THESIS

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ACCURACY OF POPULATION PROJECTION MODELS
FOR LOCAL GOVERNMENTS IN MICHIGAN

BY

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ABSTRACT

ACCURACY OF POPULATION PROJECTION METHODS
FOR LOCAL GOVERNMENTS IN MICHIGAN

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Trend extrapolation is a popular method for population projection in local planning offices because it is simple and inexpensive. However, its accuracy is often open to question because users have no way to determine the quality of their projections. There is a need to identify the conditions and models that make trend extrapolation methods acceptable for planning practice.

Six extrapolation models are tested with data for 172 cities and towns in Michigan. The models were calibrated on population data from 1940 to 1970, and projections were made for 1980. To test accuracy, the projected populations were then compared with actual 1980 census data. Based on this accuracy testing, recommendations are proposed for selecting the most accurate models for various types of cities, according to their growth rate change.

The findings of the research have been used to develop a composite method that comprises the models yielding the minimum mean error for each of the various types of cities. In addition, a computer program has been developed using the composite method.

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INTRODUCTION

Introduction

Information about future population is essential to the functioning of a local planning office, because population projections provide quantitative yardstick of future demandes for public facilities and services. Reliable projections thus provide a foundation for sound planning decisions.

Population trend extrapolation techniques are popular methods of that extend past and present trends of population growth or decline against time into the future for projecting the total population. Although population trend extrapolation has weak theoretical foundations and lacks of component detail, projections are ppularly used as simple and inexpensive techniques of forecasting local population in the short-term (Greenberg, Michael and Krueckeberg, 1978).

The accuracy of such projections, however, is often open to question because users have few ways to determine the quality of their projections. Therefore, there is a need to identify the accuracy of widely used trend extrapolation techniques with certain city characteristics, to make them acceptable as planning models.

To meet this need, Isserman (1977) carried out accuracy tests of a wide range of extrapolation methods and communities. He used census data from 1930 to 1950 to project 1960 population and data from 1930 to 1960 to project 1970 population for townships in Illinois and Indiana. To reduce projection errors, he used different methods for different townships. Two characteristics were used to define the township types ; namely their population levels and growth rates (Isserman, 1977 : 253). The population growth pattern of cities in the United States, however, was changed over the 1970s in many places. The trend of migration into cities started to decelerate. Many Michigan cities, especially, showed of decrease in population growth rates during the 1970s. It is, therefore, necessary to retest and determine the accuracy of popular extrapolation techniques for use with the changed trends.

This study tested the accuracy of six popular trend extrapolation models with data from 172 cities and towns in Michigan. Three of the six models were selected from Isserman's study (1977) : the linear model, the exponential model and the double logarithmic regression model. The Gompertz model, the modified exponential model, and the second degree polynomial model were added to test whether or not they are appropriate for the changed population trends. Growth rate change was used to identify and designate types of cities.

The six models were calibrated on population data from 1940 to 1970, and projections made for 1980. The projections were subsequently compared with actual 1980 census data. The accuracy of each model was then examined using the cities' growth rate change classifications. Based on accuracy tests, a proposal is made to create a projection method by selecting specific models for different types of cities and towns, as no one model yielded the minimum mean error across city types. This method is called the composite method in this study.

Purposes of the Study

Usually, the local planning offices have a responsibility to provide population projection data for various purposes (Goodman and Freund, 1968 : 51). In many cases, however, a local planning office lacks data and funds, and has a limited amount of the time and expertise needed to project future population through sophisticated techniques (Isserman, 1977). In some cases, all that is riquired only a quick and rough future population projection, in which a certain range of projection error is acceptable (Pittenger, 1976 : 29).

To satisfy the limits, trend extrapolation models have been popular in local planning office because the models are simple and inexpensive. Local planners do not have any information about the accuracy of trend extrapolation models because there is no way to measure the quality of their projections by mathematical functions.

In two previous studies, using linear and exponential regression models,Schmitt and Crosetti (1953), and Greenberg (1972) provided information about projection accuracy for communities in general, but not for specific types of communities, (Isserman, 1977 : 253). Only Isserman (1975) did accuracy test with a wide range of extrapolation methods and communities. The hybrid methods which he developed are a mixture of extrapolation techniques to fit communities with particular characteristics called composite methods in this study.

Isserman tested accuracy using 1930 to 1970 census data. Until 1970, the trend of migration into the cities was seen throughout the United States. This trend of population migration, however slowed and changed, during the 1970s in Michigan (Rathge, Wang and Beegle, 1981 : 2). Only 19 out of 172 cities in this study show a net population decrease from 1940 to 1950. Population losses in 25 cities from 1950 to 1960, in 52 cities from 1960 to 1970, and in 101 out of 172 cities from 1970 to 1980 show net population decrease. These changes probably affected the accuracy of the extrapolation models, because trend extrapolation techniques are dependent on past population changes. One objective of this study is to deternube whether or not the accuracy of each model assessed by Isserman's test is still

reliable in Michigan under changed trends, and, if not, to find a model which is appropriate changed trends.

In addition to providing information about the accuracy of various extrapolation models for each type of city, another purpose of this study is to develop a micro-computer program for local planners. The program will make population projection easier, faster, less expensive, and probably more accurate than arbitrarily selecting an extrapolation model.

In summary, the three major purposes of this study are as follows :

(1) To provided information about the accuracy of the six major extrapolation models;

(2) To test the accuracy of each of the six models and developing the composite methods for minimizing error.

(3) To develop a micro-computer program of population projection for local planners to project population easily, fast, and inexpensively.

Data and Methodology

Cities and towns with a population over 2,500 were selected from 1970 census data. Data for 172 cities and towns were available, back to 1940. All of those cities were used for testing the accuracy of the six trend extrapolation models instead of using a sample of cities and towns in Michigan. The number of 172 excluded Detroit, because of its size inequity and probability that Detroit has different migration trends than other cites.

Population projections for the accuracy tests were carried out through unadjusted projection, rather than adjusted projection as in stepdown projections. For regression models, the point projection, which involves a parallel shift of line passing the last observed population point, were used as Isserman (1977) used it. He argued that the point projection may correct for accumulated errors in the regression lines.

The census data from 1940 to 1970 were used to project 1980 population for 172 cities and towns, using six trend extrapolation models. These projected populations were then compared to actual 1980 census data in order to estimate the percentage error as a measure of the accuracy of each model. The percentage error was calculated by the following equation.

$$\text{Percent Error} = [(\text{projected} - \text{actual}) / \text{actual}] \times 100$$

Average percentage errors were calculated for all cities and for a certain city type with the percentage errors between the actual population and the projected population Projected calculated by the six models. The mean percentage error for all of cities or for a city type were then compared among the six models to determine which model yielded the minimum mean percentage error for a certain type of city.

Because minimizing the mean percentage error is not adequate as a decision-making rule for choosing among alternative methods, standard deviation was used as the secondary criterion.

In order to develop the composite method, which is the method where different models are applied to different types of citis to minimize error, two characteristics were used to define the city or town types : growth rate change and population size. The growth rate change was calculated by the growth rate from 1960 to 1970, minus the growth rate from 1950 to 1960 in this study. The growth rate change was used as a major characteristic to classify cities, instead of the growth rate that Isserman (1977) used to classify the township types, because the growth rate change implies change of the past three point observations with one figure, while the growth rate represents just change of the past two point observations. For example, the growth rate and the growth rate change for each city can be calculated with census data up to 1970 by the following equation.

$$R1 = (P2 - P1) / P1$$

$$R2 = (P3 - P2) / P2$$

$$RC = R2 - R1$$

where P1 = 1950 population
P2 = 1960 population
P3 = 1970 population
R1 = growth rate from 1950 to 1960
R2 = growth rate from 1960 to 1970
RC = growth rate change between '50s and '60s

If the growth rate is used to classify city types, the value of R2 will be used as represent the populaton trend in 1960 and 1970. If the growth rate change is used, the value of RC will represent the value of R1 and R2. Consequently, the growth rate change implies the population trend in 1950, 1960 and 1970. Because population extrapolation models merely extend past population trends into the future, the depth of the historical periodic population data may influence projections. The deeper historical data may be the better indicator of city types. Therefore, in this study, seven types of cities were identified according to growth rate change.

Population size was considered a less important characteristic than growth rate change, because population size gives a less clear distinction of accuracy among the models. Isserman (1977 : 256) also came to the same conclusion about populatin size. Other characteristics were not considered, because Isserman tested with several characteristics of cities (Greenberg, Krueckeberg and Mautner : 16) and found that growth rate was the better criterion for discriminating between city types (Isserman, 1977).

The mean percentage error and standard deviation of percentage error drived from the six extrapolation models were compared for each type of city. Certain models were chosen as the best models for particular types of cities, according to minimum mean percentage error. If the mean error of two or more models was very close, the model that had the smallest standard deviation was considered best. A composite method was, then, developed comprising the best model for each type of city meaning that different methods are applied to different types of cities.

CHAPTER 2

POPULATION TREND EXTRAPOLATION MODELS

The basic notion of population extrapolation models and models used for extrapolation provide the theoretical foundation for this research. Therefore, basic notions of extrapolation and fundamental equations for each model are described briefly in this chapter.

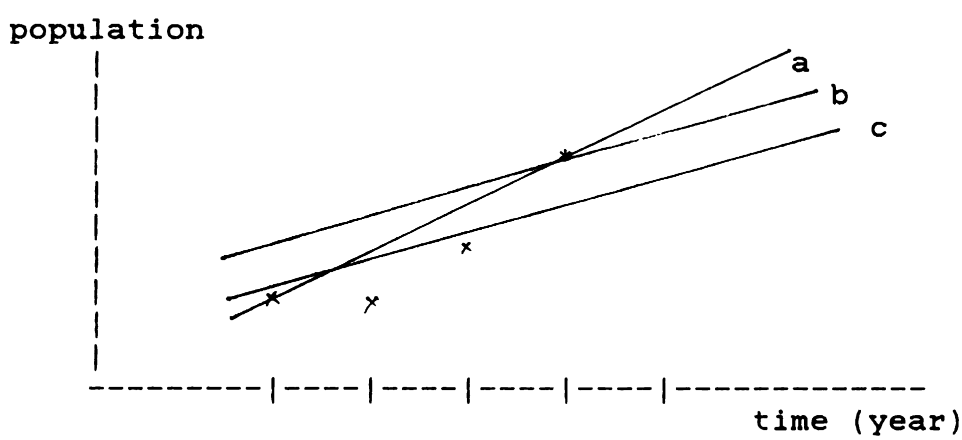
As mentioned before, population trend extrapolation models are simply the placement of lines or curves on a graph from past observed population points to the future. Therefore, extrapolation models involve nothing more than the extension of past trends over time. The lines or curves used to depict the past population trends can be interpreted by certain mathematical equations. The equation is a kind of extrapolation model.

All of these models deal with total population against time. They never deal with any other factors. Social, economic and political factors are not considered; nor are the details for sex-age breakdown or the three components of population (fertility, mortality and migration). The six most common extrapolation models will be described in this chapter.

Linear Extrapolation

The linear equation can be derived in two ways that are graphically presented here in figure 1. One (a in Figure 1) is by calculating the average population change in past time intervals and another (b and c in Figure 1) is by regressing past population data against time. The regression technique has two variations, namely, line projection and point projection which can also be applied to the exponential and the double logarithmic regression models.

A line projection (c in Figure 1) is simply a method that extends the regression line into the future. A point projection (b in the figure 1) ia a parallel shift of the



(Note) a = linear average model
b = linear regression model with point projection
c = linear regression model with line projection

FIGURE 1 : LINEAR EXTRAPOLATION MODELS

regression line of line projection, so that it passes through the last observed population point (Isserman, 1977 : 250). In this study, point projection was used for linear, exponential and double logarithmic regression. Isserman also used this technique for his accuracy test of extrapolation models and argued that it might correct for cumulated errors in the regression line (Isserman 1977 : 250). This seems reasonable ; the last observed population is the latest actual value, because it probably corrects the mathematical error up to the last observation point.

The linear regression model, with point projection, can be represented by the following mathematical equations (Isserman, 1977 : 259).

$$P_{t+n} = P_t + bn \qquad \text{............ (Equation 1)}$$

where : P = population
t = terminal year in historical data base
n = number of years from t to projection year
b = regression coefficient

The value of coefficient "b", that is drived from the least square method, is the slop. The slope represents the population growth increment per of unit of time. Therefore, the larger value of coefficient "b" indicates the larger net population growth per unit time as shown. The negative value of coefficient "b" means that the population decreases as the value of parameter "b" decreases per unit of time.

The linear regression model with point projection assumes that a population will grow or decrease by the same number of persons in each future time interval and the projection line will include the last observed population point.

Exponential Extrapolation

The exponential equation can be derived by caculating the average population growth rate in past time intervals or by regressing the logarithm of past population data against time. The exponential regression with point technique was used in this study for the same reason previously stated in the linear regression model with point projection. The exponential regression with point projection take following form (Krueckeberg and Silvers, 1974 : 262).

$$P_{t+n} = P_t (1+r)^n$$

where : P = population
t = terminal year in historical data
r = the rate of population change
n = the number of year from t to projection year calculated.

The (1+r) in the above equation can be replaced by "b", Thus, the equation can be transformed to logarithmic form as follows (Isserman, 1977 : 259) :

$$\ln P_{t+n} = \ln P_t + n \ln b \quad \text{......... (Equation 2)}$$

where : P = population
t = terminal year in historical data
r = the rate of population change
n = the number of year from t to projection year
ln b = regression coefficient

The exponential model equation is actually the same as the linear regression model with addition of the logarithm transformation of population. If the coefficient "ln b", calculated by the least square method, is positive, the population will grow at a constant rate per unit of time. If the coefficient "ln b" is negative, the population will decrease at a constantly decreasing rate over time. The larger the absolute value of coefficient "ln b" shown, the faster the growth or reduction rate.

The exponential regression model with point projection assumes that a population will increase or decrease at a constant rate over time, and the projected exponential curve will pass the last observed population point.

Double Logarithmic Extrapolation

Double logarithmic equations are derived by regressing logarithms of past population data against logarithms of time. The point projection technique was also used for this model. The formula is as follows (Isserman, 1977 : 259) :

$$\ln P_{t+n} = \ln P_t + b\,(\ln (t+n) - \ln t) \quad \text{... (Equation 3)}$$

where : P = population
t = terminal year in historical data base
n = number of years from t to projection year
b = regression coefficient

The double logarithmic equation is very similar to the exponential model with the addition of the logarithm transformation of time.

Modified Exponential Curve

The modified exponential curve is a curve of the exponential family of mathematical functions, which shows a continuously declining proportional population growth at constant rate, but a continuously increasing net population growth until the curve approaches an upper capacity limit. The modified exponential function can be presented as the following equation (Pittenger, 1976 : 67) :

$$P_{i+m} = k + a\,b^{m}$$

where : P = population
i = initial year in historical data base
m = number of years from i to the projection year
k = the upper limit or maximum population
a and b = parameter

In this equation, the parameter "a" is the difference between the value of "k" and the value of the population at origin, so the parameter "a" should be negative. The parameter "b" is the ratio of change, and the value of it should be positive and less than one (Pittenger, 1976 : 67). The parameters "k", "a" and "b" can be derived by the least square method, but a selected points technique from Croxton and Cowden (1945) was used to get two parameter in this study as Pittenger (1976) did.

The following equations are a summary of the selected points technique to get the values of parameters "k", "a" and "b". If Y1, Y2 and Y3 are the partial population totals

-- three equal groups of population data from initial year to terminal year -- and n is the number of elements comprising the partial total, then ;

$$b^n = (Y3 - Y2) / (Y2 - Y3)$$
$$\ln b = (\ln b^n) / n$$
$$a = (Y2 - Y1)(b-1) / (b^n - 1)^2$$
$$k = [Y1 - a(b^n - 1) / (b - 1)] / n$$

If parameter "b" is negative, then parameter "a" will be negative through the selected points method. In that case, the value of "b^m" in the equation will be changed in the negative and positive range according to the value of "m". If the value of "m" is even, the value of "b^m" will be positive and if the value of "m" is odd, then it will be negative. The projection curve with the negative value of the parameter "b" will produce the curve of a zig-zag form. It is not the modified exponential curve.

If parameter "b" is larger than one, then parameter "a" will be positive through the selected points technique. This means that the projected population is the upper limit plus the value of "ab^m", which is positive in the case of parameter "b" larger than one. Therefore, the projection curve does not show any upper limit population while the upper limit is the assumption of the modified exponential model.

If parameter "b" is zero or one, then the value of

"ab^m" becomes zero. In that condition, the projected population will be the same as the upper limit population "k" through time.

Therefore, it is necessary to satisfy the indispensable condition of the modified exponential model. The indispensible condition is that the parameter "b" is positive and less then one, and that the parameter "a" is negative. Graphically the curve looks like that shown in Figure 2.

The modified exponential curve assumes that there is some upper limit to the population, which may be set by zoning, subdivision control or any other public policy, and the population growth rate will decline at a constant ratio until it approaches the upper limit.

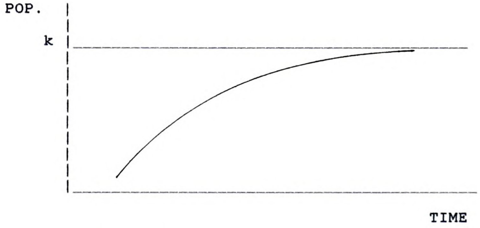


FIGURE 2 : MODIFIED EXPONENTIAL CURVE

S-Shape Curve

There are tow common "S" shaped exponential formulas, namely the Gompertz curve and the Logistic curve. The S-shape curves have a lower limit and an upper limit. The Gompertz curve was used for testing accuracy in this study.

By the S-shape curves, population growth starts from no net change in the early stage, and then begin to increase in rapid growth rate and net population and, at certain point, starts to decrease growth rate while still increasing net population, until there is finally no net change in population again at the upper limit. The mathematical formulas of the Gompertz curves are shown as following equations (Pittenger, 1976 : 57) :

$$P_{i+m} = k\, a^{b^{m}}$$

If the equation is transformed to logarithm form (Pitternger, 1976 : 58),

$$\ln P_{i+m} = \ln k + (\ln a)\, b^{m} \quad \text{..... (Equation 5)}$$

where : P = population
i = initial year in historical data base
m = number of years from i to the projection year
k = the upper limit or maximum population
a and b = parameter

The parameter "k", "a" and "b" was also derived in this study by the selected points method shown by Pittenger

(1976). The equation of the Gompertz curve is basically the same as the modified exponential model except for the logarithmic transformation of population. Therefore, the value of parameter "b" should also be positive and less than one and the value of "ln a" should be negative for the same reasons as described for the modified exponential model. The Gompertz curve's graphic equivalent is pictured in Figure 3.

The S-shape curve assumes that population growth begins slowly and gains momentum until it reaches an inflection point, after which it begins to decrease proportionately to population growth.

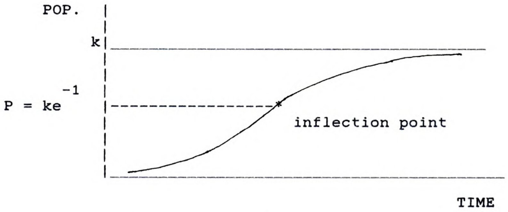


FIGURE 3 : GOMPERTZ CURVE

Polynomial Curve

The polynomial curve gives possibilities for the application of the various shape of curves. The curve shapes are dependent on the degree of the polynomial and the value of parameters. The general formula is represented as following (Pittenger, 1976 : 53) ;

$$P_{i+m} = a + bm + cm^2 + dm^3 + \ldots + qm^n \quad \ldots \text{(Equation 6)}$$

where : P = population
m = number of years from initial year to projection year
a, b, c, .. q = polynomial coefficients

In this study, the second and third degree of polynomial were tested. The second degree polynomial model was then selected to test the accuracy of the model, because

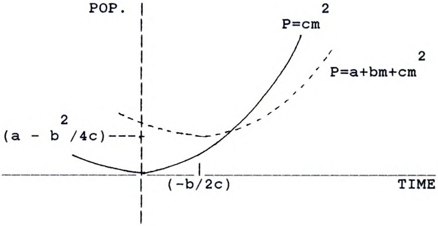


FIGURE 4 : THE SECOND DEGREE POLYNOMIAL CURVE

the second degree polynomial model showed less projection error in the Michigan case.

The coefficient "a", "b" and "c" were derived from the least square method. The second degree polynomial equation can be expressed to as following :

$$P_{i+m} = c\,(m + b/2c\,)^2 + (a - b^2/4c)$$

This formula is shown graphically in Figure 4.

The coefficient "c" decides the form of the curve. The value of "b/2c" shows the distance of the horizontal parallel shift of the equation cm^2 and the value of $(a + b^2/4c)$ shows the distance of the vertical parallel shift of the equation cm^2.

The polynomial model makes no assumption about growth pattern. It merely fits the curve to past population data and extends the curve into the future. But this model has more various curve forms than other models to fit past population data which are seldom linear through time.

CHAPTER 2

ACCURACY TEST

Accuracy for Overall Cities and Towns

Each of the six extrapolation models was used to project the 1980 population with census data from 1940 to 1970 for 172 cities and towns in Michigan. The projected population was then compared with the actual population in 1980 census data, and the percentage error for each of the cities and towns was calculated. To compare the accuracy of the six models, the mean and the standard deviation of the absolute value of percentage error were calculated.

The results of the accuracy tests are presented in Table 1. In the table, the positive value of the percentage errors represents the over-projection and the negative value represents the under-projection.

As shown in the Table 1, all six models tend to over-estimate the population of 1980 in Michigan. For instance, the 4.1 percent of the cities was under-projected more than 10 percentage with the linear model, otherwise, the 50.0 percent of the cities was over-projected more than 10 percent, although the linear model tend less to over-estimatation than the exponential and double logarithmic models. The three other models also show the tendency to over-estimation.

The linear model had the smallest mean percentage error and standard deviation. Conversely, the exponential regression model had the largest mean percentage error and standard deviation for the 172 tested cities.

The populations of 79 out of 172 cities in Michigan were projected within 10 percent of their actual values with the linear model, which was the highest percentage among four models. The linear model had the smallest range between the highest positive and the hightest negative percentage error. The range with the linear model is from -38.3 percent to 60.4 percentage error. Other models showed a more extreme range from the highest positive to the highest negative percentage errors. Through the analysis of

table 1

Frequency of Percentage Errors

for 172 Cities and Towns in Michigan

% ERROR	LIN	EXP	DLOG	POLY	GOM	MOD
25 +	36	57	57	40	4	2
10 TO 25	50	51	51	35	24	24
-10 TO 10	79	59	59	71	45	44
-25 TO -10	6	4	4	21	8	6
< -25	1	1	1	5	0	0
MEAN ERROR	16.6	34.9	34.5	18.4	10.6	10.3
SD.	14.7	77.9	76.2	17.4	7.9	7.8
TOTAL CITY	172	172	172	172	81	76

mean percentage error and standard deviation, we find that the linear model seems to be relatively the best model among these four models for 172 cities.

The Gompertz and the modified exponential model were not applicable to all cities because the two models require a certain indispensible condition. The indispensible condition is that the parameter "b" in the equations 4 and 5 should be positive and less than one, while the parameter "a" in the equations should be negative. The accuracy of the two models was tested separately for those types of cities. The past population growth pattern of the 81 of the 172 cities satisfied the indispensable condition for the Gompertz model, 76 of the 172 cities satisfied the modified exponential model. The projected 1980 population for those cities, using these two models, war relatively accurate.

The Gompertz model yielded 10.6 mean percentage error, with 7.9 standard deviation of percentage errors. The populations of 45 out of 85 cities were projected within a 10 percent error. The modified model had 10.3 mean percentage error, with 7.8 standard deviation of percentage error. The populations of 44 out of 81 cities were projected within 10 percentage error. These models also showed a relatively small range from the highest positive to the highest negative percentage error. The error range with the Gompertz model was from -21.5 to 33.5 percent and the range with the modified exponential model was from -21.4 to

41.6 percent.

These two models showed more accurate population projection than the linear model for specific cities, but require a certain condition not met by all cities.

Overall, this accuracy test for all cities showed that no model was best for all cities and towns, although the linear model and the second degree polynomial model yield comparatively better results than others. Some models are better suited for particular types of cities, such as the Gompertz and the modified exponential models, which produced more accurate projections for a particular type of past population growth pattern.

City Types According to Growth Rate Change

All of the extrapolation models project future population dependant only on past population growth trend. Therefore, the past population growth pattern inherently affects the accuracy of an extrapolation projection model.

In this section, it is assumed that different population trend extrapolation models can be applied to different types of cities classified by past growth rate changes. The hypothesis will be tested with six models.

To classify past population growth patterns for cities and towns in Michigan, changes in the population growth rate during two past time spans were used ; 1950 to 1960 and 1960 to 1970. Thus growth rate change implies the population trends in these two time spans, although it does not give any information about net population change or proportional change during the last span period. The value of the growth rate change is calculated by the growth percentage from 1960 to 1970, minus the growth percentage from 1950 to 1960. As mentioned before, the value implies the population changes over the past three observation points, while the growth rate or net population change shows just the population changes over the last two observation points. That means the growth rate change characteristic can be used for simplification of past growth patterns while it implies deeper past population trends. The value of the growth rate change indicates the degree of growth rate change. A positive value indicates a degree of growth rate increase and a negative value indicates a degree of decrease of the proportional population growth.

In this study, seven types were clasified according to past growth rate changes between the 1950s and 1960s as follows : (1) Great increase type in which percentage growth rate changes ranged more than 25 percent between the years from 1960 to 1970 and the years from 1950 to 1960; (2) Medium increase type in which the percentage growth rate changes ranged from 10 to 25 percent; (3) Moderate increase type in which the percentage growth rate changes ranged from 0 to 10 percent difference; (4) Moderate decrease type in which the percentage growth rate changes ranged from -10 to 0 percent; (5) Medium decrease type in which the percentage growth rate changes ranged from -25 to -10 percent; (6) Great decrease type in which the percentage growth rate changes ranged from -50 to -25 percent; (7) Extreme decrease type which the percentage growth rate changes were less then -50 percent.

If a local planner can roughly estimate the future growth rate for a city, he may obtain better accuracy by selecting a model according to type of future population growth, because the future population changes actually affect the accuracy of the population projections. To improve the accuracy of projection, each of the seven types was classified again by the growth rate change between 1960 to 1970 and 1970 to 1980. Growth rate changes for 1960 to 1970 and 1970 to 1980 were calculated in the same way as previously used. The actual census population data of 1980 were used to get the growth rates from 1970 to 1980 for each city, because accuracy information should be provided through known rather than uncertain data. It is assumed that the future growth rates can be perfectly anticipated. Accordingly, three types of the future growth rate changes were classified and the accuracy of the six models was tested for each of the three types.

The three types of future growth rate changes are : (1) a heavy increase in which the changes in percent growth rate are larger than 10 percent. (2) a light change in which the percentage growth rate changes ranged from -10 to 10 percent. (3) a heavy decrease that has the percentage growth rate change of less than -10 percent.

From here on, changes of percentage growth rate from 1950 to 1960 and 1960 to 1970 will be termed as "the past rate change", and "the future rate change" will designate the growth rate changes from 1960 to 1970 and 1970 to 1980. In following sections, accuracies are first tested with the seven types of the past rate change. Then the accuracies for the three types of future rate change in each of the seven types of past rate change are tested to provide more detailed information on each models' accuracy and to develop composite methods.

Great Increase in Growth Rate

Cities were classified for this type by past rate changes which were larger than 25 percent. For instance, the population of Saline in Pashtenaw county grew at a 52 percent growth rate from 1950 to 1960 and grew at a 106 percent growth rate from 1960 to 1970. Thus growth rate change, the past rate change, is 54 percent. Therefore, Saline falls into this category.

Only seven towns were of the great increase type in Michigan. While it is statistically difficult to test which model is reliable for this type on the basis of only seven cases, the fitness of a model to this pattern may be logically described by combining the results of this analysis and the assumption of each model.

All seven cities did not meet the indispensable condition for the Gompertz or the modified exponential models. The linear model showed a better mean percentage error and standard deviation than other models (see Table 2). Just 2 cities had a percentage error within 10 percent. That means the majority of cities projected had large errors. Therefore, the linear model should be used for this type, with some care. For a more detailed classification, this type can be classified again with the future rate

Table 2

Frequency of Percentage Errors for
Cities of Great Increase Type in Michigan

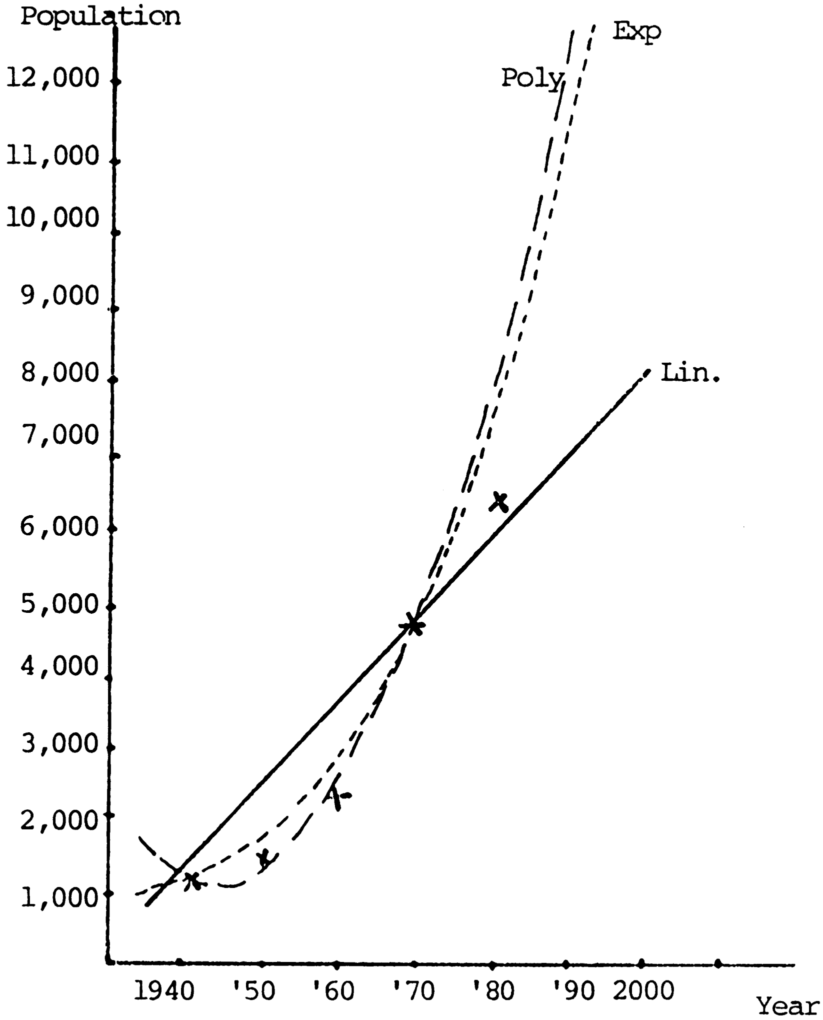
% ERROR	LIN	EXP	DLOG	POLY
25 +	2	3	3	4
10 TO 25	1	2	2	3
-10 TO 10	2	2	2	0
-25 TO -10	2	0	0	0
< -25	0	0	0	0
MEAN ERROR	19.7	26.0	25.8	33.2
SD.	14.2	21.2	21.0	14.8
TOTAL CITY	7	7	7	7

change, but the population growth rate of all seven cities was decreased more than 10 percent from 1960 to 1970 and 1970 to 1980. Therefore, the results would be the same as the results for the great increase type of past rate change.

The projection line and curves can be analyzed as an example of this type. The accuracy of each model can then be reviewed. City of Saline, a case in Pashtenaw county,can serve as an illustration of this type. Saline showed 52 percent population growth from 1950 to 1960, 106 percent growth from 1960 to 1970 and 54 percent growth from 1970 to 1980. With the pre-1970 data, all of the exponential family models draw projection curves with fast growth rates in the future, while the linear model draws a projection line with the same net population per decade in the future.

Actually, the population of Saline grew more than the average population, but the growth rate declined during the 1970s. Therefore, the linear model yields a slight under-estimation and other exponential family models yield over-estimations for the Saline population of 1980, as represented in Figure 5. If the rapid growth rate had continued during 1970s in Saline, the exponential or the double logarithmic models would probably have produced better projections than the linear model.

As an other example, three cities of this type in Michigan showed net population decrease during 1970s,



Linear Regression Model

Exponential & Double Logarithmic Model

Second Degree Polynomial Model

Actal Population

FIGURE 5 : Example of the Great Increase Type
(City of Saline)

although rapid population growth had been shown earlier. All extrapolation models yielded high over-estimation for those cities. In this case, it is probably better to use another method rather than any of the trend extrapolation models.

The exponential and double logarithmic models seemed to be more suitable for cities with continuous growth rate increase in the future, and the linear model seemed to be more suitable for cities in which net population will grow at a declining rate in the future. All of the seven cities in this type had declining growth rates during the 1970s. In conclusion, the linear model seems to be the best model for this type in Michigan cases.

To summarize, the linear model can generally be used for cities with great increases in growth rates although it did not yield satisfactory results in this accuracy test. If a planner can roughly estimate future growth rate, he can apply the exponential model to cities with future rate changes of more than 10 percent, the double logarithmic model to cities with future rate changes between -10 to 10 percent, and apply the linear model to cities with the future rate change of less than -10 percent. However, a planner could better project population on the basis of local knowledge in cases where the population grew quickly in the past, but net population is expected to decline in the future.

Medium Increase In Growth Rate

This type of city has the past rate change that range from 10 to 25 percent. For example, the population of Grand Rapids increased about 0.5 percent from 1950 to 1960, and increased again about 11.5 percent from 1960 to 1970. The growth rate change between the 1950s and the 1960s was thus 11 percent (11.5 percent - 0.5 percent = 11 percent). Therefore, Grand Rapids is a city of medium increase in growth rate.

Cities of this type comprised nine cases in Michigan. It is difficult to test which model is best for this type of city with only nine cases, but an accuracy test with nine cases can indicate some tendencies of each model.

With the nine cases of this type, the linear regression model showed the minimum percentage error and the highest percentage of cities within a 10 percent error. (See Table 3) The mean percentage error with the linear model is 11.2 percent and the populations of six out of the nine cities were projected within a 10 percentage error. As a result, it seems reasonable to use the linear model as a projection model for this type. The exponential model and the double logarithmic model are the next best models for this type. The two models yielded more over-estimation of population for 1980 than the linear model. None of cities met the indispensible condition for the Gompertz and the modified exponential model.

If the future rate changes were considered for this type, the six cases in the nine cities showed negative growth rate changes more than 10 percent. The mean percentage error was 7.7 percent with the linear model. Otherwise, the mean percentage error was 14.7 percentage error with the exponential model, 14.5 percentage error with the double logarithmic model and 22.3 percentage error with the polynomial model. Just one city had a future rate change from -10 to 10 percent. For that city, the polynomial model yielded minimum error (29.4 percent under-estimation), but the result is not significant with one case. Two cities had future rate change more than 10 percent and the linear model yielded the minimum mean error

Table 3

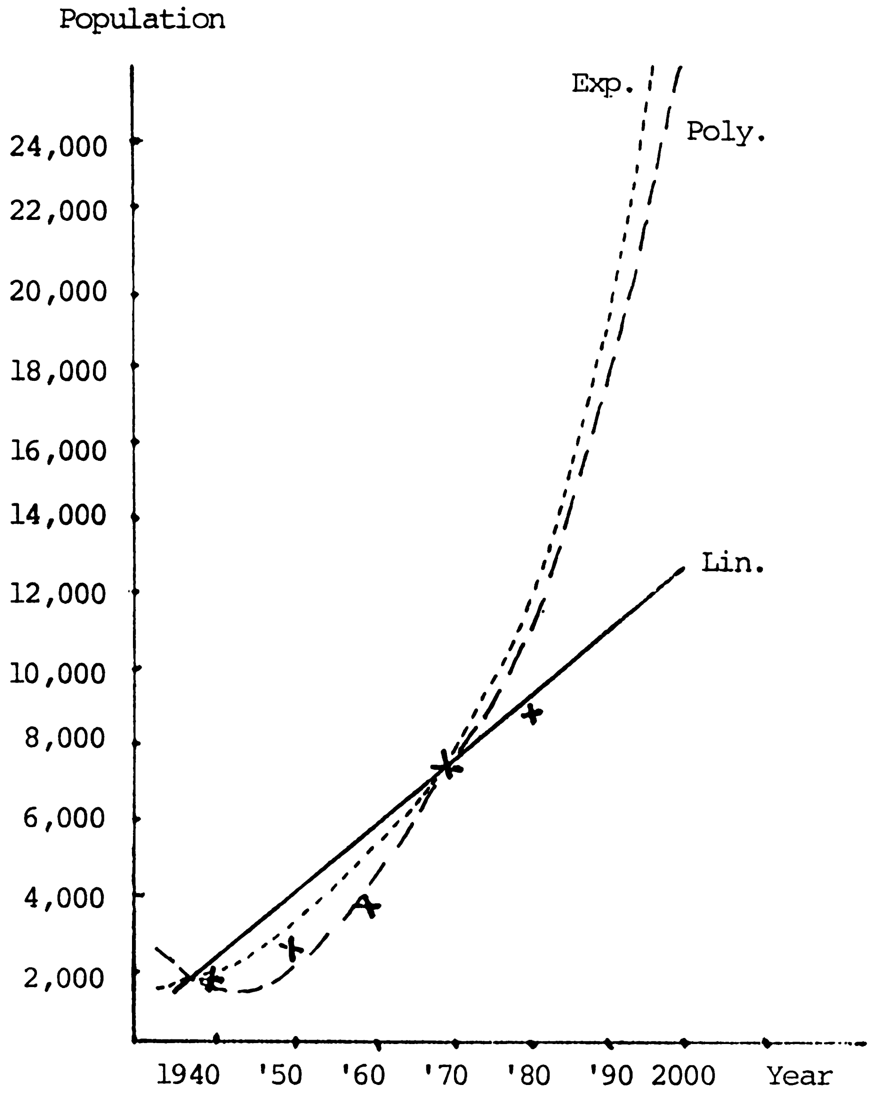
Frequency of Percentage Errors for
Cities of Medium Increase Type in Michigan

% ERROR	LIN	EXP	DLOG	POLY
25 +	0	1	1	2
10 TO 25	1	3	3	4
-10 TO 10	6	3	3	1
-25 TO -10	1	1	1	1
< -25	1	1	1	1
MEAN ERROR	11.2	14.9	14.7	20.0
SD.	10.8	10.1	10.1	9.8
TOTAL CITY	9	9	9	9

for these cities. In conclusion, the linear model can be considered as the best among the six models for this type of future rate change, with less than 10 percent in the medium increase type.

The city of Flushing in Genesee county is an illustration of the medimum increase type. In the Flushing case, the population grew 69 percent from 1950 to 1960, and grew 91 percent from 1960 to 1970. The growth rate change was thus 21 percent, which is the medimum increase type.

In the Flushing case, the exponential model and the double logarithmic model extended projection curves with the assumption that the growth rate would be kept continuous in the future as past data showed. But the actual growth rate from 1970 to 1980 did not increased, although the net population increased. As a result, the exponential model and the double logarithmic model produced over-estimations for the Flushing population of 1980 as presented in Figure 6. Otherwise, the linear regression line assumes that the same amount of the population will increase as much as the number of people estimated by regression per unit of time. The actual population increment from 1970 to 1980 was very close to the predicted number by linear regression in the Flushing case. If the population of Flushing increased rapidly with the growth rate, as in the past, the exponential model might be better for that case than the linear model.



Linear Regression Model

Exponential & Double Logarithmic Model

Second Degree Polynomial Model

X Actual Population

FIGURE 6 : Example of the Medium Increase Type
(City of Flushing)

In summary of the accuracy test for the medium increase type, the linear model can be used with relatively better accuracy for this type of city. If a planner can predict rough growth rate into the future, he can apply the exponential model for the type of the future rate change of more than 10 percent, and the linear model can be still used for the rest of the cities in this type.

But a planner has to execise caution in using this extrapolation model for cities which showed a continuous increase or decrease of population in the past, and the population trends are expected to be reversed in the future, when the net population decreases or the increases suddenly.

Moderate Increase in Growth Rate

This type of city has the past rate change that ranged from zero to 10 percent. For example, the population of Ann Arbor showed a 40 percent increase from 1950 to 1960 and a 48 percent increase from 1960 to 1970. The growth rate change between the 1950s and the 1960s is 8 percent. Therefore, Ann Arbor is a city of this type.

27 cities in Michigan were categorized in this class. Except the Gompertz and the modified exponential model, the mean percentage errors with four models are ranged from 10.1 to 15.2 percent error. The mean errors seem to be satisfiable as the degree of accuracy with the extrapolation models.

The linear model yielded the minimum percentage error and the minimum standard deviation of percent error for this type (See Table 4). The mean percentage error is 10.1 percent and 17 out of 27 cities were projected within 10 percent of their actual census populations. The polynomial model comes out as the next best model from the accuracy test.

No city satisfied the indispensible condition of the Gompertz model or the modified exponential model for this type. The exponential model and the double logarithmic model yielded extreme percentage errors in some cities,

Table 4

Frequency of Percentage Errors for
Cities of Moderate Increase Type in Michigan

% ERROR	LIN	EXP	DLOG	POLY
25 +	3	5	5	5
10 TO 25	7	7	7	6
-10 TO 10	17	14	14	13
-25 TO -10	0	1	1	2
< -25	0	0	0	1
MEAN ERROR	10.1	15.2	15.1	14.2
SD.	9.4	18.2	18.0	13.7
TOTAL CITY	27	27	27	27

compared with the linear model. The highest error with the exponential model was 80.5 percent error, while 39.1 percent error was the highest with linear model. The two models also have a higher mean percentage error than the linear model.

It is useful to consider the accuracies for this type with the future rate changes. There are two cases for the type over 10 percent of the future rate changes (heavy increase type of the future rate change), 14 cases for the type with the future rate change ranged from -10 to 10 percent (light change type), and 11 cases for the type with the future rate change less than -10 percent (heavy decrease type).

For the heavy increase and the light change types of the future rate change, the double logarithmic model and the exponential model yielded the minimum percentage error. The exponential model showed slightly better results in this analysis.

The mean percentage errors are 4.3 percent with the two models for the light change type of the future rate change, while it is 4.9 percent with the linear model and 5.2 percent with the polynomial model. The populations of 13 out of 14 cities in the light change type were projected within 10 percent error with the linear model, the exponential model and the double logarithmic model. The polynomial model projected population of 11 out of 14 cities

within a 10 percent error. All of the four models project the population for this type relatively very well.

For the 11 cities of the heavy decrease type in the future rate change, the linear model turned out as the best model among four models. But the mean percentage error is a little high at 16.5 percent, and the standard deviation of percent error is not good, either. Only two out of 11 cities are projected within 10 percent error. Other models produced a quite high mean percentage error, for example, 30.1 percent with the exponential model and 24.5 percent with the polynomial model.

Each model can be discussed with a graphic illustration. The Ann Arbor case was used as an example of the moderate increase type in the past rate change. The population of Ann Arbor grew with 40 percent growth rate from 1950 to 1960, increased with 48 percent growth rate from 1960 to 1970, and increased with 8 percent growth rate from 1970 to 1980.

Because the similar curves were depicted by the exponential model and the double logarithmic model, the double logarithmic model only was represented in Figure 7. As depicted in Figure 7, the double logarithmic model, the exponential model and polynomial model keep the fast growth rate into the future as like the past population trend until 1970. Those curves are very well fitted to the past population points. However, the actual population growth

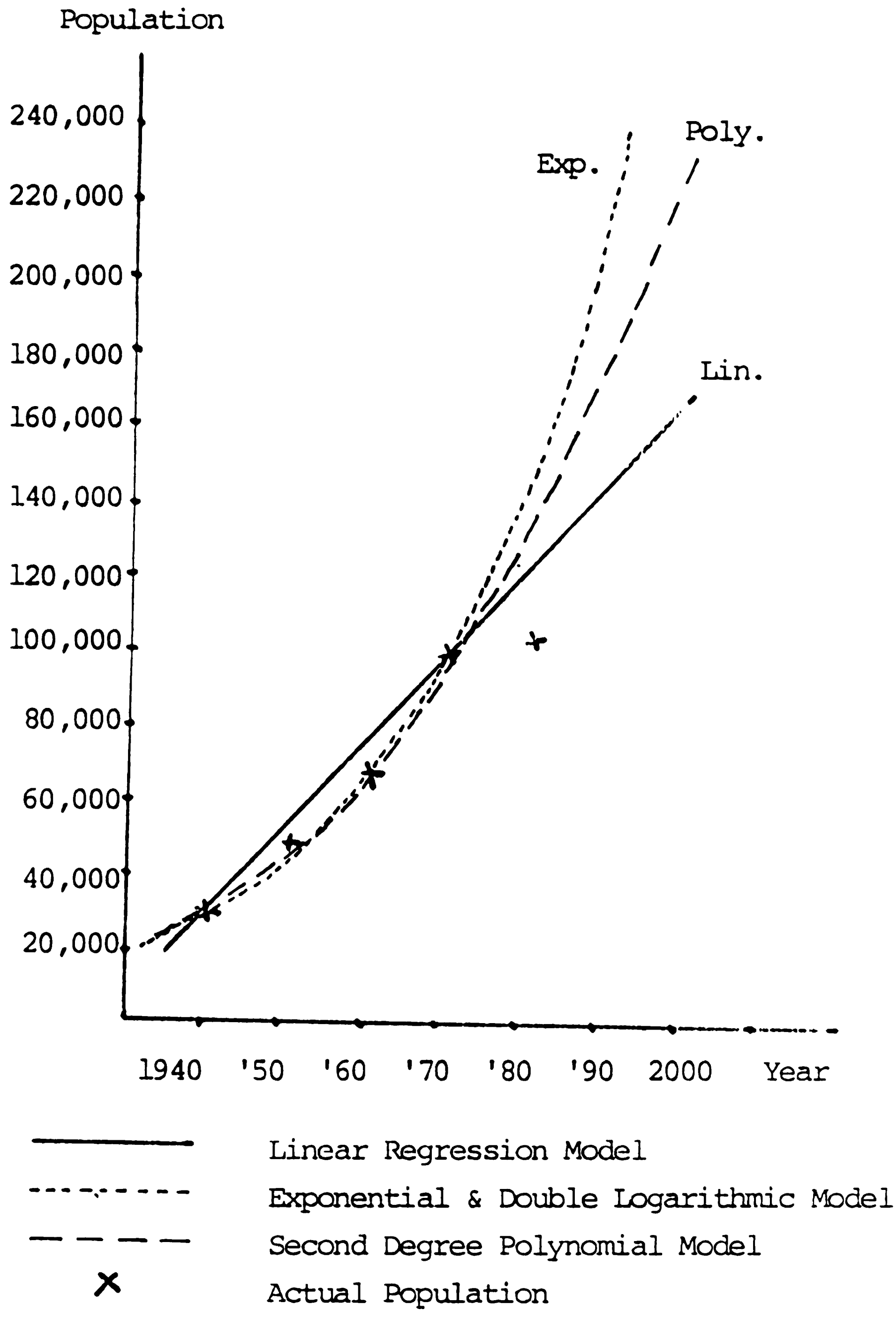


FIGURE 7 : Example of the Moderate Increase Type
(City of Ann Arbor)

rate decreased during 1970s. The future rate change for Ann Arbor is -38 percent. Therefore, the three models yielded high over-estimation for the population of 1980.

The linear model projected population with the same population increment calculated by regression. With the population increment about 22,903 persons per decade, the linear model extends the line into future. But the 8,169 persons actually increased during the 1970s in Ann Arbor. The linear model also produced over-estimation. The linear model projected population more accurately for Ann Arbor than other models did, however.

The city of Durand in Shiawassee county has a different future rate change (9 percent) than the future rate change (-38 percent) of Ann Arbor, though the past population patterns of the two cities are similar. The population of Durand grew 3.7 percent during the 1950s, 11.0 percent during the 1960s and 15.3 percent during the 1970s. In this case, the four models produced under-estimations because the actual population grew with a faster growth rate than expected by the extrapolation models.

The polynomial model was best for Durand. It drew a steeper curve than the exponential model because it reflected the rapid growth rate increase from the 1950s to the 1960s. The exponential model and the double logarithmic model were the next best models for the city of Durand. Those models were better than the linear model in the Durand

case. The exponential model is better for the cities that are expected to grow or decrease continuously while keeping the past growth rate, because the exponential model projects population with the same growth rate while the linear model projects population with the same amount of population growth. The linear model is better for cities that will grow or decrease with approximately the same increments of net population as estimated by the linear regression. Because of these reasons, the linear model is better in Ann Arbor and the exponential model is better in Durand.

In summary of the accuracy test for the moderate increasing type, it can be concluded that the linear model is generally best for this type. If a local planner can roughly estimate growth rate in future, he can use the exponential model for the heavy increase type and the light change type. If the future rate changes are expected to be less than -10 percent, then he can still use the linear model. He should be careful to use those models for the cities that are expected to decrease or increase suddenly as the opposite direction with the past trends during the next time span, as illustrated with Ann Arbor.

Moderate Decrease in Growth Rate

The cities of this type were categorized by the past rate changes which are ranged from -10 to zero percent. The past rate changes were calculated by the percent growth rate from 1960 to 1970 minus the percent growth rate from 1950 to 1960.

There are 54 cities of this type in Michigan. That is the largest number among seven types of past rate changes. The 54 cases may be statistically enough to generalize the results of this accuracy test.

The linear model and the polynomial model have minimum mean percentage error. The mean percentage error with the polynomial model is slightly lower than with the linear model. The mean percentage error is 9.097 percent with the polynomial model, and 9.128 percent with the linear model. Therefore, it can be regarded that the two models actually yield the same result in the aspect of mean error. As another aspect of the accuracy measure, the standard deviation of the percentage error can be considered to compare the accuracy of two models. The standard deviation of the percent errors with the linear model is 6.5, and 7.5 with the polynomial model (See Table 5). The linear model shows a better result in the aspect of error distribution.

The highest error with the two models is considered the accuracy measure. The highest error with the linear model was -18.0 precent error in the aspect of under-estimation and 30.2 percent error in the aspect of over-estimation, while there was a -27.7 percent error of under-estimation and 29.6 percent error of over-estimation with the polynomial model. The linear model is slightly better in

the viewpoint of error range, too. Therefore, the linear model was considered the best model for this type in this study.

Other models also show good accuracy for this type. The exponential model produces 10.1 mean percentage error and the double logarithmic model shows 10.0 mean percentage error. 31 cities for the Gompertz model and 30 cities for the modified exponential model out of 54, are satisfied with the indispensible condition of the two models. The Gompertz model shows 7.5 mean percentage error and the modified exponential model shows 7.0 mean percentage error. The populations of 24 in 31 cities were projected within 10

Table 5

Frequency of Percentage Errors for
Cities of Moderate Decrease Type in Michigan

% ERROR	LIN	EXP	DLOG	POLY	GOM	MOD
25 +	2	3	3	4	0	0
10 TO 25	14	20	20	7	5	4
-10 TO 10	36	29	29	35	24	24
-25 TO -10	2	2	2	6	2	2
< -25	0	0	0	2	0	0
MEAN ERROR	9.1	10.1	10.0	9.1	7.5	7.0
SD.	6.5	7.5	7.4	7.5	6.7	5.9
TOTAL CITY	54	54	54	54	31	30

percent error with the Gompertz model and 24 in 30 cities with the modified exponential model. It seems best to use the modified exponential model or the Gompertz model for these types of cities if those cities are proper to the two models. If the past population trend of a city can not satisfy the indispensible condition, the linear model is the optimum model for the moderate decrease type.

This type can be further classified according to the future rate change. The first type is the heavy increase type of the future rate change that is larger than 10 percent. Three cities out of 54 cities are categorized for this type. The range of the mean percentage without the Gompertz and modified exponential models is from 11.2 to 15.9 percentage error. The models tend to under-estimate the populations. Though the exponential model yields minimum mean percentage error and standard deviation of percent error, populations of two out of three cities were under-estimated with more than 10 percent error. Only one in the three cities satisfy the indispensible condition for the Gompertz and modified exponential models, but the percentage errors are very high.

The second type is the light change type of the future rate changes which are ranged from -10 to 10 percent. 41 out of 52 cities fall into this type. The linear model has a minimum mean percentage error which is 6.9 percent error. Other models also show fairly good results. The mean percentage error is 7.2 percent with the polynomial model, 7.3 percent with the double logarithmic model, and 7.4 percent with the exponential model. The populations of 33 out of 42 cities were projected within 10 percent error with the linear model and 31 cities with the polynomial model.

The Gompertz model and the modified exponential model show very accurate projection for the cities which satisfy the indispensible condition for two models. There are 22 cities for each of two models in 41 cities. The mean percentage error for those cities is 4.6 percent with the Gompertz model and with the modified exponential model. The populations of 21 out of 22 cities were projected within 10 percent error with the two models. These two model can be used as the best model for this type of the cities if those cities meet the indispensible condition of two models. If not, then, the linear model can be used for this type.

Third type is the heavy decrease type of the future rate changes which are less than -10 percent. There are 10 cities for this type. All models show a little high mean percentage error. The polynomial model represents the best results for the 10 cities. The mean percentage error is 14.8 percent with the polynomial model and 17.3 percent with the linear model. All of six models tends to over-estimate the populations. The polynomial model also over-projected the populations for 7 out of 10 cities with more than 10 percent error, but it is still best for this type among four

models except the Gompertz model and the modified exponential model.

The 8 cities for the Gompertz model and the 7 cities for the modified exponential model are satisfied with the indispensible condition of two models. The mean percentage error for those cities is 13.8 percent with the Gompertz model and 12.4 percent with the modified exponential model. This results with two models are slightly better than the polynomial model.

The graphical example will explain the tendency of each models for the moderate decrease type of the past rate change. Pontiac are illustrated for the type. The population of Pontiac showed 11.6 percent increase from 1950 to 1960 and 3.7 percent increase from 1960 to 1970.

The past growth pattern of Pontiac meet the indispensible condition for the gompertz model and the modified exponential model. The projection curve with the Gompertz model showes the rapidly increasing growth rate in early stages and the rapidly decreasing growth rate after inflection point, and then the projection curve approaches the upper limit. The modified exponential model drew the projection curve with the same decreasing growth rate continuously until approaching the upper limit. The figure 8 depicts how models are fitting the past observed population points. And it also represent why the projection errors with six models are small or high.

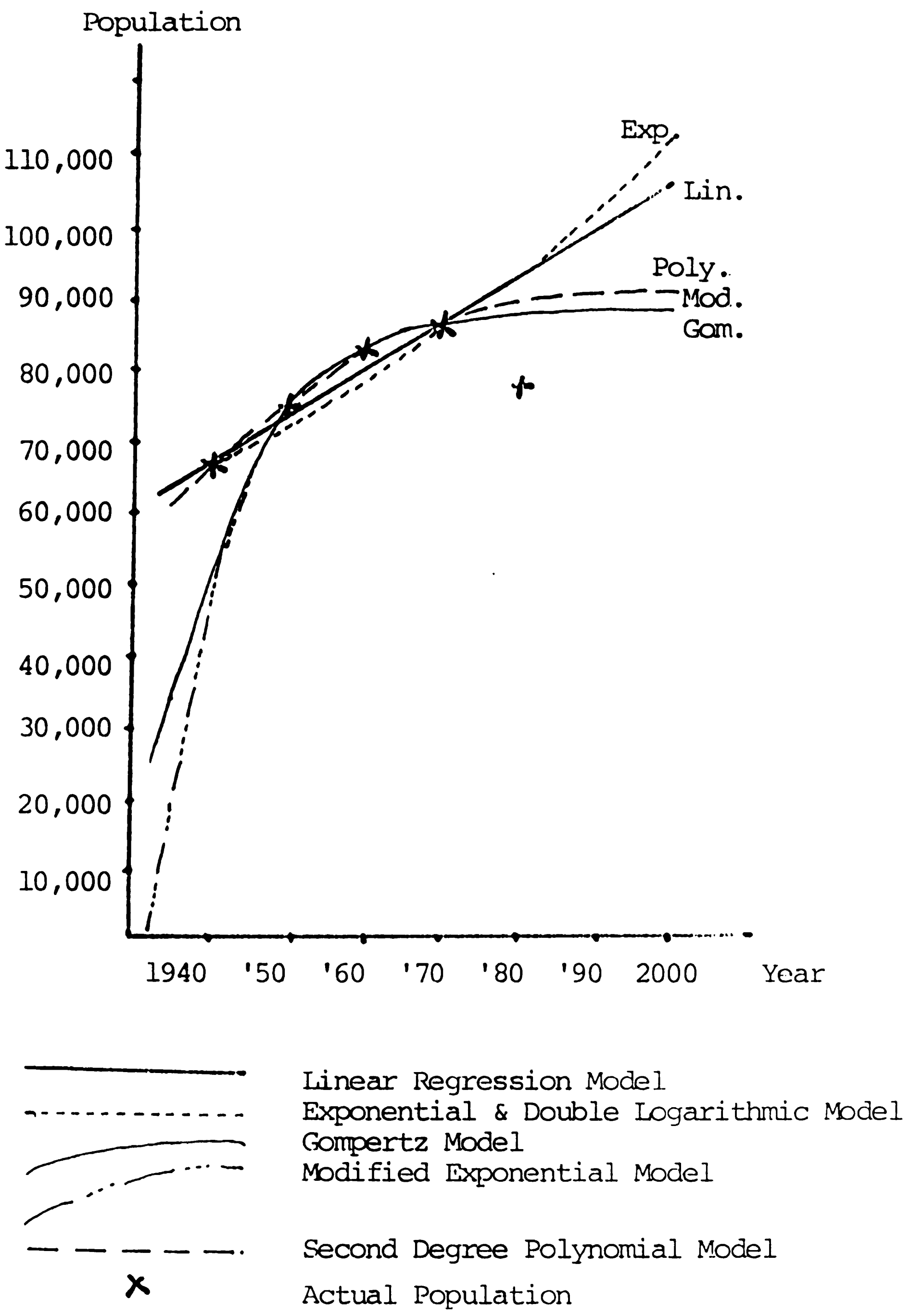


FIGURE 8 : Example of the Moderate Decrease Type
(City of Pontiac)

The Pontiac showed the net population decreased from 1970 to 1980 though the population had increased from 1940 to 1970. All of models drew the projection line or curves reflecting the past population trend, but the population change during 1970s in Pontiac does not follow the past trend. Therefore, all of model over-estimate the population for 1980. However, the Gompertz model showes relatively better accurate projection for Pontiac, because it drew projection curve with the assumption of rapid decreasing growth rate until approaching upper limit. If the population of Pontiac were stable during 1970s which may be classified as the light change type, the Gompertz or modified exponential model may yield a better accurate projection.

In summary of this section, the linear model yields the minimum mean error for cities in this type overall. But, if cities in this type satisfy the indispensible condition of two models, the Gompertz model and the modified exponential model can be used as the best model for the moderate decrease type in growth rate. If not, the linear model is the optimum model for the type.

If a local planner can roughly estimate future growth rate, he can use different models according to the future rate change. If the future rate change is larger than 10 percent, he can use the double logarithmic model. If the changes are ranged from -10 to 10 percent, the linear model can be used for this type. It is recommended to, however, use the Gompertz model when the cities satisfy the indispensible condition of that model. If the future rate change is less than -10 percent, the modified exponential model can be used when the cities meet the indispensible condition of it, otherwise, the polynomial will be the best model for this type.

Medimum Decrease in Growth Rate

These cities have past rate changes that range from -25 to -10 percent. For example, Adrian, in Lenawee county, grew at 10.6 percent growth rate from 1950 to 1960 and at 0.2 percent growth rate from 1960 to 1970. The change in the growth rate between the 1950s and the 1960s is -10.4 percent. Therefore, Adrian falls into this type.

There are 32 cities of this type in Michigan. With these 32 cities, the polynomial model generates a minimum percentage error of 14.0 percent (See Table 6). The linear model yields results very close to the polynomial model. The polynomial model and the linear model projected the populations of 14 out of 32 cities with less than 10 percent error. But the linear model projected the populations of 4 out of 32 cities with an error greater than 25 percent of actual population, while it was only 3 out of 32 cities with the polynomial model. In the aspect of standard deviation of percent error, the linear model is slightly better than the polynomial model. Since the two models yielded very similar results, either can probably be used for this type. In this study, the model that yielded the minimum mean error was selected as the best model. If the mean error was same, then the standard deviation of percent error was used as the measure of accuracy. Therefore, the polynomial model is selected as the best model for the medium decrease type in this study.

The exponential and the double logarithmic models tended to over-estimate the population more than the linear model. When the accuracy of the Gompertz model and the modified exponential model were tested with the cities that

Table 6

Frequency of Percentage Errors for
Cities of Medimum Decrease Type in Michigan

% ERROR	LIN	EXP	DLOG	POLY	GOM	MOD
25 +	4	7	7	1	1	0
10 TO 25	14	14	14	6	2	2
-10 TO 10	14	11	11	14	10	10
-25 TO -10	0	0	0	9	3	2
< -25	0	0	0	2	0	0
MEAN ERROR	14.3	16.8	16.7	14.0	9.0	7.8
SD.	9.5	11.1	11.0	10.5	8.0	6.1
TOTAL CITY	32	32	32	32	16	14

satisfied their indispensible condition, the accuracy of these model was better than any other models. The condition was satisfied with 16 cities for the Gompertz model and with 14 cities for the modified exponential model.

With those cities satisfing the condition, the Gompertz model had 9.0 mean percentage error and the modified exponential model has 7.8 mean percentage error. In the aspect of standard deviation of percent error, the 10 out of the 16 cities had the projection errors within 10 percent with the Gompertz model and 10 out of the 14 cities with the modified exponential model.

Therefore, if the cities in the medium decrease type satisfy the indispensible condition of these two models, their populations can be projected better by these two models than by the polynomial model. If a city does not meet the condition, the polynomial model can be used.

The accuracy of the models also depends on the type of future population changes. Therefore, the medium decrease type was classified into three types by the future rate change. The first type was the heavy increase type in which the future rate changes are greater than 10 percent. Only three out of 32 cities fell into this category in Michigan. The exponential model, the double logarithmic model and linear model projected population very accurately for these three cities with less than 10 percent error. The exponential model and the double logarithmic model had 5.3

mean percentage error for the three cities and the linear model had 5.9 mean percentage error.

The second type is the light change type in which future rate changes range from -10 to 10 percent. This type comprised 20 out of 32 cities. The polynomial model showed the best result for these cities. The mean percentage error was 9.5 percent with the polynomial model, while it was 11.5 percent with the linear model. The Gompertz model and the modified exponential model yield a lower mean percentage error if the models are applied to cities which satisfy the indispensible condition of the two models. With these cities, the Gompertz model had 4.6 mean percentage error with 9 cities, and the modified exponential model had 4.3 mean percentage error with 8 cities. Therefore, the modified exponential model can best be used for this type if a city satisfies its indispensible condition. If it does not satisfy this condition but satisfies the condition of the Gompertz model, the Gompertz model can best be used for the city. If the city is not appropriate for either of those, then the polynomial model can be applied.

The third type of the future rate change is the heavy decrease type in which future rate changes are less than -10 percent. There were 9 cases of this type in Michigan. Four models, except the Gompertz model and the modified exponential model, had high mean error for this type. The modified exponential model and the Gompertz model yielded

better results for specific cities. Among 9 cities, 6 cities satisfied the indispesible condition of the Gompertz model and 5 cities for the modified exponential model. The modified exponential model had 12.0 mean percentage error and projected the population of 3 out of 5 cities within 10 percent error. The Gompertz model had 14.7 mean percentage error. The other four models tend to over-estimate the populations. The mean percentage error was 18.4 percent with the polynomial model, 23.3 percent error with the linear model, 28.4 percent error with the double logarithmic model and 28.5 percent error with the exponential model. Therefore, if a city satisfied the indispensible condition of the modified model, its population can be projected by the modified exponential model. If it did not satisfy the condition for the modified model but satisfied the condition for the Gompertz model, the Gompertz model could be used for the city. If it did not satisfy the conditions of either of these, the polynomial model could be used, with care.

Adrian was graphically illustrated as one of the medium decrease cities of the past rate change in Figure 9. Adrian grew quickly from 1940 to 1960, but the population changed little from 1960 to 1980. The exponential model and double logarithmic model drew projection curves into the future at a certain population growth rate. The polynomial model drew a projection curve which showed the rapid population increase at a decreasing growth rate until the mid-1960s and

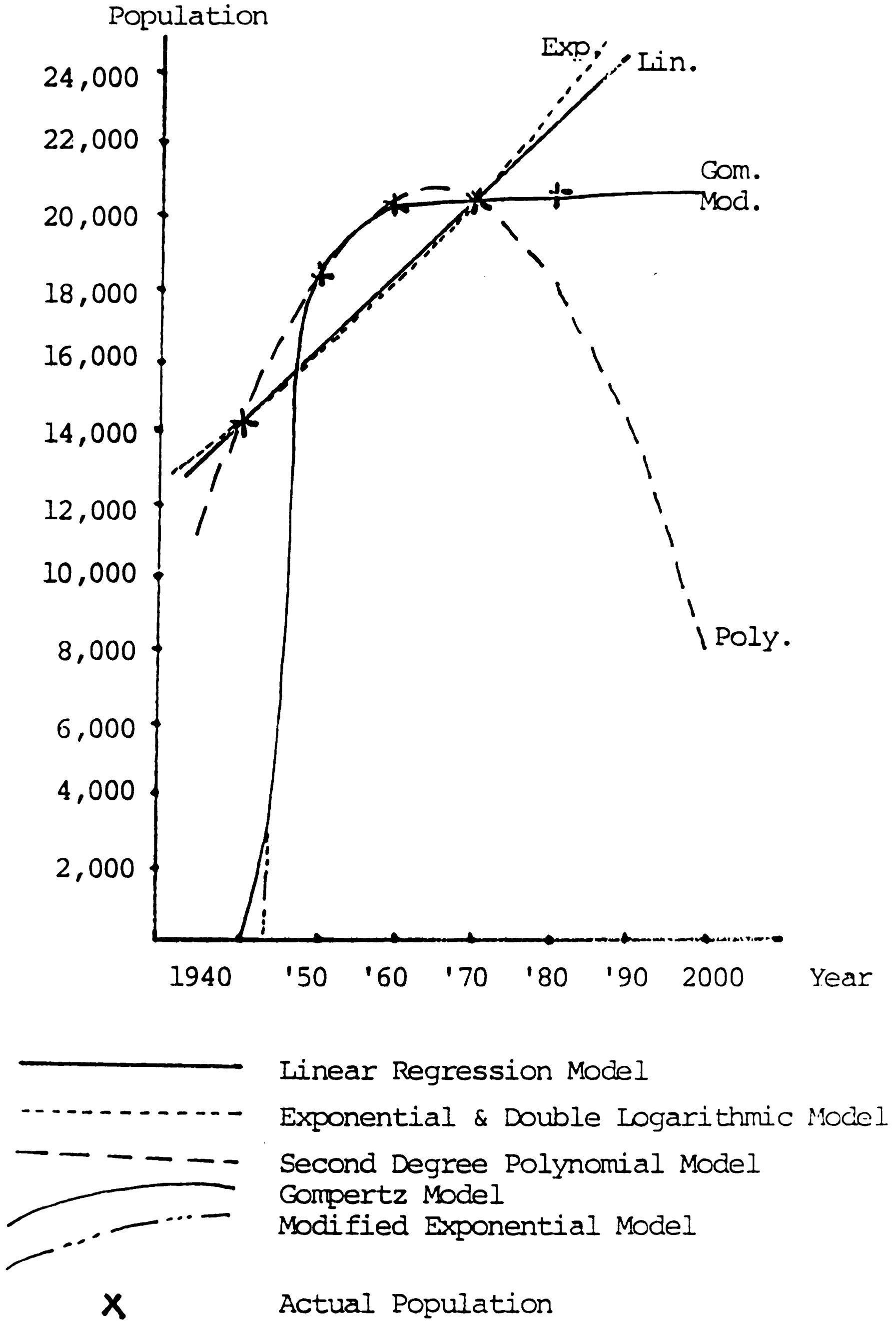


FIGURE 9 : Example of the Medium Decrease Type
(City of Adrian)

showed a rapid population decrease after the mid-1960s.

Adrian's past population trends satisfied the indispensible condition for the Gompertz and the modified exponential models. The two models drew projection curves that indicated extreme population growth in the 1940s and then a rapidly decreasing growth rate in the 1950s and little change in population after 1960. Actually, the population of Adrian increased by only 804 persons during the 1970s. Therefore, the Gompertz model and the modified exponential model projected populations very close to the actual populations of 1980. The exponential model, the double logarithmic model and the linear model yielded over-estimations, and the polynomial model under-estimated the population of Adrian for 1980.

As a summary of this section, the accuracy test of the polynomial model showed the best overall results for cities in the medimum decrease type, although the accuracy of the linear model was very close to that of the polynomial model. But, if the cities satisfy the indispensible condition of the Gompertz model or the modified exponential model, one of those models can best be used for this type.

If a planner can roughly estimate future growth rate, he is advised to use the exponential model when the future rate changes are greater than 10 percent. When the future rate changes are less than 10 percent, the modified exponential model yields the minimum mean error if the cities satisfy the indispensible condition of the model. If the modified model or the Gompertz model cannot be applied, the polynomial model seems the best alternative for these cities.

Great Decrease in Growth Rate

This type of city has past rate changes that ranged from -50 to -25 percent. For example, the population of Holland grew at a 56 percent growth rate from 1950 to 1960, and at a 7 percent growth rate from 1960 to 1970. The growth rate change was -49 percent.

Fifteen cities of this type were found in Michigan. Four models, expecting the Gompertz model and the modified exponential model, had comparatively high mean percentage errors for this type. The Gompertz model and the modified exponential model yielded better accuracy the 9 cities that satisfied the indispensible condition of the Gompertz model and 8 cities that met the condition of the modified exponential model (See Table 7). The mean percentage error was 11.5 with the modified exponential model and 13.3 with the Gompertz model.

The polynomial model was the next best method for the 15 cities of this type. It had 18.5 mean percentage error. The polynomial model have greater frequency in the category of "lower than 10 percentage error" than the Gompertz model and the modified exponential model, but the polynomial model produced more extreme error than the two models. The modified exponential model or the Gompertz model can be used if the cities meet their indispensible condition. Otherwise, the polynomial model can best be used for the cities which can not meet the condition. The other models showed highly over-estimations for this type of city. The linear, exponential and double logarithmic models yielded more than 10 percent over-estimations for all 15 cities.

If future rate changes are considered, there are three types within the great decrease cities in growth rate. The first type is characterized by heavy increase in future rate changes, but there were no cases of that type in Michigan.

Table 7

Frequency of Percentage Errors for Cities of Great Decrease Type in Michigan

% ERROR	LIN	EXP	DLOG	POLY	GOM	MOD
25 +	8	11	11	4	1	0
10 TO 25	7	4	4	3	5	5
-10 TO 10	0	0	0	6	3	3
-25 TO -10	0	0	0	1	0	0
< -25	0	0	0	1	0	0
MEAN ERROR	26.4	40.6	40.2	18.5	13.3	11.5
SD.	8.8	18.9	18.7	14.6	8.8	5.2
TOTAL CITY	15	15	15	15	9	8

The second type is the light change type in which future rate changes are between -10 and 10 percent. There are seven cities of this type in Michigan. Although that is a statistically small number of cases, the polynomial model seemed to be the best method for the type. The mean percentage error with the model was 11.0 percent error. The populations of four out of seven cities were projected within 10 percent of actual population and all cities had errors less than 25 percent. Only the city of Holland met the indispensible condition of the Gompertz and modified exponential models. These two models project the population of Holland very accurately with about 2 percent error. But one case is not sufficient to generalize the accuracy of the two models for the type. Three other models: the linear model, the exponential model and the double logarithmic model, projected all cities with over-estimations of more than 10 percent.

The third type is the heavy decrease type, in which the future rate changes are less than -10 percent. There are eight cities of this type in Michigan and all eight cities satisfied the indispensible condition of the Gompertz model, and seven cities satisfied the modified exponential model. The modified exponential model yielded the minimum mean error for the seven cities. The mean percentage with the model was 12.8. The Gompertz model model was the next best model for the type and had 14.7 mean percentage error.

However, only two cities were projected with less than 10 percent error. The other four models had high mean percentage errors. The polynomial model was slightly better than other three models. The exponential model and the double logarithmic model projected populations for all eight cities with higher than 25 percent error.

A graphical example can explain how each of models projects population for a city of the great decrease type. In the case of the Holland, rapid growth in population between 1950 and 1960 was reflected in equations. Therefore, the four models, excepting the Compertz model and the modified exponential model, drew projection line or curves extending the rapid growth into the future (See Figure 10). The Gompertz model and the modified exponential model are applicable to the city and drew projection curves to show rapid growth in population until 1960, with little growth in population after 1960. The actual net population of Holland was decreased by very few, about 200, persons. Therefore, the Gompertz model and the modified exponential model projected the 1980 population relatively well. The other four models highly over-estimated the population. If the population of 1980 had been increased at a rapid growth rate, one of the four models might have projected the population more accurately than the Gompertz model and the modified exponential model.

To summarize this section, the modified exponential

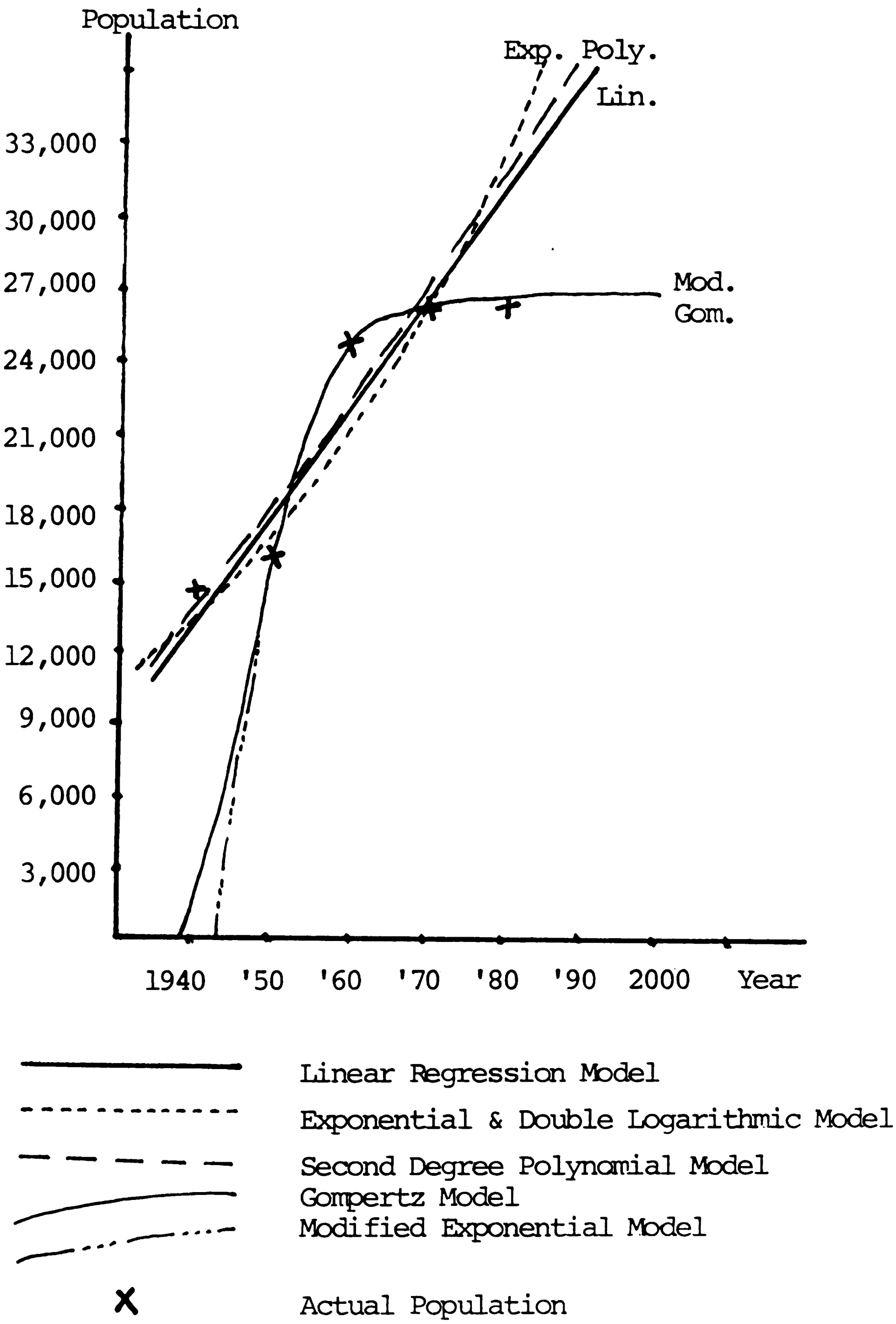


FIGURE 10 : Example of Great Decrease Type
(City of Holland)

model is the best method for the great decrease type, if the cities meet the indispensible condition of this model. If a city cannot do so, but is appropriate for the Gompertz model, the Gompertz model can be applied. If the city is not appropriate for either of these models, the polynomial model can be used.

When a planner can roughtly predict a future growth rate for a city, he can use different models for different types according to future rate change. The polynomial model turned out to be the best model for the cities which had future rate changes from -10 to 10 percent. There were not enough cases of cities which satisfied the indispensible conditions of the Gompertz and modified exponential models. those models may have better accuracy for cities that meet their indispensible conditions. When the future rate changes are less than -10 percent, the Gompertz model is the best method among the six models. For cities which cannot meet the indispensible condition of the Gompertz model, the polynomial model can be used. A planner should also keep in mind that any of these models has the potential to produce high projection errors for this type of city.

Extreme Decrease in Growth Rate

Cities classified as this type are those which have past rate changes less than -50 percent. For instance, the population of Midland, in Midland county, increased at a 94

percentage growth rate from 1950 to 1960, and at a 27 percent growth rate from 1960 to 1970. The growth rate change from the 1950s to the 1960s was thus -67 percent. Therefore, Midland is a city of the extreme decrease type.

There are 28 cities for this type in Michigan. Of these 28 cities, 25 satisfied the indispensible condition of the Gompertz model and 24 satisfied the modified exponential model. The Gompertz model yielded the best accuracy for cities that satisfied its indispensible condition. The mean percentage error was 14.4 for 25 cities with the Gompertz model. The next best model is the modified exponential model if cities satisfy its indispensible condition. The mean percentage error with the modified exponential model

Table 8

Frequency of Percentage Errors for
Cities of Extreme Decrease Type in Michigan

% ERROR	LIN	EXP	DLOG	POLY	GOM	MOD
25 +	17	27	27	21	1	1
10 TO 25	6	1	1	5	13	15
-10 TO 10	5	0	0	2	8	6
-25 TO -10	0	0	0	0	3	2
> -25	0	0	0	0	0	0
MEAN ERROR	35.3	127.6	125.5	39.8	14.4	15.5
SD.	19.5	162.7	158.7	22.9	7.5	8.9
TOTAL CITY	28	28	28	28	25	24

was 15.5 for 24 cities. The other four models had extremely high mean percentage errors. The mean percentage error was 35.3 percent error with the linear model and 39.8 percent with the polynomial model. The exponential and double logarithmic models had higher than 100 mean percentage error. All six models tended to over-estimate the populations for this type. Although the Gompertz model was the best method for this type, only 8 out of 28 cities were projected within 10 percent error (See Table 8).

The linear model was slightly better than the other four models, but its mean percentage error was 35.3 percent. Only 5 of 28 cities were projected within 10 percent of their actual population. The exponential model and the double logarithmic model had extremely high error rate (i.e., 904 percent error with the exponential model).

Therefore, the Gompertz model can best be applied to those cities that satisfy the indispensible condition of the model. For the others, the linear model may be used with the realization that, in this type of city, there is the potintial for severe over-estimation with the linear model.

If the growth rate can be roughly estimated, accuracy might be improved through using different models to project different types of future rate changes. The first type of future rate change is a heavy increase with future rate change more than 10 percent. There is no case of this type in Michigan. The second type is a light change which affects

the group of cities with future rate changes from -10 to 10 percent. There are five such cases in Michigan. The Gompertz model and the modified exponential model yielded relatively good projections for the four of the five cities that satisfied their conditions. The modified exponential model yielded 9.6 mean percentage error and the Gompertz model had 10.8 mean percentage error.

The polynomial model is the next best model for this type. The mean percentage error was 14.2 percent and two out of five cities were projected with less than 10 percent error. Other three models tended to highly over-estimate the population and while the linear model was slightly better than the others, it still had a high (18.0) mean percentage error.

The third type is a heavy decrease type in which future rate changes are less than -10 percent. There are 23 cases of this type in Michigan. All six models showed high mean percentage errors, and tended to over-estimate populations of this type. The linear model and the polynomial model had 39.1 and 45.4 mean percentage errors, respectively, and the exponential model and the double logarithmic model had extremely high errors (145.3 and 143.0 mean percentage error).

In the 23 cities, 21 satisfied the indispensible condition of the Gompertz model and 20 met the condition of the modified exponential model. With those specific cities,

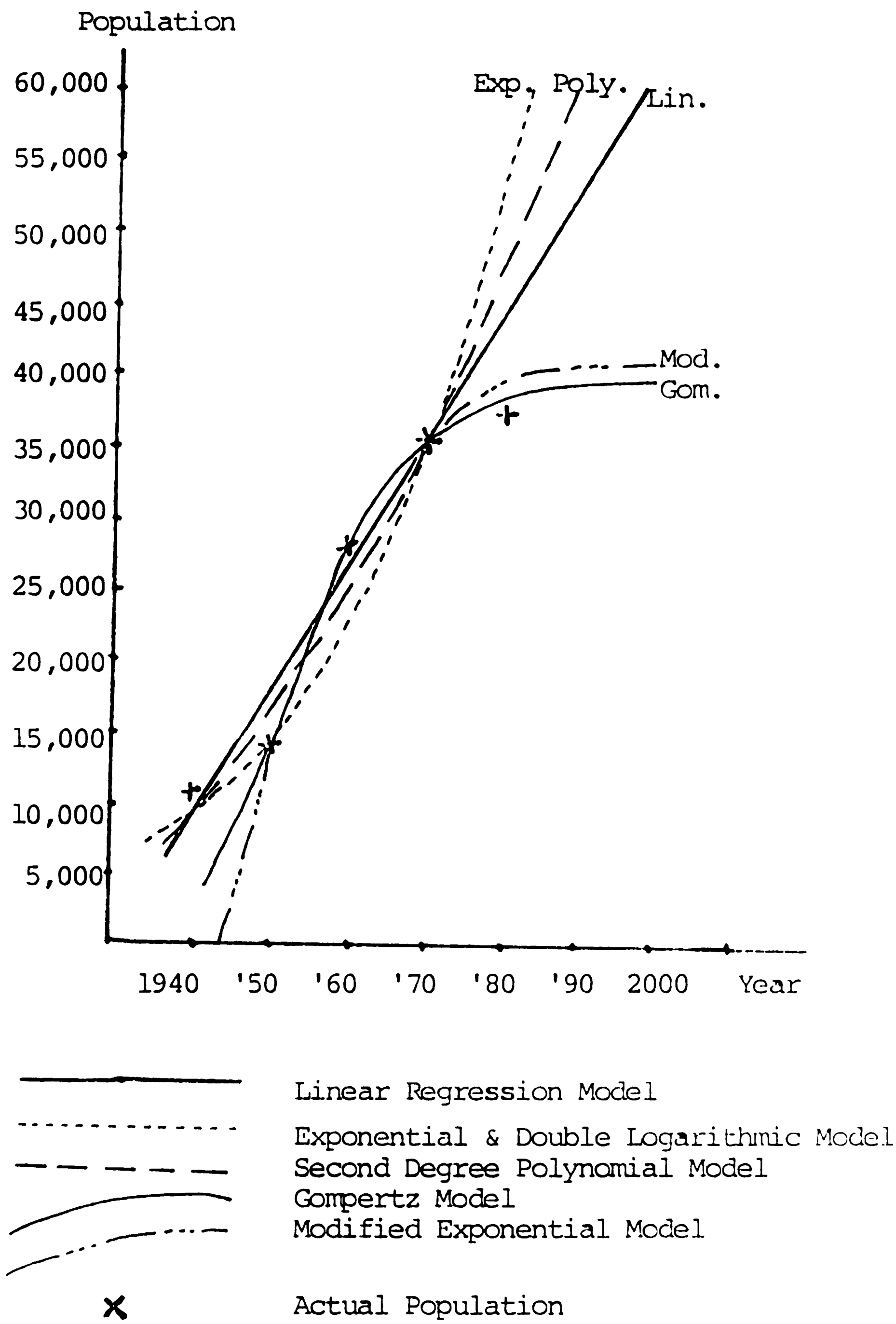


FIGURE 11 : Example of the Extreme Decrease Type
(City of Midland)

the Gompertz model had 15.0 mean percentage error and the modified exponential model had 16.7 mean percentage error.

Midland, in Midland county, is illustrated as an example of the extreme increase type of past rate change in Figure 11. The population of Midland increased at a rapidly increasing growth rate in the 1950s, but the growth rate rapidly declined through the 1960s and 1970s although the net population continuously increased. The growth pattern of Midland is consistent with the assumptions of the Gompertz model. As shown in Figure 11, the Gompertz model projects Midland's population very accurately. The modified exponential model yielded more over-estimation than the Gompertz model.

The exponential model and the double logarithmic model drew the projection curves with the tendency of the high growth rates in 1950s and 1960s and thus greatly overestimated the population. The linear model drew a projection line with an excessively larg increment per unit of time, because it also reflected the fast growth in an earilier time. The polynomial model also showed a rapid growth projection curve. If the population of the Midland had increased from 1970 to 1980 at the same growth rate as it had the previous decade, the linear model might have produced minimum error for the 1980 population of the city.

To summarize this section, none of the six models yielded good accuracy for this type. The Gompertz model was

the best method for this type and the modified exponential model was next. If cities cannot meet the indispensible condition of these two models, the linear model may be used, with caution, because of the potential for high error with the model for this type.

With specified types of future rate changes, the accuracy was slightly improved for cases of light change in which the growth rate change was between -10 and 10 percent from the 1950s to the 1960s. The modified exponential model was the best method for this type and the Gompertz was next. If a city cannot meet the indispensible condition of those two models, the polynomial model can be used. For cities in which future rate changes are less than -10 percent, all six models showed high mean percentage errors. The Gompertz model yielded the best accuracy for this type among the six models and the modified exponential model was next. When a city cannot meet the indispensible conditions of those models, the linear model may used with great care.

Cities Classified according to 1970 Population

Number of population at a given census data is another characteristic that can be used to classify cities. Four population categories, according to 1970 population figures, were used to test the accuracy of the six models (See Table 9). The linear model yielded relatively good results for cities in all four population categories. For cities

between 10,000 and 50,000 population, however, the polynomial model showed slightly better results. Among the four models tested (excepting the Gompertz and modified exponential models), the linear model was dominant in accuracy.

The modified exponential model is more accurate for smaller cities than the Gompertz model, if the cities satisfy its indispensible condition. The Gompertz model produced the lowest error for larger cities, if the cities are appropriate for this model.

All six models showed similar projection accuracy tendencies for each population size category. Each yielded

Table 9

Mean Percentage Error of Each Model
for City Types Based on 1970 Population

Model	population (thousand)			
	50 +	10 to 50	5 to 10	2.5 to 5
LIN	32.6 (14)	23.3 (54)	9.3 (41)	11.8 (63)
EXP	122.3 (14)	50.1 (54)	15.0 (41)	15.2 (63)
DLOG	120.1 (14)	49.6 (54)	14.8 (41)	15.1 (63)
POLY	35.4 (14)	21.3 (54)	12.9 (41)	15.1 (63)
GOM	17.9 (7)	11.5 (24)	7.4 (19)	9.9 (31)
MOD	20.1 (6)	12.0 (24)	6.6 (17)	8.7 (29)

(Note) number in parentheses indeicates number of cities of that type in Michigan.

its best result for the cities between 5,000 and 10,000 population, and next for the cities between 2,500 and 5,000 population. Each of the six models yielded its highest mean percentage error for cities over 50,000 population.

The results of this study are not consistent with those obtained by Isserman (1977) who found relatively high projection errors for small townships and relatively small errors for the large townships (Isserman, 1977 : 256). In contrast, the results of this study showed that the populations of smaller cities were projected with greater accuracy than those of larger cities. This difference, however, was likely due, in part, to changes in population trends in large and small cities.

However, past population size was not used as a criterion to classify the cities in this study for several reasons. First, population size does little reflect the peculiar nature of each model. The extrapolation models originally extrapolated cities' historical population growth trends into the future, whatever the size of their populations. If there is a correlation between a city's population size and its population growth pattern, it may be possible to judge the accuracy of an extrapolation model by the population size in some cases. but I believe its past population growth pattern is a better criterion than population size, because the origin of extrapolation models is based in the past population trends.

The second reason, for not using population size as a classifier, was to avoid complexity in classification. If a combination of seven types of growth rate change and four categories of population size were used, the combination would result in 28 types of cities. Considering future growth rate change, 84 city types would be produced from all possible combinations of each characteristic. When criteria become so complex, the point of clear and simple classification is lost.

The third reason is apparent the results of the accuracy test for cities of varied population sizes. The linear model dominates for cities of almost all population sizes. The Gompertz model or the modified exponential model will be used for all cities that satisfy the models' indispensible condition. There is no point to classification of city types if a model that is applied to all fails to discriminate between the categories. Therefore, I think the growth rate change is a better basis for classifying cities than population size to apply extrapolation model according to certain city types.

Summary

Overall, the linear model was, generally, the most accurate method for projecting the 1980 populations of cities in Michigan, but it is preferable that specific extrapolation models be applied to those types of cities for which they aremost accurate, rather than relying on one method. This is especially true when the Gompertz and modified exponential models are appropriate for those cities that satisfy their indispensible conditions, because these two models yield excellent quality population projections for those cities.

To classify cities, growth rate change was used. The differential pattern of errors for types of growth rate change made it a more useful classification criterion than population size. Since growth rate change during 1960s was found to be related to population size in 1970, it might also reflect this characteristic some extent.

Past and future growth rate changes were considered in classifying cities by type for this study. The results of the accuracy tests for growth rate change are presented in Table 10. Given past growth rate changes, the linear model is generally well suited to the increasing or moderate type of change in growth rate.

The polynomial model is relatively accurate for the rapidly decreasing type of growth rate, but the Gompertz or the modified exponential model are more accurate for cities of this type if the cities meet the indispensible conditions of one of the two models.

For cities with a dissimilar combination of past and future growth rate changes, the exponential model and the double logarithmic model seem to be appropriate to cities

with rapidly increasing future growth rates, but where the growth rate decreased more than 25 percent in past. The exponential model tends to work well with rapidly increasing growth rates. These two models generally produce very similar population projection in most cases.

Table 10

Population Projection Model
for Cities Based on Growth Rate Change

Past Growth Rate Change	All	Future Growth Rate Change		
		10 +	-10 to 10	< -10
25 +	Lin	-	-	Lin
10 to 25	Lin	Exp	Lin	Lin
0 to 10	Lin	Exp	Exp	Lin
-10 to 0	Lin * Mod	Dlog	Lin * Gom	Poly * Mod
-25 to -10	Poly * Mod	Dlog	Poly * Mod	Poly * Mod
-50 to -25	Poly * Gom	-	Poly * Gom	Poly * Mod
< -50	Lin * Gom	-	Poly * Mod	Lin * Gom

(Note) * indicates that the Gompertz model or the modified exponential model is the best method for that type if the cities satisfy the indispensible conditions of one of them.

The linear model yields the accurate projections for city populations that have shown rapid growth rate increases in the past and rapid growth rate decrease for the future or those that have shown moderate growth rate change in the past and for the future. The polynomial model seems to be the best model for cities with growth rate decreases in the past and moderate or decreasing growth rate change in the future. However, if cities satisfy the indispensible condition of the Gompertz or the modified exponential model, those models are the best methods for those types of cities.

CHAPTER 4

DEVELOPMENT OF COMPUTER PROGRAM

Criteria Basis to Select Best Model

A composite method which applies different extrapolation models to different types of cities has resulted from the accuracy test used with Michigan cases.

TABLE 11

Composite Method

Past Growth Rate Changes	Composite I	Composite II Future Growth Rate Change 10 +	-10 to 10	< -10
25 +	Lin	Exp	Dlog	Lin
10 to 25	Lin	Exp	Lin	Lin
0 to 10	Lin	Exp	Exp	Lin
-10 to 0	Mod * Lin	Dlog	Gom * Lin	Mod * Poly
-25 to -10	Mod * Poly	Exp	Mod * Poly	Mod * Poly
-50 to -25	Gom * Poly	Lin	Gom * Poly	Mod * Poly
< -50	Gom * Lin	Lin	Mod * Poly	Gom * Lin

(Note) * indicates that, if a city does not satisfy the indispensible condition of the Gompertz model or the modified exponential model, the designated model will be used for that type.

In order to develop a composite method, cities were classified according to their growth rate changes in the same manner used for accuracy testing in the previous chapter.

Following this, a two-part composite method was developed. Composite I uses the most accurate method employing past growth rate changes only. This can be applied to cases in which a planner cannot anticipate a future growth rate. Composite II uses the most accurate method employing the appropriate combination of past and future growth rate changes.

Seven types of past growth rate change and three types of future growth rate change were used, resulting in a total of twenty-one types for the composite II. The composite method selects a model exactly as shown in Table 11. The composite method was derived from the results of this study as presented in Table 10. For some of the types, there were few or no cases. To develop the composite method for those types, the tendencies and basic assumptions of the six models were comsidered.

Micro-computer Program with BASIC Language

The computer program provides alternatives. Users can chose the model they prefer to use or can allow the selection to be made by the program. If a user decide to use a composite method and cannot anticipate future growth rate, a model will be selected through the Composite I. If a user can make a rough estimate of a future growth rate, Composite II will select an appropriate model for the city.

If a user runs the population projection program, it will ask the user to enter data (i.e., number of cities, number of data points, time of the data points, city names and population data). Once the user enters the data, the program will ask for a projection year, and then display the alternatives (seven methods and the composite method).

If a user chooses one of the seven models, the program will immediately project the population to the desired time for those cities. If a user chooses the composite method, the program asks whether or not a rough future growth rate can be given.

When a user chooses the composite method, the program will decide to use whether Composite I or Composite II, according to the user's answer. Then, the program will calculate the growth rate over the past two time spans and the growth rate change for each city. If a user answers that he cannot anticipate the future growth rate, the program will select a model with a past growth rate change according to the Composite I, as shown in table 11. Then, the program will immediately project the population of the cities for the desired projection time with the model selected. If a user answers that he can give rough future growth rate, the program will call up future growth rates,

such as -10, 0, 10, 15, etc. Once a user enters a rough growth rate, the program will calculate growth rate change between the growth rate in last time span and the rough growth rate entered.

Then the program will select a model with the proper combination of past growth rate changes and future growth rate changes for each city according to Composite II. And the program will immediately project the populations of the cities for the projection time with the selected model. After finishing the projections, the program will ask whether or not a user wants to project the population of the cities again, with another model.

There are other facilities in the program. A user can save his data base in his disk through the program and he can use the data base again and again later for projecting the populations of the cities in the data set. He can also save the results of projections into his disk.

The program will reject requests to project population if a user does not enter the same time intervals as the times of past population observation points. The program may also recommend population projections for the short-term, so that it automatically suggests the year of the next span from the last population observation time as the projection year. If a user still wants to project populations over a longer term than one more year span, the program will project the population for the long-term using the projection year that a user types in.

In choosing the Gompertz model or the modified exponential model, the program will test the appropriateness of the city to that model. If a city cannot meet the indispensible condition of the modified exponential model, it will test for appropriateness with the Gompertz model. In some cases, it will meet the indispensible condition of the Gompertz model even though it could not satisfy the indispensible condition of the modified exponential model. If the city is not appropriate for either of these models, the program will select the next best model according to the composite method.

CHAPTER 5

CONCLUSION

This study was done in an effort to meet the needs for the information about the accuracy of extrapolation models which are the popular techniques used to project population in local planning offices. Any given population extrapolation model may be inappropriate or inapplicable to the cities for which a planner seeks information, because each of models projects populations with different assumptions and there are various population growth patterns in cities. Therefore, specific models should be used for different type of cities.

To develop the composite method, the accuracies of six extrapolation models were tested for various types of cities with varied growth rate changes. The composite method was based on the results of this accuracy testing and employs the most accurate among the six models for particular types of cities, according to the growth rate changes.

The exponential model and the double logarithmic model are generally the best method for cities of which the growth rate increased or changed moderately in the past and the growth rate in future is expected to rapidly increase.

The linear model is generally the best technique for the cities of which growth rate have changed moderately or increased in the past, or will moderately change or decrease with an earlier pattern of increasing growth rate.

The Gompertz model, the modified exponential model and the second degree polynomial model are generally the best methods for the type of cities which have had decreasing growth rates in past, or which will moderately change or decrease in growth rates, following a past growth pattern of decreasing growth rate. But the Gompertz model and the modified exponential model require certain conditions. The parameter "b" in the equations should be positive and less than one and the parameter "a" in the equations should be negative. Once cities satisfy the condition, the two models provide good quality projection for those cities.

It is remarkable that the Gompertz model and the modified exponential model played an important role in projecting 1980 populations of cities in Michigan. Isserman (1977) found that one of three methods (the linear model, the exponential model and the double logarithmic model) was the most accurate method for certain types of townships classified by growth rate. His findings can still be useful except in the case of the double logarithmic model for cities losing more than 25 percent of their population. For instance, he recommends using the exponential model for townships increasing more than 25 percent, or decreasing less than 25 percent in populaton during the last decade. This is also recommended by this study, but this study added another condition. The added condition is that cities are expected to increase more than 10 percent in growth rate, along with the previous conditions, in order to use the exponential model.

Because of the changes in population trends during the 1970s in Michigan, the recommendations of Isserman (1977) are not sufficient for cities which more recently show decreasing growth rate. This study found models for those types of cities. It is recommended that the Gompertz model or the modified exponential model be used if cities satisfy the indispensible condition of those two models. It was also found that the polynomial model yields good projections for that type of cities, if cities are not appropriate for the Gompertz model or the modified exponential model.

However, a planner has to use extrapolation models with some care. All six models yielded relatively high errors for the cities which had rapid growth rate changes. For example, the great increase type or the extreme decrease type. All of extrapolation models have especially extreme errors with cities which have had sudden reversals of population trends for the future. For instance, when the population of a city shows highly increasing population through an earlier time but suddenly decreases for the future, all of models will have high errors. Therefore, in such a case, the local planner should use a population extrapolation model with caution. Although the extrapolation models yield high errors in some case, they are still useful when data, time and funds are limited.

APPENDICES

APPENDIX 1

RESULTS OF ACCURACY TEST FOR EACH TYPE

BASED ON PAST GROWHT RATE CHANGE

APPENDIX 1 : RESULTS OF ACCURACY TEST FOR EACH TYPE BASED ON PAST GROWTH RATE CHANGE

Past Growth Rate Change		Lin	Exp	Dlog	Poly	Gom	Mod
	mean % error	19.7	26.0	25.8	33.2	-	-
25 +	sd.	14.2	21.2	21.0	14.8	-	-
	# of cases	7	7	7	7	0	0
	mean % error	11.2	14.9	14.7	20.0	-	-
10 to 25	sd.	10.8	10.1	10.1	9.8	-	-
	# of cases	9	9	9	9	0	0
	mean % error	10.1	15.2	15.1	14.2	-	-
0 to 10	sd.	9.4	18.2	18.0	13.7	-	-
	# of cases	27	27	27	27	0	0
	mean % error	9.1	10.1	10.0	9.1	7.5	7.0
-10 to 0	sd.	6.5	7.5	7.4	7.5	6.7	5.9
	# of cases	54	54	54	54	31	30
	mean % error	14.3	16.8	16.7	14.0	9.0	7.8
-25 to -10	sd.	9.5	11.1	11.0	10.5	8.0	6.1
	# of cases	32	32	32	32	16	14
	mean % error	26.4	40.6	40.2	18.5	13.3	11.5
-50 to -25	sd.	8.8	18.9	18.7	14.6	8.8	5.2
	# of cases	15	15	15	15	9	8
	mean % error	35.3	127.6	125.5	39.8	14.4	15.5
< -50	sd.	19.5	162.7	158.7	22.9	7.5	8.9
	# of case	28	28	28	28	25	24

APPENDIX 2

RESULTS OF ACCURACY TEST FOR EACH TYPE BASED ON THE COMBINATION OF PAST GROWTH RATE AND FUTURE GROWTH RATE

APPENDIX 2 : RESULTS OF ACCURACY TEST FOR EACH TYPE BASED ON THE COMBINATION OF PAST GROWTH RATE CHANGE AND FUTURE GROWTH RATE CHANGE

GREAT INCREASE TYPE

Future Growth Rate Change		Lin	Exp	Dlog	Poly
	mean % error	-	-	-	-
10 +	sd.	-	-	-	-
	# of cases	0	0	0	0
	mean % error	-	-	-	-
-10 to 10	sd.	-	-	-	-
	# of cases	0	0	0	0
	mean % error	19.7	26.0	25.8	33.2
< -10	sd.	14.2	21.2	21.0	14.8
	# of cases	7	7	7	7

MEDIUM INCREASE TYPE

Future Growth Rate Change		Lin	Exp	Dlog	Poly
	mean % error	38.3	29.4	29.6	26.6
10 +	sd.	-	-	-	-
	# of cases	1	1	1	1
	mean % error	7.9	7.9	7.9	9.8
-10 to 10	sd.	5.1	5.1	5.1	2.1
	# of cases	2	2	2	2
	mean % error	7.7	14.7	14.5	22.3
< -10	sd.	4.1	9.8	9.7	9.8
	# of cases	6	6	6	6

APPENDIX 2 : Continued

MODERATE INCREASE TYPE

Future Growth Rate Change		Lin	Exp	Dlog	Poly
	mean % error	11.3	10.0	10.1	20.8
10 +	sd.	2.2	1.3	1.3	7.5
	# of cases	2	2	2	2
	mean % error	4.9	4.3	4.3	5.2
-10 to 10	sd.	3.1	3.2	3.1	5.4
	# of cases	14	14	14	14
	mean % error	16.5	30.1	29.8	24.5
< -10	sd.	11.5	20.9	20.6	14.2
	# of cases	11	11	11	11

MODERATE DECREASE TYPE

Future Growth Rate Change		Lin	Exp	Dlog	Poly	Gom	Mod
	mean % error	12.4	11.2	11.2	15.9	21.5	21.4
10 +	sd.	5.0	4.8	4.9	7.1	-	-
	# of cases	3	3	3	3	1	1
	mean % error	6.9	7.4	7.3	7.2	4.6	4.6
-10 to 10	sd.	4.4	4.9	4.9	6.1	2.4	2.4
	# of cases	41	41	41	41	22	22
	mean % error	17.3	20.9	20.8	14.8	13.8	12.4
< -10	sd.	7.7	7.3	7.3	9.0	8.6	7.8
	# of cases	10	10	10	10	8	7

APPENDIX 2 : Continued

MEDIUM DECREASE TYPE

Future Growth Rate Change		Lin	Exp	Dlog	Poly	Gom	Mod
	mean % error	5.9	5.3	5.3	30.4	14.5	14.5
10 +	sd.	1.9	1.3	1.4	14.4	-	-
	# of cases	3	3	3	3	1	1
	mean % error	11.5	13.2	13.1	9.5	4.6	4.3
-10 to 10	sd.	7.8	8.5	8.4	6.3	3.9	3.9
	# of cases	20	20	20	20	9	8
	mean % error	23.3	28.5	28.4	18.4	14.7	12.0
< -10	sd.	8.5	8.4	8.3	10.4	9.3	5.9
	# of cases	9	9	9	9	6	5

GREAT DECREASE TYPE

Future Growth Rate Change		Lin	Exp	Dlog	Poly	Gom	Mod
	mean % error	-	-	-	-	-	-
10 +	sd.	-	-	-	-	-	-
	# of cases	0	0	0	0	0	0
	mean % error	23.7	31.0	30.8	11.0	1.8	2.0
-10 to 10	sd.	10.2	19.8	19.6	9.1	-	-
	# of cases	7	7	7	7	1	1
	mean % error	28.8	49.0	48.5	25.1	14.7	12.8
< -10	sd.	7.3	14.4	14.2	15.9	8.2	3.7
	# of cases	8	8	8	8	8	7

APPENDIX 2 : Continued

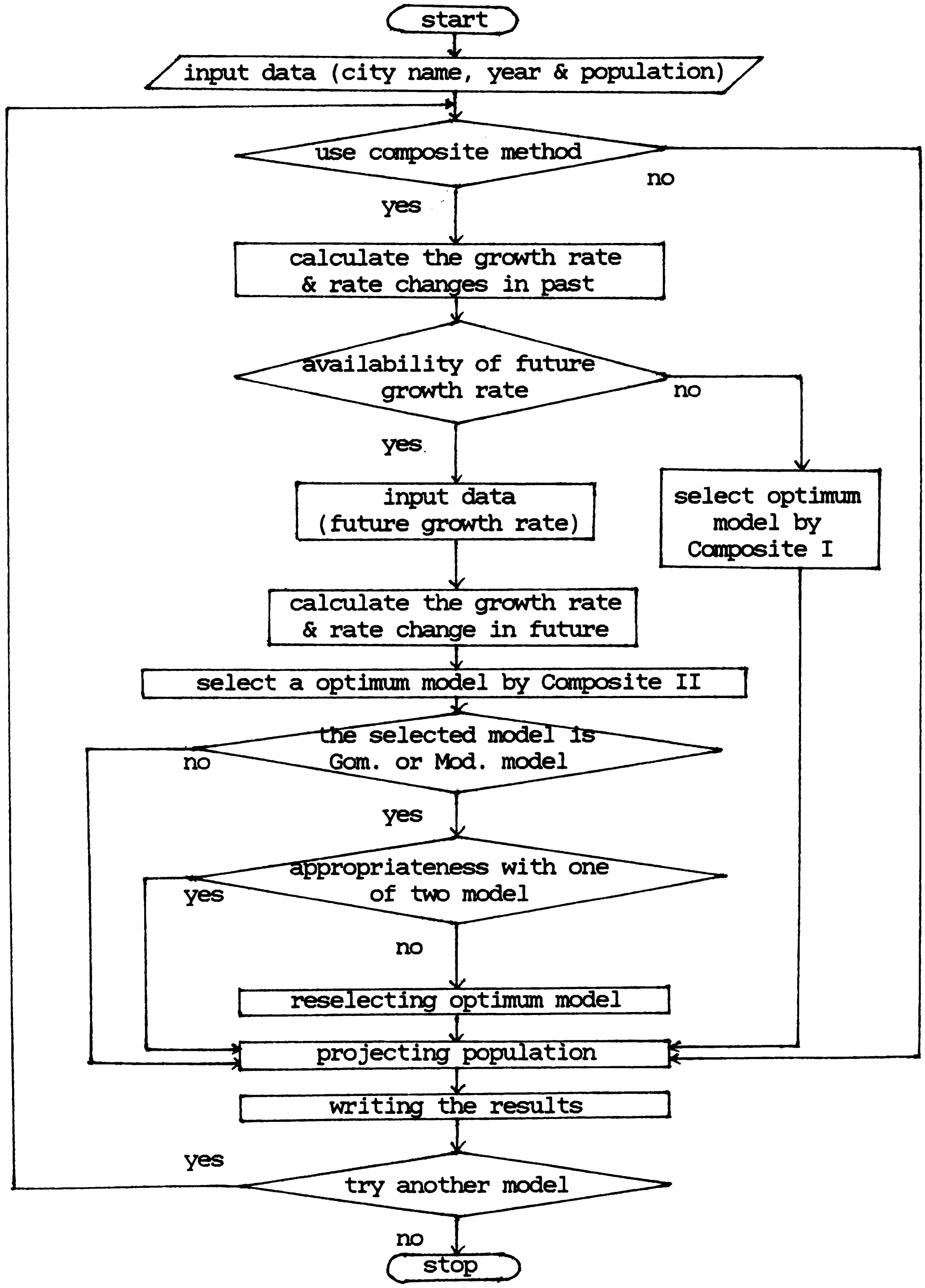
EXTREME DECREASE TYPE

Future Growth Rate Change		Lin	Exp	Dlog	Poly	Gom	Mod
	mean % error	-	-	-	-	-	-
10 +	sd.	-	-	-	-	-	-
	# of cases	0	0	0	0	0	0
	mean % error	18.0	45.8	45.1	14.2	10.8	9.6
-10 to 10	sd.	15.1	30.3	30.1	6.6	8.0	6.6
	# of cases	5	5	5	5	4	4
	mean % error	39.1	145.3	143.0	45.4	15.0	16.7
< -10	sd.	18.5	174.6	170.2	21.3	7.5	9.0
	# of cases	23	23	23	23	21	20

APPENDIX 3

COMPUTER PROGRAM FLOWCHART

APPENDIX 3 : COMPUTER PROGRAM FLOWCHART



APPENDIX 4

COMPUTER PROGRAM WITH BASIC

APPENDIX 4 : COMPUTER PROGRAM WITH BASIC

```
10 REM POPMODEL
20 REM PROJECTING POPULATION WITH THE POPULATION TREND
30 REM EXTRAPOLATION MODELS
40 REM PROGRAMMED BY IKKI KIM, AUGUST 5, 1985
50 REM VARIABLE (ALPHABETICALLY)
60 REM        A() - MATRIX OF PARAMETERS FOR POLYNOMIAL EQUATION
70 REM        A$,A2$,A3$ - USER RESPONSES
80 REM        A,B,NB - COEFFICIENT IN EQUATION
90 REM        AV1,AV2 - AVERAGE
100 REM       C() - CLASSIFYING USER'S CHOICE
110 REM       C$() - CITY NAME
120 REM       CL() - CLASSIFICATION OF CITY TYPE
130 REM       COV - COVARIANCE
140 REM       D - DEGREE OF POLYNOMIAL EQUATION
150 REM       DIF(),DIF2() - DIFFERENCE OF GROWTH RATE
160 REM       E - EXPONENT IN THE GOMPERTZ & THE MODIFIED MODEL
170 REM       F1$,F2$,F3$ - FILE NAME
180 REM       G - NUMBER OF POINTS IN GROUPED PERIOD FOR USING
190 REM           THE SELECTED POINTS TECHNIQUE
200 REM       H,I,J,L - FOR/NEXT VARIABLES
210 REM       K - UPPER LIMIT POPULATION
220 REM       M - NUMBER OF CITIES
230 REM       N - NUMBER OF DATA POINTS
240 REM       P - PROJECTION YEAR
250 REM       P$,P2$ - USER RESPONSES RELATED TO PROJENCTION YEAR
260 REM       Q,R,T - INITIALIZING NUMBER
270 REM       Q1$,Q2$,Q3$ - USER RESPONSES
280 REM       R1(),YR1(),LP - TRANSFORMED POPULATION OR YEAR
290 REM       S - PERIOD SPAN
300 REM       VAR - VARIANCE
310 REM       X(),Y() - MATRIX FOR SOLVING POLYNOMIAL EQUATION
320 REM ********************************************************
330 REM           []==============================[]
340 REM           []        ENTERING DATA         []
350 REM           []==============================[]
360 PRINT "NUMBER OF CITIES AND TOWNS FOR PROJECTION";
370 INPUT M
380 PRINT
390 PRINT "NUMBER OF DATA POINTS";
400 INPUT N
410 PRINT
420 PRINT "DO YOU HAVE A DATA-FILE (Y/N)";
430 INPUT A$
440 PRINT
450 IF A$="N" OR A$="n" THEN 510
460 PRINT "NAME OF INPUT DATA-FILE";
470 INPUT F1$
```

APPENDIX 4 : Continued

```
480 PRINT
490 OPEN "I", #1, F1$
500 INPUT #1, M,N
510 DIM YR(N),POP(M,N),C$(M),CL(M),P1(N),YR1(N),C(M)
520 DIM X(6,6),Y(6),A(6),DIF(M),DIF2(M)
530 IF A$="Y" OR A$="y" THEN 1130
540 REM ***************************************************
550 REM ENTERING DATA DRING RUNNING PROGRAM
560 PRINT "DO YOU WANT TO CREAT A DATA-FILE (Y/N)";
570 INPUT A2$
580 PRINT
590 IF A2$="N" OR A2$="n" THEN 640
600 PRINT "NAME FOR THE DATA-FILE";
610 INPUT F2$ : PRINT
620 OPEN "O",#2,F2$
630 PRINT #2, M,N
640 FOR I=1 TO M
650     PRINT "NAME OF CITY OR TOWN  :    NO.";I;
660     INPUT C$(I)
670     IF A2$="N" OR A2$="n" THEN 690
680     PRINT #2, CHR$(34);C$(I);CHR$(34);
690 NEXT I
700 PRINT : PRINT
710 FOR J=1 TO N
720     PRINT "TIME (YEAR) FOR DATA POINT #";J;
730     INPUT YR(J)
740     IF A2$="N" OR A2$="n" THEN 760
750     PRINT #2, YR(J);
760 NEXT J
770 PRINT : PRINT
780 FOR I=1 TO M
790     PRINT "-----------------------------------------------"
800     PRINT "ENTER POPULATION DATA OF ";C$(I)
810     PRINT "-----------------------------------------------"
820     FOR J=1 TO N
830         PRINT "POPULATION OF ";YR(J);
840         INPUT POP(I,J)
850     NEXT J
860     PRINT "-----------------------------------------------"
870     PRINT:PRINT
880 NEXT I
890 IF M<3 THEN 1040
900 PRINT "DO YOU WISH TO REVIEW THE POPULATION DATA (Y/N)";
910 INPUT A$
920 PRINT
930 IF A$="N" OR A$="n" THEN 1040
940 FOR I=1 TO M
```

APPENDIX 4 : Continued

```
950     PRINT " CITY NAME : ";C$(I)
960     FOR J=1 TO N
970 PRINT "FOR ";YR(J);" --- ";" POPULATION =";POP(I,J);"   OK (Y/N)";
980         INPUT A$ : PRINT
990         IF A$="Y" OR A$="y" THEN 1020
1000        PRINT "CORRECTED POPULATION :   ";
1010        INPUT POP(I,J)
1020      NEXT J
1030 NEXT I
1040 IF A2$="N" OR A2$="n" THEN 1260
1050 FOR I=1 TO M
1060    FOR J=1 TO N
1070        PRINT #2, POP(I,J);
1080    NEXT J
1090 NEXT I
1100 GOTO 1260
1110 REM ************************************************************
1120 REM LOADING DATA FROM DATA-FILE
1130 FOR I=1 TO M
1140      INPUT #1, C$(I)
1150 NEXT I
1160 FOR J=1 TO N
1170      INPUT #1, YR(J)
1180 NEXT J
1190 FOR I=1 TO M
1200      FOR J=1 TO N
1210          INPUT #1, POP(I,J)
1220      NEXT J
1230 NEXT I
1240 REM ************************************************************
1250 REM JUSTIFICATION OF PROJECTING YEAR
1260 S=YR(2)-YR(1)
1270 P=YR(N)+S
1280 PRINT "PROJECTION YEAR IS ";P;"  (Y/N)";
1290 INPUT P$ : PRINT
1300 IF P$="Y" OR P$="y" THEN 1390
1310 PRINT "PROJECTING  POPULATION FOR THE SHORT-TERM IS"
1320 PRINT "RECOMMANDED  SUCH AS FOR NEXT SPAN POINT(YEAR)."
1330 PRINT:PRINT:
1340 PRINT "DO YOU WANT TO PROJECT POP. FOR ";P;" (Y/N)";
1350 INPUT P2$ : PRINT
1360 IF P2$="Y" OR P2$="y" THEN 1390
1370 PRINT "TIME (YEAR) FOR PROJECTING POPULATION";
1380 INPUT P : PRINT
1390 FOR J=2 TO N-1
1400      IF S=YR(J+1)-YR(J) THEN 1420
1410      PRINT "YOU NEED THE CONTINUOUS EQUAL PERIOD SPAN":STOP
```

APPENDIX 4 : Continued

```
1420 NEXT J
1430 REM ******************************************************
1440 REM          []================================[]
1450 REM          []      SELECTING A MOEDL         []
1460 REM          []================================[]
1470 PRINT "DO YOU WANT TO SAVE THE RESULTS (Y/N)";
1480 INPUT Q$ : PRINT
1490 IF Q$="N" OR Q$="n" THEN 1530
1500 PRINT "FILE NAME FOR SAVING THE RESULTS";
1510 INPUT F3$ : PRINT
1520 OPEN "O",#3,F3$
1530 FOR I=1 TO M
1540     IF I=1 THEN 1570
1550     IF A3$="N" OR A3$="n" THEN 1570
1560     CL(I)=CL(I-1) : GOTO 1740
1570     PRINT "-----------------------------------------------"
1580     PRINT " WHICH MODEL DO YOU WANT TO USE FOR PROJECTION ?"
1590     PRINT : PRINT "      1. LINEAR REGRESSION MODEL"
1600     PRINT "      2. EXPONENTIAL REGRESSION MODEL"
1610     PRINT "      3. DOUBLE LOGARITHMIC REGRESSION MODEL"
1620     PRINT "      4. GOMPERTZ CURVE"
1630     PRINT "      5. MODIFIED EXPONENTIAL CURVE"
1640     PRINT "      6. SECOND DEGREE POLYNOMIAL CURVE"
1650     PRINT "      7. THIRD, FORTH OR FIFTH DEGREE POLYNOMIAL"
1660     PRINT "      8. COMPOSITE METHOD (TESTED IN '85, MICH.)"
1670     PRINT "-----------------------------------------------"
1680     PRINT
1690     PRINT "ENTER THE NUMBER OF SPECIFIC METHOD FOR ";C$(I);
1700     INPUT CL(I) : PRINT : PRINT
1710     IF M=1 THEN 1740
1720     PRINT "USE SAME METHOD FOR REST OF OTHER CITIES (Y/N)";
1730     INPUT A3$ : PRINT
1740     IF CL(I)=8 THEN C(I)=1
1750     IF CL(I)<8 THEN C(I)=2
1760 NEXT I
1770 REM **********************************************************
1780 REM SELECTING A MODEL BY PROGRAM
1790 FOR I=1 TO M
1800     IF CL(I)<8 THEN 2230
1810     R1=(POP(I,N)-POP(I,N-1))/POP(I,N-1)*100
1820     R2=(POP(I,N-1)-POP(I,N-2))/POP(I,N-2)*100
1830     DIF(I)=R1-R2
1840     IF I>1 THEN 1880
1850     PRINT "CAN YOU GIVE THE ROUGH % GROWTH RATE FOR SOME OR ALL"
1860     PRINT "OF CITIES FROM ";YR(N);" TO ";YR(N)+S;" (Y/N)";
1870     INPUT Q2$ : PRINT
1880     IF Q2$="N" OR Q2$="n" THEN 2200
```

APPENDIX 4 : Continued

```
1890      PRINT "ROUGH % GROWTH RATE FOR ";C$(I);" (..-10..0..10..)";
1900      INPUT R3 : PRINT
1910      DIF2(I)=R3-R1
1920      IF DIF(I)>10 AND DIF2(I)>10 THEN CL(I)=2
1930      IF DIF(I)>25 AND DIF2(I)>=-10 AND DIF2(I)<=10 THEN CL(I)=3
1940      IF DIF(I)<=25 AND DIF(I)>10 AND DIF2(I)<=10 THEN CL(I)=1
1950      IF DIF(I)>25 AND DIF2(I)<-10 THEN CL(I)=1
1960      IF DIF(I)>0 AND DIF(I)<=10 THEN 1980
1970      GOTO 2000
1980      IF DIF2(I)>=-10 THEN CL(I)=2
1990      IF DIF2(I)<-10 THEN CL(I)=1 : GOTO 2230
2000      IF DIF(I)>=-10 AND DIF(I)<=0 THEN 2020
2010      GOTO 2050
2020      IF DIF2(I)>10 THEN CL(I)=3
2030      IF DIF2(I)>=-10 AND DIF2(I)<=10 THEN CL(I)=4
2040      IF DIF2(I)<-10 THEN CL(I)=5 : GOTO 2230
2050      IF DIF(I)>=-25 AND DIF(I)<-10 THEN 2070
2060      GOTO 2090
2070      IF DIF2(I)>10 THEN CL(I)=2
2080      IF DIF2(I)<=10 THEN CL(I)=5 : GOTO 2230
2090      IF DIF(I)>=-50 AND DIF(I)<=-25 THEN 2110
2100      GOTO 2140
2110      IF DIF2(I)>10 THEN CL(I)=1
2120      IF DIF2(I)<=10 AND DIF2(I)>=-10 THEN CL(I)=4
2130      IF DIF2(I)<-10 THEN CL(I)=5 : GOTO 2230
2140      IF DIF(I)<-50 THEN 2160
2150      GOTO 2230
2160      IF DIF2(I)>10 THEN CL(I)=1
2170      IF DIF2(I)<=10 AND DIF2(I)>=-10 THEN CL(I)=5
2180      IF DIF2(I)<-10 THEN CL(I)=4
2190      GOTO 2230
2200      IF DIF(I)>=0 THEN CL(I)=1
2210      IF DIF(I)<0 AND DIF(I)>=-50 THEN CL(I)=5
2220      IF DIF(I)<-50 THEN CL(I)=4
2230 NEXT I
2240 REM ************************************************************
2250 REM SEND A TYPE OF CITY TO A SPECIFIC MODEL CALCULATION
2260 FOR I=1 TO M
2270      IF CL(I)<=3 THEN 2430
2280      IF CL(I)=4 OR CL(I)=5 THEN 3090
2290      IF CL(I)=6 OR CL(I)=7 THEN 3880
2300 NEXT I
2310 PRINT "------------------------------------------------------------"
2320 PRINT : PRINT
2330 PRINT "PROJECT AGAIN WITH OTHER ALTERNATIVE MODEL (Y/N)";
2340 INPUT Q3$
2350 PRINT
```

APPENDIX 4 : Continued

```
2360 IF Q3$="N" OR Q3$="n" THEN END
2370 GOTO 1530
2380 REM ********************************************************
2390 REM     []==============================================[]
2400 REM     []  LINEAR,EXPONENTIAL AND DOUBLE LOG. MODELS  []
2410 REM     []==============================================[]
2420 REM CALCULATING REGRESSION AND PROJECTING POPULATION
2430 FOR J=1 TO N
2440      IF CL(I)=1 THEN 2490
2450      P1(J)=LOG(POP(I,J))
2460      IF CL(I)=2 THEN 2500
2470      YR1(J)=LOG(YR(J))
2480      GOTO 2510
2490      P1(J)=POP(I,J)
2500      YR1(J)=YR(J)
2510 NEXT J
2520 IF CL(I)=3 THEN 2550
2530 LP=P
2540 GOTO 2560
2550 LP=LOG(P)
2560 T=0
2570 FOR J=1 TO N
2580      T=T+P1(J)
2590 NEXT J
2600 AV1=T/N
2610 R=0
2620 FOR J=1 TO N
2630      R=R+YR1(J)
2640 NEXT J
2650 AV2=R/N
2660 VAR=0 :COV=0
2670 FOR J=1 TO N
2680      VAR=VAR+((YR1(J)-AV2)^2)
2690      COV=COV+(P1(J)-AV1)*(YR1(J)-AV2)
2700 NEXT J
2710 B=COV/VAR
2720 Y=P1(N)+B*(LP-YR1(N))
2730 REM ************************************************
2740 REM PRINTING RESULTS
2750 PRINT
2760 PRINT "------------------------------------------------------"
2770 IF CL(I)=1 THEN 2960
2780 Y=EXP(Y)
2790 IF CL(I)=3 THEN 2880
2800 PRINT "PROJECTED POP. OF ";C$(I);" FOR ";P;" ..... ";Y
2810 PRINT " BASED ON THE EXPONENTIAL MODEL."
2820 PRINT
```

APPENDIX 4 : Continued

```
2830 IF Q$="N" OR Q$="n" THEN 2300
2840 PRINT #3, "PROJECTED POP. OF ";C$(I);" FOR ";P;" ..... ";Y
2850 PRINT #3," BASED ON THE EXPONENTIAL MODEL."
2860 PRINT #3,
2870 GOTO 2300
2880 PRINT "PROJECTED POP. OF ";C$(I);" FOR ";P;" ..... ";Y
2890 PRINT " BASED ON THE DOUBLE LOG. MODEL."
2900 PRINT
2910 IF Q$="N" OR Q$="n" THEN 2300
2920 PRINT #3,"PROJECTED POP. OF ";C$(I);" FOR ";P;" ..... ";Y
2930 PRINT #3," BASED ON THE DOUBLE LOG. MODEL."
2940 PRINT #3,
2950 GOTO 2300
2960 PRINT "PROJECTED POP. OF ";C$(I);" FOR ";P;" ..... ";Y
2970 PRINT " BASED ON THE LINEAR MODEL."
2980 PRINT
2990 IF Q$="N" OR Q$="n" THEN 2300
3000 PRINT #3,"PROJECTED POP. OF ";C$(I);" FOR ";P;" ..... ";Y
3010 PRINT #3," BASED ON THE LINEAR MODEL."
3020 PRINT #3,
3030 GOTO 2300
3040 REM ******************************************************
3050 REM    []========================================[]
3060 REM    []  GOMPERTZ AND MODIFIED EXPONENTIAL MODEL  []
3070 REM    []========================================[]
3080 REM GROUPING POINTS FOR USING THE SELECTED POINT TECHNIQUE
3090 G=N/3
3100 G=INT(G)
3110 N2=(N-G+1):N3=(N-G):N4=(N-G-G+1):N5=(N-G-G):N6=(N-G-G-G+1)
3120 FOR J=1 TO N
3130     IF CL(I)=5 THEN 3160
3140     P1(J)=LOG(POP(I,J))
3150     GOTO 3170
3160     P1(J)=POP(I,J)
3170 NEXT J
3180 T=0:R=0:Q=0
3190 FOR J=N2 TO N
3200     T=T+P1(J)
3210 NEXT J
3220 FOR J=N4 TO N3
3230     R=R+P1(J)
3240 NEXT J
3250 FOR J=N6 TO N5
3260     Q=Q+P1(J)
3270 NEXT J
3280 REM ****************************************************
3290 REM TEST FEASIBILITY FOR USING TWO MODEL
```

APPENDIX 4 : Continued

```
3300 NB=(T-R)/(R-Q)
3310 IF NB<0 THEN 3350
3320 B=EXP(LOG(NB)/G)
3330 A=(R-Q)*(B-1)/((NB-1)^2)
3340 IF NB>0 AND NB<1 AND A<0 THEN 3590
3350 IF C(I)=2 AND CL(I)=5 THEN 3390
3360 IF C(I)=2 AND CL(I)=4 THEN 3440
3370 IF CL(I)=4 THEN 3500
3380 CL(I)=4 : GOTO 3120
3390 PRINT " ********************************************************"
3400 PRINT "    THE PAST POPULATION TREND OF ";C$(I)
3410 PRINT "    CAN NOT BE FITTED TO THE MODIFIED MODEL"
3420 PRINT " ********************************************************"
3430 GOTO 2300
3440 PRINT " ********************************************************"
3450 PRINT "    THE PAST POPULATION TREND OF ";C$(I)
3460 PRINT "    CAN NOT BE FITTED TO THE GOMPERTZ MODEL"
3470 PRINT " ********************************************************"
3480 GOTO 2300
3490 REM RESELECTING OPTIMUM MODEL
3500 IF Q2$="N" OR Q2$="n" THEN 3550
3510 IF DIF(I)>=-10 AND DIF(I)<=0 AND DIF2(I)>=-10 THEN 3540
3520 IF DIF(I)<-50 AND DIF2(I)<-10 THEN 3540
3530 GOTO 3880
3540 CL(I)=1 : GOTO 2430
3550 IF DIF(I)>=-50 AND DIF(I)<-10 THEN 3880
3560 GOTO 3540
3570 REM ****************************************************
3580 REM PROJECTING POPULATION
3590 K=(Q-((NB-1)*A/(B-1)))/G
3600 E=G+G+G+(P-YR(N)-S)/S
3610 Y=K+(A*(B^E))
3620 REM ****************************************************
3630 REM PRINTING RESULTS
3640 PRINT
3650 PRINT "------------------------------------------------------------"
3660 IF CL(I)=4 THEN 3740
3670 PRINT "PROJECTED POP. OF ";C$(I);" FOR ";P;" ..... ";Y
3680 PRINT " BASED ON THE MODIFIED MODEL." : PRINT
3690 IF Q$="N" OR Q$="n" THEN 2300
3700 PRINT #3,"PROJECTED POP. OF ";C$(I);" FOR ";P;" ..... ";Y
3710 PRINT #3," BASED ON THE MODIFIED MODEL."
3720 PRINT #3,
3730 GOTO 2300
3740 Y=EXP(Y)
3750 PRINT "PROJECTED POP. OF ";C$(I);" FOR ";P;" ..... ";Y
3760 PRINT " BASED ON THE GOMPERTZ MODEL."
```

APPENDIX 4 : Continued

```
3770 PRINT
3780 IF Q$="N" OR Q$="n" THEN 2300
3790 PRINT #3,"PROJECTED POP. OF ";C$(I);" FOR ";P;" ..... ";Y
3800 PRINT #3," BASED ON THE GOMPERTZ MODEL."
3810 PRINT #3,
3820 GOTO 2300
3830 REM ******************************************************
3840 REM            []============================[]
3850 REM            []      POLYNOMIAL MODEL      []
3860 REM            []============================[]
3870 REM GERNERATING MATRIX FOR USING THE LEAST SQUARE METHOD
3880 IF CL(I)=6 OR C(I)=1 THEN 3910
3890 PRINT "DEGREE OF POLYNOMIAL FOR ";C$(I);"  (TYPE 3, 4 OR 5)";
3900 INPUT D : GOTO 3920
3910 D=2
3920 FOR H=1 TO D+1
3930     FOR L=1 TO D+1
3940         T=0
3950         IF (H+L)=2 THEN 4010
3960         FOR J=1 TO N
3970             T=T+((J-1)^(H+L-2))
3980         NEXT J
3990         X(H,L)=T
4000         GOTO 4020
4010         X(H,L)=N
4020     NEXT L
4030 NEXT H
4040 FOR H=1 TO D+1
4050     Q=0
4060     FOR J=1 TO N
4070         IF H=1 THEN 4100
4080         Q=Q+POP(I,J)*((J-1)^(H-1))
4090         GOTO 4110
4100         Q=Q+POP(I,J)
4110     NEXT J
4120     Y(H)=Q
4130 NEXT H
4140 REM ******************************************************
4150 REM PROJECTING POPULATION
4160 GOSUB 4350
4170 R=0
4180 FOR H=1 TO D+1
4190     R=R+A(H)*(((P-YR(1))/S)^(H-1))
4200 NEXT H
4210 REM ******************************************************
4220 REM PRINTING RESULTS
4230 PRINT
```

APPENDIX 4 : Continued

```
4240 PRINT "----------------------------------------------------------"
4250 PRINT "PROJECTED POP. OF ";C$(I);" FOR ";P;" ..... ";R
4260 PRINT "BASED ON THE ";D;"-DEGREE POLYNOMIAL MODEL."
4270 PRINT
4280 IF Q$="N" OR Q$="n" THEN 2300
4290 PRINT #3,"PROJECTED POP. OF ";C$(I);" FOR ";P;" ..... ";R
4300 PRINT #3,"BASED ON THE ";D;"-DEGREE POLYNOMIAL MODEL."
4310 PRINT #3,
4320 GOTO 2300
4330 REM *****************************************************
4340 REM MATRIX INVERSION AND MULTIPLACATION
4350 FOR L=1 TO D+1
4360     B=X(L,L)
4370     IF B=0 THEN STOP
4380     X(L,L)=1
4390     FOR J=1 TO D+1
4400         X(L,J)=X(L,J)/B
4410     NEXT J
4420     FOR H=1 TO D+1
4430         IF H=L THEN 4490
4440         B=X(H,L)
4450         X(H,L)=0
4460         FOR J=1 TO D+1
4470             X(H,J)=X(H,J)-X(L,J)*B
4480         NEXT J
4490     NEXT H
4500 NEXT L
4510 FOR H=1 TO D+1
4520     A(H)=0 : REM INITIALIZE EACH ELEMENT OF B
4530     FOR J=1 TO D+1
4540         A(H)=A(H)+X(H,J)*Y(J)
4550     NEXT J
4560 NEXT H
4570 RETURN
```

APPENDIX 5

EXAMPLE OF PROJECTING POPULATION WITH COMPUTER PROGRAM

APPENDIX 5 : EXAMPLE OF PROJECTING POPULATION WITH COMPUTER PROGRAM

```
RUN

NUMBER OF CITIES AND TOWNS FOR PROJECTION?  1

NUMBER OF DATA POINTS?  4

DO YOU HAVE A DATA-FILE  (Y/N)?  N

DO YOU WANT TO CREAT A DATA-FILE  (Y/N)?  N

NAME OF CITY OR TOWN    :      NO. 1  ?  SAGINAW

TIME (YEAR) FOR DATA POINT # 1 ?  1940
TIME (YEAR) FOR DATA POINT # 2 ?  1950
TIME (YEAR) FOR DATA POINT # 3 ?  1960
TIME (YEAR) FOR DATA POINT # 4 ?  1970


---------------------------------------
ENTER POPULATION DATA FOR SAGINAW
---------------------------------------
POPULATION OF  1940  ?  82794
POPULATION OF  1950  ?  92918
POPULATION OF  1960  ?  98265
POPULATION OF  1970  ?  91849
---------------------------------------


YOUR PROJECTION YEAR IS 1980 (Y/N)? Y

DO YOU WANT TO SAVE THE RESULTS (Y/N)? N


-----------------------------------------------------
WHICH MODEL DO YOU WANT TO USE FOR PROJECTION?

      1. LINEAR REGRESSION MODEL
      2. EXPONENTIAL REGRESSION MODEL
      3. DOULE LOGARITHMIC REGRESSION MODEL
      4. GOMPERTZ MODEL
      5. MODIFIED EXPONENTIAL MODEL
      6. SECOND DEGREE POLYNOMIAL MODEL
      7. THIRD, FORTH OR FIFTH DEGREE POLYNOMIAL
      8. COMPOSITE METHOD (TESTED IN '85, MICH.)
-----------------------------------------------------
```

APPENDIX 5 : Continued

```
ENTER THE NUMBER OF SPECIFIC METHOD FOR SAGINAW? 8

CAN YOU GIVE THE ROUGH % GROWTH RATE FOR SOME OR ALL
OF CITIES FROM 1970 TO 1980 (Y/N)?  Y

ROUGH % GROWTH RATE FOR SAGINAW?  -15

---------------------------------------------------------
PROJECTED POP. OF SAGINAW FOR 1980  ........  78909.72
BASED ON THE POLYNOMIAL MODEL
---------------------------------------------------------

PROJECT AGAIN WITH OTHER ALTERNATIVE MODEL (Y/N)? N

OK
```

APPENDIX 6

POPULATION DATA

APPENDIX 6 : POPULATION DATA

CITY	1940	1950	1960	1970	1980
Adrian	14230	18393	20347	20382	21186
Albion	8345	10406	12749	12112	11059
Algona	1931	2639	3190	3684	4412
Allega	4526	4801	4822	4516	4576
AllenP	3487	12329	37494	40747	34196
Alma	7202	8341	8978	9611	9652
Alpena	12808	13135	14682	13805	12214
AnnArb	29815	48251	67340	99797	107966
BadAxe	2624	2973	2998	2999	3184
Battle	43453	48666	44168	38931	35724
BayCit	47956	52523	53604	49449	41593
Beldin	4089	4436	4887	5121	5634
Benton	16668	18769	19136	16481	14707
Berkle	6406	17931	23275	21879	18637
Bessem	4080	3509	3304	2805	2553
BigRap	4987	6736	8686	11995	14361
Birmin	11196	15467	25525	26170	21689
Blissf	2144	2365	2653	2753	3107
Boyne	2904	3028	2797	2967	3348
Buchan	4056	5224	5341	4643	5142
Cadill	9855	10425	10112	9990	10199
Caro	3070	3464	3534	3701	4317
Center	3198	7659	10164	10379	9293
Charle	2299	2695	2751	3519	3296
Charlo	5544	6606	7657	8244	8251
Cheboy	5673	5687	5859	5553	5106
Chelse	2246	2580	3355	3858	3816
Chesan	1807	2264	2770	2876	2656
Clare	1844	2440	2442	2639	3300
Clawso	4006	5196	14795	17617	15103
Coldwa	7343	8594	8880	9155	9461
Corunn	2017	2358	2704	2829	3206
Daviso	1397	1745	3761	5259	6087
Dearbo	63587	94994	112007	104199	90660
Dowagi	5007	6542	7208	6583	6307

APPENDIX 6 : Continued

CITY	1940	1950	1960	1970	1980
Durand	3127	3194	3312	3678	4241
EDetro	8584	21461	45756	45756	38280
EGrand	4899	6403	10924	12565	10914
ELansi	5839	20325	30198	47540	51392
EatonR	3060	3509	4052	4494	4510
Ecorse	13209	17948	17328	17515	14447
Escana	14830	15170	15391	15368	14355
Essexv	2390	3167	4590	4990	4378
Farmin	1510	2325	6881	10329	11022
Fenton	3377	4226	6142	8284	8098
Fernda	22523	29675	31347	30850	26227
FlatRo	1467	1931	4696	5643	6853
Flint	151543	163143	196940	193317	159611
Flushi	1806	2226	3761	7190	8624
Franke	1100	1208	1728	2834	3753
Fraser	747	1379	7027	11868	14560
Fremon	2520	3056	3384	3465	3672
Garden	4096	9012	38017	41864	35640
Gaylor	2055	2271	2568	3012	3011
Gd.Blan	1012	998	1565	5132	6848
Gladst	4972	4831	5267	5237	4533
GrandH	8799	9536	11066	11844	11763
GrandL	3899	4506	5165	6032	6920
Gr.Po.Sh	801	1032	2301	3042	3122
GrandR	164292	176515	177313	197649	181843
Grandv	1566	2022	7975	10764	12412
Greenv	5321	6668	7440	7493	8019
Grosse	6197	6283	6631	6637	5901
GPFar	7217	9410	12172	11701	10551
GPPark	12646	13075	15457	15641	13639
GPWood	2805	10381	18580	21878	18886
Hamtra	49839	43355	34147	27245	21300
Hancoc	5554	5223	5022	4820	5122
Hartford	1694	1838	2305	2508	2493
Hastin	5175	6096	6375	6501	6418

APPENDIX 6 : Continued

CITY	1940	1950	1960	1970	1980
Highla	50810	46393	38063	35444	27909
Hillsd	6381	7297	7629	7728	7432
Hollan	14614	15858	24777	26479	26281
Holly	2343	2663	3269	4355	4874
Hought	3693	3829	3393	6067	7512
Howell	3748	4353	4861	5224	6976
Hudson	2426	2773	2546	2618	2545
Hudsvi	837	1101	2649	3523	4844
Huntin	1705	4949	8746	8536	6937
Inkste	7044	16728	39097	38595	35190
Ionia	6393	6412	6754	6361	5920
IronMo	11080	9679	9299	8707	8341
Ironwo	13369	11406	10265	8711	7741
Ishpem	9491	8962	8857	8245	7538
Ithaca	2000	2377	2611	2749	2950
Jackso	49656	51088	50720	45484	39739
Kalama	54097	57704	82089	85555	79722
Kingsf	5771	5038	5084	5276	5290
LakeOr	1933	2385	2698	2921	2907
L'Anse	2564	2376	2397	2538	2500
Lansin	78753	92129	107807	131403	130414
Lapeer	5365	6143	6160	6314	6198
Lauriu	3929	3211	3058	2868	2678
Lineol	15236	29310	53933	52984	45105
Livoni	8728	17534	66702	110109	104814
Lowell	1944	2191	2545	3068	3707
Luding	8701	9506	9421	9021	8937
Manist	8694	8642	8324	7723	7566
Maniqu	5399	5086	4875	4324	3962
Marine	3633	4270	4404	4567	4414
Marque	15928	17202	19824	21967	23288
Marsha	5253	5777	6736	7253	7201
Marysv	1777	2534	4065	5610	7345
Mason	2867	3514	4522	5468	6019
Melvin	4764	9483	13089	13862	12322

APPENDIX 6 : Continued

CITY	1940	1950	1960	1970	1980
Menomi	10230	11151	11289	10748	10099
Midlan	10329	14285	27779	35176	37250
Milan	2340	2768	3616	3997	4182
Milfor	1637	1924	4323	4699	5041
Monroe	18478	21467	22968	23894	23531
MoClem	14389	17027	21016	20476	18806
MoMorr	2237	2890	3484	3778	3246
MoPlea	8413	11393	14875	20524	23746
Munisi	4409	4339	4228	3677	3083
Muskeg	47697	48429	46485	44631	40823
MuskHe	16047	18828	19552	17304	14611
Negaun	6813	6472	6126	5248	5189
NewBal	1434	2043	3159	4132	5439
New Buff	1190	1565	2128	2784	2821
Niles	11328	13145	13842	12988	13115
NorthM	1694	2424	3855	4243	4024
Northv	3032	3240	3967	5400	5698
Norway	3728	3258	3171	3033	2919
OakPar	1169	5267	36632	36762	31537
Otsego	3428	3990	4140	3957	3802
Owosso	14424	15948	17006	17179	16455
Oxford	2144	2305	2357	2536	2746
PawPaw	1910	2382	2970	3160	3211
Petosk	6019	6468	6138	6342	6097
Plainw	2424	2767	3125	3195	3751
Pleasa	3391	3594	3807	3989	3217
Plymou	5360	6637	8766	11758	9986
Pontia	66626	73681	82233	85279	76715
PoartH	32759	35725	36084	35794	33981
Portla	2247	2807	3330	3817	3963
Richmo	1722	2025	2667	3234	3536
RivRou	17008	20549	18147	15947	12912
Riverv	804	1432	7237	11342	14569
Roches	3759	4279	5431	7054	7203
Rockwood	1147	1044	2026	3225	3346

APPENDIX 6 : Continued

CITY	1940	1950	1960	1970	1980
Rogers	3072	3873	4722	4275	3923
Romeo	2627	2985	3327	4012	3509
Rosevi	9023	15816	50195	60529	54311
RoyalO	25087	46898	80612	86238	70893
Sagina	82794	92918	98265	91849	77508
StClai	3471	4098	4538	4770	4780
StClSh	10405	19823	76657	88093	76210
StIgna	2669	2946	3334	2892	2632
StJohn	4422	4954	5629	6672	7376
StJose	8963	10223	11755	11042	9622
StLoui	3039	3347	3808	4101	4107
Saline	1227	1533	2334	4811	6483
SaultS	15847	17912	18722	15136	14448
SouthH	4745	5629	6149	6471	5943
S. Lyon	1017	1312	1753	2675	5214
Sparta	1945	2327	2749	3094	3373
Spring L.	1329	1824	2063	3034	2731
Sturgi	7214	7786	8915	9295	9468
Tecums	2921	4020	7045	7120	7320
ThreeR	6710	6785	7092	7355	7015
Traver	14455	16974	18432	18048	15516
Trento	5284	6222	18439	24127	22762
Utica	1022	1196	1454	3504	5282
Vassar	2154	2530	2680	2802	2727
Wakefi	3591	3344	3231	2757	2591
Warren	582	727	89246	179260	161134
Wayne	4223	9409	16034	21054	21159
Whiteh	1407	1819	2590	3017	2856
Willians	1704	2051	2214	2600	2981
Wyando	30618	36846	43519	41061	34006
Ypsila	12121	18302	20957	29538	24031
Zeelan	3007	3075	3702	4734	4764

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