ESTIMATION OF BETA RISK COMPONENTS FOR REDUCTION OF PREDICTION ERROR IN PORTFOLIO MODELS

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ABSTRACT

ESTIMATION OF BETA RISK COMPONENTS FOR REDUCTION
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By

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Two primary uses have been made of systematic risk as it applies to common stocks: (1) the evaluation of historical performance of common stock portfolios (2) the construction of common stock portfolios according to the risk-return (RR) criteria. This research focuses on the construction of common stock portfolios according to the RR criteria.

In order to estimate the worth of a common stock by the RR criteria, some ex ante estimate of the systematic risk of the stock is required. Typically it has been assumed that systematic risk is stationary hence a computation of ex post systematic risk can serve very well as an estimate of ex ante systematic risk. This assumption, however, still leaves the question of how to compute ex post systematic risk e.g., should the historical estimate be computed over three years, five years, ten years or some other time period and should observations of return within this period (used to compute the systematic risk) be taken weekly, monthly, quarterly, etc. Furthermore, should the relationship of risk and return be assumed to be $R_i = \alpha_i + \beta_i R_m + e$ or should some alternative market model be used?

Two market models were considered in this paper: (1) the ordinary

least squares equation presented above and (2) the market model presented by Lawrence Fisher and Jules Kamin at the Midwest Finance Meetings in March 1972 which depicts the relationship of risk and return as $R_i = \beta_i R_m$. Twenty-eight estimates of ex post systematic risk were computed for each of the thirty-five securities in the sample in each of two different (but overlapping) test periods. These twenty-eight estimates were then compared to the volatility (R_i/R_m) of the individual securities in each of twelve different holding periods in test period I and each of eight different holding periods in test period II. These differences were defined as prediction error and the mean absolute value of prediction errors was computed for all securities for each measurement period, observation interval and holding period. Prediction errors were computed for both market models and these results also compared.

The results indicated that there is not a great deal of consistency between securities i.e., the estimate of ex post systematic risk which minimizes prediction error for security i will not necessarily minimize prediction error for security j. The results also indicated that the minimum mean prediction error is most decidedly a function of the time period examined. In test period I the minimum mean prediction error arose from 69 observations of monthly return used to estimate volatility in a fifteen month holding period while in test period II the minimum mean prediction error arose from three observations of quarterly returns used to estimate volatility in a three month holding period. In general and on average, it was noted that prediction error was positively correlated with holding period length and that between 75 and 100 observations of return provided the minimum mean prediction error. It was also noted that the ordinary least squares procedure of computing ex post systematic risk provided a smaller mean prediction error than the Fisher-Kamin procedure in the majority of instances.

The results further indicated that highly accurate estimates of ex ante systematic risk were difficult to obtain e.g., the mean absolute prediction error averaged across all holding periods and measurement periods ranged between 0.45 (quarterly observation interval) and 1.20 (weekly observation interval) in test period I and between 0.37 (weekly observation interval) and 0.48 (quarterly observation interval) in test period II. Given the magnitude of these errors and the inability of a particular measurement period or observation interval to consistently provide superior prediction results, it would appear that without improved techniques to estimate ex ante systematic risk, the Risk-Return programming model is of very limited use in the real world.

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CHAPTER I

INTRODUCTION

Background and Objectives

In finance assets are frequently evaluated using quantitative measures for the expected return and anticipated risk associated with the asset.[1] The relationship of these two parameters is frequently written in the following form,

$$R_{it} = a_{it} + b_{it}R_{mt} \tag{I-1}$$

where

R_{it} = the historical return during period t on an individual asset i

$a_{it} = 1/n \sum_t R_{it} - b/n \sum_t R_{mt}$

$b_{it} = \dfrac{n \sum_t R_{mt} R_{it} - \sum_t R_{it} \sum_t R_{mt}}{n \sum R_{mt}^2 - (\sum_t R_{mt})^2}$ = the systematic risk component of asset i

R_{mt} = the historical return on a portfolio of assets similar to asset i during period t

n = the number of historical observations of R_{it} and R_{mt}

In this expression the systematic risk component, b_{it}, describes the total risk of the asset so long as the asset is held in a portfolio of

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[1]See for example, Chris A. Welles, "The Beta Revolution: Learning to Live with Risk", The Institutional Investor, Vol. V, No. 9 (September 1971), pp. 21-26 and following. See also, Frank E. Block, "Beta Evaluation", Wall Street Transcript, July 3, 1972, p. 29,056.

similar assets numbering at least twenty.[2]

The financial community uses the two parameter model in the following two ways: (1) for the evaluation of historical investment performance and (2) for the prediction of the future relationship between the return on asset i (R_i) and the portfolio of similar assets (R_m). Since it is easier to predict the behavior of a portfolio of assets than an individual asset (errors tend to cancel), and since the relationship is assumed to be stationary over time, the model can be used to predict the behavior of an individual asset with historical information and some estimate of the behavior of the portfolio. The historical information used consists of the n observations of R_{it} and R_{mt} made during time interval t.

The purpose of this research is to examine how the parameter, b_{it}, can best be estimated to provide the model user with the minimum prediction error when predicting systematic risk in some future time interval.

Problem Identification

There are several factors which will influence the predictability of future systematic risk. Equation I-2 below provides a model which measures prediction error as predicted less actual return of the individual security and shows the several factors which influence the computation of b_{it}.[3] In equation I-2, the size of the prediction error

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[2] J. L. Evans and Stephen H. Archer, "Diversification and the Reduction of Dispersion: An Empirical Analysis", Journal of Finance, December 1968, pp. 761-769.

[3] A. M. Mood and F. M. Graybill, Introduction to the Theory of Statistics, McGraw-Hill, 1963, pp. 335-343.

will be a function of the ability to correctly estimate b_{it}.

$$EP_i = (R_p - R_a) = \sigma_{ei} \left\{ 1 + 1/n + \left| \frac{(R_{mk} - \bar{R}_{mt})^2}{\sum_t (R_{mt} - \bar{R}_{mt})^2} \right| \right\} \qquad \text{(I-2)}$$

where

R_{mk} = the observed return on the portfolio of assets in time period k

$$\sigma_{ei} = \frac{1}{n-2} \left\{ \sum_t (R_{it} - a_{it} - b_{it} R_{mt})^2 \right\}$$

and n, a_{it}, and b_{it} are as described in equation (I-1).

The problem of estimating bit breaks down into two major components: (1) establishing the optimal market model and (2) specifying the parameters of the market model selected.

Market Model

There exist an infinite number of potential market models. The market model specifies the computational equation of the historical beta coefficient, b_{it}, and correction factors of any sort can be included in the computation methodology. Thus there would exist, in theory, as many market models as correction factors. Until recently however, there was basic agreement that the proper (or at least suitable) relationship between the return on an individual security and the return on the market could be expressed as in equation (I-1).

At the Midwestern Finance Association Meetings in 1972, Professor Lawrence Fisher and Jules H. Kamin found that unbiased estimates of b_{it} developed in equation (I-1) provided predictions of future systematic risk, b_{ik}, which were inferior to those developed from equation (I-3) below.[4]

$$b_{it} = \frac{\sum_t R_{it} R_{mt}}{\sum_t R_{mt}} \qquad \text{(I-3)}$$

This development of the historical beta coefficient assumes an alternative relationship between risk and return <u>viz</u>,[5]

$$R_{it} = b^F_{it} R_{mt} \qquad \text{(I-4)}$$

Equation (I-3) is an unbiased estimator of beta <u>if</u> the functional form exhibited in equation (I-4) is assumed.[6] What is at issue here is whether the relationship of risk and return for an individual security is best represented as

$$R_{it} = b_{it} R_{mt}$$

or

$$R_{it} = a_{it} + b_{it} R_{mt}$$

Under the functional form assumed by Fisher,

$$b^F_{it} = R_{it}/R_{mt}$$

whereas under the normal linear form,

$$b_{it} = (R_{it} - a_{it})/R_{mt} = R_{it}/R_{mt} - a_{it}/R_{mt}$$

Clearly, if $a_{it} = 0.0$, both estimates of systematic risk will be the

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[4] Lawrence Fisher and Jules H. Kamin, "Good Betas and Bad Betas; How to Tell the Difference", A Presentation at the Midwest Finance Association Meeting, St. Louis, Missouri, April, 1972.

[5] All statistics generated by Fisher and Kamin will be denoted by superscript F.

[6] Harold J. Larson, <u>Introduction to Probability Theory and Statistical Inference</u>, John Wiley and Sons, New York, 1969, p. 320.

same. While a_{it} typically approaches zero, it is seldom identically zero.[7] This suggests that the two systematic risk measures will vary by a factor of a_{it}/R_{mt}. Which of the two forms is more correct will depend on which is the more useful i.e. which produces the least prediction error.

Model Specification

Once the market model has been selected, the parameters of the model must still be specified. The prediction error associated with either equation (I-1) or (I-4) will depend on (1) the measurement period over which returns are taken, t, (2) the number of observations of return within the measurement period, n, and (3) the period over which the systematic risk to be predicted is computed. The importance of parameter specification to the selection of the market model is shown below.

The prediction error model of equation (I-2) can be applied to the Fisher market model as follows:[8]

$$EP_i^F = \sigma_{ei}^F \left\{ 1 + \left[(R_{mk})^2 \Big/ \sum_t (R_{mt} - \bar{R}_{mt})^2 \right] \right\} \qquad \text{(I-6)}$$

It can also be shown that[9]

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[7] Nancy L. F. Jacob, "Theoretical and Empirical Aspects of the Measurement of Systematic Risk for Securities and Portfolios", Unpublished Ph.D. dissertation, University of California - Irvine, 1970, p. 74.

[8] $$EP_i^F = \sigma_{ei} + R_{mt}^2 \hat{\sigma}_b^2 + 2R_k E\,[(\hat{a} - a)(\hat{b} - b)] + \hat{\sigma}_a^2$$

But $$E\,[(\hat{a} - a)\,(\hat{b} - b)] = \hat{\sigma}_a^2 = 0.0$$

Hence $$EP_i^F = \hat{\sigma}_{ei} + R_{mt}^2 \hat{\sigma}_b^2 = \hat{\sigma}_{ei} + R_{mt}^2 \sigma_{ei}^2 \left[1/\sum_t (R_i - \bar{R}_{mt})^2\right] \quad \text{Q.E.D.}$$

[9] H. F. Larson, op. cit., p. 339.

$$(\hat{\sigma}^F_{ei})^2 = \frac{1}{n-1} \sum_t (R_{it} - b^F_{it} R_{mt})^2 \tag{I-6}$$

This means that the squared prediction error associated with the market model of equation (I-1) will be less than the prediction error associated with the Fisher formulation of equation (I-4) for any observation of R_{mk} when

$$\hat{\sigma}_{ei} \left[1 + \frac{1}{n} + \frac{(R_{mk} - \bar{R}_{mt})^2}{\sum_t (R_{mt} - \bar{R}_{mt})^2}\right] < (\hat{\sigma}^F_{ei})^2 \left[1 + \frac{(R_{mk})^2}{\sum_t (R_{mt} - \bar{R}_{mt})^2}\right] \tag{I-7}$$

Breaking the inequality into factors it can be shown that $\hat{\sigma}^2_{ei} < (\sigma^F_{ei})^2$ when

$$\bar{R}_{mt} > \frac{R_{mt}(b_{it} + b^F_{it})}{R_{it}} - \frac{b_{it}}{n} R_{it} \tag{I-7A}$$

Further,

$$1 + \frac{1}{n} + \frac{(R_{mk} - \bar{R}_{mt})^2}{\sum_t (R_{mt} - \bar{R}_{mt})^2} < 1 + \frac{(R_{mk})^2}{\sum_t (R_{mt} - \bar{R}_{mt})^2} \tag{I-7B}$$

when

$$\bar{R}_{mt} > \frac{n+1}{2n} \left[\frac{\sum_t (R_{mt} - \bar{R}_{mt})^2}{R_{mk}}\right] \tag{I-7C}$$

Taken together, inequalities (I-7A) and (I-7C) guarantee that the prediction error associated with the market model of equation (I-1) will be less than that associated with equation (I-4) if

$$\bar{R}_{mt} > \frac{R_{mt}(b_{it} + b^F_{it})}{R_{it}} - \frac{b_{it}}{n} R_{it} \ \underline{\text{and}} \ \bar{R}_{mt} > \frac{n+1}{2n} \left[\frac{\sum_t (R_{mt} - \bar{R}_{mt})^2}{R_{mk}}\right]$$

It is not obvious when these conditions will prevail. It can be agreed, however, that the quantity $(R_{mt} - \bar{R}_{mt})^2$ will depend in part upon the observation interval selected. A priori, a larger absolute percentage could be expected if daily rather than annual returns were used because of the greater variability of returns associated with shorter time periods.[10] Similarly, the variability of b_{it} (σ_b) will depend upon the length of the observation interval. (The observation interval being that length of time between price observations used to compute returns). If b_{it} and b^F_{it} are independent of the observation interval, an assumption used in some applications, then R_{mt} $(b_{it} + b^F_{it})$ will be positively correlated with the observation interval. Since the observation interval is determined in part by the total period over which observations are taken (the measurement period), a seven year period, for example, would virtually eliminate annual observations of return since the significance of the statistical parameters would hold only for very large confidence intervals.

The prediction results obtained by Fisher and Kamin may be due to the measurement period and/or observation interval employed rather than to any difference in the functional form of risk and return exhibited in equations (I-1) and (I-4). The anticipated holding period, discussed later, can also influence these results.

Portfolio Analysis Using Volatility as a Measure of Risk

The application of the two parameter model was originally presented by Harry Markowitz in his 1952 article.[11] Because Markowitz utilized

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[10] Lawrence Fisher and James C. Lorie, "Some Studies of the Variability of Returns on Investments in Common Stock", Journal of Business, April 1970, p. 110.

[11] Harry Markowitz, "Portfolio Selection", Journal of Finance, March 1952, pp. 77-91.

variance of return as a measure of risk, it has frequently been argued that the attendant co-variance matrix required to evaluate combinations of securities is both too costly and too time consuming to be useful in the selection of portfolios.[12,13] The substitution of the beta coefficient for variance of return to measure risk was introduced by Professor Sharpe in 1967.[14] This substitution reduced the information required to obtain estimates of risk for combinations of securities and provided a low cost, viable alternative to the Markowitz mean-variance approach. The Sharpe model is described here as it applies to the evaluation of individual securities.

The Sharpe model requires two inputs:

(1) expected return for each security = E_i

(2) expected systematic risk of each security = b_i

By plotting ex ante estimates of E_i and b_i for each security and combination of securities, a feasible region of combinations would exist similar to Figure I-1 below. Efficient combinations (maximum return for any given level of risk) are indicated by the darker (upper left hand) border. The goal is to determine this set of efficient portfolios given the input requirements mentioned above.

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[12] William F. Sharpe, <u>Portfolio Theory and Capital Markets</u>, McGraw-Hill, 1970, p. 118.

[13] John Clark Francis and Stephen H. Archer, <u>Portfolio Analysis</u> Prentice-Hall, Englewood Cliffs, New Jersey, 1971, pp. 95-96.

[14] William F. Sharpe, "A Linear Programming Algorithm for Mutual Fund Portfolio Selection", <u>Management Science</u>, March 1967, pp. 499-510.

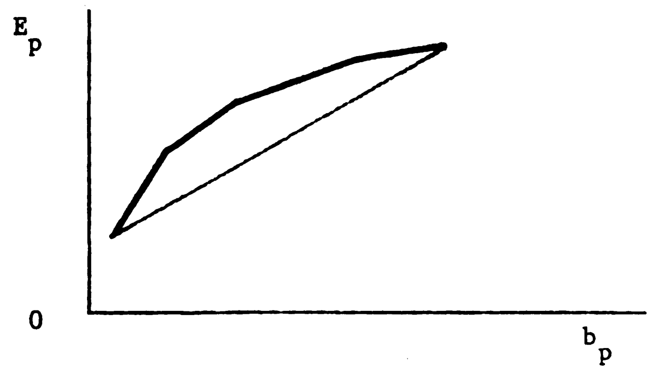

FIGURE I-1

Linear programming can be used to accomplish this goal. Since the objective is to maximize portfolio return for any given level of risk, the following notation is in order:

$$E_p = \sum_{i=1}^{n} X_i E_i$$

$$b_p = \sum_{i=1}^{n} X_i b_i$$

where

n = the total number of securities under consideration

X_i = the portion of the total portfolio commited to security i

The objective function can be stated as

$$\text{Maximize } Z_p = (1 - q)E_p - qb_p$$

subject to

$$0 \leq q \leq 1$$

$$\sum_i X_i = 1$$

$$X_i \geq 0 \text{ for all } i$$

$$L_i \leq X_i \leq U_i$$

and

q = the relative importance of risk vs. return

The constraints can be interpreted as follows:

$X_i > 0$ dis-investment is not allowed

$L_i \leq X_i \leq U_i$ at least L proportion and not more than U proportion of total funds must be held in security i

$\sum_i X_i = 1$ 100% of the available money shall be invested

$0 \leq q \leq 1$ not more than 100% risk nor less than 0.0% risk can be assumed.

For each arbitrary q selected ($0 \leq q \leq 1$), the objective function will determine an optimal combination of risky securities i.e., an optimum portfolio.

If q = 0, indicating risk (b_p) is to be totally disregarded, then maximizing Z_p is equivalent to maximizing E_p. Figure I-2 shows that the linear programming algorithm would determine A as the optimum portfolio for q = 0.

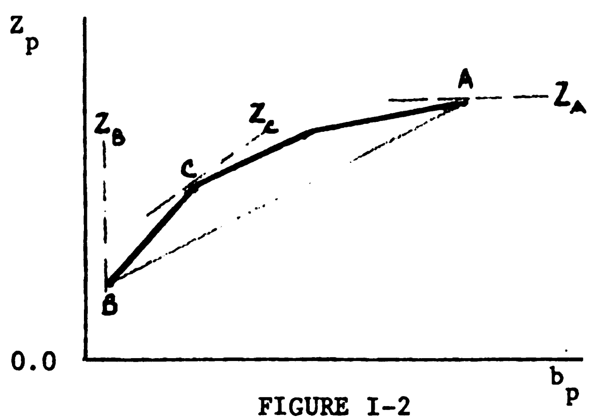

FIGURE I-2

For q = 1, the optimum portfolio would be that portfolio which minimizes risk regardless of return. In this case, maximizing Z_p is equivalent to minimizing b_p. Portfolio B in Figure I-2 depicts this situation.

For 0 > q > 1, say q = .5, the appropriate Z_p line slopes upward at .5 and denotes C as the optimum portfolio in Figure I-2.

When q is chosen such that the slope of Z_p is parallel to a linear segment of the efficient border, then any linear combination of the corner portfolios is equally acceptable and optimal. In Figure I-3 this situation is shown. If q = .4 and is parallel to line segment CD as in Figure I-3, then any linear combination of portfolios C and D which satisfies the expression $X_1D + X_2D = 1$ is equally optimal.

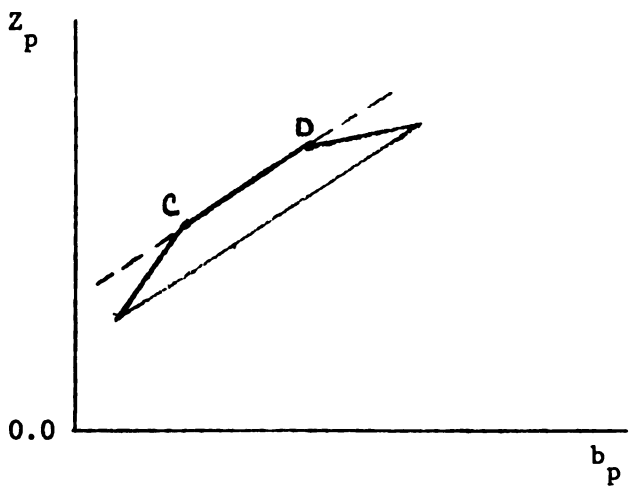

FIGURE I-3

Model Limitations

Two inputs are required to use the model, E_i and b_i. From a conceptual standpoint, E_i and b_i are <u>ex ante</u> measures. As was pointed out earlier, historical estimates of b_i are assumed to be the best estimates of future b_i. The reasoning proceeds as follows. By arguing that b_p is relatively stable over time, many authors have assumed that <u>ex post</u> estimates of b_i will provide the best estimate of future b_i.[15] The generally accepted methodology of determining historical b_i has been to use the linear regression analysis associated with the market model of equation (I-1).

This reasoning suffers from two important limitations. First, stability of portfolio volatility, b_p, is no assurance of the stability of individual security volatility, b_i. Also, the stability of the beta coefficient is not necessarily equivalent to the predictability of the beta coefficient if, indeed, the slope of the least squares regression line is what should be predicted. Finally, no specifications have been set down as to the measurement period or observation interval which the model user should consider in determining the historical b_i.[16]

Justification for Research

Incorrectly estimating the systematic risk of an individual security may cause the model user to incorrectly rank securities hence include in the portfolio securities which are less desirable than others which have been excluded. An examination of the linear programming model will demonstrate this argument.

Just as the value of a particular combination of securities was expressed as $Z_p = (1 - q)E_p - qb_p$, the value of any particular security can be expressed as

$$z_i = (1 - q)E_i - qb_i \tag{I-9}$$

[15] See for example, William F. Sharpe, Portfolio Theory and Capital Markets, McGraw-Hill, 1970, p. 142.

[16] Nicholas J. Gonedes, "Evidence on the Information Content of Accounting Numbers: Accounting-Based and Market-Based Estimates of Systematic Risk", Journal of Financial and Quantitative Analysis, June 1973, pp. 407-443.

For any given value of $q = q_o$, z_i becomes a function of E_i and b_i. For example, let $q_o = 1/2$ and $E_i = 0.10$, then z_i assumes different values depending upon the estimate of b_i. Table I-1 provides an illustration.

$$z_i = (1-q)E_i - qb_i$$

$$q_o = 1/2$$

$$E_i = 0.10$$

b_i	z_i
.1	0.00
.3	-0.10
.7	-0.30
1.0	-0.45
1.5	-0.70

TABLE I-1

From Table I-1, it is apparent that z_i is a decreasing function of b_i. This is reasonable since the relationship between value and risk is negative by assumption i.e., individuals are assumed to be risk averters. Figure I-4 plots the data in Table I-1.

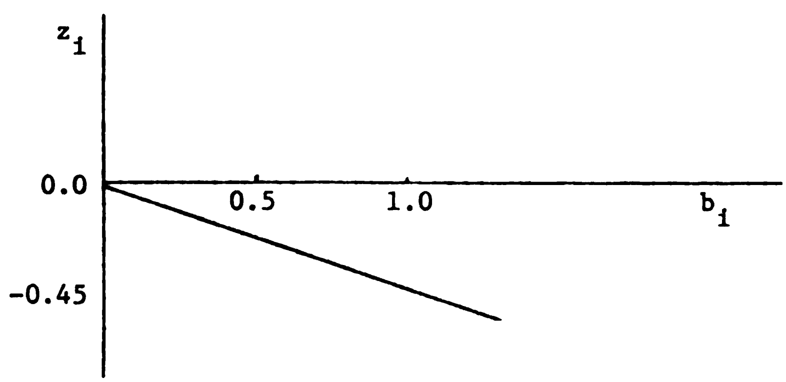

FIGURE I-4

Figure I-5 below demonstrates a simple four security universe. Each security is assumed to have a different expected value but the same systematic risk. To simplify the example, it is assumed that each security's b_i has been estimated at 1.00. A ranking of the four securities according to the z_i criteria would indicate security 1 as

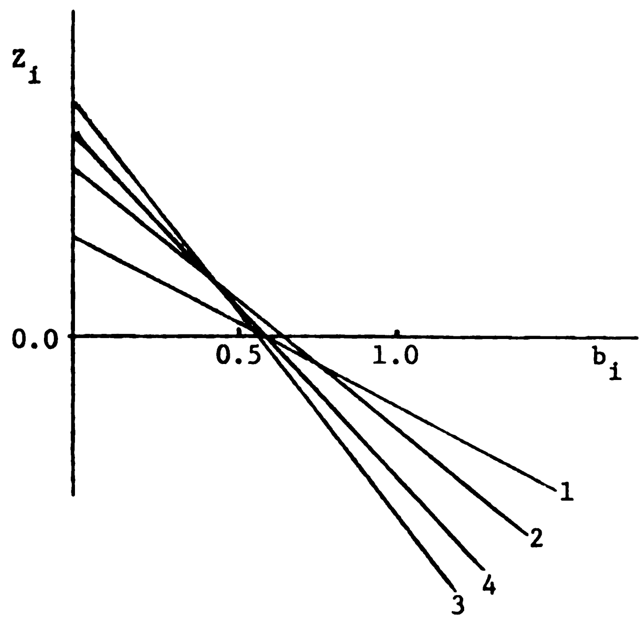

FIGURE I-5

the first choice, security 2 as the second choice, security 4 as the third choice, and security 3 as the fourth choice. Satisfying the constraint that two securities must be held would indicate holding securities 1 and 2 in equal proportions in the portfolio. If however, the correct values of b_i are 0.5 not 1.0, then the proper ranking of the securities would change. Now the highest ranking securities are 2 and 4, but not 1. In fact, security 1 now ranks last rather than first. With a large sample of securities and a proportionately smaller number of securities held, the change in rankings could be even more significant.

CHAPTER II

PAST STUDIES

The literature on portfolio theory is abundant. For the most part this literature has focused on the evaluation of historical performance of securities and portfolios or on implications of the theory itself. Little attention has been given to the problem of specification of the risk-return model parameters when the model is to be used for predictive purposes.

Michael C. Jensen's work with the portfolio model can be generally cast into that category which attempts to measure the *ex post* performance of portfolios.[1] Specifically, he set out to determine whether the returns achieved by mutual funds were consistent with their systematic risk exposure. He did, however, partially test his input data by comparing one year return with two year return (for use in computing his beta co-efficients) over twenty years for each portfolio. Regressing the betas so derived, Jensen found a correlation co-efficient of 0.89 which he interpreted to mean that use of one year observations or two year observations was largely immaterial in computing returns for use in estimating beta co-efficients.

Marshall E. Blume, II, recognizing that the results obtained by

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[1]Michael C. Jensen, "Risk, Capital Asset Prices and Evaluation of Portfolios", *Journal of Business*, April 1969, pp. 167-245.

Jensen need not apply to individual securities, tested 251 securities listed continuously on the NYSE over the period 1927-1960, for stationarity.[2] He divided the period into four equal sub-periods, computed a beta co-efficient for each security in each sub-period, and regressed the 251 individual betas from each sub-period against the respective beta in each subsequent sub-period. His average correlation coefficient was approximately 0.72. Blume interpreted this to mean that individual security betas were stationary over time.[3]

Both Jensen and Blume tested the stationarity property of the beta co-efficient; Blume for individual securities, and Jensen for groups of securities. These tests are not really a specification of how the beta co-efficient should be computed. Blume for example, arbitrarily selected one month as the observation interval and seven years as the measurement period. Jensen did compare beta co-efficients developed from one and two year observation intervals but he did not attempt to reconcile differences nor determine which of the methods provided the best results.[4]

As part of her 1970 dissertation entitled <u>Theoretical and Empirical Aspects of the Measurement of Systematic Risk for Securities and</u>

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[2]Marshall E. Blume II, "The Assessment of Portfolio Performance: An Application of Portfolio Theory", Unpublished Ph.D. dissertation, University of Chicago, 1968.

[3]The stationarity property of the beta co-efficient and the predictability of the beta co-efficient are similar but not identical. If the beta co-efficient is stationary then using historical information to estimate the future will produce small prediction errors. On the other hand, a particular beta co-efficient could be predictable but not stationary.

[4]Jensen showed that in the absence of measurement error the period over which these returns is computed is immaterial.

Portfolios,[5] Professor Nancy L. Jacob examined various properties of the beta co-efficient for 593 securities on the NYSE listed continuously between 1945-1965. To determine the predictive ability of beta, she compared adjacent time periods of identical length, utilized the same observation interval for measurement of return, and regressed the individual security's betas so derived. She used three distinct time period lengths: one year, five years and ten years and three observation intervals: monthly, quarterly and annually to compute beta for the regression test. This procedure was employed for individual securities and for random and sorted portfolios of various size. Using the co-efficient of determination (R^2) as her measure of predictive ability, she found that it was extremely difficult to predict betas for one year periods (average R^2 of approximately .100).[6] As the period lengthened, however, from one to five years, the predictive ability substantially improved (R^2 of approximately .370). Dr. Jacob did not test the predictive ability of beta between one and five years.[7]

As a result of these findings, Dr. Jacob argued "... an investor cannot use the past average volatility... of a portfolio as a best guess of its future value...."[8] In other words, the beta co-efficient

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[5]Nancy L. F. Jacob, Theoretical and Empirical Aspects of the Measurement of Systematic Risk for Securities and Portfolios, Unpublished Ph.D. dissertation, University of California - Irvine, 1970.

[6]The figures cited for one year were reported for monthly observations of return only.

[7]Nancy L. F. Jacob, op. cit., p. 91.

[8]Ibid., p. 93.

of an individual security or portfolio is not a particularly reliable estimate of either volatility or expected return of the same security or portfolio in future time periods.

In the design of her experiment, Dr. Jacob specified the computation of beta for individual securities as

$$\frac{\sum_t (mr_{i,t} - \bar{mr}_i)(mr_{m,t} - \bar{mr}_{m,t})}{\sum_t (Mr_{m,t} - \bar{mr}_m)} \qquad \text{(II-1)}$$

where mr_i and mr_m are excess returns on the individual security and the market respectively.

The market index used was unique in that it was comprised of the 593 securities which she studied. In addition, returns on the index were computed under the assumption that funds were re-allocated equally to each of the 593 securities in the market portfolio at each observation.

This technique of utilizing precisely the 593 securities in the sample as the market index may cause the residual risk of any security, e_i, to be correlated with the market index.[9] Portfolio theory typically assumes that $E(e_i) = 0$. The effect on Jacob's results is uncertain. Because of the large number of securities, it may be insignificant.

Professor Jacob found some results which do have implications for the specification of the market model of equation I-1. First, she found that the standard error of beta, $\hat{\sigma}_b$, is inversely related to the

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[9]Michael C. Jensen, "The Performance of Mutual Funds in the Period 1945-1964", Journal of Finance, May 1968, p. 392.

length of the period over which beta is measured.[10] For example, systematic risk exhibited less dispersion when measured with annual returns over five years than when measured with annual returns over two years. Secondly, she found $\hat{\sigma}_b$ was directly related to the length of the observational interval.[11] That is, greater dispersion of the beta coefficient was evident for annual and quarterly observations of return than for monthly observations of return for any given period. She did not, however, reach any conclusion as to which observation interval provided the best estimates of beta (minimum $\hat{\sigma}_b$) nor over how long a period returns should be observed.

One additional observation made by Professor Jacob is worth noting here, Capital market theory specifies a perfect linear relationship between R_i and R_m. She found this relationship less than perfect but most in evidence when R_m was significantly different from zero.

Another 1970 dissertation, this by F. B. Campanella of Harvard University, addressed itself to three specific questions concerning the measurement of beta.[12] First, Campanella asked, "Do computed values of beta vary with the length of the time period considered, e.g., five years vs. ten years; or are the values of beta sensitive to the time interval over which security returns are calculated, e.g., monthly vs. quarterly returns?"[13] He also asked, "...would beta

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[10] Nancy L. F. Jacob, op. cit., p. 61.

[11] Nancy L. F. Jacob, op. cit., p. 61.

[12] F. B. Campanella, Measurement and Use of Portfolio Systematic Risk, Unpublished DBA dissertation, Harvard University, 1970.

[13] Ibid., p. 29.

computed for the time interval t=1951-60 be a good estimate of the appropriate beta for time interval t=1961-70?"[14]

To test the stationarity of beta, Campanella regressed $(b_i)_t$ where t= 1956-66 against $(b_i)_t$ where t=1946-56 and found a correlation co-efficient of .513. For the identical regression but t=1946-56 and 1936-46, he found a correlation co-efficient of .707. And for 1936-46 vs. 1926-36, he found a correlation co-efficient of .655. To make these results similar to Jacob's, the corresponding R^2 has been computed and is shown below in Table II-1. Campanella's conclusion was that a relatively high correlation exists between the betas computed in different time periods and there is stationarity over time.

In testing the effects of varying the observation intervals and time period lengths, Campanella used five, ten and twenty year periods and computed beta from both monthly and quarterly data. He used 1956-66 betas (computed from monthly observation intervals) as a standard and correlated all other betas with these. He found that betas computed over ten years are highly correlated with betas computed over either five years or twenty years for both monthly and quarterly

TABLE II-1

Regression Study	R^2
1936-46 vs. 1926-36	.429
1946-56 vs. 1936-46	.500
1956-66 vs. 1946-56	.263

[14] loc cit.

observations. His conclusion was that security betas are not sensitive to either the length of the time period used or the observational interval used.[15]

While Campanella argues that betas are largely stationary over time, he calls for more work to be done in this area. He specifically says, "... (we) need more research in the area of measuring the systematic risk of individual securities. The need is to test that stationarity in individual security betas over the most recent decade and further to test for within period stationarity."[16]

One recent study which did examine the short term stationarity property of beta over time is described below.

Robert A. Levy developed a beta co-efficient for each of 500 NYSE securities in each of the ten years 1961-70.[17] He then compared the betas so developed in adjacent one year intervals to determine the predictability of $(b_i)_{t+1}$ using $(b_i)_t$ via regression analysis. The quadratic mean of the nine correlation co-efficients so obtained was .486. Levy went on to test the predictability of both thirteen week and twenty-six week betas using the immediately preceding 52 weeks as a measurement period for $(b_i)_t$. His correlation co-efficient was lower in each case. It should be noted that Levy used one week differencing intervals in his computation of the beta co-efficients.

An interesting result is obtained if the Jacob and Levy studies

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[15] F. B. Campanella, op. cit., p. 5.

[16] Ibid., p. 39.

[17] Robert A. Levy, "On the Short Term Stationarity of the Beta Co-efficient", Financial Analysts Journal, November/December 1971, pp. 55-62.

are compared. Dr. Jacob found that for each of the three years 1963, 1964, and 1965, she was able to obtain an R^2 of about .10.[18] This R^2 was obtained, recall, by regressing $(b_i)_{t+1}$ on $(b_i)_t$ for about 600 securities where $(b_i)_t$ was computed over the immediately preceding one year time period using monthly data. Levy was able to obtain an R^2 of more than .20 or twice that of Jacob by using the measurement period of one year immediately preceding the year he was attempting to predict but using weekly rather than monthly observations of return.[19] This suggests that Professor Jacob's contention regarding the inability of historical betas to predict future betas may have been premature. If predictive results can be improved 100% (.20/.10) by altering the specifications by which beta is computed, then perhaps further modification would provide even better results. Additionally, this result stands in direct contradiction to Campanella's claim that the observation interval is irrelevant to the stationarity property of beta. It would appear that additional empirical testing is desirable in order to resolve some of these controversies.

One additional point should be made here since it applies to all of the studies cited above. The experimental design utilized in all cases involved regressing the beta co-efficient of one time period against the beta co-efficient of an adjacent time period. As pointed out earlier, this test of stationarity is not identical to the test of predictability. Conceptually, beta is an indicator of a security's systematic risk i.e., its volatility vis a vis the market. For any

[18] Nancy L. F. Jacob, op. cit., p. 91.

[19] Robert A. Levy, op. cit., p. 57.

given period of time, the real variable of interest (in a prediction sense) is the volatility of the security with the market: R_i/R_m. Examining the predictability of beta as constructed through regression analysis in adjacent time periods is equivalent to examining the predictability of the predictor itself. Hence the tests have examined the predictability of the next predictor, not the predictability of volatility. Utilizing the beta co-efficient as the variable to be predicted requires an estimate of the quantity $\frac{R_i - a_i - e_i}{R_m}$. If volatility or $\frac{R_i}{R_m}$ is the variable of interest, then predicting a beta co-efficient will provide a prediction error which includes both a_i and e_i and thus differs from the variable of interest by a factor of $\frac{-(a_i + e_i)}{R_m}$ since $\frac{R_i - (a_i + e_i)}{R_m} - \frac{R_i}{R_m} = \frac{-(a_i + e_i)}{R_m}$. It may be argued that this will not be of great concern since the expected value of the error term is zero, i.e., $E(e_i) = 0.0$ and a_i will typically be near zero. However, e_i will typically not be zero as evidenced by the need to combine securities to eliminate residual risk. This suggests that to measure the predictive ability of beta an improved experimental design is required which utilizes volatility or R_i/R_m as the variable to be predicted.

Appropriate Time Interval Considered by Investors

The agreement reached by Blume and Jensen would seem to indicate that specification of the beta co-efficient is not of critical importance so long as the use of beta is limited to longer time periods and the historical evaluation of performance. For shorter time periods, predictive purposes, and individual securities, the evidence is contradictory and the results of previous studies are inconclusive. Recent evidence which indicates both the normative and positive appropriateness of these shorter time periods is discussed below.

Friend, Blume, and Crockett have found that institutional turnover of securities within portfolios has been increasing steadily over the last twenty years.[20] In the case of mutual funds they found turnover approximated 47% in 1968 and nearly 56% in the second quarter of 1969.[21] This turnover rate implies an expected average holding period for any security in the portfolio of less than one year; since 100% turnover would occur in (1/.56) 1.79 years, the expected holding period for any security would be 1.79/2 or .895 years (assuming a uniform distribution of the anticipated holding periods).

This positive evidence is further augmented in a recent article by Ralph A. Bing. Bing found that nearly 80% of the institutional investors whom he surveyed were concerned with time horizons of three years or less; 60% were concerned with time horizons of two years or less; and 40% were concerned with less than one year.[22] His sample included not only mutual funds, but insurance companies and in-house pension funds as well.

From a normative standpoint, the work done by Evans indicates that this orientation toward shorter holding periods may well have theoretical justification.[23] Evans suggests that by re-allocating

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[20]Irwin Friend, Marshall Blume and Jean Crockett, Mutual Funds and Other Institutional Investors; A New Perspective, New York, McGraw-Hill, 1970.

[21]Ibid., p. 9.

[22]Ralph A. Bing, "A Survey of Practioners' Stock Evaluation Methods," Financial Analysts Journal, May/June 1971, pp. 55-60.

[23]John L. Evans, "An Analysis of Portfolio Maintenance Strategies", Journal of Finance, June 1970.

portfolio dollars periodically, one could expect a higher return on the portfolio than by simply buying and holding. Presumably the random nature of stock price changes is responsible for this phenomenon, but for this paper that is unimportant. What is important is that from both a normative and a positive standpoint, investors consider substantially shorter holding periods than the five to ten year periods for which studies of beta stationarity (predictive) properties are conclusive.

The Gonedes Paper

Before introducing the Gonedes paper it will be instructive to point out the relationship between the prediction error associated with equation I-2 and the number of observations of R_i and R_m used to compute beta.

From equation I-2 recall,

$$(EP_i)^2 = E(R^P_{i,k} - R_{i,k})^2 = \hat{\sigma}_{e_i} \left[1 + \frac{1}{n} + \frac{(R_{m,k} - \bar{R}_{m,t})^2}{\sum_t (R_{m,t} - \bar{R}_{m,t})^2} \right]$$

Notice that the prediction error is a decreasing function of n since $\hat{\sigma}_{e_i}$ is a decreasing function of n. This inverse relationship between the prediction error and the number of observations suggests that to minimize the prediction error it is desirable to observe the relationship between return on the security and return on the market for as long a period as possible e.g., since the inception of the company. Alternatively, for some given length of time, the observations of return should be for as short a differencing interval as possible e.g., hourly.

It seems reasonable to expect that companies will change their management, marketing, and/or financial policies over time thus providing investors with different information upon which to base expectations regarding the company. If this is the case, then it seems likely that the relationship between the return on the security and the return on the market will change over time. It would also seem reasonable that returns computed from very short observational intervals would be less than meaningful and subject to random disturbances of a large magnitude.[24]

Professor Nicholas Gonedes has suggested that "structural changes" will occur to the extent necessary to offset any benefits from increasing the number of observations or R_i and R_m after about seven years.[25] The observation interval used by Gonedes was one month, hence 84 observations provided the best estimates of a_i and b_i for use in predicting R_i given R_m.

Gonedes first reserved the initial six monthly returns of each of 99 securities in 1960 and 1968 as returns to be predicted. He then constructed a set of beta co-efficients for each firm by ordinary least squares procedures over the period 1946-67. The 99 firms were selected according to the following criteria: (1) monthly price-relative data were available for the firm from the CRSP tape for the period January 1946 to June 1968 and that selected annual data were available from the CØMPUSTAT tapes for the same period; (2) the firm

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[24]Jack A. Treynor, W. W. Priest, L. Fisher, and C. A. Higgins, "Using Portfolio Composition to Estimate Risk," Financial Analysts Journal, September/October 1968, pp. 93-100.

[25]Nicholas J. Gonedes, "Evidence on the Information Content of Accounting Numbers: Accounting Based and Market Based Estimates of Systematic Risk", Journal of Finance and Quantitative Analysis (forthcoming).

was a member of a two-digit S&P industry grouping with at least fifteen member firms. The set of beta co-efficients for each firm was created by breaking down the total twenty-one year period into sub-periods. Gonedes selected the sub-periods just prior to the years of the reserved returns and divided the sub-periods into three, five, seven, ten, and twenty-one years of length. Figure II-1 shows the schematic approach utilized by Gonedes for establishing sub-periods and reserved returns.

FIGURE II-1

b(1957-59)		b(1965-67)	
b(1955-59		b(1963-67)	
b(1953-59)	Reserved Return (1960)	b(1961-67)	Reserved Return (1968)
b(1950-59)		b(1958-67)	
b(1946-67)		b(1946-67)	

The prediction error was computed as $EP_i^2 = (R_{ij}^p - R_{ij})^2$ where R_{ij}^p is the predicted one period rate of return for the ith firm at time j and R_{ij} is the actual rate of return.

The predicted return R_{ij}^p was computed by first observing the market return at time j, then using this to estimate the return on the individual security using one of the available historical betas and the relationship $R_{ij}^p = a_i+b_iR_{mj}$. Since there are six reserved returns in each of two years to be predicted and five betas to be used for predictive purposes, there are (6x2x5) 30 prediction errors to be recorded for each of the ninety-nine firms in the sample.

Cross sectional summary statistics indicated, as mentioned earlier, that the seven year period with 84 monthly observations used to estimate beta provided the minimum prediction error.

Gonedes concluded that "... for the aggregate and 'on average' the seven year observation period provides a (relatively) better set of estimates".[26]

The Observation Interval Problem

Using monthly observations of R_{it} and R_{mt} to construct the optimum beta co-efficient as Gonedes has done implicitly assumes that the one month observation interval is optimal. Jacob, however, suggests that the variability of beta, $\hat{\sigma}^2_b$ will be a function of the observation interval.[27] Note that

$$EP = \hat{\sigma}^2_\varepsilon \left[1 + \frac{1}{n} + \frac{(R_{mk} - \bar{R}_{mt})^2}{\sum_t (R_{mt} - \bar{R}_{mt})^2} \right] \qquad \text{(II-2a)}$$

$$= \hat{\sigma}_\varepsilon \frac{(n+1)}{n} + \hat{\sigma}^2_2 \left[\frac{(R_{mk} - \bar{R}_{mt})^2}{\Sigma (R_{mt} - \bar{R}_{mt})^2} \right] \qquad \text{(II-2b)}$$

but, $$\hat{\sigma}^2_\varepsilon \;/\; \sum_t (R_{mt} - \bar{R}_{mt})^2 = \hat{\sigma}^2_b \qquad \text{(II-2c)}$$

so, $$EP = \hat{\sigma}^2_\varepsilon \left(\frac{n+1}{n}\right) + \hat{\sigma}^2_b \, (R_{mk} - \bar{R}_{mt})^2 \qquad \text{(II-2d)}$$

hence the prediction error, since it is positively related to $\hat{\sigma}^2_b$ will be positively related to the observation interval. The failure of Gonedes to test alternative observation intervals suggests that he may

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[26] Nicholas J. Gonedes, op. cit., p. 14.

[27] Nancy L. F. Jacob, op. cit., p. 61.

have sub-optimized and in fact only determined the optimal observation period for the case of the one month observation interval.

It is also important to note that the squared difference between the average return on the market and any specific return on the market, $(R_{ik} - \bar{R}_{it})^2$ will be affected by both the size of R_{ik} and $\bar{R}_{it}$. From the model it can be seen that the prediction error will be a function of this difference. In fact, as $R_{ik} \rightarrow \bar{R}_{it}$, the quantity

$$\hat{\sigma}_\varepsilon \sqrt{1 + \frac{1}{n} + \frac{(R_{mk} - \bar{R}_{mt})^2}{\sum_t (R_{mt} - \bar{R}_{mt})^2}} \longrightarrow \sigma_\varepsilon \left(1+\frac{1}{n}\right) \equiv \sigma_\varepsilon + \frac{\sigma_\varepsilon}{n} \qquad \text{(II-3)}$$

Since for large n, σ_ε/n approaches 0, then the prediction error approaches σ_ε as $R_{ik} \rightarrow \bar{R}_{it}$.

Fisher and Lorie have shown that the variability of rates of return will be a function of the length of time over which the returns are observed.[28] The greater the variability of R_{mk} the less likely is R_{mk} to be near $\bar{R}_{mt}$. On the other hand, for any given length of time a larger observation interval implies a larger standard error of beta.[29] Some trade off function is clearly implied with respect to the prediction error which indicates the reduction in error from making $\bar{R}_{mt}$ consistent with R_{mk} versus the increase in prediction error resulting from fewer observations of R_{mt} used to compute $\bar{R}_{mt}$.

A final point regarding the observation interval concerns the $\hat{\sigma}_\varepsilon$ term in the prediction equation. Recall,

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[28] Lawrence Fisher and James C. Lorie, "Some Studies of Variability of Returns on Investments in Common Stock", Journal of Business, April 1970, p. 110.

[29] Nancy L. F. Jacob, op. cit., p. 61.

$$\hat{\sigma}_{\varepsilon}^{2} = \frac{1}{n-2}\Sigma(R_{it} - \alpha_{it} - b_{it}R_{mt}) \qquad \text{(II-4)}$$

In the computation of b_{it} the first term of the numerator and denominator are $(n\cdot\sum_t R_{it}R_{mt})$ and $(n\cdot\Sigma_t R_{mt}^2,)$ respectively.[30] These terms will dominate in the computation of b_{it} because of the n term. For $n\to\infty$

$$\frac{\Sigma_t R_{mt} \; \Sigma_t R_{it}}{n\Sigma_t R_{it} \; R_{mt}} \to \frac{(\Sigma_t R_{mt})^2}{n\Sigma_t R_{mt}^2} \to 0. \qquad \text{(II-5)}$$

Now if the return on the market index and the return on the individual security are perfectly positively correlated, then the product $(R_{mt})(R_{it})$ should always be positive. Specifically, $R_{mt} > 0$ implies $R_{it} > 0$ and $R_{mt} < 0$ implies $R_{it} < 0$. If, however, measurement error exists, this relationship will not hold. Professor Jacob has pointed out that the observation interval will influence this measurement error.[31] This suggests that the interval of time used to observe R_{it} and R_{mt} will also affect the prediction error through the determination of b_{it}, hence σ_ε. _A priori_, one might expect that returns measured over a short interval of time would be more vunerable to random effects than returns measured over a longer time period. The expected return for say, a one week observation interval will be very near zero. Ignoring discounting, an upper bound for this weekly return might be the Fisher and Lorie average annual return for securities of about

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[30]From equation I-1

$$b_{it} = \frac{n\Sigma_t R_{it}R_{mt} - \Sigma_t R_{mt} \; \Sigma_t R_{it}}{n\Sigma_t R_{mt}^2 - (\Sigma_t R_{mt})^2}$$

[31]Nancy L. F. Jacob, _op. cit._, p. 63.

.09 divided by 52 or .00167. Very little random disturbance could cause $R_{it} > 0$ and $R_{mt} < 0$ or vice versa. If this occurs then the computation of b_{it} would be biased downward (due to the monotonically increasing nature of $\sum_t R_{mt}^2$) relative to b_{it} computed for longer observation intervals. It may be that monthly or quarterly observations are less vunerable to small random fluctuations and in addition, would provide more opportunity for random disturbances to cancel out. But longer observation intervals imply fewer total observations for any given period of time. Some trade off function is suggested for evaluating the effect on the prediction error of increasing the length of the observation interval to reduce the random 'noise' versus the effect of reducing the number of observations.

The Confidence Interval Problem

In linear bivariate regression analysis the standard error of an estimated value of the dependent variable, x_1 given some value of the independent variable, x_2 is given by the expression[32]

$$\hat{\sigma}\,(x_1|x_2) = \hat{\sigma}_{12}\left[1 + \frac{1}{n} + \frac{(x_2 - \bar{x}_2)}{\Sigma x_2^2 - \frac{(\Sigma x_2)^2}{n}}\right] \qquad \text{(II-6)}$$

For any given value of x_2 it is possible to compute the standard error of the estimated value of x_1 where x_1 is given by $x_1 = a + bx_2 + e$. More importantly, the set of confidence limits established for x_1, given x_2, are non-linear.[33] Figure II-2 on the following page provides

[32] A. M. Mood and Franklin A. Graybill, Introduction to the Theory of Statistics, New York, McGraw-Hill, 1963, p. 336.

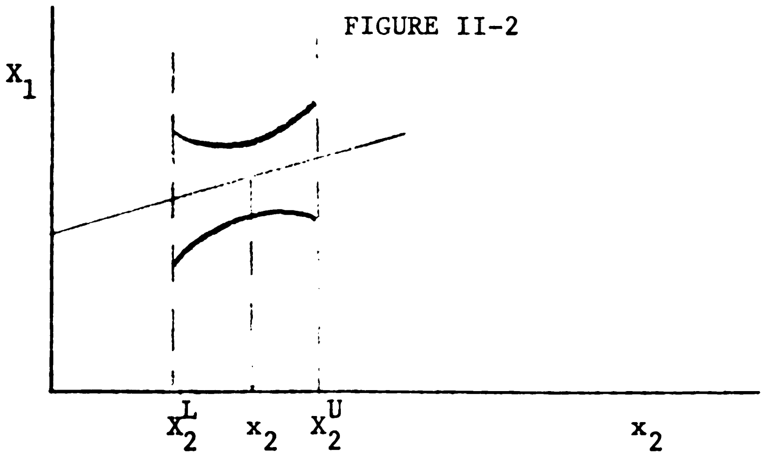
[33] Ibid., p. 337.

an illustration of a typical set of confidence limits for $X_2^L \leq x_2 \leq X_2^U$ where

X_2^L = a lower bound for the observed values of x_2

X_2^U = an upper bound for the observed values of x_2

FIGURE II-2



Note that equation II-6 is precisely the square root of the Gonedes prediction error model. What Professor Gonedes has done then is establish confidence intervals for the predicted values of return on individual securities given some return on the market. The market returns observed by Gonedes are presented in Table II-2.[34] In relation to Figure II-2 above, these returns correspond for the most part to values of x_2 near either X_2^L or X_2^U.

Averaging the prediction error (confidence intervals) across all observed values of R_{mk} for each number of observations, n will produce a mean prediction error for each n viz, $\overline{EP}(n) = \frac{1}{12} \sum_t EP_k(n)$

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[34]The returns reported here are those for the Dow Jones 65 Composite Average. These returns would be similar to the returns observed by Gonedes because of the high correlation between returns on indicies. See Sharpe, op. cit., p. 114.

TABLE II-2

Month	Annualized Monthly Observations of Return 1960	1968
January	-.716	-.413
February	.089	-.302
March	-.216	-.082
April	-.228	.871
May	.258	.222
June	.380	.216

where

$EP_k(n)$ = the total prediction error for all companies given beta co-efficients computed from n observations of return and market observation, R_{mk}.

For a larger n say $n^* = n+c$, it would be expected that $\overline{EP}(n^*) < \overline{EP}(n)$ since a larger number of observations of return will, ceterus paribus, reduce the prediction error. If the differences in individual values of $EP_k(n)$ and $EP_k(n^*)$ arise primarily in conjunction with observations of very large and very small values of R_{mk} then the difference in the mean values $\overline{EP}(n)$ and $\overline{EP}(n^*)$ need not hold for values of R_{mk} near the expected value of R_{mk}. Based upon historical evidence, the expected value of an annualized observation of R_{mk} would approximate 0.09.[35] The one observation by Gonedes of annualized R_{mk} near 0.09 (February 1960) would not seem sufficient to justify his conclusion that the seven year measurement period (n=84) is optimal for values of R_{mk}

[35] Lawrence Fisher and James H. Lorie, "Rates of Return on Investments in Common Stocks", Journal of Business, January 1964, pp. 1-17.

near 0.09. An illustration follows: suppose for $n = 24$ that R_{mk} is observed at .5, .1, and -.6 and the corresponding $EP_k(n)$ values are .8, .1, and 1.2. These observations provide a mean prediction error of $0.7=(1/3(.8+.1+1.2)=0.7)$. For $n^*=n+36$, let the observations of R_{mk} provide $EP_k(n^*)$ values of .7, .12, and 1.10. The mean prediction error would be $0.64=(1/3(.7+.12+1.1)=0.64)$. This illustration demonstrates that while $\overline{EP}(n^*) < \overline{EP}(n)$, $EP_k(n) < EP_k(n^*)$ for $R_{mk} = .10$.

Observation Periods up to Three Years

In selecting sub-periods for the determination of mean prediction error, Gonedes arbitrarily selected three, five, seven, ten, and twenty-one year periods for testing purposes. In the analysis of time series data a frequently utilized technique is that of weighting recent data more heavily than earlier data.[36] This often gives better prediction results than simply considering all data equally. This suggests that future results may be more closely related to recent events than to earlier events. The presence of weighting devices would seem to lend validity to the idea of testing sub-periods up to three years since in the prediction of monthly reserved returns (annualized), it may well be that a sub-period within the most recent three years provided the minimum prediction error. Figure II-4 on the following page shows the Gonedes assumption (line A) while lines B and C show potential alternative possibilities.

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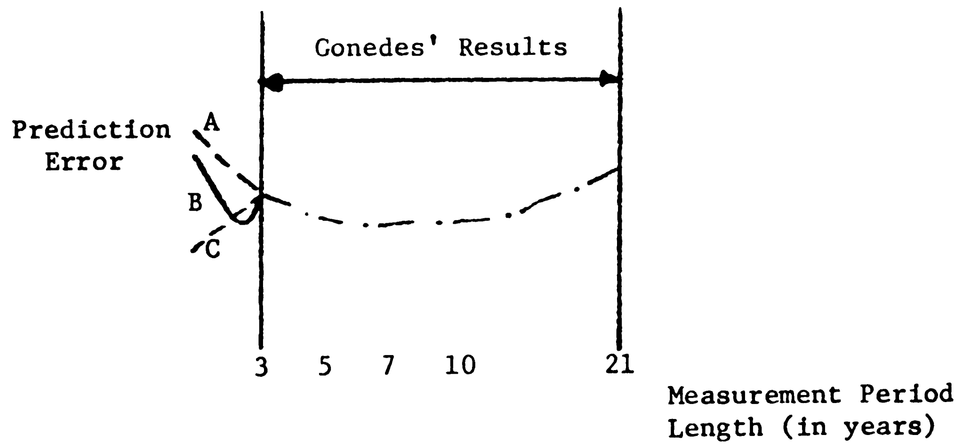
[36] Charles T. Clark and Lawrence L. Schkade, Statistical Methods for Business Decisions, South-Western Publishing Co., 1969, pp. 702-711.

Before any judgement can be exercised with respect to the appropriateness of line segments A, B, or C, further empirical work must be done in order to observe points between zero and three years.

FIGURE II-4



Summary

The number of observations of R_{it} and R_{mt}, the length of the period over which these observations are taken, and the variability of the returns will all affect the predictive ability of a security's future volatility. These items comprise the essentials of the specification problem for the risk return model.

Previous studies have been shown deficient in methodology and contradictory in results when attempting to predict the volatility of securities' returns in short time periods. It has also been shown that the market model itself <u>i.e.</u>, the ordinary least squares regression procedures for computing beta co-efficients has also recently come under attack.

What is required is a simultaneous attention to the number of observations of R_{it} and R_{mt}, the length of the period over which these observations are taken, and the variability of these returns as they affect both the market models introduced in order to consider the influences on the prediction error of the various trade-off functions which exist. Only by dropping the *ceterus paribus* assumption can the combined effect of all these factors be ascertained and consequently the optimal market model and specification of parameters of the market model be determined.

CHAPTER III

RESEARCH DESIGN

General Approach

The purpose of this research is to examine the two market models presented in Chapter I utilizing alternative parameter specifications to determine which model and which set of specifications provide the minimum prediction error when estimating the future volatility of a security based upon its historical beta co-efficient.

Unfortunately, a mathematical solution to the prediction error problem is not possible because the relationships (parameters) of the prediction error model are subject to change. Ideally, one could establish for an equation in several variables the set of partial derivatives at zero and solve the resulting series of equations simultaneously for the minimum value of the dependent variable. This minimum value of the dependent variable would then depict the optimum relationship of the independent variables. In the prediction error model, however, the question is not of establishing the optimal relationship of the independent variables, rather it is to specify the parameters of the model i.e., what are the relationships which subsequently can be optimized? Accordingly, an empirical approach will be used in order to estimate the parameters of the model and the relationship of the independent variables.

The Evaluation Model

This paper utilizes prediction error defined as in equation III-1.

$$EP_i = (b_{it} - \frac{R_{ik}}{R_{mk}}) \qquad \text{III-1}$$

where R_{ik} = the return on security i in time period k

R_{mk} = the return on the market surrogate in time period k.

The quantity R_{ik}/R_{mk} is the volatility of security i, will be defined as the systematic risk of security i, and is the measure of concern to investors. This is the relationship which will reflect the volatility of an investor's portfolio, not the regression equation of $a_{it} + b_{it}R_{mt} + e_{it}$.[1] The regression equation is the estimating device

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[1]In the prediction model utilized by Gonedes (see equation II-2a) the predicted return, R_{it} will be a function of the historical beta used in the market model. The observed return, R_{ik} will be conditional upon the market return, R_{mk} and the volatility of the security at that time (k). Implicitly this measure incorporates the slope of the regression equation at time k and as such represents an attempt to predict the future predictor rather than systematic risk only. Rewriting the equation will illustrate this point:

$$\hat{\sigma}^2_\varepsilon \quad 1 + \frac{1}{n} + \frac{(R_{mk} - \bar{R}_{mt})^2}{\sum_t (R_{mt} - \bar{R}_{mt})^2}{}^2 = (R_{mk} - R_{mt})^2\hat{\sigma}^2_b + (\frac{n+1}{n})\hat{\sigma}^2_\varepsilon \qquad \text{(III-2)}$$

$$\text{but } (R_{mk} - R_{mt})^2\hat{\sigma}^2_b + (\frac{n+1}{n})\hat{\sigma}^2_{\varepsilon i} = E\,[(\alpha_{it} - \alpha_{ik}) + (b_{it} - b_{ik})R_{mk} - \varepsilon_{ik}]^2 \qquad \text{(III-3)}$$

From equation III-3 the prediction error will be a function of the difference in the expected value of the a parameter from time t to time k, and the residual error at time k as well as the change in the beta parameter from time t to time k. If the objective is to determine the best predictor for a firm's systematic risk behavior, then the error measurement should include only the systematic risk component of the model as in equation III-1.

for volatility; R_{ik}/R_{mk} is the measure of interest. The term R_{ik}/R_{mk} will be referred to as the sensitivity of the security throughout the remainder of this paper. The quantity b_{it} is, of course the historical beta co-efficient computed from the least squares procedures of linear bivariate regression analysis.

It has been shown that average prediction error, when computed from returns to be predicted which are very different from their expected value, will not assure the model user that the beta value is optimal because the average could conceal information to the contrary. A more representative sample of returns would be generated if longer holding periods were considered and the observed returns adjusted to a basis consistent with those used in the computation of the historical beta co-efficient, b_{it}. The model used in this paper allows this flexibility since the quantity R_{ik}/R_{mk} can be observed over a period of any length desired. In addition, R_{ik}/R_{mk} need not be adjusted to a shorter time basis (weekly, monthly, quarterly) since it is a ratio and the absolute difference in return is not measured. This procedure allows a greater range of returns to be observed without the additional time and expense for adjustment to a basis consistent with the returns used in the computation of the historical beta co-efficient while at the same time providing returns nearer those an investor might expect on an *ex ante* basis *i.e.*, nearer the expected value of the market return distribution.

Computational Procedures

Two test periods were selected to observe the sensitivity measure. The first was the three year period 1/1/67 - 12/31/69, the second was the two year period 1/1/70 - 12/31/71. Allowing the first holding period to correspond to the subperiod 1/1/67 - 4/1/67, the second holding period to 1/1/67 - 7/1/67 etc., Table III-1A shows the twelve holding periods corresponding to test period one (TP1) and Table III-1B shows the eight holding periods corresponding to test period two (TP2).

A value of R_{ik}/R_{mk} was observed for each holding period, for each security. Thus there were (20x35) 700 sensitivities computed for the sample.[1]

These sensitivities were computed as in equation III-4 below.

$$\frac{P_{it+1} D_{it}}{P_{it}} - 1 \quad \frac{P_{mt+1}}{P_{mt}} - 1 + D_{mt} = \frac{R_{ik}}{R_{mk}} \qquad \text{III-4}$$

P_{it} = the price of security i at time t

P_{it+1} = the price of security i at time t+1

D_{it} = the dividends declared on security i during time t

P_{mt} = the value of the S&P 500 index at time t

P_{mt+1} = the value of the S&P 500 index at time t+1

D_{mt} = dividends (in per cent) on the S&P 500 index during time t

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[1]See subsequent section on Sample Size, p. 43

TABLE III-1

A		B	
Designation	Time Period	Designation	Time Period
HP(1)	1/1/67 - 3/31/67	HP(13)	1/1/70 - 3/31/70
HP(2)	1/1/67 - 6/30/67	HP(14)	1/1/70 - 6/30/70
HP(3)	1/1/67 - 9/30/67	HP(15)	1/1/70 - 9/30/70
HP(4)	1/1/67 - 12/31/67	HP(16)	1/1/70 - 12/31/70
HP(5)	1/1/67 - 3/31/68	HP(17)	1/1/70 - 3/31/71
HP(6)	1/1/67 - 6/30/68	HP(18)	1/1/70 - 6/30/71
HP(7)	1/1/67 - 9/30/68	HP(19)	1/1/70 - 9/30/71
HP(8)	1/1/67 - 12/31/68	HP(20)	1/1/70 - 12/31/71
HP(9)	1/1/67 - 3/31/69		
HP(10)	1/1/67 - 6/30/69		
HP(11)	1/1/67 - 9/30/69		
HP(12)	1/1/67 - 12/31/69		

The importance of using dividends (and the benefits of using longer holding periods) in the computation of sensitivity is evidenced here by examination of Table III-2 which shows the S&P 500 return for each of the twelve holding periods in TP1. Table III-2 indicates that over longer periods the dividends act as a smoothing device for returns. In addition, since dividend yield differs substantially among individual securities, ignoring dividends will bias upward the variability of returns (relative to the market) of those companies following a stable, high-payout dividend policy.

TABLE III-2

Holding Period	Return	Return + Dividends	Annualized Return + Dividend
HP(1)	13.2%	14.0%	56.0%
HP(2)	14.5	16.1	32.2
HP(3)	20.7	23.1	30.8
HP(4)	18.5	21.7	21.7
HP(5)	10.1	14.1	11.3
HP(6)	25.3	30.1	20.1
HP(7)	27.4	33.0	18.9
HP(8)	30.2	36.5	18.3
HP(9)	28.3	35.3	15.7
HP(10)	20.4	28.2	11.3
HP(11)	17.2	25.8	9.4
HP(12)	2.9	12.4	4.1

In summary, since the holding period is important for obtaining estimates of return consistent with investors' expectations and since typical holding periods may be thought of as one-three years or longer, sensitivity was computed for periods up to three years and included the return provided by dividends.

An additional advantage to this method of testing was that it provided insight into the length of the holding period required before returns were sufficiently stable for reliable estimates of sensitivity and prediction error to be obtained.

Returns used in estimating b_{it} for both TP1 and TP2 were computed over twenty-eight periods of different length utilizing weekly, monthly and quarterly observation intervals. The observation periods extended

from three months to eighty-four months and each period represented a three month increment over the previous observation period. Thus there were twenty-eight observation periods for each test period ranging from three months to seven years. Designating the first observation period corresponding to TP1 as OP(11) and the first observation period corresponding to TP2 as OP(21), Table III-3 indicates the observation periods and corresponding time interval for each test period.

A regression equation was computed for each OP(ij) using weekly, monthly, and quarterly observations of return. The particular observational interval being used was represented by its first letter. Thus $OP^{w}(ij)$ corresponded to weekly observations of return taken in TP(i) in observation period j, used in estimating the historical beta coefficient.

Weekly price observations used to compute weekly returns were taken as the closing Friday prices. When holidays or other events caused the New York Exchange to be closed on Friday, the nearest preceding day's closing price was used.

Monthly price observations were taken as every fourth weekly price. Thus in this paper there are thirteen observations of monthly return per year and 30 measurement periods over the total seven year period. The effect of obtaining thirteen rather than twelve monthly observations per year was not considered important since the focus of this paper is on the effect of lengthened observation intervals not the specific interval of month end prices for use in the computation of the historical beta co-efficients.

TABLE III-3

TP1		TP2	
Designation	Time Period	Designation	Time Period
OP(11)	10/1/66 - 12/31/66	OP(21)	10/1/69 - 12/31/69
OP(12)	7/1/66 - 12/31/66	OP(22)	7/1/69 - 12/31/69
OP(13)	4/1/66 - 12/31/66	OP(23)	4/1/69 - 12/31/69
OP(14)	1/1/66 - 12/31/66	OP(24)	1/1/69 - 12/31/69
OP(15)	10/1/65 - 12/31/66	OP(25)	10/1/68 - 12/31/69
OP(16)	7/1/65 - 12/31/66	OP(26)	7/1/68 - 12/31/69
OP(17)	4/1/65 - 12/31/66	OP(27)	4/1/68 - 12/31/69
OP(18)	1/1/65 - 12/31/66	OP(28)	1/1/68 - 12/31/69
OP(19)	10/1/64 - 12/31/66	OP(29)	10/1/67 - 12/31/69
OP(110)	7/1/64 - 12/31/66	OP(210)	7/1/67 - 12/31/69
OP(111)	4/1/64 - 12/31/66	OP(211)	4/1/67 - 12/31/69
OP(112)	1/1/64 - 12/31/66	OP(212)	1/1/67 - 12/31/69
OP(113)	10/1/63 - 12/31/66	OP(213)	10/1/66 - 12/31/69
OP(114)	7/1/63 - 12/31/66	OP(214)	7/1/66 - 12/31/69
OP(115)	4/1/63 - 12/31/66	OP(215)	4/1/66 - 12/31/69
OP(116)	1/1/63 - 12/31/66	OP(216)	1/1/66 - 12/31/69
OP(117)	10/1/62 - 12/31/66	OP(217)	10/1/65 - 12/31/69
OP(118)	7/1/62 - 12/31/66	OP(218)	7/1/65 - 12/31/69
OP(119)	4/1/62 - 12/31/66	OP(219)	4/1/65 - 12/31/69
OP(120)	1/1/62 - 12/31/66	OP(220)	1/1/65 - 12/31/69
OP(121)	10/1/61 - 12/31/66	OP(221)	10/1/64 - 12/31/69
OP(122)	7/1/61 - 12/31/66	OP(222)	7/1/64 - 12/31/69
OP(123)	4/1/61 - 12/31/66	OP(223)	4/1/64 - 12/31/69
OP(124)	1/1/61 - 12/31/66	OP(224)	1/1/64 - 12/31/69
OP(125)	10/1/61 - 12/31/66	OP(225)	10/1/63 - 12/31/69
OP(126)	7/1/61 - 12/31/66	OP(226)	7/1/63 - 12/31/69
OP(127)	4/1/61 - 12/31/66	OP(227)	4/1/63 - 12/31/69
OP(128)	1/1/61 - 12/31/66	OP(228)	1/1/63 - 12/31/69

Quarterly price observations were taken as every thirteenth weekly price observation hence, the observation interval was thirteen times the weekly interval. Again, for purposes of this study, this was considered acceptable since quarterly prices (or price changes) were not directly under study.

Returns were computed for individual securities and the market index according to equation III-4. Continuously compounded returns were not utilized for the computation of historical betas because returns for periods up to thirteen weeks were typically very small. For small returns the difference between return as computed by equation III-4 and the natural log function of the price relative is also very small (near zero). For example, a return of 3% computed by equation III-4 would provide a continuously compounded return of 2.956%, a difference of .044% or .00044. Recalling that the expected weekly return using the Fisher-Lorie results[2] would be (8.7%/52) = .167% = .00167, then all differences on weekly returns would be expected to be on the order of magnitude of (.00167/.03 = .0556) .0556 x .00044 = .00002446. Differences of this magnitude (or even thirteen times this magnitude in the case of quarterly price observations) were considered insignificant for purposes of this study.

Returns used to compute sensitivities were also taken as absolute rather than continuously compounded returns. Absolute returns were used because using continuously compounded returns for computing sensitivities while using absolute returns for computing historical

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[2]Lawrence Fisher and James H. Lorie, "Rates of Return on Investments in Common Stocks", Journal of Business, January 1964, pp. 1-17.

beta co-efficients seemed a priori unreasonable. Secondly, since sensitivities are ratios, sensitivity computed from continuously compounded returns was very similar to sensitivity computed from absolute returns. As an example, if $R_{ik} = 0.15$ and $R_{mk} = 0.20$ on an absolute basis, sensitivity would be

$$R_{ik}/R_{mk} = .15/.20 = 0.75.$$

On a continuously compounded basis, sensitivity would be

$$\frac{\ln(R_{ik} + 1.0)}{\ln(R_{mk} + 1.0)} = \frac{\ln(1.15)}{\ln(1.20)} = \frac{.14}{.18} = .77.$$

Differences of the order of magnitude of 0.02 (.77 - .75), for observations of R_{ik} and R_{mk} at 0.15 and 0.20, respectively, were not considered significant for purposes of this study. Finally, the lower sensitivities computed from absolute returns are consistently lower than those computed with continuously compounded returns. While the absolute value of the prediction error might be different, the relative difference (rankings) between individual prediction errors would be equal. Thus the same ranking for alternative specifications of the historical beta co-efficient should occur regardless of how sensitivities were computed.

Quarterly dividends for individual companies were added to price in the week, month, or quarter in which the stock went ex-dividend. For the S&P 500, annual percentage rates were observed quarterly and divided by fifty-two to estimate weekly dividend return, thirteen to estimate monthly dividend return, and four to estimate quarterly dividend return.

The S&P 500 Index was selected as the surrogate for the market.

While all indicies are highly correlated,[3] it is argued that the S&P 500 is more representative of the entire market than say the Dow Jones 65 Composite Average.[4] The S&P 500 was preferred to Moody's 300 Average because of the availability of data and to the NYSE Index because of its (NYSE's) shorter history.

An absolute prediction error defined as the absolute value of the difference between the beta coefficient and sensitivity corresponding to every combination of observation period, observation interval, and holding period was computed for each security using the above techniques. In TP1, the twenty-eight observation periods, twelve holding periods and three different observation intervals provided (3x12x28) 1008 absolute prediction errors for each company in the sample. In TP2 the three observation intervals, twenty-eight observation periods and eight holding periods provided 672 absolute prediction errors for each security in the sample. The total number of absolute prediction errors computed was ((672+1008)35)=58,800.[5]

The absolute prediction errors were averaged across all securities in the sample by observation period, by observational interval and by length of holding period. Designating the absolute prediction error across all 35 securities as $\bar{d}$, then $(\bar{d}_1)^w_{1,2}$ would correspond to the mean absolute prediction error across all 35 securities for observation period one and holding period two computed from weekly returns in TP1. Similarly,

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[3] William F. Sharpe, Portfolio Theory and Capital Markets, McGraw-Hill, 1970, p. 148.

[4] Jerome Cohen and Edward Zinbarg, Investment Analysis and Portfolio Management, Richard D. Irwin, p. 631.

[5] See the subsequent section on Sample Size.

$(\bar{d}_2)^q_{8,10}$ would correspond to the mean absolute prediction error across all 35 securities for observation period eight and holding period ten computed from quarterly returns in TP2. Table III-4 provides a schematic plan for the methodology.

TABLE III-4

$(\bar{d}_1^w)11$		$(\bar{d}_1^w)_{1,12}$	$(\bar{d}_2^w)_{11}$		$(\bar{d}_2^w)_{1,8}$
$(\bar{d}_1^m)_{11}$		$(\bar{d}_1^m)_{1,12}$	$(\bar{d}_2^m)_{11}$		$(\bar{d}_2^m)_{1,8}$
$(\bar{d}_1^q)_{11}$		$(\bar{d}_1^q)_{1,12}$	$(\bar{d}_2^q)_{11}$		$(\bar{d}_2^q)_{1,8}$
	$(\bar{d}_1^w)_{i,j}$			$(\bar{d}_2^w)_{i,j}$	
	$(\bar{d}_1^m)_{i,j}$			$(\bar{d}_2^m)_{i,j}$	
	$(\bar{d}_1^q)_{i,j}$			$(\bar{d}_2^q)_{i,j}$	
$(\bar{d}_1^w)_{28,1}$		$(\bar{d}_1^w)_{28,12}$	$(\bar{d}_2^w)28,1$		$(\bar{d}_2^w)_{28,8}$
$(\bar{d}_1^m)_{28,1}$		$(\bar{d}_1^m)_{28,12}$	$(\bar{d}_2^m)_{28,1}$		$(\bar{d}_2^m)_{28,8}$
$(\bar{d}_1^q)_{28,1}$		$(\bar{d}_1^q)_{28,12}$	$(\bar{d}_2^q)_{28,1}$		$(\bar{d}_2^q)_{28,8}$

Alternative Market Model

In Chapter I it was pointed out that Fisher and Kamin have argued for an alternate form of the market model: $R_{it} = b_{it}R_{mt}$. They have not however, specified which data should be used in the computation of the historical beta co-efficient to obtain optimal results when considering the model in a predictive sense.[7]

[7]Lawrence Fisher and Jules H. Kamin, "Good Betas and Bad Betas; How to Tell the Difference", a handout to accompany the presentation at The Meeting of the Midwest Finance Association, St. Louis, Missouri, 1972.

The same tests and methodology specified in Chapter III will be utilized in evaluating alternative specifications of the "Fisher beta". Finally, absolute prediction errors will be compared utilizing the "Fisher beta" and the "ordinary beta" to determine which provides the minimum absolute prediction error for all holding periods on average and for specific holding periods.

The Sample

Sample Size

The problem of determining sample size was essentially a statistical problem. The population was considered to be all stocks listed continuously on the New York Stock Exchange from 1960-1972. The objective was to draw from this population a random sample of sufficient size in order to make inferences about the population.

It was known that according to the index model approach to portfolio theory, any representative sample of securities will have a beta of 1.0, where $b_p = \sum_i X_i b_i$ and $X_i = 1/n$. Hence a sample of securities which would allow inferences to be made about the population should also, when combined into a portfolio, have a beta of 1.0.

As a first approximation, sixteen securities were selected at random from the population.[8] Betas were constructed for each security.[9] A portfolio beta, b_p, was then computed by the technique shown above.

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[8]See following section on Selection Procedures.

[9]The betas constructed for testing purposes were computed according to the optimal Gonedes results i.e., one month observation interval and a seven year observation period.

The standard deviation of the sample betas was also computed and after adjusting for small sample methods, taken as the population standard deviation. At this point, available information consisted of the following:

μ = population mean = 1.0

$\bar{x}$ = sample mean = 1.1304

$\sigma_{\bar{x}}$ = the sample standard deviation = .2404

S = the estimated population standard deviation
$= \sigma_{\bar{x}}/\bar{n}$ = .015

n = the number of elements in the sample = 16

The objective was to determine if μ and $\bar{x}$ were significantly different or alternatively, if the quantity $(\bar{x} - \mu)$ was significantly different from zero. The hypothesis to be tested was taken as H_o: $(\bar{x} - \mu) = 0.0$. Again, standard statistics texts demonstrate that the two-tailed T-test will provide a confidence interval for accepting this hypothesis. The confidence interval selected was .01 i.e., the results can be accepted with 99% confidence. The results indicated that the sample of sixteen was not sufficiently large to accept with 99% probability the hypothesis under consideration. The next step was to determine how large a sample was necessary.

Letting

$$S = \sigma_{\bar{x}}/\sqrt{n}$$

then

$$t_{.99} \leq \left|\frac{\bar{x} - \mu}{\sigma_{\bar{x}}/\sqrt{n}}\right|$$

and for $t_{.99} = 2.947$ a value of n can be determined which will satisfy the criteria. Here, n = 29.512. To ensure results, an additional 19 companies were selected making the total sample 35 companies.

Selection Procedures

The number of securities which appeared in the Wall Street Journal under the New York Exchange heading were counted as of the first trading day in 1960. The total number which appeared on that date was 1472. Each security was then assigned a number between 1 and 1472. Random numbers (250) between zero and one were then generated from the Michigan State University - CDC 6500 random number generator program. Multiplying each random number generated by the total number of securities available provided an adjusted random number corresponding to the number of one of the securities in the population.

Each security was then reviewed for preferred status. All preferred stocks were eliminated from consideration. The remaining common stocks were then compared to a list of the common stocks appearing in the Wall Street Journal on the first trading day of 1972. Any stocks which did not appear in this edition of the Journal were also excluded from further consideration. The remaining securities were then considered to conform to the criteria of common stocks, continuously listed and the first thirty-five chosen as the sample. Table III-5 lists the securities selected for the study.

TABLE III-5

Admiral Corporation	Getty Oil	Phelps-Dodge
Alleghany Ludlum Steel	Granby Mining	Public Service Electric & Gas
American Electric Power	Harsco Corp.	Scott Paper
American Metal Climax	International Mining	J. P. Stevens
Boeing Company	Joy Manufacturing Co.	Sunshine Mining
Borden	S. S. Kresge Co.	Tri-Continental
Continental Can	Kroehler Mfg.	TRW Inc.
Emerson Electric Co.	Lone Star Gas	U. S. Tobacco
Falstaff Brewing Co.	McAndrews & Forbes	Upjohn
Ferro Corp.	McGraw-Edison	Walgreen Inc.
General Cigar	National Can	Washington Water and Power
General Tire and Rubber	Owens-Illinois	

CHAPTER IV

ANALYSIS OF RESULTS

Ordinary Least Squares (OLS) Beta Coefficients Computed from Weekly Observations of Return

Beta coefficients computed from weekly observations of return exhibited nearly without exception a hump-backed shape over time *i.e.*, as more observations of return were included in the computation of the beta coefficient, the beta value first increased, then decreased. This would not be surprising if the value of the beta coefficient approached 1.0 as the measurement period increased for it would tend to indicate that periods of instability had cancelled over time and with enough observations of return (a sufficiently long measurement period) the security moved very much like the market. This hump-backed nature of the beta values over time, however, held even in cases where the beta value of MP(1) was near 1.0 and thus approached 1/2 or 1/4 as the measurement period lengthened.

This characteristic of the beta values over time held for both test period one (TP1) and test period two (TP2). Typically, the maximum beta value occurred between MP(5) and MP(12) indicating that the value taken on by the beta coefficient computed from weekly returns is generally increasing between zero and three years and declining thereafter. The actual number of observations of weekly returns during this interval ranged from sixty five to 156. As evidenced in column one of Table IV-1A and IV-1B, the maximum values of the beta coefficient observed between MP(5) and MP(12) typically provided the minimum mean absolute prediction error (MAPE) across securities.[1]

TABLE IV-1A

MAPE (minimum values)

TP1

Ordinary least squares beta co-efficients

Weekly	(MP)	Monthly	(MP)	Quarterly	(MP)	HP
.50891	(6)	.03320	(9)	.01874*	(3)	1
.93445	(6)	.45874	(9)	.13840*	(8)	2
.99483	(6)	.51911	(9)	.19877*	(8)	3
1.16025	(6)	.68454	(9)	.36420*	(8)	4
1.51196	(6)	1.04394	(9)	.72360*	(8)	5
1.39540	(6)	.91968	(9)	.59934*	(8)	6
1.25708	(6)	.78137	(9)	.46103*	(8)	7
1.53797	(6)	1.06225	(9)	.74191*	(8)	8
1.28177	(6)	.80605	(9)	.48571*	(8)	9
1.05982	(6)	.58411	(9)	.26377*	(8)	10
1.31285	(6)	.83714	(9)	.51680*	(8)	11
1.43040	(6)	.95468	(9)	.63434*	(8)	12

*Minimum row entry.

[1]Results shown in Tables IV-1A and IV-1B are computed from OLS beta coefficients using weekly observations of return (column 1), monthly observations of return (column 2) and quarterly observations of return (column 3). Each value corresponds to the minimum MAPE value from all twenty-eight measurement periods (i) for the holding period shown (j). Thus in column 2 for TP1 a value of .03320 was the minimum MAPE found from all 28 measurement periods when attempting to predict systematic risk in HP1 (the first 3 months subsequent to the computation of the beta coefficient) and occurred in MP(9) <u>i.e.</u>, with 27 observations of monthly return used in the historical estimate. The average MAPE values for both TP1 and TP2 are shown in Table IV-11 at the end of Chapter IV.

MAPE values for each measurement period are shown in Appendix A.

TABLE IV-1B

MAPE (minimum values)

TP2

Ordinary least squares beta co-efficients

Weekly	(MP)	Monthly	(MP)	Quarterly	(MP)	HP
.34211*	(21)	.90994	(30)	.98726	(16)	1
.08094	(1)	.06154*	(30)	.13886	(16)	2
.12355	(1)	.00377	(14)	.00123*	(25)	3
.62614*	(21)	1.19397	(30)	1.27128	(16)	4
.04049	(1)	.00086*	(23)	.01743	(16)	5
.01109*	(7)	.21877	(30)	.29609	(16)	6
.00811*	(14)	.46891	(30)	.54623	(16)	7
.01651*	(14)	.44429	(30)	.52160	(16)	8

*Minimum row entry.

In TP1, seventy-eight observations of return (MP(6)) provided beta coefficients which minimized MAPE for every one of twelve holding periods considered. In TP2 the results were not so consistent, but of the eight holding periods considered, an optimum beta fell within thirteen to 182 observations of return in seven instances. Thus in nineteen of the twenty holding periods considered in both TP1 and TP2, the minimum MAPE was produced by beta coefficients computed over a measurement period of less than three and one-half years.

a

nu

in

su

The observations made by Professor Gonedes that it is advantageous to increase the number of observations to eighty-four (seven years for monthly observations of return) seems to hold here even though eighty-four observations of weekly returns covers a period of only about 1.6 years and hence could not encompass sufficient time for structural changes to occur as he suggests.

A potential explanation of the declining value of the weekly beta coefficient after about MP(12) and the similarity of the Gonedes optimum number of observations to the weekly optimum number of observations reported here lies in a closer examination of the formula for computing beta coefficients. The regression coefficient for a particular security i can be written for computational purposes as in Equation IV-1 below. As n increases the term $n\sum_t R_{mt}R_{it}$ in the

$$b_{it} = \frac{n\sum_t R_{mt}R_{it} - \sum_t R_{it}\sum_t R_{mt}}{n\sum_t R_{mt}^2 - (\sum_t R_{mt})^2}$$

and

R_{it} = historical return during period t for security i

R_{mt} = historical return during period t for the S&P 500 Index

n = the number of historical observations of R_{it} and R_{mt}.

numerator becomes very large relative to $(\sum_t R_{mt}\sum_t R_{it})$. The same is true in the denominator, $n\sum_t R_{mt}^2$ becomes large relative to $(\sum_t R_{mt})^2$. After a sufficient number of observations, the value of the beta coefficient is largely a function of the expression: $n\sum_t R_{mt}R_{it}/n\sum R_{mt}^2$.[2] Now since R_{mt}^2 is always positive for any R_{mt} value, $\sum_t R_{mt}^2$ is a monotonically increasing function. The product $(R_{it}R_{mt})$ will not necessarily always be positive and in fact will be negative whenever R_{mt} and R_{it} are of opposite sign. Thus, the function $\sum R_{mt}R_{it}$ is not necessarily monotonically increasing.

It seems reasonable to assume that R_{mt} and R_{it} will most often be of opposite sign when index return, R_{mt}, is near zero. If so, small returns on the market index may introduce a downward bias into the regression equation viz, the monotonically increasing nature of the denominator $(n\sum R_{mt}^2)$ vs, the fluctuating positive and negative behavior of the numerator $(n\sum R_{mt}R_{it})$.[3]

To determine whether this potential bias had entered the computation of beta values computed from weekly observations of return, a runs test was performed on weekly individual security returns given both positive and negative returns on the market index for the 364 week period: 1960-1967.

A runs test is designed to determine whether the number of observations of a particular statistic are sufficiently different (in a two alternative situation) to be considered random. The statistic under consideration here was the sign of the return for an individual security given the sign of the return of the market index. If the sign of the return on the index is positive, then so should be the sign of the

[2]In the limit as $n\to\infty$, beta $= \sum R_{mt}R_{it}/\sum R_{mt}^2$.

[3]This assumes, of course, that n is sufficiently large to effectively eliminate the influence of the other term in both numerator and denominator.

return on an individual security (assuming a positive relationship). If, however, a positive return on the market index brings a sufficient number of negative returns for a particular security, then the relationship between the market returns and individual security returns is not strongly positive and the test would indicate that the return on the security is randomly positive and negative when the return on the index is positive.

The runs test was performed twice, once for all observations of weekly returns on the index such that $0.0<R_{mt}\leq.005$ and once for $-.005\leq R_{mt}<0.0$. For each observation of index return in this range the corresponding individual security return was examined for sign. Any values which the index return assumed greater than $|.005|$ were discarded and the conclusion applies only to weekly observations of index return such that $-.005\leq R_{mt}\leq.005$ and $x \neq 0.0$.[4] The information required for the test was as follows:

Z = the number of times the return on the individual security shifted from positive to negative.

n_1 = the number of times the return on the individual security was greater than zero.

n_2 = the number of times the return on the individual security was less than zero.

The results of the test indicated that thirty-two of the thirty-five companies in the sample evidenced returns which were randomly positive and negative around zero for $0.0<R_{mt}\leq.005$. When $-.005\leq R_{mt}<0.0$, thirty-one of the thirty-five companies evidenced returns which were randomly

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[4]A weekly index return such that $-.005<x\leq.005$ corresponds to an approximate annual return range of ±26%.

positive and negative around zero. Only one company, Washington Water and Power, exhibited a non-random movement with the market in both tests i.e., index return greater than zero and less than zero.

This lack of strong positive correlation for index returns of small magnitude combined with the declining values of the beta coefficients observed after about MP(12) would indicate that observing more than 156 weekly returns is of very little value and in fact may actually impair the calculation of an accurate historical estimate of a security's beta value or systematic risk component. These results also suggest that any optimum measurement period or number of observations of return used to compute historical beta coefficients are not necessarily due to changes in the nature of the company i.e., structural changes, but simply due to the lack of a strong relationship between the price of the individual companies and the market when the market exhibits small price movements.

The fact that <u>different</u> companies, Washington Water and Power notwithstanding, had non-random movement with the market when market return was greater and less than zero may indicate that the degree of responsiveness of individual companies differs as a result of a positive or negative return from the market i.e., the beta coefficient for a particular security may be different during "up" markets than during "down" markets.

<u>Ordinary least squares (OLS) beta coefficients computed from monthly observations of return</u>

Beta coefficients computed from monthly observations of return were less consistent in their behavior over time than beta coefficients computed from weekly observations of return and did not exhibit the humpbacked nature of the latter. Less consistency might be expected from monthly returns used in the computation of beta coefficients since estimates of the beta value in early measurement periods contain very few observations of return. In several instances in this study companies had a beta value not significantly different from zero (5% level) in early measurement periods.

Most of the beta coefficients computed from monthly returns tended toward 1.0 as the measurement period lengthened. This is evidenced by the range across all thirty-five securities of beta values in MP(1) of (-1.495 to 5.514) while in MP(30) the range had diminished to (.4085 to 1.650). Thus companies which exhibited a low beta value in early measurement periods tended to exhibit a higher beta value in later measurement periods and vice versa.

Beta coefficients computed from eighty-four observations of monthly returns tended to provide the minimum MAPE in TP2 while beta coefficients computed from twenty-seven observations of return provided the minimum MAPE in TP1 (see column 2 in Table IV-1A and IV-1B). This disparity in the optimum number of observations of monthly return serves to emphasize that the validity of Professor Gonedes results need not hold in time periods other than that for which his study was conducted. In addition, TP1 in this study conforms quite closely to the Gonedes' test period of 1959-1965. The optimum measurement period in Gonedes test period was MP(30) or eighty-four observations of return while in this paper the optimum number of observations was twenty-seven or MP(9). It may be important that Gonedes minimum number of observations tested was thirty-six or MP(12).

This disparity in optimum measurement period lengths between TP1 and TP2 is substantial: two years vs seven years. It was first thought that perhaps the optimum measurement period was in some way associated with the behavior of the index during the different holding periods. An examination of the relationship between the index and the mean prediction error proved of little value.

Ordinary least squares (OLS) beta coefficients computed from quarterly observations of return

Beta coefficients computed from quarterly observations of return were typically not significantly different from zero (5% level) until MP(12) or MP(13). Thus of the twenty-eight beta coefficients computed for each security, only about one-half were useful, and it was necessary to go back in time nearly three years before significance was generally obtained. Once significance was obtained, however, there was less tendency for the beta value to change as additional returns were considered. Any changes were, as in the monthly case, typically changes toward 1.0.

In TP1, optimum beta coefficients computed from quarterly observations of return came from MP(8) or eight observations of quarterly returns. This two year period also proved optimum for beta coefficients computed from weekly and monthly observations of return even though the number of observations of return was different in each case. In TP2, the optimal measurement period was MP(16) or four years of quarterly observations.

In general it appears that with the exception of beta coefficients computed from monthly returns in TP2, the measurement period which provided the minimum MAPE was typically less than five years and in TP1, closer to two years. As Table IV-1A and IV-1B indicate, however, the measurement period which will produce the minimum MAPE is _a priori_, extremely difficult to know.

A comparison of weekly, monthly, and quarterly observation intervals

A comparison of weekly, monthly, and quarterly observation intervals and their influence on MAPE reveals conflicting results. Table IV-1A and IV-1B show that in TP1 beta coefficients computed from quarterly observations of return consistently provided a smaller minimum MAPE than either the weekly or monthly observation intervals. In TP2, however, nearly the opposite is true. For five of the eight holding periods, weekly observations of return provided the smallest MAPE while quarterly observations of return provided only one. The absence of consistency is similar to the results observed with the measurement period length. Also worth noting is the fact that optimum beta coefficients in TP1, (quarterly observations of return) were found for the most part in MP(8) and with only eight observations of return available for the computation, many of the beta coefficients were not significantly different from zero (5% level). Given these results, it was felt that perhaps some artificial estimate of the beta coefficients might provide equally good predictive estimates without the necessity of many computations or the purchasing of the necessary historical information. Accordingly, beta coefficients were allowed to assume values of 0.0, 0.5, 1.0, 1.5, and 2.0 for each security and all measurement periods, and the MAPE for each holding period computed. Table IV-3 shows the results for TP1 and Table IV-4 shows the results for TP2.

Tables IV-3 and IV-4 are comparable to Tables IV-1A and IV-1B respectively. Notice that artificial beta values of 2.0 in TP1 and 0.5 in TP2 provided the minimum MAPE across all holding periods and that these particular artificial beta values provided MAPE values similar

TABLE IV-3

MAPE FOR ARTIFICIAL BETA VALUES IN TP1

Beta = 0.0	Beta = 0.5	Beta = 1.0	Beta = 1.5	Beta = 2.0	HP
1.17045	.67045	.17045	.32945	.82954	1
1.59599	1.09600	.59600	.09600	.40400	2
1.65637	1.15637	.64637	.15637	.34363	3
1.82179	1.32180	.82180	.32180	.17820	4
2.18119	1.62180	1.12180	.62180	.18120	5
2.05694	1.55694	1.05694	.55694	.05694	6
1.91862	1.41862	.91862	.41862	.08137	7
2.19951	1.69951	1.19951	.69951	.19951	8
1.94331	1.44331	.94331	.44331	.05669	9
1.72136	1.22137	.72137	.22137	.27863	10
1.97439	1.47440	.97440	.47440	.02560	11
2.09194	1.59194	1.09194	.59194	.09194	12
μ = 1.86098	1.36095	.76103	.41592	.22727	

TABLE IV-4

MAPE FOR ARTIFICIAL BETA VALUES IN TP2

Beta = 0.0	Beta = 0.5	Beta = 1.0	Beta = 1.5	Beta = 2.0	HP
.10874	.39126	.89126	1.39125	1.89125	1
.95714	.45714	.04286	.54285	1.04285	2
1.16162	.66163	.16163	.33837	.83837	3
.17529	.67529	1.17528	1.67528	2.17528	4
1.07857	.57857	.07857	.42143	.92143	5
.79991	.29991	.20009	.70008	1.20008	6
.54977	.04977	.45023	.95023	1.45023	7
.57440	.07440	.42560	.92560	1.42560	8
μ = .67568	.39850	.42819	.80028	1.36811	

to the minimum row values in Tables IV-1A and IV-1B.

The weekly, monthly and quarterly MAPE results were averaged separately across all holding periods and these figures are shown in Table IV-5.

Comparing the minimum MAPE across all holding periods with the MAPE from arbitrarily setting the beta coefficient equal to 1.0 produces some surprising results. In TP1 setting beta equal to 1.0 provides an overall MAPE for all holding periods of .76103 while the minimum MAPE for beta coefficients computed from weekly returns was 1.19945, from monthly returns was .72373, and quarterly returns was .45125. In TP2, an arbitrary beta value of 1.0 produced an overall MAPE of .42819 while the minimum MAPE for weekly returns was .36745, for monthly returns was .43753, and for quarterly returns was .48055. The results of combining TP1 and TP2 are also shown in Table IV-5 on the following page. The results indicate that if an investor could ex ante determine which of the differencing intervals and measurement periods would provide the minimum MAPE he could obtain superior results relative to arbitrarily selecting a beta coefficient of 1.0. If he is unable to ex ante determine which combination of differencing interval and measurement period will provide the minimum MAPE consistently using a particular observational interval may provide inferior results to an arbitrary selection of the beta value as 1.0.

TABLE IV-5

A COMPARISON OF OLS BETA COEFFICIENT VALUES AND AN ARBITRARY BETA VALUE OF 1.0 FOR ALL HOLDING PERIODS

	TP1	TP2	(TP1+TP2)/2
W	1.19945	.36745	.78798
M	.72373	.43753	.64274
Q	.45125	.48055	.54529
Beta = 1.0	.76103	.42819	.59461

Implications for Portfolio Construction

Portfolio theory suggests that the investor should purchase the market (or some reasonable facsimile thereof) and borrow or lend to obtain his desired risk exposure. If the investor draws a random selection of securities from the market sufficiently large (about twenty securities) the results of the preceding section do not apply.

In practice, however, investors still appear to evaluate and rank securities in an attempt to out-perform the market.[5] For individuals who are attempting to construct a portfolio of securities which exhibits a beta value near 1.0 by the inclusion of some high and low beta securities, the results may be important. They suggest that the ranking of securities might most efficiently be carried out (in a cost-benefit sense) by simply assuming the systematic risk component of each security to be 1.0, hence, ranking securities on an expected return basis.

[5]Chris A. Welles, "The Beta Revolution: Learning to Live with Risk", The Institutional Investor, Vol. V, No. 9.

For individuals who desire a portfolio which exhibits a beta value dissimilar to 1.0, the results, again, need not apply. The sample under consideration here was random and hence has no implications for groups of securities which all exhibit a high or low historical beta value. These portfolios may exhibit a beta value which is sytematically higher or lower than predicted, whereas the sample here consisted of a random selection of securities and presumably errors which were randomly positive and negative.

The effect of Holding Period length on Minimum MAPE

Increasing the holding period length tended to decrease the minimum MAPE in TP2 and increase the minimum MAPE in TP1. Combining the minimum MAPE for TP1 and TP2 by holding period indicates that overall the error tends to increase as the holding period increases. All three observation intervals provided essentially the same results, however, weekly observations of return were more consistent in exhibiting a higher prediction error as the holding period lengthened than prediction error from monthly observations of return which in turn were more consistent than prediction errors from quarterly observations of return.

Apparently, in general, the longer one allows the period to be predicted extend, the more difficult it becomes to obtain good predictions. The idea that the security's behavior will be easier to predict if a sufficiently long term interval is examined i.e., an opportunity for "normal" behavior to occur is allowed, does not appear valid when predicting an individual security's responsiveness with the market for periods up to three years. For periods longer than three years this statement may not be valid.

The conclusions must be qualified by the fact that in the two test periods examined only one indicated a higher prediction error corresponding to increased holding period length. The higher prediction error associated with increased holding period length experienced in TP1 was, however, sufficiently large to influence the combined results of TP1 and TP2.

The Fisher-Kamin Beta

The characteristics of the Fisher-Kamin (F-K) beta coefficient are nearly identical to the ordinary least squares beta coefficients and results of the prediction tests are listed in Table IV-6A and IV-6B.

A Comparison of F-K and Ordinary Least Squares (OLS) beta Coefficients

For each test period and holding period, the minimum MAPE was taken for both OLS and F-K beta coefficients. The summary of results are shown in Tables IV-7 through IV-9. The tables show that in four of the six separate cases (two test periods and three observation intervals) OLS beta coefficients provided a smaller MAPE than the F-K beta coefficients. Only the F-K beta coefficients computed from quarterly observations of return in TP1 provided a smaller MAPE (see Table IV-9B).

MAPE computed from OLS beta coefficients using monthly observations of return were consistently lower in both TP1 and TP2 than the F-K beta coefficients. This appears to be in direct contradiction to the Fisher-Kamin results cited in Chapter I. There are two potential explanations for this event. First, Fisher-Kamin used a five year measurement period to compute beta coefficients. This measurement

TABLE IV-6A

MAPE (minimum values)

TP1

F-K beta coefficients

Weekly	(MP)	Monthly	(MP)	Quarterly	(MP)	HP
.50891	(6)	.08169	(4)	.00926*	(3)	1
.93445	(6)	.50723	(4)	.10029*	(10)	2
.99483	(6)	.56760	(4)	.16066*	(10)	3
1.16025	(6)	.73302	(4)	.32609*	(10)	4
1.51965	(6)	1.09242	(4)	.68548*	(10)	5
1.39540	(6)	.96817	(4)	.56123*	(10)	6
1.25708	(6)	.82985	(4)	.42291*	(10)	7
1.53797	(6)	1.11074	(4)	.70380*	(10)	8
1.28177	(6)	.85454	(4)	.44760*	(10)	9
1.05982	(6)	.63260	(4)	.22566*	(10)	10
1.31285	(6)	.88563	(4)	.47868*	(10)	11
1.43040	(6)	1.00317	(4)	.59623*	(10)	12

*Minimum row entry.

The results for each measurement period appear in Appendix B.

TABLE IV-6B

MAPE (minimum values)

TP2

F-K beta coefficients

Weekly	(MP)	Monthly	(MP)	Quarterly	(MP)	HP
.34534*	(21)	.92800	(30)	.99120	(16)	1
.06343*	(1)	.07960	(30)	.14280	(16)	2
.14106	(1)	.00748	(16)	.00440	(18)	3
.62937*	(21)	1.21202	(30)	1.27523	(16)	4
.05800	(1)	.00151*	(24)	.02137	(16)	5
.00703*	(12)	.23683	(30)	.30003	(16)	6
.00971*	(14)	.48697	(30)	.55017	(16)	7
.01491*	(14)	.46234	(30)	.52554	(16)	8

*Minimum row entry.

The results for each measurement period appear in Appendix B.

TABLE IV-7A

MAPE (weekly observations)

TP1**

F-K and OLS beta coefficients compared

Ordinary least squares	(MP)	Fisher	(MP)	HP
.50891	(6)	.50891	(6)	1
.93445	(6)	.93445	(6)	2
.99483	(6)	.99483	(6)	3
1.16025	(6)	1.16025	(6)	4
1.51965	(6)	1.51965	(6)	5
1.39540	(6)	1.39540	(6)	6
1.25708	(6)	1.25708	(6)	7
1.53797	(6)	1.53797	(6)	8
1.28177	(6)	1.28177	(6)	9
1.05982	(6)	1.05982	(6)	10
1.31285	(6)	1.31285	(6)	11
1.43040	(6)	1.43040	(6)	12

**No minimum in TP1; both ordinary least squares and Fisher betas are equal.

TABLE IV-7B

MAPE (weekly observations)

TP2

F-K and OLS beta coefficients compared

Ordinary least squares	(MP)	Fisher	(MP)	HP
.34211*	(21)	.34534	(21)	1
.08094	(1)	.06343*	(1)	2
.12355*	(1)	.14106	(1)	3
.62614*	(21)	.62937	(21)	4
.04049*	(1)	.05800	(1)	5
.01109	(7)	.00703*	(12)	6
.00811*	(14)	.00971	(14)	7
.01651	(14)	.01491*	(14)	8

*Minimum row entry.

TABLE IV-8A

MAPE (monthly observations)

TP1

F-K and OLS beta coefficients compared

Ordinary least squares	(MP)	Fisher	(MP)	HP
.03320*	(9)	.08169	(5)	1
.45874*	(9)	.50723	(4)	2
.51911*	(9)	.56760	(4)	3
.68454*	(9)	.73302	(4)	4
1.04394*	(9)	1.09242	(4)	5
.91968*	(9)	.96817	(4)	6
.78137*	(9)	.82985	(4)	7
1.06225*	(9)	1.11074	(4)	8
.80605*	(9)	.85454	(4)	9
.58411*	(9)	.63260	(4)	10
.83714*	(9)	.88563	(4)	11
.95468*	(9)	1.00317	(4)	12

TABLE IV-8B

MAPE (monthly observations)

TP2

F-K and OLS beta coefficients compared

Ordinary least Squares	(MP)	Fisher	(MP)	HP
.90994*	(30)	.92800	(30)	1
.06154*	(30)	.07960	(30)	2
.00377	(14)	.01691	(14)	3
1.19397*	(30)	1.21202	(30)	4
.00086*	(23)	.00151	(24)	5
.21877*	(30)	.23683	(30	6
.46891*	(30	.48697	(30)	7
.44429*	(30)	.46234	(30)	8

TABLE IV-9A

MAPE (quarterly observations)

TP1

F-K and OLS beta coefficients compared

Ordinary least squares	(MP)	Fisher	(MP)	HP
.01874	(3)	.00926*	(3)	1
.13840	(8)	.10029*	(10)	2
.19877	(8)	.16066*	(10)	3
.36420	(8)	.32609*	(10)	4
.72360	(8)	.68548*	(10)	5
.59934	(8)	.56123*	(10)	6
.46103	(8)	.42291*	(10)	7
.74191	(8)	.70380*	(10)	8
.48571	(8)	.44760*	(10)	9
.26377	(8)	.22566*	(10)	10
.51680	(8)	.47868*	(10)	11
.63434	(8)	.59623*	(10)	12

TABLE IV-9B

MAPE (Quarterly observations)

TP2

F-K and OLS beta coefficients compared

Ordinary least squares	(MP)	Fisher	(MP)	HP
.98726*	(16)	.99120	(16)	1
.13886	(16)	.14280	(16)	2
.00123*	(25)	.00440	(18)	3
1.27128*	(16)	1.27523	(16)	4
.01743*	(16)	.02137	(16)	5
.29609*	(16)	.30003	(16)	6
.54623*	(16)	.55017	(16)	7
.52160*	(16)	.52554	(16)	8

period and the sixty monthly observations of return could have been optimal for the F-K beta but not the OLS beta; this would provide superior prediction results for the F-K beta. A second possibility is that neither the F-K nor the OLS beta were optimal with sixty monthly observations. Under this assumption, the difference between the five year measurement period beta and the optimal beta value is important. But this is equivalent to examining the variance between the estimates of the beta coefficients computed from the twenty-eight different measurement periods. If less variability exists between estimates of the beta coefficient computed according to the Fisher-Kamin criteria then it is possible that the F-K nonoptimum beta estimate utilizing sixty observations of return would be closer to the optimum estimate than the OLS beta estimate even though the OLS beta is a better estimate of the security's behavior with the market.

Accordingly, the variance of the different beta estimates was computed for each of the holding periods and averaged together for all measurement periods for both Fisher-Kamin and OLS beta coefficients. The results are shown in Table IV-10. In both TP1 and TP2, the F-K

TABLE IV-10

VARIANCE OF BETA ESTIMATES

	TP1		TP2	
	OLS	F-K	OLS	F-K
W	.09843	.06730	.07331	.07308
M	.07620	.07620	.15643	.14480
Q	.34900	.32700	7.32551	.36143

beta provided a minimum mean error greater than the corresponding OLS beta but with less variance between estimates. This suggests a situation in which the relationship $R_i = a_i + b_i R_m$ describes more accurately a security's behavior with the market than $R_i = b_i R_m$ but produces worse estimates of the security's future behavior. Alternatively, it may be said that OLS beta coefficients provide better estimates of a security's behavior with the market (smaller minimum prediction error) but F-K beta coefficients are more useful in a prediction sense because there exists less variance in the estimates of the beta coefficients and from the results of the previous section, it is extremely difficult, *a priori*, to select that measurement which will provide the optimum estimate of the beta coefficient for prediction purposes.

TABLE IV-11

AVERAGE MAPE FOR TP1 AND TP2 BY HOLDING PERIOD*
OLS BETA COEFFICIENTS

W	M	Q	HP
.42551	.47517	.50300	1
.50720	.26014	.13863	2
.55919	.26144	.10000	3
.89319	.93925	.81774	4
.71794	.46027	.37052	5
.63408	.56922	.44771	6
.63259	.62514	.50363	7
.77724	.75327	.63176	8
1.28177	.80605	.48571	9
1.05982	.58411	.26377	10
1.31285	.83714	.51680	11
1.43040	.95468	.63434	12

*Minimum prediction errors for holding periods 9 through 12 are TP1 only.

CHAPTER V

SUMMARY AND CONCLUSIONS

The purpose of the present research has been to accurately specify the market model for use in making predictions about an individual security's systematic risk behavior. The specification problem was considered in two areas: the market model, and the parameters of the market model. Specification of the market model was limited to the choice of two available alternatives, (1) $R_{it} = a_{it} + b_{it}R_{mt}$ which is called the ordinary least squares procedure and (2) $R_{it} = b_{it}R_{mt}$ which is called the Fisher-Kamin procedure. The parameters of the market model to be specified included the measurement period, the observation interval, and the length of the holding period. The proper specification of the market model and the parameters of the model are necessary if investors are to accurately rank securities by means of the programming model introduced by Professor Sharpe and presented in Chapter I.

Specification of the Market Model

As was pointed out in Chapter I, the OLS beta coefficient will generate less prediction error than the Fisher beta coefficient for any given combination of measurement period, holding period, and observation interval when:

$$\bar{R}_{mt} > \frac{R_{mt}(b_{it}+b^{F}_{it})}{R_{it}} - \frac{b_{it}R_{it}}{n} \qquad \text{V-1}$$

and

$$\bar{R}_{mt} > \frac{n+1}{2n} \left[\frac{(R_{mt}-R_{mt})^2}{R_{mk}} \right] \qquad \text{V-2}$$

In general these two conditions appear to have been met since the OLS beta coefficients produced lower mean absolute prediction error (MAPE) values for most measurement periods and observation intervals. In only one instance, MAPE computed with quarterly observations of return in test period two (TP2), did Fisher beta coefficients consistently provide lower MAPE values than the OLS beta coefficients.

Table V-1 indicates that the returns observed on the S&P 500 Index were sufficiently similar in TP1 and TP2 to be unable to explain why Fisher beta coefficients produced less prediction error in TP2 but not TP1. That the Fisher beta coefficient provided less prediction error using quarterly observations of return rather than monthly or weekly observations is not surprising. Condition two (equation V-2) is influenced to a large degree by the number of observations of return, (n) hence fewer returns associated with the quarterly observation interval tends to prohibit condition (2) from being satisfied. When n=1, the expression ((n+1)/2n) is maximized with a value of 1.0. As n increases, ((n+1)/2n) approaches 1/2, its minimum value. Fewer observations of return would quite naturally tend to maintain ((n+1)/2n) near its maximum value, thereby decreasing the probability that condition (2) will be satisfied.

A second factor which will influence the prediction errors associated with the two models is the sensitivity of both the average historical return of the market index and the individual security beta values to changes in the observation interval. One might expect average

TABLE V-1

RETURNS FOR S & P 500 CORRESPONDING TO MEASUREMENT PERIODS IN TP1 & TP2

MP	TP1	TP2
1	.05600	-.00500
2	-.09900	-.04900
3	-.04070	-.04500
4	-.01760	.02300
5	.03640	.02500
6	.06340	.04400
7	-.00840	-.12400
8	.03570	-.07000
9	.00690	-.00100
10	.04120	.08700
11	.03600	.01300
12	.07110	.13000
13	.03940	.05600
14	.04730	-.09900
15	.04980	-.04100
16	.06530	-.01800
17	.12700	.03600
18	.03630	.06300
19	-.20350	-.00800
20	-.02110	.03600
21	.08390	.00700
22	.03910	.04100
23	-.00110	.03600
24	.12810	.07100
25	.11120	.03900
26	-.06660	.04700
27	.03720	.05000
28	-.05810	.06500

historical returns of the market to become proportionately smaller as the observation interval is reduced. Weekly returns would be expected to be about one-thirteenth of quarterly returns and one-fourth of monthly returns. Beta coefficients although biased downward by the use of weekly data, are not so sensitive to changes in the observation interval because beta coefficients are ratios and ratios express one variable in terms of another. As a result, decreasing the length of the observation interval reduces nearly proportionately the left hand side of the inequality, $\bar{R}_{mt}$ of condition (1). As $\bar{R}_{mt}$ decreases, b_{it} and b^F_{it} decrease to a lesser extent thus the probability that $\bar{R}_{mt}$ will exceed the quantity $\frac{R_{mt}\ (b_{it}+b^F_{it})}{R_{it}} - \frac{b_{it}}{n} R_{it}$ also decreases.

The ability of the OLS beta coefficient to produce in most instances smaller MAPE values does not mean that the OLS formulation is most useful in a prediction sense. The wider variability of OLS beta estimates among the various measurement periods coupled with the difficulty in <u>a priori</u> selecting the optimal measurement period suggests that the Fisher beta may be more useful in making predictions.

Holding Period Length

The empirical evidence indicated that for periods ranging from three months to three years, the prediction error increased as the holding period length increased (Table IV-11). Contrast these results with the results of Blume who, when comparing beta coefficients in adjacent periods of seven years in length, found a high degree of predictability. These results suggest that prediction error may, over a lengthening holding period, exhibit a hump-backed shape i.e., first increasing, then decreasing. In order for the results of this study and Blume's study to be consistent, it appears that sometime after three years and before seven years, the prediction error turns downward and the effect of lengthening the holding period on prediction error becomes favorable. Unfortunately, investors seem to anticipate a holding period of one to three years in length, in most instances. This holding period is precisely that interval when the effect of the holding period on prediction error is most unfavorable. It would appear that in order to minimize the prediction error arising from the use of beta coefficients, one must either (a) pick those securities which can comfortably be held for at least three years or (b) continuously revise and update the historical beta coefficients for all securities under observation.

Measurement Period and Observation Interval

There existed no one best combination of measurement period length and observation interval which consistently provided maximum MAPE (Tables IV-1A and IV-1B), rather, the optimum combiantion seemed to change between the two test periods. The difficulty in *a priori* selecting a combination which will provide an acceptable prediction error may suggest the use of an arbitrary value of 1.0 assigned to each security for the investor who wishes to assemble a portfolio such that the portfolio beta approaches 1.0.

It would appear from the results obtained that both weekly and monthly observations of return are acceptable if no more than 75 - 100 observations of return are collected for each beta estimate. The absence of benefits from collecting weekly returns past 100 suggests that the "structural changes" hypothesized by Professor Gonedes are in reality a tendency of the individual security to exhibit random positive and negative price movement for small changes in the value of the market index. Thus the lack of a sufficiently strong relationship which exists for most securities when the market exhibits a low absolute value return is sufficient to offset any benefits of increasing the number of observations used to compute the beta coefficient.

Quarterly observations of return used to compute beta coefficients did not provide, in all instances, a sufficient number of observations over seven years to consistently insure the statistical significance of the beta estimate. It may be possible that over a longer measurement period a sufficient number of observations could be obtained which combined with the tendency of quarterly returns of individual securities to exhibit more reliable co-movement with the market (less random fluctuation) would provide a beta coefficient which produces consistently less prediction error than beta coefficients computed from either weekly or monthly observations of return. This would appear to be a promising area for future research.

At this stage it seems fair to say that the beta coefficient is an average relationship between the behavior of an individual security and the market of all securities. An average necessarily aggregates behavior and thus may hide important deviations viz, the potential dual response of a security in response to upward and downward movements in the index.

The size of the prediction errors obtained suggests that making the assumption that a particular historical relationship between an individual security and the market will continue to hold in the future is a risky business. The size of the prediction errors obtained also suggests that highly accurate estimates of a security's systematic risk behavior are extremely difficult to obtain. Given this difficulty it would appear that the programming model proposed by Sharpe which utilizes as an input the future systematic risk behavior of individual securities is of only limited usefulness for ranking securities in the real world. Perhaps, if ranking must be done, fundamental analysis should be utilized. Both techniques require estimates to be made; the programming technique requires estimates of return and systematic risk; fundamental analysis requires estimates of earnings, dividends, etc. If more reliable estimates of fundamental factors can be made than can be made for return and systematic risk then there may be justification in utilizing a fundamental rather than a programming approach.

Alternatively it appears that some combination of the programming approach and the fundamental approach may be useful. If those fundamental factors which determine systematic risk can be identified, then it may be possible to predict systematic risk more accurately from estimates of better understood fundamental factors. Some research has already appeared with this thrust.[1]

Limitations and Suggestions for Further Research

Perhaps the single largest and most important limitation to any empirical study is that data is gathered from a specific time interval and any results generated are limited in applicability to that time

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[1]See for example, William J. Breen and Eugene M. Lerner, "Corporate Financial Strategies and Market Measures of Risk and Return," Journal of Finance, May 1973, pp. 339-351.

interval. It may be useful, however, to determine empirically precisely that fact: results are in fact limited to a specific time interval and will not hold in general. Witness for example the results in TP1 of this research vs. the results of TP2. An optimum measurement period was found for each observation interval to be between two and three years in TP1 while in TP2 the results were more variable but nevertheless tended toward a longer measurement period. This inconsistency seems important for individuals attempting to use estimates of systematic risk generated from historical information to construct portfolios.

A second limitation is the blue chip bias inherent in requiring firms in the sample to be continuously listed on the NYSE during the period 1960-1972. Many firms have joined the NYSE since 1960 but were not considered in this study because data for these firms was not readily available or because of the impact on the market model which listing itself may contain.

Finally, the portfolio systematic risk component of all thirty-five securities in combination was very near 1.0. This does not allow generalization of the results to securities which in combination exhibit a systematic risk component very much different from 1.0. As mentioned earlier, these securities may exhibit a systematic prediction error which invalidates the results found here. This attribute of the sample does allow future research to be conducted which uses as a sample securities that exhibit either a very high or a very low historic beta value when combined.

Other suggestions for future research might include Runs Test for observation intervals other than one week for values of the S&P 500 index different from $|0.005|$. A repeated testing procedure might provide the researcher with information as to the size of the return of the index required to provide a reliable response in the price of the individual security. This might suggest an alteration of the methodology associated with the computation of the historic beta coefficient. In particular, it might suggest eliminating all observations of return for an individual security given that the return on the market index was less than some predetermined absolute value.

APPENDICES

APPENDIX A-1

Mean Absolute Prediction Error for each Measurement Period Computed Using Ordinary Least Squares Beta Coefficients in Test Period 1.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 1

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	0.62288	0.47700	
2	0.59546	0.23797	
3	0.54220	0.16597	0.01874*
4	0.53057	0.12900	0.02751
5	0.51583	0.08311	0.12720
6	0.50891*	0.07609	0.21317
7	0.52217	0.03674	0.23511
8	0.51528	0.04346	0.28714
9	0.67168	0.03320*	0.28034
10	0.67986	0.04371	0.27297
11	0.70445	0.07831	0.25177
12	0.69937	0.09603	0.20603
13	0.72148	0.11660	0.16171
14	0.72417	0.12891	0.11100
15	0.72654	0.14066	0.08160
16	0.72002	0.13640	0.03614
17	0.68968	0.14860	0.13720
18	0.67917	0.15857	0.14243
19	0.70165	0.14809	0.13277
20	0.70154	0.14857	0.13709
21	0.69454	0.10943	0.14460
22	0.70740	0.13671	0.14563
23	0.70728	0.12494	0.14811
24	0.70317	0.16380	0.11086
25	0.70228	0.15943	0.16011
26	0.69425	0.14014	0.17060
27	0.70768	0.15657	0.17277
28	0.70651	0.15037	0.17214
29		0.16957	
30		0.16923	

*Minimum column element. This value appears in Table IV-1A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 2

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	1.04843	0.90254	
2	1.02100	0.66351	
3	0.96774	0.59151	0.44429
4	0.95611	0.55454	0.39803
5	0.94137	0.50866	0.29834
6	0.93445*	0.50163	0.21237
7	0.94771	0.46229	0.19043
8	0.94083	0.46900	0.13840*
9	1.09722	0.45874*	0.14520
10	1.10540	0.46926	0.15257
11	1.13000	0.50386	0.17377
12	1.12491	0.52157	0.21951
13	1.14702	0.54214	0.26383
14	1.14971	0.55446	0.31454
15	1.15208	0.56620	0.34394
16	1.14557	0.56194	0.38940
17	1.11522	0.57414	0.56274
18	1.10471	0.58411	0.56797
19	1.12720	0.57363	0.55831
20	1.12708	0.57411	0.56263
21	1.12008	0.53497	0.57014
22	1.13294	0.56226	0.57117
23	1.13282	0.55048	0.57365
24	1.12871	0.58934	0.53640
25	1.12782	0.58497	0.58565
26	1.11980	0.56568	0.59614
27	1.13322	0.58211	0.59831
28	1.13205	0.57591	0.59768
29		0.59511	
30		0.59477	

*Minimum column element. This value appears in Table IV-1A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 3

Measurement Period	Observation Interval Weekly	Monthly	Quarterly
1	1.10880	0.96291	
2	1.08137	0.72388	
3	1.02811	0.65188	0.50466
4	1.01648	0.61491	0.45840
5	1.00174	0.56903	0.35871
6	0.99483*	0.56200	0.27274
7	1.00808	0.52266	0.25080
8	1.00120	0.52937	0.19877*
9	1.15760	0.51911*	0.20557
10	1.16577	0.52963	0.21294
11	1.19037	0.56423	0.23414
12	1.18528	0.58194	0.27989
13	1.20739	0.60251	0.32420
14	1.21008	0.61483	0.37491
15	1.21245	0.62657	0.40431
16	1.20594	0.62231	0.44977
17	1.17559	0.63451	0.62311
18	1.16508	0.64448	0.62834
19	1.18757	0.63400	0.61868
20	1.18745	0.63448	0.62300
21	1.18045	0.59534	0.63051
22	1.19331	0.62263	0.63154
23	1.19319	0.61086	0.63403
24	1.18908	0.64971	0.59677
25	1.18820	0.64534	0.64603
26	1.18017	0.62605	0.65651
27	1.19359	0.64248	0.65868
28	1.19242	0.63628	0.65806
29		0.65548	
30		0.65514	

*Minimum column element. This value appears in Table IV-1A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 4

Measurement Period	Observation Interval Weekly	Monthly	Quarterly
1	1.27422	1.12834	
2	1.24680	0.88931	
3	1.19354	0.81731	0.67008
4	1.18191	0.78034	0.62383
5	1.16717	0.73446	0.52414
6	1.16025*	0.72743	0.43817
7	1.17351	0.68808	0.41623
8	1.16663	0.69480	0.36420*
9	1.32302	0.68454*	0.37100
10	1.33120	0.69506	0.37837
11	1.35580	0.72966	0.39957
12	1.35071	0.74737	0.44531
13	1.37282	0.76794	0.48963
14	1.37551	0.78025	0.54034
15	1.37788	0.79200	0.56974
16	1.37137	0.78774	0.61520
17	1.34102	0.79994	0.78854
18	1.33051	0.80991	0.79377
19	1.35299	0.79942	0.78411
20	1.35288	0.79991	0.78843
21	1.34588	0.76077	0.79594
22	1.35874	0.78805	0.79697
23	1.35862	0.77628	0.79945
24	1.35451	0.81514	0.76220
25	1.35362	0.81077	0.81145
26	1.34560	0.79148	0.82194
27	1.35902	0.80791	0.82411
28	1.35785	0.80171	0.82348
29		0.82091	
30		0.82057	

*Minimum column element. This value appears in Table IV-1A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 5

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	1.63362	1.48774	
2	1.60620	1.24871	
3	1.55294	1.17671	1.02948
4	1.54131	1.13974	0.98322
5	1.52657	1.09385	0.88354
6	1.51965*	1.08683	0.79757
7	1.53291	1.04748	0.77563
8	1.52602	1.05420	0.72360*
9	1.68242	1.04394*	0.73040
10	1.69060	1.05445	0.73777
11	1.71520	1.08905	0.75897
12	1.71011	1.10677	0.80471
13	1.73222	1.12734	0.84903
14	1.73491	1.13965	0.89974
15	1.73728	1.15139	0.92914
16	1.73077	1.14714	0.97460
17	1.70043	1.15934	1.14794
18	1.68991	1.16931	1.15317
19	1.71239	1.15883	1.14351
20	1.71228	1.15931	1.14783
21	1.70528	1.12017	1.15534
22	1.71814	1.14745	1.15637
23	1.71802	1.13568	1.15885
24	1.71391	1.17454	1.2160
25	1.71302	1.17017	1.17085
26	1.70500	1.15088	1.18134
27	1.71842	1.16731	1.18351
28	1.71725	1.16111	1.18288
29		1.18031	
30		1.17997	

*Minimum column element. This value appears in Table IV-1A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 6

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	1.50937	1.36348	
2	1.48194	1.12445	
3	1.42868	1.05245	0.90522
4	1.41705	1.01548	0.85897
5	1.40231	0.96960	0.75928
6	1.39540*	0.96257	0.67331
7	1.40865	0.92322	0.65137
8	1.40177	0.92994	0.59934*
9	1.55817	0.91968*	0.60614
10	1.56634	0.93020	0.61351
11	1.59094	0.96480	0.63471
12	1.58585	0.98251	0.68045
13	1.60797	1.00308	0.72477
14	1.61065	1.01540	0.77548
15	1.61302	1.02714	0.80488
16	1.60651	1.02288	0.85034
17	1.57617	1.03508	1.02368
18	1.56565	1.04505	1.02891
19	1.58814	1.03457	1.01925
20	1.58802	1.03505	1.02357
21	1.58102	0.99591	1.03108
22	1.59388	1.02320	1.03211
23	1.59377	1.01143	1.03460
24	1.58965	1.05028	0.99734
25	1.58877	1.04591	1.04660
26	1.58074	1.02662	1.05708
27	1.59417	1.04305	1.05925
28	1.59299	1.03685	1.05862
29		1.05605	
30		1.05571	

*Minimum column element. This value appears in Table IV-1A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 7

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	1.37105	1.22517	
2	1.34362	0.98614	
3	1.29037	0.91414	0.76691
4	1.27874	0.87717	0.72065
5	1.26400	0.83128	0.62097
6	1.25708*	0.82425	0.53500
7	1.27034	0.78491	0.51306
8	1.26345	0.79163	0.46103*
9	1.41985	0.78137*	0.46783
10	1.42802	0.79188	0.47520
11	1.45262	0.82648	0.49640
12	1.44754	0.84420	0.54214
13	1.46965	0.86477	0.58646
14	1.47234	0.87708	0.63717
15	1.47471	0.88882	0.66657
16	1.46820	0.88457	0.71203
17	1.43785	0.89677	0.88537
18	1.42734	0.90674	0.89060
19	1.44982	0.89625	0.88094
20	1.44971	0.89674	0.88525
21	1.44271	0.85760	0.89277
22	1.45557	0.88488	0.89380
23	1.45545	0.87311	0.89628
24	1.45134	0.91197	0.85903
25	1.45045	0.90760	0.90828
26	1.44242	0.88831	0.91877
27	1.45585	0.90474	0.92094
28	1.45468	0.89854	0.92031
29		0.91774	
30		0.91740	

*Minimum column element. This value appears in Table IV-1A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 8

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	1.65194	1.50605	
2	1.62451	1.26702	
3	1.57125	1.19502	1.04780
4	1.55962	1.15805	1.00154
5	1.54488	1.11217	0.90185
6	1.53797*	1.10514	0.81588
7	1.55122	1.06580	0.79394
8	1.54434	1.07251	0.74191*
9	1.70074	1.06225*	0.74871
10	1.70891	1.07277	0.75608
11	1.73351	1.10737	0.77728
12	1.72842	1.12508	0.82303
13	1.75054	1.14565	0.86734
14	1.75322	1.15797	0.91805
15	1.75559	1.16971	0.94745
16	1.74908	1.16545	0.99291
17	1.71874	1.17765	1.16625
18	1.70822	1.18762	1.17148
19	1.73071	1.17714	1.16182
20	1.73059	1.17762	1.16614
21	1.72360	1.13848	1.17365
22	1.73645	1.16577	1.17468
23	1.73634	1.15400	1.17717
24	1.73222	1.19285	1.13991
25	1.73134	1.18848	1.18917
26	1.72331	1.16920	1.19965
27	1.73674	1.18562	1.20182
28	1.73557	1.17942	1.20120
29		1.19862	
30		1.19828	

*Minimum column element. This value appears in Table IV-1A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 9

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	1.39574	1.24985	
2	1.36831	1.01083	
3	1.31505	0.93882	0.79160
4	1.30342	0.90185	0.74534
5	1.28868	0.85597	0.64565
6	1.28177*	0.84894	0.55968
7	1.29502	0.80960	0.53774
8	1.28814	0.81631	0.48571*
9	1.44454	0.80605*	0.49251
10	1.45271	0.81657	0.49988
11	1.47731	0.85117	0.52108
12	1.47222	0.86888	0.56683
13	1.49434	0.88945	0.61114
14	1.49702	0.90177	0.66185
15	1.49940	0.91351	0.69125
16	1.49288	0.90925	0.73671
17	1.46254	0.92145	0.91005
18	1.45202	0.93142	0.91528
19	1.47451	0.92094	0.90563
20	1.47440	0.92143	0.90994
21	1.46739	0.88228	0.91745
22	1.48025	0.90957	0.91848
23	1.48014	0.89780	0.92097
24	1.47602	0.93665	0.88371
25	1.47514	0.93228	0.93297
26	1.46711	0.91300	0.94345
27	1.48054	0.92943	0.94563
28	1.47937	0.92322	0.94500
29		0.94242	
30		0.94208	

*Minimum column element. This value appears in Table IV-1A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 10

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	1.17380	1.02791	
2	1.14637	0.88888	
3	1.09311	0.71688	0.56965
4	1.08148	0.67991	0.52340
5	1.06674	0.63403	0.42371
6	1.05982*	0.62700	0.33774
7	1.07308	0.58765	0.31580
8	1.06620	0.59437	0.26377*
9	1.22260	0.58411*	0.27057
10	1.23077	0.59463	0.27794
11	1.25537	0.62923	0.29914
12	1.25028	0.64694	0.34489
13	1.27240	0.66751	0.38920
14	1.27508	0.67983	0.43991
15	1.27745	0.69157	0.46931
16	1.27094	0.68731	0.51477
17	1.24059	0.69951	0.68811
18	1.23008	0.70948	0.69334
19	1.25257	0.69900	0.68368
20	1.25245	0.69948	0.68800
21	1.24545	0.66034	0.69551
22	1.25831	0.68762	0.69654
23	1.25819	0.67585	0.69902
24	1.25408	0.71471	0.66177
25	1.25320	0.71034	0.71103
26	1.24517	0.69106	0.72151
27	1.25860	0.70748	0.72368
28	1.25742	0.70128	0.72305
29		0.72048	
30		0.72014	

*Minimum column element. This value appears in Table IV-1A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 11

Measurement Period	Observation Interval: Weekly	Monthly	Quarterly
1	1.42682	1.28094	
2	1.39940	1.04191	
3	1.34614	0.96991	0.82268
4	1.33451	0.93294	0.77643
5	1.31977	0.88705	0.67674
6	1.31285*	0.88003	0.59077
7	1.32611	0.84068	0.56883
8	1.31922	0.84740	0.51680*
9	1.47563	0.83714*	0.52360
10	1.48380	0.84766	0.53097
11	1.50840	0.88225	0.55217
12	1.50331	0.89997	0.59791
13	1.52542	0.92054	0.64223
14	1.52811	0.93285	0.72234
15	1.53048	0.94460	0.72234
16	1.52397	0.94034	0.76780
17	1.49362	0.95254	0.94114
18	1.48311	0.96251	0.94637
19	1.50560	0.95202	0.93671
20	1.50548	0.95251	0.94103
21	1.49848	0.91337	0.94854
22	1.51134	0.94065	0.94957
23	1.51122	0.92888	0.95205
24	1.50711	0.96774	0.91480
25	1.50622	0.96337	0.96405
26	1.49820	0.94408	0.97454
27	1.51162	0.96051	0.97671
28	1.51045	0.95431	0.97608
29		0.97351	
30		0.97317	

*Minimum column element. This value appears in Table IV-1A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 12

Measurement Period	Observation Interval: Weekly	Monthly	Quarterly
1	1.54437	1.39848	
2	1.51694	1.15945	
3	1.46368	1.08745	0.94023
4	1.45205	1.05048	0.89397
5	1.43731	1.00460	0.79428
6	1.43040*	0.99757	0.70831
7	1.44365	0.95822	0.68637
8	1.43677	0.96494	0.63434*
9	1.59317	0.95468*	0.64114
10	1.60134	0.96520	0.64851
11	1.62594	0.99980	0.66971
12	1.62085	1.01751	0.71545
13	1.64297	1.03808	0.75977
14	1.64565	1.05040	0.81048
15	1.64803	1.06214	0.83988
16	1.64151	1.05788	0.88534
17	1.61117	1.07008	1.05868
18	1.60065	1.08005	1.06391
19	1.62314	1.06957	1.05425
20	1.62302	1.07005	1.05857
21	1.61602	1.03091	1.06608
22	1.62888	1.05819	1.06711
23	1.62877	1.04642	1.06960
24	1.62465	1.08528	1.03234
25	1.62377	1.08091	1.08160
26	1.61574	1.06162	1.09208
27	1.62917	1.07805	1.09425
28	1.62800	1.07185	1.09362
29		1.09105	
30		1.09071	

*Minimum column element. This value appears in Table IV-1A.

APPENDIX A-2

Mean Absolute Prediction Error for each Measurement Period Computed Using Ordinary Least Squares Beta Coefficients in Test Period 2

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 2

HOLDING PERIOD 1

Measurement Period	Observation Interval Weekly	Monthly	Quarterly
1	0.92934	1.60065	
2	0.73048	1.68071	
3	0.74851	1.16848	4.68282
4	0.71886	1.16680	4.17877
5	0.74086	1.15491	2.64202
6	0.72414	1.15394	1.86882
7	0.68008	1.15097	1.61511
8	0.66205	1.10574	1.46637
9	0.65968	1.08580	1.46731
10	0.67563	1.08711	1.49594
11	0.66185	1.10011	1.49925
12	0.67826	1.09105	1.30168
13	0.53611	1.07743	1.12248
14	0.44914	1.05665	1.00325
15	0.42314	1.03577	0.99900
16	0.41848	1.05920	0.98726*
17	0.41766	1.03414	1.00128
18	0.40863	1.03520	1.03465
19	0.39160	1.03162	1.04811
20	0.38877	1.02648	1.06202
21	0.34211*	1.02525	1.05897
22	0.34511	1.00791	1.06445
23	0.34831	0.96897	1.06294
24	0.35020	0.96005	1.06768
25	0.34974	0.96177	1.05411
26	0.35197	0.96223	1.04400
27	0.35180	0.95568	1.04360
28	0.34743	0.93877	1.00614
29		0.93420	
30		0.90994*	

*Minimum column element. This value appears in Table IV-1B.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 2

HOLDING PERIOD 2

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	0.08094*	0.75225	
2	0.11791	0.83231	
3	0.09989	0.32009	3.83442
4	0.12954	0.31840	3.33037
5	0.10754	0.30651	1.79362
6	0.12426	0.30554	1.02042
7	0.16831	0.30257	0.76671
8	0.18634	0.25734	0.61797
9	0.18871	0.23740	0.61891
10	0.17277	0.23871	0.64754
11	0.18654	0.25171	0.65086
12	0.17014	0.24266	0.45328
13	0.31229	0.22903	0.27409
14	0.39926	0.20826	0.15486
15	0.42526	0.18737	0.15060
16	0.42991	0.21080	0.13886*
17	0.43074	0.18574	0.15289
18	0.43977	0.18680	0.18626
19	0.45680	0.18323	0.19971
20	0.45963	0.17809	0.21363
21	0.50628	0.17686	0.21057
22	0.50328	0.15951	0.21606
23	0.50008	0.12057	0.21454
24	0.49820	0.11166	0.21929
25	0.49866	0.11337	0.20571
26	0.49643	0.11383	0.19560
27	0.49660	0.10729	0.19520
28	0.50097	0.09037	0.15774
29		0.08580	
30		0.06154*	

*Minimum column element. This value appears in Table IV-1B.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 2

HOLDING PERIOD 3

Measurement Period	Observation Interval: Weekly	Monthly	Quarterly
1	0.12355*	0.54776	
2	0.32240	0.62782	
3	0.30437	0.11560	3.62994
4	0.33403	0.11391	3.12588
5	0.31203	0.10203	1.58914
6	0.32874	0.10203	1.58914
7	0.37280	0.09808	0.56222
8	0.39083	0.05285	0.41348
9	0.39320	0.03291	0.41442
10	0.37726	0.03423	0.44305
11	0.39103	0.04723	0.44637
12	0.37463	0.03817	0.24880
13	0.51677	0.02454	0.06960
14	0.60374	0.00377*	0.04963
15	0.62974	0.01712	0.05389
16	0.63440	0.00631	0.06563
17	0.63523	0.01875	0.05160
18	0.64426	0.01769	0.01823
19	0.66129	0.02126	0.00477
20	0.66411	0.02640	0.00914
21	0.71077	0.02763	0.00608
22	0.70777	0.04497	0.01157
23	0.70457	0.08392	0.91005
24	0.70268	0.09283	0.01480
25	0.70314	0.09112	0.00123*
26	0.70091	0.09112	0.00123*
27	0.70109	0.09066	0.00889
28	0.70546	0.11412	0.04675
29		0.11412	
30		0.11868	

*Minimum column element. This value appears in Table IV-1B.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 2

HOLDING PERIOD 4

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	1.21337	1.88468	
2	1.01451	1.96474	
3	1.03254	1.45251	4.96685
4	1.00288	1.45083	4.46279
5	1.02488	1.43894	2.92605
6	1.00817	1.43797	2.15285
7	0.96411	1.43500	1.89914
8	0.94608	1.38977	1.75040
9	0.94371	1.36983	1.75134
10	0.95965	1.37114	1.77997
11	0.94588	1.38414	1.78328
12	0.96228	1.37508	1.58571
13	0.82014	1.36145	1.40651
14	0.73317	1.34068	1.28728
15	0.70717	1.31980	1.28302
16	0.70251	1.34322	1.27128*
17	0.70169	1.31817	1.28531
18	0.69266	1.31922	1.31868
19	0.67563	1.31565	1.33214
20	0.67280	1.31051	1.34605
21	0.62614*	1.30928	1.34300
22	0.62914	1.29194	1.34848
23	0.63234	1.25300	1.34697
24	0.63423	1.24408	1.35171
25	0.63377	1.24580	1.33814
26	0.63600	1.24625	1.32802
27	0.63583	1.23971	1.32763
28	0.63146	1.22280	1.29017
29		1.21823	
30		1.19397*	

*Minimum column element. This value appears in Table IV-1B.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 2

HOLDING PERIOD 5

Measurement Period	Observation Interval: Weekly	Monthly	Quarterly
1	0.04049*	0.63083	
2	0.23934	0.71088	
3	0.22131	0.19866	3.71299
4	0.25097	0.19697	3.20894
5	0.22897	0.18509	1.67220
6	0.24569	0.18411	0.89900
7	0.28974	0.18114	0.64528
8	0.30777	0.13591	0.49654
9	0.31014	0.11597	0.49748
10	0.29420	0.11729	0.52611
11	0.30797	0.13029	0.52943
12	0.29157	0.12123	0.33186
13	0.43371	0.10760	0.15266
14	0.52068	0.08683	0.03343
15	0.54668	0.06594	0.02917
16	0.55134	0.08937	0.01743*
17	0.55217	0.06431	0.03146
18	0.56120	0.06537	0.06483
19	0.57823	0.06180	0.07829
20	0.58105	0.05666	0.09220
21	0.62771	0.05543	0.08914
22	0.62471	0.03809	0.09463
23	0.62151	0.00086*	0.09311
24	0.61962	0.00977	0.09786
25	0.62008	0.00806	0.08429
26	0.61785	0.00760	0.07417
27	0.61803	0.01414	0.07377
28	0.62240	0.03106	0.03631
29		0.03563	
30		0.05989	

*Minimum column element. This value appears in Table IV-1B.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 2

HOLDING PERIOD 6

Measurement Period	Observation Interval: Weekly	Monthly	Quarterly
1	0.23817	0.90948	
2	0.03931	0.98954	
3	0.05734	0.47731	3.99165
4	0.02769	0.47563	3.48760
5	0.04969	0.46374	1.95085
6	0.03297	0.46277	1.17765
7	0.01109*	0.45980	0.92394
8	0.02911	0.41457	0.77520
9	0.03149	0.39463	0.77614
10	0.01554	0.39594	0.80477
11	0.02931	0.40894	0.80808
12	0.01291	0.39989	0.61051
13	0.15506	0.38626	0.43131
14	0.24203	0.36449	0.31209
15	0.26803	0.34460	0.30783
16	0.27269	0.36803	0.29609*
17	0.27351	0.34297	0.31011
18	0.28254	0.34403	0.34349
19	0.29957	0.34046	0.35694
20	0.30240	0.33531	0.37086
21	0.34906	0.33409	0.36780
22	0.34606	0.31674	0.37329
23	0.34286	0.27780	0.37177
24	0.34097	0.26889	0.37651
25	0.34143	0.27060	0.36294
26	0.33920	0.27106	0.35283
27	0.33937	0.26451	0.35243
28	0.34374	0.24760	0.31497
29		0.24303	
30		0.21877*	

*Minimum column element. This value appears in Table IV-1B.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 2

HOLDING PERIOD 7

Measurement Period	Observation Interval: Weekly	Monthly	Quarterly
1	0.48831	1.15903	
2	0.28946	1.23968	
3	0.30749	0.72746	4.24180
4	0.27783	0.72577	3.73774
5	0.29983	0.71388	2.20099
6	0.28311	0.71291	1.42780
7	0.23906	0.70994	1.17408
8	0.22103	0.66471	1.02534
9	0.21866	0.64477	1.02628
10	0.23460	0.64608	1.05491
11	0.22083	0.65908	1.05823
12	0.23723	0.65003	0.86065
13	0.09509	0.63640	0.68146
14	0.00811*	0.61563	0.56223
15	0.01789	0.59474	0.55797
16	0.02254	0.61817	0.54623*
17	0.02337	0.59311	0.56026
18	0.03240	0.59417	0.59363
19	0.04943	0.59060	0.60708
20	0.05226	0.58546	0.62100
21	0.09891	0.58423	0.61794
22	0.09591	0.56688	0.62343
23	0.09271	0.52794	0.62191
24	0.09083	0.51903	0.62666
25	0.09129	0.52074	0.61308
26	0.08906	0.52120	0.60297
27	0.08923	0.51466	0.60257
28	0.09360	0.49774	0.56511
29		0.49317	
30		0.46891*	

*Minimum column element. This value appears in Table IV-1B.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 2

HOLDING PERIOD 8

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	0.46369	1.13500	
2	0.26483	1.21505	
3	0.28286	0.70283	4.21716
4	0.25320	0.70114	3.71311
5	0.27520	0.68926	2.17636
6	0.25849	0.68828	1.40317
7	0.21443	0.68531	1.14945
8	0.19640	0.64009	1.00071
9	0.19403	0.62014	1.00166
10	0.20997	0.62146	1.03028
11	0.19620	0.63446	1.03360
12	0.21260	0.62540	0.83603
13	0.07046	0.61177	0.65683
14	0.01651*	0.59100	0.53760
15	0.04251	0.57011	0.53334
16	0.04717	0.59354	0.52160*
17	0.04800	0.56848	0.53563
18	0.05703	0.56954	0.56900
19	0.07406	0.56597	0.58246
20	0.07689	0.56083	0.59637
21	0.12354	0.55960	0.59331
22	0.12054	0.54226	0.59880
23	0.11734	0.50331	0.59728
24	0.11546	0.49440	0.60203
25	0.11591	0.49611	0.58846
26	0.11369	0.49657	0.57834
27	0.11386	0.49003	0.57794
28	0.11823	0.47311	0.54048
29		0.46854	
30		0.44429*	

*Minimum column element. This value appears in Table IV-1B.

APPENDIX B-1

Mean Absolute Prediction Error for each Measurement Period Computed Using Fisher-Kamin Beta Coefficients in Test Period 1.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 1

Measurement Period	Observation Interval Weekly	Monthly	Quarterly
1	0.60463	0.19451	
2	0.58823	0.12200	
3	0.53451	0.08777	0.00926*
4	0.52948	0.08169*	0.07289
5	0.51860	0.08323	0.03803
6	0.50891*	0.08451	0.21114
7	0.52237	0.28391	0.22651
8	0.51337	0.14840	0.31331
9	0.66994	0.11409	0.30600
10	0.67574	0.11829	0.32526
11	0.69834	0.12883	0.31854
12	0.68863	0.12731	0.29031
13	0.71051	0.09817	0.23520
14	0.71231	0.11709	0.17263
15	0.71285	0.10449	0.13769
16	0.70405	0.10389	0.08657
17	0.67188	0.10740	0.08137
18	0.66317	0.11709	0.10131
19	0.69674	0.12563	0.11911
20	0.69731	0.12580	0.12229
21	0.68877	0.11446	0.12883
22	0.70111	0.12749	0.12937
23	0.70111	0.12843	0.12949
24	0.69320	0.15151	0.08634
25	0.69125	0.14577	0.14060
26	0.68534	0.12134	0.15017
27	0.69854	0.12223	0.15451
28	0.69922	0.11989	0.15603
29		0.12660	
30		0.13720	

*Minimum column element. This value appears in Table IV-6A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 2

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	1.03017	0.62006	
2	1.01377	0.54754	
3	0.96005	0.51331	0.41629
4	0.95503	0.50723*	0.49843
5	0.94414	0.50877	0.38751
6	0.93445*	0.51006	0.21440
7	0.94791	0.70945	0.19903
8	0.93891	0.57394	0.11223
9	1.09548	0.53963	0.11954
10	1.10128	0.54383	0.10029*
11	1.12388	0.55437	0.10700
12	1.11417	0.55286	0.13523
13	1.13605	0.52371	0.19034
14	1.13785	0.54263	0.25291
15	1.13840	0.53003	0.28786
16	1.12959	0.52943	0.33897
17	1.09742	0.53294	0.50691
18	1.08871	0.54263	0.52686
19	1.12228	0.55117	0.54466
20	1.12285	0.55134	0.54783
21	1.11431	0.54000	0.55437
22	1.12665	0.55303	0.55491
23	1.12665	0.55397	0.55503
24	1.11874	0.57706	0.51188
25	1.11679	0.57131	0.56614
26	1.11088	0.54689	0.57571
27	1.12408	0.54777	0.58006
28	1.12477	0.54543	0.58157
29		0.55214	
30		0.56274	

*Minimum column element. This value appears in Table IV-6A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 3

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	1.09054	0.68043	
2	1.07414	0.60791	
3	1.02042	0.57368	0.47666
4	1.01540	0.56760*	0.55880
5	1.00451	0.56914	0.44789
6	0.99483*	0.57043	0.27477
7	1.00828	0.76983	0.25940
8	0.99928	0.63431	0.17260
9	1.15585	0.60000	0.17991
10	1.16165	0.60420	0.16066*
11	1.18425	0.61474	0.16737
12	1.17454	0.61323	0.19560
13	1.19642	0.58408	0.25071
14	1.19822	0.60300	0.31329
15	1.19877	0.59040	0.34823
16	1.18997	0.58980	0.39934
17	1.15779	0.59331	0.56728
18	1.14908	0.60300	0.58723
19	1.18265	061154	0.60503
20	1.18322	0.61171	0.60820
21	1.17468	0.60037	0.61474
22	1.18702	0.61340	0.61528
23	1.18702	0.61434	0.61540
24	1.17911	0.63742	0.57226
25	1.17717	0.63168	0.62651
26	1.17125	0.60726	0.63608
27	1.18445	0.60814	0.64043
28	1.18514	0.60580	0.64194
29		0.61251	
30		0.62311	

*Minimum column element. This value appears in Table IV-6A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 4

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	1.25597	0.84585	
2	1.23957	0.77334	
3	1.18585	0.73911	0.64208
4	1.18083	0.73302*	0.72423
5	1.16994	0.73457	0.61331
6	1.16025*	0.73586	0.44020
7	1.17371	0.93525	0.42483
8	1.16471	0.79974	0.42483
9	1.32128	0.76543	0.34534
10	1.32708	0.76962	0.32609*
11	1.34968	0.78017	0.33280
12	1.33997	0.77865	0.36103
13	1.36185	0.74951	0.41614
14	1.36365	0.76843	0.47871
15	1.36419	0.75582	0.51366
16	1.35539	0.75523	0.56477
17	1.32322	0.75874	0.73271
18	1.31451	0.76843	0.75265
19	1.34808	0.77697	0.75265
20	1.34865	0.77714	0.77362
21	1.34011	0.76580	0.78017
22	1.35245	0.77883	0.78071
23	1.35245	0.77977	0.78082
24	1.34454	0.80285	0.73768
25	1.34260	0.79711	0.79194
26	1.33668	0.77268	0.80151
27	1.34988	0.77357	0.80585
28	1.35057	0.77123	0.80737
29		0.77794	
30		0.78854	

*Minimum column element. This value appears in Table IV-6A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

TEST PERIOD 5

Measurement Period	Observation Interval: Weekly	Monthly	Quarterly
1	1.61537	1.20525	
2	1.59897	1.3274	
3	1.54525	1.09851	1.00148
4	1.54023	1.09242*	1.08362
5	1.52934	1.09397	0.97271
6	1.51965*	1.09525	0.79960
7	1.53311	1.29465	0.78423
8	1.52411	1.15914	0.69743
9	1.68068	1.12482	0.70474
10	1.68648	1.12902	0.68548*
11	1.70908	1.13957	0.69220
12	1.69937	1.13805	0.72043
13	1.72125	1.10891	0.77554
14	1.72305	1.12782	0.83811
15	1.72359	1.11522	0.87305
16	1.71480	1.11462	9.92417
17	1.68262	1.11814	1.09211
18	1.67391	1.12782	1.11205
19	1.70748	1.13637	1.12985
20	1.70805	1.13654	1.13303
21	1.69951	1.12520	1.13957
22	1.71185	1.13822	1.14011
23	1.71185	1.13917	1.09708
24	1.70394	1.16225	1.14022
25	1.70200	1.15651	1.15134
26	1.69608	1.13208	1.16091
27	1.70928	1.13297	1.16525
28	1.70997	1.13062	1.16677
29		1.13734	
30		1.14794	

*Minimum column element. This value appears in Table IV-6A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 6

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	1.49111	1.08100	
2	1.47471	1.00848	
3	1.42100	0.97425	0.87723
4	1.41597	0.96817*	0.95937
5	1.40508	0.96971	0.84845
6	1.39540*	0.97100	0.67534
7	1.40885	1.17040	0.65997
8	1.39985	1.03488	0.57317
9	1.55642	1.00057	0.58048
10	1.56223	1.00477	0.56123*
11	1.58483	1.01531	0.56794
12	1.57511	1.01380	0.59617
13	1.59700	0.98465	0.65128
14	1.59879	1.00357	0.71385
15	1.59934	0.99097	0.74880
16	1.59054	0.99037	0.79991
17	1.55837	0.99388	0.96785
18	1.54965	1.00357	0.98780
19	1.58322	1.01211	1.00560
20	1.58379	1.01228	1.00877
21	1.57525	1.00094	1.01531
22	1.58759	1.01397	1.01585
23	1.58759	1.01491	1.01597
24	1.57968	1.03800	0.97283
25	1.57774	1.03225	1.02708
26	1.57182	1.00782	1.03665
27	1.58502	1.00871	1.04100
28	1.58571	1.00637	1.04251
29		1.01308	
30		1.02368	

*Minimum column element. This value appears in Table IV-6A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 7

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	1.35280	0.94268	
2	1.33640	0.87017	
3	1.28268	0.83594	0.73891
4	1.27765	0.82985*	0.82105
5	1.26677	0.83140	0.71014
6	1.25708*	0.83268	0.53703
7	1.27054	1.03208	0.52166
8	1.26154	0.89657	0.43486
9	1.41811	0.86225	0.44217
10	1.42391	0.86645	0.42291*
11	1.44651	0.87700	0.42963
12	1.43680	0.87548	0.45786
13	1.45868	0.84634	0.51297
14	1.46048	0.86525	0.57554
15	1.46102	0.85265	0.61048
16	1.45222	0.85205	0.66160
17	1.42005	0.85557	0.82954
18	1.41134	0.86525	0.84948
19	1.44491	0.87380	0.86728
20	1.44548	0.87397	0.87045
21	1.43694	0.86263	0.87700
22	1.44928	0.87565	0.87754
23	1.44928	0.87660	0.87765
24	1.44137	0.89968	0.83451
25	1.43942	0.89394	9.88877
26	1.43351	0.86951	0.89834
27	1.44671	0.87040	0.90268
28	1.44739	0.86805	0.90420
29		0.87477	
30		9.88537	

*Minimum column element. This value appears in Table IV-6A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 8

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	1.63368	1.22357	
2	1.61728	1.15105	
3	1.56357	1.11682	1.01980
4	1.55854	1.11074*	1.10194
5	1.54765	1.11228	0.99102
6	1.53797*	1.11357	0.81791
7	1.55142	1.31297	0.80254
8	1.54243	1.17745	0.71574
9	1.69900	1.14314	0.72305
10	1.70480	1.14734	0.70380*
11	1.72740	1.15788	0.71051
12	1.71768	1.15637	0.73874
13	1.73957	1.12722	0.79385
14	1.74137	1.14614	0.85643
15	1.74191	1.13354	0.89137
16	1.73311	1.13294	0.94248
17	1.70094	1.13645	1.11042
18	1.69222	1.14614	1.13037
19	1.72579	1.15468	1.14817
20	1.72637	1.15485	1.15134
21	1.71782	1.14351	1.15788
22	1.73017	1.15654	1.15842
23	1.73017	1.15748	1.15854
24	1.72225	1.18057	1.11540
25	1.72031	1.17482	1.16965
26	1.71440	1.15040	1.17922
27	1.72760	1.15128	1.18357
28	1.72828	1.14894	1.18508
29		1.15565	
30		1.16625	

*Minimum column element. This value appears in Table IV-6A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 9

Measurement Period	Observation Interval: Weekly	Monthly	Quarterly
1	1.37748	0.96737	
2	1.36108	0.89485	
3	1.30737	0.86062	0.76360
4	1.30234	0.85454*	0.84574
5	1.29145	0.85608	0.73482
6	1.28177*	0.85737	0.56171
7	1.29522	1.05677	0.54634
8	1.28622	0.92125	0.45954
9	1.44280	0.88694	0.46686
10	1.44860	0.89114	0.44760*
11	1.47120	0.90168	0.45431
12	1.46148	0.90017	0.48254
13	1.48337	0.87103	0.53765
14	1.48517	0.88994	0.60023
15	1.48571	0.87734	0.63517
16	1.47691	0.87674	0.68628
17	1.44474	0.88025	0.85423
18	1.43602	0.88994	0.87417
19	1.46960	0.89848	0.89197
20	1.47017	0.89865	0.89514
21	1.46162	0.88731	0.90168
22	1.47397	0.90034	0.90222
23	1.47397	0.90128	0.90234
24	1.46605	0.92437	0.91345
25	1.46411	0.91863	0.91345
26	1.45820	0.89420	0.92302
27	1.47140	0.89508	0.92737
28	1.47208	0.89274	0.92888
29		0.89945	
30		0.91005	

*Minimum column element. This value appears in Table IV-6A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 10

Measurement Period	Observation Interval Weekly	Monthly	Quarterly
1	1.15554	0.74543	
2	1.13914	0.67291	
3	1.08543	0.63868	0.54165
4	1.08040	0.63260*	0.62380
5	1.06951	0.63414	0.51288
6	1.05982*	0.63543	0.33977
7	1.07328	0.83483	0.32440
8	1.06428	0.69931	0.23760
9	1.22085	0.66500	0.24491
10	1.22665	0.66920	0.22566*
11	1.24925	0.67974	0.23237
12	1.23954	0.67823	0.26060
13	1.26143	0.64908	0.31571
14	1.26322	0.66800	0.37829
15	1.26377	0.65540	0.41323
16	1.25497	0.65480	0.46434
17	1.22280	0.65831	0.63228
18	1.21408	0.66800	0.65223
19	1.24765	0.67654	0.67003
20	1.24822	0.67671	0.67320
21	1.23968	0.66537	0.67974
22	1.25202	0.67840	0.68028
23	1.25202	0.67934	0.68040
24	1.24411	0.70243	0.63725
25	1.24217	0.69668	0.69151
26	1.23625	0.67226	0.70108
27	1.24945	0.67314	0.70543
28	1.25014	0.67080	0.70694
29		0.67751	
30		0.68811	

*Minimum column element. This value appears in Table IV-6A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 11

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	1.40857	0.99845	
2	1.39217	0.92594	
3	1.33845	0.89171	0.79468
4	1.33342	0.88563*	0.97683
5	1.32254	0.88717	0.76591
6	1.31285*	0.88845	0.59280
7	1.32631	1.08785	0.57743
8	1.31731	0.95234	0.49063
9	1.47388	0.91802	0.49794
10	1.47968	0.92223	0.47868*
11	1.50228	0.93277	0.48540
12	1.49257	0.93125	0.51363
13	1.51445	0.90211	0.56874
14	1.51625	0.92103	0.63131
15	1.51680	0.90843	0.66625
16	1.50800	0.90783	0.71737
17	1.47582	0.91134	0.88531
18	1.46711	0.92103	0.90525
19	1.50068	0.92957	0.92305
20	1.50125	0.92974	0.92623
21	1.49271	0.91840	0.93277
22	1.50505	0.93143	0.93331
23	1.50505	0.93237	0.93343
24	1.49714	0.95545	0.89028
25	1.49520	0.94971	0.94454
26	1.48928	0.92528	0.95411
27	1.50248	0.92617	0.95845
28	1.50317	0.92382	0.95997
29		0.93054	
30		0.94114	

*Minimum column element. This value appears in Table IV-6A.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 1

HOLDING PERIOD 12

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	1.52611	1.11600	
2	1.50971	1.04348	
3	1.45600	1.00925	0.91222
4	1.45097	1.00317*	0.99437
5	1.44008	1.00471	0.88345
6	1.43040*	1.00600	0.71034
7	1.44385	1.20540	0.69497
8	1.43485	1.06988	0.60817
9	1.59142	1.03557	0.61548
10	1.59722	1.03977	0.59623*
11	1.61982	1.05031	0.60294
12	1.61011	1.04880	0.63117
13	1.63200	1.01965	0.68628
14	1.63380	1.03857	0.74885
15	1.63434	1.02597	0.78380
16	1.62554	1.02537	0.83491
17	1.59337	1.02888	1.00285
18	1.58465	1.03857	1.02280
19	1.61822	1.04711	1.04060
20	1.61880	1.04728	1.04377
21	1.61025	1.03594	1.05031
22	1.62260	1.04897	1.05085
23	1.62259	1.04991	1.05097
24	1.61468	1.07300	1.00782
25	1.61274	1.06725	1.06208
26	1.60682	1.04282	1.07165
27	1.62002	1.04371	1.07600
28	1.62071	1.04137	1.07751
29		1.04808	
30		1.05868	

*Minimum column element. This value appears in Table IV-6A.

APPENDIX B-2

Mean Absolute Prediction Error for each Measurement Period
Computed Using Fisher-Kamin Beta Coefficients in Test Period 2.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 2

HOLDING PERIOD 1

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	0.91182	1.58605	
2	0.73151	1.62485	
3	0.75985	1.19140	1.43597
4	0.73388	1.22314	1.78842
5	0.74414	1.16805	2.01319
6	0.72565	1.15708	1.80794
7	0.68040	1.15231	1.61422
8	0.66160	1.10511	1.45685
9	0.65931	1.08554	1.45568
10	0.67648	1.08714	1.50308
11	0.66326	1.10114	1.51208
12	0.68414	1.09382	1.32908
13	0.54297	1.08685	1.13645
14	0.45074	1.06980	1.01160
15	0.42374	1.03837	1.00311
16	0.41883	1.06037	0.99120*
17	0.41877	1.03500	1.01057
18	0.41174	1.03714	1.05728
19	0.39389	1.03825	1.06383
20	0.39211	1.03125	1.08602
21	0.34534*	1.03122	1.08234
22	0.34903	1.01337	1.09400
23	0.35323	0.97768	1.09800
24	0.35683	0.97448	1.11305
25	0.35663	0.97448	1.10008
26	0.35966	0.97837	1.09108
27	0.36006	0.97297	1.09425
28	0.35717	0.95677	1.05923
29		0.95203	
30		0.92800*	

*Minimum column element. This value appears in Table IV-6B.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 2

HOLDING PERIOD 2

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	0.06343*	0.73765	
2	0.11689	0.77645	
3	0.08854	0.34300	0.58757
4	0.11451	0.37474	0.94002
5	0.10426	0.31966	0.16479
6	0.12274	0.30869	0.95954
7	0.16800	0.30391	0.76583
8	0.18680	0.25671	0.60845
9	0.18909	0.23714	0.69728
10	0.17191	0.23874	0.65468
11	0.18514	0.25274	0.66368
12	0.16426	0.24543	0.48068
13	0.30543	0.23846	0.28806
14	0.39766	0.22140	0.16320
15	0.42466	0.18997	0.15471
16	0.42957	0.21197	0.14280*
17	0.42963	0.18660	0.16217
18	0.43666	0.18874	0.20889
19	0.45451	0.18986	0.21543
20	0.45629	0.18286	0.23763
21	0.50306	0.18283	0.23394
22	0.49937	0.16497	0.24560
23	0.49517	0.12929	0.24960
24	0.49157	0.12294	0.26466
25	0.49177	0.12609	0.25169
26	0.48874	0.12997	0.24269
27	0.48834	0.12457	0.24586
28	0.49123	0.10837	0.21083
29		0.10363	
30		0.07960*	

*Minimum column element. This value appears in Table IV-6B.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 2

HOLDING PERIOD 3

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	0.14106*	0.53317	
2	0.32137	0.57197	
3	0.29303	0.13851	0.38308
4	0.31900	0.17025	0.73554
5	0.30874	0.11517	0.96031
6	0.32723	0.10420	0.75505
7	0.37249	0.09942	0.56134
8	0.39129	0.05222	0.40397
9	0.39357	0.03265	0.40279
10	0.37640	0.03425	0.45020
11	0.38963	0.04825	0.45919
12	0.36874	0.04094	0.27620
13	0.50991	0.03397	0.08357
14	0.60214	0.01691	0.04129
15	0.62914	0.01452	0.04977
16	0.63406	0.00748*	0.06169
17	0.63411	0.01789	0.04232
18	0.64114	0.01575	0.00440*
19	0.65900	0.01463	0.01094
20	0.66077	0.02163	0.03314
21	0.70754	0.02166	0.02945
22	0.70386	0.03952	0.04111
23	0.69966	0.07520	0.04511
24	0.69606	0.08155	0.06017
25	0.69626	0.07840	0.04720
26	0.69323	0.07452	0.03820
27	0.69283	0.07992	0.04137
28	0.69571	0.09612	0.00634
29		0.10086	
30		0.12489	

*Minimum column element. This value appears in Table IV-6B.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 2

HOLDING PERIOD 4

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	1.19585	1.87008	
2	1.01554	1.90888	
3	1.04388	1.47542	1.72000
4	1.01791	1.50717	2.07245
5	1.02817	1.45208	2.29722
6	1.00968	1.44111	2.09197
7	0.96443	1.42634	1.89825
8	0.94563	1.38914	1.74088
9	0.94334	1.36957	1.73971
10	0.96051	1.37117	1.78711
11	0.94728	1.38517	1.79611
12	0.96817	1.37785	1.61311
13	0.82700	1.37088	1.42048
14	0.73477	1.35383	1.29562
15	0.70777	1.32240	1.28714
16	0.70286	1.34440	1.27523*
17	0.70280	1.31903	1.29460
18	0.69577	1.32117	1.34131
19	0.67791	1.32228	1.34785
20	0.67614	1.31528	1.37005
21	0.62937*	1.31525	1.36637
22	0.63306	1.29740	1.37802
23	0.63726	1.26171	1.38202
24	0.64086	1.25537	1.39708
25	0.64066	1.25851	1.38411
26	0.64369	1.26240	1.37511
27	0.64408	1.25700	1.37828
28	0.64120	1.24079	1.34325
29		1.23605	
30		1.21202*	

*Minimum column element. This value appears in Table IV-6B.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 2

HOLDING PERIOD 5

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	0.05800*	0.61623	
2	0.23831	0.65503	
3	0.20997	0.22157	0.46614
4	0.23594	0.25331	0.81860
5	0.22569	0.19823	1.04337
6	0.24417	0.18726	0.83811
7	0.28943	0.18249	0.64440
8	0.30823	0.13529	0.48703
9	0.31051	0.11571	0.48586
10	0.29334	0.11731	0.53326
11	0.30657	0.13131	0.54226
12	0.28569	0.12400	0.35926
13	0.42686	0.11703	0.16663
14	0.51908	0.09997	0.04177
15	0.54608	0.06854	0.03329
16	0.55100	0.09054	0.02137*
17	0.55105	0.06517	0.04074
18	0.55808	0.06731	0.08746
19	0.57594	0.06843	0.09400
20	0.57771	0.06143	0.11620
21	0.62448	0.06140	0.11251
22	0.62080	0.04354	0.12417
23	0.61660	0.00786	0.12817
24	0.61300	0.00151*	0.14323
25	0.61320	0.00466	0.13026
26	0.61017	0.00854	0.12126
27	0.60977	0.00314	0.12443
28	0.61266	0.01306	0.12443
29		0.01780	
30		0.04183	

*Minimum column element. This value appears in Table IV-6B.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 2

HOLDING PERIOD 6

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	0.22066	0.89488	
2	0.04034	0.93368	
3	0.06869	0.50023	0.74480
4	0.04371	0.53197	1.09725
5	0.05297	0.47689	1.32203
6	0.03449	0.46591	1.11677
7	0.01077	0.46114	0.92306
8	0.02957	0.41394	0.76568
9	0.03186	0.39437	0.76451
10	0.01469	0.39597	0.81191
11	0.02791	0.40997	0.82091
12	0.00703*	0.40266	0.63791
13	0.14820	0.39569	0.44529
14	0.24043	0.37863	0.32043
15	0.26743	0.34720	0.31194
16	0.27234	0.36920	0.30003*
17	0.27240	0.34383	0.31940
18	0.27943	0.34597	0.36611
19	0.29729	0.34709	0.37266
20	0.29906	0.34009	0.39486
21	0.34583	0.34006	0.39117
22	0.34214	0.32220	0.40283
23	0.33794	0.28651	0.40683
24	0.33434	0.28017	0.42189
25	0.33454	0.28331	0.40891
26	0.33151	0.28720	0.39991
27	0.33111	0.28180	0.40309
28	0.33400	0.26560	0.36806
29		0.26086	
30		0.23683*	

*Minimum column element. This value appears in Table IV-6B.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 2

HOLDING PERIOD 7

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	0.47080	1.14502	
2	0.29049	1.18382	
3	0.31883	0.75037	
4	0.29286	0.78211	1.34740
5	0.30311	0.72703	1.57217
6	0.28463	0.71606	1.36691
7	0.23937	0.71128	1.17320
8	0.22057	0.66408	1.01583
9	0.21829	0.64451	1.01465
10	0.23546	0.64611	1.06205
11	0.22223	0.66011	1.07106
12	0.24311	0.65280	0.88805
13	0.10194	0.64583	0.69543
14	0.00971*	0.62877	0.57057
15	0.01729	0.59734	0.56208
16	0.02220	0.61934	0.55017*
17	0.02226	0.59397	0.56954
18	0.02929	0.59611	0.61626
19	0.04714	0.59723	0.62280
20	0.04891	0.59023	0.64500
21	0.09569	0.59020	0.64131
22	0.09200	0.57234	0.65297
23	0.08780	0.53666	0.65697
24	0.08420	0.53031	0.67203
25	0.08440	0.53346	0.65906
26	0.08137	0.53734	0.65006
27	0.08097	0.53194	0.65323
28	0.08386	0.51574	0.61820
29		0.51100	
30		0.48697*	

*Minimum column element. This value appears in Table IV-6B.

ABSOLUTE VALUE OF
AVERAGE PREDICTION ERROR FOR 35 COMPANIES

TEST PERIOD 2

HOLDING PERIOD 8

Measurement Period	Observation Interval		
	Weekly	Monthly	Quarterly
1	0.44617	1.12040	
2	0.26586	1.15920	
3	0.29420	0.72574	0.97031
4	0.26823	0.75748	1.32277
5	0.27849	0.70240	1.54754
6	0.26000	0.69143	1.34228
7	0.21474	0.68666	1.14857
8	0.19594	0.63946	0.99120
9	0.19366	0.61988	0.99003
10	0.21083	0.62148	1.03743
11	0.19760	0.63548	1.04643
12	0.21849	0.62817	0.86343
13	0.07731	0.62120	0.67080
14	0.01491*	0.60414	0.54594
15	0.04191	0.57271	0.53746
16	0.04683	0.59471	0.52554*
17	0.04689	0.56934	0.54491
18	0.05391	0.57148	0.59163
19	0.07177	0.57260	0.59817
20	0.07354	0.56560	0.62037
21	0.12031	0.56557	0.62834
22	0.11663	0.54771	0.62834
23	0.11243	0.51203	0.63234
24	0.10883	0.50568	0.64740
25	0.10903	0.50883	0.63443
26	0.10600	0.51271	0.62543
27	0.10560	0.50731	0.62860
28	0.10849	0.49111	0.59357
29		0.48637	
30		0.46234*	

*Minimum column element. This value appears in Table IV-6B.

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