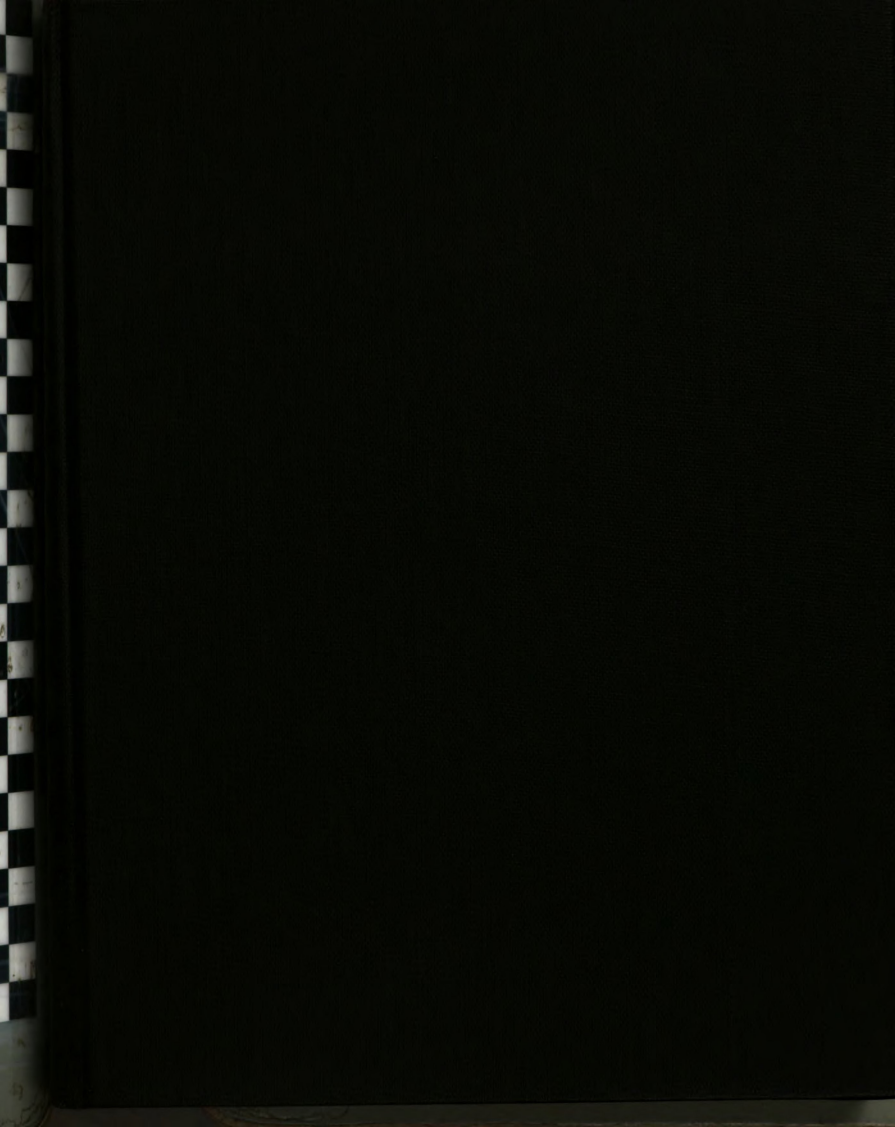
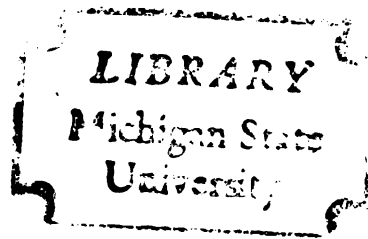






This is to certify that the

thesis entitled

THE DETERMINATION OF REDUCED ORDER MODELS FOR
LOCAL AND GLOBAL ANALYSIS OF POWER SYSTEMS

presented by

John Frederic Dorsey

has been accepted towards fulfillment

of the requirements for

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Major professor

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THE DETERMINATION OF REDUCED ORDER MODELS FOR
LOCAL AND GLOBAL ANALYSIS OF POWER SYSTEMS

By

John Frederic Dorsey

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Electrical Engineering
and Systems Science

1980

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ABSTRACT

THE DETERMINATION OF REDUCED ORDER MODELS FOR LOCAL AND GLOBAL ANALYSIS OF POWER SYSTEMS

By

John Frederic Dorsey

The nonlinear model of a power system is divided into two parts called the study group (system) and the specified group. Structural conditions on the power system are determined which cause the specified group to remain strictly coherent and respond effectively as a single generator for disturbances within the study group. Three conditions called strict synchronizing coherency (SSC), strict geometric coherence (SGC), and pseudo-coherency (PC) are identified.

The analysis is repeated for the linear model and it is shown that if the structural conditions for any of the conditions SSC, SGC, or PC hold in the linear model, then the coherent equivalent for the specified group is identical with the equivalent determined by applying the rules of modal analysis.

The singular perturbation model for the power system is then subsumed into the present theory by showing that the structural conditions on the specified group of n generators necessary to apply the singular perturbation method are exactly the structural conditions for strict synchronizing coherency.

The concept of linear decoupling is introduced as the final structural condition under which the modal and coherency equivalents

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of the specified group are identical. Three types of linear decoupling are determined. Two are classified as weak and one as strong.

A computationally efficient algorithm is found for detecting the conditions strict synchronizing coherency, strict geometric coherency and strict strong linear decoupling, using an r.m.s. coherency measure in consort with a Zero Mean, Independent, Inertially Weighted (ZMIIW) disturbance. Weak linear decoupling and pseudo-coherency conditions are intentionally not detected.

The computational algorithm distinguishes two levels of models by applying two ZMIIW disturbances. The first, a general ZMIIW disturbance of all generators detects the principal groups of machines satisfying strict synchronizing coherency, and provides a global model. The second, a ZMIIW disturbance of a subset of generators detects strict geometric coherency and strict strong linear decoupling and provides a local model for disturbances confined to the subset of generators used in the ZMIIW disturbance.

The theory and computational algorithm are satisfactorily verified by testing on the 39 Bus New England System.

For John Gauw and Thelma Gladys

The longer a man
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usually, have come about
the travelers in life

I owe a great deal
Intellectually I can never
many years of encourage
life would have been
for the last four years
Bern Schlueter, Gerald
Dr. John Kreer, Herman
Holland for continuing
me when the old dog died

The preparation of
about the excellent help
my friend David W.
graphs and figures.

ACKNOWLEDGEMENTS

The longer a man lives the more he realizes how little of his **success** is due to his own efforts, and how much his victories, large **and small**, have come about from the help of his friends, family and **fellow** travelers in life.

I owe a great deal to my family for getting me started right. **Intellectually** I can never repay Professor George H. Meyer, for his **twenty** years of encouragement, guidance and friendship, without which **my life** would have been immeasurably poorer. But for helping me **through** the last four years of labor, I would especially like to thank **Robert** Schlueter, Gerald Park, James Resh, Donald Reinhard, Robert **Barr**, John Kreer, Herman Koenig, Ronald Rosenberg, and Bruce **McFarland** for continuing to believe an old dog could learn new tricks, **even** when the old dog didn't believe it himself.

The preparation of the manuscript could not have been done **without** the excellent help of my typist Noralee Burkhardt. A **Kudu** **is due** my friend David Wilson for doing such a fine job helping with **the** graphs and figures.

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1. The Basic Problem

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CHAPTER 1

IMPROVING THE THEORETICAL BASIS FOR THE COHERENCY METHOD OF PRODUCING DYNAMIC EQUIVALENTS

I. The Basic Problem

The study of power system disturbances for the purpose of planning or security assessment requires the solution of potentially over a thousand coupled differential equations. To obtain the time domain response of these equations, even in linearized form, for as little as one second, is a very expensive computational task. Therefore, there has been a considerable effort over the last ten years to reduce the number of equations required to perform a satisfactory analysis of a power system's response to a disturbance.

Historically there have been two major approaches to this problem. In the coherency method, the full set of equations is solved for a test disturbance. The accelerations of all the generators are then checked, to see if a group of generators accelerate at the same rate, thereby maintaining their initial angle differences with respect to each other. Such a group of generators is called coherent. Each coherent group is then replaced by one equivalent, or "aggregated" generator, thus reducing the number of equations by reducing the number of generators. This reduced order model is then used to analyze the power system response for all disturbances that occur in the general area of the test disturbance used to derive the equivalent.

The modal analysis of the power system is as the external generator is retained; it is done by finding a normal form, which has no impact on variability, and fast standard operating

Both methods have advantages and certain disadvantages. The first one has an intuitive appeal because of the use of equivalent lines and generators. The second one has a degree of detail and generality which is not available to the first one.

The major shortcoming of the first method is the lack of a theoretical foundation. The second method, on the other hand, is based on the use of the transfer function method with using the understanding of the system to a better means of analysis.

The modal analysis has some shortcomings. The first one is the lack of a theoretical point of view. The second one is the lack of a better means of analysis in terms of the use of linear differential equations.

The modal analysis method defines the internal system as the area of the power system where the disturbances will occur, and everything else as the external system. A detailed model of the internal system is retained; it is the external system that is reduced. This is done by finding a linear model of the external system, transforming it to canonical form, and then eliminating canonical states, or modes, that have no impact on the internal system, using controllability, observability, and fast eigenvalue arguments. That is, one follows the standard operating procedure of linear systems theory.

Both methods have advocates; both methods have certain advantages and certain disadvantages. The coherency method has great intuitive appeal because it yields a reduced order model composed of equivalent lines and generators. Further, the models for the equivalent lines and generators can be either linear or nonlinear, and of any degree of detail desired. As will be shown, this degree of flexibility is not available in the modal analysis method.

The major shortcoming of the coherency method is that its theoretical foundations are incomplete. This causes no operational problems with using the method in its present form, but a more complete understanding of the theory of coherency equivalents would undoubtedly lead to a better means of implementing the coherency method.

The modal analysis method of producing reduced order equivalents has no shortcomings theoretically, but it has some drawbacks from the functional point of view. First of all, the reduced order model is not formulated in terms of equivalent lines and generators, but in terms of a set of linear differential equations. This is not a serious problem

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III. Prior Work that

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to those who formulate the model, but it is to the operating personnel of the utility who have to use it. The second shortcoming of the modal analysis approach is that it cannot provide a nonlinear reduced order model of the external system.

If one method of forming equivalents produced good results and the other did not, the issue would be settled. As it would happen, both methods have been used successfully. This argues strongly that both methods are utilizing the same fundamental set of structural properties of the power system to form equivalents.

If the last supposition is correct, then what is required is to show that the coherency method can be used to produce equivalents that are modally correct, that is, equivalents that will preserve both coherent properties and also retain the same modes as the modal analysis method. This is the basic problem that will be addressed in the present work. The details of the analytical approach to the problem are given in the next section.

II. Prior Work that Helps Show the Way

There are several recent developments that point to strong connections between the modal and coherency methods of forming equivalents. The most significant is the work by Schlueter [5, 6, 7, 8] which shows that by using an r.m.s. coherency measure and the appropriate statistical disturbance, rules for aggregating coherent generators can be stated that are also rules for properly eliminating modes in terms of controllability and observability arguments.

Significantly, for the appropriate statistical disturbance, the inter-generator coherency measures can be shown to depend only on the plant

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A second important result obtained by Diekmann is that the conditions on the matrix $A(t)$ because a specified t_0 may $t \geq 0$, for any t_0 incoherency can be shown to be a powerful and significant

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The results of the standpoint of the system of conditions equivalent to the modal equivalent conditions that the initial thrust structural conditions for equivalent are identical

matrix, $\underline{A}$, of the linear state model of the power system,

$$\dot{\underline{X}} = \underline{A} \underline{X} + \underline{B} \underline{U} .$$

A second important pre-cursor to the present work is the results obtained by Dicaprio and Marconato [10, 11]. These results state conditions on the structure of the power system at time $t = 0^-$ that cause a specified group of generators to remain perfectly coherent for any $t \geq 0$, for any disturbance that occurs outside the group. This coherency can be shown to hold for the nonlinear model, making it a very powerful and significant result.

Translated to the linearized model these conditions decouple the equations for the specified group of generators from those for the rest of the power system, and thereby divide the eigenvalues of the system into two groups, one associated with the equations for the specified group, the other set with the equations for the remainder of the system. This separation of eigenvalues is strong intuitive proof that when the structural conditions of Dicaprio and Marconato are satisfied, the coherency equivalent and the modal analysis equivalent are the same. This is, in fact, shown in Chapter 4.

The results of Dicaprio and Marconato have importance also from the standpoint of the strategy of the present research. The fact that one set of conditions exist which identify the coherency equivalent with the modal equivalent, leads one to speculate as to whether there are other conditions that produce the same result. In fact there are, and the initial thrust of this research is to find the complete set of structural conditions for which the modal equivalent and the coherency equivalent are identical.

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III. A Structural Outline

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- (1) Strict Symmetry
- (2) Strict Geometric
- (3) Strict Structural
- (4) Pseudo-Consistency
- (5) Weak Linearity

These conditions
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The other current work of importance is the application of singular perturbation techniques to power systems due to Kokotovic, Winkelman, Chow, Allemong, et al. [12, 13, 15]. It will be shown in Chapter 4 that this work has a natural place in the present research.

III. A Structural Outline of the Present Research

The first phase of the analysis develops a set of structural conditions on the power system, which if true, cause the modal and coherency equivalents to be identical. These conditions are called:

- (1) Strict Synchronizing Coherency
- (2) Strict Geometric Coherency
- (3) Strict Strong Linear Decoupling
- (4) Pseudo-Coherency
- (5) Weak Linear Decoupling

These conditions are all hypothetical in the sense they are never exactly satisfied in a real power system. This hardly diminishes their importance, because these five conditions are really archetypes for system coherency. Actual coherency in a real power system will be caused by near approximations to one of the five conditions or by combinations of these conditions. Thus the five archetypes have great value in conceptualizing coherent behavior in a power system.

Geometric coherency is simply a renaming of the Dicaprio-Marconato conditions. The name is chosen to reflect the structural conditions that cause the condition. Further, it is shown in Chapter 4, that in the limit as the parameter $\mu \rightarrow 0$, the singular perturbation model satisfies the condition for strict synchronizing coherency. Thus

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The second phase of the analysis demonstrates that the most important subset of the five conditions, namely strict synchronizing coherency, strict geometric coherency and strict strong linear decoupling can be detected by the r.m.s. coherency measure, in consort with a properly selected statistical disturbance. These three conditions are the most important because there are strong guarantees that if any one of these conditions exist in the linear model then that condition will also exist in the nonlinear model. The same cannot be said for pseudo-coherency and weak linear decoupling.

Further, by using the right sequence of statistical disturbances it is possible to distinguish between two levels of aggregation. A general statistical disturbance of all generators detects synchronizing coherency and determines the principle, system-wide coherent groups. This is called the global model or aggregation. A second statistical disturbance of only selected generators detects geometric coherency and strong linear decoupling. This second disturbance can be associated with a local or parochial model. The ability to distinguish global and local levels of aggregation has broad consequences that are largely beyond the reach of this present work, but have great promise for future research.

The third phase of the analysis is the integration of the theory into a formal reduction algorithm. This is done in the latter part of Chapter 5. The results of testing the algorithm on the 39 Bus New England system are excellent and follow the theory very well, indicating that the analysis is sound.

1. Background Preparation

IV. Background Preparation

The next chapter is wholly a review, in some detail, of the coherency and modal methods of producing dynamic equivalents. The reader will see some of the same phrases he has seen here. The redundancy is regrettable, unavoidable, and to some extent beneficial. To understand Chapters 3, 4 and 5 requires a good understanding, in some detail, of both the coherency and modal equivalencing techniques. Hopefully, that is what Chapter 2 provides.

1. Introduction

The construction of a model provides an excellent means of analyzing large scale electric power distribution systems. In the eastern half of the United States, the increasing problems of managing power distribution systems have placed itself in a position to meet the demand of increasing power from its users. In order to spare the risk of a major disaster with a more

Part of the problem is an increase in the difficulty of the system. There is a need for a more sophisticated model of the eastern half of the United States. The model is a first order nonlinear system. In order to use computers, the model is a first order coupled differential

CHAPTER 2

A REVIEW OF METHODS OF PRODUCING DYNAMIC EQUIVALENTS

I. Introduction

The construction of dynamic equivalents for power systems provides an excellent example of the difficulties inherent in modeling and analyzing large scale systems. To improve the reliability of the electric power distribution system, all the generators in the eastern half of the United States are interconnected. Thus if operating problems develop in one area and a particular utility finds itself in a position where it cannot produce sufficient power to meet the demand of its users, that utility can "borrow" power temporarily from its neighbors through interconnecting lines. By thus sharing the risk of operating problems the utilities provide the user with a more reliable source of power.

Part of the price paid for this improved reliability is an increase in the difficulty of analyzing the dynamic behavior of the power system. There are approximately two thousand generators in the eastern half of the United States, all connected in parallel. Modeled in the simplest way possible, every generator requires a second order nonlinear differential equation. Even with modern high speed computers, the prospect of numerically evaluating four thousand coupled differential equations is not very appetizing.

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Traditionally, the analysis of the dynamic behavior of power systems has been reduced to a manageable level by finding some simplified or lumped model for a large part of the system. This approach has great appeal in the sense that it is consistent with the usual questions asked about the stability of power systems. That is, stability is normally analyzed from the perspective of a particular utility. That utility is interested in the dynamic stability of its own generation and transmission network to disturbances, and primarily to disturbances that occur within its own network. A particular utility's interest in how disturbances impact the remainder of the system is quite secondary to its interest in how its own equipment is affected.

Because the perspective on stability is largely parochial, the natural approach to analyzing dynamic behavior has been to divide the overall system into an internal system, whose detailed behavior is of interest, and an external system whose detailed behavior is of no interest but whose effect on the internal system must be accounted for. The second step is to find a simplified model of the external system that faithfully preserves the interaction between the internal and external systems. Thus the simplification of the analysis is done at the expense of losing information about the detailed behavior of the external system. The emphasis is on the interaction between the internal and external system. Such an approach has many ramifications. Some of these will be discussed at the close of this chapter. However, the immediate task is not to question the approach but to outline how the approach has typically been implemented.

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Historically, there have been two distinct approaches to finding simplified models of power systems. One approach, called the coherency method, is based on the empirical observation that when a power system is disturbed, groups of generators tend to accelerate together maintaining the same relative voltage angles with respect to each other. This is particularly true for generators electrically distant from the disturbance. In the coherency method the response of the system to a particular disturbance is simulated and the groups of generators that accelerate together are identified. These "coherent groups" are then replaced by a single equivalent generator. The size and response characteristics of the equivalent generator are chosen to best represent the aggregate behavior of the group. The most obvious, and glaring, defect of the coherency method is that it requires the numerical integration of the differential equations of the entire system, which is precisely the problem one wishes to avoid. Some of the curse is removed by the empirical observation that the coherency information can be obtained from a linearized model of the system.

The other approach to forming dynamic equivalents for power systems is to obtain a linearized model of the external system and then reduce the order of this linear model by bringing to bear the results of linear systems theory, in particular the concepts of fast modes, observability and controllability. This approach to the problem of power system equivalents is called the modal method, since the model reduction is accomplished by discarding canonical state variables or "modes". The modal method has the obvious defect

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The two methods of constructing power system equivalents cursorily outlined above represent the two main lines of research. They have developed in almost complete isolation, and appear to be unrelated. However, the fact that they both yield good equivalents might lead one to speculate that the two methods are related. And, in fact, some recent work [5, 6, 7, 8] indicates that, for the proper choice of coherency measure, the two methods are indeed closely related.

One of the goals of the present research is to further strengthen the connection between these two methods of forming dynamic equivalents, and to show that, under certain circumstances, the two methods yield identical equivalents. To reach this goal requires a thorough understanding of both the coherency and modal methods of producing power system dynamic equivalents. The rest of this chapter provides this necessary background. In Section II a linear model of the power system is developed. This linear model is used throughout the subsequent development. Section III describes the coherency approach to dynamic equivalents and Section IV does the same for the modal analysis method.

II. The Form of the Linear Model

A linear power system model is introduced at this point. This model will be used throughout the subsequent analysis. It represents the power system in terms of a set of ordinary, linear differential equations for the electromechanical motion of the generators plus a

algebraic equation

of the

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s$$

$$\frac{d\omega_i}{dt} = \frac{1}{H_i} (P_{mi} - P_{ei})$$

where

i is the subscript

δ_i indicates the

specified (or

H_i is the inertia

ω_i is the speed

ω_s is the rotor

P_{mi} is the damping

P_{ei} is the synchron

$\Delta\omega_i$ is the change

$\Delta\delta_i$ is the change

The equation

of the network are:

$$\begin{bmatrix} \Delta P_G \\ \Delta P_L \end{bmatrix} = \begin{bmatrix} \frac{\partial P_G}{\partial \delta} \\ \frac{\partial P_L}{\partial \delta} \end{bmatrix} \Delta \delta$$

where

$$\underline{P}_G^T = [P_{G1}, P_{G2}, \dots, P_{GN}]$$

$$\underline{P}_L^T = [P_{L1}, P_{L2}, \dots, P_{LN}]$$

set of algebraic equations for the power flows among the generator and load busses of the system. The differential equations are

$$M_i \frac{d \Delta \omega_i}{dt} = \Delta PM_i - \Delta PG_i - D_i \Delta \omega_i \quad i = 1, 2, \dots, N \quad (2.1a)$$

$$\frac{d \Delta \delta_i}{dt} = 2\pi f_0 \Delta \omega_i \quad (2.1b)$$

where

- i is the subscript for generator i in an N generator system
- Δ indicates that the variable is a small deviation about some specified (pre-calculated) steady-state operating point
- M_i is the inertia constant of generator i in p.u.
- $\Delta \omega_i$ is the speed deviation of generator i
- $\Delta \delta_i$ is the rotor angle deviation of generator i (in radians)
- D_i is the damping constant of generator i (in p.u.)
- f_0 is the synchronous frequency of the power system
- ΔPM_i is the change in mechanical input power at generator i in p.u.
- ΔPG_i is the change in electrical output power at generator i in p.u.

The equations that represent the power flows in the power system network are:

$$\begin{bmatrix} \underline{\Delta PG} \\ \underline{\Delta PL} \end{bmatrix} = \begin{bmatrix} \frac{\partial \underline{PG}}{\partial \underline{\delta}} & \frac{\partial \underline{PG}}{\partial \underline{\theta}} \\ \frac{\partial \underline{PL}}{\partial \underline{\delta}} & \frac{\partial \underline{PL}}{\partial \underline{\theta}} \end{bmatrix} \begin{bmatrix} \underline{\Delta \delta} \\ \underline{\Delta \theta} \end{bmatrix} \quad (2.1c)$$

where

$$\underline{PG}^T = [PG_1, PG_2, \dots, PG_N]$$

$$\underline{PL}^T = [PL_1, PL_2, \dots, PL_Q]$$

$$\dot{\underline{x}} = [\dot{x}_1, \dot{x}_2]$$

$$\dot{\underline{x}} = [\dot{x}_1, \dot{x}_2]$$

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$$\underline{\delta}^T = [\delta_1, \delta_2, \dots, \delta_N]$$

$$\underline{\theta}^T = [\theta_1, \theta_2, \dots, \theta_Q] .$$

Some comments are in order about these equations. First, there are two first order differential equations required to represent the dynamics of one generator. This is essentially the simplest model of these dynamics that there is. The behavior of the generator can be modeled in much greater detail to account for all the electrical and mechanical phenomena at work. Even a modest attempt at accounting for these phenomena results in a seventh or eighth order differential equation for each generator. In dealing with large systems of a thousand or more generators this obviously results in an enormous number of differential equations. Thus for the analysis of large power systems there is really no choice but to use the simplest representation possible. The simpler representation used here neglects the effects of exciter and turbine governor control, at least in detail. The damping constant D_i serves to represent in a general way the overall effect of these control systems. In a power system the various control systems tend to dampen the response of the power system without greatly affecting its natural frequencies [2]. In Chapter 4, it will be shown that the assumption of uniform damping, i.e. the ratio D_i/M_i is the same for all generators, leads to a precise formulation of this empirical idea. The assumption of uniform damping will be made shortly when the equations for the power system are put in state-space form.

The second observation worth noting is that the real power equations can be decoupled from the reactive power equations. This

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$$\begin{bmatrix} \frac{\partial P_G}{\partial \delta} \\ \frac{\partial P_L}{\partial \delta} \\ \frac{\partial P_G}{\partial \delta} \\ \frac{\partial P_L}{\partial \delta} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_G}{\partial \delta} & \frac{\partial P_G}{\partial \delta} \\ \frac{\partial P_L}{\partial \delta} & \frac{\partial P_L}{\partial \delta} \\ \frac{\partial P_G}{\partial \delta} & \frac{\partial P_G}{\partial \delta} \\ \frac{\partial P_L}{\partial \delta} & \frac{\partial P_L}{\partial \delta} \end{bmatrix}$$

are

$\underline{P}_G, \underline{\delta}$ real
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$\underline{P}_L, \underline{Q}_L$ real
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$$\underline{P}_L = \frac{\partial P_L}{\partial \delta} \underline{\delta} +$$

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is a commonly made approximation, based on the following reasoning.

The changes in the complex voltages and power injections at the network generator and load buses may be expressed as:

$$\begin{bmatrix} \Delta \underline{PG} \\ \Delta \underline{PL} \\ \Delta \underline{QG} \\ \Delta \underline{QL} \end{bmatrix} = \begin{bmatrix} \frac{\partial \underline{PG}}{\partial \underline{\delta}} & \frac{\partial \underline{PG}}{\partial \underline{\theta}} & \frac{\partial \underline{PG}}{\partial \underline{E}} & \frac{\partial \underline{PG}}{\partial \underline{V}} \\ \frac{\partial \underline{PL}}{\partial \underline{\delta}} & \frac{\partial \underline{PL}}{\partial \underline{\theta}} & \frac{\partial \underline{PL}}{\partial \underline{E}} & \frac{\partial \underline{PL}}{\partial \underline{V}} \\ \frac{\partial \underline{QG}}{\partial \underline{\delta}} & \frac{\partial \underline{QG}}{\partial \underline{\theta}} & \frac{\partial \underline{QG}}{\partial \underline{E}} & \frac{\partial \underline{QG}}{\partial \underline{V}} \\ \frac{\partial \underline{QL}}{\partial \underline{\delta}} & \frac{\partial \underline{QL}}{\partial \underline{\theta}} & \frac{\partial \underline{QL}}{\partial \underline{E}} & \frac{\partial \underline{QL}}{\partial \underline{V}} \end{bmatrix} \begin{bmatrix} \Delta \underline{\delta} \\ \Delta \underline{\theta} \\ \Delta \underline{E} \\ \Delta \underline{V} \end{bmatrix}$$

where

$\underline{PG}$, $\underline{QG}$ real and reactive power injections at internal generator buses - p.u.

$\underline{PL}$, $\underline{QL}$ real and reactive power residuals at the load buses - p.u.

$\underline{E}$, $\underline{\delta}$ voltages and angles at generator internal buses

$\underline{V}$, $\underline{\theta}$ voltages and angles at load buses.

Now the first two equations can be written in the form

$$\Delta \underline{PG} = \frac{\partial \underline{PG}}{\partial \underline{\delta}} \Delta \underline{\delta} + \frac{\partial \underline{PG}}{\partial \underline{\theta}} \Delta \underline{\theta} + \frac{\partial \underline{PG}}{\partial \underline{E}} \Delta \underline{E} + \frac{\partial \underline{PG}}{\partial \underline{V}} \Delta \underline{V}$$

$$\Delta \underline{PL} = \frac{\partial \underline{PL}}{\partial \underline{\delta}} \Delta \underline{\delta} + \frac{\partial \underline{PL}}{\partial \underline{\theta}} \Delta \underline{\theta} + \frac{\partial \underline{PL}}{\partial \underline{E}} \Delta \underline{E} + \frac{\partial \underline{PL}}{\partial \underline{V}} \Delta \underline{V}$$

To first order, the power flows are largely dependent on the voltage angles at the generator and load buses, not the voltage magnitudes.

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$$\frac{\partial \mathcal{L}}{\partial \mathbf{g}} = \frac{\partial \mathcal{L}}{\partial \mathbf{g}} \cdot \mathbf{1}$$

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$$\hat{\omega}_i = \omega_i$$

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$$\hat{\omega}_i - \omega_N = \left(\frac{\partial \mathcal{L}}{\partial \mathbf{M}_i} \right)$$

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 the right-hand side

This is the same as saying $\partial \underline{PG} / \partial \underline{V} = \partial \underline{PL} / \partial \underline{V} = 0$. Further, it can be assumed that the generator voltages behind the transient reactances are constant. That is, $\Delta \underline{E} = 0$. These two approximations result in

$$\Delta \underline{PG} = \frac{\partial \underline{PG}}{\partial \underline{\delta}} \Delta \underline{\delta} + \frac{\partial \underline{PG}}{\partial \underline{\theta}} \Delta \underline{\theta}$$

$$\Delta \underline{PL} = \frac{\partial \underline{PL}}{\partial \underline{\delta}} \Delta \underline{\delta} + \frac{\partial \underline{PL}}{\partial \underline{\theta}} \Delta \underline{\theta}$$

These are the decoupled equations for real power flow.

With this background information, and assuming uniform damping, the equations for the power system can now be put in state-space model form as follows.

First, a reference frame is chosen for the angles and speeds of the generators. The reference frame chosen is the generator angle of the Nth machine, $\Delta \delta_N$. That is, one establishes N-1 angle differences:

$$\Delta \hat{\delta}_i = \Delta \delta_i - \Delta \delta_N, \quad i = 1, 2, \dots, N-1$$

and N-1 speed differences

$$\Delta \hat{\omega}_i = \Delta \omega_i - \Delta \omega_N, \quad i = 1, 2, \dots, N-1.$$

Next consider the N equations of the form represented by (2.1a), and subtract equation N from equation i to get:

$$\dot{\Delta \omega}_i - \dot{\Delta \omega}_N = \left(\frac{\Delta PM_i}{M_i} - \frac{\Delta PG_i}{M_i} \right) - \left(\frac{\Delta PM_N}{M_N} - \frac{\Delta PG_N}{M_N} \right) - \left(\frac{D_i}{M_i} \Delta \omega_i - \frac{D_N}{M_N} \Delta \omega_N \right).$$

Now the left hand side of this equation is simply $\dot{\Delta \hat{\omega}}_i$. However, the right-hand side contains all N speeds. Making the assumption

uniform damping

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$$\frac{d}{dt}$$

the $k-1$ equation

$$\frac{d}{dt} \hat{\omega}_1 = \frac{1}{M_1} (\omega_1^2 M_1)$$

where $\alpha =$

To make the

written in terms of

$$\begin{bmatrix} \omega_1^2 M_1 \\ \omega_1^2 M_1 \end{bmatrix}$$

where, notational

$$\hat{\omega}_1$$

$$\hat{\omega}_k$$

and

$$\hat{\omega}^T = \omega \hat{\omega}_1,$$

of uniform damping allows the last term on the right hand side to be written $\sigma(\Delta\omega_i - \Delta\omega_N) = \sigma(\Delta\hat{\omega}_i)$, where $\sigma = D_i/M_i = D_N/M_N$. Thus under the assumption of uniform damping, the power system can be represented by $2N-2$ differential equations, specifically the $N-1$ defining equations of the form

$$\frac{d}{dt} \Delta\hat{\omega}_i = \Delta\hat{\omega}_i \quad i = 1, 2, \dots, N-1 \quad (2.2a)$$

and $N-1$ equations of the form,

$$\frac{d}{dt} \Delta\hat{\omega}_i = \frac{1}{M_i} (\Delta PM_i - \Delta PG_i) - \frac{1}{M_N} (\Delta PM_N - \Delta PG_N) - \sigma \Delta\hat{\omega}_i, \quad i = 1, 2, \dots, N-1$$

where $\sigma = \frac{D_i}{M_i}$, $i = 1, 2, \dots, N$.

To make the model complete, the power equation must be written in terms of the new variables, i.e.

$$\begin{bmatrix} \Delta PG \\ \Delta PL \end{bmatrix} = \begin{bmatrix} \frac{\partial PG}{\partial \hat{\delta}} & \frac{\partial PG}{\partial \hat{\theta}} \\ \frac{\partial PL}{\partial \hat{\delta}} & \frac{\partial PL}{\partial \hat{\theta}} \end{bmatrix} \begin{bmatrix} \Delta\hat{\delta} \\ \Delta\hat{\theta} \end{bmatrix} \quad (2.2c)$$

where, notationally,

$$\hat{\delta}_i = \delta_i - \delta_N \quad i = 1, 2, \dots, N-1$$

$$\hat{\theta}_k = \theta_k - \delta_N \quad k = 1, 2, \dots, Q$$

and

$$\underline{\hat{\delta}}^T = [\hat{\delta}_1, \hat{\delta}_2, \dots, \hat{\delta}_{N-1}], \quad \underline{\hat{\theta}}^T = [\hat{\theta}_1, \hat{\theta}_2, \dots, \hat{\theta}_Q].$$

The next step
 can be accomplished
 (iii) for $\underline{\hat{x}}$ and
 the equation in (2)

$$\underline{\dot{x}} = \frac{\partial \underline{PG}}{\partial \underline{\hat{x}}} \underline{\hat{x}} + \frac{\partial \underline{PG}}{\partial \underline{u}}$$

can be rewritten

$$\underline{\dot{x}} = \left(\frac{\partial \underline{PG}}{\partial \underline{\hat{x}}} - \frac{\partial \underline{PG}}{\partial \underline{u}} \left[\frac{\partial \underline{u}}{\partial \underline{\hat{x}}} \right] \right) \underline{\hat{x}} + \frac{\partial \underline{PG}}{\partial \underline{u}} \left[\frac{\partial \underline{u}}{\partial \underline{\hat{x}}} \right] \underline{\hat{x}}$$

or

$$\underline{\dot{x}} =$$

The state model

$$\dot{\underline{x}}(t) =$$

or follows immediately

$$\dot{\underline{x}} = \begin{bmatrix} \underline{\dot{x}} \\ \underline{\dot{u}} \end{bmatrix}$$

$$\dot{\underline{x}} = \begin{bmatrix} \underline{A}^{(N-1) \times (N-1)} \\ \underline{0} \quad \underline{I} \end{bmatrix} \underline{x}$$

The next step is to express $\Delta \underline{PG}$ in terms of $\Delta \hat{\underline{\delta}}$ and $\Delta \underline{PL}$. This can be accomplished by solving the second power equation in (2.2c) for $\Delta \hat{\underline{\theta}}$ and substituting this expression for $\Delta \hat{\underline{\theta}}$ into the first equation in (2.2c). The result is:

$$\Delta \underline{PG} = \frac{\partial \underline{PG}}{\partial \hat{\underline{\delta}}} \Delta \hat{\underline{\delta}} + \frac{\partial \underline{PG}}{\partial \hat{\underline{\theta}}} \left[\frac{\partial \underline{PL}}{\partial \hat{\underline{\theta}}} \right]^{-1} \left[\Delta \underline{PL} - \frac{\partial \underline{PL}}{\partial \hat{\underline{\delta}}} \Delta \hat{\underline{\delta}} \right].$$

This can be rewritten as

$$\Delta \underline{PG} = \left\{ \frac{\partial \underline{PG}}{\partial \hat{\underline{\delta}}} - \frac{\partial \underline{PG}}{\partial \hat{\underline{\theta}}} \left[\frac{\partial \underline{PL}}{\partial \hat{\underline{\theta}}} \right]^{-1} \frac{\partial \underline{PL}}{\partial \hat{\underline{\delta}}} \right\} \Delta \hat{\underline{\delta}} + \frac{\partial \underline{PG}}{\partial \hat{\underline{\theta}}} \left[\frac{\partial \underline{PL}}{\partial \hat{\underline{\theta}}} \right]^{-1} \Delta \underline{PL}$$

or,

$$\Delta \underline{PG} = \underline{T} \Delta \hat{\underline{\delta}} - \underline{L} \Delta \underline{PL}.$$

The state model,

$$\dot{\underline{X}}(t) = \underline{A} \underline{X}(t) + \underline{B} \underline{U}(t) \quad (2.2d)$$

then follows immediately with

$$\underline{X} = \begin{bmatrix} \Delta \hat{\underline{\delta}} \\ \text{---} \\ \Delta \hat{\underline{\omega}} \end{bmatrix} \quad \underline{U} = \begin{bmatrix} \Delta \underline{PM} \\ \text{---} \\ \Delta \underline{PL} \end{bmatrix}$$

$$\underline{A} = \begin{bmatrix} \underline{0}^{(N-1) \times (N-1)} & 2\pi f_0 \underline{I} \\ -\hat{\underline{M}} \underline{I} & -\sigma \underline{I}^{(N-1) \times (N-1)} \end{bmatrix} \quad \underline{B} = \begin{bmatrix} \underline{0} & \underline{0} \\ \text{---} \\ \hat{\underline{M}} & \hat{\underline{M}}_L \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

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$$\hat{\underline{M}} = \begin{bmatrix} \frac{1}{M_1} & & & & -\frac{1}{M_N} \\ & \frac{1}{M_2} & & & \vdots \\ & & \ddots & & \vdots \\ & & & \frac{1}{M_{N-2}} & -\frac{1}{M_N} \\ 0 & & & \frac{1}{M_{N-1}} & -\frac{1}{M_N} \end{bmatrix}$$

$$\underline{T} = \begin{bmatrix} \frac{\partial \underline{PG}}{\partial \underline{\hat{\theta}}} - \frac{\partial \underline{PG}}{\partial \underline{\hat{\theta}}} \left[\frac{\partial \underline{PL}}{\partial \underline{\hat{\theta}}} \right]^{-1} & \frac{\partial \underline{PL}}{\partial \underline{\hat{\theta}}} \end{bmatrix} \quad \underline{L} = -\frac{\partial \underline{PG}}{\partial \underline{\hat{\theta}}} \left[\frac{\partial \underline{PL}}{\partial \underline{\hat{\theta}}} \right]^{-1}$$

This is the basic linear model of the power system that will be used throughout the following development. In Chapter 4, it will be shown that there is a more convenient reference than the Nth generator, but this change of reference will not alter the basic form of the linear model. With rare exceptions, the first m generators will be considered the internal system, and the last $n = N - m$ generators, the external system. That is, the first m generators correspond to the subsystem where the disturbances occur and whose detailed behavior is to be studied. The last n generators correspond to the remaining part of the grid to which the internal system is connected.

The linear model plays an important part in both coherency and modal analysis, although the role in the two methods is quite different. If the dynamic equivalent is produced using the modal analysis technique, the internal system is represented in detail, with each generator being described by high order (seven or eight)

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III. Reduced Order

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linear or nonlinear differential equations. The external system is divided into subsections and each subsection is represented by a linear model. Linear system theory, in particular the concepts of fast modes, controllability and observability, is then used to reduce the order of each linearized subsection of the external system. In the coherency method, a linearized model of the entire power system, internal system and external system, is subjected to a disturbance and the coherent generator groups determined. Those generators that are coherent are then replaced by a single "equivalent" machine, and a linear or nonlinear model can be produced. Thus in the coherency approach the linearized model is used only to determine which generators swing together in response to a disturbance. This coherent behavior could just as well be determined using a nonlinear model of the system, but this would, of course, be computationally much more expensive. The justification for assuming that the linear model of the power system captures the coherency behavior of the system is given in reference [2] and is discussed in the next subsection.

III. Reduced Order Equivalents by the Method of Coherency

The coherency method of forming a dynamic equivalent of a power system is based on the intuitive idea that a group of generators that maintain the same relative voltage angles to each other in response to a disturbance act, in principle, like one large generator. In the strictest sense, two generators, or buses, are defined as coherent if the ratio of their complex bus voltages is constant over time. In practice this definition of coherency has been relaxed to only requiring that the voltage angle between two

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buses remain constant over time. The principal work in coherency has been done by Podmore and Germond [2, 3, 14]. This section contains a brief overview of the main points of reference [2], as they pertain to the present research.

The procedure for forming power system dynamic equivalents as outlined in reference [2] can be summarized into the following steps.

- (1) A disturbance is applied to the power system and the coherent groups determined. The determination of coherency is done at the terminal buses of the generator. It is assumed that the fictitious internal generator buses are also coherent. This assumption is crucial to the dynamic aggregation of the generators into a single equivalent generator.
- (2) The static network equations are reduced in order by first replacing all the coherent generator terminal buses by a single equivalent bus, before eliminating load buses. This replacement allows the order of the network representation to be greatly reduced. And the network reduction and dynamic equivalencing problems are decoupled. In addition, the reduced network representation is applicable to whatever generator model, complicated or simple, that one cares to choose.
- (3) The generators at the coherent buses are replaced by one or a small number of equivalent machines at the equivalent terminal bus. One equivalent machine will be used at the bus if the set of coherent generators are similar enough in response characteristics. A "small number" of generators will be used if the coherent generators are of very different response characteristics. For instance, if the set of coherent generators includes both steam and hydro units, then two equivalent generators will be used at the bus since it has been found empirically that a satisfactory single machine equivalent for a group of generators that include both steam and hydro cannot, usually, be found. The equivalencing procedure assumes that each generator in the group (or subgroup) to be equivalenced can be represented by a block diagram of transfer functions of the same form. The equivalent machine is assumed to have a block diagram of the same form and an identification procedure,

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based on least square error is used to identify the parameters that best match the frequency response of the equivalent machine to the cumulative frequency response of the coherent group (or subgroup). [2]

The determination of the coherent groups is done using the linearized model of the power system described in Section II of this chapter. In early work on coherency, Podmore and Germond used a complete nonlinear model of the power system to determine the coherent groups. However, experience showed that a linearized model of the power system would suffice. The justification, based on this practical experience is the following. The assumption that a linear model can be used implies that coherency can be detected by small disturbances, or viewed another way, that the length of the disturbance does not change the coherent groups. This assumption is based on the observation that if a certain bus is faulted, the coherency behavior is not significantly changed by increasing the fault clearing time. Since the length of the fault essentially determines the size of the disturbance by determining how much energy is put into the acceleration of the generator, the linear model will suffice. The second important assumption is that in the linear model, the very simple constant voltage behind a transient reactance model of the generators can be used. The justification for this is based on the empirical evidence that the amount of detail in the generating unit model has some effect upon the swing curve, particularly the damping, but does not radically affect the more basic characteristics such as natural frequencies and mode shapes. Reference [2] illustrates this argument by showing that good estimates of system modes and mode shapes result from considering the classical constant voltage behind transient reactance

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$$\begin{bmatrix} V_1 \\ \vdots \\ V_m \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} Y_{11} \\ \vdots \\ Y_{m1} \\ \hline Y_{(m+1)} \\ \vdots \\ Y_{n1} \end{bmatrix}$$

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 be coherent group

model of the generators and ignoring excitation system and turbine-governor representations. This greatly simplifies the differential equation representation by reducing the number of first order differential equations per machine to two, whereas with the excitation and turbine-governor represented the number of first order equations per machine might very well be seven to ten.

Once the coherent groups are determined, step 2 is the reduction of the network. This proceeds in two stages. First, each coherent group of buses is replaced by a single equivalent bus. Second, node elimination techniques are used to remove as many load buses as possible.

The replacement of a group of coherent buses by a single coherent bus is done under a power conservation assumption. That is, **the power flow** at each boundary bus of the coherent group is **conserved**, and the power production of the coherent group is also **conserved**.

Consider the following algebraic network equations

$$\begin{bmatrix} I_1 \\ \vdots \\ I_m \\ I_{m+1} \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & Y_{1m} & | & Y_{1(m+1)} & \cdots & Y_{1n} \\ \vdots & & & & & & \\ Y_{m1} & \cdots & Y_{mm} & | & Y_{m(m+1)} & \cdots & Y_{mn} \\ \hline Y_{(m+1)1} & \cdots & Y_{(m+1)m} & | & Y_{(m+1)(m+1)} & \cdots & Y_{(m+1)n} \\ \vdots & & & & \vdots & & \\ Y_{n1} & \cdots & Y_{nm} & | & Y_{n(m+1)} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_m \\ V_{m+1} \\ \vdots \\ V_n \end{bmatrix}$$

where the first m equations refer to the buses on the boundary of the coherent group and the last $n-m$ equations are for the buses of

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$$\begin{bmatrix} \vdots \\ \vdots \\ \vdots \\ \vdots \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ \vdots & \vdots \\ Y_{m1} & Y_{m2} \\ Y_{t1} & Y_{t2} \end{bmatrix}$$

decreasing the area

leads to

the coherent group. (Only one coherent group is considered because the extension to several coherent groups is obvious and straightforward.) The current I_b , for a boundary bus is:

$$I_b = \sum_{k=1}^m Y_{bk} V_k + \sum_{k=m+1}^n Y_{bk} V_k$$

the complex power injection at bus b is given by

$$S_b = I_b^* V_b, \text{ where } * \text{ denotes the complex conjugate.}$$

Substituting the expression for I_b into the formula for the complex power results in:

$$S_b = \sum_{k=1}^m V_k^* Y_{bk}^* V_b + \sum_{k=m+1}^n V_k^* Y_{bk}^* V_b. \quad (2.3a)$$

The first term is the contribution to the power at bus b by other buses in the boundary, and the second term is the power contribution from the generators of the coherent group.

Now, the algebraic network equations with the coherent buses replaced by one equivalent bus are:

$$\begin{bmatrix} I_1 \\ \vdots \\ I_m \\ \hline I_t \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1m} & | & Y_{1t} \\ \vdots & & & & & \vdots \\ Y_{m1} & \cdots & Y_{mm} & | & Y_{mt} \\ \hline Y_{t1} & \cdots & Y_{tn} & | & Y_{tt} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_m \\ \hline V_t \end{bmatrix}$$

Repeating the analysis which was done for the unreduced equations leads to

$$S_b = \sum_{k=1}^m V_k$$

the first term in

uses in the bound

the second term in

the power conserva

that is,

$$V_t^* Y_{bt}^* V_b =$$

the reduces to:

$$Y_{bt}$$

is the equivalent

is at the equivalent

the

$$|V_t| = \frac{1}{n-1}$$

is for the equivalent

determined. The n

number of the buses

is Y_{tt} . Let c

number of the coherent

the unreduced

$$S_b = \sum_{k=1}^m V_k^* Y_{bk}^* V_b + V_t^* Y_{bt}^* V_b . \quad (2.3b)$$

The first term in (2.3b) is the power contributed at b by the other buses in the boundary and is identical to the first term in (2.3a).

The second term in (2.3b) is the power of the equivalent bus and for power conservation must be set equal to the second term in (2.3a).

That is,

$$V_t^* Y_{bt}^* V_b = \sum_{k=m+1}^n V_k^* Y_{bk}^* V_b$$

which reduces to:

$$Y_{bt} = \sum_{k=m+1}^n \frac{V_k}{V_t} Y_{bk} . \quad (2.3c)$$

Thus the equivalent impedance Y_{bt} can be determined once the voltage, V_t , at the equivalent bus is selected. This voltage is normally taken to be

$$|V_t| = \frac{1}{n-m} \sum_{k=m+1}^n |V_k| ; \quad \theta_t = \frac{1}{n-m} \sum_{k=m+1}^n \theta_k .$$

So far the equivalent Y -bus elements Y_{bt} , $b = 1, \dots, m$ have been determined. The next step is to make use of the conservation of power of the buses in the coherent group to determine Y_{tb} , $b = 1, \dots, m$ and Y_{tt} . Let c denote a bus in the coherent group. Then the total power of the coherent group is:

$$S_c = \sum_{c=m+1}^n I_c^* V_c .$$

Using the unreduced network equations I_c can be written as:

$$I_c = \sum_{b=1}^m \sum_{c=n+1}^n$$

Substituting this e

rearranging the o

$$S_c = \sum_{b=1}^m \sum_{c=n+1}^n$$

Substituting this same

is

$$S_c = \sum_{b=1}^m \sum_{c=n+1}^n Y_{bt}^* Y_{ct}^*$$

The first

inherent group

the terms gives

$$\sum_{b=1}^m \sum_{c=n+1}^n$$

which reduces to:

$$Y_{tb}$$

The second term in

inherent group, or

leads to:

$$Y_{tt} = \sum_{b=n+1}^n$$

Equations (2.3c), (

effects of the red

$$I_c = \sum_{b=1}^m Y_{cb} V_b + \sum_{b=m+1}^n Y_{cb} V_b .$$

Substituting this expression into the power equation for S_c , and interchanging the order of summation results in

$$S_c = \sum_{b=1}^m \sum_{c=m+1}^n V_b^* Y_{cb}^* V_c + \sum_{b=m+1}^n \sum_{c=m+1}^n V_b^* Y_{cb}^* V_c . \quad (2.4a)$$

Repeating this same line of analysis for the reduced network results in

$$S_c = \sum_{b=1}^m V_b^* Y_{bt}^* V_t + V_t^* Y_{tt}^* V_t . \quad (2.4b)$$

The first term in (2.4a) or (2.4b) is the power flow from the coherent group, or the equivalent, to the boundary. Equating these terms gives

$$\sum_{b=1}^m \sum_{c=m+1}^n V_b^* Y_{cb}^* V_c = \sum_{b=1}^m V_b^* Y_{tb}^* V_t$$

which reduces to:

$$Y_{tb} = \sum_{c=m+1}^n Y_{cb} \left(\frac{V_c}{V_t} \right)^* , \quad b = 1, 2, \dots, m. \quad (2.4c)$$

The second term in (2.4a) or (2.4b) is the internal power of the coherent group, or the power of the equivalent. Equating these terms leads to:

$$Y_{tt} = \sum_{b=m+1}^n \sum_{c=m+1}^n \frac{V_b}{V_t} Y_{cb} \left(\frac{V_c}{V_t} \right)^* . \quad (2.4d)$$

Equations (2.3c), (2.4c) and (2.4d) give expressions for the equivalent elements of the reduced bus admittance matrix, under the assumptions

power conservati

In fact, the
transformer
equivalent to each
angular difference

Podmore and
frequency reduction
total results just
it is used to ill

Step 1. The
through V_t is e
previous analysis
test group) is c
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in Figure 2.
operations, the ra
and no circulating

Step 2. The
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the rest of the
may have been
the generator
total buses may e
affect them. Thu
between bus
equivalent s

of power conservation. Note that $Y_{tb} \neq Y_{bt}$.

In fact, the form of Y_{bt} and Y_{tb} indicates that a phase shifting transformer has been introduced into the line from the equivalent to each boundary bus b . The phase shift is half the angular difference of Y_{bt} and Y_{tb} [2].

Podmore and Germond provide a physical interpretation of the coherency reduction that gives some valuable insight into the mathematical results just derived. The simple network of Figures 2.1a to 2.1e is used to illustrate the procedure.

Step 1. The voltage, V_t , at the equivalent bus is defined. Although V_t is essentially arbitrary the definition given in the previous analysis is usually used. Each terminal bus (of the coherent group) is connected through an ideal transformer with complex turns ratio to the equivalent bus. The turns ratio is directed as shown in Figure 2.1b and calculated as $\tilde{a}_k = V_k/V_t$. Under coherent conditions, the ratio $\tilde{a}_k$ is a constant for each bus in the group and no circulating power flows through any of the phase shifters.

Step 2. The generator terminal buses, of the coherent group will generally be connected radially through a step-up transformer to the rest of the network. However, in some cases the low voltage bus may have been eliminated by combining the transformer reactance with the generator internal reactance. In this circumstance non-radial buses may exist in the coherent group and a common branch may connect them. Thus, any intragroup branch, in this example the branch between buses 2 and 3 in Figure 2.1b, is removed by replacing it by equivalent shunt admittances. Consider the current flow in

branch between

Because of
constant and
of either V_2

Effect of the
 $\frac{1}{2} V_2 V_{23}$ at
the network

Step 3. T
buses are
in Figure 2.
transfer. The
tap ratio

Step 4. T
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the ideal

At this po
delete it, load
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The unreduce

the branch between buses 2 and 3.

$$I_{23} = (V_2 - V_3)Y_{23}.$$

Because of the assumption of coherency in the group, V_2/V_3 is a constant and the current, I_{23} , can be written as a linear function of either V_2 or V_3 . That is,

$$I_{23} = V_3 \left(\frac{V_2}{V_3} - 1 \right) Y_{23} \quad \text{or}$$

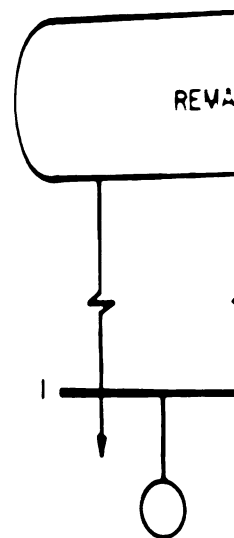
$$I_{23} = V_2 \left(1 - \frac{V_3}{V_2} \right) Y_{23}.$$

The effect of the branch can be replaced by a shunt admittance $(1 - V_3/V_2)Y_{23}$ at bus 2 and $(1 - V_2/V_3)Y_{23}$ at bus 3. Figure 2.1c shows the network after the intragroup branch is removed.

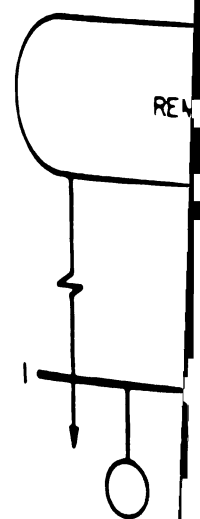
Step 3. The generation, load and shunt admittances on the coherent buses are transferred to the equivalent bus and summed as shown in Figure 2.1d. The generation and load are not modified by the transfer. The shunt admittance is scaled to account for the off-nominal tap ratio of the ideal transformer.

Step 4. The original coherent buses are eliminated by a series combination of the original branch and the ideal transformer. If several original branches connect to the eliminated bus, (see bus 2), the ideal transformer is combined with each of them.

At this point the network reduction is half complete. To complete it, load buses are eliminated. Algebraically, the elimination of load buses reduces the order of the bus admittance matrix Y_{Bus} . The unreduced Y_{Bus} is a very sparse matrix. Initially, the



CONFIGURATION OF COHERE



COHERENT GENERATOR E
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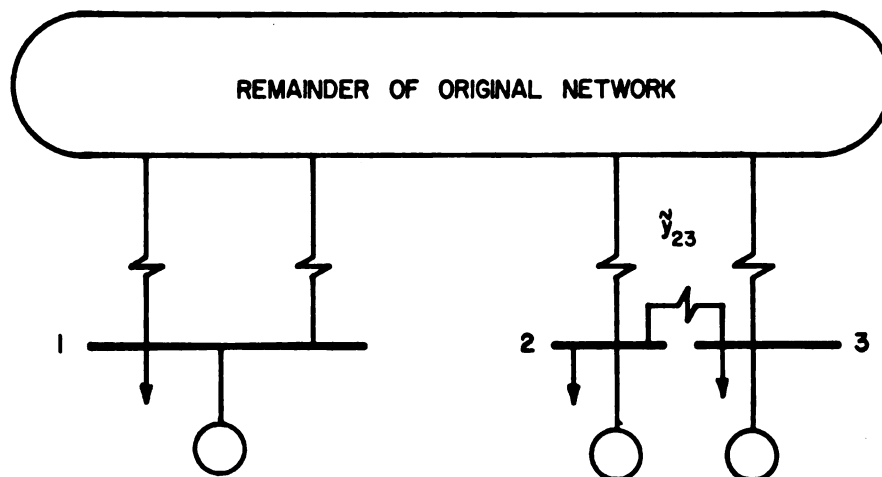


FIGURE 2-1a
CONFIGURATION OF COHERENT GENERATOR BUSES IN ORIGINAL NETWORK

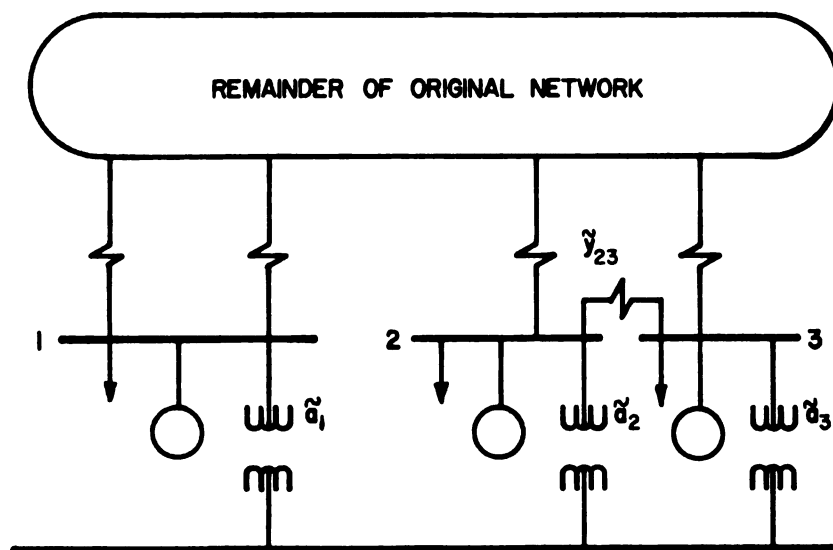
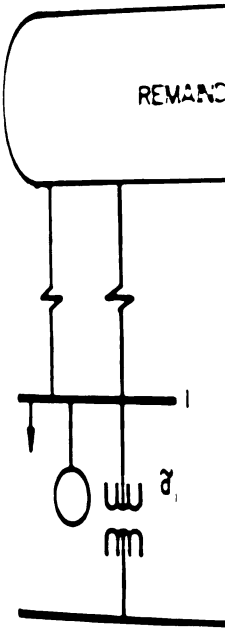
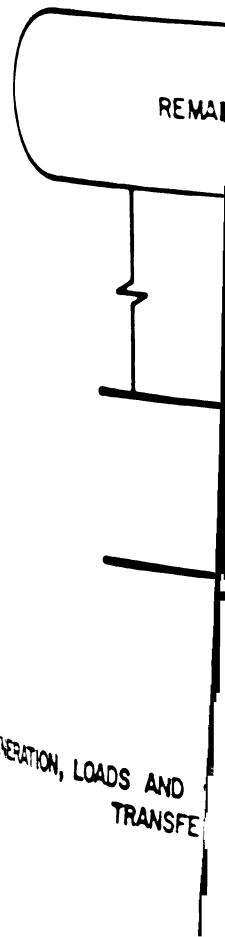


FIGURE 2-1b
COHERENT GENERATOR BUSES ARE CONNECTED TO AN EQUIVALENT BUS
THROUGH IDEAL TRANSFORMERS WITH COMPLEX RATIO



SWITCH BETWEEN COHERENT
SHUNT ADM



GENERATION, LOADS AND
TRANSFER

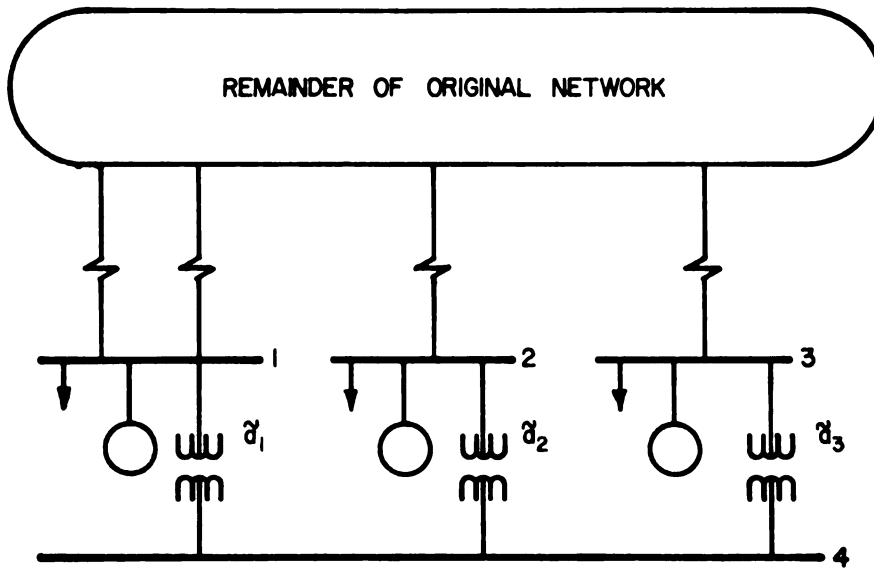


FIGURE 2-1c

BRANCH BETWEEN COHERENT BUSES 2 AND 3 IS REPLACED BY EQUIVALENT SHUNT ADMITTANCE ON BUSES 2 AND 3

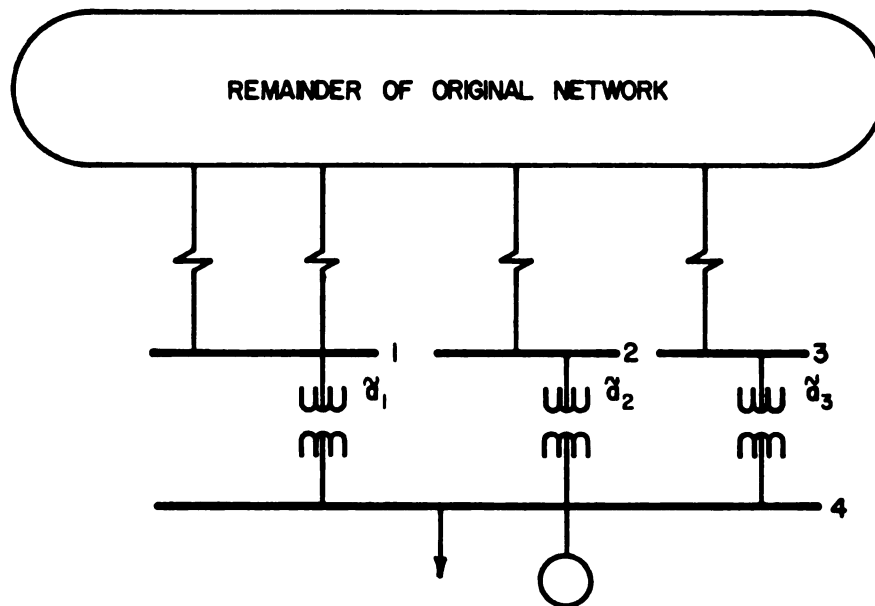


FIGURE 2-1d

GENERATION, LOADS AND SHUNT ADMITTANCES ON ORIGINAL BUSES ARE TRANSFERRED TO THE EQUIVALENT BUS

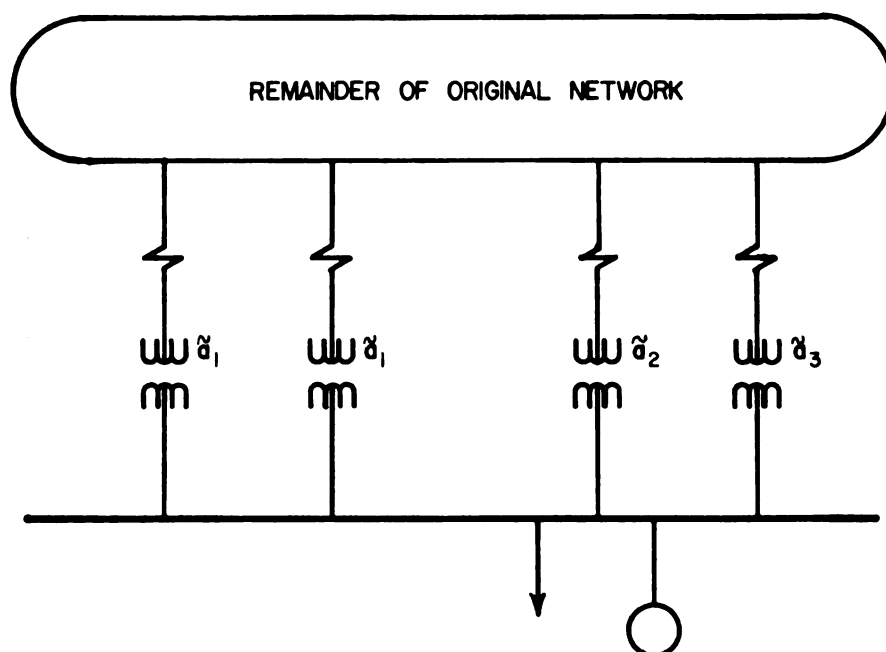


FIGURE 2-1e

ORIGINAL GENERATOR TERMINAL BUSES ARE ELIMINATED BY SERIES
COMBINATION OF IDEAL TRANSFORMERS WITH ORIGINAL BRANCHES

jection of load buses from
in Y_{Bus} . However, a
error. That is, terms h
any of two buses that we
indirectly to each other
elimination of additional
use of non-zero terms in
highly proportional to the
bus elimination is end
terms in Y_{Bus} begin
reducing based reduction to
the load bus eliminat
the network can be great
non-essential nodes
the number of branches
requires two passes. A fi
reduction process at w
retrieved. A second pas
terms [2].

With the coherent
state, the next step is
to represent each coherent

In reference [2]
tion of the identific
equivalent generator.
classical model of the ge
the dynamics of the ge

elimination of load buses further reduces the number of non-zero terms in Y_{Bus} . However, as more load buses are eliminated fill-in can occur. That is, terms have to be added to account for the interaction of two buses that were both connected to a bus being eliminated, but not directly to each other. Thus a point is reached at which the elimination of additional load buses actually increases the number of non-zero terms in Y_{Bus} . Since the computation time is roughly proportional to the number of non-zero elements in Y_{Bus} , load bus elimination is ended at the point where the number of non-zero terms in Y_{Bus} begins to increase. By first applying the coherency based reduction to the coherent generator buses, and then doing the load bus elimination, the number of branches in the equivalent network can be greatly reduced. To guard against fill-in, certain non-essential nodes are selected for retention, to help minimize the number of branches in the reduced Y_{Bus} . The node reduction requires two passes. A first pass is made to determine the point in the reduction process at which the minimum number of terms of Y_{Bus} is achieved. A second pass is then made and terminated at the point of minimum terms [2].

With the coherent groups determined and the network reduction complete, the next step is the modeling of the equivalent generators that represent each coherent group.

In reference [2] Podmore and Germond give a detailed description of the identification procedure used to produce the model of the equivalent generator. In the present research, only a very simple classical model of the generator is used. This involves only the rotor dynamics of the generator. The rotor dynamics for the

generator are e

interested in how n

lated should consult

The basic differ

series used throughout

$$2H_j \frac{d\omega_j}{dt} = P_m - P_e$$

17

1. p.u. speed devi

2. inertia constan

3. mechanical powe

4. electromagnetic

5. damping constan

6. machine subscri

change of the coherency

the same speed deviation

1.5) of the group resul

$$2 \frac{d\omega}{dt} \sum_j H_j = \sum_j (P_m - P_e)$$

1.5) for the equivalent

(a) The inertia

(b) The damping

(c) The mechan

(d) The electr

powers of t

equivalent generator are easily derived, and are shown below. The reader interested in how more detailed equivalent generators are formulated should consult reference [2].

The basic differential equation representing the rotor dynamics used throughout this research is:

$$2H_j \frac{d\Delta\omega_j}{dt} = \Delta P_{M_j} - \Delta P_{G_j} - D_j \Delta\omega_j \quad (2.5)$$

with

- $\Delta\omega$ p.u. speed deviation from synchronous speed
- H inertia constant (generator + turbine) in MWs/MVA
- P_M mechanical power in p.u.
- P_G electromagnetic power in p.u.
- D damping constant in p.u.
- j machine subscript

Because of the coherency assumption, all the machines of a group have the same speed deviation. Thus summing over the machine equations (2.5) of the group results in

$$2 \frac{d\Delta\omega}{dt} \sum_j H_j = \sum_j \Delta P_{M_j} - \sum_j \Delta P_{G_j} - \Delta\omega \sum_j D_j$$

Thus for the equivalent machine:

- (a) The inertia constant is the sum of the inertia constants of the machines of the coherent group.
- (b) The damping factor is the sum of the damping factors of the machines of coherent group.
- (c) The mechanical power is the sum of the mechanical powers of the machines of the coherent group.
- (d) The electrical power is the sum of the electrical powers of the machines of the coherent group.

Results (c) and (d) are based on the assumption already made that the details of the other transfer functions are not included in the subsequent analysis.

The method of forming equivalents has been studied in detail in the literature; this equivalent is sometimes referred to as the equivalent formed by the transfer function. The equivalent formed by the transfer function has been studied both of the transfer functions, it will then be possible to make comparisons between the two.

3. Forming Equivalents

The main work in the analysis has been done by the author in a form that will be of reference [1]. The reader who wishes to refer to the literature should refer to the literature.

In modal analysis (see, for example, [2] systems): 1) The structure and which will be the internal area, or areas, of the study area to be studied, and will be modelled.

Results (c) and (d) are in agreement with the power conservation assumption already made for the coherent bus reduction. The details of the other transfer functions are given in reference [2]. Those details are not included here because they are not needed in the subsequent analysis.

The method of forming equivalents by the use of coherency has been studied in detail in this section. The motivation for doing so is that this equivalent is the one most widely accepted and used. It is sometimes referred to as the "averaged equivalent" or the equivalent formed by the "method of averaging". In the next section, the equivalent formed by using modal analysis will be investigated. Having studied both of the primary methods of forming power system equivalents, it will then be possible to begin investigating connections between the two equivalents.

IV. Forming Equivalents by Modal Analysis

The main work in forming power system equivalents by modal analysis has been done by Undrill [1]. This section summarizes that work in a form that will be useful in the subsequent analysis. The notation of reference [1] is retained for the convenience of the reader who wishes to refer to this work.

In modal analysis the power system is divided into three areas (or systems): 1) The study area where the disturbance is assumed to occur and which will be studied (and modeled) in detail; 2) The external area, or areas, that part of the power system close enough to the study area to be influenced by and in turn influence the study area, and will be modelled in some degree of detail; 3) The

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electrically distant part of the power system which is first identified and then represented by effective inertias and impedances. Both the identification and representation of the "electrically distant" part of the power system is done by experience and engineering judgement.

Thus the models of two of the three partitions of the power system distinguished by modal analysis are determined at the outset of the analysis. That is, the study area, or system, is modelled in great detail, and the electrically distant part of the power system is immediately reduced to a very simple equivalent. Thus the thrust of modal analysis is determining the level of detail required in the model of the external area, or system. For the moment it will be assumed that there is only one external system. The generalization to several external systems can easily be made once the basic ideas have been elucidated for a single external system.

The key step in the modal analysis approach is to define the buses that connect the external system to the study (internal) system and the electrically distant parts of the power system. Since the models for the study system and the electrically distant part of the power system are well defined at this stage of the analysis, the electrically distant part can with no loss of generality be made part of the study system, and hereafter will be considered so. The linear differential and algebraic equations necessary to represent the external system can be formulated as:

$$\dot{\underline{y}} = \underline{A} \underline{y} + \underline{B} \Delta \underline{V}_T \quad (2.6a)$$

$$\Delta \underline{I}_T = \underline{\eta} \underline{y}' + \underline{\zeta} \Delta \underline{V}_T \quad (2.6b)$$

where $\underline{y}$ is a vector of state variables sufficient to describe the behavior of the external system, $\underline{y}'$ is a subvector of $\underline{y}$ and $\Delta \underline{I}_T$, $\Delta \underline{V}_T$ are vectors of the current and voltage changes at the boundary nodes between the study system and the external system.

Equations (2.6) are derived in reference [1] both for the case where the generators are represented in detail and for the case where the generators are represented in the classical form as a constant voltage behind a transient reactance. The derivation for the case where the generators are represented as a constant voltage behind a transient reactance is outlined below.

The internal voltage of generator i is determined relative to a network reference frame D-Q. That is,

$$\begin{aligned} V_{D_i} &= -E' \sin \delta_i \\ V_{Q_i} &= E' \cos \delta_i . \end{aligned}$$

Taking small perturbations gives

$$\begin{aligned} \Delta V_{D_i} &= -\Delta E' \sin \delta_i - E' (\cos \delta_i) \Delta \delta_i \\ \Delta V_{Q_i} &= \Delta E' \cos \delta_i - E' (\sin \delta_i) \Delta \delta_i . \end{aligned}$$

But the classical generator model of constant internal voltage implies that $\Delta E' = 0$, reducing the expressions for ΔV_{D_i} and ΔV_{Q_i} to:

$$\begin{aligned} \Delta V_{D_i} &= (-E' \cos \delta_i) \Delta \delta_i \\ \Delta V_{Q_i} &= (-E' \sin \delta_i) \Delta \delta_i . \end{aligned}$$

This can be put in matrix form for a system of m generators as

$$\underline{\Delta V_I} =$$

where $\underline{ED}$ is a $(2m \times m)$

type diagonal for each

The basic algebra

$$\begin{bmatrix} \underline{\Delta I_G} \\ \underline{\Delta V_I} \end{bmatrix} = \begin{bmatrix} \underline{C'} & \underline{D'} \\ \underline{A'} & \underline{E'} \end{bmatrix}$$

where $\underline{\Delta I_G}$ and $\underline{\Delta V_I}$ are

the internal voltage, and

current and voltage at

the boundary between

Substituting (2.7) into

$$\underline{\Delta I_G} = \underline{C'} \underline{ED} \underline{\Delta V_I}$$

The electrical power of

$$P_i = V_{D_i} \Delta I_{D_i}$$

which leads to the pert

$$P_i = V_{D_i} \Delta I_{D_i} + I_{D_i} \Delta V_{D_i}$$

$$= \begin{bmatrix} I_{D_i} & I_{Q_i} \end{bmatrix} \begin{bmatrix} \Delta V_{D_i} \\ \Delta V_{Q_i} \end{bmatrix}$$

This can be put into ma

$$\underline{\Delta V_I} = \underline{ED} \underline{\Delta \delta} \quad (2.7)$$

where $\underline{ED}$ is a $(2m \times m)$ matrix with n non-zero 2×1 submatrices on the diagonal for each generator being equivalenced.

The basic algebraic network equations are

$$\begin{bmatrix} \underline{\Delta I_G} \\ \underline{\Delta I_T} \end{bmatrix} = \begin{bmatrix} \underline{C'} & \underline{D'} \\ \underline{A'} & \underline{B'} \end{bmatrix} \begin{bmatrix} \underline{\Delta V_I} \\ \underline{\Delta V_T} \end{bmatrix} \quad (2.8)$$

where $\underline{\Delta I_G}$ and $\underline{\Delta V_I}$ are vectors of changes in generator current and internal voltage, and $\underline{\Delta I_T}$ and $\underline{\Delta V_T}$ are vectors of changes in current and voltage at the equivalent terminals, i.e. the terminals on the boundary between the study system and the external system.

Substituting (2.7) into (2.8) gives

$$\underline{\Delta I_G} = \underline{C'} \underline{ED} \underline{\Delta \delta} + \underline{D'} \underline{\Delta V_T} . \quad (2.9)$$

The electrical power of each generator is given by

$$P_i = V_{D_i} I_{D_i} + V_{Q_i} I_{Q_i}$$

which leads to the perturbation equation

$$\begin{aligned} \Delta P_i &= V_{D_i} \Delta I_{D_i} + I_{D_i} \Delta V_{D_i} + V_{Q_i} \Delta I_{Q_i} + I_{Q_i} \Delta V_{Q_i} \\ &= \begin{bmatrix} I_{D_i} & I_{Q_i} \end{bmatrix} \begin{bmatrix} \Delta V_{D_i} \\ \Delta V_{Q_i} \end{bmatrix} + \begin{bmatrix} V_{D_i} & V_{Q_i} \end{bmatrix} \begin{bmatrix} \Delta I_{D_i} \\ \Delta I_{Q_i} \end{bmatrix} \end{aligned}$$

This can be put into matrix form for the m generators as

$$\underline{E} = \underline{I}_0 \underline{A}$$

are $\underline{I}_0$ and $\underline{V}_0$ are

ports. The values

are the generator into

terms of the network

Substituting

$$\underline{E} = \underline{I}_0 \underline{E}_0$$

$$= \underline{A} \underline{P} \underline{A}^{-1}$$

or

$$\underline{A} \underline{P} =$$

Now, the system

$$\underline{I}_m \dot{\underline{S}} = -\underline{D}$$

$$\dot{\underline{S}}_1 = \underline{A} \underline{P} \underline{A}^{-1}$$

where $\underline{I}_m = \text{diagonal}$

$\underline{D} = \text{diagonal}$

Equation (2.11) is

Using (2.11) in

$$\underline{\Delta P} = \underline{I}_0 \underline{\Delta V}_I + \underline{V}_0 \underline{\Delta I}_M \quad (2.10)$$

where $\underline{I}_0$ and $\underline{V}_0$ are $(m \times 2m)$ matrices of (1×2) diagonal elements. The values of $\underline{I}_{D_i}$, $\underline{I}_{Q_i}$, $\underline{V}_{D_i}$ and $\underline{V}_{Q_i}$ can be calculated once the generator internal voltage phasor $\underline{E}_i' / \delta_i$ has been specified in terms of the network D-Q reference frame.

Substituting (2.7) and (2.9) into (2.10) yields

$$\begin{aligned} \underline{\Delta P} &= \underline{I}_0 \underline{ED} \underline{\Delta \delta} + \underline{V}_0 \underline{C'ED} \underline{\Delta \delta} + \underline{V}_0 \underline{D' \Delta V}_T \\ &= \underline{AP} \underline{\Delta \delta}, \underline{BP} \underline{\Delta V}_T \end{aligned} \quad (2.11)$$

with

$$\underline{AP} = (\underline{I}_0 + \underline{V}_0 \underline{C'}) \underline{ED}, \underline{BP} = \underline{V}_0 \underline{D'}.$$

Now, the system differential equations are:

$$\begin{aligned} \underline{T}_m \dot{\underline{S}} &= -\underline{DP} \underline{S} - \underline{\Delta P} \\ \dot{\delta}_i &= \omega_0 (S_i - S_R) \quad i = 1, 2, \dots, m \end{aligned} \quad (2.12)$$

where $\underline{T}_m$ = diagonal matrix of inertia constants

$\underline{DP}$ = diagonal matrix of damping coefficients.

Equation (2.11) is an expression for $\underline{\Delta P}$ in terms of $\underline{\Delta \delta}$ and $\underline{\Delta V}_T$.

Putting (2.11) into (2.12) results in

$$\begin{bmatrix} \dot{\underline{y}} \\ \dot{\underline{z}} \end{bmatrix} = \begin{bmatrix} \underline{A} & \underline{B} \\ \underline{C} & \underline{D} \end{bmatrix} \begin{bmatrix} \underline{y} \\ \underline{z} \end{bmatrix} + \begin{bmatrix} \underline{E} \\ \underline{F} \end{bmatrix} \underline{u}$$

Substituting (2.7) into (2.6)

$$\dot{\underline{z}} = \underline{A}' \underline{E} \underline{D} \underline{z} + \underline{B}' \underline{E} \underline{D} \underline{z}$$

Equations (2.13) are of the form

Equations that are subject to

The equations (2.14)

known state-space in

whose columns are eigenvectors

transformation $\underline{y} = \underline{S} \underline{p}$ in

the form

$$\dot{\underline{p}} = \underline{\Lambda} \underline{p} + \underline{g}$$

$$\dot{\underline{z}} = \underline{R}'' \underline{p} + \underline{h}$$

$$\underline{S}_R = \underline{R}''' \underline{p}$$

where,

$$(1) \underline{\Lambda} = \underline{S}^{-1} \underline{A} \underline{S}$$

$$(2) \underline{R}'' \text{ is the speed dev}$$

$$(3) \underline{R}''' \text{ is the speed dev}$$

$$\begin{bmatrix} \dot{\Delta\delta} \\ \vdots \\ \dot{\underline{S}} \end{bmatrix} = \begin{bmatrix} & & \omega_0 & & -\omega_0 \\ & & \omega_0 & & -\omega_0 \\ & 0 & & \ddots & -\omega_0 \\ & & & & \vdots \\ & & & & 0 \\ \hline -\underline{T}_m^{-1} \underline{AP} & & -\underline{T}_m^{-1} \underline{DP} & & \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \vdots \\ \Delta\underline{S} \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ -\underline{T}_m^{-1} \underline{BP} \end{bmatrix} \Delta V_T \quad (2.13a)$$

Substituting (2.7) into (2.8) and solving for $\Delta\underline{I}_T$ gives

$$\Delta\underline{I}_T = \underline{A}' \underline{ED} \Delta\delta + \underline{B}' \Delta V_T \quad (2.13b)$$

Equations (2.13) are of exactly the same form as (2.6). It is these equations that are subjected to modal analysis.

The equations (2.6) are next transformed to canonical form by well known state-space analysis techniques. That is a matrix $\underline{S}$ whose columns are eigenvectors of the matrix $\underline{A}$ is found. Then the transformation $\underline{y} = \underline{S}\underline{p}$ is made. This transforms equations (2.6) to the form

$$\dot{\underline{p}} = \underline{\Lambda}\underline{p} + \underline{\theta}' \Delta V_T \quad (2.14a)$$

$$\Delta\underline{I}_T = \underline{\eta}\underline{R}''\underline{p} + \underline{\xi} \Delta V_T \quad (2.14b)$$

$$S_R = \underline{R}''' \underline{p}$$

where,

- (1) $\underline{\Lambda} = \underline{S}^{-1} \underline{A} \underline{S}$ is the diagonal matrix whose nonzero diagonal elements are the eigenvalues of $\underline{A}$.
- (2) $\underline{R}''$ is the rows of $\underline{S}$ corresponding to the subvector $\underline{y}'$ of $\underline{y}$
- (3) $\underline{R}'''$ is the row of $\underline{S}$ corresponding to S_R , the reference speed deviation of the external system.

(4) $\underline{g}'$ is given

Note that (1) ass
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matrix $(\underline{I} - e^{\underline{a}t})$
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 the natural modes of

(4) $\underline{\theta}'$ is given by

$$\underline{\theta}' = \underline{S}^{-1} \underline{B}$$

Note that (1) assumes no zero or repeated eigenvalues. This is a legitimate assumption for power systems. The algebraic equations are obtained by extracting the rows from $\underline{S}$ that correspond to the subvector $\underline{y}'$ of $\underline{y}$ and to the state S_R which is the speed deviation of the reference machine in the external system. S_R must be retained because it is required when the external system equations are solved in combination with those of the study system.

The method used for the deletion of unimportant response modes follows from an explanation of the transient and steady-state solutions of (2.14), to the step input $\Delta \underline{V}_T$.

Assuming all the eigenvalues are distinct and have negative real parts the steady-state solution is

$$\underline{p}_{ss} = -\underline{\Lambda}^{-1} \underline{\theta}' \Delta \underline{V}_T,$$

and the transient solution is

$$\begin{aligned} \underline{p}(t) &= (\underline{I} - e^{-\underline{\Lambda}t}) \underline{\Lambda}^{-1} \underline{\theta}' \Delta \underline{V}_T \\ &= (\underline{I} - e^{-\underline{\Lambda}t}) \underline{p}_{ss}. \end{aligned}$$

The matrix $(\underline{I} - e^{-\underline{\Lambda}t})$ is diagonal and thus the transient response of each element $\underline{p}$ is independent of all the others. The response of each element of $\underline{p}$, therefore, has a steady-state component and a decaying transient component. These exponentially decaying components are the natural modes of response of the system. The actual response

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of the system, $\underline{y}$, is built up of linear combinations of the natural modes. The order of the electromechanical equivalent can be reduced by rejecting from (2.14) those rows and columns corresponding to elements of $\underline{p}$ which do not contribute significantly to $\underline{y} = \underline{S}\underline{p}$.

Some of the grounds for deleting specific modes are

- a) The real part of the eigenvalue is such a large negative number in relation to other eigenvalues that the mode may be assumed to jump instantly to its final steady-state value in response to a step disturbance.
- b) The corresponding row of $\underline{\Lambda}^{-1}\underline{\theta}'$ contains such small numbers in relation to other rows that the mode may be assumed not to be excited by the input $\Delta\underline{V}_T$.
- c) The corresponding column of $\underline{R}''$ contains such small numbers in relation to other columns that the mode may be assumed to contribute nothing to the state vector $\underline{y}$.
- d) The corresponding column of $\underline{\eta}\underline{R}''$ contains such small numbers in relation to other columns that the mode may be assumed to have negligible effect on the equivalent output vector $\Delta\underline{I}_T$.

In terms of linear system theory nomenclature these four conditions can be categorized as follows. Condition a) is a fast mode condition. Condition b) is a controllability condition. That is the natural modes discarded by this condition are discarded because they cannot be controlled by the input $\Delta\underline{V}_T$. Condition c) is an observability argument that says the modes do not observably affect the measured state of the external system. Condition d) is an observability condition in the sense that modes discarded under this condition have no observable effect on the output vector $\Delta\underline{I}_T$.

The application of labels such as "observability" and "controllability" to the four conditions of mode elimination may seem

trial and at best reduction
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Once the require
 equation (2.7) can

$$\begin{bmatrix} q \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 \\ 0 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_2 \end{bmatrix} \begin{bmatrix} q \\ p_1 \\ p_2 \end{bmatrix}$$

$$\begin{bmatrix} q \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} q & p_1 & p_2 \\ p_3 & p_4 & p_5 \end{bmatrix}$$

here,
 q = subvector
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trivial and at best redundant. However, in Chapter 4 it is shown that if certain structural conditions are satisfied, the modal and coherency methods yield an identical dynamic equivalent. The arguments for this claim rest on the concepts of observability and controllability. It is for this reason that the relationship between these concepts and the four conditions for mode elimination is so carefully, perhaps tediously, drawn.

Once the required selection of modes to be retained has been made equation (2.7) can be rearranged in the form

$$\underline{p} = \begin{bmatrix} \underline{q} \\ \underline{p}_1 \\ \underline{p}_2 \end{bmatrix} \quad (2.15a)$$

$$\begin{bmatrix} \dot{\underline{q}} \\ \dot{\underline{p}}_1 \\ \dot{\underline{p}}_2 \end{bmatrix} = \begin{bmatrix} \underline{\gamma} & \underline{0} & \underline{0} \\ \underline{0} & \underline{\phi}_1 & \underline{0} \\ \underline{0} & \underline{0} & \underline{\phi}_2 \end{bmatrix} \begin{bmatrix} \underline{q} \\ \underline{p}_1 \\ \underline{p}_2 \end{bmatrix} + \begin{bmatrix} \underline{\theta} \\ \underline{\rho}_1 \\ \underline{\rho}_2 \end{bmatrix} \Delta V_{\underline{I}} \quad (2.15b)$$

$$\begin{bmatrix} \underline{\alpha} & \underline{n}_1 & \underline{n}_2 \\ & & \\ \underline{n}_3 & \underline{n}_4 & \underline{n}_5 \end{bmatrix} \begin{bmatrix} \underline{q} \\ \underline{p}_1 \\ \underline{p}_2 \end{bmatrix} = \begin{bmatrix} \underline{y}_1 \\ \\ \underline{y}_2 \end{bmatrix} \quad (2.15c)$$

where,

$\underline{q}$ = subvector of $\underline{p}$ to be retained

$\underline{p}_1$ = subvector of $\underline{p}$ assumed to jump immediately to steady state

$\underline{p}_2$ = subvector of $\underline{p}$ assumed to be zero

$$\mathbf{y}, \mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3, \mathbf{z}_4$$

For a step input

we yield

$$\mathbf{p}_1 = -\mathbf{z}_1^{-1}$$

or

$$\dot{\mathbf{z}}_1 = \mathbf{z}_1 \mathbf{Q}$$

Equation (2.15c) yields

$$\mathbf{y}_0 = \mathbf{z}_1 \mathbf{Q}$$

$$= \mathbf{z}_1 \mathbf{Q}$$

$$= \mathbf{z}_1 \mathbf{Q}$$

Since the speed
is retained in $\mathbf{y}_0$ the

we combine (2.14b)

$$\mathbf{z}_1 \mathbf{T} = \mathbf{z}_1 \mathbf{Q} +$$

$$\mathbf{S}_R = \mathbf{Q}_2 \mathbf{Q} + \mathbf{z}_1$$

where $\mathbf{z}_1, \mathbf{z}_2$ are a
the last rows of

$\underline{\gamma}, \underline{\phi}_1, \underline{\theta}, \underline{\rho}_1, \underline{\alpha}, \underline{n}_1$ = submatrices of $\underline{\Lambda} \underline{\theta}'$ and $\underline{S}$ obtained after reordering rows and columns.

For a step input $\dot{\underline{p}}_1 = \underline{0}$ and $\underline{p}_2 = \underline{0}$. Hence multiplying out (2.15b) yields

$$\underline{p}_1 = -\underline{\phi}_1^{-1} \underline{\rho}_1 \Delta \underline{V}_T$$

and

$$\dot{\underline{q}}_1 = \underline{\gamma} \underline{q} + \underline{\theta} \Delta \underline{V}_T.$$

Equation (2.15c) yields

$$\begin{aligned} \underline{y}_0 &= \underline{\alpha} \underline{q} + \underline{n}_1 \underline{p}_1 \\ &= \underline{\alpha} \underline{q} + \underline{n}_1 (-\underline{\phi}_1^{-1} \underline{\rho}_1 \underline{V}_T) \\ &= \underline{\alpha} \underline{q} + \underline{\beta} \Delta \underline{V}_T \end{aligned} \quad (2.16)$$

Since the speed deviation S_R is one of the states that must be retained in $\underline{y}_0$ the vector $\underline{y}_0$ becomes

$$\underline{y}_0 = \begin{bmatrix} \underline{y}' \\ \text{---} \\ S_R \end{bmatrix}$$

The combining (2.14b) with (2.16) yields

$$\Delta \underline{I}_T = \underline{n} \underline{\alpha}_1 \underline{q} + (\underline{n} \underline{\beta}_1 + \underline{\epsilon}) \Delta \underline{V}_T$$

$$S_R = \underline{\alpha}_2 \underline{q} + \underline{\beta}_2 \Delta \underline{V}_T$$

where $\underline{\alpha}_1, \underline{\beta}_1$ are all rows of $\underline{\alpha}, \underline{\beta}$ except the last, and $\underline{\alpha}_2, \underline{\beta}_2$ are the last rows of $\underline{\alpha}, \underline{\beta}$. Then the equations become:

$$\dot{q} = \sum q +$$

$$\frac{1}{S_1} = \frac{1}{S_2}$$

$$S_R = \frac{1}{S_2} S$$

Equations (2.17) a
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$$\dot{\underline{q}} = \underline{\gamma} \underline{q} + \underline{\theta} \Delta \underline{V}_T \quad (2.17a)$$

$$\Delta \underline{I}_T = \underline{\eta} \underline{\alpha}_1 \underline{q} + (\underline{\eta} \underline{\beta}_1 + \underline{\zeta}) \Delta \underline{V}_T \quad (2.17b)$$

$$\underline{S}_R = \underline{\alpha}_2 \underline{q} + \underline{\beta}_2 \Delta \underline{V}_T \quad (2.17c)$$

Equations (2.17) are the final form of the reduced equivalent of the external system.

There are several points worth noting about the model equivalent. First, the dynamic simulation of the power system requires the simultaneous integration of the differential equations of the study system and the external system. The integration of these equations is straightforward, once the input $\Delta \underline{V}_T$ has been determined. The determination of the $\Delta \underline{V}_T$ requires the combined solution of the algebraic network equation for the study system and the external system.

Second, the modal analysis technique determines a linear equivalent for the external system. In the coherency method, by contrast, the equivalent can be either linear or nonlinear. In one respect this is a drawback. However, an advantage is that a reduced order equivalent can be found for a group of generators that are not necessarily coherent.

Third, the input to the modal equivalent model, is the voltage differences $\Delta \underline{V}_T$ at the boundary. This is in contrast to the linear model developed in Section 2 of this chapter, where the input is the step input in mechanical powers. The choice of the model in Section 2 results from the intent to relate coherency and modal analysis, but from the coherency perspective.

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There are some real disadvantages to the modal approach that also deserve some attention. First, the mode elimination procedure requires the calculation of eigenvalues and eigenvectors. This is a computationally expensive step. Even if one is willing to accept this expense there is another difficulty. That is the fact that it is not practical to compute eigenvalues for a system of more than 100th order. Using the simple classical model of each generator this means that the external system cannot have over 50 generators. For a large system the approach adopted by Undrill [1] is to break the external system into sections and construct a linear model of each section. Implicit in this approach is the assumption that specific eigenvalues can be associated with specific sections. This automatically introduces another approximation.

A second difficulty with the modal equivalent is that the reduced order model of the external system is not expressed in terms of equivalent lines and generators but in terms of retained canonical states. This robs the equivalent of much of its physical insight. It is particularly unappealing to power system operators and planners who are used to thinking in terms of lines and generators.

This second disadvantage of the modal method is closely related to a third disadvantage which is that the available transient stability programs are all written to accept dynamic equivalents expressed in terms of equivalent lines and generators. To implement modal equivalent procedures involves modifying the existing transient stability programs.

The fourth and most telling disadvantage of the modal approach is that rules exist for aggregating on the basis of coherency while

carefully preserving

LITERATURE

This chapter has
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simultaneously preserving modal properties [5, 6, 7].

V. Summary

This chapter has established a linearized model for a power system and then described, in some detail, the two primary methods of forming power system equivalents, namely coherency and modal analysis. Although lengthy, this chapter has provided the basic understanding necessary to uncover the connections between these two equivalencing procedures. Chapter 3 takes the first step in that process by examining those system structure conditions that result in a group of generators behaving as a single generator.

1. Introduction

The concept of coherence was first introduced by R. A. Silverman [1] in the context of two machine systems. In this paper, we shall consider the problem of coherence in the context of a single machine system. If the input and output signals are denoted by $x(t)$ and $y(t)$ respectively, then the system is said to be coherent if the cross-correlation function $c_{xy}(t) = c_{yx}(t)$ is equal to the product of the auto-correlation functions $c_{xx}(t)$ and $c_{yy}(t)$.

In the case of a single machine system, the input and output signals are distinguished. The input signal is the generator output and the output signal is the generator input. The difference between the two signals is only a phase shift. The generator output is denoted by $x(t)$ and the generator input is denoted by $y(t)$.

Chang and Hsiao [2] have derived an equivalent power spectrum for a single machine system. However, this method is not applicable to a single machine system. In [3], [4] defines a method for determining the acceleration of a single machine system. In addition, it can be used to determine the acceleration of a generator. The method is based on the assumption that the generator output is a random signal.

CHAPTER THREE

STRUCTURAL CONDITIONS UNDER WHICH A GROUP OF MACHINES BEHAVES AS A SINGLE MACHINE

I. Introduction

The concept of forming a reduced order dynamic equivalent using coherency was first introduced by Chang and Adibi [4]. They defined two machines to be coherent, "to oscillate together" in their terminology, if there exists a constant c_{ij} such that $\delta_i(t) - \delta_j(t) \approx c_{ij}$ for $0 < t < t_0$. A group of generators is said to be coherent if each pair of generators in the group is coherent.

In the subsequent development two gradations of coherency are distinguished. If $\delta_i(t) - \delta_j(t) = c_{ij}$ for $0 < t < t_0$ then the generators will be said to be strictly coherent. If the angle difference is only approximately true, i.e. if $\delta_i(t) - \delta_j(t) \approx c_{ij}$ then the generators will be said to be coherent.

Chang and Adibi modeled the generators as current sources and derived an equivalent that could not be expressed in terms of equivalent power system components, i.e. equivalent lines and generators. However, some more recent work by Dicaprio and Marconato [10, 11] defines a structural condition under which a group of generators accelerate together and remain strictly coherent. Under this condition it can be shown that the coherent group behaves as a single generator. The work of Dicaprio and Marconato is described in detail

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in the next section because it is fundamental to an understanding of the relationship between coherency and modal dynamic equivalents.

However, the structural condition of Dicaprio and Marconato is only one of three rather hypothetical conditions under which a group of generators behaves as if it were a single generator. Two other structural conditions can be found which lead to this same result. These conditions are explored in two subsequent sections of this chapter.

A word here about nomenclature and notation. In this chapter the coherency of a specific group of n generators will be considered. This group of generators will most often be referred to as "the specified group of n generators".¹ To be perfectly correct, the group of generators should always be referred with this phrase. However, in consideration of the reader's ears, "the specified group of n generators" will frequently be referred to as "the group of n generators" and occasionally as simply "the group". (Somewhere right now Mary McCarthy is smiling.) Hopefully the meaning will be clear from the context.

In terms of notation, the frequently encountered expression

$$i = 1, 2, \dots, n$$

$$k = 1, 2, \dots, m$$

means that i is an index over a set of n elements and k is an index over a different set of m elements.

¹In discussing pseudo-coherency in Section IV, it will also be convenient to call the generators external to the specified group the study group.

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$$\frac{|E_3|}{M_3} Y_3$$

II. Strict Geometric Coherency

Consider the simple power system model of Figure 3.1a. Let generators G_3 and G_4 and the admittances y_1 through y_8 be the external system, with buses 1 and 2 the boundary between the internal and external system. The generators G_3 and G_4 in the external system will be considered constant voltages, E_3 and E_4 , behind transient reactances, y_7 and y_8 , respectively.

In Figure 3.1b the transient reactances y_7 and y_8 have been eliminated. This can be accomplished in the simple example of Figure 3.1 by a series of star-mesh transformations, or in a more general setting by writing a set of node equations $\underline{I} = \underline{Y}\underline{V}$ for the buses 1,2,3,4,3',4' and then eliminating buses 3' and 4'.

Then the equations for buses 1,2,3 and 4, with I_1 and I_2 the equivalent current injections at buses 1 and 2, are

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ E_3 \\ E_4 \end{bmatrix}$$

Dicaprio shows [10, 11] that if certain structural conditions exist in the power system at time $t = 0^-$, then no matter what disturbance occurs in the internal system at time $t \geq 0$, generators G_3 and G_4 will remain strictly coherent, for all $t \geq 0$. The conditions that must exist in the system of Figure 3.1 at $t = 0^-$ are:

$$\frac{|E_3|}{M_3} Y_{31} e^{-j(\delta_3^0 - \delta_4^0)} = \frac{|E_4|}{M_4} Y_{41} \quad (3.1a)$$

$$\frac{E_2}{M_2}$$

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$$\frac{E_1}{M_1} = \frac{E_2}{M_2}$$

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$$\frac{|E_3|}{M_3} Y_{32} e^{-j(\delta_3^0 - \delta_4^0)} = \frac{|E_4|}{M_4} Y_{42} \quad (3.1b)$$

where, M_3 and M_4 are the inertias of generators 3 and 4 and δ_3^0 and δ_4^0 the phase angles of internal generator voltages E_3 and E_4 respectively, at time $t = 0^-$.

Now, express the admittances Y_{ij} in polar form as $Y_{ij} = |Y_{ij}| e^{j\gamma_{ij}}$. Multiply equation (3.1a) by $|E_1| e^{j(\delta_1^0 - \delta_4^0)}$ and equation (3.1b) by $|E_2| e^{j(\delta_2^0 - \delta_4^0)}$ to obtain:

$$\frac{|E_1| |E_3| |Y_{31}| e^{j(\delta_1^0 - \delta_3^0 + \gamma_{31})}}{M_3} = \frac{|E_1| |E_4| |Y_{41}| e^{j(\delta_1^0 - \delta_4^0 + \gamma_{41})}}{M_4} \quad (3.2a)$$

$$\frac{|E_2| |E_3| |Y_{32}| e^{j(\delta_2^0 - \delta_3^0 + \gamma_{32})}}{M_3} = \frac{|E_2| |E_4| |Y_{42}| e^{j(\delta_2^0 - \delta_4^0 + \gamma_{42})}}{M_4} \quad (3.2b)$$

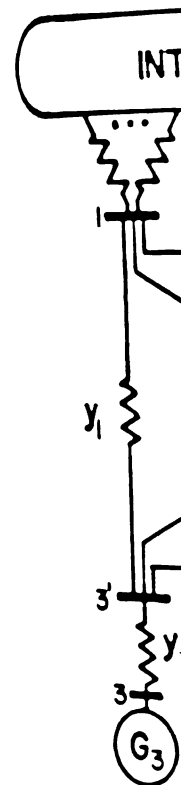
Now equation (3.2a) can be rewritten as

$$\begin{aligned} \frac{|E_1| |E_3| |Y_{31}|}{M_3} \cos(\delta_1^0 - \delta_3^0 + \gamma_{31}) + j \sin(\delta_1^0 - \delta_3^0 + \gamma_{31}) \\ = \frac{|E_1| |E_4| |Y_{41}|}{M_4} \cos(\delta_1^0 - \delta_4^0 + \gamma_{41}) + j \sin(\delta_1^0 - \delta_4^0 + \gamma_{41}) \end{aligned}$$

Equating the real parts of this expression yields

$$\frac{|E_1| |E_3| |Y_{31}| \cos(\delta_1^0 - \delta_3^0 + \gamma_{31})}{M_3} = \frac{|E_1| |E_4| |Y_{41}| \cos(\delta_1^0 - \delta_4^0 + \gamma_{41})}{M_4}$$

Now $|E_1| |E_3| |Y_{31}| \cos(\delta_1^0 - \delta_3^0 + \gamma_{31})$ and $|E_1| |E_4| |Y_{41}| \cos(\delta_1^0 - \delta_4^0 + \gamma_{41})$ are the synchronizing power coefficients between bus 1 and bus 3 and between



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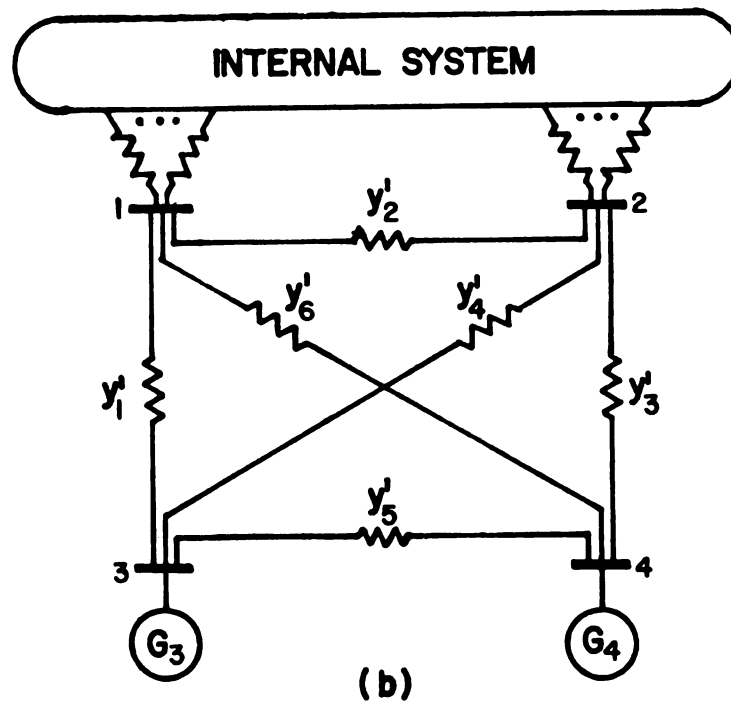
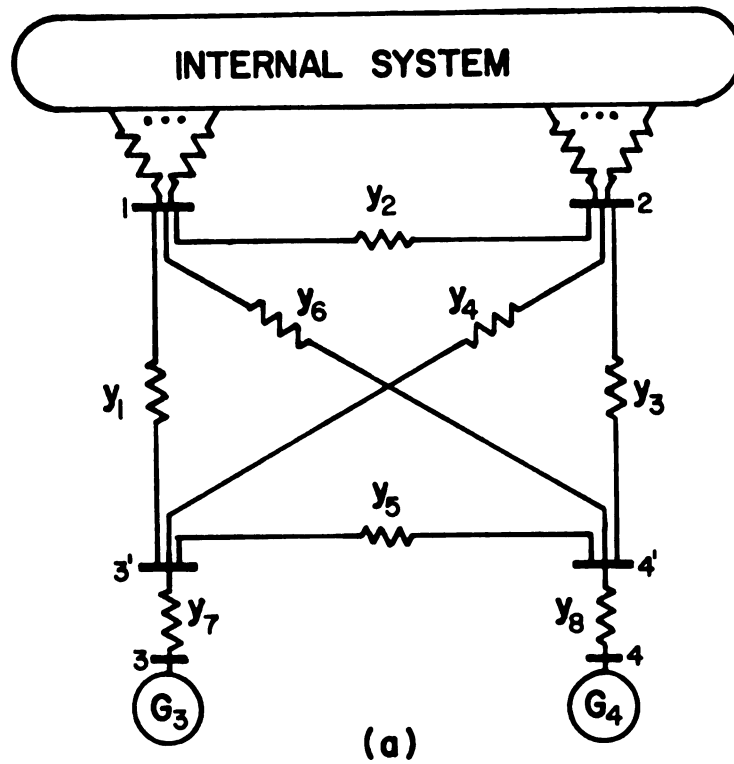


FIGURE 3-1
TWO GENERATOR EXTERNAL GROUP
(a) BEFORE AND (b) AFTER AGGREGATION OF GENERATOR
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$$\begin{bmatrix} I_k \\ I_n \end{bmatrix} = \begin{bmatrix} Y_k \\ Y_n \end{bmatrix}$$

bus 1 and bus 4, respectively, at time $t = 0^-$. The conditions of (3.1) show that for generators 3 and 4 to remain coherent after the disturbance occurs, the synchronizing power coefficients between bus 1 and buses 3 and 4 are proportional to each respective generator's inertia. The same is true for the synchronizing power coefficients between bus 2 and buses 3 and 4. The result is that when a disturbance occurs in the internal system, the amount of that disturbance energy delivered by each boundary bus to the generators of the external system is prorated to each generator's inertia such that all the generators of the external group accelerate at the same rate and remain perfectly coherent for any $t \geq 0$.

It is easy enough to generalize from the simple example of Figure 3.1 to the case of n generators in the external system and m boundary buses. In the general case Dicaprio's conditions for perfect coherency become

$$\frac{E_i}{M_i} Y_{ik} e^{-j(\delta_1^0 - \delta_n^0)} = \frac{E_n}{M_n} Y_{nk} \quad (3.3)$$

for: any $i = 1, 2, \dots, n-1$

any $k = 1, 2, \dots, m$

Dicaprio calls the conditions specified in (3.3) the conditions for "theoretical coherency in the large", with "large" meaning that the conditions imply coherency for the nonlinear representation of the system used to derive (3.3), namely the algebraic equations

$$\begin{bmatrix} \underline{I}_k \\ \underline{I}_n \end{bmatrix} = \begin{bmatrix} Y_{kk} & Y_{kn} \\ Y_{nk} & Y_{nn} \end{bmatrix} \begin{bmatrix} \underline{V}_k \\ \underline{E}_n \end{bmatrix} \quad (3.4a)$$

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Abstract

plus a second order differential equation of the form

$$PM_i = PG_i + M_i \ddot{\delta}_i \quad i = 1, 2, \dots, n \quad (3.4b)$$

for each generator in the system with

$\underline{I}_k$ a $k \times 1$ vector of the currents injected at the boundary buses

$\underline{I}_n$ a $n \times 1$ vector of the currents injected at the internal buses of the n generators of the coherent group

$\underline{V}_k$ a $k \times 1$ vector of the voltages at the k boundary buses

$\underline{E}_n$ a $n \times 1$ vector of the voltages at the internal buses of the n generators of the coherent group each with magnitude E_i , phase angle δ_i , $i = 1, 2, \dots, n$.

$\underline{Y}_{kk}$ a $k \times k$ matrix of the admittances between the k boundary buses

$\underline{Y}_{kn}$ a $k \times n$ matrix of the admittances between the k boundary buses and the n internal generator buses of the coherent group

$\underline{Y}_{nn}$ an $n \times n$ matrix of the admittances between the n internal generator buses of the coherent group

$$\underline{Y}_{kn} = \underline{Y}_{nk}^T$$

PM_i the constant mechanical input power of the generator i of the coherent group.

$$PG_i = \text{Re}\{E_i \cdot I_i^*\}$$

M_i the inertia constant of generator i of the coherent group

Although Dicaprio calls the coherency that results from the satisfaction of conditions (3.3) "Theoretical Coherency", in the

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remainder of this work it will be referred to as strict geometric coherency (SGC). The reason for the name change is to distinguish the coherency that results from the structural condition of strict geometric coherency from the coherency that results when the generators of a group are strongly interconnected electrically.

A proof that Dicaprio's condition (3.3) is both necessary and sufficient for strict geometric coherency is now given. First, assume

- (1) buses $k = 1, 2, \dots, m$ are the boundary buses connecting a group of n generators to the remainder of the power system.
- (2) buses $i = 1, 2, \dots, n$ are the internal voltage buses of the specified group of n generators. That is, assume that the terminal buses of the n generators, and all load buses, have been removed by a node elimination procedure.
- (3) That the generators of the specified group are referenced to generator n of the group.
- (4) That the generator internal voltage magnitudes are constant in the specified group.

To prove necessity, assume the n generators of the specified group are strictly coherent. Then all the generators of the specified group accelerate at the same rate, so that, using equation (3.4b) it is possible to write

$$\ddot{\delta}_i = \frac{PM_i - PG_i}{M_i} = \ddot{\delta}_n = \frac{PM_n - PG_n}{M_n} \quad i = 1, 2, \dots, n-1 \quad (3.5)$$

Now $PM_i - PG_i$ is the difference between the mechanical power input to generator i and the electrical power output of generator i . Thus if the n machines are strictly coherent then the ratio of generator power difference to generator inertia will be the same for all n generators in the group.

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$$\frac{PG_i(0^-) - PG_i(0^+)}{M_i}$$

The complex power generated by a particular machine in the specified group goes two places, to the other generators in the group and to the m boundary buses. For a disturbance that occurs outside the specified group the amount of power transmitted among the generators of the group is the same before and after the disturbance. This results from 1) the fact that the voltage magnitudes at the internal generator buses are constant, and 2) the assumption that the voltage angles at these internal buses are strictly coherent.

As a result any change in the power distributed by a generator of the specified group after a disturbance occurs will be a change in the power distributed by that generator to the m boundary buses. This can be formalized as follows.

Let $PG_i + jQG_i$ be the power injected by generator i of the group. Then

$$PG_i + jQG_i = PG_i' + jQG_i' + PG_i'' + jQG_i''$$

where $PG_i' + jQG_i'$ is the complex power transmitted from generator i to the other generators of the group and $PG_i'' + jQG_i''$ is the complex power transmitted from generator i to the m boundary buses. For a disturbance at time $t = 0$, the system is in equilibrium at time $t = 0^-$. That is $PM_i(0^-) = PG_i(0^-)$. Putting this information into (3.5) gives

$$\frac{PG_i(0^-) - PG_i(t)}{M_i} = \frac{PG_n(0^-) - PG_n(t)}{M_n} \quad \text{or,}$$

$$\frac{e_i(t) - PG_i(t)}{n_i} +$$

$$= \frac{PG_i(t)}{n_i}$$

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$$M_i = PG_i(0^-) = \text{Re}\{E_i^0 e^{j\omega t}\}$$

$$= \text{Re}\{E_i^0\}$$

$$\frac{PG_i'(0^-) - PG_i'(t) + PG_i''(0^-) - PG_i''(t)}{M_i} = \frac{PG_n'(0^-) - PG_n'(t) + PG_n''(0^-) - PG_n''(t)}{M_n}$$

Under the assumption of strict coherency among the n internal generator buses, this reduces to

$$\frac{PG_i''(0^-) - PG_i''(t)}{M_i} = \frac{PG_n''(0^-) - PG_n''(t)}{M_n} \quad (3.6)$$

for $i = 1, 2, \dots, n-1$, since the result is true at any generator bus in the specified group.

The Dicaprio condition (3.3) is a restatement of equations (3.6) in terms of network parameters and the steady-state load flow conditions at time $t = 0^-$. To see this write the dynamic equation for generator i as

$$\ddot{\delta}_{in} = \ddot{\delta}_i - \ddot{\delta}_n = \frac{PM_i - PG_i}{M_i} - \frac{PM_n - PG_n}{M_n}.$$

Now express the PM_i in terms of the steady state load flow conditions existing at time $t = 0^-$. That is

$$\begin{aligned} PM_i = PG_i(0^-) &= \operatorname{Re}\{E_i^0 e^{j\delta_i^0} \sum_{k=1}^m Y_{ki}^* V_k^0 e^{-j\delta_k^0} + E_i^0 e^{j\delta_i^0} \sum_{j=1}^n Y_{ji}^* E_j^0 e^{-j\delta_j^0}\} \\ &= \operatorname{Re}\{E_i^0 e^{j(\delta_i^0 - \delta_n^0)} \sum_{k=1}^m Y_{ki}^* V_k^0 e^{-j(\delta_k^0 - \delta_n^0)} \\ &\quad + E_i^0 e^{j(\delta_i^0 - \delta_n^0)} \sum_{j=1}^n Y_{ji}^* E_j^0 e^{-j(\delta_j^0 - \delta_n^0)}\} \end{aligned}$$

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The first summation represents the power flows between generator i of the specified group and the m boundary buses. The second summation gives the power flows between generator i and the other $n-1$ generators of the group. The superscript o designates the steady state values at time $t = 0^-$. Similar expressions can be written for PG_i , PM_n , and PG_n , namely,

$$PG_i = \operatorname{Re}\{E_i e^{j(\delta_i - \delta_n)} \sum_{k=1}^m Y_{ki}^* V_k e^{-j(\delta_k - \delta_n)} + E_i e^{j(\delta_i - \delta_n)} \sum_{j=1}^n Y_{ji}^* E_j e^{-j(\delta_j - \delta_n)}\}$$

$$PM_n = \operatorname{Re}\{E_n^o \sum_{k=1}^m Y_{kn}^* V_k^o e^{-j(\delta_k^o - \delta_n^o)} + E_n^o \sum_{j=1}^n Y_{jn}^* E_j^o e^{-j(\delta_j^o - \delta_n^o)}\}$$

$$PG_n = \operatorname{Re}\{E_n \sum_{k=1}^m Y_{ki}^* V_k e^{-j(\delta_k - \delta_n)} + E_n \sum_{j=1}^n Y_{ji}^* E_j e^{-j(\delta_j - \delta_n)}\}.$$

Defining $\delta_{in} = \delta_i - \delta_n$, and expressing $\ddot{\delta}_{in} = \ddot{\delta}_i - \ddot{\delta}_n$ in terms of these four power expressions, results after some manipulation in,

$$\begin{aligned} \ddot{\delta}_{in} = & \operatorname{Re}\left\{ \sum_{k=1}^m \left\{ \left[\frac{E_n}{M_n} Y_{kn}^* - \frac{E_i}{M_i} Y_{ki}^* e^{j(\delta_i - \delta_n)} \right] V_k e^{j(\delta_n - \delta_k)} \right. \right. \\ & \left. \left. - \left[\frac{E_n^o}{M_n} Y_{kn}^* - \frac{E_i^o}{M_i} Y_{ki}^* e^{j(\delta_i^o - \delta_n^o)} \right] V_k^o e^{j(\delta_n^o - \delta_k^o)} \right\} \right\} \\ & - \operatorname{Re}\left\{ \sum_{j=1}^n \left\{ \left[\frac{E_n}{M_n} Y_{jn}^* - \frac{E_i}{M_i} Y_{ji}^* e^{j(\delta_i - \delta_n)} \right] E_j e^{j(\delta_n - \delta_j)} \right. \right. \\ & \left. \left. - \left[\frac{E_n^o}{M_n} Y_{jn}^* - \frac{E_i^o}{M_i} Y_{ji}^* e^{j(\delta_i^o - \delta_n^o)} \right] E_j^o e^{j(\delta_n^o - \delta_j^o)} \right\} \right\} \end{aligned} \quad (3.7)$$

for all $i = 1, 2, \dots, n-1$.

Recall that the
 n generators are
 were eliminated, leaving
 that is, $E_i^0 = E_i$
 and these facts in mind

$$\frac{E_n}{M_n} v_{kn}^* = \frac{E_i}{M_i} v_{ki}^*$$

$$\frac{E_n}{M_n} y_{jn} = \frac{E_i}{M_i} y_{ji}^*$$

the terms E_i, E_j ,
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 Write:

$$a_{ki}(\varepsilon_{in}) = \frac{E_n}{M_n}$$

$$b_{ji}(\varepsilon_{in}) = \frac{E_n}{M_n}$$

Substituting the expressions

$$v_k = \operatorname{Re} \left\{ \sum_{k=1}^m a_{ki}(\varepsilon_{in}) v_k \right\}$$

$$= \operatorname{Re} \left\{ \sum_{j=1}^n [b_{ji}(\varepsilon_{in})] \right\}$$

for $i = 1, 2, \dots, n-1$.

Recall that the disturbances occur outside the specified group of n generators and that the n terminal buses of the group have been eliminated, leaving the internal bus voltages of expression (3.7). That is, $E_i^0 = E_i$ for any generator in the specified group. Keeping these facts in mind, consider the expressions

$$\frac{E_n}{M_n} Y_{kn}^* - \frac{E_i}{M_i} Y_{ki}^* e^{j(\delta_i - \delta_n)} \quad \begin{matrix} k = 1, 2, \dots, m \\ i = 1, 2, \dots, n-1 \end{matrix} \quad (3.8a)$$

$$\frac{E_n}{M_n} Y_{jn} - \frac{E_i}{M_i} Y_{ji}^* e^{j(\delta_i - \delta_n)} \quad \begin{matrix} j = 1, 2, \dots, n \\ i = 1, 2, \dots, n-1 \end{matrix} \quad (3.8b)$$

Since the terms E_i , E_j , E_n in (3.8) refer to constant internal bus voltages within the specified group of n generators it is possible to write:

$$a_{ki}(\delta_{in}) = \frac{E_n}{M_n} Y_{kn}^* - \frac{E_i}{M_i} Y_{ki}^* e^{j(\delta_i - \delta_n)} \quad \begin{matrix} k = 1, 2, \dots, m \\ i = 1, 2, \dots, n-1 \end{matrix} \quad (3.9a)$$

$$b_{ji}(\delta_{in}) = \frac{E_n}{M_n} Y_{jn} - \frac{E_i}{M_i} Y_{ji}^* e^{j(\delta_i - \delta_n)} \quad \begin{matrix} j = 1, 2, \dots, n \\ i = 1, 2, \dots, n-1 \end{matrix} \quad (3.9b)$$

Substituting the expressions (3.9) into (3.7) yields

$$\begin{aligned} \ddot{\delta}_{in} = & \operatorname{Re} \left\{ \sum_{k=1}^m a_{ki}(\delta_{in}) V_k e^{j\delta_{nk}} - a_{ki}(\delta_{in}^0) V_k^0 e^{j\delta_{nk}^0} \right\} \\ & - \operatorname{Re} \left\{ \sum_{j=1}^n [b_{ji}(\delta_{in}) e^{-j\delta_{jn}} - b_{ji}(\delta_{in}^0) e^{j\delta_{jn}^0}] E_j \right\} \end{aligned} \quad (3.10)$$

for $i = 1, 2, \dots, n-1$.

the assumption of

$$\delta_{in} = \delta_{in}(t)$$

equation (3.10) we

$$U_k = \operatorname{Re} \left[\sum_{k=1}^N a_{ik} e^{i\delta_{in}} \right]$$

$$U_k = \operatorname{Re} \left[\sum_{k=1}^N a_{ik} e^{i\delta_{in}} \right]$$

Because the
mutually coherent, δ_{in}
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$$a_{ik}(\delta_{in}) = 0$$

proves necessity.

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Under the assumption of strict coherency in the specified group,

$$\delta_{in} \triangleq \delta_{in}(t) = \delta_{in}(0^-) \triangleq \delta_{in}^0,$$

and equation (3.10) reduces to

$$\ddot{\delta}_{in} = \text{Re} \left\{ \sum_{k=1}^m a_{ki}(\delta_{in}) [V_k(t) e^{j\delta_{nk}(t)} - V_k(0^-) e^{j\delta_{nk}(0^-)}] \right\}$$

or,

$$\ddot{\delta}_{in} = \text{Re} \left\{ \sum_{k=1}^m a_{ik}(\delta_{in}) U_k(t) \right\} \quad i = 1, 2, \dots, n-1 \quad (3.11)$$

Because the n generators of the specified group are strictly coherent, $\ddot{\delta}_{in} = 0$, $i = 1, 2, \dots, n-1$. Since the voltage differences $U_k(t)$ at the m boundary buses are arbitrary, the satisfaction of (3.11) requires

$$a_{ik}(\delta_{in}) = 0 \quad \text{for} \quad k = 1, 2, \dots, m \quad (3.12)$$

This proves necessity.

The proof of sufficiency proceeds in a straightforward way from equation (3.10). Assuming that the conditions (3.3) are satisfied or equivalently that conditions (3.12) are satisfied, consider that since the disturbance occurs outside the specified group of n generators, the angles and speeds of this group cannot change instantaneously. That is,

$$\delta_{in}(0^+) = \delta_{in}(0^-) = \delta_{in}^0 \quad (3.13a)$$

$$i = 1, 2, \dots, n-1$$

$$\dot{\delta}_{in}(0^+) = \dot{\delta}_{in}(0^-) = 0$$

eq. (3.13a) implies

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Further (3.13a) implies that

$$a_{ik}(\delta_{in}(0^-)) = a_{ik}(\delta_{in}(0^+)) = 0 \quad k = 1, 2, \dots, m.$$

Substituting this result into (3.11) yields

$$\ddot{\delta}_{in}(0^+) = 0 \quad i = 1, 2, \dots, n-1 \quad (3.14)$$

Relationships (3.13) and (3.14) guarantee that

$$\begin{aligned} \ddot{\delta}_{in}(t) &= \ddot{\delta}_{in}(0^+) = 0 \\ \dot{\delta}_{in}(t) &= \dot{\delta}(0^+) = 0 \\ \delta_{in}(t) &= \delta_{in}(0^+) = \delta_{in}(0^-) \end{aligned} \quad (3.15)$$

for all $t > 0$, completing the proof of sufficiency.

Dicaprio generalizes his results somewhat by eliminating all load buses and the terminal buses of all generators in the power system, both in the specified group and outside. This leaves only m internal generator buses outside the specified group plus the n internal generator buses of the group itself. Under this configuration Dicaprio uses deviations in the admittances to represent the application of a disturbance. This is necessary since the internal bus voltages are by assumption constant for all time. Dicaprio defines a general "disturbance external to the group" of generator $i = 1, 2, \dots, n$ and then shows that conditions (3.3), with k representing now the m generators external to the specified group, are necessary and sufficient conditions for the specified group to be strictly coherent in response to a disturbance "external to the group" [11]. The generalization is Dicaprio's way of trying to identify coherent groups without having to

specify the coherent group in advance. It is worth noting that this is exactly the function fulfilled by a coherency measure.

The discussion of coherency in Chapter 2 established that if a group of generators were strictly coherent, then it was possible to replace the group by a single equivalent generator and perfectly preserve the response of the system to a disturbance outside the group. The equivalent derived by Podmore [2] required phase shifting transformers in the equivalent lines connecting the single equivalent buses to the boundary buses. That is $Y_{ke} \neq Y_{ek}$ where e represents the equivalent bus and $k = 1, 2, \dots, m$ is one of the boundary buses.

Dicaprio shows [11] that it is possible to replace the coherent group by a single equivalent machine, that perfectly preserves the dynamic response, but does not require the phase shifter, i.e. $Y_{ke} = Y_{ek}$, $k = 1, 2, \dots, m$. The choice of which equivalent to use is a matter of taste. Dicaprio's equivalent has some advantage in that it does not require phase shifting transformers. The important point is that if conditions (3.3) are satisfied then from the perspective of the rest of the power system, the strictly coherent group of n generators looks like one single machine.

The coherency that results from the satisfaction of conditions (3.3) has been termed strict geometric coherency because it results from the structural geometry (or topology) of the network and load-flow conditions. This terminology distinguishes strict geometric coherency from another type of coherency, namely the coherency that results from two generators being very tightly interconnected. Another look at Figure 3.1 shows that generators 3 and 4 can be

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coherent without satisfying the conditions for SGC. Specifically, these two generators can be coherent if the admittance Y_5' of Figure 3.1b is very large in relation to the other admittances Y_1' , Y_3' , Y_4' and Y_6' . In fact, in the limit as Y_5' gets infinitely large, generators 3 and 4 become strictly coherent. The reader may find this to be patently obvious, and hardly worthy of consideration. In addition, actual admittances are not infinitely large in actual power systems. However, in that regard, it is worth pointing out that the structural conditions (3.3) for strict geometric coherency will never be satisfied exactly in any real power system either. And, in point of fact, it is far more likely that in a real power system generators 3 and 4 are coherent because admittance Y_5' is very large compared to the other admittances than because conditions (3.3) are exactly, or even approximately, satisfied. For completeness, it is shown in the next section that if $n-1$ lines connecting all n generators of a group are made infinitely stiff then the n generators are strictly coherent. The reader who considers this to be carrying coals to Newcastle can proceed immediately to Section IV, which deals with the less obvious data of pseudo-coherency.

III. Strict Synchronizing Coherency

Consider again a specified group of n generators connected to the remainder of the power system through a set of m boundary buses, and assume that the disturbances that occur are external to the specified group. Consider the particular equation from (3.7) for generator 1 in the specified group. Now divide through this equation by Y_{21}^* , the admittance connecting generator 1 to generator 2 in the group,

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and let $|Y_{21}^*| \rightarrow \infty$. The left hand side of (3.7) goes to zero and all terms on the right hand side vanish except those containing Y_{21}^* . Thus equation (3.7) reduces to

$$0 = \text{Re}\left\{-\frac{E_1 E_2}{M_1} e^{j(\delta_1 - \delta_2)} + \frac{E_1^0 E_2^0}{M_1} e^{j(\delta_1^0 - \delta_2^0)}\right\}$$

Since $E_i = E_i^0$, $i = 1, 2, \dots, n$ this expression reduces to

$$0 = e^{j(\delta_1 - \delta_2)} - e^{j(\delta_1^0 - \delta_2^0)},$$

or equivalently,

$$\delta_1 - \delta_2 = \delta_1^0 - \delta_2^0.$$

Thus generators 1 and 2 are strictly coherent. As a next step take the particular equation from (3.7) for generator 2 and repeat the steps above this time letting $|Y_{32}^*| \rightarrow \infty$. This yields,

$$\delta_2 - \delta_3 = \delta_2^0 - \delta_3^0$$

so that generators 1, 2 and 3 are now coherent. Proceeding in this way, at step i adding generator $i+1$ to the group by letting $|Y_{i+1,i}^*| \rightarrow \infty$ makes, after $n-1$ steps the entire group or n generators strictly coherent.

Next take the particular equation from (3.7) for generator i of the specified group of n generators and let

$$|Y_{ki}^*| \rightarrow \infty, \quad k = 1, 2, 3, \dots, n, \quad k \neq i, \quad (3.16)$$

in such a way that

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$$\frac{|Y_{ji}^*|}{|Y_{ki}|} = C_{jk} \quad \text{for } j, k = 1, 2, \dots, n \quad j, k \neq i$$

Then dividing through by $|Y_{ki}^*|$ and invoking (3.16) reduces the equation for $\ddot{\delta}_{in}$ to

$$0 = \operatorname{Re} \left\{ \sum_{j=1}^n \left\{ \frac{E_i^0 E_j^0}{M_i} C_{jk} [e^{j(\delta_i - \delta_j)} - e^{j(\delta_i^0 - \delta_j^0)}] \right\} \right\}$$

This expression is true for any $i = 1, 2, \dots, n$ and for any steady state load flow conditions $E_j^0 e^{j\delta_j^0}$, $j = 1, 2, \dots, n$. Hence it must be true that

$$(\delta_i - \delta_j) = (\delta_i^0 - \delta_j^0) \quad i = 1, 2, \dots, n$$

Thus the n generators of the specified group are strictly coherent.

It is easy to see at this point that choosing any set of $n-1$ lines that connect all n machines and letting these lines become infinitely stiff results in the group of n generators being strictly coherent.

The practical case, of course, does not allow for infinitely stiff connections. However, from equations (3.7) it is clear that if a group of n machines can be found whose interconnections are very large relative to the interconnections between the group and the rest of the power system, then the analysis carried through above for strict synchronizing coherency is approximately true. It is also clear from (3.7) that the approximation will be better if the number

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of boundary buses between the group and the rest of the power system is small.

All of this may seem almost trivial. Part of the reason for this is that most of the ideas presented here have been in the folklore of power systems for a long time, and the ideas seem so obvious that they hardly need justification. Nonetheless, Chapters 4 and 5 will prove this formalization to be a powerful conceptual tool.

The coherency due to a group of machines being tightly bound will be called synchronizing coherency, and in the case of $n-1$ infinitely strong interconnections, strict synchronizing coherency (SSG). Synchronizing coherency is the predominant cause of coherency in real power systems, and the time devoted to it here is probably well spent.

The analysis of this section has resulted in a second set of conditions that result in a specified group of n machines being strictly coherent and appearing to the remainder of the power system to be one single machine. In the next section conditions are formulated under which the n machines of the specified group are no longer coherent, but still appear to the remainder of the power system to be one single generator.

IV. Pseudo-Coherency

In Section II, strict geometric coherency resulted from network structure and loadflow conditions that prorateded the disturbance energy in such a way that all the generators of the specified group accelerated at the same rate, causing them to appear to be a single generator. A natural question to ask is whether this

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$$\dot{z}_j = -2\sigma_j \sum_{j=1}^n z_j$$

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Equation (3.17)
 (3.7).

phenomena could work in reverse. That is, suppose a disturbance occurs outside a specified group of n machines, and propagates into the group, causing the generators to accelerate at different rates. Can conditions be found such that the specified group although not coherent, still "appears" to be coherent to the rest of the system?

Suppose, now, there are n generators in the study group, that is the part of the power system external to the specified group, and m boundary buses between the study group and the rest of the power system, i.e. the specified group. Let i be one of the n generators in the study group. Then equation (3.7) can be interpreted as the acceleration of generator i ,

$$\begin{aligned} \ddot{\delta}_{in} = & -\text{Re}\left\{ \sum_{j=1}^n \left\{ \left[\frac{E_n}{M_n} Y_{jn}^* - \frac{E_i}{M_i} Y_{ji}^* e^{j(\delta_i - \delta_n)} \right] E_j e^{j(\delta_n - \delta_j)} \right. \right. \\ & \left. \left. - \left[\frac{E_n^0}{M_n} Y_{jn}^* - \frac{E_i^0}{M_i} Y_{ji}^* e^{j(\delta_i^0 - \delta_n^0)} \right] E_j^0 e^{j(\delta_n^0 - \delta_j^0)} \right\} \right\} \\ & + K_i(t) \quad i = 1, 2, \dots, n-1 \end{aligned} \quad (3.17)$$

where

$$\begin{aligned} K_i(t) = & \text{Re}\left\{ \sum_{k=1}^m \left\{ \left[\frac{E_n}{M_n} Y_{kn}^* - \frac{E_i}{M_i} Y_{ki}^* e^{j(\delta_i - \delta_n)} \right] V_k e^{j(\delta_n - \delta_k)} \right. \right. \\ & \left. \left. - \left[\frac{E_n^0}{M_n} Y_{kn}^* - \frac{E_i^0}{M_i} Y_{ki}^* e^{j(\delta_i^0 - \delta_n^0)} \right] V_k^0 e^{j(\delta_n^0 - \delta_k^0)} \right\} \right\} . \end{aligned} \quad (3.18)$$

Equation (3.17) is simply a rearrangement of the terms of equation (3.7).

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$$\frac{1}{k} \frac{d}{dt} \left(\frac{1}{k} \right) = -j \left(\frac{1}{k} \right) - \delta$$

The first summation in (3.17) is the contribution to $\ddot{\delta}_{in}$ due to the energy transfers among the n generators of the study group. The second term is the acceleration of generator i of the study group due to the interaction at the boundary between the n generators of the study group and the generators of the specified group. If the generators of the specified group are to appear as a single generator to the study group, then this second summation, which is the acceleration of generator i of the study group relative to generator n of the study group caused by the specified group must be zero, i.e.

$$K_i(t) = 0, \quad i = 1, 2, \dots, n-1, \quad (3.19)$$

for all $t > 0$. In other words, the acceleration caused by the generators of the specified group is the same for every machine of the study group.

It is assumed that the angles of the n generators of the study group are not coherent. Therefore it is not possible to factor the expression on the right hand side of (3.18) as was done for the case of strict geometric coherency. It is apparent that the conditions of (3.19) cannot be satisfied exactly for any arbitrary disturbance, if they can be satisfied at all.

Now consider the following. Suppose that at $t = 0^-$ the following relationships exist.

$$\frac{E_i}{M_i} Y_{ik} e^{-j(\delta_i^0 - \delta_n^0)} = \frac{E_n}{M_n} Y_{nk} \quad \text{for } i = 1, 2, \dots, n-1, k = 1, 2, \dots, m \quad (3.20)$$

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Further suppose that the disturbance that occurs in the area containing the n generators of the study group is not large, so that the angle deviations between the n generators are only a few degrees. Assuming further the angle differences among the n generators and the m boundary buses at steady state are not large, it follows that the conditions of (3.20) are approximately satisfied for all $t > 0$. It is then possible to write

$$K_i(t) \approx \operatorname{Re} \left\{ \sum_{k=1}^m \left\{ \left[\frac{E_n}{M_n} Y_{kn}^* - \frac{E_i}{M_i} Y_{ki}^* e^{j(\delta_i^0 - \delta_n^0)} \right] [V_k e^{j(\delta_n - \delta_k)} - V_k e^{j(\delta_n^0 - \delta_k^0)}] \right\} \right\}$$

$$\approx 0, \quad i = 1, 2, \dots, n-1.$$

Thus if the conditions of (3.20) are satisfied, then the relative acceleration of the generators of the study group does not depend on the generators of the specified group.

The analysis above indicates that if the specified group of n machines is not strictly coherent, then it cannot under any conditions be represented by a single equivalent generator without distorting to some extent the response of the remainder of the system. However, the analysis also indicates that for small disturbances within the study group, the specified group may, under conditions (3.20), be quite adequately represented by a single generator. In fact, it will be shown in Chapter 4, that the conditions (3.20) are sufficient to decouple the linearized equations for the power system, so that for the linearized model, the n generators of the

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specified group behave exactly as a single generator. Since, from the perspective of the remainder of the power system, the behavior of the specified group of generators is identical to that of a group of strictly coherent generators, the conditions (3.20) will be called the conditions for pseudo-coherency. The absence of the adjective strict means that these conditions result in pseudo-coherency only for the linearized model of the power system.

V. Some Observations of the Relative Utility of the Three Types of Coherency

This chapter has explored conditions under which a specified group of n generators responds to a disturbance outside the group as if it were a single generator. The first set of conditions (3.3) which lead to strict geometric coherency are purely hypothetical in the sense that they could probably never be satisfied exactly in any real power system. The utility of an approximate satisfaction of these conditions can only be answered empirically. It is interesting to note that the most important use of the conditions (3.3) comes from their trivial satisfaction when the Y_{ik} are very, very small. That is, conditions (3.3) explain conceptually the well known empirical fact that generators a long electrical distance from a disturbance accelerate together, even if their inertias are widely different.

Strict synchronizing coherency is purely hypothetical for a different reason, namely that real power systems do not have infinite admittances. It is, however, by far the most important set of conditions for coherent behavior of group. Its utility when approximately satisfied has been well established by Podmore and

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others. Further, as will be shown in Chapter 4, in the linearized model, the synchronizing coherency can lead to a separation of the power system into fast and slow subsystems, through the techniques of singular perturbation theory.

The third condition, pseudo-coherency, is by far the weakest, applying only to small disturbances.

VI. Implications of the Coherency Conditions

The three types of coherency discussed in this chapter have important implications for understanding the connection between the modal and coherency methods of forming dynamic equivalents.

For strict geometric and strict synchronizing coherency, the fact that the specified group of n generators acts like a single generator really means that the internal behavior of the specified group is beyond the control of disturbances outside the specified group. For the case of pseudo-coherency the internal behavior of the specified group is undetectable by the rest of the power system, that is, unobservable.

It was shown in Chapter 2, that observability and controllability conditions are used to discard canonical states in forming the dynamic equivalent of the external system. In Chapter 4, it will be shown that for a linearized model of the power system, if the conditions for SGC, SSG or PC are satisfied for a group of n machines then the modal and coherency dynamic equivalents are identical. The results in this chapter make this result seem fairly evident and it takes only a little work to establish it.

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CHAPTER 4

THE LINEAR MODEL IDENTIFYING THE COHERENCY EQUIVALENT WITH THE MODAL EQUIVALENT

I. Introduction

In Chapter 3 conditions were found which caused a specified group of n generators to behave, from the point of view of the rest of the power system, like a single generator. This analysis was done using a nonlinear model of the power system. One would expect those conditions to yield the same result if the equations for the power system are linearized about some stable operating point.

Section II is devoted to showing that this is in fact the case, for a modified form of the linear equations developed in Chapter 2.

Section III then formalizes the concepts about controllability and observability discussed in a qualitative way at the end of Chapter 3. Specifically, it is shown that if the conditions for strict geometric coherency or strict synchronizing coherency or pseudo-coherency are met by a specified group of n generators, then the coherency and modal analysis methods yield the same equivalent for the specified group of generators. Section IV then extends these results by showing that the structural conditions necessary to apply the techniques of singular perturbation theory to a specified group of generators are, in the limit as the parameter $\mu \rightarrow 0$, the conditions for strict synchronizing coherency of the specified group. Section V introduces the concept of linear decoupling.

An aside is required here about notation and nomenclature. The notation used here follows that of Chapter 3, but perhaps needs some clarification, because it is not precisely the orthodox notation used with power systems; it has, however, some advantages which will become clearer in Chapter 5. In the typical linearized, N generator, power system model, the generators are numbered sequentially and the last, or the N -th, generator is taken as the reference. That notation will be followed here, but with a second notation superimposed over it. The first m generators will be termed the "study group", and the last $n = N - m$ generators will be termed "the specified group of n generators", as in Chapter 3. In general, the first m generators, or study group, corresponds to the area of the power system where the disturbances occur. It can be identified with the "study system" or "internal system" nomenclature typically found in the power system literature. The specified group of n generators is the group of generators whose behavior is being investigated for the purpose of finding some property which will allow the group to be replaced by a reduced order model. In this chapter the analysis is primarily aimed at showing conditions under which the specified group can be represented by a single generator. As in Chapter 3, in order to save the reader's sanity the term "specified group of n generators" will occasionally be shortened to "the specified group" or just "the group". The meaning should be clear from the context.

This chapter also introduces the concept of an archetype. An archetype is a hypothetical set of structural conditions on a group of generators. The conditions are hypothetical in the sense that

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they are never exactly satisfied in an actual power system, although they are assumed to hold for the archetype. In some cases the structural conditions are achievable but have a probability of occurring that is so small as to be considered zero. Dicaprio's conditions (3.3) for strict geometric coherency are an example of this kind of archetype.

In other cases the archetype is achieved by a limiting process, such as making synchronizing power coefficients infinitely large. In the linear model this limiting process will result in a matrix going to zero or the product of two matrices going to zero. The notation adopted to distinguish this case is $\underline{A} \rightarrow \underline{0}$ or $\underline{AB} \rightarrow \underline{0}$. This will avoid having to write $\lim_{t_k \rightarrow \infty} \underline{A}^{-1} = \underline{0}$. It is simply a shorthand notation employed to make the monograph smoother for the reader. It has the added advantage of helping remind the reader that a hypothetical configuration is being considered.

The reader should not be concerned if the concept of an archetype seems a little vague. That vagueness will dissipate as various archetypes are considered.

II. Decoupling the Linear Model

The conditions found in Chapter 3, which cause a specified group of n generators to behave as a single generator were formulated using a second order nonlinear representation for the generators in the system, and the nonlinear, algebraic, power equations for the system. The same result can be shown for a linearized version of the equations, as follows.

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The linear model that will be used in the following discussion is a slight modification of the model developed in Chapter 2. That model was a state-space representation of order $2N-2$. Half of the equations in that model are simply defining equations that relate the generator angle excursions $\Delta\delta_i$ to the generator speed excursions $\Delta\omega_i$, $i = 1, 2, \dots, N-1$. For the present analysis, it is more convenient to rewrite the $2N-2$ state equations as $N-1$ second order equations,

$$\Delta\ddot{\delta}_{N-1} = -\hat{\underline{M}} \underline{T} \Delta\delta_{N-1} - \sigma \dot{\Delta\delta}_{N-1} + \underline{B} \underline{U} \quad (4.1)$$

where,

$$\delta_{N-1} = [\delta_1, \delta_2, \dots, \delta_{N-1}]^T$$

$$\sigma = \frac{D_i}{M_i}, \quad i = 1, 2, \dots, n-1$$

$$\underline{B} = [\hat{\underline{M}} \quad \hat{\underline{M}} \underline{L}]$$

$$\underline{U} = [\Delta PM_1, \Delta PM_2, \dots, \Delta PM_N; \Delta PL_1, \Delta PL_2, \dots, \Delta PL_Q]^T$$

$$\hat{\underline{M}} = \begin{bmatrix} \frac{1}{M_1} & & & & -\frac{1}{M_N} \\ & \frac{1}{M_2} & & & -\frac{1}{M_N} \\ & & \ddots & & \vdots \\ & & & \ddots & \vdots \\ & & & & \frac{1}{M_{N-1}} & -\frac{1}{M_N} \end{bmatrix}$$

$$\underline{L} = -\frac{\partial \underline{PG}}{\partial \hat{\underline{\theta}}} \left[\frac{\partial \underline{PL}}{\partial \hat{\underline{\theta}}} \right]^{-1}$$

$\underline{T} = N \times N-1$ matrix of synchronizing power coefficients.

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The modification required concerns the representation of the forcing function. The linear model of Chapter 2 assumes that load buses can be in the following locations:

- (1) In the part of the power system outside the specified group of n -generators and not contiguous to this group.
- (2) On the boundary between the group of n generators and the rest of the power system and contiguous to both areas.
- (3) Inside the specified group of n generators and not contiguous to the power system outside the group.

The analysis of Chapter 3 assumes that only buses of categories (1) and (2) remain. That is, that all the load buses and generator terminal buses of the specified group have been eliminated and only the generator internal buses remain. In the following discussion, the standard $2N-2$ model will be rearranged into this semi-reduced form to show how satisfaction of the conditions of Chapter 3 decouples the linearized equations.

First, with $N = m+n$, let,

- 1) δ_{n_i} , $i = 1, 2, \dots, n$, be the internal generator angles of the specified group.
- 2) δ_{m_k} , $k = 1, 2, \dots, m$, be the internal generator angles of the m generators of the study group, i.e. the generators outside the specified group.

Partition the matrices of equation (4.1) to match the dimensions of the vectors

$$\underline{\delta}_m = [\delta_{m_1}, \delta_{m_2}, \dots, \delta_{m_k}, \dots, \delta_{m_m}]$$

$$\underline{\delta}_{n-1} = [\delta_{n_1}, \delta_{n_2}, \dots, \delta_{n_i}, \dots, \delta_{n_{n-1}}]$$

That is, the angles δ_{m_k} of the vector $\underline{\delta}_m$ are the internal generator angles of the first m generators of the power system; the angles

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

$$\text{here } \hat{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$$= \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$$

and

δ_{n_i} of vector δ_{n-1} are generator angles $\delta_{m+1}, \dots, \delta_{N-1}$ of the overall power system. Generator N is the reference.

Then the equations (4.1) can be written

$$\begin{bmatrix} \ddot{\Delta\delta_m} \\ \vdots \\ \ddot{\Delta\delta_{n-1}} \end{bmatrix} = \begin{bmatrix} (-\hat{\underline{M}} \underline{T})_{11} & (-\hat{\underline{M}} \underline{T})_{12} \\ \vdots & \vdots \\ (-\hat{\underline{M}} \underline{T})_{21} & (-\hat{\underline{M}} \underline{T})_{22} \end{bmatrix} \begin{bmatrix} \Delta\delta_m \\ \vdots \\ \Delta\delta_{n-1} \end{bmatrix} - \begin{bmatrix} \sigma \underline{I}_m & \underline{0} \\ \vdots & \vdots \\ \underline{0} & \sigma \underline{I}_{n-1} \end{bmatrix} \begin{bmatrix} \dot{\Delta\delta_m} \\ \vdots \\ \dot{\Delta\delta_{n-1}} \end{bmatrix} + \begin{bmatrix} \hat{\underline{M}}_1 & (\hat{\underline{M}} \underline{L})_1 \\ \vdots & \vdots \\ \hat{\underline{M}}_2 & (\hat{\underline{M}} \underline{L})_2 \end{bmatrix} \begin{bmatrix} \Delta \underline{PM} \\ \vdots \\ \Delta \underline{PL} \end{bmatrix} \quad (4.2a)$$

(4.2b)

where $\hat{\underline{M}}$, in partitioned form, is,

$$\begin{bmatrix} \hat{\underline{M}}_{11} & \hat{\underline{M}}_{12} \\ \vdots & \vdots \\ \underline{0} & \hat{\underline{M}}_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{M_1} & & & & & -\frac{1}{M_N} \\ & \frac{1}{M_2} & & & & -\frac{1}{M_N} \\ & & \ddots & & & \vdots \\ & & & \frac{1}{M_m} & & \vdots \\ \hline & & & & \frac{1}{M_{m+1}} & \vdots \\ \underline{0} & & & & & \vdots \\ & & & & & \frac{1}{M_{m+n-1}} & -\frac{1}{M_N} \end{bmatrix}$$

$$\underline{L} = \begin{bmatrix} \underline{L}_{11} & \underline{L}_{12} \\ \vdots & \vdots \\ \underline{L}_{21} & \underline{L}_{22} \end{bmatrix}$$

$\underline{I}_m$ and $\underline{I}_{n-1}$ are identity matrices of dimension m and n-1 respectively.

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$\hat{\underline{M}}_1 = [\hat{\underline{M}}_{11} \mid \hat{\underline{M}}_{12}]$ is $m \times N$ and consists of the first m rows of $\hat{\underline{M}}$

$\hat{\underline{M}}_2 = [0 \mid \hat{\underline{M}}_{22}]$ is $n-1 \times N$ and consists of the last $n = N-m$ rows of $\hat{\underline{M}}$

$(\hat{\underline{M}} \underline{L})_1 = \hat{\underline{M}}_{11} \underline{L}_{11} + \hat{\underline{M}}_{12} \underline{L}_{21}$ is $m \times Q$ and consists of the first m rows of $(\hat{\underline{M}} \underline{L})$

$(\hat{\underline{M}} \underline{L})_2 = \hat{\underline{M}}_{22} \underline{L}_{22}$ is $n-1 \times Q$ and consists of the last $n-1 = N-m-1$ rows of $\hat{\underline{M}} \underline{L}$.

If we assume that equations (4.2) are in the semi-reduced form, then the conditions for strict geometric coherency amount to the condition $(\hat{\underline{M}} \underline{L})_2 = \underline{0}$. To see why this is so consider the following.

Assume that the system is in the semi-reduced form of Chapter 3, i.e. the load buses have been eliminated from the group of n generators, and that there are q_m load buses left in the study group, and q_b load buses on the boundary between the study group and the specified group of n generators. Consider the power equation at generator i of the specified group of n generators.

$$PG_i = \operatorname{Re} \left\{ \sum_{j=1}^n E_i E_j y_{ij}^s e^{j(\delta_i^0 - \delta_j^0 - \gamma_{ij})} + \sum_{k=1}^{q_b} E_i V_k y_{ik}^s e^{j(\delta_i^0 - \theta_k - \gamma_{ik})} \right\}$$

where $y_{ij}^s = |Y_{ij}^s|$ is the magnitude of the admittance Y_{ij}^s between buses i and j and γ_{ij} is the phase angle of Y_{ij}^s . The superscript s indicates the network is in the semi-reduced form.

The first summation represents the power exchanged between generator i and the other generators of the group. The second summation is the power exchange between generator i and the m boundary buses.

Taking the partial derivative of PG_i with respect to θ_h , $h = 1, 2, \dots, q_b$ results in

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$$\frac{\partial PG_i}{\partial \theta_h} = E_i V_h y_{ih}^s \sin(\delta_i - \theta_h - \gamma_{ih}), \quad i = 1, 2, \dots, n; \quad h = 1, 2, \dots, q_b$$

which are the synchronizing power coefficients between the boundary buses and generator i .

Now take the partial derivative of PG_i with respect to θ_ℓ where θ_ℓ is the voltage angle at one of the load buses not contiguous to the specified group of n generators, then

$$\frac{\partial PG_i}{\partial \theta_\ell} = 0 \quad \begin{array}{l} i = 1, 2, \dots, n \\ \ell = 1, 2, \dots, q_m \end{array} \quad (4.8)$$

Let,

- 1) $\underline{\theta}_{q_b} = [\theta_1, \theta_2, \dots, \theta_h, \dots, \theta_{q_b}]$ be the vector of voltage angles at the load buses of boundary between the study group and the specified group.
- 2) $\underline{\theta}_{q_m} = [\theta_1, \theta_2, \dots, \theta_\ell, \dots, \theta_{q_m}]^T$ be the vector of voltage angles at the load buses internal to the study group.
- 3) $\underline{PG}_m = [PG_1, PG_2, \dots, PG_k, \dots, PG_m]^T$ be the vector of electrical powers injected by the generators of the study group.
- 4) $\underline{PG}_n = [PG_1, PG_2, \dots, PG_i, \dots, PG_n]^T$ be the vector of electrical powers injected by the generators of the specified group.
- 5) $\underline{\theta} = [\underline{\theta}_{q_m} ; \underline{\theta}_{q_b}]^T$, $\underline{PG} = [\underline{PG}_m ; \underline{PG}_n]^T$.

Then,

$$\frac{\partial \underline{PG}}{\partial \underline{\theta}} = \begin{bmatrix} \underline{P}_{11} & \underline{P}_{12} \\ \underline{P}_{21} & \underline{P}_{22} \end{bmatrix} = \begin{bmatrix} \underline{P}_{11} & \underline{P}_{12} \\ \underline{0} & \underline{P}_{22} \end{bmatrix}$$

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$$\underline{P}_{11} \text{ is } m \times q_m$$

$$\underline{P}_{12} \text{ is } m \times q_b$$

$$\underline{P}_{21} \text{ is } n \times q_m$$

$$\underline{P}_{22} \text{ is } n \times q_b$$

The matrix $\underline{P}_{21} = \underline{0}$ by (4.8).

Now assume the conditions (3.3) are satisfied for the q_b boundary buses and premultiply $\partial \underline{PG} / \partial \underline{\theta}$ by $\hat{\underline{M}}$, to obtain

$$\begin{aligned} \hat{\underline{M}} \frac{\partial \underline{PG}}{\partial \underline{\theta}} &= \begin{bmatrix} \hat{\underline{M}}_{11} & \hat{\underline{M}}_{12} \\ \underline{0} & \hat{\underline{M}}_{22} \end{bmatrix} \begin{bmatrix} \underline{P}_{11} & \underline{P}_{12} \\ \underline{0} & \underline{P}_{22} \end{bmatrix} \\ &= \left[\begin{array}{c|c} \hat{\underline{M}}_{11} \underline{P}_{11} & \hat{\underline{M}}_{11} \underline{P}_{12} + \hat{\underline{M}}_{12} \underline{P}_{22} \\ \hline \underline{0} & \hat{\underline{M}}_{22} \underline{P}_{22} \end{array} \right] \end{aligned}$$

where

$$\hat{\underline{M}}_{11} \text{ is } m \times m$$

$$\hat{\underline{M}}_{12} \text{ is } m \times n$$

$$\hat{\underline{M}}_{22} \text{ is } (n-1) \times m$$

$$\underline{0} \text{ is } (n-1) \times n$$

The elements in $\hat{\underline{M}}_{22} \underline{P}_{22}$ are of the form

$$\{\hat{\underline{M}}_{22} \underline{P}_{22}\}_{ij} = \frac{t_{ij}}{M_i} - \frac{t_{nj}}{M_n} \quad i = 1, 2, \dots, n-1; j = 1, 2, \dots, q_b$$

where the index i runs over generators of the specified group and the index j runs over the boundary buses of the group.

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$$\frac{t_{ij}}{M_i} = \frac{\partial \overline{PG_i}}{\partial \theta_j} = \frac{E_i V_j y_{ij}^s \sin(\delta_i^0 - \theta_j^0 - \gamma_{ij})}{M_i} \quad (4.3)$$

Assume the conditions (3.3) for strict geometric coherency are satisfied, namely

$$\frac{E_i}{M_i} y_{ij}^s e^{-j(\delta_i^0 - \delta_n^0)} = \frac{E_n}{M_n} y_{nj}^s \quad \text{or,}$$

$$\frac{E_i y_{ij}^s e^{-j(\delta_i^0 - \delta_n^0 - \gamma_{ij})}}{M_i} = \frac{E_n}{M_n} y_{nj}^s e^{j\gamma_{nj}}$$

where

$$i = 1, 2, \dots, n \quad \text{and} \quad y_{ij}^s = y_{ij}^s e^{j\gamma_{ij}}$$

$$j = 1, 2, \dots, q_b$$

Now, moving all terms to the left-hand side of equation (4.3), conjugating, and multiplying by $E_j e^{j(\delta_n^0 - \delta_j^0)}$ results in the expression

$$\frac{E_i E_j y_{ij}^s e^{j(\delta_i^0 - \delta_j^0 - \gamma_{ij})}}{M_i} - \frac{E_n E_j y_{nj}^s e^{j(\delta_n^0 - \delta_j^0 - \gamma_{nj})}}{M_n} = 0 \quad (4.4)$$

Now note that the elements $\{\hat{M}_{22}^{P_{22}}\}_{ij}$ are simply the imaginary part of the expressions in equation (4.4). That is,

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$$\begin{aligned}
& \text{Imag} \left\{ \frac{E_i E_j y_{ij}^s}{M_i} e^{j(\delta_i^0 - \delta_j^0 - \gamma_{ij})} - \frac{E_n}{M_n} E_j y_{nj}^s e^{j(\delta_n^0 - \delta_j^0 - \gamma_{nj})} \right\} \\
&= \frac{E_i E_j y_{ij}^s \sin(\delta_i^0 - \delta_j^0 - \gamma_{ij})}{M_i} - \frac{E_n E_j}{M_n} y_{nj}^s \sin(\delta_n^0 - \delta_j^0 - \gamma_{nj}) \\
&= \frac{t_{ij}}{M_i} - \frac{t_{nj}}{M_n} \quad i = 1, 2, \dots, n; \quad j = 1, 2, \dots, q_b \\
&= 0.
\end{aligned}$$

Therefore, if the conditions (3.3) for strict geometric coherency hold, then the matrix $\hat{M}_{22}^{P_{22}} = \underline{0}$. As a consequence, it is now possible to write, for the semi-reduced model, i.e. with the load buses in the specified group or n generators eliminated,

$$\frac{\underline{M}}{\underline{\partial \theta}} \frac{\partial \underline{P}_G}{\partial \underline{\theta}} = \left[\begin{array}{c|c} \hat{M}_{11}^{P_{11}} & \hat{M}_{12}^{P_{11}} + \hat{M}_{12}^{P_{22}} \\ \hline \underline{0} & \underline{0} \end{array} \right] \quad (4.5)$$

Equation (4.5) in turn implies that

$$-\hat{\underline{M}} \frac{\partial \underline{P}_G}{\partial \underline{\theta}} \frac{\partial \underline{P}_L}{\partial \underline{\theta}}^{-1} = \hat{\underline{M}} \underline{L} = \left[\begin{array}{c} (\hat{\underline{M}} \underline{L})_1 \\ \hline (\hat{\underline{M}} \underline{L})_2 \end{array} \right] = \left[\begin{array}{c} (\hat{\underline{M}} \underline{L})_1 \\ \hline \underline{0} \end{array} \right]$$

Now, if the disturbances are confined to the study group then equations (4.2b), the equations of the specified group of n generators, are unforced. If it can now be shown that $(-\underline{M} \underline{T})_{21} = \underline{0}$ then, under SGC, the equations for the specified group are completely decoupled from the equations for the study group.

The condition $(-\hat{\underline{M}} \underline{T})_{21} = \underline{0}$ can be shown to result from the condition $\hat{M}_{22}^{P_{22}} = \underline{0}$ as follows. Consider the unreduced equations

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$$\begin{bmatrix} \Delta \underline{PG}_1 \\ \Delta \underline{PG}_2 \\ \Delta \underline{PL}_1 \\ \Delta \underline{PL}_2 \\ \Delta \underline{PL}_3 \end{bmatrix} = \begin{bmatrix} \frac{\partial \underline{PG}_1}{\partial \Delta \delta_1} & \frac{\partial \underline{PG}_1}{\partial \Delta \delta_2} & \frac{\partial \underline{PG}_1}{\partial \Delta \theta_1} & \frac{\partial \underline{PG}_1}{\partial \Delta \theta_2} & \frac{\partial \underline{PG}_1}{\partial \Delta \theta_3} \\ \frac{\partial \underline{PG}_2}{\partial \Delta \delta_1} & \frac{\partial \underline{PG}_2}{\partial \Delta \delta_2} & \frac{\partial \underline{PG}_2}{\partial \Delta \theta_1} & \frac{\partial \underline{PG}_2}{\partial \Delta \theta_2} & \frac{\partial \underline{PG}_2}{\partial \Delta \theta_3} \\ \frac{\partial \underline{PL}_1}{\partial \Delta \delta_1} & \frac{\partial \underline{PL}_1}{\partial \Delta \delta_2} & \frac{\partial \underline{PL}_1}{\partial \Delta \theta_1} & \frac{\partial \underline{PL}_1}{\partial \Delta \theta_2} & \frac{\partial \underline{PL}_1}{\partial \Delta \theta_3} \\ \frac{\partial \underline{PL}_2}{\partial \Delta \delta_1} & \frac{\partial \underline{PL}_2}{\partial \Delta \delta_2} & \frac{\partial \underline{PL}_2}{\partial \Delta \theta_1} & \frac{\partial \underline{PL}_2}{\partial \Delta \theta_2} & \frac{\partial \underline{PL}_2}{\partial \Delta \theta_3} \\ \frac{\partial \underline{PL}_3}{\partial \Delta \delta_1} & \frac{\partial \underline{PL}_3}{\partial \Delta \delta_2} & \frac{\partial \underline{PL}_3}{\partial \Delta \theta_1} & \frac{\partial \underline{PL}_3}{\partial \Delta \theta_2} & \frac{\partial \underline{PL}_3}{\partial \Delta \theta_3} \end{bmatrix} \begin{bmatrix} \Delta \delta_1 \\ \Delta \delta_2 \\ \Delta \theta_1 \\ \Delta \theta_2 \\ \Delta \theta_3 \end{bmatrix} \quad (4.6)$$

where

$\underline{PG}_1$ and $\Delta \delta_1$ are the internal generator power and angles for the study group

$\underline{PG}_2$ and $\Delta \delta_2$ are the internal generator power and angles for the specified group of n generators

$\underline{PL}_1$ and $\Delta \theta_1$ are the load power and angles for the load buses of the study group

$\underline{PL}_2$ and $\Delta \theta_2$ are the load power and angles for the boundary buses

$\underline{PL}_3$ and $\Delta \theta_3$ are the load power and angles for the load buses of the specified group of n generators.

It is assumed here that the generator terminal buses have all been eliminated and that there are no connections between the generators of the study group and the specified group, and no connections between the load buses of the study group and the load buses of the specified group.

It is implicitly assumed that no study group internal generator buses are boundary buses. This causes no lack of generality because

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$$\begin{bmatrix} \underline{PG}_1 \\ \underline{PG}_2 \\ \underline{PL}_1 \\ \underline{PL}_2 \\ \underline{PL}_3 \end{bmatrix}$$

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$$\begin{bmatrix} \underline{PG}_1 \\ \underline{PG}_2 \\ \underline{PL}_1 \\ \underline{PL}_2 \end{bmatrix} =$$

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if a study group generator were a boundary bus, the corresponding column in $(-\hat{\underline{M}} \underline{I})_{21}$ would, assuming conditions (3.3) are true, be zero.

As a consequence equations (4.6) can be put in the form:

$$\begin{bmatrix} \Delta \underline{PG}_1 \\ \Delta \underline{PG}_2 \\ \Delta \underline{PL}_1 \\ \Delta \underline{PL}_2 \\ \Delta \underline{PL}_3 \end{bmatrix} = \begin{bmatrix} \underline{J}_{11} & \underline{0} & \underline{J}_{13} & \underline{J}_{14} & \underline{0} \\ \underline{0} & \underline{J}_{22} & \underline{0} & \underline{J}_{24} & \underline{J}_{25} \\ \underline{J}_{31} & \underline{0} & \underline{J}_{33} & \underline{J}_{34} & \underline{0} \\ \underline{J}_{41} & \underline{J}_{42} & \underline{J}_{43} & \underline{J}_{44} & \underline{J}_{45} \\ \underline{0} & \underline{J}_{52} & \underline{0} & \underline{J}_{54} & \underline{J}_{55} \end{bmatrix} \begin{bmatrix} \Delta \underline{\delta}_1 \\ \Delta \underline{\delta}_2 \\ \Delta \underline{\theta}_1 \\ \Delta \underline{\theta}_2 \\ \Delta \underline{\theta}_3 \end{bmatrix} \quad (4.7)$$

To put (4.7) in the semi-reduced form assumed for equations (4.2)

requires the elimination of the load buses of the specified group.

Setting $\Delta \underline{PL}_3 = \underline{0}$, solving for $\Delta \underline{\theta}_3$, and eliminating $\Delta \underline{\theta}_3$ from the first four equations of (4.6) yields

$$\begin{bmatrix} \Delta \underline{PG}_1 \\ \Delta \underline{PG}_2 \\ \Delta \underline{PL}_1 \\ \Delta \underline{PL}_2 \end{bmatrix} = \begin{bmatrix} \underline{J}_{11} & \underline{0} & \underline{J}_{13} & \underline{J}_{14} \\ \underline{0} & \underline{J}_{22} & \underline{0} & \underline{J}_{24} \\ \underline{J}_{31} & \underline{0} & \underline{J}_{33} & \underline{J}_{34} \\ \underline{J}_{41} & \underline{J}_{42} & \underline{J}_{43} & \underline{J}_{44} \end{bmatrix} \begin{bmatrix} \Delta \underline{\delta}_1 \\ \Delta \underline{\delta}_2 \\ \Delta \underline{\theta}_1 \\ \Delta \underline{\theta}_2 \end{bmatrix} \quad (4.8)$$

where

$$\tilde{\underline{J}}_{22} = \underline{J}_{22} - \underline{J}_{25} \underline{J}_{55}^{-1} \underline{J}_{52}$$

$$\tilde{\underline{J}}_{24} = \underline{J}_{24} - \underline{J}_{25} \underline{J}_{55}^{-1} \underline{J}_{54}$$

$$\tilde{\underline{J}}_{42} = \underline{J}_{42} - \underline{J}_{45} \underline{J}_{55}^{-1} \underline{J}_{52}$$

$$\tilde{\underline{J}}_{44} = \underline{J}_{44} - \underline{J}_{45} \underline{J}_{55}^{-1} \underline{J}_{54}$$

Next eliminate the load buses $\Delta \underline{PL}_1$ and $\Delta \underline{PL}_2$ from equations (4.8)

by the same procedure to get:

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$$\begin{aligned}
\begin{bmatrix} \Delta \underline{PG}_1 \\ \Delta \underline{PG}_2 \end{bmatrix} &= \left\{ \begin{bmatrix} \underline{J}_{11} & \underline{0} \\ \underline{0} & \tilde{\underline{J}}_{22} \end{bmatrix} - \begin{bmatrix} \underline{J}_{S11} & \underline{J}_{S12} \\ \underline{J}_{S21} & \underline{J}_{S22} \end{bmatrix} \right\} \begin{bmatrix} \Delta \delta_1 \\ \Delta \delta_2 \end{bmatrix} \\
&= \begin{bmatrix} (\underline{J}_{11} - \underline{J}_{S11}) & -\underline{J}_{S12} \\ -\underline{J}_{S21} & (\tilde{\underline{J}}_{22} - \underline{J}_{S22}) \end{bmatrix} \begin{bmatrix} \Delta \delta_1 \\ \Delta \delta_2 \end{bmatrix}
\end{aligned} \tag{4.9a}$$

where

$$\begin{aligned}
\underline{J}_{S11} &= \underline{J}_{11}(\underline{Z}_{11}\underline{J}_{31} + \underline{Z}_{12}\underline{J}_{41}) + \underline{J}_{14}(\underline{Z}_{21}\underline{J}_{31} + \underline{Z}_{22}\underline{J}_{41}) \\
\underline{J}_{S12} &= \underline{J}_{13}\underline{Z}_{12}\underline{J}_{42} + \underline{J}_{14}\underline{Z}_{22}\tilde{\underline{J}}_{42} \\
\underline{J}_{S21} &= \tilde{\underline{J}}_{24}(\underline{Z}_{21}\underline{J}_{31} + \underline{Z}_{22}\underline{J}_{41}) \\
\underline{J}_{S22} &= \tilde{\underline{J}}_{24}\underline{Z}_{22}\tilde{\underline{J}}_{42} \quad \text{and}
\end{aligned} \tag{4.9b}$$

$$\begin{bmatrix} \underline{Z}_{11} & \underline{Z}_{12} \\ \underline{Z}_{21} & \underline{Z}_{22} \end{bmatrix} = \begin{bmatrix} \underline{J}_{33} & \underline{J}_{34} \\ \underline{J}_{43} & \tilde{\underline{J}}_{44} \end{bmatrix}^{-1}$$

Now $\tilde{\underline{J}}_{24}$ is the matrix $\underline{P}_{22}$, i.e. the synchronizing torque coefficients between the boundary buses and the generators of the specified group of n generators. If the conditions (3.3) for strict geometric coherency are satisfied then

$$\hat{\underline{M}}_{22} \tilde{\underline{J}}_{24} = \underline{0}, \text{ and}$$

$$\hat{\underline{M}}_{22} \underline{J}_{S21} = \underline{0} \text{ from (4.9b).}$$

Now $-\underline{J}_{S21}$, in turn, is the matrix of synchronizing torque coefficients between the generators of the study group and the generators of the specified group for the fully reduced model. That is $-\underline{J}_{S21} = -\underline{I}_{21}$

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where $\underline{T}$ is the $N \times N-1$ matrix of synchronizing torque coefficients for the fully reduced model, with all load buses eliminated, i.e.,

$$\begin{aligned} \hat{\underline{M}} \underline{T} &= \begin{bmatrix} \hat{\underline{M}}_{11} & \hat{\underline{M}}_{12} \\ \hline \underline{0} & \underline{M}_{22} \end{bmatrix} \begin{bmatrix} \underline{T}_{11} & \underline{T}_{12} \\ \hline \underline{T}_{21} & \underline{T}_{22} \end{bmatrix} \\ &= \begin{bmatrix} (\hat{\underline{M}}_{11}\underline{T}_{11} + \hat{\underline{M}}_{12}\underline{T}_{21}) & \hat{\underline{M}}_{11}\underline{T}_{12} + \hat{\underline{M}}_{12}\underline{T}_{22} \\ \hline \hat{\underline{M}}_{22}\underline{T}_{21} & \hat{\underline{M}}_{22}\underline{T}_{22} \end{bmatrix} \end{aligned}$$

Thus if condition (3.3) is satisfied then $\hat{\underline{M}}_{22}\tilde{\underline{J}}_{24} = \underline{0}$ which in its turn implies that $\hat{\underline{M}}_{22}\underline{J}_{S_{21}} = \hat{\underline{M}}_{22}\underline{T}_{21} = \underline{0}$. Thus it has been shown that the conditions (3.3) for strict synchronizing coherency completely decouple the equations for the specified group from the equations for the study group. That is, under the conditions of SGC, $(\hat{\underline{M}} \underline{L})_2 = (\hat{\underline{M}} \underline{T})_{21} = \underline{0}$ so that equations (4.2) take the form

$$\begin{bmatrix} \ddot{\Delta\delta}_{-m} \\ \vdots \\ \ddot{\Delta\delta}_{-n} \end{bmatrix} = \begin{bmatrix} \underline{\Sigma}_{11} & \underline{\Sigma}_{12} \\ \hline \underline{0} & \underline{\Sigma}_{22} \end{bmatrix} \begin{bmatrix} \Delta\delta_{-m} \\ \vdots \\ \Delta\delta_{-n} \end{bmatrix} - \begin{bmatrix} \sigma \underline{I}_m & \underline{0} \\ \hline \underline{0} & \sigma \underline{I}_n \end{bmatrix} \begin{bmatrix} \dot{\Delta\delta}_{-m} \\ \vdots \\ \dot{\Delta\delta}_{-n} \end{bmatrix} + \begin{bmatrix} \underline{B} \underline{U} \\ \hline \underline{0} \end{bmatrix} \quad (4.10a)$$

where

$$\underline{B} \underline{U} = [\hat{\underline{M}} \underline{T}]_1 \begin{bmatrix} \underline{\Delta PM} \\ \underline{\Delta PQ} \end{bmatrix} \quad \text{and} \quad \underline{\Sigma}_{ij} = (-\hat{\underline{M}} \underline{T})_{ij}, \quad i, j = 1, 2.$$

The term $\hat{\underline{M}}_2 \underline{\Delta PM}$ does not appear in equation (4.10b) because disturbances on $\underline{\Delta PM}$ are restricted to generators of the study group, making $\hat{\underline{M}}_2 \underline{\Delta PM} = \underline{0}$.

The initial conditions $\Delta\ddot{\delta}_{-n}(0^-) = \dot{\Delta\delta}_{-n}(0^-) = \underline{0}$ combined with conditions (3.3), the conditions for strict geometric coherency, are sufficient to yield $\Delta\delta_{-n}(t) = \underline{0}$ for all $t > 0$. Thus equations (4.10)

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reduce to the differential equations of the m generators outside the specified group of n generators, referenced to generator n of that group. Since the differential equations in (4.10) now only involve the first m generators, or the study group, and the last generator of the specified group, the network power equations can be reduced by setting $\Delta P_{Gi} = 0$, $i = 1, 2, \dots, n-1$, and eliminating $\Delta \delta_i$, $i = 1, 2, \dots, n-1$. The resulting equations represent a system of $m+1$ generators, i.e. the m generators of the study group referenced to generator n of the specified group. That is, the specified group of n generators acts like a single generator, for disturbances outside that group. In (4.2) the system as a whole is referenced to generator n of the specified group, resulting in the inertia of the equivalent machine in equations (4.10) being the inertia of machine n of the original group. If the inertia is large, then this choice of reference is probably adequate. It may be advantageous, however, to use a reference that yields an equivalent generator whose inertia is more representative of the group as a whole, such as the sum of the inertias of the group. This is the approach taken in the next section.

The analysis so far has established that if the conditions (3.3) for strict geometric coherency hold, then in the linear model, the specified group of coherent generators act like a single generator. An analogous line of reasoning shows that the same result is true if the conditions (3.20) for pseudo-coherency are satisfied. This time the structural conditions exist between the boundary buses and the generator buses of the study group. Since the proof is very similar to that just provided for SGC, it will not be given in full detail. Rather

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a careful outline of the steps will be provided. As will be discussed later in the chapter, pseudo-coherency is a conceptual property whose primary function is to lend completeness to the theory. It has very little practical utility, at least in the present research.

Consider the matrix

$$\frac{\partial \underline{PG}}{\partial \underline{\theta}_1} = \begin{bmatrix} \underline{P}_{11} & | & \underline{P}_{12} \\ \hline \underline{P}_{21} & | & \underline{P}_{22} \end{bmatrix}$$

where $\underline{PG} = [\underline{PG}_m \mid \underline{PG}_n]^T$ as before, but now, the load buses are eliminated from the study group so that

$$\underline{\theta}_1 = \begin{bmatrix} \underline{\theta}_{q_b} \\ \hline \underline{\theta}_{q_n} \end{bmatrix}$$

where q_b is the number of buses in the boundary and q_n is the number of load buses in the specified group. As before assume all generator terminal buses are eliminated. The dimensions of the sub-matrices of $\partial \underline{PG} / \partial \underline{\theta}$ are then

$$\underline{P}_{11} \quad m \times q_b$$

$$\underline{P}_{12} \quad m \times q_n$$

$$\underline{P}_{21} \quad n \times q_b$$

$$\underline{P}_{22} \quad n \times q_n$$

Then the $\underline{P}$ matrix has the form

$$\underline{P} = \begin{bmatrix} \underline{P}_{11} & | & \underline{0} \\ \hline \underline{P}_{21} & | & \underline{P}_{22} \end{bmatrix},$$

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$$\left[-\frac{1}{M} \right]$$

$$\left[-\frac{1}{M} \right]$$

$$\dot{V} = \dots$$

Then,

$$\frac{\dot{M}_{EPG}}{32} =$$

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since, this time, the load buses in the specified group have no direct effect on changes in generation in the study group. Next, change the reference from the last generator of the specified group to the first generator of the study group. This gives $\hat{\underline{M}}$ the form

$$\hat{\underline{M}} = \left[\begin{array}{ccc|ccc} -\frac{1}{M_1} & \frac{1}{M_2} & & & & \\ -\frac{1}{M_1} & & \frac{1}{M_3} & & & \\ & & & \ddots & & \\ & & & & \frac{1}{M_{m-1}} & \\ \hline & & & & \frac{1}{M_{m+1}} & \\ & & & & & \ddots \\ -\frac{1}{M_1} & & & & & \frac{1}{M_{m+n}} \end{array} \right] = \left[\begin{array}{cc} \hat{\underline{M}}_{11} & \underline{0} \\ \hline \hat{\underline{M}}_{21} & \hat{\underline{M}}_{22} \end{array} \right]$$

Then,

$$\begin{aligned} \hat{\underline{M}} \frac{\partial \underline{PG}}{\partial \underline{\theta}} &= \left[\begin{array}{cc} \hat{\underline{M}}_{11} & \underline{0} \\ \hline \hat{\underline{M}}_{12} & \underline{M}_{22} \end{array} \right] \left[\begin{array}{cc} \underline{P}_{11} & \underline{0} \\ \hline \underline{P}_{24} & \underline{P}_{22} \end{array} \right] \\ &= \left[\begin{array}{cc} \underline{M}_{11} \underline{P}_{11} & \underline{0} \\ \hline \underline{M}_{12} \underline{P}_{11} + \underline{M}_{22} \underline{P}_{21} & \underline{M}_{22} \underline{P}_{22} \end{array} \right] \end{aligned}$$

where

$\underline{M}_{11}$ is $(m-1) \times m$

$\underline{M}_{21}$ is $n \times m$

$\underline{M}_{22}$ is $n \times n$

and the $\underline{0}$ is $(m-1) \times n$.

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The conditions (3.20) can be manipulated as was done in the case of conditions (3.3) to yield

$$\frac{t_{ik}}{M_i} - \frac{t_{1k}}{M_1} = 0 \quad \text{for} \quad \begin{matrix} i = 2, \dots, m \\ k = 1, 2, \dots, q_b \end{matrix}$$

That is the subscript i runs over generators $2, \dots, m$ of the study group, and the subscript k over the q_b buses of the boundary. But

$$\{\hat{M}_{11}P_{11}\}_{ik} = \frac{t_{ik}}{M_i} - \frac{t_{1k}}{M_1} = 0$$

so that the conditions (3.20) yield

$$\hat{M}_{11}P_{11} = \underline{0},$$

Just as the conditions (3.3) led to

$$\hat{M}_{22}P_{22} = \underline{0}.$$

Now eliminate the load buses within the study group by setting

$\Delta PL_1 = \underline{0}$, in equation (4.7), solving for $\Delta \theta_1$ and back substituting to get:

$$\begin{bmatrix} \Delta PG_2 \\ \Delta PG_2 \\ \Delta PB_2 \\ \Delta PL_3 \end{bmatrix} = \begin{bmatrix} \tilde{J}_{11} & \underline{0} & \tilde{J}_{14} & \underline{0} \\ \underline{0} & J_{22} & J_{24} & J_{25} \\ \tilde{J}_{41} & J_{42} & \tilde{J}_{44} & J_{45} \\ \underline{0} & J_{52} & J_{54} & J_{55} \end{bmatrix} \begin{bmatrix} \Delta PG_1 \\ \Delta PG_2 \\ \Delta \theta_2 \\ \Delta \theta_3 \end{bmatrix}$$

$\tilde{J}_{14}$ is the matrix of synchronizing torque coefficients between the generators of the study group and the boundary buses where the network has been reduced by eliminating the load buses of the study group.

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That is $\tilde{J}_{14}$ is the matrix P_{11} . Hence, if conditions (3.20) hold, then

$$\hat{M} \tilde{J}_{14} = 0$$

Now eliminate $\underline{PL}_2$ and $\underline{PL}_3$ to obtain

$$\begin{bmatrix} \Delta \underline{PG}_1 \\ \Delta \underline{PG}_2 \end{bmatrix} = \begin{bmatrix} (\underline{J}_{11} - \underline{J}_{S11}) & -\underline{J}_{S12} \\ -\underline{J}_{S21} & (\underline{J}_{22} - \underline{J}_{S22}) \end{bmatrix} \begin{bmatrix} \Delta \underline{\delta}_1 \\ \Delta \underline{\delta}_2 \end{bmatrix}$$

where

$$\underline{J}_{S11} = \tilde{J}_{14} \underline{Z}_{11} \tilde{J}_{41}$$

$$\underline{J}_{S12} = \tilde{J}_{14} (\underline{Z}_{11} \underline{J}_{42} + \underline{Z}_{12} \underline{I}_{52})$$

$$\underline{J}_{S21} = \underline{J}_{24} \underline{Z}_{11} \tilde{J}_{41} + \underline{J}_{25} \underline{Z}_{21} \tilde{J}_{41}$$

$$\underline{J}_{S22} = \underline{J}_{24} (\underline{Z}_{11} \underline{J}_{42} + \underline{Z}_{12} \underline{J}_{52}) + \underline{J}_{25} (\underline{Z}_{21} \underline{J}_{42} + \underline{Z}_{22} \underline{J}_{52})$$

$$\begin{bmatrix} \underline{Z}_{11} & \underline{Z}_{12} \\ \underline{Z}_{21} & \underline{Z}_{22} \end{bmatrix} = \begin{bmatrix} \tilde{J}_{44} & \underline{J}_{45} \\ \underline{J}_{54} & \underline{J}_{55} \end{bmatrix}^{-1}$$

But $\underline{J}_{S12} = \underline{I}_{12}$ the submatrix of synchronizing torque coefficients of the fully reduced model, with the reference generator 1 of the study group. That is

$$\begin{bmatrix} \hat{M}_{11} & 0 \\ \hat{M}_{21} & \hat{M}_{22} \end{bmatrix} \begin{bmatrix} \underline{I}_{11} & \underline{I}_{12} \\ \underline{I}_{21} & \underline{I}_{22} \end{bmatrix} = \begin{bmatrix} \hat{M}_{11} \underline{I}_{11} & \hat{M}_{11} \underline{I}_{12} \\ \hat{M}_{21} \underline{I}_{11} + \hat{M}_{22} \underline{I}_{21} & (\hat{M}_{21} \underline{I}_{12} + \hat{M}_{22} \underline{I}_{22}) \end{bmatrix}$$

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and

$$\begin{aligned}\hat{\underline{M}}_{11}\underline{I}_{12} &= -\hat{\underline{M}}_{11}\underline{J}_{S_{12}} = -\hat{\underline{M}}_{11}\underline{J}_{14}(\underline{Z}_{11}\underline{J}_{42} + \underline{Z}_{12}\underline{J}_{52}) \\ &= \underline{0}\end{aligned}$$

when $\hat{\underline{M}}_{11}\underline{J}_{14} = \underline{0}$. Hence the conditions (3.20) are sufficient to decouple the equations for the study group from the equations for the specified group of m generators. Thus, if the conditions of (3.20) hold, equation (4.2) can be written.

$$\Delta\ddot{\delta}_{-m} = (-\hat{\underline{M}}_{-1}\underline{I})_{12}\Delta\delta_{-m} - \sigma\Delta\dot{\delta}_{-m} + [\underline{M}_{-1} \mid (\hat{\underline{M}}_{-1}\underline{L})_1][\Delta\underline{P}\underline{M} \mid \Delta\underline{P}\underline{L}]^T \quad (4.11a)$$

$$\Delta\ddot{\delta}_{-n} = (-\hat{\underline{M}}_{-1}\underline{I})_{22}\Delta\delta_{-n} - \sigma\Delta\dot{\delta}_{-n} + (-\hat{\underline{M}}_{-1}\underline{I})_{21}\Delta\ddot{\delta}_{-m} + [\hat{\underline{M}}_{-1}\underline{L}]_2[\Delta\underline{P}\underline{L}] \quad (4.11b)$$

Once again the term $\hat{\underline{M}}_2 \Delta\underline{P}\underline{M}$ does not appear in (4.11b) because the disturbances are confined to generators of the study group. Equation (4.11b) shows that the specified group of n generators is definitely **not** coherent. But (4.11a) shows that the motion of the specified **group** is decoupled from that of the study group, so that the specified **group** appears, to the study, group to be a single generator.

Next assume that the conditions for strict synchronizing coherency are satisfied, and that $(-\hat{\underline{M}}_{-1}\underline{I})_{22}^{-1}$ exists. The inverse of $(-\hat{\underline{M}}_{-1}\underline{I})_{22}$ can be written

$$(-\hat{\underline{M}}_{-1}\underline{I})_{22}^{-1} = \frac{1}{\text{Det}(-\hat{\underline{M}}_{-1}\underline{I})_{22}} \begin{bmatrix} \text{Cof}_{11} & \text{Cof}_{12} & \dots & \text{Cof}_{1n} \\ \text{Cof}_{21} & \text{Cof}_{22} & \dots & \text{Cof}_{2n} \\ & & \cdot & \cdot & \cdot \\ \text{Cof}_{n1} & \text{Cof}_{n2} & \dots & \text{Cof}_{nn} \end{bmatrix}$$

where Cof_{ij} is the ij th cofactor of $(-\hat{\underline{M}}_{-1}\underline{I})_{12}$

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Assume as was done in Chapter 3 that $n-1$ of the interconnections that link all n generators of the specified group are made infinitely stiff. In the linear model, the corresponding $n-1$ elements of $(-\hat{\underline{M}} \underline{I})_{22}$ become infinitely large. Now $(-\hat{\underline{M}} \underline{I})_{22}$ is $(n-1) \times (n-1)$ so that $\text{Det}(-\hat{\underline{M}} \underline{I})_{22}$ is the summation of $(n-1)!$ terms, each term being the product of $n-1$ elements of $(-\hat{\underline{M}} \underline{I})_{22}$. One of these terms is the product of all $n-1$ elements that are being allowed to become infinitely large. Now each cofactor Cof_{ij} is, in turn, the summation of $(n-2)!$ terms each term being the product of $(n-2)$ elements of $(-\hat{\underline{M}} \underline{I})_{22}$. Hence no term in Cof_{ij} can be the product of more than $n-2$ of the elements that are becoming infinitely large. As a result $\text{Det}(-\hat{\underline{M}} \underline{I})_{22}$ dominates every term in the summation of Cof_{ij} , $i, j = 1, 2, \dots, n-1$. Thus in the limit all terms of $(-\hat{\underline{M}} \underline{I})_{22}^{-1}$ tend to zero.

Now assume that $(-\underline{M} \underline{I})_{22}$ is finite and rewrite equation (4.2b) as

$$\begin{aligned} (-\hat{\underline{M}} \underline{I})_{22}^{-1} \Delta \ddot{\delta}_{n-1} &= (-\hat{\underline{M}} \underline{I})_{22}^{-1} (-\hat{\underline{M}} \underline{I})_{21} \Delta \delta_m + \Delta \delta_{n-1} - \sigma (-\hat{\underline{M}} \underline{I})_{22}^{-1} \Delta \dot{\delta}_{n-1} \\ &\quad + (-\hat{\underline{M}} \underline{I})_{22}^{-1} [\underline{M}_2 \Delta \underline{P} \underline{M} + (\hat{\underline{M}} \underline{L})_2 \Delta \underline{P} \underline{L}] \end{aligned}$$

Now letting the $n-1$ elements of $(-\hat{\underline{M}} \underline{I})_{22}$ become infinitely large results in

$$\underline{0} = \Delta \delta_{n-1} \text{ for all } t > 0$$

which says that there is no perturbation of the specified group.

This result in turn reduces equation (4.2a) to the form

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$$\ddot{\Delta\delta}_m = (-\hat{M}^{-1} \underline{I})_1 \Delta\delta_m - \sigma \dot{\Delta\delta}_m + [\underline{M}_1 \Delta P_M + (\hat{M}^{-1} \underline{L})_1 \Delta PL].$$

Thus once again, assuming zero initial conditions, the specified group of n generators behaves, from the point of view of the remainder of the system, like a single equivalent generator.

The analysis to this point can be summarized as the following,

Result 1: Given the linearized model of an N -generator power system (4.2) in the reduced or semi-reduced form, if any of the three conditions SGG, SSG, or PC holds for a specified group of n generators, that group of n generators can be replaced by a single equivalent generator, and the response of the remainder of the system to a disturbance outside the specified group of n generators will be perfectly preserved.

If the proper reference frame is chosen, the form of the single equivalent generator is exactly that proposed by Podmore. However, the N -th generator reference frame is not the proper one. The proper reference frame is defined in the next section.

Result 1 is the first important step in reaching the goal stated at the end of Chapter 3, namely connecting the modal and coherency equivalents via the concepts of controllability and observability. That goal is realized in the next section.

III. Identifying the Coherency Equivalent with the Modal Equivalent

This section uses the results of Section II, to establish conditions under which the modal and coherency equivalents are identical. As one might suspect the conditions that lead to identical equivalents are the three conditions of strict geometric coherency, strict synchronizing coherency and pseudo-coherency. There are two intermediate steps that simplify the proof of the main result. The

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first intermediate step is a lemma that gives expressions for the eigenvalues of the model of equations (4.2) in terms of the damping coefficient σ , and the elements of the matrix $-\hat{\underline{M}} \underline{I}$. The second step is to establish a more general referencing scheme for the model of (4.2).

The following lemma provides some useful insight into the way the system structure affects the eigenvalues of the linear model of the power system.

Lemma. Let $\underline{A}$ be the plant matrix in the state space representation of equations (4.2), with the frequency expressed in per unit. If $\lambda_i, \lambda_i^*, i = 1, 2, \dots, n-1$, are the $2N-2$ eigenvalues of $\underline{A}$ and $\gamma_i = 1, 2, \dots, N-1$ are the $N-1$ eigenvalues of $-\hat{\underline{M}} \underline{I}$, then

$$\lambda_i = \frac{-\sigma}{2} + \frac{1}{2} \sqrt{\sigma^2 + 4\gamma_i}, \quad \lambda_i^* = \frac{-\sigma}{2} - \frac{1}{2} \sqrt{\sigma^2 + 4\gamma_i}$$

Proof: Putting the frequency in per unit results in

$$\underline{A} = \left[\begin{array}{c|c} \underline{0} & \underline{I}_{N-1} \\ \hline -\hat{\underline{M}} \underline{I} & -\sigma \underline{I} \end{array} \right]$$

Then

$$\text{Det}[\lambda \underline{I} - \underline{A}] = \text{Det} \left[\begin{array}{c|c} \lambda \underline{I} & -\underline{I} \\ \hline \hat{\underline{M}} \underline{I} & (\lambda + \sigma) \underline{I} \end{array} \right]$$

Using the identity,

$$\text{Det} \left[\begin{array}{c|c} \underline{A}_{11} & \underline{A}_{12} \\ \hline \underline{A}_{21} & \underline{A}_{22} \end{array} \right] = \text{Det}\{[\underline{A}_{11} - \underline{A}_{12} \underline{A}_{22}^{-1} \underline{A}_{21}] [\underline{A}_{22}]\}$$

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$$\begin{aligned}
\text{Det}\{\lambda \underline{I} - \underline{A}\} &= \text{Det}\{[\lambda \underline{I} - (-\frac{1}{\lambda + \sigma} \hat{\underline{M}} \underline{I})][(\lambda + \sigma)\underline{I}]\} \\
&= \text{Det}\{(\lambda)(\lambda + \sigma)\underline{I} - (-\hat{\underline{M}} \underline{I})\} \\
&= \text{Det}\{\gamma \underline{I} - (-\hat{\underline{M}} \underline{I})\},
\end{aligned}$$

the last step indicates that if γ_i , $i = 1, 2, \dots, N-1$ are the eigenvalues of $-\hat{\underline{M}} \underline{I}$, then the eigenvalues of $\underline{A}$ are the solutions of the equation $\lambda^2 + \lambda\sigma - \gamma = 0$, or

$$\left. \begin{aligned} \lambda_i &= -\frac{\sigma}{2} + \frac{1}{2} \sqrt{\sigma^2 + 4\gamma_i} \\ \lambda_i^* &= -\frac{\sigma}{2} - \frac{1}{2} \sqrt{\sigma^2 + 4\gamma_i} \end{aligned} \right\} \quad i = 1, 2, \dots, N-1$$

It is the usual case with power systems that σ is small.

Thus in most analysis, including Dicaprio [10, 11] and Kokotovic [15], σ is set equal to zero. For the present analysis σ will be retained, but the results of the lemma make it clear that the eigenvalues of the linearized model are predominately imaginary and depend almost exclusively on the structure of the matrix $-\hat{\underline{M}} \underline{I}$.

The second intermediate step modifies the reference frame of the linear model. To this point, the reference for the $2N-2$ state equations has been δ_n , the angle of generator n of the specified group, which corresponds to generator N in the state space representation. A reference that is more in line with the aggregation of generators using in coherency equivalents is a modification of the reference frame that Meisel [9], calls the Uniform Center of Angle (UCA) reference frame. The reference frame used here will be referred to as the UCA_n reference frame. The

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subscript n denotes the fact that the summations involved are taken over the n generators of the specified group.

Define $\delta_e = \sum_{i=1}^n \frac{M_i}{M_e} \delta_i$, where the summation runs over the n generators of the group and $M_e = \sum_{i=1}^n M_i$. Assume the same convention as before, namely that there are $N = m+n$ generators in the power system, the first m being the study group and the last $n = N-m$ being the specified group. Then in the notation of the state model the summation above is over generators $m+1, \dots, N$.

Then, using the fictitious angle δ_e as the reference, the system of N generators can be expressed as,

$$\dot{\Delta\tilde{\delta}}_i = 2\pi f_0 \Delta\tilde{\omega}_i \quad (4.12a)$$

$$\Delta\tilde{\omega}_i = \frac{1}{M_i} (\Delta PM_i - \Delta PG_i) - \frac{1}{M_e} (\Delta PM_e - \Delta PG_e) - \sigma \Delta\tilde{\omega}_i \quad (4.12b)$$

$$\begin{bmatrix} \Delta \underline{PG} \\ \text{---} \\ \Delta \underline{PL} \end{bmatrix} = \begin{bmatrix} \frac{\partial \underline{PG}}{\partial \underline{\tilde{\delta}}} & \frac{\partial \underline{PG}}{\partial \underline{\tilde{\theta}}} \\ \frac{\partial \underline{PL}}{\partial \underline{\tilde{\delta}}} & \frac{\partial \underline{PL}}{\partial \underline{\tilde{\theta}}} \end{bmatrix} \begin{bmatrix} \Delta \underline{\tilde{\delta}} \\ \text{---} \\ \Delta \underline{\tilde{\theta}} \end{bmatrix} \quad (4.12c)$$

where

$$\left. \begin{aligned} \tilde{\delta}_i &= \delta_i - \delta_e \\ \tilde{\theta}_i &= \theta_i - \delta_e \\ \tilde{\omega}_i &= \omega_i - \tilde{\omega}_e \\ \sigma &= \frac{D_i}{M_i} \end{aligned} \right\} \quad i = 1, 2, \dots, N-1$$

$$\underline{\tilde{\delta}}_{N-1} = [\tilde{\delta}_1, \tilde{\delta}_2, \dots, \tilde{\delta}_{N-1}]^T \quad \underline{\tilde{\theta}} = [\tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_Q]^T$$

$$\underline{PG} = [PG_1, PG_2, \dots, PG_N]^T \quad \underline{PL} = [PL_1, PL_2, \dots, PL_Q]^T$$

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$$\underline{T} = \frac{\partial \underline{PG}}{\partial \underline{\delta}} - \frac{\partial \underline{PG}}{\partial \underline{\theta}} \left[\frac{\partial \underline{PL}}{\partial \underline{\theta}} \right]^{-1} \frac{\partial \underline{PL}}{\partial \underline{\delta}} \quad \underline{L} = - \frac{\partial \underline{PG}}{\partial \underline{\theta}} \left[\frac{\partial \underline{PL}}{\partial \underline{\theta}} \right]^{-1}$$

Note that the form of the equations in the UCA_n reference is exactly the same, as it was for the N -th generator reference frame, with $\underline{\tilde{M}}$ substituted for $\underline{\hat{M}}$. Thus equations (4.2) will be used as the standard form of the power system model, and the designation of the reference frame will dictate whether $\underline{\tilde{M}}$ or $\underline{\hat{M}}$ is used in the equations.

There are two important observations to be made about these equations. First the matrices $\underline{T}$ and $\underline{L}$ shown above are identical to the $\underline{T}$ and $\underline{L}$ in the state model referenced to δ_N . That $\underline{T}$ and $\underline{L}$ are invariant under a change of reference can be shown by considering the electrical power output of generator i .

$$PG_i = E_i^2 G_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^N E_i E_j (B_{ij} \sin \delta_{ij} + G_{ij} \cos \delta_{ij}), \quad (4.13)$$

where,

$$\delta_{ij} = \delta_i - \delta_j$$

$$E_i = \text{voltage at generator } i$$

$$Y_{ii} = G_{ii} + jB_{ii}, \text{ a diagonal element of YBUS.}$$

$$Y_{ij} = G_{ij} + jB_{ij}, \text{ an off-diagonal element of YBUS.}$$

Note that the sine and cosine expressions in (4.13) are expressed in terms of angle differences. Thus for $\hat{\delta}_i = \delta_i - \delta_n$ and $\tilde{\delta}_i = \delta_i - \delta_e$, the two referencing systems under consideration,

$$\begin{aligned} \tilde{\delta}_i - \tilde{\delta}_j &= (\delta_i - \delta_e) - (\delta_j - \delta_e) = \tilde{\delta}_{ij} = \delta_{ij} = \hat{\delta}_{ij} = \hat{\delta}_i - \hat{\delta}_j = \\ &(\delta_i - \delta_n) - (\delta_j - \delta_n) \end{aligned}$$

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Thus the power equations are independent of the reference frame and further,

$$\frac{\partial \sin \tilde{\delta}_{ij}}{\partial \tilde{\delta}_i} = \cos \tilde{\delta}_{ij} = \cos \hat{\delta}_{ij} = \frac{\partial \sin \hat{\delta}_{ij}}{\partial \hat{\delta}_i}, \quad i = 1, 2, \dots, N-1$$

$$\frac{\partial \cos \tilde{\delta}_{ij}}{\partial \tilde{\delta}_i} = -\sin \tilde{\delta}_{ij} = -\sin \hat{\delta}_{ij} = \frac{\partial \cos \hat{\delta}_{ij}}{\partial \hat{\delta}_i}, \quad i = 1, 2, \dots, N-1$$

Consequently

$$\frac{\partial \underline{PG}}{\partial \underline{\tilde{\delta}}} - \frac{\partial \underline{PG}}{\partial \underline{\tilde{\theta}}} \left[\frac{\partial \underline{PG}}{\partial \underline{\tilde{\theta}}} \right]^{-1} \frac{\partial \underline{PG}}{\partial \underline{\tilde{\delta}}} = \underline{I} = \frac{\partial \underline{PG}}{\partial \underline{\hat{\delta}}} - \frac{\partial \underline{PG}}{\partial \underline{\hat{\theta}}} \left[\frac{\partial \underline{PG}}{\partial \underline{\hat{\theta}}} \right]^{-1} \frac{\partial \underline{PL}}{\partial \underline{\hat{\delta}}}$$

A similar argument holds for the matrix $\underline{L}$.

The second observation is that, even though a fictitious reference angle, δ_e , has been used, the linear model for the N generator system has only $2N-2$ states. This is possible because the angles of the specified group of n generators are not independent. This is not a property peculiar to the reference frame. It is true for any reference frame, including the N -th reference frame used previously in this work. This angle dependence results from the fact that the sum of the power changes generated by the machines of the system must equal zero [16].

The dependence of the $n = N-m$ generator angles of the specified group means that one of these angles can be eliminated as a state. A second state can be eliminated, namely the speed of one of the generators, under the assumption of uniform damping. For uniform damping the n dynamic equations for the generators of the specified group can be written:

$$\ddot{\tilde{x}}_i = \frac{1}{M_i} (\Delta P M_i)$$

take each equation

from equation (3)

and

$$\ddot{\tilde{x}}_e = \frac{1}{M_e} \sum_{i=1}^n \frac{M_i}{M_e} \ddot{\tilde{x}}_i = \frac{1}{M_e} \sum_{i=1}^n M_i \ddot{\tilde{x}}_i$$

since $\sum_{i=1}^n M_i = M_e$

$$\sum_{i=1}^n \ddot{\tilde{x}}_i = \ddot{\tilde{x}}_e$$

so that equation (4)

$$\ddot{\tilde{x}}_j = \frac{1}{M_j} (\Delta P M_j)$$

where $\Delta P M_e = \sum_{i=1}^n \Delta P M_i$

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$$\ddot{\Delta\delta}_i = \frac{1}{M_i}(\Delta PM_i - \Delta PG_i) - \sigma \dot{\Delta\delta}_i \quad i = 1, 2, \dots, n.$$

Now take each equation i and multiply it by M_i/M_e and subtract it from equation j where j is a specific generator in the specified group,

$$\begin{aligned} \ddot{\Delta\delta}_j - \sum_{i=1}^n \frac{M_i}{M_e} \ddot{\Delta\delta}_i &= \frac{1}{M_j}(\Delta PM_j - \Delta PG_j) - \sum_{i=1}^n \frac{M_i}{M_e} \left(\frac{\Delta PM_i - \Delta PG_i}{M_i} \right) - \sigma \dot{\Delta\delta}_j \\ &\quad - \sigma \sum_{i=1}^n \frac{M_i}{M_e} \dot{\Delta\delta}_i. \end{aligned} \quad (4.14)$$

Now since $\sum_{i=1}^n M_i \ddot{\Delta\delta}_i = 0$, then also

$$\sum_{i=1}^n \dot{\Delta\delta}_i = 0, \quad \sum_{i=1}^n \ddot{\Delta\delta}_i = 0$$

so that equation (4.14) reduces to

$$\ddot{\Delta\delta}_j = \frac{1}{M_j}(\Delta PM_j - \Delta PG_j) - \frac{1}{M_e}(\Delta PM_e - \Delta PG_e) - \sigma \dot{\Delta\delta}_j \quad (4.15)$$

where $\Delta PM_e = \sum_{i=1}^n \Delta PM_i$, $\Delta PG_e = \sum_{i=1}^n \Delta PG_i$. Multiplying the right hand side of (4.15) by M_j and then summing over $j = 1, 2, \dots, n$ gives

$$(\Delta PM_e - \Delta PG_e) - (\Delta PM_e - \Delta PG_e) - \sigma \sum_{j=1}^n M_j \dot{\Delta\delta}_j = 0.$$

Thus the dynamic equations of the specified group are dependent and one generator speed can be eliminated. In equations (4.12) the two states eliminated are $\ddot{\Delta\delta}_n$ and $\dot{\Delta\delta}_n$ where n refers to generator n of the specified group.

The conditions (3.3) for perfect coherency extends, as would be expected, to the UCA_n reference frame. Recall that in the last

section it was shown that, condition (3.3) implies

$$\frac{t_{ik}}{M_i} = \frac{t_{nk}}{M_n} \quad \begin{array}{l} i = 1, 2, \dots, n-1 \\ k = 1, 2, \dots, m \end{array} \quad (4.16)$$

where the subscript i is over the generators of the specified group and the subscript k can be over either the m generators of the reduced system or the m boundary buses of the semi-reduced system, as shown in the previous section.

Now consider an element of the submatrix $(-\tilde{\underline{M}} \underline{T})_{21}$ and use (4.16) to show, that if condition (3.3) is true then,

$$\begin{aligned} \{(-\tilde{\underline{M}} \underline{T})_{21}\}_{ik} &= \frac{t_{ik}}{M_i} - \frac{1}{M_e} \sum_{j=1}^n t_{jk} \\ &= \frac{t_{ik}}{M_i} - \frac{1}{M_e} \sum_{j=1}^n \frac{M_j}{M_n} t_{nk} \\ &= \frac{t_{ik}}{M_i} - \frac{t_{nk}}{M_n} \frac{1}{M_e} \sum_{j=1}^n M_j \\ &= \frac{t_{ik}}{M_i} - \frac{t_{nk}}{M_n} = 0 \quad \begin{array}{l} i = 1, 2, \dots, n-1 \\ j = 1, 2, \dots, m. \end{array} \end{aligned}$$

Thus the submatrix $(-\tilde{\underline{M}} \underline{T})_{21}$ is zero. Using arguments completely parallel to those given above for $(\hat{\underline{M}} \underline{L})_2$, it can be shown that the corresponding matrix $(\tilde{\underline{M}} \underline{L})_2$ is zero. The arguments that infinitely stiff interconnections, and pseudo coherency decouple the linear equations to produce a single machine equivalent are also exactly parallel to those presented in the last section with $\tilde{\underline{M}}$ substituted for $\hat{\underline{M}}$ in equations (4.2).

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Thus it is clear at this point that the UCA_n reference frame will yield all the same results as the single machine reference frame, and the following convention is adopted. When the discussion specifically involves the system referenced to generator N or the UCA_n reference frame, $\hat{M}$ or $\tilde{M}$, respectively, will be used. When the discussion is not concerned with a specific reference frame plain M will be used.

The UCA_n reference frame is the most convenient for understanding the connection between modal equivalents and the particular coherency equivalents derived by Podmore [2]. Assume that the specified group of n machines satisfies the conditions for either strict synchronizing coherency, strict geometric coherency, or pseudo-coherency. Then the dynamic equations of the specified group can be eliminated as discussed in Section II. In the UCA_n reference frame, the dynamic equations for the generators of the study group are of the form,

$$\ddot{\Delta\delta}_k = \frac{1}{M_k} (\Delta PM_i - \Delta PG_i) - \frac{1}{M_e} (\Delta PM_e - \Delta PE_e) - \sigma \Delta\dot{\delta}_k, \quad k = 1, 2, \dots, m \quad (4.17)$$

where

$$\Delta PM_e = \sum_{i=1}^n \Delta PM_i, \quad \Delta PG_e = \sum_{i=1}^n \Delta PG_i, \quad M_e = \sum_{i=1}^n M_i,$$

and

$$\tilde{\delta}_k = \delta_k - \delta_e, \quad \delta_e = \sum_{i=1}^n \frac{M_i}{M_e} \delta_i.$$

This says that the specified group has been replaced by a single generator satisfying the following conditions

- (1) The inertia of the equivalent machine is the sum of the inertias of the n machines of the group.

- (2) The mechanical power of the equivalent machine is the sum of the mechanical powers of the n machines of the group.
- (3) The electrical power of the equivalent machine is the sum of the electrical powers of the n machines of the group.
- (4) The angle of the equivalent generator is arbitrarily established as the weighted average of the angles of the n generators of the group.

The equivalent generator is, in fact, the same equivalent that would be formed by Podmore's techniques as presented in Chapter 2. The important point is that if any of the conditions SGC, SSG, or PC holds for a specific group of n generators, then the application of the rules of modal analysis to the equations for the group yields this same single machine equivalent. That is under SGC, SSC, or PC, the modal and coherency equivalents for the group of n generators are identical. This idea is implicit in Dicaprio's work [10, 11] and can be shown explicitly as follows.

Assume first that the conditions for strict geometric coherency are true for a specified group of n generators, in an N generator system, and use the model (4.12) in the UCA_n reference frame. Then the conditions for SGC cause the submatrix $(-\tilde{M} \underline{I})_{21}$ to be zero. Therefore,

$$\begin{aligned} \text{Det}[\lambda \underline{I} - (-\tilde{M} \underline{I})] &= \text{Det} \begin{bmatrix} \lambda \underline{I} - (-\tilde{M} \underline{I})_{11} & (-\tilde{M} \underline{I})_{12} \\ \underline{0} & \lambda \underline{I} - (-\tilde{M} \underline{I})_{22} \end{bmatrix} \\ &= \{\text{Det}[\lambda \underline{I} - (\tilde{M} \underline{I})_{11}]\} \{\text{Det}[\lambda \underline{I} - (-\tilde{M} \underline{I})_{22}]\}. \end{aligned}$$

This result shows that there are $n-1$ eigenvalues γ_i , $i = 1, 2, \dots, n-1$ depending only on $(\tilde{M} \underline{I})_{22}$ and therefore strictly associated with the

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specified group of n generators. Using the lemma proved earlier in this section this translates in the state model (4.12) to $n-1$ eigenvalue pairs of the form,

$$\lambda_i, \lambda_i^* = \frac{-\sigma}{2} \pm \frac{1}{2} \sqrt{\sigma^2 + 4\gamma_i} \quad i = 1, 2, \dots, n-1.$$

These eigenvalues correspond to modes associated with inter-machine oscillations within the specified group. However under SGC, and for a disturbance outside the group, these modes are not excited. That is the disturbance cannot "control" the intermachine oscillations. It was shown in Chapter 2 that this condition of uncontrollability was precisely one of the rules for mode elimination in forming the modal equivalent. Thus if the state space model is transformed to canonical form, under SGC, $2n-2$ of the eigenvalues, representing canonical modes, or states, that were uncontrollable for a disturbance outside the specified group would be eliminated. One pair of eigenvalues in the reduced system represents the motion of the group against the remainder of the system.

The elimination of modes under the conditions for SSC, is almost identical to that for SGC. Assume that the conditions for SSC are satisfied by the specified group of n generators and note that

$$\text{Det}[\underline{I} - (-\underline{M} \underline{I})] = \text{Det}\{[\lambda \underline{I} - (-\underline{M} \underline{I})_{11}] - (\underline{M} \underline{I})_{12}[\lambda \underline{I} - (-\underline{M} \underline{I})_{22}]^{-1} \underline{M}_{21}\} \times \\ \text{Det}\{\lambda \underline{I} - (-\underline{M} \underline{I})_{22}\}.$$

$$\begin{aligned} \text{Now } [\lambda \underline{I} - (-\underline{M} \underline{I})_{22}]^{-1} &= \lambda [I + \frac{1}{\lambda} (\underline{M} \underline{I})_{22}]^{-1} \\ &= \lambda \{ \underline{I} - [\underline{I} + \lambda (\underline{M} \underline{I})_{22}]^{-1} \} \end{aligned}$$

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so that in the limit, when $n-1$ connections are allowed to grow infinitely large,

$$[\lambda \underline{I} - (-\underline{M} \underline{I})_{22}]^{-1} \rightarrow \lambda [\underline{I} - \underline{I}] = \underline{0}$$

and

$$\text{Det}[\lambda \underline{I} - (-\underline{M} \underline{I})] = \text{Det}[\lambda \underline{I} - (-\underline{M} \underline{I})_{11}] \text{Det}[\lambda \underline{I} - (-\underline{M} \underline{I})_{22}]$$

Thus, once again, the eigenvalues are segregated precisely as they were in the discussion of SGC, and there are no oscillations within the group due to disturbances in the study group. Hence, as before the modes eliminated as uncontrollable are those associated with $(-\underline{M}\underline{I})_{22}$. Note that plain $\underline{M}$ has been used to emphasize the eigenvalues are not reference dependent.

If the conditions for pseudo-coherency are satisfied for the group of n generators then once again mode elimination leads to an equivalent identical to the equivalent obtained by the coherency method. Under PC the submatrix $(-\underline{M} \underline{I})_{12} = \underline{0}$, so that

$$\begin{aligned} \text{Det}[\lambda \underline{I} - (\underline{M} \underline{I})] &= \text{Det}[\lambda \underline{I} - (\underline{M} \underline{I})_{11} - (-\underline{M} \underline{I})_{12} [\lambda \underline{I} - (-\underline{M} \underline{I})_{22}]^{-1} (-\underline{M} \underline{I})_{21}] \\ &\quad \times \text{Det}[\lambda \underline{I} - (-\underline{M} \underline{I})_{22}] \\ &= \text{Det}[\lambda \underline{I} - (-\underline{M} \underline{I})_{11}] \text{Det}[\lambda \underline{I} - (-\underline{M} \underline{I})_{22}]. \end{aligned}$$

The eigenvalues λ_i, λ_i^* of the system model (4.12) that represent intermachine oscillations within the specified group are again decoupled from the eigenvalues for the study group.

In pseudo-coherency, the n generators of the specified group are no longer coherent but still appear to the remainder of the

system to behave as a single machine. This implies that transformed to the canonical form, the modes represented by the eigenvalues of $(-M \ I)_{22}$ will be eliminated as unobservable. The m eigenvalue pairs associated with $(M \ I)_{11}$ represent the intermachine oscillations among the $m+1$ generators of the reduced system which consists of the m generators of the study group and the reference generator which is just the single generator equivalent of the specified group.

The analysis to this point has now formally established the following important relationship.

Result 2. Given a linear model of an N generator power system (4.2), if any of the three conditions SGC, SSC, or PC hold for a specified group of n generators, then the equivalents formed by modal techniques and coherency techniques are identical. Further in the UCA₀ reference frame, the equivalent for the specified group is of the exact form proposed by Podmore.

Result 2 provides some very important insight into the relationship between modal and coherency techniques by showing three structural conditions under which the two methods produce identical equivalents. Further each condition could be related to a controllability or observability condition for mode elimination. Chapter 2 also listed "fast" eigenvalues as a rule for mode elimination. For the second order generator models used in this research, fast eigenvalues, in the classical sense of modes that decay rapidly to zero are not present. This is apparent from the lemma proved earlier in this section which showed that all the eigenvalues have the same small real part, that is $-\sigma/2$. Fast eigenvalues do occur in this model, however, if fast is interpreted as high frequency. The results of references [5, 6, 7] indicate that for a group of n generators

$n-1$ eigenvalue pairs can be associated with the intermachine oscillations of the group. These oscillations are of high frequency, relative to other eigenvalues in the system. Intuitively the high frequency of these eigenvalues would seem to be due to strong interconnections between machines of the group. But strong relative to what? The next section provides some analysis of the case where fast (high frequency) modes are present. The results of this analysis help quantify the meaning of "strong" interconnections.

IV. Decoupling High Frequency Modes

The analysis to this point has all been aimed at decoupling the differential equations for the study group from those for the specified group of n generators. A principal tool for decoupling differential equations is singular perturbation theory. This section investigates how singular perturbation theory can be applied to decoupling the equations of our example system (4.2).

Singular perturbation in the usual sense means that a system contains a set of canonical modes or states that are highly damped and decay rapidly to zero in a short boundary layer of time after a disturbance has been applied to the system. The solution technique is to set the derivatives of these fast modes to zero, changing a subset of the dynamic equations for the system from differential to algebraic form. These algebraic equations are then solved for the fast modes and back substituted into the remaining equations to eliminate the fast variables. The formalities of the procedure are as follows.

Consider the system

$$\ddot{\underline{x}}_1 = \underline{C}_{11}\underline{x}_1 + \underline{C}_{12}\underline{x}_2 + \underline{D}_1\dot{\underline{x}}_1 + \underline{B}_1\underline{u}$$

$$\mu^2\ddot{\underline{x}}_2 = \underline{C}_{21}\underline{x}_1 + \underline{C}_{22}\underline{x}_2 + \mu\underline{D}_2\dot{\underline{x}}_2$$

where μ is a sufficiently small scalar. The form of the equations shown here is often referred to as the two-time scale form. It will subsequently be shown that this form can be obtained not only for the usual interpretation of singular perturbation but also for systems with slightly damped, high frequency modes. The vector $\underline{x}_1$ can be identified with the study group of the example system used throughout this chapter, and $\underline{x}_2$ with the specified group of n generators.

In the singular perturbation approach the effect of the fast transients is neglected by setting $\mu = 0$ in the second set of equations. This makes these equations algebraic. Solving them for $\underline{x}_2$ and back substituting into the first set of equations yields

$$\ddot{\underline{x}}_1 = (\underline{C}_{11} - \underline{C}_{12}\underline{C}_{22}^{-1}\underline{C}_{21})\underline{x}_1 + \underline{D}_1\dot{\underline{x}}_1 + \underline{B}_1\underline{u}.$$

This provides a reduced order model of the overall system. Notice that the aggregation is not equivalent to replacing the specified group of n machines by one equivalent machine because there is a change in the equations describing the relative motion of the machines of the study group.

This analysis is not directly applicable to the power system model, because, as was shown in the lemma of Section III, all the eigenvalues of the model have the same, small real part. Thus

the power system model has no modes that are highly damped and decay rapidly to zero. However, Chow, Allemong and Kokotovic have shown [15] that this same approach can be applied to the case where a system contains a set of lightly damped high frequency states. An outline of the analysis of reference [15] is given here. The notation follows that of the reference for the benefit of those readers who may wish to examine the subject in more detail.

Consider a system of first order differential equations of the form

$$\underline{\dot{x}} = \underline{A} \underline{x} + \underline{B} \underline{z} \quad (4.18a)$$

$$\mu \underline{\dot{z}} = \underline{C} \underline{x} + \underline{D} \underline{z} \quad (4.18b)$$

with initial conditions $\underline{x}(t_0) = \underline{x}_0$, $\underline{z}(t_0) = \underline{z}_0$, and assume

- (1) The norms of the matrices $\underline{A}, \underline{B}, \underline{C}, \underline{D}$ are bounded about $\mu = 0$ and the state $\underline{z}$ is of even dimension, that is $\underline{z} \in \mathbb{R}^{2m}$
- (2) The matrix $\underline{D}$ is of the form

$$\underline{D} = \begin{bmatrix} \mu \underline{D}_1 & \underline{D}_2 \\ \underline{D}_3 & \mu \underline{D}_2 \end{bmatrix}$$

where $\underline{D}_2, \underline{D}_3$ are $m \times m$ non-singular matrices and the matrix $\underline{D}_2 \underline{D}_3$ has simple and negative eigenvalues $-\omega_i^2$.

In reference [15], Chow, et al. first show that the eigenvalues of the matrix $\frac{1}{\mu} \underline{D}$ are of the form

$$\sigma_i \pm j\omega_i/\mu \quad i = 1, 2, \dots, m$$

where σ_i is the i -th diagonal element of the matrix

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$\underline{R}_1 = \underline{I}(\underline{D}_4 + \underline{D}_3^{-1}\underline{D}_1\underline{D}_3)\underline{I}^{-1}$ and $\underline{I}$ is such that $\underline{I}\underline{D}_2\underline{D}_3\underline{I}^{-1} = \text{Diag}(-\omega_1^2, -\omega_2^2, \dots, -\omega_m^2)$. This establishes that as $\mu \rightarrow 0$ the eigenvalues of the system

$$\mu \dot{\underline{w}} = \underline{D} \underline{w} + \underline{u} \quad (4.19)$$

approach infinity along asymptotes parallel to the imaginary axis.

This guarantees that the states associated with the subsystem

$$\dot{\underline{z}} = \underline{D} \underline{z} + \underline{C} \underline{x}$$

will be of high frequency, with $\underline{C} \underline{x}$ playing the role of $\underline{u}$.

Chow next examines the system

$$\mu \dot{\underline{w}} = \underline{D} \underline{w} + \underline{u}$$

where $\underline{D}$ satisfies assumption (2) given above, and shows that if

$\underline{u}(t) = \underline{\bar{u}}(t) + \underline{\tilde{u}}(t)$ with $\underline{\bar{u}}(t)$ the slowly varying part of $\underline{u}(t)$ and

$\|\dot{\underline{\tilde{u}}}\| \leq C_1, \|\underline{\tilde{u}}\| < C_2$ for some fixed C_1, C_2 , then there is a finite

$T_1(\mu)$ such that the slowly varying part $\underline{\bar{w}}(t)$ of $\underline{w}(t)$ is

$$\underline{\bar{w}}(t) = -\underline{\bar{D}}^{-1} \underline{\bar{u}}(t) + O(\mu)$$

where

$$\underline{\bar{D}}^{-1} = \begin{bmatrix} \underline{0} & \underline{D}_3^{-1} \\ \underline{D}_2^{-1} & \underline{0} \end{bmatrix}.$$

This result is then applied to equation (4.18b), with $\underline{C} \underline{x}$ playing the role of $\underline{u}(t)$, to obtain the slowly varying part $\underline{\bar{z}}$ of $\underline{z}$ as

$$\underline{\bar{z}} = -\underline{\bar{D}}^{-1} \underline{C} \underline{x} + O(\mu).$$

Next the slowly varying part of $\underline{Z}$ is separated from $\underline{Z}$ by introducing the change of variables

$$\underline{\eta} = \underline{Z} + \underline{D}^{-1} \underline{C} \underline{X} + \underline{\mu} \underline{G} \underline{X} \equiv \underline{Z} + \underline{L} \underline{X} \quad (4.19)$$

and determining $\underline{G}$ such that equations (4.18) become

$$\dot{\underline{X}} = (\underline{A}_0 - \underline{\mu} \underline{B} \underline{G}) \underline{X} + \underline{B} \underline{\eta} \quad (4.20a)$$

$$\underline{\mu} \dot{\underline{\eta}} = (\underline{D} + \underline{\mu} \underline{L} \underline{B}) \underline{\eta} \quad (4.20b)$$

That is, the slowly varying part of $\underline{Z}$ has been transferred to equation (4.20a) in the sense that (4.20b) now only involves the fast variable $\underline{\eta}$.

To obtain the form (4.20) requires that $\underline{G}$ satisfy the relation

$$-\underline{D} \underline{G} + (\underline{D}^{-1} \underline{C} + \underline{\mu} \underline{G})(\underline{A}_0 - \underline{\mu} \underline{B} \underline{G}) = \underline{0}.$$

Invoking the implicit function gives the solution to this equation as

$$\begin{aligned} \underline{G} &= \underline{D}^{-2} \underline{C} \underline{A}_0 + \underline{O}(\underline{\mu}) \\ &= \underline{\bar{D}}^{-2} \underline{C} \underline{\bar{A}}_0 + \underline{O}(\underline{\mu}) \end{aligned}$$

where

$$\underline{\bar{A}}_0 = \underline{A} - \underline{B} \underline{\bar{D}}^{-1} \underline{C}.$$

The fast varying part of $\underline{X}$ is then separated by introducing the change of variables

$$\underline{\xi} = \underline{X} - \underline{\mu}(\underline{C} \underline{D}^{-1} + \underline{\mu} \underline{N}) \underline{\eta} = \underline{X} - \underline{\mu} \underline{H} \underline{\eta} \quad (4.21)$$

and choosing $\underline{N}$ such that

$$\underline{B} + \mu(\underline{A}_0 - \mu \underline{B} \underline{G}) \underline{H} - \underline{H}(\underline{D} + \mu \underline{L} \underline{B}) = \underline{0}$$

which by invoking the implicit function theorem is

$$\begin{aligned} \underline{N} &= \underline{A}_0 \underline{B} \underline{D}^{-2} - \underline{B} \underline{D}^{-2} - \underline{B} \underline{D}^{-2} \underline{C} \underline{B} \underline{D}^{-2} + \underline{O}(\mu) \\ &= \underline{A}_0 \underline{B} \underline{D}^{-2} - \underline{B} \underline{D}^{-2} \underline{C} \underline{B} \underline{D}^{-1} + \underline{O}(\mu). \end{aligned}$$

This completes the transformation of variables given by (4.19) and (4.21)

which can be put in the form

$$\begin{bmatrix} \underline{\xi} \\ \underline{\eta} \end{bmatrix} = \begin{bmatrix} \underline{I} - \mu \underline{H} \underline{L} & -\mu \underline{H} \\ \underline{L} & \underline{I} \end{bmatrix} \begin{bmatrix} \underline{X} \\ \underline{Z} \end{bmatrix}$$

with inverse transformation

$$\begin{bmatrix} \underline{X} \\ \underline{Z} \end{bmatrix} = \begin{bmatrix} \underline{I} & \mu \underline{H} \\ -\underline{L} & \underline{I} - \mu \underline{L} \underline{H} \end{bmatrix} \begin{bmatrix} \underline{\xi} \\ \underline{\eta} \end{bmatrix}$$

Using the full transformation takes the equations (4.18) to the form

$$\dot{\underline{\xi}} = \underline{A} \underline{\xi} \quad (4.22a)$$

$$\mu \dot{\underline{\eta}} = \underline{D} \underline{\eta} \quad (4.22b)$$

where $\underline{A} = \underline{A}_0 - \mu \underline{B} \underline{G}$, $\underline{D} = \underline{D} + \mu \underline{L} \underline{B}$.

Thus the slowly varying parts of $\underline{X}$ and $\underline{Z}$ have been separated in the sense that equation (4.22a) depends only on $\underline{\xi}$ and equation (4.22b) depends only on $\underline{\eta}$. Thus $\underline{\xi}$ embodies the slowly varying parts of $\underline{X}$ and $\underline{Z}$ and $\underline{\eta}$ the fast varying; or high frequency parts.

Neglecting the $\underline{O}(\mu)$ terms in (4.22b), define the slowly varying subsystem of (4.18) as

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$$\dot{\bar{X}} = \bar{A}_0 \bar{X} \quad \bar{X}(t_0) = \bar{X}_0 \quad (4.23a)$$

$$\bar{Z} = -\bar{D}^{-1} \bar{C} \bar{X} . \quad (4.23b)$$

The oscillatory subsystem

$$\dot{\underline{Z}} = \underline{D}, \quad \underline{Z}(t_0) = \underline{Z}_0 + \underline{D}^{-1} \underline{C} \underline{X}_0 \quad (4.24)$$

is obtained from (4.22b) by neglecting $O(\mu)$ terms in $\underline{D}$.

It is interesting to note that the final results in equation (4.23) are the same as would be obtained by setting $\mu = 0$ in (4.18b) solving for $\underline{Z}$ and substituting into (4.18a) as if the states of $\underline{Z}$ were classic fast decaying states.

This overview of reference [15] highlights the main points germane to the following discussion of high frequency modes in the example power system. It has been included to provide continuity and to give the reader without immediate access to the reference some feel for singular perturbation techniques.

The equations (4.23) show that if the conditions for "fast eigenvalues" exist then the original system can be represented by a reduced set of dynamic equations for the lower frequency states and a set of algebraic equations. Further, the subsystem (4.24) provides the dynamics for the fast eigenvalues, if this information becomes significant. The form of the singular perturbation equivalent (4.23a) is not the Podmore or "averaged" equivalent, since $\underline{A}_0 = \underline{A} - \underline{B} \underline{D}^{-1} \underline{C}$.

However, it will be shown next that when these results are applied to a power system, the term $\underline{B} \underline{D}^{-1} \underline{C}$ is $O(\mu^2)$ so that $\underline{A}_0 = \underline{A}$, and the singular perturbation and averaged equivalents are identical.

To put the example power system equations in the form of (4.18), let

$$\begin{aligned}\underline{x}_1 &= \Delta \delta_{-m} \\ \underline{x}_2 &= \Delta \dot{\delta}_{-m} = \Delta \omega_{-m} \\ \underline{z}_1 &= \Delta \delta_n / \mu^2 \\ \underline{z}_2 &= \Delta \dot{\delta}_n / \mu.\end{aligned}$$

Then equations (4.2) can be rewritten in the form

$$\begin{aligned}\begin{bmatrix} \dot{\underline{x}}_1 \\ \dot{\underline{x}}_2 \end{bmatrix} &= \begin{bmatrix} \underline{0} & \underline{I} \\ (-\tilde{\underline{M}} \underline{T})_{11} & -\sigma \underline{I} \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} + \begin{bmatrix} \underline{0} & \underline{0} \\ \mu^2 (-\tilde{\underline{M}} \underline{T})_{12} & \underline{0} \end{bmatrix} \begin{bmatrix} \underline{z}_1 \\ \underline{z}_2 \end{bmatrix} \\ &+ [\tilde{\underline{M}}_1 \vdots (\hat{\underline{M}} \underline{L})_1] \begin{bmatrix} \Delta PM \\ \dots \\ \Delta PL \end{bmatrix}\end{aligned}\quad (4.25a)$$

$$\begin{bmatrix} \mu \dot{\underline{z}}_1 \\ \mu \dot{\underline{z}}_2 \end{bmatrix} = \begin{bmatrix} \underline{0} & \underline{0} \\ (-\tilde{\underline{M}} \underline{T})_{21} & \underline{0} \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} + \begin{bmatrix} \underline{0} & \underline{I} \\ \mu^2 (-\tilde{\underline{M}} \underline{T})_{22} & \mu(-\sigma \underline{I}) \end{bmatrix} \begin{bmatrix} \underline{z}_1 \\ \underline{z}_2 \end{bmatrix}\quad (4.25b)$$

Then the matrices $\underline{A}, \underline{B}, \underline{C}, \underline{D}$ of (4.18) are

$$\begin{aligned}\underline{A} &= \begin{bmatrix} \underline{0} & \underline{I} \\ (-\tilde{\underline{M}} \underline{T})_{11} & -\sigma \underline{I} \end{bmatrix} & \underline{B} &= \begin{bmatrix} \underline{0} & \underline{0} \\ \mu^2 (-\tilde{\underline{M}} \underline{T})_{12} & \underline{0} \end{bmatrix} \\ \underline{C} &= \begin{bmatrix} \underline{0} & \underline{0} \\ (-\tilde{\underline{M}} \underline{T})_{21} & \underline{0} \end{bmatrix} & \underline{D} &= \begin{bmatrix} \underline{0} & \underline{I} \\ \mu^2 (-\tilde{\underline{M}} \underline{T})_{22} & -\mu \sigma \underline{I} \end{bmatrix}\end{aligned}\quad (4.26)$$

To insure that these matrices satisfy the conditions necessary for applying the singular perturbation transformations, it is necessary to do the following.

(1) Insure that $\mu^2(-\tilde{\underline{M}} \underline{I})_{12}$ and $(-\tilde{\underline{M}} \underline{I})_{21}$ are finite for $\mu \rightarrow 0$.

(2) Find the conditions on $(-\tilde{\underline{M}} \underline{I})_{22}$ to insure that

$$(\tilde{\underline{M}} \underline{I})_{22} = \frac{1}{\mu} \underline{D}_3$$

where $\underline{D}_3$ is nonsingular so that $\underline{D}$ of (4.26) satisfies the conditions on $\underline{D}$ in equations (4.18). To this end consider the four machine system of figure 4.1. Let generator 1, be the study group and generators 2,3,4 be the specified group of n generators. Use the UCA_n reference frame over the three generators of the specified group, and discard the equation for generator 4 as redundant. The matrix $(-\underline{M} \underline{I})_{22}$ can be written

$$(-\tilde{\underline{M}} \underline{I})_{22} = - \begin{bmatrix} \left(\frac{1}{M_2} - \frac{1}{M_e} \right) & -\frac{1}{M_e} & -\frac{1}{M_e} \\ -\frac{1}{M_e} & \frac{1}{M_3} - \frac{1}{M_e} & -\frac{1}{M_e} \end{bmatrix} \begin{bmatrix} t_{22} & t_{23} \\ t_{23} & t_{33} \\ t_{24} & t_{34} \end{bmatrix}$$

$$= - \begin{bmatrix} \left\{ \frac{t_{22}}{M_2} - \left(\frac{t_{22} + t_{23} + t_{24}}{M_e} \right) \right\} \left\{ \frac{t_{23}}{M_2} - \left(\frac{t_{23} + t_{33} + t_{34}}{M_e} \right) \right\} \\ \left\{ \frac{t_{32}}{M_3} - \left(\frac{t_{22} + t_{23} + t_{24}}{M_e} \right) \right\} \left\{ \frac{t_{33}}{M_3} - \left(\frac{t_{23} + t_{33} + t_{34}}{M_e} \right) \right\} \end{bmatrix}$$

The matrix $(-\tilde{\underline{M}} \underline{I})_{12}$ can be written

$$(-\tilde{\underline{M}} \underline{I})_{12} = -[\tilde{\underline{M}}_{11} \underline{I}_{12} + \tilde{\underline{M}}_{12} \underline{I}_{22}]$$

$$= -\frac{1}{M_1} \begin{bmatrix} t_{12} & t_{13} \end{bmatrix} - \begin{bmatrix} -\frac{1}{M_e} & -\frac{1}{M_e} & -\frac{1}{M_e} \end{bmatrix} \begin{bmatrix} t_{22} & t_{23} \\ t_{23} & t_{33} \\ t_{24} & t_{34} \end{bmatrix}$$

so that,

$$(-\tilde{\underline{M}} \underline{I})_{12} = -\left[\left\{\frac{t_{12}}{M_1} - \frac{(t_{22} + t_{23} + t_{24})}{M_e}\right\}\left\{\frac{t_{13}}{M_1} - \frac{(t_{23} + t_{33} + t_{34})}{M_e}\right\}\right] \quad (4.23)$$

Consider first $(-\tilde{\underline{M}} \underline{I})_{12}$. Making the substitutions

$$t_{22} = -t_{12} - t_{23} - t_{24} \quad t_{33} = -t_{13} - t_{23} - t_{34} ,$$

it is possible to write $(-\tilde{\underline{M}} \underline{I})_{12}$ as

$$(-\tilde{\underline{M}} \underline{I})_{12} = -\left[\left\{\frac{t_{12}}{M_1} + \frac{t_{12}}{M_e}\right\}\left\{\frac{t_{13}}{M_1} + \frac{t_{13}}{M_e}\right\}\right].$$

Here the symmetry of the matrix $\underline{I}$ has been used plus the fact that the elements of any column or row of $\underline{I}$ sum to zero. Note that $(-\tilde{\underline{M}} \underline{I})_{12}$ does not depend on the elements of the submatrix $\underline{I}_{22}$ of synchronizing power coefficients between the machines of the specified group. The same is true for $(-\tilde{\underline{M}} \underline{I})_{21}$. Thus these matrices will always be finite, since only elements $\{\underline{I}_{22}\}_{ij}$ of $\underline{I}_{22}$ will be set equal to $\frac{1}{\mu}$ in the subsequent development.

Now consider the matrix $(-\tilde{\underline{M}} \underline{I})_{22}$. Make the substitution

$$t_{22} = -t_{12} - t_{23} - t_{24}$$

$$t_{33} = -t_{13} - t_{23} - t_{34}$$

then

$$(-\tilde{\underline{M}} \underline{I})_{22} = -\begin{bmatrix} \left\{-t_{12}\left(\frac{1}{M_e} - \frac{1}{M_2}\right) - \frac{t_{23}}{M_2} - \frac{t_{24}}{M_2}\right\} & \left\{\frac{t_{23}}{M_2} + \frac{t_{13}}{M_e}\right\} \\ \left\{\frac{t_{23}}{M_3} + \frac{t_{12}}{M_e}\right\} & \left\{t_{13}\left(\frac{1}{M_e} - \frac{1}{M_3}\right) - \frac{t_{23}}{M_3} - \frac{t_{34}}{M_3}\right\} \end{bmatrix}$$

Suppose that only the interconnection between generators 2 and 3 is strong and let $t_{23} = \frac{1}{\mu^2}$, and factor out $\frac{1}{\mu^2}$. Then in the limit as $\mu^2 \rightarrow 0$, the matrix $(-\tilde{M} \underline{I})_{22}$ approaches

$$-\frac{1}{\mu^2} \begin{bmatrix} -\frac{1}{M_2} & \frac{1}{M_2} \\ \frac{1}{M_3} & -\frac{1}{M_3} \end{bmatrix}$$

that is $(-\tilde{M} \underline{I})_{22} \rightarrow -\frac{1}{\mu^2} \underline{D}_3$ where $\underline{D}_3$ is singular. The conditions for applying the singular perturbation transformations are that $\underline{D}_3$ be nonsingular. Now let both the interconnections between generators 2 and 3 and generators 2 and 4 be strong by letting $t_{24} = t_{23} = \frac{1}{\mu^2}$. In this case, factoring out $\frac{1}{\mu^2}$ and letting $\mu^2 \rightarrow 0$ cause the matrix $(-\tilde{M} \underline{I})_{22}$ to approach

$$-\frac{1}{\mu^2} \begin{bmatrix} -\frac{2}{M_2} & \frac{1}{M_2} \\ \frac{1}{M_3} & -\frac{1}{M_3} \end{bmatrix}$$

so that $\underline{D}_3$ is nonsingular.

Generalized to an n dimensional case, it is easy enough to show that the requirement for the non-singularity of $\underline{D}_3$ is that a set of $n-1$ interconnections, linking all the machines of the specified group, be set equal to $\frac{1}{\mu^2}$. This is the same specification for stiff interconnections required by strict synchronizing coherency.

Now, returning to equations (4.23), and letting $n-1$ interconnections among the machines of the specified group equal $\frac{1}{\mu^2}$ puts the equations (4.23) in the form

$$\begin{aligned}
 \begin{bmatrix} \dot{\underline{x}}_1 \\ \dot{\underline{x}}_2 \end{bmatrix} &= \begin{bmatrix} \underline{0} & \underline{I} \\ (-\tilde{\underline{M}} \underline{I})_{11} & -\sigma \underline{I} \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} + \begin{bmatrix} \underline{0} & \underline{0} \\ \mu^2 (-\tilde{\underline{M}} \underline{I})_{12} & \underline{0} \end{bmatrix} \begin{bmatrix} \underline{z}_1 \\ \underline{z}_2 \end{bmatrix} \\
 &\quad + [\underline{M}_1 \quad ; (\underline{M} \underline{L})_1] \begin{bmatrix} \underline{\Delta PM} \\ \text{---} \\ \underline{\Delta PL} \end{bmatrix}
 \end{aligned} \tag{4.27a}$$

$$\begin{bmatrix} \underline{\mu} \dot{\underline{z}}_1 \\ \underline{\mu} \dot{\underline{z}}_2 \end{bmatrix} = \begin{bmatrix} \underline{0} & \underline{0} \\ (-\tilde{\underline{M}} \underline{I})_{21} & \underline{0} \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} + \begin{bmatrix} \underline{0} & \underline{I} \\ \underline{D}_3 & \mu(-\sigma \underline{I}) \end{bmatrix} \begin{bmatrix} \underline{z}_1 \\ \underline{z}_2 \end{bmatrix} \tag{4.27b}$$

which is the form necessary to apply the singular perturbation approach.

The form of $\underline{D}$ in (4.27b) makes it easy to calculate the eigenvalues of $\frac{1}{\mu} \underline{D}$

$$\begin{aligned}
 \text{Det}\{\lambda \underline{I} - \frac{1}{\mu} \underline{D}\} &= \text{Det} \begin{bmatrix} \lambda \underline{I} & -\frac{1}{\mu} \underline{I} \\ -\frac{1}{\mu} \underline{D}_3 & (\lambda + \sigma) \underline{I} \end{bmatrix} \\
 &= \text{Det}\{[\lambda \underline{I} - \frac{1}{\mu^2} \frac{1}{\lambda + \sigma} \underline{D}_3][(\lambda + \sigma) \underline{I}]\} \\
 &= \text{Det}\{[(\lambda^2 + \lambda \sigma) \underline{I} - (\frac{1}{\mu^2} \underline{D}_3)]\} \\
 &= \text{Det}\{\gamma \underline{I} - \frac{1}{\mu^2} \underline{D}_3\}, \quad \underline{D}_3 \text{ nonsingular.}
 \end{aligned}$$

Thus the eigenvalues of $\underline{D}$ are $\gamma_i, \gamma_i^* = \pm \sqrt{\lambda_i}$ where $\lambda_i, i = 1, 2, \dots, n-1$ are the eigenvalues of $\frac{1}{\mu^2} \underline{D}_3 = (-\tilde{\underline{M}} \underline{I})_{22}$ which are known to have infinitely large imaginary parts as $\mu \rightarrow 0$.

The process of letting $\mu \rightarrow 0$ corresponds to letting $n-1$ interconnections in the specified group become infinitely stiff. That is $\mu \rightarrow 0$ is the "vehicle" by which one "travels" back to the results of Section II, namely that for $n-1$ infinitely stiff connections among

the members of the specified group of n generators, the specified group behaves precisely as a single generator. That is, the process of sending μ to zero blends the singular perturbation concept of fast eigenvalues into the concept of strict synchronizing coherency.

In addition (4.23) shows that the approximation to a single machine is very good even for μ small, since it is only necessary to drop terms $O(\mu^2)$ in order to have $\underline{A}_0 = \underline{A}$. Thus it is possible to state the following result.

Result 3. If the states of a specified group of n generators can be identified as high frequency, then the specified group of n generators can be replaced by a single generator without affecting the response of the remainder of the system. Further, in the limit as $\mu \rightarrow 0$, the singular perturbation concept of high frequency modes merges with the concept of strict synchronizing coherency. The singular perturbation equivalent is then identical to the modal and coherency equivalent, and the equivalencing has not introduced any error in the response of the study group.

At this point four conditions have been examined, strict geometric coherency (SGC), strict synchronizing coherency (SSC), pseudo-coherency (PC) and fast eigenvalues (FE). FE, however can be identified with synchronizing coherency and hereafter the term synchronizing coherency will stand for both these concepts. In the next section, one last method of decoupling the linear model is investigated. At that point the theory of Chapters 3 and 4 will begin to clearly point the way towards a theoretically sound, and computationally viable algorithm for generating reduced order dynamic equivalents.

V. Linear Decoupling

So far in this chapter, three archetypal conditions have been considered, each of which when satisfied causes a specified group of

machines to behave, from the perspective of the rest of the power system, like a single generator. Each of the archetypes, in turn, has been associated with conditions on one of the submatrices of $-\underline{M} \underline{T}$. Strict geometric coherency causes $(-\underline{M} \underline{T})_{21} = \underline{0}$. Strict synchronizing coherency causes $(-\underline{M} \underline{T})_{22}^{-1} \rightarrow \underline{0}$ in the limit as the inter connections between machines are progressively stiffened. Pseudo-coherency has been shown to cause $(-\underline{M} \underline{T})_{12} = \underline{0}$. All of these conditions, in turn, decouple the differential equations for the study group from those for the specified group to allow a reduction in the order of the system of equations that need to be analyzed for disturbances that occur within the study system. As has been pointed out before, all of the archetypes are hypothetical in the sense that they are never satisfied exactly in any real power system. It is possible to achieve near approximations to these conditions. That is, $\|(-\underline{M} \underline{T})_{22}\| < \epsilon$, or $\|(\underline{M} \underline{T})_{12}\| < \epsilon$ or $\|\underline{M} \underline{T}_{21}\| < \epsilon$, where ϵ is small.

Having been able to show that each archetype causes a submatrix of $-\underline{M} \underline{T}$ to either be zero or go to zero in the limit, it is natural enough to ask if structural conditions can exist in the power system that cause the multiplicative product of two of these submatrices to be zero, and if so, does this result in a decoupling of the equations for the specified group from those for the study group? In particular, consider the following example system where

$$(\hat{\underline{M}} \underline{T})_{22}^{-1} (\hat{\underline{M}} \underline{T})_{21} \rightarrow \underline{0}.$$

Figure 4.1 shows a four generator system. Let generator 1 be the study group and generators 2, 3 and 4 the specified group, with generator 4, the reference. First, multiply out the $\hat{\underline{M}} \underline{T}$ matrix.

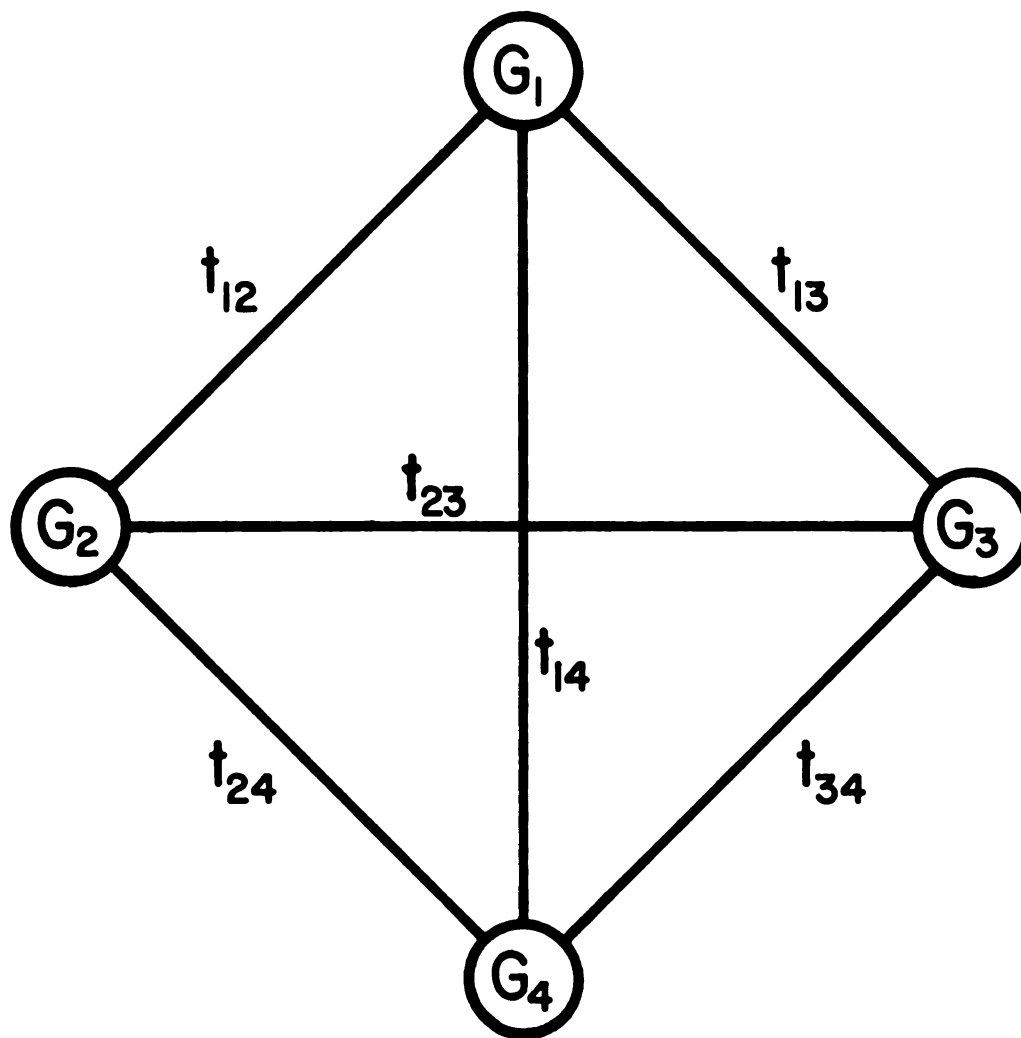


FIGURE 4-1
FOUR GENERATOR SYSTEM EXHIBITING
STRONG LINEAR DECOUPLING

$$\hat{\underline{M}} \underline{I} = \begin{bmatrix} \frac{1}{M_1} & & & -\frac{1}{M_4} \\ & \frac{1}{M_2} & & -\frac{1}{M_4} \\ & & \frac{1}{M_3} & -\frac{1}{M_4} \\ & & & \end{bmatrix} \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{12} & t_{22} & t_{23} \\ t_{13} & t_{23} & t_{33} \\ t_{14} & t_{24} & t_{34} \end{bmatrix}$$

$$= \begin{bmatrix} (\frac{t_{11}}{M_1} - \frac{t_{14}}{M_4}) & (\frac{t_{12}}{M_1} - \frac{t_{24}}{M_4}) & (\frac{t_{13}}{M_1} - \frac{t_{34}}{M_4}) \\ \hline (\frac{t_{12}}{M_2} - \frac{t_{14}}{M_4}) & (\frac{t_{22}}{M_2} - \frac{t_{24}}{M_4}) & (\frac{t_{23}}{M_2} - \frac{t_{34}}{M_4}) \\ \hline (\frac{t_{13}}{M_3} - \frac{t_{14}}{M_4}) & (\frac{t_{23}}{M_3} - \frac{t_{24}}{M_4}) & (\frac{t_{33}}{M_3} - \frac{t_{34}}{M_4}) \end{bmatrix}$$

Now let $t_{24} = \frac{1}{\mu}$, $t_{23} = t_{34} = \mu$, and $\frac{t_{12} + t_{14}}{M_2 + M_4} = \frac{t_{13}}{M_3}$.

Then,

$$\begin{aligned} \frac{t_{22}}{M_2} - \frac{t_{24}}{M_4} &= \frac{-t_{12}}{M_2} - \frac{t_{23}}{M_2} - \frac{t_{24}}{M_2} - \frac{t_{24}}{M_4} \\ &= \frac{-t_{12}}{M_2} - \frac{\mu}{M_2} - \frac{1}{\mu} \left(\frac{1}{M_2} + \frac{1}{M_4} \right) = a \end{aligned}$$

$$\frac{t_{23}}{M_2} - \frac{t_{34}}{M_4} = \mu \left(\frac{1}{M_2} - \frac{1}{M_4} \right) = b$$

$$\frac{t_{23}}{M_3} - \frac{t_{24}}{M_4} = \frac{\mu}{M_2} - \frac{1}{\mu M_4} = c$$

$$\begin{aligned}\frac{t_{33}}{M_3} - \frac{t_{34}}{M_4} &= \frac{-t_{13}}{M_3} - \frac{t_{23}}{M_3} - \frac{t_{34}}{M_3} - \frac{t_{34}}{M_4} \\ &= \frac{-t_{13}}{M_3} - \mu\left(\frac{2}{M_3} + \frac{1}{M_4}\right) = d\end{aligned}$$

So that

$$(\hat{M} \underline{I})_{22} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{and} \quad (\hat{M} \underline{I})_{22}^{-1} = \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$$

Stiffen the connection between generators 2 and 4 by letting $\mu \rightarrow 0$. Then note that

$$\lim_{\mu \rightarrow 0} \mu(ad-bc) = \left(\frac{1}{M_2} + \frac{1}{M_4}\right)\left(\frac{t_{13}}{M_3}\right) = K_d$$

This means that²

$$\lim_{\mu \rightarrow 0} \frac{d}{ad-bc} = \lim_{\mu \rightarrow 0} \frac{\mu d}{\mu(ad-bc)} = \lim_{\mu \rightarrow 0} \frac{\frac{\mu t_{13}}{M_3} - \mu^2\left(\frac{2}{M_2} + \frac{1}{M_4}\right)}{K_d} = 0$$

$$\lim_{\mu \rightarrow 0} \frac{-b}{ad-bc} = \lim_{\mu \rightarrow 0} \frac{-\mu b}{\mu(ad-bc)} = \lim_{\mu \rightarrow 0} \frac{-\mu^2\left(\frac{1}{M_2} - \frac{1}{M_4}\right)}{K_d} = 0$$

$$\lim_{\mu \rightarrow 0} \frac{-c}{ad-bc} = \lim_{\mu \rightarrow 0} \frac{-\mu c}{\mu(ad-bc)} = \frac{\frac{1}{M_4}}{\left(\frac{1}{M_2} + \frac{1}{M_4}\right)\left(\frac{t_{13}}{M_3}\right)} = \frac{M_2}{(M_2 + M_4)} \frac{M_3}{t_{13}}$$

$$\lim_{\mu \rightarrow 0} \frac{a}{ad-bc} = \lim_{\mu \rightarrow 0} \frac{\mu a}{\mu(ad-bc)} = \frac{\left(\frac{1}{M_2} + \frac{1}{M_4}\right)}{\left(\frac{1}{M_2} + \frac{1}{M_4}\right)\frac{t_{13}}{M_3}} = \frac{M_3}{t_{13}}$$

The matrix $(\hat{M} \underline{I})_{21}$ is,

²The condition $t_{23} = t_{34} = \mu$ is not necessary to the final results of this example as shown by a similar example in Chapter 5, but does simplify the algebra considerably.

$$(\hat{\underline{M}} \underline{T})_{21} = \begin{bmatrix} \frac{t_{12}}{M_2} - \frac{t_{14}}{M_4} \\ \frac{t_{13}}{M_3} - \frac{t_{14}}{M_4} \end{bmatrix}$$

but $\frac{t_{13}}{M_3} = \frac{t_{12} + t_{14}}{M_2 + M_4}$ so that

$$\begin{aligned} \frac{t_{13}}{M_3} - \frac{t_{14}}{M_4} &= \frac{t_{12} + t_{14}}{M_2 + M_4} - \frac{t_{14}}{M_4} = \frac{M_4 t_{12} + M_4 t_{14} - M_2 t_{14} - M_4 t_{14}}{M_4 (M_2 + M_4)} \\ &= \frac{t_{12}}{(M_2 + M_4)} - \frac{M_2 t_{14}}{M_4 (M_2 + M_4)} \\ &= \left(\frac{M_2}{M_2 + M_4} \right) \left(\frac{t_{12}}{M_2} - \frac{t_{14}}{M_4} \right) . \end{aligned}$$

Then in the limit as $\mu \rightarrow 0$, the matrix product $(\hat{\underline{M}} \underline{T})_{22}^{-1} (\hat{\underline{M}} \underline{T})_{12}$ can be written

$$\begin{aligned} (\hat{\underline{M}} \underline{T})_{22}^{-1} (\hat{\underline{M}} \underline{T})_{21} &= \begin{bmatrix} 0 & 0 \\ \left(\frac{M_2}{M_2 + M_4} \right) \frac{M_3}{t_{13}} & - \frac{M_3}{t_{13}} \end{bmatrix} \begin{bmatrix} \left(\frac{t_{12}}{M_2} - \frac{t_{14}}{M_4} \right) \\ \left(\frac{M_2}{M_2 + M_4} \right) \left(\frac{t_{12}}{M_2} - \frac{t_{14}}{M_4} \right) \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} . \end{aligned}$$

This result is hypothetical in the sense that letting $\mu \rightarrow 0$ means $t_{24} \rightarrow \infty$ causing the matrix $(\hat{\underline{M}} \underline{T})_{22}^{-1}$ to be singular which is impossible.

But the important point to note here is what happens by virtue of the limiting process. As $\mu \rightarrow 0$, generators 2 and 4 merge into one machine, as predicted by strict synchronizing coherency. This causes the synchronizing torque coefficient between generator 1 and the

composite machine 2-4 to become $t_{12} + t_{14}$, while the inertia of the composite generator is $M_2 + M_4$. In this particular system

$\frac{t_{12} + t_{14}}{M_2 + M_4} = \frac{t_{13}}{M_3}$. Thus in the limit as $\mu \rightarrow 0$ the composite generator 2-4 and generator 3 satisfy the conditions for strict geometric coherency, and hence the whole group 2,3,4 acts like a single generator. In this particular case the phenomenon of $(\hat{M} \underline{I})_{22}(\hat{M} \underline{I})_{12} \rightarrow 0$ combines two archetypal structure conditions, namely SGC, and SSC. Whether this is always the case, cannot be answered definitely here, but is an interesting consideration for future research. What the example does point out is that the condition $(\hat{M} \underline{I})_{22}(\hat{M} \underline{I})_{12} \rightarrow 0$ can cause the generators of the specified group to behave as a single generator. This is the condition that always previously has led to decoupling the differential equations of the specified group from those of the study system with the consequent separation of eigenvalues that allows the model and coherent equivalents to be identical. Thus the example points to a new archetype that will cause strict coherency of the specified group. In a real system $(\hat{M} \underline{I})_{22}^{-1}$, of course, would never be singular, but could have the form

$$(\hat{M} \underline{I})_{22}^{-1} = \begin{bmatrix} \epsilon_1 & \epsilon_2 \\ \{(\frac{M_2}{M_2 + M_4})\frac{M_3}{t_{13}} + \epsilon_3\} & (-\frac{M_3}{t_{13}} + \epsilon_4) \end{bmatrix}$$

where $\epsilon_i < \epsilon$, $i = 1,2,3,4$ are all small. For instance, consider the following data for the system of figure 4.1.

$$\begin{array}{ll}
 \mu = .1 & M_2 = 2 \\
 t_{12} = .1 & M_3 = 2 \\
 t_{13} = .2 & M_4 = 1 \\
 t_{14} = .2 &
 \end{array}$$

which yields

$$a = -.05 - .05 - 10(1.5) = -15.1$$

$$b = .1(-.5) = -.05$$

$$c = .05 - 10 = -9.95$$

$$d = -.1 - .1(2) = -.3$$

$$\frac{t_{12}}{M_2} - \frac{t_{14}}{M_4} = .05 - .2 = -.15$$

$$\frac{t_{13}}{M_3} - \frac{t_{14}}{M_4} = .1 - .2 = -.1$$

and results in

$$(\hat{M} \ I)_{22}^{-1} (\hat{M} \ I)_{21} = \begin{bmatrix} -.0744 & .0124 \\ 2.467 & -3.745 \end{bmatrix} \begin{bmatrix} -.15 \\ -.1 \end{bmatrix} = \begin{bmatrix} .0099 \\ -.0045 \end{bmatrix}.$$

Note that in this example t_{23} and t_{34} are moderately weak and t_{24} only moderately stronger than the other synchronizing power coefficients, yet using the largest element as the norm for the matrices involved, the norm of $(\hat{M} \ I)_{22}^{-1} (\hat{M} \ I)_{21}$ is an order of magnitude smaller than the norm of $(\hat{M} \ I)_{22}^{-1}$ or $(\hat{M} \ I)_{12}$. This may be a significant difference in

the practical situation where coherent a group is determined by setting a threshold value for some coherency measure.

The next question to be asked is whether the condition $(\underline{M} \ \underline{I})_{22}^{-1}(\underline{M} \ \underline{I})_{21} = \underline{0}$ results in the modal and coherency equivalents being identical. Certainly the hypothetical example indicates that this is the case. The answer is yes. The proof that is offered here is not done in the most general setting, but it is general enough for the use that will be made of this decoupling concept in Chapter 5. The proof is analogous in most respects to those given previously for the other three archetypal conditions.

Assume first that the disturbances occur only at generators of the study group. This results in

$$[\underline{M}_2 \mid (\underline{M} \ \underline{L})_2] \begin{bmatrix} \underline{\Delta PM} \\ \text{---} \\ \underline{\Delta PL} \end{bmatrix} = \underline{0}$$

and,

$$[\hat{\underline{M}}_1 \mid (\hat{\underline{M}} \ \underline{L})_1] \begin{bmatrix} \underline{\Delta PM} \\ \text{---} \\ \underline{\Delta PL} \end{bmatrix} = [\hat{\underline{M}}_1 \ \underline{\Delta PM}] ,$$

since $\underline{\Delta PL} = \underline{0}$. To simplify the notation in what follows, let

$$\underline{M}_1 \ \underline{\Delta PM} = \underline{B}_1 \underline{U}_1 \quad \text{and}$$

$$- \begin{bmatrix} (\underline{M} \ \underline{I})_{11} & (\underline{M} \ \underline{I})_{12} \\ (\underline{M} \ \underline{I})_{21} & (\underline{M} \ \underline{I})_{22} \end{bmatrix} = \begin{bmatrix} \underline{A}_{11} & \underline{A}_{12} \\ \underline{A}_{21} & \underline{A}_{22} \end{bmatrix}$$

Then the equations (4.2) for the example power system can be written

$$\ddot{\Delta\delta}_{-m} = \underline{A}_{11}\Delta\delta_{-m} + \underline{A}_{12}\Delta\delta_{-n} - \sigma\dot{\Delta\delta}_{-m} + \underline{B}_1\underline{U}_1 \quad (4.28a)$$

$$\dot{\Delta\delta}_{-n} = \underline{A}_{21}\Delta\delta_{-m} + \underline{A}_{22}\Delta\delta_{-n} - \sigma\dot{\Delta\delta}_{-n} \quad (4.28b)$$

Introduce the change of variables

$$\underline{Z} = \Delta\delta_{-n} + \underline{A}_{22}^{-1}\underline{A}_{21}\Delta\delta_{-m}$$

Then equations (4.28) can be put in the form

$$\ddot{\Delta\delta}_{-m} = \underline{A}_0\Delta\delta_{-m} + \underline{A}_{12}\underline{Z} - \sigma\dot{\Delta\delta}_{-m} + \underline{B}_1\underline{U}_1 \quad (4.29a)$$

$$\ddot{\underline{Z}} = \underline{A}_{22}^{-1}\underline{A}_{21}\underline{A}_0\Delta\delta_{-m} + (\underline{A}_{22} + \underline{A}_{22}^{-1}\underline{A}_{21}\underline{A}_{12})\underline{Z} - \sigma\dot{\underline{Z}} + \underline{A}_{22}^{-1}\underline{A}_{21}\underline{B}_1\underline{U}_1 \quad (4.29b)$$

where

$$\underline{A}_0 = (\underline{A}_{11} - \underline{A}_{12}\underline{A}_{22}^{-1}\underline{A}_{21}).$$

If the structural conditions on the power system are such that

$(\underline{A}_{22}^{-1}\underline{A}_{21}) \rightarrow \underline{0}$ then equations (4.29) reduce to

$$\ddot{\Delta\delta}_{-m} = \underline{A}_0\Delta\delta_{-m} + \underline{A}_{12}\underline{Z} - \sigma\dot{\Delta\delta}_{-m} + \underline{B}_1\underline{U}_1 \quad (4.30a)$$

$$\ddot{\underline{Z}} = \underline{A}_{22}\underline{Z} + \dot{\underline{Z}} \quad (4.30b)$$

If $\Delta\delta_{-m}(0) = \Delta\delta_{-n}(0) = \underline{0}$, then $\underline{Z}(0) = \underline{0}$. Hence, for zero initial con-

ditions, the system (4.30) becomes

$$\begin{bmatrix} \ddot{\Delta\delta}_{-n} \\ \ddot{\underline{Z}} \end{bmatrix} = \begin{bmatrix} (-\underline{M} \ \underline{I})_{11} & (-\underline{M} \ \underline{I})_{12} \\ \underline{0} & (-\underline{M} \ \underline{I})_{22} \end{bmatrix} \begin{bmatrix} \Delta\delta_{-m} \\ \underline{Z} \end{bmatrix} - \begin{bmatrix} \sigma \ \underline{I} & \underline{0} \\ \underline{0} & \sigma \ \underline{I} \end{bmatrix} \begin{bmatrix} \dot{\Delta\delta}_{-m} \\ \dot{\underline{Z}} \end{bmatrix} + \begin{bmatrix} \underline{M}_1 \Delta \underline{PM} \\ \underline{0} \end{bmatrix} \quad (4.31a)$$

$$(4.31b)$$

This is precisely the form attained for the previous three archetypal conditions. That is, the transformed specified group requires no dynamics to represent it. Its effect upon the study group

is perfectly preserved by the reference machine which is assumed to be either a generator in the specified group, or the inertial average of the specified group if the reference frame is UCA_n .

It has already been shown for the other archetypal conditions that the form of the equations (4.31) results in two sets of eigenvalues, one set associated with $(-\underline{M} \underline{I})_{11}$ and the other set associated with $(-\underline{M} \underline{I})_{22}$. As before, rules of modal reduction will discard the set of eigenvalues associated with $(-\underline{M} \underline{I})_{22}$ using the argument that the states associated with these eigenvalues are uncontrollable, and thus, once again, in the limiting condition of structural conditions that cause $(\underline{M} \underline{I})_{22}^{-1}(\underline{M} \underline{I})_{11} \rightarrow \underline{0}$, the modal and coherency equivalents for the specified group are identical. These results can be summarized as

Result 4. For the model of equations (4.2) with the disturbance confined to the generators of the study group, the condition $(-\underline{M} \underline{I})_{22}^{-1}(-\underline{M} \underline{I})_{21} \rightarrow \underline{0}$ causes the specified group of n generators to appear, to the study group, as a single generator, and the modal and coherency equivalents for the specified group are identical.

The condition $(\underline{M} \underline{I})_{22}^{-1}(\underline{M} \underline{I})_{21} \rightarrow \underline{0}$ will be called strict strong linear decoupling (SSLD). The adjective strong is used to distinguish this type of linear decoupling, from the linear decoupling obtained by analyzing the conditions $(\underline{M} \underline{I})_{12}(\underline{M} \underline{I})_{22}^{-1} \rightarrow \underline{0}$ and $(\underline{M} \underline{I})_{12}(\underline{M} \underline{I})_{22}^{-1}(\underline{M} \underline{I})_{21} \rightarrow \underline{0}$.

The analysis for these types of linear decoupling is not presented because these conditions, and the condition of pseudo-coherency, are not tested in the formal algorithm for producing dynamic equivalents presented in the next chapter. The analysis of Chapters 3 and 4 has shown that pseudo-coherency is a concept that can only be strictly

true in the linear model. The conditions for pseudo-coherency depend upon the structure of the study group, or internal system, at time $t = 0^{-1}$. This is reflected in the linear model by the fact that the conditions for pseudo-coherency is $(-\underline{M} \underline{T})_{12} = \underline{0}$. Since the disturbances occur in the study group these conditions may very well be destroyed by the disturbance. As a consequence, the presence of pseudo-coherency in the linear model does not strongly guarantee that the condition will persist in the nonlinear model. The two types of linear decoupling not analyzed here depend, like pseudo-coherency, on the conditions within the study system, through the matrix $(-\underline{M} \underline{T})_{12}$. Hence, they are discarded along with pseudo-coherency.

In contrast, the conditions of strict synchronizing coherency and strict geometric coherency transfer from the linear model quite strongly because they are not directly dependent on structural conditions within the study group. This is reflected in the linear model by the fact that conditions for synchronizing and geometric coherency are expressed in terms of the matrices $\underline{M} \underline{T}_{12}$ and $(\underline{M} \underline{T})_{22}$, namely as $(\underline{M} \underline{T})_{21} = \underline{0}$ and $(\underline{M} \underline{T})_{22}^{-1} \rightarrow \underline{0}$.

Strict strong linear decoupling depends on these same two matrices. Thus it is reasonable to suppose that the structural conditions for strict strong linear decoupling if present in the linear model will also be present in the nonlinear model. The hypothetical example adds credibility to this argument since SSLD appears in many cases to be a combination of strict synchronizing coherency and strict geometric coherency.

VI. Establishing a Hierarchy of Structural Conditions for Coherency

This very lengthy chapter, has traveled over a lot of material and concepts. It is necessary at this point to summarize the results of this chapter and point out how they might be used in producing dynamic equivalents.

This chapter has used a basic example system consisting of a study group and a specified group of n generators. The basic plan of attack has been to establish those conditions that cause the specified group to be strictly coherent, or appear to be strictly coherent to the study group. In other words, conditions were sought under which the specified group could be replaced by a single equivalent machine without changing the response of the study group to disturbances within the study group. Those conditions can be summarized as

- (1) Synchronizing Coherency
- (2) Geometric Coherency
- (3) Pseudo-Coherency
- (4) Linear Decoupling

In the process of establishing these various conditions, their relative merits, in producing dynamic equivalents have been discussed in an informal way. In the preceding section of this chapter, the ranking of these conditions became less informal when pseudo-coherency and two types of linear decoupling were discarded as conditions for determining dynamic equivalents.

The process of ranking these structural conditions is completely formalized by figure 4.2. The position in the table indicates

the value assigned to the condition in forming dynamic equivalents.

As can be seen strict synchronizing coherency is ranked highest. It is easily the single most important condition to be tested for in a power system, for the purpose of forming equivalents. Tightly interconnected machines are coherent under a wide variety of disturbances, because the tight interconnections force the machines to remain in synchronism.

Geometric coherency and strong linear decoupling are ranked second in importance. As will become evident in the next chapter, these conditions will be of use in forming equivalents where the disturbances are assumed to occur only in a particular area of the power system. This is almost self-evident, because there is an immediate and natural identification of the area in the power system where the disturbances occur, with the study group of this chapter.

The concepts of pseudo-coherency and weak linear decoupling rank lowest in importance. As discussed in the preceding section, these conditions are not used in the algorithm presented in Chapter 5 for determining coherent groups.

The dotted lines in figure 4.2 that connect the two types of linear decoupling to the other three conditions implicitly categorize linear decoupling as a derivative of the other three conditions. This is really an artificial choice. The example used to introduce strong linear decoupling was a combination of both synchronizing coherency and geometric coherency. Probably most cases of linear decoupling can be broken down in this fashion, but there are potentially many cases of strong linear decoupling that cannot be categorized in this way.

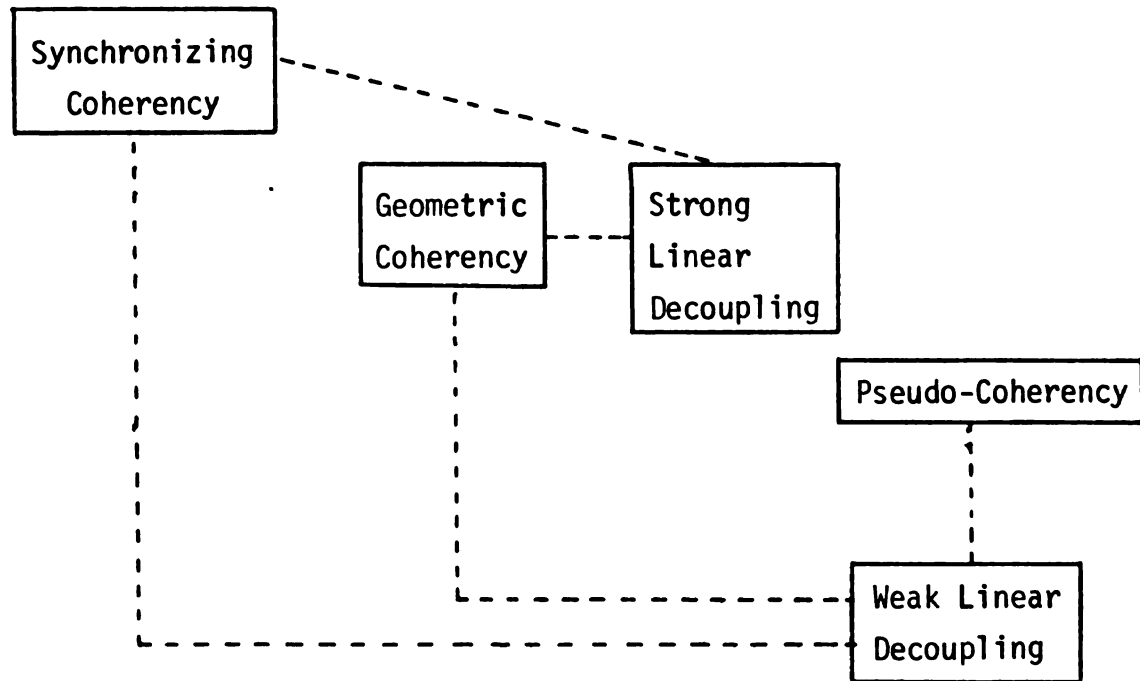


Figure 4.2. Relative Ranking of Structural Conditions for Coherency

Thus one could just as well categorize SLD as an independent condition.

It is worth noting at this point that what has been termed synchronizing coherency includes the singular perturbation ideas of high frequency eigenvalues that can be associated with inter-machine oscillations of the group, allowing the group to be represented as a decoupled subsystem. As has been shown in this chapter the singular perturbation approach is, in the limit of infinitely stiff connections, indistinguishable from the previously introduced idea of strict synchronizing coherency. The choice of terminology is arbitrary, and those who prefer to use the terminology of singular perturbation theory to describe this condition should do so. The writer wishes in no way to obscure or detract from the very fine work of Mssrs. Chow, Allemong, Kokotovic, Winkelman, et al.

Finally the reader may have noticed that the adjective strict no longer modifies the conditions for declaring a group of machines coherent. The idea of strict coherency has been very useful in establishing some conceptual classes of conditions under which groups of generators are either coherent, or appear to some other part of the system to be coherent.

In reality strict coherency is never achieved, only approximate coherency. In Chapter 5, a method is developed for measuring the structural conditions presented in this chapter. Where it is a useful aid to the analysis process, the conceptual idea of strict coherency will be re-introduced. The goal, however, is a practical scheme for measuring coherency conditions. The reader should recognize that the

coherency that is actually measured is seldom, if ever, purely synchronizing coherency or geometric coherency although in many cases one or the other predominates. In all cases the coherency is never perfect and one encounters the sticky task of establishing aggregation thresholds. That is, how big can the measure of coherency become, before the group is no longer considered coherent?

CHAPTER 5

A REDUCTION ALGORITHM FOR DETERMINING DYNAMIC EQUIVALENTS

I. Introduction

Chapter 4 established five hypothetical conditions on the structure of a power system namely,

- (1) Strict Synchronizing Coherency
- (2) Strict Geometric Coherency
- (3) Strict Strong Linear Decoupling
- (4) Pseudo-Coherency
- (5) Weak Linear Decoupling

that allow a specified group of generators to be replaced by a single generator. For brevity these five conditions will be given the collective name "structural conditions for coherency", even though for conditions (4) and (5) the specified group may not be coherent, but only appear to be coherent.

Although it is not feasible to satisfy any of these conditions exactly in a real power system, the assumption is that near satisfaction will still preserve modal and coherent properties. There is already empirical evidence to indicate that this assumption is true [6, 7].

The conditions for structural coherency can be considered rules for aggregation, and the next task is to find a means of identifying these conditions when they are satisfied or nearly

satisfied in an actual power system. Chapter 4 showed that all five conditions could be expressed in terms of the submatrices of $-M^{-1}I$. The most direct approach would be to test the submatrices themselves. While this is the most direct approach, it may not be the best, or the most convenient to implement computationally. An alternative is to find some other measure that can also detect the structural conditions for coherency. As it turns out, the r.m.s. coherency measure is one such measure. The first part of this chapter shows that the r.m.s. coherency measure, when used with the proper statistical disturbances, can identify a major subset of the structural conditions discussed in Chapter 4. It is then a straightforward task, one in fact that has already been accomplished, to modify the software developed by Podmore and Germond for the Electric Power Research Institute (EPRI) [2], to use the r.m.s. coherency measure. This modified software is used extensively in the study of the 39 Bus New England System discussed in Chapter 6.

II. The R.M.S. Coherency Measure

The possibility of using an r.m.s. coherency measure to determine coherent groups of generators which could be aggregated into single generators to form reduced order power system models, was initiated by Schlueter [5]. Three subsequent papers [6, 7, 8] strengthened the connection between modal and coherency equivalents. An optimum form of disturbance for determining coherent groups was also established. That is, disturbances were found that form coherent groups that depend on the dynamic structure of the power system but do not depend on the disturbance used to determine them.

In this section the r.m.s. coherency measure is defined, and the results of references [5, 6, 7, 8] pertinent to the present discussion are reviewed.

The r.m.s. coherency, $c_{k\ell}$, between generators k and ℓ of a power system is defined as

$$c_{k\ell} = \sqrt{\frac{1}{T} \xi \left\{ \int_0^T [\Delta\delta_k(t) - \Delta\delta_\ell(t)]^2 dt \right\}}$$

where ξ is the expectation operator. The expectation operator appears, because as shown in [7], the optimum disturbance for detecting coherent groups that depend on the power system structure is not deterministic. In fact, it is shown in [7] that there is no single deterministic disturbance that will adequately detect structural coherency.

The next task is to define the form of the probabilistic input $\underline{u}(t)$. This $\underline{u}(t)$ will then be used as an "input disturbance" to drive the linear power system model of Chapter 2, for determining coherent groups.

First, decompose $\underline{u}(t)$ into two functions $\underline{u}_1(t)$ and $\underline{u}_2(t)$, i.e. $\underline{u}(t) = \underline{u}_1(t) + \underline{u}_2(t)$. The function $\underline{u}_1(t)$ is defined as

$$\underline{u}_1(t) = \begin{cases} \underline{u}_1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

That is, $\underline{u}_1(t)$ is a vector step function, initiated at time $t = 0$.

In the linear model $\underline{u}(t)$ represents deviations in mechanical power on the generators and deviations in electric power at load buses. Since $\underline{u}_1(t)$ is a step function, non zero entries in $\underline{u}_1(t)$ will

model

- (1) Loss of generation
- (2) Loss of load due to load shedding
- (3) Line switching

If $\underline{u}_1(t)$ is to represent the random occurrence of such events, then it is necessary to define,

$$\xi\{\underline{u}_1(t)\} = \begin{bmatrix} \underline{M}_{11} \\ \underline{M}_{12} \end{bmatrix} = \underline{M}_1$$

and

$$\xi\{[\underline{u}_1(t) - \underline{M}_1][\underline{u}_1(t) - \underline{M}_2]^T\} = \begin{bmatrix} \underline{R}_{11} & 0 \\ 0 & \underline{R}_{22} \end{bmatrix} = \underline{R}_1$$

The reader should note that the vector matrix $\underline{M}_1$ is not the same as the submatrix $\underline{M}_1$ of the $(N-1) \times N$ matrix $\underline{M}$ that appears in $-\underline{M} \underline{T}$.

The matrices $\underline{M}_{11}$ and $\underline{R}_{11}$ describe the uncertainty in the location and magnitude of generation changes $\Delta \underline{P}_M$. The matrices $\underline{M}_{12}$ and $\underline{R}_{22}$ describe the uncertainty in the location and magnitude of power injections on buses due to either loads being shed or lines being switched.

The function $\underline{u}_1(t)$ can only model disturbances that resemble step changes. To model a fault, define

$$\underline{u}_2(t) = \begin{cases} 0 & t > T_1 \\ \underline{u}_2 & 0 \leq t \leq T_1 \\ 0 & t < 0 \end{cases}$$

That is $\underline{u}_2(t)$ represents a pulse of duration T_1 , occurring at time $t = 0$. Recall from Chapter 2 that faults were represented by changing the mechanical power to a generator. Thus

$$\underline{u}_2(t) = \begin{bmatrix} \underline{\Delta PM} \\ \underline{0} \end{bmatrix}$$

so that the last Q elements of $\underline{u}_2(t)$ are zero, where Q is the number of load buses in the power system. If $\underline{u}_2(t)$ is to be probabilistic, then define

$$\xi\{\underline{u}_2(t)\} = \begin{bmatrix} \underline{M}_{21} \\ \underline{0} \end{bmatrix} = \underline{M}_2$$

and

$$\xi\{[\underline{u}_2(t) - \underline{M}_2][\underline{u}_2(t) - \underline{M}_2]^T\} = \begin{bmatrix} \underline{R}_{21} & \underline{0} \\ \underline{0} & \underline{0} \end{bmatrix} = \underline{R}_2$$

The initial conditions are also assumed to be random,

$$\xi\{\underline{X}(0)\} = \underline{0}$$

$$\xi\{\underline{X}(0) \underline{X}(0)^T\} = \underline{V}_X(0)$$

This assumption reflects the idea that for a given steady-state operating point, the power system is expected, on the average, to be right at the operating point, although instantaneously it may be subject to transient fluctuations. The initial conditions are assumed to be uncorrelated with $\underline{u}_1(t)$ and $\underline{u}_2(t)$, i.e.

$$\xi\{\underline{X}(0) \underline{u}_1^T(t)\} = \underline{0}$$

$$\xi\{\underline{X}(0) \underline{u}_2^T(t)\} = \underline{0}$$

Finally, it is assumed that $\underline{u}_1(t)$ and $\underline{u}_2(t)$ are uncorrelated with respect to one another. This assumption is based on the fact, that the model is only used to represent one type of contingency at a time.

For the linear model of Chapter 2, the r.m.s. coherency measure, $c_{k\ell}$ can now be written

$$\begin{aligned} c_{k\ell} &= \left[\frac{1}{T^n} \xi \left\{ \int_0^T [\Delta\delta_k(t) - \Delta\delta_\ell(t)]^2 dt \right\} \right]^{1/2} \\ &= \left[\frac{1}{T^n} \xi \left\{ \int_0^T [(\Delta\delta_k(t) - \Delta\delta_N(t)) - (\Delta\delta_\ell(t) - \Delta\delta_N(t))]^2 dt \right\} \right]^{1/2} \\ &= \left[\underline{e}_{k\ell}^T \underline{S}_X(t) \underline{e}_{k\ell} \right]^{1/2} \end{aligned}$$

where $\underline{X}(t)$ is the state vector of the linear model, of the power system, and

$$\underline{S}_X(t) = \frac{1}{T^n} \int_0^T \underline{X}(t) \underline{X}(t)^T dt \quad (5.1)$$

is a $(2N-2) \times (2N-2)$ square matrix, with $\underline{e}_{k\ell}$ a $2N-2$ vector defined by

$$\{\underline{e}_{k\ell}\}_j = \begin{cases} 1 & j = k \\ -1 & j = \ell \\ 0 & j \neq k, \end{cases} \quad \text{for } k \neq N, \ell \neq N$$

$$\begin{cases} 1 & j = k \\ 0 & j \neq k \end{cases} \quad \text{for } k \neq N, \ell = N$$

$$\begin{cases} 1 & j = \ell \\ 0 & j \neq \ell \end{cases} \quad \text{for } k = N, \ell \neq N$$

For the input function $\underline{u}(t) = \underline{u}_1(t) + \underline{u}_2(t)$, $\underline{X}(t)$ has the form

$$\underline{x}(t) = \begin{cases} \underline{e}^{At} \underline{x}(t_0) + \int_0^t \underline{e}^{A(t-v)} \underline{B}(\underline{u}_1 + \underline{u}_2) dv & \text{for } t < T_1 \\ \underline{e}^{At} \underline{x}(0) + \int_0^t \underline{e}^{A(t-v)} \underline{B} \underline{u}_1 dv + e^{A(t-T_1)} \int_0^{T_1} \underline{e}^{A(T_1-v)} \underline{B} \underline{u}_2 dv & \text{for } t > T_1 \end{cases}$$

Substituting this expression for $\underline{x}(t)$ into (4.1), carrying out the expectation operation, and utilizing the assumptions that $\underline{u}_1(t)$, $\underline{u}_2(t)$ and $\underline{x}(t)$ are uncorrelated, leads to the expression

$$\begin{aligned} \underline{S}_x(t) = & \frac{1}{T^n} \int_0^T \underline{e}^{A\tau} \underline{v}_x(0) \underline{e}^{A\tau} d\tau \quad (5.2) \\ & + \frac{1}{T^n} \int_0^{T_1} \left\{ \left[\int_0^\tau \underline{e}^{A(\tau-v)} \underline{B} dv \right] [\underline{R}_1 + \underline{R}_2 + \underline{m}_1 \underline{m}_1^T + \underline{m}_2 \underline{m}_2^T + \underline{m}_1 \underline{m}_2^T + \underline{m}_2 \underline{m}_1^T] \left[\int_0^\tau \underline{e}^{A(\tau-v)} \underline{B} dv \right]^T \right\} d\tau \\ & + \frac{1}{T^n} \int_{T_1}^T \left\{ \left[\int_0^\tau \underline{e}^{A(\tau-v)} \underline{B} dv \right] [\underline{m}_1 \underline{m}_1^T + \underline{R}_1] \left[\int_0^\tau \underline{e}^{A(\tau-v)} \underline{B} dv \right]^T \right\} d\tau \\ & + \frac{1}{T^n} \int_{T_1}^T \left\{ \left[\underline{e}^{A(\tau-T_1)} \int_0^{T_1} \underline{e}^{A(T_1-v)} \underline{B} dv \right] [\underline{m}_2 \underline{m}_2^T + \underline{R}_2] \left[\underline{e}^{A(\tau-T_1)} \int_0^{T_1} \underline{e}^{A(T_1-v)} \underline{B} dv \right]^T \right\} d\tau \\ & + \frac{1}{T^n} \int_{T_1}^T \left\{ \left[\underline{e}^{A(\tau-T_1)} \int_0^{T_1} \underline{e}^{A(T_1-v)} \underline{B} dv \right] [\underline{m}_2 \underline{m}_1^T] \left[\int_0^\tau \underline{e}^{A(\tau-v)} \underline{B} dv \right]^T \right\} d\tau \\ & + \frac{1}{T^n} \int_{T_1}^T \left\{ \left[\int_0^\tau \underline{e}^{A(\tau-v)} \underline{B} dv \right] [\underline{m}_1 \underline{m}_2^T] \left[\underline{e}^{A(\tau-T_1)} \int_0^{T_1} \underline{e}^{A(T_1-v)} \underline{B} dv \right]^T \right\} d\tau. \end{aligned}$$

The integer n is chosen to be one if a load shedding, line switching, or generator dropping contingency occurs and zero for a fault. These choices guarantee a finite non-zero value of $\underline{S}_x(t)$ for an infinite observation interval.

Equation (5.2) gives the form of $\underline{S}_x(t)$ for a very general stochastic disturbance. However (5.2) is not much help analytically

because it is not easy to get a closed form expression for $\underline{S}_X(t)$. The major cause of this difficulty is the pulse portion $\underline{u}_2(t)$. If the disturbances are restricted to step functions, i.e. $\underline{u}(t) = \underline{u}_1(t)$, then as shown in reference [7] $\underline{S}_X(t)$ can be put in closed form by letting $T \rightarrow \infty$. That is

$$\lim_{T \rightarrow \infty} \underline{S}_X(t) = \underline{S}_X(\infty) = [\underline{M} \ \underline{I}]^{-1} \underline{K}_U [\underline{M} \ \underline{I}]^{-T} \quad (5.3)$$

where,

$$\begin{aligned} \underline{K}_U = & [\underline{R}_{11} + \underline{m}_{11} \underline{m}_{11}^T + \underline{L} \underline{m}_{12} \underline{m}_{11}^T + \underline{m}_{11} \underline{m}_{12}^T \underline{L}^T \\ & + \underline{L}(\underline{R}_{22} + \underline{m}_{22} \underline{m}_{22}^T) \underline{L}^T] \end{aligned} \quad (5.4)$$

Equation (5.3) reveals that, for step disturbances the r.m.s. coherency measure depends only on power system structure through the matrix $[\underline{M} \ \underline{I}]^{-1}$. However, it is also dependent on the nature of the disturbance through the matrix $\underline{K}_U$ which contains the statistics of the disturbance. The next section investigates some fundamental properties of the r.m.s. coherency measure for the general case where $\underline{K}_U$ depends on the statistics of $\underline{u}(t)$. In the subsequent section it is shown that the proper selection of $\underline{u}(t)$ causes $\underline{K}_U$ not to be dependent on $\underline{u}(t)$. That is, the elements of $\underline{K}_U$ will no longer be values specific to the statistics of $\underline{u}(t)$.

III. General Properties of $\underline{S}_X(\infty)$.

Assume, as in Chapter 4, a power system with $N = m+n$ generators, where the first m generators constitute the study group of interest and the last n generators constitute a group of generators whose coherency is to be investigated. For notational convenience let

$$\begin{bmatrix} \underline{g}_{11} & \underline{g}_{12} \\ \underline{g}_{21} & \underline{g}_{22} \end{bmatrix} = \begin{bmatrix} (\underline{M}\underline{I})_{11} & (\underline{M}\underline{I})_{12} \\ (\underline{M}\underline{I})_{21} & (\underline{M}\underline{I})_{22} \end{bmatrix}$$

where the partitioning is conformal with the dimensions m and n of the study group and the specified group of n generators.

Similarly, partition

$$\underline{K}_u = \begin{bmatrix} \underline{K}_{11} & \underline{K}_{12} \\ \underline{K}_{21} & \underline{K}_{22} \end{bmatrix}$$

where the submatrices $\underline{k}_{ij}$ contain values that depend on the mean and covariance matrices $\underline{M}_1$ and $\underline{R}_1$ of the statistical step disturbance vector $\underline{u}(t) = \Delta \underline{P} \underline{M}$. To make $\underline{K}_u$ independent of these matrices would mean that the $\underline{k}_{ij}$ submatrices always had the same constant value, no matter what the statistics of $\underline{u}(t)$.

Assuming that $\underline{g}_{11}$ and $\underline{g}_{22}$ are nonsingular, the inverse of $\underline{M}\underline{I}$ can be written as

$$[\underline{M}\underline{I}]^{-1} = \begin{bmatrix} \underline{Q}^{-1} & -\underline{g}_{11}^{-1} \underline{g}_{12} \underline{P}^{-1} \\ -\underline{g}_{22}^{-1} \underline{g}_{21} \underline{Q}^{-1} & \underline{P}^{-1} \end{bmatrix}$$

where

$$\underline{Q} = \underline{g}_{11} - \underline{g}_{12} \underline{g}_{22}^{-1} \underline{g}_{21}$$

$$\underline{P} = \underline{g}_{22} - \underline{g}_{21} \underline{g}_{11}^{-1} \underline{g}_{12}$$

Then the r.m.s. coherency measure $\underline{S}_x^{(\infty)}$ can be written

$$\begin{aligned}
\underline{S}_X(\infty) &= [\underline{M} \ \underline{I}]^{-1} \underline{K}_u [\underline{M} \ \underline{I}]^{-T} \\
&= \begin{bmatrix} \underline{Q}^{-1} & -\underline{g}_{11}^{-1} \underline{g}_{12} \underline{P}^{-1} \\ -\underline{g}_{22}^{-1} \underline{g}_{21} \underline{Q}^{-1} & \underline{P}^{-1} \end{bmatrix} \begin{bmatrix} \underline{K}_{11} & \underline{K}_{12} \\ \underline{K}_{21} & \underline{K}_{22} \end{bmatrix} \begin{bmatrix} \underline{Q}^{-T} & -\underline{Q}^{-T} \underline{g}_{21} \underline{g}_{22}^T \\ -\underline{P}^{-T} \underline{g}_{12} \underline{g}_{11}^T & \underline{P}^{-T} \end{bmatrix} \\
&= \begin{bmatrix} \underline{S}_{X11} & \underline{S}_{X12} \\ \underline{S}_{X21} & \underline{S}_{X22} \end{bmatrix}
\end{aligned}$$

where

$$\begin{aligned}
\underline{S}_{X11} &= \underline{Q}^{-1} \underline{K}_{11} \underline{Q}^{-T} - \underline{Q}^{-1} \underline{K}_{12} \underline{P}^{-T} \underline{g}_{12} \underline{g}_{11}^{-T} - \underline{g}_{11} \underline{g}_{12} \underline{P}^{-1} \underline{K}_{21} \underline{Q}^{-T} \\
&\quad + \underline{g}_{11} \underline{g}_{12} \underline{P}^{-1} \underline{K}_{22} \underline{P}^{-T} \underline{g}_{12} \underline{g}_{11}^{-T} \quad (5.5a)
\end{aligned}$$

$$\begin{aligned}
\underline{S}_{X12} &= -\underline{Q}^{-1} \underline{K}_{11} \underline{Q}^{-T} \underline{g}_{21} \underline{g}_{22}^T + \underline{Q}^{-1} \underline{K}_{12} \underline{P}^{-T} + \underline{g}_{11}^{-1} \underline{g}_{12} \underline{P}^{-1} \underline{K}_{21} \underline{Q}^{-T} \underline{g}_{21} \underline{g}_{22}^T \\
&\quad - \underline{g}_{11}^{-1} \underline{g}_{12} \underline{P}^{-1} \underline{K}_{22} \underline{P}^{-T} \quad (5.5b)
\end{aligned}$$

$$\begin{aligned}
\underline{S}_{X21} &= -\underline{g}_{22}^{-1} \underline{g}_{21} \underline{Q}^{-1} \underline{K}_{11} \underline{Q}^{-T} + \underline{g}_{22}^{-1} \underline{g}_{21} \underline{Q}^{-1} \underline{K}_{12} \underline{P}^{-T} \underline{g}_{12} \underline{g}_{11}^{-T} \\
&\quad + \underline{P}^{-1} \underline{K}_{21} \underline{Q}^{-T} - \underline{P}^{-1} \underline{K}_{22} \underline{P}^{-T} \underline{g}_{12} \underline{g}_{11}^{-T} \quad (5.5c)
\end{aligned}$$

$$\begin{aligned}
\underline{S}_{X22} &= \underline{g}_{22}^{-1} \underline{g}_{21} \underline{Q}^{-1} \underline{K}_{11} \underline{Q}^{-T} \underline{g}_{21} \underline{g}_{22}^T = \underline{g}_{22}^{-1} \underline{g}_{21} \underline{Q}^{-1} \underline{K}_{12} \underline{P}^{-T} - \underline{P}^{-1} \underline{K}_{21} \underline{Q}^{-T} \underline{g}_{21} \underline{g}_{22}^T \\
&\quad + \underline{P}^{-1} \underline{K}_{22} \underline{P}^{-T} \quad (5.5d)
\end{aligned}$$

Now consider $\underline{S}_{X22}$, the submatrix that contains the information on the coherency between the n generators of the specified group whose coherency is being investigated. In the linear model for

the power system, the condition of strict geometric coherency (SGC) corresponds to $\underline{g}_{21} = \underline{0}$. If $\underline{g}_{21} = \underline{0}$, then

$$\underline{S}_{x_{22}} = \underline{P}^{-1} \underline{K}_{22} \underline{P}^{-T} = \underline{g}_{22}^{-1} \underline{K}_{22} \underline{g}_{22}^{-T}$$

so that $\underline{S}_{x_{22}}$ depends only on the structure of the specified group of n generators and not on the study group. This is exactly what is implied by SGC. This means that for a disturbance within the study group, that is, outside the specified group, $\underline{K}_{22} = \underline{0}$ which in turn causes $\underline{S}_{x_{22}} = \underline{0}$. Thus the r.m.s. coherency measure will capture the condition of strict geometric coherency. The only defect is that the expression for $\underline{S}_{x_{22}}$ is dependent on the nature of the statistics of the disturbance that occurs in the study group. That is, it is possible that terms in $\underline{S}_{x_{22}}$ would go to zero not because $\underline{g}_{21} = \underline{0}$ but because the particular statistics of the disturbance cause $\underline{K}_{ij} = \underline{0}$. What is needed is a type or category of disturbance that yields the same constant $\underline{K}_{ij}$ no matter how the details of its statistics change. This type of disturbance is formulated in the next section.

A parallel argument can be carried out on $\underline{S}_{x_{22}}$ for the condition of strict synchronizing coherency (SSC), which in the linear model corresponds to $\underline{g}_{22}^{-1} \rightarrow \underline{0}$. For this case, using the matrix identity (5.9),

$$\underline{P}^{-1} = \underline{g}_{22}^{-1} + \underline{g}_{22}^{-1} \underline{g}_{21} [\underline{g}_{11} - \underline{g}_{12} \underline{g}_{22}^{-1} \underline{g}_{21}]^{-1} \underline{g}_{12} \underline{g}_{22}^{-1} \rightarrow \underline{0} ,$$

so that

$$\underline{S}_{x_{22}} \rightarrow \underline{0}$$

for a disturbance either in the study group or within the specified group of n generators.

Now consider the condition of pseudo-coherency, which corresponds in the linear model to $\underline{g}_{12} = \underline{0}$. Examining $\underline{S}_{x_{11}}$ in equation (5.5a), which measures the coherency of the generators of the study group, shows that if the conditions for pseudo-coherency are satisfied by the power system structure then

$$\underline{S}_{x_{11}} = \underline{Q}^{-1} \underline{K}_{11} \underline{Q}^{-T} = \underline{g}_{11}^{-1} \underline{K}_{11} \underline{g}_{11}^{-T}$$

which depends only on the structure of the study group. Thus for a disturbance that occurs outside the study group, i.e. in the specified group of n generators, $\underline{K}_{11} = \underline{0}$ which, in turn, causes $\underline{S}_{x_{11}} = \underline{0}$, so that the r.m.s. coherency measure will detect the condition of pseudo-coherency if it exists in the structure of the power system, but only by a ZMIIW disturbance of the specified group. It is worth noting that pseudo-coherency does not eliminate terms in $\underline{S}_{x_{22}}$. This is as expected since in the case of pseudo-coherency the specified group is not truly coherent, but only appears coherent to the study group.

Finally the case of strict strong linear decoupling (SSLD) is detected by the r.m.s. coherency measure. In the linear model the conditions for SSLD imply $\underline{g}_{22}^{-1} \underline{g}_{21} \rightarrow \underline{0}$ which causes

$$\underline{S}_{x_{22}} \rightarrow \underline{P}^{-1} \underline{K}_{22} \underline{P}^{-T}$$

so that $\underline{S}_{x_{22}}$ depends only on the structure of the specified group and $\underline{S}_{x_{22}} \rightarrow \underline{0}$, for a disturbance confined to the study group, as was the case with strict geometric coherency. The weak linear decoupling conditions $(\underline{g}_{12} \underline{g}_{22}^{-T}) \rightarrow \underline{0}$ and $(\underline{g}_{12} \underline{g}_{22}^{-T} \underline{g}_{21}) \rightarrow \underline{0}$ are not

detected by the r.m.s. coherency measure, for a disturbance confined to the study group. Chapter 4 concluded by ranking the five structural conditions for coherency and intimating that pseudo-coherency and weak linear decoupling were not conditions that were necessarily worth detecting. The foregoing analysis indicates that PC and WLD can be left undected by using only 1) disturbances of the whole system to detect synchronizing coherency and 2) disturbances confined to the study system to detect geometric coherency and strong linear decoupling. This is the strategy adopted in the reduction algorithm formulated at the end of this chapter.

It is apparent at this point that the r.m.s. coherency measure is capable of detecting all the major, and most of the minor, system structure conditions of Chapter 4, that permit generators to be aggregated while preserving the modal and coherency properties of the power system. The results in this section have been established using a very general stochastic disturbance. As a result $\underline{S}_X(\infty)$ is disturbance dependent. In the next section, a particular stochastic disturbance is chosen, which makes $\underline{S}_X(\infty)$ independent of the disturbance. That is, for the chosen disturbance $\underline{S}_X(\infty)$ depends only on the structure of the power system as embodied by the matrix $-\underline{M} \underline{I}$. At the same time the next section takes the first step towards the formulation of a general algorithm for determining modal-coherent dynamic equivalents for a power system.

IV. The ZMIIW Disturbance

In Section II, the general form of $\underline{S}_X(\infty)$ for step disturbances was found to be

$$\underline{S}_X(\infty) = [\underline{M} \ \underline{I}]^{-1} \underline{K}_u [\underline{M} \ \underline{I}]^{-T}$$

where the expression for

$$\underline{K}_u = \begin{bmatrix} \underline{K}_{11} & \underline{K}_{12} \\ \underline{K}_{21} & \underline{K}_{22} \end{bmatrix}$$

as given by (5.3) depends on the statistics of $\underline{u}(t)$.

Suppose that a type of disturbance could be found that yielded the same constant $\underline{K}_{ij}$ submatrices when applied to different power system models. That is, the specific details of the $\underline{M}_1$ and $\underline{K}_1$ matrices would be particular to the power system, but the resulting $\underline{K}_{ij}$ would always be the same constant matrices that did not depend on $\underline{M}_1$ and $\underline{R}_1$.

For such a disturbance the expressions (5.5) for the submatrices of $\underline{S}_X(\infty)$ would be ideal measures for determining aggregation conditions that depend solely on system structure. There are, in fact, as shown in reference [7] two particular disturbances that will accomplish this very goal. One of these disturbances, called a ZMIID disturbance has

$$\underline{R}_{11} = \underline{I}, \quad \underline{R}_{22} = \underline{0}, \quad \underline{M}_{11} = \underline{0}, \quad \underline{M}_{12} = \underline{0}, \quad \underline{M}_{22} = \underline{0}$$

and results in

$$\underline{K}_u = \underline{I}.$$

This disturbance has the potential liability that it is reference dependent [7]. The second disturbance results in

$$\underline{K}_u = \underline{K}_{IW} \tag{5.7}$$

where

$$\{\underline{K}_{IW}\}_{ij} = \begin{cases} 2 & \text{for } i = j \\ 1 & \text{for } i \neq j \end{cases}$$

The disturbance that yields the $\underline{K}_{IW}$ of (5.7) is a zero mean, independent disturbance over all the generators of the system with

$$\underline{R}_{11} = \text{diag}\{M_1^2, M_2^2, \dots, M_1^2, \dots, M_{m+n}^2\}, \underline{R}_{22} = \underline{0}, \underline{M}_{11} = \underline{0}, \underline{M}_{12} = \underline{0}.$$

where the M_i 's are the generator inertias of the system. For expository convenience, this disturbance will be called a zero mean, independent, inertially weighted (ZMIW) disturbance.

It is worth noting at this point that the effect of the stochastic ZMIW disturbance can be obtained as the summation of $n + m$ deterministic disturbances [7]. That is, let $\underline{S}_{X_i}(\infty)$ be the matrix that results from a disturbance of M_i^2 per unit on bus i only. Then

$$\underline{S}_X(\infty) = \sum_{i=1}^{m+n} \underline{S}_{X_i}(\infty).$$

For the ZMIW disturbance, the resulting matrix $\underline{K}_{IW}$ can be partitioned, conformal with the dimensions m and n of the study group and specified group, respectively, as

$$\underline{K}_{IW} = \begin{bmatrix} \underline{K}_{IW11} & \underline{K}_{IW12} \\ \underline{K}_{IW21} & \underline{K}_{IW22} \end{bmatrix}$$

Then substitution of the $\underline{K}_{IW_{ij}}$ for the $\underline{K}_{ij}$ in equations (5.5) produces expressions for the submatrices of $\underline{S}_X(\infty)$ that depend exclusively on the structure of the power system, as embodied in the matrix $\underline{M} \underline{T}^{-1}$. Having established that $\underline{K}_{IW}$ is disturbance independent, the subscript IW is now omitted and $\underline{K}_{IW}$ will subsequently be referred to as $\underline{K}$.

The disturbance independent form of the equations (5.5) can now be used to begin establishing an algorithm for producing dynamic equivalents. Consider again the expression (5.5d) for $\underline{S}_{X_{22}}$ where the $\underline{K}_{ij}$ are now the $\underline{K}_{IW_{ij}}$. It was shown earlier that for a general disturbance of all generators $\underline{S}_{X_{22}}$ can be small only if $\underline{P}^{-1}$ is small. Recall that

$$\underline{P} = \underline{g}_{22} - \underline{g}_{21} \underline{g}_{11}^{-1} \underline{g}_{12} \quad (5.8)$$

Applying the matrix identity

$$[\underline{a}_{11} - \underline{a}_{12} \underline{a}_{22}^{-1} \underline{a}_{21}]^{-1} = \underline{a}_{11}^{-1} + \underline{a}_{11}^{-1} \underline{a}_{12} [\underline{a}_{22} - \underline{a}_{21} \underline{a}_{11}^{-1} \underline{a}_{12}]^{-1} \underline{a}_{21} \underline{a}_{11}^{-1} \quad (5.9)$$

to (5.8) yields

$$\underline{P}^{-1} = \underline{g}_{22}^{-1} [\underline{I} + \underline{g}_{21} [\underline{g}_{11} - \underline{g}_{12} \underline{g}_{22}^{-1} \underline{g}_{21}]^{-1} \underline{g}_{12} \underline{g}_{22}^{-1}] = \underline{g}_{22}^{-1} [\underline{I} + \underline{K}_a] \quad (5.10)$$

Then,

$$\underline{P}^{-1} \underline{K}_{22} \underline{P}^{-T} = \underline{g}_{22}^{-1} [\underline{I} + \underline{K}_a] [\underline{K}_{22}] [\underline{I} + \underline{K}_a]^T \underline{g}_{22}^{-T} \quad (5.11)$$

Equation (5.11) indicates that the fourth term in the expression for $\underline{S}_{X_{22}}$ depends on $\underline{g}_{22}^{-1}$ and $\underline{K}_{22}$, and for $\|\underline{K}_{22}\|$ not small, $\underline{S}_{X_{22}} \rightarrow \underline{0}$ only if $\underline{g}_{22}^{-1} \rightarrow \underline{0}$. For the ZMIIW disturbance over all generators $\|\underline{K}_{22}\|$ is not small. Hence $\underline{S}_{X_{22}} \rightarrow \underline{0}$ only if $\underline{g}_{22}^{-1} \rightarrow \underline{0}$. A similar argument holds for $\underline{S}_{X_{11}}$ for $\underline{g}_{11}^{-1} \rightarrow \underline{0}$.

The condition $g_{22}^{-1} \rightarrow 0$ is the condition for strict synchronizing coherency, i.e. tight interconnections among a group of generators. Thus a ZMIW disturbance over all the generators of a system has the effect of identifying groups of generators that are tightly interconnected because neither strict geometric coherency or strict strong linear decoupling can cause $S_{x_{22}} \rightarrow 0$ for such a general ZMIW disturbance. Only $g_{22}^{-1} \rightarrow 0$ will cause $S_{x_{22}} \rightarrow 0$. Stated another way, a ZMIW over all the generators of both the study group and the specified group of n generators would identify the study group and the specified group as two tightly bound subsystems only if the norms of $S_{x_{11}}$ and $S_{x_{22}}$ were both small or zero.

Identifying the tightly interconnected groups of generators is a primary step in forming dynamic equivalents since generators that are very tightly interconnected tend to remain coherent in the face of very strong disturbances. Further, tightly bound groups are more impervious to the location of the disturbance than groups formed by satisfying the other structural conditions for coherency. For instance, the specified group of n generators in the example system would have a much greater tendency to remain coherent in response to a disturbance within the specified group if the group coherency were due to synchronizing coherency, as opposed to geometric coherency or strong linear decoupling. Thus a ZMIW disturbance of all generators in the system would be the ideal first step in determining dynamic equivalents.

If the first step in forming dynamic equivalents is to find the tightly bound group, the obvious second step is to test for the other structural conditions for coherency, namely geometric coherency and strong linear decoupling. How that step can be accomplished is the subject of the next section.

V. The Three Part Partition of the Example System

Consider again the expression (5.5d) for $\underline{S}_{x_{22}}$. The first three terms of $\underline{S}_{x_{22}}$ depend on $\underline{g}_{22}^{-1}\underline{g}_{21}$, the fourth term on $\underline{g}_{22}^{-1}$. If a disturbance could be found which eliminated the fourth term in (5.5d) then that disturbance would detect structural conditions where $\underline{g}_{22}^{-1} \rightarrow \underline{0}$ is not satisfied, but $\underline{g}_{22}^{-1}\underline{g}_{21} \rightarrow \underline{0}$ is. Since strict geometric coherency implies $\underline{g}_{12} = \underline{0}$ and strict strong linear decoupling results from $\underline{g}_{22}^{-1}\underline{g}_{21} \rightarrow \underline{0}$, a disturbance that eliminates the fourth term in $\underline{S}_{x_{22}}$ could thus be used to detect these two structural conditions. The requisite disturbance is a zero mean, independent, inertially weighted disturbance such that

$$\underline{R}_{11} = \text{diag}\{\underline{M}_1^2, \underline{M}_2^2, \dots, \underline{M}_m^2, 0, \dots, 0\}, \underline{R}_{22} = \underline{0}, \underline{m}_{11} = \underline{0}, \underline{m}_{12} = \underline{0}$$

that is a ZMIW disturbance over the generators of the study group.

Such a disturbance gives $\underline{K}_u$ the form

$$\underline{K}_u = \begin{bmatrix} \underline{I}_{m \times m} & \underline{0}_{m \times n} \\ \underline{0}_{m \times n} & \underline{0}_{n \times n} \end{bmatrix} \quad (5.12)$$

that is $\underline{K}_{12} = \underline{K}_{21} = \underline{K}_{22} = \underline{0}$. This reduces $\underline{S}_{x_{22}}$ to the form

$$\underline{S}_{x_{22}} = \underline{g}_{22}^{-1} \underline{g}_{21} \underline{Q}^{-1} \underline{Q}^{-T} \underline{g}_{21}^T \underline{g}_{22}^{-T} \quad (5.13)$$

Before showing how this disturbance over the study group can be used to detect strict geometric coherency and strict strong linear decoupling, first consider a partition of the example system into three parts as show below.

$$\begin{bmatrix} \ddot{\underline{x}}_1 \\ \vdots \\ \ddot{\underline{x}}_2 \\ \vdots \\ \ddot{\underline{x}}_3 \end{bmatrix} = - \begin{bmatrix} \underline{h}_{11} & \underline{h}_{12} & \underline{h}_{13} \\ \vdots & \vdots & \vdots \\ \underline{h}_{21} & \underline{h}_{22} & \underline{h}_{23} \\ \vdots & \vdots & \vdots \\ \underline{h}_{31} & \underline{h}_{32} & \underline{h}_{33} \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \vdots \\ \underline{x}_2 \\ \vdots \\ \underline{x}_3 \end{bmatrix} - \sigma \begin{bmatrix} \dot{\underline{x}}_1 \\ \vdots \\ \dot{\underline{x}}_2 \\ \vdots \\ \dot{\underline{x}}_3 \end{bmatrix} + [\underline{M} \mid \underline{M} \mid \underline{L}] \begin{bmatrix} \underline{\Delta PM} \\ \vdots \\ \underline{\Delta PL} \end{bmatrix}$$

The inverse of the matrix $\underline{h} = \underline{M} \underline{I}$ can be written

$$[\underline{M} \mid \underline{I}]^{-1} = \begin{bmatrix} \underline{f}_{11} & \underline{f}_{12} & \underline{f}_{13} \\ \underline{f}_{21} & \underline{f}_{22} & \underline{f}_{23} \\ \underline{f}_{31} & \underline{f}_{32} & \underline{f}_{33} \end{bmatrix}$$

where,

$$\underline{f}_{11} = \underline{V}_{11}^{-1}$$

$$\underline{f}_{21} = (-\underline{V}_{21}^{-1} \underline{h}_{21} + \underline{h}_{22}^{-1} \underline{h}_{23} \underline{V}_{31}^{-1} \underline{h}_{31}) \underline{V}_{11}^{-1}$$

$$\underline{f}_{31} = (-\underline{V}_{31}^{-1} \underline{h}_{31} + \underline{h}_{33}^{-1} \underline{h}_{32} \underline{V}_{21}^{-1} \underline{h}_{21}) \underline{V}_{11}^{-1}$$

$$\underline{f}_{21} = (-\underline{V}_{12}^{-1} \underline{h}_{12} + \underline{h}_{11}^{-1} \underline{h}_{13} \underline{V}_{32}^{-1} \underline{h}_{32}) \underline{V}_{22}^{-1}$$

$$\underline{f}_{22} = \underline{V}_{22}^{-1}$$

$$\underline{f}_{32} = (-\underline{V}_{32}^{-1} \underline{h}_{32} + \underline{h}_{33}^{-1} \underline{h}_{31} \underline{V}_{12}^{-1} \underline{h}_{12})$$

$$\underline{f}_{13} = (-\underline{V}_{13}^{-1} \underline{h}_{13} + \underline{h}_{11}^{-1} \underline{h}_{12} \underline{V}_{23}^{-1} \underline{h}_{23}) \underline{V}_{33}^{-1}$$

$$\underline{f}_{23} = (-\underline{V}_{23}^{-1} \underline{h}_{23} + \underline{h}_{22}^{-1} \underline{h}_{21} \underline{V}_{13}^{-1} \underline{h}_{13})$$

$$\underline{f}_{33} = \underline{v}_{33}^{-1}$$

$$\begin{aligned} \underline{v}_{11} = & [\underline{h}_{11} - \underline{h}_{12}\underline{v}_{21}^{-1}\underline{h}_{21} + \underline{h}_{12}\underline{h}_{22}^{-1}\underline{h}_{23}\underline{v}_{31}^{-1}\underline{h}_{31} - \underline{h}_{13}\underline{v}_{31}^{-1}\underline{h}_{31} \\ & + \underline{h}_{13}\underline{h}_{33}^{-1}\underline{h}_{32}\underline{v}_{21}^{-1}\underline{h}_{21}] \end{aligned}$$

$$\underline{v}_{12} = [\underline{h}_{11} - \underline{h}_{13}\underline{h}_{33}^{-1}\underline{h}_{31}]$$

$$\underline{v}_{13} = [\underline{h}_{11} - \underline{h}_{12}\underline{h}_{22}^{-1}\underline{h}_{21}]$$

$$\begin{aligned} \underline{v}_{22} = & [\underline{h}_{22} - \underline{h}_{21}\underline{v}_{12}^{-1}\underline{h}_{12} + \underline{h}_{21}\underline{h}_{11}^{-1}\underline{h}_{13}\underline{v}_{32}^{-1}\underline{h}_{32} - \underline{h}_{23}\underline{v}_{32}^{-1}\underline{h}_{32} \\ & + \underline{h}_{23}\underline{h}_{33}^{-1}\underline{h}_{31}\underline{v}_{12}^{-1}\underline{h}_{12}] \end{aligned}$$

$$\underline{v}_{21} = [\underline{h}_{22} - \underline{h}_{23}\underline{h}_{33}^{-1}\underline{h}_{32}]$$

$$\underline{v}_{23} = [\underline{h}_{22} - \underline{h}_{21}\underline{h}_{11}^{-1}\underline{h}_{12}]$$

$$\underline{v}_{33} = [\underline{h}_{33} - \underline{h}_{31}\underline{v}_{13}^{-1}\underline{h}_{13} + \underline{h}_{31}\underline{h}_{11}^{-1}\underline{h}_{12}\underline{v}_{23}^{-1}\underline{h}_{23} + \underline{h}_{32}\underline{h}_{22}^{-1}\underline{h}_{21}\underline{v}_{13}^{-1}\underline{h}_{13} - \underline{h}_{32}\underline{v}_{23}^{-1}\underline{h}_{23}]$$

$$\underline{v}_{31} = [\underline{h}_{33} - \underline{h}_{32}\underline{h}_{22}^{-1}\underline{h}_{23}]$$

$$\underline{v}_{32} = [\underline{h}_{33} - \underline{h}_{31}\underline{h}_{11}^{-1}\underline{h}_{13}]$$

First consider a general ZMIIW disturbance over all the generators of both the study group and the specified group. For this disturbance, $\underline{S}_X(\infty)$ can be written

$$\underline{S}_X(\infty) = [\underline{M} \ \underline{I}]^{-1} \underline{K} [\underline{M} \ \underline{I}]^{-T} \quad (5.15)$$

$$= \begin{bmatrix} \underline{f}_{11} & \underline{f}_{12} & \underline{f}_{13} \\ \underline{f}_{21} & \underline{f}_{22} & \underline{f}_{23} \\ \underline{f}_{31} & \underline{f}_{32} & \underline{f}_{33} \end{bmatrix} \begin{bmatrix} \underline{K}_{11} & \underline{K}_{12} & \underline{K}_{13} \\ \underline{K}_{21} & \underline{K}_{22} & \underline{K}_{23} \\ \underline{K}_{31} & \underline{K}_{32} & \underline{K}_{33} \end{bmatrix} \begin{bmatrix} \underline{f}_{11}^T & \underline{f}_{21}^T & \underline{f}_{31}^T \\ \underline{f}_{12}^T & \underline{f}_{22}^T & \underline{f}_{32}^T \\ \underline{f}_{13}^T & \underline{f}_{23}^T & \underline{f}_{33}^T \end{bmatrix}$$



where

$$\{K_{mn}\}_{ij} = \begin{cases} 2 & i = j \\ 1 & i \neq j \end{cases}, \quad m = 1, 2, 3$$

$$\{K_{mn}\}_{ij} = \{1 \text{ for all } i, j, \quad m, n = 1, 2, 3, \quad m \neq n.$$

Consider $S_{X_{33}}$ which measures the coherency between angles of the vector X_3 . Carrying out the multiplication in (5.13) yields

$$\begin{aligned} S_{X_{33}} &= [\underline{f}_{31}K_{11} + \underline{f}_{32}K_{21} + \underline{f}_{33}K_{31}]\underline{f}_{31}^T \\ &+ [\underline{f}_{31}K_{12} + \underline{f}_{32}K_{22} + \underline{f}_{33}K_{32}]\underline{f}_{32}^T \\ &+ \underline{f}_{33} [\underline{f}_{33}^{-1}\underline{f}_{31}K_{13} + \underline{f}_{33}^{-1}\underline{f}_{32}K_{23} + K_{33}]\underline{f}_{33}^T. \end{aligned} \quad (5.16)$$

It is possible to make the first two terms vanish. Assume that the group of generators represented by X_3 satisfy the conditions of strict geometric coherency. Then $\underline{h}_{31} = \underline{h}_{32} = \underline{0}$, which implies $\underline{f}_{31} = \underline{f}_{32} = \underline{0}$. For the third term in (5.16) to be small $\underline{f}_{33}$ must be small. The matrix $\underline{f}_{33}$ can be written

$$\begin{aligned} \underline{f}_{33} &= [\underline{h}_{33} - \underline{h}_{31}\underline{V}_{13}^{-1}\underline{h}_{13} + \underline{h}_{31}\underline{h}_{11}^{-1}\underline{h}_{12}\underline{V}_{23}^{-1}\underline{h}_{23} + \underline{h}_{32}\underline{h}_{22}^{-1}\underline{h}_{21}\underline{V}_{13}^{-1}\underline{h}_{13} - \underline{h}_{32}\underline{V}_{23}^{-1}\underline{h}_{23}]^{-1} \\ &= \left[\underline{h}_{33} - [\underline{h}_{31} \quad \underline{h}_{32}] \begin{bmatrix} \underline{V}_{13}^{-1} & -\underline{h}_{11}^{-1}\underline{h}_{12}\underline{V}_{23}^{-1} \\ -\underline{h}_{22}^{-1}\underline{h}_{21}\underline{V}_{13}^{-1} & \underline{V}_{23}^{-1} \end{bmatrix} \begin{bmatrix} \underline{h}_{13} \\ \underline{h}_{23} \end{bmatrix} \right]^{-1} \\ &= \left[\underline{h}_{33} - [\underline{e}_1 \underline{e}_2 \underline{e}_3]^{-1} \right] \end{aligned}$$

Applying the matrix identity (5.9) yields

$$\underline{f}_{33} = \underline{h}_{33}^{-1} + \underline{h}_{33}^{-1}\underline{e}_1[\underline{e}_2^{-1} - \underline{e}_3\underline{h}_{33}^{-1}\underline{e}_1]^{-1}\underline{e}_3\underline{h}_{33}^{-1} = \underline{h}_{33}^{-1} [\underline{I} + \underline{K}_b].$$

Thus the third term in $\underline{S}_{x_{33}}$ depends on $\underline{h}_{33}^{-1}$ and $\underline{S}_{x_{33}} \rightarrow 0$ only if $\underline{h}_{33}^{-1} \rightarrow 0$. It is obvious that the same analysis holds for any $\underline{S}_{x_{kk}}$, $k = 1, 2, 3$, using the other two groups as the "study group". This establishes for the three way partition of the model, the same result shown for the two way partition, namely that for a ZMIW disturbance over all the generators of the system, the principal structural condition detected will be the identification of tightly interconnected groups of generators. This repetition of the analysis already done is somewhat redundant, but it provides a very casual inductive proof that the analysis can be extended to an n-way partition with the same result.

Next, assume that the ZMIW disturbance is applied to the generators of group 1, and that the reference is taken over a machine or a group of machines in either group 2 or group 3 so that

$$\underline{K}_u = \begin{bmatrix} \underline{I}_{m \times m} & \underline{0} & \underline{0} \\ \underline{0} & \underline{0}_{n_1 \times n_1} & \underline{0} \\ \underline{0} & \underline{0} & \underline{0}_{n_2 \times n_2} \end{bmatrix}$$

where m, n_1, n_2 are the number of generators in groups 1, 2, 3, respectively. Then the expression (5.16) for $\underline{S}_{x_{33}}$ reduces to

$$\underline{S}_{x_{33}} = \underline{f}_{31} \underline{k}_{11} \underline{f}_{31}^T = 2(\underline{f}_{31} \underline{f}_{31}^T) \quad (5.17d)$$

Similar expressions can be written for $\underline{S}_{x_{22}}$, $\underline{S}_{x_{23}}$, and $\underline{S}_{x_{32}}$ namely,

$$\underline{S}_{x_{22}} = \underline{f}_{21} \underline{k}_{11} \underline{f}_{21}^T = 2(\underline{f}_{21} \underline{f}_{21}^T) \quad (5.17a)$$

$$\underline{S}_{x_{23}} = \underline{f}_{21} \underline{k}_{11} \underline{f}_{31}^T = 2(\underline{f}_{21} \underline{f}_{31}^T) \quad (5.17b)$$

$$\underline{S}_{x_{32}} = \underline{f}_{31} \underline{k}_{11} \underline{f}_{21}^T = 2(\underline{f}_{31} \underline{f}_{21}^T) \quad (5.17c)$$

Note that for this disturbance, $\underline{S}_{x_{23}}$ and $\underline{S}_{x_{32}}$ provide the same information as $\underline{S}_{x_{22}}$ and $\underline{S}_{x_{33}}$. In expanded form

$$\underline{S}_{x_{33}} = (-\underline{v}_{31}^{-1}\underline{h}_{31} + \underline{h}_{33}^{-1}\underline{h}_{32}\underline{v}_{21}^{-1}\underline{h}_{21})\underline{v}_{11}^{-1}\underline{v}_{11}^{-T}(-\underline{v}_{31}^{-1}\underline{h}_{31} + \underline{h}_{33}^{-1}\underline{h}_{32}\underline{v}_{21}^{-1}\underline{h}_{21})^T \quad (5.18a)$$

$$\underline{S}_{x_{22}} = (-\underline{v}_{21}^{-1}\underline{h}_{21} + \underline{h}_{22}^{-1}\underline{h}_{22}\underline{v}_{31}^{-1}\underline{h}_{31})\underline{v}_{11}^{-1}\underline{v}_{11}^{-T}(-\underline{v}_{21}^{-1}\underline{h}_{21} + \underline{h}_{22}^{-1}\underline{h}_{22}\underline{v}_{31}^{-1}\underline{h}_{31})^T \quad (5.18b)$$

The expressions for $\underline{f}_{21}$ and $\underline{f}_{31}$ can be written as

$$\begin{bmatrix} -\underline{f}_{21} \\ -\underline{f}_{31} \end{bmatrix} = \begin{bmatrix} \underline{v}_{21}^{-1} & -(\underline{h}_{22}^{-1} \underline{h}_{23} \underline{v}_{31}^{-1}) \\ -(\underline{h}_{33}^{-1} \underline{h}_{32} \underline{v}_{21}^{-1}) & \underline{v}_{31}^{-1} \end{bmatrix} \begin{bmatrix} \underline{h}_{21} \\ \underline{h}_{31} \end{bmatrix} \quad (5.19)$$

Note that the left matrix on the right hand side of equation (5.19) is the inverse of the matrix

$$\begin{bmatrix} \underline{h}_{11} & \underline{h}_{12} \\ \underline{h}_{32} & \underline{h}_{33} \end{bmatrix} \quad (5.20)$$

In the present analysis the disturbances are confined to group

1. That is group 1 can be identified as the "study group", and groups 2 and 3 are a two-way partition of the "specified group of n generators". Equation (5.19) is then very important because it establishes that for a ZMIIW disturbance over the study group $\underline{S}_x(\infty)$ provides exactly the same information as $\underline{M} \underline{I}$. The conceptual power of (5.19) is shown by some examples. Consider first that a ZIIW disturbance of all the generators has shown groups 2 to be a tightly interconnected group. Equation (5.19) indicates that if the generators of groups 2 and 3 are, together, satisfying the conditions of

geometric coherency or strong linear decoupling, then a ZMIW disturbance over the generators of the study group will show this, since the matrix (5.20) is the $\underline{g}_{22}$ and the matrix $[\underline{h}_{21} \ \underline{h}_{31}]^T$ the $\underline{g}_{21}$ of the analysis done for the two part partition. If the primary interest were in disturbances within the study group (group 1), then groups 2 and 3 could be aggregated into a single equivalent generator.

Now suppose that the ZMIW disturbance over all generators does not indicate that either groups 2 or 3 are separate, tightly interconnected groups. The ZMIW disturbance over the generators of the study group will still indicate whether groups two or three are together, or individually, exhibiting strict geometric coherency (SGC) or strict strong linear decoupling (SSLD). If all generators of the two groups are exhibiting either SGC or SSLD, then equation (5.19) shows directly that these coherency conditions are satisfied.

If either of the two groups is exhibiting SGC or SSLD by itself, this will also be detected. Consider the expression for $\underline{S}_{X_{33}}$. The matrix $\underline{V}_{31}^{-1}$ can be written in the form

$$\underline{V}_{31}^{-1} = [\underline{h}_{33}^{-1} + \underline{h}_{33}^{-1} \underline{h}_{22} - \underline{h}_{23} \underline{h}_{33}^{-1} \underline{h}_{32}] \underline{h}_{23} \underline{h}_{33}^{-1}$$

This allows $\underline{S}_{X_{33}}$ to be written as

$$\begin{aligned} \underline{S}_{X_{33}} = & \{ [\underline{h}_{33}^{-1} + \underline{K}_c] \underline{h}_{33}^{-1} \underline{h}_{31} + \underline{h}_{33}^{-1} \underline{h}_{32} \underline{V}_{21}^{-1} \underline{h}_{21} \} \underline{V}_{11}^{-1} \underline{V}_{11}^{-T} \{ [\underline{h}_{33}^{-1} + \underline{K}_c] \underline{h}_{33}^{-1} \underline{h}_{31} \\ & + \underline{h}_{33}^{-1} \underline{h}_{32} \underline{V}_{21}^{-1} \underline{h}_{21} \}^T \end{aligned}$$

Thus any one of the conditions

$$(1) \ \underline{h}_{33}^{-1} \rightarrow 0 \quad (\text{strict synchronizing coherency})$$

$$(2) \begin{bmatrix} \underline{h}_{31} \\ \vdots \\ \underline{h}_3 \end{bmatrix} = \begin{bmatrix} \underline{0} \\ \vdots \\ \underline{0} \end{bmatrix} \quad (\text{strict geometric coherency})$$

$$(3) \underline{h}_{33}^{-1} \begin{bmatrix} \underline{h}_{31} \\ \vdots \\ \underline{h}_{32} \end{bmatrix} \rightarrow \underline{0} \quad (\text{strict strong linear decoupling})$$

will cause $\underline{S}_{x_{33}} = \underline{0}$ or $\underline{S}_{x_{33}} \rightarrow \underline{0}$. In this case, $\underline{h}_{33}$ is identified with $\underline{g}_{22}$ and, $\begin{bmatrix} \underline{h}_{31} & \underline{h}_{32} \end{bmatrix}$ with $\underline{g}_{21}$, so that groups 1 and 2 are the "study system" and group 3 "the specified group of n generators."

One might very well ask how the last result can be obtained when the ZIIW disturbance is over group one, but both groups one and two together are being identified as the study group. Suppose that the ZIIW disturbance is done over both groups one and two. $\underline{S}_{x_{33}}$ will be modified by the addition of the term

$$\begin{bmatrix} -\underline{V}_{32}^{-1} \underline{h}_{32} + \underline{h}_{33}^{-1} \underline{h}_{31} \end{bmatrix} \underline{V}_{12}^{-1} \underline{h}_{12} \quad \underline{V}_{22}^{-1} \underline{V}_{22}^{-T} \begin{bmatrix} -\underline{V}_{32}^{-1} \underline{h}_{32} + \underline{h}_{33}^{-1} \underline{h}_{31} \end{bmatrix} \underline{V}_{12}^{-1} \underline{h}_{12}$$

This term can be rewritten in the form

$$\begin{aligned} & \begin{bmatrix} \underline{h}_{33}^{-1} + \underline{K}_a \end{bmatrix} \underline{h}_{33}^{-1} \underline{h}_{32} + \underline{h}_{33}^{-1} \underline{h}_{31} \underline{V}_{12}^{-1} \underline{h}_{12} \underline{V}_{22}^{-1} \underline{V}_{22}^{-T} \begin{bmatrix} \underline{h}_{33}^{-1} + \underline{K}_a \end{bmatrix} \underline{h}_{33}^{-1} \underline{h}_{32} \\ & + \underline{h}_{33}^{-1} \underline{h}_{31} \underline{V}_{12}^{-1} \underline{h}_{12} \end{bmatrix}^T \end{aligned}$$

This additional term is also zero under precisely the same conditions just analyzed. Hence the ZIIW disturbance of group 1 is sufficient to identify group 3 as coherent. Intuitively this seems to say that ZIIW disturbance of group 1 disturbs group 2 strongly enough to detect if either the conditions for GC or SLD are satisfied between groups 2 and 3.

An example of strong linear decoupling was considered in Chapter 4; another is considered here. Figure 5.1a shows a four generator system. Let generator 1 be the study group, generators 2 and

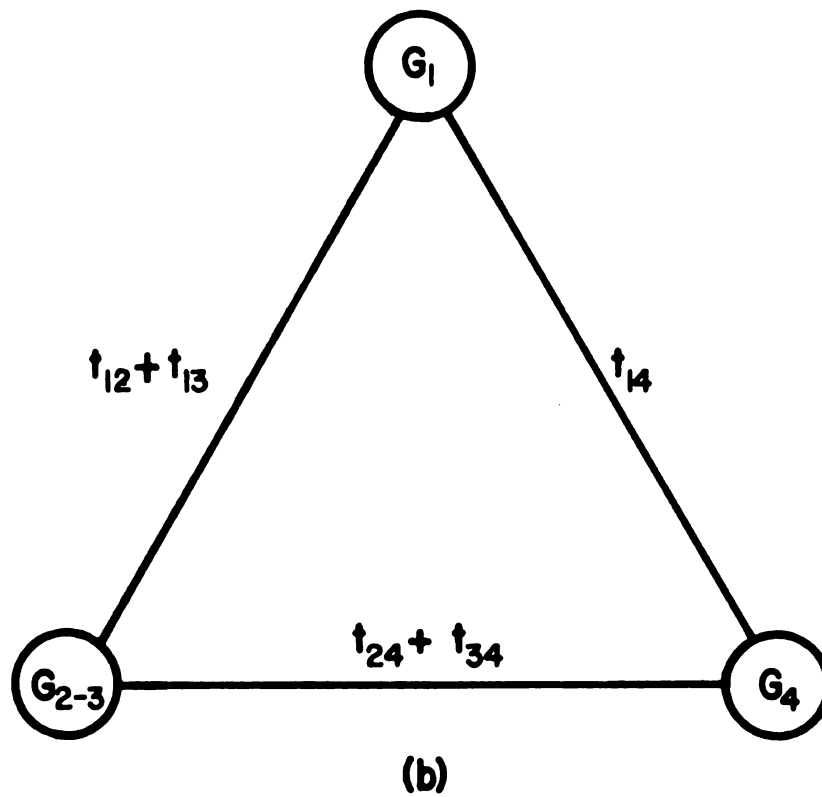
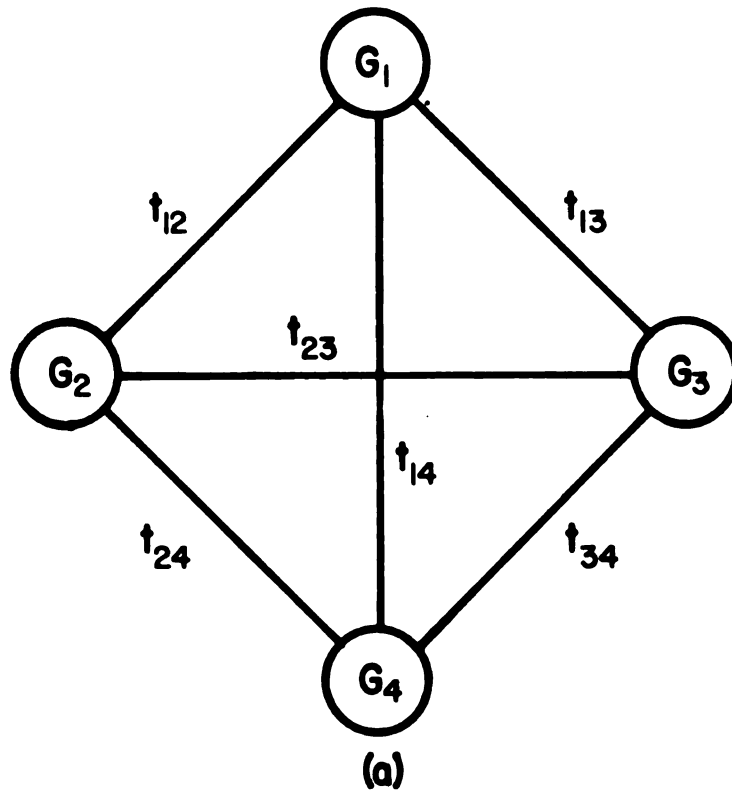


FIGURE 5-1
FOUR GENERATOR SYSTEM
(a) BEFORE AND (b) AFTER AGGREGATION OF GENERATORS 2 AND 3

3 be group 2 and generator 4 group 3. Let the reference be generator

2. Then

$$\hat{\underline{M}} \underline{T} = \begin{bmatrix} \frac{1}{M_1} & & -\frac{1}{M_2} \\ & \frac{1}{M_3} & -\frac{1}{M_2} \\ & & \frac{1}{M_4} & -\frac{1}{M_2} \end{bmatrix} \begin{bmatrix} t_{11} & t_{13} & t_{14} \\ t_{12} & t_{23} & t_{24} \\ t_{13} & t_{33} & t_{34} \\ t_{14} & t_{34} & t_{44} \end{bmatrix}$$

$$= \begin{bmatrix} (\frac{t_{11}}{M_1} - \frac{t_{14}}{M_2}) & (\frac{t_{13}}{M_1} - \frac{t_{23}}{M_2}) & (\frac{t_{14}}{M_1} - \frac{t_{24}}{M_2}) \\ (\frac{t_{13}}{M_3} - \frac{t_{12}}{M_2}) & (\frac{t_{33}}{M_3} - \frac{t_{23}}{M_2}) & (\frac{t_{34}}{M_3} - \frac{t_{24}}{M_2}) \\ (\frac{t_{14}}{M_4} - \frac{t_{12}}{M_2}) & (\frac{t_{34}}{M_4} - \frac{t_{23}}{M_2}) & (\frac{t_{44}}{M_4} - \frac{t_{24}}{M_2}) \end{bmatrix}$$

$$\text{Let } t_{23} = \frac{1}{\mu} \text{ and } \frac{t_{12} + t_{13}}{M_2 + M_3} = \frac{t_{14}}{M_4}$$

Then

$$\begin{aligned} \frac{t_{33}}{M_3} - \frac{t_{23}}{M_2} &= -\frac{t_{13}}{M_3} - \frac{t_{23}}{M_3} - \frac{t_{24}}{M_3} - \frac{t_{23}}{M_2} \\ &= -\frac{1}{\mu} \left(\frac{M_2 + M_3}{M_2 M_3} \right) - \frac{t_{13}}{M_3} - \frac{t_{24}}{M_3} = h_{22} \end{aligned}$$

$$\frac{t_{34}}{M_3} - \frac{t_{24}}{M_2} = h_{23}$$

$$\frac{t_{34}}{M_4} - \frac{t_{23}}{M_2} = -\frac{1}{\mu} \frac{1}{M_2} + \frac{t_{34}}{M_4} = h_{32}$$

$$\frac{t_{44}}{M_4} - \frac{t_{24}}{M_2} = h_{33}$$

Next write

$$\begin{bmatrix} h_{22} & h_{23} \\ h_{32} & h_{33} \end{bmatrix}^{-1} = \begin{bmatrix} \frac{h_{33}}{d} & -\frac{h_{23}}{d} \\ -\frac{h_{32}}{d} & \frac{h_{22}}{d} \end{bmatrix}$$

where $d = h_{22} h_{33} - h_{32} h_{23}$, and $\lim_{\mu \rightarrow 0} \mu d = \left(-\frac{M_2 + M_3}{M_2 M_3} h_{33} + \frac{h_{23}}{M_2}\right) = d'$

Then

$$\lim_{\mu \rightarrow 0} \frac{h_{23}}{d} = \lim_{\mu \rightarrow 0} \frac{\mu h_{23}}{d'} = 0$$

$$\lim_{\mu \rightarrow 0} \frac{-h_{23}}{d} = \lim_{\mu \rightarrow 0} \frac{-\mu h_{23}}{d'} = 0$$

$$\lim_{\mu \rightarrow 0} \frac{-h_{32}}{d} = \lim_{\mu \rightarrow 0} \frac{-\mu h_{32}}{d'} = \frac{1}{d'} = \frac{1}{K}$$

$$\lim_{\mu \rightarrow 0} \frac{h_{22}}{d} = \lim_{\mu \rightarrow 0} \frac{\mu h_{22}}{d'} = \frac{-\frac{1}{M_2} \left(\frac{M_2 + M_3}{M_3}\right)}{d'} = \frac{-\left(\frac{M_2 + M_3}{M_3}\right)}{K}$$

The condition $(t_{12} + t_{13})/(M_2 + M_3) = t_{14}/M_4$ can be used to show that

$$\left(\frac{t_{13}}{M_3} - \frac{t_{12}}{M_2}\right) = \left(\frac{M_2 + M_3}{M_3}\right) \left(\frac{t_{14}}{M_4} - \frac{t_{12}}{M_2}\right) = \left(\frac{M_2 + M_3}{M_3}\right) c$$

Combining the above results gives

$$\begin{bmatrix} h_{22} & h_{23} \\ h_{21} & h_{33} \end{bmatrix}^{-1} \begin{bmatrix} h_{21} \\ h_{23} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \frac{1}{K} & -\left(\frac{M_2 + M_3}{M_3}\right) \frac{1}{K} \end{bmatrix} \begin{bmatrix} \left(\frac{M_2 + M_3}{M_3}\right) c \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (5.21)$$

where $C = (\frac{t_{14}}{M_4} - \frac{t_{12}}{M_2})$.

Figure 5.1b gives a physical interpretation of this result. Letting $\mu \rightarrow 0$ causes $t_{23} \rightarrow \infty$, making the line between generators 2 and 3 infinitely stiff. This corresponds to aggregating generators 2 and 3 into one generator, called 2-3 in Figure 5.1b, whose inertia is $M_{23} = M_2 + M_3$. The synchronizing torque coefficient becomes $t_{1,2-3} = t_{12} + t_{13}$. Hence, in the limit as $\mu \rightarrow 0$ generator 2-3 and generator 4 satisfy the conditions for strict geometric coherency with respect to generator 1.

The four generator model of Figure 5.1 is an example of the case where group 2 satisfies strict synchronizing coherency, while group 3 and the single generator equivalent for group 2 satisfy strict geometric coherency with group 1. In the example group 3 contains only one generator, but the generalization is clear. It is an instructive exercise to consider a five generator model where generator 1 is the study group, generators 2 and 3, group 2 and generators 4 and 5, group 3, where generator 2 is the reference and the power system structure satisfies, $t_{14}/M_4 = t_{15}/M_5 = t_{12} + t_{13}/M_2 + M_3$. The general form of equation (5.21) can be achieved without doing all the algebra, which involves two 2×2 matrix inversions.

A completely parallel line of analysis can be developed for the conditions of pseudo-coherency and weak linear decoupling of the types $(\underline{M} \ \underline{T})_{12}(\underline{M} \ \underline{T})_{22}^{-1}M_{21} \rightarrow 0$ and $(\underline{M} \ \underline{T})_{12}(\underline{M} \ \underline{T})_{22} \rightarrow 0$. This analysis is not presented here for two reasons. First, it is completely analogous to what has already been done. Second, the formal algorithm for producing dynamic equivalents which is presented in the

next section does not test for these coherency conditions. The rationale for omitting these conditions goes as follows.

It was shown in Chapters 3 and 4 that the pseudo-coherency condition could only be exactly satisfied in the linear model and that the transference of pseudo-coherent behavior observed in the linear model to the nonlinear model could not be strongly guaranteed. The weak linear decoupling conditions, like pseudo-coherency depend on the submatrix $(\underline{M} \ \underline{I})_{12}$. They are therefore, categorized with pseudo-coherency and not detected. On the other hand, all the coherency conditions that will be tested for in the formal algorithm are strong conditions in the sense that satisfaction of any of the conditions in the linear model strongly guarantees its satisfaction in the nonlinear model.

The detection of coherency conditions presented in terms of the three-way partition of the example system has used a sequence of two disturbances, the first a ZMIW disturbance over all the generators of the system, the second, a ZMIW disturbance over just the generators of the study group. The ZMIW disturbance is designed to detect what might be termed, global or principal groups, i.e. groups of generators that are tightly interconnected and will remain coherent for a large class of disturbances. The second disturbance detects a more parochial kind of coherent behavior, that is, generators that are coherent in response to disturbances located in a certain region of the power system. This general procedure is the basis of the formal algorithm for producing dynamic equivalents presented in the next section, and is integral to the overall philosophy of viewing the power system at two levels; the more general or global level,

associated with the ZMIW disturbance over all generators and the specialized or parochial level, associated with the ZMIW disturbance over only a subset of generators.

VI. A Formal Algorithm for Producing Reduced Order Dynamic Equivalents

The analysis of the previous sections has laid the ground-work necessary to formulate a specific procedure for producing reduced order dynamic equivalents of power systems. This section proposes a formal algorithm for producing these equivalents.

Consider the power system, shown in Figure 5.2. Note that three zones have been identified, the internal system, or zone, a buffer zone, and a far system, or zone. The internal system represents those generators about which detailed response information is desired. It can be identified with what has been termed the study group in the example system used in the previous sections of this chapter. The buffer zone includes those generators that are electrically close enough to the internal system to warrant some level of detailed representation. The buffer zone can be generally associated with the "specified group of n generators" of the previous sections. The far system represents those generators that are electrically distant enough from the internal system to be lumped into a single equivalent machine.

A general outline of the overall algorithm can be stated as follows. The three zones are defined precisely and the far system is aggregated into a single machine. A ZMIW disturbance is then applied to all the generators of both the internal and buffer zones to determine the tightly interconnected groups. These groups then

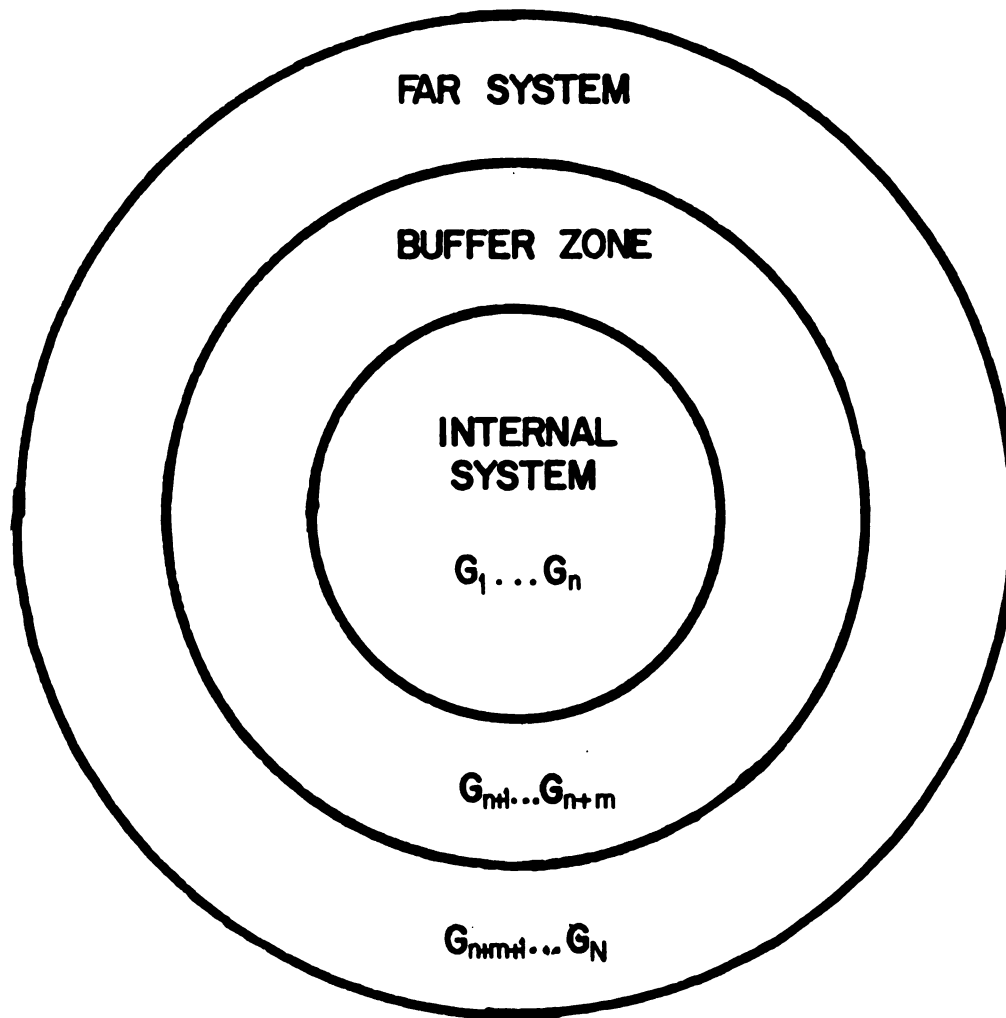


FIGURE 5-2
THREE ZONE PARTITION OF POWER SYSTEM

represent what is termed the global model of the power system. If interest centers on disturbances in a specific area, generally identified with the internal system or zone, a ZMIW disturbance is then conducted at the buses of the internal system. The purpose of the second disturbance is to determine if further aggregation beyond that indicated by the global model is possible for studying disturbances confined to the internal system. There are a number of questions associated with this general outline of the algorithm that need to be answered in some detail before the algorithm can be stated in a step by step manner. These questions are dealt with below, and the algorithm is then stated in terms of a set of operational steps.

One of the first questions of interest is how the boundary between the buffer zone and the far system can be determined. The boundary between the internal system, or zone, and the buffer zone is well defined, initially, although as will be shown, this boundary may be subject to redefinition, depending on the results of the ZMIW disturbance over all generators of the internal and buffer zones. The boundary between the buffer zone and the far system is not clearly defined. Determining this boundary precisely involves an iterative combination of experiment and engineering judgement.

It is a well established empirical fact that generators that are, electrically, a long distance from the source of a disturbance can be lumped into one machine. The analysis of this chapter and the last helps explain this behavior, and also provides a constructive experimental procedure for locating the boundary in question exactly.

Consider the example system of previous sections partitioned into three subgroups. Identify group one with the internal system, group two with the buffer zone and group three with the far system. It was argued in Chapter 4 that generators a long electrical distance from a disturbance satisfy the conditions for strict geometric coherency trivially, in the sense that the synchronizing power coefficients, t_{ij} , between a bus i near the disturbance and a bus j electrically distant are small. Hence these t_{ij} are all about equal, in the sense that they are all almost zero. This is precisely the condition that makes the submatrix $[C_{31} \ C_{32}] = [0 \ 0]$, implying strict coherency of the far system (group three). Now the boundary between the buffer zone and the far system can be determined, approximately, by inspection of the network, aided by any operating experience with disturbances that may be available. If a ZIIW disturbance is now conducted over the buses of the internal system, the generators electrically near the preliminary boundary can be checked for coherency. If they are coherent then the chosen boundary is adequate. In fact it may be possible to improve the boundary by checking the coherency of additional generators electrically closer to the internal system. If the generators are not coherent in a neighborhood around the preliminary boundary then it will be necessary to check the coherency of generators electrically more distant from the internal system. In either case it should be possible in a few iterations to define quite accurately the boundary between the buffer zone and the far system. The principle point, here, is that it is not necessary to check the coherency of all the machines in the overall

system, only those generators fairly close, electrically, to the preliminary boundary.

Having established the boundary between the buffer zone and far system, it is then possible to perform a ZMIIW disturbance over all the generators of both the internal system and buffer zone, to determine the tightly interconnected groups. These groups are based on the greatest aggregation of machines that can be achieved and still "adequately" reproduce the behavior of the unreduced system. More will be said about the qualitative term adequately in the next chapter. Once the level of aggregation is established and the coherent groups determined, it may happen that a tightly interconnected group contains generators from both the internal system and the buffer zone. If that is the case then a redefinition of the boundary between the internal system and buffer zone is required. There are two choices, either include the affected generators of the buffer zone as part of the internal system or include the affected generators of the internal system as part of the buffer zone and aggregate the group. The third choice of aggregating only those machines of the group that belong to the buffer zone will not result in good equivalents as will be shown in Chapter 6.

Based on the foregoing discussion, it is now possible to state a formal algorithm for producing dynamic equivalents. The steps are the following.

- (1) Define the boundary between the internal system and the buffer zone formally.
- (2) Find the boundary between the buffer zone and far system. If necessary use a ZMIIW disturbance of the

generators of the internal system to do an iterative check of the coherency of generators near an estimated boundary until the boundary is exactly defined.

- (3) Use a ZMIW disturbance of all the generators of the internal system and buffer zone to determine the tightly interconnected groups. If necessary, based on the group membership, redefine the boundary between the internal system and buffer zone.
- (4) (Optional) Use a ZMIW disturbance of all the generators of the internal system alone to see if any further aggregation is possible.

VII. The Global and Local Perspectives

The four step algorithm proposed in the preceding section, produces a reduced order dynamic equivalent for investigating disturbances that occur within a pre-selected region of the power system, the region designated in the algorithm as the internal system. In other words, it is explicitly assumed at the outset that the only generators whose detailed response is required are those generators in the internal system. The behavior outside the internal system is of no interest and any model of the buffer zone and far system that accurately represents the effect of these zones on the internal system would be acceptable.

This local or parochial view of disturbances is the traditional view which has concentrated on the transient stability problem from the perspective of one generator. It is possible to take a broader view of the problem, what might be called a global perspective. Step (3) of the formal algorithm is a ZMIW disturbance over all the generators of both the internal system and buffer zone, that is over all the generators that are initially unaggregated. The purpose of this disturbance is to find the principal groups of generators, generators

that depend only upon tight intermachine connections. These groups are called principal, because they are the groups that will remain coherent in response to the greatest variety of disturbances both in terms of the size of the disturbances and the location of the disturbances. Therefore, if interest centers on the large scale effects of system behavior the best approach would be to do a ZMIW over all the generators of the system and find the principal groups. For a particular disturbance, the principal group (or groups) nearest the disturbance will have to be left disaggregated. The remainder of the principal groups are aggregated into single machines. The response to the disturbance then will indicate how the principal groups of the overall system interact with each other. This type of global analysis is increasingly an area of interest both for system planning and security assessment. Performing the ZMIW disturbance to obtain the principal groups may seem like a formidable task. Actually it is not so formidable. It is true that for a system of n generators n deterministic disturbances are needed. However these disturbances are not difficult to compute and in particular require no matrix inversions. Further the n -disturbance sequence only has to be done once.

Thus the algorithm of the previous section is really an appropriate analysis tool for two very different problems. If the perspective is global, then the first three steps of the algorithm are used. If the perspective is local, the fourth step is added.

VIII. Summary

This chapter has shown how the r.m.s. coherency measure, in conjunction with the appropriate ZMIIW disturbance can be used to detect all the important coherency conditions that lead to reduced order equivalents which preserve both modal and coherency properties. A formal algorithm was developed which can be used to produce both global and local equivalents. In the next chapter the methods developed here will be applied to an analysis of a linearized version of The 39 Bus New England System.

CHAPTER 6

TESTING THE REDUCTION ALGORITHM ON THE 39 BUS NEW ENGLAND SYSTEM

I. Introduction

To test the ideas formalized in Chapter 5 as a reduction algorithm, simulations were made using the 39 Bus New England System. A schematic of this system, also referred to as just the "New England System", is shown in figure 6.1. This system is one frequently used as an example in the literature, and has been used for basic software testing by Podmore [2].

As a first step to test the reduction algorithm, a ZMIIW disturbance of all ten generators was used to identify the tightly interconnected groups, i.e. the principal groups exhibiting synchronizing coherency. The output of the software routine used to do the ZMIIW disturbance is a ranking table of the coherency measures between each pair of generators, ordered from the most coherent pair of generators to the least coherent. Table 6.1 is an example of such a ranking table.

The order of aggregation of generators is determined by applying a commutative rule to this ranking table. The commutative rule means that a generator must be coherent with all generators in an existing group before it can be added to that group. To combine two coherent groups requires that every member of the first group be coherent with

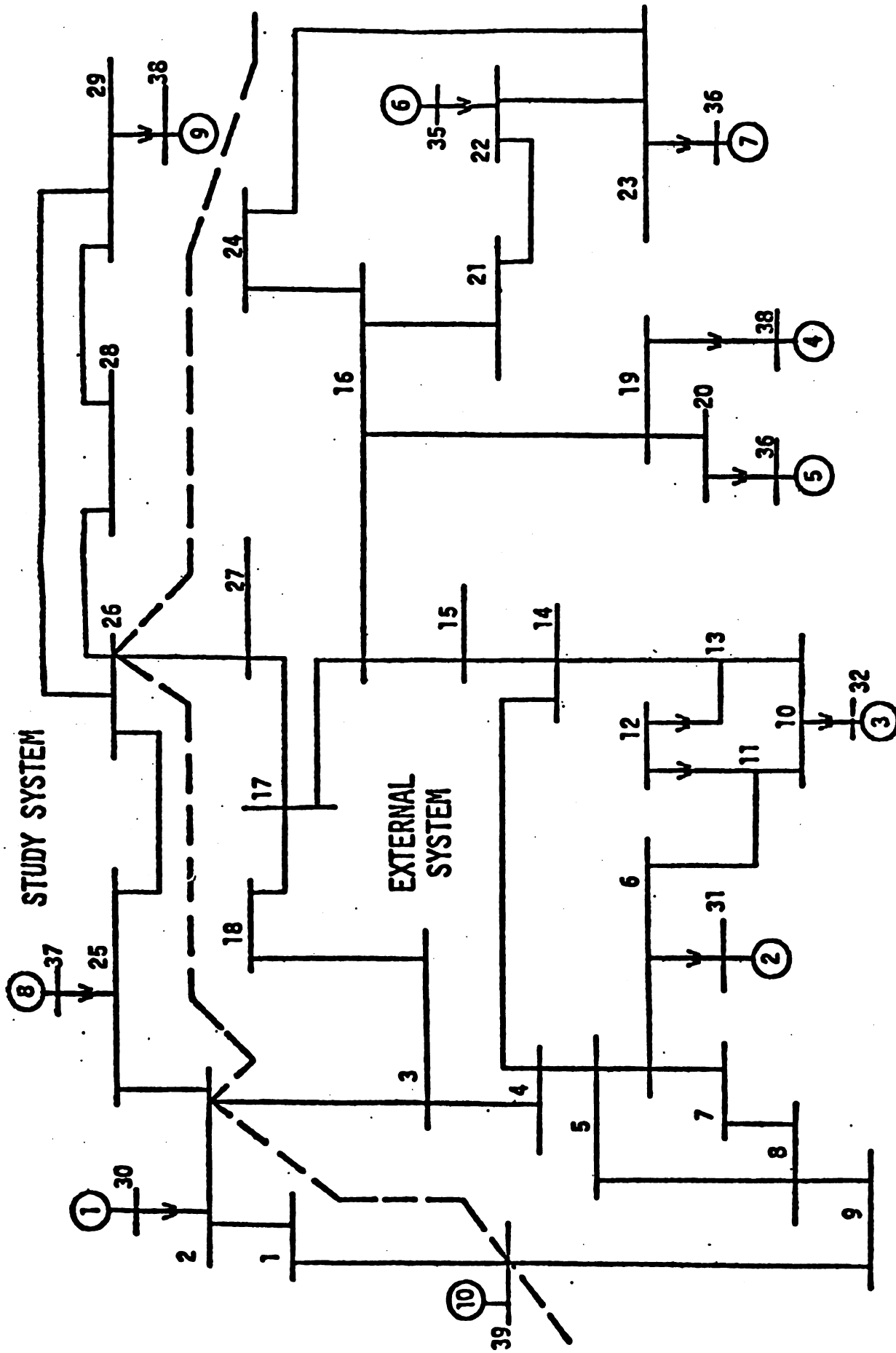


Figure 6.1. Line Diagram of 39 Bus New England System

every member of the second group. Beginning at the top of the ranking table with the two most coherent generators and moving downward, applying the commutative rule at each row of the ranking table, generates a set of aggregated models of the full system. Each level of aggregation reduces the number of generators by one and the order of the state model by two. Moving down far enough will result in the aggregation of all ten generators into a single generator.

Having identified the tightly inter-connected groups for all levels of aggregation, a series of ZMIW disturbances, over the internal system generators and subsets of these generators, were used to see if further aggregation was possible for disturbances confined to the internal system. This is a test of the optional fourth step of the reduction algorithm. The internal system of the 39 Bus New England System is generators 1, 8, 9 and 10. Generator 10 is used throughout as the reference.

For each test disturbance, the first step in the data collection and analysis procedure was to determine the coherent groups at each level of aggregation. Then for each level of aggregation eigenvalues and coherency measures based on the matrix $\underline{M} \underline{T}$ of the aggregated model are calculated. Finally, computer simulations are made, comparing the response of the system at each level of aggregation to the response of the full 39 Bus System, for various disturbances. Hopefully, the information provided by the eigenvalues, the coherency measures and the simulations at each level of aggregation will be consistent.

All of the analysis in this chapter is based on a linearized model of the New England System. For the eigenvalue and coherency analysis phases of the data analysis the linearized model is a prerequisite. For the simulations, however, a nonlinear model would seem to be preferable. However, a nonlinear simulation is more expensive and in many situations may only show minor differences from the linear simulation. It has to be remembered that a power system is nonlinear but not pathologically nonlinear. A power system, operating at a stable equilibrium point, when subjected to a typical disturbance, oscillates for a period of time, and then settles out to a new stable equilibrium point. This line of argument is born out by Podmore's comparison of linear and nonlinear simulation results for the New England System [2]. Linear simulations were, therefore, considered adequate.

II. General ZMIW Disturbance of the New England System

All the ZMIW disturbances of the New England System were made using a modified version of Podmore's linear simulation program, LINSIM [2]. The modified version uses the r.m.s. coherency measure in place of the min-max coherency measure used in the original version of LINSIM.

Table 6.1 is the ranking table for a ZMIW disturbance of all ten generators, sometimes referred to as just a general ZMIW disturbance.

Table 6.2 is a compilation of the magnitude of the imaginary part of the system eigenvalues at each level of aggregation. It was shown in Chapter 4, that the system eigenvalues are complex conjugate

Table 6.1. Ranking Table of R.M.S. Coherency Measures for ZMIIW
Disturbance of all Ten Generators

<u>Ranking</u> <u>Ranking</u>	<u>Generator Pair</u> <u>Generator Pair</u>	<u>Coherency Measure</u> <u>Coherency Measure</u>	<u>Aggregation</u> <u>Level</u>
1.	C(6,7)	2.6626	1
2.	C(1,8)	2.8395	2
3.	C(4,7)	3.1125	
4.	C(4,6)	3.1926	3
5.	C(4,8)	3.7503	
6.	C(7,8)	3.8427	
7.	C(2,3)	3.8812	4
8.	C(3,8)	3.9151	
9.	C(2,8)	3.9396	
10.	C(6,8)	3.9563	
11.	C(1,2)	3.9836	
12.	C(1,3)	3.9965	5
13.	C(1,4)	4.0120	
14.	C(4,5)	4.0761	
15.	C(1,7)	4.1062	
16.	C(3,4)	4.1539	
17.	C(1,6)	4.2192	
18.	C(3,7)	4.2348	
19.	C(3,6)	4.3382	
20.	C(2,4)	4.3477	
21.	C(2,7)	4.4290	
22.	C(2,6)	4.5348	6
23.	C(5,7)	4.7821	
24.	C(5,6)	4.8373	
25.	C(4,9)	5.4666	
26.	C(8,9)	5.5026	
27.	C(7,9)	5.5085	
28.	C(6,9)	5.5801	
29.	C(5,8)	5.6626	
30.	C(1,9)	5.8902	
31.	C(3,5)	5.8976	
32.	C(1,5)	5.9590	
33.	C(2,5)	6.1038	7
34.	C(3,9)	6.2177	
35.	C(2,9)	6.3621	
36.	C(5,9)	6.5344	8
37.	C(1,10)	8.4657	
38.	C(8,10)	8.8583	
39.	C(2,10)	9.0565	
40.	C(3,10)	9.3828	
41.	C(4,10)	10.4496	
42.	C(7,10)	10.5294	
43.	C(6,10)	10.6120	
44.	C(9,10)	11.9256	
45.	C(5,10)	12.1987	

Table 6.2. Eigenvalue Data for ZMIW Disturbance of all Ten Generators

<u>Aggregation</u>	<u>Magnitudes of Imaginary Parts of System Eigenvalues for all Levels of Aggregation</u>										<u>Level</u>
None	4.461	6.296	6.921	7.676	8.287	8.315	9.541	9.811	9.984		0
6-7	4.464	6.297	6.922	7.678	8.288	8.315	9.542	9.811			1
1-8 6-7	4.465	6.300	6.922	7.678	8.288	8.315	9.542				2
1-8 4-6-7	4.466	6.307	6.979	7.872	8.314	8.372					3
1-8 2-3 4-6-7	4.467	6.307	6.980	7.879	8.372						4
1-2-3-8 4-6-7	4.469	6.341	7.175	8.142							5
1-2-3-4-6-7-8	4.550	6.437	7.324								6
1-2-3-4-5-6-7-8	4.595	6.539									7
1-2-3-4-5-6-7-8-9	4.637										8

pairs of the form $\lambda_i, \lambda_i^* = -\frac{\sigma}{2} \pm j b_i$. For the linearized model of the New England System used in the present data collection σ was set at .275. Since $-\frac{\sigma}{2}$ is the same for all eigenvalues and very small, all that is required to characterize the eigenvalue pair λ_i, λ_i^* is b_i , the magnitude of the imaginary part. Therefore, for expository convenience, the imaginary part of the eigenvalue may occasionally be loosely referred to as an eigenvalue.

At each level of aggregation in Table 6.2 the system model has one less generator and one less pair of eigenvalues, until finally at level 8 the system model consists of two generators, the aggregate of generators 1 through 9, and the reference, generator 10.

The eigenvalue information shown for each level of aggregation in Table 6.2 is calculated from $\hat{M}_i \underline{T}_i$, the $\hat{M} \underline{T}$ matrix for the level i aggregation. An explanation is given here of how $\hat{M}_i \underline{T}_i$ is obtained from the matrices $\hat{M}$ and $\underline{T}$ of the Full New England System.

Table 6.3 gives the matrix $-\underline{T}$ for the New England system, with all load buses and all generator terminal buses eliminated. Only the internal generator buses remain. The matrix $\hat{M}$ for the New England System, with generator 10 the reference is

Table 6.3. The Matrix $-T$ for the 39 Bus New England System

-16.153	1.078	1.035	1.317	.531	1.365	1.057	3.068	1.344
1.078	-9.261	2.038	.732	.295	.759	.588	.558	.372
1.348	2.038	-10.851	1.011	.408	1.048	.812	.704	.484
1.317	.732	1.011	-12.036	2.417	2.045	1.584	.769	.673
.531	.295	.408	2.417	-6.329	.828	.642	.312	.273
1.365	.759	1.048	2.045	.828	-13.677	3.859	.757	.663
1.0573	.588	.812	1.584	.642	3.859	-10.808	.622	.544
3.068	.558	.704	.769	.312	.757	.622	-10.624	1.102
1.344	.372	.484	.673	.273	.663	.544	1.102	-6.846
5.044	2.841	2.998	1.611	.624	2.354	1.101	2.733	-20.697

$$\begin{bmatrix} \frac{1}{M_1} & & & & & & & & & -\frac{1}{M_{10}} \\ & \frac{1}{M_2} & & & & & & & & -\frac{1}{M_{10}} \\ & & \frac{1}{M_3} & & & & & & & -\frac{1}{M_{10}} \\ & & & \frac{1}{M_4} & & & & & & -\frac{1}{M_{10}} \\ & & & & \frac{1}{M_5} & & & & & -\frac{1}{M_{10}} \\ & & & & & \frac{1}{M_6} & & & & -\frac{1}{M_{10}} \\ & & & & & & \frac{1}{M_7} & & & -\frac{1}{M_{10}} \\ & & & & & & & \frac{1}{M_8} & & -\frac{1}{M_{10}} \\ & & & & & & & & \frac{1}{M_9} & -\frac{1}{M_{10}} \end{bmatrix}$$

where,

$$M_1 = .2228$$

$$M_6 = .1846$$

$$M_2 = .1607$$

$$M_7 = .1400$$

$$M_3 = .1899$$

$$M_8 = .1289$$

$$M_4 = .1517$$

$$M_9 = .1830$$

$$M_5 = .1379$$

$$M_{10} = 2.6525$$

Then $-\hat{M} \underline{I}$ is, of course, just the multiplication of these two matrices.

To find the level 1 aggregation where generators 6 and 7 are combined into a single generator, first form $-\underline{I}_1$ by 1) adding column 7 of $\underline{I}$ to column 6, and then 2) adding row 7 of this intermediate matrix to row 6. The result is $-\underline{I}_1$, shown in Table 6.4. To form $\hat{M}_1$

Table 6.4. The Matrix $-T_1$ for the 6-7 Aggregation of the 39 Bus New England System

-16.153	1.078	1.348	1.317	.531	2.422	3.068	1.344
1.078	-9.261	2.038	.732	.295	1.347	.558	.372
1.348	2.038	-10.851	1.011	.408	1.859	.704	.484
1.317	.732	1.011	-12.036	2.417	3.629	.769	.673
.531	.295	.408	2.417	-6.329	1.47	.312	.273
2.422	1.347	1.859	3.629	1.47	-16.768	1.379	1.207
3.068	.558	.704	.769	.312	1.378	-10.624	1.102
1.344	.372	.484	.673	.273	1.206	1.102	-6.846
5.044	2.841	2.998	1.611	.624	3.455	2.733	1.391

add the inertias of generators 6 and 7 to get the inertia of the equivalent machine. Put the inertia of the equivalent machine in place of the inertia for generator 6 in the matrix $\hat{\underline{M}}$ and eliminate the row of $\hat{\underline{M}}$ that contains the inertia for generator 7. The result is

$$\underline{\underline{M}}_1 = \begin{bmatrix} \frac{1}{M_1} & & & & & & & & & -\frac{1}{M_{10}} \\ & \frac{1}{M_2} & & & & & & & & -\frac{1}{M_{10}} \\ & & \frac{1}{M_3} & & & & & & & -\frac{1}{M_{10}} \\ & & & \frac{1}{M_4} & & & & & & -\frac{1}{M_{10}} \\ & & & & \frac{1}{M_5} & & & & & -\frac{1}{M_{10}} \\ & & & & & \frac{1}{M_6 + M_7} & & & & -\frac{1}{M_{10}} \\ & & & & & & \frac{1}{M_8} & & & -\frac{1}{M_{10}} \\ & & & & & & & \frac{1}{M_9} & & -\frac{1}{M_{10}} \end{bmatrix}$$

The matrix $-\hat{\underline{M}}_1 \hat{\underline{T}}_1$ is then just the multiplication of the matrix $\hat{\underline{M}}_1$ above by the matrix $-\underline{T}_1$ in Table 6.4. Note that the $\underline{T}$ and $\underline{T}_1$ matrices have one less column than row. The $\underline{T}$ matrix is actually square, but the last column is always deleted because it contains redundant information (found in the last row) about the reference machine.

Returning now to Table 6.2, the arrangement of the eigenvalue data shows clearly why the aggregation scheme used by Podmore is often called an "averaged" equivalent. Each entry in row i of the table

can be thought of as a weighted average of the entries directly above it in row $i-1$. Table 6.2 can be interpreted as meaning that at each level of aggregation a pair of eigenvalues has to be eliminated. The new eigenvalues are a "best fit weighting" of the information available from the level above.

A careful look at Table 6.2 indicates that the weighting can heavily favor particular pairs of eigenvalues. For instance, the eigenvalues in level 1 of the table are all very close to the eigenvalues above to the left in level 0. That is, the eigenvalue pair with imaginary part 9.984 is essentially discarded at the first level of aggregation. Note that this is the highest frequency pair. This means that the eigenvalue pair $-\frac{\sigma}{2} \pm 9.984$ can be closely identified with the intermachine oscillations between generators 6 and 7. At other levels, as in going from aggregation level 4 to 5 the weighting is such that no single eigenvalue absolute value is discarded. That is, there is no single eigenvalue pair that can be associated, in this case, with the oscillation between group 1-8 and group 2-3. Note that at level 1, 9.984 is discarded, at level 2, 9.811 is discarded. At level 3 all the eigenvalues are retained except 8.288 and 9.542 which are averaged into 8.373. At level 4, the eigenvalue 8.314 is discarded and at level 5, 8.372. The general pattern is to always discard the eigenvalue pair of highest frequency. Since the ZMIW disturbance of all ten generators is designed to detect tightly interconnected groups, this pattern makes sense, since what should be discarded are the high frequency oscillations within the groups.

It can be expected, then, that when an eigenvalue absolute value is essentially discarded in going from level $i-1$ to level i that the resulting model at level i will be almost as good as the model at level $i-1$. If this is not the case one might expect a noticeable degradation in the model's ability to reproduce the actual system response. Specifically, for the information about the New England system contained in Table 6.2, one would expect the model to be very good through aggregation level 4, i.e. the 1-8 2-3 4-6-7 grouping, but to deteriorate at about level 5. These predictions are generally born out by the data presented in subsequent sections, but before examining those results, it is shown that the eigenvalue information of Table 6.3 is contained in the coherency measure matrix $S_X(\infty)$.

Table 6.5 shows the r.m.s. coherency measures between the reference, generator 10, and the individual machines at each level of aggregation. Generator 10 is a very "large" generator in the sense that its inertia is approximately ten times that of any other generator in the system. It is also the last generator to be aggregated so that r.m.s. coherency measures are available between it and all other equivalent generators at each level of aggregation.

A few words of explanation of how to read Table 6.5 is perhaps in order. The top row of the table contains the coherency measures between generator 10 and the other nine generators of the full New England System. That is, the number directly under (1,10) is the coherency measure between generators 1 and 10, etc.

At the first level of aggregation the remaining machines, in order, are 1,2,3,4,5,6-7,8,9,10. The entries in row two, from left

Table 6.5. Coherency Data for ZMIIW Disturbance of All Ten Generators

<u>Aggregation</u>	<u>Coherency Measures Between Generator 10 and Each System</u> <u>Generator at all Levels of Aggregation</u>									<u>Level</u>
	<u>(1,10)</u>	<u>(2,10)</u>	<u>(3,10)</u>	<u>(4,10)</u>	<u>(5,10)</u>	<u>(6,10)</u>	<u>(7,10)</u>	<u>(8,10)</u>	<u>(9,10)</u>	
None	.04591	.04888	.05051	.05616	.06520	.05418	.05653	.04635	.06214	0
6-7	.04611	.04904	.05071	.05664	.06553	.05728	.04655	.06229		1
1-8 6-7		.04759	.04916	.05084	.05679	.06563	.05743	.06245		2
1-8 4-6-7			.04804	.04955	.05129	.05964	.06653	.06281		3
1-8 2-3 4-6-7				.04817	.05222	.05979	.06663	.06288		4
1-2-3-8 4-6-7					.05208	.06043	.06706	.06360		5
1-2-3-4-6-7-8						.05957	.06647	.06494		6
1-2-3-4-5-6-7-8							.06234	.06576		7
1-2-3-4-5-6-7-8-9								.06571		8

to right, correspond to the coherency measures between generator 10 and each of the generators, in the order given. That is, the first entry on the left is the coherency measure between generator 1 and 10, the second that between 2 and 10, the sixth entry that between the aggregated generator 6-7 and 10, the seventh that between generators 8 and 10, and so on.

As another example at level 5 the generators are the aggregate 1-2-3-8, the aggregate 4-6-7, and generators 5 and 9. The four entries in this row, from left to right, correspond to the coherency measures between generators 10 and generators 1-2-3-8, 4-6-7, 5 and 9, in that order. The rule is to aggregate to the lowest machine in the group and to list the coherency measures, left to right, from the lowest numbered machine to the highest.

For the ZMIW disturbance of all 10 generators, the coherency measure information in Table 6.5 is essentially the same as the eigenvalue information in Table 6.2. For instance, the coherency measures in the level 1 aggregation are all very close to the corresponding values in level 0, with the exception of generator 6-7 which has a larger coherency measure, with generator 10, than that for either machine 6 or machine 7 at level 0. The change in the coherency measure is an order of magnitude greater for 6-7 than it is for any other machine at that level. At lower levels there is an averaging of coherency measures just as there was an averaging of eigenvalues in Table 6.2. It is more difficult to detect because the entries in Table 6.5 are not rank ordered as they were in Table 6.2.

The general interpretation of the shifts in coherency measures is the following. If two generators with nearly the same coherency

measure are aggregated the coherency measure for the aggregate is generally larger, but the behavior of the aggregate should follow that of either of the generators aggregated.

If the coherency measures of the two generators being aggregated are quite different the coherency measure of the aggregate may be close to one of the two initial coherency measures or it may be somewhere in between. If the coherency measure of the aggregate is close to one of the two initial coherency measures, the aggregate's behavior will probably be close to that of the generator whose coherency measure it favors. If the aggregate's coherency measure is somewhere in between, its behavior may follow that of neither of the generators being aggregated. This is, generally, the same line of reasoning followed in discussing eigenvalue averaging in Table 6.2.

Since each row i of Table 6.5 is generated by forming $\hat{M}_i T_i$, finding its inverse and then generating $S_{X_i}(\infty) = (\hat{M}_i T_i)^{-1} K (\hat{M}_i T_i)^{-T}$, the information in this table is, computationally, no easier to obtain than the eigenvalue information in Table 6.2. All that has been done is verify computationally, what has been shown theoretically in Chapters 3 through 5 of this present work and in references [5-8], namely that the coherency information available from the $S_X(\infty)$ matrix is the same information available from the calculation of eigenvalues.

If coherency measure information is to be useful in assessing the loss of system model accuracy at each level of aggregation, that information must come from the initial coherency calculation in the LINSM program, modified to use the r.m.s. coherency measure. This

computer program generates the matrix $(\hat{\underline{M}} \underline{T})^{-1}$ of the unaggregated system directly, i.e. without an inversion. The information available from $(\hat{\underline{M}} \underline{T})^{-1}$ is the ranking Table 6.1. Note that to the right of this table is the point at which each level of aggregation occurs. Since the numbers in the table are all of roughly the same order of magnitude there is no dramatic jump in the threshold level of the coherency measure required to reach an additional level of aggregation. Intuitively Table 6.1 seems to indicate that the decay in the quality of the system model will be fairly uniform, rather than taking dramatic jumps.

Figure 6.2a shows a plot of the magnitude of the coherency measure at each level of aggregation. Note the plateau at levels 4 and 5. This contradicts only slightly the qualitative assessment made above, and seems to indicate that there might be a drop in model validity around aggregation level 4, 5, or 6. Figure 6.2b is a similar plot for the ZMIIW disturbance of generators 1, 8, 9 and 10 discussed in the next section. It's plateau is not as distinct. These results indicate that it may be possible to detect the aggregation cut-off level directly from the ranking table.

III. The ZMIIW Disturbance of the Internal System

The internal system of the New England System consists of generators 1, 8, 9 and 10 [2]. Table 6.6 is the ranking table for a ZMIIW disturbance of these four generators.

The coherency measures thresholds for the first two levels of aggregation are both very small in absolute value and of the same order of magnitude. One would expect the system models at these two levels

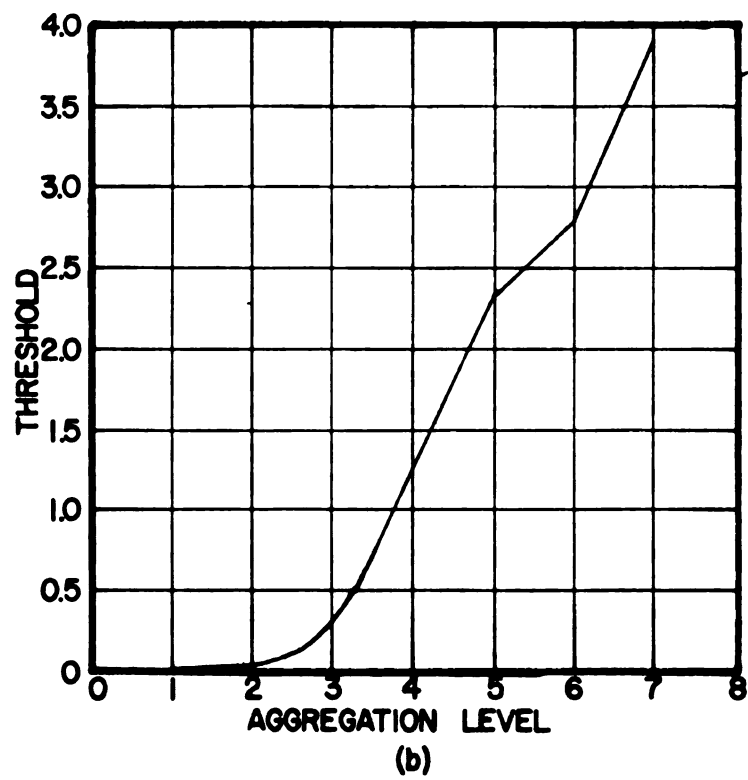
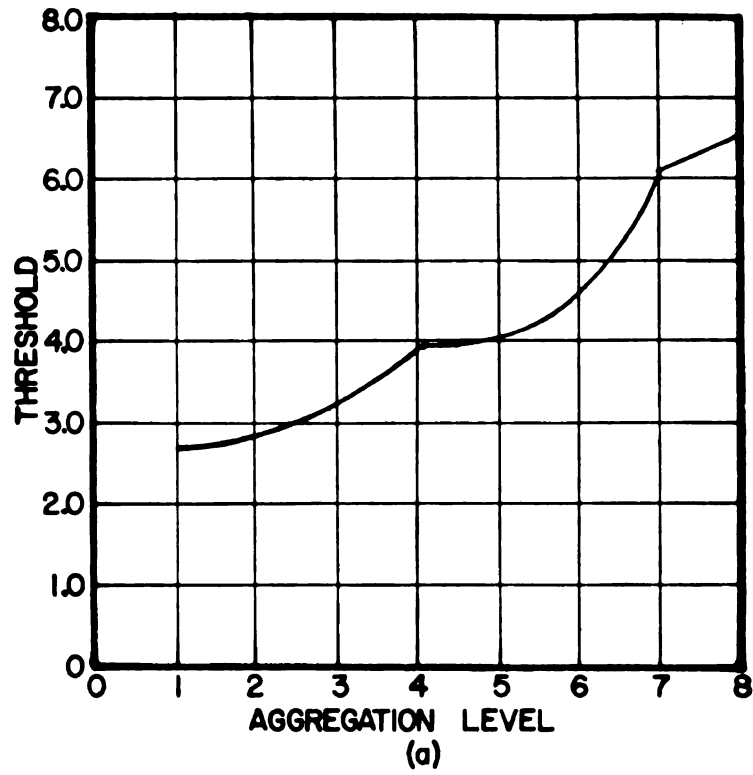


FIGURE 6-2

RELATIVE MAGNITUDE OF COHERENCY MEASURE VS. AGGREGATION LEVEL
(a) ZMIW OF ALL GENERATORS (b) ZMIW OF GENERATORS 1, 8, 9, 10

Table 6.6. Ranking of Coherency Measures for ZMIW Disturbance of Generators 1, 8, 9, 10

Ranking	Generator Pair	Coherency Measure	Aggregation Level
1.	C(6,7)	.0253	1
2.	C(4,7)	.0607	
3.	C(4,6)	.0861	
4.	C(2,3)	.3069	2
5.	C(5,6)	1.1886	
6.	C(5,7)	1.2140	
7.	C(3,4)	1.2446	3
8.	C(4,5)	1.2748	
9.	C(3,7)	1.3023	
10.	C(3,6)	1.3264	4
11.	C(2,4)	1.5513	
12.	C(2,7)	1.6090	
13.	C(2,6)	1.6332	5
14.	C(3,8)	2.3847	
15.	C(2,8)	2.4088	
16.	C(1,2)	2.4794	6
17.	C(3,5)	2.4855	
18.	C(1,3)	2.5064	
19.	C(4,8)	2.6775	7
20.	C(7,8)	2.7145	
21.	C(6,8)	2.7302	
22.	C(2,5)	2.7888	8
23.	C(1,8)	2.8385	
24.	C(1,4)	2.9826	
25.	C(1,7)	3.0262	9
26.	C(1,6)	3.0446	
27.	C(5,8)	3.5920	
28.	C(1,5)	3.9924	10
29.	C(6,9)	4.8028	
30.	C(7,9)	4.8045	
31.	C(4,9)	4.8090	11
32.	C(5,9)	4.8748	
33.	C(3,9)	5.3451	
34.	C(8,9)	5.5017	12
35.	C(2,9)	5.5044	
36.	C(1,9)	5.8884	
37.	C(2,10)	8.2068	13
38.	C(1,10)	8.3528	
39.	C(3,10)	8.5122	
40.	C(8,10)	8.7396	14
41.	C(4,10)	9.7495	
42.	C(7,10)	9.8086	
43.	C(6,10)	9.8333	15
44.	C(5,10)	10.9923	
45.	C(9,10)	11.8382	

to be very near that of the full New England System. The coherency measure threshold used to reach level 3 is 3.56 times that at level 2, but still rather small in magnitude, so that the model at level 3 could still be expected to be quite good. The coherency measure threshold for the level 4 aggregation is 4.15 times that for level 3 and 14.8 times that for level 2. In addition, its relative size is the same as that for the coherency measures of the general ZMIW disturbance of all ten generators. Therefore, a significant decay in the quality of the system model might be expected to begin about level 4. The plot in Figure 6.2b of the coherency thresholds for the levels of aggregation neither confirms nor denies this supposition.

Since the coherency measure threshold to reach levels 5,6,7 and 8 are all of the same order of magnitude as the threshold for level 4, the system model might be expected to decay rather slowly and uniformly from level 4 onwards. Essentially the same information can be derived from either the eigenvalue information of Table 6.7 or the coherency information of Table 6.8.

Table 6.7 shows that the imaginary part 9.984 is essentially discarded at level 1, and 9.543 at level 2, and 8.314 at level 3. Note, however, that the imaginary part 9.811 persists to level 4 and is finally averaged into 9.054 at level 5. This is a somewhat different pattern of eigenvalue elimination than was found previously where it was generally the highest frequency eigenvalues that were eliminated first. In this case the modes being eliminated are those not excited by the partial disturbance, which will not necessarily be the high frequency, intermachine oscillations of tightly interconnected groups.

Table 6.7. Eigenvalue Data for ZMIW Disturbance of Generators 1, 8, 9, 10

<u>Aggregation</u>	<u>Magnitudes of Imaginary Parts of System Eigenvalues for Six Levels of Aggregation</u>											<u>Level</u>
None	4.461	6.296	6.921	7.676	8.287	8.315	9.541	9.811	9.984			0
6-7	4.464	6.297	6.922	7.678	8.288	8.315	9.543	9.811				1
4-6-7		4.466	6.304	6.978	7.873	8.314	8.373	9.811				2
2-3 4-6-7		4.467	6.304	6.979	7.878	8.372	9.811					3
2-3 4-5-6-7		4.497	6.384	7.287	8.254	9.811						4
2-3-8 4-5-6-7		4.498	6.444	7.546	9.054							5
1-2-3-8 4-5-6-7		4.500	6.449	7.597								6

Table 6.8. Coherency Data for ZMIIW Disturbance of Generators 1, 8, 9, 10

<u>Aggregation</u>	<u>Coherency Measures Between Generator 10 and Each System Generator at Six Levels of Aggregation</u>											<u>Level</u>
	<u>(1,10)</u>	<u>(2,10)</u>	<u>(3,10)</u>	<u>(4,10)</u>	<u>(5,10)</u>	<u>(6,10)</u>	<u>(7,10)</u>	<u>(8,10)</u>	<u>(9,10)</u>			
None	.04591	.04888	.05051	.05616	.06520	.05418	.05653	.04634	.06214	0		
6-7	.04611	.04904	.05071	.05663	.06553	.05728	.04655	.06229		1		
4-6-7		.04657	.04944	.05117	.05950	.06643	.04701	.06265		2		
2-3 4-6-7			.04671	.05210	.05964	.06653	.04715	.06272		3		
2-3 4-5-6-7			.04731	.05264	.06355	.04775	.06320			4		
2-3-8 4-5-6-7				.04783	.05184	.06377	.06352			5		
1-2-3-8 4-5-6-7					.05262	.06426	.06407			6		

At level 5 the imaginary parts 7.546 and 9.054 are more clearly averages of 7.287, 8.254, and 9.811. At level 6, 9.054 is essentially discarded, indicating that it is representative of the oscillations between generator 1 and the aggregate 2-3-8.

Table 6.8, the coherency measure data for the ZMIW disturbance of generators 1,8,9,10 yields the same information as Table 6.7. The coherency measures at level 1 are slightly larger, but essentially the same as the coherency measures for the corresponding machines at level 0. The biggest increase, as might be expected is the coherency measure of the aggregate generator 6-7 that replaces generators 6 and 7 of level 0. The same analysis can be made of levels 2 and 3.

At level 4, the coherency measure for the aggregate 4-5-6-7 is very clearly an average of the coherency measures for generator 5 and the aggregate 4-6-7 at level 4. This could mean that the behavior of the aggregate generator 4-5-6-7 will portray the average response of all four generators, rather than favoring either generator 5 or the aggregate 4-6-7.

By contrast at level 5 the coherency measure for the aggregate generator 2-3-8 is significantly closer to that of the aggregate 2-3 at level 4 than to the coherency measure of generator 8 at level 4. Thus the behavior of the aggregate 2-3-8 may follow the behavior of generator 2 or generator 3 more closely than that of generator 8. The same analysis applies to generator 1 when it is aggregated at level 6

Figures 6.3a through 6.3f are simulations comparing the response of the full 39 Bus New England System to the system models at the first 6 levels of aggregation dictated by the ZMIW disturbance over generators

Figures 6.3. Simulations of System Response to One Per Unit Step Disturbances on Generators 8 and 9.

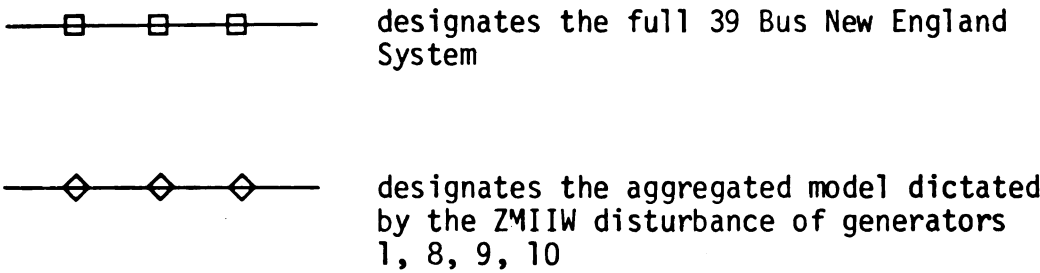
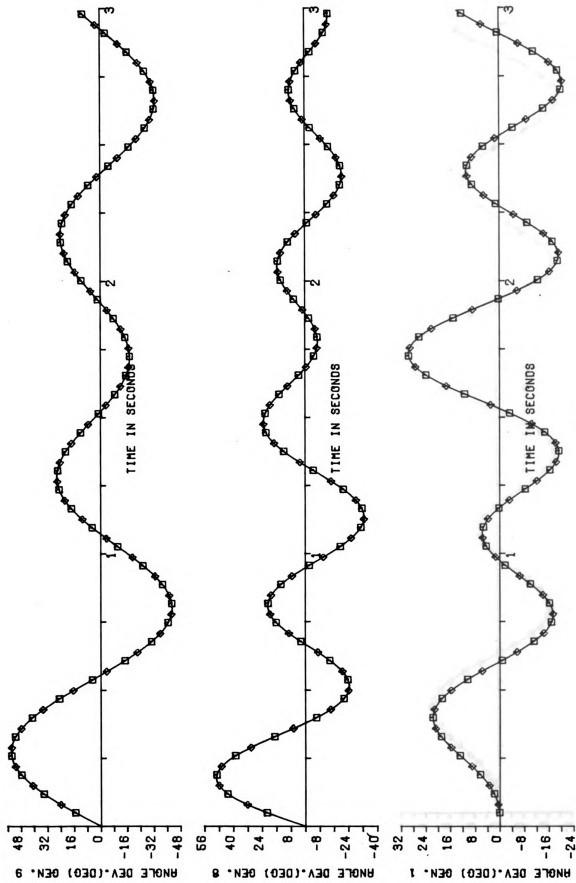
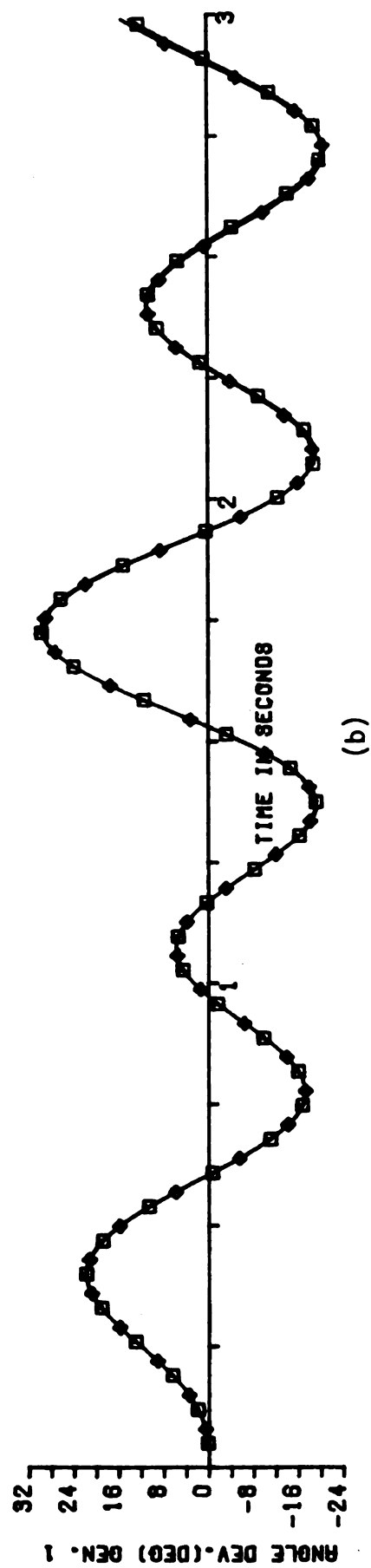
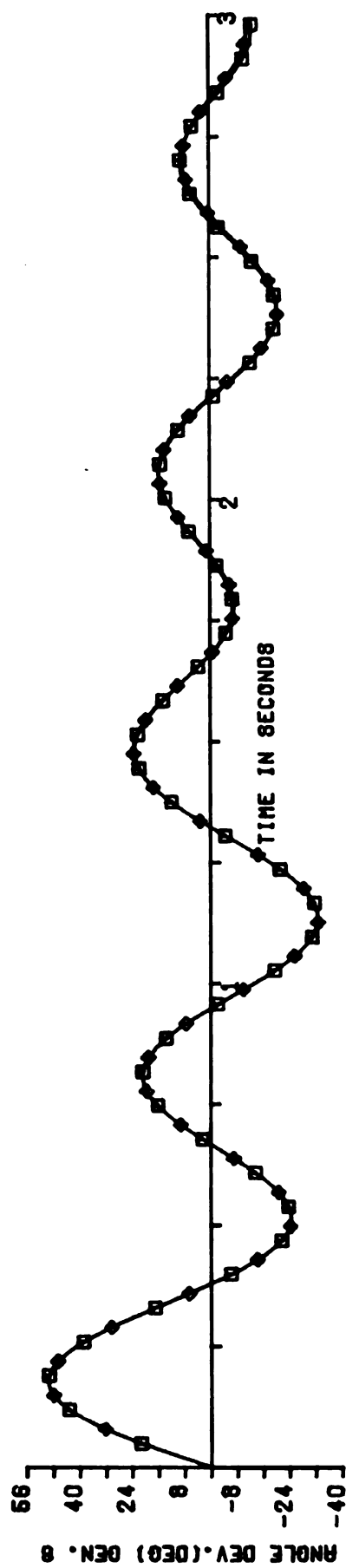
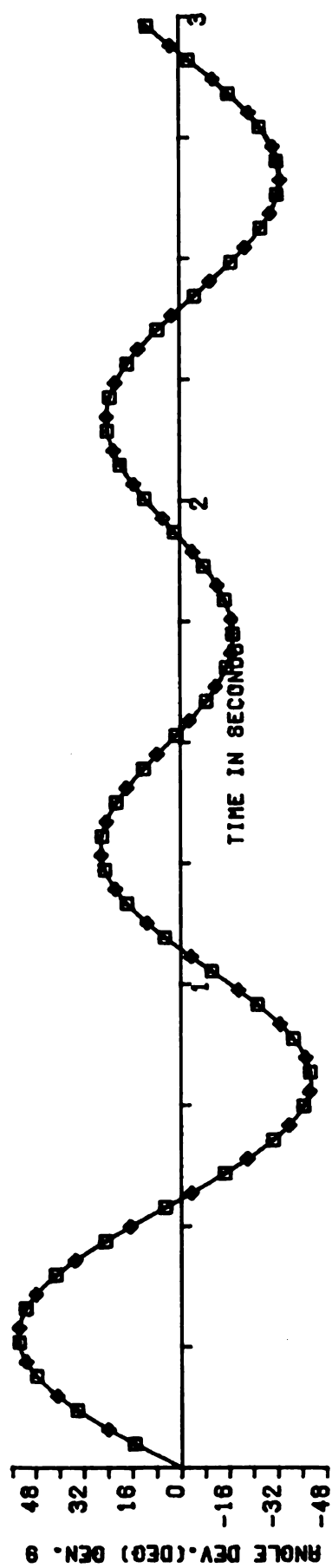


Figure 6.3	Aggregation Level	System Generators
(a)	1	1 2 3 4 5 6-7 8 9 10
(b)	2	1 2 3 4-6-7 5 8 9 10
(c)	3	1 2-3 4-6-7 5 8 9 10
(d)	4	1 2-3 4-5-6-7 8 9 10
(e)	5	1 2-3-8 4-5-6-7 9 10
(f)	6	1-2-3-8 4-5-6-7 9 10

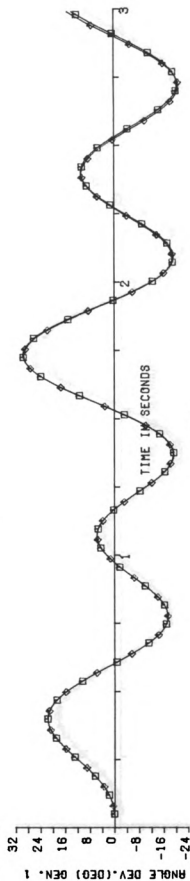
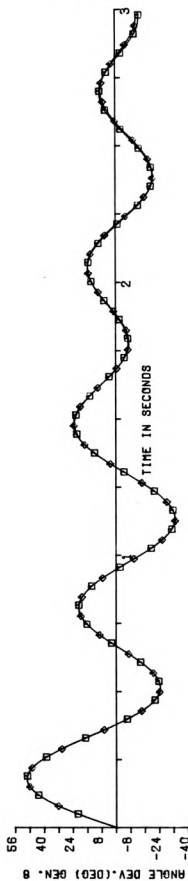
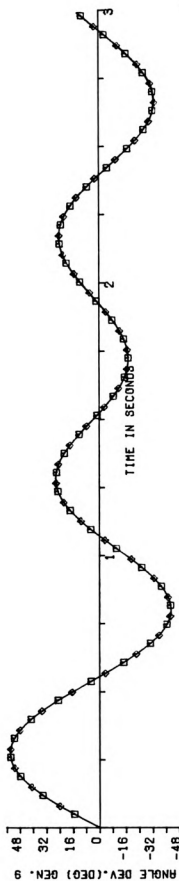
Generator 10 is the reference



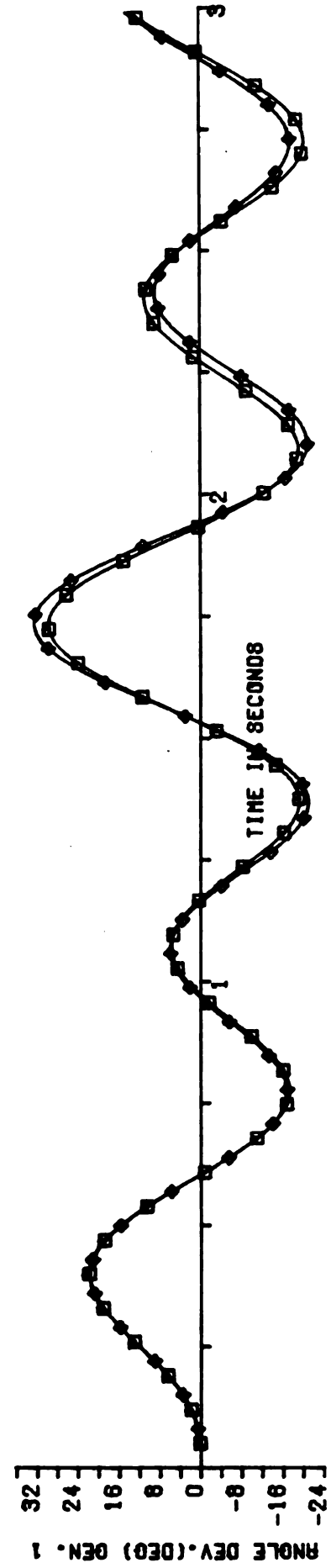
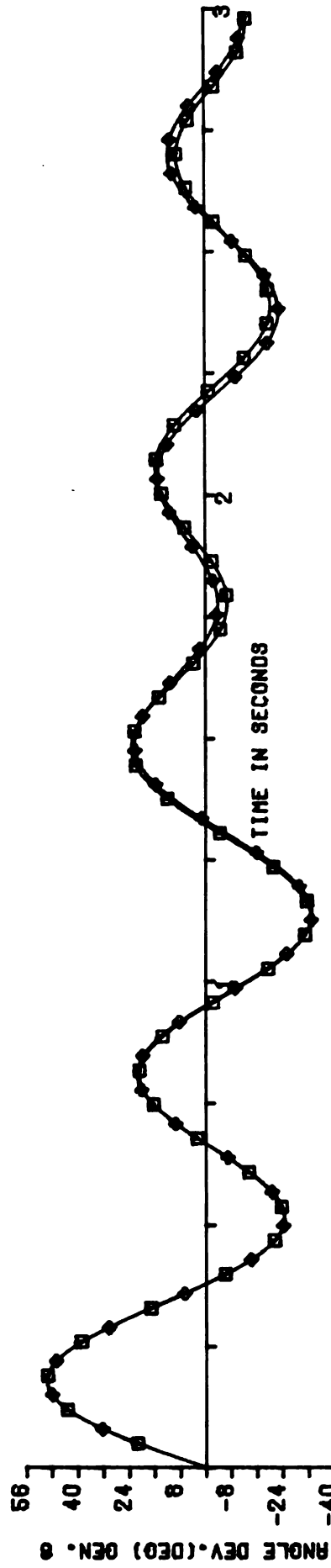
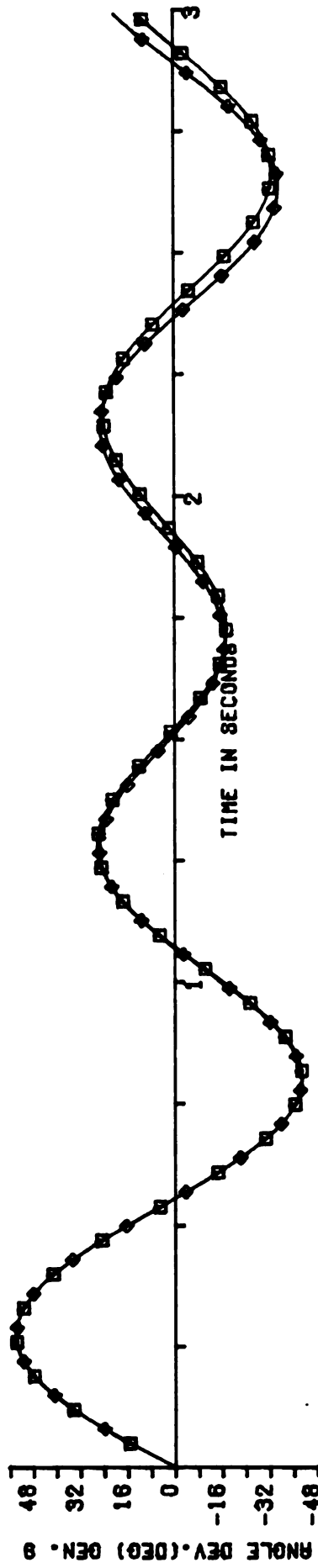
(a)



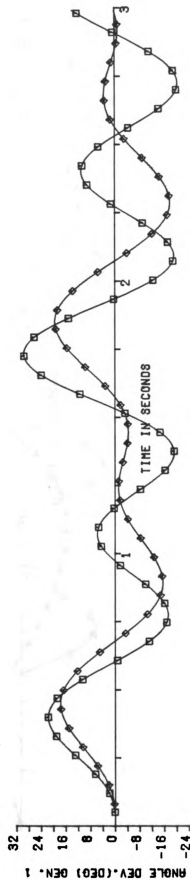
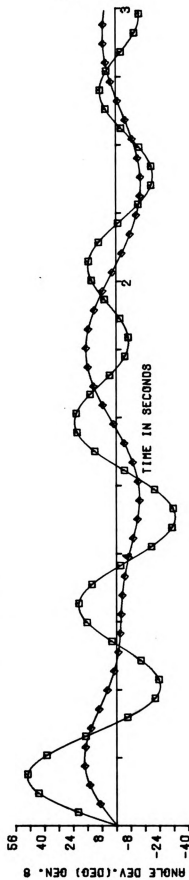
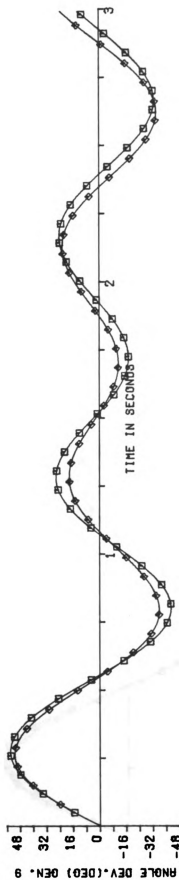
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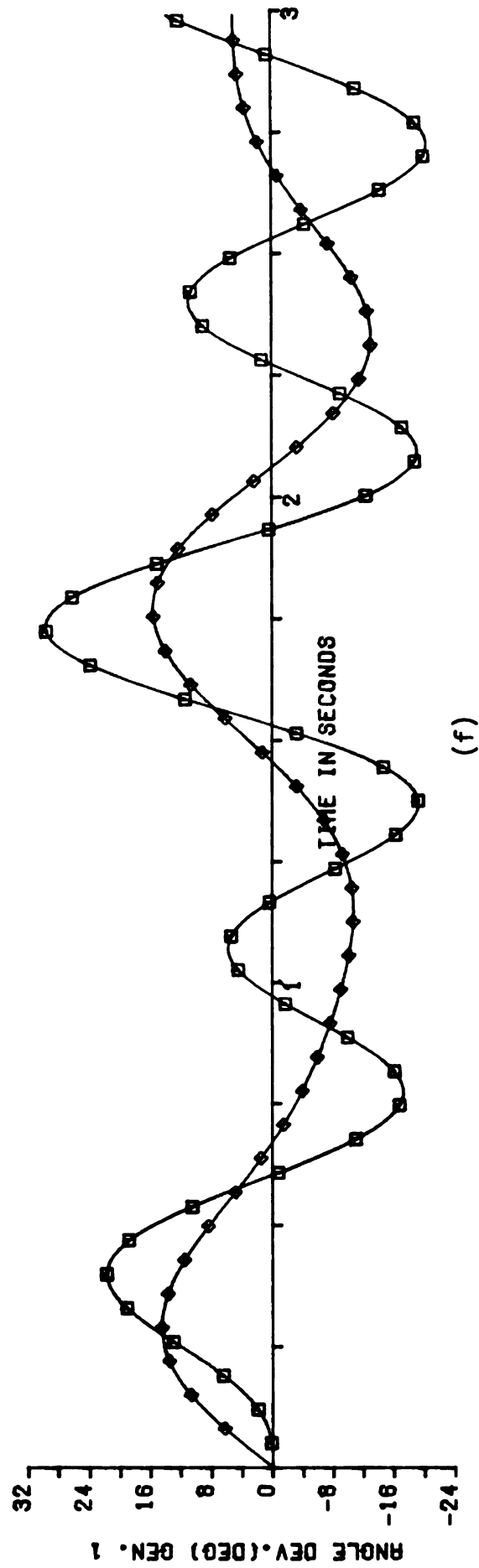
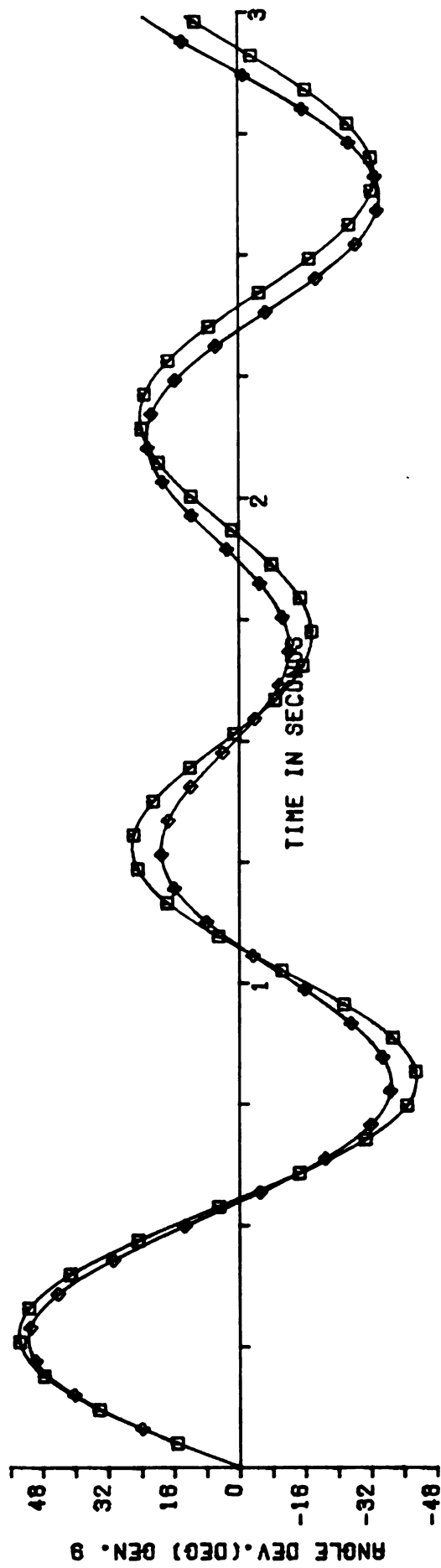
(c)



(d)



(e)



1,8,9,10. The disturbance used in the simulation is one p.u. steps on both generators 8 and 9. In all the simulation curves of this chapter the curve designated by $\square$ is the New England System and the curve designated by $\diamond$ the aggregated model.

These simulations verify the conclusions reached by analyzing the data of Tables 6.6 through 6.8. The system model starts to deviate from that of the full system at level 4, although the model at level 4 is still very good for 3 seconds.

At level 5, the response quality has decayed badly on generator 8 and is not much better on generator 1. By analyzing Table 6.8 it was predicted that the response of the aggregate generator would follow that of generator 2 or 3 more than that of generator 8. Figure 6.3e seems to verify this. The same result occurs, as hypothesized, for the aggregate 1-2-3-8 in Figure 6.3f. Note that the main degradation of the model occurs when a generator is aggregated into a group with which it is not very coherent. Note, in contrast, that the response of generator 9, is quite accurate through six levels of aggregation.

Tables 6.9, 6.10, 6.11 and Figures 6.4a, and 6.5a through 6.5f provide the standard data for a ZMIW disturbance of generators 8 and 9. The same sort of analysis performed above can be performed here, with about the same result.

Table 6.9 shows that the progression in the size of the coherency measures is almost linear through the middle aggregation levels. It takes a dramatic jump between levels 6 and 8 as shown by the plot of coherency measure threshold magnitude versus aggregation level shown in Figure 6.4a. This curve has a somewhat different shape than the corresponding curves for the previous two disturbances.

Table 6.9. Ranking of Coherency Measures for ZMIIW Disturbance of Generators 8 and 9

Ranking	Generator Pair	Coherency Measure	Aggregation Level
1.	C(6,7)	.0021	1
2.	C(4,7)	.0050	
3.	C(4,6)	.0072	2
4.	C(2,3)	.0500	3
5.	C(5,6)	.0996	
6.	C(5,7)	.1017	
7.	C(4,5)	.1068	4
8.	C(3,5)	.1346	
9.	C(1,4)	.1630	
10.	C(1,7)	.1655	
11.	C(1,6)	.1666	
12.	C(2,5)	.1833	
13.	C(3,6)	.2299	
14.	C(3,7)	.2320	
15.	C(1,5)	.2338	5
16.	C(3,4)	.2369	
17.	C(2,6)	.2797	
18.	C(2,7)	.2818	
19.	C(2,4)	.2868	
20.	C(1,3)	.3229	
21.	C(1,2)	.3693	6
22.	C(2,10)	.6290	
23.	C(3,10)	.6789	
24.	C(5,10)	.8079	
25.	C(6,10)	.9068	
26.	C(7,10)	.9090	
27.	C(4,10)	.9140	
28.	C(1,10)	.9878	7
29.	C(1,8)	2.0472	
30.	C(4,8)	2.2072	
31.	C(7,8)	2.2104	
32.	C(6,8)	2.2118	
33.	C(5,8)	2.2764	
34.	C(3,8)	2.3241	
35.	C(2,8)	2.3549	
36.	C(8,10)	2.8131	8
37.	C(4,9)	4.7526	
38.	C(7,9)	4.7569	
39.	C(6,9)	4.7587	
40.	C(1,9)	4.7694	
41.	C(5,9)	4.8421	
42.	C(3,9)	4.9745	
43.	C(2,9)	5.0197	
44.	C(8,9)	5.0326	
45.	C(9,10)	5.5806	

Table 6.10. Eigenvalue Data for ZMIW Disturbance of Generators 8 and 9

<u>Aggregation</u>	<u>Magnitudes of Imaginary Parts of System Eigenvalues at Six Levels of Aggregation</u>											<u>Level</u>
None	4.461	6.296	6.921	7.676	8.287	8.315	9.541	9.811	9.984			0
6-7	4.464	6.297	6.922	7.678	8.288	8.315	9.542	9.811				1
4-6-7		4.466	6.304	6.978	7.873	8.314	8.373	9.811				2
2-3 4-6-7			4.467	6.304	6.979	7.879	8.372	9.811				3
2-3 4-5-6-7				4.497	6.384	7.287	8.254	9.811				4
1-4-5-6-7 2-3					4.576	6.475	7.447	9.384				5
1-2-3-4-5-6-7						4.589	6.493	9.358				6

Table 6.11. Coherency Measure Data for ZMIW Disturbance of Generators 8 and 9

<u>Aggregation</u>	<u>Coherency Measures between Generator 10 and Each System Generator at Six Levels of Aggregation</u>									<u>Level</u>
	<u>(1,10)</u>	<u>(2,10)</u>	<u>(3,10)</u>	<u>(4,10)</u>	<u>(5,10)</u>	<u>(6,10)</u>	<u>(7,10)</u>	<u>(8,10)</u>	<u>(9,10)</u>	
None	.04591	.04888	.05051	.05616	.06520	.05418	.05653	.04635	.06214	0
6-7	.04611	.04904	.05071	0.5663	.06553	.05728	.04655	.06229		1
4-6-7		.04657	.04944	.05117	.05950	.06643	.04701	.06265		2
2-3 4-6-7		.04671	.05210	.05964	.06653	.04715	.06272			3
2-3 4-5-6-7			.04731	.05264	.06335	.04775	.06320			4
1-4-5-6-7 2-3				.05933	.05260	.04986	.06373			5
1-2-3-4-5-6-7					.06155	.05151	.06460			6

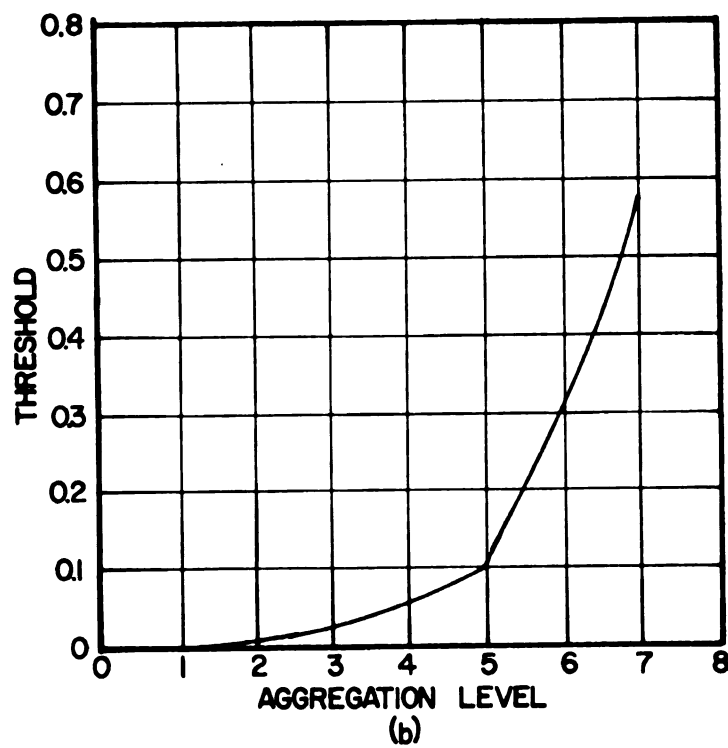
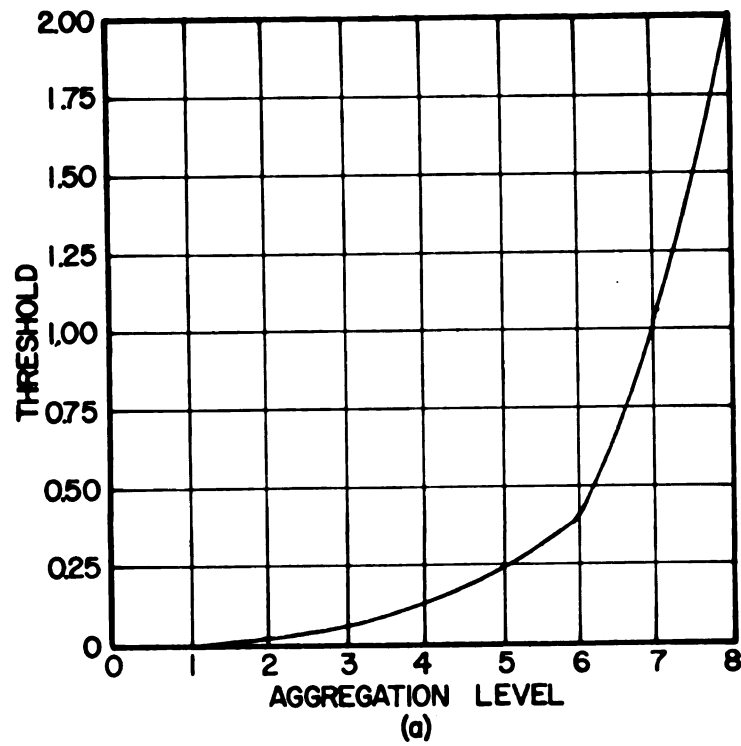


FIGURE 6-4

RELATIVE MAGNITUDE OF COHERENCY MEASURE VS. AGGREGATION LEVEL
(a) ZMIIW OF GENERATORS 8,9 (b) ZMIIW OF GENERATOR 8

Figures 6.5. Simulations of System Response to One Per Unit Step Disturbances on Generators 8 and 9



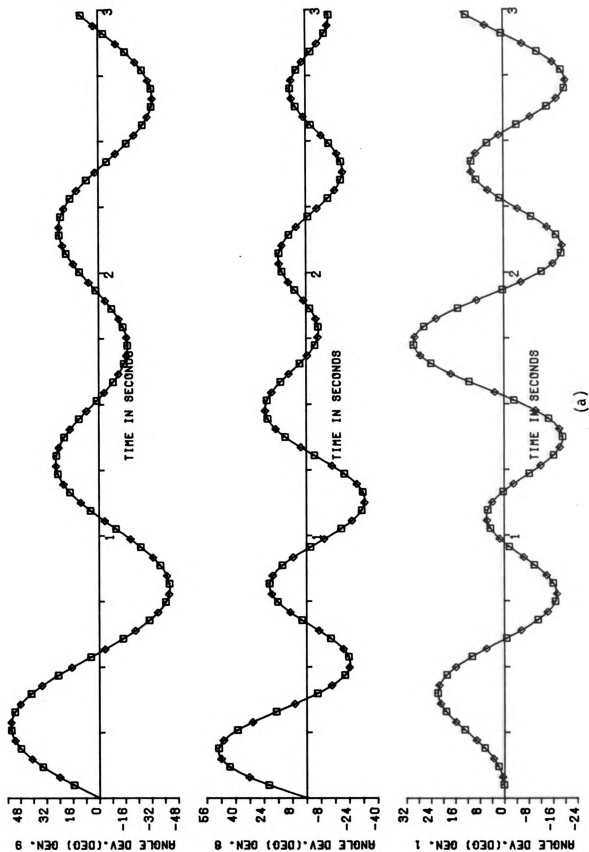
designates the full 39 Bus New England System

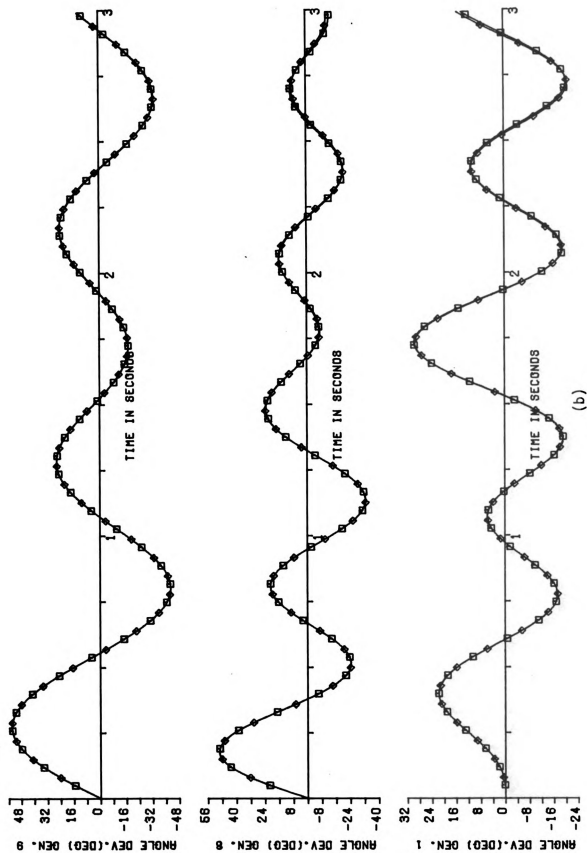


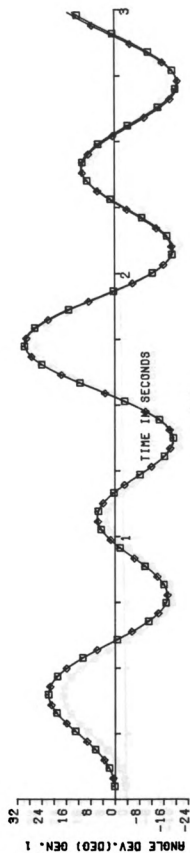
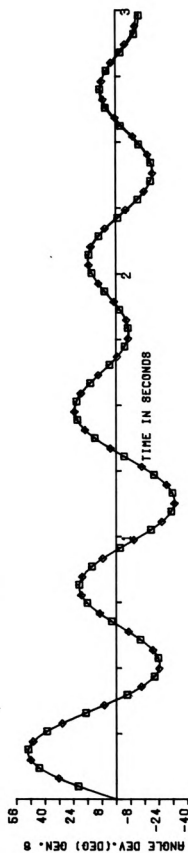
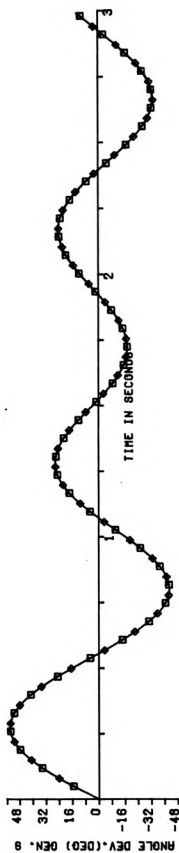
designates the aggregated model dictated by the ZMIIW disturbance of generators 8 and 9

Figure 6.5	Aggregation Level	System Generators									
(a)	1	1	2	3	4	5	6-7	8	9	10	
(b)	2	1	2	3	4-6-7	5		8	9	10	
(c)	3	1	2-3		4-6-7	5		8	9	10	
(d)	4	1	2-3		4-5-6-7			8	9	10	
(e)	5	1-4-5-6-7		2-3				8	9	10	
(f)	6	1-2-3-4-5-6-7						8	9	10	

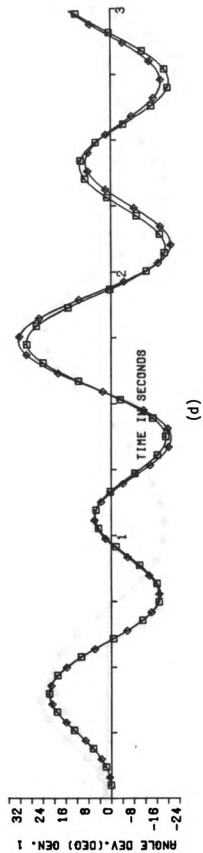
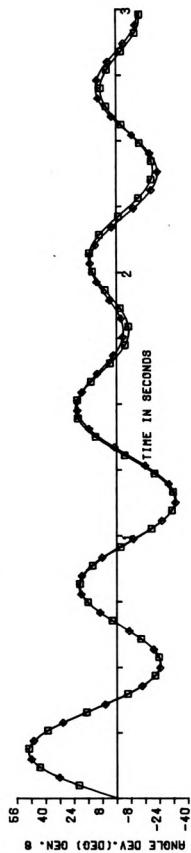
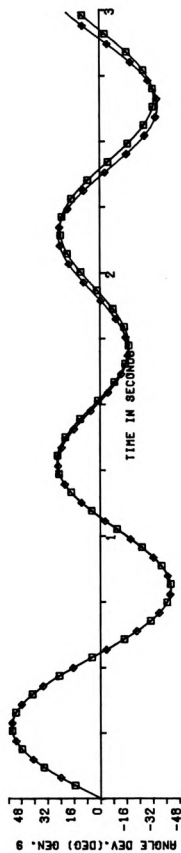
Generator 10 is the reference



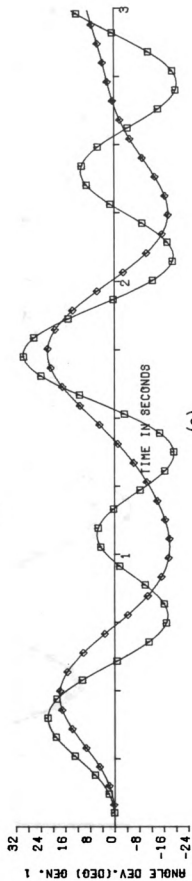
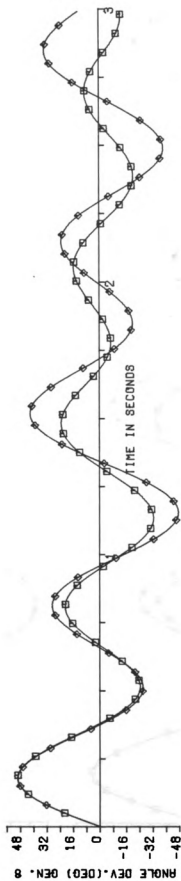
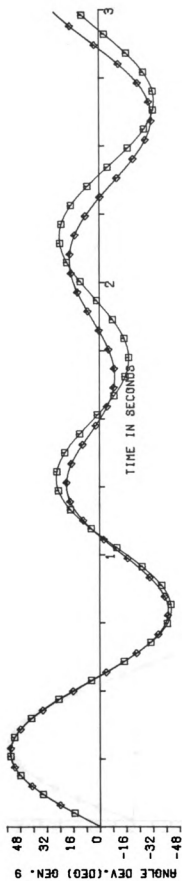




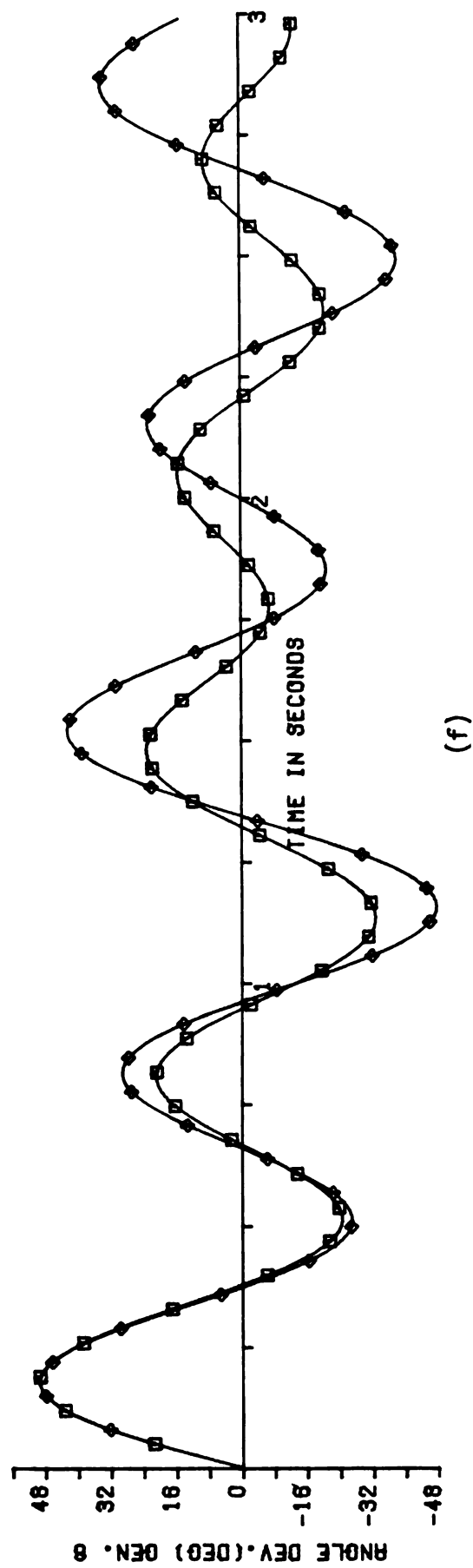
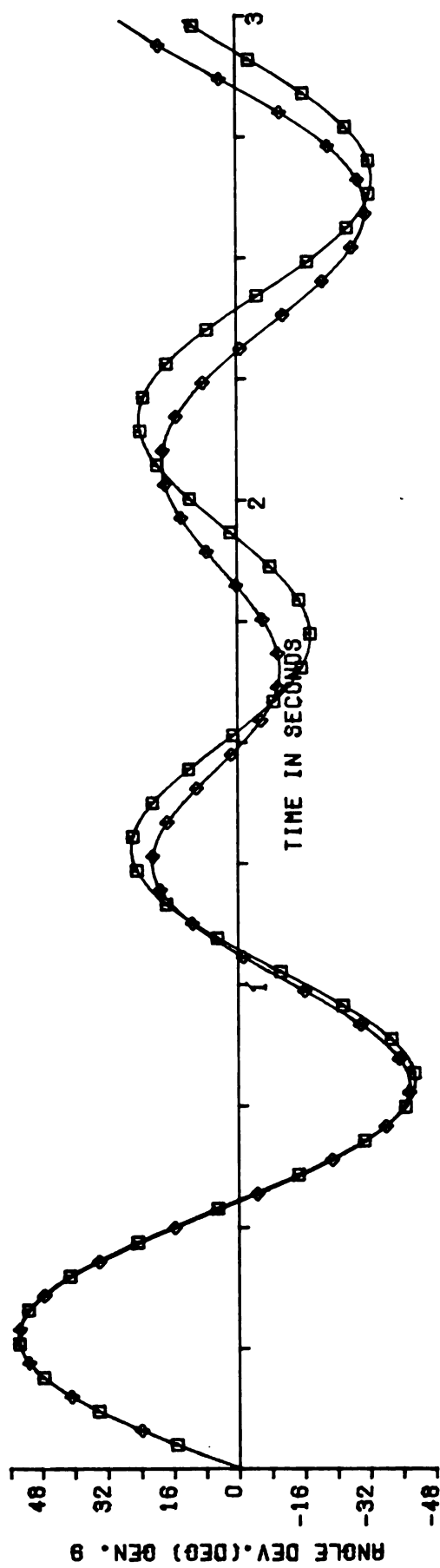
(c)



(d)



(e)



This same information is available from the eigenvalue data of Table 6.10. Note that the imaginary parts 4.589 and 9.358 at level 6 are fairly close to the 4.461 and 9.541 values of level 0. The value 6.493 at level 6 appears to be an equal weighting of the values 6.296 and 6.921 at level 0.

The analysis of the data in Table 6.11 is almost a repeat of that done for Table 6.10. Note in particular that when generator 1 is added to 4-5-6-7 at level 5, that the resulting coherency measure for the aggregate 1-4-5-6-7 is much closer to the measure for 4-5-6-7 at level 4 than to the measure for generator 1 at level 4. This seems to indicate that the behavior of the aggregate 1-4-5-6-7 will be closer to that of 4-5-6-7 than to that of generator 1.

The main reason for including the data for the ZMIW disturbance of generators 8 and 9, can be seen by comparing the eigenvalue imaginary parts retained at levels of 5 and 6 of Tables 6.7 and 6.10. The aggregations in these two tables are identical through level 4. At level 5, however the 8,9 disturbance retains a slightly higher frequency eigenvalue, 9.384 versus 9.054 for the 1,8,9,10 disturbance. The 9.054 eigenvalue is discarded at level 6 by the 1,8,9,10 disturbance when generator 1 was added to the group 2-3-8. This, as stated earlier, identifies the eigenvalue pair $-\frac{\sigma}{2} \pm j 9.054$ with the oscillations between generators 1 and 8. Now note that for the 8,9 disturbance generator 1 is added to the group 4-5-6-7 at level 5 and the group 2-3 is added to the group 1-4-5-6-7 at level 6. That is, generator 1 is not added to a group containing generator 8 and this is reflected by the retention through level 6 of the eigenvalue pair $-\frac{\sigma}{2} \pm j 9.358$

which represents the oscillations between generators 1 and 8. The eigenvalue retention is clearly dependent on the geography of the partial ZMIW disturbance. Thus applying the commutative aggregation rule to the r.m.s. coherency ranking table seems to provide selective eigenvalue retention, based on the geography of the partial ZMIW disturbance. This is a rather remarkable result.

The 8,9 disturbance also touches on another issue, namely the question of disturbing part of a tightly interconnected group. The ZMIW disturbance of all ten generators showed generators 1 and 8 to be tightly interconnected. Tables 6.7 and 6.10 both show that the greatest averaging of eigenvalues occurs when one of these generators is aggregated into a group that does not contain the other. More will be said about this in the next section.

Figures 6.5a through 6.5f are the simulation results for one per unit step disturbances on generators 8 and 9 for the first six levels of aggregation as dictated by the ZMIW disturbance of generators 8 and 9. The simulation results show that the system model response has its first noticeable decay at level 4. The level 5 and level 6 models are marginally better than those at the same level for the ZMIW disturbance of buses 1,8,9,10. But it is clear from Figure 6.5e that the degradation is pronounced once generator 1 is aggregated with 4-5-6-7. This speaks directly to the matter of disturbing only some of the generators in a tightly interconnected group. It is the reason why the reduction algorithm contains a step in which the boundary between the internal system and buffer zone is redefined in order to avoid disturbing only part of a group of generators that are tightly interconnected and cross the boundary between the internal

system and the buffer zone. Recall that the ZMIW disturbance of all ten generators showed generators 1 and 8 to be tightly interconnected at aggregation level 2. If generators 8 and 9 are considered to be the internal system then, then group 1-8 crosses the boundary between internal system and the buffer zone. The consequences of forming equivalents under these conditions are explored in greater detail in the next section.

IV. ZMIW Disturbances of Buses 1 and Bus 8

Tables 6.12, 6.13, 6.14, Figure 6.4b and Figures 6.6a through 6.6f provide the standard information for a ZMIW disturbance of generator 8. An examination of Figures 6.6a through 6.6d shows that the quality of the system models response decays rapidly at level 4 when generator 1 is aggregated with generator 9. A look at Table 6.13 shows that the absolute values of the eigenvalue imaginary parts at level 4 are truly averages of those at level 3. This means that there is no eigenvalue pair that can be associated closely with the intermachine oscillation of generators 1 and 9. This is born out by Table 6.14 which shows that the coherency measures for generators 1 and 9 are significantly different from each other.

All of this analysis seems to indicate that the process of re-defining the boundary between the internal system and buffer zone is a necessary step in the reduction algorithm for producing dynamic equivalents. This conclusion is supported by a similar analysis for ZMIW disturbances of generators 8 and 10, and generator 9. However, there are cases where the aggregation does seem to work, even though a tightly interconnected group has been broken up.

Table 6.12. Coherency Measure Ranking for ZMIIW Disturbance at Generator 8

Ranking	Generator Pair	Coherency Measure	Aggregation Level
1.	C(6,7)	.0012	1
2.	C(4,7)	.0029	
3.	C(4,6)	.0041	2
4.	C(2,3)	.0235	3
5.	C(1,9)	.0268	4
6.	C(3,5)	.0300	
7.	C(2,5)	.0536	5
8.	C(5,6)	.0573	
9.	C(5,7)	.0585	
10.	C(4,5)	.0615	
11.	C(3,6)	.0873	
12.	C(3,7)	.0886	
13.	C(3,4)	.0915	
14.	C(2,6)	.1109	
15.	C(2,7)	.1121	
16.	C(2,4)	.1151	6
17.	C(4,9)	.1351	
18.	C(7,9)	.1380	
19.	C(6,9)	.1393	
20.	C(1,4)	.1620	
21.	C(1,7)	.1649	
22.	C(1,6)	.1661	
23.	C(5,9)	.1966	
24.	C(1,5)	.2235	
25.	C(3,9)	.2267	
26.	C(2,9)	.2502	
27.	C(1,3)	.2535	
28.	C(1,2)	.2771	7
29.	C(2,10)	.3272	
30.	C(3,10)	.3508	
31.	C(5,10)	.3808	
32.	C(6,10)	.4382	
33.	C(7,10)	.4394	
34.	C(4,10)	.4423	
35.	C(9,10)	.5775	8
36.	C(1,10)	.6044	
37.	C(1,8)	2.0392	
38.	C(8,9)	2.0661	
39.	C(4,8)	2.2012	
40.	C(7,8)	2.2042	
41.	C(6,8)	2.2054	
42.	C(5,8)	2.2628	
43.	C(3,8)	2.2928	
44.	C(2,8)	2.3164	
45.	C(8,10)	2.6436	

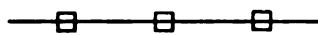
Table 6.13. Eigenvalue Data for ZIIW Disturbance of Generator 8

<u>Aggregation</u>	<u>Magnitudes of Imaginary Parts of System Eigenvalues at Six Levels of Aggregation</u>											<u>Level</u>
None	4.461	6.297	6.921	7.676	8.287	8.315	9.541	9.811	9.984			0
6-7		4.464	6.297	6.922	7.678	8.288	8.315	9.542	9.811			1
4-6-7			4.466	6.304	6.979	7.873	8.314	8.373	9.811			2
2-3 4-6-7				4.467	6.304	6.979	7.879	8.372	9.811			3
1-9 2-3 4-6-7					4.520	6.823	7.367	8.059	9.530			4
1-9 2-3-5 4-6-7						4.598	7.274	7.965	9.530			5
1-9 2-3-4-5-6-7							4.601	7.346	9.529			6

Table 6.14. Coherency Data for ZMIIW Disturbance of Generator 8

<u>Aggregation</u>	<u>Coherency Measures between Generator 8 and Each System Generator at Six Levels of Aggregation</u>										<u>Level</u>
	(1,8)	(2,8)	(3,8)	(4,8)	(5,8)	(6,8)	(7,8)	(8,9)	(8,10)		
None	.01473	.02077	.02075	.02040	.03051	.01995	.02092	.02857	.04635	0	
6-7	.01473	.02077	.02075	.02061	.03059	.02274	.02857	.04655		1	
4-6-7	.01473	.02078	.02075	.02369	.03076	.02857	.04701			2	
2-3 4-6-7	.01473	.02225	.02369	.03075	.02857	.04715				3	
1-9 2-3 4-6-7	.01693	.02246	.02369	.03075	.04760					4	
1-9 2-3-5 4-6-7	.01693	.02385	.02302	.04751						5	
1-9 2-3-4-5-6-7	.01693	.02546	.04906							6	

Figures 6.6. Simulations of System Response to a One Per Unit Step Disturbance on Generator 8



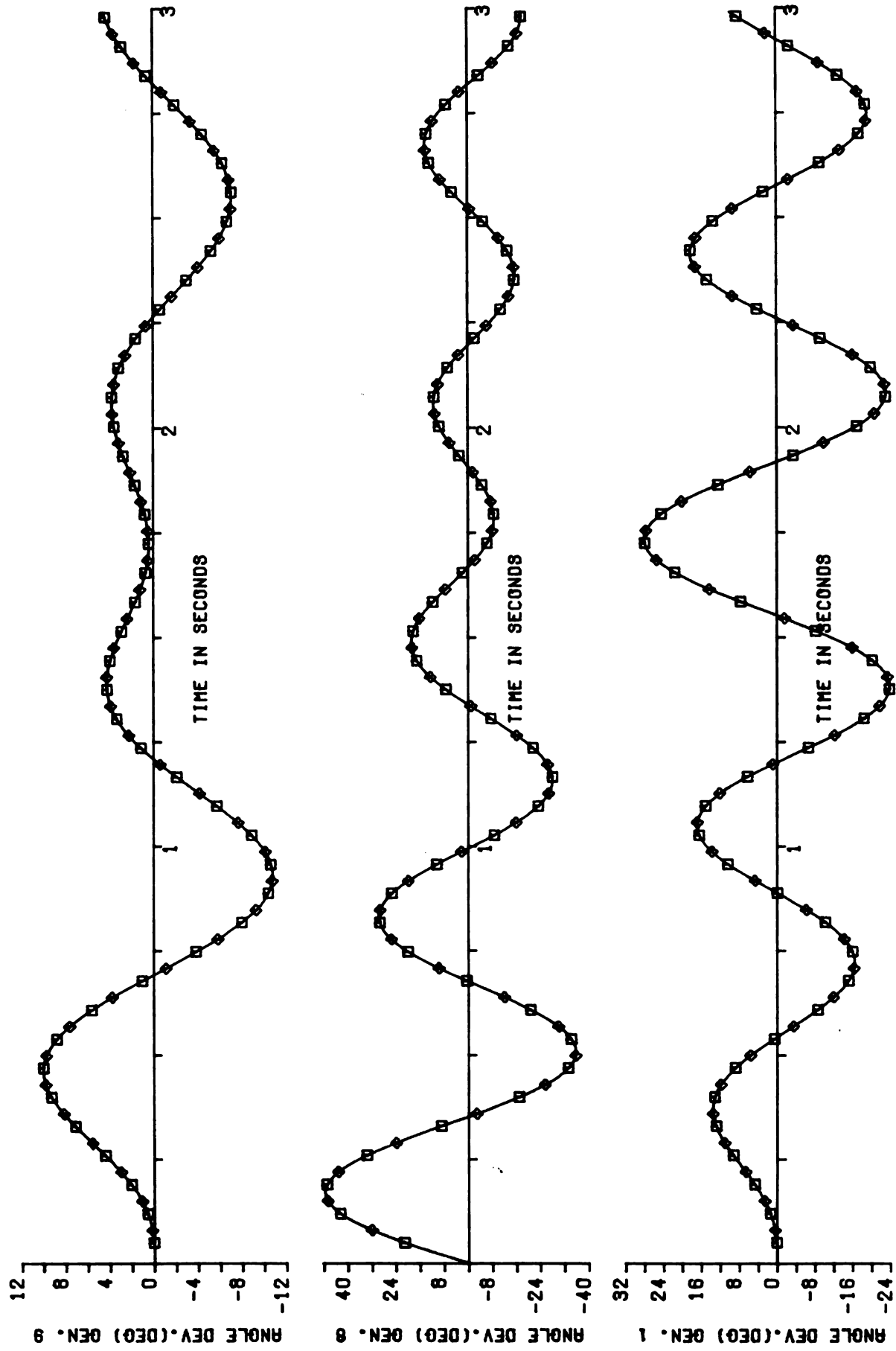
designates the full 39 Buss New England System



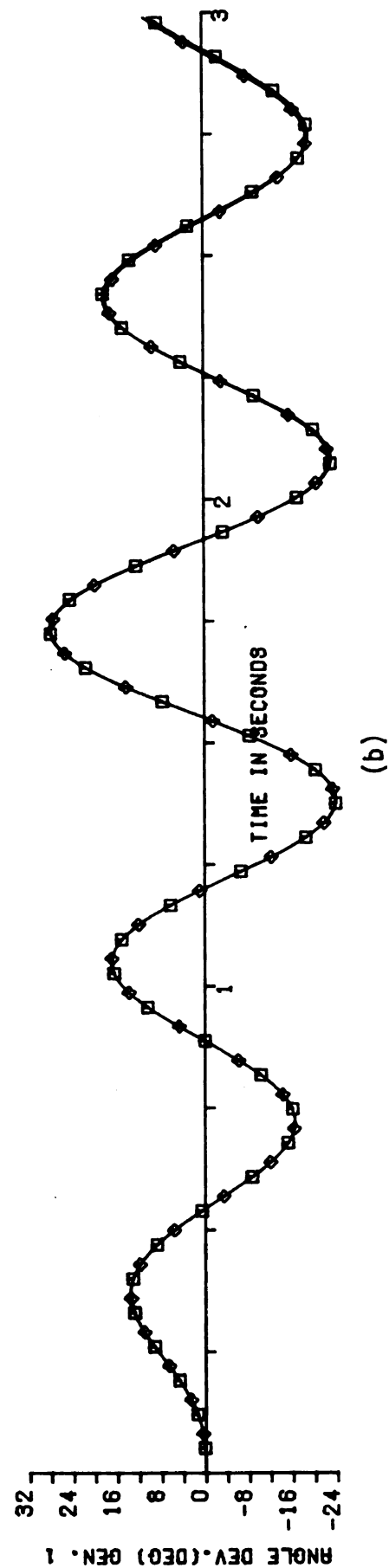
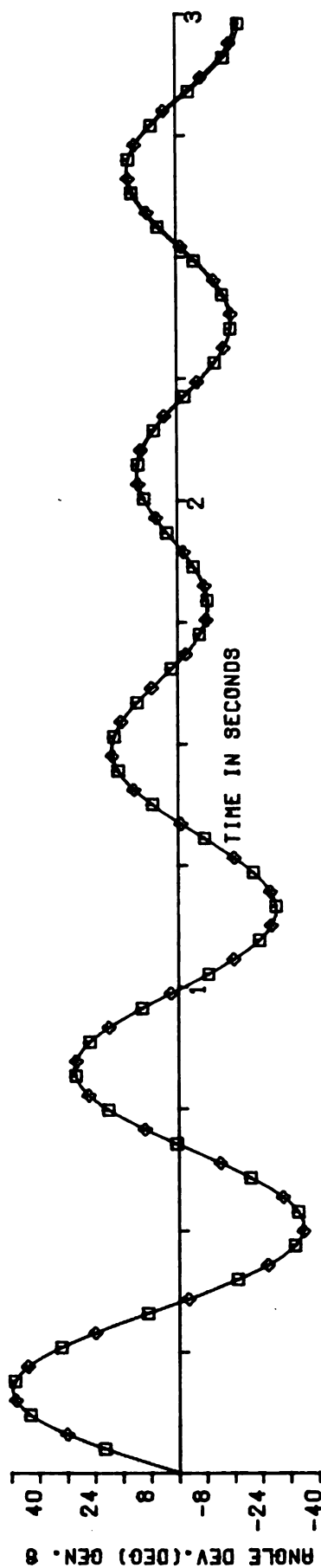
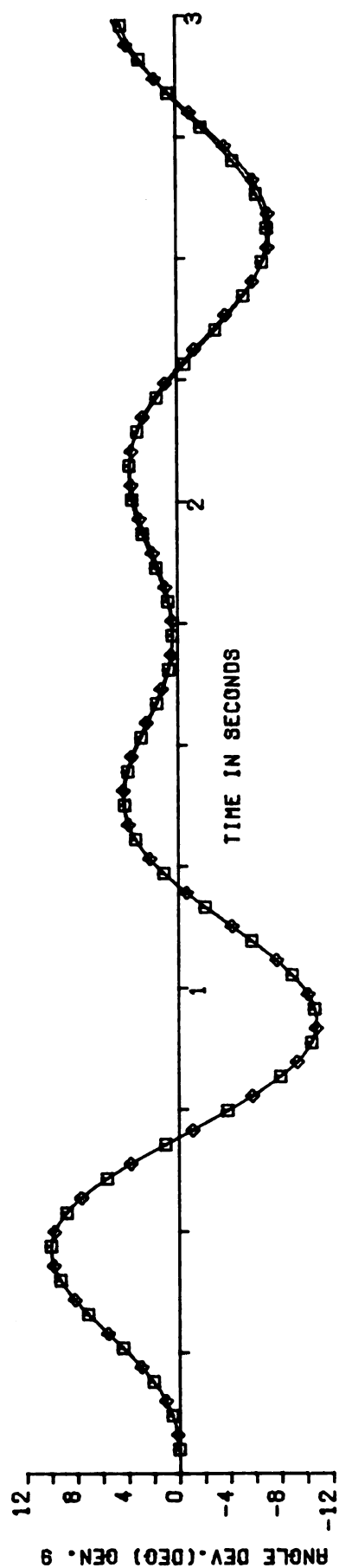
designates the aggregated model dictated by the ZMIW disturbance of generator 8

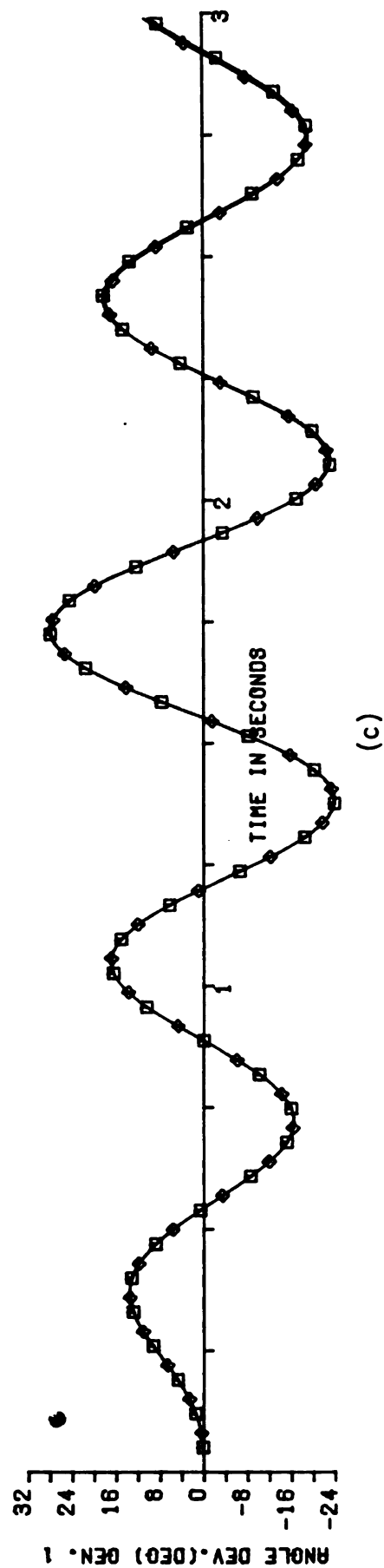
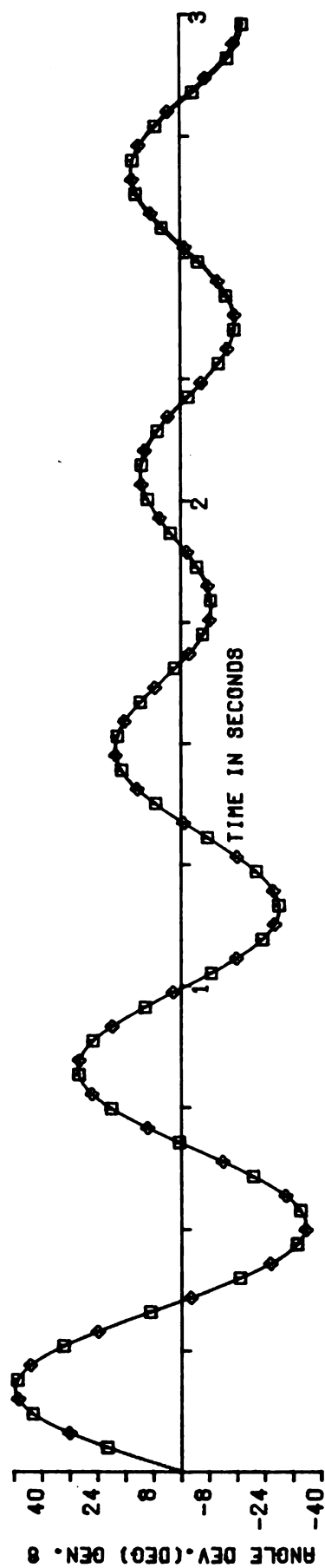
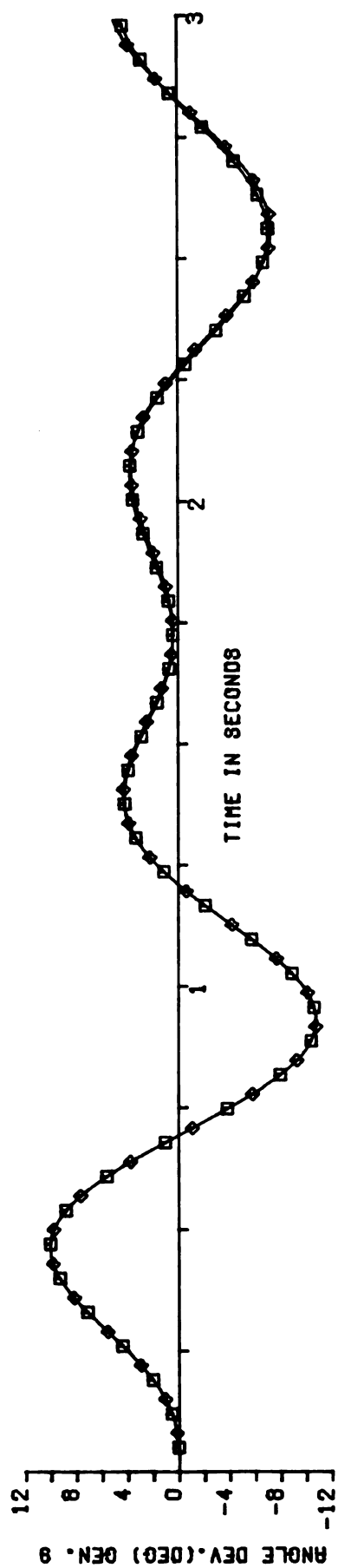
Figure 6.6	Aggregation Level	System Generators								
(a)	1	1	2	3	4	5	6-7	8	9	10
(b)	2	1	2	3	4-6-7	5	8	9	10	
(c)	3	1	2-3	4-6-7	5	8	9	10		
(d)	4	1-9	2-3	4-6-7	5	8	10			
(e)	5	1-9	2-3-5	4-6-7	8	10				
(f)	6	1-9	2-3-4-5-6-7	8	10					

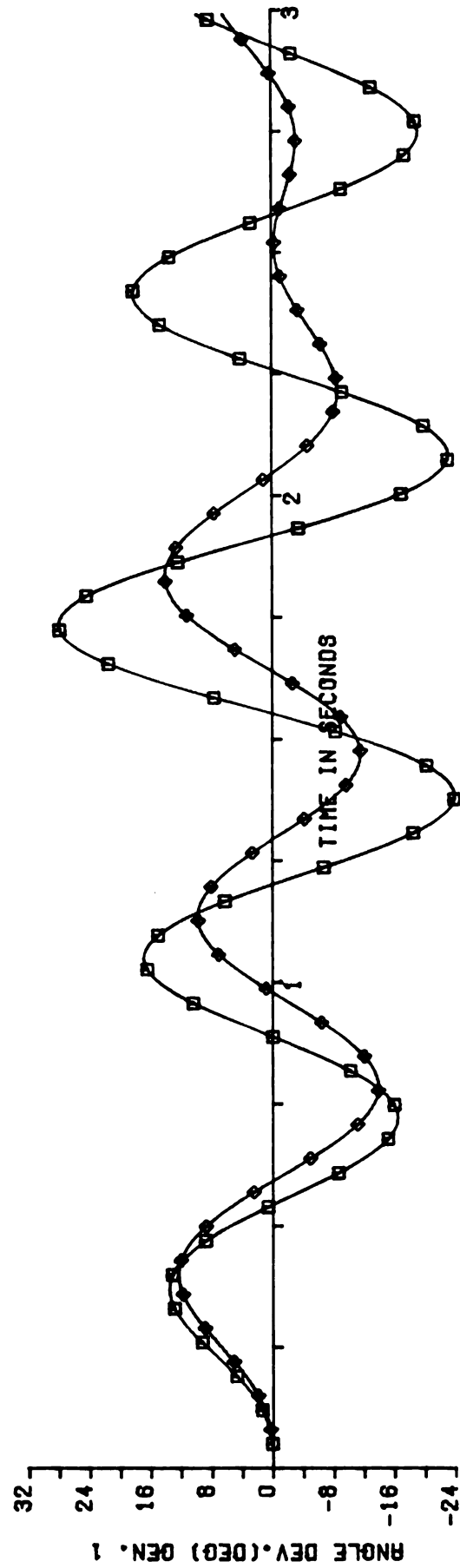
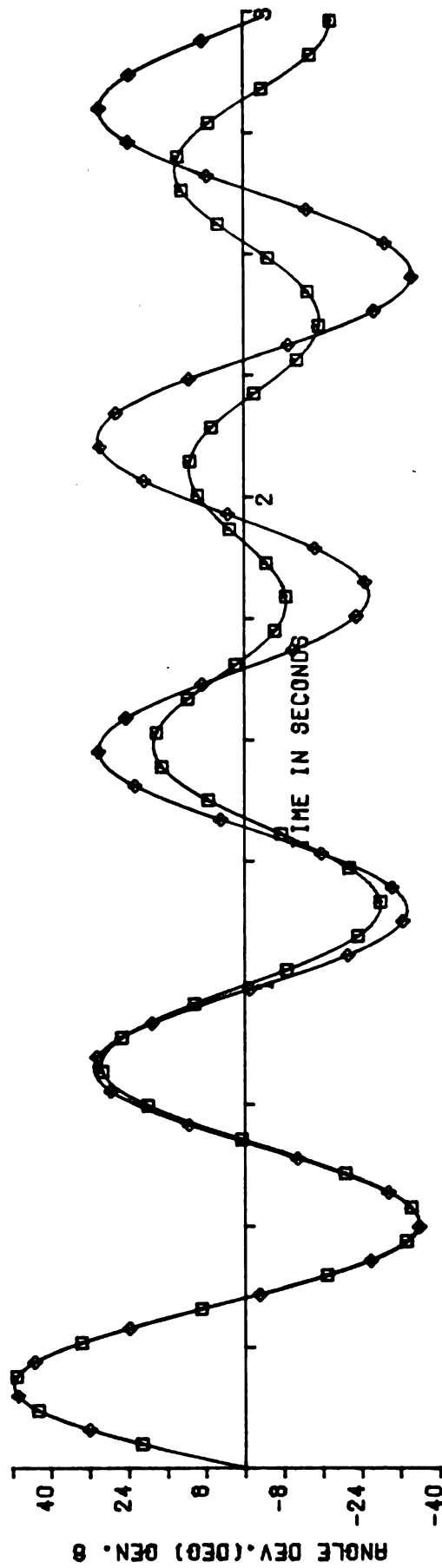
Generator 10 is the reference



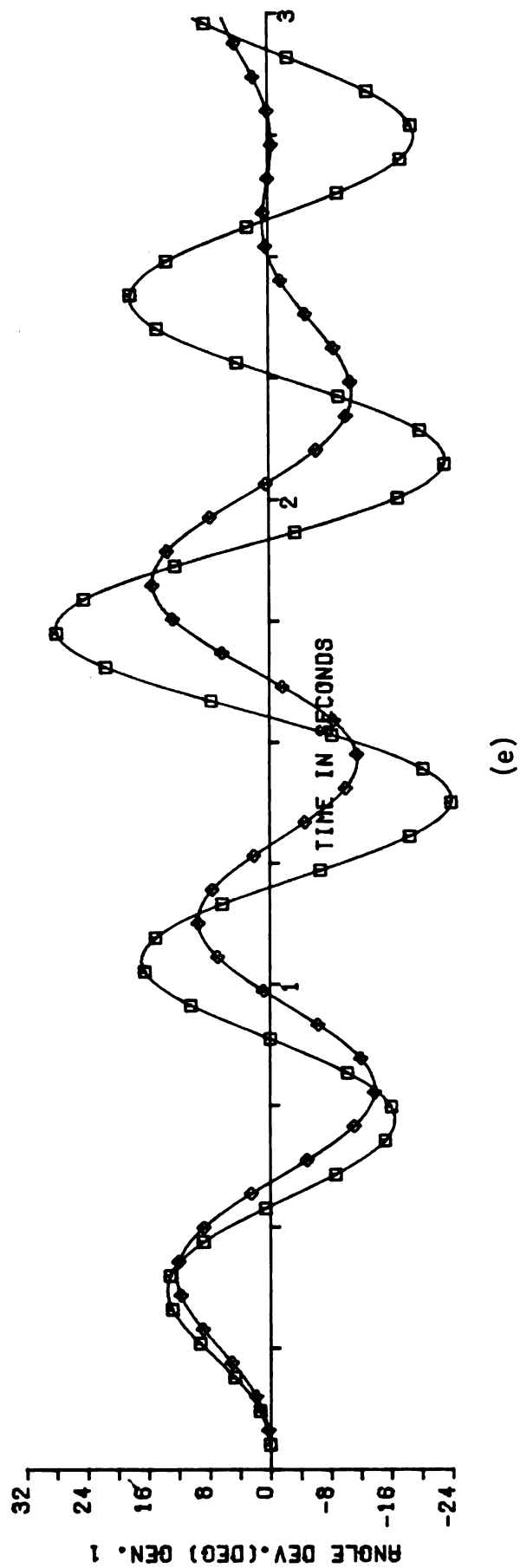
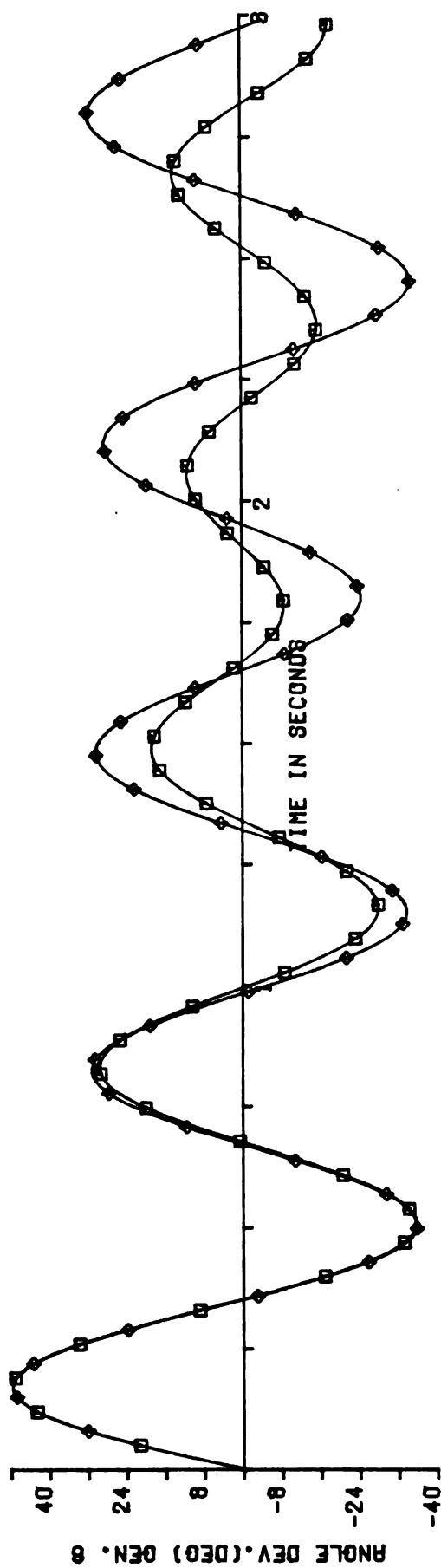
(a)

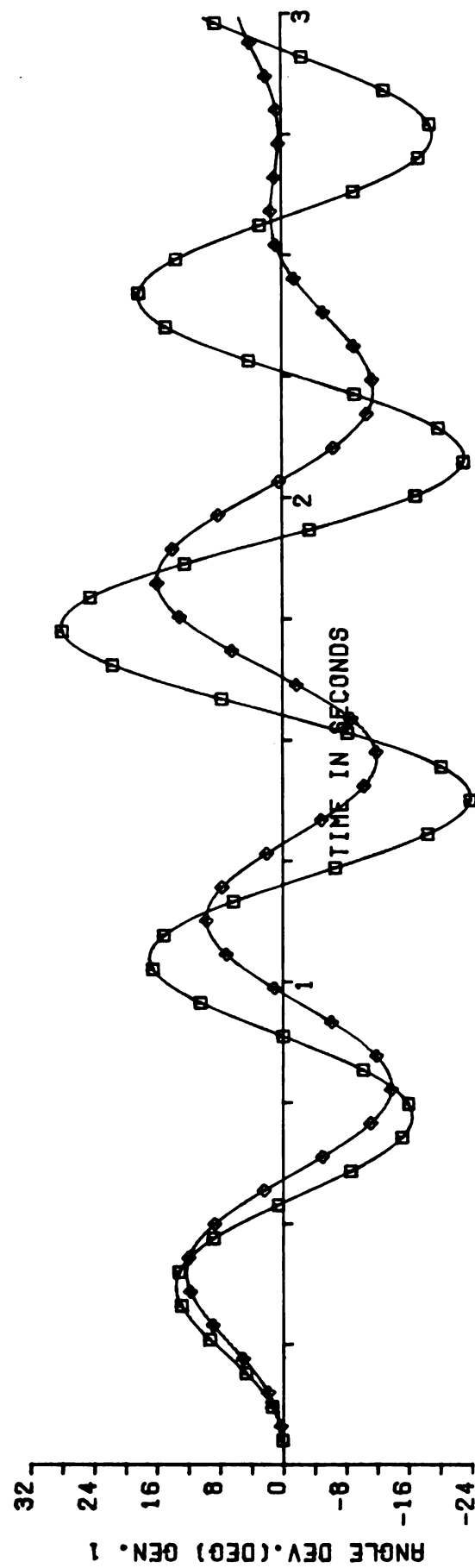
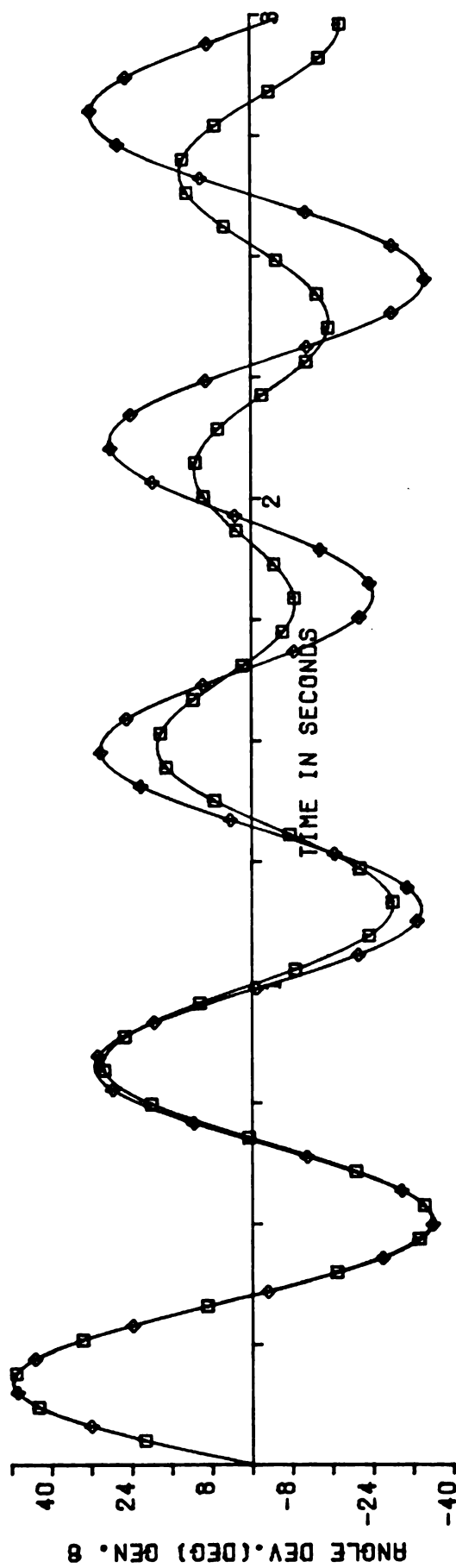






(d)





(f)

Tables 6.15, 6.16, 6.17, Figure 6.7, and Figures 6.8a through 6.8f provide the standard data for a ZMIW disturbance of generator 1. A quick look at Figures 6.5a to 6.5f shows that the response of the model matches that of the full system through six levels of aggregation.

A look at Table 6.15 shows that 5 levels of aggregation can be achieved from the first 9 coherency measures in the table, and that 7 levels of aggregation can be achieved before the absolute value of the coherency measure becomes very large.

An examination of Table 6.16 shows that there is definitely an averaging of eigenvalues at levels 4, 5, and 6. However, the result at level 6 is imaginary eigenvalue parts 4.558, 8.248 and 9.808 that are very close to the values 4.461, 8.287, and 9.541 of level 0. Note also that a value near 4.5, one near 8.3 and one near 9.8 are preserved through all levels of aggregation. Thus it appears that these latter three values are the imaginary parts of three eigenvalue pairs that represent the intermachine oscillations between generators 1, 8, and 2-3-4-5-6-7-9 that are very close to the oscillations between generators 1, 8, and 2 at level 0. This leads one to suspect that conditions for geometric coherency, strong linear decoupling or a combination thereof are satisfied for the aggregate 2-3-4-5-6-7-9, for a disturbance at generator 1.

An examination of Table 6.17 shows that the coherency measures between generator 1 and generators 2,3,4,6,7 are very close to the same at level 0, with coherency measures between 1 and 9 and 1 and 5 somewhat higher. At level 6 the coherency measure between generator 1 and the aggregate 2-3-4-5-6-7-9 is close to the coherency measure between

Table 6.15. Ranking of Coherency Measures ZMIW Disturbance of Bus 1

Ranking	Generator Pair	Coherency Measure	Aggregation Level
1.	C(6,7)	.0021	1
2.	C(4,7)	.0050	
3.	C(4,6)	.0071	2
4.	C(3,5)	.0139	3
5.	C(2,3)	.0377	
6.	C(2,5)	.0516	4
7.	C(4,9)	.0711	
8.	C(7,9)	.0762	
9.	C(6,9)	.0783	5
10.	C(5,6)	.0991	
11.	C(5,7)	.1012	
12.	C(4,5)	.1063	
13.	C(3,6)	.1130	
14.	C(3,7)	.1151	
15.	C(3,4)	.1202	
16.	C(2,6)	.1508	
17.	C(2,7)	.1529	
18.	C(2,4)	.1580	
19.	C(5,9)	.1775	
20.	C(3,9)	.1914	
21.	C(2,9)	.2291	6
22.	C(8,9)	.2754	
23.	C(4,8)	.3466	
24.	C(7,8)	.3517	
25.	C(6,8)	.3538	
26.	C(5,8)	.4529	
27.	C(3,8)	.4668	
28.	C(2,8)	.5046	7
29.	C(2,10)	.5813	
30.	C(3,10)	.6191	
31.	C(5,10)	.6330	
32.	C(6,10)	.7321	
33.	C(7,10)	.7342	
34.	C(4,10)	.7393	
35.	C(9,10)	.8105	
36.	C(8,10)	1.0859	8
37.	C(1,8)	1.9096	
38.	C(1,9)	2.1850	
39.	C(1,4)	2.2562	
40.	C(1,7)	2.2613	
41.	C(1,6)	2.2634	
42.	C(1,5)	2.3625	
43.	C(1,3)	2.3764	
44.	C(1,2)	2.4142	
45.	C(1,10)	2.9955	

Table 6.16. Eigenvalue Data for ZMW Disturbance on Generator 1

<u>Aggregation</u>	<u>Magnitudes of Imaginary Parts of System Eigenvalues for Six Levels of Aggregation</u>											<u>Level</u>
None	4.461	6.296	6.921	7.676	8.287	8.315	9.541	9.811	9.984			0
6-7	4.464	6.927	6.922	7.678	8.288	8.315	9.542	9.811				1
4-6-7	4.466	6.304	6.978	7.873	8.314	8.373	9.811					2
3-5 4-6-7	4.526	6.365	7.793	8.047	8.368	9.811						3
2-3-5 4-6-7	4.541	6.368	7.859	8.303	9.811							4
2-3-5 4-6-7-9	4.55	7.564	8.291	9.809								5
2-3-4-5-6-7-9	4.558	8.248	9.808									6

Table 6.17. Coherency Measure Data for ZMIIW Disturbance of Generator 1

<u>Aggregation</u>	<u>Coherency Measures between Generator 1 and Each System</u> <u>Generator at Six Levels of Aggregation</u>									<u>Level</u>
	<u>(1,2)</u>	<u>(1,3)</u>	<u>(1,4)</u>	<u>(1,5)</u>	<u>(1,6)</u>	<u>(1,7)</u>	<u>(1,8)</u>	<u>(1,9)</u>	<u>(1,10)</u>	
None	.02113	.02115	.02116	.03130	.02063	.02167	.01473	.02989	.04591	0
6-7	.02113	.02115	.02136	.03138	.02341	.01473	.02988	.04611		1
4-6-7		.02113	.02115	.02438	.03156	.01473	.02988	.04657		2
3-5 4-6-7			.02116	.02436	.02457	.01506	.03115	.04551		3
2-3-5 4-6-7			.02411	.02361	.01473	.02979	.04668			4
2-3-5 4-6-7-9				.02389	.02483	.01471	.04748			5
2-3-4-5-6-7-9					.02594	.01471	.04968			6

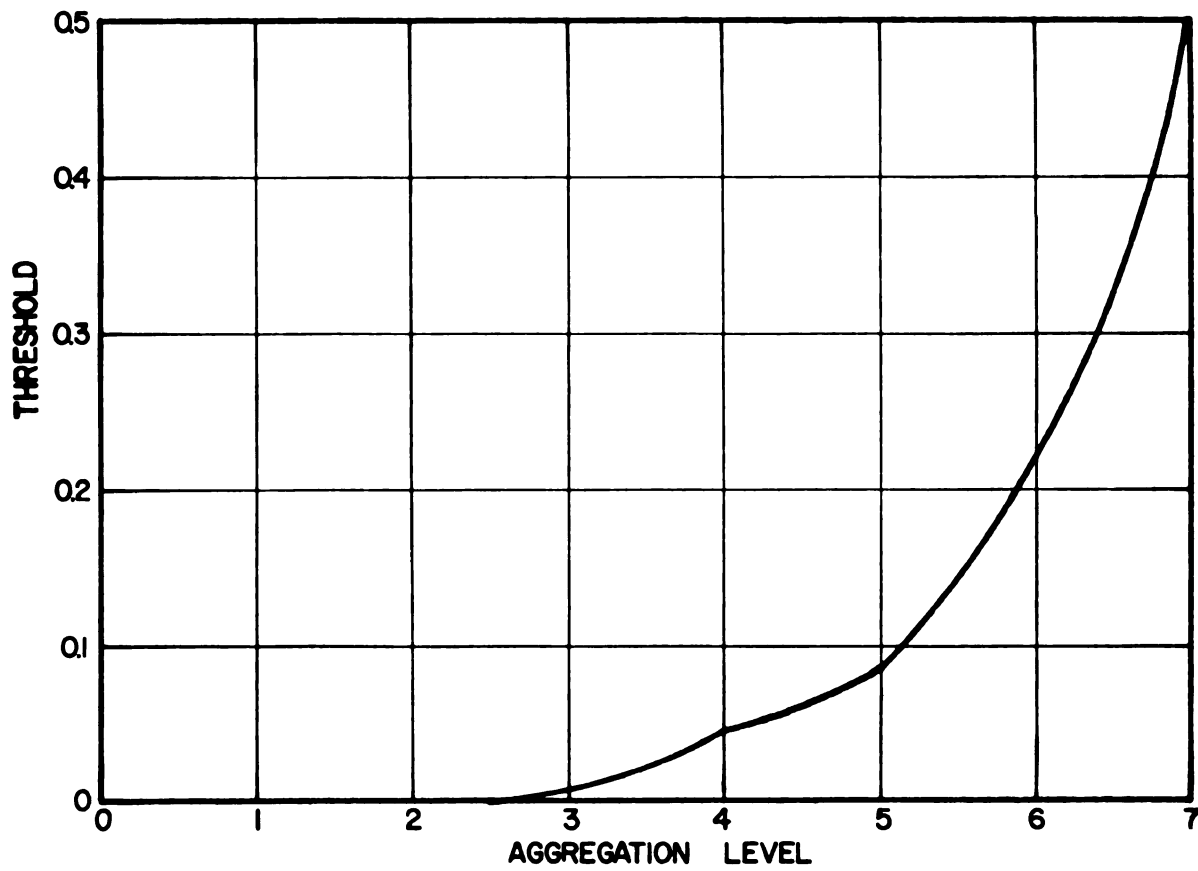
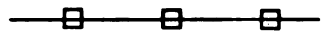


FIGURE 6-7

MAGNITUDE OF COHERENCY MEASURE THRESHOLD VS. AGGREGATION LEVEL

Figure 6.8. Simulations of System Response to a One Per Unit Step Disturbance on Generator 1

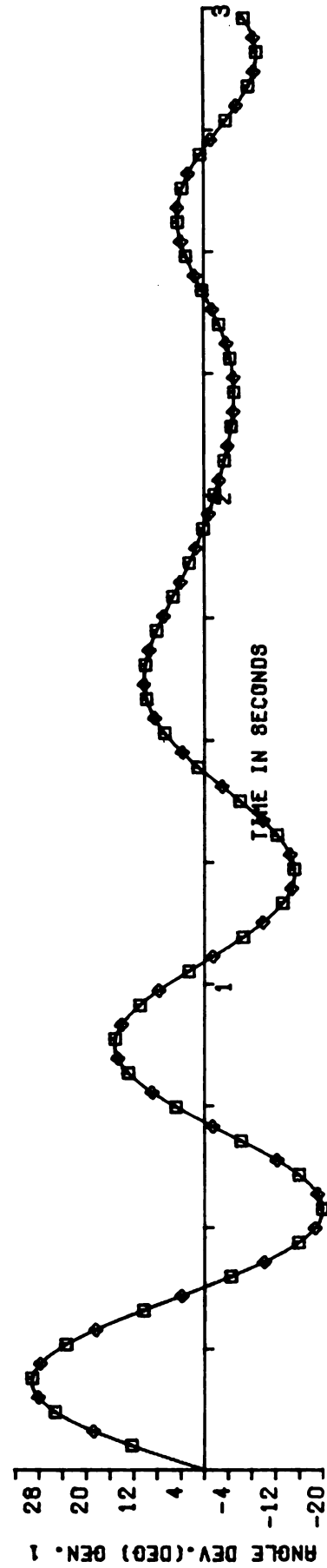
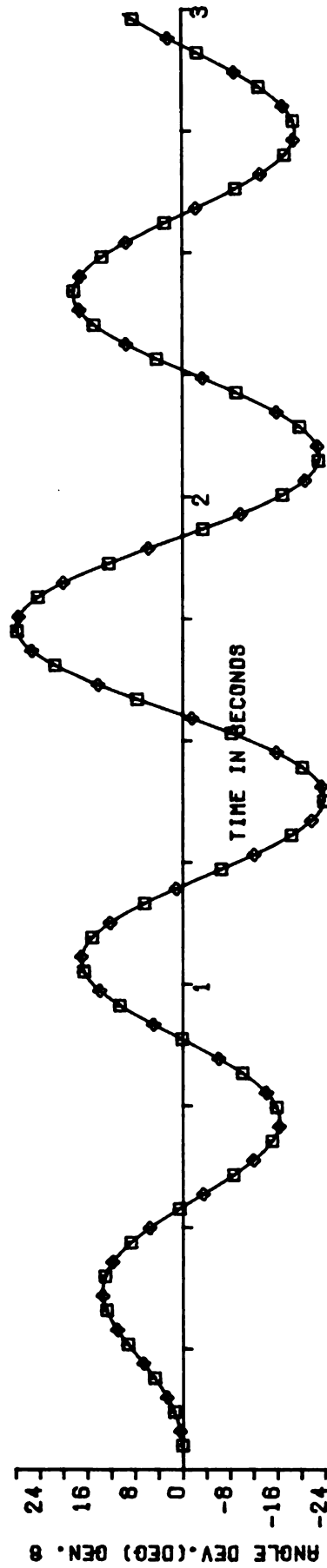
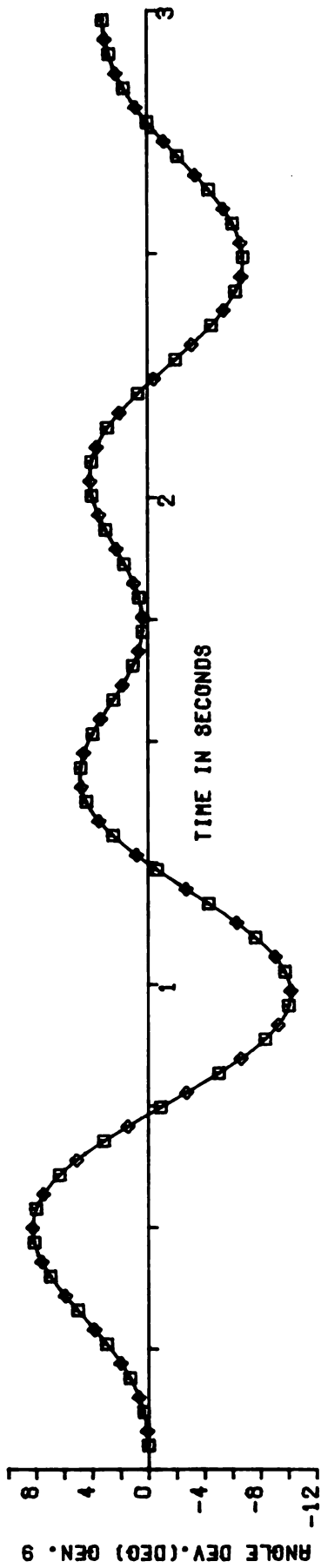


designates the full 39 Bus New England System

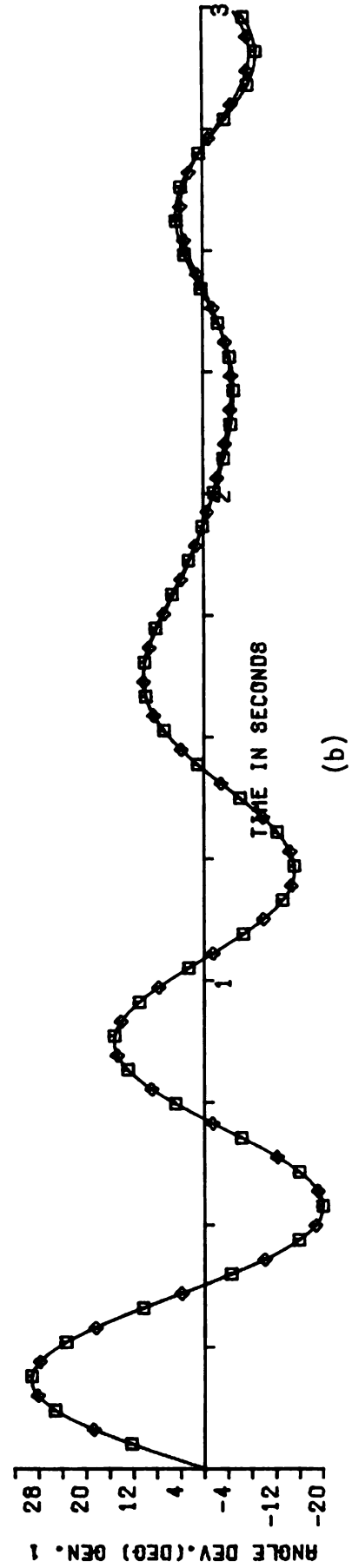
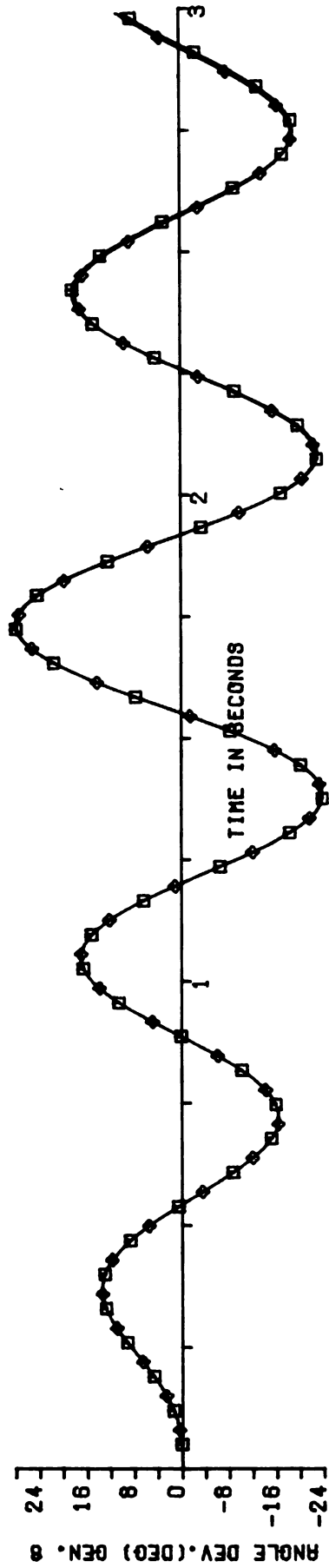
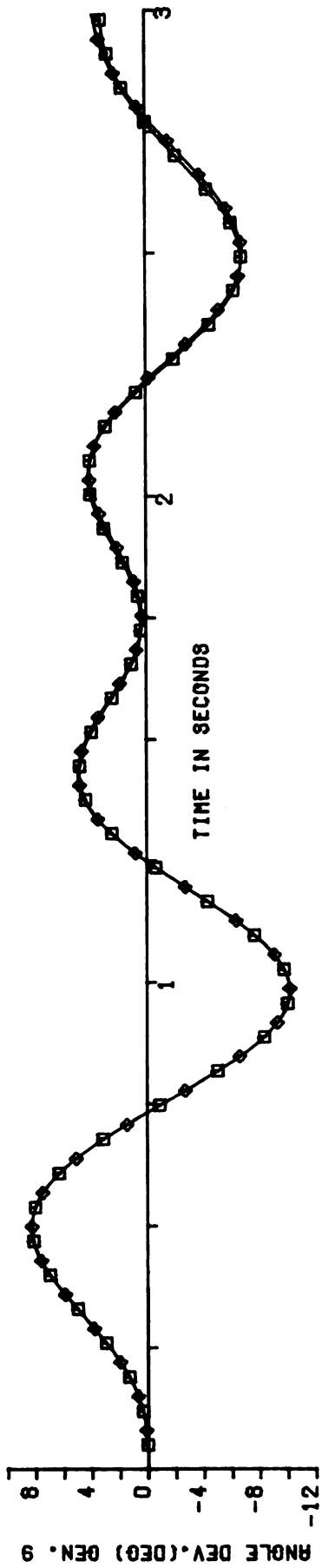


designates the aggregated model dictated by the ZMIW disturbance of generator 1

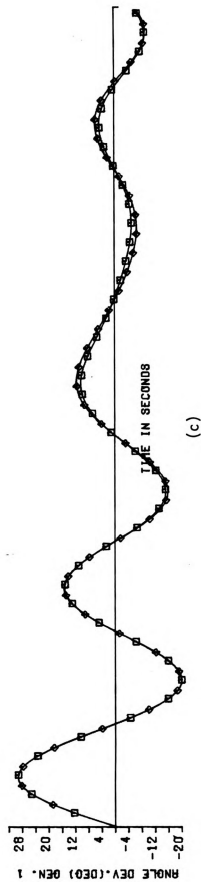
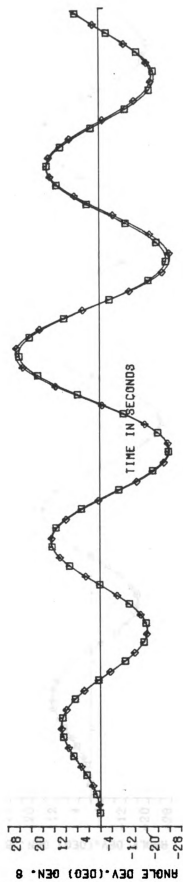
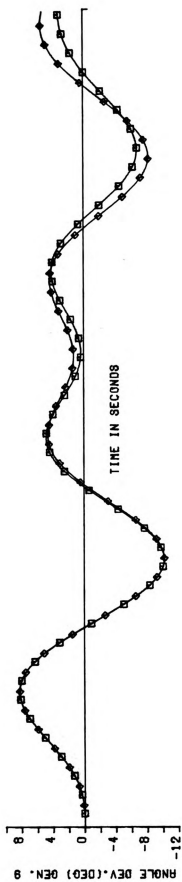
Figure 6.8.	Aggregation Level	System Generators								
(a)	1	1	2	3	4	5	6-7	8	9	10
(b)	2	1	2	3	4-6-7	5	8	9	10	
(c)	3	1	2	3-5	4-6-7	8	9	10		
(d)	4	1	2-3-5	4-6-7	8	9	10			
(e)	5	1	2-3-4	4-6-7-9	8	9	10			
(f)	6	1	2-3-4-5-6-7-9	8	10					

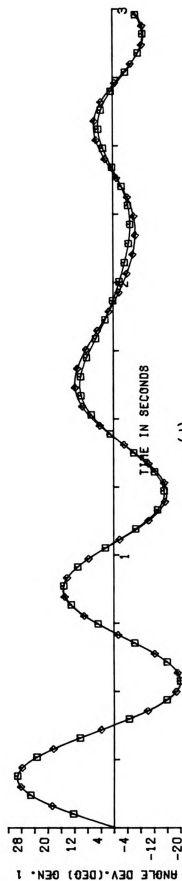
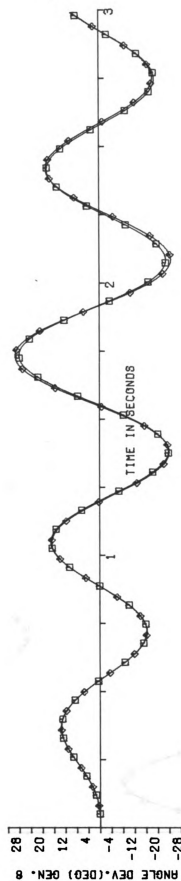
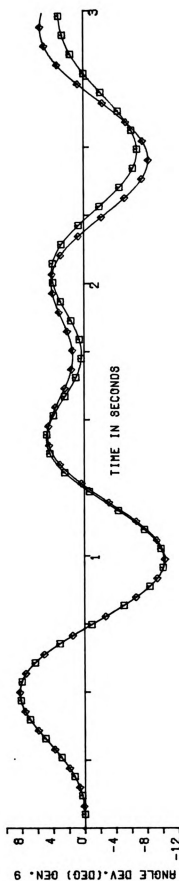


(a)

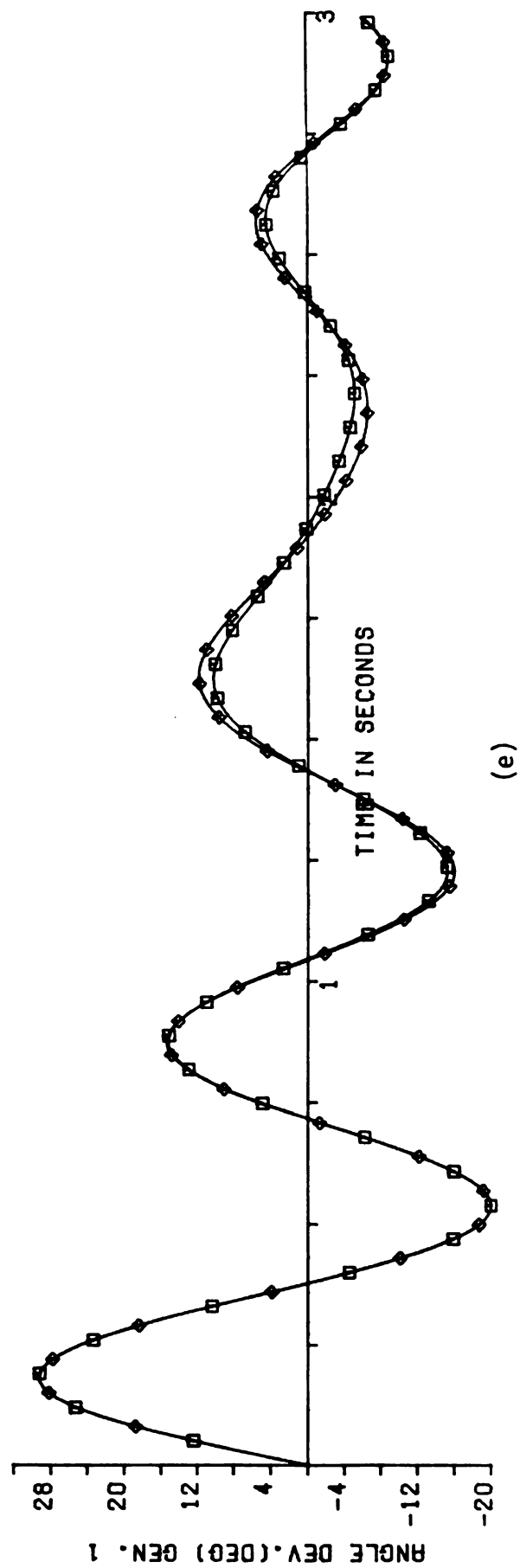
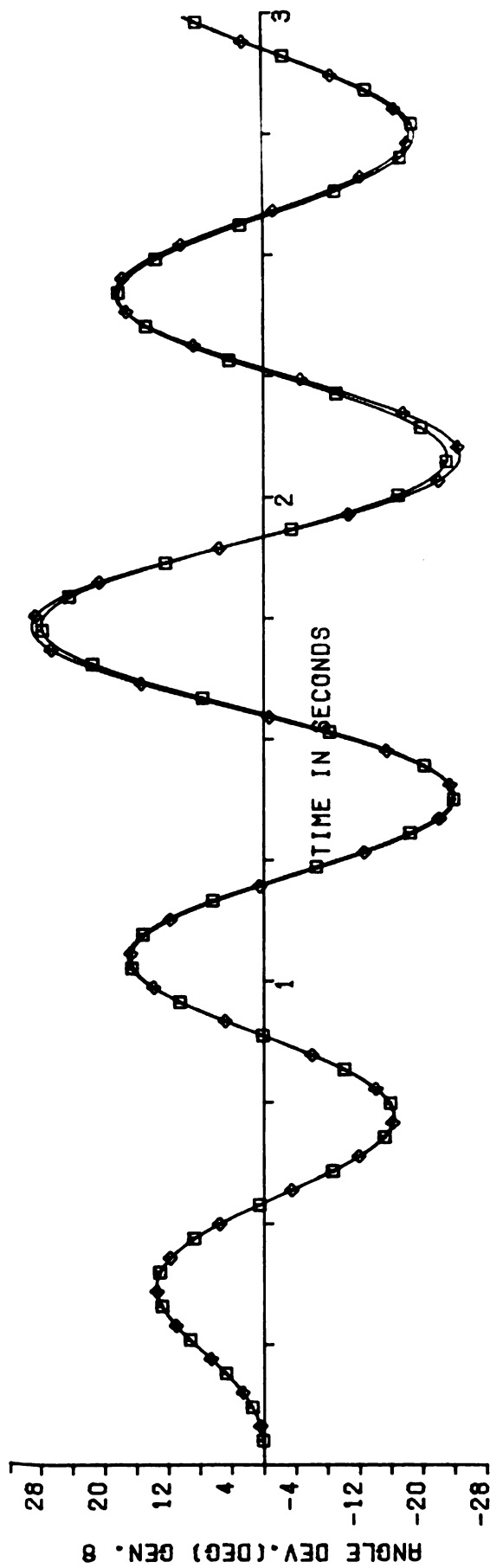


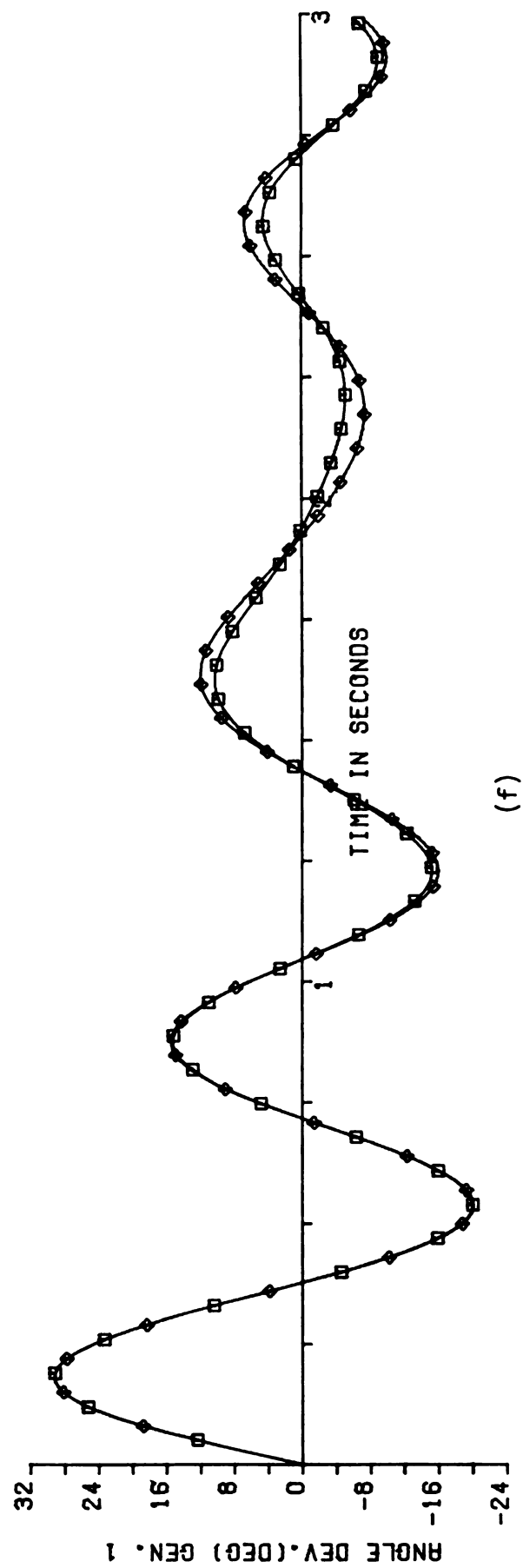
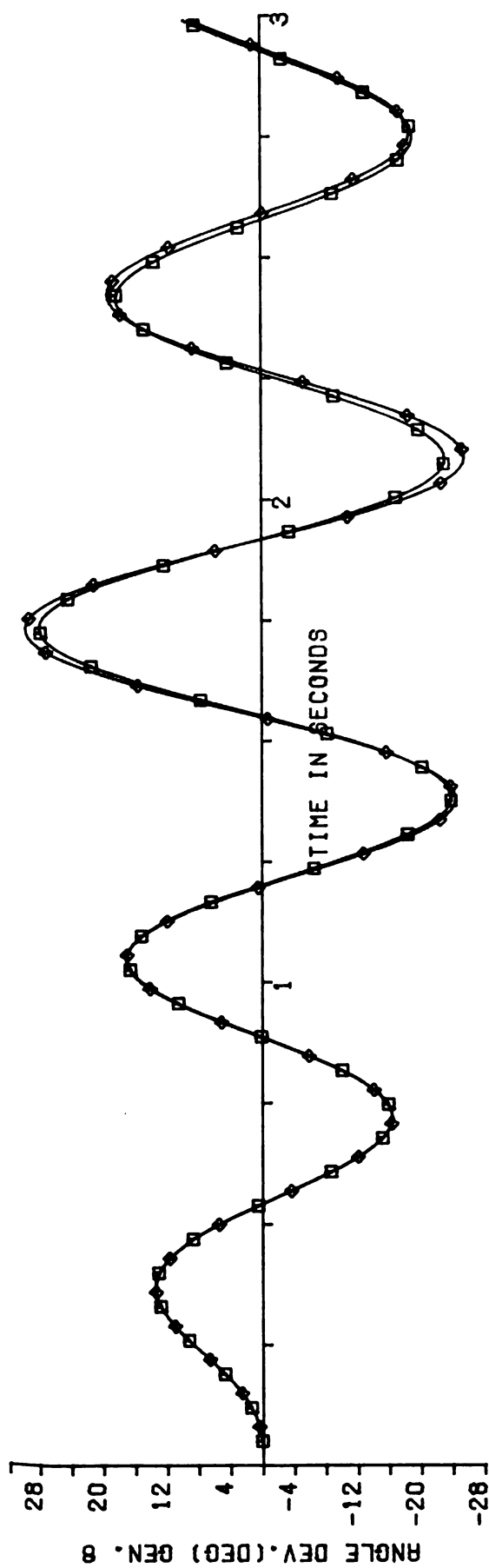
(b)





(d)





(f)

1 and the individual machines of the aggregate at level 0. The coherency measure between generators 1 and 8 are almost identical at levels 0 and 6, while the coherency measure between generators 1 and 10 changes only moderately (about 8%) between levels 0 and 6.

This seems to verify the supposition that for a disturbance at generator 1 either geometric coherency or strong linear decoupling is at work. This says that the partial disturbance step of the reduction algorithm can pay a handsome return if the structural conditions of the power system are satisfying geometric coherency or strong linear decoupling. The other results in this section, however, offer a strong caveat against ignoring the boundaries of tightly interconnected groups of generators when doing ZMIIW disturbances over a subset of generators. Despite the very good results for the ZMIIW disturbance of generator 1, the best rule would be to disturb all generators of a tightly bound group when determining a local model for a particular generator within the group.

V. Summary of Results

The results in this Chapter verify quite conclusively that the reduction algorithm for producing dynamic equivalents proposed in Chapter 5 is a reasonable and viable approach to producing dynamic equivalents. The importance of the general ZMIIW disturbance of all generators in determining tightly interconnected groups has been substantiated, and the validity of the partial ZMIIW disturbance for detecting the conditions of geometric coherency and strong linear decoupling have been verified. Embedded in this verification is the necessity to be cautious in forming local equivalents based on the

disturbance of only some of the generators within a tightly interconnected group.

The ZMIW disturbance over a subset of generator coupled to the commutative rule for aggregation of the r.m.s. coherency ranking table also showed the ability to do eigenvalue retention in a "geographic" way. That is, the eigenvalue retention seems to be based on the modes most excited by a disturbance located in a certain area of the power system. Hence, overall, the ZMIW disturbance of only part of the power system has proved to be a viable step in the overall reduction algorithm.

The question of whether the coherency measure thresholds in the ranking table can predict the point of overaggregation is not answered definitively by the data presented in this chapter. Most of the plots of threshold magnitude versus aggregation level swing upward rapidly at or after level 6, while the proper aggregation level for valid system response seems to be level 4 or 5. More data collection may help prove or disprove this trend.

Although the data presented in this chapter is more than adequate to verify the analysis of Chapters 3, 4 and 5 some important work remains to be done. One interesting and necessary follow-on would be to test the algorithm on a "large" system, one with at least fifty generators. One reason for doing this would be to test the proposed method of refining the boundary between the buffer zone and the far system. This has not been possible in a ten generator system where realistically all that is present is the internal system and the buffer zone. Such a study would also determine if a general ZMIW disturbance of all generators will identify the same principle groups of tightly

interconnected groups as the algorithm based on singular perturbation theory proposed by Kokotovic and Winkelman [12, 13].

It would also be worthwhile to build some nonlinear system models based on the aggregations dictated by the ZMIW disturbances. The easiest place to do this would probably be on the New England System by implementing the software package developed for EPRI by Podmore and Germond [2]. This would silence some of the criticism that inevitably results from using linear system theory to analyze nonlinear systems. It would help test the supposition put forward in Chapter 4 that synchronizing coherency, geometric coherency and strong linear decoupling are structural power system conditions whose presence in the linear model strongly guarantees their presence in the nonlinear model.

CHAPTER 7

REVIEW, CONTRIBUTIONS, AND TOPICS FOR FUTURE RESEARCH

I. Overview of Thesis

This research was initiated primarily to establish a stronger theoretical connection between the two traditional methods of producing reduced order dynamic equivalents for power systems, namely coherency equivalents and modal equivalents. These two methods had both been used successfully, and both had their proponents. The fact that both methods could produce good equivalents was strong, intuitive evidence that both methods must be utilizing the same fundamental properties of the power systems's structure to produce dynamic equivalents. The evidence that this was the case was strong [5, 6, 7, 8, 10, 11], but far from complete.

The review of the two methods of producing equivalents in Chapter 2 while aimed at delineating the differences between the two methods also pointed up one similarity. In both methods the search for an equivalent began by assuming that the disturbances would occur in a particular area of the power system, called the internal system. Everything else fell into the category of the external system. The perspective, then, for both methods was to look outward from the internal system and form a reduced order model of the external system. One might call this a local or parochial perspective on the dynamic equivalents problem. Through the course of this research, aimed

primarily at linking the two traditional techniques of forming equivalents, a broader, more global, perspective on the problem of dynamic equivalents emerged. More will be said about this in later sections of the chapter.

Chapter 3 began the hunt for theoretical connections between the modal and coherency techniques, by reviewing one of the strongest clues, namely the work of Dicaprio and Marconato, on what was to be eventually termed, in this present work, "Strict Geometric Coherency" [10, 11]. Dicaprio and Marconato divided their example power system into a study group and a specified, or in their terminology an "evidenced", group. They then stated structural conditions between the study group and the specified group, which if true at time $t = 0^-$, caused all the generators of the specified group to accelerate at the same rate in response to any disturbance within the study group.

This meant that, from the viewpoint of the study group, the specified group appeared to be one generator.

The striking feature of Dicaprio and Marconato's result is that it holds for the nonlinear model. The rest of Chapter 3 investigated other conditions that, like the Dicaprio-Marconato condition, might cause the specified group to behave, from the perspective of the study group, like a single generator; two were identified. The first was called strict synchronizing coherency and depended upon progressively stiffening, at least, $n-1$ interconnections linking all n generators of the specified group, until these interconnections were infinitely strong (zero impedance). The other condition called, pseudo-coherency, was a mirror image of the Dicaprio-Marconato conditions for strict

geometric coherency, in that it also relied upon structural conditions between the study group and the specified group of n generators at time $t = 0^-$. In pseudo-coherency, however, the specified group was not coherent but only appeared to be coherent to the study group.

Thus Chapter 3, determined three hypothetical conditions under which the specified group could be replaced by a single machine. The three conditions are called hypothetical in the sense that they could never be perfectly satisfied in a real power system. Strict synchronizing coherency relies upon infinitely stiff (zero impedance) interconnections between generators. Strict geometric coherency and pseudo-coherency conditions can be realized with real components, but the probability of the conditions being satisfied for a sizeable group of machines is effectively zero.

It could be argued that strict synchronizing coherency is hypothetical in a different sense than the other two conditions because it relies upon non-finite components. This argument has philosophical but not practical merit, because near approximations to strict synchronizing coherency are more common in power systems than are near approximations of the other two conditions. Further, the argument becomes irrelevant by the end of Chapter 4, since at that point it is evident that the real power of these three conditions is in their use as conceptual tools for understanding the combination of conditions that lead to coherency in a real power system. That is, the three conditions, SSC, SGC, and PC can be viewed as archetypes for group coherency. In a real system a combination of these archetypes may be at work simultaneously to cause group coherency.

Chapter 4 re-examined the three conditions for coherency for the linear model. One of the results of Chapter 3 had been that satisfaction of the conditions for strict synchronizing or strict geometric coherency at time $t = 0^-$, guaranteed coherency of the specified group for all $t > 0$. This could not be shown for pseudo-coherency and in that sense pseudo-coherency was a far weaker condition than the other two.

All three conditions did however decouple the linear equations, leading to the specified group behaving as a single machine, for disturbances within the study group. This decoupling provided the key to showing that if any one of the conditions SSG, SGC or PC were satisfied, the modal and coherency methods produced the same equivalent for the specified group. This result depends on the decoupling of the linear equations which in turn separates the eigenvalues for the system model into two sets, one set (of eigenvalues) associated with the equations for the study group (of generators) through the matrix $(-\underline{M} \ \underline{I})_{11}$ and the other set with the equations for the specified group through the matrix $(-\underline{M} \ \underline{I})_{22}$. The coherency method of finding an equivalent replaced the specified group by a single machine because it behaved as a single machine from the perspective of the study group, for disturbances within the study group. The modal analysis method produced the identical equivalent by using controllability and observability arguments to discard the modes (canonical states) associated with the specified group of n generators, through the matrix $(-\underline{M} \ \underline{I})_{22}$. In the case of strict synchronizing and strict geometric coherency, the modes were discarded as uncontrollable. In the case of strict (linear)

pseudo-coherency the modes were discarded as unobservable. Thus Chapter 4, established the conditions under which the modal and coherency equivalents for a specified group of generators were identical.

This picture was made even more complete by 1) incorporating into the three hypothetical conditions for coherency the work of Chow, Kokotovic, Allemong, Winkelman, et al., on singular perturbation equivalents, and 2) by introducing the idea of linear decoupling. It was shown that the singular perturbation model, which discards high frequency modes as unobservable is almost perfectly congruent with the concept of strict synchronizing coherency. In fact, the limiting process of sending the parameter μ to zero in the singular perturbation model was shown to coincide with the process of letting $n-1$ interconnections linking all n machines become infinitely stiff, so that, in the limit, strict synchronizing coherency and the two time scale separation of singular perturbation become identical.

The concept of linear decoupling was then introduced to account for those cases where the linear equations were essentially decoupled, but the decoupling could not be attributed completely to any one of the three conceptual conditions for coherency, i.e. SSC, SGC or PC.

Once again, the concept of linear decoupling was introduced as a conceptual aid by showing how the specified group could be perfectly decoupled by a combination of strict synchronizing coherency and strict geometric coherency. Three types of linear decoupling were identified. Two of these types were classified as "weak", since like pseudo-coherency the presence of the conditions in the linear model did not strongly guarantee the presence of the conditions in the nonlinear model.

One type of linear decoupling, however, did carry this guarantee and could be classified with strict synchronizing coherency and strict geometric coherency. This type of linear decoupling was termed strict strong linear decoupling. It is best understood by an example.

Consider the five generator example of figure 7.1a and suppose the following structural conditions hold.

$$(1) \quad \frac{t_{14}}{M_4} = \frac{t_{15}}{M_5}$$

$$(2) \quad \frac{t_{12} + t_{13}}{M_2 + M_3} = \frac{t_{14}}{M_4}$$

where the t_{ij} 's are synchronizing power coefficients and the M_i 's are machine inertias.

Let generator 1 be the study group, generators 2 and 3 group 2, and generators 4 and 5 group 3. The conditions (1) and (2) are not sufficient to cause groups 2 and 3 to behave like one large coherent group. That requires

$$(3) \quad \frac{t_{14}}{M_4} = \frac{t_{15}}{M_5} = \frac{t_{12}}{M_2} = \frac{t_{13}}{M_3}$$

or a tree of stiff interconnections among generators 2,3,4 and 5.

Neither of these conditions is implied by conditions (1) and (2). Thus, the generators 2, 3, 4 and 5 are satisfying neither the conditions for strict geometric coherency or strict synchronizing coherency.

Now let the interconnection between generators 2 and 3 become infinitely stiff. This causes generators 2 and 3 to act like a single machine of inertia $M_2 + M_3$. The synchronizing power coefficients between the aggregate generator 2-3 and generators 1, 4, and 5 are as shown in figure 7.1b.

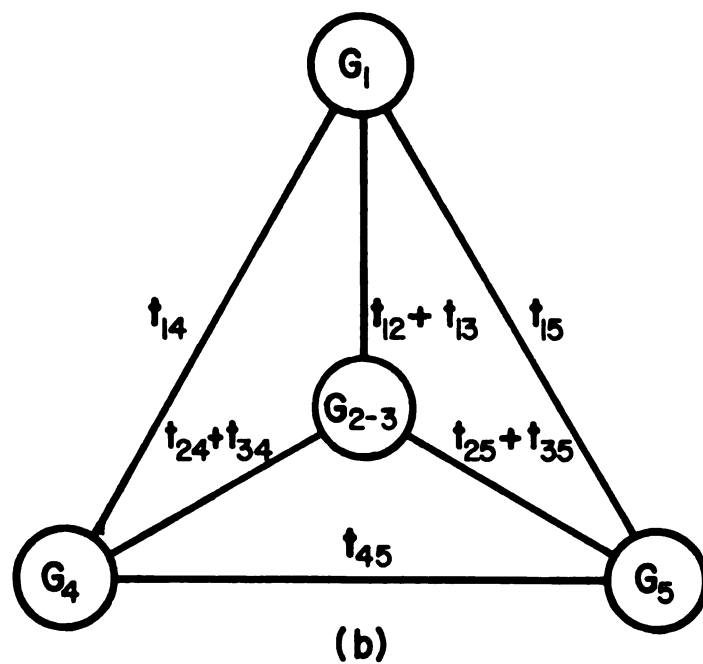
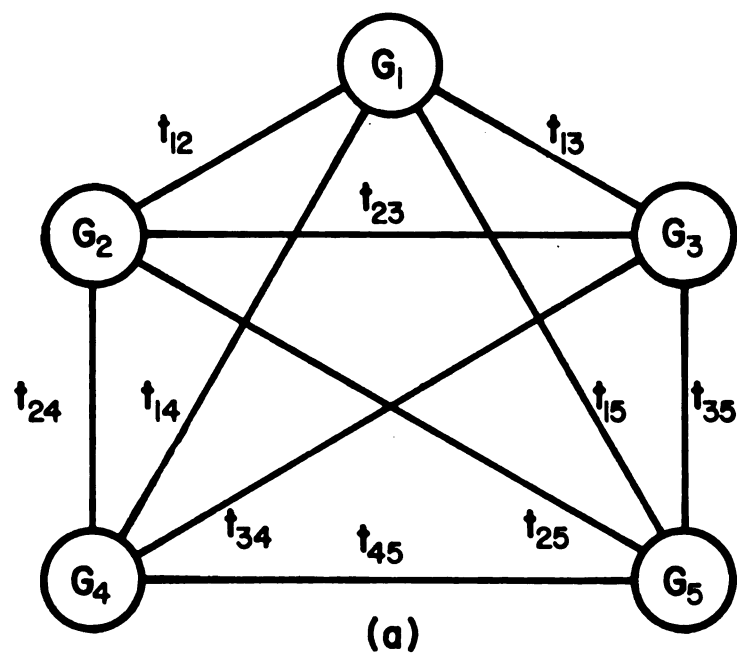


FIGURE 7-1
 FIVE GENERATOR SYSTEM
 (a) BEFORE AGGREGATION OF GENERATORS 2 AND 3
 (b) AFTER AGGREGATION OF GENERATORS 2 AND 3

Since $\frac{t_{12} + t_{13}}{M_2 + M_3} = \frac{t_{14}}{M_4} = \frac{t_{15}}{M_5}$, generators 4, 5, and the aggregate 2-3 satisfy the conditions for strict geometric coherency for disturbances at generator 1. Thus a combination of strict synchronizing coherency and strict geometric coherency will cause generators 2, 3, 4, and 5 to act as a single generator for disturbances at generator 1. This example also illustrates the great conceptual power of the archetypal conditions, strict synchronizing coherency and strict geometric coherency.

Chapter 4, then, contains two major accomplishments, 1) a set of conditions under which the modal and coherency equivalents were identical and 2) a determination of which of these conditions were worthy of being used to form a dynamic equivalent. Those chosen for use in forming the dynamic equivalent were synchronizing coherency, geometric coherency and the strong type of linear decoupling (SLD).

Chapter 5 next provided the means of detecting the selected conditions. It was shown that a particular type of disturbance, called a ZMIW disturbance made the r.m.s. coherency measure depend only on the structure of the linear model, i.e. on the $-M^{-1}I$ matrix. Further by using different ZMIW disturbances one could distinguish synchronizing coherency from geometric coherency and strong linear decoupling. Synchronizing coherency was detected by a general ZMIW disturbance of all generators. The other two types by a ZMIW disturbance over a specific subset of generators called the internal system. This internal system can be identified with the study group of Chapters 4 and 5.

The distinction between the types of disturbances leads in a very natural way to a distinction between two types of reduced order

models of a power system. The general ZMIW disturbance over all generators detects those groups of generators that are tightly interconnected. That is, it divides the overall power system into areas, called principal groups, that react in consort to disturbances anywhere in the system. Thus, if the main concern is how a disturbance propagates among the principle groups, then the general ZMIW disturbance can provide the proper model. This model might be thought of as a global model.

The ZMIW disturbance over a specific subset of machines, on the other hand, provides a means of finding what is coherent, looking outward from that subset of machines. It is not hard to see that this is congruent with the traditional perspective on forming equivalents, discussed in Chapter 2.

Chapter 5, concludes by incorporating both the general and the specific ZMIW disturbances into a reduction algorithm for producing dynamic equivalents. The results of testing the algorithm on the 39 Bus New England System were summarized in Chapter 6, and indicated that the algorithm worked very well.

II. Contributions

The ideas of pseudo-coherency and linear decoupling are new. In some limited sense strict synchronizing coherency is also new. The knowledge that tightly interconnected generators remain coherent has existed for a long time, but it was never formalized into a theoretical concept requiring $n-1$ infinitely stiff interconnections among n generators to make them strictly coherent. It was this formulation that led to the result that, in the limit when the parameter $\mu \rightarrow 0$,

the singular perturbation concept of two time scale separation is identical to strict synchronizing coherency.

As important as the individual ideas are, the significant contribution has been the integration of these individual ideas into a general theory that provides a good conceptual understanding of how a power system responds to a disturbance, both at the global level and the local level. The generality of this theory is demonstrated by its ability to encompass all the current methods employed in constructing dynamic equivalents, including the singular perturbation approach.

This conceptual understanding can be directly applied to the problems of security assessment and planning. The global modeling level is of great interest for on-line system monitoring and control, since it provides some insight into how major portions of the power system interact in response to a disturbance. The r.m.s. coherency measure may be capable of serving as a security measure that would indicate when the system is vulnerable to an unstable condition (called a contingency), so that corrective action can be taken. For power system planning, both the global and local modeling aspects can be utilized, since in planning, both global stability and transient stability from the standpoint of a single machine are of interest.

The reduction algorithm itself has already been implemented by making appropriate changes to the EPRI software package [2]. This modified software package provides a computationally efficient means of producing dynamic equivalents for systems of any size. It's main virtue is that it does not require the calculation of eigenvalues, which is the main drawback to almost any modal analysis scheme for producing equivalents, including the singular perturbation approach.

The data collection done on the 39 Bus New England System not only verified the analysis of Chapters 3, 4, and 5, it added some new information, in particular, the idea of eigenvalue retention tuned to the location of the disturbance. This is in itself a very interesting result.

III. Topics for Future Research

The analysis in Chapter 4, indicates that the singular perturbation concept coincides with the concept of synchronizing coherency. This means that the principle groups identified by the reduction algorithm of Chapter 5 should coincide with the major groups determined by Winkelman, Kokotovic, et al. in references [12, 13]. This is purely computational work to be done on a system with at least fifty or sixty generators. Two prime candidates are the 64 generator system used by Winkelman, Kokotovic, et al. [13], and the model of the western grid used by Podmore [2].

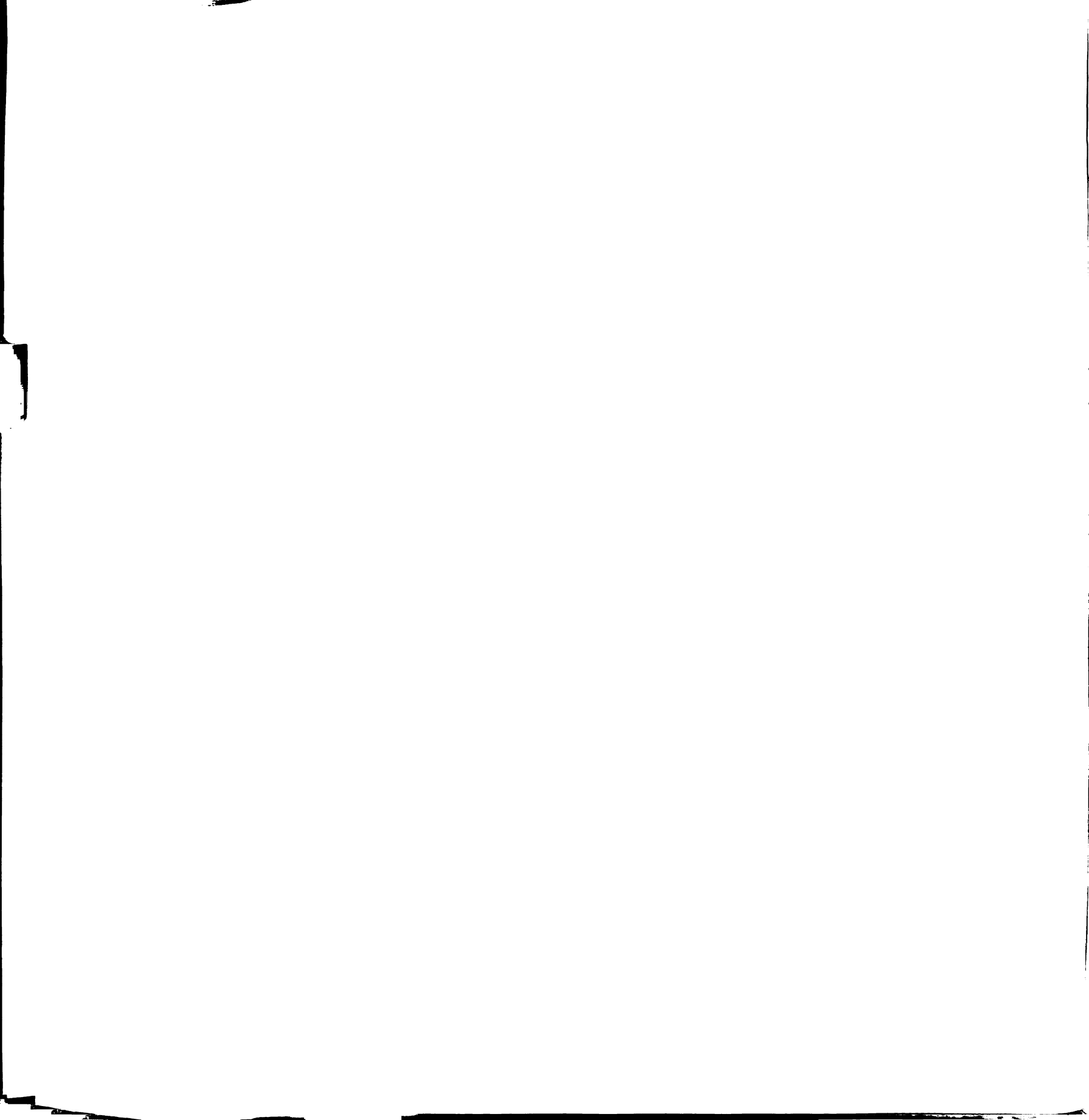
Another useful investigation is the comparison of the performance of the coherency equivalent, i.e. the averaged equivalent machine, with the singular perturbation equivalent for the same generator aggregation. The conjecture is that the coherency equivalent will perform better for short intervals and the singular perturbation equivalent will be better for long intervals.

Some computational research can also be done to compare Podmore's and Germond's results with the proposed reduction algorithm. Podmore and Germond use a different coherency measure, a different aggregation rule for determining coherent groups, and disturbances that are not ZMIIW. It might be useful to use the model of the western grid provided

by reference [2] and perform ZMIW disturbances from the same locations as the disturbances in Podmore and Germond's work and compare the equivalents.

The present work also provides new insight into the problem of on-line identification of the external system. The identification problem requires an assumption about the structure of the external system. That structure can be obtained from a general ZMIW disturbance of all the generators. It would provide first of all the order of the state model of the external system and second, the appropriate locations at which to make the measurements. That is, measurements would be taken at machines that belong to the principal groups determined by the general ZMIW disturbance. It may even be possible to do the identification based only on measurements taken within the internal system and at the boundary between the internal and external systems.

Th future research possibilities out of the current work seem fairly rich. The ones discussed here by no means exhaust the list, but only serve to indicate some of the more promising avenues of exploration.



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