THE MEASUREMENT OF THE SURFACE STRAIN IN GLACIERS USING EMBEDDED RESISTANCE STRAIN GAGES

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ABSTRACT

THE MEASUREMENT OF SURFACE STRAIN IN GLACIERS
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By

Gordon G. Warner

Geologists, physicists and engineers are continuing to seek an understanding of the mechanism of deformation and flow of glaciers. A necessary component of such basic research is the accurate determination of strain rates in stressed polycrystalline glacier ice. The purpose of this investigation was to develop a method of measuring the strain in glacier ice which is dependable and precise under difficult field conditions.

The measuring technique developed employed embedded electrical resistance strain gages consisting of single strands of five mil Constantan wire ten feet long. Six strain gages are embedded in the surface ice of a glacier in the form of two delta rosettes such that the strain rate at a point could be determined with some redundancy of data in this two dimensional problem. The rigid body rotation of the anchor posts to which the gages were attached was measured by sensitive inclinometers in order to assess the influence of pressure melting on the strain data. Instrumentation consisted of a switching unit and a conventional strain indicator. The data from a series of field studies

are interpreted using the least-squares technique to curve fit the strain-time curves. The slopes of these curves are then fitted to the strain transformation equations to yield the maxiumum shear strain rates and the average normal strain rates.

The first series of strain readings was obtained over a period of ten days on the Ptarmigan Glacier, a maritime-temperate ice mass of the Juneau Icefield near Juneau, Alaska. The maximum shear strain rate at the surface ranged from 0.25 to 1.2 microstrain per hour, which agrees with estimates derived from known flow rates of this glacier. The Ptarmigan Glacier had a snow-cover which served as a protective insulation for the gages. The gages were found to adhere to the ice well enough to render anchor posts unnecessary. Pressure melting is therefore considered to be insignificant. The strain gages as transducers were found to have a tolerance of ±6.0 microstrain.

Another series of strain readings was obtained over a period of five days in late summer on the Cathedral Glacier, a continental sub-temperate to sub-polar glacier near Atlin, British Columbia. The Cathedral Glacier had no snow cover so that the anchor posts were required. After five days ablation had loosened the anchor posts sufficiently to prohibit further study. It was found that the strain gages in this application had a tolerance of ±60.0 microstrain. A protective covering, such as snow, is essential for accurate determination of strain rates by this method.

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ACKNOWLEDGMENTS

The author feels fortunate to have had the opportunity to apply present engineering techniques to a unique problem in glaciology. In solving this problem in Glacier Mechanics, the author has had the opportunity to investigate or consider the effects of climate and solar radiation, celestial mechanics, basic properties of ice, and the general theory of glacier flow. The study has proved to be personally broadening and extremely interesting.

As the study is interdisciplinary in nature, requiring extensive field work, special instrumentation, and much hard physical labor involving the help of many persons, it would be impossible to give credit to each individual who had a part in its successful completion.

Dr. Gary L. Cloud of the Department of Metallurgy, Mechanics and Material Science, Michigan State University, has been very helpful, patient, and understanding as the major advisor on the project. His enthusiasm and insight into the problem were extremely helpful. Not enough can be said about the contribution made by Dr. Maynard M. Miller of the Geology Department, Michigan State University. Without his efforts, the project could never have been started. It is he who is responsible for arranging the financial support, the field research facilities, the moral support, the glaciological background, and the desire to see the project completed. Joan Miller deserves recognition for coordinating the logistics support including the essential shipment of 100 pounds of dry ice from Seattle to the Juneau Icefield.

The National Science Foundation in its support of the Glaciological Institute and the Army Research Office-Durham via its contract with the Foundation for Glacier and Environmental Research provided the basic financial support. Members of the Foundation for Glacier and Environmental Research and the Glaciological Institute spent many hours of assistance under extreme conditions on the Juneau Icefield, Juneau, Alaska. Their efforts can be appreciated only by one who was there to observe and by me to whom they gave such unstinting help.

General Motors Institute, including many of the faculty and staff, provided useful suggestions, materials, equipment and facilities in preparation for the field research and subsequently in the data analysis. The author's wife and children tolerated the author's absence during the two summers of field research and many weeks of planning and data evaluation. Their patience and sacrifice is especially appreciated.

The amount of technical and personal knowledge gained during the pursuit of this project can never be measured, but the learning experience and professional growth will last the author a life-time.

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INTRODUCTION

A detailed understanding of the fundamental flow mechanism of glacier ice remains today a most significant and still only partially answered question in glaciology and ice physics. Serious studies of glaciers were begun approximately two hundred years ago, but for the most part, remained descriptive until the late 1940's. Because of the effects of glaciers on their environment, the Germans, French, and Swiss were among the first to develop a keen interest in glacier flow mechanics (Bader, 1949; Haefeli, 1948; Embleton and King, 1968). However, some useful early work was conducted by British and American scientists (Demorest, 1943; and Sharp, 1954). The introduction of a rheological approach and the application of sophisticated scientific techniques and computational methods over the past three decades have made possible a major thrust in the study of glaciology (Orowan, 1949; Haefeli, 1952; and Paterson, 1969).

Previous Studies

Presently geologists, glaciologists, physicists, geophysicists, and mechanicians are studying the properties of glacier ice and the forces at work on glaciers as well as the response of the glaciers in the form of flow dynamics and mass balance. The British physicists, J. F. Nye (1951, 1953) and J. W. Glenn (1952), have developed a partial set of constitutive equations for ice. S. Steinmann (1954) paralleled some of this research as it relates to flow-recrystallization. M. F. Perutz (1950), R. P. Sharp (1953), and M. M. Miller (1958) have made studies of englacial temperatures and deformations using embedded steel and aluminum pipes in deep bore-holes. M. M. Miller (1954) has allied the formation studies to field measurements of thermal anomalies in Alaskan glaciers in the area of this study. B. Kamb and E. LaChapelle (1964) and others have made direct observations of basal sliding by tunneling into active glaciers. J. G. McCall (1952), M. F. Meier (1960) and R. P. Sharp (1960) have attempted to describe the general flow mechanisms in survey articles based on extensive field observations. Many articles in the Journal of Glaciology and the books by P. A. Shumskiy (1964) and W.S.B. Paterson (1969) have summarized much of the progress to date.

J. W. Glen (1955) and especially J. F. Nye (1952, 1957, 1959, 1965) have been particularly active in building a mathematical theory of glacier flow. Nye's work has been largely based on the application of continuum mechanics to ice flow. Eventually, it is hoped, there will result a more detailed theoretical model of glacier flow which more fully explains the behavior of these complex systems as they are observed in nature.

Requirements in Field Measurements

One important difficulty in the larger problem has been an inability to make precise experimental measurements of the strain rates in existing glaciers. Such measurements are needed to guide the thinking of theoreticians as well as to check the predictions of their models. Strain measurement techniques on elastic bodies using a Lagrangian (material) coordinate system is a well developed science and the evaluation of strain rates in Newtonian fluids using Eulerian (spatial) coordinates is also well-known to mechanicians. Glacier ice, being a visco-elastic substance when thermally temperate and elasto-plastic when thermally polar, is complex in nature (Pounder, 1965) and field measurements of strain rate have been frustrating. Ideally, an Eulerian system should be used since the rate of strain is both a function of time and position. However, the most feasible techniques for measuring strain rates utilize the Lagrangian system. This presents a direct conflict in concept. Given the fact that glacier flow rates on temperate Alaskan glaciers have been measured in excess of one meter per day (Miller, 1958), the motion of the gage site over a long period of time is too great to be ignored. The method developed in this investigation employs the Lagrangian system. However, the time duration of the studies was short and the absolute displacement of the gage site was "small." As extended studies are conducted, several gage sites will be required. The positions of the sites will have to be tracked in such a manner that the strain rate as a result of position may be incorporated into the analysis.

The classical method of surface strain measurement has involved setting flagged stakes in the ice and determining the strain history through periodic measurement of the distance between pairs of stakes using theodolite, tape, and motion parallux photogrammetric- techniques (Paterson, 1969). When using the taping method (Wu and Christensen, 1964), the stakes are ordinarily set up in a diamond configuration with each gage length recommended to be approximately equal to the ice thickness, i.e., in a valley glacier the thickness is typically 200 - 1200 feet thick (60 - 360 m.). A surveyor's tape of temperature compensated invar is usually used for distance measurements. This method provides greater accuracy than theodolite surveying techniques. The strain is obtained by computing the change of distance divided by the original distance.

While this method is simple and straight-forward, its precision is greatly impaired by several factors. Problems result from (1) the low sensitivity of the tape; (2) large gage lengths and the long periods of time required to obtain deformations great enough to read; (3) long gage lengths which often cross crevasses producing extraneous data; (4) the long gage lengths ignoring the possibilities of a strain gradient in the region of the study; and (5) the exclusion of short term phenomena such as diurnal variations from the overall strain rate measurements. Variations of technique and changes in ambient conditions which may occur over several weeks or months of reading further decrease precision. Since physical access to the stakes is necessary, a less than ideal location is often required. Ablation will also cause the stakes to loosen and tilt. Consideration of all these factors leads to the conclusion that the taping method can, at best, yield only approximate average values for glacier strain rate.

In order to establish accurately the surface strain rates in glaciers and to assess high-frequency phenomena such as diurnal variations, the apparatus must have a sensitivity of approximately 10 microstrain. Since strain at a point is desired, gage lengths must be as small as possible - certainly less than 30 feet (9 m.). The system has to be self contained and as simple as possible. Ease of installation with minimum disturbance of the ice is critical. Any sensing device must be like a spy, measuring the event without letting the system know it is being measured. This is true of any and all measuring systems.

Environmental Considerations

The environmental conditions in glacierized regions are major factors to be considered. The weather is often rainy and windy with poor visibility. The logistics of glacier research require that equipment be light in weight, portable, and rugged. Electrical equipment usually has to be battery powered, but ambient conditions are usually such that battery life is short. Solar radiation even on cloudy days is intense and soon melts the ice immediately adjacent to objects embedded in the surface. Thus, posts and stakes tend to become loose in a short time unless protected by some sort of cover. During the summer season, the ice near a glacier surface is saturated with water. The surface of temperate glaciers is also at the melting temperature; hence, the ice is very sensitive to regelation (pressure melting). Areas of interest are often heavily crevassed, and because the ice is relieved of stress near a crevasse, the presence of such fissures places limits on site location and gage length. Also, much of a glacier is covered with snow or firn which is granular, wet, soft, and of no use in deformation research except to limit access to the ice.

Given the conditions discussed above, embedded wire resistance strain gages were chosen as the best transducer. The technology of resistance strain measurement is well developed in other fields and it seemed feasible that this technology could be applied to glacier strain mechanics. Constantan wire, normally used in resistance strain gage applications, can be obtained in many sizes and lengths giving versatility in the choice of system configuration. Constantan wire also has a constant gage factor in both the elastic and plastic range allowing accurate data acquisition over relatively large ranges of strain. If small diameter wire were used, very small forces would be exerted against the anchor system. The instrumentation associated with resistance strain gages is also simple and accurate. Temperature compensation can easily be achieved in the basic Wheatstone bridge circuit.

In this study, the technique developed for measuring glacier surface strain rates using embedded resistance strain gages is described. A discussion of representative data obtained from use of this method on the Ptarmigan and Cathedral Glaciers in the Juneau Icefield is presented. Special consideration is given to the application of the technique in additional research being planned.

INITIAL INVESTIGATION

Research Location

The experimental portion of the project was planned for two summer field seasons in the Juneau Icefield (Figure 1) located in the Northern Boundary Range of southeastern Alaska and northern British Columbia. The coastal section of this region is often referred to as the "Alaskan Panhandle." Of the several icefields along Alaska's rugged coast, this one is the fourth largest, and embraces the glacierized region to the north and east of the capital city of Juneau. The Foundation for Glacier and Environmental Research, headquartered at the Pacific Science Center in Seattle, Washington, has a number of permanent field camps in the Juneau Icefield. Through a cooperative arrangement with Michigan State University, these facilities are available for the use of teaching and research programs of the Glaciological and Arctic Sciences Institute. The camps encompass an area of 5,000 square miles and are within ± 25 miles either side of a transverse line between Juneau, Alaska and Atlin, British Columbia. The research camps are thus located in a transection region of the icefield and include glaciers in both the U.S.A. and Canada.

The objectives of the first season were: (1) gather general background and first-hand information about the glaciology of the Juneau Icefield; (2) become acquainted with the conditions under which the

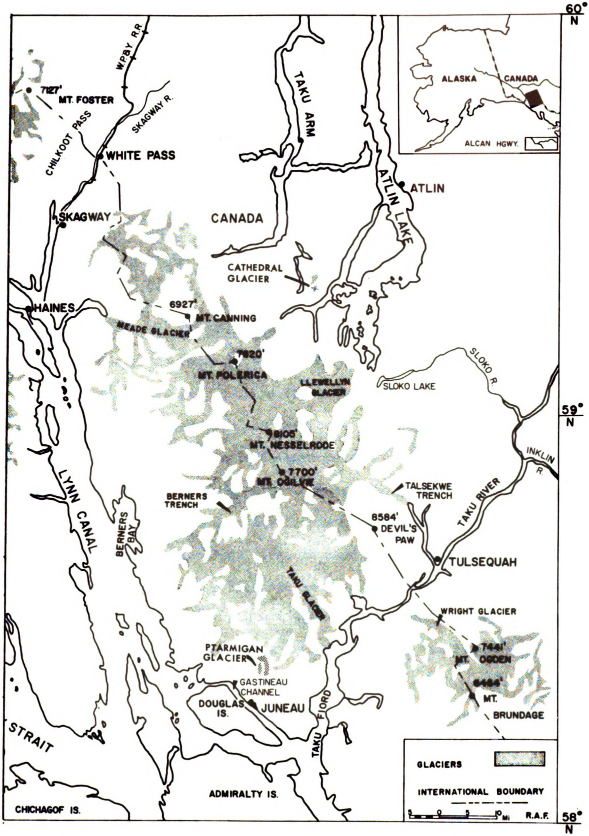

Figure 1 - Northern Boundary Range, Alaska - Canada

anticipated field research must be conducted; and (3) determine the feasibility of using resistance strain gages. The second field season was to be used for testing a prototype system and, if possible, obtain some field data of interpretive significance to glaciologists.

It was immediately clear that a practical strain measuring system in a field application on glaciers had to be as simple as possible. Therefore, recourse was made to conventional strain indicators, which are rugged, reliable, and portable. The only probable causes for failure in a conventional strain measuring system are tubes and batteries. To rectify this, extra tubes and batteries can be carried.

Instrumentation and Installation Procedures

Constantan wire five mil. (0.127 mm.) in diameter and ten feet (3.0 m.) in length has a resistance of 120 ohms, which is the standard impedance for wire and foil strain gages. The wire used in this research was Teflon coated thermocouple wire manufactured by Omega Engineering, Stamford, Connecticut. Since the gages were to be embedded in the glacier surface, long lead wires were required. Standard 20 gage instrumentation lead wire was used to minimize the effect of lead resistance changes. Each strain gage had an independent set of lead wires 15 feet long. A BLH Model M strain indicator was modified with an Amphenol connector and a rotary switching device for gage selection.

Two delta rosette gage configurations were prewired with 3 x 5 in. plexiglass anchor plates at the ends of each gage. Extraneous data were anticipated; so, six gages were used to provide cross-correlation.

Three glaciers on the Juneau Icefield were observed as possible sites for the initial investigation. The Taku Glacier in the Camp 10 region was the first observed, but it was found to have an excessively deep snow and firn cover. The Gilkey Glacier near Camp 18 was ruled out because of being so heavily crevassed in the more accessible regions. The Ptarmigan Glacier (Figure 2) seemed the most suitable choice because of its configuration and its accessibility to the well-equipped field station at Camp 17. The Ptarmigan Glacier is a small northerly flowing glacier about 2.5 miles (4.0 km) in length, and 0.5 miles (800 m.) in width. The maximum ice depth is approximately 400 feet (125 m.). The ice at the head wall at an elevation above sea level of 4,500 feet (1370 m.) and the terminus rests at 2,500 feet (760 m.) giving an average surface slope of nine degrees; however, the glacier does have a step-like character of the slope. The glacier has a light to moderate firn pack in midsummer, and it is relatively free of crevasses.

The strain gage site was located 0.5 mile (800 m.) west of Camp 17 at approximately 3,500 feet elevation in a sector where the surface is slightly concave and presumably in a state of compressive flow. The surface slope at this site is six degrees. The midsummer firn pack at the site was initially two feet (0.6 m.) deep. The site was close to the transient snow-line and some distance below the 1971 seasonal névé line. Because the author lacked field experience in glaciology, his original method of gage installation was completely impossible and is not worthy of further discussion. This method consisted of trying to resistance melt the strain gage wires into the ice and anchor them with 5 x 7 inch plastic plates.

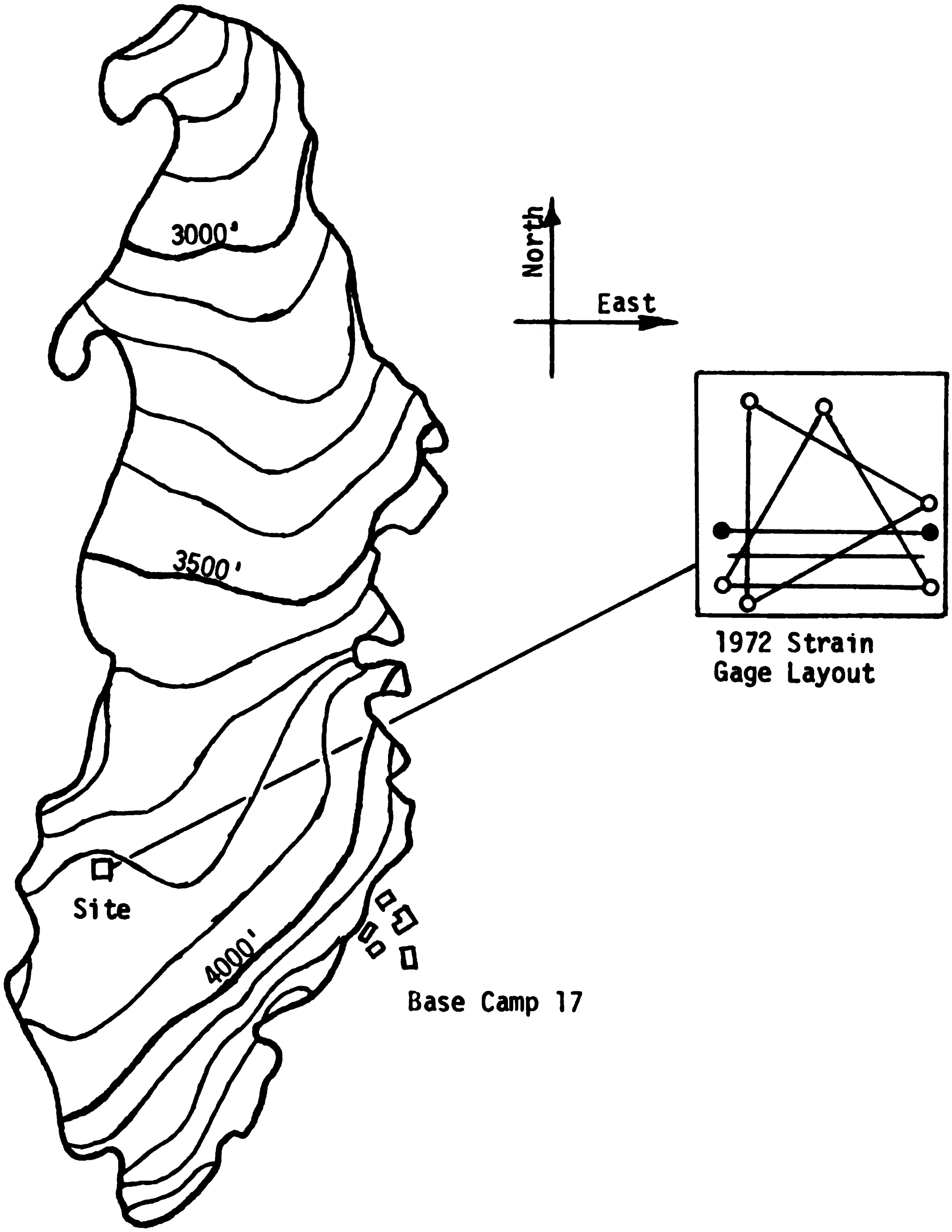

Figure 2
Map of Ptarmigan Glacier
and Site Location

An expedient method was devised on the ice field. Two equilateral triangles, 125 inches (320 cm.) per side, as shown in Figure 3, were laid out on the firn. One triangle had its base on an east-west line and the other triangle had its base in the north-south direction. The gages were thus oriented at 0, 30, 60, 90, 120 and 150 degrees with respect to east, thus providing strain readings in six independent directions. Slots two inches (5 cm.) wide were dug in the firn along the sides of the triangles. A partially threaded, aluminum anchor pole, one inch (2.5 cm.) in diameter and ten feet (3 m.) in length, was screwed into the bubbley glacier ice at the vertex of each triangle until only four inches (10 cm.) were exposed. The strain gage lead wires were tied to the anchor poles with a tensile prestrain ranging from 2000 to 4000 microstrain units. The prestrain was placed on the gages to allow for a compressive strain rate. It was assumed that the Teflon coated strain wire would not bond or adhere to the ice. A tensile prestrain of 1500 microstrain was a desirable maximum because the yield strain was 2000 microstrain. Obtaining an accurate prestrain was impossible because of the crude method of fastening. A sealed compensating gage was placed on the ice near one of the poles and firn was carefully placed around the gage wires filling the slots. Firn was then shoveled over the system to return the depth to two feet. Figures 4 through 6 illustrate the installation.

Strain data were taken twice per day for four days. The Model M strain indicator weighs 27 pounds. As this was a sizable load to carry one-half mile to the site for each set of measurements, the indicator was left at the site, protected inside a plastic bag. Three people were stationed at Camp 17 with several research projects to be

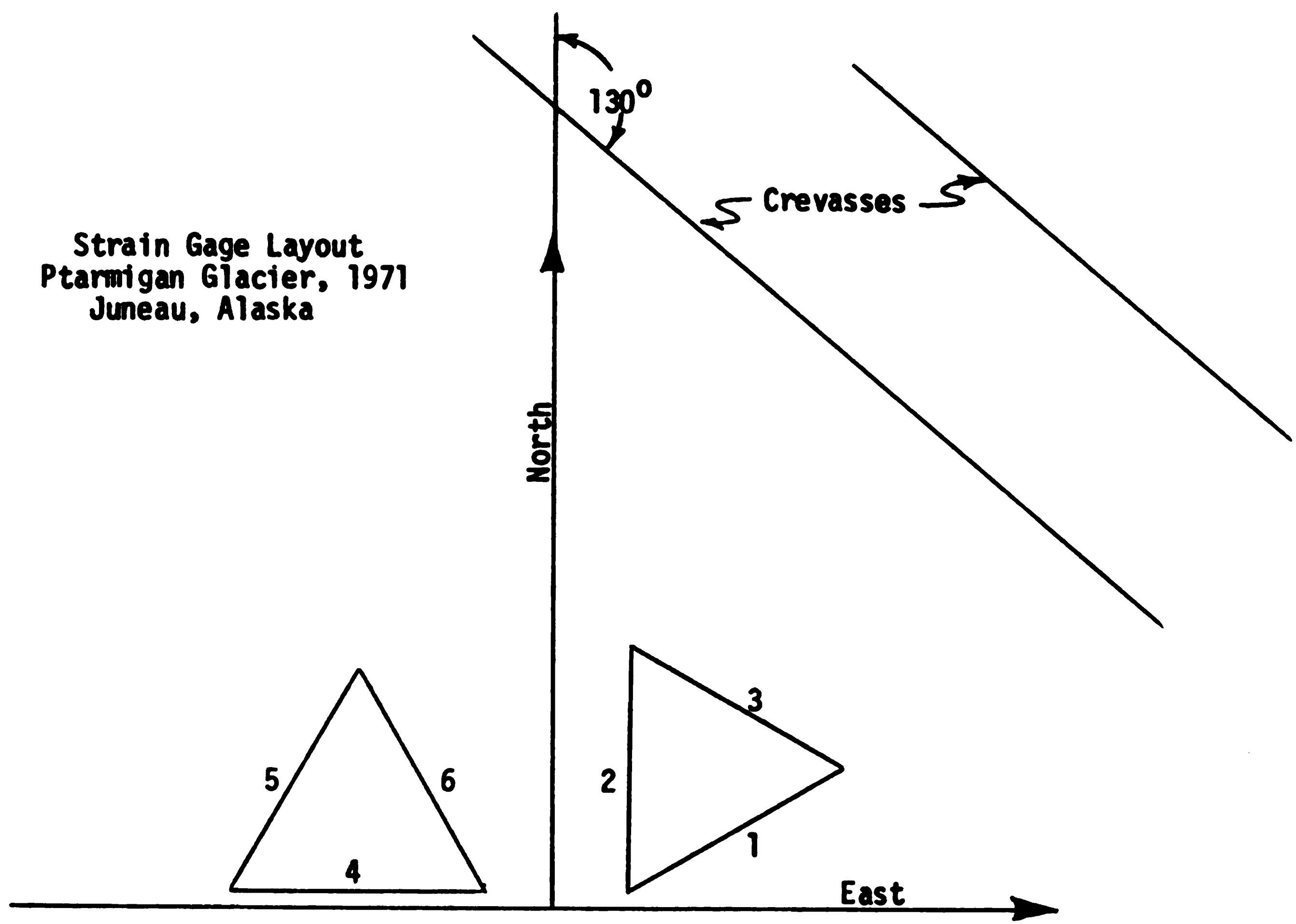

Figure 3 - Strain Gage Layout - 1971

Figure 4 - Anchor Pole Installation
August 1971

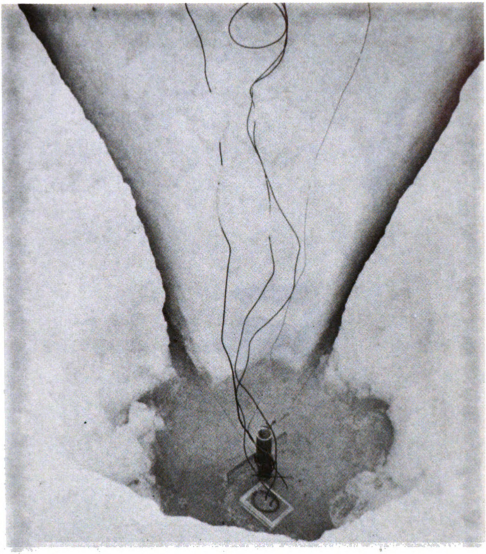

Figure 5 - Attachment of Strain Gages to Poles
August 1971

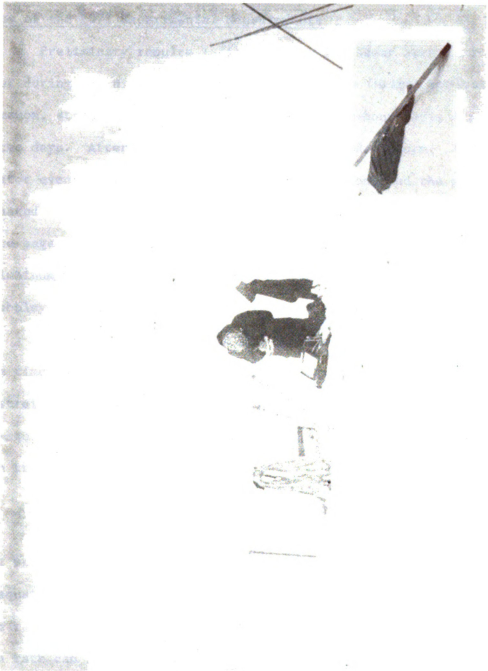

Figure 6 - Completed Installation
August 1971

maintained; thus, measurements were not collected at uniform time intervals. Firn was periodically shoveled onto the system to maintain the insulating cover.

Results of the 1971 Experimental Measurements

Preliminary results indicated that the shear strain rate was greater during the night than during the day. To further evaluate this phenomenon, strain data were collected on a three-hour basis for the next two days. After 27 hours, there was a four-day storm. The strain indicator eventually got wet and ceased to function, and the project was terminated. After the storm had passed, the firn cover had melted away and the gage wires were exposed as shown in Figure 7. The ice around the aluminum poles had ablated to a depth of two feet (0.6 m.) below the bubbley ice surface.

Strain rates were determined by dividing the strain differences by the time interval between data sets. The average strain rate in microstrain units per hour for each interval was obtained. Using a least-squares-best-fit computer program, the data were fitted to the strain transformation equation:

$$\varepsilon_i = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta_i + \frac{\gamma_{xy}}{2} \sin 2\theta_i$$

Mohr's strain circle can be derived directly from the strain transformation equations. The average normal strain rate and the maximum shear strain rate for each time interval was calculated. The average normal strain rate can be visualized as the rate of shift of the center of Mohr's circle while the maximum shear strain rate can be thought of as the rate of change of the diameter of Mohr's circle. Extraneous data were eliminated with the 85 percent confidence level (1.5 times the standard

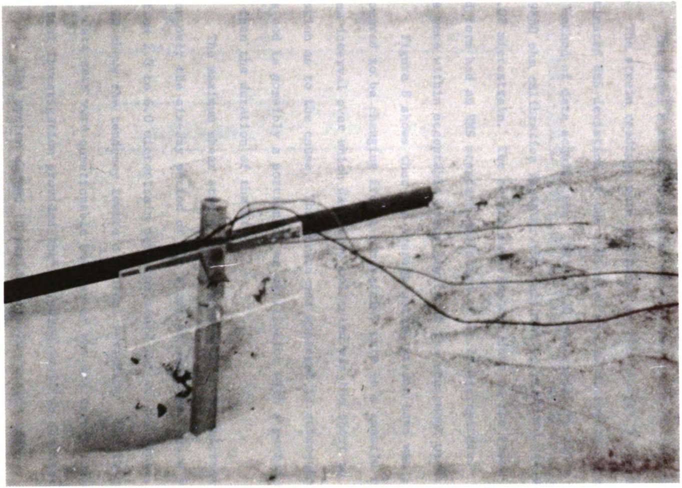

Figure 7 - Exposed System After Completion of Study
August 1971

deviation) used as the criteria. Gage five at 60 degrees from east was found to be out of limits for 60 percent of the time intervals. The final results were calculated by omitting gage five.

The Model M strain indicator has a least count of 10 microstrain. The strain readings were estimated to the nearest 5 microstrain. The estimated RMS deviation of the indicator was 10 microstrain based on the method of data acquisition. Using 120 ohm resistors and a precision 59880 ohm calibration resistor the calibrating standard deviation was 10.08 microstrain. The actual experimental data in the final reduced form had an RMS standard deviation of 11.72 microstrain. The results were within acceptable error limits of the equipment used.

Figure 8 shows that the average normal strain rate was linear and appeared to be changing from a compressive rate to a tensile rate. The time interval over which data were collected was too short to allow speculation as to the cause. This trend was apparent in subsequent studies and is possibly a portion of an oscillation with a period much longer than the duration of the study.

The maximum shear strain rate averaged about 3.5 microstrain per hour over the six-day period of the study as shown in Figure 9. The range from 2.0 to 6.0 microstrain per hour could not be explained. Figure 10 shows the tendency toward a diurnal variation. The last several data sets were questionable because of the wet strain indicator. The initial investigation provided the following positive results:

1. The system seemed to yield reliable results and was worth further development.
2. Lightweight, rugged instrumentation was essential.

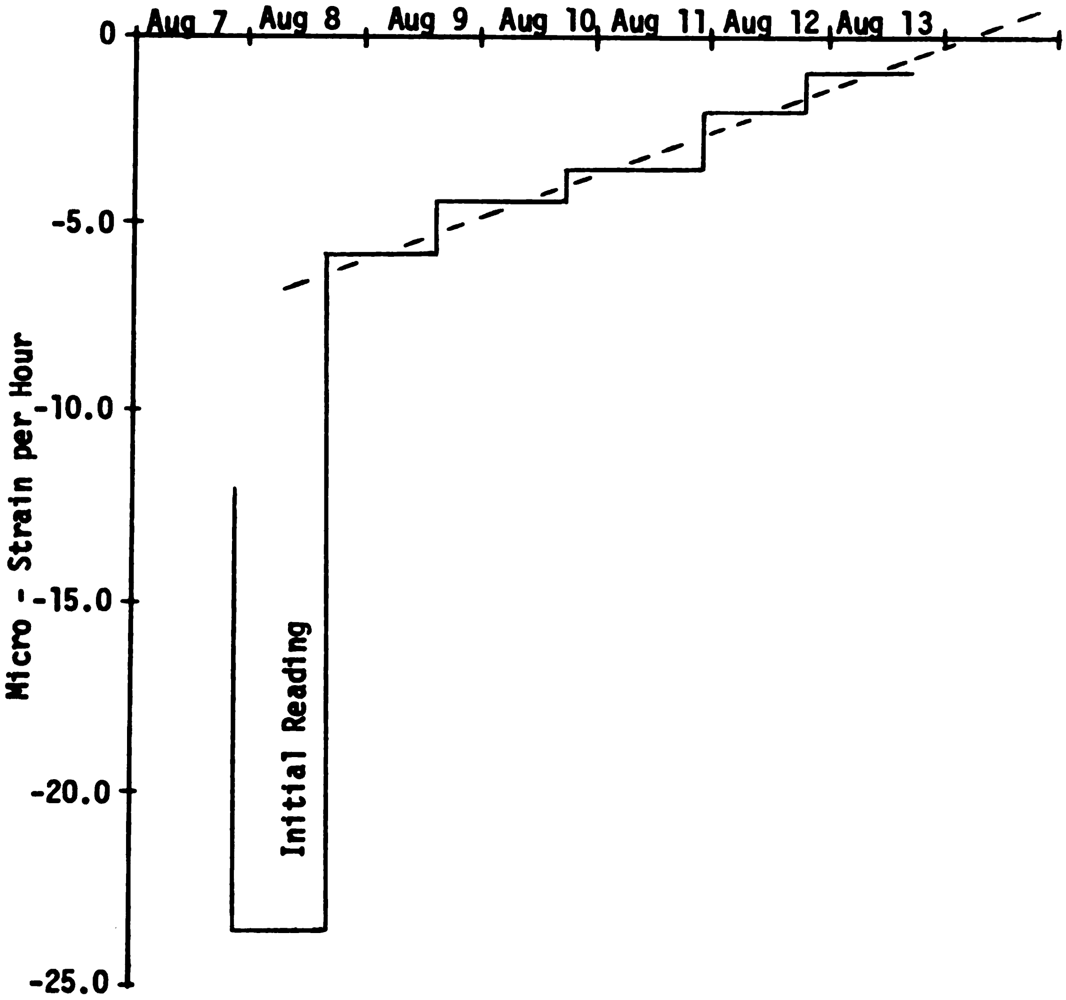

Figure 8 - Average Normal Strain Rate - 1971

Maximum Shear Strain Rate
as Measured Daily
Ptarmigan Glacier, Juneau, Alaska

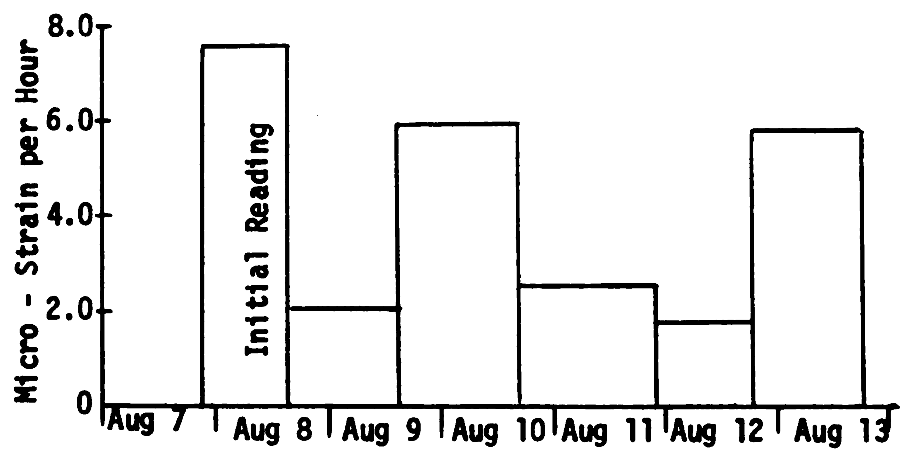

Figure 5 - Maximum Daily Shear Strain Rate - 1971

Maximum Shear Strain Rate
as Measured Tri-hourly
Ptarmigan Glacier, Juneau, Alaska

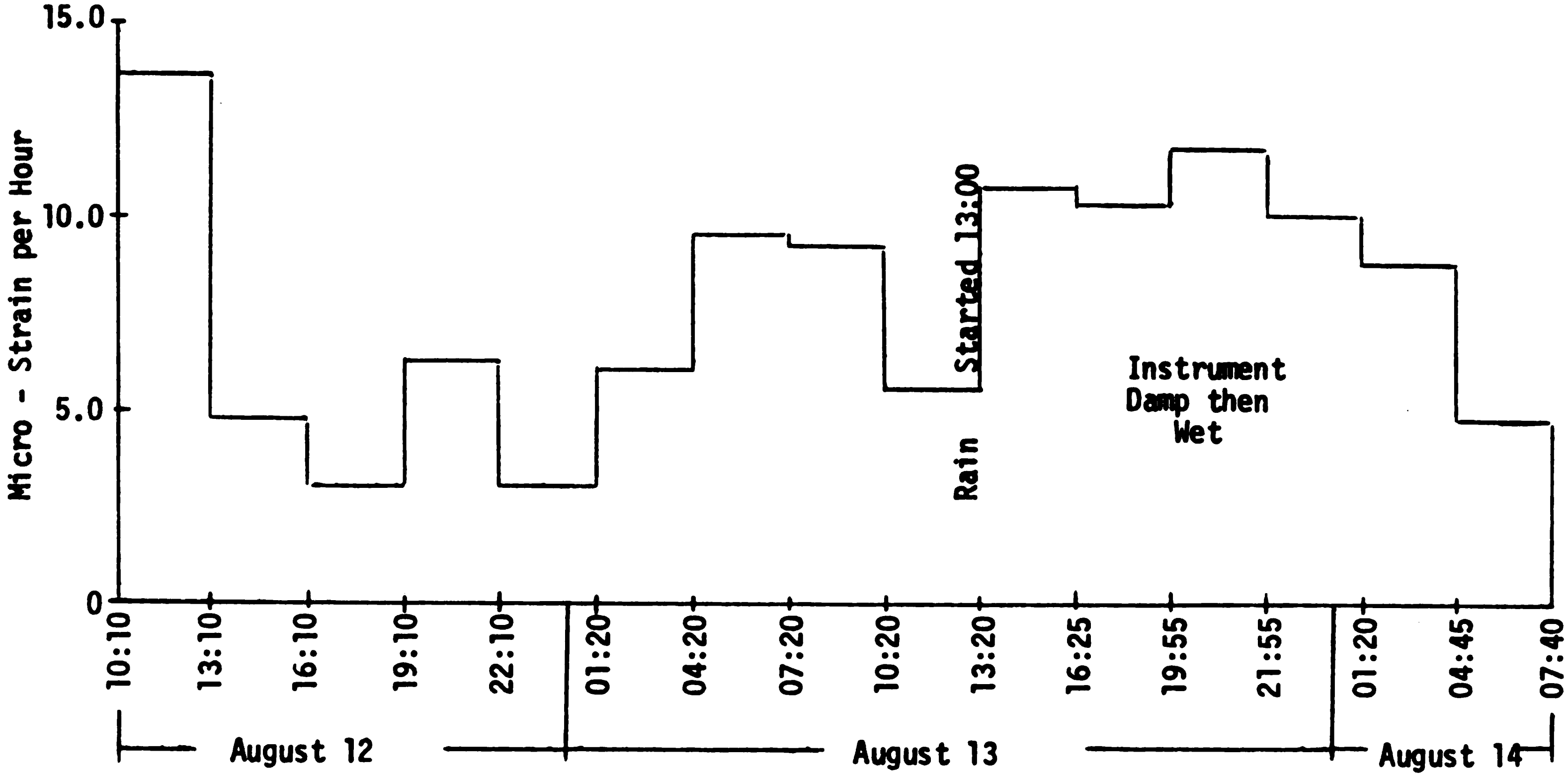

Figure 10 - Tri-hourly Maximum Shear Strain Rate - 1971

3. A method of retarding ablation of the snow or firn cover was necessary.
4. Metal anchor posts were undesirable.
5. An evaluation of the pressure melting effect on the system was required.

DESCRIPTION OF EQUIPMENT AND INSTALLATION PROCEDURES

Based on the 1971 initial investigation, it was decided to install strain measuring systems on two glaciers during the 1972 field season. An improved installation was planned for the Ptarmigan Glacier, and a similar installation was built for the Cathedral Glacier near Atlin, British Columbia. The Cathedral instrumentation system will be discussed in a later chapter.

Protection From Ablation

The prevention of excessive ablation of the firn cover was accomplished using a tarpaulin to shield the surface from solar radiation. The following are considerations for the selection of the tarpaulin:

1. The tarp has to be transported to the strain site in a remote area, so it must be of minimal weight.
2. The amount of rain on the Ptarmigan Glacier in the summer tends to compact the snow; the tarp must be waterproof.
3. The long-wave solar radiation is best reflected by a light-colored, high albedo material.
4. The short-wave solar radiation is best resisted by an opaque material.

A yellow, 5 oz. per yard, polymer coated nylon was selected. Since the installation comprised two superimposed rosettes, the installation size would be 10 x 10 feet. Each tarp was made 13.5 feet square. After the nylon tarps were completed, it was noted that they were somewhat translucent. To further block solar radiation, particularly that of short-wave length, 10 feet square tarps were made of dark green six mil polyethelene. The plastic tarp was to be placed under the nylon tarp.

Anchor Post Installation

Solar radiation tends to be absorbed by objects, particularly objects of low albedo. This absorbed radiation increases the thermal energy within the object, causing it either to warm up or to dissipate the energy to its surroundings. Thus, an object embedded in the surface of ice at 32° will melt the ice at depth depending on the thermal conductivity of the object. Ablation (thermal melting) around the anchor poles makes aluminum with a relatively high thermal conductivity an undesirable material. Schedule 40 PVC tubes with 4.5 inch O.D. and 0.25 inch wall thickness were selected for use as anchor posts. The posts were cut to 40 inch (1 m.) lengths.

Rigidly anchoring posts into temperate, isothermal (32° F.) ice is difficult. The glacial surface has a substantial quantity of surface water and the ice is permeable; so, a hole drilled in the ice quickly fills with water. This condition was used to advantage by inserting the posts into holes slightly larger than the posts and freezing them in place using dry ice as a heat sink. A 40 inch deep, five inch diameter hole with a 4.5 inch diameter post in it required a minimum of five pounds of dry ice to freeze the surrounding water. At least eight pounds of dry ice had to be used. To minimize the amount of water to be frozen and to prevent the dry ice from contacting the water, the bottoms of the posts were sealed with plexiglass caps. This method of post installation had been tested on a frozen lake near Lansing, Michigan in early March and was found to be very effective. Plexiglass caps were made for the tops of the posts to seal the inside and to provide a means of fastening the strain gages. An installed post illustrating the fastening technique is shown in Figure 11.

Pressure Melting Considerations

Ice at 32°F. is very sensitive to regelation (pressure melting). Each gage was to have a prestrain of 1500 microstrain which would put a 0.5 pound pull on the top of each anchor post. The ice, therefore, had to constrain the post from rotating by a distributed force on the side of the post. In other words, the post had to be maintained in static equilibrium. By assuming several forms of the distribution of the constraining force between the ice and the post, the force system acting on the post to maintain equilibrium could be estimated. For each form of the distributed force (square, triangular, polynomial) the strain and deformation in the post were predicted. It was determined that the 0.5 pound tensile pull of the strain gage would produce a maximum bending strain in the post of 0.2 microstrain. Thus the deformation of the post is extremely small for the more probable constraining force distributions, and the post can be treated as a rigid body. If pressure melting did occur, the post would rotate with a rigid body motion, hence, a triangular resisting force distribution on the side of the post was assumed.

The rigid body rotation caused by pressure melting had to be measured. Using the assumed triangular pressure distribution of the ice on the post, it was calculated that a 20 microradian (4 second)

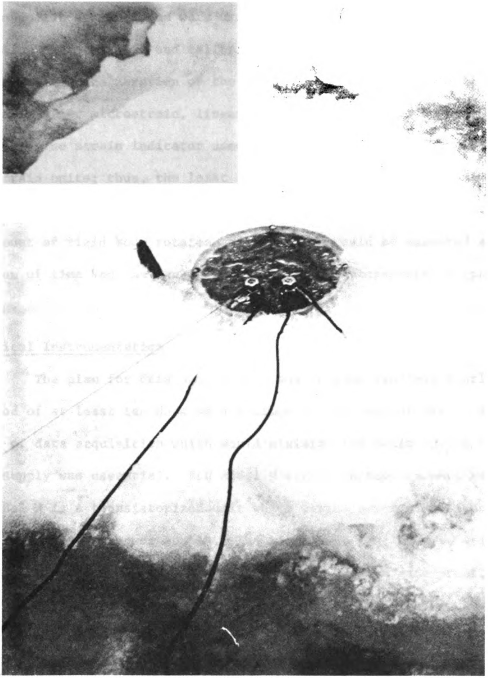

Figure 11 - Anchor Post with Gages Attached
August 1972

rotation of the posts at each end of the gage would cause a 10 micro-strain change on the wire strain gage. Two inclinometers were constructed to mount in the tops of two of the posts to evaluate rigid body rotation. Semi-conductor strain gages were used as secondary transducers. A discussion of the design and calibration of the inclinometers is in Appendix A. The calibration of the inclinometers was 10.5 and 11.2 microradians per microstrain, linear over a range of $\pm$ 15 degrees rotation. The strain indicator used could be read to the nearest five microstrain units; thus, the least increment of angle was approximately 55 microradians, equivalent to 23 microstrain on the strain gage wire. The amount of rigid body rotation of the posts could be measured as a function of time and the strain data adjusted to compensate for pressure melting.

Electrical Instrumentation

The plan for data acquisition was to take readings hourly for a period of at least ten days or a minimum of 120 sets of data. A method of data acquisition which would minimize the drain on the battery power supply was essential. BLH Model N strain indicators were used. The Model N is a transistorized unit which weighs seven pounds including the 15 volt battery power supply. The instrument was light in weight and rugged, but there was no convenient way to make it waterproof, a source of major difficulty to be discussed in a later chapter.

A data set consisted of measuring 13 strain values:

1. Initial zero.
2. Instrument calibration check.

3.-8. Strain gage rosettes.

9. A gage which was not anchored to posts.

10. Strain gage between pressure melting posts.

11. Gages at maximum bending moment location of one pressure melting post to measure any non-rigid deflection.

12.-13. Two inclinometers.

A standard switch and balance unit would have excessive drift over a period of ten days in probable inclement weather; therefore, a special switching unit for each installation was built. Each switching unit was fabricated from 24 gage sheet steel. They were made small, 3-1/4 x 3-1/4 x 9 inches and 3-1/4 x 3-1/4 x 5 inches respectively, so that they could be transported easily to the location. Five of the sides were welded for water tightness and the bottom was fastened with sheet metal screws and sealed with GE silicone rubber. Silicone rubber was used to seal the amphenol connectors entering the box and around the mountings of the controls. Dessicant packets were put inside the switching units.

The wiring diagram for the switching unit used on the Ptarmigan Glacier is shown in Figure 12. The strain indicator uses a two-arm Wheatstone bridge circuit. Initial zero balance and temperature compensation was accomplished using two lengths of gage wire sealed inside a small plastic box. One wire was for the compensating arm of the Wheatstone bridge circuit and the other for the active arm. A 59880 ohm precision resistor in series with a momentary contact switch was wired across the compensating bridge arm. A double wafer, 12 position rotary switch was used for switching. The first position was for the zero reference check. When the momentary contact switch was pressed, a

Table for Wiring between Rotary Switch And Five Pin Amphenol Connectors

Pin No.	Amphenol #1		Amphenol #2		Amphenol #3		Amphenol #4		Amphenol #5	
	Rotary Switch Post	Gage	Rotary Switch Post	Gage	Rotary Switch Post	Gage	Rotary Switch Post	Gage	Rotary Switch Post	Gage
a	Top 0	Calb Zero	Top 1	1	Top 5	5	Top 9	Post Ten Side	Top 11	Incl Ten Side
b	Bottom 0	Temp Comp	Top 2	2	Top 6	6	Bottom 9	Post Comp Side	Bottom 11	Incl Comp Side
c	------	----	Top 3	3	Top 7	7	Top 10	Incl Ten Side	------	----
d	------	----	Top 4	4	Top 8	8	Bottom 10	Incl Comp Side	------	----
e	Common	All	Common	All	Common	All	Common	All	Common	All

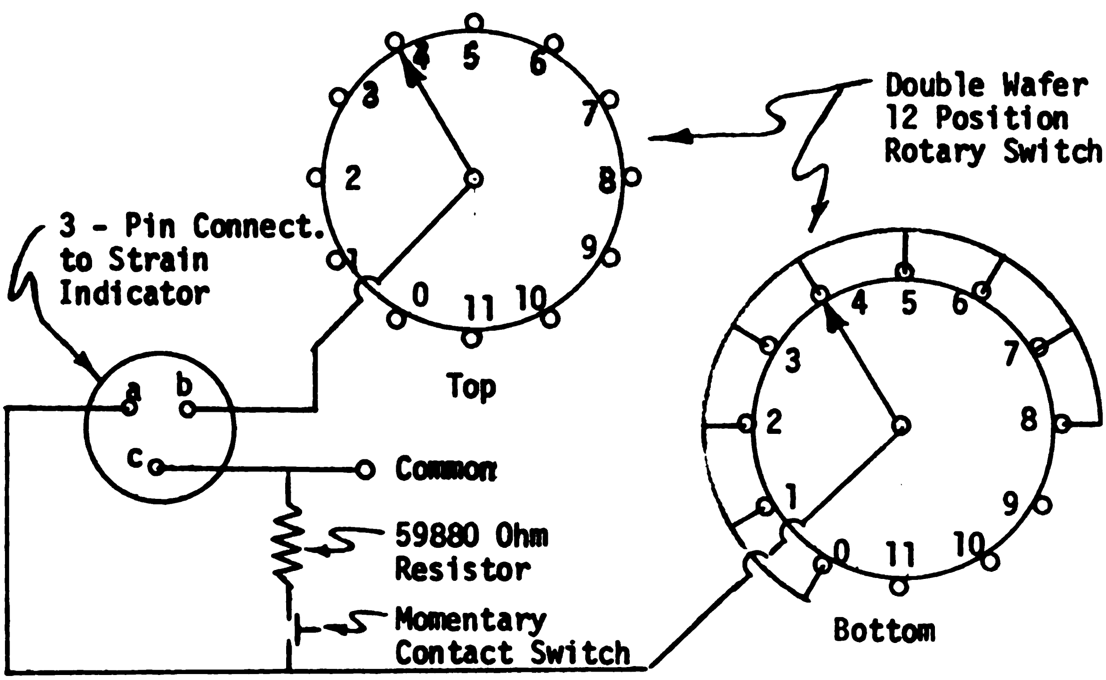

Figure 12 - Switching Unit Wiring Diagram - 1972

calibration check could be made. Positions 2 through 12 were for each of the remaining items listed above.

Fifteen feet of 20 gage, single element lead wire were soldered to each end of the Constantan strain gages. These lead wires were soldered to ten foot lengths of plastic-shielded, five conductor, 20 gage cable. The cables were attached to the switching unit with amphenol type connectors. The switching unit was wired to the strain indicator with an 18-inch piece of cable. The total length of the leads from gage to indicator was 27 feet. Care was taken to insulate and waterproof all of the connections.

Figure 13 illustrates the instrumentation package used on the Ptarmigan Glacier. The equipment for the Cathedral Glacier installation was identical, except that it was designed to measure only items 1 through 8 as listed above.

Supply and Storage of Dry Ice

The nearest source for the dry ice to be used for freezing in the plastic anchor posts was located in Seattle, Washington. The dry ice had to be air-freighted to Juneau and then transported to Camp 17 on the Ptarmigan Glacier by helicopter. Since helicopters can fly only in clear weather, there was a possibility of having to store the dry ice in Juneau for four to five days. Approximately 60 pounds of dry ice was required, so 120 pounds was ordered to allow for losses. A storage container was built to hold 120 pounds of dry ice. The container was made of an inner lining of 1/4-inch plywood, an insulation layer of 2-inch thick styrofoam, and an outer shell of 1-inch pine boards. Each layer was separated with a vapor seal of light-weight plastic sheet. The

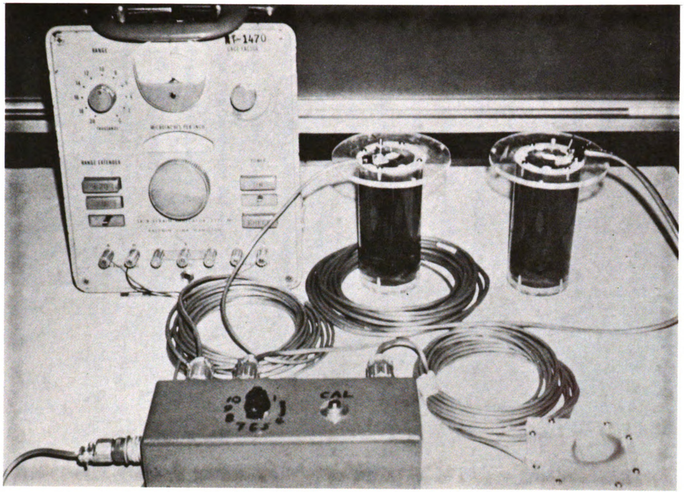

Figure 13 - Instrumentation Package - 1972

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design of the container was suggested by Dr. Gunnar Östrem, Chief Glacial Hydrologist for the Norwegian Hydroelectricity Board.

INSTALLATION AND DATA ACQUISITION ON THE PTARMIGAN GLACIER

Installation Procedures

The 1972 installation site was 40 feet (12 m.) up glacier (south) from the site of the 1971 initial investigation. The firn pack was one meter deep when the installation was started on July 29. In Figure 2 and 14, the site location is noted with respect to Camp 17. The aluminum poles were still in place from the previous year. Incidentally, these aluminum poles protruded 40 inches (one meter) out of the ice revealing a one meter ablation of the glacier's surface from August 7, 1971 to July 30, 1972. A 12 foot square hole was dug in the firn pack to the surface of the bubbley glacier ice. The locations of the gages were laid out: one delta rosette with its base in an east-west direction, the other rosette with a side in the north-south direction. The two pressure melting posts were planted in an east-west direction two feet north of the rosette gage in the same direction.

A 120 mm (5 inch) diameter Norwegian ice drill was used to bore eight 40-inch holes at the anchor post locations. This phase of the installation was completed by 1700, July 31. The dry ice was scheduled to arrive on August 1.

The dry ice arrived in Juneau on schedule at 1830, August 1, and was flown to Camp 17 at noon on August 2. There was an estimated 14 percent loss of dry ice in transit. During the afternoon, the

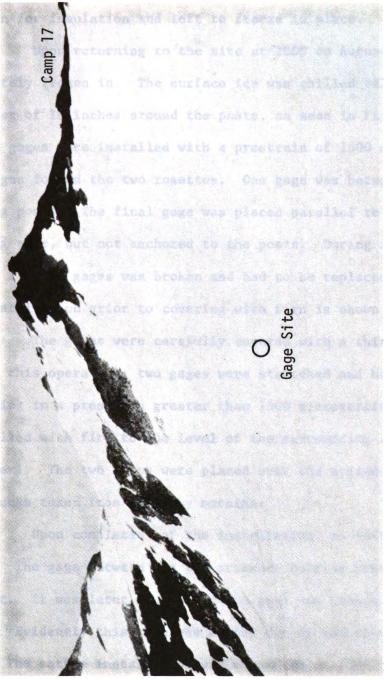

Figure 14 - Upper Ptarmigan Glacier from West Edge
July 1972

anchor posts were filled 2/3 full of dry ice and inserted in the boreholes. Within five minutes, the posts were sufficiently frozen to prevent movement. At 1600, the posts were covered with a couple inches of firn for insulation and left to freeze in place.

Upon returning to the site at 2000 on August 2, the posts were completely frozen in. The surface ice was chilled below freezing to a diameter of 18 inches around the posts, as seen in Figure 11. Eight strain gages were installed with a prestrain of 1500 microstrain units. Six gages formed the two rosettes. One gage was between the pressure melting posts. The final gage was placed parallel to the pressure melting gage, but not anchored to the posts. During installation, one of the rosette gages was broken and had to be replaced with a spare. The installation prior to covering with firn is shown in Figure 15.

The gages were carefully covered with a thin layer of firn. During this operation, two gages were stretched and had to be remounted resulting in a prestrain greater than 1500 microstrain units. The hole was filled with firn to the level of the surrounding region (36 inches of cover). The two tarps were placed over the system and held in place with rocks taken from a nearby moraine.

Upon completion of the installation, an initial data set was taken. The gage between the two pressure melting posts had an open circuit. It was later found that the gage was broken at one of the posts. Evidently this gage was bumped during the firn covering operation. The entire installation was completed at 2300, August 2, eleven hours after receipt of the dry ice.

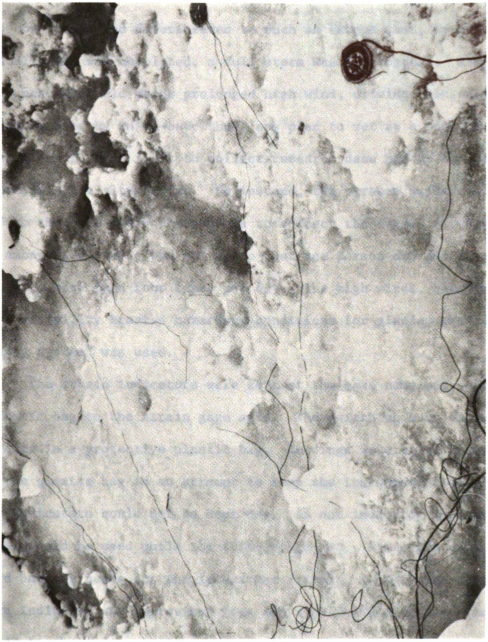

Figure 15 - Completed Installation Prior to Covering
Ptarmigan Glacier - August 1972

Data Acquisition

The original plan for data acquisition was to establish a temporary tent camp at the strain gage site and collect data on an hourly basis, but circumstances made it impossible to implement this plan. The weather conditions had deteriorated to such an extent that, by the time the installation was completed, a full storm was in progress. A storm on the Juneau Icefield means prolonged high wind, driving rain, and sleet often accompanied by white-out; thus, any plan to set up a tent camp and expect someone to stay in it to collect research data had to be modified to a more sensible alternative. So instead, two persons walked from Camp 17 to the site every three hours to collect the data. A rotating system among five people was set up so that one person did not have to collect data more than four times per day. The high winds, heavy rain and poor visibility created hazardous conditions for glacier travel, so the "buddy system" was used.

The strain indicators were kept at the base camp and carried in a plastic bag to the strain gage site. The switching unit was kept at the site in a protective plastic bag. Readings were taken from inside the plastic bag in an attempt to keep the instrumentation dry, but the indicators could not be kept dry. As one indicator got wet, the other would be used while the first dried out. After two days, the standard battery packs for the indicators failed. A new power supply for each indicator was improvised from ten alkaline D-cell batteries and taped to the indicator in a sealed plastic bag. (One indicator was set aside as a spare to be used during the repeat of the pressure melting experiment.) In spite of the new power supply, the indicator wa

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was providing such erratic measurements that the experiement was stopped on the sixth day.

Inspection of the switching unit revealed that it had stayed dry inside; however, the momentary contact switch in the calibration circuit was shorted from the outside. Calibration readings had been stopped on the third day because of erratic readings.

During the evening of the seventh day, the weather cleared. On the morning of the eighth day, August 10, a tent was set up at the site and data were collected on an hourly basis. The experiment was terminated after two days, so that preparations could be made to move the equipment to the Cathedral Glacier on the continental side of the Juneau Icefield.

Pressure Melting Experiment

Logistics in a glacial environment often become complicated. After several days' delay, it was decided to repeat the pressure melting experiment on the Ptarmigan Glacier rather than move it to the Cathedral Glacier. Two posts aligned in an east-west direction were installed in the ice surface using the same method as the first rosette installation. This time, the posts were frozen into place using an ice-brine solution as the heat sink. The ice-brine method was as satisfactory as dry ice, except the top six inches (15 cm.) of the posts were not frozen solidly into the glacier ice. An inclinometer was mounted on each post and a strain gage wire was stretched between the posts. The system had a firn covering of 15 inches (38 cm.) and a plastic tarp was employed as a radiation shield. The tarp was held in place by rocks. Figure 16 shows the installation.

Figure 16 - Pressure Melting Experiment
Ptarmigan Glacier - August 1972

Data were taken twice per day at approximately 0900 and 2100. The system appeared to respond normally after the usual 24-hour seating time. On the fifth day, the data appeared erratic and the experiment was terminated on the sixth day. When the tarp was removed, it became apparent that, as the firn covering ablated, the tarp had been pulled taut against the anchor posts causing them to rotate via pressure melting toward each other (in a positive direction). The cause of the erratic data was obvious.

The apparent failure of the pressure melting experiment illustrates the kind of practical problems encountered in field research. One must invoke a constant awareness and caution when conducting such research, noting all unusual problems and events and being prepared to interpret them scientifically if the occasion arises. As will be shown later, this particular event was crucial to the interpretation of pressure melting.

INSTALLATION AND DATA ACQUISITION ON THE CATHEDRAL GLACIER

Physical Setting

The Cathedral Glacier is a cirque type glacier nestled near the 7000 foot (2100 m.) peak of the Cathedral Mountain massif on the north (continental) side of the Juneau Icefield. The glacier's terminus lies at about 3000 feet (900 m.) above Lake Atlin, British Columbia. The glacier is approximately 90 miles (150 km.) north-northeast of Juneau, Alaska. Its shape is essentially an equilateral triangle with each side about one mile (1.6 km.) in length and with one vertex at the terminus. The glacier appears to flow in a direction 60° east of north. The summer of 1972 was the first season any field research has been conducted on the Cathedral Glacier except for aerial surveys in 1971 (Miller, 1973).

The site picked for the strain gage installation, as shown in Figure 17, was 300 yards (270 m.) up glacier from Camp 29 (elevation 5400 feet (1600 m.)) where a small permanent research station was constructed in 1972. The strain site was 100 yards (90 m.) from the northern side-wall. The site was about one-third the distance from the terminus toward the head-wall. The surface configuration was relatively smooth at the site with an inclination of 15° from horizontal. Fifty yards up glacier from the site the surface was convex, but as in the rest of the glacier, there were no open crevasses. There were many supraglacial

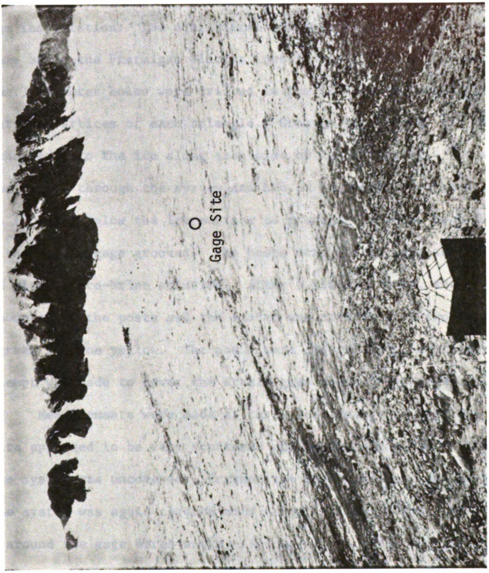

Figure 17 - Head-wall of Cathedral Glacier from Camp 29
August 1972

streams flowing down the glacier at approximately 30 feet (9 m.) intervals. Being late summer (August 27 - September 2), the glacier had no snow or firn pack except in the vicinity of the head-wall (Figure 17).

Installation and Data Acquisition

The lack of snow or firn pack cover necessitated some changes in gage installation. Two equilateral triangles were laid out exactly the same as in the Ptarmigan Glacier installation. Six 4-5/16 inch (110 mm.) diameter holes were drilled in the surface 44 inches (110 cm.) deep at the vertices of each triangle. Grooves four inches (10 cm.) deep were chopped into the ice along each side of the triangles. Two small streams flowed through the system and had to be deepened to about eight inches (20 cm.) below the ice surface to prevent the water from flowing along the strain gage grooves. The posts were forced into the holes and filled with an ice-brine solution. After a six-hour wait, the gages were mounted to the posts and the system was covered with two tarps, one dark green and one yellow. The posts were covered with ice chips, but no attempt was made to cover the strain gage wires (Figure 18).

Measurements were made at three-hour intervals for five days. The data appeared to be very scattered and inconsistent, so on the third day the system was uncovered. Crushed ice was packed around the gages and the system was again covered with tarps. It was hoped that the ice chips around the gage wires would eliminate any scatter resulting from temperature changes. Measurements were made for two and a half more days. High winds during the last two days of the experiment caused a greater than normal surface ablation on the glacier. When the tarps were removed on September 2, the posts were found to be ablated and quite loose in their holes, as shown in Figure 19.

Figure 18 - Initial Cathedral Installation
August 1972

Figure 19 - Cathedral System after Completion of Study
September 1972

DATA ANALYSIS

During the 1971 initial investigation on the Ptarmigan Glacier, strain rates were determined by dividing the change in strain by the time interval between readings. The 1972 data were much too scattered to yield meaningful results using the 1971 data reduction technique. A method of obtaining smooth curves with continuous first derivatives was required. An IBM 360 Model 40 computer was used for data reduction.

The first approach was to fit polynomial curves,

$$\varepsilon = \sum_{n=1}^{N} A_n t^{n-1} ,$$

to the net strain data using the least-squares-best-fit technique. In general, the Ptarmigan Glacier data appear to be a parabolic with a superimposed diurnal oscillation, while the Cathedral Glacier data appear linear with superimposed diurnal oscillations. A ninth order polynomial gave the best fit for the Ptarmigan Glacier data, but close inspection of the fit revealed that the curves tended to smooth out the oscillations. The curves would be about 10 microstrain inside the peaks; hence, the strain rates were not as great as the data would indicate. Higher order curves up to 20 degrees were tried, but the fit was poor.

The second approach was a least-squares-best-fit Fourier expansion,

$$\varepsilon = \varepsilon_{avg} + \sum_{n=1}^{N} (A_n \sin \frac{n\pi}{P} t + B_n \cos \frac{n\pi}{P} t),$$

where P was the time interval over which the measurements were collected and N was the number of Fourier terms. Two difficulties were encountered with this approach:

1. The computer program was very slow and the maximum number of terms which could be used was N = 12; a 24 x 24 matrix inversion was not very accurate.
2. The Fourier technique forced the curve to be the same at the extreme ends of the data.

The primary objective of this study was to demonstrate a reliable technique for measuring glacier strain rates; therefore, an evaluation of the apparent diurnal variation was not deemed essential at this time. It was decided to ignore the diurnal variation and evaluate only the overall trend. The results from this approach are discussed in the next chapter.

The rosette gage data from the Ptarmigan Glacier were fitted with parabolic curves. A 95 percent confidence level ($\pm$ 2 times the standard deviation) was used to determine extraneous data points. After several runs, eliminating out-of-limits data points, the slopes of the finalized curves were determined. The strain rates (slopes of the strain curves) for the six gages were substituted into the strain transformation equation:

$$\varepsilon = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta_i + \frac{\gamma_{xy}}{2} \sin 2\theta$$

Using the least-squares technique, the average normal strain rate and the maximum shear strain rate for each hour of the study were determined

The rosette gage data from the Cathedral Glacier were analyzed in like manner; however, straight line curves were used instead of parabo

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parabolas. The Cathedral Glacier data were divided into two sets. The first set included data obtained when the gages were not covered with ice chips, and the second set when the gages were covered with ice chips. The results are considered in the next chapter.

Appendix B describes a step function technique of curve fitting. The results of this technique are presented in Appendix C.

RESULTS

This chapter is divided into three sections: The results of the pressure melting experiment, the results of the study on the Ptarmigan Glacier, and the results of the study on the Cathedral Glacier.

Pressure Melting Evaluation

Two entirely separate experiments were conducted to evaluate the effect of pressure melting on the anchor posts. One experiment was to measure the rigid body rotation of the anchor posts caused by pressure melting. The second experiment was to evaluate the strain in a gage which had no anchor posts. Both experiments led to the same conclusion; the gages become bonded to the ice when a snow or firn cover is used.

No quantitative results were obtained for the effect of pressure melting on strain gage data. Instead, it was found that pressure melting was not a significant consideration. Figure 20 illustrates the strain gage and post rotational trends for the five days of the study during August, 1972. Initial seating of the installation took approximately 28 hours as indicated by inclinometer 11. For the first four days (100 hours), there appeared to be no significant rotation of the anchor posts as shown by the inclinometers. The strain gage, however, had a straight line response with a slope of 0.7 microstrain per hour. Everything seems to have reacted normally.

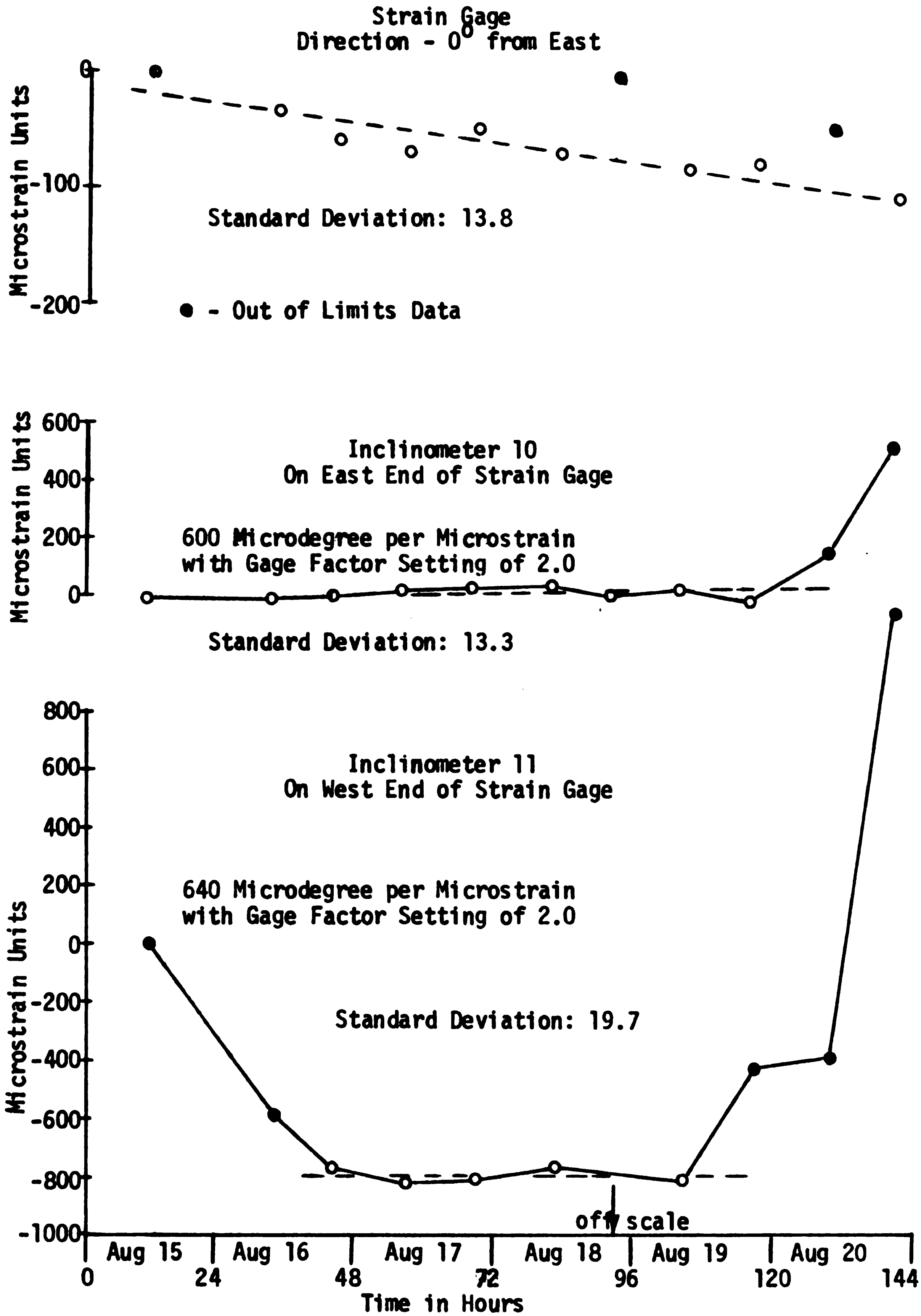

Figure 20 - Plot of Pressure Melting Results

At midnight of August 20, the inclinometers started to rotate toward each other at a surprising rate of 0.013 degree per hour for inclinometer no. 10 and 0.514 degree per hour for inclinometer no. 11. As previously stated on page 41, the rotation was caused by the tarp pressuring against the anchor posts. Calculations based on the triangular pressure assumption indicate that this amount of post rotation should have caused a 5000 compressive microstrain change in the wire strain gages. However, the strain gage did not react at all to the rotation of the posts. The trend of the strain gage remained constant at 0.7 microstrain per hour. This experiment demonstrates that the inward rotation of anchor posts does not affect the strain gages.

The second experiment on pressure melting consisted of comparing the trends of two wire gages in parallel directions; one anchored to posts and the other with no anchor. Both gages produced the same results over a ten-day period. After extraneous data were removed, the standard deviation on the difference between the two gages was 11.0 microstrain which is within the capability of the instrumentation used. Figure 21 shows the net strain of both parallel gages. This experiment demonstrates that there is no need for anchor posts.

Both pressure melting experiments lead to the same conclusion. The Teflon-coated strain gage wires become bonded to the ice surface when a protective snow or firn cover is used. The bond is sufficiently strong to measure changes in surface strain. If the bond were not strong, the gage without anchor posts would have had the same trends, but not the same magnitudes of change as the anchored gage.

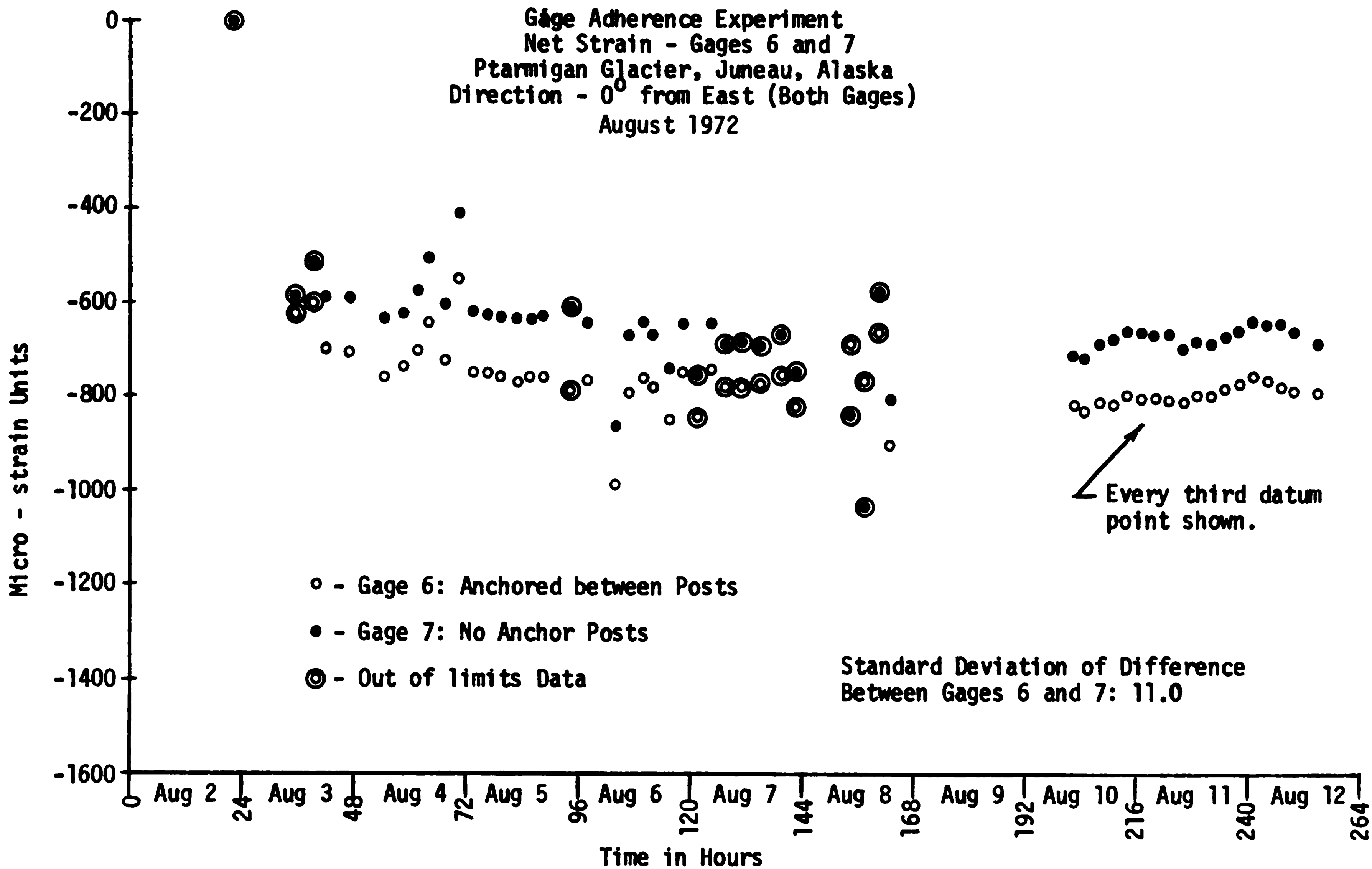

Figure 21 - Net Strain of Gages 6 and 7

Strain Results From the Ptarmigan Glacier

The Ptarmigan strain data have provided some very interesting results. The net strains for each of the six gages, Figures 22 through 27, lead to several interpretations.

Twenty to twenty-four hours are required for the gages to bond to the ice before useful data can be taken. The gages were mounted with approximately 1500 microstrain prestrain. Evidently, there is a relaxation in the system before the gages bond to the ice. The relaxation ranges from 600 to 1400 microstrain. Since the gages become bonded to the ice, the prestrain is not required.

Approximately one-third of the data taken during the first six days is extraneous. Much of the scatter occurs between midnight and 0600, when observers tend to be less alert. About half of the extraneous data points are either 100 or 2000 microstrain units from the normal trend of surrounding data, indicating that the strain indicator was misread. Such mistakes are easy to make with the Model N indicator because of poor numerical identification in the fine scale and parallax on the coarse scale. This problem is magnified when reading by lantern light on a stormy night. A relevant fact is that icefield personnel who were untrained in experimentation aided in this project by taking several of the readings. The strain indicator was behaving erratically during the wet weather further adding to the possibility of erroneous readings. Of the data taken during good weather, only one reading was extraneous.

Even with the scatter present in the data, there appears to be a diurnal variation of strain rate. The cause of diurnal variations

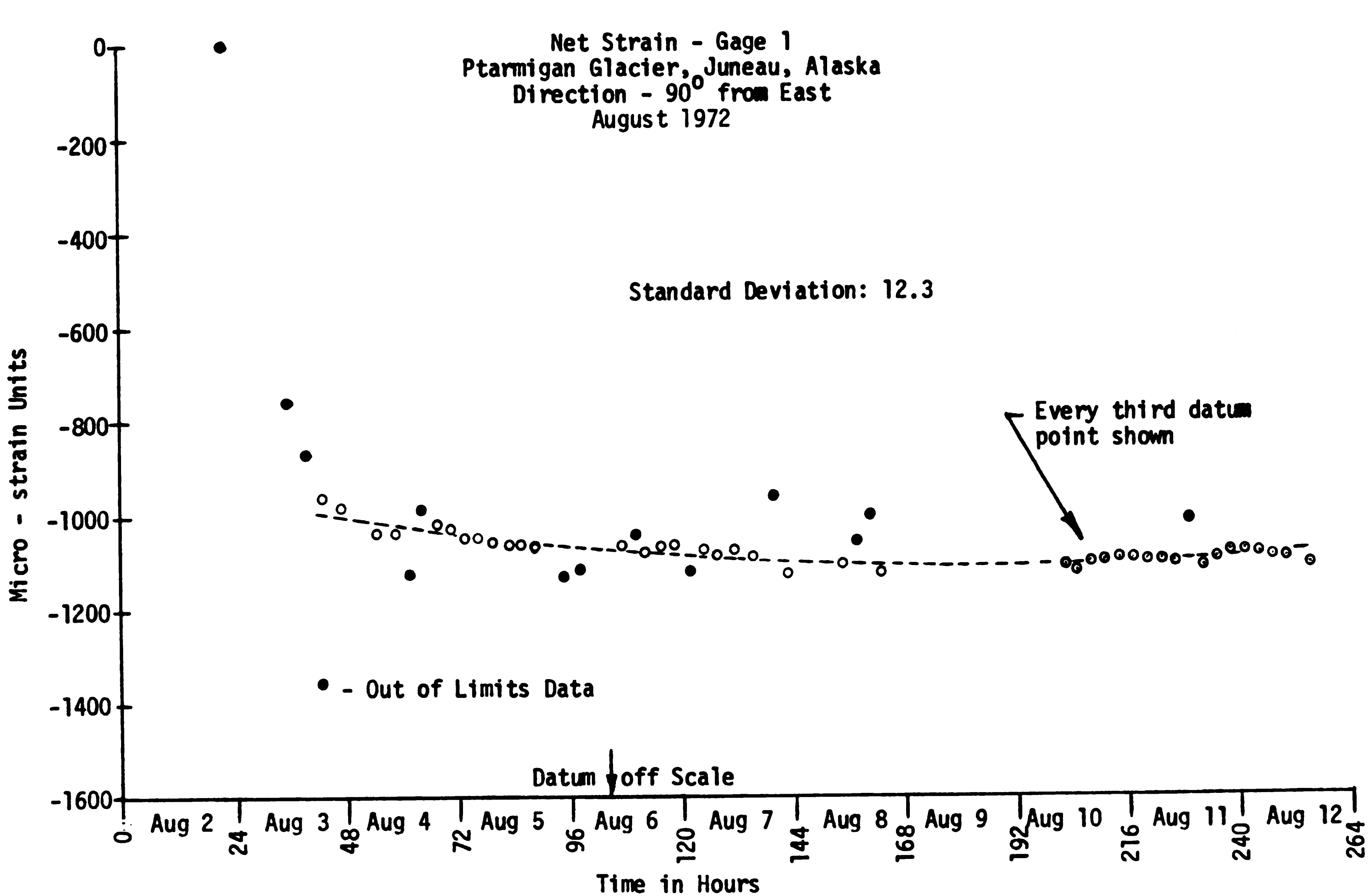

Figure 22 - Net Strain of Ptarmigan Gage 1

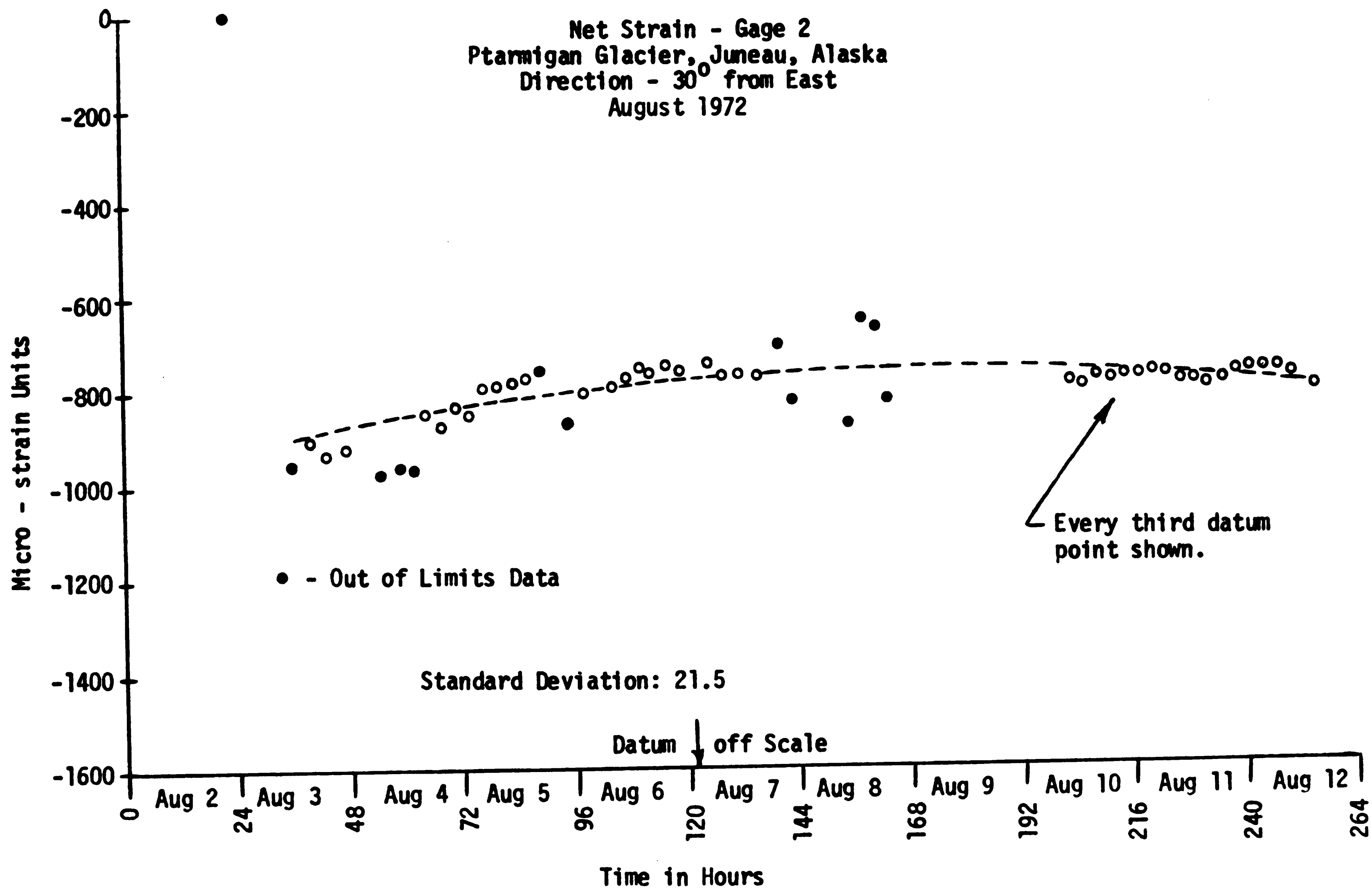

Figure 23 - Net Strain of Ptarmigan Gage 2

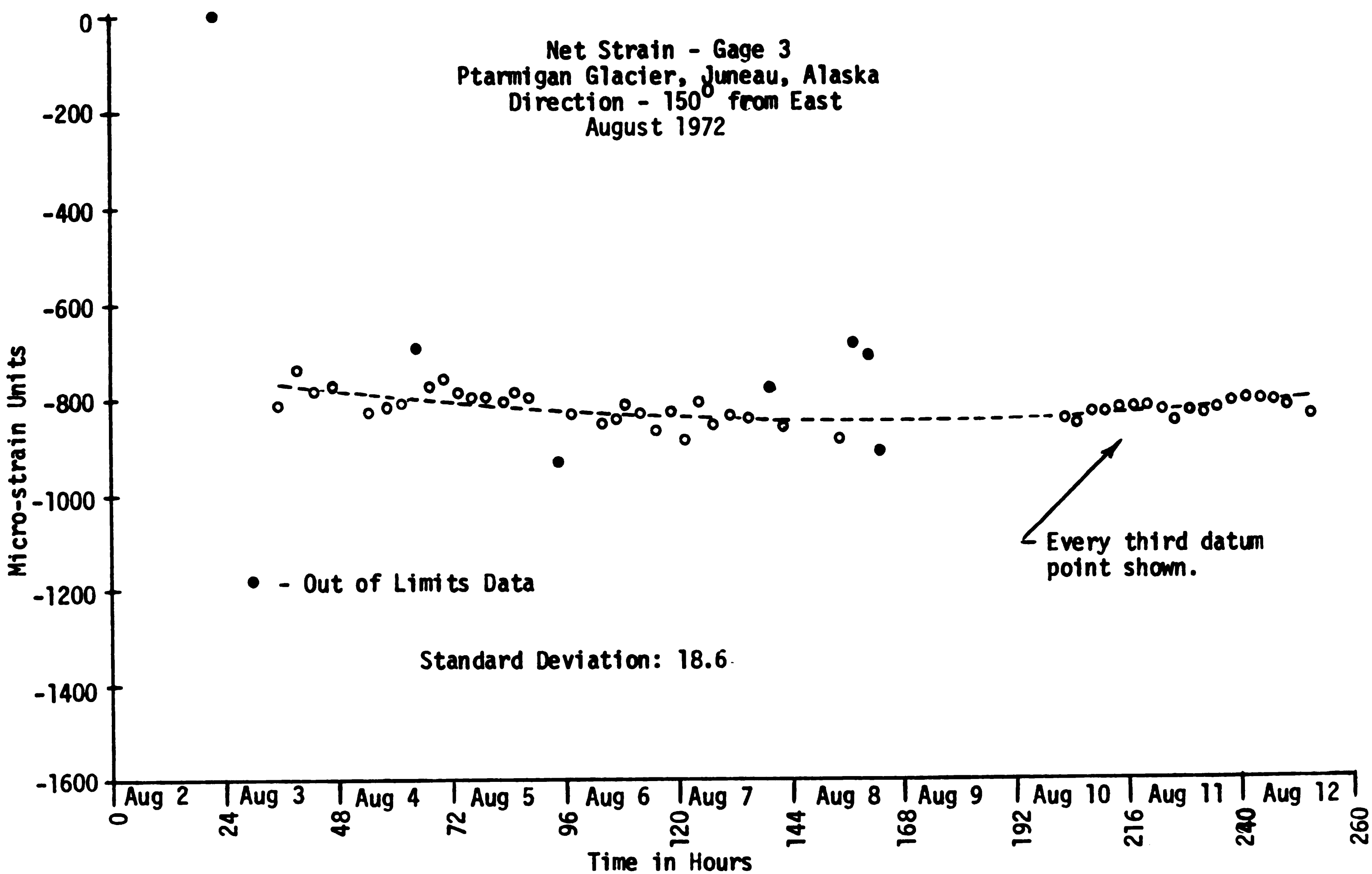

Figure 24 - Net Strain of Ptarmigan Gage 3

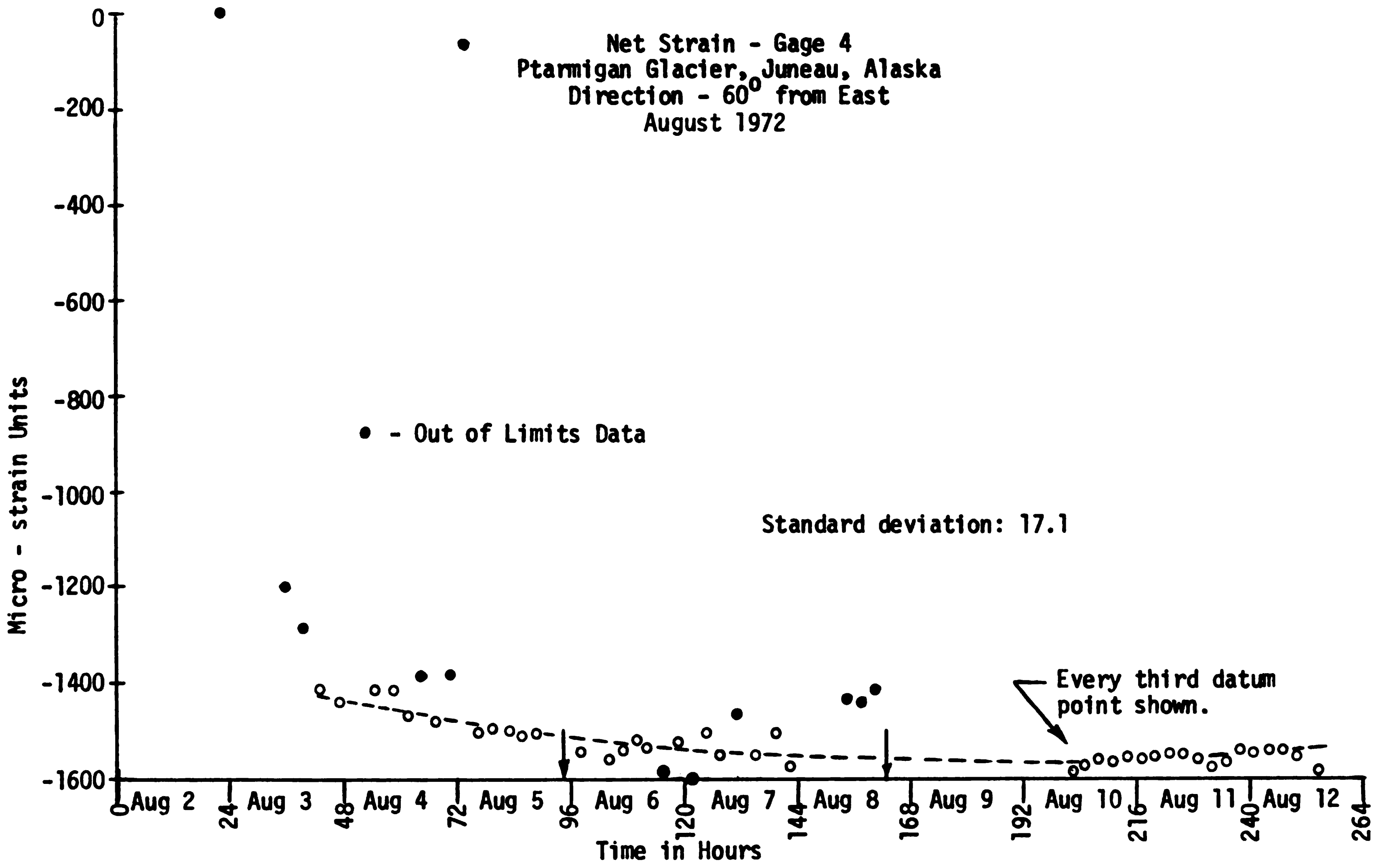

Figure 25 - Net Strain of Ptarmigan Gage 4

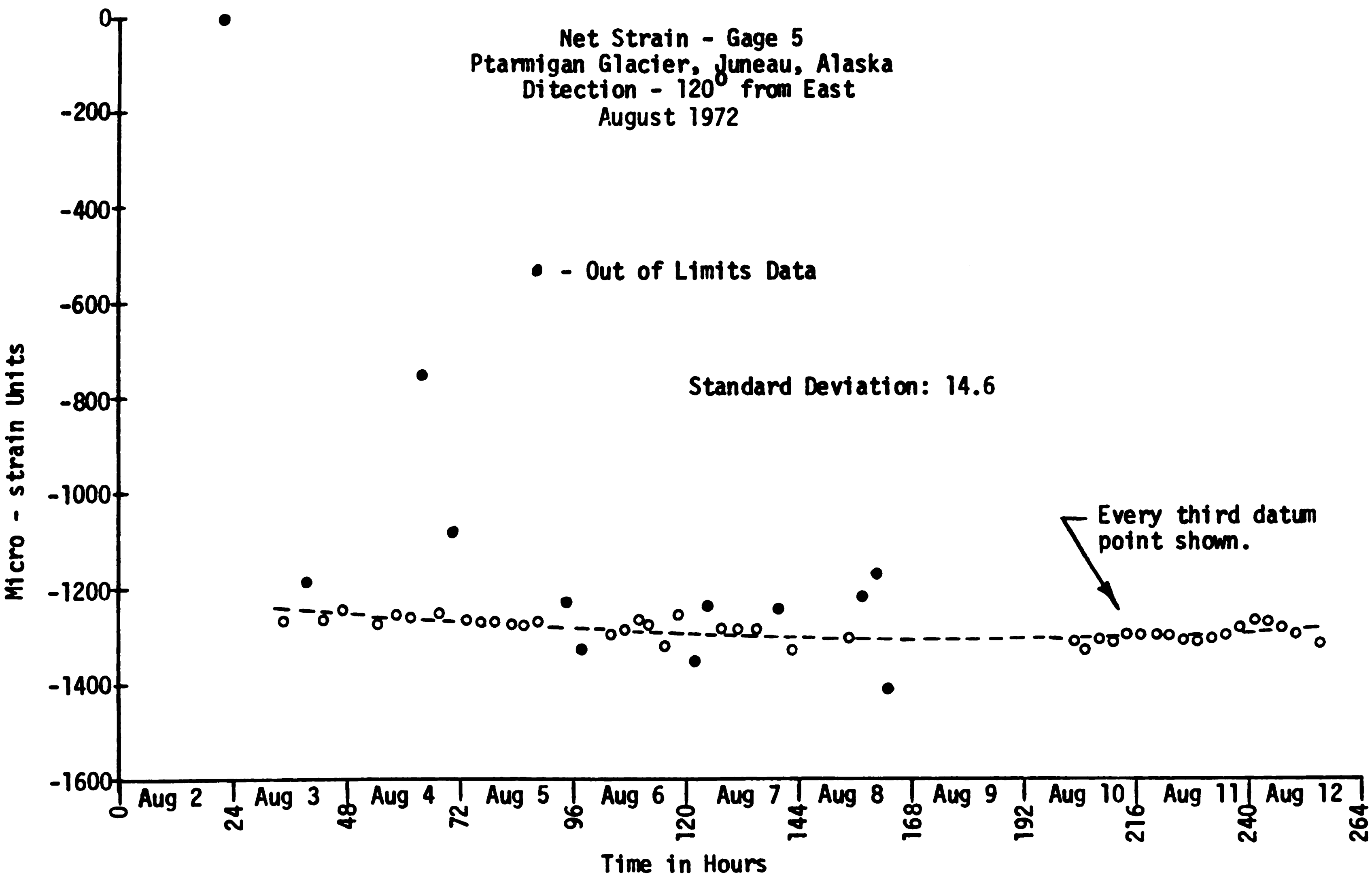

Figure 26 - Net Strain of Ptarmigan Gage 5

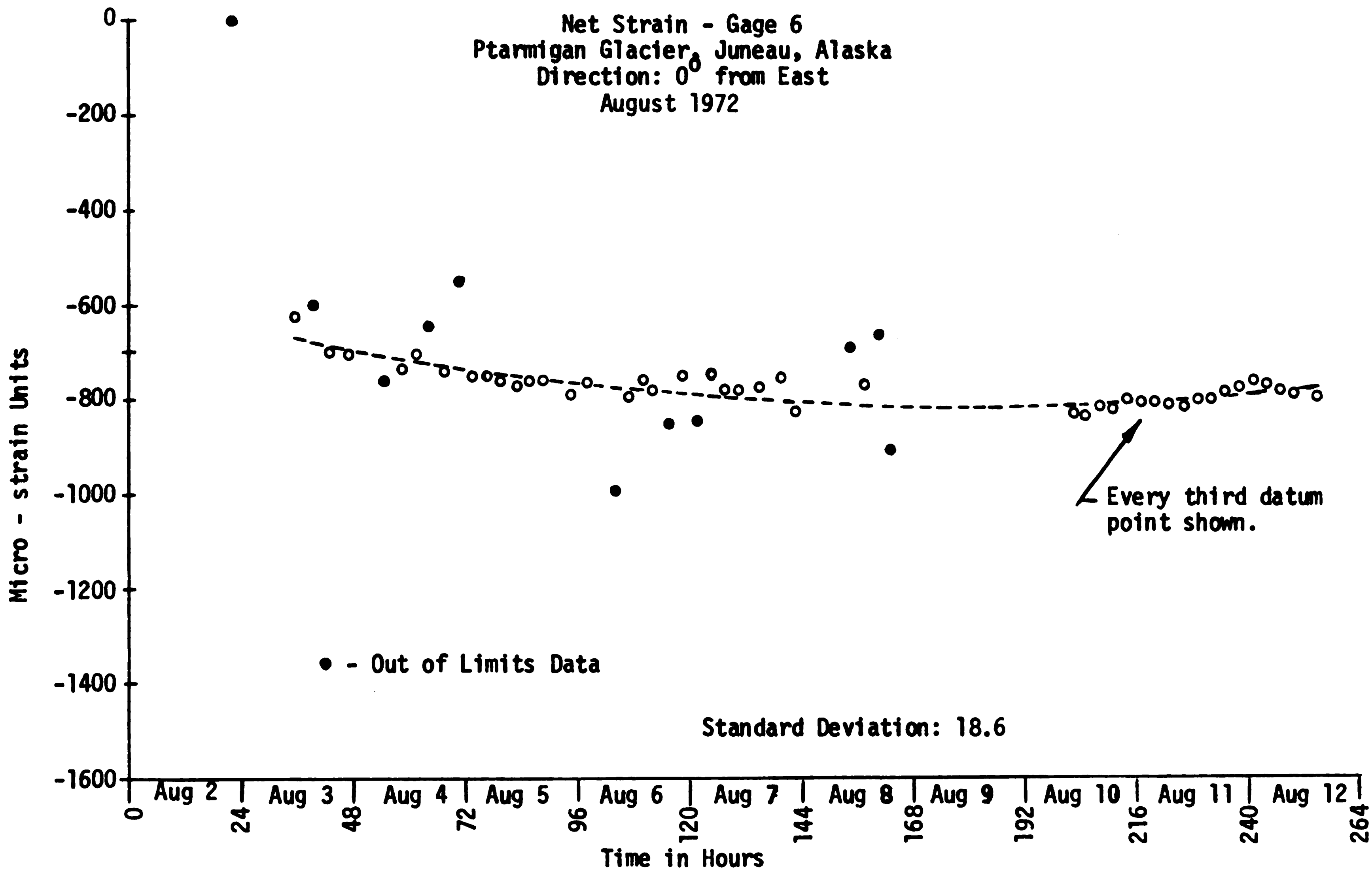

Figure 27 - Net Strain of Ptarmigan Gage 6

is unknown, but will be a subject of future studies. The diurnal variation appears to be sinusoidal with an amplitude of about ± 25 microstrain.

The outputs of the six rosette gages as functions of time suggest a parabolic relationship. The least-squares parabola for each gage is shown in Figures 22 through 27. The standard deviation for each gage, including the diurnal variation, ranged from 12.3 to 21.5 microstrain. Combining the results of all six gages, the overall standard deviation is 17.4 microstrain. If the effect of the diurnal variation is subtracted from this value, the net standard deviation is 9.1 microstrain. However, the theoretical standard deviation expected from the ability to read the strain indicator accurately is 7.1 microstrain. It can be concluded that the transducers and instrumentation are contributing a standard deviation of only 2.0 microstrain. In other words, the tolerance which can be placed on the strain gages is ± 3 times the standard deviation or ± 6.0 microstrain units.

The time derivatives of the parabolic curves were best fitted into the strain transformation equations to determine the maximum shear strain rate and the average normal surface strain rate. Figure 28 shows that the maximum shear strain rate ranged from 0.25 to 1.2 microstrain per hour. These values are smaller than expected; however, data obtained from annual photographs show that the average annual flow rate at the gage site amounts to only 0.03 feet per day. By assuming a parabolic velocity profile across the glacier normal to the flow direction and no flow in the transverse direction, one can calculate an approximate shear strain rate near the surface of 1.6 microstrain per hour (Paterson, 1969, p. 106; and Nye, 1952). One must remember, the surface motion of

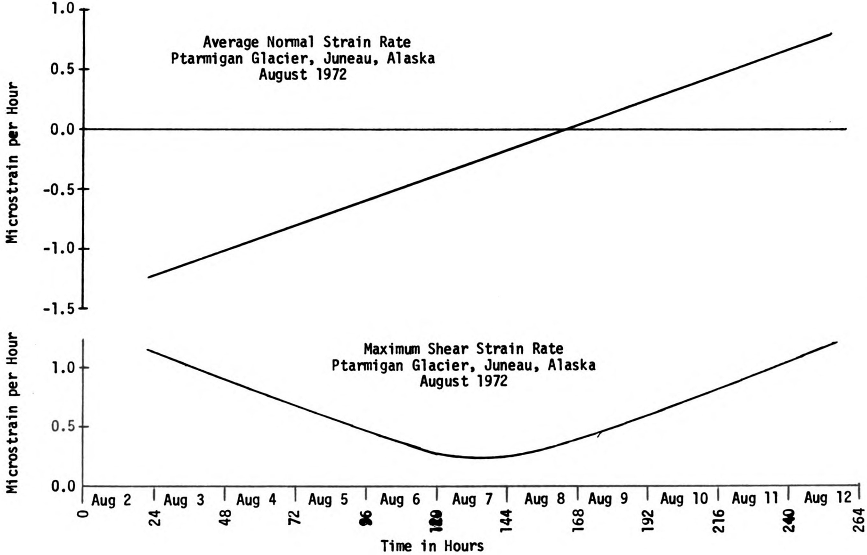

Figure 28 - Maximum Shear Strain and Average Normal Strain Rates

0.03 feet per day included any basal slipping; thus, it is not surprising that the calculated shear strain rate is larger than the experimental values. However, for a glacier of diminishing mass balance, such as the Ptarmigan in its current state, little basal slipping should be expected (Miller, 1972). It seems that a rather dead area of the glacier was chosen for the gage site. By accident, the experiment served as an especially severe test of the technique. The evidence demonstrates that the embedded strain gages did yield a valid measurement of the surface strain rate.

Strain Results from the Cathedral Glacier

The strain data from the Cathedral Glacier indicate that the system does not work well in temperate bubbley glacier ice where there is no snow or firn cover. The installation on the Cathedral Glacier was entirely dependent on the anchor posts since no firn cover was available. From the graphs of the net strain data, Figures 29 through 34, it is apparent that approximately twenty-four hours is required to freeze the anchor posts into place. In this case, the posts were installed eight hours before the initial data reading was made.

There were a few extraneous data points during the study but no pattern can be determined. During the last day, all of the gages produced erroneous data and the last three data sets were not included in the analysis (Figures 29-34). The posts had ablated to such an extent that further study was unwarranted.

Diurnal variation was evident in this study. During the first half of the experiment, the variation was about ± 90 microstrain and during the last half of the experiment with crushed ice around the gages,

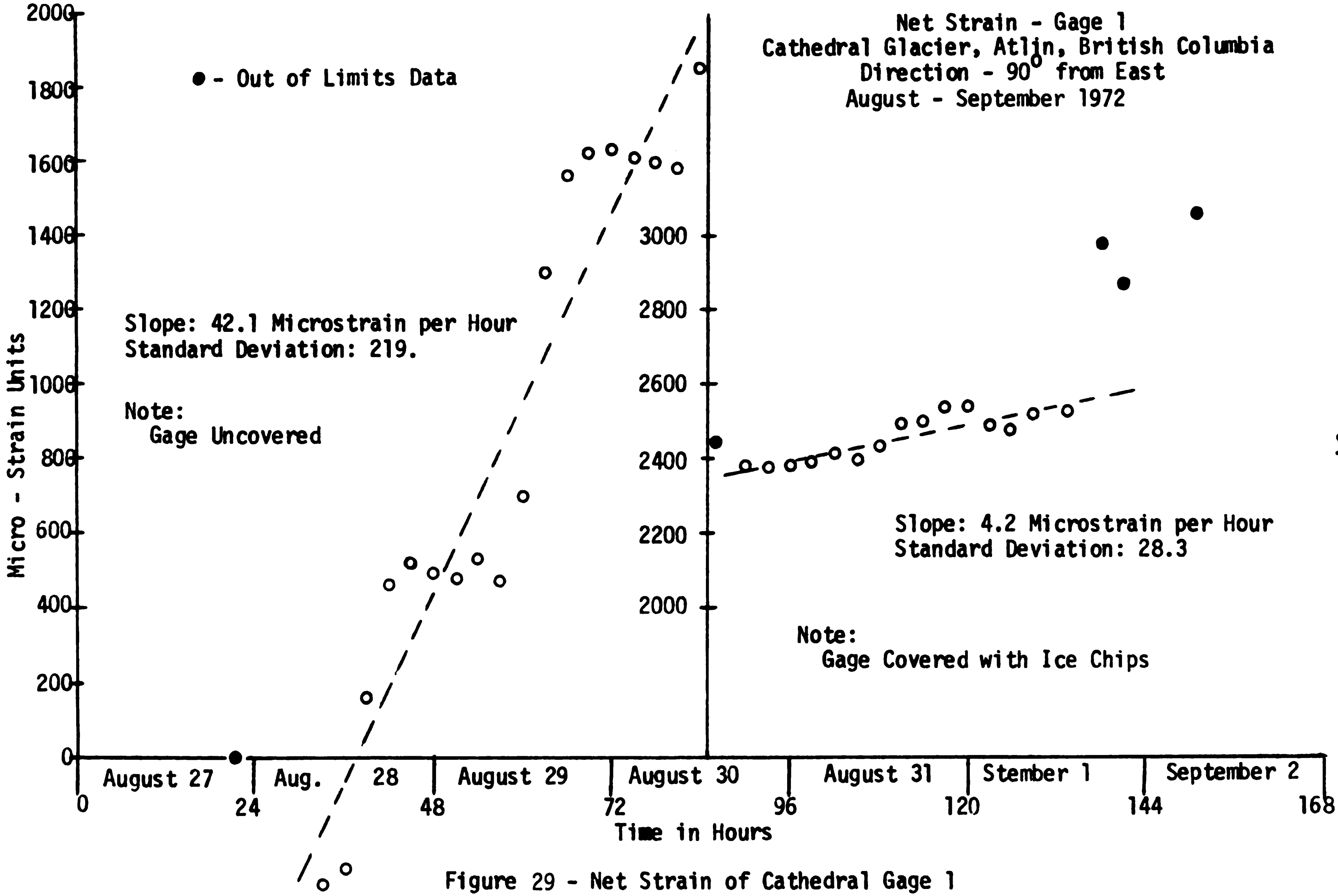

Figure 29 - Net Strain of Cathedral Gage 1

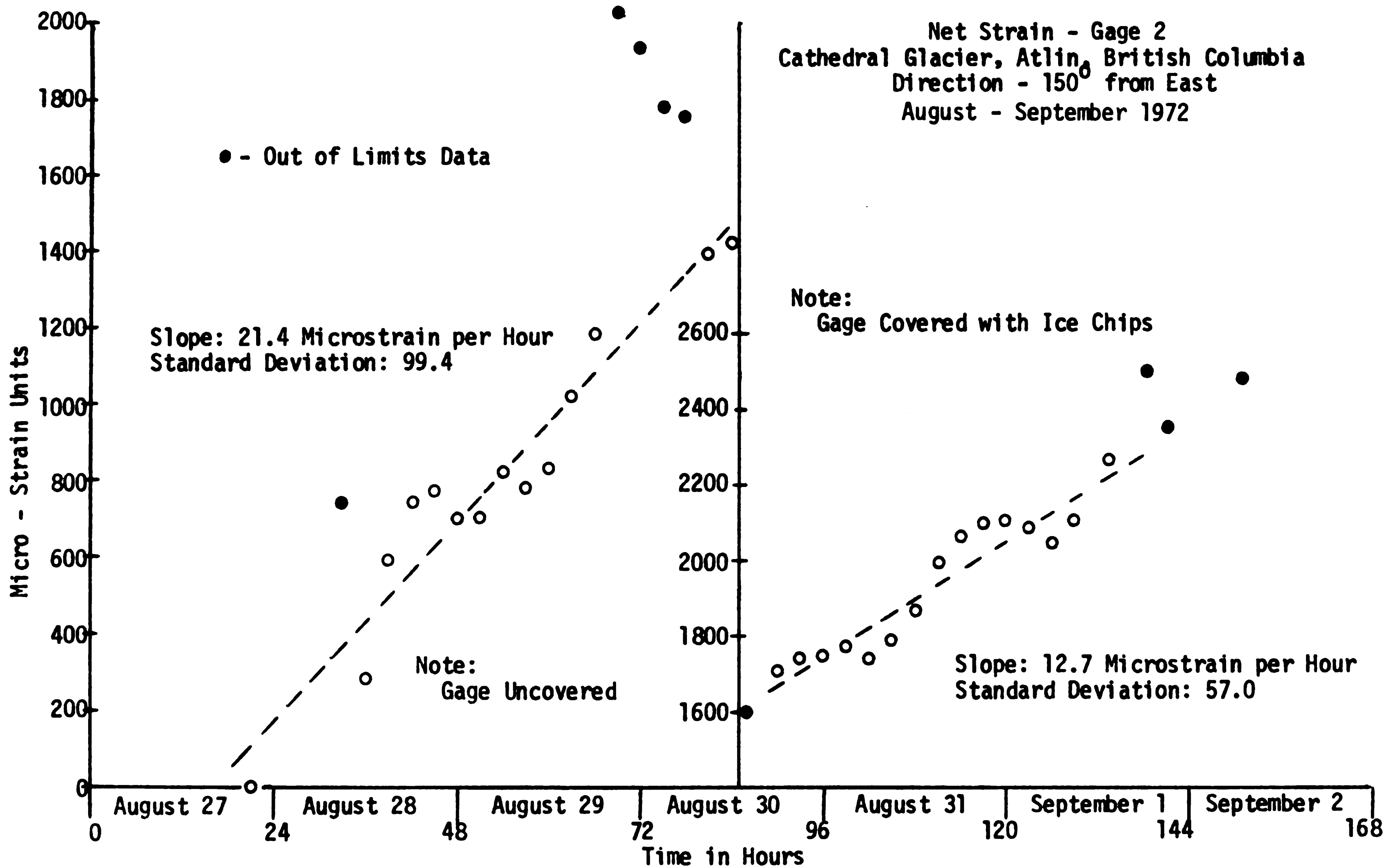

Figure 30 - Net Strain of Cathedral Gage 2

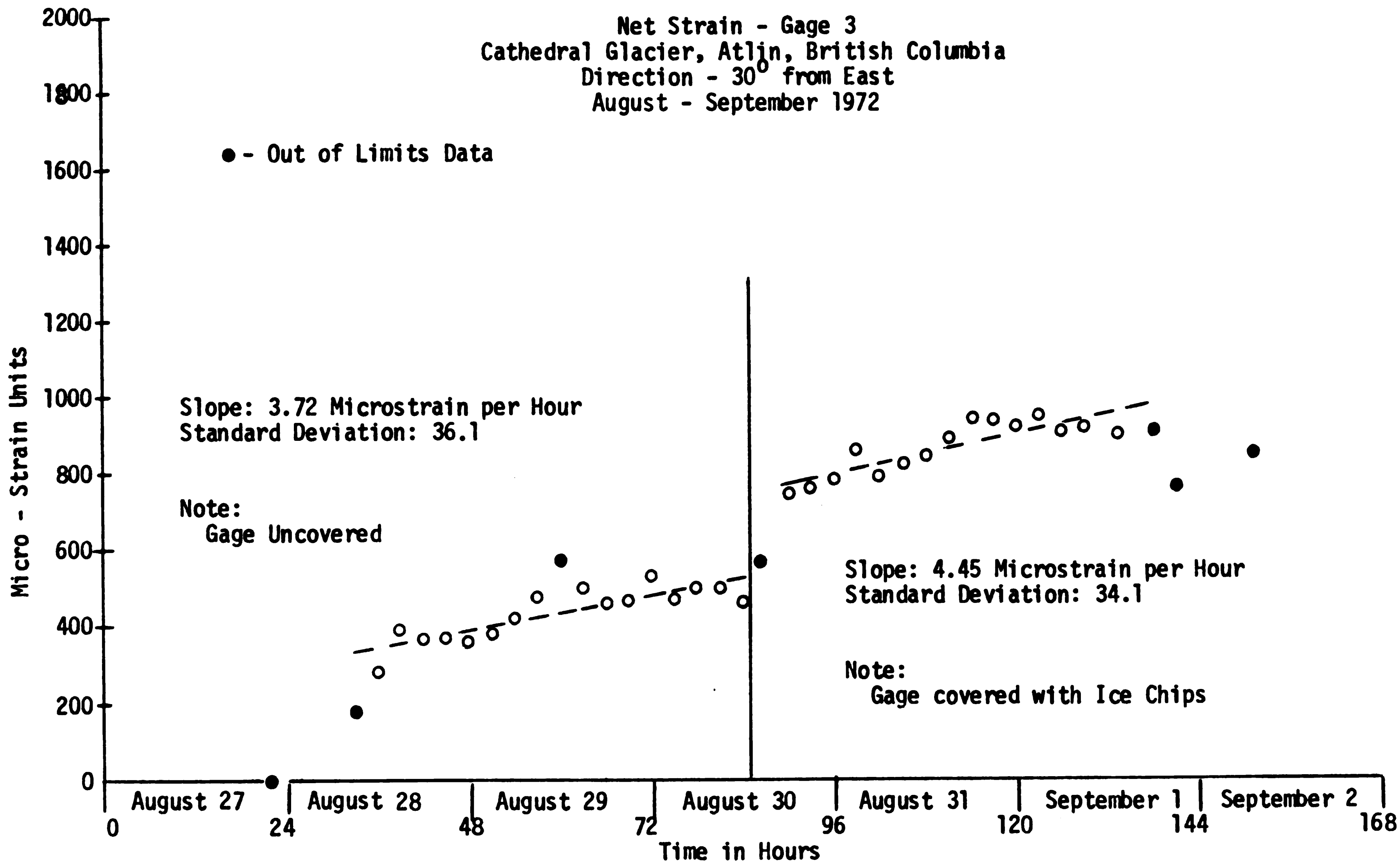

Figure 31 - Net Strain Cathedral Gage 3

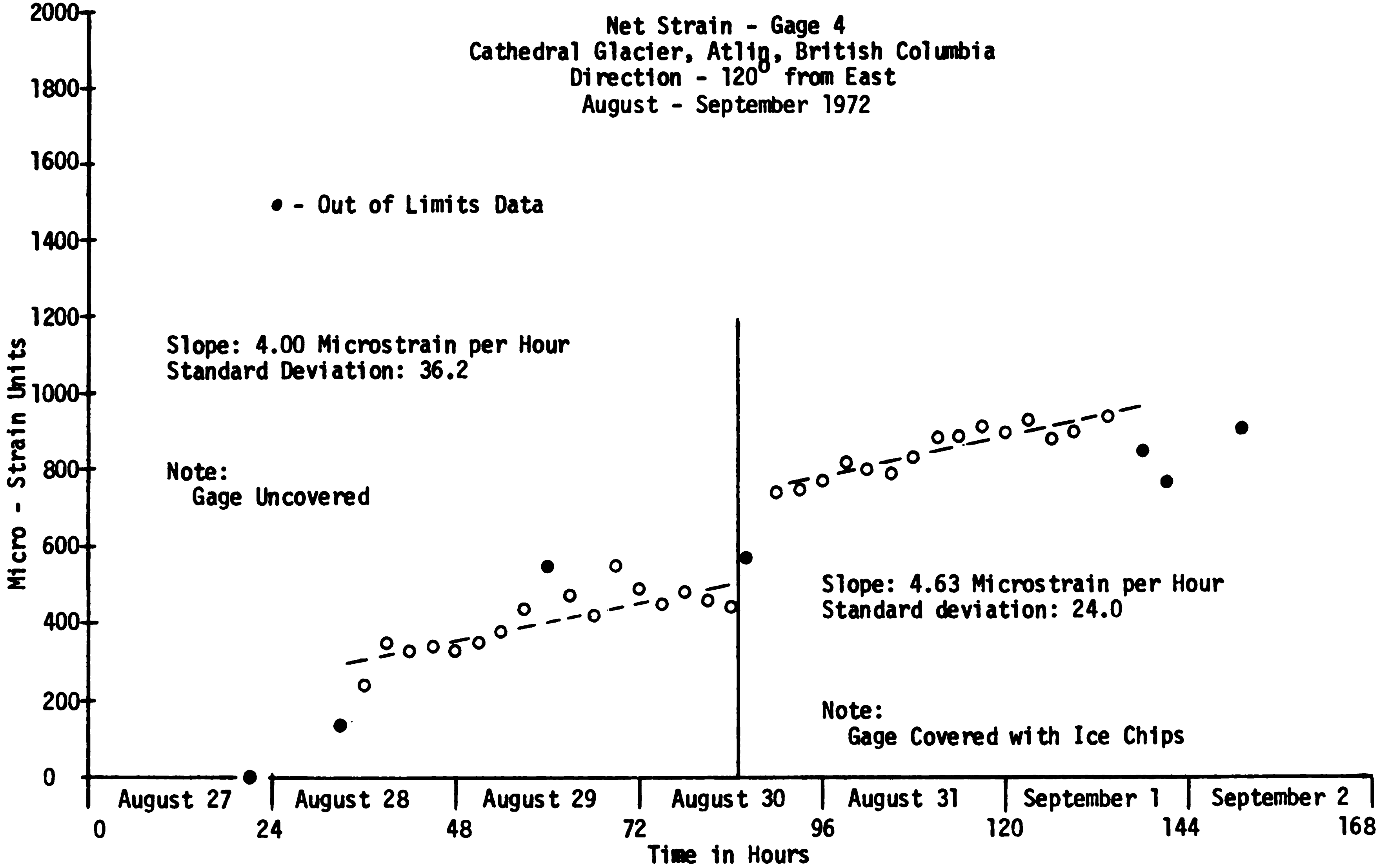

Figure 32 - Net Strain of Cathedral Gage 4

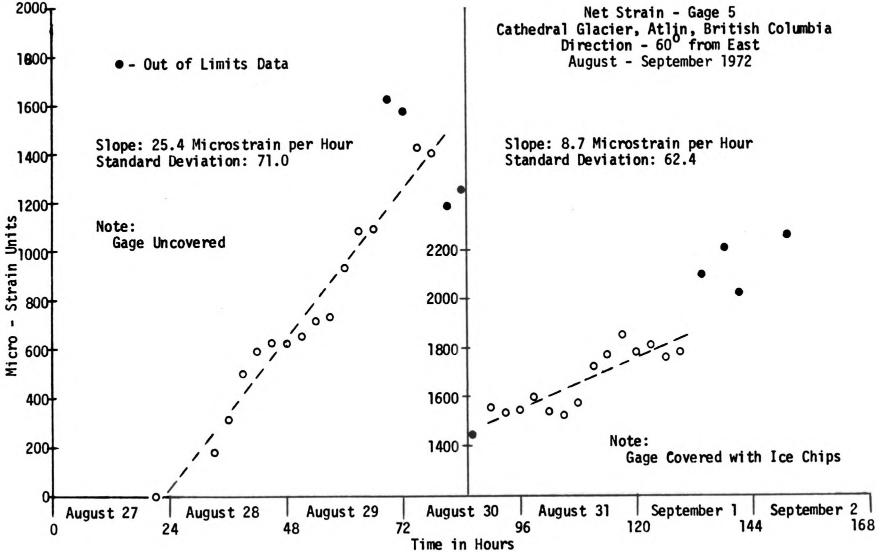

Figure 33 - Net Strain of Cathedral Gage 5

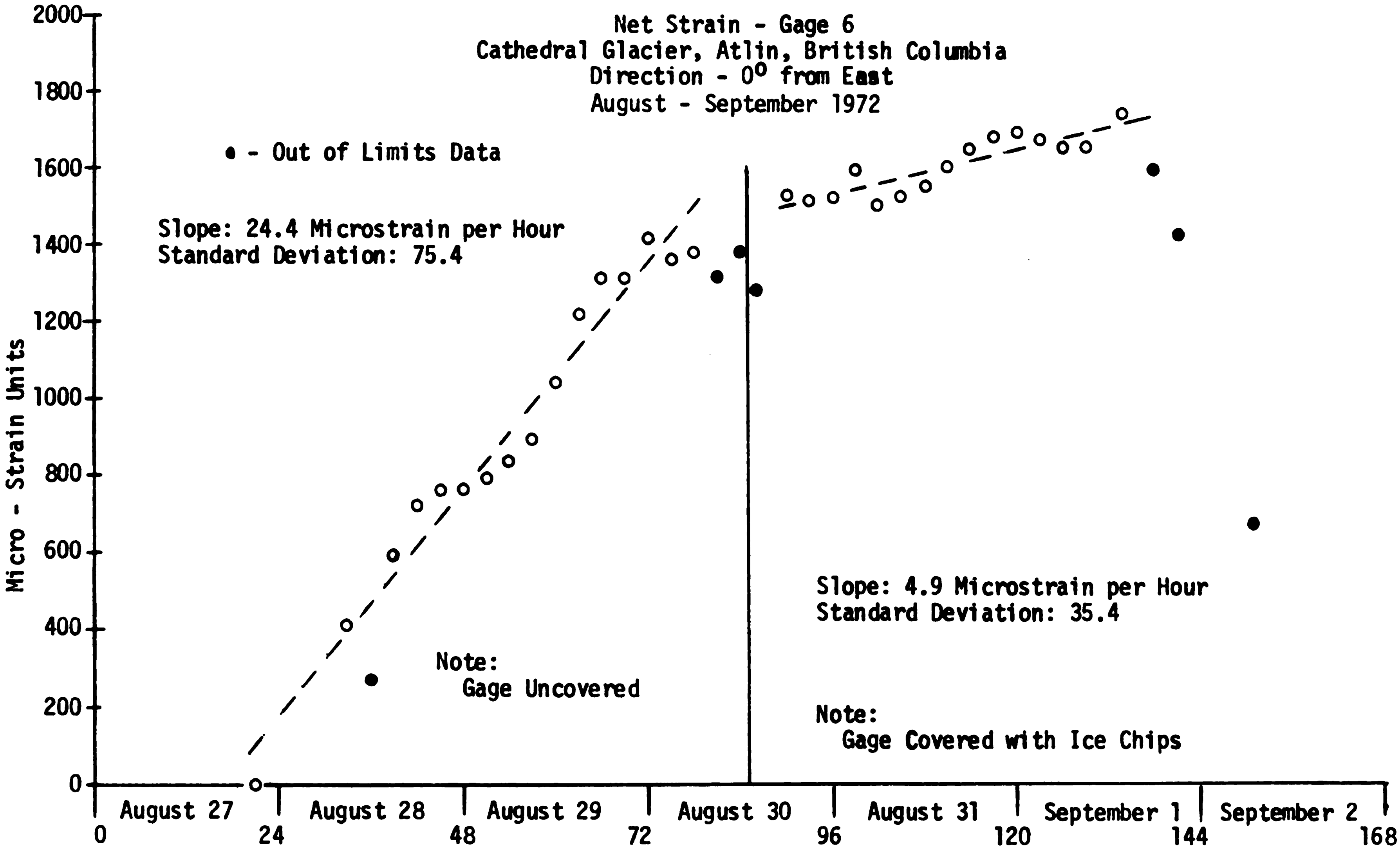

Figure 34 - Net Strain of Cathedral Gage 6

the variation was about ± 51 microstrain. The variation is substantially smaller when the gages are covered with either firn or ice, leading to the possibility that diurnal variation in strain is caused by the daily change in either temperature or radiation. It has been demonstrated by Miller (1963) that daily ambient temperature fluctuations are directly correlated with diurnal variations in total sky and solar radiation.

The graphs of strain as a function of time suggest a linear relationship. The standard deviation during the first half of the study ranged from 35.2 to 219 microstrain. The overall standard deviation obtained from the six gages was 57.5 microstrain. By subtracting the effect of diurnal variation and readability of the strain indicator, the standard deviation of the gages and instrumentation was 20.4 microstrain units. Likewise, during the last half of the study, the standard deviation ranged from 24.0 to 62.4 with a combined value of 42.4 microstrain. Subtracting the diurnal variation and instrument readability the standard deviation of the gages and instrumentation is 18.3 microstrain. The tolerance of the gages in this application is ± 60.0 microstrain based on the above standard deviations.

The conclusion is that, when the gages do not become bonded to the ice and are dependent on anchor posts, the accuracy of the data is only one-tenth as good as the firn-covered system. The system in this application is useful only for overall strain rates taken over an extended period of time. However, it appears that the anchor posts will loosen as a result of ablation in about five days. The loosening of the anchor posts is further evidence that the use of poles or posts in the bubbley surface of temperate ice should be avoided if at all possible.

Values for the maximum shear strain rate and average normal strain rate were calculated from the strain transformation equation using the slopes of the linear curve fits in Figures 29-34. The maximum shear strain rate was found to be 17.9 and 9.2 microstrain per hour while the average normal strain rate was 14.4 and 9.2 microstrain per hour respectively for parts one and two of the study. The shear strain rate of the Cathedral Glacier is approximately ten times that measured on the Ptarmigan Glacier. This would be expected since the Cathedral Glacier has a greater slope than the Ptarmigan Glacier at the respective site locations. The site picked on the Ptarmigan Glacier was in an apparent compressive zone while the site on the Cathedral Glacier was in a tensile zone further explaining the difference in the localized shear strain rates.

SUMMARY

Conclusions

1. The embedded strain gage technique is valid, simple and useful for measuring the localized strain rates on the surface of glaciers.

2. The gages become bonded to the ice when there is a snow or firn cover. Hence, anchor posts are not necessary when five mil Constantan gages are used. Gages which adhere to the ice do not need a tensile prestrain since the gages will compress when the surface of the glacier is experiencing compressive strain rates.

3. The accuracy of the wire strain gage as a transducer is $\pm$ 6.0 microstrain when a snow or firn cover is used.

4. Anchor posts are required and the system will loosen as a result of ablation within five days if there is no snow or firn cover on the temperate ice surface. In this situation the strain gage transducer accuracy is $\pm$ 60.0 microstrain. the study demonstrates that anchor posts be used only when the ice temperature is below zero degrees Centigrade.

5. Sealed weatherproofed instrumentation is required. The BLH Model N strain indicator is not adequate for the rugged field conditions encountered on the Juneau Icefield. Automatic data acquisition is advisable to eliminate the human error.

6. There is an apparent diurnal variation in the measured surface strain rate of glaciers. The diurnal variation of the normal strain was measured to be $\pm$ 25 microstrain on the Ptarmigan Glacier. On the Cathedral Glacier the variation was measured at $\pm$ 90 and $\pm$ 50 microstrain.

7. The maximum shear strain rate for the chosen site on the Ptarmigan Glacier ranged from 0.25 to 1.25 microstrain per hour during the time of the study. This is very close to the 1.6 microstrain per hour shear strain rate approximated from annual flow studies.

8. The maximum shear strain rate at the gage site on the Cathedral Glacier was 9.2 microstrain per hour.

Recommendations

1. Future studies involving the embedded strain gage techniques developed in this study should be conducted in a crevasse free region where there is at least two feet (0.60 m.) of snow or firn cover. This protective cover must be maintained for the duration of the study.

2. If the region to be studied is not normally snow or firn covered, the project should be planned for a season when snow or some other suitable cover is available or snow or firn must be moved to the system from another location.

3. Ideally, if time permits, the gages should be implaced during the early accumulation season in a region just above the seasonal névé line. Measurements could be taken over an extended period of time in all seasons. The researcher is cautioned that, for studies of long duration, he is collecting data which are in a Lagrangian coordinate system that is moving with the glacier.

4. All instrumentation must be weatherproofed and have an ample or spare power supply to operate in rugged glacierized environments.

5. This study demonstrates only the ability to measure the strains on the surface of glaciers using embedded strain gages. The opportunity is now available for extensive experimental research into deformable glacier mechanics using the techniques developed. A study to determine the magnitude and cause of diurnal variation in strain rates is particularly desirable.

6. Strain rate gradients can be determined experimentally using embedded strain gages as background in the further development of a mathematical theory of glacier flow.

7. Since the wire strain gages were found to adhere to the polycrystalline glacier ice surface, the development of a three-dimensional strain transducer seems to be technically possible. With such a system, strain rates could be measured at depth.

APPENDICES

APPENDIX A

INCLINOMETER DESIGN

The pressure melting experiment required very precise measurement of the inclination of the anchor posts. Preliminary calculations of the effect of post deformation and rotation on the strain gage results indicated that changes in inclination had to be measured to the nearest 100 microradians (20 sec.). Several transducers were investigated including LVDT's, potentiometers, proximity detectors, and strain gages.

Transducers which employed the strain gage principle were desirable so that the inclinometers would be compatible with the rest of the instrumentation system. A pendulum arrangement appeared the most advantageous means of sensing changes in inclination. A vertically supported cantilever beam with a weight on the free end was investigated as a deformable type of pendulum. A computer program which would evaluate the output strain as a function of support rotation was written. This was an interesting problem, since the differential equation for beam bending is a function of the deflection of the beam,

$$\frac{d^4\delta}{dz^4} = A\ f(\delta,\theta)\ ,$$

where δ is the deflection of the beam, A is a constant which includes the beam parameters, θ is the rotation of the support, and z is the distance measured along the beam.

A Fourier expansion was used to solve the differential equation. This solution was computerized to give the stress, strain, and deflection for various rotations as the beam parameters and pendulum weights were changed.

The solution was iterated until the desired results were obtained. The endurance stress of the beam material could not be exceeded. Maximum sensitivity was desired. The dimensions and deflection had to be contained inside the anchor post. A practical system was a 4.5 inch long steel beam with a thickness of 0.010 inch and a width of 0.25 inch having a 0.04 pound weight on the end. The strain levels of the beam were so small that standard foil strain gages could not be used. P-type semi-conductor gages were selected. With one gage on each side of the beam located near the support, a maximum sensitivity could be obtained which would be compatible with the two arm wheatstone bridge circuit used for the rosette strain gages.

Figure 12 illustrates the completed assembly. Ten mil shim stock was used for the beams. Solder was used for the weights. The beams were cemented into slits in the plastic anchor post caps. The BLH SPB 3-07-35 semi-conductor strain gages were cemented to the beam with Micro-Measurements, Inc., AE-10 epoxy cement at 3/8 inch from the support. A room temperature adhesive was required because of the plastic anchor cap. The system was protected by encasing it in a plexiglas cylinder with a 2-inch I.D. and 5 inches long. The system was so sensitive that the balancing meter strain indicator could not be kept from vibrating. To protect the transducer from random variation, the cylinder was filled with SAE-10 engine oil for damping.

Calibration of the transducers proved to be a problem. The semi-conductors are extremely temperature sensitive. Since the transducers were to be embedded with the anchor posts in ice at 32°F., calibration was necessary at the same temperature. Arrangements were made to use the General Motors Institute cold storage refrigerator which operates at 33 to 36 degrees. The transducers were mounted in an optical indexing head which had a least count of five seconds. The whole system (transducers, indexing head, switching unit, and strain indicator) was soaked in the refrigerator for twelve hours prior to calibration. Eighty data points were obtained for each inclinometer over a $\pm$ 15 degree rotation. The data points plotted as a linear curve over the whole range. The least-squares-best-fit lines for inclinometers 10 and 11 had slopes of 10.5 and 11.2 microradius per microstrain, respectively, with a gage factor setting of 2.0 on the strain indicator.

APPENDIX B

STEP FUNCTION CURVE FITTING

As mentioned in the chapter entitled Data Analysis, neither the polynomial nor the Fourier methods of curve fitting were accurate enough to analyze the diurnal variation. Recent publications on spline functions indicate that piecewise curve fitting might give the desired results. Berghaus and Cannon (1973) and Rowlands, et al (1973) have presented papers involving the use of smoothed-spline functions to obtain derivatives from experimental data. Both papers require knowing the approximate standard deviation of the data and have difficult methods of matching boundary conditions. This author has developed a method which has neither of these difficulties. The method is simple in concept but difficult to computerize.

Since a continuous first derivative is desired, the expression for the slope of the experimental data cannot be less than a second order polynomial.

$$\frac{dy}{dx} = \sum_{i=1}^{3} A_i \, X^{i-1}$$

Based on this presumption, the third derivative will be a constant.

$$\frac{d^3y}{dx^3} = A$$

To obtain a system of functions which will fit the data, the third derivative is expressed as a series of step functions,

$$\frac{d^3y}{dx^3} = \sum_{i=1}^{n} A_i \left\langle x - a_i \right\rangle^0 ,$$

where n is the number of steps and $\left\langle x - a_i \right\rangle^0$ are the steps which are defined as zero if $x < a_i$.

To obtain the form of the equation which is to fit the data, the step functions are integrated three times,

$$y = \sum_{i=1}^{n} A_i \left\langle x - a_i \right\rangle^3 + \sum_{j=1}^{3} C_j \, X^{j-1} ,$$

where the C_j's are the constants of integration. The data are expressed as discrete points, so the calculated value of the ordinate yc would become:

$$yc_i = \sum_{j=1}^{3} c_j \, X_i^{\,j-1} + \sum_{j=4}^{n+3} c_j \left\langle X_i - a_{j-1} \right\rangle^3$$

The normal method of least squares can be applied where the c_j are the unknown coefficients. The matching of boundary conditions across the steps is automatic and the standard deviation (σ) can be calculated by the usual method:

$$\sigma = \sqrt{\frac{(y_i - yc_i)^2}{N}}$$

This method works well if the data is taken over short intervals and the change in slope of the curve is not severe. The major difficulties of the system are in determining the interval of the steps and the ascending magnitudes of the coefficients. Each successive coefficient must not only correct for the change in shape of the curve but must also correct for the effect of the previous coefficients.

Another method is to express the coefficients as being effective only over a short interval of the curve. This can be done by writing the third derivative as:

$$\frac{d^3y}{dx^3} = \sum_{i=1}^{n} A_i \left[\langle x - a_i \rangle^0 - \langle x - a_{i+1} \rangle^0 \right]$$

In this form, the effect of each step will be zero outside the range $a_i > x > a_{i+1}$. Each coefficient A_i will have to fit only the data points within its interval.

The calculated value of the ordinate becomes:

$$yc_i = \sum_{j=1}^{3} c_j X_i^{j-1} + \sum_{j=4}^{n+3} c_j \left[\langle X_i - a_{j+3} \rangle^3 - \langle X_i - a_{j+4} \rangle^3 \right]$$

The solution of this expression is the same as described for the first method.

APPENDIX C

DIURNAL VARIATION ANALYSIS

The initial investigation conducted during the 1971 field season indicated the possibility of a diurnal variation in the surface shear strain rate of glaciers. The only reference to a diurnal variation was found in Paterson (1969) which states that "Fluctuations of 100 percent or more have been observed in measurements made every few hours." Hence, very little is known about short time phenomena associated with glacier flow. Precise measurement of the diurnal variation was planned for the 1972 field season.

During the 1972 field season, the plan was to measure strain at hourly intervals for ten days on both the Ptarmigan Glacier and the Cathedral Glacier. The mid-August storm on the Ptarmigan Glacier made hourly data acquisition impossible; however, consecutive hourly readings were taken for 48 hours after the storm subsided. The installation and logistics on the Cathedral did not lend themselves to hourly readings, but measurements were obtained at three-hour intervals for five days.

The methods described in Appendix B were used to analyze the strain-time data of each strain gage. The step-function curve fitting technique conformed to the data but the derivative was not acceptable near the beginning and ending data points.

Ptarmigan Glacier

The data from the Ptarmigan Glacier was divided into two groups: (1) The data collected at three-hour intervals during the storm, and (2) the data collected hourly during fair weather. The unit-step interval for the first group of data included five data points. The standard deviation on the curve fit was 8 to 13 microstrain. The reliability of the fit is questionable since the elimination of extraneous data left large intervals to be spanned by the smooth curves. The second group of data, taken during fair weather, was analyzed using a step interval which included eight data points. The standard deviation ranged from 5 to 6 microstrain, a good fit since the RMS readability of the strain indicator was 7.1 microstrain.

Figure 35 shows that the shear strain rate of the Ptarmigan Glacier did tend to oscillate with a maximum rate of 6.1 and a minimum rate of -2.5 microstrain per hour. The negative rate was not anticipated since a negative value would indicate that the localized flow of the glacier was backward for part of the day. The gaging site was located on a relatively "dead" region of the glacier. Evidently, superimposed oscillations on the general flow trend caused negative values of the shear strain rate for short periods of time.

Cathedral Glacier

The measurements from the Cathedral Glacier were divided into two groups: (1) The data obtained when the gage wires were not packed in crushed ice, and (2) the data collected when the gages were covered with ice. Since the accuracy of this system was ± 60 microstrain (see Conclusions), the data were difficult to curve fit using the step method.

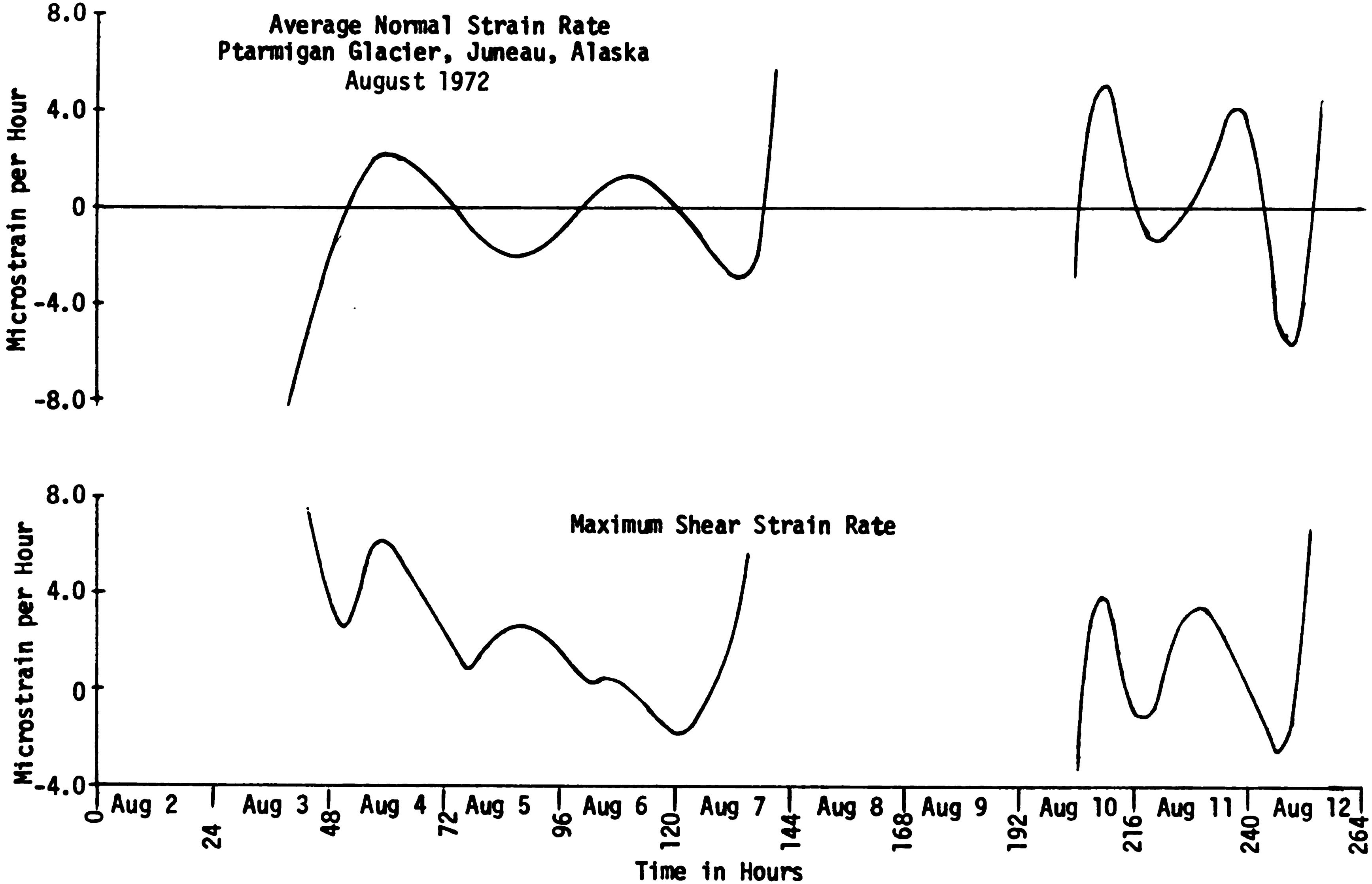

Figure 35 - Diurnal Variation of Ptarmigan Glacier

The first group of data could not be evaluated. The second group of data was analyzed using an interval of four data points. The standard deviation was 15 to 20 microstrain units.

Oscillations of the shear strain rate appeared to diminish toward the end of the study as shown in Figure 36. This may be the result of the loosening anchor posts. The values ranged from a minimum of 3.5 to a maximum of 20 microstrain per hour. The average rate of 9.89 is comparable to the 9.2 rate as determined previously.

The results of this study verify that a diurnal variation does exist. However, the author is reluctant at this time to formulate any conclusions or hypotheses based on the data currently available. Further study is indicated with the following modifications:

1. A maximum of one hour between readings.
2. More sensitive strain gages.
3. Automated data acquisition.
4. At least ten consecutive days of data.

Plans are presently underway to conduct further research involving diurnal strain trends in the Juneau Icefield during the 1973 summer field season. The project will be a part of the total research program of the Foundation for Glacier and Environmental Research and supported by the Army Research Office-Durham.

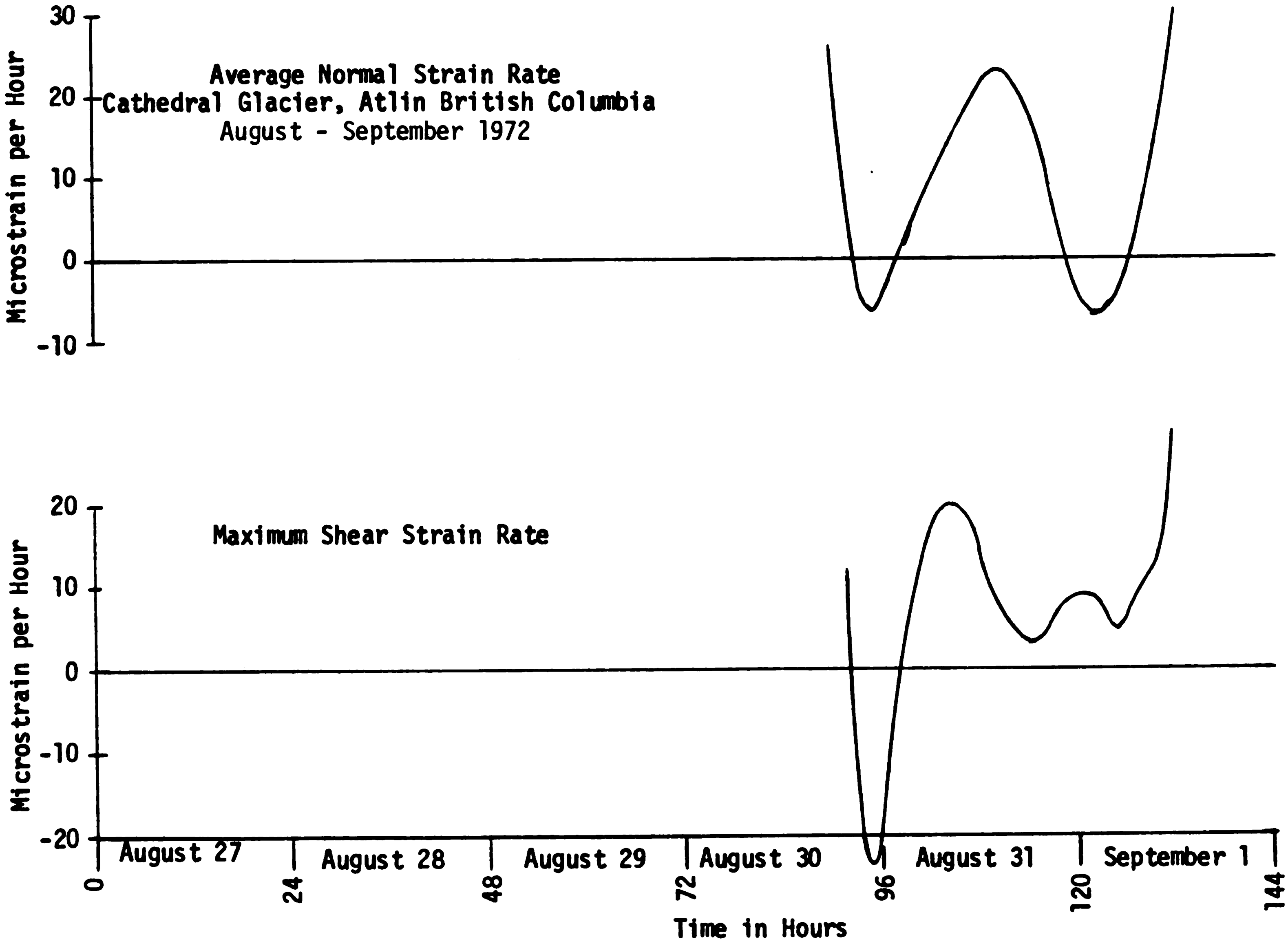

Figure 36 - Diurnal Variation of Cathedral Glacier

APPENDIX D

GLOSSARY OF TERMS

Ablation - Total radiation and warm summer temperatures increase the energy level on the surface of glaciers causing sublimation and melting of either the firn-pack or bubbley glacier ice, whichever is exposed. Any body lying on or embedded in the surface will absorb radiation at a faster rate causing melting in the immediate vicinity of the body.

Albedo - Albedo is a measure of the diffused reflectivity of long wave electromagnetic energy by various materials. A light-colored (ideally white) material has a higher albedo than a dark-colored (black) material.

Bubbley glacier ice - The ice near the surface of a temperate glacier does not have a solid crystalline structure because of entrapped pockets of air and water. This nonhomogeneous, bubbley structure is found to depths in excess of five meters, having a density of 0.9.

Crevasse - If the stress near the surface of a glacier is sufficiently tensile, tension cracks called crevasses will develop normal to the surface. Theoretically, crevasses can be as much as 40 meters deep; however, in temperate glaciers, these fissures are rarely found deeper than 30 meters.

Deformable body - In the study of continuum mechanics, an object which deforms when subjected to external forces is said to be a deformable body.

Elastic - If material, when subjected to external loads, returns to its original state after the loads are removed, it is said to be elastic. Ice tends to be elastic when rapidly loaded and unloaded.

Eulerian coordinates - A coordinate system which is fixed in space is referred to as a spatial or Eulerian system. Eulerian coordinates are normally used in fluid flow analysis since it references a point in space as opposed to a point moving with the fluid. The Eulerian system can be thought of as a window approach, where the fluid is observed flowing past a point or window.

Firn - Old snow which has been rounded by partial melting and compacted by pressure is called firn. Quantitatively, firn is the state in the metamorphosis of old snow to ice at which the specific gravity is between 0.5 and 0.7. The term firn usually implies a depth connotation when expressed as a firn pack, i.e., the firn pack was one meter deep.

Gage factor - The calibration constant relating the change in resistance per unit resistance of a wire strain gage to the strain being sensed is the gage factor

$$\varepsilon = \frac{1}{G.F.} \left(\frac{\Delta R}{R}\right)$$

For the Constantan wire used in this study the gage factor was 1.96.

Lagrangian coordinates - A coordinate system which is fixed to the body being studied is a material or Lagrangian system. Lagrangian coordinates are normally used in the study of solid deformable bodies to evaluate the state of stress or strain at the point being studied. Since Lagrangian coordinates are body-fixed, they are time dependent necessitating extreme caution when used on a flowing material such as a glacier.

Mass balance - When used in glaciology, the mass balance is an indicator of the state of health of a glacier. Mass balance is positive if total accumulation exceeds ablation. In the terminal region the localized mass balance is normally negative while the local mass balance is usually positive in the source area. If the total net mass balance on a glacier is changing, it can be presumed that the flow rate will be affected.

Moraine - During its advance a glacier accumulates and carries a large quantity of debris consisting of rocks, gravel, sand, etc. This debris is eventually deposited in the proximity of the glacier forming moraines. The three basic types of moraines are: (1) terminal moraines formed by the advancing snout; (2) lateral moraines formed from deposits along the periphery; and (3) medial moraines formed within a glacier longitudinal to its path as a result of the junction of two glaciers having lateral moraines.

Névé - This French term meaning "consolidated snow not yet changed to ice" is used by the glaciologist with an areal connotation. The névé region refers to the area covered by perennial snow. Thus the seasonal névé line is the lower boundary of snow cover at the end of the ablation season averaged over several years.

Plastic - When a material is deformed beyond the elastic limit, there is a permanent deformation as the load is removed. In such an instance the material has plastically deformed. Creep in metals at temperatures near melting and similarly in ice is a form of plastic deformation.

Polar ice - Glaciers which have mean perennial temperatures below freezing (0°C.) are considered to be made of polar ice. The colder the ice the greater the resistance to plastic flow. During the ablation season, the surface of the glacier may be temperate, but at depth the ice remains polar.

Pressure melting - The melting temperature of ice is reduced if subjected to pressures greater than atmospheric. Thus in temperate ice any object which puts a pressure on the ice will cause melting. If heat energy is not available, this supercooled water will freeze when the pressure is removed, or the water may flow around the object to a region of lower pressure and regelate. An object can be displaced in the ice via the pressure melting process. The pressure melting temperature connotates the depression of the melting temperature.

Prestrain - In experimental strain analysis, an initial strain is frequently placed on the gaging device to enable the researcher to alter the upper and lower limits of the gage without changing the range.

Regelation - The process of refreezing after pressure melting has occurred.

Rigid-body-motion - If a body or mass does not deform or change shape as it moves, the motion is rigid-body-motion. Normally, infinitesimal deformations do occur, but are sufficiently small as to be negligible.

Rosette - A rosette as used in this study refers to a combination of three strain gages arranged in such a manner as to measure the normal strain in three different directions. In experimental strain analysis, three basic configurations are used: (1) The delta rosette which forms an equilateral triangle; (2) the Y rosette in which each gage is at an angle of 120° from its neighbor; and (3) the rectangular rosette in which two gages are perpendicular and the third is at a 45° angle between them. In this study, the delta configuration was employed.

Standard deviation - When reducing large quantities of data it is frequently convenient to use curve fitting techniques to determine a mathematical expression which will model the data. A means of determining the degree of fit is expressed by the standard deviation. Standard deviation is the square root of the sum of the squares of the difference between the mathematical value subtracted from each datum value divided by the total number of data points. Statistically, it can be shown that 68 percent of the data will fall within $\pm$ one times the standard deviation; 95 percent will be within $\pm$ twice the standard deviation; and 99-3/4 percent will be within $\pm$ three times the standard deviation. Thus, a tolerance can be placed on the range of the data to determine a normal spread.

Strain - To the mechanician, normal strain is $\frac{de_x}{d_x}$, where e_x is the change in length over a given length x. Shear strain as used in this study is $\left[\frac{\partial e_{sx}}{\partial y} + \frac{\partial e_{sy}}{\partial x}\right]$ where e_{sx} is the displacement of a line segment y units long in a direction normal to the initial y direction and e_{sy} is the same type of displacement with respect to the initial x direction. It can be seen that strain is a unitless quantity. The glaciologist normally expresses strain rate as strain units per year. In this study time was measured in hours and it was more convenient to express the strain rate in hour units; however, the magnitudes were so small that millionths of a unit deformation per unit length per hour was used. This is expressed more easily as microstrain per hour.

Strain gradient - The change in strain as a function of position is the strain gradient. Strain gradients are essential in the conversion of strain rates from a Lagrangian system to an Eulerian system.

Temperate ice - Ice which is at its melting temperature is considered to be temperate. A temperate glacier is one in which the ice at depth is perennially temperate.

Theodolite - A theodolite is a surveying instrument which measures both vertical and horizontal angles. Precise positions can be determined by triangulation techniques.

Transducer - A transducer is any device which senses an event or phenomenon and converts the signal to another energy form. The wire strain gages used in this study sensed change in length per unit length and produced an electrical response as a change in resistance (see gage factor).

Visco-elastic - Some materials react in an elastic manner when subjected to certain types of loading and in a plastic manner when acted upon by other types of loading. Ice is visco-elastic because it responds elastically to rapidly changing loads, but it has a time dependent creep when loaded statically.

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