A MONTE CARLO INVESTIGATION OF THE ANALYSIS OF VARIANCE APPLIED TO NON-INDEPENDENT BERNOULLI VARIATES

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This is to certify that the

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ABSTRACT

A MONTE CARLO INVESTIGATION OF THE ANALYSIS
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By

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The analysis of data from a repeated measures type of experimental design was considered for the case in which each repeated measure score was obtained as the sum of a set of evaluations for responses to a set of items. The analysis was considered separately for the case in which the items were included as a factor in the design and for the situation in which the items were ignored as a factor in the design. It was shown that whether items were considered as a factor in the experimental design or not, they could provide a non-null source of variation, which if present would be confounded with the source of variation for repeated measures effects. The suggestion was made that the inclusion of items as a factor in the design and employment of the "quasi-F" statistic might result in an appropriate test for repeated measures effects, if the response evaluations could be considered independent and normally distributed. However, it was noted that a series of evaluations on a

single subject are seldom independent and are frequently zeros and ones corresponding to incorrect and correct responses, that is, data which could be modeled by a vector of non-independent Bernoulli variates rather than a vector of independent normal variates. Because others had had success in the application of the Analysis of Variance (ANOVA) to independent Bernoulli variates, it was considered interesting and important to attempt to determine if ANOVA could be appropriately applied to the analysis of non-independent Bernoulli variates, particularly with respect to the subjects by repeated measures design with items either nested within or crossed with repeated measures.

A mathematical modeling of the situation of interest was undertaken, an algorithm was devised, and a computer program was written to simulate zero-one type data with any given desired consistent covariance structure and parameter configuration. For a given parameter configuration a Monte Carlo procedure was employed to determine the appropriateness of ANOVA for the analysis. Then the obtained empirical distributions of variance ratio tests were compared to Theoretical F-distributions, with respect to the probability of a type I error or relative power, for null effect or non-null effect conditions respectively. Particular attention was paid to the empirical distributions of the regular variance ratio test for repeated measures, the "quasi-F" test for repeated measures, and the regular variance ratio test for the subjects by repeated measures interaction.

There were 720 cases or parameter configurations which were investigated. For all cases investigated the number of items associated with a repeated measure and the number of repeated measures were fixed at three and four respectively. The items provided either a null source of variation or a non-null source of variation and were either crossed with or nested within the repeated measures. The number of subjects varied from 4 to 12. The probability of a one in the zero-one data was either .5, .2, or .1. The degree of subject heterogeneity was one of four values. And the effects of repeated measures and the subject by repeated measure interaction were either, both null or separately non-null.

The results indicate that the "quasi-F" should not be applied to the type of data investigated, that the power of variance ratio test on non-independent zero-one data is approximately half that for normal data, that in the absence of a confounded non-null source of variation the regular variance ratio test for repeated measures is appropriate given the number of subjects is large enough, and that the variance ratio test for the subjects by repeated measures interaction is appropriate only when the probability of a one in the data is close to .5.

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CHAPTER I

INTRODUCTION

An experimental design which is often employed in psychological and educational research is the repeated measures design.[1] The essential characteristic of the repeated measures design is that each subject is evaluated more than once. Thus the simplest of repeated measure designs would represent a situation in which n subjects were evaluated t times, resulting in a data matrix with n rows and t columns. Variations on the simple repeated measures design include assigning subjects to treatment groups and classifying the repeated measures into levels of independent variables associated with them. In many instances an experimenter will, by necessity, have to evaluate the subjects on each of the t times in a manner such that the evaluations may take on only two values corresponding to, for example, correct or incorrect responses. When the evaluations can only take on two values (e.g. zero or one), a data matrix of dichotomous values results. Can the familiar analysis of variance, ANOVA, procedure be usefully applied to a data matrix of

[1]The repeated measures design is sometimes referred to as a split plot design.

such dichotomous values? Of the assumptions on which the ANOVA procedure is based, clearly the assumption of normality is violated, and it will be shown that it is likely that the assumption of independence will be violated as well. The problem with which this paper will be concerned is, in general, the analysis of dichotomous repeated measure data, and specifically the applicability of the ANOVA procedure to the analysis of dichotomous repeated measure data.

The repeated measures type of experimental design allows not only for the experimental investigation and analysis of events over time, but also offers promise to serve as a vehicle for the investigation of individual differential response on the part of the subjects or experimental units with respect to the variables of experimental intervention (see Cronbach, Jensen, and others in Gagné, 1967). The importance of determining if it is likely that experimental units have differential responses with respect to the variables of experimental intervention is illustrated in an example referred to by Jensen (1967), of a study by Hovland (1939), who performed an experiment in which no statistically significant differences were found between massed and distributed practice on paired associate learning tasks. After reporting the above mentioned nonsignificance, Hovland went on to report that 44 per cent of the subjects in his study improved more rapidly with distributed practice, 28 per cent learned faster with massed practice, and 28 per cent showed no effect due to the type of practice. The

percentages which Hovland reported suggest the possibility of a significant subject by type of practice interaction. A subject by type of practice interaction would indicate that the effect of type of practice was not null but rather different for different types of subjects.

In the Hovland study all subjects were measured on learning trials when given massed practice and on learning trials when given distributed practice. Table 1 presents the sources of variation, degrees of freedom, and expected mean squares for data from a study such as Hovland's. Note, there are tests for the type of practice main effects and the trials main effects, but no test for the subjects by type of practice interaction effects. Thus there is no test for what would appear to be an important source of variation in the Hovland data.

Examine an experimental design the Hovland study might have employed. In the suggested design subjects are arrayed in two groups (a massed practice group and a distributed practice group) where each subject is given five trials on four randomly selected sets of paired associates. Groups and trials have two and five fixed levels respectively and thus represent fixed sources of variation. Subjects and paired associates are randomly selected from supposedly infinite populations and therefore represent random sources of variation.

Table 2 contains the sources of variation, degrees of freedom and expected mean squares for a design such as that

Table 1

Sources of variation, degrees of freedom and expected mean squares for data from a design such as Hovland's

Source	df	E(MS)
A (subjects)	s-1	$2t\sigma^2_A$
B (type practice)	1	$t\sigma^2_{AB} + st\sigma^2_B$
C:B (trials)	2(t-1)	$\sigma^2_{AC:B} + s\sigma^2_{C:B}$
AB	1(s-1)	$t\sigma^2_{AB}$
AC:B	2(s-1)(t-1)	$\sigma^2_{AC:B}$

Note, not all of the above are variances, since there are some fixed effects.

Table 2

Sources of variation, degrees of freedom, and expected mean squares for the suggested design

Source	df	E(MS)
A (groups)	1	$5\sigma^2_{CS:A} + 5n\sigma^2_{AC} + 20\sigma^2_{S:A} + 20n\sigma^2_{A}$
S:A (subjects: groups)	(n-1)2	$5\sigma^2_{CS:A} + 20\sigma^2_{S:A}$
B (trials)	4	$\sigma^2_{BCS:A} + 4\sigma^2_{BS:A} + 2n\sigma^2_{BC} + 8n\sigma^2_{B}$
C (PA's)	3	$5\sigma^2_{CS:A} + 10n\sigma^2_{C}$
BC	12	$\sigma^2_{BCS:A} + 2n\sigma^2_{BC}$
AB	4	$\sigma^2_{BCS:A} + 4\sigma^2_{BS:A} + n\sigma^2_{ABC} + 4n\sigma^2_{AB}$
AC	3	$5\sigma^2_{CS:A} + 5n\sigma^2_{AC}$
ABC	12	$\sigma^2_{BCS:A} + n\sigma^2_{ABC}$
BS:A	(n-1)8	$\sigma^2_{BCS:A} + 4\sigma^2_{BS:A}$
CS:A	(n-1)6	$5\sigma^2_{CS:A}$
BCS:A	(n-1)24	$\sigma^2_{BCS:A}$

Note, not all of the above are variances since there are some fixed effects.

mentioned in the previous paragraph, given that the assumptions on which the analysis of variance procedure is based, hold. Inspection of the expected mean squares in Table 1 indicates that the ratio,

$$\frac{MS_{BS:A}}{MS_{BCS:A}},$$

would provide a test for the source of variation--subjects by trials interaction. The source of variation--subjects by trials interaction--would reflect the type of individual differential response to massed or distributed practice that was suggested in the Hovland data, that is the differences between the trial curves for subjects would be greater if there was a significant subjects by type of practice interaction than would be expected otherwise. Thus had the experimental design suggested in this paper been employed and the ANOVA consistent with it been used to analyze the data obtained, a test for the subjects by trials source of variation would have been available, which if significant would indicate the possibility of a subjects by type of practice interaction.

At least at first glance the ANOVA which Table 2 suggests as a possible means of analysis, appears to be a reasonable way to insure a means of testing for a differential response on the part of the two groups[2] as well as a means of testing for a subjects by trials interaction.

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[2]By means of a synthetic variance ratio due to Satterthwaite (1941).

However, closer examination will indicate violations of the ANOVA assumptions.

In order to explain the nature and sources of the violations a digression must be indulged. The digression is necessary in order to discuss population distributions. Consider any population of entities, each entity may have as many as an infinity of attributes and if the number of entities in a population is so few as to be countable in a finite time, a frequency distribution of the entities with respect to any one attribute may be constructed. If one value can represent all of the entities in a population with respect to one attribute, the population can be said to have a point distribution with respect to that attribute. Note that the above same population may not necessarily have a point distribution with respect to some other attribute. Thus it is important when speaking of the distribution of a population to specify with respect to what attributes the population is distributed.

One of the assumptions of an ANOVA is that the response evaluations on dependent variables are independent for each subject or experimental unit. Another assumption of an ANOVA is that the dependent variables have a normal distribution. If the dependent variables have a multivariate normal distribution a zero correlation between them is necessary and sufficient to establish their statistical independence.

It will now be shown that it is likely that data

obtained from a design implied by Table 2 will violate the assumption of independence on which the ANOVA is based. Note in Table 2 that in order to have a test for the source of variation--subjects by trials interaction--it is necessary to regard the sets of paired associates as a random sample of sets of paired associates sampled from some population of sets of paired associates. Had the sets of paired associates been regarded as all of the sets of interest, "sets" would not have provided a random source of variation and the Table of sources of variation, degrees of freedom and expected mean squares would have been as in Table 3. An inspection of Table 3 indicates no test for the source--subjects by trials interaction. If the source of variation, sets, is random, it is unlikely that the effect on the dependent variable would be null. In order for the effect of sets to be null when sets are random the distribution of sets would have to be a point distribution with respect to the attribute-effect on the dependent variable. If the effect of sets is non-null it is likely that there will be a positive correlation between subjects across sets. Similarly, if the effect of subjects within groups is non-null, which is very likely, it is likely that there will be a positive correlation between sets across subjects.

Why is it likely that there would be a positive correlation between sets, for example, if the effect of subjects on the dependent variable is non-null? The reason can be stated generally in terms of experimental design. In the

Table 3

Sources of variation, degrees of freedom, and expected mean squares for the suggested design when paired associates are fixed

Source	df	E(MS)
A (groups)	1	$20\sigma^2_{S:A} + 20n\sigma^2_{A}$
S:A (subjects:groups)	(n-1)	$20\sigma^2_{S:A}$
B (trials)	4	$4\sigma^2_{BS:A} + 8n\sigma^2_{B}$
C (PA's)	3	$5\sigma^2_{CS:A} + 10n\sigma^2_{C}$
BC	12	$\sigma^2_{BCS:A} + 2n\sigma^2_{BC}$
AB	4	$4\sigma^2_{BS:A} + 4n\sigma^2_{AB}$
AC	3	$5\sigma^2_{CS:A} + 5n\sigma^2_{AC}$
ABC	12	$\sigma^2_{BCS:A} + n\sigma^2_{ABC}$
BS:A	(n-1)8	$4\sigma^2_{BS:A}$
CS:A	(n-1)6	$5\sigma^2_{CS:A}$
BCS:A	(n-1)24	$\sigma^2_{BCS:A}$

Note, not all of the above are variances, since there are some fixed effects.

absence of a disordinal interaction between two independent variables in an experimental design whose levels are completely "crossed," a non-null effect of one independent variable, say A, will result in positive correlation between the levels of the other independent variable, say B, across the levels of the non-null independent variable, A. If the previous sentence is not intuitively sufficient, consider the following example. Let A and B be two independent variables whose levels are completely crossed. Allow the variance of A, B and error to be defined as, $\sigma_A^2 > 0$, $\sigma_B^2 = 0$, and $\sigma_e^2 > 0$, respectively. If A and B are the only independent variables, then any observation is a function of the effect of a level of A and error. Then the covariance of two levels of B, for example level one and level two, across the levels of A will be equal to

$E[(A+e_1)(A+e_2)] - E[A+e_1]\ E[A+e_2]$, which is equal to the variance of A plus the covariances of A and e_1, A and e_2, and e_1 and e_1. If the preceding three covariances are assumed to be equal to zero, the covariance of two levels of B is equal to the variance of A. Since $\sigma_A^2 > 0$ and the variances of the errors is greater than zero the correlation will be greater than zero.

In case the reader is not familiar with what is meant by the term crossed, note that crossed as used above means that all the levels of factor A occur in combination with all of the levels of factor B so that the number of combinations is equal to the Cartesian product of the number of levels of A and the number of levels of B. The term crossed is used in contradistinction to nested which implies a two stage selection procedure. Where, if the levels of A were nested within the levels of B, the levels of B would be selected first and the levels of A could then be freely selected (without replacement) within each of the levels of B. As a matter of definition, the levels of the factor which represents the subjects or experimental units are crossed with the levels of the factor which represents repeated measures in a repeated measures experimental type of design. Recognize at this point that all "within" type factors have levels crossed with or nested within the levels of the factor for repeated measures and have levels which are crossed with the levels of the factor which represent the subjects or experimental units. Thus in order to not violate the assumption of independence between the response evaluations for subjects or the experimental units the factor for repeated measures must not represent a non-null source of variation nor may any factor with levels crossed with or nested within the levels of the factor for repeated measures.

As previously mentioned only in the trivial case of a point distribution would a random source of variation be a null source of variation. If a factor does not represent a random source of variation, no generalization can be made to the levels of that factor which did not occur in a given experiment. This is a severe limitation, in that items are almost always crossed with or nested within repeated measures and experimenters seldom wish only to make a conclusion that is restricted to the items which they actually use in an experiment.

Next consider the type of response which must be evaluated in an experiment designed to be consistent with Table 1. In a paired associate task a stimulus "word" is paired with a response "word," by the experimenter, for the subject, in the initial phase of the experiment. Thereafter when presented with the stimulus word a subject is to respond with the response word, which the experimenter has indicated should be associated with the stimulus word. It would often be very difficult for an experimenter to evaluate a response in any other than a dichotomous fashion. That is the subject either recalled correct response word or the subject did not.

Next consider the number of paired associates in a set. If the number is as small as two the evaluation of the set would be a discrete variable which could take on the values 0, 1, or 2, corresponding to both incorrect, one correct and both correct respectively. Thus the dependent variable would be a discrete variable with a three point distribution rather than a normal random variable with a continuous normal variate as assumed by the ANOVA.

In order to circumvent the problem associated with the random non-null source of variation, items, crossed with the source of variation due to subjects or experimental units, a typical experimenter will sum the response evaluations for the stimuli to form a repeated measures score. He will then do his analysis on those repeated measures scores. In doing so, he has eliminated one problem, but caused another. In order to demonstrate this problem three repeated measures experimental designs will be defined and the analysis for them will be discussed when stimuli provide a random source of variation. It will be shown that when the response evaluations are simply summed and ignored as a factor in the design, the test within the ANOVA framework for a repeated measures effect is not correct. The first of the three designs is the simplest form of a repeated measures experimental design. It has only two factors: subjects and repeated measures. For the purposes of this paper this simple design will be called Design 1. If the repeated measures scores in Design 1 are formed as the sums of response evaluations to items, two more designs can be considered. The first of these two will be called Design 2 in which the factor items is crossed with the factor repeated measures. In the second which will be called Design 3, the levels of the factor items are nested within levels of the factor repeated measures.

Design 2 may be termed a three way factorial design with subjects crossed with repeated measures and items. Design 3 is a three way factorial with subjects crossed with repeated measures and items and items are nested within repeated measures. Tables 4, 5, and 6 represent

Table 4

Sources of variation, degrees of freedom, and expected mean squares for Design 1

Source	df	E(MS)
A (subjects)	s-1	$\sigma_e^2 + r\sigma_A^2$
B (repeated measures)	r-1	$\sigma_e^2 + \sigma_{AB}^2 + s\sigma_B^2$
AB	(s-1)(r-1)	$\sigma_e^2 + \sigma_{AB}^2$

Note, not all of the above are variances, since there are some fixed effects.

Table 5

Sources of variation, degrees of freedom, and expected mean squares for Design 2

Source	df	E(MS)
A (subjects)	s-1	$\sigma_e^2 + r\sigma_{AC}^2 + tr\sigma_A^2$
B (repeated measures)	r-1	$\sigma_e^2 + \sigma_{ABC}^2 + t\sigma_{AB}^2 + s\sigma_{BC}^2 + st\sigma_B^2$
AB	(s-1)(r-1)	$\sigma_e^2 + \sigma_{ABC}^2 + t\sigma_{AB}^2$
C (items)	t-1	$\sigma_e^2 + r\sigma_{AC}^2 + rs\sigma_C^2$
AC	(s-1)(t-1)	$\sigma_e^2 + r\sigma_{AC}^2$
BC	(r-1)(t-1)	$\sigma_e^2 + \sigma_{ABC}^2 + s\sigma_{BC}^2$
ABC	(s-1)(r-1)(t-1)	$\sigma_e^2 + \sigma_{ABC}^2$

Note, not all of the above are variances, since there are some fixed effects.

Table 6

Sources of variation, degrees of freedom, and expected mean squares for Design 3

Source	df	E(MS)
A (subjects)	s-1	$\sigma_e^2 + \sigma_{AC:B}^2 + tr\sigma_A^2$
B (repeated measures)	r-1	$\sigma_e^2 + \sigma_{AC:B}^2 + t\sigma_{AB}^2 + s\sigma_{C:B}^2 + s\sigma_B^2$
AB	(s-1)(r-1)	$\sigma_e^2 + \sigma_{AC:B}^2 + t\sigma_{AB}^2$
C:B (items)	(t-1)r	$\sigma_e^2 + \sigma_{AC:B}^2 + s\sigma_{C:B}^2$
AC:B	(s-1)(t-1)r	$\sigma_e^2 + \sigma_{AC:B}^2$

Note, not all of the above are variances, since there are some fixed effects.

respectively for Designs 1, 2, and 3 their sources of variation, degrees of freedom, and expected mean squares. For Design 1, it can be seen by inspection of expected mean squares that a test statistic may be formed as the ratio

$$\frac{MS_B}{MS_{AB}} \tag{1.1}$$

to test the effect due to repeated measures. The computational formulas for these two mean squares with respect to Design 1 are

$$MS_B = \frac{\dfrac{\sum_{j=1}^{r} (\sum_{i=1}^{s} X_{ij})^2}{s} - \dfrac{(\sum_{j=1}^{r} \sum_{i=1}^{s} X_{ij})^2}{rs}}{r - 1}$$

and

$$MS_{AB} = \frac{\dfrac{\sum_{j=1}^{r}\sum_{i=1}^{s} X_{ij}^{2}}{1} - \dfrac{\sum_{j=1}^{r}(\sum_{i=1}^{s} X_{ij})^2}{s}}{(r - 1)(s - 1)} - \frac{\dfrac{\sum_{i=1}^{s}(\sum_{j=1}^{r} X_{ij})^2}{r} + \dfrac{(\sum_{j=1}^{r}\sum_{i=1}^{s} X_{ij})^2}{rs}}{(r - 1)(s - 1)}$$

where X_{ij} is an observation for repeated measure j on subject i, j = 1, 2, ... r, i = 1, 2, ... s. Noting the above formulas examine the computational formulas for the same mean squares with respect to Designs 2 and 3 (the formulas are identical for Designs 2 and 3). These formulas are

$$MS_B = \frac{\dfrac{\sum_{j=1}^{r}(\sum_{i=1}^{s}\sum_{k=1}^{t} X_{ijk})^2}{st} - \dfrac{(\sum_{j=1}^{r}\sum_{i=1}^{s}\sum_{k=1}^{t} X_{ijk})^2}{rst}}{r - 1}$$

and

$$MS_{AB} = \frac{\dfrac{\sum_{j=1}^{r}\sum_{i=1}^{s}(\sum_{k=1}^{t} X_{ijk})^2}{t} - \dfrac{\sum_{j=1}^{r}(\sum_{i=1}^{s}\sum_{k=1}^{t} X_{ijk})^2}{st}}{(r - 1)(s - 1)} - \frac{\dfrac{\sum_{i=1}^{s}(\sum_{j=1}^{r}\sum_{k=1}^{t} X_{ijk})^2}{rt} - \dfrac{(\sum_{j=1}^{r}\sum_{i=1}^{s}\sum_{k=1}^{t} X_{ijk})^2}{rst}}{(r - 1)(s - 1)}$$

where X_{ijk} is an observation on subject i for repeated measures j and item k. i = 1, 2, ... s, j = 1, 2, ... r, and k = 1, 2, ... t. By inspection of the two sets of computational formulas, it can be observed that the mean squares MS_b and MS_{AB} for Design 1 are linear transformations of the same mean squares for Designs 2 and 3, that is of

course given that the formulas are applied to the same data such as the X_{ijk} observations defined above and given that the repeated measures scores in the case of Design 1 were formed as a simple sum of response evaluations to the items which are associated with the repeated measures. Since the linear transformation for each mean square was the same, ratio 1.1 will be invariant across Designs 1, 2, and 3.

The invariance of ratio 1.1 provides interesting implications for the case in which items are a random non-null source of variation, and yet Design 1 was considered appropriate and the analysis consistent with it was employed. Inspecting the expected mean squares for the source--repeated measures and the source--subjects by repeated measures interaction in Tables 5 and 6, it can be seen that ratio 1.1 is not the appropriate statistic to test for the effect due to repeated measures. Therefore, it should be apparent that if repeated measures scores are formed as a simple sum of response evaluations for items, the analysis which is implied by Design 1 is inappropriate. If the expected values for the mean squares are substituted in ratio 1.1 for Designs 2 and 3 the kind of errors which can arise becomes apparent. For Design 2 the substitution results in the ratio

$$\frac{2\sigma_e^2 + \sigma_{ABC}^2 + t\sigma_{AB}^2 + s\sigma_{BC}^2 + st\sigma_B^2}{2\sigma_e^2 + \sigma_{ABC}^2 + t\sigma_{AB}^2} .$$

For Design 3 the substitution produces

$$\frac{2\sigma_e^2 + \sigma_{AC:B}^2 + t\sigma_{AB}^2 + s\sigma_{C:B}^2 + st\sigma_B^2}{2\sigma_e^2 + \sigma_{AC:B}^2 + t\sigma_{AB}^2} .$$

Thus, when ratio 1.1 is employed to test for an effect due to repeated measures an implicit assumption has been made. If Design 2 is appropriate the assumption is that $\sigma_{BC}^2 = 0$. If Design 3 is appropriate the assumption is that $\sigma_{C:B}^2 = 0$. If the implicit assumptions are not valid it is possible to obtain a spuriously high value of ratio 1.1. Because of the possibility that σ_{BC}^2 is unequal to zero or $\sigma_{C:B}^2$ is unequal to zero it appears important to include items as a factor in the design. When items are random, if the factor for items is included in a repeated measures experimental design, a result due to Satterthwaite (1941) provides that quasi-F ratios may be formed to test hypotheses concerning an effect due to repeated measures. To illustrate, if items are crossed with repeated measures the ratio $\frac{MS_B + MS_{ABC}}{MS_{AB} + MS_{BC}}$ will have a sampling distribution that approximates a central F distribution, when the effect of B is null. If on the other hand, items are nested within repeated measures the ratio $\frac{MS_B + MS_{AC:B}}{MS_{AB} + MS_{C:B}}$ has a sampling distribution which approximates a central F distribution, for a null situation with respect to the effect of B. Hudson and Krutchkoff (1968) did a Monte Carlo study of the empirical probability of a Type I error and the empirical power of the quasi-F test based on

normal variates and concluded that the quasi-F had properties that were generally similar to the F-test.

When dependent variables are formed as the sum of response evaluations to several items, an approximation to normality may be pleaded on the basis of the central limit theorem. But, when the response evaluations to items themselves are the dependent measures it is then the response evaluations which must be distributed normally in order to meet the assumption. In many instances a response to an item is evaluated as either a one or a zero corresponding to either an acceptable response or one which is unacceptable, in which case the dependent measure would have a two point discrete Bernoulli distribution rather than a continuous normal distribution. The problem has now been shown. The experimenter who would like to regard the items he employs in an experiment as a random sample from a population of items which has other than a point distribution with respect to the attribute of the items which effects the response evaluations, who sometimes must evaluate responses as zeros or ones, and who would like to employ the analysis of variance as a means of analysis is in a difficult position. If he fails to include items as a factor in his design one test in the ANOVA will be questionable. If he includes items as a factor he has two violations of the ANOVA model with which to concern himself.

Thus it would appear that an attempt at a resolution of the above dilemma is justified. Two possible means of

dealing with the dilemma occurred to this investigator. The first would ignore the ANOVA and attempt to formulate a new model and analysis to fit the situation and Chapter II will follow this line of investigation. The second would be to demonstrate that the ANOVA is robust to violations of independence and normality and Chapters III, IV, V and VI will examine this possibility.

CHAPTER II

MODEL DEVELOPMENT

A series of models are developed in this chapter. The models are developed for two reasons. The main reason was to get a better understanding of the experimental situation and the insight necessary to develop the data simulation algorithms discussed later. The decisions which were made with respect to the techniques of data simulation, as outlined in Chapter IV, were directly influenced by the discussion and development presented in this chapter. The secondary reason was to examine the models with the hope that a method of data analysis would become apparent which had the advantages that an ANOVA would have when the ANOVA assumptions are met, but which did not require assumptions of independence and normality.

The first model to be developed in this chapter will represent the response evaluation of the response of a random subject to a random stimulus when the evaluation can only take on the values zero or one. The second model will be an extension of the first so as to represent the response evaluations of the responses of a random subject to a series of random stimuli. Then the model will be extended to

represent possible experimental intervention between any two stimuli in the series which are presented to a subject. Finally a model will be considered to represent the situation in the previous sentence for an arbitrary number of subjects.

To set the background for the first model attention must be given to specification of the nature of the population of subjects and of the population of stimuli. First consider the population of subjects. Each of the subjects in the population will be allowed infinitely many specific abilities. All subjects will be allowed all abilities albeit each in varying amounts. Thus the subjects in the population may be distributed with respect to each of infinitely many abilities associated with subjects and the distribution of the population with respect to each will have some density. Therefore, if ability u is of interest in any particular situation the subjects can be considered as distributed in the population with respect to u and have density f_u. Now consider the population of stimuli. Each stimulus may be considered to have a potential with respect to eliciting a response within the domain of responses in which the experimenter is interested. The potential that a stimulus has to elicit a response within the domain of responses in which an experimenter is interested is in part conditioned on the understanding a subject has with respect to what is expected of him. The understanding or expectation that a subject has is a function of the instructions

which the experimenter provides and the atmosphere of the experimental situation. For example, if a subject, because of the instructions an experimenter gives him, understands that it is his ability with respect to u which is to be brought to bear in responding to say stimulus A, the potential of A may be different than if the subject thought that an ability other than u should be brought to bear in responding. A more concrete example would be represented by a situation in which the stimuli are words. If the subject thinks he is to give a free associate to, for example, the word girl when the experimenter wants a definition, the response potential of the stimulus word girl might be different with respect to the experimenter's criterion of an acceptable response than if the subject thought he should give a definition. Thus the stimuli in the population of stimuli may be distributed in infinitely many ways each with respect to the understanding respondents have about which of their infinitely many abilities should be brought to bear in responding. Let the respondent and the experimenter both understand that it is ability u which is to be brought to bear in responding to all stimuli. And, let p be the response potential of a stimulus with respect to ability u. Stimuli in the population of stimuli may be distributed with respect to p and have a density f_p.

One further parameter may effect a response evaluation. Since, given a subject, a stimulus, and a joint understanding between experimenter and subject the response evaluation

is not determined. The remaining parameter is the criterion the experimenter has with respect to what is an acceptable response. Let the experimenter's criterion be a constant c for a given experiment.

The background concepts for modeling have now been established. Before the first model is developed, however, it will be useful to establish some conventions. For the balance of this chapter unless otherwise noted capital Roman letters will be used to represent variates.[1] The corresponding lower case Roman letters will represent an evaluation or obtained value (a constant) of that variate. $\Phi(u)$ will represent the standard normal distribution function[2] evaluated at u. $\phi(u)$ will represent the standard normal density at u. $F_V(x)$ is the cumulative distribution function of the variate V evaluated at x; and $f_D(y)$ is the density of the variate D at y. With these conventions established the modeling of the response evaluations may begin.

Let the variate U represent a random subject, then u is the ability of a given subject which has been sampled. Let P represent a random stimulus and p a given one. Then let b represent the response evaluation of the response of subject u to stimulus p made by an experimenter with criterion c. The response evaluation b may be defined as a

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[1]The term variate will be considered synonymous with random variable, as it is in much of the statistical literature.

[2]That is the cumulative up to u.

function of the three values u, p, and c, conditioned on no error in the evaluation and no misunderstanding of the subject. That is

$$b = f_1(u,p,c).$$

Analogously a random response evaluation may be defined as

$$B = f_1(U,P,c)$$

where B is a variate which may take on only two values (zero or one). B is thus a Bernoulli variate and has the familiar probability function

$$P_B(b) = \theta^b(1-\theta)^{1-b}$$

where θ is the probability that B = 1.

The value p has been defined as the potential that a particular stimulus has with respect to eliciting an acceptable response. It will be useful to define a value g', where g' = 1-p. Let the range of g' be specified as $0 \leq g' \leq 1$, where if g' = 0 no matter what u and c are, b will equal one and where if g' = 1 no matter what u and c are, b will equal zero. Thus g' is in some sense the potential difficulty of a stimulus, with respect to an acceptable response. Let the range of u be $-\infty < u < \infty$ and let the density of u be $\phi(u)$. The quantile rank, r, of u is given by $r = \Phi(u)$, and has a uniform density on the interval (0,1). Note that r is the probability that the variate U is less than the value u, or in symbols,

$$\Phi(u) = \Pr(U \leq u), \quad \text{where } -\infty < u < \infty .$$

Note that the inverse exists as well, $u = \Phi^{-1}(r)$, $0 \leq r \leq 1$. With the above values defined the function which will

represent one response evaluation, f_1, may be defined

$$b = f_1(r,g',c) = \begin{cases} 0, & r \leq g' + c \\ 1, & r > g' + c \end{cases} .$$

The value c is really a part of the definition of an acceptable response, and so may be incorporated into the value which represents the stimulus' difficulty. So let

$$g = g' + c ,$$

then

$$b = f_1(r,g) = \begin{cases} 0, & r \leq g \\ 1, & r > g \end{cases} .$$

Observe that f_1 is an expression of conditional probability if the variates corresponding to b, r, and g are substituted in the expression. Assuming that R and G are stocastically independent the conditional probabilities are

$$P_r(B = 0|G = g) = P_r(R \leq g) = F_R(g) = g$$ [3]

and

$$P_r(B = 1|G = g) = 1-P_r(R \leq g) = 1-F_R(g) = 1-g$$

from which it is apparent that

$$P_r(B = b|G = g) = (1-g)^b \, g^{1-b}, \; b=0,1 \text{ and } 0 \leq g \leq 1 ,$$

or more simply written in the shorthand notation

$$p_{B|G}(b|g) = (1-g)^b \, g^{1-b} .$$

The joint probability density of B and G is

$$p_{B,G}(b,g) = p_{B|G}(b|g) \; f_G(g)$$

and the probability function of B is then

[3]Because r has a uniform probability density.

$$P_B(b) = {}_0\int^1 P_{B,G}(b,g)\,dg \;.$$

It will be possible to complete the development of the first model once $f_G(g)$ is specified. We will assume that $f_G(g)$ belongs to a family of Beta distributions (this family will be large enough to contain nearly all kinds of distributions for G that are encountered in practice). Thus we assume

$$f_G(g) = \frac{(s+f+1)!}{s!f!} g^s (1-g)^f, \; 0 \le g \le 1 \;,$$

where the parameters s and f are non-negative integers. Then

$$P_B(b) = \frac{(s+f+1)!}{s!f!} {}_0\int^1 (1-g)^b \, g^{1-b} \, g^s \, (1-g)^f dg$$

or

$$P_B(b) = \frac{(s+f+1)!}{s!f!} {}_0\int^1 g^{s+1-b} \, (1-g)^{b+f} dg = \begin{cases} \frac{s+1}{s+f+2}, & b = 0 \\ \frac{f+1}{s+f+2}, & b = 1 \end{cases}$$

which models the response evaluation of the response of one random subject to one random stimulus.

Observe that the expression f_1 which was shown capable of representing a probability conditioned on g may represent a probability conditioned on r as well. That is the

$$P_r(B = 1|R = r) = F_G(r)$$

and the

$$P_r(B = 0|R = r) = 1-F_G(r),$$

from which the conditional probability function

$$P_{B|R}(b|r) = (F_G(r))^b \; (1-F_G(r))^{1-b}$$

may be written. Let $\underline{G}$ represent a vector variate $\lceil G_1, G_2, \ldots, G_n \rceil'$, where the G_i, $i = 1, 2, \ldots, n$, are representative of n stimuli sampled independently from the population of stimuli. Thus $F_{\underline{G}}(g) = \prod_{i=1}^{n} F_{G_i}(g_i)$. Let $\underline{B}$ represent a vector variate $\lceil B_1, B_2, \ldots, B_n \rceil'$. Then the conditional probability function for $\underline{B}$ given R may be written as

$$P_{\underline{B}|R}(\underline{b}|r) = \prod_{i=1}^{n} (F_{G_i}(r))^{b_i} (1-F_{G_i}(r))^{1-b_i} ,$$

and since

$$f_R(r) = 1, \ 0 \leq r \leq 1 ,$$

the joint probability density function for $\underline{B}$ and r is

$$P_{\underline{B},R}(b,r) = P_{\underline{B}|r}(\underline{b}|r)$$

and the probability function for $\underline{B}$ is

$$P_{\underline{B}}(\underline{b}) = \int_0^1 \prod_{i=1}^{n} (F_{G_i}(r))^{b_i} (1-F_{G_i}(r))^{1-b_i} \, dr ,$$

which models the response evaluations of a series of n responses made by a random subject to a series of n random stimuli.

In order to extend the model to take into account the effects of experimental intervention, consideration must be given to what the effects of intervention would be. It would be possible to conceive of the situation as one in which the stimulus potentials are changed by experimental intervention, but that did not appeal to this investigator. Experimental intervention was instead conceived of as something that could change the subject. For example, if no

intervention occurred the values which may represent a subject ability when the subject encountered each of n stimuli in a series of n stimuli, would all be u, $u_1 = u_2 = \ldots = u_n = u$. However, if an experimental intervention occurred between the presentation of stimulus j and stimulus j+1, the subject's ability at each of the n points in time may be represented as $u_1 = u_2 = \ldots u_j$ and $u_{j+1}+K = u_{j+2}+K = \ldots = u_n+K$, where K is the change brought about in the subject's ability. Since r is a strictly monotone increasing function of u, $(r = \Phi(u))$ the same situation could be expressed in terms of r, but because of the wealth of knowledge about variates with normal or multivariate normal distribution it was considered advisable to discuss the model in terms of B, U, and a new variate V. The variate V will be defined as $V = \Phi^{-1}(g)$. The nature and distribution of the variate V may be shown by the following series of formulae. By definition $F_V(v) = P_r(V \leq v)$ and by substitution we may express $F_V(v) = P_r(\Phi^{-1}(G) \leq \Phi^{-1}(g))$. Then since $\Phi^{-1}(G)$ is a differentiable strictly monotonically increasing function the

$$P_r(\Phi^{-1}(G) \leq \Phi^{-1}(g)) = P_r(G \leq g) = F_G(g)$$

(Wadsworth and Bryan, 1960), from which we may write

$$F_V(v) = P_r(G \leq \Phi(v)) = F_G(\Phi(v))$$

or

$$F_V(v) = {}_0\!\int^{\Phi(v)} f_G(t)dt \ .$$

Then taking the derivative of both sides

$$f_V(v) = f_G(\Phi(v))\ d(\Phi(v))\ ,$$

by the chain rule and thus

$$f_V(v) = f_G(\Phi(v))\ \phi(v)\ .$$

Recall at this point that $f_G(p)$ is a member of the Beta family of distributions, that is

$$f_G(\Phi(v)) = \frac{(s+f+1)!}{s!f!} (\Phi(v))^s\ (1-\Phi(v))^f\ .$$

Let n = s+f, then f = n-s thus we may write

$$f_V(v) = \frac{(n+1)!}{s!(n-s)!} (\Phi(v))^s\ (1-\Phi(v))^{n-s}\ \phi(v)\ .$$

Let γ = s+1 and m = n+1 then

$$f_V(v) = \frac{m!}{(\gamma-1)!(m-\gamma).} (\Phi(v))^{\gamma-1}\ (1-\Phi(v))^{m-\gamma}\ \phi(v)\ .$$

We observe that f_V is the density of the γ^{th} order statistic of a sample of size M sampled from a population with standard normal distribution. Values of the first two moments of f_V have been given by Tiechroew (1956) for small values of γ and m.

Recall that the previous modeling was based on the function f_1 in such a manner so as to obtain a probability statement conditioned on g and r. Now that it is wished to account for a systematic change in u, it will be helpful to define a function which will lead to a probability conditioned on u. Before the new function is defined, note that since $\Phi^{-1}$is a strictly monotone function, that

$$g \leq r \longleftrightarrow^{4} \Phi^{-1}(g) \leq \Phi^{-1}(r) \; .$$

Therefore the function f_2 defined as

$$b = f_2 \; (\Phi^{-1}(r), \quad \Phi^{-1}(g')),$$

$$= \begin{cases} 0, & \Phi^{-1}(g) > \Phi^{-1}(r) \\ 1, & \Phi^{-1}(g) \leq \Phi^{-1}(r) \end{cases}$$

$$= \begin{cases} 0, & v > u \\ 1, & v \leq u, \end{cases}$$

is strictly analogous to the function f_1. Again the conditional probabilities may be written,

$$P_r(B = 0|U = u) = 1-P_r(V \leq u) = 1-F_V(u)$$

and

$$P_r(B = 1|U = u) = P_r(V \leq u) = f_V(u)$$

or

$$P_{B|U}(b|u) = (F_V(u))^b \; (1-F_V(u))^{1-b}.$$

The joint probability density of B and U is

$$P_{B,U}(b,u) = (F_V(u))^b \; (1-F_V(u))^{1-b} \; \phi(u)$$

and the probability function of B is

$$P_B(b) = {}_{-\infty}\int^{\infty} (F_V(u))^b \; (1-F_V(u))^{1-b} \; \phi(u) \; du \; .$$

For a series of n stimuli sampled independently

$$F_{\underline{V}}(u) = \prod_{i=1}^{n} F_{V_i}(u)$$

and

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[4]If and only if.

$$P_{\underline{B}}(\underline{b}) =$$

$$_{-\infty}\int^{\infty}{}_{\infty}\int^{\infty}\cdots{}_{-\infty}\int^{\infty}\prod_{i=1}^{n}(F_{V_i}(u_i))^{b_i}(1-F_{V_i}(u_i))^{1-b_i}\phi(u_i)du_1du_2\ldots du_n.$$

Note that

$$F_{V_i}(u_i) = {}_{-\infty}\int^{u_i} f_V(t)\ dt$$

or

$$F_{V_i}(u_i) = K\ {}_{-\infty}\int^{u_i} (\Phi(v_i))^{\gamma-1}\ (1-\Phi(v_i))^{m-\gamma}\ \phi(v_i)\ dv_i\ ,$$

where

$$K = \frac{m!}{(\gamma-1)!\,(m-\gamma)!}\ .$$

Also note that

$$\Phi(v_i) = {}_{-\infty}\int^{v_i} \phi(t)\ dt$$

or

$$(v_i) = \frac{1}{\sqrt{2_\Pi\ \sigma^2}}\ {}_{-\infty}\int^{V_i} \quad -\frac{t^2}{2}\quad dt\ .$$

Observe that a complete expression of $P_{\underline{B}}(\underline{b})$ would involve a very complicated set of multiple integrals and could not be easily set upon one page. In any event the above expression for $P_{\underline{B}}(\underline{b})$ allows for a multiple value of a subject's ability with respect to forming acceptable responses to a series of n stimuli.

If a treatment intervention did occur between the presentation of stimulus V_j and stimulus V_{j+1}, it would be of interest to test a hypothesis about a change in the subject's ability with respect to responding in an acceptable

manner. Symbolically, if the subject's ability on n series of occasions were represented as $u_1, u_2, \ldots, u_j,$ $u_{j+1}+K, u_{j+2}+K, \ldots, u_n+K$; one hypothesis would be

$$H_0: \quad K = 0$$

to be tested against the alternative

$$H_1: \quad K \neq 0 \ .$$

If K = 0 there should be no systematic difference between the first j response evaluations and the remaining n-j response evaluations under the above model, $P_{\underline{B}}(\underline{b})$. On the other hand if $K \neq 0$, the model below for $P'_{\underline{B}}(\underline{b})$ would apply.

$$P'_{\underline{B}}(\underline{b}) = {}_{-\infty}\int^{\infty} {}_{-\infty}\int^{\infty} \cdots {}_{-\infty}\int^{\infty} \prod_{i=1}^{j} P_{B_i, U_i}(b_i, u_i)$$

$$du_1,\ du_2,\ \ldots,\ du_j$$

$${}_{-\infty}\int^{\infty} {}_{-\infty}\int^{\infty} \cdots {}_{-\infty}\int^{\infty} \prod_{i=j+1}^{n} [P_{B_i, u_i+k}(b_i, u_i+K)]$$

$$du_{j+1},\ du_{j+2},\ \ldots,\ du_n \ .$$

Under the model for $P'_{\underline{B}}(\underline{b})$, if $K \neq 0$, there would be a systematic difference between the first j response evaluations and the remaining n-j. Thus in order to test the hypothesis that K = 0, first assume the model for $P'_{\underline{B}}(\underline{b})$. Second, select a statistic which contrasts the first j with the remaining n-j response evaluations. Third, determine the distribution of the selected statistic under the condition of K = 0 and under the condition of $K \neq 0$. Fourth, determine some decision rule and then perform the experiment and let the results decide.

One statistic which has some appeal is n times the difference between the mean of the first j response evaluations and the mean of all n response evaluations. This statistic can be shown to have a binomial distribution with parameters p and n, if $j = \frac{n}{2}$, the evaluations are independent, and $p = \Phi(v_i)$, for all i, i = 1, 2, ..., n. The above restriction on v_i would be true only in what was previously called (in Chapter I) a trivial case, the trivial case occurring only when the variance of v is zero. However, the binomial gives a "good" approximation when the variance of v is "small." Unfortunately the binomial distribution does not hold up for a situation in which there is more than one experimental intervention. As the number of interventions increase the distribution of the above suggested statistic becomes more platykurtic. The short consideration of n times the "effect" statistic was given at this point to show that one promising statistic which this investigator tried, proved to be less than ideal.

Further complications in modeling occur, when the situation is considered in which ℓ random subjects respond to the same series of n random stimuli. Now a probability function for an ℓ by n matrix of response evaluations is desired. Let $[B_{ij}]$ represent the ℓ by n matrix of response evaluations.

$$P_{[B_{ij}]}([b_{ij}]) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} P_{[B_{ij}]|\underline{U}}([b_{ij}]|\underline{u})\ \phi(\underline{u})\ du$$

could express the desired probability function. The density $\phi(\underline{u})$ presents no problem for it is simply the multivariate

standard normal density, that is $\underline{U}$ is distributed as a standard normal vector variate with mean vector $\underline{U}$ and covariance matrix $\sigma^2 I$ (since subjects are sampled independently). However, the conditional probability density function $P_{[B_{ij}]|\underline{U}}$ can not be expressed as the product $\prod_{i=1}^{\ell} P_{\underline{B}_i|U_i}(\underline{b}|u_i)$, since all subjects must respond to the same series of n stimuli. Note: if each subject responded to a series of n stimuli sampled for and only for that subject $P_{[B_{ij}]|\underline{U}}([b_{ij}]|\underline{u})$ could be expressed as the product $\prod_{i=1}^{\ell} P_{\underline{B}_i|U}(\underline{b}_i|u_i)$. Note, however, that experimenters are usually reluctant to sample a separate set of stimuli for each subject.

Because of the inequality

$$P_{[B_{ij}]|\underline{U}} \neq \prod_{i=1}^{\ell} P_{\underline{B}_i|u_i} ,$$

this investigator felt that the development of a new model and concomitant analysis was approaching a complexity which would preclude it as a practical means of dealing with the problems laid out in Chapter I. Although the attempts at modeling provided insight, which proved to be very helpful in selecting an algorithm to simulate the data considered in the following chapters, it would appear to this investigator at this point that a demonstration of robustness of the ANOVA model would prove a more fruitful means of finding an analysis for the situation in which several random subjects respond to a series of random stimuli with experimental intervention.

CHAPTER III

A CONSIDERATION OF THE ROBUSTNESS OF THE F-TEST WITH RESPECT TO DEPENDENT BERNOULLI VARIABLES

Several investigators have considered the problem of the application of the ANOVA techniques to the analysis of categorical data. Hsu and Feldt (1969) generated empirical sampling distributions of F-statistics calculated from ANOVA procedures for a simple randomized design with respect to four different discrete dependent variables. The four different discrete dependent variables had either five, four, three, or two point distributions. Hsu and Feldt concluded from their empirical data which was generated for a true null hypothesis, that in most of the cases they investigated, the probability of a Type I error, α, is very close to the tabled values of the F-distribution, for $\alpha = .1$, .05, and .01. Lunney (1968) generated empirical sampling distributions of F-statistics calculated from ANOVA procedures for one and two way factorial fixed models under central (null hypothesis true) and non-central conditions, with respect to Bernoulli type dependent variables. Lunney concluded that in situations where the independent variables were fixed, the probability of a "one" was between .2 and .8, and the

degrees of freedom for mean square error were 20 or more; a good fit of the empirical sampling distribution of the F-statistic to the F-distribution had been obtained. Lunney also concluded that for situations in which the independent variables were fixed, the probability of a one was between .1 and .9, and there were 40 or more degrees of freedom for the mean square error, a good fit of empirical to theoretical F-values was obtained. For cases where the degrees of freedom were less than the above limits, Lunney indicated that the F-test was generally conservative.

Donaldson (1966) considered the power of the F-test for continuous non-normal dependent variables and for normal dependent variables with unequal cell error variances. Donaldson generated empirical central distributions and empirical power curves for the F-test, calculated with respect to fixed effects models where the dependent variable was either a normal, lognormal or an exponential variate. The reasonably good fit of empirical to theoretical F-distribution for the situations investigated was explained by Donaldson in terms of the effect of the central limit theorem on the mean square for a hypothesis, MS_h, and the correlation between the MS_h and the mean square error, MS_e, in the F-ratio:

$$F_h = MS_h(MS_e)^{-1}. \tag{3.1}$$

Donaldson pointed out that the calculation of MS_h is based on averages. Thus as the number of values being averaged increases the distribution of MS_h becomes less

sensitive to non-normality, by the central limit theorem. The effects of non-normality, number of groups, and group size on the variance of MS_h, will be enlightening with respect to the statement made in the preceding sentence. Let k be the number of experimental classifications or treatment groups and n, the number of experimental units per group for a balanced fixed effects design. When the null hypothesis is true the variance of MS_h is given by the expression

$$\text{Var } (MS_h) = \frac{2\sigma^4}{k-1} \left[1 + \frac{1}{2} \gamma_2 \left(\frac{k-1}{nk}\right)\right] ,$$

where σ^2 is the variance of the dependent variable and γ_2[1] is the kurtosis of the dependent variable. When $\gamma_2 = 0$, as for the normal variate, the var $(MS_h) = 2\sigma^4 (k-1)^{-1}$. Observe that as $n \to \infty$, var $(MS_h) \to 2\sigma^4 (k-1)^{-1}$. Thus the effect of non-normality on the variance of (MS_h) is diminished with an increase in n.

The situation with respect to the distribution of MS_e is very different than that for the distribution of MS_h. The variance of MS_e is given as

$$\text{Var } (MS_e) = \frac{2\sigma^4}{k(n-1)} \left[1 + \frac{1}{2} \gamma_2 \left(\frac{n-1}{n}\right)\right] .$$

Observe that as n becomes large the effect of γ_2 becomes maximal (Scheffé, 1959). Thus as n becomes large, when $\gamma_2 \neq 0$, the variance of MS_h for non-normal variates becomes closer to the variance of MS_h for normal variates, but as n

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[1] $\gamma_2 = \sigma^{-4} \Sigma[(x - \Sigma[x])^4] - 3.$

becomes large the variance of MS_e for non-normal variates ($y_2 \neq 0$) becomes less like the variance of MS_e for normal variates. It may be shown that when MS_h and MS_e are statistically independent, the variance of (3.1) is approximated by a function of σ and the sum of the variances of MS_h and MS_e. Therefore, if σ and var (MS_h) are held constant and the variance of MS_e is increased the variance of (3.1) will be increased. Thus the apparent result of an increase in var (MS_e), with no other concomitant change, would be to cause the distribution of (3.1) to become more platykurtic than the corresponding F-distribution (with k-1 and n(k-1) degrees of freedom). A statistic with a distribution similar to F, but more platykurtic than F, would give too many values in a tail rejection region based on the F-distribution. It will be shown, however, that the correlation between MS_h and MS_e is principally a function of y_2 and that for $y_2 \neq 0$, MS_h and MS_e are not independent. Note that a positive correlation between MS_h and MS_e would cause the distribution of (3.1) to be more leptokurtic than a corresponding F-distribution. If the leptokurtosis due to a positive correlation were to balance the platykurtosis due to an increase in the var (MS_e) a robust situation with respect to the variance of F would be evident.

From Donaldson (1966) the

$$\text{corr}\ (MS_h,\ MS_e) = y_2\ \left[\frac{4n^2k}{(k-1)(n-1)} + \frac{2n(nk-1)}{(k-1)(n-1)}\, y_2 + y_2^2\right]^{-\frac{1}{2}}$$

and the

$$\text{cov}\ (MS_h,\ MS_e) = y_2\sigma^4\ (nk)^{-1}.$$

Then from Hansen, Hurwitz, and Madow (1953), the
$\mathrm{var}(MS_h(MS_e)^{-1}) \simeq \sigma^{-4}[\mathrm{var}(MS_h) + \mathrm{var}(MS_e) - 2\ \mathrm{cov}(MS_h, MS_e)]$.

Then by substitution

$$\mathrm{var}(MS_h(MS_e)\ ^{1}) \simeq \sigma\ ^{4}[(\frac{2\sigma^4}{k-1} + \frac{y_2\sigma^4}{nk}) + (\frac{2\sigma^4}{k(n-1)} + \frac{y_2\sigma^4}{nk}) - \frac{2y_2\sigma^4}{nk}].$$

Observe that the terms containing y_2 cancel (Donaldson, 1966). Thus the

$$\mathrm{var}\ (MS_h(MS_e)^{-1}) \simeq \frac{2}{k-1} + \frac{2}{k(n-1)}\ .$$

Also observe that k-1 and k(n-1) are v_1 and v_2 the degrees of freedom for MS_h and MS_e respectively. Thus

$$\mathrm{Var}\ (MS_h(MS_e)^{-1}) \simeq \frac{2}{v_1} + \frac{2}{v_2} + \frac{2(v_1+v_2)}{v_1 v_2}\ .$$

The variance of F is given by,

$$\mathrm{var}\ (F) = \frac{2(v_1 + v_2 - 2)}{v_1 v_2 - 8 + 16v_2^{-4} - 16v_2^{-2}}\ .$$

Observe that as v_2 increases the approximate variance of (3.1) approaches the var (F). As an example the var (F) for five groups with 20 experimental units each is .556+ and the approximate variance of (3.1) is .520.

It would appear that the F-test for fixed effects and balanced designs, is in general robust with respect to non-normality given the other assumptions on which the F-test is based, when n is sufficiently large. Recall that the question of robustness which is of concern in this paper is with respect to non-normal non-independent dependent variables, particularly non-independent Bernoulli dependent variables. Recently Seeger and Gabrielsson (1968) and Mandeville (1970) have considered the problem of the applicability of the F-test to the analysis of zero-one data which arise from a situation which may be represented by a repeated measures design. Seeger and Gabrielsson employed simulation techniques to obtain empirical sampling distributions for Cochran's Q statistic (Cochran, 1950), the F-statistic (3.1), and the F-statistic calculated following an arc sin transformation on the dependent variables, all with respect to a true null hypothesis of no repeated measure differences. The Seeger and Gabrielsson simulations were divided into 60 sets with respect to the degree of dependence between repeated measures, degree or absence of subject by repeated measures interaction, the number of independent observations on a subject repeated measure combination,[2] and the number of subjects. The number of repeated measures was fixed at five for all of the above 60 sets of simulations. The conclusions reached by Seeger and Gabrielsson may be summarized as follows:

1. For all cases investigated the arc sin transformation produced no significant improvement in the fit of the empirical F for repeated measures to the corresponding F-distribution.

2. The Q test is applicable to the situations investigated only if no interaction of subjects by repeated measures is present.

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[2]That is Seeger and Gabrielsson's data were simulated in such a way that a repeated measure score could be thought of as the sum of independent item (zero-one) scores.

3. In the presence or absence of an interaction the F-test gives a generally good fit to theoretical values, and a better fit than does Q in all but a couple of the 60 cases investigated.

Mandeville (1970) simulated data for a repeated measures design with subjects nested within groups, where the dependent variable was a Bernoulli variate. In his study Mandeville allowed the number of groups to be two or four, the number of repeated measures to be two, three, six or ten, the number of subjects per group to be five, ten or twenty, and the correlation between repeated measures to be 0, .2, .5, or .8. For the central cases he generated, Mandeville found generally good agreement between the corresponding F-distribution and the empirical sampling distributions of the F-statistics for groups, repeated measures and groups by repeated measures interaction tests. For the non-central cases he investigated (non-null repeated measure by group interaction and non-null group effects) Mandeville found consistently less power than the nominal power of the F-test.

Although the Mandeville and the Seeger and Gabrielsson studies dealt with dependent Bernoulli variates with respect to experimental designs very similar to those which are of interest to this paper, they did not deal with precisely the situation of interest; that is the three (or more) way factorial design with two random non-null main sources of variation, subjects and items. Bradley (1968) has indicated that the robustness of a parametric test is idiosyncratic rather than general with respect to any violation or set of violations. For example the F-test has been shown to be robust to heteroscedasticity for balanced experimental designs (equal cell "n's"), but generally not robust to heteroscedasticity for unbalanced designs. Thus the robustness or lack thereof, of the F-test, to the particular situation of interest in this paper, should be examined although the studies cited above indicate there is some hope that the F-test may be robust under the conditions of interest to this paper. In order to further examine the robustness of the F-test to non-independent Bernoulli variates, data were simulated and empirical sampling distributions of F-statistics were determined. The cases investigated and the means of data generation will be discussed in the next chapter.

CHAPTER IV

METHOD OF DATA GENERATION AND CASES GENERATED

Data were generated to simulte the results of experiments conforming to Designs 2 and 3 in Chapter I. The data were first generated as s·r·t pseudo random standard normal numbers (mean zero and unit variance) and then subsequently dichotomized. The above procedure allowed for the analysis of the normal data before dichotomization, which enabled the examination of the effects of item and subject heterogeneity[1] on the ANOVA statistics based on the normal data and comparisons between the ANOVA statistics based on normal data and the ANOVA statistics based on Bernoulli type data. First, the method used to obtain pseudo random standard normal numbers will be discussed. Then it will be shown how the standard normal numbers were manipulated to obtain the desired data simulations.

Many methods for the generation of random, pseudo random, and quasi-random numbers have been proposed. A book

[1]Heterogeneity is used here in the strong sense implying that the subject or item effect with respect to the dependent variable is non-null.

by Hammersley and Handscomb (1964) discusses several of the methods and a review by Greenberger (1961) considered pseudo random number generators in detail. More recently Marsaglia and Bray (1968) have suggested fast composite generators which employ two stage procedures.

Although ANOVA statistics based on the normal data were calculated, the main interest of this paper concerned statistics based on the dichotomized data. Because of the main concern of the paper, the algorithm selected to generate the pseudo random normal deviates was chosen for speed and efficiency over other algorithms which give a slightly better fit to the normal density. In the first phase of the number generation algorithm a series of numbers which had a uniform distribution were obtained by the congruential method (Lehmer, 1951). The congruential method requires an initial value w_i and the series is then generated by the recurrence relation, $w_{i+1} = a\ w_i +$ (modulo m), where m is a large integer and a, c, and w_i are integers between 0 and m-1. Once a series of 16 values had been obtained by the congruential method, their sum was formed and a linear transformation was performed on the sum to result in the value Z such that the $E(Z) = 0$ and $E(Z^2) = 1$. By the central limit theorem, the distribution of Z will be approximated by the standard normal density. Henceforth, the variate Z will be referred to as though it did have a standard normal probability density.

Now, the algorithm with respect to the establishment of a s·r·t three dimensional array of normal numbers which could represent the results on experiments conforming either to Design 2 or Design 3 will be discussed. In the first step of the algorithm, a normal deviate was generated for each of the s subjects. Then each of the s normal deviates was rescaled to obtain the desired subject variance. In the second step r·t pseudo random normal deviates were generated to represent error scores, for the r·t observations made on each subject. These error scores were then rescaled to obtain the desired error variance. Then for each subject the subject deviate was added to each of the r·t error deviates associated with that subject to obtain r·t values which could be termed r·t "observed" normal deviates for that subject with variance equal to the sum of error and subject variances. Four different values of subject variance were employed in the series of simulations for this paper. In each case the error variance was selected to compliment the subject variance so as to obtain "observed" deviates with unit variance. The subject and error variances for four levels of subject heterogeneity were as given below. These ratios of subject to error variance were deemed by the author as similar to those that often occur in practice.

Level 1:	Subject variance = .2	Error variance = .8
Level 2:	Subject variance = .3	Error variance = .7
Level 3:	Subject variance = .4	Error variance = .6
Level 4:	Subject variance = .5	Error variance = .5.

At this point in the algorithm a branching situation was dealt with, with respect to whether items were fixed and homogeneous or random and heterogeneous. If items were to be fixed there need be no changes made in the values with respect to the item. If items were random the algorithm dealt with another branching situation with respect to whether items were crossed with or nested within the repeated measures. If items were to be crossed, t deviates were obtained and rescaled to have the desired item variance. Then each of the t deviates was added to each of the r·s deviates with which it was associated. If items were to be nested t·r deviates were obtained and rescaled, then each of the t·r deviates was added to the s deviates with which it was associated. Three different values of item variance were employed in the simulations for this paper, they were .0808, .1514 and .1739. These variance values corresponded to the variances of certain order statistics. The reason for the selection of the variances of order statistics will be given later in this chapter.

The next step in the algorithm was with respect to the null or non-null effects of repeated measures and the null or non-null effects of subjects by repeated measures interaction. Three possibilities were allowed for in the algorithm:

(1) null effects for both the subjects by repeated measures interaction and the repeated measures main effect;

(2) non-null repeated measure main effects, but null interaction effects;

(3) non-null interaction effects, and null repeated measure main effects.

The preceding three options provided the means of generating both central and non-central sampling distributions of the F-statistics for the repeated measure and the subject by repeated measure interaction tests. If both interaction and repeated measure main effects were null, no further changes had to be made. If the repeated measures effect were non-null each of r repeated measure effects was added to the s·t values with which it was associated. If the interaction was to be non-null the s·r·t values were divided into two arrays with respect to subjects; then r repeated measure effects were established for one array and r different repeated measure effects were established for the other array. The effects were established in such a way as to cancel each other out with respect to repeated measures main effect. At this point in the algorithm the s·r·t values were considered to represent simulated data from Design 2 or Design 3 with normally distributed response evaluations.

The next section of the algorithm dealt with the establishment of Bernoulli type data to represent data from Design 2 or Design 3. In order to do this another array was formed of the same dimensions of the array discussed above. Let the array with normal values be represented as $[X_{ijk}]$ and let the array with zeros and ones be $[y_{ijk}]$. The y_{ijk} elements of $[y_{ijk}]$ were formed by the rule

$$\text{If } \begin{cases} (X_{ijk}-h) \leq 0, \; y_{ijk} = 0 \\ (X_{ijk}-h) > 0, \; y_{ijk} = 1 \, . \end{cases}$$

Following the example of Lunney (1969) the probability of a one in $[y_{ijk}]$ was either .5, .2, or .1 (given all null effects) for which h took on the values of 0, .84, or 1.28 respectively.

Recall from Chapter II that the Beta family of variates may represent most of the distributions of random item difficulties that are encountered in practice. Also recall the relationship between a Beta variate and an Order variate, that is, that an item effect for normal data which is Order (γ, m) can produce an item effect for the same data after dichotomization, that is, Beta $(\gamma-1, m-\gamma)$.[2] In practice, this investigator has observed that tests which have mean item difficulties of .5, .2 and .1 often have a distribution of item difficulties which may be approximated by the densities of Beta (9,9), Beta (2,10) and Beta (1,14)

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[2]Recall from Chapter II that $s = \gamma-1$ and $f = m-s-2$.

respectively. The corresponding order variates would have densities of Order (10,19), Order (3,13) and Order (2,16) and would have variances .0808, .1514 and .1739 respectively, the variances employed in this study.

In review, first subject heterogeneity was taken into account by the algorithm and four levels were employed. Next, items were allowed to be fixed and null in effect or random and non-null in effect and crossed or nested. Then, the null or non-null effects of repeated measures and the null or non-null effects of subject by repeated measures interaction were allowed. Following the above, the normal data were dichotomized and three levels of a probability of a one in $[y_{ijk}]$ were employed.

In this study interest did not center on the effect of varying the number of items or on the effect of varying the number of repeated measures; thus r and t were set at four and three respectively, values which were as small as would usually be found in practice. Five levels of s were investigated beginning with s = 4, a very small value, and then s = 6, 8, 10 and 12: values which were not unusual in practice. Three sets of expected repeated measure mean scores for the dichotomous data were selected for the non-null repeated measure effect cases. The means were: (1) .75, .50, .50, .50; (2) .36, .20, .20, .20; and (3) .19, .10, .10, .10. Note that since for a Bernoulli variate the variance is p(1-p) where p is the probability of a one, that for h = 0, .84 and 1.28 the expected variances are .25, .16

and .09 respectively. Then note that for the non-null repeated measures cases defined on the dichotomized data the first expected repeated measure mean exceeded the others by the amount of the null case variance. Thus the non-centrality parameter given by the expression

$$\delta = \sqrt{\frac{N \sum_{j=1}^{r} \beta_j^2}{\sigma}} ,$$

where β_j is the effect of the j^{th} repeated measure, will be the same for all h.

For the non-null situation with respect to the interaction the same values were used to deviate the repeated measure scores from the rest as were used for the repeated measure main effect non-null cases. The first half of the subjects' scores were deviated in one direction and the other half were deviated in the other direction producing a null effect for repeated measures main effect, but a non-null interaction effect for which the non-centrality parameter should be the same on the dichotomous data for all values of h.

Once both $[x_{ijk}]$ and $[y_{ijk}]$ were in their final configuration an analysis of variance was performed on both. The values of the statistics obtained from each ANOVA were then used to increment various sums, sums of squares, sums of products, and counters associated with critical values of the statistics in question. Then the entire generation process was repeated 999 more times so as to result in the

generation of 1000 samples for each case of interest. Once 1000 samples had been generated the correlation between all mean squares for the dichotomous data were calculated and printed. Then for the statistics which were of interest the frequency of values larger than α.1000 were printed for α = .05, .025 and .01.

Four levels of subject heterogeneity, three levels of a probability of a one, five levels of the number of subjects, items fixed or random, crossed or nested and one null and two non-null situations resulted in a $4 \times 3 \times 5 \times 2 \times 2 \times 3$ array of 720 cases for which data were generated. The results of those generations will be presented in the next chapter.

The entire list of the generation program may be seen in Appendix C.

CHAPTER V

RESULTS OF THE MONTE CARLO SIMULATIONS

In this chapter data will be presented with respect to the fit of F-distributions to the empirical sampling distributions of variance ratio test statistics for tests of repeated measures main effects[1] and subjects by repeated measures interaction effects, under all of the conditions and parameter configurations outlined in Chapter IV. For each condition and parameter configuration the data will consist, in part, of the frequencies of 1000 test statistics based on 1000 simulated data samples which had values occurring in α = .05, .025, and .01 rejection regions.[2] The presentation of the empirical correlations between the mean squares in the variance ratio tests for repeated measures main effects and the subjects by repeated measures interaction effects will complete the presentation of raw data. For each data table the significant trends within the table will be indicated, significant interactions graphed and

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[1]Both for the "ordinary" variance ratio tests and for the "quasi-F" variance ratio tests.

[2]The rejection regions being defined by an F-distribution with the same degrees of freedom as the variance ratio test concerned.

summary mean statistics reported where appropriate. Because the correlations between the frequencies in $\alpha = .05$, .025 and .01 rejection regions across all cases in each table of frequencies were generally greater than .9, detailed summary mean statistics will only be reported for frequencies in the $\alpha = .05$ rejection region. Similarly significant interactions will only be graphed with respect to frequencies in $\alpha = .05$ rejection regions.

The purpose of the Monte Carlo simulations discussed in Chapter IV was to determine if it was likely that ANOVA procedures could be "appropriately" employed for the analysis of data such as was simulated for this study. In order to give a reasonable consideration to the results of the simulations, it is necessary to have some criterion for "appropriate employment" of ANOVA procedures. For the purposes of this paper "appropriate employment" will be adjudged in terms of hypothesis testing and two conditions will be considered as necessary and sufficient for it. The first of the above two conditions requires that the empirical probability of a type I error, for a given hypothesis testing situation, is "reasonably close" to the nominal probability of a type I error as determined by ANOVA considerations. The second of the above two conditions requires that the power of a test of a true non-null hypothesis is not "unreasonably small." The phrases "reasonably close" and "unreasonably small" will be discussed below.

If 1000 samples are simulated so that a null hypothesis

is true and so that the assumptions of an ANOVA are met, the number of F-statistics testing the above hypothesis which have values which exceed the $F_{1-\alpha}$ quantile of a corresponding F-distribution[3] will be approximately 1000α. If 1000 samples of data are simulated so that a null hypothesis is not true and so that the assumptions of an ANOVA are met, the number of F-statistics testing the above hypothesis which have values which exceed the $F_{1-\alpha}$ quantile of a corresponding F-distribution will be approximately 1000 times the nominal power (1-β) for the situation simulated.

Let Xi be defined according to the rule

$$Xi = \begin{cases} 1, & F_i > \mathcal{F}_{1-\alpha} \\ 0, & \text{otherwise} \end{cases}$$

where F_i is the F-variate calculated on the i^{th} sample, $i = 1, 2, \ldots, 1000$, and $\mathcal{F}_{1-\alpha}$ is the 1-α quantile of the corresponding F-distribution. The variate Xi is then an indicator variable, which takes on the value one when F_i is in the α rejection region and which takes on the value zero when F_i does not occur in the rejection region. Note that the frequency of F_i's which fall in the α rejection region, F, may be represented by the expression $f = \sum_{i=1}^{1000} Xi$.

Observe that f is a binomial variate with parameters p = α and n = 1000. The variance of f is then np(1-p) or for n = 1000 and α = .05, the var (f) = 47.5. Therefore a .95

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[3]An F-distribution with the same degrees of freedom as the variance ratio for the test.

probability interval may be formed about the expected value of f, $E(f) = 1000\ \alpha$, which for $\alpha = .05$ is $Pr(36.5 < f < 63.5) = .95$.

We now have a basis on which to define what "reasonably close" to the nominal probability of a type I error may be. If the empirical frequency of 1000 variance ratio test statistics, testing a true null hypothesis, which fall in an α rejection region as defined by ANOVA considerations, is a frequency which occurs in a .95 probability interval about 1000α, the empirical probability of a type I error may be considered reasonably close to the nominal α.

The problem of what an unreasonably small value of empirical power would be is more difficult to resolve than the question concerning the empirical probability of a type I error which was considered above. Clearly an empirical power less than or equal to α would be unreasonably small (a test with this property is sometimes called a biased test), but beyond that clear lower limit, the matter must in this investigator's judgment, be arbitrary. For the purposes of this paper it was arbitrarily decided that an empirical power of less than one third the nominal power would be unreasonably small.

Table 7 contains frequencies in $\alpha = .05$, .025 and .01 rejection regions of variance ratio tests of repeated measure effects under conditions in which the repeated measure effects were null, items were crossed with repeated measures and there was no interaction between subjects and

Table 7. The Empirical Frequencies, in α-1000 Rejection Regions, of the F for Repeated Measures, Based on Normal and Dichotomous Data, Items Crossed, Interaction and Repeated Measure Effects Both Null

Probability of a One			.5						.2						.1					
α · 1000			50		25		10		50		25		10		50		25		10	
Dichotomous or Normal			D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N
No. of Sub.s.	Items	Level of Sub. Het.																		
4	Fixed	1	51	44	26	20	7	8	41	41	21	24	12	11	41	67	15	42	7	15
		2	47	55	28	27	9	9	22	33	9	17	7	6	10	53	5	25	4	14
		3	39	46	18	27	4	16	14	56	6	33	2	15	12	50	2	21	2	7
		4	45	45	22	23	8	10	12	55	6	37	3	24	9	49	5	15	5	11
	Random	1	47	50	28	25	3	11	27	56	9	30	1	19	26	44	14	20	7	12
		2	40	52	20	33	2	14	29	44	7	15	4	10	18	44	10	15	4	9
		3	27	50	18	29	6	11	27	51	11	25	3	14	13	38	1	14	1	6
		4	38	45	15	30	7	17	20	40	6	24	0	17	12	49	4	19	0	9
6	Fixed	1	51	40	25	23	8	8	50	32	16	15	7	6	42	30	13	17	9	7
		2	66	50	31	22	6	6	30	32	19	14	6	6	37	24	6	12	6	3
		3	52	51	34	28	10	14	35	34	20	12	10	5	22	29	3	14	0	5
		4	61	57	38	27	14	12	39	32	21	15	2	8	16	25	6	12	0	5
	Random	1	48	58	22	28	5	12	41	25	23	14	15	5	22	23	14	8	7	3
		2	46	49	21	27	6	10	30	29	12	16	5	7	18	29	9	12	7	4
		3	53	34	26	14	9	9	30	31	17	19	6	10	28	24	8	11	3	4
		4	51	39	15	13	5	6	38	28	20	13	3	5	32	22	11	4	0	1
8	Fixed	1	48	36	30	19	11	8	34	36	16	25	7	4	31	29	19	16	1	4
		2	47	39	22	27	8	9	42	41	18	31	10	4	30	45	22	30	0	4
		3	59	47	29	26	14	13	35	41	17	32	12	6	40	38	8	30	0	0
		4	61	46	38	23	14	11	29	44	13	26	6	8	16	48	0	34	0	5
	Random	1	53	52	26	30	10	11	52	34	15	24	7	4	39	45	19	31	9	7
		2	47	62	16	35	5	9	32	43	16	28	3	2	16	54	16	38	4	6
		3	66	51	40	37	19	9	55	32	7	27	3	3	26	54	9	37	2	4
		4	57	38	30	29	17	2	39	44	22	33	6	7	28	45	8	27	1	5
10	Fixed	1	46	55	30	29	9	15	36	49	21	26	5	16	28	49	22	25	13	10
		2	42	46	16	21	5	10	45	59	19	28	4	13	42	57	18	30	8	8
		3	55	63	32	28	5	16	44	65	15	36	3	14	25	51	3	23	0	11
		4	53	52	16	28	7	13	64	53	16	29	4	14	20	65	1	40	1	13
	Random	1	51	60	21	40	12	22	32	46	19	22	1	9	41	58	33	35	12	18
		2	57	55	30	29	12	17	47	62	20	35	8	16	36	43	12	25	7	15
		3	53	55	29	34	8	17	59	64	23	27	6	13	23	55	3	20	0	9
		4	52	59	29	37	12	21	30	59	18	40	5	14	23	43	0	35	0	12
12	Fixed	1	46	53	28	27	11	13	44	62	23	35	2	8	60	53	19	36	4	9
		2	45	63	25	39	15	16	70	49	28	29	7	4	39	47	16	35	2	4
		3	65	55	23	27	8	8	37	50	15	26	4	11	36	48	14	36	8	12
		4	63	53	24	33	10	12	58	46	21	29	9	5	17	50	4	34	4	9
	Random	1	59	57	29	33	7	18	47	56	35	37	13	14	31	45	25	28	10	6
		2	46	50	18	22	4	6	71	65	50	39	25	10	39	41	28	29	18	3
		3	53	80	28	43	12	18	60	54	33	34	22	18	45	31	25	22	11	5
		4	49	63	33	40	14	8	48	48	21	30	10	7	64	50	19	30	4	4

repeated measures. Table 7 is laid out as a six-dimensional array with three left margins and three upper margins. The left-most margin indicates the number of subjects employed with respect to a given simulation of data. Proceeding from left to right the next margin indicates the nature of the items, whether fixed and null in effect or random and non-null, and the right-most of the left margins indicates the level of subject heterogeneity. The upper margins from top to bottom indicate: (1) the probability of a one with respect to a given simulation of data, (2) one thousand times nominal α, (3) an indication of whether the 1000 variance ratios with respect to a given cell were calculated from the dependent variables when they were variates with normal density (N) or from the subsequently dichotomized normals (D).

In order to test for trends in the data reported in Table 7, the margins were considered as fixed sources of variation and a multivariate analysis of variance was performed on the frequency data three-tuples[4] within the table, employing the highest order interaction mean products as an estimate of error under the assumption that the highest order interaction is truly null.

From the first analysis of the data in Table 7 it was concluded that the frequencies for tests based on the dichotomous variates with overall mean vector 39.9, 18.5,

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[4]For α = .05, .025, and .01.

6.7[5] were significantly different from the frequencies for tests based on the normal variates with overall mean vector 46.9, 26.4, 9.7. Then subsequent analyses were performed on frequencies with respect to dichotomous and normal variates separately.

With respect to the frequencies based on dichotomous variates, it was concluded that there were significant main effects due to the probability of a one and the number of subjects as well as an interaction between the two significant main effects. The significant interaction is represented in Figure 1. In the figure the two horizontal lines represent .95 probability limits for mean frequencies such as those graphed, given an expected value of 50. Observing Figure 1, it appears that a favorable comparison of nominal α and empirical probabilities of a Type I error occurred when the probability of a one was .5 and there were six or more subjects. Also a favorable comparison occurred when the probability of a one was .2 and there were ten or more subjects. However when the probability of a one was .1 no favorable comparisons occurred, although the graph suggests that a favorable comparison might occur given more subjects. The marginal mean frequencies for the α = .05 regions for probabilities of a one equal to .5, .2, and .1 were 50.9, 39.9, and 28.8 respectively, and for numbers of subjects equal to 4, 6, 8, 10, and 12 they were 27.8, 39.2, 40.9,

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[5]Ordered for frequencies in α = .05, .025, and .01 rejection regions, respectively.

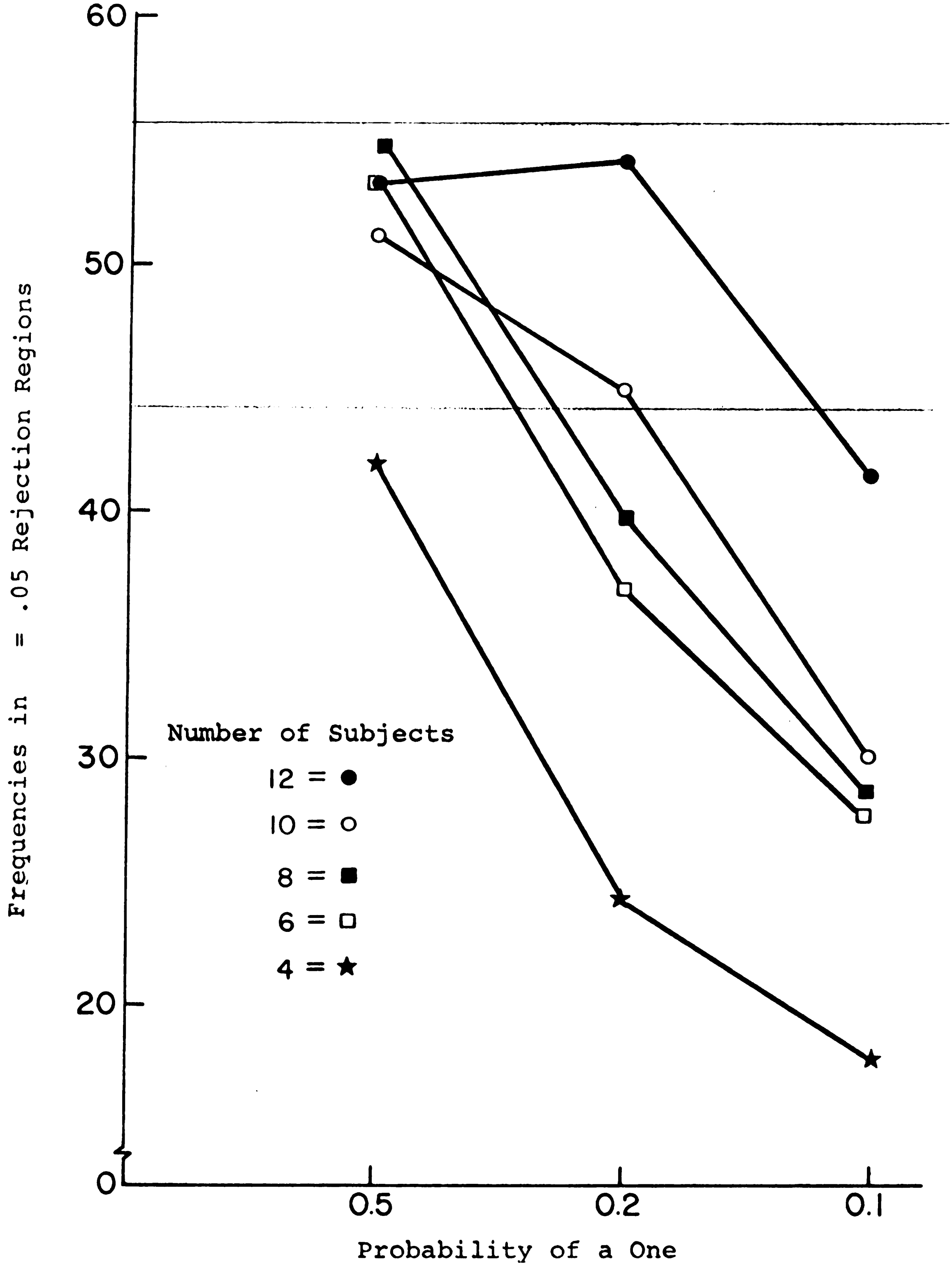

Figure 1. The interaction of the probability of a one and the number of subjects, with respect to the data in Table 7.

42.0, and 49.7 respectively.

With respect to the frequencies based on normal variates there was an unexpected effect due to the number of subjects. The marginal mean frequencies for $\alpha = .05$ regions were 48.2, 34.4, 43.5, 55.1 and 53.0 for numbers of subjects 4, 6, 8, 10 and 12 respectively, where the mean frequency for six subjects differs significantly from the other mean frequencies. The only reason this investigator can give for the unexpected low mean frequency for six subjects is that it was a peculiar occurrence which would not reoccur if the simulations were repeated with a different starting value for the random number generator.

Table 8 is analogous to Table 7 in that it differs from Table 7 in only two aspects: (1) the values in the table were obtained by simulating data which could have arisen from Design 2 rather than Design 3, and (2) no frequencies appear with respect to tests based on normal variates. The reason that frequencies with respect to tests based on normal variates do not appear in Table 8 is that because of a programming problem, the early simulation runs on which the frequencies with respect to 4, 6, 8, and 10 subjects in Table 8 are based, did not allow for the influence of non-null items effects to show itself in the tests based on the normal variates. The simulation runs for 12 subjects, however, did not suffer from the above limitation and those data will be presented in Table 16. From the analysis of the frequencies in Table 8, it was

Table 8. The Empirical Frequencies, in α·1000 Rejection Regions, of the F for Repeated Measures, Based on the Dichotomous Data Items Nested, Interaction and Repeated Measure Effects Both Null.

Probability of a One			.5			.2			.1		
α · 1000			50	25	10	50	25	10	50	25	10
No. of Sub.s	Items	Level of Sub. Het.									
4	Fixed	1	51	26	7	41	21	12	41	15	7
		2	47	28	9	22	9	7	10	5	4
		3	39	18	4	14	6	2	12	2	2
		4	45	22	5	12	6	3	9	5	5
	Random	1	73	38	14	115	61	22	44	17	6
		2	58	30	15	72	47	14	19	5	0
		3	74	23	14	59	26	6	54	32	8
		4	62	26	5	69	28	9	29	10	0
6	Fixed	1	51	25	8	50	16	7	42	13	9
		2	66	31	6	30	19	6	37	6	6
		3	52	34	10	35	20	10	22	3	0
		4	61	38	14	39	24	2	18	6	0
	Random	1	100	68	37	120	63	26	69	36	21
		2	99	56	20	119	41	18	87	51	36
		3	125	67	24	119	48	24	104	56	45
		4	134	64	19	119	63	31	78	34	19
8	Fixed	1	48	30	11	34	16	7	31	19	1
		2	47	22	8	42	18	10	30	22	0
		3	59	29	14	35	17	12	40	8	0
		4	61	38	14	29	13	6	16	0	0
	Random	1	127	63	39	131	86	38	133	64	33
		2	125	78	30	153	105	64	82	23	11
		3	122	76	36	171	115	56	90	47	24
		4	115	56	31	156	101	57	105	71	21
10	Fixed	1	46	30	9	38	21	5	28	22	13
		2	42	16	5	45	19	4	42	18	8
		3	55	32	5	44	15	3	25	3	0
		4	53	16	7	64	16	4	20	1	1
	Random	1	145	83	42	155	110	77	200	122	60
		2	172	106	58	204	135	69	153	96	48
		3	162	89	46	207	141	67	111	50	30
		4	173	104	49	211	148	94	108	42	24
12	Fixed	1	46	28	11	44	23	2	60	19	4
		2	45	25	15	70	28	7	39	16	2
		3	65	23	8	37	15	4	36	14	8
		4	63	24	10	56	21	9	17	4	4
	Random	1	206	143	80	258	160	101	177	129	50
		2	188	126	61	253	185	123	213	108	54
		3	202	148	81	206	159	121	174	97	48
		4	174	128	84	268	164	107	160	90	55

concluded that there were significant main effects due to: (1) the probability of a one, (2) the number of subjects, and (3) items fixed and null in effect versus items random and non-null in effect. There were also significant interactions between the probability of a one and items fixed vs. random and a significant interaction between the number of subjects and items fixed vs. random. Both of these interactions were represented on one graph, Figure 2. The horizontal lines in Figure 2 are .95 probability limits about 50 for means such as those graphed. An inspection of Figure 2 indicates that a favorable comparison of nominal α and empirical probabilities of a Type I error occurred when items were fixed and null in effect and the probability of a one was .5. Also a favorable comparison occurred when the items were fixed and null, the probability of a one was .2 and there were 10 or more subjects. In the absence of the above conditions, however, only unfavorable comparisons resulted.

The overall vector of mean frequencies for Table 8 was 86.6, 48.5, 24.0. The marginal mean vectors for items fixed-null vs. random--non-null were 40.0, 18.0, 6.3 and 133.2, 78.0, 41.7 respectively. The large mean frequencies for the items random non-null conditions confirm the contention made in Chapter I that non-null items effects associated with a design in which items are nested within repeated measures will cause the regular variance ratio test for repeated measures to have too many Type I errors.

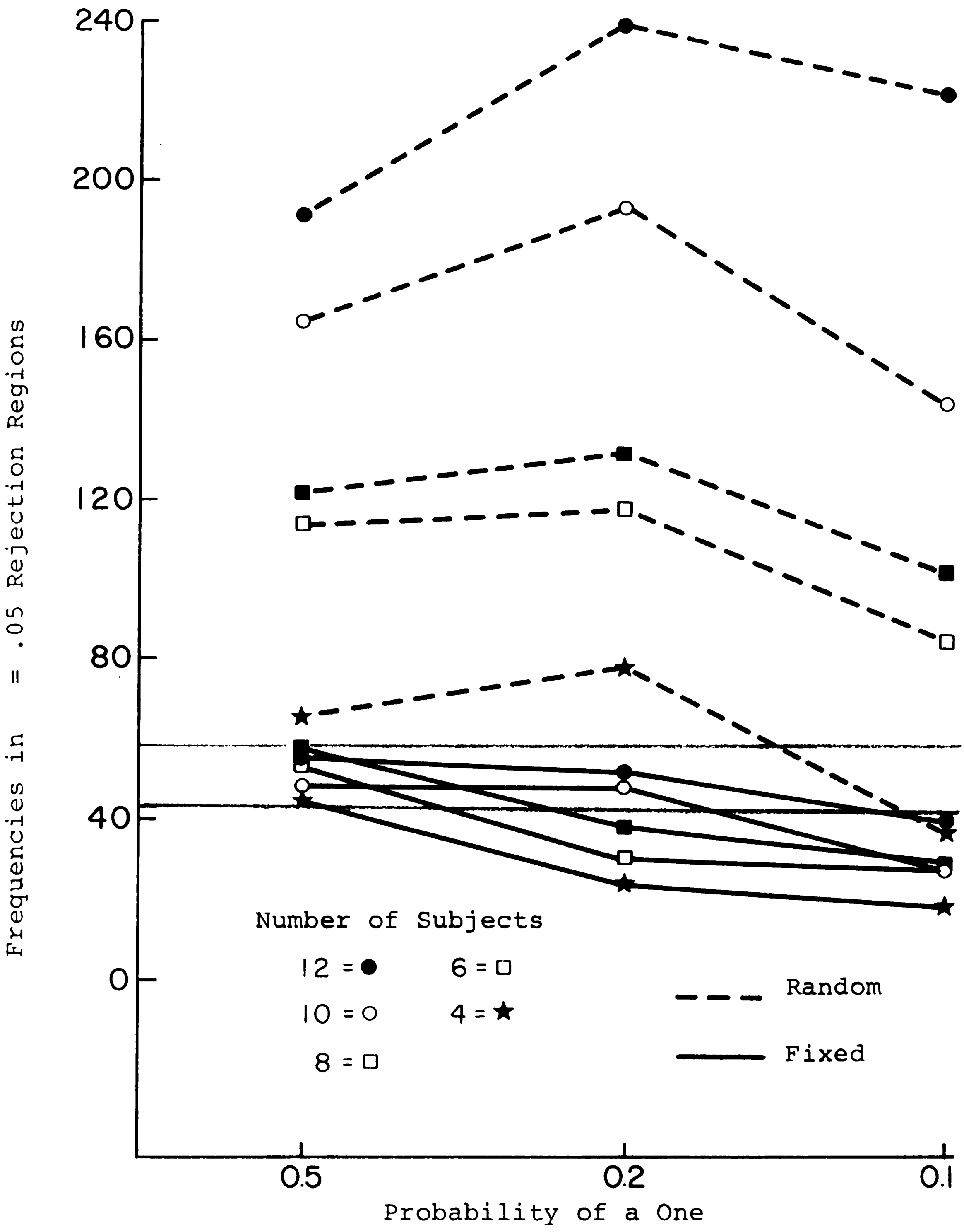

Figure 2. The interaction of the probability of a one, the number of subjects, and items fixed vs. random, with respect to the data in Table 8.

Note that when items are fixed-null the results in Table 8 are generally the same as those in Table 7, but when the item effect is random non-null the situation is quite different.

The marginal mean frequencies in $\alpha = .05$ rejection regions for the probability of a one equal to .5, .2, and .1 were 91.9, 98.7, and 69.1 respectively, and for the number of subjects equal to 4, 6, 8, 10, and 12 they were 44.6, 74.0, 82.6, 104.3, and 127.5 respectively.

Tables 9 and 10 contain the frequencies in $\alpha = .05$, .025, and .01 rejection regions of the variance ratio test statistics for tests of repeated measures effects when the data were simulated under the non-null repeated measures effect conditions indicated in Chapter IV. Table 9 contains frequencies with respect to tests based on both normal and dichotomous data, whereas the frequencies with respect to normal data were omitted from Table 10 for the same reason that frequencies with respect to normal data were omitted from Table 8.

The data in Table 9 which were with respect to normal variates with overall mean vector 517.5, 418.6, 308.7 were significantly different from the data in Table 9 which were with respect to dichotomous variates and which had an overall mean vector of 262.3, 185.6, 114.2. The data in Table 9 with respect to normal and dichotomous data were subjected to separate multivariate analyses of variance and very similar significant effects were found. For both

Table 9. The Empirical Frequencies, in $\alpha \cdot 1000$ Rejection Regions, of the F for Repeated Measures, Based on Normal and Dichotomous Data, Items Crossed, Repeated Measure Effects Non-null, Interaction Null.

Probability of a One			.5						.2						.1					
$\alpha \cdot 1000$			50		25		10		50		25		10		50		25		10	
Dichotomous or Normal			D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N
No. of Sub.s	Items	Level of Sub. Het.																		
4	Fixed	1	199	329	121	220	59	104	90	197	43	95	25	36	52	132	22	71	1	34
		2	177	342	106	222	53	119	127	231	63	134	27	57	22	124	9	56	3	36
		3	174	411	102	270	45	151	106	266	42	145	18	69	24	159	10	78	5	33
		4	204	516	113	356	61	195	70	290	33	179	7	74	27	147	15	73	5	41
	Random		163	320	99	196	44	105	130	198	67	124	33	59	39	135	21	68	8	44
		2	181	367	111	257	39	143	104	209	54	113	19	56	32	118	13	61	6	32
		3	172	372	113	253	50	131	114	247	64	138	29	72	39	139	16	73	11	35
		4	189	509	96	352	42	196	104	307	72	200	39	89	37	146	5	76	5	46
6	Fixed	1	320	521	218	388	127	253	175	313	90	192	48	100	39	173	25	105	19	46
		2	326	600	217	475	124	308	143	335	85	207	36	126	57	198	24	110	11	47
		3	346	567	232	537	116	367	141	408	96	266	38	146	62	221	9	141	2	73
		4	388	757	266	634	156	475	175	507	104	358	42	199	59	282	16	166	3	60
	Random	1	302	526	180	368	91	257	158	301	80	190	47	105	64	164	29	107	10	49
		2	290	562	202	438	93	259	152	350	97	236	43	127	47	175	17	116	4	61
		3	349	675	246	543	138	349	164	397	99	253	53	139	60	216	18	152	15	63
		4	348	751	238	602	145	452	201	516	119	383	63	221	61	287	27	177	0	95
8	Fixed	1	410	696	296	561	186	417	262	429	153	325	99	201	87	264	41	174	16	100
		2	462	790	330	679	200	521	284	478	185	352	116	233	55	294	43	178	11	96
		3	465	839	340	733	206	590	250	548	160	426	64	308	75	316	41	242	12	157
		4	510	899	371	831	223	704	273	650	198	526	99	383	74	384	35	298	5	202
	Random	1	424	717	307	594	171	444	256	419	179	327	108	201	67	246	33	163	17	93
		2	461	766	350	609	231	510	245	508	177	358	98	263	43	303	29	202	10	122
		3	467	816	341	716	227	573	230	559	163	435	96	306	75	340	28	229	8	123
		4	480	892	360	821	233	668	247	650	179	538	117	376	82	410	50	291	27	169
10	Fixed	1	516	819	401	736	262	613	327	561	245	462	162	344	121	362	72	279	25	149
		2	546	870	436	799	292	680	336	607	263	500	196	376	143	398	70	290	34	168
		3	586	927	462	876	321	759	370	716	282	591	185	454	106	388	48	317	30	252
		4	662	971	511	934	355	854	414	809	306	707	167	539	136	499	75	403	30	278
	Random	1	536	830	428	746	297	600	360	579	247	442	163	314	152	367	80	275	38	155
		2	564	886	460	811	311	697	332	643	246	525	159	375	138	411	96	313	45	203
		3	670	946	527	899	360	773	361	729	252	587	152	443	122	443	76	366	47	276
		4	681	975	535	941	396	856	345	620	251	700	157	527	107	482	65	404	29	301
12	Fixed	1	615	898	494	820	351	699	365	609	263	507	174	349	158	343	85	244	45	142
		2	623	938	500	878	356	746	343	653	250	556	133	407	125	379	94	292	44	158
		3	661	977	532	929	397	852	396	771	269	656	191	512	147	464	75	357	36	227
		4	751	958	636	969	448	936	454	854	320	754	186	614	176	469	102	405	54	292
	Random	1	611	915	472	835	333	719	365	558	255	462	157	349	150	350	117	255	54	171
		2	588	933	495	873	349	766	376	664	287	558	185	409	166	308	115	294	65	157
		3	666	966	538	930	387	858	405	752	293	642	176	486	157	487	119	348	41	220
		4	696	992	582	970	428	941	402	639	303	712	201	615	160	491	92	435	45	294

Table 10. The Empirical Frequencies, in $\alpha \cdot 1000$ Rejection Regions, of the F for Repeated Measures, Based on the Dichotomous Data, Items Nested, Repeated Measure Effects Non-null, Interaction Null.

Probability of a One			.5			.2			.1		
$\alpha \cdot 1000$			50	25	10	50	25	10	50	25	10
No. of Sub.s	Items	Level of Sub. Het.									
4	Fixed	1	199	121	59	90	43	25	52	22	1
		2	177	106	53	127	63	27	22	9	3
		3	174	132	45	106	42	18	24	10	5
		4	204	113	61	70	33	7	27	15	5
	Random	1	229	143	74	205	137	61	67	39	11
		2	215	131	56	177	136	53	87	43	14
		3	227	126	67	141	87	35	63	39	12
		4	252	166	65	171	125	58	55	32	15
6	Fixed	1	320	218	127	175	90	48	39	25	19
		2	326	217	124	143	88	36	57	24	11
		3	346	232	116	141	96	38	62	9	2
		4	398	266	156	175	134	42	59	19	3
	Random	1	374	244	138	272	177	103	116	67	49
		2	355	271	150	255	176	95	119	65	46
		3	404	269	161	240	172	73	131	69	41
		4	437	314	200	268	265	106	130	67	30
8	Fixed	1	410	296	186	262	153	99	57	41	16
		2	462	330	200	284	185	116	55	43	11
		3	465	340	206	250	160	64	75	41	12
		4	510	371	223	273	198	99	74	35	5
	Random	1	463	370	259	342	245	149	141	87	47
		2	521	435	285	355	259	157	135	79	40
		3	535	399	290	359	272	155	149	110	78
		4	575	481	323	420	292	171	141	91	38
10	Fixed	1	516	401	262	336	263	196	121	72	25
		2	546	436	292	327	245	162	143	70	34
		3	586	462	321	370	282	185	106	48	30
		4	662	511	355	414	306	187	136	75	30
	Random	1	588	505	337	440	367	269	284	215	111
		2	596	473	367	426	348	255	260	181	114
		3	661	544	397	448	363	252	246	154	88
		4	685	568	432	459	377	247	237	140	73
12	Fixed	1	615	494	351	365	263	174	158	85	45
		2	623	500	356	363	250	133	125	94	44
		3	661	632	397	396	269	191	147	75	36
		4	751	636	448	452	320	185	176	102	54
	Random	1	655	550	447	492	484	293	311	265	199
		2	686	590	479	499	484	317	272	214	182
		3	734	646	528	516	422	313	317	226	170
		4	750	667	541	515	445	333	294	221	136

dichotomous and normal there were significant main effects due to the probability of a one (which you may recall is confounded with degree of non-null effects for repeated measures), the number of subjects, and the level of subject heterogeneity. Also in both there were significant interactions between the probability of a one and the number of subjects as well as between the probability of a one and the level of subject heterogeneity. In addition within the data with respect to the normal variates, there was a significant first order interaction between the number of subjects and the level of subject heterogeneity, and a second order interaction of the probability of a one, the number of subjects, and the level of subject heterogeneity.

To find very similar effects in the data with respect to both normal and dichotomous data is somewhat reassuring, but the relationship of the empirical power of a test based on normal variates to the power of that same test based on the subsequently dichotomized variates requires further analysis.

Each frequency in Table 9 which was based on dichotomous variates was divided by the corresponding frequency in Table 9 which was based on normal variates to form a new variable which can be considered as the relative power of a test based on subsequently dichotomized variates with respect to the power of the same test based on normal variates before they were dichotomized. The relative power data were subjected to a multivariate analysis of variance and

three significant main effects and no interactions were found. The significant effects were (1) probability of a one, (2) the number of subjects, and (3) the level of subject heterogeneity. The overall mean vector of relative power variables was .45, .38, .32, which indicates that the relative power of the variance ratio test for repeated measures effects decreases as the nominal α level decreases, from .05, to .025, to .01. There was no interaction between nominal α level and the above significant effects so marginal mean relative power will be reported for $\alpha = .05$ only. For probabilities of a one equal to .5, .2, and .1 the mean relative powers for $\alpha = .05$ were .58, .49, and .28 respectively. For numbers of subjects 4, 6, 8, 10, and 12, the means were .39, .41, .43, .50, and .54. For the four levels of subject heterogeneity 1, 2, 3, and 4 the means were .52, .46, .43, and .41 respectively.

The differences in mean relative power show clear trends. As the probability of a one becomes smaller so does the relative power, which is the opposite of the trend which might have been expected, since as you may recall from Chapter IV the degree of non-null effect in the simulated data was selected to counter the effect of decreased variance corresponding to a decreased probability of a one. Thus the power of tests based on the dichotomous variates should not have changed across levels of a probability of a one, whereas the power of the test based on normals should have and did decrease across levels of a probability of a

one (and decreasing non-null effects). The results indicate however, that the power of tests based on the dichotomous variates fell off more rapidly across levels of the probability of a one than did the power of tests based on the normals. The explanation of the above may be that as the probability of a one becomes smaller the point of dichotomization is such that more of the "information" carried in the normals is lost.[6] Another clear trend is that as the number of subjects increases, the relative power does so as well. The trend with respect to the number of subjects is most likely a function of the effect of the central limit theorem. The third trend indicates a loss in relative power with an increase in subject heterogeneity, a trend for which this investigator has at present no explanation.

A multivariate analysis of variance of the frequencies in Table 10 disclosed significant differences for almost all sources of variation. The interaction of items fixed vs. random and the level of subject heterogeneity was not significant nor were the two second order interactions which included the above two sources, the probability of a one by items fixed vs. random by subject heterogeneity and number of subjects by items fixed vs. random by subject heterogeneity, but tests of all other sources indicated significant differences. Relative power ratio variables were formed for

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[6]Recall from Chapter IV that the non-null repeated measure effects were added in before rather than after dichotomization.

all of the situations with respect to Table 10 for which there were frequencies based on normal variates that were not inappropriate.[7] The relative power ratio variables that could be formed were analyzed by means of a multivariate analysis of variance with empty cells. The deficiency of the above analysis compared to the like analysis performed for Table 8 can be expressed in terms of which sources of variation are untestable due to empty cells. The untestable sources of variation are: all of the second order interactions and the first order interaction of number of subjects and items fixed vs. random. Of the testable sources of variation significant differences were found for: (1) the probability of a one, (2) the number of subjects, (3) items fixed vs. random, and (4) the level of subject heterogeneity. The overall mean vector of relative power was .47, .41, .34. The marginal mean relative powers for $\alpha = .05$ regions for probabilities of a one .5, .2, and .1 were .62, .51, and .30 respectively; for numbers of subjects 4, 6, 8, 10, and 12 they were .37, .41, .44, .50, and .54 respectively; for items fixed .45, for items random, .59; and for levels of subject heterogeneity 1, 2, 3, and 4 they were .53, .48, .45, and .44 respectively. The same trends in relative power were found in this analysis as were found in the above mentioned analysis of relative power plus an

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[7] Inappropriate frequencies based on normal variates occurred where the number of subjects was fewer than 12 and items were random--non-null.

effect for items fixed vs. random. The evidence that the relative power is greater for items random than for items fixed under Design 3 is unimportant, however, since other evidence strongly indicates the variance ratio test for repeated measures is too liberal under Design 3 when the null hypothesis is true and items are random.

Recall at this point that in Chapter I it was indicated that Satterthwaite's "synthetic variance ratios" or "quasi"-F tests could provide an appropriate test for repeated measures effects when in Design 2 there was a non-null items by repeated measures interaction or when in Design 3 there was a non-null items effect. The most startling result of this study concerned the empirical testing of the above contention with respect to dichotomous data.

Tables 11 and 12 contain the frequencies in $\alpha = .05$, .025, and .01 rejection regions of the quasi-F ratio test statistics for tests of repeated measures effects when the data were simulated under null repeated measures effect conditions and null subject by repeated measure interaction effect conditions. That is, Tables 11 and 12 are the quasi-F analogs to Tables 7 and 8.

The data in Table 11 which were with respect to normal variates with overall mean vector 36.5, 16.6, 5.6, were significantly different from the data in Table 11 which were with respect to dichotomous variates with overall mean vector 65.9, 38.1, 18.2. The data in Table 11 with respect

Table 11. The Empirical Frequencies, in $\alpha \cdot 1000$ Rejection Regions, of the Quasi-F for Repeated Measures, Based on Normal and Dichotomous Data, Items Crossed, Interaction and Repeated Measures Effects Both Null.

Probability of a One			.5						.2						.1					
$\alpha \cdot 1000$			50		25		10		50		25		10		50		25		10	
Dichotomous or Normal			D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N
No. of Sub.s	Items	Level of Sub. Het.																		
4	Fixed	1	75	45	37	25	15	12	45	59	23	29	9	15	20	60	6	33	2	22
		2	63	47	33	26	19	12	61	42	39	17	15	6	18	54	4	29	2	14
		3	63	47	36	30	12	8	30	66	20	27	8	14	6	46	3	20	0	8
		4	51	64	26	32	10	17	27	56	19	33	6	18	6	61	3	33	0	14
	Random	1	68	57	43	25	17	9	61	71	28	33	9	11	3	55	0	23	0	14
		2	55	57	28	27	15	12	34	47	26	23	10	10	8	59	5	33	2	17
		3	76	39	25	12	12	8	31	52	23	22	13	10	22	52	11	27	9	12
		4	57	44	26	25	12	12	23	62	12	38	3	19	18	50	14	23	4	13
6	Fixed	1	78	39	45	18	18	8	74	35	36	17	21	3	42	55	19	27	11	5
		2	81	34	47	13	24	5	80	37	50	17	29	5	43	38	16	21	3	3
		3	73	45	46	25	18	9	71	32	31	19	16	5	26	52	6	29	1	4
		4	87	56	45	26	19	7	49	48	25	30	14	5	12	48	7	21	0	4
	Random	1	77	39	41	20	13	7	66	43	28	19	14	7	43	41	7	21	0	2
		2	60	44	31	21	10	2	40	46	33	27	16	6	50	45	22	16	4	1
		3	83	43	49	21	26	6	60	49	44	26	13	7	38	37	15	13	0	0
		4	67	50	45	27	27	6	55	48	30	19	12	3	24	39	16	10	2	0
8	Fixed	1	102	24	57	12	29	5	57	17	40	11	15	0	48	14	30	6	11	0
		2	81	30	52	17	30	7	71	16	29	9	13	0	41	26	21	14	9	0
		3	73	38	44	19	14	8	72	25	46	15	14	0	30	27	24	5	15	0
		4	67	23	47	17	23	7	80	25	51	16	17	0	37	25	26	13	6	0
	Random	1	62	32	30	18	16	8	51	18	34	8	19	0	82	27	53	9	42	0
		2	72	36	43	21	26	10	54	19	38	5	16	0	96	33	56	12	35	0
		3	77	27	42	10	17	2	58	13	34	3	9	0	46	34	38	15	12	0
		4	64	14	40	5	16	0	60	20	38	9	19	0	53	21	30	5	0	0
10	Fixed	1	98	37	59	19	25	10	94	29	76	6	31	6	43	24	14	3	7	2
		2	93	27	63	10	33	7	94	36	53	8	32	8	110	38	81	8	53	3
		3	92	38	53	15	27	8	104	42	51	12	29	9	58	21	29	6	12	3
		4	80	36	46	14	23	4	78	34	41	5	31	5	53	39	35	11	8	4
	Random	1	92	34	62	12	26	7	70	29	39	4	5	2	62	31	33	10	18	7
		2	89	27	52	11	25	7	75	36	32	7	15	4	72	24	42	6	10	3
		3	84	29	44	12	24	6	98	27	63	2	39	2	92	30	51	8	15	4
		4	92	37	50	13	17	9	112	24	76	9	53	5	81	41	43	12	21	7
12	Fixed	1	102	25	63	12	35	7	90	25	59	10	27	1	87	30	54	16	41	5
		2	83	46	50	20	27	5	74	24	51	10	26	2	89	27	63	22	27	2
		3	94	20	50	9	23	2	91	25	44	10	23	2	92	21	35	14	17	6
		4	80	28	59	6	35	0	79	13	39	7	11	0	74	31	53	19	27	5
	Random	1	90	39	56	19	34	3	66	35	37	24	28	4	121	33	74	15	40	6
		2	94	22	41	10	24	1	67	35	49	20	19	1	101	17	49	10	30	3
		3	78	45	33	23	21	4	87	37	47	21	26	7	95	19	54	14	33	5
		4	71	21	47	10	27	2	96	34	57	12	40	3	68	34	38	15	22	4

Table 12. The Empirical Frequencies, in $\alpha \cdot 1000$ Rejection Regions. of the Quasi-F for Repeated Measures, Based on Dichotomous Data, Items Nested, Interaction and Repeated Measure Effects Both Null.

Probability of a One			.5			.2			.1		
$\alpha \cdot 1000$			50	25	10	50	25	10	50	25	10
No. of Sub.s	Items	Level of Sub. Het.									
4	Fixed	1	61	37	14	27	15	9	8	1	0
		2	59	35	17	50	32	13	6	4	0
		3	62	29	11	25	12	6	4	4	3
		4	47	25	4	24	9	5	6	3	0
	Random	1	59	38	14	45	25	16	18	4	4
		2	68	40	19	50	28	13	27	19	7
		3	64	37	13	42	13	5	32	18	11
		4	58	32	17	35	14	9	24	17	4
6	Fixed	1	77	41	17	61	36	13	51	24	3
		2	62	39	22	68	38	20	47	20	10
		3	68	39	15	70	31	23	35	13	5
		4	85	43	16	52	21	11	16	10	2
	Random	1	108	61	28	67	44	21	46	24	11
		2	86	55	28	61	35	23	35	17	8
		3	84	30	13	61	42	23	34	12	9
		4	72	46	16	64	37	21	47	25	11
8	Fixed	1	92	52	23	54	19	16	70	25	11
		2	73	44	18	51	29	9	54	20	15
		3	66	38	14	65	35	14	53	30	22
		4	67	41	16	83	47	24	37	18	1
	Random	1	102	65	33	74	45	17	44	23	8
		2	92	55	34	85	49	33	46	32	11
		3	140	101	51	113	62	38	85	36	25
		4	119	85	54	86	46	27	64	38	20
10	Fixed	1	82	49	22	90	57	35	71	27	13
		2	100	51	23	78	49	28	93	66	41
		3	87	48	25	80	50	31	66	29	12
		4	88	53	21	73	50	35	67	38	11
	Random	1	106	59	32	96	63	52	50	21	12
		2	102	65	48	134	82	52	56	34	24
		3	113	70	28	114	85	50	61	41	24
		4	87	46	28	113	75	44	45	34	10
12	Fixed	1	101	67	39	102	54	29	94	46	31
		2	78	48	19	92	53	28	96	59	33
		3	77	49	27	82	47	26	105	58	30
		4	97	69	25	85	48	12	77	52	16
	Random	1	123	77	40	85	63	29	103	82	41
		2	122	87	68	72	45	28	213	108	54
		3	128	89	58	81	53	37	102	68	44
		4	174	128	84	115	60	31	101	43	31

to normal and dichotomous data were then put to separate multivariate analyses of variance.

The analysis of the data with respect to the dichotomous variates indicated significant effects due to: (1) the probability of a one, (2) the number of subjects, the interaction of (1) and (2), and the interaction of (1) and items fixed vs. random. The marginal mean frequencies in $\alpha = .05$ regions for probabilities of a one equal to .5, .2, and .1 were 77.7, 67.1, and 52.8 respectively and for the number of subjects equal to 4, 6, 8, 10, and 12 the marginal mean frequencies were 38.6, 57.5, 63.9, 83.6, and 85.8 all respectively. The above two significant interactions are represented in Figures 3 and 4 respectively. Both figures indicate that the empirical probability of a Type I error is not in general close to .05. Figure 3 indicates that although the frequency in the rejection region is less affected by the probability of a one as the number of subjects increases, the tests become rather liberal. Figure 4 is self explanatory.

The analysis of the data in Table 11 with respect to the normal variates was interesting in that it tends to contradict earlier findings. The analysis indicated a strong significant effect due to the number of subjects. The marginal mean frequencies in $\alpha = .05$ regions were 53.8, 43.5, 24.3, 32.2 and 28.8 for 4, 6, 8, 10, and 12 subjects respectively, which indicates a general downward trend in the empirical probability of a Type I error with an increase

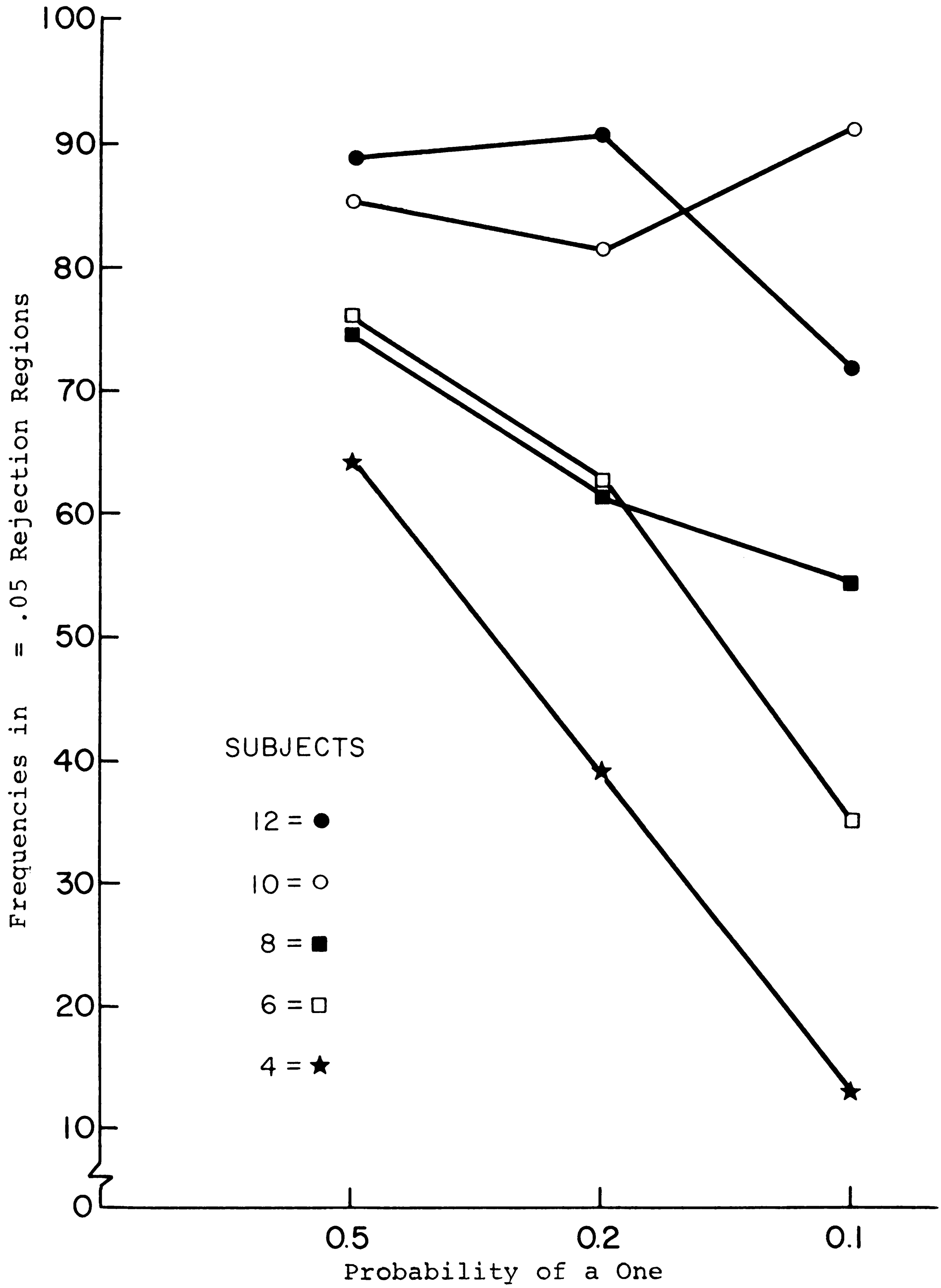

Figure 3. The interaction of the probability of a one and the number of subjects, with respect to the data in Table 11.

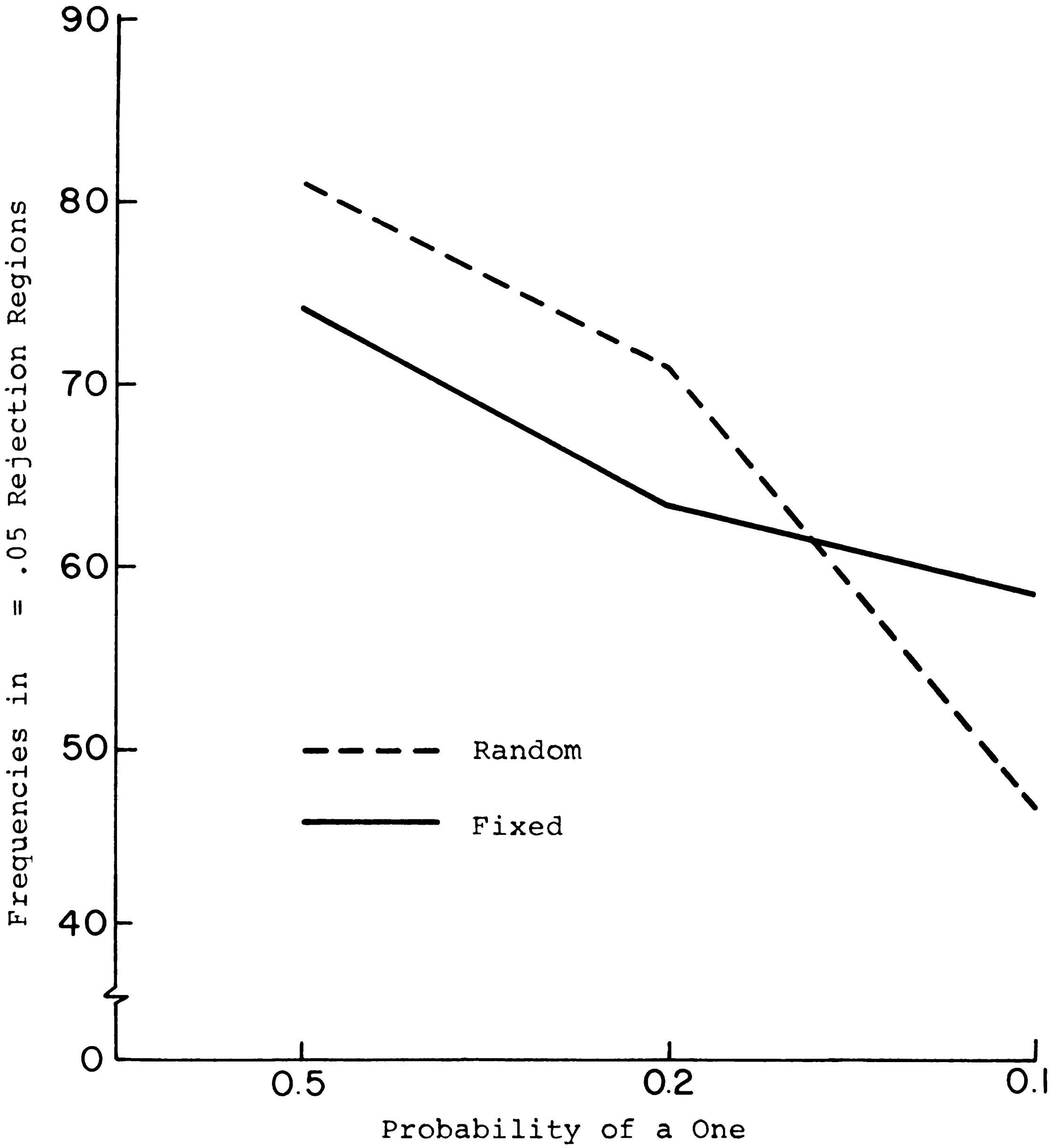

Figure 4. The interaction of the probability of a one and items fixed vs. random, with respect to the data in Table 11.

in number of subjects.

The data in Table 12 were analyzed by means of a multivariate analysis of variance and significant effects were found with respect to: (1) the probability of a one, (2) the number of subjects, (3) items fixed vs. random, and the interaction of (1) and (2). The overall mean vector for Table 11 was 72.8, 42.5, 22.3. For α = .05 regions the marginal mean frequencies for probabilities of a one equal to .5, .2, and .1 were 88.4, 72.5, and 57.2 respectively, for numbers of subjects 4, 6, 8, 10, and 12 they were 37.5, 50.7, 75.6, 85.5, and 104.4 respectively, for items fixed and null the marginal mean frequency was 65.0 and for items random non-null it was 80.5. The significant interaction is represented in Figure 5 which is somewhat similar to Figure 3 and which in general lends itself to the same interpretation.

Tables 13 and 14 contain frequencies in α = .05, .025, and .01 rejection regions of the quasi-F test statistics for tests of repeated measure effects when the data were simulated under non-null repeated measures effect conditions. Thus Tables 13 and 14 are the quasi-F analogs of Tables 8 and 9. The results in Tables 13 and 14 are most startling for it is apparent that although the quasi-F tests based on normal variates responded in an appropriate manner to non-null effects that the quasi-F tests based on dichotomous variates did not. The quasi-F test based on dichotomous variates has significantly fewer frequencies in rejection

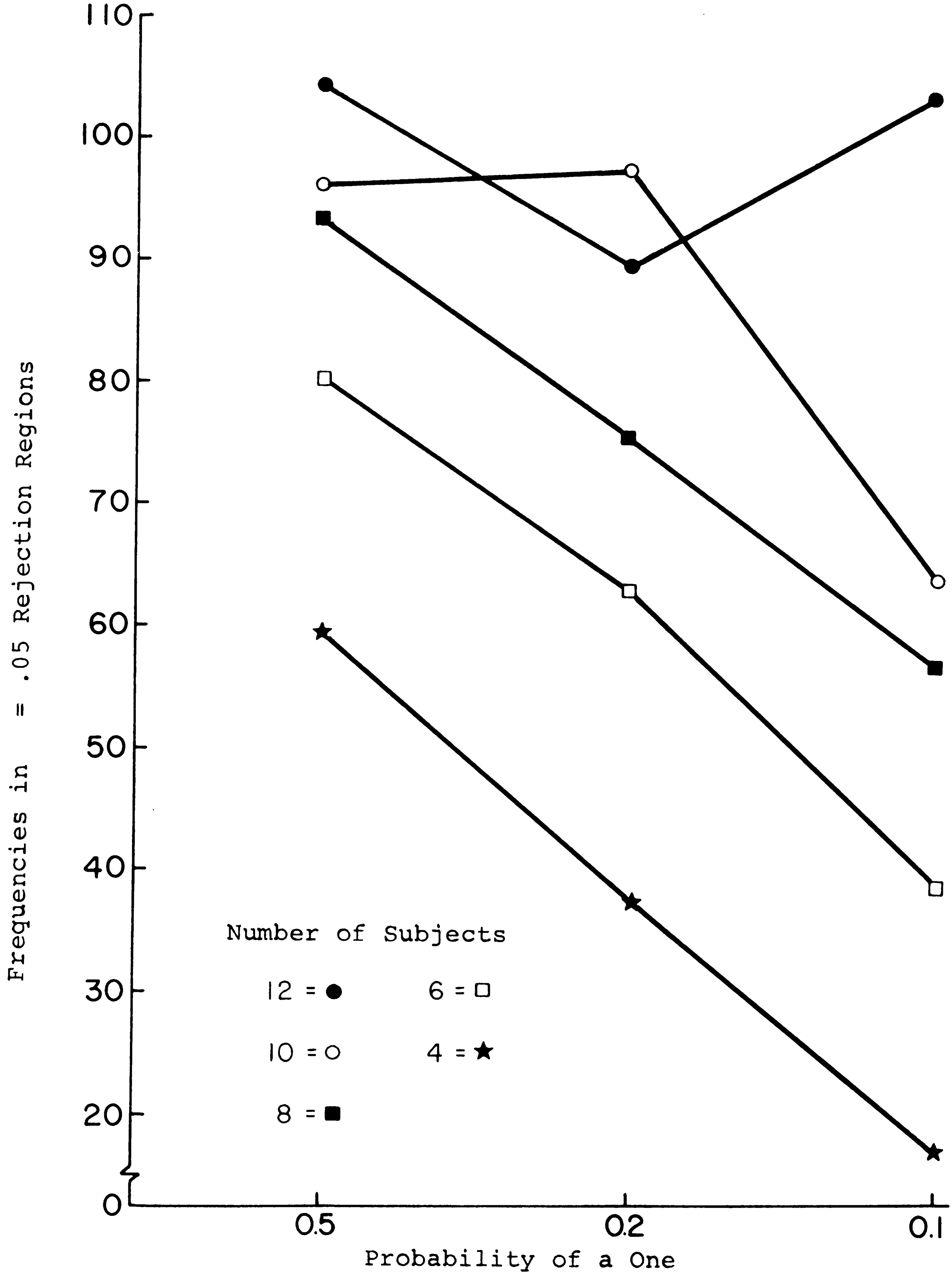

Figure 5. The interaction of the probability of a one and the number of subjects, with respect to the data in Table 12.

Table 13. The Empirical Frequencies, in the $\alpha \cdot 1000$ Rejection Region, of the Quasi-F for Repeated Measures, Based on Normal and Dichotomous Data, Items Crossed, Repeated Measure Effects Non-null, Interaction Null.

Probability of a One			.5						.2						.1					
$\alpha \cdot 1000$			50		25		10		50		25		10		50		25		10	
Dichotomous or Normal			D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N
No. of Sub.s	Items	Level of Sub. Het.																		
4	Fixed	1	28	298	13	189	6	107	20	186	16	91	7	43	29	111	17	67	12	34
		2	20	328	9	225	3	129	26	208	16	126	3	65	8	109	6	66	6	36
		3	11	384	7	258	4	155	18	244	15	133	8	62	7	140	6	68	2	35
		4	6	430	2	358	1	216	15	289	7	184	3	82	5	151	5	66	0	44
	Random	1	24	288	10	205	5	122	33	183	19	101	10	56	14	113	4	55	1	32
		2	17	342	11	237	3	125	14	193	2	109	1	55	12	118	3	61	2	32
		3	12	346	4	232	1	126	25	244	11	141	5	78	13	136	9	67	5	25
		4	13	463	5	345	1	216	11	280	1	194	0	94	5	151	1	100	1	47
6	Fixed	1	14	412	6	275	2	150	22	232	11	168	4	61	43	142	25	65	10	30
		2	5	474	2	343	1	222	23	266	8	183	5	93	45	166	11	79	5	44
		3	11	545	6	406	3	277	28	289	7	202	7	123	25	177	10	91	4	51
		4	6	656	3	516	2	354	13	386	6	264	2	149	14	219	9	118	1	64
	Random	1	9	403	3	286	0	182	22	236	15	155	8	73	51	127	19	64	10	39
		2	8	439	4	309	3	202	18	257	8	189	7	84	48	142	20	67	0	47
		3	7	544	4	415	2	276	29	265	24	197	19	98	36	178	16	69	10	48
		4	7	602	1	472	1	319	23	401	19	300	11	193	34	221	27	126	14	69
8	Fixed	1	10	544	9	389	4	233	21	308	10	219	8	112	48	218	19	113	1	48
		2	5	629	1	497	0	299	32	342	17	241	6	145	42	209	22	109	13	58
		3	4	697	2	567	0	393	20	420	9	276	9	173	30	253	12	176	5	92
		4	2	792	1	690	1	529	22	515	14	376	7	228	31	295	25	238	20	131
	Random	1	5	582	4	443	1	273	34	315	23	218	14	118	41	207	24	112	16	53
		2	11	614	5	487	3	314	18	353	4	241	3	147	57	233	34	133	24	76
		3	15	672	9	579	7	399	24	438	12	298	0	194	57	245	20	151	20	91
		4	8	790	2	653	0	512	16	520	14	388	0	237	37	304	14	223	0	109
10	Fixed	1	9	686	6	559	4	371	9	379	5	291	2	207	56	234	16	163	16	77
		2	5	732	4	582	4	436	18	441	10	307	6	229	39	249	21	189	14	107
		3	2	915	2	702	1	525	13	525	6	377	2	269	7	294	1	203	0	122
		4	2	910	0	804	0	658	11	620	2	475	0	299	20	343	7	231	5	141
	Random	1	3	689	1	534	1	347	24	356	18	287	2	197	41	223	18	160	11	62
		2	5	760	3	620	0	440	10	449	10	314	1	230	43	266	30	196	13	118
		3	2	827	0	726	0	532	21	526	12	357	4	255	30	330	14	225	0	142
		4	2	911	2	784	0	644	22	611	5	465	0	311	23	343	13	231	5	151
12	Fixed	1	3	734	0	622	0	444	10	432	6	313	3	173	46	353	32	119	24	53
		2	1	802	1	679	0	497	12	462	6	331	3	208	38	254	25	139	22	82
		3	0	880	0	777	0	625	4	569	4	404	2	274	40	319	31	221	18	115
		4	0	937	0	878	0	728	0	666	0	519	0	307	47	330	31	267	17	141
	Random	1	0	514	0	337	0	278	9	402	4	308	0	163	52	261	38	131	21	63
		2	1	528	0	420	0	279	13	494	8	360	0	221	45	267	22	146	6	71
		3	1	591	1	447	0	331	6	548	5	391	0	246	33	319	9	239	7	110
		4	2	603	2	479	2	354	6	616	2	497	2	345	35	329	35	271	24	150

Table 14. The Empirical Frequencies, in $\alpha \cdot 1000$ Rejection Regions, of the Quasi-F for Repeated Measures, Based on Dichotomous data, Items Nested, Repeated Measure Effects Non-null, Interaction Null.

Probability of a One			.5			.2			.1		
$\alpha \cdot 1000$			50	25	10	50	25	10	50	25	10
No. of Sub.s	Items	Level of Sub. Het.									
4	Fixed	1	25	14	8	17	8	4	15	10	4
		2	12	4	2	28	13	2	5	3	0
		3	8	4	2	16	13	5	13	1	0
		4	4	2	0	8	4	0	7	7	0
	Random	1	30	17	8	38	25	12	16	3	1
		2	36	13	6	47	26	17	15	9	5
		3	30	12	5	24	9	9	21	8	5
		4	16	9	5	23	13	6	23	7	3
6	Fixed	1	13	6	4	26	13	4	45	32	8
		2	10	4	1	25	6	2	39	31	8
		3	15	5	3	29	16	3	28	23	6
		4	10	3	3	11	4	2	18	12	1
	Random	1	27	17	7	44	25	12	40	23	13
		2	13	4	2	38	31	10	25	8	4
		3	13	6	2	44	33	16	36	24	8
		4	14	6	3	26	22	11	50	20	4
8	Fixed	1	9	7	2	17	8	3	59	20	7
		2	3	1	0	32	9	3	49	17	11
		3	4	4	2	22	6	1	33	11	4
		4	2	1	0	32	10	6	37	24	10
	Random	1	24	14	2	60	43	32	72	34	12
		2	28	12	4	39	34	16	54	30	10
		3	17	14	4	45	29	16	53	28	15
		4	22	17	9	46	37	24	79	26	15
10	Fixed	1	5	5	5	18	9	5	47	21	14
		2	4	3	3	9	5	2	32	21	9
		3	3	2	0	17	11	1	20	6	0
		4	0	0	0	10	3	0	18	14	12
	Random	1	19	14	5	38	29	14	51	32	11
		2	9	8	3	52	39	23	49	33	22
		3	4	2	1	42	31	13	35	26	17
		4	6	3	1	14	10	6	7	7	3
12	Fixed	1	3	0	0	8	3	3	37	32	11
		2	2	1	0	6	3	3	62	29	4
		3	1	1	0	8	3	2	55	31	6
		4	0	0	0	0	0	0	43	24	17
	Random	1	8	4	1	41	20	11	51	39	27
		2	8	5	3	38	19	12	91	56	40
		3	10	10	6	33	20	5	85	57	37
		4	10	8	4	69	48	8	99	42	24

regions when the effect it is testing is non-null than when that effect is null and in the cases investigated the empirical power of the quasi-F test is consistently less than the nominal α level!

The data in Table 13 with respect to normal variates has an overall mean vector of 406.6, 300.2, 195.4 which is significantly less than the mean vector of the data with respect to normal variates in Table 9. The relative power of the quasi-F based on normal variates to the regular F was not analyzed, but it can be seen that it is somewhwere between .8 and .6 depending on conditions. The frequencies in Table 13 based on normal variates were analyzed in the same manner as in Table 9 and the same significant effects were found.

Relative power ratio variables were formed with respect to the data in Table 13 based on dichotomous frequencies and an overall mean vector was calculated. It was .10, .08, .06. Further analysis of the data in Table 13 or analysis of the data in Table 14 appeared superfluous and was not done.

Table 15 is a rearrangement of data which has been previously presented. The arrangement of data in Table 15 was established to allow easy contrast of data with respect to regular F and quasi-F tests based on normal and dichotomous variates under null and non-null repeated measures conditions.

Table 16 is laid out in the same manner as Table 15

Table 15. The Empirical Frequencies, in $\alpha \cdot 1000$ Rejection Regions, of the F and Quasi-F for Repeated Measures, Based on Normal and Dichotomous Data, Items Crossed, Twelve Subjects.

Probability of a One				.5						.2						.1					
$\alpha \cdot 1000$				50		25		10		50		25		10		50		25		10	
Normal or Dichotomous				D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N
Repeated Measures	F	Items	Level of Sub. Het.																		
Central	B	Fixed	1	45	53	28	27	11	13	44	62	23	35	2	8	60	53	19	36	4	9
			2	45	68	25	39	15	16	70	49	28	29	7	4	39	47	16	35	2	4
			3	65	55	23	27	8	8	37	50	15	26	4	11	36	48	14	36	3	12
			4	63	53	24	33	10	12	58	46	21	29	9	5	17	50	4	34	4	9
		Random	1	59	57	29	33	7	18	47	55	35	37	13	14	31	45	25	28	10	6
			2	46	50	18	22	4	6	71	65	50	39	25	10	39	41	25	29	18	3
			3	53	80	28	43	12	18	60	54	33	34	22	18	45	31	25	22	11	5
			4	49	63	33	40	14	8	48	48	21	30	10	7	64	50	19	30	4	4
	Quasi-B	Fixed	1	102	25	53	12	35	7	90	25	59	10	27	1	87	30	54	16	41	5
			2	83	46	50	20	27	5	74	24	51	10	26	2	69	27	63	22	27	2
			3	94	20	50	9	23	2	91	25	44	10	23	2	92	21	35	14	17	6
			4	80	28	59	6	35	0	79	18	39	7	11	0	74	31	53	19	27	5
		Random	1	90	39	56	19	34	3	66	35	37	24	28	4	121	33	74	15	40	6
			2	84	22	41	10	24	1	67	35	49	20	19	1	101	17	49	10	30	3
			3	78	45	33	23	21	4	87	37	47	21	26	7	95	19	54	14	33	5
			4	71	21	47	10	27	2	96	34	57	12	40	3	68	34	38	15	22	4
Non-Central	B	Fixed	1	615	898	494	820	351	699	365	609	263	507	174	349	158	343	85	244	45	142
			2	623	938	500	878	356	746	343	653	250	556	133	407	125	379	94	292	44	158
			3	661	977	532	929	397	852	396	771	269	656	191	512	147	464	75	357	36	227
			4	751	988	636	969	448	936	454	854	320	754	186	614	176	489	102	405	54	292
		Random	1	611	915	472	835	333	719	365	558	255	462	157	349	150	350	117	255	54	171
			2	583	933	495	878	349	766	376	664	267	558	185	409	136	368	115	294	65	157
			3	666	966	538	930	387	858	405	752	298	642	176	486	157	487	119	348	41	220
			4	696	992	582	970	428	941	402	839	308	712	201	615	160	491	92	435	45	254
	Quasi-B	Fixed	1	3	734	0	622	0	444	10	432	6	313	3	173	46	353	32	119	24	53
			2	1	802	1	679	0	497	12	462	6	331	3	208	38	254	25	139	22	62
			3	0	880	0	777	0	525	4	569	4	404	2	274	40	319	31	221	18	115
			4	0	937	0	878	0	728	0	666	0	519	0	367	47	330	31	267	17	141
		Random	1	0	514	0	387	0	273	9	402	4	338	0	163	52	261	38	131	21	63
			2	1	528	0	420	0	279	13	494	8	360	0	221	45	267	22	146	6	71
			3	1	591	1	447	0	331	6	548	5	391	0	246	33	318	9	239	7	110
			4	2	603	2	479	2	364	6	616	2	497	2	345	35	329	35	271	24	150

Table 16. The Empirical Frequencies, in α·1000 Rejection Regions, of the F and Quasi-F for Repeated Measures, Based on the Normal and Dichotomous Data, Items Nested, Twelve Subjects.

Probability of a One				.5						.2						.1					
α·1000				50		25		10		50		25		10		50		25		10	
Normal or Dichotomous				D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N
Repeated Measures	F	Items	Level of Sub. Het.																		
Central	B	Fixed	1	46	53	28	27	11	13	44	62	23	35	2	8	60	53	19	36	4	9
			2	45	68	25	39	15	16	70	49	28	29	7	4	39	47	16	35	2	4
			3	65	55	23	27	8	8	37	50	15	26	4	11	36	48	14	36	8	12
			4	63	53	24	33	10	12	58	46	21	29	9	5	17	50	4	34	4	9
		Random	1	206	276	143	207	80	127	258	445	160	339	101	266	177	486	129	369	50	291
			2	198	309	126	217	61	154	253	484	185	410	123	313	213	515	106	454	54	353
			3	202	334	148	245	81	181	206	534	159	476	121	352	174	546	97	459	48	380
			4	174	393	128	304	84	204	268	571	164	495	107	413	160	652	90	550	55	405
	Quasi-B	Fixed	1	101	40	67	14	39	8	102	34	54	14	29	4	94	46	46	20	31	9
			2	78	55	48	27	19	13	92	39	53	14	28	4	96	41	59	17	33	12
			3	77	29	49	7	27	3	82	38	47	13	26	4	105	40	58	14	30	9
			4	97	40	69	11	25	2	85	31	48	9	12	3	77	46	52	20	16	10
		Random	1	123	94	77	44	40	20	85	104	63	89	29	51	103	93	82	71	41	50
			2	122	75	87	52	68	17	72	113	45	82	26	48	213	119	108	100	54	66
			3	126	89	89	66	58	28	81	109	53	77	37	47	102	127	66	107	44	68
			4	174	95	128	73	94	36	115	109	60	88	31	49	101	144	43	101	31	59
Non-Central	B	Fixed	1	615	898	494	820	351	699	365	609	263	507	174	349	158	343	85	244	45	142
			2	623	938	500	878	356	746	363	653	250	556	133	407	125	379	94	292	44	158
			3	661	977	632	929	397	852	396	771	269	656	191	512	147	464	75	357	36	227
			4	751	988	636	969	448	936	452	654	320	734	186	614	176	469	102	405	54	292
		Random	1	655	885	550	827	447	744	492	764	404	681	293	593	311	650	265	584	199	480
			2	686	909	590	863	479	795	499	818	404	720	317	604	272	707	214	593	182	479
			3	734	928	546	931	508	849	516	638	422	792	313	705	317	774	226	699	170	574
			4	750	933	667	907	541	876	515	907	445	871	333	791	234	834	221	749	136	676
	Quasi-B	Fixed	1	3	791	0	639	0	541	8	499	3	360	3	250	37	259	32	206	11	99
			2	2	849	1	745	0	600	6	525	3	413	3	273	62	262	29	195	4	91
			3	1	904	1	828	0	708	8	610	3	513	2	341	55	317	31	254	6	153
			4	0	948	0	914	0	842	0	698	0	591	0	466	43	366	24	306	17	197
		Random	1	8	553	4	446	1	319	41	245	20	173	11	130	51	167	39	134	27	110
			2	9	577	5	469	3	354	38	278	17	189	12	137	91	179	56	124	40	66
			3	10	593	10	524	6	391	33	321	20	217	5	144	85	209	57	153	37	111
			4	10	632	8	523	4	416	69	321	48	229	8	143	99	224	42	157	24	132

and includes some new data, that with respect to normal variates under Design 3.

Tables 17 and 18 present frequencies in α = .05, .025, and .01 rejection regions for variance ratio tests of subject by repeated measure interaction effects, when the data were simulated under null repeated measure and null subject by repeated measure interaction effect conditions. The frequency data in Table 17 are with respect to Design 2 and the frequency data in Table 18 with respect to Design 3.

A multivariate analysis of variance indicated a significant difference between the data in Table 17 based on normal variates with overall mean vector 48.4, 24.9, 12.2 and the data in Table 17 based on dichotomous data with overall mean vector 73.3, 45.4, 25.5. Subsequent analysis of the data in Table 17 based on normal variates indicated a significant effect due to the number of subjects and a significant interaction of the number of subjects and the probability of a one, results that were unexpected. A series of post hoc comparisons indicated that the unexpected results occurred only when the data were simulted for 8 subjects and the probability of a one for the dichotomous variates was .1. Since the probability of a one for the dichotomous variates cannot affect the data in Table 17 based on normal variates (this was checked very carefully) it must be concluded that the significant effects found in the frequencies based on normal variates are Type I errors and if the suspect simulations were rerun with a different

Table 17. The Empirical Frequencies, in $\alpha \cdot 1000$ Rejection Regions, of the F for the Subjects by Repeated Measures Interaction, Based on the Normal and Dichotomous Data, Items Crossed, Interaction and Repeated Measures Effects Both Null.

Probability of a One			.5						.2						.1					
$\alpha \cdot 1000$			50		25		10		50		25		10		50		25		10	
Dichotomous or Normal			D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N
No. of Sub.s	Items	Level of Sub. Het.																		
4	Fixed	1	54	62	31	30	16	10	47	50	24	28	11	13	38	40	25	27	17	14
		2	54	40	26	25	7	10	62	65	37	33	24	20	46	57	30	20	27	13
		3	71	48	40	22	25	7	71	35	56	16	31	13	64	58	47	19	28	9
		4	54	41	26	20	12	8	65	35	50	14	36	10	47	35	32	13	21	7
	Random	1	55	68	35	35	14	12	53	45	30	21	15	9	51	54	33	18	16	10
		2	49	62	29	29	22	13	49	43	24	17	13	8	57	46	36	19	29	12
		3	62	71	37	36	26	21	44	45	25	18	13	9	61	43	[illegible]	18	27	11
		4	75	51	46	32	26	18	47	52	24	25	21	15	64	53	37	27	32	18
6	Fixed	1	58	43	29	20	10	7	46	46	50	23	12	5	56	66	31	39	12	18
		2	63	53	30	20	18	5	65	49	37	22	12	11	101	51	70	27	57	14
		3	65	34	41	19	17	3	81	62	57	29	23	8	116	55	63	27	52	10
		4	68	57	39	28	18	15	87	52	59	31	23	12	113	60	72	34	35	16
	Random	1	39	37	23	15	10	10	61	48	36	24	25	8	52	45	33	29	17	20
		2	54	43	27	14	14	6	48	58	31	34	19	21	85	49	51	27	28	12
		3	63	45	43	26	15	9	51	51	41	31	34	11	81	56	49	33	37	20
		4	69	50	44	27	11	7	43	51	37	32	22	14	75	39	47	23	37	13
8	Fixed	1	52	52	28	24	8	5	53	36	39	20	15	7	78	33	35	18	18	4
		2	58	58	31	31	14	11	66	40	41	16	28	7	111	43	79	27	67	7
		3	44	45	15	25	5	12	65	55	52	22	22	11	130	29	76	19	51	2
		4	73	47	36	19	22	10	90	53	49	30	28	13	122	43	63	22	49	7
	Random	1	55	38	36	21	11	11	60	39	44	23	25	6	67	31	46	22	29	10
		2	49	42	23	18	10	10	70	33	45	19	19	11	74	38	70	25	36	4
		3	56	52	31	33	14	12	83	42	45	28	33	13	75	33	47	14	26	3
		4	60	49	35	22	17	13	84	38	38	26	25	8	76	36	49	19	15	4
10	Fixed	1	68	60	41	29	15	15	64	49	45	16	15	15	63	51	54	26	25	22
		2	56	41	23	17	13	10	93	49	47	21	22	19	115	35	87	11	51	9
		3	65	45	31	23	13	11	112	52	57	20	38	17	142	47	109	25	63	20
		4	67	42	35	18	19	10	103	62	67	24	43	18	150	55	118	27	61	20
	Random	1	48	41	18	21	8	11	66	61	28	17	20	14	112	60	47	29	30	26
		2	47	46	22	15	9	8	59	58	24	24	18	19	77	55	42	26	39	18
		3	49	49	21	18	10	12	88	52	58	26	24	21	130	52	64	23	37	21
		4	53	43	22	14	12	8	99	48	60	30	52	26	124	54	64	21	51	17
12	Fixed	1	45	51	20	28	6	9	84	42	47	25	15	11	130	46	92	30	50	12
		2	40	52	23	26	11	16	110	52	72	35	38	19	121	58	92	35	55	20
		3	74	56	34	28	20	11	80	49	45	30	20	12	76	51	72	22	57	11
		4	68	46	42	31	12	7	108	50	80	35	24	9	153	50	129	34	51	16
	Random	1	54	45	32	29	10	12	56	55	30	38	17	16	60	56	36	28	27	12
		2	62	59	27	40	15	19	61	44	51	34	26	8	104	53	76	26	50	10
		3	46	48	19	36	11	16	62	47	32	32	4	15	92	40	55	29	35	10
		4	65	46	29	24	14	12	121	47	82	26	42	14	100	47	83	28	45	15

Table 18. The Empirical Frequencies, in $\alpha \cdot 1000$ Rejection Regions, of the F for the Subjects by Repeated Measures Interaction, Based on the Normal and Dichotomous Data, Items Nested, Interaction and Repeated Measure Effects Both Null.

Probability of a One			.5						.2						.1					
$\alpha \cdot 1000$			50		25		10		50		25		10		50		25		10	
Dichotomous or Normal			D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N
No. of Sub.s.	Items	Level of Sub. Het.																		
4	Fixed	1	59	57	32	22	16	10	50	53	29	20	9	12	46	38	31	26	25	14
		2	55	44	27	16	8	7	63	65	45	28	28	14	52	51	41	29	28	9
		3	64	52	34	24	18	6	61	42	47	18	24	7	71	46	55	26	30	12
		4	59	51	33	21	15	10	68	43	48	11	32	6	60	29	39	17	18	6
	Random	1	55	49	26	28	11	14	50	42	13	19	9	8	54	49	42	22	23	10
		2	65	63	32	26	9	10	58	42	20	18	8	11	68	34	44	25	32	11
		3	54	55	28	29	9	13	76	52	39	19	14	4	61	39	50	26	21	14
		4	57	60	37	31	17	17	52	47	42	15	25	6	70	29	51	20	25	7
6	Fixed	1	55	49	24	23	12	8	61	43	44	18	14	8	65	65	36	35	19	18
		2	69	50	42	28	22	8	70	47	41	23	21	16	99	59	64	29	57	16
		3	66	45	39	22	21	9	73	52	59	23	26	11	108	57	79	38	63	17
		4	85	64	42	32	21	17	81	48	69	30	36	17	106	54	61	31	58	14
	Random	1	64	44	30	27	13	14	84	38	40	19	21	11	55	52	37	24	17	14
		2	69	47	39	25	16	10	80	44	56	18	35	10	73	42	51	23	51	9
		3	78	50	44	28	18	11	81	44	53	27	40	12	80	50	58	37	48	17
		4	72	57	37	30	25	14	102	52	70	34	52	15	86	52	60	33	39	13
8	Fixed	1	61	49	36	30	12	8	51	44	31	14	10	2	55	52	40	18	21	6
		2	52	55	35	31	17	6	70	44	38	15	20	6	114	62	66	16	56	10
		3	57	45	26	27	9	10	60	58	51	21	26	5	123	48	65	17	56	8
		4	85	49	55	28	31	4	106	53	64	24	29	3	126	51	71	20	50	9
	Random	1	61	43	30	22	9	7	69	43	37	22	18	7	77	68	44	17	35	6
		2	61	50	31	17	10	7	81	47	41	15	25	3	83	55	61	16	28	2
		3	51	59	36	23	18	2	76	48	46	14	22	4	92	62	51	13	35	0
		4	62	43	30	21	17	7	95	49	56	17	29	5	81	43	52	6	43	0
10	Fixed	1	69	62	38	30	12	14	70	56	55	28	21	12	88	55	59	32	29	16
		2	67	50	24	25	7	12	85	49	42	30	28	15	112	45	98	17	59	7
		3	68	60	32	25	13	13	119	59	70	37	36	16	138	55	110	31	76	17
		4	59	46	36	23	18	13	104	56	67	34	46	17	146	69	109	36	73	18
	Random	1	63	54	34	26	14	17	64	65	45	45	19	15	71	76	51	37	26	12
		2	76	56	51	33	22	14	88	62	61	34	35	15	106	73	71	40	45	16
		3	67	46	36	23	6	10	100	67	52	32	34	17	128	62	96	35	61	13
		4	99	54	62	25	24	13	95	65	77	37	28	19	141	61	118	38	92	18
12	Fixed	1	50	53	21	28	10	14	82	47	49	28	19	17	119	40	67	33	41	17
		2	44	59	19	28	10	15	105	49	74	39	54	24	108	47	79	37	56	28
		3	84	72	47	35	17	12	90	50	61	37	21	18	93	48	70	26	41	14
		4	79	52	45	23	18	12	106	39	74	34	41	14	143	52	117	33	86	21
	Random	1	62	79	42	42	25	16	91	39	41	34	17	20	70	49	51	36	46	13
		2	44	52	25	34	13	14	76	48	40	37	16	20	103	35	64	26	51	12
		3	46	50	33	30	12	8	56	56	26	40	3	23	110	42	100	25	64	14
		4	60	47	35	29	19	18	76	28	41	16	19	14	101	33	66	20	34	7

starting point for the number generator the significant differences would vanish.

The multivariate analysis of the data in Table 17 based on dichotomous variates indicated significant effects due to: (1) the probability of a one, (2) the number of subjects, (3) items fixed vs. random, (4) the level of subject heterogeneity, the interaction of (1) and (2) and the interaction of (1) and (3). The marginal means for α = .05 regions for the probabilities of a one equal to .5, .2 and .1 were 57.7, 72.0, and 90.3 respectively; for numbers of subjects 4, 6, 8, 10, and 12 they were 55.8, 68.3, 73.9, 86.3, and 82.3 respectively; for items fixed 79.4, for items random 67.2, and for levels of subject heterogeneity 1, 2, 3, and 4 they were 61.5, 70.3, 77.4, and 84.1 respectively. The two interactions are displayed graphically in Figures 6 and 7.

Inspection of Figure 6 indicates that a favorable comparison of empirical probability of a Type I error to nominal α occurred only for 6 subjects and a probability of a one equal to .5, and for 4 subjects and a probability of a one equal to .2 or .1. For all other conditions the test is too liberal. Figure 7 is self explanatory.

A multivariate analysis of the data in Table 18 indicated the same trends and significant effects that were found in Table 17, therefore only the overall mean vectors for the data based on normal and dichotomous variates in Table 18 will be reported in order to avoid tedious

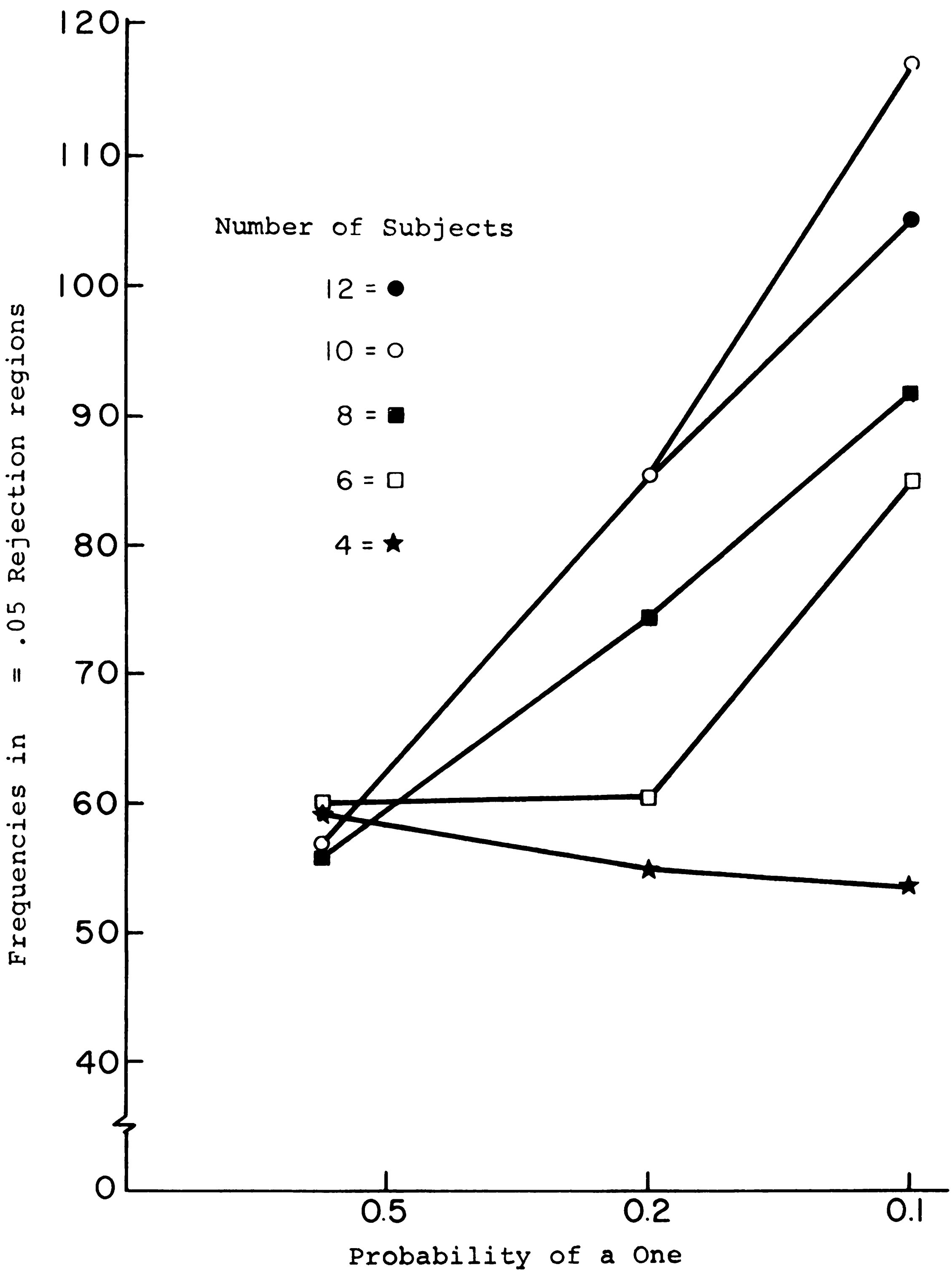

Figure 6. The interaction of the probability of a one and the number of subjects, with respect to the data in Table 17.

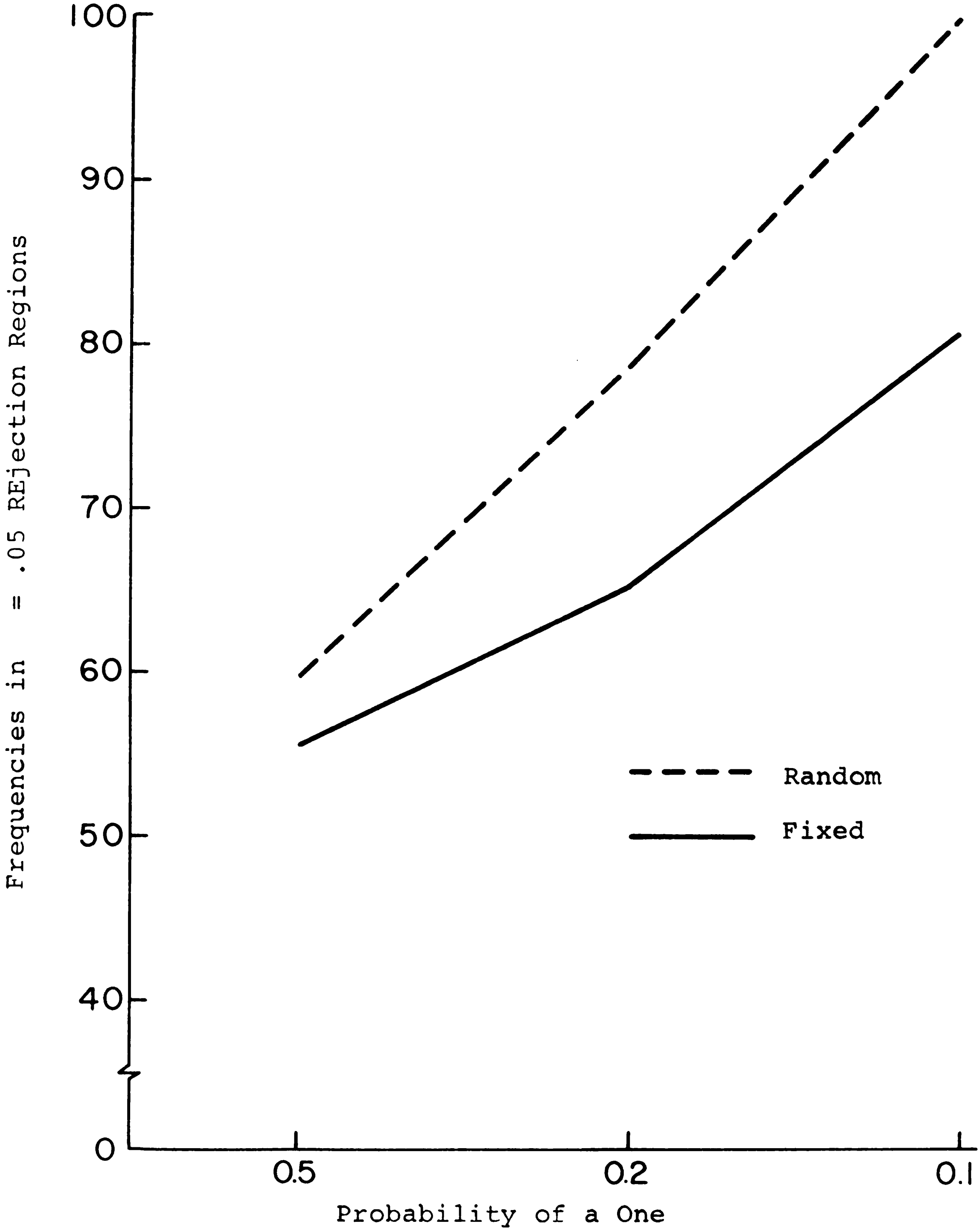

Figure 7. The interaction of the probability of a one and items fixed vs. random, with respect to the data in Table 17.

repetition. For the data based on normal variates the overall mean vector was 51.1, 26.3, 11.7 and for the data based on the dichotomous variates the overall mean vector was 78.3, 49.8, 28.2.

Tables 19 and 20 contain frequencies in α = .05, .025, and .01 rejection regions of variance ratio tests for subject by repeated measure interaction effects, when the data were simulated under the non-null interaction effect conditions indicated in Chapter IV.

Relative power variables were formed for both Table 19 and Table 20 and separate multivariate analyses of variance were performed on the relative power variables for both tables. In both analyses significant effects were found due to: (1) the probability of a one, (2) the number of subjects, the interaction of (1) and (2), the interaction of (1) and items fixed vs. random, and the interaction of (1) and the level of subject heterogeneity. In addition the analysis of the relative power variables from Table 19 disclosed a significant main effect due to items fixed vs. random.

For Table 19 the marginal mean relative powers for α = .05 regions for the probabilities of a one equal to .5, .2, and .1 were .57, .70, and .81 respectively and for numbers of subjects 4, 6, 8, 10, and 12 they were .67, .69, .64, .79, and .69 respectively. The three interactions are represented graphically in Figures 8, 9, and 10.

For Table 20 the marginal mean relative powers for

Table 19. The Empirical Frequencies, in $\alpha \cdot 1000$ Rejection Regions, of the F for the Subjects by Repeated Measures Interaction, Based on the Normal and Dichotomous Data, Items Crossed, Interaction Effects Non-null, Repeated Measures Null.

Probability of a One			.5						.2						.1					
$\alpha \cdot 1000$			50		25		10		50		25		10		50		25		10	
Dichotomous or Normal			D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N
No. of Sub.s.	Items	Level of Sub. Het.																		
4	Fixed	1	156	230	91	137	50	79	107	107	64	66	45	34	63	86	34	59	25	22
		2	155	223	104	125	58	74	115	150	87	94	53	50	73	114	46	65	42	27
		3	172	257	108	180	57	98	108	165	66	91	41	50	88	99	62	62	50	22
		4	187	308	107	198	64	111	116	212	86	106	52	56	72	117	29	87	27	35
	Random	1	125	209	63	110	32	58	83	136	45	68	19	36	40	112	27	62	12	23
		2	126	232	78	136	34	75	93	144	56	91	26	55	62	99	39	66	30	25
		3	156	296	107	191	55	103	91	159	41	105	25	53	64	123	36	68	33	35
		4	179	321	110	227	65	135	77	208	47	125	32	73	69	124	47	82	44	41
6	Fixed	1	147	250	75	151	41	76	111	155	72	89	25	48	74	108	50	69	13	42
		2	171	279	122	182	66	106	109	171	77	112	40	85	100	113	57	65	31	41
		3	176	347	117	226	58	123	137	152	99	117	65	82	112	128	63	78	43	49
		4	229	406	162	294	102	165	156	190	114	129	68	79	137	139	62	104	35	55
	Random	1	133	231	90	147	44	77	81	127	45	87	20	44	80	113	55	69	32	41
		2	153	262	92	159	46	84	75	161	41	114	23	66	98	90	58	56	30	39
		3	150	319	83	223	50	123	115	153	70	109	38	65	127	125	75	66	32	46
		4	196	395	112	272	68	162	119	182	89	119	43	64	97	130	65	96	42	56
8	Fixed	1	184	287	112	167	62	90	118	153	71	95	36	43	70	92	35	62	16	38
		2	182	359	126	250	71	126	141	204	84	448	53	59	106	119	60	79	34	41
		3	199	400	127	276	84	151	140	229	95	132	50	73	131	126	101	69	54	26
		4	228	480	157	354	106	239	194	310	107	194	72	98	161	184	133	83	90	52
	Random	1	169	288	110	190	57	111	129	188	61	112	31	62	82	109	58	78	29	39
		2	201	361	146	247	82	165	96	195	45	110	24	60	108	118	79	74	35	38
		3	229	415	150	309	104	172	103	230	61	132	33	63	114	111	60	69	29	39
		4	217	453	151	334	95	210	134	291	96	159	59	84	107	167	78	79	36	56
10	Fixed	1	181	337	112	229	60	135	120	156	81	127	48	77	104	112	71	67	29	38
		2	208	372	137	252	81	157	152	187	7	133	51	86	115	118	92	68	53	31
		3	222	437	138	319	76	197	179	216	124	146	74	109	155	121	132	94	88	34
		4	267	534	189	409	114	287	181	291	131	196	91	143	163	132	145	94	111	44
	Random	1	168	324	111	201	58	119	112	159	71	117	38	70	108	107	73	49	19	35
		2	191	364	123	252	68	157	109	137	70	102	42	65	128	98	80	57	46	34
		3	215	421	137	311	63	198	172	202	92	128	54	89	162	104	115	62	60	36
		4	226	513	146	398	74	244	150	272	115	164	66	118	153	118	114	91	60	49
12	Fixed	1	213	403	135	283	79	174	155	221	89	140	61	93	102	122	60	65	24	46
		2	269	463	176	362	103	236	155	234	117	154	66	90	144	129	96	82	44	41
		3	250	490	165	362	101	243	150	281	118	196	72	91	172	141	121	110	67	48
		4	210	641	234	503	156	375	212	327	134	212	89	135	153	209	119	120	60	64
	Random	1	189	396	119	233	62	181	109	193	52	112	35	57	101	107	62	73	37	44
		2	221	407	143	323	79	204	138	228	77	151	36	92	151	134	103	93	63	43
		3	250	516	169	413	114	297	150	292	103	194	54	102	143	137	92	103	54	71
		4	310	621	223	434	153	373	175	371	100	259	59	153	123	178	106	98	54	55

Table 20. The Empirical Frequencies, in α·1000 Rejection Regions, of the F for the Subjects by Repeated Measures Interaction, Based on the Normal and Dichotomous Data, Items Nested, Interaction Effects Non-null, Repeated Measures Null.

Probability of a One			.5						.2						.1					
α·1000			50		25		10		50		25		10		50		25		10	
Dichotomous or Normal			D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N
No. of Sub.s	Items	Level of Sub. Het.																		
4	Fixed	1	164	258	93	169	46	89	111	125	72	70	44	23	75	86	63	58	25	15
		2	166	239	103	149	65	86	133	154	86	100	53	48	86	122	66	73	36	25
		3	190	284	116	203	57	124	100	182	74	108	40	52	90	97	78	66	42	15
		4	199	349	131	231	67	138	114	222	92	145	61	63	75	121	51	81	27	46
	Random	1	132	237	89	163	46	82	93	162	47	100	31	43	58	95	33	54	16	16
		2	154	265	88	130	50	109	115	150	65	102	34	43	55	120	33	73	19	23
		3	173	314	109	230	62	143	154	163	101	113	55	62	81	116	69	74	18	22
		4	187	367	126	245	74	154	139	171	119	130	69	73	84	107	70	65	33	37
6	Fixed	1	149	272	90	167	38	94	128	169	90	105	36	53	76	109	48	65	18	49
		2	180	313	121	218	63	126	124	183	75	122	55	77	91	118	63	38	33	45
		3	200	394	134	268	79	153	153	187	106	123	70	77	102	127	61	73	52	46
		4	260	432	171	324	107	204	169	221	117	156	79	87	104	146	76	93	47	52
	Random	1	162	296	102	194	45	107	126	154	77	110	44	56	86	133	57	60	39	47
		2	173	292	110	215	57	126	144	179	115	123	62	63	93	108	58	59	38	31
		3	176	362	106	252	61	169	176	209	122	145	92	84	109	122	71	77	48	41
		4	194	425	129	304	78	190	175	210	132	141	114	83	96	134	64	79	65	37
8	Fixed	1	175	307	114	201	63	109	115	181	69	105	40	48	86	94	45	67	24	34
		2	196	382	122	271	66	154	124	232	74	125	47	63	118	123	52	79	31	35
		3	216	419	148	317	86	194	138	250	96	134	55	70	135	145	99	82	75	36
		4	256	514	170	401	103	297	175	326	111	218	63	127	176	160	130	91	117	55
	Random	1	204	343	126	232	49	130	127	214	77	121	33	65	63	97	45	71	26	36
		2	208	377	149	282	65	146	115	237	86	132	34	75	92	116	57	63	37	25
		3	219	443	144	332	90	216	132	251	100	130	62	65	95	120	81	62	44	30
		4	250	533	163	398	108	282	137	272	86	174	57	105	130	178	89	96	64	55
10	Fixed	1	186	362	109	261	60	144	123	163	87	109	47	70	112	99	66	61	39	43
		2	215	399	142	301	80	181	148	246	97	146	61	86	121	100	102	71	64	43
		3	232	470	164	337	95	234	172	266	139	165	76	92	178	119	129	81	90	45
		4	271	579	190	446	129	330	179	320	138	227	86	131	173	149	138	83	114	56
	Random	1	197	354	123	245	71	148	149	177	87	111	50	81	93	92	50	67	23	38
		2	219	410	129	275	70	172	132	216	88	129	64	88	119	109	84	89	54	52
		3	163	450	170	356	107	234	143	253	90	176	63	92	121	100	93	69	75	51
		4	339	569	247	437	152	323	167	297	127	209	77	107	183	138	144	68	111	55
12	Fixed	1	247	428	152	303	81	198	163	234	94	153	60	77	102	145	66	61	32	41
		2	288	499	194	376	117	265	169	246	119	175	76	116	147	144	111	105	65	54
		3	256	540	191	410	113	263	170	298	126	207	76	124	177	170	126	98	93	53
		4	345	695	271	557	185	419	204	329	148	245	94	129	156	215	116	141	80	72
	Random	1	198	453	116	328	74	213	123	279	77	137	36	89	121	127	64	70	42	40
		2	259	469	155	339	94	238	133	289	81	194	52	102	121	115	90	76	61	50
		3	296	565	176	412	106	298	175	314	122	226	32	121	122	149	77	95	47	59
		4	358	683	252	554	175	384	169	364	126	258	81	157	156	168	75	117	60	74

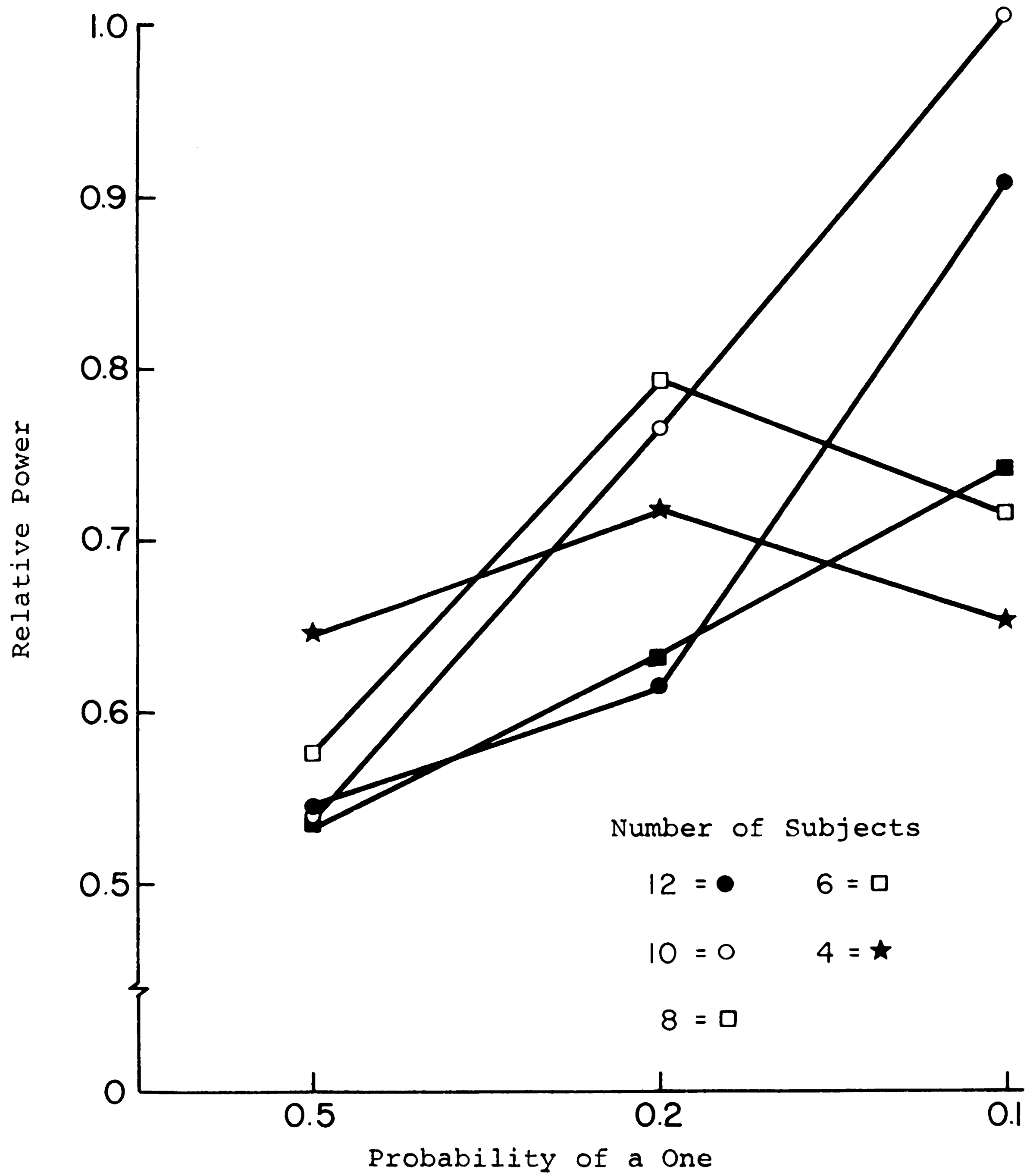

Figure 8. The interaction of the probability of a one and the number of subjects, with respect to the relative power of the test for the subjects by repeated measures interaction based on dichotomous data under Design 2.

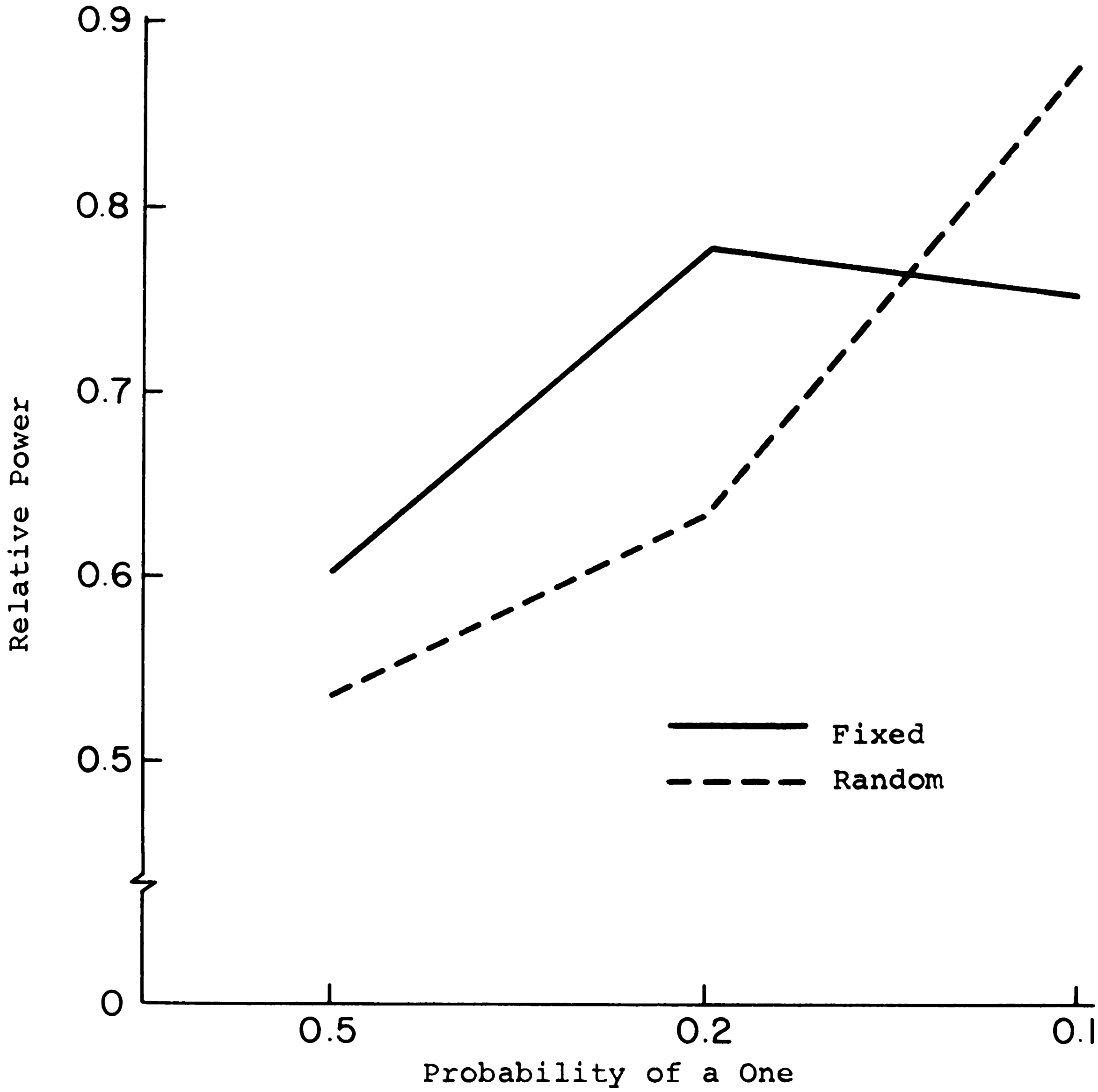

Figure 9. The interaction of the probability of a one and items fixed vs. random, with respect to the relative power of the test for the subjects by repeated measures interaction based on dichotomous data under Design 2.

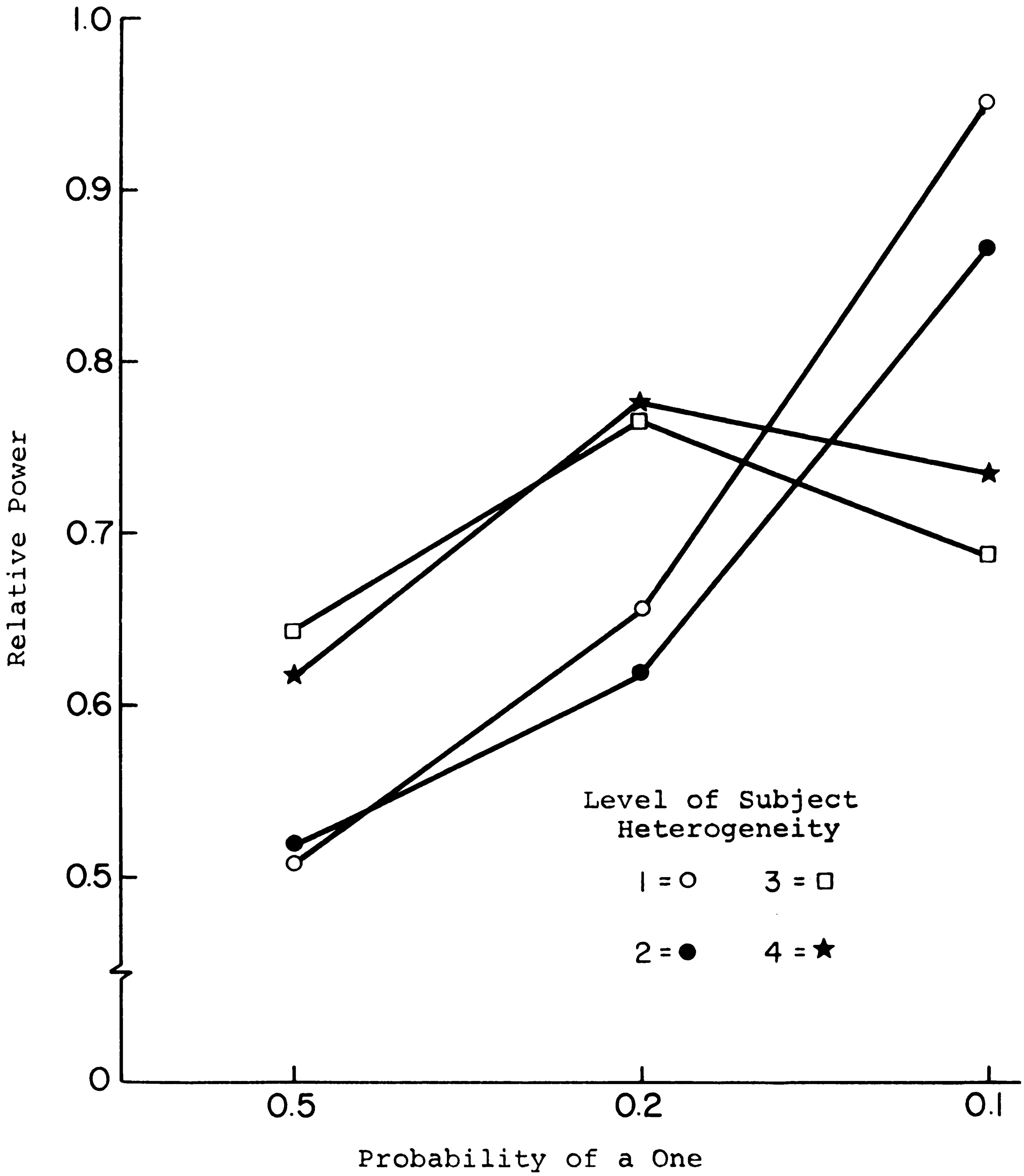

Figure 10. The interaction of the probability of a one and the level of subject heterogeneity, with respect to the relative power of the test for the subject by repeated measures interaction based on dichotomous data under Design 2.

$\alpha = .05$ regions for the probabilities of a one equal to .5, .2, and .1 were .53, .65, and .91 respectively, for numbers of subjects equal to 4, 6, 8, 10, and 12 they were .52, .71, .67, .82 and .68 respectively, and for items fixed the mean was .73 while for items random the mean was .67. The three interactions are graphed in Figures 11, 12, and 13.

A detailed interpretation of Figures 8 through 13 and the other results of the analysis of the relative power variables for Tables 19 and 20 will not be attempted because the results of the analyses of the frequencies in Tables 17 and 18 have indicated that the variance ratio tests for repeated measure by subject interaction effects are in most of the instances simulated, too liberal. In general it may be observed, however, that higher relative powers correspond to greater "liberalness" in the test of a true null hypothesis. Thus the general increases in relative power observed in Figures 8 through 13 across levels of a probability of a one .5 to .2 to .1 are in some sense specious.

Tables of frequency data for other tests under Designs 2 and 3 can be found in Appendix A.

Two tables of correlations between mean squares have been included in this chapter. The importance of the correlation between mean squares in a variance ratio test of a source of variation based on variates with a non-zero kurtosis, was indicated in Chapter III. As was indicated, the correlation between a mean square for a hypothesis and the associated mean square error in a ratio of mean squares

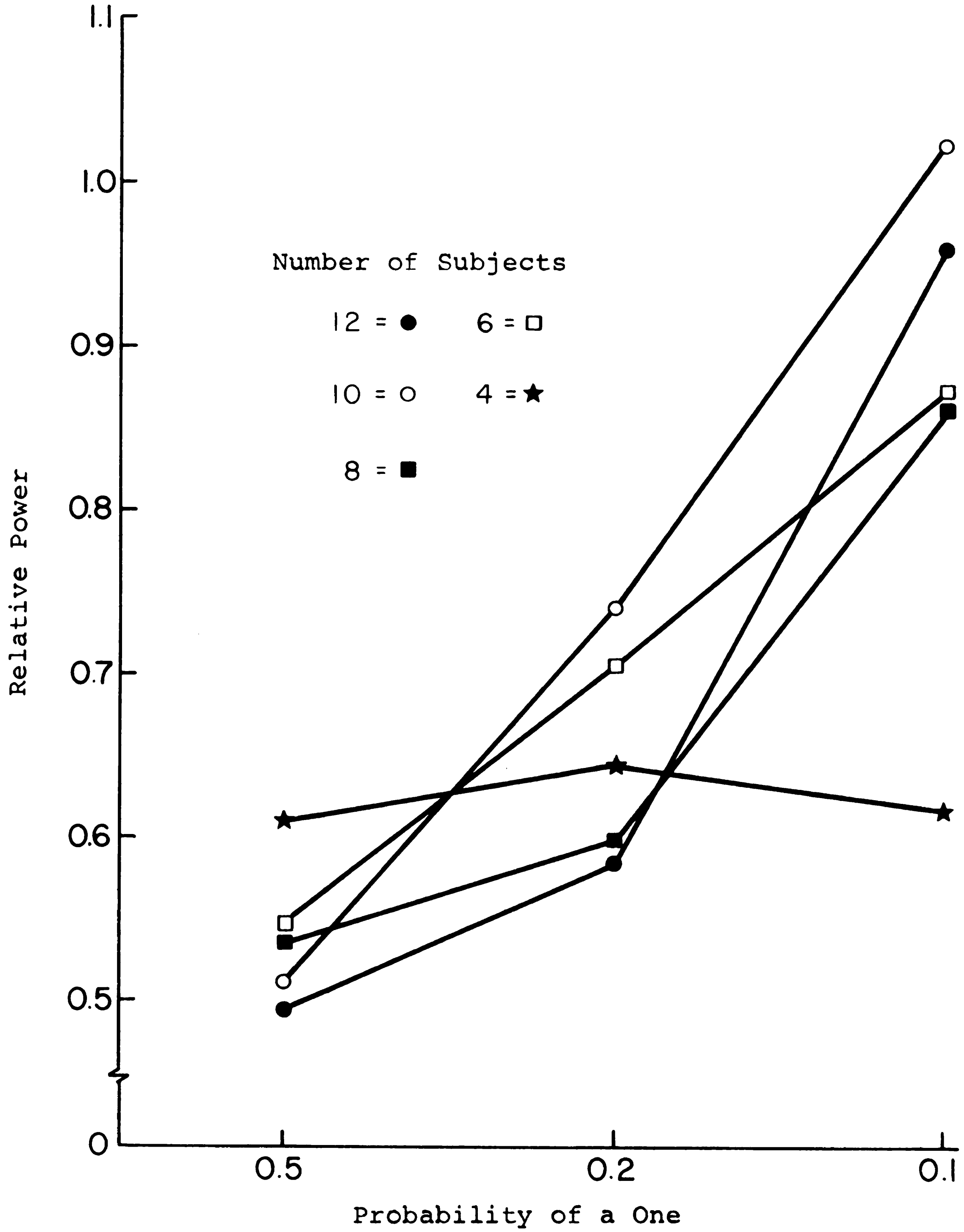

Figure 11. The interaction of the probability of a one and the number of subjects, with respect to the relative power of the test for the subjects by repeated measures interaction based on dichotomous data under Design 3.

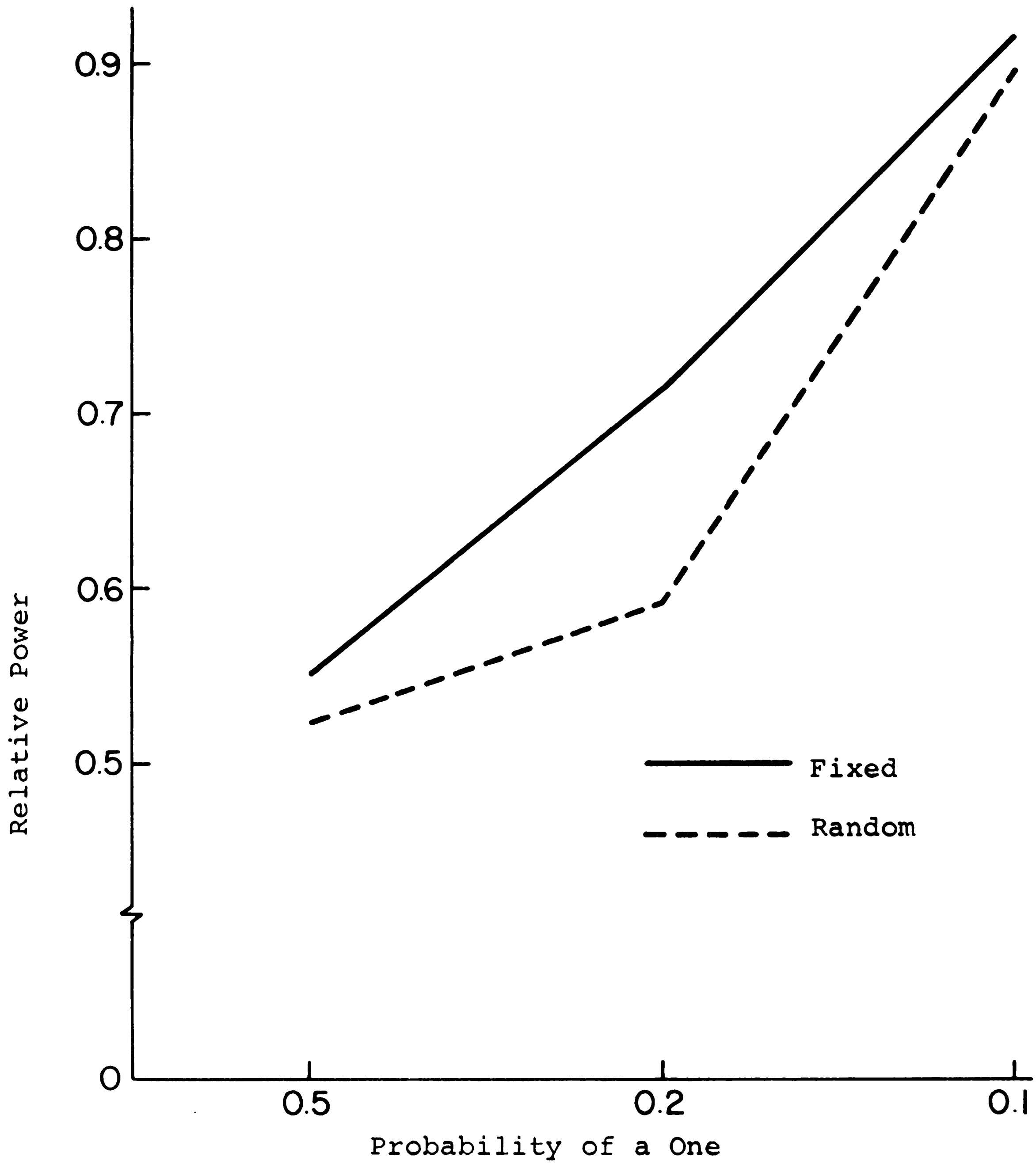

Figure 12. The interaction of the probability of a one and items fixed vs. random, with respect to the relative power of the test for the subjects by repeated measures interaction based on dichotomous data under Design 3.

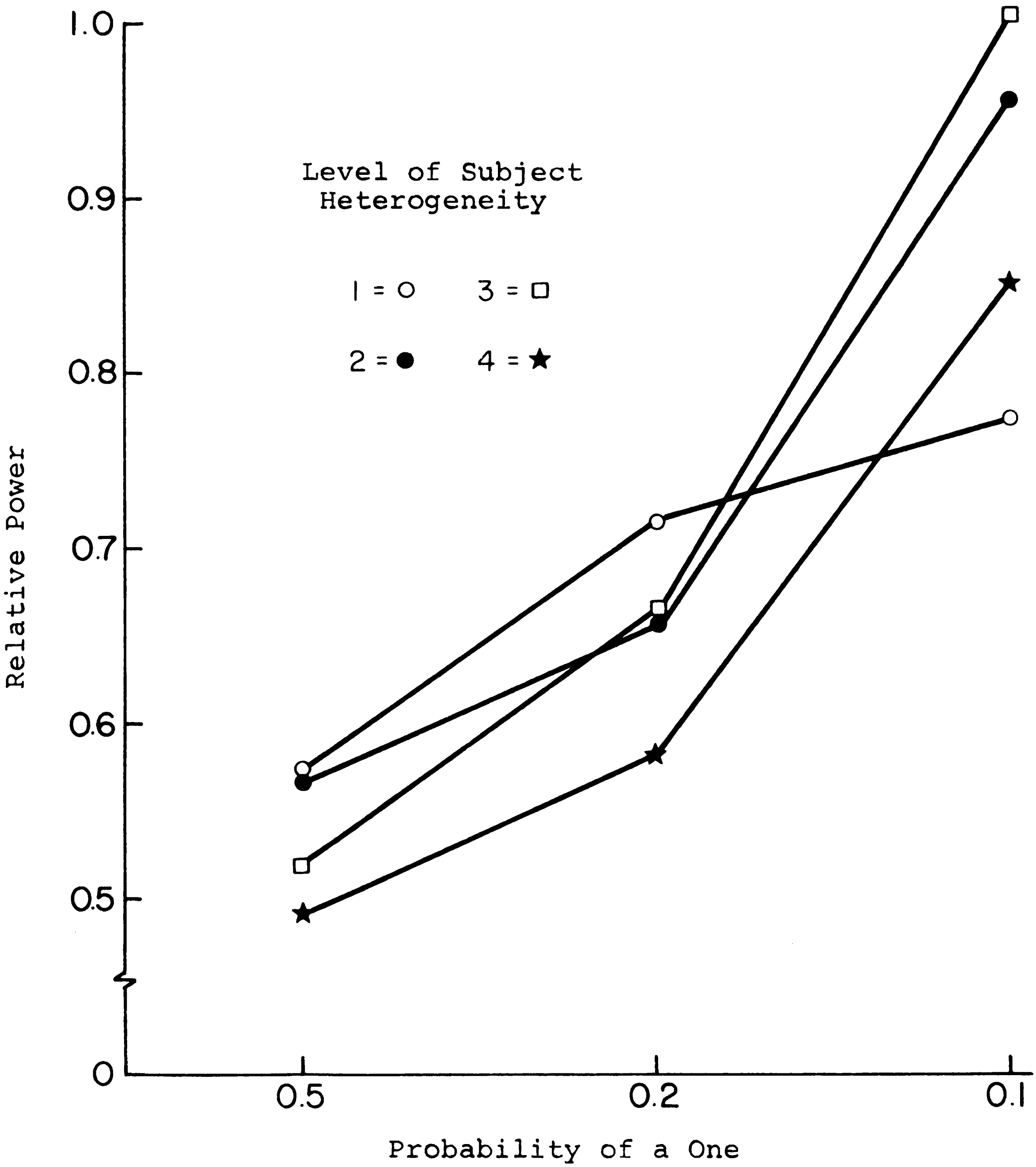

Figure 13. The interaction of the probability of a one and the level of subject heterogeneity, with respect to the relative power of the test for the subjects by repeated measures interaction based on dichotomous data under Design 3.

was not expected to be necessarily zero, when the dependent variable was not a variate with a normal probability density. The correlation between MS_h and an associated MS_e in a variance ratio would be expected to be zero only if the kurtosis of the dependent variable was zero and the dependent variables were independent from each other. The kurtosis of the binomial variate is given by the expression: $y_2 = (1-6pq)(npq)^{-1}$, where p is the probability of a one on any trial, q = 1-p, and n is the number of trials. The Bernoulli variate is a binomial variate where n = 1. Thus the kurtosis of a Bernoulli variate is $(pq)^{-1} - 6$. Bernoulli variates with parameters p = .1, .2, and .5 would have kurtoses 5.11, 0.25, and -2.00 respectively. If the dependent variables in the cells of Designs 2 and 3 were independent of each other, the expected values of the correlations between various mean squares could be calculated from the above information and the expression for a correlation,

$$\text{corr } [MS_h, MS_e] = y_2 \left[\frac{4n^2k}{(k-1)(n-1)} + \frac{2n(nk-1)}{(k-1)(n-1)} y_2 + y_2^2\right]^{-\frac{1}{2}} .$$

However, since the dependent variables in the cells of Designs 2 and 3 were not generated in a manner so that they would be independent the expression for the correlation between mean squares given above is not appropriate.

The expression for the correlation between two mean squares for non-independent data involves complicated terms with many cross expectations which are non-zero. No attempt will be made in this paper to represent the expressions

mentioned in the previous sentence, but the empirical values of the correlations between mean squares in variance-ratio tests of repeated measures effects and subjects by repeated measure interaction effects will be presented.

Although the precise values of expected correlations between mean squares of interest were not calculated, it could be predicted that the correlations between two mean squares of interest would have a rank ordering with respect to the kurtosis of the dependent variable. That is, for three different data simulations where the kurtosis of the dependent variable for simulation i, y_{2_i}, i = 1, 2, 3; has rank ordering $y_{2_1} > y_{2_2} > y_{2_3}$, the rank ordering of the correlations, corr $(MS_h, MS_e)_i$, i = 1, 2, 3; may be expected to be

$$\text{corr } (MS_h, MS_e)_1 > \text{corr } (MS_h, MS_e)_2 > \text{corr } (MS_h, MS_e)_3 .$$

Recall that the kurtosis of the dependent variable is a function of p, the probability of a one in the data and observe that the expected rank ordering is borne out in Tables 21 and 22.

Tables 21 and 22 are six dimensional arrays of correlations with marginal labels. The labels crossed and nested in the far left margin of Tables 21 and 22 indicate respectively whether Design 2 (items crossed with repeated measures) or Design 3 (items nested within repeated measures) was the model for the data simulation. The next left-most margin indicates the number of subjects associated with a

Table 21. The Correlations Between Mean Squares for Repeated Measures and the Mean Square Error Associated, Based on Dichotomous Data, Interaction and Repeated Measure Effects Both Null.

Probability of a One				.5				.2				.1			
Level of Subject Heterogeneity				1	2	3	4	1	2	3	4	1	2	3	4
Items	Repeated Measures	No. of Sub.s	Items												
Crossed	Central	4	Fixed	-0.034	-0.005	0.044	0.202	0.254	0.309	0.396	0.413	0.363	0.487	0.588	0.612
			Random	-0.039	0.078	0.144	0.110	0.211	0.353	0.413	0.435	0.422	0.446	0.529	0.547
		6	Fixed	-0.007	-0.011	0.024	0.110	0.165	0.175	0.265	0.300	0.363	0.400	0.511	0.582
			Random	-0.008	0.090	0.062	0.175	0.192	0.274	0.246	0.297	0.415	0.348	0.407	0.496
		8	Fixed	-0.125	0.040	0.022	0.136	0.178	0.219	0.237	0.340	0.423	0.438	0.433	0.554
			Random	-0.024	0.073	0.049	0.141	0.290	0.340	0.282	0.297	0.401	0.403	0.419	0.451
		10	Fixed	-0.059	0.001	0.005	0.127	0.082	0.180	0.170	0.262	0.304	0.346	0.426	0.480
			Random	-0.075	-0.065	-0.002	0.127	0.167	0.195	0.174	0.239	0.383	0.333	0.457	0.459
		12	Fixed	-0.074	0.076	0.011	0.120	0.092	0.170	0.274	0.258	0.171	0.300	0.308	0.507
			Random	-0.001	0.035	0.024	0.134	0.172	0.152	0.255	0.253	0.438	0.447	0.362	0.395
	Non-central	4	Fixed	-0.093	-0.014	0.060	0.184	0.194	0.251	0.340	0.471	0.382	0.524	0.554	0.512
			Random	-0.072	-0.023	0.089	0.155	0.128	0.232	0.258	0.396	0.349	0.458	0.435	0.560
		6	Fixed	-0.128	0.021	0.062	0.152	0.129	0.194	0.228	0.268	0.338	0.367	0.433	0.525
			Random	-0.043	0.108	0.035	0.146	0.226	0.325	0.311	0.436	0.403	0.422	0.365	0.569
		8	Fixed	-0.101	-0.045	0.011	0.190	0.070	0.159	0.330	0.281	0.449	0.470	0.492	0.526
			Random	0.009	0.004	0.145	0.191	0.125	0.165	0.205	0.241	0.381	0.385	0.390	0.372
		10	Fixed	-0.086	-0.032	0.116	0.111	0.053	0.125	0.216	0.326	0.289	0.410	0.445	0.450
			Random	-0.075	-0.046	0.138	0.131	0.169	0.177	0.188	0.296	0.375	0.418	0.367	0.414
		12	Fixed	-0.060	-0.044	0.163	0.149	0.026	0.076	0.172	0.270	0.238	0.347	0.452	0.423
			Random	-0.009	0.010	0.089	0.241	0.025	0.150	0.293	0.310	0.330	0.358	0.456	0.450
Nested	Central	4	Fixed	-0.034	-0.005	0.044	0.202	0.254	0.309	0.396	0.413	0.363	0.487	0.588	0.612
			Random	-0.070	-0.029	0.044	0.203	0.111	0.230	0.304	0.308	0.443	0.467	0.408	0.502
		6	Fixed	-0.007	-0.011	0.024	0.110	0.165	0.175	0.265	0.300	0.363	0.400	0.511	0.582
			Random	-0.052	0.019	0.120	0.240	0.149	0.190	0.278	0.289	0.321	0.264	0.411	0.442
		8	Fixed	-0.125	0.040	0.022	0.136	0.178	0.219	0.237	0.340	0.423	0.438	0.433	0.554
			Random	0.001	0.049	0.021	0.092	0.166	0.174	0.214	0.400	0.484	0.495	0.527	0.501
		10	Fixed	-0.059	0.001	0.005	0.127	0.082	0.180	0.170	0.262	0.304	0.346	0.426	0.430
			Random	-0.019	-0.074	0.038	0.113	0.065	0.144	0.200	0.315	0.407	0.462	0.477	0.522
		12	Fixed	-0.074	0.076	0.011	0.120	0.092	0.170	0.274	0.258	0.171	0.300	0.308	0.507
			Random	-0.098	0.039	0.063	0.150	0.253	0.248	0.282	0.372	0.348	0.398	0.514	0.553
	Non-central	4	Fixed	-0.093	-0.014	0.004	0.184	0.194	0.250	0.340	0.471	0.382	0.524	0.554	0.512
			Random	-0.137	-0.044	0.053	0.126	0.018	0.126	0.293	0.197	0.376	0.365	0.431	0.529
		6	Fixed	-0.128	0.021	0.062	0.152	0.129	0.194	0.228	0.268	0.338	0.367	0.433	0.525
			Random	-0.057	-0.077	0.095	0.217	0.125	0.139	0.262	0.296	0.343	0.394	0.534	0.463
		8	Fixed	-0.101	-0.045	0.011	0.190	0.070	0.159	0.330	0.281	0.449	0.470	0.492	0.526
			Random	-0.101	-0.001	0.124	0.144	0.076	0.046	0.227	0.235	0.330	0.452	0.392	0.549
		10	Fixed	-0.086	-0.032	0.116	0.111	0.053	0.125	0.216	0.326	0.289	0.410	0.445	0.450
			Random	-0.151	-0.092	-0.038	0.156	0.083	0.004	0.069	0.242	0.178	0.362	0.446	0.566
		12	Fixed	-0.060	-0.044	0.163	0.149	0.026	0.076	0.172	0.270	0.238	0.347	0.452	0.423
			Random	-0.143	-0.017	0.064	0.130	0.070	0.111	0.217	0.299	0.267	0.325	0.375	0.524

Table 22. The Correlations Between the Mean Squares for Subjects by Repeated Measures Interaction and the Mean Square Error Associated, Based on the Dichotomous Data, Interaction and Repeated Measures Effects Both Null.

Probability of a One				.5				.2				.1			
Level of Subject Heterogeneity				1	2	3	4	1	2	3	4	1	2	3	4
Items	Inter-action	No. of Sub.s	Items												
Crossed	Central	4	Fixed	-0.212	-0.093	-0.049	0.157	0.263	0.266	0.343	0.419	0.584	0.639	0.580	0.634
			Random	-0.172	-0.063	0.044	0.049	0.335	0.408	0.399	0.430	0.542	0.498	0.509	0.578
		6	Fixed	-0.260	-0.044	0.006	0.123	0.280	0.282	0.351	0.374	0.538	0.544	0.584	0.572
			Random	-0.159	-0.093	0.035	0.041	0.334	0.452	0.455	0.464	0.602	0.540	0.527	0.568
		8	Fixed	-0.190	-0.085	0.058	0.117	0.342	0.323	0.394	0.497	0.540	0.477	0.499	0.563
			Random	-0.151	-0.069	0.067	0.182	0.431	0.484	0.478	0.501	0.611	0.596	0.620	0.668
		10	Fixed	-0.270	-0.117	0.033	0.147	0.158	0.227	0.289	0.367	0.483	0.480	0.485	0.550
			Random	-0.105	-0.057	0.054	0.161	0.476	0.467	0.484	0.488	0.584	0.557	0.644	0.617
		12	Fixed	-0.222	-0.052	0.018	0.169	0.171	0.191	0.327	0.368	0.426	0.490	0.555	0.495
			Random	-0.165	-0.066	0.178	0.185	0.456	0.483	0.491	0.485	0.757	0.651	0.707	0.597
	Non-central	4	Fixed	-0.260	-0.093	-0.037	0.159	0.330	0.320	0.407	0.470	0.606	0.623	0.574	0.648
			Random	-0.223	-0.139	0.005	0.111	0.306	0.328	0.281	0.382	0.461	0.479	0.536	0.547
		6	Fixed	-0.293	-0.058	0.019	0.132	0.257	0.281	0.388	0.408	0.568	0.548	0.598	0.571
			Random	-0.282	-0.154	-0.028	0.090	0.221	0.237	0.304	0.315	0.471	0.404	0.494	0.506
		8	Fixed	-0.245	-0.095	0.047	0.117	0.361	0.352	0.415	0.515	0.590	0.526	0.546	0.607
			Random	-0.242	-0.098	0.020	0.082	0.305	0.334	0.357	0.426	0.455	0.533	0.556	0.605
		10	Fixed	-0.306	-0.130	0.041	0.083	0.177	0.284	0.309	0.388	0.512	0.517	0.524	0.599
			Random	-0.319	-0.253	-0.111	-0.035	0.281	0.200	0.303	0.343	0.557	0.552	0.526	0.531
		12	Fixed	-0.321	-0.049	0.010	0.187	0.186	0.243	0.385	0.419	0.484	0.526	0.610	0.545
			Random	-0.218	-0.066	0.064	0.087	0.283	0.333	0.430	0.418	0.517	0.530	0.447	0.505
Nested	Central	4	Fixed	-0.244	-0.201	-0.054	0.091	0.193	0.258	0.307	0.387	0.496	0.571	0.403	0.553
			Random	-0.157	-0.009	-0.009	0.091	0.301	0.435	0.441	0.514	0.579	0.540	0.527	0.511
		6	Fixed	-0.239	-0.144	-0.104	0.037	0.260	0.295	0.356	0.397	0.516	0.550	0.559	0.500
			Random	-0.262	-0.172	-0.021	0.103	0.407	0.375	0.420	0.462	0.581	0.561	0.594	0.590
		8	Fixed	-0.263	-0.165	0.004	0.082	0.223	0.292	0.370	0.401	0.548	0.549	0.565	0.537
			Random	-0.217	-0.114	-0.110	-0.040	0.443	0.479	0.511	0.497	0.631	0.638	0.603	0.621
		10	Fixed	-0.283	-0.170	-0.012	0.127	0.123	0.196	0.266	0.355	0.142	0.230	0.480	0.555
			Random	-0.223	-0.091	0.076	0.178	0.412	0.452	0.478	0.471	0.645	0.635	0.629	0.657
		12	Fixed	-0.289	-0.232	-0.015	0.018	0.148	0.244	0.302	0.329	0.529	0.494	0.511	0.537
			Random	-0.246	-0.115	-0.018	0.047	0.447	0.462	0.484	0.557	0.721	0.704	0.644	0.669
	Non-central	4	Fixed	-0.266	-0.232	-0.049	0.099	0.232	0.305	0.370	0.435	0.485	0.583	0.496	0.605
			Random	-0.269	-0.192	-0.055	-0.040	0.191	0.189	0.238	0.272	0.505	0.496	0.472	0.527
		6	Fixed	-0.264	-0.163	-0.114	0.027	0.204	0.272	0.359	0.421	0.580	0.584	0.575	0.593
			Random	-0.337	-0.114	-0.079	0.132	0.193	0.180	0.215	0.241	0.385	0.463	0.440	0.528
		8	Fixed	-0.299	-0.168	-0.024	0.070	0.252	0.311	0.416	0.437	0.569	0.603	0.623	0.566
			Random	-0.385	-0.261	-0.079	0.033	0.201	0.286	0.351	0.424	0.531	0.593	0.621	0.545
		10	Fixed	-0.303	-0.162	-0.025	0.131	0.160	0.254	0.291	0.392	0.168	0.295	0.536	0.576
			Random	-0.362	-0.201	-0.060	-0.049	0.180	0.225	0.230	0.284	0.505	0.517	0.491	0.414
		12	Fixed	-0.331	-0.224	-0.012	0.015	0.160	0.275	0.339	0.360	0.544	0.524	0.524	0.572
			Random	-0.244	-0.165	-0.018	0.050	0.242	0.326	0.275	0.324	0.564	0.490	0.507	0.576

given data simulation. The final column margin indicates whether items were considered fixed and null in effect or random and non-null in effect. The top row margin refers to the overall probability of a one in the data given no interaction or repeated measures effect. The second row margin refers to the level of subject heterogeneity. Each element in these two 2×2×5×2×4×3, six dimensional arrays is a correlation between 1000 pairs of mean squares calculated from simulated data in dichotomous (zero-one) form. The correlations in Table 21 are correlations between the mean square for the repeated measures source of variation and the mean square error associated. The correlations in Table 22 are correlations between the mean square for the subjects by repeated measure interaction source of variation and the mean square error associated.

The 480 correlations in each of Tables 21 and 22 were transformed by the "Fisher r to Z" procedure and then the transformed variables were considered as dependent variables in a six way factorial design with six fixed main effects and an ANOVA was performed on the transformed data from each table, 21 and 22, separately.

The analysis of the transformed data from Table 21 indicated significant effects due to the probability of a one, the number of subjects and the level of subject heterogeneity. For Table 21 the marginal mean correlations (reconverted from Fisher Z means) for the probabilities of a one .5, .2, and .1 were .04, .22, and .44 respectively; for

the numbers of subjects 4, 6, 8, 10, and 12 they were .27, .24, .24, .21, and .20 respectively and for the levels of subject heterogeneity 1, 2, 3, and 4 they were .14, .20, .26, and .33 respectively.

The analysis of the transformed data from Table 22 indicated significant effects for all of sources found significant with respect to Table 21 plus significant effects due to items fixed vs. random, the interaction of probability of a one and level of subject heterogeneity which is represented graphically in Figure 14, and three interactions involving the distinction items crossed vs. nested (see Figures 15, 16, and 17 in Appendix B) which were considered of marginal importance. For Table 22 the marginal mean correlations for probabilities of a one .5, .2, and .1 were -.07, .34, and .55 respectively, for numbers of subjects 4, 6, 8, 10, and 12 they were .27, .26, .30, .25, and .28 respectively, for levels 1, 2, 3, and 4 of subject heterogeneity they were .18, .24, .30 and .36, for items fixed the mean was .26, and for items random the mean was .29.

As was expected the correlations increase across levels .5, .2 and .1 of the probability of a one (which corresponds to increasing kurtosis) in both tests of repeated measures (Table 21) and tests of subject by repeated measure interactions (Table 22). Also in both tables there is an increase in correlation as the subjects become more heterogeneous.

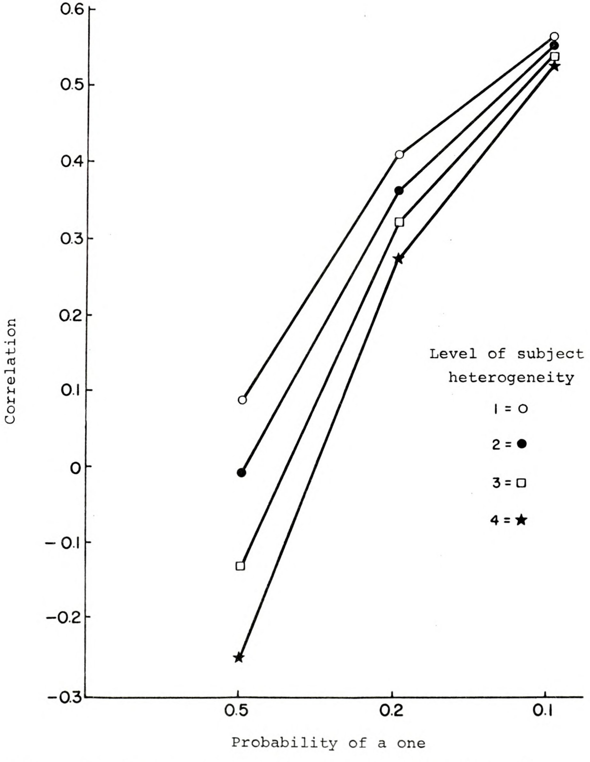

Figure 14. The interaction between the probability of a one and the level of subject heterogeneity, with respect to the correlations in Table 21.

Other tables of correlations between mean squares can be found in Appendix B.

CHAPTER VI

IMPLICATIONS AND CONCLUSIONS

In general the implications of this study are, that although many individuals have had some success in demonstrating that variance ratio tests based on dichotomous data can be assumed to have probabilities of a Type I error very close to α when the $1-\alpha^{th}$ quantile of a corresponding F-statistic is employed as a critical value to define a rejection region, severe caveats should be issued to potential employers of analyses of variance to dichotomous data in a repeated measures design. The most important warning concerns the use of the quasi-F test when the dependent variables are zero-one data; not only was the probability of a Type I error extremely variant in the data in this study, but the frequency of the quasi-F test based on dichotomous data in any rejection region was always less when a null hypothesis was false, than it was when the null hypothesis was true! Further, the "reverse power" of the quasi-F test based on dichotomous data was not a function of the presence or absence of a confounding source of variation.

In the absence of a confounding source of variation the usual variance ratio test of repeated measures based on

dichotomous data gave a good fit of a probability of a Type I error to the corresponding nominal α given a large enough number of subjects, a number that was somewhat fewer than usually occur in practice. When there was a confounding source of variation which was non-null (as was the case with random items in Design 3) the usual test of repeated measures based on either normal or dichotomous data was inappropriate just as was suggested in Chapter I. The power of the variance ratio tests for repeated measures based on dichotomous data were approximately half the power of the same tests based on normal data, where such tests were appropriate.

Although the results concerning the quasi-F were the most startling of the results of this study, the most disappointing results of the study were those concerning the tests of subjects by repeated measures interactions based on the dichotomous data. A good fit of a probability of a Type I error to a corresponding nominal α and a set of reasonable power characteristics for the variance ratio test of the subjects by repeated measures interaction based on dichotomous data would have been most useful in the investigation of individual differences and as an indication of the need of additional individual difference blocking variables, as was indicated in Chapter I.

In order to examine the implications of this study more extensively, consider what the discussion and results in this paper might suggest to an experimenter who would like to do

a repeated measures type of experiment and analyze his results with hypothesis testing in mind. It has been suggested that it is important for the experimenter to decide whether the experimental response evokers (i.e. items) he employs in his experiment are all of the response evokers of interest (that is all of the response evokers to which he would like to generalize his results) or a random sample of those response evokers he might employ. The above importance results from the likelihood that if the response evokers are a random sample, a non-null source of variation will be associated with them. A non-null source of variation associated with response evokers (i.e. items nested within repeated measures or an items by repeated measure interaction) may be confounded with the repeated measures source of variation in the ordinary analysis of variance tests for repeated measures effects (Chapter I, pages 17-18). Further, an experimenter may be unaware of the above confounding if he doesn't include the response evokers as a factor in his design (Chapter I, pages 13-18).

The above contentions are supported by the results (Chapter V, pages 59 through 64) and are important considerations whether the dependent variables may be expected to have a normal probability density or not. Thus an experimenter who is considering doing a repeated measures type of experiment should consider the nature of the response evokers that will be employed in his experiment and examine for possible confounding, a design and analysis which includes the response

evokers as a factor.

If an experimenter must have confounding in the analysis consistent with the design of his repeated measures experiment, he is in a somewhat difficult position with regard to analysis of variance testing of the source of variation with which confounding is present. For even if he can expect his dependent variables to have a normal probability density, the results of this study suggest that the quasi-F test will not have particularly good properties (Chapter V, pages 76, 78). If, on the other hand, the experimenter must evaluate responses in such a manner that his dependent variables are dichotomous, the quasi-F test is completely unacceptable.

If an experimenter finds that he can expect to have no confounding such as that mentioned above, but must have dichotomous dependent variables the results suggest, he can expect to appropriately employ the ordinary analysis of variance, variance ratio test for the repeated measure effects under the following conditions. Appropriate employment is suggested, when there are more than three response evokers associated with a repeated measure, and when (1) the probability of one is close to .5 and there are six or more subjects in an experiment, or (2) the probability of a one is between .2 and .8 and there are ten or more subjects in the experiment, or (3) the probability of a one is between .1 and .9 and there are more than 20 subjects in the experiment. The results also suggest that the above experimenter

should expect the power of analysis of variance tests based on dichotomous data to be between one third and one half the power he could expect if his dependent variables had a normal probability density. The practical suggestion implied by the results of this study is, that if the above experimenter must have dichotomous dependent variables he should employ a larger number of subjects than he would if he could expect his dependent variables to have a normal probability density.

The results of this study with respect to the test of the subject by repeated measure interaction when based on dichotomous data, suggest that even a very large variance ratio statistic may not indicate that the null hypothesis is false if there are a large number of subjects and the probability of a one is not close to .5 (see Figure 6). For a probability of a one close to .5, however, it would appear that the probability of a Type I error is only approximately 1.2 times the nominal α level. Since the power of the test of the subjects by repeated measures interaction based on dichotomous data with a .5 probability of a one was greater than half the power of the test based on normal data, the results suggest that when the probability of a one is close to .5 the test may be appropriately employed. If the probability of a one is not close to .5, however, the results suggest the above test may not be appropriately employed. As a practical suggestion, an experimenter who could expect possible subject by repeated measure

interaction and who must have dichotomous dependent variables should endeavor to employ response evokers which will give him an overall .5 probability of a one.

This investigator has no practical implications or suggestions to present with regard to the correlations presented in Chapter V. As far as he knows, the control of the correlations between mean squares is not in the hands of the experimenter. The results with respect to the correlations were interesting, however, and something of which many readers may have been unaware.

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APPENDICES

APPENDIX A

Frequency data for variance ratio tests other than those included in Chapter V.

Table 22. The Empirical Frequencies, in $\alpha \cdot 1000$ Rejection Regions, of the F for Subjects Based on Normal and Dichotomous Data, Items Crossed, Interaction and Repeated Measures Effects Both Null.

Probability of a One			.5						.2						.1					
$\alpha \cdot 1000$			50		25		10		50		25		10		50		25		10	
Dichotomous or Normal			D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N
No. of Sub.s	Items	Level of Sub. Het.																		
4	Fixed	1	298	412	192	301	100	182	194	431	143	288	73	172	88	391	67	252	42	139
		2	439	572	329	462	199	325	294	593	215	466	130	303	146	572	121	434	71	276
		3	519	684	412	581	274	421	359	673	280	595	170	461	197	678	161	618	83	464
		4	636	813	522	729	373	569	423	775	341	669	249	571	226	766	195	691	133	555
	Random	1	263	415	170	257	89	158	97	405	73	259	43	155	62	422	46	261	34	166
		2	402	575	292	434	155	279	176	583	131	454	73	306	69	598	50	460	27	281
		3	523	687	420	582	241	432	237	684	172	616	118	474	144	660	108	573	67	432
		4	614	808	495	712	369	585	327	805	259	725	182	591	166	763	134	678	101	545
6	Fixed	1	365	546	267	431	176	303	280	576	212	444	144	315	197	538	148	422	87	285
		2	553	774	442	684	328	528	421	761	325	681	210	530	289	739	209	648	162	493
		3	718	862	614	810	483	709	521	864	440	796	322	671	385	870	275	789	214	688
		4	832	933	733	891	624	814	573	926	499	887	401	801	424	942	341	916	267	634
	Random	1	390	587	281	463	155	335	258	534	159	403	101	271	176	521	116	392	63	261
		2	557	754	437	663	299	509	371	730	258	625	185	466	229	714	184	629	133	459
		3	694	873	608	806	460	691	429	852	340	794	247	683	294	868	256	773	186	681
		4	601	939	711	904	551	817	532	923	422	889	329	804	345	936	255	894	224	782
8	Fixed	1	490	676	379	581	262	441	333	674	261	561	146	362	275	675	225	588	158	414
		2	653	850	557	792	410	673	517	859	398	755	321	644	407	856	308	763	217	664
		3	843	960	758	927	648	856	637	950	528	904	442	840	510	951	425	916	322	853
		4	921	986	864	973	792	944	714	988	618	975	533	935	597	990	541	977	419	956
	Random	1	449	696	347	576	221	428	331	647	239	565	162	404	242	651	200	565	117	382
		2	692	855	587	785	442	698	408	873	337	747	247	649	325	852	244	752	203	667
		3	818	946	730	908	599	843	563	955	495	918	396	852	449	966	379	917	283	858
		4	886	988	826	976	731	942	680	989	599	971	503	947	531	956	467	968	394	944
10	Fixed	1	601	809	491	724	359	604	474	736	351	636	244	532	291	761	207	642	154	540
		2	771	913	679	866	540	781	675	931	580	896	435	796	448	909	375	883	299	605
		3	890	963	831	945	743	918	780	966	697	941	592	910	545	980	497	949	398	912
		4	958	993	923	984	881	969	867	989	798	983	704	967	669	985	600	972	525	960
	Random	1	563	804	453	713	324	577	387	752	303	655	226	546	283	777	190	692	141	571
		2	768	900	659	865	535	800	533	910	450	872	367	787	355	908	268	830	203	787
		3	876	967	808	948	728	911	688	970	619	943	511	904	449	966	373	936	276	910
		4	972	995	922	986	670	972	787	996	682	986	563	973	493	983	432	964	377	947
12	Fixed	1	646	838	516	751	387	636	498	852	412	758	284	624	340	862	232	792	162	692
		2	828	956	729	922	601	870	716	954	606	926	490	886	495	947	408	913	329	675
		3	935	987	908	983	845	961	837	986	774	973	693	956	635	991	538	988	466	909
		4	973	998	958	996	915	992	891	995	846	995	809	988	763	994	681	994	603	992
	Random	1	657	854	533	774	420	671	438	835	384	766	273	650	317	845	213	770	138	648
		2	825	958	756	924	654	869	663	951	565	923	443	896	407	968	348	935	291	690
		3	925	993	880	987	794	970	775	985	639	984	579	965	473	984	392	976	349	932
		4	977	1000	959	999	895	993	854	997	775	997	657	989	602	999	525	999	446	999

Table . The Empirical Frequencies, in α-1000 Rejection Regions, of the F for Subjects, Based on Normal and Dichotomous Data, Items Nested, Interaction and Repeated Measure Effects Both Null.

Probability of a One			.5						.2						.1					
α-1000			50		25		10		50		25		10		50		25		10	
Dichotomous or Normal			D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N
No. of Sub.s	Items	Level of Sub. Het.																		
4	Fixed	1	422	550	326	468	236	366	227	560	178	462	107	348	126	555	101	457	52	333
		2	548	694	456	631	372	522	351	717	306	649	219	565	188	698	163	632	93	522
		3	660	803	596	740	481	677	427	630	375	774	282	678	232	828	197	753	137	660
		4	778	873	719	838	619	793	518	851	463	832	394	791	243	863	223	845	166	766
	Random	1	400	559	322	462	211	366	228	538	170	438	117	326	133	554	96	460	46	334
		2	562	717	473	644	366	533	319	708	259	638	176	538	209	674	181	625	103	523
		3	699	834	612	768	524	706	421	820	357	757	276	668	261	805	211	738	141	622
		4	703	865	628	839	577	790	527	882	466	663	366	824	286	884	249	867	192	807
6	Fixed	1	497	662	408	584	292	481	365	704	261	627	191	508	230	659	185	592	106	500
		2	713	868	629	811	532	734	517	868	420	808	335	743	351	865	310	789	200	710
		3	829	924	776	899	695	856	639	940	526	904	437	847	367	936	340	904	261	859
		4	912	961	877	953	810	935	720	979	637	966	537	932	453	979	415	960	354	934
	Random	1	506	715	419	629	303	527	344	662	227	590	174	486	253	674	164	595	39	481
		2	704	860	602	788	487	721	494	867	395	783	309	710	318	855	241	775	163	718
		3	834	933	766	904	656	865	620	928	534	895	406	855	388	918	301	876	242	823
		4	889	961	857	947	803	928	679	976	617	963	516	935	481	978	416	958	320	945
8	Fixed	1	519	787	428	731	311	641	439	774	336	692	289	620	364	771	259	680	177	613
		2	772	926	706	895	618	838	630	927	532	911	430	840	492	953	399	941	343	861
		3	924	981	890	975	832	957	745	983	680	964	608	951	569	979	503	966	408	957
		4	970	994	953	987	921	981	816	997	763	993	691	980	634	998	589	987	513	975
	Random	1	676	797	509	736	693	660	440	767	348	683	244	630	328	781	278	933	206	835
		2	801	940	734	910	631	841	629	944	559	919	449	856	460	986	370	963	315	959
		3	927	980	875	966	826	945	760	981	687	966	603	959	557	1000	463	985	403	972
		4	972	998	940	994	893	991	839	998	765	983	730	964	617	793	571	720	503	621
10	Fixed	1	702	876	625	827	514	773	521	859	429	791	332	701	350	846	285	794	196	719
		2	855	959	804	938	733	897	732	965	661	950	530	924	490	947	430	934	355	896
		3	940	990	912	984	873	971	843	982	793	980	721	963	603	988	539	988	476	970
		4	982	995	963	991	944	987	893	990	655	989	827	989	699	983	643	979	605	979
	Random	1	652	865	555	815	437	744	542	865	445	814	325	749	404	848	294	786	192	707
		2	857	955	794	939	747	912	714	953	644	944	517	913	567	952	485	939	388	901
		3	937	989	916	985	877	977	828	992	788	990	717	971	614	985	565	985	483	966
		4	979	997	962	995	942	994	899	991	865	988	801	988	705	993	663	993	562	993
12	Fixed	1	757	908	675	868	574	811	575	921	469	857	387	808	431	924	333	886	246	830
		2	914	983	867	969	801	940	796	975	733	954	629	950	590	993	522	966	386	962
		3	969	994	961	993	934	986	897	991	846	991	809	985	711	998	655	998	564	993
		4	993	1000	982	1000	977	1000	943	1000	917	1000	882	997	789	1000	743	1000	690	994
	Random	1	756	911	674	866	551	808	621	911	531	875	404	832	408	908	307	865	237	614
		2	926	992	885	979	818	967	771	983	685	963	614	959	606	986	513	960	453	960
		3	965	996	943	996	914	989	883	997	832	997	746	989	690	994	613	994	582	984
		4	994	1000	986	1000	979	1000	924	1000	895	1000	855	998	770	1000	715	1000	652	995

Table [illegible]. The Empirical Frequencies, in α-1000 Rejection Regions, of the F for Subjects, Based on Normal and Dichotomous Data, Items Crossed, Repeated Measure Effects Non-null, Interaction Effects Null.

Probability of a One																				
α-1000			50		25		10		50		25		10		50		25		10	
Dichotomous or Normal			D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N
No. of Sub.s	Items	Level of Sub. Het.																		
4	Fixed	1	292	411	199	289	108	171	218	416	139	280	85	165	116	378	92	259	50	149
		2	422	573	308	464	171	315	318	599	228	467	136	287	164	557	120	418	73	277
		3	526	706	413	620	296	457	379	678	281	578	176	435	229	677	174	612	103	449
		4	649	820	529	744	379	611	480	795	409	709	303	603	301	760	233	704	139	573
	Random	1	267	395	167	253	83	160	176	399	103	270	39	168	88	432	66	273	57	153
		2	406	596	302	462	179	298	209	572	151	442	75	272	90	578	85	446	47	280
		3	523	710	367	611	237	462	307	685	197	598	134	450	163	650	138	593	74	455
		4	639	794	500	697	347	578	379	792	276	699	201	568	203	772	157	686	113	562
6	Fixed	1	390	575	283	465	166	329	308	591	224	464	151	304	216	498	150	595	82	272
		2	517	740	410	653	293	515	414	744	330	669	240	517	334	745	252	552	175	478
		3	696	863	590	795	461	681	549	876	460	798	373	680	424	876	309	788	230	583
		4	797	933	701	895	592	828	641	945	535	911	408	817	503	943	407	964	329	817
	Random	1	366	539	249	416	159	313	277	557	194	426	121	282	176	500	122	392	65	263
		2	540	755	422	655	284	526	329	708	248	630	179	462	276	742	205	553	149	482
		3	664	837	557	764	419	655	504	867	401	788	265	670	329	652	246	755	138	607
		4	765	939	714	899	570	825	589	934	496	896	373	797	395	939	308	912	259	815
8	Fixed	1	458	661	346	572	228	450	346	658	254	559	150	390	241	669	176	575	134	385
		2	671	866	558	785	418	654	585	866	441	757	345	676	426	875	304	764	243	658
		3	810	950	727	915	607	844	692	958	565	909	467	834	561	944	451	910	363	839
		4	898	985	861	974	745	944	760	989	687	980	583	945	639	984	565	975	459	954
	Random	1	450	692	346	587	219	443	336	666	235	573	150	418	237	680	191	564	122	398
		2	666	870	553	784	405	694	480	863	365	754	243	649	417	876	315	771	220	674
		3	784	947	699	908	548	832	640	947	534	908	430	822	476	957	303	926	291	857
		4	878	987	925	973	733	948	747	985	621	973	532	953	609	986	521	971	399	949
10	Fixed	1	551	785	433	687	287	556	483	750	367	659	254	533	323	758	237	662	154	531
		2	789	940	677	889	546	814	700	911	613	885	510	819	464	876	354	855	307	760
		3	911	975	937	965	717	941	761	970	714	940	587	899	599	971	521	943	433	922
		4	959	994	932	988	873	975	867	987	843	977	758	957	718	987	643	973	567	964
	Random	1	560	814	452	723	312	608	450	776	344	701	212	568	327	729	195	652	110	523
		2	721	900	627	865	472	780	608	910	518	878	369	811	422	905	320	875	208	796
		3	673	975	920	956	726	931	736	971	633	952	542	915	509	966	431	943	326	916
		4	935	989	907	980	846	965	825	984	748	971	622	962	649	990	521	981	424	970

Table . The Empirical Frequencies, in $\alpha \cdot 1000$ Rejection Regions, of the F for the Items by Repeated Measures Interaction, Based on the Normal and Dichotomous Data, Items Nested, Interaction and Repeated Measure Effects Both Null

Probability of a One			.5						.2						.1					
$\alpha \cdot 1000$			50		25		10		50		25		10		50		25		10	
Dichotomous or Normal			D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N
No. of Sub.s	Items	Level of Sub. Het.																		
4	Fixed	1	62	54	30	31	13	13	22	72	5	45	0	20	19	59	8	31	4	13
		2	54	43	29	19	7	6	26	56	5	27	1	9	23	46	6	25	0	14
		3	45	43	22	26	6	11	34	52	5	25	0	12	17	49	0	24	0	12
		4	42	36	20	16	7	6	26	48	9	27	2	12	7	53	2	31	2	20
	Random	1	54	44	27	29	9	14	52	46	28	25	7	7	44	59	9	33	2	16
		2	52	48	26	20	11	2	42	48	17	29	4	15	39	41	7	26	2	15
		3	47	60	23	30	3	9	23	46	11	23	7	17	23	48	3	25	0	20
		4	47	51	20	29	7	9	29	52	11	38	3	21	14	45	5	30	4	17
6	Fixed	1	61	54	31	26	13	11	28	56	12	32	5	6	19	51	11	29	3	15
		2	57	54	27	30	10	8	33	50	14	24	4	11	14	46	3	30	0	8
		3	42	43	17	23	4	7	21	56	13	28	4	12	16	57	0	29	0	13
		4	49	45	13	28	4	6	11	54	8	34	3	15	13	52	7	26	2	18
	Random	1	47	42	27	28	5	11	44	62	15	30	10	9	43	53	15	31	11	16
		2	39	52	23	33	8	11	32	60	21	39	5	19	25	35	15	18	7	3
		3	43	55	18	30	8	16	44	66	26	49	7	25	17	48	9	25	9	12
		4	36	48	21	30	7	15	39	49	12	32	6	14	25	39	16	13	16	7
8	Fixed	1	59	46	41	26	10	14	26	39	20	19	11	8	46	28	13	16	0	4
		2	50	39	23	22	13	11	28	42	14	21	6	4	18	36	6	22	2	9
		3	39	39	24	19	12	9	37	41	20	15	15	9	15	30	3	22	3	5
		4	40	33	17	15	10	5	29	63	18	30	11	14	31	37	5	20	0	3
	Random	1	42	53	17	26	10	14	52	44	27	20	13	7	35	41	4	26	1	6
		2	46	46	23	34	10	13	23	41	9	17	7	5	35	35	16	22	4	8
		3	32	39	24	25	14	10	39	44	9	23	4	10	24	28	14	18	4	7
		4	40	37	12	21	8	4	29	34	5	19	5	3	11	30	6	21	0	5
10	Fixed	1	47	57	17	22	4	10	28	38	16	12	9	8	17	34	9	10	8	6
		2	53	42	32	21	10	5	47	34	20	11	14	8	25	35	17	18	8	13
		3	51	41	28	22	14	11	42	31	14	12	4	9	9	29	5	8	5	8
		4	39	44	26	18	10	12	33	37	19	16	12	11	31	52	9	21	5	16
	Random	1	40	45	15	24	5	15	45	25	5	7	5	5	36	42	20	18	11	13
		2	43	38	22	18	9	7	30	38	22	12	5	11	36	31	5	14	5	12
		3	32	26	10	7	6	4	56	34	17	20	8	18	8	37	8	15	3	12
		4	43	28	23	10	14	7	49	33	10	16	5	13	24	38	16	17	2	15
12	Fixed	1	46	38	27	19	15	12	44	46	20	17	10	8	36	31	15	14	4	11
		2	44	39	19	19	10	7	32	38	17	11	8	4	47	32	16	14	9	10
		3	84	51	47	23	17	9	22	37	10	13	5	4	20	31	1	14	1	11
		4	40	49	21	24	6	8	52	50	23	31	8	10	23	39	17	15	5	11
	Random	1	53	46	30	20	13	5	54	60	28	25	14	11	26	20	18	16	12	12
		2	56	33	28	16	8	7	47	52	28	27	9	12	10	27	7	20	7	13
		3	51	45	23	25	11	10	59	29	22	11	9	7	41	25	29	18	11	12
		4	49	38	24	26	8	9	63	33	36	17	17	13	9	28	9	15	5	7

Table . The Empirical Frequencies, in α·1000 Rejection regions, of the F for Items, Based on the Normal and Dichotomous Data, Items Crossed, Interaction and Repeated Measure Effects Both Null.

Probability of a One			.5						.2						.1					
α·1000			50		25		10		50		25		10		50		25		10	
Dichotomous or Normal			D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N
No. of Sub.s	Items	Level of Sub. Het.																		
4	Fixed	1	50	47	18	26	6	2	38	54	13	32	7	11	26	45	13	34	2	6
		2	52	65	31	38	17	8	33	51	10	34	4	13	13	42	7	29	1	9
		3	54	46	22	26	11	12	29	52	13	31	8	13	14	48	1	30	0	7
		4	38	46	17	21	8	8	20	43	9	29	0	10	5	56	0	37	0	5
	Random	1	151	56	83	30	37	5	139	42	88	27	40	7	101	44	50	28	20	4
		2	139	48	61	22	29	8	108	44	57	28	30	11	66	47	26	32	10	6
		3	161	39	61	17	31	4	104	41	42	31	23	5	68	41	27	28	6	6
		4	151	45	76	26	42	8	101	46	56	32	26	3	65	62	17	32	2	7
6	Fixed	1	49	35	16	18	6	2	45	41	19	24	4	8	25	37	17	23	4	9
		2	52	50	24	22	5	9	17	44	3	27	1	12	38	42	17	18	1	5
		3	40	42	14	26	2	12	38	37	11	17	4	6	17	38	1	21	0	9
		4	52	51	19	26	5	16	20	42	6	23	1	8	4	46	3	22	1	10
	Random	1	234	54	143	25	85	9	253	45	162	29	103	15	210	56	112	36	72	10
		2	262	36	180	12	99	5	286	33	170	18	103	6	176	42	97	31	59	6
		3	267	48	181	23	98	8	231	31	132	19	80	5	151	54	61	30	40	10
		4	287	56	197	25	97	8	251	30	144	15	54	4	140	36	68	16	33	1
8	Fixed	1	59	39	24	24	11	9	23	26	13	18	5	6	31	28	5	19	0	5
		2	45	49	16	24	8	10	40	25	23	17	8	5	41	28	16	19	0	2
		3	52	46	29	28	13	13	18	27	9	15	0	8	13	33	6	24	6	4
		4	48	48	25	26	10	14	21	25	6	11	0	4	6	26	2	13	0	1
	Random	1	290	53	220	30	147	13	317	34	221	18	145	5	234	27	167	18	112	2
		2	350	53	246	35	165	8	312	24	220	18	136	6	218	29	147	22	97	4
		3	344	52	260	33	184	9	327	26	207	15	125	2	224	36	134	24	86	5
		4	336	43	254	29	176	5	323	20	250	12	144	0	214	24	103	13	51	2
10	Fixed	1	61	66	31	27	8	9	48	67	34	27	12	2	35	62	13	44	6	3
		2	44	41	22	15	4	6	47	41	28	24	19	0	40	61	17	40	17	5
		3	39	40	25	24	8	10	51	54	24	32	19	4	29	50	12	35	8	7
		4	52	63	27	30	7	15	28	50	13	26	3	1	1	54	0	31	0	1
	Random	1	332	48	253	18	175	6	457	53	360	30	252	4	299	49	211	31	131	5
		2	381	56	284	30	193	15	415	55	333	37	255	6	368	52	266	29	167	5
		3	378	60	290	32	194	8	417	56	337	36	248	1	346	53	258	30	127	4
		4	445	53	361	26	256	0	466	66	340	41	250	3	305	57	194	31	111	0
12	Fixed	1	43	40	17	19	8	9	47	33	21	15	4	0	42	41	14	22	5	3
		2	50	58	24	29	6	17	30	27	20	15	4	2	30	37	14	16	14	3
		3	44	51	22	27	13	9	42	30	17	15	14	6	55	41	25	26	0	4
		4	44	43	23	27	13	17	57	32	25	15	7	4	31	47	4	33	0	2
	Random	1	460	594	346	515	267	421	458	697	382	644	280	562	409	720	297	664	199	573
		2	465	635	357	551	278	458	493	742	399	687	259	594	422	746	308	702	217	629
		3	481	685	387	596	285	493	461	759	383	690	261	624	390	783	271	726	177	632
		4	468	727	394	655	326	570	473	797	374	761	275	689	335	829	250	758	164	680

Table . The Empirical Frequencies, in $\alpha \cdot 1000$ Rejection Regions, of the F for Items, Based on the Dichotomous Data, Items Crossed, Repeated Measure Effects Non-null, Interaction Null.

Probability of a One			.5			.2			.1		
$\alpha \cdot 1000$			50	25	10	50	25	10	50	25	10
No. of Sub.s	Items	Level of Sub. Het.									
4	Fixed	1	45	15	6	39	15	9	31	15	4
		2	43	19	10	17	4	2	16	7	2
		3	44	16	4	10	6	1	13	2	0
		4	32	13	3	7	1	0	2	0	0
	Random	1	160	100	47	164	87	58	114	56	19
		2	154	87	40	155	92	41	91	37	15
		3	160	73	36	133	69	45	68	32	11
		4	154	87	41	138	65	39	49	16	7
6	Fixed	1	50	25	12	38	14	5	15	8	2
		2	44	18	9	31	11	0	23	17	2
		3	47	24	9	37	20	6	18	1	0
		4	53	22	9	32	14	2	23	6	0
	Random	1	218	143	85	290	175	116	225	126	93
		2	259	172	93	231	204	129	176	102	73
		3	269	181	101	313	196	105	164	105	55
		4	289	198	88	297	169	109	141	68	35
8	Fixed	1	41	16	6	65	29	15	31	13	2
		2	34	13	4	31	10	5	24	4	3
		3	41	19	9	40	16	7	15	7	0
		4	49	28	14	27	11	0	27	7	7
	Random	1	281	199	113	368	258	170	289	181	153
		2	272	181	105	369	263	162	247	131	90
		3	308	219	146	350	249	157	214	139	75
		4	339	287	185	367	269	172	234	153	88
10	Fixed	1	46	29	11	32	24	8	43	34	10
		2	48	25	4	29	7	4	50	19	10
		3	46	18	5	36	18	6	44	28	4
		4	49	22	3	38	19	3	34	12	0
	Random	1	347	274	186	451	383	247	346	247	159
		2	374	287	211	457	366	262	369	270	152
		3	373	290	183	507	398	271	374	250	186
		4	437	341	256	532	421	255	325	258	161

Table . The Empirical Frequencies, in $\alpha \cdot 1000$ Rejection Regions, of the F for Items, Based on the Dichotomous Data, Items Nested, Interaction and Repeated Measure Effects Both Null.

Probability of a One				.5			.2			.1		
No. of Sub.s	Items	Level of Sub. Het.	$\alpha \cdot 1000$	144	37	9	144	37	9	144	37	9
4	Fixed	1		145	49	11	119	21	4	106	16	3
		2		142	40	10	133	29	2	74	13	1
		3		131	28	9	116	23	8	62	17	4
		4		130	32	11	101	21	2	22	6	2
	Random	1		233	75	17	305	110	49	226	77	38
		2		249	56	20	295	105	56	219	69	29
		3		276	98	29	271	86	44	200	56	26
		4		286	85	25	289	94	32	231	58	16
			$\alpha \cdot 1000$	134	30	6	134	30	6	134	30	6
6	Fixed	1		136	36	7	94	18	3	59	12	4
		2		150	37	5	86	15	4	66	5	2
		3		115	22	1	100	10	0	37	7	0
		4		129	29	3	85	11	1	47	8	0
	Random	1		285	108	40	331	120	55	261	112	45
		2		279	96	26	268	101	44	236	93	37
		3		312	128	29	289	101	36	234	92	32
		4		310	118	40	309	116	26	257	76	37
			$\alpha \cdot 1000$	127	27	5	127	27	5	127	27	5
8	Fixed	1		142	45	5	92	17	8	95	16	0
		2		112	28	3	95	18	9	105	18	11
		3		104	25	5	112	24	4	80	6	0
		4		95	17	5	88	23	2	86	10	0
	Random	1		365	156	46	382	156	52	293	84	26
		2		405	172	51	402	161	64	311	95	29
		3		389	144	49	436	162	68	316	97	18
		4		414	171	87	412	137	62	307	109	35
			$\alpha \cdot 1000$	127	25	4	127	25	4	127	25	4
10	Fixed	1		129	31	3	104	18	12	75	20	4
		2		126	20	4	117	20	3	106	18	5
		3		127	34	11	124	13	6	75	15	0
		4		120	23	8	117	14	0	67	11	5
	Random	1		435	177	47	484	228	96	349	161	53
		2		437	179	69	514	268	122	379	142	37
		3		403	153	63	480	242	107	351	159	46
		4		426	192	84	504	289	131	392	181	51
			$\alpha \cdot 1000$	126	24	4	126	24	4	126	24	4
12	Fixed	1		100	27	7	118	17	1	111	17	0
		2		129	20	4	112	17	2	128	6	0
		3		116	21	5	108	17	1	92	0	0
		4		103	21	4	125	12	5	98	11	5
	Random	1		470	231	82	586	273	135	523	241	95
		2		500	254	95	562	267	131	417	247	121
		3		518	265	103	623	360	130	461	215	81
		4		561	287	120	649	320	162	483	205	104

Table . The Empirical Frequencies, in $\alpha \cdot 1000$ Rejection Regions, of the F for Items, Based on the Normal and Dichotomous Data, Items Crossed, Interaction and Repeated Measure Effects Both Null, Twelve Subjects.

Probability of a One		.5						.2						.1					
$\alpha \cdot 1000$		50		25		10		50		25		10		50		25		10	
Normal or Dichotomous		D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N
Items	Level of Sub. Het.																		
Fixed	1	43	40	17	19	8	9	47	33	21	15	4	0	42	41	14	22	5	3
	2	50	58	24	29	6	17	30	27	20	15	4	2	30	37	14	16	14	3
	3	44	51	22	27	13	9	42	30	17	15	14	6	55	41	25	26	0	4
	4	44	43	23	27	13	17	57	32	25	15	7	4	31	47	4	33	0	2
Random	1	460	594	346	515	267	421	458	697	382	644	280	562	409	720	297	664	199	579
	2	465	635	357	551	278	458	493	742	399	687	289	594	422	746	308	702	217	629
	3	481	695	387	596	285	493	461	759	383	690	261	624	390	783	271	726	177	682
	4	468	727	394	655	326	570	473	797	374	761	275	689	335	829	250	758	164	660

Table . The Empirical Frequencies, in 1000 Rejection Regions, of the F for Items, Based on Normal and Dichotomous Data, Items Nested, Interaction and Repeated Measure Effects Both Null, Twelve Subjects.

Probability of a One		.5						.2						.1					
1000		126		24		4		126		24		4		126		24		4	
Normal or Dichotomous		D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N
Items	Level of Sub. Het.																		
Fixed	1	100	102	27	19	7	5	118	94	17	22	1	6	111	63	17	18	0	11
	2	129	115	20	27	4	6	112	88	17	21	2	6	128	77	6	16	0	10
	3	116	109	21	24	5	7	108	94	17	24	1	4	92	93	0	20	0	11
	4	103	117	21	28	4	4	125	87	12	33	5	6	98	104	11	20	5	11
Random	1	470	634	231	417	82	246	586	851	273	645	135	472	523	866	241	740	95	529
	2	500	680	254	421	95	268	562	664	267	694	131	522	417	860	247	759	121	631
	3	518	740	265	505	103	330	623	910	360	793	130	630	461	935	215	827	81	675
	4	561	776	287	561	120	380	649	950	320	855	162	717	483	979	206	869	104	746

APPENDIX B

Empirical values of correlations between mean squares in variance ratios other than those presented in Chapter V.

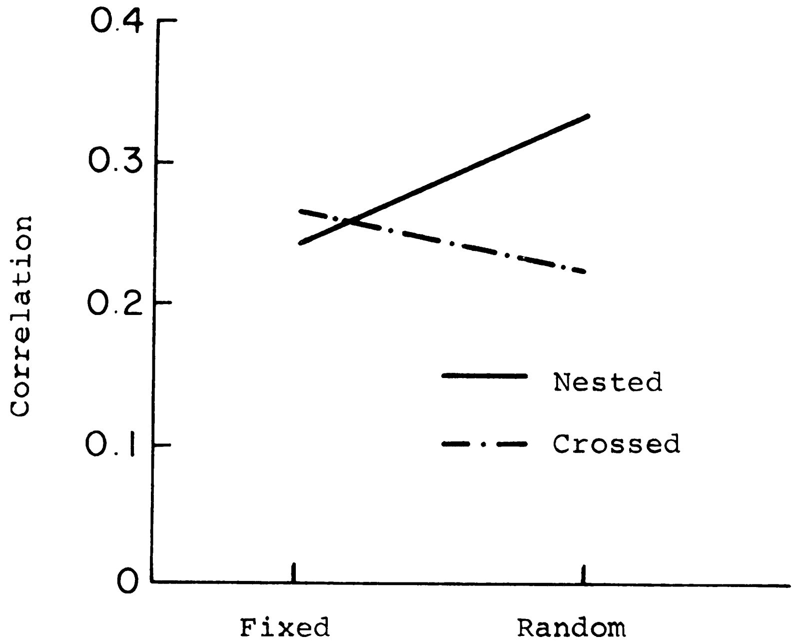

Figure 15. The interaction between items fixed vs. random and items crossed vs. nested, with respect to the correlations in Table 21.

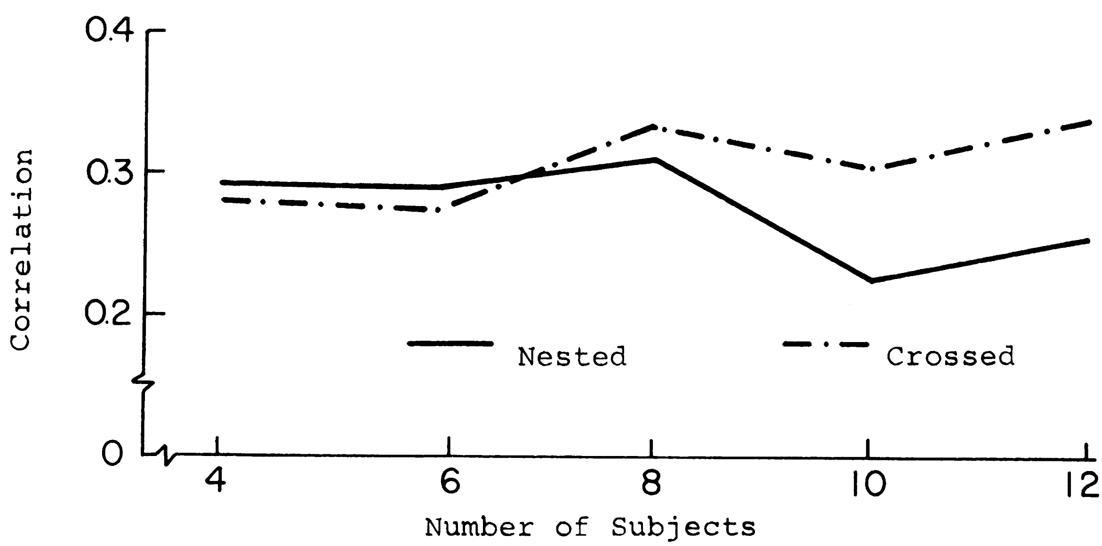

Figure 16. The interaction between the number of subjects and items crossed vs. nested, with respect to the correlations in Table 21.

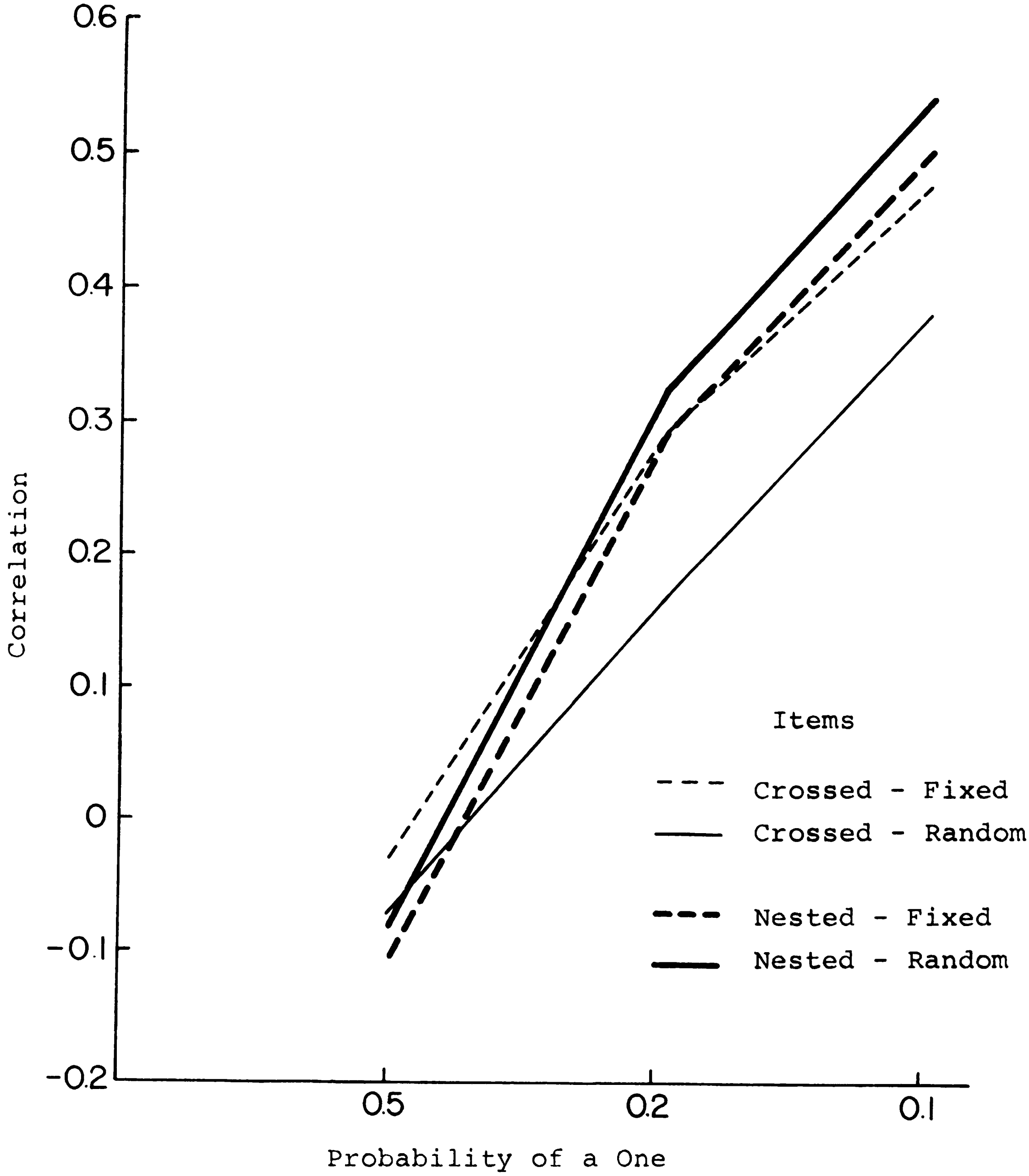

Figure 17. The second order interaction between the probability of a one, items fixed vs. random, and items crossed vs. nested, with respect to the correlations in Table 21.

Table 32. The Correlations Between the Mean Square Items by Repeated Measures Interaction and the Mean Square Subjects by Items by Repeated Measures Interaction Based on the Dichotomous Data, Interaction and Repeated Measure Effects Both Null.

Probability of a One		.5				.2				.1			
Level of Subject Heterogeneity		1	2	3	4	1	2	3	4	1	2	3	4
No. of Sub.s	Items												
4	Fixed	-0.200	-0.029	0.003	0.147	0.382	0.432	0.477	0.541	0.658	0.656	0.769	0.789
	Random	-0.162	-0.045	0.059	0.137	0.352	0.438	0.527	0.568	0.572	0.596	0.609	0.696
6	Fixed	-0.144	-0.038	0.015	0.086	0.267	0.299	0.421	0.527	0.480	0.578	0.650	0.640
	Random	-0.052	-0.002	0.123	0.186	0.274	0.387	0.383	0.465	0.498	0.546	0.568	0.525
8	Fixed	-0.140	0.020	0.045	0.106	0.164	0.274	0.268	0.412	0.486	0.573	0.591	0.651
	Random	0.064	0.016	0.065	0.107	0.281	0.404	0.353	0.422	0.480	0.540	0.540	0.687
10	Fixed	-0.048	-0.047	0.046	0.151	0.250	0.265	0.381	0.359	0.533	0.488	0.589	0.597
	Random	-0.025	0.004	0.044	0.049	0.331	0.373	0.305	0.378	0.587	0.521	0.535	0.587
12	Fixed	-0.110	0.035	0.023	0.126	0.172	0.206	0.298	0.360	0.417	0.418	0.434	0.488
	Random	-0.051	-0.030	0.058	0.074	0.298	0.347	0.346	0.290	0.510	0.517	0.495	0.513

Table . The Correlations Between the Mean Squares for Items and the Mean Square Error Associated, Based on the Dichotomous Data, Interaction and Repeated Measure Effects Both Null.

Probability of a One			.5				.2				.1			
Level of Subject Heterogeneity			1	2	3	4	1	2	3	4	1	2	3	4
Items	No. of Sub.s	Items												
Crossed	4	Fixed	-0.026	-0.014	0.037	0.176	0.299	0.374	0.344	0.418	0.286	0.488	0.534	0.620
		Random	-0.052	0.073	0.140	0.148	0.230	0.382	0.426	0.407	0.520	0.584	0.665	0.721
	6	Fixed	-0.022	0.007	0.103	0.094	0.210	0.306	0.228	0.302	0.301	0.315	0.518	0.509
		Random	-0.091	-0.049	0.071	0.110	0.257	0.290	0.396	0.414	0.401	0.402	0.491	0.534
	8	Fixed	0.012	0.023	0.005	0.083	0.190	0.217	0.354	0.412	0.349	0.375	0.475	0.589
		Random	0.032	0.006	0.103	0.169	0.285	0.337	0.404	0.419	0.411	0.481	0.482	0.623
	10	Fixed	-0.020	0.048	0.138	0.073	0.055	0.152	0.221	0.333	0.267	0.337	0.406	0.432
		Random	-0.023	-0.063	0.092	0.063	0.191	0.268	0.360	0.462	0.492	0.619	0.650	0.726
	12	Fixed	-0.036	0.041	0.005	0.105	0.188	0.180	0.253	0.258	0.223	0.268	0.320	0.390
		Random	-0.083	-0.082	0.065	0.093	0.186	0.283	0.315	0.465	0.329	0.370	0.462	0.539
Nested	4	Fixed	-0.248	-0.048	0.022	0.194	0.414	0.469	0.549	0.606	0.688	0.725	0.766	0.627
		Random	-0.226	-0.044	0.029	0.226	0.224	0.309	0.408	0.559	0.590	0.584	0.641	0.745
	6	Fixed	-0.189	-0.053	0.051	0.151	0.332	0.370	0.482	0.575	0.600	0.651	0.751	0.695
		Random	-0.265	-0.105	0.032	0.182	0.224	0.319	0.454	0.553	0.451	0.499	0.599	0.665
	8	Fixed	-0.181	0.019	0.056	0.182	0.206	0.289	0.351	0.455	0.571	0.609	0.661	0.717
		Random	-0.208	-0.147	-0.024	0.100	0.331	0.408	0.488	0.556	0.534	0.631	0.639	0.665
	10	Fixed	-0.117	-0.055	0.068	0.172	0.270	0.302	0.440	0.441	0.567	0.569	0.618	0.656
		Random	-0.219	-0.106	0.012	0.098	0.270	0.264	0.437	0.438	0.494	0.544	0.570	0.641
	12	Fixed	-0.076	-0.057	-0.012	0.058	0.232	0.201	0.275	0.288	0.256	0.340	0.323	0.351
		Random	-0.165	-0.014	0.090	0.138	0.162	0.142	0.193	0.276	0.328	0.321	0.354	0.330

Table [illegible]. The Correlations Between the Mean Squares for Subjects and the Mean Square Error Associated, Based on the Dichotomous Data, Interaction and Repeated Measure Effects Both Null.

Probability of a One			.5				.2				.1			
Level of Subject Heterogeneity			1	2	3	4	1	2	3	4	1	2	3	4
Items	No. of Sub.s	Items												
Crossed	4	Fixed	-0.244	-0.274	-0.284	-0.250	0.155	0.171	0.178	0.232	0.373	0.419	0.437	0.341
		Random	-0.132	-0.190	-0.268	-0.224	0.316	0.293	0.237	0.230	0.507	0.445	0.383	0.415
	6	Fixed	-0.270	-0.339	-0.355	-0.370	0.156	0.195	0.167	0.102	0.354	0.363	0.438	0.487
		Random	-0.135	-0.199	-0.258	-0.298	0.198	0.186	0.276	0.280	0.389	0.406	0.413	0.384
	8	Fixed	-0.255	-0.311	-0.345	-0.390	0.272	0.183	0.109	0.172	0.331	0.333	0.389	0.366
		Random	-0.204	-0.249	-0.269	-0.316	0.281	0.282	0.216	0.205	0.439	0.440	0.491	0.481
	10	Fixed	-0.269	-0.315	-0.362	-0.343	0.103	0.139	0.141	0.220	0.374	0.321	0.343	0.421
		Random	-0.164	-0.251	-0.302	-0.293	0.265	0.264	0.359	0.262	0.509	0.480	0.457	0.378
	12	Fixed	-0.267	-0.350	-0.328	-0.443	0.129	0.123	0.136	0.141	0.359	0.259	0.354	0.378
		Random	-0.288	-0.293	-0.321	-0.402	0.219	0.271	0.278	0.243	0.564	0.473	0.411	0.441
Nested	4	Fixed	-0.497	-0.592	-0.562	-0.571	0.270	0.289	0.264	0.337	0.539	0.586	0.584	0.502
		Random	-0.407	-0.478	-0.524	-0.521	0.246	0.271	0.251	0.292	0.490	0.512	0.551	0.453
	6	Fixed	-0.577	-0.650	-0.669	-0.644	0.185	0.192	0.214	0.150	0.477	0.497	0.536	0.523
		Random	-0.476	-0.510	-0.589	-0.599	0.184	0.256	0.220	0.160	0.456	0.394	0.448	0.405
	8	Fixed	-0.610	-0.677	-0.701	-0.711	0.259	0.208	0.161	0.207	0.465	0.399	0.451	0.408
		Random	-0.504	-0.596	-0.606	-0.658	0.187	0.149	0.222	0.270	0.538	0.443	0.427	0.435
	10	Fixed	-0.639	-0.662	-0.739	-0.734	0.169	0.244	0.203	0.213	0.498	0.430	0.384	0.427
		Random	-0.477	-0.641	-0.652	-0.639	0.220	0.193	0.227	0.304	0.470	0.446	0.488	0.484
	12	Fixed	-0.630	-0.737	-0.745	-0.809	0.133	0.156	0.154	0.157	0.483	0.472	0.457	0.426
		Random	-0.577	-0.676	-0.725	-0.707	0.234	0.198	0.207	0.199	0.576	0.534	0.531	0.466

Table 35. The Correlations Between the Mean Squares for Items and the Mean Squares for Subjects by Items Interaction Based on the Dichotomous Data, Items Crossed, Repeated Measure Effects are non-null and Interaction Effects are Null.

Probability of a One		.5625				.2400				.1225			
Level of Subject Heterogeneity		1	2	3	4	1	2	3	4	1	2	3	4
No. of Sub.s	Items												
4	Fixed	0.016	-0.038	-0.004	0.086	0.189	0.317	0.321	0.436	0.326	0.389	0.539	0.575
	Random	-0.051	0.023	0.019	0.117	0.156	0.208	0.300	0.384	0.440	0.559	0.625	0.705
6	Fixed	0.001	0.048	0.075	0.027	0.115	0.259	0.264	0.275	0.392	0.364	0.488	0.492
	Random	-0.094	-0.069	0.031	0.056	0.161	0.141	0.189	0.236	0.295	0.348	0.432	0.523
8	Fixed	0.011	0.066	-0.024	0.006	0.002	0.221	0.287	0.262	0.264	0.285	0.410	0.431
	Random	0.018	0.061	0.041	0.106	0.186	0.263	0.314	0.425	0.409	0.508	0.460	0.601
10	Fixed	-0.024	-0.032	0.025	-0.018	0.090	0.195	0.186	0.249	0.146	0.221	0.248	0.324
	Random	-0.077	-0.066	0.104	0.163	0.164	0.249	0.290	0.400	0.399	0.469	0.567	0.625

Table 36. The Correlations Between the Mean Squares for Subjects and the Mean Squares for Subjects by Items Interaction Based on the Dichotomous Data, Items Crossed, Repeated Measure Effects are Non-null and Interaction Effects are Null.

Probability of a One		.5				.2				.1			
Level of Subject Heterogeneity		1	2	3	4	1	2	3	4	1	2	3	4
No. of Sub.s	Items												
4	Fixed	-0.239	-0.240	-0.246	-0.245	0.122	0.089	0.133	0.057	0.289	0.345	0.428	0.377
	Random	-0.138	-0.182	-0.206	-0.257	0.216	0.154	0.124	0.158	0.374	0.371	0.334	0.398
6	Fixed	-0.218	-0.283	-0.343	-0.361	0.070	0.059	0.034	0.075	0.321	0.323	0.330	0.382
	Random	-0.149	-0.211	-0.274	-0.276	0.130	0.143	0.168	0.129	0.398	0.429	0.374	0.367
8	Fixed	-0.170	-0.266	-0.284	-0.323	0.042	0.076	0.111	0.078	0.275	0.280	0.275	0.297
	Random	-0.196	-0.231	-0.207	-0.274	0.159	0.171	0.114	0.150	0.473	0.433	0.356	0.381
10	Fixed	-0.251	-0.227	-0.226	-0.376	0.005	-0.005	0.037	0.063	0.418	0.314	0.340	0.333
	Random	-0.183	-0.186	-0.322	-0.361	0.176	0.221	0.179	0.214	0.456	0.482	0.446	0.383

APPENDIX C

The listing of the computer program employed in the Monte Carlo procedure.

```
PAGE   1

// JOB T

LOG DRIVE    CART SPEC    CART AVAIL  PHY DRIVE
  0000         1111         1111        0000

V2 M09   ACTUAL  8K  CONFIG  8K

// FOR
* IOCS(1132 PRINTER,CARD,DISK)
* LIST SOURCE PROGRAM
      DIMENSION A(80)
      DO 5 J=1,50
      WRITE(3,1)
      WRITE(3,2)
      DO 5 I=1,45
      READ(2,3)(A(K),K=1,80)
    5 WRITE(3,4)(A(K),K=1,80)
    1 FORMAT(1H1)
    2 FORMAT(8(/))
    3 FORMAT(80A1)
    4 FORMAT(20X,80A1)
      CALL EXIT
      END

FEATURES SUPPORTED
 IOCS

CORE REQUIREMENTS FOR
 COMMON      0  VARIABLES    168  PROGRAM    132

END OF COMPILATION

// XEQ
```

```
      PROGRAM GENN3 (INPUT,OUTPUT,TAPE60=INPUT,TAPE61=OUTPUT,PUNCH,TAPE6GEN
     12=PUNCH)                                                          GEN
      DIMENSION H(3,4,12), H1(12,12)                                    GEN
      DIMENSION Z0(144), Z1(12,12), Z2(3,4,12)                          GEN
      DIMENSION Y0(144), Y1(12,12), Y2(3,4,12)                          GEN
      DIMENSION XXX(9,9), YYY(9), RHO(9,9), VAR(9), F(14)               GEN
      DIMENSION ICOUNT(20,13), CNTILE(10,12), POINT(19), EFCT(13), CM(4)GEN
      DIMENSION COUNT(10,12), JCOUNT(10,12), LCOUNT(2,12), NCOUNT(2,12) GEN
      DIMENSION PT(12), POIN(12), A(9), FMT3(8), FMT4(8), DF(9), RM(9)  GEN
      DIMENSION XX4(4,12), XX5(4,4), SX5(4,4), SX6(4,4), XX(12,12)      GEN
      DIMENSION SX(12,12), CORR(12,12), CHOL1(12,12)                    GEN
      DIMENSION SUMRT(12), SUMST(4), SUMSR(3), SUMT(12,4), SUMR(12,3), SGEN
     1UMS(4,3), MEANV1(4), MEANV2(4), MEANS(12)                         GEN
      EQUIVALENCE (H,H1), (PT,POIN), (Z0,Z1,Z2), (Y0,Y1,Y2)             GEN
      INTEGER COUNT,DF                                                  GEN
      REAL MEANV1,MEANV2,MEANS                                          GEN
      DATA (PT(I),I=1,12)/.1,.2,.3,.4,.5,.6,.7,.8,.9,.95,.975,.99/      GEN
      DATA (POINT(I),I=1,19)/-.9,-.8,-.7,-.6,-.5,-.4,-.3,-.2,-.1,.0,.1,.GEN
     12,.3,.4,.5,.6,.7,.8,.9/                                           GEN
      READ (60,106) NS,NR,NT,NUMSAM,INISH                               GEN
      READ (60,107) FMT3                                                GEN
      READ (60,107) FMT4                                                GEN
      READ (60,108) EPS,START                                           GEN
      READ (60,106) NU1,NU2,NU3,NU4                                     GEN
      CALL RANSET (START)                                               GEN
      A(1)=FMT3(1)                                                      GEN
      A(2)=FMT3(2)                                                      GEN
      A(3)=FMT3(4)                                                      GEN
      A(4)=FMT3(5)                                                      GEN
      A(5)=FMT4(1)                                                      GEN
      A(6)=FMT3(6)                                                      GEN
      A(7)=FMT4(2)                                                      GEN
      A(8)=FMT4(3)                                                      GEN
      A(9)=FMT4(4)                                                      GEN
C                                                                       GEN
C     DEFINITIONS OF DO PARAMETERS AND DEGREES OF FREEDOM               GEN
C                                                                       GEN
      NMID=NS/2                                                         GEN
      NMIDS=NMID+1                                                      GEN
      RNT=NT                                                            GEN
      RNR=NR                                                            GEN
      RNS=NS                                                            GEN
      RMS=NS-1.                                                         GEN
      RMR=NR-1.                                                         GEN
      RMT=NT-1.                                                         GEN
```

```
      NSR=NS*NR
      NST=NS*NT
      NRT=NR*NT
      NTOT=NS*NR*NT
      RNSR=NSR
      RNST=NST
      RNRT=NRT
      RNTOT=NTOT
      DF(1)=RMS
      DF(2)=RMR
      DF(3)=RMT
      DF(4)=(RMS*RMR)
      DF(5)=(RMS*RMT)
      DF(6)=(RMR*RMT)
      DF(7)=(RMS*RMR*RMT)
      DF(8)=(RMT*RNR)
      DF(9)=(RMS*RMT*RNR)
      DO 1 I=1,9
      CHANG=DF(I)
      DF(I)=((CHANG*EPS)+.5)*10.
    1 DF(I)=DF(I)/10
      IF (INISH.LE.0) GO TO 4
      DO 2 I=1,10
      DO 2 J=1,12
    2 CNTILE(I,J)=0.
      DO 3 J=1,12
      CALL FDIST (DF(1),DF(5),CNTILE(1,J),POIN(J))
      CALL FDIST (DF(1),DF(9),CNTILE(2,J),POIN(J))
      CALL FDIST (DF(2),DF(4),CNTILE(3,J),POIN(J))
      CALL FDIST (NU1,NU2,CNTILE(4,J),POIN(J))
      CALL FDIST (NU3,NU4,CNTILE(5,J),POIN(J))
      CALL FDIST (DF(3),DF(5),CNTILE(6,J),POIN(J))
      CALL FDIST (DF(8),DF(9),CNTILE(7,J),POIN(J))
      CALL FDIST (DF(4),DF(7),CNTILE(8,J),POIN(J))
      CALL FDIST (DF(4),DF(9),CNTILE(9,J),POIN(J))
    3 CALL FDIST (DF(6),DF(7),CNTILE(10,J),POIN(J))
      GO TO 6
    4 CONTINUE
      DO 5 I=1,10
    5 READ (60,75) (CNTILE(I,J),J=1,12)
      GO TO 8
    6 CONTINUE
      DO 7 I=1,10
    7 WRITE (62,75) (CNTILE(I,J),J=1,12)
    8 CONTINUE
```

```
      WRITE (61,77)
      WRITE (61,75)
      DO 9 I=1,10
    9 WRITE (61,74) (CNTILE(I,J),J=1,12)
      SEC=SECOND(SEC)
      WRITE (61,76) SEC
   10 CONTINUE
      DO 11 I=1,2
      DO 11 J=1,12
   11 NCOUNT(I,J)=0.
      DO 12 J=1,4
      DO 12 I=1,4
   12 SX6(I,J)=0.
      DO 13 I=1,20
      DO 13 J=1,13
   13 ICOUNT(I,J)=0
      DO 14 J=1,10
      DO 14 I=1,12
   14 COUNT(J,I)=0.
      DO 15 I=1,9
      DO 15 J=1,9
      XXX(I,J)=0.
   15 YYY(I)=0.
      DO 16 J=1,12
      DO 16 I=1,2
   16 LCOUNT(I,J)=0
      DO 17 J=1,12
      DO 17 I=1,10
   17 JCOUNT(I,J)=0
      READ (60,106) NEST
      IF (EOF(60)) 71,18,71
   18 CONTINUE
      READ (60,108) SIGMAV,SIGMAD,SIGMAM,DIFF
      READ (60,108) (MEANV1(J),J=1,NR)
      READ (60,108) (MEANV2(J),J=1,NR)
      READ (60,108) EL1,EL2,EL3,EL4
C
C
      DO 19 I=1,3
      DO 19 J=1,3
      CORR(I,J)=EL1
      CORR(I+3,J+3)=EL1
      CORR(I+6,J+6)=EL1
      CORR(I+9,J+9)=EL1
      CORR(I+3,J)=EL2
```

```
      CORR(I+6,J+3)=EL2
      CORR(I+9,J+6)=EL2
      CORR(I,J+3)=EL2
      CORR(I+3,J+6)=EL2
      CORR(I+6,J+9)=EL2
      CORR(I,J+6)=EL3
      CORR(I+3,J+9)=EL3
      CORR(I+6,J)=EL3
      CORR(I+9,J+3)=EL3
      CORR(I,J+9)=EL4
      CORR(I+9,J)=EL4
      DO 19 K=1,NRT
   19 CORR(K,K)=1.0
      CALL CHOLM (CORR,CHOL1,NRT,NRT)
C
C      BEGIN SAMPLING ITERATIONS
C
      DO 56 NITER=1,NUMSAM
      SEC=SECOND(SEC)
      PPPP=RANF(-1)
      PPPP=PPPP*999999
      SEC=PPPP/SEC
      SEC=(SEC*2.)+1.
      LLL=SEC
      CALL RANST (LLL)
      DO 20 I=1,NS
   20 MEANS(I)=RANN(-1)*SIGMAM
      DO 21 I=1,NS
      DO 21 J=1,NRT
   21 Z1(J,I)=RANN(-1)
      DO 22 J=1,NS
      DO 22 I=1,NRT
      Y1(I,J)=0.
      DO 22 K=1,NRT
   22 Y1(I,J)=Y1(I,J)+CHOL1(I,K)*Z1(K,J)
      DO 23 J=1,NS
      DO 23 I=1,NRT
      Y1(I,J)=Y1(I,J)*SIGMAD
   23 Y1(I,J)=Y1(I,J)+MEANS(J)
      DO 24 I=1,NT
      DO 24 J=1,NR
      DO 24 K=1,NMID
   24 Y2(I,J,K)=Y2(I,J,K)+MEANV1(J)
      DO 25 I=1,NT
      DO 25 J=1,NR
```

```
      DO 25 K=NMIDS,NS
   25 Y2(I,J,K)=Y2(I,J,K)+MEANV2(J)
      IF (NEST.EQ.1) GO TO 27
C             NON-CENTRAL ITEMS AS CROSSED
      DO 26 K=1,NT
      H(K,1,1)=(SIGMAV*RANN(-1))+DIFF
      DO 26 J=1,NR
      DO 26 I=1,NS
   26 H(K,J,I)=H(K,1,1)
      GO TO 29
C       NON-CENTRAL ITEMS AS NESTED
   27 CONTINUE
      DO 28 K=1,NT
      DO 28 J=1,NR
      H(K,J,1)=(SIGMAV*RANN(-1))+DIFF
      DO 28 I=1,NS
   28 H(K,J,I)=H(K,J,1)
   29 CONTINUE
C       DICOTOMIZATION PROCEDURE H(K,J,I) IS A QUANTILE AT WHICH DICTOMY
      DO 32 K=1,NT
      DO 32 J=1,NR
      DO 32 I=1,NS
      Y2(K,J,I)=Y2(K,J,I)-H(K,J,I)
      IF (Y2(K,J,I)-0.) 31,31,30
   30 Z2(K,J,I)=1.
      GO TO 32
   31 Z2(K,J,I)=0.
   32 CONTINUE
C
      DO 56 NTIM=1,2
      NTIME=NTIM
      IF (NTIME.EQ.1) GO TO 34
      DO 33 I=1,NTOT
   33 Z0(I)=Y0(I)
   34 CONTINUE
C
C         BEGIN THE ANALYSIS OF VARIANCE
C
      SUMALL=0.
      SQ=0.
      SQA=0.
      SQB=0.
      SQC=0.
      SQAB=0.
      SQAC=0.
```

```
      SQBC=0.
      SQABC=0.
      DO 35 I=1,NS
      DO 35 J=1,NR
      DO 35 K=1,NT
      SUMRT(I)=0.
      SUMST(J)=0.
      SUMSR(K)=0.
      SUMT(I,J)=0.
      SUMR(I,K)=0.
   35 SUMS(J,K)=0.
      DO 36 I=1,NS
      DO 36 J=1,NR
      DO 36 K=1,NT
      P=Z2(K,J,I)
      SUMALL=SUMALL+P
      SUMRT(I)=SUMRT(I)+P
      SUMST(J)=SUMST(J)+P
      SUMSR(K)=SUMSR(K)+P
      SUMT(I,J)=SUMT(I,J)+P
      SUMR(I,K)=SUMR(I,K)+P
      SUMS(J,K)=SUMS(J,K)+P
   36 SQ=SQ+P*P
      BY=Z2(1,1,1)
      BRRT=SUMRT(1)/RNRT
      BRST=SUMST(1)/RNST
      BRSR=SUMSR(1)/RNSR
      BRT=SUMT(1,1)/RNT
      BRR=SUMR(1,1)/RNR
      BRS=SUMS(1,1)/RNS
      BALL=SUMALL/RNTOT
      EFCT(1)=BRRT-BALL
      EFCT(2)=BRST-BALL
      EFCT(3)=BRSR-BALL
      EFCT(4)=(BRT-BRST)-EFCT(1)
      EFCT(5)=(BRR-BRSR)-EFCT(1)
      EFCT(6)=(BRS-BRSR)-EFCT(2)
      EFCT(7)=((BY+BRSR)-(BRR+BRS))-EFCT(4)
      EFCT(8)=BRS-BRST
      EFCT(9)=(BY+BRST)-(BRT+BRS)
      SQALL=SUMALL*SUMALL
      DO 37 I=1,NS
   37 SQA=SQA+SUMRT(I)*SUMRT(I)
      DO 38 J=1,NR
   38 SQB=SQB+SUMST(J)*SUMST(J)
```

```
      DO 39 K=1,NT
   39 SQC=SQC+SUMSR(K)*SUMSR(K)
      DO 40 I=1,NS
      DO 40 J=1,NR
   40 SQAB=SQAB+SUMT(I,J)*SUMT(I,J)
      DO 41 I=1,NS
      DO 41 K=1,NT
   41 SQAC=SQAC+SUMR(I,K)*SUMR(I,K)
      DO 42 J=1,NR
      DO 42 K=1,NT
   42 SQBC=SQBC+SUMS(J,K)*SUMS(J,K)
      SQALL=SQALL/RNTOT
      SQA=SQA/RNRT
      SQB=SQB/RNST
      SQC=SQC/RNSR
      SQAB=SQAB/RNT
      SQAC=SQAC/RNR
      SQBC=SQBC/RNS
      SSA=SQA-SQALL
      SSB=SQB-SQALL
      SSC=SQC-SQALL
      SSAB=SQAB-SQA-SQB+SQALL
      SSAC=SQAC-SQA-SQC+SQALL
      SSBC=SQBC-SQB-SQC+SQALL
      SSABC=SQ-SQAB-SQAC-SQBC+SQA+SQB+SQC-SQALL
      SSACNB=SSAC+SSABC
      SSCNB=SSC+SSABC
      RMSA=SSA/RMS
      RMSB=SSB/RMR
      RMSC=SSC/RMT
      RMSAB=SSAB/(RMS*RMR)
      RMSAC=SSAC/(RMS*RMT)
      RMSBC=SSBC/(RMR*RMT)
      RMSABC=SSABC/(RMS*RMR*RMT)
      RCNB=SSCNB/(RMT*RNR)
      RACNB=SSACNB/(RMS*RMT*RNR)
      RM(1)=RMSA
      RM(2)=RMSB
      RM(3)=RMSC
      RM(4)=RMSAB
      RM(5)=RMSAC
      RM(6)=RMSBC
      RM(7)=RMSABC
      RM(8)=RCNB
      RM(9)=RACNB
```

```
      DO 43 I=1,9                                                       GEN 3260
   43 IF (RM(I).LE.0.0) RM(I)=1.0E-100                                  GEN 3270
      RMSA=RM(1)                                                        GEN 3280
      RMSB=RM(2)                                                        GEN 3290
      RMSC=RM(3)                                                        GEN 3300
      RMSAB=RM(4)                                                       GEN 3310
      RMSAC=RM(5)                                                       GEN 3320
      RMSBC=RM(6)                                                       GEN 3330
      RMSABC=RM(7)                                                      GEN 3340
      RCNB=RM(8)                                                        GEN 3350
      RACNB=RM(9)                                                       GEN 3360
      FAC=RMSA/RMSAC                                                    GEN 3370
      FAN=RMSA/RACNB                                                    GEN 3380
      FB=RMSB/RMSAB                                                     GEN 3390
      QFBC=(RMSABC+RMSB)/(RMSAB+RMSBC)                                  GEN 3400
      QFBN=(RACNB+RMSB)/(RMSAB+RCNB)                                    GEN 3410
      FCC=RMSC/RMSAC                                                    GEN 3420
      QFBN=(RACNB+RMSB)/(RMSAB+RCNB)                                    GEN 3410
      FCC=RMSC/RMSAC                                                    GEN 3420
      FCN=RCNB/RACNB                                                    GEN 3430
      FABC=RMSAB/RMSABC                                                 GEN 3440
      FABN=RMSAB/RACNB                                                  GEN 3450
      FBC=RMSBC/RMSABC                                                  GEN 3460
      F(1)=FAC                                                          GEN 3470
      F(2)=FAN                                                          GEN 3480
      F(3)=FB                                                           GEN 3490
      F(4)=QFBC                                                         GEN 3500
      F(5)=QFBN                                                         GEN 3510
      F(6)=FCC                                                          GEN 3520
      F(7)=FCN                                                          GEN 3530
      F(8)=FABC                                                         GEN 3540
      F(9)=FABN                                                         GEN 3550
      F(10)=FBC                                                         GEN 3560
      RNU1=(RMSABC+RMSB)**2                                             GEN 3570
      RNU2=RMSABC**2                                                    GEN 3580
      RNU3=RMSB**2                                                      GEN 3590
      RNU2=RNU2/(RMS*RMR*RMT)                                           GEN 3600
      RNU3=RNU3/RMR                                                     GEN 3610
      RNU2=RNU2+RNU3                                                    GEN 3620
      RNU1=RNU1/RNU2                                                    GEN 3630
      NU1=(RNU1+.5)*10.                                                 GEN 3640
      NU1=NU1/10                                                        GEN 3650
      RNU1=(RMSAB+RMSBC)**2                                             GEN 3660
      RNU2=RMSAB**2                                                     GEN 3670
      RNU3=RMSBC**2                                                     GEN 3680
```

```
      RNU2=RNU2/(RMS*RMR)
      RNU3=RNU3/(RMR*RMT)
      RNU2=RNU2+RNU3
      RNU1=RNU1/RNU2
      NU2=(RNU1+.5)*10.
      NU2=NU2/10
      RNU1=(RACNB+RMSB)**2
      RNU2=RACNB**2
      RNU3=RMSB**2
      RNU2=RNU2/(RMS*RMT*RNR)
      RNU3=RNU3/RMR
      RNU2=RNU2+RNU3
      RNU1=RNU1/RNU2
      NU3=(RNU1+.5)*10.
      NU3=NU3/10
      RNU1=(RCNB+RMSAB)**2
      RNU2=RCNB**2
      RNU3=RMSAB**2
      RNU2=RNU2/(RMT*RNR)
      RNU3=RNU3/(RMR*RMS)
      RNU2=RNU2+RNU3
      RNU1=RNU1/RNU2
      NU4=(RNU1+.5)*10.
      NU4=NU4/10
      RRNU1=NU1
      RRNU2=NU2
      RRNU3=NU3
      RRNU4=NU4
      PRO1=FPROB(RRNU1,RRNU2,F(4))
      PRO2=FPROB(RRNU1,RRNU2,F(5))
      IF (NTIME.EQ.1) GO TO 45
      DO 44 J=1,10
      DO 44 I=1,12
   44 IF (F(J).GE.CNTILE(J,I)) JCOUNT(J,I)=JCOUNT(J,I)+1
   45 CONTINUE
      IF (NTIME.EQ.1) GO TO 47
      DO 46 I=1,12
      IF (PRO1.GE.PT(I)) LCOUNT(1,I)=LCOUNT(1,I)+1
   46 IF (PRO2.GE.PT(I)) LCOUNT(2,I)=LCOUNT(2,I)+1
   47 CONTINUE
      IF (NTIME.EQ.2) GO TO 55
      DO 48 I=1,12
      IF (PRO1.GE.PT(I)) NCOUNT(1,I)=NCOUNT(1,I)+1
   48 IF (PRO2.GE.PT(I)) NCOUNT(2,I)=NCOUNT(2,I)+1
      CALL COVAR (Z1,NRT,NS,XX,SX,NRT,NS)
```

```
      EFCT(10)=SX(1,2)                                                  GEN 4140
      EFCT(11)=SX(1,4)                                                  GEN 4150
      EFCT(12)=SX(1,7)                                                  GEN 4160
      EFCT(13)=SX(1,10)                                                 GEN 4170
      DO 49 J=1,10                                                      GEN 4180
      DO 49 I=1,12                                                      GEN 4190
   49 IF (F(J).GE.CNTILE(J,I)) COUNT(J,I)=COUNT(J,I)+1                  GEN 4200
      DO 50 I=1,9                                                       GEN 4210
      DO 50 J=1,9                                                       GEN 4220
   50 XXX(I,J)=XXX(I,J)+RM(I)*RM(J)                                     GEN 4230
      DO 51 I=1,9                                                       GEN 4240
   51 YYY(I)=YYY(I)+RM(I)                                               GEN 4250
      DO 52 I=1,19                                                      GEN 4260
      DO 52 J=1,13                                                      GEN 4270
   52 IF (EFCT(J).GE.POINT(I)) ICOUNT(I,J)=ICOUNT(I,J)+1                GEN 4280
      DO 53 J=1,NR                                                      GEN 4290
      DO 53 K=1,NS                                                      GEN 4300
   53 XX4(J,K)=Z2(1,J,K)+Z2(2,J,K)+Z2(3,J,K)                            GEN 4310
      CALL COVAR (XX4,NR,NS,XX5,SX5,NR,NS)                              GEN 4320
      DO 54 I=1,4                                                       GEN 4330
      DO 54 J=1,4                                                       GEN 4340
   54 SX6(I,J)=SX6(I,J)+SX5(I,J)                                        GEN 4350
   55 CONTINUE                                                          GEN 4360
   56 CONTINUE                                                          GEN 4370
      DO 57 I=1,4                                                       GEN 4380
      DO 57 J=1,4                                                       GEN 4390
      REAN=NUMSAM                                                       GEN 4400
   57 SX6(I,J)=SX6(I,J)/REAN                                            GEN 4410
      RN=RNR*RNR                                                        GEN 4420
      DO 58 I=1,NR                                                      GEN 4430
      DIAG=0.                                                           GEN 4440
   58 CM(I)=0.                                                          GEN 4450
      SQQ=0.                                                            GEN 4460
      AAL=0.                                                            GEN 4470
      CSQ=0.                                                            GEN 4480
      DO 59 I=1,NR                                                      GEN 4490
      DIAG=DIAG+SX6(I,I)/RNR                                            GEN 4500
      DO 59 J=1,NR                                                      GEN 4510
      AAL=AAL+SX6(I,J)/RN                                               GEN 4520
      CM(J)=CM(J)+SX6(I,J)/RNR                                          GEN 4530
   59 SQQ=SQQ+SX6(I,J)*SX6(I,J)                                         GEN 4540
      DO 60 I=1,NR                                                      GEN 4550
   60 CSQ=CSQ+CM(I)*CM(I)                                               GEN 4560
      CSQ=CSQ*RNR+2.                                                    GEN 4570
      RNUM=RN*(DIAG-AAL)*(DIAG-AAL)                                     GEN 4580
```

```
      RDEM=SQQ-CSQ                                                      GEN 4590
      RDEM=RDEM+(RN*AAL*AAL)                                            GEN 4600
      RDEM=RDEM*(RNR-1.)                                                GEN 4610
      EPS=RNUM/RDEM                                                     GEN 4620
      DO 61 I=1,13                                                      GEN 4630
   61 ICOUNT(20,I)=ICOUNT(19,I)                                         GEN 4640
      DO 62 I=1,18                                                      GEN 4650
      DO 62 J=1,13                                                      GEN 4660
   62 ICOUNT(20-I,J)=ICOUNT(19-I,J)-ICOUNT(20-I,J)                      GEN 4670
      DO 63 I=1,13                                                      GEN 4680
   63 ICOUNT(1,I)=NUMSAM-ICOUNT(1,I)                                    GEN 4690
      DO 64 I=1,9                                                       GEN 4700
      DO 64 J=1,9                                                       GEN 4710
      RHO(I,J)=REAN*XXX(I,J)                                            GEN 4720
      RHO(I,J)=RHO(I,J)-(YYY(I)*YYY(J))                                 GEN 4730
      VAR(I)=REAN*XXX(I,I)                                              GEN 4740
      VAR(I)=VAR(I)-(YYY(I)*YYY(I))                                     GEN 4750
      VAR(J)=REAN*XXX(J,J)                                              GEN 4760
      VAR(J)=VAR(J)-(YYY(J)*YYY(J))                                     GEN 4770
      VARR=VAR(I)*VAR(J)                                                GEN 4780
      VARR=SQRT(VARR)                                                   GEN 4790
   64 RHO(I,J)=RHO(I,J)/VARR                                            GEN 4800
      DO 65 I=1,9                                                       GEN 4810
   65 YYY(I)=YYY(I)/REAN                                                GEN 4820
      WRITE (61,77)                                                     GEN 4830
      WRITE (61,79)                                                     GEN 4840
      WRITE (61,81) NS,NR,NT,NUMSAM,NEST,SIGMAV,SIGMAD,SIGMAM,DIFF      GEN 4850
      WRITE (61,78)                                                     GEN 4860
      WRITE (61,83) NMID,NMIDS,NS                                       GEN 4870
      WRITE (61,78)                                                     GEN 4880
      WRITE (61,84) (MEANV1(J),J=1,NR),(MEANV2(J),J=1,NR)               GEN 4890
      WRITE (61,78)                                                     GEN 4900
      WRITE (61,85)                                                     GEN 4910
      WRITE (61,86) (A(I),I=1,9)                                        GEN 4920
      WRITE (61,87) (YYY(I),I=1,9)                                      GEN 4930
      DO 66 I=1,9                                                       GEN 4940
   66 VAR(I)=SQRT(VAR(I))                                               GEN 4950
      WRITE (61,88) (VAR(I),I=1,9)                                      GEN 4960
      WRITE (61,89)                                                     GEN 4970
      WRITE (61,86) (A(I),I=1,9)                                        GEN 4980
      DO 67 I=1,9                                                       GEN 4990
   67 WRITE (61,90) A(I),(RHO(I,J),J=1,9)                               GEN 5000
      WRITE (61,77)                                                     GEN 5010
      WRITE (61,79)                                                     GEN 5020
      WRITE (61,81) NS,NR,NT,NUMSAM,NEST,SIGMAV,SIGMAD,SIGMAM,DIFF      GEN 5030
```

```
      WRITE (61,80)                                                 GEN 5040
      WRITE (61,83) NMID,NMIDS,NS                                   GEN 5050
      WRITE (61,80)                                                 GEN 5060
      WRITE (61,84) (MEANV1(I),I=1,NR),(MEANV2(I),I=1,NR)           GEN 5070
      WRITE (61,82)                                                 GEN 5080
      WRITE (61,92)                                                 GEN 5090
      WRITE (61,91)                                                 GEN 5100
      WRITE (61,92)                                                 GEN 5110
      WRITE (61,93)                                                 GEN 5120
      WRITE (61,94)                                                 GEN 5130
      WRITE (61,97) FMT3(1),(COUNT(1,J),J=1,12)                     GEN 5140
      WRITE (61,97) FMT3(2),(COUNT(3,J),J=1,12)                     GEN 5150
      WRITE (61,97) FMT3(3),(NCOUNT(1,J),J=1,12)                    GEN 5160
      WRITE (61,97) FMT3(4),(COUNT(6,J),J=1,12)                     GEN 5170
      WRITE (61,97) FMT3(5),(COUNT(8,J),J=1,12)                     GEN 5180
      WRITE (61,97) FMT3(6),(COUNT(10,J),J=1,12)                    GEN 5190
      WRITE (61,95)                                                 GEN 5200
      WRITE (61,77)                                                 GEN 5210
      WRITE (61,79)                                                 GEN 5220
      WRITE (61,81) NS,NR,NT,NUMSAM,NEST,SIGMAV,SIGMAD,SIGMAM,DIFF  GEN 5230
      WRITE (61,80)                                                 GEN 5240
      WRITE (61,83) NMID,NMIDS,NS                                   GEN 5250
      WRITE (61,80)                                                 GEN 5260
      WRITE (61,84) (MEANV1(I),I=1,NR),(MEANV2(I),I=1,NR)           GEN 5270
      WRITE (61,82)                                                 GEN 5280
      WRITE (61,82)                                                 GEN 5290
      WRITE (61,91)                                                 GEN 5300
      WRITE (61,96)                                                 GEN 5310
      WRITE (61,93)                                                 GEN 5320
      WRITE (61,94)                                                 GEN 5330
      WRITE (61,97) FMT3(1),(COUNT(2,J),J=1,12)                     GEN 5340
      WRITE (61,97) FMT3(2),(COUNT(3,J),J=1,12)                     GEN 5350
      WRITE (61,97) FMT3(3),(NCOUNT(2,J),J=1,12)                    GEN 5360
      WRITE (61,97) FMT3(4),(COUNT(7,J),J=1,12)                     GEN 5370
      WRITE (61,97) FMT3(5),(COUNT(9,J),J=1,12)                     GEN 5380
      WRITE (61,95)                                                 GEN 5390
      WRITE (61,77)                                                 GEN 5400
      WRITE (61,79)                                                 GEN 5410
      WRITE (61,81) NS,NR,NT,NUMSAM,NEST,SIGMAV,SIGMAD,SIGMAM,DIFF  GEN 5420
      WRITE (61,80)                                                 GEN 5430
      WRITE (61,83) NMID,NMIDS,NS                                   GEN 5440
      WRITE (61,80)                                                 GEN 5450
      WRITE (61,84) (MEANV1(I),I=1,NR),(MEANV2(I),I=1,NR)           GEN 5460
      WRITE (61,82)                                                 GEN 5470
      WRITE (61,98)                                                 GEN 5480
```

```
      WRITE (61,99)
      DO 68 J=1,9
   68 WRITE (61,100) A(J),(ICOUNT(I,J),I=1,20)
      WRITE (61,77)
      WRITE (61,79)
      WRITE (61,81) NS,NR,NT,NUMSAM,NEST,SIGMAV,SIGMAD,SIGMAM,DIFF
      WRITE (61,80)
      WRITE (61,83) NMID,NMIDS,NS
      WRITE (61,80)
      WRITE (61,84) (MEANV1(I),I=1,NR),(MEANV2(I),I=1,NR)
      WRITE (61,82)
      WRITE (61,101)
      WRITE (61,99)
      WRITE (61,100) FMT4(5),(ICOUNT(I,10),I=1,20)
      WRITE (61,100) FMT4(6),(ICOUNT(I,11),I=1,20)
      WRITE (61,100) FMT4(7),(ICOUNT(I,12),I=1,20)
      WRITE (61,100) FMT4(8),(ICOUNT(I,13),I=1,20)
      WRITE (61,102) EPS
      WRITE (61,103).
      DO 69 I=1,4
   69 WRITE (61,104) (SX6(I,J),J=1,4)
      WRITE (61,32)
      WRITE (61,77)
      WRITE (61,79)
      WRITE (61,81) NS,NR,NT,NUMSAM,NEST,SIGMAV,SIGMAD,SIGMAM,DIFF
      WRITE (61,80)
      WRITE (61,83) NMID,NMIDS,NS
      WRITE (61,80)
      WRITE (61,84) (MEANV1(I),I=1,NR),(MEANV2(I),I=1,NR)
      WRITE (61,82)
      DO 70 I=1,10
   70 WRITE (61,72) (JCOUNT(I,J),J=1,12)
      WRITE (61,82)
      GO TO 10
   71 CONTINUE
      WRITE (61,105)
C
C
   72 FORMAT (10X,12I9//)
   73 FORMAT (46X,13HCENTILE CHECK,/)
   74 FORMAT (6X,12F9.5/)
   75 FORMAT (8F10.6/4F10.6)
   76 FORMAT (///40X,F20.10)
   77 FORMAT (1H1)
   78 FORMAT (//)
```

```
79 FORMAT (20X,4(10HNUMBER OF ),10H  ONE IF  ,3(10H STANDARD ),21H   GEN 5940
  1MEAN LEVEL OF ITEM,/20X,50H SUBJECTS  REPEATED    ITEM    SAMPLES GEN 5950
  2   NESTED  ,3(11H DEVIATIONS),20H  DIFFICULTY FOR A  ,/30X,10H MEAGEN 5960
  3SURES ,20X,10H  ZERO IF ,3(10H    OF      ),18H   STANDARD NORMAL,/6GEN 5970
  40X,10H  CROSSED ,43H  DICOTO     ERROR      TRUE         POPULATION GEN 5980
  50F,/,70X,10H  POINTS  ,2(10H  SCORES  ),14H      SUBJECTS,/)      GEN 5990
80 FORMAT (/)                                                          GEN 6000
81 FORMAT (15X,3I10,2X,I10,I8,5X,3(1X,F7.5,2X),6X,F7.5)                GEN 6010
82 FORMAT (///)                                                        GEN 6020
83 FORMAT (12X,118HTHE REPEATED MEASURE MEAN SCORES ANTECEDENT TO     GEN 6030
  1                       THE REPEATED MEASURE MEAN SCORES ANTECEDENT GEN 6040
  2TO,/11X,45HDICOTOMAZATION FOR SUBJECTS NUMBER 1 THROUGH ,I2,23X,34GEN 6050
  3HDICOTOMAZATION FOR SUBJECTS NUMBER,I3,8H THROUGH,I3)              GEN 6060
84 FORMAT (9X,4(2X,F8.5,2X),22X,4(2X,F8.5,2X))                         GEN 6070
85 FORMAT (33X,54HTHE MEANS AND STANDARD DEVIATIONS OF THE MEAN SQUARGEN 6080
  1ES ,/)                                                              GEN 6090
86 FORMAT (25X,9A10//)                                                 GEN 6100
87 FORMAT (13X,10H  MEANS   ,9(F9.3,1X)//)                             GEN 6110
88 FORMAT (14X,10HSTD. DEV.S,9F10.2//)                                 GEN 6120
89 FORMAT (46X,46HTHE INTERCORRELATIONS BETWEEN THE MEAN SQUARES,/)   GEN 6130
90 FORMAT (15X,A10,9(F8.3,2X)//)                                       GEN 6140
91 FORMAT (42X,36HTHE EMPIRICAL SAMPLING DISTRIBUTION ,/)              GEN 6150
92 FORMAT (39X,40HFOR ITEMS CROSSED WITH REPEATED MEASURES,/)         GEN 6160
93 FORMAT (115H FOR ALPHA =  .9        .8        .7        .6        .5 GEN 6170
  1      .4        .3        .2        .1       .05      .025      .01,/) GEN 6180
94 FORMAT (1X,117(1H-)/)                                               GEN 6190
95 FORMAT (/112(1H-)/112(1H-)/)                                        GEN 6200
96 FORMAT (38X,41HFOR ITEMS NESTED WITHIN REPEATED MEASURES,/)        GEN 6210
97 FORMAT (/A10,12(1X,I5,3X)/)                                         GEN 6220
98 FORMAT (39X,42HTHE EMPIRICAL DISTRIBUTIONS OF THE EFFECTS,/)       GEN 6230
99 FORMAT (13X,51H-1.0  -.9  -.8  -.7  -.6  -.5  -.4  -.3  -.2  -.1   GEN 6240
  1,53H 0    .1   .2   .3   .4   .5   .6   .7   .8   .9  1.0,/)       GEN 6250
100 FORMAT (5X,A10,20I5//)                                             GEN 6260
101 FORMAT (38X,42HEMPIRICAL DISRIBUTIONS OF THE CORRELATIONS,/)      GEN 6270
102 FORMAT (/40X,34HTHE VALUE OF THE BOX CORRECTION IS,F6.3/)         GEN 6280
103 FORMAT (47X,26HCALCULATED FROM THE MATRIX,/)                      GEN 6290
104 FORMAT (40X,4(F8.3,2X)//)                                         GEN 6300
105 FORMAT (55X,10HEND OF JOB)                                        GEN 6310
106 FORMAT (16I5)                                                     GEN 6320
107 FORMAT (8A10)                                                     GEN 6330
108 FORMAT (16F5.3)                                                   GEN 6340
    END                                                               GEN 6350
```

```
      SUBROUTINE FDIST (MM,NN,FX,PROBX)                                 DIS  110
C     CLARK HOLLOWAY AND W.B.CAPP, AUGUST 31,1959                       DIS  120
C     REVISED APRIL 1,1961    R.J.MCKELVEY                              DIS  130
      DIMENSION B(2)                                                    DIS  140
      NOUT=6                                                            DIS  150
      SF=0.0                                                            DIS  160
      SPROB=0.0                                                         DIS  170
      F=FX                                                              DIS  180
      PROB=PROBX                                                        DIS  190
      M=MM                                                              DIS  200
      N=NN                                                              DIS  210
      IF (F) 48,5,1                                                     DIS  220
    1 SF=F                                                              DIS  230
      IF (F-1.0) 2,2,4                                                  DIS  240
    2 XM=M                                                              DIS  250
      XN=N                                                              DIS  260
      LOW=1                                                             DIS  270
      FLO=F                                                             DIS  280
    3 PLO=0.0                                                           DIS  290
      DELTA=FLO/500.0                                                   DIS  300
      GO TO 11                                                          DIS  310
    4 FLO=1.0/F                                                         DIS  320
      XM=N                                                              DIS  330
      XN=M                                                              DIS  340
      LOW=0                                                             DIS  350
      GO TO 3                                                           DIS  360
    5 SPROB=PROB                                                        DIS  370
      IF (PROB) 48,48,6                                                 DIS  380
    6 IF (PROB-0.5) 7,7,9                                               DIS  390
    7 XM=M                                                              DIS  400
      XN=N                                                              DIS  410
      LOW=1                                                             DIS  420
      PLO=PROB                                                          DIS  430
    8 FLO=0.0                                                           DIS  440
      DELTA=PLO/200.0                                                   DIS  450
      GO TO 11                                                          DIS  460
    9 IF (PROB-1.) 10,48,48                                             DIS  470
   10 XM=N                                                              DIS  480
      XN=M                                                              DIS  490
      LOW=0                                                             DIS  500
      PLO=1.0-PROB                                                      DIS  510
      GO TO 8                                                           DIS  520
   11 FACTL=0.0                                                         DIS  530
      FACT=1.0                                                          DIS  540
      B(1)=(XM-2.0)/2.0                                                 DIS  550
```

```
      B(2)=(XN-2.0)/2.0                                                 DIS  560
      A=(XM+XN-2.0)/2.0                                                 DIS  570
      IF (A-0.2) 12,48,13                                               DIS  580
   12 FACT=0.31830989                                                   DIS  590
      GO TO 27                                                          DIS  600
   13 IF (A-0.7) 14,48,15                                               DIS  610
   14 FACT=0.5                                                          DIS  620
      GO TO 27                                                          DIS  630
   15 DO 21 I=1,2                                                       DIS  640
      IF (B(I)-0.7) 16,48,21                                            DIS  650
   16 IF (B(I)-0.2) 19,48,17                                            DIS  660
   17 FACT=FACT/0.886226925                                             DIS  670
   18 B(I)=1.0                                                          DIS  680
      GO TO 21                                                          DIS  690
   19 IF (B(I)+0.2) 20,48,18                                            DIS  700
   20 FACT=FACT/1.772453850                                             DIS  710
      GO TO 18                                                          DIS  720
   21 CONTINUE                                                          DIS  730
      IF (A-0.7) 25,48,22                                               DIS  740
   22 FACT=FACT*A/(B(1)*B(2))                                           DIS  750
      IF (FACT-99999999.) 23,27,27                                      DIS  760
   23 IF (FACT-1.0E-8) 27,27,24                                         DIS  770
   24 A=A-1.0                                                           DIS  780
      B(1)=B(1)-1.0                                                     DIS  790
      B(2)=B(2)-1.0                                                     DIS  800
      GO TO 15                                                          DIS  810
   25 IF (A-0.2) 27,48,26                                               DIS  820
   26 FACT=FACT*0.886226925                                             DIS  830
   27 FACTL=FACTL+ALOG(FACT)                                            DIS  840
      FACT=1.0                                                          DIS  850
      IF (A-0.7) 28,48,24                                               DIS  860
   28 Y1=FACTL+(XM/2.0)*ALOG(XM/XN)                                     DIS  870
      Y2=(XM-2.0)/2.0                                                   DIS  880
      Y3=(XM+XN)/2.0                                                    DIS  890
   29 F=DELTA/2.0                                                       DIS  900
      CUM=0.0                                                           DIS  910
C                                                                       DIS  920
   30 HFDL=Y1+Y2*ALOG(F)-Y3*ALOG(1.0+XM*F/XN)+ALOG(DELTA)               DIS  930
      IF (HFDL+20.) 31,32,32                                            DIS  940
   31 HFD=0.0                                                           DIS  950
      GO TO 33                                                          DIS  960
   32 HFD=EXP(HFDL)                                                     DIS  970
   33 CUM=CUM+HFD                                                       DIS  980
      F=F+DELTA                                                         DIS  990
      IF (F-FLO) 30,30,34                                               DIS 1000
```

```
34 IF (PLO) 48,39,35                                                  DIS 1010
35 IF (HFD) 48,38,36                                                  DIS 1020
36 IF (ALOG(PLO)-HFDL-4.604) 37,38,38                                 DIS 1030
37 DELTA=DELTA/2.0                                                    DIS 1040
   GO TO 29                                                           DIS 1050
38 IF (CUM-PLO) 30,39,39                                              DIS 1060
39 FLO=F-DELTA                                                        DIS 1070
   IF (SF) 48,43,40                                                   DIS 1080
40 F=SF                                                               DIS 1090
   IF (LOW) 48,42,41                                                  DIS 1100
41 PROB=CUM                                                           DIS 1110
   GO TO 46                                                           DIS 1120
42 PROB=1.0-CUM                                                       DIS 1130
   GO TO 46                                                           DIS 1140
43 PROB=SPROB                                                         DIS 1150
   IF (LOW) 48,45,44                                                  DIS 1160
44 F=FLO                                                              DIS 1170
   GO TO 46                                                           DIS 1180
45 F=1.0/FLO                                                          DIS 1190
46 PROBX=PROB                                                         DIS 1200
   FX=F                                                               DIS 1210
47 RETURN                                                             DIS 1220
48 GO TO 47                                                           DIS 1230
   END                                                                DIS 1240
```

```
      SUBROUTINE CHOLM (R,F,NP,N)                                       CHO  110
      DIMENSION R(N,N), F(N,N)                                          CHO  120
      DO 1 I=1,NP                                                       CHO  130
      DO 1 J=1,NP                                                       CHO  140
    1 F(I,J)=0.                                                         CHO  150
      DO 4 I=1,NP                                                       CHO  160
      K=I+1                                                             CHO  170
      F(I,I)=SQRT(R(I,I))                                               CHO  180
      DO 2 J=K,NP                                                       CHO  190
    2 F(J,I)=R(J,I)/F(I,I)                                              CHO  200
      K=I                                                               CHO  210
      DO 3 L=K,NP                                                       CHO  220
      DO 3 M=K,NP                                                       CHO  230
    3 R(L,M)=R(L,M)-F(L,I)*F(M,I)                                       CHO  240
    4 CONTINUE                                                          CHO  250
      RETURN                                                            CHO  260
      END                                                               CHO  270
```

```
      SUBROUTINE COVAR (Y,NP,NR,XX,SX,NV,NS)                            COV  110
      DIMENSION Y(NV,NS), YR(100), XX(NV,NV), SX(NV,NV)                 COV  120
      R=NR                                                              COV  130
      RR=R*R-R                                                          COV  140
      DO 1 I=1,NP                                                       COV  150
      YR(I)=0.                                                          COV  160
      DO 1 K=1,NR                                                       COV  170
    1 YR(I)=YR(I)+Y(I,K)                                                COV  180
      DO 3 I=1,NP                                                       COV  190
      L=I                                                               COV  200
      DO 3 J=L,NP                                                       COV  210
      XX(I,J)=0.                                                        COV  220
      DO 2 K=1,NR                                                       COV  230
    2 XX(I,J)=XX(I,J)+Y(I,K)*Y(J,K)                                     COV  240
      XX(I,J)=(R*XX(I,J)-YR(I)*YR(J))/RR                                COV  250
      XX(J,I)=XX(I,J)                                                   COV  260
    3 CONTINUE                                                          COV  270
      DO 4 I=1,NP                                                       COV  280
      DO 4 J=1,NP                                                       COV  290
    4 IF (XX(I,J).EQ.0.0) XX(I,J)=1.0E-100                              COV  300
      DO 5 I=1,NP                                                       COV  310
      L=I                                                               COV  320
      DO 5 J=L,NP                                                       COV  330
      SX(I,J)=XX(I,J)/(SQRT(XX(I,I)*XX(J,J)))                           COV  340
      SX(J,I)=SX(I,J)                                                   COV  350
    5 CONTINUE                                                          COV  360
      RETURN                                                            COV  370
      END                                                               COV  380
```

```
      FUNCTION FPROB(U,V,F)
      A=.5*U
      B=.5*V
      TEMP=B+A*F
      X=A*F/TEMP
      IF (X) 1,1,2
    1 FPROB=1.
      GO TO 7
    2 XC=B/TEMP
      AB=A+B
      CON=0.
      SGN=+1.
      IF (F-1.) 3,4,4
    3 TEMP=A
      A=B
      B=TEMP
      TEMP=XC
      XC=X
      X=TEMP
      CON=1.
      SGN=-1.
C     CONVERGENT SERIES EXPANSION - SEE AMS-55 PG. 944
    4 TOP=AB
      BOT=B+1.
      SUM=1.
      TERM=1.
    5 TEMP=SUM
      TERM=TERM*(TOP/BOT)*XC
      SUM=SUM+TERM
      TOP=TOP+1.
      BOT=BOT+1.
      IF (SUM-TEMP) 6,6,5
    6 FPROB=CON+SGN*EXP(A*ALOG(X)+B*ALOG(XC)+GAML(AB)-GAML(A)-GAML(B))*S
     1UM/B
    7 RETURN
      END
```

```
      FUNCTION GAML(A)
C     ...................................................................
C
C        FUNCTION GAML
C
C        PURPOSE
C           COMPUTES THE NATURAL LOGARITHM OF THE GAMMA FUNCTION
C           OF A NUMBER.
C
C        USAGE
C           G = GAML(A)
C
C        DESCRIPTION OF PARAMETERS
C           A - - - INPUT
C           G - - - NATURAL LOGARITHM OF THE GAMMA FUNCTION OF A
C
C        REMARKS
C           NONE
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C     ...................................................................
C
      W=A
      TEMP=0.
      IF (W-13.) 1,1,3
    1 N=14.-W
      TEMP=1.
      DO 2 I=1,N
      TEMP=TEMP*W
    2 W=W+1.
      TEMP=ALOG(TEMP)
    3 W2=W*W
      GAML=(.0833333333-(.00277777777-.000793650793/W2)/W2)/W+.918938533
     1-W+(W-.5)*ALOG(W)-TEMP
      RETURN
      END
```

```
         IDENT    RANN
         ENTRY    RANN,RANST
RANN     BSSZ     1
         SA2      RANDOM
         SA3      RANMLT
         SX0      60
         DUP      16,2
         DX2      X3*X2
         IX0      X2&X0
         AX0      4
         SA5      TW47
         IX0      X0-X5
         PX0      B0,X0
         NX0      X0
         BX7      X2
         SA5      SIGMA
         FX6      X5*X0
         SA7      A2
         EQ       RANN
RANST    BSSZ     1
         SA2      X1
         MX0      59
         NZ       X2,RR
         SA2      RAND
RR       PX5      B0,X2
         BX6      -X0&X5
         MX0      1
         BX6      -X0*X6
         SA6      RANDOM
         EQ       RANST
RAND     DATA 20001274321477413155B
RANDOM   DATA 20001274321477413155B
RANMLT   DATA 20000000000000553645B
SIGMA    DATA 16436733172720541145B
TW47     DATA 00004000000000000000B
         END
```