled antennas other than for two-port terminal measurements is as an absorption measurer. The radiation, waveguide transmission, and reception of energy for absorption studies is used in electrochemistry analysis of absorption spectra. In a similar manner, the data for received voltage was taken for different separations between two antennas in the cavity filled with sewage plant effluent. The case chosen was for a frequency of 1.0

GHz with the cavity and antenna having dimensions:

$$a = 2b = 16.51 \text{ cm}$$

$$l = 60.96 \text{ cm}$$

$$d_1 = 8.08 \text{ cm}; d_2 = \text{parameter}$$

$$h_1 = 6.0 \text{ cm}; h_2 = 5.08 \text{ cm} .$$

The theoretical data points were obtained by making individual program runs for each separation and noting the input current at the second antenna when loaded with a 50 ohm load. The results are presented in Figure 5.53 where in 5.53(a) data is included for distilled water and in 5.53(b) the sodium chloride solution results are included.

The data was taken for the active antenna located as close as experimentally possible to one shorting plate ($d_1 = 8.08 \text{ cm} = 0.133\lambda$). The receiving antenna was then moved in increments of 3.2 cm (approximately 0.052λ) and the received voltage noted. This voltage was measured on the vector voltmeter with reference taken to one arbitrary side of the admittance sensing block. The data presented in Figure 5.53 is only the magnitude of the complex voltage received. The phase information involves multiples of 2π ambiguities, because for water ($\epsilon_r = 83$) the 3.2 cm increment is nearly equal to a full wavelength. Furthermore, the magnitude information is normalized to the voltage received at the closest separation to simplify comparison.

Although this set of single frequency data is only part of the measurements that are necessary for swept frequency absorption studies, they show the range of dissipation predicted (and measurable) for these sample media over an increasing path length.

It is interesting to note that of the three water samples studied only the distilled water results tend to approximate the theory. It is also interesting that the sewage results are smoother with less fluctuation than the sodium chloride results. Of course this data is insufficient to suggest explanations, but the differences seen are significant enough to warrant further investigation.

This concludes Chapter 5 and the presentation of experimental and numerical results of this report. The highlights of these results are summarized in the final chapter which follows.

TABLE 5.34

Analysis of Sewage Sample

(sample taken 20 November 1974)

pH	7.9
Dissolved oxygen	12.6 mg/l - D.O.
Total phosphorus	0.80 mg/l - P
Soluble phosphorus	0.65 mg/l - P
Nitrite	0.078 mg/l - N
Nitrate	5.1 mg/l - N
Ammonia	9.3 mg/l - N
Total Carbon	48 mg/l - C
Organic Carbon	6 mg/l - C
Alkalinity	179 mg/l CaCO_3

Conductivity measurements using method of Figure 4.8

frequency, MHz	σ , mhos/m
1.5	0.104
2.0	0.100
5.0	0.100
10.0	0.106
15.0	0.096

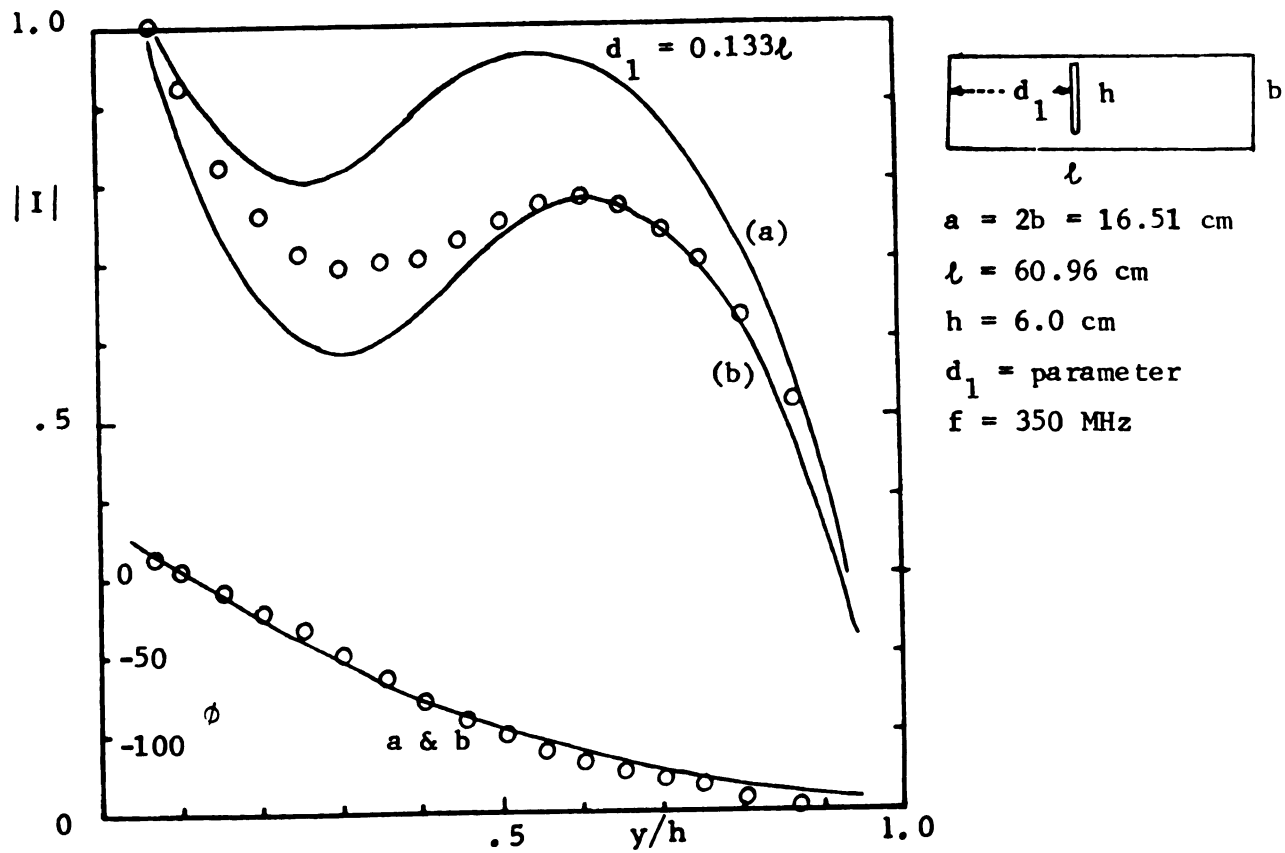
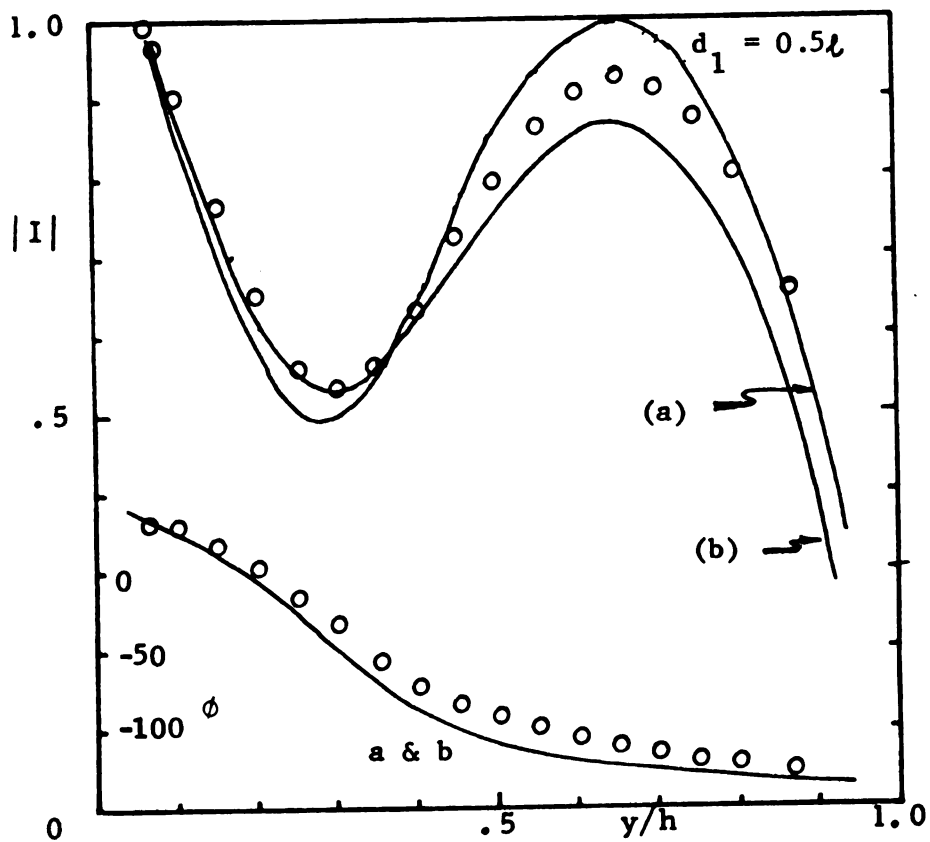
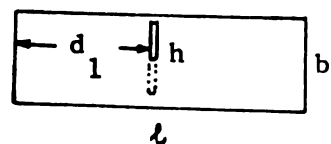
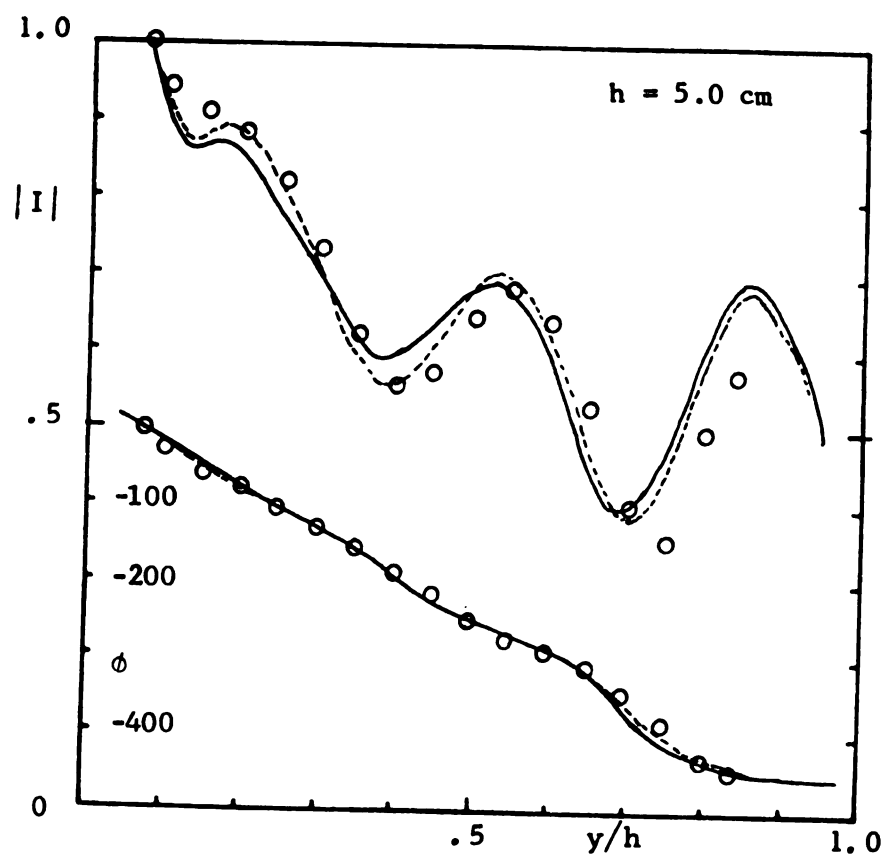


FIGURE 5.46
Current for Single
Antenna in Sewage
(350 MHz)



Theory:

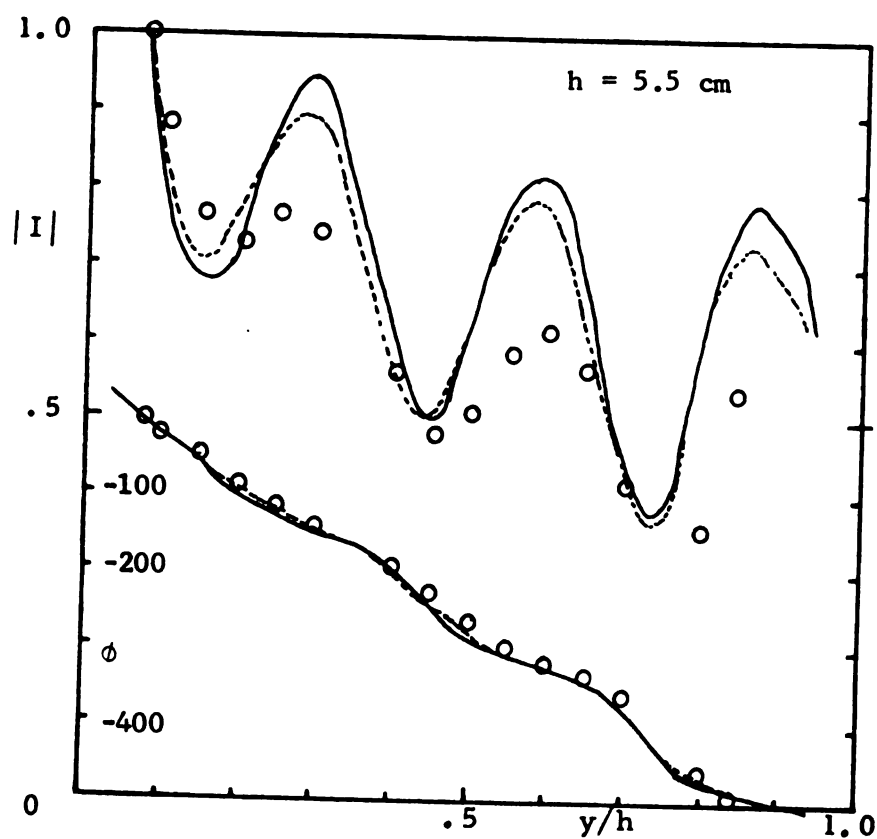
- (a) $\epsilon_r = 83.0$
 $\sigma = 0.1 \text{ mho/m}$
 (b) $\epsilon_r = 83.5$
 $\sigma = 0.2 \text{ mho/m}$



$a = 2b = 16.51 \text{ cm}$
 $l = 60.96 \text{ cm}$
 $d_1 = 0.5l$
 $h = \text{parameter}$
 $f = 1.0 \text{ GHz}$

FIGURE 5.47

Current as Function
of Height for Single
Antenna in Sewage
(1.0 GHz)



Theory:

— $\epsilon_r = 83$
 $\sigma = 0.1 \text{ mho/m}$
 --- $\epsilon_r = 83.5$
 $\sigma = 0.2 \text{ mho/m}$

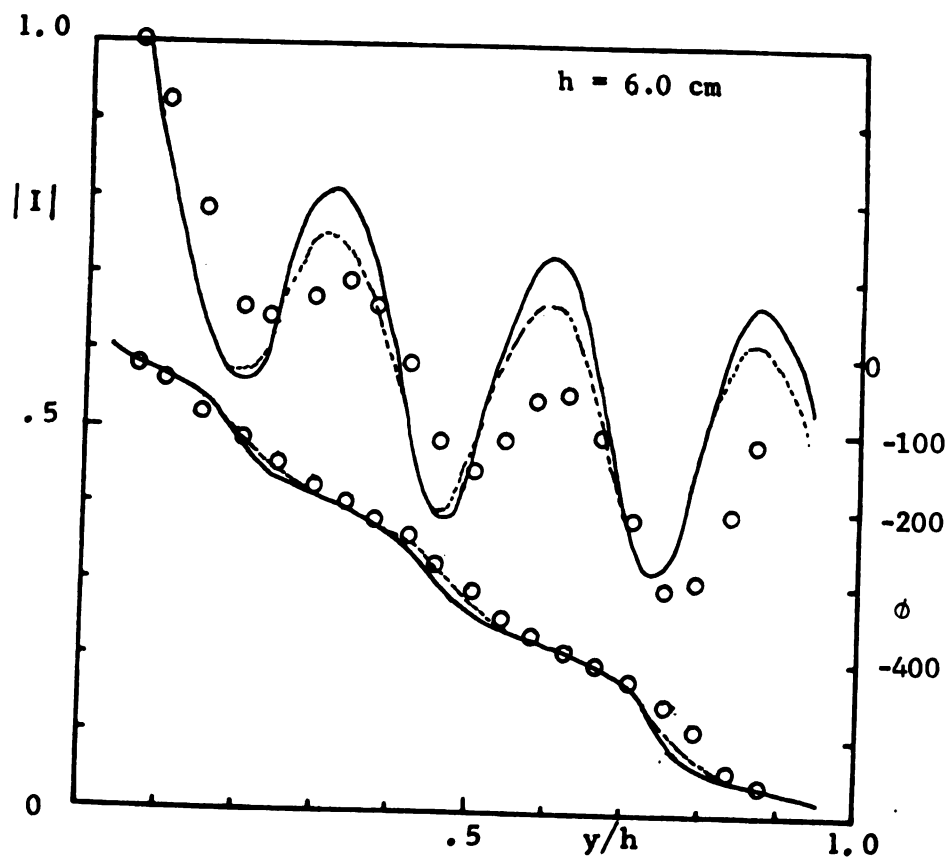
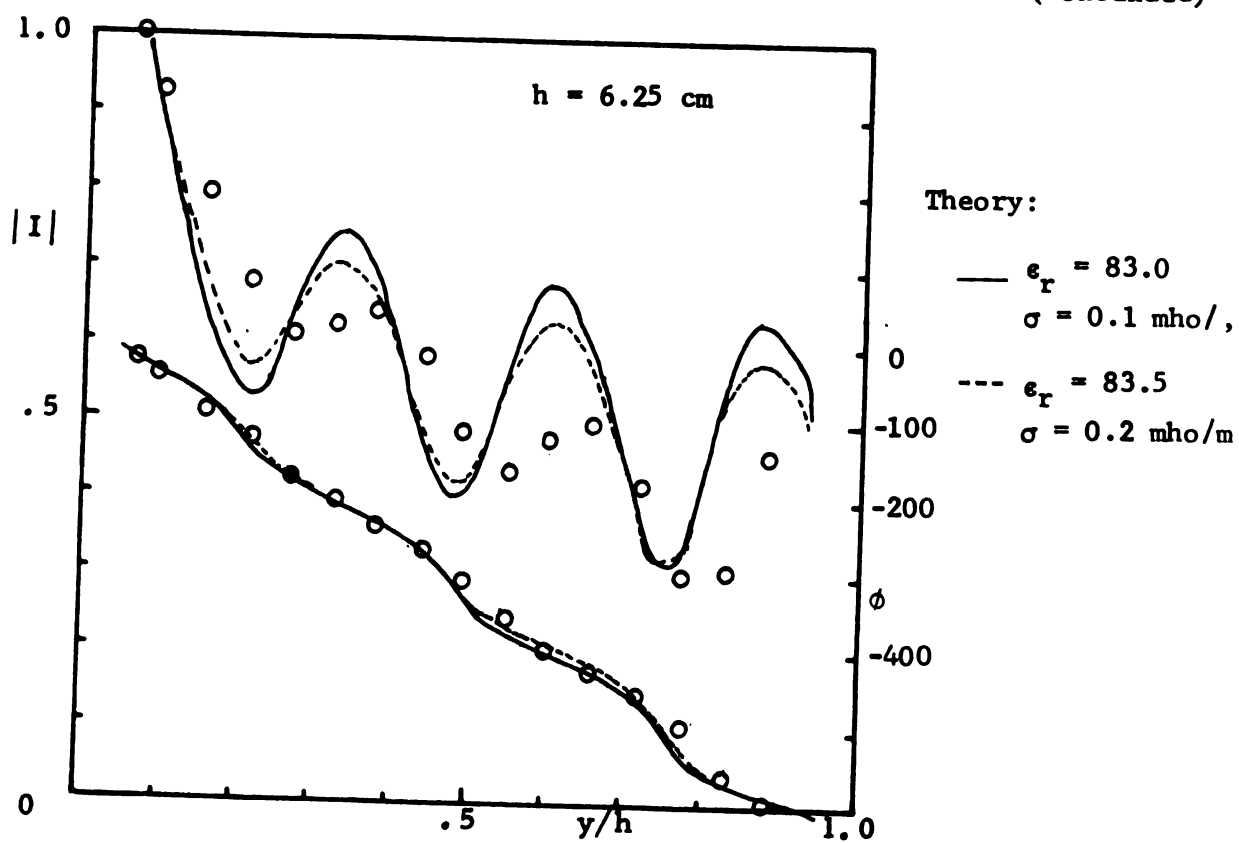


FIGURE 5.47
(continued)



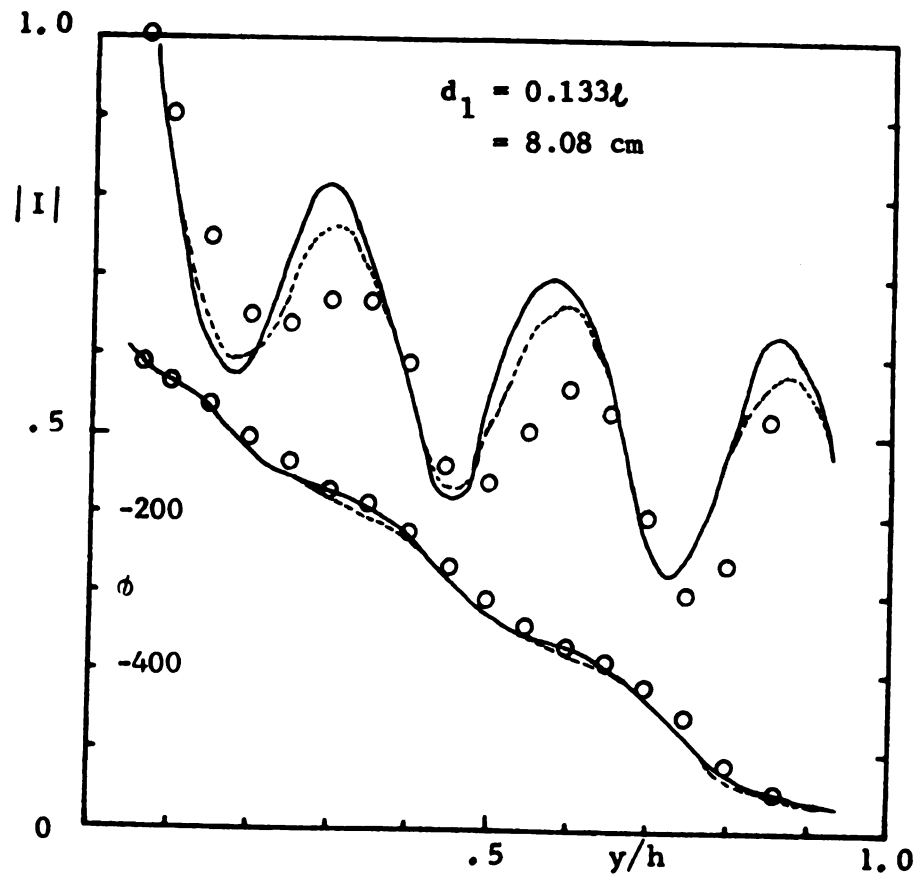
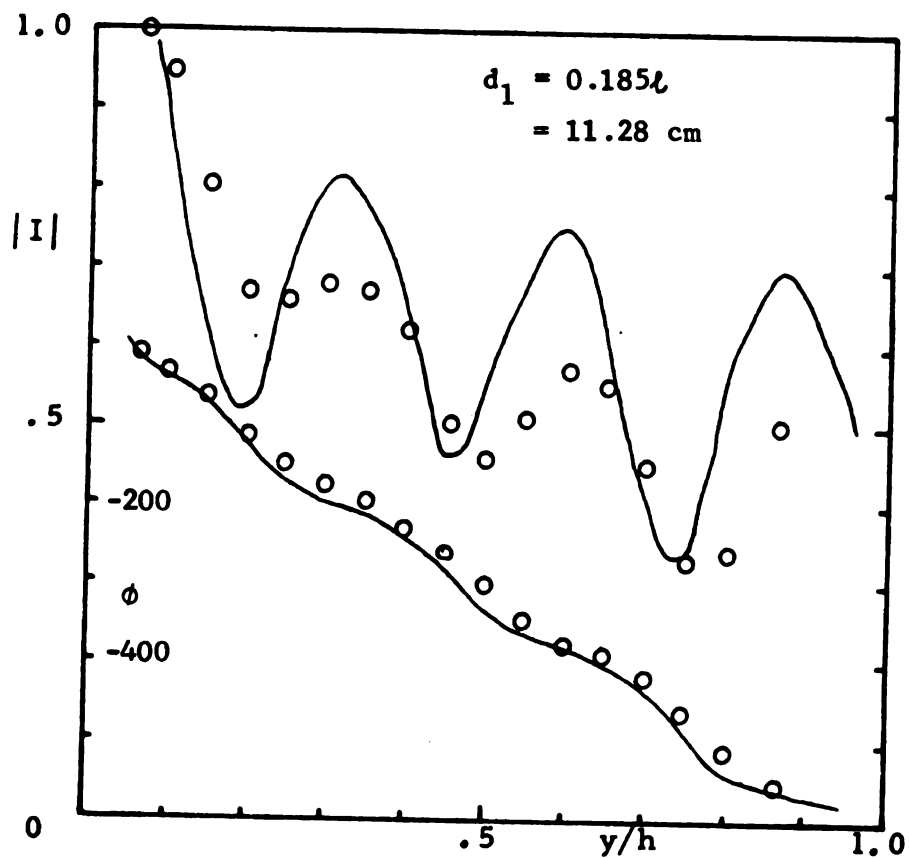


FIGURE 5.48
Current as Function
of Location for
Single Antenna in
Sewage (1.0 GHz)



Theory:

- $\epsilon_r = 83.0$
 $\sigma = 0.1 \text{ mho/m}$
- $\epsilon_r = 83.5$
 $\sigma = 0.2 \text{ mho/m}$

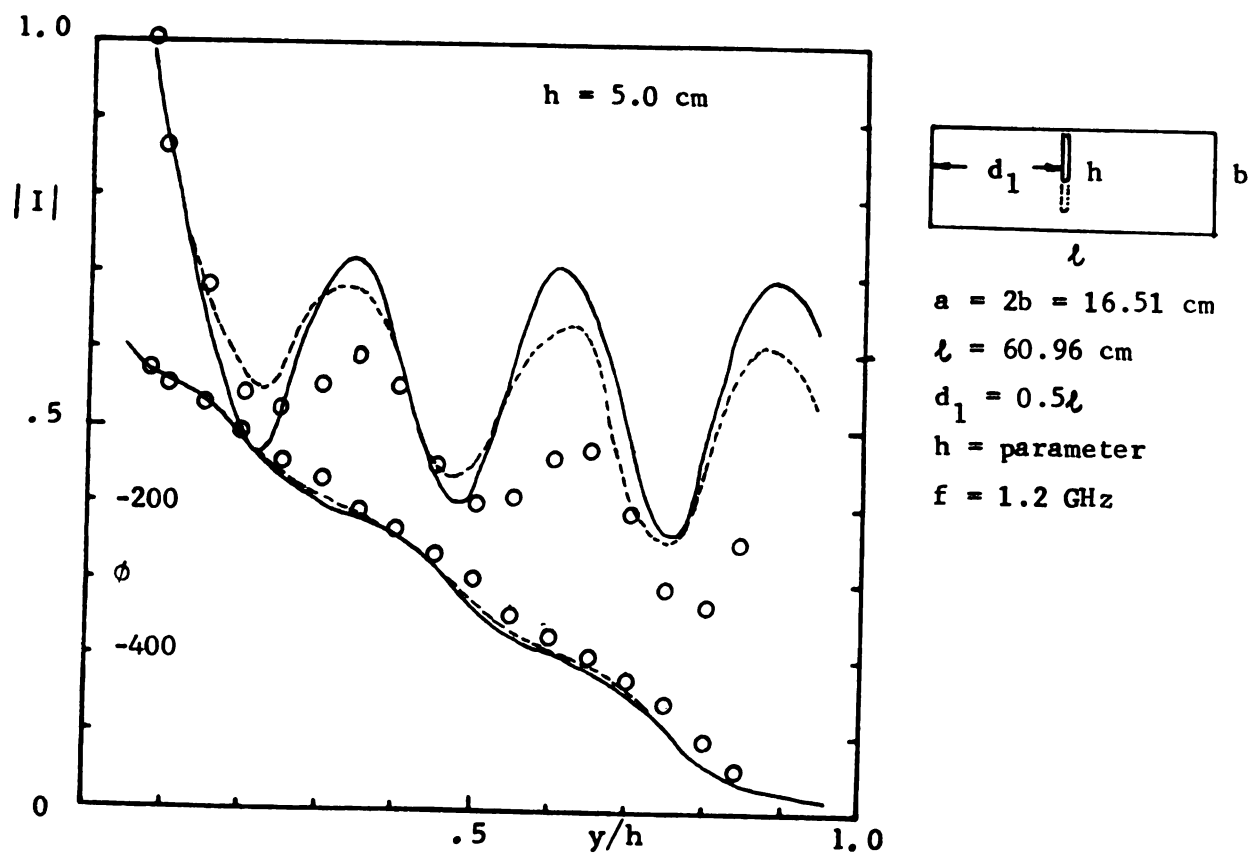
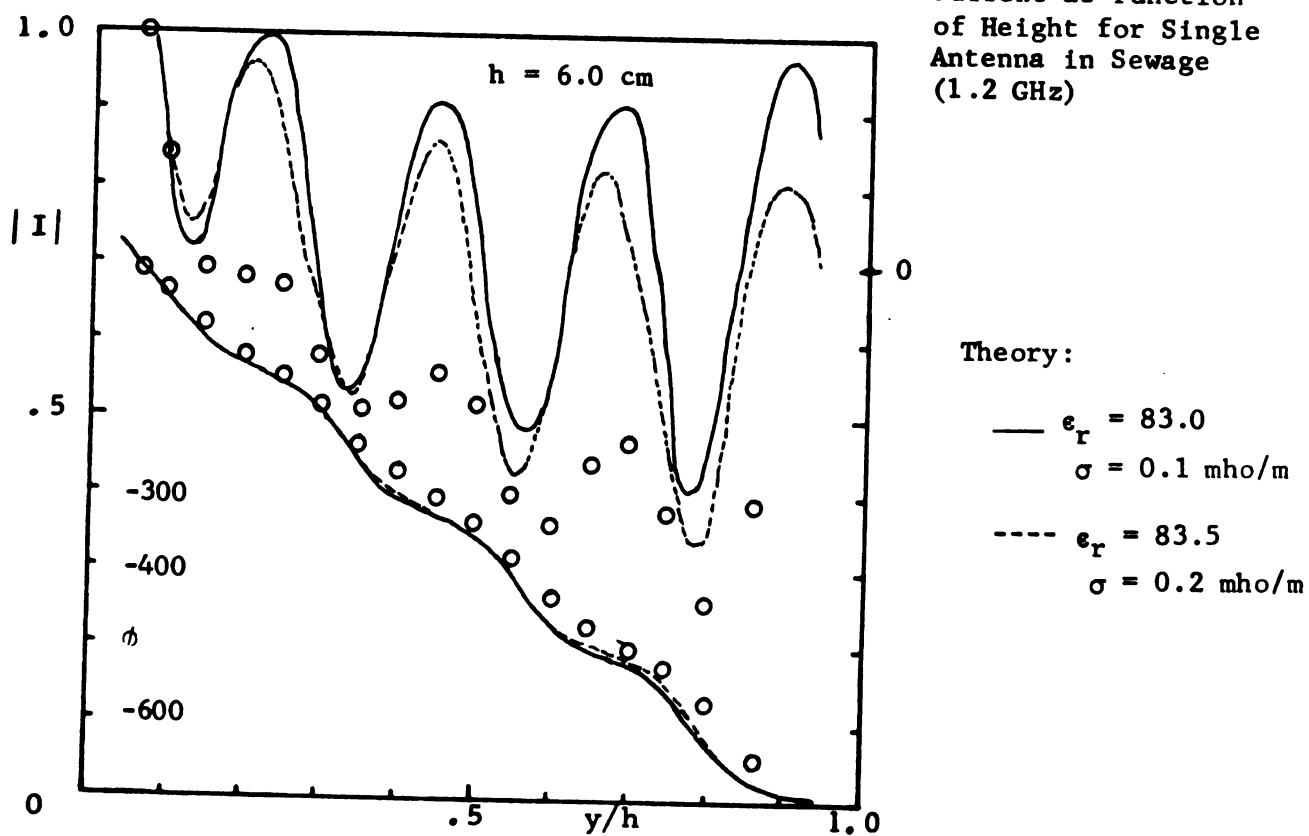
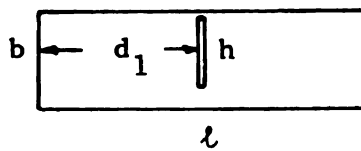
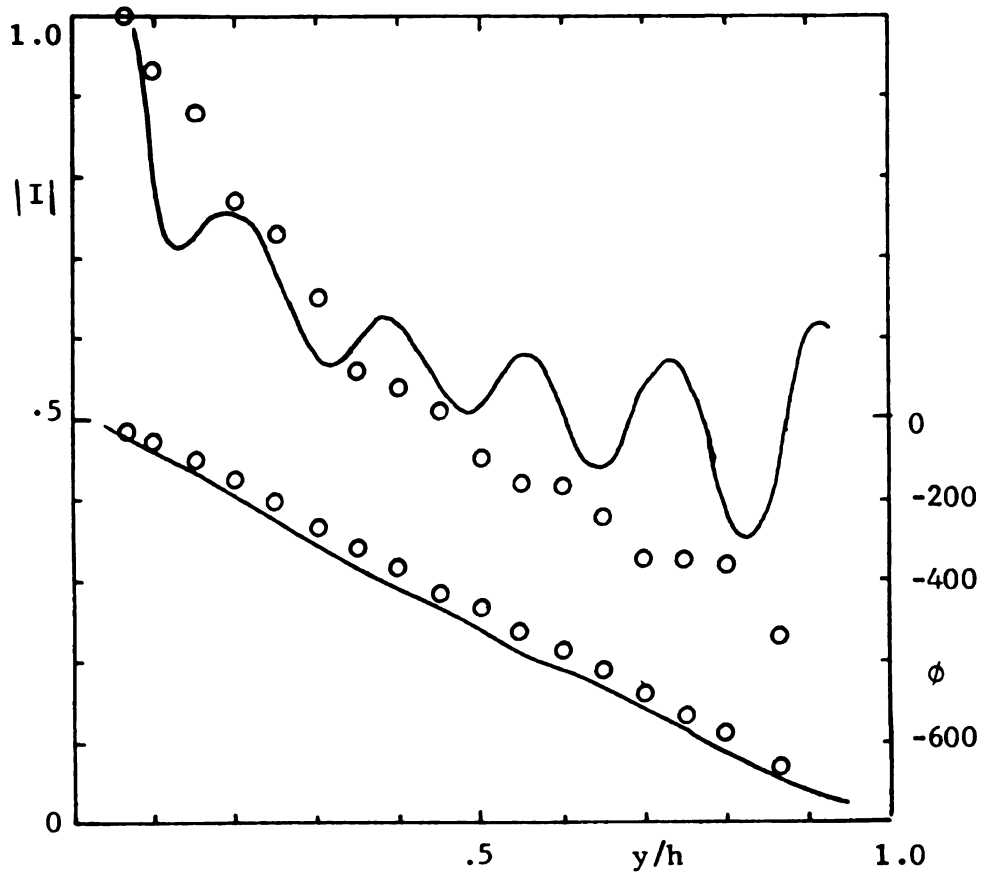


FIGURE 5.49

Current as Function
of Height for Single
Antenna in Sewage
(1.2 GHz)





$$a = 2b = 16.51 \text{ cm}$$

$$l = 60.96$$

$$d_1 = 0.5l$$

$$h = 6.0 \text{ cm}$$

Theory: $\epsilon_r = 83.0$, $\sigma = 0.1 \text{ mho/m}$

Input Admittance: $Y_{\text{theory}} = 119 + j75 \text{ millimhos}$

FIGURE 5.50

Current for Single Antenna in Sewage at 1.6 GHz

TABLE 5.35

Admittance for Single Antenna in Sewage
(350 MHz)

see Figure 5.46 for configuration and current

theoretical admittance, millimhos

	$\epsilon_r = 83$ $\sigma = 0.1 \text{ mho/m}$	$\epsilon_r = 83.5$ $\sigma = 0.2 \text{ mho/m}$
$d_1 = 0.133\lambda$	$43.3 + j39.6$	$49.5 + j49.3$
$d_1 = 0.5\lambda$	$44.4 + j59$	$50 + j56.5$

TABLE 5.36
Admittance as Function of Height of
Single Antenna in Sewage
(1.0 Ghz)

see Figure 5.47 for configuration and current

theoretical admittance, millimhos

h, cm	$\epsilon_r = 83$ $\sigma = 0.1 \text{ mho/m}$	$\epsilon_r = 83.5$ $\sigma = 0.2 \text{ mho/m}$
5.0	$78 + j89$	$77.8 + j90.5$
5.5	$104 + j97$	$96.4 + j87.8$
6.0	$104 + j50$	$93.0 + j51.3$
6.25	$86 + j46$	$81.0 + j48.6$

TABLE 5.37

Admittance as Function of Location for

Single Antenna in Sewage

(1.0 Ghz)

see Figure 5.48 for configuration and current

theoretical admittance, millimhos

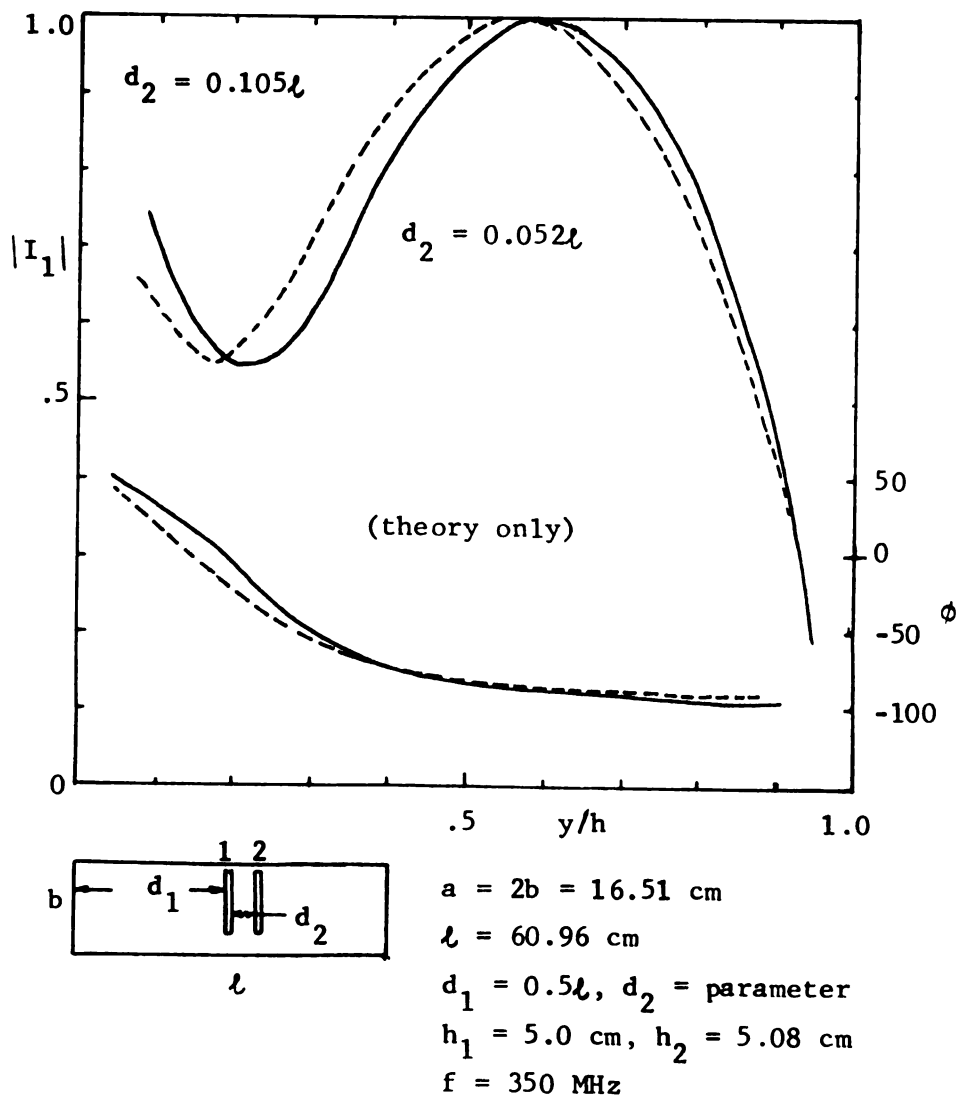
	$\epsilon_r = 83.0$ $\sigma = 0.1 \text{ mho/m}$	$\epsilon_r = 83.5$ $\sigma = 0.2 \text{ mho/m}$
$d_1 = 0.133\lambda$	$105 + j65$	$96 + j58$
$d_1 = 0.185\lambda$	$85 + j53$	-

TABLE 5.38
 Admittance as Function of Height of
 Single Antenna in Sewage
 (1.2 GHz)

see Figure 5.49 for configuration and current

theoretical admittance, millimhos

h, cm	$\epsilon_r = 83.0$	$\epsilon_r = 83.5$
	$\sigma = 0.1 \text{ mho/m}$	$\sigma = 0.2 \text{ mho/m}$
5.0	$97 + j70$	$96.7 + j82$
6.0	$114 + j90$	$103 + j85$

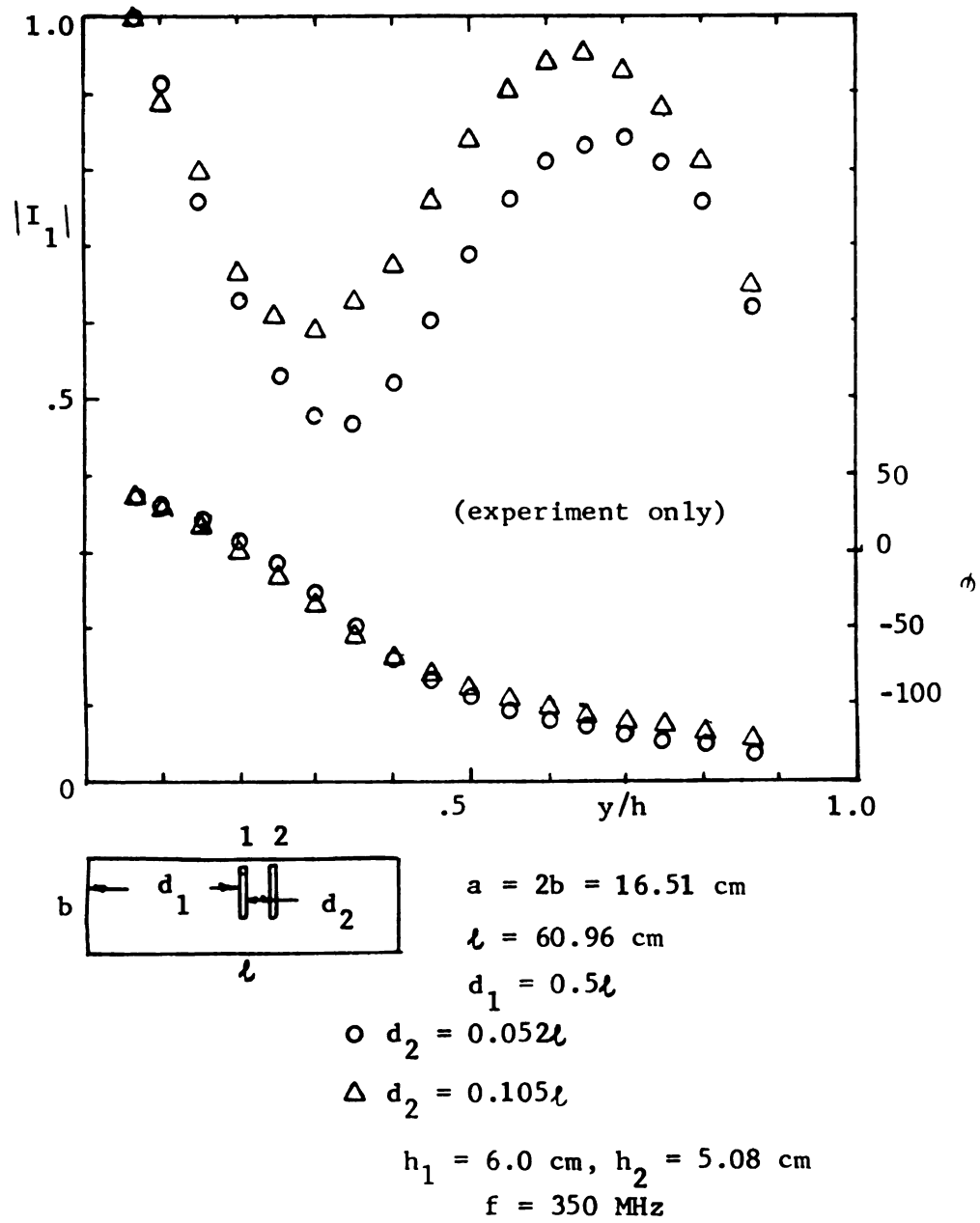


Theory: $\epsilon_r = 83.0, \sigma = 0.1 \text{ mho/m}$

50 ohm load on parasite (#2)

FIGURE 5.51

Theoretical Driven Antenna Current for Coupled
Antennas in Sewage (350 MHz, $h_1 = 5 \text{ cm}$)



50 ohm load on parasite (#2)

FIGURE 5.52

Experimental Driven Antenna Current for Coupled
Antennas in Sewage (350 MHz, $h_1 = 6 \text{ cm}$)

TABLE 5.39
Theoretical Admittance for Coupled Antennas
in Sewage at 350 MHz
($h_1 = 5$ cm)

see Figure 5.51 for configuration and current
(theory only)

for $d_2 = 0.052\lambda$

$$[Y] = \begin{bmatrix} 24 + j42.5 & 8.1 - j2.4 \\ 8.1 - j2.4 & 28 + j41.5 \end{bmatrix} \text{ millimhos}$$

$$Y_{50} = 24 + j43.8 \text{ millimhos}$$

for $d_2 = 0.105\lambda$

$$[Y] = \begin{bmatrix} 23 + j38.7 & -1.9 - j9.0 \\ -1.9 + j9.0 & 28.5 + j35.6 \end{bmatrix} \text{ millimhos}$$

$$Y_{50} = 23.6 + j37.2 \text{ millimhos}$$

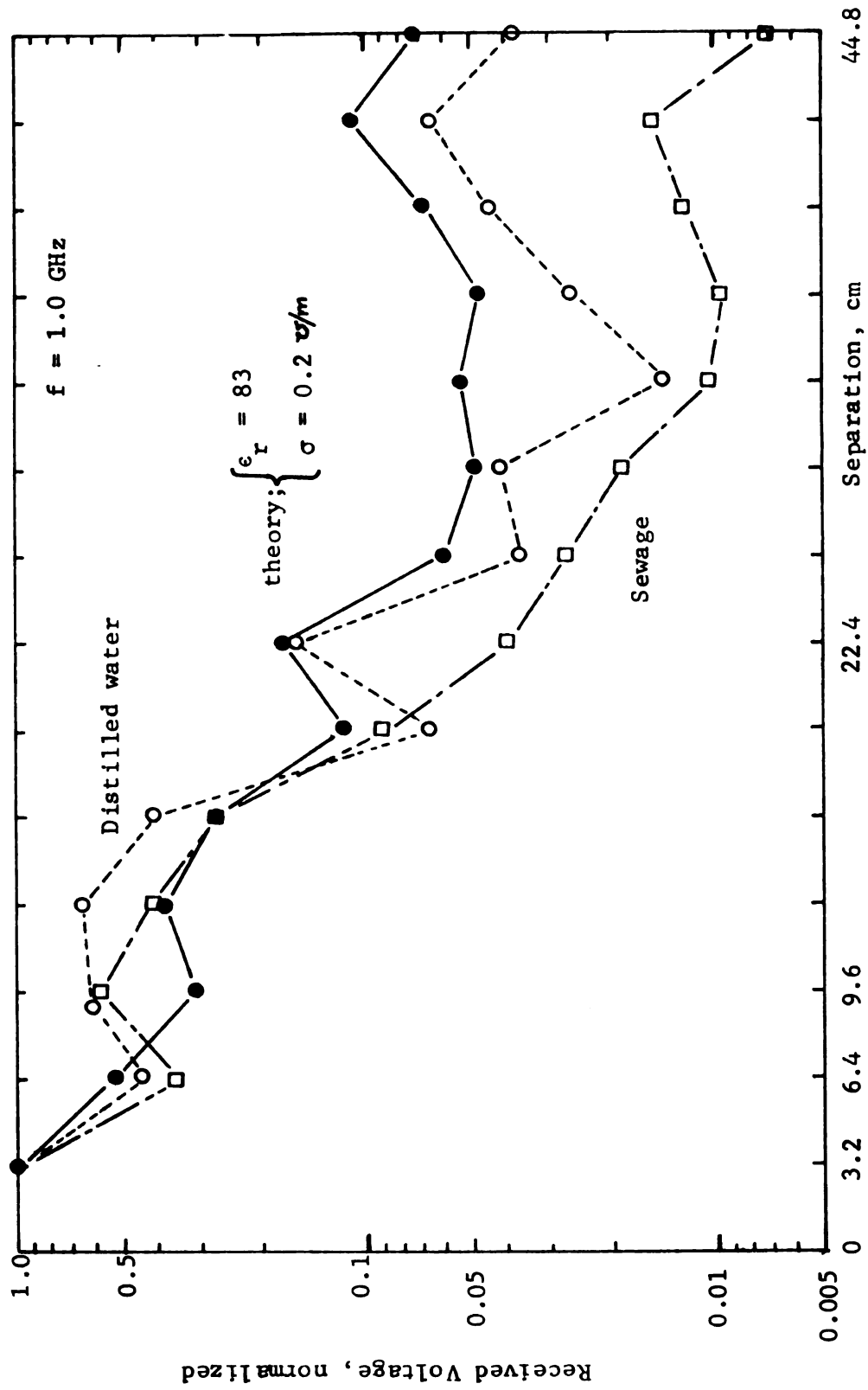


FIGURE 5.53(a)
Received Voltage versus Separation for Coupled Antennas in Cavity Filled with
Dissipative Media

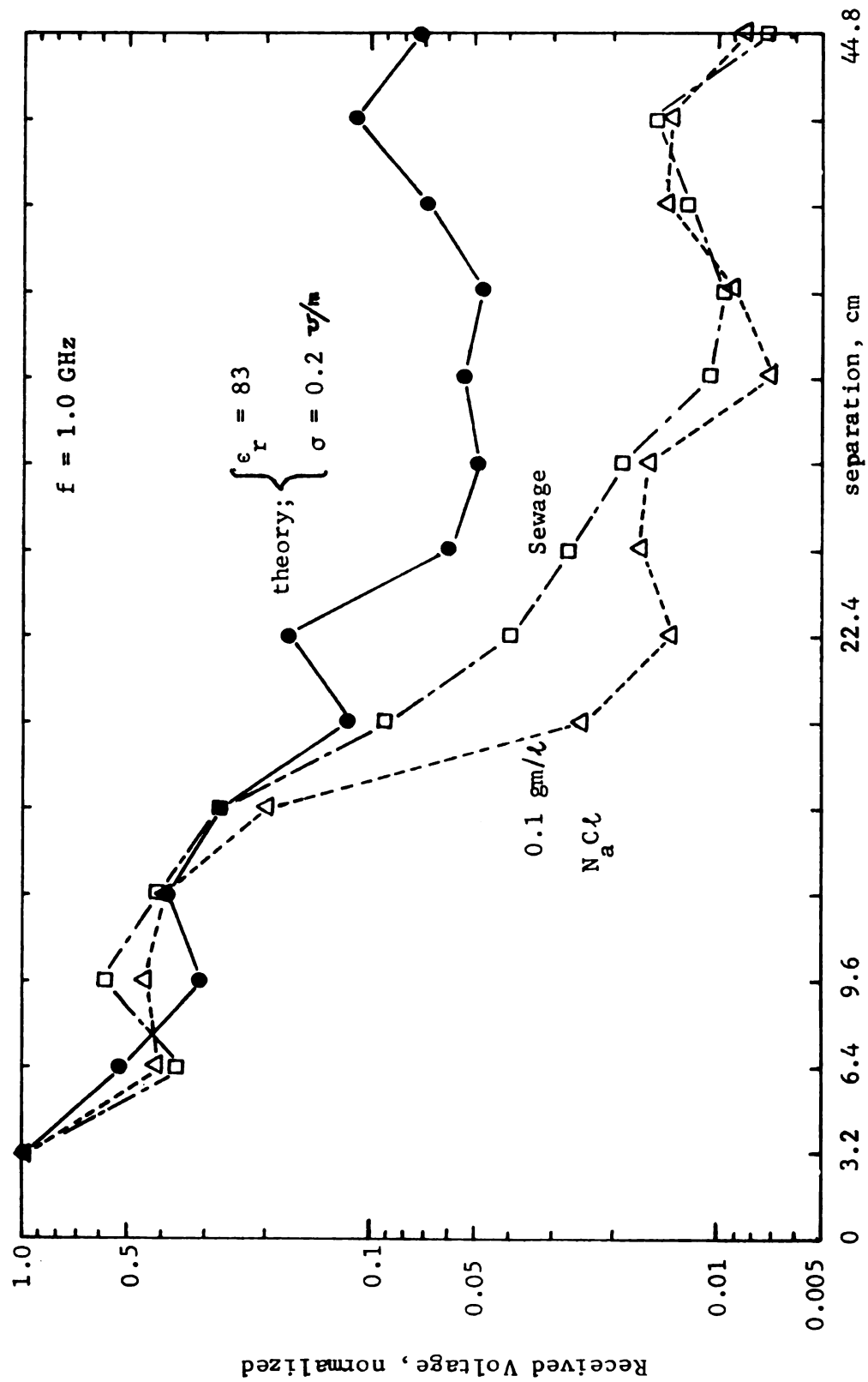


FIGURE 5.53(b)
Received Voltage versus Separation for Coupled Antennas in Cavity Filled With
Dissipative Media

CHAPTER 6

CONCLUSION; RECOMMENDATIONS

This report has described the theoretical and experimental investigation of antennas in waveguide boundaries with general homogeneous isotropic media. With an emphasis towards practical utilization of the relation between antenna characteristics and electrical properties of the medium, results have been presented for the antenna current distribution and input admittance as a function of both the waveguide configuration and the conductivity, permittivity and permeability of the medium.

The theoretical development formally presents the mathematical model of N linear antennas in a rectangular waveguide with arbitrary reflection coefficients and medium. The set of coupled integral equations for the unknown current distributions are found by appropriate application of the Lorentz reciprocity relation to a modal analysis of the induced fields in the waveguide region. Parallel developments are presented which consider the antennas under the thin-wire and the strip approximations for an assumed delta-gap excitation. The single component of the dyadic Green's function which is derived by this method for the interior of the waveguide illustrates the modification necessary for consideration of arbitrary waveguide terminations compared to the infinite (matched) waveguide case. The integral equations developed are reduced to

Hallen-type equations in order to facilitate solution by numerical methods. The antenna problem is then reconsidered analytically for the case of a coaxial gap TEM mode excitation. For a single antenna under the thin-wire approximation the integral equation is developed by evaluation of definite integrals over the coaxial region in the rectangular coordinate system of the waveguide. The theoretical development is concluded by showing the agreement between suitable special cases of the general results found here and results which have been published in the literature.

The numerical solution of the general coupled set of integral equations is accomplished by the method of subsections and point matching according to the typical moment method approach. Results of this simplest approach are then shown to agree with solutions obtained by the more accurate Galerkin's method based on pulse functions. Attention is paid to the convergence properties of the numerical solution and it is shown that the strip approximation of the cylindrical antenna exhibits improved convergence properties over the conventional thin-wire approximation. Criteria are derived for optimally truncating the involved modal series and a correspondence is established between these criteria and sampling theory. Finally, representative cases are presented which compare the results for the various theoretical approximations. Relative current distributions are shown to be the same for all approximations with the input impedance varying between approximations.

The experimental investigation of the antenna-waveguide-medium system is described in Chapter 4. Details are given of the construction of the probes for measuring antenna current distributions

and antenna input impedance. The design of the impedance sensing block for measuring input impedance with a vector voltmeter is discussed and shown to be an effective, accurate, and convenient method compared with the conventional slotted line techniques. Descriptions are also given of the methods used to measure the electrical properties of the dissipative media studied; conductivity is measured via a low-frequency coaxial capacitor and permittivity by perturbation measurements in a cylindrical cavity.

Included in the discussion of experimental equipment are considerations which proved significant in the measurement techniques. In particular, the orientation of the current sensing probe in the waveguide is shown to affect the voltage induced in the probe. The orientation in a transverse waveguide plane which most truly represents the current on the antenna is determined from a discussion of the dominant waveguide fields and supportive experimental data. Techniques are also described for calibration of the impedance sensing block which are unique to the set-up used. Discussions of the predominant sources of error in measurement of phase angle are included with an accuracy of $\pm 2^\circ$ being typical.

The numerical solutions of the theory and experimentally measured values of current distributions and antenna input admittance are compared in Chapter 5. The correlation of experimental and theoretical input admittance is shown to depend on the infinite susceptance introduced by the delta-gap model, whereas this effect is not seen in the coaxial gap model. Attempts to obtain satisfactory agreement between theoretical and experimental admittance typically involve terminal zone correction networks. The analysis

of a simple two element lumped equivalent network is presented as it applies to this problem. Of particular interest in this analysis is the unique relationship for determination of the optimum matching network based on simple curve fitting techniques. The correction network is not used to alter the experimental data, however, for two reasons. First, the interaction that exists between the antenna feed and the cavity and the antenna itself is extremely complex, which requires a different correction network for virtually every configuration. Second, the presentation of results which have been made to agree by some correction network offers very little insight into either the true operation of the system or the importance of the antenna feed configuration in the cavity excitation process.

The antenna-waveguide-media system is studied in Chapter 5 by systematically varying the waveguide/cavity, antenna height, antenna location, and the operating frequency for both the single antenna and two coupled antennas. For the open ended (i.e., semi-infinite) waveguide the results for the single antenna current are shown to differ significantly with the typically assumed zeroth order shifted sine distribution. The possibility of applying this theoretical development to characterizing coaxial line to rectangular waveguide couplers is well demonstrated. For the cavity configuration the air medium is stressed to show the interaction of the antenna with the resonant cavity. The most significant results are best summarized in Figures 5.23 and 5.24 for the dependence of the current distribution on the cavity resonance. This relationship forms the basis for understanding the effect of other parameter variations from a cavity perturbation viewpoint. That is, as the antenna height, location, or

cavity dimensions are varied the current distribution changes accordingly as the cavity resonance is shifted with respect to the operating frequency. For the study of the effect of the dissipative media on the antenna-cavity coupling, excellent agreement was obtained between the theory and both published and independently measured values. The ability to obtain measurements easily for the water medium supports the practicality of this system for potential application in this area.

The study of coupled antennas for a representative subset of cases covered for the single antenna was primarily seen as a perturbation of the cavity by the second antenna. The coupling between antennas, however, was more dependent on element spacing and the distribution of fields in the cavity as was expected. The study of dissipative media for coupled antennas showed a much reduced coupling effect due to the high losses at the L-band frequencies. Reduction of the operating frequency by a factor of about 3 to 350 MHz resulted in increased coupling and stronger interaction with the cavity.

The final study was done for sewage treatment plant effluent as the medium filling the cavity. As a feasibility study the results were highly successful showing that this extreme case exhibited losses sufficiently low to render meaningful measurements. The effects seen were primarily related to the macroscopic properties (conductivity and dielectric constant) rather than the microscopic electro-chemistry as would be expected. However, a more detailed study of other cases might easily depend on the microscopic properties. In particular the absorption-type results presented show

a feasibility for this type of correlation. The differences noted between sewage, sodium chloride solution, and distilled water seemed to indicate the possibility of a relationship between differences in type or concentration of ions present and the energy transmitted through the solution

As a brief summary of the conclusions of this report, the following main points stand out. First and foremost, the theory which was developed gives excellent agreement with all experimentally determined current distributions including those special cases published in the literature. The agreement obtained with the input admittance is only qualitative, however, with the absolute differences being relateable through terminal zone correction network analysis. In the numerical analysis, knowledge was gained of optimum truncation of series in modal analysis problems solved by the method of moments.

The usefulness of the findings to the initial motivating problem of developing a measurement tool is based primarily on the current distribution as a function of cavity resonance. The newly determined fact is that the purely quadrature current on an antenna in a cavity exhibits a zero crossing which is a strong function of the operating frequency compared to the cavity resonance frequency. Since the zero crossing is easy to detect, and since it is extremely sensitive to the cavity resonance (which is easily perturbed), the possibility exists for exploiting this feature as a detection or monitoring tool. The development of this relationship is certainly worth further study.

Recommendations for Further Study

Besides developing the relationship between current and cavity resonances just mentioned, some further study is also recommended in other areas that have been discussed in this report. In particular, the connection noted between sampling theory and the optimum numerical truncation offers the possibility of giving further understanding of these types of modal analysis problems. Also, the observations and relationships noted between the experimental coaxial gap and the theoretical delta-gap model could be systematically investigated using the rectangular cavity as a control on the type of coupling between transmission line and antenna. Such work would be invaluable in the development of well understood terminal zone correction networks.

Areas which this author feels deserve continuation or new effort are both in the purely theoretical as well as the applications study areas. It is possible that a reworking of the coaxial theory developed here (with the integral solutions in Appendix II) to include more than the TEM coaxial mode could result in better agreement in absolute input admittance values. The coaxial theory could obviously also be expanded to include coupling between multiple antennas. Further, the theory could be simplified in an effort to obtain an approximate analytical, closed form, two or three term expression for the current distribution to aid in separating the interactions between antenna and cavity.

The experimental or applications work begun here bears continuation in at least two areas. First, the short study of semi-infinite waveguides could be made complete to advance the knowledge

beyond the approximate analysis typically used today. Second, the application to pollution or dissipative media studies should be expanded based on the following areas. Electrochemistry should be included in the theory at least in the form of the frequency dependent relation between complex permittivity and frequency for dipolar liquids. Studies should be expanded using controlled salt concentration and various salt types to look for possible correlation. Microscopic investigations should be continued of the two antenna absorption type done at the end of Chapter 5. And finally, and perhaps most interestingly, there remains a great deal of potential in the area of air pollution studies where low dissipation and low dielectric constant maintain the ability for reliable measurements in the 1.0 to 2.0 GHz range.

BIBLIOGRAPHY

BIBLIOGRAPHY

1. Chang, D.C., "End correction of a monopole antenna with arbitrary radius," IEEE Trans. on Antennas and Propagation, Vol. 16, pp. 606-607, September, 1968.
2. Collin, R.E., Field Theory of Guided Waves, McGraw Hill, New York, 1960.
3. Collin, R.E., Foundations of Microwave Engineering, McGraw Hill, New York, 1966.
4. Cooper, R., "The electrical properties of salt-water solutions over the frequency range 1-4000 Mc/s.," IEE Journal, Vol. 93, Part III, No. 22, pp. 69-75, March 1946.
5. Dorsey, N.E., Properties of Ordinary Water Substance, American Chemical Society Monograph, Hafner Publishing Co., Inc., New York, 1968.
6. Duncan, R.H., "Theory of the infinite cylindrical antenna including the feedpoint singularity in antenna current," Journal of Research, N.B.S., Vol. 66D, No. 2, pp. 181-188, March-April 1962.
7. Eisenhart, R.L. and P.J. Khan, "Theoretical and experimental analysis of a waveguide mounting structure," IEEE Trans. MTT, Vol. 19, No. 8, pp. 706-719, 1971.
8. Galejs, J., "Admittance of a rectangular slot which is backed by a rectangular cavity," IEEE Trans. A.P., pp. 119-126, March 1963.
9. Gradshteyn, I.S. and I.M. Ryzhik, Table of Integral Series and Products, Academic Press, New York, 1965.
10. Grant, E.H., T.J. Buchanan and H.F. Cook, "Dielectric behavior of water at microwave frequencies," Journal of Chemical Physics, Vol. 26, No. 1, January, 1957.
11. Harrington, R.F., Field Computation by Moment Methods, MacMillan Company, New York, 1968.

12. Iizuka, K., "An experimental study of the insulated dipole antenna immersed in a conducting medium," IEEE Trans. A.P., pp. 518-532, September, 1963.
13. Iizuka, K. and R.W.P. King, "The dipole antenna immersed in a homogeneous conducting medium," IRE Transactions on Antennas and Propagation, pp. 384-392, July, 1962.
14. Iizuka, K. and R.W.P. King, "An experimental study of the half-wave dipole antenna immersed in a stratified medium," IRE Trans. A.P., pp. 393-399, July, 1962.
15. Jasik, H., Antenna Engineering Handbook, McGraw Hill, New York, 1961.
16. Johnson, C.C., Field and Wave Electrodynamics, McGraw Hill, New York, 1965.
17. King, R.W.P., Theory of Linear Antennas, Harvard University Press, Cambridge, Massachusetts, 1965.
18. King, R.W. P. and C.W. Harrison, Antennas and Waves, MIT Press, Cambridge, Massachusetts, 1969.
19. King, R.W.P. and K. Iizuka, "The complete electromagnetic field of a half-wave dipole in a dissipative medium," IEEE Trans. A.P., pp. 275-285, May, 1973.
20. King, R.W.P., R.B. Mack and S.S. Sandler, Arrays of Cylindrical Dipoles, Cambridge University Press, Cambridge, Massachusetts, 1968.
21. King, R.W.P. and L.D. Scott, "The cylindrical antenna as a probe for studying the electrical properties of media," IEEE Trans. A.P., Vol. 19, No. 3, pp. 406-416, May, 1971.
22. Lee, S.W., W.R. Jones and J.J. Campbell, "Convergence of numerical solutions of iris-type discontinuity problems," IEEE Trans. MTT, Vol. 19, No. 6, pp. 528-536, June 1971.
23. Lepley, L.K. and W.M. Adams, "Electromagnetic dispersion curves for natural waters," Water Resources Research, Vol. 7, No. 6, pp. 1538-1547, December, 1971.
24. Lewin, L., Advanced Theory of Waveguides, Iliffe and Sons, Ltd., London, 1951.
25. Lewin, L., "A contribution to the theory of probes in waveguides," Proc. of I.E.E. (London) Part C, pp. 105-116, 1958.
26. McLachlan, N.W., Bessel Functions for Engineers, Oxford Press, 1934.

27. Mishra, S.R., L.D. Scott and R.W.P. King, "Currents, charges, and admittance of linear antennas in dissipative media," *Radio Science*, Vol. 9, No. 4, pp. 487-495, April, 1974.
28. Papoulis, A., The Fourier Integral and Its Applications, McGraw Hill, New York, 1962.
29. Rao, B. Rama, "A two-element Yagi-type array in a parallel-plate waveguide - theoretical and experimental studies," *IEEE Transactions on Antennas and Propagation*, Vol. 13, pp. 675-682, September, 1965.
30. Rogers, J.C. and I.C. Peden, "The electrically short sheathed dipole: Experimental relationship between its measured admittance and the permittivity of the external medium," *IEEE Trans. A.P.*, Vol. 21, No. 6, pp. 857-862, November, 1973.
31. Sauter, G.D. and E.G. Chilton, "Air improvement recommendations for the San Francisco Bay Area," NASA contract NGR-05-020-409, School of Engineering, Stanford University, 1970.
32. Saxton, J.A., "Electrical properties of water," *Wireless engineer*, pp. 288-292, September, 1949.
33. Saxton, J.A. and J.A. Lane, "Electrical properties of sea-water," *Wireless Engineer*, pp. 269-275, October, 1952.
34. Scaife, B.K.P., Complex Permittivity, The English University Press Ltd., London, 1971.
35. Scott, L.D., "Apparatus for studying the properties of antennas in an effectively infinite dissipative medium," NASA Scientific Report No. 6, Division of Engineering and Applied Physics, Harvard University Press, Cambridge, Massachusetts, December, 1969.
36. Scott, L.D. and Mishra, S.R., "Theoretical and experimental investigations on coupled antennas in homogeneous isotropic media," Technical Report No. 641, Division of Engineering and Applied Physics, Harvard University, Cambridge, Massachusetts, April, 1973.
37. Sucher, M. and J. Fox (eds.), Handbook of Microwave Measurements, Volume II, Polytechnic Press, New York, 1963.
38. Tai, C.T., Dyadic Green's Functions in Electromagnetic Theory, Scranton, Pa., International Text Book Co., 1971.
39. VanBladel, J., "Some remarks on Green's dyadic for infinite space," *IRE Trans. on Antennas and Propagation*, pp. 563-566, November, 1961.
40. Wu, T.T., "Input admittance of linear antennas driven from a coaxial line," *J. Res. NBS*, Vol. 67D, pp. 83, 1963.

GENERAL REFERENCES

- Ament, W.S., J.C. Katzin, M. Katzin, and B.Y. Koo, "Impedance of a cylindrical dipole having a sinusoidal current distribution in a homogeneous anisotropic ionosphere," *Radio Science J. Res. N.B.S.* 68D, 4, 379-405 (1964).
- Biggs, A.W., "Dipole antenna radiation field in stratified Antarctic media," *Trans. IEEE AP-16*, 445 (1968).
- Blaedel, W.J. and H.V. Malmstadt, "High frequency titrations," *Analytical Chemistry*, 22, 734 (1950).
- Blair, W.E., "Experimental verification of dipole radiation in a conducting half-space," *Trans. IEEE AP-11*, 269 (1963).
- Brekhovskikh, L.M., Waves in Layered Media, Academic Press (1960).
- Ginzburg, V.L., The Propagation of Electromagnetic Waves in Plasmas, 2nd Edition, (Translated from Russian), Pergamon Press Ltd. (1970).
- Golterman, H.L., Methods of Chemical Analysis of Fresh Waters, International Biological Programme Handbook No. 8, Blackwell Scientific Publications, Oxford (1969).
- Grant, F.S. and G.F. West, Interpretation Theory in Applied Geophysics, McGraw-Hill Book Company (1965).
- Iizuka, K., "An experimental investigation on the behavior of the dipole antenna near the interface between the conducting medium and free space," *Trans. IEEE AP-12*, 27 (1964).
- Iizuka, K., "Research on the properties of antennas in a conducting media," Scientific Report No. 8, Gordon McKay Laboratory, Harvard University, Cambridge, Massachusetts (1964).
- Iizuka, K., "The circular loop antenna immersed in a dissipative medium," *Trans. IEEE AP-13*, 1, (1965).
- Iizuka, K., "Experimental study on the circular loop antenna immersed shallowly in a conducting medium," *Radio Science J. of Res. N.B.S.*, 69D, 9, (1965).

- Johnson, P.L. (Ed.), Remote Sensing in Ecology, Univ. of Georgia Press, Athens (1969).
- Johnson, D.E. and C.G. Enke, Analytical Chemistry, Vol. 42, 329, (1970).
- King, R.W.P., C. Harrison, Jr., and D.H. Denton, Jr., "The electrically short antenna as a probe for measuring free electron densities and collision frequencies in an ionized region," J. Res. of N.B.S. Radio Propag., 65, 4, (1961).
- Kraichman, M.B., "Basic experimental studies of the magnetic field from electromagnetic sources immersed in a semi-infinite conducting medium," J. Res. NBS 64D, 21 (1960).
- Muller, O.H., The Polarographic Method of Analysis, Chem. Ed. Pub. Co., Easton, Pa. (1956).
- Olson, T.A. and F.J. Burgess (editors), Pollution and Marine Ecology, Interscience Publishers, New York, (1967).
- Saran, G.S. and Held, "Field strength measurements in fresh water," J. Research NBS 64D, 435 (1960).
- Siegel, M., "Linear antennas and dipole arrays in a dissipative half-space," Technical Report No. 608, Division of Engineering and Applied Physics, Harvard University, Cambridge, Massachusetts (July 1970).
- Tannenbaum, B.S., Plasma Physics, McGraw Hill Book Company, Inc., New York (1967).
- Wait, J.R., Electromagnetic Waves in Stratified Media, Pergamon Press, Oxford (1962).

APPENDICES

APPENDIX I

DESCRIPTION AND DEFINITION OF NOTATION

A list of the frequently used notation for this report is given in this appendix. Symbols which are not used repeatedly and those whose use is readily implied from the context are not included. If a symbol has been used for two different quantities, only the primary usage is given here. Where possible, the units are included as well as reference to defining equations or figures in the text.

a, b	transverse waveguide dimensions in the $\hat{x}$ and $\hat{y}$ directions, respectively. (meters) Figure 2.1.
B	antenna input susceptance. (mhos)
A_v, B_v, C_v, D_v	waveguide mode expansion coefficients, Equation (2.7).
d	spacing between antennas. (meters)
d_1	positive distance from origin to waveguide termination in negative $\hat{z}$ direction. (meters) Figure 2.2.
d_2	positive distance from origin to waveguide termination in positive $\hat{z}$ direction. (meters) Figure 2.2.

DBES(p)	$J_p(r_b \xi) - J_p(r_a \xi)$ difference of p-th order Bessel functions with specified arguments.
DSi	$Si(r_b \xi) - Si(r_a \xi)$ difference of sine integrals with specified arguments. (Appendix II)
$e_{x,v}(x,y)$	normalized v -th mode $\hat{x}$ -component of electric field involving only x and y dependence. Equation (2.5).
$\vec{e}_{t,v}(x,y)$	transverse component of normalized v -th mode electric field. Equation (2.6).
e	superscript for TM mode.
$\vec{E}_v^{\pm}(x,y,z)$	total electric field for v -th eigenmode, propagating in $\pm \hat{z}$ direction. (volts/ meter) Equation (2.6).
$f_v(z z')$	reflection coefficient factor. Equation (2.13).
G	antenna input conductance. (mhos).
G_{ij}	kernel of integral equation, "Green's function".
h	magnetic field with usage analogous to electric field, e . Equation (2.5).
h	superscript for TE mode.
h	height of antenna (monopole). (meters).
H	total magnetic field with usage analogous to electric field, E . Equation (2.6).

i	subscript for number of antenna used as <u>source</u> location.
$I(y)$	antenna line current density. (amps).
$\mathcal{J}_0$	zeroth order modified Bessel function. Equation (2.33).
j	complex number, $(-1)^{\frac{1}{2}}$.
j	subscript for number of antenna used as <u>field</u> location.
$J(x,y,z)$	antenna volume current density. (amps/m ²).
$\mathcal{J}_p$	p-th order Bessel function
k_0	$\omega(\mu\epsilon)^{\frac{1}{2}}$, wavenumber for general medium. (meters ⁻¹).
k_x	$n\pi/a$, waveguide mode eigenvalue for x coordinate. (meters ⁻¹).
k_y	$m\pi/b$, waveguide mode eigenvalue for y coordinate. (meters ⁻¹).
k_c	$(k_x^2 + k_y^2)^{\frac{1}{2}}$, waveguide cutoff wavenumber. (meters ⁻¹).
K_i	number of subsections on i-th antenna. Equation (3.2).
l	length of cavity or cavity resonance mode index.
$\mathcal{L}_0$	zeroth order modified Struve function. Equation (2.33) and following Equation (3.21).
m	waveguide mode index for y coordinate.

M _{MAX}	maximum value of m for truncation of mode summations. Equation (3.36).
n	waveguide mode index for x coordinate.
N _{MAX}	maximum value of n for truncation of mode summations. Equation (3.42).
p	subscript for number of subsection of antenna current. Equation (3.2).
q	subscript for number of subsection of antenna for point matching. Equation (3.9).
$Q(y)$	complex antenna line charge density. (coulombs/m) Equation (3.24).
$\bar{Q}(y)$	$-jQ(y)$, complex antenna line charge density referenced in phase to the antenna driving voltage. (coulombs/m) Equation (3.25).
r_a	radius of antenna, radius of inner conductor of coaxial line. (meters) Figure (2.1).
r_b	radius of outer conductor of coaxial line. (meters) Figure (2.1).
R_{in}	approximate antenna input resistance for semi-infinite waveguide configuration. (ohms) Equation (5.5).
s_l	superscript for longitudinal strip approximation. Section 2.5.2.
s_t	superscript for transverse strip approximation. Section 2.5.2.
S	surface area.

S_{pq}	matrix representing discretized form of kernel of integral equation. Section 3.1.4.
$Si(x)$	$\int_0^x \sin u/u \, du$, sine integral. (Appendix II).
tw	superscript for thin wire approximation. Section 2.5.1.
$u_{i,p}(y')$	pulse function for method of subsections. Equation (3.3).
V_o	voltage across coaxial line at antenna terminals. Equation 2.35.
V_j	driving voltage for j-th antenna.
V_i	volume around i-th antenna for Lorentz reciprocity relation. Figure 2.3.
x	transverse waveguide coordinate. Figure 2.1.
X_L	theoretical antenna input reactance. (ohms) Section 5.1.2.
X_l	reactance of terminal zone inductor. (ohms) Equation (5.3).
y	transverse waveguide coordinate. Figure 2.1.
Y_a	apparent (experimental) antenna input admittance. (mhos)
Y_{in}	general antenna input admittance. (mhos)
Y_{50}	input admittance of driven antenna with second parasitic antenna loaded with 50 ohms. (mhos)

Y_{sc}	input admittance of driven antenna with second parasitic antenna short circuited. (mhos)
Y_{oc}	input admittance of driven antenna with second parasitic antenna open circuited. (mhos)
z	longitudinal waveguide coordinate. Figure 2.1.
Z_a, Z_L, Z_{in}	antenna input impedance analogous to admittance. (ohms).
Z_v^e	$-j\gamma_v/\omega\epsilon$, wave impedance for v -th waveguide TM mode. (ohms)
Z_v^h	$+j\omega\mu/\gamma_v$, wave impedance for v -th waveguide TE mode. (ohms)
$\alpha_{i,p}$	amplitude of current at p -th subsection on i -th antenna. (amps) Equation (3.3).
γ_{nm}	$(k_c^2 - k_o^2)^{\frac{1}{2}}$, (n,m) -th mode propagation constant in waveguide. (meters ⁻¹).
Γ_{1L}	complex reflection coefficient of waveguide termination at location of load, $z = -d_1$.
Γ_{1v}	$\Gamma_{1L} e^{-2\gamma_v d_1}$, complex reflection coefficient of waveguide termination referred in phase to the origin of coordinates. Equation (2.7).
$\delta(y - \tau/2)$	Dirac's delta function. Equation (2.2).

ϵ	$\epsilon_0 \epsilon_r - j\sigma/\omega$, complex permittivity of dissipative medium. (farads/m).
ϵ_0	permittivity of free space (8.85×10^{-9} farads/m).
ϵ_r	relative dielectric constant.
ϵ_{on}	1 for $n = 0$; 2 for $n \neq 0$. Neumann's number.
ξ^2	$k_x^2 - \gamma_v^2$, dummy variable. following Equation (II.28).
μ	permeability of general medium. (henry/m).
μ_0	permeability of free space ($4\pi \times 10^{-7}$ henry/m).
ν	general mode index used as shorthand notation to imply <u>both</u> n and m indices for TE <u>and/or</u> TM modes.
σ	conductivity of medium. (mhos/m).
τ	infinitesimal gap width for delta gap approximation. Equation (2.2).
ω	radian frequency. (sec.^{-1}).

APPENDIX II

EVALUATION OF DEFINITE INTEGRALS

From Section 2.5.2

In Section 2.5.2, the development of the longitudinal strip approximation requires evaluation of the following integral:

$$F_v(z_j|z_i) = \int_{z_i-2r_a}^{z_i+2r_a} \frac{f_v(z_j|z') dz'}{\pi \sqrt{(2r_a)^2 - (z' - z_i)^2}} \quad (\text{II.1})$$

where z_j is the center of the antenna on which the boundary condition is satisfied, and z_i is the center of the antenna carrying the source current. With the expression for $f_v(z_j|z')$ given in Equation (2.13) the integral becomes

$$\begin{aligned} F_v(z_j|z_i) = \frac{1}{\pi(\Gamma_{1v}\Gamma_{2v}-1)} \{ & \int e^{-\gamma|z_j-z'|} / \sqrt{(2r_a)^2 - (z' - z_i)^2} dz' + \\ & + \Gamma_{1w} \int e^{-\gamma(z_j+z')} / \sqrt{(2r_a)^2 - (z' - z_i)^2} dz' + \\ & + \Gamma_{2w} \int e^{+\gamma(z_j+z')} / \sqrt{(2r_a)^2 - (z' - z_i)^2} dz' + \\ & + \Gamma_{1v}\Gamma_{2w} \int e^{+\gamma|z_j-z'|} / \sqrt{(2r_a)^2 - (z' - z_i)^2} dz' \} \end{aligned} \quad (\text{II.2})$$

with the limits of integration the same as in Equation (II.1).

To evaluate Equation (II.2), consider first the middle two integrals. They may be written as

$$e^{\pm \gamma z_j} \int_{z_i - 2r_a}^{z_i + 2r_a} e^{\pm \gamma z'} / \sqrt{(2r_a)^2 - (z' - z_i)^2} dz' \quad (\text{II.3})$$

which with the change of variables

$$z' - z_i = 2r_a \cos \theta$$

results in

$$e^{\pm \gamma z_j} \int_0^\pi e^{\pm \gamma z_i} e^{\pm 2\gamma r_a \cos \theta} d\theta \quad (\text{II.4})$$

Now, from a table of integrals (e.g., formula 3.915.4 in Gradshteyn

[9]) one obtains:

$$\int_0^\pi e^{\pm \beta \cos \theta} d\theta = \pi \mathcal{J}_0(\beta)$$

since the zeroth order modified Bessel function ($\mathcal{J}_0$) is an even function. Therefore, the integral (II.3) is evaluated as

$$\begin{aligned} e^{\pm \gamma z_j} \int_{z_i - 2r_a}^{z_i + 2r_a} e^{\pm \gamma z'} / \sqrt{(2r_a)^2 - (z' - z_i)^2} dz' &= \\ &= e^{\pm \gamma (z_j + z_i)} \pi \mathcal{J}_0(2\gamma r_a) \end{aligned} \quad (\text{II.5})$$

Now, consider the first and fourth integrals in Equation (II.2). They may be written as

$$\int_{z_i - 2r_a}^{z_i + 2r_a} e^{\pm \gamma |z_j - z'|} / \sqrt{(2r_a)^2 - (z' - z_i)^2} dz' \quad (\text{II.6})$$

In order to handle the absolute value, three cases must be considered:

- (1) the source point antenna is to the left of the field point antenna; i.e., $i < j$ or $z_i < z_j$ which implies $z' < z_j$.

- (2) the source point antenna is to the right of the field point antenna; i.e., $i > j$ or $z_i > z_j$ which implies $z' > z_j$.
- (3) the source point and field point are on the same antenna; i.e., $i = j$ or $z_i = z_j$ which implies two cases:
- (a) $z' < z_j$ for $z_i - 2r_a \leq z' < z_i$
- (b) $z' > z_j$ for $z_i < z' \leq z_i + 2r_a$.

Case (1): $i < j$; $z' < z_j$

The integral in (II.6) becomes

$$\int_{z_i - 2r_a}^{z_i + 2r_a} e^{\pm \gamma(z_j - z')} / \sqrt{(2r_a)^2 - (z' - z_i)^2} dz'$$

With Equation (II.5), this evaluates as

$$\begin{aligned} e^{\pm \gamma z_j} \int_{z_i - 2r_a}^{z_i + 2r_a} e^{\mp \gamma z'} / \sqrt{(2r_a)^2 - (z' - z_i)^2} dz' = \\ = e^{\pm \gamma(z_j - z_i)} \pi J_0(2\gamma r_a) \quad \text{for } i < j. \end{aligned} \quad (\text{II.7})$$

Case (2): $i > j$; $z' > z_j$

The integral in (II.6) becomes

$$\int_{z_i - 2r_a}^{z_i + 2r_a} e^{\mp \gamma(z_j - z')} / \sqrt{(2r_a)^2 - (z' - z_i)^2} dz'.$$

With Equation (II.5), this evaluates as

$$\begin{aligned} e^{\mp \gamma z_j} \int_{z_i - 2r_a}^{z_i + 2r_a} e^{\pm \gamma z'} / \sqrt{(2r_a)^2 - (z' - z_i)^2} dz' = \\ = e^{\mp \gamma(z_j - z_i)} \pi J_0(2\gamma r_a) \quad \text{for } i > j. \end{aligned} \quad (\text{II.8})$$

The results of Case (1) and Case (2) may now be recombined by using the absolute value. Thus, Equation (II.6) becomes

$$\int_{z_i - 2r_a}^{z_i + 2r_a} e^{\pm \gamma |z_j - z'|} / \sqrt{(2r_a)^2 - (z' - z_i)^2} dz' = e^{\pm \gamma |z_j - z_i|} \pi J_0(2\gamma r_a) \quad \text{for } i \neq j \quad (\text{II.9})$$

Case (3): $i = j$

The integral given in (II.6) must be divided into two intervals as follows:

$$\begin{aligned} \int_{z_i - 2r_a}^{z_i + 2r_a} e^{\pm \gamma |z_j - z'|} / \sqrt{(2r_a)^2 - (z' - z_i)^2} dz' &= \\ &= \int_{z_i - 2r_a}^{z_i} e^{\pm \gamma (z_i - z')} / \sqrt{(2r_a)^2 - (z' - z_i)^2} dz' + \\ &+ \int_{z_i}^{z_i + 2r_a} e^{\mp \gamma (z_i - z')} / \sqrt{(2r_a)^2 - (z' - z_i)^2} dz' \quad (\text{II.10}) \end{aligned}$$

Making the change of variables $z_i - z' = u$ in the first integral, but $z' - z_i = u$ in the second gives

$$\begin{aligned} - \int_{2r_a}^0 e^{\pm \gamma u} / \sqrt{(2r_a)^2 - u^2} du + \int_0^{2r_a} e^{\pm \gamma u} / \sqrt{(2r_a)^2 - u^2} du &= \\ &= 2 \int_0^{2r_a} e^{\pm \gamma u} / \sqrt{(2r_a)^2 - u^2} du \quad (\text{II.11}) \end{aligned}$$

This is evaluated with the aid of formula 3.387.5 in Gradshteyn

[9] which gives

$$\int_0^\alpha e^{\beta u} (\alpha^2 - u^2)^{-\frac{1}{2}} du = (\pi/2) (J_0(\alpha\beta) + \mathcal{L}_0(\alpha\beta)) \quad (\text{II.12})$$

Since the zeroth order modified Bessel function ($\mathcal{I}_0$) is even, and the zeroth order modified Struve function ($\mathcal{L}_0$) is odd, the integral in (II.10) becomes

$$\begin{aligned} \int_{z_i - 2r_a}^{z_i + 2r_a} e^{\pm \gamma |z_i - z'|} / \sqrt{(2r_a)^2 - (z' - z_i)^2} dz' &= 2 \int_0^{2r_a} e^{\pm \gamma u} / \sqrt{(2r_a)^2 - u^2} du = \\ &= \pi (\mathcal{I}_0(2\gamma r_a) \pm \mathcal{L}_0(2\gamma r_a)) \quad \text{for } i = j \end{aligned} \quad (\text{II.13})$$

Combining (II.9) and (II.13), the integral in (II.6) is completely evaluated as

$$\begin{aligned} \int_{z_i - 2r_a}^{z_i + 2r_a} e^{\pm \gamma |z_j - z'|} / \sqrt{(2r_a)^2 - (z' - z_i)^2} dz' &= \\ &= \begin{cases} \pi e^{\pm \gamma |z_j - z_i|} \mathcal{I}_0(2\gamma r_a) & \text{for } i \neq j \\ \pi (\mathcal{I}_0(2\gamma r_a) \pm \mathcal{L}_0(2\gamma r_a)) & \text{for } i = j \end{cases} \end{aligned} \quad (\text{II.14})$$

All the integrals in Equation (II.2) for $F_v(z_j|z_i)$ have now been evaluated. Substituting the results (II.5) and (II.14) back into the desired integral (II.2) gives:

For $i \neq j$,

$$\begin{aligned} F_v(z_j|z_i) &= \frac{1}{\Gamma_{1v}\Gamma_{2v}-1} \{ e^{-\gamma_v |z_j - z_i|} \mathcal{I}_0(2\gamma_v r_a) + \\ &\quad + \Gamma_{1v} e^{-\gamma_v (z_j + z_i)} \mathcal{I}_0(2\gamma_v r_a) + \\ &\quad + \Gamma_{2v} e^{+\gamma_v (z_j + z_i)} \mathcal{I}_0(2\gamma_v r_a) + \\ &\quad + \Gamma_{1v}\Gamma_{2v} e^{+\gamma_v |z_j - z_i|} \mathcal{I}_0(2\gamma_v r_a) \} \end{aligned}$$

or, upon factoring and identifying the function $f_v(z_j|z_i)$ from Equation (2.13),

$$F_v(z_j|z_i) = \mathcal{J}_0(2\gamma_v r_a) f_v(z_j|z_i) \quad \text{for } i \neq j \quad (\text{II.15a})$$

For $i = j$,

$$\begin{aligned} F_v(z_i|z_i) = & \frac{1}{\Gamma_{1v}\Gamma_{2v}-1} \{ \mathcal{J}_0(2\gamma_v r_a) - \mathcal{L}_0(2\gamma_v r_a) + \\ & + \Gamma_{1v} e^{-2\gamma_v z_i} \mathcal{J}_0(2\gamma_v r_a) + \\ & + \Gamma_{2v} e^{+2\gamma_v z_i} \mathcal{J}_0(2\gamma_v r_a) + \\ & + \Gamma_{1v}\Gamma_{2v}(\mathcal{J}_0(2\gamma_v r_a) + \mathcal{L}_0(2\gamma_v r_a)) \} \end{aligned}$$

or, upon factoring as above

$$F_v(z_i|z_i) = \mathcal{J}_0(2\gamma_v r_a) f_v(z_i|z_i) - \mathcal{L}_0(2\gamma_v r_a) \quad \text{for } i = j \quad (\text{II.15b})$$

This completes the evaluation of the integrals in Section 2.5.2.

From Section 2.6

For the development of the coaxial gap solution in Section 2.6, there are eight integrals which must be evaluated. They are

$$\mathcal{J}_{(1)}^{e+} = \int_{S_g(1)} (\hat{y} \times \vec{E}_g) \cdot \vec{H}_v^{e+} dS \quad (\text{II.16a})$$

$$\mathcal{J}_{(1)}^{h+} = \int_{S_g(1)} (\hat{y} \times \vec{E}_g) \cdot \vec{H}_v^{h+} dS \quad (\text{II.16b})$$

$$\mathcal{J}_{(2)}^{e+} = \int_{S_g(2)} (\hat{y} \times \vec{E}_g) \cdot \vec{H}_v^{e+} dS \quad (\text{II.16c})$$

$$\mathcal{J}_{(2)}^{h+} = \int_{S_g(2)} (\hat{y} \times \vec{E}_g) \cdot \vec{H}_v^{h+} dS \quad (\text{II.16d})$$

In order to evaluate these, the integrals must first be reduced to functional form. Since $\vec{E}_g$ and S_g are referenced to an origin of coordinates at the center of the coaxial gap ($y = 0$), the magnetic field $\vec{H}_v(x, 0, z)$ (referenced to a coordinate system centered at the waveguide corner) must be translated in the x coordinate by the amount $a/2$. Therefore, the magnetic field to be used is $\vec{H}_v(\tilde{x}, 0, z)$ where,

$$\tilde{x} = x + a/2 \quad (\text{II.17})$$

so that x is centered at the coaxial gap and $\tilde{x}$ is the usual coordinate for waveguide fields. The coaxial gap field, $\vec{E}_g$, is radially directed and may be broken into $\hat{x}$ and $\hat{z}$ components as

$$E_g \hat{r} = E_g ((x/r)\hat{x} + (z/r)\hat{z})$$

where

$$r = (x^2 + z^2)^{1/2} . \quad (\text{II.18})$$

Then, the cross-product with the unit vector $\hat{y}$ yields

$$\hat{y} \times \vec{E}_g = E_g ((z/r)\hat{x} - (x/r)\hat{z}) .$$

Since E_g varies inversely with r , from Equation (2.35), this becomes

$$\hat{y} \times \vec{E}_g = \frac{V_0}{\ln(r_b/r_a)} \left(\frac{z}{r} \hat{x} - \frac{x}{r} \hat{z} \right) . \quad (\text{II.19})$$

Considering first the integrals in Equation (II.16a),

$$\begin{aligned}\mathcal{F}_{(1)}^{e+} &= \int_{S_g(1)} (\hat{y} \times \vec{E}_g) \cdot \vec{H}_v^{e+} dS = \\ &= \frac{v_o}{\ln(r_b/r_a)} \int_{S_g(1)} \pm (z/r^2) h_{xv}^e(\tilde{x}, 0) e^{\mp \gamma z} dS\end{aligned}$$

since $h_{zv}^e \equiv 0$, and, using Equations (2.5b) this is

$$\mathcal{F}_{(1)}^{e+} = \frac{v_o}{\ln(r_b/r_a)} \sqrt{\frac{2}{Z_{ab}^e}} \frac{k_y}{k_c} \int_{S_g(1)} \pm (z/r^2) \sin(k_x \tilde{x}) e^{\mp \gamma z} dx dz \quad (\text{II.20})$$

Using (II.17) for $\tilde{x}$ and (II.18) for r in the above yields

$$\mathcal{F}_{(1)}^{e+} = \begin{cases} (-1)^{n/2} \mathcal{J} \int_{S_g(1)} \pm \sin(k_x x) e^{\mp \gamma z} z/(x^2 + z^2) dx dz & \text{for } n \text{ even} \end{cases} \quad (\text{II.21a})$$

$$\mathcal{F}_{(1)}^{e+} = \begin{cases} (-1)^{\frac{n-1}{2}} \mathcal{J} \int_{S_g(1)} \pm \cos(k_x x) e^{\mp \gamma z} z/(x^2 + z^2) dx dz & \text{for } n \text{ odd} \end{cases} \quad (\text{II.21b})$$

Now, since the integrand for even numbered modes n , above, is an odd function about $x = 0$, and $S_g(1)$ is symmetric about $x = 0$, Equation (II.21a) evaluates to zero. Applying the Euler Identity to Equation (II.21b) gives

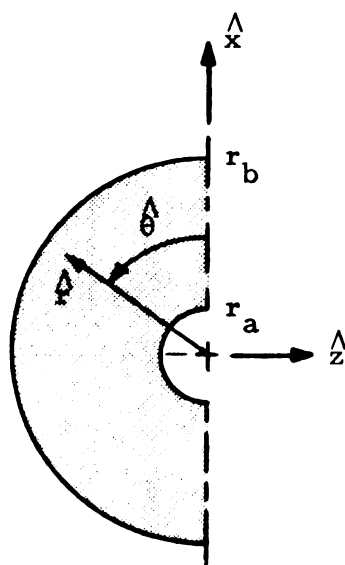
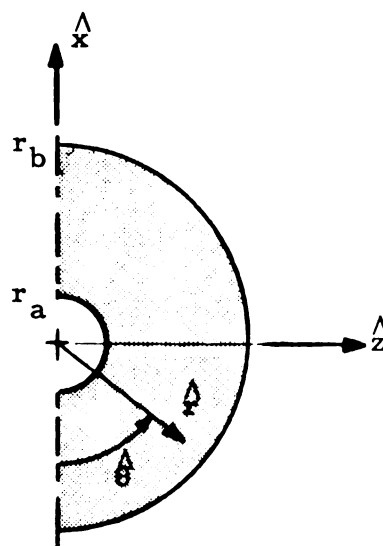
$$\begin{aligned}\mathcal{F}_{(1)}^{e+} &= \pm (-1)^{\frac{n-1}{2}} \mathcal{J} (1/2) \int_{S_g(1)} [e^{-jk_x x + \mp \gamma z} + e^{+jk_x x + \mp \gamma z}] \times \\ &\quad \times z/(x^2 + z^2) dx dz \quad \text{for } n \text{ odd}\end{aligned}$$

Because of the shape of the region $S_g(1)$ (see Figure II.1) it is best to transform to polar coordinates as shown.

$$\begin{aligned}x &= -r \cos \theta & dx dz &= -r dr d\theta \\ z &= r \sin \theta\end{aligned}$$

Thus,

$$\begin{aligned}x &= -r \cos \theta \\z &= r \sin \theta \\dx dz &= -r dr d\theta\end{aligned}$$



$$\begin{aligned}x &= r \cos \theta \\z &= -r \sin \theta \\dx dz &= -r dr d\theta\end{aligned}$$

FIGURE II. 1

Transformation to Polar Coordinates for Gap Regions

$$\begin{aligned}
\frac{e^{\pm}}{(1)} = \pm (-1)^{\frac{n-1}{2}} \mathcal{D}(-1/2) \int_{r_a}^{r_b} \int_0^{\pi} \sin \theta [e^{jr(k_x \cos \theta \pm j\gamma \sin \theta)} + \\
+ e^{jr(-k_x \cos \theta \pm j\gamma \sin \theta)}] dr d\theta \quad \text{for } n \text{ odd}
\end{aligned} \quad (\text{II.22})$$

From the above, now consider the single integral:

$$I' = \int_{r_a}^{r_b} \int_0^{\pi} \sin \theta e^{jr(k_x \cos \theta + j\gamma \sin \theta)} dr d\theta \quad (\text{II.23})$$

This is evaluated by considering two other integrals^{*}; namely,

$$\begin{aligned}
I'_1 &= \int_{r_a}^{r_b} \int_0^{\pi} (k_x \cos \theta + j\gamma \sin \theta) e^{jr(k_x \cos \theta + j\gamma \sin \theta)} dr d\theta \\
&= \int \int g(\theta) e^{jr g(\theta)} dr d\theta
\end{aligned} \quad (\text{II.24})$$

$$\begin{aligned}
I'_2 &= \int_{r_a}^{r_b} \int_0^{\pi} (-k_x \sin \theta + j\gamma \cos \theta) e^{jr(k_x \cos \theta + j\gamma \sin \theta)} dr d\theta \\
&= \int \int \frac{d g(\theta)}{d\theta} e^{jr g(\theta)} dr d\theta
\end{aligned} \quad (\text{II.25})$$

These two integrals combine to yield the desired integral I' as:

$$I' = (1/(k_x^2 - \gamma^2)) [j\gamma I'_1 - k_x I'_2] \quad (\text{II.26})$$

The integrals I'_1 and I'_2 are now evaluated as follows:

* The author is indebted to Dr. J.S. Frame, Department of Electrical Engineering, Michigan State University, for his many suggestions incorporated in the evaluation of this integral.

$$\begin{aligned}
I_2' &= \int_{r_a}^{r_b} \int_0^\pi \frac{d g(\theta)}{d \theta} e^{j r g(\theta)} dr d\theta \\
&= \int_{r_a}^{r_b} (1/jr) (e^{j r g(\pi)} - e^{j r g(0)}) dr = -2 \int_{r_a}^{r_b} \frac{\sin r k_x}{r} dr \\
&= -2 \left[\int_0^{r_b k_x} \sin u/u du + \int_0^{r_a k_x} \sin u/u du \right] \\
&= -2 [Si(r_b k_x) - Si(r_a k_x)]
\end{aligned}$$

$$I_2' = -2 \text{DSi} \quad (\text{II.27})$$

where DSi is used to denote the difference of sine integrals.

$$\begin{aligned}
I_1' &= \int_{r_a}^{r_b} \int_0^\pi g(\theta) e^{j r g(\theta)} dr d\theta \\
&= \int_0^\pi -j (e^{j r_b g(\theta)} - e^{j r_a g(\theta)}) d\theta
\end{aligned} \quad (\text{II.28})$$

This may be simplified by defining new constants in terms of the old as:

$$\begin{aligned}
j\gamma &= \xi \cos \psi & \xi^2 &= k_x^2 - \gamma^2 \\
&& \text{such that} \\
k_x &= \xi \sin \psi & \psi &= \arctan(k_x / j\gamma)
\end{aligned}$$

then

$$\begin{aligned}
g(\theta) &= k_x \cos \theta + j\gamma \sin \theta = \xi (\sin \psi \cos \theta + \cos \psi \sin \theta) \\
&= \xi \sin(\theta + \psi)
\end{aligned}$$

and

$$I_1' = -j \int_0^\pi e^{j r_b \xi \sin(\theta + \psi)} d\theta + j \int_0^\pi e^{j r_a \xi \sin(\theta + \psi)} d\theta$$

or

$$I_1' = -j \int_0^{\pi+\psi} e^{jr_b \xi \sin \varphi} d\varphi + j \int_0^{\pi+\psi} e^{jr_a \xi \sin \varphi} d\varphi$$

Now, the exponential integrands may be expressed as a series of Bessel functions (e.g., see McLachlan [26], pg. 42),

$$e^{ju \sin \varphi} = J_0(u) + 2 \sum_{p=0}^{\infty} J_{2p+2}(u) \cos(2p+2)\varphi + \\ + 2j \sum_{p=0}^{\infty} J_{2p+1}(u) \sin(2p+1)\varphi$$

and the integration is easily performed over the trigonometric functions. The series of even multiple angle cosine functions integrates to zero since their period is π , but the odd multiple angle sine functions have period 2π and thus have non-zero integrals. The result obtained for I_1' is thus,

$$I_1' = -j\pi [J_0(r_b \xi) - J_0(r_a \xi)] + 4 \sum_{\substack{p \\ \text{odd}}} [\ J_p(r_b \xi) - J_p(r_a \xi) \] \frac{\cos(p\psi)}{p}$$

or

$$I_1' = -j\pi \text{DBES}(0) + 4 \sum_{\substack{p \\ \text{odd}}} \text{DBES}(p) \cos(p\psi) / p \quad (\text{II.29})$$

where $\text{DBES}(p)$ represents the difference of p -th order Bessel functions. Combining (II.27) and (II.29) into (II.26), the integral I' is finally found to be

$$I' = \int_{r_a}^{r_b} \int_0^{\pi} \sin \theta [e^{jr(k_x \cos \theta + j\gamma \sin \theta)}] dr d\theta$$

$$I' = (1/\xi^2) [\pi\gamma \text{DBES}(0) + j4\gamma \sum_{\substack{p \\ \text{odd}}} \text{DBES}(p) \cos(p\psi) / p + 2k_x \text{DSi} \quad (\text{II.30})$$

where

$$\text{DBES}(p) = J_p(r_b \xi) - J_p(r_a \xi)$$

$$\xi^2 = k_x^2 - \gamma^2$$

$$\psi = \arctan(k_x / j\gamma)$$

$$\text{DSi} = \text{Si}(r_b k_x) - \text{Si}(r_a k_x)$$

Equation (II.30) is now one of the integrals necessary to evaluate $\mathcal{F}_{(1)}^{e+}$ in Equation (II.22). The others can be obtained by appropriate sign changes on k_x and γ . This is done by observing that the zeroth order Bessel function is an even function while the odd order Bessel functions are odd. Also, the cosine function is even and the sine integral is odd. The arguments of the functions depend on ξ and ψ . The angle ψ is seen to change sign when either k_x or γ changes sign (the range $-\pi/2 \leq \text{Re}(\psi) \leq \pi/2$ is assumed). The number ξ changes sign when γ changes sign but not when k_x changes sign since

$$\xi \cos \psi = j\gamma \qquad \xi \sin \psi = k_x.$$

When both γ and k_x change sign, ψ remains unchanged but ξ changes sign.

With the above observations, the result (II.30) can be used to evaluate $\mathcal{F}_{(1)}^{e+}$ in Equation (II.22). Thus,

$$\begin{aligned} \mathcal{F}_{(1)}^{e+} = & \pm (-1)^{\frac{n-1}{2}} \mathcal{A}^e \frac{-1}{2} \left[\pm \pi \gamma \text{DBES}(0) + j4\gamma \sum_{\substack{p \\ \text{odd}}} \text{DBES}(p) \cos(p\psi) / p + \right. \\ & \left. + 2k_x \text{DSi} \right] \quad \text{for } n \text{ odd} \end{aligned} \quad (\text{II.31})$$

where

$$\mathcal{D}^e = \frac{V_o}{\ln(r_b/r_a)} \sqrt{\frac{2}{Z_{ab}^e}} \frac{k_y}{k_c}$$

$$\text{DBES}(p) = \mathcal{J}_p(r_b \xi) - \mathcal{J}_p(r_a \xi)$$

$$\xi^2 = k_x^2 - \gamma^2$$

$$\text{DSi} = \text{Si}(r_b k_x) - \text{Si}(r_a k_x)$$

$$\Psi = \arctan(k_x/j\gamma)$$

There are now six other integrals to be evaluated in (II.16) of a similar nature. The integrals in (II.16b) are

$$\mathcal{F}_{(1)}^{h+} = \int_{S_g(1)} (\hat{y} \times \vec{E}_g) \cdot H_{\nu}^{h+}(\tilde{x}, 0, z) dS \quad (\text{II.16b})$$

Using (II.19) for $(\hat{y} \times \vec{E}_g)$ and H_{ν}^{h+} from Equation (2.6), this becomes

$$\mathcal{F}_{(1)}^{h+} = \frac{V_o}{\ln(r_b/r_a)} \int_{S_g(1)} [\pm (z/r^2) h_{x\nu}^h(\tilde{x}, 0) e^{\mp \gamma z} + (x/r^2) h_{z\nu}^h(\tilde{x}, 0) e^{\mp \gamma z}] dS$$

and, with Equations (2.5a)

$$\begin{aligned} \mathcal{F}_{(1)}^{h+} = \frac{V_o}{\ln(r_b/r_a)} \sqrt{\frac{\epsilon_{on} \epsilon_{om}}{2Z_{ab}^h}} \frac{k_x}{k_c} \int_{S_g(1)} [\pm (z/r^2) \sin(k_x \tilde{x}) e^{\mp \gamma z} + \\ - \frac{k_c^2}{k_x \gamma} (x/r^2) \cos(k_x \tilde{x}) e^{\mp \gamma z}] dS \end{aligned}$$

With $\tilde{x} = x + a/2$ and $\mathcal{D}^h$ for the coefficient, this becomes, for n even

$$\begin{aligned} \mathcal{F}_{(1)}^{h+} = (-1)^{n/2} \mathcal{D}^h \int_{S_g(1)} [\pm \sin(k_x x) e^{\mp \gamma z} (z/r^2) + \\ - \frac{k_c^2}{k_x \gamma} \cos(k_x x) e^{\mp \gamma z} (x/r^2)] dS \end{aligned}$$

and for n odd

$$\mathcal{F}_{(1)}^{h+} = (-1)^{\frac{n-1}{2}} a^h \int_{S_g(1)} \left[\pm \cos(k_x x) e^{\mp \gamma z} (z/r^2) + \frac{k_c^2}{k_x \gamma} \sin(k_x x) e^{\mp \gamma z} (x/r^2) \right] dS.$$

The integral for n even is seen to be zero because the integrand is odd with respect to x and the region of integration is symmetrical about $x = 0$, as seen in Figure II.1. The integrand for odd n is expanded using $r^2 = x^2 + z^2$ to give,

$$\begin{aligned} \mathcal{F}_{(1)}^{h+} = & \pm (-1)^{(n-1)/2} a^h \int_{S_g(1)} \cos(k_x x) e^{\mp \gamma z} z/(x^2 + z^2) dx dz + \\ & + (-1)^{(n-1)/2} a^h \int_{S_g(1)} \sin(k_x x) e^{\mp \gamma z} x/(x^2 + z^2) dx dz \end{aligned} \quad (\text{II.32})$$

for n odd

The first integral above is seen to be the same as the integral considered in Equation (II.21b) for $\mathcal{F}_{(1)}^{e+}$. Replacing $\pm a^h$ for a^e , the result is identical to that of Equation (II.31). The second integral is evaluated below following the same steps as for evaluating $\mathcal{F}_{(1)}^{e+}$. Thus, consider the integral

$$\int_{S_g(1)} \sin(k_x x) e^{\mp \gamma z} x/(x^2 + z^2) dx dz \quad (\text{II.33a})$$

With Euler's identity, this is

$$(-j/2) \int_{S_g(1)} \left[e^{jk_x x + \gamma z} - e^{-jk_x x + \gamma z} \right] x/(x^2 + z^2) dx dz$$

Making the transformation to polar coordinates as in Figure II.1 gives

$$(-j/2) \int_{r_a}^{r_b} \int_0^\pi \cos \theta [e^{jr(-k_x \cos \theta + j\gamma \sin \theta)} - e^{jr(k_x \cos \theta + j\gamma \sin \theta)}] dr d\theta \quad (\text{II.33b})$$

Equation (II.33a) and (II.33b) are equivalent. Now from (II.33b) consider the single integral

$$I'' = \int_{r_a}^{r_b} \int_0^\pi \cos \theta e^{jr(k_x \cos \theta + j\gamma \sin \theta)} dr d\theta \quad (\text{II.34a})$$

This is evaluated by considering two other integrals

$$I_1'' = \int_{r_a}^{r_b} \int_0^\pi (k_x \cos \theta + j\gamma \sin \theta) e^{jr(k_x \cos \theta + j\gamma \sin \theta)} dr d\theta$$

$$I_2'' = \int_{r_a}^{r_b} \int_0^\pi (-k_x \sin \theta + j\gamma \cos \theta) e^{jr(k_x \cos \theta + j\gamma \sin \theta)} dr d\theta$$

These two integrals combine to yield the desired integral I'' as:

$$I'' = (1/(k_x^2 - \gamma^2)) [k_x I_1'' + j\gamma I_2''] \quad (\text{II.34b})$$

The integral I_2'' is of the form

$$I_2'' = \int_{r_a}^{r_b} \int_0^\pi \frac{dg(\theta)}{d\theta} e^{jr g(\theta)} dr d\theta$$

which, as in Equation (II.27), integrates to a difference of sine integrals as

$$I_2'' = -2[\text{Si}(r_b k_x) - \text{Si}(r_a k_x)] = -2 \text{DSi} \quad (\text{II.35})$$

The integral I_1'' is of the form

$$\begin{aligned}
I_1'' &= \int_{r_a}^{r_b} \int_0^\pi g(\theta) e^{jr} g(\theta) dr d\theta \\
&= -j \int_0^\pi e^{jr_b g(\theta)} - e^{jr_a g(\theta)} d\theta
\end{aligned} \tag{II.36}$$

This is identical to Equation (II.28) for I_1' and thus the results are given by Equation (II.29):

$$I_1'' = -j\pi \text{DBES}(0) + 4 \sum_{\substack{p \\ \text{odd}}} \text{DBES}(p) \cos(p\psi) / p \tag{II.37}$$

The desired integral I'' is now found by using Equation (II.34b) to combine the results for I_1'' and I_2'' above. Thus

$$\begin{aligned}
I'' &= \int_{r_a}^{r_b} \int_0^\pi \cos \theta e^{jr(k_x \cos \theta + j\gamma \sin \theta)} dr d\theta = \\
&= \frac{1}{\xi^2} [-j\pi k_x \text{DBES}(0) + 4k_x \sum_p \text{DBES}(p) \cos(p\psi) / p - 2j\gamma \text{DSi}]
\end{aligned} \tag{II.38}$$

where

$$\begin{aligned}
\text{DBES}(p) &= \mathcal{J}_p(r_b \xi) - \mathcal{J}_p(r_a \xi) \\
\xi^2 &= k_x^2 - \gamma^2 \\
\psi &= \arctan(k_x / j\gamma) \\
\text{DSi} &= \text{Si}(r_b k_x) - \text{Si}(r_a k_x)
\end{aligned}$$

This is one of the integrals in Equation (II.33b) above. The other integrals may be evaluated by appropriate sign changes on the terms in Equation (II.38) for corresponding sign changes on k_x and γ . The symmetry observations made in the discussion following Equation (II.30), above, still hold. Thus, for example, from (II.33b), the result is

$$\begin{aligned}
& \int_{r_a}^{r_b} \int_0^\pi \cos \theta e^{jr(-k_x \cos \theta \pm j\gamma \sin \theta)} dr d\theta = \\
& = \frac{1}{\xi^2} \left[+j\pi k_x \text{DBES}(0) \pm 4k_x \sum_{\substack{p \\ \text{odd}}} \text{DBES}(p) \cos(p\psi)/p \pm 2j\gamma \text{DSi} \right] \quad (\text{II.39})
\end{aligned}$$

Using Equations (II.38) and (II.39) in (II.33b), the integrals in (II.33a) are thus

$$\begin{aligned}
& \int_{\xi}^{\infty} \sin(k_x x) e^{\pm \gamma z} x/(x^2 + z^2) dx dz = \\
& = \frac{1}{\xi^2} \left[\pi k_x \text{DBES}(0) \pm 4jk_x \sum_{\substack{p \\ \text{odd}}} \text{DBES}(p) \cos(p\psi)/p \pm 2\gamma \text{DSi} \right] \quad (\text{II.40})
\end{aligned}$$

All the integrals involved in computing $\mathcal{F}_{(1)}^{h+}$, Equation (II.32), are now evaluated. Combining (II.31) and (II.40) appropriately in (II.32) gives

$$\begin{aligned}
\mathcal{F}_{(1)}^{h+} = & \pm (-1)^{\frac{n-1}{2}} d^h \frac{-1}{\xi^2} \left[\pm \pi \gamma \text{DBES}(0) + j4\gamma \sum_{\substack{p \\ \text{odd}}} \text{DBES}(p) \cos(p\psi)/p + 2k_x \text{DSi} \right] + \\
& + (-1)^{\frac{n-1}{2}} d^h \frac{k_c^2}{k_x \gamma \xi^2} \left[\pi k_x \text{DBES}(0) \pm j4k_x \sum_{\substack{p \\ \text{odd}}} \text{DBES}(p) \cos(p\psi)/p + \right. \\
& \left. \pm 2\gamma \text{DSi} \right] \quad \text{for } n \text{ odd} \quad (\text{II.41})
\end{aligned}$$

where

$$\begin{aligned}
d^h &= \frac{V_o}{\ln(r_b/r_a)} \sqrt{\frac{\epsilon_{on} \epsilon_{om}}{2Z_v^h ab}} \frac{k_x}{k_c} \\
\text{DBES}(p) &= \mathcal{J}_p(r_b \xi) - \mathcal{J}_p(r_a \xi) \\
\xi^2 &= k_x^2 - \gamma^2 \\
\psi &= \arctan(k_x/j\gamma) \\
\text{DSi} &= \text{Si}(r_b k_x) - \text{Si}(r_a k_x) \\
\text{Si} &= \text{sine integral}
\end{aligned}$$

To this point, four of the integrals in (II.16a,b) have been evaluated. Four more remain in (II.16c,d), but, fortunately, these may be found from the first four by symmetry arguments; the integrands are identical but the region of integration changes from $S_g(1)$ to $S_g(2)$. This amounts to a change of sign of the variable z , with all else remaining unchanged. More explicitly, from (II.16c),

$$\mathcal{F}_{(2)}^{e+} = \int_{S_g(2)} (\hat{y} \times \vec{E}_g) \cdot \vec{H}_v^{e+} dS$$

and from (II.20),

$$\mathcal{F}_{(2)}^{e+} = \frac{V_o}{\ln(r_b/r_a)} \sqrt{\frac{2}{Z_v^{e+} ab}} \frac{k_y}{k_c} \int_{S_g(2)} \pm (z/r^2) \sin(k_x \tilde{x}) e^{\mp \gamma z} dS \quad (\text{II.42})$$

Changing the sign of z in the integrand (with the element of area, dS , unchanged) converts the area to $S_g(1)$ so that the following relation is obtained:

$$\mathcal{F}_{(2)}^{e+} = \mathcal{F}_{(1)}^{e+} \quad (\text{II.43})$$

The result is then obtainable from Equation (II.31). Similarly, from (II.16d),

$$\mathcal{F}_{(2)}^{h+} = \int_{S_g(2)} (\hat{y} \times \vec{E}_g) \cdot \vec{H}_v^{h+} dS$$

and from (II.32),

$$\begin{aligned} \mathcal{F}_{(2)}^{h+} = & \pm (-1)^{\frac{n-1}{2}} d^h \int_{S_g(2)} \cos(k_x x) e^{\mp \gamma z} z/(x^2 + z^2) dS + \\ & + (-1)^{\frac{n-1}{2}} d^h \int_{S_g(2)} \sin(k_x x) e^{\mp \gamma z} x/(x^2 + z^2) dS . \end{aligned}$$

Changing the sign on z , as above, introduces a sign change on the first integral but not on the second; while changing the exponential factors of both. The region is then effectively $S_g(1)$ and the results in Equation (II.41) can be applied to give:

$$\begin{aligned} \mathcal{F}_{(2)}^{h+} = & \bar{+}(-1)^{\frac{n-1}{2}} \mathcal{A}^h \frac{1}{\xi^2} [\bar{+}\pi\gamma \text{DBES}(0) + 4j\gamma \sum_{\substack{p \\ \text{odd}}} \text{DBES}(p) (\cos(p\psi)/p + 2k_x \text{DSi})] + \\ & +(-1)^{\frac{n-1}{2}} \mathcal{A}^h \frac{k_c^2}{k_x \gamma \xi^2} [\pi k_x \text{DBES}(0) + j4k_x \sum_{\substack{p \\ \text{odd}}} \text{DBES}(p) \cos(p\psi)/p + \\ & \quad \bar{+} 2\gamma \text{DSi}] \quad \dots \quad \text{for } n \text{ odd} \end{aligned} \quad (\text{II.44})$$

where the notations are described following Equation (II.41).

This concludes the evaluation of the original eight integrals shown in Equations (II.16). Although it has been convenient in this presentation to arrive at the results by abstraction or occasional hand waving, the reader is assured that all the lengthy details for each of the different integrals have been carried out to verify the results shown.